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A CONTRIBUTION
TO THE PSYCHOLOGY
OF LOGIC

AUGUSTUS D. WALLER

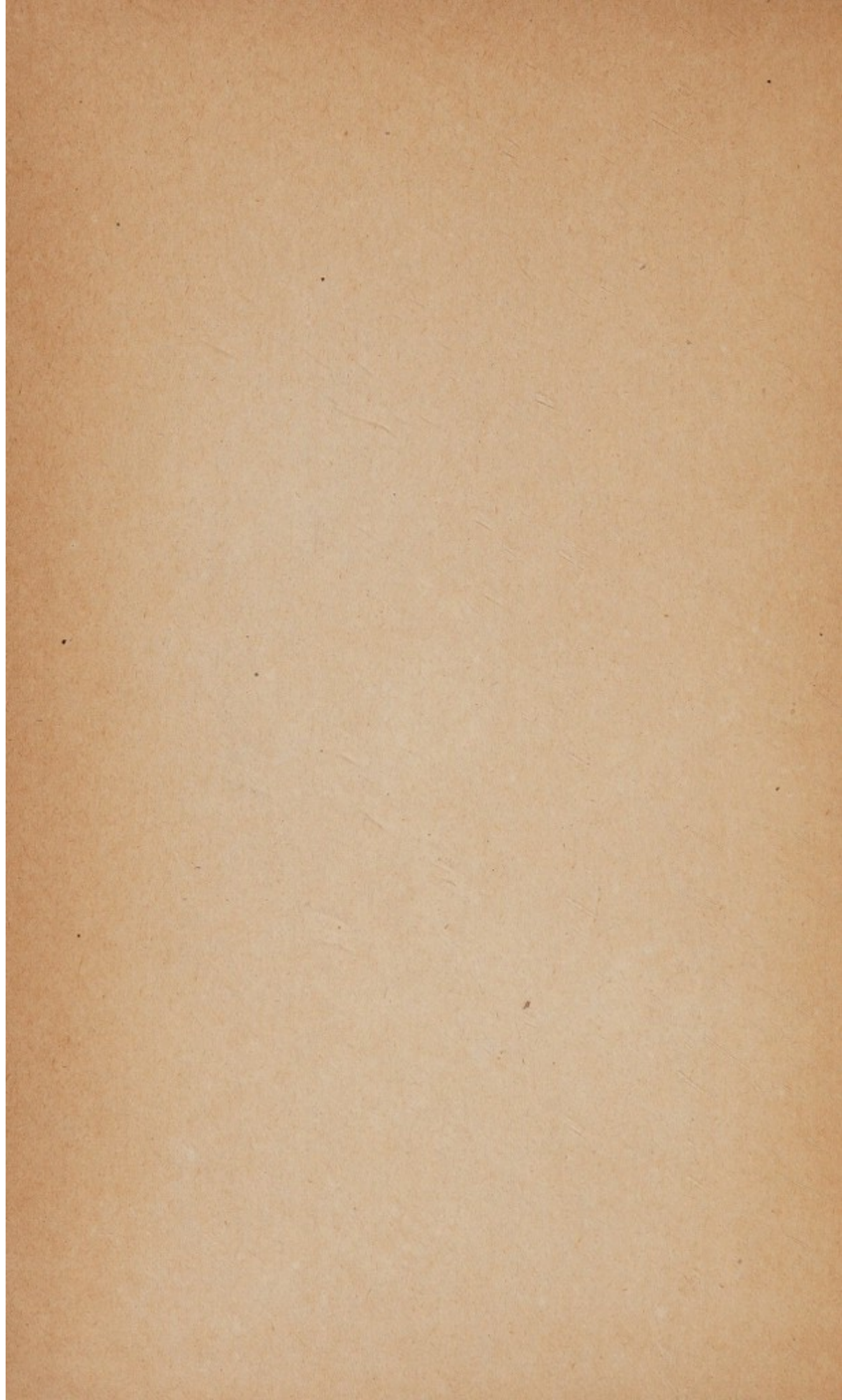


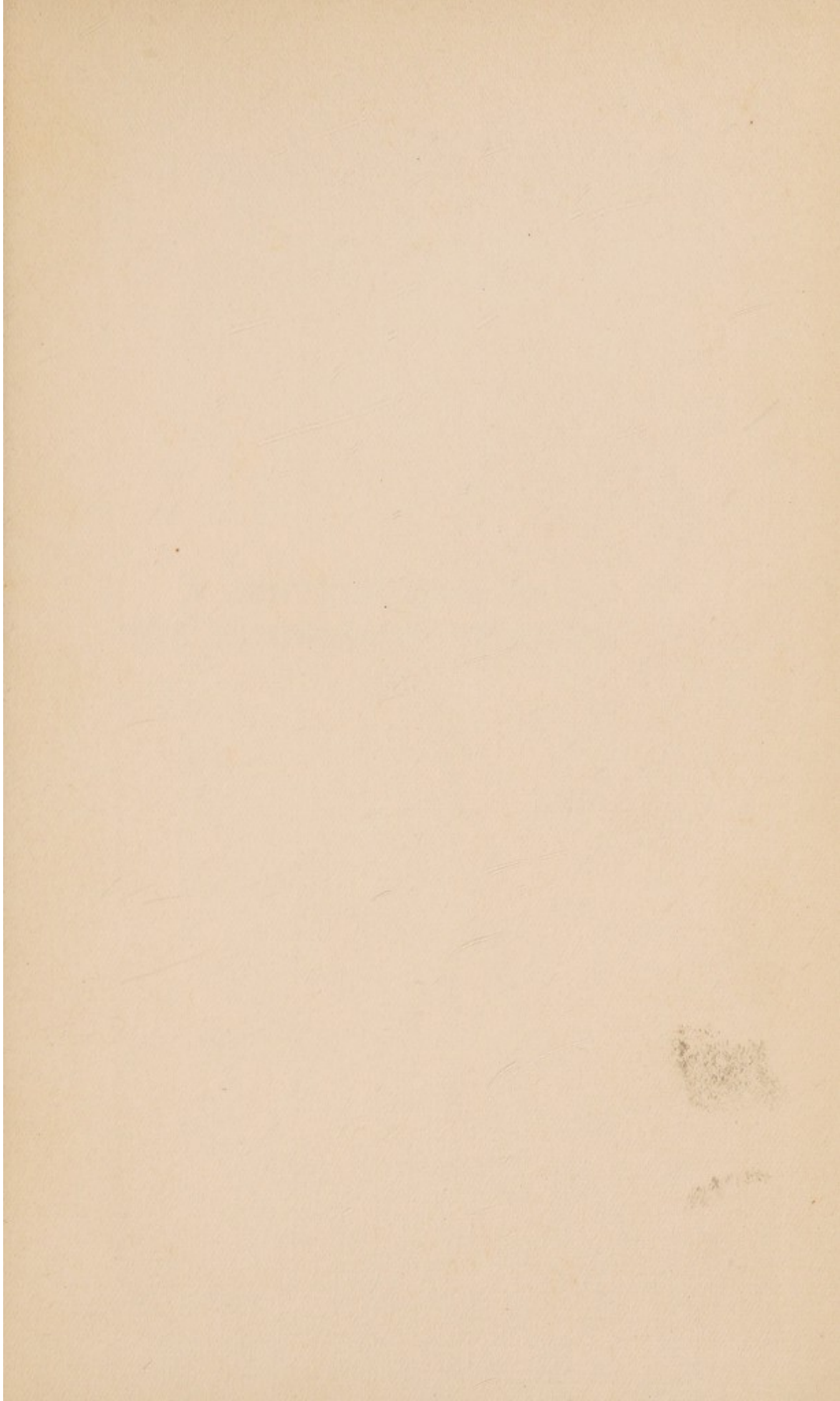
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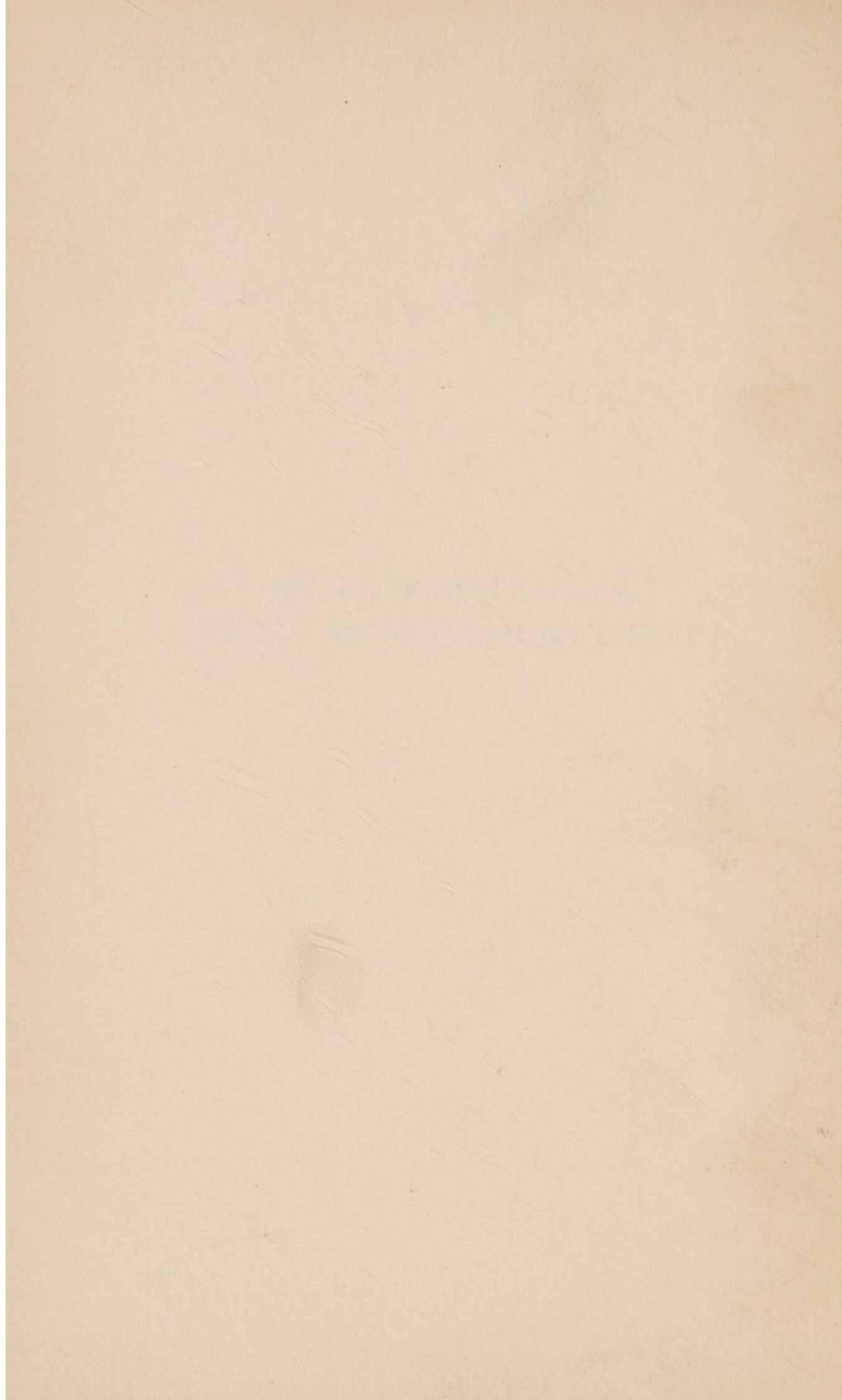




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A CONTRIBUTION TO
THE PSYCHOLOGY OF LOGIC



A CONTRIBUTION TO THE PSYCHOLOGY OF LOGIC

CONSIDERED FROM A
PHYSIOLOGICAL STANDPOINT

BY

AUGUSTUS D. WALLER

M.D., LL.D., F.R.S., DIRECTOR OF THE PHYSIOLOGICAL
LABORATORY OF THE UNIVERSITY OF LONDON



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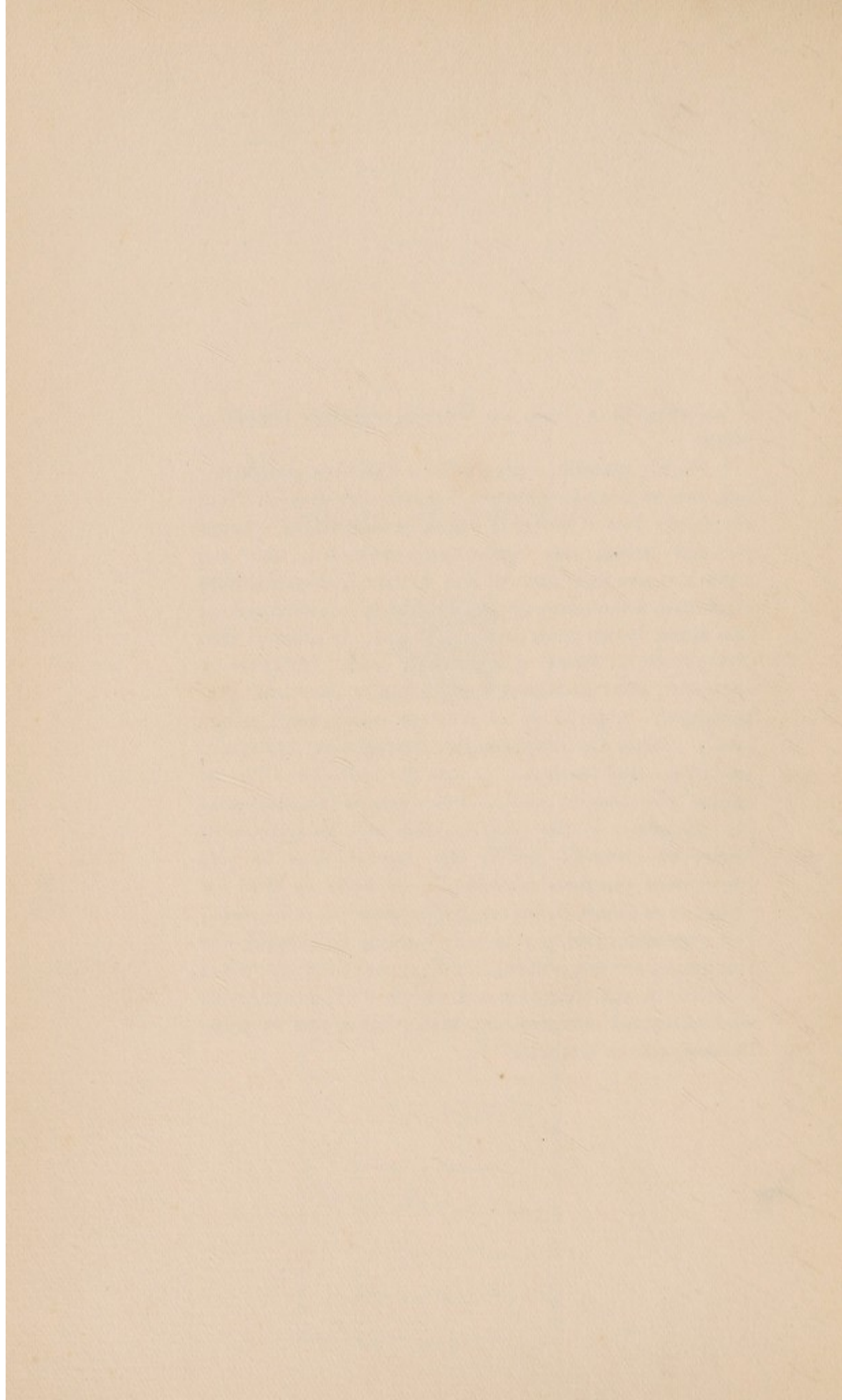
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Leibnitz, in a letter to Wagner, expresses himself as follows :

“ J’ai fait autrefois, à propos d’une discussion mathématique, avec un fort savant homme, l’expérience que voici. Nous cherchions l’un et l’autre la vérité, et nous avons échangé plusieurs lettres, avec beaucoup de courtoisie, mais non cependant sans nous plaindre l’un de l’autre, chacun de nous reprochant à son adversaire de dénaturer, involontairement sans doute, le sens et les paroles de l’autre. Je proposai alors d’employer la forme syllogistique : mon adversaire y consentit ; nous poussâmes l’essai jusqu’au douzième prosyllogisme. A partir de ce moment même, toute plainte cessa ; chacun des deux comprit l’autre, non sans grand profit pour tous les deux. Je suis persuadé que si l’on en agissait plus souvent ainsi, si l’on s’envoyait mutuellement des syllogismes et des prosyllogismes avec les réponses en forme, on pourrait par là très souvent, dans les plus importantes questions scientifiques, en venir au fond des choses, et se défaire de beaucoup d’imaginationes et de rêves ; l’on couperait court, par la nature même du procédé, aux répétitions, aux exagérations, aux divagations, aux expositions incomplètes, aux réticences, aux omissions involontaires ou volontaires, aux désordres, aux malentendus, aux émotions fâcheuses qui en résultent.”



THE PSYCHOLOGY OF LOGIC

CHAPTER I

INTRODUCTORY

§ 1. IN ordinary language there is antithesis between the adjectives "natural" and "artificial"; we say, for instance, that a product is natural or that it is artificial, implying thereby that "natural" and "artificial" are contrary qualities. But to the student of biological science there is no fundamental antithesis between such qualities, every possible event, from the simplest and most "natural" to the most complicated and "artificial," is essentially natural, and sometimes phenomena that seem at first blush to be the most sophisticated and "unnatural," are in his eyes most pregnant of natural meanings. The reflex adjustments of the central nervous system reach their highest degree of complication in Man, and if in the march of evolution, complication be in the direction of perfection, then we must recognise in the highly "artificial" products and customs and conventions of societies of men, the highest expression of natural laws of organic reaction.

Formal logic, sometimes placed in express antithesis to natural logic, is ordinarily regarded and treated as a highly artificial dialectical exercise, and altogether remote from the subject-matter of psychology and of physiology. The purpose I have now in view is to show that a study of formal logic can be helpful to a physiologist, and

that a knowledge of physiology and psychology is not altogether foreign to the interests of the scientific logician. And if the aspect in which I am about to present the case be in reasonable measure a suitable verbal representation of fact in physiology and in logic, then I shall urge further that such representation must be admitted to rank as a real extension of horizon—to the logician by bringing to mind that his classical processes conform with psychological law—to the physiologist by showing how such formal logical processes bear witness to some of the more obscure working of the central nervous system. Such relation is indeed not now stated for the first time; it is very clearly implied in the very title—*Laws of Thought*—applied by Boole, logician as well as mathematician, to one of his principal works. But the relation has, so far as I know, never been presented in the light it has taken in the mind of an empirical student of experimental science.

§ 2. Approaching the subject from a physiologist's standpoint, we recognise at the outset that the most elementary single sensation in consciousness is the resultant of a ratio between two sensificatory objective data, and that it has as its outward and visible representation in formulated logic a *judgment* or *conclusion*, which is the resultant of a ratio between two verbal *premises*.¹

¹ "Nothing can be perceived otherwise than by comparison with something else; the most elementary sensation is the resultant of a *ratio* between *two* sensificatory terms *a* and *b*, and not that of a single term *a* or *b*.

"Something is perceived in comparison with nothing—when it is a '*minimum perceptible*' thing—or with something else, and the resultant of the ratio in consciousness is a sensation. The highest forms of sensation are equally reducible to a ratio between *two* terms, each of which may, however, be a highly compound ratio."—From Waller: *An Introduction to Human Physiology*, 3rd edition, 1896, p. 554.

The simplest possible sensation, such as the consciousness that the skin has been touched, is not the isolated consequence of one isolated objective stimulus, but always and of necessity the result of a contrast phenomenon, an unconsciously effected conclusion in consciousness arising out of the comparison and compounding of two states. Sensation is the result, not of a state of matter, but of a change of that state, and proceeds from two successive sensificatory states fusing into a resultant state, the actual sensation in consciousness being the subjective concomitant of the process of fusion. Full and overt sensation is a conscious act of comparison. Let us take a simple case of a single sensation—*e. g.* the sensation (or judgment) that the skin has been pricked by a pin. The objective stimulus is the pin-prick. One sensificatory term is the state of brain (and corresponding neural ray formed by nerve-tract and sense-organ) before the pin-prick ; this we may take as the analogue of a major premise. The second sensificatory term is the changed molecular brain state brought about by the stimulus ; this we may take as the analogue of a minor premise. And as the subjective concomitant of these two superposed different states we have in consciousness, until the ripple of difference has subsided, the sensation of a pin-prick ; this may be taken as the physiological analogue of the conclusion of the syllogism ; it is the momentary state compounded of two antecedent states. Viewed thus the mechanism of the simplest sensation is objectively illustrated by the familiar logical process of compounding two component propositions (the premises) into one resultant proposition (the conclusion) ; or by the still simpler and similar mental process of compounding into one concept or proposition two component concepts, the subject and the predicate.

§ 3. Thus the simplest sensation is in its essential mechanism an act of *judgment*, and as such involves always two terms, analogous with minor and major of a syllogism, or put more simply, with subject and predicate of a proposition. It is in every instance reducible to a *ratio* between two sensificatory terms, and if symbolically represented should therefore be cast in the form $\frac{a}{b}$.

Similarly, a logical proposition expresses a ratio between two terms, and is to be symbolised in the form $\frac{S}{P}$ to signify that a subject S forms part of or belongs to the denomination P. And as a sensation or judgment can be complex, composed of two simple sensations, or of two groups of more or less compound sensations, we shall recognise as its most suitable mode of formulation a ratio between two ratios.¹ Similarly, two logical pro-

¹ For the present argument the ratio between two terms may equally well be expressed as the product of two factors. A pair of sensificatory data may be written indifferently a/b or ab . A pair of compound data may then be given in the form $(ab)(cd)$. Similarly, the logical proposition may be equally well expressed as $\frac{S}{P}$ or as SP . And the logical syllogism can be put either as a ratio between minor and major, or as a product of these factors. We shall in the following exposition use the fractional form $\frac{S}{P}$ to denote a premise, and the form of multiplication

$\frac{S}{M} \times \frac{M}{P} = \frac{S}{P}$ to denote the syllogism $SM. MP : SP$.

There is, however, no essential distinction implied in the two forms of expression. A premise $\frac{S}{P}$ expresses a simple judgment that a Subject is related to a Predicate and exhibits the elements of a simple immediate judgment. The formula $\frac{S}{M} \times \frac{M}{P}$ sets forth a judgment that a minor premise is related to a major premise. The multiplication form $\frac{S}{M} \times \frac{M}{P}$ exhibits the elements of a syllogistic judgment, and indicates plainly the middle term eliminated to form the conclusion $\frac{S}{P}$.

positions are naturally cast into the form of a ratio between two ratios; as we shall see later such a compound ratio between two propositions simplifies itself whenever a factor common to both can be eliminated, *i.e.* when there is a valid syllogism, with disappearance of a middle term.

The product of repeated sensation is *experience*, and the characteristic event in the formation of experience is *simplification*. By the repetition of similar stimuli and of similar sensations a functional organisation is established—an instruction of neural centres resultant upon the repeated similar sensations that have been aroused. A modified state of excitability is gradually established by repeated similar centripetal stimuli, as in the process of logical induction a generalised formula is established from particular similar cases; a state of brain is constituted that forms what in formal logic is the major premise, acting as the principal determinant factor of further judgments. This *personal ratio* of the individual—animal or man—that colours all his further judgments and actions, since it is their major premise—is formed by slow (or hasty) process of induction, with much or little error in the retention and elimination of factors presented by the minor premises of daily life. A natural process of simplification—of generalisation from particulars—takes place, of which in formal logic the outward and visible sign consists in elimination of a common middle term; centrifugal deduction springing from and consequent upon centripetal induction, becomes the increasingly effective factor in the further acquisition of experience and in the guidance of conduct—the instructed individual consciously or unconsciously imagines necessary consequences of a principle, acts upon previsions, predicts the particular effects of general law, is directed by theory and

doctrine. Reflex reasoning—centripetal inductive and centrifugal deductive—informed from full (or scanty) experience, operating in varying degree of accuracy, becomes gradually established as the universal *modus vivendi*, not only as regards the properly speaking rational conduct of life, but also as regards the instinctive reactions of any living matter. This is indeed to use the word “reasoning” in a most extensive sense, and quite emptied of its ordinary connotation, to embrace not only the perfected special thought of the human brain, but even the lowliest organic tropisms by which the simplest living thing has survived, in ratio and harmony with its surrounding medium. The liberty of language will perhaps be held excusable if it serves to indicate that there is an essential similarity of mechanism and a unity of principle between what may be termed the real logic of organic life, and the formal logic of the verbalising organism.

§ 4. An important and at first sight fundamental point of difference between real or organic and formal or verbal logic, consists in the fact that the latter admits negative as well as positive premises, whereas in the logic of organic life we can have to do only with positive premises. For every sensation or judgment, although it may be aroused by stimulation in a “negative” direction as well as by stimulation in a “positive” direction—by the removal of some *thing* as well as by the application of some *thing*—is of necessity a positive phenomenon. Negative sensation—sensation less than no sensation—is psychologically impossible.

On closer examination the discrepancy turns out to be more apparent than real, and to depend upon a verbal misunderstanding that can easily be made evident. The

question in psychology as to whether there can be "negative" sensations soon found as its answer that a sensation is necessarily a positive phenomenon, and that the expression "negative sensation" is merely a loose and misleading term to denote a sensation caused by a negative stimulus. The physiological sensificatory change caused by the sudden commencement or the sudden cessation of a continuous stimulation is a positive phenomenon ; a muscle gives contraction at make and at break of a constant current ; a man feels loss as well as gain.

Every concept must accordingly be positive, a negative concept in the sense of a concept less than no concept cannot have existence, and the expression "negative concept" is a loose term used to denote a concept expressed verbally in negative form. The negative pole of a magnet is as much a positive concept as is its positive pole. The negative expression "not-something" denotes in reality the positive concept "everything else" as being within the field of discourse. Unity or the universe of discourse consists of "something + everything else," and for the purposes of formal logical discourse we shall find it necessary to enumerate as our four positive elements :—

S the subject, "something" and S' the co-subject, "everything else" within the field of thought $S + S'$.

P the predicate, "something" and P' the co-predicate, "everything else" within the field of thought $P + P'$.
(v. *infra*, p. 24.)

§ 5. From the present point of view it is the admission of negative premises as fundamental elements of formal logic, and the rejection of negative sensations as fundamental elements in physiological psychology, that forms the principal feature of divergence between the two

systems of knowledge. For the ordinary requirements of logical discourse negative words and negative propositions are as fundamentally essential as are positive words and positive propositions.

The admission of negative words as symbols for negative concepts as the equals and opposites of positive words as symbols for positive concepts, brings logic out of harmony with psychology, and raises difficulty within the boundaries of logic itself.

It is hardly possible to devise any consistent scheme of symbolic logic unless we adopt at the very outset the principle of the Universe of Discourse, and with it as its necessary consequence the principle that the negative concept has no real and circumscribed existence, and that the universal negative is essentially an universal affirmation. This principle is implicitly, if not explicitly, adopted when we have admitted the essential validity of the universe of discourse as being in each instance composed of something *plus* everything else not that thing, of $S + S'$, $P + P'$, men *plus* not-men, mortalia et immortalia.

§ 6. There is in subordination to this chief difference a further point in which the logical presentation and the psychological are not parallel.

As presented by logicians propositions are of the four kinds :—

A	I	E	O
(affirmo)		(nego)	

but as presented psychologically in order of development of thought the order is

first the particular affirmative I then the general affirmative A

first the particular negative O then the general negative E

that is the particular (observation and) affirmation that *this* thing S is included or excluded from a given area of thought P, is antecedent to the generalisation that *all* things like this thing are so included or excluded.

Between the initial observation that this thing is so-and-so and the final generalisation that all things like this thing are so-and-so there occur (or should occur) the intervening steps that some things—many things—all things we have seen of the kind—are so-and-so.

And in passing we may frankly admit the finite and imperfect character of all such empirical induction, without envying the perfect cogency of syllogistic deduction where the major premise has not arisen from sufficient previous induction.

§ 7. It will be advisable at this stage to consider our four kinds of propositions, translating them from words to symbols conformably with the axiom that all sensations and symbols of sensations are essentially positive phenomena.

1. The universal affirmative proposition A offers no ambiguity, whether expressed by words or by symbols.

All slaves are protected $\frac{S}{P}$

2. The particular affirmative I is plain in words and in symbols. But an implied ambiguity has to be guarded against by previous convention. "Some" may imply "*some but not all*" or "*some or all*." We shall adopt this second reading "*some or all*," and indicate the limitation by prefixed letters *a* or *b*.

Some slaves are protected $\frac{aS}{P}$ *i. e.* we have observed

that some slaves are protected, and do not imply that this may or may not be the case for all slaves.

3. The universal negative E is very ambiguous where expressed in words, and some care is necessary to express it correctly in symbols. "No slaves are patriotic" is a proposition negating the quality of patriotism in the case of all slaves. The subject of discourse proposed in the formula "*no slaves are patriotic*" is not "*no slaves*" but "*all slaves*" (to be represented by the symbol S), and the negative must be transferred from the subject slaves to the predicate patriotic, to signify that all *slaves* are outside the concept *patriotic*, *i. e.* are non-patriotic.

No slaves are patriotic signifies the fact that

$$\text{All slaves are non-patriotic}^1 = \frac{S}{\bar{P}}$$

4. The particular negative proposition O, "*some slaves are not patriotic*," is not liable to ambiguity provided the conventional meaning of some = some (or may be all)² is adhered to, and is at once expressed in symbols without difficulty as $\frac{aS}{\bar{P}}$. So that in summary we find that the four positive and negative propositions A E I O are to be symbolised as four positive relations thus :—

¹ This form of negative universal, with the negative attached to the predicate is not open to the ambiguous implication sometimes contained in the verbal form "all slaves are not patriotic" (*i. e.* but some slaves are patriotic), which is in reality a particular negative involving a particular positive.

² This meaning of "some" forbids the implication "some slaves are not patriotic" (but others are). This mistake is prevented by attaching the negative to the predicate. The conventional meaning of "some" is consonant with the psychological fact that the observation of "some cases" precedes that of "all similar cases."

A	all slaves are protected . . .	S (total inclusion of \bar{P} S in P).
E	no slaves are patriotic . . . (= all slaves are non-patriotic)	S (total exclusion of \bar{P}' S from P).
I	some slaves are protected . . .	aS (partial inclusion \bar{P} of S in P).
O	some slaves are non-patriotic . . .	aS (partial exclusion \bar{P}' of S from P).

Psychologically the order is the order of observation ; first the particular propositions I and O, then the general propositions A and E. We first observe that *some* S is included in P or in P' ; we subsequently observe that *all* S is included in P or in P'.

The particular negative O is arrived at just like the particular positive I by positive observation of the presence of some S in not-P (involving the absence of at least some S from P). The universal negative E should be an extension of the previous particular observation of some S in not P to become all S in not P, but is often arrived at directly by the observation of P itself in full extension and a failure to detect any S in that P, a result that finds natural expression in the verbal formula : No S is P ; and in the phrase employed by journalists "conspicuous by absence." But again we repeat that the correct formula of such universal negative, symmetrical with the universal affirmative is all S in not P or $\frac{S}{\bar{P}'}$. In the normal course

of sensation and judgment we learn that "some roses have thorns," before generalising "all roses have thorns," and when we say "no roses are blue," we do so from our experience that all the roses we have seen have been not-blue. The particular I is predecessor of the universal A, the particular O is predecessor of the universal E, and

the four statements, symbolically expressed, must be expressed throughout on the positive type :—

$$\begin{array}{cc} \frac{aS}{P} & \frac{S}{P} \end{array} \qquad \begin{array}{cc} \frac{aS}{P'} & \frac{S}{P'} \end{array}$$

All statements, whether positive or negative verbally, are *psychological* affirmations. "*Negatio affirmatio contraria est.*"

To pass from I to A and from O to E by observation is to follow the method of Induction from observed data, which may be empirically insufficient or sufficient, but can never be absolutely and logically perfect. The universal resultant is a principle or a theory, that plays the part of major premise in further scientific ratiocination.

The predicate is of necessity taken in full extension when its quantity is not specified.

§ 8. Further study of the essential homology that underlies the most elemental neural process as well as the most formal and artificial processes of scholastic logic will go far to convince us that the principal laws and formulas embodied in classical logic may be regarded as in a sense the outward and visible verbal crystallisation of neural processes that are most clearly presented to our intelligence in terms borrowed from the vocabulary of the logician. The formal syllogism may not be a very useful instrument of research to the empirical investigator and discoverer in the domain of natural science ; but we should not therefore ignore the very considerable assistance rendered to our elementary physiological concepts of neural processes by the recognition of the syllogism as a concrete objective type, whose sharply chiselled terms and rigid processes conform with and represent the workings of the rational human brain. The terms "major premise," "minor

premise " and " conclusion " form a group of terms that can be usefully employed in a far more general sense than that embraced by verbal logic. They may be applied to the data of human consciousness, whether we take such data in the subjective form of sensation, or in the objective form of organic states and sensificatory changes. The first neural process in the life of the individual is to be excited by the environment, to feel and to recognise objects in the crowd of material phenomena. When such objects, recognised and remembered, are referred to by sign or symbol or gesture or word, their mental representations form the subjects of propositions. From simple sensificatory ratios is formed the compound individual ratio that plays the part of major premise throughout the whole mental and organic life of each individual. The series of sensations are the series of minor premises which compounded with that major premise form the series of conclusions that are his actual thoughts.

A first step in the natural logic of sensation is the qualification of the subject, and the instinctive classification of subjects possessing similar attributes : stones are hard ; iron is heavy ; fire burns ; dogs bark, &c., &c. A second step is to promote qualification into a rudimentary quantification : horses are large ; cats are small ; the next village is a long way off, &c.

The third step consists in the closer quantification of the predicates : this stick is a yard long ; the distance between sun and earth is 92,000,000 miles ; a litre of oxygen weighs 1.43 gramme.

We are not content to have learned that " something " belongs to or forms a part of " something else," we ask to know to *how much* of that something else our something is equal or equivalent. In the proposition that " Bread is dear " the subject is qualified, the predicate is

unquantified ; the predicate (and in this respect the subject also) is quantified when the price of bread has been stated. Such quantification of predicates (and *ipso facto* of the related subjects) in given cases of greatest general significance, and with greatest attainable practical exactness, is the highest task of practical science. By induction from particular cases empirical observation has furnished us with an abstract principle. By deduction from that principle, assumed as correct, we predict and look for the presence and dimensions of particular instances conceived to fall under that principle. The law of gravitation is arrived at by induction, the diameter of the sun has to be predicated with the greatest attainable exactness, and measured whenever opportunity presents itself, by the aid of more and more perfected instruments.

The premises of a logical syllogism may be represented as a ratio between two ratios—major and minor—each containing a common term that cancels out. In the ordinary logical proposition with unquantified predicate the operation is explicitly laid out in words by which we normally proceed in the conscious or unconscious observation of material phenomena, viz. a named subject is placed at the centre of our field of attention and qualified or classified by a named attribute or class. "Grass is green," "sheep are mammals," are propositions of this low rank, in which a subject is merely qualified without any attempted quantification of its predicate.

"Grass," the named subject or centre of the field of attention, arouses in consciousness an idea or sensation of which there are two component terms (distinguished from each other by the mere fact that "grass" is thus centralised), viz. grass and a surrounding dimly-felt portion of the universe within which grass may be thought

about. The sensation or idea of "grass," is compounded from the objective data "grass" and other things "not-grass." The universe of thought in this connection is composed of the subject "grass" and of its complement or co-subject, "all things except grass," or symbolically of S and S' .

The sensation or idea aroused by simple propositions is evidently dependent upon *two* sensificatory terms that are respectively subject and predicate of the proposition.

It may be expressed in the form of the ratio $\frac{S}{P}$ or of the product SP . The proposition "Grass is green" put into the form $\frac{S}{P}$ is to indicate or enumerate *grass* as belong-

ing to the denomination of *green* things. "Grass is green" put into the form SP is to qualify the concept *grass* by the concept *green*. There is no valid distinction either logical or psychological between these two connections of S and P . The two data are combined whether they be considered as a ratio or as a compound. Psychologically, and therefore logically, the algebraic symbols of division and of multiplication are of identical significance.

The major and minor premises of a valid syllogism are to be considered on the same lines as a ratio or combination between two ratios (or two pairs of ratios since each term of each premise is in itself a ratio) in which a factor—the middle term—being common to both ratios cancels out leaving a resultant proposition—the conclusion—composed of one term of the major ratio and one term of the minor ratio.

For the sake of convenience of manipulation we shall express the proposition in the fractional form $\frac{S}{P}$ and shall

indicate the combination between component propositions of a syllogism as a multiplication of two fractions with elimination of a common factor above and below the line. The familiar syllogism in Barbara :—

M	F	K	M
All men are fallible.		All kings are men.	
	K	F	
∴ All kings are fallible.			

becomes on this notation : $\frac{M}{F} \times \frac{K}{M} = \frac{K}{F}$

§ 9. Reviewing the position thus far taken up we note as its chief points :—

That we treat words as the symbols of sensations which are essentially positive phenomena.

That we recognise the psychologically positive nature of negative words and concepts.

That the universe or sphere of discourse essentially consists of some thing + every thing else within that universe or sphere, *i. e.* of $S + S'$.

That in the psychological genesis of the proposition, the particular and limited forms I and O precede the general and unlimited forms A and E.

That the predicate when unquantified must be taken as being of full extension.

That the negative quality is essentially exclusion from the predicate, *i. e.* inclusion in the co-predicate.

That the negative quality of a proposition whether the negating word be attached to the subject, copula, or predicate, belongs psychologically to the predicate.

That in the symbolic representation of the verbal proposition the negating word whether attached to the subject, copula, or predicate, must always appear in qualification of the predicative symbol.

NOTE.—In setting out the symbols of a syllogism, it will be found inconvenient to restrict ourselves to the letters S and P for subjects and predicates, since this would oblige us to use a different S for a different subject, or to use the letter S in the position of a predicate when the subject of the first premise is the predicate of the second.

We shall take as far as possible the initial capital letters of leading words above and below the line in the subject and predicate position and reserve the small letters a. b. c. as limiting prefixes, indicative of "some," "many," "few," or other qualifying adjective; these prefixed letters may be used indifferently except when it is desirable to indicate specifically that the limitation has or has not changed its value.

In giving examples we have chosen words with different initial letters for the sake of simplicity. In the actual working out of verbal arguments, of which the form is generally deductive and in which the different leading words often have the same initial letter, distinctive capitals, such as Y and Z, are required to prevent mistakes, and to bring out whatever may be the middle term by the identical letter presenting itself twice.

I have not developed the argument in favour of the essentially affirmative character of negative propositions in ignorance of the consensus of logicians against that point of view. It is, in fact, that very consensus that has led me to present as clearly as possible and with some reiteration what I consider to be the psychological justification of the point of view. The transfer from subject and copula of the negative quality when present to the predicate is only an incident of the general point of view.

In order that the reader may at once have under his eyes the opposite side of the case, I have selected from the works of Lotze, Mill, Bain and Venn, passages in which that side is most explicitly and forcibly presented. I have to confess that none of these authors has carried conviction to my mind.

"... more appropriately than this limitative or infinite judgment, which is supposed to attribute a negative predicate to the subject by a positive copula and is usually expressed in the formula S is not-P. Much acumen has been expended even in recent times in vindicating this form of judgment, but I can only see in it an unmeaning product of pedantic ingenuity. Aristotle himself saw clearly enough that such expressions as 'not-man' are no concepts; they are not even apprehensible ideas.

"It is not worth while to spend more words on this point; obvious vagaries in science must be propagated even by an elaborate polemic."—Lotze, *Logic*, I. p. 64.

and J. S. Mill (Cap. iv. § 2, *Logic*, 6th Ed., 1865)—

“Some logicians, among whom may be mentioned Hobbes, state this distinction differently; they recognise only one sort of copula, *is*, and attach the negative sign to the predicate. Cæsar is dead, and Cæsar is not dead, according to these writers, are propositions agreeing not in the subject and predicate, but in the subject only. They do not consider “*dead*” but “*not-dead*” to be the predicate of the second proposition, and they accordingly define a negative proposition to be one in which the predicate is a negative name. The point, though not of much practical moment, deserves notice as an example (not unfrequent in logic) where by means of an apparent simplification, but which is merely verbal, matters are made more complex than before. The notion of these writers was, that they could get rid of the distinction between affirming and denying, by treating every case of denying as the affirming of a negative name. But what is meant by a negative name? A name expressive of the *absence* of an attribute.”

and Bain (*Deduction*, p. 83)—

“Some logicians have proposed to do away with the distinction between affirmative and negative by transferring the sign of negation from the copula to the predicate. A is-not B. A is not-B. There is then the appearance, but only the appearance, of making all propositions affirmative. The attempt is illusory. Affirmation and Denial belong to the very nature of things; and the distinction, instead of being concealed or disguised to make an imaginary unity should receive the utmost prominence that language can bestow.”

To this I should make the remark that I fully realise the impossibility of doing away with the distinction between affirmative and negative, which is psychologically a fundamental opposition between *inclusion* and *exclusion*. To affirm is to assert inclusion; to deny is to assert exclusion. Both are positive assertion of the existence of the subject, and it is beside the mark to argue whether such existence be real or imagined. For the purposes of discourse dragons and unicorns are just as existent as are dogs and cats.

In formal or verbal logic, negative as well as positive subjects, predicates and premises are the elements of discourse. But to take such negative elements as representing negative concepts is in my view to confuse thought by means of words with the words themselves. We may have a negative word, we cannot have a

negative thought. The words "no men" denote in thought "all men"; the words "not men" denote in thought "all the universe except men." From a psychological standpoint I am wholly unable to concur with the view expressed by Venn¹ that "the attempt to do away with the form of negation by transferring it from copula to the predicate in the hope that it could better be stowed away out of sight in the latter than in the former, is utterly trivial."

In my view the negative is essentially positive, and must belong to S or to P, converting these concepts to the complementary positive concepts, "not-S" or "not-P." The copula "is" is an unessential element of the proposition "S is P," to be represented by the horizontal line in our symbol $\frac{S}{P}$, and that copula is incapable of containing its own negation since it is posited.

Psychologically and logically negation belongs to the predicate, to which therefore the negating sign should be attached when required. A given subject has or has not a given property. S is either P, *i.e.* included in P, or P', *i.e.* excluded from P. To attach a negating sign to S, the subject of discourse, involves the psychological contradiction that we are undertaking to talk about S', *i.e.* not the subject of discourse, but something else—which is absurd. Of course it is permissible to talk about something else, S', other than the something named S, but that something else S' is now the positive subject of discourse, and is thereby entitled to receive the logical symbol S.

In illustration of the essential character of the negative concept, of which the expression "conspicuous by its absence" is representative, a friend calls my attention to the familiar lines "Old Mother Hubbard went to the cupboard to get her poor dog a bone" and insists that the central idea is the absent bone, and we are not thinking about all the bones in the Universe as would be implied by the formula $\frac{B}{C}$, "All bones are outside this cupboard."

To this objection I reply that psychologically the subject of thought and of discourse is the cupboard which after examination is discovered to be empty, as would be implied by the formula $\frac{C}{B}$.

This cupboard is boneless.

¹ Venn's *Empirical Logic*, p. 215.

CHAPTER II

THE INSTINCTIVE STATEMENT—CLASSIFICATION OF THE SUBJECT—THE CLASSIFICATORY SYLLOGISM

§ 10. THE simplest possible verbal statement is an affirmation of relation between two selected and named concepts S and P both taken in full extension. S is P implies that "*all S*" as regards the quality selected or understood, is part of "*all P*," or that "*all S*" is co-extensive with "*some or all P*."

This is what may be characterised as the primitive and instinctive statement. At a much later stage the affirmation that "*all S is co-extensive with some or all P*" is limited and made more precise when we are able to state that the co-extension of S with P is with "*all P*" or with "*some P*" or with "*so much P*," i.e. when we have quantified P with reference to S.

Statements of the type "*S is P*" are CLASSIFICATORY only; they constitute the premises of the classical or Aristotelian syllogism.

Statements of the type "*S = aP*," affirming how much P is co-extensive with S, are QUANTIFICATORY; they fulfil, in my view, the psychological system foreshadowed by Hamilton, Boole and De Morgan in connection with the "quantification of the predicate" as dealt with in verbal logic.

I shall accordingly divide the further subject-matter of this essay into two chapters, dealing respectively with the classificatory and the quantificatory syllogism.

§ 11. In classificatory premises composing classificatory syllogisms, the extension of the subject is always stated. We know whether we are talking about all S or some S. But as regards the predicate we do not distinguish any limitation of extension. When we say that all S or that some S is P we take P in full extension. By affirmation or denial that all S or some S is P we form the four kinds of proposition :—

- A. *Unlimited affirmative* (= universal or total)
- I. *Limited affirmative* (= particular or partial)
- E. *Unlimited negative* (= universal or total)
- O. *Limited negative* (particular or partial).

I do not intend to re-open now the question of the meaning, value and position of the negating word ; I have already indicated the position taken up throughout to the effect that psychologically and symbolically we have to deal with only positive elements ; and the question sometimes debated as to whether the negating word belongs logically to the copula or to the predicate is on those lines altogether empty. I do not regard the copula as an essential element of the proposition, which in my view consists only of subject and predicate brought into relation with each other in sensation and in discourse. I am of course unable to connect a negating sign with the copula since I take the connection between subject and predicate as conveyed by their juxtaposition in thought or in discourse or on paper. The proposition "S is P" is to my mind stated in the formula $\frac{S}{P}$; the dividing line may, if desired, be termed the copula, but we shall hardly be tempted to devise any sort of dividing line to denote that S is not P ; we shall take S (if it exists) as existing in the category of things other than P, and write

"S is not P" by the formula $\frac{S}{P'}$. Even if verbalised thought the copula need not appear as a definite element of the proposition, nor was it treated by Aristotle himself as such. The proposition "Man is good" was adequately stated by the words $\delta \alpha \nu \theta \rho \omega \pi \omicron \varsigma \alpha \gamma \alpha \theta \omicron \varsigma$.¹

§ 12. It is the indeterminate character of so-called negative propositions that constitutes the most fruitful source of doubt or of actual fallacy in logical processes effected by means of words. The doubt or the fallacy becomes most apparent as soon as an attempt is made to convert more or less elastic words into more or less rigid symbols. The proposition "*all men are not wise*" might logically mean that "*all men are foolish*," but is generally taken to signify that "*some men are not wise*." The proposition "*no men are immortal*" really signifies that "*all men are not-immortal*" (=mortal).

An unlimited negative statement such as "No lawful action is dishonourable" brings into the field the subject of discourse not "no lawful action" but "every lawful action." Although the negating sign is attached to the subject, it is clear that the subject is not negative but positive. All lawful actions are felt or imagined as being outside the category "dishonourable." And that this is so becomes apparent as soon as we attempt to cast the proposition into a symbolic form. We find ourselves debarred from indicating by any negative symbol the negative verbal subject "no lawful action." We may not write it as S' , which would signify the equally positive co-subject or "Every thing except all lawful actions"; we are forced to write it S , signifying "all lawful actions," and to place the negating sign with

¹ $\delta \kappa \iota \nu \delta \upsilon \nu \omicron \varsigma \mu \epsilon \gamma \alpha \varsigma \kappa \alpha \lambda \omicron \varsigma \delta \epsilon \delta \alpha \gamma \omega \nu$.—Plat *Phaedo*.

the predicate. The proposition then has as its proper symbolic form $\frac{S}{\bar{P}}$, by which is asserted the existence of all legal actions in the category complementary of P, *i. e.* in the co-predicate, *i. e.* outside the category of the predicate *i. e.* outside the category dishonourable.

There was no actual ambiguity of meaning in the proposition "No lawful action is dishonourable," but in the proposition "All lawful actions are not honourable" while the form is that of an unlimited negative proposition E, the meaning is ambiguous, and the words generally contain and convey a spurious implication. If, as is not the case, the proposition were stated thus "All lawful actions are not-honourable" it would be a true unlimited negative E, with a positive subject assigned to a predicate with the negating sign. We should symbolise the statement correctly in the form $\frac{S}{\bar{P}'}$, where P' is the category "not-honourable" complementary to P denoting the category "honourable." And it is also obvious that we should have been entitled to take the proposition as an unlimited affirmative A, with a positive subject assigned to a predicate without negating sign, to be written P, provided that we understood P as indicating the category "not-honourable," where of course P', the co-predicate, would have to stand for the category "honourable."

But as commonly used the proposition "All lawful action is not honourable" is not uttered as an unlimited E (or A) with its negating sign in the predicate, but as a spurious phrase with the negating sign in the copula, to contain and convey a meaning belonging properly to the two limited propositions O and I, "some lawful actions are not honourable," "some lawful actions are honourable." And this proper meaning could be properly stated only

by two such propositions, and properly symbolised only by two corresponding formulas : viz. $\frac{aS}{P'}$ and $\frac{bS}{P}$.

§ 13. It is a first step towards the clearing up of ambiguities of this character to recognise that every proposition, whether affirmative or negative in form, is psychologically an affirmation, *negatio affirmatio contraria*. The subject of any proposition is necessarily positive. To name a subject is necessarily to place something in the field of discourse; there is no subject of discourse if nothing (or *per impossibile* less than nothing) is placed in the field.

We cannot have a non-existent¹ or negative subject. We can only have a subject, or some of a subject, or everything except a subject, or something except a subject, viz. S, or aS, or S', or aS'; hence we have as a first rule to be observed in the reduction of propositions from words to symbols that :—

Unlimited (= universal) negative propositions E, of the form "No S is P" must be converted into unlimited affirmative propositions A, of the form "All S is not-P" by transferring the negating word from the verbal subject to the verbal predicate.

Thus the symbolic form of "No S is P" is $\frac{S}{P'}$.

No such conversion is required in the case of the limited (= particular) proposition, O, of the form "Some S is not P" since the verbal subject is already positive in form, and the negating word is attached to the predicate.

The symbolic form of "Some S is not P" is $\frac{aS}{P'}$.

The positive propositions A and I, "All S is P,"

¹ For the purpose of logical discourse, dragons and devils and angels are as existent as horses and dogs and sheep.

"Some S is P," offer no difficulty or ambiguity, either in verbal or in symbolic form.

The unlimited affirmation "All S is P" is $\frac{S}{P}$. The limited affirmation "Some S is P" is $\frac{aS}{P}$.

Up to this point it will have become apparent that in the symbolic treatment of the four propositions A, I, E, O, the only one offering the slightest difficulty has been the universal negative E. I attach considerable importance to the argument by which I have sought to show that this "Dictum de Nullo" is substantially and psychologically a "Dictum de Omnibus," and I shall repeat the argument in this sense in a slightly different form.

Before making the statement that "None of these sheep are black," we must have seen that "All of these sheep are not-black." "These sheep" being our subject of discourse in full extension, it is clear that the two propositions are psychologically identical, and correctly symbolised by the formula $\frac{S}{P'}$ where S stands for "all these sheep," and P for the category of "black things."

The field of thought—or universe of discourse—in this, as in any other case, consist of *something* S, plus *everything else* S', or symbolically $1 = S + S'$.

Similarly the universe of discourse with reference to the predicate consists of P (= all things possessing a given attribute) + P' (= all things whatsoever except things possessing that given attribute).

We have found it convenient, if not necessary, to call S', the co-subject, or complement of S; P' the co-predicate or complement of P. We shall shortly find it convenient, and necessary for the symbolic treatment of the syllogism in its several forms, to substitute S and

S' for each other, and similarly P and P' . This proceeding will be referred to as the "reversal of indices."

All propositions, whether negative or positive in form, are psychologically (and objectively) substantiated by positive relations between positive phenomena (and symbols) S , S' , P , P' .

Keeping a given subject and predicate in the field of thought, we may have four classificatory relations :—

(I) $\frac{S}{P}$ signifying that *all S is P*.

(II) $\frac{S}{P'}$ signifying that *all S is something other than P*.

i. e. in both cases we state the existence of S , and taking it in its entire extension, we include or exclude it from all things of the denomination P ; these are the universal or unlimited propositions A and E of formal logic.

(III) $\frac{aS}{P}$ signifying that *some S is P*.

(IV) $\frac{aS}{P'}$ signifying that *some S is something other than P*,

which are the particular or limited propositions I and O .

If instead of affirming a given subject S , we affirm "everything except that subject" or "something except that subject," viz. S' or aS' , we have another group of the following four relations :—

(V) $\frac{S'}{P}$ signifying that *everything except S is P*.

(VI) $\frac{S'}{P'}$ signifying that *everything except S is something other than P*.

(VII) $\frac{aS'}{P}$ signifying that *something except S is P*.

(VIII) $\frac{aS'}{P'}$ signifying that *something except S is something other than P*.

The above eight formulas include all the relations that it is possible to posit between subject, co-subject, predicate and co-predicate, as positive numerator and denominator of a positive fraction, the line between numerator and denominator standing for the copula "is" or for the words "*forms part of*," and the prefixed letter *a* standing for the word "*some*"; thus $\frac{S}{P}$, signifying that S is P, is more precisely "*All S (or every S) forms part of things possessing the attribute P.*" Limiting prefixes are used only in the numerator, the predicate or denominator being always taken in full extension; obviously if S forms part of or is equal to *aP*, it forms part of P, if S is a particular sort of P, it is part of P without reservation. In a later part of this paper, when we shall deal with quantified predicates, we shall find it convenient to adopt an equation as the notation for a co-extension, the equating sign to stand for the words "*is co-extensive with*"; the fractional form is appropriated to the unquantified propositions of ordinary language, the fractional line standing for the words "*forms part of*." Before entering upon the detailed consideration of the syllogism we should take a preliminary survey of the possible significations and of the necessary implications contained in each of the four propositions A, E, I, O. This survey will guide us to the substitutions that will be required in the symbolisation and reduction of premises to their logical conclusion—substitutions that will be found to be in substance a simplified form of the logical processes of limitation, conversion and obversion.

§ 14. The meanings and implications of the four verbal propositions A, E, I, O, are as follows :—

I **A.** ALL S IS P

May signify that : S is some P

or that : S is all P

and necessarily implies that :—

Some S is P

Some P is S

All not-P is not-S

Some not-P is not-S

Some not-S is not-P

II **E.** No S is P (= All S is not-P)

May signify that : S is some not-P

or that : S is all not-P

and necessarily implies that :—

Some S is not-P

Some not-P is S

All P is not-S

Some P is not-S

Some not-S is P

III **I.** SOME S IS P ("some" = "some or all")

May signify that : S is some P

or that : S is all P

or that : Some S is some P

or that : Some S is all P

and necessarily implies that : Some P is S

IV **O.** SOME S IS NOT P

May signify that : S is some not-P

or that : S is all not-P

or that : Some S is some not-P

or that : Some S is all not-P

and necessarily implies that : Some not-P is S

The implied propositions contained in each of these four original propositions are brought into a clearer light by a tabular summary of the facts, the several propositions being cast in the fractional form S/P to signify that S is or forms part of P .

Original Propositions	Necessarily Implied Propositions					
A $\frac{S}{P}$		$\frac{aS}{P}$	$\frac{aP}{S}$	$\frac{P'}{S'}$	$\frac{aP'}{S'}$	$\frac{aS'}{P'}$
E $\frac{S}{P'}$		$\frac{aS}{P'}$	$\frac{aP'}{S}$	$\frac{P}{S'}$	$\frac{aP}{S'}$	$\frac{aS'}{P}$
I $\frac{aS}{P}$	$\frac{aP}{S}$					
O $\frac{aS}{P'}$	$\frac{aP'}{S}$					
	Conversion	Limitation	Limitation Conversion	Conversion Obversion	Limitation Conversion Obversion	Limitation Obversion

The careful inspection of this table can teach us in many ways. It exhibits at a glance the cases where the conversion and obversion of a proposition are legitimate and where limitation of the transformed proposition is required. It affords us forms from which it is an easy matter to pick out and tabulate all possible contrary and contradictory cases. But for our present purpose what is of more immediate interest is that we can derive from it two simple rules that will serve us for the reduction of any two premises to their valid logical conclusion, and lead us away from all danger of fallacy by "illicit extension" and "undistributed middle."

§ 15. The list of contrary and contradictory propositions drawn up on this basis is as follows :—

Original Propositions		Contrary Propositions (with excluded middle)	Contradictory Propositions
A	$\frac{S}{P}$	$\frac{S}{\overline{P}}$	$\frac{aS}{\overline{P}}$
E	$\frac{S}{\overline{P}}$	$\frac{S}{P}$	$\frac{aS}{P}$
I	$\frac{aS}{P}$	$\frac{aS}{\overline{P}}$	$\frac{aS}{\overline{P}}$
O	$\frac{aS}{\overline{P}}$	$\frac{aS}{P}$	$\frac{aS}{P}$

The interpretation of the previous symbolic list will be helped by the following verbal list of examples :—

Original	Contrary	Contradictory
A All slaves are black	No slaves are black	Some slaves are not black
E No slaves are black	All slaves are black	Some slaves are black
I Some slaves are black	Some slaves are not black	Some slaves are not black
O Some slaves are not black	Some slaves are black	Some slaves are black

§ 16. The rules derivable from the table are as follows:—

RULE 1. The subject and predicate of a universal proposition, A or E, can be—

(a) **Transposed with reversal of indices** (e.g. for $\frac{S}{P}$ we may substitute $\frac{P'}{S'}$).

(b) **Transposed with limitation of the new subject** (e.g. for $\frac{S}{P}$ we may substitute $\frac{aP}{S}$).

RULE 2. The subject and predicate of a particular proposition, I or O, can be **transposed, with limitation of the (new) subject** (e.g. for $\frac{aS}{P}$ we may substitute $\frac{aP}{S}$); or, if it be desirable to indicate that the limiting expression has changed its value, the prefixed letter may be changed, and the substituted expression given as $\frac{bP}{S}$.

These two rules, by aid of which we shall find that any pair of premises capable of reduction to valid conclusion, can be dealt with without danger of fallacy, are in themselves no great burden upon the memory, being in themselves obviously most simple and rational. They can however be memorised if desired by this single tag, of which I presume that any explanation would be superfluous:—

“Totales totaliter vel partialiter, partiales partialiter solum.”

The new predicate of a transposed relation cannot preserve any limiting symbol, since, as stated on p. 26, the predicate of the simple proposition is given in full extension.

The object to be attained by the two rules given above is to bring an identical symbol (representing a middle term to be cancelled out) above and below the line in each of the two premises. Evidently the symbol to be eliminated must be unlimited ; *no limited term can be eliminated.*

THE CLASSICAL SYLLOGISM.

§ 17. We may now proceed to apply this fractional system of notation to the Syllogism, regarding its two premises as two factors compounded together into a resultant, *i. e.* multiplied out so as to give a conclusion by the elimination of a middle term occurring twice, once in a numerator and once in a denominator. We shall find that on this system there is no danger of "illicit process," *i. e.* of an undue extension in the conclusion of a term limited in the premises ; the fallacy of "undistributed middle" will also be very clearly apparent, and therefore easily avoided. In order to test the system upon a great variety of forms, we will first apply it to the nineteen classical moods of valid syllogism, applying our two rules for the reduction when necessary.

The plan of procedure, which will indeed be sufficiently apparent on inspection, is as follows : the two subjects and the two predicates of the premises are represented by any suitable letters ; one such letter must occur twice, and is the middle term to be eliminated ; when necessary, *i. e.* when this term is in both premises on the same side of the line, a substitution is made in accordance with our two rules, so as to make the term identical above and below the line ; it is then eliminated, leaving two terms as the conclusion. We begin by translating the verbal terms of the premises into symbols, we end by retranslating the symbols of the conclusion into

their original words. We shall follow out in detail the resolution of examples of all the nineteen moods in the four classical figures, in order to prove the general applicability of the notation to any form of syllogism without reference to mood or figure or rules of conversion other than the rule explained on the previous page.

FIG. I. MP . SM : SP

1. *Barbara*.

All acts are statements. All statements are facts.

$$\begin{array}{cccc} A & S & S & F \\ \frac{A}{S} \times \frac{S}{F} = \frac{A}{F} = \text{All acts are facts. (Concl.)} \end{array}$$

2. *Celarent*.

No statement is necessary. Every truth is a statement.

$$\begin{array}{cccc} S & N & T & S \end{array}$$

The negative quality of the first proposition has to be transferred from subject to predicate. The subject of discourse is not "no statement" but "every statement," and the proposition signifies that "all statements are non-necessary."

$$\begin{array}{l} \frac{S}{N'} \times \frac{T}{S} = \frac{T}{N'} = \text{Every truth is non-necessary.} \\ = \text{No truth is necessary. (Concl.)} \end{array}$$

3. *Darii*.

All statements are deceptive. Some actions are statements.

$$\begin{array}{cccc} S & D & aA & S \\ \frac{S}{D} \times \frac{aA}{S} = \frac{aA}{D} = \text{Some actions are deceptive. (Concl.)} \end{array}$$

4. *Ferio*.

No statement is necessary. Some actions are statements.

$$\begin{array}{cccc} S & N & aA & S \\ \frac{S}{N'} \times \frac{aA}{S} = \frac{aA}{N'} = \text{Some actions are unnecessary. (Concl.)} \\ D \end{array}$$

FIG. II. PM . SM : SP

5. *Cesare*.

No conclusions are direct. All truths are direct.

C D T D

$\frac{C}{D'} \times \frac{T}{D}$ where it is evident that one of the premises must be transposed *totaliter* to give an identical middle term above and below the line.

$\frac{D}{C'} \times \frac{T}{D} = \frac{T}{C'} =$ All truths are non-conclusions.
 $=$ No truths are conclusions. (*Concl.*)

6. *Camestres* is obviously the same as *Cesare* with the premises stated in reverse order.

All truths are direct. No conclusions are direct.

T D C D

$\frac{T}{D} \times \frac{C}{D'}$

$\frac{D'}{T} \times \frac{C}{D} = \frac{C}{T'} =$ All conclusions are non-truths.
 $=$ No conclusions are truths. (*Concl.*)

Obviously this result $\frac{C}{T'}$ is obtainable by the transposition *totaliter* of the previous result $\frac{T}{C'}$.

7. *Festino*.

No conclusions are direct. Some truths are direct.

C D aT D

$\frac{C}{D'} \times \frac{aT}{D}$ by transposition applied to the first premise becomes

$\frac{D}{C'} \times \frac{aT}{D} = \frac{aT}{C'} =$ Some truths are not conclusions. (*Concl.*)

8. *Baroko*.

All truths are direct. Some conclusions are not direct.

T D aC D

$$\frac{T}{\overline{D}} \times \frac{aC}{\overline{D'}}$$

$$\frac{D'}{\overline{T}} \times \frac{aC}{\overline{D}} = \frac{aC}{\overline{T}} = \text{Some conclusions are not truths. (Concl.)}$$

FIG. III. MP. MS : SP

9. *Darapti*.

All comments are painful. All comments are odious.

C P C O

$\frac{C}{\overline{P}} \times \frac{C}{\overline{O}}$ where it is evident that one of the premises must be transposed *partialiter* to give an identical middle term above and below the line.

$$\frac{C}{\overline{P}} \times \frac{aO}{\overline{C}} = \frac{aO}{\overline{P}} = \text{Some odious things are painful. (Concl.)}$$

10. *Disamis*.

Some comments are painful. All comments are odious.

aC P C O

$$\frac{aC}{\overline{P}} \times \frac{C}{\overline{O}}$$

$$\frac{aP}{\overline{C}} \times \frac{C}{\overline{O}} = \frac{aP}{\overline{O}} = \text{Some painful things are odious.}$$

$$= \frac{aO}{\overline{P}} = \text{Some odious things are painful. (Concl.)}$$

11. *Datisi* is merely *Disamis* with reversed premises.

All comments are odious. Some comments are painful.

C O aC P

$$\frac{C}{\overline{O}} \times \frac{aC}{\overline{P}}$$

$$\frac{C}{\overline{O}} \times \frac{aP}{\overline{C}} = \frac{aP}{\overline{O}} = \text{Some painful things are odious. (Concl.)}$$

This conclusion, which is orthodox, is of course equivalent with the transposed form $\frac{aO}{\overline{P}}$ i.e. some odious things are painful.

12. *Felapton.*

No comment is possible. All comments are odious.

C P C O

$$\frac{C}{P'} \times \frac{C}{O}$$

$$\frac{aP'}{C} \times \frac{C}{O} = \frac{aP'}{O} = \text{Some things that are not possible are odious.}$$

$$= \frac{aO}{P'} = \text{Some odious things are not possible.}$$

(Concl.)

The second is the orthodox form of the conclusion ; the same result would be arrived at directly if transposition were applied to the second instead of to the first premise, thus :—

$$\frac{C}{P'} \times \frac{aO}{C} = \frac{aO}{P'} = \text{Some odious things are not possible.}$$

13. *Bokardo.*

Some comments are not possible. All comments are odious.

aC P' C O

$$\frac{aC}{P'} \times \frac{C}{O}$$

$$\frac{aP}{C} \times \frac{C}{O} = \frac{aP'}{O} = \text{Some impossible things are odious.}$$

$$= \frac{aO}{P'} = \text{Some odious things are impossible.}$$

(Concl.)

14. *Ferison.*

No comment is possible. Some comments are odious.

C P aC O

$$\frac{C}{P'} \times \frac{aC}{O}$$

$$\frac{C}{P'} \times \frac{aO}{C} = \frac{aO}{P'} = \text{Some odious things are impossible.}$$

(Concl.)

FIG. IV. PM . MS : SP

15. *Bramantip*.

All facts are useful. All useful things are valuable.

$$\begin{array}{cccc} F & U & U & V \\ \frac{F}{U} \times \frac{U}{V} = \frac{F}{V} = \text{All facts are valuable. (Concl.)} \end{array}$$

This general conclusion, although unorthodox in this mood, is obviously valid, and includes the limited conclusions :—

$$\frac{aF}{V} ; \text{ or, Some facts are valuable ;}$$

$$\text{and } \frac{aV}{F} ; \text{ or, Some valuable things are facts,}$$

which last is the orthodox logical conclusion.

16. *Camenes*.

All facts are valuable. No valuable things are superfluous.

$$\begin{array}{cccc} F & V & V & S \\ \frac{F}{V} \times \frac{V}{S'} = \frac{F}{S'} = \text{All facts are non-superfluous.} \\ = \text{No facts are superfluous. (Concl.)} \end{array}$$

This general conclusion, correct but not orthodox, is obviously equivalent to the transposed conclusion :—

$$\frac{S}{F'} ; \text{ or, All superfluous things are not-facts ;}$$

$$\text{or, No superfluous things are facts,}$$

which last is the orthodox logical conclusion in this mood.

17. *Dimaris*.

Some facts are valuable. All valuable things are desirable.

$$\begin{array}{cccc} aF & V & V & D \\ \frac{aF}{V} \times \frac{V}{D} = \frac{aF}{D} = \text{Some facts are desirable.} \\ = \frac{aD}{F} \text{ i.e. Some desirable things are facts,} \end{array}$$

which last is the orthodox logical conclusion, while obviously the former is equally correct.

18. *Fesapo*.

No facts are negligible. All negligible things are trifles.

F N N T

$$\frac{F}{N'} \times \frac{N}{T}$$

Here it is necessary to apply transposition to both pre-mises, *totaliter* to the first, *partialiter* to the second, in order to obtain an identical middle term above and below the line.

$$\frac{N}{F'} \times \frac{aT}{N} = \frac{aT}{F'} = \text{Some trifles are not facts. (Concl.)}$$

19. *Fresison*.

No facts are negligible. Some negligible things are trifles.

F N aN T

$$\frac{F}{N'} \times \frac{aN}{T}$$

Double transposition has to be applied here as in the previous case.

$$\frac{N}{F'} \times \frac{aT}{N} = \frac{aT}{F'} = \text{Some trifles are not facts. (Concl.)}$$

§ 18. If we inspect any or all of the cases we may choose to formulate thus we shall find that there is no danger of committing the fallacy of "ILLICIT PROCESS"; we evidently may not eliminate any limiting symbol from a numerator, we are not liable to do so inadvertently, and *the limitation in a premise will* therefore always appear in the conclusion. Nor is there much danger of the accidental extension of a term in the process of conversion; this simple, yet far too common source of fallacy, is forbidden by the obvious consideration that $\frac{S}{P}$ does not involve $\frac{P}{S}$ but only $\frac{aP}{S}$ and

the fallacy is too glaring to slip in accidentally. The only remaining fallacy is that of "UNDISTRIBUTED MIDDLE," and it may perhaps be well to follow its possible origin by the detailed consideration of a syllogism in *Disamis*.

aF	P	F	E
Some fruits	are poisonous.	All fruits	are edible.

∴ some edibles are poisonous

$$\frac{aF}{P} \times \frac{F}{E} \text{ or } \frac{aP}{F} \times \frac{F}{E} = \frac{aP}{E} \text{ or } \frac{aE}{P}$$

We might perhaps have been tempted to try substitution of the second instead of the first premise, writing $\frac{aF}{P} \times \frac{bE}{F}$ and then, by cancelling F, putting as the conclusion $\frac{abE}{P}$ viz. Some of some edibles are poisonous. To

cancel F in this way is not legitimate, and would in other cases draw us into the fallacy of undistributed middle. The expression aF must be treated as a single undecomposable expression, and not as if composed of two separable algebraic factors. Substitution applied to the second premise in this example does not yield a removable middle term. This fact is of sufficient importance for us to recognise it in the prohibitive rule: *no limited term can be eliminated*.

The elegance with which a valid conclusion comes out from premises thus treated, as compared with the cumbersome and imperfect processes based upon the well-known mnemonic lines *Barbara*, *Celarent*, &c., will be particularly obvious if we arrange each of the sixteen possible pairs of premises (affirmative and negative, universal and particular, A E I O) in each of the four classical figures of the syllogism.¹

¹ Perhaps the best case for the comparison is 8. *Baroko*. Cf. Bain's *Logic* (Deduction), pp. 140-1. The affirmative major is treated as a

On reducing the premises to their conclusion (applying our two rules where necessary) we find that in any valid syllogism the middle term is easily eliminated, and a fractional expression with an obvious verbal interpretation on its face, obtained as the conclusion. Where no conclusion can be drawn from the premises, the middle term cannot be got rid of by either of the substitutions which it is allowable to make. Take, for instance, the premises $\frac{A}{M} \times \frac{B}{M}$ and it will be found at once that an identical middle cannot be placed above and below the line by either of our legitimate transpositions of premises; we obtain the irreducible expressions $\frac{M'}{A'} \times \frac{B}{M}$ or $\frac{aM}{A} \times \frac{B}{M}$. Whereas with the premises $\frac{A}{M} \times \frac{B}{M'}$ we see at once that there is valid syllogism since we may write $\frac{A}{M} \times \frac{M}{B'}$ or $\frac{M'}{A'} \times \frac{B}{M'} = \frac{A}{B'}$ or $\frac{B}{A'}$, so as to cancel an identical middle term.

On inspection of the Table on page 46, it may be seen in what respect the nineteen Aristotelian syllogisms are deficient and redundant.

The mood *Bramantip* is merely an unnecessarily weakened *Barbara*, for its premises $A A$ carry a conclusion A , and not merely the conclusion I . They are deficient in so far as the repetition of similar syllogisms in different figures is not fully carried out, *e. g.* the syllogism $A O O$ (*Baroko* of the second figure) is also valid in the third figure, and the syllogism $O A O$ (*Bokardo* of the third

disguised negative, the negative minor as a disguised positive. The distortion is remedied by obverting and converting the major, and by obverting the minor. *Baroko* was proved and reduced to a mood of the first figure by the cumbrous process known as "*reductio ad impossibile*."

figure) is also valid in the second figure. The rule that there is no conclusion from particular premises is seen to hold good throughout, *but the rule that there can be no conclusion from negative premises is seen to be subject to restriction*, in consequence of our fundamental principle that propositions, whether formally negative or positive, are really positive relations between positive concepts $SS'PP'$.

Moreover, I think now as I thought many years ago on my first introduction to the nineteen classical moods in the four classical figures of formal logic, that the system was not thoroughly consistent throughout. My introduction to the subject occurred at Aberdeen, so that I naturally turn to Bain's *Logic* for an illustration of my meaning. I take as my examples the following pair of premises (*Deduction*, pp. 175-7) changing one of the words in order to have a convenient initial letter:—

“All men are liars,”
“No dogs are men.”

These are premises A E in Fig. I, and are given as affording no valid conclusion.

But taking the same premises in reversed order, viz. :—

“No dogs are men,”
“All men are liars,”

they become premises E A in Fig. IV, and are given as affording valid conclusion by a syllogism in *Fesapo*, viz. :—

“Some liars are not dogs.”

The inconsistency is obvious ; the premises in question must be invalid in both cases or valid in both cases. Their transposition does not affect the psychological

significance of their combination. We cannot have the same premises furnishing a conclusion in one case and not the other.

Worked out symbolically, the premises give conclusion as follows :—

$$\frac{M}{L} \times \frac{D}{M'} \text{ or } \frac{aL}{M} \times \frac{M}{D'} = \frac{aL}{D'} \text{ and } \frac{aD'}{L}$$

or, "Some liars are not dogs,"
and, "Some not-dogs are liars."

Of these two conclusions the former comes out in Fig. IV by *Fesapo*, the latter in Fig. I by *Baleno*.

My contention is that, logically, if the conclusion is valid in *Fesapo* it is valid in *Baleno*; if it is not valid in *Baleno* it is not valid in *Fesapo*.

Another obvious weak spot in the formal logic of the nineteen classical moods is the syllogism in *Bramantip* of Fig. IV, where from two premises A A the conclusion drawn is I, where it might be A. Taking, for example, the following premises :—

{ "All oranges are fruits."
{ "All fruits are juicy."

Only the limited conclusion is drawn that "Some juicy things are oranges," when obviously the unlimited conclusion that "all oranges are juicy" can be drawn from the premises given.

Simple reversal of their order to—

"All fruits are juicy,"
"All oranges are fruits,"

gives premises in *Barbara* of Fig. I, from which the orthodox conclusion is that : "All oranges are juicy."

Worked out symbolically, the premises give conclusion thus in whatever order we choose to write them down :—

$$\frac{O}{F} \times \frac{F}{J} = \frac{O}{J}$$

In the actual working out of premises to their conclusion, I pay no heed to mood or figure.

Taking, *e. g.* the premises :—

“All comments are odious,”

“Some comments are necessary.”

I write $\frac{C}{O} \times \frac{aC}{N}$

Reversing the second premise I have $\frac{C}{O} \times \frac{aN}{C}$

From which by cancelling C the conclusion is $\frac{aN}{O}$

Which in words is “Some necessary things are odious.”

This happens to be a syllogism in the third figure MP . MS : SP in *Datisi*, but the fact is of no concern, since quite apart from it the syllogism is obviously valid.

Take, however, the same premises reversed, *i. e.*—

“Some comments are necessary.”

“All comments are odious.”

The conclusion comes out as before $\frac{aN}{O}$, “some necessary things are odious”; but it is quite superfluous to learn that this is now a syllogism in *Disamis*.

Take another case :—

“All comments are odious.”

“All comments are necessary.”

I write $\frac{C}{O} \times \frac{C}{N}$; by reversing the first premise I get $\frac{aO}{C} \times \frac{C}{N} = \frac{aO}{N}$ which reversed = $\frac{aN}{O}$

i. e. “Some odious things are necessary,”

and, “Some necessary things are odious.”

It is pure pedantry to say that with the second of these two correct conclusions the syllogism is correct (of the third figure in *Darapti*) and with the first conclusion incorrect (*i.e.* of no recognised figure and mood).

If I had reversed the second instead of the first premise I should have got $\frac{C}{O} \times \frac{aN}{C} = \frac{aN}{O}$ or $\frac{aO}{N}$ *i.e.* correct conclusions as before ; and it does not matter at all that one of them (*i.e.* the first) is orthodox, and the other not.

Take one more pair of premises, to be worked out by our two rules (if required), without any reference to formal rules, figure or mood :—

“ No selfish people are contented,”

“ All lucky people are contented.”

$$\frac{S}{C'} \times \frac{L}{C}$$

Reversing $\frac{L}{C}$ (Rule 1*a*, p. 31) we have $\frac{S}{C'} \times \frac{C'}{L'}$

Cancelling C' we have the conclusion

$$\frac{S}{L'} \text{ or reversing } \frac{L}{S'}$$

Translating back into words we have our conclusion :—

All selfish people are	}	or,	No selfish people are
unlucky			lucky
All lucky people are			No lucky people are
unselfish			selfish.

The conclusion is correct and complete apart from any reference to figure or mood ; there is no need to refer to Cesare and to remember that to get the orthodox result the first premise (and not the second) should be simply transposed ; the order of terms in the premises and conclusion is indifferent, we need not remember that it should

be PM . SM : SP nor that the only orthodox conclusion of these premises in Cesare is that "No lucky people are selfish," nor that in using the converse form "No selfish people are lucky" we are no longer strictly formal logicians.

I do not regard the distinction of syllogisms into Figures in accordance with the position of the middle term as possessing any psychological significance whatever, nor do I value the 19 classical moods that have survived the ordeal of scholastic disputation, otherwise than as supplying a store of variously twisted verbal syllogisms that can be reduced to one simple type from which a middle term common to both premises naturally disappears. That type is represented by *Barbara*, Fig. I, or symbolically by the formula :—

$$\frac{M}{P} \times \frac{S}{M} = \frac{S}{P}^1$$

But in the process of arriving at this conclusion, I thought it worth while to make a complete review of all possible cases of pairs of premises under the four figures, and to examine on the symbolic plan adopted in this essay each case as to its syllogistic validity.

§ 19. The results of that review and examination are summarised in the accompanying table (p. 46).

There are 16 possible pairs of premises in each of the four figures, making up a grand total of 64 possible pairs as shown in the table.

According to my analysis the 64 pairs are divisible into two groups : a group of 28 cases in which the premises yield no conclusion, in which, therefore, there can be no syllogism ; and a group of the remaining 36 cases in which, according to the principles set forth in this

¹ Or preferably $\frac{S}{M} \times \frac{M}{P} = \frac{S}{P}$ which, however, is not strictly *Barbara* although psychologically its equivalent.

I. MP. SM:SP 4 classical 4 Spanish 8 invalid	AAA Barbara AEO Baleno AII Darii AO —	EAE Celarent EEO Celebro EIO Ferio EO —	IA — IEO Silento II — IO —	OA — OEO Bolero OI — OO —
II. PM. SM:SP 4 classical 4 Spanish 8 invalid	AAO Fandango AEE Camestres AI — AOO Baroko	EAE Cesare EEO Celebro EIO Festino EO —	IA — IEO Silento II — IO —	OA — OEO Bokardo OI — OO —
III. MP. MS:SP 6 classical 6 Spanish 4 invalid	AAI Darapti AEO Baleno AII Datisi AOO Baroko	EAO Felapton EEO Celebro EIO Ferison EOO Hesnogo	IAI Disamis IEO Silento II — IO —	OA — OEO Bokardo OI — OO —
IV. PM. MS:SP 5 classical 3 Spanish 8 invalid	AAI Bramantip AEE Camenes AI — AO —	EAO Fesapo EEO Celebro EIO Fresison EOO Hesnogo	IAI Dimaris IEO Silento II — IO —	OA — OEO — OI — OO —
Normal Type. SM. MP:SP 4 normal 4 Spanish 8 invalid	AAA normal AEE " AI — AO —	EAO "Spanish" EEO " EIO " EOO "	IAI normal IEO " II — IO —	OA — OEO — OI — OO —

essay, the premises are capable of yielding a conclusion by elimination of a middle term common to both premises, in which, therefore, there is valid syllogism. These 36 cases comprise two groups : the classical group of 19 valid syllogisms accepted by the great majority of logicians (and indicated in the table by capital letters) ; and an additional group of the remaining 17 pairs (indicated in the table by small letters) and which, for reasons that will presently appear, I designate as the Spanish group.

So that finally the table comprises three groups :—

- A group of 28 admittedly invalid pairs of premises,
- A group of 19 admittedly valid pairs of premises,
- A group of 17 debateable pairs of premises, *i.e.* the Spanish group.

It will be noticed on inspection of the table that in all cases of this last group, the conclusion is a particular negative proposition O. This, coupled with the fact that I happened, at the time of studying them, to be travelling in Spain, led me to designate and memorise them by aid of the Spanish sounding jargon :—

Celebro, Silento, Baleno, Bolero, Fandango, Hesnogo.

On inspection of the table it will be evident that—

Celebro occurs in	Figures	I.	II.	III.	IV.
Silento	„ „ „	I.	II.	III.	IV.
Baleno	„ „ „	I.	—	III.	—
Bolero	„ „ „	I.	—	III.	—
Fandango	„ „ „	—	II.	—	—
Hesnogo	„ „ „	—	—	III.	IV.

i.e. in 15 cases. The other two cases (making up the total of the “Spanish group” to 17) are Bokardo II.

and Baroko III. which are validated if the arguments put forward in this essay are valid.

But as stated above I do not regard the distinction of moods and figures as possessing any psychological basis. I view the whole system of moods and figures merely as a temporary scaffolding of 64 pigeon-holes containing 64 possible pairs of premises inclusive of the 19 pairs recognised as valid in formal logic. On the principle set forth in this essay that all premises must be regarded as psychologically positive, I urge that the 64 cases contain 36 valid pairs (inclusive of the orthodox 19) from which a syllogistic conclusion can be drawn. Finally, neglecting all reference to mood or figure, I symbolise all propositions as positive relations between positive symbols S, S', P and P', and reduce two pairs of relations (the major and minor premises) to one relation (the conclusion) applying when necessary the rule of transposition, *Totales totaliter vel partialiter, partiales partialiter solum*, so as to obtain an identical term (the middle term) above and below the line. Wherever this cannot be done (*i.e.* in 28 cases of the 64) there is no syllogism. Where this can be done there is syllogism of the form—

$$\frac{S}{M} \times \frac{M}{P} = \frac{S}{P}$$

§ 20. Following the ordinary mode of notation this syllogism is to be figured as

$$SM . MP : SP$$

and it appears to me that this unauthorised figure is a more natural as well as a more convenient type or standard than that generally adopted by logicians as the normal syllogism. The premises begin with the subject and end with the predicate of the conclusion ; the middle

term to be eliminated appears twice as the intermediate between these two limits.

It may be objected to this point of view that the adoption of this as the normal syllogism involves a transposition of the premises of the classical Barbara, and a departure from strict deductive reasoning which takes for its point of departure the universal proposition as the major premise, and proceeds to apply to it some particular proposition as the minor premise.

To this I should urge that while no doubt that is so in verbal exposition and argument which commonly opens fire by the *petitio principii*, it is not the order in which conclusions and judgments are psychologically generated. Sensation and observation take origin in the particular facts and phenomena by which our attention is solicited. The minor premise or stimulus is the first fact. Our state of sensibility, our quality of judgment, our state of mind, our character and principle, our *personal ratio*, are the major premise, compounded with which the particular fact gives particular conclusion. Induction is antecedent to deduction.

§ 21. I advance two pleas in justification of the symbolic method presented in this essay—first, that it is psychologically sound ; second, that it is practically convenient. Of these two pleas the first is the more serious, but the more indefinite and less susceptible of proof or disproof ; the second is more superficial and tangible, and can be tested at once by the trial of actual cases.

In support of the first plea I could only plead justification by analogy with certain doctrinal aspects of physiological psychology. I do not propose at this juncture to labour this plea. All that I wish to do is to indicate that the plea exists, by one or two brief quotations from

physiological literature—selecting for this purpose the third edition (1896) of my *Introduction to Human Physiology*.

SENSORY JUDGMENTS. ILLUSIONS.

Comparisons and judgments, unconscious as well as conscious, are performed in the brain, and consist in the collation and comparison of sensations which may be simultaneous or successive.

The standard of reference that we unconsciously or consciously employ in forming a conclusion from “distance” to “size,” or from “size” to “distance,” is the cerebral resultant of previously felt relations between “size” and “distance”—the *personal ratio* drawn from our stock of similar experience. Formally expressed, the major or general premise in our cerebral ratio of previous similar *felt size : felt distance* : the minor premise is one or other of these magnitudes in the particular case under observation ; if we make *distance* the minor premise, *size* is the conclusion ; if we make *size* the premise, *distance* is the conclusion.

Or, expressed as a proportion, a particular relation $s : d$ is compared with the general ratio $S : D$ with s as the datum and d as the conclusion or vice versa.

Practically we carry out this proportion every day, more or less correctly in our visual judgments. In any given case we form a cerebral minor premise by observation of a particular magnitude ; we refer that premise to the standard of past similar experience as its cerebral major premise, and by inference form our cerebral conclusion. If, for instance, we have formed a cerebral datum from a retinal image cast by an object of known

magnitude at an unknown distance, or of unknown magnitude at a known distance, we infer distance from size, or size from distance—more or less accurately according as our personal ratio between size and distance has been more or less accurately formed by previous training. A man who has been accustomed to express in numbers lengths that he has seen and measured, can put a more exact value upon the size of a house or field than a man who has not been so trained.

We are liable to be more or less grossly deceived by appearances. A man accustomed to hilly landscape will, if set down among mountains, over-estimate size and under-estimate distance. Having thus unconsciously prejudged magnitudes, he may be surprised to find that men and cattle are not really smaller than usual, and that distances are much greater than he had at first imagined. A hazy atmosphere makes hills look like mountains; a clear atmosphere makes mountains look like hills. A Swiss travelling in Scotland commented on the great size of the cattle; a Scotchman remarked that Swiss cattle were very small.

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The cerebral representation of a sensation or group of sensations may, quite independently of any external distorting cause, be modified by the voluntary attitude and focalisation of attention. A perfectly cylindrical glass can be imagined and "*seen*" at will with its open mouth directed either towards or from the observer, especially if one eye be closed to eliminate collateral items that tend to correct the illusion. We can even so forcibly focalise our attention as to *see* erect a cylindrical glass that is in reality lying on its side, and *not see* the perspective that contradicts the illusion. We can wilfully persuade ourselves of the fiction, and thereafter having formed it, we

may consistently use real objective data in further distortion of the reality. This last capacity, which is more or less obviously manifested in every man's daily life, is admirably illustrated by the closer consideration of this experiment. Taking a cylindrical glass placed erect and seen as erect, or placed on its side and seen on its side, the glass in either case seems to be cylindrical ; but if by a forcible self-suggestion the observer "*sees*" the erect glass tilted away from him, the apparent nearer end looks narrower than the apparent further end ; or if he forcibly sees the overturned glass as if erect, then the apparent upper diameter seems to be smaller than the apparent lower diameter. In both cases the really cylindrical object appears to be conical. The observer, having first formed his false notion (= a major premise), makes further use of the real retinal magnitude of a diameter as a minor premise, which, conjoined with the first false notion, gives birth to a conformably false conclusion with respect to the dimensions and shape of the glass. In the first fiction the retinal image of the upper diameter is imagined to be the more distant and therefore appears to be the larger ; in the second fiction the further end is imagined as the nearer, and appears therefore to be the smaller end.

Contrast.—Any quality, optical or other, of an object or person, causes in the observer an impression that is intensified if the surroundings are of an opposite quality, weakened if they are of a similar quality. Any fixed quantity is over-estimated when its surroundings are unusually small, under-estimated when its surroundings are unusually large. "Rest can only be enjoyed after labour ; sound, to be heard clearly, must rise out of silence ; light is exhibited by darkness ; darkness by light ; and so on in all things." Stated more generally,

the value of any given constant quantity is estimated to be + or -, according as its surroundings are - or +. A person of average height appears taller among short people, shorter among tall people.

Formally expressed, if $A : B$ is the personal ratio or major premise, $a : b$ the minor premise formed by an object in its surroundings, then the object a varies inversely as the surrounding b . If by the focussing of attention b is made the premise, a becomes the conclusion, and vice versa.

Judgment, in its simplest form, or inference, is the resultant of compared sensations, a consciousness of their identity and of their difference, *i. e.* of their *ratio*. In its higher and more compound forms, judgment is the resultant of compared ratios, *i. e.* of a synthetic process in which the major premise is the personal ratio of the thinker.

The illustration just used containing the *two* elements—*size* and *distance*—is, however, not the simplest conceivable ; it is in reality composite of the first degree ; the elementary judgment is one of size or one of distance.

I have seen many men and my idea of men is associated with an idea of their size. I have formed a general major premise from these elementary ideas that can be represented by the ratio $\frac{\text{MEN}}{\text{SIZE}}$. I meet a particular man—

“John” and he acts on my consciousness with respect to his size—John’s size is the acting minor premise. My judgment or conclusion about John with respect to size is resultant from the two premises

$$\frac{\text{MEN}}{\text{SIZE}} \times \frac{\text{John}}{\text{size}}$$

“I” am Gulliver in Lilliputia, and on meeting John for the

first time I call him "small," he calls me "large" ; or I am Gulliver in Brobdingnag, and then I call John big.

And if I, Gulliver, am a science man, I shall be more accurate and say in some measure or other how much smaller or bigger John is than my normal average man.

I have classified John as to size when I have seen and said that "John is tall," "John is short," I have quantified the predicate about John when I have said that "John is 6 ft. 8 in.," John is 4 ft. 10 in.

I know, let us say, that an average man is 6 feet high. I learn that 6 feet at a distance of 1 mile gives a retinal image 0.017 mm. high—*i.e.* at 176 yards an image 0.17 mm. ; at 17.6 yards an image 1.7 mm. Of course, when I meet John and find that he is tall or short, I do not consciously translate his height into a retinal magnitude ; I unconsciously take his size as it stands among familiar magnitudes. The minor premise impinging on my brain (and "mind") is $\frac{\text{John}}{\text{in the surroundings}}$. The major premise already present in my brain (and mind) is $\frac{\text{ordinary men}}{\text{in ordinary surroundings}}$.

"I" am a normal person who has lived in a normal environment and my major premise or personal ratio is normal. I shall therefore in the ordinary way judge "John" to be tall or short according as he is above or below the "usual height."

or

"I" am a Swiss peasant accustomed to see my cows and sheep in large scenery ; if I should go to Wales, and should see sheep on the hill-side, I shall probably, if asked whether they are large or small, say that they are large.

or

vice versa, "I" am a Welshman watching a flock of sheep coming down the opposite slope of a Swiss valley, and I shall very probably think the sheep are or seem to be very small sheep indeed. Formulating as we best can we have our resultant conclusion or judgment made up of its premises as follows :—

<i>Major.</i>	<i>Minor.</i>	<i>Conclusion.</i>
Ordinary things	this thing	
Ordinary surroundings	this surrounding	
$\frac{S}{P}$	$\frac{s}{p}$	

§ 22. The second plea, that the method of notation is practically convenient, can readily be proved, or the reverse, by its application to actual cases, where conclusions do or do not follow from given premises. The foregoing pages contain many such cases selected on account of their simplicity and obviousness. But it will be useful to supplement these by a few less obvious instances in which the method is tested upon premises that less obviously betray their conclusion or that actually mislead to a false conclusion. The instances I am about to consider are selected from the works of Bain, De Morgan, Delbœuf, and of Lewis Carrol.

An example from Bain :—

All comments are odious A.

No odious things are necessary E.

The premises are A . E.

The middle term, "odious things," is predicate of the major and subject of the minor, so that the form of the syllogism is PM . MS : $\frac{1}{2}$ SP *i.e.* in Fig. IV, in which the conclusion from A . E is E.

The mood therefore is *Camenes*, and the orthodox conclusion is that :—

No necessary things are comments.

Dealt with symbolically, without reference to figure or mood, we say :—

All comments
are
odious = $\frac{C}{O}$

No odious things
are
necessary = All odious things
are
not-necessary = $\frac{O}{N'}$

$\frac{C}{O} \times \frac{O}{N'} = \frac{C}{N'} =$ All comments
are
not-necessary = No comments
are
necessary.

This has been an instance where the premises could be brought into a conventional mood.

But let us take premises that do not come into any conventional mood, *e.g.* the following :—

Some comments are necessary I.

No necessary things are agreeable E.

The middle term “necessary things” is predicate of major and subject of minor, *i.e.* the premises are in the Fig. IV of which the form is PM . MS : SP, *i.e.* we want in the conclusion “agreeable things” as the subject, and “some comments” as the predicate. Which is impossible. There is in orthodox logic no conclusion from a particular affirmative I and a universal negative E by this or any other figure.

But dealt with symbolically without reference to figure or mood we have—

Some comments
are
necessary = $\frac{aC}{N}$

$$\begin{array}{rcccl}
 \text{No necessary things} & & \text{All necessary things} & & \\
 \text{are} & = & \text{are} & = & \frac{N}{A'} \\
 \text{agreeable} & & \text{not-agreeable} & & \\
 \\
 \frac{aC}{N} \times \frac{N}{A'} = \frac{aC}{A'} = & \text{Some comments} & & & \\
 & \text{are} & & & \\
 & \text{not-agreeable.} & & &
 \end{array}$$

This is an obviously rational if unorthodox conclusion, viz. a particular negative O from premises I and E, viz. in *Silento* of the normal figure SM . MP : SP.

An example from Delbœuf:—

Les animaux ne sont pas insensibles
 Les êtres insensibles ne sont pas vivants,

∴ Les animaux sont vivants.

I hold that this is a false conclusion from the premises given, and that the proper conclusion is—

Quelques êtres non-vivants ne sont pas des animaux,
 or Quelques êtres qui ne sont pas des animaux, ne sont pas vivants.

This conclusion is obtained as follows, symbolising by their initial letters Delbœuf's original premises :—

$$\begin{array}{l}
 \frac{A}{I'} \times \frac{I}{V'} \\
 \frac{I}{A'} \times \frac{aV'}{I} = \frac{aV'}{A'} \text{ or } \frac{aA'}{V'}.
 \end{array}$$

I recognise, of course, the weakness of the conclusion to be drawn from two universal negative premises. But that is not the point. The point aimed at by the example chosen, and worked out by Delbœuf himself, at p. 84 of his *Logique Algorithmique*, and debated between us at some length in 1892, is whether from these premises Delbœuf's conclusion or mine is the right one. I recognise further that in orthodox formal logic, apart from all

argument, and merely on the authority of the canon that "there can be no conclusion from two negative premises," both conclusions are wrong. But apart from canons, I submit on the basis of the arguments advanced in this essay that my conclusion is valid.

An example from De Morgan.

Men eat cheese. Mice eat cheese. \therefore ?

Now whether we assume all men or only some men, all mice or only some mice, there is, on the principles of this essay, as well as upon orthodox principles, no conclusion to be drawn from the premises. Writing M for men, R for mice (rats), and E for eaters of cheese, we have—

$$\frac{M}{E} \times \frac{R}{E} \quad \text{or} \quad \frac{aM}{E} \times \frac{bR}{E}$$

which are insoluble expressions.

De Morgan, by his system of logic, draws as the conclusion that: There are things which are neither mice nor men, *i. e.* all which do not eat cheese.

Again, the somewhat trivial case: "All is not gold that glitters." Sovereigns glitter \therefore Sovereigns are not gold

is very simply exposed in symbols (taking M for "glitters").

$$\frac{aG'}{M} \times \frac{S}{M}$$

are irreducible premises from which no conclusion can be drawn.

On the other hand, the following premises are readily found to contain a conclusion :—

No philosophers are conceited.

Some conceited persons are not gamblers.

$$\frac{P}{C'} \times \frac{aC}{G'} \quad \frac{C}{P'} \times \frac{aG'}{C} = \frac{aG'}{P'} \quad \text{or} \quad \frac{aP'}{G'}$$

or in words :—

Some persons who are not gamblers are not philosophers.
and—

Some persons who are not philosophers are not gamblers.

Here is another example in which De Morgan by his symbolic method draws a conclusion which, according to my method, is not valid. It occurs in an apparently perfectly serious letter to Sir John Herschell (*Memoir of Augustus De Morgan*, p. 209). The syllogism, so-called, is as follows :—

For every Z there is an X which is not Y.

Some Y's are Z's

∴ Some X's are Z's.

According to me there is here no syllogism at all. The first premise is $\frac{Z}{XY'}$ the second $\frac{aY}{Z}$ so that Z is the middle term which cancels out, giving as the conclusion $\frac{aY}{XY'}$ i. e. $\frac{aY}{Y'}$; or, "Some Y's are not Y's," which is absurd.

NOTE.—It is obvious in this example that if $\frac{Z}{XY'}$ it follows that $\frac{Z}{X}$ and $\frac{Z}{Y'}$.

Now take a somewhat more involved case, viz. the syllogism given by "Carroll" in the frontispiece of his book on *Symbolic Logic*, 2nd edition, 1896.

That story of yours about your once meeting the sea-serpent always sets me off yawning.

I never yawn unless when I'm listening to something totally devoid of interest.

The subject of the first proposition is "story, etc.," which call S. The predicate is "things that make me yawn," which call Y. The first premise is $\frac{S}{Y}$.

The second proposition is verbally distorted so as to conceal the subject. When the words have been untwisted, they are found to contain the proposition that "All my yawns occur when I'm listening to something totally devoid of interest," or $\frac{Y}{L}$.

The premises are now found to present a typical syllogism :—

$$\frac{S}{Y} \times \frac{Y}{L} = \frac{S}{L}$$

of which the conclusion, translated into words, becomes—

That story of yours about your once meeting the sea-serpent is (one of the things I listen to that are) totally devoid of interest.

"Carroll" gives as the conclusion : "That story of yours about your once meeting the sea-serpent is totally devoid of interest," and obviously includes the previous conclusion ; but it does not give all the information contained in the premises, since it does not say whether I listen to the story or not. If objection be taken that no yawns are affirmed in the second proposition, the reply is that the second premise is then $\frac{L'}{Y'}$ which by transposition gives $\frac{Y}{L}$.

§ 23. It would be tedious to write out at length illustrative examples of the 64 syllogisms tabulated

THE NORMAL TYPICAL FIGURE SM.MP:SP.

SM.MP:SP (8 valid cases, 8 invalid)	$\frac{S}{\bar{M}} \times \frac{M}{P} = \frac{S}{P}$	$\frac{S}{\bar{M}'} \times \frac{M}{P} = \frac{aP}{S'} \text{ or } \frac{aS'}{P}$	$\frac{aS}{\bar{M}} \times \frac{M}{P} = \frac{aS}{P}$	$\frac{aS}{\bar{M}'} \times \frac{M}{P}$
	$\frac{S}{\bar{M}} \times \frac{M}{P'} = \frac{S}{P'}$	$\frac{S}{\bar{M}'} \times \frac{M}{P'} = \frac{aP'}{S'} \text{ or } \frac{aS'}{P'}$	$\frac{aS}{\bar{M}} \times \frac{M}{P'} = \frac{aS}{P'}$	$\frac{aS}{\bar{M}'} \times \frac{M}{P'}$
	$\frac{S}{\bar{M}} \times \frac{bM}{P} = \frac{bM}{P}$	$\frac{S}{\bar{M}'} \times \frac{bM}{P} = \frac{bP}{S'} \text{ or } \frac{bS'}{P}$	$\frac{aS}{\bar{M}} \times \frac{bM}{P} = \frac{bM}{P}$	$\frac{aS}{\bar{M}'} \times \frac{bM}{P}$
	$\frac{S}{\bar{M}} \times \frac{bM}{P'} = \frac{bM}{P'}$	$\frac{S}{\bar{M}'} \times \frac{bM}{P'} = \frac{bP'}{S'} \text{ or } \frac{bS'}{P'}$	$\frac{aS}{\bar{M}} \times \frac{bM}{P'} = \frac{bM}{P'}$	$\frac{aS}{\bar{M}'} \times \frac{bM}{P'}$

This is merely Fig. I with the order of the premises transposed, and there are of course three other possible combinations, viz. SM.PM:SP, MS.MP:SP, and MS.PM:SP, which I have not given *in extenso*, as it was not my object to add to logical formalities, but only to justify the conclusion that the normal syllogism is best expressed by the form SM.MP:SP.

THE FOUR CLASSICAL FIGURES

(64 possible moods, viz. 19 classical, 17 "Spanish," 28 inconclusive.)

I MP . SM : SP (4 classical, 4 "Spanish," 8 invalid)	Barbara.	Celarent.		
	$\frac{M}{P} \times \frac{S}{M} = \frac{P}{P}$	$\frac{M}{P'} \times \frac{S}{M} = \frac{P}{P'}$	$\frac{aM}{P} \times \frac{S}{M}$	$\frac{aM}{P'} \times \frac{S}{M}$
	<i>Baleno.</i>	<i>Celebro.</i>	<i>Silento.</i>	<i>Bolero.</i>
	$\frac{M}{P} \times \frac{S}{M'} = \frac{P}{P}$	$\frac{M}{P'} \times \frac{S}{M'} = \frac{P}{P'}$	$\frac{aM}{P} \times \frac{S}{M'} = \frac{P}{P}$	$\frac{aM}{P'} \times \frac{S}{M'} = \frac{P}{P'}$
	<i>Darii.</i>	<i>Ferio.</i>		
	$\frac{M}{P} \times \frac{bS}{M} = \frac{P}{P}$	$\frac{M}{P'} \times \frac{bS}{M} = \frac{P}{P'}$	$\frac{aM}{P} \times \frac{bS}{M}$	$\frac{aM}{P'} \times \frac{bS}{M}$
	$\frac{M}{P} \times \frac{bS}{M'} = \frac{P}{P}$	$\frac{M}{P'} \times \frac{bS}{M'} = \frac{P}{P'}$	$\frac{aM}{P} \times \frac{bS}{M'}$	$\frac{aM}{P'} \times \frac{bS}{M'}$
	<i>Fandango.</i>	<i>Cesare.</i>		<i>Bokardo.</i>
	$\frac{P}{M} \times \frac{S}{M} = \frac{P'}{P'}$	$\frac{P}{M'} \times \frac{S}{M} = \frac{P'}{P'}$	$\frac{aP}{M} \times \frac{S}{M}$	$\frac{aP}{M'} \times \frac{S}{M} = \frac{P}{P}$
	<i>Camestres.</i>	<i>Celebro.</i>	<i>Silento.</i>	
	$\frac{P}{M} \times \frac{S}{M'} = \frac{P}{S'}$	$\frac{P}{M'} \times \frac{S}{M'} = \frac{P'}{P'}$	$\frac{aP}{M} \times \frac{S}{M'} = \frac{P}{P}$	$\frac{aP}{M'} \times \frac{S}{M'}$
	$\frac{P}{M} \times \frac{bS}{M} = \frac{P}{M}$	<i>Festino.</i>		
		$\frac{P}{M'} \times \frac{bS}{M} = \frac{P'}{P'}$	$\frac{aP}{M} \times \frac{bS}{M}$	$\frac{aP}{M'} \times \frac{bS}{M}$
	<i>Baroko.</i>			
	$\frac{P}{M} \times \frac{bS}{M'} = \frac{P'}{P'}$	$\frac{P}{M'} \times \frac{bS}{M'} = \frac{P'}{P'}$	$\frac{aP}{M} \times \frac{bS}{M'}$	$\frac{aP}{M'} \times \frac{bS}{M'}$
II. PM . SM : SP (4 classical, 4 "Spanish," 8 invalid)				

III MP. MS : SP (6 classical, 6 "Spanish," 4 invalid)	Darapti. $\frac{M}{P} \times \frac{M}{S} = \frac{aS}{P}$	Felapton. $\frac{M}{P'} \times \frac{M}{S} = \frac{aS}{P'}$	Disamis. $\frac{aM}{P} \times \frac{M}{S} = \frac{bP}{S} = \frac{cS}{P}$	Bokardo. $\frac{aM}{P'} \times \frac{M}{S} = \frac{bP'}{S} = \frac{cS}{P'}$
	Baleno. $\frac{M}{P} \times \frac{M}{S'} = \frac{aS'}{P}$	Celebro. $\frac{M}{P'} \times \frac{M}{S'} = \frac{aS'}{P'}$	Silento. $\frac{aM}{P} \times \frac{M}{S'} = \frac{cP}{S'} = \frac{bS'}{P}$	Bolero. $\frac{aM}{P'} \times \frac{M}{S'} = \frac{cP'}{S'} = \frac{bS'}{P'}$
	Datisi. $\frac{M}{P} \times \frac{bM}{S} = \frac{aS}{P}$	Ferison. $\frac{M}{P'} \times \frac{bM}{S} = \frac{aS}{P'}$		
	Baroko. $\frac{M}{P} \times \frac{bM}{S'} = \frac{aS'}{P}$	Hesnogo. $\frac{M}{P'} \times \frac{bM}{S'} = \frac{aS'}{P'}$		
	Bramantip. $\frac{P}{M} \times \frac{M}{S} = \frac{aS}{P}$	Fesapo. $\frac{P}{M'} \times \frac{M}{S} = \frac{aS}{P'}$	Dimaris. $\frac{aP}{M} \times \frac{M}{S} = \frac{bS}{P} = \frac{aS}{P}$	
	Camenes. $\frac{P}{M} \times \frac{M}{S'} = \frac{aS'}{P'}$	Celebro. $\frac{P}{M'} \times \frac{M}{S'} = \frac{aS'}{P'}$	Silento. $\frac{aP}{M} \times \frac{M}{S'} = \frac{bS'}{P} = \frac{aS'}{P'}$	
	$\frac{P}{M} \times \frac{bM}{S}$	Fresison. $\frac{P}{M'} \times \frac{bM}{S} = \frac{aS}{P'}$		
	$\frac{P}{M} \times \frac{bM}{S'}$	Hesnogo. $\frac{P}{M'} \times \frac{bM}{S'} = \frac{aS'}{P'}$		
IV PM. MS : SP (5 classical, 3 "Spanish," 8 invalid)				

on p. 46. The reader who has followed the argument so far, and who admits the psychological validity of the axiom *Negatio affirmatio contraria*, will easily invent such examples and resolve or fail to resolve them by applying the rules given on p. 31 for the elimination of an identical middle term above and below the line. The operation in any particular case can readily be checked by referring to the tables on pp. 46 and 62-3 in which the 64 cases are given in their symbolic form. It will be realised on trial or by reference to these tables, that these 64 cases include—

- 19 valid or classical syllogisms,
- 17 debateable or "Spanish" syllogisms,
- 28 cases where the premises gives no conclusion,

and that the 17 Spanish cases, giving a particular negative conclusion O, contradict the logical canon to the effect that there can be no conclusion from negative premises.

I do not, however, offer this "Spanish" group as possessing a logical validity equal to that of the 19 classical cases. Their conclusions are defective in that a negating sign is attached to the subject, and psychologically we may not admit as the subject of discourse something which is *not* that subject. They are valid only in the sense that S' is taken to indicate something other than the something taken as the original subject (v. *supra*, p. 19).

CHAPTER III

THE SCIENTIFIC STATEMENT—QUANTIFICATION OF THE PREDICATE IN RELATION TO THE SUBJECT

THE classificatory statement "*S is P*" symbolised by the relation $\frac{S}{P}$ signifies that "*all S*" is co-extensive with "*some P*" or with "*all P*." These possible co-extensions are to be represented symbolically by the equations $S = aP$, $S = P$ which indicate that the predicate *P* is quantified with reference to the subject *S*. And obviously the subject *S* is quantified with reference to the predicate *P*.

The eight classificatory relations specified above on p. 26 afford sixteen co-extensions, in which the predicate (and the subject) are quantified in relation to each other, as follows :—

Classificatory Propositions.		Quantificatory Co-extensions. ¹	
$\frac{S}{P}$. signifies .	(1) $S = P$	or (2) $S = bP$
$\frac{S}{P'}$. " .	(3) $S = P'$	or (4) $S = bP'$
$\frac{aS}{P}$. " .	(5) $aS = P$	or (6) $aS = bP$
$\frac{aS}{P'}$. " .	(7) $aS = P'$	or (8) $aS = bP'$
$\frac{S'}{P}$. " .	(9) $S' = P$	or (10) $S' = bP$
$\frac{S'}{P'}$. " .	(11) $S' = P'$	or (12) $S' = bP'$
$\frac{aS'}{P}$. " .	(13) $aS' = P$	or (14) $aS' = bP$
$\frac{aS'}{P'}$. " .	(15) $aS' = P'$	or (16) $aS' = bP'$

¹ The continuous line includes Hamilton's eight propositions ; the dotted line includes Boole and De Morgan's eight propositions. This

On examination of these sixteen propositions of co-extension they will be found to include (a) the group of eight propositions with quantified predicate as given by Hamilton, viz. :—

- | | |
|--------------------|--------------------------|
| 1. All S is all P | 2. All S is some P |
| 3. No S is P | 4. All S is some not-P |
| 5. Some S is all P | 6. Some S is some P |
| 7. Some S is not-P | 8. Some S is some not-P. |

(b) the somewhat different group of eight propositions with quantified predicate, as given by Boole and by De Morgan, viz. :—

summary is based upon the original writings of Hamilton, De Morgan and Boole. I must confess, however,—or claim—that its actual presentation in the form of a series of equations, and the sense in which “quantification of the predicate” is developed in this essay are not to be found as such in these sources. The expression does not denote an identical doctrine in the minds of Hamilton and De Morgan, as is evident on study of the polemic between them in 1847. I have studied that controversy with some care, and should find it wholly impossible to give any intelligible account of it in brief compass. All that I think is useful to say is that whereas, for Hamilton, quantification already applied to the subject was also to be applied to the predicate, for De Morgan quantification was to be applied to the middle term whether subject or predicate. The latter gives *e.g.* the following extra-Aristotelean syllogism. Most of these men wear coats. Most of these men wear waistcoats. Therefore some of these men wear coats and waistcoats. This sort of quantification naturally resulted in the arithmetical syllogism of De Morgan. Sixty of these one hundred men wear coats. Sixty of these one hundred men wear waistcoats. Therefore, at least ten of these one hundred men wear coats and waistcoats. Hamilton would call this “ultradimidiated quantification of the middle term.” I have not followed De Morgan’s developments of his symbolic notation of syllogisms (nor I believe has anyone else done so). As to the quantification controversy between Hamilton and De Morgan, it is curious to realise how each of the disputants, while failing to understand the other, had his views modified more or less consciously by the effort to understand his opponent. Thirteen years later De Morgan printed a “*Syllabus of a Proposed System of Logic*,” London, 1860, in the preface of which he abandons the formulae and notation of his writings of 1847.

- | | |
|--------------------------|------------------------------|
| 2. All S is some P | 4. All S is some not-P |
| 6. Some S is some P | 8. Some S is some not-P |
| 10. All not-S is some P | 12. All not-S is some not-P |
| 14. Some not-S is some P | 16. Some not-S is some not-P |

and (c) four others which are not included in either of the above two groups (a) and (b), viz.:—

- | | |
|-------------------------|------------------------------|
| 9. All not-S is all P | 11. All not-S is all not-P |
| 13. Some not-S is all P | 15. Some not-S is all not-P. |

Neither Hamilton's group (a) (1. 2. 3. 4. 5. 6. 7. 8.) nor Boole and De Morgan's group (2. 4. 6. 8. 10. 12. 14. 16.) contains, as far as I can see, any complete enumeration of propositions with quantified predicate. The fact that only four propositions (2. 4. 6. 8.) are common to both groups, while four (1. 3. 5. 7.) are included by Hamilton, excluded by Boole and De Morgan, and four (10. 12. 14. 16.) are included by Boole and De Morgan, excluded by Hamilton, is an indication that each of these groups is logically incomplete, and that a complete list must include both groups as well as the four propositions added above that were not included in either.

And if I have not mistaken the statements advanced by Hamilton, Boole and De Morgan, the completed list of propositions with quantified predicate (and subject) affords in itself an illustration of the way in which a simple system of logical symbols conduces to clearness and completeness of thought.

Ordinary language and the classical logic which is its reflection, deal principally with classificatory statements or relation of the type "S is P" or $\frac{S}{P}$. We say that "something" forms part of "something else" without defining "what part" or "*how much*," i.e. without quantifying the predicate with regard to the subject.

We represent the ordinary classical syllogism as composed of the relations or fractional expressions—the premises—compounded as if by a multiplication into a single resultant fraction—the conclusion—in consequence of the elimination of an identical middle term appearing in a numerator and in a denominator of our two original fractions or premises. And we shall find in conclusion of the survey upon which we are entering that all possible cases of valid syllogism, however they may be verbally twisted and disguised, are by the application of two simple rules reducible to the type : Premises $\frac{S}{M} \times \frac{M}{P} = \frac{S}{P}$
Conclusion.

A scientific statement—and in rudimentary degree the particular observations that go to form our generalised judgments and statements—are essentially the result of attempts, more or less conscious and more or less precise, to circumscribe and to quantify the predicate of each particular field of thought and of discourse—to define and *measure* what part and how large a part of the predicate “something else” is co-extensive with the thing investigated that constitutes the subject. We may indeed paraphrase all scientific and exact inquiry as an “endeavour to quantify our predicates in relation to our subjects.” And as the appropriate form by which to represent a proposition of co-extension between quantified predicate and subject, we naturally take the equation $S = P$ which is one of the two definite and quantified cases contained in the less definite classificatory expression $\frac{S}{P}$.

To represent the syllogism with propositions composed of quantified terms we naturally adopt the form of addition, as being the simplest and the most closely

representative of the psychological process by which premises containing quantified terms are accumulated to a general conclusion. On this basis the ordinary syllogism :—

$$\frac{S}{M} \times \frac{M}{P} = \frac{S}{P} \text{ becomes } \begin{array}{l} S = M \\ M = P \\ \hline \therefore S = P \end{array}$$

or things that are equal to the same thing are equal to each other.

But this conclusion is itself in anticipation of the argument by which it has to be justified.

[We may usefully recognise at this stage that the quantified proposition, and the syllogism composed of quantified premises, have in general an extension or scope far more limited than that embraced by the ordinary classificatory proposition and the corresponding syllogism, since of necessity the predicate is taken in full extension. An ordinary syllogism is in effect a deduction, from a general principle assumed in the major premise, and a particular case observed and specified in the minor premise, to a third particular case specified in the conclusion ; this conclusion is necessarily correct whether it has been observed or not. If the major principle assumed is correct, if the minor case has been correctly observed, the conclusion of a valid syllogism by which the principle and the case have been fused together, names or predicts an additional case within the scope of our original assumption. But while assisting us to classify our thoughts, deduction by way of the orthodox syllogism does little or nothing in the augmentation of our real knowledge of phenomena.

$$\frac{S}{M} \times \frac{M}{P} = \frac{S}{P} \text{ whereas the syllogism } \begin{array}{l} S = M \\ M = P \\ \hline S = P \end{array} \text{ where the}$$

terms of all the propositions are fully quantified applies

naturally to such limited phenomenal fields as can be mentally embraced through our sensory organs.]

The sixteen co-extensions given in the Table above are logically and psychologically redundant, by reason of the limitation of scope introduced by the quantified predicate.

Whereas the classificatory statement S is P or $\frac{S}{P}$ cannot be simply transposed to give the statement P is S , a quantified statement S is co-extensive with P or $S = P$ can be so transposed to give the statement P is S or $P = S$. Again if $S = aP$ necessarily $aP = S$ where logically considered aP is the subject and S the predicate of the verbalised proposition. And now we realise that this new subject aP , in spite of its formal limitation, is psychologically an entire and unlimited subject.

A verbal example may help to make this point clear :—

An original proposition }	All slaves	are co-extensive with	certain men
	S	=	aP
Simply transposed }	Certain men	are co-extensive with	all slaves
	aP	=	S

In the original proposition the subject S is unlimited or total, and the predicate aP is quantified as being equal to S .

In the transposed proposition the new subject aP , although formally a particular or partial expression, is psychologically a subject taken in its totality, *i.e.* the entire group of men quantified in relation to all slaves.

Or again, taking a co-extension with both terms particular :—

This Space	is co-extensive	with that Pond
aS		bP

it is clear that as both subject and predicate, although formally partial, and formulated logically as particular expressions, are psychologically limited so as to constitute total concepts in the field of thought and of discourse.

Thus we are led to recognise that the quantification of the predicate, followed out completely, results in the abolition of distinction between particular or limited and universal or unlimited. Quantification of terms of the syllogism amounts to a delimitation of parts in fact, thought and words, whereby such parts are mentalised and presented as total subjects and predicates of observation, thought or discourse.

It follows, therefore, that in propositions and syllogisms composed of fully quantified terms, all terms are taken as total; limited terms and the symbols of limitation have disappeared.

It follows further that in such propositions, all terms are positive. Subject and co-subject, predicate and co-predicate apply to positive concepts within particular limited fields of observation and discourse.

Thus the entire list of sixteen propositions is reducible to the single type $S = P$. The subject, however limited it be by words, is taken in totality. The predicate, whether unlimited or limited in form in co-extension with the subject, is likewise taken in totality. Subject and predicate of any such proposition of co-extension are convertible. The syllogism that results from two such propositions is : $(S = M) + (M = P) = (S = P)$, which is equivalent to the statement that if two things S and P are equal to a third thing M , they are equal to each other.

The manner in which this result has been arrived at appears to me to afford a solution of the logical difficulty of bringing this axiom to the form of the syllogism.

We have reached it from the ordinary classificatory syllogism $\frac{S}{M} \times \frac{M}{P} = \frac{S}{P}$ where a middle term taken in full extension but unquantified with reference to S or P, has been eliminated.

When that middle term has been quantified we have formed the more precise syllogism $S = M, M = P$ $\therefore S = P$, *i. e.* the axiom in question.

It is evident that this more precise pair of premises with quantified terms, implies the earlier and less precise pair of premises of an ordinary syllogism :—

$$\frac{S}{M} \times \frac{M}{P} = \frac{S}{P} \text{ or } S \text{ is a part of } M$$

M is a part of P

therefore S is a part of P.

The conception of the "Universe of Discourse" plays a very considerable part in the psychological aspect of Logic, and is at the root of all systems of logical symbolisation. We have already had recourse to that conception in the argument presented above as to the psychologically affirmative nature of negative propositions ; we may recognise, indeed, that the principle *negatio affirmatio contraria* is essentially bound up with, if not actually identical with, the principle that the universe of discourse consists of Something + Every other thing, viz. $S + S'$ (*v. supra*, p. 8 and p. 22).

Universe or Field of Discourse is the enunciated form of Universe or Field of Thought. The expression *Field of Discourse*, *Field of Thought*, which to a physiologist naturally offers itself by reason of its obvious analogy with Field of Vision, very aptly conveys what is an essential character of the logical Universe or Field, viz. its shifting and variable position and extension.

When some thing S is indicated, everything else S' is

implied as the Field within which that something is distinguished. $S + S' = \text{the Universe}$.

Instinctively, or of set purpose, thought and language are restricted to any given field which constitutes the universe of the moment. We think and speak of Englishmen in a Field composed of $E \times E'$, where E' may signify other white men, or Continentals, or Asiatics, or Americans. The complementary concepts "not-Englishmen" or E' has a value that shifts and varies with the context and import of each particular proposition and argument.

Exact thought and scientific discourse consist essentially in particularisation and measurement in first instance—a subject is isolated and delimited in thought and language. Then the subject is qualified by some comprehensive predicate, placed in some class of similar subjects, with predicate taken in full extension. Finally the subject or a portion of the subject isolated to form a new subject is equated to a definite quantity of its predicate. And when there is co-extension between subject and predicate, the predicate is quantified in relation to the subject. The field of thought and of discourse is anything we choose to make it. A comprehensible subject of concentrated attention is analogous with an image limited to the area of distinct vision, it is for a time our universe of thought and of observation and of discourse. To study that subject in a scientific sense is to quantify its predicates, *i.e.* to measure it. And clearly any subject, however fragmentary, is taken in full extension, and its fragmentary predicate is also taken in full extension. The whole of S , or all S , or every S is co-extensive with so much P or with a certain kind of P , can be written $S = aP$ or $aP = S$ which amounts to affirming that so much P or a certain kind of P is co-extensive with all or

every S, and obviously the transposed partial P is essentially now a total subject of the proposition in its transposed form of statement.

If we draw up in tabular form all the possible pairs of premises containing four terms $S = M$, $M = P$, varied as to quantity and quality, we shall find ourselves in possession of 16×12 or 192 cases. But when we reflect that inasmuch as every fully quantified term is of necessity positive and total, our original sixteen cases are psychologically redundant and reducible to one common form $S = P$, we realise that the distinctions apparent in such a table are logically fictitious. The whole of our apparently distinct cases of syllogism with fully quantified terms are recognised as reducible to the single form

$$S = M \quad M = P \quad \therefore S = P$$

In conclusion I venture to submit as two principal results of the foregoing study :—

That by the simple and natural method of symbolisation employed therein and more especially by aid of the two simple and natural rules given on page 31, the operations of ordinary scholastic logic are very clearly exposed and examined.

That the thorough realisation of the “quantification of terms” and of the “Universe of Discourse” affords a bridge of identification between the logic of instinctive reasoning represented on the type $\frac{S}{M} \times \frac{M}{P} = \frac{S}{P}$ and the logic of scientific reasoning represented on the type

$$S = M \quad M = P \quad \therefore S = P.$$



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