Short, but yet plain elements of geometry; shewing how by a brief and easy method, most of what is necessary and useful in Euclid, Archimedes, Appollonius, and other excellent geometricians, both ancient and modern, may be understood / written in French by F. Ignat. Gaston Pardies; And render'd into English, by John Harris.

#### **Contributors**

Pardies, Ignace Gaston, 1636-1673 Harris, John, 1667?-1719 Euclid Archimedes Apollonius.

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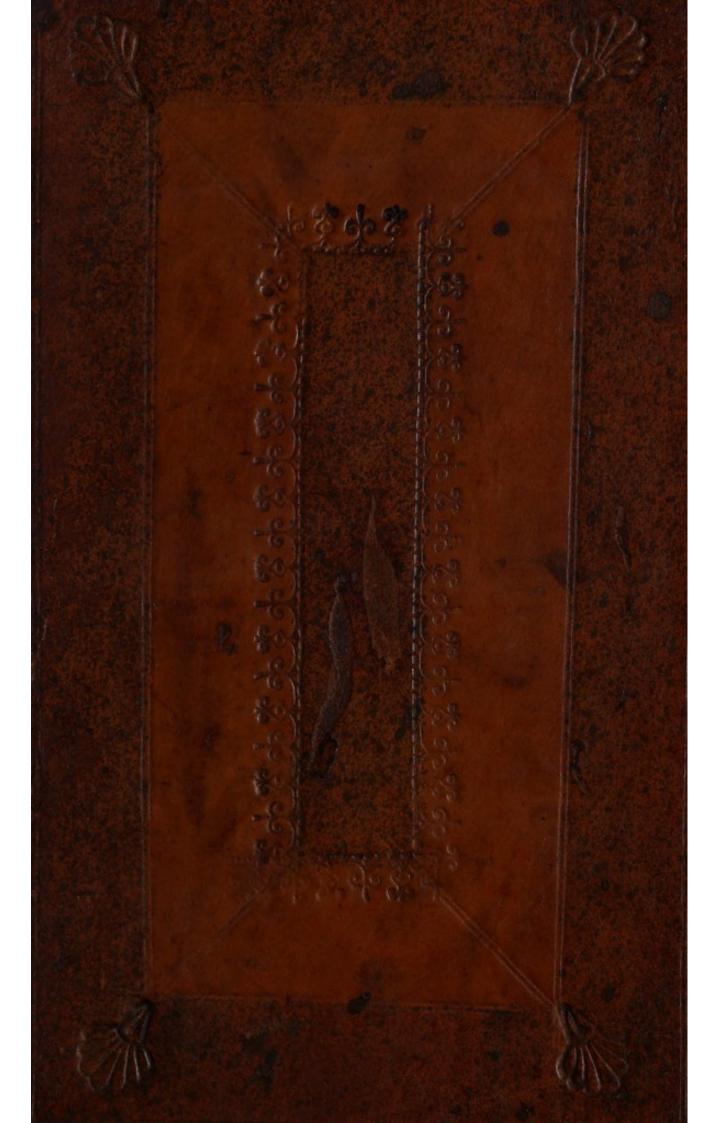
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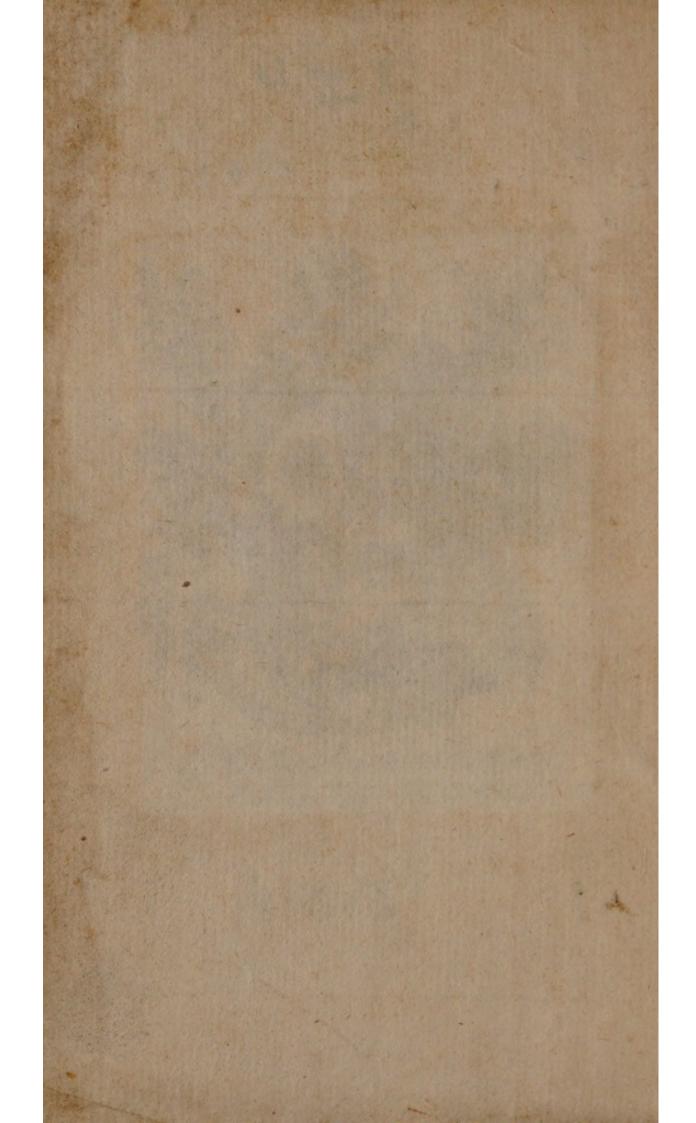


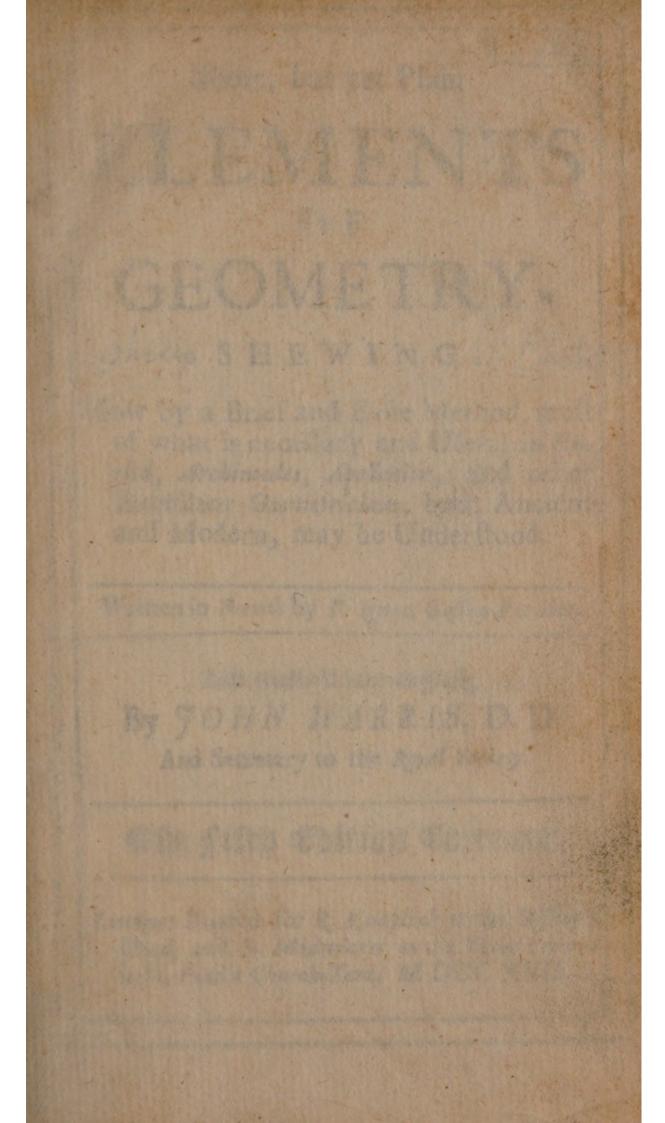
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PARDIES









Short, but yet Plain

# ELEMENTS

OF

GEOMETRY.

Shilip SHEWING (): Chefn

How by a Brief and Easie Method, most of what is necessary and Useful in Euclid, Archimedes, Apollonius, and other Excellent Geometricians, both Ancient and Modern, may be Understood.

Written in French by F. Ignat. Gaston Pardies.

And render'd into English,

By JOHN HARRIS, D. D.

And Secretary to the Royal Society.

The Kifth Edition Corrected.

London: Printed for R. Knaplock at the Bishop's Head, and D. Midwinter at the Three Crowns in St. Paul's Church-Yard. M DCC XVII.

Short, but yet Plain

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Leader's Valued for R. Magnisok as the Editop's

House and D. Midmines and Store Three Cresses
in St. Parch Church Town M. D.C. N. VII.



The Epittle Dedicatory.

Mathematick Lecture which longs

#### aftention O. T. posts of your fineer

# My Worthy Friend

# Charles Cox Esquire,

Member of Parliament for the Burgh of Southwark.

## Dear SIR,

A MONG the many Obligations You have conferr'd upon me, I account it not the least, that you have given me a Rise to Revive my Mathematical Studies; in which as I have formerly spent some time, so I know of no more Useful way of employing my leisure Hours.

And indeed, Sir, the Diversion and Advantage I have lately reaped from them, hath (by the Divine Blessing) both supported me under, and in a good Measure carried me through such Pressures and Difficulties, as I once almost despaired of Surmounting.

A 2 The

## The Epistle Dedicatory.

The Mathematick Lecture which Tou at first set up Gratis in your Burgh, and which out of an uncommon Generosity, You did asterwards remove into the City of London, is a Demonstrative Proof both of your sincere Endeavour to promote the good of your Country, and also of your Capacity to do it the best way. And as I have already in a good degree, so I hope to see such Effects from so Noble a Design, as will render your Name justly Honourable to Posterity, as well as to this present Age. Sir, You know your Self and Me too well, to take this for Flattery. 'Tis what Truth, Justice and Gratitude Oblige me to say.

I shall only add, That I am again glad of this opportunity to shew the Just Esteem I have of your Merit, and the equal Regard

I have for your Friendship. I am,

SIR,

Your most Obliged

Humble Servant

John Harris.

#### THE

# TRANSLATOR

TOTHE

# READER.

A fter frequent Perusal, and mature Deliberation on this Book; I judge it to be the plainest, shortest, and yet easiest Geometry I have ever
seen Publish'd: And therefore I thought it very well
worth my while to let it appear a sifth time in our
own Language, as it had already done twice before
in the Latin Tongue. And 'tis so well esteemed
of, by very Competent Judges amongst Us, as to
be read in our Universities, by Tutors to their Pupils: And also, which is not usual with Books of
this kind, there hath been three entire Impressions
sold off in a little more than as many Years time.

As to the Translation; I have by no means Obliged my self servilely to follow the French way of Expression; for indeed a Literal Version of a Book out of any Language will be scarce Intelligible in English. I have therefore all along aimed rather to give you F. Pardies's Sense, than his Words; and have made him speak what I judge be would have done, had he wrote in our Language. I have made no Scruple to add any thing that I saw necessary to render him clear and intelligible; and particularly

# The Translator, &c.

ticularly what follows, which was not in some of

the former Editions.

As the second Book of Euclid about the Power of Lines; The Mensuration of the Surfaces of Solids, Archimedes bis Proportion of the Inscribed Cone and Sphere to the Circumscribing Cylinder; the Figures of the & Regular Bodies; several Additions and Improvements in the Doctrine of Proportion: The Mensuration of the Frustums of Pyramids and Cones; some new Properties of a Rightangled Triangle, and of the Circle, &c. I have also left out some more of Pardies's Propositions, which on repeated Experience in Teaching, I have found less useful; as also all the Elements of plain Trigonometry, which I had before added to his Ninth Book; because I have publish'd a small Treatise on that Subject by it self; and my chief Aim now bath been to lead the Learner into a little more Abstracted and Concise, tho' a most useful and Universal Method of Demonstration; introducing now and then a little of Algebra, that I may thereby engage the Reader in a Love of, and Value for that most Noble and Wonderful Science: And to give him a good Foundation to Build upon, and a sufficient Rise thereby to carry bim into Fluxions, and the new Methods of Investigation and Demon-Stration, where be will find sufficient Satisfaction. Nor need be be discouraged at the Attempt, for tis well known, that I have taught several Persons to understand the Elementary Parts of all Mathematicks so well, that they have been able to go on every where, without the Assistance of any Master, in less than a Year's Time. Pardie's



# Pardie's Advice to those who would Understand Geometry.

There will be fome Labour and Difficulty at first, but they will break thro' it in two or three Days.

2. They ought not to be discouraged, if they meet with some things which they do not understand at first; Geometry is not so easie to be attained, as History.

g. If, after they have Read and Considered attentively any Proposition, they find they don't understand it; let it be passed over, it will probably be Intelligible by reading farther, or at least when they have gone over the whole, and have began to Read it over a-new. There are indeed many things in Geometry, that will

# Advice to those, &c.

will never be well understood at first Rea-

ding over.

4. The Numbers which are within the Parenthesis, v. gr. (3.14.) shew that the Matter there spoken of, hath been proved elsewhere, viz. in this Instance, in the fourteenth Article of the Third Book; And they ought always to mind the Number of the Article, and to consult the Places referred to, that so they may gain the Demonstration of what they Read.

5. When they meet with any Words which they don't understand, they may consult the Table at the End of the Book.

6. 'Tis good to have a Master at first, to Explain to them the Nature and Manner of the Demonstrations: For by that means they will Understand the thing much easier and much sooner, than they can do by Reading by themselves.



ELE-



# ELEMENTS

OF

# GEOMETRY.

## BOOK I.

## Of Lines and Angles.



Y the Word Quantity, which in the general is the Subject of Geometry, we mean that whereby one Thing, being compared with another of the same Nature, may be said to be Greater or Less than,

Equal or Unequal to it: As Extension, i.e. Length, Breadth or Thickness, Number, Weight, Time, Motion, and all those things, which are capable of being so compared as to More or Less, are the

Object of Geometry.

2. We design nevertheless to consider now only Extension; as being that which serves for an Example and Rule to Measure all other Quantities by.

3. That

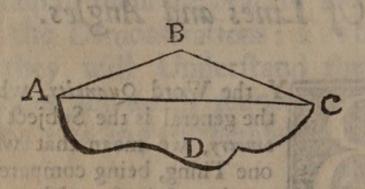
3. That Quantity which, being supposed without any Breadth or Thickness, is extended only in Length, is called a Line. That which hath both Length and Breadth, (but is supposed to have no Thickness) is called a Surface, or Superficies: And that which hath Length, Breadth and Thickness, is called a Body, or Solid.

4. A Point is that, which is confider'd as having no manner of Dimensions; and as being Indivisible in every respect. The Ends or Extremities of

Lines, as also the Middle of them, are Points.

or Curved Ones: Also there are Even and Plain Surfaces which are called Plans; and there are Crooked or Curved Ones: Which like a Vault, (or the Tilt of a Boat or Waggon,) are Convex above, and Concave below.

The Generation of Lines may easily be conceived to be made by the Motion or Fluxion of a Point, as A.



Which if it move directly from the Term A, to the Term C, or go the nearest or shortest way possible, it then forms what Geometers call a Right, or Strait Line.

If it go first directly to B, and then also the nearest way to C; it forms two right Lines, AB and BC, which taken together, are longer than the Line AC; and consequently, two sides of any Triangle must be longer than the Third.

200 2 48

If

If the Point A, move not in one or more right or strait Lines rowards C, it must go Crooked, and so will form a Curve or crooked Line, as A D C.

And from hence also 'tis plain, That any two Points moving with equal Velocity, will in the same

Time generate equal Lines.

6. When two Lines meet in a Point, The Aperture, Distance or Inclination between them, is call'd an Angle. Which, when the Lines forming it are right or strait Ones, is called a Rectilineal Angle; as A. But if they are crooked, 'tis called a Curvilineal One; as B. And when one is strait and the other crooked, 'tis called a Mixt Angle; as C. the Leas of Blying on it. For its

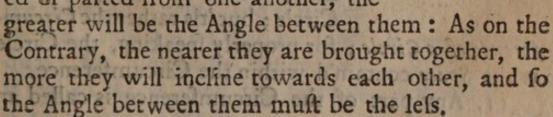
N. B. The Dines forming any Angle, are called its Logs are much igs Lestichned to one another, than those of B, and therefore it is lefs than B.



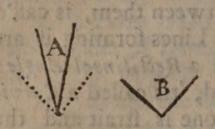
7. That Angle is faid to be less than another, whose Legs are more enclined to (or nearer to) each other. Let there be two Lines AB and A C meeting in the Point A. If you ima-

gine those Lines to be moveable like the Legs of a pair of Compasses, and yet fastened together in A, as with a Toint; 'tis easie then to conceive, that the further they are opened or parted from one another, the

Line de (pessing eineugh ehe Center) and



8. It must therefore be observed, that the Quantity of Angles, is by no means Measured by the Length of their Legs, but by their Inclination. Thus, v. gr. The Angle B is bigger than A; tho' the Legs of the Angle B, are much shorter than those

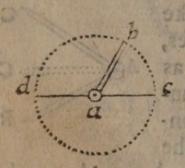


of A: But then those of A are much more inclined to each other, than those of B. And to apprehend this the better, imagine the Angle B to be put upon A, as you may conceive by the prickt Lines about A,

which represent the Legs of B lying on it. For 'tis plain the Angle A will be easily contained within B: and that its Legs are much more inclined to one another, than those of B, and therefore it is less than B.

9. An Angle is usually marked by three Letters, of which the middlemost, and which always is placed at the Angular Point where the Lines meet, denotes the Angle. As in the Figure following, b a c denotes the Angle made by the two Lines b a and ca meeting in the Point a.

10. If we imagine the Line a b fastened by its end a, in the middle of the Line d c, but yet so as to be



moveable on a, as on a Center: If then you conceive it to be moved quite round, till it arrive at the place where it began, the Point b will describe a Curve Line, which is usually called a Circle; but 'tis rather the Circum-

ference of a Circle; for properly speaking, the Circle is the Space contain'd within the Circumference.

11. Any part of the Circumference is called an

Ark, as bc.

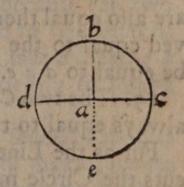
12. The Line d c (passing through the Center) and terminated by the Circumference, is called the Diameter. meter, and divides the Circle into two equal parts. Also every right Line passing thro' the Center a (and Terminated at each end by the Circumference) divides the Circle into two equal parts, and will be a Diameter.

13. The Line a b or a c, or any other drawn from the Center to the Circumference, is called the Radius or Semi-diameter.

14. All Radius's or Semi diameters (of the same or equal Circles,) are equal. (As is plain from the Genesis of a Circle given in Art. 10.)

15. When the end b of the Radius a b is equally Distant from the two ends of the Diameter d c;

That is, when the Point b is in the very middle of the Semi circumference d b c; then will b a make two Angles with d c that are called Right Ones: Which are equal to one another, that is, the Angle d a b is equal to b a c. And if the Line b a be produced below

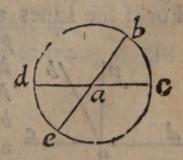


gles; and it will be another Diameter; which with the former de will divide the Circle into four equal Parts.

one to another, viz. b a to d c, and d a to b e.

17. But if b be nearer to one End of the Diame-

ter (or Right Line) dc, than it is to the other, it is then said to fall on the other obliquely; and it makes with dc two Angles that are Unequal: Of which the Lesser bac is called Acute, and the Greater dab is called Obtuse.



If the Line a b be produced to e, it will be a new Diameter, and will make below two other Angles:

d a c

So that in the whole there will be four Angles; of which those two that touch only in the Angular Point, as b a c and e a d; as also, d a b and e a c, are called Vertical, or Opposite Angles. But those that have one Leg common to both, as

dab, and bac; and bac, and eac, are called Ad-

joining or Contiguous Angles.

18. Those Angles which (at equal Distances from the Angular Point) are subtended by equal Arks, are also equal themselves. As if the Ark b c be proved equal to the Ark d e, then will the Angle b a c be equal to d a e.

19. The two Contiguous Angles taken together are

always equal to two Right Ones.

For as the Line d c is a Diameter, and therefore, cuts the Circle into two equal Parts, the two Arks d b and b c taken together, will be equal to a Semi-circle. Wherefore the two Angles d a b and b a c together, will be equal to two Right Ones, because they compleat the whole Semi-circle, as two Right Ones do. (Art. 15.).

20. So that this Proposition is of Universal Truth, That one Right Line falling on another, makes the Contiguous Angles (together) equal to two Right Ones. For if the Lines are Perpendicular to each other, as

p a is to d c. Then tis plain the Angles must be Right (by the 15.) And if the Line fall Obliquely, as b a doth, then indeed the Angles are Unequal: But as much as the Obtuse one d c b exceeds one Right

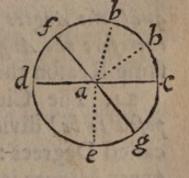
Angle, by so much is the Acute one bac exceeded

by the other Right One. So that the Smallness of one is compensated by the Greatness of the other.

21. Hence also it follows Conversely, for (whereever the Property is found, there the Thing is, in Geometry,) that if two Angles which have one Leg Common to both, do make Angles equal to two Right Ones, their other Legs do make but one Right Line. Let the Angles d a b and b a c be (together) equal to two Right Ones. Then I say, that the Lines d a and a c do join so together, as to make one Right Line (vid. Fig. in Art. 17.) which is clear from what hath been said. For if on the Center a you describe a Circle d b c e, the two Arks d b and b c will be equal to a Semi-circle, because the two Angles dab and bac are supposed to be equal to two Right Ones. Wherefore the Lines d a and c a will make a Diameter, and consequently be joined into one Right Line.

22. If from the Point a you draw several Lines, as ad, af, ab, ah, ag, &c. they will make Diverse Angles; and all those Angles taken together, be they

more or less, will be equal to four Right Ones. For 'tis clear, all these Angles together do compleat the Circle d b c e, whose Circumference they divide into as many Arks as there are Angles. Now all these together are equal to four Quarters of a Circle; which is

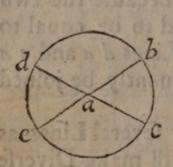


as much as to say, that all the Angles are equal in the whole to four Right Ones; for so many Right Angles do also compleat the Circle.

### AXIOM I.

If to, or from equal Things, you Add or Substract Equals, the Sums or Remainders will be equal.

23. The Vertical or Opposite Angles are equal. Let there be two Right Lines d a c and b a e (crossing or cutting one another in the Point a,) I say, the Angle d a e is equal to b a c. For the Ark b d with the



Ark b c makes a Semi-circle; and fo doth the same Ark b d with the Ark d e. Therefore the Ark b c must be equal to de; because the Ark d b continues the same, whether it help to compleat the Semi-circle with de or bc: (wherefore being taken away from both, it must

leave the Ark de equal to bc. But if the Arks be equal, the Angles subtended by them must be so too, and therefore the Angle dae is equal to bac) And by the same Reason the Angle dab will be equal to eac.

24. The Circumference of every Circle is (supposed to be) divided into 360 equal Parts, which are called Degrees: And every Degree into 60 Minutes, every Minute into 60 Seconds, every Second into 60 Thirds, and so on infinitely. And to determine the Quantity of every Angle, we compute the Degrees that (the Ark, which is its Measure) doth contain, v. gr. When we speak of an Angle of 90 Deg. we mean a Right Angle; because the Right Angle contains the fourth Part of the whole Circumference, which is 90 Deg. the fourth Part of 360. So an Angle of 60 Deg. is an Angle that contains two Thirds of a Right One.

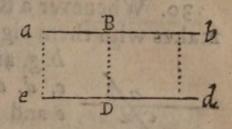
25. (Deg.)

9

25. (Degrees are marked either with Degr. or usually with a small Cypher over the last Figure, as 60°.) Minutes with a small Line, as 50', Seconds with two such, as 20", Thirds with three such, as 25", &c. So that 25° 32' 43", is to be read, 25 Degrees, 32 Minutes, 43 Seconds.

26. Two Lines are faid to be Parallel, when they

run always equi distant from each other. Thus the two a Lines a b and e d are Parallel, if they are equally distant from each other in a e, in B D, in b d, and in all other Places.



27. This Distance is always Measured by a Perpendicular; as if from the Point a you imagine the Line a e to fall Perpendicular on e c; as also doth the Line b d on the same Line; we naturally conceive that if those two Perpendiculars are of the same Length, or equal: the two Lines a b and e d are equally distant from each other in those two Places, which is self-evident and needs no Proof.

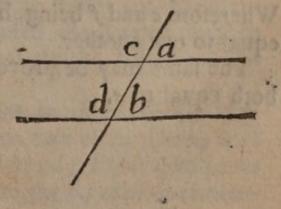
28. Two Parallel Lines being continued infinitely, yet can never meet. For being always equally distant, there may any where be drawn between them a Perpendicular equal to a e or b d, and consequent-

ly they can never meet.

29. Two Parallel Lines, have the same Inclinati-

on, one as the other, to any right Line that croffes them both.

That is, the Angle a will always be equal to b, and c to d; for the Intersecting Line being supposed inflexible, as is the Case of all Ma-



thematick Lines, it cannot bend to, or from one Parallel

Parallel more than it doth to or from the other: And neither of these Lines can alter its Position in respect of the Crossing Lines, for then the Parallelism would be destroyed, which Contradicts the Supposition.

And this is the first Property of Parallel Lines.

30. Whenever a Right Line cuts two Parallels, it makes with them eight Angles: Of which, four a. b.

b. g. are External; and the other four c. d. e. f. are Internal. The Angles c and f, as also d and e, are called Alternate. The Angles e and a, as also f and b, are called the Internal, and Opposite on the same side. And the es d f, as also c and e, are called the Internal

Angles d f, as also c and e, are called the Internal Angles on the same side.

#### AXIOM II.

Things equal to a Third, are equal to one another.

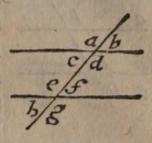
31. The Alternate Angles c and f must be equal; as also e and d; for c is equal to the Vertical Angle b, and b is equal to the Internal one f, by the last Prop. Wherefore c and f being both equal to b, must be equal to one another.

The same may be proved of c and d, which are

both equal to a.

32. When a Line falls on two Parallel Ones, it makes the Internal Angles on the same side equal to two Right Ones.

I say, the Angle d with f, is equal to two Right Ones: Because f is equal to c (by 31) and c and d together are equal to two Right Ones (by 20.) Therefore f and d together must be equal to two Right Ones, which was to be provid.



(The same way may c and e together be proved equal to two Right Ones; for c and d taken together are so (by 20) but d is equal to e (31.) Therefore c and e are

equal to two Right Ones.)

33. One Proposition is called the Converse of another; when after a Conclusion is drawn from fomething Supposed, in the Converse Proposition that Conclusion is Supposed; and then that which was in the other Supposed, is now drawn as a Conclusion from it: For Example: We say here, if two Lines are Parallel, (and another cross them,) the Angles d and f together, are equal to two Right Ones: Where we suppose the Lines to be Parallel, and from thence conclude those Angles must be equal to two Right Ones: But the Converse is thus, If the Internal Angles on the same side, d and f together, are equal to two Right Ones; Then those Lines are Parallel: Where after we have supposed the Angles equal to two Right Ones, we conclude the Lines are Parallel.

34. Converse Propositions in this Case are very True; as that, if a Line cut two other Lines, and makes the Alternate Angles equal; Those two Lines are Parallel: Which I desire the Reader to remember.

#### 12 ELEMENTS, &c.

35. If two Lines are Parallel to a third Line, they are so to one another.

Let the Line a b be Parallel to c d; and let e f also be Parallel to the same Line c d; I say, a b is Pa-

rallel to ef: For if you draw a Line as b d f cutting them all Three; the Angle b will be equal to d (by 31.) and the same d will be equal to f (by (31.) because ef is also Parallel to ed. Wherefore the Angle b must be equal

to f: Because by Axiom 2, if two things are equal to a third, they are so one to another. But if the Angle be = f, then the Line a b is Parallel to e f (by 34.)





# ELEMENTS

# GEOMETR

#### BOOK



Figure is a Space compassed round on all Sides. And if the Lines which Terminate it, are all Right Ones 'tis called a Rectilineal (or Right Lined) Figure: If they are Crooked, 'tis called a Curviline-

al; and if they are partly Right Lines, and partly Crooked, 'tis called a Mixt Figure.

2. There are Plane Figures, which are Plane Surfaces, and there are Solid Ones, which have three Dimensions. But we speak here only of Plane Surfaces, or Plane Figures.

#### ELEMENTS

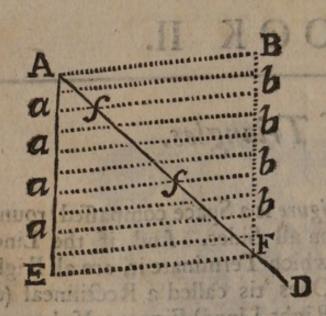
3. All the Lines which encompass any Figure taken together, make that which is called the Circumference, Perimeter, or the Compass of the Figure.

4. Of all Curvilineal or Mixt Plane Figures, in Common Geometry we consider properly only the Circle, or a part of a Circle terminated on one fide by an Ark, and on the other by one or more Right Lines.

5. Of Rectilineal Figures, the most Simple are Triangles, which are Terminated by three Right

Lines (and no more) making as many Angles.

If a Right Line, (A B) having one of its Ends or Points (as A) in the Vertex or Top of the Angle, E A D, be moved downwards, with a Motion always Parallel to its felf, so that the Point A shall always keep in or touch the Line AE, until it come to be



all of it within the Legs of the Angle EAD; that is, till it come to be in the Situation EF; that I ine shall in its Motion continually cut the Line A D, and at length describe the Triangle EAF within the Legs of the Angle; as also another equal to it

(AFB) on the other fide of the Line AD. The Parts of which latter Triangle shall continually Decrease, as those of the former A E F, do continually Increase. And the Line A B shall also describe with its whole length the Quadrilateral Figure A E F B; which will be divided into two equal Parts by the

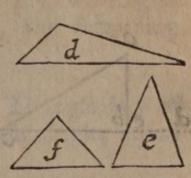
Diagonal Line A F.

MA S

N. B. The Line A B may be called the Describent, and A E the Dirigent, because the latter directs the Motion of the former.

6. A Triangle as a, which hath one Right Angle, is a Right-angled Triangle; if it have one Angle Obtuse, 'tis called an Obtuse-angled One, as b; and if all its three Angles are Acute, 'tis called an Acute-angled Triangle, as c.

7. If a Triangle have all its three sides Unequal, 'tis called a Scalene, as d. If it hath two sides equal, 'tis called an Isosceles, as e; And if all the three sides are equal, 'tis called an Equilateral one, as f.



a

8. When two fides of a Triangle are confider'd, they may be called its Legs, and the third fide may then be called the Base. But any one fide may be called the Base, tho we usually and most properly call that so, which lies Parallel to the Horizon, and which is next to us.

9. In every Triangle, the three Angles taken to-

gether are equal to two Right Ones.

Let the Triangle be a b c: I say, that the Angle a added to the Angle c, added to the Angle b (or the Sum of all three) are equal to two Right Ones. For let de be drawn Parallel to the Base a c, then will those two Parallel Lines be cut by the Line b c; and consequently the Alternate Angles c and d will be equal to each other (by 1. 31.) Moreover the Line b a falling on, or cutting the same Parallels d c and a c, will make the two Internal Angles on the

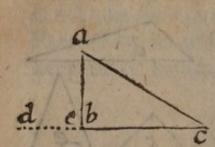
the same side equal to two Right Ones; that is, a aded to a b e are equal to two Right Angles (by 1.

Bd e

the two Angles b and d. So that the Angle a added to b added to d, will be equal to two Right Ones. But c being equal to d, it will follow, that a added to b added to c, or the Sum of all three together,

must be equal to two Right Ones: Which was to be Proved.

10. If any fide of a Triangle be produced, or



ne Angle a

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drawn out, the external Angle will be equal to the two Internal Opposite Angles, (taken together.) Let the Triangle be a b c, whose Base c b draw out to d, by which means a New Angle as e will be made, which

is called the External Angle of that Triangle. Then I say, that That External Angle e, is equal to both

the Internal and Opposite Ones a and c.

For those Angles a and c together with b are equal to two Right Ones (by the Precedent,) and so also are e and b by (1. 20.) wherefore e must be equal to a added to c, because together with b, it makes two Right Angles, as they do. Q. E. D.

#### COROLLARIES.

- is the same.
- 2. No Triangle can have above one Right, or Obtuse Angle.

- 3. If in any Triangle one Angle be Right, the other two must be Acute.
- 4. If in any Triangle there be one Angle equal to both the others, that must be a Right One.
- Triangle, you know the Sum of the other two; for its what is wanting of 180°, and if the Sum of any two be known, the Quantity of the Remainder is known.
- 6. Hence if two Triangles have any two Angles respectively equal to one another, the remaining Angles must also be equal.
- 7: The Angle of an Equilateral Triangle is \(\frac{7}{3}\) of two Right Angles, or \(\frac{2}{3}\) of one Right Angle, equal to 60°.
- 8. Hence 'tis very easy to Trisect a Right Angle, by making on one of the Legs an Equilateral Triangle.
- A C, equal to two others a b and a c in another Triangle, and if also the Angle A be equal to a; I say, the third Side B C shall be equal to b c; the Angle B equal to b, the Angle C to c, and the B whole Triangle A B C to a b c.

For if we imagine the Triangle a b c to be placed upon ABC, so that be the Side a b shall lie exactly on its e-

qual AB: Then must the Side a c sall on its equal AC, because the Angle a is equal to A, and so the Point c will sall on C, and b upon B, and the whole

Tri-

Triangle a b c on the Triangle A B C; because all things so exactly answer, that nothing of the Upper

Triangle can fall besides the Under one.

12. Figures which do thus meer, fit, or answer to each other exactly, when they are placed one upon the other are called Congruous Figures, Quia mutuo fibi Congruunt.

And therefore the third Axiom is, Que sibi mutuo Congruunt sunt Aqualia; i.e. Those Figures which placed one upon another, do answer to, and cover

one another exactly, are equal.

13. It is also true, That, if a Triangle hath all its three Sides equal to the three Sides of another Triangle, all the Angles also in one, shall be equal

> to those in the other: And all the Space which one Triangle contains, shall be equal to that contained in the other: As if A B be equal to ab, A C to ac, and BC to bc: I say, that the Angle A shall be equal to a, B to b, and C to c; and the whole Triangle ABC, to

a b c; this needs no other Proof.

14. If the Angle A be equal to a, the Angle B to b, and the Side A B to ab: Then shall the Side A C be always equal to ac, BC to bc; and the whole Triangle ABC to abc: Which is easy to prove by the precedent Propositions.

15. In every Isosceles Triangle, the Angles at the

Base, opposite to the equal Legs, are equal.

Let the Triangle be a b c, whose Legs a b and a c are equal: I say, the Angle b is also equal to c. For imagine the Base be divided into two equal Parts c in the Point d, then will the Line ad (which let be drawn) make of the whole two Triangles, abd and dac, which will have all three Sides in one, equal to those in the other: For

06

qual to dc, and a d is common to both. Wherefore (by 2. 13.) the whole Triangle b a d is equal to dac, and the Angle b is equal to c; which was to

be proved.

16. In an Isosceles Triangle, if a Line drawn from the Angle at the Top do (bisself or) divide the Base into two equal Parts, it is both Perpendicular to the Base, and also bisselfs the Angle at the Top. For (vid. Fig. pracedent.) the Angle adc is equal to the Angle adb (by the last) and consequently they must be both Right ones; and therefore the Line ad is perpendicular to the Base bc (1.15.) and the Angle dac will be equal to dab (by the last Prop.)

17. In every Triangle the Greater Side is always

opposite to, or subtends the Greater Angle.

In the Triangle a b c, let the Side b c be longer than b a; then I say, the Angle b a c subtended by the Greater Side b c, is bigger than the Angle c, which is subtended by the lesser Side. For let b d be taken equal to b a, then will a b d be an

Isosceles Triangle; whose Angle bad will be equal to bda (2.

ger than bad; (The whole being be

greater than the Part) and therefore must be bigger than b d a (which is equal b a d.) Now the Angle a d b is an External Angle in respect of the little Triangle a d c; and therefore must be bigger than the Internal one c (by 2.10.) Wherefore the Angle b a c being bigger than d, must certainly be bigger than c; which was to be proved. 18. Of all Lines that can be drawn from a Point given to a Line given, the Shortest is the Perpendicular; and they are all Longer, according as they

are farther distant from it. Let the given Line be ad, and the Point given b; let ba be perpendicular to da; let also be and bd be drawn. I say, that ba is the shortest Line

that can possibly be drawn from b; and (for instance) is shorter than bc (or any other that can be assigned): And I say also, that bd is longer than bc.

For in the Triangle bac, the Angle a is a Right One, and consequently, bigger than either of the other; because they must necessarily be both Acute (by Cor. 3. of Art. 10). Therefore the Side bc is longer than ba(2.17.) as subtending a greater Angle.

So also in the Triangle d b c, the Angle d c b is Obtuse, because the Angle b c a is Acute: And consequently the Side d b must be longer than c b, as

subtending a greater Angle (2. 17.)

19. In every Triangle any two Sides taken together are longer than the Third; because a Right Line is the nearest Distance between any two Points.

ever than the twitte is court to a di

### PROBLEM I.

On a Line given a d, to make an Angle B, equal to a given one Z.

Place the Compasses in c the Vertex of the given Angle, and describe the Ark Rr; then keeping them

at the same distance, set one Foot in a, one end of the given Line. and with the other describe the Ark obd; fet R r from d to b; C And draw a b, so shall the Angle bad or B be equal to Z.

qual Circles, and the

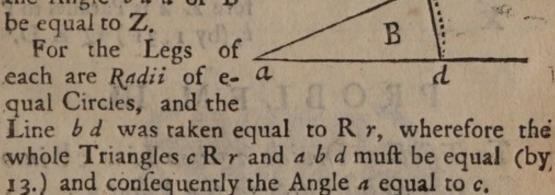
For the Triangles hed and decare

h d and a w o will be equal alfo (by

Therefore the Triangles

OB9

equal (by 2.13.) Wherefore the Angle



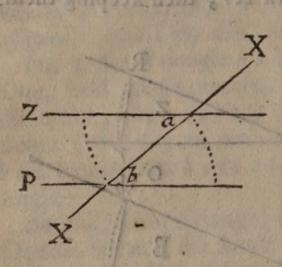
#### PROBLEM II.

Hence the Practice of making all forts of Triangles, Equilateral, Isoscelar; or without any given Angles or Sides, will easily appear.

### PROBLEM III.

A Right Line, as P, being given, to draw thro'a, a Point given, the Line Za, Parallel to it.

Through a draw any Line, as XX, making any



Angle, as b, with the given Line, Then make the Angle Z a X = to b, and Z a shall be the Parallel sought.

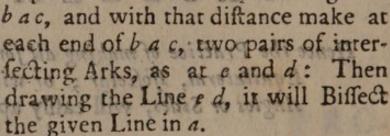
For the Alternate Angles a and b are equal by

Construction; Wherefore Z a is Parallel to P b, (by 1.31) Q E D.

### PROBLEM III.

To Bissect or Divide a given Line c binto two equal Parts in the Point a.

Open the Compasses to more than the length of



For the Triangles bed and dec are equal (by 2.13.) Wherefore the Angle adb = adc. Therefore the Triangles abd and adc will be equal also (by

2) 11.) and consequently a b is = to a c. Q. E. D. PRO-

#### PROBLEM V.

By much the same Method may a Perpendicular, as ad, be raised in the middle of any given Line, or one may be let fall from the Point e or d, to the given Line a b c, And the Demonstration is the same in all:

And after the same way of Practice may the given Angle bdc be Bissected.

If setting one Foot of the Compasses in d, you take d b equal to d c. And then setting the Compasses in b and c, strike the Arks intersecting each other in e : So shall d e bissect the Angle required.



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called a Rectangled Parallete-

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the middle of any given Line, or one may

L. CEDMETRY.

# ELEMENTS

And after the famil Or of Practice may

# GEOMETRY.

BOOK III.

# Of Quadrilateral Figures and Polygons.



HOSE Figures, whose Sides are four Right Lines, and those making four Angles, are called Quarteral, or four-sided Figures.

2. When the opposite Sides are Parallel, the Quadrilateral Figure

is called a Parallelogram, as a; but if not, 'tis called a Trapezium, as B.

Right, 'tis called a Rectangled Paralleloa gram; or for brevity's fake a Rectangle, as c: And if the Angles are right, and the Sides are all equal, 'tis called a Square, as d.

4. If

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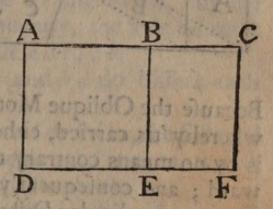
4, If a Parallelogram hath all its Sides equal, but its Angles unequal, then 'tis called a Rhombus, as e.

5. If a Parallelogram hath neither its Angles nor Sides all equal, 'tis cal-

led a Rhomboides, as a.

The Generation of all Parallelogramick Figures

will be eafily conceived, if you suppose the Desoribent AC, to be carried or moved along the Dirigent A D, in a Position of Association of the always Parallel to it self in its first Situation. For then, if the Angle A which D E F the Describent makes



with the Dirigent, be a right One, and A B be equal to AD: The Figure produced will be a Square. If AC be longer or shorter than A D, the Figure will be an Oblong, or a Rectangle.

If the Angle at A be Oblique, only a Parallelogram at large will be described: Which when the Describent is equal to the Dirigent, the Figure will be a Rhombus; if unequal to it, a Rhomboides. Angle, o, is equal to dis

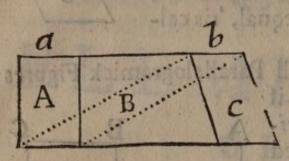
### COROLLARIES.

I. Hence 'tis natural to suppose, that equal Lines moving thro' the same or equal Spaces, will describe equal Surfaces.

II. Equal Lines, with uniform or equable Motions (i. e. being neither accelerated nor retarded) in equal Times, will describe equal Surfaces: And if they do thus describe equal Surfaces, it must be in equal Times.

III. Hence

III. Hence also, if the Line a in a given Timede-scribe the Parallelogram A, and the equal Line b in

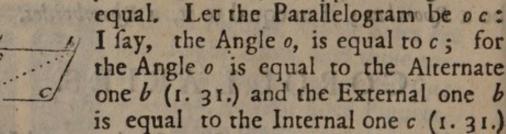


the fame Time describe the Oblique Parallelogram B or C, whose Perpendicular Altitude is the same with that of A: Those Parallelograms will be all three equal one to another.

Because the Oblique Motion, which the Line b hath, whereby 'tis carried, either to the right or lest Hand, is by no means contrary to the direct Motion downward; and consequently, the Line b will move the same perpendicular Distance in the same time, with an equable Motion, whether the latter Motion be impressed upon it or not. Wherefore,

IV, All Parallelogramick Figures, with equal Bases and equal Perpendicular Altitudes, must be equal,

6. In every Parallelogram, the opposite Angles are



wherefore o is equal to c.

Figure from Angle to Angle, is called the Diagonal, and by some, the Diameter.

8. Every Parallelogram is divided into two equal Parts by the Diagonal. The Diagonal b d divides the Parallelogram o c, into the two equal Triangles o b d and b c d. For, 1. The Angle o is equal to c (3.6)

2. The Angle ob d is equal to cdb (1.31.) and the fide bd is common to both these Triangles, wherefore the Triangle obd is equal to cdb (by 2.14.)

9. In every Parallelogram, the opposite Sides are

always equal.

For (drawing the Diagonal db) the whole Triangle dob will be equal to the Triangle bcd, by the foregoing Prop. And consequently, the Side cd must be equal to ob, and the Side od to cb.

10. Two Diagonals ac and bd do biffect each

other in the middle at e.

For in the two Triangles a e d and bec, the Side a d is equal to b c (3.9.) The Angle e a d is equal to e e b (1.31.) and moreover the (Vertical)

Angles a e d and c e b are equal also a (1.23.) Wherefore the whole Triangle a e d is respectively equal to the Triande c gle bec (2.14.) And consequently, the side d e is equal to e b, and the Side a e to the Side e c. The two Diagonals therefore bissect each other in the middle. Q. E. D.

11. Every Right Line, as fg, passing through the middle of a Diagonal, divides the Parallelogram into

two Equal Paris.

To demonstrate which, the Trapezium or Irregular Quadrilateral Figure f g d a must be proved equal to the Trapezium f g c b. And that is thus done. 1. The Triangle b e f is equal to the Triangle d e g: For the Side d e is equal to e b by the Supposition; and the Angle e f b is equal to a f b e g d (1.31.) and the opposite Angles at e are equal; wherefore the Triangle d e f b is equal to e d g (2.14.) 2. The great Triangle a b d is equal to b d c (3.8.) wherefore if from the Triangle a b d you take away the little Triangle fe b, and instead of it put the Triangle e d g (which

(which is equal to feb) you will have the Trapezium fadg, which will be equal to the Triangle adb: That is, to just one half of the whole Parallelogram: (3. 8.) which was to be proved.

12. If in the Diagonal db you take a Point as e, and thro it draw two Lines bi and fg Parallel to the

two Sides of the Parallelogram, it will be divided by them into four i lesser Parallelograms, i. e. f i, h g:
(which two are called the Parallelograms about the Diameter) and a e, e c: which other two are called the Complements with

the Complements. And those two Complements with either of the Parallelograms about the Diameter, make a Figure that is called a Gnomon. As you see in the Figure, where the Gnomon is distinguished by being shaded.

13. In every Parallelogram the Complements are Equal. We must prove that ea is equal to e c.

### DEMONSTRATION.

The whole Triangle a b d is equal to the whole b d c (3.8.) And the little Triangle ef b is (for the same Reason) equal to e b i. And a f b the Triangle b e d is also (by the lame) equal to e d g. Wherefore if, from the two equal Triangles c a b d and b d c, we take away equal things, viz if from one we take a-

way e f b and d b e, and from the other e b i and e g d, there will remain on one Side the Parallelogram e a, equal to the Parallelogram e c, which remains on the other; which was to be proved.

14. Parallelograms having the same Base, and be-

ing between the same Parallels, are equal.

Let

Let there be a Parallelogram b c, and another af, both on the same Base a b; and let the Line c d, when produced, be supposed to c de f pass by ef; So that the two Parallelo-

grams shall be between the same Paral- all bels, and Terminated by them; that is,

between the two Parallels of and a b. I say then,

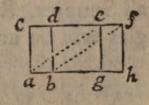
that the Parallelogram c b is equal to af.

For c a is equal to b d, and a e equal to b f, because opposite Sides of Parallelograms, and the Angles at c and d equal (by 29. 1.) wherefore the Triangle c a e is equal to the Triangle d b f. Now if from each of these equal Triangles be taken the little Triangle d o e, and to the remainders be added the Triangle a o b, the Parallelogram a d will be equal to the Parallelogram a f. Q. E. D.

between the same Parallels a b and c f, are equal.

For if we imagine the third Parallelogram f a to be drawn; that shall be equal to the Parallelogram

ad, because on the same Base ab with it, and between the same Parallel Lines ab and cf. And that Parallel lelogram will also be equal to eb, because it hath the same Base ef with it (it matters not whether you reckon



Parallels. Therefore b e and b c, being both equal to the third Parallelogram f a, must be equal to each other.

16. Triangles on the same Base a b, and being be-

tween the same Parallels of and ab,

are always equal.

The Triangle a b c is equal to a e b:

Because if you imagin a Line b d drawn a

Parallel to a c, and another as b f, drawn

Parallel to ae; there will be made two Parallelograms

acdb and aefb; which being on the same Base, and between the same Parallels, will be equal to one

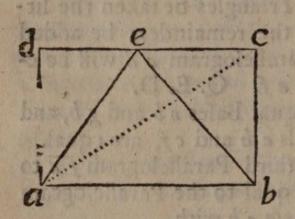
another (3. 14.)

But the Triangle a b c is the half of the Parallelogram a c d b, and the Triangle a b e is the half of the Parallelogram a e f b (3.8); wherefore, (fince the wholes are equal, the halves must) and consequently the Triangle a c b is equal to the Triangle a e b

17. Triangles on equal Bases, and between the same Parallels, are also equal; as is very easy to

prove from (3. 15).

18. If a Triangle a e b have the same Base with a

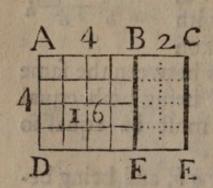


Parallelogram, and be also between the same Parallels, it shall be just the half of that Parallelogram. For it will still be equal to abc, which is just half (3.8.) of abcd.

The Menfuration of

all Squares, Rectangles, Parallelograms and Triangles will be understood from what hath been deli-

ver'd above. If you suppose,



or A C, before its motion, be divided into any determinate Number of equal Parts; and the Dirigent (now supposed to stand at Right Angles with it) into the same or any other Number of such Parts; for then the

Motion of the Describent Line, thus mark'd out by Points into Units will describe a Square (if the Dirigent be equal to it,) and a Rectangle if it be unequal. Which Square or Rectangle will be divided into as many little Squares, as there are Units in

the

the Product of the Number of the Divisions, or equal parts in one Line, Multiply'd by those in the other. That is A B 4 Multiply'd by A D 4, produces 16, The Square of 4. And A C 6 Multiply'd by A D 4 produces 24; the Rectangle under A C and A D. So that what is a Product in Numbers or in Arithmetick; in Lines, or in Geometry, is called a Rectangle. And therefore you will find the Latin Writers of Geometry, when A C is to be Multiply'd by A D, not saying Multiplica, but Duc A C in A D. That is, carry the Line A C along the Dirigent A D, in a Normal Position to it, till it come to end, and then it will form the Rectangle A F=24; wherefore the Area of a Square is found, by Multiplying the Side A B into it self.

The Area of any Rectangle, as AF, is found by

Multiplying the Side A C

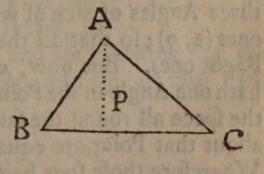
And fince a Rectangle on the same Base and of the same Altitude with a Parallelogram is equal

any Parallelogram as A B, you must multiply the Side A C by a Perpendicular as P, let fall from the other

Side to it.

And fince every right-lined Triangle is the half

of a Parallelogram or Rectangle of the same Base and Altitude: To find the Area of the Triangle A B C, you must Multiply any Side as B C, by a Perpendicular as P, let fall to it from an oppo-



site Angle, and take half the Product: or if either P or B happen to be even Numbers, Multiply one by

of the other, the Product is the Area of the Triangle.

19. A Pentagon is a Figure having five Sides and

five Angles.

If all the Sides are equal, and confequently the

Angles, 'tis called a Regular Pentagon.

20. An Hexagon is a Figure of fix Sides and Angles, an Heptagon of seven, an Odagon of eight, Sc. which are all called Regular when they have equal Sides and Angles.

21. A Polygon in general fignifies any Figure of many Sides and Angles; but no Figure is called by this Name, unless it have more than four or five Sides.



into as many Triangles as it hath Sides, if any where within the Polygon you take a Point, as a, and from thence draw Lines to every Angle ab, ac, ad, &c. they shall make as many

Triangles as the Figure hath Sides.

23. The Angles of any Polygon taken all together, will make twice as many Right ones, except four, as the Figure hath Sides, v.gr. If the Polygon have fix Sides, the double of that is 12, from whence take four, there remains eight. I say, that all the Angles of that Polygon, viz. b, c, d, e, f, g, taken together, are equal to eight Right Angles. For the Lines ab, ac, ad, &c. do divide the Figure into fix Triangles; the three Angles of each of which are equal to two Right ones (2. 9); so that all their Angles together make 12 Right ones. But now, each of these fix Triangles hath one Angle in the Point a, and by it they compleat the space all round the said Point. And all the Angles about that Point, are equal to four Right ones (1.22.) Wherefore those four being taken from 12, (The Sum of the Right Angles of all the six Triangles) leaves eight, the Sum of the Right Angles of the Hexagon, which

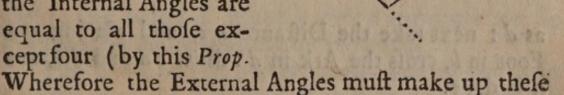
which make 8 times 90, or 720 Degrees; and therefore each Angle must be 3 of that, viz. 120 Degrees.

So that the Figure hath plainly twice as many right Angles as it hath Sides, except four; which was to be proved.

### COROLLARY.

All the external Angles of any right-lined Figure,

are equal to just four Right Ones: For drawing out the Sides as in the Figure, 'tis plain the Internal and External Angles together will make twice as many right Ones as the Figure hath Sides; but the Internal Angles are equal to all those except four (by this Prop.



four, and no more.

24. A Polygon may be divided also into Triangles, by drawing Lines from Angle to Angle. But then the Number of the Sides will exceed that of the Triangles. And hence the Area of any

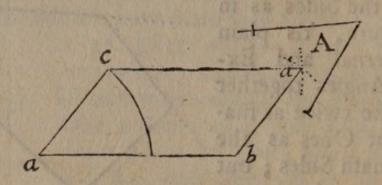
right-lined Figure may be found, by reducing it into Triangles, and then finding the A-rea of each Triangle severally, adding all into one

Sum.

### PROBLEM I.

On a given Line a b, to make a Parallelogram, having an Angle equal to a given Angle A.

Make the Angle c = b = A. Then take a b in your Compasses, and setting one Foot in c, strike an Ark



as d: next take the Distance a c, and placing one Foot in b, cross the Ark in d: draw c d and d b, and it is done.

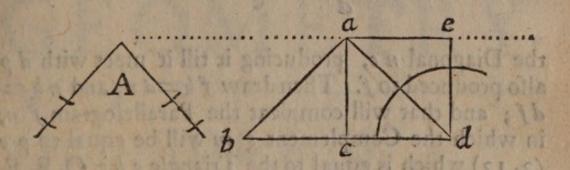
And thus also may the Line cd be drawn Parallel to ab, thro' a Point assigned, and any Parallelogram readily be described.

#### PROB. II.

Then produce of till o as become

A Triangle a b d being given, to make a Parallelogram equal to it, which shall have a given Angle equal to A.

Bissect the Base of the Triangle in c: Make the Angle c de=A, thro' the Vertex a draw a e Paral-



lel to the Base b d. Make a = c d and draw a c. So

will c e be the Parallelogram required.

For being on but half the Base, and of the same height with the Triangle, it will be equal to it, by the 18th of this Book, and its Angle c de is equal to A. Q. E. F.

### PROB. III.

On a Line given as L, to make a Parallelogram, equal to a given Triangle c b e, and having an Angle equal to an Angle given, as A.

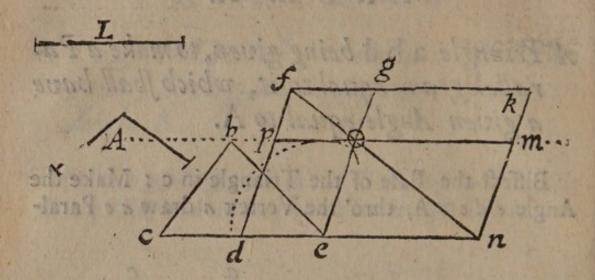
Make the Parallelogram do equal to the Triangles and having its Angle e = A, by Problem the last.

D 2

Then

ELEMENTS, &c.

Then produce potillom become equal to L, and draw out de till en be also equal to L. Then draw



the Diagonal no, producing it till it meet with dp also produced to f. Then draw f = d n, and n = d ndf; and that will compleat the Parallelogram f n, in which the Complement g m will be equal to p e (3. 13) which is equal to the Triangle c be, Q. E. F.

And thus 'tis easie to make a Parallelogram equal to any Right-lined Figure given: By reducing that

beight contained Triangle, it will be equal to le, by

the 18th of this Book, and its Angle c of circulal to

Figure into Triangles, &c.

A by Problem the laft,



E. H. Farallelogeam & e. equal to the Triangle,



# ELEMENTS

OF

# GEOMETRY.

### BOOK IV.

## Of a Circle.

Line is said to Touch (or to be a Tangent to) a Circle, when the point of Contact, it will only touch it, and debate and

not cut or enter within it. Thus the Line a touches the Circle C, as that Circle C doth the Circle D; but d enters within the Circle, and cuts it, and is called a Secant.

2. If a right Line enter within a Circle and cut it into two Parts, those Parts are called Segments: b is a less Segment and D a greater: That part of the D 3

Line cutting the Circle (and which is within it) is called a Chord as e f. And the Parts of the Circle (or rather Circumference) cut off, are called Arks: The Chord with the Ark makes two mix'd Angles as e and f, and they are called Angles of a Segment.

3. If you take a Point as c, in the Ark of any Seg-

ment, and from thence draw two Lines c a and c b (to the ends of the Chord) they shall make an Angle a c b; which is call'd an Angle in a Segment: And that Angle a c b is said to insist or stand on a b d, the Ark of the other Segment below.

4. A Sector of a Circle is a mix'd Triangle comprehended between two Radii, a b, a c, and the Ark of the Circle b c; 'tis mark'd in the Figure by being shaded.

5. If at the end of any Radius, or Semidiameter, a b, you draw a Perpendicular, as d b, it shall touch the Circle but in one Point. And all the

Points of the Line b d shall be without the Circle, v. g. I say, the Point d ( or any other assignable) is without: For if you draw the Line a d from the Center, and that shall cut the Circle in the Point c, that Line a d will be longer than a b;

(2.17.) and consequently longer than a c, which is equal to a b (1.14.) Wherefore the Point d is without the Circle. Q. E.D.

6. A Chord, as b c is divided into two equal Parts

(or Biffected) by a Perpendicular d a,

drawn from the Center a. For the Tri
angle a b c is an Isosceles, because b a is

equal to c a (1. 14.) and therefore the

Perpendicular a d Bissects the Base b c (2.16.) The Ark b c is also by this means

Biflected

7. Two

7. Two Tangents, c b and cd, drawn from the same Point without a Circle, are equal one to another. For, draw from the Center to the Points of

Contact, b a and a d. Then will those Lines be Perpendiculars to the Tangents (by 4.5.) Then if you draw also the Line b d, the Angle a b d will be equal to b a d b (2. 15.) Wherefore if from the Right, and (consequently) equal Angles

c b a and c d a, you take away the equal One a b d and a d b, the remaining Angles c b d and c d b will be equal: Wherefore their opposite Sides must also be equal by the Converse of (2. 15.) That is, cb is e-

qual to c d. Q. E. D.

8. Equal Chords as b c and f b, do cut off equal Segments b d c and f g h. And the Perpendiculars ae and a i, drawn to them from the Center, are also equal, as is easily proved; (saith Pardie, but he gives us no Demonstration.) Yet'tis plainly thus proved: The Chords and Arks are both Biffetted by the Perpendicu-

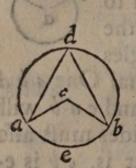
lars (4. 6.) And therefore the Sectors c a d dab, fag and gah, must be all equal; as also will all the Triangles x, z, o and k, by (2. 11.) Therefore their Doubles will also be equal, i.e. The Sector ba c will be equal to fah: And the Triangle bac to the Triangle f a h. And if these last Tri-

angles are taken from the equal Sectors haf and bac, the Segments b d c and h g f must remain equal. That the Perpendiculars are equal, is plain from the Equali-

ties of the Triangles z and o, or X and K.

9. Let there be a Semidiameter R c, and a Perpendicular (to it without the Circle) R T. another Line cutting the Circle in S, and a Perpendicular (let fall from thence) to the Radius R C in n ( a Point within the Circle.) All these Lines have Artisi-

cial Names. The Line TR is called the Tangent of the Ark RS (which suppose 30°.) T C is called the Secant of the same Ark of 30°, and the Line S n is called the (Right) Sine of the same Ark. R C is by some called the whole Sine, but most usually the Kadius. And nR is called the Versed Sine of the same Ark.



Circle, you take two Points, as a and b, and from thence draw two Lines to the Center c, and two others to any Point, as d in the Circumference; they will make two Angles, of which a c b is called an Angle at the Center, and

a d b an Angle at the Circumference.

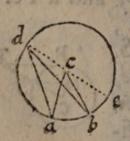
11. The Angle at the Center a c b is always double to one at the Circumference a d b (in
sisting with it on the same Ark a b)

Of which there are three Cases.

a b I. If one of the Lines, as d b, pais thro' the Center c, then 'tis plain the external Angle a c b (2. 10.) will be equal to both the internal and opposite Ones a and d taken together.

But the two Angles d and a are equal, because a c d is an Isosceles Triangle, whose Side a c is equal to c d (2. 15.) Therefore the Angle c at the Center being equal to both, is double of either alone: That is, double to d. Q. E. D.

II. If neither of the Lines d b, d e (which form



the Angle at the Circumference) pass thro' the Center c: (But fall both on the same side of the Diameter) Let the Diameter d c e be drawn. Then will the whole Angle a c e (at the Center) be double to the Angle a d e (at the Circumference)

cumference) by what was proved in the first Case. Also the Angle b c e is double to b d e, by the same. Wherefore if from the Angle a c e, we take away that b c e, and from the Angle a d e, which is the half of a c e, we take away b d e, which also is the half of b c e, the remaining Angle a d b must be just the half of a c b. For 'tis as plain as an Axiom, that if one Quantity be double to another, and you take away from the bigger just the double of what you take from the other, the remainder of the Bigger must be double to the remainder of the Lesser.

III. If the Diameter fall between the Lines form-

Then will, as before, the Angle a c e be double to a b e (by Case 1. of this) and the Angle e c d will be double to e b d by the same; therefore the whole Angle a c d must be double to a b d. (So that in all Cases, the Angle at the Center

is double to one at the Circumference if they both stand on the same Ark, or (which is all one) are in the same Segment.

12. All Angles (in the same Segment or) Infisting on

the same Ark a b, are equal, let them Terminate in any part of the Circum ference whatsoever.

For the Angle a d b will be equal to a e b, because each is the half of the

Angle at the Center a c b (4 11.)

13, An Angle at the Center b ce standing on half of the Ark a e b, is equal to the Angle a d b at the Circumference, standing on the whole Ark, For c is equal to twice x; (by 4. 11.) and x is equal to 0, that is to half a b d

(4. 6, and 4. 8.) Wherefore c is equal to 2 d b. Q.E.D.

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14. The Angle adb standing on the Semi-circumference aeb (or being in the Semi-circle adb) is a

a c b

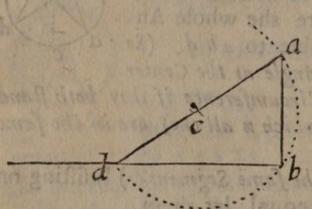
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Right One. Let c e be drawn Bissecting the Semi-circumserence a e b, then is (by the Precedent) the Angle a c e at the Center, standing on half a Semi-circle (or on a Quadrant) equal to a d b at the Circumserence, which stands on twice that Ark or on a Semi-

circle. But a c e is a Right Angle, wherefore a d b, (it's equal) must be so too.

### COROLLARY I.

Hence is derived the Common Practice of Erecting a Perpendicular, as a b, at the end of a given Line,



For, opening the Compasses to any convenient distance, set one Point in c, and with the other draw the Ark d b a, cutting the given Line in d; then a Ruler laid from d to c shall find the Point a,

at the Center a c. b

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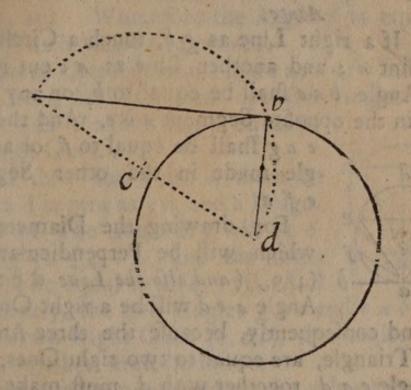
on half of the Ark a c

which is Perpendicularly over b: For the Angle d b a, being in a Semi-circle, is a Right One.

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### COROL. II.

Hence also arises this expeditious Practice of drawing from a Point given as a, a Tangent as a b, to a given Circle. For joining the Points a and d, the

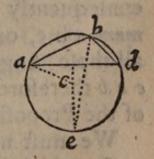


Center of the Circle, Bissect their Distance a din the Point c: On c, as a Center, describe the Semi-circle a b d: So shall a b be a true Tangent, because the Angle a b d being in a Semi-circle is a Right One.

15. The Angle a b d in a Segment less (than a Se-

mi-circle) is Obtuse: Because the Ark a e d being more than half the Circumserence, its half, the Ark a e, must be more than 90°, therefore the Angle a b d, which is equal to a c e, (4. 13.) must also be more than 90°, that is Obtuse.

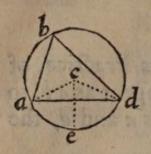
fab: by the first part of this Pro-



### ELEMENTS

16. The Angle a b d made in a Segment greater

than a Semicircle, is Acute.



For 'tis equal to the Angle a c e (4. 13.) whose Measure a e being the half of a e d, an Ark less than a Semi-circle, must be less than 90°. And therefore a b d is less than 90°. (i.e.)

Acute.

17. If a right Line as g b, touch a Circle, as in the Point a; and another Line as a e cut it there. The Angle bae shall be equal to b, or any Angle made in the opposite Segment a b e. And the Angle

e ag shall be equal to f, or any Angle made in the other Segment,

efa.

b

For, drawing the Diameter a d, which will be Perpendicular to a b (4 9.) (and also the Line de:) The Angle a e d will be a right One; (4.

14.) and consequently, because the three Angles of every Triangle, are equal to two right Ones, (2. 9.) the Angle e a d, together with d, must make just a-

nother right Angle.

But that Angle dae together with eab doth make also a right One, because the Radius ca is Perpendicular to the Tangent ab; wherefore take away ead from both and then eab will remain equal to d. And consequently to b, or to any other Angle in that Segment ahe, or that stands on the same Ark efa. For all those Angles are equal (by 4. 12). The Angle eab therefore is equal to b: Which is the sirst part of the Proposition.

We must next prove the Angle gae to be equal

to f; which is the other part.

In the Triangle a fe all the three Angles e, f and a are equal to two right Ones (2 9). And the Angle c is equal to fab; by the first part of this Proposition

18. Every Quadrilateral Figure as defa Inscribed in a Circle, hath its two opposite Angles taken

together (as d added to f) equal to

two Right Ones.

For if thro' the Point a, there be drawn a Tangent as gb, and a Diagonal as ea: The Angle at f will be equal to gae (4.17.) and the Angle eab will be equal to d (4.17). And

consequently, the two Angles g a e and e a b being equal to two Right Ones (1. 20), the Angles d and

f taken together must be so too.

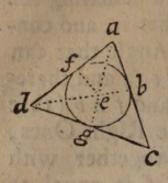
After the same manner might the other two opposite Angles daf and def be proved equal to two Right Ones; by drawing another Tangent through

the Point f.

19. The Converse of this Proposition is also manifest; viz. That if any Quadrilateral Figure have its opposite Angles equal to two Right Ones, it may then be inscribed in a Circle; That is, a Circle may be made, that shall touch or pass through all its four Angular Points.

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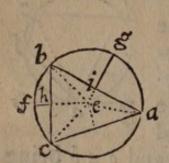
one sheet Loines, if they is not able to



20. A Rectilineal Figure is said to be Circumscribed about a Circle, when all its Sides touch the Circle, without cutting it. Thus the Triangle dae is Circumscribed about the Circle bgf; because every side of the Triangle touches the Circle, in b, g and f.

21. A Figure is said to be Inscribed in a Circle when all its Angles are in the Circumference of that Circle, as the Triangle a b c in the following Figure.

22. Every Triangle, a b c may be Inscribed in a



Circle; for if two Lines as e b and e i, are drawn Perpendicularly Biffecting the fides b a and c b, they will cross or meet each other in the point e, on which as on a Center, a Circle may be drawn, which shall pass through b. And I say also,

That that Circle shall pass through a and c.

For I. The two Triangles e i b and e i a are equal; because i b is equal to i a by the supposition, the side e i is common to both, and the Angles at i are Right. Wherefore the side e b is also equal to e a (2.11.)

II. And for the same Reason The Triangles e h c and e h b may be proved equal, and consequently, the side e c also will be equal to e b and to e a. But if those three Lines are all equal, the Point e, where they meet, must be the Center of a Circle of which they are Radii: And therefore the Triangle is Circumscribed by a Circle. Q. E. D.

And thus may a Circle be made to pass through any three Points, if they be not all in a Right Line.

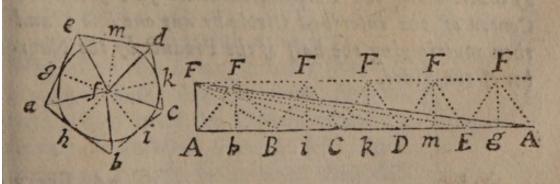
23. Every Triangle may (have a Circle inscribed in it, or) be circumscribed about one, vid. Fig. in Art. 20.

For drawing the Lines a e and e d Bissecting the Angles a and d, and from the Point e, where they cross, letting fall the Perpendiculars (to the Sides of the Triangle) e b, e f and e g; I say, that if you draw a Circle on the Center e through b; that Circle shall touch all the Sides of the Triangle in the Points b, f and g.

For I. The two Triangles a ef and a e b are equal, as having the Side a e common, the Angles at f and b Right, and those at a equal (by the Supposition:) wherefore e b is equal to ef. (2.14.)

II. By the same Method, e g may be proved equal also to e f, (that is to e b) so that these three Lines being all equal, a Circle will pass through their three Extremities, of which Circle they will be Radii, and being also all Perpendicular to the Sides of the Triangle, the said Sides are Tangents to that Circle (4. 5), and therefore do Circumscribe it (by 4.18.)

24. Every Polygon circumscribed about a Circle is equal to a Rectangled Triangle, one of whose Legs, shall be the Radius of the Circle, and the other the Perimeter (or the Sum of all the Sides) of the Polygon.



Let the Line F A be equal to the Radius f b, and to it at Right Angles draw the infinite Line ABCD, &c. out of which take A b equal to a b, b B equal to b b, B i equal to b i, and i C equal to i c, &c. So that the whole Line ABCDEA, may be equal to the whole Compass or Perimeter of the Polygon a b c de a. Also draw F F Parallel to A A, so that all the Perpendiculars F b, F i, F k, &c. may be equal to the Radius f b, or f i, &c. Tis then plain, that the Triangle A F B will be equal to the Triangle a f b in the Polygon, and the Triangle B F C to b f c, and also C F D to c f d, &c. So that all these Triangles taken together will be equal to all these in the Polygon, or to the whole Polygon.

But the Triangle F A A is equal to all the five Triangles within the Parallels; because drawing the Lines, B F, C F, D F, &c. The Triangle FAB will be equal to F A B, F B C to F B C, &c. (3. 16.); wherefore the Triangle F A A is equal to the Poly-

gon, which was to be proved.

25. Every regular Polygon is equal to a Rectangled Triangle, one of whose Legs is the Perimeter of the Polygon, and the other a Perpendicular drawn from the Center to one of the Sides of the Polygon. The Proof of which is the same as that in the precedent Proposition; For all the Perpendiculars f b, f i, f k, &c. are equal, &c. See the last Figure.

Wherefore the Area of every regular Polygon, is found by multiplying the Perpendicular let fall from the Center of the inscribed Circle by any one side; and then multiplying the half of the Product, by the Number of the Sides. Book IV. of GEOMETRY.

26. Every Polygon Circumscribed about a Circle. is bigger than it; and every Polygon Inscribed, is less than the Circle; As is manifest, because the thing containing, is always greater than the thing contained.

27. The Perimeter, or (as some call it, tho' improperly) the Circumference of every Polygon Circumscribed about a Circle, is greater than the Circumference of that Circle; and the Perimeter of e-

very Polygon Inscribed, is less.

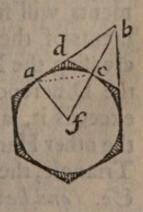
28. If in any little Segment of a Circle, you Inscribe an Isosceles Triangle, as a b c; so that a b, be

equal to bc; I say, that Triangle shall be greater than half that Segment. e. For if you draw a Tangent e b d, which shall be Parallel to ca; and which shall be, as cais, Perpendicular to the Radius b f; (4. 5.) (4. 6.) And then Compleat the Rectangle a d e c;

That Rectangle will be greater than the whole Segment a c b: But the Triangle a b c, is the half of that Parallelogram (3. 18.) And therefore must be

greater than half the Segment a b c. 29. Let there be a Tangent a d b, a Secant f c b,

a Chord a c, and another Tangent c d; I say, that the Triangle d b c is more than half the mixt Triangle a c b, comprehended between the Lines a b, b c, and the Ark of the Circle a c. For in the Triangle d b c, the Angle c, being a Right one (4. 5.) the Side d b, is longer than d c (2. 17.) That is, than da; which is equal to dc (4.7.) Wherefore the Triangle b d c (baving a longer Base, but the same Height with a d c) must be greater than it; (as may be collected from (3.7.) And therefore it must be greater than the



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half of the whole Triangle a c b. But the Triangle a c b, is greater than the mixt Triangle, made by the Ark a c, and the Right Lines, a b and a c; and therefore the Triangle b d c (which is more than half of a c b) must be greater than the half of the mixt Tri-

angle a b c. Q E. D.

30. From these two last Positions, it follows, that by multiplying the Sides of Polygons, you may make them fo Circumscribed about, or Inscribed in Circles, that the difference by which the Circumscribed exceeds, or the Inscribed wants of the Circle, shall be as small as you will; Because if from any Quantity whatever, you take more than the half, and from the Remainder more than its half, and again from that Remainder more than its half: You may by doing this very often, at last come to leave a Remainder as small as you please; as is Self-evident. Thus (See the 28th Figure) after a Triangle is inscribed in a Circle that shall be less than it by the three great Segments, you may inscribe an Hexagon that shall exceed the Triangle by those three Segments, but shall be less than the Circle, by the fix little Segments, that are left white in the Figure. I all med med to the

But those six white Segments taken together, do not contain so much Space as the half of the three former shaded ones, (4. 28.) After this you may also Inscribe a Duodecagon; which will be lesser than the Circle by 12 smaller Segments; which 12 Segments will still be less than the half of the six Segments of the Hexagon: And thus may you by increafing the Number of Sides of the Polygon, leffen the Difference by which the Circumscribing Circle exceeds it, as much as you please. So likewise on the other Hand, you might have first Circumscribed a Triangle, then an Hexagon, and then a Duodecagon, &c. (and have made, that way, the Difference between the Circumscribing Polygon and the Circle, as small as you would.) 31. Every

Book IV. of GEOMETRY.

31. Every Circle is equal to a Rectangled Triangle, one of whose Legs is the Radius, and the other a Right Line equal to the Circumference of the Circle. For such a Triangle will be greater than any Polygon Inscribed, and less than any Polygon Circumscribed, by (24, 25, 26, and 27 of this fourth Book.) And therefore must be equal to the Circle.

For should it be greater than the Circle, be the Excess as little as it will, a Polygon may be Circumscribed, whose Difference from the Circle shall be yet less than the Difference between that Circle and the Rectangled Triangle: And that Polygon will be less than the Triangle; which is absurd. And if it be said that this Rectangled Triangle is less than the Circle; an Inscribed Polygon may be made, which shall be greater than that Triangle, which is

impossible.

This kind of Demonstration which we here use, and which is called Reductio ad Absurdum, five ad ' Impossibile, is one of the finest Inventions of the ' Ancients: And on it is founded all the Geometry of ' Indivisibles; so that I cannot but much wonder ' fome of our Modern Authors should reject it as indirect and deficient. But if we must arrive to such a Point of Niceness, that we can't bear any De-'monstration, unless it be Direct and Positive; 'tis easie enough to give this before us such a turn, as

fhall render it Regular and Direct.

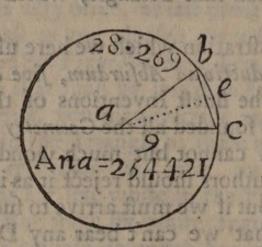
For this cannot but be admitted as a Principle; That if two determinate Quantities a and b are such, that every other imaginable Quantity, which is greater or less than a, is also greater or less than b; These two Quantities a and b must be equal. And this Principle being granted, which is in a manner selfevident, it may directly be proved that the Triangle (before-mentioned) is equal to the Circle: Because every Imaginable inscribed Figure, which is f less Less than the Circle, is also less than the Triangle:
And every Circumscribed Figure greater than the

Circle, is also greater than the Triangle.

This is that which is called the Quadrature of (or Squaring) the Circle, which consists in finding a Square, Triangle, or any other Rectilineal Figure exactly equal to a Circle. And this would easily be done, could we find a Right Line equal to the Circumference; as is plain from this last Proposition. But such an Equality is not to be found Geometrically.

### To find the Area of a Circle.

Since the Circle is equal to a Right-angled Triangle, whose Base is the Radius, and the Perpendicu-



lar a Line equal to the Circumference; half the Product of the Radius into the Periphery, will give the Area of the Circle.

In Practice, therefore say, either as 7: to 22: So is the Diameter in Inches, equal Parts, &c.

Division, say, as 1000 is to 3141:: So is the Radius of any Circle in Inches (suppose 9 Inches) to 28. 269, which therefore will be Semi-circumserence: And this multiplied by 9 Radius, gives 254. 421, for the Area required.

For the Area of a Sector or Segment of any Circle.

Since a Circle may be conceived as an Aggregate of an infinite Number of Isosceles Triangles, whose common Vertex is the Center: Any Portion of the Periphery, as b c, being considered as a strait Line,

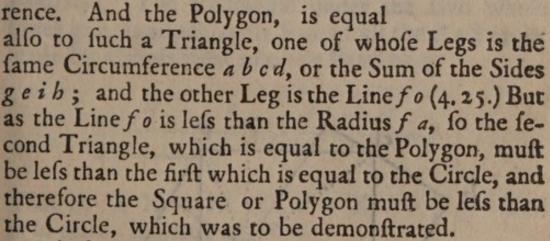
and

and the Perpendicular a e let fall, the Area of the Sector must be half the Product of the Ark b c into the Radius a e; and if from the Sector you take the Area of the Right-Lined Triangle a b c, there will

remain the Area of the Segment bec.

32. If a right Line could be dispos'd into the Form of the Circumference of a Circle, it would contain more Space than any other Figure, or Regular Polygon whatsoever: Suppose the Circumference of the Circle, a b c d, to be disposed into the Form of a Square, or into any other Regular Polygon: So that all the Sides e g, g h, b i and i e to-

gether may be equal to the Circumference a b c d; I say, the Circle is greater than that Square. For the Circle is equal to a Rectangled Triangle, one of whose Legs is the Radius f a, and the other the Circumse-



'And this is what we mean, when we usually 'say, that of Isoperimetrical Figures (or which have equal Perimeters or Circumferences) the greatest is

the Circle.

Before we go to Solids, I thought it proper to give the Learner here, this most Noble Theorem of Pythagoras; because, tho' it be indeed demonstrated in the sixth Book, yet nearly after Euclid's manner, it may also be done here: Thus,

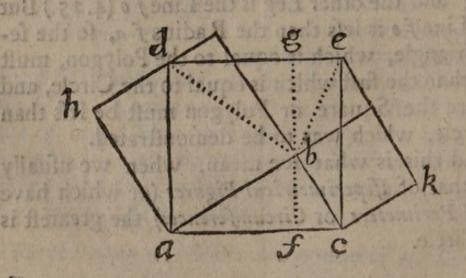
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In every Right-angled Triangle as a b c. The Square of the Hypothenuse a c, is equal to the Sum of the Squares of the Legs a b and b c; For,

I. The Square of c a, is equal to the two Rectangles df and fe.

II. The Rectangle d f is double of the Triangle a b d, being of the same Base and Altitude; and the Rectangle f e is for the same Reason, double of the Triangle b e c. (by 3. 18.)

III. But those Triangles, being of the same Base and Altitude with, will be equal also to one, half of the Squares b b and b k; Wherefore the Square of a c is equal to the Sum of the Squares of the Legs.



sadl cone here: That

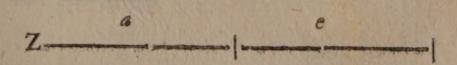
I have here added also the Substance of the second Book of Euclid, about the Power of Lines, &c. And I would advise the Young Geometrician before he proceeds any farther (and if not done already) to begin the Study of Algebra; a little of which will be of excellent Use to him in facilitating the Demonstrations in Geometry, and in preparing the Mind, and enuring of it to Abstraction, before he come to the Doctrine of Proportion. And the four first Rules of Addition, Substraction, Multiplication and Division, in Integers and Fractions, will be sufficient to enable him to understand the following Propositions. As also the most useful ones, which he will find added (in this Edition) in all the following Books of these Elements.

I. If there be two Lines Z and X; one of which, as z, is divided into any Number of Parts, as into a+e+i+o. The Rectangle under the two whole Lines z x, is equal to the Sum of all the Rectangles made by x multiplied into the Parts of z.

$$Z = \begin{bmatrix} a & e & i & o \\ X & & & & \end{bmatrix}$$

That is, Z X = X a X e + X i + X o. This is fo plain, it needs no Proof.

If a Right Line, as Z, be divided into two Parts, a+e. The Rectangles made by the whole Line, and both its Parts are equal to the Square of the whole Line.



## 56 ELEMENTS

That is, za + ze = zzFor za = aa + ae. And ze = ae + ee. That is,

 $z_{a+z_{e}=a_{a+2}a_{e+e_{e}}=Q.a_{e}$ . Q. E. D.

III. Let the Line Z be cut into a + e; then shall the Rectangle under the whole Line (Z) and the Part (a) be equal to the Square of that Part a, together with the Rectangle made by the two Parts a and e.

That is, Za = aa + ae.

$$Z = a + e$$

And 
$$a + e \times a = aa + ae$$
. Q. E. D.

IV. The Square of any Line, as Z, divided into any two Parts a and e, is equal to both the Squares of those Parts, together with two Rectangles made out of those Parts.

That is, 77 = a a + 2 a e + e e.

Multiply a + e by its self, and the thing is plain.

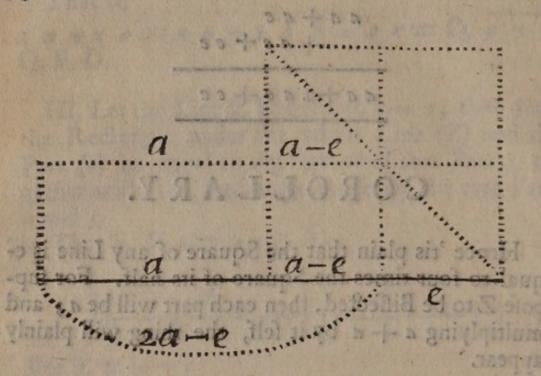
#### COROLLARY.

Hence 'tis plain that the Square of any Line is equal to four times the Square of its half. For suppose Z to be Bissected, then each part will be a; and multiplying a + a by it self, the thing will plainly appear.

V. If a Line be divided into two Parts equally, and in two other Parts unequally, the Rectangle under the unequal Parts, together with the Square of (the intermediate Part) the Difference between the equal and unequal Parts, is equal to the Square of half that Line.

#### 8 ELEMENTS

Let the whole Line be 2 a, then each Part will be a. Let the lesser unequal Part be e, then the Intermediate Part will be a—e, and the greater unequal Part will be 2 a—e; which multiplied by



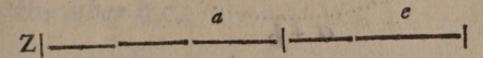
e, produces 2 a e - e e; To which adding the Square of the Difference, or intermediate part a - e, which is a a - 2 a e + e e, the Sum will be only a a, the Square of half the Line.

VI. If a Line be Biffected, and then another Right Line be added to it, the Rectangle or Product of the whole Augmented Line, multiplied by the Part added, together with the Square of the half Line, is equal to the Square of the half Line and part added, confider'd as one Line.

Let the first Line be 2 a, and the Part added e, then the whole will be 2 a+e; which multiplied by e, produces 2 a e+e e, and the Square of half the Line

Line a a being added to it, it will be 2 a e+e e+ a a, which is equal to the Square of a+e, by Prop. 4.

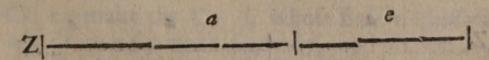
VII. If a Quantity or Line be divided any how into two Parts, the Square of the whole added to the Square of one of the Parts, shall be equal to two Rectangles contained under the whole Line and that Part added to the Square of the other Part.



Let a be one Part, and e the other. The Square of the whole and of the lesser Part e, makes a a+2 a e+2 e e, Then if the whole a+e be multiplied twice by e, it will produce 2 a e+2 e e; and if to this be added the Square of the other Part a a, the Sum will be

a a+2 a e+2 e e, equal to the former.

VIII. If a Line be cut any how into two Parts, the Quadruple Rectangle under the whole Line and one of the Parts, added to the Square of the other Part, is equal to the Square of the whole and the other Part added to it, as if it were but one Line.



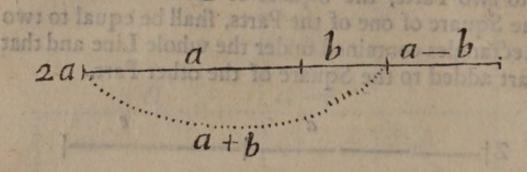
Let the whole Line be  $a \rightarrow e$ , then four times that multiplied by e (or the Quadruple Rectangle under that and e) will be  $4 \cdot a \cdot e + 4 \cdot e \cdot e$ ; to which adding the Square of the other Part  $a \cdot a$ , the Sum will be  $a \cdot a - 4 \cdot a \cdot e + 4 \cdot e \cdot e$ .

And if you square a+2 e, which expresses the whole Line, with e added to it, the Product will be

the former Sum of a a+4 a e+4 e e.

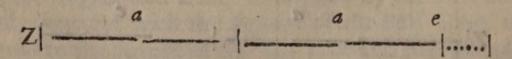
IX. If

IX. If a Line be biffected, and also cut into two other unequal Parts, the Sum of the Squares of the unequal Parts will be double to the Sum of the Squares of the half Line, and of the Difference between the two unequal Parts.



Let the whole Line be 2a; and the Difference between the equal and unequal Parts, b; then the greater unequal Part will be a + b, and the leffer a - b: The Sum of the Squares of the unequal Parts will be 2aa + 2bb, which is double to the Square of half the Line added to the Square of the Difference. Q. E. D.

X. If a Line be biffected, and then another Line added to it; the Square of the whole encreased Line, together with the Square of the Part added, is double the Sum of the Squares of the half Line, and of the half Line and Part added, taken as one Line.

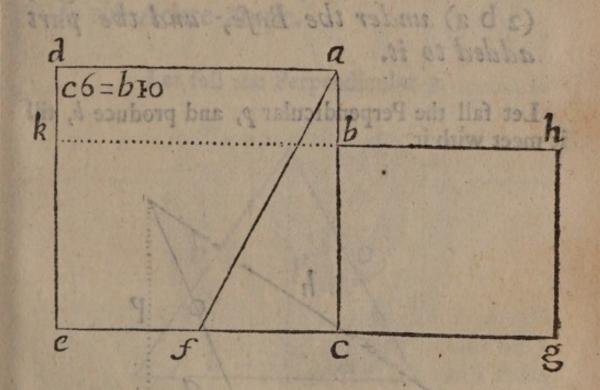


Let the whole Line be 2 a, and the Part added e; then the whole encreased Line will be 2 a + e; and the half Line and Part added will be a + e. The Sum of the Squares of 2 a + e, and of e, is 4 a a + 4 a e + 2 e e; which is plainly double to a a, and a a + 2 a e + ee, added together. Q E. D.

This Problem is also of frequent Use.

#### PROBLEM.

To divide a Line so, as that the Rectangle under the whole Line a c, and one Segment a b, shall be equal to the Square of the other b c.



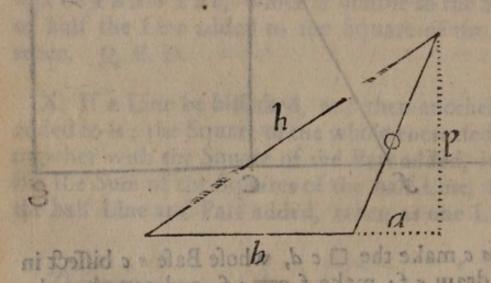
On a c make the  $\Box$  c d, whose Base e c bissect in f, and draw a f; make f g = a f, and compleat the  $\Box$  b g, producing b b to k; Then is a c truly divided in b; for the Line e c being bissected in f, and the Part c g added to it, the (by Prop. 6. of the Power of Lines) Rect kg + fc q = fg q = fa q = acq + fcq: Wherefore taking fcq from both, the Rect. kg = acq, and taking the Rect. kc from both the Rect. kg = acq, and taking the Rect. kc from both the Rect.

N.B. This is called dividing a Line according to Extream and Mean Proportion; which Proportion cannot be express'd in Numbers.

#### PROP. I.

In an Obtuse-Angled Triangle, the Square of the Side subtending the Obtuse Angle, exceeds the Sum of the Squares of the other two Sides by the double Rectangle, (2 b a) under the Base, and the part added to it.

Let fall the Perpendicular p, and produce b, till it meet with it.



#### DEMONSTRATION.

1. bb=bb+2 ba+a a+pp.

2. And 0 0 = p p+a a.

3. But b b+0 0=b b+a a+p p.

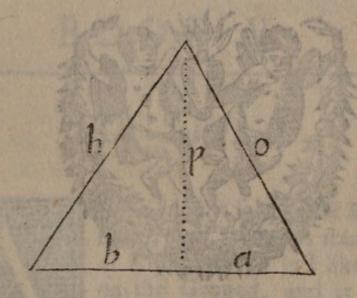
Wherefore b b exceeds the last Step by 2 b a. Q. E. D. PROP.

### PROP. II. on to country?

4. bb+pp+2aa+2ab is the Sum of the

In an Acute-angled Triangle, the Square of the Side (h) subtending an Acute-Angle, is less than the Sum of the Squares of the other two Sides, by double the Rectangle under the whole Base, (b+a) and the Segment of the Base (a) which is next to the Acute-angle.

Let fall the Perpendicular p.



#### DEMONSTRATION.

1. bb = bb + pp.

2.00 = pp + aa.

3, Q. b + a = b b + 2 b a + a a.

64 ELEMENTS, &c.

4. bb+pp+2 a a+2 a b, is the Sum of the Squares of the Legs.

Wherefore b b is less than that by 2aa+2ab, which is plainly equal to the Double Rectangle under the whole Base, and the Part a.

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ender the revole Befor (1-42) and the

. Sogment of the Baja (a) rebiches a next

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DEMONSTRATION.

39+ 49= 99 1

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# ELEMENTS

OF

# GEOMETRY.

## BOOK V.

### Of Solids.



Right Line is said to be Right up on a Plane, when it stands on it at Right Angles, just like a Pillar on the Ground, and is inclined no more to any one side of the Plane, than to the other.

2. Two lanes are Parallel to each other, when all the Perpendiculars that can be drawn between them, are equal. (That is, when they every where

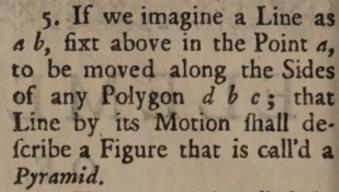
are equally distant.)

3. One Plane is Right or Perpendicular to another Plane, when, like a well-made Wall, it inclines and leans on one fide no more than it does on the other.

#### ELEMENTS

4. A folid Angle is made by the meeting of three

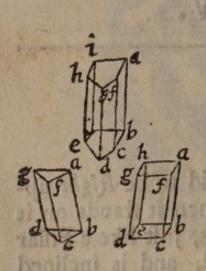
or more Planes, and those joining in a Point: Like the Point of a Diamond well cur.



6. The Polygon is called the

Base of the Pyramid.

7. If a Line fastened, as before, move round a Circle, as d b c, it will describe a Cone; and the Circle is its Base. And a Line drawn from the Center e to a, is call'd its Axis.



66

8. If a Line a b move uniformly about two Polygons gf a and d c b, which are every way equal, having their Sides and Angles mutually Parallel and corresponding exactly to one another, as a f to b c, f g to d c, & c. then that Line shall by its Motion describe a Figure which is call'd a Prism, and the Polygon is its Base.



Book V. of GEOMETRY.

If all the Sides of a Prism be a Parallelogram. then that Prism is call'd a Paralle-

lopiped.

10. If a Line a b move uni- 8 ea formly round two equal and Parallel Circles, it shall describe or a b generate a Cylinder.

11. The Line joining the Centers ee, in the two

Bases, is call'd the Axis.

'There is no need of conceiving two Bases, Equal, Parallel and Opposite, for the Genesis of Prisms and Cylinders. For they will be describ'd as well by Imagining a Line moving round the Circumference of any plane Figure with a Motion always Parallel to its felf in its first Position. ab be supposed to be carried round any of the Bases d c b, keeping always the same Angle with the Plane, which it first had, it will describe a Triangular, Quinquangular, or Circular Prism, according to the Figure of the Base. And the upper end of the Line will describe a Base (as you may call it) at the Top, equal and parallel to that below.

#### COROLLARY.

The Solid Content of all Isosceles Prisms and Cylinders (as also, of all Parallelopipeds), is had by Multiplying their Height into the Area of their Base.

And if they are Scalenous Prisms or Cylinders, by Multiplying the Base by the perpendicular Altitude.

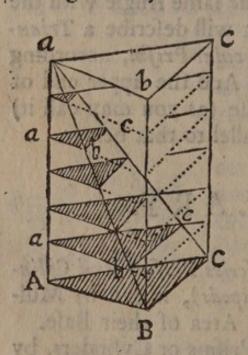
But after all, this Genesis of Prisms and Pyramids, of Mr. Pardie, respects only their Surfaces. And therefore, the most proper way to conceive the Genesis of all Kinds of Prisms, is to imagine a Triangle, Quadrilateral Figure, or Polygon, or the Plane of a Circle to be moved in a Position always Paral-

lel to it self; as suppose from b to e, or from g to d (in the preceding Figures) according to the Direction of the Line be or g d. Or according to Euclid, a Cylinder will be generated by the Revolution of the Parallelogram g e d e (See Fig. in Art. 8.) round about the Axis a e.

### COROLLARY.

And from hence (as was observed before of Lines) tis plain that equal Surfaces moved uniformly over equal Places or Intervals, will describe or generate equal Solids.

And as for the Genesis of Pyramids, suppose the Triangle a b c, to move downwards from the Top of



Shiron always Paral-

a Plane Angle, determined by the two Planes a A B, a A C: Let this Motion be always Parallel to it self, and let the Angular Point of the moving Triangle a, be supposed always to keep in the Line a A.

'Tis plain, as this Triangle moves farther downwards, it will still get more and more within the Solid Angle, and at last will come to be all of it within it, and

to lie in the Position A B C, which will be the Base of a Triangular Pyramid, whose Vertex is at a.

The same Triangle a b c, will also, by its Motion, describe another Pyramid, whose Base shall be the Parallelogram b c B C, and its Vertex a, as before.

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on its Diameter a b, it will describe a

Sphere or Globe, whose Axis will be a b,
and its Center c, the same with the Semicircle. Every Line passing through the
Center c, and terminated at each end by
the Surface of the Sphere, is called a Diameter, and may be cail'd an Axis.

14. All Lines drawn from the Center c to the Surface, are call'd Radii, and are all equal to one ano-

ther.

#### To find the Surfaces of Solids.

#### I. For all Prisms, Parallelopipeds and Cylinders.

Find the Perimeter of the Base (which in Practice is done by girting it with a String) and Multiply that by the perpendicular Height, the product is the Surface without the Base, (i.e. without the top and bottom Planes) and the Bases may be found by the Rules given in Plain Mensuration: The Reason of which is, because a Rectangle of that Form and Dimensions will just cover the outside of the Body.

#### II. For Pyramids and Cones.

The Surface of a Pyramid, is only an Aggregate of Triangles, which therefore must be found severally, and then added up into one Sum.

The Surface of Scalenous Cones cannot be found exactly; but for Right Ones Multiply the Circumference of the Base, by half of the Side of the Cone, the Product is the Area of the Convex Surface. Because the Curve Surface of a Cone is equal to a Tri-

F 3

angle,

#### ELEMENTS

angle, whose Base is the Periphery of the Base, and its height the side of the Cone; such a Figure being capable of exactly covering it.

#### 111. For the Surface of the Sphere.

Multiply the Diameter by the Periphery of any great Circle, or by such a Circle as hath the Diameter of the Sphere for its Diameter, the Product is the Surface. As appears from what will be prov'd below, after Art. 34.

IV. The Surface of the five Regular Bodies, is easily had, by the Principles of Plain Mensuration.

15. Two Right Lines if they meet so as to cut or cross each other, are in the same Plane: Wherefore all the Angles and Sides of every Triangle are in the same Plane.

16. If two Planes e b d and a g f cut or Intersect

one another, they shall do so in a Right Line, as bd; which is call'd their common Section.

17. If a Right Line de be Perpendicular to two Lines df and dg, which are in the same Plane, that Line is also Perpendicular to that Plane.

18. If a Right Line d c be Perpendicular to three Right Lines df, dg and da, they are all three in the same Plane.

19. If two Lines d c, b i are Perpendicular to the same Plane f g a, they will be Parallel to one another.

20. If two Lines dc, bi are Parallel, and you draw another Line, from any Point in one to the other, as b d, those three will be all in the same Plane.

cause the Curve Surface of a Cone is equal to a Tri-

21. If two Lines d c, b i are Parallel to a third a k, though that third Line be not in the same Plane with them, yet they shall be Parallel to each other.

22. If a right Line a b be perpendicular to (or make

any other equal Angles with) two Planes fe and cd, those Planes are Parallel.

23. If two Parallel Planes, d h g and a f e, are cut by a third i i i, the common Sections f e and h g are Parallel.

by three Plane Angles, any two of those are always greater than the third.

All these Propositions are somanifest to one that will but consider them with a little Attention, that 'tis needless to stay to Demonstrate them. (And indeed the Solemn and Regular Demonstration of a thing Plain

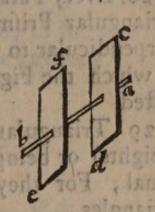
in its felf, always makes it more Obscure.)

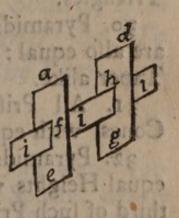
One, taken all together, are always less than four Right Ones. For if they should make four Right Angles, they would form a Plane and not an Angle. Wherefore, that they may make a Solid Angle, they must be less than four Right Ones.

'Tis a very good way in order to gain a clear Idea of Solids and their Angles, to make the Regular Bodies out of thick Paper or Past-board, and after the Description of every Body, you will see the Figure, which being fold-

ed up together, will express the Solid:

26. In all Parallelopipeds, the Opposite Planes are equal; as is easie to conceive (from 5. 9)





27. All Parallelopipeds having equal Bases (and Heights) or being between the same Parallels are equal, for they are equal Aggregates of equal Paral-

lelograms. (3. 14.)

28. Every Parallelopiped is divided into two equal Triangular Prisms, by a Diagonal Plane, which is Perpendicular to its Base: For every Parallelogram of which the Figure is composed, is equally Bissected.

29. Triangular Prisms, having equal Bases (and Heights) or being between the same Parallels, are equal; For they are equal Aggregates of equal Triangles.

30. Pyramids having equal Bases and Heights, are also equal: For they are all supposed to grow

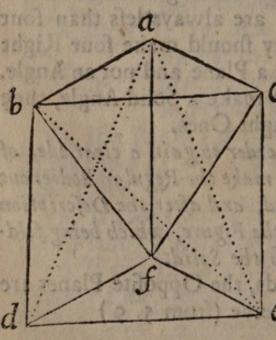
Taper alike.

31. All Prisms in general, all Cylinders and Cones, with equal Bases and Heights, are equal.

32. Pyramids and Cones on equal Bases, and of equal Heights with Prisms and Cylinders, are one third of such Prisms and Cylinders:

In a Triangular Prism and Pyramid of the same

Base and Altitude, it is thus prov'd.



Pyramid a c e f b is divided into two equal Triangular Ones, by the Triangular Plane f b c, and the Pyramid f c a b, is the very same with b a c f; and this is equal to the Pyramid d f e a: As having an equal Base and the same Altitude with it, and therefore

the whole Prism is divided into three equal Pyramids.

And

And since all Multangular Prisms can be divided into Triangular ones, and that Cylinder is only a Multangular Prism of infinite Sides, the Proposition is Universally true; That Pyramids and Cones, &c.

N. B. A piece of Cork or Wood, in the Form of a Triangular Prism, may be cut into three equal Pyramids.

# COROLLARY I.

Hence the way of finding the Solidity of a Pyramid or Cone is discover'd, viz. To Multiply the Base by \( \frac{1}{3} \) of the Perpendicular Altitude.

33. Every Sphere is equal to a Cone whose Perpendicular Axis is the Radius of the Sphere, and its Base a Plane, equal to all the Convex Surface of it.

For you may conceive the Sphere to consist of an infinite Number of Cones, whose Bases taken all together compose the Surface, and whose Vertex's meet all together in the Center of the Sphere: Just as a Circle may be imagined to be composed of an infinite Number of Isosceles Triangles, the Aggregate of whose Bases makes the Circumserence, and their common Vertex is at the Center.

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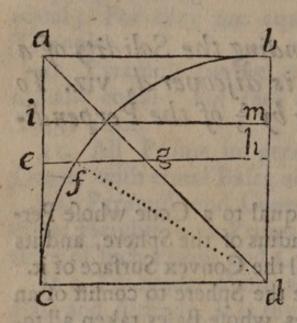
wherever you draw the Line a b, or i m. &cc.

II. If you take the Circle made by the Revolution

#### COROL. II.

Hence the Solidity of the Sphere will be gain'd by Multiplying its Surface by \{\frac{1}{3}} of its Radius.

Let the Square a d, the Quadrant c b d, and the right-angled Triangle a b d, be supposed all three to



revolve round the Line b d as an Axis: Then will the Square generate a Cylinder, the Quadrant an Hemisphere, and the Triangle a Cone, all of the same Base and Altitude.

I. Then the Square of e b (which is equal to the Square of f d, which is equal to the Square of

f h, together with that of h d (or its equal g h) will be equal to the Square of g h (=b d) together with the Squares of f h. And fince Circles are as the Squares of their Diameters (which must be now taken for granted, but will be proved in the 6th Book) the Circle made by the Revolution of e h, must be equal to the two Circles made by the Motion of f h and h g. Wherefore,

II. If you take the Circle made by the Revolution of f b from both, there will remain the Circle made by the Motion of g b equal to the Ring described by the Motion of e f: And thus, it must always be wherever you draw the Line e b, or i m, &c.

III. There-

III. Therefore the Aggregate of all the Rings made by the Revolution of the ef's, must be equal to that of all the Circles made by the Motion of the g h's: (i. e.) the Dish-like Solid, formed by the Revolving Rings, will be equal to the Cone formed by the Revolution of the g h's, which are the Elements of the Triangle abd. That is, the Dish-like Solid will be, as the Cone is, \frac{1}{3} of the Circumscribing Cylinder, and consequently the Hemisphere must be \frac{2}{3} of it: Wherefore the Sphere is \frac{2}{3} of the Circumscribing Cylinder.

IV. Let then the Radius of the Sphere be r = c d=b d, then the Diameter will be 2r; let the Surface of the Sphere generated by the Revolving Semicircle be called S; and that of the Cylinder, formed by the Revolution of 2 a c=2 r= Diameter, be called f. Wherefore in what was just now proved; (by Art. 33. of this Book) the Expression for the Solidity of the Sphere in this Notation will be r S and putting c equal to the Circumference of the Base, or for the Periphery of a great Circle of the Sphere, the Curve Surface of the Cylinder (by Multiplying the Altitude into the Periphery of the Base) will be 2 re; also will be the Area of a great Circle (by Prop. 26. of Book 4.) and this multiplied by 2 r, makes 2 r r c, which is the Solidity of the Cylinder (by Cor. Art. 11.) Now fince f was put equal to 2 r c = to the Curve Surface of the Cylinder 1 (by Substituting of for 2 rc) will be also = to the Solidity of the Cylinder. Now fince the Sphere is  $=\frac{2}{3}$  of the Cylinder;  $\frac{rS}{r}$   $= \frac{2}{3} \frac{rf}{2}. \text{ That is } \frac{rS}{3} = \frac{2rf}{6} = \frac{rf}{3}. \text{ Where-}$ 

fore rS = rf; that is, dividing by r, S = f; or the Surface of the Sphere is equal to the Curve Surface of the Cylinder: But the Curve Surface of the

Cylinder was 2 r c.

Wherefore to find the Area of the Surface of either Sphere or Cylinder, you must Multiply the Diameter (= 2 r) by the Circumserence of a great Circle of the Sphere, or by the Periphery of the Base. From

this Notation also "c, the Area of a great Circle of

the Sphere, is plainly  $\frac{1}{2}$  of 2 r c the Surface of the Sphere. That is, the Surface of the Sphere is Quadruple of the Area of a great Circle of it.

V. Wherefore to 2 r c the Convex Surface of the Cylinder, add r c equal to the Area of both its Bases (each of which is  $\frac{r c}{2}$ ) you will have 3 r c; which

shews you that the Surface of the Cylinder (including its Bases) being 3 r c, is to the Surface of the Sphere, which is, 2 r c, as three is to two: Or that the Sphere is  $\frac{2}{3}$  of the Circumscribing Cylinder, in Area, as well as Solidity.

34. Of all Solid Figures that can be encompass'd or determinated by the same Surface, the greatest is a Spherical One, by Art. 13th of this Book, and Art.

the last of Book the 4th.

35. That is call'd a Regular Body, whose Surface is composed of Regular and Equal Figures. And whose Solid Angles are all equal, as are—

ill be also Tro the Soliding of the Cylinics.

36. The Tetrabedron, which is a Pyramid, com-

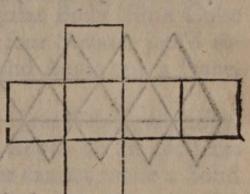
prehended under four equal and equilateral Triangles; so that its Base is equal to each Side.

Wherefore its Solidity will be found by Multiplying the Base by  $\frac{1}{3}$  of the Altitude; which is the general way for all Pyramids.

37. The Hexahedron or Cube, whose Surface is compos'd of six equal Squares, like Dice which are us'd in play.

Its Solidity will be found

by Cor. of Art. 12.



38. The Octabedron, which is bounded by eight equal and equilateral Tri-

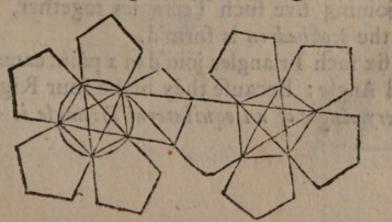
angles.

This Figure is two Pyramids put together at their Bases: Wherefore its Solidity is had by Multiplying the Quadrangular Base of either (here they are both join'd together in the middle

of the Figure) by one third of the Perpendicular Altitude of one of the joined Pyramids, and then doubling

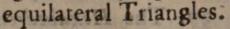
the Product.

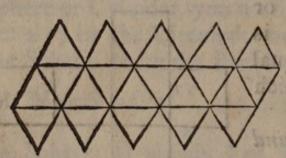
39. The Dodecahedron, which is contained under twelve equal and equilateral Pentagons.



This Figure consists of twelve Pyramids, with Pentagonal Bases, whose common Vertex is the Center of a Circumscribing Sphere: Wherefore any one of these twelve Pentagonal Bases Multiplied by; of the Distance between the Center of that Base, and the Center of the Sphere; and then that Product Multiplied by twelve, gives the Solid Content of this Regular Body.

40. The Icosihedron, confisting of twenty equal and





This Figure is compofed of 20 Triangular Pyramids, all equal to one another, and whose Vertex is the Center of a Circumscribing Sphere:

Wherefore any one of the 20 Triangular Faces, Multiplied by \frac{1}{2} of the Distance between the Center of the Face and the Center of the Sphere, and that Product Multiplied again by 20, gives its Solid Content.

41. Besides these five Regular Bodies, 'tis not possible to find any others that shall correspond to the

Definition; which is thus Demonstrated.

To begin with equilateral Triangles, which are the most simple of all Rectilineal Figures. Of these there must be three at the least to make a Solid Angle, and three of them join'd together will just make the Tetrabedron. For those three Triangles meeting in a point do form a Triangular Base similar and equal to the Sides; as appears by the bare Composition of the Figure. Four Triangles join'd together in a Point make the Angle of the Ostabedron.

By joining five such Triangles together, the An-

gle of the Icosihedron is form'd.

But six such Triangles join'd in a point cannot make a Solid Angle: Because they make four Right Ones (for every Angle of an equilateral Triangle is \frac{1}{3} of two,

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Book V. of GEOMETRY.

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or  $\frac{2}{3}$  of one Right Angle, either of which Fractions Multiplied by fix, gives four right Angles.) Whereas every Solid Angle is made up of such plane Angles as all together must be less than four Right ones (5. 25.) So that with Triangles 'tis impossible to form any more Regular Bodies than these three.

Next, if you take Squares and join three of them together, they will make the Angle of the Cube: And there can no other Regular Body but a Cube be made with Squares, for four Squares join'd together, will not make a Solid Angle but a Plane.

(5. 25.)

If you join the Angles of three Pentagons together, you will constitute the Angle of the Dodecahedron: But four such Angles cannot make a Solid One.

And lastly, Three Hexagons joined together do make just four Right Angles, and therefore they cannot make a Solid Angle: And as for three Heptagons, or other Figures of yet more Sides, they can much less do it; (because their Angles being very Obtuse, three of them will exceed four Right Ones.) So that upon the whole its plain, that of these five Regular Bodies, three are made of Triangles, one of Squares, and one of Pentagons, and there can be no other.



-ELE-what Magnitude it hath in compari-



## ELEMENTS

OF

# GEOMETRY.

#### BOOK VI.

## Of Proportion.



HEN we speak of Magnitude, and say, that any Quantity is great, we always make a Comparison between that Quantity and some other of the same Nature, in respect to which we say that it is

Great.

Thus we say of an Hill, that 'tis Little; or of a Diamond, that 'tis Large; because we compare that Hill with others that are Higher, and in respect of them 'tis Little; and we compare that Diamond with others that are Little, and in respect of them, we say 'tis a Large one.

2. When we consider one Quantity in respect of another, to see what Magnitude it hath in compari-

Con

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on of that other: The Magnitude so found, is call'd ts Ratio or Reason; tho' it would be more intelligible if it were call'd Comparison.

3. That Quantity which is compar'd with another s call'd the Antecedent; and that other with which

t is compar'd'is call'd the Consequent.

4. When we consider four Quantities, and compare them (by Pairs) two with two; as a four with b 2, and c 6, with d 3. If we find that a hath as much Magnitude (or is as big) in

comparison of b, as c hath in comparison of d; (i. e. When we find that a is contained in; or doth contain b:: as often as c is contained in, or doth contained in in comparison of d; (i. e. When we find that a doth contained in, or doth contained in in comparison of d; (i. e. When we find that a doth contained in, or doth contained in in comparison of d; (i. e. When we find that a doth contained in in comparison of d; (i. e. When we find that a doth contained in in comparison of d; (i. e. When we find that a doth contained in in contained in conta

tio's are equal; that is, the Ratio that a hath to b; is equal to the Ratio of c to d: For as a is twice as

big as b; so c is twice as big as d.

5. But if a 4 hath more Magnitude in respect of b, than c 6 hath in respect of e 5. That is, if as a 4 is twice as big as b 2, c 6 be found not to be twice as big as e 5: Then the Ratio's are unequal: And we say, a hath a greater Ratio to b, than c hath to e. So that to have a greater Ratio, is nothing but to have more Magnitude, or to be bigger, in respect of a second Term, than a third is in respect of a fourth.

6. The Equality of Ratio's is call'd Proportion; and when we find that of four Quantities or Numbers, the first hath as much Magnitude (or is as big) in respect of the second; as the third is in respect of the fourth; then we say, that those four Quantities

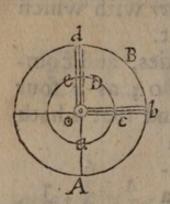
ate Proportionals.

The better to make the Mysteries of Proportion comprehended, which pass for the most difficult things in Geometry, as unquestionably they are most Important, I will explain them by an Example; which (in my Opi-

ed slive es an ion)

nion) will render all those things very Intelligible, which otherwise appear very perplexed.

7. Let us imagin the Circle b A d to be describ'd by the Motion of the Line ob, round the Center o:



And at the same time, let the Circle c a e be described by the Motion of a Point c, in the Line ob: Let us suppose also that the Line b ob be moved once round again, and at last to stand in the Position o d. Let the Ark d B b be called B, and the Ark e D c, be call'd D. Let A

be put for the whole outer Circle, and a for the whole inner one.

Now if we compare the whole Circle A with its Ark B, and the whole other Circle a with its Ark D. We shall find plainly, that the Circle A is just as big in respect of the Ark B, as the inner one a is in respect of the Ark D; and therefore if B be a fourth, or any other part of the Circle A, D also will be a fourth, or the same proportional part of its Circle a... Which we usually express by saying, as A is to B, so is a to D. And write it thus A. B:: a. D. or 24. 6 :: 8. 2.

8. If you should change the Order of the Terms, and compare B with A, and D with a ; you will find plainly that B. A :: D. a. So that supposing A. B :: a. D. we cannot but presently conclude by Inverse

Proportion, that B. A :: D. a.

9. If you change them so as to compare Antecedent with Antecedent, and Confequent with Consequent; you will find Alternately, that A. a. :: B. D. And this is very plain; for if the whole Circle A be double, triple of, or in any other Proportion, to the Circle a, the Ark B must be also double, triple of, or in the same Proportion to the Ark D; for Aliquot Parts will be as their Wholes. This I say is plain.

plain, because the two Circles A and a are described by the Motion of the Line o c b; so that while b describes the Circle A, c describes the inner Circle a; and while b describes the Ark B; c also describes the Ark D. And this by one common circular Motion; only the Point c moving much slower than the Point b, describes a Circle much less, in proportion to the slowness of its Motion: Thus also when the Point b shall have described the Ark B, the Point c in like manner will have described the Ark D, which will be much less than B; in proportion to the Slowness of its Motion; in Numbers 24. 8:: 6. 2.

10. If we compare the Differences between the

Antecedents and Consequents, with their Consequents; as for Instance, A less B with B, and a less D with D, we shall find they also are Proportional: And that A less B. B:: a less D. D. 18. 6.:: 6.2.

For 'tis manifest that the Ark b A d (which is A less B) is to B.

as the Ark cae (which is a less D) is to D. And this is call'd Proportion by Division.

together; we shall find that A more B, is to B:: as a more D is to D. Which is call'd Composition. In Numbers 30. 6.:: 10. 2.

12. And if we should say, that A A less B:: a. a. less D. This kind of Proportion is call'd Conversion. You may also infer by way of Mixing the Terms, as some call it, That A + B. A - B:: a + D. a - D. or that A + B. a + D:: A - B. a - D. &c. That is 30, 18:: 10.6 and 30. 10:: 18.6, &c.

And it will be very convenient for the Learner to enure himself to all the Changes and Varieties of Proportion, and to have them ready in his Mind; Because a great many Propositions in Geometry, as

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they have been delivered by the Ancients, and purfued by the Moderns that have trod in their Steps, are demonstrated by Composition, Division, Alternation, and intermixing of Proportion.

13. If never so many Quantities are thus Proportional: It will be as any one Antecedent to its Confequent:: So is the Sum of all the Antecedents to

the Sum of all the Consequents. v. gr.

If 4 12:: 2.6.:: 3.9:: 5.15: then shall 14.42:: 4. 12.

14. If a. b:: c. d.
4. 12:: 3.9. and also,

b. f: d.g. 12.36:: 9.27.

Then will it be by Proportion of Equality.

a.f:: c.g. 4.36::3.27.

The Reason of which is plain, if you consider, That since b. f:: d. g: f and g must needs be either Similar aliquot Parts, or Equimultiples of b and d. And therefore since a and c are to b and d, in the same Ratio as b and d are to f and g, a must also be in the same Ratio to f. the Part or Multiple of b:: as c is to g. the Part or Multiple of d.

If a.b:: c.d.
12.4:: 9.3 and then,

b. f :: b. c. 4. 2. :: 18.9. 12. 2:: 18. 3.

Which is called, Proportio ex aque perturbata; and this must be true: because 12 containing 4 as oft as 9 contains 3: and 4 containing 2, as oft as 18 contains 9; 12 must contain 2 as often as 18 contains 3. Wherefore this is only the orderly Proportion of Equality disturbed, and therefore is by some called

Inordinate Proportion.

3 D. we may conclude that B. D:: 3 B. 3 D. or as 10 B to 10 D. also 12 B. to 12 D. And so on in whatever Proportion the two Magnitudes B and D are multiplied, so they are multiplied equally, or that you take one as often as you take the other. For then there will be the same Proportion between the Magnitudes thus equally multiplied, as there was between the simple Magnitudes, before such multiplication. And these Magnitudes thus equally multiplied, are call'd Equimultiples of the simple Magnitudes B and D; and we say that Equimultiples are in the same Proportion as such simple Magnitudes, out of which they are compounded.

16. If B be divided in the same manner as D is; and ex gr. you take a fourth Part of B, and the like of D. or the tenth, or any other Part of B, and the same of D. Then will these Parts be Proportional to their wholes, B. D:: B (or B) is to A, or D.

All which is Self-evident.

Rectangled Parallelogram, whose two Contiguous Sides shall be the two Lines given. Thus, if you multiply the Line A by B; 'tis the same thing as to make the Rectangle a b c d; whose Side a b is equal to A, and a c to B.

18. To multiply a Rectangle, or any other Surface

by a Right Line, is to make a Rectangled Parallelopiped (or Prism) (5.9) whose Base shall be the Surface given, and its Perpendicular

Height the Line given.

Thus to multiply the Surface a b

d c by the Line E, sis the same thing
as to make a Solid a b f g b e, whose Base is the Surface given a d, and its Height ae or b f, equal to E,

the Line given.

As if one Magnitude be double or triple of another; or in any other Ratio, two Lines may easily be taken, of which one shall be double or triple of the other, or in any other like Proportion with those Magnitudes: So for Instance, to express two times, as one Hour and two Hours; or two Velocities, of which one shall be double to the other; you need only take two Lines, as a double of b; and then you may say that a represents two Hours or Velocities, and b answers to one of each; and then you may proceed to compute with those two Lines, as with the Hours and Velocities themselves, &c.

20. To know the Proportion of Rectangles, the Ratio of the Length of One to the Length of the Other, and moreover the Ratio of the Breadth of One to the Breadth of the Other must be known.

For Example; To know what Proportion the

Rectangle a c hath to e g: 'Tis not enough only to know that the Length c ab is triple of eb; but it must be known also, that a d is double of ef. For if a i be taken equal to ef, the

Rectangle b i will be triple of eg, because a b is triple of e b, and a i equal to e f. And moreover, because i d is also equal to a i, or ef (for

Book VI. of GEOMETRY.

a d is supposed to be double of a i, and of e f) the Rectangle i c shall also be triple of e g; so that the whole Rectangle ac is twice triple of the Rectangle e g; that is, sextuple of it, or containing it six times. And what we say now only of the double or triple Ratio of their Breadths and Lengths, is also to be understood of any other Ratio, be it what it will: For if a b be quadruple of e b, and a d triple of e f, the Rectangle a c, will be three times quadruple of the Rectangle e g; that is duodecuple of it, or doth contain it twelve times.

But if a b be duodecuple of e b, and at the same time ef be Triple of ad, then there is a certain Compensation made: For if Respect were had to their Breadths a b and e b only, the Rectangle a c would

exceed the other, nay indeed contain it 12 times: Nevertheless this Excess is lost (in some Measure) in respect of their Altitudes or Heights a d and ef, which if only consider'd, the Rectangle eg would be Triple of a c. But then when we come to compare these several Ex-

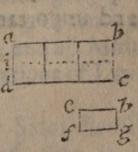
cesses and Desiciencies together; we shall find that the Rectangle ac being one way 12 times greater, and the other way three times less than eg, will be

at last but only four times as great.

all Rectangles are to each other in a Ratio Compounded

of that of their Sides; for if ab be triple of eb, and ad double of ef, the Rectangle ac, shall be to the Rectangle e g in a Ratio compounded of the triple and the double, that is, it shall be thrice double, or twice triple, or in one Word Sex-

tuple. So also if a b were quadruple of eb, and a d triple



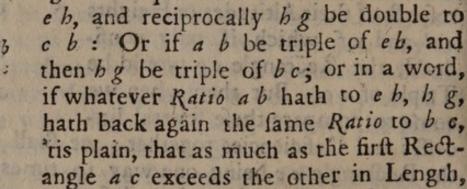
triple of ef; the Rectangle ac would then be to eg in a Ratio compounded of the Quadruple and the Triple; so that it would have been three times Quadruple, or four times Triple, or in one word Duodecuple of eg.

Moreover, if a b were Duodecuple of e b, and a d Subtriple of e f, (that is, if e f be Triple of a d) the Ratio of the Rectangle a e to e g would be compounded of the Duodecuple and Subtriple Ratio; so that ac, would have been 12 times subtriple of, or in one word Quadruple of e g.

If you take the third part of a Crown 12 times, it will make, or be equal to four whole Crowns: So that four Crowns are 12 times subtriple of one Crown; that

is, do make 12 thirds of a Crown.

of two Rectangles are reciprocally Proportional, those two Rectangles are equal: For if a b be double to



or set light, a a planting to

the triple and the double,

just so much is it exceeded by the other in Breadth; so that the Length of one Compensates for the Breadth of the other, and consequently they must be equal. And from hence is deduced this most useful and important Proposition: That,

23. If four Quantities (or Numbers) be proportional, the Product arising from the Multiplication of the two middle Terms, is al-

of the two middle Terms, is always equal to that which is made by the Multiplication of the two Extreams. As if a b. e b:: b g. b c.

I say, from the Multiplication of the Extreams a b by b c there is produced the Rectangle a c:

And by multiplying the middle Terms e h and h g, there is produced the Rectangle e g; and those two Rectangles a c and e g are equal. (6. 22.) (Because, as much longer as a b is than e h just so much longer is h g than b c). On which is founded the Reason of the Golden Rule.

#### COROLLARY.

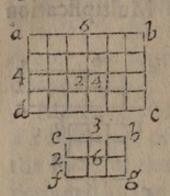
Hence, if in two Ranks of Discrete Proportionals, the four middle Terms are the same: As if a. b:: c. d, and then also e. b:: c. f. I say it will be as a. e:: So will reciprocally f. be to d: For since the middle Terms are the same in both, the Rectangle a d will be equal to e f, and consequently their Sides must be reciprocally proportional; that is a. e:: f. d.

What is thus done by Lines and Rectangles, may be done by any Quantity whatsoever; because all Quantities can be express'd by Lines, and all Multiplications of Magnitudes by Multiplications of Lines,

i. e. by Rectangles (6. 24.)

Proportional, so that a b. e h:: a d. ef. then is the Rectangle a c to the Rectangle e g, in a Duplicate Ratio, to that of their Sides: For the Ratio of a c to

eg, is compounded of the Ratio of a b to eb, and of



the Ratio of a d to e f. (6. 26). But the Ratio of a b to eb is in this Case (by the Supposition) the same as the Ratio of a d to ef; fo that to gain the Ratio which the Rectangle a c hath to eg, we need only take twice the Ratio of ab to eh. For Example, if as here a b be double to e b,

and ad double to ef, the Rectangle ac shall be twice double, that is, Quadruple of the Rectangle eg. And if ab had been triple of eb, and consequently a d triple of ef: Then the Rectangle a c would have been three times triple, that is nine times as big as eg; Or if a b had been Quadruple of eb, a c would have been 16 times as great as e g.

25. If a third Line be taken as no; and it be so proportional that a b. e b :: e b. no. Then n .... o shall the two Rectangles a c and e g be to one another as the two Lines ab and no:

(vid. Fig. Preced.)

For ab is to no in a duplicate Ratio of ab to eb. And if a b had been (as it is double,) triple or quadruple of e b: Then would a b have been in a Ratio three times Triple; or four times quadruple of (that is nine or 16 times as great as) the third Proportional n o. al : Indoinogo

26. Those Rectangles which have their Sides thus Proportional: That a b. e h:: a d. e f, are called Similar, whose Homologous Sides are those which an-Iwer each to other in the Proportion, as a b and e b, or a d and ef: For as a b is the greatest Side of the Rectangle ac, so eh is also the greatest Side of the Rechangle eg. wed so made A ded W .as

Proportional, fo that a b. e b :: a d. e f. then is the Rechangle, a c to the Rechangle e g, in a Duplicare All A. 7210 that of their Sides: For the forth of a co

27. All Squares are fimilar Rectangles. For 'tis plain that if a b be double or triple of e b, a m must also be double or triple of bi:

Because am is equal to a b, and bitoeh.

28. All fimilar Rectangles are to each other as the Squares of their Homologous Sides. I

fay the Rectangle a c is to the

Rectangle eg:: as the Square b m to the Square ei. For as well Squares as Rectangles are to one another

in a Duplicate Ratio of a b to e b (6. 29. 30).

29. To know the Ratio between two Solid Rectangles or Parallelopipeds, there ought to be known the several Ratio's that their Bases and Heights have to each other; because the Ratio of one Solid to another is compounded of the Ratio's of their Lengths and Breadths and Thicknesses or Heights; as is eafie to conceive, if that be well understood which hath been said about the Proportions of Rectangles. For if one Parallelopiped hath its Base double to the Base of another, and its Height, triple of the Height of the other: The former will be twice triple, or three times double, or in one word Sextuple of the latter.

30. If the Bases of two Parallelopipeds be Reciprocally as their Heights, those Parallelopipeds are equal: Which is proved by the 27th of this Book; for as much as one exceeds the other in Breadth and Length, fo much doth the other exceed it in Height.

31. When Parallelopipeds have all their fides Proportional, they are called Similar; and they are in a Triplicate Ratio of their Sides, as it hath been proved of Rectangles, that they are in a Duplicate Ratio of their Sides.

32. Similar Parallelopipeds are to one another as the Cubes of their Homologous Sides; for both Cubes and Parallelopipeds are in a Triplicate Ratio of their Homologous Sides. 34. All

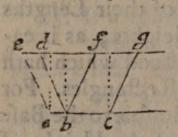
34. All Rectangles having the same or equal Heights, are to one another as their Bases, and having the same Bases their Heights are equal.

Let the Rectangles A and B be between the same Parallel Lines df and ca; so that a d be equal to cf:

then do I say, that A. B :: a b. b c. That the Rectangle A is to the Rectangle B. as the Base ab. to the Base bc: And that if, for Instance, ab be double to bc, then shall A be double to B. For A is

nothing but the Line b a multiply'd by da. (6. 17); and B is nothing but the Line c b multiply'd by the fame Line a d,or (which is all one) be or f c. Wherefore (6. 15) A. B :: a b. b c.

35. All Parallelograms which are between the same Parallels (or which have the same Height) are as their



Bases. I say the Parallelogram e b is to the Parallelogram bg:: as the Base ab is to the Base b c. For having made the two Prick'd Rectangles on the same Bases, those will be equal to the Parallelograms,

(by 3. 14). But those Rectangles are as their Bases (by the Precedent). Wherefore the Parallelograms must also be as their Bases; That is e b. b g :: a b. be.

36. All Triangles (which have the same Heights) or are between the same Parallels, are as their Bafes; For they are halves of Parallelograms (3. 8).

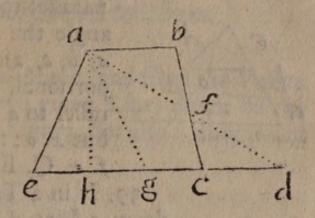
37. When Triangles (as those in the following Figure) have their Bases on one and the same Line, and their Vertices or tops meeting in the same Point: They are taken to be between the same Parallels, as a de and e de, and a de and b de (because they have the same Perpendicular Height.)

#### PROBLEM I.

Hence may a Trapezium as a b c e, whose two

sides a b and e c are parallel, be divided in any given Ratio.

For take cd = ab and draw ad, then will the  $\triangle sabf$  and fcd be equal (by 14. 2.) and consequently the  $\triangle cad$ 

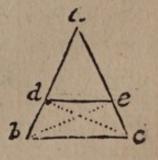


= Trapez. e a b c. Wherefore if you divide e d the Base of the Δ e a d into any number of Parts or according to any Ratio, Lines drawn from the Vertex to such Divisions of the Base, will divide the Δ e a d, and consequently the Trapezium, in the same Ratio.

38. If in any Triangle a Line be drawn Parallel to the Base, that Line shall cut the Legs Proportio-

nally. Let the Triangle be a be, and let the Line de be Parallel to b c:

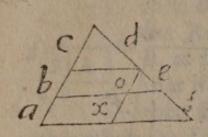
I say that ad. ae:: ab. ac::
db. ec, &c. Draw the Lines dc and
eb, then shall the Triangle ecd be
to ead, as the Base ec, is to ae. (6.
40.46). So also the Triangle deb is
to ead:: as the Base db is to da.



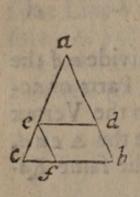
But the Triangle e c d is equal to d e b (3.15); wherefore the Triangle b d e (or c e d) is to the Triangle
e a d:: as b d. is to d a:: or as c e to c a. Therefore
also must b d. d a:: c e. e a, because both the Ratio
of b d. to d a, and also that of e c. e a, are the very
same with that of the Triangle b e d or c e d, to the
Triangle a d e.

COROL-

### COROLLARY.



If many Lines are drawn parallel to the Base of any Triangle the Segments of the Sides a, b, c, and d, e, f, will be proportional, for drawing ox parallel to abc: b = o and a = x, but x. o:: f. e: wherefore a. b:: f. e. Q. E. D.



39. If in a Triangle, as a c b, you draw a Line d e Parallel to the Base cb, I say, that ed. cb:: ae. ac:: or as a d. a b. For drawing ef Parallel to a b; f b will be equal to e d (3. 9) But by the Precedent f b. c b:: a e a c. wherefore e d. (f b) e b :: a e. a c. or as a d. to a b.

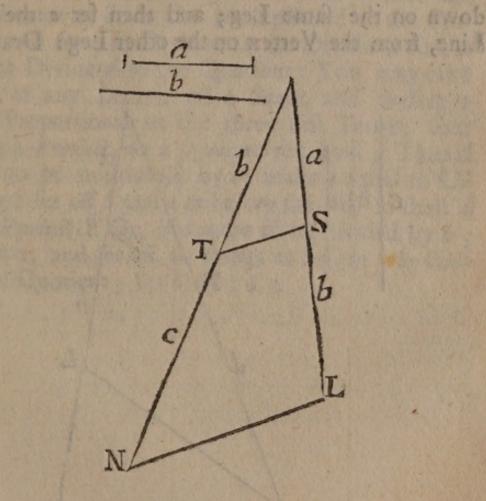


dello ther or ext ore ore the very of the Triangle be a or e al, to the

### PROBLEM I.

Two Lines a and b being given, to find c, a third Proportional to them.

Make any Rectilineal Angle, and from the Vertex or Top of it, set the two given Lines down on the Legs, as you see in the Figure. Set also b downward



from S to L, join S T, and draw N L Parallel to it; fo shall T N be c, the Line sought; For a. b::b.c. by this Proposition.

to it; So thail TM be equal to d the fourth Pro-

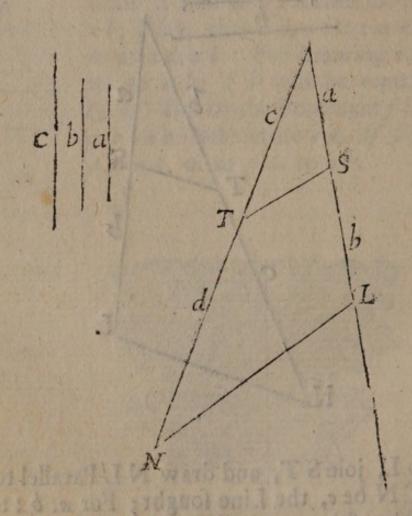
portional fought; for a. b :: c. d. by the Precedent

P R O-

### PROBLEM II.

If three Lines as a,b and c, had been given, to find a fourth Proportional (as d) to them, or to work the Rule of Three in Lines, you must proceed thus.

Set the two first Lines a and b from the Vertex down on the same Leg; and then set c the third Line, from the Vertex on the other Leg: Draw the



Line TS, and thro' the Point L draw LN Parallel to it; So shall TN be equal to d the fourth Proportional sought; for a. b:: c. d. by the Precedent Propositions.

PRO-

#### PROBLEM III.

And this way 'tis very easie to find a Line that shall express the Product of any two Numbers or Quantities: Or the Quotient of one divided by the other.

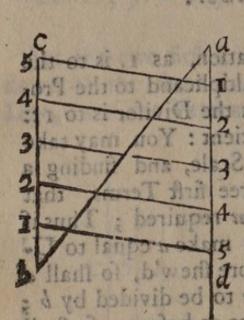
For fince in all Multiplication, as 1. is to the Multiplicator: So is the Multiplicand to the Product: And fince in Division as the Divisor is to 1:: So is the Dividend to the Quotient: You may take your 1. of any Length off a Scale, and finding a fourth Proportional to the three first Terms, that shall be a Product, or a Quotient required; Thus if b were to be multiply'd by c: make a equal to Us nity, and set off b and c as before shew'd, so shall d be the Product. Or, if d were to be divided by b; take a=1, and set off all things as before; so shall c be the Quotient; for b. a::d.c.

divide a b in the fame Proportion as will and be are

### PROBLEM IV.

To divide a given Line a b into any Number of equal Parts: As suppose into Six.

Make at a and b any two equal Angles, and on



the Legs da and e b run with a pair of Compasses five equal Divisions (for they must be always one less in Number than the required Division or Parts of the Line 3 given) drawing also Lines across from one Point to the other, as you fee in the Figure; fo shall those Lines d divide the given Line a b ved habivib ad of into the fix Parts required: good a For the croffing Lines being Parallel one to another, must

divide a b in the same Proportion as a d and be are

divided.

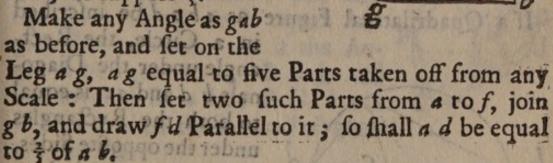
# PROBLEM V.

To divide a given Line a b into two Parts, fo that they shall be to each other as the Line C to D; or in any given Ratio.

Make any Angle with the given Line ab, and set the Line c from its Vertex a, to f. And set the Line

D from f to g; draw the Line g b, and thro'f, a Parallel to it, as f d: So shall the Point d divide ab in the Ratio required: For C. D:: a d. d b.

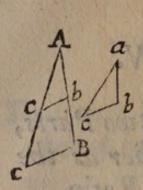
And much the same way may you cut off from any given Line ab, any part or parts required; as suppose 2.



baxed to adxbe,
make the Anglebace
cad; and then adding
the Anglecaf to both,
the Anglebafead;
And the Aced finit

sidryd) ad a d :: ba a a H 2 mad T : a 38. Those

### ELEMENTS



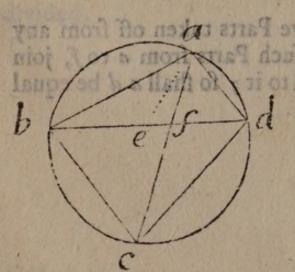
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Like or Similar, which have all their three Angles respectively equal to one another, or which are Equiangular: v. gr. If the Angle A be equal to a, the Angle B to b, and C to c, then the whole Triangle ABC is Like or Similar to the Triangle a b c.

39. All similar Triangles have their Sides about the equal Angles Proportional. I say, A B. a b:: A C. a c:: B C, b c, &c. For take in the greater Triangle ABC, A b equal to a b, and A c equal to a c; then will the Triangle A b c be every way equal to a b c (2.11.) and the Angle A b c is equal to the Angle abc: Wherefore it will be also to B, which by the supposition was equal to a b c, and therefore c b is Parallel to C B (1.31) and consequently (by 6.42,43), A b. A B:: A c. A C:: c b. C B.

### COROLLARY I.

If a Quadrilateral Figure as a b c d be inscribed



in a Circle, the Rectangle under the Diagonals b d and a c, is equal to both the Rectangles under the opposite Sides.

That is,  $ac \times bd = ba \times cd + ad \times bc$ , make the Angle bac = cad; and then adding the Angle eaf to both, the Angle baf = ead: And the  $\Delta ead$  fimi-

lar to  $\triangle b$  a c: Then will a c. c d:: b a. be. (by this Prop.) Wherefore  $a \ c \times b = c \ d \times b$  a. Again also,

Book VI. of GEOMETRY. 101

ad. de::ac.cb. Wherefore  $ad \times cb = de \times ac$ .

But  $ac \times be + ac \times ed = ac \times bd$ . Wherefore  $ac \times bd = ba \times dc + da \times bc$ . Q. E. D.

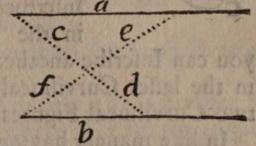
### COROLLARY II.

The Segments of Lines intersecting each other be-

tween two Parallels, are

Proportional:

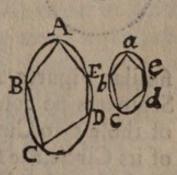
That is, c.d.::e.f. for by fimilar Triangles c.e::d.f. wherefore alternately c.d::e.f. wherefore c.f. wherefore c.f.



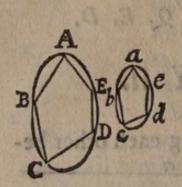
40. All similar Triangles are in a Duplicate Ratio of, or as the Squares of their Homologous Sides; for similar Triangles are the Halves of similar Parallelograms, wherefore they must be as their Wholes.

41. Similar Polygons are those which having an

equal Number of Sides have all the several Angles in one, equal to those in the other, and also the sides about those equal Angles Proportional. As if the Angles Proportional. As if the Angle A be equal to a, B to b, and moreover A B. a b:: B C. b c:: C D. c d, then those two Polygons are Similar.



42. And among Curvilineal or Mixt Figures, those are Similar in which you may inscribe, or about which

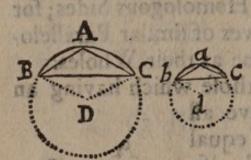


you may Circumscribe similar Polygons; so that any Polygon being Inscribed or Circumscribed about one Figure, you may Inscribe or Circumscribe a Similar one about the other. For instance, if having Inscribed any Polygon as ABCDE in the greater Curvilineal Figure,

you can Inscribe another in all respects Similar to it in the lesser Curvilineal Figure a b c d e, then those

two Curvilineal Figures are Similar.

In like manner having taken two mixt Figures as the two Segments of Circles B A C and b a c; and



Triangle at Pleasure as B A C, if then you can Inscribe in the other Segment another Triangle b a c, that shall be Similar to the former; then shall those two Segments be

segments be compleated, they shall be similar Parts of those two Circles, so that if BAC be a third part of its Circle, bac shall also be a third part of its Circle. And if to the Centers you draw the Lines BD and CD, and also b d and cd; the Angles D and d shall be equal. (See 4. 11. and the following Propositions.)

#### COROLLARY.

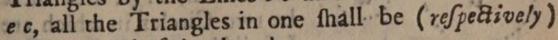
The Peripheries of Circles are as their Diameters: For as B A. ba:: B D. bd:: 2 B D. 2 bd:: so it will be of every side of the Inscribed or Circumscribed Polygon; wherefore the Sum of them all, that is the Peripheries, must be in the same Ratio.

43. AU

43. All Circles are similar Figures.

44. All similar Polygons may be divided into an equal Number of similar Tri-

angles. Let the similar Polygons be ABCDE and abcde;
and let the first be divided into Triangles by the Lines E B
and E B (3. 14.) I say, that if
the other be also divided into
Triangles by the Lines e b and



Similar to those in the other.

For instance, I say, the Triangle abe is Similar to ABE: for the Angle a is equal to A (by the supposition) and also AB. ab:: AE. ae (by the same); wherefore the Triangle ABE is Similar to abe. (6.46.) Again the Angle EBC may be proved equal to ebc, because the Angle ABC is (by the Supposition) equal to abc, and it was proved (in the last step, where the Triangle ABE was proved Similar to abe) that the Angle abe is equal to ABE; wherefore from equal things taking away equal, the Angle EBC remains equal to the Angle ebc. In like manner the Angle ecb, is proved equal to ECB, and consequently (6.45.) the whole Triangle ebc will be Similar to EBC; and so of the rest.

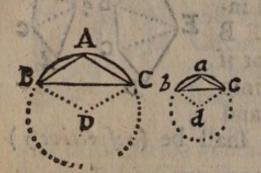
Hence the Practice of making on a Line given a Polygon Similar to one assigned, is derived. For dividing the given Polygon into Triangles, make a Figure consisting of a like Number of similar Tri-

angles, on the given Line.

Duplicate Ratio of, or as the Squares of their Homologous Sides. I say, as the Square of AB is to the Square of ab:: so is the whole Polygon ABCDE to the Polygon ab cde. For since all the Triangles in one Polygon, are Similar to those H

in the other (6.51.) All in one Polygon will be to all those in the other in a Duplicate Ratio of any of their Homologous Sides; that is, as the Square of A B is to the Square of a b.

46. All similar Figures, even Curvilineal ones, are



of any fide of any fimilar Figures, which can be Inferibed or Circumscribed about them, v. gr. Let there be two Circles, in which are Inscrib'd two similar Triangles abc and ABC: I

fay, as the whole Circle ABC, is to the Circle abc: fo is the square of BC, to the Square of bc, or which is the same thing, as the Square of the Radius BD, to the Square of the Radius bd. For in or about the Circle a bc may be Inscribed or Circumscribed any Polygon you please; (or at least such an one may be imagin'd) (4.30.) But every Polygon inscrib'd in a bc will have a less Ratio to the Circle ABC, than the Square of bc hath to the Square BC: and every one Circumscribed about a bc will have a greater Ratio to the Circle ABC, as is easie to prove by the Precedent, and from what hath been said of Circles in the fourth Book. Wherefore all similar Figures, &c.

### COROL. I.

I. Circles are to each other as the Squares of their Radii or Diameters: For suppose a Circle whose Radius is r, and then another Circle greater (or less) than that; and call its Radius R; then will its Diameter be 2 R: Then whatever the Ratio of the Diameter (2 R) be to the Periphery, let it be expressed by

II. Hence tis plain that, the Square of the Diameter of any Circle is to the Area of it: as the Diameter is to 4 Part of the Periphery.

For 4RR, RR e:: 2 R. 
$$\frac{2Re}{4}$$
 (=  $\frac{1}{2}Re$ )

As is plain by multiplying the Extreams and mean Terms by one another.

III. Hence also 'tis plain (again) that the Peripheries of Circles are as their Diameters.

That is, 2 R. 2 r: : 2 R e. 2 re.

IV. And fince the Area of every Circle is rre, (That is, The Product of the Square of the Radius multiplied into the Name of the Ratio, between its Diameter and Periphery.) A very ready way (for common use) to find the Area of a Circle whose Radius is given, will be to multiply the Square of the Radius into this or such like Decimal 3.1. Thus suppose the Radius 9 Inches:  $81 \times 3.1 = 251.1.$  which is very nearly the Area in square Inches, tho something less.

47. All this may be apply'd to Solids. And therefore Similar Solids are such as have their Angles all equal, and the Sides about those Angles Proportional; or (if they are of a Spherical or of any Spheroidical Figure) such as can have similar Soli's Inscrib'd or

Circumscrib'd in, or about them, &c.

48. Simi-

#### 106 ELEMENTS

48. Similar Solids are to one another in a (Triplicate Ratio of, or) as the Cubes (of their Homologous

Sides, &c.) See 6. 36, 37, &c.

(And therefore all Spheres must be to one another as the Cubes of their Diameters, &c.) Which may be easily thus proved; The Solidity of the Sphere may be expressed after this manner; by what is said in

the Corollaries in p. 75, 76.

The Area of a great Circle of the Sphere whose Radius is R or r, being RR e or rre (by Cor. 4. p. 75, 76.) 4 times that will be the Surface of each Sphere; That is, 4 R R e the Surface of the greater, and 4 rre the Surface of the lesser: and multiplying the Surface by of the Radius, the Solidities will be

4 R R R e and 4 rrre: Which two Quantities being multiplied and divided by the same, will be in

and Division: That is 4RRe. 4rre::RRR.

Radii, and consequently, as the Cubes of their Diameters. Q. E. D.

Lacounterfield in, or showe them, 6

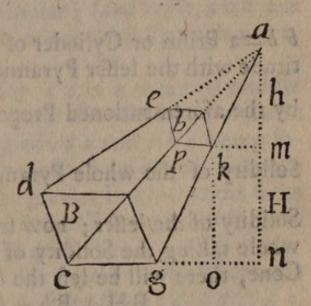
Radius p linches: Br My. 1 = 251. 1.

#### PROBLEM.

To find the Solidity of the Frustum of a Pyramid or Cone, cut by a Plain parallel to the Base, having given the two Bases together with the Height of the Frustum.

Solution. By Prop. 32. of Solids, a Pyramid or Cone is equal  $\frac{1}{3}$  of a Prism or Cylinder, of the same Base and Altitude. Let m = k o the Altitude of the

Frustum, be called H, and m a the height of the Top piece wanting, b; the greater Base of the Frustum B, and the lesser b: the Triangles apk and acg are Similar, (cg and pk being parallelex Hyp.) Wherefore cg. ga::pk.ka, and alternately cg.pk::ga.ka.But ga.ka::



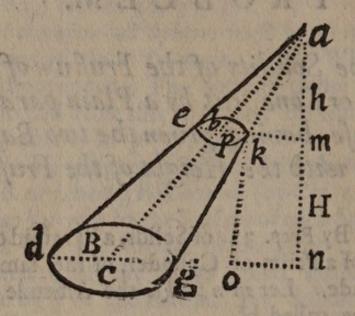
n a. ma (from the Similarity of the Triangles g a n and k a m) wherefore ex agus e g. p k:: n a. m a, and by Division e g — p k. p k:: n a — m a ( = m n.) m a; which put into Symbols (putting e g the side of the Base=S, and p k=s) will stand thus, S—

s.s:: H.  $\frac{sH}{S-s} = b$ . Wherefore having found

the Height of the little Pyramid or Cone which is wanting; I say, having found it in known Terms, it will be easie to find the Solidity of the Frustum; for multiplying the Base B into the whole height H+b,

### 108 ELEMENTS

the Product BH + Bb = a Prism of the same Base and Altitude with the whole Pyramid or Cone; and

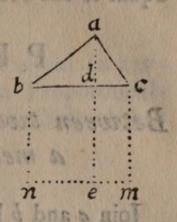


b h=a Prism or Cylinder of the same Base and Altitude with the lesser Pyramid or Cone. Wherefore by the aforementioned Proposition,  $\frac{BH+Bb}{3} = \frac{Bh}{3}$  Solidity of the whole Pyramid or Cone, and  $\frac{bh}{3} = \frac{bh}{3}$  Solidity of the lesser; now from the Solidity of the whole taking the Solidity of the lesser Pyramid or Cone, there will be less the Solidity of the Frustum required, viz.  $\frac{BH+Bb-bh}{3} = Solidity$  of the

Frustum.

The Theorem in words is this; Multiply the greater Base by the whole Height, and from the Product subtract the Upper Base multiply'd by the Height of the Top piece wanting, and \(\frac{1}{3}\) of the Remainder will give the Frustum.

49. If in a Rectangle Triangle a b c, a Line as ad be drawn from the Vertex or top of the Right Angle, Perpendicular to the Base, Hypothenuse, or longest side be: It shall divide the Triangle a b c into two other Rect- b angled ones, a b d, and d a e, which will be Similar to each other, and to the whole bac. For 1. All the Angle. 2. The Triangles a b c and



a b d have the Angle b common to both: Wherefore they are Similar (6. 45) 3. The Triangles abc and a d c have also the Angle c common to both; therefore they two are Similar; (and lastly, a b d and a d c being both Similar to one third Triangle a b c,

will be so to each other.)

PROB.

50. The Perpendicular a d is a mean or middle Proportional between bd and dc. That is, cd. da:: d a. d b. For the Triangles c d a and b d a being Similar (by the last) c d (the lesser Leg of the Triangle c d a) shall be to a d (the greater Leg): : as the same ad (the lesser Leg of the other Triangle adb) is to

b d the greater Leg. (6.46.)

51. The Square of a d is equal to the Rectangle made between c d and d b. For, fince c d. da:: da. d b. (by the last,) the Rectangle of the Extreams c d and db is equal to the Rectangle of the mean Terms d a and d a (6, 28.) But the two sides of that Rectangle being equal, because its only d a taken twice; that Rectangle must be the Square of da; and so it may be laid down as an universal Theorem, that, &c. (the Square of the Perpendicular drawn from the Vertex of any Restangle Triangle to the Hypothenuse, is equal to the Restangle under the Segments of that Hypothenuse,

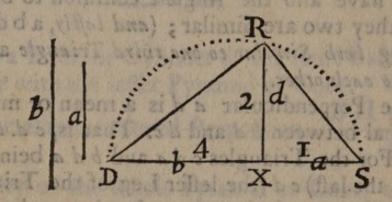
### 110 ELEMENTS

52. The Square of a mean Proportional is always equal to the Rectangle of the Extreames.

### PROBLEM I.

Between two given Lines a and b, to find a mean Proportional, as d.

Join a and b both in one Line which make the Diameter of a Circle'; and then at the Point x, where the given Lines join, erect a Perpendicular as d; that



Thall be the mean Proportional required. For the Angle DRS being a right one (as being in a Semicircle) b. d:: d. a, by Prop. 51.

#### PROB. II.

And thus may you find a Line equal to the Square Root of any Number or Quantity, by finding a mean Proportional between it and 1. For if b=4, and a=1; then will d=2, equal to the Square Root of b.

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ya, The

### PROBLIII.

To exacels a Reclangle you need use bus

b d, and the other d c. Bucil we lay the Rectangle Thus also may a Square be found equal to any Rectangle given, by finding a mean Proportional between its Sides, which shall be the Side of the Square required. of the) Squares of the two other fides

# ada to command Part O B. ada IV.

# To find a Square equal to any Triangle.

Find a mean Proportional between a Perpendicular let fall from any Angle to an opposite Side, and the half of that Side; and that shall be the Side of

the Square required. on or laups si as a staupe slodw 53. A Restangle being given to make another Rest angle equal to it, which shall have a Length given. Let the Rectangle given be a c, and let it be required to make another equal to ait, the Length of one of whose Sides dshall be the Line of: here are now three entry Lines given, viz. a b and b c (which are the fides of the Rectangle given) and fig ef, which must be one side of the Rectangle required. Therefore a fourth Line must be found, which shall be the other side of the Rectangle. fought: Which is done by finding a fourth Proportional to the three given Lines (6. 43.) which let be eb. So that ef. ab: bc. eb. and then, I say, the Rectangle f b is equal to d b, and is the Rectangle required. (6.27.) a bata , a d to red seed a sand 2 odt

N. B. This is called Application of the Rectangle, equal to a Right Line ab, vid. Eucl.p. 6. e. 6.

### 112 ELEMENTS

54. To express a Rectangle you need use but three Letters, v. gr. When we say the Rectangle b d c: We mean a Rectangle one of whose sides is b d, and the other d c. But if we say the Rectangle b c d, we then mean a Rectangle one of whose sides is b c, and the other c d.

55. In every Rectangled Triangle, the Square of

the Hypothenuse is equal to the (Sum of the) Squares of the two other sides (or to the Sum of the Squares of the

c Legs.)

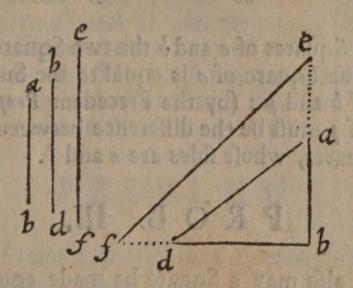
Let the Square b m, be divided by the Perpendicular a d e into the two Rectangles d m and d n. I say, that the Rectangle d m is equal to the Square of a c, and the Rectangle d n.

to the Square of a b; and that by consequence, the whole Square b m is equal to the Sum of the Squares of a b and a c. For 1. The two Triangles a d c and a b c being Similar (6.56.) d c. c a (in the lesser Triangle d c a):: as the same a c. c b in the greater Triangle a c b. Wherefore a c is a mean Proportional between d c and c b (or c m) and consequently the Square of a c is equal to the Rectangle b c d, or d c m, that is d m.

And after the very same manner may a b be prov'd to be a mean Proportional between b d and b c (that is b n, &c.) (for the Triangle a b d being Similar to a b c; d b the lesser side in one will be to b a the greater side; as that b a (now the lesser side in the other Triangle a b c) is to b c the greater side; That is d b. b a:: b a. b e (or b n) and consequently the Square of a b is equal to the Rectangle d b n or d n. And so both the Squares together, of b a, and a c, or their Sum, is equal to the Square of the Hypothenuse. Q. E. D.

#### PROBLEM I.

Hence any two, or more Squares may easily be added together into one Sum. Let a b, b d, and e f, be the Sides of three given Squares, place ab and bd



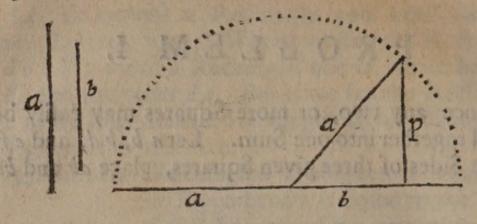
at Right Angles, and draw the Hypothenuse a d, whose Square will be equal to the Sum of the Squares of db and ab. Then set da from b to e, and the given line ef, from b to f; So shall the Hypothenuse fe, be the side of a Square equal to the Sum of the three given Squares.

# PROB. II.

Or if two Squares be given, you may Subtract one from the other, and find a Square equal to the Difference between them.

Let a and b be the Sides of the given Squares; make (the longest Line) the Radius of a Circle, and set b from the Center on the same Right Line with a; at the end of b, erect the Perpendicular p, which will be the side of a Square equal to the Difference between

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tween the Squares of a and b the two Squares given; for fince the Square of a is equal to the Sum of the Square of b and p: (by the Precedent Prop.) The Square of p must be the difference between the two given Squares, whose sides are a and b.

### PROB. III.

Hence also may a Square be made equal to any given Polygon, or irregular Right-lined Figure: By reducing the Figure into Triangles; finding Squares equal to those Triangles; and then one Square at last equal to the Sum of all those Squares.

Or by making Rectangles equal to those Triangles, which shall have all the same height; then joining those Rectangles together, so as to make one great one equal to them all; and lastly, make a

tide of a Somme equal to the Diffie

Square equal to that Rectangle.

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two Society be given, you may Subited

### Book VI. of GEOMETRY.

59. If upon the three sides of a Rectangled Triangle are made three similar Figures, and those similarly Posited, the greatest shall

be equal to the other two.

For the three Figures being Similar are as the Squares of their Homologous sides (6.53.)

And therefore the Figure A countries finall be to B and C, as the Square of b c is to the Squares of b and a countries of b c is to the Square of b c

of a b and a c. But the Square of b c is equal to those two Squares (by the last) therefore (the Figure

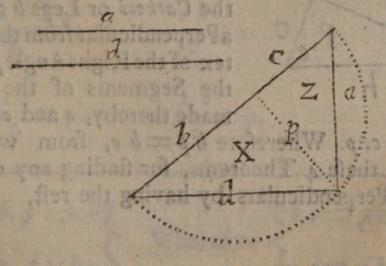
A is equal to both B and C together.)

### PROBLEM I.

To find two Lines b and c, which shall have the same Ratio to one another, as two given Squares, Similar Triangles, Similar Polygons, or Circles.

Let a and d be the Sides of the two given Squares, Triangles, Polygons; or the Diameters or Radius's of the Circles given: Set them at Right Angles to

Curve as preben draw dand mandelic



one another, as you see, and draw the Hypotheruse b+c, to which let fall the Perpendicular p, which shall divide the Hypotheruse into two Parts b and c, the Lines required. For the Triangles Z and X being Similar (by 56. of the 6.) will be to one another as the Squares of their Homologous Sides a and d, (6. 47.) These Triangles also having the same height, will be as their Bases (6. 42.) wherefore their Bases b and c, are as the Squares of d and a. Q. E. D.

# or lamps aid PROB. II. who has de lo

This Problem may be Inverted, thus; To make two Squares, Triangles, &c. having the Ratio of two given Lines, b and c, or in any given Ratio.

Join the Lines into one continued Line, and then make that the Diameter of a Circle, from the Point where b and c join; erect a Perpendicular to the Curve as p, then draw d and a, and they shall be the Sides of the Squares, Triangles, similar Polygons, or the Diameters of the Circles required.

b Pec

In a Right-Angled Triangle let the Hypothenuse be h, the Catheti or Legs b and c, a Perpendicular from the Vertex of the Right Angle p, and the Segments of the Base made thereby, a and e. Then

T. h. b:: c. p. Wherefore hp = bc, from whence will arise these 4 Theorems, for finding any of the Sides or Perpendiculars by having the rest.

1. 
$$b = \frac{bc}{p}$$
2.  $b = \frac{bp}{c}$ 
3.  $c = \frac{bp}{b}$ 
4.  $p = \frac{bc}{b}$ 

2. a.b::b.b\\ e: c.:: c.b\\ Wherefore 
$$\begin{cases} b & b = ab \\ c & c = eb \end{cases}$$

Whence will arise these two Theorems for finding the Segments from the 3 sides.

$$1.\frac{bh}{b} = a. \quad 2.\frac{cc}{b} = e$$

$$3. a.p::b.c$$
Wherefore 
$$\begin{cases} ac = pb \\ b = ac \end{cases}$$

From whence these Theorems will arise for finding the Segments, the Sides or the Perpendiculars.

1. 
$$\frac{pb}{c} = a$$
. 2.  $\frac{pc}{b} = c$ .

3.  $\frac{ac}{p} = b$ . 4.  $\frac{eb}{p} = c$ .

4. b. 
$$p::c.e$$

Wherefore
$$\begin{cases} b = pc \\ c = pb \end{cases}$$

I 3

And confequently,
$$1. e = \frac{pc}{b}.$$

$$2. a = \frac{pb}{c}.$$

5. And fince  $e \equiv \frac{b}{b}$  and (by 3)  $\frac{c}{b} = e$ .

Therefore  $\frac{b}{b} = \frac{c}{b}$ . Wherefore b = bcc. and dividing both by 8, ph = bonon anamged edit

That is, the Rectangle under the Legs, is equal to that of the Perpendicular into the Hypothenule, &c.

For, by proceeding after this Method, the Reader may easily discover many such Propositions as these: Which I leave to exercise his Skill and Diligence this way.

J. That the Rectangle under either Leg of a right Angled A, and the opposite Segment of the Bale, is equal to that under the Perpendicular into the other Leg.

II. The Square of the Hypothenuse is to that of either Leg: : as the Rectangle under the Hypothenuse, and the Segment of it, opposite to that Leg, is to the Square of the Perpendicular.

III. The Solid under the Perpendicular into the Rectangle of the Legs, is equal to that under the Hypothenuse into the Rectangle of its Segments.

IV. The Square of the Perpendicular, is to the Square of any Leg, as the Segment opposite to the Leg, is to the whole Hypothenuse. V. The Book VI. of GEOMETRY.

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V. The Square of one Leg into the opposite Segment of the Hypothenuse equal to the Square of the other into its opposite Segment. Wherefore,

VI. The Squares of the Sides are as those Seg-

ments.
56. If on the Hypothenuse b c of a Rectangled

Triangle, there be made a Semicircle b a c, and on the other two Sides a b and a c, two more Semicircles b n a and a m c, that great Semicircle will be equal to the other two



(by the last Proposition.) And if from the greater Semicircle, and the two lesser ones, you take away what is common to both; which are the two shaded Segments a b and a c; what remains of each must be equal, i. e. the Triangle a b c is equal to both the Lunes b n a and a m c.

And this is the Quadrature of the Lunes of Hip-

pocrates of Scio,

of Real County

the Lunes will be equal, and then also the Triangle a b o, being the half of a b c, will be equal to each Lune. But if the Triangle be a Scalene as in this Figure, the Lunes are unequal; and 'tis as difficult to divide the Triangle a b c into two parts by the Line a o, so as to be able to prove the Triangle a b o to be equal to the Lune b n a, and the Triangle o a c to be equal to the other Lune a m c; this is, I say, as difficult as to find the Quadrature of the Circle.

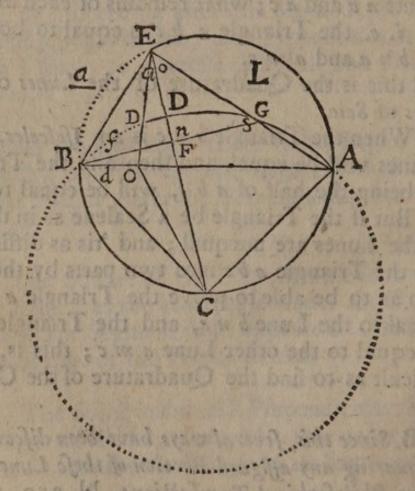
N.B. Since this, several ways have been discovered of squaring any assigned Portion of these Lunes (See the Philosophical Transactions, N. 259. pag. 43 11.) Of which this is one.

Let there be a greater Circle BG AC, on whose Quadrantal Arch BA, let the Lune BEAGB, or L, be drawn by describing the Semicircular Arch BEA, which is one half of the lesser Circle BCAE. Let then a Line as CE, be drawn from the Center of the greater Circle, cutting off any Portion or Segment of the Lune, as BED: 'Tis required to Square that Segment.

Draw BG at right Angles to EC; So shall the Chord BG be Perpendicularly Bissected in the Point F or n, draw also BE and EGA. I say, that the Right-Lined Triangle BEF, is equal to the Part

of the Lune BED.

For FG being equal to FB, EF common to both, and the Angles at F equal, because both Right, the



Triangle EFG will be equal to BEF: Wherefore the Angle o being equal to a, they must be both Semi-right; And consequently, f and S must be also Se-

Semi-right: Therefore the three Triangles E B G, E B F and E F G, must be each one the half of a Square. And consequently, GB. EB::  $\sqrt{:2.} \sqrt{:1.}$  for the Square of G B is double the Square of E B, and since Similar Segments are as the Squares of their Chords, the Segments B G must be double of B E: Wherefore the half of one will be equal to all the other; that is, BDF equal to the Segment B E. And therefore the Rectilineal Triangle B E F, exceeding the Portion of the Lune by the half Segment B E, and falling short of the Lune by the Segment B E, which is equal to that former half Segment BDF, the Triangle is exactly equal to the Portion of the Lune. Q. E. D.

And the ground of all is this, that the Angle BCE being at the Center of one Circle, and at the Circumference of the other, must divide the Quadrantal Arch BGA, in the same Proportion'as it doth the Semi-circular one BEA: On which depends the

Equality of the Segments BE, and BDF.

And fince the Triangle BCA is equal to the Lune L (as is apparent by taking the common Segment BGAB, from the Semi-circle BEAB, and from the Quadrant B G A C.) It will be easie to take from thence a Part, as the Triangle B O C, equal to the Affigned Portion of the Lune. For having let fall a Perpendicular from E, to find the Point O, draw OC; and then will the Triangle BOC, be equal to the Triangle BEF, before proved equal to the Segment of the Lune. For the Triangles B C A and B E F are fimilar, as being each the half of a Square: And therefore the former to the latter will be as the Square of B A, to the Square of B E (6.47.) their Homologous Sides. That is as B A is to B O (6. 25.) for BE is a mean Proportional between B A and BO. Farther, the Triangle BAC, having the same Height with BOC, will be to it as the Base A B to BO. WhereWherefore the two Triangles BEF and BCO, being proved to have the same Ratio to one and the

same thing, must be equal. Q. E. D.

And therefore to divide the Lune according to amy given Ratio, you need only divide the Diameter AB, according to that Ratio in the Point O, and from thence erect a Perpendicular to find the Point E: then draw EC, which shall cut off the assigned Portion of the Lune.

58. Two Chords cutting or croffing each other in

a Circle, have the Segments Reciprocal-

ly Proportional.

I say, that a c. b e :: d e. e c, and consequently the Rectangle a e c is equal

to the Rectangle de b.

For draw the prickt Lines a b and

d c, and the two Triangles a b e and d c e will be Similar: Because 1. The Vertical or opposite Angles at e are equal (1, 23.) 2. The Angle b is equal to c, because standing both on the same Ark a d, and being in the same Segment (4.12.) wherefore the two Triangles are Similar, and conquently a e. e b:: d e. e c. (6.46.) Q. E. D.

59. If a c be the Diameter of a Circle, and b da

Perpendicular to it, de or be will be a mean Proportional between the Segments of the Diameter a e and ec. Because de is equal to eb (by 4. 6.) and therefore since (by the last) the Rectangle bed (that is be Square) is equal to a ec. as the Rectangles of

the Parts of all crossing Chords are; the Line be or ed, must be a mean Proportional between a e and

ec. Q. E. D.

60. Two Lines a c and a d, drawn from a Point

a, without a Circle, to the Internal and opposite Part of its Circumserence; are to each other Reciprocally as their external Segments. I say, a c, a d: a e. a b. and consequently the Rectangle c a b is equal to d a e. For supposing the Lines c e and b d to be drawn, the Triangles a e c and a d b



will be similar, because the Angle a is common to both and the Angle c is equal to d, because standing on the same Ark be (4. 12.) wherefore da. ab:: ca. ae; and Alternately, da. ca:: ab. ae. and by Inversion ca. da:: ae. ab (6. 45.) And therefore the Rectangle cab is equal to dac. Q. E. D.

If one Line as a b, touch a Circle, as in the Point b, and another Line a d, drawn from the same Point

a, do cut it; Then is a b (the Tangent) a mean Proportional between a
a d and a e (i. e. between the whole
Secant, and the Part of it without the
Circle.)

For drawing the Lines be and bd, the Triangles ae b and bad will be Similar, because the Angle a, is com-

equal to e a c by the Supposition,

convenely ba. ad: : a c. d c. (and therefore

fore, the Triangles are Similar,

mon to both, and the Angle a be (made by the Tangent, and Secant e b) is equal to d (an Angle in the opposite Segment) (4.17.) therefore they are similar, and consequently e a (in the little Triangle) will be to a b:: as that same a b is to a d, in the greater Triangle: i. e. e a. a b:: a b. a d. Q. E. D.

61. Let there be a Diameter ab cut in c by an Infi-

nite Perpendicular e e, whether within the Circle, as in Fig. 1. at the Circumference, as in Fig. 2. or without
the Circle, as in Fig. 3. Let there
be drawn also from the Point a, any
Right Line, as a e, cutting the Perpendicular in e, and the Circle in d. I
say, it shall always be as a d. ac::ab.

For drawing the Line b d, there will be made two Triangles that are Similar, as e a c and d a b; which will be so, because they have one Angle as a, common to both, and the

Angle d equal to c, because both are Right ones (for d is Right by 4. 14.) as being an Angle in a Semicircle, and c is Right by the Supposition. Wherefore the Triangles are similar, and consequently a d. ac:: a b. a e. Q E. D.

62. In the second Figure, ad is always a mean Proportional between a e and a d; in the first, the middle Proportional is a E, drawn from a, to the Place where the Line e c cuts the Circle.

63. If of a Triangle inscribed in a Circle, the An-

gle bac be Biffected by the Line a e d.

I say, then ba. ae:: ad. ao. For drawing the

Line e b, there will be made two Triangles a b d and a e c, which are Similar; because the Angle d is equal to c (4.12.) as (being in the same Segment) or insisting on the same Ark, and b a d is equal to e a c by the Supposition, Wherefore the Triangles are Similar,

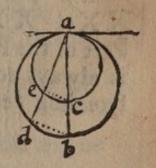
and consequently b a. a d: : a e. a c. (and therefore alternately b a. a e: : a d. a c.) Q. E. D.

64. When the Angle at the Vertex is thus biffected the Segments of the Base b c, are also proportional to the Legs of the Triangle (i.e.) b e. e c:: b a. a e. For supposing ef drawn Parallel to b a. Then will b a. a c:: ef. f c (6. 40.) But ef is equal to a f: because the Angle a ef is equal to e a b, (as being Alternate Angles 1. 31.) and consequently to e af (by the Supposition) wherefore the Triangle a e f is an Isosceles (2. 15.) And therefore instead of putting of it as before b a. a c:: ef. f c, we may say b a. a c:: a f. f. c. But as af. f c:: so is b e. e c (6. 42.) wherefore b a. a c:: b e. e c. Or, which is all one, b c. e c:: a b a. a c. Q. E. D.

(N. B. This Proposition is Universal; and if any Angle of a Triangle be Bissected, the Legs about that Angle are proportional to the Segments of the opposite side made by the Line Bissecting the Angle.)

65. If two Circles touch one another (in a Point

within) as a, and if to that Point you draw a Tangent and a Perpendicular a c b (which will pass through both their Centers) (4. 5.) and if also you draw any Secant from the same Point, as a e d. I say, 'twill always be a e. a d:: a c. a b. For having drawn the Lines e c and d b



the Triangles ae c and ab d will be Similar, as having the Angle at a, common; and e and d both right ones; (by 4. 14.) and consequently a e. a d:: a c. a b. Q. E. D.

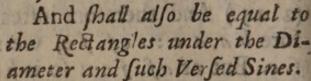
66. The Ark e c is to the Ark d b, as the whole Circle a e c, to the whole Circle a d b. (6.49. and 4.11.)

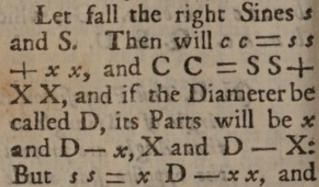
### PROP.

67. If two (or more) Chords, as c, C, issue from the same end of any Diameter of a Circle; Their

Squares shall be directly, as the

Versed Sines x X.





SS = DX - XX (by 66 of this Book) wherefore Substitute those two last Quantities instead of
the Equals s s and S S; and you will have c c=Dx

- xx + xx (that is) D x and C C = DX 
XX + XX (that is) D X which proves the latter
Part of the Proposition, that the Square of the Chord
is always equal to the Rectangle under the corresponding versed Sine, and the whole Diameter.

And 'tis plain that,

Dx. DX :: x. X. Q. E. D.

#### PROP.

Surface of a given Cone, will have its Radius a mean Proportional between the side of the Cone and Radius of its Base.

Let

Let the Side of the Cone be = a, the Radius of the Base = r; then the Diameter will be 2 r, and the Periphery = 2 r e = c. But half the Periphery into the Side of the Cone is = to the Convex Surface of the Cone (by ....) that is, are expresses the Area of the Cone. Now fince V: ar is a mean proportional between a and r (for a. V: ar:: V: ar r) I imagine \( \cdot : a r to be the Radius of the Circle whose Area = Area of the Cone. Then will its Diameters be 2 V: ar and its Periphery 2 V: ar e: and by Multiplication of 2 V: ar e, the Periphery into ½ V: rathe half Radius: or V: are into V: ra the Radius of the Circle will be are = b, the Surface of the Cone. Q. E. D.

69. The Convex Surface of a right Cone is to the Area of its Base :: as the Side of the Cone is to the

Radius of the Base.

For fince the Convex Surface of the Cone, (by what is said after 14. Book 4.) is equal to a Triangle whose Base is equal to the Periphery of the Circular Base of the Cone, and its Height the Side of that Cone, call the Periphery c, and the Side of the Cone a, then will  $\frac{ac}{2}$  express the Area of the

Convex Surface, and the Area of the Base will be -(by Art 26. Book 4.) But there is no doubt but ac re :: a.r. Wherefore, &c.

70. A Circle whose Radius is equal to the Diameter of the Sphere, will have its Area equal to the Sphere's Surface.

# 128 ELEMENTS, &c.

Let the Radius of such a Circle be 2 r, then its Diameter is 4 r, and its Periphery will be 4 r e, and by multiplying that by r=half of Radius, the Area is 4 r r e. Let then the Radius of the Sphere be r, then will its Diameter be 2 r, and the Periphery of a great Circle 2 r e, which being multiplied by the Radius r, makes 2 r r e; the half of which is r r e, the Area of a great Circle; but the Area of 4 such Circles is equal to the Sphere's Surface (by Cor. V. p. 76.) that is, 4 r r e = to the Sphere's Surface; which was above proved equal to the Area of the Circle, whose Radius was equal to the Sphere's Diameter. Wherefore, &c.



hole Radius is could to the I

ELE-



# ELEMENTS

OF

# GEOMETRY.

# BOOK VII.

# Of Incommensurables.



Lesser Quantity is said to Measure a greater, when being taken a certain number of Times, it is exactly equal to the greater. V. gr. Suppose a Fathom to contain six Feet; then may one Foot be said to Measure that Fa-

thom, because being taken or repeated six times, it

will be exactly equal to the Fathom.

2. The Quantity which is thus a Measure to a greater Quantity, is called a Part of that greater; and the greater Quantity is call'd the Multiple of the lesser. So a Foot is the Part of a Fathom, and a Fathom is the Multiple of a Foot.

3. If you take the Quantity (of a common French Pace) which is two Foot and a half, and try with that

you add that Pace only twice, it will make but five Foot, which are less than the Fathom; and if you take it three times, it makes seven Foot and a half, which are more than the Fathom; so that this Quantity of two Foot and a half cannot measure the Fathom, and therefore properly speaking is not a Part of it. But nevertheless they may be said to be Parts of the Fathom, because this Quantity contains sive half Feet; for an half Foot is a Part of a Fathom, because being taken 12 times, it will just Measure it; so therefore this Pace contains Parts of the Fathom, because it contains sive half Feet which are that is sive twelsths of a Fathom.

4. When two Quantities are such, that a third can be found which shall be an (Aliquot or Even) Part of both, that is, which shall measure them both exactly: Then those Quantities are said to be commensurable: As for instance, a Pace and a Fathom are two commensurable Quantities, because we can find a third Quantity, viz. half a Foot, which will measure them both; For if the half Foot be taken sive times it makes the Pace, and taken 12 times, it

makes the Fathom.

5. But when it is not possible to find any third Quantity which can measure two others, then those two Quantities are called Incommensurables.

Number, that is, those Quantities are as Number to Number, that is, those Quantities can be expressed by Numbers, so that as one Quantity is to the other, so shall one Number be to the other. Thus a Line of six Foot or a Fathom, and a Line of two Foot and a half, as a Pace, are to one another as Number to Number. For half a Foot measuring them both, the latter by being taken 5 times, and the former by being taken 12 times; it's plain that one Line contains 5 half Feet, and the other 12, and therefore they are as 5 to 12, or as Number to Number.

7. It

7. If two Quantities are not as Number to Number, that is, if it be impossible to express their Magnitudes by two Numbers, they are Incommensurable: As is plain from the last.

8. We ought then to see whether there are in Reality any such Quantities whose Magnitude cannot be express'd by Numbers, and if there be any such, we must say that there are Incommensurable Quantities.

9. A plane Number is that which may be produced by the Multiplication of two Numbers (one into another,) v. g. 6 is a plane Number, because it may be produced by the Multiplication of 3 by 2: For twice 3 makes 6: So also 15 is a plane Number, arising from 5 being Multiplied by 3; and 9 is a plane Number, produced by the Multiplication of 3 by 3.

by another, do produce a plane Number, are called the Sides of that Plane, as 2 and 3 are the sides of

the Plane 6; and 3 and 5 are the sides of 15.

those Squares may be formed into a Rectangle, if their Number be a Plane. V. gr. 12 Squares may be placed in the form of a Rectangle, one of whose sides may be 6 and the other 2, and 48 will make a Rectangle whose two sides may be 12 and 4. See the following Figures B and C.

equal; as 4 arising from the Multiplication of 2 by 2; as 9, the Product of 3 by 3: And 16 made by

4 Multiplied by 4, &c.

13. A Square Number may be ranged into the form of a Square, and that Number which can be ranged into the Form of a Square is a Square Number, and that which cannot be ranged into the Form of a Square, is not a Square Number.

### 132 ELEMENTS

14. Similar plane Numbers are those which may

2 | 12 | B

be ranged into the form of similar Rectangles; that is into Rectangles, whose Sides are proportional; such are 12 and 48; For the Sides of 12 are 6 and 2 (See Fig. B) and the Sides of 48 are 12 and 4

(See Fig. C.) But 6. 2::12. 4. and therefore those

Numbers are Similar.

15. All square Numbers are similar Planes (6. 32.)

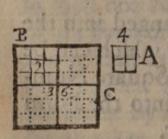
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C	S. A.	101				4	8			13/6	100	127
4		100			th it		·			37	100	100

ber may be placed in the Form of a Right line, and in that disposition may be taken for a Plane. Thus 3 (in Fig. A)

may be conceived as a Plane Similar to 12 or B; For the sides of the Plane 3, are 1 and 3, (because once 3 is 9) and the sides of 12 are 2 and 6. But as 1. 3:: 2.6.

Planes: As if you examin from 1. to 10. you will find indeed that 1. 4. 9. being Squares are similar, and so are 2 and 8, which have one side double to the other. But the rest as 3. 5. 6. 7. are by no means similar Planes.

12. If one Square Number be Multiplied by another, the Product will be a third Square Number.



Thus A 4. and B 9. being both Squares do, when Multiplied into one another, produce the Number 36 or C: And I say that third Number is a Square. For the meaning of Multiplying B by A, is to take B as often as there are Unites in A. But

But I may consider the whole Number B. 9. as one only Square, and I can take that as often as there are Unites or little Squares in A. And as the Unites in A are ranged into a Square, so I can range the Square B as often into a Square Form, just as if it were an Unite. So that there will be four such Squares of B, which being placed as you see in the Figure, will make the Square C or 36.

19. If two Numbers are similar Planes, the greater may be divided into as many Squares as there are U-

nites in the lesser. A. 3. and B.

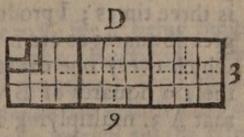
12. are similar Planes; so that the side 3. is to 6: as the side

1. is to 2. Wherefore I can divide the Plane B, 12 into 3

Squares placed just in such a manner as those 3 little Squares in the Plane A. And every one of the great Squares of B shall answer to 4 of those

in A. So also if the Planes
had been 8 and 72; I can
Divide 72 into eight Squares

of which every one shall contain 9 of those in the lesser Plane 8. The same



would come to pass also, if either one, or both the Numbers had been Fractions. As if A contain 3 and 1, and B 14. I can divide 14 into three Squares and an half, disposed just like those in A. as may be seen by the Partitions in the Figure, and by the half Square added in Prick'd Lines.

In like manner if the Planes were B 12, and D 27, I can divide 27. not only into three Squares, disposed after the same manner as those in A: But also into 12 Squares, so ranged as those in B, as the prick'd Lines in the Figure D do shew. The way to do which is to divide the Sides of the greater Plane into as many Parts as the Homologous sides of the lesser Plane

K 3

are divided into; the Figure shews the thing, and makes it easie.

ded as that there are as many Squares in the greater Plane, as there are Units in the lesser, are Similar;

this is the Converse of the former.

into another do produce a Square. For having divided the greater Plane into as many Squares as there are Unites in the lesser (7.19.) One Plane will be multiplied by the other, if the greater Squares of the greater Plane be taken as often as there are Unites or little Squares in the lesser Plane: But to multiply any Number of Squares, by the same Number, is to

make one Square out of all those Squares.

For Instance, A 3. and B 27. being Similar Planes, I consider B. 27. as a Plane compos'd of three great Squares, as A 3. is a Plane compos'd of three Unites, or three little Squares. So that if I take all these three great Squares, as often as there are Unites in A, that is three times; I produce then three times three such great Squares as are in B, that is, 9 such Squares; of which every one contains 9 of those in A, and all these 9 Squares of B contain 81 of those of A; so that A 3. multiplying B 27. produces 81. which is a

Number of the lesser Squares rang'd into a Square Figure; and by conse-

like manner if the Planes were B. 12.
and D. 27. I divide 27 into 12
Squares, which I multiply by 12,
and there are produced 144 greater
Squares rang'd into the Form of a

Square, which do contain in all 324 of those of the lesser Plane. (N. B. To divide 27 into 12 Squares, each Square must be 2. 25. (or two and a quarter) as you may see it is in the Figure D. No. 19]

29.11

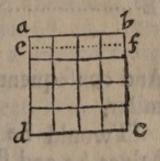
29. If two Plane Numbers are Similar, after what form soever you range one, the other also may be so disposed. Let 3 and 12 be similar Planes. If 12 be so rang'd in a Right line that it will make a Rectangle, one of whose sides shall be 12, and the other 1. I say that 3 may be so disposed as to make a similar Rectangle, one of whose sides will be 6, and the other the half of one, &c.

23. If one number divide another that is a Square one, a third shall be produced which will be a Plane

Similar to the Divisor.

Let there be a Square a c 16, and let it be divided by any Number, as suppose by 8. which is done if

you take the eighth Part of the side ad, viz. ae. and thro'e draw the Parallel a ef. For by that means you will have the Plane af, which will be the eighth Part of the Square ac. But to divide a Number or a Plane by 8. is to take at the eighth Part of that Number or



I say the Plane af, is similar to 8; for 8 being ranged into a Right line, so as to make a Rectangle, one of whose sides shall be 8. and the other 1. shall be Similar to it, because ae was taken the eighth Part of a d or a b: Wherefore as 8. 1:: (which are the sides of the Plane 8 the Divisor) so shall a b. ae (which are the sides of the Plane of the Quotient arising when the Square ac was divided by 8.) Therefore if one Number divide another that is a square, &c. Q. E D.

24. If two Planes multiplying one another do

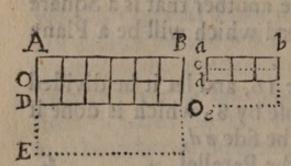
produce a Square, whose Planes are Similar.

25. Two Plane Numbers which are not Similar, if they are multiplied into one another, cannot produce a Square. These two Propositions are Consectaries from the foregoing ones.

26. If

26. If two Numbers are similar Planes, their Equimultiples and any of their (Respectively) equal Parts,
are also Similar Planes. Let the Planes be a b c d. 3.
and ABCD. 12. so that ab. AB::b c. BC. I say,
if you take the double of the one and the double of
the other (or any other Equi-multiple, be it what
you please) those doubles shall be Similar.

For having taken a e double to ad, and A E dou-



ble to AD: in order

to make the Plane be
double to bd, and BE
double to BD: Tis
clear that ad. AD:

a e. AE. But ad. AD

: a b. AB. Wherefore
also a e. AE: a b. AB.

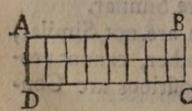
And consequently the Planes be and BE are Si-

milar.

'Twould be the same thing had you taken their halves bo and BO, or any other equal Parts of each.

27. If two Numbers are not similar Planes, their Equi-multiples, and all their (respectively) equal Parts will also be not Similar, which follows from the last.

28. Between any two similar plane Numbers whatsoever, there is to be found a mean Proportional. Let the two Numbers be 2 and 8, I say it is possible to find a Number which shall be a mean Proportional between them. For if we imagin the Plane 8 to be ranged in a right Line A B, and the Plane 2, also to be ranged in another right Line as A D. and that out of those two right Lines there be



That Plane A C. 16. Will be produced by the Multiplication of the two Numbers 2 and 8 (6. 17, and the following Pro-

positions) and consequently the Number of the lit-

tle Squares of the whole Plane A C. 16. shall be a square Number (7. 21.) and they

may be ranged into the form of a Square (7.13.) Let them then be disposed into the Square ac. So shall the Square ac be equal to the Plane A C, for 'tis only the same Number

dispos'd or rang'd after another manner. Wherefore (6, 59.) the side a b 4 shall be a mean Proportional

between A. D.2. and AB8.

Proportional can't be found. Let the Numbers be 4. and 6. Range each of them into a Right-line, and multiply them, they will produce the Plane 24. But this Plane 24 is not a square Number (7.25.) and consequently cannot be ranged into a square Form. Wherefore its impossible to have any Mean between 4. and 6. For such a pretended mean Proportional must, multiplied by it self, produce a Square, which (as hath been prov'd elsewhere) will be equal to the Plane made between 4. and 6. (6.59.) which is impossible, because this Plane 24, made out of 4 and 6. is not a square Number.

30. Let there be two Lines ae and ec, so to one

another, as one Number to another Nonsimilar. V. gr. as 1 to 2. Let also eb be a mean Proportional, so that ac. eb:: eb. ec. I say, that eb is Incommensurable with the two Extremes ae and ec. For a e and ec being as 1

to 2. (i. e.) as Numbers Non-similar (by the Supposition) as also are all their Equimultiples (7. 27.) 'tis impossible to find a mean Proportional between ae and ec (by the Precedent) and consequently, eb cannot be to ae, or to ec, as Number to Number. Wherefore it is Incommensurable with them.

31. The Diameter of a Square ab is Incommen-

furable to the fide ac. For taking ad double to ac, and making the Triangle abd, it shall be Similar to the Triangle abc; because cd being equal to cbd (2.15.) and the Angle d must be a Semi-right one as well as cab;

wherefore ab d is a Right angle; and consequently a c. a b:: a b. a d. That is, a b. is a mean Proportional between a c 1. and a d 2. and therefore Incommensurable (by the Precedent.)

#### COROLLARY.

Hence 'tis impossible to express one Square that shall be Double of another in Rational Numbers.

32. The Power of a Line is the Square which is made upon it, Thus the Power of the Line a c (Fig. preced.) is the Square aebc; and the Power of the Line ab is the Square abdf. And we say that Line ab is double in Power (in Latin bis potest) to the Line ac, which is a manner of Speaking borrowed from the Greeks, and generally received amongst Geometers.

33. The Diameter ab is Commensurable in Power to the side ac: That is, its Square abdf is Commensurable to the Square aebc, for 'tis indeed double to it.

A and earthyodie Pr

34. But if you take ao, a mean Proportional between ab and ac, that mean ao shall be Incommensurable to them even in Power; i.e. the Square of ao is Incommensurable to the Square of ab, or to the Square of ae. for the Square of ac a o to the Square of ab. is in a Duplicate Ratio of ac to ao (6.22); that is, as ac to ab (6.30.) But ac is Incommensurable to ab (7.31.) wherefore the Square of ac is Incommensurable to the Square of ao.

35. There is a Second Power of a Line which is called the Cube, which is made by multiplying the

Square by that first Line, or Root.

36. If two mean Proportionals an and am be

taken between a c and a b; so that a c.

an:: am, ab; the Line an will be In- a - n commensurable in this second Power to a - m a c (i. e.) The Cube of ac will be In-

commensurable to the Cube of an, because the Cube of ac to the Cube of an is in a Triplicate Ratio of the side ac to the side an; i.e. as ac to ab. But ac and ab are Incommensurable, wherefore, &c. However ac and ab are Commensurable in the second Power, for the Cube of ab is double to the Cube of ac.

37. Tis easie to apply to Solid Numbers what hath here been said of Plane ones. And those are called Solid Numbers which arise from the Multiplication of a Plane Number by any other whatsoever. V. gr. 18 is a solid Number made of 6 (which is a Plane) multiplied by 3; or of 9 multiplied by 2.

38. Similar Solid Numbers are those, whose little Cubes may be so ranged as to make similar and

rectangular Parallelopipeds.

39. Cubick Numbers are such as can be ranged into the form of Cubes as 8, or 27, whole sides are 2 and 3, and their Bases 4 and 9.

40. Every Cubick Number multiplying another Cubick Number produces a third Cubick Number.

41. Between two similar solid Numbers there may

be found two mean Proportionals.

That which hath been demonstrated in respect to

Plane Numbers, may be applied to Solids.

that there are Incommensurable Lines and Magnitudes shew also that a Continuum is not compos'd of Finite Points: For if the Diameter as well as the side of a Square were compos'd of Finite Points, a Point would measure both the Side and the Diameter, for that Point would be found a certain Number of Times in the side, and another determinate Number of Times in the Diameter, which the preceding

Propositions prove impossible.

43. Because in a Rectangle Triangle the Square of the Hypothenuse is equal to the Sum of the Squares of the Legs; (6. 61.) we have always used this Triangle for the discovery of Incommensurables. For if all the three fides are Commensurable, they may be all three express'd by three Numbers, and then the Square of the greatest Number will be equal to the Sum of the Squares of the other two. As if the greatest side or Hypothenuse be 5 Feet, the least fide 3, and the middle one 4: The Square of 5 will be 25, the Square of 3, 9, and the Square of 4 will be 16: And 9 and 16 added together do make the great Square 25. But if the least side of such a Triangle be 2. and the middle one 3. then the greatest side cannot be express'd in Numbers, because the Square of the least side 4 added to the Square of the middle side 9 makes 13, which express the Square of the greatest side. But as that Number 13 is not a square Number, so its side or Root cannot be express'd by any Number.

44. At all times Men have been Sollicitous to find out some Method of discovering proper Numbers to

express the three sides of a Rectangle Triangle, so as to be assured that all the three sides are Commensurable. Therefore I here shew you such a Method, by which you may find out all the possible Num-

bers that are proper for this purpose.

45. If you take any two Numbers (even Unity it self) differing but by an Unite, and add the Squares of them together, the Sum will be a Number which shall be the Root of a Square equal to two Squares; And that number will express the greatest side of a Rectangle Triangle, whose middle side shall be that Number lessen'd by Unity, and the least side shall be the Sum of the two first Numbers. V. gr. Having taken 1 and 2, and Squared each of them, you have I and 4; Add those two Squares together, and the Sum is 5. I say 5 will express the greatest side; and then 4 will be the middle one, and 3 the least; and 25 the Square of the Hypothenuse will be equal to the Sum of the other two Squares. In like manner if you take 2 and 3, and add the Squares 4 and 9 together, the Sum is 13. Then I say, will 13, 12 and 5 be three sides of a Rectangle Triangle; so that 169 the Square of 13, shall be equal to 144 and 25, the Squares of 12 and 5. Moreover if you take 3 and 4: The Sum of their Squares 9 and 16, makes 25, wherefore I say 25 may be the greatest fide of a Rectangle Triangle, whereof 24 will be the middle fide, and 7 the least fide.

It must be observ'd also, that the Equimultiples of any 3 Numbers thus sound will do the same thing: Thus, having sound 5, 4 and 3 their doubles 10. 8. and 6. will represent the three sides of a Rectangle Triangle, so that 100 the Square of 10 shall be equal to the Sum of 64 and 36 the two Squares of 8 and 6. And their Triples also 15. 12. and 9. will do the same thing: For any one may see that all these Numbers still having the same Proportion, do as it

## 142 ELEMENTS, &c.

were constitute but one only Triangle, viz, that which is express'd by 5.4. and 3. And therefore all those Numbers may be taken for the same.

N.B. The three Sides of a Rectangled Triangle will then only be Commensurable, when they are in this Proportion, viz. as a a -- ee, a a -- ee, and 2 a e. That is, the Sum of two Square Numbers, the difference of their Squares, and the double Rectangle of their Roots.





# ELEMENTS

OF

# GEOMETRY.

### BOOK VIII.

Of Progressions and Logarithms.



Rogression is a Series or Rank of Quantities which keep between one another any kind of similar Relation or Proportion; and every one of the Quantities is called a Term.

2. When the Terms which so follow one another do equally Increase or Decrease, the Progression is called Arithmetical; as are all Numbers proceeding according to the natural Order of the Figures, as 1, 2, 3, 4, 5, 6, &c. As also all odd Numbers, as 1, 3, 5, 7, 9, 11, &c. or as 4, 8, 12, 16. or as 20, 15, 10, and the like.

3. Arithmetical Progression may be increased Infinitely, but not diminished.

4. If

11 4

4. If in an Arithmetical Progression there be four Terms, the Difference between the two first of which is equal to the difference between the other two, those four Terms are said to be Arithmetically Proportional: As in the Progression of the natural Numbers, 1, 2, 3, 4, 5, 6, 7, 8, 9, &c. If you take four as 2. 3 ::: 9. 10 (This mark :: : I shall for the future use to signifie Arithmetical Proportion) there will be the same Arithmetical Proportion between 2 and 3 as there is between 9 and 10; that is, 10 exceeds 9, as much as 3 doth 2: So also 3.5::: 8.10. are in Arithmetical Proportion; and so are 1.5:::5.9. where 5 being taken twice, is an Arithmetical mean Proportional between 1 and 9.

5. In Arithmetical Proportion the Aggregate or Sum of two extreams is equal to the Aggregate of the two Means, as in 2.3::: 9. 10. the Sum of 2 and 10 is equal to the Sum of 3 and 9, that is 12; so also in 3.5::: 8. 10. The Aggregate of 3 and 10 is 13, which is equal to the Aggregate of 5 and 8. And the reason of this is self-evident. For tho' 10 exceeds 8, yet that which is added to 8 (viz. 5) doth just as much exceed 3, which is added to 10, and so there necessarily arises an Equality between them.

6. The Sum of the first and last Terms in any Arithmetical Proportion is equal to the Sum of the second and the last save one; or to the Sum of the third from the first Term, added to the third, accounted backward from the last, &c. as in the first Example, I and 9 make 10; and so do 2 and 8, 3 and 7, or 6 and 4 always make 10. And in the middle remains 5, which being taken twice (as if it were equivalent to two Terms, because 'tis equally distant from the first and last Term) makes also 10.

7. If you add the first Term to the last, and multiply that Sum by half the Number of the Terms, the Product shall be equal to the Aggregate or Sum

of all the Terms. As in the former Example, 1 added to 9, makes 10, and 10 multiplied by  $4\frac{1}{2}$  (or 4, 5) for there are 9 Terms, produces 45, which is the Sum of all the Terms from 1 to 9. As is manifest from the Precedent.

8. When the Terms of the Progression are continual Proportionals; that is, when the first is to the second as that is to the third Term, as the third is to the fourth, and as the fourth is to the fifth, &c. then the Progression is call'd Geometrical, as 1, 2, 4, 8, 16, 32::; Or as 1, 3, 9, 27, 81:: or again, as 3, 12, 48, 192, 768, or descending, as 8, 4, 2, 1::; or lastly as \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{6} \frac{1}{4} \cdot \frac{1}{6}

9 Geometrical Progression may be encreas'd and

diminish'd infinitely. [941 941 yd 9541 104

10. When the Progression begins with 1, the second Term is call'd the Root, Side or first Power; the third is call'd the Square or second Power; the fourth, the Cube or third Power; the fifth, the Biquadrate or fourth Power; the sixth, the Sur-solid or fifth Power; the seventh the Quadrato-Cube or sixth Power, &c.

the former two of which are as much distant from each other, as the two latter are: Those are simply Proportional, and the Rectangle of their extreams is

equal to that of their two middle Terms.

E and F, &c. that A B. A C:: A C. A D:: A D. A E, &c. Then I say, B C. C D. D E. E F, &c. are in continual Geometrical Proportion; and also that A B. A C:: B C. C D:: C D. D E, &c. for because A B. A C:: A C. A D. it will follow by Division of Proportion, that A B. less A C. (that is C B.) A C:: as A C less A D. (that is D C) A D. and consequently Alternately C B. C D:: A C. A D. or as AB. AC. and so of all others it may be proved:: DC. ED:: E F:: G F, &c. L

13. Let there be a Progression of Quantities in a Right-line B C, C D, D E, E F, &c. let C d be equal to the second Term CD, that so we may have B d the difference between the first and second Terms: And let it be made as B d. B C :: B C. to a fourth Line, viz. B A. I say, that if the Number of the Terms BC. CD. DE, &c. be Finite, tho' never fo great, all those Terms taken together, although there be an hundred thousand Millions of them, shall be less than B A. But if we suppose the Progression infinite, or that the Terms are infinitely many, then shall all de of them taken together be exactly equal to BA. For fince by the supposition B d. (that is BC. less C d or CD) is to B C: BC. (that BL is A B less A C) A B. it may easily be found that as B C. C D :: A B. A C :: A C. A D. &c. and consequently all the Terms CD, DE, EF, &c. will always be found within, or be hither the Point A. To which it Approaches the nearer, the more the number of the Terms is increas'd. So that we see plainly, that all these Terms (which in Books are usually call'd Parts Proportional) tho' they be actually infinite, cannot make an Infinite Length, because they will all be included within the Line BA. 14. This Demonstration will appear much more easie and sensible by the Example of a particular Progression, where the Terms are in a double Ratio v. gr. Let CB be double to DC, and DC double to DE, &c. For if the Number of the Terms be here

Progression, where the Terms are in a double Ratio v. gr. Let CB be double to DC, and DC double to DE, &c. For if the Number of the Terms be here Finite, tho' it be an hundred thousand Millions, and you take the last and least Term, for Example FE, and add to it another Quantity, as suppose AF, equal to it: It is then plain, that EA must be equal to the Term E D, which is the last save one; For E D is double to E F by the Supposition (the Ratio being

being every where double) and E A is also double to EF by the Construction, it having been made so, by taking F A equal E F. In like manner A E with D E, that is A D, shall be equal to the following Term C D, and at last A C will be equal to B C. So that from hence it appears, that the first or greatest Term is always equal to all the others taken together, provided there be added to them but a Quantity equal to the last and least Term; but if nothing be added, the first Term is always greater than the Sum of them all.

If these Terms are supposed to be actually Infinite, then the greatest BC will be exactly equal to all those Infinite others taken together CD, DE, EF, &c. For any one may easily discern, that the more there are of such Terms, the more you approach towards A. by cutting off still the half of the Remainder: But when any Quantity is thus lessen'd by half, and the Remainder again by half, and then the half of that third Remainder taken and so on: Tis plain, that by supposing the Diminution to be made an Infinite number of times, nothing at last will Remain.

This also might be demonstrated by a Reduction ad Impossibile, by shewing that all those infinite Terms taken together, are neither greater nor less than

A B.

Schoolmen against the (Infinite) Divisibility of a Continuum be solved, the Persons Ignorant of Geometry they appear Unsolvible: But indeed at the Bottom they are nothing but meer Paralogisms.

16. If two Progressions are supposed, one Geometrical beginning with 1, and the other Arithmetical beginning with 0, so that the Terms in one shall be placed over and answer Respectively to those in the

A. 2

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other; The Arithmetical ones are call'd Loga ithms, Exponents, (or Indexes) as in the following Ranks.

0. 1. 2. 3. 4. 5. 6. 7. 8. 1. 2. 4. 8. 16. 32. 64. 128. 256.

17. That which is produced in a Geometrical Progression by Multiplication and Divisior, is effected in the Logarithms by Addition and Subtraction: As, if having three Numbers given; 2.8:: 64; You would find a fourth Proportional to them in Geometrical Progression: You must multiply 64. by 8. (which are the two middle Terms). For the Product 512. shall be equal to the Product made by 2. and the fourth Number sought, they being the two extreams of four Proportionals. And to find this sourth Number, you need only divide 512 by 2, and the Quotient will be 256. So that 2.8:: 64.256. and 64 and 256. will be just as far distant from one another in the order of the Progression, as 2. and 8 are.

But if instead of the Geometrical Numbers 2. 8:: 64. you had used their Logarithms 1. 3:: 5, which answer to them in the Progression, and were minded to find a fourth Logarithm, then you must have added 3 and 6 which make 9, and from thence have Subtracted 1. there would remain 8. The Logarithm answering to the Geometrick Number 256.

18. So also if there be two Geometrick Numbers 4 and 8, to which the Logarithms 2 and 3 do answer; by multiplying 4 by 8 you produce 32; the Number under the Logarithm 5, which is the Sum of the Logarithms of 2 and 3.

16. In like manner by multiplying 16 by it felf, there will be produced 256 which stands under the Logarithm of 8, the Sum of 4 added to it self.

128;

20. So if the Geometrical Number were required that shall answer to or stand under the Logarithm 16; you must take 256 which stands under 8, and multiply it by it self, and it will produce 65536 the Number requir'd.

21. If moreover the Geometrical Number answering to the Logarithm 23 were requir'd, you may take any two Logarithms whose Sum is 23, as suppose 7 and 16, and multiplying the Geometrical Number under them, viz. 128 and 65536 one by another, the Product will be 8388608. The number which ought to stand under the Logarithm 23, or in the 23 place of a Series of Geometrical Proportionals be-

ginning from 1.

of will discover Terior 22. From hence appears the way of Answering that ordinary Question, how much a Horse would cost, if bought on this Condition: That for the first Nail in his Shoe a Farthing were to be paid, for the second Nail two Farthings for the third Nail four Farthings, for the fourth Nail eight Farthings, and so on, still doubling for 24 times: For the 23d place in such a Progression would be the last Number 8388608 Farthings, which being reduced is 8738 l. 2 s. 8 d. and being doubled according to (8. 14.) gives the whole Price of the Horse 17476 1. 5 s. 4 d.

23. Where two compleat Progressions are fitted lo as to answer one to another, the Geometrical to the Arithmetical; as suppose in Tables for that purpose calculated in Books, there abundance of Pains and Labour is spared, in finding the Geometrical Numbers: For instance, let those three Numbers 32. 64. 128. be given, and that a fourth Proportional were required: Instead of multiplying 64 by 128, and dividing the Product by 32 (which way is very tedious in great Numbers): you need only take the I ogarithms of the three given Numbers, viz. 32. 64.

L 3

Sum Subtract the first, the Difference will be the Logarithm of the corresponding Geometrick Num-

ber 256:

24. But because in such a Geometrical Progression all Numbers will not be found, this Medium hath been discovered; they have calculated two Progressions, one of which contains all Numbers 1.2.3.4.5.6.7.8. So. which seems to be an Arithmetical Progression, but yet hath in reality the Properties of a Geometrical one. And the other which contains Numbers in appearance the most Irregular, is nevertheless a true Arithmetical Progression. See here a Line, which will discover Perfectly all these Mysteries.

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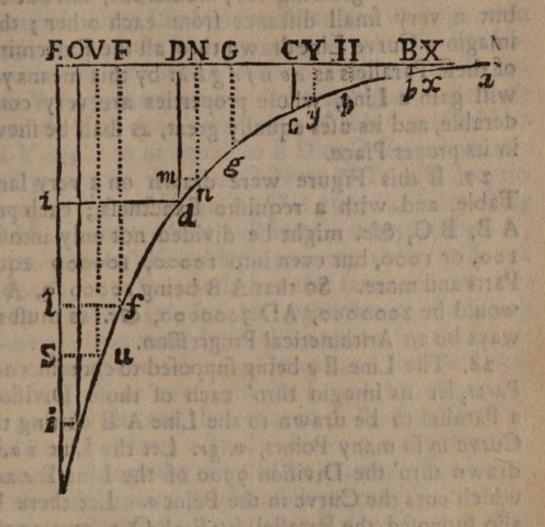
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deviced: loftead of multiplying 4. by 4.3 h, and devicting the Product by 3.2 (which way is very reduced on an great Numbers): you need only take the 1.0-

ves the whole Frice of the Horfe are 78 4.

Book VIII. of GEOMETRY.

25. Let the Right-line A E be divided into the equal Parts A B, B C, C D, D E, &c. from the Points A, B, C, D, E, &c. let the Lines A a, B b, C c, D d, and E e, be drawn all (Perpendicular to AE and consequently) Parallel to one another: And let them be all in a Geometrical Progression: As let A a be 1, B b 10, Cc 100, Dd 1000,



Ee 10000, &c. Then shall we have two Progressions of Lines, the one Arithmetical and the other Geometrical: For the Lines AB, AC, AD, AE, are in Arithmetical Progression, or as 1.2.3.4.5, &c. and so do represent the Logarithms; to which the Geometrical Lines A a, B b, C c, &c. do correspond, by this moons a Table of Lo

26. Let each of the equal Parts E D, D C, C B, &c. be divided equally again in F, G H, and let the Parallels Ff. G g, &c. be drawn, and be mean Proportionals between the Collateral ones; that is, E e. Ff: Ff. D d:: D d. G g. Let there also be more mean Proportionals, drawn from the middle of each Sub division E F, F D, D G, and so on, till these Parallel Lines growing very numerous, have at last but a very small distance from each other; then imagin a Curve Line drawn thro' all the Extremities of these Parallels as e o u f d g b a: by this means you will gain a Line, whose properties are very considerable, and its uses equally great, as shall be shewn in its proper Place.

Table, and with a requisite Exactness; each part A B, B C, &c. might be divided not only into an 100, or 1000, but even into 10000, 100000 equal Parts and more. So that A B being 1000000, A C would be 2000000, AD 3000000, &c. as must al-

ways be an Arithmetical Progression.

28. The Line E e being supposed to contain 1000 Parts, let us imagin thro' each of those Divisions a Parallel to be drawn to the Line A E cutting the Curve in so many Points, v. gr. Let the Line i o be drawn thro' the Division 9900 of the Line E e and which cuts the Curve in the Point o. Let there be also supposed the Parallel (to E e) O o, cutting the Line A E in the Division 399563. Then any one may know that 399563 is the Logarithm of the Number 90000. In like manner if S u passed thro' the Division 9000 of the Line E e, and the Line u v were drawn cutting AE in 395424, then would that Line u v be the Logarithm of 9000, &c.

19. So that by this means a Table of Logarithms from 1 to 10,000 may easily be made; and farther,

by producing the Line AE.

30. Note,

30. Note, to obtain all the Logarithms from 1 to 10000; 'twill be enough to feek the Logarithms from 1000 to 10000: That is (having drawn the Parallel d t) to take the Logarithms of all the Divisions from t to e, which Logarithms are all contained between E and D. For by this you will have the Logarithms of all the Parts that are between t and E; and whose Logarithms lie between D and A: For Example, fince Oo is 9900 Parts, and its Logarithm 399563, the same Number may be taken for the Logarithm of 990 which is Nn; as also of the Number Y y 99, changing only the first Figure 3. Because according to the Composition of this Line, O N or N Y ought to be equal to E D or D C, as one may eafily prove. So that O N or NY will contain 100, 000; and because AO is 399563, subtracting ON 100, 000, there will remain 299563, for AN, from whence also taking 100,000, there will rest 199563 for AY. And after the same manner, having AY 3995424 for the Logarithm of Vu which is 9000; you may have also 095424 for the Logarithm of X x which is 9. Or 195424 for the Logarithm of 90, or 29524 for the Logarithm of 900.

culation, without actually drawing these Figures, but only Imagining them to be drawn. For by the Rules of Common Arithmetick we may find out Ff, the mean Proportional between D d and Ee, and after that, another Mean between D d and Ff, or between F f and Ee, &c. But what we have here explained is sufficient to gain as much Knowledge as is necessary for us to have of the Nature and Composition of Logarithms: There being no need for us to undergo the Labour of Calculating Tables of Logarithms; since it's already so well and so often done to our Hands. God, for the Publick Good, having raised some Persons, whom he was pleased to endow with sufficient Patience to surmount so tedious and

laborious

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laborious a Work, as one would think to be insuperable. For we know that above 20 Men were engaged in such a Calculation, for above 20 Years together, with indefatigable Industry and Assiduity.

(Pardie speaks here a little Covertly, seeming willing to insinuate that this most useful and admirable Work was done first in his own Country, whereas the Logarithms were the Invention of my Lord Neper a Scotch Baron, and the first Tables were Calculated by him, with the Assistance of our Country man, Mr. Henry Briggs.)

Of late several Improvements have been made in this Matter: As by Nicholas Mercator, of which see Dr. Wallis's Thoughts, in Philosoph. Transact. 38. John Gregory bath given us a way to find Logarithms to 25 Places by help of the Hyperbola. But Doctor Halley in Philos. Transact. N°. 216. Shews a way from the bare Consideration of Numbers, and withall by the help of Mr. Newton's Way to find the Uncix of the Numbers of a Binomial Power, &c. By which you may find compendiously the Logarithms of all Numbers to above 30 Places. And he gives there several Series for this purpose, some Universal, and some appropriated to a peculiar sort of Logarithms.



ELE-



# ELEMENTS

OF

# GEOMETRY.

BOOK IX.

Problems or Practical Geometry.



HAT Proposition is called a Problem in (Geometry) which teaches us how to do any thing, and demonstrates also the Practice of it: Whereas Theorems are Speculative Propositions, in which are considered the Affections and Properties of Things

already done.

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2. To Divide a Circle into four and into six, and all Arks into two equal Parts. To divide it into four

d a c

draw two Lines as da c and Bac at Right Angles to each other. To divide it into eight Parts; Bissect the four Arks Bc, ce, &c. which is done by striking (without the Ark Bc) two other Arks, with the same opening from the Points B, and C, for if a Line be drawn

from the Point where those Arks cross each other, to the Center a, it shall Bissect the Ark BC. The like

is to be done for the other Arks.

To divide a Circle into fix equal Parts; you need only take the Length of the Radius: and applying it fix times about the Circle, it will exactly divide the Circumference into fix equal Parts, and thus by a new Bissection, may a Circle be divided into 12, 24, 48, or into any Number of equal Parts, &c.

3. To divide a Circle into five, into fifteen, and into other Equal Parts: This may be done thus; (as I demonstrate in Algebra) Make a Rectangled Triangle, one of whose Legs shall be the Radius of the Circle, and the other half the Radius From the Hypothenuse of this Triangle, take half the Radius the Remainder shall be the Chord of 36 deg. and the fide of a Decagon. Double that Ark, you have the Ark of 72 deg. (whose Chord is the side of a Pentagon) and it is the fifth part of the Circumference; and the same Chord shall be also the Hypothenuse of a Rectangled Triangle, one of whose sides is the Radius and the other the fide of a Decagon. And as by the last was found the Chord of 60 deg. so by subtracting the Chord of 36 deg. from 60 deg. you may have the Chord of 24 deg. which is the 15th part of the Circumference. But for Practice, the shortest and surest way, is by repeated Tryals with the Compasses to find a Distance

Distance that will go precisely five times about the Circle: Then divide, after the same manner (by Tryals) that distance into three equal parts exactly. So shall you gain a Chord that will divide the Circumference into 15 equalParts, and then dividing each of those 15 Chords into sour equal Parts, and each of those into six; you will divide the whole Circumference into 360 deg. And this Division is most Commodious for Practice and Use. Note, that the way to divide a Circle into 3, 5, 7, or into any other odd Number of Parts, is not yet sound Geometrically; Geometrically I say, that is, by making Use only of a strait Line and a Circle.

"This Division of a Circle into 360 deg. is very useful, when a Person understands how to use the

" Compasses of Proportion (or

" Sector.) 'Tis so called be-

" cause 'tis a kind of Com-

" passes with broad Legs:

" As a B. a C. on which are

" described divers Lines and

" Divisions, but those which

" are most in use, are of two sorts. On one side of this Sector, and on each Leg, is a Line a e B and a e C, which serves to divide a Circle into 360 deg. at once, and also to take at any time as many may be be be as you please: And this Line on the

" Sector is thus divided.

4. To divide and graduate the Sector, that it may ferve for the Division of a Circle. Imagine a Semi-circle a EDB accurately divided into 180 deg. it then from the Point a, as from a Center, you transfer the Divisions of the Semicircle into a Line a B. v. gr. If from E, 60 deg. you draw the Ark E e. and if from D 90 deg. in the Semicircle, you draw the Ark D d, &c. Then ought 60 deg. on that Leg. of the Sector, to be placed at the Point e, and 90 deg.

deg. at the Point d, &c. And if you Transfer the same Degrees after the same manner into the other Leg a C, you will graduate the Lines a B and a C (on the Sector) as they ought to be for this purpose,

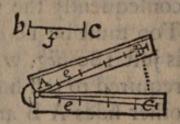
and will they be two similar Lines of Chords.

5. To explain the Use of the Sector as far as it serves for the Division of a Circle. Let there be a Circle given Af; take with your Compasses the Radius Af; and (keeping that Distance) set one Foot of them in e or 60 deg. on one Leg of the Sector; move the other Leg of the Sector to and fro so long, till the other Point of the Compasses falls exactly on e or 60 deg. in that Leg of the Sector: So that the Distance e e be exactly equal to the Radius Af: Then if you would have readily 90 degrees of that Circle; (Letting the Sector lie still, and always keeping the same Angle) Open your Compasses till the Points fall exactly on d and d or 90 deg. on each fide of the Sector: And then that Distance transferred into the Circle, in f. g. gives you the Ark of 90 deg. f. g. So also if you would have had 35 deg. you need only apply your Compasses to 35. deg. and 35 deg. on each Leg of the Sector in the Lines (of Chords) a B and a C: and that distance transferred into the Circle, shall cut off the Ark of 35 deg. and thus may you proceed to find any Degrees you please. All which is grounded on the 42, 43, 49, and 50 Propositions of the VI. Book. For fince all Circles are similar Figures, (6. 50.) the Chord f g will be to the Radius f A :: as the Chord d d to the Radius e e; that is, as a d is to de. Now 'tis plain, from what hath been proved elsewhere, that the Triangles a d d and ne e are similar; and therefore d d. e e:: a d. ae. But d d is by the Construction equal to f g. and e e to Af. wherefore fg. Af:: da.ae.Q. E. D.

of bus secured at the Pourte and go

6. To divide the Line of Equal Parts or Lines on the Sector, for the Division of any Right-lines given There being two Right-lines drawn from the Center of the Sector on the Legs as a B and a C: Let each be divided into 100 or 200 equal Parts: And then they

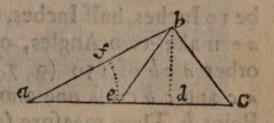
will serve to divide any Line given into any Number of equal Parts; As for Instance, let the Line given be c b, and that you were required to take  $\frac{25}{27}$  parts of it. Now to divide the whole



Line cb into 97 equal Parts, and then to take 25 of them according to the common way of dividing Lines, would be very tedious: But by the Sector its done easily and speedily thus; Take the length of the whole Line cb in your Compasses, and sit or apply it over in your Sector between 97 and 97 in each Leg, from B, suppose to C. Then letting the Sector lie open'd at that Angle, take in your Compasses, the distance between 25 and 25 in each Leg, or between e and e, which transfer into the given Line from b to f; so shall bf be just \frac{1}{25} of the whole Line cb: As is plain from the Triangles ABC and Aee being Similar.

7. On a Line given to make an Angle that Shall con-

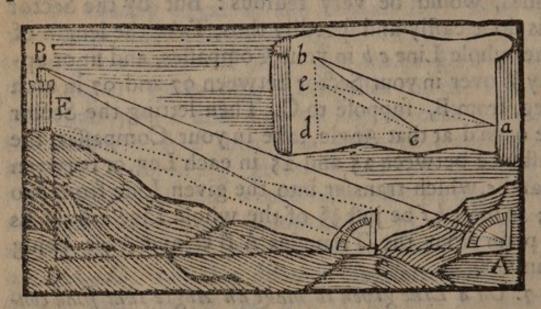
as from a Center strike



the Ark fe, from which take by the Sector, or otherwise, 30 deg. from e to f; then thro' f draw the Line a f, which with the Line a e will make an Angle of 30 deg.

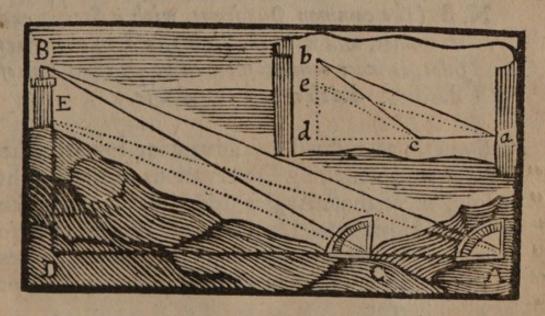
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8. Having the Angles of any Triangle and one side given to find the other two sides. Suppose you are told there is a certain Triangle some where, whose Base AC is 10 Fathom; and that the two Angles at the Base are ACB 150 deg. and CAB 20 deg. (and consequently the remaining Angle at the Vertex or Top must be 10 deg. for the Sum of 150, 20 and 10, is just 180 deg. which is two Right Angles). You are required to tell how many Fathom there are in the other sides AB and AC. Make on Paper, or rather on fine Pastboard, a Triangle a b c Similar to the propos'd one, after this manner. Take a Base at pleasure a c, and from any Scale of equal Parts let it



be to Inches, half Inches, &c. in Length. On this Line ae make two Angles, one cab of 20 deg. and the other acb of 150 (9.7.) Then will the two Lines ac and cb cross one another when produced in the Point b. Then measure (on the same Scale you took the Base ac from) how many Inches, &c. the Lines ab and cb are in Length; And you may be assured that there are just so many Fathom in the Lines AB and CB sought, as you find Inches, &c. or any equal Parts, in the Lines ab and cb. For since the Triangles are Equiangular, they are Similar, and therefore a c. ab:: AC. AB, &c.

9. To measure Distances, Heights, Depths, and in general, the Dimensions and Magnitudes of all Remote and Inaccessible Places. If on the Top of any Hill appearing at a Distance, there were a Tower, as BE, and its Distance from us and its Height, were required: You must first with some Instrument (as with a Quadrant, that is the fourth pert of a Circle divided into 90 deg. and furnished with a Ruler, or Label with Sights, and moveable on the Center) you must, I say, with some such Instrument, take two Angles at two several Stations in this manner: If you are in the Station A, place your Instrument so, that one fide of it may answer exactly to the Horizontal Line AD; and keep it without raising or depressing it in this Position. Then place your Eye at A, (that is at the Center of the Instrument) and turn the Label till it point to the Top of the Tower B, and that



looking through the Sights you can see the Top of the Tower exactly; then will the Label cut in the Limb of the Quadrant the Degrees of the Angle B A D, for the Limb is supposed to be graduated for this Purpose: Then change your Station, moving in a Rightline forwards 10 Fathom (or it might have been any other Distance, and backward as well as forward) to C, and there take after the same manner the An-

gle BCD: By which means you will have also the Angle BCA, because those two together make 180 deg. or two Right ones. So that in the Triangle ACB you have now found the Base AC, which is 10 Fathom, and also the two Angles at the Base; and Consequently, the Sides C B, and AB, may be known: (9. 18.) And then you may have the Height DB, or the Distance A D, if you make a little Triangle Similar to that, and there from the Point b, let fall a Perpendicular b d, to the Base Line A C continued to d. For B D, or A D, will be just as many Fathoms as b d, and a d will be equal Parts measur'd on the Scale, (as in the last.) And if after you have thus gain'd the Height B D, you find, by the same Method, the Height ED also, you may (by Subtracting this Altitude from the former) find the Height of the Tower E B.

N.B. (The common Quadrant, with a String and Plummet, and with the Sightsfix'd on one of its sides, is more convenient and ready than this of Pardies, which is now out of Use.

"Sometimes instead of advancing towards the "Tower, and of making Observations of the Height below, or of those Angles the visual Rays make "with the Horizontal Line, it is convenient to take two Stations side-ways of each other; But it comes all to the same, and the Practices in reality are not at all different.

"And by this Means, as any one may see, may all imaginable Heights and Distances, and other Dimensions be taken; provided we can but come

" to observe their Extremities, from two different Places. I shall not stay now to describe the Par-

"ticular ways of doing this; nor to enumerate the great Advantages that would accrue from the use

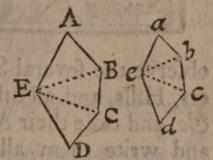
" of Telescopical Sights six'd on the Label, or on the fide of the Instrument used in taking Angles; which

" indeed is an Invention of inestimable Benefit to

"Surveyors.

be a City, or any other Place, and you were required to take the Plane, and to make a Draught of it. Take all the Length of its sides, and of Lines drawn from Angle to Angle: And transfer all these upon Paper, laying them down according to their true Proportion. For Instance, having found A B to be

30 Paces, BC to be 59, CD to be 50, B E to be 67, and A E 49, So and having ready drawn on Paper a plain Scale divided into 100 equal Parts. Make the Line ab 30 of such Parts; be, 67; and ae, 49,



then those Lines drawn and join'd together will make the Triangle abeevery way Similar to the Triangle ABE. And if you go on thus, and make the Triangle bec, Similar to BEC, &c. you will form the Figure abcde every way Similar to the Plane of the Place ABCD E.

it, and to measure the Distance between the Angles EB and EC, you must take the several Angles of the Plane, and transfer them into your Draught; so that if the Angle BAE be 66 deg. the Angle bae must

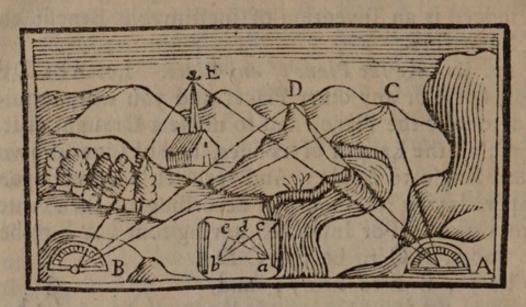
also be 66 deg. and so of all the rest.

12. To make a Draught of any City or Country. Afeend up into any two elevated Places, from whence
you can plainly fee the City or Country, whose Delineation you would make. And having with you a
Quadrant, whole Circle, or Semicircle well divided
into Degrees, together with its Label (with Sights)
and its Center: Place your Instrument at A, and

M 2

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so that one of its Sides may lie in a Line between A and B, which done, and the Instrument fix'd there,



observe the several Steeples, eminent Houses, Towers, Hills, and all other remarkable Places, as EDC, &c. and take their Angles with the Label and Sights, and write them all down to help your Memory, Thus, let the Angle C A B be 50 deg. 30 min. the Angle DAB, 45 deg. 8 min. &c. Proceed after the same manner at the Station B; noting down the Angle ABC to be 40 deg. 10 min. the Angle ABD 47 deg. 28 min. &c. After which, draw on Paper any Line at Pleasure, as a b, and make at each End of it, Angles equal to those which you found, ca b equal to CAB, da b equal to DAB, and a bc equal to ABC, &c. And by this Means, you will have the Points c, d and e, &c. which will be in the same Position to one another as the Steeples, or other eminent Places CDE, &c. are. And thus having drawn the most Conspicuous and Principal Places, the rest may easily be taken by the Eye. But to make this Operation very exact, 'tis convenient to take the Angles also at a third or fourth Station, and then if they all agree, any one will know that the Work was well done.



#### A

# TABLE

Of the Words and Terms of Arts explained in this Geometry.

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The Characters, Marks, Signs or Symbols here used are only these.

= F Qual to. + More, or Adding. -Less or Subtracting.

:: The Mark of four Quantities being discretely

Proportional Geometrically.

The Mark for Continual Proportion, or Geometrick Progression.

::: The Mark for Arithmetical Proportion.

× The Mark for Multiplication.

☐ Square.
☐ Rectangle.
△ Triangle.
᠘ Angle.
☐ Parallel.



Mary Clark Market Hills Topical Colquestically. LE NATIONAL CAMPBELLE BUILDING







