Special Report Series, No. 60.

Privy Council

MEDICAL RESEARCH COUNCIL

The Use of Death-rates as a Measure of Hygienic Conditions

By John Brownlee, M.D., D.Sc.

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PUBLISHED BY HIS MAJESTY'S STATIONERY OFFICE
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Price 3s. net.

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15 Buckingham Street,
Strand, W.C. 2.

THE USE OF DEATH-RATES AS A MEASURE OF HYGIENIC CONDITIONS

BY

John Brownlee, M.D., D.Sc., Director of Statistics, Medical Research Council.

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(6965). Ps. 6145. Wt. 1349·315/952. 1500. 1/22. O.U.P.

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I. Introduction.

'Those to whom the King had entrusted me observing how ill I was clad ordered a tailor to come next morning and take measure for a suit of clothes. This operator did his office after a different manner from those of his trade in Europe. He first took my altitude by a quadrant and then with rule and compasses described the dimensions and outlines of my whole body, all which he entered upon paper: and in six days brought my clothes, very ill made, and quite out of shape by happening to mistake a figure in the calculation. But my comfort was, that I observed such accidents very frequent and little regarded.'

In the following pages it is proposed to criticize the methods that have been and may be employed to draw conclusions from the manner in which death-rates experienced among those who live in different environments or those who work in different industries may vary, and, if possible to discover what are the canons of just inference from such data as are at our disposal. This seems to be specially important at the present time, a time when administrative measures competent to ameliorate unfavourable social conditions are expected in reply to instant demand. To determine how to act best requires exact knowledge, a knowledge not to be attained without a full appreciation of the error which may arise in making deductions from the different measures in which the data are recorded.

The data which are at our disposal to determine many of these questions unfortunately relate only to deaths and not to incapacity. In the best class of such evidence the actual death-rates can be calculated for deaths against known populations. In another class of statistics, however, the deaths alone are known and the only comparison possible is between the number of deaths from any cause and the total number of deaths from all causes. These two categories obey different laws and require quite different methods of statistical treatment. It is commonly believed that deductions based upon data of the first kind are unassailable while deductions based upon data of the second must be regarded with distrust. The problem, however, is not nearly so simple as this. In both cases there are many fallacies which must be avoided. These will be discussed in the appropriate place. For the present it is sufficient to give two examples.

The mean age at which deaths from measles occur in towns is considerably lower than that in the country. As nearly every one exposed to measles develops the disease an adequate explanation is at once suggested. The phenomenon is due to the fact that epidemics occur at shorter intervals in the towns than in the country, and in consequence there is on the average a larger susceptible population at higher ages in the latter. On the other hand a disease such as cancer, which many hold to be degenerative and not parasitic, has a similar difference in its age incidence, deaths in towns occurring on the average at an earlier age than deaths in the rural districts. An explanation of the difference cannot in this case be sought directly from the side of infection. The difference will be shown later to be quite independent of this theory of causation.

Thus even when the numbers of deaths and of the corresponding populations at each age period are known it is often difficult to determine the significance of the figures. When the number of deaths alone is known the matter is still more complex. In both cases it is nearly true to say that each disease requires special consideration, but though this may be or may not be required in the first category, it is always required in the second. It is to examine how far the different measures of the death-rates are measures of comparison, and to discuss the range in which each of these measures is of practical importance, that the following pages have been put together.

The method of presenting the subject-matter adopted is in the first place to discuss all the essential facts as clearly as possible in the text, adding at the same time a series of tables and diagrams. The description of the important technical methods is not placed in the text but in the subsidiary explanation attached to these tables and diagrams. Purely mathematical processes are placed in Part II. It is hoped that this arrangement will enable each reader to find what he wants. I do not know whether such a subdivision of the discussion will be approved, but at the present moment with the great difference of mathematical attainment in the health services it seems necessary to grade the discussion of any technical matter on a series of levels, (1) for the generally informed person, (2) for the executive official such as the medical officer of health, (3) for the statistician. To write for each severally is impossible. It is further undesirable as the knowledge of most persons is not limited to one range. The test of the art of any author at present is therefore his capacity to write for all three at once.

In certain places where a passage of my own already published seems to describe some phenomenon more concisely than is in my power to improve, this passage has been transferred to the present text without the addition of marks of quotation. After all, it cannot reasonably be expected that a purple patch can be written more than once. I have to thank Dr. Major Greenwood for reading the proof, and also am under great obligation to Sir Alfred Watson and Mr. A. Henry for the care with which they investigated a number of special points.

PART I. DEATH-RATES IN GENERAL

II. Crude Death-rates.

The first measure of mortality introduced statistically was the ratio of the number of deaths in each year to the average number of persons living during that year. This was commonly expressed as a fraction. Thus a mortality of $\frac{1}{20}$ signified that out of every 1,000 persons constituting the population 50 died in each year, while a mortality of $\frac{1}{50}$ signified that out of each 1,000 persons 20 died. This method of describing the mortality by a fraction has grave inconveniences. It is inverse in character, difficult to print and to read, while the comparison of the hygiene of one district with another involves undue mental effort.

At the present moment the death-rate is stated as a proportion of the number dying per annum to the number living, and is expressed in units per thousand. Thus, if 33 persons die in each 1,000 of the population, the death-rate is said to be 33 per 1,000. This method is direct and much more convenient both from the point of view of printing and of thinking. It is in addition, in spite of its drawbacks, which will presently be discussed, a measure which in general is not far from representing the facts in a manner easily understood.

When the change took place from the one system to the other I do not know. The data of Dr. Heysham, 1780–8, on which the Carlisle Life Table was founded, are expressed in the first method. In 1829 Dr. Hawkins still employed it in his Goulstonian Lectures on Medical Statistics. In 1837, however, in Dr. Farr's creative article on 'Vital Statistics' in McCulloch's *Statistical Survey of the British Empire* the new method is employed. As Dr. Farr immediately thereafter became medical adviser to the Registrar-General, it is no wonder that the method finds its place in the publications issued by that newly-founded office. The formation of this convenient comparative standard must thus, I think, be ascribed to Dr. Farr. In some important statistical returns, however, such as those issued by the City Chamberlain in Glasgow, Mr. Watt, both

TABLE I. *Illustrating the proportionate distribution of the population in each age and sex group.*

Age Period.	*Nine Rural[1] Registration Districts of Yorkshire.* (1891–1900)		*Doncaster.* (1891–1900)		*Liverpool Registration District.* (1891–1900)		*Health Districts in Glasgow.* (1899–1903) *Pollokshields.*		*Springburn.*		*Cowcaddens.*	
	Males.	*Females*	*Males.*	*Females.*	*Males.*	*Females.*	*Males.*	*Females.*	*Males.*	*Females.*	*Males.*	*Females.*
0–5	5·91	5·89	6·31	6·43	5·41	5·43	2·53	2·67	7·35	7·37	5·62	5·55
5–10	5·92	5·78	5·95	5·98	5·00	5·10	2·82	2·54	6·22	6·21	5·39	5·20
10–15	5·71	5·60	5·47	5·31	4·95	4·98	3·25	3·55	5·23	5·23	5·13	5·28
15–20	5·41	4·90	5·15	4·60	4·77	5·17	3·87	6·56	4·69	4·24	5·01	5·07
20–25	4·06	4·33	4·69	4·32	4·65	5·41	3·74	8·61	4·40	4·38	4·73	4·70
25–35	6·56	7·05	7·93	7·43	8·31	8·54	6·51	12·70	8·96	8·59	8·63	8·12
35–45	5·50	5·68	6·05	5·69	6·73	6·54	5·02	8·44	6·58	5·95	6·40	6·40
45–55	4·46	4·37	4·40	4·16	5·03	4·89	4·90	7·24	4·10	3·79	4·63	4·85
55–65	3·25	3·25	2·79	2·85	2·86	2·94	3·83	4·67	2·15	2·14	2·67	3·08
65–75	2·19	2·18	1·51	1·69	1·19	1·41	2·06	2·88	0·91	1·05	1·07	1·77
75+	0·96	1·04	0·57	0·72	0·25	0·44	0·64	0·97	0·16	0·30	0·25	0·45
Total	49·93	50·07	50·82	49·18	49·15	50·85	39·17	60·83	50·75	49·25	49·53	50·47

[1] Sedbergh, Settle, Great Ouseburn, Patrington, Driffield, Malton, Easingwold, Guisborough, Bedale.

DESCRIPTION OF TABLE I

In this table the variations which may take place from district to district in the relative age and sex constitution of the population is shown. In the first half of the table the figures are given for three districts in the north of England. It will be noted how in the rural districts the proportion of persons at upper ages is very high as compared with what is found in Liverpool, and that in the rural districts the defect of the population is found between the ages of 25 and 55 years. In the second half of the table three health districts of Glasgow are compared. One of these is Pollokshields representing the type of residential villa population. In place of the sexes being nearly equal in number, females are in great excess, the ratio being three females to two males. A great part of this excess occurs between the ages of 15 and 45 years, the ages at which domestic service is most common. The very small proportion of children is also to be noted. The second of the districts is Springburn, a good working-class district. Here we have a large proportion of persons at early adult ages, a large number of children and relatively few persons at the higher ages; the proportion above 75 years for the males and females being respectively 0·16 and 0·30 as against 0·96 and 1·04 in the rural districts of Yorkshire. The third district, Cowcaddens, is a slum. The proportions of the population do not greatly differ from those in Springburn except in the ages of childhood and in old age, there being a defect in the former and an excess in the latter.

methods are employed till the middle forties of last century, and even the shipping health returns of the Board of Trade employ the earlier method to the present day.

For its immediate purposes this death-rate, now termed the crude death-rate, was quite sufficient. Its disadvantages as a measure appeared later. These disadvantages, now well known to every schoolboy, as Macaulay would have said, are that from place to place the population varies in constitution of age and sex. Extending manufacturing towns such as Doncaster (Table I) are peopled largely by young married couples, and though the mortality of infants is very high, yet, as normally the chief number of deaths takes place at the later ages of life, and as the numbers living at these ages in a population such as just described are in defect, the crude death-rate as a measure of the deleterious effects of city life is unduly small. Rural districts (Table I) show the opposite condition, elderly persons being in great excess. Still more artificially small death-rates are found in city suburbs where, in addition to the proportion of children being unfortunately progressively smaller and smaller, the number of maid-servants at healthy ages of life employed is in great excess. These if mortally affected as a rule return home to die. The result of such a concatenation of individuals is to produce, as not unusual, crude death-rates of 7, 8, or 9 per thousand. The manner in which the different districts of a town may vary is shown for Glasgow (Table I). Pollokshields is a residential district such as has just been referred to, Springburn a good working-class district, and Cowcaddens a slum.

Whatever error arises in forming crude death-rates when whole populations are taken into account, it is nothing to what may arise when individual trades or professions are compared. Of this an example is given in the accompanying table (Table II), in which

TABLE II. *Showing a comparison between the health of clergymen and railwaymen, 1900–02.*

		Age Periods.			
		45–55.	*55–65.*	*65 and upwards.*	*45 and upwards.*
Clergymen	Average number living	8,453	6,886	6,310	21,649
	„ „ of deaths	83	161	521	766
	Death-rate per 1,000 .	9·82	23·43	82·62	35·37
Railway engine-drivers, &c.	Average number living	25,858	11,793	4,540	42,191
	„ „ of deaths	345	351	423	1,119
	Death-rate per 1,000 .	13·33	29·79	93·17	26·52

it is shown, comparing mortalities among railwaymen and clergymen, that the death-rates at each age period above 45 years are very much higher among railwaymen than among clergymen. Yet when the totals at all ages are taken it is found that the death-rate among clergymen is very much higher than among railwaymen. This result is obviously due to the fact, that the proportion of railwaymen living above 65 years is very much smaller than that of clergymen. The retired engine-driver, if still fit, drifts into some other employment and disappears from view, while the clergyman remains longer at duty and even if retired retains his position among the members of his profession.

In this connexion the controversy initiated in the eighties of last century by the late Sir B. W. Richardson may be recalled. He claimed that death-rates should in no district be more than eight per thousand. Such a crude death-rate may exist but it means nothing. If the population were stationary, as will presently be seen, the average life would be 125 years. I do not think the author of this dictum was convinced of his error, and though he be dead he yet speaketh. I have had the greatest difficulty in convincing members of borough councils in England that such death-rates were death-rates that could only be attained if the Creator reconsidered the principles on which he had changed the dust of the earth into our first parent.

III. Standardized Death-rates.

To meet the difficulties inherent in any method which does not correct for age and sex distribution, standardized death-rates were introduced more than forty years ago. Two methods of standardizing, termed respectively the direct and the indirect methods, are in regular employ. Both are apparently due to Dr. Ogle, and were described at first as ‘ corrected ’ death-rates. At the present moment the term in fashion is ‘ standardized ’. The indirect method is first published in the annual summary of the Registrar-General for 1883, in which the death-rates for the twenty-eight great towns of England and Wales are so calculated. Two years later, 1885, the direct method of comparison appears in the decennial supplement relating to the decade 1871–1880.

The direct method, which seems the most natural, is calculated in the following manner. A population arranged in selected age and sex groups, commonly that of the community which comprehends the whole areas to be examined, is taken as a standard population. Thus in correcting the death-rates for the towns or rural districts of England the population of England and Wales in its age and sex constitution has been taken by the Registrar-General as the standard population. Also in the case of London, where comparison is made between the individual boroughs, the population of London as a whole has been chosen by the Medical Officer of Health as a suitable criterion.

The standard population for practical purposes is arranged in groups of five years of age from birth to twenty-five years of age, and thereafter in groups of ten years. Finer divisions fail to increase the accuracy and greatly increase the labour of calculation. The death-rates for the districts examined are calculated for the same selected age-groups and for each sex separately. These death-rates are multiplied into the respective groups in the standard population. The total number of deaths found is that which would occur under the conditions descriptive of the district examined, that is, if the population in the district were distributed in the same age and sex proportion as that of the whole community (Table III (A)).

The differences produced by local variations in the age and sex of the population described in the discussion just given may be illustrated from the decade 1891–1900. The crude death-rate in Sedburgh in Yorkshire was in this decade 15·35 per thousand, whereas when standardized it was only 12·72. This is the general rule, as in rural districts there is generally a large proportion of persons living at high ages. The opposite condition obtains in towns, for example, in the Manchester Registration District the crude death-rate in the same decade was 26·4 ; standardized it became 28·3.

The indirect method of calculating a standard death-rate depends on the death-rates in the standard population for each age and sex group (Table III (B)). These rates are multiplied into the corresponding age and sex population groups of the district in which the health conditions are to be investigated. When the calculations have been made a death-rate is found equivalent to that which would obtain if the distribution of death-rates at each age group were the same as that in the standard population. The ratio to this death-rate of the death-rate in the population chosen as a standard will in some cases be greater and in some cases be less than unity. The crude death-rate of the district examined must be multiplied by this ratio. The number thus obtained is the standardized death-rate corrected in the manner described as 'indirect'.

TABLE III. *Illustrating the method of standardizing death-rates by* (a) *the direct,* (b) *the indirect method. The example chosen refers to the registration district of Liverpool, 1891–1900, and to nine rural registration districts of Yorkshire, 1891–1900.* (*Males.*)

(A)

Age Period.	*Number of males in standard population.*	*Death-rates among males in Liverpool.*	*Number of deaths obtained.*	*Death-rates among males in the nine rural districts of Yorkshire.*	*Number of deaths obtained.*
0–5	59,052	121·49	7,174	42·71	2,522
5–10	56,000	9·42	528	3·21	180
10–15	53,521	4·64	248	1·97	105
15–20	49,986	7·43	371	2·99	149
20–25	44,106	9·78	431	3·89	172
25–35	74,159	16·63	1,233	4·69	348
35–45	57,412	28·89	1,659	7·04	404
45–55	41,980	44·95	1,887	11·17	469
55–65	27,212	71·00	1,932	22·55	614
65–75	15,026	116·52	1,751	58·32	876
75+	5,603	199·47	1,118	154·09	863
Total	484,057		18,332		6,702

Standardized death-rate $= \frac{18,332}{484,057} \times 1,000 = 37·87$ as against a crude death-rate of 35·77.

Standardized death-rate $= \frac{6,702}{484,057} \times 1,000 = 13·85$ as against a crude death-rate of 15·68.

DESCRIPTION OF TABLE III (A)

In this table the method of formation of a standard death-rate is shown. In the first column the average population of all England, 1891–1900, is given for each age period. In the second column the death-rates in Liverpool at the same ages are shown. The third column is obtained by multiplying the figures in the first two columns together. Thus the population between the ages of 0–5 years, 59,052, with a death-rate of 121·49 per thousand gives in each year 7,174 deaths. In the same way 56,000 persons living between 5–10 years with a death-rate of 9·42 provide 528 deaths, and so on for each age. The figures thus obtained are summed and the sum divided by the total number in the standard population. In this way a standardized death-rate of 37·87 is obtained against a crude death-rate of 35·77. A second example is given for the rural districts of Yorkshire in columns four and five. In this case a standardized death-rate of 13·85 is found against a crude death-rate of 15·68.

(B)

Age Period.	*Death-rates among males in standard population.*	*Male population in Liverpool.*	*Number of deaths expected.*	*Male population in the nine rural districts of Yorkshire.*	*Number of deaths expected.*
0–5	62·71	8,261	518	8,049	505
5–10	4·31	7,640	33	8,063	35
10–15	2·45	7,550	18	7,778	19
15–20	3·79	7,278	28	7,366	28
20–25	5·06	7,105	36	5,528	28
25–35	6·76	12,683	86	8,938	60
35–45	11·50	10,265	118	7,496	86
45–55	18·95	7,676	145	6,072	115
55–65	34·95	4,372	153	4.434	155
65–75	70·39	1,810	127	2,987	210
75+	160·09	380	61	1,307	209
Total		75,020	1,323	68,018	1,450

Death-rate = 19·32

Death-rate found in Liverpool $= \frac{1,323}{75,020} \times 1,000 = 17·64$

Death-rate found $= \frac{1,450}{68,018} \times 1,000 = 21·33$

Correcting factor $= \frac{19·32}{17·64} = 1·095$

Correcting factor $= \frac{19·32}{21·33} = ·9058$

Standardized death-rate
= Crude death-rate × 1·095
= 35·77 × 1·095 = 39·18

Standardized death-rate
= 15·68 × ·9058 = 14·20

TABLE III (B)

The indirect method of standardizing the death-rate is shown. In the first column the death-rates at each age group for England as a whole are given, and in the second column the numbers living at each age period in Liverpool. These two columns are multiplied together severally as in the previous example and the number of deaths at each age that would occur if the population were as given in column two and the death-rate as in column one are shown in column three. When these deaths are summed and divided by the total population a death-rate of 17·64 is obtained. This would be the death-rate in Liverpool with its age constitution of population if the same death-rates held as are found for the country as a whole. The constitution of the population of Liverpool thus gives a death-rate of 17·64 as compared with that in England as a whole, which is 19·32. To obtain the standardized death-rate, the crude death-rate as found in Liverpool is thus multiplied by the factor obtained by dividing 19·32 by 17·64, or 1·095. The result is to raise the crude death-rate of Liverpool from 35·77 to 39·18. The same process is also shown for the rural districts of Yorkshire.

A comparison of crude death-rates for different conditions of life and of death-rates standardized by the two methods is given in the accompanying table (Table IV) taken from the report by Dr. Stevenson for the year 1911. It will be noted that though the difference between the crude and the standardized death-rates may be fairly large the difference between the two standardized death-rates is very small.

When, however, smaller districts are considered there may not be the same correspondence. In Table III, in which the death-rates of nine rural districts in Yorkshire and the registration district of

Liverpool are compared, the correspondence, though close, differs in the first case by 1·31 per 1,000 and in the second by 0·35 per 1,000. The numbers involved in both cases are small, so that concordance by the two methods does not necessarily follow. In fact, large populations are always necessary in such biological problems if anything approaching complete correlation is to be attained. However, if a large number of small districts are combined, closely corresponding results can be expected from the application of both methods.

Table IV. *Comparison of results of standardization by direct and indirect methods.*

	Crude death-rate per thousand population, 1911.	*Standardizing factor.*	*Standardized Death-rates.*	
			By indirect method.	*By direct method.*
England and Wales .	14·595	0·9790	14·289	14·307
London	15·152	1·0000	15·152	15·254
County Boroughs .	16·113	1·0263	16·537	16·606
Other Urban Districts .	14·035	0·9944	13·956	13·996
Rural Districts . .	13·083	0·8882	11·620	11·390

IV. Life Table Death-rates.

The life table death-rate is the third measure in use. For reasons which follow from the assumptions on which life tables are constructed this measure must be taken as the most adequate at our disposal. It is assumed that an equal number of persons are born in each year and that the subsequent history of these persons is recorded year by year till all have died, uniform conditions as regards environment being postulated. As every one ultimately dies, the number of deaths in any one year is equal to the number of births, or in a very important extension of this statement, the number of persons dying above any individual age, in each year, is equal to the number attaining that age during the same period.

This ideal in practice is only approximately attained. If a population could be observed in which the same diseases were present in the same amount year by year, and if the births were regularly equal to the deaths and if there was no migration it might be possible, possessing statistics of such a community for a series of years, to deduce direct conclusions. But the conditions under which statistics can be collected never have presented such favourable features. Thus environment varies from time to time according to the zeal of the administrator, and during the present generation there has been on the whole an immense improvement in hygiene. Persons born in one environment have lived their lives in constantly changing circumstances and consequently the data collected do not necessarily describe the effect of the conditions present at the epoch of collection, but the sum of the effects of a changing complex. In practice, however, such data alone are at our disposal.

The term life table death-rate, as used in the succeeding pages, must be strictly defined. It is taken to signify the ratio of the number of deaths of persons above any defined age to the number living above that age in a stationary population. If the whole population be considered it refers to all persons from birth and upwards. It is easy to see that the healthier the district, correspondingly fewer deaths occur at each of the earlier ages of life, and consequently a small death-rate implies a relatively larger population at high ages. The converse of this is that in an unhealthy district the number of persons reaching high ages is small. It is immediately obvious that the number of the younger individuals in such a population bears a higher ratio to the number of the elderly than in the previous case. The difficulties introduced by standard populations where the proportion of the young on account of the high birth rates in last century in general exceeds the proper proportion of the old is thus avoided. Had the construction of a life table been easy no one would have used a standardized death-rate. The latter can at best be described as no more than a *pis aller*.

A life table death-rate has, however, other properties. It is obvious that the average number of years of life after birth or after any age, known technically as the expectation of life, will be the reciprocal of the life table death-rate. Death-rates, life table or other, are usually defined by a number per 1,000, hence the average life or the expectation of life above any age will be given by dividing the number 1,000 by the life table death-rate. The lower the life table death-rate therefore the greater the mean age at which the individuals die. To return to Sir B. W. Richardson: his death-rate of 8 per thousand means that those born reach an average age at death of one hundred and twenty-five years. That such a demand suggests the superman seems sufficiently evident.

The definition of a life table death-rate as given above requires some comment. The definition is true even if the actual distribution of deaths in age periods be assumed to vary according to the most fantastic rules. In a population distributed according to experience the average age at death in many cases is about fifty years. This average age, however, is equally easily attained if half the persons born die at birth and the other half live to the age of 100 years. No one, however, could take a life table death-rate as a criterion of comparison did such distributions of deaths as that just suggested occur. What is actually found is that certain limited variations about a norm are possible and that these variations obey very definite laws. The chief life tables used in the discussion are given in Table V.

This section cannot close without some comparison between standardized death-rates and life table death-rates to illustrate one specific disadvantage of using standardized death-rates. This is best seen by taking specific examples (Table V). The standardized death-rate of the males in the healthy district life table (H_3) is observed to be 13·49. The life table death-rate is 18·91. A death-rate of 13·49 gives, however, on a stationary population a mean life of $\frac{1,000}{13\cdot49}$ or 74 years, a figure hardly conceivable if the observed properties of life represent anything which is fundamental. Manchester (M_1), on the other hand, has a corrected death-rate of 27·94 per 1,000. The expectation of life at birth is 34·71 years, giving a death-rate of 28·81 on a stationary population. In other words, the variation of the standardized death-rate is from 13·5 to 27·94, the extremes being in the ratio of nearly one to two. The corresponding range of

TABLE V. *Showing the districts and dates of the life tables used in the discussion with symbols of reference and with the standardized and life table death-rates in each district for comparison.*

No.	District		Author	Dates	Symbol	Standardized death-rates. Males.	Standardized death-rates. Females.	Life table death-rates. Males.	Life table death-rates. Females.
B 1	England	. . . . .	Farr	1838–1851	E_3	22·37	21·14	25·06	23·90
B 2	”	Healthy Districts	”	1849–1851	H_1	16·03	15·95	20·59	20·22
B 3	England	. . . . .	Ogle	1871–1880	E_4	21·64	19·40	24·18	22·41
B 4	England	. . . . .	Tatham	1881–1890	E_5	19·79	17·74	22·90	21·19
B 5	”	Healthy Districts	”	” ”	H_2	14·26	13·40	19·42	18·50
B 6	Brighton	. . . . .	Newsholme	” ”	B_1	19·70	16·05	22·94	20·41
B 7	Manchester	. . . .	Tatham	” ”	M_1	27·94	24·46	28·81	26·02
8	”	Township . .	”	” ”	M_2	35·90	30·36	34·75	30·61
9	”	Outlying Districts	”	” ”	M_3	24·85	22·21	26·24	24·05
10	Oldham	. . . . .	Tattersall	” ”	O	27·91	24·27	27·12	24·54
B*11	England	. . . . .	Tatham	1891–1900	E_6	19·32	17·14	22·66	20·93
B*12	”	Healthy Districts	”	” ”	H_3	13·49	12·49	18·91	17·95
B*13	London	. . . . .	Murphy	” ”	L_1	21·82	18·70	24·40	22·06
14	Brighton	. . . . .	Newsholme	” ”	B_2	18·62	15·43	22·26	19·93
15	Salford	. . . . .	Tattersall	” ”	S_1	27·42	24·63	30·03	27·17
*16	England	. . . . .	King	1901–1910	E_7	16·35	14·32	20·61	19·09
*17	London	. . . . .	Murphy	” ”	L_2	17·42	14·70	21·39	19·45
18	Salford	. . . . .	Tattersall	” ”	S_2	21·54	19·19	24·83	22·42
*19	England	. . . . .	King	1910–1912	E_8	14·55	12·70	19·42	18·07
*20	London	. . . . .	”	1911–1912	L_3	Information not given.			
*21	County Boroughs	. .	”	” ”	C				
*22	Urban Districts	. . .	”	” ”	U				
*23	Rural Districts	. . .	”	” ”	R				

Index for Easy Reference.

Symbol.	No.	Symbol.	No.	Symbol.	No.	Symbol.	No.
B_1	6	E_6	11	L_1	13	O	10
B_2	14	E_7	16	L_2	17	R	23
C	21	E_8	19	L_3	20	S_1	15
E_3	1	H_1	2	M_1	7	S_2	18
E_4	3	H_2	5	M_2	8	U	22
E_5	4	H_3	12	M_3	9		

NOTE.—The tables marked with an asterisk are those used by Mr. Finch and myself for the calculation of the expectation of life. Those marked B are tables used in my first investigation (see Sect. VII).

the life table death-rates is 18·9 to 28·8, a difference very much less. As will be seen later, there is good reason for considering the second measure as much the most useful. In both these cases the population has a higher birth-rate than death-rate. If the converse be the case, the life table death-rate is lower than the standardized death-rate. Thus in the Liverpool registration district 1891–1900 the life table death-rate is 35·3 as against a standardized death-rate of 37·9 per thousand. The standardized death-rate as an independent measure of health is thus open to some objection except in the hands of those who know its limitations.

V. Methods suggested by Farr and Pearson.

There is, however, another method of approaching the problem used so far back as 1875 by Dr. Farr and that is to calculate the actual number of deaths due to disease in stationary populations. This assumes that life as represented by a large corpus of individuals obeys laws which show their existence by calculable responses to different hygienic circumstances. In the supplement for the decade 1861–1870 Dr. Farr gives a series of tables showing how persons living in certain different environments would die. This information is given for the whole of England and Wales ; for the healthy districts of England and Wales, and also for the Liverpool district. A specimen of his work is given in the accompanying table (Table VI). This method presents great advantages. The difficulty of the application of the method, a difficulty already alluded to, is that the necessary tables required for the discrimination of the different types of response to disease at different ages and in different environments had not been constructed. Now, however, that many life tables have been calculated and the process of making rough life tables has been so much improved, it is quite certain that the use of this method will supply in many cases the only figures on which it is safe to found a theory of mortalities. For instance, it will be possible to eliminate to a great extent the difficulties which arise on account of the different rates at which individuals grow old in town and country, differences which determine great alterations in the age incidence of disease. It is little use to apply statistical tests to decide whether differences in the age incidence of a disease in different environments are real or not. What is important is to settle whether such differences in age incidence imply that a disease is present in a greater or lesser amount.

The same salient of attack nearly thirty years ago appealed to Professor Karl Pearson, though his method of considering the problem at that moment was very different. Professor Pearson analysed the distribution of the deaths on the theory that special groups of deaths properly belong to corresponding periods of life. Thus some diseases are specially characteristic of infancy, some of childhood, some of adolescence, some of middle life, and some of old age. These groups overlap. In this way he describes in close accord with the facts how every one is subject to a series of specific age group risks. Professor Pearson's method introduced a new conception, but it must be criticized in that the analysis used by him referred only to the types of curves he thought fundamental. These curves can be shown to be insufficient to describe the known processes of life. Though the method fails in this regard, yet when the necessary clearing of the ground has been accomplished and the lines on which it may be developed become better defined, and in addition when some agreement has been established as to what constitute age group risks of life, the mechanism which underlies the changes in the number of deaths in different environments at each age period will come within the range of competent mathematical analysis.

Professor Pearson's diagram (Diagram I) is rather inaccessible. It is reproduced here on a somewhat different scale, which I think illustrates the theory more clearly than the original diagram.

There is, however, one other measure derivable directly from Farr's conception which requires to be considered. It might be proposed to estimate how many persons living at a definite age in a given environment may be expected to die of some specific disease during their future life. Thus we might compare different districts by stating the percentage of persons reaching the age of 15 years who afterwards die of any disease. An example of this method of estimating the amount of disease in relation to cancer will be given later, but as yet none of the necessary spade work has been done, and until the possibilities of this criterion have been explored its use will be quite unsafe.

VI. Comparison of Death-rates at Different Ages.

The comparison of death-rates at different ages was first introduced by Dr. Farr in his article on 'Vital Statistics' in McCulloch's *Statistical Survey of the British Empire,* and much further developed in the Decennial Supplement relating to the years 1861–1870.

In this supplement Dr. Farr collected the registration districts into seven groups, starting with 53 healthy districts in which the crude death-rate ranged from 15 to 17 per 1,000 and ending up with the Liverpool district in which the crude death-rate was 39 per 1,000. For each of these groups of districts he gives full details of the populations and deaths. Choosing the group of 53 districts with the lowest mortalities as the standard, he assumed that at each age period the death-rates in the first group of districts should be represented by 100 and with such a basis he calculates the relative mortalities in each several district in relation to the mortalities in the healthy districts. It is now recognized that in a matter like this it is much better to keep the sexes separate. While, therefore, Dr. Farr's figures are reproduced, the figures for the two sexes calculated from the original data are shown separately (Table VII).

The criticism of the results divides itself into two parts. The first concerns the ages from birth to 15 years, the second from 15 years and upwards. The first division will be more fully considered later, but one point must be noted. Under 5 years of age females are more affected by unhealthy surroundings than males, though from 5 years and upwards the converse phenomenon is observed.

Starting at the age of 15 years, it is to be observed that among males the maximum effect produced by unhealthy conditions occurs between the ages of 45 and 55 years, the mortality increasing with

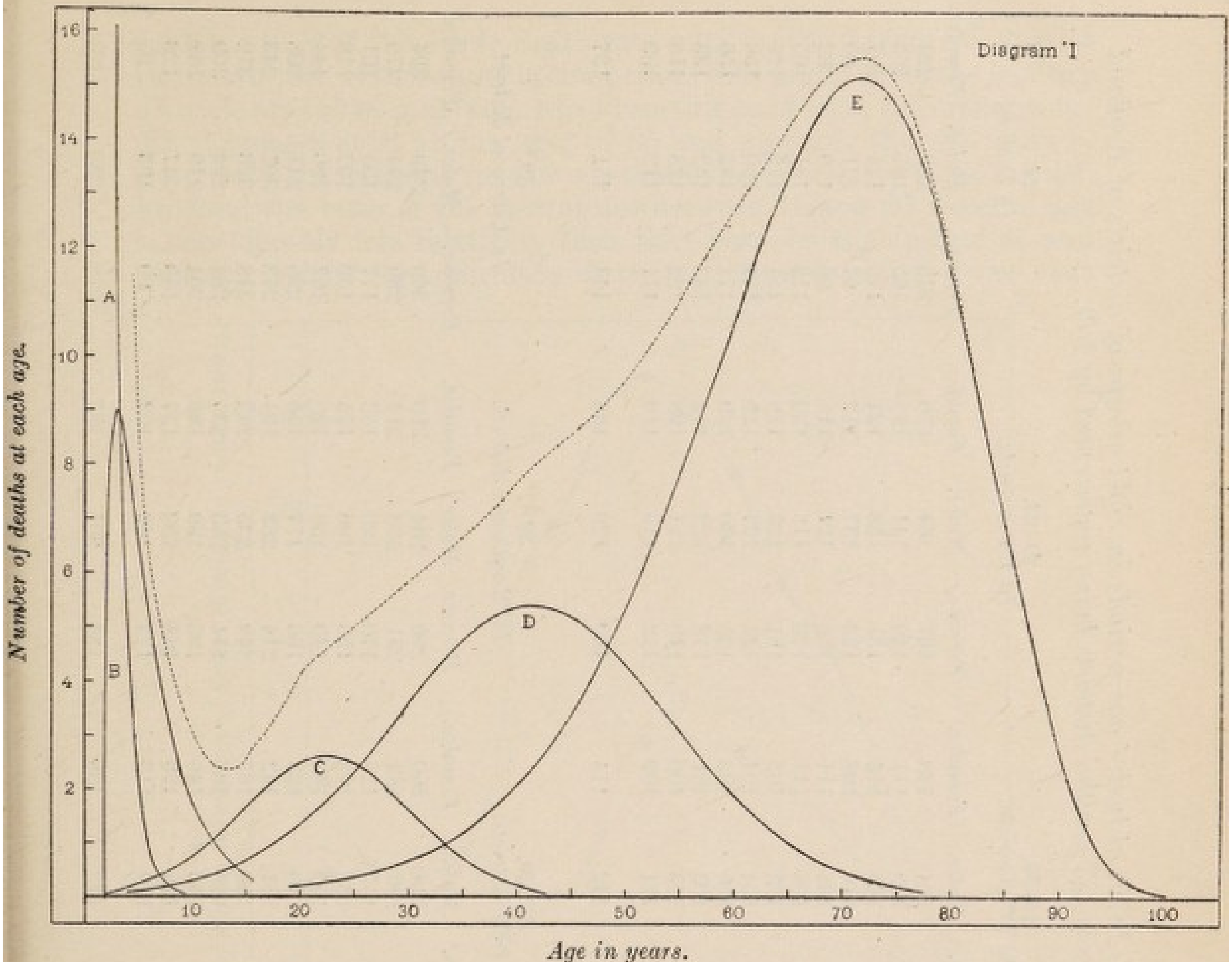
TABLE VI. *Table of mortality derived from the English life table, showing of what diseases and at what ages 1,000,000 liveborn children may be expected to die.*

The Table shows also, according to the English Life Table, the Annual Births being 1,000,000; the Annual Deaths 1,000,000; the Deaths at the respective Ages and of the respective Causes out of a Population of 40,858,184, enjoying a Mean Life-time of 40·858 years.

AGES	*All Ages.*	*0–*	*5–*	*10–*	*15–*	*20–*	*25–*	*35–*	*45–*	*55–*	*65–*	*75–*	*85 and upwards.*
DEATHS FROM ALL CAUSES	1,000,000	263,182	34,309	17,946	21,813	28,705	62,052	69,078	81,800	112,086	147,905	122,559	38,565
TOTAL ZYMOTIC DISEASES. Order 1	175,619	87,099	19,256	6,555	4,717	4,554	7,918	7,616	8,101	9.795	11,256	7,229	1,523
ZYMOTIC DISEASES. Order 1:													
1. Small-pox	6,521	3,331	833	244	291	456	624	347	210	113	53	16	3
2. Measles	12,865	11,507	1,080	127	39	29	41	23	11	5	2	1	—
3. Scarlatina	30,021	17,959	8,743	1,901	493	283	330	162	80	45	19	6	—
4. Diphtheria	4,945	2,425	1,364	464	165	100	125	91	74	68	47	19	3
5. Whooping-cough	15,161	14,424	682	35	6	2	4	2	2	2	2	—	—
6. Typhus	38,107	5,401	4,036	2,842	2,907	2,696	4,197	3,777	3,749	3,822	3,233	1,287	160
7. Diarrhoea and Dysentery	34,366	20,344	427	154	141	244	664	776	1,115	2,162	3,883	3,551	905
8. Cholera	6,155	1,129	382	173	134	215	640	781	829	865	688	281	38
9. Other Zymotic Diseases. Order 1	27,478	10,579	1,709	615	541	529	1,293	1,657	2,031	2,713	3.329	2,068	414
FIVE CONSTITUTIONAL DISEASES:													
10. Cancer	21,311	71	35	31	58	100	682	2,290	4,583	5,998	5,122	2,040	301
11. Scrofula and Tabes	14,106	8,115	1,100	811	633	541	815	576	542	511	361	93	8
12. Phthisis	114,417	4,469	2,139	3,526	9,074	13,785	27,134	22,404	16,468	10,445	4,294	627	52
13. Hydrocephalus	11,252	9,296	1,363	362	84	32	36	24	20	18	12	5	—

DESCRIPTION OF TABLE VI

This Table is a portion selected from a series of tables given by Dr. Farr in the Supplement of the Registrar-General for the decade 1861–1870, to illustrate his method of comparing different districts by following from birth to death a definite number of individuals and showing of what diseases they may be expected to die. A development of this method seems to be the only way to obtain accurate information with regard to the amount of some diseases.



This diagram is a re-drawing of that given by Professor Pearson in the *Transactions of the Royal Society* in 1894. The dotted curve shows the number of deaths at each age as given by the life table for England and Wales for the decade 1881–1890. The curves in continuous line represent the distribution of the deaths as suggested by Professor Pearson. The first curve (A) describes the mortality due to infantile diseases. The commencement is not shown, on account of the scale of the diagram. The great bulk of deaths in this group occur before the age of two years, and it ceases to be a factor of any importance by the age of nine years. The second curve (B) shows the mortality due to diseases of childhood; the maximum mortality in this group is at the age of three years: the diseases comprehended in this group cease to be of any importance by the age of fifteen years. The third curve (C) shows the mortality due to diseases of youth: the maximum number of deaths occur here at the age of twenty-two and a half years. The mortality due to diseases of middle life (D) has its maximum at forty-one and a half years, and the mortality due to diseases of old age (E) at sixty-seven years. When deaths in these different groups are added together the sums approach with extreme closeness the observations.

The equations of the curves are as follows:

A.	Infantile Mortality.	$y = 236(x + \cdot 75)e^{-\cdot 75x}$	origin at birth.
B.	Mortality of Childhood.	$y = 9(1 + x)^{\cdot 3271}e^{-\cdot 3271x}$	origin at 3 years.
C.	Mortality of Youth.	$y = 2\cdot 6e^{\frac{-x^2}{121\cdot 68}}$	origin 22·5 years
D.	Mortality of Middle Age.	$y = 5\cdot 4e^{\frac{-x^2}{327\cdot 68}}$	origin 41·5 years.
E.	Mortality of Old Age.	$y = 15\cdot 2(1 - \frac{x}{35})^{7\cdot 7525}e^{\cdot 2215x}$	origin 71·5 years.

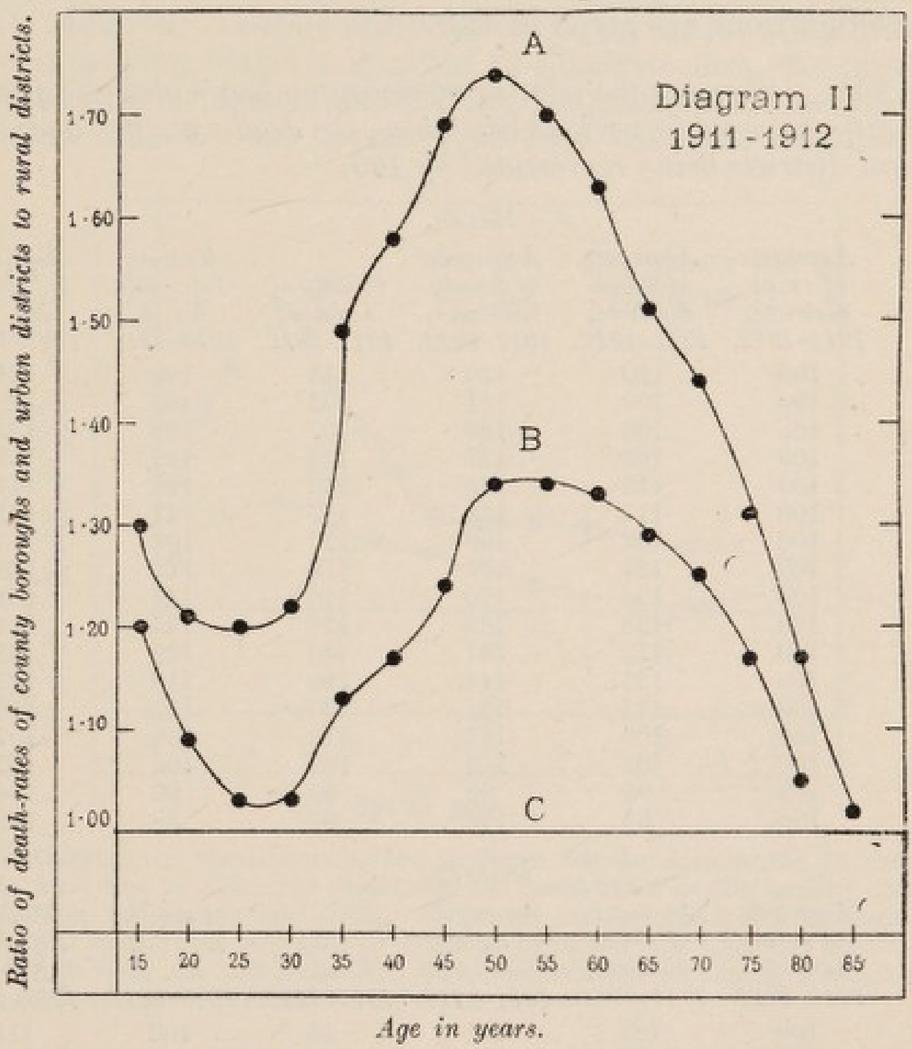
where y is the number of deaths and x the distance from the origin measured in years.

TABLE VII. *Showing the relative mortality, 1861–1870, at twelve age periods in eight groups of districts of England and Wales, the deaths at each age in 53 healthy districts being represented by 100.*

Mortality range:	*15–17. In 53 healthy districts.*			*18–20. In 345 districts.*			*21–23. In 137 districts.*			*24. In London.*		
Age period.	*Persons.*	*Males.*	*Females.*	*Persons.*	*Males.*	*Females.*	*Persons.*	*Males.*	*Females.*	*Persons.*	*Males.*	*Females.*
0–5	100	100	100	126	124	128	167	164	170	216	210	224
5–10	100	100	100	110	109	111	140	141	139	165	167	163
10–15	100	100	100	111	109	112	131	138	124	115	124	107
15–20	100	100	100	108	110	108	123	137	113	97	121	80
20–25	100	100	100	109	108	111	118	121	116	95	110	82
25–35	100	100	100	109	106	111	120	118	123	118	131	105
35–45	100	100	100	104	103	105	121	123	119	151	172	129
45–55	100	100	100	104	104	104	123	127	119	174	190	156
55–65	100	100	100	105	106	103	125	130	120	171	185	156
65–75	100	100	100	102	103	101	118	122	114	143	152	133
75–85	100	100	100	102	103	100	110	111	108	122	128	116
85 +	100	100	100	110	111	109	112	113	110	111	112	109
All ages	100	100	100	113	112	114	129	130	128	144	151	137

Mortality range:	*24–26. In 47 districts.*			*27–30. In 9 districts.*			*32. In Manchester district.*			*39. In Liverpool district.*		
Age period.	*Persons.*	*Males.*	*Females.*	*Persons.*	*Males.*	*Females.*	*Persons.*	*Males.*	*Females.*	*Persons.*	*Males.*	*Females.*
0–5	217	212	223	251	244	260	296	284	310	369	349	394
5–10	166	168	164	189	196	183	231	250	212	286	294	279
10–15	141	158	125	140	157	125	170	190	152	177	195	161
15–20	127	150	111	142	187	108	150	196	115	152	197	118
20–25	116	117	115	124	139	109	140	156	125	167	190	144
25–35	120	117	123	141	159	123	166	177	156	225	241	208
35–45	132	137	127	162	181	142	214	233	194	272	302	243
45–55	146	152	139	184	199	167	249	255	242	318	323	312
55–65	154	161	146	177	186	167	244	253	235	284	299	269
65–75	138	144	132	155	163	146	193	199	186	204	219	189
75–85	124	129	119	132	138	126	148	159	137	139	147	129
85 +	116	120	112	116	119	112	123	142	102	99	103	95
All ages	146	149	144	165	172	158	194	201	187	228	233	223

each increase of the crude death-rate until in the Liverpool district the death-rate at these ages is three times that found when the healthy districts are taken as a standard. From this age period the ratio gradually decreases until at the ages of 85 and upwards the mortality is essentially the same no matter whether town or country be examined. Further, the ratio of the mortalities between 15 and 20 years of age is considerably less relatively than that between the ages of 45 and 55 years, yet in the unhealthy districts it rises to nearly twice that



In this diagram the mortalities of the rural, urban, and county death-rates at ages 15 and upwards are compared for the years 1911–1912. The death-rate at each age period in the rural districts has been taken as the standard and is denoted by the line marked unity. The curve (B) represents the ratio of the death-rates in the urban districts to that of the rural districts. Thus at the age period 50–55 years the reading 1·34 from the scale indicates that the death-rate at these ages in the urban districts is 1·34 times that in the rural. The curve (A) represents the same phenomena for the county boroughs. The death-rate at the same age period is found to be 1·74 times or nearly twice what it is in the rural districts.

in the healthy. Females considered separately show some differences : unlike males it is found that at the age of 15–20 years there is little difference between town and country. The relative mortality progressively increases until between 45 and 55 years the difference between the healthy and the unhealthy districts is as one to three. From this age-period the ratio regularly declines till it becomes, within the limits of random error, unity at the ages of 85 years and upwards.

These figures refer to data collected more than fifty years ago. A corresponding series of ratios, however, calculated from the figures given in the supplement of life tables constructed by Mr. King and issued by the Registrar-General, demonstrate that the same conditions still hold. In this case the life table death-rates for the rural districts selected by Mr. King have been taken as the norm of life in healthy conditions. The age periods illustrated range from 15 to 95 years. The results are given in the accompanying table (Table VIII) and in part illustrated in the diagram (Diagram II). Other comparisons are given in Table IX.

TABLE VIII. *Showing the relative mortality among males and females at different ages in various life tables, the deaths at each age in the rural districts being represented by 100.*

MALES.

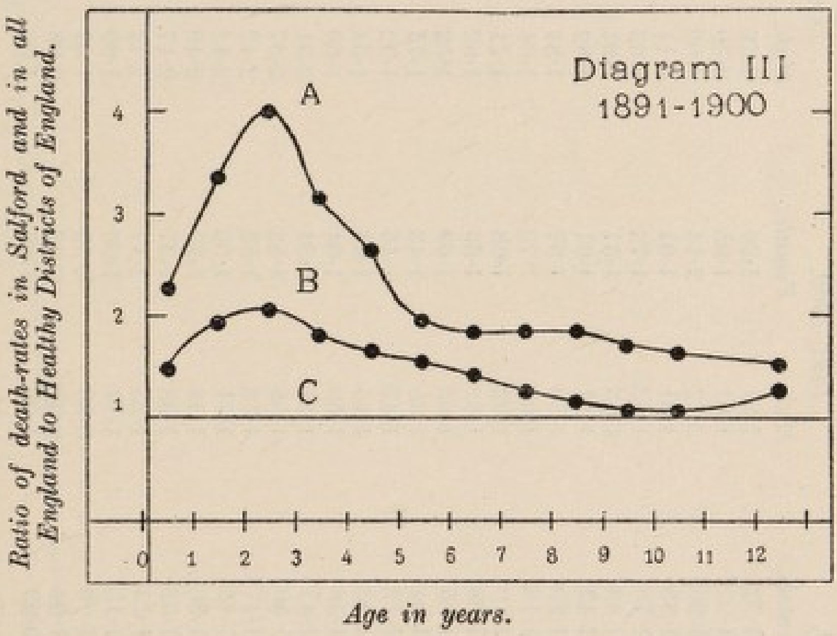
Age.	*Aggregate of rural districts, 1911–1912.*	*Aggregate of urban districts, 1911–1912.*	*Aggregate of county boroughs, 1911–1912.*	*County of London, 1911–1912.*	*English Life table No. 8, 1910–1912.*	*English Life table No. 7, 1901–1910.*
15	100	120	130	113	109	121
20	100	109	121	103	112	122
25	100	103	120	107	108	122
30	100	103	122	124	112	132
35	100	113	149	150	126	147
40	100	117	158	167	131	151
45	100	124	169	177	138	155
50	100	134	174	178	141	158
55	100	134	170	170	141	154
60	100	133	163	157	135	144
65	100	129	151	143	129	135
70	100	125	144	130	119	123
75	100	117	131	120	112	116
80	100	106	117	113	108	107
85	100	102	102	102	102	102
90	100	95	99	97	97	104
95	100	84	100	96	88	103

FEMALES.

Age.	*Aggregate of rural districts, 1911–1912.*	*Aggregate of urban districts, 1911–1912.*	*Aggregate of county boroughs, 1911–1912.*	*County of London, 1911–1912.*	*English Life table No. 8, 1910–1912.*	*English Life table No. 7, 1901–1910.*
15	100	103	115	95	103	111
20	100	93	106	80	96	106
25	100	92	103	83	96	109
30	100	93	104	90	99	116
35	100	101	123	109	108	128
40	100	110	139	132	120	139
45	100	123	152	151	130	147
50	100	122	150	145	128	142
55	100	117	144	135	123	137
60	100	123	143	129	122	134
65	100	116	138	122	120	127
70	100	117	133	115	114	123
75	100	113	125	107	108	115
80	100	108	118	107	108	108
85	100	104	110	107	102	107
90	100	94	101	98	96	104
95	100	102	98	104	99	105

It will be noticed that the maximum ratios between the death-rates occur about 45 to 50 years of age, and that though in different districts there are slight differences in the curve, essentially the same phenomena are found throughout. It is on individuals at middle age that environment tells most severely. The reactions to the surroundings at different ages are, however, very closely correlated, as will be shown in Section X.

The behaviour of life in different environments under 15 years of age also obeys laws of its own. Here quinquennial groups are no longer sufficient. The data must be examined from year to year; while in the earliest period of life, differentiation from month to month is required. Dealing first with the larger aspect of the matter, the accompanying diagram is given to illustrate how environment affects life at these ages (Table IX and Diagram III). The healthy district life table for the decade 1891–1900 has been taken as the



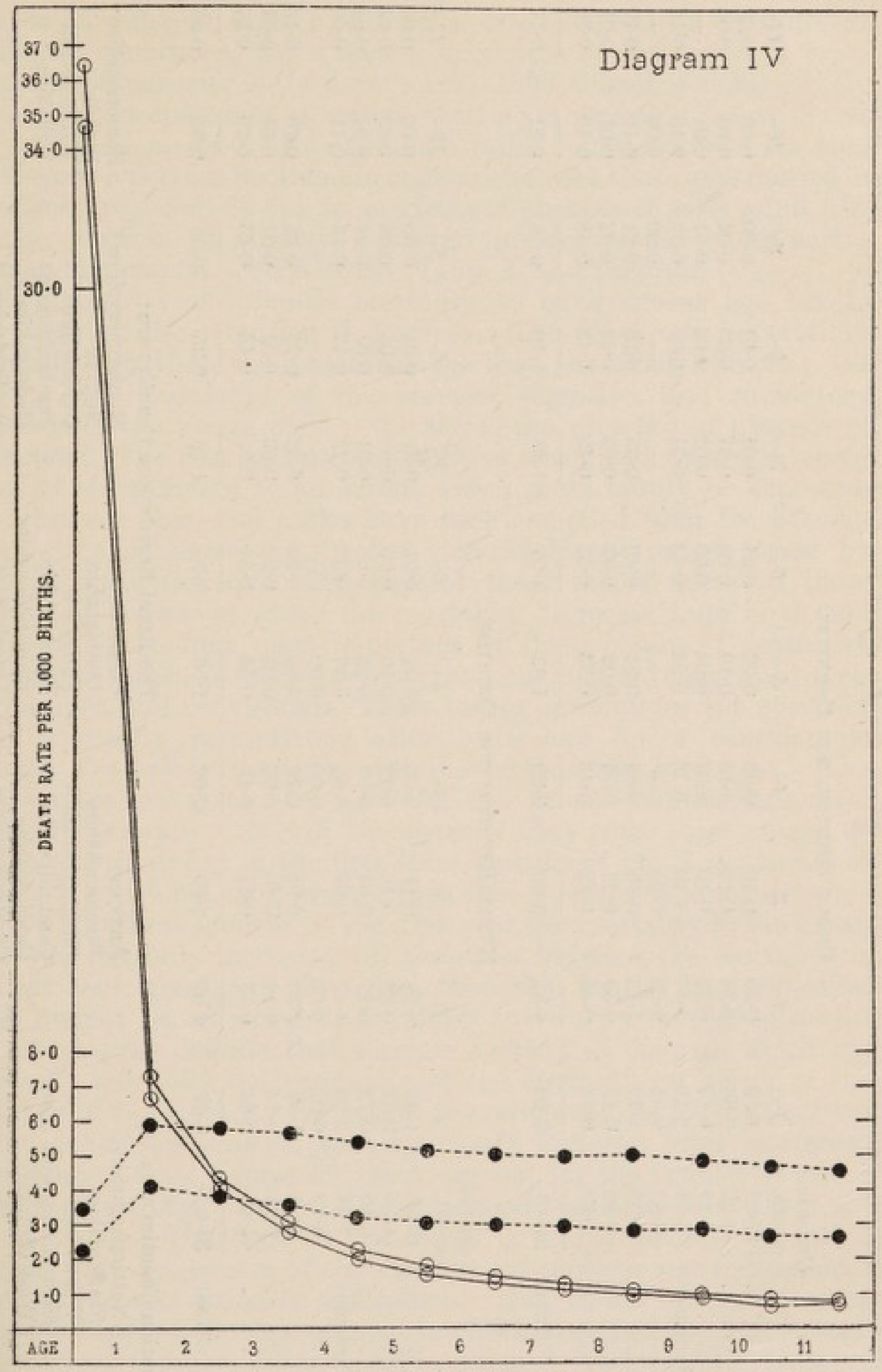
In this diagram the same phenomenon is shown for the ages under 15 years of age. The base line in this case illustrates the death-rates in the healthy districts of England for the decade 1891–1900. The curve marked (B) is the comparison of England as a whole with the healthy districts, and the curve (A) the same comparison for Salford. It will be noticed that it is between the ages of 2 and 3 years that the greatest amount of relative injury is done.

standard since Mr. King does not give any information in his rural district life table for the years 1910–1912 for ages below that of 12 years; the first death-rate has been taken as 100 as in the case of adult life. From the comparison it appears that child life is unequally affected by the environment. Though the large infantile mortality in cities impresses on account of its size it is not at this age that the chief relative damage is to be seen. Environment acts progressively more and more unfavourably until the age of 2 years is reached, as may be medically verified by those who see large numbers of young children drawn from slum life. In all the curves of comparison, both those shown in the diagram and those not shown, the same phenomenon is observable. The ratio rises from birth to 2 years of age and then falls until in the neighbourhood of the age of 5 years a ratio is reached approximately equivalent to that experienced

TABLE IX. *Showing the relations of death-rates at different ages in different districts to those in the healthy districts of England, 1891–1900.*

	Death-rates at different ages.		*Ratio of death-rates in several life tables to those of H_3.*							
	H_3. 1891–1900.		E_{6} 1891–1900.		Scotland. 1891–1900.		Salford. 1891–1900.		Manchester. 1881–1890.	
Age.	*Male.*	*Female.*	*Male.*	*Female.*	*Male.*	*Female.*	*Male.*	*Female.*	*Male.*	*Female.*
0	132·074	101·327	1·46	1·52	1·21	1·26	2·29	2·42	1·66	1·73
1	28·500	26·421	1·92	1·92	1·91	1·95	3·39	3·59	3·26	3·11
2	10·100	10·355	2·08	1·96	2·22	2·14	4·00	3·67	3·72	3·50
3	7·386	7·093	1·80	1·89	1·81	1·91	3·11	3·14	3·31	3·28
4	5·787	5·657	1·68	1·70	1·62	1·72	2·63	3·21	2·95	3·08
5	4·551	4·489	1·57	1·58	1·45	1·51	1·94	1·76	2·86	3·06
6	3·680	3·662	1·42	1·43	1·51	1·61	1·82	1·92	3·01	3·17
7	2·999	3·037	1·28	1·29	1·59	1·67	1·84	1·99	3·16	3·23
8	2·504	2·588	1·17	1·18	1·61	1·73	1·84	1·93	3·22	3·21
9	2·165	2·300	1·10	1·10	1·62	1·68	1·71	1·83	3·21	3·06
10	1·966	2·139	1·09	1·08	1·62	1·69	1·62	1·64	3·08	2·83
15	2·376	2·913	1·29	1·05	1·63	1·60	1·52	1·24	1·79	1·36
20	3·793	3·981	1·21	1·04	1·65	1·43	1·74	1·13	1·46	1·27
25	4·939	4·491	1·15	1·12	1·47	1·46	1·52	1·25	1·69	1·66
30	5·286	5·021	1·27	1·23	1·45	1·63	1·54	1·58	2·23	2·03
35	6·261	5·757	1·44	1·36	1·50	1·64	1·59	1·90	2·50	2·22
40	7·569	6·823	1·58	1·47	1·54	1·54	2·51	2·14	2·65	2·26
45	9·323	7·834	1·60	1·50	1·58	1·52	2·67	2·41	2·70	2·45
50	12·545	10·215	1·56	1·47	1·52	1·53	2·42	2·39	2·53	2·47
55	16·551	14·108	1·57	1·45	1·71	1·50	2·42	2·41	2·53	2·42
60	24·946	21·894	1·47	1·36	1·48	1·35	2·49	2·27	2·30	2·16
65	36·387	32·355	1·40	1·32	1·38	1·37	2·17	2·04	2·20	2·12
70	58·545	52·270	1·28	1·23	1·23	1·17	1·72	1·80	1·90	1·82
75	95·095	85·099	1·17	1·15	1·17	1·05	1·99	1·98	1·60	1·56
80	151·442	134·717	1·09	1·09	0·96	0·99	1·07	0·96	1·36	1·32
85	234·018	206·615	1·02	1·04	0·95	0·99	0·98	1·52	1·18	1·13
90	349·364	306·353	0·98	0·99	1·08	1·06	1·06	0·78	1·06	0·96
95	503·268	439·205	—	—	—	—	—	—	—	—
100	694·444	608·696	—	—	—	—	—	—	—	—

The death-rates in the first column are obtained by dividing the number of deaths occurring at each year of age by the mean population living in that year.



In this diagram two sets of curves are given. The first in continuous line shows the curve of the death-rate from those diseases which are specially due to instability—premature birth, convulsions, &c., and the second in dotted line to the diseases which are due to infection. It will be observed that in the first class the phenomenon observed in the town and in the country differ very slightly. The relation of the deaths from infectious diseases in the two classes of districts are, however, very different, the figures for the town being 60 per cent. on the whole more than those of the country.

TABLE X. *Showing the mortality per thousand living births of children during the first twelve months of life in the urban and rural registration counties from the diseases which chiefly affect the first few months of life, 1906–1910.*

URBAN REGISTRATION COUNTIES.

	Under one month.	*1–*	*2–*	*3–*	*4–*	*5–*	*6–*	*7–*	*8–*	*9–*	*10–*	*11–*
Premature Birth . . .	17·80	1·29	0·36	0·10	0·05	0·03	0·01	0·01	0·01	0·00	0·00	0·00
Congenital Defects . .	4·98	0·68	0·36	0·19	0·15	0·10	0·08	0·07	0·05	0·04	0·04	0·04
Starvation	0·15	0·17	0·13	0·10	0·07	0·04	0·03	0·02	0·02	0·01	0·01	—
Atrophy, Debility, Marasmus	6·38	2·26	1·57	1·16	0·76	0·54	0·42	0·31	0·27	0·22	0·16	0·16
Injury at Birth	0·89	0·01	0·00	0·00	—	—	—	—	—	0·00	0·00	—
Convulsions	4·25	1·73	1·18	0·90	0·68	0·51	0·40	0·35	0·25	0·21	0·19	0·17
Suffocation	0·58	0·47	0·35	0·22	0·15	0·08	0·05	0·03	0·02	0·02	0·01	0·01
Other Causes	1·40	0·68	0·42	0·42	0·43	0·53	0·52	0·49	0·48	0·48	0·40	0·39
Total	36·43	7·29	4·37	3·09	2·29	1·83	1·51	1·28	1·10	0·98	0·81	0·77

RURAL REGISTRATION COUNTIES.

	Under one month.	*1–*	*2–*	*3–*	*4–*	*5–*	*6–*	*7–*	*8–*	*9–*	*10–*	*11–*
Premature Birth . . .	16·53	1·07	0·29	0·11	0·06	0·01	0·01	0·01	0·01	0·00	0·00	0·00
Congenital Defects . .	4·19	0·63	0·29	0·18	0·12	0·10	0·09	0·08	0·05	0·05	0·03	0·05
Starvation	0·10	0·13	0·11	0·04	0·03	0·03	0·02	0·01	0·01	0·01	0·01	0·01
Atrophy, Debility, Marasmus	7·31	2·25	1·42	0·99	0·55	0·37	0·32	0·22	0·17	0·16	0·11	0·09
Injury at Birth	0·78	0·01	0·00	—	0·00	0·00	—	—	—	—	—	—
Convulsions	4·04	1·59	1·28	0·91	0·70	0·48	0·39	0·32	0·25	0·22	0·20	0·16
Suffocation	0·24	0·14	0·14	0·10	0·07	0·03	0·02	0·02	0·01	0·01	0·01	0·01
Other Causes	1·43	0·80	0·53	0·47	0·49	0·54	0·51	0·53	0·45	0·43	0·29	0·34
Total	34·62	6·62	4·06	2·80	2·02	1·56	1·36	1·19	0·95	0·88	0·65	0·66

The Urban Registration Counties are :

Glamorganshire
Lancashire
London
Middlesex
Monmouthshire
Northumberland
Nottinghamshire
Staffordshire
Warwickshire
East Riding with York
West Riding.

The Rural Registration Counties are :

Anglesey
Brecknockshire
Buckinghamshire
Cambridgeshire
Cardiganshire
Carmarthenshire
Cornwall
Carnarvonshire
Denbighshire
Flintshire
Herefordshire
Huntingdonshire
Lincolnshire
Merionethshire
Montgomeryshire
Norfolk
Oxfordshire
Pembrokeshire
Radnorshire
Rutlandshire
Shropshire
Somersetshire
Suffolk
Westmorland
Wiltshire.

under the age of one year. The actual minimum occurs in different places at different ages, a difference often depending on different epidemic conditions, but a second maximum at the age of 8 years found in Manchester in the decade 1881–1890, though possibly largely due to severe epidemics of scarlet fever, is yet probably due to defect in statistical method, while the high ratios found between the ages of 10 and 15 years in Scotland in comparison with those experienced in England are definitely due to an excess of phthisis of early adult life.

The problem of infancy, however, presents some other points demanding careful consideration (Table X and Diagram IV).

The relation of infantile mortality to environment has hardly yet received the attention it deserves. The death-rate is certainly dependent on three main factors: the first, the shock of birth; the second, the instability of the nervous, digestive, and circulatory systems of the young child; the third, the presence or absence of infection. The first and second of these two divisions are independent of environment to an extent which could hardly be expected. To illustrate this, two tables have been compiled from Dr. Stevenson's data contained in the last decennial supplement issued by the Registrar-General. The first of these tables refers to those groups of disease in which the mortality decreases from birth and upwards. The four most important of these causes of death are premature birth, atrophy, debility and marasmus, congenital malformations and convulsions. These causes account for the specially high mortality immediately after birth and for a considerable though decreasing mortality till the end of the first year. The death-rates are given per 1,000 for the urban counties and rural counties severally. It will be observed that from these causes of death the mortality in the first three months of life is in the urban counties only 6 per cent. greater than that in the rural counties, while in the last three months of the first year the mortality in the urban counties has only increased till the ratio between the mortality is 17 per cent. greater in the urban than that in the rural counties. This cannot be esteemed as anything but a very surprising result. It seems quite definite that there is a group of diseases which apparently kill quite independently of the environment either of the child or of the mother. In fact, in the rural counties the deaths from these conditions amount to very nearly half the total mortality. Considering these figures for the town and for the country it seems obvious that this group of deaths depends on influences which act accidentally. The child in the womb is a true parasite protected against the vicissitudes of the mother and more or less independent even of her starvation or dissipation. How to act on such obscure relationships is a problem of great intellectual attraction, but, though the study of these conditions must be pursued, it affords little prospect of immediate profit.

With regard to the third group of diseases there is, I think, a quite different outlook. Examining the table (Table XI) which describes the range of deaths due to infection proper a totally different series of relationships is seen. Here in the first three months of life the mortality in the urban counties is 48 per cent. greater than in the rural counties, while in the last three months of the first year it is 75 per cent. greater. It is this group of diseases that sanitation should attack with immediate effect.

In comparing the death-rates at different ages, however, some care is necessary. As has been seen in the diagrams and tables in later life the maximum effect of unhygienic surroundings occurs about the age of 50 years. Taking the data of life tables where large populations are dealt with, it is found that there is an almost perfect correlation between the death-rate at ages 50 to 55 years and the life table death-rate for all ages above 35 years: in other words the death-rate in that quinquennium may be taken as representing accurately the whole effect of environment on health in middle and advanced ages. When smaller populations are used the correlations are still large. For instance, taking the data referring to the counties of England for the years 1891–1900 the correlation between the death-rates at the age periods 45–55 years and 55–65 years and the life table death-rates at the ages above 35 years are respectively 0·967 and 0·980. It is thus possible from the death-rates at either of these age periods to calculate the life table death-rates within a range of ½ per cent. Though this is true with regard to the later ages of life the same cannot, however, be said of the earlier ages. As we have seen, the maximum effect of bad environment in depressing health is between the ages of 2 and 3 years. But the death-rate at this age is no longer linearly related in the same way to the life table death-rate. For instance, if the infantile mortality be taken as the standard of health in a district, the death-rate at 2 years of age while increasing with the infantile mortality does not rise in such a manner that the relationship can be expressed by a straight line. The increase is slow at first, then much more rapid. It is, therefore, not safe in investigating the statistics of children's diseases to use age mortality figures for rigid correlation without making an initial investigation.

It would not be fair to pass to the next part of the argument without making a short historical note. Dr. Farr's last great work was his supplement to the thirty-fifth Annual Report of the Registrar-General, which was published in 1875. In this supplement practically every method which has been found of importance in the use of health statistics since that date is contained. It might have been expected that having done so much he would have carried his investigations further, but before there was time for that further collection of data necessary for the development of his ideas, he was driven by ill health to retirement and thus robbed of the fruit of his vision.

VII. The Connexion between Life Table and Standardized Death-rates.

From what has been said it is to be inferred that a life table death-rate is the criterion of ultimate importance, and that for many purposes unless such a death-rate is obtainable it is impossible to draw valid conclusions.

To demand life table death-rates, however, if these could only be obtained by calculating the required life table would be futile, as

TABLE XI. *Showing the death-rates per thousand living births among infants during the first twelve months of life in the urban and rural registration counties from the diseases which are due to infection, 1906–1910.*

URBAN REGISTRATION COUNTIES.

	Under one month.	*1–*	*2–*	*3–*	*4–*	*5–*	*6–*	*7–*	*8–*	*9–*	*10–*	*11–*
Measles	0·02	0·04	0·02	0·02	0·05	0·11	0·19	0·32	0·44	0·52	0·60	0·66
Scarlet fever	0·00	0·00	0·00	0·00	0·01	0·01	0·01	0·02	0·02	0·02	0·03	0·03
Diphtheria	0·01	0·01	0·01	0·01	0·01	0·01	0·03	0·03	0·05	0·06	0·07	0·07
Whooping-cough	0·11	0·42	0·42	0·38	0·36	0·37	0·40	0·42	0·47	0·45	0·45	0·47
Enteritis	0·25	0·38	0·42	0·38	0·34	0·29	0·24	0·19	0·17	0·15	0·13	0·12
Gastro-enteritis	0·20	0·29	0·31	0·30	0·28	0·23	0·20	0·16	0·14	0·13	0·11	0·09
Gastro-intestinal catarrh	0·07	0·11	0·10	0·09	0·07	0·06	0·04	0·03	0·03	0·03	0·03	0·02
Diarrhoea	0·53	1·26	1·54	1·71	1·59	1·41	1·28	1·14	1·02	0·91	0·77	0·70
Tuberculous meningitis	0·02	0·03	0·07	0·10	0·12	0·16	0·18	0·18	0·20	0·17	0·16	0·16
Tuberculous peritonitis	0·04	0·12	0·19	0·22	0·22	0·20	0·16	0·14	0·15	0·14	0·12	0·12
Other tuberculous diseases	0·03	0·06	0·10	0·13	0·13	0·12	0·12	0·12	0·12	0·12	0·12	0·14
Syphilis	0·26	0·27	0·19	0·13	0·08	0·05	0·03	0·02	0·02	0·02	0·02	0·01
Erysipelas	0·06	0·06	0·03	0·02	0·02	0·01	0·01	0·01	0·01	0·01	0·01	0·00
Rickets	0·00	0·01	0·02	0·03	0·03	0·04	0·04	0·05	0·06	0·06	0·07	0·07
Meningitis	0·09	0·11	0·13	0·16	0·19	0·22	0·22	0·22	0·20	0·21	0·18	0·17
Laryngitis	0·01	0·01	0·01	0·01	0·01	0·01	0·01	0·02	0·01	0·03	0·02	0·02
Bronchitis	0·99	1·49	1·06	0·92	0·78	0·73	0·68	0·66	0·64	0·58	0·54	0·49
Pneumonia	0·53	0·94	0·94	0·90	0·91	1·00	1·09	1·16	1·20	1·19	1·17	1·17
Gastritis, Gastric catarrh	0·23	0·29	0·27	0·22	0·19	0·13	0·11	0·10	0·07	0·07	0·05	0·04
Total	3·45	5·90	5 83	5·73	5·39	5·16	5·04	4·99	5·02	4·87	4·65	4·55

Table XI—*continued.*

Rural Registration Counties.

	Under one month.	*1–*	*2–*	*3–*	*4–*	*5–*	*6–*	*7–*	*8–*	*9–*	*10–*	*11–*
Measles	0·01	0·02	0·02	0·03	0·03	0·06	0·10	0·11	0·17	0·20	0·23	0·24
Scarlet fever	0·00	0·01	0·00	0·00	0·01	0·00	0·01	0·01	0·01	0·01	0·00	0·01
Diphtheria	0·00	0·01	0·01	0·00	0·01	0·01	0·01	0·02	0·02	0·02	0·02	0·04
Whooping-cough	0·13	0·49	0·44	0·44	0·35	0·33	0·35	0·36	0·36	0·39	0·33	0·38
Enteritis	0·18	0·28	0·26	0·25	0·16	0·17	0·13	0·13	0·09	0·09	0·07	0·06
Gastro-enteritis	0·16	0·26	0·28	0·25	0·22	0·18	0·14	0·12	0·13	0·11	0·07	0·07
Gastro-intestinal catarrh	0·04	0·06	0·06	0·04	0·04	0·03	0·04	0·01	0·02	0·01	0·01	0·00
Diarrhoea	0·31	0·63	0·56	0·64	0·57	0·54	0·42	0·40	0·29	0·30	0·24	0·22
Tuberculous meningitis	0·01	0·02	0·03	0·07	0·06	0·10	0·10	0·11	0·12	0·13	0·14	0·07
Tuberculous peritonitis	0·03	0·10	0·15	0·14	0·13	0·12	0·10	0·09	0·08	0·06	0·04	0·05
Other tuberculous diseases	0·02	0·06	0·08	0·08	0·12	0·09	0·07	0·09	0·11	0·09	0·12	0·11
Syphilis	0·12	0·10	0·09	0·06	0·04	0·04	0·03	0·02	0·01	0·01	0·01	0·01
Erysipelas	0·04	0·06	0·03	0·01	0·02	0·01	0·00	0·01	0·00	0·00	0·00	0·00
Rickets	0·00	0·00	0·02	0·03	0·02	0·02	0·04	0·05	0·04	0·03	0·08	0·09
Meningitis	0·02	0·04	0·09	0·11	0·11	0·11	0·10	0·12	0·09	0·13	0·10	0·08
Laryngitis	0·02	0·01	0·01	0·01	0·01	0·01	0·01	0·02	0·03	0·02	0·02	0·01
Bronchitis	0·66	1·01	0·82	0·63	0·58	0·46	0·48	0·46	0·47	0·42	0·40	0·40
Pneumonia	0·36	0·68	0·69	0·59	0·62	0·64	0·79	0·74	0·72	0·78	0·71	0·66
Gastritis, Gastric catarrh	0·17	0·25	0·24	0·18	0·13	0·09	0·09	0·04	0·07	0·05	0·03	0·04
Total	2·28	4·09	3·88	3·56	3·18	3·01	3·01	2·91	2·83	2·85	2·62	2·54

Note.—See Table X for list of Urban and Rural Registration Counties.

TABLE XII. *Giving the values of the constants required for obtaining the life table death-rates or the expectations of life from the standardized death-rates at each age and upwards, the standardized death-rates being calculated from the population given in the table.*

Age period.	*Males.* m	*Males.* c	*Males.* Δ	*Females.* m	*Females.* c	*Females.* Δ
0	0·68168	9·60	0·14	0·67905	9·37	0·11
5	0·47450	12·46	0·10	0·48971	11·89	0·12
10	0·45126	13·58	0·09	0·45154	13·15	0·12
15	0·45352	14·58	0·11	0·45469	14·09	0·12
20	0·47543	15·33	0·10	0·47696	14·81	0·12
25	0·49846	16·13	0·08	0·48785	15·76	0·10
35	0·55201	17·87	0·12	0·51554	17·89	0·15
45	0·59426	20·54	0·19	0·56960	20·16	0·33
55	0·67588	22·09	0·55	0·65433	21·81	0·64

Age period.	*Numbers in standard population, 1891–1900.* *Males.*	*Females.*
0–5	59,052	59,468
5–10	56,000	56,289
10–15	53,521	53,550
15–20	49,986	50,814
20–25	44,106	49,419
25–35	74,159	81,938
35–45	57,412	61,276
45–55	41,980	45,629
55–65	27,212	31,184
65–75	15,026	18,596
75 and upwards	5,603	7,780

Age.	*Population at each age and upwards.* *Males.*	*Females.*
0	484,057	515,943
5	425,005	456,475
10	369,005	400,186
15	315,484	346,636
20	265,498	295,822
25	221,392	246,403
35	147,233	164,465
45	89,821	103,189
55	47,841	57,560
65	20,629	26,376
75	5,603	7,780

Equation of relation $D_2 = mD_1 + c$
where D_2 is the life table death-rate, D_1 the standardized death-rate, and Δ the square root of the mean of the squares of the differences of the actual and theoretical values.

NOTE.—The method of using this table is fully described in Sect. XXII.

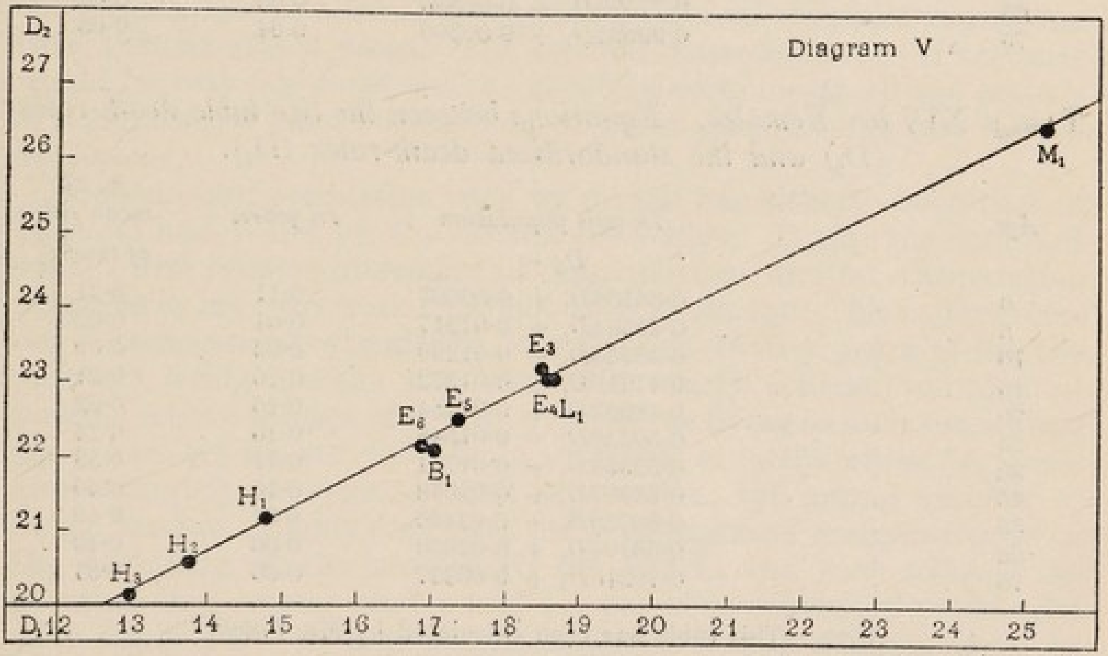
The values of the constants are not those already given in the original paper but have been specially recalculated as some errors in the death-rates on which the original calculations were based have been discovered. The fits of the formulae are considerably more accurate than those originally given.

TABLE XIII. *Showing the comparison of the actual and theoretical life table death-rates obtained from the life tables and from the short method described in the text.*

	Age 0–				Age 15–				Age 35–				Age 55–			
	Males.		*Females.*		*Males.*		*Females.*		*Males.*		*Females.*		*Males.*		*Females.*	
	Act.	*Theo.*	*Act.*	*Theo.*	*Act.*	*Theo.*	*Act.*	*Theo.*	*Act.*	*Theo.*	*Act.*	*Theo.*	*Act.*	*Theo.*	*Act.*	*Theo.*
H_3	18·91	18·75	17·95	17·86	20·13	20·28	19·58	19·74	30·01	30·01	28·74	28·92	55·19	54·59	52·30	52·00
H_2	19·43	19·32	18·50	18·47	20·57	20·65	20·13	20·15	30·58	30·51	29·27	29·29	55·56	55·19	52·47	52·08
H_1	20·59	20·52	20·22	20·20	21·19	21·20	21·26	21·12	30·40	30·44	29·89	30·00	54·08	54·34	51·98	52·60
E_6	22·66	22·77	20·93	21·01	22·12	22·16	21·00	21·07	34·20	34·22	31·73	31·73	63·33	63·13	58·00	57·68
E_5	22·90	23·09	21·20	21·42	22·49	22·46	21·48	21·44	34·59	34·52	32·09	32·00	63·53	63·25	58·04	57·77
E_4	24·18	24·35	22·41	22·55	23·04	23·00	21·92	21·94	34·92	35·05	32·36	32·53	62·70	63·68	57·70	58·68
E_3	25·06	24·85	23·89	23·73	23·16	22·97	22·78	22·60	34·01	34·10	32·69	32·85	60·79	61·98	57·37	58·74
B	22·94	23·03	20·41	20·27	22·39	22·28	20·38	20·34	34·46	34·22	30·79	30·44	60·68	60·63	54·11	53·57
L	24·40	24·49	22·06	22·07	23·04	23·08	21·23	21·35	36·70	36·61	32·87	32·79	67·75	67·40	59·81	59·55
M	28·81	28·61	26·01	25·94	25·79	25·95	24·10	24·26	42·09	42·32	38·02	38·04	80·06	79·84	71·89	71·51

the construction of a life table requires both skill and time. Two questions therefore arise: first, is there any relation between standardized death-rates and life table death-rates; second, can life tables sufficient for the necessary practical purposes be easily constructed? In this section the first will be considered. The second is discussed later.

This subject I entered upon for the first time in 1913, when I discovered that standardized death-rates and life table death-rates were directly related. In fact, life table death-rates can be at once calculated within a very small range of error if the standardized death-rates are known.



In this diagram the relationship of the life table death-rate to the standardized death-rate is shown for the ages of 15 years and upwards with regard to ten life tables. The very close agreement of the formula to the fact is to be noted.

It may be useful to recapitulate this work briefly. Certain life tables (Table V, marked with the letter B) were chosen referring to districts with a large range of mortality. The standard population chosen was the mean population of England between 1891 and 1900. Standardized death-rates were calculated for a series of ages from birth and upwards: at birth, at five years, at ten years, &c., and compared with the life table death-rates. It was found that at each age linear relationships correct within a very small margin of error held between the two series of death-rates. It is not necessary to reproduce all the figures originally given, but the equations of relation are reproduced (Table XII), and the close correspondence between the standardized and the life table death-rates (Table XIII) is shown at birth and at the ages of 15, 35, and 55 years for both males and females. A diagram of the data (Diagram V) is also given for the age of 15 years in males, one of the closest correspondence, though in the majority of cases the error is very little greater.

Mr. Finch of Somerset House, repeating this work and using the more recent life tables, deduces a new series of equations (Table XIV).

TABLE XIV *a*. *Equations between the life table death-rates* (D_2) *and the standardized death-rates* (D_1).

Age.	*In unit population* D_2 =	*In years.*	*% of mean value of true* $\overset{o}{e}_x$
0 . . .	$0{\cdot}66986D_1 + 0{\cdot}00982$	0·17	0·36
5 . . .	$0{\cdot}46088D_1 + 0{\cdot}01249$	0·05	0·09
10 . . .	$0{\cdot}46547D_1 + 0{\cdot}01329$	0·05	0·11
15 . . .	$0{\cdot}48464D_1 + 0{\cdot}01402$	0·09	0·19
20 . . .	$0{\cdot}50105D_1 + 0{\cdot}01484$	0·09	0·20
25 . . .	$0{\cdot}52042D_1 + 0{\cdot}01569$	0·09	0·23
35 . . .	$0{\cdot}56361D_1 + 0{\cdot}01772$	0·08	0·27
45 . . .	$0{\cdot}60613D_1 + 0{\cdot}02039$	0·07	0·30
55 . . .	$0{\cdot}66883D_1 + 0{\cdot}02319$	0·06	0·35
65 . . .	$0{\cdot}77593D_1 + 0{\cdot}02336$	0·05	0·49
75 . . .	$0{\cdot}99695D_1 + 0{\cdot}00390$	0·04	0·65

TABLE XIV *b*. *Females. Equations between the life table death-rates* (D_2) *and the standardized death-rates* (D_1).

Age.	*In unit population* D_2 =	*In years.*	*% of mean value of true* $\overset{o}{e}_x$
0 . . .	$0{\cdot}66556D_1 + 0{\cdot}00967$	0·11	0·21
5 . . .	$0{\cdot}45584D_1 + 0{\cdot}01217$	0·01	0·02
10 . . .	$0{\cdot}45553D_1 + 0{\cdot}01299$	0·05	0·09
15 . . .	$0{\cdot}47319D_1 + 0{\cdot}01372$	0·10	0·21
20 . . .	$0{\cdot}48993D_1 + 0{\cdot}01454$	0·10	0·22
25 . . .	$0{\cdot}50238D_1 + 0{\cdot}01545$	0·10	0·24
35 . . .	$0{\cdot}52935D_1 + 0{\cdot}01761$	0·11	0·33
45 . . .	$0{\cdot}55907D_1 + 0{\cdot}02068$	0·10	0·40
55 . . .	$0{\cdot}60759D_1 + 0{\cdot}02469$	0·07	0·40
65 . . .	$0{\cdot}68104D_1 + 0{\cdot}02950$	0·06	0·49
75 . . .	$0{\cdot}78216D_1 + 0{\cdot}03337$	0·06	0·81

NOTE.—This table has been calculated by Mr. Finch.

DESCRIPTION OF TABLE XIV

This table gives the values of the constants when the Census population for 1901 for England and Wales is used as a standard population. The figures have been calculated with the help of the data obtained from the life tables marked with an asterisk in Table V. The errors are given in years of life in the expectations.

The standard population is in this case taken as that given by the census of 1901 for England and Wales. The life tables used by Mr. Finch are of a higher order of accuracy, and are those marked with an asterisk in Table V. The errors existing in this case between fact and theory are very much smaller than those found in the original investigation.

VIII. THE CHOICE OF A STANDARD POPULATION.

As it has been found, first by myself and later by Mr. Finch, that the population of England and Wales at any date between 1890 and 1901 might be taken as a standard population to determine the true or life table death-rate, a discussion of the nature of the requirements of a standard population is obviously demanded.

It might be thought in the first instance that it would be possible. taking a selection of the most accurately constructed life tables as the criterion of the facts, to assume that a series of standard populations at each age, taken at first as unknown, might be evaluated by the process of least squares in comparison with the criterion selected. This would necessarily be laborious, but that would not be an objection as it could be done once for all. It is found, however, on trial that this method gives rise to negative populations at certain ages, a finding previously recognized by Professor Pearson and Dr. Tocher in their work on cancer death-rates. Now it is not I think advisable to work with negative populations. The assumption is too artificial. Further, in calculating life table death-rates for each different age from birth and upwards, a different set of age period groups in the standard population is found necessary. A life table death-rate at birth thus requires a different standard population than that at fifteen years. The method, therefore, even if accurate would be very laborious, and as absolute accuracy at all age periods is impossible, a laborious method does not seem to offer any special advantage.

The standard population used by myself has hitherto been that of England and Wales for the decade 1891–1900. It was the standard when I first became interested in these matters, and for comparative purposes in my own work I did not wish to change. This is, however, not a homogeneous standard. For the first twenty years of life the numbers living are the survivors of a nearly constant number of births. Above the age of twenty years those living in each succeeding age period are the survivors from a number of births which decreases approximately in a geometrical progression, the actual number of births having roughly increased in a geometrical progression from the beginning of the nineteenth century to the year 1875. The question thus arises, what would happen if the standard population were really a life table population derived from a number of births increasing uniformly in a geometrical progression? Such a standard population was accordingly calculated by multiplying each age group in the life table population for the decade 1891–1900 by a factor representing the numbers of births at each age period. It was found using this standard that some improvement in the prediction of the life table death-rates was obtained. The further question then seemed to arise, did the method of King and Newsholme, in which a specific life table population was chosen as a standard population, offer a better approximation? To answer this a trial was made using the life table population of England and Wales for the decade 1891–1900 as a standard population. Results, however, of inferior accuracy were obtained. It was then suggested that as part of the mechanism of the standard population which had given the best results consisted of the geometric ratio into which a life table population had been multiplied at each respective age group, perhaps a standard population in which the numbers at each age period bore a constant geometric ratio to those of each preceding age period might be used advantageously. Such a population has a constant death-rate, a fact which might have some bearing on the relationships found. A standard population was therefore calculated so that the number of survivors at 60 years of age was equal to one-tenth of the persons born. It was found using this population that the results were much less accurate than those based on the life table population. What was remarkable, however, in all these trials was the small difference made by these several changes.

Here the matter might well have rested had not the spirit of unrest demanded further inquiry. An arithmetical progression was therefore assumed as the law of decrease of population. With an arithmetical progression the slope of the progression does not matter; if death-rates are applied to ordinates in the same position relatively to the time abscissae the magnification of populations and deaths is in an equal ratio. The standardized death-rates are thus not affected. The datum of importance is the age of the ultimate limit of life. Three limits of life were chosen—77½, 82½, and 87½ years respectively: for the first of these the populations at each age group of five years are in the ratio—15, 14, 13, &c., for the second 16, 15, 14, &c., leaving one survivor after the age of 75 years; for the third 17, 16, 15, &c. Using these populations as standard populations, it is found that the most accurate results are obtained when the upper limit of life is taken as 82·5 years. The mean errors found in each case when life table death-rates with regard to the ten life tables given in Table V (marked B) are compared with the death-rates standardized in all the ways above described, are shown in the accompanying table (Table XV). It is curious to observe that De

TABLE XV. *Giving the standard error of the life table death-rates calculated upon different populations as described in the text.*

Standard populations.	*Age.* 0–	15–	35–	55
Population of England, 1891–1900 .	0·14	0·11	0·12	0·55
Life table population of England, 1891–1900	0·29	0·19	0·16	0·59
Life table population multiplied by the appropriate term of a geometrical progression . . .	0·17	0·12	0·11	0·54
Population in arithmetical progression	0·17	0·13	0·08	0·43
Population in geometrical progression	0·48	0·43	0·20	0·83

Moivre's arbitrary population designed in 1725 for insurance purposes, namely, that one person died in each year of life up to the limit of the age of 86 years, so closely corresponds to the type of standard population required to equate the standard death-rate and the life table death-rate. For the purposes for which the method was devised it proved a poor guide, but his intuition has not been in vain. His hypothesis comes back surrounded with a nimbus of convenience, and is thus a fine example of a solar myth.

The life tables, however, on which this investigation was first based are not wholly satisfactory, as they are not strictly comparable in method of construction. For inquiries into the length of the expectation of life based on data prior to 1890, however, the first found equations (Table XII) had best be used. Health conditions have, however, changed since last century in a manner which cannot be directly calculated. In the early days among other things there were large epidemics of cholera and typhus. Further bad watersupplies, over-crowding, and dissipation were much more common during most of last century. In framing then a new set of equations to obtain the life table death-rates, it has been thought better to limit the calculations to the more recent life tables which Mr. Finch selected as the basis of his work (Table XVI). The standard errors

Table XVI. *Giving the constants required to calculate the life table death-rates from the number of deaths obtained by applying the death-rates at each age period to a standard population adjusted so as to be in arithmetical progression.*

	Males.			Females.				Population on Arithmetical Basis.	
Age.	*m*	*c*	Δ	*m*	*c*	Δ	*Age.*	*Population at each age.*	*Population at each age and upwards.*
0–	0·0047986	10·052	0·09	0·0046168	10·604	0·05	0–	16	136
5–	0·0036317	12·756	0·03	0·0039159	12·460	0·02	5–	15	120
10–	0·0044503	13·210	0·06	0·0043761	13·395	0·03	10–	14	105
15–	0·0050789	14·303	0·07	0·0052281	14·202	0·03	15–	13	91
20–	0·0061455	15·254	0·06	0·0063082	15·132	0·03	20–	12	78
25–	0·0076268	16·204	0·07	0·0077535	16·154	0·03	25–	21	66
35–	0·012322	18·546	0·09	0·012239	18·759	0·03	35–	17	45
45–	0·021530	22·017	0·12	0·020931	22·606	0·05	45–	13	28
55–	0·044486	26·720	0·13	0·043032	27·667	0·05	55–	9	15
65–	0·127236	31·954	0·25	0·121408	34·123	0·13	65–	5	6
75+	1·003635	2·823	0·99	0·876036	20·901	1·03	75–	1	1

Note.—It is to be noted that in this table the constants have been adjusted to obtain a life table death-rate directly from the number of deaths, thus saving the labour of calculating a standard death-rate for each age (Sect. XXII).

are added for comparison. It will be observed how small these become when the standard population formed in an arithmetical progression is used in combination with the most accurate life tables. This must be held to be a somewhat astonishing result. A population decreasing in a uniform manner has a law of increase of death-rate which has no analogy in life, yet it is found possible to obtain from this population, by a series of simple equations, the same results as by the more elaborate method of calculating a life table. The success of the method is, however, in keeping with the discovery that a life table death-rate could be calculated from a specific standard population. This standard, the mean population of England and Wales for the decade 1891–1900, is very nearly in an arithmetical progression, and that calculated from the life table population of the same epoch multiplied into a suitable geometrical progression still more so. It would seem, therefore, that Providence had a hand in furnishing at the proper time the data in the form most suitable to this work.

There is, however, one further point of interest in the matter. Examining the constants of the equations connecting the two death-rates, it is obvious that they do not progress from quinquennium to quinquennium in a uniform manner. There is between one five years and another a more rapid increase than between the succeeding similar periods. The differences fall to rise again. This is analogous to what is found in the life tables themselves as constructed. In these the third and fourth differences between the numbers surviving at successive ages oscillate slowly between positive and negative values.

It may be asked why, when such great accuracy can be obtained, a still greater accuracy is not possible ? The reasons for this I think have already been explained in the introduction, but there is one special reason which vitiates all the statistics, even those for the whole country. Between the ages of 15–30 years there is a great influx of young persons into the towns, from a healthy to unhealthy environment. When town or country districts are considered separately this migration tends to raise the country death-rates at these ages. To take a special example, referring to the decade 1861–70, the death-rates at the ages from 10–25 years in London are lower than that of all England and Wales as a whole. It is not possible at present to make any attempt to evaluate the effect of this numerically, but the formulae as given do not in any case make a very great error in the expectation.

IX. Farr's Law. The Relation of Density to Death-rate.

In Section IV the life table death-rate was selected as the best measure inasmuch as it described the actual life-history of a large number of individuals from birth to death. Whether, however, the life table death-rate is an absolute measure, that is to say, whether a district with a death-rate of 24 per 1,000 is 30 per cent. more unhealthy than a district with a death-rate of 18 per 1,000, has not been settled, an actual unit of ill-health not being yet established. The life table death-rate, however, has one property which places it as a measure above either the standardized death-rates or the crude death-rates, inasmuch as it has been found for England and Wales to be very closely connected with the density of population.

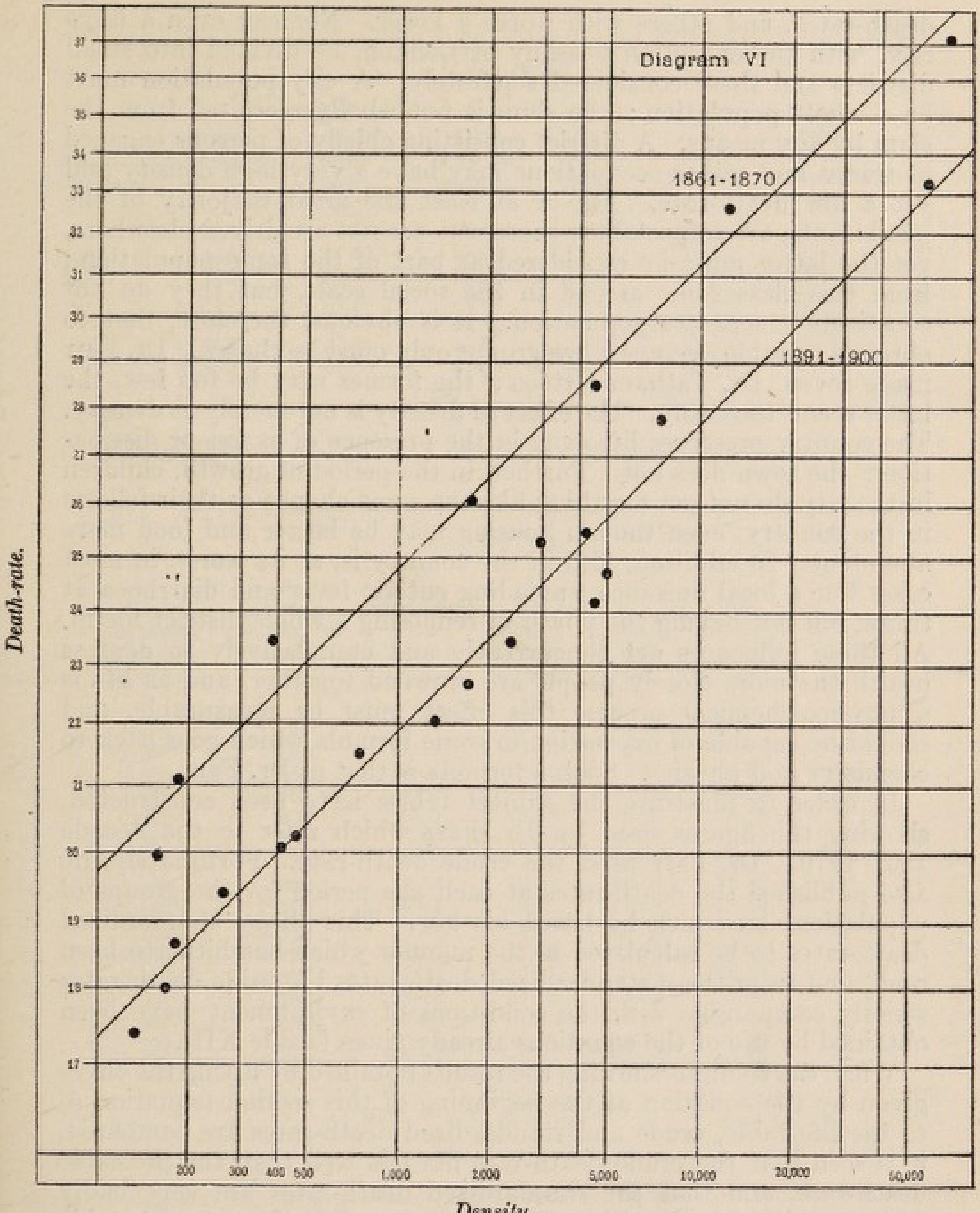
This subject was first considered statistically by the late Dr. Farr. His treatment of it is one of the brilliant attempts to extract the real meaning of figures so frequent in his work, but though this theory has not shared in the complete neglect that has been the lot of his attempt to put a quantitative measure to the course of epidemics, it has suffered as much from the kind of patronage with which it is usually discussed. On one at least of the great medical officers of health of his time, however—the late Dr. J. B. Russell of Glasgow—the theory exercised a strong fascination. My own copy of Farr's *Vital Statistics* came from Dr. Russell's library, and the whole passage referring to the law is lined with his characteristic nervous pencil marks, and in much of his work on vital statistics the influence can be easily traced.

The law itself, if the death-rate be denoted by D and the density of population (say the number of persons per square mile) by δ, is that

$$D = c\delta^m \quad . \; . \; . \; . \; (a)$$

where c and m are constants.

By Farr the crude death-rate was used and found to give a good measure of the facts. When later, the standardized death-rate being introduced, it seemed to be the proper course to adopt this measure the law obviously did not hold. Even with crude death-rates, its success as a descriptive formula was not nearly so marked. Thus in the absence of any *a priori* justification the law was relegated to a somewhat obscure position.



In this diagram the relationship of density to death-rate (Farr's Law) is shown for two epochs, 1861–1870 and 1891–1900. The ordinates are the logarithms of the life table death-rates, and the abscissae the logarithms of the number of persons living on each square mile. The observations are indicated by black circles and the theoretical straight line has been drawn for both cases. It will be observed that the two lines are parallel and that for each of the decades the observations group themselves very closely on the line.

Before proceeding to the justification of the law, however, it is necessary to have a clear idea of the kind of evidence necessary to establish it. The law must be a law of average, for on account of the arbitrary nature of persons living on an acre it is merely a rough approximation. The groups of localities which supply the figures must further be large, as some with better conditions will have higher death-rates, and others with worse a lower. Nor can even a large city, with the exception possibly of London, be divided into small districts and these considered separately. A city population must be a whole population; the slum is not wholly recruited from the slum by any means. A district consisting chiefly of persons engaged in trades and minor occupations may have a very high density and yet a low death-rate. All, or at least the great majority of the inhabitants, are respectable; those who are not are driven elsewhere, yet the latter must be considered as part of the same population; from this class some ascend in the social scale, but they do not constitute a separate population. It is obvious, therefore, that to obtain a suitable average a few groups only must be chosen. Dr. Farr made seven, Dr. Tatham sixteen; the former may be too few, the latter seems too many. The effect of density is not merely as density. The country preserves life even in the presence of excess or dissipation: the town does not. Further, in the period of growth, children in the city do not get anything like the same chance as their fellows in the country, even though housing may be better and food more abundant. In addition, filth in the country is, at its worst, in most cases but a local nuisance, spreading enteric fever and diarrhoea at times, but not having the power of rendering a whole district foetid. All these influences act concurrently and cumulatively to depress health the more closely people are crowded together, and as life is a physico-chemical process this effect must be measurable, and should be capable of expression in some formula which goes back to chemistry and physics. Such a formula is that of Dr. Farr.

In order to illustrate the subject tables have been constructed, showing the figures used by Dr. Farr which refer to the decade 1861–1870. Dr. Farr used the crude death-rate. Fortunately, he also published the death-rates at each age period for the groups of populations on which he based his law. This allows standardized death-rates to be calculated in the manner which has hitherto been used, and from these standardized death-rates life table death-rates strictly comparable with the conditions of environment have been obtained by use of the equations already given (Table XII).

When the columns showing the results obtained by fitting the curve given by the equation at the beginning of this section (equation a) to the life table, crude and standardized death-rates are compared, it is seen that the crude death-rate fits less well than the life table death-rate, and that the standardized death-rates are very badly represented by the formula. The excellence of the fit of the life table death-rate to the formula is shown in Diagram VI. It will be noticed that the crude death-rate curve of Dr. Farr has an exponent of 0·1199. This value is much nearer the probable true exponent 0·100 than that found by Dr. Tatham to most nearly graduate the crude death-rates for the decade 1891–1900, namely 0·1276. This is explained by the fact that in the earlier period the crude death-rate was 22·42 as against a life table death-rate of 24·06, while in the later period the corresponding figures are 18·19 and 21·77. Dr. Farr had thus a better opportunity of formulating a law than his successors. Using the crude death-rate it became more and more difficult to accept the relationship demanded by the formula (Tables XVII and XVIII).

TABLE XVII. *Showing the figures relating to density and death-rate, 1861–1870.*

No. of Districts.	Density (persons per square mile).	Standardized death-rate. (1)	Do. fitted by least squares.	Crude death-rate. (2)	Do. fitted by Farr.	Life table death-rate. (3)	Do. fitted by least squares.
53	166	15·30	16·70	16·75	18·90	19·90	20·73
345	186	17·02	17·00	19·16	19·16	21·07	20·96
137	379	20·52	18·99	21·88	20·87	23·47	22·51
47	1,718	24·35	24·03	24·90	25·02	26·09	26·19
9	4,499	27·94	27·92	28·08	28·08	28·54	28·84
1	12,357	33·98	32·67	32·49	32·70 [1]	32·67	31·92
1	65,823	40·55	42·39	38·62	38·74	37·17	37·74
			E% = 3·79 Δ = 1·17		E% = 2·70 Δ = 0·90		E% = 2·01 Δ = 0·61

(1) $R = 7{\cdot}534\, D^{0{\cdot}15571}$ (2) $R = 10{\cdot}234\, D^{0{\cdot}11998}$ (3) $R = 12{\cdot}419\, D^{0{\cdot}10018}$

TABLE XVIII. *Showing the figures relating to density and death-rate, 1891–1900.*

1 No. of Districts.	2 No. of inhabitants divided by 1,000.	3 Density (persons per square mile).	4 Standardized death-rate.	5 Do. fitted by least squares. (1)	6 Crude death-rate.	7 Do. fitted by least squares. (2)	8 Life table death-rate.	9 Do. fitted by least squares. (3)
27	305	136	11·63	13·06	14·20	14·16	17·38	17·18
112	1,676	161	12·54	13·43	15·05	14·51	18·01	18·12
121	2,496	181	13·44	13·70	15·44	14·68	18·62	18·33
92	2,849	261	14·52	14·56	15·46	15·38	19·36	19·02
53	2,272	407	15·53	15·68	16·08	16·28	20·05	19·90
56	2,577	457	16·53	15·99	16·67	16·52	20·24	20·13
31	1,839	737	17·58	17·32	17·64	17·56	21·45	21·12
40	3,690	1,303	18·53	19·05	18·04	18·88	22·10	22·31
31	3,159	1,705	19·42	19·93	18·61	19·54	22·71	22·99
21	2,240	2,339	20·37	21·00	19·50	20·35	23·36	23·72
18	2,777	4,424	21·56	23·37	20·21	22·08	24·18	25·31
13	2,119	4,884	22·36	23·76	20·69	22·35	24·72	25·56
6	801	4,194	23·48	23·16	22·05	21·93	25·49	25·10
5	762	2,925	24·33	21·80	23·29	20·94	26·07	24·21
5	791	7,480	26·54	25·51	24·74	23·60	27·58	26·68
4	288	55,563	34·82	35·66	32·67	30·49	33·25	32·58
				E% = 4·3 Δ = 1·05		E% = 3·8 Δ = 1·14		E% = 2·03 Δ = 0·63

(1) $R = 12{\cdot}40\, D^{0{\cdot}16715}$ (2) $R = 13{\cdot}57\, D^{0{\cdot}12755}$ (3) $R = 10{\cdot}83\, D^{0{\cdot}10078}$

NOTE.—Where E is the mean percentage error and Δ the square root of the mean of the squares of the actual errors, R the death rate and D the number of persons living per square mile.

After this full statement it is only necessary to detail the results obtained when life table death-rates have been taken as the measure of health. The formulae have been calculated for the four decades for which data exist, and the results are given in the accompanying table:

1861–1870 . . . . $D = 12{\cdot}42\,\delta^{0{\cdot}1001}$
1881–1890 . . . . $D = 11{\cdot}45\,\delta^{0{\cdot}0985}$
1891–1900 . . . . $D = 10{\cdot}83\,\delta^{0{\cdot}1008}$
1901–1910 . . . . $D = 9{\cdot}90\,\delta^{0{\cdot}1023}$

A misprint in the original of 37·7 has been corrected

Thus it is found that though the general health has improved, the power of the density has stood unchanged for forty years. That is to say, that the death-rate and density remained related in essentially the same way in the counties of England and Wales in 1905 as in 1865. It is the constant multiplier that has been affected by hygienic measures and not the law of the power. Hygiene acts surely all round but still is subjected to fundamental laws.

In correlating density and death-rate the stumbling-block has been London. Considering the density of the city the death-rate was nothing like so high as it should have been by Farr's law. The differences were really very great; for instance, in the decade 1861–1870 the formula as adjusted by myself gave a death-rate of 32 per 1,000 as against 26 per 1,000 actually found. The meaning of this exception was difficult to ascertain. In a former paper I ascribed it as probably due to the results of the extreme selection established by city life, as only those fitted to survive could hope to procreate children. While this might be expected to have such an effect, it was theorizing without proof. It was not possible to make any test of this matter till 1911. Prior to this the deaths occurring in institutions were not distributed to the registration districts to which they properly belonged, and some of the institutions, such as the London Hospital, were very large. In his annual report for London in 1911, however, Sir Shirley Murphy published standard death-rates for the different boroughs of London, for the five years 1907–1911, the deaths in institutions being ascribed to the districts to which they belonged. It was thus possible to calculate the life table death-rates for each borough.

The relationship between the density and the death-rate calculated for these life table death-rates has been found to be

$$D = 6{\cdot}33\delta^{0{\cdot}1045}.$$

Here again we see that the relationship which has been found to hold for England and Wales holds also for London, the death-rate for each district in London being proportional to the tenth root of the density. The constant multiplier is, however, much smaller than that for England and Wales as a whole, showing that the discrepancy observed in the decade 1861–1870 still persists. London has thus achieved in some way or other a greater absolute healthiness than its density would suggest. This greater degree of health has not, however, absolved it from obedience to the general relationship between density and death-rate, the relation between the death-rates of different boroughs being determined by the general principle.

X. Fitting the Life Table Death-rates to the Formula $a(c-x)^{-n}$.

In Section VI some evidence was given as to the manner in which the death-rate varied after the age of 10 years. Taking the healthy districts as the standard, the mortalities in the more unhealthy districts and in cities were shown to vary so that the ratio of the death-rate in the more unhealthy districts to that of the healthy

TABLE XIX. *Showing the values of n, c, and log a obtained when fitting the expectations at 15, 45, and 75 to the formula* $a(c-x)^{-n}$.

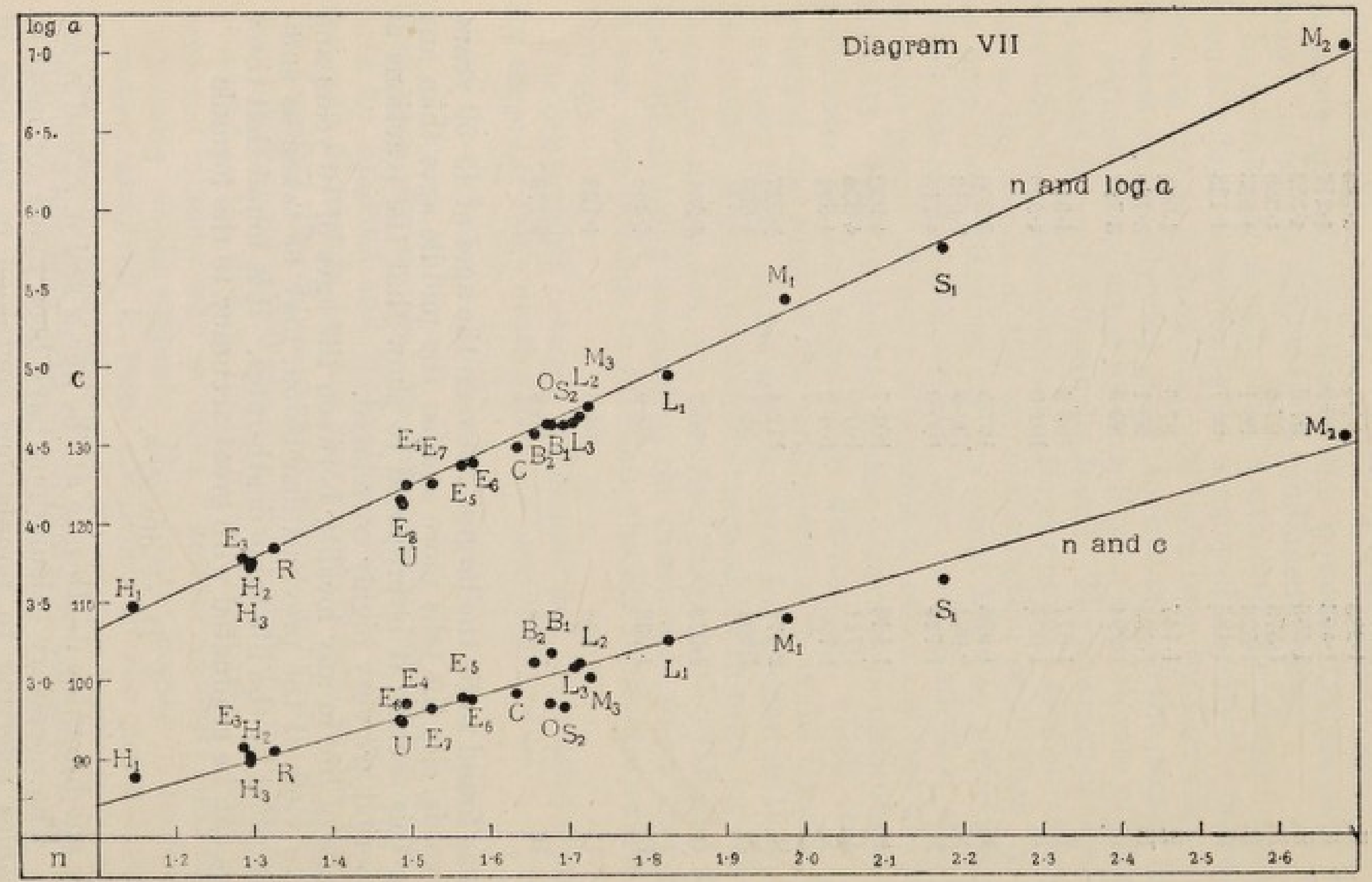
	n	*c*	*log a.*
E_2	1·285	91·4	3·783
E_4	1·493	96·9	4·217
E_5	1·566	97·6	4·352
E_6	1·576	97·6	4·366
E_7	1·525	96·1	4·235
E_8	1·485	94·7	4·138
H_1	1·147	87·6	4·051
H_2	1·295	90·1	3·769
H_3	1·294	89·6	3·782
B_1	1·677	103·2	4·611
B_2	1·657	102·0	4·556
M_1	1·976	107·5	5·300
M_2	2·684	130·6	7·013
M_3	1·724	100·0	4·715
L_1	1·826	104·7	4·930
L_2	1·711	101·9	4·654
L_3	1·705	101·2	4·631
S_1	2·176	112·4	5·727
S_2	1·691	96·1	4·600
O	1·673	96·7	4·601
C	1·632	98·0	4·468
U	1·488	94·6	4·139
R	1·325	90·8	3·779

districts was found to be the largest between the ages of 45–50 years. In other words, city life tells more against the middle ages than any other. In this section it is proposed to show that the reactions at each age period are very highly correlated.

The range of life in any locality between the ages of 10 years and 75 years, in view of the previous discussion, may be taken as sufficiently described by the life table death-rates. It is found that these death-rates can be graduated with great accuracy to the formula

$$D_x = a(c-x)^{-n},$$

where D is the life table death-rate at the age x and a, c, and n are constants. The necessary calculations have been made for most of the life tables referring to populations contained in the area of England and Wales, and the values of the constants are given in the accompanying table (Table XIX). It will be, perhaps, however, more intelligible to exhibit the relationship of the different constants diagrammatically. Explanatory diagrams have accordingly been constructed (Diagrams VII and VIII). It will be observed that the quantities c, n, and *log* a, taken pair by pair are so related that their values are in linear relationship. The correlation between each pair is singularly high, being in all cases over 0·97, so that given one of

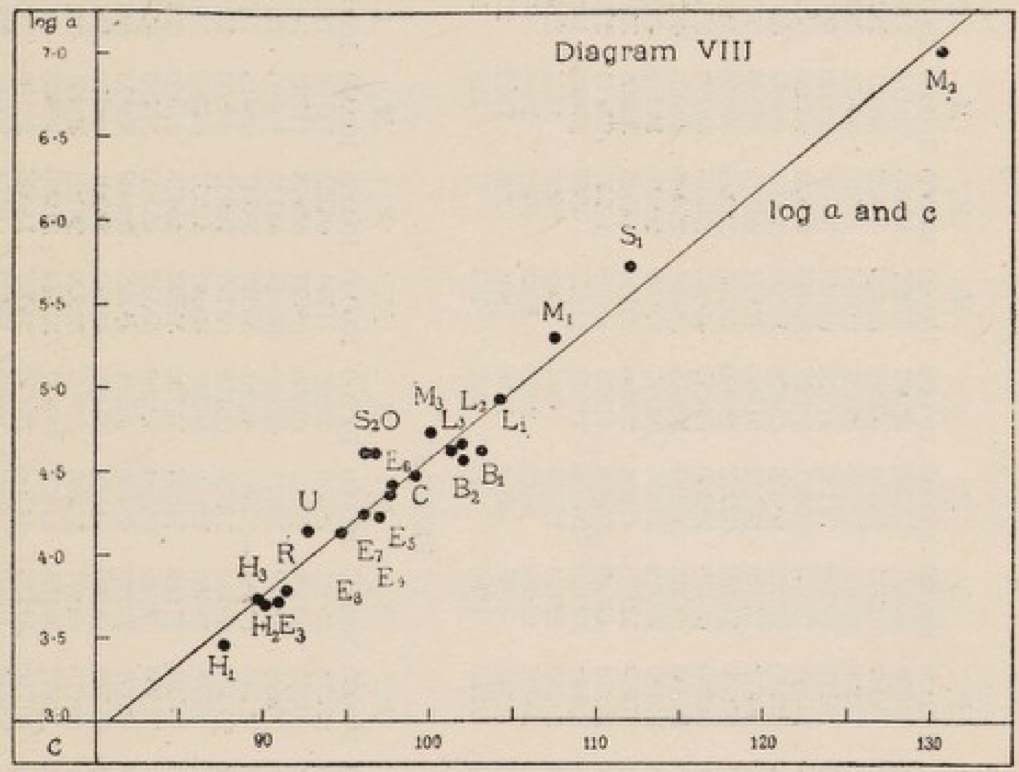


In this diagram the relationship found from twenty-three life tables between *n* and *log a* and *n* and *c* when the life table death-rates have been graduated to the formula $a(c-x)^{-n}$ is shown: it will be noted how closely the points group themselves on the straight lines, the chief divergence being found in tables based on small populations such as those for Brighton, Oldham, and Salford.

the constants the other two are practically determined. The relationships between each pair are given by the equations,

$$log\ a = \cdot07751\ c - 3\cdot149$$
$$n = \cdot44393\ log\ a - \cdot380$$
$$n = \cdot03523\ c - 1\cdot860.$$

The reaction of life to environment is thus very rigidly limited, and though in one place or another there may be small deviations, yet the deviation in no place can be very great. The larger deviations, however, to be observed in the diagrams are in general, though not always, most evident in those cases in which the life table refers to



In this diagram the relation between log a and c is shown for the same graduation. It will be noted again that the divergence chiefly occurs in the same life tables referring to small districts.

a small number of persons. Considering the character of the inhabitants of towns such as Salford or Brighton, though the deviations may be real, it is more likely they are due to the error of small numbers. Even if such slight differences occur it seems no less true that a fundamental relationship of environment to life table death-rates exists.

XI. Comparison of Life as found in various Life Tables.

It is intended in the next section to give some consideration to life tables as a guide to the use of death-rates. The whole columns contained in a life table are not of equal importance for this purpose. I have therefore made a selection from the tables to show the important facts ascertained for a wide range of environment and for different epochs. Further, in Part II I give some notes on the theory of life tables and also directions for the rapid calculation of those parts of a life table useful for the discussion of health problems.

Table XX *a*. *Giving the number of survivors at each age out of 100,000 born in England and Wales at different periods, and for some of the healthy and unhealthy districts.*

MALES.

Age.	H_1	H_2	H_3	E_3	E_4	E_5	E_6	E_7	E_8	B_2	S_1	M_1	M_2	M_3	L_1	L_2	L_3	$C.B.$	$U.$	$R.$
0 .	100000	100000	100000	100000	100000	100000	100000	100000	100000	100000	100000	100000	100000	100000	100000	100000				
5 .	81409	82686	83342	72372	73407	75149	75028	79398	82627	75790	62092	67897	62326	70414	71898	77792				
10 .	78570	81182	82028	68986	70899	73348	73430	78082	81241	74550	60296	64676	59323	68361	70152	76472				
15 .	77109	80306	81221	67278	69642	72619	72537	77296	80464	73785	59402	63076	57540	66656	69296	75717	100000	100000	100000	100000
20 .	75086	78960	80031	65190	68003	71256	71171	76112	79343	72430	57979	61644	55646	65046	68072	74634	98630	98408	98517	98730
25 .	72384	77144	78284	62422	65708	69381	69389	74546	77869	70551	56013	59645	52924	63220	66555	73230	96908	96423	96763	97064
30 .	69558	74903	76325	59509	63004	66928	67320	72740	76222	68352	53878	56824	49099	60862	64602	71482	94770	94126	94818	95176
35 .	66698	72385	74200	56444	59886	63965	64817	70472	74219	65732	51553	53174	44315	57765	61742	69131	91909	91230	92562	93030
40 .	63782	69585	71734	53166	56308	60492	61596	67668	71672	62373	48360	48753	38996	53816	57941	65948	88052	87484	89690	90552
45 .	60697	66389	68856	49577	52237	56444	57701	64230	68459	58260	43491	43665	33319	49133	53471	61904	83025	82757	86102	87494
50 .	57266	62674	65311	45573	47698	51764	53089	59903	64333	53284	38018	38031	27095	44029	48272	56923	76699	76651	81328	83714
55 .	53280	58159	60923	40946	42468	46298	47585	54439	59012	47535	32117	31859	20680	38318	42224	50925	68921	68970	74905	78768
60 .	48501	52507	55197	35633	36501	39840	40952	47563	52110	41041	25206	25116	14945	30308	35325	43872	59426	59324	66489	72024
65 .	41940	45212	47777	29459	29716	32248	33234	39278	43523	33836	17895	18068	10117	23111	27764	35689	48315	47783	55531	62872
70 .	33536	36096	38286	22349	22206	23863	24663	29898	33430	25808	11594	11423	6028	15015	19828	26626	36233	35071	42604	51043
75 .	23822	25362	26732	14808	14496	15389	15861	19753	22424	17655	6064	6069	2906	8327	12199	17449	23432	21495	27838	36166
80 .	14121	14349	14972	8034	7735	8002	8230	10607	12193	9935	2428	2563	1087	3688	6038	9377	12286	10727	14918	20819
85 .	6415	5702	6008	3298	3079	2987	3132	4348	4957	4012	1118	804	323	1162	2217	3721	4815	4065	6177	8801
90 .	2006	1308	1488	932	802	697	772	1116	1360	745	212	173	77	219	551	937	1349	1048	1678	2417

FEMALES.

Age.	H_1	H_2	H_3	E_3	E_4	E_5	E_6	E_7	E_8	B_2	S_1	M_1	M_2	M_3	L_1	L_2	L_3	$C.B.$	$U.$	$R.$
0 .	100000	100000	100000	100000	100000	100000	100000	100000	100000	100000	100000	100000	100000	100000	100000	100000				
5 .	83556	85517	86120	75055	76262	78324	78214	82178	85005	78656	65867	71791	66323	74163	78615	80479				
10 .	80524	84006	84746	71577	73838	76615	76527	80755	83598	77273	63909	68257	62873	71738	76636	79107				
15 .	78640	82927	83795	69692	72496	75906	75550	79898	82774	76273	62880	66614	61115	70105	75693	78305	100000	100000	100000	100000
20 .	76138	81313	82324	67412	70795	74432	74177	78755	81681	75307	61681	65219	59341	68610	74579	77370	98828	98542	98694	98671
25 .	73142	79303	80628	64434	68486	72479	72539	77390	80412	74274	60179	63299	56857	66803	73306	76312	97546	96871	97219	97063
30 .	70036	76933	78756	61277	65842	70005	70582	75778	78953	72899	58337	60647	53597	64380	71631	74991	95965	94998	95528	95254
35 .	66895	74313	76698	57991	62884	67099	68215	73769	77183	71044	55726	57336	49673	61309	69181	73135	93935	92713	93522	93156
40 .	63729	71554	74360	54584	59611	63891	65301	71308	74988	68584	52380	53512	45122	57783	65914	70571	91106	89644	91046	90804
45 .	60502	68654	71744	51040	56017	60401	61918	68359	72283	65450	48241	49192	39962	53873	62102	67407	87390	85949	88012	88160
50 .	57138	65406	68690	47325	52090	56430	58032	64742	68880	61619	43487	44197	34301	49279	57602	63487	82612	81145	84043	84952
55 .	53542	61409	64837	43333	47744	51638	53311	60179	64477	56984	37840	38324	28164	43691	52163	58591	76752	75100	78935	80602
60 .	49226	56264	59535	38397	42284	45768	47304	54157	58659	51472	30997	31544	21596	36963	45635	52563	69282	67200	71988	74597
65 .	43003	49407	52403	32417	35617	38550	39830	46716	51187	44766	23473	24027	15096	29109	37948	45106	60136	57288	62731	66684
70 .	35055	40425	43019	25316	27723	29922	30917	37645	41687	36002	16075	16323	9474	20383	29066	36034	48965	45299	51462	56090
75 .	25661	29410	31204	17480	19057	20421	21069	26418	30090	25837	8786	9459	5275	11995	19505	25724	35290	30773	36446	41838
80 .	15860	17688	18601	10039	10894	11454	11807	15544	18086	15476	3961	4468	2548	5640	10725	15387	21435	17277	21636	26103
85 .	7602	7912	8281	4442	4763	4813	4993	7093	8324	6439	1707	1653	1029	2037	4443	7010	9755	7382	9911	12616
90 .	2536	2276	2429	1380	1423	1342	1433	2158	2764	1517	449	463	329	540	1252	2123	3033	2258	3262	4152

TABLE XX *b*. *Showing the expectation of life in England and Wales at different periods, and for some of the healthy and unhealthy districts.*

MALES.

Age.	H_1	H_2	H_3	E_3	E_4	E_5	E_6	E_7	E_8	B_2	S_1	M_1	M_2	M_3	L_1	L_2	L_3	C.B.	U.	R.
0 .	48·56	51·48	52·87	39·91	41·35	43·66	44·13	48·53	51·50	44·92	33·30	34·71	28·78	38·11	40·98	46·74				
5 .	54·39	57·05	58·26	49·71	50·87	52·75	53·50	55·90	57·14	53·94	48·02	45·59	40·53	48·65	51·60	54·82				
10 .	51·28	53·03	54·16	47·05	47·60	49·00	49·63	51·81	53·08	49·80	44·39	42·75	37·47	45·03	47·84	50·73				
15 .	47·20	48·62	49·67	43·18	43·41	44·47	45·21	47·31	48·57	45·29	40·00	38·78	33·56	41·12	43·40	46·21	46·74	46·28	49·01	51·45
20 .	43·40	44·41	45·37	39·48	39·40	40·27	41·02	43·01	44·21	41·09	35·93	34·62	29·61	37·08	39·13	41·84	42·35	41·99	44·71	47·08
25 .	39·93	40·39	41·32	36·12	35·68	36·28	37·01	38·86	40·00	37·12	32·10	30·69	26·00	33·08	34·96	37·59	38·06	37·80	40·48	42·84
30 .	36·45	36·52	37·32	32·76	32·10	32·52	33·07	34·76	35·81	33·23	28·28	27·08	22·82	29·26	30·94	33·45	33·87	33·66	36·25	38·64
35 .	32·90	32·70	33·32	29·40	28·64	28·91	29·24	30·79	31·71	29·45	24·44	23·76	20·01	25·69	27·25	29·50	29·84	29·65	32·07	34·47
40 .	29·29	28·92	29·37	26·06	25·30	25·42	25·64	26·96	27·74	25·90	20·87	20·68	17·39	22·38	23·87	25·80	26·03	25·80	28·02	30·35
45 .	25·65	25·19	25·49	22·76	22·07	22·06	22·20	23·27	23·92	22·54	17·92	17·80	14·93	19·27	20·65	22·31	22·45	22·13	24·07	26·31
50 .	22·03	21·53	21·74	19·54	18·93	18·82	18·90	19·76	20·29	19·41	15·13	15·06	12·77	16·21	17·60	19·04	19·09	18·69	20·33	22·39
55 .	18·49	18·00	18·12	16·45	15·95	15·74	15·79	16·48	16·89	16·44	12·45	12·49	10·96	13·25	14·76	15·98	15·95	15·48	16·85	18·63
60 .	15·06	14·66	14·72	13·53	13·14	12·88	12·93	13·49	13·78	13·65	10·16	10·16	9·24	10·63	12·14	13·14	13·09	12·57	13·66	15·13
65 .	12·00	11·60	11·61	10·82	10·55	10·31	10·34	10·80	10·99	11·01	8·30	8·15	7·48	8·50	9·76	10·56	10·51	9·99	10·84	11·95
70 .	9·37	8·88	8·84	8·45	8·27	8·04	8·05	8·39	8·53	8·66	6·51	6·48	5·91	6·77	7·66	8·30	8·17	7·69	8·35	9·11
75 .	7·15	6·56	6·56	6·49	6·34	6·10	6·15	6·41	6·49	6·49	5·19	5·11	4·74	5·28	5·91	6·35	6·28	5·99	6·45	6·81
80 .	5·37	4·70	4·78	4·93	4·79	4·52	4·62	4·86	4·90	4·63	4·89	4·02	3·98	4·02	4·53	4·75	4·79	4·64	4·96	5·01
85 .	4·00	3·30	3·45	3·73	3·56	3·29	3·45	3·53	3·72	3·01	3·04	3·16	3·45	2·98	3·48	3·48	3·71	3·55	3·72	3·67
90 .	2·99	2·31	2·50	2·84	2·66	2·37	2·58	2·56	2·87	2·06	1·79	2·48	2·94	2·16	2·69	2·52	2·75	2·71	2·98	2·66

FEMALES.

Age.	H_1	H_2	H_3	E_3	E_4	E_5	E_6	E_7	E_8	B_2	S_1	M_1	M_2	M_3	L_1	L_2	L_3	C.B.	U.	R.
0 .	49·45	54·04	55·71	41·85	44·62	47·18	47·77	52·38	55·35	50·19	36·81	38·44	32·67	41·58	45·33	51·41				
5 .	53·93	58·01	59·53	50·33	53·08	54·92	55·79	58·53	59·94	58·52	50·31	48·06	43·66	50·64	55·12	58·63				
10 .	50·88	54·01	55·46	47·67	49·76	51·10	51·97	54·53	55·91	54·53	46·79	45·43	40·94	47·28	51·49	54·61				
15 .	47·04	49·68	51·06	43·90	45·63	46·55	47·61	50·08	51·44	50·21	42·51	41·50	37·05	43·32	47·10	50·14	51·13	49·64	51·76	53·08
20 .	43·50	45·62	46·93	40·29	41·66	42·42	43·44	45·77	47·10	45·82	38·29	37·33	33·08	39·21	42·77	45·72	46·73	45·34	47·41	48·76
25 .	40·17	41·71	42·86	37·04	37·98	38·50	39·37	41·54	42·80	41·42	34·18	33·38	29·41	35·20	38·46	41·32	42·29	41·08	43·09	44·52
30 .	36·85	37·91	38·82	33·81	34·41	34·76	35·39	37·36	38·54	37·15	30·18	29·73	26·04	31·43	34·30	37·00	37·94	36·84	38·81	40·32
35 .	33·46	34·16	34·79	30·59	30·90	31·16	31·52	33·31	34·37	33·06	26·47	26·30	22·90	27·87	30·42	32·87	33·71	32·68	34·58	36·17
40 .	30·00	30·38	30·81	27·34	27·46	27·60	27·82	29·37	30·30	29·15	23·00	22·99	19·95	24·42	26·80	28·97	29·67	28·71	30·45	32·04
45 .	26·46	26·56	26·84	24·06	24·06	24·05	24·20	25·53	26·34	25·42	19·75	19·79	17·20	21·01	23·29	25·21	25·82	24·84	26·41	27·93
50 .	22·87	22·75	22·92	20·75	20·68	20·56	20·64	21·81	22·51	21·84	16·63	16·74	14·62	17·72	19·91	21·60	22·17	21·15	22·54	23·88
55 .	19·24	19·06	19·12	17·43	17·33	17·23	17·24	18·27	18·87	18·41	13·73	13·91	12·25	14·67	16·72	18·19	18·66	17·64	18·83	20·03
60 .	15·69	15·56	15·59	14·34	17·24	14·10	14·10	15·01	15·48	15·10	11·19	11·35	10·21	11·87	13·74	14·98	15·39	14·41	15·39	16·43
65 .	12·58	12·36	12·36	11·51	11·42	11·26	11·27	11·99	12·36	11·98	8·98	9·11	8·54	9·38	11·01	12·03	12·34	11·46	12·28	13·07
70 .	9·85	9·53	9·48	9·02	8·95	8·77	8·78	9·25	9·58	9·26	6·97	7·25	7·18	7·31	8·59	9·42	9·57	8·80	9·39	10·04
75 .	7·52	7·14	7·10	6·93	6·87	6·68	6·70	7·10	7·29	6·91	5·69	5·76	6·03	5·72	6·57	7·17	7·29	6·77	7·21	7·58
80 .	5·64	5·23	5·24	5·26	5·20	5·00	5·05	5·36	5·49	4·87	5·09	4·62	5·02	4·53	4·97	5·34	5·39	5·16	5·47	5·65
85 .	4·19	3·77	3·83	3·98	3·88	3·71	3·80	3·94	4·19	3·36	3·40	3·76	4·13	3·64	3·75	3·90	4·02	3·98	4·16	4·15
90 .	3·11	2·71	2·80	3·01	2·90	2·75	2·87	2·94	3·16	2·34	2·85	3·11	3·34	2·96	2·84	2·82	3·10	3·10	3·18	3·06

TABLE XX c. *Showing the population living in each age group of five years in England and Wales at different periods, and for some of the healthy and unhealthy districts.*

MALES.

Age.	H_1	H_2	H_3	E_3	E_4	E_5	E_6	E_7	E_8	B_2	S_1	M_1	M_2	M_3	L_1	L_2	L_3	C.B.	U.	R.
0 . .	88172	83567	81573	98569	96925	92015	90400	85459	83175	89523	104694	108236	122117	101099	94770	87714				
5 . .	82118	79452	78107	88154	86922	84773	83895	80978	79446	83608	91640	95178	105383	90998	86384	82361				
10 . .	80215	78444	77205	85374	85038	83598	82710	80059	78512	82594	89863	91929	101466	88554	85086	81413				
15 . .	78421	77393	76296	83028	83154	82450	81457	79057	77617	81468	88245	89867	98406	86408	83825	80444	106260	107214	101298	96603
20 . .	75873	75846	74902	79934	80981	80569	79656	77628	76339	79639	85591	87447	94463	84191	82150	79109	104607	105270	99630	95163
25 . .	73086	73870	73118	76464	77880	78103	77464	75896	74829	77364	82503	83984	88788	81473	80063	77431	102557	102968	97745	93432
30 . .	70228	71550	71205	72679	74315	74991	74910	73812	73074	74709	79187	79319	81260	77912	77173	75272	99913	100187	95623	91475
35 . .	67172	68975	69040	68641	70264	71313	71680	71203	70872	71392	75250	73496	72421	73295	73087	72319	96338	96614	93021	89242
40 . .	64106	66071	66518	64428	65712	67012	67630	68004	68093	67231	69042	66630	62885	67599	68019	68456	91600	92061	89747	86566
45 . .	60813	62730	63514	59603	60447	62021	62837	64025	64546	62175	61224	58896	52566	61155	62142	63637	85540	86236	85526	83263
50 . .	56924	58757	59757	54356	54548	56230	57113	59003	59993	56205	52713	50397	41464	54127	55286	57770	78016	78814	79842	79067
55 . .	52455	53854	55053	47970	47822	49424	50272	52662	54079	49379	43187	41086	30818	45835	47384	50792	68795	69481	72323	73442
60 . .	46776	47598	48817	40934	40174	41373	42103	44836	46554	41758	32258	31091	21660	35774	38534	42649	57727	57978	62425	65745
65 . .	38931	39630	40879	32547	31405	32176	32863	35725	37467	33238	21933	21120	13921	24922	29046	33377	45314	44890	50253	55602
70 . .	29634	29944	30859	23239	22186	22438	22922	25585	27143	24198	13221	12382	7593	15118	19453	23537	31882	30463	35948	42549
75 . .	19460	19216	19626	14156	13266	13230	13468	15454	16652	15285	5885	5972	3288	7653	10923	14192	18863	17072	21522	27583
80 . .	10332	9446	9608	6842	6310	6034	6171	7448	8032	7551	2543	2238	1116	2977	4794	6757	8763	7631	10408	13988
85 . .	4049	3071	3220	2419	2135	1881	1998	2577	2819	2344	907	608	307	785	1519	2265	3031	2504	3667	5032
90 & upwards	1236	586	703	663	516	369	451	589	758	342	114	124	78	125	362	505	794	613	1022	1248
Total population .	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000

FEMALES.

Age.	H_1	H	H_3	E_3	E_4	E_5	E_6	E_7	E_8	B_2	S_1	M_1	M_2	M_3	L_1	L_2	L_3	C.B.	U.	R.
0 . .	88741	81979	79717	97367	92786	88267	86664	81748	79446	82673	99791	102434	113456	96740	88883	82236				
5 . .	82733	78324	76592	87322	83776	81926	80793	77629	76056	77607	87954	90763	98445	87605	81365	77473				
10 . .	80452	77254	75650	84253	82068	80882	79604	76690	75157	76501	86088	87628	94750	85242	80085	76555				
15 . .	78306	76028	74582	82067	80384	79699	78390	75740	74299	75518	84604	85772	92190	83412	79005	75710	97224	100010	95994	93603
20 . .	75610	74333	73137	78708	78042	77779	76802	74539	73228	74542	82787	83682	88967	81463	77762	74748	96030	98427	94646	92206
25 . .	72252	72307	71536	75235	75185	75683	74927	73127	72002	73363	80530	80690	85182	78939	76236	73602	94643	96649	93123	90599
30 . .	69266	69987	69777	71166	72279	72629	72675	71407	70553	71763	77534	76793	79056	75624	74100	72089	92897	94586	91349	88761
35 . .	66014	67493	67813	67292	68622	69397	69923	69275	68767	69619	73494	72141	72577	71637	71077	69934	90540	91895	89191	86664
40 . .	62889	64882	65581	63155	64803	65864	66605	66702	66564	66838	68426	66854	65142	67176	67336	67143	87350	88483	86549	84320
45 . .	59482	62062	63069	58788	60634	61985	62830	63583	63821	63371	62360	60821	56852	62107	62997	63721	83200	84229	83187	81598
50 . .	55935	58741	59990	54170	55988	57329	58351	59715	60326	59164	55330	53762	47834	55989	57786	59447	78025	78813	78836	78084
55 . .	52132	54546	55952	48908	50489	51801	52784	54682	55732	54114	46884	45525	38089	48588	51507	54147	71532	71828	73077	73232
60 . .	46790	49034	50359	42412	43786	44777	45714	48250	49743	48075	36977	36186	28005	39834	44038	47623	63414	62848	65219	66709
65 . .	39572	41726	43010	34592	35550	36383	37140	40436	42127	40415	26797	26198	18639	29802	35321	39577	53549	51893	55425	58090
70 . .	30803	32432	33451	25619	26266	26707	27232	30662	32522	30862	16867	16596	11092	19351	25541	30096	41314	38386	42626	46356
75 . .	20934	21756	22303	16328	16646	16774	17073	19909	21718	20558	8100	8805	5814	10345	15747	19922	27675	24000	27921	31975
80 . .	11648	11593	11790	8393	8554	8354	8526	10567	11646	10701	3901	3754	2610	4364	7712	10661	14940	12035	14886	17937
85 . .	4846	4382	4470	3232	3217	3002	3105	4126	4717	3608	1229	1241	963	1399	2755	4153	5826	4508	5967	7469
90 & upwards	1595	1141	1221	993	925	782	862	1212	1576	706	347	375	337	384	747	1163	1841	1410	2005	2397
Total	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000

XII. Effect of Environment on Length of Life.

A case which has hitherto not been considered now presents itself. The expectation of life is well known. There is, however, a different aspect of expectation. If a person living in an unhealthy environment die at an early age, how much longer would he have lived had he been able to live in a healthy environment? Before proceeding to inquire into this, I wish to make a few remarks on the use of the word 'age'. The phrase 'high age' is often used confusedly in common parlance, the degree of physical weakening due to advance in life, and the number of years lived being imperfectly distinguished. To save this ambiguity, 'high age' will imply the number of years lived, 'old age' will imply the physical state of the persons involved. Thus, I am going to postulate as a general principle that though there are a great many more persons living at higher ages in the country than in the town, the number of 'old aged' persons is the same in both places. This implies that life in the environment of the town brings about senility of the tissues at an earlier age than life in a rural district. If this be not granted then, the succeeding argument must not be taken as proved.

It is not apparently possible by any means at our disposal to make an exact estimate of this ageing process. It can, however, be illustrated graphically (Diagram IX), and I think the approximation obtained by this graphical representation is sufficiently close for the purpose of argument; even though the error be 15 to 20 per cent., the argument remains essentially the same. A comparison is made between the data given in the life tables for the decade 1891–1900 for England and Wales as a whole and for the healthy districts. The age groups are five yearly groups, beginning at the age of 9 years, as at that age the death-rate in England and Wales as a whole and in the healthy districts of England and Wales is identical; so that a person brought up in a town to the age of 9 years and then transferred to the country will have no immediate benefit for the change. This test is rough but sufficient.

In the diagram in the lower part each rectangle gives the number of deaths which occur at each five yearly period of life from 9 years and upwards for England and Wales as a whole. The mean age at which such groups of persons die is assumed to lie half-way along the rectangle; the approximation is sufficiently accurate.

To compare this graph with the graph of the healthy district life table requires a new postulate. This postulate is, that persons dying at any specific age are, taking a considerable average, likely to be a group. Thus persons dying between the ages of 10 and 15 years in the town are likely to die at approximately the same age in the country—the death-rates being essentially the same at these ages: further, persons dying, say, between the age of 50 and 55 years in the town would, on the whole, assuming these to be a group, had they lived in the country, lived six or seven years longer. The graph in the upper part of the diagram, based on the healthy districts for the years 1891–1900, has been constructed on this assumption. The groups of

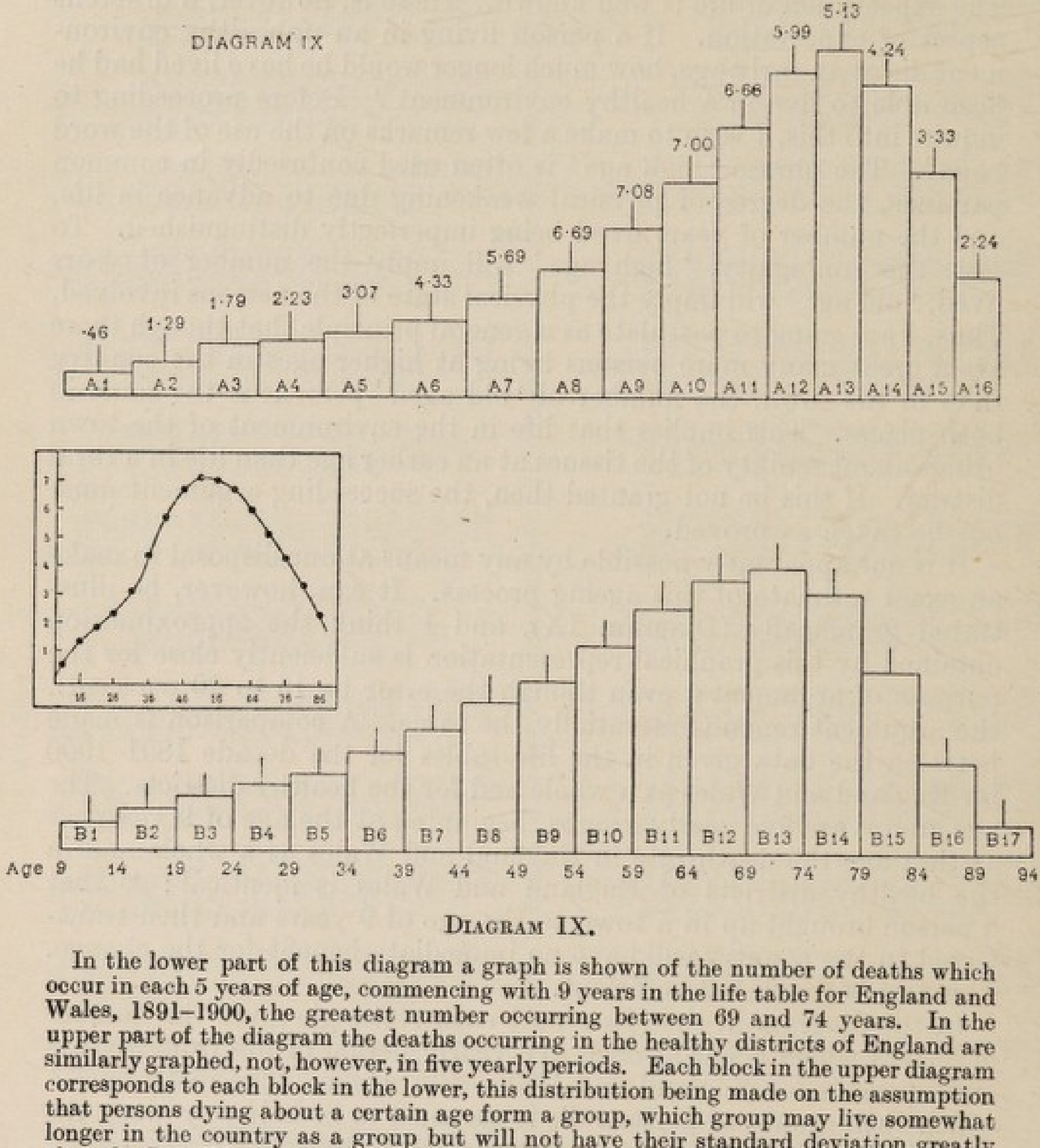


Diagram IX.

In the lower part of this diagram a graph is shown of the number of deaths which occur in each 5 years of age, commencing with 9 years in the life table for England and Wales, 1891–1900, the greatest number occurring between 69 and 74 years. In the upper part of the diagram the deaths occurring in the healthy districts of England are similarly graphed, not, however, in five yearly periods. Each block in the upper diagram corresponds to each block in the lower, this distribution being made on the assumption that persons dying about a certain age form a group, which group may live somewhat longer in the country as a group but will not have their standard deviation greatly altered. It will be observed that in the first part of this graph the bases of each block are longer than in the lower graphs and that this persists until the age of 50 years when the base line of each block becomes smaller, the result being that a like number of deaths has occurred in 5 years in the lower graph and 4 years in the upper. Taking the mean of each block as the average age at death, the differences in length of life are shown by the figures above the blocks in the upper part of the diagram, the maximum increase of life occurring about 50 years. These figures are graphed in an inset between the diagrams to illustrate the manner in which they vary.

deaths which are considered to correspond are marked by numerals. Thus the number of deaths in Block A1 cover a period of nearly a year longer in the healthy districts and correspond to Block B1. In the same way, the number of deaths in Block A11 are equivalent to those in Block B 11. The process is continued to the end of life. It is found that for a considerable time the blocks of equal area have much larger bases in diagram A than in diagram B. About the age of 60 years the bases of the two become approximately equal. After this epoch, however, the bases become much narrower; when Block A is compared with Block B at these ages the same number of deaths occur in a very much shorter time. This relation holds until the end of life.

On the assumption then that each of these blocks of persons correspond, the difference of age at which the same person would die in the two environments corresponds roughly with the difference between the curves placed at the middle point of the rectangle in the diagram. Thus, persons dying at the age of 51 years in the average environment would have, had they lived in the country, a mean life of seven years longer. This is the maximum difference. At higher ages the difference is less. The person who has the potentiality of living to the age of 80 years has a force of life which is more or less independent of environment. His system can stand the 'whips and scorns of time'. It is on the person who is likely to die at the age of 50 to 55 years that the storm of life is most likely to have the effect of producing premature senility.

To supplement this it would be natural to use the life tables constructed by Mr. King for the years 1910–12 which differentiate the rural, urban, and county borough areas of England and Wales more perfectly than the life tables just used. Unfortunately Mr. King does not begin these life tables until the age of 15 years. The start is then later than the period of equal death-rates. The process just shown has, however, been applied to Mr. King's tables and with the same result. The difference between the rural areas and the county boroughs are very much the same as that between the two life tables for the decade 1891–1900, while the data referring to urban districts fall between.

XIII. Phthisis.

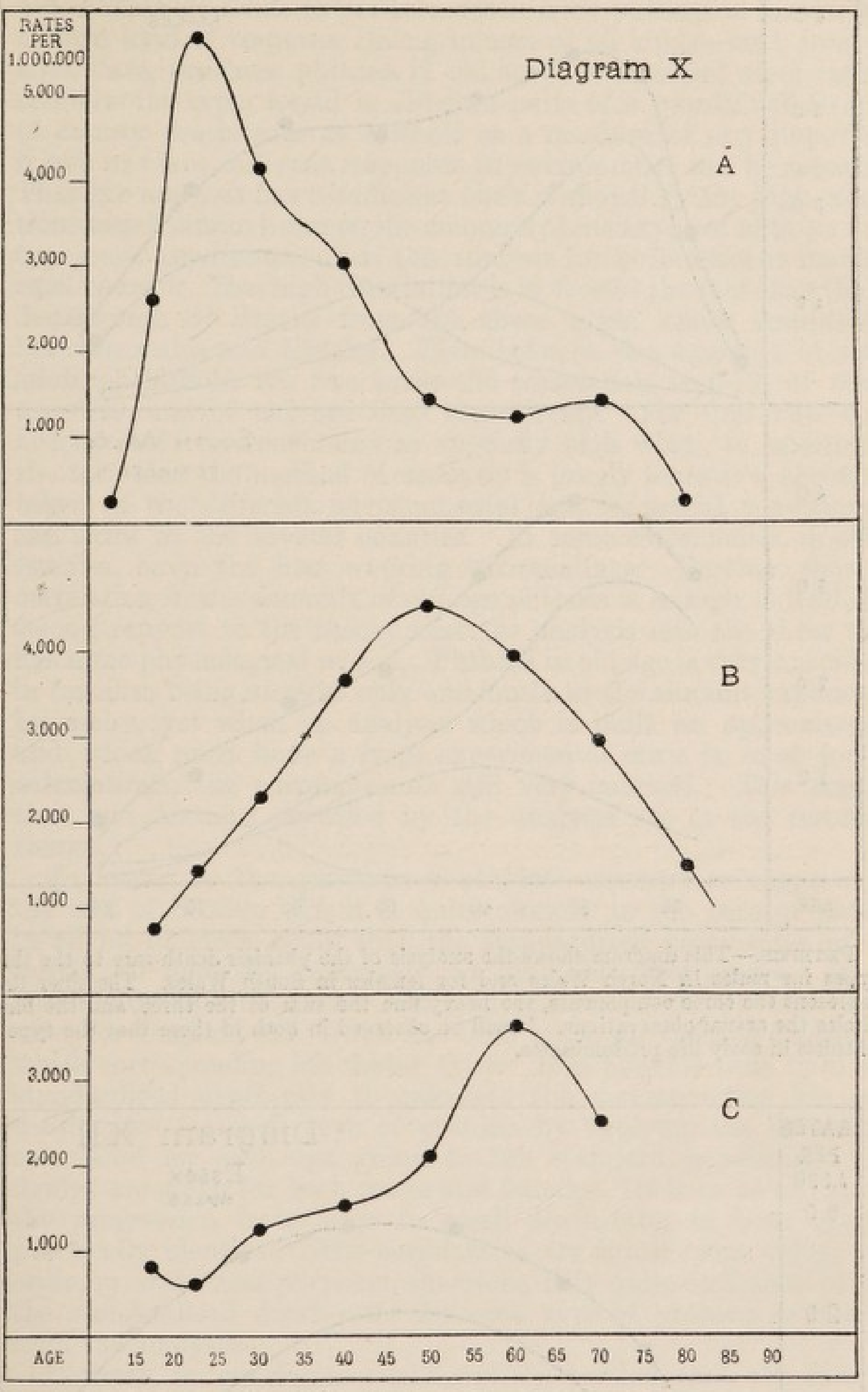
'The consumption a flattering disease cozening men into hope of long life at the last gasp.'

Up to the present the discussion of the matter at issue has been general. It is now necessary to proceed to the particular. The disease phthisis offers one of the best tests of the validity of rough comparisons. It is selected because it is still the battle-ground of the clinical statisticians. Every medical journal contains pages in which crude death-rates from phthisis calculated against the whole population are offered for purposes of comparison. I think it is fair to remark that on whatever theory the prevalence of phthisis at different ages and in different environments is to be explained such comparisons are of little value. Consider, for instance, the age distribution of the death-rates among males from phthisis in Shetland, in London, and among ironstone miners. These are shown in the accompanying diagram (Diagram X). It is obvious at a glance that the greatest death-rate in Shetland is between the ages of 20 and 25 years: in London between the ages of 45 and 55: among ironstone miners between 55 and 65 years. How the understanding of such phenomena is helped by expressing death-rate differences by any single unit passes my comprehension. Poincaré has remarked that mathematics are sometimes a nuisance: the epigram may certainly be justly applied to such statistical methods.

But the matter goes much deeper than figures. The phthisis death-rates in the young adult are distributed and react to environment in complete contrast to the death-rates due to phthisis in middle age. The former tends to be a disease of rural districts, the latter one of the towns. Further, as far as present evidence goes, bad feeding, sitting in wet clothing, and such-like causes are the predisposing elements in young adult phthisis. On the other hand, middle age phthisis is associated with depression of health due to industrialism. I have made elsewhere an attempt to distinguish statistically what I think different types of phthisis, namely, those of youth, middle age, and old age. The method is given in Section XXVII.

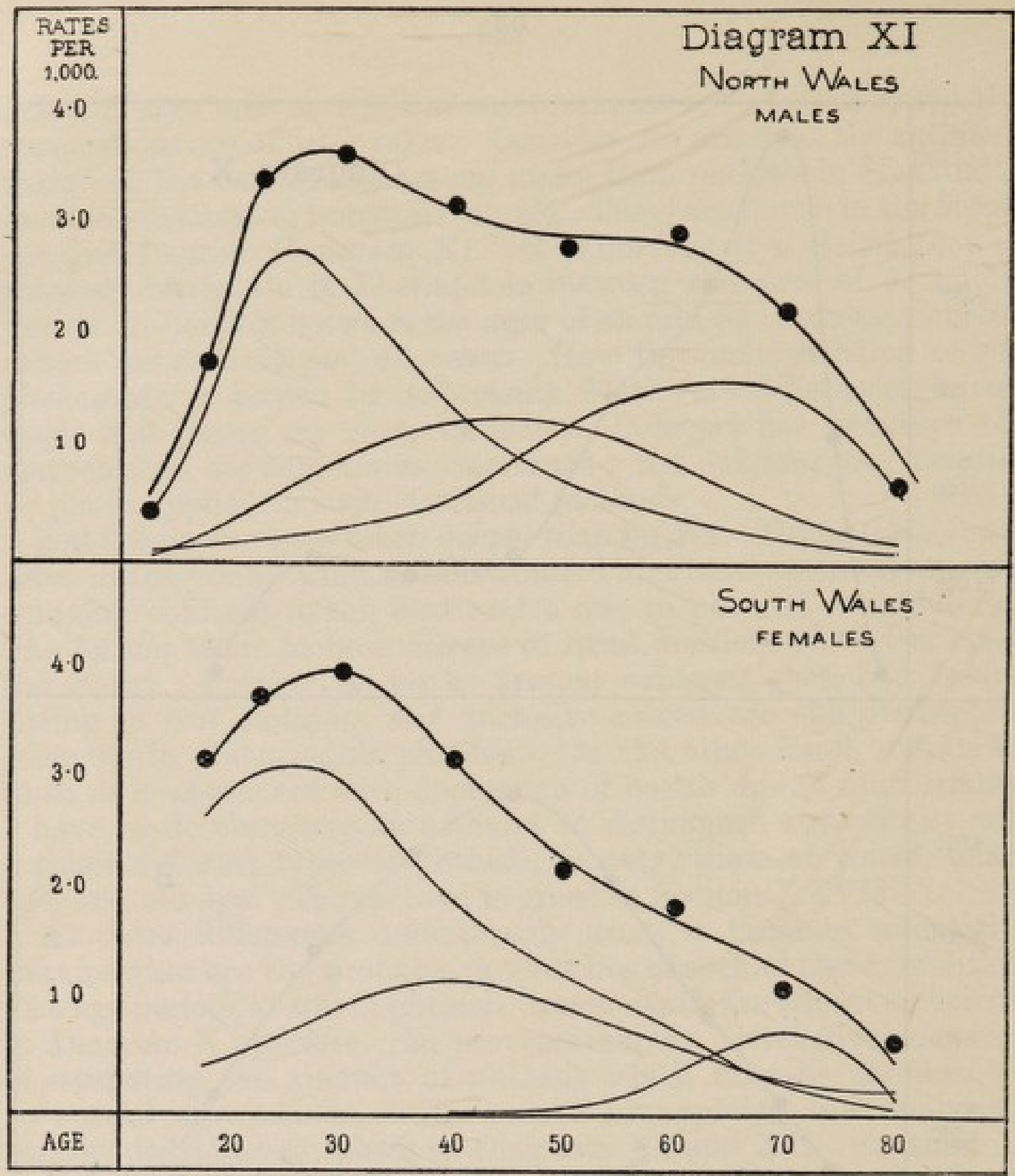
As these differences undoubtedly exist, it becomes essential to inquire what are the probable proportions of each of these conditions. The age periods at which phthisis causes death are shown sufficiently in Diagram X opposite. In previous papers I have given a method of estimating the amount of phthisis which may be assumed due to special age periods. The results of applying this analysis to certain districts are shown in Diagrams XI and XII. In these the actual death-rates are shown by solid black circles, the death-rates due to each component by thin lines, and the theoretical complete death-rate, the sum of the components at each age period, by a continuous heavy line. It will be observed that in districts of very various life conditions the analysis holds. In North Wales the chief amount of phthisis among males is in early adult life and in old age. In South Wales the same holds among females except that phthisis among them is much more uncommon in old age. Essex, on the other hand, may be taken as a type of county in which the amount of phthisis among young adult and elderly persons is small, the bulk of the deaths occurring in middle age. The method of analysis is given in an improved form in Section XXVII. As now offered it permits the direct calculation of the standardized death-rates from each type of phthisis in any district.

The question as to whether the death-rate can be split up in this manner is one which of course demands a good deal of justification. But the diagrams show how the phenomena in different places and districts can vary. The separation appears therefore essential. One kind of condition, namely a life in rural environment, is observed to be associated with death from phthisis at young adult ages; another responding to environmental conditions such as those in

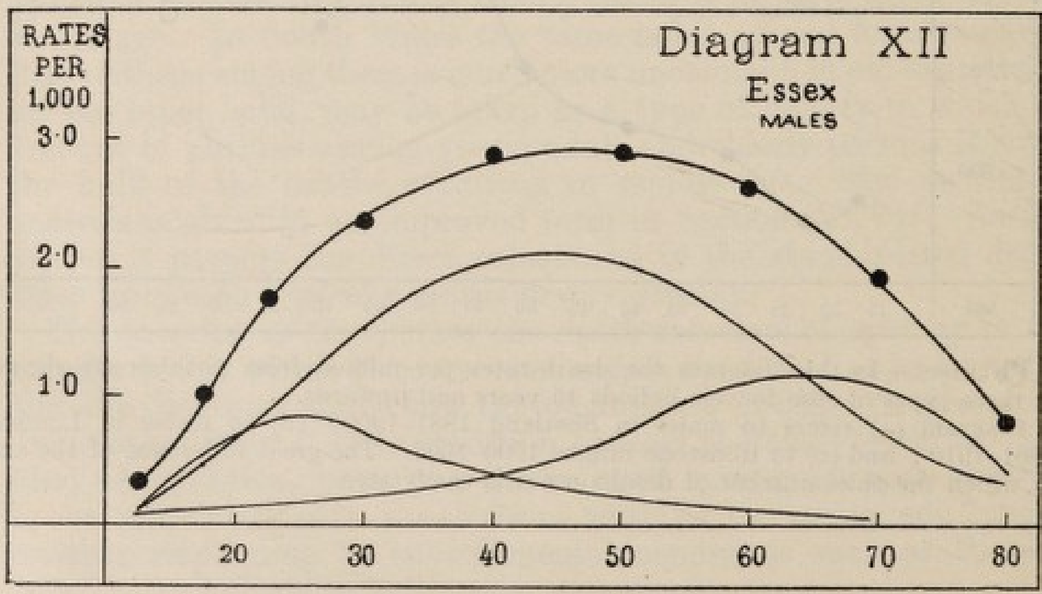


PHTHISIS.—In this diagram the death-rates per million from phthisis are shown in three types of case for age periods 15 years and upwards.

Diagram (A) refers to males in Shetland 1881–1900; (B) to males in London 1901–1910; and (C) to ironstone miners 1900–1902. The great difference of the age at which the chief number of deaths occur is easily seen.



Phthisis.—This diagram shows the analysis of the phthisis death-rate to the three types for males in North Wales and for females in South Wales. The finer lines represent the three components, the heavy line the sum of the three, and the black circles the actual observations. It will be observed in both of these that the type of phthisis in early life predominates.



Phthisis.—This diagram shows the same analysis for phthisis among males in Essex. In this case much the greatest amount of phthisis is at middle age.

urban districts tends to produce death from phthisis in middle age; a third kind of response among miners of all kinds—coal, iron, tin, lead, slate, produces phthisis in old age. It does not seem right to combine the types found in different parts of a county and to speak of county death-rates as a whole as a measure of any importance if two or three different responses to environment can be separated. That the analysis has a sufficient basis is shown by the high correlations which obtain between the amounts of each type of phthisis in the two sexes ascertained when the analysis for both sexes is made for each county. This high correlation is in face of the fact that the age distribution of deaths from the three types varies considerably between males and females. Thus between the amounts of young adult phthisis in the two sexes the correlation is 0·79, of middle age 0·76, and of old age 0·60 respectively. The first two correlations must be considered as specially high when, in addition to the fact that the method of analysis is purely tentative, account is taken of the different environmental and industrial conditions of the sexes in the several counties. In some cases males, in others females, have the best working surroundings. Further, that the correlation in the amounts of old age phthisis is as high as 0·60 lends strong support to the theory that the analysis into the three types has some physiological reason. Phthisis in old age is very uncommon in females, being at most only one-fourth of the amount experienced by males, yet when an analysis which is built on approximation and which must have a large experimental error is used for the calculations, the correlation is still very marked. This suggests that the features revealed by the analysis are in the nature of things.

To return to the question of absolute measure as suggested at the end of Section VII, it is quite obvious in the case of phthisis in all its different types that the standardized death-rates are for nearly all purposes equivalent to life table death-rates. In the accompanying table (Table XXI) an analysis of the phthisis death-rates for England and Wales has been made for the five decades for which corresponding life tables exist. It is possible thus from each standardized death-rate to calculate the corresponding life table death-rates for each type of phthisis by applying the death-rates calculated for each age group to the standard population. The figures are given for both males and females. It is to be noted that the progression from large to small death-rates in both cases is practically identical. The correlations are in all cases unity. For ordinary statistical purposes, therefore, it is quite sufficient to take the standardized death-rates for each type of phthisis as derived from the analysis.

It must, in conclusion, be definitely posited that the lumping together of the whole phthisis deaths independently of their age grouping is in the present state of knowledge unjustifiable.

Table XXI. *Showing the standard death-rates and the life table death-rates for each type of phthisis among males at ten years and upwards, and females at fifteen years and upwards, in England and Wales, 1851–1910.*

MALES

	'Young Adult' Type.		'Middle Age' Type.		'Old Age' Type.	
Date.	*Standard death-rate.*	*Life table death-rate.*	*Standard death-rate.*	*Life table death-rate.*	*Standard death-rate.*	*Life table death-rate.*
1851–1861 . .	1·52	1·40	1·12	1·20	0·49	0·61
1871–1880 . .	0·88	0·81	1·71	1·84	0·24	0·29
1881–1890 . .	0·55	0·51	1·58	1·72	0·24	0·30
1891–1900 . .	0·32	0·30	1·45	1·59	0·21	0·26
1901–1910 . .	0·24	0·21	1·16	1·27	0·26	0·33
	$r = 1{\cdot}000$		$r = 0{\cdot}999$		$r = 1{\cdot}000$	

FEMALES

1851–1861 . .	2·14	1·93	1·42	1·41	0·124	0·178
1871–1880 . .	1·38	1·24	1·36	1·35	0·068	0·098
1881–1890 . .	0·93	0·82	1·22	1·19	0·065	0·096
1891–1900 . .	0·52	0·45	1·05	1·03	0·058	0·087
1901–1910 . .	0·41	0·35	0·79	0·76	0·064	0·102
	$r = 1{\cdot}000$		$r = 1{\cdot}000$		$r = 0{\cdot}995$	

Equations connecting the life table and the standard death-rates.

	Males.	*Females.*
'Young Adult' type	$y = 0{\cdot}9233x + 0{\cdot}004$	$y = 0{\cdot}9144x - 0{\cdot}028$
'Middle Age' type	$y = 1{\cdot}0798x + 0{\cdot}008$	$y = 1{\cdot}0296x - 0{\cdot}055$
'Old Age' type	$y = 1{\cdot}2505x - 0{\cdot}002$	$y = 1{\cdot}3615x + 0{\cdot}085$

where y denotes the life table death-rate, and x denotes the standardized death-rate.

XIV. Sarcoma and Cancer.

'Some sicknesses besot, others enrage men, some are too swift, others too slow.'

The mortality from cancer affords a second illustration of the difficulties which arise in dealing with death-rates. A very cursory inspection of the figures shows that their interpretation is not easy. Thus considering the figures in Table XXII, figures calculated from the data given by the Registrar-General for the years 1911–1914, it is found that in the county boroughs, the urban districts, and the rural districts, the age period at which the highest cancer death-rates occur is quite significantly different. The figures in this table relate to years in which the deaths from malignant disease were carefully distributed to the districts to which they belong. The town rate is therefore not altered by the fact that many residents in rural districts go to city or urban hospitals for operation. The figures must therefore be taken as representing the truth so far as the data available at present suffice for its discovery. Considering the figures it is at once obvious that the ages at which the highest death-rates are observed to occur from cancer are very markedly differentiated between the town and the country. These differences found to exist between county boroughs and rural districts have been, after a close criticism by Professor Pearson and Dr. Tocher,

TABLE XXII. *Showing the death-rates per million at five yearly age periods from certain diseases on the data given by the Registrar-General for the years 1911–1914 among males. The figures are given separately for county boroughs, urban districts, and rural districts. The standardized death-rate is based upon the mean population 1891–1900, and the life table death-rate on Mr. King's tables.*

	Standardized death-rate. 15+	*Life table death-rate. 15+*	Death-rate per 1,000,000. *15–*	*25–*	*35–*	*40–*	*45–*	*50–*	*55–*	*60–*	*65–*	*70–*	*75–*	*80–*	*85+*
CANCER:															
County Boroughs	1391	1985	46	120	308	737	1472	2576	4069	6100	7622	9398	9861	8971	6966
Urban Districts	1260	2014	41	107	277	584	1180	2159	3627	5382	7258	9062	9824	8802	8856
Rural Districts	1106	1993	38	97	209	499	998	1685	2938	4721	6327	8436	9846	9260	8218
VALVULAR DISEASE OF HEART:															
County Boroughs	848	1142	171	216	400	603	884	1326	1963	2958	3861	4981	5968	4967	4390
Urban Districts	698	1069	127	145	247	417	615	960	1503	2444	3693	4902	5640	5411	5382
Rural Districts	617	1096	87	116	205	327	488	767	1236	2019	3228	4994	6147	6068	6385
DIABETES:															
County Boroughs	178	239	40	61	93	94	130	230	404	611	1028	1042	1211	842	902
Urban Districts	170	249	42	61	77	83	131	198	376	703	866	1060	1037	913	651
Rural Districts	151	224	57	75	91	103	115	186	287	493	616	829	823	708	359
PNEUMONIA:															
County Boroughs	1030	1254	320	608	931	1111	1413	1615	2077	2540	2886	3253	3563	3562	4150
Urban Districts	680	888	228	370	571	680	845	1104	1292	1748	2011	2598	2518	2658	3634
Rural Districts	470	661	164	216	425	493	625	760	884	1034	1327	1794	2174	2399	3234
CIRRHOSIS OF LIVER:															
County Boroughs	189	256	2	23	113	188	379	485	628	685	784	699	535	302	120
Urban Districts	170	246	2	17	80	193	298	435	559	684	739	643	462	286	274
Rural Districts	128	198	3	22	51	142	212	283	400	558	543	484	494	348	194
BRIGHT'S DISEASE:															
County Boroughs	651	893	59	138	277	478	795	1197	1683	2467	3189	3693	3870	3282	3849
Urban Districts	538	820	49	108	202	379	574	924	1322	1985	2663	3565	3680	3380	3839
Rural Districts	403	686	44	84	146	258	360	693	907	1456	1917	2974	3150	2770	3234
PHTHISIS:															
County Boroughs	2129	2253	1259	2026	2644	2901	3163	3315	3041	2727	2118	1169	755	453	361
Urban Districts	1363	1406	925	1401	1670	1725	1787	1851	1791	1735	1182	780	456	245	171
Rural Districts	1099	1082	793	1368	1321	1312	1203	1181	1223	1049	867	713	375	252	138

considered to represent real differences quite beyond the error of random sampling. It is impossible to contradict the results of this investigation. But the problem as they treat it is stated in terms which concern statistical and not biological requirements. The effect of different environment on the physiology of the inhabitant has not been considered. The fact that the death-rate from cancer at ages 70 to 75 years is very much higher in the county boroughs than in the rural districts does not permit of any deduction until the question has been settled how far its occurrence is due to ageing of the tissues. As it has been seen that in towns the same degree of senescence is found at an earlier age than in the country, until this degree of difference has been measured, it is impossible to settle at what ages a comparison should be made between city and country death-rates.

To advance beyond this stage of the argument it is necessary to criticize the results obtained when the routine methods of calculating death-rates are applied. The standardized death-rates, taking the mean population of England between 1891–1901 as the standard population, have thus far been calculated. The values are given in Table XXII. It will be observed that judging by this measure there is considerably more cancer in towns than in country. But I do not think this measure can be considered valid. As we have already seen, a person dying in the city at the age of 50, dies on an average about six years earlier than he would have died had he lived in the country. Either this is due to premature ageing or due to greater exposure to the infective diseases of later age, namely pneumonia and bronchitis. Now cancer is commonly believed to be a degenerative disease, that is, a disease to which the ageing of the tissues predisposes. Chronic irritation rarely produces cancer in youth, but chronic irritation tends to produce cancer in middle age. In fact, the chief sites of cancer are those regions of the human body most subject to chronic irritation. The necessary allowance for the ageing of the body experienced in towns over that experienced in the country must thus first be made before any differentiation between the amounts of cancer present in different districts is justifiable, though here the possibility of local ageing of tissues must not be overlooked, a phenomenon well known when problems of local immunity are the seat of question. Thus cancer of the stomach and of the uterus are more common between the ages of 65 and 75 years than above that age, while cancer of the lip is three times more prevalent at ages over 75 years than in the previous decade.

In the case of cancer a method of comparison is, however, in our hands. Mr. King has constructed life tables for London, for the county boroughs, for the urban districts, and for the rural districts, and these tables are based on the same data as are used by Dr. Stevenson in calculating the cancer rates for each age period. A comparison between a standard and a stationary population is thus immediately possible. When the appropriate death-rates for each age group are applied to the respective age groups in a stationary population the true life table death-rates from cancer are discovered. Contrary to what we have already had reason to believe was the case, namely that cancer was more prevalent in towns, it is found that the life table death-rate is constant. This means that, taking an equal number of persons living above the age of 15 years, exactly the same proportion is destined to die from cancer whether the individuals live in a rural district, an urban district, or in a county borough. A common criticism of such a result is that all the deaths from cancer have been grouped together and that is not medically admissible. To meet this objection the death-rates from cancer of the stomach have been taken out separately. The result is the same.

There is, however, another way of looking at the subject. It may be asked what proportion of persons attaining the age of 15 years of age in different environments are likely to die of cancer in after-life. This question can be quite easily answered from the figures given in the life tables. From the life table population as the cancer death-rates at each age are known the number of deaths above 15 years of age can at once be obtained. The life table also gives the number of persons attaining the age of 15 years. It is thus at once calculated that out of 10,000 people attaining the age of 15 years 919 are destined to die of cancer in the county boroughs, 987 in the urban districts, and 1,025 in the rural districts. Thus the paradoxical conclusion is arrived at that more cancer, an equal amount of cancer, or less cancer exists in the county boroughs than in the rural districts according as the standardized death-rate, the life table death-rate, or the survivors of those who attain the age of 15 years is taken as the criterion. This example makes plain the great care necessary in drawing conclusions from figures.

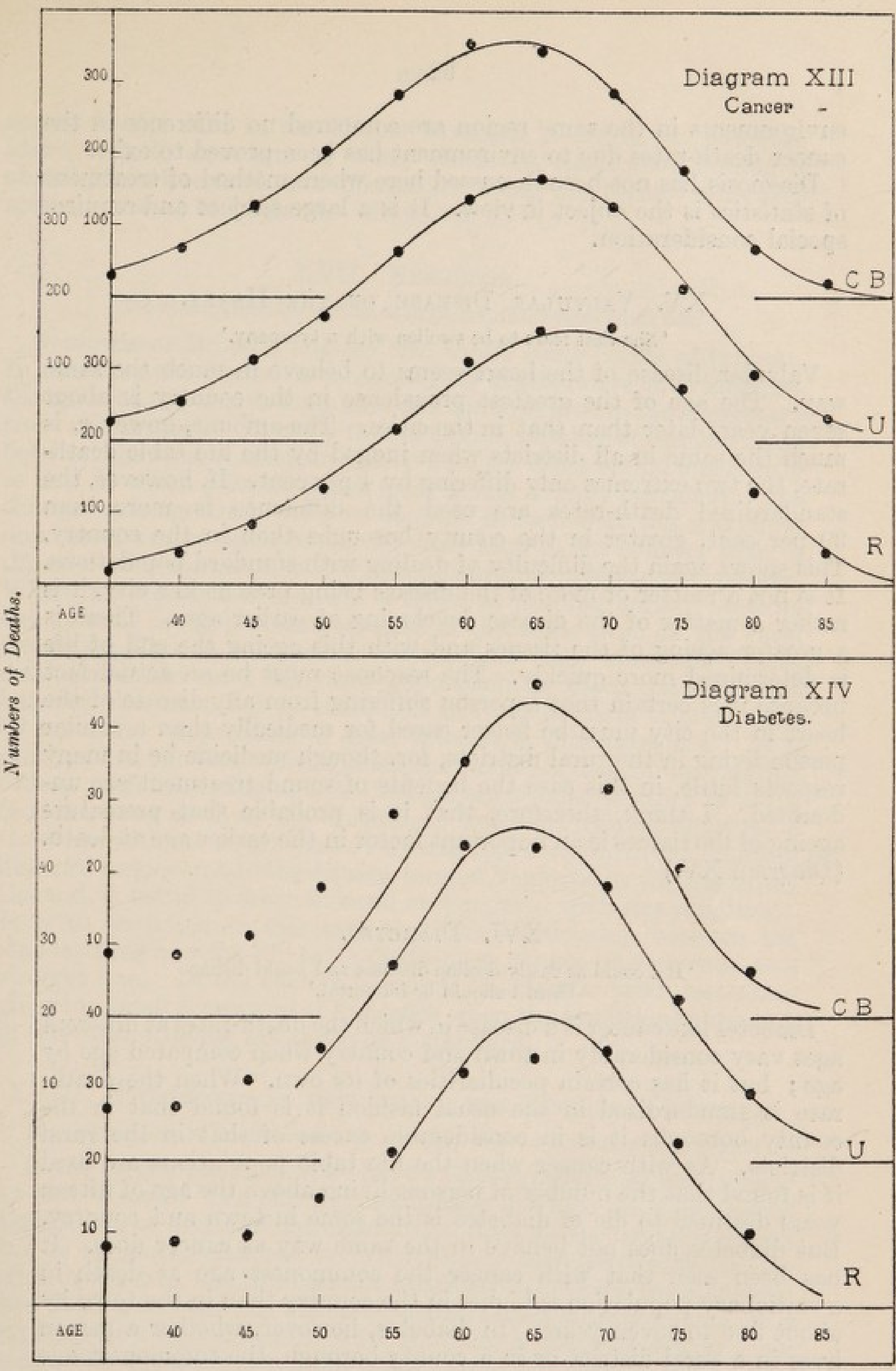
The figures given in the preceding paragraphs refer to the average of the rural districts and county boroughs of the whole country. Thus the rural districts of the south are combined with the rural districts of the north, and it is the same with the urban districts and the county boroughs. That is, when the average of the whole country is taken for rural districts, urban districts, and county boroughs, there is an exact equality in the number of persons living above the age of 15 years destined to die of cancer in these different environments. The figures for London to a certain extent represent an exception, the rates being considerably higher than those seen in the other three districts. But London is not an average of the whole of England. Therefore before theorizing it is necessary to make further inquiry. Either London has a cancer rate of its own or a large part of the underlying aetiology has not been revealed by the method by which the statistics have just been examined.

The further examination of this point would have been impossible had not the department of the Registrar-General kindly placed at my disposal a series of life table data calculated for different districts of the country but not published. We have thus information for the north of England, for Wales, for the midland, and for the southern counties of England. For the present inquiry such large districts are sufficient. In this case the rural districts in each group of counties are not separated from the included towns and county boroughs: each average is for the whole of the district. When the cancer death-rates at each age period calculated from the population and deaths in these groups are applied to the corresponding life table populations it is found that the discrepancy observed with regard to London when it was compared with the average of the rural districts and county boroughs for the whole of England is in accordance with a second general law. It is found as we pass from the north to the south that the amount of cancer very considerably increases. In the north the life table death-rate at ages above 15 is 1·78 ; in Wales and the midlands respectively 1·97 and 1·95 ; in south England 2·24 and in London 2·16. Though London thus presents a slightly lower rate than the south of England it falls in line with that district. The death-rate from cancer thus increases as the latitude decreases.

It has already been noted that an earlier age incidence of a disease such as measles in a town might be associated with more frequent epidemics of disease. Even in an adult disease such as cancer if that were due to an infection a similar phenomenon might also be present. It has been shown that the age at which the greatest number of deaths from cancer occurs in county boroughs is earlier than in the rural districts. But the argument from measles does not hold. Association with infection is not required, as in their susceptibility to the disease which is due to increasing senility persons living in towns must on the average succumb at an earlier age than persons living in the country. In consequence of this earlier senility an earlier mean age at which the disease occurs must be expected. To argue from an earlier average age at death from cancer in towns than in the country to the fact that cancer is an infectious disease is therefore impossible.

It is necessary, however, to try to estimate more correctly the difference between the age of the maximum number of deaths from cancer in the three districts. In Diagram XIII, therefore, the number of deaths occurring at each age in a stationary population is shown. It is easily observed that the age of maximum mortality in the county boroughs is at least five years earlier than the maximum age at death in the rural districts and that the form of the curves is nearly identical. The graph of urban districts mortality lies almost exactly between that of the county boroughs and that of the rural districts. The exact difference between the age of maximum fatality in city boroughs and rural districts is, however, not directly measurable. The form of the distribution of deaths, however, very closely agrees with the curve called by Professor Pearson, Type III. Such curves have accordingly been calculated and their course is shown in the diagram (Diagram XIII) by a continuous line. The difference between the maxima given by the curves for the county boroughs and the rural districts is seven and a half years. ' High aged ' persons are more common in the country, ' old age ' equally common.

The conclusion seems to be that the frequency of cancer is a function of senescence and can only be estimated by the method originally introduced by Dr. Farr where the appropriate stationary populations are taken as the standard population and used to calculate the respective total death-rates. It is to be distinctly understood that this function of senescence may differ in different regions, and all that has been shown is that when different



CANCER.—This diagram illustrates for the rural districts, the urban districts, and the county boroughs the number of deaths due to cancer occurring at each age period above the age of 35 years in the life table populations appropriate to each group of districts. These deaths have been graduated to the curve called Type III by Professor Pearson, the actual observations being shown by black circles. The total number of deaths is in each group of districts the same. It will be observed that the age at which the largest number of deaths occur is progressively later in life the healthier the conditions of life.

DIABETES.—This diagram shows the same facts for diabetes, the graduation curve being a normal curve of error. The deaths in the life table population are again the same in the groups of districts but differ from cancer in that the age period at which diabetes is most common is constant.

C.B. = county boroughs. U = urban districts. R = rural districts.

environments in the same region are compared no difference in the cancer death-rates due to environment has been proved to exist.

Diagnosis has not been discussed here where method of treatment of statistics is the object in view. It is a large subject and requires special consideration.

XV. Valvular Disease of the Heart.

'She that fears to be swollen with a tympany.'

Valvular disease of the heart seems to behave in much the same way. The age of the greatest prevalence in the country is about seven years later than that in the cities. The amount, however, is much the same in all districts when judged by the life table death-rate, the two extremes only differing by 4 per cent. If, however, the standardized death-rates are used the difference is more than 30 per cent. greater in the county boroughs than in the country. This shows again the difficulty of dealing with standard populations. It is not a matter of more of the disease being present in a city, it is rather a matter of the disease developing at earlier ages. There is a greater ageing of the tissues and with this ageing the end of life is determined more quickly. The response must be an actual fact because it is certain that a person suffering from any disease of the heart in the city must be better cared for medically than a similar person living in the rural districts, for, though medicine be in many respects futile, in this case the benefits of sound treatment are undoubted. I think, therefore, that it is probable that premature ageing of the tissues is an important factor in the earlier age at death. (Diagram XV.)

XVI. Diabetes.

'If I could as easily decline diseases as I could dislike them I should be immortal.'

Diabetes like cancer is a disease in which the death-rates at different ages vary considerably in town and country when compared age by age ; but it has certain peculiarities of its own. When the death-rate is standardized in the usual fashion it is found that in the county boroughs it is in considerable excess of that in the rural districts. As with cancer when the life table populations are used it is found that the number of persons living above the age of fifteen years destined to die of diabetes is the same in town and country. But diabetes does not behave in the same way as cancer does. It has been seen that with cancer the commonest age at death in a stationary population is higher in the country than in the town by about five to seven years. In diabetes, however, whether a person lives in a rural district or in a county borough, the commonest age at death is the same. The degenerative factors which dispose to diabetes are thus of a different kind from those which predispose to cancer. This difference is in accord with what is found in other branches of the theory of immunity. It tends, however, to make the association of the two diseases in the same conditions somewhat doubtful. This association, shown to exist by Dr. Maynard with regard to the data in the United States of America, has not been found to hold elsewhere by other observers, Greenwood finding it absent in Switzerland, and Claremont, using the variate difference method, absent in England. (Diagram XIV.)

XVII. Nephritis.

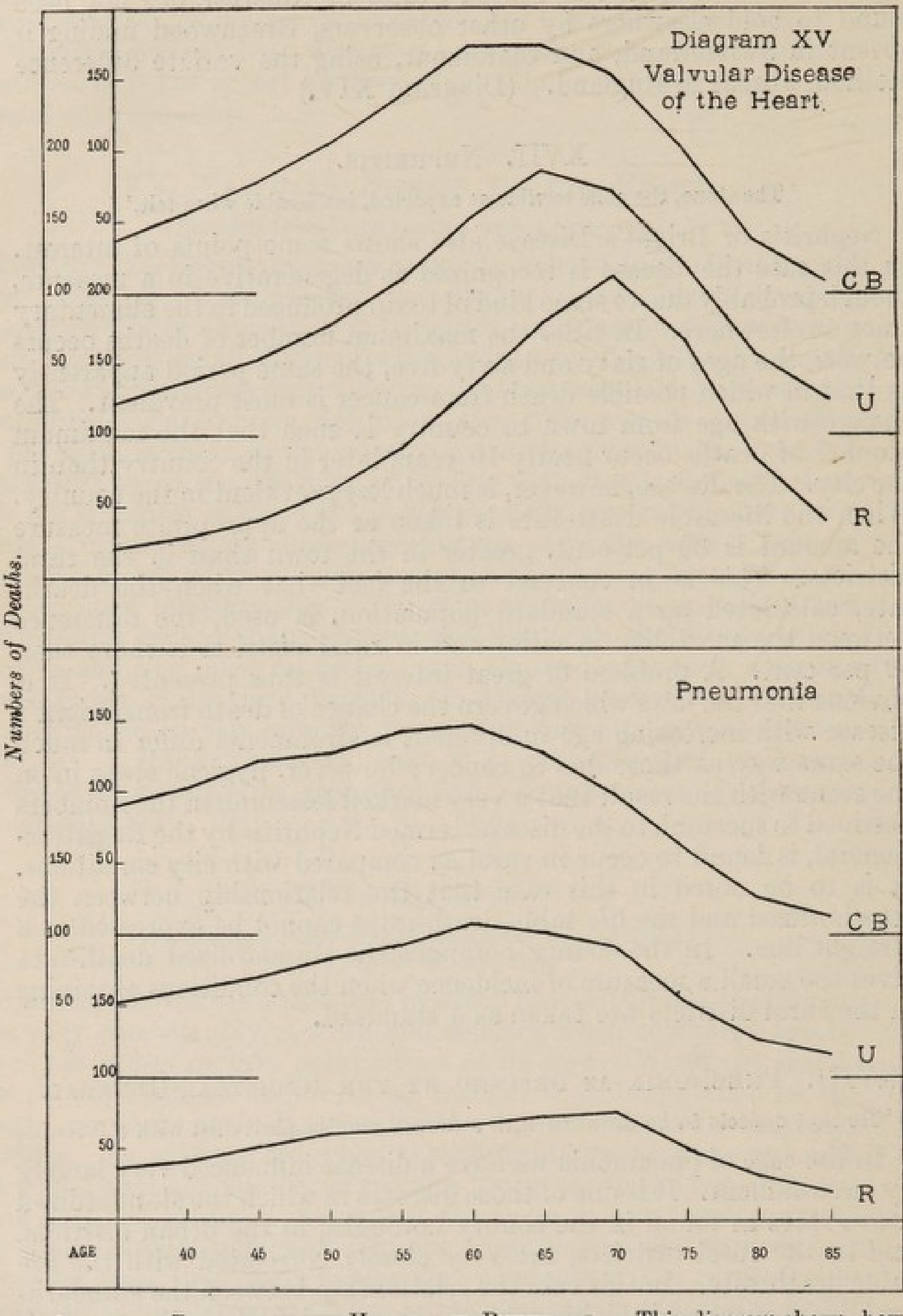
'The stone, the colic terrible as expected, intolerable when felt.'

Nephritis or Bright's Disease also shows some points of interest. In this case the disease is recognized as degenerative in a measure, though probably due to some kind of toxin produced in the alimentary tract or elsewhere. In cities the maximum number of deaths occurs between the ages of sixty and sixty-five, the same period apparently as that in which possible death from cancer is most prevalent. The change with age from town to country is such that the maximum number of deaths occur nearly 10 years later in the country than in the city. The disease, however, is much less prevalent in the country. When the life-table death-rate is taken as the appropriate measure the amount is 30 per cent. greater in the town than in the rural districts. This is in contrast to the fact that when the death-rate, calculated on a standard population, is used, the difference between the mortality in cities and in rural districts is more than 50 per cent. A problem of great interest is thus presented. It is obvious that the laws which govern the chance of death from Bright's disease with increasing age in different environments differ in much the same way as those due to cancer; however, hygiene steps in on the scene with the result that a very marked lessening in the numbers destined to succumb to the disease, termed Nephritis by the Registrar-General, is found to occur in rural as compared with city conditions. It is to be noted in this case that the relationship between the standardized and the life table death-rates cannot be expressed as a straight line. In the county boroughs the standardized death-rate gives too small a measure of incidence when the conditions obtaining in the rural districts are taken as a standard.

XVIII. Pneumonia as defined by the Registrar-General.

'He that expects to be drowned with a dropsy may be shrivelled with a fever.'

In the case of pneumonia we have a disease influenced very largely by environment. It is one of those diseases in which the standardized death-rates as found in the county boroughs, in the urban districts, and in the rural districts, are very closely correlated with the life table death-rate. In this case the relationship between the standardized death-rates and the life table death-rates is strictly linear. It is expressed by $D_2 = 1{\cdot}0589D_1 + 163{\cdot}3$, which is accurate within $\frac{1}{3}$ per cent. The disease thus resembles phthisis in this characteristic. The manner in which the age incidence of this disease varies is shown in the diagram (Diagram XV). The difference between the ages of maximum susceptibility in town and country is about 7–10 years. The maximum occurs in the county boroughs about 60 years of age and in the country at nearly 70. At the earlier ages of life the disease is about twice as common in the town as in the country, and



Valvular Disease of the Heart and Pneumonia.—This diagram shows how the number of deaths vary in a stationary population in the same groups of districts for valvular disease of the heart and pneumonia. In each of these diagrams the maximum number of deaths occur progressively later as we pass from county boroughs to urban and rural districts. The total death-rates from valvular disease of the heart is practically the same in the three groups of districts. In pneumonia, however, it is very much less in the rural districts than in the urban districts and county boroughs.

this relationship holds till the age of 60, when the numbers become nearly equal, while after 75 years of age the numbers dying in the rural districts are nearly thrice those in the towns. With such relationships of the number of the deaths it is obviously not possible to compare the death-rates at different ages in different conditions, so far as these are expressed quantitatively by means of 'crude' or 'standardized death-rates'.

XIX. General Survey.

The facts given in the previous pages regarding the death-rates in different environments may now be surveyed. I think it may be taken as most probable that the most satisfactory measure of the amount of disease is a life table death-rate. The effect of two such different environments as that of a city or a rural district cannot be compared directly without considering how far the influence of the surroundings has changed the average vital processes of the individual. What happens is not known, but I think it must be admitted that a person living in city conditions tends to age more rapidly than a person living in rural surroundings. It is not merely that the infective processes, which determine deaths in old age, are more generally found in the town. It is that the conditions which tend to depress vitality also tend to produce premature senility. The range in age vitality between town and country in the years 1910–1912 is equivalent to a difference of six years. Very nearly the same figures are found to describe what happens in cancer and nephritis in the two types of districts. In both of these diseases the maximum age of death is about seven years earlier in the town than in the country, a number identical with that already found when all diseases are considered together.

If the life table death-rate be thus taken as the standard it requires to be considered how far standardized death-rates can be justified for ordinary use. It has been found that the standardized death-rate from all causes is in direct linear relation with the life table death-rates within an error not more than ½ per cent. For general hygiene, therefore, the standardized death-rate may be considered accurate.

When special diseases are considered, however, a very different problem has to be faced. In such diseases as phthisis and pneumonia the standardized death-rates are in linear relationship with the life table death-rates. The one can be calculated directly from the other. The standardized death-rates thus fulfil their purpose. With regard to cancer, diabetes, and nephritis, it is found that the life table death-rate is constant in county boroughs, urban or rural districts. The standardized death-rate having thus no relation to the life table death-rate, cannot therefore be used as a criterion. In other words, if the correlation between the standard and life table death-rate is high, the standardized death-rate is a sound basis of deduction. If, on the other hand, it is low, no conclusion can safely be drawn.

I think, further, that comparison of death-rates from one disease at a definite age cannot be employed. The whole change in the age distribution of disease is part of a general process, and to compare, for instance, the death-rate from cancer in town and country, between the ages of 55 and 65 years is to compare sections of the population which consist of different elements, as it has been rendered probable that the inhabitant of the country is at these ages biologically about six to seven years younger than the inhabitant of the town.

PART II. MATHEMATICAL TREATMENT

XX. Introduction.

Having discussed the main points with reference to death-rates and the connexion between hygienic conditions and the death-rate, it remains briefly to give a description of the different methods by which those desirous of making inquiry into health conditions may most conveniently work. For many purposes it is necessary to calculate one or other of the life table data, and the more easily this can be done the more likely is investigation to be undertaken. It is proposed in this section to give first a general theory of life table mathematics and to follow this by examples of the method of calculation. The mathematical theory is given in a form which I think is new, but the equations are so simple that it is highly probable that they have been published already. Methods of calculating devised by Dr. Snow and myself follow. Some of the methods given by Dr. Snow are essentially the same as those used by myself, though arrived at independently, but there are differences. Dr. Snow's method is to calculate, first the number of survivors at any age ; from thence derive the population and the expectations. I prefer to calculate the expectations and the number of survivors separately, and obtain the population by multiplication. For an individual life table, probably my method will take longer than Dr. Snow's, but if a large number are required, and if half a dozen or more are calculated at the same time, the fact that constant multipliers are used will probably render my method the more convenient. The relative convenience of both must, however, be tested in practice. It is to be noted that my method has been devised to use the data in the form in which they are recorded by the Registrar-General. The populations, deaths and death-rates are given by him for the registration districts in separate years of life up to 5 years : in five yearly age groups of life up to 25 years : in ten yearly age groups from this to the age of 75 years, while above this age no further age distinction is made. The mathematical section which follows need not be read except by those interested in the subject as the description of the technique is practically quite independent of the theory.

XXI. Mathematical Theory.

The general theory of the life table when the expectation of life is taken as the fundamental function is much more easy than when the death-rate is chosen. The former leads to very simple formulae, the latter to a mathematic which is itself cumbrous and also unsuitable for the purpose of calculation. In what follows the common notation with one exception is used. It is a matter of regret that actuaries have chosen the letter e to denote the expectation. It is already reserved by long convention to denote the base of the natural logarithms, and as these occur largely in the theory, some other symbol for the expectation must be chosen.

I

Notation.

x = age.

l_x = number of survivors at age x.

μ_x = instantaneous death-rate

$$= -\frac{1}{l_x}\frac{dl_x}{dx}.$$

E_x = expectation of life at age x

$$= \frac{\int_x l_x dx}{l_x}.$$

D_x = life table death-rate, i.e. the death-rate of all persons living in a stationary population above the age x.

$$= \frac{1}{E_x}.$$

T_x = the total population living above the age x.

$L_{x \cdot y}$ = the population living between the ages x and y.

$$= T_x - T_y.$$

II

The mathematical relationships of the quantities just defined, when the expectation of life is taken as the fundamental function, can be shown in a few lines.

Since
$$E_x = \frac{\int_x l_x dx}{l_x} \tag{1}$$

$$\frac{dl_x E_x}{dx} = -l_x$$

and
$$E_x \frac{dl_x}{dx} + l_x \frac{dE_x}{dx} = -l_x$$

whence
$$\mu_x = -\frac{1}{l_x}\frac{dl_x}{dx} = \frac{1}{E_x} + \frac{1}{E_x}\frac{dE_x}{dx}. \tag{2}$$

Integrating
$$l_x = \frac{1}{E_x} e^{k - \int \frac{dx}{E_x}} \quad \text{where } k \text{ is a constant.} \tag{3}$$

Integrating a second time we have

$$T_x = e^{k - \int \frac{dx}{E_x}}. \tag{4}$$

If the life table death-rate D_x or the reciprocal of the expectation be substituted, these formulae become respectively :

$$\mu_x = D_x - \frac{1}{D_x}\frac{D_x}{dx}. \tag{2^1}$$

$$l_x = D_x e^{k - \int D_x dx} \tag{3^1}$$

$$T_x = e^{k - \int D_x dx} \tag{4^1}$$

XXII. On the Calculation of Life Table Death-rates and Expectations.

The method of calculating the life table death-rates and the expectations has been described in the Sections VII and VIII, but to make this technical part complete, an example of the use of the method is given. As will be remembered, it essentially depended on the fact that if the death-rate of a standard population is known for all individuals above a definite age a suitable series of relationships can be found, which will give the life table death-rate and immediately the expectation by taking the reciprocal. The standard population chosen for which the equations of relationship are given is that which is in arithmetical progression. The example chosen for illustration (Table XXIII) is the registration district of Liverpool for the decade 1891–1900: the males alone are employed. The standard population of those living at each age is given in the first column; the death-rates in the Liverpool registration district in the second column; the number of deaths obtained by multiplying the death-rates into the standard population in the third column; the sum of these totals from each age and upwards in the fourth. Thus the figure opposite 55 is the sum of the deaths occurring above the age of 55 years, in the three age groups 55–65 years, 65–75 years, and 75 years and upwards. It is only necessary to put the figures in the fourth column in the appropriate equation given previously in Table XVI (p. 37), to obtain the life table death-rates. These are given in the fifth column, and in the sixth column the expectations or the reciprocals. It is not necessary, in view of the length of discussion previously given, to say any more upon this subject.

Table XXIII. *Showing the method of calculating life table death-rates and expectations.*

Age group.	*Population.*	*Death-rate Liverpool, 1891–1900.*	*Deaths.*	*Age.*	*Sum of Deaths above each age.*	*L. T. death-rate.*	*Expectation.*
0–5	16	121·19	1939·04	0	5205·03	35·029	28·55
5–10	15	19·42	141·30	5	3265·99	24·617	40·62
10–15	14	4·64	64·96	10	3124·69	27·116	36·88
15–20	13	7·43	96·59	15	3059·73	29·843	33·51
20–25	12	9·78	117·36	20	2963·14	33·464	29·88
25–35	21	16·63	349·23	25	2845·78	37·908	26·38
35–45	17	28·89	491·13	35	2496·55	49·308	20·28
45–55	13	44·95	584·35	45	2005·42	65·194	15·34
55–65	9	71·00	639·00	55	1421·07	89·938	11·12
65–75	5	116·52	582·60	65	782·07	131·461	7·61
75–85	1	199·47	199·47	75	199·47	203·018	4·93

XXIII. On Graduation of the Life Table Death-rates and Expectations.

For the purpose of observing the relationship between different life tables, the life table death-rates were graduated in Section X to the formula $a(c-x)^{-n}$, or in the form used for calculation

$$\log D = \log a - n \log (c-x) \tag{5}$$

or

$$\log E = -\log a + n \log (c-x) \tag{5'}$$

The method by which this is done is to choose the three values of the expectations at equal age distances and then solve for the constants. For example the expectations in the Manchester life table at 15 years, 45 years, and 75 years of age are respectively, 38·78 years, 17·80 years, and 5·11 years. Taking the logarithms of the expectations we obtain the following three equations :

$$1{\cdot}58861 = -\log a + n \log (c-15) \quad (6)$$
$$1{\cdot}25042 = -\log a + n \log (c-45) \quad (7)$$
$$0{\cdot}70842 = -\log a + n \log (c-75) \quad (8)$$

Subtracting (7) from (6) and (8) from (7), we have

$$0{\cdot}33819 = n \log (c-15) - n \log (c-45) \quad (9)$$
$$0{\cdot}54200 = n \log (c-45) - n \log (c-75) \quad (10)$$

Dividing (9) by (10)

$$0{\cdot}62397 = \frac{\log (c-15) - \log (c-45)}{\log (c-45) - \log (c-75)} \quad (11)$$

This equation is difficult to solve : the value of c can be found to be 112·164. It is hoped soon to publish a table by which this and similar equations can be solved by reference to a series of calculated values. From the value we obtain immediately

$$n = 2{\cdot}1088$$
$$\log a = 2{\cdot}60265$$
$$a = 400{\cdot}54$$

A second method of graduating the expectation is to consider the life table death-rate given by the formula :

$$D_x = al^x + bm^x \quad (12)$$

This formula graduates many life tables with extreme accuracy, and the solution is quite easy. Taking four values A, B, C, and D of the life table death-rates at equal intervals of age, say p years, and forming the equations, we have

$$\left.\begin{aligned} A &= a+b \\ B &= al^p + bm^p \\ C &= al^{2p} + bm^{2p} \\ D &= al^{3p} + bm^{3p} \end{aligned}\right\} \quad (13)$$

The solution of these equations is found at once to be given by :

$$l^p m^p = \frac{BD-C^2}{AC-B^2} = K. \quad (14)$$

$$l^p + m^p = \frac{AD-BC}{AC-B^2} = L. \quad (15)$$

so that $$l^p - m^p = \sqrt{L^2-4K}. \quad (16)$$

whence l^p and m^p are at once determined, and thence a and b from the original equations :

$$a = \frac{B-Am^p}{l^p-m^p} \qquad b = A-a. \quad (17$$

An example of the working is shown in the next section.

XXIV. Construction of a Life Table from the Graduated Expectations or Life Table Death-rates.

In this section a short note will be given on the method of calculating the different life table constants by means of these graduations. The first method is that in which the life table death-rate is graduated to $a(c-x)^{-n}$. This method of graduation has a great defect in that it holds only between 10 and 75 years, and consequently from the information at our disposal beyond these years no extrapolation is possible. The formula, however, for T_x is as follows :

$$T_x = e^{k-\frac{a}{(n-1)(c-x)^{n-1}}}. \qquad (18)$$

The second method of graduation affords more extensive information. To illustrate this the whole process of calculating a life table has been worked out and compared with Mr. King's Table E_8. In the first instance the life table death-rates have been calculated by the method given in Section XXII. These death-rates are shown in the first column of Table XXIV. The next step is to take the life table death-rates at 15, 35, 55, and 75 years as basis of the calculations.

Using equation (13) we have

$$\begin{aligned} 0{\cdot}02061 &= a + b \\ 0{\cdot}03159 &= al^{20} + bm^{20} \\ 0{\cdot}05930 &= al^{40} + bm^{40} \\ 0{\cdot}15324 &= al^{60} + bm^{60} \end{aligned} \qquad (19)$$

whence

$$\begin{aligned} l^{20}m^{20} &= 5{\cdot}905870 \\ l^{20}+m^{20} &= 5{\cdot}730294 \end{aligned} \qquad (20)$$

whence

$$\begin{aligned} l^{20} &= 4{\cdot}382776 \\ m^{20} &= 1{\cdot}347518 \end{aligned} \qquad (21)$$

From these

$$\begin{aligned} a &= 0{\cdot}00125777 \\ b &= 0{\cdot}01935223 \end{aligned} \qquad (22)$$

As the quantities to be calculated proceed by intervals of ten and five years the second and fourth roots of l^{20} and m^{20} are required :

$$\begin{aligned} l^{10} &= 2{\cdot}093508 \qquad & m^{10} &= 1{\cdot}160826 \\ l^{5} &= 1{\cdot}446895 \qquad & m^{5} &= 1{\cdot}077416 \end{aligned} \qquad (23)$$

Multiplying a and b consecutively by l^5 and m^5 and adding, we get the life table death-rates at 20 and 25 years ; following this by multiplying by l^{10} and m^{10} the values at 35, 45 years, &c. It is well to do this throughout, and thus the values calculated can then be checked by the original equations at ages 35, 55, and 75 years. The graduated values are given in the second column of Table XXIV.

In the subsequent discussion, Naperian logarithms are used. This saves a good deal of trouble, and as a table quite efficient for the purpose exists in *Dale's Five Figure Mathematical Tables* (London : Edwin Arnold, price 3*s*. 6*d*.), no difficulty lies in the way of their use.

To obtain T_x we have

$$T_x = e^{k - \int D_x dx}$$
$$= e^{k - a\frac{l^x}{\log l} - b\frac{m^x}{\log m}}. \quad (24)$$

Log l and log m are immediately obtained from the table of Naperian logarithms. It is to be noted that the values already found are l^5 and m^5, and not l and m, so that if the logarithms of these quantities are looked up they must be divided by 5.

Taking logarithms, we have

$$\log T_x = k - \frac{al^x}{\log l} - \frac{bm^x}{\log m} \quad (25)$$

where it is to be noticed that al^x and bm^x have already been calculated in graduating the life table death-rate, and have only to be multiplied by reciprocals of log l and log m.

The next stage in the calculation is carried out by assuming a value of T_x. In this case as we are comparing the values obtained by this method and by the ordinary actuarial method, with regard to the data on which E_8 was constructed, Mr. King's value of T_x at the age of 15 years is selected as the starting-point. To obtain the constant k, the value of the second part of the expression at the age of 15 years must be added to log T_x, and the subsequent values of the function subtracted at each successive age.

$L_{x \cdot x+5}$, $L_{x \cdot x+10}$ are obtained by subtracting the value T_{x+5} or T_{x+10} from T_x : l_x by multiplying a value of T_x by the corresponding life table death-rate. The whole results are given in the table. The correspondence of fact and theory is very close. For nearly all the purposes for which a life table is required in hygiene they are more than sufficiently accurate. The extrapolation is found to be of very considerable accuracy. In fact the expectation at 95 years, namely 1·90 years, is, I think, more accurate than Mr. King's value of 2·43 years. The data at these high ages tend to have an error which renders Mr. King's values subject to suspicion. The tendency at such high ages seems to be for living persons to record a higher age in the census paper than their friends give to the registrar on the act of death. As Mr. King's project was to graduate the data as he found them, this consideration was not introduced. It is not necessary to present further comparisons. The figures relating to many tables have been examined and the fitting found good. The rural districts and county boroughs life tables, for instance, have been graduated in exactly the same way, and show very nearly as close a correspondence.

A third method, however, can be used to calculate the T_x and l_x columns from the expectations or life table death-rates by the use of the formula :

$$cx = \log a - nE - \log E \quad (26)$$

where log E is the natural logarithm. This formula I have fully discussed elsewhere, and it is not necessary to repeat the argument

TABLE XXIV. *Construction of a life table by means of the graduation* $D_x = al^x + lm^x$.

Age	Life Table Death-rates.		Expectations.			T_x.		l_x.		$L_{x \cdot x+5}$ or $L_{x \cdot x+10}$.	
Period.	Calculated.	Graduated.	Calculated.	Graduated.	Mr. King's.	Calculated.	Mr. King's.	Calculated.	Mr. King's.	Calculated.	Mr. King's.
15	0·020610	0·020610	48·52	48·52	48·57	390,781	390,781	8,054	8,047	40,029	39,970
20	0·022655	0·022670	—	44·11	44·21	350,752	350,811	7,952	7,935	39,429	39,311
25	0·025047	0·025097	—	39·85	40·00	311,323	311,500	7,813	7,787	76,139	76,163
35	0·031590	0·031590	31·66	31·66	31·71	235,004	235,337	7,424	7,422	71,514	71,560
45	0·041886	0·041812	—	23·92	23·92	163,490	163,777	6,836	6,846	64,052	64,132
55	0·059300	0·059300	16·86	16·86	16·89	99,438	99,645	5,897	5,901	51,831	51,822
65	0·091146	0·091371	—	10·94	10·99	47,607	47,823	4,350	4,352	33,083	33,270
75	0·153240	0·153240	6·52	6·52	6·49	14,524	14,553	2,226	2,243	12,708	12,711
85	—	0·276645	—	3·61	3·72	1,816	1,842	503	496	1,774	1,785
95	—	0·527890	—	1·90	2·43	42	57	22	24	42	57

by means of which it was deduced. Its convenience must be its only recommendation. Differentiating (26) we have:

$$cdx = -ndE - \frac{dE}{E} \qquad (27)$$

so that the formula (4) for a number of persons living above any age becomes

$$T_x = e^{k+\int\left(\frac{n}{c}\frac{dE}{E}+\frac{1}{c}\frac{dE}{E^2}\right)}$$

$$= e^{k+\frac{n}{c}\log E - \frac{1}{cE}} . \qquad (28)$$

As an example of the method the expectations in E_8 have been chosen at the ages 15, 45, and 75, as calculated by the method in Section XXII.

This gives the equations:

$$\begin{aligned} 15c &= \log a - n\ 48{\cdot}520 - 3{\cdot}88198 \quad (1) \\ 45c &= \log a - n\ 23{\cdot}872 - 3{\cdot}17271 \quad (2) \\ 75c &= \log a - n\ \ 6{\cdot}523 - 1{\cdot}87533 \quad (3) \end{aligned} \qquad (29)$$

Subtracting (1) from (2) and (2) from (3)

$$30c = 24{\cdot}648\ n + 0{\cdot}70927 \quad (5)$$

$$30c = 17{\cdot}349\ n + 1{\cdot}29738 \quad (6)$$

Subtracting (5) from (6)

$$0 = 7{\cdot}299\ n - 0{\cdot}58811$$

Whence $n = 0{\cdot}080574$

and $c = 0{\cdot}089842$

whence $\frac{n}{c} = 0{\cdot}89685$

$$\frac{1}{c} = 11{\cdot}1307$$

Taking logarithms in equation (28)

$$\log T_x = k + \frac{n}{c}\log E - \frac{1}{cE}$$

$$= k + 0{\cdot}89685 \log E - 11{\cdot}1307\ D.$$

The value of k is chosen to give the initial value desired. In this case, putting the expectation at 15 years into the equation and taking $\log T_x = \log 39078$, the value of T_x in Mr. King's table, we get

$$k = 7{\cdot}32112.$$

The values of T_x obtained from the graduation, multiplied by the corresponding life table death-rates, give the value of l_x. The comparisons are given in Table XXV.

TABLE XXV.

	T_x.			l_x.		
		Graduated			*Graduated*	
Age.	*Mr. King's.*	*in one stage.*	*in two stages.*	*Mr. King's.*	*in one stage.*	*in two stages.*
15	39,078	39,078	39,078	80,465	80,540	80,540
20	35,081	35,089	35,047	79,344	79,494	79,381
25	31,150	31,224	31,143	77,870	77,959	78,013
35	23,534	23,580	23,515	74,219	74,489	74,284
45	16,378	16,323	16,385	68,459	68,377	68,637
55	9,965	9,846	9,930	59,012	58,387	58,885
65	4,782	4,698	4,732	43,523	42,820	43,127
75	1,455	1,477	1,462	22,425	22,634	22,404

The error is in most places less than 1 per cent. The greatest at 65 years is about 2 per cent. It is to be observed, however, that a very severe test of the graduation has been made by fitting to these values of the expectations separated by 30 years each. When the process is continued in two stages an almost perfect fit is obtained. The stages chosen in this case were 15–35 years and 35–75 years. Extrapolation here cannot be easily used, as the expectation can only with difficulty be calculated from the age in formula (26).

XXV. The Calculation of Life Tables by other Short Methods.

For general hygienic purposes there are two parts of a life table which are of the greatest importance. One is that which concerns the age periods from birth to 10 years, and the other for ages over 10 years. This age marks a dividing period at which the diseases of childhood have largely disappeared to give way to the causes of death which affect young adults and the later ages of life. The age period over 10 years is of great importance. These are the ages at which the problems of industrial hygiene arise. Some easy method of examining the effects of industry on health are thus necessary. The method proposed here is essentially the same as that given by Dr. Snow. It was, however, arrived at independently, and the method of construction has a slightly different principle.

The method proposed is as follows. It is assumed that if r be the mean death-rate between two ages, separated by a number of years n, l_{x+n} can be obtained from l_x by multiplying by a factor of the form

$$1-nr+bn^2r^2,$$

where b is a constant to be determined by the comparison of the data in life tables constructed by the usual method. The calculations are based on five life tables: C, E_6, E_8, H_2, and R. The constant b has been calculated by means of least squares, and its value is given in the accompanying table for both males and females. All that is necessary is to choose an appropriate value of l_x and apply the formula.

Table XXVI. *Showing the values of b for use at different ages in the coefficient* $1-nr+bn^2r^2$.

Age period.	*Males. b.*	*Females. b.*
10	0·0	0·0
15	0·0	0·0
20	0·0	0·0
25	0·4012	0·3611
35	0·3768	0·3894
45	0·3853	0·3763
55	0·3951	0·3868
65	0·3599	0·3660

The method of writing is shown in Table XXVII,

TABLE XXVII.

Age period.	nr.	n^2r^2.	bn^2r^2.
25–35	0·0551	0·00304	0·00122
35–45	0·0960	0·0922	0·00347
45–55	0·1784	0·03182	0·01226
55–65	0·3560	0·1267	0·05006
65–75	0·7424	0·5512	0·19848

taking the county borough life table C. The first column contains the death-rates multiplied by ten as the interval in the calculations is ten years; the second the square of this or a hundred times the square of the death-rate; the third column the values of the squares multiplied by the appropriate values of b. The procedure for this is as follows. Starting with 95757, the value of l_x at 25 years of age in life table C, this number is multiplied by the corresponding number in the first column, the product subtracted from 95757 and to the resulting number the product of 95757 and the corresponding number in the third column is added. Thus l_{35} is obtained from l_{25} by subtracting $l_{25} \times 0{\cdot}0551$ and adding $l_{25} \times 0{\cdot}00122$, or the change may be made in one operation. The same process is repeated. With a calculating machine the process is rapid and continuous. T_x is obtained by multiplying l_x by the life table death-rate and $L_{x \cdot x+10}$ by subtraction. The result of the process is given and compared with Mr. King's in Table XXVIII. In the second and third columns the l_x is compared; in the fourth and fifth T_x, and in

TABLE XXVIII.

Age period.	L.T.D.R. based on arithmetical progression.	l_x. Mr. King's.	l_x. Calculated.	T_x. Mr. King's.	T_x. Calculated.	$L_{x \cdot x+10}$. Mr. King's.	$L_{x \cdot x+10}$. Calculated.
25	0·026610	95,759	95,759	361,949	359,862	93,372	92,957
35	0·033944	90,598	90,599	268,577	266,905	86,717	85,783
45	0·045393	82,183	82,216	181,860	181,122	75,859	75,133
55	0·064682	68,492	68,557	106,001	105,989	58,581	58,358
65	0·099899	47,452	47,583	47,420	47,631	34,633	34,560
75	0·166030	21,346	21,701	12,787	13,071	12,787	13,071

the sixth and seventh columns the corresponding values of $L_{x \cdot x+10}$ or the populations living during each ten-yearly period are shown. A test of the use of this method is given with regard to cancer. If the death-rates from cancer as found to exist in the county boroughs of England between the years 1911 and 1914 are multiplied into the corresponding populations, both as given by Mr. King and as calculated here, the values of the death-rates from cancer per 1,000 of persons living above 25 years are respectively 2·75 and 2·73. As no statistical conclusion could be drawn from such a difference, it is obvious that the method is sufficiently accurate. The example given, that of the county boroughs, is of those tested, the one in which the methods give the greatest divergence. The method is even sufficiently accurate if applied to Manchester Township (M_2) 1881–90.

XXVI. Dr. Snow's Method of Calculating Life Tables.

Dr. Snow's method of forming a life table proceeds very much on the same assumptions as those given in the preceding paragraph. Basing his calculations on the data of the life table for England and Wales for the years 1910–12, he forms a series of multipliers by which l_{x+5} or l_{x+10} can be obtained from l_x. These multipliers denoted by p are in general quadratic functions of the mean death-rate at each age group, denoted by r. The expression for calculating p are, however, complex, and had calculations to be made in each individual case there would still be a good deal of work to do. To obviate this a series of tables have been constructed, from which, given the value of r, the appropriate value of p can at once be read off. So far there is agreement between Dr. Snow's method and my own. The next step is different: Dr. Snow has invented a new function which he denotes by the letter k. Knowing the value of p, the appropriate value of k is obtained again from a table of the function. When k is multiplied into the corresponding value of l_x the product is $\frac{1}{2}l_x + L_{x \cdot x+5} - \frac{1}{2}l_{x+5}$ or $\frac{1}{2}l_x + L_{x \cdot x+10} - \frac{1}{2}l_{x+10}$ according as the group is quinquennial or decennial. From this Dr. Snow obtains the expectation by summing, dividing by l_x and subtracting 0·5. The working is shown in the accompanying table which illustrates the construction of a life table for the county boroughs of England and Wales. The correspondence of the expectations obtained by this method and those in Mr. King's extended table is very close, the error never amounting to more than a month.

Table XXIX.

Age group. (1)	*Death-rate* (r). (2)	*Chance of surviving throughout Period* (n^px). (3)		l_x. (4)	k_x. (5)	$l_x \times k_x = L'_x$. (6)	$S(L'_x) = T'_x$ (7)	$E_x = \frac{T'_x}{l_x} - \frac{1}{2}$ (8)	*E_x in extended Table.* (9)
10–15	0·00215	0·9894	(10)	10,000·0	4·9796	49,796	512,866	50·78	—
15–20	0·00321	0·9835	(15)	9,894·0	4·9683	49,156	463,070	46·30	46·28
20–25	0·00407	0·9798	(20)	9,730·7	4·9613	48,277	413,914	42·04	41·99
25–35	0·00511	0·9463	(25)	9,534·1	9·7770	93,215	365,637	37·85	37·80
35–45	0·00960	0·9074	(35)	9,022·1	9·6204	86,796	272,422	29·69	29·65
45–55	0·01784	0·8337	(45)	8,186·7	9·3225	76,321	185,626	22·17	22·13
55–65	0·03560	0·6922	(55)	6,825·3	8·7118	59,461	109,305	15·51	15·48
65–75	0·07424	0·4559	(65)	4,724·5	7·602	35,916	49,844	10·05	9·99
75–85	0·1508	0·1808	(75)	2,153·9	5·746	12,376	13,928	5·97	5·99
85 and upwards	0·2764	0·0417	85	389·4	3·860	1,503	1,552	3·49	3·55
	—	—	95	16·2	3·00	49	—	—	—

The ease of applying Dr. Snow's method depends on the series of tables which have been constructed and which are published by the Registrar-General. Any one, therefore, desiring to use it must procure these tables. In many ways they represent a great advance. One disadvantage is that it does not give the T_x in the form suitable for calculating life table death-rates for different diseases, another operation being required, namely, the subtraction of half of l_x. The method of working is as follows. (Table XXIX.)

The death-rates (r) in the different age groups are tabulated in column (2). The values of p corresponding to these values of r are then taken from the tables and entered in column (3). Beginning at the age of 10 years and taking l_{10} equal to 10,000, successive multiplications by the values of p in column (3) are carried out and entered in column (4). For example, l_{20} is obtained by multiplying l_{15} by 0·9835 and so on. The values in brackets are the values of x in l_x. The values of k corresponding to the values of p in column (3) are now taken from the tables and entered in column (5). Column (6) contains the products of the numbers in columns (4) and (5), and column (7) the successive sums for each age and upwards. Column (8) is obtained by dividing the numbers in column (7) by those in column (4) and subtracting 0·5. These numbers are the complete expectations of life at the ages shown in brackets in column (4). Column (9) gives for comparison the expectations in Mr. King's extended table.

XXVII. Analysis of Phthisis Death-rate.

In Section XIII, referring to phthisis, a note was made upon the analysis of the disease into several components, each of which might be found to exist almost alone in different environments or industries. The method of analysis is given here with the figures adjusted so as to permit the standard death-rate from each type of phthisis to be calculated directly. It is to be noted that though applied to phthisis the method is also applicable to other health problems. There are some classes of disease which are mixed up in the returns given by the Registrar-General, and which cannot well be separated. Thus many cases of osteo-arthritis may be grouped with rheumatoid arthritis. There is no reason, however, why, if two diseases have different age distributions, and if they occur in varying amounts in different places, their statistical separation on the lines developed should not be possible. The problem of deaths from heart disease suggests particular analysis. Such questions, however, are complex, and no one need enter on such an investigation unless he is willing to face a good deal of arithmetic.

In Table XXX the coefficients a, b, and c denote respectively the age period death-rates from 'young adult', 'middle age', and 'old age' phthisis, adjusted so as to give, on solution of the equation, the standardized death-rate for each type of the disease. The values on the right-hand side of the equation are the phthisis death-rates. The working of the left-hand side of the equation is identical in every instance, while the numbers to be analysed are given by the figures on the right. To reduce the nine equations to three, namely those denoted by (1), (2), and (3), the procedure is as follows. To obtain the number in equation (1), the figure on the right-hand side of each of the nine equations is multiplied by the corresponding coefficient of a. Thus 31 is multiplied by 31, 100 multiplied by 100, and 178 by 192, and so on. The numbers are then added. With any calculating machine the process is continuous. To obtain the figures in equation (2) the same numbers on the right hand in the nine equations are multiplied by the coefficients of b, and summed. Equation (3) is obtained likewise by multiplying by the coefficients of c. Having obtained these numbers, the subsequent arithmetic is plain sailing.

Table XXX.

Analysis of Death-rates among Males.

Age Group.	*Young Adult.*		*Middle Age.*		*Old Age.*		*Phthisis Death-rate, Essex.*
10–15	$31a$	+	$8b$	+	$22c$	=	31
15–20	$100a$	+	$35b$	+	$31c$	=	100
20–25	$192a$	+	$74b$	+	$39c$	=	178
25–35	$176a$	+	$129b$	+	$61c$	=	236
35–45	$97a$	+	$175b$	+	$109c$	=	287
45–55	$51a$	+	$173b$	+	$191c$	=	290
55–65	$27a$	+	$126b$	+	$305c$	=	264
65–75	$14a$	+	$64b$	+	$307c$	=	193
75+	$8a$	+	$26b$	+	$128c$	=	81

The solution in one case, namely, that for Essex, is given. The method of forming equations (1), (2), (3), is described in the text.

$$91{,}800a + 70{,}964b + 55{,}877c = 139{,}780 \quad (1)$$
$$70{,}964a + 104{,}608b + 125{,}540c = 195{,}481 \quad (2)$$
$$55{,}877a + 125{,}540b + 258{,}707c = 261{,}932 \quad (3)$$

Dividing each equation by the coefficients of a.

$$a + 0{\cdot}773028b + 0{\cdot}608682c = 1{\cdot}522658 \quad (I)$$
$$a + 1{\cdot}474100b + 1{\cdot}769066c = 2{\cdot}754650 \quad (II)$$
$$a + 2{\cdot}246720b + 4{\cdot}629937c = 4{\cdot}687653 \quad (III)$$

Subtracting (I) from (II), and (II) from (III)

$$0{\cdot}701072b + 1{\cdot}160384c = 1{\cdot}231992$$
$$0{\cdot}772620b + 2{\cdot}860871c = 1{\cdot}933003$$

Dividing each equation by the coefficients of b.

$$b + 1{\cdot}655157c = 1{\cdot}757297 \quad (IV)$$
$$b + 3{\cdot}702818c = 2{\cdot}501881 \quad (V)$$

Subtracting $\quad 2{\cdot}047661c = 0{\cdot}744584$

$$\therefore c = 0{\cdot}363627$$

To obtain b, put the value of c in equation (IV),

giving $b = 1{\cdot}155435$

To obtain a, put the values of b and c in equation (I),

giving $a = 0{\cdot}408141$

The standard death-rates due to each type of phthisis are thus found to be:

Young adult = a = 0·408141
Middle age = b = 1·155435
Old age = c = 0·363627

Analysis of Death-rates among Females.

Age Group.	*Young Adult.*		*Middle Age.*		*Old Age.*	
15–20	$126a$	+	$57b$			=
20–25	$145a$	+	$76b$			=
25–35	$140a$	+	$123b$			=
35–45	$90a$	+	$154b$	+	$22c$	=
45–55	$60a$	+	$121b$	+	$67c$	=
55–65	$37a$	+	$75b$	+	$400c$	=
65–75	$11a$	+	$25b$	+	$867c$	=
75+	$3a$	+	$19b$	+	$200c$	=

$$69{,}700a + 59{,}649b + 30{,}937c =$$
$$59{,}649a + 69{,}122b + 66{,}970c =$$
$$0.937a + 66{,}970b + 956{,}662c =$$

Dividing each equation by the coefficients of a.

$$\begin{array}{rrrrrl} a & + & 0{\cdot}855796b & + & 0{\cdot}443859c & = \\ a & + & 1{\cdot}158812b & + & 1{\cdot}122735c & = \\ a & + & 2{\cdot}164722b & + & 30{\cdot}922908c & = \end{array}$$

Subtracting (I) from (II), and (II) from (III)

$$\begin{array}{rrrrl} + & 0{\cdot}303016b & + & 0{\cdot}678876c & = \\ + & 1{\cdot}005910b & + & 29{\cdot}800173c & = \end{array}$$

Dividing each equation by the coefficients of b.

$$\begin{array}{rrrl} b & + & 2{\cdot}240397c & = \\ b & + & 29{\cdot}625089c & = \end{array}$$

Subtracting $\quad + \; 27{\cdot}384692c =$

$$\therefore c =$$

To obtain b, put the value of c in equation (IV),

giving $b =$

To obtain a, put the values of b and c in equation (I),

giving $a =$

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Printed under the authority of HIS MAJESTY'S STATIONERY OFFICE
by Frederick Hall, at the University Press, Oxford