TRIGONOMETRIE.—NORWOOD.

1661

~~[illegible]~~. 3265

Packing Case No 181.

ames | When the Extreames Adjacent fall
ajacent | upon the Angle at Bass hipothenusall or angle at perpendiculer they are all ways Tangent Complements
But upon the Bass or perpendiculer Tangents

etreames | When the Extreames Opposett fall upon the
oposett | Angle at Bass hipothenusall or angle at perpen they are always Sines
But if ye fall upon ye Bass or perpendiculer ye are Sines Complements

middle | The midde part fall upon ye Angle at Bass
Part | hipothenusall or angel at perpendiculer is always Sines Complements
But upon ye Bass or perpendiculer Sines

When the midde part is Required Radius must be ye first Number Otherwise one of the Extreames

TRIGONOMETRIE,
OR,
THE DOCTRINE OF TRIANGLES:
Divided into two Books.

The first ſhewing the menſuration of right lined Triangles, the ſecond of Spherical, with the grounds and demonſtrations thereof.

Both performed by that late and excellent invention of *Logarithms*, after a more eaſie and compendious manner, than hath been formerly taught.

Whereunto is annexed (chiefly for the uſe of *Sea-men*) A Treatiſe of the application thereof in the three principal kinds of Sailing.

With exact Tables of the *Suns Declination*, newly calculated: and Tables of the right Aſcenſion and Declination of ſome eminent *Fixed Stars*, with the true times of the coming to the *Meridian* at four of the clock in the morning, fitted for the preſent ſeaſon, and may ſerve for many years without any alteration.

Alſo other neceſſary Tables uſed in NAVIGATION.

By *Richard Norwood*, Reader of the Mathematicks.

This fourth Edition being diligently corrected, in divers difficult places explained; New Tables of the Stars right Aſcenſions and Declinations added, and the whole Work very much enlarged, by the Authour himſelf.

LONDON,
Printed by *Robert* and *William Leybourn*, for *George Hurlock*, and are to be ſold at his ſhop at *Magnus* Church corner, in *Thames*-Street. *MDCLXI.*

TO THE RIGHT HONORABLE *Francis* Earl of *Bedford*, Lord *Ruſſel*, Baron *Ruſſel* of *Thournhaughe*, Lord Lievtenant of the County of *Devon:* and City of *EXETER.*

Right Honourable:

AS it hath pleaſed the ſoveraigne Fountain of Light to ſhine upon the World in theſe later Times, by a more clear manifeſtation of thoſe heavenly myſteries, that concern eternal life and bleſſedneſſe: ſo he hath alſo enlightened the minds of men with knowledge in humane Arts and Sciences, and diſcovered many profitable inventions unknown to former ages. To ſpeak of all, were a ſubject deſerviug of it ſelf a peculiar Treatiſe. To ſpeak of thoſe that have reference to the *Mathematicks*, would require a larger diſcourſe than becomes this place. Amongſt the reſt, and of the higheſt rank, is that admirable invention of Logarithms, by the famous *John Nepair*, late Baron of *Marchifton*: which hath been further perfected by the labours of *Mr. Henry Brigs*. And although the maturity of this invention was prevented in them both, by their feveral and moft happy changes, from this life to a better; yet they proceeded fo far, as to lay a verie good foundation for fundrie conclufions Mathematical. Upon which foundation chiefly, I have grounded this prefent Treatife of the *Doctrine of Plain and Spherical Triangles*; annexing an application thereof in the three principal kinds of failing. And howfoever (being rudely compofed) it may feem unworthy the protection of one fo eminent in place, and of fuch ripeneffe and judgement in all kind of learning: Yet I am bold to prefent it to your Lordfhip, in confidence of your favourable acceptance, according to that noble refpect you are accuftomed to manifeft towards all good endeavours. The moft High God and Lord of all things, increafe and continue unto your Lordfhip, all his bleffings temporal and eternal.

Your Honours moft devoted,

Richard Norwood.

TO THE READER.

HOw necessary, and of what excellent use the Doctrine of Triangles is, in Astronomy, Geography, Navigation, Fortification; and other parts of Architecture, in all the kinds of Perspective, in Dialling, and in the practice of other parts of the Mathematicks: is so much the better known unto every man, by how much he hath been more exercised in these Arts. For which cause there hath been for many former ages, much time and diligence bestowed by most industrious and learned men, to reduce it to as great perfection as they could; and much hath been done to this purpose of late years. But all that hath been done these many hundred years, is not comparable to that which hath been effected in our times, by the Honourable Lord John Nepair Baron of Marchiston: who by an invention of Logarithms, takes away those difficulties that were in the practice thereof. Which invention hath been illustrated and much perfected by the labours of M. Henry Brigs. Neither is M. Edward Wright to be forgotten, though his endeavours were soonest prevented. And these were the first that communicated their labours on this subject to the world; being men, as of singular piety and integrity of life, so of that excellent knowledge in the Mathematicks, as few ages afford the like. Of the construction and divers application of Logarithmes, Mr. Brigs hath written a book called Arithmetica Logarithmica. And since again began another excellent work of like nature, entituled, Trigonometria Britannica. I have onely seen (in the hands of a friend of his) a printed Copy of so much as he had done, namely, the Tables, and some part of the Treatise, touching the construction of those Tables: but whilest he was in hand with the rest, he departed this life. Wherefore having my self some years past (but especially this last winter) bestowed more than ordinary pains in conforming the Doctrine of Triangles, to the nature of Logarithms now in use; and yet so, as the rules might likewise be applyed to natural Sines, Tangents and Secants, and also to instrumental operations: and considering the present want of directions, and of ordinary Tables in

this

this kind, I have thought good to publish these. If any man think it should be a hinderance to them who have been at the charge to print that which Mr. Brigs *hath begun to write upon this subject, he may be pleased to take notice, that though we both handle the same thing, yet it is in such a different manner, that there is scarce any one proposition handled by us both; besides his in Latine, mine in English: so that though his were finished, according to his intent and method, the one would little or nothing prejudice the other. I rather hope, as the case now stands, that this will further the sale of his; forasmuch as the rules here delivered may very aptly be applyed to his Tables, and almost to any other. And they are such (especially for spherical triangles) as I doubt not will be found more easie for memory, and more ready for practice, than those that have been formerly used. If in some things you find me too brief, or otherwise faulty, I hope you will pardon it; so much the rather, because all this Summer, whilest this work was printing, I was absent upon necessary occasions above an hundred miles. And to make some part of amends, I shall (God willing) be ready to give further satisfaction herein, by word of mouth, or otherwise to those that desire it. As touching others that are bent to detraction, and will be glad to snatch at every occasion for that purpose; I could wish them of a better mind, and to remember, that it is much easier to find faults in another mans work, than without the light thereof to make the like. I have detracted no man, but have freely attributed to them whose works I have used, that which is due unto them; desiring so to be dealt withal as I deal by others. It may haply be expected, that I should have shewed the application of the* Doctrine of Triangles, *in the Mathematical Arts before mentioned, &c. But other necessary occasions withdrawing me, I had rather leave that untouched, than by making an imperfect application in every of them, heap together many titles, with little or no profit to the Reader. Yet I have been perswaded to annex hereunto certain Problemes, touching the three principal kinds of sailing. Which with the rest I commend to your friendly acceptance. Farewell.*

Tower-hill, Anno 1631. Novemb. 1.

THE

THE DOCTRINE OF PLAIN TRIANGLES.

CHAP. I.

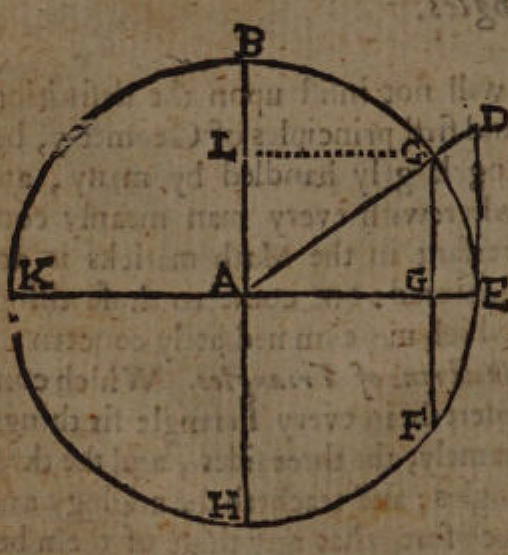
Of the lines used in measuring Plain and Spherical Triangles.

WE will not insist upon the definitions and first principles of Geometry, being largely handled by many, and wherewith every man meanly conversant in the Mathematicks is acquainted: but come to those things which more immediately concern the *Doctrine of Triangles*. Which considereth in every Triangle six things, namely, the three sides, and the three angles; and teacheth the anology and proportionality of these six, in such sort, that any three of them being known, the other three may by the rule of proportion be discovered. But seeing the sides of a spherical Triangle are arches of a Circle, and the angles both of plain and spherical Triangles are measured by arches of a Circle, therefore the proportions of all these parts one to another cannot be declared, unlesse these arches be after a sort reduced to right lines; because the proportions of arches one to another, and of an arch to a right line, is not to this day found out.

These arches of a Circle are after a sort reduced to right lines, by defining the quantity which the right lines to them applyed have, in respect of Radius or the Semidiameter of the Circle. And it is to be understood, that every arch of a Circle is measured by degrees, minutes, seconds, thirds, &c. a degree being such a part of a Circle as the whole circumference whether great or little conteins 360. A degree is measured by minutes, and every degree is supposed to contein 60 minutes. In like sort, every minute conteins 60 seconds, and every second 60 thirds, &c.

And although the measure of every arch cannot be exactly expressed by these parts, yet it may be so neerly expressed, that all sensible errour in ordinary use and application shall be avoided, which is esteemed sufficient.

1. *The right lines applyed to a Circle are Chords, Sines, Tangents, and Secants.*



2 *A Chord is a right line drawn in a circle, from one part of the circumference to another.* Thus CF is the Chord of the arches CEF, and CKF, also BH the diameter, is the Chord of the semicircles BEH, and BKH.

3 *The right Sine of an arch, is half the Chord of twice that arch.* As CG being half the Chord, CF is the right sine of the arch CE, also of the arch CBK: which arch CE is the half of CEF.

Whence first it is manifest, that the right sine of an arch lesse than a quadrant; is also the right sine of an arch as much greater than a quadrant: For as the arch CE is lesse than a quadrant by the arch BC, so the arch CK doth as much exceed a quadrant, CG being the right sine to them doth.

So that properly the sine complement of an arch is the sine of the complement of a lesser arch unto a quadrant. As the complement of the lesser arch C E, unto a quadrant, is the arch C B, the sine whereof is C L, wherefore C L is properly said to be the sine of the complement of the arch C E.

Secondly, that the right sine of any arch, is a line falling from one end of that arch perpendicularly upon the diameter drawn to the other end of that arch. As C G, is perpendicular to K E.

Thirdly, that the right sine of the complement of an arch, is equal to that part of the diameter, which lieth between the right sine of that arch and the center. As C L, the sine of the complement of C E, is equal to A G.

4. *The versed sine of an arch, is that part of the Diameter which lieth between the right sine of that arch, and the circumference.* Thus G E is the versed sine of the arch C E: and G K the versed sine of the arch C B K.

5. *If unto one end of an arch there be drawn a diameter, and to the other end a right line from the center cutting the circle; and if from the end of the diameter be raised a perpendicular till it concur with the line cutting the circle, that perpendicular is the tangent of that arch.* As D E is the tangent of the arch C E.

6. *The foresaid right line cutting the circle, is the secant of that arch.* Thus A D is the secant of the arch C E.

7. *Now to define or expresse in numbers, the quantity that these right lines have in respect of the semidiameter of the circle, is the constructions of the tables of natural sines, tangents and secants.*

Thus supposing the semidiameter of the circle A E to be 1000000 parts, and the arch C E to be 30 degrees, the right sine of that arch C G will be 500000 parts, the tangent E D 577350 parts, and the secant A D 1154701 such parts. The quantities of versed sines, and of the chords of arches, are not usually expressed in the Tables, because they are easily found by the right sines: As the versed sine of the arch C E, namely, G E, is found by substracting the sine complement of C E, namely A G, from the semidiameter A E: also the versed sine of the arch K B C, is found by adding the same A G, to the semidiameter A K. Also the chord of the arch C E F, namely C F, is found, by doubling the sine of half that arch, namely, by doubling

CG. So that in the tables, there are onely expressed the right Sines, Tangents, and Secants of every arch of a circle not exceeding a quadrant. Which how to finde is largely shewed by *Lansbergius*, *Petiscus*, Mr. *Henry Briggs*, (which I have not yet read) and by others, therefore we passe over that. And intending to shew the resolution of plain & spherical triangles, after a more easie & compendious way, by Logarithms lately invented by the Honourable Lord *John Nepair*, Baron of *Marchiston*, and since further perfected by the late learned Mathematician Mr. *Henry Briggs*, (both of ever worthy memory :) we come in the next place to speak something of the nature and affections of those numbers wherein I shall (as occasion requireth) follow Mr. *Briggs* in his *Arithmetica Logarithmica*.

CHAP. II.

Of the nature and affections of Logarithms.

Logarithms are numbers, so fitted to proportional numbers, that themselves retein equal differences.

As let there be a rank of numbers how many soever in continual proportion, namely, 1. 2. 4. 8. 16. 32. 64. 128. 256. and let there be as many other numbers in any progression arithmetical, as 3. 5. 7. 9. 11. 13. 15. 17. 19. then forasmuch as these later are equidifferent (for every one differs from his next by 2) therefore they are logarithms to the former each to his correspondent. As 3 being the Logarithme of 1, and 5 of 2: 7 is the Logarithm of 4, and 9 of 8: and the like is to be understood of the rest.

Numb. proport.	Log	Log	Log
1	0	3	0
2	1	5	3
4	2	7	6
8	3	9	9
16	4	11	12
32	5	13	15
64	6	15	18
128	7	17	21
256	8	19	24

So likewise 0. 1. 2. 3. 4. 5. 6. 7. 8. are Logarithms to the same numbers, and so are 0. 3. 6. 9. 12. 15. 18. 21. 24. And so infinite others.

thers might be found, obſerving that where numbers are in like proportion, the differences of their logarithms muſt be equal.

And as any of theſe three rowes may be logarithms to the firſt, ſo they may be logarith. to any other numbers in continual proportion.

2 *If of four numbers, the firſt exceed the ſecond as much as the third exceeds the fourth: then the ſumme of the firſt and fourth is equal to the ſumme of the ſecond and third, and the contrary.*

As 8, 5, 6, 3, here 8 exceeds 5, as much as 6 exceeds 3, therefore the ſum of the firſt and fourth, namely, of 8 and 3 is equal to the ſum of the ſecond and third, namely, of 5 and 6. And ſo 9, 18, 15, 24, where the ſum of the extreams is 33, and ſo of the two middle ones. *Bachetus in Diophantum.*

3 *If four numbers be proportional, the Logarithm of the firſt ſubſtracted from the ſum of the Logarithmes of the ſecond and third, leaves the Logarithmes of the fourth.*

As if the proportion be. As 256 to 32: ſo 64 to a fourth number: here adding 5 and 6 the logarithmes of the ſecond and third, the ſumme is 11, from which ſubſtracting 8, the logarithme of the firſt, the remainder is 3. the logarithme of the fourth proportionall 8.

Abſolute numbers.	Logarith.
256	8
32	5
64	6
	11
8	3

For ſeeing (by ſuppoſition) the firſt number is in proportion to the ſecond, as the third is to the fourth, therefore (by the firſt definition of this ſecond chapter) the logarithmes of the firſt and ſecond differ as much as the logarithmes of the third and fourth, therefore (by the ſecond prpposition) the ſumme of the logarithmes of the firſt and fourth, is equal to the ſumme of the logarithmes of the ſecond and third; therefore if from the ſumme of the logarithmes of the ſecond and third, be taken the logarithme of the firſt, there remains the logarithme of the fourth.

Corollary. *Hence it is evident, that if four numbers be proportionall, the ſum of the Logarithms of the firſt and fourth is equal to the ſum of the Logarithms of the ſecond and third. And if the ſum of the Logarithms of the firſt and fourth, be eqnal to the ſum of the Logarithms of the ſecond and third, then is the firſt in proportion to the ſecond, as the third is to the fourth.*

Let the proportion be

As	256	*Logar:*	8
to	32		5
ſo	64		6
to	8		3

Here the ſumme of the Logarithmes of the firſt and fourth, namely, 8 + 3 that is 11, is equall to the ſum of the Logarithmes of the ſecond and third, namely, of 5 + 6 that is 11.

4. *If in ſtead of ſubſtracting the foreſaid Logarithmes of the firſt, we add his complement arithmetical to any number: the total abating that number, is as much as the remainer would have been.*

The complement arithmetical of one number to another (as here we take it) is that, which makes that firſt number equal to the other; thus the complement arithmetical of 8 to 10 is 2, becauſe 8 and 2 are 10. And ſo the complement arithmetical of 9,76144 to 20,00000 is 10,23856, becauſe 10,23856, and 9,76144 added together, are 20,00000.

Now then whereas (in the example of the third propoſition before going) ſubſtracting 8 from 11, there remained 3; if in ſtead of ſubſtracting 8, we add his complement arithmetical to 10, which is 2, the toal is 13, from which abating 10, there remains 3 as before, and the like is to be underſtood of any other.

The reaſon is manifeſt, for whereas we ſhould have abated 8 out of 11, we did not onely not abate it, but added moreover his complement to 10, which is 2, wherefore the total is more than it ſhould be by 8 and 2, that is by 10; wherefore abating 10 from it, we have the Logarithme deſired.

Which rule, although it be general, yet we ſhall ſeldome have occaſion to uſe any other complements, than ſuch as are complements of the Logarithmes given, either to 10, 0000000, or to 20, 0000000, as ſhall hereafter appear in due place.

And thus much of Logarithmes in generall, whereof (as is before noted) there might be fitted divers kinds, but we intend to uſe onely that kind which were framed by Mr. *Henry Briggs*, at the requeſt of the Baron of *Marchiſton*; where a cypher is made the Logarithme of Unite or 1, and an Unite with many Cyphers, the Logarithme of 10, and the reſt fitted accordingly: theſe being the beſt kind, and the ground of all the beſt tables of Logarithms hitherto put forth by any.

And of this kind are the tables to this book annexed, which wanting,

ing leasure to calculate my self, I conferred together such as were formerly extant, and out of them have drawn these. It is true that the *first* of these differs in form from all others, but I have ordered it thus, esteeming it most convenient and ready for ordinary use. The *later* sheweth the Logarithms of absolute numbers from 1 to 10000, and may be used for numbers far greater; the *first* sheweth the Logarithms of the Sines and Tangents of every degree and minute of the Quadrant, and also the complements Arithmetical of the Logarithm of every Sine, which may serve as a Table of Secants. Which Logarithms of absolute numbers, Sines and Tangents, we may call Logarithmetical numbers, Sines and Tangents, or (with their first Inventour) Artificial Sines and Tagents, as being used for, and in stead of the natural. And thus if you enter the *later* of these Tables with any absolute number, you finde against it his Logarithme, if you enter the *first* with any number of degrees and minutes, you find against it his artificial sine and tangent, each under his proper title. As entring the Table with an arch of 30 degrees, 00 minutes, I finde the artificial sine thereto answering to be 9,6989700, and the tangent 9,7614394, which are the Logarithms of the natural sine 500000, and of the natural tangent, 577350 And contrariwise a Logarithme being given, you may finde the arch thereto answering.

Of artificial secants we make little use, but if you desire the artificial secant of an arch, substract the artificial sine of the complement of that arch from twice radius, or 20,0000000, the remainer is the secant required. As if I desire the secant of 22 deg. 37′: I finde the sine of his complement to be 9,9652480, which substracted from 20,0000000, there remains 10,0347520, the secant of 22 deg. 37′: the reason whereof is evident by the Corollary of the first Theoreme of Variety hereafter following, Chap. 4. in stead of these secants we have set in the two last columns of the second Table the complements arithmetical of the sines, to every of which if you add radius or 10,0000000, they become secants: these being more necessary than the secants, and by which the secant of any arch is most readily found; for if the sine of an arch be in the first column, his secant is in the last, (adding as aforesaid radius) if the sine be in the second, the secant is in the last but one. As if I would have the secant

of

of 22 deg. 37′, the ſine thereof is in the firſt column, therefore I look for the ſecants in the laſt, where I finde 0347520, to which adding 10,0000000 or 10, it is 10, 0347520, the ſecant of 22 deg. 37′.

5 *Of the Character of Logarithmes.*

The Character or Characterical note of every Logarithme in theſe tables, is the firſt figure or figures towards the left hand, diſtinguiſhed from the reſt by a Comma: and it ſheweth of how many places above the place of unites, the abſolute number to that logarithme belonging doth conſiſt. And thus the character of the logarithmes of every number leſſe than 10 is 0, but the character of the logarithme of 10 is 1, and ſo of all other numbers to 100: but the character of the logarithme of 100 is 2, and ſo of the reſt to 1000, and the character of the logarithme of 1000 is 3, and ſo of the reſt to 10000; and ſo forward. Wherefore, by the character of a logarithme you may know of how many places the abſolute number anſwering to that logarithme doth conſiſt.

6. *To find readily the complement arithmetical of a logarithme.*

The complement arithmetical of a logarithme (as it is moſt uſually taken) is the reſidue of that logarithme unto 10,000000, As the complement arithmetical of 7, 1079054 is that which makes it up 10, 0000000: if therefore 7, 1079054 be ſubſtracted from 10,0000000 the remainer is his complement arithmetical.

But to ſubſtract it readily, I begin (contrary to the ordinary courſe) with the firſt figure toward the left hand, and write the complement or reſidue thereof unto 9, and ſo I do with the reſt, till I come to the laſt figure towards the right hand, and thereof I ſet down the reſidue unto 10. Thus for the complement arithmetical of 7, 1079054 I write, for 7 his reſidue unto 9 which is 2; for 1,8; for 0,9; for 7,2; for 9,0; for 0, 9; for 5, 4; and for 4, 6: and ſo I have this number 2, 8920946, which is the complement arithmetical of 7, 1079054 unto 10, 0000000.

So if I deſire the complement arithmetical of 9,9652480, unto 20, 0000000: I write for 9, 0: for 9 again 0; for 6, 3; for 5,4; for 2, 7: for 4,5; for 8,2; and the cypher; and ſo I have 0, 0347520; and before all putting an unite it is 10, 0347520, the complement arithmetical required.

The complements arithmetical of the artificial ſines are expreſſed in in the tables; and the complements arithmetical of the tangents are the tangents of their complements: as we shall further shew hereafter.

7. *To find the Logarithme of a number that hath a fraction annexed, as also of a proper fraction.*

Reduce your number that hath a fraction annexed into an improper fraction, and substract the logarithme of the denominator from the logarithme of the numerator, the remainer is the logarithme of the whole number and fraction proposed. As if I desire the logarithm of $13\frac{1}{3}$, I reduce it into an improper fraction making it $\frac{40}{3}$ and finding the logarithme of 40 to be 1,6020600, and the logarithme of 3 to be 0,4771212, I substract the later from the former, the remainer is 1,1249388, which is the logarithme of $13\frac{1}{3}$, required.

Numerator	40.	1,6020600
Denominator	3.	0,4771212
	$13\frac{1}{3}$.	1,1249388

The reason is, for that every fraction (whether proper or improper) signifies some part or parts of an unite, the denominator shewing into how many parts the unite is divided, and the numerator shewing how many of those parts are by that fraction signified: Wherefore, as the denominator is in proportion to the numerator, so is 1 to the value of that fraction; therefore (by the *corr:* of 3 *prop: chap:* 2.) the summe of the logarithms of the denominator and of the fraction, is equal to the summe of the logarithms of the numerator and of 1; but the logarithme of 1 being 0 the logarithme of the numerator alone, is equal to the summe of the logarithmes of the denominator and of the fraction. Therefore if from the logarithm of the numerator be substracted the logarithme of the denominator, the remainer is the logarithme of the fraction. Thus in the foregoing examples.

		Logarithmes.
As the Denominator	3.	0,4771212
to the Numerator	40.	1,6020600
So is	1.	0,0000000
to $\frac{40}{3}$: or to	$13\frac{1}{3}$.	1,1249388

And for the same reason we may in like manner finde the logarithme of a proper fraction. Where it is to be noted, that seeing the logarithme of the unite 1, is 0, and every proper fraction is lesse than

an

an unite; therefore the logarithme of every proper fraction is lesse than o. As if we desire the logarithme of this proper fraction $\frac{2}{3}$; I find the logarithme of its numerator 2 to be 0,3010300, and of its denominator 3 to be 0,4771213, and substracting the later from the former, there remains —0,1760913, for the logarithme of $\frac{2}{3}$ that is 0,1760913, lesse than 0: which though it may seem strange to some, yet being a thing well understood by the skilful in Arithmetick, and of no great use here, I passe it over without further explanation.

2.	0,3010300
3.	0,4771213
$\frac{2}{3}$.	0,1760913

8. *To correct any number found in these tables, by the part proportional.*

I put these things here at the beginning as the fittest place for them, not that I esteem it necessary for young beginners to have them all perfectly before they passe any further; for, for ordinary occasions the numbers in the Tables may (for the most part) satisfie without correction by the part proportional; especially if in plain triangles you reduce the measures of the sides into their smallest parts: as if a side be given in paces, you may reduce it into feet or inches, (keeping within the compasse of the Table:) if in poles, you may reduce it into yards or feet; if in miles, you may reduce it into furlongs, poles or paces. Or, which is most easie and ready, you may reduce all measures into decimal parts, as into tenths and hundredth parts, putting behind the number given a cypher or two. As if a side of a plain triangle be 57 leagues, if we put a cypher behind, it will be 570 tenths of a league: if two cyphers, it will be 5700 centesmes or hundredths of a league; and so for any other measures. And the question being wrought, the answer will come forth in the like parts, which are easily reduced again to integers with their parts.

As suppose the side of a plain triangle given be 57 leagues, and we desire to find one of the other sides to the hundredth part of a league. I put behind it two cyphers, and so it becomes 5700, and working as you shall hereafter be directed, admit there come forth for the side required 3475, then I say, that the side required is 3475 centesmes or hundredth parts of a league, that is $34\frac{75}{100}$ leagues, or 34 leagues and 75 centesmes of a league.

If there be a fraction annexed to your number given. As if you would reduce $57\frac{1}{3}$ leagues to centesmes, I put behind 57 two cyphers, (that

(that is, I multiply it by 100) and so it becomes 5700: also I put behind the numerator of the fraction, namely, behind 1, two cyphers, and so it becomes 100, which divided by the denominator 3, the quotient is 33, (omitting the fraction) which added to 5700, the summe is 5733: And so much is $57\frac{1}{3}$ leagues in centesmes of a league. If you would have it onely in tenths, you put behind the whole number, and likewise behind the numerator of the fraction, onely one cypher, and in all things else do as before: which being easie and common, I forbear to be large therein.

But when more exactnesse is required, you may attein to it by the part proportional, after the form of these examples following.

Example 1.

Let there be required the absolute number answering to this logarithme 1,9369826. Looking for this Logarithme in the Chiliads, I find not the same, but the neerest lesse than it is, 1, 9344984, against which I find 86, which you may correct by the part proportional thus. I change the character given, making it to be 3, and so it becomes 3, 9369826, for this I look in the Chiliads, but not finding the same, I find the neerest lesse than it to be 3.9369659, and against it this absolute number 8649; whence it appears, that the number answering to the logarithme proposed, is $86\frac{49}{100}$, and something more.

But if you desire more exactnesse, as to correct it two places further: substract 3,9369659, the neerest lesser logarithme, from 3,9370161, the neerest greater, noting the difference which is here 502: Also substract the lesser 3,9369659, out of the logarithme given 3.9369826, noting the difference which is here 167. Then say by the rule of proportion,

3.9369826	} *differ.* 167
3.9369659	
3.9370161	} *differ.* 502

As the greater difference 502, is to the lesser 176:

So is 100 to 33, (and somewhat more, which we omit) which put behind 8649 towards the right hand, shews the number required to be $86\frac{4933}{10000}$, and so is it vertified to 6 places.

Example 2.

Let there be required the absolute number answering to this logarithm 5.9369826.

Because the character or characteristick is here 5, therefore the

 absolute

absolute number answering to this Logarithme must consist of *6* places: whereas the absolute numbers in these *Chiliads* consist but of four places, therefore changing the character to 3, I look for 3,9369826, and finde the neerest in the Table lesse than it to be 3,9369659, differing from it 167, and against it I finde the absolute number 8649, which I note: and the neerest greater than the Logarithme given is 3,9370161, differing from his next before found 502; therefore I say by the rule of proportion,

3,9369826 } *differ.* 167
3,9369659
3,9370161 } *differ.* 502

As the greater difference 502, is to the lesser 167:

So is 100 to 33, which put behind 8649 towards the right hand, shews the number answering to the Logarithm given to be 864933. and so may you finde any number not exceeding 6 places, answering to any Logarithme proposed.

If in either of these examples you desire it but to 5 places, then for the third number in the rule of proportion (which is here 100) put 10, and so the quotient will come out in one figure, which put towards the right hand as before.

Example 3.

Let it be required to find the Logarithm answering to this absolute number 864933.

I finde in the Chiliads the Logarithme of the first four figures 8649 to be 3,9369659, and because the number given consists of 6 places, the characteristick must be 5, therefore 5,9369659 is the Logarithme of 864900. But to finde the part proportional to be added to this Logarithme for the 33 remaining: I substract the Logarithme of 8649 from the Logarithme 8650, and finde the difference to be 502: therefore I say by the rule of proportion,

As 100 is in proportion to 33:

So is the difference 502, to 166 *fere*.

Which 166 added to 5,9369659, the summe is 5,9369825, the Logarithme of the absolute number 864933 required: if the absolute number consist but of 5 places; then for the first number in the rule of proportion (which here is 100) put 10) and proceed as before.

And although in these three examples, we have verified but to the sixth place of the absolute number; yet might we by these Tables

proceed

proceed to the seventh place, seldome erring one whole unite: the operation is after the same manner, save onely in stead of 100 used in the rule of proportion we put 1000.

And thus much touching the part proportional in the use of the first Table of Chiliads. Now for the second Table of Artificial Sines and Tangents.

Example 4.

Let there be required the arch answering to this artificial tangent 9,6197888.

Looking in the columne of Tangents, I finde not exactly the same, but the neerest lesse than it is 9,6197205, being the tangent of 22 deg. 37′: therefore the arch required is 22 deg. 37′, and some part of a minute more. Now if you desire to know what part of a minute, namely, how many seconds it is more, we may find it thus, I find the next greater than the tangent given to be 9,6200762, from which substracting the next lesser, namely, 9,6197205, the difference is 3557, also substracting this least from the tangent given 9,6197888 the difference is 683: I say therefore by the rule of proportion,

9,6197888 }	*differ.* 683
9,6197205	
9,6200762 }	*differ.* 3557

As the greater difference 3557, to the lesser 683:

So is 60 seconds, to 11 seconds and something more.

Therefore the arch required, answering to this tangent given 9,6197888 is 22 deg. 37′, 11″, and some part of a second more; but thus it is verified to a second.

And in like sort you may deale with any other, whether it be sine or tangent.

Example 5.

Let there be required the artifical tangent for this arch 22 deg. 37′, 11″. I find in the table the tangent 22 deg. 37′ to be 9,6197205, and the tangent of 22 deg. 38′ to be 9,6200762, the difference of these two is 3557, for one minute or 60 seconds: therefore by the rule of proportion.

As 60 seconds to 11 seconds: so the difference 3557, to 652; which added to the lesser 9,6197205, the summe is 9,6197857, the artificial tangent of 22 deg. 37′. 11″. And in like sort you may find the artificial sines or tangents of other arches consisting of degrees, minutes, and seconds.

The general rule and reason for all these examples may briefly be this:

As the difference of any two next logarithms in the tables, is to any part of that difference:

So is the difference of the two numbers to which they belong, to the proportional part of that difference; and the contrary.

But because this holds truer in the later part of the Chiliads where the numbers are great, than in the former; therefore we have shewed in the examples (as occasion requires) how to bring the numbers proposed to the later part of the Chiliads. And thus much touching the part proportional.

9. If one number multiply another, the summe of their Logarithms is equal to the Logarithm of their product.

As let the two numbers multiplied together be 36 and 15, the product is 540. I say then that the summe of the logarithmes of 36 and 15, is equal to the logarithme of 540, as here you may see.

36.	1.5563025
15.	1.1760913
540.	2.7323938

The reason is, for that (by the ground of multiplication.) As a unite, is in proportion to the multiplier: so is the multiplicand, to the product: therefore (by the *Coroll*: of the 3. *Prop*: *Chap*: 2) the sum of the logarithms of unite and of the product, is equal to the summe of the logarithms of the multiplier and multiplicand: but the logarithme of unite is 0, therefore the logarithme of the product alone, is equal to the summe of the logarithms of the multiplier and multiplicand.

And by the like reason, if three or more numbers be multiplied together, the summe of all their logarithms is equal to the logarithme of the product of them all.

Corollary. *Whence it is manifest, that the logarithme of a number doubled, is the logarithme of the square of that number: and the logarithme of a number trebled, is the logarithme of the cube of the same number, &c.*

Thus the logarithme of 4 being doubled, is the logarithme of 16, which is the square of 4; and being trebled, it is the logarithme of 64, which is the cube of 4; as is here to be seen.

	Logarithms.
4.	0.6020600
16.	1.2041200
64.	1.8061800

10. If

10. *If one number divide another, the logarithme of the divisor substracted from the logarithme of the dividend, leaves the logarithme of the quotient.*

		Logarithms.
As let 540 be divided by 36, the quotient will be 15: I say then if the logarithme of 36, be substracted from the logarithme of 540, there will remain the logarithme of 15, as is here to be seen.	540.	2.7323938
	36.	1.5563025
	15.	1.1760913

For seeing the quotient multiplied by the divisor produceth the dividend, therefore by the last prop: the summe of the logarithms of the quotient, and of the divisor, is equal to the logarithme of the dividend: if therefore from the logarithme of the dividend, be substracted the logarithme of the divisor, there remains the logarithme of the quotient.

And by the like reason, if the first quotient be divided by a second divisor, and the second quotient by a third divisor, &c. the summe of the logarithmes of all the divisors, substracted from the logarithme of the first dividend, leaves the logarithme of the last quotient.

As if 540 be divided by 36, the quotient is 15, which again divided by 5, the quotient is 3: I say then, that if the summe of the logarithms of the divisors 36 and 5, be substracted from the logarithme of the dividend 540, there will remain the logarithme of the last quotient 3.

Corol: *Hence it is manifest, that the half of the logarithme of any number, is the logarithme of the square root of that number, and that the third part of the logarithme of any number, is the logarithme of the cubique root of the same number.*

		Logarithms.
Thus half the logarithme of 64, is the logarithme of 8, which is the square root of 64: and the third part of the logarithme of 64, is the logarithm of 4, which is the cubique root of 64, as by this example may be seen.	64.	1.8061800
	8.	0.9030900
	4.	0.6020600

And thus much for a taste of the nature and affections of logarithms, sufficing for our present occasion: he that desires to be further acquainted with the construction and diverse applications of them, may read Mr. *Briggs* in his *Arithmetica Logarithmica.*

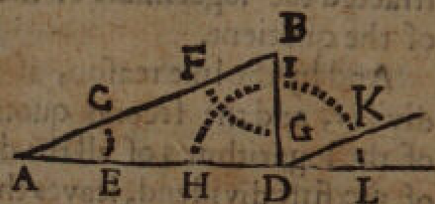
CHAP.

CHAP. III.

Of the four fundamental Axiomes of the Doctrine of Plain Triangles, *and of the cases deduced from them.*

LEMMA.

The three Angles of a right lined Triangle, are equal to two right Angles: Euclid, Lib.1. Prop.32.



THe Angles of a Triangle are measured(as we have said) by arches of a circle, the arch being described on an angular point as on a center: thus the arch C E is the measure of the angle at A, so that look how many degrees, minutes, seconds, *&c.* are in the arch C E, so much is the measure of the angle at A. In like sort, the arch F G is the measure of the angle at B, and I H the measure of the angle B D A: and these three arches C E, F G, and I H are 180 degrees, which is the measure of two right angles, (90 degrees being the measure of one right angle) for these three arches C E, F G, and I H, are equal to the semicircle H I K L: F G being equal to I K, and C E, to K L.

If therefore a triangle be right angled, one of its acute angles is the complement of the other to 90 degrees.

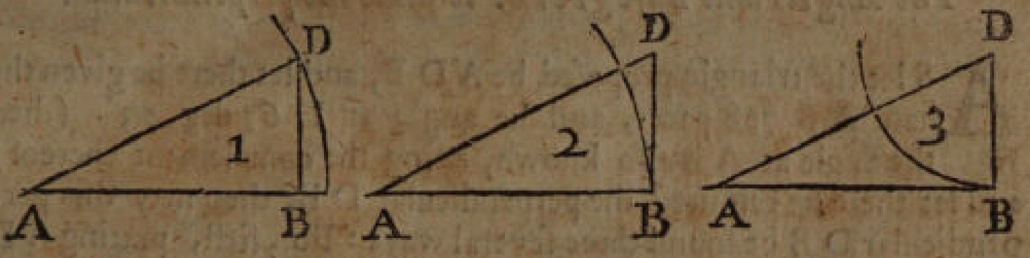
If it be an oblique angled Triangle, yet one of his angles substracted from two right angles, (that is from 180 degrees) the remainer is the summe of the other two, or if the summe of two of its angles be substracted from 180 degrees, the remainer is the third angle.

AXIOME

AXIOME I.

Of right angled Triangles.

In a plain right angled Triangle, any of the three sides may be put as Radius: and the other sides will be as Sines, Tangents or Secants.



As if A D be Radius, or the semidiameter of the circle, or the whole sine, (for by these several names it is called) then B D is the sine of the angle at A, and A B the sine of the angle at D.

If A B be Radius, (as in the second figure) then B D is the tangent, and A D the secant of the angle at A.

If D B be Radius, (as in the third figure) then A B is the tangent, and A D the secant of the angle at D.

And what proportion the side put as Radius, hath unto Radius: the same proportion hath the other sides, unto the sines, tangents or secants by them represented.

As in the third figure, look what proportion D B hath unto Radius: the same proportion hath A B, to the tangent of the angle at D, and the same hath A D to the secant of that angle: and the like is to be understood of the rest.

And from this ground are deduced the Corollaries or Cases following for the resolution of plain right angled triangles, by three things known several ways.

And for distinction sake, we call the side subtending the right angle, the Hypothenusal: and one of the sides conteining the right angle, we call the Base; and the other side the Perpendicular. As in these triangles, the hypothenusal is marked with A D, the base with A B, and the perpendicular with D B: and it will not be amisse to mark them always so. The right angle is always one of the three things given.

In

In the examples, *s* ſtands for *ſine*: *t* for *tangent*: *ſc.* for *ſine complement*: *tc.* for *tangent complement*: *ſec.* for *ſecant.*

Case I.

The Angles and Baſe given: to finde the perpendicular.

As let the triangle propoſed be A D B, and let there be given the baſe A B 768 paces, and the angle at D 67 deg. 23′. (then the angle at A is alſo known, being the complement thereof) and let there be required the perpendicular D B, then may this perpendicular D B be found three ſeveral ways: For, firſt, putting A D as radius, it followeth that

As ſine the angle at the perpendicular	*s.* D 67 deg. 23′	9.9652480
is in proportion to the baſe:	A B 768 paces	2.8853612
ſo is ſine the angle at the baſe,	*s.* A 22-37.	9.5849685
		12.4703297
In proportion to the perpendicular,	D B 320 paces,	2.5050817

Here (according to the 3 *Prop. Ch.* 2) I add the Logarithms of the ſecond and third, and from that ſum ſubſtract the Logarithme of the firſt, and the remainer which is 2,5050817 is the Logarithme of the fourth: Wherefore looking in the table for the abſolute number anſwering thereto, I finde the neereſt to be 320 which is the fourth number required. It is ſomething more than 320, but for brevity, and the eaſe of the Learner, I omit the fraction, having before ſhewed how to finde it: And if (according to the *Corol.* of 3 *Prop. Chap.* 2) in ſtead of ſubſtracting the Logarithme of the firſt, I adde his complement Arithmetical, that totall abating Radius is alſo 2,5050817 as before. And the work ſtands in this manner.

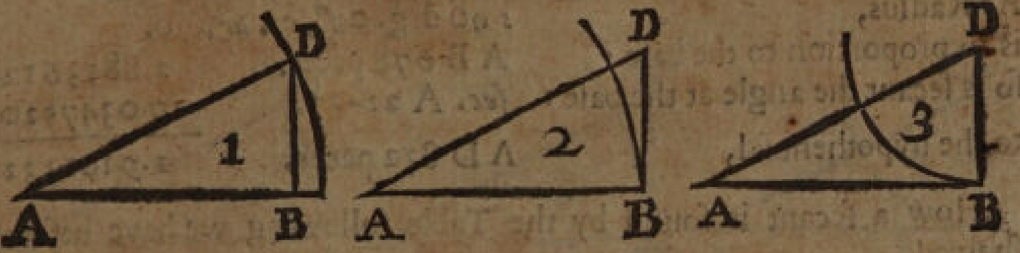
s, D 67 deg. 23′ *comp. ar.*	0.0347520
A B 768 paces.	2.8853612
s, A 22-37.	9.5849685
D B 320 paces.	2,5050817

Thus having ſufficiently explaned the operation in this firſt example, we ſhall be briefer in the reſt that follow, underſtanding the like in them alſo.

2. *If we make* A B *Radius, the proportion holds thus.*

As Radius,	*rad. s.* 90 deg. *co. ar.*	
to the base :	A B 768 paces.	2.8853612
so tang. the angle at the base,	*t* A. 22-37.	9.6197205
to the perpendicular :	D B. 320 paces.	2.5050817

Here because the compl. arith. of Radius (which is in the first place) is 0, therefore I set down in the first place onely cyphers or nothing.



3. *If we make* D B *Radius, then*

As tang. the angle at the perpen.	*t* D 67 deg. 23′ *co. ar.*	9.6197205
is to the base :	A B 768 paces.	2.8853612
so is Radius	*s* 90—00	10.
to the perpendicular.	D B 320 paces.	2.5050817

Because the Arithmetical complement of a tangent to twice Radius or 20.0000000. is the tangent of his complement, (as hereafter shall be shewed) therefore in the former example we have put for the complement arithmetical of tang. D, *his tangent complement, and so abate twice Radius : and the like you may always do when you have a tangent in the first place.*

Case 2.

The Angles and Base given : to finde the Hypothenusal.

Let there be given A B 768 paces, and the angle D 67 deg. 23′, and let there be required the hypothenusal A D.

 1. *Making*

1. *Making* AD *Radius.*

As sine the angle at the perpen.	*s.* D 67 deg. 23′ *co. ar.*	0.0347520
is in proportion to the base:	AB 768 paces.	2.8853612
so is Radius,	*s.* 90—00	10.
to the hypothenusal.	AD 832 paces.	2.9201132

almost 832 paces.

2. *Making* AB *Radius.*

As Radius,	*s.* 90 deg. 00′, *co. ar.*	0.
is in proportion to the base:	AB 678 paces.	2.8853612
so is secant the angle at the base:	*sec.* A 22-37.	10.0347520
to the hypothenusal,	AD 832 paces.	2.9201132

How a secant is found by the Table following we have before shewed.

3. *Making* DB *Radius.*

As tang. the angle at the perpen.	*t* D. 67 deg. 23′ *c. a.*	9.6197205
is in proportion to the base:	AB. 768 paces.	2.8853612
so is the secant of the same angle,	*sec.* D 67-23	10.4150315
to the hypothenusal.	AD 832 paces.	2.9201132

CASE 3.
The Angles with the Hypotehenusal given: to finde the Base.

Dat. AD 832 *paces*, D 67 *deg.* 23′. *Required* AB.

1. *Making* AD *Radius.*

As Radius,	*s.* 90 deg. 00′ *co. ar.*	0.
to the hypothenusal:	AD 832 paces.	2.9201233
so sine the angle at the perpen.	*s.* D 67-23	9.9652480
to the base.	AB 768 paces.	2.8853713

2. *Making* AB *Radius.*

As the secant of the angle A, is unto the hypothenusal AD: so is Radius, to the base AB.

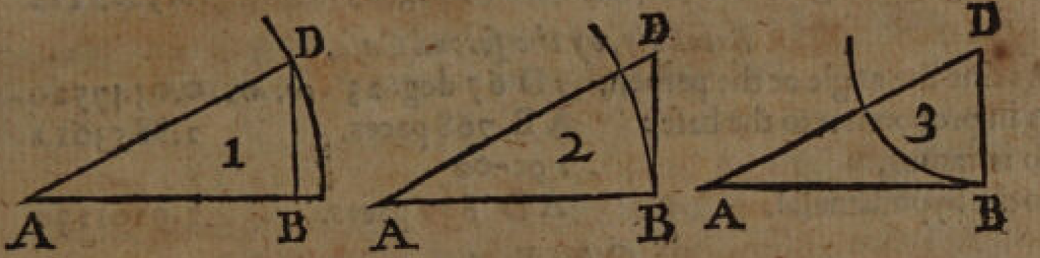
3. *Making* D B *Radius.*

As the secant of the angle D, is unto the hypothenusal A D: so is the tangent of the angle D, to the base A B.

CASE 4.

The Base and Perpendicular given: to finde an Angle.

Dat. A B 768 *paces,* D B 320 *paces. Required* A *or* D.



1. *Making* A B *Radius.*

As the base,	A B 768 paces. *co. ar.*	7.1146388
is in proportion to Radius:	*s* 90 deg. 00′.	10.
so is the perpendicular,	D B 320 paces.	2.5051500
to tang. the angle at the base.	*t* A 22-37.	9.6197888

2. *Making* D B *Radius.*

As the perpendicular	D B 320 pac. *co. ar.*	7.4948500
is in proportion to Radius;	*s* 90 deg. 00′	10.
so is the base,	A B 768 paces.	2.8853612
to tang. the angle at the perpen.	*t* D 67-23.	10.3802112

And thus are these angles found with lesse than a minute errour, he that desires exactnesse, may use the ways we have before shewed, Cap. 2. Prop. 8. It shall suffice in the examples of this book to set down the measure of arches and angles in degrees and minutes: as well for brevity, as not to burthen young beginners with all things at the first.

CASE 5.

The Base and Perpendicular given: to finde the Hypothenusal.

Dat. A B 768 *paces*, D B 320. *Required* A D.

First, by the fourth Case.

As the perpendicular,	D B 320 paces *co. ar.*	7.4948500
is in proportion to Radius:	*s* 90-00	
so is the base,	A B 768 paces.	2.8853612
to tang. the angle at the perpen.	*t* D 67 deg. 23′	10.3802112

Secondly, by the second Case.

As sine the angle at the perpen.	*s* D 67 deg. 23′. *co. ar.*	0.0347520
is in proportion to the base:	A B 768 paces.	2.8853612
so is radius,	*s* 90-00	
to the hypothenusal.	A D 832 paces.	2.9201132

CASE 6.

The Base and Hypothenusal given: to finde an angle.

Dat. A B 768 *paces*, A D 832 *paces, required* D.

1. *Making* A D *Radius.*

As the hypothenusal,	A D 832 pac. *co. ar.*	7.0798767
is in proportion to Radius:	*s* 90-00′	
so is the base,	A B 768 paces.	2.8853612
to sine the angle at the perpen.	*s* D 67 deg. 23′	9.9652379

2. *Making* A B *Radius.*

As the base A B, is in proportion to Radius: so is the hypothenusal A D, to the secant of the angle at the base A.

CASE 7.

The Base and Hypothenusal given, to find the Perpendicular.

Dat. A B 768 *paces*, A D 832 *paces, required* D B.

First, by the sixth Case.

As the hypothenusal,	A D 832 pac. *co. ar.*	7.0798767
is in proportion to Radius:	*s* 90-00′	
so is the base,	A B 768 paces.	2.8853612
to sine the angle at the perpen.	*s* D 67 deg. 23′	9.9652379

Secondly,

Secondly, by the first Case.

As Radius,	s 90 deg. 00′ *co. ar.*	c.
is in proportion to the base:	A B 768 paces.	2.8853612
so is tang. the angle at the base:	t A 22-37	9.6197205
to the perpendicular	D B 320 paces.	2.5050817

Mr. *Briggs* in his *Arithmetica Logarithmica* C. 19. but in the second edit. C. 17. resolves this Case more readily thus.

Take the Logarithmes of the summe and difference of the Hypothenusal and side given, half the summe of those two Logarithms, is the Logarithme of the Perpendicular or side required.

As let

		Logarithm.
the side given be 768 }	{ the summe 1600	3.2041200
the hypothenusal 832 }	{ difference 64	1.8061800
	summe	5.0103000
the side required 320	½ summe	2.5051500

The difference between this Logarithm here found, and that which was found by the former operation, ariseth chiefly by neglecting certain seconds in the angle D, and consequently in the angle A; for the angle A is indeed 22 deg. 37′ 11″, and somewhat more.

And thus may right angled triangles be distinguished into 7 Cases, though the resolution of all these Cases depends upon one rule, which is the axiome before put.

The three axiomes following are true in all plain triangles, but are chiefly intended for the oblique angled; which now we come to handle.

AXIOME

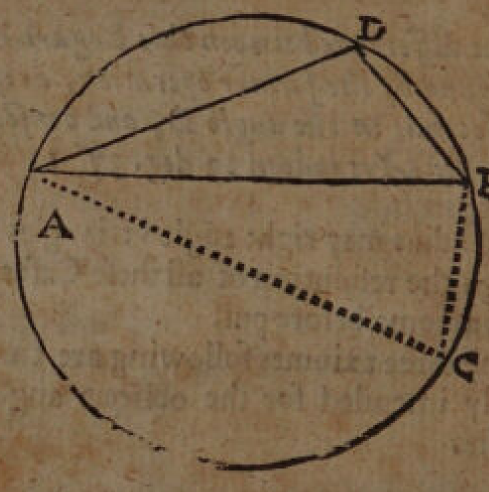
AXIOME II.

In all plain Triangles, the ſides are in ſuch proportion one to another, as are the ſines of their oppoſite angles.

AS *in the triangle* ADE. *As the ſide* AD *is in proportion to* ED : *ſo is the ſine of the angle at* E, *to the ſine of the angle at* A. *And ſo of the reſt.*

Conſt. *About the triangle* ADE, *deſcribe the circle* ADEC, *by* 5. 4. Euclid.

Demonſt. *Then are the ſides of the triangle* ADE, *as ſubtendents or chords in the circle* ADEC. *So that as the chord of the arch* AD *is in proportion to the chord of the arch* ED; *ſo is the ſide of the triangle* AD, *to the ſide* ED; (*and the like is to be underſtood of* AE) *But the half chords are ſines of half the arches ſubtended by thoſe chords, and as the whole is to the whole, ſo is the half to the half: Therefore as the ſine of half the arch* AD, *is in proportion to the ſine of half the arch* ED : *ſo is the ſide* AD, *to the ſide* ED. *But half the arch* AD, *is the meaſure of the angle at* E ; *and half the arch* ED *is the meaſure of the angle at* A, (*by* 20. 3. Euclid.) *therefore as the ſine of the angle at* E, *is in proportion to the ſine of the angle at* A ; *ſo is the ſide* AD *to the ſide* ED. *And the like is to be underſtood of the ſide* AE, *and his oppoſite angle at* D. *Therefore in all plain triangles*, &c. *which was to be proved.*



And ſeeing as the ſine of E *to the ſine of* A : *ſo is* AD *to* ED, *therefore alſo alternately* (*by* 16. 5. Euclid.) *As the ſine of the angle at* E, *is in proportion to* AD ; *ſo is the ſine of the angle at* A *to the ſide* ED, &c. Therefore,

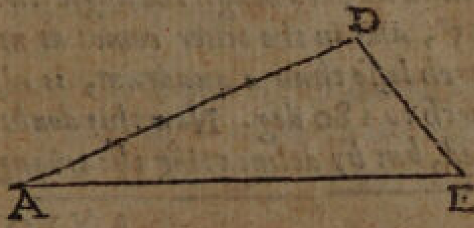
CASE

CASE 8.

The angles of a triangle, with one of the sides being given to finde any of the other two sides.

Let there be given the angle at A, 22 degrees 37'. and the angle at E 53 degr. 08'. and the side A D 780 paces.

And let there be required the side E D. Then by this Axiome



As the sine of an angle,	s E 53 deg. 08' *co. ar.*	0.0968917
is to his opposite side given:	A D 780 paces.	2.8920946
so is the sine of another angle,	s A 22-37	9.5849685
to his opposite side required.	E D 375 paces *ferè.*	2.5739548

Here it is not full 375 paces, but wants about four inches, but 375 is the number in the Table neerest agreeing to the Logarithme 2.5739548 without a fraction, and I would not trouble beginners with fractions at first; having spoken sufficiently of them, Chap.2. Sect.7. and 8.

CASE 9.

Two sides being given, with an angle opposite to one of them: to finde an angle opposite to the other of them.

Dat. A D 780, E D 375, the angle at E 53 deg. 08'.
Required the angle at A.

As one of the sides given,	A D 780 par. *c.a.*	7.1079054
to the sine of his opposite ang. given,	s E 53 deg. 08'.	9.9031083
so is the other side given,	ED 375 parts.	2.5740313
to the sine of his opposite angle req.	s A 22 deg. 37'.	9.5850450

In the use of this last Case, the angle opposite to the greatest side being required, it will be sometimes doubtfull whether it be acute or obtuse; for in the triangle A D E, in the scheme of the second Axiome.

As E D 375 *parts, to* s A 22 deg. 37'.
so A E 945 *parts, to* s D 104 deg. 15'.

And

And in the triangle A E C,

As E C 375 *parts, to* s E A C, 22 deg. 37′.
so A E 945 *parts, to* s C, 75 deg. 45′.

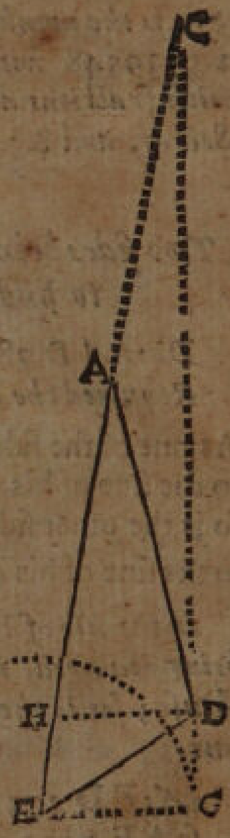
In either of which the operation is one and the same, and the sine found all one, though the angle in the one exceed a quadrant by 14 *deg.* 15′, *and in the other comes as much short: Because every sine of an arch lesse than a quadrant, is also the sine of the complement of that arch to* 180 *deg. Now this doubt cannot sometimes be otherwise cleared, but by delineating the triangle as exactly as you can.*

AXIOME III.

In all plain triangles, as the summe of two sides, is to their difference: so is the tangent of the half summe of their two opposite angles, to the tangent of the difference of either of them, above or under the half summe.

Let A D E *be an oblique triangle,*

Conſt. *Make* A C *and* A H *each equal to* A D, *and draw* D H, *and parallel thereto draw* E G; *and draw a line from* C *to* D, *extending it to* G.



Demonſt. *And forasmuch as* A H *is equal to* A D, *therefore (by* 5.1. Euclid) *the angle* AHD *is equal to* A D H, *and by the like reason the angle* ACD *is equal to* ADC, *therefore the whole angle* HDC *is equal to both these angles* CHD *&* HCD; *therefore (by the* corr. 31.3. Euclid) *the angle* H D C *is a right angle. And forasmuch as* E G *is parallel to* H D, *therefore (by* 29.1. Euclid) *the angle* E G C *is also a right angle: for it is equal to* HDC, *and (by the same) the angle* CEG *is equal to* CHD, *and* EDH *to* DEG. *But (by* 32.1. Euclid) *the outward angle* AHD *is equal to the two inward angles* HED *and* EDH, *put* ADH *common to both: then these two angles* AHD *and* ADH, *are equal to these two* AED *and* ADE; *therefore either of these two angles* AHD *and* ADH *is half the sum of these two angles* AED *and* ADE, *therefore also the angle* C E G *is half the sum of the same angles* A E D *and* A D E.

Now if to one of the sides of a triangle there be drawn a parallel, it divides the other sides proportionally (*by* 2.6. Euclid) *therefore as* C H *is in proportion to* H E, *so is* C D *to* D G: *therefore also composed* (*by* 18.5. Euclid) *As* C E *to* H E, *so is* C G *to* D G: *that is,*

As C E *the summe of the sides* A E *and* A D,
is in proportion to H E *their difference:*
so is CG *the tangent of half the sum of the angles* AED *and* ADE,
to D G *the tangent of the angle* D E G, *being that which the angle* A E D *comes short of the half summe: as* H D E *is the excesse of the angle* A D E *above the half summe.*

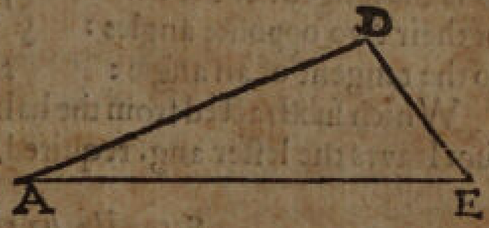
Therefore in all plain triangles, as the summe of two sides, is to their difference; so, &c. which was to be proved.

Therefore in any plain oblique triangle:

CASE 10.

Two sides, with their conteined angle being given: to finde the other angles.

Dat. { A E 189 *paces* } *Sum* 345
{ AD 156 *paces* } *differ.* 33
{ A 22 deg. 37′



Requ. D *or* E; *which together are* 157 *deg.* 23′, *being the complement of the angle* A *to* 180 *deg.* 00′, *by the first Lemma.*

As the sum of the sides given; (AE+AD)	345 parts.	c.a.	7.4621810
is in propor. to their difference; (AE—AD)	33		1.5185139
so is the tang. of the half summe of their two opposite angles;	t ½(E+D)	78 deg. 41′ ½	10.6090331
to the tangent of an angle	t F	25—33 ½	9.6797280
Which added to the half sum makes the greater of the angles required.	D	104—15	
Or substracted leaves the lesser	E	53—08	

Here (A E+A D) signifies A E more A D, or the summe of them added together (A E—A D) A E lesse A D, or the remainer of A E when A D is substracted: $t\frac{1}{2}$ E+D the tangent of half the summe of the angles E and D.

The angle found we mark with F onely for distinction sake; and the like is to be understood when we meet with the like notes.

CASE II.

Two sides and their conteined angle given: to finde the third side.

Dat. A E 189 *paces*: A D 156 *paces*: A 22 *deg.* 37′.
Req. E D.

First by the tenth Case.

As the sum of the sides given;	(AE+AD) 345 par. *c.a.*		7.4621810
is in propor. to their difference:	(E—AD) 33 parts		1.5185139
so is the tangent of the half sum of their two opposite angles:	$t\frac{1}{2}$(E+D) 78 de. 41$\frac{1}{2}$		10.6990331
to the tangent of an angle:	t F	25—33$\frac{1}{2}$	9.6797280
Which substracted from the half sum leaves the lesser ang. required.	E	53—08	

Secondly, by the eighth Case.

As the sine of the angle found,	s E 53 deg. 08′. *c.a.*	0.0968917
is in proportion to his opposite side	A D 156 paces	2.1931246
so is the sine of the angle given;	s A 22 37	9.5849685
to this opposite side required	E D 75 paces	1.8749848

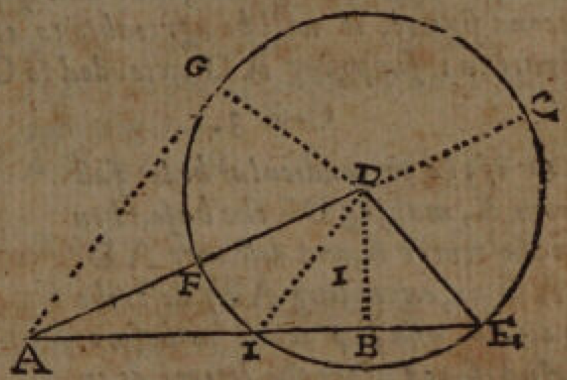
AXIOME

AXIOME IV.

In oblique triangles, as the true base is in proportion to the summe of the sides: so is the difference of the sides, to the alternate base.

As in the oblique triangle A D E.

CASE 1.



Admit A E *to be the true base;* Const. *Upon the point* D, *and distance* D E, (D E *not exceeding* D A) *describe the circle* IEFG; *and producing* AD *to* C, *let fall the perpendicular* DB, *and draw the touch line* AG. *Then* DC *and* DF, *being each of them equal to* D E; A C *is the sum and* AF *the difference of the sides* D E *and* D A; *and* A E *is the true base, and* A I *the alternate base.*

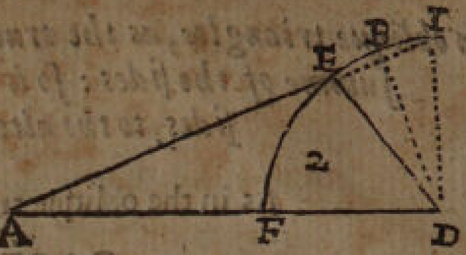
I say then, as the true base A E, *is to the summe of the sides* A C. *So is the difference of the sides* A F, *to the alternate base* A I.

Demonst. *For seeing that from a point without the circle* A, *there is drawn the line* A C *cutting the circle, and the line* A G *touching the circle, therefore (by* 36 prop. 3 Euclid) *the rectangle figure of* A C *and* A F, *is equal to the square of* A G: *and by the like reason, the rectangle of* A E *and* A I, *is equal to the square of* A G. *Therefore the rectangle of* A C *and* A F *is equal to the rectangle of* A I *and* A E. *But equal rectangles have their sides reciprocally proportional, (by* 14 *pro.* 6 Euclid.) *Therefore as* A E *is in proportion to* A C, *so is* A F *to* A I. *Which was to be proved.*

And this Case might suffice, there are two others, which are as followeth.

Case 2.

The like demonstration serves for the other two cases in this Axiome, namely, if we let fall the perpendicular from E, *making* A D *the base. For then upon the center* E, *and distance* E D. (E D *not exceeding* E A) *describe the circle*, &c. *proceeding as before: and if you place the letters as in this second figure, it will be agreeable to the words of the former demonstration: supposing* A D *extended to* C.



Case 3.

Or if the perpendicular be let fall from A, *making* DE *the base, then on the center* A, *and distance* A D, (A D *not exceeding* AE) *describe a circle, and so proceed as before. And if you place the letters as in the third figure, it will be agreeable to the former demonstration: supposing* A D *extended to* C.

In every of these three triangles A E *is the true base, and* A I *is the alternate base.*

Case 12.

Three sides of an oblique triangle being given: to finde an angle.

Dat. A E 189 *paces*, A D 156 *paces*, E D 75 *paces. Required* A. } In the first triangle.

First, (by this fourth Axiome) I resolve it into two right angled triangles, thus.

As the true base,	A E	189 pac. *co. ar.*	7.7235382
is to the summe of the sides	(A D+ED)	231 pac.	2.3636120
so is the difference of the sides	(AD—ED)	81 pac.	1.9084850
to the alternate base,	A I	99 pac.	1.9956352

Having thus the true and alternate base, substract the lesser from the greater, and in the middle of the remainer fals the perpendicular:

(by

by 3. *prop.* 3. *Euclid*) resolving the oblique triangle into two right an-gled triangles, in either of which the hypothenusal and base is known.
As the difference of the true and alternate base being 90 *paces, the half is* E B 45 *paces*; (serving to find the angle at E) *being the base in the right angled triangle* E B D.
Which half here substracted from the true base A E 189 *paces, leaves the base in the other right angled triangle* A B D, *namely*, A B 144 *paces*; serving to find the angle at A.

Then in the right angled triangle *A D B*, having the base *A B* 144 paces, and the hypothenusal *A D* 156 paces; we may finde the angle at *A*, (by the 6 Case before going) thus.

As the hypothenusal,	A D 156 parts, *co. ar.*	7.8068754
is in proportion to Radius :	s 90 deg. 00′.	
so is the base found,	A B 144 parts.	2.1583625
to the sine of the complement of the angle at the base.	*sc.* A 67 23	9.9652375

the complement whereof 22 deg. 37′ is the angle at A required.

In like manner might be found the angle at E.

In setting down this fourth Axiome I have followed the Lord *Nepair*: *Pitiscus* and others have it thus.

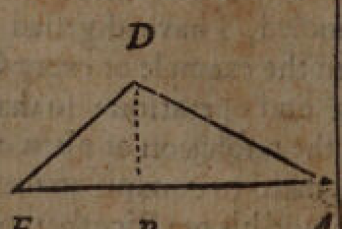
As the greatest side is to the sum of the other two sides; so is the differerence of these two, to a part of the greatest: which taken from the greatest, the perpendicular falls in the middle of the remainer.

As in the first figure before going; as the greatest side A E, is to the summe of the other sides A D and E D, (that is AC :) so is the difference of those sides A F, to a part of the greatest A I: which taken from the greatest, the remainer is I E, in the middle whereof at B, fals the perpendicular.

Which differs little from the former, and is demonstrated in the same manner.

Now that you may at once have a view of that which we have before in this Chapter more largely handled, I have digested into this Table the things given and required in the example of every Case, expressing also briefly their proportion and operation; so that hereby you may be sufficiently directed for the resolution of plain triangles. Though I would rather advise every man to commit to memory the four Axiomes before going, and to ground his practice thereon.

An Exemplary Table of Plain Triangles.

	Case.	Dat.	Req.	Proportionality.
In right angled triangles { The ſide given is marked with A B, or if none be given, the ſide required is marked with A B; placing B always at the right angle, and A D to the hypothenuſal.	1	AB. D	D B	Ra. t A. A B. D B.
	2		A D	s D. Ra. A B. AD.
	3	A D. D	A B	Ra. s D. A D. A B.
	4	AB. DB	D	D B. A B. Ra. t D.
	5		A D	{ DB.AB. Ra. t D. / s D. R a. AB. AD. }
	6	AB. AD	D	A D. A B. Ra. s D.
	7		D B	{ AD. AB. Ra. s D. / Ra. t A. AB. DB. }
		Or for this laſt		{ A D + AB } DB.DB { A D - A B }
In oblique angled triangles, Mark the things given and required with the letters here given and required in that Caſe. Oppoſite ſides and angles given and required	8	A. E. A. D.	E D.	s E. s A. AD. E D.
	9	AD. E. E. D.	A	AD. ED. s E. s A.
Two ſides and their contein-ed angle gi-ven: to finde { An angle.	10	A E the longer, AD the ſhorter	E or D	As (AE + AD) to (AE-AD ſo t $\frac{1}{2}$ (E + D) to t F $\frac{1}{2}$ E + D { + F is D / -- F is E }
The third ſide. }	11	A	ED	Find by the laſt caſe E, then having A: E. A D. finde by the 8 Caſe E D.
Three ſides given: to finde an angle. 	12	AE baſe A D the longer, E D the ſhorter ſide	A	As A E to (A D + E D) ſo A D--E D) to AI. $\frac{1}{2}$ differ. A E and AI is EB A E + or--E B is A B. Then by the 6 Caſe; As AD to AB: ſo Ra: to ſc: A. or As E D to E B; ſo Ra: to ſc: E.

CHA

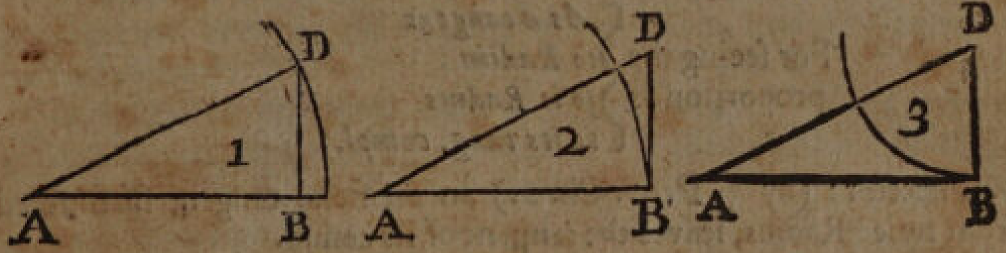
CHAP. IV.

PITISCUS in his *Trigonometria*, hath four Theoremes for the varying of proportions, and for the finding ou: the thing required in a plain or ſpherical triangle ſeveral ways: which briefly are in effect as followeth

The Grounds or Theoremes for varying the terms of the proportions of Sines, Tangents and Secants.

Theoreme 1.

The proportion of Radius to a ſine, tangent or ſecant; and contrariwiſe the proportion of a ſine, tangent or ſecant to Radius: may be varied three ways, by the firſt Axiome of plain Triangles.



For,

As ſine D B, *to Radius* A D; *in the firſt triangle,*
ſo Rad. D B, *to ſecant* A D; *in the third triangle, &*
ſo tang. D B, *to ſecant* A D; *in the ſecond triangle.*
} And the converſe.

Again.

As tangent D B, *to Rad.* A B; *in the ſecond triangle,*
ſo is Radius D B, *to tang.* A B; *in the third triangle, &*
ſo is ſine D B, *to ſine* A B; *in the firſt triangle.*
} And the converſe.

And the like is to be underſtood of ſecants, but this may ſuffice.

Hence then,

As the ſine of an arch or angle, is to Radius:
ſo is Radius, to the ſecant compl. of that arch;
and ſo is the tangent of that arch, to his ſecant;
} And the converſe.

Alſo

Alſo

As the tangent of an arch or angle, is to Radius: ſo is Radius, to the tangent complement thereof: and ſo is the ſine thereof, to the ſine of its compl. } And the converſ.

Corollary.

Hence it is evident, that Radius a mean proportional between the ſine of an arch, and the ſecant of the complement of the ſame arch: alſo between the tangent of an arch, and the tangent of the complement of the ſame arch.

And hence it is, that the complement arithmetical of the artificial ſine of an arch, is the artificial ſecant of that arches complement. And the complement arithmetical of the artificial tangent of an arch, is the tangent of the complement of that arch. (Here you are to underſtand the complement arithmetical to twice Radius, or to 20.0000000.)

For ſeeing the proportion is { *As a tangent to Radius; ſo is Radius to its tang. compl.*

Therefore (by 3. Prop. Ch. 2.) an artificial tangent ſubſtracted from twice Radius, leaves the tangent of its compl.

Or (by the Corollary of the 3. Prop. Ch. 2.) a tangent added to the tangent of his complement is equal to twice Radius.

And the like is to be underſtood of the ſine of an arch, and the ſecant of the complement thereof.

Theoreme 2.

The ſines of ſeveral arches, and the ſecants of their complements, are reciprocally proportional. That is,

As the ſine of an arch or angle,
is to the ſine of another arch or angle:
ſo is the ſecant of the complement of that other,
to the ſecant of the complement of the former.

Demonſt. For (by the foregoing Corollary) Radius is the mean proportional between the ſine of any arch, and the ſecant of the complement of the ſame arch.

Therefore

Therefore the rectangle of any sine, and of the secant of his complement, is equal to the square of Radius, (by 17. 6. *Euclid.*) so that all rectangles made of the sines of arches, and of the secants of their complements, are equal one to another.

But equal rectangles haue their sides reciprocally proportional (by 14. 6. *Euclid*) Therefore, &c.

Theoreme 3.

The tangents of several arches, and the tangents of their complements, are reciprocally proportional. That is,

As the tangent of an arch or angle,
is to the tangent of another arch or angle:
so is the tangent of the complement of that other,
to the tangent of the complement of the former.

Demonst. For (by the fore-going Corollary) Radius is the mean proportional between the tangent of every arch, and the tangent of his complement.

Therefore the rectangle made of any tangent, and of the tangent of his complement is equal to the square of Radius, (by 17.6. *Euclid*) so that all rectangles made of the tangent of arches, and of the tangents of their complements, are equal one to another.

But equal rectangles, &c. as before.

Theoreme 4.

If four magnitudes be proportional: then alternately also they are proportional: 16 pro. 5 Euclid.

And the like is to be understood of numbers.

As if 3 be in proportion to 4; as 9 to 12. then also,
As 3 is in proportion to 9; so is 4 to 12.

And hence (whereas we have before throughout this book compared sides, to the sines and tangents of angles, &c.) we may compare sides to sides, and angles to angles, as in the exemplary Table we have done.

And thus much touching the Doctrine of plain Triangles.

THE DOCTRINE OF SPHERICAL TRIANGLES.

CHAP. I.

Of Circles of the Sphere, and their interſections; and of the kinds and affections of Spherical Triangles in general.

To define in this place the ſeveral circles of the Sphere were ſuperfluous, becauſe they are beſt underſtood in the *uſe of the ſphere or globe*, wherewith it is requiſite the Reader ſhould be acquainted (at leaſt in part) before he apply himſelf to the *Doctrine of Spherical Triangles.* Therefore paſſing by theſe, we come to thoſe things which more immediately concern our preſent purpoſe.

Prop. 1. *The ſides of a ſpherical triangle are three arches of great circles; every arch being leſſe than a ſemicircle.*

Therefore the arches of parallels, or other leſſer circles of the ſphere, are not to be taken as the ſides of a ſpherical triangle.

2. *A great circle of the Sphere, is that which divides the Sphere equally into two Hemiſpheres: and is every where diſtant from its own poles, by a quadrant or fourth part of a great circle.*

Thus the Equinoctial is a great circle of the Sphere, dividing it equally into the Northern and Southern Hemiſpheres, and it is every where diſtant from its own poles, (namely, from the North and South poles of the world) by a quadrant, or 90 degrees. The like is to be underſtood of the Eclipick, and of all Horizons, Meridians, Azimuths, and of all other great circles of the Sphere.

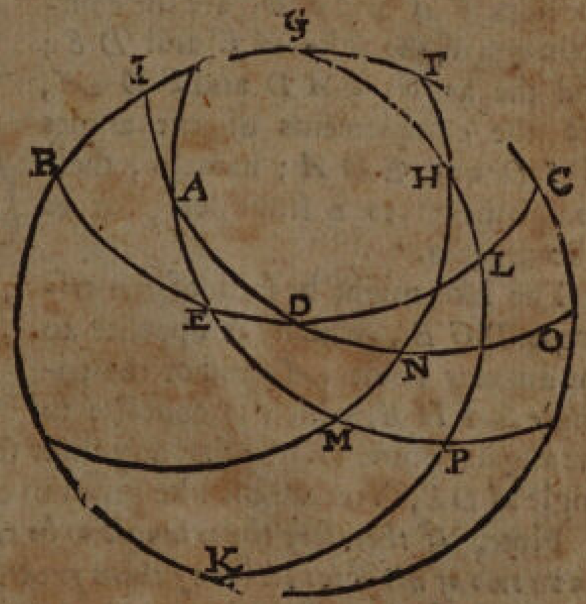
3. A

3. *A spherical angle is measured by the arch of a great circle, described on the angular points as a center, between the sides being extended to quadrants.*

Thus in the scheame next following, the angle A D E is not measured by the arch A E, but by the arch I B: because I B is described on the angular point D, as a center, between the sides D A and D E, being extended to quadrants.

4. *Any two great circles of the sphere, intersect one another in two opposite points, making the angles at those points equal one to another, and either of them equal to the distance of the poles of the same circle.*

As the Equinoctial and Ecliptick intersect one another in the points of *Aries* and *Libra*, which points are directly opposite one to another, being distant a semicircle or 180 degrees; and the angle by them comprehended at the beginning of *Aries*, is equal to that by them comprehended at the beginning of *Libra*: And either of these angles is equal to the distance of their poles, namely, 23 deg. 31′.

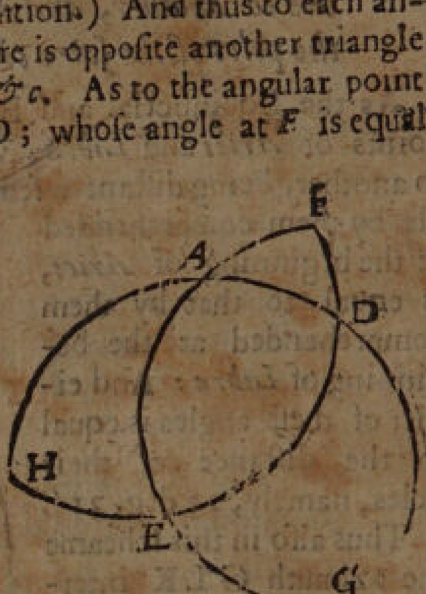


Thus also in this scheame the azimuth G L K intersects the meridian G C K in the opposite points G and K (that is in the Zenith and Nadir) the angle of their intersection at G, being equal to that at K; either of which angles is measured by the arch of the horizon C L, which is equal to E D, the distance of the poles of the same circles.

Corol. *Therefore if a great circle of the sphere passe by the poles of another great circle, it divides the same at right angles: and the converse.*

5. *Every spherical triangle hath opposite to each angular point another triangle, having the same base with the former, and the angle opposite thereto equal, the other parts of it are the complements of the several parts of the former to a semicircle.*

Let *A D E* be a spherical triangle, and extend the sides thereof *D A* and *D E*, till they concurre at *H*, also *A D* and *A E*, till they concurre at *G*; and lastly, *E A* and *E D*, till they concurre at *F*. Then are the arches *D A H*, *D E H*, *A E G*, *A D G*, *E D F*, and *E A F*, semicircles (by the fourth Proposition.) And thus to each angular point of the triangle *A D E*, there is opposite another triangle having the same base with the former, &c. As to the angular point *E*, there is opposite the triangle *A F D*; whose angle at *F* is equal to the angle at *E*, and the base *A D* is common to both triangles; and the sides *F A* and *F D*, are the complements of the sides *A E* and *D E*; and the angles *F A D* and *F D A*, are the complements of the angles *E A D* and *E D A*; namely, their complements to a semicircle, or to 180 degrees.



The like might be said of the triangle *D G E*, which is opposite to the angular point *A*, and of the triangle *A H E*, which is opposite to the angular point *D*. So that any three things being given in the triangle *A D E*, there are the like given in every of these triangles.

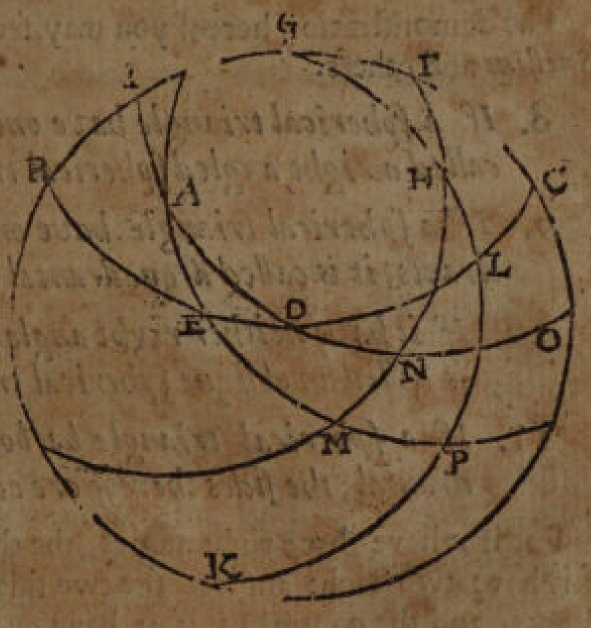
Note. *If therefore the triangle to be resolved be obtuse angled, or have two of his sides either of them greater than a quadrant*: though you might find out the thing required in that, yet it will be more convenient to resolve one of the least of the three triangles opposite to his angular points. As if a question were proposed in the triangle *A D E*, it may more conveniently be wrought in the triangle *A F D*.

6. If

6. If three great circles make by their intersections a spherical triangle; and if the poles of those circles be the angular points of another spherical triangle: the angles of the first triangle shall be equal to the sides of the second, and the sides of the first, to the angles of the second. If only in stead of the greatest side or greatest angle, you take the complement thereof to a semicircle.

This is apparent by the fourth Proposition of this Chapter, and both this, and the latter part of that may be further manifested thus.

Let A D be an arch of the equinoctial, A E an arch of the eclip-tick, E D an arch of the horizon, making the triangle A D E; and let G be the pole of the horizon, F the pole of the equinoctial, and H the pole of the ecliptick. Then on the point A as a center, and at the distance of a quadrant or 90 degrees A M or A N, describe the arch M N, which (by the third Proposition) is the measure of the angle at A, and in like sort O C, the measure of the angle at D, and P L, the measure of the complement of the greatest angle A E D to a semicircle. And forasmuch as the arch M N is distant from A 90 degr.es, and the poles of the arches A D and A E, namely, F and H, are also (by the second Proposition) distant from the same point A 90 deg. therefore the arch M N being produced, will passe by the poles H and F. And for the like reason the arch O C, will passe by the poles F and G. And P L by the poles H and G, so making the triangle G H F. I say then that the sides of the triangle G H F, are equal to the angles of the triangle A E D.



For the quadrant F N is equal to the quadrant M H, and taking away

way N H, which is common to them both, there remains the ſide F H, equal to M N; which arch M N is the meaſure of the angle at A. And by the like reaſon G F is equal to C O, the meaſure of the angle at D, and G H is equal to L P, the meaſure of the complement of the greateſt angle A E D to the two right angles. And in like ſort we may prove that the ſide A E, is equal to M P, the meaſure of the angle at H; and E D equal to L C, the meaſure of the angle at G; and A D equal to N O, the meaſure of the complement of the greateſt angle G F H to 180 degrees. Therefore, *If three great circles make by their interſections a triangle, &c.* which was to be proved.

Corrollary.

Hence it is evident, that the angles of a ſpherical triangle, may be changed into ſides, and the ſides into angles.

7. *The three angles of every ſpherical triangle, are greater than two right angles.*

The demonſtration hereof you may ſee in *Regiomontanus*, *Pitiſcus*, *Snellius* and others.

8. *If a ſpherical triangle have one or more right angles, it is called a right angled ſpherical triangle.*

9. *If a ſpherical triangle have one or more of his ſides quadrants, it is called a quadrantal triangle.*

10. *If it have neither right angle, nor any ſide a quadrant, it is called an oblique ſpherical triangle.*

11. *If a ſpherical triangle be both right angled, and quadrantal, the ſides thereof are equal to the oppoſite angles.*

For if it have three right angles, the three ſides of it are quadrants, if it have two right angles, the two ſides ſubtending them are quadrants, and the contrary: if it have one right angle, and one ſide a quadrant, it hath two right angles, and two quadrantal ſides: All which is evident by the Corrollary of the fourth propoſition. But if two ſides be quadrants, the third meaſureth their conteined angle by the third propoſition. Therefore for the ſolution of theſe kinds of triangles there needs no further rule.

To

To these we may adde three Propositions set down by the *Baron of Merchistone* in his book of the use of the admirable Table of Logarithms: being as followeth.

12. *Two oblique angles of a sphericall triangle, are either of them of the same kinde of which their opposite sides are.*

Therefore knowing of what kinde the one is, it appeareth also of what kinde the other is.

13. *If any angle of a triangle be neerer to a quadrant than his opposite side: two sides of that triangle shall be of one kinde, and the third lesse than a quadrant.*

14. *But if any side of a triangle be neerer to a quadrant than his opposite angle, two angles of that triangle shall be of one kinde, and the third greater than a quadrant.*

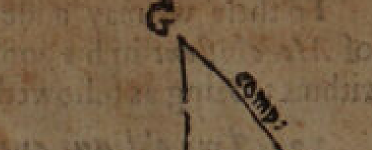
CHAP. II.

Of the first fundamental Axiome for sphericall Triangles: and of the solution of right angled and quadrantal Triangles thereby.

PITISCUS, and others to these times, for the solution of right angled spherical triangles, (not medling with quadrantals) have delivered two Axiomes; by help whereof two things given, (besides the right angle) a third may be found. But for the most part; the sides of the triangle must be produced, that so there may be divers triangles made by their intersections, consisting of the parts of the first, or of the complements of those parts diversly. And then it must be considered, to which of all those triangles one of the said Axiomes may aptly and immediately be applied, for finding the thing required, or the complement thereof. But the Honourable Lord *Nepair*, amongst many excellent propositions by him framed in the Doctrine of Triangles, hath two, which we intend to make use of, as fundamental Axiomes for the solution of all the cases of spherical triangles. The first serving for the solution of right angled and quadrantal triangles without producing any side, which after some preparation thereunto, we will set down with some little alteration answerable to the nature of the Logarithms now in use.

It

It is firſt to be underſtood, that right angled ſpherical triangle hath five parts beſides the right angle; which he calls the natural parts: As the triangle A B D, right angled at B, hath the ſide A B, the angle at A, and the hypothenuſal A D, the angle A D B, and the ſide D B. Three of theſe parts which are fartheſt from the right angle, namely, the angle B A D, the hypothenuſal A D, and the angle A D B, we mark or note by their complements to a quadrant. As the angle



B A D we account as the complement of the ſame angle, and ſo write *compl:* B A D; for A D *compl:* A D; for A D B *compl.* A D B. But the two ſides D B and A B, being next to the right angle, are not noted by their complements. And theſe parts thus noted or accounted, he calls the five circular parts of a right angled triangle. Namely, 1 A B, 2 *compl:* B A D, 3 *compl:* A D, 4 *compl:* A D B, 5 D B.

Likewiſe the quadrantal triangle A D G, (whoſe ſide D G is a quadrant) hath five parts beſides the quadrantal ſide. Namely, the ſide A G, the angle at G, the angle G D A, the ſide A D, and the angle D A G, which we may call his natural parts. But three of theſe parts, which are furtheſt from the quadrantal ſide, namely, the ſide G A, and the angle G A D, & the ſide A D, we account as the complements of the ſame parts, and ſo note them by their complements. As complement G A, complement G A D, or D A B (which is all one) complement A D. The other two angles A D G and D G A, being next to the quadrantal ſide are not noted by their complements. And theſe 5 parts thus noted or accounted, he cals the 5 circular parts of a quadrantal triangle. Namely, 1 complement A G, 2 A G D, 3 G D A, 4 complement A D, 5 complement D A G or D A B.

Now of theſe five parts, two are always given to find a third; and of theſe three one is in the middle, and the other two are extreams, either adjacent to that middle one; or oppoſite to it. As in the triangle A D B, A B and A D lying next to the angle D A B, are ſaid to be adjacent extreams to A; and for the like reaſon the angles

gles B D A and B A D, are extreams adjacent to the hypothenusal A D, and so are A D, and D B, to D; and D and A B, to D B: and lastly, D B and A to A B, for the right angle at B, is not reckoned amongst the five circular parts. So also A B and A, are said to be opposite extreams to the angle A D B, because neither of them are adjoyning to it: also *A* and *A D*, are opposite extreams to *D B*; *A D* and *D*, to *A B*; *D* and *D B*, to *A*; *D B* and *A B* to *A D*.

And the like is to be understood in the quadrantal triangle *D A G*; namely, that the angles at *D* and *A*, are extreams adjacent to *A D*; *A D* and *A G*, to *A*; A and G, to A G: A G and D, to G; G and D A, to D. And in like manner A G and G, are opposite extreams to A D: G and D to A; D and A D, to A G; A D and A, to G, A and A G to D.

1. *Fundamental* AXIOME.

Of the five circular parts in a spherical Triangle, right angled or quadrantal.

THe sine of a middle part with Radius, is equal to the tangents of the extreams adjacent; or to the sines complement of the opposite extreams.

Understanding by sines and tangents, the artificial sines and tangents, that is, the Logarithms of the natural sines and tangents.

For,

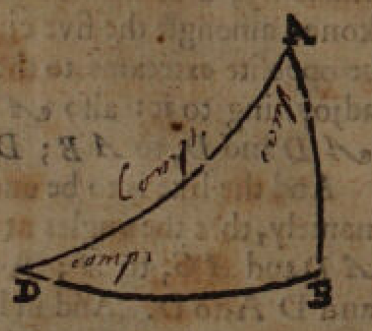
As Radius, to the tangent of one of the extreams adjacent: so is the tangent of the other extream adjacent, to the sine of the middle part.

And,

As Radius, to sine complement of one of the opposite extreams: so is sine complement the other opposite extream, to the sine of the middle part.

The demonstration hereof he hath briefly shewed in his book of the use of the admirable Table of the Logarithms: and we more fully at the end of this book. Therefore we will here onely illustrate it by examples, as followeth.

In the right angled Triangle A B D, we have shewed before how A B and A D are extreams adjacent to the angle at A: and that A D, as also the angle A, are noted by their complements. Therefore by this Axiome, The sine of the complement of the angle at *A*, added to Radius; is equal to the tangent of the complement of *A D*, added to the tangent *A B*, which we may briefly expresse thus. *s c. A + Radius = tc. A D + t A B*: which is as much as to say; sine complement *A*, more Radius, is equal to tangent complement *A D*, more tangent *A B*; this signe + signifying more, or addition, this = equality, this — lesse, or substraction, as we have before noted upon the third Axiome of plain triangles.



Now admit — Then is

A D 74 deg. 50′; *tc* 9.4330804, = *A* 70 deg. 03′ 03″ *sc* 9.5329939
A B 51 32; *t* 10.0999135, — *Radius* 10.0000000
19.5329939 — 19.5329939

Here the tangent of the complement of *A D*, being added to the tangent of *A B*, the summe is 19.5329939; so also the sine of the complement of *A*, added to Radius, the totall is 19.5329939, as the other: And here the angle *A* is 70 deg. 03′, 02″, 35‴, but we neglect the thirds. Again (by the later part of this Axiome) the sine of the complement of the angle *A D B*, more Radius, is equal to the sine of the angle at *A*, more the sine of the complement of the side *A B*, which we expresse thus: *sc* D + *Rad.* = *s* A + *sc* A B; which may thus appear.

Admit — Then is

A 70 deg. 03′ 03″; *s* 9.9731255 = D 54 deg. 12′ 58″ *sc* 9.7669572
AB 51 32 00″; *sc* 9.7938317 — *Radius* 10.0000000
19.7669572 — 19.7669572

So also (by this Axiome) in the quadrantal triangle A D G, the sine of the complement of the angle at A more Radius, is equal to the tangent of the complement of A G, more the tangent of the complement of A D; which we expresse thus; *sc* A + *Rad.* = *tc* A G + *tc* A D; which may thus appear.

Admit

Admit

AG 38 d. 28′; *tc*	10.0999135	=	A 70 deg. 03′ 03″ *sc*	9.5329979
AD 74 50 *tc*	9.4330804		*Rad.*	10.0000000
	19.5329939			19.5329939

And the like is to be underſtood of the reſt, as by this Table following may appear.

1 *s* D B + Rad = *s* A D + *s* A
2 *ſc* D + Rad = *ſc* A B + *s* A
3 *ſc* AD + Rad = *ſc* DB + *ſc* AB
4 *ſc* AD + Rad = *tc* D + *tc* A
5 *ſc* A + Rad = *t* AB + *tc* AD
6 *s* AB + Rad = *tc* A + *t* DB

Or in ſtead of the ſecond we may ſay 2 *ſc* A + Rad = *ſc* DB + *s* D And in like ſort he that liſteth may ſet down the equality of the ſines and tangents of the other ſides and angles; and ſo there will be 10 of theſe, of every of which according to the things given and required he may make 3 caſes, and ſo 30 in all, anſwerable to the ſeveral poſitions of the letters; as is done by the Honourable Lord *Nepair*: If in ſtead of the Equality of the two terms on the one ſide of the Equation, to the two terms on the other, you would expreſſe the proportion of the four terms, it ſufficeth to put the terms reciprocally. As whereas in the firſt it is ſaid *s* D B + Radius = *s* A D + *s* A, we may put the terms reciprocally, and ſay, As Radius, to ſine AD, ſo is *s* A to *s* D B, or as Rad. to *s* A, ſo is *s* AD to *s* D B, putting alwayes the term which is ſigned with the term required, for the firſt number in the rule of proportion, and the two terms that are on the other ſide of the Equation, the one in the ſecond place, and the other in the third place of the Rule of Three. But this may here ſuffice; for to theſe may the ſixteen caſes of a right angled ſpherical triangle be reduced, namely, 3 to the firſt, 3 to the ſecond, 2 to the third, 2 to the fourth, 3 to the fifth, and 3 to the ſixth.

As admit there were given the hypothenuſal A D and the angle at A, and required the ſide D B; then by the firſt, ſeeing that *s* A D + *s* A, is equal to *s* D B + Rad. Therefore if from the ſumme of the ſines of A D and A, we ſubſtract Radius, the remainer is the ſine of D B.

Secondly, admit there were given A D and D B, and required the angle at A, then ſeeing *s* D B + Rad = *s* A D + *s* A; therefore if from *s* D B + Rad. we ſubſtract *s* A D, the remainer is *s* A.

Or thirdly, if there were given D B, and the angle at A, and required the hypothenusal A D; then forasmuch as *s* D B + Rad = *s* A D + *s* A; therefore if from the summe of *s* D B + Rad, we substract *s* A, the remainer is *s* A D. And the like is to be understood of the rest.

For if from equal things, we take equal things, the remainers are equal,

As if 6 + 10 be equal to 9 + 7; then if from 6 + 10, that is, from 16, we take 9, the remainer is 7; or if we take away 7, the remainer is 9, *&c.*

So also in the quadrantal triangle A D G, (whose side D G is a quadrant) the equality of the artificial sines and tangents of the parts, is such as here appeareth.

1 *s* G + Rad = *s* A + *s* A D
2 *sc* A G + Rad = *sc* D + *s* A D
3 *sc* A + Rad = *sc* G + *sc* D

---

4 *s* G + Rad = *tc* A G + *t* D
5 *sc* A G + Rad = *tc* A + *t* G
6 *sc* A + Rad = *tc* A G + *tc* A D

And to these 6 may the 16 cases of a quadrantal triangle be reduced, in such sort as we have before touched in right angled triangles, and shall further manifest in the Table following, though we do not in all places retein the same letters.

But because the work being thus ordered, would for the most part be performed by substraction, whereas it is something easier to adde than to substract; therefore you may in stead of substracting a sine or tangent, adde his complement arithmetical, whereof we have before spoken; and so the work may be conformable to these Tables following; whereof one serveth for right angled triangles, the other for quadrantal.

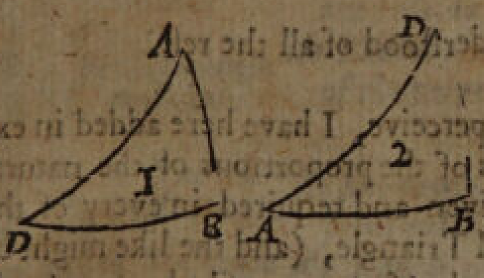
In the use of these Tables you are to mark the things given and required, with the letters in that Case given and required; and you must cut off from every summe, Radius or 1 in the first place towards the left hand, for indeed *s* A B + *t* A, is equal *t* D B + Radius, and so of the rest; except where you have the complement arith. of a sine; as your own reason in the use of this Table will direct you.

An

An exemplary Table

of the resolution of the several Cases of right angled spherical Triangles.

Dat	Req	Operation or equality.	Cases
AB A	DB	s AB + t A = t DB	1
	D	sc AB + s A = sc D	2
	AD	sc A + tc AB = tc AD	3
DB A	D	co. ar. sc DB + sc A = s D	4
	AD	co. ar. s A + s DB = s AD	5
	AB	tc A + t DB = s AB	6
AD A	AB	sc A + t AD = t AB	7
	DB	s AD + s A = s DB	8
	D	sc AD + t A = tc D	9
AB DB	AD	sc DB + sc AB = sc AD	10
	A	s AB + tc DB = tc A	11
AB AD	DB	co. ar. sc AB + sc AD = sc DB	12
	A	t AB + tc AD = sc A	13
AD DB	A	co. ar. s AD + DB = s A	14
A D	DB	co. ar. s D + sc A = sc DB	15
	AD	tc D + tc A = sc AD	16

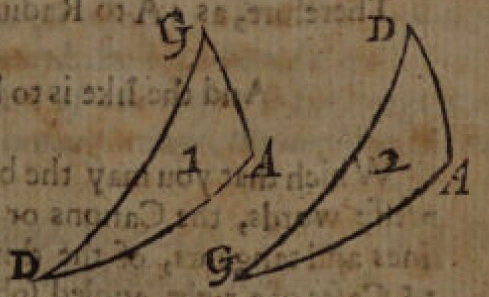


In all these Cases the angle given is marked with A, or if none be given, the angle required is marked with *A*, the right angle with B, the hypothenusal with A D.

An exemplary Table

of the resolution of quadrantal Triangles, one of whose sides is a quadrant.

Dat	Req	Operation or equality.	Cases
AG A	G	sc AG + t A = t G	1
	D	s AG + s A = s D	2
	AD	sc A + t AG = tc AD	3
A G	D	co. ar. sc G + sc A = sc D	4
	AD	co. ar. s A + s G = s AD	5
	AG	tc A + t G = sc AG	6
AG G	D	s G + t AG = t D	7
	A	sc AG + tc G = tc A	8
	AD	s AG + sc G = sc AD	9
AG D	G	t AG + t D = s G	10
	A	co. ar. s AG + s D = s A	11
	AD	co. ar. sc D + sc AG = s AD	12
AG AD	A	tc AG + tc AD = sc A	13
	G	co. ar. s AG + sc AD = sc G	14
D G	A	sc D + sc G = sc A	15
	AG	s G + tc D = tc AG	16



In all these Cases the quadrantal side is marked with D G, and the opposite angle with A.

And

And thus we have ſhewed in theſe Tables, the equality of the artificial ſines and tangents of the things given and required in all ſuch ſpherical triangles as have either a right angle, or one of their ſides a quadrant: But if you deſire the proportion of their natural ſines and tangents; it is

As radius, to the firſt of the three:
ſo is the ſecond to the third.

Except there be the complement arithmetical of a ſine, for then

As that ſine is to Radius:
ſo is the ſecond in theſe tables, to the rhird.

Example of right angled Triangles.

1 *Caſe.* $s\,AB ; + t\,A = t\,DB$,

Therefore, as Radius, to $s\,AB$: ſo $t\,A$ to $t\,DB$.

5 *Caſe. Compl. Arith.* $s\,A + s\,DB = s\,AD$.

Therefore, as $s\,A$ to Radius; ſo $s\,DB$, to $s\,AD$.

And the like is to be underſtood of all the reſt.

Which that you may the better perceive, I have here added in expreſſe words, the Canons or Rules of the proportions of the natural ſines and tangents, of the things given and required in every of the 16 Caſes of a right angled ſpherical Triangle, (and the like might be done for Quadrantals) all which rules (as may eaſily be perceived) depend upon the fundamental Axiome before going, and the Corollary of the third Propoſition of the ſecond Chapter of plain Triangles. And here the ſide ſubtending the right angle we call the hypothenuſal, the other two conteining the right angle we may call ſimply the ſides, but for farther diſtinction we call one of theſe conteining ſides (it matters not which) the baſe, and the other the perpendicular.

The

Given	To finde	Rule
The baſe and angle at the baſe given: to finde	1 The Perpendicular.	*As Radius, to the ſine of the baſe: ſo is the tangent of the angle at the baſe, to the tangent of the perpendicular.*
	2 The angle at the perpendic.	*As Radius, to ſine compl. the baſe: ſo is ſine the angle at the baſe, to ſine compl. the angle at the perpendic.*
	3 The hypothenuſal.	*As Radius, to ſine compl. the angle at the baſe: ſo is tang. compl. the baſe, to tang. compl. the hypothenuſal.*
The perpendicular & angle at the baſe given: to finde	4 The angle at the perpendic.	*As ſine compl. the perpendicular, to Radius: ſo ſine compl. the angle at the baſe, to ſine the angle at the perpend.*
	5 The hypothenuſal.	*As ſine the angle at the baſe, to Radius: ſo is the ſine of the perpendicular, to the ſine of the hypothenuſal.*
	6 The baſe	*As Radius, to tang. compl. the angle at the baſe: ſo is the tangent of the perpendicular, to the ſine of the baſe.*
The hypothenuſal, & angle at the baſe given: to finde	7 The baſe	*As Radius to ſine compl. the angle at the baſe: ſo is the tangent of the hypothenuſal, to the tangent: of the baſe.*
	8 The perpendicular.	*As Radius, to the ſine of the hypothenuſal: ſo is the ſine of the angle at the baſe, to the ſine of the perpendic.*
	9 The angle at the perpendic.	*As Radius, to ſine compl. the hypothenuſal: ſo tang. the angle at the baſe, to tang. compl. the angle at the per.*
The baſe & perpend. given: to finde	10 The hypothenuſal	*As Radius to ſine compl. the perpendicular: ſo ſine compl. the baſe, to ſine compl. the hypothenuſal:*
	11 The angle at the baſe	*As Radius, to the ſine of the baſe: ſo the tangent compl. of the perpendic. to tang. compl. the angle at the baſe.*
The baſe and hypothenuſal given: to finde	12 The perpendicular.	*As ſine compl. the baſe, to Radius; ſo ſine compl. the hypothenuſal, to ſine compl. the perpendicular.*
	13 The angle at the baſe.	*As Radius to the tangent of the baſe: ſo tangent compl. the hypothenuſal, to ſine compl. the angle at the baſe.*
	14 The angle at the perpen.	*As the ſine of the hypothenuſal, to Radius: ſo is the ſine of the baſe, to ſine the angle at the perpendicular.*
The angles at the baſe and perpendic. given: to finde	15 The perpendicular.	*As ſine the angle at the perpendicular, is to Radias: ſo ſine compl. the angle at the baſe, to ſine compl. the perpendic.*
	16 The hypothenuſal.	*As Radius, to tangent compl. the angle at the perpendicular; ſo tangent complement the angle at the baſe, to ſine complement the hypothenuſal.*

And

And becauſe by the third Prop. Chap.4. of plain Triangles,

As the tangent of an arch,
is to the tangent of another arch :
ſo is the tangent of the complement of that other,
to the tangent of the complement of the former.

And by the Corollary of the firſt Propoſition of the ſame Chapter.

Radius is a mean proportional between the tangent of an arch,
and the tangent complement of the ſame arch :
ſo that as Radius, is to the tangent of an arch:
ſo is the tangent complement of that arch, to Radius.

Therefore if any man deſire either for variety, or conveniencie, to alter the propoſitions wherein there are tangents, he may eaſily do it.

As in ſtead of the firſt he may ſay.

As the ſine of the baſe, is to Radius : ſo is the tangent complement of the angle at the baſe, to the tangent complement of the perpendicular.

For the third,

As ſine complement the angle at the baſe, to Radius : ſo is the tangent of the baſe, to the tangent of the hypothenuſal.

For the ſixth,

As tangent the angle at the baſe, to tangent the perpendicular : ſo is Radius, to the ſine of the baſe.

For the ſeventh,

As ſine complement the angle at the baſe, to Radius : ſo tangent complement the hypothenuſal, to tangent complement the baſe.

For the ninth,

As ſine complement the hypothenuſal, to Radius : ſo tangent complement the angle at the baſe, to tangent the angle at the perpendicular.

For the eleventh,

As the ſine of the baſe, is to Radius : ſo is the tangent of the perpendicular, to tangent the angle at the baſe.

For the thirteenth,

As the tangent of the hypothenuſal, to the tangent of the baſe : ſo is Radius, to ſine complement the angle at the baſe.

For the ſixteenth,

As tangent the angle at the baſe, to tangent complement the angle at the perpendicular : ſo is Radius, to ſine complement the hypothenuſal.

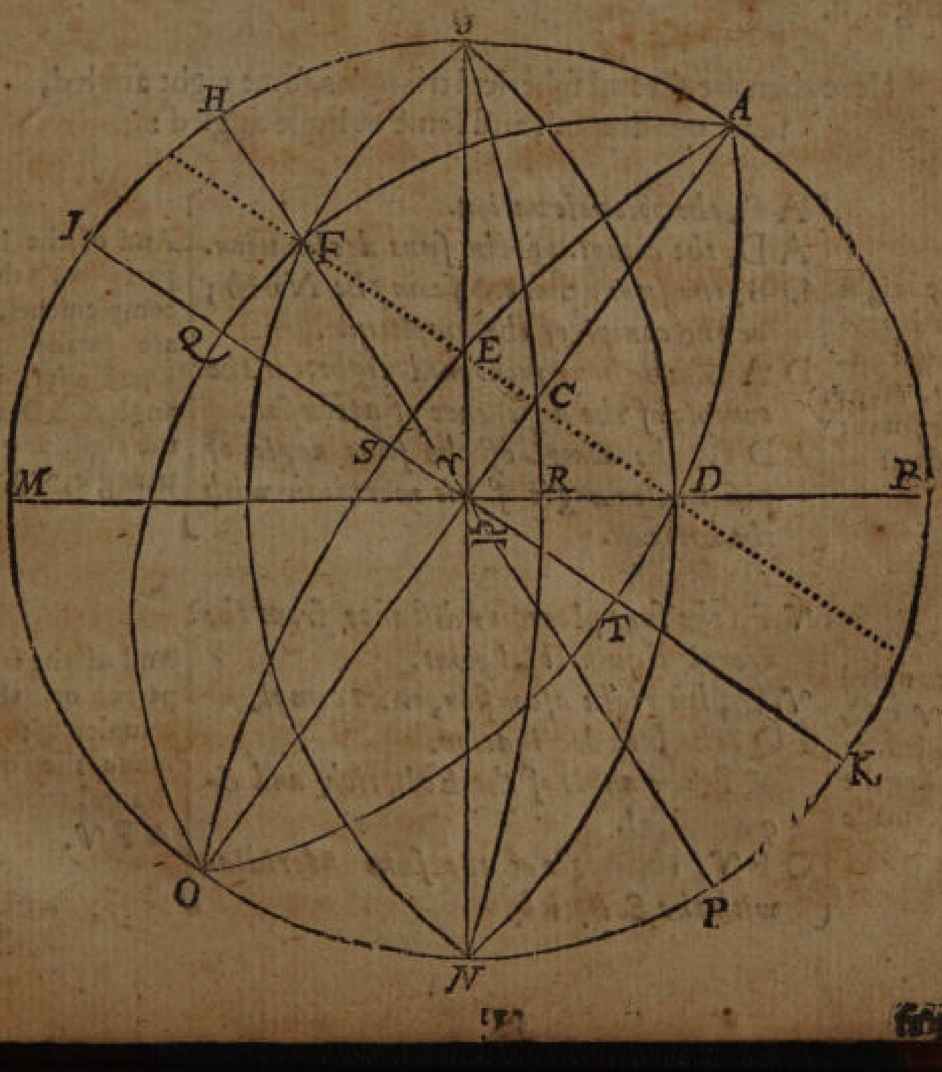
Many

Many other wayes might theſe propoſitions be varied, by the foreſaid Corollary, and third and fourth prop. of the fourth Chapter of plain triangles. And not onely theſe, but the reſt wherein there are onely ſines, are varied by theſe, and by the ſecond propoſition of the ſame Chapter. The varieties thence ariſing being very abundant and of no great uſe, I rather leave to your own practice at your leaſure, than beſtow further time therein.

CHAP. III.

Of the Caſes and Queſtions incident in every ſpherical Triangle, right angled or quadrantal in general. And of the examples of the ſixteen caſes of a right angled Triangle in particular.

WE come now to give examples of every of theſe caſes in the reſolution of ſome problemes of the Sphere. And ſuppo-



ſing the Reader to be already acquainted with the principal circles of the ſphere or globe, we will forbear their definitions.

Let G M N B, repreſent the meridian of the place, L K, the Equinoctial, H P the Eclipt ick, ♈ ♎ the points of Aries and Libra, A the North pole, O the South pole, A O the axis of the world; or meridian of the Sun at ſix a clock, M B, the Horizon, G, the Zenith, N, the Nadir, G N, the azimuth of Eaſt and Weſt, or the firſt vertical. F D, a parallel of declination, A D O an arch of a meridian paſſing by the center of the Sun at his riſing or ſetting. A E O, the meridian of the Sun being in the Eaſt or Weſt azimuth, A F O the Suns meridian being at F, G D N the Suns azimuth at his riſing, G C N the Suns azimuth at ſix of the clock, G F N, the Suns azimuth being at F.

Note. All the inward arches are indeed (in this kind of projection) Elliptical, though for readineſſe ſake we deſcribe them circular, and ſo alſo they do ſufficiently repreſent the things intended.

Here then are ſeveral ſpherical triangles, ſome right angled, ſome quadrantal, and ſome oblique angled:

Thus the right angled triangle A B D, right angled at B, (ſuppoſing the Sun at D) is made of	A B, *the poles elevation.* A D, *the compl. of the ſuns declination.* D B, *the ſuns azimuth from the North; or the compl. of the amplitude.* D A B, *the hour from mid-night, or the compl. of the difference of aſcenſion.* A D B, *the compl. of the ſuns angle of poſition, or angle of his meridian with the horizon.*	And of the like parts or their complements, are made the Quadrantal triangle, GAD, and the right angled triangle, ♈ T *D*.
The right angled triangle ♈ F Q. right angled at Q (ſuppoſing the Sun at F) is made of	♈ F, *the ſuns place, or diſtance from the neereſt Equinoctial point.* ♈ Q, *his right aſcenſion, or its compl.* F Q, *the ſuns declination.* ♈ F, *the angle of the Eclip tick and Equinoctial.* Q F ♈, *the angle of the ſuns Meridian with the Ecliptick.*	And of the ſame parts or their complements, is made the quadrantal triangle, A F ♈.

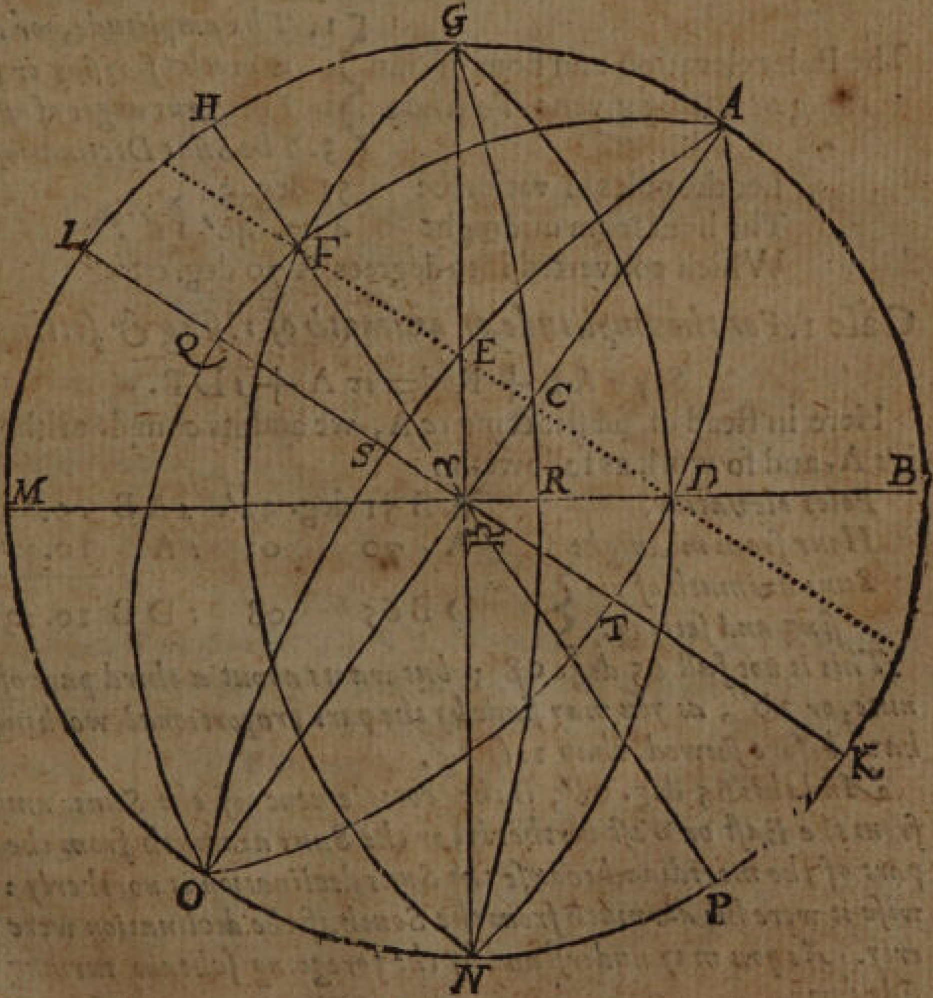
The

The right angled Triangle R♈C, right angled at R (if we suppose the Sun at C) is made of
- ♈C, *the suns declination.*
- CR, *the suns height at the hour of 6.*
- ♈R, *the suns azimuth from East or West at the hour of 6.*
- C♈R, *the poles elevation.*
- ♈CR, *the angle of the suns position.*

And of the same parts or their complements is made the quadrantal triangle, ♈GC.

The right angled triangle ♈SE, right angled at S, (if we suppose the sun at E) is made of
- SE, *The suns declination.*
- S♈, *the hour from 6.*
- ♈E, *the suns height being East or West.*
- S♈E, *the latitude.*
- SE♈, *the angle of the suns position.*

And of the like parts, or their complements, is made the quadrantal triangle, ♈EA.



The oblique angled triangle A G F, having neither right angle, nor any side a quadrant, (if we suppose the Sun at F) is made of
- G A, *the complement of the poles elevation,*
- F A, *the complement of the Suns declination,*
- G F, *the complement of the Suns height,*
- G A F, *the angle of the hour from noon,*
- A F G, *the angle of the Suns position.*
- F G A, *the azimuth of the Sun from the north part of the meridian.*

Other Triangles are represented in this scheam, but these I thought good to note, to give occasion to young beginners to exercise themselves.

Now we will shew the solution of one of the right angled triangles, namely, A D B, also of the oblique angled triangle A G F, whereby you may understand the like operation in all others.

The Poles elevation and hour of Sun rising or setting given: To finde
1. *The amplitude, or Suns azimuth of rising or setting.*
2. *The Suns angle of Position.*
3. *The Suns Declination.*

Let the poles elevation be 51 deg. 32′,
The hour from midnight 4 ho. 40′ 12″.
Which converted into degrees is 70 deg. 03′.

Case 1. *For the amplitude, or azimuth of rising & setting* D B.

Say *s* A B + Rad = *tc* A + *t* D B.

Here in stead of substracting *tc* A, we add its compl. arith. which is *t* A, and so work as followeth:

Poles elevation,	A B	51 deg.	32′.	*s* A B	9.8937452
Hour from midnight	A	70	03	*t* A	10.4401146
Suns azimuth of rising and setting.	D B	65	08	*t* D B	10.3338598

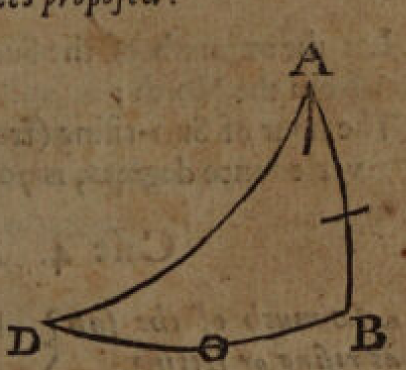
This is not full 65 deg. 08′; but wants about a third part of a minute, or 20″, as you may finde by the part proportional, working as we have before shewed, chap. 2. sect. 8.

And this 65 deg. 08′, is the complement of the Suns amplitude from the East or West northerly, or the Suns azimuth from the North part of the meridian, because the Suns declination is northerly: otherwise it were his azimuth from the South, if the declination were southerly. As you may understand by the foregoing scheam turning it upside down.

And

And whereas it is said, Hour from midnight 70 deg. 3′, it is to be understood, the hour converted into deg. and min. which is done by allowing 15 degrees for an hour, and one degree for four minutes of time, and 15 minutes of a degree for one minute of time, &c. Or saying by the Rule of Three, If one hour, or 60 minutes, give 15 degrees, what gives the time proposed? And so the contrary: if you would convert degrees into hours, say, If 15 degrees give one hour, or 60 minutes of time, what gives the degrees proposed?

Note that for your ease in resolving questions, whether in plain or spherical Triangles, it will be expedient to mark the things given and required as in this example, where the side A B, and the angle A being given, are each marked with a dash thus—, and the side required D B, with an o or cypher thus, o.



Case 2. *For the Suns angle of position.*

Say *sc* D + Rad = *s* A + *sc* A B.

Therefore the operation is thus.

Poles elevation,	A B	51 deg.	32′	*sc*	A B	9.7938317
Hour from midnight	A	70	03	*s*	A	9.9731236
Suns angle of position is the complement of	D	35	47	*sc*	D	9.7669553

Which 35 deg. 47 is the angle of the Suns position.

Case 3. *For the Suns declination.*

Say *sc* A + Rad = *t* A B + *tc* A D.

Here in stead of substracting *t* A B, we add its compl. arith. which is *tc* A B, and the like is to be understood in the rest that follow.

Hour from midnight	A	70 deg.	03′	*sc*	A	9.5330090
Poles elevation	A B	51	32	*tc*	A B	9.9000865
Suns declination is the complement of	A D	15	10	*tc*	A D	9.4330955

Which 15 deg. 10′ is the Suns declination towards the North pole (or elevated pole) because the hour from midnight is lesse than six; if it were more than six, the declination should be southerly; as is evident by the scheam before-going turned up-side down.

After the form of these three examples: If there were given the amplitude, and angle of the suns position, we might find the poles elevation, the hour of sun-rising or setting; and the suns declination: and if you use the exemplary table, you may use the second triangle under the table.

The Amplitude, or Azimuth of the Suns rising or setting, with the hour given: To find { 4 *The angle of position.* 5 *The suns declination.* 6 *The poles elevation.* }

Let the azimuth of the Sun at his rising or setting be 65 deg. 08′ from the North;

The hour of Sun-rising (from mid-night) 4 ho. 40′ 12″, which converted into degrees, is 70 deg, 03′.

Case 4. *For the angle of position.*

Azimuth of the sun at rising or setting	D B	65 d.	08′	*co. ar. sc.* D B	0.3762257
Hour of sun-rising	A	70	03	*sc* A	9.5330090
Angle of position compl.	D	35	46	*s* D	9.9092347

This *s* D, 9.9092347, gives an arch or angle of 54 deg. 14′; which is the angle that the Suns meridian makes with the horizon; but the angle of the Suns position is the complement thereof, namely, 35 deg. 46′.

Case 5. *For the suns declination.*

Hour of Suns rising	A	70 deg.	03′. *co. ar.*	*s A*	0.0268764
Azimuth of rising	D B	65	08	*s* DB	9.9577455
Suns declination compl.	AD	15	10	*s* AD	9.9846219

Here (as we noted before) the arch answering to *s AD* 9.9846219, is 74 deg. 50′, but the Suns declination is the complement thereof, that is 15 deg. 10′, and so of others.

Case 6. *For the Poles elevation.*

Hour of sun-rising	*A*	70 deg.	03′	*tc A*	9.5598854
Suns azimuth of rising	*DB*	65	08	*t DB*	10.338.712
Poles elevation	*AB*	51	33	*t AB*	9.8938566

Thus

Thus D B being 65 deg. 08′, we finde A B to be 51 deg. 33′ but if we should take D B to be but 65 deg. 07′ 40″, as before we found it, then A B the poles elevation would be but 51 deg. 32′, as before.

And (after the form of these three examples) if there were given the poles elevation, and the angle of the suns position, we might finde the hour of sun-rising, the suns declination, and the amplitude, or azimuth of rising and setting.

The Suns declination, and the hour of the Suns rising or setting given: to find
- 7 *The Poles elevation.*
- 8 *The amplitude, or the suns azimuth.*
- 9 *The angle of position.*

Let the Suns declination be 15 deg. 10′ northerly,
The hour of Sun-rising 4 ho. 40′ 12″,
Which converted into degrees is 70 deg. 03′.

Case 7. *For the poles elevation.*

Hour of sun-rising,	A	70 deg. 03′	sc A	9.5330090
Suns declination compl.	AD 15	10	t AD	10.5669196
Poles elevation,	AB 51	32	t AB	10.0999286

Case 8. *To find the suns azimuth.*

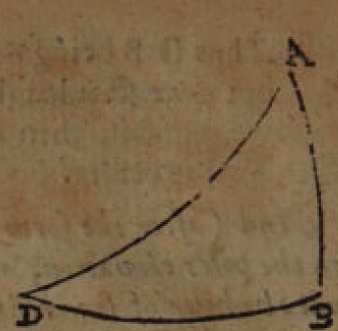
Suns declination compl.	AD 15 deg.	10′	s AD	9.9846033
Hour of sun-rising	A 70	03′	s A	9.9731236
Suns azimuth from the north part of the meridian	DB 65	08	s DB	9.9577269

The complement hereof 24 deg. 52′, is the amplitude of the Suns rising and setting from the East and West northerly; because the declination is northerly.

Case 9. *To find the angle of the suns position.*

Suns declination compl.	AD 15 deg.	10′	sc AD	9.4176837
Hour of sun-rising,	A 70	03	t A	10.4401146
The angle of position compl.	D 35	47	tc D	9.8577983

And



And (after the form of these three examples) if there were given, the Suns declination, and the angle of the Suns position at his rising, we might finde the Suns azimuth, the Poles elevation, and the hour of Sun-rising.

The Poles elevation, and amplitude of Sun-rising or setting given: To find { 10 *The Suns declination.* 11 *The hour of Sun-rising, or setting.*

Let the Poles elevation be 51 deg. 32′.
Suns amplitude of rising and setting 24 deg. 52′ northerly.

Case 10. *To finde the Suns declination.*

The amplitude is the complement of	DB 24 deg.	52′	*sc*	DB	9.6237743
Poles elevation,	AB 51	32	*sc*	AB	9.7938317
Suns declination compl.	AD 15	10	*sc*	AD	9.4176060

This declination 15 deg. 10′ is northerly, because the amplitude given is northerly, and when the one is southerly, so is the other.

Case 11. *To finde the hour of Sun-rising and setting.*

Poles elevation,	AB 51 deg.	32′	*s*	AB	9.8937452
Suns amplitude compl.	DB 24	52	*tc*	DB	9.6660287
Hour of sun-rising	A 70	03	*tc*	A	9.5597739

Which 70 deg. 03′ converted into time is 4 ho. 40′ 12″, which is the time of Sun-rising: But if the amplitude had been southerly, the arch thus found had been the hour of Sun-setting, as is evident by the first general scheame turned up-side down.

And after the form of this last example, we might by the same things given, find the angle of the suns position.

The

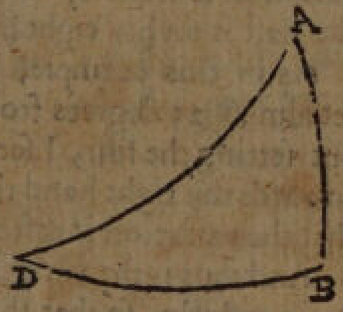
The elevation of the pole, and declination of the sun given: To find	12 *The amplitude.* 13 *The hour of Sun-rising and setting.*

Let the elevation of the pole be 51 deg. 32′,
Suns declination northerly 15 deg. 10′.

Case 12. *To find the amplitude.*

The poles elevation,	A B 51	deg. 32′	*co.ar.*	*sc* A B	0.2061683
Suns declination compl.	A D 15	10		*sc* AD	9.4176837
The Amplitude, the complement of	D B 24	52		*sc* D B	9,6238520

And this amplitude 24 deg. 52′ is northerly, because the suns declination is northerly: That is, the sun riseth 24 deg. 52′ to the northwards of the East, and sets as much to the northwards of the West. When the declination is southerly, the amplitude thus found is southerly, as may appear by the first general scheam turned up-side down.



Of the amplitude thus found there is often use made at sea, for finding the variation of the Compasse: which is done after this manner, if you do it by the Compasse.

Supposing the circumference or outermost edge of the card or flie of the Compasse to be divided into 360 degrees, and the points of the needles to be placed directly under the Flower-de-luce, or north and south points: you are to observe at sun-rising or setting, how many degrees the sun is from the east or west points of the Compasse, which number of degrees, if they agree with the amplitude found by this proposition, as is before shewed, and be on the same side; then hath the Compasse no variation: but if they differ, look how many degrees that difference is, so much is the variation.

As for example, admit I find the amplitude (as before) to be 24 degrees, 52 minutes northerly, then I know that the sun should set almost 25 degrees from the West to the northwards, but observing at sun-setting with my compasse, admit I find it to set but 19 degrees from the west point of my compasse to the northwards, then hereby I gather that the variation of my Compasse is almost six degrees. And thus you may finde how much the variation of the Compasse is. Now,

To finde which way the Compasse varieth.

If the degree of the compasse, which should directly respect the sun at his rising or setting, (namely, the degree of amplitude found as before) be more towards the right hand than the sun-rising or setting, the variation is easterly; but if it be more towards the left hand, the variation is westerly. Because when a mans face is towards the North, the East is on his right hand, and the West on his left.

As in this example, I find by the amplitude, that the sun should set almost 25 degrees from the west point of my Compasse northerly, but setting the sun, I see that the 25 degree of my Compasse is more towards the right hand than the place of sun-set; therefore I conclude, that the variation is easterly.

And thus is the variation of the Compasse found to be almost 6 degrees easterly, so that the north point of the Compasse shews not the true North, but points almost 6 degrees to the Eastward of the true North, and consequently all the other points of the Compasse direct more toward the right hand than they should do, by almost 6 degrees. And the like in all points is to be understood, if the observation had been made at sun-rising.

Note. It is fittest to make these observations when the sun seems to be a little above the horizon, namely, when the lower edge of the sun seems almost to touch the horizon, for then the sun is in the horizon, though by reason of his refraction and parallax he seem to be above it.

Case 13. *To finde the hour of sun-rising and setting.*

Poles elevation	AB 51 deg. 32′	*t*	AB	10.0999135
Suns declination compl.	AD 15	10	*tc* AD	9.4330804
Hour of sun-rising	A 70	03	*sc* A	9.5329939

This 70 deg. 03′ converted into time, is 4 ho. 40′ 12″, which is the time of sun-rising after midnight: But if the declination

clination had been ſoutherly, this 4 ho. 40′ 12″ thus found, had been the time of Sun-ſetting after noon, as may appear by the general ſcheam turned up-ſide down.

And after the form of this laſt example, if there were given the azimuth of the ſuns riſing or ſetting, and the ſuns declination, we might finde the angle of the ſuns meridian with the horizon: or the poles elevation, after the form of the laſt but one.

Caſe 14. *The Declination of the Sun, and his azimuth of riſing and ſetting given: to find the hour.*

Let the Suns declination be 15 deg. 10′ northerly,

His azimuth at his riſing or ſetting 65 deg. 08′ from the North.

To finde the hour.

Suns declinat. compl.	A D 15 deg.	10′ *co. ar.*	*s* A D	0.0153967
Suns azimuth	D B 65	08	*s* D B	9.9577455
Hour of ſun-riſing	A 70	03	*s* A	9.9731422

Which 70 deg. 03′ converted into time, is 4 ho. 40′ 12″, the hour of ſun-riſing: but if the declination had been ſoutherly, this arch thus found had been the hour of ſun-ſetting.

And after the form of this example, if there were given (as in the thirteenth Caſe) the latitude, and ſuns declination, we might find the angle of the ſuns poſition, or the complement thereof, which is the angle of the ſuns meridian with the horizon.

The hour of Sun-riſing or ſetting, and the angle of the Suns meridian, with the horizon given: to finde } 15. *The amplitude.* 16. *The ſuns declination.*

Let the hour of ſun-riſing be 4 ho. 40′ 12″,

Which converted into degrees is 70 deg. 03′.

The angle of the ſuns meridian with the horizon 54 deg. 13′.

Caſe 15. *To find the amplitude.*

Angle of meridian & horizon	D 54 deg.	13′ *co. ar.*	*s* D	0.0908579
Hour of ſun-riſing in deg.	A 70	03′	*sc* A	9.5330090
Amplitude compl.	DB 24	52	*sc* DB	9.6238629

 Caſe

Case 16. *To find the Suns declination.*

Angle of merid. and horizon	D	54 deg. 13′	*ts* D	9.8578031
Hour of Sun-rising	A	70 03	*ts* A	9.5598854
Suns declination compl.	A D	15 10	*sc* AD	9.4176885

Which declination 15 deg. 10′ is northerly, because the hour of sun-rising is before 6: otherwise the said hour being after 6, the declination should be southerly.

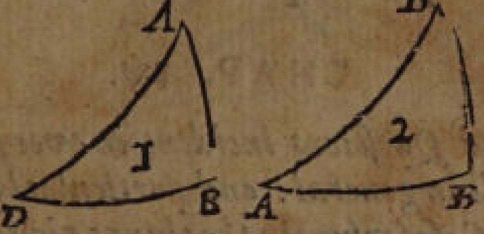
And after the form of the last Case but one, we may by the same things given find the Poles elevation.

And thus it is evident, that of the five circular parts of this right angled sphericall triangle, namely, of the two oblique angles, the two sides, & the hypothenusal, there may be framed 30 problemes or questions of the sphere, and these 30 problemes are reduced to 16 Cases, and these 16 Cases to that one fundamental Axiome before set down; and the like is to be understood in other right angled spherical triangles.

The same 30 questions might also have been moved and resolved in the quadrantal triangle A G D, *and they are also reduced to 16 Cases, and those 16 Cases to the afore-said fundamental Axiome. Of which things having before given sufficient light, we will leave the practice thereof to the industrious Reader.*

And it will not be amisse, when there is a question proposed in a right angled spherical triangle, to mark it with the letters A B D; *setting* B *at the right angle, and* A D *to the hypothenusal; Or if it be a quadrantal triangle, set* D G *to the quadrantal side, and* A *at angle thereto opposite.*

As (if in the general scheam of the sphere before going) I would resolve the triangle ♈ Q F, *right angled at* Q. *I put for* ♈, A, *for* Q, B, *and for* F, D, *as in the first of these triangles: Or I put for* F, A, *for* Q, B, *and for* ♈, D, *as in this second triangle.*



In this first triangle. *In this second triangle.*

A, *is The Equinoctial point of ♈ and ♎, which in this second triangle is* D.

AB, *is An arch of the Equinoctial, which in the second is* DB.

DB, *is An arch of the Suns meridian, which in the second is* AB.

And thus,

AD, *is The Suns place or distance from the neerest Equinoctial point, which in the second triangle is also* DA.

AB, *is The Suns right ascension from the neerest Equinoctial point, which in the second triangle is* DB.

DB, *is The Suns declination, which in the second triangle is* AB.

A, *is The angle of the Ecliptick with the Equinoctial, which in the second triangle is* D.

D, *is The angle of the Suns Meridian with the Ecliptick, which in the second triangle is* A.

And any two of these being given, we may find any third required; and so frame 30 several questions, every of which in one of these triangles will be conformable to the exemplary table of right angled triangles before set down.

And the like is to be understood in the other two triangles before mentioned R♈C, *and* ♈SE: *so that in these four right angled triangles, you may frame* 120 *questions of the sphere, and their resolutions. And the like you may do in their quadrantals: all which I leave to your own practice, desiring to use as much brevity as conveniently I may.*

And thus much touching the resolution of such spherical triangles as are either right angled or quadrantal: Now we come to those that are oblique, which have 12 *Cases, ten whereof do also depend upon the first general axiome afore-going, and might be thence deduced. But that all things may be the more easie and perspicuous, we will lay down two Consectaries following of the said first Axiome, after we have declared in general the Cases of an oblique triangle.* Of

Of Oblique Spherical Triangles.

CHAP. IV.

Of the Cases and Questions incident in every oblique spherical Triangle in general: and particularly of those two Cases wherein the things given and required are opposites.

TO the intent the application of the doctrine of Spherical Triangles may be the better underſtood, we will here (as we have before in right angled Triangles) give examples of the ſeveral Caſes of an oblique Triangle in the actual reſolution of ſome known Triangle of the ſphere. And we have before noted in the general ſcheam of the ſphere, chap. 3. that A G F is an oblique angled Triangle. Let us ſuppoſe the firſt of theſe Triangles following marked with A D E to be the ſame (where we change the letters, not of any neceſſity, but for the better conformity of all the examples.) So that A here may be in place of A there: namely, at the pole of the world; D here, in place of G there, namely, at the Zenith; and E here, in place of F there, namely, at the Sun. Then is,

A D, the complement of the Poles elevation, or the diſtance of the Pole from the Zenith.

A E, the complement of the Suns declination, or the diſtance of the Sun from the Pole.

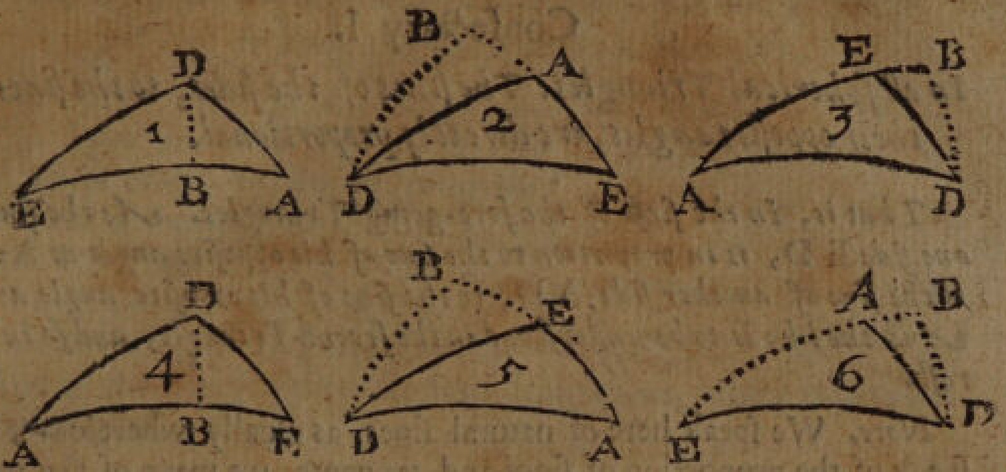
E D, the complement of the Suns height, or the Suns diſtance from the Zenith.

A, the angle of the hour from noon, or the angle of the meridian of the Sun, with the meridian of the place.

E, the angle of the Suns poſition in reſpect of the Pole and Zenith.

D, the ſuns azimuth from the north part of the meridian.

And



And any three of theſe being given, the other three may be found. So that of theſe ſix parts conferred together, there ariſe in this Triangle, and ſo in others, ſixty queſtions or problemes of the ſphere: which all are reduced to 12 Caſes, the reſolution whereof we intend now to ſhew, and exemplifie in this Triangle, and withall to point out the ſaid ſixty queſtions here incident, referring every of them to their proper Caſes.

And that theſe ſixty problemes may be the more conformable to the 12 Caſes whereunto they are referred, I have marked this Triangle ſix ſeveral wayes: that ſo the things given and required in every of theſe ſixty problemes, (and ſo in all others) may in one of theſe Triangles be noted by the ſame letters, as are uſed in the Caſe and Example whereunto that probleme is referred; whereunto I am the rather induced by the example of the Honourable Lord *Nepair* in his 12 Caſes of an oblique Triangle, ſet forth in his book of the Conſtruction of Logarithms.

But every man is at liberty to do herein as he thinks good, for the rules are general, howſoever the Triangles or their parts are marked.

And thus having ſhewed in general, what Caſes and queſtions are incident in an oblique ſpherical Triangle, we come now to handle them particularly; laying for the two firſt Caſes this ground.

Conſectary

Consectary I.

In all spherical Triangles: The sines of the sides, to the sines of their opposite angles, are directly proportional.

That is, In the first of the fore-going Triangles. As the sine of one side E D, *is in proportion to the sine of his opposite angle at* A: *so is the sine of another side* A D, *to the sine of his opposite angle at* E. *And the like is to be understood in the second Triangle, and so in the rest.*

Note. We speak here of natural sines, as usually wheresoever we speak of the proportion of sines and tangents, we mean of the naturall sines and tangents, and where we speak of the equality of sines and tangents, we mean of the artificial sines and tangents, that is, of the Logarithmes of the natural sines and tangents: *For where there is an equality of the artificial, there is a reciprocal proportionality of the natural,* as is evident by the Corol. of 3 Prop. of 2 Chap of Plain Triangles.

Construct. *Now touching this Consectary, let* A D E *be an oblique angled triangle, if then we let fall the perpendicular* D B, *it is resolved into two right angled triangles,* A D B, *and* E D B.

Demonst. *Wherefore by the fundamental Axiome of right angled triangles, if we take the perpendicular* B D *for the middle part, and* A D *and* A *for his opposite extreams, in the triangle* A D B; *and* E D, *and* E *for his opposite extreams, in the triangle* E D B, *then*

Rad + *s* D B *is equal to s* A D + *s* A, *also*
Rad + *s* D B *is equal to s* E D + *s* E.

But things that are equal to one and the same thing, are equal one to another: therefore s A D + *s* A, *is equal to s* E D + *s* E. *Therefore by the Coroll. of* 3 *Prop.* 2 *Chap. of plain triangles, the proportion of their natural sines is reciprocal, thus:*

As the sine of E D, *is to the sine of the angle* A:
so is the sine of A D, *to the sine of the angle at* E.

And the like is to be understood in the second triangle. Therefore in all spherical triangles, &c. which was to be proved.

And hence may two Cases in an oblique triangle be resolved. As, Case 1. *Two angles, with a side opposite to one of them given: to finde the side opposite to the other.*

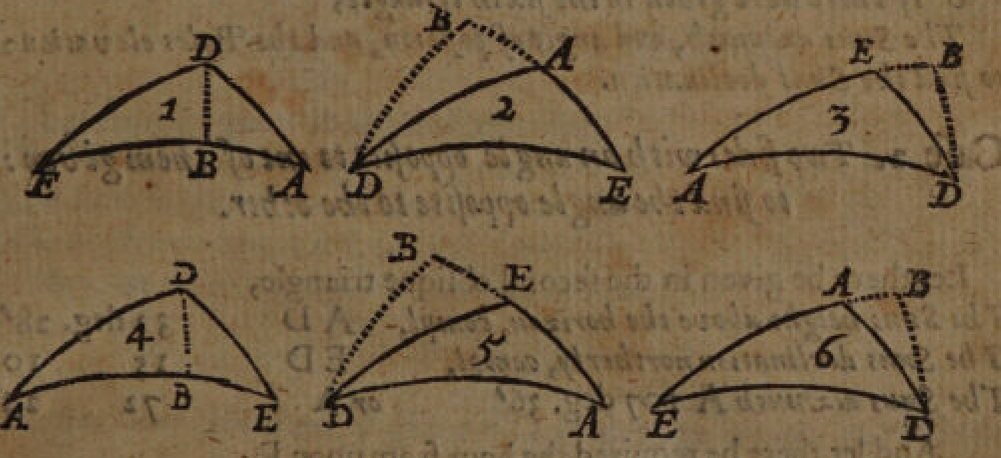
As in the second oblique Triangle, Let there be given

The Suns azimuth from the North	DAE,	107 deg.	36′
whose compl. to 180 deg. being the Suns azimuth from the South, is	BAD,	72	24
The hour from noon E 3 *ho.* 45′ 44″, *which converted into degrees, is*		56	26
The Suns height being the complement of AD,		32	28

And let there be required the Suns declination, which is the complement of ED.

As the sine of the hour from noon,	*s* E 56 d. 26′, *co.ar.*	0.0792283
to the sine compl. of the suns height:	*s* AD 57 32	9.9261900
so the sine of the suns azimuth,	*s* A 72 24	9.9791798
to the sine of the compl. of the suns declination.	*s* ED 74 50	9.9845981

Whereby the Suns declination appears to be 15 deg. 10′.



Another Example of this Case.

Let there be given in the fifth Triangle.

The suns azimuth from the North	DEA,	107 deg.	36′
whose complement to 180 deg. is	BED,	72	24
The hour from noon A 3 *ho.* 45′ 44″, *which converted into degrees is,*		56	26
The suns declination, the complement of AD		15	10

And

And let there be required the Suns height, being the compl. of ED.

As the fine of the Azimuth,	*s* E 72 deg. 24′ *co. ar.*		0.0208202
to fine compl. the funs declinat.	*s* AD74	50	9.9846033
fo the fine of the ho. from noon,	*s* A 56	26	9.9207717
to fine compl. the funs height.	*s* ED57	32	9.9261952

Whereby the Suns height appears to be 32 deg. 28′.

Note. *By the imitation of either of thefe examples, there may four other queftions in this Triangle, and fo of any other, be refolved:*

As 3. If (in the firft Triangle) there be given,

The hour of the day, the angle of the Suns pofition, and the height of the Pole: To find the height of the Sun.

4 *If there were given in the fourth Triangle,*

The hour of the day, the angle of the Suns pofition, and the height of the Sun: To find the height of the Pole.

5 *If there were given in the third Triangle,*

The Suns azimuth, and angle of pofition, and declination: to find the elevation of the Pole.

6 *If there were given in the fixth triangle,*

The Suns azimuth, and angle of pofition, and the Poles elevation: to find the Suns declination.

Cafe 2. *Two fides with an angle oppofite to one of them given: to find the angle oppofite to the other.*

Let there be given in the fecond oblique triangle,

The Suns height above the horizon, compl.	A D	32 deg.	28′
The Suns declination northerly, compl.	ED	15	10
The Suns aizmuth A 107 deg. 36′	*or* A	72	24

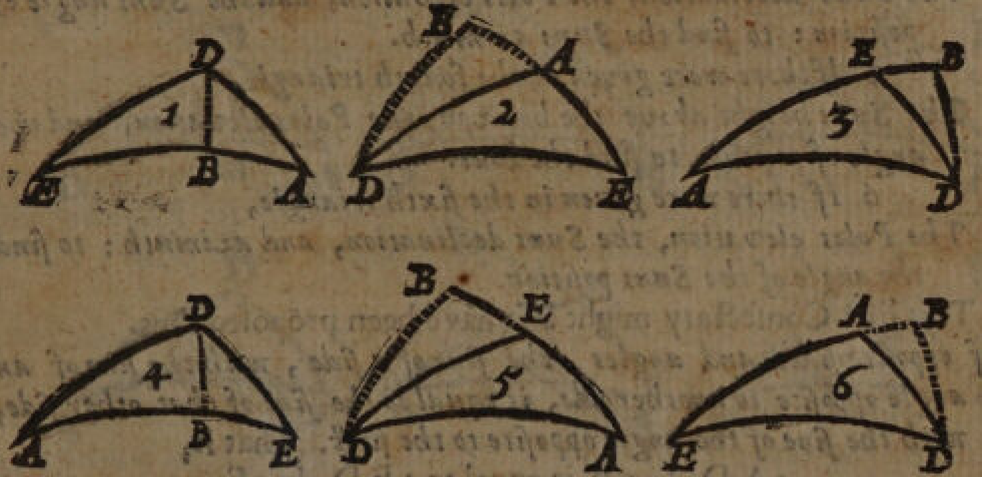
And let there be required the hour from noon E.

As fine compl. the Suns declinat.	*s* E D 74 deg. 50′ *co. ar.*		0.0153967
to the fine of the azimuth;	*s* A 72	24	9.9791798
fo fine compl. the Suns height,	*s* A D 57	32	9.9261900
to the fine of the ho. from noon.	*s* E 56	26	9.9207665

Which 56 deg. 26′ converted into time, is 3 ho. 45′ 44″, which in the forenoon is 14′ 16″ after 8 of the clock, but in the afternoon 45′ 44″ after three of the clock.

Note.

Note. *The arch or angle answering to 9.9207665, is not full 56 deg. 26′ but wants almost a fifteenth part of a minute, or four seconds, but for the more facility and readinesse, it shall suffice to give the examples to a minute, such as desire more precisenesse may do as we have shewed in the second chapter of plain triangles, sect. 8.*



Another Example of this second Case.

Let there be given in the fifth triangle,

The Suns declination northerly, complement	A D	15 deg.	10′
The Suns height above the horizon, complement	E D	32	28
The ho. from noon, 3 ho. 45′ 44″, which in deg. is	A	56	26

And let there be required the Suns azimuth E.

Proportion.

As sine compl. the Suns height,	s E D	57 deg.	32′. co. ar.	0.0728100
to the sine of the hour;	s A	56	26	9.9207717
so sine compl. the Suns declinat.	s A D	74	50	9.9846033
to the sine of the Azimuth	s E	72	24	9.9791850

Which 72 deg. 24′ is here the Suns azimuth from the South, the complement whereof to 180 degrees is 107 deg. 36′ the Suns azimuth from the north.

By imitation of either of these examples, there may foure other questions in this triangle, and so of any other be resolved: As,

3 *If there were given in the first triangle,*
The Poles elevation, the Suns height above the horizon, and the hour from noon: to find the Suns angle of position.

4 *If there were given in the third triangle,*
The Suns declination, the Poles elevation, and the Suns angle of position: to find the Suns azimuth.

5 *If there were given in the fourth triangle,*
The Suns height above the horizon, the Poles elevation, and the angle of position, to find the hour.

6 *If there were given in the sixth triangle,*
The Poles elevation, the Suns declination, and azimuth: to find the angle of the Suns position.

This first Consectary might also have been proposed thus.

Of opposite sides and angles, the sine of a side, with the sine of an angle opposite to another side, is equal to the sine of that other side, with the sine of the angle opposite to the first. That is,

$$s\,AD + s\,A \text{ is equal to } s\,ED + s\,E$$

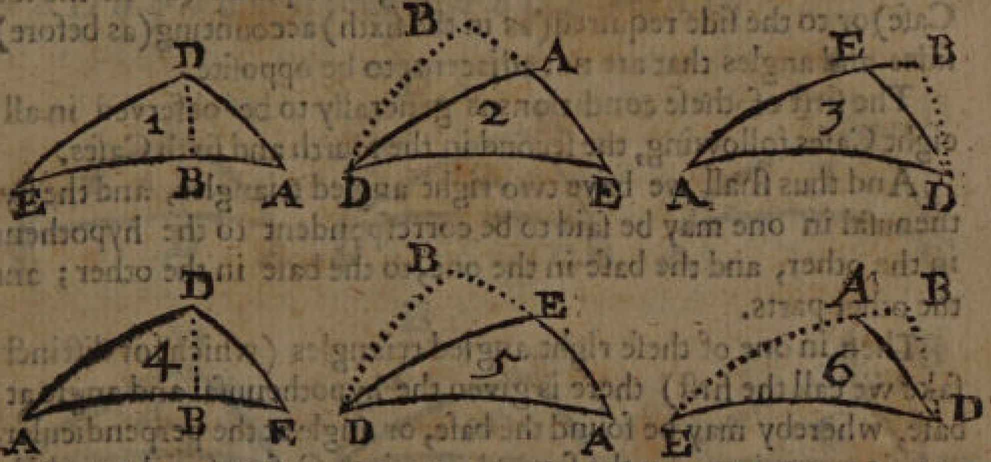
Which in effect is the same with the former, and in like sort demonstrated. But the former is to be preferred being brief, perspicuous, and well known to such as have been conversant in sphericall triangles.

But in the use of this Consectary, and of the two last Cases, there happens the like doubt, as we have noted upon the ninth Case of plain Triangles. Namely, in sphericall Triangles it is doubtfull, whether the angle neerest to a right angle, and his opposite side be both of one and the same, or of divers kinds, unlesse you discover it by your work, or that it be a thing given by supposition.

This doubt may (for the most part) be removed by the exact delineation of the scheam or figure: whereby you shall perceive whether a sphericall angle be acute or obtuse, and a side greater or lesse than a quadrant. But you may be further directed herein, by the three propositions of the Baron of *Marchiston*, which I have for that purpose set down in the first chapter of sphericall triangles.

As in this last example, seeing the side AD 74 deg. 50′ is neerer to a quadrant than his opposite angle at E being 72 deg. 24′, or 107 deg. 36′ therefore (by the last of those three) two angles of that triangle

angle are of one kind, and the third greater than a quadrant. That is, the two angles at A and D, are acute, and the third at E, namely, A E D is greater than a quadrant: therefore the angle there found A E D is 170 deg. 36′. And the like judgement is to be given of others.



CHAP. V.

Of eight other Cases of an oblique spherical Triangle, resolved at two operations by a perpendicular let fall.

NOte. If this way of resolving these eight Cases at two operations seem hard, you may more easily resolve them at three operations, as is shewed in the sixth Chapter next following; but here we shew their resolution at two operations onely, thus;

In the eight Cases next following there are also three things (in an oblique triangle) given to find a fourth; for the finding whereof it is requisite, that this triangle proposed be reduced to two right angled triangles, by a perpendicular let fall from one of the angles to his opposite side; which perpendicular fals sometimes within the triangle, sometimes without.

If the angles at the base be both of one kind (that is both obtuse or both acute) the perpendicular fals within the triangle, if of divers kinds, without: and the converse.

In

In letting fall the perpendicular observe, that forasmuch as in every of these Cases there is given a side with an angle adjoyning;

1 Let fall the perpendicular from the end of that side opposite to that adjacent angle:

And further, when that sufficeth not,

2 Let it fall also opposite to the angle required (as in the fourth Case) or to the side required (as in the sixth) accounting (as before) the sides and angles that are not adjacent, to be opposite:

The first of these conditions is generally to be observed in all the eight Cases following, the second in the fourth and sixth Cases.

And thus shall we have two right angled triangles, and the hypothenusal in one may be said to be correspondent to the hypothenusal in the other, and the base in the one, to the base in the other; and so the other parts.

Then in one of these right angled triangles (which for distinction sake we call the first) there is given the hypothenusal and angle at the base, whereby may be found the base, or angle at the perpendicular, as occasion requires; by the seventh or ninth Cases of right angled triangles. And this is the first operation.

For the second, there must (of the things thus given and required) two things in one triangle, be compared to two correspondent things in the other triangle, which two in each, with the perpendicular make three things in each triangle, either adjacent (that is lying together) or opposite; of which three the perpendicular is alwayes one of the extreams, and the thing required, one of the other extreams. All which may appear in every of these six triangles.

So that by the first general Axiome of right angled sperical triangles.

1 { *Radius* + *sc* AD / *sc* DB + *sc* EB } is equal to { *sc* AB + *sc* DB. / *sc* ED + *Radius*. }

2 { *Radius* + *s* AB / *t* DB + *tc* E } is equal to { *tc* A + *t* DB. / *s* EB + *Radius*. }

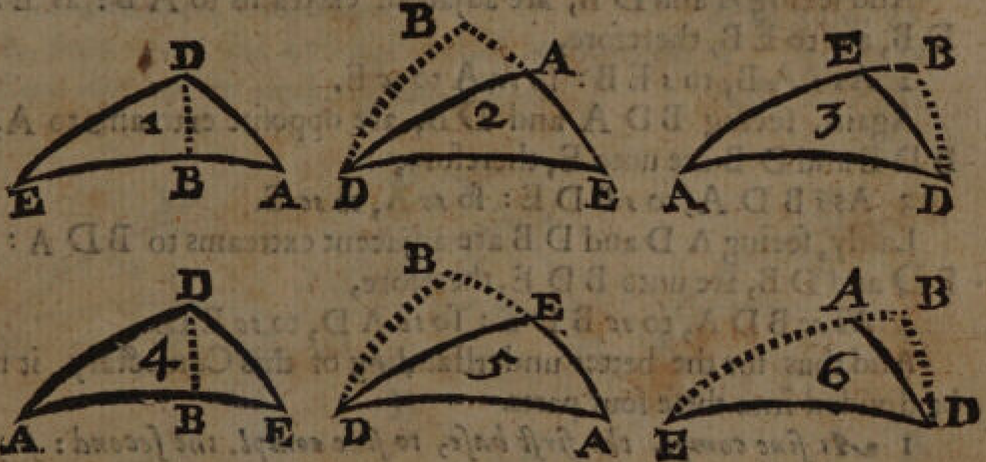
3 { *Radius* + *sc* A / *sc* DB + *s* BDE } is equal to { *s* BDA + *sc* DB. / *sc* E + *Radius*. }

4 { *Radius* + *sc* BDA / DB + *tc* ED } is equal to { *tc* AD + *t* DB. / *sc* BDE + *Radius*. }

But

But if from equal things we take away equal things, the things remaining are equal. Therefore from either side taking *t* or *sc* D B and *Radius*, it follows that

1 *sc* A D + *sc* E B is equal to *sc* E D + *sc* A B.
2 *s* A B + *tc* E is equal to *s* E *B* + *tc* A.
3 *sc* A + *s* *B* D E is equal to *sc* E + *s* *B* D A.
4 *ss* *B* D A + *tc* E D is equal to *sc* *B* D E + *tc* A D.



Wherefore in each right angled triangle, supposing the three parts more remote from the right angle, to be noted as is aforesaid, with their complements, and using (as is expressed in the fundamental Axiome) the sines of the middle parts, and the tangents of the extreams adjacent, or the sines compl. of the opposite extreams, you may observe, that

The middle part in the first triangle, with the extream in the second : is equal to the middle part in the second, with the extream in the first.

And by help of this Consectary might these eight Cases be resolved, which also by the Corollary of 3 prop. chap. 2. of plain triangles, may be proposed as followeth ; in which form we intend to use it.

Consectary 2.

As the middle part in the first Triangle, is in proportion to the middle part in the second : so is the extream in the first, to the extream in the second.

Though

Though the perpendicular be alwayes one of the extreams in either Triangle, (as is before noted) yet we use not that, but the other extream in both.

Wherefore in any of the six oblique Triangles, seeing A B and D B, are opposite extreams to A D, as E B and D B are to E D, therefore,

1 As *sc* A B to *sc* E B: so *sc* A D to *sc* E D.

And seeing A and D B, are adjacent extreams to A B: as E and D B, are to E B, therefore,

2 As *s* A B, to *s* E B: so *tc* A to *tc* E.

Again, seeing B D A and D B, are opposite extreams to A, as B D E and D B are unto E, therefore,

3 As *s* B D A, to *s* B D E: so *sc* A, to *sc* E.

Lastly, seeing A D and D B are adjacent extreams to B D A: As E D and D B, are unto B D E, therefore,

4 As *sc* B D A, to *sc* B D E: so *tc* A D, to *tc* E D.

And thus for the better understanding of this Consectary, it may be divided into these four parts.

1 *As sine compl. the first base, to sine compl. the second: so sine compl. the first hypothenusal, to sine compl. the second.*

And this serves for the 3 and 7 Cases following.

2 *As the sine of the first base, to the sine of the second: so tangent compl. the first angle at the base, to tangent compl. the second.*

Which serves for 4 and 10 Cases.

3 *As the sine of the first angle at the perpendicular, to the sine of the second: so sine compl. the first angle at the base, to sine complement the second.*

Which serves for the 5 and 9 Cases.

4 *As sine compl. the first angle at the perpendicular, to sine complement the second: so tangent compl. the first hypothenusal, to tangent compl. the second.*

And this serves for the 6 and 8 Cases following.

The words (*first* and *second*) we here use to distinguish the two right angled Triangles.

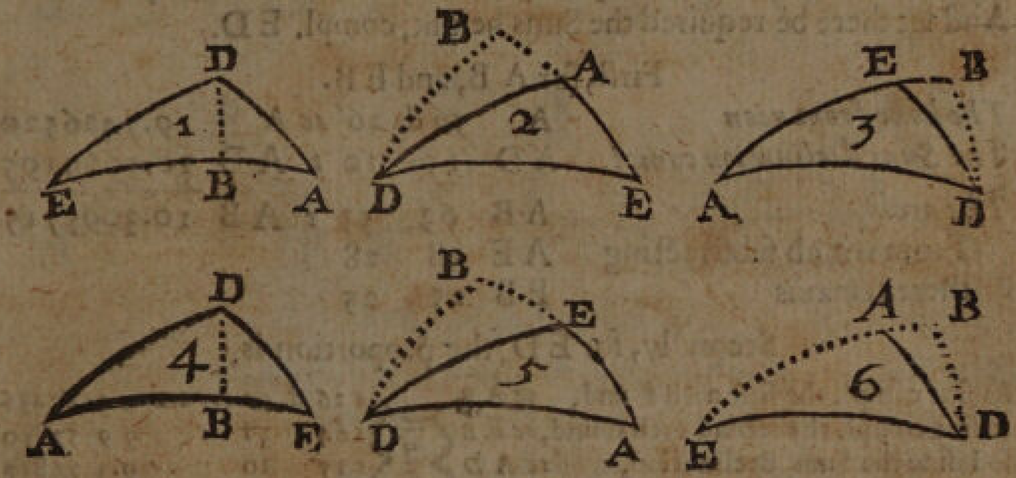
This Consectary might have been otherwise demonstrated, as by producing the sides of the oblique Triangle to Quadrants, &c. But I have the rather used this form, that so the deduction thereof from the

the first fundamental Axiome before going might the better appear. And this ground thus laid, we come now to the eight Cases thereon depending.

Case 3. *Two sides, and their conteined angle given, to finde the third side.*

Let there be given in the first oblique triangle,

The Poles elevation, complement	A D,	51 d.	32′
The ho. from noon 3 ho. 45′-44″, which in degrees is	A	56	26
The Suns declination northerly, complement	A E,	15	10



And let there be required the Suns height, complement E D.

First, By the seventh Case of right angled triangles, to find A B and E B.

The hour from noon,	A	56 deg. 26′	sc A	9.7426520
The Poles elevation compl.	A D 51	32	t A D	9.9000865
The arch	A B 23	43	t A B	9.6427385

The summe or remainer of A B and A E is E B.

But here from A E 74 d. 50′	Or if to compl. A E 15 deg. 10 min.	
subtracting A B 23 43	we add A B 23 43	
there remains E B 51 07	we have compl. E B 38 53 and so of the rest.	

 Secondly,

Secondly, for E D, by the ſecond Confectary, the proportion is,

As ſine compl. the firſt arch found,	*sc* A B	That is	*s* 66 d.	15 *co. ar.*		0.0383200
to ſine compl. the ſecond arch found,	*sc* E B		*s* 38	53		9.7977775
ſo is the ſine of the Poles elevation	*sc* A *D*		*s* 51	32		9.8937452
to the ſine of the Suns altitude	*sc* E *D*		*s* 32	28		9.7298427

2 *Example.*

Let there be given in the fifth Triangle,

The Suns declination northerly, compl.	A D	15 deg.	10′
The hour from noon, 3 ho. 45′ 44″, which in degrees is the angle,	A	56	26
The Poles elevation complement	A E	38	28

And let there be required the Suns height, compl. E D.

First, for A B, and E B.

The hour from noon	A	56 d.	26′	*sc* A	9.7426520
The Suns declination compl.	A D	15	10	*t* A D	10.5669195
The arch	A B	63	53	*t* A B	10.3095715
From which ſubſtracting	A E	38	28		
there remains	E B	25	25		

Secondly, for E D, the proportion is,

As ſine compl. the firſt arch found,	*sc* A *B*	That is	*s* 26 d.	07 *co. ar.*	0.3563496
to ſine compl. the ſecond arch found,	*sc* E *B*		*s* 64	35	9.9557890
ſo is ſine the Suns declination,	*sc* A *D*		*s* 15	10	9.4176837
to ſine the Suns altitude.	*sc* E *D*		*s* 32	28	9.7298223

Note. *Although there be a difference between the artificial ſine here found : and the former, yet the difference of their arches is little more than one tenth part of a min. which ariſeth by neglecting the ſeconds and thirds in the arch firſt found* A B. *He that deſires to work to ſeconds, may do it as we have ſhewed chap.2. ſect.8. of plain triangles. But in theſe examples, we would not trouble beginners with them at the firſt, it being ſufficient for ordinary occaſions, if the work be true to a minute.*

And after the form of either of theſe examples, we may calculate tables of the Suns height for every hour and minute of the day. By which tables may be made the Quadrants and Ring Dials, and other inſtrumental and fixed Dials, that give the hour of the day by the Suns height.

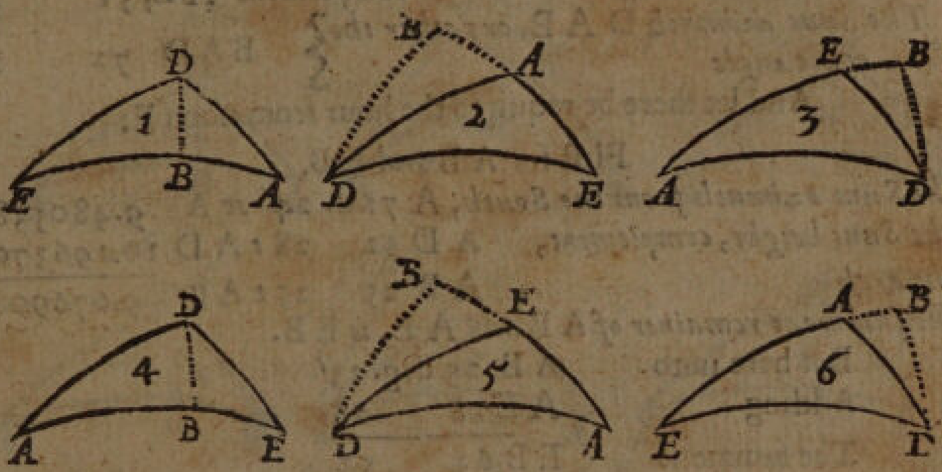
3 *Example.*

3 Example.

Let there be given in the second oblique Triangle.

The Suns height above the horizon, complement	AD 32 d.	28′
The Suns azimuth D A E, *or rather the acute ang.* B A D 72		24
The Poles elevation *complement,*	AE 51	32

And let there be required the Suns declin. compl. E D.



First, for A B and E B.

The Suns azimuth from the South,	A 72 d. 24′	*sc* A	9.4805385
The Suns height, complement,	A D 32 28	*t* AD	10.1963704
The arch	A B 25 25	*t* A B	9.6769089
Whereto adding	A E 38 28		
The summe is	E B 63 53		

Secondly, for E D, the proportion is,

As sine compl. the first arch found,	*sc* A B	That is	*s* 64 d.	35′ *co. ar.*	0.0442110
to sine compl. the second arch found,	*sc* E B		*s* 26	07	9.6436504
so is the sine of the Suns height,	*sc* A D		*s* 32	28	9.7298197
to the sine of the Suns declination,	*sc* E D		*s* 15	10	9.4176811

The same might be found by the same things given in the sixth triangle, where the perpendicular falls from the Pole.

And after the form of any of these three examples, there may a third question in this triangle, and so in any other be resolved:

As 3. If in the third or fourth triangle there be given,

The Suns declination, the Suns height above the horizon, and the angle of the Suns position: to find the Poles elevation.

Case

Case 4. *Two sides, and their contained angle given: to find one of the other angles.*

Let there be given in the second oblique triangle,

The Suns height above the horizon complement	A D,	32 deg.	28′
The Poles elevation, *complement*	A E,	51	32
The Suns azimuth D A E, *or rather the acute angle*	B A D	72	24

And let there be required the hour from noon E.

First for A B and E B.

The Suns azimuth from the South,	A 72 d.	24′	*sc* A	9.4805385
The Suns height, complement,	A D 32	28	*t* A D	10.1963704
The arch	A B 25	25	*t* A B	9.6769089

The summe or remainer of A B *and* A E, *is* E B.

But here unto	A B 25 deg.	25′
Adding	A E 38	28
The summe is	E B 63	53

Secondly, for E, by the second Consectary the proportion is,

As the sine of the first arch found,	*s* A B	That is	*s* 25 d.	25 *co. ar.*	0.3673424
to sine the second arch found :	*s* E B		*s* 63	53	9.9532277
so tang. compl. the azimuth	*tc* A		*t* 17	36	9.5013588
to tang. compl. the hour	*tc* E		*t* 33	34	9.8219289

Whose compl. 56 deg. 26′ converted into time, is 3 ho. 45′ 44″ before or after noon.

Or the proportion is
As the sine of the first arch found,
to the sine of the second arch found :
so is the tang. of the azimuth from East or West,
to the tangent of the hour from six.

Or by the 3 Theoreme of ch. 4. of plain Triangles.
As the sine of the second arch found,
is to the sine of the first arch found :
so is the tang. of the azimuth from the meridian,
to the tangent of the hour from noon.

Note. The like variety may be used in the next example, and also in the examples of the 6, 8, and 10 Cases, and partly in every Case; which having here briefly noted, we shall leave to your own practice, as your occasion requires.

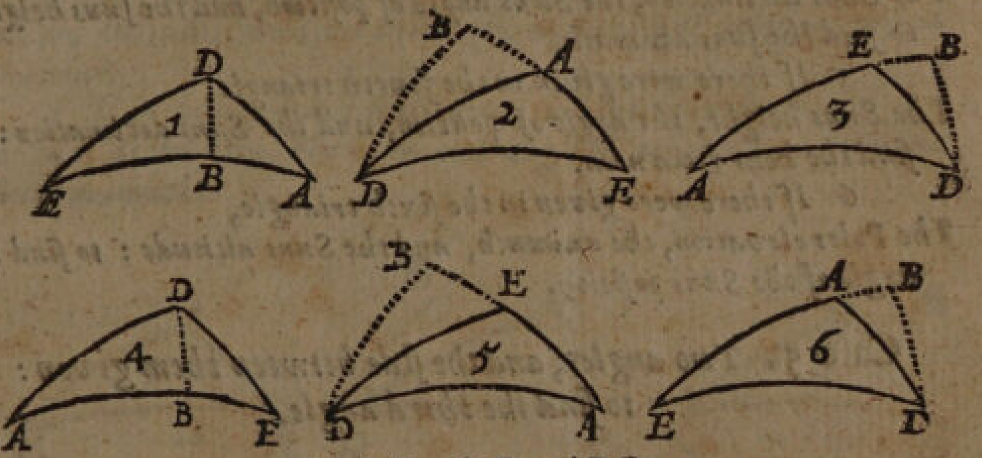
2 *Example.*

2 Example.

Let there be given in the fifth Triangle,

The Suns declination northerly, complement	A D	15 deg.	10′
The ho. from noon 3 ho. 45′ 44″, which in degrees is	A	56	26
The Poles elevation, complement	A E	51	32

And let there be required the Suns azimuth E,



First, for A B and E B.

The ho. from noon in degrees,	A	56 deg. 26′	*sc* A	9.7426520
The Suns declination compl.	A D 15	10	*t* A D	10.5669195
The arch	A B 63 53		*t* A B	10.3095715

From which substracting	A E 38 28	Or unto	A B 63 53
		Adding com.	A E 51 32
The remainer is	E B 25 25	Summe is	E B 25 25

Secondly, for E.

As the sine of the first arch found,	*s* A B	That is	*s* 63 d. 53′ *co. ar.*	0.0467723
to sine the second arch found,	*s* E B		*s* 25 25	9.6326176
so is tangent compl. the hour,	*tc* A		*t* 33 34	9.8218803
to tangent compl. the azimuth,	*tc* E		*t* 17 36	9.5013102

Which 17 deg. 36′ is the Suns azimuth from the East or West, and the complement thereof 72 deg. 24′, is the Suns azimuth from the South, whose complement to 180 deg. that is, 107 deg. 36′ is his azimuth from the North.

Hence might tables be framed shewing the Suns azimuth for every hour of the day, and for several seasons of the year, whereby may be made the Dials rendring the hour by the Suns azimuth.

By imitation of either of these examples, there may four other questions in this triangle, and so of any other be resolved: As,

3 *If there were given in the first oblique triangle,*
The Poles elevation, the hour, and the Suns declination: to find the Suns angle of position.

4 *If there were given in the third triangle,*
The Suns declination, the Suns angle of position, and the suns height: to find the suns azimuth.

5 *If there were given in the fourth triangle,*
The Suns height, the angle of position, and the Suns declination: to find the hour from noon.

6 *If there were given in the sixth triangle,*
The Poles elevation, the azimuth, and the Suns altitude: to find the angle of the Suns position.

Case 5. *Two angles, and the side between them given: to find the third angle.*

Let there be given in the first oblique Triangle,

The Poles elevation, complement	A D	51 deg.	32′
The ho. from noon 3 ho. 45′ 44″, which in degrees is	A	56	26
The Suns azimuth,	D	107	36

And let there be required the angle of position, E.

First, for the angles B D A and B D E, by the ninth Case of right angled triangles.

The Poles elevation, compl.	A D	51 d.	32′	*sc* A D	9.8937452
The hour from noon	A	56	26	*t* A	10.1781197
The angle	B D A	40	17	*tc* BDA	10.0718649

The summe or remainer of B D A and D, is B D E.

But here from	D	107 deg.	36′
substracting	B D A	40	17
remainer is	B D E	67	19

Secondly, for the angle E, by the second Consectary.

As the sine of the first angle found,	*s* B D A	That is *s* 40 17′	*co.ar.*	0.1893859
to the sine of the second angle found,	*s* B D E	*s* 67 19		9.9650371
so is the sine compl. the hour,	*sc* A	*s* 33 34		9.7426520
to sine compl. the angle of position.	*sc* E	*s* 52 05		9.8970750

There-

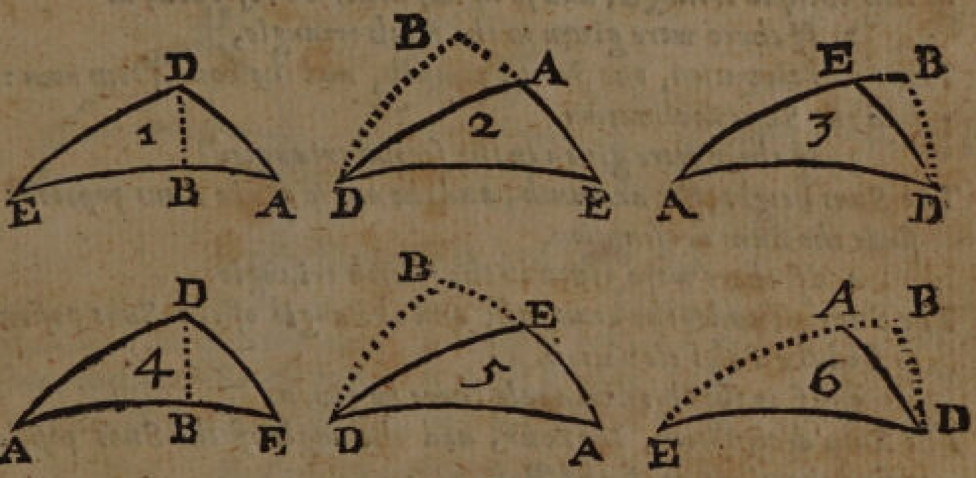
Therefore the angle of position E, is 37 deg. 55′.

The same might be found by the same things given in the sixth Triangle, where the perpendicular falls from the Pole, as here from the Zenith.

And after the form of this example there may two other questions in this triangle, and so in any other be resolved, As,

2 If in the second and fourth triangle there were given,
The Suns altitude, the Suns azimuth, and angle of position : to find the hour.

3 If in the third and fifth triangle there were given,
The Suns declination, the hour and angle of position : to finde the Suns azimuth.



Case 6. *Two angles, and the side between them given : to find one of the other sides.*

Let there be given in the first oblique Triangle,

The Poles elevation, complement	A D,	51 d.	32
The ho. from noon, 3 ho. 45′ 44″, which in degres is	A,	56	26
The Suns azimuth from the North, the obtuse angle	D,	107	36

And let there be required the Suns height, compl. E D.

First, for the angles B D A and B D E,

The Poles elevation, compl.	A D	51 d. 32′	*sc* A D	9.8937452
The hour from noon in deg.	A	56 26	*t* A	10.1781197
The angle	B D A	40 17	*tc* B D A	10.0718649

The

The summe or remainer of B D A and D, is B D E.

But here from	D	107 deg.	36′
substracting	B D A	40	17
The remainer is	B D E	67	19

Secondly, for E D,

As sine compl. the first angle found,	*sc* B D A	That is	*s* 49 43′ *co.ar.*	0.1175572
to sine compl. the second angle found,	*sc* B D E		*s* 22 41	9.5861794
so is tangent the Poles elevation	*tc* A D		*t* 51 32	10.0999135
to the tang. of the Suns altitude,	*tc* E D		*t* 32 28	9.8036501

Note. *By imitation of this example there may five other questions in this oblique triangle, and so in any other be resolved, as*

2 *If there were given in the sixth triangle,*
The Poles elevation, the Suns azimuth, and the hour from noon: to find the Suns declination.

3 *If there were given in the second triangle,*
The Suns height, the azimuth, and the angle of the Suns position: to finde the Suns declination.

4 *If there were given in the fourth triangle,*
The Suns altitude, the azimuth, and the angle of the Suns position: to find the Poles elevation.

5 *If in the third triangle there be given,*
The Suns declination, the hour, and the angle of the Suns position: to find the altitude of the Pole.

6 *If in the fifth triangle there be given,*
The Suns declination, the hour, and the angle of the Suns position: to find the Suns altitude.

Case 7. *Two sides, with an angle opposite to one of them given: to find the third side.*

Let there be given in the second oblique triangle,

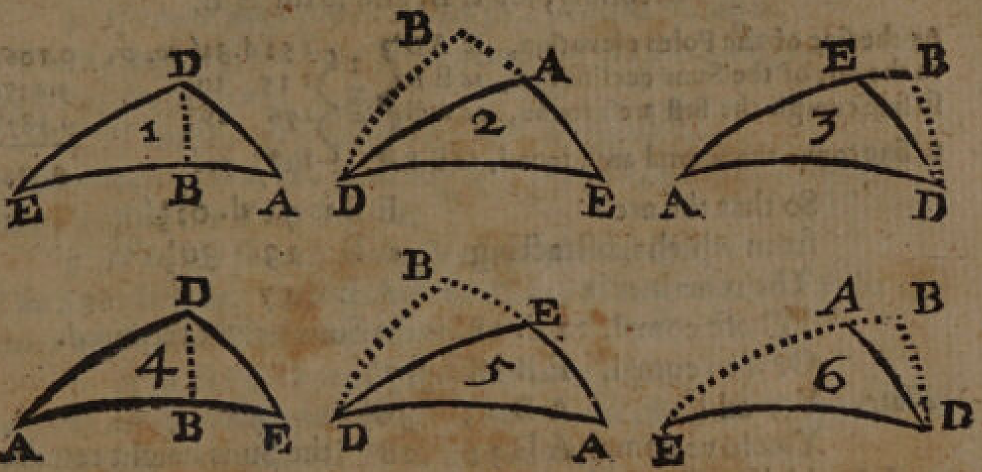
The Suns height above the horizon, complement,	A D,	32 deg.	28′
The Suns azimuth, namely, the acute angle at	A,	72	24
The Suns declination northerly complement	E D,	15	10

And let there be required the Poles elevation compl. A E.

First,

First, for the arch A B.

The Suns azimuth,	A 72 deg.	24′	*sc* A	9.4805385
The Suns height, complement,	A D 32	28	*s* A D	10.1963704
The arch first found	A B 25	25	*s* A B	9.6769089



Secondly, for E B, and so for A E.

As the sine of the Suns height,	*sc* A D	That is	*s* 32 deg.	28	*co. ar.*	0.2701803
to the sine of the Suns declinat.	*sc* E D		*s* 15	10		9.4176837
so sine comp. the first arch found,	*sc* A B		*s* 64	35		9.9557890
to sine com. the second arch found,	*sc* E B		*s* 26	07		9.6436530

So that the arch E B, is 63 deg. 53′.

The summe or remainer of A B and E B, is A E.

But here from	E B 63 deg.	53′	
substracting	A B 25	25	
The remainer is	A E 38	28	the side required.

2 Example.

Let there be given in the sixth Triangle,

The Poles elevation, *complement*	A D,	51 deg.	32′
The Suns azimuth from the meridian the acute angle at	A	72	24
The Suns declination northerly, complement	E D	15	10

And let there be required the Suns height, compl. A E.

 First,

First, for the arch A B.

The Suns azimuth,	A	72 deg. 24′	*sc* A	9.4805385
The Poles elevation compl.	A D 51	32	*t* A D	9.9000865
The arch first found,	A B 13	30½	*t* A B	9.3806250

Secondly, for E B, and so for A E.

As the sine of the Poles elevation, *sc* A D	That is	*s* 51 d. 32′ *co. ar.*	0.1062548
to the sine of the Suns declination, *sc* E D		*s* 15 10	9.4176837
so sine compl. the first arch found, *sc* A B		*s* 76 29½	9.9878163
to sine comp. the second arch found, *sc* E B		*s* 18 57½	9.5117548

So that the arch E B is 71 d. 02½′
from which substracting A B 13 30½
The remainer is A E 57 32
Whose compl. 32 d. 28′ is the Suns height required.
Or if to compl. E B 18 d. 57½
Note. You add A B 13 30½
You have comp. A E 32 28 the Suns height required.

I should digresse too much, if I should shew all the uses whereunto the questions falling out in this one triangle might be applyed: some of the principal I thought good to point at, that I might give occasion of exercise, especially in these later Cases, being something harder than the rest.

Thus by this proposition you may for one day, in any latitude, find how many degrees above the horizon the Sun will be upon any point of the Compasse; and thereby the variation of the Compasse.

As admit, being in the latitude of 51 deg. 32′ northerly, I find by the tables for that purpose the suns declination northerly, for some day, to be 15 deg. 10′. And I would know how high the sun will be that day, being upon the east southeast point of the compasse, that is 67 deg. 30′ from the meridian. Here working according to the former example, I find the suns height to be about 35 deg. 33′, therefore I observe with staffe, quadrant, or other instrument, till I find the sun to be 35 deg. 33′ high, and then is the sun east southeast. Wherefore at that instant setting the sun with my compasse, if I find it to be upon the east southeast point, then hath it no variation: if it differ, look how much that difference is, so much is the variation. Which whether it be easterly or westerly, may be known by the rule before given after the 12 Case of the third chapter of right angled spherical triangles.

By

By this proposition also are the azimuths drawn on those quadrants that give the Suns azimuth by his altitude, and so on those Dials that do the like.

And after the form of either of these examples there may four other questions in this oblique triangle, and so in any other be resolved. As,

3 *If there were given in the first oblique triangle,*

The Poles elevation, the hour from noon, and the Suns height : to find the Suns declination.

4 *If there were given in the third triangle,*

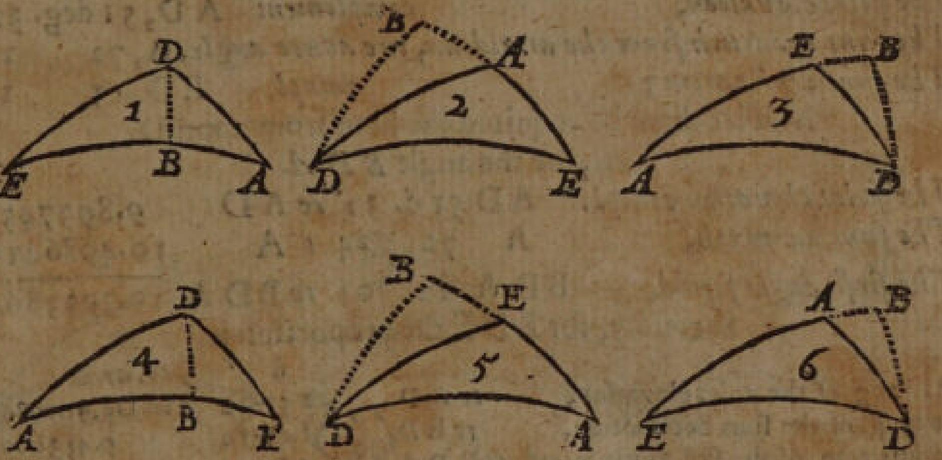
The Suns declination, the Suns angle of position, and the Poles elevation : to find the Suns height.

5 *If in the fourth triangle there were given,*

The Suns height, the angle of the Suns position, and the Poles elevation : to find the Suns declination.

6 *If in the fifth triangle there were given,*

The Suns declination, the hour from noon, and the Suns height above the horizon : to find the Poles elevation.



Case 8. *Two sides, with an angle opposite to one of them given : to find their conteined angle.*

Let there be given in the first oblique Triangle,

The Poles elevation,	*compl.* A D, 51 deg.	32′
The ho. from noon 3 *ho.* 45′-44″, *which in degrees is*	A, 56	26
The Suns altitude above the horizon,	*compl.* E D, 32	28

And

And let there be required the Suns azimuth from the North, D.

First, for the angle B D A.

The Poles elevation compl.	A D	51 deg. 32′	*sc* A D	9.8937452
The hour from noon,	A	56 26	*t* A	10.1781197
The first angle found	B D A	40 17	*tc* B D A	10.0718649

Secondly, for B D E the proportion is,

As tang. of the Poles elevation,	*tc* A D	That is	*t* 51 d. 32′	co. ar. or *t* A D 9.9000865
to tang. of the Suns altitude,	*tc* E D		*t* 32 28	9.8036196
so sine com. the first angle found,	*sc* B D A		*s* 49 43	9.8814428
to sine compl. the second,	*tc* B D E		*s* 22 41	9.5861589

The summe or remainer of the first and second angles found, namely, of B D A *and* B D E, *is the angle required* D.

But here to B D A 40 d. 17′
adding B D E 67 19
The summe is D 107 36 the Suns azimuth required.

2 *Example*

Let there be given in the sixth Triangle.

The poles elevation,	*complement*	A D, 51 deg.	32′
The suns azimuth from the meridian, the acute angle,		A, 72	24.
The suns declination;	*compl.*	E D, 15	10

And let there be required the hour from noon D.

First, for the angle *B D A.*

The poles elevation compl.	A D	51 d. 32′	*sc* A D	9.8937452
The suns azimuth,	A	72 24	*t* A	10.4986412
The first angle found,	B D A	22 03	*tc* B D A	10.3923864

Secondly, for B D E the proportion is,

			d ′	
As tang. of the poles elevation,	*tc* A D	That is	*t* 51-32	co. ar. or *t* A D 9.9000865
to tang. of the suns declination,	*tc* E D		*t* 15-10	9.4330804
so sine com. of the first angle found,	*sc* B D A		*s* 67-57	9.9670125
to sine compl. the second.	*sc* B D E		*t* 11-31	9.3001794.

So that *B D E* is 78 d. 29′
From which take *B D A* 22 03
The remainer is *D* 56 26

Or if unto *B D A* 22-03. You add *co. B D E* 11-31. The sum is *co. D* 33-34 the ho. from 6.

Which 56 deg. 26′ converted into time, is 3 ho. 45′-44″, from noon, that is, 14′ 16″ after 8 of the clock in the forenoon, or 45′-44″ after 3 of the clock in the afternoon.

And thus in any place, for any day, you may frame a table of the hour and minute of the Suns position upon every point of the Compasse: Whereby you shall manifestly see the errour of the common rule, of bringing two and thirty to four and twenty.

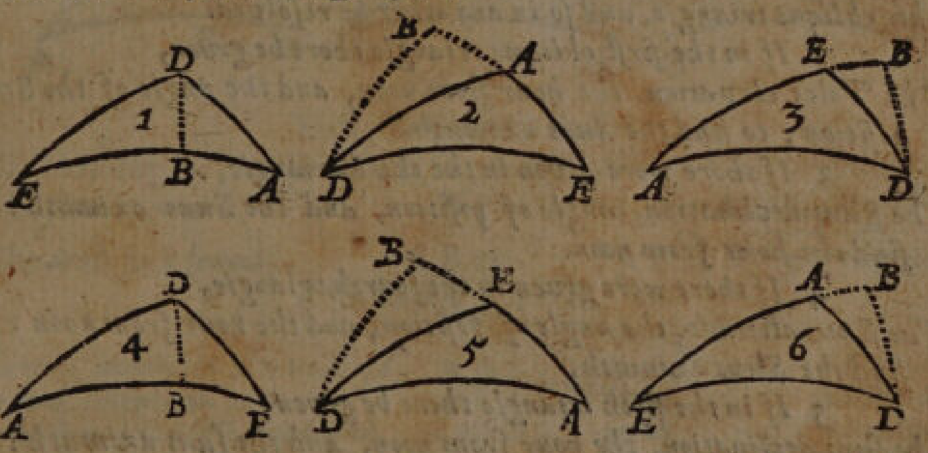
By imitation of either of these examples, there may four other questions in this triangle, and so of any other be resolved. As,

3 *If in the second oblique triangle there were given,*
The altitude of the Sun, the azimuth, and the Suns declination: to find the angle of the Suns position.

4 *If in the third triangle there were given,*
The suns declination, the angle of position, and the poles elevation: to find the hour.

5 *If in the fourth triangle there were given,*
The suns altitude, the angle of position, and the poles elevation: to find the suns azimuth.

6 *If in the fifth triangle there be given,*
The suns declination, the hour, and the altitude of the sun above the horizon: to find the angle of position.



Case 9. *Two angles, and a side opposite to one of them given: to find the third angle.*

Let there be given in the second oblique Triangle,

The suns height above the horizon, complement	A D 32 d.	28′
The suns azimuth from the meridian, the acute angle	A 72	24
The hour from noon, 3 ho. 45′-44″, which in deg. is	E 56	26

And let there be required the angle of position, D.

First,

First, for the angle B D A.

The Suns altitude compl.	A D,	32 d. 28′	*sc* A D	9.7298197
The Suns azimuth,	A	72 24	*t* A	10.4986412
The angle first found,	B D A,	30 35	*tc* B D A	10.2284609

Secondly, for B D E, the proportion is,

		That is		
As sine compl. the azimuth,	*sc* A	*s* 17 d. 36′	*co. ar.*	0.5194615
to sine compl. the hour from noon,	*sc* E	*s* 33 34		9.7426520
so the sine of the first angle found,	*s* B D A	*s* 30 35		9.7065394
to the sine of the second,	*s* B D E	*s* 68 30		9.9686529

The summe or remainder of the first and second angle found B D A and B D E, is the angle D required.

But in this example, From B D E 68 d. 30′
subtracting B D A 30 35
The remainer is D 37 55 the angle of position re-
(quired.

After the form of this example, there may five other questions in this oblique triangle, and so in any other be resolved.

2 *If in the first oblique triangle there be given,*
The Poles elevation, the hour from noon, and the angle of the Suns position: to find the Suns azimuth.

3 *If there were given in the third triangle,*
The Suns declination, angle of position, and the Suns azimuth: to find the hour from noon.

4 *If there were given in the fourth triangle,*
The Suns altitude, the angle of position, and the hour from noon: to find the Suns azimuth.

5 *If in the fifth triangle there be given,*
The suns declination, the hour from noon, and the suns azimuth: to find the angle of the suns position.

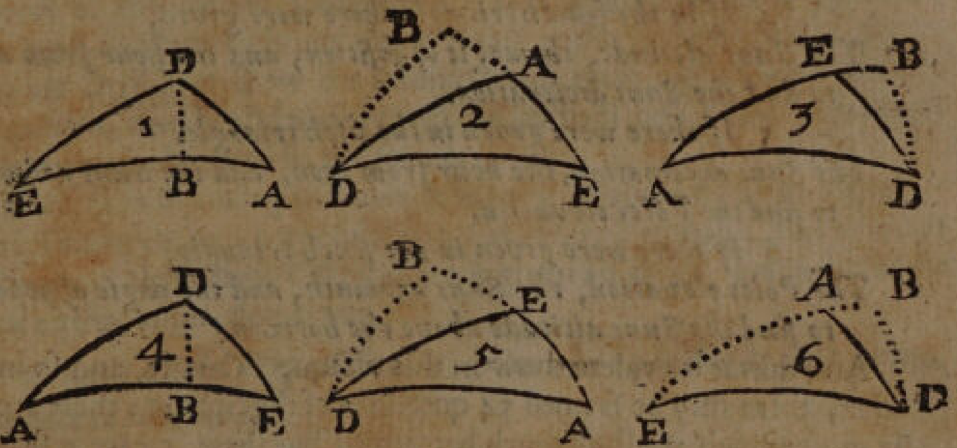
6 *If there were given in the sixth triangle,*
The poles elevation, the suns azimuth, and the angle of the suns position: to find the hour from noon.

Case 10. *Two angles, and a side opposite to one of them given: to find the side between them.*

Let there be given in the second oblique Triangle,

The suns height above the horizon complement	A D,	32 deg.	28′
The suns azimuth from the merid. the acute angle	A,	72	24
The ho. from noon 3 ho. 45′-44″, which in deg. is	E,	56	26

And let there be required the Poles elevation compl. A E.



First, for the arch A B.

The suns height, complement,	A D 32 deg.	28′	t A D	10.1963704
The suns azimuth,	A 72	24	sc A	9.4805385
The arch first found,	A B 25	25	t A B	9.6769089

Secondly, for E B.

		That is d. ′	co. ar. or	
As tang. compl. the suns azimuth,	tc A	t 17-36	t A	10.4986412
to tang. compl. the hour,	tc E	t 33-34		9.8218803
so the sine of the first arch found,	s AB	s 25-25		9.6326576
to the sine of the second arch found,	s E B	s 63-53		9.9531791

The summe or remainer of the first and second arch found, (A B and E B) is the side required A E.

But here from E B 63 deg. 53′
substracting A B 25 25
The remainer is A E 38 28

Which is the complement of the Poles height required, 51 deg. 32′.

By imitation of this example, there may five other questions in this oblique triangle, and so of any other be resolved: As,

2 If in the first oblique triangle there were given,

The Poles elevation, the hour from noon, and the angle of the Suns position: to find the Suns declination.

3 If in the third triangle there were given,

The Suns declination, the angle of position, and the azimuth: to find the Suns height above the horizon,

4 If in the fourth triangle there were given,

The Suns altitude, the angle of position, and the hour from noon: to find the Suns declination.

5 If there were given in the fifth triangle,

The Suns declination, the hour from noon, and the Suns azimuth: to find the Poles elevation.

6 If there were given in the sixth triangle,

The Poles elevation, the Suns azimuth, and the angle of position: to find the Suns altitude above the horizon.

And thus it is evident how in this oblique Triangle, and so in any other, there may be framed 54 questions of the Sphere; there are also six more which we shall touch hereafter; but these 54 are reduced as we have shewed to ten Cases, and those ten Cases to two Consectaries, which two Consectaries are deduced from the first fundamental Axiome; so that the resolution of all the Cases and questions hitherto handled, whether in right or oblique angled spherical triangles, depend upon that one Axiome; and may be thereunto reduced. There remains (as is said) six other questions in this oblique Triangle, (and the like in any other) which are reduced to two Cases, namely, when three sides are given, to find an angle; or three angles given to find a side. And these also might well be resolved by the grounds before laid, without adding any more, but because the wayes devised by the Lord *Nepair* are more apt for this purpose, we will make use of them.

And as we have shewed the resolution of the 8 Cases last beforegoing, by help of a perpendicular; the same might have been done by drawing in stead of the perpendicular, a quadrantal side: so reducing the Triangle given to two quadrantal Triangles. But this we must now leave to your practice.

CHAP.

CHAP. VI.

The eight last Cases of an Oblique Triangle, resolved by finding the Perpendicular.

THe resolution of these eight Cases, hath usually been at three operations, though (as we have shewed) they may be done at two. Yet because the way at three operations is more easily understood, and is more immediately performed by the first general Axiome, without respect to the second Consectary; and because in many questions you have occasion to know the quantity of the perpendicular: therefore we shall here shew the resolution of these eight Cases at three operations, briefly and plainly; to the satisfaction (I hope) of such as complain of obscurity in the former.

First, then the oblique Triangle is to be resolved into two right angled Triangles by a perpendicular (as before) namely, that whereas in every of these eight Cases, there is given an angle, and a side adjacent to that angle, you are to

1 *Let fall the perpendicular from the end of that side given, opposite to that angle.*

Aud further, when that sufficeth not,

2 *Let it fall also opposite to the angle required:* (as in the fourth Case.) *Or opposite to the side required* (as in the sixth.)

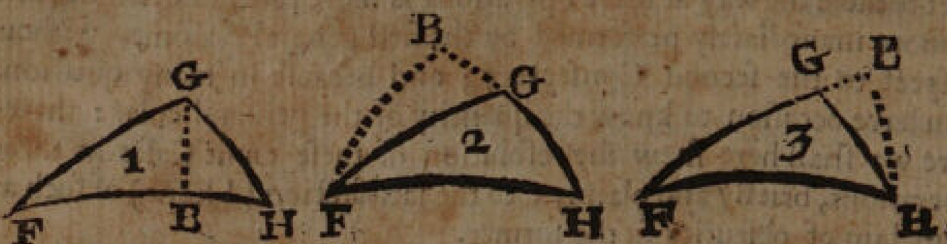
Accounting (as before) the sides and angles that are not adjacent, to be opposite.

And for helping your memory, you may (as is afore-said in the first Case of spherical Triangles) mark the sides or angles given with a dash thus —, and that required with an o or cypher, or with three pricks thus ∴

And here we might mark the Triangles with the same letters as before, namely, that whereas there is alwayes (as I have said) an angle given, and a side adjacent to that angle, you may mark the said angle given with A, and the adjacent side given with A D, and the angle remaining with E, and the perpendicular with D B. But because there is no necessity that a man should hold himself alwayes to that form (as I have before noted) we will here mark the same Triangle with other letters at adventure, as

 with

with F G H, then the perpendicular falling from G, or F, or H, (as the propofition in hand ſhall require) reſolves it into two right angled Triangles, in one of which there is given the hypothenuſal and angle at the baſe, whereby you may find firſt the perpendicular; ſecondly, the baſe or angle of the perpendicular: Or you may firſt find the baſe or angle at the perpendicular; ſecondly, the perpendicular.



The firſt fundamental axiome we will here again repeat, being as followeth:

Of the five circular parts in a right angled ſpherical Triangle.

The ſine of a middle part, with Radius, is equal to the tangent of the adjacent extreams, or to the ſines complement of the oppoſite extreams.

Hence we reſolve theſe eight Caſes in manner following.

Caſe 3. *Two ſides, and their conteined angle being given: to find the third ſide.*

Dat. G H 38. d. 28′, F H 74 d. 50′, H 56 d. 26′, required F G.

Here in the firſt Triangle, from G the end of the ſide G H given, being adjacent to an angle given H, I let fall the perpendicular G B oppoſite to that angle: And ſo we have two right angled Triangles, G B H, and G B F; and in the firſt there is given the hypothenuſal H G, and the angle at the baſe H, by which to find the perpendicular G B. I ſay by this axiome,

s G B + Rad. = *s* G H + *s* H, therefore from *s* G H + *s* H ſubſtracting Radius, the remainer is *s* G B; or to avoid ſubſtraction, add unto *s* G H + *s* H the complement arithmetical, which for Radius is is oo) leaving out Rad. or the first unite in the summe, and so the work is as followeth:

s G H 38 deg. 28′		9.7938317
s H 56	26	9.9207717
s G B 31	13	19.7146034

Secondly, for the base B H, say sc H + Rad = tc G H + t B H, and so the operation stands thus as beneath.

t G H 38	28	9.9000065
sc H 56	26	9.7426520
t B H 23	43	9.6427385

The summe or remainer of B H and F H is F B.

But here from F H 74 deg. 50′
substracting B H 23 43
there remains F B 51 07

Thirdly, for the side required F G, having G B and F B. Say sc F G + Rad = sc G B + sc F B, and so the operation stands thus.

G B 31 deg. 13′	sc G B	9.9320746
F B 51 07	sc F B	9.7977775
F G 57 32	sc F G	9.7298521

Which 57, 32, is the side F G required.

And as in this first Example the perpendicular was let fall from the angle at G, so it might in this Case have been let fall from the angle at F, as in the second Triangle, and in this second Example.

Dat. F H 74 d. 50′, H 56 d. 26′, G H 38 28, required F G.
First, for the perpend. *F B* say, *s F B* + Rad = *s F H* + *s H.*
Secondly, for the base, B H say, *sc H* + Rad = *tc F H* + *t B H,*
And *B H* — *G H* = *B G.*
Thirdly, for the side required *FG* say, *sc FG* + Rad = *sc FB* + *sc BG.*
The operations are as followeth.

I. *F H* 74 d. 50′	*s F H*	9.9846033
H 56 26	*s H*	9.9207717
F B 53 32	*s F B*	9.9053750

II. *F H* 74 50	*t F H*	10.5669196
H 56 26	*sc H*	9.7426520
B H 63 53	*t B H*	10.3097516
— *G H* 38 28		
B G 25 25		

III. *F B* 53 deg. 32′	*sc F B*	9.7740459
BG 25 25	*sc BG*	9.9557090
FG 57 32	*sc FG*	9.7298349

Which 37 32′ is the side required as before.

Third Example.

Dat. F G 57 d. 32′, G H 38 d. 28′; G, or rather the acute angle B G F 72 d. 24′ required F H.

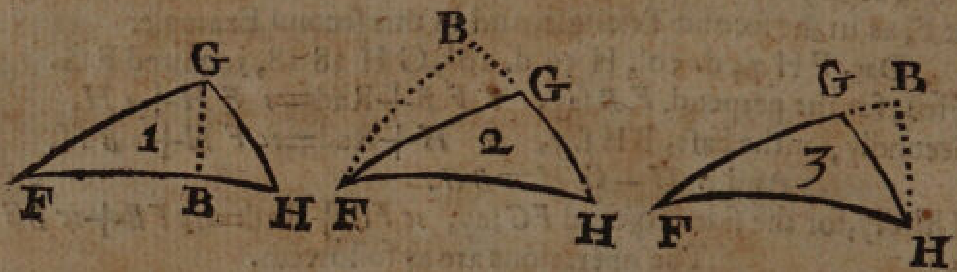
The perpendicular may fall from F or H, but here we let it fall from F as in the second Triangle.

Then for the perpendicular F B, say, F B + Rad = *s* F G + *s* G.
Secondly, for the base B G, say, *sc* G + Rad = *tc* F G + *t* B G.
Thirdly, for the side required F H, say, *sc* F H + Rad = *sc* B F + *sc* B H.

The operations are as followeth.

s F G	9.9261901	*t* F G	10.1963704	*sc* B F	9.7740459
s G	9.9791798	*sc* G	9.4805385	*sc* B H	9.6436504
s F B	9.9053699	*t* B G	9.6769089	*sc* F H	9.4176963
F B 53 d. 32′		B G 25 d. 25′		F H 74 deg. 50′	
		G H 38 28		the side required.	
		B H 63 53			

In this and the other Cases following, having by the first operation found the perpendicular, you may use it in the second operation as one of the two things given, which we shall not need to exemplifie.



Case 4. *Two sides, and their conteined angle given: to find one of the other angles.*

Dat. F G 57 d. 32′, G H 38 d. 28′, G 107 d. 36′, or G acute 72 d. 24′ required the angle H.

In this Case the perpendicular falls onely from F (as in the second Triangle) and so is opposite to the angle given G, and to the angle required H.

First,

First, then for the perpend. F B, say, s F B+Rad=s FG+s G.
Secondly, for the base, B G, say, sc G+Rad=tc F G+t B G,
The summe or remainer of B G and G H, is B H.
Thirdly, for the angle required H, say, s B H+Rad=t F B+tc H.

And accordingly we order the operations as followeth.

	d. ′				d. ′	
I. F G	57 32	s F G 9.9261901	II. F G	57 32	t FG	10.1963704
G	72 24	s G 9.9791798	G	72 24	sc G	9.4805385
F B	53 32	s F B 9.9053699	B G	25 25	t B G	9.6769089
			+G H	38 28		
			B H	63 53		

III. F B 53 32 tc F B 9.8686804
BH 63 53 s BH 9.9532278
H 56 26 tc H 9.8219082, which 56 deg. 26′ is the angle required.

Second Example.

Dat. F H 74 d. 50, G H 38 d. 28′, H 56 d. 26′, required G, here in the second Triangle as before.

First, for the perpendicular, F B, say, s F B+Rad=s F H+s H.
Secondly, for the base, B H, say, sc H+Rad=tc F H+t B H,
And B H—G H=B G.
Thirdly, for the angle required G, say, s B G+Rad=t F B+tc G.

And accordingly the operations are as follow.

I. s F H 9.9846037
s H 9.9207717
s F B 9.9053750
So is F B 53 d. 32′

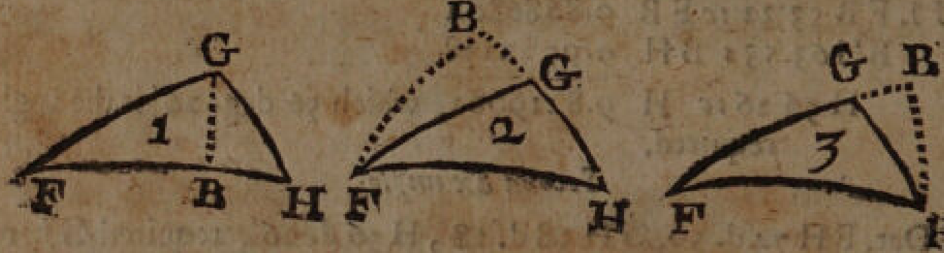
II. t FH 10.5669196
sc H 9.7426520
t BH 10.3095716
B H 63 53
—G H 38 28
B G 25 25

III. tc F B 9.8686804
s BG 9.6326576
tc G 9.5013380
So is G 72 d. 24′, or G obtuse 107 d. 36′ the angle required.

Case

Case 5. *Two angles and the side between them given: to find the third angle.*

Dat. H 56 d. 26′, *G* 107 d. 36, *G H* 38 d. 28′, required the angle *F.*
In this case the perpendicular may fall from *G or H*, as here from *G.*
First then for the perpend. *G B*, say *s GB*+Rad=*s GH*+*s* H.
Secondly, for the angles at the perpend. *BG H* and *BGF* } say *sc GH*+Rad=*tc H*+*tc BGH.*
The summe or remainer of *G* and *B G H*, is *B G F.*
Thirdly, for the angle required *F*, say *sc F*+Rad=*s BGF*+*sc BG.*
The operations are as followeth.



I. *s* G H	38 d.	28′	9.7938317	
s H	56	26	9.9207717	
s G B	31	13	9.7146034	

II. *t* H	56 d.	26′	10.1781197
sc GH	38	28	9.8937452
tc BGH	40	17	10.0718649
from *G*	107	36	
leaves *B G F*	67	19	

III. *s B G F*	67	19	9.9650371
sc B G	31	13	9.9320746
sc F	37	55	9.8971117

Which 37 d. 55′ is the angle at *F* required.

Case 6. *Two angles and the side between them given: to find one of the other sides.*

Dat. *G H* 38 d. 28′, H 56 d. 26′, *G* 107 d. 36′, required *F G.*
Let fall the perpendicular, from G as in the first Triangle; for so it fals from the end of the side given *G H*, opposite to its adjacent angle given *H*, and also opposite to the side required *F G*, as in this Case it ought to do. Then,

First,

First, for the perpend. *G B*, say, *s G B*+Rad=*s* H+*s G* H.
Secondly, for the angle at the perpend. *B G* H, say, *sc G* H+Rad=
tc H+*tc B G H*, the summe or remainer of *G*, and B G H is B G F.
Thirdly, for the side required *F G*, say, *sc* B G F+Rad=*t* B G+
tc F G.

The operations are as followeth.

I. *s* H	56 d. 26′	9.9207717	II. *t* H	56 d. 26′	10.1781197
s GH 38	28	9.7938317	*sc* GH 38	28	9.8937452
s GB 31	13	9.7046034	*tc* BGH 40	17	10.0718649
			from G	107 36	
			rests BGF	67° 19	

III. *tc* B G	31 13	10.2175136
sc B G F 67	19	9.5861795
tc F G	57 32	9.8036931

Which 57 deg. 32 is the side required.

Case 7. *Two sides with an angle opposite to one of them given: to find the third side.*

Dat. *F* G 57 d. 32′, *F* H 74 d. 50′, *F* G H 107 d. 36′, or its complement to 180 degrees 72 d. 24′ required G H: here letting fall the perpendicular from *F*, as in the second Triangle.
First, for the perpendicular *F B*, say, *s F B*+Rad=*s FG*+*sFGB*.
Secondly, for the base B G, say, *sc FGB*+Rad=*tc F G*+*t B G*.
Thirdly, for the base B H, say, *sc F* H+Rad=*sc FB*+*sc* B H.
The summe or remainer of B G and B H is G H, the side required.

The operations are these.

I. *s FG*	57 d. 32	9.9261901	II. *t FG*	57 d. 32′	10.1963704
s FGB 72	24	9.9791798	*sc F G B* 72	24	9.4805385
s FB	53 32	9.9053699	*t* BG	25 25	9.6769089

III. *Co. ar. sc F B*	53 d. 32	0.2259541
sc FH 74	50	9.4176963
sc BH 63	53	9.6436404
—BG 25	25	
rest *GH* 38	28, which is the side required.	

Second

Second Example.

In the third Triangle:

Dat. GH 38 d. 28′, FH 74 d. 50′, G or rather BGH 72 d. 24′ required FG.

Firſt, for the perpendicular *BH*, ſay, *s BH*+Rad=*s GH*+*s BGH*.

Secondly, for rhe baſe *BG*, ſay, *sc BGH*+Rad=*tc GH*+*t GB*.

Thirdly, for the baſe *BF*, ſay, *sc FH*+Rad=*sc BH*+*sc BF*.

And here from *BF*, ſubſtracting *BG*, there remains *FG* required.

The operations.

I. *s GH*	38 d.	28′	9.7938317	II. *t GH*	38 d.	28′	9.9000965
s BGH	72	24	9.9791798	*sc BGH*	72	24	9.4805385
s BH	36	22	9.7730215	*t BG*	13	30½	9.3806250

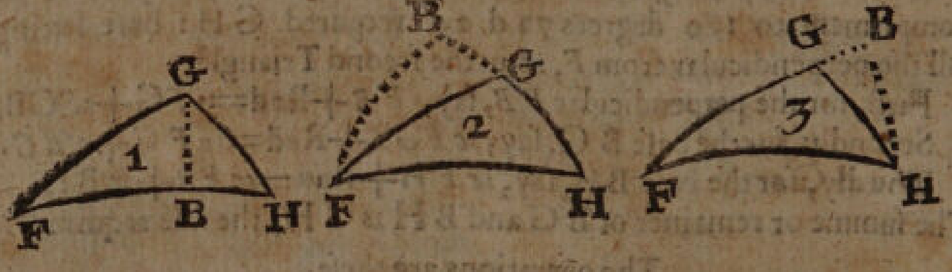
III. *Co. ar. sc BH* 36 d. 22′ 0.0940753

sc FH 74 50 9.4176963

sc BF 71 0½ 9.5117716

—*BG* 13 30½

reſt *FG* 57 32, which is the ſide required.



Caſe 8. *Two ſides with an angle oppoſite to one of them given: to find their conteined angle.*

Dat. *GF* 57 d. 32′, *GH* 38 d. 28′, *H* 56 d. 26′ required *G*.

Let fall the perpendicular from *G*, as in the firſt Triangle.

Firſt, for the perpendicular *GB*, ſay, *s GB*+Rad=*s* GH+*s H*.

Secondly, for the angle BGH, ſay, *sc* GH+Rad=*tc* H+*tc* BGH.

Thirdly, for the angle BG*F*, ſay, *sc* BG*F*+Rad=*t* BG+*tc* G*F*.

The ſum or remainer of BH and BG*F* is the angle at G required.

The

The operations are as followeth.

I. *s* G H	38 d. 28′	9.7938317	II. *t* H	56 d. 26′	10.1781197
s H	56 26	9.9207717	*sc* G H	38 28	9.8937452
s G B	31 13	9.7146034	*tc* B G H	40 17	10.0718649

III. *t* B G	31 d. 13′	9.7824064
tc G F	57 32	9.8036296
sc B G F	67 19	9.5861160
+ B G H	40 17	
G	107 36, which is the angle required.	

2 *Example*, in the third Triangle.

Dat. F H 74 d. 50′, G H 38 d. 28′, the acute angle at G 72 d. 24′, required the angle at H, that is, F H G.

First, for the perpend. B H, say, *s* B H + Rad = *s* B G H + *s* G H.

Secondly, for the angle B H G, say, *sc* G H + Rad = *tc* B G H + *tc* B H G.

Thirdly, for the angle B H F, say, *sc* BHF + Rad = *t* BH + *tc* FH.

The operations follow.

I. *s* BGH	72 d. 24′	9.9791798	II. *t* BGH	72 d. 24′	10.4986413
s G H	38 28	9.7938317	*sc* G H	38 28	9.8937452
s B H	36 22	9.7730115	*tc* BHG	22 $03\frac{1}{2}$	10.3923865

III. *t* B H	36 d. 22′	9.8670937
tc F H	74 50	9.4330804
sc B H F	78 $29\frac{1}{2}$	9.3001741
— B H G	22 $03\frac{1}{2}$	
rest G H F	56 26, the angle required.	

Case 9. *Two angles and a side opposite to one of them given: to find the third angle.*

Dat. F G 57 d. 32′, G acute 72 d. 24′, H 56 d. 26′, required F.

Let fall the perpend. from F, as in the second Triangle.

First, for the perpend. F B, say, *s* F B + Rad = *s* F G + *s* F G B.

Secondly, for the angle B F G, say, *sc* F G + Rad = *tc* G + *tc* BFG.

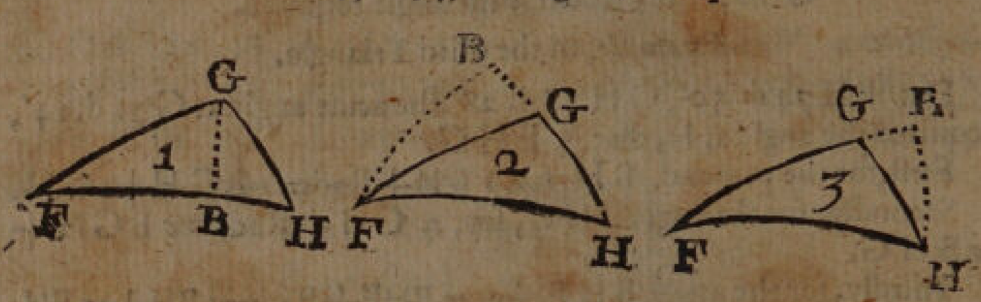
Thirdly, for the angle B F H, say, *sc* H + Rad = *sc* F B + *s* B F H.

The summe or remainer of B F G and B F H, is G F H required.

The operations.

I. *s F G* 57 d. 32′	9.9261901	II. *t*	*G* 72 d. 24′	10.4986413
s G 72 24	9.9791798	*sc F G* 57 32		9.7298197
s F B 53 32	9.9053699	*tc BFG* 30 35		10.2284610

III. *Co. ar. sc F B* 53 d. 32′ 0.2259541
sc H 56 26 9.7426520
s B F H 68 29 9.9686061
—*B F G* 30 35
G F H 37 54, the angle at *F* required.



Case 10. *Two angles and a side opposite to one of them given: to find the side between them.*

Dat. H. 56 d. 26′, G acute 72 d. 24′, F G 57 d. 32′, required G H.
Here the perpendicular must fall from F, as in the second Triangle.
First, for the perpend. F B, say, *s* F B + Rad = *s* F G + *s* G.
Secondly, for the base B G, say, *sc* G + Rad = *tc* F G + *t* B G.
Thirdly, for the base B H, say, *s* B H + Rad = *t* F B + *tc* H.
The summe or remainer of B G and B H is G H.

The operations.

I. *s F G* 57 d. 32′	9.9261901	II. *t F G* 57 d. 32′	10.1963704
s G 72 24	9.9791798	*sc G* 72 24	9.4805385
s F B 53 32	9.9053699	*t B G* 25 25	9.6769039

III. *t F B* 53 d. 32′ 10.1313196
tc H 56 26 9.8218803
s BH 63 53 9.9532999
—*BG* 25 25
G H 38 28, the side required.

And

And this may suffice touching these eight Cases of an oblique spherical Triangle, there remain two other Cases; namely, when the three sides are given to find an angle, or the three angles to find a side. The first of which is of frequent use, and therefore (though in the former Editions of this book) we have shewed the resolution thereof three several wayes, as may appear in the Chapter following; yet I conceive it will not be superfluous, to give here an example or two more in that third way of application: which as it is easily wrought arithmetically, so it is the aptest for instrumental operations, whether you use Mr. *Gunters* Logarithmical Ruler, or Mr. *Wingates*, or any other right lined, or other circular or serpentine projection of the Logarithms, or the Sector, &c.

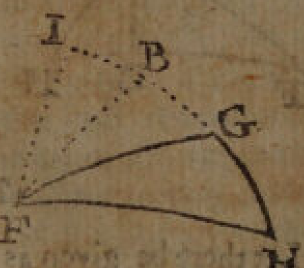
Case 11. *Three sides of an oblique Triangle given: to find an angle.*

The rule and ground for the Solution of this Probleme is shewed in the Chapter following, therefore we come to Examples.

The first Example may be this

Let there be given the latitude of the place, or poles elevation 52 deg. 32′, the Suns altitude 32 deg. 28′: the Suns declination northerly 15 deg. 10′. And let there be required the Suns azimuth.

In this triangle F G H, let G represent the zenith, H the North pole, F the Sun, then the complement of the poles elevation is GH 38 d. 28′
the com. of the suns altitude G F 57 32
the com. of the suns declin. F H 74 50
or the suns distance from the north pole.



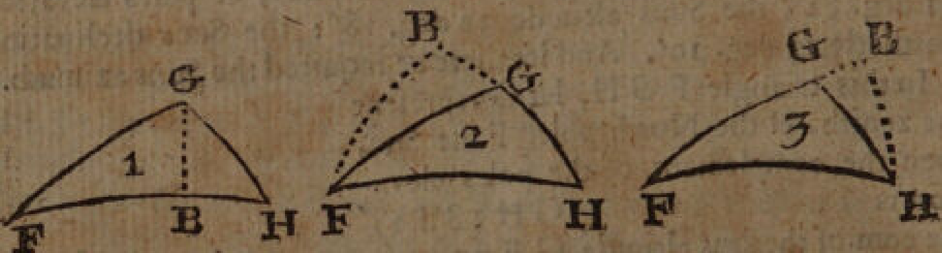
Then is the angle F G H the Suns azimuth from the North, or F G B the Suns azimuth from the South, either of which is the angle required, and therefore opposite thereto I let fall the perpendicular F B, and make B I equal to B G, ordering the work as followeth.

		d. ′		d. ′	
Poles elevat. 51 32	whose comp. GH	38 28	halved	19 14 *tc*	10.4573123
Suns altitude 32 28	whose comp. FG	57 32		28 46	
Suns declin. 15 10	whose comp. *FH*	74 50		37 25	
the summe of half the sides				66 11 *t*	10.3551676
the difference of half the sides				08 39 *t*	9.1822106
Gives the half of the alternate base		½ H I		44 39	9.9946905
from which take half the true base		½ G H		19 14	
there remains		B G		25 25	

Thus in the right angled Triangle B G F, having found the base *B G*, and the hypothenusal *F G*, being before at first given, we may find the angle at *G*, saying,

sc G + Rad = *t B G* + *tc F G*, that is, *t B G* 25 d. 25′ 9.6768686
tc F G 57 . 32
sc G 72 24

And thus have we found the acute angle at *G*, namely, *B G F*, to be 72 deg. 24′; which is the Suns azimuth from the South, which was required.



The Second Example.

Let there be given as before, the latitude 51 deg. 32′, the suns altitude 32 deg. 28′, the suns declination northerly 15 deg. 10′. And let there be required the hour from noon, or angle at H.

We order the operation as followeth.

Latitude

	d. ′		d. ′		d. ′		
Latitude	51 32	whose com. is	*GH* 38 28	halved	19 14	*tc*	10.4573123
Suns altitude	32 28	whose com. is	*FG* 57 32	halved	28 46		
Suns declin.	15 10	whose com. is	*FH* 74 50	halved	37 25		
the summe of half the sides					66 11	*t*	10.3551676
the difference of half the sides					08 39	*t*	9.1822106
gives the half the alternate base			½ H I		44 39	*t*	9.9946905
To which adding half the true base			½ G H		19 14		
the summe is the base			BH		63 53		

Thus in the right angled Triangle BHF, having the base BH 63 d. 53′, and hypothenusal FH, being before at first given 74 d. 50′, we may find the angle at H, saying,

sc H + Rad = *t* BH + *tc* FH, that is,	*t*	BH 63 53	10.3095777
	tc	FH 74 50	9.4330804
	sc	H 56 26	9.7426581

And thus have we found the angle of the hour from noon H, to be 56 deg. 26′, which converted into time is 3 ho. 45′ 44″, before or after noon.

In like sort you may find the hour of night by any known star, for the same things being given, namely, the latitude of the place, the altitude of the star, and its declination, you may find the angle at H, as before; and so the true hour and minute (if it were the sun) which note. Then from that stars right ascension substract the suns right ascension, and the remainer converted into time, add to the hour and minute before noted, that total is the true hour and minute of the night.

But in gathering the suns right ascension, you must remember that it differs every day about one degree or four minutes of time (as more exactly in the Table appears) and so you must allow proportionably, for the time that the sun is past the meridian of the place for which your Tables were made: as if it be six hours past it, the right ascension is increased by about one minute of time; if twelve hours, then about two minutes; if eighteen hours, then about three minutes of time, &c.

CHAP.

CHAP. VII.

Of the second Fundamental Axiome, and of the Cases thereon depending: with two other Axiomes to the same purpose.

2 *Fundamental Axiome.*

IN a spherical Triangle, if half the difference of the sides conteining an angle, be added to half the side opposite to that angle, and likewise substracted from the same, and the sum and remainer noted:

Then as the rectangle of the sines of the conteining sides, is to the square of Radius:

So is the rectangle of the sines of the foresaid summe and remainer to the square of the sine of half the conteined angle.

As in the Triangle *A D E.*

Let D *be the conteined angle, and let* A B *be the difference of the conteining sides* A D *and E D (for D B is equal to E D) and let* A E, *that is,* A S: *be the side opposite to the angle at* D. *Then making S K equal to* A B: *draw the subtendents* A K *and B S: and dividing the arch* A K *or B S equally in R, draw from the center the line H R. Then drawing Q X parallel to H P, and B L and G O to* A H, *&c.*

G Q *is the versed sine of the angle* A D E *as also of the arch* G X. *Therefore the arch* G X, *is the measure of the angle* A D E: *But* Q X *is the right sine of the arch* G X, *therefore* Q X *is also the right sine of the angle* A D E.

And seeing A S *is equal to the opposite side* A E, *and S K to* A B, *the difference of the conteining sides, therefore the whole arch* A K, *is equal to* A E *and* A B; *therefore the half thereof* A R, *is the summe of the halves of* A E *and* A B, *that is, of half the opposite side, and of half the difference of the conteining sides; the sine whereof is* A W. *And if the difference* A B, *be taken from the side* A E, *that is from* A S, *the remainer is B S, the half whereof is* B R: *so that if the half of* A B *be substracted from the half of* A E *or* A S, *the remainer is B R. And seeing* G N *is equal to* A D, G O *the sine of* G N, *is also the sine of* A D, *and* B C *is the sine of* D E *or* D B. *So that* B C *and* G O *are the sines of the conteining sides* A D *and* E D, *and* A W *and* B R *are the sines of the foresaid summe and remainer, and* G Y *the sine of half the angle at* D. *I say then that,*

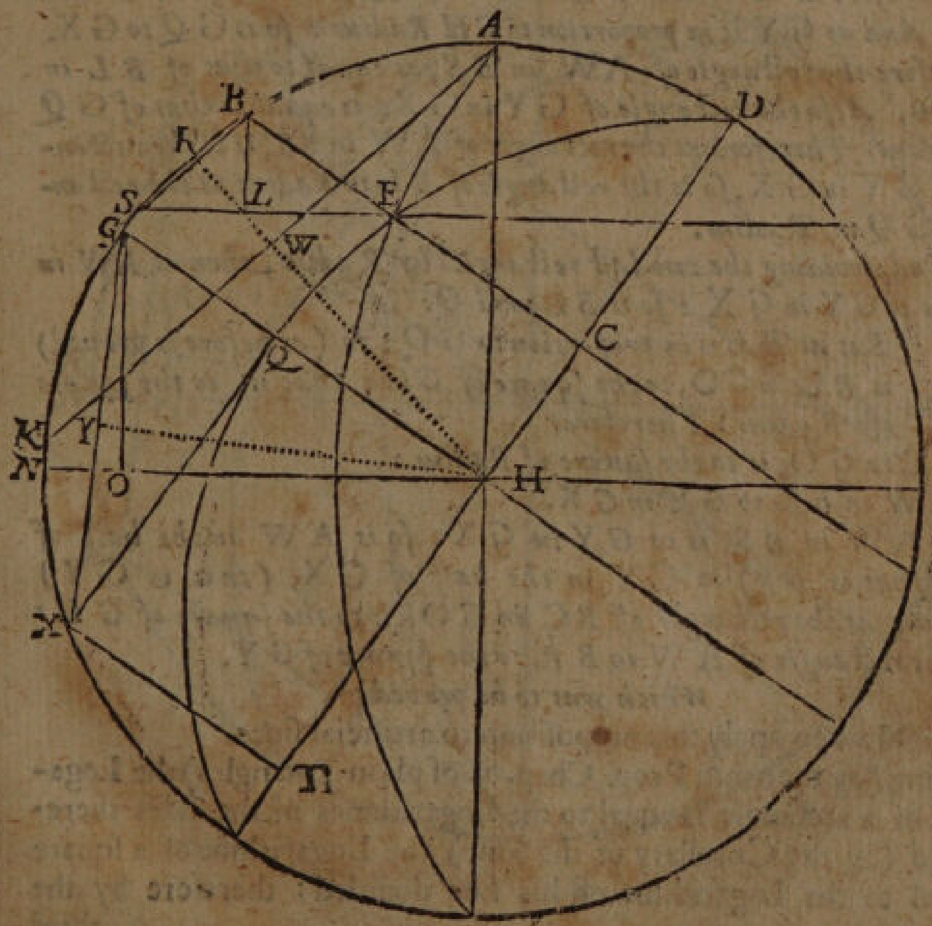
As

As the rectangle of the sines of the conteining sides A D *and* E D, *is to the square of Radius:*
So is the rectangle of the sines of the sum and remain: AR *and* BR, *to the square of the sine of half the angle* A D E, *namely, to the square of the sine of half the arch* G X.

That is,

As the rectangle of G O *and* B C, *is to the square of* G H, *so is the rectangle of* A W *and* B R, *to the square of* G Y.

Demonstr. *For as* G H, *the semidiameter of a great circle, is in proportion to* B C *the semidiameter of a lesser: so is* Q H *the sine of a certain arch in the greater, to* E C *the sine of the like arch in the lesse: and so is* G Q *the versed sine in the one, to* B E *the versed sine in the other. Which is more largely demonstrated by* Pitiscus, lib. 5. *and by others.*



Therefore

Therefore as G H *is in proportion to* B C, *so is* G Q *to* B E.

And as G H *is in proportion to* G O, *so is* B E *to* B L. *For the triangles* G O H *and* B L E *are equiangled. Therefore as the square of* G H, *is to the rectangle of* B C *in* G O: *so is the rectangle of* G Q *in* B E, *to the rectangle of* B L *in* B E.

And dividing the two last rectangles by B E, *then as the square of* G H, *is to* B C *in* G O: *so is* G Q *to* B L.

Or the Converse, namely,

As B C *in* G O, *is to the square of* G H: *so is* B L *to* G Q.

Again, seeing that A K *is parallel to* B S, *and* B L *to* A H: *therefore the angle* S B L, *is equal to the angle* H A W: *therefore the right angled triangles* S B L, *and* H A W, *are equiangled. Likewise seeing the right angled triangles* Y G H, *and* Q G X, *have the angle* Y G H *common to them both, therefore they are also equiangle.*

Therefore as A W *is in proportion to* A H *Radius: so is* B L *to* B S. *And as* G Y *is in proportion to* G H *Radius: so is* G Q *to* G X. *Therefore the rectangle of* A W *in* B S, *is equal to that of* B L *in Radius. Also the rectangle of* G Y *in* G X, *is equal to that of* G Q *in Radius. Therefore as the rectangle of* A W *in* B S, *is to the rectangle of* G Y *in* G X, *so is the rectangle of* B L *in Radius, to the rectangle of* G Q *in Radius.*

And dividing the two last rectangles by Radius, then as A W *in* B S, *is to* G Y *in* G X: *so is* B L *to* G Q.

But as B L *is in proportion to* G Q: *so (as before is proved) is* B C *in* G O, *to the square of* G H; *that is, to the square of Radius: Therefore,*

As B C *in* G O, *is to the square of Radius:*
so is A W *in* B S, *to* G Y *in* G X.

But as A W *in* B S, *is to* G Y *in* G X: *so is* A W *in the half of* B S, *(that is* B R*) to* G Y *in the half of* G X, *(that is* G Y*) therefore as the rectangle of* B C *in* G O, *is to the square of* G H: *so is the rectangle of* A W *in* B R, *to the square of* G Y.

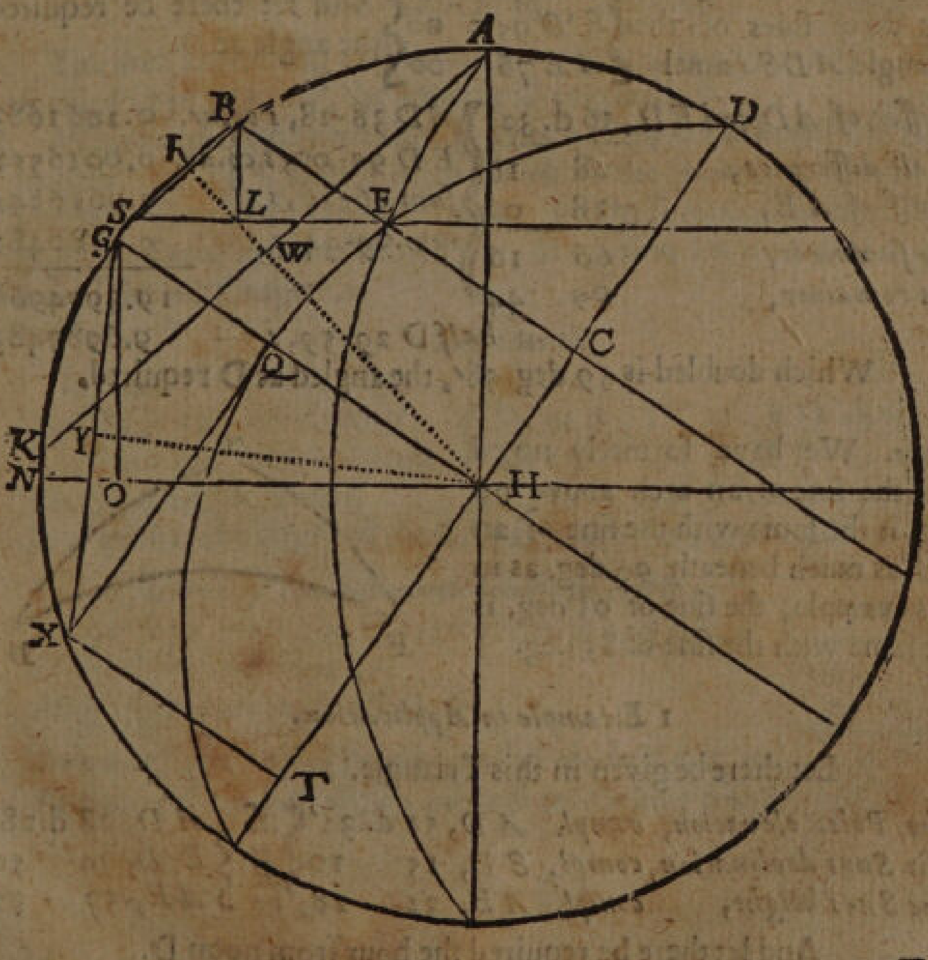
Which was to be proved.

Now to apply this proposition to artificial sines.

Seeing (by the ninth Prop. Chap. 2. of plain Triangles) the Logarithme of a rectangle is equal to the Logarithmes of the sides thereof: and (by the Corollary of the same) the Logarithme of a square is equal to the Logarithme of his side doubled: therefore by the third

third Prop. of the same Chap.) If unto the artificial sines of the foresaid sum and remainer, be added twice Radius; and from that total be substracted the sines of the conteining sides: half the remainer is the sine of half the conteined angle required. Or, (by the 4 Prop. of that Chapter) If in stead of substracting the sines of the conteining sides, we add their several Complements arithmetical, the total is more than the remainer would have been by twice Radius. Therefore leaving out twice Radius: if to the several Complements arithmetical of the sines of the conteining sides, be added the sines of the afore-said summe and remainer, half that total is the sine of half the conteined angle required.

This ground thus laid, we come to that two Cases thereon depending.



Case 11. *The three sides of a spherical Triangle being given: to find an angle.*

Take half the difference of the sides conteining the angle required, and add it to half the side opposite to that angle; and likewise substract it from the same, noting the summe and remainer.

Then to the complements arithmetical of the artificial sines of the conteining sides, add the artificial sines of the fore-said summe and remainer, and the half of that total is the artificial sine of half the angle required.

This being before proved, we proceed to Examples.

Let there be given the three sides of the triangle *ADE*, namely, { *AD* 38 d. 28, *ED* 95 00, *AE* 76 00 } And let there be required the angle at *D*.

Differ. of AD and ED,	56 d.	32′	*AD* 38-28, *s co.ar.*	0.2061683
Half difference,	28	16	*ED* 95-00, *s co.ar.*	0.0016558
Half of AE,	38	00	*sum* 66-16, *s*	9.9616244
the summe is,	66	16	*rem.* 09-44, *s*	9.2280481
the remainer,	09	44		19.3974966
			half D 29-59, *s*	9.6987483

Which doubled is 59 deg. 58′, the angled at D required.

Note. We have formerly noted that the sine of an arch above 90 deg. is the same with the sine of an arch as much beneath 90 deg. as in this example, the sine of 95 deg. is the same with the sine of 85 deg.

1 *Example in Application.*

Let there be given in this Triangle.

The Poles elevation,	*compl.*	*AD*,	51 d. 32′	That is,	*AD*,	38 d.	28′
The Suns declination,	*compl.*	*ED*,	15 10		*ED*,	74	50
The Suns height,	*compl.*	*AE*,	32 28		*AE*,	57	32

And let there be required the hour from noon D.

The

The difference of the sides A D *and* E D *conteining the angle required,*	*is* 36 deg.	22′
The half of that difference is	18	11
The half of the opposite side A E, *is*	28	46
The summe of the half difference and of the half side is	46	57
The remainer of the half diff. taken from the half side is	10	35

Which thus ordered, we resolve the Probleme thus.

The Poles elevation compl. A D	38 deg.	28′	*s co. ar.*	0.2061683
The Suns declinat. compl. E D	74	50	*s co. ar.*	0.0153967
The aforesaid summe,	46	57	*s*	9.8637737
The aforesaid remainer,	10	35	*s*	9.2640274
			Summe	19.3493661
	28 deg.	13′.	*The half*	9.6746830

The arch answering to this sine 9.6746830, is 28 deg. 13′, which doubled is 56 deg. 26′, the angle at D required.

Which converted into time is 3 ho. 45′ 44″, the hour from noon, namely, 14′ 16″, after 8 of the clock in the morning, or 45′ 44″, after 3 of the clock in the afternoon. In like sort may the hour of the night be found by some known star, as we have before touched at the end of the last Chapter.

2 *Example.*

Let there be given,

The Poles elevation, compl.	A D, 51 d, 32′	That is,	A D 38 deg.	28′
The Suns declination, compl.	E D, 15 10		E D 74	50
The Suns height, compl.	A E, 32 28		A E 57	32

And let there be required the Suns azimuth from the North *A*.

The difference of the sides conteining the angle required, namely, the difference of A D *and* A E,	*is* 19 deg.	04′
Half of that difference, is	09	32
Half of the opposite side E D, *is*	37	25
Summe of the half difference, and of the half side, is	46	57
Remainer of the half differ. taken from the half side, is	27	53

Which thus ordered, we resolve the Probleme thus.

Poles elevation, compl.	A D, 38 deg. 28′		*s co. ar.*	0.2061983
Suns altitude, compl.	A E, 57	32	*s co. ar.*	0.0738100
Aforesaid summe,	46	57	*s*	9.8637737
Aforesaid remainer,	27	53	*s*	9.6696420
			Summe	19.8136940
	53 deg. 48′		*the half*	9.9068470

The arch answering to this sine 9.9068470 is 53 deg. 48′, which doubled is 107 deg. 36′, the angle at A, which is the Suns azimuth from the North part of the Meridian.

Otherwise the operations in this Probleme may be thus ordered.

	d.	′			d.	′	The halfe	d.	′
Poles elevat. No.	51	32	*Co. of the poles elevat.*	A D	38	28		19	14
Suns altitude	32	28	*Co. of the Suns altitude*	A E	57	32		28	46
Suns declin. No.	15	10	*Compl. thereof*	E D	74	50		37	25
					difference			9	32
					summe			46	57
					remainer			27	53

Hence we resolve the probleme, as before.

	A D 38 deg. 28′		*s co. ar.*	0.2061683
	A E 57	32	*s co. ar.*	0.0738099
summe	46	57	*s*	9.8637737
remainer	27	53	*s*	9.6699420
				19.8136939
	53	48	*s*	9.9068469
which doubled, is	107	36, the Suns azimuth from (the North.		

Again, for the South declination.

	d.	′			d.	′	The halfe	d.	′
Poles elevat. No.	51	32	*Co. of the poles elevat.*	A D	38	28		19	14
Suns altitude	05	10	*Co. of the Suns altitude*	A E	84	50		42	25
Suns declin. So.	15	10	*Suns dist. from no. pole*	E D	105	10		52	35
					difference			23	11
					summe			75	46
					remainer			29	24

Hence

Hence we resolve the question thus.

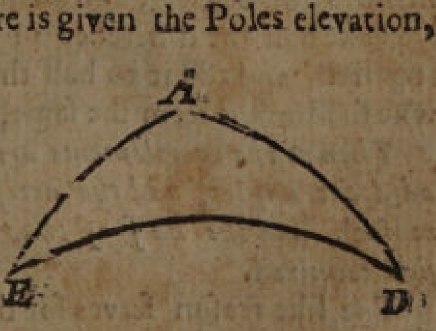
A D 38 deg.	28′	*s co. ar.*	0.2061683
A E 84	50	*s co. ar.*	0.0017682
summe 75	46	*s*	9.9864593
remainer 29	24	*s*	9.6909964
			19.8853022
61	13	*s*	9.9426511
which doubled, is 122	25, the Suns azimuth from (the North.		

Note. *And after the form of either of these examples, we may by the same things given, find the angle of the same position.*

3 *As if there were given, The Poles elevation, the Suns declination, and the Suns height: to find the angle of the Suns position.*

This eleventh Proposition is often used by Sea-men, especially the second Example, for finding the azimuth, whereby the variation of the Compasse may be known at Sea, after this manner.

About the middle of the forenoon or afternoon, the height of the Sun above the Horizon is to be taken by some instrument for that purpose, which being noted down, you are at the same instant (so neer as may be) to set the Sun with your Compasse (fitted for that purpose, the outward circumference of the Fly or Card divided into degrees, and the needle placed under the North and South points of the Card) and note down likewise upon what degree of the Compasse (reckoning from the North) you found the Sun. Then knowing by your former observations and reckoning, your latitude, and by your tables for that purpose the Suns declination, there is given the Poles elevation, the Suns declination, and the Suns height above the Horizon, whereby, according to the second example last before going, you may find the Suns true azimuth in degrees and minutes from the North; which compared with the degrees before found by the Compasse, if both agree;



the Compasse hath no variation; if there be any difference, that difference is the variation. Which variation, whether it be easterly or westerly, wefterly, may be known by the rule before given upon the twelfth Cafe of the third Chapter of right angled Triangles.

As in the fecond example laft before going. Admit that at the fame inftant when I obferved the height of the Sun in the morning to be 32 deg. 28′, I fet the Sun by my Compaffe, and found it to be from the eaft point towards the fouth 12 degrees, that is, from the north 102 degrees. But the Suns true azimuth from the north found by calculation, is 107 deg. 36′, the difference between thefe two is 5 deg. 36′, which is the variation of the Compaffe.

But to know whether this variation be eafterly or wefterly, I confider that by the Suns true azimuth found by calculation, the Sun fhould have been from the north 107 deg. 36′, that is, from the eaft point of the Compaffe to the fouthwards 17 deg. 36′. Whereas fetting it with my Compaffe, it was from the eaft to the fouthwards but 12 deg. So that the degree whereon the Sun fhould have been, was more toward the right hand than the degree whereon it was: therefore I affirm the variation to be eafterly 5 deg. 36 minutes. By the fame Probleme may the variation of a Needle be found on the land.

Cafe 12. *The three angles of a fpherical Triangle given: to find a fide.*

This is performed by the laft Axiome, the angles being converted into fides, and the fides into angles, (as we have fhewed Chap. 1. of fpherical Triangles) taking in ftead of the greateft angle his complement to 180 degrees.

Wherefore having taken in ftead of the greateft angle his complement to 180 degrees, and all things elfe remaining as before.

Take half the difference of the angles that are adjacent to the fide required, and add it to half the angle oppofite to that fide; and likewife fubftract it from the fame, noting the fum and remainer.

Then to the complements arithmetical of the artificial fines of the adjacent angles, add the artificial fines of the forefaid fum and remainer, and the half of that total is the artificial fine of half the fide required.

The like reafon ferves for this, as for the laft Cafe before going. We come therefore to examples.

Let there be given the three angles of the triangle A D E, namely,	A 107 d. 36′, that is	72 d. 24′	And let there bee required the fide E D.
	D	56 26	
	E	37 55	

Differ.

Differ. of E *and* D *is,*	18 d. 31′	D 56-26	*s co. ar.*	0.0792283
The half difference,	09 15½	E 37-55	*s co. ar.*	0.2114677
The half of A, *is*	36 12	*sum* 45-27½	*s*	9.8529314
The summe	45 27½	*rem.* 26-56½	*s*	9.6561780
The remainer	26 56½		*summe*	19.7998054
		52-35	*half*	9.8999027

Which doubled is 105 deg. 10 min. the complement whereof to 180 deg. is 74 deg. 50 min. which is the side required, E D.

1 *Example in application.*

Let there be given.

The Suns azimuth,	A. 107 deg. 36′, or 72 deg.	24	A.
The hour from noon	D, *in degrees* 56	26	D.
The angle of position,	E 37	55	E.

And let there be required the Suns height, being the complement of *A E.*

The difference of the adjacent angles A *and* E, *is*	34 d.	29′
The half of that difference is	17	14½
The half of the angle D, *opposite to the side required, is*	28	13
The summe of the half difference and half angle, is	45	27½
The remainer of the half differ. taken from the half angle, is	10	58½

Then for the resolution of this Probleme.

The Suns azimuth A	72 deg. 24′	*s Compl. ar.*	0.0208202
The angle of position E	37 55	*s Compl. ar.*	0.2114677
The foresaid summe	45 27½	*s*	9.8529314
The foresaid remainer	10 58½	*s*	9.2796227
		summe	19.3648420
	28 46	*half*	9.6824210

Which doubled is 57 deg. 32 min. the side *A E,* the complement whereof 32 deg. 28 min. is the height of the Sun required.

And after the form of this example the same things being given: namely, the Suns azimuth, the hour from noon, and the angle of the Suns position being given: we may find 2 The Suns declination (as in the former example) 3 Or the Poles elevation.

Note. *Although in the conversion of angles into sides, you may alwayes (as is aforesaid) take in stead of the greatest angle, his complement to 180 deg. yet you are not so to do of necessity, for you may take the complement of one of the lesser angles, to 180 degrees: As*

Let

Let there be given the three angles of the triangle A D E, namely, { A 107 d. 36′; D 56 deg. 26′, or 123 34; E 037 55 } And let there be required (as before) the side E *D*.

Differ. of E *and* D, *is*	85 deg.	39′			
The half difference	42	49½	D, 56-26	*s co.ar.*	0.0792283
The half of A *is*	53	48	E, 37-55	*s co.ar.*	0.2114677
The summe,	96	58½	*sum.* 83-22½	*s*	9.9971562
The remainer,	10	37½	*rem.* 10-58½	*s*	9.2796227
					19.5674749
			37-25		9.7837374

Which doubled is 74-50, *the side* ED *required*

He hath another way very little inferiour to the former, for the solution of the two last Cases, which Mr. *Gunter* makes use of, As if *Three sides be given to find an angle.*

Add the three sides together, noting half that summe, and from that half, substract the side opposite to the angle required, and note the remainer. Then,

As the rectangle of the sines of the conteining sides,
is to the square of Radius :
so is the rectangle of the sines of the foresaid sum and remainer,
to the square of a sine; whose arches complement doubled is the angle sought.

By conteining sides, we mean the sides conteining the angle required.

Therefore working by artificial sines,

Add to the complements arithmetical of the sines of the conteining sides, the sines of the foresaid sum and remainer, half that total is the sine of an arch, whose complement doubled, is the angle sought.

Let the example be here as before, namely,

Let there be given { A E 57 d. 32′, and let there be required the angle at D.

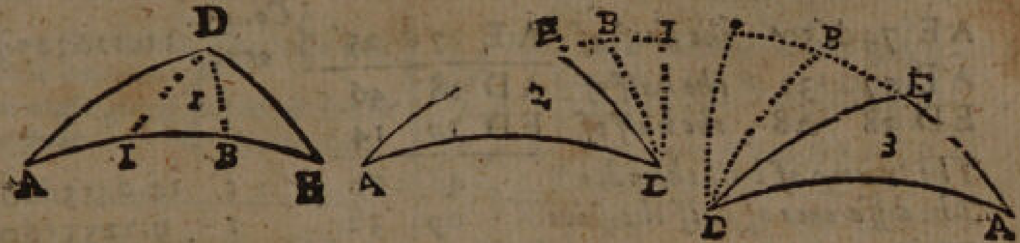
A D	38	28	*s Compl. arith.*	0.2061683
E D	74	50	*s Compl. arith.*	0.0153967
	170	50		
The half summe	85	25	*s*	9.9986090
The remainer	27	53	*s*	9.6699420
				19.8901160

The complement of this sines arch is 28 *deg.* 13′. 9.9450580
Which doubled is 56 26. *the angle at* D. (*required.*

If

If the three angles be given to find a side, you may convert the angles into sides, &c. as before is shewed.

Although either of these two last axiomes are very sufficient for the solution of the two last Cases of an oblique sphericall triangle arithmetically; yet neither of them can so aptly be applyed instrumentally, We will therefore here set down the third axiome, which he hath to the same purpose.



The three sides of a triangle being given, and an angle required, let fall a perpendicular opposite to that angle, the side whereon that perpendicular fals we call, for distinction sake, the base, and the other two the sides: thus in every of these triangles A E *is the base,* A D *and* E D *the sides,* D B *the perpendicular,* B *being placed at the right angle, and* B I *alwayes made equal to* B E: *Thus in every of them* A E *being the true base,* A I *is the alternate base, whose end* I *is as far from the perpendicular* B *one way, as the end of the true base* E, *is from the perpendicular the other way. Which things thus conceived. I say,*

As the tangent of the true semibase given,
is to the tangent of half the summe of the sides; 1
So is the tangent of half the difference of the sides,
to the tangent of the alternate semibase.
That is,
As the tangent of the half of A E, 2
to the tangent of half the summe of A D *and* E D.
So is the tangent of half the difference of A D *and* E D,
to the tangent of the half of A I.

The demonstration whereof you may see in his second book of triangles.

Therefore adding the half of the true base A E, *to the half of the alternate base* A I; *the summe is* A B, *the base of the right angled triangle* A B D: *also the difference of the halves of* A E *and* A I, *is* E B, *the base of the other right angled triangle* E B D.

And thus in either of the right angled triangles A B D *and* E B D, *we have the base and hypothenusal, whereby at one other operation either of the angles opposite to the perpendicular, namely, the angle at* A, *or that at* E, *may be found by the* 13 *Case of right angled triangles. Therefore, the three sides being given, we may find an angle.*

As for example, in the first of these triangles, let there be required the angle at *A*, the three sides being given, namely,

A E 74 d. 50′	*the half of*	A E	37 d.	25′	*Co. ar. or sc.*		10.1163279
A D 57 32	*the half of*	A D	28	46			
E D 38 28	*the half of*	E D	19	14			
The summe of half the sides			48	00		*t*	10.0455626
The difference of half the sides			09	32		*t*	9.2251560
The half of A I			13	42		*t*	9.3870465
to which adding half of A E			37	25			
The summe is A B			51	07			

Secondly,

A B 51 deg.	07′	*the tangent of A B*	10.0934397
A D 57	32	*tan. compl. of A D*	9.8036296
A 37	55	*make sine compl. A*	9.8970693

And thus we have found the angle at *A* to be 37 deg. 55′, and in like manner we might have found any of the other angles.

Note. For the resolution of questions of this nature instrumentally; Mr. *Gunter* (an ingenious man in contriving and applying of Instruments) makes use of the right and versed sines, and so resolves them at two operations, and sometimes he useth the right sines onely, but then he hath three operations. Notwithstanding they may also be performed at two operations without versed sines, using onely the tangents, as we have here shewed.

Now, as we have before for right angled triangles, so we will here for oblique represent in a Table the operations used in every Case, by the view of which Table you may be directed in the resolution of any oblique sperical triangle.

Given	To finde	Dat.	Req.	*The Proportionality.*	Caſe
Two angles, and a ſide oppoſite to one of them given: to finde the ſide oppoſite to the other.		A E AD	ED.	sE, sAD, sA sED.	1
Two ſides, with an angle oppoſite to one of them given: to find the angle oppoſite to the other.		AD A ED	E.	sED sA, sAD, sE.	2
Two ſides with their contein-ed angle given: to finde	The third ſide.	AD A AE	ED	*Ra.* scA, tAD, tAB. ſum or remainer of A B & A E is E B scAB, scEB, scAD, scED.	3
	One of the other angles.		E	*Ra.* scA, tAD, tAB ſum or rem. A B & A E is E B sEB, sAB, tA, tE	4
Two angles and the ſide between them given: to finde	The third angle.	AD A D	E	*Ra.* scAD, tA tcBDA ſum or rem. B D A & D is B D E sBDA, sBDE, scA scE	5
	One of the other ſides.		ED	*Ra.* scAD, tA, tcBDA ſum or rem. B D A & D is B D E scBDE, scBDA, tAD, tED	6
Two ſides with one of their oppoſite angles given: to find	The third ſide.	AD A ED	*AE*	*Ra.* sc *A*, t*AD*, t*AB* sc *AD*, sc ED, sc *AB*, sc *EB* ſum or differ. *A B* & *E B* is *AE*	7
	Their conteined angle.		D	*Ra.* sc *AD*, t*A* tc*BDA* tc *AD*, tc *ED*, sc *BDA*, sc *BDE* ſum or differ. *BDA* & *BDE* is *D*	8
Two angles with one of their oppoſite ſides given: to finde	The third angle.	*AD* *A* *E*	D	*Ra.* sc *AD*, t*A*, tc*BDA* sc *A*, sc *E*, s*BDA*, s*BDE* ſum or differ. B*DA* & BDE is *D*	9
	The ſide between them		*AE*	*Ra.* sc *A*, t*AD*, t*AB* t*E*, t*A*, s*A*B, sEB ſum or differ. of *A*B & *E*B is *AE*	10
				The Equality	
Three ſides given; to finde	An angle.	AD ED AE	D	½AE + ½ dif. AD & ED is ſum F ½AE — ½ dif. AD & ED is rem. G c.a.sAD c.a.sED *Sum* sF *Rem.* sG ½ is s ½D	11
[T]hree angles given; to finde	A ſide.	A D E	*ED*	½A + ½ dif. E & D is ſumme F ½A — ½ diff. E & D is rem. G c.ar.sD c.ar.sE *Sum* sF *Rem* sG ½ is s.½ ED	12

This last Table might be proposed in other terms; as the first Case we might expresse thus *Co. ar.* *s*E + *s* A D + *s* A, makes E D, cutting off an unite or 1 in the first place toward the left hand, which thing being before sufficiently explained, we shall not here need to insist thereon.

Here I intended to conclude this work: but because the demonstration of the first fundamental axiome for spherical triangles, as it is delivered by the Lord of *Merchiston* is very brief, and by him applyed to another kinde of Logarithms, so that it may seem obscure, I have thought good here (though something out of place) to illustrate the same, first premising certain *Lemma's* serving to that purpose.

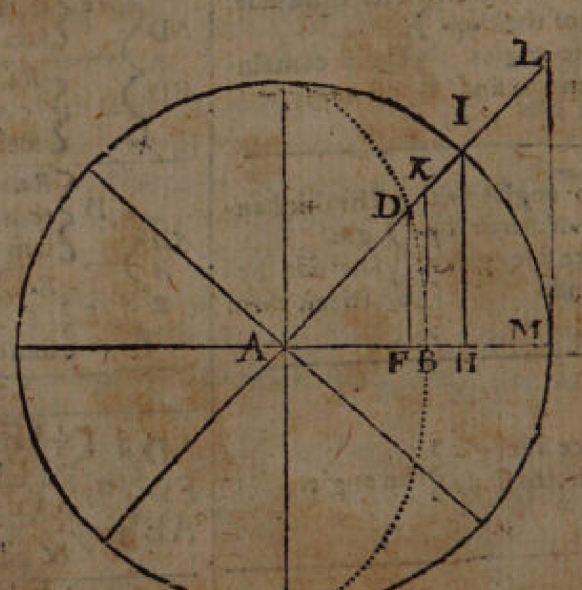
LEMMA. 1.

In a right angled spherical triangle.

As the sine of the base, is in proportion to Radius: so is the tangent of the perpendicular, to the tangent of the angle at the base.

As in this Diagram,

Let A D B represent a spherical Triangle, right angled at B; so that A D is the sine of the hypothenusal, A B the sine of the base, and D B is the perpendicular.



Then is D A B the angle at the base, and I H the sine, and L M the tangent thereof. Also D F is the sine, and K B the tangent of the perpendicular D B.

I say,

I say then,
As A B *the sine of the base,*
is in proportion to A M *Radius :*
so is B K *the tangent of the perpendicular,*
to M L *the tangent of the angle at the base.*

LEMMA 2.

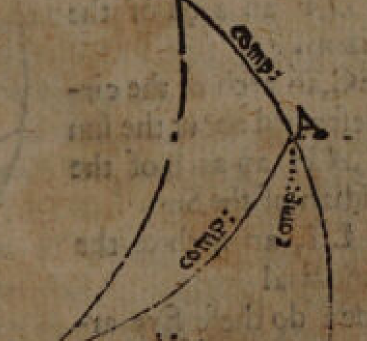
In a right angled spherical Triangle,
As the sine of the hypothenusal, is in proportion to Radius : so is the sine of the perpendicular, to the sine of the angle at the base.

That is, in the fore-going figure,
As A D *the sine of the hypothenusal,*
is in proportion to A I *Radius.:*
so is D F *the sine of the perpendicular,*
to I H *the sine of the angle at the base.*

These two *Lemma's* might be demonstrated in this Diagram, but because the same in effect are at large demonstrated by *Lansbergius*, *Pitiscus*, *Snellius*, and others, we let that passe.

LEMMA 3.

The circular parts of a right angled Triangle, are the same with the circular parts of a quadrantal Triangle adjoyning.



As let A B D be a Triangle right angled at B : and let one of the sides thereof, namely, A B, be extended till it become a quadrant, that is to G, and draw an arch from G to D. Then is G A D a quadrantal Triangle, adjoyning to the right angled Triangle A B D. I say therefore that the circular parts of the quadrantal Triangle GAD, are the same with the circular parts of the right angled Triangle ABD. For the circular parts of either of them are as here appeareth.

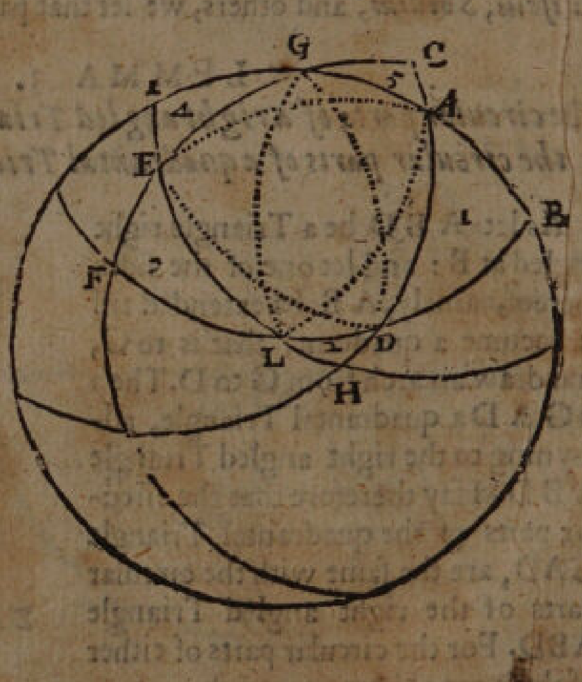
The five circular parts of the triangle. ABD, are AB. DB *com.* BDA. *com.* AD. *com.* A.
GAD, are *com.* AG. AGD. GDA. *com.* AD. *com.* A.

Where it is evident, G B and G *D* being quadrants G *D* B is a right angle, and *D* B is the measure of the angle at G: so that the side A B in the one is equal to *compl.* A G in the other; and the side *B D* in the one, equal to the angle A G *D* in the other, and *compl.* B *D* A in the one is G *D* A in the other, and *compl.* A *D* in the one is the same with *compl.* A *D* in the other; and lastly, that *compl.* *A* in the one, is the same with *compl.* *A* in the other, for the complement of the acute angle *D A* B unto a quadrant, is also the *compl.* of the obtuse angle G *A D*.

LEMMA 4.

If five circles of the sphere be so ordered, that the first intersect the second, the second the third, the third the fourth, the fourth the fifth, and the fifth, the first, at right angles: the right angled triangles made by their intersections, do all consist of the same circular parts.

As in this Scheme,



Let G represent the Zenith, *A* the north pole and D the Sun being in the Horizon. So that I G B is an arch of the Meridian of the place.

B *D* F an arch of the Horizon.

FEC, an arch of the circle described about the sun

C *A* H, an arch of the Meridian of the Sun.

H L I, an arch of the Equinoctial.

Then do these five arches retein the conditions required.

The first intersecting the second in B; the second, the third in F; the third, the fourth in C; the fourth, the fifth in H; and the fifth, the first in I. And these intersections at B, E, C, H, I, are at right angles;

angles; therefore I say the right angled triangles made by the inter-sections of these circles, namely, *A B D*, *D H L*, *L F E*, *E G I*, and *G C A*, do all consist of the same circular parts, for the circular parts in every of them are, as here appeareth,

The 5	ABD, *are*	AB,	BD,	*com.* BDA,	*com.* AD,	*com.* DAB
circular	DHL, *are*	*com.* HLD,	*com.* LD,	*com.* LDH,	DH.	HL
parts in the	LFE, *are*	*com.* ELF,	LF,	FE,	*com.* FEL,	*com.* EL
trian-	EIG, *are*	IG,	*com.* IG*E*,	*com.* G*E*,	*com.* G*EI*,	*EI*
gle.	GCA, *are*	*com.* GA,	*com.* AGC,	GC,	CA,	*com.* CAG

Where you may observe, that to the side A B in the first triangle, is equal *compl.* H L D in the second, or *compl.* *E L F* in the third, or I G in the fourth, or *compl.* A G in the fifth. In like sort, to the side D B in the first triangle, is equal *compl.* L D in the second, the side L F in the third, *compl.* I G *E* in the fourth, or *compl.* A G C in the fifth: And the like is to be seen in the rest, taken in such order as they are placed.

To expresse this more plainly: *A* B, the poles elevation in the first triangle, is the complement of the angle H L D in the second, or the complement of the angle *E L F* in the third, or the side I G in the fourth, or the complement of the hypothenusal G *A* in the fifth. And the like is to be understood of the rest.

The same uniformity of the circular parts is also apparent in quadrantal triangles.

As in the same scheame G from D, D from *E*, *E* from *A*, *A* from L, and L from G, are distant by arches each equal to a quadrant. But the arches G *A*, *A* D, D L, L *E*, and *E* G, are not quadrants. Here are therefore five quadrantal triangles G *A* D, *A* D L, D L *E*, L *E* G, and *E* G *A*: whose circular parts are as here appeareth.

The 5	GAD, *are*	*com.* AG,	AGD,	GDA,	*com.* AD,	*com.* DAG
circular	ADL, *are*	ALD,	*com.* LD	*com.* ADL,	*com* AD,	DAL
parts in	DL*E*, *are*	*com* DL*E*,	*com.* LD,	*E*DL,	D*E*L,	*com.* L*E*
the tri-	LEG, *are*	GL*E*,	LG*E*,	*com.* *E*G,	*com.* L*E*G,	*com.* L*E*
angle.	*E*GA, *are*	*com.* AG,	*com.* *E*GA,	*com*, *E*G,	G*E*A,	GA*E*

where you may observe that the circular parts in every of them remain the same unchangeable. And not onely in these ten triangles, but in all others which do arise of the other intersections of these ten arches drawn forth to whole circles: which because they are many and confused, we here let them passe, this being sufficient for our purpose.

1 Funda-

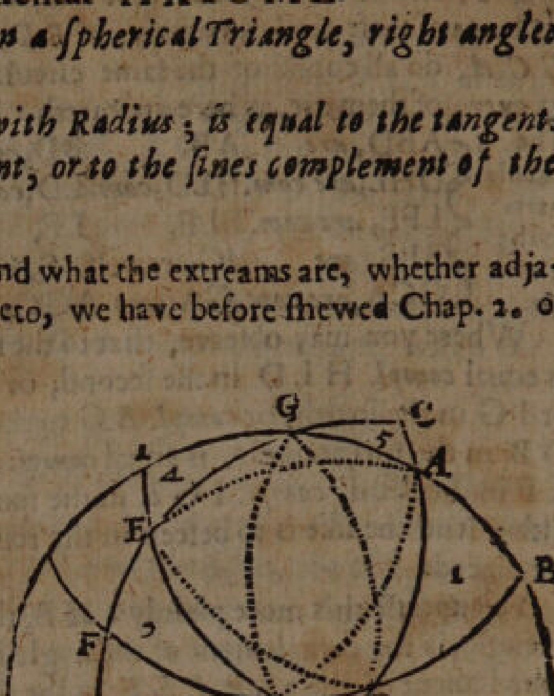
1 Fundamental AXIOME.

Of the five circular parts in a spherical Triangle, right angled or quadrantal.

The sine of a middle part with Radius; is equal to the tangents of the extreams adjacent, or to the sines complement of the opposite extreams.

WHat a middle part; and what the extreams are, whether adjacent or opposite thereto, we have before shewed Chap. 2. of spherical Triangles.

Part 1. Now touching the first part of this axiome in right angled Triangles; The middle part is either one of the sides, or one of the oblique angles, or the hypothenusal.



Case 1. Let the middle part be a side; as in the Triangle *ABD*, let *AB* be the middle part, and *DB*, and compl. *A* the extreams adjacent; then I say, that the sine of *AB* with Radius, is equal to the tangent of *DB*, with the tangent of the complement of *DAB*.

For (by the first Lemma) as the sine of *AB*, is in proportion to Radius: so is the tangent of *DB*, to the tangent of the angle at *A*; therefore also alternately, as sine *AB*, to tangent *DB*: so Radius to tangent *A*.

But (by the Corollary of the first Theoreme of the fourth Chapter of plain triangles) Radius is a mean proportional between the tangent of an arch, and the tangent of the complement of the same arch; so that as Radius, is to tangent *A*, so is tang. compl. *A*, to Radius: therefore as *s AB*, to *t DB*: so is *tc A* to Radius: therefore (by the Corollary of 3 Prop. Chap. 2. of plain triangles) *s AB* + Radius, is equal to *t DB* + *tc A*.

Case 2.

Case 2. Let the middle part be an angle; as in the triangle D H L; let *compl.* H L D be the middle part, and H L and *compl.* L D the extreams adjacent, then I say, that the sine complement of H L D, with Radius; is equal to the tangent of H L, with the tangent of the complement of L D.

For (by Lemma 4) compl. H L D is equal to A B, and compl. L D to D B, and H L to compl. D A B, and here before we have proved, that *s* A B + Radius, is equal to *t* D B + *tc* A, therefore also *sc* H L D + Radius, is equal to *tc* L D + *t* H L.

Case 3. Let the middle part be the hypothenusal; As in the Triangle G C A, let complement A G be the middle part, and complement A G C, and complement C A G, the extreams adjacent; Then also I say, *sc* A G + Radius, is equal to *tc* A G C + *tc* C A G.

For we have before proved, that *s* A B + Radius, is equal to *t* D B + *tc* A, but (by the 4 Lemma) complement A G is equal to A B, and compl. A G C to D B, and compl. C A G to compl. D A B, therefore also *sc* A G + Radius, is equal to *tc* A G C + *tc* C A G.

Therefore, in a right angled Triangle, the sine of a middle part with Radius, is equal to the tangents of the extreams adjacent.

I say further, that

Part 2. *The sine of a middle part with Radius: is equal to the sines complement of the opposite extreams.*

For here also the middle part is either one of the sides, or the hypothenusal, or one of the oblique angles.

Case 1. Let the middle part be a side. As in the triangle *A* B D, let D B be the middle part, and compl. *A D*, and compl. *A* the opposite extreams. Then I say, that the sine of B D with Radius, is equal to the sine of *A D* with the sine of *A*.

For (by Lemma 2) as *s A D* to Radius: so *s* D B to sine *A*, therefore (by Coroll. 3. Prop. 2. Chap. of plain Triangles) *s* D B + Radius, is equal to *s A D* + *s A*.

Case 2. Let the hypothenusal be the middle part. As in the triangle D H L, let compl. L D be the middle part, and D H and H L the opposite extreams, then I say, that *sc* L D + Radius; is equal to *sc* D H + *sc* H L.

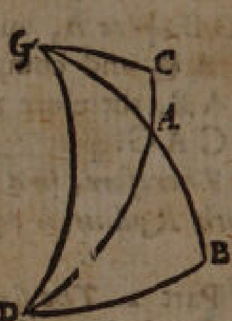
For compl. L D is equal to D B, and D H is equal to compl. A D; and H L to compl. D A B, (by the 4 Lemma) therefore, &c.

Case 3. Let one of the oblique angles be the middle part. As in the triangle E I G let compl. I G E be the middle part. Then I say, that *sc* I G E + Radius, is equal to *s* G E I + *sc* E I. For compl. I G E is equal to D B, and G E I is equal to A D, and E I to compl. D A B.

Therefore in a right angled triangle, *The sines of a middle part with Radius, is equal to the sines compl. of the opposite extreams.*

And seeing (by the third Lemma) the circular parts of a right angled triangle, are the same with the circular parts of the quadrantal triangle adjoyning; therefore, that which is here proved touching right angled triangles, is also true of quadrantals. *Therefore in a spherical triangle, right angled or quadrantal, &c.* Which was to be proved.

The same might also have been demonstrated in this Dgiagram without the fourth Lemma before going, but because that fourth Lemma is of singular invention, and of it self worthy to be k own, I have chosen rather to follow herein the invention of the noble Authour and Inventour of th s Prop. and of that third and fourth Lemma than otherwise.



And thus have we shewed the resolution of plain and spherical Triangles by this late invention of *Logarithms*, not excluding the wayes formerly used by natural sines, tangents, and secants; but delivering the rules in such sort, as they may be applyed to either. What hath been largely handled by others, I have lightly passed over; other things I have more insisted upon. In all I have endeavoured so much brevity, as might stand with perspicuity. Now touching the application hereof, I doubt not but he that is exercised in the Mathematicks, will be able to apply it divers wayes, especially to those parts wherein he is conversant; yet for their help that are but newly entered, I hope to do something in that kinde hereafter, as it shall please God to give opportunity. To whom alone is due all glory in all things.

FINIS.

AN APPENDIX.

Touching the application of the Doctrine of Triangles in the three principal kinds of sayling.

MY intent was here to have annexed a Treatise of *Navigation*, and especially of such points therein as have reference to the *Doctrine of plain and spherical Triangles.* Being the rather thereunto induced, because I had my first breeding in Mathematical Studies and practices at Sea: whereby I stand the more indebted, as to that excellent *Art*, so to the worthy Professours and Practisers thereof. But wanting time for the accomplishing of that according to my desire, by reason of my necessary absence and imployment far from home all this Summer, I have here, in stead thereof, shewed the resolution of certain Problemes, touching the three principal kinds of Sailing.

Questions of sailing by the plain or ordinary Sea-Chart.

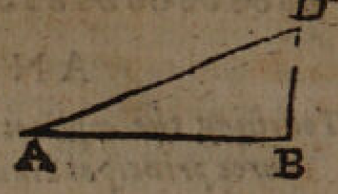
Although the ground of the projection of the ordinary Sea-Chart being false, (as supposing the Earth and Sea to be a plain Superficies) and so the conclusions thence derived must also for the most part be erroneous: yet because it is most easie, and much used, and the errours in small distances not so evident, we will not wholly neglect it.

Quest. 1. *Sailing 100 leagues upon the sixth Rumbe: how much shall I alter my parallel or Latitude?*

Note. The angle that any point of the Compasse makes with the Meridian, we call the Rumbe: but the angle that it makes with any parallel, we call the complement of the Rumbe.

And forasmuch as to every point of the Compasse there answers 11 deg. 15′, therefore the sixth Rumbe from the Meridian, (namely, *e ne*, *e s e*, *w s w*, or *w n w*) makes an angle therewith of 67 deg. 30′, whose complement 22 deg. 30′, is the angle of the same Rumbe with every parallel.

Now admit I sail from *D*, to *A*. *ene* 100 leagues; I demand the difference of latitude *DB*.



By the third Case of plain Triangles.

As Radius,		
to the distance run:	*AD* 100 leagues	2.00000
so sine compl. the Rumb,	*s A* 22 deg. 30′	9.58284
to the difference of latitude,	*DB* $38\frac{27}{100}$ leagues	1.58284

In like manner you may find the difference of latitude for any distance run upon any other point of the Compasse.

2 *Sayling* 100 *leagues upon the sixth Rumb: how far am I departed from the meridian of the place from which I came?*

That is by the same things, as before I demand *A B*.

By the third Case of plain Triangles.

As Radius,		
to the distance run,	*AD* 100 leagues,	2.00000
so is the sine of the Rumb,	*s D* 67 deg. 30′	9.96562
to the departure from the Merid.	A *B* $92\frac{39}{100}$ leagues,	1.96562

3 *Sailing upon the sixth Rumb, till I alter my latitude one deg. I demand how far I have sailed?*

As sailing from *D* to A, *ene*, till the difference of latitude *D B* be 20 leagues; I demand the distance run *A D*.

Say by the second Case of plain Triangles.

As, sine compl. the Rumb,	*s* A 22 deg. 30′ *co. ar.*	0.41716
to the difference of latitude;	*D B* 20 leagues	1.30103
so is Radius,		
to the distance run,	*AD* $52\frac{26}{100}$ leagues	1.71819

The like question might be moved by the departure from the Meridian given.

4 *Sailing*

4 *Sailing upon the sixth Rumb, till I have altered my latitude one degree: how much am I departed from my first Meridian?*

As sailing from D to A, *e n e*, till the difference of latitude D B be 20 leagues; I demand A B, my departure from the Meridian.

By the first Case of plain Triangles.

As Radius,		
to the difference of latitude;	D B 20 leagues,	1.30103
so is the tangent of the Rumb,	t D 67 deg. 30′,	10.38278
to the departure from the Merid.	A B $48\frac{28}{100}$	1.68381

In like manner by the departure from the Meridian given, you might find the difference of latitude.

5 *Sailing upon some Rumb, between the north and east $52\frac{1}{4}$ leagues; and finding that I have altered my latitude one degree: I demand upon what point I have sailed?*

As if I sail from D to A, (being some Rumb between the East and North) $52\frac{1}{4}$ leagues, and then find the difference of latitude D B, to be 20 leagues; I demand the angle A D B.

Say by the sixth Case,

As the distance run,	D A $52\frac{1}{4}$ leag. *co. ar.*	8.28191
is to Radius:		
so is the difference of latitude,	D B 20 leagues	1.30103
to sine compl. the Rumb,	A 22 deg. 30′	9.58294

Whose complement D 67 deg. 30′ is the sixth point from the Meridian, namely, *e n e*. Here we neglect some part of a minute, (as in these things not to be regarded) and so in other places.

6 *Sailing upon some Rumb between the North and the East $52\frac{1}{4}$ leagues; and finding that I have altered my latitude one degree: I would know my departure from my first Meridian.*

By

By the seventh Case.

To the distance run, add the difference of latitude, and also substract it from the same, noting the *summe* and *remainer*. Then add together the Logarithms of this summe and remainer, and half the total is the Logarithm of the distance from the first Meridian.

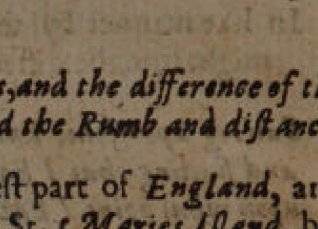
Distance run D A, $52\frac{1}{4}$ leagues	Summe $72\frac{1}{4}$ leagues.	1.85884
Differ. of latit. D B, 20 leagues	Remain $32\frac{1}{4}$ leagues.	1.50853
		3.36737
Departure from the Meridian A B, $48\frac{28}{100}$ leagues		1.68368

The same may be otherwise found by the same Case.

And in like sort might the difference of latitude be found, the departure from the Meridian being known.

7. *The distance of the meridians of two places, and the difference of the latitudes of the same places being given, to find the Rumb and distance.*

As let A represent the *Lizard* in the west part of *England*, and A B the parallel thereof, and let D represent St. *Maries Island*, being one of the *Azores*, D B the Meridian thereof.



Then is A B, the distance of the *Lizard* from the meridian of St. *Maries*, which let be 272 leagues; and D B the distance of their parallels, or difference of their latitudes 256 leagues. I demand the Rumb: namely, the angle at D, and the distance in the Rumbe A D.

First, for the Rumbe, say by the fourth Case.

As the difference of latitude, D B 256 leagues, *com. ar.*	7.59176
is in proportion to Radius :	
so is the distance of the merid. A B 272 leagues,	2.43457
to the tangent of the Rumbe, *t* D 46 deg. 44′.	10.02633

Which is the fourth Rumbe from the meridian and 1 deg. 44′ more, which shews the course from St. *Maries*, to the *Lizard*, to be Northeast 1 deg. 44′ easterly: or from the *Lizard* to St. *Maries* Southwest, 1 deg. 44′ westerly. And thus it should be by the plain Chart.

Secondly,

Secondly, for the distance A D ; *say by the second Case.*

As the sine of the Rumb, *s* D 46 deg. 44′. *co. ar.* 0.13776
to the distance of the Meridians A B 272 leagues, 2.43457
so is Radius,
to the distance of the places A D 373½ leagues, 2.57233

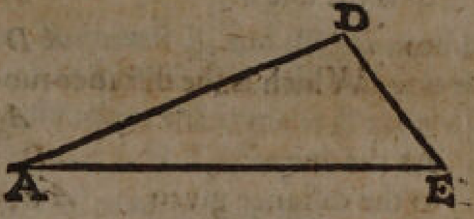
Or otherwise,

As sine compl. the Rumbe, *s* A 43 d. 16 *co. ar.* 0.16406
to the difference of latitudes : D B 256 leagues, 2.40824
so is Radius,
to the distance of the places, A D 373½ leagues, 2.57230

And such should be the distance by the plain Chart.

8 *Sailing away* w s w, *I see a point of land, which I set, and find to bear from me* w *by* n ; *and having sailed six leagues further, I find it bears from me* n w *by* w : *I would know how far it is distant.*

As let E be a point of land, which when the ship is at A, I set & find to bear from me *w* by *n*, but I hold on my course from A to D *w s w* 18 miles, and at D, I set the same point of land again, and find it to bear from me *n w* by *w* : I demand the distance thereof D E, that is, how far it was from me in my last observation.



First, I consider that between A E the *w* by *n*, and A D the *w s w* is 3 points of the Compasse, that is 33 deg. 45′, which is the angle at A : also between *E A*, the *e* by *s*, and E D the *s e* by *e* are two points, that is 22 deg. 30′.

Therefore by the 8 *Case of plain Triangles.*

As sine the angle at the point seen, *s* E 22 deg. 30′ *com. ar.* 0.41716
is to the distance run A D 18 miles, 1.25527
so sine the angle at the first place of observation, *s* A 33 deg. 45′, 9.74474
to the distance of the point seen E D 26⅛ miles, 1.41717

Whereby it appears, that the distance of the point seen from the place of your last observation is 26 miles, and a furlong. In like manner you may find the distance thereof from the place of your first observation *A*.

Admit

Admit the course from the Lizard to St. Maries be s w, *the distance* $373\frac{1}{2}$ *leagues. A certain ship bound from the Lizard to St. Maries steers away* s s w, *and afterwards* w *by* s, *and so sometimes upon one of these points, sometimes upon the other, till she arrives at St. Maries, now I demand how many leagues she hath sailed upon one of these points, and how many upon the other?*

Let *A* be the *Lizard*, *E St. Maries*, and seeing *s s w* being from *s w* two points, makes an angle therewith of 22 deg. 30′, which let be *A*; also *w* by *s* makes with *s w* an angle of 33 deg. 45′, which let be *E*; also *s s w* makes with *w* by *s* an angle of 56 deg. 15′, which let be the complement of *D* to 180 degrees.

Therefore by the 8 Case.

As the sine of	*D*, 56 deg. 15′ *compl. ar.*	0.08015
to the distance given	*A E*, $373\frac{1}{2}$ leagues,	2.57113
so is the sine of	*E*, 33 deg. 45′,	9.74474
to	*A D* $248\frac{2}{7}$ leagues.	2.39602

Which is the distance run upon the *s s w* point.

Again,

As the sine of	*D*, 56 deg. 15′. *co. ar.*	0.08015
to the distance given	*A E*, $373\frac{1}{2}$ leagues	2.57113
so is the sine of	*A*, 22 deg. 30′.	9.58284
to the way run	*E D*, $171\frac{44}{107}$	2.23412

Which is the distance run upon the *w* by *s* point.

10. *A Merchant man, being in the latitude of 43 degrees, fals into the hands of Pyrats; who amongst other things take away his sea-compasse. But when he is gotten cleer, he sails away as directly as he can, and after two dayes meets with a man of war; who also had been the day before in the latitude of 43 deg. and had sailed thence* s e *by* s *37 leagues: He desirous to find these Pyrats, the Merchant man tels him, he left them lying to & fro where they took him, and he had sailed since at least 64 leagues, between the south and west: what course shall the man of war shape to find these Pyrats?*

Let *A E* be the parallel of 43 deg. *D* the place where the ships meet. Then is there given *A D* 64 leagues, *E D* 37 leagues, and the angle *D E A* five points or 56 deg. 15′.

Therefore

Therefore by the 9 Case of plain Triangles.

As the distance run by the Merchant man,	A D 64 leagues,	*co. ar.*	8.19382
to sine the angle given:	*s* 56 deg. 15′,		9.91985
So is the distance run by the man of war,	E D 37 leagues,		1.56820
to sine an angle required,	*s* A 28 deg. 44′,		9.68187

That is *w s w* 6 degr. 14′ southerly, and so hath the Merchant man, failed; therefore to return to the same place he must shape his course *e n e* 6 degrees 14′ northerly.

11 *There are two ports lying* n e, *and* s w *one of another, a ship sails from the westermost of these ports* e s e, *47 leagues; another departing from the eastermost port sails 66 leagues, and then meets with the former; what course hath this second ship kept, and how far are these ports asunder?*

Let the Northeast port be A, the Southwest E, and the place where these ships meet at D; and forasmuch as from E to A, the course is *n e*, and from E to D East South East: therefore the angle at E is 67 deg. 30′; and the side E D, 47 leagues; and A D, 66 leagues.

Therefore by the 9 Case of plain Triangles.

As A D, 66 leagues,	*co. ar.*	8.18046
to sine E, 67 deg. 30′		9.96561
so E D, 47 leagues,		1.67210
to sine A 41 deg. 08′		9.81817

And seeing from A to E, the course is Southwest, and from A to D 41 d. 08′ more Southerly: therefore the course from *A* to *D*, is South 3 deg. 52′ westerly.

Secondly, for the distance of these ports A E, the angle at A, being 41 deg. 08′, and the angle at E 67 deg. 30′; the summe of them both is 108 deg. 38′, which substracted from 180 deg. leaves the angle at D 71 deg. 22′.

Therefore by the 8 *Case of plain Triangles.*

As sine E 67 deg. 30′,	*co. ar.*	0.03439
to A D, 66 leagues:		1.81954
so sine D 71 deg. 22′;		9.97662
to A E 67$\frac{7}{10}$ leagues.		1.83055

So that the distance between the two ports is 67$\frac{7}{10}$ leagues.

Some may think it requisite, that the latter part of this probleme should have been a distinct Case in plain Triangles: but because the same things are here given as in the 9 Case, and the operation manifest by the 8 and 9, I thought it not necessary to make another Case of it.

12 *Costing along towards the evening, I have sight of a Cape or head-land, beyond which I desire to steer in the next morning; it bears from me* S S E, *and is distant by estimation* 11 *leagues; but I steer away South, till two of the clock in the morning, about* 12 *leagues; and then would know how the Cape bears from me, and how far it is off?*

As admit at A I observe the Cape D to bear from me *s s e* 11 leagues; but I steer away south, to *E* 12 leagues. I have then A D 11 leagues, A *E* 12 leagues, the angle at A 22 deg. 30′.

First then for the angle at E by the 10 *Case.*

As A *E* + A *D*, 23 leagues,	*compl. ar.*	8.63828
to A *E* + A *D*, 01 league,		
so $t\frac{1}{2}(E+D)$ t 78 deg. 45′,	*t*	10.70134
to tang. an angle F, 12 deg. 20′,		9.33962
Which substracted, there remains } *E*, 66 deg. 25′,		

In working this example, because the angle given A is 22 deg. 30′, therefore the other two *E* and *D* are 157 deg. 30′ (by the 1 Lemma of the 3 Chapter of plain Triangles) the half whereof is 78 deg. 45′, whereby we find an angle at *F*, 12 deg. 20′, which substracted from 78 d. 45′, there remains the angle at *E* 66 d. 25′. Wherefore seeing *E* A is a North line, *E D* is almost *e n e*, namely, *e n e* 1 d. 5′ northerly.

Secondly, for the distance of the Cape E D *by the* 8 *Case.*

As sine the angle found,	*s* E 66 d. 25′ *co. ar.*	0.03788
to the distance in the evening:	A D 11 leagues,	1.04139
So the sine of the angle given,	*s* A 22 deg. 30	9.58284
to the distance in the morning	E D $4\frac{6}{10}$ leagues,	0.66211

That is above 4 leagues and an half distance.

13 *Admit*

13 *Admit I sail away from a certain port* s s w 50 *leagues, & thence again* w *by* s 30 *leagues; upon what point have I made my way good, and how far am I come from that port?*

As admit I sail from A to D *s s w* 50 leagues, and from D to E *w* by *s* 30 leagues, there is required the course A, or E, & distance A E. From the *s s w* to the *w* by *s*, are five points, that is 56 d. 15′, which is the complement of the angle at D, to 180 deg. So that the angle at D, is 123 d. 45′. Wherefore here are given the two sides A D and E D, and their conteined angle at D: Therefore,

As A D + E D 80 leagues,	*co. ar.*	8.09691
to A D — E D 20 leagues,		1.30103
so *t* ½ (A + E) *t* 28 d. 08′,		9.72810
to *t* F 07 d. 37′		9.12604
Which substracted, there remains	A 20 deg. 31′.	

Wherefore seeing the course from A to D is *s s w*, the course from A to E is 20 d. 31′ more westerly, that is *s w* two deg. southerly; so that I have made my way good *s w* two deg. southerly.

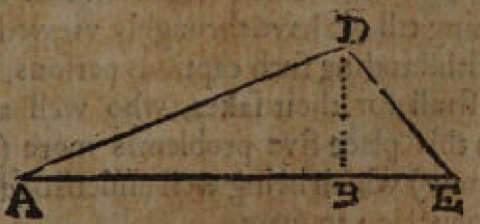
Secondly, for the distance upon that point.

As sine the angle found,	*s* A 20 deg. 31′ *co. ar.*	0.45534
to his opposite side given:	E D 30 leagues,	1.47712
so sine the angle given,	*s* D 56 deg. 15′	9.91985
to his opposite side required,	A E 71½ leagues,	1.85231

Which is the distance from that port.

14 *There are two ports in one and the same parallel or latitude, distant 64 leagues, and there is a certain Island more southerly, distant from the Eastermost of these ports 47 leagues, and from the westermost of them 34 leagues: I demand the course, from the Eastermost port to that Island?*

Let the Eastermost port be A, the westermost E, both in one and the same parallel A E, distant 64 leagues; and let the Island be D, distant from A 47 leagues, and from E 34 leagues,



there is required the course from A to D, that is the angle at A, or the complement thereof.

By the 12 Case of plain Triangles.

As the distance of the ports A E 64 leagues.		co. ar.	8.19382
to the summe of A D and E D, 81 leagues			1.90848
so is the difference of A D and E D 13 leagues			1.11394
to a certain line	A I 16 $\frac{454}{1000}$		1.21624
which added to A E is	80 $\frac{454}{1000}$		
the half whereof is	A B 40 $\frac{227}{1000}$		

Then by the 6 Case of plain Triangles.

As A D 47 leagues;	compl. arith.	8.32790
to Radius		
So A B 40 $\frac{227}{1000}$		1.60452
to sc A, s 58 degrees 51′		9.93242

That is Southwest and by West 2 degrees 36′ westerly, which is the course from the Eastermost port of the Island.

15 *A ship sails from one port to a second* SSE *76 leagues, and from thence to a third 54 leagues, and from that third to the first 85 leagues. I demand the course from the second port to the third, and from the third to the first?*

This and the like are to be wrought as the former, which therefore we leave to your own practice.

Some (as I have understood) who do little of themselves, but carp at others, and yet borrow of them: blame me for setting down so many problemes; but he that knows how to number aright the questions that might be moved, knows that I leave untouched a far greater number of the same kind than those I handle, for I desire not to be tedious at any time. Yet he that learns no more than needs must, will never be able to learn all that; for we do not fully understand a thing till we have throughly viewed it on every side. Therefore notwithstanding such captious persons, as take offence without just cause; I shall for their sakes, who well accepted my former labours, add in this place five problemes more (not handled by any other that I know) which being well understood and considered, may be as an introduction to many.

First,

First, then it is to be understerstood, that a ship sailing to wind-ward will (as I remember) usually lye within 5½ points of the wind (if it be something more or lesse it matters not) yet by reason of her Leeward way, she will scarce make her way good within 6¾ points of the wind, but sometimes more, sometimes lesse, according as the Sea is rougher or smoother, and according to the mould of the ship, and sail she bears: So that in sailing to a place directly to wind-ward, she sails usually three or four times the distance of that place, before she arrive at it. But if the place to which she sails be not directly to wind-ward, but within a point, two, three, four, five or six points of the wind, then though she turn to wind-ward, as before; yet she will sooner arrive at the place than before: but how, and in what proportion, for the one and for the other, may appear by these ensuing problemes. As,

16 *Let the position from A to B (in this next figure) be South* 100 *miles; and the wind at South, and admit the ship intending to sail from A to B, make her way good within* 75 *deg.* 31′ *of the wind (which is almost* 6¾ *points of the Compasse) I demand how far she must sail upon one tack, and how far upon the other, before she arrive at B?*

Then A C being the ships way, so neer the wind as she can make her way good, the angle B A C is 75 deg. 31′, whereto is equal the angle A B C (for A C and B C crosse the south line AB alike, or at equal angles) the summe of these two angles at A and B is 151 degr. 02′, which substracted from 180 deg. leaves the angle at C 28 degr. 58′; and thus we have the angles of the Triangle A B C, and the side A B 100 miles: Say then,

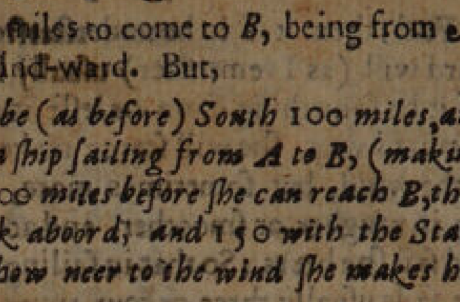
As sine the angle C 28 deg. 58′, *compl. arith.*	0.31488
to the side A B 100 miles,	2.00000
so *s* the angle A 75, 32	9.98597
to the side B C 200 miles *ferè*	2.30085

Whereto is equal the side A C (because the angles at A and B are equal) therefore I say, she must sail with her Larboord tack aboord 200 miles (whether at one or many boords) and as much with her Star-

Star-boord tack aboord, in all 400 miles to come to *B*, being from *A* onely 100 miles, but directly to wind-ward. But,

17. *Let the position from A to B be (as before) South* 100 *miles, and the wind at South, and admit a ship sailing from A to B, (making one or many boords) doth run* 300 *miles before she can reach B, that is* 150 *with the Lar-boord tack aboord; and* 150 *with the Star-boord tack aboord. I demand how neer to the wind she makes her way good?*

Let fall from *C* a perpendicular to *AB*, namely *AD*, which divides the side *AB* into two equal parts, so that *AD* and *DB* are each 50 miles, and *AC* and *BC* each 150 miles; therefore in the right angled Triangle *ACD*, Say,

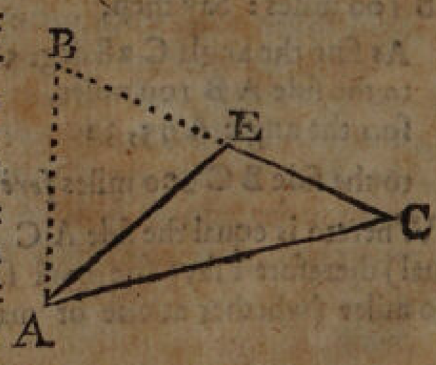


As the side *AC* 150	*compl. arith.*	7.82391
to Radius :		
so the side *AD* 50		1.69897
to *s* the angle *DCA* 19 deg. 28′		9.52288

Whose compl. 70 deg 32′ is the angle *DAC*, shewing that she makes her way good within 70 deg. 32′ of the wind.

18 *Let the distance from A to E, in this next Diagram, be* 100 *miles Southwest, the wind at South; and let the ship make her way good within* 70 *deg.* 32′ *of the wind: I demand the distances AC and CE, that is the ships way by dead reckoning upon the one tack and upon the other.*

Then *AB* being a South line (or point upon which the wind is) the angle *BAC* is 70 deg. 32′, from which substracting *BAE* 4 points, or 45 deg. there remains the angle *EAC* 25 deg. 32′. Again, the complement of 70 deg. 32′ is 19 deg. 28′, which doubled is 38 deg. 56′, the angle at *C*, and adding these two, namely 25 deg. 32′, and 38 deg. 56′ the Sum



is 64 deg. 28′, the outward angle at *E*. Thus have we the angles of the Triangle *A E C*, and the side A *E* 100 miles. Say then,

As *s C* 38 d. 56′ *co. ar.*	0.20175	As *s* C 38 d. 56′ *co. ar.*	0.20175
to A *E* 100 m.	2.00000	to A E 100 m.	2.00000
so *s E* 64 28	9.95537	so *s* EA C 25 d. 32′	9.63451
to A *C* 143 6/10	2.15712	to E C 68 6/10	1.83626

Thus it appears, that to sail from A to E, which is southwest 100 miles with the wind at South, and making her way good within 70 deg. 32′ of the wind, she must sail with her Lar-boord tack aboord neer 143 6/10 miles, and with her Star-boord tack aboord neer 68 6/10 miles: In all 212 2/10 miles, that is neer *w s w* a quarter point westerly 143 6/10 miles, and *e s e* a quarter of a point easterly 68 6/10 miles.

19 *Let the distance from A to E be* 100 *miles Southwest, the wind at South: a ship sails from A to C, so neer the wind as she can by* 143 6/10 *miles, and from C to E* 68 6/10 *miles: I demand how neer the wind she makes her way good?*

Here from A C	143. 6	
substract	*E* C	68. 6
there rests	*E* B	75 miles.

Then say,

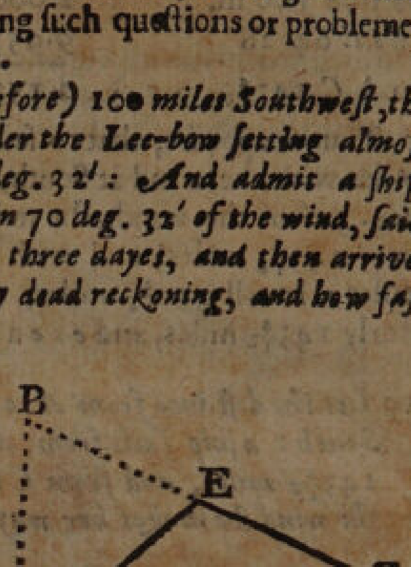
As the side *E B* 75 miles,	*compl. arith.*	8.12494
to *s* the angle *B A E* 45 deg. 00′,		9.84948
so is the side *A E* 100 miles,		2.00000
to *s* the angle at *B* 70 deg. 32′		9.97442

Whereto is equal the angle *B A C*, whence I conclude, that she makes her way good within 70 deg. 32′ of the wind, so that the wind being at South, she makes her way good upon the one tack neer *w s w* 1/4 point westerly, and upon the other *e s e* 1/4 point easterly.

I have about 22 years past, in my book entituled, *The Sea-mans Practice*, shewed (first I think of any man, though some have touched upon it since) the resolution of about a dozen questions touching *Currents*,

Currents, I have added onely this one in this place, leaving the rest (which are many) that might be moved (touching a ship sailing in a Current, or out of a Current, by a wind or large) to our ingenious young men at sea, for their exercise at their leisure; having elswhere opened cursorily the method for framing such questions or problemes, in subjects mathematical, or otherwise.

Let the distance from A to E be (as before) 100 miles Southwest, the wind at South, and a Current under the Lee-bow setting almost e s e, *namely, South easterly 70 deg. 32′: And admit a ship, which will make her way good within 70 deg. 32′ of the wind, sails close by the wind from A towards C three dayes, and then arrives at E: I demand how far she sails by dead reckoning, and how fast that Current sets?*



Here then the angle B A C is 70 deg. 32′, from which substracting B A E 4 points, or 45 deg. there remains the angle E A C 25 deg. 32′. And seeing the Current sets according to the line C E B, East southeast easterly, namely, South easterly 70 deg. 32′, therefore the angle at B, is also 70 deg. 32′, to which adding B A C, the summe is 141 deg. 04′, which taken from two right angles, or 180 deg. leaves the angle A C B or A C E 38 deg. 56′, to which adding E A C 25 deg. 32′, the sum is the outward angle B E A 64 deg. 28′. And thus we have the three angles of the Triangle A E C, and the side A E 100 miles, whereby we may find A C the distance run by dead reckoning, and C E the drift of the Current, in manner following.

As *s* ACE 38 d. 56′ *co.ar.*	20175	As *s* ACE 38 d. 56′ *co.ar.*	20175
to the side A E 100 m.	2.00000	to the side A E 100 m.	2.00000
so *s* B E A 64 d. 28′	9.95537	so *s* E A C 25 32	9.63451
to the side A C $143\frac{6}{10}$	2.15712	to the side C E $68\frac{6}{10}$	1.83626

Which $14\frac{6}{10}$ is the distance run by dead reckoning.

Which $68\frac{6}{10}$ miles is the drift of the Current in three dayes, which is almost a mile an hour, namely, $\frac{95}{100}$ of a mile, or 95 centesmes of a mile hourly.

Again,

Of

Of Sayling by Mercators Chart.

And thus much of the plain chart, which as it hath this commodity that it is most easie: so it hath some discommodities intollerable. For there be very few places that can therein be expressed according to their true situation and distance one from another. Which as it is a great impediment in the practice of Navigation; so it hath caused much confusion in the *Geographical* and *Hydrographical* descriptions of places, insomuch as there are scarce extant any descriptions of the World, or the parts thereof, that are not pestered with notorious errours: the greatest part of them hence arising. It is indeed ancient, and till the Sea Compasse was known, it was the aptest Chart that could be used, because till then men were coasters, and for the most part returned back the same way they went forth. And it may still serve without any great errour, in such places as are neer the Equinoctial, also in many other places for short voyages, and even for long voyages, provided that a man be sure to return the same way that he went, or neer the same. Otherwise if he trust to the plain Chart, he will be most grossely deceived many times in his course a point or two of the Compasse, and in his distance many hundred miles. But in this Sea-Chart called *Mercators*, all or any parts of the World may be set down, according to their longitudes, latitudes, courses, and distances, as truly and far more conveniently for the Mariners use then upon the Globe it selfe. So that it will truly shew the direction, and distance from place to place, which way soever a man goes or returns.

Some men will say, that in divers reckonings by *Mercators* chart, they have found as little certainty as by the plain chart. Which I deny not, but the reason is, because there are few or no charts made directly according to this projection. It will be said, yes, there are many; and that a man may have of them whensoever he will bespeak them. I grant a man may have those which are so called; but that which is such indeed, must not only have the meridians, parallels, and rumbes drawn according to this projection; but the sea-coasts must be inserted by the like art and means as they have formerly been inserted into the common sea-chart; otherwise he that shall transfer places out of the common sea-chart into *Mercators*, without due knowledge and respect upon what occasion, or for what reason they were so placed in the common sea-chart, he shall trasfer the errours of the one into the other, and that sometimes with increase. Wherefore it requires more than an ordinary judgment, to draw a plot directly according to this projection, for any place or places; and he must further know, or be made acquainted with the reckonings of Mariners frequenting those places; and that truly whether with allowance or without, and whether agreeing or disagreeing with their plots; and so comparing one thing with another, and weighing all in the ballance of a good judgment, he shall be able to do it. The ground of the projection of this kind of charts was pointed at by *Ptolomy*, many hundred years since; and according to that ground, *Mercator* did of late years set forth an universal map of the World, whereupon these have been called ***Mercators Charts.*** But the way how to describe them was first taught by that learned Navigatour of our times M. *Ed. Wright*, in his book of the *Corrections of errours in Navigation.* From whence also the ground and reasons of these ensuing problemes are to be taken: and if we would be as grateful to our own Countreymen as to strangers, I see not but we may ascribe as much to him in this as to any other man. Now that which he hath shewed to perform by the chart it self, we will here shew to work by the doctrine of plain triangles; using the help of his table of Latitudes: of which, as M. *Gunters* Table for the division of the meridian Line is an abridgment; consisting of the quotients of every sixth number, divided by 6, and two figures cut off: so this which I here exhibit, and call a Table of Meridional parts, is also an abridgment of that Table of M. *Wrights*; namely every sixth number, cutting off 4 figures. So that this Table sheweth how many parts every degree and every tenth part of a degree of latitude in this chart, is from the Equinoctial: namely, of such parts as a degree of the Equinoctial conteines 60; he that desires a larger Table may use M. *Wrights* extant in his book before mentioned.

A

A Table of Meridional parts.

Lat. d. m.	Mer. parts	Lat. d. m.	Mer. parts	Lat. d. m.	Mer. parts	Lat. d. m.	Mer. parts	Lat. d. m.	Mer. parts	Lat. d. m.	Mer. parts
0 00	00	3 00	180	6 00	361	9 00	542	12 00	725	15 00	910
06	06	06	186	06	367	06	548	06	731	06	917
12	12	12	192	12	373	12	554	12	738	12	923
18	18	18	198	18	379	18	560	18	744	18	929
24	24	24	204	24	385	24	567	24	750	24	935
30	30	30	210	30	391	30	573	30	756	30	942
36	36	36	216	36	397	36	575	36	762	36	948
42	42	42	222	42	403	40	585	42	768	42	954
48	48	48	228	48	409	48	591	48	774	48	960
54	54	54	234	54	415	54	597	54	781	54	966
1 00	60	4 00	240	7 00	421	10 00	603	13 00	787	16 00	973
06	66	06	246	06	427	06	605	06	793	06	979
12	72	12	252	12	433	12	615	12	799	12	985
18	78	18	258	18	439	18	621	18	805	18	991
24	84	24	264	24	445	24	627	24	811	24	998
30	90	30	270	30	451	30	634	30	818	30	1004
36	96	36	276	36	457	36	640	36	824	36	1010
42	102	42	282	42	463	42	646	42	830	42	1016
48	108	48	288	48	469	48	652	48	836	48	1023
54	114	54	294	54	475	54	658	54	842	54	1029
2 00	120	5 00	300	8 00	482	11 00	664	14 00	848	17 00	1035
06	126	06	306	06	488	06	670	06	855	06	1042
12	132	12	312	12	494	12	676	12	861	12	1048
18	138	18	318	18	500	18	682	18	867	18	1054
24	144	24	324	24	506	24	689	24	873	24	1060
30	150	30	330	30	512	30	695	30	879	30	1067
36	156	36	337	36	518	36	701	36	886	36	1073
42	162	42	343	42	524	42	707	42	892	42	1079
48	168	48	349	48	530	48	713	48	898	48	1086
54	174	54	355	54	536	54	719	54	904	54	1092
3 00	180	6 00	361	9 00	542	12 00	725	15 00	910	18 00	1098

A Table of Meridional parts.

Lat. d.	Lat. m.	Mer. parts	Lat. d.	Lat. m.	Mer. parts	Lat. d.	Lat. m.	Mer. parts	Lat. d.	Lat. m.	Mer. parts	Lat. d.	Lat. m.	Mer. parts	Lat. d.	Lat. m.	Mer. parts
18	00	1098	21	00	1289	24	00	1484	27	00	1684	30	00	1888	33	00	2100
	06	1104		06	1296		06	1491		06	1690		06	1895		06	2107
	12	1111		12	1302		12	1497		12	1697		12	1902		12	2114
	18	1117		18	1308		18	1504		18	1704		18	1909		18	2121
	24	1123		24	1315		24	1510		24	1710		24	1916		24	2128
	30	1130		30	1321		30	1517		30	1717		30	1923		30	2135
	36	1136		36	1328		36	1524		36	1724		36	1930		36	2143
	42	1142		42	1334		42	1530		42	1731		42	1937		42	2150
	48	1149		48	1341		48	1537		48	1738		48	1944		48	2157
	54	1155		54	1347		54	1543		54	1744		54	1951		54	2164
19	00	1161	22	00	1354	25	00	1550	28	00	1751	31	00	1958	34	00	2171
	06	1168		06	1360		06	1557		06	1758		06	1965		06	2179
	12	1174		12	1367		12	1563		12	1765		12	1972		12	2186
	18	1181		18	1373		18	1570		18	1772		18	1979		18	2193
	24	1187		24	1380		24	1577		24	1778		24	1986		24	2201
	30	1193		30	1386		30	1583		30	1785		30	1993		30	2208
	36	1200		36	1393		36	1590		36	1792		36	2000		36	2215
	42	1206		42	1399		42	1596		42	1799		42	2007		42	2222
	48	1212		48	1406		48	1603		48	1806		48	2014		48	2230
	54	1219		54	1412		54	1660		54	1813		54	2021		54	2237
20	00	1225	23	00	1419	26	00	1616	29	00	1819	32	00	2028	35	00	2244
	06	1232		06	1425		06	1623		06	1826		06	2035		06	2252
	12	1238		12	1432		12	1630		12	1833		12	2043		12	2259
	18	1244		18	1438		18	1637		18	1840		18	2050		18	2266
	24	1251		24	1445		24	1643		24	1847		24	2057		24	2274
	30	1257		30	1451		30	1650		30	1854		30	2064		30	2281
	36	1264		36	1458		36	1657		36	1861		36	2071		36	2288
	42	1270		42	1464		42	1663		42	1868		42	2078		42	2296
	48	1276		48	1471		48	1670		48	1875		48	2085		48	2303
	54	1283		54	1477		54	1677		54	1881		54	2092		54	2311
21	00	1289	24	00	1484	27	00	1684	30	00	1888	33	00	2100	36	00	2318

A Table of Meridional parts.

Lat. d.	m.	Mer. parts	Lat. d.	m.	Mer. parts	Lat. d.	m.	Mer. parts	Lat. d.	m.	Mer. parts	Lat. d.	m.	Mer. parts	Lat. d.	m.	Mer parts
36	00	2318	39	00	2545	42	00	2782	45	00	3030	48	00	3292	51	00	3569
	06	2325		06	2553		06	2790		06	3039		06	3301		06	3578
	12	2333		12	2560		12	2798		12	3047		12	3310		12	3588
	18	2340		18	2568		18	2806		18	3056		18	3319		18	3598
	24	2348		24	2576		24	2814		24	3064		24	3328		24	3607
	30	2355		30	2584		30	2822		30	3073		30	3337		30	3617
	36	2363		36	2592		36	2830		36	3081		36	3346		36	3627
	42	2370		42	2599		42	2839		42	3090		42	3355		42	3636
	48	2378		48	2607		48	2847		48	3098		48	3364		48	3646
	54	2385		54	2615		54	2855		54	3107		54	3373		54	3656
37	00	2393	40	00	2623	43	00	2863	46	00	3116	49	00	3382	52	00	3665
	06	2400		06	2631		06	2871		06	3124		06	3391		06	3675
	12	2408		12	2638		12	2880		12	3133		12	3401		12	3685
	18	2415		18	2646		18	2888		18	3142		18	3410		18	3695
	24	2423		24	2654		24	2896		24	3150		24	3419		24	3705
	30	2430		30	2662		30	2904		30	3159		30	3428		30	3714
	36	2438		36	2670		36	2913		36	3168		36	3437		36	3724
	42	2446		42	2678		42	2921		42	3176		42	3447		42	3734
	48	2453		48	2686		48	2929		48	3185		48	3456		48	3744
	54	2461		54	2694		54	2938		54	3194		54	3465		54	3754
38	00	2468	41	00	2702	44	00	2946	47	00	3203	50	00	3475	53	00	3764
	06	2476		06	2710		06	2954		06	3212		c6	3484		06	3774
	12	2484		12	2718		12	2963		12	3220		12	3493		12	3784
	18	2491		18	2726		18	2971		18	3229		18	3503		18	3794
	24	2499		24	2734		24	2979		24	3238		24	3512		24	3804
	30	2507		30	2742		30	2988		30	3247		30	3522		30	3814
	36	2514		36	2750		36	2996		36	3256		36	3531		36	3824
	42	2522		42	2758		42	3005		42	3265		42	3540		42	3834
	48	2530		48	2766		48	3013		48	3274		48	3550		48	3844
	54	2537		54	2774		54	3022		54	3283		54	3559		54	3855
39	00	2545	42	00	2782	45	00	3030	48	00	3292	51	00	3569	54	00	3865

A Table of Meridional parts.

Lat. d.	m.	Mer. parts	Lat. d.	m.	Mer. parts	Lat. d.	m.	Mer. parts	Lat. parts		Mer. parts	Lat. d.	m.	Mer. parts	Lat. d.	m.	Mer. parts
54	00	3865	57	00	4183	60	00	4528	63	00	4905	66	00	5324	69	00	5795
	06	3875		06	4194		06	4540		06	4919		06	5339		06	5812
	12	3885		12	4205		12	4552		12	4932		12	5354		12	5829
	18	3896		18	4216		18	4564		18	4945		18	5369		18	5846
	24	3906		24	4227		24	4566		24	4959		24	5384		24	5863
	30	3916		30	4238		30	4588		30	4972		30	5399		30	5880
	36	3927		36	4250		36	4600		36	4986		36	5414		36	5997
	42	3937		42	4261		42	4613		42	4999		42	5429		42	5914
	48	3947		48	4272		48	4625		48	5013		48	5444		48	5932
	54	3958		54	4283		54	4637		54	5026		54	5459		54	5949
55	00	3968	58	00	4295	61	00	4650	64	00	5040	67	00	5475	70	00	5967
	06	3979		06	4306		00	4662		06	5054		06	5490		06	5984
	12	3989		12	4317		12	4674		12	5067		12	5505		12	6002
	18	4000		18	4329		18	4687		18	5081		18	5521		18	6020
	24	4010		24	4340		24	4699		24	5095		24	5537		24	6038
	30	4021		30	4352		30	4712		30	5109		30	5552		30	6056
	36	4031		36	4363		38	4725		36	5123		36	5568		36	6074
	42	4042		42	4375		42	4737		42	5137		42	5584		42	6092
	48	4052		48	4386		48	4750		48	5151		48	5600		48	6110
	54	4063		54	4398		54	4763		54	5165		54	5615		54	6128
56	00	4074	59	00	4409	62	00	4775	65	00	5179	68	00	5631	71	00	6147
	06	4085		06	4421		06	4788		06	5194		06	5648		06	6165
	12	4096		12	4433		12	4801		12	5208		12	5664		12	6184
	18	4106		18	4445		18	4814		18	5222		18	5680		18	6202
	24	4117		24	4456		24	4827		24	5237		24	5696		24	6221
	30	4128		30	4468		30	4840		30	5251		30	5712		30	6240
	36	4139		36	4480		36	4853		36	5265		36	5729		36	6259
	42	4150		42	4492		42	4866		42	5280		42	5745		42	6278
	48	4161		48	4504		48	4879		48	5295		48	5762		48	6297
	54	4172		54	4516		54	4892		54	5309		54	5779		54	6316
57	00	4183	60	00	4528	63	00	4905	66	00	5324	69	00	5795	72	00	6326

A Table of Meridional parts.

Lat. d.	m.	Mer. parts	Lat. d.	m.	Mer. parts	Lat. d.	m.	Mer. parts	Lat. d.	m.	Mer. parts	Lat. d.	m.	Mer. parts	Lat. d.	m.	Mer. parts
72	00	6336	75	00	6972	78	00	7746	81	00	8742	84	00	10141	87	00	12521
	06	6355		06	6995		06	7775		06	8780		06	10199		06	12638
	12	6375		12	7018		12	7804		12	8819		12	10258		12	12759
	18	6394		18	7042		18	7834		18	8859		18	10318		18	12884
	24	6414		24	7066		24	7864		24	8899		24	10379		24	13015
	30	6434		30	7089		30	7894		30	8939		30	10441		30	13150
	36	6454		36	7114		36	7924		36	8980		36	10504		36	13291
	42	6474		42	7138		42	7954		42	9021		42	10569		42	13438
	48	6495		48	7162		48	7985		48	9063		48	10634		48	13591
	54	6515		54	7187		54	8016		54	9105		54	10701		54	13752
73	00	6535	76	00	7211	79	00	804[illegible]	82	00	914[illegible]	85	00	10770	88	00	13920
	06	6556		06	7236		06	8079		06	9192		06	10839		06	14097
	12	6577		12	7261		12	8111		12	9236		12	10910		12	14284
	18	6598		18	7287		18	814		18	9280		18	10983		18	14481
	24	6618		24	7312		24	8176		24	9325		24	11057		24	14691
	30	6640		30	7338		30	8209		30	9371		30	11133		30	14914
	36	6661		36	7364		36	8242		36	94[illegible]7		36	11210		36	15153
	42	6682		42	7390		42	8275		42	9464		42	11290		42	15409
	48	6704		48	7416		48	8309		48	9512		48	11371		48	15686
	54	6725		54	7442		54	8343		54	9560		54	11454		54	15987
74	00	6747	77	00	7469	80	00	8377	83	00	9609	86	00	11539	89	00	16318
	06	6769		06	7495		06	8412		06	9659		06	11626		06	16683
	12	6791		12	7522		12	8447		12	9709		12	11716		12	17092
	18	6813		18	7550		18	8483		18	9760		18	11808		18	17556
	24	6835		24	7577		24	8518		24	9812		24	11902		24	18293
	30	6857		30	7605		30	8555		30	9865		30	11999		30	18729
	36	6880		36	7633		36	8591		36	9918		36	12099		36	19511
	42	6903		42	7661		42	8628		42	9973		42	12202		42	20524
	48	6925		48	7689		48	8666		48	10029		48	12308		48	21967
	54	6948		54	7717		54	8703		54	10084		54	12408		54	24499
75	00	6972	78	00	7746	81	00	8742	84	00	10141	87	00	12521	90	00	Infinit

The ufe of this Table fhall partly appear in the Problemes following, and may firft be illuftrated thus.

Prob 1. *To finde by this Table, what Meridional parts are conteined in any difference of Latitude.*

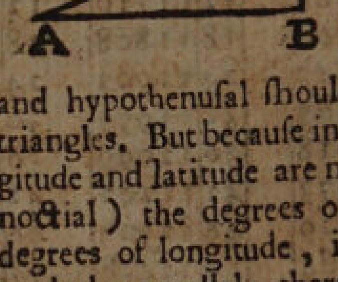
Take the meridional parts anfwering to each latitude, fubftract the leffer from the greater; the remainer is the number of meridional parts, conteined in the difference of latitude propofed.

As let the one latitude be	50 deg.	00′	3475	*Merid. parts.*
The other	32	25	2058	
The merid. parts conteined in the difference of latitude			1417	*Difference.*

Probl. 2. *The latitudes and difference of longitude of two places given: to finde the rumbe and diftance.*

To the intent the application may be the more evident, we will give examples of two places expreffed in the chart.

As admit the latitude of the *Lizard* to be 50 deg. 00′, the latitude of *Summers Ilands*, fometimes called the *Bermudas*, 32 deg. 25′, and the difference of longitude to be 70 deg. 00′; the *Summers Ilands* being fo much to the weftward of the *Lizard*: I demand the courfe and the diftance from the one to the other?



As in this right angled triangle *A D B*, Let *A* reprefent the *Lizard*, and *A B* the parallel thereof, *D Summers Ilands*, and *D B* the meridian thereof.

Then is there given *D B* the difference of latitude 17 deg. 35′, and A B, the difference of longitude 70 deg. 00′; whereby the angles and hypothenufal fhould be found, by the 4 and 2 cafes of plain triangles. But becaufe in this kind of projection, the degrees of longitude and latitude are not equal; (except in places neer the Equinoctial) the degrees of latitude at every parallel exceeding the degrees of longitude, in fuch proportion as the Equinoctial exceeds that parallel: therefore thefe differences of longitude and latitude muft firft be expreffed by fome one common meafure. And for that purpofe ferves the foregoing table, which fheweth how many ny equal parts are from the Equinoctial to every degree of latitude: namely, of such equal parts as a degree of longitude conteins 60′.

Wherefore multiplying 70 deg. 00′, the difference of longitude, by 60, I have 4200, for the meridional parts conteined in the difference of longitude; also (by the last probleme) I find the meridional parts conteined in the difference of latitude to be 1417; so that D B is 1417 parts, and A B 4200 such parts.

Therefore by the 4 Case of plain Triangles.

As the differ. of latitude in parts, D B 1417 parts.	*co. ar.*	6.84863
is in porportion to Radius:		
So is the differ. of longit. in parts, A B 4200 parts.		3.62325
to the tangent of the Rumb,	*t* D 71 deg. 21′,	10.47188

Which sheweth the course from the *Summers Ilands*, to the *Lizard* to be *e n e* 3 deg. 51′ easterly; or from the *Lizard* to the *Summers Ilands*, *w s w*, 3 deg. 51′ westerly.

Secondly, for the distance in the Rumb.

Reduce the difference of latitude into miles, (multiplying the degrees by 60, and to the product adding the minutes.)

Then by the 2 Case of plain Triangles.

As sine complement the Rumb,	*s* A, 18 deg. 39′ *co. ar.*	0.49514
to the difference of latitude:	D B, 1055 miles	3.02325
So is Radius		
to the distance	A D, 3299 miles	3.51839

Which is almost 1100 leagues, and this is the distance measured in the Rumb; there is a neerer cut between these two places, whereof we shall speak hereafter in *Great Circle sailing*; but here, whensoever we speak of the distance of two places, we mean their distance measured in their Rumb.

Probl. 3. *The latitudes of two places, and their distance given, to find the Rumb, and difference of longitude.*

Admit I sail from the *Lizard*, being in the latitude of 50 degrees, upon some point to the westward, 3299 miles; and then find my self in the latitude of 32 deg. 25′: I would know upon what point I have made my way good, and how much I have altered my longitude?

The difference of latitude D B is 17 deg. 35′, which reduced into miles is 1055 miles.

As the diſtance ſailed, is in proportion to Radius:	A D 3299 miles, *co. ar.*	6.48161
So is the differ. of latitude,	D B 1055 miles,	3.02325
to ſine compl. the Rumb,	*s* A 18 deg. 39′,	9.50486

That is *w s w* 3 deg. 51′ weſterly:

Secondly, for the difference of longitude.

Find by the firſt probleme what meridional parts are conteined in the difference of latitude, which are here 1417, then ſay,

As Radius		
to the differ. of latitude in parts:	D B, 1417 parts,	3.15137
So is the tangent of the Rumb,	*t* D, 71 deg. 21′,	10.47188
to the differ. of longitude of parts,	A B, 4200 parts,	3.62325

Which parts reduced into degrees, dividing them by 60, the quotient is 70 deg. the difference of longitude required.

Probl. 4. *By the Rumb, and latitude of two places given: to find their diſtance and difference of longitude.*

Admit I ſail from the *Lizard*, being in the latitude of 50 deg. *w s w* 3 deg. 51′ weſterly, till I find my ſelf in the latitude of 32 deg. 25′: I demand how far I have ſailed, and how much I have altered my longitude?

The diſtance is found as in the latter part of the ſecond probleme thus. The difference of latitude converted into miles is 1055 miles.

Say then,

As ſine compl. the Rumb,	*s* A, 18 deg. 39′ *co. ar.*	0.49514
to the difference of latitude,	D B, 1055 miles	3.02325
So is Radius		
to the diſtance,	A D, 3299 miles	3.51839

And ſo much is the diſtance: the difference of longitude may be found, as in the latter part of the third probleme; ſaying,

As Radius, to the difference of latitude in meridional parts: ſo is the tangent of the Rumb, to the difference of longitude in minutes.

Probl. 5. *By the difference of longitude, Rumb, and one latitude: to find the other latitude and the diſtance.*

Admit I ſail from the *Lizard*, being in the latitude of 50 d. *w s w* 3 deg. 51′ weſterly, tlll I have altered my longitude 70 deg. how much have I laid the pole, and how far am I from the *Lizard*?

Reduce the difference of longitude into minutes, by 60; and so it makes 4200; then say,

As the tangent of the Rumb,	*t* D, 71 d. 21′ *co. ar.*	9.52829
to the differ. of longitude in parts:	A B, 4200 parts	3.62325
So is Radius,		
to the differ. of latitude in parts	D B, 1417	3.15154

Now the meridional parts answering the latitude of 50 deg. 00′, are 3475, from which substracting 1417 here found, there remains 2058, against which I find in the first Column of the Table 32 deg. 25′; which is the latitude required of that other place to which I am come: so that the difference of latitude is 17 deg. 35′.

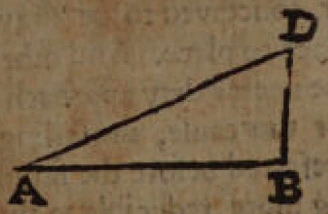
Secondly, for the distance.

Having already the Rumb, and difference of latitude, it may be found as in the second and fourth problemes; saying,

As sine compl. the Rumb,	*s* A 18 deg. 39′, *co. ar.*	0.49514
to the difference of latitude,	D B, 1055 miles	3.02325
So is Radius		
to the distance	A D, 3299 miles	3.51839

Probl. 6. *By the Rumb, the distance, and one latitude given: to find the other latitude, and the difference of longitude.*

Admit I sail *w s w* 3 deg. 51′ westerly, 3299 miles; and then find my self in the latitude of 32 deg. 25′: I demand the latitude of the place from which I came, and the difference of longitude between that and this?



First, for the difference of latitude.

As Radius,		
to the distance run:	A D 3299 miles,	3.51838
So sine compl. the Rumb,	*s* A 18 deg. 39′,	9.50486
to the difference of latitude,	D B 1055 miles,	3.02324

Which 1055 miles converted into degrees, is 17 deg. 35′, the difference of latitude required: which added to 32 deg. 25′, makes 50 deg. 00′ the latitude of the first place.

The difference of longitude is found as before in the third probleme; saying,

As Radius, to the difference of latitude in meridional parts,
so is the tangent of the Rumb, to the differ. of longit. in minutes.

And thus the difference of longitude will be found as in this example to be 70 deg. 00′.

If at any time you desire to convert this difference of longitude found in any parallel into miles, you may do it after this example.

7. Admit there be two places, both in the parallel of 50 deg. which differ in longitude 70 deg. 00′: I demand the distance of these two places?

First, it is to be understood, that the minutes of longitude in any parallel, are in proportion to the distance in miles; as the equinoctial is to that parallel; or as the semidiameter of the one is to the semidiameter of the other. That is,

As Radius is in proportion,		
to sine compl. the latitude;	sc 50 deg. 00′,	9.80807
So is the difference of longitude, 4200 minutes,		3.62325
to the distance in that parallel, 2700 miles,		3.43132

Problemes of sailing by a great Circle.

SEeing the superficies of the earth and sea is spherical, therefore the most absolute way of sailing is by the arch of a great circle, drawn or conceived to be drawn on the spherical surface of the sea from place to place. And other wayes are so much the better, by how much the neerer they approach thereto, or may be thereunto reduced. And for this cause, the sailing according to *Mercators* Chart, is to be preferred before the sailing according to the common Sea-Chart, being more reducible to the spherical Superficies of the Earth and Sea. Neither have I at any time said otherwise, whatsoever some in print would make me say without any assent or knowledge of mine. It will be said that there is some more difficulty in sailing according to *Mercators* Chart, than by the Common Sea-Chart; and somewhat more difficult sailing by the arch of a great Circle, than by either of them: No doubt, since the fall of man there are thorns and briars, or difficulties encountring our best endeavours: But truth and exactnesse, though joyned with some difficulty, is to be preferred before errour, though never so easie; alwayes endeavouring to make the way of truth as easie as we can. Therefore we come now briefly to shew the way of failing by the arch of a great Circle, by help of the *Doctrine of Spherical Triangles*, forasmuch as there is no way discovered to the world more absolute.

In the former problemes of sailing, whether by the plain Chart, or that called *Mercators*, we have used meridians, parallels, and rumbs, as the sides of every triangle. But here we use not the Rumbs so, because they are not Circles, but helispherical lines; nor the parallels, because they are not great Circles: whereas the sides of every spherical triangle must be arches of great Circles: But here we use arches of the meridians, and of the Equinoctial, and of other great Circles drawn, or imagined to be drawn from one place to another, upon the spherical superficies of the earth and sea. First, therefore,

If two places lie under the Equinoctial, their position is east and west, and the degrees of their difference of longitudes converted into leagues or miles, is their distance in leagues or miles.

If two places be in the same Meridian, their position is north and south, and the degrees of their difference of latitude, converted into leagues or miles, is their distance.

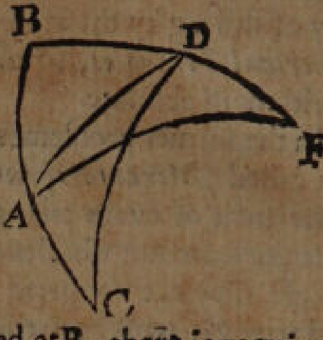
And thus far doth this kind of sailing agree with the two former; the difference between this and them may appear in the problemes following.

Probl. 1. *Two places being proposed, the one under the Equinoctial, the other in any latitude given: and the difference of the longitude of the same places being also known: to find,*

1. *Their neerest distance in a great Circle:*
2. *The direct position of the first place from the second:*
3. *And of the second place from the first.*

The angle that the Rumb leading from one place to another, makes with the meridians, is sometimes called the position of those places. But because the arch of a great Circle, drawn between two places, is the most direct way, and neerest distance from the one place to the other: therefore the angles which that arch makes with the meridians of those places we here call the angles of the *direct position* of those places one from another.

Now



Now in this Diagram, let D represent that part of the entrance of the river of *Amazones*, which lieth under the Equinoctial line; D B an arch of the Equinoctiall; and let A represent the *Lizard*, lying in the latitude of 50 deg. 00′ northerly, and A B the meridian thereof; and admit their difference of longitude D B to be 51 deg. 00′.

Then in this triangle A D B, right angled at B, there is required A D, the neerest distance of these places in the arch of a great Circle; the angle B A D, which is the angle of the direct position of the *Amazones* from the *Lizard*, and the angle B D A, being the complement of the angle of the direct position of the *Lizard* from the *Amazones*.

1 *For the neerest distance* A D. Seeing there are given the sides A B and D B: therefore by the first fundamental axiome of sphericall triangles.

sc A D + Rad. = *sc A B + sc D B*, therefore *sc A B + sc D B — Rad.* = *sc A D*, and so it fals into the 10 Case, thus.

The difference of longitude is	D B 51 deg. 00′,	*sc* D B	9.79887
The difference of latitude is	A B 50	00, *sc* A B	9.80807
The distance is	A D 66	08, *sc* A D	9.60694

Which 66 deg. 08′ converted into leagues is 1322⅔ leagues, which is the neerest distance between these two places.

2 *For the direct position from the Lizard to the Amazones*, namely, the angle B A D by the same things given.

s A B + *Rad.* = *t* D B + *tc* B A D; therefore *s* A B + *Rad.* — *t* D B = *tc* B A D, that is *s* A B + *tc* D B = *tc* B A D, abating Radius, and thus it falls into the 11 Case, and is wrought thus.

The difference of latitude is	A B 50 deg. 00′,	*s* A B	9.88425
The difference of longitude is	D B 51	00, *tc* D B	9.90837
The angle of position is	B A D 58	11, *tc* A	9.79262

3 *For the direct position from the Amazones towards the Lizard*, namely, the complement of the angle B D A.

s D B + Rad. = *t A B + tc D*, therefore *s D B + Rad.* — *t A B* = *tc D*, therefore *s D B + tc A B* = *tc D*, still abating Radius, and so it falls into the 11 Case, and is thus wrought.

The

The difference of longitude is D B 51 deg. 00′,	*s* D B,	9.89050
The difference of latitude is A B 50	00, *tc* A B,	9.92381
The angle of position is compl. B D A 33	07, *tc* A	9.81431

If you would have the Letters in all examples to agree with the exemplary Tables, you must mark your right angled Triangle two wayes, and the oblique six wayes, as we have before shewed; and it will not be amisse to do so, especially if you use those tables. But as I have before said, I would rather wish every man, to deduce his operations from the two fundamental Axiomes and their Consectaries, in such sort as I have here shewed in these three examples, for the like is to be conceived in all others, though it be not expressed. Yet I have set down those exemplary Tables for all the Cases in all kinds of Triangles; as well because some others have in part done the like before, (though in a different manner) as because a man may by them readily examine the form of his work.

The three parts of this Probleme, and so the rest that follow, might have been as well resolved in the quadrantal Triangle *A D G*. Where G represents the north pole; the angle at G, the difference of longitude, *A G* the complement of the latitude of the *Lizard*; *A D G* the angle of direct position from the *Amazones* to the *Lizard*, &c. As admit this last angle *A D G* were required: Then forasmuch as there is given the angle *G*, being the difference of longitude, and *A G*, the complement of the latitude: therefore by the first fundamental Axiome.

s G + *Rad.* = *tc A* G + *t* A D G therefore *s* G + *t* A G = *t* A D G, and thus it falls into the 7 Case of quadrantal Triangles, and is wrought as in this example.

The difference of longitude is G 51 deg. 00′,	*s* G	9.89050
The latitude is compl. A G 50 00,	*t* A G	9.92381
The angle of position is A D G 33 07,	*t* A D G	9.81431

The same might have been found in the quadrantal Triangle, A D F, all which to handle particularly would be too tedious; therefore it shall suffice hereafter to shew this application onely in right angled triangles, for by this one example of quadrantals, you may conceive the rest.

And thus it appears, that he which would fail the neerest way from the *Amazones* to the *Lizard*, ſhould at firſt ſhape his courſe 33 deg. 07′ from the meridian to the eaſtward, that is almoſt 3 points of the Compaſſe, namely, *n e* by *n*. Now admit the wind ſhould ſo ſerve that he might come away *n e* by *n*, yet it is to be underſtood, that in this kind of ſailing, he is not to continue this courſe long, but to ſhift it as often as occaſion requires, ſtill inclining more and more to the eaſtwards. Which how it may be done, we ſhall more expreſſely ſhew hereafter.

Probl. 2. *Two places being propoſed, the one under the Equinoctial, the other in any latitude given; and the neereſt diſtance in a great Circle of the ſame places being alſo known, to find,*

1 *Their difference of longitude,*
2 *The direct poſition from the firſt place to the ſecond,*
3 *And from the ſecond place to the firſt.*

Let the places be the ſame as before; and let there be given the difference of latitude A B 50 deg. 00′, and their neereſt diſtance A D 1322⅔ leagues, that is 66 deg. 08′ in the arch of a great circle.

Firſt, then for the difference of longitude D B, by the 12 Caſe of right angled triangles.

The latitude is	A B 50 d. 00′, *co. ar.*	*s* A B,	0.19193
The neereſt diſtance is	A D 66 08	*sc* A D,	9.60704
The differ. of longitude is	D B 51 00	*sc* D B	9.79897

Secondly, for the direct poſition from A *to* D, *by the* 13 *Caſe.*

The latitude is	A B 50 deg. 00′	*t* A B	10.07619
The neereſt diſtance is	A D 66 08	*tc* A D	9.64586
The poſition is	B A D 58 11	*sc* A	9.72205

Thirdly, for the direct poſition from D *to* A, *by the* 14 *Caſe.*

The neereſt diſtance is	A D 66 d. 08′ *co. ar.*	*s* A D	0.03882
The latitude is	A B 50 00	*s* A B	9.88425
The poſition is compl.	B D A 33 07	*s* BDA	9.92307

In like ſort, if there were given the latitude A B, and the angle of direct poſition B A D: we might find the difference of longitude B D, by the firſt Caſe of ſpherical triangles; the direct poſition BDA, by the ſecond Caſe, and the neereſt diſtance A D by the third Caſe. And thus we might proceed to frame in all 30 queſtions touching theſe two places; as we have before ſhewed in handling right angled ſpherical triangles. Which things I leave to your own practice, deſiring to uſe as much brevity, as I may.

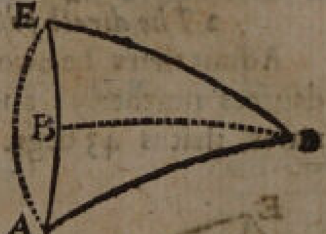
Probl.

Probl. 3. *Two places proposed, both in one and the same latitude given, and their difference of longitude being also knowne: to find*

1 The neerest distance of those two places.

2 The direct position of the one place from the other.

Admit there be two places, both in the latitude of 50 degrees, 00′, northerly, and differing in longitude 70 deg. 00′; I demand their neerest distance in the arch of a great Circle, and the direct position of the one from the other?



In the 7 probleme of sailing by *Mercators Chart*, there was required the distance of these two places measured in their parallel: but here is required their neerest distance in the arch of a great Circle.

As in this example *E A D*, let the two places be E and A, and let D be the north pole, then A D and E D are either of them 40 deg. 00′: namely, the complement of the latitude, and the angle E D A is the difference of longit. 70 deg. 00′; there is required the neerest distance E B A: and the direct position from the one to the other, D E A or D A E, for in this Case those two angles are equal.

And seeing E D is equal to A D, therefore letting fall the perpendicular D B, the triangle E D A is divided into two right angled triangles, E D B and A D B, which are every wayes equal. Wherefore,

First, for the neerest distance E A; there is given in the right angled triangle A D B, the complement of the latitude A D 40 deg. 00′, and half the difference of longitude A D B 35 deg. 00′; whereby I find A *B* agreeable to the 8 Case thus.

The compl. of the latitude is	A D 40 deg.	00′	*s* A D	9.80807
Half the differ. of longitude is	A D *B* 35	00	*s* A D *B*	9.75859
Half the distance is	A *B* 21	38	*s* A *B*	9.56666
Which doubled is	A E 43	16		

And this converted into miles is 2596 miles, the neerest distance of these two places in the arch of a great Circle, being lesse than their distance measured in their parallel by 104 miles.

Secondly, for the direct position D A B, by the 9 Case.

The compl. of the latitude is	A D 40 d.	00′	*sc* A D	9.88425
Half the differ. of longitude is	A D *B* 35	00	*t* A D *B*	9.84523
The angle of position is	D A *B* 61	48	*tc* D A *B*	97 948

Which

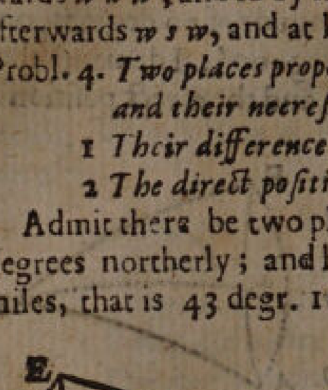
Which sheweth, that he which would go the neerest way from A to E, must not go west, though both be under one parallel; but he is at first to shape his course from A *w n w* half a point northerly; afterwards *w n w*; and so by little and little *w* by *n*; then west; then *w* by *s*; afterwards *w s w*, and at last *w s w* ½ a point southerly.

Probl. 4. *Two places proposed, both in one and the same latitude given, and their neerest distance being also known: to find,*

1 *Their difference of longitude.*

2 *The direct position of the one place from the other.*

Admit there be two places, as A and E, both in the latitude of 50 degrees northerly; and let their neerest distance be *A B E* 2596 miles, that is 43 degr. 16′: I demand their difference of longitude, which is the angle A D E, and the direct position of the one from the other, namely, the angle D A E, or D E A?



First, *for the difference of longitude*, A D E. Seeing that A B E is 43 deg. 16′, therefore A B is 21 deg. 38′: wherefore by the 14 Case of right angled spherical triangles, I find A D E thus.

The compl. of the latit. is	A D 40 deg.	00′,	*co.ar. s* A D	0,19193
Half the distance is	A B 21	38	*s* A B	9.56663
Half the differ. of long.	A D B 35	00	*s* A D B	9.75856

Which doubled is A D E 70 00, the difference of longitude required.

Secondly, for the direct position D A E *or* D A B, *by the* 13 *Case.*

The latitude is the compl. of	A D 50 deg.	00′,	*tc* A D	10.07619
Half the distance is	A B 21	38	*t* A B	9.59835
The angle of position is	D A B 61	48	*sc* D A B	9.67454

Probl. 5. *Two places proposed, both in one and the same latitude given; and the distance of those places in their parallel being also known: to find,*

1 *Their difference of longitude,*

2 *Their neerest distance in the arch of a great Circle,*

3 *The direct position of the one from the other.*

Admit there be two places, both in the latitude of 50 degrees, 00 minutes northerly; and let the distance of these places in their

parallel

parallel be 2700 miles; there is required their difference of longitude, &c.

VVe have noted before, that as the ſemidiameter of a parallel is in proportion to the ſemidiameter of the equinoctial: ſo is any number of miles in that parallel, to the minutes of longitude, anſwering to thoſe miles: and if we ſuppoſe the ſemidiameter of the equinoctial to be Radius, then the ſemidiameter of any parallel is the ſine of that parallels diſtance from the pole, that is the ſine of the complement of the latitude of that parallel. Therefore,

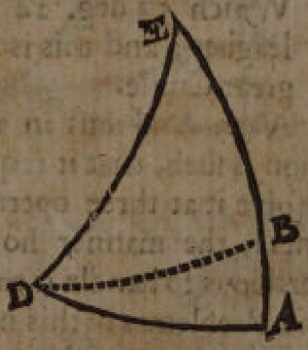
As ſine complement the latitude, sc 50 deg. 00′, co. ar. to Radius:	0.19193
So the diſtance in that parallel, 2700 miles,	3.43136
to the difference of longitude, 4200 miles,	3.62329

VVhich converted into degrees, is 70 deg. 00′, the difference of longitude, required.

And thus having found the difference of longitude. The neereſt diſtance, and the direct poſition may be found as in the third probleme before going, which with ſuch other queſtions as might be moved in this Triangle A E D, I leave to your own practice.

Probl. 6. *The latitudes of two places being given, together with their difference of longitude, to find,*

1 *Their neereſt diſtance in the arch of a great Circle.*
2 *The direct poſition from the firſt place to the ſecond,*
3 *And from the ſecond place to the firſt.*



As in the triangle ADE. Let A repreſent the north pole, D the *Lizard* lying in the latitude of 50 deg. 00 min. the complement whereof is *A D* 40 deg. 00 min.and let *E* repreſent the *Summers Iſlands*, lying in the latitude of 32 degrees, 25 min. the complement whereof is *A E* 57 degrees,35 minutes; and let their difference of longitude be 70 deg. 00 min. namely, the conteined angle *D A E*: there is required the neereſt diſtance of theſe two places *E D*, and the ſeveral poſitions of the one from the other, namely, the angles *A D E*,and *A E D*.So that here are given two ſides *A D* & *A E* with their conteined angle *D A E*: and first there is required the third side *E D*.

Wherefore according to the directions, Chap. 5. of spherical triangles, I let fall a perpendicular from *E* or *D*, for so it will fall from the end of a side given, and opposite to an angle given, &c. As first, let it fall from the point of the *Lizard* represented here by *D*, upon the meridian of *Summers Ilands* A E; and because the angles at *A* and E are both of one kind, namely, both acute; therefore the perpendicular falls within the triangle.

Then for the neerest distance required E D, the way hath been formerly to find it at three operations, thus:

First, for the perpend. D B, by the 8 Case of right angled triangles.

The compl. of latitude	A D *is* 40 deg.	00′	s A D,	9.80807
The differ. of longitude	D A B *is* 70	00	s A,	9.97298
The perpendicular	D B *is* 37	10	s D B,	9.78105

Secondly, for the dist. of the perpend. from the pole A B by the 7 Case,

The differ. of longitude	D A B *is* 70 deg.	00′	sc D A B,	9.53405
The compl. of latitude	A D *is* 40	00	t A D,	9.92381
The first arch	A B *is* 16	01	t A B,	9.45786
Which substracted from	A E 57	35,	there remains	
the second arch	E B 41	34		

3. *Having found D B, and E B, we may find E D by the 10 Case, thus,*

The perpendicular	D B *is* 37 deg.	10′,	sc D B,	9.90139
The second arch	E B *is* 41	34,	sc E B,	9.87401
The neerest distance	E D *is* 53	24,	sc E D,	9.77540

Which 53 deg. 24′ converted into miles is 3204 miles, or 1068 leagues; and this is the neerest distance required in, the arch of a great Circle.

Note. And thus in any oblique spherical triangle, when the question is such, that it requires the perpendicular to be let fall, you may resolve it at three operations, by the Cases of a right angled triangle onely, the manner how, is of it self so manifest, that it seemed superfluous to handle it particularly. Wherefore, as before in the Cases and Problemes of this nature, so in those which follow; it shall suffice to shew their resolution at two operations; which as it is much readier being well understood, so it is something harder to be understood than the former.

First,

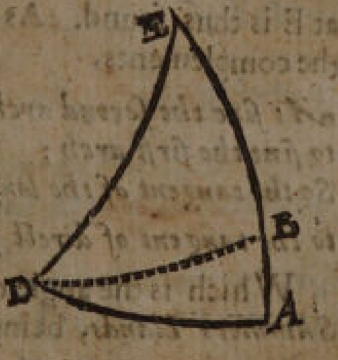
First, therefore the complement of the one latitude being A D 40 deg. and of the other A E 57 deg. 35′: and the difference of longitude D A E 70 deg. we may find the neerest distance E D at two operations agreeable to the 3 Case of oblique spherical triangles; thus,

The difference of longitude,	D A B *is* 70 d.	00′	*sc* DAB,	9.53405
The complement of latitude,	A D *is* 40	00	*t* A D,	9.92381
The first arch	A B *is* 16	01	*t* A B,	9.45786
Which substracted from	A E 57	35		there remains
the second arch	E B 41	34		
As sine compl. the first arch,	*sc* A B *sc* 16 deg.	01′		0.01719
to sine compl. the second:	*sc* E B *sc* 41	34		9.87401
So the sine of the latitude,	*sc* A D *s* 50	00		9.88425
to sine compl. the distance,	*sc* E D *s* 36	36		9.77545

Therefore the arch *E D* is 53 deg. 24′, which is the distance of these two places in the arch of a great Circle; and this converted into leagues is 1068 leagues, as before.

Secondly, by the same things given: to find the direct position of the one place from the other.

As first, to find the position from *Summers Islands*, which suppose to be at *E*, to the *Lizard* at *D*.

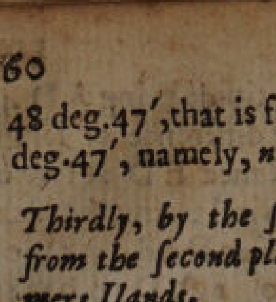


Here according to the second condition of letting fall a perpendicular, Chap. 5. I let it fall from the *Lizard* at *D*, that so it may be opposite, not onely to the angle given at A, but also to the angle required at E. And then agreeable to the fourth Case of oblique spherical triangles, I first find as before *A B* to be almost 16 deg. 1′, and *E B* 41 deg. 34′, then I say as *s A B* to *s E B*, so *tc A* to *tc E*. Or if you would not work by their complement: Say,

As sine the second arch,	*s E B* 41 d.	34′,	*co. ar.*	0.17816
to sine the first arch,	*s* A *B* 16	01		9.44044
So the tangent of the longitude,	*t* DAE 70	00		10.43893
to the tangent of direct position,	*t* AED 48	47		10.05753

Whereby is appears, that the angle of position from *E* towards *D*, is 48 deg.47′,that is from the north part of the meridian E A 4 points 3 deg.47′, namely, *n e* 3 deg. 47′ easterly.

Thirdly, by the same things given: to find the direct position from the second place to the first. As from the Lizard to Summers Ilands.



Here the work differs not from the former, provided, that you let fall the perpendicular so, as it may be opposite to the angles given and required. As in this Triangle, let *A* be the pole, *E* the *Lizard*, D *Summers Ilands*, the perpendicular I let fall from *D* to *B*, that so it may be opposite to the angle given at *A*, and to the angle required at *E*. Then is *A D* 57 degrees 35′, *A E* 40 deg. 00′, *D A E* 70 deg. 00′, therefore I say,

The differ. of longitude	D A B *is* 70 deg.	00′,	*sc* D A B	9.53405
The compl. of latitude	A D *is* 57	35	*t* A D	10.19721
The first arch	A B *is* 28	18	*t* A B	9.73126

Which taken from A E 40 00 there remains
The second arch. E B 11 42, whereby the angle at E is thus found. As *s* A B, to *s* E B: so *tc* A, to *tc* E, or to shun the complements,

As sine the second arch,	*s* E B 11 deg.	42′,	*co. ar.*	0.69296
to sine the first arch:	*s* A B 28	12		9.67445
So the tangent of the longitude	*t* A 70	00		10.43895
to the tangent of direct position	*t* E 81	08		10.80634

Which is the angle of the direct position from the *Lizard* toward *Summers Ilands*, being from the north part of the meridian to the westwards 7 points of the Compasse, and almost a quarter, that is *w* by *n* 2 deg. 23′ westerly.

And thus it appears, that he which would sail the neerest way from *Summers Ilands* to the *Lizard*, must at first shape his course *n e* easterly, afterwards by degrees *n e* by *e*, then *e n e*, then *e* by *n*, then east, then east southerly, &c. as we shall more particularly shew hereafter, and the like is to be understood of other places.

But

But here, after the first part of this probleme was wrought, namely, after the distance of the two places E and D was found, the angles of position from the one to the other, might have been more readily found, either of them at a single operation, as in this following probleme.

Probl. 7. *The neerest distance of two places, with their difference of longitude, and one of their latitudes given: to find the direct position thereof from the other.*

As admit the distance in a great Circle from the *Lizard* to *Summers Ilands*, namely, from *E* to *D*, to be as it was before found 1068 leagues, or 53 deg. 24′; and let their difference of longitude *E A D* be 70 deg. 00′; and let the latitude of the *Lizard* be 50 deg. 00′, whose complement *E* A is 40 deg. 00′; there is required the direct position from *Summers Ilands* to the *Lizard*, namely, the angle A *D E*. Then doth this probleme come under the second Case of oblique sphericall Triangles, and is thus resolved.

As the sine of the distance of the places,	*s E D* 53	deg. 24′,	*co. ar.*	0.09538
to sine their differ. of longit.	*s D A E* 70	00,		9.97298
So sine compl. the latitude of to one place given,	*s A E* 40	00,		9.80807
to the sine of the direct position from the other.	*s A D E* 48	48,		9.87643

VVhereas there is a minute difference between the arch before found, and this; it may arise by neglecting some seconds or parts of a minute in the work, which here we regard not.

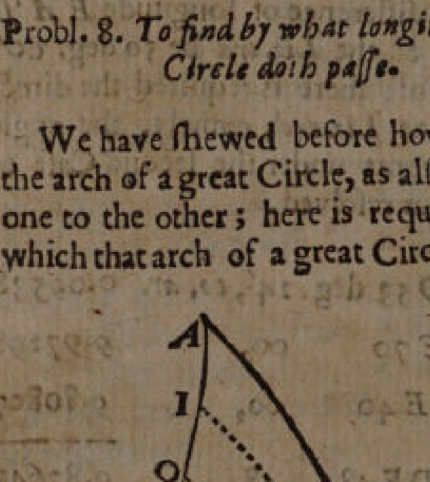
In like manner, by the complement of the other latitude given, A D, we might find the direct position from the *Lizard* to *Summers Ilands*, namely, the angle A E D.

And thus we might proceed to frame many other questions in this Triangle to the number of 60, touching the distance, difference of longitude, latitudes, and angles of position of these two places which will not be hard to him, that understandeth what we have before delivered touching oblique spherical Triangles.

And what hath been said touching these two places, the same is to be conceived of any other two places differing in their longitudes and latitudes. And though the one place should have latitude northerly, and the other southerly, yet is the operation little different, for still the arches of their meridians intercepted between them and the neerest pole, are two sides of the triangle, the arch of a great Circle intercepted between the two places is the third side; the angles conteined between that arch and the meridian of either place, are the angles of position; and the angle comprehended between their two meridians, is their difference of longitude. Therefore passing over these, we haste to such things as more necessarily concern the practice of sailing by a great Circle.

Probl. 8. *To find by what longitudes and latitudes the arch of a great Circle doth passe.*

We have shewed before how to find the distance of two places in the arch of a great Circle, as also the angles of direct position from the one to the other; here is required the longitudes and latitudes, by which that arch of a great Circle doth passe.



As in this triangle, Let A be *Summers Ilands*, E the *Lizard*, A E an arch of the great Circle passing by these places; it is required to shew the longitudes and latitudes by which this arch A E doth passe.

Here it is requisite to let fall a perpendicular from the pole D, to the arch A E (extended if need so require) which let be *D B*; then first to find the length of that perpendicular; secondly, the parts of the vertical angle *A D B* and *E D B*, for these being had, every other question will fall in right angled triangles, and so be resolved by the addition of two numbers onely.

First, then for the perpendicular D B there are given the hypothenusal A D 57 deg. 35′, and the angle of position at A was before found 48 deg. 48′: therefore by the 8 Case,

The

The complement of latitude A D *is* 57 deg.	35′ s A D,	9.92643
The angle of position A *is* 48	48 s A	9.87645
The perpendicular D B *is* 39	26 s D B,	9.80288

And this 39 deg. 26′, is the complement of the greatest latitude, by which the great Circle *A B E* doth passe, therefore the greatest obliquity or latitude from the equinoctial of that circle is 50 d. 34′.

Secondly, for the angles A D B *and* E D B, by the ninth Case.

The latitude is the compl. of A D 32 deg.	25′, sc A D,	9.72922
The angle of direct position is A 48	48, t A,	10.05778
The angle at the perpend. is A D B 58	31, tc A D B	9.78700

And seeing the whole A D E is 70 deg. 00′, therefore the angle E D B is 11 deg. 29′. So that for the greatest latitude of this Circle, which is B, we have found the difference of longitude from E to E D B 11 deg. 29′, and from A the angle A D B 58 deg. 31′.

Now the difference of longitude from A to E, namely, the angle A D E being 70 deg. 00′; let it be required to find by what latitudes the arch A E doth passe for every tenth degree of longitude from *A*. As supposing the point I, to differ in longitude from A 10 degrees; I would know the latitude of the same point I.

Here seeing we have before found the angle A D B to be 58 degr. 31′, and the angle A D I being by supposition 10 deg. therefore the angle I D B is 48 deg. 31′, and the perpendicular D B, we found before to be 39 deg. 26′: by which we may find the complement of the latitude D I according to the third Case thus.

The angle I D B 48 deg.	31′, sc I D B,	9.82112
The perpendicular D B 39	26, tc D B,	10.08492
The latitude is the compl. of D I 38	51, tc D I,	9.90604

In like manner supposing the point O, to differ in longitude from A 20 deg. 00′, V 30 deg. M 40 deg. N 50 deg. we shall find the latitude of the point O to be 43 deg. 34′, the latitude of V 46 deg. 54′, the latitude of M 49 deg. 04′, and the latitude of N 50 deg. 15′.

Note. For every of these differences of longitude proposed, we might also find the distances, and angles of position contrariwise, for any difference of latitude given, we might find the difference of longitude, the distance, and angle of position: and for any diſtance given, we might find the difference of longitude and latitude, and the angle of poſition. All which will be eaſily performed by him that is a little exercised in ſpherical Triangles.

Probl. 9. *To find how far a man ſails by the arch of a great Circle, and how much he ſhall alter his longitude and latitude, before he alter his courſe any number of degrees propoſed.*

We found before, that the angle of poſition at A was 48 deg. 48′, ſhewing that he which would ſail from *Summers Ilands*, here repreſented by A, to the *Lizard* at E, the directeſt and neereſt way, muſt at firſt ſhape his courſe from A northeaſt 3 deg. 48′ eaſterly. Yet he is not to continue this courſe, but to incline by degrees more and more to the eaſtwards, &c. Now then I demand how far a man ſails from A in the arch of a great Circle, before he alter his courſe 7 deg. 27′, that is before he may ſteer away *n e* by *e*, and how much ſhall he firſt alter his longitude and latitude?

Suppoſe he muſt firſt come to J, before he alter his courſe 7 degr. 27′ then is there required the diſtance A J, and the longitude and latitude of the point J.

Here it is requiſite, that the perpendicular D B be known, which we before found to be 39 deg. 26′, alſo the parts of the baſe A B and E B, which we may find by the ſeventh Caſe thus.

The angle of poſition given	A *is* 48 deg.	48′,	*sc* A,	9.81868
The complement of latitude	A D *is* 57	35,	*t* A D,	10.19720
The baſe	A B *is* 46	03,	*t* A B,	10.01588
Which taken from	A E 53	24		
there remains	E B 07	23		

Theſe things premiſed, we come to reſolve the queſtion. And conſidering that the courſe given at I, is *n e* by *e*, which rumb makes with the meridian an angle of 56 deg. 15′, therefore in the triangle D I B, the angle at I is 56 deg. 15′, and the perpendicular D B is 39 degr. 26′, whereby we may find I B by the ſixth Caſe thus.

The angle of poſition given	I, 56 deg.	15′,	*tc* D I B,	9.82489
The perpendicular is	D B 39	26,	*t* D B,	9.91507
The baſe is	I B 33	20	*s* I B,	9.73996
Which taken from	A B 46	03		
there remains	A I 12	43		

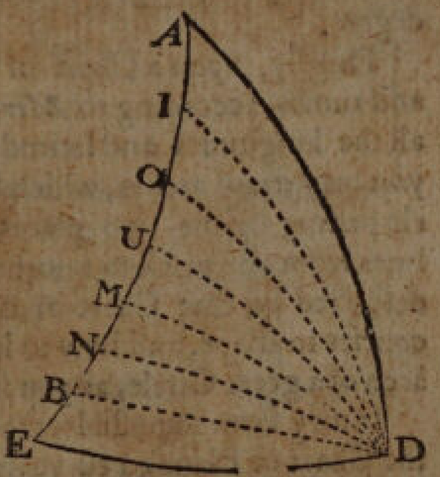
Which

Which converted into leagues, is $254\frac{1}{3}$ leagues, and so far you are to sail from A in the arch of a great Circle, before you alter your course 7 degrees 27 minutes. And in like sort you may find it for every single degree to be such as by this Table appeareth.

Angle of position.	Dist. in d, & m.
d ′	d ′
48—48	00—00
49—48	02—02
50—48	03—56
51—48	05—43
52—48	07—25
53—48	09—03
54—48	10—35
55—48	12—04
56—15	12—43

Where you may perceive, that having runne from A towards E by the arch of a great Circle 2 degrees 2 minutes, that is $40\frac{2}{3}$ leagues, you alter your course one degree more easterly than you began. When you have run 3 degrees, 56 minutes, you alter your course 2 degrees 00 minutes, &c. as in the Table.

Now for finding the longitude and latitude of any of these points, it may be done by help of the perpendicular and angle of position given. As if there were required the longitude and latitude of the point I; there is given in the Triangle I D B, the angle of position at I, and the perpendicular D B, wherefore by the 5 Case I find the complement of the latitude D I, and by the 4 Case the difference of longitude B D I, the angle B D A being before known.



But notwithstanding all that hath hitherto been said, it may seem hard to direct a ship, and to keep such a reckoning as may be agreeable to this method of sailing by a great Circle. And indeed, as it is in a manner impossible, so neither is it necessary, that a ship should alwayes persevere exactly in the arch of a great Circle. It may suffice, and is almost the same in effect, if a ship be so directed, that she go neer this arch. Which how it may be done, and that with facility, we come now to shew in this next probleme.

Probl. 10. *How a man may direct his courses, and keep his reckoning, that would sail neer the arch of a great Circle.*

That this may be the more plain, we will briefly repeat some things before handled serving for this purpose. And first, suppose the latitudes, and the difference of longitude of the two places to be given; then may you find their neerest distance in the arch of a great Circle, and the angles of the position of the one from the other; as we have shewed in the sixth and seventh problemes before going. And thus all the parts of the Triangle proposed are known, namely, the three angles, and the three sides.

Secondly, you may find (as we have before shewed in the eighth probleme) by what longitudes and latitudes this arch of a great Circle doth passe, namely, the arch that goes by the two places proposed. And this you may do for every fifth degree of longitude, or for every single degree, if you will take that pains. Or if your difference of latitude be more than your difference of longitude, you may do it for every fifth degree of difference of latitude, or for every single degree.

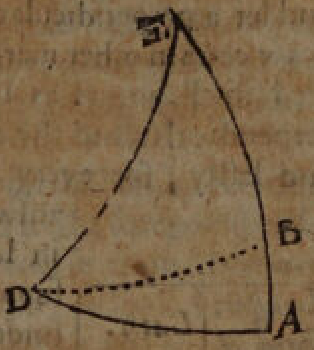
Thirdly, upon a Chart or Blank lined with meridians, parallels, and rumbs, according to *Mercators* projection; you may prick down all the longitudes and latitudes found as aforesaid; by which pricks you may draw arches, which shall represent the arch or the great Circle passing by the two places proposed: or if you onely draw right lines from one prick to another it may suffice. Which arch being thus described on that Chart or Blank, you shall easily see thereby what courses to shape, and how to keep your reckoning, sailing so neer that arch of a great Circle, as you shall think convenient.

It may seem impossible, that this arch of a great Circle, being upon the Chart or Blanck a curve line, should be a shorter passage between two places, than the right line drawn on the Chart from the one to the other. But he that well understands the ground and projection of this Chart, will be able of himself to resolve this paradox; forasmuch as the degrees of latitude by which the arch doth passe, are greater than the degrees of latitude, by which the right line doth passe: whence it is, that the degrees conteined in the arch, are fewer

fewer than those conteined in the right line; therefore to proceed.

Let us take for example the two places before mentioned, namely, *Summers Ilands*, lying in the latitude of 32 degrees 25 minutes, and the *Lizard* in the latitude of 50 degrees 00 minutes, and let their difference of longitude be 70 degrees.

As in this Diagram, let *E* represent *Summers Ilands*, *D* the *Lizard*, *A* the north pole: Then is *A E* the complement of the latitude of *Summers Ilands*, 57 degrees, 35 minutes, *A D* the complement of the latitude of the *Lizard* 40 degrees, *D A E* their difference of longitude 70 degrees, 00 minutes. By which things given we may find their neerest distance *E D*, as in the sixth probleme thus.



To sine complement	D A B, *that is, to sine complement*	70 d.	00′
Add the tangent of	A D, *that is the tangent of*	40	00
The summe is the tangent of	A B, *that is the tangent of*	16	01
Which substracted from	A E, *that is, from*	57	35
There remains	E B,	41	34

Then I say,

As sine complement	A B, *that is, sine complement*	16 deg.	01′
to sine complement	E B, *that is, sine complement*	41	34
So sine complement	A D, *that is, sine complement*	40	00
to sine complement	E D, *that is, the sine of*	36	36
Therefore the arch	E D is	53	24

Which is the distance of the *Lizard* from *Summers Ilands*, in the arch of a great Circle, namely, 1068 leagues.

This done, we may find their positions one from another, namely, the angles at *E* and *D* by the seventh probleme, saying,

As sine *E D* 53 deg. 24′, to sine *D A E* 70 deg. 00′.

So sine *AE* 57 deg. 35′, to sine *A D E* 81 deg. 08′, the direct position from the *Lizard* to *Summers Ilands*. Also,

As sine *E D* 53 deg. 24′, to sine *D A E* 70 deg. 00′:

so sine *A D* 40 deg. 00′, to sine *A E D* 48 deg. 48″; the direct

position

position from *Summers Ilands* to the *Lizard*. And thus are all the sides and angles of this triangle discovered.

Secondly, by the 8 probleme, I find by what longitudes and latitudes this arch E D must passe: For which the former perpendicular D B is not apt, therefore in the fore-going Triangle, pag. 165. Let A represent *Summers Ilands*, E the *Lizard*, D the north pole, and let a perpendicular fall from the pole D, which let be D B: and draw certain other meridians, as D I, D O, D V, &c. And so proceed in all points as in the 8 probleme, to find the length of this perpendicular, and the angles at the perpendicular A D B and E D B: and lastly, for every several longitude from A, find the latitude answerable. Thus supposing the point *I* to differ in longitude from the point A, 5 degrees, that is, supposing the angle A D I to be 5 degrees, we shall finde the the latitude of that point I to be 35 degr. 52′; or supposing that angle A D I to be 10 degrees: we shall finde the latitude of that point I to be 38 degr. 51′; and so of the rest, as by this Table appears.

Longit. from A	*Latit.*	
deg.min	*deg.*	*min*
00—00	32	25
05	35	52
10	38	51
15	41	24
20	43	34
25	45	24
30	46	54
35	48	07
40	49	04
45	49	47
50	50	15
55	50	31
60	50	33
65	50	23
70	50	00

Thirdly, I draw a blank according to *Mercators* projection, (which may be done either by Master *Wrights* own Tables, as he hath shewed in his book of the *Correction of Errours in Navigation*, Chap. 5. or by the abridgement thereof, which I have before placed, and called a Table of meridional parts) so as there may a meridian be drawn by every fifth degree of longitude. In which blank, I set down *Summers Ilands*, and the *Lizard*, according to their latitudes, and difference of longitude before given, and in the meridian, that is 5 degrees to the eastward of *Summers Ilands*, I make a prick or mark at 35 degrees 52 minutes of latitude; likewise in the meridian, that is 10 degrees to the eastwards of *Summers Ilands*, I make a mark at 38 degrees 51 minutes of latitude; and so I proceed with all the rest as as by this Table I am directed. Then by these pricks or marks thus made on the blank, I draw the arches of circles or right lines from one to another, and so shall I describe a curve line on the blank, representing so neer as shall be necessary, an arch of the great Circle passing from *Summers Ilands* to the *Lizard.* And if it were done for every single degree, (as here it is for every fifth degree) it would come neerer the exact truth. Which curve line being thus described on your blank, you shall thereby see what courses to shape, to keep as neer it as you think good; and you may set down your reckoning on that blank accordingly.

As having drawn the aforesaid curve line upon the blank, according to the several longitudes and latitudes expressed in the foregoing table; I see by that blank, that I may first shape my course from *Summers Ilands*, *n e* half a point easterly about 200 leagues; so shall I have run my self into the latitude of 38 degrees 45 minutes; and have altered my longitude 9 degrees 30 minutes: From thence again, I see I may sail away *n e* by *e*; or if I would not come neer the bank of *New-found land*, I may shape a more easterly course; but suppose I still desire to keep neer the arch of a great Circle, then I say I may sail away *n e* by *e* 200 leagues, and so should be in the latitude of 41 degrees 32 minutes, and have altered my longitude 14 degrees 56 minutes. From thence again I may sail *e n e* half a point northerly 165 leagues, and then should be in the latitude of 45 degrees 25 minutes, having altered my longitude 24 degrees 58 minutes. From thence again failing *e n e* 130 leagues, I shall be in the latitude of 47 degrees 54 minutes, and have altered my longitude 33 degrees 42 minutes. From thence *e n e* half a point easterly 88 leagues, into the latitude of 49 degrees 11 min. and difference of longitude 40 deg. 5 min. From thence again if I sail *e b n* 70 leagues, I shall be in the latitude of 49 degrees 52 minutes, and have altered my longitude from *Summers Ilands* to the eastward 45 degr. 22 minutes. And thus being neer the parallel of the *Lizard*, J keep in the same parallel, sailing east till J come right off from it, which by this reckoning should be 317 leagues. And so the whole distance from *Summers Ilands* to the *Lizard*, according to these courses should be about 1070 leagues, going over the bank of *New-found-land*. Now, J say, coming into the latitude of 49 degrees 52 minutes, or thereabouts,

abouts, though by my reckoning, well rectified by obſervations, I find my ſelf to be ſtill ſhort of the *Lizard*, about 317 leagues: yet I follow not the great Circle any further, but that I may the more certainly fall with the place intended, whether *Selly* or the *Lizard*, I keep my ſelf in that parallel. And the rather, becauſe the reckonings outward and homeward, of voyages made to this and other places of the *West Indies*, do for the moſt part diſagree much. Which diſagreement ariſeth partly by the currant ſetting homeward from thoſe parts; but chiefly becauſe thoſe reckonings are kept upon the plain or common Sea-Chart; which Chart, except a man return the ſame way home that he went out, is commonly ſubject to groſſe errours.

And whereas I know, that the moſt part are wholly addicted to the uſe of this Chart; ſome alſo deſpiſing all others, and may haply be offended that I ſhould thus tax it with groſſe errours; I ſhall make it appear (partly in this preſent example) that I do it not without juſt cauſe.

In ſailing from the *Lizard* to theſe Ilands, and ſo to other parts of the Weſt Indies; men commonly run far to the ſouthwards, as ſometimes into the latitude of 30 degrees, ſometimes more ſoutherly, to get a wind; but coming homewards, their courſes are commonly more northerly than the Rumb leading from thence home. But in this example following, let us keep a mean, and to make ſhort, ſuppoſe a man ſhould ſail from the *Lizard* ſouth weſt neer 500 leagues, and then find himſelf in the latitude of 32 degrees 20 minutes, and from thence weſt 782 leagues, till he find himſelf directly ſouth from *Summers Iſlands*, and about two leagues off. Then by this reckoning on the plain Chart, *Summers Iſlands* ſhould be diſtant from the *Lizard* 1189 leagues in a ſtraight courſe. Now admitting this reckoning outward bound to be true, and theſe places to be thus ſituated on the common Chart; let us ſuppoſe the reckoning homewards to be alſo kept on the ſame Chart. And becauſe coming home men keep to the northwards, let us ſuppoſe that he ſteers away *ne* half a point eaſterly 200 leagues; then *ne* by *e* 100 leagues; *ene* half a point northerly 165 leagues, *ene* 130 leagues; eaſt north eaſt halfe a point eaſterly 88 leagues; eaſt and by north 70 leagues; and *e* 317 leagues. Then by this reckoning upon the plain Chart: he ſhould be ſhort of the *Lizard* about 160 leagues.

leagues. Whereas by a true reckoning he should be as farre shot as the *Lizard*. And hence it is that they which come from thence and other parts of the West Indies (making no allowance) are at home before their reckonings sometimes 200 leagues and more. For a mans reckoning by the plain Chart, makes him shorter then he should be by 160 leagues; sometimes more, sometimes lesse; and the current may put him forwards 50 or 60 leagues more, so that his ship may be above 200 leagues before his reckoning.

And thus much at present, touching the three principal kinds of sayling. Which I hope I shall have opportunity to handle more fully hereafter, with some other things of like nature; and to correct such faults as may peradventure be here committed through haste.

A Table for the angles which every Rumb maketh with the Meridian.

North	*South*	D	M	*South*	*North*
		02	49		
		05	37		
		08	26		
NE *by* E	S *by* E	11	15	S*by*W	N *by*W
		14	04		
		16	52		
		19	41		
NNE	SSE	22	30	SSW	NNW
		25	19		
		28	07		
		30	56		
NE *by* N	SE *by* S	33	45	SW*by* S	NW*b*N
		36	34		
		39	22		
		42	11		
NE	SE	45	00	SW	NW
		47	49		
		50	37		
		53	26		
N E*by* E	SE *by* E	56	15	SW*b*W	NW*b*W
		59	04		
		61	52		
		64	41		
ENE	ESE	67	30	WSW	WNW
		70	19		
		73	07		
		75	56		
E *by* N	E *by* S	78	45	W *by* S	W *by* N
		81	34		
		84	22		
		87	11		
East	*East*	90	00	*West*	*West*

Of the Declination of the Sun, and fixed Stars.

BEcauſe in the practice and application of the doctrine of Triangles, it is often requiſite that the Sunnes declination be known, I have thought good here to place four Tables thereof; the firſt ſhewing the Suns declinations for every day of the firſt four years after the Leap years; namely, for the years 1649, 1653, 1657, 1661, 1665, 1669. The ſecond for the ſecond years after the Leap years, namely for 1650, 1654, 1658, 1662, 1666, 1670. The third for the third years after the Leap years, namely for 1651, 1655, 1659, 1663, 1667, 1671. And the fourth for theſe Leap years 1652, 1656, 1660, 1664, 1668, 1672, according as they are expreſſed in the head of each Table. And becauſe the obſervations of our Countreyman *Mr. Edward Wright* are not (as I take it) inferiour to any other at this day extant, therefore I have drawn theſe Tables out of his, rectifying them by Proſthaphereſis for theſe next enſuing times.

To theſe I have added (chiefly for the uſe of Sea-men) rules for finding the latitudes of places by the declination and meridian altitude of the Sunne or Starres; and a Table of the right aſcenſions and declinations of about 74 principal fixed Starres, calculated according to their longitudes and latitudes and latitudes ſet down by *Tycho Brahe Anno* 1600, with allowance for their motion of longitude, or for the preceſſion of the Equinoxes for the year 1660 compleat. I have alſo noted at what times of the year theſe *Stars* will be upon the meridian at four of the clock in the morning, whereby you may readily ſee when they are in ſeaſon to be obſerved for finding the latitude; by which alſo you may conjecture their other times of being upon the Meridian. For the Starre which in any day propoſed is upon the meridian at four of the clock in the morning, will about fifteen dayes ater be on the meridian at three of the clock in the morning, and about a moneth after at two, &c. Wherein alſo Mariners uſe to help themſelves by their Compaſſe, whereby they

they see when the Sunne or starre is neer the meridian. Such as desire the exact time of a starres comming to the meridian for any day, may substract the right ascension of the Sunne for that day from the right ascension of the Starre. (adding thereto if need require twenty four hours) the remainer shewes how many hours it will be after noon, before the starre be upon the meridian.

The Sunnes right ascension for any day may be found by his declination for that day, by the resolution of a right angled spherical triangle, as of the triangle ♈ F Q in the general scheame of the third chapter, of spherical triangles. But I have annexed an exact Table of the Sunnes right ascension at the end.

Also for the starres neer the Equinoctial, I have set down (in this fourth Edition) their longitudes and latitudes, that so the Moons place may be discovered by her longitude from any of them; especially being in her ninetieth degree: Whereby the longitude of places on sea or land may be neerly gathered. For the performance whereof I intended to have handled the Moons motion, and to have set down the best wayes, I have thought upon; but other urgent occasions have hindred me. Notwithstanding if you find by some exact Ephemerides the Moons true longitude, at the time of observation, and observing with meet instruments her longitude from some of those starres, and withal the hour and minute of the night (which by the right ascensions and declinations of the Sunne and starres may be known) you may neerly gather the difference of longitude. For which causes I thought it not superfluous to set down their longitudes and latitudes and an exact Table of the Suns right ascension, though I cannot prosecute the rest at present. Also by the longitudes and latitudes of these starres, their right ascensions and declinations may be examined.

The Table of the Sunnes Declination, for

Days	1649 January deg.	m.	dif	1653 February deg.	m.	dif	1657 March deg.	m.	dif	1661 April deg.	m.	dif.	1665 May deg.	m.	dif	1669 June deg.	m.	d.
1	21	44	10	13	40	20	03	24	24	08	36	22	18	05	15	23	12	0
2	21	34	11	13	26	21	03	06	23	08	58	22	18	20	15	23	16	0
3	21	23	10	13	05	20	02	37	24	09	20	22	18	35	15	23	19	0
4	21	13	11	12	45	20	02	13	24	09	42	21	18	50	14	23	22	0
5	21	02	12	12	25	21	01	49	24	10	03	21	19	04	14	23	25	0
6	20	50	12	12	04	21	01	25	24	10	24	21	19	18	13	23	27	0
7	20	38	12	11	43	22	01	01	23	10	45	21	19	31	13	23	29	0:
8	20	26	13	11	21	21	South	38	24	11	06	21	19	44	13	23	30	01
9	20	13	13	11	00	22		14	24	11	27	20	19	57	13	23	31	00
10	20	00	14	10	38	22	North	10	23	11	47	20	20	10	12	23	31	00
11	19	46	14	10	16	22		33	24	12	07	21	20	22	12	23	32	00
12	19	32	14	09	54	22	00	57	24	12	28	20	20	34	11	23	31	00
13	19	18	15	09	32	22	01	21	23	12	48	19	20	45	11	23	30	01
14	19	03	15	09	10	22	01	44	24	13	07	20	20	56	11	23	29	01
15	18	48	15	08	48	23	02	08	23	13	27	19	21	07	10	23	28	01
16	18	33	16	08	25	22	02	31	23	13	46	19	21	17	10	23	26	03
17	18	17	16	08	03	23	02	54	24	14	05	19	21	27	10	23	23	03
18	18	02	16	07	40	23	03	18	23	14	24	18	21	37	09	23	20	03
19	17	45	17	07	17	23	03	41	24	14	42	19	21	46	09	23	17	03
20	17	28	17	06	54	23	04	05	23	15	01	18	21	55	09	23	14	04
21	17	11	17	06	31	23	04	28	23	15	19	18	22	04	08	23	10	04
22	16	54	18	06	08	23	04	51	23	15	37	17	22	12	08	23	06	05
23	16	36	18	05	45	24	05	14	23	15	54	18	22	20	07	23	01	06
24	16	18	18	05	21	23	05	37	23	16	12	17	22	27	07	22	55	05
25	16	00	18	04	58	24	06	00	22	16	29	17	22	34	07	22	50	06
26	15	42	19	04	34	23	06	22	23	16	46	16	22	41	06	22	44	07
27	15	23	19	04	11	24	06	45	22	17	02	16	22	47	06	22	37	06
28	15	04	19	03	47	23	07	07	23	17	18	16	22	53	05	22	31	08
29	14	45	19				07	30	22	17	34	16	22	58	05	22	23	07
30	14	26	20				07	52	22	17	50	15	22	03	05	22	16	08
31	14	06	20				08	14	22				23	08	04			

Dayes	1649			1653			1657			1661			1665			1669		
	July			*August*			*September*			*October*			*November*			*December*		
	deg.	m.	dif	deg	m	dif	deg.	m.	dif	deg.	m.	dif.	deg.	m.	dif.	deg.	m	dif
1	22	08	08	15	12	18	04	24	23	07	15	23	17	46	16	23	09	04
2	22	00	09	14	54	18	04	01	23	07	38	23	17	56	16	23	13	04
3	21	51	09	14	36	19	03	38	23	08	00	22	18	12	16	23	17	03
4	21	42	10	14	17	19	03	15	23	08	22	22	18	28	15	23	20	03
5	21	32	10	13	58	19	02	52	23	08	45	23	18	43	15	23	23	03
6	21	22	10	13	39	19	02	29	23	09	07	22	18	58	15	23	26	02
7	21	12	10	13	20	19	02	05	24	09	29	22	19	13	14	23	28	02
8	21	02	11	13	01	20	01	42	23	09	51	22	19	27	14	23	30	02
9	20	51	11	12	41	20	01	19	23	10	13	21	19	41	14	23	31	01
10	20	40	12	12	21	20	00	55	24	10	35	22	19	55	12	23	31	00
11	20	28	12	12	01	20	North	32	24	10	56	21	20	08	13	23	32	01
12	20	16	12	11	41	20		08	23	11	18	22	20	21	13	23	31	01
13	20	04	13	11	21	21	South	16	23	11	39	21	20	34	12	23	30	01
14	19	51	13	11	01	21		39	24	12	00	20	20	46	12	23	29	02
15	19	38	13	10	39	21	01	03	23	12	21	21	20	58	11	23	27	02
16	19	25	13	10	18	21	01	26	24	12	41	21	21	09	11	23	25	03
17	19	12	14	09	57	21	01	50	23	13	02	20	21	20	11	23	22	03
18	18	58	15	09	36	21	02	13	24	13	22	20	21	31	10	23	19	03
19	18	43	14	09	15	22	02	37	23	13	42	20	21	41	09	23	16	04
20	18	29	15	08	53	22	03	00	23	14	02	19	21	50	10	23	12	05
21	18	14	15	08	31	22	03	23	24	14	21	20	22	00	09	23	07	05
22	17	59	15	08	09	22	03	47	23	14	41	19	22	09	08	23	02	05
23	17	14	16	07	47	22	04	10	23	15	00	19	22	17	08	22	57	06
24	17	28	16	07	25	22	04	33	24	15	19	19	22	25	08	22	51	07
25	17	12	16	07	03	22	04	57	23	15	37	18	22	33	07	22	44	07
26	16	56	17	06	41	23	05	20	23	15	55	18	22	40	06	22	37	07
27	16	39	17	06	18	23	05	43	23	16	13	18	22	46	06	22	30	08
28	16	22	17	05	56	23	06	06	23	16	31	18	22	52	06	22	22	08
29	16	05	17	05	33	23	06	29	23	16	49	17	22	58	06	22	14	09
30	15	48	18	05	10	23	06	52	23	17	06	17	23	04	05	22	05	09
31	15	30	18	04	47	23				17	23	17				21	56	09

The Table of the Sunnes Declination, for

Dayes	1650			1654			1658			1661			1666			1670		
	January			*February*			*March*			*April*			*May*			*June*		
	deg.	m.	dif.	deg.	m.	dif	deg.	m.	dic	deg.	m.	dif	deg.	m.	dif.	deg.	m.	dif.
1	21	47	10	13	51	20	03	29	23	08	31	22	18	02	15	23	11	04
2	21	37	10	13	31	21	03	06	24	08	53	22	18	17	15	23	15	03
3	21	27	11	13	10	20	02	42	24	09	15	21	18	32	14	23	18	03
4	21	16	11	12	50	20	02	18	23	09	36	22	18	46	14	23	21	03
5	21	04	12	12	30	21	01	55	24	09	58	21	19	00	14	23	24	02
6	20	53	12	12	09	21	01	31	24	10	19	21	19	14	14	23	26	02
7	20	41	12	11	48	22	01	07	24	10	40	21	19	28	13	23	28	02
8	20	29	13	11	26	21	South	43	23	11	01	21	19	41	13	23	30	01
9	20	16	13	11	05	22		20	24	11	22	20	19	54	13	23	31	00
10	20	03	14	10	43	21		04	24	11	42	21	20	07	12	23	31	00
11	19	49	14	10	22	21	North	28	23	12	03	20	20	19	12	23	32	01
12	19	35	14	10	00	22		51	24	12	23	20	20	31	11	23	31	00
13	19	21	14	09	38	23	01	15	24	12	43	20	20	42	11	23	31	01
14	19	07	15	09	15	22	01	39	23	13	03	19	20	53	11	23	30	02
15	18	52	15	08	53	22	02	02	23	13	22	19	21	04	11	23	28	02
16	18	37	16	08	31	23	02	25	24	13	41	19	21	15	10	23	26	02
17	18	21	16	08	08	23	02	49	23	14	00	19	21	25	10	23	24	03
18	18	05	16	07	45	23	03	12	24	14	19	19	21	35	10	23	21	03
19	17	49	17	07	22	23	03	36	23	14	38	18	21	44	09	23	18	03
20	17	32	17	06	59	23	03	59	23	14	56	18	21	53	09	23	15	04
21	17	15	17	06	36	23	04	22	23	15	14	18	22	02	08	23	11	05
22	16	58	17	06	13	23	04	45	23	15	32	18	22	10	08	23	06	04
23	16	41	18	05	50	23	05	08	23	15	50	18	22	18	07	23	02	05
24	16	23	18	05	27	23	05	31	23	16	08	17	22	25	07	22	57	06
25	16	05	18	05	04	24	05	54	23	16	25	17	22	32	07	22	51	06
26	15	47	19	04	40	23	06	17	23	16	42	16	22	39	06	22	45	06
27	15	28	19	04	17	24	06	40	22	16	55	16	22	45	06	22	39	07
28	15	09	19	03	53	24	07	05	23	17	14	16	22	51	06	22	32	07
29	14	50	19				07	25	22	17	30	16	22	57	05	22	25	07
30	14	31	20				07	47	22	17	46	16	23	02	05	22	18	08
31	14	11	20				08	09	22				23	07	04			

the second Years after the Leap-year, *viz.*

	1650			1654			1658			1662			1666			1670		
Dayes	*July*			*August*			*September*			*October*			*November*			*December*		
	deg.	m.	dif	deg	m.	dif	deg.	m.	dif	deg.	m.	dif.	deg.	m.	dif.	deg.	m.	dif
1	22	10	08	15	17	18	04	30	23	07	09	23	17	36	16	23	08	04
2	22	02	09	14	59	19	04	07	23	07	32	23	17	52	16	23	12	04
3	21	53	09	14	40	18	03	44	23	07	55	22	18	08	16	23	16	04
4	21	44	09	14	22	19	03	21	23	08	17	22	18	24	16	23	20	03
5	21	35	10	14	03	19	02	58	24	08	39	23	18	40	15	23	23	03
6	21	25	10	13	44	19	02	34	23	09	02	22	18	55	14	23	26	02
7	21	15	11	13	25	20	02	11	23	09	24	22	19	09	15	23	28	01
8	21	04	10	13	05	19	01	48	24	09	46	22	19	24	14	23	29	01
9	20	54	11	12	46	20	01	24	23	10	08	21	19	38	14	23	30	01
10	20	43	12	12	26	20	01	01	24	10	29	22	19	52	13	23	31	00
11	20	31	12	12	06	20	North	37	23	10	51	21	20	05	13	23	32	00
12	20	19	12	11	46	20		14	24	11	12	22	20	18	13	23	31	00
13	20	07	13	11	26	21	South	10	23	11	34	21	20	31	12	23	31	01
14	19	54	13	11	05	21		33	24	11	55	20	20	43	12	23	29	01
15	19	41	13	10	44	21	00	57	23	12	15	21	20	55	11	23	28	02
16	19	28	13	10	23	21	01	20	24	12	36	21	21	06	11	23	26	03
17	19	15	14	10	02	21	01	44	23	12	57	20	21	17	11	23	23	03
18	19	01	14	09	41	21	02	04	24	13	17	20	21	28	10	23	20	03
19	18	47	14	09	20	22	02	31	23	13	37	20	21	38	10	23	17	04
20	18	33	15	08	58	22	02	54	24	13	57	19	21	48	09	23	13	05
21	18	18	15	08	36	22	03	18	23	14	16	20	21	57	09	23	08	05
22	18	03	16	08	15	22	03	41	23	14	36	19	22	06	09	23	03	05
23	17	47	15	07	53	22	04	04	24	14	55	19	22	15	08	22	58	06
24	17	32	16	07	31	22	04	28	23	15	14	19	22	23	08	22	52	06
25	17	16	16	07	09	23	04	51	23	15	33	18	22	31	07	22	46	07
26	17	00	17	06	46	22	05	14	23	15	51	18	22	38	07	22	39	07
27	16	43	17	06	24	23	05	38	23	16	09	18	22	45	06	22	32	08
28	16	26	17	06	01	23	06	00	23	16	27	18	22	51	06	22	24	08
29	16	09	17	05	38	23	06	23	23	16	45	17	22	57	06	22	16	09
30	15	52	18	05	16	23	06	46	23	17	02	17	23	03	05	22	07	09
31	15	34	17	04	53	23				17	19	17				21	58	09

The Table of the Sunnes Declination, for

Dayes	1651 January deg. m.	dif.	1655 February deg. m.	dif	1659 March deg. m.	di:	1663 April deg. m	dif	1667 May deg. m.	dif.	1671 June d g. m.	dif.
1	21 49	10	13 56	20	03 35	24	08 26	22	17 58	15	23 10	04
2	21 39	10	13 36	20	03 11	23	08 48	21	18 13	15	23 14	04
3	21 29	11	13 16	21	02 48	24	09 09	22	18 28	15	23 18	03
4	21 18	11	12 55	20	02 24	24	09 31	22	18 43	14	23 21	03
5	21 07	11	12 35	21	02 00	23	09 53	21	18 57	14	23 24	02
6	20 56	12	12 14	21	01 37	24	10 14	21	19 11	14	23 26	02
7	20 44	12	11 53	21	01 13	24	10 35	21	19 25	13	23 28	01
8	20 32	13	11 32	22	South 49	23	10 56	21	19 38	13	23 29	01
9	20 19	13	11 10	21	26	24	11 17	20	19 51	13	23 30	01
10	20 06	13	10 49	21	North 02	24	11 37	21	20 04	12	23 31	00
11	19 53	14	10 37	22	22	24	11 58	20	20 16	12	23 32	00
12	19 39	14	10 05	22	00 46	23	12 18	20	20 28	12	23 31	00
13	19 25	15	09 43	22	01 09	24	12 38	20	20 40	11	23 31	01
14	19 10	15	09 21	23	01 33	23	12 58	19	20 51	11	23 30	01
15	18 55	15	08 58	22	01 56	24	13 17	20	20 02	10	23 29	01
16	18 40	15	08 36	22	02 20	23	13 37	19	21 12	11	23 27	02
17	18 25	16	08 14	23	02 43	24	13 56	19	21 23	10	23 25	03
18	18 09	16	07 51	23	03 07	23	14 15	19	21 33	09	23 22	03
19	17 53	17	07 28	23	03 30	23	14 34	18	21 42	09	23 19	03
20	17 36	17	07 05	23	03 53	24	14 52	18	21 51	09	23 16	04
21	17 19	17	06 42	23	04 17	23	15 10	18	22 00	08	23 12	04
22	17 02	17	06 19	23	04 40	23	15 28	18	22 08	08	23 08	05
23	16 45	18	05 56	24	05 03	23	15 46	17	22 16	08	23 03	05
24	16 27	18	05 32	23	05 26	23	16 03	18	22 24	07	22 58	05
25	16 09	18	05 09	23	05 49	23	16 21	17	22 31	07	22 53	06
26	15 51	19	04 46	24	06 12	22	16 38	16	22 38	06	22 47	06
27	15 32	18	04 22	23	06 34	23	16 54	17	22 44	06	22 41	07
28	15 14	19	03 59	24	06 57	22	17 11	16	22 50	06	22 34	07
29	14 55	20			07 19	23	17 27	16	22 56	05	22 27	07
30	14 35	19			07 42	22	17 43	15	23 01	05	22 20	08
31	14 16	20			08 04	22			23 06	04		

the third Years after the Leap-year, *viz.*

	1651			1655			1659			1663			1667			1671		
Dayes	*July*			*August*			*September*			*October*			*November*			*December*		
	deg.	m.	dif	deg.	m.	dif	deg.	m.	dif	deg.	m.	dif.	deg.	m.	dif.	deg.	m.	dif
1	22	12	8	15	21	18	04	35	23	07	04	22	17	32	16	23	06	05
2	22	04	9	15	03	18	04	12	23	07	26	23	17	48	16	23	11	04
3	21	55	9	14	45	19	03	49	23	07	49	23	18	04	16	23	15	04
4	21	46	9	14	26	18	03	26	23	08	12	22	18	20	16	23	19	03
5	21	37	10	14	08	19	03	03	23	08	34	22	18	36	15	23	22	03
6	21	27	10	13	49	19	02	40	23	08	56	22	18	51	15	23	25	02
7	21	17	10	13	30	20	02	17	24	09	18	22	19	06	14	23	27	02
8	21	07	11	13	10	19	01	53	23	09	40	22	19	20	14	23	29	01
9	20	56	11	12	51	20	01	30	24	10	02	22	19	34	14	23	30	01
10	20	45	11	12	31	20	01	06	23	10	24	22	19	48	14	23	31	00
11	20	34	12	12	11	20	North	43	23	10	46	21	20	02	13	23	32	00
12	20	22	12	11	51	20		20	24	11	07	21	20	15	13	23	31	00
13	20	10	13	11	31	21	South	04	24	11	28	21	20	28	12	23	31	01
14	19	57	12	11	10	21		28	23	11	49	21	20	40	12	23	30	02
15	19	45	13	10	49	20	00	51	24	12	10	21	20	52	11	23	28	02
16	19	32	14	10	39	21	01	15	23	12	31	21	21	04	11	23	26	02
17	19	18	14	10	08	22	01	38	24	12	52	20	21	15	10	23	24	03
18	19	04	14	09	46	21	02	02	23	13	12	20	21	25	11	23	21	04
19	18	50	14	09	25	21	02	25	23	13	32	20	21	36	10	23	17	04
20	18	36	15	09	04	22	02	48	24	13	52	20	21	46	09	23	13	04
21	18	21	15	08	42	22	03	12	24	14	12	19	21	55	09	23	09	05
22	18	06	15	08	20	22	03	36	23	14	31	19	22	04	09	23	04	05
23	17	51	16	07	58	22	03	59	23	14	50	19	22	13	08	22	59	06
24	17	35	16	07	36	22	04	22	24	15	09	19	22	21	08	22	53	06
25	17	19	16	07	14	23	04	46	22	15	28	18	22	29	07	22	47	06
26	17	03	16	06	51	22	05	09	23	15	46	19	22	36	07	22	41	07
27	16	47	17	06	29	22	05	32	23	16	05	18	22	43	07	22	34	08
28	16	30	17	06	07	23	05	55	23	16	23	18	22	50	06	22	26	08
29	16	13	17	05	44	22	06	18	23	16	41	17	22	56	05	22	18	08
30	15	56	17	05	22	23	06	41	23	16	58	17	23	01	05	22	10	09
31	15	39	18	04	59	24				17	15	15				22	01	10

The Table of the Sunnes Declination, for

Dayes	1652 *January* deg.	m.	dif	1656 *February* deg.	m.	dif	1660 *March* deg.	m.	dif	1664 *April* deg.	m.	dif	1668 *May* deg.	m.	dif	1672 *June* deg.	m.	dif.
1	21	51	09	14	01	20	03	17	23	08	42	22	18	10	15	23	13	04
2	21	42	10	13	41	20	02	54	24	09	04	22	18	25	14	23	17	03
3	21	32	11	13	21	21	02	30	24	09	26	21	18	39	15	23	20	03
4	21	21	11	13	00	20	02	06	23	09	47	22	18	54	14	23	23	02
5	21	10	11	12	40	21	01	43	24	10	09	21	19	08	14	23	25	02
6	20	59	12	12	19	21	01	19	24	10	30	21	19	22	13	23	27	02
7	20	47	12	11	58	21	00	55	24	10	51	21	19	35	13	23	29	01
8	20	35	13	11	37	21	South	32	23	11	12	20	19	48	13	23	30	01
9	20	22	13	11	16	22		08	24	11	32	21	20	01	12	23	31	00
10	20	09	13	10	54	22	North	16	24	11	53	20	20	13	12	23	31	00
11	19	56	14	10	32	22		40	23	12	13	20	20	25	12	23	32	00
12	19	42	14	10	10	22	01	03	24	12	33	20	20	37	11	23	31	01
13	19	28	14	09	48	22	01	27	24	13	53	19	20	48	11	23	30	01
14	19	14	15	09	26	22	01	50	23	13	12	20	20	59	11	23	29	02
15	18	59	15	09	04	22	02	14	23	13	32	19	21	10	10	23	27	02
16	18	44	15	08	42	23	03	38	24	13	51	19	21	20	10	23	25	02
17	18	29	16	08	19	23	03	02	23	14	10	19	21	30	10	23	23	03
18	18	13	16	07	56	23	03	25	24	14	29	19	21	40	09	23	20	04
19	17	57	17	07	33	23	03	48	23	14	48	18	21	49	09	23	16	03
20	17	40	16	07	10	23	04	11	23	15	06	18	21	58	08	23	13	04
21	17	24	17	06	47	23	04	34	23	15	24	18	22	06	08	23	09	05
22	17	07	18	06	24	23	04	57	23	15	42	17	22	14	08	23	04	05
23	16	49	17	06	01	23	05	20	23	15	59	18	22	22	07	22	59	05
24	16	32	18	05	38	23	05	43	23	16	17	17	22	29	07	22	54	06
25	16	14	19	05	15	23	06	06	23	16	34	16	22	36	07	22	48	06
26	15	59	18	04	52	23	06	29	22	16	50	17	22	43	06	22	42	06
27	15	37	19	04	29	23	06	51	23	17	07	16	22	49	05	22	36	07
28	15	18	19	04	04	23	07	14	22	17	24	16	22	54	05	22	29	08
29	14	59	19	03	41	24	07	36	22	17	39	15	22	59	05	22	21	08
30	14	40	19				07	58	22	17	54	16	23	04	05	22	13	08
31	14	21	20				08	20	22				23	09	04			

Dayes	1652			1656			1660			1664			1668			1672		
	July			*August*			*September*			*October*			*November*			*December*		
	deg.	m.	dif.	deg.	m.	dif	deg.	m.	dif	deg.	m.	dif	deg.	m.	dif.	deg.	m.	dif.
1	22	05	08	15	07	18	04	18	23	07	21	23	17	44	16	23	10	04
2	21	57	09	14	49	18	03	55	23	07	44	22	18	00	16	23	14	04
3	21	48	09	14	31	19	03	32	23	08	06	23	18	16	16	23	18	03
4	21	39	09	14	12	19	03	09	24	08	29	22	18	32	15	23	21	03
5	21	30	10	13	53	19	02	45	23	08	51	22	18	47	15	23	24	03
6	21	20	10	13	34	19	02	22	23	09	13	22	19	02	15	23	27	02
7	21	10	11	13	15	20	01	59	24	09	35	22	19	17	14	23	29	01
8	20	59	11	12	55	19	01	35	23	09	57	22	19	31	14	23	30	01
9	20	48	11	12	36	20	01	12	23	10	19	22	19	45	14	23	31	00
10	20	37	12	12	16	20	00	49	24	10	41	21	19	59	13	23	31	00
11	20	25	12	11	56	20	*North*	25	23	11	02	22	20	12	13	23	32	00
12	20	13	13	11	36	21		02	24	11	23	21	20	25	12	23	31	01
13	20	00	12	11	15	21	*South*	23	23	11	44	21	20	37	12	23	30	01
14	19	48	13	10	54	20		45	24	12	05	21	20	49	12	23	29	02
15	19	35	14	10	34	21	01	09	24	12	26	21	21	01	11	23	27	03
16	19	21	13	10	13	22	01	33	23	12	47	20	21	12	11	23	24	03
17	19	08	14	09	51	21	01	56	23	13	07	20	21	23	10	23	21	03
18	18	54	15	09	30	21	02	19	24	13	27	20	21	33	10	23	18	03
19	18	39	14	09	09	22	02	43	23	13	47	20	21	43	10	23	15	04
20	18	25	15	08	47	22	03	06	24	14	07	19	21	53	09	23	11	05
21	18	10	15	08	25	22	03	30	23	14	26	20	22	02	09	23	06	05
22	17	55	15	08	03	22	03	53	24	14	46	19	22	11	08	23	01	06
23	17	40	16	07	41	22	04	17	23	15	05	19	22	19	08	22	55	06
24	17	24	17	07	19	22	04	40	23	15	24	18	22	27	08	22	49	07
25	17	07	16	06	57	23	05	03	23	15	42	18	22	35	07	22	42	07
26	16	51	17	06	34	22	05	26	23	16	00	18	22	42	06	22	35	07
27	16	34	17	06	12	22	05	49	23	16	18	18	22	48	06	22	28	08
28	16	17	17	05	50	23	06	12	23	16	36	18	22	54	06	22	20	08
29	16	00	17	05	27	23	06	35	23	16	54	17	23	00	05	22	12	09
30	15	43	18	05	04	23	06	58	23	17	11	17	23	05	05	22	03	09
31	15	25	18	04	41	23				17	28	16				21	54	10

Stars near the Equinoctial or Declining to 50 degrees. Their Longitudes, Latitudes, Right ascensions, and Declinations.

An. Dom. 1660 compleat, with their Seasons for observation.

Mag.	*Their Names.*		*Long.* d. m.		*Latit.* d. m.	*R. Asc* d. m.		*Decli.* d. m.	*Seasons*
2	In the Whales tail the brighteſt	♓	27.48	s	20.47	6.39	s	19.52	*July* 17
2	In the girdle of Andromeda	♈	25.41	n	25.59	12.42	n	33.52	*July* 23
3	In the Whales back the weſtermoſt	♈	07.03	s	16.55	13.17	s	12.43	*July* 24
3	In the Whales back Eaſtermoſt	♈	11.34	s	15.46	16.51	s	09.55	*July* 27
3	In the Whales belly	♈	17.17	s	20.19	23.45	s	11.57	*Aug.* 03
3	In the Rams-horn the firſt	♈	28.29	n	07.08	23.47	n	17.36	*Aug.* 04
2	In the ſouth foot of Andromeda	♉	09.31	n	27.46	25.51	n	40.41	*Aug.* 06
3	In the Rams-head	♉	02.58	n	09.57	27.05	n	21.51	*Aug.* 07
3	Perſeus right ſhoulder	♉	25.18	n	34.30	40.09	n	52.09	*Aug.* 22
2	In the Whales jaw the brighteſt	♉	09.39	s	12.37	41.13	s	02.45	*Aug.* 23
3	Meduſa's head, or Algol	♉	21.29	n	22.22	41.38	n	39.37	*Aug.* 23
2	Perſeus right ſide	♉	27.09	n	30.05	44.51	n	48.33	*Aug.* 26
3	In the Pleiades the brighteſt	♉	25.16	n	04.00	51.52	n	23.01	*Sept.* 3
1	Bulls-eye, Aldebaran	♊	05.04	s	05.31	64.10	n	15.46	*Sept.* 17
1	The Goat, or Wagoners, &c.	♊	17.08	n	22.50	73.17	n	43.36	*Sept.* 27
1	In Orions left foot	♊	12.09	s	31.11	74.37	s	08.38	*Sept.* 28
2	Wagoners right foot	♊	17.52	n	05.20	76.15	n	28.15	*Sept.* 30
2	Orions left ſhoulder	♊	16.15	s	16.53	76.48	n	05.59	*Oct.* 1
2	Firſt in Orions girdle	♊	17.43	s	23.38	78.46	s	00.35	*Oct.* 3
2	Second in Orions girdle	♊	18.46	s	24.33	79.48	s	01.28	*Oct.* 4
2	Third in Orions girdle	♊	19.58	s	25.21	80.57	s	02.10	*Oct.* 5
2	Wagoners right ſhoulder	♊	25.20	n	21.28	83.53	n	44.51	*Oct.* 8
2	Orions right ſhoulder	♊	24.04	s	16.06	84.16	n	07.17	*Oct.* 9
2	The great Dogs fore-foot	♋	02.34	s	41.18	92.03	s	17.49	*Oct.* 17
2	In the bright foot of the Twins	♋	04.23	s	06.48	94.33	n	16.38	*Oct.* 19

Mag.	Their Names.		Long. d. m.		Latit. d. m.	R. Asc. d. m.		Decli. d. m.	Seasons
1	The great Dog in his mouth	♋	99.27	s	39.30	97.35	s	16.15	*Oct.* 22
2	In the upper head of the twins								
2	Castor	♋	15.33	n	10.02	108.15	n	32.34	*Nov.* 2
2	The lesser Dog	♋	21.10	s	15.57	110.27	n	06.03	*Nov.* 4
2	In the lower-head Pollux	♋	18.35	n	06.38	111.10	n	28.47	*Nov.* 5
1	Hydraes heart	♌	22.37	s	22.24	137.47	s	07.14	*Nov.*30
1	Lions heart	♌	25.09	n	00.26	147.36	n	13.35	*Dec.* 9
2	Lions neck	♌	24.51	n	08.47	150.18	n	21.31	*Dec.*11
2	Lions back	♍	06.33	n	14.20	164.00	n	22.23	*Dec.* 23
1	Lions tail	♍	16.55	n	12.18	172.58	n	16.28	*Dec.*31
1	Virgins spike	♎	19.08	s	01.59	196.53	s	09.21	*Jan.* 23
1	Arcturus	♎	19.31	n	31.22	210.15	n	21.19	*Feb* 6
2	South ballance	♏	10.23	n	00.26	218.06	s	14.34	*Feb* 14
2	North ballance	♏	14.40	n	08.35	224.45	s	08.04	*Feb.* 21
2	In the Crown the brightest	♏	07.30	n	44.23	230.06	n	27.54	*Feb.* 27
1	Scorpions heart	♐	05.05	s	04.27	242.15	s	25.34	*mar.*12
3	In Ophiucas right foot	♐	15.53	n	02.16	254.55	s	10.30	*mar.*26
1	In the Harp the brightest	♑	10.35	n	61.48	276.24	n	38.32	*Apr.*18
2	Eagles heart, *alias* Vultures.	♑	27.01	n	29.22	293.35	n	08.03	*May* 6
3	Dolphins tail	♒	09.24	n	29.08	304.18	n	10.13	*May*17
2	In the Swans tail	♓	00.46	n	59.57	307.31	n	44.07	*May*20
2	Water-pourers leg	♓	04.14	s	08.10	339.14	s	17.34	*Jun.* 20
1	Fomahant	♒	29.04	s	21.00	339.38	s	31.20	*Jun.* 20
2	In Pegasus leg	♓	24.42	n	31.08	341.54	n	26.16	*Jun.* 22
2	In Pegasus shoulder	♓	18.48	n	19.26	342.01	n	13.25	*Jun.*23
2	The head of Andromeda	♈	09.39	n	25.42	357.47	n	27.15	*July* 8
2	In Pegasus wing, the last	♈	04.30	n	12.35	359.03	n	13.18	*July* 9

Stars near the North Pole, their right *Ascensions, Declinations, and distance from the Pole, Anno Christi* 1660 *compleat, with their seasons for Observation.*

Mag.	*The Names of the Stars.*	*R. Asc.* d. m.	*Decli.* d. m	*distan* d. m.	*Seasons*
3	In the breast of Cassiopeia	005.29	54.42	35.18	*Jul.* 16
3	The North-star	008.04	87.31	02.29	*Jul.* 18
3	In the hip of Cassiopeia	009.16	58.54	31.06	*Jul.* 19
3	In Cassiopeias knee	016.05	58.27	31.33	*Jul.* 26
3	In Cassiopeias leg	022.43	61.59	28.01	*Aug.* 2
2	In Perseus right shoulder	040.09	52.09	37.51	*Au.* 20
2	In the great Bears side	160.09	58.11	31.49	*Dec.* 20
2	In the great Bears back	160.37	63.34	26.26	*Dec.* 20
2	In the great Bears thigh	173.52	55.35	34.25	*Jan.* 1
3	In Dragons tail, the last but one	175.56	71.37	18.23	*Jan.* 4
2	In the great Bears rump	179.37	58.54	31.06	*Jan.* 7
2	Between her tail and the Lions	188.58	40.37	49.23	*Jan.* 16
2	First in the great Bears tail	189.41	57.50	32.10	*Jan.* 16
2	The middlemost in her tail	197.30	56.43	33.17	*Jan.* 24
2	In the end of her tail	203.30	51.03	38.57	*Jan.* 30
2	In the bending of Dragons tail	209.34	65.59	24.01	*Feb.* 5
2	The fore-most guard	222.45	75.38	14.22	*Feb.* 19
3	The hindermost guard	231.09	73.18	16.42	*Feb.* 28
3	In Dragons head fore-most	260.44	52.35	37.25	*Apr.* 1
3	In Dragons head hindermost	267.12	51.37	38.23	*Apr.* 9
3	In Cepheus girdle	321.02	69.08	20.52	*Jun.* 2
3	In Cepheus left foot	333.53	64.15	25.16	*Jun.* 15
3	In the back of Cassiopeias chair	357.54	57.18	32.42	*Jul.* 08

Rules for finding the latitude or Poles elevation by the meridian altitude of the Sun or Stars, and by the Table of their Declinations beforegoing.

Case I. *If the Sun or Starre be on the Meridian to the southwards, and have south declination.*

Adde the Suns declination to his meridian altitude, and taking that total from 90 degrees, the remainder is the latitude, or the poles elevation northerly.

As admit upon the 10 of *January*, 1649, I finde by the foregoing tables, The Suns declination southerly

tables, The Suns declination southerly	20 deg.	00′
The Suns meridian altitude by observation	17	55.
The sum or total is	37	55.
Which substracted from	89	60.
There remains the latitude northerly	52	05.

But when you have added the Suns declination to his meridian altitude, if the total exceed 90 deg. substract from it 90 deg. and the remainder is your latitude to the southwards.

As admit the suns declination to be southerly	20 deg.	11′
The meridian altitude by observation	70	35.
The sum or total is	90	46.
From which substracting	90	00.
There remaines the latitude southerly	00	46.

Case 2. *If the sun or star be on the meridian to the southwards, and have north declination.*

Substract the suns declination from his meridian altitude, and that which remaines, substract from 90 deg. then that which remaines is your latitude or elevation northerly.

As admit upon the 20 of *April* 1649, I find

The suns declination northerly	15 deg.	01′
The meridian altitude by observation	64	22.
The remainder, substracting the declination is	49	21.
Which substracted from	89	60.
There remains the latitude northerly	40	38

Case 3. *If the sun or star be on the meridian to the northwards, and have north declination.*

Adde the Suns declination to his meridian altitude, that total take

take from 90 deg. and the remainer is your latitude southerly, or the elevation of the south pole.

But when you have added the Suns declination to his meridian altitude, if it exceed 90 degr. substract from it 90 degr. and the remainer is your latitude northerly.

Case 4. *If the Sun be to the northwards at noon, and have south declination.*

Substract the Suns declination from his meridian altitude, and that which remaines substract from 90 degr. then that which remains is your latitude southerly.

These rules might have been set down diverse other wayes, but let this suffice. And what is here said of the Sun, is also to be understood of the stars being upon the meridian.

5 *If you chance to observe when the Sun hath no declination.*

Substract his meridian altitude from 90 deg. and the remainer is your latitude.

6 *If you chance to observe when the Sun or star is in the Zenith, that is 90 deg. above the Horizon.*

Look in the Table for the declination of the Sun, or of that Star, and the same is your latitude.

7 *If the Sun come to the meridian beneath the Pole.*

If you be within the Artick or Antartick circle, and observe the Sun upon the meridian under the Pole, substract the Suns declination from 90 degrees, the remainer is the Suns distance from the Pole, which distance added to his meridian altitude, the summe or totall is your latitude or poles elevation.

And the like is to be understood of the stars; for which cause touching those stars that are neer the pole, we have expressed in the fore-going Table the complements of their declinations; that is, their distances from the north pole.

If therefore you observe any of these stars upon the meridian beneath the pole: adde to its meridian altitude found by observation, his distance from the pole, the total is the elevation of the north pole, or your latitude northerly.

If you observe any of those stars upon the meridian, above the pole, then from the meridian altitude of that star substract his distance from the pole; the remainer is the height of the north pole. Or out of the stars distance from the pole, substract his meridian altitude, the remainer is your latitude southerly.

FINIS.

POSTSCRIPT,

Containing another Method of Calculation, for Sailing by the Arch of a great Circle.

Courteous Reader:

WHereas I underſtand that ſome have complained of the difficulty and tedioufneſſe of the Calculation of the Requiſites in great Circle-ſailing, as alſo that it ſomewhat troubles them to determine the affections of the ſides and angles ſought for their ſatisfaction, I thought fit to annex another Method of Calculation than that by me already handled, which at three operations finds both the angles of Poſition and the Diſtance, and alſo determines the affections of all theſe Arks: the Proportions I ſhall make uſe of are demonſtrated in Mr. *Oughtreds* late *Trigonometry*, and other Authours, and ſerved to expedite the fourth Caſe of oblique Spherical Triangles (which I have handled in page 94) without the letting fall of a Perpendicular, which Caſe I ſhall propound otherwiſe.

Two ſides (together leſſe than a Semi-circle) with the angle comprehended given, To find both the other angles.

As the ſine of half the ſum of the ſides,
Is to the ſine of half their difference;
So is the Co-tangent of half the contained angle,
To the tangent of half the difference of the other angles.

Again,

As the Co-ſine of half the ſum of the ſides,
Is to the Co-ſine of half their difference;
So is the Co-tangent of half the contained angle,
To the tangent of half the ſum of the other angles.

Add the half difference of theſe angles to their half ſum, and you have the greater of the angles of Poſition, and if you ſubſtract the half difference from the half ſum, or the leſſer from the greater, there remains the leſſer angle of Poſition.

2 As from the Inverſe of either of the former Proportions, it follows that by having two angles and their oppoſite ſides given, the angle conteined by thoſe ſides, being the third angle,

 may

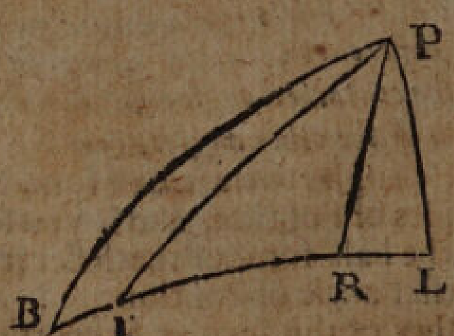
may be found: So also it follows, from either of these Proportions so varied, that they may be applied to the fifth Case of oblique Spherical triangles: that having the same *Data, viz.* two angles, and their opposite sides given, the third side may be found, and the Proportion is,

As the sine of half the difference of the angles,
Is to the sine of half their sum;
So is the tangent of half the difference of the sides,
To the tangent of half the third side.

Now these three Proportions may be easily wrought numerically, the third term in the two first Proportions being common to both, and at the same view in the Tables without turning them over for the two first Proportions, you may take out two terms at a time; and in working the last Proportion, when you search in the tangents to find the arks found by the two first Proportions, you may at the same view take out their sines, &c. to be used in the third Proportion. But to apply what hath been said to Sailing by the arch of a great Circle, the Proposition will be thus:

The Latitudes of two places being given together with their difference of Longitude, let it be required to find,

1 The angle of direct Position from the first place to the second.
2 And from the second place to the first.
3 The distance of those places in the arch of a great Circle.
4 And by what Longitudes and Latitudes it doth passe.



This we shall illustrate by example in a new Scheme: Let P represent the North pole, L the *Lizard* in Latitude 50 d. B the *Bermudas* in Latitude 22 deg. 25′, and let the angle B P L be 70 d. the difference of Longitude between the *Bermudas* and *Lizard* (though I account it to be much lesse, as I have elswhere intimated.)

Then

d.

Then is P B 57 35 } Half
PL 40 09 }
Sum 99 35 } 48 d. 47′½ } And the Complement of half
Diff. 17 35 } 8 47½ } B P L is 55 deg.

s 48 d. 47′½ ar. co.	.12359	Cofine ar. co.		.18124
8 47½ fine	9.18424	Cofine		9.99486
55 d. tang.	10.15477	Idem		10.15477
16 11 tang.	9.46260—64 d. 59′ tang.			10.33087
64 59				

Sum 81 10 } being the angle { L } for the greatest side subtends
Diff. 48 48 } of Position at { B } the greatest angle, &c.

Thirdly, to find the distance in the great Arch, to wit, the side, B L.

½ Diff. of the angles	16 d. 11′	sine ar. co.	.55484
½ Summe is	64 59	sine	9.95721
½ Diff. of the sides	8 47½	tang.	9.18937
The half distance	26 42′	tang.	9.70142

Wherefore the side B L, or the whole distance is 53 d. 24′, as we found it before.

And thus the six things in this Triangle, B P L, are all known, being the same here, as we found them in the former Method.

Fourthly, to find by what Longitudes and Latitudes this Arch of a great Circle doth passe.

First, let fall the perpendicular P R, which divides the triangle B P L into two right angled triangles, B R P and L R P, in either of which the hypothenusal and angles at the base are given. As in the triangle B R P, the angle at B is 48 d. 48′, and the hypothenusal B P 57 deg. 35′, whereby we find the perpendicular P R thus, saying, *s* P R + Rad. = *s* B + *s* B P (or if you would expresse the proportion, then, As Rad. to *s* B : so *s* B P to *s* P R) therefore *s* B + *s* B P − Rad. = *s* P R, and the operation stands thus:

s B 48 d. 48′, 9.87646
s B P 57 35, 9.92643
s P R 39 26, 9.80289

And this 59 deg. 26′, is the nearest distance of any part of this Circle from the Pole, and the complement

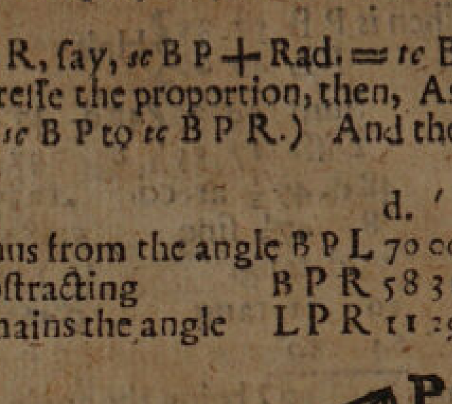
thereof

thereof 50 deg. 34′, is the greatest obliquity or latitude it hath from the Equinoctial.

Then for the vertical angle B P R, say, *sc* B P + Rad. = *tc* B + *tc* B P R (or if you would expresse the proportion, then, As *tc* B to Rad. or, As Rad. to *t* B, so *sc* B P to *tc* B P R.) And the operation is as followeth:

t	B 48 d. 48′,	10.05778		d. ′
sc	B P 57 35,	9.72922	Thus from the angle B P L	70 00
tc	B P R 58 31,	9.78700	substracting B P R	58 31
			remains the angle L P R	11 29

Now the difference of longitude from A to E, namely, the angle B P L being 70 deg. 00′. Let it be required to find by what latitude the arch B L doth passe for every fifth Meridian, or every fifth degree of longitude from B. As supposing the point I to differ in longitude from B 5 deg. I would know the latitude of the same point I.



Here seeing we have before found the angle B P R to be 58 deg. 31′, and the angle B P I is by supposition lesse by 5 degrees; therefore I P R is 53 deg. 31′, and the perpendicular P R we found to be 39 deg. 26′; therefore we shall find P I, saying, *sc* R P I + Rad. = *tc* P I + *t* P R, or (if you would expresse the proportion, then) As *t* P R to Rad. or as Rad. to *tc* P R :: so *sc* R P I to *tc* P I.

Therefore the operation is thus:

sc	R P I 53 d. 31′	9.77422	Thus we find the point I is distant
tc	P R 39 26	10.08492	from the pole, P 54 deg. 08′, and
tc	P I 54 08	9.85914	so the latitude thereof from the Equinoctial is 35 deg. 52′.

In like sort we may proceed to find the rest, which because they are many, we may for the more dispatch set the operations down in a tabular form in five Columns, as in the following Table appeareth. In the first Column is set down the longitude of every point from B, as the angles B P I, &c. In the second, their angles with the perpendicular, R P B, R P I, &c. In the third

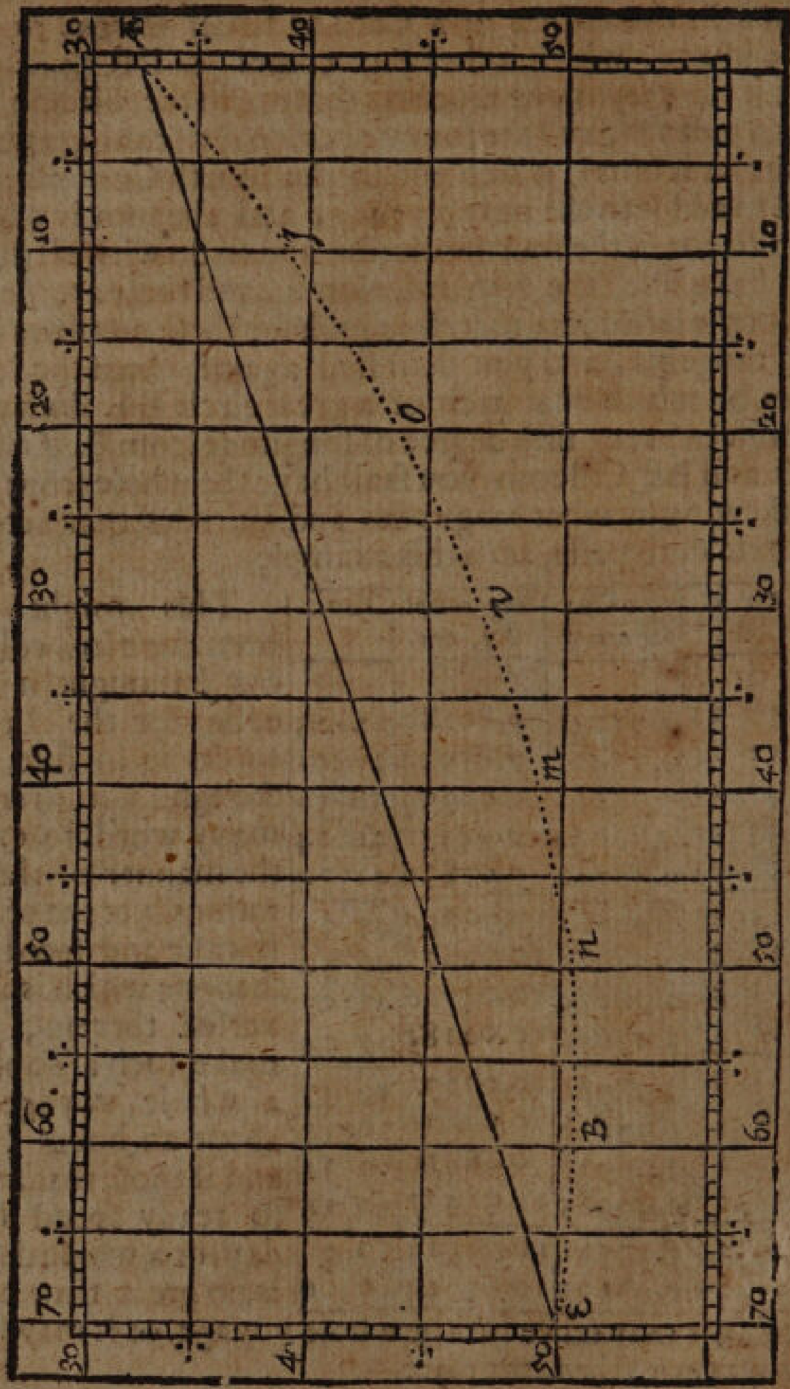
Column,

Column, the ſines complements of thoſe angles (out of the tables) then in the corner of a Card or ſmall piece of paper ſet down 10.08492, which is the tang. compl. of P R 39 deg. 26′, and add it to every of the numbers that are in the third Column; as firſt, to 9.71788, and the ſumme is 9.80280 (caſting away the firſt unite, as is uſual) which put in the fourth Column: Alſo proceed to add it to the next 9.77422, and the ſum is 9.85914, which alſo put in the fourth column; and ſo do with all the reſt, till you have filled the fourth Column, and theſe are the tang. compl. of the latitudes: therefore look for theſe numbers in the table of tangents, and you ſhall find againſt them the ſeveral latitudes by which that arch of a great circle B L doth paſſe, anſwerable to every fifth degree of longitude from B. And ſo in the firſt and laſt Column you ſhall have the whole ſcope of the work, ſhewing by what longitudes and latitudes that arch of a great circle doth paſſe, as in this example.

Longit. from B		*R P B. &c.*		*10 08492 ſine compl.*	*tang.-compl. P I, &c.*	*Latit.*	
d.	′	d.	′				
00	00	58	31	9.71788	9.80280	32	25
05	00	53	31	9.77422	9.85914	35	52
10	00	48	31	9.82112	9.90604	38	51
15	00	43	31	9.86044	9.94536	41	24
20	00	38	31	9.89344	9.97836	43	34
35	00	33	31	9.92102	10.00594	45	23
30	00	28	31	9.94383	10.02875	46	54
35	00	23	31	9.96234	10.04726	48	07
40	00	18	31	9.97691	10.06183	49	04
45	00	13	31	9.98780	10.07272	49	47
50	00	8	31	9.99518	10.08010	50	15
55	00	3	31	9.99918	10.08410	50	31
60	00	1	29	9.99985	10.08477	50	34
65	00	6	29	9.99721	10.08213	50	23
70	00	11	29	9.99122	10.07614	50	00

This might have been done in 4 columns but I thought it more eaſie for the Learner to do it in five; and though it requires many words to expreſs the manner of the operations, yet the practice is eaſie and ſpeedy, ſo that one which is a little verſed therein, may make ſuch a Table for a whole voyage, in about an hours ſpace; and if thoſe that are not ſo ready ſpend half a day, or a whole day, it is no great time nor labour, ſeeing it may ſerve for the whole voyage, or it may be for many voyages to the ſame place.

Some


30
40
50
10
20
30
40
50
60
70

Some think the Diſtance need not be found, nor doth either this or the former method of Calculation require the finding thereof, though in it ſelf deſirable. The angles of Poſition are of good uſe, for they ſhew at each place how far you muſt begin to ſhape your firſt Courſe from the Meridian, and thoſe that will find theſe things, muſt calculate for them as we have done, and theſe are the Difficulties complained of by ſome in this manner of Sailing by the arch of a great Circle, which notwithſtanding practice will render familiar. I confeſſe (as formerly I have ſaid) it would be hard to proceed in all points by the doctrine of Triangles, to calculate every courſe and diſtance that a ſhip muſt run, ſailing by the arch of a great Circle; which I did duly conſider when I wrote that Treatiſe, (as by thoſe words there uſed may appear.) And conſidering with my ſelf of ſundry wayes how thoſe difficulties may be ſhunned, I could think upon none better than by pricking out upon *Mercators* Chart, the foreſaid longitudes and latitudes by which the foreſaid arch of a great Circle doth paſſe; which may well be done within two or three minutes. And ſo the arch it ſelf being pricked or traced out in that Chart, there is no more difficulty in the traverſes and reckonings to keep neer that arch, than there is in the ordinary uſe of that Chart. Which things how to perform on the ſaid *Mercators* Chart, I have ſhewed in the tenth and laſt probleme of that Subject, and ſhall add no more here, ſave onely a blank which ſhould have been inſerted in page 168, or 169; whereby the things before ſpoken of, may be the better underſtood; this is drawn in a ſmall ſcale, that it might be conteined in a page: but in reckonings at ſea, the ſcale of the blank may be ten times ſo large, and yet not comberſome, eſpecially if it be kept on ſingle ſheets of paper, which may be joyned together, and taken aſunder at pleaſure.

I have yet hopes to do ſomething hereafter for the further Illuſtration of it, and ſome other points in the Art of Navigation, if mine other neceſſary occaſions will permit; the rather for that love and courteous enterteinment my former labours have found amongſt Sea-men eſpecially. But at preſent I ſhall add no more, leſt this come too late, the ſhip waiting only for a wind to ſet ſail. *Sommer Iſlands* R. N.

14 Feb. 1659.

To

To give a neer conjecture of the Suns place without Tables.

Though this Rule seemeth to be needlesse, being the book is furnished with tables for this purpose, yet the performing the same without tables, may be an ease to memory by observing a Rule, which I being effected with, I assayed to frame so that it might come neer the truth: And having put the letters in order, with some little consideration, this Distick was first presented to my thoughts sutable to the Rule: And the verse of twelve words relating to the twelve moneths of the year, gives you the day of the moneth on which the Sun entereth into the signe proper to the said moneth, to which purpose take the verse following, which is fitted to the Suns ingresse into

Gemini, *Cancer*, *Leo*, *Virgo* } on the { 10 *May*, 11 *June*, 12 *July*, 12 *Aug.* } which I take to come nearer to the truth, than if we should fit the same same to the following day of each of these moneths, as others have done.

Encline an ear, judicious order own,
Or virtue in its action 'soverthrown.

's overthrown, in stead of *is overthrown*, for conformity to the rule; which I note to take away the occasion of carping.

March	*April*	*May*	*June*	*July*	*August*
♈	♉	♊	♋	♌	♍
Encline	*an*	*ear*	*judicious*	*order*	*own*,
Septemb.	*Octob.*	*Novemb.*	*Decemb.*	*January*	*February.*
♎	♏	♐	♑	♒	♓
Or	*virtue*	*in*	*its*	*action*	*'soverthrown.*

It is to be understood, that as there are 12 moneths in the yeer, so the Ecliptick is divided into 12 signs, and in every of their moneths months, the Sun enters into one of theſe ſignes: As in *March* into ♈, or *Aries*; In *April* into ♉, that is *Taurus*; In *May* into ♊, that is *Gemini*; and ſo the reſt ſucceſſively. But now to know what day of the month the Sun enters the correſpondent ſign, we have ſet twelve words, each anſwering to his month and ſigne, as before you may ſee; each of theſe twelve words begin with a vowel, except the laſt for *February*, which begins with an *s*, and ſignifies 8, and ſo denotes, that upon the 8 day of *February* the Sun enters into *Piſces*. The vowels at beginning of every other word ſtand for numbers, as *A* for 1, *E* for 2, *I* for 3, *O* for 4, *V* for 5.

Now, if in any month you would know what day the Sun enters the ſigne for that month, put the number of that vowel to 8, and ſo you have the number of the day whereon the Sun enters that ſigne.

Example. I would know upon what day of July the Sun enters into ♌, that is *Leo*, I find the word anſwering to that month is [*order*] whoſe vowel *O* ſignifies 4, which put to 8 makes 12; therefore, I ſay, upon the 12 of *July* the Sun entereth into *Leo*: And ſo for any other month according to this Table.

Thus we ſhall find the Suns entrance into the reſpective ſignes to be

March	10	♈ *Aries*
April	9	♉ *Taurus*
May	10	♊ *Gemini*
June	11	♋ *Cancer*
July	12	♌ *Leo*
Auguſt	12	♍ *Virgo*
September	12	♎ *Libra*
October	13	♏ *Scorpio*
November	11	♐ *Sagitary*
December	11	♑ *Capricorn*
January	9	♒ *Aquarius*
February	8	♓ *Piſces*

Now knowing on what day of the month the Sun enters any ſigne, it will be eaſie to know for any other day of that month the degree the Sun is in:

Thus.

Subſtract the day of the Suns entrance into the ſign of the month, from the day propoſed (adding to it 30, if it be too little) the remainer is the degree wherein the Sun is, which if you borrowed 30, is the degree of the ſigne preceding; otherwiſe it is the degree of the ſigne of the month propoſed.

Example

Example 1.

July the 25, I would find in what signe and degree the Sun is, I find by the former verses, the Sun enters into *Leo* the 12 of July, which taken from 25, there remains 13, shewing that the Sun is in the thirteenth degree of *Leo* that day, that is 47 degrees from the beginning of *Libra*, being the neerest Equinoctial point.

Example 2.

July the fifth, let the Suns place be required, I find that on the 12 of July, the Sun enters *Leo*, which 12 substracted from 5, that is from 35 (borrowing 30) there remains 23, shewing that the Sun is in the 23 degree of *Cancer*, which is the signe preceding the signe of this month, because we borrowed 30.

THE

THE EPILOGUE
Or, CONCLUSION.

IN handling the Doctrine of Triangles, I have not set down all that I might, but this I have chiefly endevoured according to my slender ability, namely to found it upon such Axiomes as might be few in number, easie for memory, ready in practice, and consonant to the nature of Logarithmes: yet so as they might also direct the operations by natural sines, tangents and secants, and likewise by instruments. In the demonstration of these Axioms, I have laboured to be briefe and perspicuous. In deducing the cases from them, I have opened the method how it is done, and of all the questions incident in every case, whereby the Reader may conceive the like in any triangle proposed. The examples I have set down in such sort as might best manifest the operation, and be a most ready way of practice. The application I have partly shewed in handling the cases, but further in a subject wherein all the Problemes of plain and spherical Triangles may aptly, ordinarily, and to good purpose be used. The Tables I have so ordered, as I thought might be most easie and ready for ordinary use, according to the method I have used. Many varieties that might have been shewed, I have purposely omitted, that I might not seem tedious. As I have shewed from one ground how to resolve all the cases and questions of a plain right angled triangle as hath been usual before. Whereas for the fift case where the base and perpendicular are given; to find the hypotenusal: Mr. Briggs hath shewed a more peculiar way, in his Arithmetica Logarithmica chap. 19. So likewise I have shewed the resolution of the twelfth case, (where three sides of an oblique triangle are given: to find an angle) by an Axiome briefe, easie, and the same in effect that hath been long used for that purpose; whereas it might have been done, and that peradventure a little more speedily, by such a way as may be gathered from M. Briggs his Arithmetica Logarithmica, chap. 18. though he do not expresly handle it. But then I conceived the rule would not have been so easie for memory, nor so applyable to instrumental operations, which I intended breiefely to

touch,

touch, if other occasions had not hindred. Yet since understanding it would have been acceptable to diverse, and being very apt for the Arithmetical work, I have thought good here to place it.

Mr. *Briggs* hath shewed chap. 18. sect. 3 and 5: Having the three sides of a triangle, how to find the Semidiameter of the inscribed circle, and any of the angles thus, or to this effect.

Substract the three sides severally from half the perimeter, and note the remainers, then first.

B. As half the perimeter
C. to one of the remainers
D F. so is the rectangle of the other two
G G. to the square of the semidiameter of the inscribed circle.

Secondly,

C. As one of the foresaid remainers
G. to the semidiameter of the inscribed circle
R. so is Radius
A. to the tangent of half the angle opposite to that remainer, these he hath there demonstrated.

By the first of these it is evident, that As B to C; so is D F to G G, and multiplying the first and second by the second, As B C to C C; so is D F to G G: and alternarly, As B C to D F; so is C C to G G.

Again, by the second, As C to G; so R to A, and squaring them, as C C to G G; so is R R to A A: But as C C to G G; so is B C to D F as before was proved, therefore, as B C to D F; so is R R to A A.

That is,

B C. As the rectangle of half the perimeter in one of the foresaid remainers;
D F. is to the rectangle of the other two remainers;
R R. so is the square of Radius,
A A. to the square of the tangent of half the angle opposite to that first remainer.

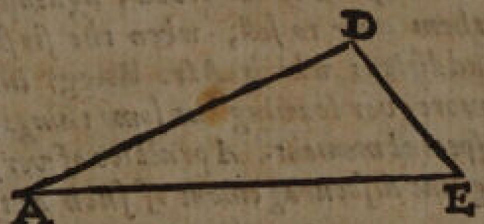
Thus being limited, that I cannot conveniently in this place demonstrate by words at large, this Algebraical deduction and demonstration may suffice, which to the learned in that kind will not be obscure: Hence then,

The

The three sides of a plain triangle being given; we may find any of the angles.

Substract the three sides severally from halfe the perimeter, noting the remainers. Then to the complements Arithmetical of half the perimeter, and of the remainer opposite to the angle required; add the Logarithmes of the other two remainers, half the summe of these four is the tangent of half the angle opposite to that first remainer.

As let A D E be the triangle whose three sides are given.



A E. 189 } And let there
A D. 156 } be required the
E D. 075 } angle at D.

The perimeter 420			
half the perimeter 210		*co. ar.*	7.6777807
from which substract *A* E 189	remains 21.	*co. ar.*	8,6777807
A D 156	remains 54, logar.		1.7323937
E D 075	remains 135, logar.		2.1303337
d. m.			20.2182888
52—07½	$t\frac{1}{2}$D.		10.1091444

which double is 104—15 the angle at D required.

Note that the side opposite to the angle required, which is here *A* E 189, being substracted from half the perimeter 210, the remainer 21 is that which we have before called, the remainer opposite to the angle required.

2 *Example.*

Let there be required the angle at E.

Half the perimeter 210		*co. ar.*	7.6777807
from which substract *A* D 156 rem.	54	*co. ar.*	8.2676063
A E 189 rem.	21	logar.	1.3222193
E D 075 rem.	135	logar.	2.1303337
d. m.			19.3979400
26—34	$t\frac{1}{2}$E		9.6989700

Which doubled is 53—08 the angle at E required.

Now whereas I have here, and in sundry places of this Book cited

M Briggs *his* Arithmetica Logarithmica, (*lest I may seem to abuse the Reader) you are to understand not the Book put forth about a moneth since in English, as a translation of his, and with the same title; being nothing like his, nor worthy his name; but the Book which himself put forth with this title in Latine, being printed at* London, Anno 1624. *And here I have just occasion to blame the ill dealings of these men, both in the matter before mentioned, and in printing a second Edition of his* Arithmetica Logarithmica *in Latine, whilst he lived, against his mind and liking; and brought them over to sell, when the first were unsold; so frustrating those additions which Mr.* Briggs *intended in his second Edition, and moreover leaving out some things, that were in the first Edition, of special moment. A practice of very ill consequence, and tending to the great discouragement of such as take pains in this kind.*

TEN
CHILIADES
OR, THE
LOGARITHMES
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