To face the Title

C. Mosley Sculp.

A PLAIN and FAMILIAR

METHOD

For Attaining the

Knowledge and Practice

OF

COMMON ARITHMETIC.

CONTAINING

All the USEFUL RULES both in Whole Numbers, and Fractions, Vulgar and Decimal, Extraction of the Square and Cube-Roots, Simple and Compound Interest, Annuities, &c. delivered in a more practical and correct Manner, than in any Work hitherto extant.

By EDMUND WINGATE, of *Gray's Inn*, Esq;

THE NINETEENTH EDITION.

Wherein the Additions and Emendations, made by Mr. *John Kersey*, in his Appendix, and Mr. *George Shelly*, in his Supplement, are introduced in their proper Places; and all the Improvements in this Science, that have appeared in other Writers since their Time, are carefully inserted: Also sundry others, that are entirely new, are added.

By *JAMES DODSON*, Accomptant, and Teacher of the Mathematicks.

LONDON:

Printed for C. HITCH and L. HAWES, and R. BALDWIN, in *Pater-noster-Row*; A. MILLER, in the *Strand*; JOHN RIVINGTON, in *St. Paul's Church-Yard*; and S. CROWDER and Co. on *London-Bridge*.

M.DCC.LX.

TO THE

RIGHT HONOURABLE

THOMAS,

Earl of *Arundel* and *Surrey*,

Earl Marſhal of

ENGLAND, &c.

Right Honourable,

THE good Affection You bear to all Kind of Learning, and in particular to the Mathematicks, makes me adventure to preſent your Lordſhip with this Tractate of Arithmetic; becauſe that Art, compared with other Mathematical Sciences, is as the *Primum Mobile*, in reſpect of the other inferior Orbs: For as the Poets uſed in Times paſt to ſay of *Venus*, *Sine Cerere & Baccho, friget Venus*; ſo may I alſo confidently aver

A of

of them, without Arithmetic they are poor, and without Motion. Preſuming therefore that your Lordſhip, loving the Art, cannot diſaffect the Artiſt, nor his Intention to do Good in that Kind; I am bold to ſhelter this TREATISE under your Lordſhip's Protection, humbly entreating your gracious Acceptation, and earneſtly deſiring for ever to remain,

Your HONOUR'S *in all Service,*

Affectionately Devoted,

EDM. WINGATE.

THE PREFACE.

THE many Editions this Work has gone through, and the Reputation it has deservedly maintained for upwards of 120 *Years, would undoubtedly have been Authority sufficient for the Publication of this Edition, without any Alteration from the former: But as several Arithmetical Improvements, both in Theory and Practice, have appeared since this Treatise received the last Hand, it has been thought convenient to insert them in this Edition, together with some which have not been published before.*

The first Edition of Wingate's *Arithmetic was printed about the Year* 1629, *by himself; in which his principal Design was to obviate the Difficulties which ordinarily occur in the using of Logarithms: To perform this, he divided his Work into two Books; the first he called* Natural, *and the second* Artificial Arithmetic. *The Basis on which the present Structure has been, at different Times, reared, is the first of those Books.*

For after the first Impression of the above two Books were disposed of, Mr. Wingate *(not having Leisure to revise the same, and to supply some Defects which too strict an Attention to his Design,* viz. *that of explaining the Use of Logarithms, had occasioned) requested Mr.* John Kersey, *an able Mathematician, to undertake the same. Accordingly Mr.* Kersey *(in several Editions) made many Improvements,*

Improvements, which take in his own Words, as they ſtood in his Preface to the former Editions.

" Firſt, For the Eaſe and Benefit of thoſe Learners, " that deſire only ſo much Skill in Arithmetic, as is " uſeful in Accompts, Trade, and ſuch like ordinary " Employments; the Doctrine of whole Numbers, " (which, in the firſt Edition, was intermingled with " Definitions and Rules concerning broken Numbers, " commonly called Fractions,) is now entirely handled " a-part. And to the end the full Knowledge of Prac- " tical Arithmetic in whole Numbers might more " clearly appear, I have explained divers of the old " Rules in the firſt five Chapters, and framed a-new " the Rules of Diviſion, Reduction, and the Golden- " Rule, in the ſixth, ſeventh, eighth, and ninth Chap- " ters: So that now, Arithmetic in whole Numbers " is plainly and fully handled before any Entrance be " made into the craggy Paths of Fractions, at the Sight " of which ſome Learners are ſo diſcouraged, that they " make a Stand, and cry out, *Non plus ultra*, There's " no Progreſs farther.

" Secondly, To aſſiſt ſuch young Students as would " lay a good Foundation for the attaining of a general " Knowledge in the Mathematics, I have in a familiar " Method delivered the entire Doctrine of Fractions, " both Vulgar and Decimal, which was omitted in the " firſt Edition; and have alſo newly framed the Ex- " traction of the Square and Cube-roots, in a Method " which by Experience is found to be much eaſier than " that commonly uſed heretofore, and is exactly ſuit- " able to the Conſtruction or Compoſition of Square " and Cube Numbers.

" Laſtly, I have added an Appendix, furniſhed with " Variety of choice and delightful Knowledge in Num- " bers, both Practical and Theoretical."

But as Mr. Kerſey *has omitted to enumerate the Particulars of which his Appendix conſiſted, the Editor has* *here given them, as printed in a Table of Contents prefixed thereto.*

The Work, thus enlarged and amended, passed through divers Editions, till about the Year 1700, *when Mr.* George Shelley, *Writing-Master of* Christ's-Hospital, *wrote a Supplement to it, containing divers practical, compendious, and easy Methods for the Performance of particular Cases in most of the Rules of Arithmetic; together with Decimal Tables useful in the Computation of Interest and Exchanges, and some useful Rules and Observations relating to Practical Measuring.*

Such was the State of this Work when it came into the present Editor's Hands, under whose Care it has met with the following Alterations and Additions:

1. *All the different Materials relating to any one Subject, which in the former Editions lay separated from each other*

other in the different Parts of the Work, the Appendix, and the Supplement, are here collected together into their proper Places; so far as the same could be done consistently with the keeping the Doctrine of whole Numbers separate from that of Fractions, before-mentioned by Mr. Kersey.

2. *Many useful Properties of Numbers, practical Observations, and Compendiums in Operations, (not mentioned in the former Editions) are here inserted in their proper Places.*

3. *The Demonstrations given by Mr.* Kersey, *which were founded on Geometrical and Algebraical Principles, are supplied by others purely Arithmetical.*

4. *The Properties of Numbers considered as Prime and Composite are delivered, as a necessary Help to the Management of Fractions.*

5. *The Operations of Vulgar Fractions are rendred much easier, by an Artifice in the Management and Abbreviation of them.*

6. *The Doctrine of Repeating or Circulating Decimals is introduced; and the Management of them, in a more general and easy Manner than hitherto taught, is scientifically explained.*

7. *An universal Rule of Proportion, which answers the Purpose of the several Rules of Three, single, double, direct or inverse, in whole Numbers or Fractions, is delivered and illustrated by many Examples; in which the great Usefulness of the above-mentioned Method of managing and abbreviating Vulgar Fractions, will abundantly appear.*

8. *The Rule of Alligation Alternate, as hitherto delivered, will give but few Answers to Questions propounded therein; most of which are capable of many, and some of innumerable Answers: To remedy this Defect, a Method is laid down to find more Answers than the common Rules will give, in those Cases which admit of more; and as many Answers as the Computer pleases, in those Cases where the Number of Answers is unlimited.*

9. *The Properties of Numbers considered as Roots and Powers, and the Nature and Properties of Logarithms are compendiously explained.*

10. *A more practical Method of extracting the Cube-Root is inserted, instead of that delivered by Mr.* Kersey; *to which is annexed a Table of the Square and Cube-Roots of all integers less than* 145.

11. *The Properties of Numbers in Arithmetical Proportion continued, are more fully explained and illustrated.*

12. *The Properties of Numbers in Geometrical Progression are also more particularly handled, with a View to the Application of such Numbers, to the Computations relating to Compound Interest.*

13. *Mr.* Shelley'*s Tables of Simple Interest are brought into a narrow Compass, and adapted to those Rates which are now commonly wanted.*

14. *The Tables for Compound Interest and Annuities, are enlarged from* 30 *to* 60 *Years, and computed to the same Rates as those of Simple Interest; the manner of making and using them is more particularly shewn; easy Methods are given to find Numbers beyond the Extent of the Tables; and some new Tables are added, relating to Half-yearly and Quarterly Payments.*

15. *A great Variety of Examples, with their Answers, are inserted; which will not only be a good Exercise for Learners, but will also serve to ease Teachers of the Burthen of writing out Questions.*

16. *A copious Index is annexed, to which the Reader may turn for more particular Information of the Contents of the whole.*

Thus altered and enlarged, the Editor conceives that this Work contains, methodically, all that is necessary to be known, or performed in Common Arithmetic; and by consequence, that the Purchasers hereof need not be at the Trouble, or Charge of looking into any other of the, almost, numberless Writers on that Subject.

If the above Improvement in the Operations of the Rule of Alligation Alternate, ſhould prove of Service in real Buſineſs; the Knowledge thereof may induce the Editor to publiſh ſome farther Thoughts upon that Subject.

Bell-Dock, Wapping,
April 4th, 1751.

The Explanation of certain Marks and Characters, which, for the Sake of Brevity and Perſpicuity, are frequently uſed in the enſuing Work.

$+$ is the Mark of Addition; and when it ſtands between two Numbers, denotes that they are to be added together.

$-$ is the Mark of Subtraction; and, when it ſtands between two Numbers, denotes that the latter is to be taken from the former.

$\times$ is the Mark of Multiplication; and, when it ſtands between two Numbers, it denotes that they are to be multiplied together.

$\div$ is the Mark of Diviſion; and, when two Numbers, are placed in the ſame manner as the two Points are here, it denotes that the Number above is to be divided by that below.

$=$ is the Mark of Equality; which, being ſet between two numerical Expreſſions, denotes that they are equal between themſelves.

$: \, :: \, :$ are the Marks of Proportionality; and denote that the Numbers, between which they are placed, are proportional Numbers.

Examples.

For $4+3=7$; read, the ſum of 4 and 3 is equal to 7.

For $4-3=1$; read, when three are taken from 4, the Remainder is equal to 1.

For $4\times3=12$; read, the Product of 4 and 3 is equal to 12.

For $\frac{12}{3}=4$; read, if 12 be divided by 3, the Quotient is equal to 4.

For $1:4::3:12$; read, as 1 is to 4, ſo is 3 to 12.

A
TREATISE
OF
Common Arithmetic.

CHAP. I.

Concerning NOTATION *of Numbers.*

1. ARITHMETIC teaches the properties of Numbers; and by them deduces the methods of calculating, or computing from certain *data*, the values, weights, measures, distances, proportions, *&c.* of things.

2. Number is that by which every thing is counted: or that which answers this question, How many? (unless it be answer'd by nothing:) So if it be asked, how many days are in a week? the answer is seven, which is therefore called the Number of days in a week.

3. The Notes or Characters, by which Number is ordinarily expressed, are these, 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 0 nothing.

4. These Notes or Characters are either significant Figures, or a Cypher.

5. The significant Figures are the first nine, *viz.* 1, 2, 3, 4, 5, 6, 7, 8, 9, usually called Digits. The first of these is more particularly called an Unit or Unity, and the rest are said to be composed of Units: So 2 is composed of two Units, 3 of three Units, *&c.* that is, 1 more 1 is equal to 2; 2 more 1 is equal to 3, *&c.*

Note, The Characters prefixed to the several Articles of this Treatise may serve for an Example of the natural rank or series of Numbers, so increasing by the continual addition of 1.

These Characters were first used in *England*, about the year 1130.

Note, also, That in the natural series of Numbers, 1, 2, 3, 4 5, 6, 7, 8, *&c.* the first, third, fifth, *&c.* Numbers, *viz.* 1, 3, 5, 7, 9, 11, *&c.* are called *odd* Numbers; and the second, fourth, sixth, *&c.* Numbers, *viz.* 2, 4, 6, 8, 10, 12, *&c.* are called even Numbers.

6. The Cypher is the last, which tho' of itself it signifies nothing, yet, being annexed after any of the rest, it increases their value; as will appear in the following Rules.

7. Arithmetick has two Parts, Notation and Numeration.

8. Notation teaches how to express, read, or declare the signification or value of any number written; and also to write down any number proposed, with proper characters, in their due places.

9. A Number is said to have so many places, as there are characters in the number, *viz.* when divers figures, whether they be intermixed with a cypher or cyphers or not, are placed together, like letters in a word, without any point, comma, line, or other note of distinction interposed; all those characters make but one number, which consists of so many places as there are characters so placed together; so this number 205 consists of 3 places, and this 30600 of 5 places, *&c.*

10. Notation consists in the knowledge of two things, *viz.* the order of places, and their values.

11. The order of the places is from the right-hand towards the left: So in this number 465, the figure 5 stands in the first place, 6 in the second, and 4 in the third; likewise in this number 7560, a cypher stands in the first place, 6 in the second, 5 in the third, and 7 in the fourth.

12. The first place of a number, which, as before, is the outermost towards the Right-hand is called the place of Units; in which place any figure signifies its own simple value: So in this

this number 465, the figure 5 ſtanding in the firſt place ſignifies five units, or five.

13. The ſecond place of a number is called the place of Tens, in which place any figure ſignifies ſo many tens as the figure contains units: So in this number 465, the figure 5 in the firſt place ſignifies ſimply 5, but the figure 6 in the ſecond place denotes ſix tens, or ſixty.

14. The third place of a number is called the place of Hundreds; in which place any figure ſignifies ſo many hundreds as there are units contained in the figure: So in this number 465, the figure 4 in the third place expreſſes four hundreds: Wherefore if it be required to read or pronounce this number 465, you are to begin on the left-hand; and, according to the aforeſaid rules, to pronounce it thus, four hundred ſixty-five; likewiſe this number 315, is to be pronounced thus, three hundred and fifteen; and this number 205, two hundred and five; alſo this number 500, five hundred. Whence it is manifeſt, that although a cypher of itſelf ſignifies nothing, yet being placed on the right-hand of a figure it increaſes the value of it, by advancing ſuch figure to an higher place, than that wherein it would be ſeated, if the cypher were abſent.

The true reading or pronouncing the value of any number written, as alſo the writing down any number propoſed, depends principally upon a right underſtanding of the three firſt places before-mentioned, and therefore the learner ſhould be well exerciſed therein, before he proceeds to the following Rules.

15. The fourth place of a number is called the place of Thouſands (that is,any number of thouſands under ten thouſand;) the fifth place Tens of Thouſands; the ſixth place Hundreds of Thouſands; the ſeventh place Millions; (a million being ten hundred thouſand;) the eighth place Tens of Millions; the ninth place Hundreds of Millions; the tenth place Thouſands of Millions; the eleventh place Tens of Thouſands of Millions; the twelfth place Hundreds of Thouſands of Millions: And, in that order, you may conceive places to be continued infinitely from the right-hand towards the left, each following place being ten times the value of the next preceding.

16. *From the Rules aforegoing, an eaſy way may be collected to read or expreſs the value of a number propounded*, viz. Let it be required to read or pronounce this number, 521426341. *Firſt*, diſtinguiſh by a comma, or point, every three places, beginning at the right-hand, and proceeding towards the left, ſo will the aforeſaid number be diſtinguiſh'd into parts, which may be called *periods*, and ſtands thus, 521,426,341. Where you may note the firſt *period* towards the right-hand conſiſts of

thefe figures, 341, the *fecond* of thefe 426, and the *third* of thefe 521. *Secondly*, read or pronounce the figures in every *period*, as if they ftood apart from the reft; fo will the firft *period* be pronounced three hundred forty-one, the fecond four hundred twenty-fix, and the third five hundred twenty-one. *Thirdly*, to every *period*, except the firft towards the right-hand, a peculiar *denomination* or *furname* is to be applied, *viz.* the *furname* of the fecond *period*, is *thoufands*; of the third, *millions*; of the fourth, *thoufands* of *millions*, &c. Therefore beginning to pronounce at the higheft *period*, which in this example is the third, and giving every *period* its due *furname*, the faid number will be pronounced thus, *five hundred twenty-one millions*, *four hundred twenty-fix thoufand*, *three hundred and forty-one*.

17. And, when 'tis required to write down or read more places than twelve, let the fifth *period* be called *billions*; the fixth, *thoufands* of *billions*; the feventh, *trillions*; the eighth, *thoufands* of *trillions*; the ninth, *quadrillions*, &c.

Note, When a number is diftinguifhed into *periods*, as before, the higheft *period* will not always compleatly confift of three places, but fometimes of one place, and fometimes of two; neverthelefs after fuch *period* is pronounced as if it ftood apart, the due *furname* is to be annexed; fo this number 3204689, after it is divided into *periods* will ftand thus, 3,204,689, and is to be pronounced thus, *three millions*, *two hundred and four thoufand*, *fix hundred eighty-nine*; and this number 17,213, is to be read, *feventeen thoufand two hundred thirteen*.

18. The aforefaid Rules for the right pronouncing or reading of a number which is written down, being well underftood, will fufficiently inform the reader how to write down any number propounded to be written.

The

The TABLE of NOTATION.

The Order of Places.			*The Value of Places.*
	&c.		&c.
Fourth Period	*Twelfth Place*	3	*Hundreds of Thousands of Millions.*
	Eleventh Place	2	*Tens of Thousands of Millions.*
	Tenth Place	1	*Thousands of Millions.*
Third Period	*Ninth Place*	9	*Hundreds of Millions.*
	Eighth Place	8	*Tens of Millions.*
	Seventh Place	7	*Millions.*
Second Period	*Sixth Place*	6	*Hundreds of Thousands.*
	Fifth Place	5	*Tens of Thousands.*
	Fourth Place	4	*Thousands.*
First Period	*Third Place*	3	*Hundreds.*
	Second Place	2	*Tens.*
	First Place	1	*Units.*

19. NOTATION of *Numbers, by* Latin *Letters.*

1	I.	21	XXI.
2	II.	30	XXX.
3	III.	40	XL.
4	IIII. or thus IV.	49	XLIX.
5	V.	50	L.
6	VI.	59	LVIIII. or thus LIX.
7	VII.	60	LX.
8	VIII. or thus IIX.	89	LXXXIX.
9	VIIII. or thus IX.	100	C.
10	X.	200	CC.
11	XI.	300	CCC.
12	XII.	400	CCCC.
18	XVIII. or thus XIIX.	500	D. or thus IↃ.
19	XVIIII. or thus XIX.	600	DC. or thus IↃC.
20	XX.	700	DCC. or thus IↃCC.

1000	CIↃ. or thus M.
2000	CIↃ. CIↃ. or MM.
3000	CIↃ. CIↃ. CIↃ. or MMM.
5000	IↃↃ.
10,000	CCIↃↃ.
50,000	IↃↃↃ.
100,000	CCCIↃↃↃ. or thus CM.
500,000	IↃↃↃↃ.
1,000,000	CCCCIↃↃↃↃ.
1750.	CIↃ, IↃCC,L, or MDCCL.

CHAP.

CHAP. II.

Concerning English *Money, Weights, Measures,* &c.

20. THE *things expressed by numbers, are Money, Weight, Measure, Time,* &c. Of the three first of these, there are infinite kinds and varieties, according to the diversity of the several *Commonwealths* wherein they are used ; all which here to produce were both endless and needless: Wherefore only such *Money, Weights, Measures,* &c. as are used in this nation, will be here treated of.

21. The least piece of money used in *England* is a Farthing, from whence this following Table is produced.

4 *Farthings*	make	1 *Penny.*
12 *Pence*	make	1 *Shilling.*
20 *Shillings*	make	1 *Pound.*

English (or *Sterling*) *Money*, is ordinarily written down with Figures, after this manner:

l.	*s.*	*d.*	*f.*
34 .	13 .	05 .	2
09 .	05 .	10 .	1
69 .	00 .	06 .	3
00 .	12 .	11 .	0
00 .	00 .	07 .	2

The first Rank of the said Numbers signifies thirty-four pounds thirteen shillings five-pence two farthings ; the second Rank expresses nine pounds five shillings ten-pence one farthing ; the third Rank, sixty-nine pounds no shillings six-pence three farthings, *&c.*

22. The smallest weight used in *England* is a Grain, that is, the weight of a grain of wheat, well dried, and gathered out of the middle of the ear, of which thirty-two make another weight, called a Penny-weight, and twenty penny-weights make an Ounce *Troy* *.

Here observe, that by the *Statutes* quoted in the margin †, the weight of two and thirty grains of wheat, make a penny-weight, which weight being once discovered by thirty-two such grains, the said penny-weight (being the twentieth part of an ounce *Troy*) is usually subdivided into twenty-four parts only, called also *Grains*, as appears by the ensuing Table.

* Vide Stat. de Compositione Ponderum, 51 *Hen.* III.
† 31 Ed. I. v. Rast. Weights. 7, & 8. 12 *Hen.* VII. 5.

A TABLE of TROY-WEIGHTS.

32 *Grains of Wheat*	make	24 *Artificial Grains.*
24 *Grains*	make	1 *Penny-weight.*
20 *Penny-weights*	make	1 *Ounce.*
12 *Ounces*	make	1 *Pound.*

Troy-weight is ordinarily ſet down with Figures, after this manner :

lb.	*oz.*	*p.w.*	*gr.*
17 .	05 .	13 .	13
00 .	11 .	07 .	06
00 .	00 .	05 .	20

The firſt Rank of the ſaid Numbers expreſſes ſeventeen pounds five ounces thirteen penny-weights, thirteen grains of *Troy-weight:* The ſecond Rank, no pounds eleven ounces ſeven penny-weights and ſix grains; and the third no pounds no ounces five penny-weights and twenty grains.

Now this *Troy-weight* ſerves only to weigh Gold, Silver, and Electuaries. And here obſerve alſo, that *Troy-weight* regulates and preſcribes a form how to keep the money of *England* at a certain *Standard:* For about two hundred years before the conqueſt, *Oſbright* a *Saxon*, being then King of *England*, cauſed an ounce *Troy* of ſilver to be divided into twenty pieces, at the ſame time called pence; and ſo an ounce of ſilver at that time was worth no more than twenty-pence, or one ſhilling eight-pence; which continued at the ſame value until the time of *Henry* the Sixth, who (in regard of the enhancing of money in foreign parts) valued the ſame at thirty-pence; ſo that then there were accordingly thirty pieces made out of the ounce, and the old pieces went then for three half-pence, 'till the time of *Edward* the Fourth, who valued the ounce at forty-pence, and then the old pieces went at two-pence apiece. After this *Henry* the Eighth valued the ounce of Sterling-ſilver at forty-five pence, which value continued 'till Queen *Elizabeth*'s time, who valued the ſame old pence at three-pence the piece; ſo that all three-pences coined by the ſame Queen weigh'd but a penny-weight, and every ſix-pence two penny-weights; and the ſhilling and other pieces accordingly; which made the ounce *Troy* of ſilver to be valued at ſixty pence, or five ſhillings, as it remains at this day without alteration.

Note,

Note, Jewels, Pearls, Diamonds, &c. are alſo weigh'd by *Troy-weight*, the Ounce being ſubdivided into 150 parts, called *Carats*, and each Carat into 4 *Grains.*

23. The weights uſed by Apothecaries are derived from a pound *Troy*, which is ſubdivided, as in the following Table.

A TABLE *of* APOTHECARIES WEIGHTS.

℔	*A Pound Troy*	is equal to	12 *Ounces.*
℥	*An Ounce*	is equal to	8 *Drams.*
ʒ	*A Dram*	is equal to	3 *Scruples.*
℈	*A Scruple*	is equal to	20 *Grains*

So that if you were to expreſs, in Figures, twelve pounds ten ounces five drams two ſcruples and ſixteen grains; alſo three pounds five ounces ſeven drams one ſcruple and two grains, the ordinary way to write them down is thus:

℔	℥	ʒ	℈	*gr.*
12 .	10 .	5 .	2 .	16
3 .	05 .	7 .	1 .	02

By theſe weights Apothecaries compound their medicines; but they buy and ſell their drugs by Avoirdupoiſe-weight.

24. Beſides *Troy-weight* before-mentioned, there is another kind of weight uſed in *England*, called Avoirdupoiſe-weight, a pound whereof is equal to fourteen ounces eleven penny-weights ſixteen grains *Troy.* Avoirdupoiſe-weight ſerves to weigh all kinds of Grocery-ware, as alſo Butter, Cheeſe, Fleſh, Tallow, Wax, Tobacco, Iron, Braſs, Lead, Tin, Soap, Pitch, Tar, Roſin, Salt, Flax, Hemp, Drugs, and every other thing that bears the name of *Garbel*, and from which iſſues a refuſe or waſte.

25. Avoirdupoiſe-weight is either greater or leſs.

26. The greater is, when one hundred and twelve pounds Avoirdupoiſe are conſidered as one entire weight, commonly called an Hundred-weight, and then ſuch Hundred-weight is ſubdivided firſt into four quarters, and each quarter into twenty-eight pounds: Again, each pound into four quarters, or (if you will be more exact) into ſixteen ounces, and if you pleaſe each ounce into four quarters. But ordinarily a pound is the leaſt quantity that is taken notice of, in Avoirdupoiſe groſs weights.

A TABLE *of* AVOIRDUPOISE *Greater* WEIGHT.

28 *Pounds*	make	*A Quarter of* 112*lb*.
4 *Quarters*	make	*An Hundred-weight*, or 112 *lb*.
20 *Hundred weights*	make	*One Ton*.

So that if you were to expreſs, by Figures, eight hundreds three quarters and five pounds ; likewiſe, ſeven hundreds one quarter and ſeventeen pounds ; the ordinary way to ſet them down is thus :

C.	*q.*	*lb.*
8 .	3 .	05
7 .	1 .	17

27. The leſſer Avoirdupoiſe-weight is, when a pound is the higheſt name or integer, each pound being ſubdivided into ſixteen ounces, and each ounce again into ſixteen drams, and if you pleaſe each dram into four quarters, as by the ſubſequent Table is manifeſt.

A TABLE *of* AVOIRDUPOISE *Leſſer* WEIGHT.

4 *Quarters of a Dram*	make	1 *Dram*.
16 *Drams*	make	1 *Ounce*.
16 *Ounces*	make	1 *Pound*.

So that if you were to expreſs, by Figures, eighteen pounds twelve ounces five drams and three quarters of a dram ; likewiſe five pounds no ounces twelve drams and one quarter of a dram ; the ordinary way to write them down is thus :

lb.	*oz.*	*dr.*	*q.*
18 .	12 .	05 .	3
5 .	00 .	12 .	1

N. B. A *Stone* of *Meat* in *London*, is 8 *lb.* Avoirdupoiſe ; but in ſome other places 14 *lb.*

A *Stone* of *Wool*, is 14 *lb.* and two Stones make a Todd.

A *Stone*, *Horſeman*'s weight, is 14 *lb.*

A Stone of Hemp, is 32 *lb.*

A *Fother* of Lead is $19\frac{1}{2}$ *C.* wt.

28. The Meaſures uſed in *England* are either of Capacity, or Length.

29. The Meaſures of Capacity are thoſe which are produced from weight, and they are either liquid or dry.

30.

30. All Meaſures of Capacity, both liquid and dry, were at firſt made from *Troy-weight:* Vide *Statutes,* 9 *Henry* III. 51 *Henry* III. 12 *Henry* VII. *&c.* wherein it is enacted, that eight Pounds *Troy-weight* of wheat, gathered out of the middle of the ear, and well dried, ſhould make one *Gallon* of *Wine-meaſure:* And that there ſhould be but one *Meaſure* for *Wine, Ale* and *Corn,* throughout this Realm. (*Vid.* Stat. 14 *Edw.* III. 15 *Richard* II.) But time and cuſtom hath altered Meaſures, as they have done Weights (and perhaps for one and the ſame reaſon); for now we have three different Meaſures, *viz.* one for *Wine,* one for *Ale* or *Beer,* and one for *Corn.*

31. The common Wine-gallon, ſeal'd at *Guild-hall* in *London,* by which all Wines, Brandies, Spirits, Strong-waters, Mead, Perry, Cyder, Vinegar, Oil and Honey, *&c.* are meaſured and ſold, is ſuppoſed to contain 231 cubic Inches; and from thence the reſt are computed, as in this Table.

A TABLE *of* WINE-MEASURE.

231 *Cubic Inches*	make	1 *Gallon.*
42 *Gallons*		1 *Tierce.*
63 *Ditto*		1 *Hogſhead.*
84 *Ditto*		1 *Puncheon.*
2 *Hogſheads*		1 *Pipe,* or *But.*
2 *Pipes*		1 *Tun.*

Note, $31\frac{1}{2}$ Gallons is a Wine or Vinegar Barrel, and 236 Gallons is a Tun of ſweet Oil.

32. The *Beer* or *Ale* Gallon (which are both one) is much larger than the *Wine* Gallon; it being probably made at firſt to correſpond with *Avoirdupoiſe*-weight, as the *Wine*-gallon did with *Troy*-weight: For one pound *Avoirdupoiſe* being nearly equal to 14 ounces 12 penny-weights *Troy:* And as one pound *Troy* is in proportion, to the cubic inches in a *Wine*-gallon, ſo is one pound *Avoirdupoiſe* to the cubic inches in an *Ale*-gallon. That is, $12 : 14\frac{12}{20} :: 231 : 282$ nearly.

A TABLE *of* ALE-MEASURE.

282 *Cubic Inches*	make	1 *Gallon.*
8 *Gallons*		1 *Firkin.*
2 *Firkins*		1 *Kilderkin.*
2 *Kilderkins*		1 *Barrel.*
3 *Ditto*		1 *Hogſhead.*

A TABLE *of* BEER-MEASURE.

282 *Cubic Inches*	make	1 *Gallon.*
9 *Gallons*		1 *Firkin.*
2 *Firkins*		1 *Kilderkin.*
2 *Kilderkins*		1 *Barrel.*
3 *Ditto*		1 *Hogshead.*
3 *Barrels*		1 *Butt.*

Note, A Firkin of Soap, and of Herrings, are the same with that of Ale.

33. This distinction or difference betwixt *Ale* and *Beer* Measure, is now only used in *London.* But in all other places of *England*, the following Table of *Beer* or *Ale*, whether it be strong or small, is to be observed, according to a Statute of Excise made in the year 1689.

282 *Cubic Inches*	make	1 *Gallon.*
$8\frac{1}{2}$ *Gallons*		1 *Firkin.*
2 *Firkins*		1 *Kilderkin.*
2 *Kilderkins*		1 *Barrel.*

Note, In all Measures *liquid* or *dry*,

2 *Pints*	make	1 *Quart.*
2 *Quarts*		1 *Pottle.*
2 *Pottles*		1 *Gallon.*

34. *Dry* measure is different both from *Wine* and *Ale* measure, being as it were a mean betwixt both, tho' not exactly so; which, upon examination, will be found to be in proportion to the aforesaid old standard *Wine*-gallon, as *Avoirdupoise*-weight is to *Troy*-weight; that is, as one pound *Troy* is to one pound *Avoirdupoise*, so are the cubic inches contained in the old *Wine*-gallon, to the cubic inches contained in the *dry* or *Corn*-gallon, *viz.* $12 : 14\frac{7}{12} :: 224 : 272\frac{1}{2}$, the common received content of a *Corn*-gallon nearly. Altho' now it is otherwise settled by an act of parliament, made in 1697, the words of that act are these: *Every round* Bushel *with a plain and even bottom, being made eighteen* Inches *and a half wide throughout, and eight* Inches *deep, should be esteemed a legal* Winchester Bushel, *according to the standard in his* Majesty's Exchequer. Now a vessel thus made, will contain 2150,42 *cubic inches*, consequently the *Corn-gallon* doth contain but $268\frac{4}{5}$ *cubic Inches.*

A

A TABLE *of* Dry *or* Corn-measure.

268$\frac{4}{5}$ *Cubic Inches*	make	1 *Gallon.*
2 *Gallons*	make	1 *Peck.*
4 *Pecks*	make	1 *Bushel*
8 *Bushels*	make	1 *Quarter.*

Note, When *Salt* and *Sea-coal* are measured by the Corn-bushel, they are heaped; 36 Bushels is a Chalder of Coals, and 21 Chalders a Score.

35. As the least part of *Weight* was originally a *Wheat-corn,* so the least part of Long-measure was a *Barley-corn,* taken out of the middle of the ear, and being well dried, three of them in length were to make one inch; and thence the rest, as in the following Table.

3 *Barley-corns in length*	make	1 *Inch.*
12 *Inches*	make	1 *Foot.*
3 *Feet,* or 16 *Nails*	make	1 *Yard.*
3 *Feet* 9 *Inches*	make	1 *Ell.*
6 *Feet,* or *two Yards*	make	1 *Fathom.*
5 *Yards and an half*	make	1 *Pole,* or *Perch.*
40 *Poles,* or *Perches*	make	1 *Furlong.*
8 *Furlongs*	make	1 English *Mile.*

ALSO,

16	*Nails* or	4	*Quarters*	1	*Yard.*
20	*Nails* or	5	*Quarters*	1	*Ell* English.
12	*Nails* or	3	*Quarters*	1	Flemish *Ell,* or *Auln.*

Note, That a Yard is usually subdivided into four Quarters, and each Quarter into four Nails.

And each Ell into four Quarters; but each Quarter of an Ell contains five Nails.

36. Superficial, or square Measures of Land, are such as are express'd in the following Table:

40 *Square Poles,* or *Perches*	make	1 *Rood,* or *Quarter of an Acre.*
4 *Roods*	make	1 *Acre.*
640 *Acres*	make	1 *Mile.*

For 40 Poles or Perches in length, and 4 in breadth do make a Statute *Acre* of Land; that is 220 yards multiplied by 22 yards,

yards, which is equal to 4840 *ſquare Yards*, are a Statute *Acre*.

Note, Land is beſt meaſured by a Chain of 4 Poles long, divided into 100 parts, called Links.

And if you would expreſs,by Figures,theſe quantities of Land, *viz.* thirty-ſix Acres three Roods twenty Perches; alſo ſeven Acres no Roods thirty-two Perches, the ordinary way to ſet them down, is thus:

A.	*R.*	*P.*
36 .	3 .	20
7 .	0 .	32

37. A TABLE of TIME is this that follows:

60 *Seconds*	make	1 *Minute.*
60 *Minutes*		1 *Hour.*
24 *Hours*		1 *Day natural.*
7 *Days*		1 *Week.*
4 *Weeks*		1 *Month of* 28 *Days.*
13 *Months* 1 *Day and* 6 *Hours*		1 *Year very near.*

But in ordinary computations of time, the whole year, conſiſting of three hundred ſixty-five days, is divided either into twelve equal parts or months; every month then containing thirty days and ten hours: or elſe into twelve unequal *Kalender*-months, according to the ancient Verſe:

Thirty Days hath September, April, June, *and* November;
February *hath twenty-eight alone, and each of the reſt thirty-one.*

Note, That every *Leap-year* (which happens once in four years) contains three hundred ſixty-ſix days; and, in ſuch year, *February* contains twenty-nine days.

38. Of things accounted by the dozen, a Groſs is the Integer, conſiſting of twelve dozens, each dozen containing twelve particulars. So that if you would expreſs, in Figures, ſeven groſs four dozens and five particulars; alſo four dozens and eight particulars, they may be written thus:

G.	*D.*	*P.*
7 .	04 .	05
0 .	04 .	08

CHAP.

CHAP. III.

ADDITION *of Whole Numbers.*

39. CONcerning Notation of Numbers, and how thereby the quantities of things are uſually expreſſed, a full declaration has been made in the preceding Chapters: Numeration follows, which comprehends all manner of operations by Numbers.

40. In Numeration, the four primary or fundamental operations are theſe, *Addition*, *Subtraction*, *Multiplication*, and *Diviſion.*

41. Addition is that, by which divers numbers are collected together, to the end that their ſum, aggregate or total, may be diſcovered.

42. In Addition, place the numbers given one above another, in ſuch ſort, that like places or degrees in every number, may ſtand in the ſame rank; that is, Units above Units, Tens above Tens, Hundreds above Hundreds, &c. So theſe numbers 1213 and 462, being given to be added together, you are to order them as appears in the margin.

```
1213
 462
```

43. Having thus placed the numbers, and drawn a line under them, add them together, beginning with the units firſt, and ſaying thus, 2 and 3 makes 5, which write under the rank of units; then proceed to the ſecond rank, and ſay, 6 and 1 make 7, which write under the ſecond rank (being the place of tens); again 4 and 2 make 6, which write under the third rank. Laſtly, write down 1, being all that ſtands in the fourth rank; ſo the ſum of the ſaid given numbers is found to be 1675, and the operation will appear as in the margin.

```
1213
 462
----
1675
```

In like manner, the numbers 2315, 7423, and 141, being given to be added together, their ſum will be found to be 9879, and the operation will ſtand as in the example.

```
2315
7423
 141
----
9879
```

44. When the ſum of the figures of any of the ranks amounts to ten, or any number of tens without any exceſs, write down a cypher under that rank; but when the ſum of any rank exceeds ten, or any number of tens, ſet down the exceſs under ſuch rank; and for every ten contained in the ſum of any rank, reſerve an unit or 1 in your mind, and add

ſuch unit or units to the figures of the next rank towards the left-hand; ſo the numbers 4937, 9878, and 394, being given to be added together, the operation will be thus, *viz.* Beginning with the rank of units, 4, 8 and 7 make 19, wherefore write down 9, the exceſs above ten, and carry 1 in mind inſtead of the ten contained in the ſaid 19: Then 1 and 9 (9 being the lowermoſt figure of the ſecond rank) make 10, which added to 7 and 3, the other figures of the ſame rank, the whole ſum of them is 20; therefore ſet down a cypher under the line in that rank, becauſe the exceſs above the two tens is nothing; next carry 2 to the third rank: 2 and 3 (3 being the lowermoſt figure of the third rank,) make 5, which being added to 8 and 9 (the other figures of the ſame rank;) the ſum of them is 22; therefore writing down 2 (being the exceſs above the two tens) under the line in the third rank, carry 2 in mind (becauſe there were two tens in 22) to the fourth rank: Laſtly, 2 and 9 make 11, which added to 4 make 15; this 15, becauſe it is the ſum of the laſt rank, write totally down under the line, on the left-hand of the figures before ſubſcribed; ſo the ſum of the three numbers given, is found to be 15209, as in the example.

```
 4937
 9878
  394
—————
15209
```

45. The reaſon of the above operations will be very evident from this undeniable maxim, *viz. that the whole is equal to all its parts*; and the method of ſetting down the total, may eaſily be accounted for, from the nature of numeration, which explains the different value of places as they proceed from the right, to the left-hand: For, as 9 is the greateſt ſimple character or figure, ſo every number exceeding 9, being compound, muſt require more places then one to expreſs it. Thus, the number 10 can no otherwiſe be expreſſed in figures, but by removing the figure into the place of tens, which is done by ſupplying the unit's place with a cypher: And as it is the ſame with every other column (ten being ſtill the proportion of increaſe) conſequently, when the ſum of any column, amounts to 10 or more, the units exceeding, if there be any, or a cypher if none, muſt be ſet under ſuch column, and the ten or tens of the amount carried on, as ſo many units, to the next column on the left.

What is here obſerved, as to carrying the tens (the proportion of increaſe) from one column to another in integers, may be as juſtly applied to the proper numbers in adding ſums of different denominations.

This demonſtration may be applied to the example work'd in Art. 44, as follows:

The

The Sum of	4,	8, and	7	is	19
The Sum of	90,	70, and	30	is	190
The Sum of	300,	800, and	900	is	2000
The Sum of		9000, and	4000	is	13000
Therefore the Sums of	394,	9878, and	4937	is	15209

46. From the ſame maxim is deduced a method of proving the truth of any operation in Addition, *viz.* by parting or ſeparating the given Numbers into two parcels (or more, according to the largeneſs of it), and then adding up each parcel by itſelf: For if thoſe particular ſums, ſo found, be added into one ſum, and that ſum prove equal, or the ſame with the total ſum firſt found, then all is right; if not, care muſt be taken to diſcover and correct the error.

E X A M P L E.

Add	5647 3289 4016	The Sum of theſe Parts is	12952
	2900 5007 1606	The Sum of theſe is	9513

The total Sum of all theſe Parts — 22465 The Sum of each Parcel put together ——— 22465

Note, This laſt method may be uſefully applied in the addition of long columns, in order to eaſe the memory; and the ſame may be proved by dividing them into different parcels.

47. Another method of proof is, to perform the operation again, beginning with the uppermoſt figure of each rank, and adding downwards.

Note, 1. The ſum of two or more *even* numbers will be an *even* number.

2. The ſum of two *odd* numbers will be an *even* number.

3. The ſum of two numbers; one *even*, the other *odd*, will be an *odd* number.

4. If more than two *odd* numbers be added together; the ſum will be *even*, when the number of its parts, ſo to be added together, is *even*; but *odd*, if the number of parts be *odd*.

5. If *even* numbers be intermixed with *odd*, the ſum will be *even* or *odd*, according as the number of *odd* parts be *even* or *odd*.

48. When numbers given to be added, expreſs things of diverſe denominations; firſt write them down orderly (according to the Examples in *Chap.* II.) then after a line is drawn under them all, begin to add the numbers of the leaſt denomination, and if the ſum of them amounts to one integer, or many integers of the next greater denomination, with ſome exceſs of the leſs denomination, ſet down that exceſs, or a cypher when there is no exceſs, under the line, ſo as it may ſtand under the leaſt denomination, and keep the ſaid integer or integers in mind, to be added to thoſe of the next greater denomination on the left-hand; but when the ſum of the numbers of the leaſt denomination does not amount to one integer of the next greater denomination, ſet down the ſum itſelf under the line: Then add the integer or integers kept in mind (when any happens) to the numbers of the next greater denomination on the left-hand, and proceed to add them, as alſo thoſe of every greater denomination, in like manner as above is directed; until you come to the numbers of the greateſt (or higheſt) denomination, which are to be added according to *Art.* 43. and 44. So theſe ſeveral ſums 24*l.* 13*s.* 5*d.* 3*f.* alſo 12*l* 0*s.* 8*d.* and 5*l.* 18*s.* 0*d.* 2*f.* being propos'd to be added, their total ſum is 42*l.* 12*s.* 2*d.* 1*f.* For having written them down orderly according to *Art.* 21. and drawn a line underneath, begin with the farthings firſt, and ſay, two farthing and three farthings make five farthings, that is one penny with a farthing over and above; wherefore ſetting down 1 under the denomination of farthings, carry one penny to the denomination of pence; then ſay, 1, 8, and 5 pence make 14 pence, which contain one ſhilling and two-pence; wherefore writing 2 under the denomination of pence, carry one ſhilling to the denomination of ſhillings; then adding the ſaid 1 ſhilling to 18 ſhillings and 13 ſhillings, the ſum will be found 1 pound and 12 ſhillings; wherefore ſetting down 12 under the denomination of ſhillings, carry 1 pound in mind to the denomination of pounds, ſaying, 1 pound in mind, together with 5, 2, and 4 pounds, which ſtand in the firſt rank of pounds, make 12 pounds; wherefore (according to *Art.* 44.) write 2, the exceſs above 10, under the ſaid firſt rank of pounds, and carry 1 in mind for the ſaid 10 to the ſecond rank of pounds; then ſaying, in like manner, 1 in mind, together with 1 and 2, which ſtand in the ſecond rank of pounds make 4, which write under the line; that done, the total of the three given ſums appears to be 42*l.* 12*s.* and 2*d.* 1*f.*

l.	*s.*	*d.*	*f.*
24 .	13 .	05 .	3
12 .	00 .	08 .	0
05 .	18 .	00 .	2
42 .	12 .	02 .	1

In like manner 3 *lb.* 05 *oz.* 19 *p.w.* 15 *gr.* Alſo 2 *lb.* 0 *oz.* 3 *p.w.* 7 *gr.* Alſo 0 *lb.* 10 *oz.* 6 *p.w.* 0 *gr.* And 0 *lb.* 9 *oz.* 0 *p.w.* 17 *gr.* being given to be added together, their ſum will be found 7 *lb.* 01 *oz.* 9 *p.w.* 15 *gr.* and the work will ſtand thus:

3 .	05 .	19 .	15
2 .	00 .	03 .	07
0 .	10 .	06 .	00
0 .	09 .	00 .	17
7 .	01 .	09 .	15

Note, In adding together the numbers in the laſt example, it muſt be remember'd, that twenty-four grains make one penny-weight; twenty penny-weights one ounce; and twelve ounces one pound *Troy* (as before is declared in *Art.* 22); and then you are to proceed according to *Art.* 48.

More Examples are theſe following, which preſuppoſe the learner to be well exerciſed in the Tables of *Chap.* II. that he may readily know what integers are to be carried from every leſſer denomination to the next greater.

Addition *of* English Money.

l.	*s.*	*d.*	*f.*	*l.*	*s.*	*d.*
230 .	17 .	10 .	1	0 .	13 .	05
175 .	12 .	11 .	3	0 .	17 .	08
052 .	05 .	06 .	0	0 .	00 .	10
009 .	00 .	08 .	1	0 .	10 .	03
506 .	13 .	07 .	2	0 .	15 .	06
974 .	10 .	07 .	3	2 .	17 .	08

Addition *of* Troy-Weight.

lb.	*oz.*	*p.w.*	*gr.*	*oz.*	*p.w.*	*gr.*
23 .	07 .	16 .	13	536 .	13 .	16
17 .	10 .	15 .	07	206 .	11 .	10
325 .	06 .	19 .	20	68 .	00 .	05
49 .	11 .	07 .	12	96 .	00 .	12
417 .	00 .	19 .	04	907 .	05 .	19

ADDITION *of* AVOIRDUPOISE-WEIGHT.

C.	*q.*	*lb.*	*lb.*	*oz.*	*dr.*
235	3	13	14	13	12
576	1	17	5	10	13
628	2	15	12	00	06
412	0	10	06	09	05
1852	3	27	39	02	04

ADDITION *of* MEASURES *of* LENGTH.

Yards.	*q.*	*nails.*	*Ells.*	*q.*	*n.*
26	3	2	15	3	2
13	1	3	16	1	3
12	0	1	09	0	1
29	1	1	12	2	1
81	2	3	53	3	3

ADDITION *of* SUPERFICIAL MEASURES *of* LAND.

Acres.	*Roods.*	*Per.*	*A.*	*R.*	*P.*
136	3	13	240	2	17
513	1	26	500	3	13
212	2	10	249	1	36
517	0	00	006	0	10
1379	3	09	996	3	36

49. The following Queſtions are deſigned to put the learner upon reflection, which will enable him the better to ſee the nature and uſe of this Rule.

1. A man was born in the year 1721; when will he be 45 years of age? *Anſw.* in the year 1766.

2. If after having paid 12*l.* 10*s.* in part of a debt, there ſtill remains due 17*l.* 10*s.* what was that debt? *Anſw.* 30*l.*

3. There are two numbers, whoſe difference is 17, and the leſſer number is 44; what is the greater number? *Anſw.* 61.

4. A man has 6 bags of Hops; the firſt weighs 2 *qu.* 14*lb.* and each of the others weighs 14*lb.* what quantity has he in all? *Anſw.* 1 *C.* 1 *qu.*

5. Re-

5. Received of *A* three half-pence, of *B* ſeven farthings, of *C* fifteen-pence, of *D* five groats, of *E* half a crown, and of *F* thirty ſhillings; how much was received? *Anſw.* 1*l.* 15*s.* $8\frac{1}{4}$*d.*

6. Bought a parcel of goods for 40*l.* 10*s.* paid for packing them 13*s.* for carriage 1*l.* 6*s.* 8*d.* and expended at making the bargin 15*s.* 6*d.* what do the goods ſtand me in? *Anſw.* 43*l.* 5*s.* 2*d.*

7. Between *London* and *Royſton* are 33 miles; from thence to *Cambridge* 10; thence to *Newmarket* 10; thence to *Bury* 10; and from thence to *Norwich* 32 miles: How many miles on this road from *London* to *Norwich*? *Anſw.* 95 miles.

8. How many days are there in the 12 Calendar months, *January* having 31 days, *February* 28, *March* 31, *April* 30, *May* 31, *June*, 30, *July* 31, *Auguſt* 31, *September* 30, *October* 31, *November* 30, and *December* 31 days? *Anſw.* 365.

9. How many days are there from the 28th day of *March* to the 17th day of *December* in the ſame year, both days incluſive? *Anſw.* 265.

10. How many days are there from the 19th day of *April*, 1748, to the 27th day of *November*, 1750, both days excluſive? *Anſw.* 951.

11. A merchant, in the year 1750, imported, by one ſhip 8 tuns of Claret, by another 12 tuns 3 hogſheads 42 gallons of Red Port, by another 14 tuns 2 hogſheads 11 gallons of Sherry, and by another 5 tuns 3 hogſheads 10 gallons of Canary; what quantity of wines did he import in all? *Anſw.* 41 tuns 1 hogſhead.

12. From the creation of the world to the beginning of the deluge is accounted 1656 years; and from the beginning of the deluge to the birth of *Arphaxad*, *Helvicus* reckons 2 years; and thence, to *Terah* 220 years; thence to the birth of *Abraham* 70 years (*Gen.* 11.); thence to the promiſe given (mentioned *Gen.* 12.) he reckons 75 years; and thence to the going out of *Egypt* 430 years; and from that going out, to the Temple of *Solomon* 480 years; and thence to the birth of *Chriſt* 1015 years; and he ſuppoſes, that from thence, to the beginning of the common Chriſtian or *Dionyſian* Æra 2 years elapſed; and thence, to the preſent year wherein this was writ, we reckon 1750 years: According to the Chronology of *Helvicus*, therefore, how long is it ſince the creation of the world? *Anſw.* 5700 years.

13. A ſurveyor, having meaſured 5 ſeveral pieces of land, finds one of them to contain 7 acres 3 roods 24 perches, another to contain 18 acres 1 rood 16 perches, another 12 acres

10 perches, and the laſt 15 acres, 2 roods; how many acres were ſurveyed in all? *Anſw.* 53 acres, 3 roods, 10 perches.

14. In the year 1563, 20000 perſons died of the plague in *London*; in 1593, 10635 died of the ſame diſtemper; in 1603, 30578; in 1625, 54265, of the plague but 35417; in 1665, 97351 died, and of them 68586 of the plague; how many died of the plague between the years 1563 and 1665, both years included? *Anſw.* 165216.

CHAP. IV.

SUBTRACTION *of Whole Numbers.*

50. SUBTRACTION is that by which one number is taken out of another, to the end that the remainder or difference between the two numbers given may be known.

51. The number out of which the Subtraction is to be made, muſt be greater, or at leaſt, equal with the other. For you may ſubtract 4347 out of 9478, but you cannot ſubtract 9478 out of 4347.

52. In Subtraction rank the two given numbers one under the other, as in Addition, with this caution, that the number placed uppermoſt may exceed, or at leaſt be equal to the other: So if the number 4347 be given to be ſubtracted from 9478, place them as in the margin: Then proceeding to the Subtraction, ſay, 7 taken out of 8, there remains 1, which place in the ſame rank under the line. In like manner, 4 being taken out of 7, the remainder is 3, which likewiſe ſet under the line in the next rank; again, taking 3 from 4, the remainder is 1, which place under the third rank: Laſtly, ſubtracting 4 from 9, there will remain 5, which ſubſcribe under the fourth rank; now the whole operation being finiſhed, it appears that if 4347 be taken out of 9478, the remainder is 5131, or (which is the ſame) the difference between the numbers 9478 and 4347 is 5131, as in the Example.

$$\begin{array}{r} 9478 \\ 4347 \\ \hline 5131 \end{array}$$

In like manner, if 106 be ſubtracted from 2856, the remainder will be found 2750; for after the numbers are orderly ranked, begin at the place of units, and ſay 6 from 6, there remains nothing; therefore ſubſcribe 0 (or nothing); then proceeding to the ſecond rank, ſay, if 0 be taken from 5, there will remain 5, which alſo ſubſcribe under the line; again, 1 from 8, there remains 7: Laſtly, 0 from 2, there remains 2. See the Work in the margin.

$$\begin{array}{r} 2856 \\ 106 \\ \hline 2750 \end{array}$$

53. When any of the figures of the number given to be ſubtracted is greater than the upper figure out of which it is to be ſubtracted,

ſubtracted, you muſt borrow 10 of the next rank towards the left-hand, and add the ſaid 10 to the ſaid upper figure; then from the ſum of ſuch Addition ſubtract the lower figure and ſet down the remainder: In this caſe the figure of the next rank which is to be ſubtracted, muſt be eſteemed an unit greater than it is; therefore keeping 1 in your mind, add it to the next figure of the number given to be ſubtracted, and deducting all out of the figure above it, proceed in like ſort, till you have finiſhed the whole operation. *Example*, Let it be required to ſubtract 374 out of 8023. Having ranked them as before, ſay, 4 out of 3, that cannot be, wherefore borrowing 10 of the next rank, and adding the ſame to the ſaid 3, ſay, 4 out of 13, there remains 9; then writing 9 under the line, and carrying 1 in mind, ſay, 1 and 7 makes 8, 8 out of 2 that cannot be, but 8 out of 12 (12 becauſe 10 being borrowed, and added to 2, makes 12) there remains 4, which ſubſcribe under the line; again 1 in mind being added to 3 makes 4, 4 out of nothing, that cannot be, but 4 out of 10 there remains 6, which likewiſe ſubſcribe under the line; laſtly, 1 in mind being taken out of 8, there remains 7. Thus you ſee that the remainder after 374 is ſubtracted from 8023, is 7649. Note diligently, that as often as 10 is borrowed, 1 muſt be kept in mind to be added to the figure ſtanding in the next place of the lower number, and the ſum of ſuch Addition muſt be ſubtracted from the upper place; but if it happen that there is no figure in the next place of the lower number, then the 1 in mind muſt be ſubtracted from the upper place; (as in the laſt rank of the laſt Example.) *Another Example*, Let it be required to ſubtract 92 from 62801. Having placed the greater number uppermoſt, and the leſſer orderly underneath, begin at the place of units, and ſay, 2 from 1 that cannot be, but borrowing 10, and adding it to the ſaid 1, ſay 2 from 11 there remains 9, which ſubſcribe under the line; then proceed and ſay, 1 in mind with 9, makes 10, 10 out of 0 that cannot be, but borrowing 10, ſay, 10 out of 10 and there remains 0, wherefore ſubſcribe 0 under the line; again, 1 in mind out of 8, there remains 7; then becauſe there are no figures in the lower number, ſay 0 out of 2 there remains 2; laſtly, 0 out of 6 there remains 6; therefore it appears that 62801 exceeds 92 by 62709.

```
8023
 374
----
7649
```

```
62801
   92
-----
62709
```

54. Thoſe who have conſidered *Art.* 45. and are thence ſatisfied of the reaſon of the operation uſed in Addition, will eaſily perceive that the ſame maxim may be applied to the work of

Subtraction, as directed in *Art.* 52. And the truth of the operations in *Art.* 53. will appear from the following

Propoſition: *If to each of two numbers, the ſame or an equal number be added, the difference between the numbers reſulting will be the ſame with the difference of the firſt two numbers.* Example:

If to 14 and 6 whoſe difference is 8,
the number 3 . . 3 be added; then,
The Reſults 17 and 9 alſo differ by 8.

Now, in *Art.* 53. by taking 4 out of 13 (inſtead of 3) 10 is added to the upper number, and by carrying in mind 1, and adding it to the figure which ſtands in the place of tens in the lower number, ten is added to that lower number, therefore the true difference will thence ariſe by the propoſition.

55. If the numbers propoſed have diverſe denominations, place them as before, and beginning with the leaſt denomination firſt, ſubtract the lower number from the upper when it may be ſubtracted, and place the remainder underneath; but if it happens that the lower number cannot be taken out of the upper, you muſt borrow an integer of the next greater denomination on the left-hand; which integer, after it is converted into the ſame denomination with the ſaid upper number, muſt be added to it: Then from the ſum of ſuch Addition, you are to ſubtract the lower number, and write down the remainder, keeping 1 (that is the integer borrowed) in your mind, to be added to the next place of the number given to be ſubtracted, as before: So 90*l.* 14*s.* 10*d.* 3*f.* being ſubtracted from 124*l.* 11*s.* 7*d.* 1*f.* the remainder is 33*l.* 16*s.* 8*d.* 2*f.* For beginning with the farthings, ſay, 3 farthings out of 1 farthing cannot be taken, therefore borrowing 1 penny (that is an integer of the next greater denomination) and having converted this penny into 4 farthings, add them to the aforeſaid 1 farthing; ſo the ſum is 5 farthings, out of which ſubtracting 3 farthings, there remain 2 farthings, which place under the denomination of farthings; then proceed to the next denomination, and ſay, 1 penny borrowed and 10*d.* make 11*d.* this 11*d.* out of 7*d.* cannot be taken; therefore borrowing 1 ſhilling, or 12*d.* and adding 12*d.* to the ſaid 7*d.* the ſum is 19*d.* from which ſubtract the ſaid 11*d.* ſo there remains 8*d.* which ſubſcribe under the denomination of pence: Again, 1 ſhilling borrowed, being added to 14*s.* makes 15*s.* which cannot be ſubtracted out of 11*s.* and therefore borrow 1 pound or 20*s.* which being added to the ſaid 11*s.* make 31*s.*

l.	*s.*	*d.*	*f.*
124	11	07	1
90	14	10	3
33	16	08	2

from which ſubtracting 15 *s.* there remains 16 *s.* which ſubſcribe under the denomination of ſhillings; then carrying 1 pound, borrowed, to the lower place of pounds, ſay, 1 in mind with 0 make 1, which taken out of 4, there remains 3; again, 9 out of 2 cannot be taken, but 9 out of 12 (10 being borrowed and added to the ſaid 2, according to *Art.* 53.) and there remains 3. Laſtly, 1 (for the 10 that was borrowed) being taken out of 1, there remains nothing; whence if *A* being indebted to *B*, in 124 *l.* 11 *s.* 7 *d.* 1 *f.* has paid in part thereof 90 *l.* 14 *s.* 10 *d.* 3 *f.* there remains yet undiſcharged 33 *l.* 16 *s.* 8 *d.* 2 *f.*

56. When many numbers are given to be ſubtracted from a number propounded, you muſt firſt add thoſe numbers together, according to the rules of the third chapter, and then the ſum found is to be ſubtracted from the number firſt given. *Example*, *A* being indebted to *B*, in 3240 *l*, paid thereof at one time 700 *l.* at a ſecond payment 1236 *l.* and at a third 305 *l.* the queſtion is, how much of the debt remain'd undiſcharged? Firſt, add together the ſeveral ſums paid, and find the total 2241 *l.* this ſubtract from 3240l. there remains 999 *l.* undiſcharged. See the operation in the margin.

l.	
3240	*The Debt.*
700	*Payments.*
1236	
305	
2241	*Total paid.*
999	*Reſt unpaid.*

Another Example of the like nature. *A* being indebted to *B* in 500 *l.* paid in part thereof at one payment 340 *l.* 12 *s* 6 *d.* at a ſecond payment 13 *l.* 18 *s.* 3 *d.* at a third 17 *l.* 16 *s* 10 *d.* the queſtion is, how much was in arrear? Here, if the operation be proſecuted as before, it will appear that there was 127 *l.* 12 *s.* 5 *d.* unpaid. See the Work in the margin.

	l.	*s.*	*d.*
The Debt	500	00	00
Payments	340	12	06
	13	18	03
	17	16	10
Paid in all	372	07	07
Reſt unpaid	127	12	05

57. Addition may be proved by Subtraction, and Subtraction by Addition. For having added divers numbers together, if you ſubtract one of them out of the ſum, the remainder muſt be equal to all the reſt, as you may obſerve by the Example following, *viz.* ſuppoſing theſe 4 numbers are given to be added, *viz.* 236, 452, 29, 217, and that their ſum is found to be 934 (by the rules of the third chapter); it is required to

prove

prove whether the said sum be true or not; to perform this, draw a line under the uppermost number 236, to separate it from the rest, and seek the sum of all the numbers given, except that uppermost, which sum is 698. Then subtract the said uppermost number 236, from 934, (the total sum of all the numbers first found) and because the remainder 698 is the same with the sum of all the numbers, excluding the uppermost, it may be concluded, that the sum of all the numbers, first found, was truly computed.

236	
452	934
29	236
217	698
934	
698	

In like manner Subtraction is proved by Addition; for if you add the remainder and the number given to be subtracted together, the sum must be equal to the number out of which the Subtraction is made; so if 4347 be subtracted from 9478, the remainder is 5131; for if 5131 be added to 4347, the sum is 9478, which is the same with the number out of which the Subtraction was made.

	Example 1.	*Example* 2.
	l.	*l. s. d.*
out of	9478	24 . 13 . 07
subtract.	4347	19 . 15 . 08
Rest	5131	04 . 17 . 11
Proof	9478	24 . 13 . 07

And again, if a servant receive 24*l.* 13*s.* 7*d.* and lay out or disburse 19 *l.* 15 *s.* 8 *d.* there must remain in his hands 4 *l.* 17 *s.* 11 *d.* for this being added to 19 *l.* 15 *s.* 8 *d.* which was the money he expended, the sum will be equal to 24 *l.* 13 *s.* 7 *d.* (being the money with which he was first charged.)

Note 1. The difference of two *even* numbers, will be an *even* number.

2. The difference of two *odd* numbers, will be an *even* number.

3. The difference of two numbers, the one *even* and the other *odd*, will be an *odd* number.

58. More Examples of Subtraction are these that follow:

SUBTRACTION *of* ENGLISH MONEY.

	l. s. d. f.	*l. s. d. f.*
Rec.	3090 . 10 . 07 . 1	24 . 00 . 00 . 0
Paid.	0099 . 14 . 08 . 3	05 . 17 . 11 . 3
Rest	2990 . 15 . 10 . 2	18 . 02 . 00 . 1
Proof	3090 . 10 . 07 . 1	24 . 00 . 00 . 0

SUBTRACTION *of* TROY-WEIGHT.

	lb.	*oz.*	*p.w.*	*gr.*	*oz.*	*p.w.*	*gr.*
Bought	352	10	13	19	205	13	19
Sold	019	11	16	18	118	16	20
Rest	332	10	17	1	86	16	23
Proof	352	10	13	19	205	13	19

SUBTRACTION *of* AVOIRDUPOISE-WEIGHT.

	C.	*q.*	*lb.*	*lb.*	*oz.*	*dr.*
Bought	256	2	23	25	13	12
Sold	079	3	26	00	14	13
Rest	176	2	25	24	14	15
Proof	256	2	23	25	13	12

SUBTRACTION *of* SUPERFICIAL MEASURES *of* LAND.

	Acres.	*Roods.*	*Per.*	*A.*	*R.*	*P.*
Bought	780	2	35	2040	1	20
Sold	090	3	36	919	3	30
Rest	689	2	39	1120	1	30
Proof	780	2	35	2040	1	20

59. Questions to exercise Addition and Subtraction.

Quest. 1. Two persons *A* and *B* are of several ages, the age of the elder, being that of *A* is 70, the difference of their ages is 19, what is the age of *B*? *Answ.* 51.

Quest. 2. What number is that which being added to 168, makes the sum to be 205? *Answ.* 37.

Quest. 3. The sum of two numbers is 517, the lesser is 40, what is the greater? *Answ.* 477.

Quest. 4. A certain person born in the year of our Lord 1616, desired to know his age in the year 1676, what was his age? *Answ.* 60.

Quest. 5. The greater of two numbers is 130, the difference is 49, what is the lesser number? *Answ.* 81.

Quest. 6. What number of pounds, shillings, and pence, added to 34 *l.* 16 *s.* 9 *d.* will make 100 *l.*? *Answ.* 65 *l.* 3 *s.* 3 *d.*

Quest. 7. How many years since the *Spanish* Invasion, it being in the year 1588, and the present year being 1750? *Answ.* 162 years.

Quest.

Quest. 8. From 100 pounds borrow'd, take 72 paid;
'Twas a Virgin that lent it, what's due to the Maid?
Answ. 28 *l.*

Quest. 9. A Miser hath three bags of money, containing in all 2984 *l.* 6 *s.* of which the first contains 324 *l.* 10 *s.* and the second 913 *l.* 0 *s.* 6 *d.* what doth the third contain? *Answ.* 1746 *l.* 15 *s.* 6 *d.*

Quest. 10. A Merchant had 5 debtors, *A*, *B*, *C*, *D*, and *E*, which, together, owe him 1156 *l.* now *B*, *C*, *D*, and *E*, together, owe him 737 *l.* what is *A*'s debt? *Answ.* 419 *l.*

Quest. 11. The three Towns of *London*, *Huntingdon*, and *York*, lie in the same road; the distance between the farthest of these Towns, *viz.* *London* and *York*, is 192 miles; now if from *London* to *Huntingdon* be 57 miles, how far is it from *Huntingdon* to *York*? *Answ.* 135 miles.

CHAP. V.

MULTIPLICATION *of Whole Numbers.*

60. MULTIPLICATION teaches how by two Numbers given to find a third, which shall contain either of the Numbers given so many times, as the other contains 1 or unity: Or Multiplication may be considered as a manifold addition, or the repeating of a given Number as often as required.

61. Of the two Numbers given in Multiplication, one (which you will) is called the Multiplicand, and the other the Multiplier, or both are called Factors.

62. The Number sought, or rising by the Multiplication of the two Numbers given, is called the Product, the Fact, or the Rectangle: So if 5 be given to be multiplied by 3, or 3 by 5, the Product is 15; that is 3 times 5, or 5 times 3 makes 15; and here 5 may be called the Multiplicand, and 3 the Multiplier, or 3 may be called the Multiplicand, and 5 the Multiplier; and as 3 (one of the two numbers given) contains 1 or unity thrice, so 15 the Product, contains 5 (the other given number) thrice; likewise as 5 (one of the given numbers) contains unity 5 times, so 15 (the Product) contains 3 (the other given number) 5 times: This same Product may be found by Addition two ways, *viz.* either by writing down the number 5,

three

three times; or the number 3, five times; and adding them together, as below

5	3
5	3
5	3
—	3
15	3
	—
	15

63. Multiplication is either single or compound.

64. Single Multiplication is, when the Multiplicand and Multiplier consist each of them of one figure only, as in the last example. In like manner if you multiply 9 by 5, the product is 45; this is likewise single Multiplication: Now the several Varieties of single Multiplication are well expressed in the following Table, usually called *Pythagoras*'s Table. The truth of which may be proved by Addition, as above.

The TABLE *of* MULTIPLICATION.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

The use of the Table is this: Having one figure given to be multiplied by another, to know the product of them, find the Multiplicand in the top of the Table, and the Multiplier in the first column thereof towards the left-hand; then the product will be found on the same line with the latter, and under the former. So 9 being given to be multiplied by 5, I find 9 in the top of the table, and 5 in the first column towards the left-hand; then carrying my eye from 5 in a right line equidistant to the upper side or top line of the Table, until I come to that square which is directly under 9, I find there 45, which is the product required. The particular varieties of this Table ought to be learned by heart, (that is, a man must be able to give the product of any single Multiplication, without the least pause

or

or ſtay) before he can readily work compound Multiplication, as will appear hereafter.

Note 1. The product of any number by an *even* number, will be an *even* number.

2. The product of an *odd* number by an *odd* number, will be an *odd* number.

3. The product of any two numbers can have, at moſt, but as many places of figures as are in both Factors; and, at leaſt, but one place fewer. *Examp.* $9\times9=81$, and $1\times1=1$.

65. Compound Multiplication, is when the Multiplier and Multiplicand, either one or both, conſiſt of more figures than one.

66. In Compound Multiplication, when the numbers given end with ſignificant figures, place them as in *Addition* and *Subtraction*. So 134 being given to be multiplied by 2, place them thus: Then proceeding to the Multiplication, ſay thus, 2 times 4 is 8, which ſet under the line in the rank of your multiplying figure; again ſay 2 times 3 is 6, which likewiſe ſet under the line in the next rank: Laſtly, 2 times, 1 is 2, which being likewiſe ſet down under the line in the next rank, the product is diſcovered to be 268, and the work will ſtand as in the margin.

```
134
  2
---
268
```

67. The truth of this proceſs may be made evident: Thus,

Since	100 multiplied by 2 produces	200
	30 multiplied by 2 produces	60
And	4 multiplied by 2 produces	8
Therefore	134 multiplied by 2 produces	268

For ſince the parts 100, 30, and 4, added together make the whole 134; therefore the products of each of thoſe parts, being added together, will be the product of the whole.

68. When the Multiplier conſiſts of more figures than one, as many figures as it has, ſo many ſeveral products muſt be ſet down under the line, which at laſt being added into one ſum, will give you the total product of all. So 1232 being given to be multiplied by 23, the operation will ſtand thus; 1232 being multiplied by 3 (according to the laſt rule) the product is 3696. Again, 1232 being multiplied by 2, the product is 2464, which ſeveral products after they are placed in their due order, (that is, the firſt figure ariſing in every product under its reſpective multiplying figure) and added together, produce 28336, the product required: In like manner 1321 being given to be multiplied by 123, the pro-

```
 1232
   23
-----
 3696
2464
-----
28336
```

```
  1321
   123
------
  3963
 2642
1321
------
162483
```

duct

duct is 162483, and the operation will appear as in the margin.

69. The product of 1232 by 23, is equal to the product of 1232 by 20 and the product of 1232 by 3, added together. See *Examp.* 1. *Art.* 68.

But 1232 multiplied by 20 produces	24640
And 1232 multiplied by 3 produces	3696
Therefore 1232 multiplied by 23 produces	28336

70. When the product of any of the particular figures exceeds ten, place the excess under the line, as before, and for every ten that it so exceeds, keep one in mind to be added to the next rank; as was taught in Addition.

Example, 3084 being given to be multiplied by 36, the work will stand thus; 6 times 4 being 24, set 4 under the line, and reserve 2 in mind for the two tens; then say 6 times 8 is 48, to which add 2 kept in mind, the whole is 50; therefore set down 0 in the next rank under the line (0 because there is no excess of 50 above 5 tens) and keep 5 in mind for the 5 tens; again, say 6 times nothing is nothing, to which adding 5 that was kept in mind, the whole will be but 5, which set down under the line in the next rank; again, 6 times 3 is 18, which (in regard 3 is the last figure of the Multiplicand) set wholly down; so that the particular product arising from the multiplying by the figure 6 is 18504: In like manner proceeding with the multiplying figure 3, the particular product arising will be 9252. Lastly, these several products being placed in due order, and added together (after the manner of the last Article) will give 111024, which is the total product arising from the Multiplication of 3084 by 36, and the operation will stand as in the margin. After the same manner, if 5073 be given to be multiplied by 256, the product will be found to be 1298688, and the operation will stand as you see in the example.

```
  3084
    36
 -----
 18504
 9252
 ------
111024
```

```
   5073
    256
  -----
  30438
 25365
10146
-------
1298688
```

71. When the two numbers given to be multiplied, do, one or both of them, end with a cypher or cyphers towards the right-hand, multiply the significant figures in both numbers, one by the other, neglecting such cyphers; and when the Multiplication of the significant figures is finished, annex, on the right-hand of the number produced by the Multiplication, the cypher or cyphers, with which one or both of the numbers first given did end, so will the whole give you the true product demanded. *Example*, 43100 being given to be multiplied by 15000, the product will be found 646500000; for, omitting the

the cyphers which ſtand in the laſt places towards the right-hand, as well in the Multiplicand as the Multiplier, multiply the ſignificant figures 431, by the figures 15 (according to the former rules) ſo there will ariſe 6465, to which annexing on the right-hand all the cyphers before omitted, the true product will be 646500000. More examples hereof are theſe following:

```
   43100
   15000
---------
 2155
431
---------
646500000
```

```
  43125          5108000
   1500              125
--------       ----------
 215625          25540
 43125          10216
--------        5108
64687500       ----------
               638500000
```

72. When, in the Multiplier, cyphers are included between ſignificant figures, multiply by the ſaid ſignificant figures, neglecting ſuch cyphers or cypher, but obſerve diligently to ſet the particular products of the ſignificant figures in their due places, according to *Art.* 68. So if 56324 be given to be multiplied by 20006, firſt multiply the whole Multiplicand 56324 by 6, and place the product orderly under the line; then paſſing over the three cyphers, multiply 56324 by 2, and place 8 (which is the firſt figure of this particular product) directly under the multiplying figure 2, and the reſt in their order; ſo at laſt the true product will be found to be 1126817944, and the Work will ſtand as in the example.

```
     56324
     20006
----------
    337944
112648
----------
1126817944
```

More Examples hereof are theſe that follow:

```
  3094          23765
   104          10302
-------       ---------
 12376          47530
3094          71295
-------      23765
321776       ---------
             244827030
```

Note, That one of the principal cautions, to be obſerved in Multiplication, is the due placing of the particular products ariſing by each multiplying figure; and that may be performed, either, by taking care to place the firſt figure or cypher which ariſes arifes in each product under the refpective multiplying figure; or at leaft the firft place arifing in the fecond product muft ftand under the fecond place of the firft product, and the firft place of the third particular product, under the third place of the firft, *&c.*

73. When a number is given to be multiplied by a number that confifts of 1 (or an unit) in the firft place towards the left-hand, and a cypher or cyphers on the right-hand of fuch unit (as are 10, 100, 1000, 10000, *&c.*) the Multiplication is performed by annexing the cypher or cyphers of the Multiplier at the end (to wit, on the right-hand) of the Multiplicand; fo if 326 be given to be multiplied by 10, the product is 3260; if by 100, the product is 32600; if by 1000, the product is 326000; in like manner if 170 be multiplied by 10, the product is 1700; if by 100, 17000, *&c.*

74. Hence if it be required to multiply by 5, add a cypher to the Multiplicand and take $\frac{1}{2}$ the refult, fo $326 \times 5 = \frac{3260}{2} = 1630$.

75. How to multiply by any number under 20, fo as to bring the whole operation into one line.

Rule. *Multiply each figure in the Multiplicand by the unit figure in the Multiplier, adding to each its back figure; as in the following Examples.*

47947	27942
13	17
623311	475014

Which are done thus; *viz.* 3 times 7 is 21, fet down 1 and carry 2; then 3 times 4 is 12, and 2 is 14, and 7 (which is the back figure to 4) is 21, 1 and carry 2; 3 times 9 is 27, and 2 is 29, and 4 (the back figure) is 33, 3 and carry 3; 3 times 7 is 21, and 3 is 24, and 9 (the back figure) is 33, 3 and carry 3; 3 times 4 is 12, and 3 is 15, and 7 is 22, 2 and carry 2, which added to 4 makes 6.

76. In like manner the whole operation of the Multiplication of Numbers between 20 and 30, are brought into one line, by taking in the double of the back figure, and between 30 and 40, the treble of it, *&c.*

By which Rule in multiplying by Numbers compounded of thofe under 20, the product is readily found thus:

By 112, either multiply by 2 and 11, or by 12 and 1, as,

94267	94267
112	112
188534	1131204
1036937	94267
10557904	10557904

Alſo in multiplying by 1614, and 1715; in the firſt multiply by 14 in one line, and by 16 in the other: And in the latter by 15 in one line, and 17 in the other, as above directed; obſerving always to place the firſt figure of a line under its own Multiplier; as

479479	479479
1614	1715
6712706	7192185
7671664	8151143
773879106	822306485

77. Otherwiſe for any number under 20, a method not ſo troubleſome to the memory.

Rule. *Multiply by the unit figure, ſetting that whole product one figure back on the right-hand, which added to the Multiplicand, gives the product*, viz.

47947	27642	479479 by 14
13	17	1917916
143841	193494	6712706
623311	469914	

Note, This will be ſtill eaſier, if the Multiplier be not wrote under the Multiplicand, as in the laſt Example.

But for 112, multiply by 12, ſetting the product two figures back, which likewiſe added to the Multiplicand, gives the product, as,

94267		94267 by 112
112	Or	1131204
1131204		10557904
10557904		

78. In

78. In like manner, if the Multiplier be 21, 31, 41, &c. let the Multiplicand ſtand for its product by unity, and write under it, in the proper place, the product by the other figure. For example, the operation of multiplying 365 by 31 may ſtand as below.

365
1095
———
11315

79. When it is required to multiply by a number conſiſting of nines, *viz.* 9, 99, 999, 9999, &c. write as many cyphers as there are nines in the Multiplier to the right-hand of the Multiplicand, and from the reſult ſubtract the Multiplicand, and the remainder will be the required product.

Examp. 1 Let it be required to multiply 456 by 9?
One cypher annex'd to 456 makes $4560 = 456 \times 10$. *Art.* 73.
From which ſubtract the Multiplicand $456 = 456 \times 1$
The remainder is the product $4104 = 456 \times \overline{10-1}$

Examp. 2. Required the product of 456 by 99?
Two cyphers annex'd to 456 make $45600 = 456 \times 100$
From which ſubtract $456 = 456 \times 1$
The remainder is the product $45144 = 456 \times \overline{100-1}$

Examp. 3. What is the product of 456 by 999?
Three cyphers annex'd to 456 make $456000 = 456 \times 1000$
From which take $456 = 456 \times 1$
The remainder is the anſwer $455544 = 456 \times \overline{1000-1}$

80. When it is required to multiply by a number conſiſting of any digit repeated *viz.* 11, 111, 1111, &c. or 22, 222, 2222, &c. or 33, 333, 3333, &c. &c. find by *Art.* 79. the product of the given Multiplicand by 99, 999, 9999, &c. and take the ninth part thereof, * that is divide the product by 9, then the reſult multiplied by the digit which repeats in the given Multiplier will be the product required.

Examp. 1. Let it be required to multiply 456 by 444?
The product of 456 by 999 is $455544 = 456 \times 999$ *Art.* 79.
The ninth part thereof is $50616 = 456 \times (\frac{222}{9})\ 111$
Multiply by the repeating digit 4
The product is the anſwer $202464 = 456 \times (111 \times 4 =) 444$

 Examp.

* If the Reader hath not learnt how to divide, let him poſtpone the practice of this Rule till he hath read the next Chapter.

Examp. 2. Multiply 4538769 by 7777777?

Seven Cyphers annexed to the Multiplicand	= 45387690000000
Subtract the Multiplicand	4538769
Remains	45387685461231
$\frac{1}{9}$ of the remainder	5043076162359
Multiply by the digit	7
The answer	35301533136513

81. To multiply by a Factor consisting of so many cyphers, between two digits, as there are places in the Multiplicand.

34567
6000005

Prod. 207402172835

In this and the like cases there is no more difficulty than in multiplying by a single digit; for the number of cyphers being equal to the places in the Multiplicand, the product by the 6 falls just to the left-hand of the product by the 5; but if the product of the digit next the left-hand by that in the unit's place, and what is carried, be less than 10, then a cypher must be put down between the two products.

82. The following is an universal method for all cases; whereby tho' there is not any contraction, and even some more to do, yet it makes the work so easy that there is no time lost, at least in large examples, and more certainty in the operation. Thus,

Write down the Multiplicand, then double it; add this sum to the Multiplicand, and this again, and so on, every sum to the Multiplicand, till you have nine numbers, and it's plain that thus you have a Table of the products of the Multiplicand by all the digits, made up by a very simple and easy operation; and then you have no more to do, but to transfer your several products out of this Table, and sum them up.

TABLE.

1	467853798
2	935707596
3	1403561394
4	1871415192
5	2339268990
6	2807122788
7	3274976586
8	3742830384
9	4210684182

EX

EXAMPLE.

467853798
6839754
1871415192
2339268990
3274976586
4210684182
1403561394
3742830384
2807122788
3200004886285692

Altho' this method is univerſal, yet we need not apply it in every caſe, for that would not always be beſt; but in ſuch Examples as this, the eaſe and readineſs with which it is done, does more than ſave the time ſpent in making the Table; with this advantage, that the work is performed with much more certainty, becauſe it is more ſimple.

83. This may be contracted in many caſes; for there is no neceſſity always to make the Table for all the nine digits. And it may happen, that by help of ſome of the preceeding methods, we can as eaſily make a Table for few, or no more than we have uſe for in the Multiplier; nor is it any great matter in what order they ſtand in the Table.

For Example, to multiply 78659 by 6897. In making this Table, firſt take 3, then double its product for 6; and do the reſt by the common way.

TABLE.

1	78659
3	235977
6	471954
7	550613
8	629272
9	707931

Again, to multiply 783596 by 3857, firſt multiply by 3, then by 5 (as in *Art.* 74.) then add theſe products for 8, then ſubtract the firſt from this for 7.

TABLE.

1	783596
3	2350788
5	3917980
8	6268768
7	5485172

84. Another Method by a ſmall moveable Table.

Make a Table of the Multiplicand only for the numbers 1, 2, 5 (uſing *Art.* 74. for 5.) and make it upon a bit of looſe paper, that it may be always applied directly and immediately over the place where every particular product is to be written down, (for much of the difficulty lies in the diſtance and croſs poſition of the Multiplicand to the ſeveral products) and out of this ſmall Table find your product thus:

Suppoſe for a Multiplicand 685497. When the figure of the Multiplier is 2 or 5, here you have the products; then for 3 add 2 to 1 (*i. e.* the numbers againſt 2 and 1,) for 4, double the number againſt 2; for 6, add 5 and 1, or multiply 2 by 3; for 7, add 5 and 2; for 8, add 5, 2, and 1; for 9, uſe the method of *Art.* 79.

TABLE.

1	685497
2	1370994
5	3427485

85. When more numbers than two are given to be multiplied one by the other, that kind of Multiplication is called *continual*, and is thus performed, *viz.* firſt multiply any two of the numbers given one by the other, then multiply the product by another of the numbers given, and this product by the fourth number given (if there be ſo many) and in that order 'till every one of the given numbers has been made a Multiplier, ſo the laſt product is the true product required. *Example*, If 4, 18, and 22, were given to be multiplied *continually*, firſt 18 multiplied by 4 produces 72, which multiplied by 22 (the third number) produces 1584, the laſt product or number required. See the Work in the margin. The proof of Multiplication is by Diviſion, as will appear by the next Chapter.

```
  18
   4
 ----
  72 Prod. 1.
  22
 ----
 144
144
 ----
1584 Prod. 2.
```

86. If one of the Factors is equal to the product of any two or more numbers, then multiply by thoſe numbers continually.

That

That is, to multiply 42991 by 56, firſt multiply it by 7, and that product by 8, then the ſame product will ariſe as if it were multiplied by 56, the product of 7 by 8. *Example.*

42991	42991
7	56
300937	257946
8	214955
2407496	2407496

Note, If Numbers are given that cannot be produced by the Multiplication of any two figures, then find out two figures whoſe product comes neareſt, either under or above it, to which add what is wanting, or from which ſubtract what is over the given number; as,

479 by 47	479	927 by 29	927
6	47	4	29
2874	3353	3708	8343
8	1916	7	1854
22992	22513	25956	26883
479 ſubtracted		927 added	
22513		26883	

87. To multiply Numbers of ſeveral denominations. For example; to multiply 3*l.* 13*s.* 7½*d.* by 8, the proceſs may ſtand as below.

	3	13	7½
			8
Anſw.	29	9	0

The Work is performed as follows, ſaying 8 half-pence make 4 pence, which reſerve in mind; again, 8 times 7 pence make 4*s.* 8*d.* (to wit, 8 ſix-pences make 4*s.* and there are 8 pence beſides) to which adding 4 pence in mind, there will ariſe 5*s.* which reſerve in mind, and ſubſcribe a cypher under the place of pence; again, ſay 8 times 13*s.* make 5*l.* 4*s.* (to wit 8 angels make 4*l.* and 8 times 3*s.* make 1*l.* 4*s.*) to which adding 5*s.* in mind, the ſum will be 5*l.* 9*s.* wherefore ſubſcribe 9*s.* (the exceſs above the pounds) under ſhillings, and keep 5*l.* in mind; laſtly, ſay 8 times 3 pounds make 24 pounds, which with

5 pounds in mind make 29 pounds; ſo that the total product or anſwer of the queſtion is found to be 29*l.* 9*s.*

More Examples of this kind are theſe:

	l.	*s.*	*d.*
	17	15	$5\frac{1}{4}$
			7
Anſw.	124	8	$0\frac{3}{4}$

	l.	*s.*	*d.*
	18	12	$6\frac{3}{4}$
			8
Anſw.	149	00	6

CHAP. VI.

DIVISION *of Whole Numbers.*

88. DIVISION is that by which we diſcover how often one number is contained in another; or (which is the ſame) it ſhews how to divide a number propos'd, into as many equal parts as you pleaſe.

89. In Diviſion there are always three remarkable numbers, which are commonly called by theſe names, the *Dividend*, the *Diviſor*, and the *Quotient*.

90. The *Dividend* is the number given to be divided into equal parts.

91. The *Diviſor* is the number by which the Dividend is to be divided; that is, it is the number which declares into how many equal parts the Dividend is to be divided.

92. The *Quotient* is the number ariſing from the Diviſion, and ſhews one of the equal parts required; ſo if 15 were given to be divided by 5, or into 5 equal parts, the number ariſing, or one of the equal parts will be 3, for 5 is found three times in 15: And here 15 is the Dividend, 5 the Diviſor, and 3 the Quotient.

93. As Multiplication is a manifold addition, ſo Diviſion is a manifold Subduction; that is, the taking of one number out of another as often as poſſible. Thus, 15 divided by 5, quotes 3. For

For if 5 be three times taken from 15, nothing will remain.

Note 1. If two numbers conſiſt of an equal number of places the leſſer cannot be contained in the greater above nine times.

For 10 times any number conſiſts of the ſaid number with a cypher annexed, by *Art.* 73. that is, it has one place more than that number.

Note 2. If two numbers be given, the greater of which has one place of figures more than the leſſer, but if the unit's place of the greater number be taken away, the remaining figures of that number be leſs than the leſſer number, then the leſſer number is not contained in the greater more than 9 times.

For if the leſſer number be contained in the greater 10 or more times, then the figures which remain in the greater number, after the unit's place is taken away, will be equal to or greater than the leſſer number.

Note 3. If an even number be divided by an odd number, the Quotient will be even.

Note 4. If an odd number be divided by an odd number, the Quotient will be odd.

Note 5. If an even number be divided by an even number, the Quotient may be either even or odd.

94. Diviſion being the hardeſt leſſon in Arithmetic, muſt be heedfully attended to by the learner, for whoſe eaſe the utmoſt endeavour is uſed to make the way ſmooth by Rules and Examples, beginning with the eaſieſt firſt, which will be in that caſe when the Diviſor conſiſts of one figure only; for example, let it be required to divide 192 by 8, or 192 pounds into 8 equal parts or ſhares; here 192 is the Dividend, 8 is the Diviſor, and the Quotient or one of the equal parts is ſought.

95. Place a crooked line at each end of the Dividend, that on the left-hand ſerving for the place of the Diviſor, and that on the right for the Quotient; then if the Diviſor be a ſingle figure, ſubſcribe a point under the firſt figure of the Dividend towards the left-hand, if ſuch firſt figure be either equal to, or greater than the Diviſor; but if ſuch firſt figure be leſs than the Diviſor put a point under the next place of the Dividend; which number ſo diſtinguiſhed by the point may be called the Dividual; ſo in the *Example* above given, 192 being the Dividend, and 8 the Diviſor, ſet a point under 9, not under 1, becauſe it is leſs than the Diviſor. This done, the Dividual, or number whereof the queſtion muſt be aſked, is 19.

8) 192 (
  ·

96. Having thus prepared the numbers, aſk how often the Diviſor is contained in the Dividual, and write the number which

which anſwers the queſtion in the Quotient; then multiply the Diviſor by the number placed in the Quotient, and ſubſcribe the product under the Dividual. Laſtly, having drawn a line under the product, ſubtract it from the Dividual, and ſubſcribe the remainder orderly under the line. So demanding how many times the Diviſor 8, is found in the Dividual 19, the anſwer is two times, therefore write 2 in the Quotient; then multiplying the Diviſor 8, by 2 (the number placed in the Quotient) the product is 16, which ſubſcribe orderly under the Dividual 19; and after a line is drawn under the product 16, ſubtract it from the Dividual 19, and place the remainder 3 under the line.

```
8) 192 (2
   .
   16
  ------
    3
```

97. Put another point under the next place of the Dividend towards the right-hand, and bring down the figure or cypher ſtanding in that place to the remainder; that is, ſet it next after it, ſo the whole will be a new Dividual: Thus a point being placed under 2, which ſtands in the next place of the Dividend, write 2 next after (to wit on the right-hand of) the remainder 3, ſo 32 is a new Dividual, or number whereof the ſecond queſtion muſt be aſked, and the Work will ſtand as you ſee in the Example.

```
8) 192 (2
   . .
   16
  ------
    32
```

98. A new Dividual being ſet apart, renew the queſtion, and proceed according to *Art.* 96. Thus demanding how often the Diviſor 8, is found in the Dividual 32, the anſwer is four times; therefore write 4 in the Quotient: Then multiplying the Diviſor 8, by 4 (the figure laſt placed in the Quotient) the product is 32, which ſubſcribe under the Dividual 32, and after a line is drawn underneath, ſubtract the product 32 from the Dividual 32, and there being no remainder, ſubſcribe 0 under the line; ſo the whole Work being finiſhed the Quotient is found to be 24, and the operation ſtands as you ſee in the Example; therefore if 192 pounds be equally divided among 8 perſons, the ſhare of each perſon will be 24 pounds.

```
8) 192 (24)
   . .
   16
  ------
    32
    32
  ------
     0
```

A ſecond example: Let it be required to divide 936 pounds into 9 equal parts: having diſtinguiſhed the firſt Dividual by a point, (according to *Art.* 95.) demand how often the Diviſor 9 is found in the Dividual 9, and finding it once contained in it, write one in the Quotient; then multiplying the Diviſor 9

```
9) 936 (1
   .
   9
  ------
   0
```

by

by 1, the product is 9, which ſubſcribe under the Dividual 9; after this, a line being drawn under the product 9, ſubtract it from the Dividual 9, and there being no remainder, place a 0 under the line, as in the *Example.*

Again, placing a point under 3 which ſtands in the next place of the Dividend, tranſcribe the ſaid 3 next after the remainder 0 for a new Dividual; then aſking how often the Diviſor 9 is contained in the Dividual 3, and not finding it once contained therein, write 0 in the Quotient: And now becauſe the product which ought to ariſe from the Multiplication of the Diviſor by 0 (the cypher laſt placed in the Quotient) amounts to 0, the Dividual 3, out of which that product ſhould have been ſubtracted, remains the ſame without Alteration; therefore after a point is ſubſcribed under 6, the next place of the Dividend, annex 6 to the Dividual 3, ſo there will be a new Dividual, to wit 36: Then demanding how often the Diviſor 9 is found in the Dividual 36, the anſwer will be 4 times; therefore place 4 in the Quotient, and multiplying the Diviſor 9 by 4, the product is 36, which ſubſcribe underneath, and ſubtract from the Dividual 36; ſo the remainder is 0; thus the whole Work being finiſhed, the Quotient is found to be 104, as in the *Example:* Wherefore conclude, if 936 *l.* be equally divided among 9 perſons, the ſhare of each will be 104 *l.* In like manner, if 296163 be divided by 7, the Quotient will be 42309.

```
9) 936 (10
   ..
   9
   ———
   03
```

```
9) 936 (104
   ...
   9
   ———
   036
    36
   ———
     0
```

99. The whole Work of Diviſion is chiefly contained in this following Verſe.

Dic quot, multiplica, ſubduc, transferque ſecundam.

Or thus,

Firſt you muſt aſk how oft, in Quotient anſwer make;
Then multiply, ſubtract, a new Dividual take.

And the reaſon of the operation will appear from the following explanation thereof.

```
Diviſor 8) 936 ( 100      the firſt Quotient.
           800 = 100×8  the product of the Diviſor and
           ———                 the Quotient.
           136 =          firſt remainder.
           ———
```

Diviſor

Divifor 8) 136 (10 the fecond Quotient.
80 = 10×8 the product of the Divifor and the Quotient.

56 = fecond remainder.

Divifor 8) 56 (7 the third Quotient.
56 = 7×8 the Product of the Divifor and the Quotient.

00

Now the fum of the feveral Quotients, *viz.* 100 + 10 + 7 = 117 the Quotient, as found by the operation above directed, *viz.*

```
8) 936 (117
   8
   --
   13
    8
   --
    56
    56
    --
    00
```

100. When in the divifion the Divifor confifts of a fingle figure only, the Quotient may be exprefs'd, and all the operation performed in mind, without writing down any part thereof; fo 82506 being given to be halfed or divided into two equal parts, the Work will be thus: The Divifor 2 is found in 8 four times; in 2 once; in 5 twice, and there will remain 1, which 1 being fuppofed to ftand before (to wit, on the left-hand of) the cypher makes 10; then fay 2 is found in 10, 5 times; and laft of all in 6, 3 times; fo that the true *Quotient* or one half of the given number 82506 is found to be 41253.

2)82506(41253

In like manner, if 82506 be given to be divided by 3 or into 3 equal parts, the Work will be thus: The *Divifor* 3 is found in 8 twice, and there will remain 2, which 2 being fuppofed to ftand before (to wit, on the left-hand of) the following 2, makes 22; then fay, 3 is found in 22, 7 times; in 15, 5 times; in 0, not at all; and laftly, in 6 twice; fo that the true *Quotient*, or one of the three equal parts required is 27502. After the fame manner may Divifion be work'd by any fingle figure, without much charge to the memory.

3)82506(27502

When

When the Diviſion is thus performed, it is uſual to write the Quotient under the Dividual: Thus

3) 82506

27502

101. Here the *Learner* may aſk, what ſhall be done with the laſt remainder, if any happen, when the Diviſion is finiſhed? A full anſwer to this will be found in *Chap.* 17. where the Doctrine of Fractions is explained; yet I ſhall here produce an *Example* where the ſaid caſe happens, *viz.* Let it be required to divide 351 by 8, or 351 pounds equally among 8 perſons; now, if the operation be proſecuted according to the former rules, the Quotient will be found to be 43, and after the Diviſion is finiſhed, there will remain 7, that is, each perſon muſt have 43 pounds, and there will be an overplus of 7 pounds, which muſt be alſo divided equally among the 8 perſons; but that cannot be done till the 7 pounds be reduced into ſhillings, and then thoſe ſhillings muſt be divided by 8, to give every perſon his due ſhare of the ſhillings contained in the ſaid 7 pounds: Again, if there yet remain any ſurpluſage of ſhillings, they muſt be reduced to pence, which alſo are to be divided by 8, to give every perſon his due ſhare of pence: So that when this queſtion is fully anſwered, each perſon's ſhare will appear to be 43 *l.* 17 *s.* 6 *d.* But how the before-mentioned *Reduction* is performed, will be made manifeſt in the next chapter.

8) 351

43 . 7

102. When the Diviſor conſiſts of two, three, or more places, the operation is more difficult than the former; but depends upon the ſame grounds; and therefore the Learner being well verſed in the preceding method, of dividing by a ſingle figure, will the more readily underſtand theſe that follow, which are two, of which the firſt is the eaſier, but the latter more expeditious: For an *Example* of the former, let it be required to divide 4112772 by 708; or, which is the ſame, to divide 4112772 into 708 equal parts.

Firſt a *Table* is to be made to ſhew at firſt ſight any *Multiple* or product of the *Diviſor*, it being taken twice, thrice, or any number of times under ten; ſo having firſt written down the *Diviſor* itſelf 708, and drawn a line on the right-hand of it, place 1 on the right-hand of the line directly againſt the *Diviſor*; then under the

The Diviſor	708	1
Multiples of the Diviſor	1416	2
	2124	3
	2832	4
	3540	5
	4248	6
	4956	7
	5664	8
	6372	9

Diviſor

Divifor 708, fubfcribe the double thereof, which is 1416, and place the figure 2 directly againft the faid double, to wit, on the other fide of the line. Again, adding 1416, (to wit, the double of the *Divifor)* to the *Divifor* itfelf 708, the fum is 2124 for the triple of the *Divifor* ; fubfcribe this under the double, and place 3 on the other fide of the line right againft it. Again, adding 2124 (the triple of the *Divifor)* to the *Divifor* 708 produces 2832 for the Quadruple of the *Divifor*, which Quadruple fubfcribe under the Triple ; and proceeding in like manner, at laft the Table is finifhed, which readily fhews the *Divifor*, with the *Duple*, *Triple*, *Quadruple*, *Quintuple*, *Sextuple*, *Septuple*, *Octuple*, and *Noncuple* of the *Divifor*.

Now for a proof of the faid Table, adding the laft number thereof, to wit, 6372 (which was found to be nine times the *Divifor)* to the *Divifor* 708, the fum is 7080, which is evidently ten times the *Divifor* ; therefore the Table is true, in regard that the laft number of it is derived from all the fuperior numbers.

The Table of Multiples or Products of the *Divifor* being thus prepared, fet down the *Dividend* on the right-hand of the *Divifor* ; then diftinguifh by a point fo many of the foremoft places of the *Dividend* towards the left-hand, as are either equal in value to the *Divifor*, or which being greater, yet come neareft to the value thereof; thus fubfcribe a point under 2, thereby fetting apart 4112, being the feweft of the foremoft places which contain the *Divifor* 708, fo is 4112 the *Dividual* or number whereof the firft queftion muft be afked ; then demanding how often the *Divifor* 708 is contained in the *Dividual* 4112. the anfwer will be found by the Table to be five times ; for looking in the Table it appears that 6 times the *Divifor* is the next greater than the *Dividual* 4112, and 5 times is the next leffer, therefore write 5 in the Quotient, and the number in the Table which ftands againft 5, to wit, 3540, fubfcribe under the *Dividual* 4112. Then having drawn a line underneath, fubtract 3540 (which is five times the *Divifor*) from the *Dividual* 4112, and fubfcribe the remainder 572 under the line ; that done, put a point under the next place of the *Dividend* towards the right-hand,

708	1)	4112772 (5809
		....
1416	2	3540
		————
2124	3	5727
2832	4	5664
		————
3540	5	6372
4248	6	6372
		————
4956	7	0
5664	8	
6372	9	

hand, and because the figure 7 ſtands in that place, tranſcribe 7 next after the remainder 572, ſo there is 5727 for a new *Dividual.*

Then demanding how often the *Diviſor* 708 is contained in the *Dividual* 5727, the anſwer will be found by the Table to be 8 times; for looking by the Table, 9 times the *Diviſor* is the next greater, and 8 times is the next leſſer than the *Dividual*; therefore write 8 in the Quotient, and the number in the Table, which ſtands againſt 8, to wit, 5664 ſubſcribe under, and ſubtract from the *Dividual* 5727, placing the remainder 63 under the line.

Again, put a point under the next place of the *Dividend*, to wit, the figure 7, and tranſcribing 7 next after the remainder 63, the new *Dividual* will be 637; then demanding how often the *Diviſor* 708 is contained in the *Dividual* 637, and not finding it once contained therein, write 0 in the Quotient, and ſince in this caſe (that is, when a cypher anſwers the queſtion) the *Dividual* remains the ſame without alteration, the figure or cypher ſtanding in the next place of the *Dividend*, is to be tranſcribed after the *Dividual* for a new *Dividual*, ſo writing 2 next after 637, the new *Dividual* is 6372: Then demanding how often the *Diviſor* 708 is contained in 6372, the Table ſhews that it is contained in it 9 times; therefore writing 9 in the Quotient, and placing the number which ſtands againſt 9 in the Table, to wit, 6372 under the *Dividual* 6372, and ſubtracting it from the *Dividual*, there will remain 0. Whence, if 4112772 be divided by 708, or into 708 equal parts, the true Quotient or one of the equal parts required is 5809. In like manner, if 20304 be divided by 188, that is into 188 equal parts, the Quotient ariſing, or one of thoſe equal parts will be 108, and the operation will ſtand as you ſee.

Diviſor	188	1)	20304 (108
Multiples of the Diviſor	376	2	· · ·
	564	3	188
			———
	752	4	1504
	940	5	1504
			———
	1128	6	0
	1316	7	
	1504	8	
	1692	9	

The preceding Method of *Diviſion* by the help of a Table of Multiples or products of the *Diviſor*, as it is moſt eaſy, ſo in ſome caſes, namely, where the *Diviſor* is great, and a Quotient of many places is required, (as in calculating Tables of Intereſt, Aſtronomical Tables, and ſuch like) it excels all other

other ways of *Diviſion*; both in reſpect of certainty and expedition; but for common practice it is too tedious.

103. *The laſt and principal method of Diviſion, when the Diviſor conſiſts of many places, which to ſuch as have the Table of Multiplication by heart, will not be difficult*: For *Example*, let 56304 be a number given to be divided by 184, that is, into 184 equal parts, and the Quotient, or one of the equal parts is required.

Firſt, Diſtinguiſh by a point (as before) ſo many of the foremoſt places of the Dividend towards the left-hand, as are either equal in value (when conſider'd apart) to the Diviſor, or elſe which being greater, yet come neareſt to it: Thus ſubſcribe a point under the figure 3, thereby ſetting apart 563, being the feweſt of the foremoſt places, which contain the Diviſor; ſo is 563 the Dividual, or number whereof the firſt queſtion muſt be aſked. Having thus prepared the numbers demand how often the Diviſor 184 is contained in the Dividual 563; and ſince to anſwer this queſtion and ſuch like, there is a neceſſity of trial, it will be requiſite to ſhew how this trial may fitly be made: Firſt, therefore, compare the number of places in the Dividual, with the number of places in the Diviſor, and when the number of places is the ſame in both, let it be aſked how often the firſt or extreme figure of the Diviſor, towards the left-hand, is contained in the firſt figure of the Dividual towards the ſame hand, ſo here demanding how often 1 is contained in 5, the anſwer is 5 times; therefore the Diviſor 184 is not contained oftner than 5 times in the Dividual 563 (for 6 times 184 is manifeſtly greater than 563,) but whether it be contained 5 times in it or not, examination muſt be made, either by multiplying (in ſome bye-place) the Diviſor 184 by the ſaid 5, and comparing the product with the Dividual 563; or elſe thus, ſaying 5 times 1 (to wit, the 1 in the Diviſor) is contained in 5, to wit, the firſt figure of the Dividual 563, 5 times; but then the 8, the following figure of the Diviſor, cannot be found 5 times in 6, the following figure of the Dividual and conſequently the Diviſor 184 is not contained 5 times in the Dividual 563; wherefore make another trial to ſee whether it may be contained 4 times in it or not; ſaying, 4 times 1 is 4, which is found in 5, and there will remain 1; but then 4 times 8, which is 32, cannot be had in 16 (for the 1 before remaining being ſuppoſed to ſtand on the left-hand of 6 makes 16); hence again, the Diviſor 184 is not contained 4 times in the Dividual 563; wherefore make another trial to ſee whether it may be contained 3 times in it or not; ſaying 3 times 1 is 3 which is found in 5,

184) 56304 (

and there will remain 2; again three times 8 is 24, which is found in 26 (for the 2 before remaining being ſuppoſed to ſtand before the 6 in the Dividual, makes 26) and there will remain 2. Laſtly, 3 times 4 is 12, which is likewiſe found in 23, (for 2 remaining in the 26 being ſuppoſed to ſtand before the 3 in the Dividual makes 23); therefore the Diviſor 184 is contained 3 times in the Dividual 563; write 3 in the Quotient, and proceeding according to *Art.* 96. multiply the Diviſor 184 by 3 (the figure placed in the Quotient) ſo the product is 552, which ſubſcribe orderly under the Dividual 563; then having drawn a line under the ſaid product, ſubtract it from the Dividual, and ſubſcribe the remainder, which is 11 under the line.

184) 56304 (3
      552
      ———
       11

Again, according to *Art.* 97. bring down 0, which ſtands in the next place of the Dividend, to the remainder 11, ſo there is 110 for a new Dividual; then demanding how often the Diviſor 184 is found in the Dividual 110, and not finding it once contained in it, write 0 in the Quotient (which is to be done as often as the queſtion is anſwered by nothing); now becauſe the product ariſing from the Multiplication of the Diviſor by 0, (the cypher laſt placed in the Quotient) amounts to 0, the Dividual 110, out of which that product ſhould be ſubtracted, remains the ſame without alteration; therefore, after a point is ſubſcribed under 4, the following place of the Dividend, annex 4 to the laſt Dividual 110, ſo there will be a new Dividual, to wit, 1104; and here the queſtion at large is, to know how often 184 is found in 1104; but to leſſen the trial, becauſe the Dividual conſiſts of one place more than is in the Diviſor, it muſt be aſked how often the firſt figure of the Diviſor, on the left-hand, is contained in the two foremoſt places of the Dividual, towards the left-hand, *viz.* demand how often 1 is contained in 11, and altho' it may be had 11 times, yet never begin the trial above 9 times, ſee *Note* 2. *Art.* 93. therefore make trial with 9, ſaying, 9 times 1 is 9, which is found in 11, and there will remain 2; but then 9 times 8, which is 72, cannot be found in 20 (20, becauſe the 2 remaining being ſuppoſed to ſtand before 0 in the Dividual makes 20,) therefore make trial with 8, ſaying, 8 times 1 is 8, which is found in 11, and there will remain 3; but then 8 times 8 cannot be had in 30 (30, becauſe the 3 remaining being ſuppoſed to ſtand before the 0 or cypher makes 30); therefore make trial with 7, ſaying, 7 times 1 is 7, which

184) 56304 (306
      552··
      ———
       1104
       1104
      ———
          0

is found in 11, and there will remain 4; but then 7 times 8 cannot be had in 40; therefore make trial with 6, ſaying, 6 times 1 is 6, which is found in 11, and there will remain 5; then 6 times 8 is 48, which is found in 50, and there will remain 2; therefore the Diviſor 184 is contained 6 times in the Dividual 1104: Wherefore write 6 in the Quotient, and proceeding according to *Art.* 96. multiply the Diviſor 184 by 6 (the figure laſt placed in the Quotient) ſo the product is 1104, which being ſubſcribed under, and ſubtracted from the Dividual 1104, the remainder is 0; therefore the Quotient ſought is 306.

104. If the figure aſſumed for the Quotient holds good upon trial, as aforeſaid, by two or three of the foremoſt places of the Dividual, it will for the moſt part hold throughout the Dividual; but this muſt be a perpetual rule, that whenſoever the product of the Multiplication of the Diviſor, by the figure placed in the Quotient, happens to be greater than the Dividual, from which it ought to be ſubtracted, ſuch product muſt be ſtruck out of the Work, and a leſſer figure is to be placed in the Quotient.

For a ſecond *Example*, Let it be required to divide 15114220 by 2987, or into 2987 equal parts.

Firſt, the Diviſor 2987 being greater than 1511, (to wit, the four foremoſt places of the Dividend) ſet a point under 4, thereby ſetting apart 15114 for a Dividual; then, becauſe the Dividual conſiſts of one place more than the Diviſor, aſk how often 2 (the firſt figure of the Diviſor towards the left hand) is contained in 15, (the two foremoſt places of the Dividual) and finding the anſwer to be 7 times, infer thence that the Diviſor 2987 cannot be contained more than 7 times in the Dividual 15114; but whether it will be contained 7 times in it or not, examination muſt be made, either by multiplying 2987 by 7 (in ſome bye-place) and comparing the product with the Dividual 15114; or elſe by the manner of trial before delivered in the laſt Example: So at length it will be diſcovered, that the Diviſor 2987 will not be found above 5 times in the Dividual 15114; wherefore (according to *Art.* 96.) writing 5 in the Quotient, and multiplying 2987 by 5, ſubſcribe the product of that Multiplication, which is 14935, under the Dividual 15114; then drawing a line under the ſaid product, and ſubtracting it from the Dividual 15114, ſubſcribe the remainder 179 under the line.

```
2987) 15114220 (5
          .
      14935
      -----
        179
```

Again,

Again, (according to *Art.* 97.) bring down 2, the next place of the Dividend, to the said remainder 179, so the new Dividual will be 1792; that done, asking how often the Divisor 2987 is contained in the Dividual 1792, and not finding it once contained in it, write 0 in the Quotient; and here, because the Question is answered by 0, the next place of the Dividend, to wit 2, is to be brought down to the Dividual 1792, so the new Dividual is 17922. Then renewing the question, and proceeding as before, at length the Division being finished, the Quotient will be found 5060 exactly, without any remainder; but if any remainder had happened, after the Subtraction of the last product, it must have been prosecuted according to *Art* 101.

```
2987) 15114220 (50
        ..
      14935
      -----
       1792
```

```
2987) 15114220 (5060
        ....
      14935
      -----
       17922
       17922
       -----
          00
```

In like manner, if 1208939550 be divided by 19999, or into 19999 equal parts, the Quotient, or one of those equal parts, will be found 60450, and the Work will stand as you see here.

This latter method of Division is to be preferred before any of the ways of dividing, by dashing out figures, where the steps of the Division are so confounded (besides the burden upon the memory, by a promiscuous Multiplication and Subtraction) that if any error happen it can hardly be corrected without beginning the Work a-new: But in the way before explained, the particular Multiplications, Subtractions, and Remainders, which belong to every figure of the Quotient, are so distinctly and clearly expres's'd, that if an error happen, the Work may easily be reformed.

```
19999) 1208939550 (60450
             .....
       119994
       ------
         89995
         79996
         -----
         99995
         99995
         -----
            00
```

105. So often as the question is required in Division, so many places there must be in the Quotient, (which may be discovered by the number of points placed under the Dividend) and so many times is one and the same kind of operation repeated, the substance whereof is contained in the Verse beforementioned in *Art.* 99.

106.

106. When the Divifor confifts of 1 or an unit in the extreme place towards the left-hand, and nothing but cyphers towards the right, the Divifion is performed by cutting off, with a line, fo many places of the Dividend towards the right-hand, as the Divifor has cyphers; fo the figures which ftand on the left-hand of the line, give the Quotient, and thofe cut off to the right (if they be fignificant figures) are to be proceeded with as a furplufage or overplus remaining, according, to *Art.* 101. So if 4720 *l.* were given to be divided equally among 10 perfons, the fhare of each would be 472 *l.* alfo if the faid 4720 *l.* were to be divided equally among 100 perfons, the fhare of each would be 47 *l.* and there would be a furplufage or remainder of 20 *l.* to be alfo fubdivided among them, after the faid 20 *l.* are converted into fhillings, according to *Art.* 119. Laftly, if the faid 4720 *l.* were to be divided among 1000 perfons, the fhare of each would be 4 *l.* and there would be a remainder of 720 *l.* to be alfo divided as aforefaid. See the form of the Work in the margin.

10)	472|0	(472
100)	47|20	(47
1000)	4|720	(4

107. When the Divifor confifts of any fignificant figure or figures in the firft or foremoft place or places, towards the left-hand, and nothing but a cypher or cyphers towards the right, cut off, by a line, fo many places of the Dividend towards the right-hand, as the Divifor has cyphers towards the right; then divide the figures of the Dividend, which ftand on the left-hand of the line, by the figures in the Divifor which remain, when the faid cypher or cyphers are omitted, remembering, after the Divifion is finifhed, to write down next after the laft remainder, the places of the Dividend which were firft cut off: So if 36732 were given to be divided by 20, the Quotient will be 1836, and there will remain 12, *viz.* if you cut off one place from the Dividend towards the right-hand (becaufe the Divifor ends with one cypher, and then divide the reft, to wit, 3673) by 2 (according to *Art.* 102.) there will arife in the Quotient 1836, and the laft remainder after fuch Divifion is finifhed, will be 1, to which if 2 (the figure firft cut off from the Dividend) be annexed, the total remainder is 12.

2|0)3673|2
1836:12

In like manner, if 7456787 were given to be divided by 304000, the Quotient will be 24, and there will remain 160787; *viz.* if you cut off 3 places from the Dividend towards the right-hand (3 places, becaufe the Divifor ends with 3 cyphers) and then divide 7456 by 304, there will arife in the Quotient

24,

24, and the last remainder, after such Division is finished, will be 160, to which if 787 (the places first cut off from the Dividend) be annexed, the total remainder or surplusage is 160787, which is to be proceeded with, as in *Art.* 101.

```
3041000) 7456|787 (24
         608
         1376
         1216
          160787
```

108. If it be required to divide by a number consisting of nines, *viz.* 9, 99, 999, 9999, &c. it may be performed by Addition, on the same principles as the multiplying those numbers was done by Subtraction, in *Art.* 79. as follows.

Divide the given Dividend into periods, of so many places of figures as there are nines in the Divisor, beginning from the left-hand, and annex as many cyphers, to the right-hand of the number, as may be wanted to compleat a period.

Then write the figures of the left-hand period under those of the second period, or that which is next thereto toward the right-hand; add these two together, and place their sum under the third period; observing if the sum of the two figures in the highest place exceed 9, to place the figure that would (in common Addition) be carried, under the lowest place of the second period; add the third period to those figures which stand under it, including the carried figure, and place them under the fourth period; and so proceed, till you have placed figures under the right-hand period, and under them place such a figure, as would have been to be there placed, had the Work been to have proceeded a period farther.

Add the whole together; and, beginning at the right-hand, cancel as many figures as there were cyphers annexed to the Dividend; then, from the figures that remain, cut off with a comma, from the right-hand towards the left, as many figures as the Divisor contained nines; so shall the figures, to the left-hand of the comma, be the Quotient; and those to the right-hand thereof the remainder.

Note, If the remainder be all nines, add 1 to the Quotient.

Examp. 1. Let it be required to divide 571665762 by 999?

The Dividend will make just 3 periods	571.665.762
The first period wrote under the second	571
The sum of 665 and 571 will be	1.236
	1
Under the last place is set 1, because	572237,999

if 762 and 1236 were to have been added together, the 1 would have been so placed; then, by adding the whole, the sum will be 572237999; and three figures being cut off, from the right-hand, will give 572237,999, that is, 572237 for the Quotient, and 999 for the remainder; but since the remainder is equal to the Divifor, the Quotient, increafed by 1, *viz.* 572238 will be the anfwer.

To prove this Work, let 572238 be multiplied by 999, according to *Art.* 79. See the Work.

```
572238000
   572238
571665762 Product.
```

Where it is evident that the figures fubtracted are the fame with the fum of thofe added, except in the unit's place.

Note, The above procefs will take up lefs room, if the carried figures be wrote in a line by themfelves, as follows:

```
571.665.762
    571.236
      1   1
572237,999
```

Examp. 2 Divide 57166576279 by 999?

The number with a cypher annex'd =	571.665.762.790
	571.236.998
	1 1 2
After the operation there arifes the number	572 238 000 790
From which cancel the right-hand figure, becaufe a cypher was added to the Dividend, and it will be — —	57 223 800 079
And this being properly feparated gives 57223800 for the Quotient, and 79 for the remainder — — —	57223800,079
Proof	57223800000
	57223800
To 57223800 × 999 =	57166576200
Add the remainder	79
The fum makes the Dividend	57166576279

Examp. 3. Divide 5710668153 by 999?

The number with two cyphers annex'd	571.066.815.300
	571.637.452
	1 1
From the operation arifes	571 638 453 753
And cancelling 2 figures	5 716 384 537
Which properly feparated will be	5716384.537

viz. 5716384 for the Quotient, and 537 for the remainder.

Proof	5716384000
	5716384
To 5716384 × 999 =	5710667616
Add the remainder	537
The fum is the Dividend	5710668153

109. If

109. If it be required to divide by a number confifting of any Digit repeated, *viz.* 11, 111, 1111, *&c.* or 22, 222, 2222, *&c.* or 33, 333, 3333, *&c.* divide the given Dividend by the Digit which repeats in the Divifor, and multiply the Quotient by nine; then divide the product by 99, 999, 9999, *&c.* as in *Art.* 108. and the refult will be the Quotient required.

Examp. Let it be required to divide 202464 by 444?

4)202464

50616 = the Quote of 202464 divided by the repeatingDigit.

9

455544 = the Product of 50616 by 9.

455

455,999 or 456, the Quotient required, by *Art.* 108.

110. Divifion and Multiplication interchangeably prove one another; for in Divifion if you multiply the Divifor by the Quotient, the product will be equal to the Dividend: So in the *Example* of *Art.* 103. if 184 the Divifor be multiplied by 306 the Quotient, the product is 56304, which is the fame with the Dividend; but when, after the whole Divifion is finifhed, any figures remain of the laft Subtraction, add them likewife to the product: So in the laft *Example* of *Art.* 107. the Divifor 304000, being multiplied by the Quotient 24, produces 7296000, unto which if you add the number remaining, to wit, 160787, the fum is 7456787, which is the fame with the Dividend. Again, in Multiplication, if the product be divided by the Multiplier, the Quotient will give you the Multiplicand, or if the product be divided by the Multiplicand, the Quotient will give you the Multiplier: So in the firft *Example* of *Art.* 70. if the product 111024 be divided by the Multiplicand 3084, the Quotient gives the Multiplier 36.

111. There is alfo a common Proof of Multiplication argued from the Multiplicand, the Multiplier and the Product, by cafting away nines; but by that way of proof (though rightly ufed) a falfe product will be affirmed to be true: *Example*, If 3462 be multiplied by 786, the true product is 2721132; but if I fay, 4953132 or 3153132 is the product (or many others which may be given) the proof by nines will confirm them to be true products, though they are falfe, as will be evident to fuch as know the *Rule*, which is mentioned here, only to fet a brand upon it, that it may be avoided by all lovers of truth.

112. Another proof of Divifion is, by adding together thofe lines which, in the following *Example*, are marked with Afte-

rifms (being the particular products of the Divifor, multiplied feverally by each figure in the Quotient, together with the remainder of the Divifion) the total of which (if right) will be the Dividend.

```
736) 863256 (1172
     736*
     ----
     1272
      736*
     ----
     5365
     5152*
     -----
     2136
      1472*
     -----
      664*
   ------
   863256  Proof.
```

113. When a whole number is given to be divided by a Divifor, which is equal to the product of the Multiplication of two fingle figures; inftead of dividing by that Divifor, you may firft divide by one of thofe fingle figures, and then divide the Quotient by the other; fo will the laft Quotient be the fame, as if the Divifion had been finifhed by the Divifor firft given: Thus, if 3456 be given to be divided by $48 = 6 \times 8$, divide firft by 8, the Quotient will be 432; and then divide that Quotient (432) by 6, and the refult 72 is the fame as will arife by dividing 3456 by 48.

114. To divide Numbers of divers denominations: For *Example*, to divide 12 *l.* 12 *s.* 6 *d.* into 5 parts, the Work may ftand as below:

```
5) 12 . 12 . 6
   -----------
    2 . 10 . 6
```

And the operation is performed in the following manner: Say the fives in 12, twice, and there remain 2, which are two pounds (for the remainder is always of the fame name with the Dividend) or 40 fhillings, and 12 *s.* in the fhillings place, make 52 fhillings; now the fives in 52, ten times, and there remain 2, which are two fhillings or 24 pence, to which adding the 6*d.* in the place of pence, make 30 pence; then the fives in 30 fix times.

A Queftion whofe Solution requires the ufe of the four preceding Rules.

A father left among five fons an eftate, confifting of 500 *l.* in Cafh, with 5 bills each of 48 *l.* 10 *s.* 6*d.* he ordered 20 *l.* to be beftowed upon his burial, and his debts to be paid, amounting

ing to 164*l.* then his free eſtate to be divided in this manner, *viz.* the eldeſt ſon to have the third part, and the other four ſons to have equal ſhares: What is the ſhare of each ſon? *Anſw.* 186*l.* 4*s.* 2*d.* to the eldeſt, and 93*l.* 2*s.* 1*d.* to each of the reſt. *See the operation.*

48 . 10 . 6	the value of each bill.
5	bills.
242 . 12 . 6	total of the bills.
500 . 0 . 0	caſh.
742 . 12 . 6	total.
184 . 0 . 0	deducted, *viz.* 20 + 164.
3)558 . 12 . 6	free eſtate.
186 . 4 . 2	eldeſt ſon's ſhare.
4)372 . 8 . 4	remains.
93 . 2 . 1	the ſhare of each of the other 4 ſons.

CHAP. VII.

REDUCTION.

115. FOraſmuch as in Money there are diverſities of kinds, *viz.* in *England*, pounds, ſhillings, pence, and farthings; alſo diverſe kinds of Weights, Meaſures, &c. as has been fully declared in the ſecond Chapter; and becauſe it is often required to find how many pieces of one kind of Money are equal in value to a given number of another, (and ſo likewiſe of Weights, Meaſures, &c.) it is requiſite in this place to ſhew how that is performed, ſince thereby the Rules of Multiplication and Diviſion before delivered will be exerciſed. This kind of operation is called *Reduction.*

116. *Reduction* is either deſcending or aſcending.

117. *Reduction* deſcending, is, when ſome number of integers of a greater denomination being given, it is required to find how many integers of a leſſer denomination are equal in value to that given number of the greater; As when it is demanded to find how many ſhillings are contained in 30*l.* Likewiſe how many pence in 320*s.* or how many hours in 365 days, &c.

118. *Reduction* aſcending is, when ſome number of integers of a leſſer denomination being given, it is required to find how

how many integers of a greater donomination are equal in value to that given number of the leſſer: As when it is propoſed to find how many pence are contained in 500 farthings; likewiſe how many ſhillings in 348 pence; or how many days in 864 hours, &c.

119. *Reduction* deſcending is performed by Multiplication; for if the given number of integers of a greater denomination, be multiplied by a number which expreſſes how many integers of the leſſer are equal to one of the integers given, the product is the number of integers of the leſſer denomination required.

So 230 *l.* of *Engliſh* money will be reduced into 4600 *s.* for if 230 be multiplied by 20, (the number of ſhillings which are equal to 1 pound) the product is 4600; in like manner 4600 *s.* will be reduced into 55200 *d.* for if 4600 be multiplied by 12 (the number of pence contained in 1 ſhilling) the product is 55200. Alſo 55200 pence being multiplied by 4 (becauſe 4 farthings make a penny) are reduced into 220800 farthings, as by the operation in the margin is evident.

```
   230 Pounds.
    20
  ----------------
  4600 Shillings.
    12
  ----------------
 55200 Pence.
     4
  ----------------
220800 Farth.
```

The like method is to be obſerved in Weights, Meaſures, &c. So 345 ounces *Troy* are reduced into 6900 penny-weights, and 6900 penny-weights, to 165600 grains, as by the operation in the margin you may ſee.

```
   345 Ounces.
    20
  ----------------
  6900 PennyW.
    24
  ----------------
   276
  138
  ----------------
165600 Grains.
```

Note, By this Rule the learner is furniſhed with ſkill to reſolve that caſe in Diviſion, when the Dividend is leſs than the Diviſor. *Example*, Let it be required to divide 7 pounds of *Engliſh* money, equally among 8 perſons; here it is evident that the Dividend 7 is leſs than the Diviſor 8; that is, the number of pounds is leſs than the number of perſons, and conſequently each ſhare muſt be leſs than a pound: So that in effect it is required to find how many ſhillings and pence belong to every perſon for his ſhare: Firſt, therefore, reduce the 7 pounds into ſhillings, which will be 140, theſe divided by 8 give 17 ſhillings to each perſon, and there will yet be a remainder of 4 ſhillings to be alſo equally divided into 8 parts; but theſe 4 ſhillings muſt be firſt reduced into pence, which will be 48, then dividing 48 by 8, the Quotient will give 6 pence more to every perſon: So at laſt it appears that if 7 pounds of *Engliſh* money be equally divided into 8 parts, the entire Quotient (or one of the equal ſhares) will be 17 ſhillings and 6 pence.

In

In like manner, if 354 pounds of *English* money be given to be divided equally amongst 125 persons, the share of each will be found to be 2 pounds 16 shillings 7 pence 2 farthings and somewhat more, but the parts of a farthing being of no moment (and not properly to be handled in this place) are neglected.

Compare these two Examples with the Example, *Art.* 101.

120. In *Reduction* descending, the learner may receive help by the subsequent Tables.

1. English Money.

Pounds	*Multiplied by*	20	*Produce*	*Shillings.*
Shillings		12		*Pence.*
Pence		4		*Farthings.*

2. Troy-Weight.

Pounds	*Multiplied by*	12	*Produce*	*Ounces.*
Ounces		20		*Penny-weights.*
Penny-weights		24		*Grains.*

Also in Apothecaries-Weights.

Ounces-Troy	*Multiplied by*	8	*Produce*	*Drams.*
Drams		3		*Scruples.*
Scruples		20		*Grains.*

3. *Of* Avordupoise-Weights.

Hundred Weights	*Multip. by*	4	*Produce*	*Quarters.*
Quarters		28		*Pounds.*
Pounds		16		*Ounces.*
Ounces		16		*Drams.*

4. *Of* Liquid Measures.

Hogsheads	*Multip. by*	63	*Produce*	*Gallons.*
Gallons		2		*Pottles.*
Pottles		2		*Quarts.*
Quarts		2		*Pints.*

5. *Of* DRY MEASURES.

	Multiplied by	Produce
Quarters	8	*Buſhels.*
Buſhels	4	*Pecks.*
Pecks	2	*Gallons.*
Gallons	2	*Pottles.*
Pottles	2	*Quarts.*
Quarts	2	*Pints.*

6. *Of* LONG MEASURE.

	Multiplied by	Produce
Engliſh Miles	8	*Furlongs.*
Furlongs	220	*Yards.*
Yards	3	*Feet.*
Feet	12	*Inches.*
Inches	3	*Barley-corus.*

ALSO,

	Mul. by	Produce
Yards, or *Ells*	4	*Quarters.*
Quarters	4	*Nails.*

7. *Of* SUPERFICIAL MEASURES *of* LAND.

	Mul. by	Produce
Acres	4	*Roods.*
Roods.	40	*Perches*, or *Poles.*

8. *Of* TIME.

	Mul. by	Produce
Weeks	7	*Days.*
Days	24	*Hours.*
Hours.	60	*Minutes.*

121. Integers of ſeveral denominations may be reduced into the leaſt of thoſe denominations, according to *Art.* 120. by deſcending orderly to the next inferior denomination; and adding to each product ſuch integers, (if there be any) as are of the ſame name.

So 12 pounds 13 ſhillings and 10 pence may be reduced into 3046 pence, in this manner, *viz.* 12 pounds multiplied by 20 (becauſe 20 ſhillings make 1 pound) produce 240 ſhillings, to which adding 13 ſhillings, the ſum is 253 ſhillings. Again, 253 ſhillings multiplied by 12 (becauſe 1 ſhilling is equal to 12 pence,) produce 3036 pence, to which, if 10 pence be added, the ſum is 3046 pence, as by the operation in the Margin is manifeſt.

	l.	*s.*	*d.*
	12 .	13 .	10
	20		
	240		
add	13		
	253 Shillings.		
	12		
	506		
	253		
	3036		
add	10		
	3046 Pence.		

But after that general method is well underſtood, the Work of the laſt *Example*, and ſuch like, may be contracted thus, *viz.* To convert 12 pounds 13 ſhillings and 10 pence, all into pence: Firſt, 12 multiplied by 0 (which ſtands in the unit's place of 20) produces 0, but inſtead of 0 write down 3 under the line (to wit, the 3 that ſtands in the unit's place of the 13 ſhillings in the ſum propoſed:) Then proceed to multiply 12 by 2, ſaying twice 2 is 4, to which adding 1 (for the ten in the ſaid 13 ſhillings) it makes 5, which ſet on the left-hand of 3 before-written: Laſtly, twice 1 is 2, which ſet on the left hand of 5; and ſo 12 pounds 13 ſhillings, are converted into 253 ſhillings.

l.	*s.*	*d.*
12 .	13 .	10
20		
253 Shillings.		
12		
516		
253		
3046 Pence.		

It remains to multiply the ſaid 253 by 12 (becauſe 12 pence make one ſhilling) and to add 10 to the product, which may be done thus: Firſt, twice 3 is 6, to which adding 10 (to wit, 10 pence in the ſum firſt propounded,) it makes 16; wherefore (according to the Rule of Multiplication) ſet 6 under the line, and keep 1 in mind: Again, twice 5 with 1 in mind making 11, write down 1, and keep 1 in mind; likewiſe twice 2 and 1 in mind making 5, write down 5: Then 253 multiplied by 1 making 253, which ſet orderly under 516: Laſtly, thoſe two products added together make 3046, which is the number of pence contained in 12 *l.* 13 *s.* 10 *d.* as before was found out by the general method.

So 35 ounces 16 penny-weights and 12 grains *Troy* will be reduced into 17196 grains.

122. *Reduction* aſcending is performed by Diviſion; for if the number of integers given, be divided by ſuch a number of the ſame

ſame integers, as are equal to one of the integers required, the Quotient is the number of integers ſought for.

So 220800 farthings being divided by 4 (the number of farthings in a penny,) give 55200 pence, in the Quotient: In like manner, if 55200 pence be divided by 12 (the number of pence in a ſhilling) the Quotient is 4600 ſhillings. Laſtly, 4600 ſhillings being divided by 20 (becauſe 20*s.* make a pound *Sterling)* the Quotient is 230 pounds *Sterling*, which are equal to 220800 farthings firſt given. The operation is as follows:

```
     4) 220800
    ----------
   12)  55200
    ----------
  2|0)  460|0
    ----------
        230 £.
```

In like manner, 34268 grains *Troy* will be reduced to 5 pounds 11 ounces 7 penny-weights and 20 grains.

123. This kind of *Reduction* may be made the eaſier to the learner by the following Tables.

1. *Of* ENGLISH MONEY.

Farthings	*Div. by*	4	*give*	*Pence.*
Pence		12		*Shillings.*
Shillings		20		*Pounds.*

2. *Of* TROY-WEIGHT.

Grains	*Div. by*	24	*give*	*Penny-weights.*
Penny-weights		20		*Ounces.*
Ounces		12		*Pounds-Troy.*

Alſo in APOTHECARIES-WEIGHTS.

Grains	*Div. by*	20	*give*	*Scruples.*
Scruples		3		*Drams.*
Drams		8		*Ounces-Troy.*

3. *Of* AVOIRDUPOISE-WEIGHTS.

Drams	*Divid. by*	16	*give*	*Ounces.*
Ounces		16		*Pounds.*
Pounds		28		*Quarters of Hund.*
Quarters		4		*Hundred Weights.*

4. *Of* LIQUID MEASURES.

	Divid. by	give
Pints	2	*Quarts.*
Quarts	2	*Pottles.*
Pottles	2	*Gallons.*
Gallons	63	*Hogsheads.*

5. *Of* DRY MEASURES.

	Divided by	give
Pints	2	*Quarts.*
Quarts	2	*Pottles.*
Pottles	2	*Gallons.*
Gallons	2	*Pecks.*
Pecks	4	*Bushels.*
Bushels	8	*Quarters.*

6. *Of* LONG MEASURES.

	Divided by	give
Barley-corns	3	*Inches.*
Inches	12	*Feet.*
Feet	3	*Yards.*
Yards	220	*Furlongs.*
Furlongs	8	*English Miles.*

ALSO,

	Div. by	give
Nails	4	*Quarters of Yards.*
Quarters	4	*Yards.*
Quarters	5	*Ells English.*

7. *Of* SUPERFICIAL MEASURES *of* LAND.

	Div. by	give
Perches, or *Poles*	40	*Roods*, or *Quarters of Acres.*
Roods	4	*Acres.*

8. *Of* TIME.

	Div. by	give
Minutes	60	*Hours.*
Hours	24	*Days.*
Days	7	*Weeks.*

Note, That if after Division is finished, in Reduction ascending, there be any remainder, it is of the same denomination with the Dividend.

Note also, That Reduction descending and ascending mutually prove one another, by inverting the question; for as in 56 pounds *Sterling*, there will be found 53760 farthings by Reduction descending: So for proof thereof 53760 farthings will be reduced to 56 pounds, by Reduction ascending.

124. Queſtions to exercise Reduction.

1. In 257 *l.* how many ſhillings? *Anſw.* 5140.

2. In 3076 *l.* how many ſhillings? *Anſw.* 61520.

3. In 902 *s.* how many pence? *Anſw.* 10824.

4. In 2179 *s.* how many farthings? *Anſw.* 104592.

5. In 49 *l.* 13 *s.* 7 d. how many pence? *Anſw.* 11923.

6. In 2053 *l.* 14 *s.* 9 *d.* 2 *f.* how many farthings? *Anſw.* 1971590.

7. In 354 *lb.* of *Troy*-weight, how many grains of Goldſmith's weight? *Anſw.* 2039040.

8. In 300 *Engliſh* miles, how many yards? *Anſw.* 528000.

9. In 1 *Engliſh* mile, how many barley corns length? *Anſw.* 190080.

10. In 560 acres, how many perches? *Anſw.* 89600.

11. In 225 acres, 3 roods, and 30 perches, how many perches? *Anſw.* 36150.

12. In 11923 pence, how many pounds? *Anſw.* 49 *l.* 13 *s.* 7 *d.*

13. In 5764684 farthings, how many pounds? *Anſw.* 6004 *l.* 17 *s.* 7 *d.*

14. In 234678 perches, how many acres? *Anſw.* 1466 acres, 2 roods, 38 perches.

15. In 525960 minutes of an hour, how many days? *Anſw.* 365 days and 6 hours, (or one year very near.)

16. In 10080 pints, how many hogſheads? *Anſw.* 20.

17. In 34678 grains of Apothecaries weight, how many ounces *Troy*? *Anſw.* 72 ounces, 1 dram, 2 ſcruples, and 18 grains.

18. In 106735 pints of wheat, how many quarters? *Anſw.* 208 quarters, 3 buſhels, 2 pecks, 1 gallon, 1 pottle, 1 quart, 1 pint.

19. In 3969301 barley-corns length, how many miles? *Anſw.* 20 miles, 7 furlongs, 12 yards, 2 feet, 4 inches, and 1 barley-corn's length.

20. In 1900800 barley-corns length, how many miles? *Anſw.* 10.

21. In 24 *C.* weight 3 quarters, 26 *lb.* how many pounds Avoirdupoiſe? *Anſw.* 2798.

Note, Hundred weights, Quarters and Pounds, may be ſpeedily reduced into pounds, thus: Set down the Hundred weights four ſeveral times in the form following, and take in the odd weight.

C. qrs.
24 . 3 . 26
24
24
24
110 odd weight.

Anſw. 2798 pounds, as above.

22. In 12 crowns, how many ſhillings and pence? *Anſw.* 60 *s.* and 720 *d.*

23. In 50 half-crowns, how many pence and farthings? *Anſw.* 1500 *d.* and 6000 *f.*

24. In 306 crowns, how many half-crowns and pence? *Anſw.* 612 half-crowns, and 18360 pence.

25. How many ſhillings and pence are in 17 guineas? *Anſw.* 357 ſhillings, and 4284 pence.

26. How many crowns and ſix-pences are in 28 *l.*? *Anſw.* 112 crowns, and 1120 ſix-pences.

27. In 6000 farthings, how many pence and half-crowns? *Anſw.* 1500 pence, and 50 half-crowns.

28. In 18360 pence, how many half-crowns and crowns? *Anſw.* 612 half crowns, and 306 crowns.

29. How many ſhillings and guineas are in 4284 pence? *Anſw.* 357 ſhillings and 17 guineas.

30. How many crowns and pounds are in 1120 ſix-pences? *Anſw.* 112 crowns, and 28 pounds.

31. In 30 chalders of coals, each 36 buſhels, how many pecks? *Anſw.* 4320.

32. If a piece of ground contains 24 acres, and an incloſure (of 17 acres 3 roods) be taken out of it, how many perches are there in the remainder? *Anſw.* 1000.

33. I have a bank note of 20 *l.* a note of hand for 6 *l.* 10 *s.* and, in ſeveral coins, as follow: In copper, 13 farthings, and 45 half-pence; in ſilver, 25 two-pences, 36 three-pences, 56 groats, 96 ſix-pences, 67 ſhillings, 97 half-crowns, 126 crowns; in gold, 25 quarter-guineas, 65 half-guineas, 77 guineas, and 34 moidores: I would know what I am worth? *Anſw.* 245 *l.* 0 *s.* $2\frac{3}{4}$ *d.*

F CHAP.

CHAP. VIII.

Of the Rule *of* Three Direct.

125. THE *Rule of Three* is ſo called, becauſe by three numbers known or given, it teaches to find a fourth unknown; it is alſo called the *Golden Rule*, for the excellency thereof: Laſtly, it is called the *Rule of Proportion*, for the reaſon hereafter declared.

126. The *Rule of Three* is either ſingle or Compound.

127. The Single Rule is, when three terms or numbers are propoſed, and a fourth proportional to them is demanded.

128. Four numbers are ſaid to be proportionals, when the firſt contains the ſecond, or is contained by the ſecond, in the ſame manner as the third contains the fourth, or is contained by the fourth. So thoſe four numbers are ſaid to be proportionals, 8, 4, 12, 6; for as 8 contains 4 twice, ſo does 12 contain 6 twice, and therefore 8 is ſaid to have ſuch proportion to 4, as 12 has to 6; likewiſe theſe are proportionals, 4, 8, 6, 12. For as 4 is the half of 8, ſo is 6 the half of 12; and therefore 4 is ſaid to have ſuch proportion to 8, as 6 has to 12.

129. The terms or numbers of the *Rule of Three* (to wit the three numbers given, and the fourth ſought) conſiſt of two different denominations, *viz.* two of the three given terms have one name, and the other given term with the term required have another: So this queſtion being demanded, if four ſtudents ſpend 19 pounds in certain months, how much money will ſerve 8 ſtudents for the ſame time, and at the ſame rate of expence? Here ſtudents and pounds are the two denominations of the terms in the queſtion, *viz.* 4 and 8 (being two of the terms propoſed) have the denomination of ſtudents, and 19 the other term given, together with the term required, have the denomination of pounds.

130. In the *Rule of Three*, two of the three given terms imply a ſuppoſition, and the third moves a queſtion: So in the afore-mentioned queſtion a ſuppoſition is made, that 4 ſtudents ſpend 19 pounds, and a queſtion is moved with the number 8, to wit, how many pounds will 8 ſtudents ſpend?

131. In the *Rule of Three*, the numbers given muſt be ſo ranked, that the known number or term upon which the queſtion is aſked, may poſſeſs the third place; alſo of the other two, that which has the ſame denomination with the third, muſt be

in the firſt place: Laſtly, the other known term, which is of the ſame denomination with the fourth term ſought, (or anſwer of the queſtion) muſt poſſeſs the ſecond place: So in the queſtion before-mentioned, the terms 4, 19, and 8, are to be thus placed, *viz.* 8 is the term upon which the queſtion is moved, and therefore to poſſeſs the third place in the rule; 4 is of the ſame denomination with 8, *viz.* of ſtudents, and therefore to be in the firſt place: Laſtly, 19 being of the ſame denomination with the term ſought for, *viz.* of money, is to be in the ſecond place: And ſo they will be placed thus:

Students. Pounds. Students.
If 4 : 19 :: 8

That is to ſay, if 4 ſtudents ſpend 19 pounds, what will 8 ſtudents ſpend? And here, for the better diſcerning of the term or number upon which the queſtion is aſked, you may obſerve, that for the moſt part it is the known number in the queſtion, which immediately follow theſe or ſuch like words, *viz. How many? how much? what will? how long? how far,* &c.

132. The *Rule of Three* is either *Direct* or *Inverſe*.

133. The *Rule of Three Direct* is, when the ſenſe or tenor of the queſtion requires that the fourth number ſought ſhould have ſuch proportion to the ſecond, as the third number has to the firſt; ſo in the aforementioned queſtion, if 4 ſtudents ſpend 19 pounds, how many pounds will 8 ſtudents ſpend at the ſame rate of expence? It is evident, that the thing required is to find a number which may have ſuch proportion to 19, as 8 has to 4; that is, as 8 is the double of 4, ſo ought the fourth number to be the double of 19; for if 19 pounds be required to maintain 4 ſtudents a certain time, as much more muſt needs be required for the maintenance of 8 ſtudents the ſame time; and therefore in this caſe, we may ſay in a direct proportion, as 4 is to 8, ſo is 19 to the number required.

134. In the *Direct Rule of Three,* if you multiply the ſecond term by the third, (or which is all one) the third term by the ſecond, and then divide the product by the firſt, the Quotient will give the fourth term or fourth proportional required. So in the queſtion before propoſed, if you multiply 19 by 8 the product is 152, which, if you divide by 4, the Quotient will give you 38, the fourth term demanded, and the Work will ſtand thus:

```
 Stud.  l.   Stud.
If 4 : 19 :: 8
           8
         ----
     4)152(38 Pounds.
       12
       ---
        32
        32
        --
         0
```

A ſecond Example may be this, if 8 yards coſt 9 pounds, how much will 3 yards coſt? *Anſw.* 3 *l.* 7 *s.* 6 *d.*

This queſtion being ſtated according to *Art.* 131. will ſtand as here you ſee; then multiplying (as before) the ſecond term 9 by the third term 3, the product is 27, which being divided by the firſt term 8, the Quotient is 3 pounds, and there is a remainder of three pounds, which muſt be reduced into ſixty ſhillings, and after thoſe ſhillings are divided by 8, and the reſt of the Work proſecuted according to *Art.* 114. the entire Quotient or anſwer of the queſtion is 3 *l.* 7 *s.* 6 *d.*

```
     y      l.      y.
If 8  :   9   ::   3
          3
       --------
       8) 27
       -------------
          3 . 7 . 6
```

A third Example, if 51 ounces of ſilver plate be ſold for 13 pounds *Sterling*, what is the price of 1 ounce of that plate? *Anſw.* 5 *s.* 1 *d.* and ſomewhat more. The operation is thus: After the three known terms of this queſtion are rightly ordered, they will ſtand as here you ſee in the Example; then multiplying the ſecond term 13 by the third term 1, the product will be alſo 13, (for Multiplication by 1 makes no alteration;) which 13 being divided by 51, after the manner of operation delivered in the Note upon *Art.* 119. the entire Quotient or anſwer of the queſtion will at length be found to be 5 *s.* 1 *d.* and ſomewhat more; but the ſurpluſage being leſs than a farthing, is omitted as uſeleſs.

```
   oz.     l.     oz.
If 51  :  13 ::  1
           1
       -----------
          13
          20
       ------
 51) 260(5 Shillings
     255
     -----
       5
      12
    --------
 51)  60 (1 Pence.
      51
     -----
       9
```

Example. 4. What muſt be paid to a Labourer for his wages for 27 weeks, at the rate of 4 *s.* for 1 week? *Anſw.* 5 *l.* 8 *s.*

After the three given terms are rightly placed they will ſtand as you ſee in the Example; then multiplying the third term 27 by the ſecond term 4, the product is 108, which divide by the firſt term 1; but ſince Diviſion by 1 makes no alteration, the Quotient is alſo 108, ſo that the fourth term ſought is 108 Shillings, which being reduced to pounds, according to *Art.* 122. gives 5 *l.* 8 *s.* for the anſwer of the queſtion.

```
Weeks. Shill. Weeks.
  1   :  4   ::  27
                  4
                ----
                 108
```

135.

135. In the *Rule of Three*, if after the queſtion is ſtated according to *Art.* 131. any of the three given terms be a compound term conſiſting of diverſe denominations; as pounds, ſhillings and pence; or weeks, days, hours, &c. ſuch compound term muſt firſt be reduced into the loweſt of thoſe denominations (by *Art.* 121.) to the end that the three given terms may be three ſingle numbers; alſo of theſe three ſingle numbers, the firſt and third muſt always be of one and the ſame denomination: For if it happen that they expreſs things of different names, that of the two which has the greater name (or denomination) is to be reduced into the ſame name with the leſſer (by *Art.* 119) Theſe preparations being obſerved, the reſt of the Work is to be proſecuted according to *Art.* 134.

Examp. What will 48 ounces 17 penny-weights and 20 grains of ſilver-plate amount to at the rate of 5*s.* 6*d.* the ounce? *Anſw.* 13*l.* 8*s.* 10*d.* 3*f.* very near.

This queſtion being ſtated according to *Art.* 131. will ſtand as you ſee in the *Example*, to wit, if 1 ounce coſt 5*s.* 6*d.* what will 48 *oz.* 17 *p. wt.* 20 *gr.* coſt? Here, becauſe the third term is compounded of diverſe denominations, it muſt be reduced into the loweſt of thoſe denominations, to wit, grains; ſo, by *Art.* 121. there will be found 23468 grains for the third term: Likewiſe becauſe the ſecond term 5*s.* 6*d.* is a compound term, whoſe loweſt name is pence, it muſt be reduced into pence (by the aforeſaid Rule:) So there will be found 66 pence for the ſecond term: Again becauſe the firſt term has the name ounce, and the third term the name grain, the firſt term 1 ounce muſt be converted into 480 grains (which are equal to 1 ounce) then will the three terms or ſingle numbers ſtand as here you ſee, *viz.* If 480 grains coſt 66 pence, how many pence will 23468 grains coſt? Now proceeding according to *Art.* 134. there will

```
oz.    s.   d.        oz.  p.w.  gr.
 1  .  5  . 6   : :  48  . 17  . 20
20    12  .          20
——    ——            ———
20    66  .         977
24        .          24
——                 ————
480                3928
                   1954
                   ————
                  23468 Grains.
```

```
gr.    pence.     gr.
480  :  66  : :  23468
```

ariſe in the Quotient 3226 pence, beſides a remainder of 408 pence, which being reduced to 1632 farthings, and thoſe divided by the firſt term 480, the Quotient will be 3 farthings: So that the entire Quotient is 3226 pence, 3 farthings, and ſomewhat more; (but the parts of a farthing being of no moment, may be neglected. Laftly, the faid 3226 pence being reduced according to *Art.* 122. give 13*l.* 8 *s.* 10 *d.* 3 *f.* fo that 13*l.* 18 *s.* 10 *d.* 3 *f.* and fomewhat more, will be the anfwer of the queftion.

136. For the proof of the *Direct Rule of Three*, multiply the fourth term by the firft; which done, if that product be equal to the product of the fecond term multiplied by the third, the Work is right, otherwife it is erroneous: So in the firft Example, 38 the fourth term being multiplied by the firft term 4, the product is 152, which is alfo the product of 19 multiplied by 8. But if it happen that after the fourth term, or anfwer of the queftion, is found in the fame denomination with the fecond term, there is yet a remainder, fuch remainder muft be added to the product of the firft term, multiplied by fuch fourth term, and then the fum muft be equal to the product of the fecond and third terms, (the fecond term confifting of the fame denomination with the fourth:) So in the laft Example the fourth term is 3226, and there happens to be a remainder of 408, which being added to the product of the Multiplication of the faid 3226 by the firft term 480, gives 1548888, which is the fame with the product of the third term 23468 multiplied by the fecond term 66, as will appear when work'd.

137. Queftions to exercife the *Rule of Three Direct.*

1. If 17 yards of cloth coft 19*l.* 2 *s.* 6 *d.* what will 35 yards coft at that rate? *Anfw.* 39 *l.* 7 *s.* 6 *d.*

2. If 35 yards coft 39*l.* 7 *s.* 6*d.* how many yards may be bought at that rate for 19*l.* 2*s.* 6*d.*? *Anfw.* 17 yards.

3. If 35 yards coft 39 *l.* 7 *s.* 6 *d.* what are 17 yards worth at that rate? *Anfw.* 19 *l.* 2 *s.* 6 *d.*

4. If 17 yards be fold for 19 *l.* 2 *s.* 6 *d.* how many yards will 39*l.* 7*s.* 6*d.* buy at that rate? *Anfw.* 35 yards.

5. What muft I pay for the carriage of 17 hundred weight, 3 quarters, and 11 pound *Avoirdupoife*, at the rate of 7 fhillings the hundred weight? *Anfw.* 6*l.* 4*s.* 11*d.* 1*f.*

6. If 6*l.* 4*s.* 11*d.* 1*f.* be paid for the carriage of 17 hundred weight, 3 quarters and 11 pounds, what was paid for the carriage of 1 pound weight? *Anfw.* 3 farthings.

7. What muft I pay for 39 ounces, 7 penny-weights, and 18 grains of white plate, at the rate of 5 *s.* and 5 *d.* the ounce? *Anfw.* 10*l.* 13*s.* 4*d.* and 3 quarters of a farthing.

8. What muft 1*l.* (or 20*s.*) pay towards a tax, when 326 *l.* 6 *s.* 8 *d.* is affeffed at 41 *l.* 16 *s.* 2 *d.* 3*f*? *Anfw.* 2 *s.* 6 *d.* 3 *f.*

9. What will the interest of 876 *l.* 17 *s.* 6 *d.* amount to for 1 year, at the rate of 6 *l.* for 100 *l.* for the same time? *Answ.* 52 *l.* 12 *s.* 3 *d.*

10. If 3 yards in length of *English* Measure be equal to 4 ells *Flemish*, how many *Flemish* ells are contained in 120 yards *English*? *Answ.* 160 *Flemish* ells.

11. If 4 *Flemish* ells in length be equal to 3 *English* yards, how many *English* yards in 300 *Flemish* ells? *Answ.* 225 *English* yards.

12. If 3 ells in length of *English* measure be equal to 5 *Flemish* ells, how many *Flemish* ells in 120 *English* ells? *Answ.* 200 *Flemish* ells.

13. If 5 *Flemish* ells in length be equal to 3 *English* ells, how many *English* ells in 145 *Flemish* ells? *Answ.* 87 *English* ells.

14. If 3 ounces of silk-weight be equal to 4 ounces of *Venice* weight, how many ounces *Venice* are equal to 60 ounces of silk-weight? *Answ.* 80 ounces *Venice.*

15. A merchant delivered at *London* 120 *l.* *Sterling* to receive 207 *l.* *Flemish* at *Amsterdam*, what was 1 *l.* *Sterling* valued at in *Flemish* money? *Answ.* 1 *l.* 14 *s.* 6 *d.*

16. If a bill of exchange be accepted at *London*, for payment of 400 *l.* *Sterling* for the value delivered at *Amsterdam*, at 1 *l.* 13 *s.* 6 *d.* *Flemish* for 1 *l.* *Sterling*, how much *Flemish* money was delivered at *Amsterdam*? *Answ.* 670 *l.* *Flemish.*

17. When the exchange from *Antwerp* to *London* is at 1 *l.* 4 *s.* 7 *d.* *Flemish* for 1 *l.* *Sterling*, how much *Sterling* must I pay at *London* to receive 236 *l.* *Flemish* at *Antwerp*? *Answ.* 192 *l.* *Sterling.*

18. A merchant delivered at *London* 370 *l.* *Sterling* by exchange for *Roan*, at 74 *d.* *Sterling* for 50 *s.* *Tournois*; how much *Tournois* ought he to receive at *Roan*? *Answ.* 60000 *s.* *Tournois.*

19. In 370 ducats, at 4 *s.* 2 *d.* the ducat, how many *French* crowns at 6 *s.* 2 *d.*? *Answ.* 250 crowns. For if 74 *d.* give 1 crown, 18500 *d.* (or 370 ducats) will give 250 crowns.

20. In 516 dollars, at 4 *s.* 5 *d.* the dollar, how many guineas at 1 *l.* 1 *s.* 6 *d.* the piece? *Answ.* 106 guineas. For if 258 *d.* give one guinea, 27348 *d.* (or 516 dollars) will give 106 guineas.

21 At a noble *per* week, how many months board may I have for 50 *l.*? *Answ.* 37 months, 2 weeks.

22. Bought a firkin of butter containing 56 *lb.* for 18 *s.* 8 *d.* what is that *per* pound? *Answ.* 4 *d.*

23. If a yard of Cambrick coſt 12 *s.* what coſt 4 pieces, each 20 yards? *Anſw.* 48 *l.*

24. If a gallon of beer coſt 4 *d.* what coſts a barrel? *Anſw.* 12 *s.*

25. How much muſt I pay for the carriage of 10 $\frac{1}{2}$ *C.* wt. at the rate of 1 *d. per lb?* *Anſw.* 4 *l.* 18 *s.*

26. The cloathing of a regiment of 740 men comes to 3000*l.* how much is that for each man? *Anſw.* 4*l.* 1*s.* 0*d.* 3*f.*

27. If a buſhel of coals coſt 10 *d.* how many chalders for 100 *l.*? *Anſw.* 66 chalders, and 24 buſhels.

28. If a man's yearly income be 300*l.* what is it *per* day? *Anſw.* 16*s.* 5*d.* $1\frac{15}{365}$*f.*

29. What coſt 49392 caſe-knives at 4*s.* 4*d. per* dozen? *Anſw.* 891*l.* 16*s.*

30. At $3\frac{1}{2}$ *d. per lb.* what comes 30 *C.* weight 3 quarters 25 *lb.* of cheeſe to? *Anſw.* 50*l.* 11*s.* $9\frac{1}{2}$*d.*

31. What is cheeſe *per C.* weight at $3\frac{1}{2}$*d. per lb.*? *Anſw.* 1*l.* 12*s.* 8*d.*

32. A man bought a piece of cloth for 16 *l.* 10 *s.* at 15*s. per* yard, how many yards did it contain? *Anſw.* 22 yards.

33. A draper bought 4 bales of cloth, each bale containing 6 pieces, and each piece 27 yards, at 16 *l.* 4 *s.* per piece, what was the price of the whole, and what the rate *per* yard? The whole coſt 388 *l.* 16 *s.* and one yard coſt 12*s.*

34. If 1 *C.* weight of cheeſe coſt 37*s.* 4*d.* what is that *per lb.*? *Anſw.* 4*d.*

35. If a yard of cloth is worth 14*s.* what is the worth of 5 pieces, each 19 yards? *Anſw.* 66 *l.* 10 *s.*

36. Bought 12 pieces of cloth, each 12 yards, at 10 *s.* 6 *d. per* yard, what come they to? *Anſwer.* 75*l.* 12*s.*

37. A debtor owing ſeveral perſons in all 1490 *l.* 5 *s.* 10 *d.* compounds with, and pays them as far as his effects will go, which amount to no more than 931 *l.* 8 *s.* $7\frac{3}{4}$ *d.* how much do the creditors by this compoſition receive *per* pound? *Anſw.* 12*s.* 6*d.*

38. If 2 *oz.* of ſilk coſt 2 *s.* 6 *d.* what coſt 7 *lb?* *Anſw.* 7 *l.*

39. If 1 *oz.* of ſilver coſt 5 *s.* 6 *d.* what is the price of a tankard that weighs 1 *lb.* 10 *oz.* 10 *penny-weights*, 4 *grains?* *Anſw.* 6*l.* 3*s.* 9*d.* $2\frac{36}{486}$*f.*

40. If an ingot of ſilver weighs 36 *oz.* 10 *p. wt.* what is it worth at 5 *s. per* ounce? *Anſw.* 9 *l.* 2 *s.* 6 *d.*

41. What will an eſtate of 4000 *l. per annum* allow a gentleman to ſpend *per* day? *Anſw.* 10*l.* 19*s.* $2\frac{10}{73}$ *d.*

42. If

42. If a gentleman has an eſtate of 245 *l.* 10 *s.* a year how, much may he ſpend one day with another, to lay up 60 guineas at the year's end? *Anſw.* 10 *s.*

43. A goldſmith ſold a tankard for 10 *l.* 12 *s.* at the rate of 5 *s.* 4 *d. per oz.* what was the weight of it? *Anſw.* 39 *oz.* 15 *p. wts.*

44. If 1 *lb.* of tobacco coſts 15 *d.* what coſt 3 hogſheads weighing (together) 15 *C.* wt. 1 quarter, 19 *lb.*? *Anſw.* 107 *l.* 18 *s.* 9 *d.*

45. If a pint of wine coſt 10 *d.* what coſts 3 hogſheads? *Anſw.* 63 *l.*

46. If 17 *C.* 3 quarters, 17 *lb.* of tobacco coſt 133 *l.* 13 *s.* 4 *d.* what coſts 1 *oz.*? *Anſw.* 1 *d.*

47. If 1 *C.* wt. of lead coſts 15 *s.* 11 *d.* what coſt 5 fothers? *Anſw.* 77 *l.* 11 *s.* $10\frac{1}{2}$ *d.*

48. If 19 dozen pair of ſhoes coſt 25 *l.* 13 *s.* what coſts one pair? *Anſw.* 2 *s.* 3 *d.*

49. If the expences in houſe-keeping ſix weeks amount to 9 *l.* 3 *s.* 6 *d.* how long will 100 *l.* laſt at that rate? *Anſw.* $65\frac{145}{367}$ weeks.

50. If a dozen ells of Holland are valued at 3 *l.* 6 *s.* how much will 8 pieces, (each piece containing 54 ells) amount to at the ſame rate? *Anſw.* 118 *l.* 16 *s.*

C H A P. IX.

Of the INVERSE RULE *of* THREE.

138. THE *Rule of Three Inverſe*, is, when the fourth term required ought to proceed from the ſecond term, according to the ſame rule or proportion that the firſt proceeds from the third: So this queſtion being propounded; if 8 horſes will be maintained 12 days with a certain quantity of provender, how many days will the ſame quantity maintain 16 horſes? Here, as 8 is half 16, ſo ought the fourth term required to be half 12; for if certain buſhels of provender ſerve 8 horſes 12 days, 16 horſes will eat as much provender in half that time: And therefore you cannot ſay here in a direct proportion (as before in the *Rule of Three Direct*) as 8 to 16, ſo is 12 to another number which ought to be in that caſe as great again as 12; but contrarywiſe by an

Horſes.	*Days.*	*Horſes.*
8 .	12 .	16

inverted

inverted proportion, beginning with the laſt term firſt; as 16 is to 8, ſo is 12 to another number, which ought to be in this caſe half 12. And by the due obſervation of this definition together with that of the *Rule of Three Direct* (laid down in *Art.* 133.) when any queſtion belonging to the ſingle *Rule of Three* is propoſed, you may readily diſcern by which of thoſe Rules it ought to be reſolved; for if the three terms given look for a fourth in a direct proportion as they ſtand ranked in the Rule, you muſt reſolve the queſtion by the direct Rule; contrarywiſe when the proportion is inverted or turned backwards, it ought to be reſolved by the *Inverſe Rule of Three*, which here follows.

139. In the *Inverſe Rule of Three*, after the three given terms are rightly placed and reduced (if there be need) according to *Art.* 135. multiply the firſt term by the ſecond, (or which is the ſame) the ſecond term by the firſt, and then divide the product by the third term, ſo the Quotient will give you the fourth term required, or anſwer of the queſtion; thus in the queſtion premiſed in the laſt *Article*, if you multiply 12 by 8, the product is 96, which if you divide by 16, the Quotient gives you 6, the fourth term required, as by the ſubſequent operation is manifeſt.

Horſes.	*Days.*	*Hrſes.*
8 :	12 ::	16
	8	
	16) 96 (6	
	96	
	0	

140. For the more ready diſcovering, whether a queſtion propounded belongs to the *Rule of Three Direct*, or to the *Rule Inverſe*, obſerve the following directions, *viz.* Firſt, by the ſenſe and tenor of the queſtion conſider, whether more be required or leſs; that is, whether the number ſought for, muſt be greater or leſs than the ſecond term: Secondly, eſteeming the firſt and third terms as extremes in reſpect of the ſecond, this will be a general Rule; namely, when more is required, the leſſer extreme is the Diviſor; but when leſs is required, the greater extreme is the Diviſor. Laſtly, the Diviſor being found out, it will be apparent whether the Rule be Direct or Inverſe; for when the Diviſor is the firſt term, it is a *Rule Direct*; but when the Diviſor is the third term, the Rule is *In-*

verse. Another Example of the *Rule Inverse* may be thus: If 12 Mowers do mow certain acres in 4 days, in what time will 23 Mowers perform the same work? *Answ.* 2 days 2 hours and somewhat more. Here, the three known terms being rightly placed, will stand, as you see in the Example; and since it is evident that 23 men will require less time than 12 men to finish the same work, therefore (by the Rule afore-going) the greater of the two extreme numbers 23 and 12 must be the Divisor; and because the Divisor 23 stands in the third place, this question is to be work'd by the *Rule Inverse*; wherefore multiplying the first term 12 by the second term 4, the product is 48, which being divided by the third term 23, the Quotient gives 2 days, and there is a remainder of 2 days, which being reduced to hours, and those divided by 23, the Quotient will be 2 hours; and there is yet a remainder of 2 hours to be subdivided into 23 parts if you please; so that the fourth term sought, or answer of the question, is, 2 days, 2 hours, and somewhat more.

```
     M.  D.  M.
If  12 : 4 :: 23
     4
-----------------
23) 48  (2 Days.
    46
    --
     2
    24
-----------------
23) 48  (2 Hours.
    46
    --
     2
```

Again, take this for a third Example: If I lend my friend 356 pounds for 1 year and 35 days, (the year being supposed to consist of 365 days) how long time ought he to lend me 500 pounds to requite my courtesy? *Answ.* 284 days, and somewhat more, there being a remainder, to wit, 400, after the Division is finished, as by the subsequent operation is manifest.

```
     l.       y.  d.    l.
If  356  :    1 . 35 :: 500
          -------
              365
        add    35
          -------
multiply  { 400
          { 356
          -------
5|00)1424|00(284 Days.
```

141. The proof of the *Inverse Rule of Three* is this, multiply the third term by the fourth, then if this product be equal to the product of the first term multiplied by the second, the Work is true, otherwise erroneous; so in the *Example* of

Art.

Art. 139. the product of 16 and 6 is equal to the product of 8 and 12. But if it happen that, after the fourth term, or anſwer of the queſtion, is found in the ſame denomination with the ſecond term, there is yet a remainder, ſuch remainder muſt be added to the product of the third term, multiplied by the fourth, and then the ſum muſt be equal to the product of the firſt and ſecond terms (ſuch ſecond term being of the ſame particular denomination with the fourth:) So in the laſt *Example*, the fourth term is 284 days, and there remain 400 after the diviſion is finiſh'd, this 400 being added to the product of the Multiplication of the third term 500 by the fourth term 284, gives 142400, which is equal to the product of the firſt term 356, multiplied by the ſecond term 400 days.

142. In the *Rule of Three*, as well *Direct* as *Inverſe*, when the Diviſor with either of the other two given numbers may be ſeverally divided by ſome common meaſure, without leaving any remainder, the quotients may be taken for new terms, and proceeding in like manner as often as poſſible, the operation will be much contracted: So if it be demanded, what 52 yards of cloth will coſt at the rate of 21*l.* for 14 yards; the anſwer will be found 78 pounds in manner following.

	y.	*l.*	*y.*	
If	14 .	21 .	52	
	2 .	3 .	52	
	1 .	3 .	26	. (78

In the firſt rank you may obſerve that the Diviſor 14 and the ſecond term 21, being ſeverally divided by their common meaſure 7, the three new terms (in the ſecond rank) will be 2, 3, 52. Again, in the ſecond rank, the Diviſor 2 and the third term 52 being ſeverally divided by their common meaſure 2, the three new terms (in the third rank) will be 1, 3, 26. Laſtly, working with theſe according to the *Rule of Three Direct*; the anſwer to the queſtion, (or fourth term) will appear to be 78.

Another Example. If 21 men can finiſh a work in 16 days, what time muſt be allowed to 12 men for the finiſhing of ſuch a work? *Anſw.* 28 days.

Men.	*Days.*	*Men.*
21 .	16 .	12 (28 Days.
7 .	16 .	4
7 .	4 .	1

In the firſt rank you may obſerve, that the Diviſor 12 (for the *Rule is Inverſe*,) and the firſt term 21 being ſeverally divided

vided by their common meaſure 3, the three new terms in the ſecond rank will be 7, 16, 4. Again, in the ſecond rank, the Diviſor 4, and the ſecond term 16 being ſeverally divided by their common meaſure 4, the three new terms in the third rank will be 7, 4, 1. Laſtly, working with theſe as the *Rule of Three Inverſe* requires, the anſwer to the queſtion (or fourth term) will be found 28.

143. Queſtions to exerciſe the *Rule of Three Inverſe.*

1. There was a certain Building raiſed in 8 months, by 120 workmen; but the ſame being demoliſhed, it is required to be rebuilt in 2 months: How many men muſt be employed about it? *Anſw.* 480 men.

2. If 28 *s.* will pay for the carriage of an hundred weight 150 miles, how far may 6*C.* wt. be carried for the ſame money? *Anſw.* 25 miles.

3. If for 5 *l.* 5 *s.* I have 14 *C.* wt. carried 136 miles, how many miles may I have 24 *C.* wt. carried for the ſame money? *Anſw.* $79\frac{1}{3}$ miles.

4. If a footman performs a journey in 3 days, when the days are 16 hours long, how many days will he require of 12 hours long to go the ſame journey in? *Anſw.* 4 days.

5. How many yards of pluſh is ſufficient to make a cloak of equal magnitude with one which hath in it 4 yards of 7 quarters wide, when the pluſh is but 3 quarters wide? *Anſw.* $9\frac{1}{3}$ yards of pluſh.

6. How many yards of canvas, that is ell-wide, will be ſufficient to line 20 yards of Say, that is three quarters wide? *Anſw.* 12 yards.

7. If a man performs a journey in 6 days, when the day is 8 hours long? in what time will he do it, when the day is 12 hours long? *Anſw.* 4 days.

8. If I lent my friend 100 *l.* for 6 months (allowing the month to be 30 days) how long ought he to lend me 1000 *l.* to requite my kindneſs? *Anſw.* 18 days.

9. If 6 mowers can mow a field in 12 days, in what time will 24 mowers do it? *Anſw.* 3 days.

10. Suppoſe 800 ſoldiers were placed in a garriſon, and their proviſions were computed ſufficient for 2 months, how many ſoldiers muſt depart, that the proviſions may ſerve them 5 months? *Anſw.* 480 men.

11. Admit that I lent to a friend on his occaſion 100 *l.* for 6 months, and he promiſed me the like kindneſs when I deſired it; but

but when I came to requeſt it, he could lend me only 75 *l.* the queſtion is, how long I may keep his money, to recompence my courteſy to him? *Anſw.* 8 months.

CHAP. X.

The Double Golden Rule Direct, performed by Two ſingle Rules.

144. THE Compound *Golden Rule* is, when more than 3 terms are propoſed; therefore, under the compound *Golden Rule* is comprehended the *Double Golden Rule*, and divers Rules of plural proportion.

145. The *Double Golden Rule* is, when five terms being given, a ſixth proportional to them is demanded: As in this queſtion, if 4 ſtudents ſpend 19 pounds in 3 months, how much will ſerve 8 ſtudents 9 months? Or this, if 9 buſhels of provender ſerve 8 horſes 12 days, how many days will 24 buſhels laſt 16 horſes?

146. The five terms given in this Rule conſiſt of two parts, *viz.* a ſuppoſition expreſſed in the three firſt terms; and a demand made in the two laſt: So in the firſt Example, this clauſe (if 4 ſtudents ſpend 19 pounds in 3 months) is the ſuppoſition, and this (how much will ſerve 8 ſtudents 9 months,) is the Demand? Likewiſe, in the other Example, this clauſe (if 9 buſhels of provender ſerve 8 horſes 12 days) is the ſuppoſition, and this (how long, or how many days will 24 buſhels laſt 16 horſes,) is the propoſed demand?

147. Here for ranking the terms given, in their due order, firſt obſerve amongſt the terms of ſuppoſition, which of them has the ſame denomination with the term required: Then reſerving that term for the ſecond place, write the other two terms of ſuppoſition one above another in the firſt place; and laſtly, the terms of demand likewiſe one above another in the third place of the Rule, in ſuch ſort that the uppermoſt may have the ſame denomination with the uppermoſt of thoſe in the firſt place. *Example,* If 4 ſtudents ſpend 19 pounds in 3 months, how much will ſerve 8 ſtudents 9 months? Here the three terms of ſuppoſition are 4, 19 and 3, and of theſe terms 19 has the ſame denomination with the term required, *viz.* of pounds;

pounds; for you are to enquire how much money is requiſite for the maintenance of 8 ſtudents 9 months: Wherefore reſerving 19 for the ſecond place, write 4 and 3 one above another, thus; then on the right-hand of 4, write 19 in the ſecond place; this done, the work will ſtand as in the Margin: Laſt of all, the terms of demand being 8 and, 9, and 8 having the denomination of ſtudents, place it in the ſame line with 4 and 19, and write 9 under it; all this performed, the terms in this queſtion rank themſelves as follow, *viz.* Thus,

4 . 19
3

4 . 19 . 8
3 9

Or thus,

3 . 19 . 9
4 8

In like manner, ff the ſecond queſtion, in *Art.* 145. were propounded; the terms thereof ought to be diſpoſed thus,

8 . 12 . 16
9 24

Or thus,

9 . 12 . 24
8 16

148. Queſtions belonging to the *Double Golden Rule* may be reſolved by two ſingle Rules of Three.

149. When Queſtions of this nature are reſolved by two ſingle Rules, the Proportions are as follow:

1. As the uppermoſt term of the firſt place, is to the middle term; ſo is the uppermoſt term of the laſt place to a fourth number.
2. As the lower term of the firſt place, is to that fourth number; ſo is the lower term of the laſt place to the term required.

So in this Example before recited uſing tacitly the lower term of the firſt place as a common number in the firſt proportion: Say thus,

4 . 19 . 8
3 9

1. If 4 ſtudents ſpend 19 pounds (in 3 months) what will ſerve 8 ſtudents the ſame time?

Or thus, If 4 ſtudents ſpend 19 pounds, what will 8 ſpend?

Which *Rule of Three* will be diſcovered to be direct (by *Art.* 140.) therefore the fourth proportional proceeding from the

the ſaid three given numbers, 4, 19, and 8 is 38, (by *Art.* 134.) Again, to find the term required, uſing tacitly the uppermoſt term of the third place as a common number in this laſt proportion; ſay as follows:

2. If in 3 months 38 pounds are ſpent (by 8 ſtudents) how much will ſerve them for 9 months?

Or thus, if 3 give 38, what will 9 yield?

Which *Rule of Three* will likewiſe be diſcovered to be direct (by *Art.* 140.) therefore the fourth proportional proceeding from the ſaid 3 numbers, 3, 38, and 9, you will likewiſe find (by *Art.* 134.) to be 114; for 38 being multiplied by 9 the product is 342, which divided by 3, yields in the quotient 114: So that if 4 ſtudents ſpend 19 pounds in 3 months, 114 pounds will ſerve 8 ſtudents 9 months.

150. In like manner, if two ſingle Rules of Three be formed (according to the preceding *Art.* 149.) out of the five numbers given in the laſt-mentioned queſtion, the ſame being ranked according to the latter manner of ordering the ſaid numbers in *Art.* 147. each of the ſaid two Rules of Three will be a Rule Direct, and the ſame anſwer of the queſtion, to wit, 114 pounds will be diſcovered, as may be ſeen by the ſubſequent operation.

```
3 . 19 . 9
4 .      8
```

3 : 19 :: 9	4 : 57 :: 8
9	8
3) 171 (57	4) 456 (114
15	4
21	05
21	4
0	16
	16
	0

151. The *Double Golden Rule* is either *Direct* or *Inverſe.*

152. The *Double Golden Rule Direct*, is, when both the ſingle Rules do each of them look for a fourth term in a direct proportion; as in the Example, *Art.* 149. where each of the two ſingle Rules of Three is Direct.

For

For another *Example* take this, If the carriage of 8 *C.* weight 128 miles cost 48 shillings, for how much may I have 4 *C.* weight carried 32 miles after the same rate? The terms of this question, according to *Art.* 147. rank themselves in this order.

$$\begin{matrix} 128 & . & 48 & . & 32 \\ 8 & & & & 4 \end{matrix}$$

Now taking tacitly the lower term of the first place as a common number, from the first Rule of Three, according to *Art.* 149. saying,

1. If the carriage of a certain weight (to wit, 8*C.*) 128 miles, costs 48 shillings, what will the carriage of the same weight 32 miles cost?

Here it is easy to discern, that the fewer miles any weight is carried, the less money will pay for the carriage of that weight; therefore the fourth number sought by the said Rule of Three must be less than the second number 48: And in regard that by *Art.* 140. when less is required, the greater extreme (whether it be the first or third number) must be the Divisor; therefore the first number 128 is the Divisor, and consequently the Rule of three above proposed is a Rule Direct; then finding out the fourth number, by *Art.* 134. to be 12 shillings, proceed to the second proportion, and say,

2. If the carriage of 8 *C.* (32 miles) cost 12 shillings, how much must I give to have 4*C.* carried the same distance?

And here likewise finding a fourth number to be looked for in a direct proportion, I discover that fourth by *Art.* 134. to be 6*s.* which is the term demanded, and the answer to the question propounded: So if the carriage of 8 *C.* 128 miles, cost 48 *s.* the carriage of 4*C.* 32 miles will cost 6 *s.* according to the same rate.

CHAP. XI.

The Double Golden Rule Inverse, performed by two single Rules.

153. THE *Double Golden Rule Inverse*, is, when one of the single Rules looks for a fourth term in an inverted proportion: As in the last *Example* proposed in *Art.* 145. For if you rank the terms of that question, according to *Art.* 147. Thus,

8 . 12 . 16
9 24

And then working by two ſingle Rules of Three, formed according to *Art.* 149. you'll find by *Art.* 140. that the firſt of the ſaid two Rules of Three will be Inverſe, and the latter Direct. For ſaying firſt, if 8 horſes be maintained 12 days (by 9 buſhels of provender,) how many days will 16 horſes be kept by ſo much provender? Here the anſwer 6 days will be found out by the Rule of Three Inverſe: *Secondly*, ſaying, if 9 buſhels of provender be eaten up (by 16 horſes) in 6 days, in how many days will 24 buſhels be ſpent? Here the anſwer 16 days will be found out by the Rule of Three Direct.

But if you order the given terms of the ſame queſtion thus,

9 . 12 . 24
8 16

And then work by two ſingle Rules of Three, formed according to *Art.* 149. you'll find by *Art.* 140. that the firſt of the ſaid two Rules of Three will be Direct, and the latter Inverſe. For ſaying, firſt, if 9 buſhels of provender laſt 12 days (to maintain 8 horſes) how many days will 24 buſhels ſerve the ſame number of horſes? The anſwer 32 days will be found out by the Rule of Three Direct. Secondly, ſaying, if 8 horſes are maintained 32 days, (by 24 buſhels of provender,) how long will 16 horſes be kept by the ſame quantity of provender? Here the anſwer 16 days will be found out by the Rule of Three Inverſe.

Therefore, whenſoever a queſtion belonging to the Double Rule of Three is ſeparated into two ſingle Rules of Three, (according to the preceding Rules) if one happens to be a Rule Inverſe, that Double Rule is called the Double Rule Inverſe.

CHAP. XII.

The RULE *of* FELLOWSHIP.

154. THE Rules of plural proportion are thoſe, by which we reſolve queſtions that are diſcoverable by more Golden Rules than one, and yet cannot be performed by the Double Golden Rule mentioned before in the two laſt chapters. Of theſe Rules there are divers kinds and varieties ac-

cording to the nature of the queſtion propoſed; for here the terms given are ſometimes four, ſometimes five, ſometimes more, and the terms required ſometimes more than one, &c.

155. Two particular Rules of plural proportion are theſe, the Rule of Fellowſhip, and the Rule of Alligation.

156. The Rule of Fellowſhip is that, by which in accounts among divers men (their ſeveral ſtocks together with the whole gain or loſs being given) the gain or loſs of each particular man may be diſcovered: As in this Example, *A* and *B* were ſharers in a parcel of merchandize, in the purchaſe of which *A* laid out 7 *l.* and *B* 11 *l.* and they having ſold this commodity, find that their clear gains amount to 54 *s.* Now here the queſtion to be reſolved by this Rule, is, what part of that 54 *s.* belongs to *A*, and what to *B*, according to the rate of the ſeveral ſums and ſtocks which they adventured? Again, *A*, *B*, and *C*, freight a ſhip from the *Canaries* for *England* with 108 tuns of wine, of which *A* had 48, *B* 36, and *C* 24, the mariners meeting with a ſtorm at ſea, where conſtrained for the ſafety of their lives, to caſt 45 tuns thereof over-board: Here the queſtion to be reſolved, is, how many of the 45 tuns every particular merchant has loſt, according to the rate of his adventure.

157. The Rule of Fellowſhip is either ſingle, or double.

158. The ſingle Rule is, when the ſtocks propoſed continue in the adventure (or common bank) equal times, to wit, one ſtock as long time as another.

159. In the ſingle Rule of Fellowſhip, take the total of all the ſtocks for the firſt term, the whole gain or loſs, for the ſecond, and the particular ſtocks for the third term; that done, repeating the Rule of Three ſo often, as there are particular ſtocks in the queſtion, the fourth terms produced upon thoſe ſeveral operations, are the reſpective gains or loſſes of thoſe particular ſtocks propounded: So in the firſt Example above-mentioned 7 *l.* and 11 *l.* are the ſtocks propoſed, whoſe total is 18 *l.* which take for the firſt term: Again, 54 *s.* the common gain, is the ſecond term, and 7 *l.* the firſt particular ſtock, is the third term of the firſt proportion; whereupon ſay, as 18 *l.* to 54 *s.* ſo 7 *l.* to another number, which by the Direct Rule of Three is 21 *s.* *viz.* the part of the gain due to *A*, who expended the 7 *l.* ſtock. Then for the ſecond proportion, ſay, as 18 *l.* to 54 *s.* ſo 11 *l.* to another number, which by the Rule of Three Direct is 33 *s.* *viz.* the part of the gain due to *B*, for his 11 *l.* ſtock.

$$\text{As } 18 : 54 :: \begin{cases} 7 : 21 \\ 11 : 33 \end{cases}$$

Again, in the other premised Example, the particular Loss that happens to *A* is 20 tuns, to *B* 15, and to *C* 10 tuns.

$$\text{For as } 108 : 45 :: \begin{cases} 48 : 20 \\ 36 : 15 \\ 24 : 10 \end{cases}$$

160. The Double Rule of Fellowship is, when the stocks proposed are double numbers, *viz.* when each stock has relation to a particular time: Example, *A*, *B*, and *C*, hold a pasture in common, for which they pay 45 *l. per Annum.* In this pasture, *A* had 24 oxen during 32 days; *B* had 12 there 48 days, and *C* fed 16 oxen there 24 days: Now the question to be resolved by this Rule, is, what part each of these tenants ought to pay of the 45 *l.* rent? And here you may observe, that the stocks propounded are double numbers, *viz.* each stock of oxen has reference to a particular time; for the respective stock of *A* is 24 oxen, and its particular time is 32 days; again, the stock of *B* is 12 oxen, and the respective time is 48 days: And lastly, the stock of *C* is 16 oxen, and its peculiar time is 24 days, which, as you see, are double numbers.

161. In the Double Rule of Fellowship, multiply each particular stock by its respective time, and take the total of their products for the first term, the whole gain or loss for the second, and the said particular products of the double numbers for the third term: This done, repeating, as before, the Rule of Three, so often as there are products of the double numbers; the fourth terms produced upon those several operations, are the numbers you look for: So in the above Example, the product of 24 and 32, is 768, the product of 12 and 48 is 576, and the product of 16 and 24 is 384, the sum of these products in 1728, which is the first term in the question; then 45 *l.* the rent, is the second term, and 768 the first product, is the third term of the first proportion: Wherefore I say, as 1728 to 45 *l.* so 768 to another number, which I find by the Direct Rule of Three to be 20 *l.* *viz.* the part of the rent that *A* ought to pay: Then for the second proportion say, as 1728 to 45 *l.* so 576 to 15 *l.* which is the part that *B* ought to pay: And lastly, as 1728 to 45 *l.* so is 384 to 10 *l.* *viz.* the part that *C* must pay.

$$\text{As } 1728 : 45 :: \begin{cases} 768 : 20 \\ 576 : 15 \\ 384 : 10 \end{cases}$$

A second Example: Three merchants *A*, *B*, and *C*, enter partnership, and agree to continue in a joint adventure 16 months; *A* puts into the common stock at the beginning of the

the ſaid term 100 pounds, at 8 months end he takes out 40 pounds, and 4 months after ſuch taking out he puts in 140 pounds. *B* puts in at firſt 200 pounds, at 6 months end he puts in 50 pounds more, and 4 months after the putting in of the 50 pounds, he takes out 100 pounds. *C* puts in at firſt 150 pounds, at 4 months end he takes out 50 pounds, and 8 months after ſuch taking out puts in 100 pounds. Now, at the end of the ſaid 16 months, they had gained 357 pounds, the queſtion is, how much of the ſaid gains belongs to every merchant for his ſhare?

In queſtions of this nature, two things are principally to be obſerved; 1. The whole time of partnerſhip. 2. The reſpective time belonging to each man's particular ſtock; ſo here, it is evident that the whole time is 16 months, and the particular ſtocks and times belonging to every merchant will be as follows, *viz.*

A. had 100*l.* in the common ſtock for 8 months, therefore 100*l.* multiplied by 8 produces————	} 800
Alſo 60*l.* for 4 months, therefore 60 multiplied by 4 produces————	} 240
Alſo 200*l.* for 4 months, therefore 200 multiplied by 4 produces————	} 800
The total of the products of money and time for *A* is————	} 1840
B, had 200*l.* in the common ſtock for 6 months, therefore 200 multiplied by 6 produces————	} 1200
Alſo 250*l.* for 4 months, therefore 250 multiplied by 4 produces————	} 1000
Alſo 150*l.* for 6 months, therefore 150 multiplied by 6 produces————	} 900
The total of the products of money and time for *B*, is————	} 3100
C, had 150*l.* in the common ſtock for 4 months, therefore 150 multiplied by 4 produces————	} 600
Alſo 100*l.* for 8 months, therefore 100 multiplied by 8 produces————	} 800
Alſo 200*l.* for 4 months, therefore 200 multiplied by 4 produces————	} 800
The total of the products of money and time for *C*, is————	} 2200

Then adding the ſaid three Totals together, to wit, 1840, 3100, and 2200, the ſum is 7140; wherefore proceeding as in the laſt Example, ſay, by the Rule of Three Direct, as 7140 is to the total gain 357 pounds; ſo is 1840 to 92 pounds the gain of *A:* Again, as 7140 is to 357; ſo is 3100 to 155 the gain of *B:* Laſtly, as 7140 is to 357; ſo is 2200 to 110 the gain of *C.*

162. The Rule of Fellowſhip is proved by addition of the terms required, whoſe ſum ought to be equal to the ſecond term in the queſtion, otherwiſe the whole Work is erroneous: So in the firſt Example in *Art*, 159. 21*s.* and 33*s.* being added together, are eqaal to 54*s.* the ſecond term in that queſtion. Likewiſe in the laſt Example of the ſame *Article*, as alſo in the firſt Example of the laſt *Article*, the ſum of 20, 15, and 10, the terms required, are equal to 45, the ſecond term propounded.

CHAP. XIII.

Of the RULE *of* ALLIGATION.

163. THE Rule of Alligation is that, by which we reſolve queſtions, that concern the mixing of divers ſimples together.

164. Alligation is either Medial or Alternate.

165. Alligation Medial, is when having the ſeveral quantities and rates of diverſe ſimples propoſed, we diſcover the rate of a mixture compounded of thoſe ſimples. So 10 buſhels of wheat at 4*s.* or (which is all one) 48*d.* the buſhel; 40 buſhels of rye at 3*s.* or 36*d.* the buſhel; and 50 buſhels of barley at 2*s.* or 24*d.* the buſhel; being mixed with 20 buſhels of oats at 12*d.* the buſhel, the Rule of Alligation Medial ſhews you the price of that mixture.

166. In Alligation Medial, firſt ſum up the given quantities, then find the total value of all the ſimples: That done, the proportion will be as follows:

As the ſum of the quantities is to the total value of the ſimples:

So is any part of the mixture propoſed, to the required rate or price of that part.

Repeating again the above Example, I demand how much one buſhel of that mixture is worth? Now the ſum of 10, 40,

50,

50, and 20, (the given quantities) is 120 buſhels; and the value of the 10 buſhels of wheat at 48*d.* the buſhel amounts to 480*d.* Again, the value of the 40 buſhels of rye at 36*d.* the buſhel, is 1440*d.* The value of the 50 buſhels of barley at 24*d.* the buſhel is 1200*d.* And the value of 20 buſhels of oats at 12*d.* the buſhel, is 240*d.* All theſe values being added together, their total is 3360*d.* Then ſay, by the Rule of Three Direct, if 120 buſhels give 3360*d.* what will 1 buſhel yield? The Rule anſwers 28*d.* Therefore a buſhel of that mixture may be afforded for 28*d.* that is, 2*s.* 4*d.* which is the reſolution of the Queſtion propoſed.

In like manner, if it be demanded, what 8 buſhels or a quarter of that mixture is worth, the anſwer will be 224*d.* which being divided by 12, and by that means reduced into ſhillings is 18*s.* 8*d.*

167. In Alligation Medial, the trial of the work is by comparing the total value of the ſeveral ſimples with the value of the whole mixture: For when thoſe ſums accord, the operation is perfect: So in the above Example,

		l.	*s.*	*d.*
The value of	10 buſhels of wheat at 4*s.* the buſhel, is ——	2	0	0
	40 buſhels of rye at 3*s.* the buſhel, is ——	6	0	0
	50 buſhels of barley at 2*s.* the buſhel, is ——	5	0	0
	And 20 buſhels of oats at 12*d.* the buſhel, is	1	0	0
All which amount to ——		14	0	0

Which is likewiſe the value of 120 buſhels, at 28*d.* or 2*s.* 4*d.* the buſhel, for that alſo amounts to 14*l.*

168. Alligation Alternate, is, when having the ſeveral rates of diverſe ſimples given, we diſcover ſuch quantities of them, as are neceſſary to make a mixture, which may bear a certain rate propounded.

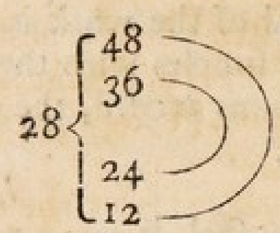
Example, A man being determined to mix wheat at 4*s.* or 48*d.* the buſhel, with rye of 3*s.* or 36*d.* the buſhel, with barley of 2*s.* or 24*d.* the buſhel, and with oats of 1*s.* or 12*d.* the buſhel; the Rule of Alligation Alternate will diſcover to you how much rye, how much barley, and how much oats he ought to add to the wheat, in ſuch ſort, that the mixture of them all together may bear a certain rate or price propoſed.

169. In queſtions of Alligation Alternate, you muſt rank the terms after ſuch a manner, that the given rate of the mixture may repreſent the root, and the ſeveral rates of the ſimples may ſtand as branches iſſuing from that root: So the above Example being laid down, demand how much rye, barley, and oats, ought to be added to the wheat, that the mixture of all together may bear the rate or price of 28*d.* or 2*s.* 4*d.* the

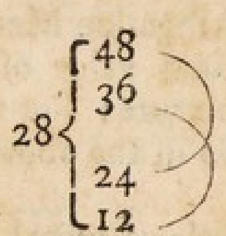
bushel: And therefore, drawing a line of connexion, place 28*d.* the given rate of the mixture, upon the left-hand thereof, by itself, representing the root, and likewise write the other rates proposed, *viz.* 48*d.* 36*d.* 24*d.* and 12*d.* one above another upon the right-hand of that line of connexion, which rates are conceived to issue from 28*d.* as branches from the root, the fabrick whereof appears plainly in the margin.

$$28\begin{cases}48\\36\\24\\12\end{cases}$$

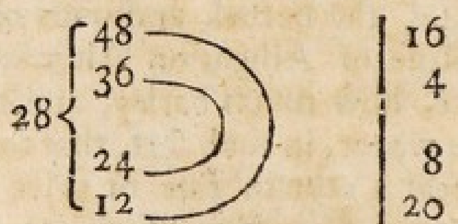
170. Having ranked the terms in their due order, link the branches together by certain arches, in such sort, that one that is greater than the root or rate of the mixture, may always be coupled with another that is less than the same: So in the premised Example, 48 may be linked with 12, and 36 with 24, or otherwise 48 may be coupled with 24, and 36 with 12, and then the work will stand thus:



Or thus,



171. Having alligated the branches and found the differences betwixt them and the root, write the differences of each branch just against its respective yoke-fellow. So the branches of the Example aforegoing being linked after the first manner, and the difference between 28 and 48 being 20, place 20 just against 12, the respective yoke-fellow of 48. Again, 16 being the difference between 28 and 12, write it just against 48. In like manner 8 being the difference between 28 and 36, place it right against 24. And lastly, 4, the difference between 28 and 24, write just against 36: In the end the whole fabrick of the Work (as the branches are thus linked) will stand as in this Example.



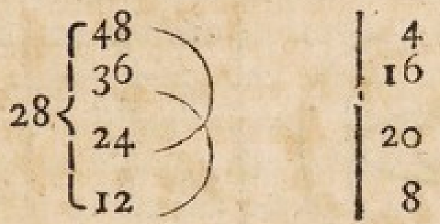
172. And the differences thus placed will answer the question; for if to 16 bushels of wheat at 4*s.* *per.* bushel, be added 4 bushels of rye at 3*s.* 8 bushels of barley at 2*s.* and 20 bushels of oats at 1*s.* the price of a bushel of the mixture will be 2*s.* 4*d.*

For	16 buſhels of	wheat	at 4 *s.*	are worth	64 *s.*
	4	rye	3 *s.*		12
	8	barley	2 *s.*		16
	20	oats	1 *s.*		20

Therefore 48 buſhels of the mixture are worth 112*s.*

And 1 buſhel thereof is worth $\frac{112}{48}$ *s.* $= \frac{7}{3} =$ 2 *s.* 4 *d.*

173. But the branches being linked after the other manner, the Work will be thus diſpoſed:



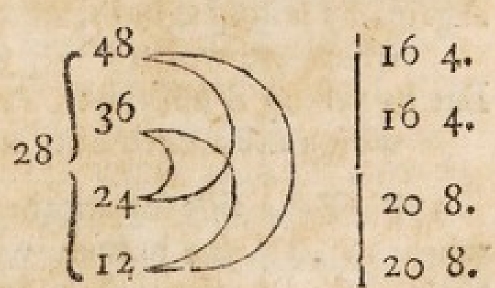
For in this Caſe 48 has 24 for its yoke-fellow, and the reſpective comrade of 36 is 12; and here the interchangeable placing of the differences (as in the premiſed Examples) is that which is more particularly termed Alternation.

174. The numbers thus found will alſo anſwer the queſtion.

For	4 buſhels of	wheat	at 4 *s.*	are worth	16*s.*
	16	rye	at 3*s.*		48*s.*
	20	barley	at 2 *s.*		40*s.*
	8	oats	at 1 *s.*		8*s.*

Therefore 48 buſhels of the mixturer ae worth 112*s.* as before.

175. When one branch is linked to diverſe other branches, and not to one alone, the differences ought to be as often tranſcribed, as it is ſo diverſely linked. So in the premiſed Example, you may (if you pleaſe) conceive 12 to be coupled both with 48 and 36; likewiſe 24 may be conceived to be linked with the ſame 48 and 36; wherefore the difference between 28 and 12 being 16, write it againſt both 48 and 36. In like manner the difference between 28 and 24 being 4, write it likewiſe over-againſt the ſame numbers 48 and 36. Again, 20 being the difference betwixt 28 and 48, place it juſt againſt 24 and 12; and 8 being the difference between 28 and 36, write it likewiſe over-againſt the ſame numbers 24 and 12: All this performed, the whole frame of the work will ſtand as in the Margin.



176. When

176. When, in one and the ſame line, there are found more differences than one, add them together, and write the ſum juſt againſt the ſame differences beyond a ſtraight line drawn towards the right-hand of the Work.

So in the laſt Example, the ſum of 16 and 4 (the differences placed juſt againſt the firſt branch) being 20, write it over-againſt the ſame differences, beyond the new line drawn upon the right-hand of the Work, and ſo conſequently the reſt in their due order, as appears by the Work hereunto annexed.

28	48	16	4.	20.
	36	16	4.	20.
	24	20	8.	28.
	12	20	8.	28.

Which numbers are a third anſwer to the queſtion.

For 20 buſhels of	wheat	at 4*s.* are worth	80*s.*
20	rye	at 3*s.*	60*s.*
28	barley	at 2*s.*	56*s.*
28	oats	at 1*s.*	28*s.*

Therefore 96 buſhels of the mixture are worth 224*s.*

And 1 buſhel is worth $\frac{224}{96}=\frac{112}{48}$ as before.

177. The reaſon of theſe combinations and the alternate placing of their differences will appear from this plain conſideration, *viz.* that whatever is loſt by ſelling any quantity whoſe given price exceeds the mean, is gained again on the quantity alligated thereto, whoſe given price is leſs than the mean.

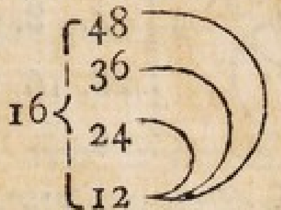
Thus, in the Example work'd in *Art.* 171, and proved in *Art.* 172.

By ſelling 16 buſh. at which are worth	28 *d.* 48 *d.*	you loſe 16 × 20 = 320*d.*
And by ſelling 20 buſh. at which are worth but	28 *d.* 12 *d.*	you gain 20 × 16 = 320*d.*
Again, by ſelling 4 buſh. at which are worth	28 *d.* 36 *d.*	you loſe 4 × 8 = 32 *d.*
But by ſelling 8 buſh. at which are worth but	28 *d.* 24 *d.*	you gain 8 × 4 = 32 *d.*

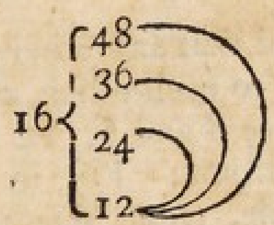
178. Take this for another Example: It is required to mix wheat at 48 *d.* the buſhel, with rye of 36 *d.* the buſhel, with barley of 24 *d.* the buſhel, and with oats of 12 *d.* the buſhel; and the queſtion now is, how much rye, barley, and oats ought

to

to be added to the wheat, that the entire mixture may be afforded at 16*d.* the buſhel? Here the branches of this queſtion (according to *Art.* 170.) ought to be linked: Thus,

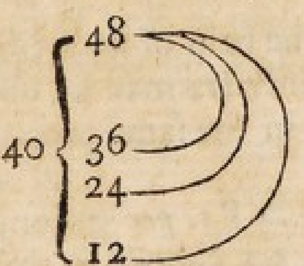


And as for the Alternation of the differences, it is evident (by *Art.* 176.) that the difference between 16 and 12 being 4, ought to be thrice tranſcribed, *viz.* firſt, juſt againſt 48, then againſt 36, and laſt of all againſt 24. Again, 32 the difference between 16 and 48, as alſo 20 the difference between 16 and 36; and laſtly, 8 the difference betwixt 16 and 24, ought all to be placed juſt againſt 12.



4	4
4	4
4	4
32. 20. 8.	60

179. Again, determining to mix wheat at 48*d.* the buſhel, with rye of 36*d.* the buſhel, with barley of 24*d.* the buſhel, and with oats of 12*d.* the buſhel, I deſire to know how much of each I ought to take, that I may afford the whole mixture at 40*d.* the buſhel: Here the whole Work being ordered according to the Rules foregoing, will ſtand as follows.



4. 16. 28.	48
8	8
8	8
8	8

180. A man intending to mix wheat at 48*d.* the buſhel, with rye of 36*d.* the buſhel, with barley of 24*d.* the buſhel, with peaſe of 16*d.* the buſhel, and with oats of 12*d.* the buſhel, deſires to know how much rye, barley peaſe, and oats he ought to add to the wheat, that the whole maſs of corn ſo mixed may be afforded at 20*d.* the buſhel. This queſtion being thus propoſed, the terms of it (by the Rules foregoing) may be alligated, and the differences of the terms alternated, as follows:

20	48	4	4
	36	4	4
	24	4. 8.	12
	16	28. 16. 4.	48
	12	4	4

181. When two kinds of things only are given to be mixed, this Rule of Alligation will give but one anſwer; for inſtance,

Suppoſe it were required to mix brandy at 8 *s. per* gallon, with cyder at 1 *s. per* gallon, ſo as to make the mixture worth 5 *s. per* gallon? The operation would ſtand as below.

5	8	4
	1	3

And the anſwer will be 4 gallons of brandy, and 3 of cyder.

182. Nevertheleſs let it be obſerved that any other two numbers that are in the ſame proportion to each other, as 4 and 3, will alſo anſwer the queſtion. Thus 8 and 6, 12 and 9, 16 and 12, *&c.* are anſwers to the queſtion: For in theſe the quantity of the mixture is the double, triple, quadruple, *&c.* of the quantity in the former; and ſo are the parts of which it is compoſed.

183. If three kinds of things are given to be mixed, the Rule of Alligation will give but one anſwer; but, then, as before, all numbers that are in the ſame proportion between, themſelves, as the numbers which compoſe that anſwer, will alſo ſatisfy the queſtion.

184. But this is not all, for by the help of an artifice, now to be explained, innumerable other anſwers may be obtained, the numbers compoſing which are not in the ſame proportion as the above.

Let it be required to mix brandy at 8 *s. per* gallon, with wine at 7 *s. per* gallon, and cyder at 1 *s. per* gallon, ſo that the mixture may be worth 5 ſhillings *per* gallon? The Work by the Rule of Alligation will ſtand as follows.

5	8	4	4
	7	4	4
	1	3, 2	5

Which ſhews that 4 gallons of brandy, 4 of wine, and 5 of cyder will anſwer the queſtion.

For

For 4 gallons of	brandy at 8 *s.* *per* gal.	are worth	32 *s.*
4 ditto	wine at 7 *s.* ditto		28 *s.*
5 ditto	cyder at 1 *s.* ditto		5 *s.*
Therefore 13 ditto of the mixture		are worth	65 *s.*
And 1 ditto		is worth	$\frac{65}{13}$ *s.* = 5 *s.*

Now let us ſuppoſe, that it ſhould be determined to uſe 5 gallons of cyder in the mixture conſtantly, but to uſe any quantities of brandy and wine that will anſwer the queſtion.

Then may the quantity of brandy be increaſed or diminiſhed by 2, the difference between the prices of the wine and mixture; if at the ſame time the quantity of the wine be diminiſhed or increaſed by 3, the difference of the prices of the brandy and mixture.

That is to ſay, to (4) the quantity of brandy given by the above anſwer, add 2; and from (4) the quantity of wine, take 3; ſo ſhall the ſum 6, and difference 1, be reſpectively quantities of brandy and wine, which mixed with 5 gallons of cyder will anſwer the queſtion.

For 6 gallons of	brandy at 8 *s.*	are worth	48 *s.*
1 ditto	wine at 7 *s.*	is worth	7 *s.*
5 ditto	cyder at 1 *s.*	are worth	5 *s.*
Therefore 12 ditto of the mixture		are worth	60 *s.*
And 1 ditto		is worth	$\frac{60}{12}$ *s.* = 5 *s.*

Again, if from (4) the quantity of brandy given in the firſt anſwer, 2 be ſubtracted; and if to (4) the quantity of wine, 3 be added; then will 2 and 7, the remainder, and ſum, be reſpectively quantities of brandy and wine, which will anſwer the queſtion.

For 2 gallons of	brandy at 8 *s.*	are worth	16 *s.*
7 ditto	wine at 7 *s.*	are worth	49 *s.*
5 ditto	cyder at 1 *s.*	are worth	5 *s.*
Therefore 14 ditto of the mixture		are worth	70 *s.*
And 1 ditto		is worth	$\frac{70}{14}$ *s.* = 5 *s.*

185. Now if inſtead of the numbers of the firſt anſwer 4, 4, and 5 larger numbers in the ſame proportion *viz.* 12, 12, and 15, were taken, the following 8 anſwers would be found by increaſing and diminiſhing the quantities of brandy and wine as above directed, the quantity of cyder remaining conſtantly 15, *viz.*

Brandy	18	16	14	12	10	8	6	4	2
Wine	3	6	9	12	15	18	21	24	27
Cyder	15	15	15	15	15	15	15	15	15

And

And if, inſtead of theſe, ſtill larger numbers in that proportion, or in proportion as any of the laſt found anſwers, be aſſumed ; a greater number of other anſwers may be found.

186. If inſtead of ſuppoſing the quantity of cyder invariable, the quantity of brandy be taken for ſuch ; then an infinite number of anſwers may be found, by continually increaſing the quantity of wine by (4) the difference beteen the prices of the cyder and mixture ; and the quantity of cyder by (2) the difference between the prices of the wine and mixture.

Thus, aſſuming the ſecond anſwer 6, 1, and 5, as the baſis of our work, and eſteeming the 6 gallons of brandy as invariable, the following ſyſtem of anſwers will ariſe.

Brandy	6 .	6 .	6 .	6 .	6 .	6 .	6 .	6 .	6	*&c.*
Wine	1 .	5 .	9 .	13 .	17 .	21 .	25 .	29 .	33	*&c.*
Cyder	5 .	7 .	9 .	11 .	13 .	15 .	17 .	19 .	21	*&c.*

187. Laſtly, eſteeming the quantity of wine as invariable, the quantity of brandy muſt be increaſed by (4) the difference of the prices of cyder and mixture ; and the quantity of cyder muſt be increaſed by (3) the difference of the prices of the brandy and mixture.

Thus taking the third anſwer 2, 7, and 5, as the baſis, and making 7 invariable, theſe anſwers ariſe.

Brandy	2 .	6 .	10 .	14 .	18 .	22 .	26 .	30	*&c.*
Wine	7 .	7 .	7 .	7 .	7 .	7 .	7 .	7	*&c.*
Cyder	5 .	8 .	11 .	14 .	17 .	20 .	23 .	26	*&c.*

188. When there are 4 kinds of things to be mixed, and two of them are of greater value, and the other two of leſſer value than the mixture ; the Rule of Alligation will give three anſwers, as in *Art.* 172, 174, 176, with either of which, or with any numbers that are in the ſame proportion among themſelves as thoſe, as a baſis, innumerable other anſwers, conſiſting of numbers which are in different proportions among themſelves, may be found, by making any two of them invariable, and changing the reſt in the manner as above.

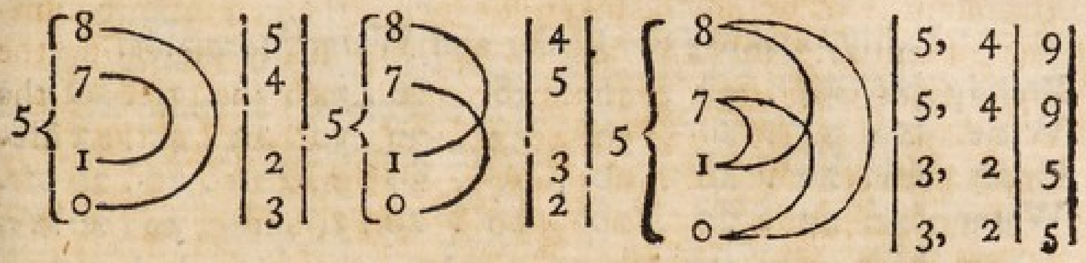
Note 1. That the number by which the quantity of any ſimple is to be varied, is always the difference between the price of the mixture, and the price of the other ſimple, which in any operation is conſidered as variable.

Note 2. That if the ſimples, which in any operation, are conſidered as variable, be both of greater, or both of leſs value than the mixture ; than while the one is increaſed, the other muſt be diminiſhed ; but if the one be of greater value than the mixture, and the other of leſſer, then they muſt be both increaſed, or both diminiſhed.

189.

189. Let it be required to mix brandy at 8 *s.* wine at 7 *s.* cyder at 1 *s.* and water at nothing *per* gallon together; ſo that the mixture may be worth 5 *s.* *per* gallon?

The three anſwers, by the Rule of Alligation, are as follow.



Now taking the laſt anſwer for the baſis of our operation, and making the quantities of wine and cyder invariable, we ſhall have

Brandy	9	14	19	24	29	34	39	*&c.*
Wine	9	9	9	9	9	9	9	*&c.*
Cyder	5	5	5	5	5	5	5	*&c.*
Water	5	8	11	14	17	20	23	*&c.*

Making the Brandy and Cyder invariable.

Brandy	9	9	9	9	9	9	9	*&c.*
Wine	9	14	19	24	29	34	39	*&c.*
Cyder	5	5	5	5	5	5	5	*&c.*
Water	5	7	9	11	13	15	17	*&c.*

Making the Wine and Water invariable.

Brandy	9	13	17	21	25	29	33	*&c.*
Wine	9	9	9	9	9	9	9	*&c.*
Cyder	5	8	11	14	17	20	23	*&c.*
Water	5	5	5	5	5	5	5	*&c.*

Making the Brandy and Water invariable.

Brandy	9	9	9	9	9	9	9	*&c.*
Wine	9	13	17	21	25	29	33	*&c.*
Cyder	5	7	9	11	13	15	17	*&c.*
Water	5	5	5	5	5	5	5	*&c.*

Making the Brandy and Wine invariable.

Brandy	9	9	9
Wine	9	9	9
Cyder	10	5	0
Water	1	5	9

Or take 4 other numbers in the ſame proportion as 9. 9. 5 and 5. *viz.* 36. 36. 20 and 20.

Brandy

Brandy	36	36	36	36	36	36	36	36
Wine	36	36	36	36	36	36	36	36
Cyder	40	35	30	25	20	15	10	5
Water	4	8	12	16	20	24	28	32

Laſtly, making the Cyder and Water invariable.

Brandy	&c.	44	42	40	38	36	34	32	30	28	&c.
Wine	&c.	24	27	30	33	36	39	42	45	48	&c.
Cyder	&c.	20	20	20	20	20	20	20	20	20	&c.
Water	&c.	20	20	20	20	20	20	20	20	20	&c.

190. Not only the ſets of numbers thus found, but their ſums and differences will alſo be anſwers:

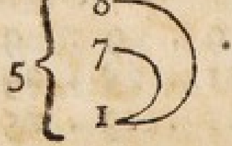
	Brandy,	Wine,	Cyder,	Water.	
Thus from or to	42	27	20	20	
Take or add	9	9	10	1	
The remainder	33	18	10	19	will be anſwers to the queſtion.
And ſum	51	36	30	21	

191. Alligation Alternate, is either Partial, or Total.

192. Alligation Partial, is, when having the ſeveral rates of diverſe ſimples, and the quantity of one of them given, we diſcover the ſeveral quantities of the reſt, in ſuch ſort, that a mixture of thoſe ſimples being made according to the quantity given, and the quantities ſo found, that mixture may bear a certain rate propoſed.

Examp. Suppoſed it were required to mix brandy at 8 *s. per* gallon, and wine at 7 *s.* with 10 gallons of cyder at 1 *s.* ſo that the mixture may be worth 5 *s. per* gallon.

Proceed as before in *Art.* 170, &c. to alligate the prices, and alternate their differences.



8	4	4
7	4	4
1	3, 2	5

Then having by this means obtained ſuch quantities of the ſeveral ſimples as will make a mixture price propoſed, ſay,

As (5) the quantity of cyder found by alternation,
Is to (10) the quantity of ditto given in the queſtion;
So is (4) the quantity of wine and brandy *per* alternation,
To (8) the quantity of ditto required.

Note, This proportion may be applied to any of the anſwers found by the other directions.

193.

193. Now having found one anſwer by the above proportion, others may be found by *Art.* 184. Thus,

Brandy	12 .	10 .	8 .	6 .	4 .	2
Wine	2 .	5 .	8 .	11 .	14 .	17
Cyder	10 .	10 .	10 .	10 .	10 .	10

By which means, other five anſwers are obtained as above. See the proof of the two extreme anſwers below.

	12 gallons	of brandy at 8 *s.*	are worth	96 *s.*
	2 ditto	wine at 7 *s.*	ditto	14 *s.*
	10 ditto	cyder at 1 *s.*	ditto	10 *s.*
Therefore	24 ditto	of the mixture	are worth	120 *s.*
And	1 ditto		is worth	$\frac{120}{24}$ *s.* = 5 *s.*
Again	2 gallons	of brandy at 8 *s.*	are worth	16 *s.*
	17 ditto	wine at 7 *s.*	ditto	119 *s.*
	10 ditto	cyder at 1 *s.*	ditto	10 *s.*
Therefore	29 ditto	of the mixture	are worth	145 *s.*
And	1 ditto		is worth	$\frac{145}{29}$ *s.* = 5 *s.*

194. Alligation Total, is, when inſtead of the quantity of any particular ſimple, the quantity of the mixture is given, together with the prices of the mixture, and the ſeveral ſimples of which it is to be compoſed.

Examp. Let it be required to mix brandy at 8 *s.* wine at 7 *s.* and cyder at 1 *s. per* gallon together, ſo that the mixture may contain 26 gallons, and be worth 5 *s. per* gallon?

The prices being alligated, and their differences being alternated as before, *viz.*

5	8	4	4
	7	4	4
	1	3, 2	5
			13

Add the numbers which compoſe the anſwer together, and ſay,
As (13) the total of the mixture *per* alternation,
To (26) the total of the mixture given in the queſtion;
So are (4, 4, and 5) the ſeveral quantities of the ſimples *per* alternation,
To (8, 8, and 10) the quantities of the ſaid ſimples required.

195. One anſwer being thus obtained, others may be found as follows:

1°. Let the quantity of that ſimple, whoſe value alone is greater, or leſs than the value of the mixture, be increaſed or

H diminiſhed,

diminiſhed, by the difference of the differences between the prices of the other two ſimples and the price of the mixture.

2°. Of the remaining two ſimples, let the quantity of that ſimple whoſe value is fartheſt from the value of the mixture, be increaſed or decreaſed (according as the former is) by the ſum of the differences, between the prices of the other two ſimples, and that of the mixture.

3°. Let the quantity of the remaining ſimple be decreaſed or increaſed, alſo, by the ſum of the differences, between the prices of the other two ſimples, and that of the mixture; but obſerve, that the quantity of this ſimple is to be decreaſed, when thoſe of the two former are increaſed; and the contrary.

In the example before us, the value of cyder is alone leſs than the value of the mixture: Alſo the differences between the prices of the other two ſimples, brandy and wine and that of the mixture are ſeverally 3 and 2, the difference of which differences is 1; therefore (10) the quantity of the cyder, may be increaſed or diminiſhed by 1, that is, it may become 11 or 9.

Of the other two ſimples; the value of the brandy is fartheſt from that of the mixture; alſo the prices of the other two ſimples, wine and cyder, differ from the price of the mixture, ſeverally, by 2 and 4, whoſe ſum is 6; therefore (8) the quantity of the brandy, may be increaſed or diminiſhed by 6, and become 14 or 2.

Laſtly, the quantity of the wine (8) may be diminiſhed or increaſed by (7) the ſum of 3 and 4, the reſpective differences between the prices of the brandy and cyder, and that of the mixture; that is, it may become 1 or 15.

Thus are two more anſwers to the queſtion obtained, which, with the former, may ſtand as below.

Brandy	14 .	8 .	2 .
Wine	1 .	8 .	15 .
Cyder	11 .	10 .	9 .

The proof of the two laſt anſwers follows:

	14 gallons of brandy	at 8 *s.*	are worth	112 *s.*
	1 ditto wine	at 7 *s.*	ditto	7 *s.*
	11 ditto cyder	at 1 *s.*	ditto	11 *s.*
Therefore	26 ditto of the mixture		are worth	130 *s.*
And	1 ditto		is worth	$\frac{130}{26}$ *s.* = 5 *s.*
Again	2 gallons of brandy	at 8 *s.*	are worth	16 *s.*
	15 ditto wine	at 7 *s.*	ditto	105 *s.*
	9 ditto cyder	at 1 *s.*	ditto	9 *s.*

Therefore 26 ditto of the mixture are worth 130 *s.* as before.

196. *Note,* That if there be 4 or more ſimples out of which the mixture is to be compounded, then 1 or more of them muſt be conſidered as invariable, ſo that there may be only 3 variable; and thoſe, ſo that one of them may be of a contrary value, with reſpect to the price of the mixture, from the other two.

Example. 'Tis required to mix ſuch a quantity of brandy at 8 *s.* wine at 7 *s.* cyder at 1 *s.* and water at 0 *s.* *per* gallon, as will make a hogſhead, or 63 gallons of the mixture, worth 5 *s.* *per* gallon?

By the proceſs in *Art.* 189. the two following proportions were found, *viz.*

	Brandy,	Wine,	Cyder,	Water.
Among the firſt found anſwers	9 .	9 .	5 .	5
Among the third . .	9 .	14 .	5 .	7
And their ſums (which are anſwers by *Art.* 190.) - -	18 .	23 .	10 .	12

make juſt 63 the quantity given.

Then 1°. making the water invariable we ſhall have *per Art.* 195.

Brandy	36 .	30 .	24 .	18 .	12 .	6.
Wine	2 .	9 .	16 .	23 .	30 .	37.
Cyder	13 .	12 .	11 .	10 .	9 .	8.
Water	12 .	12 .	12 .	12 .	12 .	12.

2°. Making the cyder invariable, will produce

Brandy	32 .	25 .	18 .	11 .	4.
Wine	7 .	15 .	23 .	31 .	39.
Cyder	10 .	10 .	10 .	10 .	10.
Water	14 .	13 .	12 .	11 .	10.

3°. Making the wine invariable, gives

Brandy	19 .	18 .	17.
Wine	23 .	23 .	23.
Cyder	2 .	10 .	18.
Water	19 .	12 .	5.

Laſtly, making the brandy invariable, we have

Brandy	18 .	18 .	18.
Wine	24 .	23 .	22.
Cyder	3 .	10 .	17.
Water	18 .	12 .	6.

197. If you are inclined to find more anſwers you may, for

Water,	make any number invariable from	5 to 19.
Cyder,	ditto - - - - -	2 to 18.
Wine,	ditto - - - - -	2 to 39.
Brandy,	ditto - - - - -	4 to 36.

For ſince, in ſome of the above anſwers, thoſe quantities have been produced; therefore 'tis highly probable, that all the intermediate numbers may have place in ſome of the anſwers, and perhaps ſome out of thoſe limits.

Example. Suppoſe it were required to make a hogſhead of mixture of the ſame ſimples, and at the ſame prices; but ſo that there ſhould be juſt 16 gallons of wine.

When the water was made invariable, this anſwer among the others aroſe.

Brandy	24	And now making 16 invariable, two other anſwers will ariſe, *viz.*	25	24	23
Wine	16		16	16	16
Cyder	11		3	11	19
Water	12		19	12	5

198. But if, inſtead of gallons, you will mix by pints; then (making the ſame number of gallons, *viz.* 24, 16, 11, 12, or their equivalent number of pints, *viz.* 192, 128, 88, 96, the baſis of the operation) more anſwers may be produced, as follow.

Brandy	*&c.*	197	196	195	194	193	192	191	190	189	188	187	*&c.*
Wine	*&c.*	128	128	128	128	128	128	128	128	128	128	128	*&c.*
Cyder	*&c.*	48	56	64	72	80	88	96	104	112	120	128	*&c.*
Water	*&c.*	131	124	117	110	103	96	89	82	75	68	61	*&c.*

The two extreme anſwers are proved below.

	197 pints of brandy at	1 *s. per* pint are worth	9*l.* 17*s.*
	128 ditto wine	0 *s.* 10½ *d.* ditto	5*l.* 12*s.*
	48 ditto cyder	0 *s.* 1½ *d.* ditto	0*l.* 6*s.*
	131 ditto water		0*l.* 0*s.*
Therefore	504 ditto or 63 gal. of the mixture are worth		15*l.* 15*s.*
And	1 gallon thereof is worth $\frac{315}{63}$ *s.* = 5 *s.*		
Again	187 pints of brandy at	1 *s. per* pint are worth	9*l.* 7*s.*
	128 ditto wine	at 0 *s.* 10½ *d.* ditto	5*l.* 12*s.*
	128 ditto cyder	at 0 *s.* 1½ *d.* ditto	0*l.* 16*s.*
	61 ditto water		0*l.* 0*s.*
Therefore	504 ditto of the mixture are worth		15*l.* 15*s.*

as before.

CHAP.

CHAP. XIV.

The Application of Alligation to the mixing of Metals, according to their different Fineneſs; and to the Compoſition of Medicines.

199. ALLOY is a ſort of coarſe ſilver or copper, or ſome baſe metal, with which goldſmiths mix gold or ſilver, to abate the fineneſs thereof.

An ounce of gold is divided into 24 parts, called carracts; and an ounce of ſilver into 20 parts, called penny-weights; therefore to diſtinguiſh fineneſs of metals, ſuch gold as will abide the fire without loſs, is accounted 24 carracts fine; if it loſe 2 carracts in trial, it will then be 22 carracts fine.

Silver is valued in ounces, and a pound of ſilver which loſeth nothing in trial, is called 12 ounces fine; but if it loſe 2 penny-weights, it is then ſaid to be 11 ounces 18 penny-weights fine.

Examp. A goldſmith hath 20 ounces of gold at 20 carracts fine, and would mix it with ſome at 22 carracts fine, and ſome of 24 carracts fine; how much of 22 and 24 carracts fine, and how much alloy, muſt he mix with the 20 ounces of 20 carracts fine, ſo as the whole maſs may be 18 carracts fine.

Firſt, ſet down the values in order as uſual, with the mean value, and in the place of the alloy, becauſe it is not accounted of any value, place a cypher; then alligate the values, and alternate their differences as uſual, *per Art.* 171, &c.

18	20	18	18
	22	18	18
	24	18	18
	0	2, 4, 6	12

Then by *Art.* 192.

As 18 : 20 ::	18 :	20	The quantity of	22 Car. fine.
	18 :	20		24 Car. fine.
	12 :	$13\frac{1}{3}$		Alloy.

The Application of Alligation to the Compoſition of Medicines.

200. Medicines and Simples, in reſpect of their qualities, are conſidered in ſome of theſe five ways, *viz.* either as they are hot or cold, moiſt or dry, or as they are temperate; ſo that ſuch ſimples or medicines as work heat in our bodies, are ſaid to be hot; ſuch cold, as are the cauſe of coldneſs, *&c.*

The mean or middle between the extreme qualities of heat and coldneſs, alſo between dryneſs and moiſture, is called temperate, or the temperature; from which every one of the ſaid qualities, hot, cold, moiſt, and dry, differs in four degrees; ſo that a medicine or ſimple is ſaid to be either temperate, or elſe hot, cold, moiſt, or dry, in the firſt, ſecond, third or fourth degree.

201. If the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, be placed as you ſee from A to B, the differences between 5, (the middle number) and the ſuperior numbers 6, 7, 8, 9, will be 1, 2, 3, 4, which may repreſent the 4 degrees of the qualities hot and dry; likewiſe the differences between 5 and the inferior numbers 4, 3, 2, 1, will be 1, 2, 3, 4, which may repreſent the 4 degrees of the qualities cold and moiſt, the temperature repreſented by 0, being the mean or middle from whence the ſaid degrees do ſwerve.

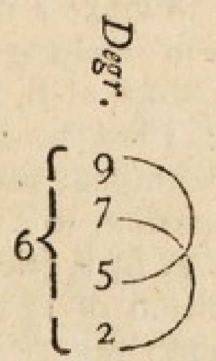
	Ind.	*Deg.*	
B	9	4	*Qualities hot and dry.*
	8	3	
	7	2	
	6	1	
	5	0	*Temperature.*
	4	1	*Qualities cold and moiſt.*
	3	2	
	2	3	
A	1	4	
	Ind.	*Deg.*	

202. Since the Rule of Alligation Alternate requires, that of two things miſcible, the one muſt exceed the mean propoſed, and the other be leſs; therefore the queſtions of Alligation in this kind are to be worked with the numbers in the aforeſaid column A B: For by them the degrees and qualities are diſcovered, being placed as you ſee in the column adjacent to A B, and

and for diſtinction's ſake, thoſe numbers in the ſaid column AB, may be called the Indices or Exponents of the degrees; which Indices are to be uſed in the ſame manner, as the prices of merchandizes in the queſtions of Alligation Alternate, and therefore thoſe Examples may be compared with theſe.

203. *Prop.* I. Having divers ſimples whoſe qualities are known, to make a compoſition or mixture of them; in ſuch manner that the quality of the medicine may be ſome mean among the qualities of the ſimples, and the quantity of it any quantity aſſigned.

Example 1. An Apothecary has four ſorts of ſimples, A, B, C, D, whoſe qualities are as follow, *viz.* A is hot in the fourth degree, B is hot in the ſecond, C is temperate, and D is cold in the third degree; the queſtion is to know what quantities of each of them ought to be taken, to make a medicine, whoſe quantity may be 12 ounces, and the quality in the firſt degree of heat? Seek in the aforeſaid column A, B, for the Indices or Exponents of the qualities of the ſimples given, *viz.* for A which is hot in the fourth degree, take 9; for B which is hot in the ſecond, take 7; for C which is temperate, take 5; and for D which is cold in the third degree, take 2: That done, rank thoſe numbers in the ſame manner as the prices of merchandizes in the queſtions of the 13th Chap. *viz.* deſcend from the higheſt degree of heat to the temperature, and ſo proceed downwards to the degrees of cold, ſetting 6 the Index or Exponent of the mean quality propounded, which is 1 degree of heat, as common to them all: Then, by crooked lines or otherwiſe, connect two ſuch Indices, whereof one may be greater than the mean, and the other leſs, and proceeding according to *Art.* 171, &c. you'll find that to make a medicine of 9 ounces, and the quality reſulting to be in the firſt degree of heat, you muſt take 1 ounce of A (being that ſimple which was hot in 4°.) 4 ounces of B, 3 ounces of C, and 1 ounce of D, as will be manifeſt by the proof.



Oun.	*Simp.*	*The Proof.*
1	A	9 × 1 = 9
4	B	7 × 4 = 28
3	C	5 × 3 = 15
1	D	2 × 1 = 2
9		9) 54 (6

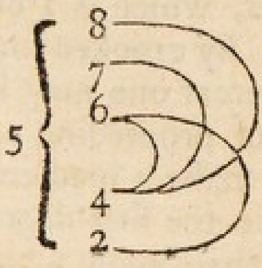
204. By the *Rule of Proportion* you may increase the medicine to the quantity of 12 ounces, and yet the quality to continue in the first degree of heat, according to the following operation.

Oun.		Oun.		Oun.		Oun.	
9	:	1	: :	12	:	$1\frac{1}{3}$	of A
9	:	4	: :	12	:	$5\frac{1}{3}$	of B
9	:	3	: :	12	:	4	of C
9	:	1	: :	12	:	$1\frac{1}{3}$	of D

The Quantity assigned 12 *Ounces.*

By the other connexions of the qualities, other quantities of every simple would arise; but that, and the method of finding answers in other proportions, have been sufficiently manifested in the last Chapter,

Example 2. Suppose there are five simples, A, B, C, D, E, whose qualities are as follows, *viz.* A is hot in 3°. B is hot in 2°. C is hot in 1°. D is cold in 1°. E is cold in 3°. and it is required to mix 4 ounces of B, with such quantities of the rest, that the quality of the medicine may be temperate?



Degr.			*Ounce*	*Simp.*	*The Proof.*
5	8	1	1	A	8 × 1 = 8
	7	1	1	B	7 × 1 = 7
	6	1 + 3	4	C	6 × 4 = 24
	4	3 + 2 + 1	6	D	4 × 6 = 24
	2	1	1	E	2 × 1 = 2
			13		13) 65(5

Proceed as before, so you'll find that to make a medicine of 13 ounces, and the quality of the form resulting to be temperate, you must take 1 ounce of A, 1 ounce of B, 4 ounces of C, 6 ounces of D, and 1 ounce of E: Then since the quantity of B, in the composition proposed, is limited, *viz.* 4 ounces, find numbers which may be in such proportion to 4 (the quantity of B assigned) as the numbers 1. 4, 6, 1, (the quantities of A, C, D, E, in the aforesaid composition of 13 ounces) are to 1 (the quantity of B in the said composition) in manner following.

Oun.

Oun.		*Oun.*		*Oun.*		*Oun.*	
1	:	1	: :	4	:	4 of A	*to be mixed with* 4 *ounces of* B.
1	:	4	: :	4	:	16 of C	
1	:	6	: :	4	:	24 of D	
1	:	1	: :	4	:	4 of E	

205. *Prop.* II. A Medicine being compounded of ſeveral ſimples, whoſe qualities and quantities are known, to find the degree of the form reſulting, *viz*, the exact temperament of the medicine.

Example 1. Suppoſe a medicine to be compounded of two ſimples, *viz.* 6 ounces of B hot in 4°. and 3 ounces of C hot in 3°. and it is required to find the temperament of the medicine, *viz.* the degree and quality reſulting from ſuch mixture? Seek in the aforeſaid column A B for the Indices, of the reſpective degrees and qualities of the ſimples given, and diſpoſe them orderly in ranks right againſt their reſpective quantities; then multiply each Index by its reſpective quantity, and divide the ſum of the products by the ſum of the quantities: So will the quotient be the Index of the degree, and quality of the medicine.

Ind.		*Oun.*		*Prod.*
9	×	6	=	54
8	×	3	=	24
		9		)78($8\frac{2}{3}$

So in the ſaid Example, the quotient will be found $8\frac{2}{3}$, which is the Index of $3\frac{2}{3}$ degrees of heat; and therefore the ſaid medicine is hot in $3\frac{2}{3}$ degrees.

Example 2. Suppoſe a medicine to be compounded of 4 ſimples, whoſe qualities and quantities are known, *viz.* 2 ounces of A hot in 3°. 3 ounces of B hot in 2°. 4 ounces of C temperate, and 5 ounces of D cold in 4°. and let it be required to find the quality reſulting from ſuch mixture. According to the aforeſaid Rule, multiply each Index by its reſpective quantity, and divide the ſum of the products by the ſum of the quantities, ſo the quotient is $4\frac{3}{7}$, which is the Index of $\frac{4}{7}$ degrees of cold (for the difference between 5 the Index of the temperature, and $4\frac{3}{7}$ the Index found, is $\frac{4}{7}$ degrees of cold) which is the quality of the ſaid medicine.

Ind.

Ind.		Oun.		Prod.
8	×	2	=	16
7	×	3	=	21
5	×	4	=	20
1	×	5	=	5
14)				62 ($4\frac{3}{7}$

Example 3. Suppose a medicine to be compounded of several simples, whose qualities and quantities are as follow, *viz.* 4 ounces of a simple which is cold in 2°. and moist in 1°; 5 ounces hot in 3°. and (in respect of dryness and moisture) temperate; 3 ounces hot in 2°. and dry in 2°; 6 ounces hot in 1°. and moist in 4°; 4 ounces cold in 3°. and moist in 2°; the question is to know the temper resulting?

In the resolution of this question there must be two distinct operations, each of them like that in the last Example, *viz.*

1. Find in the same manner as before, the degree and quality resulting from the commixture of the qualities hot and cold; so you'll discover $5\frac{7}{22}$ which is the Index of $\frac{7}{22}$ degrees of heat (for the difference between 5, the Index of the temperature, and $5\frac{7}{22}$ the Index found, is $\frac{7}{22}$ degrees of heat.)

Ind.		Oun.		Prod.
3	×	4	=	12
8	×	5	=	40
7	×	3	=	21
6	×	6	=	36
2	×	4	=	8
22)				117($5\frac{7}{22}$

Ind.		Oun.		Prod.
4	×	4	=	16
5	×	5	=	25
7	×	3	=	21
1	×	6	=	6
3	×	4	=	12
22)				80($3\frac{7}{11}$

2. Seek in the same manner, the temper resulting from the mixture of the qualities dry and moist; and you'll find $3\frac{7}{11}$, which is the Index of $1\frac{4}{11}$ degree of moisture; so that the quality of the said medicine is $\frac{7}{22}$ degree of heat, and $1\frac{4}{11}$ degree of moisture, as by the operation is manifest.

206. *Prop.* III. *To augment or diminish a Medicine in quality, according to any degree assigned.*

Suppose a medicine to be compounded as follows, *viz.* 1 dram of a simple hot in 4°. 2 drams hot in 3°. 2 drams hot in 2°. 1 dram hot

hot in 1°. 1 dram cold in 1°. and 1 dram cold in 2°. Then will the quality of the ſaid medicine be in $1\frac{1}{2}$ degree of heat (as is manifeſt by the ſecond Propoſition.) Now let it be required to augment the ſaid medicine in quality, *viz.* to add ſuch a quantity of ſome one of the ingredients (or ſome other ſimple) which may raiſe the quality of the medicine $\frac{1}{2}$ degree; ſo that the temperament of the medicine after it is increaſed in quantity, may be in 2° of heat. Make choice of ſuch a ſimple, the Index of whoſe quality may exceed the Index of the quality aſſigned, *viz*, take that ſimple which is hot in 3°. whoſe Index is 8, then proceed according to the 1 Example of the firſt Propoſition; ſo will you find that if 1 dram of the aforeſaid medicine be mixed with $\frac{1}{2}$ dram of that ſimple which is hot in 3°. the temper reſulting from ſuch mixture will be in 2°. of heat.

Laſtly, By the *Rule of Three*, ſay if 1 dram require $\frac{1}{2}$ dram, what ſhall 8 drams (the quantity of the medicine firſt given) require?

Anſw. 4 drams. So that if 4 drams of a ſimple which is hot in 3°. be mixed with 8 drams of a medicine hot in $1\frac{1}{2}$ degree, the temper reſulting will be in 2°. of heat, as by the operation is manifeſt,

	Ind.	*Drams.*
7 {	$6\frac{1}{2}$	1
	8	$\frac{1}{2}$

$1 : \frac{1}{2} :: 8 : 4$

The Proof.

Ind.	*Drams.*	*Prod.*
$6\frac{1}{2}$	$\times 8$	$= 52$
8	$\times 4$	$= 32$

12) 84 (7

If it be required to diminiſh a medicine in quality, you are to make choice of ſuch a ſimple, the Index of whoſe quality may be leſs than the Index of the quality aſſigned, and then to proceed as before.

Here obſerve, that if in queſtions of this nature, the quantities of the ſimples be expreſſed by weights of diverſe denominations, they muſt be reduced to that weight which is of the loweſt denomination in the queſtion.

The

The augmenting or diminiſhing of a medicine, in reſpect of quantity: Alſo the finding of the value of any quantity of a medicine, the prices of the ingredients being known, will be familiar to ſuch as underſtand the Rule of Proportion and therefore I ſhall not inſiſt upon them.

CHAP. XV.

The RULE *of* FALSE.

207. THE *Rule of Falſe* is always performed by falſe and ſuppoſititious numbers taken at pleaſure after the propoſition is made, and the queſtion ſtated: For things are ſaid to be found out by the *Rule of Falſe*, when by falſe terms ſuppoſed, we diſcover the true terms required.

208. The *Rule of Falſe*, is either of ſingle, or double poſition.

The Rule of ſingle poſition is, when at once, *viz.* by one falſe poſition, we have means to diſcover the true reſolution of the propoſed queſtion.

For Example: *A*, *B*, and *C*, determining to buy together a certain quantity of timber, that ſhould coſt them 36*l.* agree among themſelves that *B* ſhall pay of that ſum a third part more than *A*, and that *C* ſhall pay a fourth more than *B*. Now the queſtion is, what particular ſum each of theſe parties ought to pay of the 36*l.* To reſolve this queſtion; firſt, put the caſe that *A* ought to pay 6*l.* of the 36*l.* and then *B* muſt pay 8*l.* becauſe he pays one third part more than *A*. And laſtly, *C* ought to pay 10*l.* becauſe he is to lay out one fourth part more than *B*. This done, although by Addition of theſe three ſums, *viz.* 6, 8, and 10, I find that I have made a wrong poſition (their total amounting only to 24*l.* which ſhould have been 36*l.*) nevertheleſs by thoſe ſuppoſititious numbers, I have the means to diſcover the true ſums which the ſeveral parties ought to pay: For by the *Rule of Three Direct*.

1. As 24 to 36, ſo is 6 to 9*l.* the part that *A* muſt pay.
2. As 24 to 36, ſo is 8 to 12*l.* the part that *B* ought to pay.
3. As 24 to 36, ſo is 10 to 15*l.* the part of the 36*l.* that *C* muſt pay.

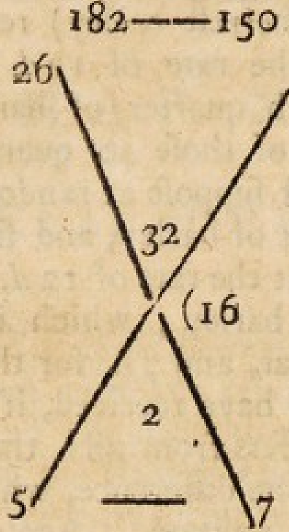
209.

209. Here for trial of this Rule, the total of the ſums found, ought to agree with the ſum given: So in the above Example, 9, 12, and 15, being added together, amount to 36, the ſum propoſed.

210. The Rule of Double Poſition, is when two falſe poſitions are ſuppoſed for the reſolution of the queſtion propounded. As in this, a workman having threſh'd out 40 quarters of grain (part of it wheat and the reſt barley) received for his labour 28 *s.* being paid after the rate of 12 *d.* for every quarter of wheat, and 6 *d.* for each quarter of barley: Now here the queſtion is, how many of thoſe 40 quarters were wheat, and how many barley? Firſt ſuppoſe at random, that there were 26 quarters of wheat and 14 of barley, and find how much money is due to the workman at the rate of 12 *d.* the quarter of wheat, and 6 *d.* the quarter of barley; which is 33 *s.* (*viz.* 26 *s.* for the 26 quarters of wheat, and 7 *s.* for the 14 quarters of barley,) which he ought to have received, if the ſuppoſition were right; but becauſe it differs from 28 *s.* the true ſum that he received, keep in mind 5 the difference, which call the firſt error, or the error of the firſt poſition: Again propound for the ſecond poſition, that there were 30 quarters of wheat, and 10 quarters of barley; and then the ſecond error will be 7; for there is then due to the workman for the 30 quarters of wheat 30 *s.* and for the 10 quarters of barley 5 *s.* in all 35 *s.* which differs from 28 *s.* the true ſum that he received, by 7 *s.* and by theſe two falſe poſitions, together with their errors may be diſcovered how many quarters of wheat, and how many of barley the workman threſh'd, as ſhall be further explained as follows.

211. In the Rule of Double Poſition, having drawn two lines acroſs, and placed the terms of the falſe poſition, (*viz.* thoſe that have the ſame denomination) at the uppermoſt end of that croſs, as alſo each error under its reſpective poſition at the lower end of the ſame croſs, multiply each error by the contrary poſition; that is to ſay, the ſecond error by the firſt poſition, and the firſt error by the ſecond poſition; this done, when both the errors are of one and the ſame kind (*viz.* both exceſſes or both defects) ſubtract the leſs product out of the greater, and then the remainder is your dividend: But if the errors be of different kinds, (*viz.* one of them an exceſs, and the other a defect) add thoſe products together, and the ſum will be your dividend, which if you divide by the difference of the errors, (when they are of one and the ſame kind) or by their ſum (when they are of different kinds) the quotient will give you the number you look for, having the ſame denomination with the falſe poſitions placed at the upper end of the croſs.

Example 1. The above queſtion being again propoſed, place theſe terms, *viz.* 26 (having the denomination of the quarters of wheat in the firſt poſition) and 30 (having the ſame denomination in the ſecond poſition) at the upper end of the croſs: As alſo 5 and 7 the two errors reſpectively under them at the lower end of the ſame croſs, as you may ſee it exemplified by the following pattern.

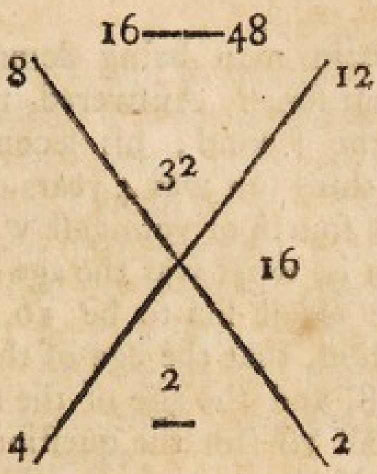


This done, having multiplied 26 by 7, the product is 182, and likewiſe 30 by 5, the product is 150, which being deducted out of 182 (becauſe the errors here are both of the ſame kind, that is, are each of them an exceſs above 28*s.* the ſum that the workman receiv'd,) the remainder is 32, which being divided by 2 (the difference between 5 and 7 the two errors,) leaves in the quotient 16, for the quarters of wheat that the workman threſh'd, whoſe complement to 40, *viz.* 24, are the quarters of barley, that he likewiſe threſh'd; ſo the workman receiving 28*s.* for his wages in threſhing out 40 quarters of grain (being part wheat, part barley) at 12*d.* the quarter of wheat, and 6*d.* the quarter of barley; threſh'd in all 16 quarters of wheat, and 24 quarters of barley.

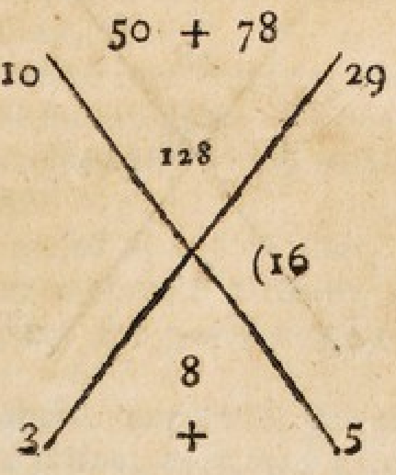
Example 2. The ſame queſtion being again propounded, ſuppoſe for the firſt poſition that there are 8 quarters of wheat, and 32 quarters of barley, and then the firſt error will be 4*s.* for 8*s.* being accounted for the 8 quarters of wheat, and 16*s.* for the 32 quarters of barley, make in all 24*s.* which want 4*s.* of 28*s.* the ſum received: Again ſuppoſing that there are 12 quarters of wheat, and 28 quarters of barley, the ſecond error will be 2*s.* for 12*s.* being allowed for the 12 quarters of wheat, and 14*s.* for the 28 quarters of barley, the ſum is 26*s.* which comes 2*s.* ſhort of 28*s.* the right ſum: Now then, 8 being multiplied by 2, the product is 16; likewiſe 12 by 4 produces

48,

48, out of which if you deduct 16 (because the errors in this case happen to be both defects under 28 *s.* the sum received,) the remainder is 32, which being divided by 2 (the difference of the errors) gives you in the quotient 16, *viz.* the quarters of wheat, as before.

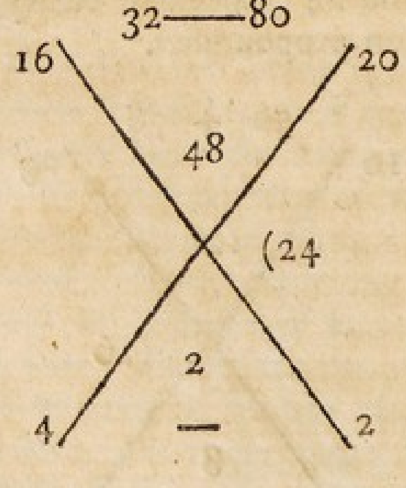


Example 3. The same demand being the third time produced, take for the first position 10 quarters of wheat, and 30 quarters of barley, and then proceeding as before, the first error will prove 3*s.* which upon that position is wanted of 28 *s.* the right sum: Again, for the second position take 26 quarters of wheat, and 14 quarters of barley, and then the second error will be 5*s.* by which that position exceeds 28*s.* the true sum: Now multiplying 10 by 5, the product is 50, and 26 by 3, the product is 78. And here (because the errors are of different kinds, one of them being a defect, and the other an excess) add 50 and 78 the two products together, whose sum is 128, which being divided by 8, the sum of 3 and 5 the two errors, gives in the quotient 16, for the quarters of wheat, as before in the former resolutions: So that what positions soever you take in this question, you will always find, that the workman thresh'd 16 quarters of wheat, and 24 quarters of barley, which is the resolution of the question propounded.



212. Here the proof is the ſame with the proceſs uſed in finding out the errors: So in the Example premiſed, 16 and 24 being the numbers found, and 16*s.* being allowed for the 16 quarters of wheat, likewiſe 12 *s.* for the 24 quarters of barley, their ſum is 28*s.* which was the ſum received by the workman.

Example 4. A certain man being demanded what was the age of each of his four ſons? Anſwered, that his eldeſt ſon was 4 years older than the ſecond; his ſecond was 4 years older than the third; his third ſon was 4 years older than the fourth, or youngeſt; and his fourth or youngeſt was half the age of the eldeſt: The queſtion is, what was the age of every ſon? Here gueſs the age of the eldeſt ſon to be 16, then it may be inferr'd from the queſtion, that the age of the ſecond ſon was 12, the age of the third 8, and the age of the fourth or youngeſt 4; this four ſhould be half 16 (for the queſtion ſays, that the age of the youngeſt was half the age of the eldeſt) but it wants 4 thereof; wherefore I make a ſecond poſition, and take 20 for the age of the eldeſt, then the age of the ſecond muſt neceſſarily be 16, the age of the third 12, and the age of the fourth 8, which ſhould be half 20, but it wants 2: Now (according to the Rule) multiplying 16 (the firſt poſition) by 2 (the ſecond error) the product is 32: Alſo multiplying 20 (the ſecond poſition) by 4 (the firſt error) the product is 80, and becauſe the errors are both of one kind, to wit, both defective; I ſubtract the leſſer product from the greater, ſo the remainder is 48 for a dividend; alſo ſubtracting the leſſer error from the greater, the remainder is 2 for a diviſor: Laſtly, dividing 48 by 2, the quotient is 24, and ſuch was the age of the eldeſt ſon; therefore the age of the ſecond was 20, the age of the third 16, and the age of the fourth 12; which is half the age of the eldeſt as was declared by the queſtion.



CHAP.

CHAP. XVI.

Of Prime and Compoſite NUMBERS.

213. ONE Number is ſaid to *meaſure*, or be a *Meaſure* of another, when it is contained in it a certain number of times preciſely; ſo that being taken out of it as often as poſſible, there ſhall nothing remain over. Thus, 4 meaſures 12; becauſe it is contained in it preciſely 3 times. Obſerve alſo, that one number is ſaid to meaſure another by that number which is the quotient: So 4 meaſures 12 by 3; and reciprocally, 3 meaſures 12 by 4: And hence any number with the quote, by which it meaſures another, may be called the reciprocal meaſure of that other number.

214. A number is called the common meaſure of two or more numbers, when it meaſures each of them; ſo 3 is a common meaſure of 6, 9, 12. And if it is the greateſt number that meaſures them, it is called their greateſt common meaſure; as unity is their leaſt.

215. A leſſer number is an aliquot part of a greater, when the leſſer exactly meaſures the greater; and the greater is then called a multiple of the leſſer. Thus 4 is an aliquot part of 12, and 12 is a multiple of 4.

216. A leſſer number is an aliquant part of a greater, when the leſſer doth not exactly meaſure the greater. Now 5 is an aliquant part of 12, becauſe 2 times 5 is 10, and 3 times 5 is 15; and conſequently 5 doth not exactly meaſure 12.

217. A number is called a prime number, which has no meaſure but itſelf and unity; and which conſequently is the product of no other numbers. So 2, 3, 5, 7, &c. are prime numbers.

218. A compoſed number, is that which ſome number meaſureth, as 4, 6, 9, 10, 12, &c.

219. Numbers prime to one another, are ſuch as have no other common meaſure but an unit. Thus 15 and 32 are prime to each other; for though 5 and 3 will meaſure 15, yet neither 5 nor 3 will meaſure 32.

220. Numbers compoſed to one another, are ſuch, as have ſome number for their common meaſure. So 15 and 21 are compoſed to one another, for 3 will meaſure 15 by 5, and 21 by 7.

221. All even numbers, except 2, are compoſite. But of odd numbers ſome are prime, as 3, 5, 7; and ſome compoſite,

as 9, 15, 21: And since the series of even and odd numbers comprehend all numbers, it follows, that

222. All prime numbers are odd, except the prime 2.

223. If a number measures each of two or more several numbers, it will measure their sum: And if it measure them all but one, it cannot measure the sum.

224. If a number measures the sum of two numbers, and also measures one of those numbers, it will measure the other; or, if it measures the sum of several numbers, and also each of the parts but one, it must measure that one also.

225. The number A, that measures another B, measures all the numbers that B measures, *i. e.* all the multiples of B; and what is measured by A, is so by all the aliquot parts of A.

226. No even number can measure any odd number.

227. If one number A, measures each of two other numbers B and C, it will also measure the remainder after B is taken out of C as oft as possible.

228. Of whatever factors any number is composed by Multiplication, it is resolvable into the same by Division, *i. e.* it is measurable by each of these factors, or the product of any two or more of them.

229. If the two right-hand figures of any number are measurable by 4, the whole number is measurable by 4.

230. If the three right-hand figures of any number are measurable by 8, the whole is measurable by 8.

231. If the sum of the digits or figures, constituting any number, is a multiple of 3 or 9, then the whole number is a multiple of 3 or 9.

232. If the right-hand place of any number be a cypher, or 5, the whole is a multiple of 5.

233. If the right-hand place of any number be a cypher, the whole is a multiple of 10.

234. If a number cannot be divided by some prime number, not greater than the square root thereof, that number is a prime.

235. The unit's place of all prime numbers, except 2 and 5, must be filled by one of the following digits 1, 3, 7, or 9; for all even numbers, and those which end with the digit 5, are composed.

236. To find which of the odd numbers ending with 1, 3, 7, or 9, under 1000, are primes; and to find the component parts of the rest, from such a Table as the following, the problem may be easily solved; for if any number not divisible by 2 or 5, and not exceeding the given number 1000, be

sought

ſought in the Table thus, *viz.* the hundreds at the top or bottom, and the tens and units in the margin; then under the hundreds, and on a line with the tens and units, will be found P, if the number be a prime; or its leaſt component prime, if the number be compoſite.

Then divide the number by its leaſt component prime, and uſe the quotient in like manner.

So 829 is found to be a prime; and 483 is found to be diviſible by 3: Now 483 divided by 3, quotes 161, which by the Table is a multiple of 7; and conſequently 161 divided by 7, quotes 23, which is a prime; whence the component primes of 483, are 3, 7, 23.

237. But to make this table uſeful for all numbers under 1000, let even numbers be divided by 2 continually, as oft as poſſible; and let thoſe numbers which end in 5, be divided by 5 as oft as poſſible, and the reſult ſought in the Table, as before.

Thus, if 616 be divided by 2 three times, it quotes 77, which by the Table gives 7 for a meaſure, and the quote is 11, which by the Table is a prime; conſequently the component primes of 616, are 2, 2, 2, 7, 11.

Again, 435 divided by 5, quotes 87, which by the Table gives 3 for a meaſure, which quotes 29 a prime; whence 435 is compoſed of 5, 3, 29.

238. The calculating of ſuch a Table being not difficult, I ſhall next ſhew the method of performing it.

Let the paper be firſt ruled with as many columns, as the limiting number contains hundreds; then number the hundreds at top and bottom, and the tens and uits in the margins.

	0	1	2	3	4	5	6	7	8	9	
1		P	3	7	P	3	P	P	3	17	1
3	P	P	7	3	13	P	3	19	11	3	3
7	P	P	3	P	11	3	P	7	3	P	7
9	3	P	11	3	P	P	3	P	P	3	9
11	P	3	P	P	3	7	13	3	P	P	11
13	P	P	3	P	7	3	P	23	3	11	13
17	P	3	7	P	3	11	P	3	19	7	17
19	P	7	3	11	P	3	P	P	3	P	19
21	3	11	13	3	P	P	3	7	P	3	21
23	P	3	P	17	3	P	7	3	P	13	23
27	3	P	P	3	7	17	3	P	P	3	27
29	P	3	P	7	3	23	17	3	P	P	29
31	P	P	3	P	P	3	P	17	3	7	31
33	3	7	P	3	P	13	3	P	7	3	33
37	P	P	3	P	19	3	7	11	3	P	37
39	3	P	P	3	P	7	3	P	P	3	39
41	P	3	P	11	3	P	P	3	29	P	41
43	P	11	3	7	P	3	P	P	3	23	43
47	P	3	13	P	3	P	P	3	7	P	47
49	7	P	3	P	P	3	11	7	3	13	49
51	3	P	P	3	11	19	3	P	23	3	51
53	P	3	11	P	3	7	P	3	P	P	53
57	3	P	P	3	P	P	3	P	P	3	57
59	P	3	7	P	3	13	P	3	P	7	59
61	P	7	3	19	P	3	P	P	3	31	61
63	3	P	P	3	P	P	3	7	P	3	63
67	P	P	3	P	P	3	23	13	3	P	67
69	3	13	P	3	7	P	3	P	11	3	69
71	P	3	P	7	3	P	11	3	13	P	71
73	P	P	3	P	11	3	P	P	3	7	73
77	7	3	P	13	3	P	P	3	P	P	77
79	P	P	3	P	P	3	7	19	3	11	79
81	3	P	P	3	13	7	3	11	P	3	81
83	P	3	P	P	3	11	P	3	P	P	83
87	3	11	7	3	P	P	3	P	P	3	87
89	P	3	17	P	3	19	13	3	7	23	89
91	7	P	3	17	P	3	P	7	3	P	91
93	3	P	P	3	17	P	3	13	19	3	93
97	P	P	3	P	7	3	17	P	3	P	97
99	3	P	13	3	P	P	3	17	29	3	99
	0	1	2	3	4	5	6	7	8	9	

First, Then in the place of the first odd prime 3, put P; and then raise 3 to all its powers under 1000, the number given, and in their places put 3.

Secondly, The next odd number is 5, which is passed by, because its multiples end in 5.

Thirdly, The next odd number is 7, whose place is yet vacant, and is therefore to have P in it; then multiply the numbers belonging to every place that is filled by 7, except such as give products greater than the limiting number, and in the place of the product put the figure that was in the place of the multiplicand, or else the multiplicand itself, if it be a prime. Then in the place of the square of 7, put 7; and by that square multiply the same multiplicands, putting in the places of the products as before. Also in the places of the powers of 7, put 7, and by these powers multiply the same multiplicands, putting in the places of the products as before.

Fourthly, The next odd number is 9, whose place is filled, and therefore to be passed over.

Fifthly, The next odd number is 11, whose place is yet vacant, and is therefore to have P in it; then by 11, multiply the numbers belonging to every place that is filled, except such as give products greater than the limiting number, and in the place of the product, put the figure that was in the place of the multiplicand, or else the multiplicand itself, if it be a prime; then in the place of the square and other powers of 11, put 11; and by the square of the other powers of 11, multiply the same multiplicands, putting in the places of the products, as before, and so use 13, 17, 19, 23, *&c.* omitting 15, 21, 25, 27, *&c.*

239. To reduce any number given into its component primes.

Rule. Divide the number given by 2 continually as oft as possible, without a remainder, and a result by 3 as oft as possible, and this last result by 5, 7, 11, 13, and all the prime numbers, 'till there arise a quotient less than the divisor; and then the divisors and the last dividend are the primes, composing the number given.

Examp. 1. Reduce 122760 to its component primes.

2)122760
2)61380
2)30690
3)15345
3)5115
5)1705
11)341
31

Answer 2,2,2,3,3,5,11,31.

In the operation I have omitted the division by 7, because it would not measure 341; and afterward omitted dividing by 13 and 17, because they are not aliquot parts of 31.

Examp. 2. Reduce 360 to its constituent primes.

2) 360
2) 180
2) 90
3) 45
3) 15
5

Answer 2,2,2,3,3,5.

240. To find all the just divisors of any number.

Rule. Make as many columns as there are different component primes in the given number: And at the head of each column place each prime, and where there are more than one prime of the same sort, place in the same column the powers of that prime, till the index of the last power be the number of those like component primes; lastly, multiply every number in the first column by every number in the second, placing each product in the second; and then every number in the first and second by every number in the third, placing each product in the third, and so with the 4th, 5th, *&c.* columns, till all are thus used.

And so shall all the numbers in these several columns, be all the divisors required.

Example.

Example. Find all the just divisors of 360.

2)	360
2)	180
2)	90
3)	45
3)	15
5)	5
	1

2	3	5
4	9	10
8	6	20
	12	40
	24	15
	18	45
	36	30
	72	60
		120
		90
		180
		360

Answer 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360.

241. To find all the aliquot parts of any number given: Find all the just divisors of the number given by *Art.* 240. excluding the number itself, which divisors with unity are all the aliquot parts.

So, all the aliquot parts of 360, are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180.

242. A number which can be measured by two or more numbers, is called their common multiple; and if it be the least number that can be so measured, it is called their least common multiple.

243. To find the least common multiple of two, or more given numbers.

Rule. Set down the given numbers on a line, and divide them severally by 2, 3, or any other number which will measure two or more of them, and place their quotes with the numbers not divided on a line beneath; divide them continually by the same or any other numbers, till the results be units, reserving their several divisors: which divisors multiply into each other continually, and the product shall be the number sought.

Note, If the numbers are prime to each other, then the continual product of the given numbers will be their least common multiple.

Examp. 1. Find the least common multiple of 2, 3, and 4.

		2
Divisors 2	2, 3, 4	2
2	1, 3, 2	4
3	3, 1	3
	1	12

Answer, 12 is their least common multiple.

Examp. 2. Find the least common multiple of 2, 4, 6, 10, 12, and 15.

		2
Divisors 2	2, 4, 6, 10, 12, 15	2
2	1, 2, 3, 5, 6, 15	4
3	1, 3, 5, 3, 15	3
5	1, 5, 1, 5	12
	1, 1	5
		60

Answer, 60 is the least common multiple.

The

The Doctrine of Vulgar Fractions.

CHAP. XVII.

NOTATION *of* VULGAR FRACTIONS.

244. THE Doctrine of Fractions depends upon this ſuppoſition, that unity, or at leaſt one whole thing, whatſoever it be, may in mind be conceived diviſible into any number of equal parts. Some will not allow 1 or unity to be a number, when it is conſidered in the abſtract, and ſeparated from matter: But ſince that prince of Arithmeticians, *Diophantus* of *Alexandria*, in ſeveral of his ſubtil problems mentions unity as a number, and propounds it to be divided into numbers; I ſhall take the like liberty to eſteem 1 or unity as a number, and likewiſe ſuppoſe it diviſible into any number of equal parts.

245. A broken number, otherwiſe called a fraction, is only part of an integer or whole number, as if you would expreſs in figures the length of a piece of cloth, that contains three fourths, or (which is all one) three quarters of a yard, you may write it thus $\frac{3}{4}$; that is, an entire yard being ſuppos'd to be divided into four equal parts, the length of the piece propoſed is three of thoſe four parts: In like manner, (a foot being divided into 12 inches) you may write 6 inches thus $\frac{6}{12}$ that is, ſix twelfth parts of a foot; or if the foot be divided into one hundred equal parts, to expreſs twenty-five of thoſe parts, ſet them down thus $\frac{25}{100}$, that is, twenty-five hundredth parts of a foot.

246. A fraction conſiſts of two parts, the Numerator and the Denominator, which are placed one above the other, and ſeparated by a little line.

247. The Numerator is the number ſet above the line, and the Denominator is the number placed underneath: So in the aforementioned fraction $\frac{3}{4}$, the number 3 placed above the line is the Numerator, and the number 4 ſet under it is the Denominator.

$$\frac{3 \text{ Numerator.}}{4 \text{ Denominator.}}$$

Alſo in this fraction $\frac{6}{12}$, the Numerator is 6, and the Denominator is 12. The Denominator

is

is ſo called, becauſe it denominates or declares into how many equal parts the integer or whole number is ſuppoſed to be divided; and the Numerator is ſo named, becauſe it numbers or expreſſes how many of theſe equal parts of the integer are ſignified by the fraction.

248. If two fractions have the ſame denominator and different numerators, that fraction is the greater whoſe numerator is the greater: But if two fractions have the ſame numerator and different denominators, that fraction is the greater whoſe denominator is the leaſt; for the fewer the parts are, into which the whole is divided, the greater muſt one of thoſe parts be.

249. Hence by multiplying the numerator, the fraction will be multiplied; and by dividing the numerator the fraction will be divided.

For inſtance, if the fraction $\frac{3}{12}$ be propoſed, and its numerator 3 be multiplied by 2, the product $\frac{3\times 2}{12}=\frac{6}{12}$ is plainly the double of $\frac{3}{12}$.

And if, in the ſame fraction, the numerator 3 be divided by 3, the quotient $\frac{1}{12}$ is evidently the third part of $\frac{3}{12}$.

250. Alſo by multiplying the denominator the fraction will be divided, and by dividing the denominator the fraction will be multiplied.

For example, if 12 the denominator of the fraction $\frac{3}{12}$ be multiplied by 3, the product ($\frac{3}{36}=$) $\frac{1}{12}$ is the third part of $\frac{3}{12}$.

And if, in the ſame fraction, the denominator 12 be divided by 2, the quotient ($\frac{3}{6}=$) $\frac{6}{12}$ is the double of $\frac{3}{12}$.

251. But if both numerator and denominator, be multiplied or divided by the ſame number, the fraction ariſing will be of the ſame value.

252. A fraction is either proper, or improper.

253. A proper fraction is that whoſe Numerator is leſs than the Denominator; ſuch are the fractions before-mentioned $\frac{3}{4}$, $\frac{6}{12}$, $\frac{25}{100}$, and the like.

254. A proper fraction is either ſingle, or compound.

255. A ſingle fraction is that which conſiſts of one Numerator, and one Denominator; ſuch are $\frac{3}{4}$, $\frac{6}{12}$, $\frac{25}{100}$, and the like.

A ſingle fraction often ariſes in diviſion of whole numbers, for when the diviſion is finiſhed, if any number remain, it is to be eſteemed as the Numerator of a fraction, which has the Diviſor for a Denominator, and is to be annexed to the integer or integers in the quotient, as part of the quotient; which fraction always expreſſes certain parts (or at leaſt a part) of an integer or entire unit, which has the ſame denomination with one

 of

of the integers in the quotient: So if 17 pounds be given to be equally divided among 5 perſons, there will ariſe 3 entire pounds in the quotient; and there will be a remainder or ſurpluſage of 2 pounds, which 2 is to be placed, as the Numerator of a fraction, over the Diviſor 5 as a Denominator; ſo will the fraction be $\frac{2}{5}$, and the complete quotient will be $3\frac{2}{5}$, that is 3 pounds and 2 fifth parts of a pound for every perſon's ſhare.

5) 17 ($3\frac{2}{5}$

A ſingle fraction does likewiſe ariſe, when a leſſer whole number is given to be divided by a greater; for in ſuch caſe the Dividend is to be made the Numerator of a fraction, and the Diviſor the Denominator; which fraction is the true quotient, and always expreſſes certain parts (or at leaſt a part) of an integer, which has the ſame name with the Dividend. So if 3 pounds Sterling be given to be divided equally among 4 perſons, the ſhare of each, that is, the quotient will be $\frac{3}{4}$, to wit, 3 fourth parts of a pound.

In like manner, if 5 be given to be divided by 8 the quotient is $\frac{5}{8}$; ſo that the Numerator of a fraction is always a Dividend, the Denominator is a Diviſor, and the Fraction itſelf is the Quotient.

256. A compound Fraction (otherwiſe called a Fraction of a Fraction) is that which has more Numerators and Denominators than one, and may be diſcovered by the word (*of*) which is interpos'd between the parts of ſuch compound Fraction: So $\frac{2}{3}$, of $\frac{3}{4}$ is a Fraction of a Fraction, or compound Fraction, and expreſſes two thirds of three fourths of an integer, *viz.* a pound Sterling being ſuppoſed the integer, and firſt divided into four parts, three of thoſe four parts are equal to 15 *s.* Again, if the ſaid 15 *s.* be divided into three parts, two of thoſe three parts are equal to 10 *s.* therefore the compound Fraction $\frac{2}{3}$ of $\frac{3}{4}$ of a pound Sterling does expreſs 10 *s.* In like manner, the compound Fraction $\frac{1}{4}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of a pound Sterling, that is, one fourth of three fourths of four fifths of a pound Sterling expreſſes 3 *s.* as will be manifeſted hereafter.

257. An improper Fraction is that, whoſe Numerator is either greater, or at leaſt equal to the Denominator: So this Fraction $\frac{16}{4}$, that is 16 fourths, is called an improper Fraction, and ſo is this $\frac{4}{4}$; and a Fraction of this kind may well be ſurnamed improper, becauſe it will not admit the definition of a true Fraction; ſince it is always greater than an entire unit, or at leaſt equal to it; ſo ſixteen farthings, or $\frac{16}{4}$ of a penny are equal to 4 entire pence; and 4 farthings or $\frac{4}{4}$ of a penny are equal to 1 penny: Therefore when the Numerator

is

is greater than the Denominator, ſuch improper Fraction ſignifies more than 1 or an integer; but when the Numerator is equal to the Denominator, (be it what number ſoever) ſuch improper fraction is always equal to unity, or 1 integer.

258. A mixt number conſiſts of entire units (or integers) or at leaſt of unity (or 1 integer) and a fraction annexed: Thus $5\frac{11}{12}$, $1\frac{3}{4}$, and ſuch like, are called mixt numbers: So that if a piece of timber be five feet and eleven inches in length, you may write that length thus, $5\frac{11}{12}$; in like manner, one mile and three quarters or fourths of a mile may be expreſſed thus $1\frac{3}{4}$.

CHAP. XVIII.

Reduction *of* Vulgar Fractions.

259. THE ſame parts of Numeration, as have been work'd in whole Numbers in the preceeding Chapters, are likewiſe to be performed in Fractions: But firſt of all Reduction of Fractions in diverſe kinds muſt be known, which being the principal ſkill in the Doctrine of Fractions, ought to be diligently obſerved by the learner.

260. Two numbers being given, their greateſt common Diviſor, that is, the greateſt number which will meaſure or divide each of the numbers given without leaving any remainder, may be found out in this manner, *viz.* Divide the greater number by the leſs, then divide the Diviſor by the remainder (if there be any) and ſo continue dividing the laſt Diviſors by the remainders, until there be no remainder (neglecting the quotients;) ſo is the laſt Diviſor the greateſt common Diviſor to the numbers given.

Thus, if the greateſt common Diviſor to the numbers 91 and 117 be ſought, divide the greater number 117 by 91, the remainder is 26; by which dividing 91, the remainder is 13; by which dividing 26 the remainder is 0; ſo is 13 the greateſt common Diviſor to the numbers 117 and 91, as is manifeſt from dividing each of them by 13; for 13 is found in 91 preciſely 7 times, and in 117 preciſely 9 times. In like manner 29 will be found a

```
91) 117 (1
    91
-----------
   26) 91 (3
       78
-----------
   13) 26 (2
       26
-----------
        0
```

common

common Divifor to 116 and 145; and 51 a common Divifor to 561 and 612.

261. A fingle Fraction may be abbreviated or reduced into the leaft terms, by dividing the Numerator and Denominator by their greateft common meafure (or Divifor;) for the quotients will be the Numerator and Denominator of a Fraction equal to the former, and in the leaft terms.

So if the Fraction $\frac{91}{117}$ be given to be reduced into the leaft terms, fearch out the greateft common Divifor to 91 and 117 by the laft Rule, which will be found 13, and then dividing 91 by 13, the quotient will be 7 for a new Numerator; alfo dividing 117 by 13, the quotient will be 9 for a new Denominator: So the fraction $\frac{91}{117}$ is reduced into the leaft terms, *viz.* into the fraction $\frac{7}{9}$. In like manner $\frac{116}{145}$ will be reduced to $\frac{4}{5}$. And $\frac{561}{612}$ to $\frac{11}{12}$: But here you are to obferve, that if the greateft common Divifor to the Numerator and Denominator be 1, fuch fraction is in its leaft terms already: So the fraction $\frac{10}{151}$ cannot be reduced into lower terms, becaufe the greateft common Divifor will be found 1; the like may happen to infinite others: And though the laft be a general Rule for Reduction of Fractions into their leaft terms, yet there are other practical Rules, which in fome cafes will be more ready, (efpecially to beginners) *viz.*

262. When the Numerator and Denominator are even numbers, they may be meafured or divided by 2. Therefore in fuch cafe you may take the half of the Numerator for a new Numerator, alfo the half of the Denominator for a new Denominator. So if $\frac{16}{64}$ be given, draw at length the line which feparates the Numerator from the Denominator, and crofs the fame with a down-right ftroke near the Fraction, as you may fee in the Margin; then take the half of 16, which is 8, for a new Numerator, and the half of 64, which is 32, for a new Denominator: Again, the half of 8 is 4, for a new Numerator, alfo the half of 32 is 16, for a new Denominator; and proceeding in the like manner, there will be found $\frac{1}{4}$ equivalent to $\frac{16}{64}$.

16	8	4	2	1
64	32	16	8	4

263. When the Numerator and Denominator each of them end with 5, or one of them ends with 5, and the other with a cypher; they may be both meafured or divided by 5. So $\frac{225}{475}$, will be reduced into $\frac{9}{19}$, and $\frac{50}{425}$ into $\frac{2}{17}$, as by the operation in the Margin is manifeft.

225	45	9
475	95	19

50	10	2
425	85	17

264. Whenever you can find any other number, which will exactly divide the Numerator and Denominator (though it be not the greatest common Divisor) you may divide the Numerator and Denominator by such number as before: So $\frac{28}{84}$ may be first reduced to $\frac{7}{21}$ by 4, and $\frac{7}{21}$ may be reduced to $\frac{1}{3}$ by 7, as by the operation is manifest.

$$\frac{28}{84}\Big|\frac{7}{21}\Big|\frac{1}{3}$$

265. When the Numerator and Denominator, each of them end with a cypher or cyphers, cut off equal cyphers in both, and the Fraction will be reduced to lesser terms: So $\frac{400}{500}$ is reduced to $\frac{4}{5}$, and $\frac{700}{9000}$ to $\frac{7}{90}$.

$$\frac{4}{5}\Big|\frac{00}{00} \qquad \frac{7}{90}\Big|\frac{00}{00}$$

266. An expression of the following form, *viz.* $\frac{4 \times 21 \times 15}{30 \times 18 \times 14}$, which denotes the continual product of 4, 21, and 15 divided by the continual product of 30, 18, and 14, may be abbreviated, by dividing any of the factors or parts of the Numerator and Denominator by some number that is a common measure of those parts: Thus because 2 will measure 4 (a part of the Numerator) and 14 (a part of the Denominator) the Fraction will be reduced to $\frac{2 \times 21 \times 15}{30 \times 18 \times 7}$; and because 5 will measure 15 and 30 it will become $\frac{2 \times 21 \times 3}{6 \times 18 \times 7}$; also because 3 will measure 21 and 18 it will be $\frac{2 \times 7 \times 3}{6 \times 6 \times 7}$: Now since the number 7 is a factor in both Numerator and Denominator, it may be farther reduced to $\frac{2 \times 3}{6 \times 6}$; and since 2 a part of the Numerator will measure 6 a part of the Denominator it will become $\frac{1 \times 3}{3 \times 6}$; and because 3 will measure 6 it will be $\frac{1 \times 1}{3 \times 2}$, or $\frac{1}{6}$.

Thus, if it were required to abbreviate $\frac{12\times35}{49\times40}$

1. Dividing by 7 reduces it to $\frac{12\times5}{7\times40}$

2. Dividing by 5 reduces it to $\frac{12\times1}{7\times8}$

3. Dividing by 4 reduces it to $\frac{3\times1}{7\times2}$, or $\frac{3}{14}$.

But the operation may be performed much ſhorter by cancelling the figures that are divided, and writing the quotients of thoſe diviſions either above or below them: Thus, in the laſt Example after dividing by 7,

the Fraction might ſtand thus $$\frac{12\times\overset{5}{\cancel{35}}}{\underset{7}{\cancel{49}}\times40}$$

After dividing by 5, thus $$\frac{12\times\overset{\overset{1}{\cancel{5}}}{\cancel{35}}}{\underset{7}{\cancel{49}}\times\underset{8}{\cancel{40}}}$$

And after dividing by 4, thus $$\frac{\overset{3}{\cancel{12}}\times\overset{\overset{1}{\cancel{5}}}{\cancel{35}}}{\underset{7}{\cancel{49}}\times\underset{\underset{2}{\cancel{8}}}{\cancel{40}}}=\frac{3\times1}{7\times2}=\frac{3}{14}.$$

Suppoſe it were required to abbreviate $\frac{15\times77\times36}{28\times45\times33}$, the reſult with all the figures uſed in the operation will ſtand as

follows $$\frac{\overset{\overset{1}{\cancel{3}}}{\cancel{15}}\times\overset{\overset{1}{\cancel{11}}}{\cancel{77}}\times\overset{\overset{1}{\cancel{9}}}{\cancel{36}}}{\underset{\underset{1}{\cancel{4}}}{\cancel{28}}\times\underset{\underset{1}{\cancel{9}}}{\cancel{45}}\times\underset{\underset{1}{\cancel{11}}}{\cancel{33}}}=\frac{1}{1}=1.$$

267. The value of a ſingle fraction in the known parts of the integer, may be found out in this manner, *viz*, multiply the Numerator of the Fraction propos'd, by the number of known parts, of the next inferior denomination, which are equal to the integer, and divide that product by the Denominator, ſo is the quotient the value of the Fraction in that inferior denomination; and if there happen to be any Fraction in the quotient, you may find the value of it, in the next inferior denomination, by the ſame Rule, and ſo proceed till you come to the leaſt known parts.

So the value of $\frac{9}{16}$ of a pound Sterling will be found 11*s.* 3*d.* *viz.* multiply the Numerator 9 by 20 (the number of ſhillings which are equal to 1 pound Sterling) the product is 180, which being divided by the Denominator 16, the quotient is $11\frac{4}{16}$ ſhillings. In like manner, the value of $\frac{4}{16}$ of a ſhilling will be found 3 pence; for multiplying the Numerator 4 by 12 (the number of pence in a ſhilling) the product is 48, which being divided by the Denominator 16, the quotient is 3 pence. Alſo the value of $\frac{7}{13}$ of a pound Sterling, will be found 10*s.* $9\frac{3}{13}$*d.*

```
          9
         20
16) 180 (11 4/16
    16
    --
     20
     16
     --
      4
     12
16)  48 (3
     48
     --
      0
```

268. A mixt number may be reduced to an improper Fraction equivalent to the mixt number, in this manner, *viz.* multiply the integer or integers in the mixt number, by the Denominator of the Fraction annexed to the integer or integers, and unto the product add the Numerator of the ſaid Fraction; ſo is the ſum, the Numerator of an improper Fraction, whoſe Denominator is the ſame with that of the ſaid Fraction annexed.

So $4\frac{11}{12}$ will be reduced to the improper Fraction $\frac{59}{12}$ for 4 being multiplied by 12 the product is 48, to which adding the Numerator, 11, the ſum is 59 for a new Numerator, which being placed over the Denominator 12, gives the improper Fraction $\frac{59}{12}$; which is equivalent to $4\frac{11}{12}$ (as will appear by *Art.* 271.) In like manner $7\frac{1}{2}$ will be reduced to $\frac{15}{2}$.

269. A whole number is reduced to an improper Fraction, by placing the whole number given as a Numerator, and 1 as a Denominator.

So 14 integers will be reduced to the improper Fraction $\frac{14}{1}$, and one integer to the improper Fraction $\frac{1}{1}$.

270.

270. A whole number is reduced to an improper Fraction, which ſhall have any Denominator aſſigned, by multiplying the whole number given by the Denominator aſſigned, and placing the product as a Numerator over the ſaid Denominator.

As, if 13 be given to be reduced to an improper Fraction, whoſe Denominator ſhall be 4, multiply 13 by 4, the product is 52, which being placed over 4, gives the improper Fraction $\frac{52}{4}$ equivalent to 13 (as will appear by the next *Art.*) In like manner 13 may be reduced to $\frac{91}{7}$.

271. An improper Fraction may be reduced to its equivalent whole number, or mixt number, in this manner, *viz.* divide the Numerator by the Denominator, and the quotient will give the whole number or mixt number ſought: So the improper Fraction $\frac{59}{12}$ will be reduced to this mixt number $4\frac{11}{12}$; for if 59 be divided by 12 the quotient is $4\frac{11}{12}$. Alſo this improper Fraction $\frac{52}{4}$ will be reduced to the whole number 13.

272. Fractions having unequal Denominators, may be reduced to Fractions of the ſame value, which ſhall have equal Denominators, by this Rule and the next following, *viz.* when two Fractions having unequal Denominators, are propoſed to be reduced into two other fractions of the ſame value, which ſhall have a common Denominator; multiply the Numerator of the firſt Fraction (that is either of them) by the Denominator of the ſecond, and the product ſhall be a new Numerator (correſpondent to the Numerator of that firſt Fraction;) alſo multiply the Numerator of the ſecond Fraction by the Denominator of the firſt, the product is a new Numerator (correſpondent to the Numerator of the ſecond Fraction;) laſtly, multiply the Denominators one by the other, and the product is a common Denominator to both the new Numerators.

Thus, if the Fractions $\frac{2}{3}$ and $\frac{4}{5}$ be propoſed, multiply 2 by 5, the product 10 is a new Numerator correſpondent unto 2: Alſo multiply 4 by 3, the product 12 is a new Numerator correſpondent to 4: Laſtly, multiply 3 by 5, and the product 15 will be a common Denominator to the new Numerators: So the Fractions $\frac{10}{15}$ and $\frac{12}{15}$ are found out, which have equal Denominators, and each of theſe new Fractions is equal to its correſpondent Fraction firſt given, *viz.* $\frac{10}{15}$ is equal to $\frac{2}{3}$, and $\frac{12}{15}$ is equal to $\frac{4}{5}$; as is manifeſt by *Art.* 261.

2	4
5	3
—	—
10	12

5
3
—
15

273. When three or more Fractions having unequal Denominators, are given to be reduced to other Fractions of the ſame value with thoſe given, but ſuch as ſhall have one common Denominator; multiply continually the Numerator of the

first Fraction into all the Denominators, except its own, *i. e.* the Denominator of that Fraction ; and reserve the last product for a new Numerator, instead of that first Numerator : In like manner, multiply continually the Numerator of the second Fraction into all the Denominators, except the Denominator of the second Fraction ; and reserve the last product for a new Numerator, instead of the second Numerator: Proceed in like manner to find out new Numerators for the rest of the given Fractions. Lastly, multiply continually all the Denominators one into another, and the last product will be a common Denominator to all the new Numerators.

As for Example, if these three Fractions $\frac{3}{8}$, $\frac{2}{5}$, $\frac{5}{7}$, having unequal (or different) Denominators, be given to be reduced to three other Fractions of the same value, which shall have equal Denominators, (or one common Denominator.) First, multiply continually the first Numerator 3 into the second and third Denominators 5 and 7 ; saying 3 times 5 makes 15, which multiplied by 7 produces 105, for a new Numerator instead of the first Numerator 3: Secondly, multiply continually the second Numerator 2 into the first and third Denominators 8 and 7 ; saying twice 8 is 16, which multiplied by 7 produces 112, for a new Numerator instead of the second Numerator 2: Thirdly, multiply continually the third Numerator 5 into the first and second Denominators 8 and 5 ; saying, 8 times 5 makes 40, which multiplied by 5 produces 200, for a new Numerator instead of the third Numerator 5: Fourthly and lastly, multiply continually all the Denominators 8, 5, and 7, one into another ; saying, 8 times 5 makes 40, which multiplied by 7 produces 280 for a Denominator to each of the three new Numerators 105, 112, and 200, before found out: And so these three Fractions $\frac{105}{280}$, $\frac{112}{280}$, and $\frac{200}{280}$ are discovered, which have one common Denominator 280; and every one of them is equal in value to its correspondent Fraction first given, *viz.* $\frac{105}{280}$ is equal to $\frac{3}{8}$: Also $\frac{112}{280}$ is equal to $\frac{2}{5}$; and $\frac{200}{280}$ is equal to $\frac{5}{7}$; as may easily be proved by *Art.* 261.

$\frac{3}{8}$	$\frac{2}{5}$	$\frac{5}{7}$
$\frac{105}{280}$	$\frac{112}{280}$	$\frac{200}{280}$

After the same manner, these four Fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$, are reducible to these $\frac{240}{360}$, $\frac{270}{360}$, $\frac{288}{360}$, and $\frac{300}{360}$, which have 360 for a common Denominator ; and are equal in value respectively to the four Fractions given to be reduced.

Note, Although by the aforegoing *Art.* 272. and 273. any multitude of Fractions may be reduced to a common Denominator ; yet because Fractions in their least terms are fittest for use, I shall shew how lesser Denominators than those that are discovered by the said Rule, may frequently be found out.

274. When 2 or more Fractions having unequal Denominators, are given to be reduced to other equal Fractions having the least common Denominator possible; find by *Art.* 243. the least common multiple of all the given Denominators, so shall that be the common Denominator required.

Divide the said common Denominator severally by the Denominators of the given Fractions, and multiply each Numerator respectively by the quotient arising from its own Denominator, and the products will be the Numerators required.

Examp. 1. Reduce $\frac{5}{12}$ and $\frac{7}{18}$ to equivalent Fractions, having the least common Denominator possible.

By *Art.* 243.

6	12 . 18
2	2 . 3
3	1 . 3
	. 1

Therefore $6 \times 2 \times 3 = 36$ the common Denominator.

Then $\frac{36}{12} = 3$. And $\frac{36}{18} = 2$.
And $(3 \times 5 =)15$. And $(7 \times 2 =)$ 14, are the new Numerators.

Therefore $\frac{15}{36}$ and $\frac{14}{36}$ are the Fractions required.

Examp. 2. Reduce $\frac{2}{3} . \frac{5}{6} . \frac{4}{9} . \frac{7}{12} . \frac{11}{18}$ and $\frac{13}{36}$ to equivalent Fractions having the least common Denominator possible.

3	3 . 6 . 9 . 12 . 18 . 36
3	1 . 2 . 3 . 4 . 6 . 12
2	2 . 1 . 4 . 2 . 4
2	1 . . 2 . 1 . 2
	1 1

Therefore $3 \times 3 \times 2 \times 2 = 36$ is the common Denominator.

Then 36 divided severally by	3 . 6 . 9 . 12 . 18 and 36.
Will quote ————	12 . 6 . 4 . 3 . 2 and 1.
Which Quotients multiplied respectively by ————	2 . 5 . 4 . 7 . 11 and 13.
Produce the new Numerators	24 . 30 . 16 . 21 . 22 and 13.

And $\frac{24}{36} . \frac{30}{36} . \frac{16}{36} . \frac{21}{36} . \frac{22}{36}$. and $\frac{13}{36}$ are the Fractions required.

Examp. 3. Reduce $\frac{5}{8} . \frac{7}{12} . \frac{4}{9} . \frac{2}{3} . \frac{5}{6}$ and $\frac{3}{4}$ to the least common Denominator possible.

2	8 . 12 . 9 . 3 . 6 . 4
2	4 . 6 . 9 . 3 . 3 . 2
2	2 . 3 . 9 . 3 . 3 . 1
3	1 . 3 . 9 . 3 . 3
3	1 . 3 . 1 . 1
	1

Therefore $2 \times 2 \times 2 \times 3 \times 3 = 72$ is the common Denominator.

Then 72, divided ſeverally by	8 . 12 . 9 . 3 . 6 . 4
Will quote	9 . 6 . 8 . 24 . 12 . 18
Thoſe multiplied by . . .	5 . 7 . 4 . 2 . 5 . 1
Produce	45 . 42 . 32 . 48 . 60 . 18

Anſwer $\frac{45}{72}$. $\frac{42}{72}$. $\frac{32}{72}$. $\frac{48}{72}$. $\frac{60}{72}$ and $\frac{18}{72}$.

275. A compound Fraction (otherwiſe called a Fraction of a Fraction) may be reduced to a ſingle Fraction in this manner, *viz.* multiply all the Numerators continually, and take the product for a new Numerator; alſo multiply all the Denominators continually, and the product will be a new Denominator.

Thus, if the compound Fraction $\frac{2}{3}$ of $\frac{3}{4}$ be given to be reduced to a ſingle Fraction; multiply the Numerators 2 and 3, one by the other, ſo is the product 6 a new Numerator. Alſo multiplying the Denominator 3 and 4 one by the other, the product 12 is a new Denominator, ſo $\frac{6}{12}$ or $\frac{1}{2}$ is the ſingle Fraction ſought, being equivalent to $\frac{2}{3}$ of $\frac{3}{4}$, the compound Fraction given to be reduced.

$\frac{2}{3}$ of $\frac{3}{4}$
$\frac{6}{12}$ or $\frac{1}{2}$

In like manner, this compound Fraction $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ will be reduced to $\frac{24}{60}$, or $\frac{2}{5}$; for the Numerators 2, 3, 4, being multiplied continually, produce the new Numerator 24; and the Denominators 3, 4, 5, multiplied continually, produce the new Denominator 60: Laſtly, the new Fraction $\frac{24}{60}$ (by *Art.* 261.) will be reduced to $\frac{2}{5}$, which is equal to $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$: But to make the meaning hereof more evident, ſuppoſe the integer to be 1 pound of *Engliſh* money; then

$\frac{4}{5}$ of 1 *l.* (*viz.* of 20 *s.*) is 16 *s.*

$\frac{3}{4}$ of thoſe (*viz.* of 16 *s.*) is 12 *s.*

$\frac{2}{3}$ of thoſe $\frac{3}{4}$ (*viz.* of 12 *s.*) is 8 *s.* or $\frac{2}{5}$ *l.* whereby 'tis manifeſt that $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ *l.* is equal to $\frac{2}{5}$ *l.*

276. In operations of this kind it will frequently happen, that the Reduction will be performed more ſpeedily, if the factors be only connected by ($\times$) the ſign of Multiplication, without actually performing the operation of multiplying: Thus, in the laſt Example ſay $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5} = \frac{2\times3\times4}{3\times4\times5}$; then this expreſſion will by *Art.* 266. be reduced to $\frac{2}{5}$, only by cancelling (3×4) the Factors common to both Numerator and Denominator.

277. By this Rule a Fraction or mix'd number of a leſſer name may be reduced to a Fraction of a greater name. As if $3\frac{1}{2}$ pence be propoſed to be reduced to a compound Fraction of a pound Sterling, the operation will be in this manner, *viz.* $3\frac{1}{2}$ or $\frac{7}{2}$ of a penny is $\frac{7}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound Sterling; which compound Fraction will, by the aforeſaid Rule, be reduced to $\frac{7}{480}$*l.* In like manner, $42\frac{3}{16}$ minutes of an hour are equal to $\frac{45}{64}$ of an hour.

For $42\frac{3}{16} = \frac{675}{16}$, by *Art.* 268.

And $\frac{675}{16}$ of $\frac{1}{60} = \frac{675}{16\times60}$, by *Art.* 275. and 276.

But $\frac{675}{16\times60} = \frac{135}{16\times12} = \frac{45}{16\times4} = \frac{45}{64}$ by *Art.* 266.

Here you may alſo obſerve, that when a compound Fraction is one of the given terms in any queſtion, it is firſt of all to be reduced to a ſingle Fraction.

278. Two or more Fractions being given, there may be whole numbers found, which ſhall have the ſame reaſon or proportion as the Fractions given, *viz.* when the Fractions given have unequal Denominators, reduce them to equivalent Fractions, which have a common Denominator (by *Art.* 274.) then, rejecting the common Denominator, the Numerators will have the ſame reaſon or proportion as the Fractions firſt propoſed.

So $\frac{3}{5}$ and $\frac{5}{8}$ being given, will firſt of all be reduced to their equivalent Fractions $\frac{24}{40}$ and $\frac{25}{40}$; then rejecting the common Denominator 40, the Numerators, 24 and 25 have the ſame reaſon with $\frac{3}{5}$ and $\frac{5}{8}$, *viz.* as $\frac{3}{5}$ is to $\frac{5}{8}$, ſo is 24 to 25: Alſo if the Fractions $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$ were propoſed, there would be found 1, 2, and 4, which are in the ſame proportion one to the other as thoſe Fractions given: In like manner, if mixt numbers be given, there may be whole numbers found which ſhall have the ſame reaſon or proportion, as the mixt numbers; ſo $5\frac{2}{3}$ and $3\frac{5}{9}$ being given, will be firſt reduced to the improper Fractions $\frac{17}{3}$ and $\frac{29}{8}$ (by *Art.* 268.) Alſo the ſaid $\frac{17}{3}$ and $\frac{29}{8}$ will be reduced to $\frac{136}{24}$ and $\frac{87}{24}$; then rejecting the common Denominator 24, the Numerators 136 and 87 will have the ſame reaſon as $5\frac{2}{3}$ and $3\frac{5}{8}$, *viz.* as 136 is to 87, ſo is $5\frac{2}{3}$ to $3\frac{5}{8}$. Alſo $16\frac{1}{2}$ and 18 being given, there will be found 33 and 36, which being divided by their common meaſure 3, will give 11 and 12, which have the ſame reaſon as $16\frac{1}{2}$ and 18.

CHAP. XIX.

ADDITION *of* VULGAR FRACTIONS, *and* MIXT NUMBERS.

279. WHEN the numbers given to be added are ſingle Fractions, and have equal Denominators, add all the Numerators together; ſo is the ſum the Numerator of a Fraction, whoſe Denominator is the ſame with the common Denominator, which new Fraction is the ſum of the Fractions given to be added.

So $\frac{3}{9}$ and $\frac{2}{9}$ being propoſed to be added, their ſum will be found $\frac{5}{9}$, *viz.* the ſum of the numerators, 3 and 2 is 5, which being placed over the common Denominator 9, gives $\frac{5}{9}$: In like manner, the ſum of theſe Fractions $\frac{7}{8}$, $\frac{5}{8}$, $\frac{3}{8}$, and $\frac{2}{8}$, will be found $\frac{17}{8}$, which (by *Art.* 271.) will be found equivalent to $2\frac{1}{8}$; ſo that $2\frac{1}{8}$ is the ſum of the Fractions given to be added.

280. When the Fractions, propoſed to be added, have unequal Denominators, they are firſt to be reduced to Fractions of the ſame value, which ſhall have a common Denominator (by *Art.* 274.) and then they may be added as above.

So if $\frac{2}{3}$ and $\frac{3}{5}$ were given to be added, their ſum will be found $1\frac{4}{15}$; for $\frac{2}{3}$ and $\frac{3}{5}$ will be reduced to their equivalent Fractions $\frac{10}{15}$ and $\frac{9}{15}$, which having equal Denominators may be added according to *Art.* 279. and ſo the ſum will be found $1\frac{4}{15}$: In like manner, the ſum of theſe Fractions $\frac{12}{28}$ and $\frac{3}{4}$ will be found $1\frac{5}{28}$. Alſo the ſum of theſe ſix Fractions $\frac{13}{16}$, $\frac{11}{18}$, $\frac{7}{12}$, $\frac{4}{9}$, $\frac{5}{6}$, $\frac{2}{3}$, after they are reduced to a common Denominator (according to Example 2d. in *Art.* 274.) will be found $\frac{126}{36}$, that is, $3\frac{1}{2}$.

281. When any of the Fractions given to be added is a compound Fraction, ſuch compound Fraction is firſt of all to be reduced to a ſingle Fraction (by *Art.* 275.) and then you may proceed as before.

So

So $\frac{3}{5}$ and $\frac{2}{3}$ of $\frac{1}{4}$ being given to be added, their ſum will be found $\frac{23}{30}$; for the compound Fraction $\frac{2}{3}$ of $\frac{1}{4}$ will be reduced to $\frac{1}{6}$; which added to the ſingle Fraction $\frac{3}{5}$ gives $\frac{23}{30}$. Here you may obſerve, that the Fractions given to be added in all the former caſes, are ſuppoſed to be Fractions of integers, which have one and the ſame particular Denomination, *viz.* if one of the fractions given to be added, be a Fraction of a pound Sterling, all the reſt ought to be Fractions of a pound Sterling, and the like is to be underſtood of other denominations.

282. When Fractions of Integers of different Denominations are given to be added, they are firſt of all to be reduced to Fractions of Integers that have one and the ſame particular Denomination, (by *Art.* 277.) and then they may be added by the firſt or ſecond Rule of this Chapter.

So if $\frac{7}{9}$ of a pound Sterling, $\frac{3}{5}$ of a ſhilling, and $\frac{5}{8}$ of a penny were given to be added, reduce the two latter into Fractions of a pound Sterling, *viz.* $\frac{3}{5}$ of a ſhilling is $\frac{3}{5}$ of $\frac{1}{20}$ of a pound Sterling, which compound Fraction being reduced to a ſingle Fraction gives $\frac{3}{100}$ *l.* Likewiſe $\frac{5}{8}$ of a penny is $\frac{5}{8}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound Sterling, which compound Fraction being reduced, gives $\frac{1}{384}$ *l.* Laſtly $\frac{7}{9}$ *l.* $\frac{3}{100}$ *l.* and $\frac{1}{384}$ *l.* being added according to *Art.* 280. their ſum will be found $\frac{23339}{28800}$ *l.*

283. When mixt numbers are given to be added, find firſt of all the ſum of the Fractions (by *Art.* 279. and 280.) then add the integer or integers (if there be any found) in the ſum of the Fractions, to the whole numbers.

So if $3\frac{1}{2}$, $4\frac{1}{3}$, and $16\frac{5}{8}$ were given to be added, their ſum will be found $24\frac{11}{24}$, *viz.* the ſum of the Fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{5}{8}$, will be found (by *Art.* 280.) to be $1\frac{11}{24}$, and the ſum of the whole numbers 3, 4, and 16, is 23, to which adding 1 (the integer found in the ſum of the Fractions) the ſum is 24; ſo that $24\frac{11}{24}$ is the ſum of the mixt numbers given to be added.

CHAP. XX.

SUBTRACTION *of* VULGAR FRACTIONS *and* MIXT NUMBERS.

284. WHEN the numbers given are both ſingle Fractions, and have equal Denominators: Subtract the leſſer Numerator from the greater, and place the remainder over the common Denominator, ſo is ſuch new Fraction the difference between the Fractions given.

Thus the difference betwixt the Fractions $\frac{9}{11}$ and $\frac{7}{11}$ is $\frac{2}{11}$, which is found by Subtracting the leſſer Numerator 7, from the greater Numerator 9, and placing the remainder 2 over the common Denominator 11: Alſo the difference between the Fractions $\frac{11}{21}$ and $\frac{17}{21}$ is $\frac{6}{21}$; that is the Fraction $\frac{17}{21}$ exceeds $\frac{11}{21}$ by $\frac{6}{21}$.

285. When the numbers given are both ſingle Fractions, and have not a common Denominator, reduce them to Fractions of the ſame value which have a common Denominator, (by *Art.* 274.) and then find their difference by the laſt Rule.

So the difference between the Fractions $\frac{6}{7}$ and $\frac{7}{8}$, will be found $\frac{1}{56}$, *viz.* reducting the Fractions given to their equivalent Fractions $\frac{48}{56}$ and $\frac{49}{56}$, which have a common Denominator, the difference ſought will be found $\frac{1}{56}$ by *Art.* 284. Likewiſe $\frac{7}{12}$ being ſubtracted from $\frac{11}{13}$ there remains $\frac{41}{156}$.

286. When one of the numbers given is a whole number or a mixt number, alſo when both of them are mixt numbers; reduce ſuch whole or mixt numbers to an improper Fraction or Fraictons, by *Art.* 268. and 269. and then the operation will be according to *Art.* 284. or 285.

So $7\frac{3}{5}$ being given to be ſubtracted from 12, the remainder will be found $4\frac{2}{5}$, *viz.* firſt $7\frac{3}{5}$ will be reduced to the improper Fraction $\frac{38}{5}$, alſo 12 will be reduced to $\frac{12}{1}$, then theſe two improper Fractions $\frac{38}{5}$ and $\frac{12}{1}$, will be reduced to their equivalent Fractions $\frac{38}{5}$ and $\frac{60}{5}$, (which have a common Denominator.) Laſtly, the difference between $\frac{38}{5}$ and $\frac{60}{5}$, is $\frac{22}{5}$, or $4\frac{2}{5}$: In like manner $9\frac{1}{2}$ being given to be ſubtracted from $12\frac{1}{5}$, the remainder will be found $2\frac{7}{10}$, as by the ſubſequent operation is manifeſt.

9

$9\frac{1}{2} = \frac{19}{2}$; and $12\frac{1}{5} = \frac{61}{5}$ by *Art.* 268.
$\frac{19}{2} = \frac{95}{10}$ and $\frac{61}{5} = \frac{122}{10}$ by *Art.* 274.
Then $\frac{122}{10} - \frac{95}{10} = \frac{27}{10} = 2\frac{7}{10}$ by *Art.* 284.

Though the three last Rules are sufficient for all cases in Subtraction of Fractions, mixt numbers, or whole and mixt; nevertheless, the following Rules will be more expeditious in the Subtraction of mixt numbers, or whole and mixt, especially when the Integers consist of many places, as is manifest by the operation, *viz.*

287. When a whole number is given to be Subtracted from a mixt number, subtract the said whole number from the integer or integers of the mixt number, and unto the remainder annex the fractional part of the mixt number given; so is the mixt number thus found, the remainder or difference sought.

As if 7 be given to be subtracted from $24\frac{5}{8}$, the remainder will be $17\frac{5}{8}$, as appears from the operation.

$$\begin{array}{r} 24\frac{5}{8} \\ 7 \\ \hline 17\frac{5}{8} \end{array}$$

288. When a Fraction is given to be subtracted from unity, subtract the Numerator from the Denominator, and place that which remains over the Denominator, which new Fraction thus found, is the remainder or difference sought.

So $\frac{3}{5}$ being subtracted from unity, or 1, the remainder is $\frac{2}{5}$: Also $\frac{13}{19}$ being subtracted from 1, the remainder is $\frac{6}{19}$.

289. When a Fraction is given to be subtracted from a whole number greater than 1, subtract the said Fraction from one (by the last Article) so the remaining Fraction being annexed to the number of integers lessened by unity or 1, gives the remainder or difference sought.

Thus $\frac{5}{7}$ being subtracted from 17, the remainder is $16\frac{2}{7}$; also $\frac{7}{12}$ being subtracted from 39, the remainder is $38\frac{5}{12}$.

290. When a mixt number is given to be subtracted from a whole number, subtract first of all (by *Art.* 288.) the fractional part of the mixt number from an unit, and set down the remaining Fraction; then adding the unit borrowed to the integer or integers of the mixt number, subtract the said sum from the whole number given (as is taught in Subtraction of whole numbers;) so that which remains, together with the remaining Fraction before found, is the remainder or difference sought.

So if $9\frac{7}{12}$ be subtracted from 50, the remainder is $40\frac{5}{12}$, as by the operation is manifest.

$$\begin{array}{r} 50 \\ 9\frac{7}{12} \\ \hline 40\frac{5}{12} \end{array}$$

291. When a Fraction is given to be ſubtracted from a mixt number, and the ſaid Fraction is leſs than the fractional part of the mixt number; ſubtract the leſſer Fraction from the greater by *Art.* 284, or 285. then the remaining Fraction being annexed to the integer or integers of the mixt number, gives the remainder or difference ſought for.

So $\frac{5}{9}$ being ſubtracted from $12\frac{7}{8}$, the remainder is $12\frac{23}{72}$, as by the operation is manifeſt.

$$\begin{array}{r} 12\frac{7}{8} \\ 0\frac{5}{9} \\ \hline 12\frac{23}{72} \end{array}$$

292. When a Fraction is given to be ſubtracted from a mixt number, and the ſaid Fraction is greater than the fractional part of the mixt number, ſubtract the ſaid greater Fraction from an unit borrowed from the mixt number (by *Art.* 288.) and add the remaining Fraction to the fractional part of the mixt number, (by *Art.* 279, or 280.) ſo the Fraction found by that addition, being annexed to the integers of the mixt number leſſened by an unit, or 1, gives the remainder or difference ſought.

Thus $\frac{5}{9}$ being ſubtracted from $13\frac{3}{8}$; the remainder is $12\frac{59}{72}$, *viz.* ſubtracting $\frac{5}{9}$ from 1, the remainder is $\frac{4}{9}$, which added to $\frac{3}{8}$ gives $\frac{59}{72}$, which being annexed to 12, (the number of integers in the mixt number leſſened by 1 or unity) gives $12\frac{59}{72}$, the remainder ſought.

$$\begin{array}{r} 13\frac{3}{8} \\ 0\frac{5}{9} \\ \hline 12\frac{59}{72} \end{array}$$

293. When a mixt number is given to be ſubtracted from a mixt number, and the fractional part of the mixt number to be ſubtracted, is leſs than the fractional part of the mixt number from which you are to ſubtract, ſubtract the ſaid leſſer Fraction from the greater (by *Art.* 284, or 285.) and ſet down the remaining Fraction: Alſo ſubtract the integers of the leſſer mixt number from the integers of the greater (as in Subtraction of whole numbers;) ſo is the mixt number thus found, the remainder or difference ſought for.

So if $17\frac{3}{8}$ be given to be ſubtracted from $20\frac{5}{7}$; the remainder will be found $3\frac{19}{56}$, *viz.* ſubtracting $\frac{3}{8}$ from $\frac{5}{7}$ the remainder is $\frac{19}{56}$, alſo ſubtracting 17 from 20, the remainder is 3.

$$\begin{array}{r} 20\frac{5}{7} \\ 17\frac{3}{8} \\ \hline 3\frac{19}{56} \end{array}$$

294. When a mixt number is given to be ſubtracted from a mixt number, and the fractional part of the mixt number to be ſubtracted, is greater than the fractional part of the mixt number from which you are to ſubtract; ſubtract the ſaid greater Fraction from an unit borrowed from the greater mixt number (by *Art.* 288.) and add the remaining Fraction to the fractional part of the greater mixt number (by *Art.* 279, or 280.) ſo is the ſum to be reſerved as the fractional part of the remainder

ſought: Then add the unit borrowed to the integer or integers of the leſſer mixt number, and ſubtract the ſum from the integers of the greater mixt number (as in Subtraction of whole numbers;) ſo that which remains, together with the Fraction before reſerved, is the remainder or difference ſought for.

Thus, if $20\frac{7}{8}$ be given to be ſubtracted from $35\frac{3}{5}$, the remainder will be found $14\frac{29}{40}$, *viz.* ſubtracting $\frac{7}{8}$ from an integer, or 1, the remainder is $\frac{1}{8}$, which added to $\frac{3}{5}$, gives $\frac{29}{40}$; then adding the integer borrowed to 20, it will be 21, which ſubtracted from 35, the remainder is 14, ſo the remainder or difference ſought, is $14\frac{29}{40}$.

$$\begin{array}{r} 35\frac{3}{5} \\ 20\frac{7}{8} \\ \hline 14\frac{29}{40} \end{array}$$

When you cannot clearly diſcern which is the greater of two Fractions, having unequal denominations, reduce them to Fractions of the ſame value which have a common Denominator, (by *Art.* 274.) and then it will be apparent which of the two Fractions is the greater. As, if it be deſired to know which of theſe two Fractions $\frac{6}{7}$ and $\frac{11}{13}$ is the greater; after they are reduced to $\frac{78}{91}$ and $\frac{77}{91}$, it is evident that the former exceeds the latter by $\frac{1}{91}$.

CHAP. XXI.

MULTIPLICATION *of* VULGAR FRACTIONS, *and* MIXT NUMBERS.

295. WHEN the numbers given to be multiplied, are both ſingle Fractions, multiply the Numerators one by the other, and take the product for a new Numerator; alſo multiply the Denominators one by the other, and the product will be a new Denominator, which new Fraction is the product ſought for.

So $\frac{7}{12}$ and $\frac{5}{8}$ being given to be multiplied, the product will be found $\frac{35}{96}$; for 7 multiplied by 5 produces 35 for a new Numerator; and 12 multiplied by 8 gives 96 for a new Denominator; alſo $\frac{5}{7}$ and $\frac{3}{7}$ being multiplied one by the other, the product will be found $\frac{15}{49}$. Here you may obſerve that in the Multiplication of proper Fractions, the product is always leſs than either of the terms given; for in multiplying by an unit, the product is equal to the Multiplicand; therefore when you multiply

multiply by a proper Fraction, that is, by leſs than an unit, the product muſt be leſs than the Multiplicand.

296. When one of the numbers given is a whole number or a mixt number; alſo when both of them are mixt numbers, reduce ſuch whole number, or mixt number or numbers to an improper Fraction or Fractions, by *Art.* 268, and 269. and then the operation will be the ſame as in the laſt Article.

So $8\frac{2}{3}$ being given to be multiplied by 5, the product will be found $43\frac{1}{3}$, *viz.* $8\frac{2}{3}$ being reduced to the improper Fraction $\frac{26}{3}$, alſo 5 into $\frac{5}{1}$, multiply 26 by 5, the product is 130 for a new Numerator: Alſo multiply 3 by 1, the product is 3, for a new Denominator, which new Fraction $\frac{130}{3}$ being reduced (according to *Art.* 271.) will be $43\frac{1}{3}$, the product ſought. In like manner, $7\frac{1}{2}$ being multiplied by $\frac{3}{5}$, the product will be found $4\frac{1}{2}$. Here obſerve, that when either of the terms given is a compound Fraction, it muſt firſt of all be reduced to a ſingle Fraction, and then the operation is as before.

297. The directions of *Articles* 276. and 266. may often be uſefully applied here: For Example, if $\frac{6}{7}$ be to be multiplied by $\frac{5}{12}$ write down the product thus $\frac{6\times 5}{7\times 12}$, which contracted by dividing both Numerator and Denominator by 6, becomes $\frac{5}{7\times 2}=\frac{5}{14}$.

298. To take any part or parts of a number propoſed, is nothing elſe but to multiply the ſaid number by the Fraction, which declares what part is to be taken: So if you deſire to know what is $\frac{5}{8}$ of 320, multiply $\frac{320}{1}$ by $\frac{5}{8}$, the product will be 200; for $\frac{320\times 5}{1\times 8}=40\times 5=200$. In like manner $\frac{2}{3}$ of $45\frac{3}{8}$ is $\frac{2}{3}\times\frac{363}{8}=\frac{2\times 363}{3\times 8}=\frac{363}{3\times 4}=\frac{121}{4}=30\frac{1}{4}$. And $\frac{1}{4}$ of $120=\frac{120}{4}=30$.

299. Hence if it be required to multiply an integer by a Fraction, multiply the integer by the Numerator of the Fraction and divide the product by the Denominator. For Example, the product of 48 by $\frac{5}{8}$ is $\frac{48\times 5}{8}=6\times 5=30$.

That the methods above directed are true, may be proved by Multiplication of whole numbers: Thus, aſſume 2 Fractions $\frac{8}{4}$ and $\frac{6}{2}$, which are ſeverally equal to 2 and 3, two whole numbers;

numbers; then it will appear that the product of the Fractions $\frac{8 \times 6}{4 \times 2} = \frac{48}{8}$ is equal to the product of the whole numbers 2×3 for $\frac{48}{8} = 6$.

300. Sometimes the Work of Multiplication, in mixt numbers, may be compendiously performed after the manner of these following Examples, *viz.* if it be required to multiply $120\frac{1}{4}$ by $48\frac{1}{2}$, first multiply the whole numbers mutually, to wit, 120 by 48, and place the particular products orderly one under the other, as in Multiplication of whole numbers; then multiply the said whole numbers first given by the Fractions alternately, *viz.* take a $\frac{1}{4}$ of 48, which is 12, also take $\frac{1}{2}$ of 120, which is 60, and place the said 12 and 60 orderly to be added to the former particular products: Lastly, add all together, and to the sum annex the product of the two Fractions; to wit, in this Example, the product of the Multiplication of $\frac{1}{4}$ by $\frac{1}{2}$, which is $\frac{1}{8}$, so the total product required will be $5832\frac{1}{8}$, as you see by the Example in the Margin. In like manner, if $18\frac{1}{2}$ be multiplied by $40\frac{1}{3}$, the product will be $746\frac{1}{6}$; and if $29\frac{1}{2}$ be multiplied by 50, the product is 1475, as you see by the Examples following.

$120\frac{1}{4}$
$48\frac{1}{2}$
960
480
12
60
$5832\frac{1}{8}$

$18\frac{1}{2}$	$29\frac{1}{2}$
$40\frac{1}{3}$	50
720	1450
20	25
6	1475
$746\frac{1}{6}$	

CHAP. XXII.

Division *of* Vulgar Fractions, *and* Mixt Numbers.

301. WHEN the numbers given are both single Fractions, multiply the Denominator of the Divisor by the Numerator of the Dividend; and take the product for a new Numerator: Also multiply the Numerator of the Divisor by

by the Denominator of the Dividend, and the product is a new Denominator; which new Fraction is the quotient sought.

So if $\frac{4}{9}$ be given to be divided by $\frac{3}{5}$, the quotient will be found $\frac{20}{27}$, *viz.* multiplying 5 by 4 the product is 20 for a new Numerator; also multiplying 3 by 9 the product is 27 for a new Denominator, so is $\frac{20}{27}$ the quotient sought. In like manner if $\frac{5}{8}$ be given to be divided by $\frac{2}{7}$, the quotient will be found $\frac{35}{16}$, that is $2\frac{3}{16}$, as you see in the Example: Here you may observe that in Division, by proper Fractions, the quotient is always greater than the Dividend; for in Division the less the Divisor is, the greater will the Quotient be; and when unity is the Divisor, the Quotient will be equal to the Dividend; therefore when the Divisor is a proper Fraction, that is less than an unit, the Quotient will be greater than the Dividend.

$\frac{3}{5})\frac{4}{9}(\frac{20}{27}$

$\frac{2}{7})\frac{5}{8}(\frac{35}{16}$

302. When one of the numbers given is a whole number, or a mixt number; also when both are mixt numbers, reduce such whole number or mixt number or numbers to an improper Fraction or Fractions, by *Art.* 268. or 269. and then the operation will be the same as in the last Article.

So if 42 be divided by $7\frac{1}{2}$, the Quotient will be found $5\frac{3}{5}$, for $7\frac{1}{2}$ and 42 will be reduced to these improper Fractions $\frac{15}{2}$ and $\frac{42}{1}$, then multiplying 42 by 2, the product 84 is a new Numerator, also multiplying 15 by 1 produces 15 for a new Denominator; so is $\frac{84}{15}=\frac{28}{5}=5\frac{3}{5}$ the Quotient sought.

303. Here also the directions of *Art.* 276. and 266. will often prove advantageous: Thus, if $\frac{25}{8}$ be to be divided by $\frac{15}{4}$, the Quotient may be represented by $\frac{4\times25}{15\times8}$, which reduced by dividing by 4 becomes $\frac{25}{15\times2}$, and that again divided by 5, becomes $\frac{5}{3\times2}=\frac{5}{6}$.

304. If a whole number be given to be divided by a Fraction, multiply the whole number by the Denominator of the Fraction, and divide the Product by the Numerator thereof: For Example, if 48 be to be divided by $1\frac{1}{7}=\frac{8}{7}$, then the Quotient will be $\frac{48\times7}{8}=6\times7=42$.

305. If two Fractions that have the same Denominator are given for a Divisor and Dividend, then the Quotient of their

Numerators

Numerators will be the number required. For Example, if $\frac{5}{8}$ be to be divided by $\frac{3}{8}$, the Quotient will be $\frac{5}{3}$.

$$\text{For } \frac{8 \times 5}{3 \times 8} = \frac{5}{3}.$$

The truth of these methods will appear by assuming 2 Fractions $\frac{16}{4}$ and $\frac{10}{5}$, which are severally equal to 2 whole numbers 4 and 2 for the Quotient of the Fractions $\frac{16}{4}$ divided by $\frac{10}{5}$, $\frac{5 \times 16}{4 \times 10}$ is equal to the Quotient of the whole number 4 divided by 2, for $\frac{80}{40} = \frac{8}{4} = \frac{4}{2} = 2$.

306. Questions to exercise the Rules of Vulgar Fractions before delivered.

Quest. 1. The difference of two numbers is $1\frac{13}{24}$, the lesser number is $2\frac{1}{8}$, what is the greater? *Answ.* $3\frac{2}{3}$ (found by Addition.)

Quest. 2. What number is that, which if added to $3\frac{5}{8}$ gives the sum $8\frac{23}{88}$? *Answ.* $4\frac{7}{11}$ (found by Subtraction.)

Quest. 3. There is in 3 bags the sum of $121\frac{9}{40}$ *l.* *viz.* in the first bag $50\frac{5}{8}$ *l.* in the second $40\frac{4}{15}$ *l.* what is in the third bag? *Answ.* $30\frac{1}{3}$ *l.* (found by Addition and Subtraction.)

Quest. 4. Two Merchants *A* and *B* having certain shares in a ship, the share of *A* is $\frac{7}{10}$ of the ship, that of *B* $\frac{9}{15}$, what is the difference between their parts? *Answ.* the share of *A* exceeds the share of *B* by $\frac{1}{10}$, (found by Subtraction.)

Quest. 5. What is $\frac{5}{8}$ of $130\frac{2}{3}$? *Answ.* $81\frac{2}{3}$ (found by Multiplication.)

Quest. 6. What number is that, which being multiplied by $\frac{3}{5}$ produces $25\frac{2}{5}$? *Answ.* $42\frac{1}{3}$. (found by Division.)

The

The Doctrine of Decimal Fractions.

CHAP. XXIII.

NOTATION *of* DECIMAL FRACTIONS.

307. IT is hard to determine, who was the firſt that brought Decimal Arithmetic to light, though it be a late invention; but without doubt it has received much improvement within the compaſs of a few years by the induſtry of Artiſts, and now ſeems to be arrived at perfection. The excellency thereof is beſt known to ſuch as can apply it to the practical part of the Mathematics, and to the Conſtruction of Tables, which depend upon ſtanding or conſtant proportions; ſuch are Trigonometrical Canons, Tables for the computing of Compound Intereſt, *&c.* in which caſes Decimal operations afford ſo great help, that, in my opinion, many ages have not produced a more uſeful invention. But it may be objected, that Decimal Arithmetic often gives an imperfect ſolution to a queſtion. This I grant, yet the anſwer ſo given may be as uſeful as that which is exactly true; for in common affairs, the loſs of $\frac{1}{1000}$ part of a grain, or of an inch, *&c.* to wit, any quantity which cannot be ſeen, is inconſiderable: But I would not be miſtaken; for in extolling Decimals, I do not cry down Vulgar Fractions; ſince experience ſhews, that Decimal Fractions are commonly abuſed, by being applied to all manner of queſtions about Money, Weights, *&c.* when indeed many queſtions may be reſolved with more facility by Vulgar Arithmetick; ſo that the right uſe of Decimals depends upon the diſcretion of the artiſt.

308. When a ſingle Fraction has for its Denominator, a number conſiſting of 1 or unity in the extreme place towards the left-hand, and nothing but a cypher or cyphers towards the right, it is more particularly called a Decimal: Of this kind are theſe that follow, *viz.* $\frac{5}{10}$, that is five tenths; $\frac{5}{100}$, five hundredth parts; likewiſe theſe are Decimal Fractions, $\frac{34}{1000}$, $\frac{205}{10000}$, $\frac{1023}{100000}$, *&c.*

309.

309. A Decimal Fraction may be expreſſed without the Denominator, by prefixing a point or comma before (to wit, on the left hand of) the Numerator; ſo $\frac{5}{10}$ may be writ thus .5, or thus ,5; and $\frac{25}{100}$ thus, .25, or thus ,25.

310. In Decimals, when the Numerator does not conſiſt of ſo many places as the Denominator has cyphers; fill up the void places, in the Numerator, with cyphers prefixed on the left-hand: So $\frac{5}{100}$ is ſet down thus, ,05; likewiſe $\frac{50}{1000}$ thus, ,050; and $\frac{205}{10000}$ thus, ,0205; likewiſe $\frac{6}{1000}$ thus, ,006.

311. In Decimals thus expreſſed, the Denominator is diſcoverable by the places of the Numerator: For if the Numerator conſiſts of one place, the Denominator conſiſts of 1 or unity with one cypher; if of two places, the Denominator conſiſts of 1, with two cyphers annexed; if of three, the Denominator conſiſts of 1 or unity, with three cyphers annexed: So the Denominator of ,25 is 100, the Denominator of .050 is 1000, and the Denominator of .096 is 1000.

312. Cyphers at the end of a Decimal do neither augment nor diminiſh the value of it: So .2, .20, .200, .2000 are Decimals, which have one and the ſame value; for $\frac{20}{100}$ being abbreviated by *Art.* 265. will be made $\frac{2}{10}$, and ſo will $\frac{200}{1000}$ or $\frac{2000}{10000}$. But cyphers placed between the ſeparating comma and the figures of the Decimal diminiſh its value: So ,5 is $\frac{5}{10}$, but ,05 is $\frac{5}{100}$.

313. Therefore Decimal Fractions are eaſily reduced to a common Denominator, (which is a troubleſome work in Vulgar Fractions;) for if all the Numerators of as many Decimal Fractions as are given, be made to conſiſt of the ſame number of places, by annexing a cypher or cyphers at the end, (that is on the right-hand) of ſuch Numerators as are defective, they will all be reduced to a common Denominator: So theſe Decimals, .2, .03, .027, which ſignify $\frac{2}{10}$, $\frac{3}{100}$, $\frac{27}{1000}$) may be reduced to theſe, .200, .030, .027, which have 1000 for a common Denominator.

314. The order of places in any Decimal proceeds from the left-hand to the right, contrary to the order of places in Integers, which is from the right-hand to the left: So in this Decimal .247 the figure 2 ſtands in the firſt place, (being the outermoſt towards the left-hand, and next to the point,) the figure 4 ſtands in the ſecond place, and 7 in the third. Alſo in this Decimal .0245 a cypher ſtands in the firſt place, 2 in the ſecond, 4 in the third, and 5 in the fourth.

315. Every place in the Numerator of a Decimal Fraction has a peculiar Denominator, or proper value, *viz.* the Deno-

minator of the firſt place is 10, of the ſecond 100, of the third 1000, *&c.* So that the firſt place of a Decimal, ſignifies tenth parts of an unit, or integer; the ſecond place, hundredth parts of an integer; the third place, thouſandth parts of an integer, *&c.* Hence it is manifeſt, that this Decimal .3254 (every place thereof being conſidered a-part by itſelf) conſiſts of .3, .02, .005, .0004, (*viz.* $\frac{3}{10}$, $\frac{2}{100}$, $\frac{5}{1000}$, $\frac{4}{10000}$, which being reduced to a common Denominator (by *Art.* 313.) will give theſe .3000, .0200, .0050, .0004, *viz.* $\frac{3000}{10000}$, $\frac{200}{10000}$, $\frac{50}{10000}$, $\frac{4}{10000}$, all which collectively make .3254 (or $\frac{3254}{10000}$.)

316. In whole numbers, the firſt place above (that is on the left-hand of) the place of units, ſignifies tens of units, but in Fractions the firſt place beneath (that is, on the right-hand of) the place of units, denotes tenth parts of 1 or unity, and is called the firſt place of Decimal parts, or place of primes; likewiſe the ſecond place above the place of units, ſignifies hundreds of units; but the ſecond place beneath the place of units expreſſes hundredth parts of 1 or unity, and is called the ſecond place of Decimals, or place of ſeconds: So that, as the value of the places in integers aſcend in a decuple proportion, from the place of units, towards the left-hand; ſo the values of the places of Decimals deſcend in a ſubdecuple proportion, beneath the place of units, towards the right-hand, *viz.* among the places of integers, every following place towards the left-hand, is ten times the value of the next preceding place: But among the places of Decimal parts, every following place towards the right-hand is one tenth part of the value of the next preceding place: All which will be evident by the following Table.

A TABLE

A TABLE *for the* NOTATION *of* Integers *and* Decimals.

INTEGERS.	*&c.*		*&c.*	
	Ten Thousands	7	*Fifth Place*	of Units.
	Thousands	3	*Fourth Place*	
	Hundreds	2	*Third Place*	
	Tens	8	*Second Place*	
	Units (under 10.)	5	*First Place*	
DECIMAL PARTS.	*Tenth Parts*	. 8	*First Place*	of 1, or Unity.
	Hundredth Parts	2	*Second Place*	
	Thousandth Parts	3	*Third Place*	
	Ten thousandth Parts	7	*Fourth Place*	
	&c.		*&c.*	

317. In the foregoing Table, you may obſerve, that the places of integers or whole numbers are ſeparated from the places of Decimal parts by a point; ſo the number on the left-hand of the point expreſſes 73285 integers or units: But the number on the right-hand of the point only ſhews 8237 parts of 1 (or an integer) ſuppoſed to be divided into 10000 equal parts. In like manner, this number 5.8 ſignifies 5 integers and eight tenth parts of an integer, and this number 285,82 denotes 285 integers (or units) and $\frac{82}{100}$ parts of an integer.

318. Hence it follows, that if the ſeparating comma, in any mix'd or fractional expreſſion, be moved one place towards the left-hand, then every figure, and conſequently the whole expreſſion, is but a tenth part of what it was before; that is, it is divided by 10; if it be moved two places, the expreſſion is divided by 100; if three places, by 1000, *&c.*

But if the ſeparating comma be moved towards the right-hand, then every figure, and the whole expreſſion, is multiplied by 10, 100, 1000, &c. according as it is moved one, two, or three places.

CHAP. XXIV.

Concerning the Reduction *of* Vulgar Fractions *to* Decimal Fractions.

319. IF the greateſt integer of money, as alſo of weight meaſure, &c. were ſubdivided decimally, to wit, a pound of *Engliſh* money into ten equal pieces of coin, and every one of theſe into ten other equal pieces, &c. and weights, meaſures, &c. after the ſame manner: The doctrine of Arithmetic would be taught, with much more eaſe and expedition than now it is; but it being improbable that ſuch a reformation will ever be brought to paſs. I ſhall proceed in directing a courſe to the ſtudious, for obtaining the frugal uſe of ſuch Decimal Fractions as are in their power.

320. Since, in Arithmetical Queſtions, ſome of the given numbers, for the moſt part, happen to be Fractions, a way muſt be ſhewed how to reduce a Vulgar Fraction to a Decimal Fraction; yet in ſome caſes there is no need of this reduction: For example, a foot in length is commonly ſubdivided into 12 inches, an inch into 4 quarters, and every quarter into 2 half quarters; but a foot may eaſily, and a great deal more commodiouſly, be divided, firſt into ten equal parts, and then every of thoſe into ten other equal parts; and each of theſe into ten other equal parts; (or at leaſt ſuch diviſion may be ſuppoſed or imagined when it cannot actually be made. This foot in length ſo divided, being applied to the ſides of Superficial Figures, or Solids, will at firſt ſight give the quantities of lines in feet and Decimal parts of a foot (as readily as a foot, vulgarly divided, will ſhew you how many feet, inches quarters, and half-quarters, are contained in any line) from whence the ſuperficial or ſolid content may be found in feet by Multiplication only; and how much this excels the Vulgar way, I ſhall partly manifeſt hereafter. The like Subdiviſion would have to be made of a yard, pearch, &c.

321

321. A Vulgar Fraction may be reduced to a Decimal of the ſame value, or infinitely near (for all Vulgar Fractions cannot be exactly reduced to Decimals) by the Rule of Three Direct; for as the Denominator of any Fraction whatſoever, is to the Numerator thereof, ſo is any other propoſed Denominator to its correſpondent Numerator: *Example*, Let it be required to reduce $\frac{5}{8}$ to a Decimal, whoſe Denominator is aſſigned to be 1000: Say, by the Rule of Three, if the Denominator 8, has 5 for a Numerator; what will the Denominator 1000 require for a Numerator? Multiply and divide as the Rule of Three Direct requires, ſo will the fourth proportional be found to be 625, which is the Numerator ſought; therefore $\frac{625}{1000}$, or ,625, is a Decimal Fraction equal in value to $\frac{5}{8}$. Another *Example*, Let it be required to reduce $\frac{7}{240}$ to a Decimal Fraction, whoſe Denominator ſhall be 100000; ſay by the Rule of Three, if 240 the Denominator give 7 for a Numerator, what will the Denominator 100000 require for a Numerator? *Anſw.* 2916, and ſomewhat more.

322. Hence may be deduced the following Rule to reduce a Vulgar Fraction to a Decimal.

To the Numerator, annex as many cyphers as may be convenient; then divide by the Denominator, and the Quotient will be the Decimal required. *Note*, There muſt always be as many figures in the quotient, as there are cyphers annexed to the Numerator; and if ſo many do not ariſe, ſupply the deficiency by placing cyphers after the ſeparating comma.

Examp. 1. What is the Decimal equal to $\frac{1}{2}$?

Denominator 2) 1,0 = the Numerator, with a cypher annexed.
,5 = the Decimal required.

Examp. 2. What is the Decimal equal to $\frac{1}{4}$?

Denominator 4) 1,00 = the Numerator with two cyphers.
,25 = the Decimal required.

Examp. 3. What is the Decimal equal to $\frac{1}{5}$?

Denominator 5) 1,0 = the Numerator with a cypher.
,2 = the Decimal required.

Examp. 4. What is the Decimal equal to $\frac{1}{25}$?

Denominator 25) 1,00 = the Numerator with two cyphers.
,04 = the Decimal required.

Examp. 5. What is the Decimal equal to $\frac{1}{9}$?

Denominator 9) 1,00000 &c. = the Numerator with cyphers.
,11111 &c. = the Decimal required.

323. If the numbers of cyphers, annexed to the Numerator of the laſt Example, had been ever ſo great, and the diviſion (by the Denominator 9) had been proſecuted till thoſe cyphers had been all exhauſted; the Quotient, or Decimal required, would conſiſt of as many ones as there had been cyphers annexed, and there would ſtill be an unit remaining; by annexing cyphers to which; the diviſion might be continued farther.

324. Thoſe Decimals which are produced from Vulgar Fractions, whoſe Numerators, with cyphers annexed, can be meaſured by their Denominators, are called finite or terminate Decimals; becauſe they conſiſt of a finite and determinate number of places: Such are the Decimals reſulting from the four firſt Examples.

325. Thoſe Decimals which are produced from Vulgar Fractions, whoſe Numerators, with cyphers annexed, cannot be meaſured by their Denominators, are called infinite, interminate, circulating, and repeating Decimals; becauſe they do not conſiſt of a finite and determinate number of places, and becauſe ſome figure or figures do continually circulate or repeat in them; ſuch is the Decimal reſulting from the fifth Example.

326. Hence it follows, that no Vulgar Fraction will produce a finite Decimal, but ſuch, whoſe Denominators are compounded of the primes 2 and 5; for no other numbers can meaſure 10, 100, 1000, 10000, &c. or their Multiples, and ſuch every Numerator becomes, by annexing cyphers thereto.

327. The finite Decimals, ariſing from ſuch Vulgar Fractions, will conſiſt of as many places of figures, as there are tens, twos or fives, in the compoſition of their Denominators. Thus $\frac{1}{10}$, $\frac{1}{5}$, and $\frac{1}{2}$, have finite Decimals of 1 place, *viz.* ,1 ,2 and ,5; $\frac{1}{100}$, $\frac{1}{25}$, and $\frac{1}{4}$, have finite Decimals of 2 places, *viz.* ,01 ,04 and ,25 &c.

328. In circulating Decimals, if one figure only repeats, it is called a ſingle Repetend: Thus in *Examp.* 5th. the Decimal, conſiſting of the repeating figure 1, is a ſingle Repetend.

329. To avoid the trouble of writing down unneceſſary figures, a ſingle Repetend is denoted by the repeating digit daſhed; that is, the Decimal ,11111 &c. conſiſting of the ſingle Repetend 1, is expreſſed by ,$\acute{1}$.

330. Since ,1 is the Decimal equivalent to the Vulgar Fraction $\frac{1}{9}$; therefore ,$\acute{2}$ will be the Decimal equal to $\frac{2}{9}$; ,$\acute{3}$, to ($\frac{3}{9}$ =) $\frac{1}{3}$; ,$\acute{4}$, to $\frac{4}{9}$; ,$\acute{5}$, to $\frac{5}{9}$; ,$\acute{6}$, to ($\frac{6}{9}$ =) $\frac{2}{3}$; ,$\acute{7}$, to $\frac{7}{9}$; ,$\acute{8}$, to $\frac{8}{9}$; and ,$\acute{9}$, to ($\frac{9}{9}$ = $\frac{1}{1}$ =) 1.

Therefore in all operations for $\dot{9}$=0,9999 &c. *ad infinitum*, put an unit.

331. *Examp.* 6. What is the Decimal equal to $\frac{1}{6}$?
Denominator 6) 1,00000 &c. = Numerator with cyphers.
,16666 &c. = ,1$\dot{6}$=the Decimal required.

Examp. 7. What is the Decimal equal to $\frac{1}{12}$?
Denominator 12) 1,000000 &c. = Numerator with cyphers.
,083333 &c. =,08$\dot{3}$=the Decimal requir'd.

Examp. 8. What is the Decimal equal to $\frac{1}{15}$?
Denominator 15) 1,000000 &c. = Numerator with cyphers.
,066666 &c. =,0$\dot{6}$= the Decimal requir'd.

Examp. 9. What is the Decimal equal to $\frac{1}{75}$?
Denominator 75) 1,000000 &c. =Numerator with cyphers.
,013333 &c. =,01$\dot{3}$=the Decimal requir'd.

The Decimals resulting from the 4 last operations are called mixt repeating Decimals, or mixt single Repetends, because they have a finite number before the single Repetend begins.

332. And, from the four last Examples, it will appear, that the finite part, of mix'd repeating Decimals, arise because the Denominators, of their corresponding Vulgar Fractions, are partly compounded of the primes 2 and 5; for in *Examp.* 6. the Denominator 6 = 2 × 3; in *Examp.* 7. 12 = 4 × 3; in *Examp.* 8. 15 = 5 × 3; and in *Examp.* 9. 75 = 25 × 3. And it is farther observable, that the finite parts of the mixt Decimals in these Examples, consist of as many places, as the finite Decimals before found in Examples 1. 2. 3. and 4. where the Denominators were respectively 2. 4. 5. and 25. which Denominators in these Examples, are severally multiplied by 3.

333. From these Examples, also, may be deduced the manner of expressing the value of mixed Repetends, in the form of Vulgar Fractions, *viz.* the finite part has a decimal Denominator, as in *Art.* 311. but the repeating part has 9 for its Denominator, by *Art.* 330. and should be esteemed only a part of the last finite Denomination.

So in the expression of Example the 6th. ,1$\dot{6}$=$\frac{1}{6}$, the finite part ,1 is $\frac{1}{10}$; and the Repetend, had it stood in the place of primes, would be $\frac{6}{9}$, but because the finite part of the expression is in that place, it must be esteemed only $\frac{6}{9}$ of $\frac{1}{10}$ or $\frac{6}{90}$: Now $\frac{1}{10}+\frac{6}{90}=\frac{9}{90}+\frac{6}{90}=\frac{15}{90}=\frac{1}{6}$.

In like manner for the expreſſion of the 7th Example, $,08\not{3} = \frac{1}{12}$ the finite part, $,08 = \frac{8}{100}$; and the Repetend, if it were in the place of primes would be $\frac{3}{9}$; but the laſt figure of the finite part being in the place of hundredths, therefore the Repetend is $\frac{3}{9}$ of $\frac{1}{100}$ or $\frac{3}{900}$: Now $\frac{8}{100} + \frac{3}{900} = \frac{72}{900} + \frac{3}{900} = \frac{75}{900} = \frac{1}{12}$.

334. *Examp.* 10. What is the Decimal equal to $\frac{1}{99}$?

```
99) 1,00000000 &c. (,01010101 &c. the Decimal required.
    99
    ---
     100
      99
     ---
      100
       99
      ---
       100
        99
       ---
         1 &c.
```

'Tis apparent, that how long ſoever the Diviſion in the laſt Example be continued, the two Quotient figures (01) will repeat once, for every two cyphers annexed to the Dividend.

335. And hence we have the generation of Repetends conſiſting of 2 figures, which (as well as thoſe of 3, 4, 5, &c. figures) are called compound Repetends.

336. Compound Repetends are diſtinguiſhed by daſhing the firſt and laſt repeating figure: Thus ,010101 &c. *ad infinitum*, may be wrote $,\not{0}\not{1}$, and ,123123123 &c. *ad infinitum*, $,\not{1}2\not{3}$.

337. Since $\frac{1}{99} = ,\not{0}\not{1}$; $\frac{2}{99}$ will be $= ,\not{0}\not{2}$; $\frac{3}{99} = (\frac{1}{33} =)\ ,\not{0}\not{3}$; $\frac{4}{99} = ,\not{0}\not{4}$, &c. $\frac{9}{99} = (\frac{1}{11} =)\ ,\not{0}\not{9}$; $\frac{10}{99} = ,\not{1}\not{0}$; $\frac{11}{99} = (\frac{1}{9} =)\ ,\not{1}\not{1} = ,\not{1}$; $\frac{12}{99} = (\frac{4}{33} =)\ ,\not{1}\not{2}$, &c.

And, in general, a Vulgar Fraction which, being reduced to a Decimal, produces a compound Repetend of 2 figures, either is, or is equal to, a Fraction, whoſe Numerator is the two repeating figures, and Denominator 99.

338. And the Denominators of thoſe Fractions which produce mixt compound Repetends of 2 figures, ſuch as $,1\not{5}\not{0}$, $,04\not{0}\not{1}$, &c. are compound of the primes 2 or 5, and the number 99.

339. *Example* 11. What is the Decimal equal to $\frac{1}{999}$?

```
999) 1,000000000 &c. (,001001001 &c.
     999
     ---
       1000
        999
       ----
        1000
         999
        ----
           1 &c.
```

Here we have the generation of a compound Repetend consisting of 3 figures, and by reasoning, as in *Art.* 337. it will appear, that the vulgar Fractions, which produce such Decimals, are, or are equal to, those, which have the 3 repeating digits for Numerators, and 999 for a common Denominator.

340. Universally, a Decimal Fraction, consisting only of a Repetend, is equal to a Vulgar Fraction whose Numerator is that Repetend, and the Denominator a number consisting of as many nines, as there are places in the Repetend.

341. *Example* 12. What is the Decimal equal to $\frac{1}{7}$?

```
7) 1,000000000000 (0,1428571 &c.
   7
   -
   30
   28
   --
    20
    14
    --
     60
     56
     --
      40
      35
      --
       50
       49
       --
        10
         7
        --
         3 &c.
```

Since

Since the number 1, the only ſignificant figure of the Dividend, becomes the remainder, after multiplying by the ſixth quotient figure, it is clear that thoſe ſix figures will continually repeat in the quotient; therefore $\frac{1}{7} = 0,\dot{1}4285\dot{7}$.

342. Hence, if the Denominator of a proper Fraction, in its leaſt terms, be not compoſed of the primes 2 and 5, that Fraction will be equal to a repeating Decimal, whoſe Repetend will have fewer figures than there are units in the Denominator.

For, in the 12th Example, where unity, with cyphers annexed, is divided by 7; 'tis obvious, that no remainder, in any part of the diviſion, can be greater than 6: Therefore if, as in that Example, all the poſſible remainders ſhould happen before any of them return, yet after that, *viz.* after 6 figures (a number leſs than the Diviſor by unity) are obtained in the Quotient, ſome one of the former remainders muſt neceſſarily recur (for by *Art.* 326. there will be a remainder, becauſe the Diviſor cannot meaſure the Dividend); to which former remainder a cypher being annexed, as in the former part of the Diviſion, that figure muſt repeat in the Quotient, which was placed there when the ſame Dividual happened before: And therefore the other figures muſt follow and continue the circulation.

343. Having any proper Fraction given, to find whether the Decimal Fraction, equal thereto, will be finite or infinite; and if infinite, whether it will conſiſt of a pure or mixed Repetend; and how many places that Repetend will conſiſt of.

Rule. Reduce the given Fraction into its leaſt terms, and divide the Denominator by 10, 2, and 5, as often as poſſible; then make the reſult a Diviſor, and let the Dividend conſiſt of as many nines as may be neceſſary; continue the Diviſion until there be no remainder; and the number of nines uſed will ſhew the number of places in the Repetend; which Repetend will begin after as many places of figures, as there were tens, twos, or fives, divided by.

If the whole Denominator vaniſhes by repeating your Diviſions by 10, 2, or 5, the Decimal will be finite, and conſiſt of as many places as you perform Diviſions, by *Art.* 327. but if not, there will be a Repetend, by *Art.* 342. Alſo a Fraction, equal to the given one, will have a number conſiſting of as many nines in the compoſition of its Denominator, as there will be figures in the Repetend, by *Art.* 340. Therefore that number of nines will be a Multiple of the given Denominator, reduced as above.

Examp.

Examp. 1. How many places will the Repetend of that Decimal consist of, which is equal to $\frac{1}{7}$?

$$7)\,\underline{999999}$$
$$142857$$ *Answ.* 6 places.

Examp. 2. Will the Fraction $\frac{1}{51200}$ produce a finite or infinite Decimal; and of how many places will it consist if finite?

Divisions	Number of Divisions.
10) 51200	
10)5120	1
2)512	2
2)256	3
2)128	4
2)64	5
2)32	6
2)16	7
2) 8	8
2) 4	9
2) 2	10
1	11

Answ. The Decimal will be finite, and consist of 11 places.

Proof 512|00) 1,000000000|00 (0,00001953125

```
512
----
4880
4608
----
 2720
 2560
 ----
 1600
 1536
 ----
  640
  512
 ----
 1280
 1024
 ----
  2560
  2560
  ----
  0000
```

Example

Example 3. Will the Fraction $\frac{1}{880}$ be finite or infinite; if infinite, how many figures will its Repetend consist of; and how many finite places will there be, before the Repetend begins?

	Number of Divisions
10) 880	
2) 88	1
2) 44	2
2) 22	3
11	4
11) 99 (9	
99	

Answ. The Decimal will be a mix'd compound Repetend of 2 places, beginning after 4 finite places.

Proof 88|0) 1,0000|0|0(0,0011 3̇6̇

```
        88
       ---
       120
        88
       ---
       320
       264
       ---
       560
       528
       ---
       320
```

344. When a Decimal expression is finite, or circulating, it is called compleat; but if it be not finite, and no law of circulation appears, it is called approximate.

345. Approximate Decimals have some places true, and the rest uncertain, and are sometimes wrote with the signs + or —, to denote whether the last figure is less or greater than just: Thus, for 327,645 +, read 327,645 more something; that is, the true Decimal exceeds 327,645 by some uncertain figures; and for 327,646 —, read 327,646 less something; that is, the true Decimal exceeds 327,645, and is less than 327,646.

346. If in reducing a Vulgar Fraction to a Decimal, or in any other process, the Repetend of the result doth not so speedily appear as might be wished for, proceed to a determined number of places, and the result will be an approximate Decimal.

347

347. Upon the aforesaid grounds, the known or accustomary parts of Money, Weight, Measure, Time, *&c.* may be reduced to Decimals: For if you desire to know what Decimal Fraction of a pound Sterling is equal in value to one shilling, consider first that a pound is the integer, and that 20 shillings are equal to that integer, therefore 1 shilling is $\frac{1}{20}$ of a pound.

In like manner, forasmuch as 240 pence are equal to a pound of *English* money, 7 pence are $\frac{7}{240}$ parts of a pound, which Fraction will be reduced to this Decimal ,0291$\dot{6}$ *l.*

After the same method, may the vulgar Sexagenary Fractions used in Astronomy be reduced to Decimals: For since a degree is usually subdivided into sixty parts, called Minutes, or primes; a Prime or Minute, into sixty parts, called Seconds; a Second into sixty Thirds; a Third into sixty Fourths, *&c.* It is evident that 7 Minutes (or Primes) are $\frac{7}{60}$ parts of a degree, which may be reduced to the Decimal. 11$\dot{6}$.

This to the ingenious will be a sufficient light for the finding of Decimals suitable to the Shillings, Pence, and Farthings, which are under a pound Sterling: Also the Decimals of the known parts of Weight, Measure, Time, *&c.* as they are expressed in the following Table,

THE

THE

TABLE

OF

REDUCTION.

TABLET I. *Of* English *Money, the Integer being a Pound.*		*Pence with Farthings.*		*Decimals of a Pound.*
Shillings.	*Decimals of a Pound.*			
			3	.048958ʒ
			2	.047916̸
			1	.046875
		11		.0458ʒ
			3	.0447916̸
19	.95		2	.04375
18	.9		1	.042708ʒ
17	.85	10		.0416̸
16	.8		3	.040625
15	.75		2	.03958ʒ
14	.7		1	.038541 6̸
13	.65	9		.0375
12	.6		3	.036458ʒ
11	.55		2	.035416̸
10	.5		1	.034375
9	.45	8		.0ʒ
8	.4		3	.0322916̸
7	.35		2	.03125
6	.3		1	.030208ʒ
5	.25	7		.02916̸
4	.2		3	.028125
3	.15		2	.02708ʒ
2	.1		1	.0260416̸
1	.05	6		.025

3	.023958ʒ
2	.02291ϕ
1	.021875
5	.02083
3	.019791ϕ
2	.01875
1	.017708ʒ
4	.01ϕ
3	.015625
2	.01458ʒ
1	.013541ϕ
3	.0125
3	.011458ʒ
2	.01041ϕ
1	.009375
2	.008ʒ
3	.007291ϕ
2	.00625
1 Pen.& 1 Far.	.005208ʒ
1 Penny.	.0041ϕ
3 Farth.	.003125
2 Farth.	.00208ʒ
1 Farth.	.001041ϕ

TABLET II.

Of Troy-*Weight, the Integer being an Ounce.*

Penny-Weights.	*Decimals of an Ounce.*
19	.95
18	.9
17	.85
16	.8
15	.75
14	.7
13	.65
12	.6
11	.55
10	.5
9	.45
8	.4
7	.35
6	.3
5	.25
4	.2
3	.15
2	.1
1	.05

Grains.	*Decimals of an Ounce.*
23	.04791ϕ
22	.0458ʒ
1	.04375
20	.041ϕ
29	.03958ʒ
18	.0375
17	.03541ϕ
16	.0ʒ
15	.03125
14	.02916
13	.02708ʒ
12	.025
11	.02291ϕ
10	.0208ʒ
9	.01875
8	.01ϕ
7	.01458ʒ
6	.0125

5	.010416
4	.0083
3	.00625
2	.00416
1	.002083

TABLET III.

Of Avoirdupoise *great Weight, the Integer being an hundred Weight, to wit,* 112 *pounds.*

Quarters of 1 hundred.	*Decimals of 1 hundred.*
3	.75
2	.5
1	.25

Pounds.	*Decimals of 1 hundred.*
27	.2410714
26	.2321429
25	.2232143
24	.2142857
23	.2053571
22	.1964286
21	.1875
20	.1785714
19	.1696429
18	.1607143
17	.1517857
16	.1428571
15	.1339286
14	.125
13	.1160714
12	.1071429
11	.0982143
10	.0892857
9	.0803571
8	.0714286
7	.0625
6	.0535714
5	.0446429
4	.0357143
3	.0267857
2	.0178571
1	.0089286

Ounces.	*Decimals of 1 hundred.*
15	.0083705
14	.0078125
13	.0072544
12	.0066964
11	.0061383
10	.0055803
9	.0050223
8	.0044643
7	.0039062
6	.0033482
5	.0027901
4	.0022321
3	.0016741
2	.0011160
1	.0005580

Quarters of 1 Ounce.	*Decimals of 1 hundred.*
3	.0004185
2	.0002790
1	.0001395

TABLET

TABLET IV.

Of Avoirdupoise *little Weight, the Integer being a Pound.*

Ounces.	*Decimals of a Pound.*
15	.9375
14	.875
13	.8125
12	.75
11	.6875
10	.625
9	.5625
8	.5
7	.4375
6	.375
5	.3125
4	.25
3	.1875
2	.125
1	.0625

Drams.	*Decimals of a Pound.*
15	.05859375
14	.0546875
13	.05078125
12	.046875
11	.04296875
10	.0390625
9	.03515625
8	.03125
7	.02734375
6	.0234375
5	.01953125
4	.015625
3	.01171875
2	.0078125
1	.00390625

Quarters of a Dram.	*Decimals of a Pound.*
3	.00292969
2	.00195313
1	.00097656

TABLET V.

Of Liquid Measures, the Integer being a Gallon.

Pints.	*Decimals of a Gallon.*
7	.875
6	.75
5	.625
4	.5
3	.375
2	.25
1	.125

Quarters of a Pint.	*Decimals of a Gallon.*
3	.09375
2	.0625
1	.03125

TABLET VI.

Of Dry Measures, the Integer being a Quarter.

Bushels.	*Decimals of a Quarter.*
7	.875
6	.75
5	.625
4	.5
3	.375
2	.25
1	.125

Pecks.	*Decimals of a Quarter.*
3	.09375
2	.0625
1	.03125

Quarters of a Peck.	*Decimals of a Quarter.*
3	.0234375
2	.015625
1	.0078125

Pints.	*Decimals of a Quarter.*
3	.005859375
2	.00390625
1	.001953125

TABLET VII.

Of Long Measures, one Yard being the Integer.

Quarters of one Yard.	*Decimals of one Yard.*
3	.75
2	.5
1	.25

Nails.	*Decimals of one Yard.*
3	.1875
2	.125
1	.0625

Quarters of one Nail.	*Decimals of one Yard.*
3	.046875
2	.03125
1	.015625

TABLET VIII.

Of the Reduction of Inches, &c. to Decimals, the Integer being a Foot in Length.

Inches.	*Decimals of a a Foot.*
11	.916
10	.83
9	.75

8	.6̸
7	.583̸
6	.5
5	.416̸
4	.3̸
3	.25
2	.16̸
1	.083̸

Quarters of an Inch.	*Decimals of a Foot.*
3	.0625
2	.0416̸
1	.02083̸
Half a Quarter of an Inch.	.010416̸

TABLET IX.

Of Dozens, the Integer being a Grofs.

Dozens.	*Decimals of a Grofs.*
11	.916̸
10	.83̸
9	.75
8	.6̸
7	.583̸
6	.5
5	.416̸
4	.3̸
3	.25
2	.16̸
1	.083̸

Parts of a Dozen.	*Decimals of a Grofs.*
11	.07638̸
10	.0694̸
9	.0625
8	.05̸
7	.04861̸
6	.0416̸
5	.03472̸
4	.027̸
3	.02083̸
2	.0138̸
1	.00694̸

TABLET X.

Of Time, a Day being the Integer.

Hours.	*Decimals of a Day.*
23	.9583̸
22	.916̸
21	.875
20	.83̸
19	.7916̸
18	.75
17	.7083̸
16	.6̸
15	.625
14	.583̸
13	.5416̸
12	.5
11	.4583̸
10	.416̸

9	.375
8	.$\dot{3}$
7	.291$\dot{6}$
6	.25
5	.208$\dot{3}$
4	.1$\dot{6}$
3	.125
2	.08$\dot{3}$
1	.041$\dot{6}$

Minutes.	*Decimals of a Day.*
59	.04097$\dot{2}$
58	.0402$\dot{7}$
57	.03958$\dot{3}$
56	.03$\dot{8}$
55	.03819$\dot{4}$
54	.0375
53	.03680$\dot{5}$
52	.036$\dot{1}$
51	.03541$\dot{6}$
50	.0347$\dot{2}$
49	.03402$\dot{7}$
48	.0$\dot{3}$
47	.03263$\dot{8}$
46	.0319$\dot{4}$
45	.03125
44	.030$\dot{5}$
43	.02986$\dot{1}$
42	.0291$\dot{6}$
41	.02847$\dot{2}$
40	.02$\dot{7}$
39	.02708$\dot{3}$
38	.0263$\dot{8}$
37	.02569$\dot{4}$
36	.025
35	.02430$\dot{5}$
34	.0236$\dot{1}$
33	.02291$\dot{6}$
32	.0$\dot{2}$
31	.02152$\dot{7}$
30	.0208$\dot{3}$
29	.02013$\dot{8}$
28	.019$\dot{4}$
27	.01875
26	.0180$\dot{5}$
25	.01736$\dot{1}$
24	.01$\dot{6}$
23	.01597$\dot{2}$
22	.0152$\dot{7}$
21	.01458$\dot{3}$
20	.013$\dot{8}$
19	.01319$\dot{4}$
18	.0125
17	.01180$\dot{5}$
16	.0$\dot{1}$
15	.01 41$\dot{6}$
14	.0097$\dot{2}$
13	.00902$\dot{7}$
12	.008$\dot{3}$
11	.00763$\dot{8}$
10	.0069$\dot{4}$
9	.00625
8	.00$\dot{5}$
7	.00486$\dot{1}$
6	.0041$\dot{6}$
5	.00347$\dot{2}$
4	.002$\dot{7}$
3	.00208$\dot{3}$
2	.0013$\dot{8}$
1	.00069$\dot{4}$

348. This Table aforegoing conſiſts of ten ſeveral Tablets, of which the firſt (intituled *Engliſh* money) contains in the firſt column of it the particular Fractions, (*viz.* the ſhillings, pence and farthings) of a pound Sterling; and in the other column the Decimals, to which they may be reſpectively reduced: So in the ſame Tablet .65 is the Decimal anſwerable to 13 *s.* .02083 to 5 *d.* and .003125 to 3 *f.* Likewiſe, .048958 is the Decimal of 11 *d.* together with 3 farthings: Alſo .03125 is the Decimal of 7 pence half-penny.

349. The next Tablet (intitled *Troy*-weight) contains in the firſt column the particular Fractions (*viz.* the penny-weights and grains) of an ounce *Troy*, and in the other their reſpective Decimals: So .6 is the correſpondent Decimal of 12 penny-weights, and .00208 of 1 grain. Likewiſe .025 is the Decimal of 12 grains.

350. The third Tablet (intitled Avoirdupoiſe great weight,) contains in the firſt column thereof the Fractions, (*viz.* the quarters, pounds, ounces, and the quarters of an ounce of an hundred, according to Avoirdupoiſe weight,) and in the other their proper Decimals: So .5 is the Decimal of two quarters or half a hundred, .1517857 of 17 pounds, .0033482 of 6 ounces, and .0004185, the Decimal of three quarters of an ounce.

351. The fourth (intitled Avoirdupoiſe little weight) ſhews the Fractions (*viz.* the ounces, drams, and quarters of a dram) of a pound Avoirdupoiſe, together with their reſpective Decimals: So the Decimal of 3 ounces is .1875, the Decimal of 9 drams is .03515625, and the Decimal of one quarter of a dram is .00097656.

352. The fifth (intitled Liquid Meaſure) has the Fractions (*viz.* the pints and quarters of a pint) of a gallon, and likewiſe their ſeveral Decimals: So the Decimal of five pints is .625, and the Decimal of two quarters or half a pint is .0625.

353. The ſixth (intitled Dry Meaſures) gives the Fractions (*viz.* the buſhels, pecks quarters of pecks, and pints,) of a quarter, together with their peculiar Decimals: So .375 is the Decimal of 3 buſhels, .03125 of one peck, .0234375 of $\frac{3}{4}$ of a peck, and .00390625 of two pints.

354. The ſeventh (intitled Yards) offers you the Fractions (*viz.* the quarters, nails, and quarters of nails) and their reſpective Decimals: So .25 is the Decimal of one quarter of a yard, .125 of two nails, and .046875 of three quarters of a nail.

355. The eighth (intitled Reduction of Inches, *&c.* to Decimals of a foot) presents you with the Fractions *(viz.* the inches, quarters, and half quarters of an inch) of a foot, together with their correspondent Decimals: So .416 is the Decimal of 5 inches, .0625 of $\frac{3}{4}$ of an inch, and .010416 of $\frac{1}{8}$, or half a quarter of an inch.

356. The ninth Tablet (intitled Dozens) yields you the Fractions *(viz.* the dozens and particulars) of a gross, as also their respective Decimals: So .25 is the Decimal of 3 Dozen, and .04861 of 7 particulars.

357. The tenth and last Tablet (intitled Time) gives you the Fractions *(viz.* the hours and minutes) of a day; So .625 is the Decimal of 15 hours, .0375 of 54 minutes, and .000694 of one minute.

358. When a single Fraction of any of the premised Tablets is proposed to be reduced to a Decimal, find it in the first column of the Tablet to which it belongs; this done, just against that Fraction so found, you'll have the Decimal required. So 13*s.* being propounded, taking the first premised Tablet, I find 13*s.* in the first column of the Tablet of Money, and just against the same 13*s.* I observe 65, before which having prefixed a point, and by that means signified it for a Decimal, I conclude the same .65 so ordered, to be the correspondent Decimal of 13*s.* the Fraction propounded. In like manner, .022916 is the Decimal of 11 grains in the Tablet of *Troy*-weight; and .0357143 the Decimal of 4*l.* in the Tablet of Avoirdupoise great weight, *&c.*

359. When two or more Fractions are propounded, and it is required to find a Decimal equivalent to the sum of them, find the Decimal of each of the Fractions given according to the last Rule; then adding together the Decimals so found, that entire sum is the Decimal sought: So 13*s.* 5*d.* being reduced to a Decimal, is .67083; for the Decimal of 13*s.* is .65, and the Decimal of 5*d.* .02083; which being added together (by *Art.* 382.) amount to .67083, *viz.* the Decimal which represents 13*s.* 5*d.* the Fraction proposed: In like manner, the Decimal of 9 penny-weights and 13 grains, is .477083, and the Decimal of $\frac{1}{2}$ *C.* 19 *lb.* 7 *oz.* is .6735491, *&c.*

13 *s.*	.65
5 *d.*	.020833
	.670833

9 *p.wt.*

9 *p.wt.*	.45
13 *gr.*	.0270833
	.4770833

$\frac{1}{2}$ *C.*	.5
19 *lb.*	.1696429
7 *oz.*	.0039062
	.6735491

And here as you see proper Fractions reduced, so likewise may the Fractions of mixt numbers be reduced to Decimals; for example, these numbers 97 *lb.* 7 ounces, $13\frac{1}{4}$ drams; 67 gallons $5\frac{3}{4}$ pints; 28 quarters, 0 bushels, and $2\frac{1}{2}$ pecks, after Reduction are severally 97.4891, 67.7187, and 28.0781.

97.4375	67.625	28.0625
.0507	.0937	.0156
.0009		
	67.7187	28.0781
97.4891		

360. When a Decimal is proposed, to know what Fraction it represents, search the same Decimal in the second column of the Tablet, to which it belongs; where if you find it expressly, the number just against it in the first column is the Fraction you look for: So .65 (representing the Fraction of a pound Sterling,) being given, I find it in the second column of the Tablet of Money, and over-against it in the first column I find 13 *s.* which is the Fraction represented by .65, the Decimal propounded. In like manner, 3.025, (representing 3 ounces and .025 of an ounce *Troy*) being given, the number represented by it, is 3 ounces, 0 *p. wt.* 12 grains.

361. When in the second column of the Tablet to which you are directed, you cannot precisely find the Decimal proposed, search that which being less, comes nearest to it, and take the number that answers it in the first column, for the greatest Fraction of the number required: Then deducting the Decimal so found, out of the Decimal given, find likewise the remainder as another Decimal, and take its correspondent number for the next Fraction of the number required; and so proceed in that order, till you have discovered the entire number represented by the Decimal propounded.

Example, .6739 being given, I demand the Fraction of a pound Sterling repreſented by it: The Decimal in the Tablet of Money, which being leſs, comes neareſt to .6739 is .65, whoſe correſpondent number in that Tablet is 13, which are the ſhillings of the number required; then ſubtracting .65 out of .6739, the remainder is .0239, and the neareſt Decimal in the ſame Tablet to .0239 is .0208, whoſe correſpondent number is 5, which are the pence of the number required. Laſt of all, deducting .0208 out of .0239, the remainder is .0031, which gives you in the firſt column 3, being the farthings of the number required: So that I conclude the entire Fraction repreſented by the Decimal .6739, is 13 *s.* 5 *d.* 3 *f.*

	.6739 *l. Sterling,*
Subtract 13 *s.*	.65
	.0239
Subtract 5 *d.* . . .	.0208
3 *f.*	.0031

In like manner, 7.359 *C.* being reduced by the Tablet of Avoirdupoiſe great weight, is $7\frac{1}{4}$ *C.* 12 *lb.* 4 ounces: and 94.58 *lb.* reduced by the Tablet of Avoirdupoiſe little weight, is 94 *lb.* 9 ounces, and 6 drams.

	7. 359 *C.*
Subtract 1 *quarter* . . .	.25
	.109
Subtract 12 *lb.*	.107
4 *oz.*	.002
	94.58 *lb.*
Subtract 9 *oz.*	.56
6 *Drams* . . .	.02

362. But when you have not ſuch Tables as the above at hand then, *to reduce the different Denominations of Money, Weight, and Meaſure,* &c. *into Decimals,* uſe the following

Rule. 1. Write the given denominations, or parts, orderly under each other; the inferior or leaſt parts, being uppermoſt: Let theſe be Dividends.

2. Againſt each part, on the left-hand, write the number thereof, contained in one of its next ſuperior: Let theſe be Diviſors.

3. Then beginning with the upper one, write the quotient of each divifion as Decimal parts, on the right hand of the Dividend, next below it; and let this mixed number be divided by its Divifor.

Examp. 1. Reduce 10 *s.* 8 *d.* to its equivalent Decimal of a pound Sterling.

1. The given fhillings and pence wrote under each other, will ftand thus:

8
10

2. Thefe with their refpective Divifors annexed, will be

12 | 08
20 | 10

3. The divifion by 12 being performed, by fuppofing cyphers to be annexed to the Dividend 8, the work will appear as below:

12 | 8,000, &c. Or 12 | 8
20 | 10,666, &c. 20 | 10,6̸

4. Then, dividing by 20 gives the anfwer ,53̸.

12 | 8,
20 | 10,6̸
,53̸

Examp. 2. What Decimal of a pound *Troy* is equivalent to 10 *oz.* 18 *p. w.* 16 *gr.*

24 | 16,
20 | 18,6̸
12 | 10,93̸
,91

363. But the Decimals of Sterling money may be wrote in one line by the following

Rule. Write half of the greateft even number, in the given fhillings, for the place of primes.

Let the number of farthings, contained in the given pence and farthings, poffefs the places of feconds and thirds; obferving, if the given fhillings are odd, to increafe the place of feconds by 5. And to encreafe the thirds by as many units as there are times 24 in the pence and farthings.

Divide half the number of farthings in the pence and farthings (rejecting 24 or 6 pence, if there is one) by 12, the quotient written after the three places before found, will give the Decimal required.

EXAMPLES.

EXAMPLES.

1. . .	10*s.*	8*d.*	is equal to ,5$\dot{3}$*l.*
2. . .	13 :	$10\frac{1}{2}$	. . . . . ,69375*l.*
3. . .	15 :	$9\frac{3}{4}$	. . . . . ,790625*l.*
4. . .	19 :	$11\frac{1}{4}$	. . . . . ,996875*l.*
5. . .	1 :	$10\frac{1}{4}$	. . . . . ,092708$\dot{3}$*l.*
6. . .	0 :	$8\frac{3}{4}$	. . . . . ,036458$\dot{3}$*l.*
7. . .	0 :	$2\frac{1}{2}$	. . . . . ,01041$\dot{6}$*l.*
8. . .	0 :	$0\frac{3}{4}$	. . . . . ,003125*l*

One of these Examples explained, will make the Rule familiar in the 5th, *viz.* 1 *s.* $10\frac{1}{4}$ *d.* Half of one shilling is 0, write 0 in the place of primes; $10\frac{1}{4}$ is 41 farthings, and 1 added (for the 24 contained in 41) makes 42, and 50 added (for the odd shilling) makes 92, therefore the three first places of the Decimal are ,092: Now 24 taken from 41 leaves 17, its half is 8,5; which divided by 12 gives 708$\dot{3}$; these wrote as they arise, after the former three places, make ,092708$\dot{3}$ for the Decimal required.

364. A Decimal of a superior Denomination being given, to find its value in the known parts of inferior denominations.

Rule. Multiply the given Decimal by the number of parts in the next lesser Denomination; from the product cut off as many places to the right-hand, as there are in the given Decimal.

Multiply the figures above cut off to the right-hand by the parts in the next lesser Denomination, and from this product cut off as before.

And thus proceed until the least Denomination is arrived at, then the several parts cut off on the left-hand, are equivalent to the given Decimal.

Example 1. What is the value of ,53334 *l.*?

```
 ,53334
     20
 ------
10,66680
     12
 ------
 8,00160
```

Answer 10 *s.* 8 *d.*

X *Example*

Example 2. What is the value of ,91112 *lb. Troy*,

```
 ,91112
     12
-------
10,93344
     20
-------
18,66880
     24
-------
 2,67520
13,3760
-------
16,05120
```

Answ. 10*oz*, 18*p.wt.* 16*gr.*

365. But the value of the decimal part of a pound Sterling may be expressed in one line, thus:

Double the place of primes for shillings, and if the second place be 5, or exceed 5, reckon 1 shilling more; the figures in the second and third places (rejecting 5 in the second place) are so many farthings, abating 1 for every 24.

Examples.

1. The value of 0,92763*l.* is 18*s.* $6\frac{1}{2}$.
2. . . . . . 0,87638*l.* is 17 : $6\frac{1}{4}$.
3. . . . . . 0,09937*l.* is 1 : $11\frac{3}{4}$.
4. . . . . . 0,0428 *l.* is 0 : $10\frac{1}{4}$.
5. . . . . . 0,0095 *l.* is 0 : $2\frac{1}{4}$.

CHAP. XXV.

Of the Management of Circulating Decimals.

366. THOSE Repetends, which consist of the same number of places, are called similiar; thus $,\acute{4}0\acute{6}$, and $,\acute{7}351\acute{4}$ are similar Repetends.

367.

367. Similar Repetends that begin at the ſame place, *viz.* at the place of units, primes, ſeconds, &c. are ſaid to be conterminous.

368. A finite expreſſion may put on the form of an infinite, by making cyphers the Repetend. And,

A ſingle Repetend may put on the form of a compound Repetend, without altering its value; thus $,\dot{4}$ may be wrote $,\dot{4}\dot{4}$; or, $\dot{4}4\dot{4}$; or $,\dot{4}44\dot{4}$. For $\frac{4}{9}=\frac{44}{99}=\frac{444}{999}=\frac{4444}{9999}$, &c.

369. Hence any given Repetend may put on the form of another Repetend, if the number of repeating figures in the latter be a multiple of the number of repeating figures in the former.

Thus the Repetend $0,\dot{4}\dot{5}$ conſiſting of two figures, may put on the forms of Repetends conſiſting of 4, 6, 8, 10, &c. figures; that is, $0,\dot{4}\dot{5}=0,\dot{4}54\dot{5}=0,\dot{4}5454\dot{5}=0,\dot{4}545454\dot{5}$, &c.

370. Any two or more diſſimilar Repetends may be made ſimilar, by transforming them into other Repetends, which ſhall conſiſt of as many places of figures, as the leaſt common multiple of the ſeveral numbers of places, found in all the Repetends, contains units.

EXAMPLE 1.

Diſſimilar,	Made ſimilar.
$0,\dot{1}$	$0,\dot{1}\dot{1}$
$0,\dot{5}\dot{4}$	$0,\dot{5}\dot{4}$

EXAMPLE 2.

$0,\dot{4}7\dot{5}$	$0,\dot{4}7547\dot{5}$
$0,3\dot{7}\dot{4}$	$0,3\dot{7}42424\dot{4}$
$0,\dot{5}\dot{9}$	$0,\dot{5}9595\dot{9}$
$0,32\dot{7}$	$0,32\dot{7}7777\dot{7}$
$0,\dot{1}$	$0,\dot{1}1111\dot{1}$

In the laſt Example 6 is the leaſt common Multiple, and therefore the ſimilar Repetends conſiſt each of 6 places.

371. A pure Repetend may put on the form of a mixt Repetend, without altering its value; thus $0,\dot{4}\dot{5}$ may be wrote $0,4\dot{5}\dot{4}$; or $0,45\dot{4}\dot{5}$; or $0,454\dot{5}\dot{4}$, &c.

For $0,4\dot{5}\dot{4}=\frac{4}{10}+\frac{54}{99}$ of $\frac{1}{10}=\frac{4}{10}+\frac{54}{990}$, by *Art.* 333.
And $\frac{4}{10}+\frac{54}{990}=\frac{396}{990}+\frac{54}{990}=\frac{450}{990}=\frac{45}{99}=0,\dot{4}\dot{5}$.
Again, $0,45\dot{4}\dot{5}=\frac{45}{100}+\frac{45}{99}$ of $\frac{1}{100}=\frac{45}{100}+\frac{45}{9900}$, by *Art.* 333.
And $\frac{45}{100}+\frac{45}{9900}=\frac{4455}{9900}+\frac{45}{9900}=\frac{4500}{9900}=\frac{45}{99}=0,\dot{4}\dot{5}$.

372. Hence any two or more similar Repetends may be made conterminous, that is, may begin at the same place.

EXAMPLE I.

Make $0,\dot{4}\dot{0}$ and $0,73\dot{5}1\dot{4}$ conterminous. This will be performed by making $0,\dot{4}\dot{0}$ put on the same form with $0,73\dot{5}1\dot{4}$, *viz.* that of a mixed Repetend having 2 finite places; thus, $0,4\dot{0}4\dot{0}$.

EXAMPLE. 2.

Similar Repetends	Made conterminous.
$0,\dot{4}7547\dot{5}$	$0,47\dot{5}4754\dot{7}$
$0,3\dot{2}4242\dot{4}$	$0,32\dot{4}2424\dot{2}$
$0,\dot{5}9595\dot{9}$	$0,59\dot{5}9595\dot{9}$
$0,32\dot{7}7777\dot{7}$	$0,32\dot{7}7777\dot{7}$
$0,\dot{1}111\dot{1}$	$0,11\dot{1}111\dot{1}$

373. The expression $\frac{14,2}{9}$ may be called a mixed Fraction; its Numerator consists of a mixed decimal number equal in value to $14\frac{2}{10}$, and the above mixed Fraction is designed to represent the quotient, made by dividing the said mixed decimal Numerator by 9 the Denominator; which Fraction, if reduced to a Decimal by *Art.* 322. will produce a mixed Repetend: See the Work.

$$9)\ \underline{14,2000\ \&c.}$$
$$1,5777\ \&c. = 1,5\dot{7}.$$

374. Hence every mixed Repetend may be reduced to a mixed Fraction, whose Denominator shall be 9, 99, 999, or 9999, &c. by multiplying the finite part of the mixt Repetend, by as many nines as there are circulating figures in the Repetend, and adding the said Repetend to the product for a Numerator.

For the mixed Repetend $1,5\dot{7} = 1,5\frac{7}{9}$ of $\frac{1}{10}$ by *Art.* 333. which being multiplied by 10 will become $15,\frac{7}{9}$, by *Art.* 318.

Now to reduce $15\frac{7}{9}$ to an improper Fraction, by *Art.* 268. the finite number 15 must be multiplied by 9 the Denominator, and the Repetend 7 must be added thereto to make the Numerator of the equivalent improper Fraction $\frac{142}{9}$; which Numerator being divided by 10 to destroy the former Multiplication becomes $\frac{14,2}{9}$ by *Art.* 318. the same as will be found from the above Rule.

375.

375. But if we make uſe of the method delivered in *Art.* 79. the above proceſs will be rendered much ſhorter.

Thus,	From 1,5 × 10	=	15,0
	Take		1,5
	Remains 1,5 × 9	=	13,5
	Add the Repetend		7
	Numerator . .		14,2

376. And ſince the Repetend 7 is to be added, it will be done with leſs trouble by annexing it to the finite part 15, inſtead of the cypher. Thus,

From 1,5 × 10 +,7	=	15,7
Take		1,5
Remains the Numerator	=	14,2

377. Therefore in order to reduce any mixt Repetend to a mixt Fraction; write the mixt Repetend down without daſhing its repeating figures, and remove the ſeparating comma as many places toward the right-hand as there are figures in the Repetend; write the finite part of the mixed Repetend under the former, placing the right-hand figure and ſeparating comma of this, under the right-hand figure and ſeparating comma of that; then ſubtract the latter from the former, and the remainder will be the Numerator of the mixed Fraction required; and its Denominator will be as many nines, as there are repeating figures in the Repetend.

Examp. 1. Reduce 0,4$\not{7}\not{5}$ to a mixed Fraction.

From 0,475 × 100	=	47,5
Take the finite part	=	0,4
Remains		47,1
Therefore 0,4$\not{7}\not{5}$	=	$\frac{47,1}{99}$

Examp. 2. Reduce 0,348$\not{5}$73$\not{4}$ to a mixed Fraction.

From 0,3485734 × 10000	=	3485,734
Take the finite part .	=	0,348
Remains		3485,386
Therefore 0,348$\not{5}$73$\not{4}$	=	$\frac{3485,386}{9999}$.

378. If all the decimal part of the expreſſion be a Repetend, the Fraction ariſing will be wholly vulgar, and not mixed.

Example.

Examp. 3. Reduce $36,\dot{7}$ to a Vulgar Fraction.

From $36,7 \times 10 = 367,0$
Take the finite part $= 36,0$

Remains . . . . . $331,$

Therefore $36,\dot{7} = \frac{331}{9}$.

Examp. 4. Reduce $3,\dot{8}4\dot{2}$ to a Vulgar Fraction.

From $3,842 \times 1000 = 3842,0$
Take the finite part $= 3,0$

Remains . . . . $3839,0$

Therefore $3,\dot{8}4\dot{2} \;.\; = \frac{3839}{999}$.

379. If the Repetend begins among the places of integers, then as many cyphers must be annexed to the mixed Repetend and finite part, as will be sufficient for the removing the separating comma, the required number of places.

Examp. 5. Reduce $57,\dot{7}$ to a Vulgar Fraction.

From $57 \times 10 = 570,0$
Take the finite part $= 50,0$

Remains . . . . . . $520,0$

Therefore $57,\dot{7} \;. \;. = \frac{520}{9}$.

Examp. 6. Reduce $42\dot{7}5,8\dot{4}$ to a Vulgar Fraction.

From $4275,84 \times 10000 = 42758400$
Take the finite part . $= 4200$

Remains 42754200

Therefore $42\dot{7}5,8\dot{4} \;.\; = \frac{42754200}{9999}$.

Examp. 7. Reduce $5\dot{7}94\dot{6}7,946$, &c. to a Vulgar Fraction.

From $579460 \times 10000 = 5794600000$
Take the finite part . . 500000

Remains 5794100000

Therefore $5\dot{7}94\dot{6}7,946$, &c. $= \frac{5794100000}{9999}$.

380. If the expression be a pure Repetend, the above Rule will give its equivalent Vulgar or mixed Fraction.

Examp.

Examp. 8. Reduce $0{,}00\dot{7}\dot{5}$ to its equivalent mixed Fraction.

From 0,0075 × 100 = 0,75
Take the finite part = 0,00

Remains 0,75

Therefore $0{,}00\dot{7}\dot{5}$. = $\frac{0{,}75}{99}$.

Examp. 9. Reduce $0{,}\dot{7}\dot{5}$ to a Vulgar Fraction.

From 0,75 × 100 = 75,0
Take 0,0

Remains 75,0

Therefore $0{,}\dot{7}\dot{5}$. = $\frac{75}{99}$. See *Art.* 337.

Examp. 10. Reduce $\dot{7}{,}\dot{5}$ to a Vulgar Fraction.

From 7,5 × 100 = 750
Take 00

Remains 750

Therefore $\dot{7}{,}\dot{5}$. = $\frac{750}{99}$.

CHAP. XXVI.

ADDITION *of* DECIMAL FRACTIONS.

381. TO ſuch as well underſtand the Notation of Decimal Fractions, all the varieties of their Numeration, to wit, Addition, Subtraction, &c. will be as eaſy as the operations by whole numbers; therefore he that would be a good proficient in Decimal Arithmetic, muſt thoroughly underſtand the Chapters aforegoing.

382. When divers Decimal Fractions are given to be added together, they muſt firſt of all be orderly placed one under another, according to the doctrine of their Notation. So if theſe Decimal Fractions, to wit, .125, .39, and .7 were given to be added, they muſt be written thus:

.125.
.39
.7
———

Or

Or if you will have the ſame number of places to be in all the Decimals given, without altering their values, they may be writ thus:

.125
.390
.700

Not thus,

.125
.39
.7

For the figures or cyphers, which are of like degrees or places, muſt be ſubſcribed directly one under another, *viz.* tenth parts or primes muſt be ſet down exactly under tenths; alſo hundredth parts or ſeconds are to be placed under hundredth parts: As you ſee in the firſt Example, where 3 or three tenth parts in the ſecond Decimal, ſtand directly under 1 or one tenth part in the firſt Decimal; likewiſe 7 or ſeven tenths in the third Decimal, ſtands directly under the tenths in the former; and ſo of the reſt.

383. In like manner, when mixt numbers, which conſiſt of Integers and Decimal parts, are given to be added, due reſpect muſt be had to their ſubſcription one under another: So if theſe mixt numbers, to wit, 32.056, 7.07, and 1.9 were given to be added, they are to be writ down thus:

32.056
7.07
1.9

384. To add finite Decimals: Having placed the Decimals, and drawn a line underneath in manner aforeſaid, add them together, beginning with the outermoſt rank towards the right-hand (as has been taught in Addition of whole numbers of one Denomination:) So if the Decimals in the firſt Example of *Art.* 382. were given to be added, firſt ſubſcribe 5, which is all that ſtands in the firſt rank towards the right-hand; then proceeding to the ſecond rank, ſay 9 and 2 makes 11; wherefore ſet down 1, which is the exceſs of 11 above 10; and for the 10 carry 1 in mind to the next rank, ſaying 1 in mind added to 7 makes 8, which added to 3 and 1 makes 12; wherefore write 2, which is the exceſs of 12 above

.125
.39
.7

1.215

10,

10, under the line, reserving 1 in mind for the 10; then prefix a point before 2, which stands in the first place of Decimals; and on the left-hand of the point, to wit, in the place of units, or first place of integers, write 1 (being the 1 in mind;) which done, the sum of the Decimals given, is, 1.215, that is one integer (whether it be a perch, yard, foot, *&c.*) and $\frac{215}{1000}$ parts of an integer, as you see in the Example. In like manner, these mixt numbers 32.056, 7.07. and 1.9 being given to be added, their sum will be found to be 41.026, that is 41 integers, and $\frac{26}{1000}$ parts of an integer, as you see in the Margin.

32.056
7.07
1.9
———
41.026

385. To add Decimals that have single Repetends.

Rule. Make the Repetends conterminous; then add up the right-hand column, and add thereto as many units as there are nines contained in the sum, so shall the figure, to be placed under that column, be a Repetend, and the rest of the process will be as in *Art.* 384.

Example 1. Numbers proposed.	Example 1. Made conterminous.	Example 2. Numbers proposed.	Example 2. Made conterminous.
2,$\dot{3}$	2,3$\dot{3}$	4,72$\dot{4}$	4,72$\dot{4}$
2,$\dot{7}$	2,7$\dot{7}$	2$\dot{8}$,	28,88$\dot{8}$
4,7$\dot{0}$	4,7$\dot{0}$	3,	3,000
0,$\dot{3}$	0,3$\dot{3}$	25,2$\dot{6}$	25,26$\dot{6}$
5,8	5,80	18,$\dot{7}$	18,77$\dot{7}$
4,7$\dot{3}$	4,7$\dot{3}$	$\dot{5}$,	5,55$\dot{5}$
	20,7$\dot{4}$		86,21$\dot{3}$

In the first Example, the sum of the right-hand column is 22, which contains 2 nines; therefore say $22 + 2 = 24$; set down $\dot{4}$ for a Repetend, and carry 2. The rest of the operation being as common.

In like manner, the sum of the right-hand column (in Example 2,) being 30, contains 3 nines; therefore $30 + 3 = 33$: Therefore $\dot{3}$ is wrote for the Repetend, and 3 carried to the next column.

386. To add Decimals having compound Repetends.

Rule. Make the Repetends similar and conterminous, and add as in *Art.* 384. then to the right-hand figure of the sum add as many units, as are carried from that column of figures, wherein all the Repetends begin together, to the column next above it: Lastly, dash off for a Repetend, as many places as were so in the numbers added together.

Example 1.		Example 2.	
Numbers propofed.	Made fimilar and conterminous.	Numbers propofed.	Made fimilar and conterminous.
$\acute{1}6\acute{2}$,	$162,\acute{1}6216\acute{2}$	$134,\acute{0}\acute{9}$	$134,09\acute{0}9090\acute{9}$
$2,\acute{0}\acute{3}$	$2,\acute{0}3939\acute{3}$	$97,2\acute{6}$	$97,26\acute{6}6666\acute{6}$
$17\acute{2}$,	$172,\acute{2}2222\acute{2}$	$99,08\acute{3}$	$9,08\acute{3}3333\acute{3}$
$3,\acute{7}6923\acute{0}$	$3,\acute{7}6923\acute{0}$	$1,5$	$1,50\acute{0}0000\acute{0}$
5,	$5,\acute{0}0000\acute{0}$	$0,\acute{8}1\acute{4}$	$0,81\acute{4}8148\acute{1}$
	346,093007		242,75572389
	2		1
Anfwer	$346,\acute{0}9300\acute{9}$		$242,75\acute{8}7239\acute{0}$

In the firft Example, the number 2, which is added to the right-hand place, is the fame which is carried from the place of primes, where the Repetends begin together, to the place next above it.

And in the fecond Example, the number 1, which is fo added, is the fame with that carried from the place of thirds.

387. To add approximate Decimals.

Rule. Place and work as in *Art.* 382 and 384. only obferve, that the certain places of the Decimal are, for the moft part, fewer by one than the Decimal places in any one of the Approximates given.

```
12.34518—
10.5
  .28
 3.45455
 1.41421 +
 1.25928 +
─────────
29.2532 .  certain.
```

This Rule is fuch, that we fhall fcarce ever err more than unity in the laft place; but by two operations, one made with more than juft, the other made with lefs than juft, we fhall always be able to judge how far is certain, *viz.* as far as they agree.

12.34518	12.34517
10.5	10.5
.28	.28
3.45455	3.45454
1.41422	1.41421
1.25929	1.25928
29.25324	29.25320

CHAP. XXVII.

SUBTRACTION *of* DECIMAL FRACTIONS.

388. TO ſubtract finite Decimals: Having firſt ſet down the greater of the two numbers given (whether it be a whole number, mixt number, or Decimal) and the leſſer under the greater, according to the directions in *Art.* 382. proceed as you are taught in Subtraction of whole numbers: So if this Decimal Fraction .784 were given to be ſubtracted from this Decimal .837, the remainder will be .053, that is, $\frac{53}{1000}$ parts of an integer: In like manner, if this mixt number 78.919 were given to be ſubtracted from 295.094, the remainder will be 216 $\frac{175}{1000}$. In each of which Examples you may obſerve that 10 is borrowed as often as need requires, according to the Rules of Subtraction of whole numbers of one Denomination: *Note alſo*, when the Decimals in both the numbers given do not conſiſt of the ſame number of places, that Decimal which is defective in places towards the right-hand, ſhould have the void places filled up with cyphers, or at leaſt cyphers muſt be ſuppoſed to be annexed: So if this Decimal .04338 be given to be ſubtracted from this .65, the remainder will be found to be .60662 and the work will ſtand as in the Margin; where you ſee the three void places are ſupplied with cyphers, and then the operation is as in whole numbers, by borrowing 10 as often as the loweſt figure cannot be ſubtracted from the upper.

```
 .837
 .784
 ----
 .053

295.094
 78.919
-------
216.175

.65000
.04338
------
.60662
```

389.

389. To subtract Decimals that have Repetends.

Rule. Make the Repetends similar and conterminous, and subtract as in *Art.* 388. observing only, if the Repetend of the number to be subtracted, be greater than the Repetend of the number it is to be taken from; that then, the right-hand figure of the remainder must be less by unity than it would be if the expressions were finite. *Note,* The Repetend in the remainder will consist of as many places as those of the other two numbers.

EXAMPLE 1.

		Made conterminous.
From	110,$\dot{6}$	110,6666$\dot{6}$
Take	94,1458$\dot{3}$	94,1458$\dot{3}$
Remains		16,5208$\dot{3}$

EXAMPLE 2.

		Made conterminous.
From	5,0$\dot{3}$	5,033$\dot{3}$
Take	3,041$\dot{6}$	3,041$\dot{6}$
		1,9917
Deduct		1
Remains		1,991$\dot{6}$

EXAMPLE 3.

		Made similar and conterminous.
From	6,$\dot{5}$7142$\dot{8}$	6,5$\dot{7}$1428$\dot{5}$
Take	3,6$\dot{4}$2$\dot{8}$	3,6$\dot{4}$2842$\dot{8}$
	Remains	2,9$\dot{2}$8585$\dot{7}$
	Deduct	1
	Remains	2,928585$\dot{6}$

EXAMPLE 4.

		Made similar and conterminous.
From	10,5	10,5$\dot{0}\dot{0}$
Take	3,$\dot{4}\dot{5}$	3,4$\dot{5}\dot{4}$
		7,046
Deduct		1
Remains		7,0$\dot{4}\dot{5}$

390. To subtract approximate Decimals.

Rule. Place as in Addition, and subtract as in *Art.* 388. and the last place will never err more than an unit, if both are made more than just, or both less than just.

10.5	84.3275 +
3.45455	2.1847 +
7.04545 —	82.1428 .

CHAP. XXVIII.

MULTIPLICATION *of* DECIMAL FRACTIONS.

391. WHEN two numbers are given to be multiplied, and are both mixt numbers, or both Decimal Fractions, or one of them a whole number, and the other a Decimal or mixt number, (which are all the cafes that can happen) there is no neceffity of writing them precifely one under the other, as in Addition and Subtraction; for the product or number fought in Multiplication depends not upon any regular placing of the two numbers given: So if this mixt number 56.3 were given to be added to this mixt number 1.30526, they ought to be fet down one under the other, as you fee (according to *Art.* 382.) but if they are to be multiplied one by the other, they may be writ thus:

1.30526
56.3

1.30526
56.3

392. To multiply finite Decimals: Multiply the numbers given as if they were whole numbers; then cut off always from the product by a point or comma, fo many places towards the right-hand, as there are places of Decimal parts in both the numbers given to be multiplied; that done, the figure or figures, if any happen to be, on the left-hand of the faid point or line of feparation, declare the integer or integers in the product, and thofe on the right hand of the point are decimal parts of an integer: So if this mixt number 1.30526 be given to be multiplied by this mixt number 56.3 (that is, 56 integers and $\frac{3}{10}$ of an integer) the product will be found 73,486138, that is, 73 integers, and $\frac{486138}{1000000}$ parts of an integer; for having chofe that to be the Multiplier, which will caufe leaft work, and fubfcribed it under the Multiplicand (to wit, 56.3 under the 1.30526) proceed according to the Rules of Multiplication of whole numbers, *viz.* having drawn a line under the numbers given, multiply all the Multiplicand, to wit, 1.30526, as if it were a whole number, by 3 the firft multiplying figure, and fubfcribe the product thereof which is 391578 under the line, and proceeding in like manner with the

the other multiplying figures 6 and 5, at laſt find the total of the particular products, *viz.* 73,486138; and becauſe there are ſix places or decimal parts in both the numbers given (to wit, 5 places of decimal parts in the Multiplicand, and one place in the Multiplier) cut off 6 places to the right-hand from the total before produced, ſo will it ſtand thus 73,486138: Wherefore the true product is $73\frac{486138}{1000000}$ or 73,486138, that is, 73 integers and almoſt one half of an integer.

```
  1.30526
    56. 3
  -------
   391578
  783156
 652630
 --------
73,486138
```

In like manner, if this mixt number 246,25 (that is $246\frac{25}{100}$) were given to be multiplied by 35 integers, the true product will be found 8618.75, that is 8618 integers, and $\frac{75}{100}$ parts of an integer, as you ſee by the operation in the Margin, where you may obſerve that two places are cut off from the number produced by the Multiplication, towards the right-hand, becauſe there are two places of Decimals in the Multiplicand (the Multiplier conſiſting of integers only;) but if there had been decimal parts alſo in the Multiplier, ſo many more places ſhould have been cut off, as we ſhewed in the firſt Example.

```
 246.25
     35
 ------
 123125
 73875
 ------
8618.75
```

Again, if theſe two Decimals .87 and .9 (to wit, $\frac{87}{100}$ and $\frac{9}{10}$) were given to be multiplied one by the other, the true product will be found to be .783, that is $\frac{783}{1000}$ parts of an integer, as you ſee in the Example; where you may obſerve that the product is a Fraction only; for after 3 places (being the number of places of Decimals in both the numbers given to be multiplied) are cut off to the right-hand, there remains no integer on the left-hand.

```
 .87
  .9
 ----
.783
```

393. When the Multiplication is finiſhed, if there ariſe not ſo many places in all as ought to be cut off by *Art.* 392. (which may often happen when the product is a Fraction;) in ſuch caſe, as many places as are wanting, ſo many cyphers muſt be perfixed to the product on the left-hand thereof, and then a point or comma muſt be perfixed to ſign the product ſo increaſed for a Decimal: So theſe Decimals .0375 and .05 being given to be multiplied one by the other, multiply 375 by 5, and there ariſes 1875: Now, according to *Art.* 392. I ſhould cut off 6 places to the right-hand, and here are but 4 in all; therefore I prefix two cyphers, to wit, as many as there are places wanting, and then perfixing a point, the

```
  .0375
    .05
 -------
.001875
 -------
  5.525
  .0026
 -------
  33150
  11050
 -------
.0143650
```

the true product will be .001875, or $\frac{1875}{1000000}$. In like manner, if this mixt number 5.525 be multiplied by this Decimal .0026, the true product will be found to be .0143650 (or $\frac{143650}{10000000}$) as you may see by the operation in the Margin, where one cypher is perfixed to the numbers arising from the Multiplication to discover the true product.

394. To multiply a finite Decimal by a Decimal which has a Repetend.

Rule. Reduce the repeating Decimal to its equivalent Vulgar or mixt Fraction, by *Art.* 377. multiply the finite Decimal by the Numerator of such Fraction, as in *Art.* 392. and divide by its Denominator, according to the method taught in *Art.* 108. so shall the result be the product required.

The reason of this operation will appear from *Art.* 299.

Example 1. Multiply 48,734 by $0{,}0\acute{4}$.

$0{,}0\acute{4} = \frac{0{,}4}{9}$ by *Art.* 380.

$48{,}734 \times 0{,}4 = 19{,}4936$ by *Art.* 392.

And $\frac{19{,}4936}{9} = 2{,}1659\acute{5}$ the Answer.

Note, When the Divisor is a single 9, it seems easier to use the common method of Division, than to proceed by *Art.* 108.

Example 2. Multiply $48{,}7\acute{5}$ by 4.

$48{,}7\acute{5} = \frac{438{,}8}{9}$ by *Art.* 377.

$438{,}8 \times 4 = 1755{,}2$ by *Art.* 392.

And $\frac{1755{,}2}{9} = 195{,}0\acute{2}$, the Answer.

Example 3. Multiply $58{,}7\acute{6}4\acute{5}$ by 7.

$58{,}7\acute{6}4{,}\acute{5} = \frac{58705{,}8}{999}$ by *Art.* 377.

58,7

And 58705,8 × 7 = 410940,6 by *Art.* 391.

7

Also 410940,600 divided by 999 = 411,3 $\frac{519}{999}$ of $\frac{1}{10}$.

410,350

1 1

$411{,}3\acute{5}1\acute{9}$

Where observe, that because there is one Decimal place in the Dividend 410940,6, therefore there is one Decimal place in the Quotient 411,3, also that the remainder (519) will always be a Repetend.

Example

Examp. 4. Multiply 48,76 by $0,1\dot{3}4\dot{5}$.

$0,1\dot{3}4,\dot{5} = \frac{134,4}{999}$ by *Art.* 377.

```
    ,1
  -----
  134,4 × 48,76 = 6553,344 by Art. 392.
  48,76
  -----
   8064
  9408
 10752
 5376
 ----------
 6553,34400 divided by 999 = 6,559 903/999 of 1/1000
    655989
         1
 ----------
 6,559.908
```

$6553,34400$ divided by $999 = 6,559\,\frac{903}{999}$ of $\frac{1}{1000}$

$6,559.\dot{9}0\dot{8}$

395. To multiply Decimals when both Factors have Repetends.

Rule. Reduce them to Vulgar or mixt Fractions, by *Art.* 377. then multiply their Numerators together, as in *Art.* 392. Lastly divide by each of their Denominators separately, according to *Art.* 108.

This process is the same as in Multiplication of Vulgar Fractions.

Example 1. Multiply $2,\dot{3}$ by $5,\dot{6}$.

$2,\dot{3} = \frac{21}{9}$ and $5,\dot{6} = \frac{51}{9}$ by *Art.* 378.

$21 \times 51 = 1071$ by *Art.* 78.

```
  105
 ------
 9)1071
 ------
  9)119
 ------
```

$13,\dot{2}$ And $\frac{1071}{9 \times 9} = 13,\dot{2}$

Example 2. Multiply $1,\dot{1}$ by $1,\dot{1}$.

$1,\dot{1} = \frac{10}{9}$ by *Art.* 378.

$10 \times 10 = 100$

$$
\begin{array}{l}
9)100 \\
\hline
9)11,\dot{1}111111 \\
\hline
\dot{1},2345679\dot{0} \text{ the Anſwer.}
\end{array}
$$

Example 3. Multiply $3,1\dot{4}\dot{5}$ by $4,\dot{2}9\dot{7}$

$3,1\dot{4}\dot{5} = \frac{311,4}{99}$ and $4,\dot{2}9\dot{7} = \frac{4293}{999}$ by *Art.* 377. and 378.

$4293 \times 311,4 = 1336840,2$ by *Art.* 392.

$$
\begin{array}{r}
4293 \\
311,4 \\
\hline
17172 \\
4293 \\
4293 \\
12879 \\
\hline
1336840,20
\end{array}
$$

$\frac{1336840,2}{999} = 1338,1\dot{7}8\dot{3}$ by *Art.* 108.

and $\frac{1338,1\dot{7}8\dot{3}}{99} = 13,5\dot{1}6953\dot{3}$

$$
\begin{array}{l}
133817 \\
\hline
1338,1\dot{7}8\dot{3}783783783783 \ \&c. \\
135\ 1685129660492764 \\
\quad\quad 1\ 2\ 2\ 3\ 4\ 4\ 5 \\
\hline
13,5\dot{1}6953\dot{3}169533169 52 \ \&c.
\end{array}
$$

The operation of the Diviſion by 99 is continued by writing (783) the Repetend of the former Quotient as often as was convenient, that the reader might be the better ſatisfied that $\dot{1}6953\dot{3}$ will repeat in the latter Quotient.

396. To multiply finite or approximate Decimals, ſo that the product ſhall conſiſt of no more than a determinate number of places.

Rule. Under that place in the Multiplicand, thought neceſſary to be retain'd in the Product, write the unit's place of the Multiplier, and invert the order of all its other places; that is, write the Decimals on the left, and the integers (if any) on the right.

In

In multiplying, omit those places in the Multiplicand which ſtand to the right of the digit multiplying by, and let the right-hand place of every line ſtand under each other.

In each line, let the loweſt place be increaſed by the carriage which would ariſe from the omitted places, carrying 1 from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, *&c.* inſtead of carrying 1 for every 10; and the ſum of theſe lines will give the product, generally exact.

Note, in multiplying approximate Decimals by the method of *Art.* 392. many figures in the product will be uncertain, the finding of which will be avoided by uſing this Rule.

Example 1. Multiply 384,672158, by 36,8345.

Now ſeeing there would be 10 decimal places in the product, whereof the greateſt part are unneceſſary; therefore keep only 4 decimal places in the product.

384,672158
5438,63
115,401647 .
23080329 . .
3077377 . . .
115402 . . . .
15387 . . . . .
1923 . . . . . .
14169,2065 . . . . . .

384,672158	
36,8345	
1923	360790
15386	88632
115401	6474
3077377	264
23080329	48
115401647	4
14169,2066	038510

Here the Example is wrought both ways, by which may be eaſily ſeen what is ſav'd by the Rule.

In this Example, becauſe it is intended to keep 4 decimal places in the product, ſet 6, the unit's place of the Multiplier under 1, the 4th place in Decimals of the Multiplicand, and invert the order of all the reſt of the figures: Then ſay three times 8 is 24, and carry 2; three times 5 is 15, and 2 is 17; now ſet down the 7 and carry 1, becauſe this is the product, ariſing by multiplying the 5 that ſtands over the 3.

Again, 6 times 8 is 48, and carry 5; 6 times 5 is 30, and 5 is 35, and carry 3; 6 times 1 is 6, and 3 is 9: Now being come to the figure over the 6, ſet down 9, *&c.*

Again, 8 times 5 is 40, and carry 4; 8 times 1 is 8, and 4 is 12, and carry 1; 8 times 2 is 16, and 1 is 17: Now being come to the figure over the 8, ſet down 7, and carry 1, *&c.* proceeding in like manner till all is done.

Example

Example 2. Multiply 3,141592 by 52,7438, so as to have only 4 Decimal places in the product.

```
3,141592
8347,25
--------
1570796
  62832
  21991
   1256
     94
     25
--------
165,6994
```

CHAP. XXIX.

DIVISION *of* DECIMAL FRACTIONS.

397. IN any of the cases which may happen in Division, if the Dividend be greater than the Divisor, the Quotient will be either a whole number, or else a mixt number: But when the Dividend is less than the Divisor, the Quotient must necessarily be a Fraction; for a lesser number is contained in a greater once at the least, but a greater is not contained once in a lesser.

398. Sometimes the Dividend, whether it be a whole number, mixt number, or decimal Fraction, is to be prepared by annexing a competent number of cyphers thereto, to make room for the Divisor: So if 32.5 were given to be divided by 17.325, the Dividend 32.5 must be increased with cyphers at pleasure after this manner 32.50000, &c. Likewise if 1 were given to be divided by 360, the Division cannot be made till the Dividend 1 be increased with cyphers, which being annexed, the Dividend will stand thus 1.000000, &c. Here note, that the cyphers annexed in manner aforesaid, supply places of decimal parts, and will be useful in discovering the quality of the Quotient, according to *Art.* 400.

399. After the Dividend is prepared by annexing cyphers, when occasion requires, (as in the last Rule,) all the places thereof

thereof must be esteemed as one whole number, (to wit, consisting of units, or integers;) and so is the Divisor to be esteemed, whether it be a Decimal Fraction, or mixt number; for in all cases the Division must be performed in every respect according to the Rules of Division of whole numbers. So if this mixt number 326.25 were given to be divided by this mixt number 12.3, you must divide in the same manner, as when you divide 32625 integers by 123 integers. Also if this Decimal .8356 were given to be divided by this Decimal .05, you are to divide in the same manner, as when you divide 8356 integers by 5 integers; and after the Quotient is found, the degree or place of the first figure which arises in the Quotient, is to be enquired after, *viz.* you must know how far such figure is distant from the place of units, to the end, that the point or comma which is used to separate between the place of units (or first place of integers) and the first place of Decimals, may be duly fixed: This is the only knot in Decimal Division, and may be resolved by the following Rule, *viz.*

400. In any of the cases which may happen in Division of Decimals, the first figure which arises in the Quotient will be always of the same place or degree with that figure or cypher of the Dividend, which at the first question stands over, or at least belongs to the place of units in the Divisor. To illustrate this Rule, I shall give Examples in all the principal cases; and first, let a mixt number be given to be divided by a mixt number, *viz.* Let it be required to divide 172.5 by 3.746, here (according to *Art.* 398.) the Dividend must be increased with cyphers at pleasure, so will it stand thus 172.500000, &c. then Division being made according to the Rules of Division of whole numbers, the Quotient arising will be 46049, &c.

3.746)172.500000(46049, &c.

Now it remains to separate the integers in this Quotient from the Decimal parts; to perform which, subscribe the Divisor 3.746, orderly under the first Dividual 172.50 (being

3.746) 172.500000 (46.049, &c.
• • • • •
3.746

that part of the Dividend, whereof the first question must be asked) or at least imagine the Divisor to be so subscribed, and the figure 3 which stands in the place of units in the Divisor will be placed under 7, which is the place of tens, (or second place of integers) in the Dividend; therefore by the

Rule

Rule before given conclude that the firſt figure ariſing in the Quotient muſt likewiſe ſtand in the place of tens (or ſecond place of integers) and conſequently the next place on the right-hand muſt be the place of units; ſo it is evident that the ſeparating point or comma muſt be placed between the figure 6 and 0 in the Quotient; that done, the true Quotient is found to be 46.049 &c. to wit, 46 integers and $\frac{49}{1000}$ parts of an integer, and ſomewhat more; for 46 $\frac{49}{1000}$ is leſs than the true Quotient; but 46 $\frac{50}{1000}$ is greater than it; and therefore, tho', after the aforeſaid Diviſion of 172,500000 by 3.746 is ended, there will be a remainder, to wit, 446, which ſeems to be greater, yet here it is leſs in value than $\frac{1}{1000}$ part of an unit or integer; and if to that remainder you annex another cypher and continue the Diviſion, you will proceed nearer the truth, and not miſs $\frac{1}{10000}$ part of an unit of the true Quotient, and in that order you may proceed until you find a Repetend.

Example 2. Suppoſe this mixt number 2.34 be given to be divided by this mixt number 52.125 (where you may obſerve that the Dividend is leſs than the Diviſor;) firſt (as before) annex cyphers at pleaſure to the Dividend, to make room for the Diviſor, then the Diviſion being proſecuted as in whole numbers, at length theſe figures will ariſe in the Quotient, to wit.

52.125) 2.3400000 (.0448, &c.

52.125

448; and to the end the degree or quality of the firſt figure 4 may be diſcovered, ſubſcribe the Diviſor 52.125 under the Dividual 2.34000 (for ſo far the firſt queſtion did extend in the Diviſion;) and thereby find that the figure 2, which ſtands in the place of units in the Diviſor, will be ſeated under 4, which is in the ſecond place of Decimals; therefore the firſt figure ariſing in the Quotient muſt alſo ſtand in the ſecond place of Decimals, and conſequently the firſt place of Decimals (which is next on the left-hand to the ſecond) muſt be ſupplied with a cypher; ſo that if a cypher be prefixed on the left-hand of 4, and then a point placed before that cypher, the Quotient will at length be diſcovered to be .0448, &c. or $\frac{448}{10000}$ and ſomewhat more, that is to ſay, $\frac{448}{10000}$ is leſs than the true Quotient, but $\frac{449}{10000}$ is greater than it; and if you would proceed nearer the truth, you may continue the diviſion, as is directed in the firſt Example of this Rule.

Example

Example 3. Where a whole number is divided by a Decimal Fraction, *viz.* ſuppoſing 82 integers were given to be divided by this Decimal .056: After cyphers are annexed to the Dividend at pleaſure, and the Diviſion proſecuted as in

.056) 82.00000 (146428 &c.

whole numbers (to wit, 8200000 being divided by 56) theſe figures 146428 will ariſe in the Quotient; now to the end the degree or ſeat of 1, the firſt figure in the Quotient, may be known, ſubſcribe the Diviſor .056 under the firſt Dividual 82 (for ſo far did the firſt queſtion in the Diviſion extend;) and becauſe the Diviſor is leſs than unity, ſupply the place of units by a cypher or 0 prefixed on the left-hand of the point of ſeparation

.056) 0082.00000 (1464.28 &c.
 · · · · · ·
 0.056

in the Diviſor; alſo prefix cyphers before, (to wit, on the left-hand of) the integers in the Dividend, to repreſent a ſucceſſion of places of integers (for the order of places in integers is from the right-hand towards the left;) then the cypher or 0 which repreſents the place of units in the Diviſor, ſtands under that cypher, which repreſents the fourth place of integers in the Dividend (as you ſee by the Example) therefore the firſt figure ariſing in the Quotient muſt alſo be ſeated in the fourth place of integers, and conſequently the 4 firſt places in the Quotient will be Integers, and the reſt a Decimal: So that the true Quotient is 1464 Integers, and $\frac{28}{100}$ parts of an Integer, and ſomewhat more, *viz.* 1464.28 is leſs than the true Quotient, but 1464.29 is greater than it.

Example 4. Suppoſe this Decimal .0125 be given to be divided by this Decimal .5; after Diviſion is finiſhed according to the Rules of Diviſion of whole numbers (to wit, after 125 is divided by 5) theſe figures 25 will ariſe in the Quotient; now to diſcover the degree or ſeat of 2 the firſt figure in the Quotient, ſubſcribe the Diviſor 5 under the firſt Dividual .012,

.5).0125(25

and having (as in the laſt Example) prefixed a cypher on the left-hand of the point of ſeparation in the Diviſor, to denote, or repreſent the place of units, ſuch cypher or place of units ſtands under the figure which is ſeated in the ſecond place of Decimals in the Dividend, therefore the firſt figure which ariſes in the Quotient muſt alſo be in the ſecond place of Decimals, and therefore prefixing a cypher to ſupply the firſt

.5) .0125 (.025
 ·
 0.5

place

place of Decimals, and putting a point before that cypher, the Quotient is at length discovered to be .025 or $\frac{25}{1000}$.

Example 5. Suppose this Decimal .8564 be given to be divided by this .008. First, annex cyphers to the Dividend at pleasure, then prosecuting the Division as in whole numbers, to wit, dividing .856400 by 8, the Quotient arising is 107050: Now to discover the degree or place of 1, the first figure in the Quotient, subscribe the Divisor .008 under the first Dividual .8, then prefix a cypher to set forth, or supply the place of units in the Divisor; also prefix cyphers to represent places of integers in the Dividend; that done, the cypher or 0, which supplies the place of the units in the Divisor, stands under the cypher which is seated in the third place of integers in the Dividend: Therefore the first figure arising in the Quotient must be also in the third place of integers, and consequently the three first places in the Quotient will be Integers, and the rest a Decimal; so that the true Quotient is 107.050 or 107 $\frac{5}{100}$.

.008) 856400 (107050

.008) 000.85640 (107.050
  0.008

Example 6. Let it be required to divide this Decimal Fraction .73952 by this .32; first, dividing 73952 by 32, as if they were whole numbers, the figures arising in the Quotient will be 2311. Now, to discover the quality or value of the said figures, subscribe the Divisor .32 under the first Dividual 73, then prefixing a cypher as well on the left-hand of the Dividend, as of the Divisor so subscribed (or imagined to be subscribed) as aforesaid, to represent the place of units in each of them, the cypher or 0, which supplies the place of units in the Divisor, stands under the 0, which represents the place of units in the Dividend; wherefore the first figure arising in the Quotient will stand in the place of units, and consequently the following places of the Quotient will be a Decimal Fraction, so that the true Quotient is 2.311 or $\frac{311}{1000}$.

.32) 0.73952 (2.311
  0.32

401. Or the number of Decimal places in the Quotient of any such Division may be found thus:

Rule. From the number of Decimal places used in the Dividend, take the number of Decimal places in the Divisor, and the remainder will be the number of Decimal places in the Quotient, which number of places mark off by a point or comma from the right-hand in the Quotient; but if there be

not

not ſo many places therein, as this Rule requires, ſupply the defect with cyphers on the left-hand.

In the firſt Example, *Art.* 400 *viz.*

3,746) 172,500000 (46049.

From (6) the number of Decimal places in the Dividend, take (3) the number of Decimal places in the Diviſor, the remainder is 3; therefore in the Quotient let 3 places be mark'd off from the right-hand, thus 46,049.

In the ſecond Example, *Art.* 400. *viz.*

52,125) 2,3400000 (448.

From (7) the number of Decimal places in the Dividend, take (3) their number in the Diviſor, there remains 4; now ſince there are but 3 places in the Quotient, therefore 1 cypher muſt be placed on the left-hand of them to make their number equal with the above remainder 4, and then prefix the ſeparating comma, thus ,0448.

From the operation of theſe two Examples it follows, that this Rule gives the ſame reſult with the former, in *Art.* 400.

This Rule is the converſe of that given for cutting off the proper number of Decimal places in Multiplication, *Art.* 392. For, ſince the product of the Diviſor and Quotient will be equal to the Dividend, it will follow from *Art.* 392. that the Decimal places in the Dividend, muſt be equal to the ſum of thoſe in the Diviſor and Quotient.

402. If the Dividend be a circulating Decimal, and the Diviſor finite, uſe the repeating figure or figures in the ſame manner as cyphers are directed to be uſed, in *Articles* 398, 399, and 400.

Examp. 1. Divide 195,0$\acute{2}$ by 4.

4) 195,022 &c.

48,75$\acute{5}$ &c. = 48,7$\acute{5}$.

Examp. 2. Divide 6,559903 by 48,76.

```
48,76) 6,559903903 &c. (,1345345 &c. = ,1345.
       4876
       ----
       16839
       14628
       -----
        22110
        19504
        -----
         26063
         24380
         -----
          16839
          14628
          -----
           22110
           19504
           -----
            26063
            24380
            -----
             1683
```

This Division is continued till the Repetend occurs twice in the Quotient for the reader's satisfaction.

403. If the Divisor be a circulating Decimal, make the Repetends of the Divisor and Dividend similar by *Art.* 370. Then, instead of them, use the Numerators of their equivalent vulgar or mixed Fractions (found by *Art.* 377.) in the same manner as finite Decimals in *Art.* 398, 399, and 400.

Note, If the Dividend be finite, make it a similar Repetend by annexing cyphers.

Example 1. Divide 2,16595 by 0,04.

```
0,4 \      21,6595
0,0  )      2,1659
---  /    --------
0,4        19,4936
       ------------
            48,734
```

Example

Example 2. Divide 54 by ,17.

1,7) 540
1) 54
1,6) 486,000(303,75
48

60
48

120
112

80
80

00

Examp. 3. Divide 411,3519 by 58,7645.

58764,5) 411351,9
58,7) 411,3
58705,8) 410940,6(7
410940,6

Examp. 4. Divide 9 by ,45.

45,) 900
0) 9
45) 891,0(19,8
45

441
405

360
360

000

Example 5. Divide 13,2' by 5,6'

56) 132
5) 13
51) 119,0 (2,3'
102
——
170
153
——
17

Example 6. Divide 1',23456790' by 1,1'.

1111111111,) 1234567901,
1,) 1,
——
1111111110,) 1234567900,0 (1,1'
1111111110
——
1234567900

Example 7. Divide 13,5169533' by 4,297'.

4297297,) 13516953,3
4,) 13,5
——
4297293,) 13516939,800 (3,145'
12891879
——
6250608
4297293
——
19533150
17189172
——
23439780
21486465
——
1953315

404. Division by finite or approximate Decimals may be contracted as follows.

Rule. Let each remainder be a new Dividend, and for each such new Dividend, point off one figure from the right-hand of the Divisor; observing at each Multiplication to have regard

gard to the increase of the figures so cut off, as in Multiplication of Approximates, *Art.* 396.

EXAMPLE 1.

```
384,672158) 14169,2066238510 (36,8345
      .....  11540,16474
             ---------------
              262904188 .
              230803295 .
             ---------------
              . 32100893 . .
                30773772 . .
             ---------------
               . 1327121 . . .
                 1154016 . . .
             ---------------
                  173105 . . . .
                  153869 . . . .
             ---------------
                   19236 . . . . .
                   19234 . . . . .
             ---------------
```

EXAMPLE 2.

```
9,365407) 87,076326 (9,297655
  ...... 84,288663
         ---------
          2,787663
          1,873081
         ---------
            914582
            842886
         ---------
             71696
             65558
         ---------
              6138
              5619
         ---------
               519
               468
         ---------
               .51
                47
         ---------
                 4
```

405. Having explained all neceſſary Rules in Diviſion, concerning Decimal Fractions, I ſhall give a taſte of their excellent uſe by the two following Queſtions, and then conclude this Chapter.

Queſt. 1. A merchant bought of gold plate 356 ounces, 13 penny-weights, and 15 grains, for 1160 pounds Sterling; the queſtion is, what he paid for an ounce? *Anſw.* 3*l.* 5*s.* ½*d.* very near. The operation by Decimals may be after this manner, *viz.*

By the ſecond Tablet of Reduction the Decimal of 13 penny-weights is — — —	.65
The Decimal of 15 grains is — — —	.03125
The ſum of theſe two Decimals is — —	.68125.
Therefore the quantity of plate in ounces and Decimal parts of an ounce is — — —	356.68125

Then by the Rule of Three, ſay, if 356.68125 ounces coſt 1160 pounds, what will 1 ounce coſt? Here 'tis evident, that if I divide 1160 by 356.68125, the Quotient will give the value of an ounce, to wit, 3.252 pounds, or 3 pounds, 5 ſhillings, and ½*d.* very near.

356.68125) 1160.00000000 (3.252, &c.

Queſt. 2. Suppoſe the length of the Tropical year, (or the ſpace of time in which the ſun running through the whole Ecliptic circle, conſiſting of 360 degrees, is returned to the ſame Equinoctial or Solſtitial point from whence he departed) to conſiſt of 365 days, 5 hours, and 49 minutes, the queſtion is to know the ſun's mean or equal motion for 1 day, to wit, what part of 360 degrees the ſun moves thro' in a whole day? The operation by Decimals, thus:

By the tenth Tablet of Reduction the Decimal correſpondent to 5 hours is — — —	.2083333
The Decimal of 49 is — — — — —	.0340277
The ſum of thoſe Decimals is — — — —	.2423611
Therefore the time given in days and Decimal parts of a day is — — — —	365.2423611

Then by the Rule of Three, if 365.2423611 days give 360 degrees (or a total circumference) what will 1 day give? Here divide 360 by 365.2423611, the Quotient will give the diurnal motion required; which will be found very near .98564, &c. or $\frac{98564}{100000}$ parts of a degree, which Decimal being reduced to the common Sexagenary parts (by *Art.* 364.) will give 59′ 8″, &c. and ſuch is the ſun's diurnal motion very near, according to the aforeſaid ſuppoſition of the length of the Tropical year.

I ſhall here add the vulgar Sexagenary reſolution of this queſtion, that by comparing both ways together, the excellency of Decimal Arithmetic in calculations of this nature may be the more perſpicuous.

The aforeſaid queſtions being ſtated according to the Rule of Three, will ſtand thus:

Days. Hours. Min. Degrees. Day.
If 365 . 5 . 49 : 360 :: 1

The firſt term in the Rule muſt be reduced into minutes (by *Art.* 121.) ſo there will be found 525649 minutes.

D. *H.* *M.*
365 : 5 : 49
24

1465
730

8765 *Hours.*
60

525949 *Minutes.*

Likewiſe the third term 1 Day muſt be reduced to Minutes, which will be found to be 1440, as you ſee by the following operation.

1 *Day*, or 24 *Hours.*
60

1440

Then multiply the third term by the ſecond, to wit, 1440 by 360, the product is 518400, which being divided by the firſt term 525949 the quotient will give $\frac{518400}{525949}$ parts of a degree, by *Art.* 255. which Fraction being reduced to the accuſtomed Sexagenary parts (by *Art.* 267.) will give as before 59′. 8″, *&c.* for the ſun's mean diurnal motion: Now which of theſe two ways is the more expeditious, I leave to him who is ſkilled in both to determine.

CHAP. XXX.

The UNIVERSAL RULE *of* PROPORTION.

406. BY *producing terms*, in the following Rule, are meant whatſoever neceſſarily and jointly produce any effect; as the cauſe and the time; length, breadth, and depth; the buyer and his money; the ſeller and his goods, &c. and by *produced terms*, ſuch as are the effects of the former, as purchaſe, produce, expence, gain, loſs, intereſt, value, &c.

RULE.

1. *Set down the terms expreſſing the condition of the queſtion in one line, and in any order.*
2. *Under each conditional term, ſet its correſponding one, in another line.*
3. *Multiply the producing terms of one line, and the produced term of the other line, continually, and take the reſult for a Dividend.*
4. *Multiply the remaining terms continually, and let their product be the Diviſor.*
5. *The Quotient of this Diviſion will be the term required.*

This Rule is ſo general, as to comprehend all caſes that come under the common Rule of Three, whether direct or inverſe; whether ſingle, or any how compounded; in integers or fractions; ſo that queſtions in any of the above Rules, where proportion is uſed, may be readily ſolved thereby.

Example 1. (Being the firſt propoſed in *Art.* 145.) If 4 ſtudents ſpend 19 pounds in 3 months, how much will ſerve 8 ſtudents 9 months?

	Students.	*£.*	*Mon.*
The terms expreſſing the condition of the queſtion ————	4	19	3
The terms correſponding to them putting A to ſignify the anſwer—	8	A	9

Here the ſtudents and months are producing terms, and the money or expence is produced.

And therefore the producing terms of the lower line, *viz.* 8 ſtudents and 9 months, are to be multiplied by 19 *l.* the produced term of the upper line for a Dividend: And the product of the remaining terms 4 ſtudents and 3 months, is to be be the Divisor: This Dividend and Divisor expressed in the manner of a vulgar Fraction, may, by *Art.* 276. stand as below.

$$\frac{8\times9\times19}{4\times3} = \text{A, or the answer.}$$

Which expression being abbreviated by *Art.* 266. will be reduced as follows:

$$\frac{8\times9\times19}{4\times3} = \frac{2\times9\times19}{1\times3} = \frac{2\times3\times19}{1\times1} = 6\times19 = 114, \text{ the answer.}$$

Examp. 2. (Being the last Example proposed in *Art.* 145.) If 9 bushels of provender serve 8 horses 12 days, how many days will 24 bushels last 16 horses?

	Bushels.	*Horses.*	*Days.*
Conditional terms ———	9	8	12
Corresponding ———	24	16	A.

Where the horses and days are producing and the consumption of provender produced.

$$\text{Therefore A} = \frac{8\times12\times24}{9\times16} = \frac{8\times4\times24}{3\times16} = \frac{8\times4\times8}{} =$$

$$\frac{4\times8}{1\times2} = \frac{2\times8}{1\times1} = 2\times8 = 16 \text{ the answer.}$$

Examp. 3. (Being the Example in *Art.* 152.) If the carriage of 8 *C.* wt. 120 miles, cost 48 shillings; for how much may I have 4 *C.* wt. carried 32 miles?

	C. wt.	*Miles.*	*Shil.*
Conditional terms ———	8	128	48
Corresponding ———	4	32	A.

Where the weight and distance are producing, and the expence produced.

$$\text{Therefore A} = \frac{4\times32\times48}{8\times128} = \frac{4\times32\times6}{1\times128} = 6\ s.$$

Examp. 4. A regiment of 136 soldiers eat up 391 quarters of wheat in 108 days, I demand how many quarters of wheat 11232 soldiers will eat in 56 days?

	Sold.	*Quarters.*	*Days.*
Conditional terms ———	136	391	108
Corresponding ———	11232	A.	56

$$\text{Therefore A} = \frac{11232\times56\times391}{136\times108} = 104\times7\times23 =$$

16744 quarters.

Examp.

Examp. 5. If 50 acres of graſs be mowed by 24 men in 28 days, how many men will do the ſame work in 24 days?

	Acres.	*Men.*	*Days.*
Conditional terms ———	50	24	28
Correſponding ———	50	A.	24

Therefore $A = \frac{24 \times 28 \times 50}{50 \times 24} = 28$ men.

Examp. 6. If 48 buſhels of corn, or other ſeed, yield 576 buſhels in a year, how many will 240 buſhels yield in 6 years at that rate? That is to ſay, if there were ſowed 240 buſhels every one of the 6 years.

	Buſhels.	*Years.*	*Produce.*
Conditional terms — —	48	1	576
Correſponding — —	240	6	A.

Therefore $A = \frac{240 \times 6 \times 576}{48 \times 1} = 30 \times 576 = 17280$ Buſh.

Examp. 7. If 40 ſhillings is the wages of 8 men for 5 days, what will be the wages of 32 men for 24 days?

	Shil.	*Men.*	*Days.*
Conditional terms ———	40	8	5
Correſponding ———	A.	32	24

Therefore $A = \frac{32 \times 24 \times 40}{8 \times 5} = 32 \times 24 = 768$ ſhillings.

Examp. 8. If, in a family conſiſting of 7 perſons, there are drank out 2 kilderkins of beer in 12 days, how many kilderkins will there be drank out in 8 days, by another family conſiſting of 14 perſons?

	Perſons.	*Kilderk.*	*Days.*
Conditional terms ———	7	2	12
Correſponding ———	14	A.	8

Therefore $A = \frac{14 \times 8 \times 2}{7 \times 12} = \frac{8}{3} = 2\frac{2}{3}$ Kilderkins.

Examp. 9. If when wine is worth 30 *l. per* tun, 20 *l.* worth is ſufficient for the ordinary of 100 men, how many men will 4 *l.* worth ſuffice, when it is worth 24 *l. per* tun? *Anſwer* 25 men.

Tun.

	Tun.	*£.*	*£.*	*Men.*
Conditional terms ———	1	30	20	100
Correſponding ———	1	24	4	A.

Therefore $A = \frac{30 \times 100 \times 4}{24 \times 20} = 5 \times 5 = 25.$

Examp. 10. If 48 pioneers in 12 days, caſt up a trench 14 yards long, how many pioneers will caſt up a trench 168 yards long in 16 days?

	Pioneers.	*Days.*	*Long.*
Conditional terms ———	48	12	14
Correſponding ———	A.	16	168

Therefore $A = \frac{48 \times 12 \times 168}{14 \times 16} = 3 \times 12 \times 12 = 432$ Pioneers.

Examp. 11. If 12*C.* weight being carried 100 miles, coſt 6*l.* 6*s.* I deſire to know how many *C.* weight may be carried 150 miles for 12*l.* 12*s.*?

6*l.* 6*s.* = 126*s.* and 12*l.* 12*s.* = 252*s.*

	C.wt.	*Miles.*	*Shillings.*
Conditional terms ———	12	100	126
Correſponding ———	A.	150	252

Therefore $A = \frac{12 \times 100 \times 252}{126 \times 150} = 4 \times 2 \times 2 = 16$ *C.* wt.

Examp. 12. If 60 ells *Engliſh* be equal to 100 ells *Flemiſh*, and each ell *Engliſh* contains 20 nails of a yard, how many ſuch nails doth the *Flemiſh* ell contain?

	Ells Eng.	*Nails.*	*Ell Flem.*
Conditional terms ———	60	20	100
Correſponding ———	1	A.	1

Therefore $A = \frac{60 \times 20 \times 1}{100 \times 1} = 6 \times 2 = 12.$

Examp. 13. If 14 men, in 15 days, build 16 rods of wall;
How many men muſt added be, to do't in 2, that's all?

	Men.	*Days.*	*Rods.*
Conditional terms ———	14	15	16
Correſponding ———	A.	2	16

Therefore $A = \frac{14 \times 15 \times 16}{2 \times 16} = 7 \times 15 = 105.$

That is 105 men will build it in 2 days; therefore (to the 14 men) 91 muſt be added.

Examp.

Examp. 14. If 8 cannons in one day spend 48 barrels of powder, I demand how many barrels 24 cannons will spend in 22 days?

		Cannons.	*Days.*	*Barrels.*
Conditional terms	———	8	1	48
Corresponding	———	24	22	A.

Therefore $A = \frac{24 \times 22 \times 48}{8 \times 1} = 24 \times 22 \times 6 = 3168$ Barrels.

Exam. 15. If 15 hundred *British* lead cost 16 pounds in gold,
How much of that Saturnian ore for 40 pounds is sold?

		C.	*£.*
Conditional terms	———	15	16
Corresponding	———	A.	40

Therefore $A = \frac{15 \times 40}{16} = \frac{15 \times 5}{2} = \frac{75}{2} = 37\frac{1}{2}C.$

Exam. 16. If 30 pence and 30 groats, buy 50 pints of wine,
What is the cost of 60 quarts in current *English* coin?

30 groats $= 30 \times 4 = 120$ pence, $120 + 30 = 150$ pence; and 50 pints $= 25$ quarts.

		Quarts.	*Pence.*
Conditional terms	———	25	150
Corresponding	———	60	A.

Therefore $A = \frac{60 \times 150}{25} = 60 \times 6 = 360d. = 1l.\ 10s.$

Exam. 17. If 9 pounds of our *English* wool, doth 90 pence require,
The value of 1*C.* weight of you I do desire?

		℔	*Pence.*
Conditional terms	———	9	90
Corresponding	———	112	A.

Therefore $A = \frac{112 \times 90}{9} = 112 \times 10 = 1120d. = 4l.\ 13s.\ 4d.$

Exam. 18. If 16*s.* and a crown, for 13 weeks supply
For meat and drink, that is my board; how much
a day gave I?

16*s.* + 1 crown $= 21s.$ and 13 weeks $= 13 \times 7 = 91$ days.

		Man.	*Days.*	*Shillings.*
Conditional terms	———	1	91	21
Corresponding	———	1	1	A.

Therefore $A = \frac{1 \times 1 \times 21}{1 \times 91} = \frac{21}{91}s. = 2\frac{3}{4}d.$ nearly.

407. When this Rule is to be applied to Fractions, after reducing mixed numbers to improper Fractions, and compound Fractions to single ones, observe,

1°. That to multiply by a Fraction is to multiply by the Numerator, and to divide by the Denominator thereof: Therefore having prepared a line to separate the Numerator of the answer from the Denominator, as below.

A = ———————————————

Write the Numerator of a Fraction, which is to be multiplied by, above that line, and the Denominator below it.

2°. To divide by a Fraction, is to multiply by its Denominator, and divide by its Numerator: Therefore write the Denominator of a Fraction which is directed to be a Divisor above the line, and its Numerator below it.

Examp. 19. (Being the same as in the old editions of this work is given as the first in the Rule of Three direct in Fractions) If $\frac{3}{4}$ of a yard of velvet be sold for $\frac{2}{3}$ of a pound, what will $\frac{5}{6}$ of a yard cost?

Conditional terms ——————— $\frac{3}{4}$ *Yard.* $\frac{2}{3}$ £.
Corresponding ——————— $\frac{5}{6}$ *Yard.*

Here the goods are producing terms, and the money produced.

Therefore the product of $\frac{5}{6}$ by $\frac{2}{3}$ is to be divided by $\frac{3}{4}$: Now the product of $\frac{5}{6}$ by $\frac{2}{3}$ will be represented by $\frac{5\times2}{6\times3}$ by the first direction above, and therefore

$$A = \frac{5\times2\times4}{6\times3\times3} \text{ by the second direction.}$$

Whence $A = \frac{5\times2\times4}{6\times3\times3} = \frac{5\times4}{3\times3\times3} = \frac{20}{27}\, l.$

And $\frac{20}{27}\, l. = 14\, s.\ 9\, d. \frac{21}{27} = 14\, s.\ 9\, d. \frac{7}{9}$ by *Art.* 267.

Examp. 20. When $\frac{2}{5}$ of $\frac{3}{4}$ of a ship is valued at 197 *l.* 11 *s.* 3 *d.* how much is the whole ship worth?

$\frac{2}{5}$ of $\frac{3}{4} = \frac{2\times3}{5\times4}$; and 197 *l.* 11 *s.* 3 *d.* $= \frac{15805}{80}\, l.$

		Ship.	*L.*
Conditional terms	———————	$\frac{2\times3}{5\times4}$	$\frac{15805}{80}$
Corresponding	———————	1	A.

Therefore

Therefore $A = \frac{1 \times 15805 \times 5 \times 4}{80 \times 2 \times 3} = \frac{15805}{4 \times 2 \times 3}\, l.$

And $\frac{15805}{4 \times 2 \times 3} = \frac{15805}{24} = 658\, l.\ 10\, s.\ 10\, d.$

Examp. 21. (Being the firſt Example in the Rule of Three Inverſe in Fractions in the old edition.) If cloth which is $1\frac{3}{4}$ yards in breadth and $3\frac{1}{2}$ yards long will make a cloak, how much in length of ſtuff which is $\frac{5}{8}$ of a yard in breadth, will make a cloak of the ſame bigneſs with the former?

$1\frac{3}{4} = \frac{7}{4}$; and $3\frac{1}{2} = \frac{7}{2}$.

	Yards long.	*Yards broad.*	*Cloaks.*
Conditional terms	$\frac{7}{2}$	$\frac{7}{4}$	1
Correſponding ——	A.	$\frac{5}{8}$	1

Here the length and breadth are producing, and the garment produced.

Therefore $A = \frac{7 \times 7 \times 1 \times 8}{2 \times 4 \times 1 \times 5} = \frac{7 \times 7}{5} = \frac{49}{5} = 9\frac{4}{5}$

Examp. 22. If $18\frac{3}{4}$ yards of ſerge coſt 2 *l.* 5 *s.* how many yards of the ſame may I have for 300 guineas at 21 *s.* 6 *d.* each?

$18\frac{3}{4} = \frac{75}{4}$ yards, 2 *l.* 5 *s.* = 45 *s.* and 21 *s.* 6 *d.* $\frac{43}{2}$ *s.*

	Yards.	*Pes. Mon.*	*Shillings.*
Conditional terms —	$\frac{75}{4}$	1	45
Correſponding ——	A.	300	$\frac{43}{2}$

Therefore $A = \frac{75 \times 300 \times 43}{4 \times 2 \times 45 \times 1} = \frac{25 \times 5 \times 43}{2} = 2687\frac{1}{2}$ yards.

Examp. 23. Bought a caſk of wine for 62 *l.* 8 *s.* how many gallons were in the ſame, when the gallon was valued at 5 *s.* 4 *d.*

62 *l.* 8 *s* = $\frac{1248}{20}$ *l.* and 5 *s.* 4 *d.* = $\frac{64}{240}$ *l.*

	Gal.	£.
Conditional terms ————	1	$\frac{64}{240}$
Correſponding ————	A.	$\frac{1248}{20}$

Therefore $A = \frac{1 \times 1248 \times 240}{20 \times 64} = 78 \times 3 = 234$ gallons.

Examp. 24. If 15 pounds coſt 16 pence, of bread that's made of rye,
How many loaves, of 6 pounds each, for four-ſcore pounds had I?

Loaf.

	Loaf.	*lb.*	*Pence.*
Conditional terms ————	1	15	$\frac{16}{240}$
Corresponding ————	A.	6	80

Therefore $A = \frac{1 \times 15 \times 80 \times 240}{6 \times 16} = 15 \times 5 \times 40 = 3000.$

Examp. 25. (Being the first Example in the Double Rule of Three in Fractions in the old edition) If I pay 28 shillings for the carriage of 3 *C.* wt. 50 miles; how much ought I to pay for the carriage of 17 *C.* wt. 84 miles?

	Shil.	*C. wt.*	*Miles.*
Conditional terms ————	28	3	50
Corresponding ————	A.	17	84

Therefore $A = \frac{17 \times 84 \times 28}{3 \times 50} = \frac{17 \times 28 \times 14}{1 \times 25} = 13$ *l.* 6*s.* 6*d.* $\frac{18}{25}$

Examp. 26. (Being the second Example in the last mentioned Rule) If $40\frac{3}{5}$*l.* in $\frac{2}{3}$ of a Year gain $2\frac{1}{2}$*l.* what will 100*l.* gain in $\frac{7}{12}$ of a year?

Now $40\frac{3}{5} = \frac{203}{5}$; and $2\frac{1}{2} = \frac{5}{2}$.

	£.	*Years.*	£.
Conditional terms ————	$\frac{203}{5}$	$\frac{2}{3}$	$\frac{5}{2}$
Corresponding ————	100	$\frac{7}{12}$	A.

Therefore $A = \frac{100 \times 7 \times 5 \times 5 \times 3}{12 \times 2 \times 203 \times 2} = \frac{25 \times 7 \times 5 \times 5}{4 \times 203}.$

And $\frac{25 \times 7 \times 5 \times 5}{4 \times 203} = \frac{4375}{812} = 5$ *l.* 7 *s.* $9\frac{3}{29}$*d.*

Examp. 27. An usurer put 75 *l.* out to receive interest for the same, and when it had continued 9 months, he received for principal and interest 78 *l.* 7 *s.* 6 *d.* I demand at what rate *per cent. per annum* he received interest?

The amount 78 *l.* 7 *s.* 6 *d.* less the principal 75 *l.* leaves the interest 3 *l.* 7 *s.* 6 *d.* $= 3\frac{3}{8}$ *l.* $= \frac{27}{8}$ *l.*

	£.	*Months.*	£.
Conditional terms ————	75	9	$\frac{27}{8}$
Corresponding ————	100	12	A.

Therefore $A = \frac{100 \times 12 \times 27}{75 \times 9 \times 8} = 2 \times 3 = 6$ *l.* the answer.

Examp

Examp. 28. If $17\frac{1}{2}$ ells of diaper, $\frac{3}{4}$ of a yard wide, cost 5 guineas; what cost $32\frac{1}{4}$ yards $\frac{2}{3}$ of an ell wide, of the same sort and fineness?

$17\frac{1}{2} = \frac{35}{2}$ ells $= \frac{35 \times 5}{2 \times 4}$ yards; $\frac{2}{3}$ ells $= \frac{2 \times 5}{3 \times 4}$ yards.

5 guineas $= 5\,l.\ 5\,s. = \frac{105}{20}\,l.$ $32\frac{1}{4} = \frac{129}{4}$.

	Yards long.	*Yards wide.*	£.
Conditional terms ———	$\frac{35 \times 5}{2 \times 4}$	$\frac{3}{4}$	$\frac{105}{20}$
Corresponding ———	$\frac{129}{4}$	$\frac{2 \times 5}{3 \times 4}$	A.

$$\text{Therefore A} = \frac{129 \times 2 \times \not 5 \times 105 \times 2 \times \not 4 \times \not 4}{\not 4 \times 3 \times \not 4 \times 20 \times 35 \times \not 5 \times 3}$$

$$= \frac{\not 1 \not 2 \not 9 \times 2 \times \not 1 \not 0 \not 5 \times 2}{\not 3 \times 20 \times \not 3 \not 5 \times 3} = \frac{43 \times \not 7 \times \not 2 \not 1 \times \not 2}{\not 2 \not 0 \times \not 7 \times 3}$$

$$= \frac{43 \times 3}{5 \times 3} = \frac{43}{5} = 8\,l.\ 12\,s.$$

CHAP. XXXI.

Of POWERS, *and their* INDICES, *or* LOGARITHMS.

408. IF unity be continually multiplied by any number whatsoever (suppose 2) the products arising are called powers of that number; thus 2. 4. 8. 16. &c. are powers of the given number 2.

409. The given number (2) is called the first power, being produced by the first Multiplication; the next number (4) is called the second (power or square) being the product of the second Multiplication; the next number (8) is called the third power (or cube) being the result of the third Multiplication, &c.

410.

410. Again, with reſpect to any of the ſuperior products 4. 8. 16. *&c.* the given number (2) is called the root; thus 2 is the ſquare root of 4; the cube root of 8; the biquadrate root of 16, *&c.*

411. The number of Multiplications uſed, in producing any power, is called its Index; thus, the index of the root 2 is 1; the index of the ſquare 4, is 2; of the cube 8 is 3, *&c.*

412. If a rank of powers be wrote, with their proper indexes over them as below:

Indexes 0 . 1 . 2 . 3 . 4 . 5 . 6 . 7, *&c.*
Powers 1 . 2 . 4 . 8 . 16 . 32 . 64 . 128, *&c.* Then,

Firſt, The ſum of the indexes of any two powers, will ſtand over the product of thoſe powers; thus 5, the ſum of 2 and 3, ſtands in the rank of indexes over 32, the product of 4 and 8 in the rank of powers; and 6, the ſum of 3 and 3, ſtands over 64, the product of 8 and 8.

413. *Secondly*, The difference of the indexes of any two powers, will be the index of the quotient ariſing by dividing one of thoſe powers by the other; thus 2, the difference of the indexes 7 and 5, is the index of 4, the quotient of 128 divided by 32.

414. Hence having a few of the initial powers given, any other power may be found, without finding all the intermediate powers.

Rule. Compoſe the index of the required power by the addition of the indexes of the given powers; then multiply the powers correſponding to thoſe indices, and the product will be the power required.

Examp. 1. Let it be required by the help of the firſt 7 powers of 2, given in *Art.* 412. to form the 20th power thereof.

This may be performed divers ways, becauſe (20) the index of the required power can be diverſely compoſed of the given indexes.

For $7+7+6=20$ the index of the required power.
$5+5+5+5=20$
$3+4+6+7=20$
&c.

And $128\times128\times64=1048576$ the power required.
$32\times32\times32\times32=1048576$
$8\times16\times64\times128=1048576$

415. If unity be continually divided by any number (2) the quotients ariſing $\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{8} \cdot \frac{1}{16}$, *&c.* may be alſo called powers of 2.

416. But, becauſe the formation of theſe laſt powers is by a proceſs directly contrary to the former, the indexes of theſe powers are generally diſtinguiſhed, from the indexes of thoſe, by placing the ſign (—) either before or under the figure thus (—1, or $\underline{1}$) and are called negative indexes.

417. A rank of theſe powers, being placed under their indexes, have the ſame properties as the former.

Indexes	$0 \cdot \underline{1} \cdot \underline{2} \cdot \underline{3} \cdot \underline{4} \cdot \underline{5} \cdot \underline{6} \cdot \underline{7}$, *&c.*
Powers	$1 \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{8} \cdot \frac{1}{16} \cdot \frac{1}{32} \cdot \frac{1}{64} \cdot \frac{1}{128}$, *&c.*

For $\underline{5}$ the ſum of $\underline{2}$ and $\underline{3}$, ſtands over $\frac{1}{32}$, the product of $\frac{1}{4}$ and $\frac{1}{8}$.

And $\underline{2}$, the difference of $\underline{7}$ and $\underline{5}$, ſtands over $\frac{1}{4}$ the quotient of $\frac{1}{128}$ divided by $\frac{1}{32}$.

418. But when the negative powers are joined in the ſame rank with the affirmative as below,

&c. $\underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} \cdot 0 \cdot 1 \cdot 2 \cdot 3 \cdot 4$ *&c.*	
&c. $\frac{1}{16} \cdot \frac{1}{8} \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot 1 \cdot 2 \cdot 4 \cdot 8 \cdot 16$, *&c.*	Then,

The negative indexes will, in all operations, have effects contrary to thoſe of the affirmative indexes, viz.

If a negative and an affirmative power be to be multiplied together, take the difference of their indexes for the index of the product required.

Thus	$16 \times \frac{1}{2} = 8$
And	$4 - \underline{1} = 3$
Alſo	$\frac{1}{8} \times 2 = \frac{1}{4}$
And	$\underline{3} - 1 = \underline{2}$
Again	$\frac{1}{4} \times 4 = 1$
And	$\underline{2} - 2 = 0$

419. Hence, if any affirmative power be multiplied by a negative power, whoſe index is the ſame number, the product will be unity.

Note, Numbers that have the above property are called reciprocals; and their indexes, complements of each other.

420. If the reciprocal of any number be decimally expreſſed, then if you multiply thereby, the ſame reſult will be produced as would ariſe from dividing by the number; and if you divide

vide thereby, the ſame figures will appear as by multiplying by the number.

Example. $\frac{1}{4}$ = ,25 is the reciprocal of the number 4: Now let it be propoſed to divide 56 by 4, or to multiply it by ,25, the reſult will be the ſame as you ſee.

```
4) 56        56
   --        ,25
   14       -----
             280
            112
            -----
            14,00
```

Again, let 9 be multiplied by 4, or divided by ,25.

```
  9        ,25) 9,00(36
  4             75
 --            ----
 36             150
                150
```

421. If a rank of numbers be calculated, having the ſame properties with thoſe of indexes, mentioned in *Art.* 412, 413, 417, and 418. and be adapted not only to a rank of powers as thoſe, but, to all numbers whatſoever within certain limits, the ſame is called a Table of Logarithms.

422. Inſtead of the rank of powers 1, 2, 4, 8, &c. aſſume 1. 10. 100. 1000, &c. and adapt the indexes to them, *viz.*

0	1	2	3	4
1	10	100	1000	10000, &c.

Then, ſince the Index or Logarithm of 1 is 0, and the Index or Logarithm of 10 is 1; it follows, that the Logarithms of all numbers, that are greater than 1 and leſs than 10, ſuch as 2. 3. 4. 4,1. 4,2. 4,3. &c. muſt be Decimal Fractions.

423. Alſo, ſince the Index or Logarithm of 10 is 1, and the Index or Logarithm of 100 is 2; therefore the Logarithms of all numbers that are greater than 10, and leſs than 100, ſuch as 20, 30, 40, 41, 42, 43, &c. will be mixed decimal expreſſions, having 1 in the place of integers.

424. In like manner, the Logarithms of all numbers greater than 100, and leſs than 1000, will be mixed decimal expreſſions, having 2 in the place of integers, &c.

425. Hence it follows, that the decimal part of the Logarithms of all numbers that have the ſame ſignificant figures

 will

will be the ſame; for the number 4,1 ſtands with regard to 1 and 10, in the ſame place that 41 does with regard to 10 and 100, and that 410 does with regard to 100 and 1000.

Therefore ſuppoſing that ,6127839 be the Decimal part of the Logarithm belonging to the ſignificant figures 41: Then

The Logarithm of	4100,	$= 3,6127839$
	410	$= 2,6127839$
	41	$= 1,6127839$
	4,1	$= 0,6127839$
	,41	$= \bar{1},6127839$
	,041	$= \bar{2},6127839$
		&c.

426. Of this kind are the Logarithms commonly put in Tables, which if rightly calculated will perform the work of Multiplication and Diviſion by Addition and Subtraction, *&c.* in the ſame manner as the Indices did.

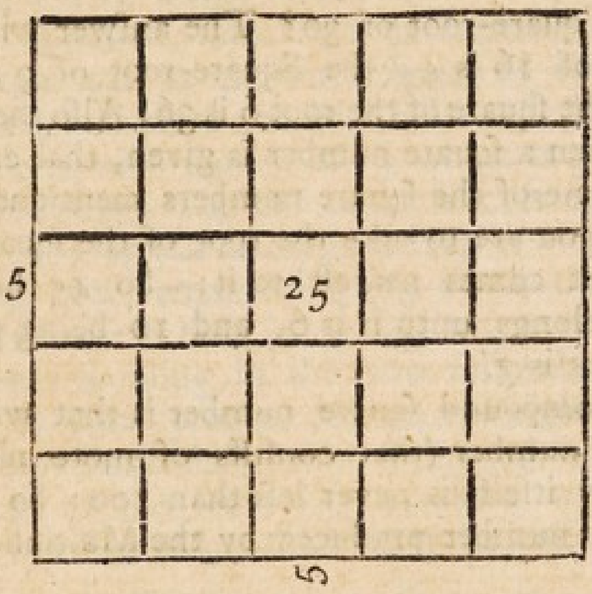
427. But as the operations in which Logarithms are chiefly uſeful in common Arithmetic, *viz.* extracting roots, and computing queſtions relating to compound intereſt and annuities, are in this treatiſe ſupplied by Tables, calculated for each purpoſe; and as the methods of calculating them depend upon algebraical and other ſuperior principles; the farther proſecution of the ſubject here is declined, it being ſufficient to have given the reader an idea of their nature and properties.

CHAP.

CHAP. XXXII.

The EXTRACTION *of the* SQUARE (*or* QUADRATE) ROOT.

428. THE extraction of the Square-root is that, by which having a number given, we find out another number, which being multiplied by itſelf, produces the number given.



429. In the extraction of the Square-root, the number propoſed is always conceived to be a ſquare number, that is, a certain number of little ſquares comprehended within one entire great ſquare, and the root or number required is the ſide of that great ſquare; as will readily appear by this Diagram, where you ſee the 25 little ſquares contained within one great ſquare; now if the ſaid content 25 be given, and the ſide or root of the ſquare containing the ſaid 25 little ſquares is required, the invention of ſuch ſide or root is called the Extraction of the Square-root; which root muſt be ſuch, that if it be ſquared, that is, multiplied by itſelf, the product will be equal to the ſquare content firſt given: So 5 is the Square-root of 25, for 5 times 5 is 25. Likewiſe this ſquare number 49 being propounded, its root is 7.

430. Square numbers are either ſingle or compound.

431. A ſingle ſquare number is that, which being produced by the Multiplication of one ſingle figure by itſelf, is always leſs than 100: So 25 is a ſingle ſquare number produced by 5; likewiſe 4 is a ſquare number produced by 2.

432. All the ſingle ſquare numbers, together with their reſpective roots are expreſſed in the following Table.

Squares.	1	4	9	16	25	36	49	64	81
Roots.	1	2	3	4	5	6	7	8	9

433. Here in the uppermoſt rank of the Table are placed the ſingle ſquare numbers of every particular figure, and in the other their reſpective roots; and therefore if it were demanded, what is the Square-root of 36? The anſwer will be 6. So the Square-root of 16 is 4; the Square-root of 9 is 3, *&c.* And contrarily, the ſquare of the root 6 is 36: Alſo the ſquare of 3 is 9.

434. When a ſquare number is given, that exceeds not 100, and yet is none of the ſquare numbers mentioned in the Table, for its root you are to take the root of the ſquare number that being leſs yet comes neareſt to it: So 45 being given, the root that belongs unto it is 6, and 10 being given, its correſpondent root is 3.

435. A compound ſquare number is that which being produced by a number (that conſiſts of more places than one) multiplied by itſelf, is never leſs than 100: So 1024 is a compound ſquare number produced by the Multiplication of 32 by itſelf.

436. To prepare any ſquare number given for Extraction, put a point over the firſt place thereof on the right-hand (being in the place of units;) then proceeding towards the left-hand, paſs over the ſecond place, and put another point over the third place; alſo paſſing over the fourth place put another point over the fifth, and ſo forward in ſuch manner that between every two points which are next to the other, one place will be intermitted: So if the Square-root of 1024 be required, the firſt point is to be placed over 4, and the ſecond over 0, and ſo many points as are in that manner placed, of ſo many figures the root demanded will conſiſt.

$1\dot{0}2\dot{4}$

437. Having thus prepared the number, it is diſtributed by the points into ſeveral ſquares: So in the laſt Example, 10 is the firſt ſquare, and 24 the ſecond; likewiſe if this number 144 were propounded for Extraction, after points are duly placed according to the laſt Rule, 1 is the firſt ſquare, and 44 the ſecond.

$\dot{1}4\dot{4}$

438. Having drawn a crooked line on the right-hand of the number given for Extraction, (after the ſame manner as is uſually done in Diviſion to denote the place of the Quotient,) find the root of the firſt ſquare, and place it in the Quotient: Now by *Art.* 433. 3 is the correſpondent root of 10; wherefore write 3 in the Quotient, and then the Work will ſtand as in the Margin.

```
 . .
1024 (3
```

```
 . .
1024 (3
9
──────
```

439. Subſcribe the ſquare of the figure placed in the Quotient under the firſt ſquare of the number given.

440. Having drawn a line under the ſquare (of the figure placed in the Quotient, ſubſcribed as aforeſaid,) ſubtract the ſame out of the firſt ſquare of the number propoſed, and place the remainder orderly under the line; ſo the ſquare of 3 which is 9, being ſubtracted from 10, the remainder is 1, and the Work will appear as in the Margin.

```
 . .
1024 (3
 9
──────
 1
```

441. To the ſaid remainder bring down the next ſquare of the given number, that is, write down the figures or cyphers ſtanding in the two following places of the number propounded on the right-hand of the ſaid remainder, ſo the ſquare 24 being placed next to the remainder 1, there will be found this number 124, which may be called the Reſolvend.

```
 . .
1024 (3
 9
──────
 124
```

442. Double the root, being the number placed in the Quotient, and place the ſaid double on the left-hand of the Reſolvend, like a Diviſor: So the double of 3 is 6, which being placed before a crooked line on the left-hand of the Reſolvend 124, the work will ſtand as in the Margin.

```
   . .
  1024 (3
   9
  ──────
6) 124
```

443. Let the whole Reſolvend, except the firſt place thereof on the right-hand (being the place of units) be always eſteemed as a Dividend, then demanding how often the Diviſor before found, is contained in the ſaid Dividend, and obſerving in that behalf the Rules before taught in Diviſion, write the anſwer in the Quotient, and alſo on the right-hand of the Diviſor, to wit, between the Diviſor and the crooked line: So if you aſk how often the Diviſor 6 is found in the Dividend 12, the anſwer is 2; therefore write 2 in the Quotient, and alſo after the Diviſor 6; as in the Margin.

```
    . .
   1024 (32
    9
   ──────
62) 124
```

444. Multiply all the figures which ſtand on the left-hand of the Reſolvend (to wit, before the crooked line) by the figure laſt placed in the Quotient, and write the product orderly under the Reſolvend, to wit, units under units tens under tens, *&c.*) then having drawn a line under the ſaid product, ſubtract it from the Reſolvend, and ſubſcribe the remainder under the line: So 62 being multiplied by 2, the product is 124, which ſubtract out of the Reſolvend 124, the remainder is 0; and thus the whole Work being finiſhed, the Square-root of 1024 (the number propoſed) is found to be 32.

```
     . .
     1024(32
     9
   -------
  62) 124
      124
   -------
        0
```

Note 1. When the product before-mentioned exceeds the Reſolvend placed above it, the Work is erroneous, and then you are to reform it by placing a leſſer figure in the Quotient.

Note 2. For every one of the particular ſquares (diſtinguiſhed by the points) except the firſt on the left-hand, a Reſolvend is to be ſet a-part, by bringing down to the remainder the congruent particular ſquare, as is directed in *Art.* 441. and as often as a Reſolvend is ſet a-part, ſo often a new Diviſor is to be found by doubling or multiplying by 2 all the root in the Quotient (conſiſting of what number of places ſoever.)

Note 3. The Work of *Art.* 438, 439, and 440. for finding out the firſt figure in the root, is but once uſed in the Extraction of the Square-root of a number conſiſting of what number of places ſoever; but the Work of *Art.* 441, 442, 443, and 444. is to be repeated for the finding of every place in the root, except the firſt.

The practice of theſe three Notes will be ſeen in the following Examples.

Example 1. Let it be required to extract the Square-root of 43623.

Having diſtributed the number propoſed into ſeveral ſquares by points, as is directed in *Art.* 436. I demand the Square-root of 4 the firſt ſquare, which by *Art.* 432. is 2; therefore placing 2 in the Quotient, and the ſquare thereof, which is 4, under the firſt ſquare 4, draw a line, and ſubtracting 4 from 4, the remainder is 0, which ſubſcribe under the line. This is always the firſt Work, which is no more repeated in the whole Extraction, (as was intimated in the third Note aforegoing.)

```
 .  . .
 43623(2
 4
 -------
 0
```

Then bringing down the next ſquare, which is 36, and placing it next after the remainder 0, the Reſolvend is 36; and

and doubling the root 2 in the Quotient, the product is 4 for a Diviſor, (by *Art.* 441, 442.) and the Dividend will be 3, (by *Art.* 443.) demand therefore how often the Diviſor 4 is contained in the Dividend 3, and not finding it once contained in it, place 0 in the Quotient, and alſo next after the Diviſor 4; and becauſe the product of 40 multiplied by 0 (the laſt character in the Quotient) is 0, the Reſolvend 36, from which the ſaid product ought to be deducted, remains the ſame without alteration; therefore bring down 23 the next ſquare, and place it after the remainder 36, ſo will 3623 be a new Reſolvend; then doubling the whole root in the Quotient, which is 20, the Diviſor will be 4 0, (according to the ſecond Note before-mentioned,) and the Dividend will be 362 to wit, all the Reſolvend except the firſt place on the right-hand (by *Art.* 443.) therefore demand how often the Diviſor 40 is contained in the Dividend 362, or how often 4 in 36, and though it be 9 times in it, yet (according to the firſt Note aforegoing) I can take but 8, (for if I ſhould take 9, and proceed according to *Art.* 443, 444. a number would ariſe greater than the Reſolvend, from which ſuch number ariſing ought to be ſubtracted:) Therefore write 8 in the Quotient, and alſo after the Diviſor 40; this done multiply 408 (the number on the left-hand of the Reſolvend) by 8 the figure laſt placed in the Quotient, and the product, to wit, 3264, ſubſcribe underneath, and ſubtract from the Reſolvend 3623, ſo there will remain 359; thus the Work being finiſhed, 208 is the number of units contained in the root ſought; and becauſe after the Extraction is ended there happens to be a remainder, to wit, 359, I conclude that the root is greater than the ſaid 208, but leſs than 209; yet how much it is greater than 208, no rules of art hitherto known will exactly diſcover, tho' we may proceed infinitely near, as in the next Article will be manifeſt.

```
     . . .
     43623 (20
     4
    ---------
 40) 036
```

```
     . . .
     43623 (20
     4
    ---------
 40) 03623
```

```
      . . .
      43623 (208
      4
     ---------
 408) 03623
      3264
     ---------
       359
```

445. To find the fractional part of the root very near, a competent number of pairs of cyphers, to wit, 00, 0000, 000000, or 00000000, *&c.* are to be annexed to the number firſt propoſed; then eſteeming the ſaid given number with the cyphers annexed, to be but one entire number, the extraction is to be made according to the preceeding Rules, and look how

many points were placed over the number firſt given, ſo many places of integers will be in the root, the reſt of the root towards the right-hand will be the Numerator of a Decimal Fraction, which Numerator conſiſts of ſo many places as there were points over the cyphers annexed: So if 43623 were given as before, to find the root thereof (according to this Rule,) annex cyphers in this manner, and then if you extract it according to the Rules aforegoing, you'll find the root ariſing in

$$\dot{4}3\dot{6}2\dot{3}.\dot{0}0\dot{0}0\dot{0}0 \;(208.861.\ \&c.$$

the Quotient to be 208.861, that is 208 $\frac{861}{1000}$; and becauſe after the Extraction is finiſhed there happens to be a remainder, I conclude that 208 $\frac{861}{1000}$ is leſs than the true or exact root, but 208 $\frac{862}{1000}$ is greater than it; ſo that by annexing three pairs of cyphers to the number propounded, you will not miſs $\frac{1}{1000}$ part of an unit of the true root; alſo by annexing four pairs of cyphers, you will not miſs $\frac{1}{10000}$ part of an unit; and in that order you may proceed infinitely near, when you cannot obtain the true root. The whole operation of the ſaid Example here follows:

```
          . . . . . . .
          43623.000000 (208.861, &c.
          4              the Root.
          -------------------------
   408) 3623
        3264
        ----
  4168) 35900
        33344
        -----
 41766) 255600
        250596
        ------
417721) 500400
        417721
        ------
         82679
```

Again, if 10 were propoſed to be extracted, you muſt prepare it thus:

$$\dot{1}0.\dot{0}0\dot{0}0\dot{0}0\dot{0}0\dot{0}0\dot{0}0\dot{0}0\dot{0}0.$$

And

And then the root thereof being extracted will be——— } $3\frac{1622776}{10000000}$, &c.

Which (according to *Art.* 309. may be writ thus——— } 3,1622776, &c.

See the Work.

```
 10.00000000000000 (3.16227, &c.
  9
 ─────────
 61)    100
         61
 ─────────
 626)    3900
         3756
 ─────────
 6322)   14400
         12644
 ─────────
 63242)  175600
         126484
 ─────────
 632447) 4911600
         4427129
 ─────────
          484471
           &c.
```

446. The number of the places in the root may be nearly doubled, if, after the above Work, you proceed with the last Divisor and remainder, according to the manner of contracted Division, in *Art.* 404.

Thus assuming the last Example.

```
632447) 484471 (76602, &c.
 .....  442713
        ------
         41758
         37947
        ------
          3811
          3795
        ------
            16
            13
        ------
             3
```

If the figures thus found (76602) be placed on the right-hand of (3,16227) thoſe before obtained, the Root will be 3,1622776602, uncertain only in the laſt figure 2.

447. Hence after it is determined to what exactneſs the Square-root of any ſurd number is to be found, the Rule for extracting the Square-root muſt be uſed, to one place of figures more than half the number of places required, and then the reſt may be found by Diviſion as above.

448. The Extraction of the Square-root is proved by multiplying the root by itſelf, for that done, the product (in ſuch caſe where there is no remainder after the Extraction is finiſhed) will be equal to the root whoſe ſquare was enquired; ſo in the firſt Example of this Chapter the root 32 being multiplied by itſelf, produces 1024 the number propounded: But when, after the Extraction is finiſhed, there happens to be a remainder, and that the root is found as near as you pleaſe, in a mixt number of Integers and Decimal parts (by annexing cyphers, as in *Art.* 445.) then ſuch mixt number being multiplied by itſelf muſt produce a mixt number leſs than the number firſt given for Extraction; yet ſo near it, that if the figure ſtanding in the laſt place of the Numerator of the Decimal Fraction in the Root, be made greater by 1, and then the mixt number ſo increaſed be multiplied by itſelf, the product muſt be greater than the number firſt propoſed: So in the Example of *Art.* 445. if 208.861 be multiplied by itſelf, it produces 43622.917, &c. which is leſs than the given number 43623; but if 208.862 be multiplied by itſelf, the product will be 43623.335, &c. which is greater than 43623.

449. But if the method of inverted Multiplication, delivered in *Art.* 396. be applied to the multiplying the Root by itſelf, the product will approximate to the ſquare very nearly.

Proof

Proof of the Example in *Art.* 446.

```
 3,1622776602
 2066772261,3
 ------------
 94868329806
 3162277660
 1897366596
  63245553
  6324555
   2213594
    221359
     18974
      1897
         6
 ------------
10,0000000000
```

450. The Square-root of a Fraction is found in this manner, *viz.* extract the root of the Numerator (by the preceding Rules of this Chapter) which root shall be a new Numerator. Also the root of the Denominator is to be taken for a new Denominator: So the new Fraction will be the Square-root of the Fraction first proposed. Thus the Square-root of $\frac{9}{16}$ is $\frac{3}{4}$, *viz.* the root of 9 is 3 for a new Numerator, also the root of 16 is 4 for a new Denominator. In like manner the Square-root of $\frac{1}{4}$ is $\frac{1}{2}$. But here note diligently, that if the Fraction, whose Square-root is required, be not in its least terms, it must first of all be reduced by *Art.* 261, *&c.* before any Extraction be made; for it frequently happens that the Fraction first given has not a perfect root, but when such Fraction is reduced to its least terms, the root thereof may be extracted: So in this Fraction $\frac{8}{18}$ each term is incommensurable to its Square-root, *viz.* neither 8 nor 18 has a Square-root expressible by any true or rational number; but the said $\frac{8}{18}$ being reduced to its least terms $\frac{4}{9}$, the root of this may be extracted; for the root of 4 is 2 for a Numerator; also the root of 9 is 3 for a new Denominator; so that $\frac{2}{3}$ is found to be the Square-root of $\frac{4}{9}$ (equivalent to $\frac{8}{18}$.)

451. When either the Numerator or Denominator of a Fraction has not a perfect Square-root, such root is usually expressed by prefixing this character, $\surd$ before the Fraction given: So the Square-root of $\frac{13}{16}$ is signified thus, $\surd\frac{13}{16}$, because the root of $\frac{13}{16}$ cannot be expres'd by any true or rational number whatsoever, yet it may be found very near, as in the next Rule.

452.

452. The Square-root of a Fraction which is incommensurable to its root, may be found near, in this manner, *viz.* reduce the Fraction proposed to a Decimal, by *Art.* 322. The more places are in the Decimal, the nearer will the root be found; but the Decimal must consist of an even number of places, *viz.* either of two, four, six, eight, or ten *&c.* places; then extract the Square-root of that Decimal, as if it were a whole number, according to the Rules aforegoing, which root found shall be a Decimal expressing near the Square-root of the given Fraction.

So if the Square-root of $\frac{13}{16}$ be required near, reduce the said $\frac{13}{16}$ to a Decimal, which is found .81250000, *&c.* then extracting the Square-root thereof as if it were a whole number, it will be found .9013 very near.

453. The Square-root of a mixt number commensurable to its root is found in the same manner, as in *Art.* 450. the mixt number being reduced to an improper Fraction, by *Art.* 268.

So the Square-root of $34\frac{33}{64}$ will be found $5\frac{7}{8}$, *viz.* $34\frac{33}{64}$ being reduced to the improper Fraction $\frac{2209}{64}$, the Square-root of the Numerator 2209 will be 47 for a new Numerator; also the Square-root of the Denominator 64 is 8, for a new Denominator; so we find $\frac{47}{8}$ which (by *Art.* 271.) is $5\frac{7}{8}$ the Square-root sought. And here the same caution is to be observed, as in *Art.* 450. *viz.* the fractional part of the mixt number, or the improper Fraction equivalent to the mixt number, must be in the least terms before any Extraction be made.

454. When the mixt number given is incommensurable to its Square-root, prefix this character before it, *viz.* $\surd$. So the Square-root of $7\frac{2}{3}$ will be thus expressed, $\surd\, 7\frac{2}{3}$: But if you desire to find the Square-root near of a mixt number incommensurable to its root, reduce the fractional part of the mixt numbea to a Decimal of an even number of places, as in *Art.* 452. and annex the Decimal so found to the whole part of the mixt number; then esteeming the said whole number and Decimal as one entire number, extract the Square-root thereof according to the preceding Rules of this Chapter, and from the root found cut off always to the right-hand, so many places as there are points over the Decimal annexed, which number so cut off shall be a Decimal, shewing the fractional part of the root, and that on the left-hand will be the whole part of the root; so the Square-root of $7\frac{2}{3}$ is found 2.7688 very near.

CHAP.

CHAP. XXXIII.

The Extraction *of the* Cube Root.

455. THE Extraction of the Cube-root is that, by which having a number given, we find another number, which being multiplied by itſelf, and then by the product, produces the number given.

456. In the Extraction of the Cube-root, the number propoſed is always conceived to be a Cube number, that is, a certain number of ſmall Cubes, comprehended within one entire great Cube, and the root or number required is the ſide of the great cube: What a Cube is may be well expreſſed by a die, which indeed is a little Cube itſelf; therefore if you place four dice in a ſquare form, that is laying two and two in a rank, you'll have a ſquare coutaining four dice, upon which if you yet erect ſuch another ſquare of dice, you ſhall have another great entire Cube comprehending two times four, that is 8 dice or ſmall cubes; and here 8 is the Cube number given, and 2 is the root or number required: In like manner, if you rank 25 dice in a ſquare form, *viz.* laying 5 in the rank, you have a ſquare containing 25 dice; now upon this ſquare of dice if you erect four other ſuch ſquares one upon another, you'll have a great entire Cube comprehending 5 times 25, that is 125 little Cubes, and in this caſe 125 is the Cube number propounded, and 5 the root or number required.

457. A Cube-number is either ſingle or compound.

458. A ſingle Cube-number is that, which being produced by the Multiplication of one ſingle figure, firſt by itſelf, and then by the product, is always leſs than 1000. So 125 is a ſingle Cube-number produced by 5 multiplied firſt by itſelf, and then by 25 the product; for 5 times 5 is 25, and 5 times 25 is 125.

459. All the ſingle Cube-numbers, and Square numbers, together with their reſpective roots, are expreſſed in the following Table.

Cubes.	1	8	27	64	125	216	343	512	729
Squares.	1	4	9	16	25	36	49	64	81
Roots.	1	2	3	4	5	6	7	8	9

Here

Here in the uppermoſt rank of the Table are placed the ſingle Cube-numbers of the particular figures, 1, 2, 3, 4, 5, 6, 7, 8, 9; in the next, the ſquares of thoſe figures, and in the loweſt rank the figures themſelves being the reſpective roots of the Cubes and Squares in the uppermoſt rank; and therefore the Cube-root of 125 being demanded, the anſwer is 5, and the Cube-root of 216 being required, the Table will give 6, and ſo of the reſt.

460. When a Cube-number is given, that exceeds not 1000, and yet is none of the Cube-numbers mentioned in the Table, for its Root you are to take the root of the Cube-number, that being leſs, yet comes neareſt to it: So 157 being given, the root that belongs to it is 5.

461. A compound Cube-number is that, which being produced by a number (that conſiſts of more places than one) firſt multiplied by itſelf, and then by the product, is never leſs than 1000. So 157464 is a compound Cube-number, produced by 54 multiplied firſt by itſelf, and then by 2916 the product, for 54 times 54 is 2916, and then 54 times 2916 is 157464, the compound Cube-number propoſed.

462. To prepare a Cube-number for Extraction, put a point over the firſt place thereof towards the right-hand (to wit, the place of units) then paſſing over the ſecond and third places, put another point over the fourth, and paſſing over the fifth and ſixth, put another point over the ſeventh, and in that order (to wit, two places being intermitted between every two adjacent points,) place as many points as the number will permit: So 157464 being given, you are to place the points as in the margin, and ſo many points as are in that manner fixed, of ſo many figures the root demanded will conſiſt.

$$15\dot{7}46\dot{4}$$

463. Having thus prepared your number, you may ſee it diſtributed by the points into ſeveral Cubes or periods.

So in the ſame Example 157 is the firſt Cube, and 464 the ſecond. In like manner, if this number 7464 were propoſed for Extraction, after points are duly placed as before, you'll ſee 7 to be the firſt Cube, and 464 the ſecond.

$$15\dot{7}46\dot{4}$$

$$\dot{7}46\dot{4}$$

464. Having drawn a crooked line on the right-hand of the number propounded to ſignify a Quotient, ſeek for the Cube-root of the firſt Cube and ſet in the Quotient: So finding (by *Art.* 460.) 5 to be the correſpondent root of 157, write 5 in the Quotient, and then the Work will ſtand as you ſee in the Margin.

$$15\dot{7}46\dot{4}\,(5$$

465.

465. Subſcribe the Cube of the root placed in the Quotient under the firſt Cube of the number given: So 125 being the Cube of 5, the root, (by *Art.* 459.) write it under 157 the firſt Cube of the number given, as you ſee in the Example.

```
157464 (5
125
```

466. Draw a line under the Cube, ſubſcribed as aforeſaid, (to wit, the Cube of the root ſet in the Quotient) and ſubtract this Cube from the firſt Cube of the number propoſed, placing the remainder orderly under the line: So 125 the Cube of 5 being ſubtracted from 157, the remainder is 32, and the Work will ſtand as you ſee.

```
157464 (5
125
------
 32
```

467. To the ſaid remainder bring down the remaining periods of the number propounded, placing theſe next after, to wit, on the right-hand of the remainder; ſo 464 being ſet after the remainder 32, there will be found this number 32464, which may be called the Reſolvend.

```
157464 (5
125
------
 32464 Reſ.
```

468. To (5) the root, or figure placed in the Quotient, annex as many cyphers as there are Cubes or periods remaining unuſed in (157464) the number whoſe root is to be extracted, which in this caſe is 1, and therefore the root will be 50.

469. Divide the Reſolvend (32464) by the root (50;) ſo will the Quotient be 649.

470. To (649) the above found Quotient, add three times the ſquare of the root (50), *viz.* 7500; ſo will the ſum be 8149. Let this number be a Diviſor.

471. Divide the Reſolvend (32464) by the Diviſor (8149;) and the Quotient figure, or figures, being annexed to (5) the figure firſt placed in the root, inſtead of the cypher or cyphers, will be the required root nearly. See the whole proceſs below.

```
          157464(50 Root            5.0) 3246.4
          125                            ------
          ---                               649
Reſolvend 32464             50×50×3=7500.
                                         ------
                                   Diviſor=8149
       8149) 32464 (4 Quotient.
```

Therefore $50+4=54$ is the root required, as will appear by cubing thereof.

Note, Altho' the Divifor 8149 cannot be had quite 4 times in the Refolvend 32464, yet as it is very near thereto, it will be fafe to take that for the Quotient.

472. The Rule above given will, if the fecond figure of the root be 1 or 0, generally give three or four figures true in the root (if it confifts of fo many); but if the fecond figure be 2, or more, it will be beft to find no more places than it, until the procefs be renewed.

473. To renew the procefs, cube the figures found in the root, that is, multiply them by themfelves, and then by the product; and place the refult under the given number, in fuch fort, that it may ftand under as many periods therein, as you have cubed figures. Subtract the lower number from the upper, and bring down the remaining periods for a Refolvend, as in *Art.* 467. Find a new Divifor, by *Art.* 468, 469, and 470. Divide the Refolvend thereby, and annex the Quotient to the former part of the root, as in *Art.* 471. fo fhall the root be found true to 5 or 6 places of figures: For each operation commonly triples the figures found in the former.

Example 2. Let it be required to extract the Cube-root of 9302348?

The given number pointed *per Art.* 462.	9302348 (2
The Cube of the root 2 is (*Art.* 465.)	8
The Refolvend, *per Art.* 466, and 467.	1302348
The root with 2 cyphers annexed by *Art.* 468.	200
The Refolvend divided thereby (*Art.* 469.) $= \frac{1302348}{200} =$	6511
Three times the fquare of the root (200)	120000
The Divifor = fum of the 2 laft, by *Art.* 470.	126511
The Refolvend divided thereby *per Art.* 471. $\frac{1302348}{126511} =$	10

Therefore 210 is the root nearly.

But,

But, to renew the operation,

From the given number	9302348 (220
Take 210×210×210	= 9261000
Resolvend	41348
The Resolvend divided by the root $=\frac{41348}{210}=$	196
Three times the square of 210. .	132300
Divisor	132496
The Resolvend with cyphers annexed divided thereby, quotes ———	,31207

See the Operation.

```
132496) 41348.00. (,31207
   ...  397488
        ------
        159920
        132496
        ------
        .27424
         26499
        ------
           925
           927
          ----
```

Therefore the true Root is 210,31207.

474. In the last Example, when the process is renewed, and 210 the first found part of the root is cubed, it is evident that the given number 9302348 has no true root in whole numbers; because there is a remainder 41348; but, by proceeding according to the Rules before given, the remaining part of the root ,31207 appears to be a Decimal Fraction.

475. If it should be thought necessary to find more places in the root, the same may be done by cubing the root 210,31207, and subtracting the cube thereof from the given number 9302348, with 3 times as many cyphers annexed as there are Decimal places in the root, and then proceeding as before.

476. The Extraction of the Cube-root is proved by multiplying the root cubically, to wit, the root being first multiplied by itself, and then the product multiplied by the root, the number arising, or last product, will be equal to the num-

ber propoſed: So in the firſt Example of this Chapter, the Cube-root 54 being multiplied firſt by itſelf produces 2916, which being multiplied again by 54 produces 157464, to wit, the number whoſe Cube-root was required. But if, as in the ſecond Example, the given number have not a perfect root in whole numbers; then the proof will be moſt expeditiouſly made by inverted Multiplication, as before in the Square-root, *Art.* 449.

477. The Cube-root of a Fraction is found in this manner, *viz.* extract the Cube-root of the Numerator (according to the foregoing Rules,) which root reſerve for a new Numerator; alſo the Cube-root of the Denominator ſhall be a new Denominator: Laſtly, this new Fraction will be the Cube-root of the Fraction firſt propoſed: So the Cube-root of $\frac{8}{27}$ is $\frac{2}{3}$; for the Cube-root of 8 is 2, for a new Numerator; alſo the Cube-root of 27 is 3, for a new Denominator. In like manner, the Cube-root of $\frac{1}{8}$ is $\frac{1}{2}$. But here note diligently, that the Fraction whoſe Cube-root is required, muſt be in its leaſt terms before any Extraction be made; for it frequently happens that the Fraction firſt given has not a perfect root, though, when ſuch Fraction is reduced to its leaſt terms, the root thereof may be extracted: So in this Fraction $\frac{16}{54}$ neither the Numerator nor Denominator has a perfect Cube-root, yet the ſaid $\frac{16}{54}$ being reduced to its leaſt terms $\frac{8}{27}$, the Cube-root of this may be extracted; for the Cube-root of 8 is 2 for a new Numerator, alſo the Cube-root of 27 is 3 for a new Denominator; ſo that the Cube-root of $\frac{8}{27}$ (which is equal to $\frac{16}{54}$) is found to be $\frac{2}{3}$.

478. The Cube-root of a Fraction, which has not a perfect Cube-root, may be found nearly in this manner, *viz.* reduce the Fraction given to a Decimal Fraction, by *Art.* 322. the more places are in the Decimal, the nearer will the root be found: But the Decimal muſt conſiſt of ternaries of places, to wit, either of three, ſix, nine, or twelve, &c. places; then extract the Cube-root of the Numerator of that Decimal, as if it were a whole number, (according to the Rule before given,) which root found, ſhall be a Decimal expreſſing nearly the Cube-root of the Fraction propoſed.

So if the Cube-root of $\frac{2}{3}$ were required, reduce the ſaid $\frac{2}{3}$ to a Decimal, whoſe Numerator may conſiſt of ternaries of places, to wit, into this, 666666666666, &c. then extracting the Cube-root thereof, I find .8735, which is very near the Cube-root of $\frac{2}{3}$.

479. The Cube-root of a mixt number commenſurable to its root may be found in the ſame manner as in *Art.* 477. the

mixt number being firſt reduced to an improper Fraction (by *Art.* 268.)

So the Cube-root of $12\frac{19}{27}$ is diſcovered to be $2\frac{1}{3}$, *viz.* reducing $12\frac{19}{27}$ to this improper Fraction $\frac{343}{27}$, the Cube-root of $\frac{343}{27}$ will be found $\frac{7}{3}$ or $2\frac{1}{3}$. And here the ſame caution is to be obſerved, as in *Art.* 477. *viz.* the fractional part of the mixt number, or the improper Fraction equivalent to the mixt number, muſt be expreſſed by a Numerator and Denominator in the leaſt terms, before any Extraction is made.

480. When the mixt number, whoſe Cube-root is required, has not a perfect Cube-root, this character, $\sqrt[3]{}$ is uſually prefixed before ſuch mixt number; ſo the Cube-root of $2\frac{3}{8}$ is thus expreſſed $\sqrt[3]{2\frac{3}{8}}$. Likewiſe $\sqrt[3]{\frac{5}{8}}$ denotes the Cube-root of $\frac{5}{8}$, which is a Fraction whoſe Cube-root is inexpreſſible by any true or rational number: But if you deſire to know the Cube-root near, of a mixt number which has not a perfect Cube-root, reduce the fractional part of the mixt number to a Decimal, (as in *Art.* 478.) and annex the Decimal ſo found to the Integers of the mixt number; then eſteeming the ſaid Integers with the Decimal ſo annexed as one entire number, extract the Cube-root thereof, and from the root found, cut off always to the right-hand ſo many places as there were points over the ſaid Decimal annexed, which places ſo cut off will be the fractional part of the root, and thoſe remaining on the left-hand ſhall be the Integers of the root: So the Cube-root of $2\frac{3}{8}$ is 1 334, and ſomewhat more.

481. A quadrate or ſquare number multiplied by itſelf produces a biquadrate number: So 4 multiplied by itſelf produces the biquadrate 16. Therefore if a number be propoſed, and the biquadrate-root of it is required, firſt extract the quadrate or ſquare-root of the ſaid given number, and then extract the Square-root of that root for the biquadrate-root ſought. Thus if 20736 be a number propounded, the biquadrate-root thereof will be found 12: For the Square-root of 20736 is 144, and the Square-root of 144 is 12. When the given number has not a perfect biquadrate-root, you are to annex quaternaries of cyphers, to wit, either 4, 8, 12, or 16, &c. cyphers, and then proceed as before; ſo you'll find the root near, whoſe fractional part will be a Decimal. Thus the biquadrate-root of 7 is near 1.62.

482. A TABLE of the Square and Cube-Roots of Numbers not exceeding 144.

N°	Squ. Root.	Cub. Root.	N°	Squ. Root.	Cub. Root.
1	1,000000	1,000000	37	6,082763	3,332222
2	1,414214	1,259921	38	6,164414	3,361975
3	1,732051	1,442250	39	6,244998	3,391211
4	2,000000	1,587401	40	6,324555	3,419952
5	2,236068	1,709976	41	6,403124	3,448217
6	2,449490	1,817121	42	6,480741	3,476027
7	2,645751	1,912933	43	6,557439	3,503398
8	2,828427	2,000000	44	6,633250	3,530348
9	3,000000	2,080084	45	6,708204	3,556893
10	3,162278	2,154435	46	6,782330	3,583048
11	3,316625	2,223980	47	6,855655	3,608826
12	3,464102	2,289428	48	6,928203	3,634241
13	3,605551	2,351335	49	7,000000	3,659306
14	3,741657	2,410142	50	7,071068	3,684031
15	3,872983	2,466212	51	7,141428	3,708430
16	4,000000	2,519842	52	7,211103	3,732511
17	4,123106	2,571282	53	7,280110	3,756286
18	4,242641	2,620741	54	7,348469	3,779763
19	4,358899	2,668402	55	7,416198	3,802953
20	4,472136	2,714418	56	7,483315	3,825862
21	4,582576	2,758923	57	7,549834	3,848501
22	4,690416	2,802039	58	7,615773	3,870877
23	4,795832	2,843867	59	7,681146	3,892996
24	4,898979	2,884499	60	7,745967	3,914867
25	5,000000	2,924018	61	7,810250	3,936497
26	5,099020	2,962496	62	7,874008	3,957892
27	5,196152	3,000000	63	7,937254	3,979057
28	5,291503	3,036589	64	8,000000	4,000000
29	5,385165	3,072317	65	8,062258	4,020726
30	5,477226	3,107232	66	8,124038	4,041240
31	5,567764	3,141381	67	8,185353	4,061548
32	5,656854	3,174802	68	8,246211	4,081656
33	5,744563	3,207534	69	8,306624	4,101566
34	5,830952	3,239612	70	8,366600	4,121285
35	5,916080	3,271066	71	8,426150	4,140818
36	6,000000	3,301927	72	8,485281	4,160168

The

The TABLE of Square and Cube Roots continued.

N°	Squ. Root.	Cub. Root.	N°	Squ. Root.	Cub. Root.
73	8,544004	4,179339	109	10,44031	4,776856
74	8,602325	4,198336	110	10,48809	4,791420
75	8,660254	4,217163	111	10,53565	4,805896
76	8,717798	4,235824	112	10,58301	4,820284
77	8,774964	4,254321	113	10,63015	4,834588
78	8,831761	4,272659	114	10,67708	4,848808
79	8,888194	4,290841	115	10,72381	4,862944
80	8,944272	4,308870	116	10,77033	4,876999
81	9,000000	4,326749	117	10,81665	4,890973
82	9,055385	4,344481	118	10,86278	4,904868
83	9,110434	4,362071	119	10,90871	4,918685
84	9,165151	4,379519	120	10,95445	4,932424
85	9,219544	4,396830	121	11,00000	4,946088
86	9,273618	4,414005	122	11,04536	4,959675
87	9,327379	4,431047	123	11,09054	4,973190
88	9,380832	4,447960	124	11,13553	4,986631
89	9,433981	4,464745	125	11,18034	5,000000
90	9,486833	4,481405	126	11,22497	5,013298
91	9,539392	4,497942	127	11,26943	5,026526
92	9,591663	4,514357	128	11,31371	5,039684
93	9,643651	4,530655	129	11,35782	5,052774
94	9,695360	4,546836	130	11,40175	5,065797
95	9,746794	4,562903	131	11,44552	5,078753
96	9,797959	4,578857	132	11,48912	5,091643
97	9,848858	4,594701	133	11,53256	5,104469
98	9,899495	4,610436	134	11,57584	5,117230
99	9,949874	4,626065	135	11,61895	5,129928
100	10,000000	4,641589	136	11,66190	5,142563
101	10,04988	4,657010	137	11,70470	5,155137
102	10,09950	4,672330	138	11,74734	5,167649
103	10,14889	4,687548	139	11,78983	5,180101
104	10,19804	4,702669	140	11,83216	5,192494
105	10,24695	4,717694	141	11,87434	5,204828
106	10,29563	4,732624	142	11,91638	5,217103
107	10,34408	4,747459	143	11,95826	5,229321
108	10,39230	4,762203	144	11,00000	5,241482

CHAP. XXXIV.

The RELATION *of* NUMBERS *in* QUANTITY.

483. THUS far ſingle Arithmetick: Comparative Arithmetick enſues, which is worked by numbers, as they are conſidered to have relation one to another.

484. This Relation conſiſts in quantity, or quality.

485. Relation in quantity is the reference or reſpect, that the numbers themſelves have one to another: As when the compariſon is made between 6 and 2, or 2 and 6; 5 and 3, or 3 and 5.

486. Here the terms or numbers propoſed are always two, of which the firſt is called the Antecedent, and the other the Conſequent: So in the firſt Example, 6 is the Antecedent, and 2 the Conſequent: And in the ſecond, 2 is the Antecedent, and 6 the Conſequent.

487. Relation in quantity conſiſts either in the difference, or elſe in the ratio or reaſon that is found between the terms propounded.

488. The Difference of two numbers is the remainder left after Subtraction of the leſs out of the greater: So 6 and 2 being the terms given, 4 is the difference between them; for if you ſubtract 2 from 6, the remainder is 4.

489. The Ratio or reaſon between two numbers is the Quotient of the Antecedent divided by the Conſequent: So if it be demanded what ratio or reaſon 6 has to 2, I anſwer triple reaſon: For if you divide 6 the Antecedent, by 2 the Conſequent, the Quotient is 3; 2 being contained juſt 3 times in 6. In like manner, there is a ſubtriple reaſon between 2 and 6; for if you divide 2 by 6, the Quotient is $\frac{2}{6}$, or (which is all one) $\frac{1}{3}$; becauſe 6 being not once found in 2, there remains 2 for the Numerator, 6 the Diviſor being the Denominator of the Fraction produced in the Quotient, by *Art.* 255.

490. This Ratio or reaſon of numbers is either equal or unequal.

491. Equal reaſon is the relation that equal numbers have to one another: As 5 to 5, 6 to 6, 7 to 7, *&c.*

492. Here the one being divided by the other, the Quotient is always an unit: For if it be demanded how often 5 is in 5, the anſwer is 1.

493. Unequal reaſon is the relation that unequal numbers have one to another; and this either of the greater to the leſs, or of the leſs to the greater.

494. Unequal reaſon of the greater to the leſs, is when the greater term is Antecedent: As of 6 to 2, 5 to 3, and the like.

495. Here the Quotient of the Antecedent divided by the Conſequent is always greater than an unit.: So 6 divided by 2, the Quotient is 3, and 5 divided by 3, the Quotient is $1\frac{2}{3}$.

496. Unequal reaſon of the leſs to the greater, is when the leſs term is Antecedent: As of 2 to 6, 3 to 5, *&c.*

497. Here the Quotient of the Antecedent divided by the Conſequent is always leſs than an unit: So 2 divided by 6, the Quotient is $\frac{2}{6}$, or $\frac{1}{3}$; and 3 divided by 5, the Quotient is $\frac{3}{5}$.

498. Each of theſe kinds of unequal reaſon is again ſubdivided into five other ſorts or varieties, of which the three firſt are ſimple, and the other two are mixt.

499. The ſimple kinds of unequal reaſon are, 1. Manifold. 2 Superparticular. 3. Superpartient.

500. Manifold Reaſon of the greater to the leſs, is, when the Conſequent is contained in the Antecedent divers times without any part remaining: As 4 to 2, 8 to 4, 16 to 8, which is alſo called double reaſon, becauſe the leſs is contained twice in the greater; ſo 6 to 2 is triple reaſon, 8 to 2 is fourfold reaſon, *&c.*

501. Here the Quotient of the Antecedent divided by the Conſequent is always a whole number: So 8 divided by 2, the Quotient is 4.

502. The oppoſite of this kind, *viz.* of the leſs to the greater, is termed Submanifold: Examples hereof are 2 to 4, 4 to 8, 8 to 16, *&c.* Likewiſe 2 to 6, 2 to 8, 2 to 10, *&c.*

503. Superparticular is, when the Antecedent contains the Conſequent once, and beſides an Aliquot part of the Conſequent; that is an, half, a third, a fourth, or a fifth part, *&c.* of the Conſequent; as 3 to 2, 4 to 3, 5 to 4, 6 to 5, and the like; here 3 divided by 2, the Quotient is $1\frac{1}{2}$, and 4 being divided by 3 the Quotient is $1\frac{1}{3}$. In like manner, 5 divided by 4, the Quotient is $1\frac{1}{4}$, and 6 divided by 5, the Quotient is $1\frac{1}{5}$; therefore I ſay 2 and half 2 (that is 1) conſtitute 3: So likewiſe 3 and one third part of 3 (*viz.* 1.) make 4, and ſo of the reſt.

504. Here the Quotient of the Antecedent divided by the Conſequent is a mixt number, whoſe whole part, as alſo the

Numerator

Numerator of the Fraction annexed, is always an unit: As is obſervable in the Examples laſt mentioned.

505. The oppoſite reaſon of this kind is Subſuperparticular, as 2 to 3, 3 to 4, 4 to 5, 5 to 6, *&c.*

506. Superpartient is, when the Antecedent contains the Conſequent once, and beſides ſeveral parts of the Conſequent: As 5 to 3, 7 to 5, 7 to 4, 8 to 5, 9 to 5, 11 to 7, *&c.* here 5 divided by 3, the Quotient is $1\frac{2}{3}$, and therefore 5 contains 3 once, and $\frac{2}{3}$ of 3; for 3 and two thirds of 3, *(viz.* 2.) conſtitute 5.

507. Here the Quotient of the Antecedent divided by the Conſequent is a mixt number, whoſe whole part being an unit, has always for the Numerator of the Fraction annexed to it a number compoſed of more units than 1: So the compariſon being made between 5 and 3, and 5 the Antecedent being divided by 3 the Conſequent, the Quotient is $1\frac{2}{3}$.

508. The oppoſite of this reaſon is Subſuperpartient: Examples hereof are 3 to 5, 5 to 7, 4 to 7, 5 to 8, 5 to 9, 7 to 11, and the like.

509. The mixt kind of unequal reaſons are Manifold Superparticular, and Manifold Superpartient.

510. Manifold Superparticular reaſon is, when the Antecedent contains the Conſequent divers times, and beſides an aliquot part of the Conſequent; as 5 to 2, 10 to 3, 17 to 4, 21 to 5, and the like.

511. Here the Quotient of the Antecedent divided by the Conſequent, is a mixt number, whoſe whole part conſiſting of more units than one, has always an unit for the Numerator of the Fraction annexed to it; ſo 5 divided by 2, the Quotient is $2\frac{1}{2}$, and 21 divided by 5, the Quotient is $4\frac{1}{5}$.

512. The oppoſite of this reaſon, is Submanifold Superparticular; as 2 to 5, 2 to 7, 3 to 7, 4 to 9, *&c.*

513. Manifold Superpartient is, when the Antecedent contains the Conſequent ſeveral times, and beſides, divers parts of the Conſequent; as 8 to 3, 17 to 5, 19 to 4, 28 to 5, *&c.*

514. Here the Quotient of the Antecedent, divided by the Conſequent, is a mixt number, whoſe whole part, as alſo the Numerator of the Fraction annexed to it, is always a number compoſed of more units than one: So 8 divided by 3, the Quotient is $2\frac{2}{3}$, and 28 divided by 5, the Quotient is $5\frac{3}{5}$.

515. The oppoſite here is Submanifold Superpartient; as 3 to 8, 5 to 17, 4 to 19, 5 to 28, and the like.

Theſe are the ſeveral kinds or varieties of the ratios, or reaſons that are found amongſt numbers; ſo that no two numbers whatſoever

whatsoever can be named, but the ratio or reason between them is comprehended under one of these five kinds.

CHAP. XXXV.

The Relation of Numbers in Quality viz. *Arithmetical and Geometrical Proportion.*

516. RELATION in Quality (otherwise called Proportion) is either the reference or respect that the reasons of numbers have one to another; or else, which the differences of numbers have one to another.

517. Therefore here the terms proposed ought always to be more than two; for otherwise there cannot be a comparison of reasons or differences in the plural number.

518. This proportion is either Arithmetical, or Geometrical.

519. Arithmetical Proportion is, when several numbers differ according to an equal difference, as 2, 4, 6, 8, 10, *&c.* here 2 is the common difference between 2 and 4, 4 and 6, 6 and 8, 8 and 10, *&c.* also 1, 2, 3, 4, 5, 6, 7, 8, *&c.* differ by Arithmetical proportion, 1 being the common difference between them.

520. Arithmetical proportion is either continued or interrupted.

521. Arithmetical proportion continued, is, when divers numbers are linked together by a continual progression of equal differences. Such are the Examples last propounded, as also these 1, 3, 5, 7, 9, 11, 13, *&c.* And 100000, 200000, 300000, 400000, *&c.* This is also called Arithmetical progression.

522. Arithmetical proportion interrupted, is, when the progression is discontinued: As in these numbers, 2, 4, 8, 10; here 2 and 4 being compared with 8 and 10, differ according to Arithmetical proportion; but so do not 4 and 8 differ; for 2 is the common difference betwixt 2 and 4, 8 and 10; whereas the difference betwixt 4 and 8 is 4. In like manner 8, 14, 17, 23, differ by Arithmetical proportion interrupted.

523. Arithmetical proportion may be continued either upwards or downwards.

524. Upwards, when the terms of the progreſſion increaſe, as theſe, 2, 4, 6, 8, 10, 12, *&c.* or theſe 1, 2, 3, 4, 5, 6, *&c.* And this laſt rank is more particularly termed Natural Progreſſion.

525. Here, when the firſt term is alſo the common difference of the terms, the laſt term being divided by the number of the terms, the Quotient will give you the firſt term of the rank: Again, in this caſe, the firſt term multiplied by the number of the terms produces the laſt term: So this rank 3, 6, 9, 12, 15, 18, 21, being propoſed, in which 3 is both the firſt term, and alſo the common difference of the terms; I ſay 21, the laſt term being divided by 7 the number of the terms, the Quotient is 3 the firſt term; contrariwiſe the firſt term multiplied by 7 produces 21 the laſt term.

526. Arithmetical proportion continued downwards, is, when the terms of the progreſſion decreaſe: Such as are 35, 32, 29, 26, 23, 20: And 40, 35, 30, 25, 20, 15, 10, 5.

527. Here, when the laſt term is alſo the common difference of the terms, the firſt term being divided by the number of the terms, the Quotient will give you the laſt term: Again, the laſt term, multiplied by the number of the terms, produces the firſt term of the rank.

For Example, This rank 40, 35, 30, 25, 20, 15, 10, 5, being propoſed, in which 5 is both the laſt term, and likewiſe the common difference of the terms; I ſay 40, the firſt term being divided by 8 the number of the terms, the Quotient is 5 the laſt term: On the other ſide, 5 the laſt term being multiplied by 8, the product is 40 the firſt term.

528. Three numbers being propoſed that differ by Arithmetical proportion continued, the mean being doubled, is equal to the ſum of the extremes: So if 5, 6, 7, be given, 6 being doubled, is equal to the ſum of 5 and 7, the two extremes.

529. Four numbers being given, that differ by Arithmetical proportion either continued or interrupted, the ſum of the two means is equal to the ſum of the two extremes: So 5, 6, 7, 8, being given, the ſum of 6 and 7, the two mean numbers, is equal to the ſum of 5 and 8, the two extremes; and 8, 14, 17, and 23, being propounded, the ſum of 14 and 17 being added together is equal to the ſum of 8 and 23.

530. In an Arithmetical progreſſion, whoſe number of terms is odd, the mean term, being doubled, is equal to the ſum of any two terms that are equally diſtant therefrom; ſo if 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. be propoſed, 6 the mean term being doubled, that is 12, is equal to $11 + 1 = 10 + 2 = 9 + 3$, *&c.*

531. Hence the ſum of an Arithmetical progreſſion, whoſe number of terms is odd, will be equal to the mean term multiplied by the number of the terms: In the above Example, the mean term (6) multiplied by (11) the number of terms produces (66) the ſum of the progreſſion.

532. Otherwiſe becauſe half the ſum of the greateſt and leaſt terms is equal to the mean: Therefore, the progreſſion will be equal to the product of the number of the terms, multiplied by half the ſum of the greateſt and leaſt terms.

533. In an Arithmetical progreſſion, whoſe number of terms is even, the ſum of the two mean terms, is equal to the ſum of any two terms equally diſtant from them; ſo if 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. be propoſed, $(5+6=)$ 11 the ſum of the two mean terms is equally to $10+1=9+2=8+3$, *&c.*

534. Hence the ſum of an Arithmetical progreſſion, whoſe number of terms is even, will be equal to the product ariſing from multiplying the ſum of the two mean terms by half the number of the terms: In the above Example 11 the ſum of the two means being multiplied by 5, half the number of the terms, produces 55 the ſum of the progreſſion.

535. Otherwiſe, becauſe the ſum of the greateſt and leaſt term is equal to the ſum of the two mean terms; therefore the progreſſion will be equal to the product of half the number of the terms, multiplied by the ſum of the greateſt and leaſt terms.

536. Therefore, by comparing *Art.* 532. and 535. it will appear, that the ſum of any Arithmetical progreſſion will be equal to half the product, made by multiplying the ſum of the greateſt and leaſt terms, by the number of the terms: For Example, 9 terms of the progreſſion 1, 3, 5, 7, 9, 11, 13, 15, 17, is equal to $\frac{\overline{1+17}\times 9}{2}=\frac{18\times 9}{2}=9\times 9=81$.

537. If from the number of the terms of an Arithmetical progreſſion, unity be ſubtracted, and the remainder be multiplied by the common difference of the terms, the product will be the difference between the greateſt and leaſt term.

This will appear from the generation of the progreſſion; ſuppoſing 6 were the firſt term, and 2 the common difference.

Then,

Then, to the firſt term	6
Adding the common difference	2
Makes the ſecond term	8
To which add the common difference	2
Makes the third term	10
To which add the common difference	2
Makes the fourth term	12 *&c.*

Where it is evident, that the common difference is added as often as the number of terms leſs 1.

538. Hence, if to the leaſt term be added the product of the common difference by the number of terms leſs 1; the ſum ariſing will be the greateſt term; and, on the contrary, if the ſaid product be ſubtracted from the greateſt term, the remainder will be the leaſt term.

For Example, In the progreſſion 3, 5, 7, 9, 11, 13, conſiſting of 6 terms; 5, the number of the terms leſs 1, being multiplied by 2, the common difference of the terms, produces 10, the difference between 3 and 13, the leaſt and greateſt terms; and therefore $3+10=13$, and $13-10=3$.

539. Hence, the ſum of an Arithmetical progreſſion will be equal to half the product of the common difference, number of the terms, and number of the the terms leſs 1, multiplied continually; and added to the product of the leaſt term and number of the terms; or ſubtracted from the product of the greateſt term, and number of the terms.

For by *Art.* 536. the ſum of the progreſſion is equal to half the product, made by multiplying the greateſt and leaſt term, by the number of the terms; and the above will appear to be half the ſaid product from *Art.* 538.

For Example, The ſum of the Arithmetical progreſſion whoſe leaſt term is 3, common difference 2, and number of terms 6, will be $\frac{1}{2} \times 2 \times 6 \times 5 + 3 \times 6 = \frac{60}{2} + 18 = 48$. And the ſum of the Arithmetical progreſſion whoſe greateſt term is 13, common difference 2, and number of terms 6, will be $6 \times 13 - \frac{1}{2} \times 2 \times 6 \times 5 = 78 - \frac{60}{2} = 48$.

540. When numbers have the ſame or an equal ratio or reaſon, they are ſaid to be in Geometrical proportion: Thus 3, 6, 12, 24, 48, *&c.* are numbers in Geometrical proportion; each Antecedent being to its Conſequent in proportion as the two firſt terms 3 and 6, *viz.* as 1 to 2.

541. Geometrical proportion is either continued or interrupted.

542. Geometrical proportion continued, otherwiſe called Geometrical progreſſion, is when every Antecedent and its Conſequent have the ſame ratio or reaſon, as in the Example quoted in *Art.* 540. 3, 6, 12, 24, 48, *&c.* each Antecedent containing its Conſequent twice. *Note*, The number (2) by which the progreſſion is continued, is called the ratio of the progreſſion.

543. Hence, in a Geometrical progreſſion, each Antecedent is to its Conſequent, as the firſt term is to the ſecond, or as unity is to the ratio of the progreſſion.

Note, All the numbers, comprehended between the firſt and laſt, are called mean proportionals; ſo 6, 12, 24, are three mean proportionals between 3 and 48.

544. Geometrical proportion interrupted, is, when the progreſſion of like reaſon is diſcontinued, in ſuch ſort, that the four numbers being given, the like reaſon is not found between the ſecond and third, that is between the firſt and ſecond, and the third and fourth; of this ſort are theſe numbers 2, 4, 16, 32; here, as 2 is to 4, ſo is 16 to 32; for they differ by double reaſon; but as 2 is to 4, ſo is not 4 to 16, for 4 and 16 differ by fourfold reaſon, 4 being contained 4 times in 16.

545. The numbers of Multiplication and Diviſion are proportional; for in Multiplication, as 1 is to the Multiplier, ſo is the Multiplicand to the product, or as 1 is to the Multiplicand, ſo is the Multiplier to the product: Again, in Diviſion, as the Diviſor is to 1, ſo is the Dividend to the Quotient: Or, as the Diviſor is to the Dividend, ſo is 1 to the Quotient.

546. If the rank of powers in *Art.* 412. *viz.* 1, 2, 4, 8, 16, *&c.* be ſeverally multiplied by the number 3, the products 3, 6, 12, 24, 48, *&c.* are a Geometrical progreſſion, whoſe ratio is equal to the root of thoſe powers.

547. Hence a rank of powers is a Geometrical progreſſion, and if a Geometrical progreſſion be divided by the firſt term, it quotes a rank of powers.

548. Hence alſo the leaſt term and ratio of any Geometrical progreſſion being given, any term (whoſe diſtance from the leaſt is known) may be found; by raiſing the ratio of the progreſſion to a power, whoſe index is the given diſtance, and multiplying that power by the firſt term of the progreſſion.

For Example, The fifth term (48) of the above progreſſion (whoſe diſtance from the firſt term is 4) is equal to (16) the

fourth

fourth power of the ratio (2) multiplied by (3) the firſt term of the progreſſion, for $3 \times 16 = 48$.

549. But if the greateſt term of the progreſſion be given; then divide it by the ſaid power of the ratio, and the Quotient will be the leaſt term.

For Example, If it be required to find the fifth term of a Geometrical progreſſion decreaſing, whoſe greateſt term is 162, and ratio 3.

The diſtance of the terms being 4, raiſe 3 to its 4th power 81. then divide 162 the greateſt term, by 81 the power, and the Quotient 2 will be the term required.

550. Any number of mean proportionals may be found between two given numbers, thus:

Divide the greater of the two numbers by the leſſer, and extract that root of the Quotient whoſe Index is greater by unity than the number of means required, ſo ſhall the reſult be the ratio of the progreſſion; from which the terms may be made by continual Multiplication. *Example*, Let it be required to find 3 mean proportionals between 2 and 162.

Then will $\sqrt[4]{\frac{162}{2}} = \sqrt[4]{81} = \sqrt[2]{9} = 3$ be the ratio.

And $2 \times 3 = 6$; $6 \times 3 = 18$; $18 \times 3 = 54$; the means required.

551. If three numbers be in Geometrical progreſſion, the ſquare of the mean term is equal to the product of the extream terms: For Example, in the three numbers 3. 6. 12; the ſquare of 6, *viz.* 36, is equal to the product of 3 and 12.

552. If four numbers be in Geometrical proportion, either continued or interrupted, the product of the two extremes will be equal to the product of the two means.

Example 1. The numbers 3. 6. 12. 24. are in Geometrical proportion continued, and $3 \times 24 = 72 = 6 \times 12$.

Example 2. The numbers 2. 4 :: 16. 32. are in Geometrical proportion interrupted, and $2 \times 32 = 64 = 4 \times 16$.

Hence, if, in any 4 numbers, the product of the two extreams, is equal to the product of the two means, thoſe numbers are proportionals.

553. If 4 numbers 2 : 6 :: 7 : 21 are proportionals, that proportionality will continue among the numbers that ariſe by thoſe changes of them in ſituation and otherwiſe, which follow, *viz.*

Caſe 1. In *Alternation*, i. e. if the two extreams, or the two means ſeverally change places between themſelves.

For

For Example $2 : 7 :: 6 : 21$ for $2 \times 21 = 7 \times 6$.
And $21 : 7 :: 6 : 2$ for $21 \times 2 = 7 \times 6$.

Case 2. In *Inversion*, i. e. if at the same time the two first and the two last change places between themselves.

For Example $6 : 2 :: 21 : 7$ for $6 \times 7 = 2 \times 21$.

Case 3. In *Composition*, i. e. if the first and second terms be added together, for a new first term; and the third and fourth terms be added together, for a new third term.

For Example $2+6 : 6 :: 7+21 : 21$.
That is $8 : 6 :: 28 : 21$ for $8 \times 21 = 6 \times 28$.

Case 4. In *Division*, i. e. if the difference of the first and second terms be made the first term; and the difference of the third and fourth terms be made the third term.

For Example $6-2 : 6 :: 21-7 : 21$.
That is $4 : 6 :: 14 : 21$ for $4 \times 21 = 6 \times 14$.

Case 5. In *Conversion*, i. e. if the sums or differences, mentioned in the two last Cases, be made the second and fourth terms, instead of the first and third.

For Example $2 : 2+6 :: 7 : 7+21$
And $2 : 6-2 :: 7 : 21-7$
That is $2 : 8 :: 7 : 28$ for $2 \times 28 = 8 \times 7$.
And $2 : 4 :: 7 : 14$ for $2 \times 14 = 4 \times 7$.

Case 6. In *Mixtion*, i. e. if the said sums, and differences, are mixed among the terms.

For Example $2+6 : 6-2 :: 7+21 : 21-7$.
That is $8 : 4 :: 28 : 14$ for $8 \times 14 = 4 \times 28$.

554. The successive differences of the terms of a Geometrical progression will form another Geometrical progression, having the same ratio.

Examp. 1. In the Geometrical progression 3. 6. 12. 24. 48. &c. if each Antecedent be subtracted from its Consequent, the remainders will be 3. 6. 12. 24. &c.

Examp. 2. In the Geometrical progression 2. 6. 18. 54. 162. 486. &c. whose ratio is 3, if each Antecedent be taken from its Consequent, the remainders will be 4. 12. 36. 108. 324. &c. in which rank of remainders the ratio is also 3.

555. The successive sums of the terms of a Geometrical progression, taken two and two, three and three, four and four, &c. will form other Geometrical progressions, having the same ratio. For Example, in the progression 1. 2. 4. 8. 16. 32. 64. 128. &c. the successive sums of the terms, taken two and two, will be 3, 6, 12, 24, 48, 96, 192, &c. the succes-

five sums of the terms, taken three and three, will be 7. 14. 28. 56. 112. 224, &c. The successive sums of the terms, taken four and four, are 15. 30. 60. 120. 240. &c. In all which ranks of successive sums, each Consequent contains its Antecedent twice; that is, each Antecedent is to its Consequent, as (1) the first term of the original progression, is to (2) its ratio.

556. Hence, let a Geometrical progression consist of never so many terms;

As the sum of the Antecedents,
To the sum of the Consequents:
So is unity,
To the Ratio of the progression.

557. Therefore, by *Case* 5. *Art.* 553.

As the sum of the Antecedents,
To the difference between the sums of the Consequents and Antecedents:
So is unity,
To the Ratio less unity.

558. But the difference between the sums of the Antecedents and Consequentss will be the difference between the last and first terms; for all the intremediate terms are included in both sums: And the sum of the Antecedents is the sum of the Progression, wanting only the last term.

559. Therefore, As the sum of the progression less the last term,
To the difference between the extreme terms:
So is unity,
To the Ratio less unity.

560. Or by *Case* 1. *Art.* 553.

As the Ratio less unity,
To unity:
So is the difference of the extreme terms,
To the sum of the progression less the last term.

561. Hence, having either the greatest or least term, and the Ratio of a Geometrical progression, given; the sum or total of any proposed number of terms thereof, may be found by the following

Rule. If the least term of the progression be given, find the greatest by *Art.* 548. If the greatest be given, find the least by *Art.* 549. Then subtract the least term from the greatest, and

and divide the remainder by the number which is leſs by one than the Ratio: Laſtly, to the Quotient add the greateſt term of the progreſſion, and the ſum will be the anſwer.

Example 1. What is the ſum of 5 terms of a Geometrical progreſſion, whoſe leaſt term is 3, and Ratio 2?

The fifth or greateſt term of this progreſſion is 48, by *Art.* 548.

Then $48 - 3 = 45 =$ the difference of the extremes;

And $\frac{45}{1} \quad = 45 =$ the Quotient of that difference, when divided by the Ratio leſs unity.

Laſtly, $45 + 48 = 93 =$ the ſum of the progreſſion 3. 6. 12. 24. 48.

Example 2. What is the ſum of 5 terms of a Geometrical progreſſion, whoſe greateſt term is 162, and Ratio 3?

The fifth or leaſt term of this progreſſion is 2, by *Art.* 549.

Then $162 - 2 = 160 =$ the difference of the extremes;

And $\frac{160}{2} \quad = 80 =$ the Quotient of that difference, when divided by the Ratio leſs unity.

Laſtly, $80 + 162 = 242 =$ the ſum of the progreſſion 162. 54. 18. 6. 2.

562. If there be two Geometrical progreſſions, whoſe Ratio's are equal: Then,

As any term of the firſt progreſſion,
To any term of the ſecond:
So is any other term of the firſt, whoſe diſtance from the former is known,
To a term equally diſtant from the former in the ſecond.

For ſince the Ratio's are equal, and the diſtance the ſame, and ſince by *Art.* 548, and 549. the reſpective terms in each progreſſion are produced by multiplying or dividing by the ſame power of the Ratio; therefore the ſaid terms muſt be in proportion to the numbers, ſo to be multiplied or divided.

563. Hence, As the leaſt or greateſt term of the firſt progreſſion,
To the leaſt or greateſt term of the ſecond:
So is the ſum of any number of terms of the firſt,
To the ſum of the like number of terms of the ſecond.

For the wholes will be in the ſame proportion as their component parts.

564. Hence, in any Geometrical progression, if the sum of any number of terms be known, the sum of a like number of terms, the first of which is situated at a given distance from the first term of the former, may be found by the following proportion.

As unity,
To that power of the Ratio, whose index is the given distance:
So is the sum of the given number of terms,
To the sum of a like number of terms, beginning at the given distance.

Example In the Geometrical progression 2. 6. 18. 54. 162. 486. 1458, *&c.* if the sum 26 of the three first terms be given, to find the sum of three other terms beginning with the fifth; then the distance of the first term from the fifth being four, it will be as 1 : 81 (the fourth power of the Ratio 3;) so is 26 to 2106, the sum of 162, 486 and 1458.

CHAP. XXXVI.

RULES *of* PRACTICE *by* ALIQUOT PARTS.

565. AN Aliquot part takes its name from the *Latin* word *aliquoties*; (for according to *Euclid*) an aliquot part is such a part of a greater number as being taken (*aliquoties* or) certain times, precisely constitutes that greater number; so 3 is an aliquot part of 12; for 3 taken 4 times, exactly makes 12, without any excess or defect: In like manner, 4 is an aliquot part of 20, because 4 taken 5 times precisely makes 20; but 7 is not an aliquot part of 20, for 7 taken twice, wants of 20; and being taken thrice, exceeds 20: This kind of part last mentioned, is by *Euclid* called *pars aliquanta*, of which there will be no use in this place.

566. When the Rule of Three Direct has 1 or an integer for the first term, it is commonly called a Rule of Practice, either from the great use and practice of it in common affairs, or else because questions of this nature may be resolved by operations more speedy and practical than those of the Rule of Three.

567. The following numbers of ſhillings are aliquot parts of a pound, *viz.*

10 *s.* = $\frac{1}{2}$ *l.*	2 *s.* = $\frac{1}{10}$ *l.*
5 *s.* = $\frac{1}{4}$ *l.*	1 *s.* = $\frac{1}{20}$ *l.*
4 *s.* = $\frac{1}{5}$ *l.*	

568. Any even number of ſhillings is either $\frac{1}{10}$ of a pound (that is 2 ſhillings) or elſe is compoſed of $\frac{1}{10}$ *l.* (to wit, 2 *s.*) taken certain times: So 8 *s.* is compoſed of $\frac{1}{10}$ *l.* (or 2 ſhillings) taken four times: In like manner, 18 *s.* is compoſed of $\frac{1}{10}$ *l.* taken 9 times.

569. When the price of 1, or an integer of what name ſoever, is 2 ſhillings, the price of as many integers as one will of that name is diſcoverable at firſt ſight; to wit, by accounting the double of the figure which ſtands in the firſt place (towards the right-hand) of the ſaid number of integers as ſhillings, and the reſt of the ſaid number as pounds: So 345 yards at 2 ſhillings the yard will coſt 34 *l.* 10 *s.* for the double of 5 is 10, which write down apart as ſhillings; then eſteeming the remaining figures towards the left-hand, to wit, 34, as pounds, the anſwer will be 34 *l.* 10 *s.* This contraction is nothing elſe, but dividing the number of integers, whoſe price is required by 10.

Yards
345

Anſw. 34 *l.* 10 *s.*

570. The reaſon of this will be evident if we argue thus, 345 yards at 20 *s.* *per* yard amount to 345 *l.*

But the given price 2 *s.* is $\frac{1}{10}$th of 20 *s.*

Therefore 345 yards at 2 *s.* *per* yard will amount to $\frac{1}{10}$ of 345 *l.*

More Examples hereof are theſe:

oz.
What coſt 2057 at 2 *s.* *per oz*?

l. *s.*
Anſwer 205 : 14

yards.
What coſt 120 at 2 *s.* *per yard*?

l. *s.*
Anſwer 12 : 0

571. When the given price of 1, or an an integer, is any even number of ſhillings greater than two ſhillings, multiply the number of integers, whoſe price is required, by half the given number of ſhillings, with this caution, that the double of the figure which ariſes, in the firſt place of the product be written apart as ſhillings, and the reſt of the product as pounds: So if it be demanded what 218 yards, at 8 ſhillings the yard, amount to, the anſwer will be found 87 *l.* 4 *s.* For I multiplied 218 by 4 (which is half 8 the given number of ſhillings) ſaying 4 times 8 is 32; here the double of 2 (to wit, of that figure which is to poſſeſs the firſt place in the product) is 4, which ſet apart as ſhillings, keeping 3 in mind for the three tens: Again, 4 times 1 is 4, which with 3 in mind makes 7: Laſtly, 4 times 2 makes 8, therefore the anſwer to the queſtion is 87 *l.* 4 *s.* The reaſon of this contraction is evident from *Art.* 568, 570. More Examples of this Rule are theſe following:

	Yards.	
	218	
	4	
	l.	*s.*
Anſwer	87	: 4

What coſt 436 yards, at 14 *s.* *per* yard?

	7	
	l.	*s.*
Anſwer	305	: 4

What coſt 320 yards, at 18 *s.* *per* yard?

	9	
	l.	*s.*
Anſwer	288	: 0

572. Any odd number of ſhillings is either compoſed of $\frac{1}{10}$ *l.* (or 2 *s.*) and of $\frac{1}{20}$, (or 1 *s.*) or elſe it is compoſed of $\frac{1}{10}$ *l.* (or 2 *s.*) taken certain times, and of $\frac{1}{20}$ *l.* (or 1 *s.*) So 3 *s.* is compoſed of 2 *s.* and 1 *s.* Alſo 7 *s.* is compoſed of 2 *s.* taken three times, and of 1 *s.* Likewiſe 13 *s.* is compoſed of 2 *s.* taken ſix times, and of 1 *s.*

573. When the given price of 1, or an integer is an odd number of ſhillings, work for the greateſt even number of ſhillings contained in that odd number, according to *Art.* 571. then for the odd ſhilling remaining, take $\frac{1}{20}$ of the number of integers whoſe price is required, (by *Art.* 107.) Theſe two reſults added together, give the anſwer to the queſtion: So, if

if it be demanded what 2344 ounces, at 13*s.* the ounce, will coſt, the anſwer will be found 1523*l.* 12*s.* For if I multiply 2344 by 6, (to wit, by half the remainder, when 1 is abated from 13, the given number of ſhillings,) there will ariſe 1406*l.* 8*s.* (by *Art.* 571.) Then taking $\frac{1}{20}$ of 2344, there will ariſe 117*l.* 4*s.* which being added to the former product, gives 1523*l.* 12*s.* for the anſwer to the queſtion.

	l.	*s.*
oz. 2344		
6		
	1406 :	8
	117 :	4
Anſ.	1523 :	12

More Examples of this Rule are theſe following:

What coſt 345 yards, at 17 *s. per* yard?

l.	*s.*	
276 :	0	$= 345 \times \frac{8}{10}$
17 ·	5	$= 345 \times \frac{1}{20}$
Anſw. 293 :	5	

What coſt 739 yards, at 19*s. per* yard?

l.	*s.*
665 :	2
36 :	19
Anſ. 702 :	1

574. This Example might have been performed otherwiſe, thus:

739 yards, at 20*s. per* yard, amount to	739*l.* 0*s.*
And 739 ditto, at 1*s. per* yard, amount to $\frac{1}{20}$ thereof	36*l.* 19*s*
Therefore 739 yards, at 19 *s. per* yard, amount to the difference of thoſe values	702*l.* 1*s.*

And by parity of reaſoning, when the given price is 18 *s.* 16*s.* or 15*s. per* yard, from as many *pounds* as there are yards, ſubtract ſeverally the $\frac{1}{10}$, $\frac{1}{5}$, or $\frac{1}{4}$ thereof, and the remainders will be the anſwers.

575. Alſo Queſtions of this kind may be ſolved, by uſing the Aliquot parts of a pound: Thus, in the ſecond Example, ſince 17 is compoſed of 10+5+2, it may be performed as below:

	345 yards at 20 *s. per* yard, amount to	345 *l.*
	345 ditto at 10 *s.* amount to $\frac{1}{2}$ of 345 *l.* =	172 : 10
	345 ditto at 5 *s.* amount to $\frac{1}{4}$ of 345 *l.* =	86 : 5
	345 ditto at 2 *s.* amount to $\frac{1}{10}$ of 345 *l.* =	34 : 10
Therefore	345 ditto at 17 *s.* amount to	293 : 5

Examp. 2. What cost 928 yards, at 5 *s. per* yard?

$\frac{1}{4}$ of 928 = 232 *l.* the answer.

Examp. 3. What cost 928 yards, at 6 *s. per* yard?

$\frac{1}{5}$ of 928 =	185 : 12	= the amount at 4 *s.*
$\frac{1}{10}$ of 928 =	92 : 16	= the amount at 2 *s.*
Therefore	278 : 8	= the amount at 6 *s.*

Note, Since 2 *s.* is the $\frac{1}{2}$ of 4 *s.* the amount at 2 *s.* might have been found, by taking the $\frac{1}{2}$ of the amount at 4 *s.*

576. When the price of 1, or an integer, consists of pounds and shillings, first multiply the number of integers, whose price is required, by the number of pounds in the said given price, and subscribe the product as pounds; then proceed with the shillings in the said given price, according to *Art.* 571, &c. and having subscribed that which arises under the aforesaid product of pounds, add them all together for the answer of the question: So, if it be demanded what 328 hundred-weight amounts to at 2 *l.* 17 *s. per* hundred weight, the answer will appear to be 934 *l.* 16 *s.* as by the operation is evident.

C. wt.
328
2

	l.	*s.*
328 × 2 =	656 :	0
328 × $\frac{8}{10}$ =	262 :	8
$\frac{1}{20}$ of 328 =	16 :	8
Answ.	934 :	16

More Examples to illustrate this Rule, are these following:

What cost 504 *C. wt.* at 7 *l.* 12 *s. per C. wt.*

	l.	*s.*
504 × 7 =	3528 :	0
504 × $\frac{6}{10}$ =	302 :	8
Answ.	3830 :	8

What

What cost 129 *C. wt.* at 5 *l.* 7 *s.* *per C. wt.*

	l.	*s.*
$129 \times 5 =$	645 :	0
$129 \times \frac{3}{10} =$	38 :	14
$\frac{1}{20}$ of 129 $=$	6 :	9
Answ.	690 :	3

577. Any number of pence under 12 is either an Aliquot part of a shilling, or else composed of Aliquot parts thereof; so 3 pence is an Aliquot part, to wit, $\frac{1}{4}$ of a shilling. Likewise 4 is $\frac{1}{3}$ of 12: But farther, 5 pence is composed of 2 Aliquot parts, to wit, of 3 pence, which is $\frac{1}{4}$ of a shilling, and of 2 pence, which is $\frac{1}{6}$ of a shilling; all which will readily appear by the following Table.

Pence.	Aliquot parts of a shilling.
1	$\frac{1}{12}$ (or $\frac{1}{3}$ of $\frac{1}{4}$)
$1\frac{1}{2}$	$\frac{1}{8}$
2	$\frac{1}{6}$
3	$\frac{1}{4}$
4	$\frac{1}{3}$
5	$\frac{1}{4} + \frac{1}{6}$
6	$\frac{1}{2}$
7	$\frac{1}{4} + \frac{1}{3}$
8	$\frac{1}{3} + \frac{1}{3}$
9	$\frac{1}{2} + \frac{1}{4}$
10	$\frac{1}{2} + \frac{1}{3}$
11	$\frac{1}{3} + \frac{1}{3} + \frac{1}{4}$

578. When the given price of 1, or an integer, is an Aliquot part of a shilling, divide the number of integers whose value is required by the Denominator of such Aliquot part; so will the Quotient be the number of shillings that answers the question;

queſtion; which number of ſhillings (when there is occaſion) may be reduced to pounds by the brief way of dividing by 20: So if it be required to know what 2686 ounces at 4 pence the ounce amount to; the anſwer will be found 44 *l.* 15 *s.* 4 *d.* For ſince 4 *d.* is an Aliquot part, to wit, $\frac{1}{3}$ of a ſhilling, divide 2686 by 3, ſo will the Quotient be $895\frac{1}{3}$. or 895 *s.* 4 *d.* which ſhillings being divided by 20, give 44 *l.* 15 *s.* 4 *d.* for the anſwer to the queſtion, as you ſee by the following operation:

	oz.		
	2686		
	s.	*d.*	
20) 89 |	5 :	4	= $\frac{1}{3}$ of 2686 *s.*
Anſwer 44 :	15 :	4	

More Examples of this Rule are theſe following:

What coſt 759 yards, at 6 *d.* *per* yard?

	s.	*d.*	
20) 37 |	9 :	6	= $\frac{1}{2}$ × 759 *s.*
Anſw. 18 :	19 :	6	

What coſt 204 yards, at 1 *d.* *per* yard?

Anſw. 17 *s.* = $\frac{1}{12}$ of 204 *s.*

579. When the given price of an integer is compoſed of Aliquot parts of a ſhilling, divide the number of integers whoſe price is required, by the ſeveral Denominators of the Aliquot parts contained in the given number of pence, then add the Quotients together, and the ſum will be the number of ſhillings which anſwer the queſtion. So if it be demanded what 2347 yards of linen cloth will coſt at 9 *d.* the yard, the anſwer will be found 88 *l.* 0 *s.* 3 *d.* For ſince 9 *d.* is compoſed of 6 *d.* and 3 *d.* to wit, of the Aliquot parts $\frac{1}{2}$ and $\frac{1}{4}$ of a ſhilling, firſt divide 2347 by 2, (the Denominator of the Aliquot part $\frac{1}{2}$,) ſo there ariſes $1173\frac{1}{2}$, or 1173 *s.* 6 *d.* Again, dividing the ſaid 2347 by 4, (the Denominator of the other Aliquot part) there will ariſe $586\frac{3}{4}$ or 586 *s.* 9 *d.* which two Quotients being added together give

	Yards.	
	2347	
	s.	*d.*
	1173 :	6
	586 :	9
20) 176 |	0 :	3
l.	*s.*	*d.*
Anſw. 88 :	0 :	3

1760 *s.*

1760 *s.* 3 *d.* or 88 *l.* 0 *s.* 3 *d.* which is the anſwer of the Queſtion.

More Examples to illuſtrate this Rule, are theſe:

What coſt 782 yards, at 8 *d. per* yard?

s.	*d.*	
260 :	8 =	$\frac{1}{3}$ of 782 *s.*
260 :	8 =	ditto.

20) 52|	1 :	4	
	l.	*s.*	*d.*
Anſw.	26 :	1 :	4

What coſt 540 *oz.* at 11 *d. per oz?*

180 = $\frac{1}{3}$ of 540 *s.*
180 = ditto.
135 = $\frac{1}{4}$ of 540 *s.*

49\|5

	l.	*s.*	*d.*
Anſw.	24 :	15 :	0

580. Or the Work of this Example might ſtand thus, *viz.*

540 yards, at 12 *d. per* yard, amount to $\frac{1}{20}$ of 540 *l.* — — —	= 27 *l.* 0 *s.*
And 540 yards, at 1 *d. per* yard, amount to $\frac{1}{12}$ of the laſt found ſum 27 *l.*	= 2 *l.* 5 *s.*
Therefore 540 yards, at 11 *d. per* yard, amount to the difference of the above values	24 *l.* 15 *s.*

After the ſame manner, when the given price is 10 *d.* 9 *d.* or 8 *d.* from the amount at 1 *s.* ſubtract ſeverally $\frac{1}{6}$, $\frac{1}{4}$, or $\frac{1}{3}$ of the ſame; and the remainders will be the anſwers.

581. 'Twill be more convenient, in ſome caſes, where the price given is not an Aliquot part of a ſhilling, to divide the ſame into two or more ſuch parts, that the firſt may be an Aliquot part of a ſhilling, and the ſecond, &c. Aliquot parts of the former: For Example, if the value of 540 yards at 7 *d. per* yard be required, the ſame may be found as follows:

540.

540 yards, at 12 *d.* *per* yard amount to	27 *l.*
540 ditto, at 6 *d.* amount to $\frac{1}{2}$ that sum	= 13 *l.* 10 *s.*
540 ditto, at 1 *d.* amount to $\frac{1}{6}$ of the the last sum	= 2 *l.* 5 *s.*
Therefore 540 ditto, at 7 *d.* amount to	15 *l.* 15 *s.*

582. And when the Price consists of pence and farthings, the process may be of the same kind: For Example, required the value of 540 yards, at $7\frac{3}{4}$ *d.* *per* yard.

540 yards, at 12 *d.* *per* yard, amount to	27 *l.*
540 yards, at 6 *d.* *per* yard, amount to $\frac{1}{2}$ of 27 *l.*	= 13 *l.* 10 *s.*
540 yards, at $1\frac{1}{2}$ *d.* *per* yard, amount to $\frac{1}{4}$ of 13 *l.* 10 *s.*	= 3 *l.* 7 *s.* 6 *d.*
540 yards, at $\frac{1}{4}$ *d.* *per* yard, amount to $\frac{1}{6}$ of 3 *l.* 7 *s.* 6 *d.*	= 11 *s.* 3 *d.*
Therefore 540 yard,, at $7\frac{3}{4}$ *per* yard, amount to	17 *l.* 8 *s.* 9 *d.*

583. When the given price of an integer consists of shillings and pence, first multiply the number of integers, whose value is required by the said given number of shillings, and subscribe the product as shillings; then divide the said number of integers by the several Denominators which are correspondent to the Aliquot parts contained in the given number of pence, and subscribe the Quotient or Quotients under the aforesaid product of shillings, all which being added together give the number of shillings which answers the question: So if it be demanded what 347 yards of cloth will cost at the rate of 7 *s.* 10 *d.* the yard, the answer will be found 135 *l.* 18 *s.* 2 *d.* For first 347 being multiplied by 7 (the given number of shillings,) produces 2429 shillings, then dividing 347 by 2 and 3 severally (because 10 *d.* is composed of $\frac{1}{2}$ and $\frac{1}{3}$ of a shilling,) the Quotients will be $173\frac{1}{2}$ and $115\frac{2}{3}$ that is 173 *s.* 6 *d.* and 115 *s.* 8 *d.* Lastly, the sum of all is 2718 *s.* 2 *d.* or 135 *l.* 18 *s.* 2 *d.*

	yards. 347	
	s.	*d.*
7 × 347 =	2429 :	
$\frac{1}{2}$ of 347 =	173 :	6
$\frac{1}{3}$ of 347 =	115 :	8
20)	271|8 :	2
	l. *s.*	*d.*
Answ.	135 : 18 :	2

More

More Examples of this kind, are these:

What cost 540 yards, at 17 *s.* 9 *d.* *per* yard?

17

$$17 \times 540 = \begin{cases} 3780 \\ 540 \end{cases} \ s.$$

$\frac{1}{2}$ of 540 = 270

$\frac{1}{4}$ of 540 = 135

20) 958|5

Answ. 479 *l.* 5 *s.* 0 *d.*

What cost 313 yards, at 14 *s.* 6 *d.* *per* yard?

14

$$14 \times 313 = \begin{cases} 1252 \\ 313 \end{cases}$$

$\frac{1}{2}$ of 313 = 156 : 6

20) 453|8 : 6

Answ. 226 *l.* 18 *s.* 6 *d.*

584. But if the given price be between one shilling and two shillings, the process may be the same as in *Art.* 578, 579, *&c.* excepting only, that the value of the given integers at 1 *s.* *per* yard, instead of being separated from the rest by a line, must be added up with the rest. For Example, required the value of 540 yards, at 1 *s.* $7\frac{3}{4}$ *d.* *per* yard?

540 yards, at 12 *d.* *per* yard, amount to 27 *l.*

540 ditto 6 *d.* *per* yard, amount to $\frac{1}{2}$ of 27 = 13 : 10

540 ditto $1\frac{1}{2}$ *d.* ditto $\frac{1}{4}$ of 13 : 10 = 3 : 7 : 6

540 ditto $\frac{1}{4}$ *d.* ditto $\frac{1}{6}$ of 3 : 7 : 6 = 11 : 3

Therefore 540 ditto $19\frac{3}{4}$ or 1 *s.* $7\frac{3}{4}$ *d.* amount to 44 : 8 : 9

Compare this Work with that in *Art.* 582.

585. When the price of an integer consists of shillings and pence, and that such shillings and pence jointly considered do make an Aliquot part of a pound, it will be a briefer way to divide the number of integers, whose value is required, by the Denominator of such Aliquot part, so will the Quotient give the answer to the question in pounds, and known parts of a pound. Thus, if it be demanded what 767 yards cost, at the

rate

rate of 6 *s.* 8 *d.* the yard, the anſwer will be found 255 *l.* 13 *s.* 4 *d.* For ſince 6 *s.* 8 *d.* is an Aliquot part, to wit $\frac{1}{3}$ of a pound, divide 767 by 3, ſo there ariſes in the Quotient $255\frac{2}{3}$, or 255 *l.* 13 *s.* 4 *d.* which is the anſwer of the queſtion. *Note,* That the Aliquot parts of a pound convenient for this Rule, are theſe expreſſed in the following Table.

	l.	*s.*	*d.*
3) 767 (	255 :	13 :	4

ſh.	*d.*	Aliquot parts of a pound.
6 :	8	$\frac{1}{3}$
3 :	4	$\frac{1}{6}$
2 :	6	$\frac{1}{8}$
1 :	8	$\frac{1}{12}$
1 :	4	$\frac{1}{15}$
1 :	3	$\frac{1}{16}$

586. And indeed every queſtion, wherein the price conſiſts of ſhillings and pence, may be ſolved by the Aliquot parts of a pound, and the Aliquot parts of them: For Example, what coſt 540 yards, at 17 *s.* 9 *d.* *per* yard?

540 yards, at 20 *s.* *per* yard, amount to	540 *l.*
540 ditto, at 10 *s.* amount to $\frac{1}{2}$ of 540 *l.* =	270
540 ditto, at 5 *s.* amount to $\frac{1}{2}$ of 270 *l.* =	135
540 ditto, at 2 *s.* 6 *d.* amount to $\frac{1}{2}$ of 135 *l.* =	67 : 10
540 ditto, at 0 *s.* 3 *d.* amount to $\frac{1}{10}$ of 67 *l.* 10 *s* =	6 : 15
Therefore 540 yards, at 17 *s.* 9 *d.* amount to	479 : 5

587. But if the price were 1 *l.* 17 *s.* 9 *d.* *per* yard, then draw no line under the 540 *l.* but add it up with the reſt; and 540 yards, at 1 *l.* 17 *s.* 9 *d.* will amount to 1019 *l.* 5 *s.*

588. When the price is ſhillings and pence, and ſuch ſhillings and pence conſiſt of the ſame figure.

Multiply the given quantity by the ſhillings, taking always the $\frac{1}{12}$ of the product for the pence, the total of which divided by 20, gives the anſwer in pounds.

Example. What comes 731 ells of Holland to, at 7*s.* 7*d.* or 11*s.* 11*d. per* Ell?

731 ells, at 7*s.* 7*d.*	731 ells, at 11*s.* 11*d.*
7	11
$\frac{1}{12}$) 5117	$\frac{1}{12}$) 8041
426 : 05	670 : 01
2|0) 554|3 : 05	2|0) 871|1 : 01
277 : 03 : 05	435 : 11 : 01

589, When the given price of 1, or an Integer, conſiſts of pounds, ſhillings, and pence reduce the pounds and ſhillings into ſhillings, then proceed according to *Art.* 583. So 517 *C.* at 3*l.* 17*s.* 5*d. per C.* will be found to amount to 2001*l.* 4*s.* 5*d.* For having reduced 3*l.* 17*s.* into 77*s.* multiply 517 by 77, and ſet down the particular products; then for the 5 pence which is compoſed of the Aliquot parts $\frac{1}{4}$ and $\frac{1}{6}$ of a ſhilling take $\frac{1}{4}$ and $\frac{1}{6}$ of 517, and ſubſcribe the Quotients orderly under the aforeſaid products: Laſtly, adding all together the ſum is 40024*s.* 5*d.* or 2001*l.* 4*s.* 5*d.* for the anſwer of the queſtion.

	C
	517
77 × 517 =	{ 3619
	{ 3619
$\frac{1}{4}$ of 517 =	129 : 3*d.*
$\frac{1}{6}$ of 517 =	86 : 2
	2|0) 4002 | 4 : 5
Anſw.	2001 : 4 : 5

More

More Examples of this Rule, are these following:

What cost 108 *C.* at 5 *l.* 13 *s.* 8 *d.* *per C.*

$$113\times108=\begin{cases}324\\108\\108\end{cases}$$

$\frac{1}{3}$ of 108 = 36

$\frac{1}{3}$ of 108 = 36

20)1227|6

	l.	*s.*	*d.*
Answ.	613 :	16 :	0

What cost 84 *C.* at 2 *l.* 10 *s.* 6 *d.* *per C.*

50 × 84 = 4200

$\frac{1}{2}$ of 84 = 42

20) 424 | 2 (212 : 2 : 0 Answ. (*l.* *s.* *d.*)

590. Questions of this kind may also be performed, by multiplying the number of integers, whose value is required, by the given number of pounds; and then proceeding according to *Art.* 586 for the shillings and pence.

Examp. 1. What cost 108 *C.* wt. at 5 *l.* 13 *s.* 8 *d.* *per C.* wt.

5 × 108 =	540 =	the amount	at 5 *l.* *per C.* wt.	
$\frac{1}{2}$ of 108 =	54 =	ditto,	at	10 *s.*
$\frac{1}{3}$ of 54 =	18 =	ditto,	at	3 *s.* 4 *d.*
$\frac{1}{10}$ of 18 =	1 : 16 =	ditto,	at	0 *s.* 4 *d.*

Therefore 613 : 16 = the amount at 5 *l.* 13 *s.* 8 *d.*

Examp. 2. What cost 306 *C.* wt. at 2 *l.* 12 *s.* 4$\frac{1}{2}$ *d.* *per C.* wt.

2 × 306 =	612	for 2 *l.*	
$\frac{1}{2}$ × 306 =	153	for	10 *s.*
$\frac{1}{5}$ × 153 =	30 : 12	for	2 *s.*
$\frac{1}{6}$ × 30 : 12 =	5 : 2	for	4 *d.*
$\frac{1}{8}$ × 5 : 2 =	0 : 12 : 9	for	0$\frac{1}{2}$ *d.*

Therefore 801 : 6 : 9 for 2 *l.* 12 *s.* 4$\frac{1}{2}$ *d.*

591.

591. The beſt way of proving the Work of Queſtions in Practice, is to perform the ſame by another method: Thus the firſt example in *Art.* 590. and the ſecond in *Art.* 589. mutually prove each other.

592. When the price of an Integer is given, and the price of many Integers of the ſame name together with $\frac{1}{4}$ or $\frac{1}{2}$ or $\frac{3}{4}$ of an Integer is required, the value of thoſe Integers may be firſt found by ſome of the preceeding Rules, and then for the price of $\frac{1}{2}$ of an Integer, take $\frac{1}{2}$ of the given price of an Integer; likewiſe for $\frac{1}{4}$ of an Integer, take $\frac{1}{4}$ of the ſaid given price; alſo for $\frac{3}{4}$ of an Integer, take the ſum compoſed of $\frac{1}{2}$ and $\frac{1}{4}$ of the ſaid given price: So if it be demanded what 134 *C.* 3 *qu.* (to wit, 134 hundred weight, and $\frac{3}{4}$ of an hundred weight) of ſugar will coſt at 4 *l.* 16 *s.* 3 *d.* *per C.* the anſwer will be found 648 *l.* 9 *s.* 8$\frac{1}{4}$ *d.* as by the ſubſequent operation is manifeſt.

134 *C. wt.* 3 *qu.*
4

4 × 134	=	536	for 4 *l.*
$\frac{1}{2}$ of 134	=	67	for 10 *s.*
$\frac{1}{2}$ of 67	=	33 : 10	for 5 *s.*
$\frac{1}{5}$ of 33 : 10	=	6 : 14	for 1 *s.*
$\frac{1}{4}$ of 6 : 14	=	1 : 13 : 6	for 3 *d.*
Add {		2 : 8 : 1$\frac{1}{2}$	for $\frac{1}{2}$ *C. wt.*
		1 : 4 : 0$\frac{3}{4}$	for $\frac{1}{4}$ *C. wt.*
Anſwer		648 : 9 : 8$\frac{1}{4}$	

593. Whenever the parts given are ſimilar to any known parts of a pound, queſtions of this kind may be then advantageouſly performed, by firſt finding what the given Integer, and the parts annexed, would amount to, at 1 *l.* for each Integer: For inſtance, in the above Example, when 1 *C. wt.* coſts 1 *l.* 3 *qu.* will coſt 15 *s.*

C. wt. qu.			*l.* *s.*
Then 134 : 3 at 1 *l.*	*per C. wt.* amount to		134 : 15
134 : 3 at 4 *l.*	= 4 × 134 : 15		= 539 : 0
134 : 3 at 10 *s.*	= $\frac{1}{2}$ of 134 : 15		= 67 : 7 : 6
134 : 3 at 5 *s.*	= $\frac{1}{2}$ of 67 : 7 : 6		= 33 : 13 : 9
134 : 3 at 1 *s.*	= $\frac{1}{5}$ of 33 : 13 : 9		= 6 : 14 : 9
134 : 3 at 3 *d.*	= $\frac{1}{4}$ of 6 : 14 : 9		= 1 : 13 : 8$\frac{1}{4}$
Therefore 134 : 3 at 4 : 10 : 3		amount to	648 : 9 : 8$\frac{1}{2}$

594. But the advantage of this method will be very conspicuous, in the computing the values of gold and silver plate: for in *Troy* weight the ounce (which is the Integer in those kinds of computation) is divided into 20 penny-weights, so that at 1 *l. per oz.* each penny-weight will be worth 1 *s.* Again the penny-weight being divided into 24 grains, every grain will be worth an half-penny, and every two grains one penny; so that the value of the given quantity, at 1 *l. per oz.* may be wrote down at sight.

Examp. 1. What cost 327 *oz.* 17 *p.w.* 20 *gr.* of gold, at 3 *l.* 17 *s.* 6 *d. per oz?*

	oz. p.w. gr.	*l.*	*s.*	*d.*
Now	327 : 17 : 20 at 1 *l. per oz.* amount to	327	17	10
Then for 3 *l.*	take 3 × 327 : 17 : 10 =	983	13	6
10 *s.*	$\frac{1}{2}$ of 327 : 17 : 10 =	163	18	11
5 *s.*	$\frac{1}{2}$ of 163 : 18 : 11 =	81	19	$5\frac{1}{2}$
2 *s.* 6 *d.*	$\frac{1}{2}$ of 81 : 19 : $5\frac{1}{2}$ =	40	19	$8\frac{3}{4}$
	Answer	1270	11	$7\frac{1}{4}$

595. *There are many persons who are absurd enough to propose, as a trial of the Arithmetical Skill of others, the multiplying of a number of pounds, shillings, and pence, by a number of pounds, shillings, and pence; the reader is therefore desired to take notice, that if any such person should propose to him the multiplying of* 327 l. 17 s. 10 d. *by* 3 l. 17 s. 6 d. *that he may proceed as in the above Example, and give the answer* 1270 l. 11 s. $7\frac{1}{4}$ d. *for the product required.*

Examp. 2. What cost 327 *oz.* 17 *p.w.* 21 *gr.* of silver, at 5 *s. per oz?*

	oz. p.w. gr.	*l.*	*s.*	*d.*
Now	327 : 17 : 21 at 1 *l. per oz.* amount to	327	17	$10\frac{1}{2}$
	Therefore for 5 *s.* take $\frac{1}{4}$ of it =	81	19	$5\frac{1}{2} + \frac{1}{8}$

596.

C. qu. lb.
596. What coſt 218 : 3 : 24 at 5 *l.* 15 *s.* $7\frac{3}{4}$ *d. per C.*

115 × 218 =	1090			
	218			
	218			
$\frac{1}{2}$ of 218	109	*d.*	*far.*	
$\frac{1}{4}$ of 109	27 :	3 :	0	
$\frac{1}{6}$ of 27 *s.* 3 *d.*	4 :	6 :	2	
The Quotients arising for by *Art.* 592. — $\frac{1}{2}$ *C.*	57 :	9 :	$3\frac{1}{2}$	
$\frac{1}{4}$ *C.*	28 :	10 :	$3\frac{3}{4}$	
14 *lb.*	14 :	5 :	$1\frac{3}{4}$	+
7 *lb.*	7 :	2 :	$2\frac{3}{4}$	+
3 *lb.*	3 :	1 :	0	+
	20) 2532|2 :	3 :	2	+
	l.	*s.*	*d.*	
	Anſw. 1266 :	2 :	$3\frac{1}{2}$	+

The Example laſt-mentioned being (of thoſe queſtions which ordinarily happen in trade) one of the hardeſt to be reſolved by the Rule of Practice; therefore in order to explain the aforegoing operation, you may obſerve, that the price of 218 *C.* 3 *qu.* is found after the manner of former Examples; then for 14 *lb.* part of the 24 *lb.* in the queſtion, take $\frac{1}{2}$ of the price $\frac{1}{4}$ *C.* Likewiſe for 7 *lb.* take half the price of 14 *lb.* and ſo there yet remains 3 *lb.* whoſe price is found by taking $\frac{3}{7}$ of the price of 7 *lb.* *viz.* the price of 7 *lb.* being very near 7 *s.* $2\frac{1}{2}$ *d.* or $86\frac{1}{2}$ *d.* Multiply $86\frac{1}{2}$ by 3, and divide the Quotient by 7, ſo there ariſes 37 *d.* or 3 *s.* 1 *d.* very near. Laſtly, all being added together, the ſum is found to be very near 25322 *s.* $3\frac{1}{2}$ *d.* or 1266 *l.* 2 *s.* $3\frac{1}{2}$ *d.*

Note, That a quarter of a farthing (or $\frac{1}{16}$ of a penny) is the ſmalleſt money expreſſed in the Example; and where any thing ariſes leſs than a quarter of a farthing, it is omitted; but it is ſuppoſed to follow this note +, for which ſurpluſages ſome reſpect ought to be had in adding all together: Now although in reſolving queſtions after this practical manner there will be ſome error, yet the loſs for the moſt part will be leſs than a farthing, which is inconſiderable.

597. Altho' queſtions of the laſt-mentioned kind cannot be performed by the method in *Art.* 593. becauſe the diviſion of a quarter of an hundred weight into 28 pounds, has nothing ſimilar to it in the ſubdiviſions of a pound Sterling; yet if the Decimal, equivalent to the given parts of an *C. wt.* (taken from

the Tablet explained in *Art.* 350. *viz.* ,75 for the 3 quarters and ,2142857 for the 24 *lb.* making together ,9642857) be annexed to 218 the number of hundred weights, the number (218,9642857) ſo ariſing will expreſs, in pounds Sterling and decimal parts of a pound Sterling, the value of 218 *C.* 3 *qr.* 24 *lb.* at 1 *l.* Sterling, *per C. wt.* and the operation may be as follows:

The given Quantity.

At 1 *l. per* hundred weight amounts to			218,9642857
For 5 *l.*	take	$5 \times 218{,}9642857 =$	1094,8214285
For 14 *s.* take $\frac{7}{10}$ thereof	$= 7 \times$	$21{,}8964286 =$	153,2750002
For 1 *s.* take $\frac{1}{20}$ thereof	$= \frac{1}{2}$ of	$21{,}8964286 =$	10,9482143
For 6 *d.*	take $\frac{1}{2}$ of	$10{,}9482143 =$	5,4741071
For $1\frac{1}{2}$ *d.*	take $\frac{1}{4}$ of	$5{,}4741071 =$	1,3685268
For $\frac{1}{4}$ *d.*	take $\frac{1}{6}$ of	$1{,}3685268 =$	0,2280878
Then will their ſum be the anſwer		$=$	1266,1153647

Which (when the decimal part is reduced to ſhillings and pence) is 1266 *l.* 2 *s.* $3\frac{3}{4}$ *d.* nearly.

598. When the price of 1 *lb.* weight is known, and the price or value of 1 *C.* (to wit 112 *lb.*) is required, the anſwer may ſometimes be given more ſpeedily, than by any of the former Rules, by this Rule which follows, *viz.* find the number of farthings contained in the given price of 1 *lb.* weight, then take twice that number of ſhillings, and once that number of groats, and having added them together, the ſum will give the value of 1 *C.* to wit, 112 *lb.* weight: So if it be demanded what 1 *C.* or 112 *lb.* weight of cheeſe will coſt at the rate of $3\frac{1}{4}$ pence the pound weight, the anſwer will be 1 *l.* 10 *s.* 4 *d.*

For according to the ſaid Rule, the number of farthings contained in $3\frac{1}{4}$ *d.* (the price of 1 pound weight) is 13, therefore the double of 13 ſhillings is —

	l.	*s.*	*d.*
the double of 13 ſhillings is —	1	6	0
13 groats make — — — —	0	4	4
Therefore the ſum (which is the price of 1 *C.* or 112 *lb.* weight) is ———	1	10	4

The reaſon of this Rule is as follows, if 1 *lb.* weight coſt 13 farthings, then 112 *lb.* muſt neceſſarily coſt 112 times 13 farthings, or (which is the ſame) 13 times 112 farthings; but 13 times 112 farthings, are equal to twice thirteen ſhillings together with once thirteen groats; becauſe 112 farthings are compoſed of twice 48 farthings (or two ſhillings) and of 16 farthings (or one groat;) wherefore the truth of the ſaid Rule is evident.

Another

Another Example, When fugar is at $5\frac{1}{2}$ *d.* the pound weight, what is the value of 1 *C.* (or 112 *lb.* weight?) *Anfwer* 2 *l.* 11 *s.* 4 *d.* For in $5\frac{1}{2}$ *d.* are contained 22 farthings,

	l.	*s.*	*d.*
therefore the double of 22 fhillings is — —	2	4	0
22 groats make — — — — — —	0	7	4
Which added together, give the price of 1 *C.* 112 *lb.* to wit — — —	2	11	4

599. Here follow the quantities of particular Goods in Wholefale-trade, found in the Book of rates, with concife and ready ways of cafting up the fame, deduced from the fame principle with that in *Art.* 598.

600. Goods fold by fixfcore to the hundred, are Lamb-fkins, Barlings, Balks great, middle, and fmall; Clap-boards, Pipe-boards, Bow ftaves, Cap-ravens, Deal-boards, Spars of all forts, Cod, Cole, Ling, and all forts of Stock-fifh; with many forts of linen, *viz.* Hamburgh, Silefia, Irifh, Mufcovia, Weftphalia, Hanover, *&c.*

If the price of one of any of thofe forts of goods be given to find the price of an hundred.

Rule. Divide the number of farthings contained in the price of one by 8, and the Quotient will be the price of 120 in pounds. *Note*, Reckon half a crown for every one that remains, after Divifion.

Example, What coft 120 ells of Canvas at $5\frac{1}{2}$ *d. per* Ell?

$5\frac{1}{2}$ *d.* is 22 farthings, and $\frac{22}{8} = 2\frac{6}{8}$; therefore 120 will coft 2 *l.* and 6 half-crowns, or 2 *l.* 15 *s.*

601. Goods fold by the thoufand, are Pafte-boards for Books, Tennis-balls, Lemons, Oranges, Teazels, Flanders-paving and pan-tiles, Lantern-horns, Barrel-hoops, and Boards; Quills, Lamperns, Squirrel-fkins, Ox-bones, Yards of Lift, *&c.*

If the price of one of any of thofe forts of goods be given to find the price of a thoufand.

Rule. To as many pounds as there are farthings in the price of 1, add the fame number of ten-pences, fo fhall the fum be the price of a thoufand.

Example. What coft a thoufand of Tiles at 3 *d.* each?

3 *d.* is 12 farthings: Then to	12 *l.*	
Add 12 ten-pences = 10 twelve-pences =	0 *l.*	10 *s.*
Therefore a thoufand will coft .	12 *l.*	10 *s.*

602. Goods fold by the great Grofs, containing 12 fmall Grofs, or 144 dozen, *viz.*

Buttons of Metal, Glafs, Thread, Silk or Hair; Beads of Bone, Box, Glafs or Wood; Cap-hooks, Chefs-men; Tobaccobacco-pipes, Combs of Light or Box-wood, Thread or Silk-points, Playing-cards, &c.

If the price of 1 dozen of any of these sorts of goods be given to find the price of a great grofs.

Rule. Multiply the farthings contained in the price of 1 dozen by 3, and the product will be the price of a great grofs in shillings.

Example. What cost a great grofs of Buttons, at $8\frac{1}{2}$ *d. per* dozen?

$8\frac{1}{2}$ *d.* is 34 farthings; and $34\times3=$ 102 *s.* = 5 *l.* 2 *s.*
Therefore a great grofs will cost 5 *l.* 2 *s.*

603. Goods sold by the small grofs of 12 dozen, *viz.* Tobacco and Pepper-boxes, Nest and Tinder-boxes, Buckles of some sorts, Wash-balls, Inkhorns, Bodkins; several sorts of Knives, Comb-cases, &c.

Goods of this quantity may be cast up by the last preceding Rule, seeing there are as many particulars in a small grofs, as dozens in a great one.

604. Goods sold by the fivescore to the hundred, are Annotto, Capers, Safflowers, Thrums, Ginger, Cloves, Indico, Crofs-bow-thread, Pack-thread, Kids or Goats-hair, Quick-silver, Cotton-wool, *English* Hard-wax, Brafs and Latten-work, as Chafing-dishes, Andirons, Laver-cocks, &c.

If the price of 1 of any of these sorts of goods be given, to find the price of an hundred.

Rule. To twice as many shillings as there are farthings in the price of one, add once as many pence, so shall the sum be the price of an hundred.

Example. What cost 100 balls of Pack-thread at $2\frac{1}{2}$ *d. per* ball?

$2\frac{1}{2}$ *d.* is 10 farthings; then to $10\times2=$ 20 *s.*
add $10\times1=$ 0 *s.* 10 *d.*
And an hundred will cost 20 *s.* 10 *d.*

605. Questions to exercise the Rules of Practice.

		l.	*s.*	*d.*
75 *C.* 2 *q.* 19 *lb.* at $4\frac{1}{2}$ *d. per lb.*	Answer	158	18	$1\frac{1}{2}$
71081 *lb.* at 35 *s.* 6 *d. per C.*	Answer	1126	10	1
67108 *oz.* at $16\frac{1}{4}$ *d. per lb.*	Answer	283	19	$8\frac{1}{2}$
71 *lb.* 10 *oz.* 15 *p.w.* at 5 *s.* 9 *d. per oz.*	Answer	248	0	$9\frac{3}{4}$
319 *yards*, at 5 *s.* 6 *d. per ell.*	Answer	70	3	$7\frac{1}{5}$
$419\frac{7}{8}$ *yards*, at 4 *s.* $10\frac{1}{4}$ *d.*	Answer	101	18	$1\frac{3}{4}$
47 *quar.* at 4 *s.* 9 *d. per bush.*	Answer	89	6	0
$715\frac{1}{4}$ *yards* at 4 *s.* $10\frac{1}{8}$ *d.*	Answer	173	4	$5\frac{1}{8}$

		l.	*s.*	*d.*
59 *oz.* 11 *p.w.* 18 *gr.* at 5 *s.* 7 *d.* *per oz.*	Anſwer	16	12	8
$50\frac{3}{4}$ *lb.* of ſilk at 20 *d.* *per oz.*	Anſwer	67	13	4
66 *groſs* at 3 *s.* 9 *d.* *per dozen.*	Anſwer	148	10	0
56 load of hay at 18 *d.* *per* truſs.	Anſwer	151	4	0

CHAP. XXXVII.

Of TARE *and* TRET, &c.

606. THESE allowances are called beyond ſea, *The courteſies of* London, becauſe not practiſed in any other place; concerning which obſerve, That,

607. Groſs is the weight of the commodity, and that which contains it without any deduction. And Tare is an allowance for that which contains the commodity, whether it be Bag, Barrel, Sack, Frail, &c. and is either,

1. At ſo much *per* Bag, Barrel, Frail, &c.
2. At ſo much *per cent.* or,
3. By ſo much of the groſs weight, called Invoice Tare.

608. When the Tare is ſo much *per* bag, as Almonds; *per* barrel, as Oil in candy barrels; *per* frail, as Raiſins, &c.

Rule. Multiply the number of the ſaid bags, barrels, frails, hogſheads, &c. by the allowance of Tare, which product ſubtract from the groſs, and (when no Tret is mentioned) the remainder is neat.

Example. What is the neat weight of 38 hogſheads of Tobacco, weighing groſs 102 : 2 : 17, Tare *per* hogſhead 70 *l.*

C. *qu.* *lb.*	38
102 : 2 : 17	2660
112	
1297	
11497 Groſs	
2660 Tare	
8837 Neat	

609. When the Tare is at ſo much *per cent.* and that,
An Aliquot part, or parts of the hundred weight;

As Figs, Almonds, Argol, &c.	14	per Cent.
Caroteels, and butts of Currants, &c.	16	
Oil in uncertain caſks, &c.	18	

Rule. Divide the whole groſs by the ſaid part or parts that the Tare is of an hundred, gives the Tare of the whole; for ſuch a part of the groſs muſt be the Tare of the whole, as the given Tare is of an hundred.

Example. What is the neat weight of 15 barrels of Argol, groſs *C.* 51 : 2 : 18. Tare 14 *per cent.* which being the 8th of an hundred, take the 8th of the groſs for Tare.

```
     C.  q.  lb.
  8) 51 : 2 : 18        Groſs
      6 : 1 : 23 :  4 Tare
     ---------------------
     45 : 0 : 22 : 12 Neat.
```

Example 2. How many neat hundreds in 18 quarter rolls of Currants, each groſs 8 : 3 : 10. Tare 21 *l.* *per cent.*

```
            C.  q.  lb.
             8 : 3 : 10 each
                      3
            -----------
            26 : 2 : 02
                      6
  lb.       -----------
  14 1/8) 159 : 0 : 12 oz. groſs
   7 1/2) ------------------
            19 : 3 : 15 :  8
             9 : 3 : 21 : 12
            ----------------
            29 : 3 : 09 : 04 Tare
            ----------------
           129 : 1 : 02 : 12 neat.
```

610. But if the Tare is not readily found to be an Aliquot part or parts of a hundred.

Rule. Multiply the pounds groſs by the allowance of Tare, dividing the product by 112, the quotient is the Tare of the whole.

Example. What is the neat produce of 12 barrels of pot-aſhes, each groſs 203 *lb.* Tare 10 *l.* *per cent.*

4 203.

```
          203
          120 = 12 × 10
       ─────────────────
  112) 24360    (217      2436      grofs
       ─────
        196                217 : ½ Tare
       ─────
        840               ──────────
       ─────
         56               2218 : ½ neat.
```

611. If the allowance for Tare is not by the hundred, but according to the grofs weight, as Sumach in bags, bales or fangots of *Aleppo*, *Cyprus*, and *Smyrna* filk, &c. fuch Tare is called Invoice Tare; of which the Book of Rates, in the Table of Allowances for Tare, fays, *viz.*

Cyprus and *Smyrna* filk.

			lb.
Bales	about or above 300 *lb.*	allowed for Tare.	16
	from 300 to 200		14
	from 200 downwards		12

Tobacco from *Virginia*.

			lb.
Hogfheads	5 *C.* and upwards	allowed for Tare.	100
	from 4 to 5,		90
	from 3 to 4,		80
	under 3 hundred		70

Example. What is the neat weight of four hogfheads of *Virginia* tobacco, weighing grofs and number'd? *viz.*

N°	*C. q. lb.*		*lb.*
27	2 : 2 : 19	Tare found as above.	70
9	5 : 0 : 15		100
4	3 : 2 : 11		80
19	4 : 2 : 19		90

```
   16 : 0 : 08    grofs 112)  340(3
    3 : 0 : 04    Tare          4
   ──────────
   13 : 0 : 04 neat, and fo of Silk, &c.
```

Alfo

Alſo Sugar from *India.*

In caſks and caniſters, } Tare { $\frac{1}{6}$
In cheſts, and caſks, *St. Thom.* } Tare { $\frac{1}{5}$

612. Tret is an allowance of 4 *lb. per* 104, on ſome ſort of goods, (to the buyer by the merchant) for the Duſt and Waſte in them; which when allowed, after you have ſubtracted the Tare from the groſs, the remainder is ſubtil, or ſuttle.

Rule. This ſuttle take out, and divide by 26, becauſe the allowance of 4 *lb. per* 104, is $\frac{1}{26}$ part, and the quote is the Tret, which Tret ſubtract from the ſuttle, and the neat remains.

Example. How many pound neat in 12 *C.* 3 *q.* 12 *lb.* groſs. Tare 2 *lb. per cent.* Tret 4 *lb. per* 104?

```
26) 1415 (54              C.  q. lb.
    ————
     115                  12 : 3  12
    ————                 112
      11                 ————
    ————              8) 1440 groſs
                      7)  180
                         ————
                           25 + Tare
                         ————
                         1415 ſuttle
                           54 Tret
                         ————
                         1361 neat.
                         ————
```

Otherwiſe, to find and deduct Tret.

Rule. Add 2 cyphers to the ſuttle pounds, which number divide by 104, and the quote is the neat pounds.

Example. What is the neat weight of 4 Bar. of *Spaniſh* Tobacco? containing, *viz.*

No

Nº	C.	q.	lb.	
4	1	0	04	Tare 14 *per cent.*
7	0	3	25	Tret 4 *per* 104 *lb.*
9	1	0	05	
3	0	2	26	

3 : 3 : 04
112

8) 424 groſs
53 Tare

104) 37100 (356 + neat.

613. Cloff, is another allowance of 2 *lb.* weight, to citizens of *London*, on every draught above three hundred weight, on ſome goods, as Galls, Madder, Sumach, Argol, &c.

614. Neat, is the pure weight of the commodity, when all given allowances are deducted.

615. As for other allowances not ſo common, as Break, Damage, &c. the firſt being generally at ſo much *per* Caſk, Bale, Bag, &c. the other ſo much in the whole, can have no difficulty.

Example. What is the neat weight of 7 hogſheads groſs, 28 *C.* 1 *q.* 19 *lb.* Tare 16 *per cent.* Break 8 *per* hogſhead, Damage $98\frac{1}{2}$?

	C.	q.	lb.		
7)	28	1	19		groſs
	4	0	06	11	Tare
	24	1	12	5	
	—	2	00	0	Break
	23	3	12	5	
		3	14	8	Damage
	22	3	25	13	neat.

616. In Oil the Tare is of two ſorts.

(1.) On Candy barrels —— — at 29 *lb. per* barrel.
(2.) On uncertain caſks — — at 18 *lb. per cent.*
The neat pounds of which are found by the foregoing Rules. But to reduce thoſe pounds into gallons;

Rule. Multiply thoſe pounds neat by 2, the product of which divide by 3, and that quote by 5, gives neat gallons, 7 *lb.* $\frac{1}{2}$ making a gallon.

Example. Oil, groſs 127 *C.* 3 *q.* 7 *lb.* Tare 18 *lb. per cent.* How many neat gallons?

```
   C.  q. lb.
   127 : 3 : 7
   112
   -----------
     1615
   -----------
7) 14315 groſs
8)  2045
     255
   -----------
    2300 Tare
   -----------
   12015 neat pounds
       2
   -----------
3) 24030
5)  8010
   -----------
    1602 gallons.
```

CHAP.

CHAP. XXXVIII.

Concerning EXCHANGES *of* COINS, WEIGHTS, *and* MEASURES.

617. THE rate or proportion between Coins, Weights, *&c.* of different kinds being known, either from some good author, or rather by experience; it will not be difficult, to such as understand the before-going Rules, to know how to exchange a given quantity of one kind, for a quantity of the same value in another kind: But since in some cases the common manner of working may be much contracted, it will be convenient to shew the most compenndious ways to perform this business.

618. In exchanging of things of different kinds, (whether they be Coins or Weights, *&c.*) when two things of different kinds are compared together, the question may be resolved by one single Rule of Three, as will be evident by the subsequent Examples, *viz.*

Quest. 1. How many Riders at 21 *s.* $2\frac{1}{2}$ *d.* Sterling the piece, ought to be received for 251 *l.* 6 *s.* $4\frac{1}{2}$ *d.* of Sterling money? Answer 237 Riders. For the first and third terms in the Rule of Three, which arise from this question, being converted into half-pence, the proportion will be this:

509 : 1 :: 120633 : 237.

Quest. 2. If 100 ells of *Antwerp* make 75 yards of *London*, how many yards of *London* measure will 27 ells of *Antwerp* make? Answer $20\frac{1}{4}$ yards.

100 : 75 :: 27 : $20\frac{1}{4}$.

619. In *London*, and throughout *England*, accounts are kept in pounds, shillings, and pence Sterling, and are cast up as *Flemish* and *French* money are, by 12 and 20, and the Exchange made with all places by pence, *i. e.* giving so many pence Sterling for the pieces on which the Exchange is made, except

1. *Portugal*, and with it in shillings and pence, on their Milrea.
2. *Antwerp*, *Hamburgh*, &c. Countries bordering upon *Flanders* and *Holland*, and with them by the pound of 20 *s.* Sterling.
3. *Ireland*, and the *American* plantations. by the hundred pound.

620.

620. In *Italy*,

Genoa, *Leghorn*, *Venice*, &c.

Genoa and *Leghorn* exchange with *London* by the dollar, or piece of eight; *Venice* by the ducat.

Genoa and *Leghorn* keep their accounts in livres, ſols, and deniers; 5 livres is a piece of eight at *Genoa*, and 6 at *Leghorn*.

621. In *Spain*,

Madrid, *Cadiz*, *Bilboa*, &c.

Exchange alſo by the piece of eight.

622. In *Portugal*,

Liſbon and *Oporto*, &c.

Exchange on the Milrea.

623. In *France*,

Paris, *Lyons*, and *Rouen*.

Exchange by the Crown.

But keep their accounts in Livres, Sols, and Deniers.

12 Deniers	make	1 Sol.
12 Sols	make	1 Livre.
3 Livres	make	1 Crown.

624. In the *Netherlands*,

Antwerp, *Bruſſels*, *Amſterdam*, *Rotterdam*, *Hamburgh*, &c.

Exchange with *London* by ſhillings and pence *Flemiſh*. Accounts are in theſe places kept in *Flemiſh* pounds, ſhillings and pence; by ſome in guilders, ſtivers, and pennicks. The *Flemiſh* pounds, ſhillings, and pence, are divided as our money, *viz.* one pound into 20 *s.* and one ſhilling into 12 *d.* But,

16 Pennicks,	one	Stiver.
6 Stivers,	one	*Flemiſh* Shilling.
20 Stivers,	one	Guilder.
6 Guilders,	one	*Flemiſh* Pound.
30 Stivers,	one	Common Dollar.
50 Stivers,	one	Specie Dollar.
63 Stivers,	one	Ducatoon.

625. *Italian*, *Spaniſh*, and *French* Exchange.

Caſe 1. The quantity of Sterling money being given.

Rule. Take the aliquot parts for the pence of the Exchange out of a pound, dividing the given crowns by thoſe parts, the total of which is the anſwer, as in *Art.* 586.

Example. How much Sterling muſt I pay here to receive in *France* 479 crowns, Exchange at 52 *d.* Sterling *per* crown?

4 *s.*

$$4\,s.\ \tfrac{1}{5}\ 479 \text{ crowns.}$$

4	$\frac{1}{5}$	95 : 16
	$\frac{1}{12}$	7 : 19 : 8
		103 : 15 : 8

Example (2.) How much Sterling for 7439 *French* crowns, Exchange, at 55 pence $\frac{5}{8}$ *per* crown?

s.	*d.*		7439
2	6	$\frac{1}{8}$	929 : 17 : 06
1	8	$\frac{1}{12}$	619 : 18 : 04
	5	$\frac{1}{4}$	154 : 19 : 07
	$\frac{5}{8}$	$\frac{1}{8}$	19 : 07 : 05 : $\frac{3}{8}$
		Anſwer	1724 : 02 : 10 : $\frac{3}{8}$

Caſe 2. The quantity of Foreign money being given.

Rule. Uſe the univerſal Rule of Proportion, *Art.* 407. as in Example 22. annexed thereto.

Example. How many *French* Crowns muſt I give for 103 *l.* 15 *s.* 8 *d.* Sterling: Exchange at 52 *d.* Sterling *per French* Crown?

	Fr. Cr.	*l.*
Conditional terms	1	$\frac{52}{240}$
Correſponding	A	$\frac{6227}{60}$. For 103 *l.* 15 *s.* 8 *d.* = 6227 groats.

$$\text{Therefore } A = \frac{1 \times 6227 \times 240}{60 \times 52} = \frac{6227 \times 4}{52} = \frac{6227}{13} = 479 \text{ crowns.}$$

626. Of *Flemiſh* and *German* Exchange.

627. *Firſt*, To reduce Sterling into *Flemiſh* pounds.

Rule. Take the aliquot parts for what the rate of the Exchange is above a pound, dividing the given Sterling by the ſaid parts, the total of which, with the given Sterling, is the *Flemiſh* money, as in *Art.* 587.

Example 1. One in *Antwerp* delivering money by Exchange for *London* at 35 *s.* 6 *d.* *Flemiſh per* pound Sterling; how much muſt he pay there to receive here 597 *l.* Sterling?

10 *s.*

10 *s.*	$\frac{1}{2}$	597
5	$\frac{1}{2}$	298 : 10
6	$\frac{1}{10}$	149 : 05
		14 : 18 : 6
		1059 : 13 : 6

Anſwer *l.* 1059 : 13 : 6 *Flemiſh.*

Example 2. A Merchant at *Rotterdam* has a bill drawn on him for the value of 673 *l.* 16 *s.* 8 *d.* Sterling, Exchange at 33 *s.* 4 *d. Flemiſh per* pound Sterling; how much muſt he pay there?

10 *s.*	$\frac{1}{2}$	673 : 16 : 8
3 4	$\frac{1}{6}$	336 : 18 : 4
		112 : 06 : 1 $\frac{1}{4}$
		1123 : 01 : 1 $\frac{1}{4}$

Anſwer *l.* 1123 : 01 : 1 $\frac{1}{4}$ *Flemiſh.*

628. *Secondly,* To reduce Sterling into Guilders.

Rule. Reduce the Sterling to *Flemiſh*, by the foregoing Rule, which *Flemiſh* money multiply by 6.

Example. If 397 *l.* 15 *s.* Sterling, Exchange at 34 *s.* 8 *d. Flemiſh per* pound Sterling, were remitted to *Amſterdam*; how many guilders, ſtivers, &c. may be received at the ſaid rate of Exchange?

10 *s.*	$\frac{1}{2}$	397 : 15
4	$\frac{1}{5}$	198 : 17 : 06
8	$\frac{1}{6}$	79 : 11 : 00
		13 : 05 : 02
		689 : 08 : 08
		6
		4136 : 12 : 00

Guil. Stiv.
Anſwer 4136 : 12

629. *Thirdly,* To reduce *Flemiſh* pounds into Sterling, uſe the univerſal Rule of Proportion, as in *Caſe* 2. *Art.* 625.

Example. A Merchant at *Bruſſels* delivers 579 *l.* 10 *s. Flemiſh*, Exchange at 35 *s.* 6 *d. per l.* Sterling; how much Sterling muſt be received?

579 *l.*

$579\,l.\ 10\,s. = \frac{1159}{2}\,l.$ and $35\,s.\ 6\,d. = \frac{71}{40}\,l.$

	Sterl.	*Flem.*
Conditional terms	1	$\frac{71}{40}$
Corresponding	A	$\frac{1159}{2}$

Therefore $A = \frac{1 \times 1159 \times 40}{2 \times 71} = \frac{1159 \times 20}{71} = 326\,l.\ 9\,s.\ 6\frac{3}{4}\,d.$

630. But those who are ready in the use of Decimal Fractions, may proceed as follows:

$579\,l.\ 10\,s. = 579{,}5$; and $35\,s.\ 6\,d. = 1{,}775$

Then $A = \frac{579{,}5}{1{,}775} = 326{,}4789$. See the operation.

```
1,775)579,5000(326,4789
  ...  5325
       -----
      . 4700
        3550
        -----
        11500
        10650
        -----
        ..8500
          7100
          ----
          1400
          1243
          ----
          .157
           142
           ---
            15
```

631. *Fourthly*, To reduce Guilders, Stivers, and Pennicks into Sterling.

Rule. Divide the Guilders, *&c.* by 6, which reduces them to *Flemish* pounds, and proceed as in *Art.* 629, and 630.

632. Of gain or loſs by Exchange.

Note, That when Exchange is made with *Italy*, *France*, *Spain*, &c. with which as aforeſaid, *London* exchanges for ſo many pence for their pieces, the gain thereby is ſo much the more, by how much the courſe of Exchange runs low; becauſe it is evident, that I can receive more ducats for 500 *l.* when the Exchange is at 53 *d.* than at 56 *per* piece.

But when I remit for *Flanders* or *Holland*, with which *London* exchanges on the pound Sterling, the higher the Exchange is above Par, the more we have the advantage of gain; for according to reaſon, I may receive more *Flemish* money for 1000 *l.* Sterling, when the Exchange is at 36 *s.* 8 *d.* than at 35 *s.* 4 *d.*

633.

633. *Decimal TABLES for* Flemish *Exchanges, at the most usual Rates of Exchange.*

Rate of Exchan.	The Value of pounds Sterling in pounds *Flemish.*	The Value of pounds *Flemish* in pounds Sterling.
s. d.		
32—	1 .6	.625 . . . .
1	1 .6041ϕ	.6233766
2	1 .608ʒ	.6217616
3	1 .6125 . . .	.6201550
4	1 .61ϕ	.6185567
5	1 .6208ʒ	.6169665
6	1 .625	.6153846
7	1 .6291ϕ	.6138107
8	1 .6ʒ	.6122449
9	1 .6375 . . .	.6106870
10	1 .641ϕ	.6091370
s. 11	1 .6458ʒ	.6075949
33—	1 .65	.6060606
1	1 .6541ϕ	.6045343
2	1 .658ʒ	.6030150
3	1 .6625 . . .	.6015037
4	1 .ϕ	.6 . . . . .
5	1 .6708ʒ	.5985037
6	1 .675	.5970149
7	1 .6791ϕ	.5955335
8	1 .68ʒ	.5940594
9	1 .6875 . . .	.5925925
10	1 .691ϕ	.5911330
11	1 .6958ʒ	.5896806
34—	1 .7	.5882352
1	1 .7041ϕ	.5867970
2	1 .708ʒ	.5853658
3	1 .7125 . . .	.5839416
4	1 .71ϕ	.5825242

Rate of Exchan.	The Value of pounds Sterling in Guilders.	The Value of Guilders in pounds Sterling.
s. *d.*		
32—	9 .6	.10416666
1	9 .625	.10389610
2	9 .65	.10362694
3	9 .675	.10335916
4	9 .7	.10309278
5	9 .725	.10282775
6	9 .75	.10256410
7	9 .775	.10230178
8	9 .8	.10204081
9	9 .825	.10178116
10	9 .85	.10152283
11	9 .875	.10126581
33—	9 .9	.10101010
1	9 .925	.10075567
2	9 .95	.10050250
3	9 .975	.10025061
4	10.0	.1 . . . . . .
5	10.025	.09975061
6	10.05	.09950248
7	10.075	.09925558
8	10.1	.09900990
9	10.125	.09876541
10	10.15	.09852216
11	10.175	.09828011
34—	10.2	.09803921
1	10.225	.09779951
2	10.25	.09756097
3	10.275	.09732360
4	10.3	.09708736

Rate

Rate of Exchan.		The Value of pounds Sterling in pounds *Flemish.*	The Value of pounds *Flemish* in pounds Sterling.
s.	*d.*		
34	5	1 .72083̇	.5811138
	6	1 .725	.5797101
	7	1 .72916̇	.5783132
	8	1 .73̇	.5769230
	9	1 .7375	.5755395
	10	1 .7416̇	.5741628
	11	1 .74583̇	.5727923
35	—	1 .75	.5714285
	1	1 .75416̇	.5700712
	2	1 .7583̇	.5687209
	3	1 .7625 . . .	.5673758
	4	1 .76̇	.5660377
	5	1 .77083̇	.5647058
	6	1 .775	.5633802
	7	1 .77916̇	.5620609
	8	1 .783̇	.5607476
	9	1 .7875 . . .	.5594405
	10	1 .7916̇	.5581395
	11	1 .79583̇	.5568445
36	—	1 .8	.5555555
	1	1 .80416̇	.5542725
	2	1 .8083̇	.5529954
	3	1 .8125 . . .	.5517241
	4	1 .816̇	.5504587
	5	1 .82083̇	.5491990
	6	1 .825	.5479452
	7	1 .82916̇	.5466970
	8	1 .83̇	.5454545
	9	1 .8375 . . .	.5442176
	10	1 .8416̇	.5429864
	11	1 .84583̇	.5417607

Rate of Exchan.		The Value of pounds Sterling in Guilders.	The Value of Guilders in pounds Sterling.
s.	*d.*		
34	5	10.325	.09685231
	6	10.35	.09661835
	7	10.375	.09638553
	8	10.4	.09615383
	9	10.425	.09592325
	10	10.45	.09569368
	11	10.475	.09546538
35	—	10.5	.09523808
	1	10.525	.09501186
	2	10.55	.09478681
	3	10.575	.09456263
	4	10.6	.09433961
	5	10.625	.09411763
	6	10.65	.09389671
	7	10.675	.09367681
	8	10.7	.09345793
	9	10.725	.09324008
	10	10.75	.09302325
	11	10.775	.09280741
36	—	10.8	.09259259
	1	10.825	.09237875
	2	10.85	.09216590
	3	10.875	.09195402
	4	10.9	.09174311
	5	10.925	.09153316
	6	10.95	.09132420
	7	10.975	.09111616
	8	11.0	.09090909
	9	11.025	.09070293
	10	11.05	.09049773
	11	11.075	.09029345

Rate of Exchan.	The Value of pounds Sterl. in pounds *Flemish*.	The Value of pounds *Flem.* in pounds Sterling.
s. *d.*		
37——	1 .85	.5405405
1	1 .85416	.5393258
2	1 .8583	.5381166
3	1 .8625	.5369127
4	1 .86	.5357143
5	1 .87083	.5345211
6	1 .875	.5333333
7	1 .87916	.5321508
8	1 .883	.5309734
9	1 .8875	.5298013
10	1 .8916	.5286343
11	1 .89583	.5274725
38——	1 .9	.5263157
1	1 .90416	.5251641
2	1 .9083	.5240174
3	1 .9125	.5228758
4	1 .916	.5217391

Rate of Exchan.	The Value of pounds Sterling in Guilders.	The Value of Guilders in poundsSterling.
s. *d.*		
37——	11.1	.09009009
1	11.125 . .	.08988763
2	11.15 . . .	.08968610
3	11.175 . .	.08948545
4	11.2	.08928571
5	11.225 . .	.08908685
6	11.25 . . .	.08888888
7	11.275 . .	.08869180
8	11.3	.08849556
9	11.325 . .	.08830021
10	11.35 . . .	.08810571
11	11.375 . .	.08791208
38——	11.4	.08771928
1	11.425 . .	.08752735
2	11.45 . . .	.08733624
3	11.475 . .	.08714596
4	11.5	.08695651

The Explication.

634. The firſt Column ſhews the Rate of the Exchange from 32*s.* to 38*s.* 4*d.* and is extended no farther; for a regulation being made of the coin, it is not probable it will riſe higher, or fall lower than thoſe limitations.

635. The ſecond Column on the left-hand, ſhews the value of pounds Sterling, expreſſed in pounds *Flemiſh*, whereby at the firſt view, you diſcern what is the amount of 1, 10, 100, 1000, &c. to ten millions of Sterling, in pounds *Flemiſh*, at any of the inſerted rates of Exchange: For example, if the Exchange is at 35*s.* 2*d.* one pound Sterling, being equivalent to one pound fifteen ſhillings and two pence at one view; therefore it appears that

l.	*l.* *s.* *d.*	
10*l.* Sterling makes	17 : 11 : 8	*Flem.*
100	175 : 16 : 8	
1000	1758 : 6 : 8 &c.	

All which *Flemiſh* monies are found in the line againſt the ſaid given rate of Exchange: But if the Sterling money, for which the value in pounds *Flemiſh* is required, conſiſts of odd pounds, or hath ſhillings, *&c.* annexed; then multiply the *Flemiſh* money, at the given Exchange, by the ſum in Sterling, and the product anſwers the queſtion. Thus 67 *l.* 10*s.* Sterling, multiplied by 1,7583 the *Flemiſh* money at 35 : 2 makes 118 *l.* 13*s.* 9*d.* Flemiſh.

636. The third Column on the left-hand page ſhews the value or amount of pounds *Flemiſh* in Sterling, after the ſame manner as the ſecond ſhews the value or amount of pounds Sterling in *Flemiſh* money:

For inſtance,

At the aforeſaid Exchange of 35*s.* 2*d.* I ſee that one pound *Flemiſh* being equivalent to .5687209, or *l.* 11*s.* $4\frac{1}{2}$*d.*
Therefore 10 *Flem.* makes 5.687209, or 5 : 13 : 9 :
100*l.* 56.87209, or 56 : 17 : 6 :
and 1000 568.7209, or 568 : 14 : 5 : } Sterling very near.

But if your *Flemiſh* money, the value of which is required in Sterling, conſiſts of odd pounds, or hath ſhillings, *&c.* annexed, multiply the ſame by the Sterling, as before directed, and the product of ſuch multiplication anſwers the queſtion; ſo if it be required to find how much Sterling for 137*l* 10*s.* *Flemiſh*, Exchange being at 35*s.* 2*d.* then 137,5 × ,5687209 = 78,19912, or 78*l.* 4*s.* Sterling nearly.

637. The fifth Column, or the ſecond on the left-hand page, ſhews the value or amount of pounds Sterling in Guilders, or what number of Guilders I ought to receive, or pay, for any ſum of Sterling, at any of the inſerted rates of Exchange:

For Example,

At the aforeſaid Exchange of 35*s.* 2*d.* it appears that 10 Guilders, 11 Stivers, are equivalent to one pound Sterling; as likewiſe that

Guild.	*Stiv.*		*l.*	
105	: 10	are to	10	Sterling.
1055		to	100	
10550		to	1000	

If the Sterling-money is in odd pounds, or hath ſhillings, *&c.* annexed, you are to multiply the Guilders correſponding to the rate of Exchange by your Sterling ſum, as above directed; ſo 132*l.* 10*s.* Sterling, Exchange at 35*s.* 2*d.* will be found to make 1397 Guilders, 17 Stivers, 8 Pennicks.

638.

638. The laſt Column ſhews the value of Guilders in pounds Sterling, or how much Sterling I ought to receive, or pay, for any number of Guilders, at any of the ſpecify'd rates of Exchange.

So at the aforeſaid rate of Exchange, *viz.* 35*s.* 2*d.* one Guilder being worth .09478681, or 1*s.* 10$\frac{3}{4}$*d.* Sterling.

Therefore,

		l.	*s.*	*d.*	
10 make	.9478681, or		18	11 : $\frac{1}{2}$	Sterling
100	9.478681, or	9	09	6 : $\frac{3}{4}$	very
1000	94.78681, or	94	15	8 : $\frac{3}{4}$	near.

If your Guilders be in odd numbers, or have Stivers annexed, multiply them alſo by the value in Sterling correſponding to the rate of Exchange, and the product will anſwer your demand.

Guild. Stiv. Penn.

So 1397 : 17 : 8, Exchange at 35*s.* 2*d.* being multiplied by .09478681, amounts to 132*l.* 10*s.* Sterling, as abovefaid.

Note, The reaſon of the Tables being calculated ſo high, is, that you may be more exact in long ſums, tho' you'll ſeldom have occaſion to make uſe of more than five or ſix figures: And the dots placed to the right-hand are inſerted for the more eaſy and ſafe obſervation of large even ſums. But if the caſting up of odd ſums ſhould be thought as troubleſome to be effected by theſe Tables, as by the common method of doing them, (tho' in all caſes performed by one ſingle Multiplication) yet even ſums moſt commonly happening in drawing and remitting by Exchange, the uſefulneſs of theſe Tables will be ſufficiently manifeſt.

639. Many queſtions propoſed in Reduction, by perſons in the common method of teaching, belong to this place: Some of the moſt uſeful follow, and by attending to the manner of ſolution of them, the reader may form Rules for other caſes.

640. To reduce guineas into pounds.

Rule. To the given number of guineas, add $\frac{1}{20}$ thereof.

Example. How many pounds in 927 guineas?

$\frac{1}{20}$ of 927 = 46 : 7*s.*

Anſwer 973*l.* 7*s.*

641. To reduce pounds into guineas.
Rule. From the given pounds ſubtract $\frac{1}{21}$ thereof.
Example. How many guineas in 973 *l.* 7 *s.*

$\frac{1}{7}$ of 973 *l.* 7 *s.* = 139 : 1

Subtract $\frac{1}{3}$ of 139 *l.* 1 *s.* = 46 : 7 = $\frac{1}{21}$ of 973 *l.* 7 *s.* by *Art.* 113.

Anſwer 927 *l.* 0 *s.*

642. To reduce moidores at 1 *l.* 7 *s.* into pounds.
Rule. To the given number of moidores add $\frac{1}{4}$ and $\frac{1}{10}$ thereof.
Example. How many pounds in 927 moidores?

$\frac{1}{4}$ of 927 = 231 : 15
$\frac{1}{10}$ of 927 = 92 : 14

Anſwer 1251 *l.* 9 *s.*

643. To reduce moidores into guineas.
Rule. To the given number of moidores add $\frac{2}{7}$ thereof.
Example. How many guineas in 924 moidores?

$\frac{1}{7}$ of 924 = 132

Anſwer 1188 guineas.

644. To reduce guineas into moidores.
From the given number of guineas take $\frac{2}{9}$ thereof.
Example. How many moidores in 1188 guineas?

$\frac{1}{9}$ of 1188 = 132
132

$\frac{2}{9}$ of ditto = 264

Anſwer 924 moidores.

645. To reduce *Portugal* pieces of 1 *l.* 16 *s.* each to pounds.
From twice the given number of pieces take $\frac{1}{10}$ thereof.
Example. How many pounds in 925 pieces, each 1 *l.* 16 *s.*

2 × 925 = 1850
$\frac{1}{10}$ of 1850 = 185

Anſwer 1665 *l.*

646.

646. To reduce *Portugal* pieces of 1 *l.* 16 *s.* to guineas.

From twice the given number of pieces take $\frac{1}{7}$ thereof.

Example. How many guineas in 924 pieces, at 1 *l.* 16 *s.*

$$2 \times 924 = 1848$$
$$\tfrac{1}{7} \text{ of } 1848 = 264$$

Anſwer 1584

647. When more than two different Coins, Weights, Meaſures, &c, are compared together, *viz.* when one kind of Coin is compared with a ſecond of another kind; that ſecond with a third; the third with a fourth; the fourth with a fifth, &c. two different caſes are ordinarily raiſed from ſuch compariſon, *viz.*

It may be required to know, 1. How many pieces of the firſt Coin are equal in value to a given number of pieces of the laſt Coin. Or,

2. How many pieces of the laſt coin are equal in value to a given number of pieces of the firſt kind of Coin.

An Example of the firſt Caſe.

If 35 ells of *Vienna* make 24 ells of *Lyons*; 3 ells of *Lyons* 5 ells of *Antwerp*; and 100 ells of *Antwerp* 125 ells at *Frankfort*: how many ells of *Vienna* are equal to 50 ells at *Frankfort?* Anſwer 35 ells of *Vienna.*

For the more eaſy underſtanding of the reſolution of this queſtion, and others of like nature: Let *a* repreſent an ell at *Vienna*, *b* an ell at *Lyons*, *c* an ell at *Antwerp*, and *d* an ell at *Frankfort*; then may the given terms in the queſtion be ſtated in the following order.

$$\text{Suppoſitions}\left\{\begin{array}{rcl} 35\,a & = & 24\,b \\ 3\,b & = & 5\,c \\ 100\,c & = & 125\,d \end{array}\right.$$
$$\text{The queſtion} \quad 50\,d = \;?\,a$$

648. This order of placing the ſaid given numbers (or terms) being obſerved, it appears, that if 35 *a* be accounted to ſtand in the firſt place; 24 *b* in the ſecond; 3 *b* in the third; 5 *c* in the fourth; 100 *c* in the fifth, &c. then all the terms which ſtand in odd places, to wit, in the firſt, third, fifth, and ſeventh places, will neceſſarily fall under the firſt row or column on the left-hand, and all the terms that ſtand in even places, to wit, in the ſecond, fourth, and ſixth places, will fall under the latter column.

649. These thing, premised, all questions which come under *Case* 1. before-mentioned, may be resolved by this Rule, *viz.*

Rule 1. Multiply all the given terms that stand in odd places (to wit, in the first column) according to the Rule of continual Multiplication, and reserve the last product for a Dividend: Again, multiply continually all the terms which stand in even places: so shall the product be a Divisor, and the Quotient arising from the said Dividend and Divisor will be the answer of the question.

650. But instead of performing the Multiplications and Division, make use of the method in *Art.* 266. applying it to the numbers, as they stand on different sides of the marks of equality, in the same manner as it is there applied to the Numerator and Denominator of a Fraction. See the above example so managed.

$$\begin{array}{rrcll}
7 & \cancel{35}\,a & = & \cancel{24}\,b & \cancel{8}\ \cancel{4}\ 1 \\
1 & \cancel{3}\,b & = & \cancel{5}\,c & 1 \\
1\ \cancel{4} & \cancel{100}\,c & = & \cancel{125}\,d & \cancel{5}\ 1 \\
5\ \cancel{10} & \cancel{50}\,d & = & ?\,a &
\end{array}$$

Therefore $7 \times 5 = 35$ the Answer.

651. The reason of this Rule will be manifest by solving the question propounded by three single Rules of Three, thus:

$$\text{I.}\quad 24\,b : 35\,a :: 3\,b : \frac{35\times3}{24}a$$

$$\text{II.}\quad 5\,c : \frac{35\times3}{24}a :: 100\,c : \frac{35\times3\times100}{24\times5}a$$

$$\text{III.}\quad 125\,d : \frac{35\times3\times100}{24\times5}a :: 50\,d : \frac{35\times3\times100\times50}{24\times5\times125}a$$

Which number at last found, to wit,

$$\frac{35\times3\times100\times50}{125\times5\times24}$$

with the before-mentioned order of placing the terms proposed in the question, gives the very Rule before expressed in words.

652. An Example of the latter of the two cases before-mentioned.

If 10 *lb.* of Avoirdupoise-weight at *London* be equal to 9 *lb.* of *Amsterdam*; 45 *lb.* at *Amsterdam*, to 49 *lb.* at *Bruges*; and 98 *lb.* at *Bruges*, equal to 116 *lb.* at *Dantzick*; how many *lb.* of *Dantzick* are equal to 112 *lb.* of Avoirdupoise-weight at *London*? Answer 129.92 *lb.* of *Dantzick.*

That

That the operation may be the more clear, let a repreſent one pound of Avoirdupoiſe-weight; b one *lb.* of *Amſterdam*; c one *lb.* of *Bruges*; and d one *lb.* of *Dantzick*; then let the queſtion be ſtated after the order in the firſt caſe, *viz.*

$$\text{Suppoſitions}\begin{cases} 10\,a = 9\,b \\ 45\,b = 49\,c \\ 98\,c = 116\,d \end{cases}$$
$$\text{The Queſtion } ?\,d \quad 112\,a$$

653. Theſe things premiſed, all queſtions which fall under *Caſe* 2. before-mentioned, may be reſolved by this Rule, *viz.*

Rule 2. Multiply all the given terms which ſtand in even places (to wit in the latter column) by the Rule of continual Multiplication, and reſerve the laſt product for a Dividend: Again, multiply continually the reſt of the terms that ſtand in odd places (to wit in the firſt column) for a Diviſor; ſo ſhall the Quotient ariſing be the anſwer of the queſtion.

654. If the method in *Art.* 266. be applied to the above Example, it may ſtand as below:

$$\begin{array}{rl} & 10a = \not{9}b \\ 5 & \not{4}\not{5}b = \not{4}\not{9}c \\ 2 & \not{9}\not{8}c = 116d \\ & ?d = 112a \end{array}$$

Therefore $\frac{116\times112}{10\times5\times2} = \frac{116\times112}{100} = 116\times1{,}12 = 129{,}92 =$ the anſwer.

655. A little practice will render the uſe of the letters and marks of equality unneceſſary, and will enable the reader to write the reſult down, with only a line between the figures as in the following Example.

A merchant having occaſion to remit 385 *l.* Sterling to *Rome*, may do the ſame two ways, as follows, *viz.* He may remit by the way of *Paris*, 56 ſhillings Sterling being equal to 12 *French* crowns, and 55 *French* crowns being worth 52 dollars at *Genoa*; and 2 *Genoa* dollars amounting to 3 Roman crowns; or he may remit by the way of *Amſterdam*, 20 ſhillings Sterling, and 33 ſhillings *Flemiſh*; 15 ſhillings *Flemiſh*, and 2 rixdollars at *Frankfort*; 11 rixdollars at *Frankfort*, and 12 *Venetion* ducats; and 11 *Venetion* ducats and 12 Roman crowns being equal: Now the queſtion is, which way he can remit to the beſt advantage? Anſwer, by the way of *Paris*.

For

For by the underwritten operation it appears that if he remits by the way of *Paris* he will receive 2340 Roman crowns.

385*l.* = 7700 ſhillings.

~~14~~ ~~56~~	~~12~~ 6
~~11~~ ~~55~~	~~52~~ 13
1 ~~2~~	3
?	~~7700~~ ~~550~~ ~~110~~ 10

And 6×13×3×10 = 2340.

But if he remits by way of *Amſterdam*, he will receive but 2016 Roman crowns.

1 ~~20~~	~~33~~ 3
~~3~~ ~~18~~	2
~~11~~	~~12~~4
~~11~~	12
?	~~7700~~ ~~385~~ ~~77~~ 7

And 2×3×4×12×7 = 2016

656. Queſtions concerning Exchanges of Coins.

1. A bill of 200*l.* is remitted to *Paris* by a merchant in *London:* What is the value in *French* crowns, at 4*s.* 6*d.* each? *Anſwer* 888 *Crowns* $\frac{48}{54} = \frac{8}{9}$.
2. A merchant at *London* received 100*l.* Sterling, for the value paid by his correſpondent, at the rate of 3*s.* 6*d.* Sterling *per* crown: How many crowns were paid at *Paris?* Anſwer 571 *crowns* $\frac{18}{42} = \frac{3}{7}$.
3. Admit a bill drawn in *Lyons*, and payable in *London* for 1510 crowns: How much *Engliſh* money comes it to, the exchange at 55*d.* $\frac{1}{8}$? Anſwer 346*l.* 16*s.* 6$\frac{3}{4}$*d.*
4. There are 2000 ducats, at 4*s.* 4*d.* each, remitted to *London*, to be paid in pounds Sterling: What is the amount? Anſwer 433*l.* 6*s.* 8*d.*
5. A bill of 220*l.* 16*s.* 8*d.* is drawn from *London:* What is the value at *Florence*, in ducatoons, at 53*d.* each? Anſwer 1000 ducatoons.
6. If 247*l.* 18*s.* 4*d.* Sterling be remitted to *Frankfort:* What is the value in Florins, at 59$\frac{1}{2}$*d.* Anſwer 1000 Florins.
7. A factor has ſold goods, at *Cadiz*, for 1468 pieces of eight, at 4*s.* 6$\frac{1}{2}$*d.* *per* piece: How much Sterling is the ſum? Anſwer 333*l.* 7*s.* 2*d.*
8. Being deſirous to remit to my correſpondent at *London*, the ſum of 2000*l.* 12*s.* 6*d.* *Flemiſh*, to diſpoſe of according to my order (Exchange at 34*s.* 6*d.* *Flemiſh per* pound Sterling:)

Sterling:) How much money Sterling ſhall I be creditor for? Anſwer 1159*l.* 15*s.* $7\frac{3}{4}$.

C H A P. XXXIX.

The Application of the Doctrine of Proportion to Queſtions relating to Loſs *and* Gain, Barter, Factorſhip, *&c.*

657. QUeſtions relating to *Loſs* and *Gain.*

1. A parcel of goods being bought for 60*l.* and ſold for 75*l.* what was the rate of gain *per cent.* Anſwer 25*l.*

For 60 : 75 :: 100 : 125

2. Suppoſe I have goods to the value of 415*l.* 12*s.* 6*d.* that come to a bad market, and know they impair by lying: What will they come to, if I am obliged to ſell them at the loſs of 12 in the 100? Anſwer 365*l.* 15*s.*

For 100—12=88 and 100 : 88 :: 415,625 : 365,75.

3. Bought 18 *C.* wt. of cheeſe at 28*s.* *per.* *C.* wt. which I ſell out again at $3\frac{1}{2}$*d.* *per lb.* What is the profit in the whole? Anſwer 4*l.* 4*s.*

$$\text{For } 1 : \tfrac{28}{20} :: 18 : \frac{18\times 28}{20} = 25\,l.\ 4\,s.$$

$$\text{And } 1 : \tfrac{7}{480} :: 18\times 112 : \frac{7\times 18\times 112}{480} = 29\,l.\ 8\,s.$$

4. Having ſold 10 yards of cloth for 4*l.* 16*s.* and thereby gained at the rate of 10 *per cent.* What was the prime coſt of 1 yard? Anſwer 8*s.* $8\frac{3}{4}$*d.*

For 110 : 100 :: 4,8 : $4{,}3\not{6}$

And 10 : $4{,}3\not{6}$ :: 1 : $0{,}43\not{6}$.

5. Having ſold 1 yard of cloth for 11*s.* 6*d.* I gained, at the rate of 15*l.* *per cent.* but, had I ſold it for 12*s.* what would be the rate of gain *per cent.* Anſwer 20*l.*

$$\text{For } : \overset{l.}{115} : \overset{l.}{100} :: \overset{l.}{\tfrac{28}{40}} : \overset{l.}{\tfrac{20}{40}} = \text{prime coſt of 1 yard.}$$

$$\text{And } (\tfrac{20}{40}=)\ \overset{l.}{\tfrac{1}{2}} : \overset{l.}{\tfrac{12}{20}} :: 100 : 120.$$

6.

6. A merchant has bought linen cloth at 11*s.* *per* ell; which proving worſe than he expected, he is willing to ſell at ſuch a price, that he may loſe preciſely after the rate of $1\frac{2}{3}$*l.* for every 20*l.* that he laid out; the queſtion is to know at what price he ought to ſell the ell, that the proportion in the ſaid loſs may be obſerved? Anſw. 10*s.* 1*d.* *per* ell.

For $20 - 1\frac{2}{3} = 18\frac{1}{3}$

And $20 : 18\frac{1}{3} :: 11 : 10\frac{1}{12}$ Shillings.

7. If 100*lb.* weight of any commodity coſt 30*s.* at what price muſt 1*lb.* weight of that commodity be ſold, to gain after the rate of 10*lb.* for every 100 laid out? Anſwer $3\frac{24}{25}$*d.* *per lb.* weight.

For 100 : 110 :: 30 : 33

And 100 : 33 :: 1 : $\frac{33}{100}$*s.* (or $3\frac{24}{25}$*d.*

8. A merchant ſells a parcel of jewels, which coſt him 250*l.* ready money, for 559*l.* payable at the end of 6 months; the queſtion is (his ſecurity being ſuppoſed to be good) what his gain was worth in ready money, upon rebate of intereſt at the rate of 6*l.* for 100*l.* for a year? Anſw. 300*l.*

For 559—250=309

And 103 : 100 :: 309 : 300.

9. Bought 3 oxen for 24*l.* 10*s.* which I ſell again for 2*s.* *per* ſtone; what ought the three oxen to weigh together, the hides and offal being the only clear again? Anſwer 245 ſtone.

	s.		*ſtone.*		*s.*		*ſtone.*
For	2	:	1	::	490	:	245

10. Suppoſe I buy 28 pieces of ſtuff at 4*l.* *per* piece, and ſell ten of the pieces at 6*l.* and eight at 5*l.* at what rate muſt I ſell the reſt, to gain 10 *per cent.* by the whole? Anſw. 2*l.* 6*s.* 5*d.*

For 28 pieces at 4*l.* are worth 112*l.*

	10 ditto	6*l.* amount to	60
	8 ditto	5*l.* ditto	40
Therefore	18	pieces were ſold for	100*l.*
But	28	pieces muſt ſell for	123*l.* 4*s.*
		For 100 : 110 :: 112 :	123,2
Therefore	10	pieces muſt ſell for	23*l.* 4*s.*
And	1	piece ditto	2*l.* 6*s.* 5*d.*

658. Questions relating to *Barter.*

1. *B* delivered 3 hogsheads of brandy at 6*s.* 8*d. per* gallon, to *C*, for 126 yards of cloth, what was the cloth *per* yard? Answer 10*s.*

For 1 gall. : $\frac{1}{3}$*l.* :: 3×63 gall. : 63*l.*
And 126 yards : 63*l.* :: 1 yard : $\frac{1}{2}$*l.*

2. How much sugar at 8*d. per lb.* weight, may be bought for 20 *C.* of tobacco, at 3*l. per C.* Answer 1800*lb.* weight of sugar.

For 1 : 3 :: 20 : 60
And $\frac{1}{30}$: 1 :: 60 : 1800

3. *C* has candles at 6*s. per* dozen ready money; but in barter he will have 6*s.* 6*d. per* dozen. *D* has cotton at 9*d. per lb.* ready money; what price must the cotton be at in barter; and how much cotton must be bartered for 100 doz. of candles? Answ. the cotton 9$\frac{3}{4}$*d. per lb.* in barter, and 7 *C. wt.* 16*lb.* of cotton must be given for 100 doz. of candles.

For 72*d.* : 78*d.* :: 9*d.* : 9$\frac{3}{4}$*d.*
Also 1 doz. : 6*s.* :: 100 doz. : 30*l.*
And $\frac{3}{80}$*l.* : 1*lb.* :: 30*l.* 800*lb.* = 7*C.* 0*qu.* 16*lb.*

4. Two merchants barter, *A* has 20*C.* wt. of Cheese at 21*s.* 6*d.* the *C.* wt. *B* has 8 pieces of *Irish* cloth, at 3*l.* 14*s. per* piece: Whether of them must receive money, and how much? Answer *A* must pay to *B* 8*l.* 2*s.*

For 8 pieces at 3*l.* 14*s.* are worth 29*l.* 12*s.*
And 20 *C.*wt. at 1*l.* 1*s.* 6*d.* 21*l.* 10*s.*

Difference 8*l.* 2*s.*

5. *A* and *B* barter, *A* has 41 *C.* of hops at 30*s. per C.* wt. for which *B* gives him 20*l.* in money, and the rest in prunes at 5*d. per* pound; how many prunes did *B* give, *A*, beside the 20*l.*? Answer 17 *C.*wt. 3*q.* 4*lb.*

For 41*C.*wt. at 1*l.* 10*s.* are worth 61*l.* 10*s.*
Deduct 20*l.*

Remains 41*l.* 10*s.*

And $\frac{5}{240}$*l.* : 1*lb.* :: $\frac{83}{2}$*l.* : 1992*lb.* = 17 *C.* wt. 3*qu.* 4*lb.*

6. *A* has 100 pieces of silks, which are worth but 3*l. per* piece in ready money, yet he barters them with *B* at 4*l. per* piece, and at that rate takes their value of *B* in wool, at 7*l.* 10*s. per C.* which is worth but 6*l. per C.* in ready money; the question is to know what quantity of wool

wool pays for the silks, and which of the two, *A* or *B*, is the gainer, and how much? Answ. $53\frac{1}{3}$C. of wool pays for the silks, and *A* gains 20*l.* by the barter.

For $7\frac{1}{2} : 1 :: 400 : 53\frac{1}{3}$

And $1 : 6 :: 53\frac{1}{3} : 320$

So it is evident, that the true worth of the wool which *B* delivered was 320*l.* for which he received only of *A* the worth of 300*l.*

7. *A* has linen cloth, worth 20*d.* an ell, ready money; but, in barter he will have 2*s.* *B* has broad cloth worth 14*s.* 6*d.* *per* yard ready money: At what price ought the broad cloth to be rated in barter? Answer 17*s.* 4*d.* $3\frac{1}{5}$*f.*

For $20 : 24 :: 174 : 208\frac{4}{5}$.

659. Questions relating to Factorship.

1. A merchant delivers to his factor 100*l.* allowing him to join to it 30*l.* and values his service worth 40*l.* what share of the gain ought the factor to have? Answer $\frac{7}{17}$ parts.

For $30+40=70$ the factor's stock.

And $170 : 1 :: 70 : \frac{7}{17}$.

2. A merchant's real stock being 100*l.* and the factor's service valued at 20*l.* who received $\frac{1}{2}$ of the gain; what was the factor's real stock? Answer 80*l.*

For $100-20=80$.

3. A merchant delivered to his factor 600*l.* upon condition that if the factor add to it 250*l.* of his own money, and bestow his pains in managing the whole stock, he should then have $\frac{2}{5}$ parts of the total gain. The question is to know what stock the factor's service was estimated at? Answer 150*l.*

The factor's part of the gain being $\frac{2}{5}$, the merchant must necessarily have the remainder, which is $\frac{3}{5}$.

Then $\frac{3}{5} : \frac{2}{5} :: 600 : 400$

And $400 - 250 = 150$.

4. The merchant's real stock being 100*l.* and the factor being allowed $\frac{1}{4}$ of the gain for his service, what real stock must he join to have $\frac{1}{3}$ of the gain? Answer $16\frac{2}{3}$*l.*

For $\frac{3}{4} : 1 :: 100 : 133\frac{1}{3}$ the whole stock.

Of which the factor's service is $33\frac{1}{3}$

But 100*l.* is $\frac{2}{3}$ of 150.

Therefore $50-33\frac{1}{3} = 16\frac{2}{3}$ the factor must advance.

5. A merchant delivers to his factor 320*l.* and permits him to add to it 64*l.* of his own money, to be employed in traffick; and by agreement between them the factor's service is estimated equivalent to a certain stock; which is such, that if the total gain be divided proportionably according to those three stocks, the factor is to receive $\frac{1}{5}$ of the total gain, in consideration of the said imaginary stock (being the value of his service;) the question is to know the full part of the gain belonging to each, and what stock the factor's service was valued at? Answer the merchant $\frac{2}{3}$ of the gain, and the factor $\frac{1}{3}$, whose service is valued at 96*l.* stock.

For $320+64=384$

And $\frac{4}{5} : \frac{1}{5} :: 384 : 96$

$$\text{Again,}\left\{\begin{array}{r}320\\64\\96\\\hline 480\end{array}\right. \quad 480 : 1 :: \left\{\begin{array}{l}320 : \frac{2}{3}\\160 : \frac{1}{3}\end{array}\right.$$

660. It will often happen, that the solution of questions in Fellowship may be facilitated by the use of Fractions, Vulgar or Decimal: For since by *Art.* 159, and 161, the first and second term are the same, in every operation of the Rule of Three, that is necessary to solve the question; it will follow, that if the second term be divided by the first, the Quotient, so obtained, may be used as a common Multiplier to each of the respective third terms: Thus, in the second Example of *Art.* 159.

$\frac{45}{108}=\frac{5}{12}$; and $\frac{5}{12}\times 48=5\times 4=20$

$\frac{5}{12}\times 36=5\times 3=15$

$\frac{5}{12}\times 24=5\times 2=10$

Again, in the following Example, *viz.* A chapman breaking, owes unto four men the following sums of money, *viz.*

To	*A*	21	9	6
	B	72	19	3
	C	114	13	9
	D	264	17	6
Sum		474	00	0

His whole estate is found to be but 148*l.* 2*s.* 6*d.* what must each have of the same, and what will it be *per* pound?

Now 148*l.* 2*s.* 6*d.* $=148{,}125$; and $\frac{148{,}125}{474}={,}3125$.

Then

Then by the method in *Art.* 396.

(21*l.* 9*s.* 6*d.*=) 21,475 ×,3125= 6,7109= 6*l.* 14*s.* $2\frac{1}{2}$*d.*
(72*l.* 19*s.* 3*d.*=) 72,9625 ×,3125=22,8008=22*l.* 16*s.* 0*d.*
(114*l.* 13*s.* 9*d.*=)114,6875 ×,3125=35,8399=35*l.* 16*s.* $9\frac{3}{4}$*d.*
(264*l.* 17*s.* 6*d.*=)264,875 ×,3125=82,7733=82*l.* 15*s.* $5\frac{3}{4}$*d.*

148,1249=148*l.* 2*s.* 6*d.*

661. Queſtions relating to Fellowſhip.

1. Three merchants trading to *Virginia*, loſt goods to the value of 800*l.* Now if *A*'s ſtock was 1200*l.* *B*'s 4800*l.* and *C*'s 2000*l.* what ſum did each man loſe? Anſwer *A* loſt 120*l.* *B* 480*l.* *C* 200*l.*

2. Four men traded with a ſtock of 800*l.* and they gained in two years time twice as much, and 40*l.* over: *A*'s ſtock was 140*l.* *B*'s 260*l.* *C*'s 300*l.* I demand *D*'s ſtock and what each man gained by trading? Anſwer *D*'s ſtock was 100*l.* and *A* gain'd 287*l.* *B* 533*l.* *C* 615*l.* and *D* 205*l.*

3. *A*, *B*, and *C* put in money together. *A* put in 20*l.* *B* and *C* put in 85*l.* they gained 63*l.* of which *B* took up 21*l.* what did *A* and *C* gain, and *B* and *C* put in?

Anſwer, *A* gained 12*l.* and *C* 30*l.*
B put in 35*l.* and *C* 50*l.*

For 105 : 63 :: 20 : 12 . and 63—12—21=30.
Alſo 12 : 20 :: 21 : 35 . and 85—35=50.

4. *A*, *B*, and *C* put in money together; *A* put in 20*l.* *B* 30*l.* *C* a ſum unknown, they gained 36*l.* *C* took up 16*l.* what did *A* and *B* gain, and *C* put in?

Anſwer *A* gained 8*l.* and *B* 12*l.*
C put in 40*l.*

For 36—16=20 the gain of *A* and *B*.

Then (20+30) 50 : 20 :: { 20 : 8*l.* *A*'s gain, 30 : 12*l.* *B*'s ditto.

Laſtly 8 : 20 :: 16 : 40 *C*'s ſtock.

5. *A*, *B*, and *C*, put in money together; *A* put in 20*l.* for 3 months, *B* put in 30*l.* for 5 months, and *C* put in 40*l.* for 7 months; they gained 60*l.* what muſt each have of the gain?

Anſwer $\begin{cases} A, 7\frac{17}{49} \\ B, 18\frac{18}{49} \\ C, 34\frac{14}{49} \end{cases}$

For $20\times3=60$
$30\times5=150$
$40\times7=280$
490

Then $490 : 60 :: \begin{cases} 60 \\ 150 \\ 280 \end{cases} : \begin{cases} 7\frac{17}{49} \\ 18\frac{18}{49} \\ 34\frac{14}{49} \end{cases}$

6. *A* and *B* are in company; *A* put in the firſt of *January* 50*l.* but *B* could not put any money in till the firſt of *May*; what muſt *B* then put in, to have an equal ſhare with *A*, at the year's end? Anſwer 75*l.*

For $50\times12=600$
And $\frac{600}{8}=75.$

CHAP. XL.

Concerning the INTEREST *of* MONEY, *and the Conſtruction of* TABLES *for that purpoſe.*

662. IN reſolving queſtions concerning Intereſt of Money, four things are to be well obſerved; to wit, firſt, the principal or money lent for gain or intereſt; ſecondly, the time for which the ſaid principal is lent; thirdly, the rate or proportion which the principal bears to the ſum of the principal and intereſt; and fourthly, the intereſt itſelf: So if 100*l.* be lent, upon condition that 106*l.* ſhall be repaid at the end of a year, the ſaid 100*l.* is called principal; the time for which the ſaid principal is lent, is one year; the proportion which the principal bears to the ſum of the principal and intereſt is ſuch, as 100 has to 106*l.* Laſtly the intereſt itſelf is 6*l.*

663. Intereſt is either Simple, or Compound.

664. Simple Intereſt is that which ariſes, or is computed from the principal only: So if 100*l.* be lent for two years, the ſimple intereſt thereof, after the rate of 6*l.* for 100*l.* for

one year, will be 12 *l.* *viz.* 6*l.* due at the first year's end, and 6*l.* due at the second year's end.

665. Compound Interest is that which arises from the principal, and also from the interest thereof, and therefore it is called Interest upon Interest: So if 100*l.* be lent and forborn three years, and the compound interest thereof is to be computed, after the rate of 6*l.* for 100*l.* for one year, there will arise, besides the simple interest of the principal for three years, the interest of 6*l.* (due at the first year's end) for two years, and the interest of 6*l.* (due at the second year's end) for one year following.

666. Rebate, or discount of money is, when a sum of money due at any time to come, is satisfied by the payment of so much present money, which if it was put forth at a certain rate of interest for the said time, would become equal to the sum first due: So if 100*l.* be due at the end of two years, and is to be satisfied by the payment of present money upon Rebate, after the rate of 6*l.* *per cent. per ann.* simple interest, there ought to be so much ready money paid, which after the said rate of interest would in two years be augmented to 100*l.* In like manner, if the Rebate or Discount were to be made after any rate of compound interest, so much ready money ought to be paid, as at such rate of compound interest for the time agreed on, would become equal to the sum first due.

667. In the taking of interest, or use-money, for the loan or forbearance of money lent, respect must be had to the rate limited by act of parliament, which hath restrained all persons from taking more than 5*l.* for the interest or use of 100*l.* lent for a year; but what part of 5*l.* may be taken for the interest of 100*l.* lent for half a year, a quarter of a year, a month, or any other part of a year, is not expressed in the Act: In this case, therefore, we must observe custom and daily practice; so we shall find that 2*l.* 10*s.* is usually taken for half a year's interest of 100*l.* and 25*s.* for a quarter of a year, *&c.* by which practice, this following analogy (which is the ground or reason of the common Rules for computing simple interest) seems to be assumed for a safe exposition of the Statute, *viz,* that such proportion as the whole year (supposed to consist of 365 days) has to any propounded space of time more or less than a year, such proportion any interest (not exceeding the rate limited by the Act) for any principal lent for a year, ought to have to the interest of the same principal for the time agreed upon: This analogy being granted, the manner of computing simple interest, for any principal lent and

forborn any time propofed, will be fuch as is expreffed in the two next fections.

668. The intereft or gain of 100*l.* principal money forborn for a year being known, the intereft of any other principal money for the fame time may be found out by one fingle operation of the Rule of Three; for as 100*l.* principal is in proportion to the intereft thereof, fo is any other principal to its intereft: So if it be demanded, what 270*l.* will gain in a year, at the rate of 5*l.* for 100*l.* for one year, the anfwer will be found to be 13*l.* 10*s.* For,

l.		*l.*		*l.*		*l.*	*l.*	*s.*	*d.*
100	:	5	::	270	:	13,5	(or 13 :	10	: 0

669. A fecond Example. What is the intereft of 246*l.* 18*s.* 10*d.* for a year, at the rate of 3*l.* for 100*l.* for a year? Anfwer 7*l.* 8*s.* $1\frac{98}{100}$*d.*

This queftion, and all others wherein the gain of (or allowance for) 100 integers is concerned, may be anfwered as follows:

Firft, Multiply 246*l.* 18*s.* 10*d.* by, 3, after the manner delivered in *Art.* 87, and write down the product which is 740*l.* 16*s.* 6*d.* Then divide the faid product by 100, in this manner, *viz.* divide 740 pounds by 100, which is performed by cutting off, towards the right-hand, the two laft places of

	l.	*s.*	*d.*
	246 :	18 :	10
			3
l. 7	40 :	16 :	06
	20		
s. 8	16		
	12		
d. 1	98		

740, fo the Quotient gives 7 pounds, and there will be a remainder of 40 pounds, which 40 pounds reduce into fhillings, fo there will arife 800*s.* to which adding the 16*s.* which ftand in the place of fhillings, the fum will be 816*s.* thefe are alfo to be divided by 100 (by cutting off two places as before, fo the Quotient will give 8*s.* and there will remain 16*s.* which being reduced to pence, and unto them 6 pence being added (to wit, the 6 pence which ftands in the place of pence) there will arife 198 pence; thefe alfo are to be divided by 100 (by cutting

cutting off two places to the right-hand as before) ſo the Quotient gives 1 penny, and there will remain 98 ; ſo the exact Quotient or anſwer of the queſtion is found to be 7*l.* 8 *s.* 1$\frac{98}{100}$*d.*

More Examples of this Rule are theſe following :

l. *l.* *l.* *s.* *d.*
100 : 6 :: 793 . 12 . 7
6

l. 47 | 61 . 15 . 6
| 20

s. 12 | 35
| 12

d. 4 | 26

l. *l.* *l.* *s.* *d.*
100 : 8 :: 43 . 14 . 3
8

l. 3 | 49 : 14 : 0
| 20

s. 9 | 94
| 12

*d.*11 | 28

670. After the ſame manner may this following queſtion and ſuch like be reſolved, *viz.* when 100 ells of linen-cloth coſts 30*l.* 18*s.* 9*d.* what is the price of 1 ell? Anſw. 6*s.* 2*d.* 1*f.*

Ells. *l.* *s.* *d.* *Ell.*
100 . 30 . 18 9 :: 1
20

Shill. 6 | 18
| 12

Pence. 2 | 25
| 4

Farth. 1 | 00

671. When the given gain of (or allowance for) 100 integers consists of some number of pounds, together with some aliquot part or parts of a pound, the operation will be little different from the last-mentioned Examples, as may appear by the resolution of the subsequent question, *viz.* what must be allowed for 2156*l.* 13*s.* 4*d.* at the rate of 6*l.* 15*s.* for 100*l.* Answer 145*l.* 11*s.* 6*d.* thus found. First multiply the said 2156*l.* 13*s.* 4*d.* by 6 (the number of pounds in the given allowance 6*l.* 15*s.*) after the manner of the last Examples, and subscribe the product which is 12940*l.* under the line; then since 15*s.* are equal to $\frac{1}{2}$*l.* together with $\frac{1}{4}$*l.* take $\frac{1}{2}$ of 2156*l.* 13*s.* 4*d.* which is 1078*l.* 6*s.* 8*d.* likewise $\frac{1}{4}$ of the said 2156*l.* 13*s.* 4*d.* to wit, 539*l.* 3*s.* 4*d.* and having subscribed these Quotients under the product first found, add them all together, and find 14557*l.* 10*s.* 0*d.* the total product, with which proceed as in the former Examples; and so at length the Answer is found to be 145*l.* 11*s.* 6*d.* View diligently the operation.

l.	*l.*		*l.*	*s.*	*d.*
100 :	$6\frac{3}{4}$ ::		2156 .	13 .	4
					$6\frac{3}{4}$
			12940 .	0 .	0
			1078 .	6 .	8
			539 .	3 .	4
		145	57 .	10 .	0
			20		
		11	50		
			12		
		6	00		

672. By this Rule, Brokage, Commission, Provision, Storage, Insurance, &c. are work'd, having no respect to time.

Example 1. Unto what comes the Commission of 359*l.* 17*s.* $11\frac{1}{2}$*d.* at $2\frac{1}{2}$ *per cent.*

$\frac{1}{2}$) 359 : 17 : 11 : $\frac{1}{2}$
2

719 : 15 : 11 : 0
179 : 18 : 11 : $\frac{3}{4}$

8 | 99 : 14 : 10 : $\frac{3}{4}$
| 20

19 | 94
| 12

11 | 38
| 4 Anſwer 8*l.* 19*s.* 11*d.* $\frac{1}{4}$.

1 | 55

Example 2. At 1$\frac{1}{4}$ *per cent.* what comes the Brokage of 198*l.* 11*s.* 7*d.* to?

$\frac{1}{4}$) 198 : 11 : 7
49 : 12 : 10 : $\frac{3}{4}$

2 | 48 : 4 : 5 : $\frac{3}{4}$
| 20

9 | 64
| 12 Anſ. 2*l.* 9*s.* 7$\frac{1}{2}$*d.*

7 | 73
| 4

2 | 95

673. But queſtions of this nature are more conciſely and readily worked by this

Rule. Divide the given ſum by the aliquot part, that the rate of the Commiſſion or Brokage, *&c.* is of 100*l.* As for inſtance, in the laſt Examples.

Example 1. The Commiſſion being 2$\frac{1}{2}$ *per cent.* which is the 40th part 100*l.* Therefore

4 | 0) 35 | 9*l.* 17*s.* 11*d.* $\frac{1}{2}$

Anſwer 8 : 09 : 11 : $\frac{1}{4}$

Example

Example 2. The Brokage being $1\frac{1}{4}$ *per cent.* which is the 80th part of 100 *l.* Therefore

8 | 0) 19 | 8 *l.* 11 *s.* 7 *d.*

Anfw. 2 : 09 : 07 : $\frac{1}{2}$

674. When Time is concerned.

Rule. Multiply the principal by the intereft of 100 *l.* for the time required, and the product (ftriking off two figures on the right-hand) is the anfwer?

Example 1. What is the intereft of 479 *l.* 18 *s.* for two years, at $4\frac{1}{2}$ *per cent. per annum?*

The intereft of 100 *l.* for two years at $4\frac{1}{2}$ *per cent.* is 9 *l.* Therefore

```
      479 : 18
             9
  43 | 19 : 02
     | 20
   3 | 82
     | 12
   9 | 84
     |  4
   3 | 36 Anfwer 43 l. 03 s. 09 3/4 d.
```

Example. What is the intereft of 571 *l.* 15 *s.* for 8 months, at 6 *per cent. per annum?*

The intereft of 100 *l.* for 8 months, at 6 *per cent.* is 4 *l.* Therefore

```
      571 l.  15 s.
               4
  22 | 87 :   00
     | 20
  17 | 40
     | 12
   4 | 80
     |  4
   3 | 20 Anfwer 22 l. 17 s. 04 3/4 d.
```

Note,

Note, If it be for 3 months, at 5 *per cent. per annum*, then you need only add $\frac{1}{4}$ of the principal to itself, working as before directed, *&c.*

675. Since the interest of any principal,

at $\left\{\begin{array}{l}5\,l.\\6\\4\\3\end{array}\right\}$ *per cent.* for $\left\{\begin{array}{l}12\\10\\15\\20\end{array}\right\}$ Months.

is just so many shillings as it has pounds. Therefore,

(1.) What is 379 *l.* 15 *s.* for 12 months, at 5 *per cent?*

(2.) What is 461 *l.* 12 *s.* 6*d.* for 10 months at 6 *per cent?*

(1.)	(2.)
2|0 37|9 : 15	2|0 41|6 : 12 : 6*d.*
Answ. 18 : 19 : 9	Answ. 20 : 16 : $7\frac{1}{2}$

(3.) What is 427 *l.* 13 *s.* 9*d.* for 20 months, at 3 *per cent?*

(4.) What is 927 *l.* 11 *s.* 3*d.* for 15 months, at 4 *per cent?*

(3.)	(4.)
2|0 42|7 : 13 : 9	2|0 92|7 : 11 : 3
Answ. 21 : 7 : $8\frac{1}{2}$	Answ. 46 : 7 : $6\frac{3}{4}$

676. Hence, to find the interest of a given principal, for any other time,

Rule. Take parts, for the time required, out of the time affixed.

Thus if the interest of 379 *l.* 15 *s.* for 12 months, at 5 *per cent.* is so many shillings, then for 6 months, it is the $\frac{1}{2}$ of so many shillings, or so many six-pences; and for 4 months the $\frac{1}{3}$ of it, or so many groats, as the principal has pounds.

Example 1. What is the interest of 279 *l.* 11 *s.* for 4 months, at 5 *per cent?*

2|0) 27|9 : 11

3) 13 : 19 : $6\frac{1}{2}$

Answ. 4 : 13 : 2

Example 2. What is the interest of 197 *l.* 11 *s.* for 2 months, at 6 *per cent?*

2|0) 19|7 : 11

5) 9 : 17 : $6\frac{1}{2}$

Answ. 1 : 19 : 6

Example

Example 3. What is 491*l.* 17 *s.* 10 *d.* for 5 months, at 4 *per cent.?*

2|0) 49|1 : 17 : 10

3) 24 : 11 : $10\frac{3}{4}$

Answ. 8 : 3 : $11\frac{1}{2}$

Example 4. What is 279*l.* 11 *s.* 8 *d.* for $7\frac{1}{2}$ months, at 4 *per cent.?*

2|0) 279 : 11 : 8

2) 13 : 19 : 7

Answ. 6 : 19 : $9\frac{1}{2}$

For what odd money is adjoined to the principal, value every 20 *s.* as 12*d.* and lesser sums in proportion.

677. Interest for days.

To find the exact number of days between any two given times, without any Book or Table.

This is done by carrying in memory that old distich, *viz.*

Thirty days has *September*, *April*, *June*, and *November*,
February has twenty-eight alone; all the rest have thirty-one.

And according to this reckoning, the year contains 365 days; therefore for instance,

To find the number of days between the 7th of *June*, 1701, and the 15th of *September*, 1703, say thus,

From the 7th of *June* 1701, to the 7th of *June* 1702, is 365
2

730 or two years.

From the 7th of *June* 1703, to the end of the same month is —— —— 23

Then in { *July* —— 31; *August* —— 31 }

From which to the 15th of *September*, is —— —— 15

So that from the 7th of *June* 1701, to the 15th of *September* 1703, is —— —— —— } days 830

Note, That 1 day must be added to every 4 years contained in the time. For every fourth year, being Leap-year, *February* has then 29 days.

After this manner may any number of days, even to $\frac{1}{2}$ or $\frac{1}{4}$ of a day, be exactly and readily discovered. Now,

678.

678. If the intereſt of 100 *l.* principal for one whole year, or 365 days be known, the ſimple intereſt of any other principal, for any number of days more or leſs than 365, may be found out by the following Rule, *viz.*

Multiply theſe three numbers, according to the Rule of continual Multiplication, to wit, the given intereſt of 100 *l.* for a year, the principal, whoſe intereſt is required, and the number of days preſcribed, reſerving the laſt product for a Dividend: Alſo multiply 365 by 100, and reſerve this product for a Diviſor: Laſtly, finiſh Diviſion, ſo ſhall the Quotient be the intereſt or gain ſought.

Note here, That the two principals, to wit 100 *l.* and the other propounded, are ſuppoſed to be of one and the ſame denomination: Alſo the intereſt required will be of the ſame denomination with the given intereſt of 100 *l.*

For an Example of this Rule, let it be required to find out the intereſt of 400 *l.* for a week or 7 days, at the rate of 6 *l.* for 100 *l.* for a year, or 365 days: Firſt multiplying theſe three numbers 6, 400, and 7 continually (*viz,* multiplying 6 by 400, and the product thence ariſing by 7) the laſt product will be 16800 for a Dividend; alſo multiplying 365 by 100, the product is 36500 for a Diviſor: Laſtly, dividing 16800 by 36500 (after cyphers at pleaſure are added to 16800) the Quotient (according to *Art.* 401.) will be diſcovered to be this Decimal ,4602, which is equal to 9 *s.* 2 *d.* 1 *f.*

679. The reaſon of the above-mentioned Rule for the computing of intereſt for days, will be manifeſt by this following way of ſolving the ſame queſtion by two ſingle Rules of Three, *viz.*

$$\text{I.} \quad 100 : 6 :: 400 : \frac{6 \times 400}{100}$$

$$\text{II.} \quad 365 : \frac{6 \times 400}{100} :: 7 : \frac{6 \times 400 \times 7}{365 \times 100}$$

680. Or the ſame will appear from the univerſal Rule of proportion, *Chap.* 30 *Art.* 406.

	Princip.	*Inter.*	*Days.*
Conditional Terms	100	6	365
Correſponding	400	A	7

Now it is evident that the principal and time produce the intereſt.

Therefore $A = \frac{400 \times 7 \times 6}{365 \times 100}$ as above.

681.

681. Hence it appears, that the above directed operation may be often contracted by the method in *Art.* 266.

682. But the most concise way of computing the interest is by using Decimal Fractions; For Example, if it be required to find the interest of 2156*l.* 13*s.* 3*d.* for $6\frac{3}{4}$ years, at $4\frac{1}{2}$ *l. per cent.*?

2156*l.* 13*s.* 3*d.* = 2156,6625 and $4\frac{1}{2}$ *l.* = 4,5
5,4 Multiplier inverted.

862665
107833

9704,98 this product divided by
100 will be 97,0498 $6\frac{3}{4}$=6,75
57,6 Multiplier inverted.

5822988
679349
48525

Answ. 655,0862 = 655*l.* 1*s.* $8\frac{3}{4}$*d.*

683. And to prevent the trouble of dividing by 365 in every computation of interest for days, the following Tables have been calculated; the first term of each of which, is the interest of 1*l.* for one day at the given rate; that is, ,000136986, the first number of the Table of 5 *per cent.* is found from $\left(\frac{1\times 1\times 5}{365\times 100}\right)$ the general Rule given in *Art.* 678, 679, and 680. And the other numbers are made by Addition; the second number being the double of the first; the third the sum of the first and second, &c.

TABLES of Simple Interest serving at any rate for any time.

	2 *per cent.*	3 *per cent.*	3½ *per cent.*
1	,0000547945	,0000821918	,0000958904
2	,0001095890	,0001643836	,0001917808
3	,0001643836	,0002465753	,0002876712
4	,0002191781	,0003287671	,0003835616
5	,0002739726	,0004109589	,0004794521
6	,0003287671	,0004931507	,0005753425
7	,0003835616	,0005753425	,0006712329
8	,0004383561	,0006575342	,0007671233
9	,0004931506	,0007397260	,0008630137
10	,0005479452	,0008219178	,0009589041
20	,0010958904	,0016438356	,0019178082
30	,0016438356	,0024657534	,0028767123
40	,0021917808	,0032876712	,0038356164
50	,0027397260	,0041095890	,0047945205
60	,0032876712	,0049315068	,0057534247
70	,0038356164	,0057534246	,0067123288
80	,0043835616	,0065753425	,0076712329
90	,0049315069	,0073972603	,0086301370
100	,0054794521	,0082191781	,0095890411
200	,0109589042	,0164383562	,0191780822
300	,0164383563	,0246575343	,0287671233
	Months.	. . . . .	. . . . .
3	,005	,0075	,00875
6	,01	,015	,0175
9	,015	,0225	,02625
	Years.		
1	,02	,03	,035
2	,04	,06	,070
3	,06	,09	,105
4	,08	,12	,140
5	,10	,15	,175
6	,12	,18	,210
7	,14	,21	,245
8	,16	,24	,280
9	,18	,27	,315
10	,20	,30	,350

	4 *per cent.*	4 ½ *per cent.*	5 *per cent.*
1	,0001095890	,0001232877	,0001369863
2	,0002191780	,0002465753	,0002739726
3	,0003287671	,0003698630	,0004109589
4	,0004383561	,0004931506	,0005479452
5	,0005479452	,0006164383	,0006849315
6	,0006575342	,0007397259	,0008219178
7	,0007671232	,0008630136	,0009589041
8	,0008767123	,0009863012	,0010958904
9	,0009863013	,0011095890	,0012328767
10	,0010958904	,0012328767	,0013698630
20	,0021917808	,0024657534	,0027397260
30	,0032876712	,0036986301	,0041095890
40	,0043835616	,0049315068	,0054794520
50	,0054794520	,0061643835	,0068493150
60	,0065753424	,0073972602	,0082191780
70	,0076712328	,0086301369	,0095890410
80	,0087671232	,0098630136	,0109589040
90	,0098630137	,0110958904	,0123287671
100	,0109589041	,0123287671	,0136986301
200	,0219178082	,0246575342	,0273972602
300	,0328767123	,0369863013	,0410958903
	Months.	. . . . . .	. . . . . .
3	,01	,01125	,0125
6	,02	,0225	,025
9	,03	,03375	,0375
	Years.		
1	,04	045	,05
2	,08	,090	,10
3	,12	,135	,15
4	,16	,180	,20
5	,20	,225	,25
6	,24	,270	,30
7	,28	,315	,35
8	,32	,360	,40
9	,36	,405	,45
10	,40	,450	,50

6 per

	6 *per cent.*	7 *per cent.*	8 *per cent.*
1	,0001643836	,0001917808	,0002191780
2	,0003287671	,0003835616	,0004383561
3	,0004931506	,0005753424	,0006575342
4	,0006575342	,0007671232	,0008767122
5	,0008219178	,0009589041	,0010958904
6	,0009863013	,0011506849	,0013150684
7	,0011506849	,0013424657	,0015342464
8	,0013150684	,0015342465	,0017534244
9	,0014794520	,0017260274	,0019726026
10	,0016438356	,0019178082	,0021917808
20	,0032876712	,0038356164	,0043835616
30	,0049315068	,0057534246	,0065753424
40	,0065753424	,0076712328	,0087671232
50	,0082191781	,0095890411	,0109589041
60	,0098630136	,0115068492	,0131506848
70	,0115068492	,0134246574	,0153424656
80	,0131506848	,0153424656	,0175342464
90	,0147945206	,0172602738	,0197260273
100	,0164383562	,0191780822	,0219178082
200	,0328767124	,0383561644	,0438356166
300	,0493150686	,0575342466	,0657534249
	Months.	. . . . . .	. . . . . .
3	,015	,0175	,02
6	,03	,035	,04
9	,045	,0525	,06
	Years.		
1	,06	,07	,08
2	,12	,14	,16
3	,18	,21	,24
4	,24	,28	,32
5	,30	,35	,40
6	,36	,42	,48
7	,42	,49	,56
8	,48	,56	,64
9	,54	,63	,72
10	,60	,70	,80

Explication.

684. The firft column of every page being time, againft it ftands the intereft of one pound for that time, at the rate *per cent.* fpecified on the top of the column, and at the fame view you have the intereft of 10 *l.* 100 *l.* 1000 *l.* and all in one line. Let the intereft of 500 *l.* for 73 Days at 5 *per cent.* be required.

Then, againft 70 days ftands	,0095890410
Againft 3 days ftands	,0004109589
Therefore the intereft of 1 *l.* for 73 days ———	is ,0099999999 or ,01

Now fince the intereft at 5 *per cent.* for 73 days,

	l.				*l. s. d.*
Of	1	is	.01	or	2 : $\frac{1}{4}$
That of	10	is	.10	or	02 : 0 : 0
And of	100	is	1.00	or	1 : 00 : 0 : 0
Alfo of	111		1.11	or	1 : 02 : 2 : $\frac{1}{4}$

So that one pound after that manner being found to be the intereft of 100 *l.* at 5 *per cent. per annum* for 73 days, 5 times that money, to wit, 5 *l.* muft therefore be the intereft of 500 *l.* Again, if the intereft of 111 *l.* at the fame rate and for the fame time were required; take the total of the above fums for an anfwer, being 1.11, or 1 *l.* 2 *s.* $2\frac{1}{4}$ *d.* So that let the given principal be what number of pounds, fhillings, and pence it will, it may be either taken in parts, as 111 *l.* above, or by multiplying the intereft of one pound by the given principal; fo that no fum, at any of the inferted rates or time, can require more than one Multiplication.

685. Although this is calculated only for ten years, beyond which time fimple intereft feldom happens, yet the intereft from thence for any longer time may as eafily and readily be found, *viz.*

Example. Unto what amounts the intereft of 79 *l.* 15 *s.* for 16 years and 73 days, at 5 *per cent. per annum?*

The intereft of 1 *l.* for	10 years is	,50
	6 years is	,30
	73 days is	,01

Therefore the intereft of 1 *l.* for 16 years 73 days } is ,81 which multiplied by the principal produces the anfwer.

79,75

```
   79,75
     ,81
  -------
    7975
  63800
  -------
  64,5975
```

Answer 64 *l.* 11 *s.* 11$\frac{1}{4}$ *d.*

686. Likewise may the interest of any sum at any other rate *per cent.* whatsoever be readily found, according to the directions in *Art.* 675.

Example 2. What comes the interest of 432 *l.* 10 *s.* for 1 year $\frac{1}{4}$, at 5$\frac{1}{2}$ *l.* *per cent. per annum?*

First take the interest of one pound for the given time at 5 *per cent. per annum*, which is —	,06250
To which adding one tenth —— ——	,00625
gives the interest of one pound at 5$\frac{1}{2}$ *per cent.* —	,06875
Multiply'd by the given principal — ——	432,5
	34375
	13750
	20625
	27500
Answ. 29 *l.* 14 *s.* 08$\frac{1}{4}$ *d.*	29,734375

687. When an annuity payable yearly is in arrear for any number of years, and it is required to know what the same will amount to, simple interest being computed for every particular yearly payment, from the time it became due, until the end of the term of years, the Work will be as in this following Example, *viz.* If an annuity, or yearly rent of 134 *l.* 10 *s.* 6 *d.* be all forborn till the end of 4 years, what will it then amount to, simple interest being allowed at the rate of 6 *per cent. per annum*, for each year's rent, from the time on which it was due, until the end of the said term of 4 years? Answ. 586 *l.* 10 *s.* 6$\frac{92}{100}$ *d.*

It is evident by the question, that at the rate of interest proposed, there must be computed the interest of 134 *l.* 10 *s.* 6 *d.* (due at the third year's end) for one year (to wit, the fourth year,) also the interest of the like sum due at the second year's end, for 2 years (to wit, the 3d and 4th years;) likewise the interest of the said sum due at the first year's end for three years, (to wit, the ſecond, third, and fourth years :) All which intereſt being added to the ſum of the four years rent, the total ſum ſhews what the ſaid annuity will amount to at the end of the ſaid term of 4 years.

Explication.

			l.	*s.*	*d.*
The intereſt of 134 *l.* 10 *s.* 6 *d.* at 6 *per cent. per ann.* for	1	is . .	8	1	5,16
	2	is . .	16	2	10,32
	3	is . .	24	4	3,48
The ſum of the 4 years rent (to wit, 4 times 134 *l.* 10 *s.* 6 *d.*)		is . .	538	2	0
All which added together, give the anſwer of the queſtion, to wit, —			586	10	6,96

688. When it is required to find out how much ready money will ſatisfy a debt due at the end of any ſpace of time to come, by rebating or diſcounting at a given rate of ſimple intereſt, it may be effected by this Rule, *viz.* firſt, find out the intereſt of 100 *l.* at the ſettled rate of intereſt, for the time which the ready money is to be paid before-hand ; then adding the intereſt ſo found to 100 *l.* make always the ſum of that addition the firſt term in an operation of the Rule of Three ; 100 *l.* the ſecond term ; and the debt propoſed to be ſatisfied the third term : laſtly, the fourth proportional found out by the ſaid operation, will be the ready money which ought to be paid in ſatisfaction of the debt propounded.

Example 1. If a debt of 100 *l.* be payable at the end of a year to come, how much ready money will diſcharge that debt, by rebating or diſcounting at the rate of 6 *per cent. per annum* ? Anſw. 94 *l.* 6 *s.* 9 *d.* 2 *far.* very near ; for by the Rule of Three,

$$106 : 100 :: 100 : 94,3396 +$$

That is to ſay, if 106 *l.* (which is compoſed of 100 *l.* principal, and 6 *l.* intereſt) proceeds from 100 *l.* principal forborn for a year, what principal forborn for a year does 100 *l.* (compoſed of principal and intereſt) proceed from ? Anſw. 94,3396 *l.* +, (or 94 *l.* 6 *s.* $9\frac{1}{2}$ *d.* very near) principal money ; therefore 94 *l.* 6 *s.* $9\frac{1}{2}$ *d.* in ready money, is of equal value with 100 *l.* due at the end of a year to come ; for if the ſaid 94 *l.* 6 *s.* $9\frac{1}{2}$ *d.* be put forth at intereſt for a year, at the rate of 6 *per cent. per annum*, it will gain 5 *l.* 13 *s.* $2\frac{1}{2}$ *d.* very near, which together with the ſaid 94 *l.* 6 *s.* $9\frac{1}{2}$ *d.* makes the 100 *l.* firſt propoſed to be diſcharged by rebate.

Example 2. If 150 *l.* 10 *s.* be payable at the end of 37 days to come, how much preſent money will diſcharge the ſaid debt, by rebating after the rate of 6 *per cent. per annum?* Anſw. 148 *l.* 14 *s.* $3\frac{1}{2}$ *d.* +, as by the following operation is manifeſt.

	days.	*l.*	*days.*	*l.*
I.	365 :	6 ::	73 :	1,2

	l.	*l.*	*l.*	*l.*
II.	101,2 :	100 ::	150,5 :	148,7154 +

That is to ſay, firſt, I ſeek the intereſt of 100 *l.* for 73 days, at the rate of intereſt propoſed, by the above proceſs, or by the Table? Anſw. $1,\frac{2}{10}$ *l.* or 1,2 *l.* Then adding the ſaid 1,2 to 100, ſay, if 101,2 *l.* principal and intereſt, payable at the end of 73 days to come, be equivalent to 100 *l.* ready money, what ready money is 150 *l.* 10 *s* (or 150,5.) payable at the end of 73 days to come equivalent unto? So by multiplying and dividing (according to the Rules of Decimal Multiplication and Diviſion explained in *Chap.* 28. and 29.) the Quotient or anſwer of the queſtion will be found 148,7154 + ; that is, 148 *l.* 14 *s.* $3\frac{1}{2}$ *d.* + ; for the Decimal ,7154 being valued according to *Art.* 365. will by inſpection only be diſcovered to be 14 *s.* $3\frac{1}{2}$ *d.* which Rule I ſhall here once for all, adviſe the learner to be well acquainted with.

The Proof.

Seek what the ready money found as aforeſaid will gain, in ſo much time as it is paid beforehand, at the rate of intereſt propounded: Then having added this gain to the ſaid ready money, if the ſum be equal to the debt firſt propoſed to be ſatisfied by rebate, the ready money was rightly found out. So the laſt Example will be thus proved:

l.	*l.*	*l.*	*l.*
100 :	1,2 ::	148,7154 :	(1,7845

Which fourth proportional 1,7845 being added to 148,7154, the ſum will be 150,4999 +, which does not want a farthing of 150 *l.* 10 *s.* the debt firſt propoſed.

689. When it is required to find the preſent worth of an annuity, by rebating or diſcounting at a given rate of ſimple intereſt, the operation will be as in the following Example, *viz.* How much preſent money is equivalent to an annuity or rent of 100 *l.* *per annum*, to continue 5 years, rebate being made at the rate of 6 *l.* for 100 *l.* for one year at ſimple intereſt? Anſw. 425 *l.* 18 *s.* $9\frac{1}{2}$ *d.* very near.

It is manifest that there must be computed the present worth of 100 *l.* due at the first year's end, also the present worth of 100 *l.* due at the second year's end, and in like manner for the third, fourth, and fifth years; all which particular present worths being added together, the aggregate or sum will be the total present worth of the annuity.

The operation by Decimals is as follows:

1.	106 : 100 :: 100 : 94,33962 +
2.	112 : 100 :: 100 : 89,28571 +
3.	118 : 100 :: 100 : 84,74576 +
4.	124 : 100 :: 100 : 80,64516 +
5.	130 : 100 :: 100 : 76,92307 +
	Answ. 425,93932 +

CHAP. XLI.

Of Compound Interest.

690. Compound Interest is derived from the following supposition, *viz.* that the interest of money ought to be punctually paid when due (which is commonly at the end of a year from the time of lending the principal) and that, if it be not so done, then the said interest should be considered as, and added to, the principal from that time; and interest be computed on their sum for the next year or other period of time.

Suppose, for instance, that 100 *l.* were lent for one year, at 5 *per cent.* then, if the 5 *l.* Interest be not paid at the year's end, there will arise a principal of 105 *l.* on which interest must be computed for the second year; which by *Art.* 668. will amount to 5 *l.* 5 *s.* now, if this be not paid at the end of the second year, then, to the second year's principal 105 *l.* add its interest 5 *l.* 5 *s.* and the sum 110 *l.* 5 *s.* will be the principal, on which interest must be computed for the third year; and so on for succeeding years.

691. But since the interest of any sum of money for one year is found by the following proportion. See *Art.* 668.

As

As 100 *l.* : to its interest : : the given principal : to its interest,
Therefore by composition, see *Art.* 553. *Case* 5.

As 100 *l.* : to 100 *l.* + its interest : : the given principal : to to the principal + its interest.

692. Whence the amounts of any given principal (suppose 300 *l.*) for any number of years (suppose 4) at any rate, suppose 5 *l. per cent.* may be found as follows, *viz.*

100 : 105 : :	300	: 315	The amount of 300 *l.* and its interest at the end of	1	Years.
	315	: 330,75		2	
	330,75	: 347,2875		3	
	347,2875	: 364,651875		4	

693. But the principal, and the several amounts, are a rank of numbers in Geometrical Progression, increasing in proportion as 100 to 105, or as 1 to 1,05.

As will appear from their production, or may be proved from the equality of the products of any extreme and mean terms.

694. Since by *Art.* 692.

As 100 *l.* : its amount : : any principal : its amount.
Therefore 100 *l.* : its amount : : 1 *l.* : its amount.
And 1 *l.* : its amount : : any principal : its amount.

695. Now the amounts of 1 *l.* for any number of years at any rate *per cent.* (suppose 5 *l.*) will be the powers of (1,05) the amount of 1 *l.* for one year by *Art.* 693. and 547.

696. Therefore, if the powers of the amount of 1 *l.* for one year be first calculated, the amonnt of any principal may be found by Multiplication only.

Thus, if the powers of 1,05 be computed, they, with their indexes, will be as follow:

0	1	2	3	4 &c.	indexes.
1 .	1,05 .	1,1025 .	1,57625 .	1,21550625 &c.	powers.

And the amount of 300 *l.* in 4 years at 5 *l. per cent.* will be the product of (1,21550625) the 4th power of 1,05, multiplied by the principal 300 *l.* See the operation.

$$\begin{array}{r} 1,21550625 \\ 300 \\ \hline 364,65187500 \end{array}$$

697. In order to facilitate the computations relating to compound interest, the powers of the amounts of 1 *l.* for one year at the several rates therein mentioned, are inserted in the following Table.

TABLE

TABLE I.

Which ſhews what one pound will amount to, being forborn to the end of any number of years under 61, compound intereſt being computed yearly, at any of theſe rates, *viz.* 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 *per cent. per annum.*

Years.	at 2 *per Ct.*	3 *per Ct.*	$3\frac{1}{2}$ *per C.*	4 *per Ct.*	$4\frac{1}{2}$ *per C.*	5 *per Ct.*	6 *per Ct.*	7 *per Ct.*	8 *per Ct.*
1	1,02000	1,03000	1,03500	1,04000	1,04500	1,05000	1,06000	1,07000	1,08000
2	1,04040	1,06090	1,07123	1,08160	1,09203	1,10250	1,12360	1,14490	1,16640
3	1,06121	1,09273	1,10872	1,12486	1,14117	1,15763	1,19102	1,22504	1,25971
4	1,08243	1,12551	1,14752	1,16986	1,19252	1,21551	1,26248	1,31080	1,36049
5	1,10408	1,15927	1,18769	1,21665	1,24618	1,27628	1,33823	1,40255	1,46933
6	1,12616	1,19405	1,22926	1,26532	1,30226	1,34010	1,41852	1,50073	1,58687
7	1,14869	1,22987	1,27228	1,31593	1,36086	1,40710	1,50363	1,60578	1,71382
8	1,17166	1,26677	1,31681	1,36857	1,42210	1,47746	1,59385	1,71819	1,85093
9	1,19509	1,30477	1,36290	1,42331	1,48610	1,55133	1,68948	1,83846	1,99901
10	1,21899	1,34392	1,41060	1,48024	1,55297	1,62890	1,79085	1,96715	2,15893
11	1,24337	1,38423	1,45997	1,53945	1,62285	1,71034	1,89830	2,10485	2,33164
12	1,26824	1,42576	1,51107	1,60103	1,69588	1,79586	2,01220	2,25219	2,51817
13	1,29361	1,46853	1,56396	1,66507	1,77220	1,88565	2,13293	2,40985	2,71962
14	1,31948	1,51259	1,61870	1,73168	1,85195	1,97993	2,26090	2,57853	2,93719
15	1,34587	1,55797	1,67535	1,80094	1,93528	2,07893	2,39656	2,75903	3,17217

TABLE

TABLE I.

Which ſhews what one pound will amount to, being forborn to the end of any number of years under 61, compound intereſt being computed yearly, at any of theſe rates, *viz.* 2, 3, 3½, 4, 4½, 5, 6, 7, and 8 *per cent. per annum.*

Years.	at 2 *per Ct.*	3 *per Ct.*	3½ *per C.*	4 *per Ct.*	4½ *per C.*	5 *per Ct.*	6 *per Ct.*	7 *per Ct.*	8 *per Cent.*
16	1,37279	1,60471	1,73399	1,87298	2,02237	2,18288	2,54035	2,95216	3,42594
17	1,40024	1,65285	1,79468	1,94790	2,11338	2,29202	2,69277	3,15882	3,70002
18	1,42825	1,70243	1,85749	2,02582	2,20848	2,40662	2,85434	3,37993	3,99602
19	1,45681	1,75351	1,92250	2,10685	2,30786	2,52695	3,02560	3,61653	4,31570
20	1,48595	1,80611	1,98979	2,19112	2,41171	2,65330	3,20714	3,86968	4,66096
21	1,51567	1,86030	2,05943	2,27877	2,52024	2,78596	3,39956	4,14056	5,03383
22	1,54598	1,91610	2,13151	2,36992	2,63365	2,92526	3,60354	4,43040	5,43654
23	1,57690	1,97359	2,20611	2,46472	2,75217	3,07152	3,81975	4,74053	5,87146
24	1,60844	2,03279	2,28333	2,56330	2,87601	3,22510	4,04894	5,07237	6,34118
25	1,64061	2,09378	2,36325	2,66584	3,00543	3,38636	4,29187	5,42743	6,84848
26	1,67342	2,15659	2,44596	2,77247	3,14068	3,55567	4 54938	5,80735	7,39635
27	1,70689	2,22129	2,53157	2,88337	3,28201	3,73346	4,82235	6,21387	7,98806
28	1,74102	2,28793	2,62017	2,99870	3,42970	3,92013	5,11169	6,64884	8,62711
29	1,77585	2,35657	2,71188	3,11865	3,58404	4,11614	5,41839	7,11426	9,31728
30	1,81136	2,42726	2,80679	3,24340	3,74532	4,32194	5,74349	7,61226	10,06266

TABLE I.

Which ſhews what one pound will amount to, being forborn to the end of any number of years under 61, compound intereſt being computed yearly, at any of theſe rates, *viz.* 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 *per cent. per annum.*

Years.	at 2 *per Ct.*	3 *per Ct.*	$3\frac{1}{2}$ *perC.*	4 *per Ct.*	$4\frac{1}{2}$ *perCt.*	5 *per Ct.*	6 *perCent.*	7 *perCent.*	8 *perCent.*
31	1,84759	2,50008	2,90503	3,37313	3,91386	4,53804	6,08810	8,14511	10,86767
32	1,88454	2,57508	3,00671	3,50806	4,08998	4,76494	6,45339	8,71527	11,73703
33	1,92223	2,65234	3,11194	3,64838	4,27403	5,00319	6,84059	9,32534	12,67605
34	1,96068	2,73191	3,22086	3,79432	4,46636	5,25335	7,25103	9,97811	13,69013
35	1,99989	2,81386	3,33359	3,94609	4,66735	5,51602	7,68609	10,67658	14,78534
36	2,03989	2,89828	3,45027	4,10393	4,87738	5,79182	8,14725	11,42394	15,96817
37	2,08069	2,98523	3,57103	4,26809	5,09686	6,08141	8,63609	12,22362	17,24563
38	2,12230	3,07478	3,69601	4,43881	5,32622	6,38548	9,15425	13,07927	18,62528
39	2,16475	3,16703	3,82537	4,61637	5,56590	6,70475	9,70351	13,99482	20,11530
40	2,20804	3,26204	3,95926	4,80102	5,81637	7,03999	10,28572	14,97446	21,72452
41	2,25220	3,35990	4,09783	4,99306	6,07810	7,39199	10,90286	16,02267	23,46248
42	2,29724	3,46070	4,24126	5,19278	6,35162	7,76159	11,55703	17,14426	25,33948
43	2,34319	3,56452	4,38970	5,40050	6,63744	8,14967	12,25046	18,34436	27,36664
44	2,39005	3,67145	4,54334	5,61652	6,93612	8,55715	12,98548	19,62846	29,55597
45	2,43785	3,78160	4,70236	5,84118	7,24825	8,98501	13.76461	21,00245	31,92045

TABLE

TABLE I.

Which ſhews what one pound will amount to, being forborn to the end of any number of years under 61, compound intereſt being computed yearly, at any of theſe rates, *viz.* 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 *per cent. per annum.*

Years.	at 2 *per Ct.*	3 *per Ct.*	$3\frac{1}{2}$ *per C.*	4 *per Ct.*	$4\frac{1}{2}$ *per C.*	5 *per Cent.*	6 *per Cent.*	7 *per Cent.*	8 *per Cent.*
46	2,48661	3,89504	4,86694	6,07482	7,57442	9,43426	14,59049	22,47262	34,47409
47	2,53634	4,01190	5,03728	6,31782	7,91527	9,90597	15,46592	24,04570	37,23201
48	2,58707	4,13225	5,21359	6,57053	8,27146	10,40127	16,39387	25,72891	40,21057
49	2,63881	4,25622	5,39607	6,83335	8,64367	10,92133	17,37750	27,52993	43,42742
50	2,69159	4,38391	5,58493	7,10668	9,03264	11,46740	18,42015	29,45703	46,90161
51	2,74542	4,51542	5,78040	7,39095	9,43911	12,04077	19,52536	31,51902	50,65374
52	2,80033	4,65089	5,98271	7,68659	9,86387	12,64281	20,69689	33,72535	54,70604
53	2,85634	4,79041	6,19211	7,99405	10,30774	13,27495	21,93870	36,08612	59,08252
54	2,91346	4,93413	6,40883	8,31381	10,77159	13,93870	23,25502	38,61215	63,80913
55	2,97173	5,08215	6,63314	8,64637	11,25631	14,63563	24,65032	41,31500	68,91386
56	3,03117	5,23461	6,86530	8,99222	11,76284	15,36741	26,12934	44,20705	74,42697
57	3,09179	5,39165	7,10559	9,35191	12,29217	16,13578	27,69710	47,30155	80,38112
58	3,15362	5,55340	7,35428	9,72599	12,84532	16,94257	29,35893	50,61265	86,81161
59	3,21670	5,72000	7,61168	10,11503	13,42336	17,78970	31,12046	54,15554	93,75654
60	3,28103	5,89160	7,87809	10,51963	14,02741	18,67919	32,98769	57,94643	101,25706

The

The Construction of the preceding Table.

698. The numbers 1, 2, 3, 4, *&c.* to 60, in the first column on the left-hand signify years; the numbers 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8, placed at the head of the rest of the columns, denote rates of interest for 100 *l.* lent for a year, and the numbers set in the several columns under those rates of interest, are the powers of the numbers which stand on the first line, which are found by continually multiplying by those numbers. Thus in the column under 5.

The number		Standing against		is the amount of 1 *l.* in		Years at 5 *l.* *per Cent.*	
	1,05		1		1		= 1,05 × 1
	1,1025		2		2		= 1,05 × 1,05
	1,15763		3		3		= 1,05 × 1,1025
	1,21551		4		4		= 1,05 × 1,15763

The calculation of which numbers will be very easy, if performed according to the following method:

To 1,05
Add 525 = 1,05 × ,05 = $\frac{1}{20}$ × 1,05

The sum = 1,1025 = 1,105 × 1,05
Add = 55125 = 1,1025 × ,05 = $\frac{1}{20}$ × 1,1025

The sum = 1,157625 = 1,1025 × ,105
Add 57881 = 1,157, *&c.* × ,05 = $\frac{1}{20}$ × 1,157, *&c.*

The sum = 1,215506 = 1,157, *&c.* × 1,05

699. PROBLEM I.

The Principal, Rate, and Time being given, to find the Amount.

Example. What will 136 *l.* 15 *s.* 6 *d.* be augmented to, being forborn 20 years, interest upon interest being computed at the rate of 6 *per cent. per annum?* Answ. 438 *l.* 13 *s.* $1\frac{1}{2}$ *d.* very near, which is thus found out.

First, looking into the 7th column of the above Table, to wit, that column which has the figure 6 placed at the head of it, right against 20 years is the number 3,20714, which shews that 1 *l.* being continued 20 years at 6 *per cent. per annum*, compound interest, and all forborn until the end of the said term, will be augmented to 3,20714 *l.* (that is, 3 *l.* 4 *s.* 1 *d.* 2 *f.* and somewhat more;) therefore after the 15 *s.* 6 *d.* in the question is reduced to the Decimal ,775, multiply the said tabular number 3,20714, by 136,775 (the principal propounded

pounded in the queſtion), ſo the product will be 438,6565, &c. that is, 438 *l.* 13 *s.* $1\frac{1}{2}$ *d.* for the anſwer of the queſtion. View the operation here following:

136,7750
41702,3
4103250
273550
9574
137
54
438,6565

700. Problem 2.

The Principal, Rate, and Amount being given, to find the Time.

Example. A perſon having lent 136 *l.* 15 *s.* 6 *d.* at 6 *l. per cent.* compound intereſt, received 438 *l.* 13 *s.* $1\frac{1}{2}$ *d.* in full payment; how many years was the money out of his hands?

Divide (438,6565) the amount, by (136,775) the principal and find (3,207, *&c.)* the Quotient, or the neareſt number to it, in the Table, in the column under (6) the rate: So ſhall (20) the number ſtanding in the left-hand column, even therewith, be the time required; exact, if the Quotient be exactly found in the Table, or nearly ſo, if otherwiſe.

701. Problem 3.

The Principal, Time, and Amount being given, to find the Rate.

Example. A perſon having lent 136 *l.* 15 *s.* 6 *d.* received in full payment thereof, at the end of 20 years, 438 *l.* 13 *s.* $1\frac{1}{2}$ *d.* at what rate *per cent.* was compound intereſt computed?

Divide (438,6565) the amount, by (136,775) the principal, and find (3,207, *&c.)* the Quotient, or the neareſt number thereto, in the Table, on a line with the given time (20); ſo ſhall (6) the number at the head of the column be the rate required; exact, if the Quotient be exactly found in the Table, or nearly ſo, if otherwiſe.

After the ſame manner, the numbers belonging to any of the other rates of intereſt mentioned in the firſt Table, are to be uſed.

702.

702. When an annuity payable yearly is in arrear for any number of years, and it is required to know what the ſame will amount to, compound intereſt being computed for each particular payment, from the time it became due, until the end of the term of years, the Work will be as in the following Example, *viz.* ſuppoſe an annuity of 300 *l.* payable at yearly payments be forborn, and all unpaid until the end of four years; the queſtion is, what will then be due, compound intereſt being computed at the rate of 5 *per cent. per annum*, for every yearly payment from the time it becomes due, to the end of the ſaid term of four years? Anſw. 1293 *l.* 0 *s.* 9 *d.*

It is evident by the queſtion, that there muſt be computed what 300 *l.* due at the third year's end will be augmented to in one year, (to wit, the fourth year) at 5 *per cent.* Alſo what 300 *l.* due at the ſecond year's end will be increaſed to in two years, (to wit, the third and fourth years;) likewiſe what 300 *l.* due at the firſt year's end will be augmented to, in the three following years, (to wit, the ſecond, third, and fourth years) all which ſums being added to 300 *l.* (the payment due at the end of the fourth year, which is incapable of any improvement,) the aggregate or ſum will be the total money in arrear at the end of the fourth year, to wit, 1293,0375 *l.* as may appear by the following operation, *viz.*

	l.
The laſt payment of the annuity due at the end of the fourth year is — — —	300
Again the 300 *l.* due at the third year's end, will in one year, after the rate of 5 *per cent.* be augmented to — — — —	315
Alſo 300 *l.* due at the ſecond year's end will in two years, at the rate of 5 *per cent. per annum*, compound intereſt, be increaſed to, (as appears by *Art.* 692.) — — —	330,75
In like manner 300 *l.* due at the firſt year's end, will in three years be augmented to —	347,2875
The ſum due at four years end is —	1293,0375

703. The invention of the numbers before-mentioned being well examined, it will appear, that if an annuity or rent payable at yearly payments be improved to the utmoſt at intereſt upon intereſt, and all forborn or reſpited to the end of certain years, the total then due will be the ſum of a Geometrical Progreſſion conſiſting of as many terms as there are yearly payments, the firſt of which proportionals is the annuity or rent:

And the second proportional proceeds from the first in the same rate as 105 proceeds from 100, if the rate of interest be 5 *per cent.* (or as 108 proceeds from 100, if the rate of interest be 8 *per cent.* &c.) and so likewise the third from the second, the fourth from the third, *&c.* after themanner of theoperation in *Art.* 692.)

704. *The Construction of the following Table* II.

Upon the aforesaid grounds, the following Table II. is calculated, to shew what one pound Annuity, payable at yearly payments, and forborn any number of years under 61 will amount to, by computing interest upon interest, at any of the rates expressed at the head of the said Table.

And the same Table may be easily composed, by the addition of the numbers in the preceding Table I. in this manner:

To	1	standing under 5 in the latter Table,
Add	1,05	the first number in the former Table,
Makes	2,05	the second number in the latter Table,
Add	1,1025	the second number in the former Table,
Makes	3,1525	the third number in the latter Table,
Add	1,15763	the third number in the former Table,
Makes	4,31013	the fourth number in the latter Table,
	&c.	

The numbers in the preceeding Table I. ought to be continued to more places than are there expressed; to prevent the errors, which may happen by the adding of defective Decimal Fractions.

TABLE II.

Which ſhews what one pound annuity, payable by yearly payments, and forborn any number of years under 61, will amount to at the end of the term, compound intereſt being computed at any of theſe rates, to wit, 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 *per cent. per annum.*

Years.	2 *per Cent.*	3 *per Cent.*	$3\frac{1}{2}$ *per Ct.*	4 *per Cent.*	$4\frac{1}{2}$ *per Ct.*	5 *per Cent.*	6 *per Cent.*	7 *per Cent.*	8 *per Cent.*
1	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000
2	2,02000	2,03000	2,03500	2,04000	2,04500	2,05000	2,06000	2,07000	2,08000
3	3,06040	3,09090	3,10623	3,12160	3,13703	3,15250	3,18360	3,21490	3,24640
4	4,12161	4,18363	4,21494	4,24646	4,27819	4,31013	4,37462	4,43994	4,50611
5	5,20404	5,30914	5,36247	5,41632	5,47071	5,52563	5,63709	5,75074	5,86660
6	6,30812	6,46841	6,55015	6,63298	6,71689	6,80191	6,97532	7,15329	7,33593
7	7,43428	7,66246	7,77941	7,89829	8,01915	8,14201	8,39384	8,65402	8,92280
8	8,58297	8,89234	9,05169	9,21423	9,38001	9.54911	9,89747	10,25980	10,63663
9	9,75463	10,15911	10,36850	10,58280	10,80211	11,02656	11,49132	11,97799	12,48756
10	10,94972	11,46388	11,73139	12,00611	12,28821	12,57789	13,18080	13,81645	14,48656
11	12,16872	12,80780	13,14199	13,48635	13,84118	14.20679	14,97164	15,78360	16,64549
12	13,41209	14,19203	14,60196	15,02581	15,46403	15,91713	16,86994	17,88845	18,97713
13	14,68033	15,61779	16,11303	16,62684	17,15991	17,71298	18,88214	20,14064	21,49530
14	15,97394	17,08632	17,67699	18,29191	18,93211	19,59863	21,01507	22,55049	24,21492
15	17,29342	18,59891	19,29568	20,02359	20,78405	21,57856	23,27597	25,12902	27,15211

TABLE

TABLE II.

Which ſhews what one pound annuity, payable by yearly payments, and forborn any number of years under 61, will amount to at the end of the term, compound intereſt being computed at any of theſe rates, to wit, 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 *per cent. per annum.*

Years.	2 *per Cent.*	3 *per Cent.*	$3\frac{1}{2}$ *per Ct.*	4 *per Cent.*	$4\frac{1}{2}$ *per Ct.*	5 *per Cent.*	6 *per Cent.*	7 *per Cent.*	8 *per Cent.*
16	18,63929	20,15688	20,97103	21,82453	22,71934	23,65749	25,67253	27,88805	30,32428
17	20,01207	21,76159	22,70502	23,69751	24,74171	25,84037	28,21288	30,84022	33,75023
18	21,41231	23,41444	24,49969	25,64541	26,85508	28,13239	30,90565	33,99903	37,45024
19	22,84056	25,11687	26,35718	27,67123	29,06356	30,53900	33,75999	37,37897	41,44626
20	24,29737	26,87037	28,27968	29,77808	31,37142	33,06595	36,78559	40,99549	45,76196
21	25,78332	28,67649	30,26947	31,96920	33,78314	35,71925	39,99273	44,86518	50,42292
22	27,29898	30,53678	32,32890	34,24797	36,30338	38,50521	43,39229	49,00574	55,45676
23	28,84496	32,45288	34,46041	36,61789	38,93703	41,43048	46,99583	53,43614	60,89330
24	30,42186	34,42647	36,66653	39,08260	41,68920	44,50200	50,81558	58,17667	66,76476
25	32,03030	36,45926	38,94986	41,64591	44,56521	47,72710	54,86451	63,24904	73,10594
26	33,67091	38,55304	41,31310	44,31175	47,57065	51,11345	59,15638	68,67647	79,95442
27	35,34432	40,70963	43,75906	47,08421	50,71132	54,66913	63,70577	74,48382	87,35077
28	37,05121	43,93092	46,29063	49,96758	53,99333	58,40258	68,52811	80,69769	95,33883
29	38,79224	45,21885	48,91080	52,96629	57,42303	62,32271	73,63980	87,34653	103,96594
30	40,56808	47,57542	51,62268	56,08494	61,00707	66,43885	79,05819	94,46079	113,28321

TABLE II.

Which ſhews what one pound annuity, payable by yearly payments, and forborn any number of years under 61, will amount to at the end of the term, compound intereſt being computed at any of theſe rates, to wit, 2, 3, 3½, 4, 4½, 5, 6, 7, and 8 *per cent. per annum.*

Years.	2 *per Cent.*	3 *per Cent.*	3½ *per Ct.*	4 *per Cent.*	4½ *per Ct.*	5 *per Cent.*	6 *per Cent.*	7 *per Cent.*	8 *per Cent.*
31	42,37944	50,00268	54,42947	59,32834	64,75239	70,76079	84,80168	102,07304	123,34587
32	44,22703	52,50276	57,33450	62,70147	68,66625	75,29883	90,88978	110,21815	134,21354
33	46,11157	55,07784	60,34121	66,20953	72,75623	80,06377	97,34317	118,93343	145,95062
34	48,03580	57,73018	63,45315	69,85791	77,03026	85,06696	104,18376	128,25877	158,62667
35	49,99448	60,46208	66,67401	73,65223	81,49662	90,32031	111,43478	138,23688	172,31680
36	51,99437	63,27594	70,00760	77,59831	86,16397	95,83632	119,12087	148,91346	187,10215
37	54,03426	66,17422	73,45787	81,70225	91,04134	101,62814	127,26812	160,33740	203,07032
38	56,11494	69,15945	77,02890	85,97034	96,13821	107,70955	135,90421	172,56102	220,31595
39	58,23724	72,23423	80,72491	90,40915	101,46442	114,09502	145,05846	185,64029	238,94122
40	60,40198	75,40126	84,55028	95,02552	107,03032	120,79977	154,76197	199,63511	259,05652
41	62,61002	78,66330	88,50954	99,82654	112,84669	127,83976	165,04768	214,60957	280,78104
42	64,86222	82,02320	92,60737	104,81960	118,92479	135,23175	175,95055	230,63224	304,24352
43	67,15947	85,48389	96,84863	110,01238	125,27640	142,99334	187,50758	247,77650	329,58301
44	69,50266	89,04841	101,23833	115,41288	131,91384	151,14301	199,75803	266,12085	356,94965
45	71,89271	92,71986	105,78167	121,02939	138,84997	159,70016	212,74351	285,74931	386,50562

TABLE II.

Which ſhews what one pound annuity, payable by yearly payments, and forborn any number of years under 61, will amount to at the end of the term, compound intereſt being computed at any of theſe rates, to wit, 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 *per cent. per annum.*

Years.	2 *per Cent.*	3 *per Cent.*	$3\frac{1}{2}$ *per Ct.*	4 *per Cent.*	$4\frac{1}{2}$ *per Ct.*	5 *per Cent.*	6 *per Cent.*	7 *per Cent.*	8 *per Cent.*
46	74,3306	96,5015	110,4840	126,8706	146,0982	168,6852	226,5081	306,7518	418,4261
47	76,8172	100,3965	115,3510	132,9454	153,6726	178,1194	241,0986	329,2244	452,9002
48	79,3535	104,4084	120,3883	139,2632	161,5879	188,0254	256,5645	353,2701	490,1322
49	81,9406	108,5406	125,6018	145,8337	169,8594	198,4267	272,9584	378,9990	530,3427
50	84,5794	112,7969	130,9979	152,6671	178,5030	209,3480	290,3359	406,5289	573,7702
51	87,2710	117,1808	136,5828	159,7738	187,5357	220,8154	308,7561	435,9860	620,6718
52	90,0164	121,6962	142,3632	167,1647	196,9748	232,8562	328,2814	467,5050	671,3255
53	92,8167	126,3471	148,3460	174,8513	216,8386	245,4990	349,9783	501,2303	726,0316
54	95,6731	131,1375	154,5381	182,8454	217,1464	258,7739	370,9170	537,3164	785,114
55	98,5865	136,0716	160,9469	191,1592	227,9180	272,7126	394,1720	575,9286	848,9232
56	101,5583	141,1538	167,5800	199,8055	239,1743	287,3482	418,8223	617,2436	917,8371
57	104,5894	146,3884	174,4453	208,7978	250,9371	302,7157	444,9517	661,4506	992,2640
58	107,6812	151,7800	181,5509	218,1497	263,2293	318,8514	472,6488	708,7522	1072,6451
59	110,8348	157,3334	188,9052	227,8757	276,0746	335,7940	502,0077	759,3648	1159,4568
60	114,0515	163,0534	196,5169	237,9907	289,4980	353,5837	533,1282	813,5204	1253,2133

705. *The Use of the preceeding Table* II.

PROBLEM 4. *The Annuity, Time and Rate being given, to find the Amount.*

Example. If an annuity of 20 *l.* payable by yearly payments for fifteen years, be all forborn or unpaid until the end of the ſaid term, what will it then amount to, upon a compution of intereſt upon intereſt, at the rate of 6 *per cent. per annum?*

In the column which has (6) the rate at the head of it, and on a line with (15) the time, you will find (23,27597) the amount of an annuity of 1 *l.* for the given time at the given rate; multiply that amount (23,27597) by the annuity (20) and the product (465,51940) or 465 *l.* 10 *s.* and $4\frac{1}{2}$*d.* will be the anſwer. View the following operation.

$$\begin{array}{r} 23{,}27597 \\ 20 \\ \hline 465{,}51940 \end{array}$$

706. PROBLEM 5. *The Annuity, Amount and Time being given, to find the Rate.*

Example. A perſon who had an annuity of 20 *l.* ſuffered it to be in arrear for 15 years, and at the end of that time received 465 *l.* 10 *s.* $4\frac{1}{2}$*d.* in full payment; at what rate was compound intereſt allowed him?

Divide (465,519, *&c.*) the amount, by (20) the annuity, and find (23,275, *&c.*) the Quotient or the neareſt number to it, on a line with (15) the time; ſo ſhall (6) the number at the head of the column be the rate required; exact, if the Quotient be exactly found in the Table, or nearly ſo, if otherwiſe.

707. PROBLEM 6. *The Annuity, Amount and Rate being given to find the Time.*

Example. In what time will an annuity of 20 *l.* amount to 465 *l.* 10 *s.* $4\frac{1}{2}$*d.* allowing compound intereſt at 6 *l. per cent.*

Divide (465,519, *&c.*) the amount, by (20) the annuity, and find (23,275, *&c.*) the Quotient, or the number neareſt to it, in the column under (6) the rate; ſo ſhall (15) the number ſtanding in the left-hand column, even therewith, be the time required; exact, if the Quotient be exactly found in the Table, or nearly ſo, if otherwiſe.

708. PROBLEM 7. *The Amount, Time and Rate being given, to find the Annuity.*

Example. A perſon received 465 *l.* 10 *s.* 4 $\frac{1}{2}$*d.* in full payment of an annuity, which had been 15 years unpaid, and was allowed 6 *l. per cent.* compound intereſt: What was the annuity?

In the column which has (6) the rate at the head of it, and on a line with (15) the time, you will find (23,27597) the amount of an annuity of 1 *l.* for the given time at the given rate; by which divide (465,519, *&c.*) the given amount, and the Quotient (20) will be the annuity required.

709. When a ſum of money is due at a time to come, and it is required to know what it is worth in ready money, rebate being made at a given rate of compound intereſt, the Work will be the reverſe of *Art.* 692. *viz.* there muſt be found a Geometrical Progreſſion whoſe firſt term is the money propoſed to be rebated, the ſecond muſt decreaſe or leſſen from the firſt, the third from the ſecond, *&c.* in ſuch manner or rate as 100 decreaſes from 105, (or as 100 from 108, if the rate of intereſt be 8 *per cent.*) then if the progreſſion be continued to a number of terms, more by one than the number of years in queſtion, the laſt term will be the required anſwer: So if 364,651875 *l.* be due at the end of four years to come, it will be found to be worth in ready money 300 *l.* rebate being made at compound intereſt at 5 *per cent.* as by the four following proportions is manifeſt. See *Art.* 692.

$$105 : 100 :: \begin{cases} 364{,}651875 : 347{,}2875 \\ 347{,}2875 : 330{,}75 \\ 330{,}75 : 315 \\ 315 : 300 \end{cases}$$

710. Upon this ground the following Table III. is calculated, to ſhew what one pound due at the end of any number of years to come, is worth in preſent money, rebate being made, at the rates of compound intereſt, mentioned in the ſaid Table; by the help of which, and of Multiplication, Queſtions of Rebate for any ſum propoſed may be performed without conſiderable error.

TABLE III.

Which ſhews what one pound, payable at the end of any number of years to come under 61, is worth in ready money; diſcount or rebate being yearly computed at any of theſe rates, to wit, 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 *per cent. per annum*, compound intereſt.

Years.	2 *per Ct.*	3 *per Ct.*	$3\frac{1}{2}$*per Ct.*	4 *per Ct.*	$4\frac{1}{2}$*per Ct.*	5 *per Ct.*	6 *per Ct.*	7 *per Ct.*	8 *per Ct.*
1	,980392	,970874	,966184	,961538	,956938	,952381	,934579	,934579	,925926
2	,961170	,942596	,933511	,924556	,915730	,907029	,889996	,873439	,857339
3	,942322	,915142	,901943	,888996	,876297	,863838	,839619	,816298	,793832
4	,923845	,888487	,871442	,854804	,838561	,822702	,792094	,762895	,735030
5	,905732	,862609	,841973	,821927	,802451	,783526	,747258	,712986	,680583
6	,887971	,837484	,813501	,790315	,767896	,746215	,704961	,666342	,630170
7	,870560	,813092	,785991	,759918	,734828	,710681	,665057	,622750	,583490
8	,853490	,789409	,759412	,730690	,703185	,676839	,627412	,582009	,540269
9	,836755	,766417	,733731	,702587	,672904	,644609	,591898	,543934	,500249
10	,820348	,744094	,708919	,675564	,643928	,613913	,558395	,508349	,463193
11	,804263	,722421	,684946	,649581	,616199	,584679	,526788	,475093	,428883
12	,788493	,701380	,661783	,624597	,589664	,556837	,496969	,444012	,397114
13	,773033	,680951	,639404	,600574	,564272	,530321	,468839	,414965	,367698
14	,757875	,661118	,617782	,577475	,539973	,505068	,442301	,387817	,340461
15	,743015	,641862	,596891	,555265	,516720	,481017	,417265	,362446	,315242

TABLE III.

Which ſhews what one pound, payable at the end of any term of years to come under 61, is worth in ready money; diſcount or rebate being yearly computed at any of theſe rates, to wit, 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 *per cent. per annum* compound intereſt.

Years.	2 *per Ct.*	3 *per Ct.*	$3\frac{1}{2}$ *per Ct.*	4 *per Ct.*	$4\frac{1}{2}$ *per Ct.*	5 *per Ct.*	6 *per Ct.*	7 *per Ct.*	8 *per Ct.*
16	,728446	,623167	,576706	,533908	,494469	,458112	,393646	,338755	,291890
17	,714163	,605016	,557204	,513373	,473176	,436297	,371364	,316574	,270269
18	,700159	,587395	,538361	,493628	,452800	,415521	,350344	,295864	,250249
19	,686431	,570286	,520156	,474642	,433302	,395734	,330513	,276508	,231712
20	,672971	,553676	,502566	,456387	,414643	,376889	,311805	,258419	,214548
21	,659776	,537549	,485571	,438834	,396787	,358942	,294155	,241513	,198656
22	,646839	,521893	,469151	,421955	,379701	,341850	,277505	,225713	,183941
23	,634156	,506692	,453286	,405726	,363350	,325571	,261797	,210947	,170315
24	,621721	,491934	,437957	,390121	,347703	,310068	,248979	,197147	,157699
25	,609531	,477606	,423147	,375117	,332731	,295303	,232999	,184249	,146018
26	,597579	,463695	,408838	,360689	,318402	,281241	,219810	,172195	,135202
27	,585862	,450189	,395012	,346817	,304691	,267848	,207368	,160930	,125187
28	,574375	,437077	,381654	,333447	,291571	,255094	,195630	,150402	,115914
29	,563112	,424346	,368748	,320651	,279015	,242946	,184557	,140563	,107328
30	,552071	,411987	,356278	,308319	,267000	,231377	,174116	,131367	,099377

TABLE

TABLE III.

Which ſhews what one pound, payable at the end of any term of years to come under 61, is worth in ready money; diſcount or rebate being yearly computed at any of theſe rates, to wit, 2, 3, 3½, 4, 4½, 5, 6, 7, and 8 *per cent. per annum*, compound intereſt.

Years.	2 *per Ct.*	3 *per Ct.*	3½ *per Ct.*	4 *per Ct.*	4½ *per Ct.*	5 *per Ct.*	6 *per Ct.*	7 *per Ct.*	8 *per Ct.*
31	,541246	,399987	,344230	,296460	,255502	,220359	,164255	,122773	,092016
32	,530633	,388337	,332590	,285058	,244501	,209866	,154957	,114741	,085200
33	,520229	,377026	,321343	,274094	,233971	,199873	,146186	,107235	,078889
34	,510028	,366045	,310470	,265552	,223896	,190355	,137912	,100219	,073045
35	,500028	,355383	,299977	,253415	,214254	,181290	,130105	,093663	,067635
36	,490223	,345032	,289833	,243669	,205028	,172657	,122741	,087535	,062625
37	,480611	,334983	,280032	,234297	,196199	,164436	,115793	,081809	,057986
38	,471187	,325226	,270562	,225285	,187750	,156605	,109239	,076457	,053690
39	,461948	,315754	,261413	,216621	,179665	,149148	,103056	,071455	,049713
40	,452890	,306557	,252572	,208289	,171929	,142046	,097222	,066780	,046031
41	,444010	,297628	,244031	,200278	,164525	,135282	,091719	,062412	,042621
42	,435304	,288959	,235779	,192575	,157440	,128840	,086527	,058329	,039464
43	,426769	,280543	,227806	,185168	,150661	,122704	,081630	,054513	,036541
44	,418401	,272372	,220102	,178046	,144173	,116861	,077009	,050946	,033834
45	,410197	,264439	,212659	,171198	,137964	,111297	,072650	,047613	,031328

TABLE

TABLE III.

Which ſhews what one pound, payable at the end of any term of years to come under 61, is worth in ready money; diſcount or rebate being yearly computed at any of theſe rates, to wit, 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 *per cent. per annum* compound intereſt.

Years.	2 *per Ct.*	3 *per Ct.*	$3\frac{1}{2}$*perCt.*	4 *per Ct.*	$4\frac{1}{2}$*perCt.*	5 *per Ct.*	6 *per Ct.*	7 *per Ct.*	8 *per Ct.*
46	,402154	,256737	,205468	,164614	,132023	,105997	,068538	,044498	,029007
47	,394268	,249259	,198520	,158283	,126338	,100949	,064658	,041587	,026859
48	,386538	,241999	,191806	,152195	,120898	,096142	,060998	,038067	,024869
49	,378958	,234950	,185302	,146341	,115692	,091564	,057546	,036324	,023027
50	,371528	,228107	,179053	,140713	,110710	,087204	,054288	,033947	,021321
51	,364243	,221463	,172998	,135301	,105942	,083051	,051215	,031727	,019742
52	,357101	,215013	,167148	,130097	,101380	,079096	,048316	,029651	,018280
53	,350099	,208750	,161496	,125093	,097041	,075330	,045582	,027711	,016925
54	,343234	,202670	,156035	,120282	,092837	,071743	,043001	,025899	,015672
55	,336504	,196767	,150758	,115656	,088839	,068326	,040567	,024204	,014511
56	,329906	,191036	,145660	,111208	,085013	,065073	,038271	,022621	,013436
57	,323437	,185472	,140734	,106930	,081353	,061974	,036105	,021141	,012441
58	,317095	,180070	,135975	,102817	,077849	,059023	,034061	,019758	,011519
59	,310878	,174825	,131377	,098863	,074497	,056212	,032133	,018465	,010666
60	,304782	,169733	,126034	,095060	,071289	,053536	,030314	,017257	,009876

711. *The Construction of the preceeding Table.*

The numbers 1, 2, 3, 4, &c. to 60, (in the first column on the left-hand) signify years; the numbers 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8, placed at the head of the rest of the columns, express rates of interest for 100 *l.* lent for a year, and the numbers set in the several columns under those rates of interest are a rank of the negative powers of the same numbers, of which the numbers in Table I. were the affirmative powers; and therefore from *Art.* 419. it is evident that the numbers in this Table are the Reciprocals of those in Table I.

712. Let it be proposed to calculate the numbers in the column of 5 *per cent.* Divide unity by 18,67919 the last number in the column of 5 *per cent.* in Table I. and the Quotient (0,053536) will be the last number of that column in this Table; or the present worth of 1 *l.* due at the end of 60 years: The remaining numbers may be found by multiplying continually by 1,05 as in Table I.

The number	standing against	is the present worth of 1 *l.* due at the end of	years at 5 *per cent.*
,053536	60	60	$=\frac{1}{18,67919}$
,056212	59	59	= ,053536 × 1,05
,059023	58	58	= ,056212 × 1,05
,061974	57	57	= ,059023 × 1,05

The

The following is the proceſs neceſſary to compute the three laſt numbers.

To	,053536	=	$\frac{1}{18,67919}$
Add	2676	=	,053536 × 0,05 = $\frac{1}{20}$ × ,053536
Sum	,056212	=	,053536 × 1,05
Add	2811	=	,056212 × 0,05 = $\frac{1}{20}$ × ,056212
Sum	,059023	=	,056212 × 1,05
Add	2951	=	,059023 × 0,05 = $\frac{1}{20}$ × ,059023
	,061974	=	,059023 × 1,05.

713. *The Uſe of the preceeding Table* III.

PROBLEM 8. *The Amount, Time, and Rate being given, to find the preſent Worth.*

Example. How much preſent money will diſcharge a debt of 356 *l.* payable at the end of 7 years to come, and allowing 7 *l. per cent.* compound intereſt?

In the column under (7) the rate, and on a line with (7) the time, you will find (,622750) the preſent worth of 1 *l.* due at the end of the given time at the given rate; multiply that preſent worth (,622750) by the given amount (356) and the product (221,699 or 221 *l.* 14 *s.*) will be the preſent worth required.

714. The method of finding the preſent worth of an annuity, may be explained by the following Example.

Let it be required to find the preſent worth of an annuity of 347,2875 *l.* to continue 3 years, allowing compound intereſt at 5 *l. per cent.*

Now the preſent worth of 347,2875 *l.* due at the end of 1 year will, by *Prob.* 8. be —	330,75
And the preſent worth of 347,2875 due at the end of 2 years, will be — — —	315,
Alſo the preſent worth of 347,2875 due at the end of 3 years, will be — — —	300,
Therefore the preſent worth of an annuity of 347,2875 to continue three years will be their ſum — — — — —	945,75

715. Hence it will be very eafy to perceive that the following Table IV. of the prefent worths of annuities of 1 *l.* may be compofed of the fucceffive fums of the numbers in Table III. in a manner fimilar to that of compofing Table II. from Table I. *viz.* the firft number in every of the columns (except the column of years) in the following Table IV. is the fame with the firft number in the like columns refpectively in the preceeding Table III. the fecond number in each of the faid columns of the fourth Table, is the fum of the firft and fecond numbers in the refpective columns of the third Table; the third number in the faid columns of the fourth Table, is the fum of the firft, fecond, and third numbers in the refpective columns of the third Table: Or, yet more eafily thus, the third number in the fourth Table, is compofed of the third in the third Table, and of the fecond in the fourth; the fourth number in the fourth Table is compofed of the fourth in the third, and of the third in the fourth; the like is to be underftood of the reft. But you are to obferve, that according to this method of compofing the fourth Table by Addition, the numbers of the third Table muft be continued to more places than are there expreffed, to prevent errors arifing by the addition of defective Decimal Fractions.

TABLE IV.

Which ſhews the preſent worth of one pound annuity, to continue any term of years under 61, and payable by yearly payments, compound intereſt being computed at any of theſe rates, *viz.* 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 *per cent. per annum.*

Years.	2 *per Cent.*	3 *per Cent.*	$3\frac{1}{2}$ *per Ct.*	4 *per Cent.*	$4\frac{1}{2}$ *per Ct.*	5 *per Cent.*	6 *per Cent.*	7 *per Cent.*	8 *per Cent.*
1	0,98039	,97087	,96618	,96154	,95694	,95238	,94340	,93458	,92593
2	1,94156	1,91347	1,89969	1,88610	1,87267	1,85941	1,83339	1,80802	1,78327
3	2,88388	2,82861	2,80164	2,77509	2,74896	2,72325	2,67301	2,62432	2,57710
4	3,80773	3,71710	3,67308	3,62990	3,58753	3,54595	3,46511	3,38721	3,31213
5	4,71346	4,57971	4,51505	4,45182	4,38998	4,32948	4,21236	4,10020	3,99271
6	5,60143	5,41719	5,32855	5,24214	5,15787	5,07569	4,91732	4,76654	4,62288
7	6,47199	6,23028	6,11454	6,00206	5,89270	5,78637	5,58238	5,38929	5,20637
8	7,32548	7,01969	6,87396	6,73275	6,59589	6,46321	6,20979	5,97130	5,74664
9	8,16224	7,78611	7,60769	7,43533	7,26879	7,10782	6,80169	6,51523	6,24689
10	8,98259	8,53020	8,31661	8,11090	7,91272	7,72174	7,36009	7,02358	6,71008
11	9,78685	9,25262	9,00155	8,76048	8,52892	8,30641	7,88688	7,49867	7,13896
12	10,57534	9,95400	9,66333	9,38507	9,11858	8,86325	8,38384	7,94269	7,53608
13	11,34837	10,63496	10,30274	9,98565	9,68285	9,39357	8,85268	8,35765	7,90378
14	12,10625	11,29607	10,92052	10,56312	10,22283	9,89864	9,29498	8,74547	8,24424
15	12,84926	11,93794	11,51741	11,11839	10,73955	10,37966	9,71225	9,10791	8,55948

TABLE IV.

Which ſhews the preſent worth of one pound annuity, to continue any term of years under 61, and payable by yearly payments, compound intereſt being computed at any of theſe rates, *viz.* 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 *per cent. per annum.*

Years.	2 *perCent.*	3 *perCent.*	$3\frac{1}{2}$ *per Ct.*	4 *perCent.*	$4\frac{1}{2}$ *per Ct.*	5 *perCent.*	6 *perCent.*	7 *perCent.*	8 *perCent.*
16	13,57771	12,56110	12,09412	11,65230	11,23402	10,83777	10,10590	9,44665	8,85137
17	14,29187	13,16612	12,65132	12,16567	11,70719	11,27407	10,47726	9,76322	9,13164
18	14,99203	13,75351	13,18968	12,65930	12,15999	11,68958	10,82760	10,05909	9,37189
19	15,67846	14,32380	13,70984	13,13394	12,59329	12,08532	11,15812	10,33560	9,60360
20	16,35143	14,87748	14,21240	13,59033	13,00794	12,46221	11,46992	10,59401	9,81815
21	17,01121	15,41502	14,69797	14,02916	13,40472	12,82115	11,76408	10,83553	10,01680
22	17,65805	15,93692	15,16713	14,55112	13,78443	13,16300	12,04158	11,06124	10,20074
23	18,29220	16,44361	15,62041	14,85684	14,14778	13,48857	12,30338	11,27219	10,37106
24	18,91393	16,93554	16,05837	15,24696	14,49548	13,79864	12,55036	11,46933	10,52876
25	19,52346	17,41315	16,48152	15,62208	14,82821	14,09395	12,78336	11,65358	10,67478
26	20,12104	17,87684	16,89035	15,98277	15,14661	14,37519	13,00317	11,82578	10,80998
27	20,70690	18,32703	17,28537	16,32959	15,45130	14,64303	13,21053	11,98671	10,93517
28	21,28127	18,76411	17,66702	16,66306	15,74287	14,89813	13,40616	12,13711	11,05108
29	21,84439	19,18846	18,03577	16,98372	16,02189	15,14107	13,59072	12,27767	11,15841
30	22,39646	19,60044	18,39205	17,29203	16,28889	15,37245	13,76483	12,40904	11,25778

TABLE IV.

Which ſhews the preſent worth of one pound annuity, to continue any term of years under 61, and payable by yearly payments, compound intereſt being computed at any of theſe rates, *viz.* 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 *per cent. per annum.*

Years.	at 2 *per Ct.*	3 *per Cent.*	$3\frac{1}{2}$ *per Ct.*	4 *per Cent.*	$4\frac{1}{2}$ *per Ct.*	5 *per Cent.*	6 *per Cent.*	7 *per Cent.*	8 *per Cent.*
31	22,93770	20,00043	18,73628	17,58849	16,54439	15,59281	13,92909	12,53181	11,34980
32	23,46834	20,38877	19,06887	17,87355	16,78889	15,80268	14,08404	12,64656	11,43500
33	23,98856	20,76579	19,39021	18,14765	17,02286	16,00255	14,23023	12,75379	11,51389
34	24,49859	21,13184	19,70068	18,41120	17,24676	16,19290	14,36814	12,85401	11,58693
35	24,99862	21,48722	20,00066	18,66461	17,46101	16,37419	14,49825	12,94767	11,65457
36	25,48884	21,83225	20,29049	18,90828	17,66604	16,54685	14,62099	13,03521	11,71719
37	25,96943	22,16724	20,57053	19,14258	17,86224	16,71129	14,73678	13,11702	11,77518
38	26,44064	22,49246	20,84109	19,36786	18,04999	16,86789	14,84602	13,19347	11,82887
39	26,90259	22,80822	21,10250	19,58449	18,22966	17,01704	14,94908	13,26493	11,87858
40	27,35548	23,11477	21,35507	19,79277	18,40158	17,15909	15,04630	13,33171	11,92461
41	27,79949	23,41240	21,59910	19,99305	18,56611	17,29437	15,13802	13,39412	11,96724
42	28,23479	23,70136	21,83488	20,18563	18,72355	17,42321	15,22454	13,45245	12,00670
43	28,66156	23,98190	22,06269	20,37080	18,87421	17,54591	15,38617	13,50696	12,04324
44	29,07996	24,25427	22,28279	20,54884	19,01838	17,66277	15,38318	13,55791	12,07707
45	29,49016	24,51871	22,49545	20,72004	19,15635	17,77407	15,45583	13,60552	12,10840

TABLE IV.

Which ſhews the preſent worth of one pound annuity, to continue any term of years under 61, and payable by yearly payments, compound intereſt being computed at any of theſe rates, *viz.* 2, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, and 8 *per cent. per annum.*

Years.	2 *per Cent.*	3 *per Cent.*	$3\frac{1}{2}$ *per Ct.*	4 *per Cent.*	$4\frac{1}{2}$ *per Ct.*	5 *per Cent.*	6 *per Cent.*	7 *per Cent.*	8 *per Cent.*
46	29,89231	24,77545	22,70092	20,88465	19,28837	17,88007	15,52437	13,65002	12,13741
47	30,28658	25,02471	22,89944	21,04294	19,41471	17,98102	15,58903	13,69161	12,16427
48	30,67312	25,26671	23,09124	21,19513	19,53561	18,07716	15,65003	13,73047	12,18914
49	31,05208	25,50166	23,27656	21,34147	19,65130	18,16872	15,70757	13,76680	12,21216
50	31,42361	25,72976	23,45562	21,48219	19,76201	18,25593	15,76186	13,80075	12,23349
51	31,78785	25,95123	23,62862	21,61749	19,86795	18,33898	15,81308	13,83247	12,25323
52	32,14495	26,16624	23,79577	21,74758	19,96933	18,41807	15,86139	13,86212	12,27151
53	32,49505	26,37499	23,95726	21,87268	20,06635	18,49340	15,90697	13,88984	12,28843
54	32,83828	26,57766	24,11330	21,99296	20,15918	18,56515	15,94998	13,91574	12,30410
55	33,17479	26,77443	24,26405	22,10861	20,24802	18,63347	15,99054	13,93994	12,31861
56	33,50469	26,96546	24,46405	22,21982	20,33303	18,69855	16,02881	13,96256	12,33205
57	33,82813	27,15094	24,50945	22,32675	20,41439	18,76052	16,06492	13,98370	12,34449
58	34,14523	27,33101	24,68642	22,42957	20,49224	18,81954	16,09898	14,00346	12,35601
59	34,45610	27,50583	24,81780	22,52843	20,56673	18,87575	16,13111	14,02192	12,36668
60	34,76089	27,67556	24,94473	22,62349	20,63802	18,92929	16,16143	14,03918	12,37655

716. *The Use of the preceding Table* IV.

PROB. 9. *The Annuity, Time and Rate being given, to find its present Worth.*

Example. What is the present worth of an annuity of 56 *l. per annum,* to continue 21 years, allowing compound interest at 6 *l. per cent. per annum?*

In the column under (6) the rate, and on a line with (21) the time, you will find (11,76408) the present worth of an annuity of 1 *l.* to continue the given time, at the given rate: Multiply that present worth (11,76408) by the annuity (56) and the product (658,78848, or 658 *l.* 15 *s.* 9 *d.*) will be the present worth required.

717. PROB. 10. *The Annuity, present Worth, and Time being given, to find the Rate.*

Example. A person sold an annuity of 56 *l.* to continue 21 years, for 658 *l.* 15 *s.* 9 *d.* at what rate was compound interest computed?

Divide (658,78848) the present worth, by (56) the annuity, and find the quotient (11,764, &c.) or the nearest number to it, on a line with (21) the time, so shall (6) the number at the head of the column, be the rate required; exact, if the quotient be exactly found in the Table; or nearly so, if otherwise.

718. PROB. 11. *The Annuity, present Worth, and Rate being given, to find the Time.*

Example. A person bought an annuity of 56 *l.* at 6 *l. per cent.* compound interest for 658 *l.* 15 *s.* 9 *d.* how long was the annuity to continue?

Divide (658,78848) the present worth, by (56) the annuity, and find the quotient (11,764, &c.) or the nearest number to it, in the Table in the column under (6) the rate; so shall (21) the number standing in the left-hand column, even therewith, be the time required; exact, if the quotient be exactly found in the Table; or nearly so, if otherwise.

719. PROB. 12. *The present Worth, Time and Rate being given, to find the Annuity.*

Example. What annuity or yearly income, to continue 21 years, can be purchased for 658 *l.* 15 *s.* 9 *d.* allowing compound interest at 6 *l. per cent.*

In the column under (6) the rate, and on the line with (21) the time, you will find (11,76408) the present worth of an annuity of 1 *l.* to continue during the given time, at the given rate; by this divide (658,78848) the given purchase-money, and the quotient (56) will be the annuity required.

720. But farther, the numbers of the ſaid fourth Table will at firſt ſight ſhew, how many years purchaſe an annuity to continue any number of years under 61 is worth, to be ſold for preſent money, compound intereſt being computed at any of the ſaid rates. So if you deſire to know how many years purchaſe an annuity iſſuing out of lands for 21 years, to begin preſently, is worth, if it were to be ſold for ready money, when the current rate of intereſt is 6 *per cent.* Seek in the firſt column of Table IV. for 21 years, and carry your eye from thence equidiſtant to the head-line of the Table, till you come under 6, which (as before has been ſaid) ſignifies 6 *per cent.* So in the fourth column you will find 11.76408, whereof you need only conſider 11,76, which ſhews that the ſaid annuity is worth 11 years purchaſe (or 11 times one year's rent whatever it be) and 76 parts of one year's purchaſe divided into 100 parts, or $11\frac{3}{4}$ years purchaſe and little more. The ſame annuity, when money was at 8 *per cent.* was worth 10 years purchaſe, and about $\frac{1}{100}$ part of a year's purchaſe more, as the number in the column of 8 *per cent.* right againſt 21 years will diſcover.

721. As it would be too operoſe to make Tables to all times and rates, the uſe of theſe may be rendered more extenſive, by following the methods, delivered in the following Auxiliary Propoſitions.

722. PROB. 13. auxiliary to PROB. 1. and 8.

To find the Amount, or preſent Worth of 1 l. *for a number of years greater than* 60, *at any rate of Compound Intereſt, ſpecified in the Tables.*

Example. What will 1 *l.* amount to, in 100 years, allowing compound intereſt at 5 *l. per cent.?*

Find in the left-hand column of Table I. two or more numbers (ſuppoſe 51 and 49) whoſe ſum will make (100) the given number of years. Then multiply (12,04077 and 10,92133) the numbers correſponding to them, in the column under (5) the rate; ſo ſhall (131,50125) the product, be the number required.

The reaſon of this proceſs will be evident from *Art.* 414.

The preſent worth will be found in the ſame manner by taking the numbers from Table III.

723. Table V. auxiliary to Table I. and Table III.

	2 *per Cent*	3 *per Cent.*	$3\frac{1}{2}$ *per Ct.*	4 *per Cent.*	$4\frac{1}{2}$ *per Ct.*
1 Quarter	1,004963	1,007417	1,008637	1,009853	1,011065
2 Quarters	1,009950	1,014889	1,017349	1,019804	1,022252
3 Quarters	1,014963	1,022416	1,026137	1,029852	1,033564
	5 *per Cent.*	6 *per Cent.*	7 *per Cent.*	8 *per Cent.*	
1 Quarter	1,012272	1,014674	1,017058	1,019426	
2 Quarters	1,024695	1,029563	1,034408	1,039230	
3 Quarters	1,037270	1,044671	1,052053	1,059419	

724. The foregoing Table contains the amounts of 1 *l.* for 1, 2, and 3 quarters of a year at the ſeveral rates therein mentioned, being ſeverally three mean proportionals between 1 and the amount of 1 *l.* in 1 year at thoſe rates: They are found by the method directed in *Art.* 550.

725. Prob. 14. auxiliary to Prob. 1. and 8.

To find the Amount or preſent Worth of 1 l. *for Years and Quarters of a Year, at any Rate of Compound Intereſt ſpecified in the Tables.*

Example. What is the amount and preſent worth of 1 *l.* for $14\frac{3}{4}$ years at 5 *l. per cent.* compound intereſt?

Find by Table I. the amount (1,979932), or by Table III. the preſent worth (,505068) of 1 *l.* for (14) the number of years given at (5) the given rate. Find alſo by Table V. the amount of 1 *l.* for (3) the number of quarters given (*viz.* 1,037270): Then if the amount be required, multiply; or if the preſent worth be required, divide the former by the latter; and the product (2,053724) will be the amount, or the quotient (,487499) will be the preſent worth required?

726. Prob. 15. auxiliary to Prob. 4. and 9.

To find the amount or preſent Worth of an Annuity of 1 l. *to continue a number of Years greater than* 60, *at any Rate of Compound Intereſt ſpecified in the Tables.*

Example, What is the preſent worth of an annuity of 1 *l.* to continue 100 years, allowing 5 *l. per cent.* compound intereſt?

From the given years (100) take 60 the extent of the beforegoing Tables; and with the remainder (40) entering

Table IV. Find (17,15909) the present worth of an annuity, to continue that number of years at the given rate: Multiply this by (,053536) the present worth of 1 *l.* due at the end of 60 years found in Table III. and to (,91883) the product add (18,92929) the present worth of an annuity of 1 *l.* to continue 60 years, found in Table IV. so shall the sum (19,84812) be the present worth required.

727. The reason of this process depends on *Art.* 564. For if the sum of 40 terms of the Geometrical Progression in Table III. beginning at the 61st term, be added to the sum of the first 60 terms; the total will be the sum of 100 terms, the number required.

If the amount be required proceed in the same manner with the numbers in Table II and I.

728. Table VI. auxiliary to Tables II. and IV.

Payments	2 *per Cent.*	3 *per Cent.*	$3\frac{1}{2}$ *per Ct.*	4 *per Cent.*	$4\frac{1}{2}$ *per Ct.*
Half yearly	1,004975	1,007445	1,008675	1,009902	1,011126
Quarterly	1,007469	1,011181	1,013031	1,014877	1,016720

Payments	5 *per Cent.*	6 *per Cent.*	7 *per Cent.*	8 *per Cent.*
Half yearly	1,012348	1,014781	1,017204	1,019870
Quarterly	1,018559	1,022227	1,025880	1,029519

The numbers in the above Table are computed by a Theorem, given by the justly celebrated Mr. *De Moivre*, in his Treatise of *Annuities on Lives*, which the Reader who understands Algebra may consult; but as it cannot easily be explained without that art, the manner of its construction must be here omitted.

729. PROB. 16. auxiliary to PROB. 4. and 9.

The Amount or present Worth of an Annuity of 1 l. *payable by yearly payments*, *for any number of years*, *at any rate* per cent. *specified in the Tables*, *being given*; *to find what the Amount or present Worth of the same Annuity would be*, *if it were payable by equal half-yearly or quarterly Payments.*

Example. What is the present worth of an annuity of 1 *l. per annum*, payable by equal (half-yearly) payments, and to continue 41 years, allowing compound interest at 5 *per cent.*?

Multiply

Multiply (17,29437) the preſent worth of the annuity payable by yearly payments found in Table IV. by (1,012348) the number which, in Table VI. ſtands on a line with the word (half-yearly) and under (5) the rate; and the product (17,507918) will be the number required.

730. PROB. 17. auxiliary to PROB. 2, 3, 5, 6, 10, and 11.

When the Time or Rate cannot be exactly obtained by the Tables, to approach nearer to the truth therein.

Example. At what rate *per cent.* compound intereſt, will an annuity of 1 *l.* to continue 8 years, be now worth 6 *l.* 10 *s.*?

Find in Table IV. on a line with (8) the time, the two neareſt numbers to (6,5) the given preſent worth; which in this caſe will be 6,59589, the preſent worth of ſuch an annuity at 4½ *per cent.* and 6,46321, the preſent worth of the ſame at 5 *per cent.* the firſt exceeding the given number (6,5) by ,09589, and the latter being ,03679 leſs than it. And the difference of the ſaid tabular preſent worths 6,59589, and 6,46321 being 0,13268; uſe the following proportion.

As (,013268) the difference between the tabular preſent worths,

To (,5) the difference between the tabular rates, *viz.* (5—4½)

So is (,09589) the difference between the tabular preſent worth at 4½ *per cent.* and the given preſent worth,

To (0,3613) the difference between the tabular rate 4½ and the required rate.

Then to the tabular rate	4,5
Add the reſult of the proportion	0,3613

The ſum is the rate required nearly 4,8613 or 4 *l.* 17 *s.* 2¾ *d.*

The ſame manner of proceeding will do in any other caſe, *mutatis mutandis.*

731. If any queſtions ſhould ariſe, in compound intereſt, at different rates from thoſe in theſe Tables, the method of their ſolution will not be difficult to a Reader, who underſtands the management of Geometrical Progreſſions, as taught in *Chap.* 35. and knows how to uſe Logarithms.

But, if that be wanting, or if ſuch queſtions frequently occur, it will be beſt to make Tables for that rate, in the manner of the above.

732. PROBLEM 18. *To find the preſent worth of the Reverſion of an Annuity; having given, the Annuity, Time of commencing, Duration, and Rate.*

Example. Having a leaſe, for 7 years to come, of ſome lands, by which I clear 32 *l. per annum*, and being deſirous to add 21 years to the ſame, in order to make my whole time therein 28 years; what fine ought I to pay to my landlord, allowing 6 *l. per cent.* compound intereſt?

Find *per* Table IV. the preſent worth of an annuity of 1 *l.* for (28) the whole duration of the intended annuity or leaſe, *viz.* ———	13,40616
Alſo find by the ſame the preſent worth of an annuity of 1 *l.* for (7) the time before the reverſionary leaſe commences, *viz.* ———	5,58238
Then ſhall the difference of theſe preſent worths be the preſent value of an annuity of 1 *l.* for 21 years to commence at the end of 7 years.	7,82378
Which being multiplied by (32) the given annuity, will produce the fine required ———	250,3610

733. *Biſhops*, *Deans* and *Chapters*, *Heads* and *Fellows* of *Colleges*, &c. follow certain cuſtoms, which have been uſed from time immemorial, in aſcertaining the fines for the renewing of leaſes, held under them.

734. The moſt general of theſe cuſtoms is, to take one year's improved rent, as a fine, for renewing, that is adding, 7 years, to 14 yet to come, of an old leaſe of twenty-one years; which is allowing the tenant about 11,56421 *l. per cent.* compound intereſt for his money.

735. The following Tables are calculated to ſuit this cuſtom, upon the ſame principles as Tables III. and IV. and are to be uſed juſt in the ſame manner.

TABLE

Years.	TABLE VII. Shewing the present worth of 1 *l.* due at the end of the years therein specified, at 11,56421 *l. per cent.* compound interest.	Years.	TABLE VIII. Shewing the present worth of an annuity of 1 *l.* to cuntinue any number of years therein specified, at 11,56421 *l. per cent.* compound interest.
1	0, 89634	1	0, 89634
2	0, 80343	2	1, 69978
3	0, 72015	3	2, 41993
4	0, 64551	4	3, 06544
5	0, 57860	5	3, 64404
6	0, 51863	6	4, 16266
7	0, 46484	7	4, 62752
8	0, 41668	8	5, 04420
9	0, 37349	9	5, 41769
10	0, 33477	10	5, 75246
11	0, 30007	11	6, 05253
12	0, 26897	12	6, 32150
13	0, 24109	13	6, 56259
14	0, 21610	14	6, 77869
15	0, 19370	15	6, 97239
16	0, 17362	16	7, 14601
17	0, 15562	17	7, 30163
18	0, 13949	18	7, 44112
19	0, 12503	19	7, 56616
20	0, 11207	20	7, 67823
21	0, 10046	21	7, 77869

736. The Uſe of Table VIII.

Queſt. 1. If a College-tenant has 7 years to come or unſpent in a leaſe of lands for 21 years, at 1 *l.* yearly rent, and deſires to have 14 years renewed or added to thoſe ſeven years, and ſo to take a new leaſe for 21 years to begin preſently, what muſt he pay for a fine? Anſw. 3 *l.* 3 *s.* 0 *d.*

The Rule for finding out the anſwer of the queſtion propoſed, and ſuch like, is this, *viz.*

From 7.77869 (being the number which anſwers to 21 years in Table VIII.) always ſubtract the tabular number which belongs to the number of years to come or unſpent in the old leaſe; ſo the remainder will ſhew what fine muſt be paid for the years to be renewed or added, to make thoſe unſpent years in the old leaſe to be 21 years compleat again, at 1 *l.* yearly rent.

So to ſolve the queſtion propoſed.

From the preſent worth of 1 *l.* yearly rent for 21 years which is	7.77869
Subtract the preſent worth of the ſame rent for 7 years, (that were unſpent in the old leaſe)	4.62752
And there will remain the fine ſought, to wit,	3.15117

That is to ſay, 3.15117 *l.* or 3 *l.* 3 *s.* $0\frac{1}{4}$ *d.* muſt be paid as a fine, for renewing or adding 14 years to 7 years, that were unſpent in the old leaſe, the yearly rent being 1 *l.* Alſo the ſaid 3.15117 ſhews, that ſuch a renewal is worth 3 years purchaſe, and $\frac{15}{100}$ parts of a year's purchaſe (whatever the rent be.)

Queſt. 2. If a tenant that has 17 years yet to come in a leaſe of lands held of a College for 21 years at 50 *l.* yearly rent, be deſirous to renew 4 years, and ſo make thoſe 17 years to be 21 years compleat again at the ſame rent, what muſt he give for a fine? Anſw. 23 *l.* 17 *s.* 2 *d.* 1 *f.* For according to the Rule before given,

From the preſent worth of 1 *l.* yearly rent for 21 years,	7.77869
Subtract the preſent worth of the ſame rent for 17 years (that were unſpent in the old leaſe)	7.30163
And there will remain	0.47706
Which multiplied by the rent	50
The product will be the fine ſought, to wit 23 *l.* 17 *s.* 0 *d.* 3 *f.*	23.85300

737. From Table VIII. by working as in the above Examples may be deduced the following Table IX. to ſhew without any operation the fine to be paid on a leaſe whoſe improved rent is 1 *l. per annum*, or rather how many years purchaſe ſhould be given for renewing, at any time before the expiration of the leaſe.

Years to be added.	TABLE IX. Shewing what fine ought to be paid for adding any number of years to a college leaſe in order to make the ſame 21 years compleat.
1	0, 10046
2	0, 21253
3	0, 33756
4	0, 47706
5	0, 63268
6	0, 80630
7	1, 00000
8	1, 21600
9	1, 45719
10	1, 72616
11	2, 02623
12	2, 36100
13	2, 73449
14	3, 15117
15	3, 61603
16	4, 13465
17	4, 71325
18	5, 35876
19	6, 07891
20	6, 88234

The uſe of this table will appear by comparing it with the two Examples in *Art.* 736. In the firſt of them, 14 years were to be added; and the anſwer 3,15117 will be found in this Table againſt the number 14. In the ſecond, 4 years were to be renewed, and the fine to be paid for a rent of 1 *l. per annum*, *viz.* 0,47706 will be found in this Table againſt the number 4.

738. *Queſtions to exerciſe the preceding Tables.*

1. If the leaſe of an houſe be worth 153 *l.* fine, and 16 *l.* yearly rent, payable yearly for 21 years, and the leſſee be deſirous to bring

bring down the fine to 50 *l.* and so to pay the more rent; the question is what rent the tenant shall pay, accounting compound interest at the rate of 8 *per cent. per annum?* Answ. 26 *l.* 5 *s.* 8 *d.*

First, find the difference between the fines, which is 103 *l.* Then by *Prob.* 12. seek what annuity or rent to continue 21 years, 103*l.* ready money will purchase at 8 *per cent.* so you will find 10 *l.* 5 *s.* 8 *d.* which being added to the old rent 16 *l.* gives 26 *l.* 5 *s.* 8 *d.* which the tenant must pay, to the end that the fine may be diminished to 50 *l.*

2. There is a lease of certain lands to be lett for 14 years for 250 *l.* fine, and 44 *l.* rent *per annum*, payable yearly; but the tenant is desirous to pay less rent, *viz.* 20 pounds *per annum*, and to give a greater fine; the question is, what fine ought to be paid to bring down the rent to 20 *l. per annum*, accounting compound interest at the rate of 6 *per cent. per annum?* Answ. 473 *l.* 1 *s.* 7 *d.*

First find the difference between the rents, which is 24 pounds *per annum.* Then by *Prob.* 9. seek what an annuity or rent 24 *l. per annum*, to continue 14 years, is worth in ready money at 6 *per cent. per annum*, so you will find 223 *l.* 1 *s.* 7 *d.* which being added to the first fine 250 pounds, gives 473 *l.* 1 *s.* 7 *d.* which the tenant must pay, to the end that the rent may be brought down to 20 *l. per annum.*

3. What annuity to continue 13 years will 1999 *l.* due 9 years hence purchase, compound interest being allowed on both sides at 8 *per cent. per annum?* Answ. 126 *l.* 10 *s.* $4\frac{3}{4}$ *d.*

By *Prob.* 8. the present worth of 1999 *l.* due at the end of 9 years is 1000 *l.*

And by *Prob.* 12. 1000 *l.* will purchase an annuity of 126 *l.* 10 *s.* $4\frac{3}{4}$ *d.* to continue 13 years.

CHAP.

CHAP. XLII.

A demonſtration of the DOUBLE RULE *of* FELLOWSHIP.

739. THE double Rule of Fellowſhip (commonly called the Rule of Fellowſhip with Time) preſuppoſes two things, *viz.* 1. That the particular ſtocks of merchants in company, have continued unequal ſpaces of time in the common ſtock. 2. That at the end of their partnerſhip, the total gain or loſs is to be divided among them, in ſuch manner, that their ſhares may have ſuch proportion between themſelves, as thoſe ſums of intereſt-money have one to another, which at any rate *per cent.* (ſimple intereſt only being computed) might be gained by the particular ſtocks, within the reſpective times of their continuance in the common ſtock: Now for the effecting of ſuch a proportional partition, the ſaid Double Rule of Fellowſhip gives this Direction, *viz.* divide the total gain or loſs into ſuch parts, which have the ſame proportion one to the other, as is between the products ariſing from the Multiplication of each particular ſtock by its correſpondent time.

For example, ſuppoſe two merchants *A* and *B*, to be partners in traffick, for a certain time firſt agreed on between them, and that *A* permits his ſtock of 100*l.* to be employed in their joint traffick three months, and *B* his ſtock of 50*l.* eight months; I ſay, (according to the ſaid Rule of Fellowſhip with Time) whatever the total gain or loſs be, that part of it which belongs to *A* muſt have ſuch proportion to the gain or loſs of *B*, as 100×3 (or 300) has to 50×8 (or 400.) The truth of this Rule, taking the two premiſed ſuppoſitions for granted, may be thus demonſtrated.

1. Suppoſing 100*l.* (the ſtock of *A*) to gain in three months any certain ſum of money, as two pounds; I ſeek how much 50*l.* (the ſtock of *B*) will gain in the ſame time, and at the ſaid rate: So find $\frac{2 \times 50}{100}l.$ For,

$$\text{As } 100 : 2 :: 50 : \frac{2 \times 50}{100} l.$$

2. Having

2. Having found what 50*l.* will gain in three months, I seek how much the said 50*l.* will gain in eight months at the same rate; and so I find $\frac{2\times50\times8}{100\times3}$ *l.*

For as $3 : 8 :: \frac{2\times50}{100} : \frac{2\times50\times8}{100\times3}$.

3. Thus it appears, that if 100*l.* in three months gains 2*l.* then 50*l.* in eight months will at the same rate gain $\frac{2\times50\times8}{100\times3}$; so that the proportion of the gain of *A* to the gain of *B*, is

As 2 is to $\frac{2\times50\times8}{100\times3}$ *l.*

4. If both the the terms (to wit, the Antecedent and Consequent) of the said proportion be severally multiplied by the said Denominator 100×3, the products will be in the same proportion with the terms multiplied, *viz.* the gain of *A* will be to the gain of *B*,

As $2 \times 100\times3$ is to $2 \times 50 \times 8$.

5. Lastly, because 2 (the suppositious gain first assumed) is a Multiplier as well in the Antecedent as in the Consequent of the last-mentioned proportion, it may be expunged out of both; and so the gain of *A*, will be to the gain of *B* in this proportion, (which was to be proved) to wit,

As 100×3 is to 50×8.

CHAP.

CHAP. XLIII.

A Collection of pleasant and subtil Questions, to exercise all the Parts of VULGAR ARITHMETIC.

740. QUEST. 1. If a wedge of gold weighing $17\frac{3}{7}$ *lb.* of *Troy* weight be worth $679\frac{5}{7}$ *l. Sterling*, what is the value of $1\frac{3}{13}$ grain of that gold? Answ. 2*d.*

1. $1\frac{3}{13}$ (or $\frac{16}{13}$) of $\frac{1}{24}$ of $\frac{1}{20}$ of $\frac{1}{12} = \frac{1}{4680}$.

2. $\frac{122}{7} : \frac{4758}{7} :: \frac{1}{4680} : \frac{1}{120}$.

Quest. 2. A man dying gave to his eldest son $\frac{2}{3}$ of $\frac{1}{4}$ of his estate, to his second son $\frac{1}{5}$ of $\frac{1}{2}$ of his estate, and when they had counted their portions, the one had 40 *l.* more than the other; the remainder of the estate was given to the wife, and younger children: The question is, what was the portion of the eldest son, also of the second and how much did belong to the wife and younger children? Answ. the eldest son's portion 100 *l.* the second son's portion 60 *l.* and 440 *l.* for the wife and younger children.

The Fractions being reduced, it will be manifest, that the eldest son had $\frac{1}{6}$, and the second $\frac{1}{10}$; also the difference of the said Fractions $\frac{1}{15}$, then say,

As $\frac{1}{15} : 40 :: 1 : 600$, the estate.

Now $\frac{1}{6}$ of 600 = 100, the eldest } son's share
And $\frac{1}{10}$ of 600 = 60, the second } son's share

Their sum 160,

Therefore there remains 440 for the wife, and younger children.

Quest. 3. A young man receives $66\frac{2}{3}$ *l.* which was $\frac{2}{3}$ of $\frac{1}{2}$ of his eldest brother's portion, and $3\frac{1}{2}$ times his eldest brother's portion was $1\frac{1}{4}$ times his father's estate; the question is, what was the father's estate? Answ. 560 *l.*

$\frac{2}{3}$ of

$\frac{2}{3}$ of $\frac{1}{2} = \frac{1}{3}$

As $\frac{1}{3} : 66\frac{2}{3} :: 1 : 200$ = the elder brother's portion.

$200 \times 3\frac{1}{2} = 700$.

As $1\frac{1}{4} : 700 :: 1 : 560$ = the father's eſtate.

Queſt. 4. If *A* can finiſh a work in 20 days, and *B* in 30 days; in what time will the work be finiſhed by *A* and *B* working together? Anſw. 12 days.

Firſt find what quantity of the work will be done by each workman in one and the ſame time; then it will be, as the ſum of thoſe quantities is in proportion to the ſaid time, ſo is 1 or the whole work to the time in which ſuch work will be finiſhed by both workmen working together.

Days. Work. Days. Work.

$30 : 1 :: 20 : \frac{2}{3}$

Add 1

Sum $1\frac{2}{3}$

Hence it appears that *A* and *B* working together 20 days, will finiſh that work once, together with $\frac{2}{3}$ of the ſame work; therefore ſay again by the Rule of Three,

Work. Days. Work. Days.

$1\frac{2}{3} : 20 :: 1 : 12$

Queſt. 5. A Gentleman a Chaiſe did buy.
An Horſe and Harneſs too;
They coſt the ſum of threeſcore Pounds,
(Upon my word 'tis true;)
The Harneſs came to half of th' Horſe,
The Horſe twice of the Chaiſe:
And if you find the Price of them,
Take them and go your ways.

Anſwer { Chaiſe——15 *l.*
Horſe——30 *l.*
Harneſs——15 *l.*

Queſt. 6. A ciſtern in a certain conduit is ſupplied with water by one pipe of ſuch bigneſs, that if the cock *A* at the end of the pipe be ſet open, the ciſtern will be filled in $\frac{1}{2}$ an hour: But at the bottom of the ciſtern two other cocks *B* and *C* are placed, whoſe capacities are ſuch, that by the cock *B* ſet open alone (all the reſt being ſtopt) the ciſtern ſuppoſed to be full, will be emptied in $1\frac{3}{7}$ hour: Alſo by the cock *C* ſet open alone, the ciſtern will be emptied in $2\frac{1}{7}$ hour: Now becauſe more water will be infuſed by the cock *A*, than can be be expelled by both the cocks *B* and *C* in one and the ſame time; the queſtion is to find in what time the ciſtern will be filled, if all the ſaid three cocks be ſet open at once? Anſwer $1\frac{9}{61}$ hour.

After the manner of the fourth queſtion of this Chapter, find how many times the ciſtern will be emptied in one and the ſame ſpace of time, by the cocks *B* and *C* running together; alſo how much the ciſtern will be filled by *A* in the ſame time; then will the difference ſhew how much of the ciſtern is gained by the filling cock in the ſaid time: Laſtly, as the ciſterns or parts gained are in proportion to the correſpondent time; ſo is the whole ciſtern, to the time wherein it will be gained or filled.

	Hou.	*Ciſt.*	*Hou.*	*Ciſt.*	
I.	$2\frac{1}{3}$:	1 ::	$1\frac{3}{7}$:	$\frac{30}{49}$	emptied by *C*
			Add	1	emptied by *B*
			Sum	$1\frac{30}{49}$	emptied by *B* and *C*
	Hou.	*Ciſt.*	*Hou.*	*Ciſt.*	
II.	$\frac{1}{2}$:	1 ::	$1\frac{3}{7}$:	($2\frac{6}{7}$ filled by *A*	
				$1\frac{12}{49}$ gained by *A* in $1\frac{3}{7}$	
	Ciſt.	*Hou.*	*Ciſt.*	*Hou.*	
III.	$1\frac{12}{49}$:	$1\frac{3}{7}$:	1 :	$1\frac{9}{61}$	

Queſt. 7. Suppoſe a dog, a wolf, and a lion, were to devour a ſheep, and that the dog could eat up the ſheep in an hour, the wolf in $\frac{3}{4}$ hour, and the lion in $\frac{1}{2}$ hour: Now if the lion begin to eat $\frac{1}{8}$ hour before the other two, and afterwards all three eat together, the queſtion is, in what time the ſheep would be devoured? Anſw. $\frac{31}{104}$ hour.

	Hou.	*Sh.*	*Hou.*	*Sh.*
I. If	$\frac{1}{2}$:	1 ::	$\frac{1}{8}$:	$\frac{1}{4}$

Thus it appears that $\frac{1}{4}$ of the ſheep would be eaten by the lion, before the dog and wolf began to eat.

2. Proceed according to the fourth queſtion, ſo will you find the remaining $\frac{3}{4}$ to be eaten by them all in $\frac{9}{52}$ hour, which added to $\frac{1}{8}$ gives $\frac{31}{104}$ hour, in which time the ſheep would be devoured.

Queſt. 8. If $120\frac{1}{3}$ *l.* is to be diſtributed among three perſons, *A*, *B*, *C*, in ſuch ſort, that as often as *A* takes 5, *B* ſhall take 4; and as often as *B takes* 3, *C* ſhall take 2; what will be the ſhare of each of them?

Anſw. *A* $51\frac{4}{7}$ *l.* *B* $41\frac{9}{35}$ *l.* *C* $27\frac{53}{105}$ *l.*

Find three numbers which may express the proportions of their shares, by the rule of Three, or (to avoid Fractions) thus;

$$\begin{array}{ccc} 5 & \cdots\cdots & 4 \\ 3 & \cdots\cdots & 2 \\ \hline 15 : & 12 & : 8 \end{array}$$

Thus found

$$5 \times 3 = 15$$
$$3 \times 4 = 12$$
$$4 \times 2 = 8$$

$$35 : 120\tfrac{1}{3} :: \begin{cases} 15 : 51\tfrac{4}{7} \\ 12 : 41\tfrac{9}{35} \\ 8 : 27\tfrac{53}{105} \end{cases}$$

Quest. 9. A governor of a certain garrison, being desirous to know how much money the port or passage of the garrison did amount to in certain months, made choice of a loyal servant, giving him orders to receive of every coachman passing with a coach 4 *d.* of every horseman 2 *d.* and of every footman $\frac{1}{2}$ *d.* Now at the year's end, the servant making his account to the governor, gives him 94 *l.* 15 *s.* 10 *d.* and lets him know, that as often as 5 passed with coaches, 9 passed on horseback; and as often as 6 passed on horseback, 10 passed on foot; the question is, how many coaches, horse-men, and footmen passed? Answ. 2500 coaches, 4500 horse-men, 7500 footmen.

Find three proportional numbers after the manner of the 8th question, which will be 5, 9, 15, then proceed as follows:

	d.
5 Coaches—	20
9 Horsemen—	18
15 Footmen—	$7\frac{1}{2}$

$$\text{If } 45\tfrac{1}{2} : 22750 :: \begin{cases} 5 : 2500 \\ 9 : 4500 \\ 15 : 7500 \end{cases}$$

Quest. 10. A factor would exchange 780 *l.* Sterling for double ducats, dollars, and *French* crowns, the ducats at 7 *s.* 6 *d.* the piece, the dollars at 4 *s.* 4 *d.* and the *French* crowns at 6 *s.* the piece, to be in such proportion, that $\frac{1}{2}$ of the number of ducats may be equal to $\frac{1}{3}$ of the number of dollars; and $\frac{1}{4}$ of the dollars equal to $\frac{3}{10}$ of the crowns; the question is, how many

many pieces of every coin he ſhall receive for his 780 pounds? Anſw. 600 ducats, 900 dollars, 1200 crowns.

Find three proportional numbers (after the manner of the eighth queſtion) which will be 6, 4, 3.

$$\begin{array}{ccccc} \frac{1}{2} & \cdots & \cdots & \cdots & \frac{1}{3} \\ \frac{1}{4} & \cdots & \cdots & \cdots & \frac{3}{10} \\ \hline \frac{1}{8} & : & \frac{1}{12} & : & \frac{1}{18} \\ 6 & : & 4 & : & 3 \end{array}$$

Thus it appears, that ſix times the number of ducats muſt be equal to four times the number of dollars, alſo equal unto three times the number of crowns. Then make choice of three numbers to anſwer thoſe proportions, ſuch as are theſe, 2, 3, 4, (for $6 \times 2 = 4 \times 3 = 3 \times 4$) with which numbers proceed as follows:

$$\begin{array}{ll} & l. \\ \text{2 ducats} & — \frac{3}{4} \\ \text{3 dollars} & — \frac{13}{20} \\ \text{4 crowns} & — 1\frac{1}{5} \end{array}$$

$$\text{ſay if } \overset{l.}{2\tfrac{3}{5}} : \overset{l.}{780} :: \begin{cases} \overset{l.}{\frac{3}{4}} : \overset{l.}{225} \\ \frac{13}{20} : 195 \\ 1\frac{1}{5} : 360 \end{cases}$$

$$\begin{array}{lllll} l. & \textit{ducat.} & & l. & \\ \frac{3}{8} : & 1 & :: & 225 : & \text{600 ducats.} \\ & \textit{doll.} & & & \\ \frac{13}{60} : & 1 & :: & 195 : & \text{900 dollars.} \\ & \textit{crown.} & & & \\ \frac{3}{10} : & 1 & :: & 360 : & \text{12000 crowns.} \end{array}$$

Queſt. 11. Twenty knights, 30 merchants, 24 lawyers, and 24 citizens, ſpent at a dinner 64 pounds, which ſum was divided among them in ſuch manner, that 4 knights paid as much as 5 merchants, 10 merchants as much as 16 lawyers, and 8 lawyers as much as 12 citizens; the queſtion is, to know the ſum of money paid by all the knights, alſo by the merchants, lawyers, and citizens?

Anſwer. The 20 knights paid 20 pounds, the 30 merchants 24 pounds, the 24 lawyers 12 pounds, and the 24 citizens 8 pounds.

Find four numbers to expreſs the proportions of their payments by the Rule of Three, or (to avoid Fractions) in manner following; ſo will the proportional numbers be 4, 5, 8, 12; *viz.* 4 knights paid as much as 5 merchants, or 8 lawyers, or 12 citizens.

4 5
10 16
8 12

320 : 400 : 640 : 960
4 5 8 12

Thus found

$4 \times 10 \times 8 = 320$
$10 \times 8 \times 5 = 400$
$8 \times 5 \times 16 = 640$
$5 \times 16 \times 12 = 960$

Then presuppoſing that a knight is to pay 4 *s.* proceed as follows, *viz.*

20 knights 4
30 merchants . . $4\frac{4}{5}$
24 lawyers . . . . $2\frac{2}{5}$
24 citizens . . . . $1\frac{3}{5}$

ſay, if $12\frac{4}{5}$: 64 :: $\begin{cases} 4 : 20 \\ 4\frac{4}{5} : 24 \\ 2\frac{2}{5} : 12 \\ 1\frac{3}{5} : 8 \end{cases}$

64

Queſt. 12. A certain man with his wife did uſually drink out a veſſel of beer in 12 days, and the huſband found by often experience, that his wife being abſent, he drank it out in 20 days; the queſtion is, in how many days the wife alone could drink it out? Anſw. 30 days.

Note, It is to be ſuppoſed that the huſband, in 12 of the 20 days in which he drank alone, did drink as much as in the 12 days, wherein he drank with his wife; hence it follows, that in the remaining 8 of the ſaid 20 days, he drank as much as his wife did in 12 days. Therefore by the Rule of Three ſay, if 8 gives 12, what 20? Anſw. 30. View the following form of the work.

From 20
Subtract 12

Then if 8 : 12 :: 20 : 30

Queſt. 13. A perſon dying left his widow 1780 *l.* and 1250 *l.* to each of his four children: He had been $25\frac{1}{2}$ years in trade, and had cleared (at an average) 126 *l.* a year. What had he to begin with? Anſw. 3567 *l.*

		l.
For he left to his wife	——	1780
to his children 1250 × 4	=	5000
Therefore he left in all	——	6780
But he gained $126 \times \frac{51}{2} = 63 \times 51$	=	3213
Therefore he began with	— —	3567

Queſt. 14. Two travellers, *A* and *B*, perform a journey to one and the ſame place, in this manner, *viz.* *A* travels 14 miles every day, and had travelled 8 days before *B* began; upon the ninth day *B* ſet forward, and travels 22 miles every day; the queſtion is, to find in what time *B* ſhall overtake *A*? Anſw. at the end of 14 days.

1. Find how many miles *A* had travelled before *B* ſet forward? Anſw. 112 miles.

Day		*Miles*		*Days*		*Miles*
1	:	14	::	8	:	112

2. Seek how many miles *B* gains of *A* in a day? Anſw. 8 miles: For,

$$22-14=8$$

	Miles		*Day*		*Miles*		*Days*
3. If	8	:	1	::	112	:	14

Queſt. 15. There is an iſland which is 36 miles in compaſs: Now, if at the ſame time, and from the ſame place, two footmen *A* and *B*, ſet forward to travel round about the ſaid iſland, and follow one another in ſuch manner, that *A* travels every day 9 miles and *B* 7 miles; the queſtion is to find in what ſpace of time they will meet again; alſo how many miles, and how many times about the iſland each footman will then have travelled?

Anſw. They will meet at the end of 18 days from their firſt parting; and then *A* will have travelled 162 miles (or $4\frac{1}{2}$ times the compaſs of the iſland;) and *B* will have travelled 126 miles (or $3\frac{1}{2}$ the compaſs of the iſland.)

	Miles.				
From	9				
Subtract	7				
		Day.	*Miles.*	*Days.*	
	2 :	1 ::	36 :	18	

mult.	18	*mult.*	18
by	9	*by*	7
	36) 162 ($4\frac{1}{2}$		36) 126 ($3\frac{1}{2}$

Quest. 16. Two footmen *A* and *B*, depart at the same time from *London* towards *York*, travelling at this rate, *viz.* *A* goes 8 miles every day, *B* goes 1 mile the first day, 2 miles the second Day, 3 miles the third day, and in that progression he goes forward, travelling in every following day 1 mile more than in the preceding day; the question is to know in how many days *B* will overtake *A?* Answ. in 15 days.

To resolve this and such like questions, double 8 (the number of miles which *A* travels daily) which make 16, from which subtract 1, the remainder is 15, the number of days sought.

Quest. 17. If *Exeter* be distant from *London* 140 miles, and that at the same time one footman *A* departed from *London* towards *Exeter*, travelling every day 8 miles; another *B* from *Exeter* towards *London*, travelling every day 6 miles, the question is, in how many days they will meet one another, and how many miles each footman will have then travelled? Answ. they'll meet at the end of 10 days, and then *A* will have travelled 80 miles, and *B* 60 miles.

Add { 8 miles travelled daily by *A*.
6 miles travelled daily by *B*.

Sum 14 miles which *A* and *B* together did travel daily.

M. *Da.* *Miles.* *Da.*
14 : 1 :: 140 : 10 in which time *A* and *B* will meet each other.

$10 \times 8 = 80$ miles travelled by *A*.
$10 \times 6 = 60$ miles travelled by *B*.

Quest. 18. A certain footman *A* sets out from *London* towards *Lincoln*, and at the same time another footman *B* departs from *Lincoln* towards *London*; also *A* travels every day $2\frac{1}{2}$ miles more than *B*. Now, supposing those two cities to be 100 miles distant one from the other, and that those two footmen do meet one another at the end of 8 days after the beginning of their journies; the question is, how many miles each will have then travelled, as alſo how many miles each travelled daily? Anſw. *A* 60 miles, *B* 40 miles: Alſo *A* travelled $7\frac{1}{2}$ miles every day, and *B* 5 miles.

Day.		*Miles.*		*Days.*		*Miles.*
1	:	$2\frac{1}{2}$	::	8	:	20

Hence it appears, that at the time of their meeting, *A* had travelled 20 miles more than *B*, which 20 miles being ſubtracted from 100 miles leave 80 miles, whereof the half is 40 miles which *B* had travelled, therefore *A* had travelled 60 miles.

Now to find how many miles each had travelled daily, ſay,

Days.		*Miles.*		*Day.*		*Miles.*
8	:	40	::	1	:	5 *Miles.*

Therefore $\left\{ \begin{matrix} A \\ B \end{matrix} \right\}$ travelled $\left\{ \begin{matrix} 7\frac{1}{2} \\ 5 \end{matrix} \right\}$ daily

Queſt. 19. There is an iſland which is 134 miles in compaſs; now at the ſame time, and from the ſame place, two footmen *A* and *B* begin a journey round about the ſaid iſland, but they travel towards contrary parts, at this rate, *viz.* *A* travels 11 miles in every 2 days, and *B* 17 miles in three days; the queſtion is to find in what ſpace of time *A* and *B* will meet one another; and how many miles each will then have travelled?

Anſw. They'll meet at the end of 12 days, and then *A* will have travelled 66 miles, and *B* 68 miles.

After the manner of the fourth queſtion of this Chapter, the time ſought will be found 12 days.

Days.		*Miles.*		*Days.*		*Miles.*
2	:	11	::	3	:	$16\frac{1}{2}$
					Add	17

Miles.		*Days.*		*Miles.*		*Days.*
$33\frac{1}{2}$	:	3	::	134	:	12

The miles travelled by each will be found in this manner.

Days.		*Miles.*		*Days.*		
2	:	11	::	12	:	66 miles travelled by *A*.
3	:	17	::	12	:	68 miles travelled by *B*.

Queſt. 20. If a clock has two indices (or hands,) one of which (to wit *A*) is carried twice round the whole circumference of the dial in one day; and the other (*B*) once in 30 days, and that both at once ſhewing the ſame point begin to be moved; the queſtion is, in what time they will be again conjoined? Anſw. $\frac{30}{59}$ day.

Day.		*Circum.*		*Days.*		*Circum.*
1	:	2	::	30	:	60
					subtract.	1
						59

Hence it appears that in 30 days *A* will have run through 60 circumferences, and *B* one circumference only in the same time; so that *A* gains of *B* 59 circumferences in 30 days, say therefore

Circum.		*Days.*		*Circum.*		*Day.*
59	:	30	::	1	:	$\frac{30}{59}$

Quest. 21. If 6 *lb.* of sugar be equal in value to 7 *lb.* of raisins; 5 *lb.* of raisins to 2 *lb.* of almonds; 3 *lb.* of almonds to 5 *lb.* of currans; 2 *lb.* of currans to 18 *d.* how many pence are the value of 3 *lb.* of sugar? Answ. 21 *d.*

1. ~~6~~ S. = 7 R.

1. ~~5~~ R. = ~~2~~ A. 1.

1. ~~3~~ A. = ~~5~~ C. 2.

1. ~~2~~ C. = ~~18~~ *d.* 3.

? *d.* = ~~3~~ S. 1.

Answer 7 × 3 = 21.

Quest. 22. If 3 dozen pair of gloves be equal in value to 2 pieces of ribbon; 3 pieces of ribbon to 7 dozen of points; 6 dozen of points to 2 yards of *Flanders'* lace; and 3 yards of *Flanders* lace to 81 shillings; how many dozen pair of gloves may be bought for 28 shillings? Answer 2 dozen pair of gloves.

1. ~~3~~ G. = ~~2~~ R. 1.

1. ~~3~~ R. = ~~7~~ P. 1.

2. ~~6~~ P. = ~~2~~ L. 1.

1. ~~3~~ L. = ~~81~~ S. ~~27~~. ~~9~~. ~~3~~. 1.

1. ~~2~~. ~~4~~. ~~28~~ S. = ? G.

Answer 2.

Quest. 23. Suppose a greyhound to be coursing a hare, in such a sort that the hare takes five leaps for every four leaps of the greyhound, and that the hare is one hundred of her own leaps distant from the greyhound; now if three of the greyhound's leaps be equal to four of the hare's; the question is, to know how many leaps the greyhound must take before he obtains his prey? Answ. 1200 leaps.

1. . If 3 : 4 :: 4 : $5\frac{1}{3}$

Thus

Thus it appears that 4 of the greyhound's leaps are equal to $5\frac{1}{3}$ of the hare's leaps; and becaufe by the queftion the greyhound takes 4 leaps for every 5 of the hare's, therefore the greyhound in every four of his leaps gains $\frac{1}{3}$ of one of the hare's leaps; therefore fay by the Rule of Three,

2. If $\frac{1}{3}$: 4 :: 100 : 1200

Queft. 24. Bought 7 tuns of wine at 17 *l. per* hogfhead, which I fell again at 1 *s. per* pint; what is the whole gain, and how much *per cent.*? Anfw. the whole gain 229 *l.* 12 *s.* the gain *per cent.* 48 *l.* 4 *s.* $8\frac{1}{2}$ *d.*

As 1 *hd.* : 17 *l.* :: 7×4 *hds.* : 476 *l.*
1 *p.* : $\frac{1}{20}$ :: 28×63×8 *p.* : 705 *l.* 12 *s.*
Therefore the whole gain = 229 *l.* 12 *s.*
As 476 *l.* 229,6 *l.* :: 100 *l.* : 48,2353 *l.*

Queft. 25. If 30 *s.* be the hire of 8 men for 3 days; how many days muft 20 men work for 15 *l.*? Anfw. 12 days.

Conditional terms	$\frac{3}{2}$ *l.*	8 *Men.*	3 *Days.*
Correfponding	15	20	A.

$$\text{Therefore A} = \frac{8\times3\times15\times2}{3\times20} = 12.$$

Queft. 26. If 10 bufhels of oats be enough for 18 horfes 20 days; how many bufhels will ferve 60 horfes, 30 days? Anfwer 50 bufhels.

	B.	*H.*	*Days.*
Conditional terms	10	18	20
Correfponding	A	60	30

$$\text{Therefore A} = \frac{60\times30\times10}{18\times20} = 50$$

Queft. 27. A man dies and leaves a legacy of 900 *l.* to be difpofed of among four of his relations, *viz.* *A*, *B*, *C*, and *D*; which legacy is to be difpofed of in this order: *B* is to have twice as much as *A*, and *C* thrice as much as *B*, and *D* is to have as much, and $\frac{1}{2}$ as much as *C*; what muft each perfon have?

Suppofe A has 1		1 : 50	
Then B . . 2	Then 18 : 900 ::	2 : 100	
. . . . . C . . 6	Or 1 : 50 ::	6 : 300	
. . . . . D . . 9		9 : 450	
18		900	

Queft. 28. A merchant would beftow 220 *l.* in cloves, mace, and nutmegs, the cloves being at 5 *s.* the pound, the mace at

11 *s.* the pound, and the nutmegs at 6 *s.* the pound; now he would have of every sort an equal quantity, the question is, how many pounds he may have of each sort? Answ. 200 *lb.*

s.
5
11
6

22 : 1 :: 4400 : 200

The Proof.

lb.		*s.*		*l.*
200	at	5	amounts to ——	50
200	at	11	amounts to ——	110
200	at	6	amounts to ——	60
				220

Quest. 29. A factor is to receive a sum of money, and is offered dollars at 4 *s.* 4 *d.* which are worth but 4 *s.* 3 *d.* or *French* crowns at 6 *s.* $1\frac{1}{2}$ *d.* which are worth but 6 *s.* the question is by which coin he shall sustain the least loss? Answ. by the dollars.

d. *d.* *d.* *d.*
$52 : 1 :: 73\frac{1}{2} : 1\frac{43}{104}$.

That is, in receiving the dollars every 6 *s.* $1\frac{1}{2}$ *d.* loses $1\frac{43}{104}$ *d.* but in receiving the crowns 6 *s.* $1\frac{1}{2}$ *d.* loses $1\frac{1}{2}$ *d.* which is a greater loss than $1\frac{43}{104}$ *d.*

Quest. 30. A *Manchester* man buys yarn for 6 *s.* the bundle, which not proving so good as he expected, he was willing to put it off again, so as to lose but 6 *per cent.* by it; the question is, at what price the bundle is to be sold for? Answ. 5 *s.* $7\frac{1}{2}$ *d.*

d. *d.*
For as 100 : (100—6=) 94 :: 72 : 67,68

Quest. 31. *A*, *B*, and *C*, put in 360 *l.* and gained 270 *l.* of which as oft as *A* took 3 *l.* *B* took up 5 *l.* and as oft as *B* took 5 *l.* *C* took up 7 *l.* what did each gain and put in?

A 3
B 5
C 7
15

As 15 : 270 ::
Or 1 : 18 ::
{ 3 : 54, 5 : 90, 7 : 126 } gain.
270

$$\begin{array}{l} \text{As } 270 : 360 :: \\ \text{Or } \ 3 : 4 :: \end{array} \left\{ \begin{array}{r} 54 : 72 \\ 90 : 120 \\ 126 : 168 \end{array} \right\} \text{sum put in.}$$

$$\overline{360}$$

Quest. 32. Five merchants, *viz.* *A*, *B*, *C*, *D*, and *E*, have gained 2025 *l.* which they divide in such sort, that $\frac{1}{2}$ of the share of *A* is equal severally to $\frac{1}{4}$ of the share of *B*, $\frac{1}{5}$ of *C*, $\frac{1}{6}$ of *D*, $\frac{1}{8}$ of *E*; the question is, what was the share of every merchant? Answ. *A* 162 *l.* *B* 324 *l.* *C* 405 *l.* *D* 486 *l.* *E* 648 *l.*

Divide a number at pleasure into several parts, which may be in such proportion as the shares required, and proceed according to the subsequent operation.

A 2
B 4
C 5
D 6
E 8

$$\begin{array}{l} \text{As } 25 : 2025 :: \\ \text{Or } \ 1 : \ \ 81 :: \end{array} \left\{ \begin{array}{l} 2 : (162 \text{ for } A, \text{ whereof } \frac{1}{2} \text{ is } 81 \\ 4 : (324 \text{ for } B, \text{ whereof } \frac{1}{4} \text{ is } 81 \\ 5 : (405 \text{ for } C, \text{ whereof } \frac{1}{5} \text{ is } 81 \\ 6 : (486 \text{ for } D, \text{ whereof } \frac{1}{6} \text{ is } 81 \\ 8 : (648 \text{ for } E, \text{ whereof } \frac{1}{8} \text{ is } 81 \end{array} \right.$$

$$\overline{2025}$$

Quest. 33. Two merchants *A* and *B* are in company, the sum of their stocks is 300 *l.* the money of *A* continuing in company 9 months, the money of *B* 11 months, they gain 200 *l.* which they divide equally; the question is to know how much each merchant did put in? Answ. *A* 165 *l.* *B* 135 *l.*

Divide 300 into two such parts which may be in proportion, as 11 to 9, so will the greater part be the stock of *A*, and the lesser the stock of *B*, which stocks being multiplied by their respective times, the products will be equal.

11
9

$$\begin{array}{l} \text{As } 20 : 300 :: \\ \text{Or } \ 1 : \ \ 15 :: \end{array} \left\{ \begin{array}{l} 11 : 165 \text{ for } A. \\ \\ 9 : 135 \text{ for } B. \end{array} \right.$$

Quest. 34. Two merchants, *viz.* *A* and *B*, are in company, *A* put in 325 *l.* more than *B*, and the stock of *A* continued in in company $7\frac{1}{2}$ months, *B* put in a certain ſum which is unknown, and it continued in company $10\frac{3}{4}$ months: After that time they divided the gain equally; the queſtion is, what each merchant put in? Anſw. *B* 750*l.* and *A* 1075*l.*

Divide the product of the difference of their ſtocks multiplied by the time of *A*, by the difference of their times, ſo will the quotient be the ſtock of *B*, which added to 325*l.* gives the ſtock of *A*.

$$325 \times 7\tfrac{1}{2} = 2437\tfrac{1}{2}$$

$3\frac{1}{4}$) $2437\frac{1}{2}$ (750 ſtock of *B*.
Add 325

1075 ſtock of *A*.

Queſt. 35. Two merchants company, *A* put in 20*l* and *B* put in 80 ducats, they gained 67*l.* 10*s.* of which *A* took up 30*l.* what is the value of a ducat? Anſw. 6*s.* 3*d.*

Firſt, find a ſtock for *B*, equivalent to *A*'s ſtock, thus,

As 30 : 20 :: (67,5 — 30 =) 37,5 : 25.
Then as 80 : 25 :: 1 : $\frac{25}{80} = \frac{5}{16}$ = 6*s.* 3*d.*

Queſt. 36. *A*, *B*, and *C*, company, and put in together 3860*l.* *A*'s money was in three months, *B*'s money was in 5 months, and *C*'s money was in 7 months; they gained 234*l.* which was ſo divided as the $\frac{1}{2}$ of *A*'s gain was equal to $\frac{1}{3}$ of *B*'s gain, and $\frac{1}{3}$ of *B*'s gain was equal to $\frac{1}{4}$ of *C*'s gain; what did each merchant gain and put in?

Suppoſe *A*'s gain was 4*l.* then muſt *B* have 6*l.* and *C* 8*l.* according to the tenor of the queſtion, which numbers added together make 18. Then ſay,

As 18 : 234 :: {4, 6, 8} : {52 *A*'s gain. 78 *B*'s gain. 104 *C*'s gain.}
Or 1 : 13 ::

234

Now ſuppoſe *A* put in 140*l.* then the product of *A*'s ſtock and time will be 3 × 140.

Then as 52 : 3 × 140 :: { 78 : $\left(\frac{3 \times 140 \times 78}{52} = \right)$ 630
104 : $\left(\frac{3 \times 140 \times 104}{52} = \right)$ 840 }

Therefore

Therefore if *A* put in 140
B put in $\frac{630}{5} = 126$
And *C* put in $\frac{840}{7} = 120$

386

Then as $386 : 3860 ::$ Or $1 : 10 ::$ $\begin{cases} 140 : 1400 \text{ } A\text{'s stock.} \\ 126 : 1260 \text{ } B\text{'s.} \\ 120 : 1200 \text{ } C\text{'s.} \end{cases}$

Quest. 37. *A* hath $\frac{1}{2}$ of a ship, *B* $\frac{1}{4}$, *C* $\frac{1}{16}$, *D* $\frac{3}{16}$, the master clears 120 *l.* how much must each owner have?

Answer $\begin{cases} A \text{ must have } 60 : 0 \\ B \text{ . . . . } 30 : 0 \\ C \text{ . . . . } 7 : 10 \\ D \text{ . . . . } 22 : 10 \end{cases}$

Quest. 38. There are 7 chests of drawers, in each of which there are 18 drawers, and in each of these there are 6 divisions; in each of which there are 16 *l.* 6 *s.* 8 *d.* how much money is there in the whole? Answ. 12348 *l.*

For $\frac{49}{3} \times 6 \times 18 \times 7 = 49 \times 2 \times 18 \times 7 = 12348$.

Quest. 39. How many men must be employed to reap 420 acres in 17 days; if there were required 34 men to reap 54 acres, in 9 days? Answ. 140 men.

$$\text{For } \frac{34 \times 9 \times 420}{54 \times 17} = 140.$$

Quest. 40. If 1000 *lb.* of beef, or pork, serve 250 seamen 7 days, how many pounds of the same will serve 550 seamen 9 weeks? Answ. 19800 *lb.*

$$\text{For } \frac{550 \times 9 \times 7 \times 1000}{250 \times 7} = 19800.$$

Quest. 41. If a footman travel 240 miles in 12 days, when the days are 12 hours long, how many days may he travel 720 miles in, of 16 hours long? Answ. 27 days.

$$\text{For } \frac{12 \times 12 \times 720}{16 \times 240} = 27.$$

Quest. 42. A carrier received 42 shillings for the carriage of 3 *C.* weight 150 miles, I demand how much he ought to receive for the carriage of 7 *C.* 3 *qrs.* 4 *lb.* 50 miles, at that rate? Answ. 36 *s.* 4 *d.*

Conditional

	L.	*C.*	*M.*
Conditional terms	$\frac{42}{20}$	3	150
Corresponding	A.	$\frac{872}{112}$	50

$$\text{Therefore } A = \frac{872 \times 50 \times 42}{112 \times 20 \times 3 \times 150} = \frac{109}{20 \times 3}$$

Quest. 43. An ancient lady, being demanded how old she was, to avoid a direct answer, said, I have 9 children, and there are three years between the birth of each of them; the eldest was born when I was 19 years old, which is exactly the age of the youngest; how old was the lady? Answ. 62 years.

Quest. 44. There are 100 stones which lie three feet or one yard upon the ground one from the other, and there is one employed to gather up the stones one by one, and bring them to a basket which standeth 3 feet from the first stone; how many yards of ground must he go backwards and forwards in all, before he hath brought the last stone to the basket?

The answer to this question may be obtained by *Art.* 536. Thus,

To fetch the first stone he goes 2 yards.
To fetch the 100th . . . 200 ditto.

Sum of the greatest and least terms 202
Multiply by 100 the number of terms.

Divide by 2) 20200

Answer 10100 yards.

Quest. 45. Lent 109 guineas at 4 *per cent.* which, by the 18th of *August* 1740, was raised by the interest to as many moidores, wanting 2 *s.* 6 *d.* on what day did the bond bear date? Answer *July* 7, 1733.

109 moidores =	147	3	0
deduct		2	6
remains	147	0	6
109 guineas	114	9	0
interest	32	11	6

	l.	*D.*	*l.*
Conditional terms	100 .	365 .	4
Corresponding	114,45 .	A	. 32,575

$$\text{Therefore } A = \frac{100 \times 365 \times 32{,}575}{4 \times 114{,}45} = 2597 \text{ days.}$$

Quest.

Quest. 46. If 3481 soldiers are to be placed in a square battle, how many are to be set in rank or in file? Answ. 59. (for the square root of 3481 is 59.)

Quest. 47. If 100 *l.* being put forth for interest at a certain rate, will at the end of two years be augmented to $112\frac{36}{100}$ *l.* (compound interest, or interest upon interest being computed) what principal and interest will be due at the first year's end?

Answ. 106*l.* (composed of 100 *l.* principal, and 6 *l.* interest) which 106 is a mean geometrical proportional between 100 and 112,36.

That is $100 \times 112{,}36 = 106 \times 106$.

Quest. 48. If 100 *l.* being put forth for interest at a certain rate, will at the end of 3 years be augmented to 115,7625 *l.* (compound interest being computed,) what principal and interest will be due at the first year's end?

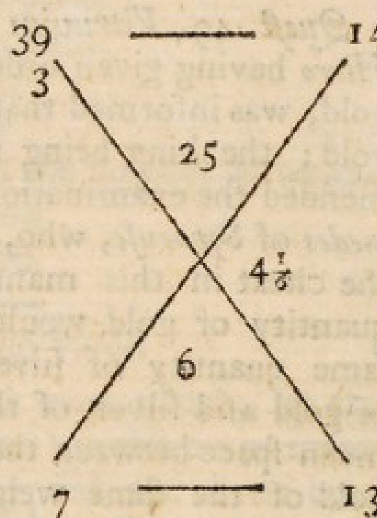
Answ. 105 *l.* (composed of 100 *l.* principal, and 5 *l.* interest) which 105 is the first of two mean proportional numbers between 100 and 115,7625 *l.*

Quest. 49. *Vitruvius* (in *lib.* 9. *cap.* 3.) reports, that king *Hiero* having given orders for the making of a crown of pure gold, was informed that the workman had detained part of the gold; the king being much displeased at the deceit, recommended the examination of the business to the famous *Archimedes* of *Syracuse*, who, without defacing the crown, discover'd the cheat in this manner, *viz.* experience telling him that a quantity of gold would possess less room or space than the same quantity of silver, and consequently that a mixt mass of gold and silver of the same quantity would take up some mean space between the two former; he made a mass of pure gold of the same weight with the crown, likewise another mass of silver of the same weight; then having put the crown, as also the other two masses severally into a vessel filled up to the brim with water, he diligently reserv'd the water flowing over into another vessel, and from those three several quantities of water so expelled, he found out the quantity of gold and of silver in the crown: But since *Vitruvius* has not deliver'd the practical operation, I shall shew the same after the manner of *Cardanus*, *Gemma Frisius*, and other Arithmeticians.

Let us therefore suppose the weight of the crown, as also of the two several masses, to have been 5 *lb.* Suppose also that by putting the mass of gold into the vessel, 3 *lb.* of water was expelled; by putting in the crown $3\frac{1}{4}$ *lb.* and by putting in the mass of silver $4\frac{1}{2}$ *lb.* The question therefore is to know how

much

much gold and how much silver the crown was composed of. This may be resolved by the Rule of False, after this manner: Suppose 3 *lb.* of gold to be in the crown, then there remained 2 *lb.* of silver; now say by the Rule of Three, if 5 *lb.* of gold expel 3 *lb.* of water, how much 3 *lb.* of gold? Answ. $1\frac{4}{5}$ *lb.* Also if 5 *lb.* of silver expel $4\frac{1}{2}$ of water, how much 2 *lb.* of silver? Answ. $1\frac{4}{5}$ *lb.* of water; add therefore the water of the silver and of the gold together, to wit, $1\frac{4}{5}$ and $1\frac{4}{5}$, so there will arise $3\frac{3}{5}$ *lb.* of water: This ought to have been $3\frac{1}{4}$ *lb.* (for so much overflow'd by putting in the crown;) but it is too much by $\frac{7}{20}$; therefore $\frac{7}{20}$ is to be noted with +, for the error of the first position 3 *lb.* Again, feign another quantity of gold to have been in the crown, to wit, 2 *lb.* therefore there remain'd 3 *lb.* of silver; then say, if 5 *lb.* of gold expel 3 *lb.* of water, how much 2 *lb.* of gold? Answ. $1\frac{1}{5}$ *lb.* of water: Also if 5 *lb.* of silver expel $4\frac{1}{2}$ *lb.* of water, how much 3 *lb.* of silver? Answ. $2\frac{7}{10}$, then add $1\frac{1}{5}$ *lb.* unto $2\frac{7}{10}$, the sum will be $3\frac{9}{10}$ of water. This ought to have been $3\frac{1}{4}$ *lb.* but it is too much by $\frac{13}{20}$. Therefore $\frac{13}{20}$ is to be noted with + for the error of the second position 2 *lb.* Here because the errors are Fractions having a common Denominator, I take their Numerators, 7 and 13 instead of the errors, then multiplying cross-wise, to wit, 3 by 13 the product is 39; also 2 by 7, the product is 14, which subtracted from the former product 39, (because the errors are alike,) leaves 25 for a Dividend; also the difference between the errors 7 and 13 is 6 for a Divisor: Lastly, dividing 25 by 6 the quotient is $4\frac{1}{6}$; so much gold, therefore, was in the crown, and consequently (because the weight of the crown was 5 *lb.*) there was $\frac{5}{6}$ of silver, which may be proved thus: Say if 5 *lb.* of gold expel 3 *lb.* of water, how much $4\frac{1}{6}$ *lb.* of gold? Answ. $2\frac{1}{2}$ *lb.* of water: Again, if 5 *lb.* of silver expel $4\frac{1}{2}$ of water, how much $\frac{5}{6}$ of silver? Answ. $\frac{3}{4}$ *lb.* of water, which being added to $2\frac{1}{2}$ *lb.* the sum is $3\frac{1}{4}$ *lb.* of water, to wit, as much as flowed over when the crown was put into the vessel.



Here note, that in making a trial of this nature, there is no necessity that the mass of gold or of silver be of the same weight with the crown, or whatsoever thing is to be examined, but of what notable part of the weight you please.

CHAP.

CHAP. XLIV.

Of Sports *and* Pastimes.

741. PROB. I.

To discover a number which any one shall have in his mind, without requiring him to reveal any part of that or any number whatsoever.

AFTER any one has thought upon a number at pleasure, bid him double it, and to that double bid him add any such even number as you please to assign; then from the sum of that addition let him reject one half, and reserve the other half: Lastly, from this half bid him subtract the number which he first thought upon; then may you boldly tell him what number remains in his mind after that subtraction is made, for it will be always half the number which you assigned him to add.

For Example: Suppose he thought upon 6, the double thereof is 12, to which bid him add some even number at your pleasure, suppose 4, so will the sum be 16, whereof the half is 8; from which if he subtract 6, (the number first thought on) the remainder is 2, (to wit, half the number 4, which was by you assigned to be added;) which remainder you discover, notwithstanding all the operation was performed in his mind, without his making known any number whatsoever. *Note*, That the adding of an even number, as aforesaid, is not of necessity, but only to avoid a fraction that will arise, by taking the half of an odd number.

742. *The Reason of the Rule.*

If to the double of any number (which number for distinction sake I call the first) a second number be added, the half of the sum must necessarily consist of the said first number, and half the second; therefore if from the said half sum the first number be subtracted, the remainder must of necessity be half the second number which was added.

743. PROB. II.

Two numbers, the one even and the other odd, being propoſed unto two perſons, to the end they may (out of your ſight) ſeverally chuſe one of thoſe numbers; to diſcover which of theſe numbers each perſon hath choſen.

Suppoſe you have propounded to *Peter* and *John* two numbers, the one even and the other odd, as 10 and 9, and that each of thoſe perſons is to chuſe one of the ſaid numbers unknown to you. Now, to diſcover which number each perſon made choice of, you muſt take two numbers, the one even and the other odd, as 2 and 3; then bid *Peter* multiply that number which he has choſen by 2; and cauſe *John* to multiply that number which he has pitched upon by 3; that done, bid them add the two products together, and let them make known the ſum to you, or elſe demand of them whether the ſaid ſum be even or odd, or by any other way more ſecret endeavour to diſcover it, by bidding them take the half of the ſaid ſum, for by knowing whether the ſaid ſum be even or odd, you obtain the principal end to be aimed at; becauſe if the ſaid ſum be an even number, then infallibly he that multiplied his number by your odd number, (to wit, by 3) did chuſe the even number (to wit, 10;) but if the ſaid ſum happen to be an odd number, then he whom you cauſed to multiply his number by your odd number, (to wit 3) did infallibly chuſe the odd number (to wit, 9.)

For example: If *Peter* had made choice of 10, and *John* 9, ſuppoſe you required *Peter* to multiply his number 10 by 2, and *John* to multiply his number 9 by 3; the products will be 20 and 27, whereof the ſum is 47, which being an odd number, you may thence conclude that *John*, whom you cauſed to multiply his number by 3, choſe the odd number 9, and therefore *Peter* took 10. But if you had ordered *John* to multiply his number 9 by 2, and *Peter* to multiply his number 10 by 3, the products would have been 18 and 30, whereof the ſum is 48, which is an even number; from whence you may infer, that he that multiplied his number by 3 pitched upon the even number, and therefore *Peter* choſe 10, and *John* 9.

The reaſon of the ſaid Rule will appear from *Note*. 1. and 2. to *Art.* 64. and *Note* 1. and 3. to *Art.* 47.

744. PROB. III.

A certain number of diſtinct things being propounded, to diſpoſe them in ſuch an order, that caſting away always the ninth, or the tenth, or any other that ſhall be aſſigned, to a certain number, thoſe remaining may be ſuch as were firſt intended to be left.

This problem is uſually propoſed in this manner, *viz.* Fifteen *Chriſtians* and fifteen *Turks* being at ſea in one and the ſame ſhip in a terrible ſtorm, and the pilot declaring a neceſſity of caſting the one half of thoſe perſons into the ſea, that the reſt might be ſaved; they all agreed, that the perſons to be caſt away ſhould be ſet out by lot after this manner, *viz.* the thirty perſons ſhould be placed in a round form like a ring, and then beginning to count at one of the paſſengers, and proceeding circularly, every ninth perſon ſhould be caſt into the ſea, until of the thirty perſons there remained only fifteen. The queſtion is, how thoſe thirty perſons ought to be placed, that the lot might infallibly fall upon the fifteen *Turks*, and not upon any of the fifteen *Chriſtians?* For the more eaſy remembering of the Rule to reſolve this queſtion, I ſhall preſuppoſe the five vowels, *a*, *e*, *i*, *o*, *u*, to ſignify five numbers, to wit, (*a*) one, (*e*) two, (*i*) three, (*o*) four, and (*u*) five; then will the Rule itſelf be briefly comprehended in the two following verſes:

From Numbers, Aid, and Art,
Never will Fame depart.

In which verſes you are principally to obſerve the vowels, with their correſpondent numbers before aſſigned; and then beginning with the *Chriſtians*, the vowel *o* (in *from*) ſignifies, that four *Chriſtians* are to be placed together; next unto them the vowel *u* (in *Num.*) imports that five *Turks* are to be placed together; in like manner *e* (in *bers*) denotes two *Chriſtians*; *a* (in *Aid*) one *Turk*; *i* (in *Aid*) three *Chriſtians*; *a* (in *and*) one *Turk*; *a* (in *Art*) one *Chriſtian*; *e* (in *ne*) two *Turks*; *e* (in *ver*) two *Chriſtians*; *i* (in *will*) three *Turks*; *a* (in *Fame*) one *Chriſtian*; *e* (in *Fame*) two *Turks*; *e* (in *de*) two *Chriſtians*; *a* (in *part*) one *Turk*.

745. The invention of the ſaid Rule, and ſuch like, depends upon the ſubſequent proceſs, *viz.* If the number of perſons be thirty, let thirty figures, or cyphers, be placed circularly, or elſe in a right line as you ſee,

oooooooooooooooooooooooooooooo

That done, begin to count from the firſt, and mark the ninth (or what other ſhall be aſſigned by putting a point or croſs over it;) then count forward from that which you have marked, and place another point over the next ninth; and continue to do the ſame, beginning again when you ſhall be at the end (if the cyphers are placed in a right line,) and paſſing over thoſe, which you had already marked, until you have marked the number required, as in the example propounded, until you have marked fifteen; for then all the cyphers marked ſhall be thoſe which muſt be caſt away, and the others thoſe that are to remain. Hence it is evident, that if you obſerve how thoſe cyphers marked, are diſpoſed among thoſe which are not marked, you will eaſily make a Rule for any number whatſoever.

By this invention (as ſome conjecture) the famous hiſtorian *Joſephus* the *Jew*, preſerved his life very ſubtilly in the cave, to which himſelf and forty of his countrymen had fled from the furious and conquering *Romans* at the ſiege of *Jotapata*: For his ſaid countrymen having moſt wickedly reſolved to kill one another, rather than yield to their enemies, he at length (when no arguments that he could uſe would diſſuade them from ſo horrid an act) prevailed with them to execute their tragical deſign by lot; and ſo by the help of the aforeſaid artifice, as we may ſuppoſe, himſelf with one other perſon only remaining alive, after the reſt were inhumanly murdered, they agreed to put an end to the lot, and thereby ſave their lives. This ſtory you may ſee at large in the fourteenth chapter of the third Book of the Hiſtory of *Joſephus*, of the Wars of the *Jews*.

746. PROB. IV.

Many numbers which proceed from 1 *or unity, in a progreſſion, according to the natural order of numbers, (ſuch as theſe,* 1, 2, 3, 4, 5, 6, &c.) *being placed in a round form like a ring; to diſcover which of thoſe numbers any one has thought upon.*

Let any multitude of numbers in the aforeſaid progreſſion, ſuppoſe theſe ten, to wit, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, be marked upon ten ivory-counters (or for want thereof upon ten ſmall pieces of paper) which may be repreſented by theſe ten letters, A, B, C, D, E, F, G, H, K, L, *viz.* ſuppoſe 1 to be writ upon the counter A, 2 upon B, 3 upon C, *&c.* Then having placed thoſe counters circularly as you ſee (with their blank faces uppermoſt, and the figures underneath, that the

A

A
1
L 10 2 B
K 9 3 C
H 8 4 D
G 7 5 E
6
F

ſubtilty of the ſport may the better be concealed) let any one think upon any number of units which does not exceed 10; that done, bid him touch one of thoſe counters at pleaſure, and to the number on the back-ſide of the counter touched (which you cannot be ignorant of, having noted well the place of 1 or A) add ſecretly in your mind, the juſt number of all the counters, and reſerve the ſum; then bid him imagine in his mind the counter touched to be the number which he thought, and from that counter to count backwards, until he has made up the aforeſaid ſum, which you reſerved, ſo will his computation infallibly end on the counter upon which the number thought of is marked.

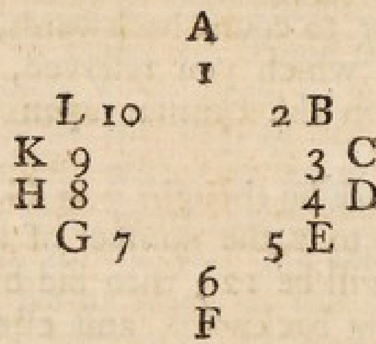
For example, ſuppoſe he thought 7 or G, and that he touched B, to wit, 2; add to 2 the number of all the counters, to wit, 10, ſo the ſum will be 12; then bid him count to 12, beginning at B and going backward, and eſteeming B to be the number thought, to wit, 7; ſo will 8 fall upon A, 9 upon L, 10 upon K, 11 upon H; and laſtly, 12 upon the counter G, which being turned up will ſhew the number thought.

747. The Reaſon of this Rule is not difficult to be apprehended, two principles being preſuppoſed, the one is this; to wit, many counters or things whatſoever being diſpoſed orderly one after the other, in one continued line, whether it be right or circular; if you value or name the firſt counter to be ſome number of units at pleaſure, and continue to count forward according to the natural order of numbers, until another number be named which falls upon the laſt counter: Or if you imagine or name the laſt counter, to be the ſame number of units as before you put upon the firſt, and continue to count backwards to the firſt counter; I ſay, that the ſame number will be named at the end of both theſe computations: For example, in theſe 9 letters, A. B. C. D. E. F. G. H. K. if the letter A be eſteemed to be 4, and from thence you count forwards unto K, according to the natural order of numbers, the letter K will fall upon the number 12. In like manner. if you eſteem

K to be 4, and count backwards from K to A, the letter A will likewiſe fall upon 12.

4.	5.	6.	7.	8.	9.	10.	11.	12.
A.	B.	C.	D.	E.	F.	G.	H.	K.
12.	11.	10.	9.	8.	7.	6.	5.	4.

The other principle is this, to wit, many counters being diſpoſed in a round form like a ring; if you eſteem any one of thoſe counters to be ſome number at pleaſure, and then from that counter if you count circularly, until you end upon the counter where you began, the number laſt named will be equal to the ſum of the number of all the counters, and of the number which you put upon the firſt counter; for example, if D be one of ten letters placed in a circumference, and that imagining D to be 7, you begin with it, and count round the whole circumference, according to the natural progreſſion of



numbers till you end with D where you began; the number 17, which is compoſed of 10 and 7, will neceſſarily fall upon D; for 9 (which is the number of letters in the circumference beſides D) being added to 7 (which was firſt put upon D) makes 16, to which 1 being added, (becauſe D ends as well as begins the circumference) the ſum is 17.

Now theſe two principles being preſuppoſed, it will not be difficult to apprehend the reaſon of the aforeſaid Rule in all caſes that can happen; for imagine that one has thought upon 7, or the counter G, then that counter which he ſhall touch muſt either be the ſame counter G, or ſome other that precedes or follows G.

Firſt therefore, ſuppoſing the counter or number touched to be the ſame with the number thought, the truth of the Rule will be then evident; for by the Rule given, he will begin to count from the ſame G to 17, putting 7 upon G, therefore by the ſecond preſuppoſition the number 17 will fall upon G.

Secondly,

Secondly, imagine that he touched a counter or number following G the number thought, as L or 10; then, according to the Rule, adding 10 (the numbers of all the counters placed circularly) to 10 or L, the (counter touched) bid him count backwards to 20 by beginning at L, and eſteem L to be 7. Now, becauſe by beginning to count at G which is 7, and proceeding to count forward, the number 10 will fall upon L; therefore by the firſt preſuppoſed principle, if we eſteem L to be 7 and count backwards, the number 10 will infallibly fall upon G, and then the number 20 ſhall alſo fall upon the ſame G by the ſecond preſuppoſed principle.

Laſtly, imagine he touched ſome number or counter which precedes 7 the number ſought, as B or 2; then adding 10 to 2, you are to bid him count unto 12, he having firſt imagined B to be the number thought 7; and going backwards to A, L, K, *&c.* Now becauſe by proceeding to count at B, which is 2, and beginning to count forward to C, D, *&c.* the number 7 falls upon G; therefore if one imagine that G is 2, and from thence count backwards towards F, E, *&c.* the number 7 will fall upon B (by the firſt preſuppoſed principle;) therefore when one aſſumes B to be 7, and counts towards A, L, *&c.* to any aſſigned number, it is in effect as much as when one imagines G to be 2, and counts towards F, E, *&c.* unto the ſaid aſſigned number, for each of thoſe computations will end in the ſame point; but it is manifeſt (by the ſecond preſuppoſed principle) that eſteeming G to be 2, and counting towards F, E, D, *&c.* round the whole circumference; the number 12 will fall upon the ſame G. And becauſe G being ſuppoſed to be 2, and counting on the ſame coaſt as before, the number 7 falls upon B; therefore if the computation be continued on the ſame coaſt from B 7, to 12, the number 12 will fall upon the ſame G. So that the practice of this ſport in all its caſes is demonſtrated.

Note, That to the number of the counter touched you may not only add the number of all the counters once (as the Rule directs) but twice, thrice, or more times: For example, B being touched, you may cauſe him to count to 12, or to 22; or to 32, 42, *&c.* the reaſon whereof is evident from the ſecond preſuppoſed principle.

748. PROB. V.

Many numbers being ſhewed by pairs, to wit, two by two, unto any perſon, that he may think upon any one of thoſe pairs at pleaſure, to diſcover the pair that was thought upon.

Let 20 numbers, ſuppoſe theſe, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, be writ upon ivory-counters, (or for want thereof upon ſmall pieces of paper) to wit, 1 upon one counter, 2 upon another, 3 upon a third, *&c.* Then diſpoſe them into pairs as you ſee, *viz.* ſuppoſe 1 and 2 to be one pair, 3 and 4 to be another pair, *&c.* and

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20

of theſe pairs let any one think upon which pair he pleaſes. That done, you are to diſtribute the ſaid 20 numbers into ranks, in form of a long ſquare, until there be 5 numbers in length, and 4 in breath, after this manner, *viz.* lay the three firſt numbers, 1, 2, and 3, in a rank (as you ſee in the ſecond figure) from A towards B; then place 4 under 1, and 5 after 3, (in the ſaid rank A B.) Again, place 6 under 4, and 7 after 5, (in the ſaid rank A B.) Then place 8 under 6, alſo 9, 10, 11, on the right-hand of 4, in the rank C D. Again, place 12 under 9, and 13 on the right hand of 11, in the rank CD, and 14 under 12. Moreover, place 15, 16, 17, on the right-hand of 12, in the rank EF. Laſtly, place 18, 19, 20, on the right-hand of 14, in the rank GH, ſo will all the numbers be ranked as you ſee in the Table. That done, you are to demand of him that thought upon two numbers as aforeſaid, in what rank or ranks the ſaid numbers happen to be found, *viz.*

A	1	2	3	5	7	B
C	4	9	10	11	13	D
E	6	12	15	16	17	F
G	8	14	18	19	20	H

in which of the ranks AB, CD, EF, GH, or in which two of the ſaid ranks: Now if he anſwer, that the two numbers he firſt thought upon are in the firſt rank AB, then 1 and 2 will be the numbers thought of, or kept in mind; if in the ſecond CD, then 9 and 10 ſhall be the numbers thought of; if in the third rank EF, then 15 and 16 will be the numbers thought: If they are in the fourth rank GH, then 19 and 20 ſhall be the numbers thought; but if he ſay that the numbers thought are in different ranks, then you are heedfully to mark the ſaid numbers 1 and 2, 9 and 10, 15 and 16, 19 and 20, which may be called the keys of the ſport, in regard they ſerve not only to diſcover the two numbers thought, when they are both in one and the ſame rank (as aforeſaid;) but even when they are in two different ranks: For in this latter caſe, as ſoon as it hath been declared to you in which two ranks the two numbers thought are placed, you are to take the key of the higheſt of thoſe two ranks, and deſcending in a down-right line from the firſt number of that key to the lower of the ſaid two ranks, you'll there find one of the two numbers thought, and upon the right-hand of the ſecond number of the ſaid key, at the ſame diſtance ſideways, from the ſecond number of the key, (as one of the numbers thought was diſtant from the firſt number of the key,) you will find the other number thought.

For example, ſuppoſe the two numbers thought are 7 and 8, and it is declared to you, that they are in the firſt and fourth ranks; take then the key of the higheſt of thoſe two ranks; to to wit, of the firſt, which is 1 and 2, and deſcending down-right from 1 to the fourth rank, you'll there find 8 one of the numbers thought: Then ſeek ſideways on the right-hand of 2 (the ſecond number of the key) a number as far ſeparated from 2, as 8 is diſtant from 1, and you'll find 7 the other number thought.

Again, ſuppoſe he ſays that the numbers thought are in the ſecond andthird ranks: Take then the key of the ſecond rank which is 9 and 10, and deſcending down-right from 9 to the third rank, you ſhall there find 12, which is one of the numbers thought; then ſeek ſideways on the right-hand of 10, (the ſecond number of the key) a number as far diſtant from 10, as 12 is from 9, and you'll find 11, which is the other number thought.

The reaſon of this will be apparent, from a ſerious conſideration of the placing of the numbers according to the Rules before given: For it is thereby evident, that of the firſt numbers coupled two by two, there can never be found more than one pair in one and the ſame rank; and of all the other pairs, one number is always found in one rank, and the other number in another rank.

Note alſo, that this ſport may be practiſed with divers perſons at once, and not only with 20 numbers, but with any ſuch multitude of numbers as is produced by the mnltiplication of any two numbers that differ by 1, or unity; as 30, which is the product of 5 multiplied by 6, and 42, which is the product of the multiplication of 6 and 7. That which is chiefly to be regarded is, the placing of the numbers in ranks according to the directions before given: And for the more eaſy comprehending of that order, I have, in the following Table, ranked 30 numbers in their due places, which being compared with the former Table, and well viewed, will be a clearer illuſtration than can be expreſſed by many words.

1	2	3	5	7	9
4	11	12	13	15	17
6	14	19	20	21	23
8	16	22	25	26	27
10	18	24	28	29	30

749. PROB. VI.

Three jealous huſbands with their wives, being ready to paſs by night over a river, do find at the river ſide a boat which can carry but two perſons at once, and for want of a waterman they are neceſſitated to row themſelves over the river at ſeveral times: The queſtion is, how thoſe ſix perſons ſhall paſs 2 by 2, ſo that none of the 3 wives may be found in the company of 1 or of two men, unleſs her huſband be preſent?

They muſt paſs in this manner, *viz.* firſt two women paſs, then one of them brings back the boat and repaſſes with the third woman; that done, one of the three women brings back the boat, and ſitting down upon the ground with her huſband, permits the other two men to paſs over to find their wives: Then one of the ſaid men with his wife brings back the boat, and placing her upon the ground, he takes the other man, and repaſſes with him: Laſtly, the woman who is found with the three men enters the boat, and at twice goes to fetch over the other two women.

750. PROB. VII.

A Country-man having a fox, a gooſe, and a peck of corn, in his journey came to a river, where it ſo happened that he could carry but one over at a time. Now, as no two were to be together that might deſtroy each other; ſo he was at his wit's end how to diſpoſe of them; for, ſays he, though the corn cannot eat the gooſe, nor the gooſe eat the fox, yet the fox can eat the gooſe, and the gooſe eat the corn: The queſtion is, how he muſt carry them over?

Firſt he muſt carry over the gooſe, leaving the fox and corn, (for the fox will not eat the corn) then, returning back, he may carry over the fox, bringing the gooſe back again; then leaving the gooſe, he may carry over the corn; laſtly, he muſt return to fetch the gooſe.

751. PROB. VIII.

Two merry companions are to have equal ſhares of eight gallons of wine, which are in a veſſel containing exactly eight gallons: now to make this equal partition they have only two other empty veſſels, of which one contains five gallons, and the other three; the

the queſtion is, how they ſhall exactly divide the wine by the help of thoſe three veſſels?

Firſt, from the veſſel which contains 8 gallons, and is full of wine, let 5 gallons be poured into the empty veſſel of 5, and from this veſſel ſo filled, let three be poured into the empty veſſel of three, ſo there will remain 2 gallons within the veſſel of 5. Then let the three gallons that are within the veſſel of 3 be poured into the veſſel of 8, which will now have 6 gallons within it; that done, let the two gallons which are in the veſſel of 5, be put into the empty veſſel of 3: Then of the ſix gallons of wine that are within the veſſel of 8, fill again the 5, and from thoſe 5 pour out one gallon into the veſſel of 3, which wanted only one gallon to fill it; ſo there will remain exactly 4 gallons within the veſſel of 5, and 4 gallons within the other two veſſels. This queſtion may be reſolved in another way, but I leave that as an exerciſe to the ſagacity of the ingenious reader.

Now though, at firſt ſight, it may be thought by ſome, that the three laſt mentioned Problems cannot be reſolved by any certain Rule, but only by many trials; yet, by infallible argumentation and diſcourſe, the ſolution of thoſe queſtions may be found out, or elſe the impoſſibility of them, if by chance they ſhould be propounded impoſſible; as the moſt ingenious *Gaſper Bachet* has manifeſted in a little book in the *French* Tongue, intitled, *Problemes plaiſans & delectables qui ſe font par les nombres*, from which book I have extracted the contents of this chapter.

CHAP.

CHAP. XLV.

Of MEASURING.

752. THE various kinds of meaſuring, are three, *viz.*
I. *Lineal*, called (by workmen) *Running Meaſure*, in that it reſpects length only, the parts of which are

12 Inches,	1 Foot,
3 Feet,	1 Yard,
$16\frac{1}{2}$ Feet,	1 Rod.

753. Superficial, or Square Meaſure, in that it reſpects Length and Breadth, the parts are, *viz.*

144 Inches,	1 Foot,
72 Inches,	$\frac{1}{2}$ a Foot,
36 Inches,	$\frac{1}{4}$ of a Foot,
48 Inches,	$\frac{1}{3}$ of a Foot,
18 Inches,	$\frac{1}{2}$ a quarter of a Foot,
$272\frac{1}{4}$ Feet,	1 Rod,
$136\frac{1}{8}$ Feet,	$\frac{1}{2}$ a Rod.
Inches 1296, or 9 Feet,	1 Yard.

Note, That in dividing by $272\frac{1}{4}$, which reduces Superficial Feet to Square Rods, workmen take no notice of the $\frac{1}{4}$ of the foot, dividing only by 272, which gives the content a little too much.

754. Solid, or Cube Meaſure, reſpecting Length, Breadth, and Thickneſs, whoſe parts are,

Inches.	
1728,	1 Foot,
1296,	$\frac{3}{4}$ of a Foot,
864,	$\frac{1}{2}$ of a Foot,
432,	$\frac{1}{4}$ of a Foot,
27 Feet,	1 Yard.

755. OF SUPERFICIAL MEASURE.

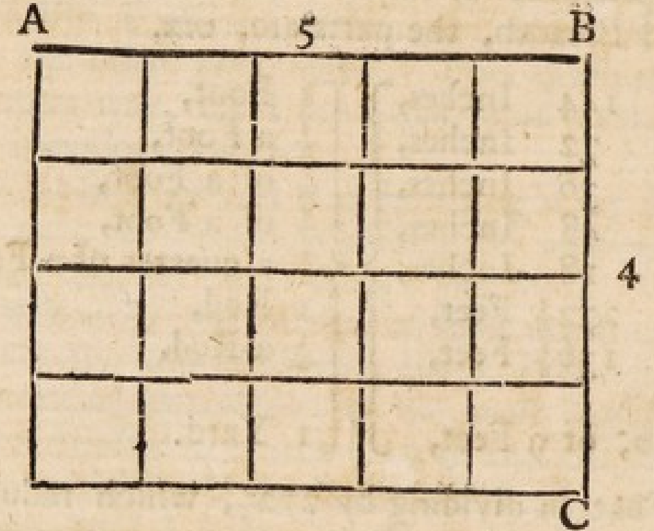
A General Rule.

Multiply the Length by the Breadth, and the Product is the Superficial Content.

Demonſtration.

Demonstration.

For if a line A B, being divided into any number of equal parts be moved conſtantly at right angles with another line, B C, divided into any other number of ſuch equal parts, as are in A B; it will, when advanced to any part of B C, deſcribe thereby as many equal ſquares as there are parts in it: And if moved on farther, to the next part, it will deſcribe as many more ſquares, and ſo the ſame number of ſquares will be as often deſcribed, as there are parts in the line, B C; and conſequently there muſt be as many ſquares in the ſurface, made by theſe lines, as there are units in the product of the parts of the two ſides.



756. Now the content of any thing may be found three ſeveral ways, *viz.*

1. By Decimals,
2. By Aliquot Parts.
3. By croſs Multiplication.

All which I ſhall promiſcuouſly uſe in the following Examples. But the latter of the three is uſed by moſt workmen, except glaziers, whoſe foot being decimally divided, they caſt up the content of their work by Decimals.

757. *Croſs Multiplication*, is performed as in the following Example.

Let it be required to multiply 47 feet 8 inches by 9 feet 4 inches: See the operation.

F.	I.	
47 :	8	
9 :	4	
423		
6 :	0	
15 :	8	
	2 :	8
444 :	10	8

Here the feet are multiplied by the feet, then (croſs-wiſe) *viz.* the 9 feet in the Multiplier by the 8 inches in the Multiplicand, gives 72 inches or 6 feet, and the 47 feet in the Multiplicand being multiplied by 4 inches in the Multiplier, give 15 feet 8 inches. Laſtly, the inches in both the factors, being multiplied one into the other, make 32 parts, or 2 inches, 8 parts, the total of which, *viz.* 444 *f.* 10 *in.* 8 *par.* is the product of 47 feet 8 inches, by 9 feet 4 inches.

758. The Content of the Work, is given either by the Foot, the Yard, the Square, or the Rod, *viz.*

759. By the Foot.

Glazing and Maſonry.

Dimenſions are taken in feet and inches, the Content given *per* Foot ſquare.

In Glazing.

Example. How many Feet of Glazing are there in that pane of glaſs which is 4 feet 5, and 2 feet, 42.

```
 2,42
  4,5
 ----
 1210
 968
 ----
10,890      Anſwer, 10 f. ,89.
```

In Maſonry.

Example. Suppoſe a yard pav'd with free-ſtone, *viz.* length 22 feet 4 inches, breadth 19 feet 7 inches; how many ſquare feet?

```
 22 : 4
 19 : 7
------
198
22
  6 : 4
 12 : 10
      2 : 4
-----------                   Feet. In. Part.
437 : 4 : 4 Facit             437 . 4 . 4
```

760. By the yard.

Painting, Joinery, Plaistering, &c.

Dimensions are taken in feet and inches; but the content given in square yards, found by dividing the superficial feet by 9.

In Painting.

Example. The height of a room being 12 feet 4 inches, and 84 feet 11 inches about, how many square yards?

```
     84 . 11 about
     12 .  4 high
   ----------------
   1019 . 0     = 84 : 11 × 12
     28 . 3 . 8 = 84 : 11 × 1/3
   ----------------
9) 1047 . 3 . 8           Ya.  Feet. Inch. Part.
    116 : 3 . 3 . 8 Answ. 116 . 3  .  3  .  8
```

Joinery.

Example. How many yards of wainscot does that room take up, whose height is 12 feet 3 inches, and compass 104 feet 6 inches?

```
   Feet. Inch.
    104 . 6
     12 . 3
   ------------
   1254 . 0     = 104 . 6 × 12
     26 . 1 . 6 = 104 . 6 × 1/4
   ------------
9) 1280 . 1 . 6
   ------------           Yar. Feet. Inch. Part.
142 . 2 . 1 . 6 Answ.     142 . 2  .  1  .  6
```

Plaistering.

Plaiſtering.

Example. How many yards in a cieling 47 feet 4 inches, 7 parts long, and 18 feet broad?

```
      47 . 4 . 7 long
               6×3=18
    -----------
     284 . 3 . 6
               3
    -----------
  9) 852 . 10 . 6
    -----------
      94 . 6 . 10 . 6    Yar. Ft. In. Part.
```

Facit 94 . 6 . 10 . 6

Under this head you may further note, that if you multiply the $\frac{1}{3}$ part of the length, by the $\frac{1}{3}$ of the breadth, the product will be the content in ſquare yards, it being the ſame to take $\frac{1}{3}$ of lineal, as $\frac{1}{9}$ of ſuperficial meaſure.

761. By the Square.

Partitioning, Flooring, Roofing, Tiling.

Dimenſions are taken in feet and inches, but the content or value is by the ſquare of 10 feet, done by cutting off two figures of the ſuperficial feet towards the right-hand.

Partitioning.

Example. How many ſquares are contained in that work which is 199 feet 10 inches in length, and 10 feet 7 inches high?

```
   199 . 10
    10 . 7
   ----------
  1998 . 4       =10 ⎫
    66 . 7 . 4 = 1/3 ⎬ ×199 : 10
    49 . 11 . 3 = 1/4 ⎭
   ----------
  21,14 . 10 . 10   Squar. Feet. Inch. Parts.
```

Anſw. 21 . 14 . 10 . 10

From which ſometimes the content of doors and other vacancies are deducted.

Flooring.

Example. A floor being 49 feet, 7 inches, 4 parts long, and 26 feet 6 inches broad, how many ſquares?

```
             49 .  7 . 4
             26 .  6 . 0
            ------------
           1289 . 10 . 8
             24 .  9 . 8
            ------------
Answ.     13,14 .  8 . 4
```

Roofing.

Example. Suppose an house 18 feet 4 inches in front, and 37 feet 10 inches in depth, how many squares in the roof?

```
                          37 . 10
                                3×6=18
                         ----------
                         113 .  6
                                6
                         ----------
⅓ of 37 . 10)            681 .  0
                          12 .  7 . 4
                         ------------
                    ½)   693 .  7 . 4
                         346 .  9 . 8 added
Squares. Feet. Inch.     ------------
  10   .  40  .  5     10,40 .  5 . 0
```

Here note, after you have multiply'd the depth by the front, and so found the content of the ground-plot in feet, half of those being added, gives the content of a pitch-roof, which reduce into squares.

762. By the rod, as in

Brick-work,

Dimensions are taken in feet and inches, and the content, given *per* the square of a rod of $16\frac{1}{2}$ feet.

The content of this work is found in the same manner as all the other, to wit, by multiplying the length by the breadth, and only differs from the former in two things.

763. When you have so found the superficial content in feet, divide it by 272, the square feet in one rod, the quote is rods, the content being always so given.

764. That when 'tis more or less than a brick and $\frac{1}{2}$ thick, it must be reduced to that thickness, being always measured by the square rod of a brick and $\frac{1}{2}$. To do which observe,

Rule.

Rule. Multiply the product of the length and height of the wall by as many half bricks as it contains in thickneſs, which product divided by 3, gives the content of the wall 1 brick and ½ thick, which ſuperficial feet divide by 272, the quote is ſquare rods reduced to 1 brick and ½; or more ſhort by the following Table.

Brick.			
For 1	ſubtract	⅓	It will reduce the ſaid thickneſs to 1 brick and ½.
2	add	¼	
3	multiply by	2	
4½	multiply by	3	
6	multiply by	4	

Example 1. Suppoſe a wall of a garden 231 feet about, and 13 feet 4 inches high, and 1 brick ½ thick, how many rods?

```
         213
          13 . 4
      ----------
        2769 . 0
          71 . 0
      ---------- Rods. Feet.
272)  2840 . 0 (10 . 120
         120
```

Example 2. How many rods of brick-work in that wall which is 40 feet 7 inches long, and 11 feet high, being one brick thick?

```
      Feet. Inches.
        40 . 7
        11 . 0
      ---------
   3) 446  . 5
      148  . 9 . 8
      ------------
272)  297  . 7 . 4               Rod. Ft. Inch. Parts.
        1  . 25 . 7 . 4  Anſwer   1 . 25 . 7 . 4
```

Example 3. If a wall be 254 feet about, and 12 feet 7 inches high, and 3 bricks thick, how many rods?

```
          254
          12 . 7
        ---------
        3048 . 0
         127 . 0
          21 . 2
        ---------
        3196 . 2
               2
        ---------   Rods. Feet. Inches.
   272) 6392 . 4    (23 : 136 : 4
```

765. Practical Questions in the Mensuration of Superficial Figures.

Quest. 1. If the side of a square Superficies be 3 feet what is the area or content of that Superficies? Or (which is the same thing) how many Squares, every one of which is a foot square are contained in that Superficies?

Answer 9 square feet, which content is found out by multiplying the given side 3 by itself, *viz.* 3 multiplied by 3 produces 9.

In like manner, if the side of a square pavement of stone be 15.7 feet, the superficial content of that pavement will be 246.49 feet, that is, 246 feet and an half very near, (for 15.7 multiplied by itself, produces 246.49.)

Likewise a square piece of wainscot, whose side is 3.24 yards, will be found to contain 10.4976 yards, or 10 yards and an half almost; for 3.24, multiplied by itself, to wit, by 3.24, will produce 10.4976.

Also if the side of a square piece of land be 37,25 perches, the content in square perches (neglecting the Fraction in the Product) will be found 1387, which being reduced will give 8 acres, 2 roods, and 27 perches for the content of that square piece of land.

Quest. 2. If a long square be 8 feet in length, and 5 feet in breadth, what is the superficial content?

Answ. 40 feet; which content is found out by multiplying the length by the breadth, *viz.* 8 multiplied by 5 produces 40. So if one of the lights of a glass-window supposed to be in form of a long square, has for its length 3.06 feet, and breadth 1.47 feet, the content of that glass will be 4.4982 feet, or 4 feet and an half almost, for 3.06 multiplied by 1.47 produces 4.4982.

In

In like manner, if there be a piece of wainſcot, plaiſtering, or any other ſuperficies in form of a long ſquare, which is in length 6.325 yards, and in breadth 3.214 yards; the ſuperficial contents will be found 20.32 + yards, that is, 20 yards, one quarter of a yard, and ſomewhat more, for 6.325 multiplied by 3.214 produces 20.32 +.

Likewiſe a piece of Tiling in form of a long ſquare, whoſe length is 18.5 feet, and breadth 11.7 feet, will be found to contain 216.45 ſquare feet, which may be reduced to 2.1645 ſquares of tiling, by allowing (according to cuſtom) 100 ſquare feet to one ſquare of tiling.

Alſo if a piece of land in form of a long ſquare be 48.75 perches in length, and 36.25 in breadth, the area or content in perches will be found 1767.18 +, which 1767 perches being reduced will give 11 acres and 7 perches for the content of that piece of ground.

Queſt. 3. If it be required to ſet forth in a meadow one acre of graſs to lie in the figure of a long ſquare, and that the length of it be limited or agreed to be 20 perches, what muſt the breadth be?

Anſw. 8 perches, which breadth is found out by dividing 160 (the number of ſquare perches contained in an acre) by the given length 20. If two acres were required, then 320 (to wit, twice 160) muſt be divided by the given ſide, whether it be the length or breadth; ſo if 7.25 perches be preſcribed for the breadth of two acres, the length muſt be 44.13 + perches.

In like manner, if the breadth of a board be 1.32 foot, and it be demanded how far one ought to meaſure along the ſide of it to have a ſuperficial foot, or a foot ſquare of that board; divide 1 by the given breadth, ſo you'll find in the quotient this decimal fraction .757 +, which repreſents three quarters of a foot, or nine inches and ſomewhat more, and ſo much in length ought to be meaſured along ſide of that board to make a ſuperficial foot. Likewiſe if the breadth of a board be given in inches, then 144 (the number of ſquare inches contained in a ſuperficial foot ſquare) being divided by the given breadth, the quotient will ſhew how many inches ought to be meaſured along the ſide of that board to make a ſuperficial foot; ſo the breadth of a board being 9 inches, the length forward to make a ſuperficial foot will be found 16 inches.

Queſt. 4. If a piece of arras-hangings, in the form of a long ſquare, has for its length 6$\frac{1}{4}$ yards *Engliſh*, and breadth 4 yards; how many ſquare ells, or ſticks *Flemiſh* are contained in that piece, when the length of a *Flemish* ell is equal to $\frac{3}{4}$ yard *English?* Answ. $44\frac{4}{9}$ square ells or sticks *Flemish.*

Forasmuch as by supposition, a *Flemish* ell in length, has such proportion to an *English* yard in length, as 3 to 4, and consequently the square of the one to the square of the other, as 9 to 16: Therefore in a direct proportion, as 9 is to 16; so is any given number of square yards *English*, to a number of square ells *Flemish*, which will take up equal space with the said square ells *English.* Also in a direct proportion as 16 is to 9, so is any given number of square ells *Flemish*, to a number of square yards *English*, which will take up an equal space with the said *Flemish* ells: Therefore to resolve the aforesaid question, first find the number of square yards *English*, contained in the said piece of arras, by multiplying the length and breadth in yards mutually one by the other, then proceed according to the aforesaid proportion; so the work will stand thus:

I. $6\frac{1}{4} \times 4 = 25$ square yards *English.*
II. $9 \; 16 :: 25 : 44\frac{4}{9}$ square ells *Flemish.*

Quest. 5. If a piece of tapestry, in the form of a long square, be in length $15\frac{1}{4}$ ells *Flemish*, and in breadth $4\frac{1}{3}$ ells *Flemish*, how many square yards *English* are contained in that piece, when 4 ells *Flemish* in length are equal to 3 yards *English?* Answ. $37\frac{11}{64}$ square yards *English.*

I. $15\frac{1}{4} \times 4\frac{1}{3} = 66\frac{1}{12}$.
II. $16 : 9 :: 66\frac{1}{12} : 37\frac{11}{64}$.

766. If the three sides of a piece of land that lies in form of a Triangle be 15 perches, 14 perches, and 13 perches, what is the area or number of square perches contained in that Triangle?

Answ. 84 perches, or half an acre and four perches, which content is found out by this Rule, *viz.*

From half the sum of the three sides of any plane Triangle, subtract each of the three sides severally, and note the three remainders; then multiply the said half sum and those three remainders one into the other (according to the Rule of continual Multiplication;) that done, extract the square root of the last product, so shall such square root be the area or content of the Triangle.

Perches.

	Perches.
The three sides of a Triangle —	15 14 13
The sum of three sides — —	42
The half of the sum — —	21
The three remainders found out by subtracting every side from the half sum — —	6 7 8
The product arising from the continual Multiplication of the four last numbers — —	7056
The square root of which product is the content required, to wit, — — —	84

Another Example.

	Perches.
The three sides of a Triangle — —	120 , 5 112 , 6 90 , 3
The sum of the three sides — —	323 , 4
The half of that sum — —	161 , 7
The three remainders found by subtracting each side from the half sum — —	41 , 2 49 , 1 71 , 4
The product arising from the continual Multiplication of the four last numbers — — —	 23355380,1096
The square root of that product —	4832,74+

Wherefore I conclude, that the content of a plain Triangle, whose three sides are 120.5 perches, 112.6 perches, and 90.3 perches, is 4832.74+ perches, which reduced, give 30 acres and 32 perches, (the fraction of a perch being neglected.)

Now since every irregular piece of ground may be divided into Triangles; for a four-sided field will be divided into two Triangles by one imaginary straight line leading overthwart from corner to corner, called a Diagonal Line; a five-sided field into three Triangles by two Diagonals; a six-sided ground into four Triangles by three Diagonals, *&c.* the Rule before given will be of excellent use to find out the contents of large fields, especially if the land be of a dear value; as also when any controversy arises by reason of the different admeasurements of surveyors of land: For if the sides of those Triangles be measured in the field, and their lengths be agreed on, all artists to whom the reason of the Rule before given is known, will agree in one and the same content. But yet this way of measuring presupposes that there is no Obstacle, as water, wood, or other impediment, to hinder the measuring of the sides of those Triangles into which the field is divided as aforesaid.

767. If the diameter of a circle be 28.25, what is the circumference?

Answ. 88.749892: For as 1 is to 3.1415927, so is the diameter to the circumference: Therefore always multiplying the diameter given by the said 3.1415927, the product will be the circumference required.

768. If the diameter of a circle be 28.25, what is the superficial content of that circle?

Answer. 626.7968: For as 1 is in proportion to .7853982, so is the square of the diameter to the superficial content. Therefore multiplying the said decimal fraction .7853982 by the square of the given diameter (which square is the product of the multiplication of the diameter by itself,) the product shall be the superficial content required.

769. If the diameter of a circle be 28.25, what is the side of a square which may be inscribed within the same circle?

Answ. 19.97577: For the square root of half the square of the diameter, or the square root of the double of the square of the semi-diameter, will be the side of the inscribed square sought. Otherwise, as 1 is to .707166, so is the diameter to the side required. Therefore if you multiply (always) the said .707166 by the diameter given, the product will be the side of the inscribed square required.

770. If the circumference of a circle be 88.75, what is the diameter?

Answ. 28.25: For as 1 is to .3183099, so is the circumference to the diameter. Therefore if .3183099 be multiplied by the given circumference, the product shall be the diameter required.

771. If the circumference of a circle be 88.75, what is the superficial content of that circle?

Answ. 626.797: For as 1 is to .0795775, so is the square of the circumference to the superficial content. If therefore .0795775 be always multiplied by the square of the given circumference, the product will be the superficial content sought.

772.

772. If the circumference of a circle be 88.75, what is the ſide of a ſquare that may be inſcribed within the ſame circle?

Anſw. 19,97566: For as 1 is to .225078, ſo is the circumference to the ſide required. Therefore if 225078 be always multiplied by the circumference given, the product will be the ſide of the inſcribed ſquare ſought.

773. If the ſuperficial content of a circle be 626.8, what is the diameter?

Anſw. 28.25: For as 1 is to 1.2732395, ſo is the content to the ſquare of the diameter. Therefore multiply 1.2732395 by the given content, the ſquare root of that product ſhall be the diameter required.

774. If the ſuperficial content of a circle be 626.8, what is the circumference?

Anſw. 88.75: For as 1 is to 12.566371, ſo is the content to the ſquare of the circumference. If therefore 12.566371 be always multiplied by the given content, the ſqaure root of the product will be the circumference required.

775. If the ſuperficial content of a circle by 626.8, what is the ſide of a ſquare equal to the ſame circle?

Anſw. 25.03598: For the ſquare root of the given content is the ſide of the ſquare required.

776. Of Solid Meaſure, which reſpects Length, Breadth, and Thickneſs.

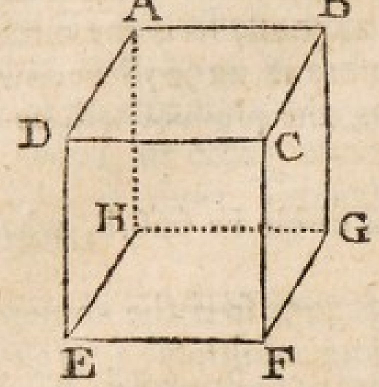
Rule. Multiply the Length by the Breadth, and that product by the Depth.

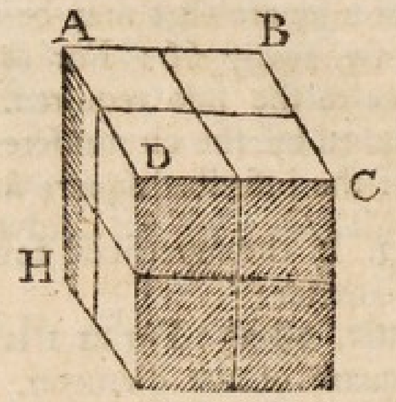
Otherwiſe, Multiply the area of the baſe by the Height, the product gives the ſolid content, which if in feet, divide by 27, the ſolid feet in one yard, which reduces them to ſolid yards, if in inches, by 1728, gives ſolid feet, *&c.*

Demonſtration.

For a ſurface ABCD being imagined to move, ſuppoſe upwards or downwards, ſo that its points A, B, C, D, deſcribe the lines AH, BG, CF, DE, and conſequently each line therein deſcribes other ſurfaces, *&c.* there will be generated by this motion, a magnitude called a ſolid, having three dimenſions, *viz.* Length, Breadth, and Thickneſs: Therefore that ſurface ABCD, being divided into little ſquare areas, and moving the height, AH, divided by a like meaſure for length, may be conceived to produce as many ſmall cubes, as is the number of the little areas, in the ſurface ABCD, multiplied by the number of the diviſions in the height AH.

Examp.





Examp. If the ſide of a cube be 12 inches, how many cubical inches are contained in that cube?

Anſwer 1728. What a cube is, may be well repreſented by a dye, which is a little cube itſelf, being a rectangular or ſquare ſolid, that has an equal length, breadth and depth, and is comprehended under ſix equal ſquares: Now if the ſide of one of thoſe equal ſquares (which is alſo the ſide of the cube,) be 12 inches, the ſuperficial content of that ſquare will be 144 ſquare inches; for 12 multiplied by 12 produces 144, which multiplied by the depth 12 inches, produces 1728 cubical inches, and ſuch is the ſolid content of that cube, whoſe ſide is 12 inches: So that by one foot of timber or ſtone, in whatſoever kind of ſolid it be found, is underſtood a cube, containing 1728 cubical or dye-ſquare inches, and conſequently half a foot ſolid contains 864 cubic inches, and a quarter of a foot ſolid contains 432 cubic inches.

In like manner, if the ſide of a cube of ſtone be 2.53 feet the ſolid content of that cube will be found 16.194 + feet; for 2.53 being multiplied by itſelf, produces 6.4009 ſuperficial feet, which product being multiplied by the ſaid 2.53, will produce 16.194 + ſolid feet.

Alſo if the ſide of a cube of ſtone or wood be 6 inches, or .5 feet, the ſolid content will be found 216 cubic inches, or .125 parts of a foot ſolid; (for 6 multiplied cubically produces 216; likewiſe . 5 multiplied cubically produces .125;) whence it may be inferr'd, that 8 little cubes of ſtone or wood, every one of which is half a foot, or 6 inches ſquare, are contained in a foot of ſtone or timber; for 8 times 216 produces 1728 (being the number of cubic inches contained in a foot ſolid;) likewiſe 8 times .125 produces 1, (to wit, one entire foot ſolid.)

777. If the breadth of a ſquared piece of timber, ſuppoſed to be ſtraight and terminated at both ends by two equal ſquares, be 1.55 feet, the depth alſo 1.55 feet, and the length 17.33 feet, how many cubic feet are contained in that piece of timber?

Anſwer. 41.635 feet; that is, 41 feet and an half, and about half a quarter of a foot: Which ſolid content is found out by this Rule, *viz.* multiply the breadth 1.55 by the depth 1.55, the product will be 2. 4025 ſuperficial feet, which is the content of the baſe, (that is, the area of either of the two equal ſquares at the ends of the piece:) Laſtly, multiplying the ſaid baſe 2.4025 by the length 17.33, the product will be 41.635+, which is the ſolid content required.

In like manner, if the breadth of a ſquare piece of timber, ſuppoſed to be ſtraight and terminated at both ends by two equal long ſquares (which are called the baſes) be 2.34 feet, the depth 1.61 feet, and the length 17.58 feet, the ſolid content will be 66.23 + feet; for (as before) multiplying the breadth by the depth, and that product by the length, the laſt product will be the ſolid content required.

778. If the breadth, as alſo the depth of a ſquared piece of timber having equal ſquare baſes, be 1.55 feet how far ought one to meaſure along the length of that piece of timber to make a foot ſolid?

Anſwer .4162 parts of a foot, or 5 inches very near; which decimal is thus diſcovered, *viz*, firſt find the ſuperficial content of the baſe, which will be 2.4025 (for 1.55 multiplied by 1.55 produces 2.4025:) Then dividing 1 (to wit, 1 ſolid foot) by the baſe 2.4025, the quotient will be .4162, or 5 inches almoſt, and ſo far ought to be meaſured along the length of the piece to make a foot ſolid. In like manner, if the breadth be 2.34 feet, and the depth 1.61 feet, the length forward along the piece to make one ſolid foot, will be found .2654 parts of a foot, or three inches and almoſt $\frac{1}{3}$ part of an inch.

779. If a ſtraight ſquared piece of timber be terminated by unequal baſes, whereof one contains 1,92 ſuperficial feet, the other .85 feet, and the length of that piece of timber be 17.4 feet, what is the ſolid content, or how many cubical feet are contained in that piece of timber?

Anſwer 23.474+feet; (found out by one of Mr. *Oughtred*'s Rules for meaſuring a ſegment of a pyramid in *Problem* 21. Chap. 19. of his *Clavis Mathemat.*) The Rule is this.

Multiply

Multiply the greater base by the less, and extract the square root of that product; then multiply the sum of the two bases and that square root by one third part of the length of the solid proposed, so will the last product be the solid content required.

Example.

The greater base — — —	1 .92
The lesser base — —	0 .85
The product of the Multiplication of those two bases — — —	1. 6320
The square root of that product — —	1 .2775
The sum of that square root and the two bases	4 .0475
One third part of the length is — —	5 .8
The product of the Multiplication of the two last numbers is the solid content required —	23 .4755

780. A pyramid is a solid comprehended under plain surfaces, and from a triangular, quadrangular, or any multangular base, diminishes equally less and less, till it finish in a point at the top; now if the superficial content of the base of a pyramid be 5.756 feet, and the height of it 14.25 feet, (which height is the length of the perpendicular line that falls from the top of the pyramid to the base;) what is the solid content of that pyramid?

Answer 27.341 + feet; for if the area of the base of a pyramid be multiplied by one third part of the height thereof, the product will be the solid content of the pyramid; therefore $5.756 \times 4.75 = 27.341$ feet = the solidity of the pyramid proposed.

Note, If a pyramid be cut into two segments by a plane parallel to the base, one of those segments will be a pyramid, and the other will have two unequal bases; for the measuring of which latter segment, a rule has been already given in *Art.* 777. the area of each base being known.

781. A Cone is a Solid, which has a circle for its base, from whence it grows equally less and less (like a round steeple of a church) till it end in a point at the top; now if the area of the base of a Cone be 5.756 feet, and the height of it be 14.25 feet, what is the solid content of that Cone?

Answ. 27.341 feet; for if the area of the base of a Cone be multiplied by one third part of the height thereof, the product shall be the solid content of the Cone.

Note, If a Cone be cut into two segments by a plane parallel to the base, one of those segments will be a Cone, and the other segment will have two unequal bases which are circles; the

the ſolidity of which latter ſegment may be found out by the rule before given in *Art.* 777, the area of each baſe (or circle) being known.

782. A Cylinder is a ſolid, which may be well repreſented by a ſtone-roll, ſuch as are uſed in gardens for the rolling of walks. Now if the circumference of a Cylinder be 4.57 feet, and the length 3.25 feet, what is the ſolid content of that Cylinder?

Anſw. 5.4 + feet, thus found out: Firſt, by the help of the given circumference 4.57, find out the ſuperficial content of that circle, (being the baſe of the Cylinder;) which content (by *Art.* 769.) will be found 1.661967 foot; then multiplying the ſaid 1.661967 by the given length 3.25, the product will be 5.401393, which is the ſolid content required.

783. If the Baſe of a Cylinder by 1.6619 feet, how much in length of that cylinder will make a foot ſolid?

Anſwer .6017 parts of a foot; for 1 (to wit, 1 ſolid foot) being divided by the baſe 1.6619, gives in the quotient the decimal .6017 for the length required.

784. A globe is a perfect round body contained under one ſurface; in the middle of the globe there is a point called the center, from whence all ſtraight lines drawn to the outſide are of equal length, and called ſemi-diameters, the double of any one of which is equal to the diameter of the globe; now if the diameter of a globe of ſtone be 1.75 foot, how many feet ſolid are contained in that globe?

Anſw. 2.8061 feet; for as 1 is to .5235988, ſo is the cube of the diameter to the ſolid content of the globe: Therefore multiplying the cube of the diameter by the ſaid decimal .5235988, the product will be the ſolid content required: So the diameter 1.75 being firſt multiplied by itſelf, the product will be 3.0625, which multiplied by the ſaid 1.75, gives in the product 5.359375, to wit, the cube of the diameter, which being multiplied by .5235988, the product thence ariſing will be 2.80616, which is the ſolidity of the globe propounded.

785. What is the diameter of a globe of ſtone, which contains 4 cubical or ſolid feet?

Anſwer 1.96 + foot; for as 1 is to 1.9098593, ſo is 4 (the ſolid content given) to a fourth proportional, to wit, 7.639437, whoſe cubic root is 1.96949, the diameter required.

786. In Timber and Stone. The Content is given by the Solid Foot.

Examp. 1. In a ſtone { Length 5 : 9 / Breadth 3 : 11 / Depth 2 : 8 } how many ſolid feet?

```
 3 . 11
 5 . 9
-----------
19 . 07
 1 . 11 . 6
 0 . 11 . 9
-----------
22 .  6 . 3
 2 .  8 . 0
-----------
45 .  0 . 6
11 .  3 . 1 . 6
 3 .  9 . 0 . 6
---------------
60 .  0 . 8 . 0
```

Anſw. | Feet. 60 | Inches. 00 | Parts. 8

787. In Digging. The Content is given by the Solid Yard.

Examp. 2. A vault digged 9 feet deep, $4\frac{1}{2}$ feet long, and 3 feet 9 inches broad, how many ſolid yards?

```
  4 . 6 Length
  9 . 0 Depth
-----------
 40 . 6
  3 . 9 Breadth
-----------
121 . 6
 20 . 3
 10 . 1 . 6
-----------
27) 151 . 10 . 6
```

Anſwer | Yards. 5 | Feet. 16 | Inches. 10 | Parts. 6

788. If a bullet of braſs of 8 inches diameter weigh 72 pounds, what ſhall a bullet of braſs weigh whoſe diameter is 4 inches?

Since like ſolids are in triple proportion to their homologous ſides, diameters, lines, &c. it holds,

As

As the cube of the diameter given,
To the weight thereof;
So is the cube of the other diameter,
To the weight thereof.

That is, As $8\times8\times8 : 72 :: 4\times4\times4 : 9$

$$\text{For } \frac{72\times4\times4\times4}{8\times8\times8}=\frac{72\times1\times1\times1}{2\times2\times2}=\frac{72}{8}=9.$$

789. If a ſhip of 100 tons be 44 feet long at the keel, of what length ſhall the keel be of a ſhip of 220 tun?

As $100 : 44\times44\times44 :: 220 : 44\times44\times44\times\frac{22}{10}$.
That is, $44\times44\times44\frac{22}{10}\times$ is the cube of the length required,

And $\sqrt[3]{44\times44\times44\times\frac{22}{10}}=44\times\sqrt[3]{2,2}=$ the length.

But $\sqrt[3]{2,2}=1,30059$; therefore $44\sqrt[3]{2,2}=57,22596$.

790. The ſide of the cube being given to find the ſide of that cube that ſhall be double, treble, quadruple, &c. in quantity to the given cube.

Multiply the cube-root of 2, 3, 4, &c. by the ſide of the given cube, and the product will be the ſide of the cube required, &c.

Examp. There is a cubical veſſel, whoſe ſide is 12 inches, and it is required to find the ſide of that veſſel which ſhall contain 3 times as much.

The cube-root of 3 is (*per* Table in *Art.* 482.)	1,442250
Multiply by	12
Anſwer	17,307000

If you would find a ſide that ſhall contain $\frac{1}{2}$ as much, $\frac{1}{3}$ as much, $\frac{1}{4}$ as much, &c. then divide the ſide of the given cube by the cube-root of 2, 3, 4, &c.

791. The concave diameter of two guns, being known, together with the quantity of gunpowder ſufficient to charge one, to find what will be ſufficient to charge the other.

Rule. The capacities are one to another, as the cubes of their diameters.

Examp. If 45 pounds of gunpowder be ſufficient to charge a gun, whoſe concave diameter is $1\frac{1}{2}$ inch, how much gunpowder will ſuffice to charge a gun whoſe concave diameter is 7 inches?

As

As $\frac{3}{2}\times\frac{3}{2}\times\frac{3}{2}$: 45 :: $7\times7\times7$: $45\times\frac{14}{3}\times\frac{14}{3}\times\frac{14}{3}$.

That is $\frac{5\times14\times14\times14}{3} = 4573\frac{1}{3}$.

792. Questions relating to *Solids*.

1. If a piece of timber 48 feet long, 6 inches broad, and 14 inches thick, cost 10*l.* what is the value of five such pieces which are 6 feet long, 4 inches broad, and 7 inches thick?

	Pieces.	*Feet.*	*Br.*	*Thickn.*	*L.*
Conditional terms	1	48	6	14	10
Corresponding	5	6	4	7	A.

Therefore $A = \frac{5\times6\times4\times7\times10}{48\times6\times14} = \frac{25}{12} = 2l.\ 1s.\ 8d.$

2. If 300 men, in 15 days, dig a trench 5600 feet long, 6 deep, and 12 wide, how long must that trench be, whose depth is 8 feet, width 14 feet, dug by 2700 men in 25 days.

	Men.	*Days.*	*Len.*	*Dep.*	*Wid.*
Conditional terms	300	15	5600	6	12
Corresponding	2700	25	A.	8	14

Therefore $A = \frac{2700\times25\times5600\times6\times12}{300\times15\times8\times14} = 54000.$

3. If 248 men, in $5\frac{1}{2}$ days, dig a trench $23\frac{1}{4}$ yards long, $2\frac{1}{3}$ deep, and $3\frac{2}{3}$ wide; in how many days will 24 men dig a trench $33\frac{3}{4}$ yards long, $3\frac{1}{2}$ deep, and $5\frac{3}{5}$ wide?

	Men.	*Days.*	*Len.*	*Dep.*	*Wid.*
Conditional terms	248	$\frac{11}{2}$	$\frac{93}{4}$	$\frac{7}{3}$	$\frac{11}{3}$
Corresponding	24	A.	$\frac{135}{4}$	$\frac{7}{2}$	$\frac{28}{5}$

$A = \frac{248\times11\times135\times7\times28\times4\times3\times3}{2\times4\times2\times5\times24\times93\times7\times11} = 189.$

4. If 24 men working 189 days, 14 hours each day, dig a trench $33\frac{3}{4}$ yards long, $3\frac{1}{2}$ deep, and $5\frac{3}{5}$ wide; how many hours per day must 217 men work to dig a trench $23\frac{1}{4}$ yards long, $2\frac{1}{3}$ deep, and $3\frac{2}{3}$ wide in $5\frac{1}{2}$ days?

	Men.	*Days.*	*Hours.*	*Len.*	*Dep.*	*Wid.*
Conditional terms	24	189	14	$\frac{135}{4}$	$\frac{7}{2}$	$\frac{28}{5}$
Corresponding	217	$\frac{11}{2}$	A.	$\frac{93}{4}$	$\frac{7}{3}$	$\frac{11}{3}$

$A = \frac{24\times189\times14\times93\times7\times11\times2\times4\times2\times5}{4\times3\times3\times217\times11\times135\times7\times28} = 16.$

So

To find the ſide of a cube that ſhall be equal in ſolidity to any given ſolid, as a Globe, Cylinder, Priſm, Cone, or ſuch like.

The Cube-root of the ſolid content of any ſolid body given, is the ſide of a Cube of equal ſolidity. So if the content of a Globe were found to be 15625 ſolid inches, ſeek the Cube-root of 15625, which is 25, which is the ſide of a Cube of equal capacity.

INDEX.

Amount

INDEX.

 Comparative

INDEX.

Meafuring

Painters-

INDEX.

The

INDEX.

Questions

I N D E X.

Table

Tare

FINIS.

Edw

Edw^d: Grant November the 2^nd In the year of our Lord 1763

A very useful Book as you will find
When you have it all Grafted in your mind

Grant