

A treatise of such mathematical instruments as are usually put into a portable case, containing their various uses. In arithmetic, geometry, trigonometry, architecture, surveying, gunnery, etc., with a short account of the authors who have treated on the proportional compasses and sector. To which is now added an appendix containing the description and use of the gunner's callipers / [J. Robertson].

Contributors

Robertson, J. 1712-1776.

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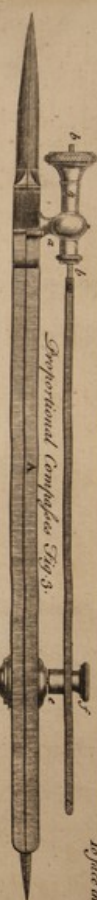
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Proportional Compass Fig. 3.

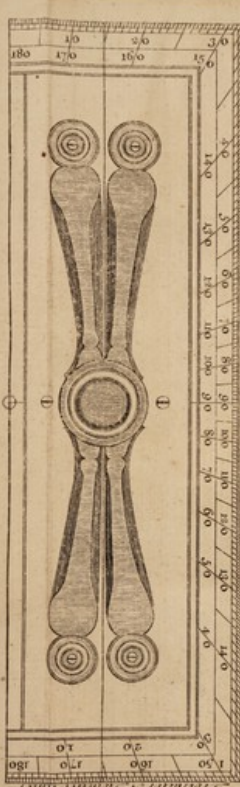


Proportional Compass Fig. 1.

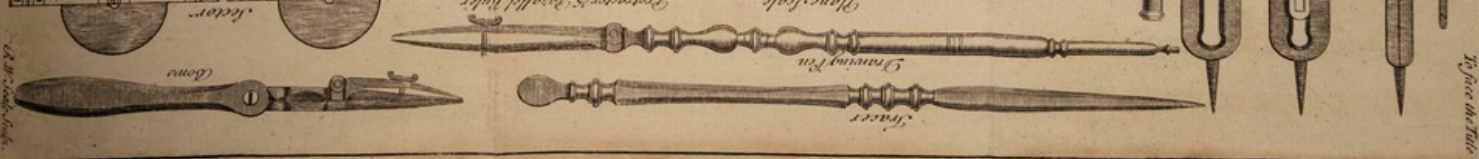


Compass

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T R E A T I S E

OF SUCH

Mathematical Instruments

As are usually put into a

P O R T A B L E C A S E,

Containing their various Uses in

ARITHMETIC,		ARCHITECTURE,
GEOMETRY,		SURVEYING,
TRIGONOMETRY,		GUNNERY, &c.

With a short Account

Of the AUTHORS who have treated on the
P R O P O R T I O N A L C O M P A S S E S
And S E C T O R.

To which is now added

An A P P E N D I X;

Containing, the Description and Use of the
G U N N E R S C A L L I P E R S.

The second Edition, with many Additions.

By J. ROBERTSON, F. R. S.
Master of the ROYAL-ACADEMY at *Portsmouth*.

L O N D O N :

Printed for T. HEATH and J. NOURSE in the
Strand; J. HODGES on *London-Bridge*, and
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ERRATA.

Page xvi. read March 5, 1757. p. 10. Line 16. for Plate iii read Plate iv. p. 12. l. 15. for KF r. KC. p. 26. l. 26. for into r. in. p. 40. l. 19. for Scale r. Scales. p. 63. l. 6. dele K. l. 29. for B3 r. B5. p. 80. l. 9. r. As the divisor, is to unity; so is the dividend, to the quotient. And as the divisor, is to the dividend; so is unity, to the quotient. p. 81. l. 19. for 25 r. 35. p. 95. l. 23. for xvii r. xviii. p. 96. l. 12. read Ex. 1. Pl. vi. Fig. 26. p. 100. l. 4. for N. r. x. l. 20. for N. r. x. p. 101. l. 34. for N r. x. l. 35. for M r. z. l. 36. for N r. x. p. 107. l. 1. for xviii r. xix. p. 109. l. 17. for on B read on P. p. 110. In the computation for the letter C read D. p. 125. l. 24. for xix read xx.

Page 128. p. 12. for $A_c = \frac{1}{n+1} N$ read $A_c = \frac{n}{n+1} N$.

Line 16 for $\frac{n}{1}$ read $\frac{1}{n}$. p. 145. l. 19. for $\frac{2}{3}$ r. $\frac{3}{2}$. p. 149. l. 24. for balks r. bulks. p. 155. l. 19 for 1. 8. r. 1. 0.



T O

PETER DAVALL, Esq;

SECRETARY to the ROYAL SOCIETY.

S I R,

IT is no new thing for a lover of Science to address his productions to a friend eminently distinguished for his general knowledge, as well as particular skill in the parts whereon the Author writes : On this account I heartily wish, that instead of the subjects contained in the following sheets, I had a work of a more elevated kind wherewith to do greater honour to the name of my friend ; however, such as they are, I hope they will, with your usual frankness and good-nature, be accepted. Indeed I must observe, that the late President of the Royal Society, MARTIN FOLKES, Esq; honoured the first Edition of this book with his Patronage ; and also, our much-esteemed and learned friend JAMES BURROW, Esq; Vice-president of the Royal Society, thought the book so worthy his perusal, as to remark all the typographical and other errors, and to make some useful observations, a list whereof he favoured me with, and for which I trust you will permit me to take this opportunity of publicly thanking him : Although I am conscious,
that

DEDICATION.

that you have the highest regard for the two respectable names, which I here mention out of gratitude ; yet I would not be understood that you are to accept hereof in this public manner, merely because those considerable personages have already favoured the Work ; I offer this as a tribute for your acquaintance and friendship, and flatter myself that you will find in this impression some things, which if they have not difficulty to recommend them, have at least, I apprehend, so much utility accompanying them, as to render the whole in some degree interesting, and perhaps not unworthy the notice of the most skilful in the Mathematical Sciences. I am,

S I R,

Your most obedient

Humble Servant,

J. ROBERTSON.

TO THE
READER.

✱✱✱✱ I ✱✱✱✱ It is needless to enumerate the many purposes, to which *mathematical instruments* serve; their use seems quite necessary to *persons* employed in most of the active stations in life.

The *Architect*, whether *civil*, *military*, or *naval*, never offers to effect any undertaking, before he has first made use of his *rule* and *compasses*; and fixed upon a scheme or drawing, which unavoidably requires those *instruments*, and others equally necessary.

The *Engineer*, cannot well attempt to put in execution any design, whether for *defence*, *offence*, *ornament*, *pleasure*, &c. without first laying before his view, the plan of the whole; which is not to be conveniently performed, but by *rulers*, *compasses*, &c.

There are indeed, very few, if any good *Artificers*, who have not in some measure, occasion for the use of one or more *mathematical instruments*; and whenever there is required, an accurate drawing of a thing to be executed, or represented; that collection of instruments, usually put in *portable cases*, is then absolutely necessary: And of these, the most common ones, or others applicable to like service, must have been in use, ever since mankind have had occasion to provide for the necessary conveniencies of life: But the *parallel ruler*, the *proportional compasses*, and the *sector*, are not of any great antiquity.

However, by means of the opportunity, which the author had of consulting most, if not all the principal
pieces,

pieces, that have been wrote on this subject † ; he thinks it will sufficiently appear from what follows, who were the inventors of these latter instruments ; and when they were first known and made use of.

I. *Gaspar Mordente*, in his book *on the compasses*, printed in folio at *Antwerp*, 1584 ; gives the construction and use of an instrument, invented by his brother *Fabricius Mordente*, in 1554 ; and by him presented to the emperor *Maximilian II.* in 1572 : *Fabricius* presented it afterwards, with some improvements, to *Rodolphus II.* the son of *Maximilian* : In 1578, *Gaspar* studied to apply the instrument to various uses by the command of the then governor of the *Netherlands*. The instrument consists of two flat legs, moveable round a joint like a common pair of compasses ; but the ends or points are turned down at right angles to the legs, so as to meet in one point when the legs are closed. In each leg there is a groove, with a slider fitted to it, carrying a perpendicular point ; so that these also appear like one point when the legs are closed, and the sliders are opposite. This compass is jointly used with a rod, containing a scale of equal parts ; whereof 30 are equal to the length of each leg. As the operations with this compass, depend on the properties of similar triangles, therefore its principles are the same with those of the sector : And most, or all the problems that are performed by the line of lines only, can with almost the same ease, be performed by these ; the transition from this instrument to the sector is very natural and easy.

The use of this instrument, is exemplified in problems concerning lines, superficies, solids, and measuring of inaccessible distances.

The author, p. 22, says, he invented an instrument there described ; which is our parallel ruler with parallel bars : The parallel ruler with cross bars, is a more modern contrivance.

† In the collection of the late *William Jones*, Esq;

II. *Daniel Speckle*, in the year 1589, published in folio, his *military architecture*, at *Strasburg*; where he was architect. In his second chapter, he takes notice of compasses then in use of a curious invention, whose center could be moved forwards or backwards, so that by the figures and divisions mark'd thereon, a right line could be readily and correctly divided into any number of equal parts, not exceeding 20. This instrument has been since called the *proportional compasses*.

In the same chapter he mentions another compasses, with an immoveable center, and broad legs, whereon were drawn lines proceeding from the center, and divided into equal parts; whereby a right line could be divided into equal parts not exceeding 20; because the divisions on the lines still kept the same proportion, to whatever distance the legs were opened. This instrument was afterwards call'd the *sector*.

III. *Dr. Thomas Hood*, printed at *London*, Anno 1598, a quarto book, intituled, *The making and use of a Geometrical Instrument called a Sector*. This instrument consists of two flat legs, moveable about a joint; on these are sectoral lines, of equal parts, of polygons, and of superficies; that is, lines so disposed, as to make all the operations that depend on similar triangles quite easy, and that without the laying down of any figure. To the legs is fitted a circular arc, an index moveable on a joint, and sights, whereby it is made fit to take angles.

IV. *Christopher Clavius*, in his *practical geometry*, printed in quarto at *Rome*, Anno 1604, in page 4, shews the construction and use of an instrument, which he calls the *instrument of parts*; it consists of two flat rulers moveable on a joint; on one side of these legs, are the sectoral lines of equal parts; on the other side, are those of the chords: After shewing some of their uses, he concludes with saying, he is sensible of many others to which it may be applied,

but leaves them for the exercise of the reader to discover.

V. *Levinus Hulsius*, in his book of *mechanical instruments*, printed in quarto at *Frankfort*, Anno 1605; gives, in the third part, the description and use of an instrument, which *Iustus Burgius* call'd the *proportional Compass*. *Hulsius* says, the use of it had not been published before, although the instrument had been long known.

VI. Anno 1605, *Philip Horschner*, M. D. published at *Mentz*, a quarto book, containing the use and construction of the *proportional compasses*. This author does not pretend to be the inventor; but that seeing such an instrument, he thought he could, from *Euclid*, shew its construction and the grounds of its operations.

VII. Anno 1606, *Galilæus* published in *Italian*, a treatise of the use of an instrument which he calls, *The geometrical and military compass*. On this instrument are described sectoral lines of equal parts, surfaces, solids, metals, inscribed polygons, polygons of given areas, and segments of circles. In the preface to an edition of this book, printed at *Padua*, Anno 1640, by *Paola Frambotti*, *Galilæus* says, that on account of the opportunity he had of teaching mathematics at *Padua*, he thought it proper to seek out a method of shortening those studies. In another part of the preface he says, that he should not have published this tract, but in vindication of his own reputation; for he was informed that a person had by some means or other, got one of his instruments, and pretended to be the inventor, although himself had taught it ever since the year 1597.

VIII. Anno 1607, *Baldessar Capra*, published a treatise of the construction and use of the *compass of proportion*, (or sector.) He claims the invention of this instrument; and hence arose a dispute between *Galilæus* and *Capra*; some particulars of which have been

been mentioned by several, and particularly by *Thomas Salusbury, Esq;* in his life of *Galilæus*, published at the end of the second volume of his *mathematical collections and translations*, at *London*, in fol. Anno 1664.

It appears from these accounts that one *Simon Marius* a German who was in *Padua* about the year 1607, translated into latin, the book published the year before by *Galilæus*, and caused his disciple *Capra* to print it as his own: *Marius* dreading a prosecution, retired, and left *Capra* in the lurch, who was proceeded against. At that time *Galilæus* published an apology, intituled, “ *The defence of Galilæus Galilæi, a Florentine gentleman, reader in the university of Padoua, against the calumnies and impostures of Baldeffar Capra a Milanese, divulged against him as well in his consideratione astronomica upon the new star of 1604, as (and more notoriously) in lately publishing for his invention the construction and uses of the geometrical and military compass, under the title of Usus & Fabrica circini cujusdam proportionis, &c.*” *Galilæus* begins with an address to the reader, wherein he concludes, that a person robbed of his inventions, suffers the greatest loss that can be sustained, because it despoileth him of honour, fame and deserved glory:” He proceeds, and says, “ *into this ultimate of miseries and unhappiness of condition, Baldeffar Capra, a Milanese, with unheard of fraud, and unparallel’d impudence hath endeavoured to reduce me, by lately publishing, and committing to the press my geometrical and military compass, as his proper invention, and as a production of his own wit, (for so he calls it in the work itself) when it was I alone, that ten years since (viz. Anno 1597) thought of, found and compleated the same, so as that no one else hath any share in it; and I alone from that time forward imparted, discovered and presented it unto many great princes, and other noble lords; and in fine, only that I a year since caused the operations thereof to be printed, and consecrated to the glorious name of the most serene prince of Tuscany, my*

lord. *Of which said instrument the above-named Capra, bath not only made himself the author, but reports me for its shameless usurper, (these are his very words) and consequently bound to blush within my self with extream confusion, as unworthy to appear in sight of learned and ingenuous men."* Galilæus then proceeds, among other things, to produce the attestations of four considerable persons, shewing that ten years before that time, he had taught the use of the instrument, and that *Capra* who had for four years past seen them making at the workman's house, had never challenged the invention, as his own.

Galilæus after this, says that he went to *Venice*, and laid the affair before the lords reformers of the university of *Padoua*, on the 8th of *April* 1607, at the same time shewing them his own book, published *June* the 10th 1606; and that of *Capra's*, published *March* the 7th 1607. The lords thereupon cited *Capra* to appear before them on the 18th of *April*; the next day the cause was heard and the parties dismissed: But on the 4th of *May* following, their lordships pronounced sentence, and sent it to *Padoua* to be put in execution; the amount of their sentence was, that having fully considered the affair, it appeared to them that *Galilæus* had been abused, and that all the remaining copies of *Capra's* book should be "*brought before their lordships to be suppressed in such fashion as they shall think fit, reserving to themselves to proceed against the printer and bookseller, for the transgressions they may have committed against the laws of printing; ordering the same to be made known accordingly.*

The same day all the copies of the said book were sent to Venice unto the lords reformers; there being found 440 with the bookseller, and 13 with the author, he having distributed 30 of them into sundry parts of Europe, &c."

IX. Anno 1610, *John Remmelin*, M. D. published at *Frankfort*, a quarto edition of two tracts of *John Faulhaber*; one of these contains the use of the *sector*, on which are lines of equal parts, superficies, solids, metals, chords, &c. He says, that *G. Brendel*, a painter, used this instrument in perspective painting.

X. *D. Henrion*, in his *mathematical memoirs*, Anno 1612, gave a short tract of the use of the compass of proportion (or *sector*.) In 1616 he printed a book of the use of the *sector*; and a fifth edition, in the year 1637, the preface to which, seems to be wrote in the year 1626, wherein he says, that about the year 1608, he had seen in the hands of *M. Alleaume*, engineer to the king of *France*, one of these *sectors*; whereupon he wrote some uses of it, which he published in his memoirs, as above. He also declares, that before his first publication, he had not seen any book on the use of a *sector*, and therefore calls what he publishes his own. He charges *Mr. Gunter* with having used many of his propositions. This author printed at *Paris* 1626, an octavo book of logarithms, at the end of which is a tract call'd *logocanon*, or the proportional ruler; which is a description and use of an instrument, he calls a *lattice*, (perhaps from the chequer-work made by lines drawn thereon) which operates the problems performed by the *french* *sectors* very accurately.

XI. Anno 1615, *Stephen Michael-Spackers*, published in quarto at *Ulm*, a treatise of the *proportional rule and compass* of *G. Galgemeyer*, revised by *G. Brendel*, a painter at *Laugingen*. On these proportional compasses, are lines of equal parts, of polygons, superficies, solids, ratio of the diameter to the circumference; reduction of planes, and reduction of solids. The use and construction of these lines, are shewn by a great variety of examples.

XII. *Benjamin Bramer*, in his book of *the description of the proportional ruler and parallelogram*, printed in

in quarto at *Marpurg*, Anno 1617; says, his ruler is applicable to the same uses as *Justus Burgius's* instrument. *Bramer's* instrument consists of a ruler, on which are lines of equal parts, of superficies, of solids, of regular solids, of circles, of chords, and of equal polygons; at the beginning of each scale, is a pin-hole, whereby he can apply the edge of another ruler, and so constitute a sector for each scale.

XIII. Anno 1623, *Adriano Metio Alcmariano*, printed at *Amsterdam* a quarto book, shewing the use of an instrument called the *rule of proportion*. In his dedication, he says, that whilst he was reviewing some things relating to practical geometry, he met with *Galileo's* book of the use of the sector, which gave him opportunity to improve on it, and occasioned the publishing of this book.

XIV. Mr. *Edmund Gunter*, professor of astronomy in *Gresham college*, printed at *London*, Anno 1624, a quarto book, called *the description and use of the sector*; on which are sectoral lines, 1st. of equal parts; 2d. superficies; 3d. solids; 4th. sines and chords; 5th. tangents; 6th. rhumbs; 7th. secants: Also lateral lines of, 8th. quadratures; 9th. segments; 10th. inscribed bodies; 11th. equated bodies; 12th. metals: On the edges are a line of inches and a line of tangents.

Mr. *Gunter* does not say any thing concerning the invention, and has no preface; but at the end of the tract, in a conclusion to the reader, he says, that the sector was thus contrived, most part of the book written, and many copies dispersed, more than sixteen years before, &c. this article being written May 1, 1623, brings the time he speaks of to about the year 1607, which was before the time *Henrion* says he first saw the sector.

The scales of logarithm numbers, sines, and tangents, were first published in 1624, in *Gunter's* description of the cross staff.

XV. *Mutio Oddi* of *Urbino* printed at *Milan*, An.

1633, a quarto book, called *the construction and use of the compasso polimetro*, (or sector.) The lines on this instrument, were such as were common at that time: He says in the dedication to his friend *Peter Linder* of *Nuremberg*, he first taught the use of it.

In the preface he says, that about the year 1568, *Commandine*, who then taught at *Urbino*, did contrive a pair of compasses with a moveable centre, to divide right lines into equal parts; which was done at the request of a gentleman named *Bartholomew Eustachio*, who wished to avoid the trouble of the common methods, or of being obliged to have many compasses for such divisions of right lines.

He farther says, that about that time, *Guidibaldo*, marquis of *Monte*, who lived at *Urbino* for the sake of *Commandine's* company, being frequently at the house of *Simone Boraccio*, who made *Commandine's* proportional compasses, did contrive, and cause to be made, an instrument with flat legs, (like the sector) which performed the operations of the compass more easily. *Oddi* says also, that great numbers were made, and in few years, had many useful and curious additions, with treatises written on its use in diverse languages, and called by different names, which occasioned the doubt of who was the true author, every one having found means to support his cause: But *Oddi* says, he not intending to decide the dispute, leaves it to time to discover; and seems contented to have pointed out who was the first inventor; his chief intention being that of making the use public, and the construction easy to workmen.

The following authors have also wrote on the sector, and sectoral lines.

XVI. *Anno* 1634, *P. Petit*, printed in 8vo. at *Paris*, a treatise on the sector. He thinks *Galilæus* was the inventor.

XVII. *An.* 1635, *Matthias Berneggerus* printed at *Strasburg* a 4to. edition of *Galilæus's* book on the sector, which consists of two parts: To this is added
a third

a third part, shewing the construction of *Galilæus's* lines, and some additional uses and tables.

XVIII. *An.* 1639, *Nicholas Forest Duchesne* printed at *Paris*, in 12mo. a book of the sector. He seems to be little more than a copier of *Henrion*.

XIX. *An.* 1645, *Bettinus* in his *Apiaria universa*, &c. *apiar.* 3d. p. 95, and *apiar.* 12, p. 4. In his *Ærarium philo. math.* 4to. *an.* 1648, vol. I. p. 262. In his *Recreationum math. appiariæ*, &c. 12mo. *an.* 1658, p. 75, applies the sector to music.

XX. *John Chatfield* printed at *London*, in 12mo. his *trigonal sector*, *anno* 1650.

XXI. *An.* 1656, *Nicholas Goldman* printed at *Leyden*, in folio, his *treatise on the sector*. He says that *Galilæus* was the first who published the description of the sector, an invention useful in all parts of the mathematics, and other affairs of life.

XXII. *John Collins* printed at *London*, in 4to. his book of *the sector on a quadrant*, *an.* 1659.

XXIII. *Pietro Ruggiero*, in his *military architecture*, in 4to. printed at *Milan*, *an.* 1661, p. 230, applies the sector to the practice of fortification.

XXIV. *An.* 1662, *Gaspar Schottus* printed at *Strasburgh* his *matheſis caſaræa*, in 4to. in which he gives a description and use of the sector: In the preface he mentions *Galilæo* as the inventor of the sector.

XXV. *J. Templar* printed in 12mo. at *London*, *an.* 1667, a book called *the semicircle on a sector*. He says, the applying of Mr. *Fcrſter's* line of verſed ſines to the sector, was first published *an.* 1660, by *John Brown*, mathematical instrument-maker in *London*.

XXVI. *Daniel Schwenter* in his *præctical geometry*, revised and augmented by *George Andrew Bocklern*, printed in 4to. at *Nuremberg*, *an.* 1667, treats on the description and use of the sector.

XXVII. *John Caramuel* printed at *Campania*, *an.* 1670, his *matheſis nova*, in 2 vols. folio. In the 2d vol.

vol. p. 1158, he treats on the sector, relates the contest between *Galilæus* and *Capra*, and thinks the same might have been objected against others, as well as against *Capra*: He also says, that *Clavius* had such an instrument before that of *Galilæus* appeared; and *Clavius* having taught for a long time at *Rome*, had many scholars, some of whom might have carried his instruments to several countries. *Caramuel* mentions a story of a *Hollander* shewing to *Galilæus* an instrument of this sort, that he had brought from his country, and of which *Galilæus* took a copy.

XXVIII. *John Brown*, in his book on *the triangular quadrant*, printed in 8vo. at *London*, an. 1671.

XXIX. *John Christopher Roblhans*, in his *math. and optical curiosities*, printed in 4to. at *Leipsic*, an. 1677, p. 216.

XXX. An. 1683, *Stanislawa Solskiego* printed at *Kracow*, his *geometria et architectura Polski*, in folio. p. 69, treats on some sectoral lines.

XXXI. *Henrick Jasper Nuis*, printed at *Texwolde*, in 4to. his *Rectangulum catholicum geometrico astronomicum*, an. 1686.

XXXII. *De Chales*, in his *cursus mathem.* printed at *Leyden*, in 2 vols. fol. an. 1690. Vol. 2d. p. 58, relates the contest between *Galilæus* and *Capra*, and ascribes the invention of the proportional compass to *Dr. Horscher*, or *Justus Burgius*.

XXXIII. An. 1691, an edition in 8vo. of *Mr. Ozanam's* treatise of the sector, was printed at the *Hague*.

XXXIV. *P. Hoste* printed at *Paris* his course of mathematics, in 3 vols. 8vo. an. 1692. In vol. 2d. p. 27. he gives a tract on the sector.

XXXV. *Thomas Allingham* in his *short treatise on the sector*, in 4to. *London*, 1698.

XXXVI. *J. Good*, in his *treatise on the sector*, in 12mo. *London*, 1713.

XXXVII. *Christian Wolfius*, in his *math. lexicon*, 8vo. printed at *Leipfic*, an. 1716, under the word *circinus proportionum*, relates, that *Levinus Hulsius*, in his treatise on the proportional compasses, printed at *Frankfort* the 10th of *May*, 1603, says, that he first saw the said instrument at *Ratisbon*, on the day of the imperial dyet: That he had sold them far and near before 1603; and that it had been inaccurately copied in several places: *Wolfius* says farther, that *Justus Burgius* was certainly the inventor, but used to let his inventions lye unpublished.

He then relates the contest between *Galilæus* and *Capra*, and ends with shewing the difference between the instruments of *Burgius* and *Galilæus*.

XXXVIII. *M. Bion*, in his construction of mathematical instruments, translated by *Edmund Stone*, fol. *London*, 1723.

XXXIX. *Mr. Belidor*, in his *new course of math.* in 4to. p. 364, *Paris*, 1725.

XL. *Roger Rea*, in his *sector and plane scale compared*, 8vo. *London*, 1727, 2d edition.

XLI. *Vincent Tosco*, in his *compendium of the math.* in 9 vols. 8vo. *Madrid*, 1727, vol. I. p. 359.

XLII. *Jacob Leupold*, in his *theatrum arithmetico-geometricum*, in fol. *Leipfic*, 1727. p. 86, gives a detail of the inventors of the proportional compasses and sector, which goes on to p. 121, and then he gives a list of the authors who have wrote on proportional instruments, viz. *Bramer*, 1617; *Capra*, 1607; *Casati*, 1664; *Conette*, 1626; *Dechales*, 1690; *Dolz*, 1618; *Faulhaber*, 1610; *Galgemeyer*, 1615; *Brendell*, 1611; *Galilæus*, 1612; *Goldman*, 1656; *Horfcher*, 1605; *Horen*, 1605; *Hulsius*, 1604; *Clavius*, 1615; *Lockmann*, 1626; *Metius*, 1623; *Patridge*—; *de Saxonica*, 1619; *Scheffelts*, 1697; *Steymann*, 1624; *Uttenboffers*, 1626.

XLIII. *Samuel Cunn*, in his *new treatise on the sector*, 8vo. *London*, 1729.

XLIV. *William Webster*, in his appendix to a translation of *P. Host's* mathematics, 8vo. 2 vols. London, 1730.

There may be several other authors who have wrote on the construction and use of the sector, or on some of the sectoral lines; but those above, are all that have come to hand; and indeed these are many more than are wanted to determine this enquiry; which may be collected chiefly, from *Mordente*, *Speckle*, *Hood*, *Clavius*, *Hulsius*, *Galilæus*, *Oddi*, *Sallusbury*, *Caramuel*, *Dechales*, *Wolffius*, and *Leupold*; the others serving only to inform the reader what works are extant on this subject. From the whole he may observe, that there are few countries in *Europe*, but have one or more treatises on the proportional compasses and sector, in their own language; and this is sufficient to shew, that these instruments have been in universal esteem.

As the publication of *Mordente's* book was in 1584, it is not improbable, as *Caramuel* relates, that a *Hollander* (or one from the neighbourhood of *Antwerp*) might shew one of *Mordente's* instruments to *Galilæus*: Neither is it improbable that *Galilæus* had seen both *Mordente's* and *Speckle's* books, the former having been published thirteen years, and the latter eight years, before *Galilæus*, by his own accounts, thought of his instrument.

As *Mutio Oddi*, was a native of *Urbino*, and from what he says in his dedication, it is not improbable but he was acquainted with one or more of the persons he mentions in his preface, or at least with some of their acquaintance, from whom he might gather the particulars he relates; to which, if any credit may be given, *Commandine* was the inventor of the proportional compasses, and *Guidobaldo* of the sector: And in the intercourse between *Italy* and *Germany*, some of *Simone Borackio's* work might get into the hands of many ingenious *Germans*, and give *Justus Burgius*,

Burgius, to whom the proportional compass is usually ascribed, opportunity of getting an early copy; and also put into *Speckle's* way, the instrument he mentions to have seen: His description pretty nearly agreeing with what *Oddi* says was contrived by *Guidobaldo*.

But while we are searching among foreigners for the inventor of the sector, what are we to think of our countryman *Dr. Hood*? who in 1598 published his account of an instrument which he really calls a sector: And though we should allow that *Hood* as well as *Galilæus* might have seen *Mordente's* and *Speckle's* books; and both of them might have seen some of *Borrachio's* work, yet it is not very probable that *Hood* could have got the form of his instrument from *Galilæus* the year after he thought of it; and as *Hood* published eight years before *Galilæus*, *Hood* certainly has an equal right with *Galilæus*, if not a greater, to the honour of the invention of the sector.

After all, it may be said, that it is not impossible for the same thing to be discovered by different persons who have no connexion with one another; examples of a like coincidence of thoughts being known on other subjects.

To the present edition, there is added an appendix on the gunners callipers, which was promised to the public in the former impression, published at the beginning of the year 1747; and beside this, the body of the book has been augmented by more than three sheets of additional illustrations and problems, and another plate: By all these additions, it is conceived the book is now rendered more generally useful.

What is done in the foregoing essay, and in the following work, is submitted to the reader's judgment; the author intending no more than to have the honour of invention ascribed to whom it is due; and also to give some assistance to beginners in the mathematical studies.

Royal Academy Portsmouth
March 5, 1755.

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THE
DESCRIPTION *and* USE
OF A
C A S E,
OR
PORTABLE COLLECTION,
Of the most Necessary
Mathematical Instruments.



SECT. I.



ASES of *Mathematical Instruments* are of various sorts and sizes; and are commonly adapted to the fancy or occasion of the persons who buy them.

THE smallest collection put into a case, commonly consists of,

I. A *pair of compasses*, one of whose points may be taken off, and its place supplied with,

B

A *crayon*

A *crayon* for lead or chalks.

A *drawing-pen* for ink.

II. A *plane scale*.

WITH these instruments only a tolerable shift may be made to draw most mathematical figures.

BUT in sets, called *complete pocket-cases*, beside the instruments above, are the following.

III. A smaller pair of *compasses*.

IV. A pair of *bows*.

V. A *black-lead pencil*, with a *cap* and *feeder*.

VI. A *drawing-pen* with a *protracting-pin*.

VII. A *protractor*.

VIII. A *parallel-ruler*.

IX. A *sector*.

IN some cases, the *plane scale*, *protractor*, and *parallel-ruler*, are included in one instrument.

THE common, and most esteemed size of these instruments, is six inches; though they are sometimes made of other sizes, and particularly of four inches and a half.

Note, the size of a case is named from the length of the *scale* or *sector*.

SOME artists have contrived a very commodious flat case, or box, where the inside of the lid or top contains the rulers and scales: The *compasses*, *drawing-pen*, &c. lie in the partitions of a drawer, that drops into the bottom part of the case, but not quite to the bottom; leaving room under it for *black-lead pencils*, *hair pencils*, *Indian ink*, *colour cells*, &c. and beside the instruments already enumerated, in boxes or cases of this sort are put

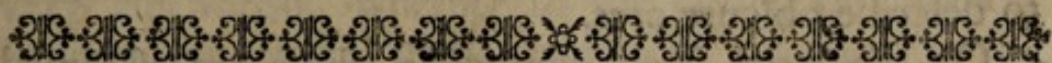
X. A *tracing-point*.

XI. A pair of *proportional compasses*.

XII. A *gunner's callipers*.

BUT the case of instruments called the *magazine*, is the most complete collection; for this contains whatever can be of use in the practice of *drawing*, *designing*, &c. and as the greatest part of these instruments
are

are scarcely ever used but in the studies or chambers of those who have occasion for them; therefore it will be useless to insist on pocket cases; for few persons care to load themselves with the carriage of what is called a *complete set*.



SECT. II.

Of the COMPASSES and BOWS.

COMPASSES are usually made of silver or brass, and those are reckoned the best, part of whose joint is steel; and where the *pin* or *axle* on which the joint turns, is a steel screw; for the opposition of the metals makes them wear more equable: and by means of the screw axle, with the help of a *turn-screw*, (which should have a place in the case) the compasses can be made to move in the joint, stiffer or easier, at pleasure. If this motion is not uniformly smooth, it renders the instrument less accurate in use. Their points should be of steel, and pretty well hardened, else in taking measures off the scales, they will bend, or be soon blunted. They also should be well polished, whereby they will be preserved free from rust a long time.

To one point of the smaller compasses, it is common to fix in the shank a spring, which by means of a screw, moves the point; so that when the compass is opened nearly to a required distance, by the help of the screw the points may be set exactly to that distance; which cannot be done so well by the motion in the joint.

To use the spring point.

HOLD the compasses in the left hand with the screw turned towards the right; turn the screw towards you,

or slacken it, and the spring point will be brought nearer to the other point: On the contrary, by turning the screw from you, or tightening it, the spring point will be set farther from the other point.

THE use of these lesser compasses, is to transfer the measures of distances from one place to another; or, to describe obscure arcs.

OF the large sized compasses, those are esteemed the best, whose moveable points are locked in by a spring and catch fixed in the shank; for if this spring be well effected, the point is thereby kept tight and steady; the contrary of which frequently happens, when the point is kept in by a screw in the shank.

THE use of these compasses is to describe arcs or circumferences with given radius's: and it is easy to conceive, that these arcs or circumferences can be described, either obscurely by the steel point; in ink, by the ink point; in black-lead or chalks, by the crayon; and with dots, by the dotting-wheel; for either of them may be fixed in the shank in the place of the steel point.

As the dotting-wheel has not hitherto been effected, so as to describe dotted lines or arcs, with any tolerable degree of accuracy, it seems therefore to be useless: and, indeed, dotted lines of any kind are much better made by the drawing-pen.

THE drawing-pen point, and crayon, have generally (in the best sort of cases) a socket fitted to them: so that they occupy but one of the holes, or partitions, in the case.

THE ink, and crayon points, have a joint in them, just under that part which locks into the shank of the compasses; because the part below the joint should stand perpendicular to the plane on which the lines are described, when the compass is opened.

If instead of the larger compass being made with shifting points, there were two pair put into the case; to one of which the ink point was fixed, and to the

the other the crayon point ; this would save the trouble of changing the points in the compass at every time they were used ; and would increase the expence, or bulk of the case, but a trifle.

MOST persons at first, handle a pair of compasses very awkwardly, whether in the taking of distances between the points, or describing of circles. To be sure long practice brings on easy habits in the use of things, however a caution or two may be serviceable to beginners.

To open and work the compasses.

WITH the thumb and middle finger of the right hand pinch the compasses in the hollow part of the shank, and it will open a little way ; then the third finger being applied to the inside of the nearest leg, and the nail of the middle finger acting against the farthest, will open the compasses far enough to introduce the fingers between the legs : then the hither one being held by the thumb and third finger, the farther leg may be moved forwards and backwards very easily by the fore and middle fingers, the fore finger pressing on the outside to shut, and the middle one acting on the inside to open, the compasses to any desired extent. In this manner the compasses are manageable with one hand, which is convenient when the other hand is holding a ruler or other instrument.

To take a distance between the points of the compasses.

HOLD the compasses upright, set one point on one end of the distance to be taken, there let it rest ; and (as before shewn) extend the other point to the other end.

ALWAYS take care to avoid working the compasses with both hands at once ; and never use them otherwise than nearly upright.

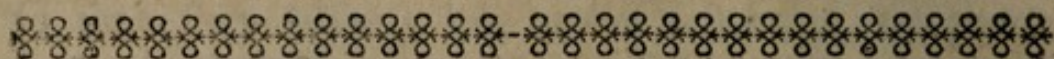
To describe circles or arcs with the compasses.

SET one foot of the compasses on the point designed for the centre, hold the head between the thumb and middle finger, and let the fore finger rest on the head, but not to press it: then by rolling the head between the finger and thumb, and at the same time touching the paper with the other point, a circle or arc may be described with great ease, either in lead or ink.

IN describing of arcs it should be observed, that the paper be not pressed at the centre, or under the foot, with more weight than that of the compasses; for thereby the great holes and blots may be avoided, which too frequently deface figures when they are made by those who are awkward or careless in the use of their instruments.

Of the B o w s.

The bows are a small sort of compasses, that commonly shut into a hoop, which serves as a handle to them. Their use is to describe arcs, or the circumferences of circles, whose radius's are very small, and could not be done near so well by larger compasses.



S E C T. III.

Of the Black-lead Pencil, Feeder, and Tracing Point.

THE *Black-lead Pencil* is useful to describe the first draught of a drawing, before it is marked with ink; because any false strokes, or superfluous lines,

lines, may be rubb'd out with a handkerchief or piece of bread.

THE *Feeder* is a thin flat piece of metal, and is sometimes fixed to a cap that slips on the top of the pencil, and serves either to put ink between the blades of the drawing-pen, or to pass it between the points, when the ink by drying, does not flow freely.

THE *Tracing Point* is a pointed piece of steel; and commonly has the feeder fixed to the other end of the handle. Its use, is to mark out the outlines of a drawing or print when an exact copy thereof is wanted, which may be done as follows.

ON a piece of paper, large enough to cover the thing to be copied, let there be strewn the scrapings of *red chalk*, or of *black chalk*, or of *black lead*; rub these on the paper, so that it be uniformly covered; and wipe off, with a piece of muslin, as much as will come away with gentle rubbing. Lay the coloured side of this paper, next to the vellum, paper, &c. on which the drawing is to be made: on the back of the colour'd paper, lay the drawing, &c. to be copied. Secure all the corners with weights, or pins, that the papers may not slip: trace the lines of the thing to be copied, with the *tracing point*; and the lines so traced will be impress'd on the clean paper.

AND thus, with care, may a drawing or print, be copied without being much damaged.

Note, The coloured paper will serve a great many times.

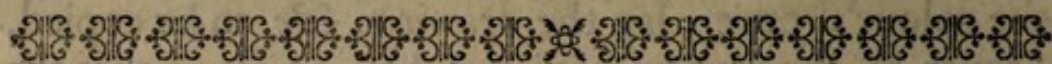
THERE is not perhaps, a more useful instrument in being for ready service in making of sketches or finished plans; whether of architecture, fortification, machines, landfkips, ornaments, &c. than a black-lead pencil; and therefore it may be proper to give a few hints concerning this excellent mineral.

BLACK-LEAD is produced in many countries, but the best yet discovered is found in the north of England: it is dug out of the ground in lumps, and sawed out

into scantlings proper for use: the kinds most proper to use on paper must be of an uniform texture, which is discoverable by paring a piece to a point with a penknife; for if it cuts smooth and free from hard flinty particles, and will bear a fine point, it may be pronounced good.

THERE are three sorts of good black-lead; the soft, the midling, and the hard: the soft is fittest for taking of rough sketches, the midling for drawing of landscape and ornaments, and the hard for drawing of lines in mathematical figures, fortification, architecture, &c. The indifferent kinds, or those which in cutting are found flinty, are useful enough to carpenters or such artificers who draw lines on wood, &c.

THE best way of fitting black-lead for use, is first to saw it into long slips about the size of a crow-quill, and then fix it in a case of soft wood, generally cedar, of about the size of a goose-quill, or larger; and this case is cut away with the lead as it is used.



S E C T. IV.

Of the Drawing-Pen, and Protracting-Pin.

THE *Drawing-pen* is an instrument used only for drawing of right lines; and consists of two blades, with steel points, fix'd to a handle. The blades by being a little bent, cause the steel points to come nearly together; but by means of a screw passing thro' both of them, they are brought closer at pleasure, as the line to be drawn should be stronger or finer.

IN using this instrument, put the ink between the blades with a common *pen*, or with the *feeder*; and (by the screw) bring them to a proper distance for drawing the intended line: hold the *pen* a little inclined,

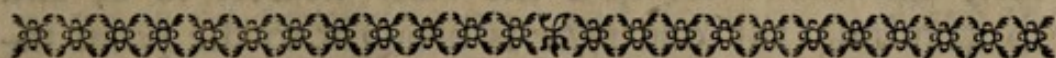
clined, but so that both blades touch the paper ; then may a line be drawn very smooth, and of equal breadth, which could not be done so well with a common pen.

Note, BEFORE the *drawing-pen* is put into the case, the ink should be wiped from between the blades ; otherwise they will soon rust and spoil, especially with common ink. And that they may be clean'd easily, one of the blades should move on a joint.

THE directions given about this *drawing-pen*, will serve for the *drawing-pen* point, used with the compasses. But it must be observed, that when any arc is described of more than an inch radius, then the ink point should be bent in the joint so that both the blades of the pen touch the paper, otherwise the arc described will not be smooth.

THE *Protracting-pin* is a piece of pointed steel (like the point of a needle) fixed into one end of a part of the handle of the *drawing-pen* ; into which, the piece with the *pin* in it, generally screws. Its use is to point out the intersections of lines ; and to mark off the divisions of the protractor, as hereafter directed.

SOMETIMES on the top of the *drawing-pen* is a socket, into which a piece of black-lead pencil may be put.



SECT. V.

Of the PARALLEL-RULER.

THIS instrument consists of two *Rulers*, connected together by two metal bars, moving easily round the rivets which fasten their ends ; these bars are so placed that both have the same inclination to

to each *Ruler*; whereby they will be *Parallel* at every distance, to which the bars will suffer them to recede.

BUT the best *Parallel-Rulers* are those, whose bars cross each other, and turn on a joint at their intersection; one end of each bar moving on a centre, and the other ends sliding in grooves as the *Rulers* recede.

THIS instrument is very useful in delineating civil and military architecture, where there are many *Parallel* lines to be drawn; and also in the solution of several geometrical *Problems*; some of which are as follows.

P R O B L E M I.

A right line AB being given, to draw a line parallel thereto, that shall pass through a given point c (Fig. 1. Pl. III.)

CONSTRUCTION. Apply one edge of the *parallel-ruler* to the given line AB; press one *ruler* tight against the paper, and move the other untill its edge cuts the point c; there stay that *ruler*, and by its edge draw a line through c, then this line will be *parallel* to AB.

IF the point c happens to be farther from the line AB, than the *rulers* will open to; stay that *ruler* nearest to c, and bring the other close to it, where let it rest, and move forward the *ruler* nearest to c, and so continue till one *ruler* is brought to the point intended.

THE manner of using the *parallel-ruler* as here directed, is understood to be the same in the solution of the following PROBLEMS.

P R O B L E M II.

A right line AB being given, to divide it into any propos'd number of equal parts; suppose 5. (Fig. 2.)

CONSTRUCTION. Draw the indefinite right line BC, so as to make with AB, any angle at pleasure; with
any

any convenient opening of the compasses, lay off on BC, the required number of equal parts, viz. 1, 2, 3, 4, 5; lay the edge of the parallel-ruler by the points 5 and A, and parallel thereto, through the points 4, 3, 2, 1, draw lines; then AB, by the intersection of those lines will be divided into 5 equal parts.

P R O B L E M III.

Any right lined quadrangle or polygon being given, to make a right lin'd triangle of equal area.

EXAM. I. *To make a triangle of equal area to the quadrilateral ABDC. (Fig. 3.)*

CONSTRUCTION. Prolong AB; draw CB; and through D, draw DE parallel to CD, cutting AE in E; then a line drawn from C to E forms the triangle ACE, of equal area to the quadrangle ABDC.

EXAM. II. *Given the pentagon ABCDE; requir'd to make a triangle of equal area. (Fig. 4.)*

CONSTRUCTION. Produce DC towards F; draw AC; through B, and parallel to AC draw BF cutting DC in F; and draw AF. Then the area of the trapezium AFDE will be equal to the area of the pentagon ABCDE.

Again. Produce ED towards G; draw AD; through F, draw FG parallel to AD, and draw AG. Then the area of the triangle AGE, will be equal to that of the trapezium AFDE; and consequently, to that of the pentagon ABCDE.

EXAM. III. *To make a triangle equal in area to the Hexagon, ABCDEF. (Fig. 5.)*

CONSTRUCTION. Draw FD, and parallel thereto, through E, draw EG meeting CD produced in G, and draw GF. Then the triangle FGD is equal to the triangle FED, and the given Hexagon is reduced to the Pentagon ABCGF equal in area.

Again,

Again. Draw AG; through F, draw FH *parallel* to AG, meeting CG produced in H; draw AH, and the *pentagon* is reduced to the trapezium ABCH.

Lastly, Draw AC, and *parallel* thereto, through H; draw HI, meeting BC produced in I, and draw AI. Then the trapezium is reduced to the triangle ABI, which is equal in area to the given *Hexagon* ABCDEF.

EXAM. IV. *Given the nine sided figure* ABCDEFGHI, *to make a triangle of equal area.* (Fig 6.)

CONSTRUCTION. 1st, Draw IB, and through A draw AK *parallel* to IB, meeting HI produced in K, and draw BK; so the three sides HI, IA, AB, are reduced to the two sides HK, KB.

2d, Draw KC, and through B draw BL *parallel* to KC, meeting CD in L; draw KL, and the three sides DC, CB, BK, are reduced to the two sides DL, LK.

3d, Draw KG; through H, draw HM, *parallel* to KG, meeting GF in M, and draw KM; so the three sides KH, HG, GF, are reduced to the sides KM, and MF.

4th, Draw KF; through M, draw MN, *parallel* to KE, meeting FE in N, and draw KN; so the three sides KM, MF, FE, are reduced to two sides KN, NE.

5th, Draw LN, and through K, draw KO, *parallel* to LN, meeting EF produced in O, and draw LO; so the three sides EN, NK, KL, are reduced to the two sides EO, OL.

Lastly, Draw LE, and through D, draw DP *parallel* to LE, meeting OE produced in P, and draw LP; so shall the triangle OLP be equal in area to the given nine sided figure.

PROCEEDING in the same manner; a figure of any number of sides may be reduced to a triangle of equal area.

S E C T. VI.

Of the PROTRACTOR.

THE *Protractor*, is an instrument of a semicircular form; being terminated by a right line representing the diameter of a circle, and a curve line of half the circumference of the same circle. As at Fig. 7. The point *c*, (the middle of *AB*) is the centre of the semicircumference *ADB*, which semicircumference is divided into 180 equal parts call'd degrees; and for the convenience of reckoning both ways, is numbered from the left hand towards the right, and from the right hand towards the left, with 10, 20, 30, 40, &c. to 180, being the half of 360, the degrees in a whole circumference. The use of this instrument is to *protract*, or lay down an angle of any number of degrees, and to find the number of degrees contained in any given angle.

BUT this instrument is made much more commodious, by transferring the divisions on the semicircumference, to the edge of a *ruler*, whose side *EF* is *parallel* to *AB*; (see Fig. 7.) which is done by laying a *ruler* on the centre *c*, and the several divisions on the semicircumference *ADB*, and marking the intersections of that *ruler* on the line *EF*, which may easily be conceiv'd by observing the lines drawn from the centre *c* to the divisions 90, 60, 30; so that a *ruler* with these divisions mark'd on 3 of its sides and numbered both ways, as in the *Protractor*, (the fourth or blank side representing the diameter of the circle) is of the same use as a *Protractor*, and is much better adapted to a case.

THAT side of the instrument on which the divisions are mark'd, is call'd the graduated side, or limb of the instrument, which should be sloped away to an edge,

edge, whereby the divisions on the limb will be much easier pointed off.

P R O B L E M IV.

A number of degrees being given; to protract, or lay down an angle whose measure shall be equal thereto. And an angle being protracted, or laid down, to find what number of degrees measures that angle.

EXAM. I. *To draw a line from the point A, that shall make an angle with the line AB of 48 deg. Fig. 8.*

APPLY the blank edge of the protractor to the line AB, so that the middle or centre thereof (which is always mark'd) may fall on the point A; then with the protracting-pin, make a mark on the paper against the division on the limb of the instrument numbered with the degrees given; (*viz.* 48.) counting from the right hand towards the left; a line drawn from A, through the said mark, as AC, shall with AB, form the angle required, *viz.* 48 degrees.

IF the line had been to make an angle with AB, at the point B; then the centre must have been laid on B, and the divisions counted from the left hand towards the right.

EXAM. II. *To find the number of degrees which measure the angle ABC. Fig. 9.*

APPLY the blank edge of the protractor to the line AB, so that the centre shall fall on the point B; then will the line BC cut the limb of the instrument in the number expressing the degrees that measure the given angle; which in this *example* is 125 degrees, counting from the left hand towards the right.

P R O B L E M

P R O B L E M V.

From any given point A, in a line AB, to draw a line perpendicular to AB. Fig. 10.

LAY the protractor across the line AB in such a manner that the centre on the blank edge, and the division numbered with 90, on the limb, may both be cut by the given line; then keeping the ruler in this position, slide it along the line, till one of these points touch the given point A, draw the line CA, and it will be perpendicular to AB.

IN the same manner, a line may be drawn, perpendicular to a given line, from a given point out of that line.

P R O B L E M VI.

In a circle given to inscribe any regular Polygon. Suppose an octagon. Fig. 11.

CONSTRUCTION. Apply the blank edge of the protractor to AB the diameter of the Circle, so that their centres shall coincide; set off a number of degrees from B to D equal to an angle at the centre of that polygon, (viz. 45.) and through that mark draw a radius CD; then shall BD the chord of the arc expressing those degrees, be the side of the intended polygon; which chord taken between the compasses, and applied to the circumference will divide it into as many equal parts as the polygon has sides, viz. 8; and the several chords being drawn will form the polygon required.

IT will rarely happen that this operation, though true in theory, will give the side of the polygon exact; for when the chord of the arc prickt off from the protractor, is taken with the compasses and applied to the circle, it generally falls beyond, or short, of the point set out from: for it must be observed that the point where two lines intersect

perfect one another is not to be readily determined in a practical manner; and a very small error in the taking the length of the chord, being several times repeated becomes considerable at last. Here the compasses with the spring point will be found of great use.

A TABLE, shewing the Angles at the Centres and Circumferences of regular Polygons from three to twelve Sides inclusive.

Names.	Sides.	Angles at Center	Angles at Cir.
Trigon	3	120° 00'	60° 00'
Square	4	90 00	90 00
Pentagon	5	72 00	108 00
Hexagon	6	60 00	120 00
Heptagon	7	51 25 $\frac{5}{7}$	128 34 $\frac{2}{7}$
Octagon	8	45 00	135 00
Nonagon	9	40 00	140 00
Decagon	10	36 00	144 00
Endecagon	11	32 43 $\frac{7}{11}$	147 16 $\frac{4}{11}$
Dodecagon	12	30 00	150 00

THIS table is constructed, by dividing 360, the degrees in a circumference, by the number of sides in each polygon; and the quotients are the angles at the centers; the angle at the center subtracted from 180 degrees, leaves the angle at the circumference.

P R O B L E M VII.

Upon a given right line AB, to describe any regular polygon. Fig. 12.

CONSTRUCTION. From the ends of the given line, draw the lines AD, BC; so that the angles BAD, ABC, may each be equal to the angle at the circumference in that polygon; make AD, BC, each equal to AB; from the points D and C, draw lines that shall make with DA, CB, angles equal to the former; make these lines each equal to AB; and so continue, till a polygon is form'd of as many sides as required.

EXAM. I. *Upon the line AB to describe an hexagon.*
Fig. 12.

DRAW AD, BC, so that the angles BAD, ABC, may be each 120 degrees; make AD, BC, each equal to AB: also, make the angles ADF, BCE, each equal to 120 degrees, and make DF, CE, each equal to AB; draw FE and 'tis done.

OR it may be done by the help of the parallel ruler, when the polygon has an even number of sides, Thus,

HAVING form'd the three sides AD, AB, BC, as before directed; through D, draw DF parallel to BC; make DF equal to AB; through F draw FE parallel to AB: make FE equal to AB and join CE.

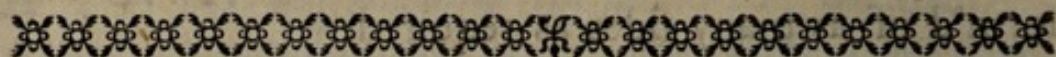
EXAM. II. *Upon the line AB to describe a pentagon,*
Fig. 13.

DRAW AC, BD, that each may make with AB, an angle of 108 degrees. Make AC, BD, each equal to AB; on the points C and D, with the compasses opened to the distance AB, describe arcs to cross each other in E; draw EC and ED, and 'tis done.

IN any regular polygon, having found all the sides but two, as above directed; those may be found as the last two in the pentagon were.

BUT a regular polygon described upon a given line AB may be constructed with more accuracy, thus. See Fig. 12, 13.

MAKE an angle BAP, and another ABP, each equal to half the angle of the required polygon; on the point P, where the lines AP, BP, cut one another, and with the radius PA describe a circle, in which if the given line AB be applied, the polygon sought will be formed.



S E C T. VII.

Of the Plain Scale.

THE lines generally drawn on the plane scale, are these following :

		Marked
I.	Lines of equal parts.	E P.
II.	Chords.	Cho.
III.	Rhumbs.	Ru.
IV.	Sines.	Sin.
V.	Tangents.	Tan.
VI.	Secants.	Sec.
VII.	Half Tangents.	S.T.
VIII.	Longitude.	Lon.
IX.	Latitude.	Lat.
X.	Hours.	Ho.
XI.	Inclinations.	In. Mer.

Of the Lines of equal Parts.

LINES of equal parts are of two sorts, *viz.* simply divided, and diagonally divided. Pl. V.

I. *Simply divided.* Draw 3 lines parallel to one another, at unequal distances, (Fig. 14.) and of any convenient length; divide this length into what number of equal parts is thought necessary, allowing some certain number of these parts to an inch, such as 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, &c. which divisions distinguish by

lines drawn across the three parallels. Divide the left hand division into 10 equal parts, which distinguish by lines drawn across the lower parallels only; but, for distinction sake, let the 5th division be somewhat longer than the others: and it may not be inconvenient to divide the same left-hand division into 12 equal parts, which are laid down on the upper parallel line, having the 3d, 6th, and 9th divisions distinguished by longer strokes than the rest, whereof that at the 6th division make the longest.

THERE are, for the most part, several of these simply divided scales put on rulers one above the other, with numbers on the left hand, shewing in each scale, how many equal parts an inch is divided into; such as 20, 25, 30, 35, 40, 45, &c. and are severally used, as the plan to be expressed should be larger or smaller.

THE use of these lines of equal parts, is to lay down any line expressed by a number of two places or denominations, whether decimally, or duodecimally divided; as leagues, miles, chains, poles, yards, feet, inches, &c. and their tenth parts, or twelfth parts: thus, if each of the divisions be reckoned 1, as 1 league, mile, chain, &c. then each of the subdivisions will express $\frac{1}{10}$ part thereof; and if each of the large divisions be called 10, then each small one will be 1; and if the large divisions be 100, then each small one will be 10, &c.

THEREFORE to lay off a line $8\frac{7}{10}$, 87, or 870 parts, let them be leagues, miles, chains, &c. set one point of the compasses on the 8th of the large divisions, counting from the left hand towards the right, and open the compasses, till the other point falls on the 7th of the small divisions, counting from the right hand towards the left, then are the compasses opened to express a line of $8\frac{7}{10}$, 87 or 870 leagues, miles, chains, &c. and bears such proportion in the plan, as the line measured does to the thing represented.

BUT if a length of feet and inches was to be expressed, the same large divisions may represent the feet, but the inches must be taken from the upper part of the first division, which (as before noted) is divided into 12 equal parts.

THUS, if a line of 7 feet 5 inches was to be laid down; set one point of the compasses on the 5th division among the 12, counting from the right hand towards the left, and extend the other to 7, among the large divisions, and that distance laid down in the plan, shall express a line of 7 feet 5 inches: and the like is to be understood of any other dimensions.

II. *Diagonally divided.* Draw eleven lines parallel to each other, and at equal distances; divide the upper of these lines into such a number of equal parts, as the scale to be expressed is intended to contain, and from each of these divisions draw perpendiculars through the eleven parallels, (Fig. 15.) subdivide the first of these divisions into 10 equal parts, both in the upper and lower lines; then each of these subdivisions may be also subdivided into 10 equal parts, by drawing diagonal lines; viz. from the 10th below, to the 9th above; from the 9th below, to the 8th above; from the 8th below, to the 7th above, &c. till from the 1st below to the 0th above, so that by these means one of the primary divisions on the scale, will be divided into 100 equal parts.

THERE are generally two diagonal scales laid on the same plane or face of the ruler, one being commonly half the other. (Fig. 15.)

THE use of the diagonal scale is much the same with the simple scale; all the difference is, that a plan may be laid down more accurately by it: because in this, a line may be taken of three denominations; whereas from the former, only two could be taken.

Now from this construction it is plain, if each of the primary divisions represent 1, each of the first subdivisions will express $\frac{1}{10}$ of 1; and each of the
second

second subdivisions, (which are taken on the diagonal lines, counting from the top downwards) will express $\frac{1}{10}$ of the former subdivisions, or a 100th of the primary divisions; and if each of the primary divisions express 10, then each of the first subdivisions will express 1, and each of the 2d, $\frac{1}{10}$; and if each of the primary divisions represent 100, then each of the first subdivisions will be 10; and each of the 2d will be 1, &c.

THEREFORE to lay down a line, whose length is express'd by 347, $34\frac{7}{10}$ or $3\frac{47}{100}$ whether leagues, miles, chains, &c.

ON the diagonal line, joined to the 4th of the first subdivisions, count 7 downwards, reckoning the distance of each parallel 1; there set one point of the compasses, and extend the other, till it falls on the intersection of the third primary division with the same parallel in which the other foot rests, and the compasses will then be opened to express a line of 347, $34\frac{7}{10}$; or $3\frac{47}{100}$, &c.

THOSE who have frequent occasion to use scales, perhaps will find, that a ruler with the 20 following scales on it, viz. 10 on each face, will suit more purposes than any set of simply divided scales hitherto made public, on one ruler.

One Side	}	The divisions	{	10, 11, 12, $13\frac{1}{2}$, 15, $16\frac{1}{2}$, 18, 20, 22, 25.
Other Side				}

THE left hand primary division, to be divided into 10 and 12 and 8 parts; for these subdivisions are of great use in drawing the parts of a fortrefs, and of a piece of cannon.

IT will here be convenient to shew, how any plan expressed by right lines and angles, may be delineated by the scales of equal parts, and the protractor.

P R O B L E M VIII.

Three adjacent things in any right lined triangle being given, to form the plan thereof.

EXAM. Suppose a triangular field, ABC, (Fig. 16.) the side $AB=327$ yards; $AC=208$ yards; and the angle at $A=44\frac{1}{2}$ degrees.

CONSTRUCTION. Draw a line AB at pleasure; then from the diagonal scale take 327 between the points of the compasses, and lay it from A to B; set the center of the protractor to the point A, lay off $44\frac{1}{2}$ degrees, and by that mark draw AC: take with the compasses from the same scale 208, lay it from A to c, and join CB; so shall the parts of the triangle ABC, in the plan, bear the same proportion to each other, as the real parts in the field do.

THE side CB may be measured on the same scale from which the sides AB, AC, were taken: and the angles at B and c may be measured by applying the protractor to them as shewn at problem IV.

If two angles and the side contained between them were given.

DRAW a line to express the side; (as before) at the ends of that line, point off the angles, as observed in the field; lines drawn from the ends of the given line through those marks, shall form a triangle similar to that of the field.

P R O B L E M IX.

Five adjacent things, sides and angles, in a right lin'd quadrilateral, being given, to lay down the plan thereof, Fig. 17.

EXAM.

EXAM. Given $\angle^* A = 70^\circ$; $AB = 215$ links; $\angle B = 115^\circ$; $BC = 596$ links; $\angle C = 114^\circ$.

CONSTRUCTION. Draw AD at pleasure; from A draw AB , so as to make with AD an angle of 70° : make $AB = 215$ (taken from the scales); from B , draw BC , to make with AB an angle of 115° : make $BC = 596$; from C , draw CD , to make with CB an angle of 114° , and by the intersection of CD with AD , a quadrilateral will be form'd similar to the figure in which such measures could be taken as are expressed in the example.

If 3 of the things were sides, the plan might be formed with equal ease.

FOLLOWING the same method, a figure of many more sides may be delineated; and in this manner, or some other like to it, do some surveyors make their plans of surveys.

The Construction of the remaining Lines of the PLAIN SCALE.

PREPARATION. Fig. 18. Pl. VI.

DESCRIBE a circumference with any convenient radius, and draw the diameters AB , DE , at right angles to each other; continue BA at pleasure towards F ; through D , draw DG parallel to BF ; and draw the chords BD , BE , AD , AE . Circumscribe the circle with the square HMN , whose sides HM , MN , shall be parallel to AB , ED .

* This mark or character \angle , signifies the angle.

This mark $=$ signifies equal to.

By links, is meant the $\frac{1}{100}$ th part of a chain of four poles or of 66 yards long.

*I. To construct the Line of Chords **

DIVIDE the arc AD into 90 equal parts; mark the 10th divisions with the figures 10, 20, 30, 40, 50, 60, 70, 80, 90; on D, as a center, with the compasses, transfer the several divisions of the quadrantal arc, to the chord AD, which marked with the figures corresponding, will become a line of chords.

Note, In the construction of this, and the following scales, only the primary divisions are drawn; the intermediate ones are omitted, that the figure may not appear too much crowded.

* The chord of an arc, is a right line drawn from one end of the arc to the other end.

II. The Line of Rhumbs †.

DIVIDE the arc BE into 8 equal parts, which mark with the figures 1, 2, 3, 4, 5, 6, 7, 8; and divide each of those parts into quarters; on B, as a center, transfer the divisions of the arc to the chord BE, which marked with the corresponding figures, will be a line of rhumbs.

† The rhumbs here, are the chords answering to the points of the mariners compass, which are 32 in the whole circle, or 8 in the quarter circle.

III. The Line of Sines ‡.

THROUGH each of the divisions of the arc AD, draw right lines parallel to the radius AC; and CD will be divided into a line of sines which are to be numbered

‡ The *sine of an arc*, is a right line drawn from one end of an arc perpendicular to a radius drawn to the other end.

And the *versed sine*, is the part of the radius lying between the arc and its right sine.

from c to D for the right lines ; and from D to c for the versed lines. The versed lines may be continued to 180 degrees by laying the divisions of the radius CD, from c to E.

IV. *The Line of Tangents* *.

A RULER on c, and the several divisions of the arc AD, will intersect the line DG, which will become a line of tangents, and is to be figured from D to G with 10, 20, 30, 40, &c.

* The *tangent of an arc*, is a right line touching that arc at one end, and terminated by a secant drawn through the other end.

V. *The Line of Secants* †.

THE distances from the center c to the divisions on the line of tangents being transferred to the line AF from the centre c, will give the divisions of the line of secants ; which must be numbered from A towards F, with 10, 20, 30, &c.

† The *secant of an arc*, is a right line drawn from the centre through one end of an arc, and limited by the tangent of that arc.

VI. *The Line of Half-Tangents (or the Tangents of half the Arcs).*

A RULER on E, and the several divisions of the arc AD, will intersect the radius cA, in the divisions of the semi, or half tangents ; mark these with the corresponding figures of the arc AD.

THE semi-tangents on the plane scales are generally continued as far as the length of the ruler they are laid on will admit ; the divisions beyond 90° are found by dividing the arc AE like the arc AD, then laying a ruler by E and these divisions of the arc AE, the divisions

sions of the semi-tangents above 90 degrees will be obtained on the line CA continued.

VII. *The Line of Longitude.*

DIVIDE AH, into 60 equal parts; through each of these divisions, parallels to the radius AC, will intersect the arc AE, in as many points; from E as a centre, the divisions of the arc EA, being transferred to the chord EA, will give the divisions of the line of longitude.

VIII. *The Line of Latitude.*

A RULER on A, and the several divisions of the lines on CD, will intersect the arc BD, in as many points; on B as a centre, transfer the intersections of the arc BD, to the right line BD; number the divisions from B to D, with 10, 20, 30, &c. to 90; and BD will be a line of latitude.

IX. *The Line of Hours.*

BISECT the quadrantal arcs BD, BE, in a , b ; divide the quadrantal arc ab into 6 equal parts, (which gives 15 degrees for each hour) and each of these into 4 others; (which will give the quarters.) A ruler on c , and the several divisions of the arc ab , will intersect the line MN in the hour, &c. points, which are to be marked as in the figure.

X. *The Line of Inclinations of Meridians.*

BISECT the arc EA into c ; divide the quadrantal arc bc into 90 equal parts; lay a ruler on c and the several divisions of the arc bc , and the intersections of the line HM will be the divisions of a line of inclinations of meridians.

SECT. VIII.

The uses of some of the Lines on the Plain Scale.

I. *Of the Line of Chords.* Pl. VI.

ONE of the uses of the line of chords is *to lay down a proposed angle, or to measure an angle already laid down.* Thus, to draw a line AC, that shall make with the line AB an angle containing a given number of degrees. (suppose 36.) Figure 19.

ON A, as a centre, with a radius equal to the chord of 60 degrees, describe the arc BC; on this arc, lay the chord of the given number of degrees from the intersection B, to c; draw AC, and the angle BAC will contain the given number of degrees.

Note, Degrees taken from the chords are always to be counted from the beginning of the scale.

The degrees contained in an angle already laid down, may be measured thus. Fig. 19.

ON A as a centre, describe an arc BC with the chord of 60 degrees; the distance BC, measured on the chords, will give the number of degrees contained in the angle BAC.

IF the number of degrees are more than 90; they must be taken from, or measured by the chords, at twice; thus if 140 degrees were to be protracted, 70° may be taken from the chords, and those degrees laid off twice upon the arc described with a chord of 60 degrees.

Note, Degrees are generally denoted by a small ° put over them.

II. *Of the Line of Rhumbs.*

THEIR use is to delineate or measure a ship's course; which is the angle made by a ship's way and the meridian.

Now

Now having the points and $\frac{1}{4}$ points of the compass contained in any course; draw a line AB (fig. 19.) for the meridian; on A as a centre, with a chord of 60° describe an arc BC; take the number of points and $\frac{1}{4}$ points from the scale of rhumbs, counting from 0, and lay this distance on the arc BC, from the intersection B to C; draw AC, and the angle BAC shall represent the ship's course.

III. *The use of the Line of Longitude.*

If any two meridians be distant one degree or 60 geographical miles, under the equator, their distance will be less than 60 miles in any latitude between the equator and the pole.

Now let the line of longitude be put on the scale close to the line of chords, but inverted; that is, let 60° in the scale of longitude be against 0° in the chords, and 0° degrees longitude against 90° chords. Then mark any degree of latitude counted on the chords; and opposite thereto, on the line of longitude, will be the miles contain'd in one degree of longitude, in that latitude.

Thus 57,95 miles, make 1 degree of longitude in the latitude of 15 degrees; 45,97 miles, in latitude 40 degrees; 36,94 miles, in latitude 52 degrees; 30 miles, in latitude 60 degrees, &c.

BUT as the fractional parts are not very obvious on scales, here follows a table shewing the miles in one degree of longitude to every degree of latitude.

THIS table is computed, upon the supposition of the earth being spherical, by the following proportion.

As the radius is to the cosine of any latitude, so is the miles of longitude under the equator to the miles of longitude in that latitude.

EVERY person who is desirous of acquiring mathematical knowledge, should have a table of the logarithms of numbers, sines, tangents, and secants; most
of

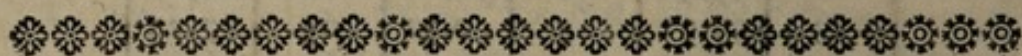
of the treatises of navigation, and some other books, have these tables; but the most useful and esteemed are *Sherwin's* mathematical tables.

A TABLE, shewing the Miles in one Degree of Longitude to every Degree of Latitude.

D. L.	Miles.	D. L.	Miles.	D. L.	Miles.
1	59,99	31	51,43	61	29,09
2	59,96	32	50,88	62	28,17
3	59,92	33	50,32	63	27,24
4	59,85	34	49,74	64	26,30
5	59,77	35	49,15	65	25,36
6	59,67	36	48,54	66	24,41
7	59,56	37	47,92	67	23,44
8	59,42	38	47,28	68	22,48
9	59,26	39	46,63	69	21,50
10	59,09	40	45,97	70	20,52
11	58,89	41	45,28	71	19,53
12	58,69	42	44,59	72	18,54
13	58,46	43	43,88	73	17,54
14	58,22	44	43,16	74	16,54
15	57,95	45	42,43	75	15,53
16	57,67	46	41,68	76	14,52
17	57,38	47	40,92	77	13,50
18	57,06	48	40,15	78	12,48
19	56,73	49	39,36	79	11,45
20	56,38	50	38,57	80	10,42
21	56,02	51	37,76	81	9,38
22	55,63	52	36,94	82	8,35
23	55,23	53	36,11	83	7,32
24	54,81	54	35,27	84	6,28
25	54,38	55	34,41	85	5,23
26	53,93	56	33,55	86	4,18
27	53,46	57	32,68	87	3,14
28	52,96	58	31,79	88	2,09
29	52,47	59	30,90	89	1,05
30	51,96	60	30,00	90	0,00

THE uses of the scales of fines, tangents, secants, and half tangents, are to find the poles and centers of the several circles represented in the orthographical and stereographical projection of the sphere; which are reserved until the explanation and use of the lines of the same name on the sector are shewn.

THE lines of latitudes, hours, and inclinations of meridians, are applicable to the practice of dialling; on which there are several treatises extant, which may be consulted.



S E C T. IX.

Of the S E C T O R.

A Sector is a figure form'd by two radius's of a circle, and that part of the circumference comprehended between the two radius's.

THE instrument called a sector, consists of two rulers moveable round an axis or joint, from whence several scales are drawn on the faces of the rulers.

THE two rulers are called legs, and represent the radii, and the middle of the joint expresses the center.

THE scales generally put on sectors, may be distinguished into single, and double.

THE single scales are such as are commonly put on plain scales, and from whence dimensions or distances are taken as have been already directed.

THE double scales are those which proceed from the center; each scale is laid twice on the same face of the instrument, *viz.* once on each leg: From these scales, dimensions or distances are to be taken, when the legs of the instrument are in an angular position, as will be shewn hereafter.

The Scales commonly put on the best Sectors, are

Single	{	1 2 3 4 5 6 7 8 9 10 11 12 13 14	{	a line of	{	Inches, each Inch divided into 8 and 10 parts.	{	mark'd	{	Cho.
						Decimals, containing an 100 parts.				Sin.
						Chords,				Tang.
						Sines,				Rum.
						Tangents,				Lat.
						Rhumbs,				Hou.
						Latitude,				Lon.
						Hours,				In. Me
						Longitude,				Num.
						Inclin. Merid.				Sin.
						the				V. Sin.
						Loga- rithms				Tan.
						of				

Double	{	1 2 3 4 5 6 7	{	a line of	{	Lines, or of equal parts,	{	mark'd	{	Lin.
						Chords.				Cho.
						Sines.				Sin.
						Tangents to 45°				Tan.
						Secants,				Sec.
						Tangents to above 45°				Tan.
						Polygons,				Pol.

THE manner in which these scales are disposed of on the sector, is best seen in the plate fronting the title page.

THE scales of lines, chords, sines, tangents, rhumbs, latitudes, hours, longitude, incl. merid. may be used, whether the instrument is shut or open, each of these scales being contained on one of the legs only. The scales of inches, decimals, log. numbers, log. sines, log. verfed sines and log. tangents, are to be used with the sector quite opened, part of each scale lying on both legs.

THE double scales of lines, chords, sines, and lower tangents, or tangents under 45 degrees, are all of the same radius or length; they begin at the center of the instrument, and are terminated near the other extremity of each leg; viz. the lines at the division

division 10, the chords at 60, the fines at 90, and the tangents at 45; the remainder of the tangents, or those above 45° , are on other scales beginning at $\frac{1}{4}$ of the length of the former, counted from the center, where they are marked with 45, and run to about 76 degrees.

THE secants also begin at the same distance from the center, where they are marked with 10, and are from thence continued to as many degrees as the length of the sector will allow, which is about 75° .

THE angles made by the double scales of lines, of chords, of fines, and of tangents to 45 degrees, are always equal.

AND the angles made by the scales of upper tangents, and of secants, are also equal; and sometimes these angles are made equal to those made by the other double scales.

THE scales of polygons are put near the inner edge of the legs, their beginning is not so far removed from the center, as the 60 on the chords is: Where these scales begin, they are mark'd with 4, and from thence are figured backwards, or towards the center, to 12.

FROM this disposition of the double scales, it is plain, that those angles which were equal to each other, while the legs of the sector were close, will still continue to be equal, although the sector be opened to any distance it will admit of.

S E C T. X.

Of the Construction of the Single Scales.

I. The Scale of Inches.

THIS scale, which is laid close to the edge of the sector, and sometimes on the edge, contains as many inches as the instrument will receive when opened: Each inch is usually divided into 8 equal parts, and also into 10 equal parts.

II. The Decimal Scale.

THIS scale lies next to the scale of inches; it is of the same length of the sector when opened, and is divided into 10 equal parts, or primary divisions; and each of these into 10 other equal parts; so that the whole is divided into 100 equal parts. And where the sector is long enough, each of the subdivisions is divided into two, four, or five parts; and by this decimal scale, all the other scales, that are taken from tables, may be laid down.

THE length of a sector is usually understood when it is shut, or the legs closed together. Thus a sector of six inches when shut, makes a ruler of twelve inches when opened, and a foot sector, is two feet long when quite opened.

III. The Scales of Chords, Rhumbs, Sines, Tangents. Hours, Latitudes, Longitudes, and Inclination of Meridians;

ARE such as have been already described in the account of the plane scale.

IV. *The Scale of Logarithmic Numbers.*

THIS scale, commonly called the artificial numbers, and by some the *Gunter's* scale, or *Gunter's* * line, is a scale expressing the logarithms of common numbers, taken in their natural order. To lay down the divisions in the best manner, there is necessary a good table of logarithms, (suppose *Sherwin's*,) and a scale of equal parts, accurately divided, and of such a length, that 20 of the primary divisions shall make the whole length of the intended scale of numbers, or logarithm scale.

The Construction.

1. FROM the scale of equal parts, take the first 10 of the primary divisions, and lay this distance down twice on the log. scale, making two equal intervals; marking the first point 1, the second 1, (or rather 10) and the third 10, (or rather 100.)

2. FROM the scale of equal parts, take the distances expressed by the logs. of the numbers, 2, 3, 4, 5, 6, 7, 8, 9, respectively, (rejecting the indices :) lay these distances on each interval of the log. scale, between the marks 1 & 10, 10 & 100, reckoning each distance from the beginning of its interval, viz. from 1, and from 10, and mark these distances with the figures 2, 3, 4, 5, 6, 7, 8, 9, in order.

THUS the first three figures of the logarithms of 2, 3, 4, 5, 6, 7, 8, 9, are, 301, 477, 602, 699, 778, 845, 903, 954; these are the numbers that are to be taken from the scale of equal parts, and laid

* From Mr. *Edmund Gunter*, the Inventor: *Astronomy-Professor in Gresham College*, Anno 1624.

down in each interval, observing that the extent for each is to be applied from the beginning of the intervals.

3. THE distances expressing the logs. of the numbers between 10 & 20, 20 & 30, 30 & 40, 40 & 50, 50 & 60, 60 & 70, 70 & 80, 80 & 90, 90 & 100, (rejecting the indices) are also to be taken from the scale of equal parts, and laid on the log. scale, in each of the primary intervals, between the marks 1 & 2, 2 & 3, 3 & 4, 4 & 5, 5 & 6, 6 & 7, 7 & 8, 8 & 9, 9 & 10, respectively; reckoning each distance from the beginning of its respective primary interval.

4. THE last subdivisions of the second primary interval are to be divided into others, as many as the scale will admit of, which is done by laying down the logarithms of such intermediate divisions, as it shall be thought proper to introduce.

V. The Scale of Logarithm Sines.

1. FROM the scale of equal parts, take the distances expressed by the arithmetical complements * of the logarithmic fines, (or the secants of the complements) of 80, 70, 60, 50, 40, 30, 20, 10, degrees respectively; rejecting the indices; and these distances, lay on the scale of log. fines, reckoning each from the mark intended to express 90 degrees.

THUS. To the fines of 80° , 70° , 60° , 50° , 40° , 30° , 20° , 10° , the three first figures of the arithmetical complements of their logarithms, are, 007, 026, 063, 115, 192, 301, 466, 760; these are the numbers to be taken from the scale of equal parts, used for

* By the arithmetical complement of any fine, tangent, &c. is meant the remainder, when that fine, tangent, &c. is subtracted from radius, or 10,000000, &c.

laying down the logarithms of numbers, and every extent of the compasses is to be laid from the right hand towards the left, beginning at the point chose for 90° , which usually stands directly under the end of the line of numbers.

2. IN the same manner, lay off the degrees under 10; and also, the degrees intermediate to those of 10, 20, 30, &c.

3. LAY down as many of the multiples of 5 minutes, as may conveniently fall within the limits of those degrees which will admit of such subdivisions of minutes.

VI. *The Scale of Logarithmic Tangents.*

1. THIS scale, as far as 45 degrees, is constructed, in every particular, like that of the log. sines; using the arithmetical complements of the log. tangents.

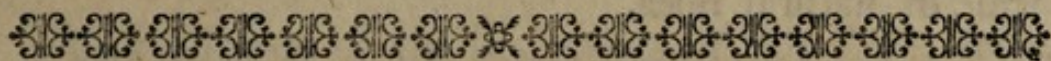
2. THE degrees above 45, are to be counted backwards on the scale: Thus 40 on the scale, represents both 40 degrees, and 50 degrees; 30 on the scale, represents both 30 degrees, and 60 degrees; and the like of the other mark'd degrees, and also of their intermediate ones.

VII. *The Logarithmic versed Sines.*

1. FROM the scale of equal parts, take the arithmetical complements of the logarithm co-sines, (or the secants of the complements) of 5, 10, 15, 20, 25, 30, 35, 40, &c. degrees; (rejecting the indices,) and the double of these distances, respectively, laid on the scale (intended) for the log. versed sines, will give the divisions expressing 10, 20, 30, 40, 50, 60, 70, 80, &c. degrees; to as many as the length of the scale will take in.

2. BETWEEN every distance of 10 degrees, introduce as many degrees, $\frac{1}{2}$ degrees; $\frac{1}{3}$ degrees; $\frac{1}{4}$ degrees, &c. as the intervals will admit. THE

THE scales of the logarithms of numbers, fines, verfed fines, and tangents, should have one common termination to one end of each scale; that is, the 10 on the numbers, the 90 on the fines, the 0 on the verfed fines, and the 45 on the tangents, should be opposite to each other: The other end of each of the scales of fines, verfed fines, and tangents, will run out beyond the beginning (mark'd 1) of the numbers; nearly opposite to which, will be the divisions representing 35 minutes on the fines and tangents, and $168\frac{1}{2}$ degrees, on the verfed fines.



SECT. XI.

Of the Construction of the Double Scales.

I. *Of the Line of Lines.*

THIS is only a scale of equal parts, whose length is adapted to that of the legs of the sector: Thus in the six inch sector, the length is about $5\frac{3}{4}$ inches.

THE length of this scale is divided into 10 primary divisions; each of these into 10 equal secondary parts; and each secondary division, into 4 equal parts.

HENCE on any sector it will be easy to try if this line is accurately divided: Thus. Take between the compasses any number of equal parts from this line, and apply that distance to all the parts of the line; and if the same number of divisions are contained between the points of the compasses in every application, the scale may be received as perfect.

II. *Of the Line of Sines.*

1. MAKE the whole length of this scale, equal to that of the line of lines.

2. FROM the scale of the line of lines, take off severally, the parts expressed by the numbers in the tables (suppose *Sherwin's*) of the natural sines, corresponding to the degrees, or to the degrees and minutes, intended to be laid on the scale.

3. LAY down these distances severally on the scale, beginning from the center; and this will express a scale of natural sines,

EXAM. To lay down $35^{\circ} 15'$; whose natural sine found in the tables is 57714, &c.

TAKE this number as accurately as may be, from the line of lines, counting from the center; and this distance will reach from the beginning of the sines, at the center of the instrument, to the division expressing $35^{\circ} 15'$; and so of the rest.

IN scales of this length, it is customary to lay down divisions, expressing every 15 minutes, from 0 degrees to 60 degrees; between 60 and 80 degrees, every half degree is expressed; then every degree to 85; and the next, is 90 degrees.

III. *Of the Scale of Tangents.*

THE length of this scale is equal to that of the line of lines, and the several divisions thereon (to 45 degrees) are laid down from the tables and line of lines, in the same manner as has been described in the sines; observing to use the natural tangents in the tables.

IV. *Of the Scale of upper Tangents.*

THIS scale is to be laid down, by taking $\frac{1}{4}$ of such of the natural tabular tangents above 45 degrees, as are intended to be put on the scale.

ALTHOUGH the position of this scale on the sector respects the center of the instrument, yet its beginning, at 45 degrees, is distant from the center, $\frac{1}{4}$ of the length or radius of the lower tangents.

V. *Of the Scale of Secants.*

THE distance of the beginning of this scale, from the center, and the manner of laying it down, is just the same as that of the upper tangents; only in this, the tabular secants are to be used.

VI. *Of the Scale of Chords.*

1. MAKE the length of this scale, equal to that of the fines; and let the divisions to be laid down, express every 15 minutes from 0 degrees to 60 degrees.

2. TAKE the length of the sine of half the degrees and minutes, for every division to be laid down, (as before directed in the scale of fines;) and twice this length, counted from the center, will give the divisions required.

THUS, twice the length of the sine $18^{\circ} 15'$, will give the chord of $36^{\circ} 30'$; and in the same manner for the rest.

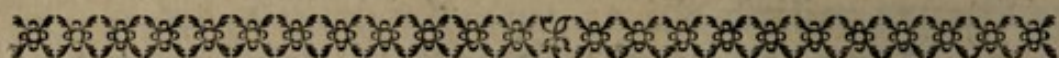
VII. *Of the Scale of Polygons.*

THIS scale usually takes in the sides of the polygons from 6 to 12 sides inclusive: The divisions are laid down, by taking the lengths of the chords of

the angles at the center of each polygon; and these distances are laid from the center of the instrument.

BUT it is best to have the polygons of 4 and 5 sides also introduced; and then this line is constructed from a scale of chords, where the length of 90 degrees is equal to that of 60 degrees of the double scale of chords on the sector.

IN the place of some of the double scales here described, there are found other scales on the old sectors, and also on some of the modern *French* ones, such as, scales of superficies, of solids, of inscribed bodies, of metals, &c. But these seem to be justly left out on the sectors, as now constructed, to make room for others of more general use: However, these scales, and some others, of use in gunnery, shall hereafter be described in a tract on the use of the gunners callipers.



S E C T. XII.

Of the Uses of the Double Scale.

IN the following account of the uses, as there will frequently occur the terms *lateral distance*, and *transverse distance*; it will be proper to explain what is meant by those terms.

Lateral distance, is a distance taken by the compasses on one of the scales only, beginning at the center of the sector.

Transverse distance, is the distance taken between any two corresponding divisions of the scales of the same name, the legs of the sector being in an angular position: That is, one foot of the compasses is set on a division in a scale on one leg of the sector, and the other foot is extended to the like division in the
the

the scale of the same name on the other leg of the sector.

It must be observed, that each of the sectoral scales have three parallel lines, across which the divisions of the scale are marked : Now in taking transverse distances, the points of the compasses must be always set on the inside line, or that line next the inner edge of the leg ; for this line only in each scale runs to the center.

Some Uses of the Line of Lines.

P R O B L E M X.

To two given lines $AB = 2$, $BC = 6$; to find a third proportional. Plate VI. Fig. 20.

OPERATION. 1. Take between the compasses, the lateral distance of the second term, (*viz.* 6.)

2. SET one point on the division expressing the first term (*viz.* 2.) on one leg, and open the legs of the sector till the other point will fall on the corresponding division on the other leg.

3. KEEP the legs of the sector in this position ; take the transverse distance of the second term, (*viz.* 6.) and this distance is the third term required.

4. THIS distance measured laterally, beginning from the center, will give (18) the number expressing the measure of the third term : For $2 : 6 :: 6 : 18$.

OR, Take the distance 2 laterally, and apply it transversely to 6 and 6 (the sector being properly opened), then the transverse distance at 2 and 2 being taken with the compasses and applied laterally from the center of the sector on the scale of lines, will give $66\frac{2}{3} = \frac{2}{3}$, the third term when the proportion is decreasing : For $6 : 2 :: 2 : \frac{2}{3}$.

Note, If the legs of the sector will not open so far as to let the lateral distance of the second term fall between the divisions expressing the first term ; then take

take $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, or any aliquot part of the second term, (such as will conveniently fall within the opening of the sector) and make such part, the transverse distance of the first term; then if the transverse distance of the second term be multiplied by the denominator of the part taken of the second term, the product will give the third term.

P R O B L E M XI.

To three given lines $AB = 3$, $BC = 7$, $CD = 10$; to find a fourth proportional. Plate VI. Fig. 21.

OPERATION. Open the legs of the sector, until the transverse distance of the first term, (3) be equal to the lateral distance of the second term, (7) or to some part thereof; then will the transverse distance of the third term, (10) give the fourth term, ($23\frac{1}{3}$) required; or, such a submultiple thereof as was taken of the second term: For $3 : 7 :: 10 : 23\frac{1}{3}$

OR, Set the lateral distance 7 transversely from 10 to 10 (opening the sector properly); then the transverse distance at 3 and 3 taken and applied laterally, will give $2\frac{1}{10}$: For $10 : 7 :: 3 : 2\frac{1}{10}$.

FROM this problem is readily deduced, how to increase or diminish a given line, in any assigned proportion.

EXAM. *To diminish a line of 4 inches, in the proportion of 8 to 7.*

1. OPEN the sector until the transverse distance of 8 and 8, be equal to the lateral distance of 7.
2. MARK the point to where 4 inches will reach, as a lateral distance taken from the center.
3. THE transverse distance, taken at that point, will be the line required.

IF the given line, suppose 12 inches, should be too long for the legs of the sector, take $\frac{1}{2}$, or $\frac{1}{3}$, or $\frac{1}{4}$, &c. part of the given line for the lateral distance; and

and the corresponding transverse distance, taken twice, or thrice, or four times, &c. will be the line required.

P R O B L E M XII.

To open the sector so, that the two scales of lines shall make a right angle.

OPERATION. Take the lateral distance from the center to the division marked 5 between the points of the compasses, and set one foot on the division marked 4 on one of the scales of lines, and open the legs of the sector till the other foot falls on the division marked 3 on the other scale of lines, and then will those scales stand at right angles to one another.

For the lines 3, 4, 5, or any of their multiples, constitute a right angle triangle.

P R O B L E M XIII.

To two right lines given, to find a mean proportional. Suppose the lines 40 and 90.

OPERATION. 1st. Set the two scales of lines at right angles to one another.

2^d. FIND the half sum of the given lines ($= \frac{90+40}{2} = 65$); also find the half difference of those lines ($= \frac{90-40}{2} = 25$).

3^d. TAKE, with the compasses, the lateral distance of the half sum (65), and apply one foot to the half difference (25), the other foot transversely will reach to (60) the mean proportional required: For 40 : 60 :: 60 : 90.

P R O B L E M XIV.

To divide a given line into any proposed number of equal parts : (suppose 9).

MAKE the length of the given line, or some known part thereof, a transverse distance to 9 and 9 : Then will the transverse distance of 1 and 1, be the $\frac{1}{9}$ part thereof ; or such a submultiple of the $\frac{1}{9}$ part, as was taken of the given line.

OR the $\frac{1}{9}$ part, will be the difference between the given line, and the transverse distance of 8 and 8.

THE latter of these methods is to be preferred when the part required falls near the center of the instrument.

To this problem may be referred the method of making a scale of a given length, to contain a given number of equal parts.

THE practice of this is very useful to those who have occasion to take copies of surveys of lands ; draughts of buildings, whether civil or military ; and in every other case, where drawings are to be made to bear a given proportion to the things they represent.

EXAM. Suppose the scale to the map of a survey is 6 inches long, and contains 140 poles ; required to open the sector so, that a corresponding scale may be taken from the line of lines.

SOLUTION. Make the transverse distance 7 and 7 (or 70 and 70, viz. $\frac{140}{2}$) equal to three inches ($= \frac{6}{2}$) ; and this position of the line of lines will produce the given scale.

If it was required to make a scale of 140 poles, and to be only two inches long.

SOLUTION. Make the transverse distance of 7 and 7 equal to one inch, and the scale is made.

EXAM.

EXAM. II. *To make a scale of 7 inches long contain 180 fathoms.*

SOLUTION. Make the transverse distance of 9 and 9 equal to $3\frac{1}{2}$ inches, and the scale is made.

EXAM. III. *To make a scale which shall express 286 yards, and be 18 inches long.*

SOLUTION. Make the $\frac{1}{3}$ of 18 inches (or 6 inches) a transverse distance to the $\frac{1}{3}$ of 286 ($= 95\frac{1}{3}$) and the scale is made.

OR, Make the $\frac{1}{4}$ of 18 inches ($= 4\frac{1}{2}$ inches) a transverse distance to $\frac{1}{4}$ of 286 ($= 71\frac{1}{2}$), and the scale is made.

EXAM. IV. *To divide a given line (suppose of 5 inches) into any assigned proportion (as of 4 to 5).*

SOLUTION. Take (5 inches) the length of the given line, between the compasses, and make this a transverse distance to (9 and 9) the sum of the proposed parts; then the transverse distances of the assigned numbers (4 and 5) will be the parts required.

P R O B L E M XV.

The use of the line of lines in drawing the orders of Civil Architecture.

IN this place it is intended to give so much of Architecture as may enable a beginner to draw any one of the orders; but that the following precepts may be rightly understood, it will be proper to explain a few of the terms.

DEFINITIONS.

1. ARCHITECTURE is the art of building well; and has for its object the Convenience, Strength, and Beauty of the building.

2. ORDER in Architecture, is generally understood as Ornament, and consists of three grand parts, namely ;

3. THE ENTABLATURE, which represents, or is, the weight to be supported.

4. THE COLUMN, that which supports any weight.

5. THE PEDESTAL or foot whereon the Column is set for its better security.

EACH of these parts consists also of three parts.

6. THE *Pedestal* is composed of a BASE, or lower part, a DIE, and a CORNICE, or upper part.

7. THE *Column* is made up of a BASE, a SHAFT, which is a middle part, and a CAPITAL, the upper part.

8. THE *Entablature* consists of an ARCHITRAVE, or lower part, a FREEZE, the middle part, and a CORNICE, the upper part.

So that an Order may be said to consist of nine large parts, each of which is made up of smaller parts called Members; whereof some are *Plane*, some *Curved*, either convex or concave, or convexo-concave.

PLANE members of different magnitude have different names.

9. A FILLET or *list* is the least plane or flat member.

10. A PLINTH is that flat member at the bottom of the Pedestal, or of the base of the Column.

11. A PLATEBAND, that at the top of the Pedestal, or the upper member of the Architrave in the Entablature.

12. AN ABACUS, that at the top of the capital.

13. THE FACIÆ or faces are flat members in the Architrave.

14. THE CORONA is a large flat member in the Cornice.

THE Convex members are,

15. AN ASTRAGAL of a small semicircular convexity.

16. THE

16. A FUSAROLE when an Astragal is cut into parts like beads.

17. A TORUS a large semicircular convexity.

18. AN OVOLA nearly of a quadrantal convexity.

THE Concave members are,

19. A CAVETTO nearly of a quadrantal concavity.

20. A SCOTIE of a concavity nearly semicircular.

THE Convexo-Concave members are a Cymaife and a Cima.

21. A CYMAISE or *Ove*, that whose convex part projects most; and by workmen is usually called an OGEE.

22. A CIMA that whose concave part projects most.

23. SOFFIT is the under part of the Crown of an Arch, or of the Corona of an Entablature.

24. TRIGLIPHS (*i. e.* three channels) is an Ornament in the Freeze of the Doric Order.

25. METOPS (*i. e.* between three's) is the space of the Freeze between two Triglyphs.

26. MODILIONS, or MUTULES, are the brackets or ends of beams supporting the Corona. In the Corinthian Order they are generally carved into a kind of Scrol.

27. DENTELS are an Ornament looking somewhat like a row of teeth; and are placed in the Cornice of the Entablature.

It is customary among Architects to estimate the heights and projections of all the parts of every order by the diameter of the column at the bottom of the shaft, which they call a module; and suppose it to consist of 60 equal parts, which are called minutes.

Of the TUSCAN ORDER.

THIS order, which some writers liken to a strong robust labouring man, is the most simple and undorned of any of the orders: The places most recommended to use it in, are country farm-houses, stables,

stables, gateways to inns, and places where plainness and strength are reckoned most necessary : Though there are instances where this order has been applied to buildings of a more public and elegant nature.

THE general proportions assigned by *Palladio*.

1. HEIGHT of the column equal to seven diameters, or modules.

2. HEIGHT of the entablature equal to one fourth of the column, wanting half a minute.

3. HEIGHT of the pedestal equal to one module.

4. THE capital and base, each half a module.

5. BREADTH of the base on a level is $1\frac{1}{6}$ module.

6. BREADTH of the capital equal to one module.

7. DIMINISHING of the column is $\frac{1}{4}$ module.

8. PROJECTION of the beams supporting the eaves is $1\frac{3}{4}$ modules.

9. IN colonades, the distance of the columns in the clear is 4 modules.

10. IN arches, and the columns set on pedestals,

The distance of the columns from middle to middle is $6\frac{5}{12}$ modules.

Height of the arch is $7\frac{2}{3}$ modules.

Breadth of the pilaster between the column and passage is 26 minutes.

THE ovolo under the corona, in the cornice of the entablature, is commonly continued within the corona, giving it a reverse bending in the soffit, something like a cyma.

Of the DORIC ORDER.

THIS order, supposed to be invented by *Dorus* a king of *Achaia*, may be likened to a well limbed genteel man ; and although of a bold aspect, yet not so sturdy and rustically clad as the Tuscan. Architects place this order indifferently in towns : But when they would decorate a country seat with it, the open campaign situation seems best for the reception of the
Doric

Doric order; notwithstanding which, there are many fine buildings of this order in other situations, where they have a very pleasing effect.

THE following general proportions are given by *Palladio*.

1. HEIGHT of the column from $7\frac{1}{2}$ to 8, and $8\frac{2}{3}$ modules.

2. HEIGHT of the entablature is one fourth of the column.

3. HEIGHT of the pedestal equal to $2\frac{1}{3}$ modules.

4. THE Attic base is used with this order, it is half a module in height, and so is the capital.

5. BREADTH of the column's base is $1\frac{1}{6}$ module.

6. BREADTH of the capital is 1 module 17 $\frac{1}{2}$ minutes.

7. DIMINISHING of the column is 8 minutes.

8. IN colonades, the distance of the columns in the clear is $2\frac{3}{4}$ modules.

9. IN arches, and the column set on pedestals,

Distance of the columns from middle to middle is $7\frac{1}{2}$ modules.

Height of the arch to its soffit is $10\frac{1}{4}$ modules.

Breadth of the pilasters is 26 minutes.

IN the Doric order the architrave has two faces and a plinth; the upper face is ornamented with rows of six *drips* or *bells*, covered with a plain cap: The freeze is divided into triglyphs and metops: The breadths of the drips, cap and triglyphs are each $\frac{1}{2}$ module: The triglyphs consist of two channels, two half channels, and three voids; the breadths of the channels and voids are each 5 minutes: The axis of the column continued, runs through the middle void, leaving the drips three on each side: The metops, or distances between the triglyphs, are equal to the height of the freeze, and are commonly ornamented with trophies, arms, roses, &c.

HERE follows a table for the particular construction of the ornaments with which the architrave and freeze are enriched.

	Altitude.	Projection	Profile.
	Min.	Min.	Min.
Capital	5	16	3
Freeze	45	—	—
Triglyphs	40	15	$\frac{1}{2} + 2\frac{1}{2}$
Plinth	$4\frac{1}{2}$	16	3
Cap	$1\frac{2}{3}$	15	2
Drips	$3\frac{1}{3}$	15	2

THE column signed altitude gives the heights of the particular parts.

THAT signed projection shews the breadths of those parts on each side of the middle line of the column continued.

AND under the word profile stand the numbers shewing how far the several parts project beyond the planes or faces of the members on which they are made.

THE soffit of the corona in the cornice of the entablature, is usually ornamented with drips corresponding to the triglyphs, and roses, arms, &c. over the metops.

THE shaft of the column is sometimes fluted; that is, cut into channels from top to bottom, the channels meeting one another in an edge, and are in number twenty.

Of the IONIC ORDER.

THIS order, which is taller and slenderer than the Doric, does not appear with such a masculine strength; and

and is by some writers compared to the figure of a grave matron. The *Ionians* who invented this order, applied it chiefly to decorate their temples: But when applied to the ornamenting a country palace, the rich and extended vale seems a proper site: Workmen indeed use it indifferently in every place.

Palladio gives to the Ionic order the following general proportions.

1. HEIGHT of the column to be 9 modules.
2. THE altitude of the entablature is equal to $\frac{2}{5}$ that of the column, and divided for the architrave, freeze, and cornice, in the proportion of 4, 3, 5.
3. THE height of the pedestal equal to 2 modules $37\frac{1}{2}$ minutes; or $\frac{8}{27}$ of the column.
4. HEIGHT of the base $\frac{1}{2}$ module; its breadth 1 module $22\frac{1}{2}$ minutes.
5. HEIGHT of the capital and volute is $31\frac{2}{3}$ minutes, and the breadth of its abaco is 1 module $3\frac{1}{3}$ minutes.
6. DIMINUTION of the column is $7\frac{1}{4}$ minutes.
7. IN colonades, the distance of the columns in the clear is $2\frac{1}{4}$ modules.
8. IN arches, and the columns set on pedestals,
Distance of the columns from middle to middle is $7\frac{7}{24}$ modules.
Height of the arch to its soffit is 11 modules.
Breadth of the pilasters is $26\frac{1}{2}$ minutes, between the column and arch.

THE distance of the modillions in the entablature is 22 minutes, and the breadth of each modilion is 10 minutes; the axis of the column produced always passes through the middle of a modilion, which in this order is a plain block representing the end of a beam. The three most elegant remains of the ancient Ionic order in *Rome* have their cornice ornamented with dentels instead of modillions; and it is the opinion of some, eminent for their taste in Architecture, that in this order dentels would have a better effect than modili-

ons; the heights of these dentels were usually twice their breadth, and their distances half their breadth.

THE freeze of this order is usually made swelling, and is formed by the segment of a circle, whose chord is parallel to the axis of the column, and the swelling projecting as far as the plateband of the architrave.

THE volutes of the capital are now made to project in the directions of the diagonals of the square cap, or abaco, over the volutes, so that their drawing should be expressed like the volutes in the Roman order: They are much better drawn by an easy hand, than by any rules for describing them with the compasses, observing the limits of their altitude and projection: But the volutes in the ancient examples of this order were curled in a plane parallel to the architrave. These volutes are supposed to represent the plaited tresses in which the Grecian women used to dress their hair.

THE shaft of the column is sometimes fluted, leaving a fillet or list between each channel: In this order there are 24 flutes and fillets.

Of the CORINTHIAN ORDER.

THIS order, the most elegant of all, is by some compared to a very fine woman clad in a wanton sumptuous habit: It was invented at *Corinth*, and soon spread into other places to adorn their public buildings. A proper rural situation for this order, seems to be a spot commanding a rich and beautiful prospect in a fine watered vale.

THE general proportions assigned by *Palladio* are;

1. THE height of the column to be $9\frac{1}{2}$ modules.
2. HEIGHT of the entablature equal to $\frac{1}{3}$ that of the column; the architrave, freeze and cornice to be in the proportion of 4, 3, 5; and the projection of the cornice equal to its height.

3. HEIGHT

3. HEIGHT of the pedestal equal to $\frac{1}{4}$ of the column.

4. THE height of the capital to be $1\frac{1}{6}$ module; of which the abaco is $\frac{1}{6}$ of a module; its horns projecting over the bottom of the column $\frac{1}{4}$ of a module.

5. THE height of the base equal to $\frac{1}{2}$ module; and its greatest breadth to be one module and a fifth.

6. THE diminution of the column to be 8 minutes.

7. IN colonades, the intercolumniation is 2 modules.

8. IN arches, and the columns set on pedestals,

The distance of the columns, from middle to middle, to be $6\frac{1}{2}$ modules.

Height of the arch equal to $11\frac{1}{6}$ modules.

Breadth of the pilaster, between the column and sides of the passage, to be 27 minutes.

IN this order, the shaft is frequently cut into 24 flutes, which are separated from one another by as many fillets.

THE capital is composed of three tiers of leaves, eight leaves in a tier, with their stalks or scrolls, encircling the body of the capital, which represents a basket, whose bottom is just as broad as the diameter of the top of the column within the channels: The ornaments of this capital are best done by hand, without rule or compass, observing the proper altitudes and projections of the parts.

THE architrave consists of three faciæ, three fufaroles, an ogee, and a plateband; the first, or lower faciæ projects the same as the top of the shaft.

THE freeze, which projects the same as the top of the shaft, has its lower part turned into a kind of cavetto, terminating with the extremity of the plateband of the architrave.

THE breadths of the dentels are $3\frac{1}{3}$ minutes, and their distance $1\frac{2}{3}$ minutes.

THE breadths of the modillions are $11\frac{1}{3}$ minutes, and their distance in the clear $23\frac{1}{4}$ minutes.

THE middle of a dentel should be under the middle of a modilion, and the axis of the column passes through the middles of both dentel and modilion.

Of the COMPOSITE ORDER.

THIS order (the poor invention of the *Romans*, and therefore frequently called the Roman order), is usually composed of the Corinthian and Ionic; the Ionic capital being set over the two lower rows of leaves in the Corinthian capital.

Palladio gives us the following general proportions.

1. THE height of the column to be 10 modules.
2. THE height of the entablature equal to $\frac{1}{3}$ of the column; the architrave, freeze, and cornice, in the proportion of 4, 3, 5; the freeze swelling like that of the Ionic.
3. HEIGHT of the pedestal to be $\frac{1}{3}$ of the column.
4. HEIGHT of the capital equal to $1\frac{1}{6}$ module; of which the abaco is $\frac{1}{6}$ module, its horns projecting from the center of the column 1 module.
5. HEIGHT of the base $31\frac{1}{2}$ minutes, and its greatest breadth $1\frac{1}{3}$ modules.
6. DIMINUTION of the column equal to 8 minutes.
7. IN colonades, the intercolumniation is $1\frac{1}{2}$ modules.
8. IN arches, and the columns set on pedestals,
Distance of the columns from middle to middle is $7\frac{1}{4}$ modules.
Height of the arch equal to $12\frac{1}{3}$ modules: In the clear, the height is to the span as 5 to 2.
The breadth of the pilasters between the column and arch is $\frac{7}{16}$ modules, or 42 minutes.

IN this order the shaft, if fluted, is to have 24 channels and 24 fillets, one between each two flutes.

THE volutes of the capital are angular, to have the same appearances on every side, and they are drawn like those in the Ionic.

THE modillions in this order are worked into two faces, with an ogee between them; the breadth of the lower face $9\frac{1}{2}$ minutes, that of the upper $12\frac{1}{2}$; the distance of two modillions at the upper faces is 20 minutes, and at the lower faces 23 minutes; the axis of the column passing through the middle of a modillion.

To draw the Mouldings in Architecture.

THE terminations or ends of flat members, are right lines.

THE astragal, fufarole, and torus, are terminated by a semicircle.

To describe the Torus. Fig. 1. Plate I.

ON AB, its breadth, describe a semicircle.

To make an Ovolo, whose breadth is AB. Fig. 2.

MAKE $AC = \frac{2}{3}$ or $\frac{3}{4}$ of AB, and draw CB.

MAKE the angle CBD equal to the angle BCD.

THEN the intersection of BD with CA will give D the center of the arc BC.

OR, Describe on BC an equilateral triangle; and make the vertex the center.

THE former of these methods is the most graceful.

To make a Cavetto, whose breadth is AB. Fig. 3.

MAKE $AC = \frac{2}{3}$ or $\frac{3}{4}$ of AB; draw BC, and produce the bottom line towards D.

MAKE an angle BCD equal to the angle CBD.

THEN D, the intersection of CD with BD, is the center sought.

OR, On BC describe an equilateral triangle, and the vertex will be the center.

To make a Scotia, whose breadth is AB. Fig. 4.

MAKE AF equal to $\frac{1}{3}$ of AB.

ON AF describe the square AC, and on BF describe the square BD.

THEN C is the center of the arc EF, and D the center of the arc FG.

To make a Cima, whose breadth is AB. Fig. 5.

MAKE AC equal to about $\frac{7}{6}$ of AB.

DRAW the right line CB, which bisect in D.

ON CD and DB, make isosceles triangles, whose legs DE, DF, may be each $\frac{6}{7}$ of the base CD, DB; and the vertexes E and F will be the centers of the arcs CD, DB.

OR, The centers of the arcs CD, DB, may be found by describing equilateral triangles on the right lines CD, DB.

To make a Cymaise, or Ogee, whose breadth is AB. Fig. 6.

MAKE AC equal to about $\frac{7}{6}$ of AB.

DRAW the right line CB, which bisect in D.

THROUGH D draw the right line EF, so, that the angle CDE may be equal to the angle DCE; meeting the upper and lower lines in E and F.

THEN E is the center of the arc CD, and F the center of the arc DB.

To describe the curve joining the shaft of a column with its upper or lower fillet, the projection of AB being given. Fig. 7.

MAKE AC equal to twice AB.

DRAW CD parallel to AB, and equal to $\frac{5}{4}$ of AC.

THEN D is the center of the arc CB.

To draw the gradual diminution of a Column. Fig. 8.

DRAW the axis AB of the intended length of the shaft; and parallel thereto, at half a module distance, draw CD; make CE equal to half the proper diminution, and draw EF parallel to BA.

MAKE

MAKE AG equal to one third of AB; and so high is the shaft to be parallel to its axis; through G draw HI at right angles to AB.

ON HI describe a semicircumference cutting the line EF in the point 4; divide the arc H4 into equal parts at pleasure, suppose 4; and through those points draw the lines 11, 22, 33, 44.

DIVIDE the line GB into a like number of equal parts, as at the points *a, b, c*; and through these points draw lines parallel to IH; making $aa = 11$, $bb = 22$, $cc = 33$.

THEN a curved line drawn through the extremities H, *a, b, c, E*, will limit the gradual diminution required.

Palladio describes another method, which is more ready in practice.

LAY a thin ruler by the points D, H, E, and the bending of the ruler will give the gradual diminution required.

To describe the Volute of the Ionic order. Figs. 9, 10.

THE altitude AB, which is $\frac{4}{9}$ of a module, or $26\frac{2}{3}$ minutes, is divided into 8 equal parts, viz. 4 from C to A, and 4 from C to B; upon $CD = 3\frac{1}{3}$, one of these parts, a circle is described, and called the eye of the volute, which corresponds with the astragal of the column.

PALLADIO gives the following manner of finding the 12 centers of the volute, which he discovered on an old unfinished capital. Fig 9.

WITHIN the eye of the volute inscribe a square, whose diagonal is CD; in this square draw the two diameters 13, 24, and these four points 1, 2, 3, 4, are the centers of the arcs AI, IB, B3, 34, which forms the first revolution.

THE centers of the arcs forming the second and third revolutions are thus found; see the eye of the volute drawn at large. Fig. 9.

DIVIDE

DIVIDE the radii $o1$, $o2$, $o3$, $o4$, each into 3 equal parts, as at the points 5, 6, 7, 8, 9, 10, 11, 12, and these will be the centers of the remaining arcs, the last of which is to coincide with the point c , in the eye.

GOLDMAN observing that in this construction the ends and beginnings of the arcs were not at right angles to the same radii, contrived the following construction. See Fig. 10. and its eye drawn at large.

UPON one half of cd , describe the square 1, 2, 3, 4; and draw the lines $o2$, $o3$; divide $o1$, $o4$, each into 3 equal parts; then lines drawn through those points parallel to 1, 2, their intersections with 14, $o2$, $o3$, will be centers of the volute.

So the points, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, will be the centers of the twelve arcs which together form the outward curve of the volute.

IN either method, the centers of the inner curve may be thus found.

TAKE oa equal to $\frac{7}{8}$ of $o1$; divide oa into three equal parts, and these divisions will give centers of the inner curve; the two eyes drawn at large will shew how the 12 inner centers are found, where they are distinguished by large points; the 12 centers of the outward curve being marked by the figures.

IN the describing of these volutes, it will frequently happen, that the last quadrant will not fall on its true termination, occasioned by the radii of the several quadrants not being exactly taken by the compasses: In order to avoid this inaccuracy, at least in some degree, here is subjoined a table shewing the length of each radius, computed from *Goldman's* method: But it may also be applied to *Palladio's*, the radius of the largest quadrant not differing $\frac{3}{1000}$ of a minute, or $\frac{1}{20000}$ of a module from the truth; and excepting the arc described from the first center, the rest may be made quadrants in the same manner as shewn in *Goldman's* method.

A TABLE of the lengths, in minutes, of the several radii of the outward and inner volutes.

N ^o Rad.	Outward Curve.	Inward Curve.	In parts of 1st rad.	
			Outward.	Inward.
1	$\frac{85}{6} = 14,166$	$\frac{605}{48} = 12,604$	100,000	88,969
2	$\frac{75}{6} = 12,500$	$\frac{535}{48} = 11,146$	88,235	78,677
3	$\frac{65}{6} = 10,833$	$\frac{465}{48} = 9,687$	76,468	68,379
4	$\frac{55}{6} = 9,166$	$\frac{395}{48} = 8,229$	64,705	58,087
5	$\frac{70}{9} = 7,777$	$\frac{1010}{144} = 7,014$	54,901	49,510
6	$\frac{60}{9} = 6,666$	$\frac{870}{144} = 6,041$	47,058	42,642
7	$\frac{50}{9} = 5,555$	$\frac{730}{144} = 5,069$	39,215	35,781
8	$\frac{40}{9} = 4,444$	$\frac{590}{144} = 4,097$	31,372	28,920
9	$\frac{65}{18} = 3,611$	$\frac{485}{144} = 3,368$	25,490	23,774
10	$\frac{55}{18} = 3,055$	$\frac{415}{144} = 2,882$	21,568	20,343
11	$\frac{45}{18} = 2,500$	$\frac{345}{144} = 2,395$	17,647	16,906
12	$\frac{35}{18} = 1,944$	$\frac{275}{144} = 1,909$	13,725	13,475

To use this table, a scale of $\frac{1}{4}$ of a module should be made, and divided into 15 minutes, and the extrem division decimally divided, whereby the lengths of the several radii may be taken: But as the sector is an universal scale, there are two other columns added,

ed, applicable to the sector ; where the longer radius 14,166 is made a transverse distance to 10 and 10, or 100 and 100, on the line of lines, and all the other radii of both curves are proportioned thereto : Now the centers of the curves being found as shewn in the eyes of the volute, the several radii may be taken from the sector, and the curves more accurately described than by any other method.

To describe the Flutings and Fillets in channelled columns. Fig. 11.

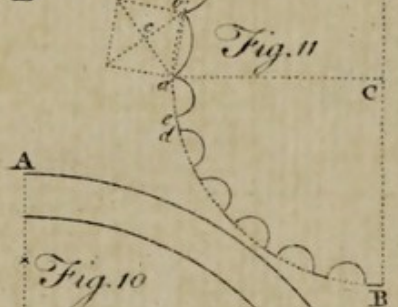
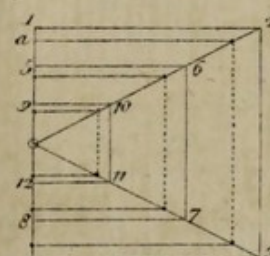
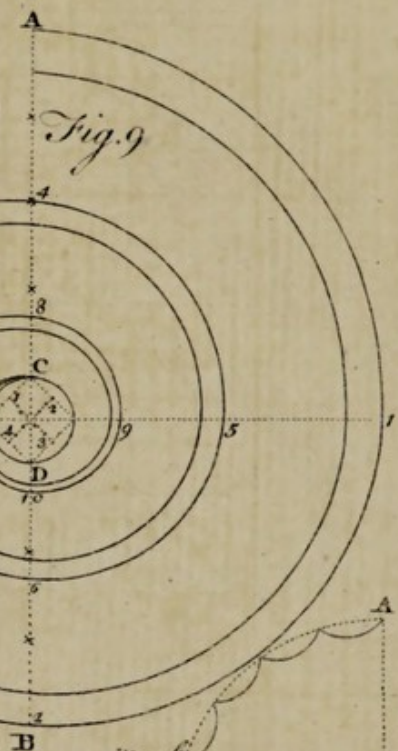
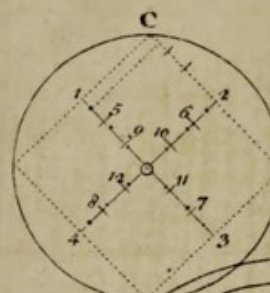
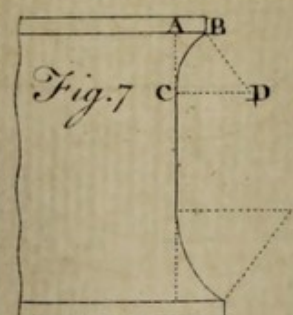
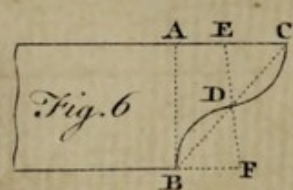
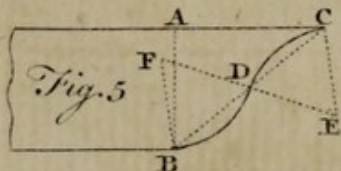
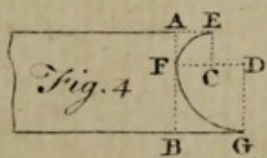
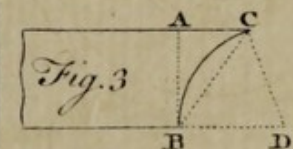
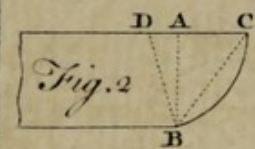
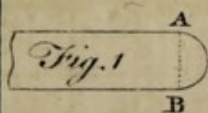
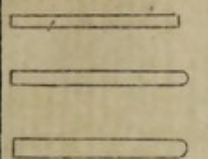
IN the Doric, the circumference of the column being divided into 20 equal parts (here the $\frac{1}{4}$ circumference is divided into 5), of which *ab* is one ; on *ab* describe a square, and the center *c* of that square is the center of the channel or flute required.

IN the Ionic, and Corinthian, divide the circumference of the column into 24 equal parts (here the $\frac{1}{4}$ circumference is divided into 6), of which *ad* is one ; divide *ad* into 4 equal parts ; then *ae* = $\frac{3}{4}ad$ is the breadth of the flute, and *ed* = $\frac{1}{4}ad$ is the breadth of the fillet.

THE flutes are semicircles described on the chords of their arcs in the column.

IN the three following tables are contained the heights and projections of the parts of each order, according to the proportions given by *Palladio* ; the orders of this architect were chosen, because the *English*, at present, are more fond of copying his productions, than those of any other architect.

THE first table serves for the pedestal, the second for the column, and the third for the entablature, of each order. Each table is divided into seven principal columns : In the first, beginning at the left hand, is contained the names of the primary divisions ; in the second those of the several divisions and members in the orders ; and the other five, titled with *Tuscan*,
Doric





Doric, Ionic, Corinthian, Roman, contain the numbers expressing the altitudes, and projections taken from the axis, or middle of the column, of the several members belonging to their corresponding orders.

THE column containing each order, is divided, first into two other columns, one shewing the altitudes, and signed Alt. and the other, the projections, and signed Proj. Each of these is also divided into two other columns, one containing modules, and marked Mo. and the other, the minutes and parts, and marked Mi.

UNDER the table of the pedestals there is another table, shewing the general proportions for the heights of the orders.

IN each of the orders of architecture, the height of the order, and the diameter of the column, have a constant relation to one another.

THEREFORE, if the diameter of the column be given, the height of the order is given also: And having determined by what scale the order is to be drawn, such as $\frac{1}{2}$ inch, 1 inch, 2 inches, &c. to a foot or yard, &c. Take from such scale, the part or parts expressing the diameter of the column, and make this extent a transverse distance to 6 and 6 (*i. e.* 60 and 60) on the scales of lines, and the sector will be opened so, that the several proportions of the order may be taken from it.

EXAM. Suppose the diameter of a column is to be 18 inches; and the drawing of the order is to be delineated from a scale of an inch to a foot: that is, the diameter of the column in the drawing is to be an inch and half.

MAKE the transverse distance of 6 and 6, on the scales of lines, equal to $1\frac{1}{2}$ inch, and the sector is fitted for the scale.

IF the height of the order is given, divide this height, by the height of the order in the table; and the quotient will be the diameter of the column.

EXAM. *What must be the diameter of the column in the Ionic order, when the whole height of the order is fixed at 18 feet 6 inches.*

THE height of the order in the table is 13 mo. $29\frac{1}{4}$ mi. $= 13\frac{29,25}{60} = 13,4875$ modules: And 18 f. 6 in. $= 18,5$ feet. Therefore $\frac{18,5}{13,4875} = 1,3709$ feet $= 1$ f. $4\frac{1}{2}$ inches nearly: And the sector may be fitted to this, as before directed, according to the intended size of the draught.

To delineate an Order by these Tables.

HAVING determined the diameter of the column at bottom, and set the sector to the intended scale, draw a line to represent the axis or middle of the order.

ON this line, lay the parts for the heights of the pedestal, column, and entablature, taken from the table of general proportions.

WITHIN each of these parts respectively, lay the several altitudes taken from the tables of particulars, under the word Alt. Through each of the points marked on the axis, draw lines perpendicular to the axis, or draw one line perpendicular, and the others parallel thereto.

ON the lines drawn perpendicular to the axis, lay the projections corresponding to the respective altitudes; these projections are to be laid on both sides of the axis, for the pedestal and column; and only on one side, for the entablature, join the extremities of the projections with such lines as are proper to express the respective mouldings and parts: And the order, exclusive of its ornaments, will be delineated.

As the altitudes of many of the parts are very small, it will not be convenient, if possible, to take from the scale of lines, such small parts alone; therefore it may be

be best to proceed as in the following example of the *Ionic* order.

To construct the Pedestal. Plate II.

IN the line AD, which represents the axis of the order, take the base $Aa = 42\frac{1}{2}$ min., the die $ad = 1$ mod. κ 35 min.; and the capital $dD = 22\frac{3}{4}$ min. Then to draw the small members in the base and cornice, proceed thus.

To the minutes in the base, $42\frac{1}{2}$, add some even number of minutes, suppose $30 = aB$, and the sum $72\frac{1}{2}$ is equal to AB; then compose a table, such as the following one, wherein the alt. of the plinth is subtracted out of the No. $72\frac{1}{2}$; then the torus out of this remainder; then the cyma out of this remainder; then the fillet out of this; and lastly, the cavetto out of this remainder. Thus,

	Min.
Base with 30 minutes	$72\frac{1}{2}$
This less by the plinth, $28\frac{1}{2}$, remains . . .	44
This less by the torus, 4, remains . . .	40
This less by the fillet, $0\frac{3}{4}$, remains . . .	$39\frac{1}{4}$
This less by the cyma, 5, remains . . .	$34\frac{1}{4}$
This less by the fillet, $0\frac{3}{4}$, remains . . .	$33\frac{1}{2}$
This less by the cavetto, $3\frac{1}{2}$, remains . . .	30,
the minutes first added.	

THEN the several numbers in the table may be taken from the line of lines on the sector, and applied from B towards A. Thus,

MAKE $BI = 44$, $B2 = 40$, $B3 = 39\frac{1}{4}$, $B4 = 34\frac{1}{4}$, $B5 = 33\frac{1}{2}$; draw lines through these points at right angles to AD, and on these lines lay the respective projections, as shewn in the general table; then the proper curvature or figure being drawn at the extremities of the numbers, the base of the pedestal will be made.

IT will be found most convenient to lay off the numbers from the greater to the lesser ones; for then there is only one motion required in the joints of the
com-

compasses, which is, to bring them closer and closer every distance laid down.

AND in the same manner, for the cornice of the pedestal, take a point C, 30 minutes below the cornice; and tabulate as before.

Cornice with 30 min.	52 $\frac{3}{4}$	= CD
This less by the fillet or cap, 2 $\frac{1}{2}$,	leaves 50 $\frac{1}{4}$	= CI
Ditto ogee 3 $\frac{1}{2}$,	ditto 46 $\frac{3}{4}$	= C2
Ditto corona 4 $\frac{1}{2}$ 42 $\frac{1}{4}$	= C3
Ditto fillet 1 $\frac{3}{4}$ 40 $\frac{1}{2}$	= C4
Ditto cyma 5 $\frac{1}{4}$ 35 $\frac{1}{4}$	= C5
Ditto fillet 1 $\frac{3}{4}$ 33 $\frac{1}{2}$	= C6
Ditto cavetto 3 $\frac{1}{2}$ 30	= cd.

THESE numbers laid from c towards D, gives the altitudes of the members of the cornice.

IN like manner the mouldings about the base and capital are laid down, by taking 30 minutes in the shaft both above the base and below the capital; having first set on the axis, the respective heights of the base, shaft, and capital.

Thus for the Base.

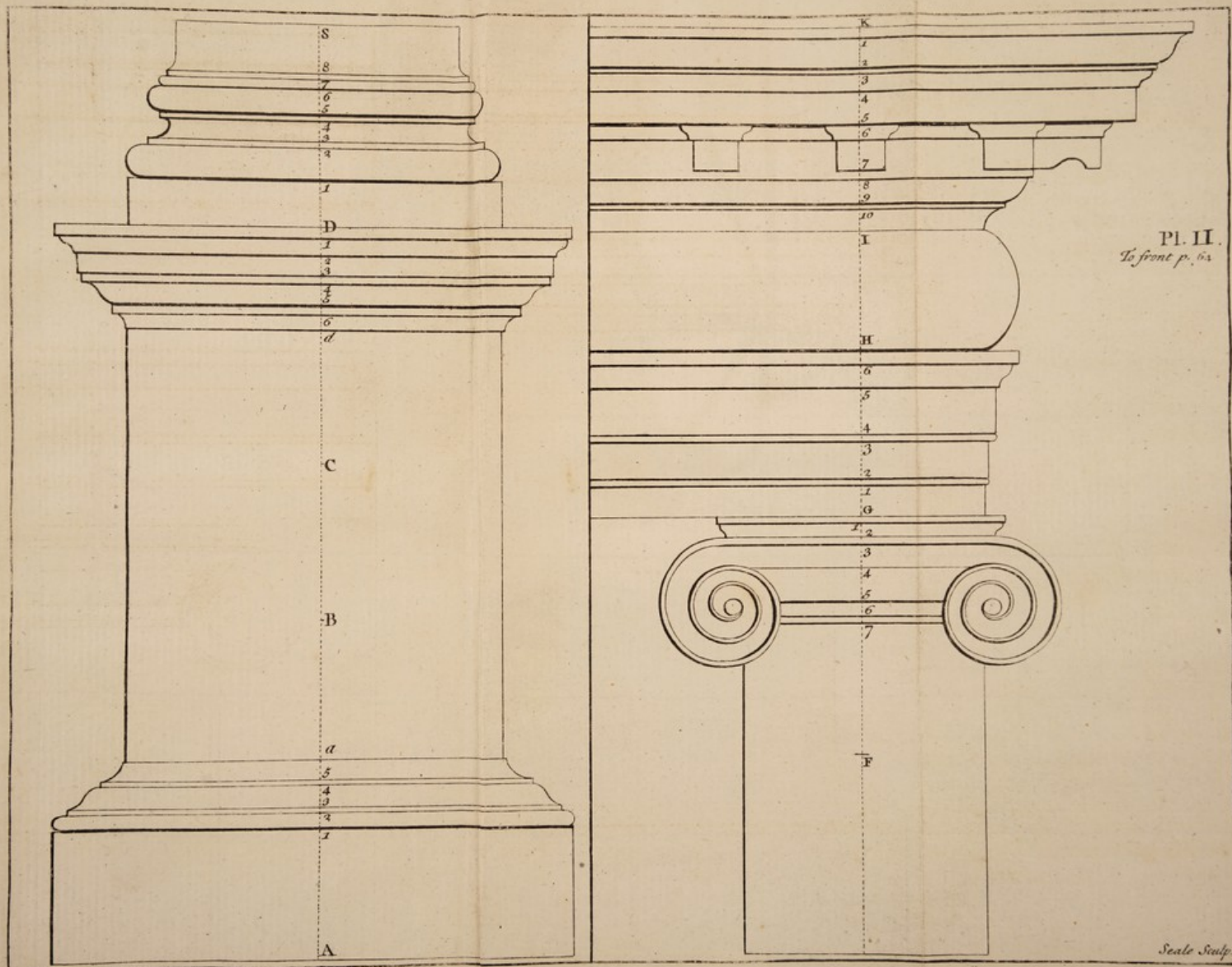
THE base 33 $\frac{1}{2}$ min. with 30 added	= 63 $\frac{1}{2}$	= SD
This less by the plinth 10 min. leaves	53 $\frac{1}{2}$	= SI
Ditto torus 7 $\frac{1}{2}$ 46	= S2
Ditto fillet 1 $\frac{1}{4}$ 44 $\frac{3}{4}$	= S3
Ditto scotia 4 $\frac{2}{3}$ 40 $\frac{1}{12}$	= S4
Ditto fillet 1 $\frac{1}{4}$ 38 $\frac{5}{6}$	= S5
Ditto torus 5 $\frac{1}{3}$ 33 $\frac{1}{2}$	= S6
Ditto astragal 2 $\frac{1}{4}$ 31 $\frac{1}{4}$	= S7
Ditto fillet 1 $\frac{1}{4}$ 30	= S8.

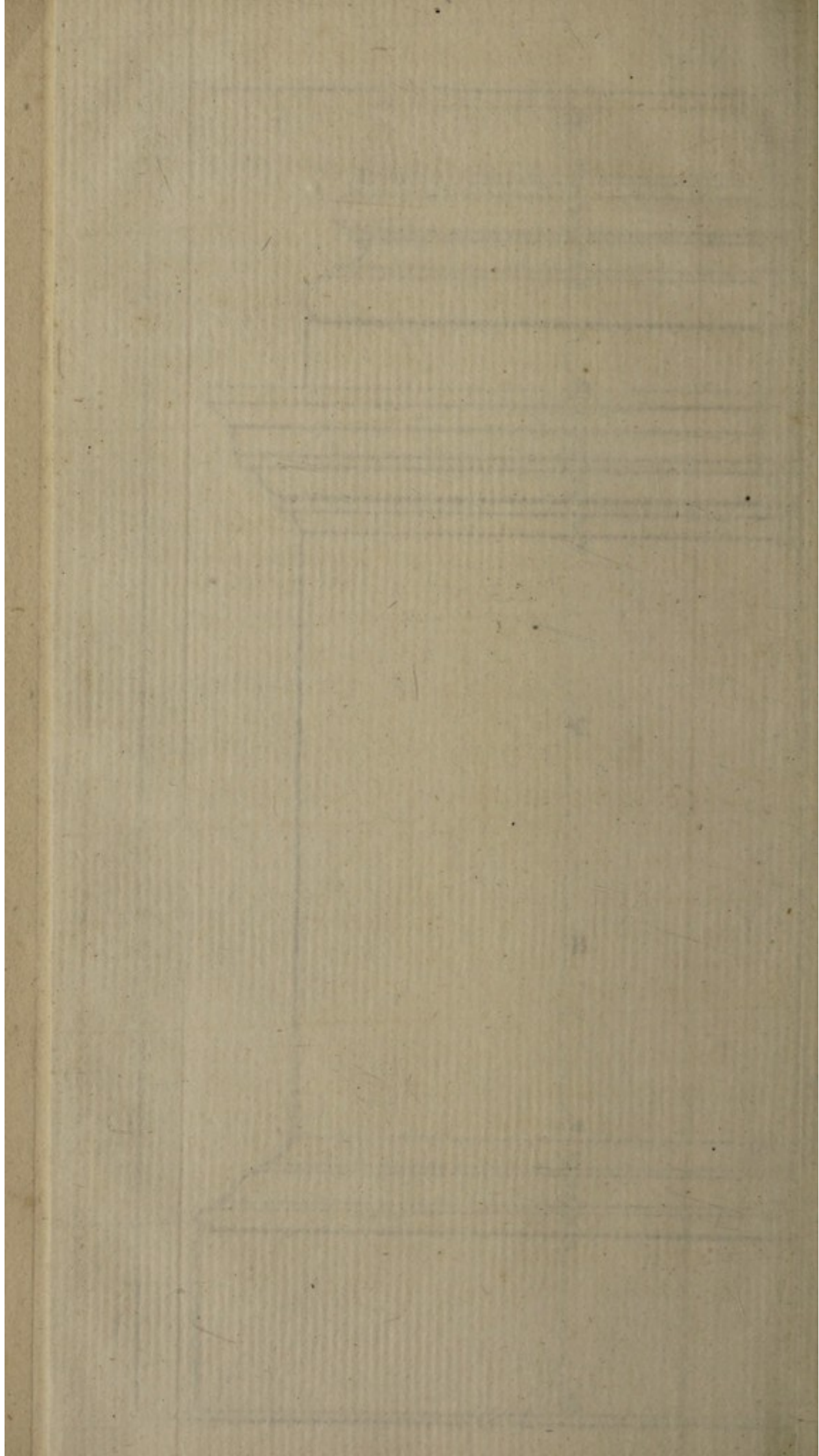
s8 is here supposed to be 30, though the plate is not high enough to admit 30 minutes to be laid in the shaft of the column.

For the Capital.

THE capital 24 $\frac{1}{4}$ with 30 added, gives	54 $\frac{1}{4}$	= FG
This less by the plateband 1 $\frac{3}{4}$, leaves	52 $\frac{1}{2}$	= FI
Ditto ogee 3 $\frac{1}{3}$ 49 $\frac{1}{6}$	= F2

This





This less by the rim of volute	$1\frac{1}{3}$	leaves	$47\frac{5}{6}$	= F3
Ditto hollow	$5\frac{1}{3}$	$42\frac{1}{2}$	= F4
Ditto ovolo . .	$7\frac{1}{2}$	35	= F5
Ditto astragal	$3\frac{1}{3}$	$31\frac{2}{3}$	= F6
Ditto fillet . .	$1\frac{2}{3}$	30	= F7.

To construct the Cornice.

IN the axis take GH = 36 for the architrave, HI = 27 for the freeze, and IK = 46 for the cornice. Then,

For the parts of the Architrave.

To the freeze 27 add HG 36, gives	63	= IG
This less by the first face	$6\frac{1}{2}$	leaves $56\frac{1}{2}$ = I1
Ditto fufarole	$1\frac{1}{4}$ $55\frac{1}{4}$ = I2
Ditto 2d face .	$8\frac{1}{3}$ $46\frac{1}{3}$ = I3
Ditto fufarole	2 $44\frac{1}{2}$ = I4
Ditto 3d face	$10\frac{1}{2}$ $34\frac{5}{2}$ = I5
Ditto ogee . .	$4\frac{3}{4}$ $29\frac{2}{3}$ = I6
Ditto fillet . .	$2\frac{2}{3}$ 27 = IH.

For the Cornice.

To the freeze 27 add the cornice 46, gives	73	= HK
This less by the fillet . .	$2\frac{1}{2}$	leaves $70\frac{1}{2}$ = HI
Ditto cima . .	7 $63\frac{1}{2}$ = H2
Ditto fillet . .	1 $62\frac{1}{2}$ = H3
Ditto ogee . .	$3\frac{1}{2}$ 59 = H4
Ditto corona	8 51 = H5
Ditto ogee . .	3 48 = H6
Ditto modillion	$7\frac{1}{2}$ $40\frac{1}{2}$ = H7
Ditto fillet . .	$1\frac{1}{2}$ 39 = H8
Ditto ovolo . .	6 33 = H9
Ditto fillet . .	1 32 = H10
Ditto cavetto .	5 27 = H1.

TABLES may be made in like manner for either of the orders, to be taken from the sector: The projections from the axis being all of them large numbers, they may be taken from the sector easily enough after it is set to the diameter of the column, as before shewn.

A LITTLE reflection will make this very clear, and perhaps more so, than by bestowing more words thereon.

TABLE

A TABLE shewing the Altitudes and Projections of Order ; according to the

Names of the Members.		Tuscan.				Doric.			
		Alt.		Proj.		Alt.		Proj.	
		Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
CORNICE.	Fillet - - - -	—	—	—	—	0	3 $\frac{1}{2}$	0	56
	Ogee - - - -	—	—	—	—	—	—	—	—
	Corona - - - -	—	—	—	—	—	—	—	—
	Fillet - - - -	—	—	—	—	—	—	—	—
	Cima - - - -	—	—	—	—	0	9	—	—
	Fillet - - - -	—	—	—	—	0	$\left\{ \begin{array}{l} 1\frac{1}{4} \\ 1\frac{3}{4} \end{array} \right.$	0	$\left\{ \begin{array}{l} 47 \\ 45\frac{3}{4} \end{array} \right.$
	Astragal - - -	—	—	—	—	—	—	—	—
	Ogee - - - -	—	—	—	—	—	—	—	—
	Cavetto - - - -	—	—	—	—	0	5	0	41 $\frac{1}{4}$
	Fillet - - - -	—	—	—	—	—	—	—	—
	The Cornice - -	—	—	—	—	0	26 $\frac{1}{6}$	—	—
	THE DIE - - -	1	0	0	42	1	20	0	40
BASE.	The Base - - -	—	—	—	—	0	40	—	—
	Fillet - - - -	—	—	—	—	—	—	—	—
	Cavetto - - - -	—	—	—	—	0	5	0	41 $\frac{1}{4}$
	Ogee - - - -	—	—	—	—	—	—	—	—
	Astragal - - -	—	—	—	—	—	—	—	—
	Fillet - - - -	—	—	—	—	0	$\left\{ \begin{array}{l} 1\frac{1}{4} \\ 1\frac{3}{4} \end{array} \right.$	0	$\left\{ \begin{array}{l} 46 \\ 47\frac{1}{4} \end{array} \right.$
	Cima - - - -	—	—	—	—	—	—	—	—
	Fillet - - - -	—	—	—	—	—	—	—	—
	Torus - - - -	—	—	—	—	—	5	0	50
	Plinth - - - -	—	—	—	—	—	27 $\frac{1}{2}$	0	50

A TABLE of general

The Order - -	9	44 $\frac{1}{2}$	—	—	12	13 $\frac{1}{6}$	—	—
The Entablature	1	44 $\frac{1}{2}$	—	—	1	53	—	—
The Column - -	7	0	—	—	8	0	—	—
The Pedestal - -	1	0	—	—	2	20 $\frac{1}{6}$	—	—

FIRST.

every Moulding and Part in the Pedestals of each Proportions given by Palladio.

Ionic.				Corinthian.				Roman.			
Alt.		Proj.		Alt.		Proj.		Alt.		Proj.	
Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
0	$2\frac{1}{2}$	0	$56\frac{1}{4}$	0	$2\frac{1}{2}$	0	57	0	$2\frac{1}{2}$	0	57
0	$3\frac{1}{2}$	0	$55\frac{1}{4}$	0	$3\frac{1}{2}$	0	$56\frac{1}{4}$	0	$3\frac{1}{2}$	0	$56\frac{1}{4}$
0	$4\frac{1}{2}$	0	$53\frac{1}{4}$	0	$4\frac{1}{4}$	0	$54\frac{1}{4}$	0	$5\frac{1}{2}$	0	$53\frac{1}{4}$
0	$1\frac{3}{4}$	0	$52\frac{1}{4}$	0	$4\frac{1}{4}$	0	$53\frac{1}{4}$	0	1	0	$52\frac{1}{4}$
0	$5\frac{1}{4}$	—	—	0	$4\frac{1}{4}$	0	$49\frac{1}{4}$	0	$8\frac{1}{2}$	—	—
0	$1\frac{3}{4}$	0	$44\frac{3}{4}$	0	$0\frac{3}{4}$	0	46	—	—	—	—
—	—	—	—	—	—	—	—	0	3	0	$46\frac{1}{4}$
—	—	—	—	0	$3\frac{3}{4}$	0	$45\frac{1}{4}$	—	—	—	—
0	$3\frac{1}{2}$	0	$41\frac{3}{4}$	—	—	—	43	—	—	—	—
—	—	—	—	—	—	—	—	0	$1\frac{3}{4}$	0	$44\frac{3}{4}$
0	$22\frac{3}{4}$	—	—	0	19	—	—	0	$25\frac{1}{4}$	—	—
1	35	0	$41\frac{1}{4}$	1	36	0	42	2	$6\frac{1}{2}$	0	42
0	$42\frac{1}{2}$	—	—	0	38	—	—	0	50	—	—
—	—	—	—	—	—	—	—	0	1	0	$45\frac{1}{2}$
0	$3\frac{1}{2}$	0	$41\frac{3}{4}$	—	—	—	—	—	—	—	—
—	—	—	—	0	4	0	$43\frac{1}{4}$	—	—	—	—
—	—	—	—	—	—	—	$46\frac{1}{4}$	0	3	0	47
0	$0\frac{3}{4}$	0	$47\frac{1}{4}$	0	$0\frac{3}{4}$	0	47	—	—	—	—
0	5	—	—	0	5	—	—	0	$7\frac{1}{2}$	0	$45\frac{1}{4}$
0	$0\frac{3}{4}$	0	$53\frac{3}{4}$	0	$0\frac{3}{4}$	0	55	0	1	0	$54\frac{3}{4}$
0	4	0	$56\frac{1}{4}$	0	4	0	57	0	$4\frac{1}{2}$	0	57
0	$28\frac{1}{4}$	0	56	0	$23\frac{1}{2}$	0	57	0	23	0	57

Proportions for the Orders.

13	$29\frac{1}{4}$	—	—	13	27	—	—	15	$22\frac{1}{4}$	—	—
1	49	—	—	1	54	—	—	2	0	—	—
9	0	—	—	9	30	—	—	10	0	—	—
2	$40\frac{1}{4}$	—	—	2	33	—	—	3	$22\frac{1}{4}$	—	—

TABLE

A TABLE, shewing the Altitudes and Projections of
according to the Propor-

Names of the Members.	Tuscan.				Doric.				
	Alt.		Proj.		Alt.		Proj.		
	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	
CAPITAL.	Angular Volutes	—	—	—	—	—	—	—	
	Abacus { Ovolo	—	—	—	—	—	—	—	
	{ Fillet	—	—	—	0	1 $\frac{3}{4}$	0	38 $\frac{3}{4}$	
	{ Cavetto	—	—	—	—	—	—	—	
	Basket Rim	—	—	—	—	—	—	—	
	Ogee	—	—	—	0	2 $\frac{1}{2}$	0	{ 37 $\frac{3}{4}$	
	Abacus	0	10	0	30	0	6 $\frac{1}{4}$	0	{ 36 $\frac{1}{4}$
	Vo- { fillet or rim	—	—	—	—	—	—	{ 35 $\frac{3}{4}$	
	lute { chan. or hollow	—	—	—	—	—	—	—	
	Ovolo	0	10	0	29	0	6 $\frac{1}{2}$	0	34 $\frac{1}{2}$
	Astragal	—	—	—	—	—	—	—	—
	Fillet	0	1 $\frac{1}{2}$	0	24 $\frac{1}{2}$	{ 0	1 $\frac{1}{9}$	0	29 $\frac{3}{4}$
	Collarino	0	8 $\frac{1}{2}$	0	2 $\frac{1}{2}$	{ 0	1 $\frac{1}{10}$	0	28 $\frac{1}{2}$
	Middle Volute	—	—	—	—	{ 0	1 $\frac{1}{9}$	0	27 $\frac{1}{4}$
	Cours. of leaves, } 3d	—	—	—	—	0	10	0	26
folding half } 2d	—	—	—	—	—	—	—	—	
their height } 1st	—	—	—	—	—	—	—	—	
SHAFT.	Astragal	0	4	0	27	0	3 $\frac{1}{2}$	0	30
	Fillet	0	1 $\frac{1}{2}$	0	24 $\frac{1}{2}$	0	1 $\frac{1}{2}$	0	28 $\frac{1}{4}$
	Body of the Column	5	54 $\frac{1}{2}$	0	{ 22 $\frac{1}{2}$	6	53 $\frac{3}{4}$	0	{ 26
	Fillet	0	2 $\frac{1}{2}$	0	30	0	1 $\frac{1}{4}$	0	{ 30
	Astragal	—	—	—	33 $\frac{3}{4}$	0	—	0	33 $\frac{1}{2}$
	—	—	—	—	—	—	—	—	—
BASE.	Torus	—	—	—	—	0	5 $\frac{1}{2}$	0	36 $\frac{2}{3}$
	Astragal	—	—	—	—	—	—	—	—
	Fillet	—	—	—	—	0	1 $\frac{1}{4}$	0	35
	Scotia	—	—	—	—	0	4 $\frac{1}{2}$	0	33 $\frac{1}{3}$
	Fillet	—	—	—	—	0	1 $\frac{1}{4}$	0	36 $\frac{2}{3}$
	Astragal	—	—	—	—	—	—	—	—
	Fillet	—	—	—	—	—	—	—	—
	Scotia	—	—	—	—	—	—	—	—
	Fillet	—	—	—	—	—	—	—	—
	Torus	0	12 $\frac{1}{2}$	0	40	0	7 $\frac{1}{2}$	0	40
	Plinth	0	15	0	40	0	10	0	40
	Base	0	2 $\frac{1}{2}$	—	—	0	30	—	—
Shaft	6	2 $\frac{1}{2}$	—	—	7	0	—	—	
Capital	0	30	—	—	0	30	—	—	

S E C O N D.

every Moulding and Part in the Columns of each Order ;
tions given by Palladio.

Ionic.				Corinthian.				Roman.			
Alt.		Proj.		Alt.		Proj.		Alt.		Proj.	
Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
o	$26\frac{2}{3}$	o	$41\frac{2}{3}$	o	12	o	41	o	$25\frac{2}{3}$	o	35
—	—	—	—	o	3	o	45	o	3	o	44
o	$1\frac{3}{4}$	o	$31\frac{1}{2}$	o	$1\frac{1}{2}$	o	42	o	$1\frac{1}{3}$	o	$42\frac{1}{2}$
—	—	—	—	o	$5\frac{1}{2}$	o	39	o	$5\frac{2}{3}$	o	41
—	—	—	—	o	$2\frac{1}{2}$	—	—	—	—	—	—
o	$3\frac{1}{3}$	o	$30\frac{3}{4}$	—	—	—	—	—	—	—	—
—	—	—	29	—	—	—	—	—	—	—	—
o	$1\frac{1}{3}$	—	—	—	—	—	—	—	—	—	—
o	$5\frac{1}{2}$	—	—	—	—	—	—	—	—	—	—
o	$7\frac{1}{2}$	o	35	—	—	—	—	o	$5\frac{1}{2}$	o	32
—	—	—	—	—	—	—	—	o	3	o	26
—	—	—	—	—	—	—	—	o	$1\frac{1}{2}$	o	24
—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	o	$9\frac{1}{2}$	—	—	—	—	—	—
—	—	—	—	o	8	—	—	o	10	—	—
—	—	—	—	o	20	o	41	o	20	o	39
—	—	—	—	o	20	o	35	o	20	o	35
o	$3\frac{1}{3}$	o	30	o	$3\frac{2}{3}$	o	30	o	4	o	30
o	$1\frac{2}{3}$	o	$28\frac{1}{3}$	o	$1\frac{1}{3}$	o	28	o	$1\frac{1}{2}$	o	28
8	$2\frac{1}{4}$	o	$26\frac{1}{3}$	7	$40\frac{3}{4}$	o	$26\frac{1}{3}$	8	9	o	$26\frac{1}{3}$
o	$1\frac{1}{4}$	o	33	o	$1\frac{3}{4}$	o	$33\frac{1}{2}$	o	1	o	34
o	$2\frac{1}{4}$	o	$34\frac{1}{2}$	o	$2\frac{1}{2}$	o	$35\frac{1}{2}$	o	3	o	$35\frac{1}{2}$
o	$5\frac{1}{3}$	o	37	o	5	o	$37\frac{1}{2}$	o	$4\frac{1}{2}$	o	37
—	—	—	—	o	$12\frac{1}{2}$	o	$35\frac{1}{2}$	—	—	—	—
o	$1\frac{1}{2}$	o	$34\frac{1}{2}$	o	$0\frac{3}{4}$	o	34	o	$0\frac{2}{3}$	o	$35\frac{1}{2}$
o	$4\frac{2}{3}$	—	—	o	3	—	—	o	3	—	—
o	$1\frac{1}{4}$	o	37	o	$0\frac{3}{4}$	o	37	o	$0\frac{1}{2}$	o	$36\frac{1}{2}$
—	—	—	—	o	$1\frac{3}{4}$	o	$38\frac{1}{2}$	o	1	o	37
—	—	—	—	—	—	—	—	o	$0\frac{1}{2}$	o	$37\frac{1}{2}$
—	—	—	—	—	—	—	—	o	3	—	$36\frac{1}{2}$
—	—	—	—	—	—	—	—	o	$0\frac{2}{3}$	o	$38\frac{1}{2}$
o	$7\frac{1}{2}$	o	$41\frac{1}{4}$	o	7	o	42	o	7	o	42
o	10	o	$41\frac{1}{2}$	o	$9\frac{2}{3}$	o	42	o	$9\frac{2}{3}$	o	42
o	30	—	—	o	30	—	—	o	$1\frac{1}{2}$	—	—
8	$10\frac{1}{4}$	—	—	7	—	—	—	8	$18\frac{1}{2}$	—	—
o	$19\frac{1}{4}$	—	—	1	10	—	—	1	10	—	—

TABLE

A TABLE, shewing the Altitudes and Projections of Order; according to the Pro-

Names of the Members.				Tuscan.				Doric.			
				Alt.		Proj.		Alt.		Proj.	
				Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
CORNICE.	Fillet	-	-	o	$3\frac{1}{2}$	1	6	o	$2\frac{1}{2}$	1	10
	Cima	-	-	o	10	—	—	o	$6\frac{1}{2}$	—	—
	Fillet	-	-	o	2	o	$54\frac{1}{4}$	o	$o\frac{3}{4}$	1	8
	Ogee	-	-	—	—	—	—	o	$3\frac{1}{4}$	} 1	7
	Corona	-	-	o	10	o	$52\frac{1}{4}$	o	8		$5\frac{1}{2}$
	Ovolo	-	-	o	9	o	42	o	6		$4\frac{1}{2}$
	Fillet or Astragal	-	-	o	$1\frac{1}{2}$	o	32	o	1	o	$39\frac{1}{2}$
	Ogee	-	-	—	—	—	—	—	—	—	$35\frac{1}{2}$
	Modillion	2d Face	—	—	—	—	—	—	—	—	—
		Ogeee	—	—	—	—	—	—	—	—	—
		1st Face	—	—	—	—	—	—	—	—	—
	Fillet	-	-	—	—	—	—	—	—	—	—
	Ovolo	-	-	—	—	—	—	—	—	—	—
	Ogee	-	-	—	—	—	—	—	—	—	—
	Fillet	-	-	—	—	—	—	—	—	—	—
	Dentel	-	-	—	—	—	—	—	—	—	—
	Astragal	-	-	—	—	—	—	—	—	—	—
ARCHITRAVE.	Fillet	-	-	—	—	—	—	—	—	—	—
	Ogee	-	-	—	—	—	—	—	—	—	—
	Cavetto	-	-	o	$7\frac{1}{2}$	o	$23\frac{1}{2}$	o	5	o	31
	Triglyphs Capital	-	-	—	—	—	—	o	5	o	$30\frac{1}{2}$
	The Cornice	-	-	o	$43\frac{1}{2}$	—	—	o	38	—	—
	THE FREEZE	-	-	o	20	o	$22\frac{1}{2}$	o	45	o	26
	The Architrave	-	-	o	35	—	—	o	30	—	—
	Fillet	-	-	o	5	o	$27\frac{1}{2}$	o	$4\frac{1}{2}$	o	28
	Cavetto	-	-	—	—	—	—	—	—	—	—
	Ogee	-	-	—	—	—	—	—	—	—	—
	Astragal or Fufarole	Third Face	—	—	—	—	—	—	—	—	—
		Second Face	-	o	$17\frac{1}{2}$	o	24	o	$14\frac{1}{2}$	o	27
		Ogee	-	—	—	—	—	—	—	—	—
	Astragal or Fufarole	First Face	-	o	$12\frac{1}{2}$	o	$22\frac{1}{2}$	o	11	26	26

T H I R D.

every Moulding and Part in the Entablature of each portions given by Palladio.

Ionic.				Corinthian.				Roman.			
Alt.		Proj.		Alt.		Proj.		Alt.		Proj.	
Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
o	$2\frac{1}{2}$	1	$1\frac{1}{2}$	o	$2\frac{1}{4}$	1	14	o	$2\frac{1}{2}$	1	$18\frac{1}{2}$
o	7	—	—	o	$6\frac{1}{2}$	—	—	o	8	—	—
o	1	1	4	o	$o\frac{2}{3}$	1	$6\frac{1}{2}$	o	1	1	10
o	$3\frac{1}{2}$	{	3	o	3	{	$5\frac{1}{2}$	o	$3\frac{3}{4}$	{	9
o	8		$o\frac{1}{2}$	o	$7\frac{1}{3}$		4	o	$9\frac{1}{4}$		6
—	—	—	$59\frac{1}{2}$	—	—	—	3	o	$2\frac{1}{2}$	o	55
—	—	—	—	o	$o\frac{2}{3}$	1	2	o	$1\frac{1}{4}$	o	54
o	3	o	{	o	$2\frac{1}{3}$	{	1	—	—	—	—
{	—	—		—	—		59	o	$6\frac{1}{2}$	o	53
	$7\frac{1}{2}$	o	52	o	$7\frac{1}{4}$	o	$40\frac{1}{4}$	o	$1\frac{1}{4}$	o	$52\frac{1}{2}$
o	$1\frac{1}{2}$	o	37	o	1	o	40	o	$3\frac{1}{4}$	o	51
o	6	o	36	o	$4\frac{1}{2}$	o	39	o	1	o	51
—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	o	1	o	36	o	5	o	{
—	—	—	—	o	$5\frac{1}{2}$	o	35	—	—	—	
—	—	—	—	—	—	—	—	o	2	o	30
o	1	o	$31\frac{1}{2}$	o	1	o	32	o	2	o	$28\frac{1}{2}$
—	—	—	—	o	$4\frac{1}{2}$	o	{	—	—	—	—
o	5	o	27	—	—	—		—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—
o	46	—	—	o	$47\frac{1}{2}$	—	—	o	50	—	—
o	27	o	34	o	$28\frac{1}{2}$	o	26	o	50	o	35
o	30	—	—	o	38	—	—	o	40	—	—
o	$2\frac{2}{3}$	o	34	o	$2\frac{1}{2}$	o	$34\frac{1}{2}$	o	$2\frac{1}{6}$	o	35
—	—	—	—	—	—	—	—	o	4	o	32
o	$4\frac{3}{4}$	o	{	o	5	o	{	o	$3\frac{2}{3}$	o	{
—	—	—		o	2	o		—	—	—	
o	$10\frac{1}{2}$	o	29	o	$10\frac{1}{2}$	o	28	—	—	—	—
o	2	o	29	o	$1\frac{1}{4}$	o	28	o	$1\frac{5}{12}$	—	29
o	$8\frac{1}{2}$	o	$27\frac{1}{2}$	o	$8\frac{1}{4}$	o	27	o	15	o	28
—	—	—	—	—	—	—	—	o	$2\frac{2}{3}$	o	{
o	$1\frac{1}{4}$	o	$27\frac{1}{2}$	o	1	o	27	—	—	—	
o	$6\frac{1}{2}$	o	$26\frac{1}{2}$	o	6	o	26	o	11	o	26

S E C T. XIII.

Some Uses of the Scales of Polygons. Pl. VI.

P R O B L E M XVI.

In a given circle, whose diameter is AB, to inscribe a regular octagon. Fig. 22.

SOLUTION. OPEN the legs of the sector, till the transverse distance of 6 and 6, be equal to AB: Then will the transverse distance of 8 and 8, be the side of an octagon which will be inscribed in the given circle.

IN like manner may any other polygon not exceeding 12 sides, be inscribed in a given circle.

P R O B L E M XVII.

On a given line AB, to describe a regular pentagon. Fig. 23.

SOLUTION. 1st. Make AB a transverse distance to 5 and 5.

2d. AT that opening of the sector, take the transverse distance of 6 and 6; and with this radius, on the points A, B, as centers, describe arcs cutting in c.

3d. ON c as a center, with the same radius, describe a circumference passing through the points A, B; and in this circle may the pentagon, whose side is AB, be inscribed.

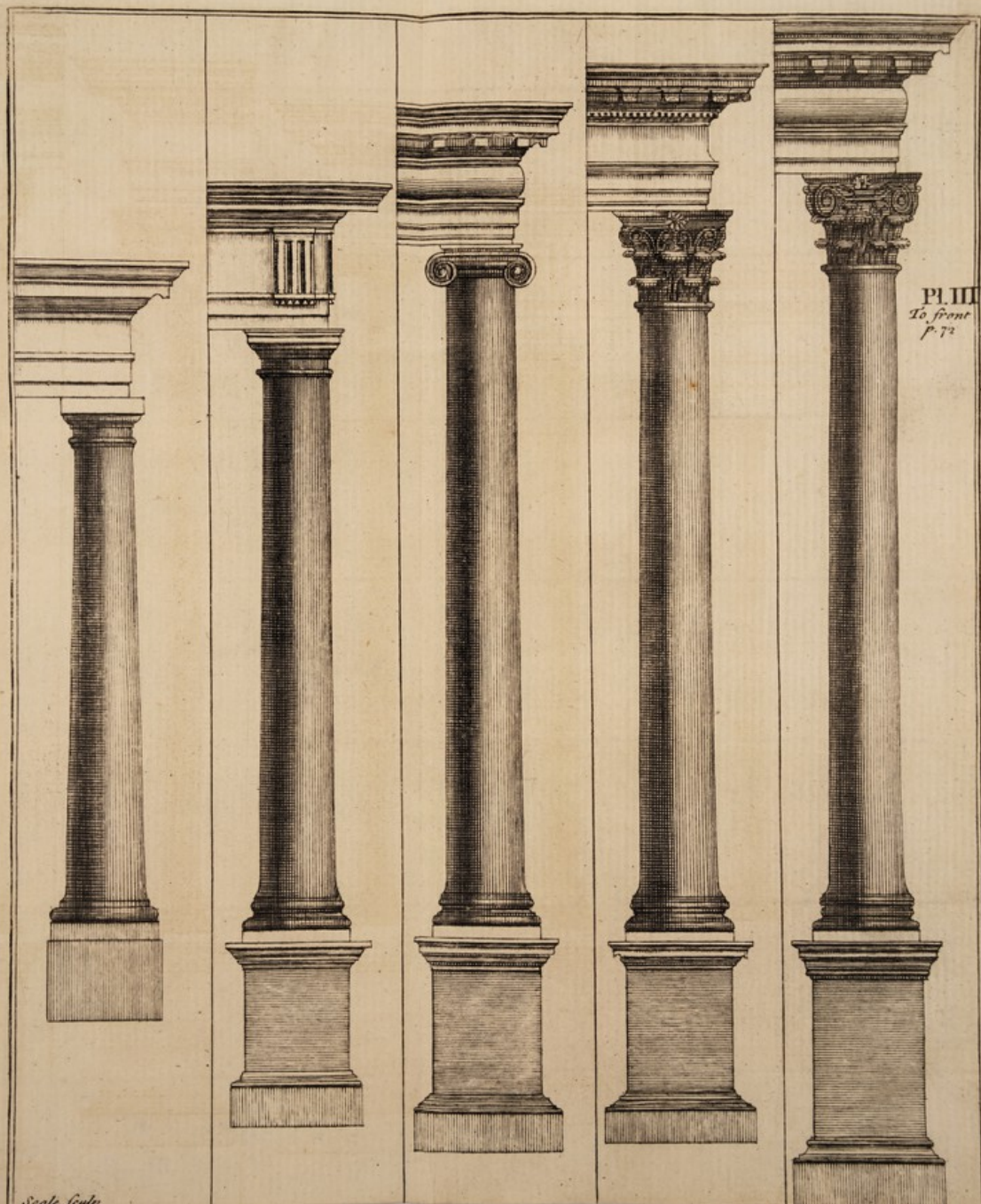
BY a like process may any other polygon, of not more than 12 sides, be described on a given line.

THE scales of chords will solve these two problems, or any other of the like kind: Thus,

In a circle whose diameter is AB, to describe a regular polygon of 24 sides. Fig. 24.

SOLUTION. 1st. Make the diameter AB, a transverse distance to 60 and 60, on the scales of chords.

2d.



PL III
No. 10

2d. DIVIDE 360 by 24; the quotient gives 15.

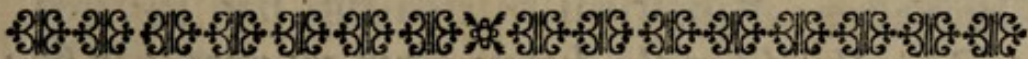
3d. TAKE the transverse distance of 15 and 15, and this will be the chord of the 24th part of the circumference.

As there are great difficulties attending the taking of divisions accurately from scales; therefore in this problem, where a distance is to be repeated several times, it will be best to proceed thus.

WITH the chord of 60 degrees, divide the circumference into six equal parts.

IN every division of 60 degrees, lay down, 1st. The chord of 15 degrees. 2d. The chord of 30 degrees. 3d. The chord of 45 degrees, beginning always at the same point.

IF methods like this be pursued in all similar cases, the error in taking distances, will not be multiplied into any of the divisions following the first.



SECT. XIV.

Some Uses of the Scales of Chords.

THESE double scales of chords, are more convenient than the single scales, such as described on the plain scale; for on the sector, the radius with which the arc is to be described, may be of any length between the transverse distance of 60 and 60, when the legs are close, and that of the transverse distance of 60 and 60, when the legs are opened as far as the instrument will admit of. But with the chords on the plain scale, the arc described, must be always of the same radius.

PROBLEM

P R O B L E M XVIII.

To protract, or lay down, a right lined angle, BAC , which shall contain a given number of degrees, Pl. VI.

CASE I. When the degrees given are under 60 : Suppose 46. Fig. 25.

1st. AT any opening of the sector, take the transverse distance of 60 and 60, (on the chords;) and with this opening, describe an arc BC .

2d. TAKE the transverse distance of the given degrees 46, and lay this distance on the arc from any point B , to c ; marking the extremities B , c , of the said distance.

3d. FROM the center A of the arc, draw two lines AC , AB , each passing through one extremity of the distance BC , laid on the arc; and these two lines will contain the angle required.

CASE II. When the degrees given are more than 60 : Suppose 148.

1st. DESCRIBE the arc BC as before.

2d. TAKE the transverse distance of $\frac{1}{2}$ or $\frac{1}{3}$, of the given degrees 148; suppose $\frac{1}{3} = 49\frac{1}{3}$ degrees; lay this distance on the arc thrice; viz. from B to a , from a to b , from b to D .

3d. FROM the center A , draw two lines AB , AD ; and the angle BAD will contain the degrees required.

When an angle containing less than 5 degrees, suppose $3\frac{1}{2}$, is to be made, it is most convenient to proceed thus.

1st. DESCRIBE the arch DG with the chord of 60 degrees.

2d. FROM some point D , lay the chord of 60 degrees to G ; and the chord of $56\frac{1}{2}$ degrees ($= 60^\circ - 3\frac{1}{2}^\circ$) from D to E .

3d. LINES drawn from the center A , through G and E , will form the angle AGE , of $3\frac{1}{2}$ degrees.

IF

If the radius of the arc or circle is to be of a given length; then make the transverse distance of 60 and 60, equal to that assigned length.

EITHER of these scales of chords, may be used singly in the manner directed in the use of chords on the plane scale.

FROM what has been said about the protracting of an angle to contain a given number of degrees, it will be easy to see how to find the degrees which are contained in a given angle already laid down.

P R O B L E M XIX.

To delineate the visual lines of a survey; by having given, the bearings and distances from each other, of the stations terminating those visual lines.

EXAM. Suppose in the field book of a survey, the bearings and distances of the stations were expressed as follows:

⊙ signifies Station.

B ——— Bearing.

D ——— Distance.

⊙ 1. B 70° 50' D 1080 links.

⊙ 2. B 128 10 D 580.

⊙ 3. B 32 15 D 605.

⊙ 4. B 287 30 D 766.

⊙ 5. B 50 45 D 940.

⊙ 6. B 273 55 D 1085.

⊙ 7. B 183 25 D 700.

Return to D 314 in ⊙ 7. ⊙ 8. B 133 30 D 510 to ⊙ 5.

⊙ 9. B 186 30 D 390 to ⊙ 2.

Return to D 700 in ⊙ 7. ⊙ 10. B 209 20 D 668 cutting
[ft D.

Return to ⊙ 10. ⊙ 11. B 275 30 D 800.

⊙ 12. B 171 50 D 784 to ⊙ 1.

THE bearings are counted from the North, Eastward. Therefore all the bearings under 90 degrees, fall between the N. and E. or in the 1st quadrant.

BEARINGS between 90° and 180° , fall between the E. and S. or in the 2d quadrant.

THOSE between 180° and 270° , fall between the S. and W. or in the 3d quadrant.

AND those between 270° and 360° , fall between the W. and N. or in the 4th quadrant.

SOLUTION. 1st. Take from the chords the transverse distance of 60 and 60, (the sector being opened at pleasure,) with this radius describe a circumference, and draw the diameters NS. WE. at right angles. PL. VI. FIG. 31.

2d. The first bearing $70^{\circ} 50'$ is in the first quadrant, but being more than 60° , take the transverse distance of the half of $70^{\circ} 50'$, and apply this extent in the circumference twice from N. towards E, and the point corresponding to the 1st bearing will be obtained, which mark with the figure 1.

3d. The second bearing $128^{\circ} 10'$, falls in the second quadrant; its supplement to 180° is $51^{\circ} 50'$, that is $51^{\circ} 50'$ from the S. point. Now take the transverse distance of $51^{\circ} 50'$, and apply it in the circumference from S. towards E, and the point corresponding to the second bearing will be found, which mark with the figure 2.

4th. The 3d bearing $32^{\circ} 15'$, is to be applied from N. to 3: The 4th bearing $287^{\circ} 30'$, is in the 4th quadrant; therefore take it from 360° , and the remainder $72^{\circ} 30'$, is to be applied from N. towards W. and the point 4 representing the 4th bearing will be known.

IN this manner proceed with all the other bearings, and mark the corresponding points in the circumference with the numbers 5, 6, 7, &c. agreeable to the number of the bearing or station.

5th.

5th. Chuse some convenient point on the paper to begin at, as at the place markt $\odot 1$. Lay a parallel ruler by c the centre of the circle, and the point in its circumference marked 1, and (by the help of the ruler) draw a parallel line thro' $\odot 1$, the point chose for the first station, in the direction of the (supposed) radius $C1$; and on this line lay the first distance; that is, take from a convenient sized scale of equal parts the extent of 1080, and transfer this extent from $\odot 1$ to $\odot 2$; and this line will represent the first distance measured, laid down according to its true position in respect to the circle first described.

6th. Lay the ruler by the centre C , and the point in the circumference noted by the figure 2, and parallel to this position of the ruler, draw thro' the point $\odot 2$ a line $\odot 2 \odot 3$, in the direction of the (supposed) radius $C2$, and on this line lay from $\odot 2$ to $\odot 3$ the extent 580 taken from the same scale of equal parts the 1080 was taken from, and this line shall represent the second measured distance laid down in its true position relative to the first distance.

PROCEED in this manner from station to station until the line $\odot 7 \odot 10$ is drawn.

7th. Take from the scale of equal parts 314, and apply this extent in the line $\odot 7 \odot 10$ from $\odot 7$ to $\odot 8$, and the relative point, where the eighth station was taken, will be represented by the point $\odot 8$; then by the parallel ruler draw the line $\odot 8 \odot 5$, in the direction of, and parallel to, the (supposed) radius $C8$; and if the preceding work is accurately performed, this line will not only pass thro' the point $\odot 5$, but the length of the line $\odot 8 \odot 5$ will be equal to 510, as the station line was measured in the field.

8th. Now as the 9th station falls on the same point as the 5th station did, draw the line $\odot 9 \odot 2$, and this line will not only be parallel to the (supposed) radius $C9$, but will also measure on the scale of equal parts

parts 390, the length measured in the field from the 9th station.

9th. The 10th station is taken from the end of the line 700 measured from the 7th station; therefore drawing from $\odot 10$ a line parallel to the (supposed) radius $C 10$, this line will concur with the first measured line at the distance of 668 from the point $\odot 10$.

10th. Returning to $\odot 10$ again, the same point is taken for the 11th station, and the line $\odot 11 \odot 12$ is to be drawn parallel to the (supposed) radius $C 11$, and to be made of the length of 800 from the scale of equal parts; and this will give the point $\odot 12$ for the 12th station: Then drawing the line $\odot 12 \odot 1$, if the operation is every where truly done, this line will not only be parallel to the (supposed) radius $C 12$, but will also measure on the scale of equal parts 784, the same as was measured in the field in proceeding from $\odot 12$ to $\odot 1$.

By such methods as these, the surveyor obtains a cheque on his work, and can make his *survey close* (as 'tis called) as he proceeds.

THE drawing of the visual lines of a survey is, tho' an essential part, but a small step towards the making a plan; for the remaining work the reader is refer'd to the treatises already extant on that subject.

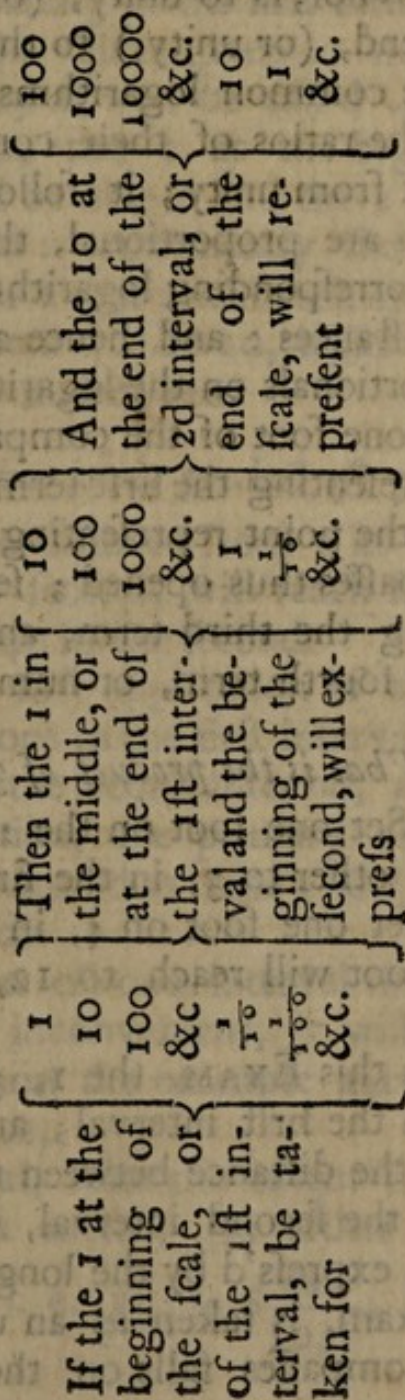
WHAT has been said about the delineating of the visual lines of a survey, may be applied to navigation in the construction of a figure to represent the various courses and distances a ship has sailed in a given time, called *traverse sailing*; for the courses are the bearings from the Meridian, and the distances sailed are of the same kind as the distance between station and station in a survey.

SECT. XV.

Some Uses of the Logarithmic Scale of Numbers.

BEFORE any operations can be performed by this scale, the notation, or the estimating of the values of the several divisions, must be well known.

Therefore, the Sector being quite opened,



And

And the primary and intermediate divisions in each interval, must be estimated according to the values set on their extremities, *viz*, at the beginning, middle and end of the scale.

In arithmetical multiplication, or division; the parts may be considered as proportional terms; for in simple multiplication; as unity or 1, is to one factor; so is the other factor, to the product: And in division; as the divisor, is to unity; (or to the dividend,) so is the dividend, (or unity,) to the quotient.

Now as the common logarithms of numbers, express how far the ratios of their corresponding numbers are distant from unity; it follows, that of those numbers which are proportional, that is, have equal ratios; their corresponding logarithms will have equal intervals, or distances: and hence arises the rule for working proportionals on the logarithmic scale.

RULE. Set one foot of the compasses on the point or division representing the first term, and extend the other foot to the point representing the second term: Keep the compasses thus opened; set one foot on the point expressing the third term, and the other foot will fall on the fourth term, or number sought.

EXAM. I. *What is the product of 3 by 4?*

SOLUTION. Set one foot on the 1 at the beginning, and extend the other to 3, in the first interval; with this opening, set one foot on 4, in the first interval, and the other foot will reach to 12, found in the second interval.

Observe. In this **EXAM.** the 1, 3, and 4, are valued as units in the first interval; and the one in the middle is 10; the distance between this 1 or 10, and the 2 or 20, in the second interval, is divided into 10 principal parts, express'd by the longer strokes; every one in this Exam. is taken as an unit; now as the point of the compasses falls on the second of these principal

principal parts, that is on 2 units beyond 10; therefore this point is to be esteemed in this Exam. as 12.

EXAM. II. *What is the product of 40 by 3?*

SOLUTION. In the first interval, take the distance between 1 and 3; and this distance will reach from (4 or) 40 in the first interval to (12 or) 120 in the second interval.

Observe. The 1 and 3 in the first interval, are taken as units: but as the values given to the divisions in either interval, may as well be call'd 40, as 4; and being taken as 40, the 1 at the beginning of the second interval will be 100; and the 2 in the second interval will be 200: consequently the principal divisions between this 1 and 2 will each express 10; and so the second of them will be 20, which with the 100, express'd by the 1, makes 120.

EXAM. III. *What is the product of 35 by 24?*

SOLUTION. The distance from 1 in the first interval, to 24 in the second, will reach from 25 in the first interval, to 840 in the second.

Observe. In the first application of the compasses, the primary divisions in the first interval are taken as units, and those in the second interval, as tens: But in the second application, the primary divisions in the first interval are reckon'd as tens; and those in the second, as hundreds.

As the extent out of one interval into the other, may sometimes be inconvenient, it will be proper to see in such cases, how the example may be solved in one interval. Thus,

In either interval, take the extent from 1 to $2\frac{4}{10}$ (i. e. 24) and this extent, (in either interval) will reach from $3\frac{5}{10}$ (i. e. 35) to $8\frac{40}{100}$; (i. e. 840.)

IN this operation ; the second term is reckoned a tenth higher than the first term ; therefore, as it falls in the same interval, the fourth term must be a tenth higher than the third term.

EXAM. IV. *What is the product of 375 by 60 ?*

SOLUTION. The extent from 1 to 6, (or 60) in the first interval will reach from $3\frac{7\frac{1}{2}}{10}$ ($=3\frac{75}{100}$ or 375) in the first interval, to $2\frac{25}{100}$ in the second interval ; which division must be reckoned 22500 : For had the point fell in the first interval, it would have been one place more than the 375, because 60 is one place more than 1 ; but as it falls in the second interval, every of whose divisions is one place higher than those in the first interval, therefore, it must have two places more than 375, which is taken in the first interval.

IF the operations in these examples be well considered, it will not be difficult to apply others to the scale, and readily to assign the value of the result.

EXAM. V. *What will be the quotient of 36 divided by 4 ?*

SOLUTION. The extent from 4 to 1, in the first interval, will reach from 36 in the second interval to nine in the first.

IT is to be observed, that when the second term is greater than the first term ; the extents are reckoned from the left hand towards the right : and when the second term is less than the first, the extents are taken from the right hand towards the left : that is, the extents are always counted the same way towards which the terms proceed.

EXAM.

EXAM. VI. *If 144 be divided by 9 ; what will be the quotient ?*

SOLUTION. The extent from 9 to 1, will reach from 144 to 36.

EXAM. VII. *If 1728 be divided by 12 ; what will be the quotient ?*

SOLUTION. The extent from 12 to 1, will reach from 1728 to 144.

EXAM. VIII. *To the numbers 3, 8, 15 ; find a fourth proportional.*

SOLUTION. The extent from 3 to 8 ; will reach from 15 to 40.

EXAM. IX. *To the numbers 5, 12, 38 ; find a 4th proportional.*

SOLUTION. The extent from 5 to 12, will reach from 38 to $91\frac{1}{5}$.

EXAM. X. *To the numbers 18, 4, 364 ; find a 4th proportional.*

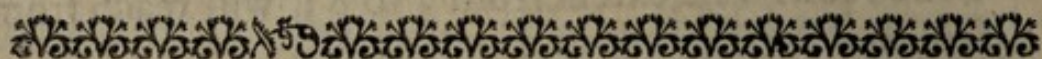
SOLUTION. The extent from 18 to 4 ; will reach from 364 to $80\frac{8}{9}$.

EXAM. XI. *To two Numbers 1 and 2 ; to find a series of continued proportionals.*

SOLUTION. The extent from 1 to 2, will reach from 2 to 4 ; from 4 to 8 in the first interval ; from 8 to 16 in the second interval ; from 16 to 32 ; from 32 to 64 ; &c. Also the same extent will reach from $1\frac{1}{2}$ to 3 ; from 3 to 6 ; from 6 to 12 ; from 12 to 24 ; from 24 to 48 ; &c. And the same extent will reach from $2\frac{1}{2}$ to 5 ; from 5 to 10 ; from 10 to 20 ; from 20 to 40 ; &c. And in a like manner proceed, if any other ratio was given besides that of 1 to 2.

THIS Example is of use, to find if the divisions of the line of numbers, are accurately laid down on the scale.

THERE are many other uses to which this scale of log. numbers are applicable, and on which several large treatises have been wrote ; but the design here, is not to enter into all the uses of the scales on the sector, only to give a few examples thereof: but after all that has been said, when examples are to be wrought whose result exceeds three places, 'tis best to do it by the pen, for on instruments, altho' they be very large ones, the lowest places of the answers, at best, are but guess'd at.



S E C T. XVI.

Some uses of the Scales of Log. Sines and Log. Tangents.

THESE scales are chiefly used in the solution of the cases of plain and spherical trigonometry, which will be fully exemplified hereafter : But in this place, it will be proper to shew, how proportional terms are applied to the scales.

IN plane trigonometrical proportions, there are always four terms under consideration ; suppose two sides and two angles, whereof, only three of the terms are given, and the fourth is required: Now the sides in plane trigonometry, are always applied to the scale of log. numbers ; and the angles are either applied to the log. sines, or to the log. tangents ; according as the sines or tangents are concerned in the proportion. Therefore, when among the three things given, if two of them be sides, and the other an angle ; or if two terms be angles, and the other a side.

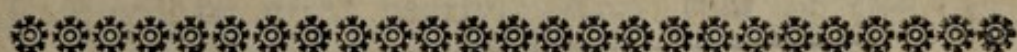
RULE.

RULE. On the scale of log. numbers, take the extent between the divisions expressing the sides; and this extent applied from the division expressing the angle given, will reach to the division shewing the angle required.

OR, the extent of the angles taken, will reach from the side given to the side required, on the line of numbers.

So in spherical trigonometry, where some of the cases are worked wholly on the sines, others partly on sines, and partly on tangents; the extent taken with the compasses, between the first and second terms, when those terms are of the same kind, will reach from the third term to the fourth.

OR, the extent from the first term to the third, when they are of the same kind, will reach from the second term to the fourth.



S E C T. XVII.

Some uses of the double Scales of Sines, Tangents, and Secants.

P R O B L E M XX.

Given the radius of a circle (suppose equal to 2 inches) required the sine, and tangent of $28^{\circ} 30'$ to that radius.

SOLUTION. Open the sector so that the transverse distance of 90 and 90, on the sines; or of 45 and 45 on the tangents; may be equal to the given radius; viz. two inches: Then will the transverse distance of $28^{\circ} 30'$, taken from the sines, be the length of that sine to the given radius; or if taken from the tangents, will be the length of that tangent to the given radius.

But if the secant of $28^{\circ} 30'$ was required?

MAKE the given radius two inches, a transverse distance to 0 and 0, at the beginning, of the line of secants; and then take the transverse distance of the degrees wanted, viz. $28^{\circ} 30'$.

A Tangent greater than 45 degrees (suppose 60 degrees) is found thus.

MAKE the given radius, suppose 2 inches, a transverse distance to 45 and 45 at the beginning of the scale of upper tangents; and then the required degrees $60^{\circ} 00'$ may be taken from this scale.

The scales of upper tangents and secants do not run quite to 76 degrees; and as the tangent and secant may be sometimes wanted to a greater number of degrees than can be introduced on the sector, they may be readily found by the help of the annexed table of the natural tangents and secants of the degrees above 75; the radius of the circle being unity.

Degrees.	Nat. Tangent.	Nat. Secant.
76	4,011	4,133
77	4,331	4,445
78	4,701	4,810
79	5,144	5,241
80	5,671	5,759
81	6,314	6,392
82	7,115	7,185
83	8,144	8,205
84	9,514	9,567
85	11,430	11,474
86	14,301	14,335
87	19,081	19,107
88	28,636	28,654
89	57,290	57,300

Measure

Measure the radius of the circle used, upon any scale of equal parts. Multiply the tabular number by the parts in the radius, and the product will give the length of the tangent or secant sought, to be taken from the same scale of equal parts.

EXAM. *Required the length of the tangent and secant of 80 degrees to a circle whose radius, measured on a scale of 25 parts to an inch, is $47\frac{1}{2}$ of those parts?*

	tangent.	secant.
Against 80 degrees stands	5,671	5,759
The radius is	47,5	47,5
	<hr/>	<hr/>
	28355	28795
	39697	40313
	22684	23036
	<hr/>	<hr/>
	269,3725	273,5525

So the length of the tangent on the twenty-fifth scale will be $269\frac{1}{3}$ nearly. And that of the secant about $273\frac{1}{2}$.

OR thus. The tangent of any number of degrees may be taken from the sector at once; if the radius of the circle can be made a transverse distance to the complement of those degrees on the lower tangent.

EXAM. *To find the tangent of 78 degrees to a radius of 2 inches.*

MAKE two inches a transverse distance to 12 degrees on the lower tangents; then the transverse distance of 45 degrees will be the tangent of 78 degrees.

IN like manner the secant of any number of degrees may be taken from the lines, if the radius of the circle can be made a transverse distance to the cosine of those degrees. Thus making two inches a transverse distance to the sine of 12 degrees; then the transverse distance of 90 and 90 will be the secant of 78 degrees.

FROM hence it will be easy to find the degrees answering to a given line, expressing the length of a

tangent or secant, which is too long to be measured on those scales, when the sector is set to the given radius.

THUS, For a tangent, make the given line a transverse distance to 45 and 45 on the lower tangents; then take the given radius and apply it to the lower tangents; and the degrees where it becomes a transverse distance is the cotangent of the degrees answering to the given line.

AND for a secant. Make the given line a transverse distance to 90 and 90 on the sines. Then the degrees answering to the given radius applied as a transverse distance on the sines, will be the co-sine of the degrees answering to the given secant line.

P R O B L E M XXI.

Given the length of the sine, tangent, or secant, of any degrees; to find the length of the radius to that sine, tangent, or secant.

Make the given length, a transverse distance to its given degrees on its respective scale: Then,

In the sines. The transverse distance of 90 and 90 will be the radius sought.

In the lower tangents. The transverse distance of 45 and 45 near the end, of the sector will be the radius sought.

In the upper tangents. The transverse distance of 45 and 45 taken towards the centre of the sector on the line of upper tangents, will be the centre sought.

In the secants. The transverse distance of 0 and 0, or the beginning of the secants, near the centre of the sector, will be the radius sought.

P R O B-

PROBLEM XXII.

Given the radius and any line representing a sine, tangent or secant; to find the degrees corresponding to that line.

SOLUTION. Set the sector to the given radius, according as a sine, or tangent, or secant is concerned.

TAKE the given line between the compasses; apply the two feet transversely to the scale concerned, and slide the feet along till they both rest on like divisions on both legs; then will those divisions shew the degrees and parts corresponding to the given line.

PROBLEM XXIII.

To find the length of a versed sine to a given number of degrees, and a given radius.

MAKE the transverse distance of 90 and 90 on the sines, equal to the given radius.

TAKE the transverse distance of the sine complement of the given degrees.

IF the given degrees are less than 90, the difference between the sine complement and the radius, gives the versed sine.

IF the given degrees are more than 90, the sum of the sine complement and the radius, gives the versed sine.

PROBLEM XXIV.

To open the legs of the sector, so that the corresponding double scales of lines, chords, sines, tangents, may make, each, a right angle.

On the lines, make the lateral distance 10, a distance between 8 on one leg, and 6 on the other leg.

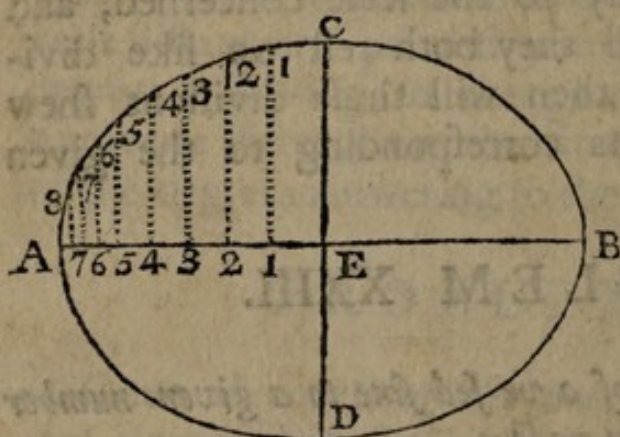
On the sines. make the lateral distance 90, a transverse distance from 45 to 45; or from 40 to 50; or from

from 30 to 60 ; or from the sine of any degrees, to their complement.

Or on the *sines*, make the lateral distance of 45 a transverse distance between 30 and 30.

PROBLEM XXV.

To describe an Ellipsis, having given AB equal to the longest diameter ; and CD equal to the shortest diameter.



SOLUTION. 1st.

Set the two diameters AB, CD, at right angles to each other in their middles at E.

2d. MAKE AE a transverse distance to 90 and 90 on the

sines ; and take the transverse distances of 10° , 20° , 30° , 40° , 50° , 60° , 70° , 80° , successively, and apply those distances to AE from E towards A, as at the points 1, 2, 3, 4, 5, 6, 7, 8 ; and thro' those points draw lines parallel to EC.

3d. MAKE EC a transverse distance to 90 and 90 on the *sines* ; take the transverse distances of 80° , 70° , 60° , 50° , 40° , 30° , 20° , 10° , successively, and apply those distances to the parallel lines from 1 to 1, 2 to 2, 3 to 3, 4 to 4, 5 to 5, 6 to 6, 7 to 7, 8 to 8, and so many points will be obtained thro' which the curve of the ellipsis is to pass,

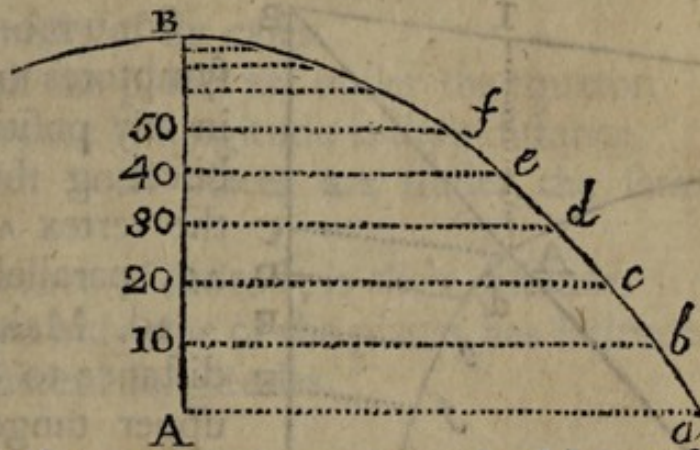
The same work being done in all the four quadrants, the elliptical curve may be compleated.

This Problem is of considerable use in the construction of solar Eclipses ; but instead of using the *sines* to every ten degrees, the *sines* belonging to the degrees and minutes corresponding to the hours, and quarter hours are to be used.

PROBLEM XXVI.

To describe a Parabola whose parameter shall be equal to a given line.

SOLUTION 1st.
 Draw a line to represent the axis, in which make AB equal to half the given parameter; divide AB like a line of fines to every ten degrees, as at the points 10, 20, 30, 40, 50, &c. and thro' these points draw lines at right angles to the axis AB.



2^d. MAKE the lines Aa, 10b, 20c, 30d, 40e, &c. respectively equal to the chords of 90° , 80° , 70° , 60° , 50° , &c. to the radius AB, and the points a, b, c, d, e, &c. will be in the curve of a parabola.

Therefore a smooth curve line drawn thro' those points and the vertex B, will represent the parabolic curve required.

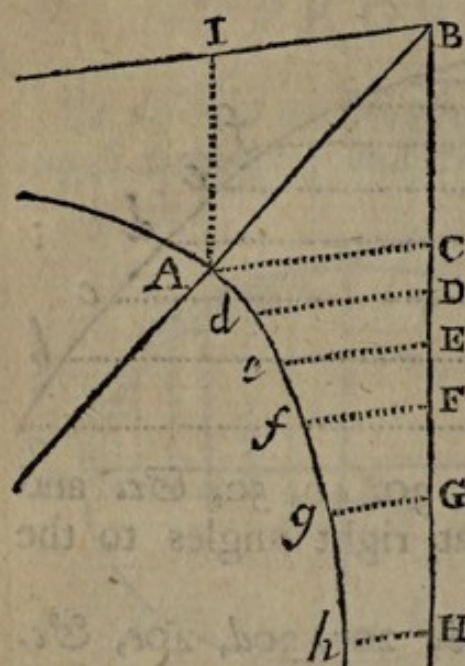
The like work may be done on both sides of the axis when the whole curve is wanted.

As the chords on the sector run no farther than 60, those of 70° , 80° and 90° may be found by taking the transverse distance of the fines of 35° , 40° , 45° to the radius AB, and applying those distances twice along the lines 20c, 10b, &c.

PRO-

PROBLEM XXVII.

To describe an Hyperbola, the vertex A and asymptotes BH, BI , being given.



SOLUTION 1st. The asymptotes BH, BI , being drawn in any position, the line BA , bisecting the angle IBH , and the vertex A taken, draw AI , AC , parallel to BH, BI .

2d. Make AC a transverse distance to 45 and 45 on the upper tangents, and apply to the asymptotes from B , so many of the upper tangents taken transversely as may be thought convenient, as $BD 50^\circ, BE 55^\circ, BF 60^\circ, BG 65^\circ, BH 70^\circ$, &c. and draw Dd, Ee, Ff, Gg, Hh , &c. parallel to AC .

3d. Make AC a transverse distance to 45 and 45 on the lower tangents, take the transverse distance of the co-tangents before used, and lay them on those parallel lines; thus make $Dd=40^\circ, Ee=35^\circ, Ff=30^\circ, Gg=25^\circ, Hh=20^\circ$, &c. and thro' the points A, d, e, f, g, h , &c. If a curve line be drawn it will be the hyperbola required.

There are many other methods of constructing the curves in the three last problems, and a multitude of entertaining and useful properties which subsist among the lines drawn within and about these curves, which the inquisitive reader will find in the treatises on conic sections.

P R O B L E M XXVIII.

To find the distance of places on the terrestrial globe by having given their latitudes and longitudes.

This problem consists of six cases.

CASE I. If both the places are under the equator.

Then the difference of longitude is their distance.

CASE II. When both places are under the same meridian.

Then the difference of latitude is their distance.

CASE III. When only one of the places has latitude, but both have different longitudes.

EXAM. *Island of Bermudas, lat. $32^{\circ} 25' N.$ longit. $68^{\circ} 38' W.$ Island of St. Thomas, lat 00 , longit. $1^{\circ} 0 E.$*

Required their distance.

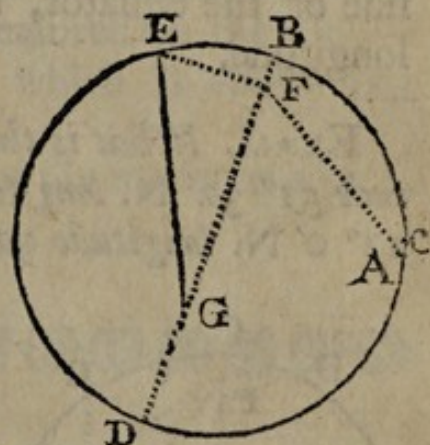
SOLUTION 1st. With the chord of 60° describe a circle representing the equator, wherein take a point c to represent the beginning of longitude.

2d. From c apply the chord of *Bermudas* longitude $68^{\circ} 38'$ to B, and that of *St. Thomas's* longitude to A, the arc AB, being the difference of longitude.

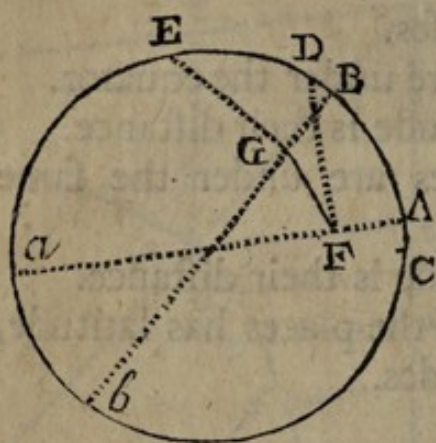
3d. From B, the place having latitude, draw the diameter BD, apply the chord of the latitude $32^{\circ} 25'$ from B to E, and draw EF at right angles to BD.

4th. Draw FC, make FG, equal to FC, and draw EG; then EG measured on the chords will give the distance sought, about 73 degrees.

CASE IV. When the given places are in the same parallel of latitude.



EXAM. Required the distance between the Lizard and Penguin Island, both in latitude $49^{\circ} 56' N.$ the longitude of the Lizard being $5^{\circ} 14' W.$ and that of Penguin Island $50^{\circ} 32' W.$

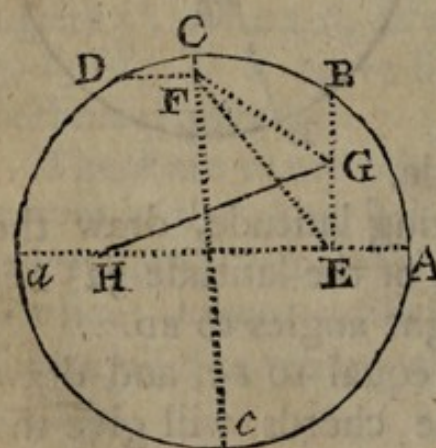


SOLUTION 1st. From c , the commencement of the longitude, apply the chord of the Lizard's longitude to A , and of Penguin's longitude to B , and draw the diameters Aa , Bb .

Apply the chord of the common latitude $49^{\circ} 56'$ from A to D , and from B to E ; draw DF and EG at right angles to Aa , Bb , and join GF ; then GF measured on the chords will give the distance sought, about 29 degrees.

CASE V. When the given places are on the same side of the equator, but differ both in latitude and longitude.

EXAM. What is the distance between London in latitude $51^{\circ} 32' N.$ longitude, $0^{\circ} 0'$ and Bengal in latitude $22^{\circ} 0' N.$ longitude $92^{\circ} 45' E.$



SOLUTION. From A , London's longitude, apply Bengal's longitude $92^{\circ} 45'$ to c , taken from the chords; also apply the chord of London's latitude from A to B , and of Bengal's latitude from c to D .

2d. Draw the diameters Aa , cc , and BE , DF , at right angles to Aa , cc , and join FE .

3d. Make BG equal to DF , and EH equal to EF , join GH ; Thus GH measured on the chords will give the distance required, which is about 72 degrees.

CASE VI. When the places are on contrary sides of the equator, and differ both in latitude and longitude.

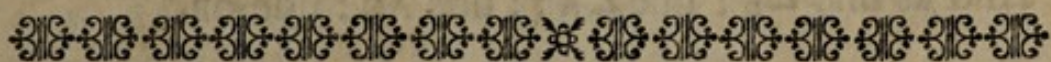
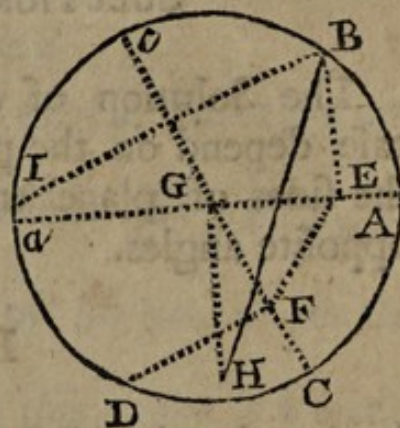
EXAM. *What is the distance between London in latitude $51^{\circ} 32'$ N. longitude $0^{\circ} 0'$ and Cape-Horn in latitude $55^{\circ} 42'$ S. longitude $66^{\circ} 00'$ W.*

SOLUTION 1st. From A, London's longitude, apply the chord of Cape-Horn's longitude to c, draw the diameters Aa, cc; also apply the chords of London's latitude from A to B, and of Horn's latitude from c to D.

2d. Draw BE and DF at right angles to Aa, cc; join EF and make EG equal to EF.

3d. At right angles to Aa, draw GH, and make it equal to DF; join BH, which measured on the chords will give the distance required, which is about 123 degrees.

To measure BH on the chords; apply BH from B to I, and measure the arc Bc I.



SECT. XVII.

The Use of some of the single and double Scales, applied in the Solution of the Cases of plain Trigonometry.

PROBLEM XXIX.

IN any right lin'd plane triangle, any three of the six terms, viz. sides and angles, (provided one of them be a side) being given, to find the other three.

This

This problem consists of three cases.

CASE I. when among the things given, there be a side and its opposite angle.

CASE II. When there is given two sides and the included angle.

CASE III. When the three sides are given.

SOLUTION of C A S E I.

The Solution of the examples falling under this case depend on the proportionality there is between the sides of plane triangles, and the sines of their opposite angles.

EXAMPLE I.

In the triangle ABC : Given $AB=56$
 $AC=64$ } equal parts.
 $\angle B=46^{\circ} 30'$

Required $\angle C, \angle A$, & BC.

The proportions are as follow,

As side AC : side AB :: sine $\angle B$: sine $\angle c$.

Then the sum of the angles B and c being taken from 180° , will leave the angle A.

And as sine $\angle B$: sine $\angle A$:: side AC : side CB.

First by the logarithm scales.

To find the angle c.

The extent from 64 ($=AC$) to 56 ($=AB$) on the scales of logarithm numbers, will reach from $46^{\circ} 30'$ ($=\angle B$) to $39^{\circ} 24'$, ($=\angle c$.) on the scale of logarithm sines.

And the sum of $46^{\circ} 30'$ and $39^{\circ} 24'$ is $85^{\circ} 54'$

Then $85^{\circ} 54'$ taken from 180° , leaves $94^{\circ} 6'$ for the angle A.

To find the side BC.

The extent from $46^{\circ} 30'$ ($=\angle B$) to $85^{\circ} 54'$ the supplement of $94^{\circ} 6'$ ($=\angle A$) on the scale of log. fines, will reach from 64 ($=AC$), to 88 ($=BC$), on the scale of logarithm numbers.

Secondly by the double Scales.

To find the Angle c.

1. Take the lateral distance of 64 ($=AC$) from the lines.
2. Make this a transverse distance of $46^{\circ} 30'$ ($=\angle B$) on the fines.
3. Take the lateral distance of 56 ($=AB$) on the lines.
4. Find the degrees to which this extent is a transverse distance on the fines, viz. $39^{\circ} 24'$; and this is the angle sought.

To find the side BC.

1. Take the lateral distance of 64 ($=AC$) from the lines.
2. Make this a transverse distance of $46^{\circ} 30'$ ($=\angle B$) on the fines.
3. Take the transverse distance of $85^{\circ} 54'$ (the supplement of $94^{\circ} 6' = \angle A$) on the fines.
4. Find the lateral distance this extent is equal to, on the lines; and this distance, viz. 88, will be the side required.

Ex. II. In the triangle ABC Pl. VI. Fig. 27.

$$\begin{aligned} \text{Given } BC &= 74 \\ \angle B &= 104^{\circ} 0' \\ \angle C &= 28^{\circ} 0' \end{aligned}$$

Required AB & AC.

Now the sum of $104^{\circ} 0'$ and $28^{\circ} 0'$ is $132^{\circ} 0'$.

And $132^{\circ} 0'$ taken from 180, leaves $48^{\circ} 0'$ for the angle A.

H

The

The proportions are,

As fine $\angle A$: fine $\angle C$:: side BC : side AB.

And as fine $\angle A$: fine $\angle B$:: side BC : side AC.

First, by the Logarithm Scales.

To find AB.

THE extent from $48^\circ 0'$ ($= \angle A$) to $28^\circ 0'$ ($= \angle C$) on the scale of logarithm lines, will reach from 74 ($= BC$) to 46, 75, ($= AB$), on the scale of logarithm numbers.

To find AC.

THE extent from $48^\circ 0'$ to $76^\circ 0'$ ($=$ supplement of $104^\circ 0'$) on the scale of log. lines, will reach from 74 to 96, 6 ($= AC$) on the scale of logarithm numbers.

Secondly by the double Scales.

To find AB.

1. TAKE the lateral distance 74 ($= BC$) on the lines.
2. MAKE this extent a transverse distance to $48^\circ 0'$ ($= \angle A$) on the lines.
3. TAKE the transverse distance of $28^\circ 0'$ ($= \angle C$) on the lines.
4. To this extent find the lateral distance on the lines, viz. 46, 75 and this will be the length of AB.

To find AC.

1. TAKE the lateral distance 74 ($= BC$) on the lines.
2. MAKE this extent a transverse distance to $48^\circ 0'$ ($= \angle A$) on the lines.

3. TAKE the transverse distance to $76^{\circ} 0'$ the supplement of $104^{\circ} 0'$ ($= \angle B$) on the lines.

4. To this extent, find the lateral distance on the lines, viz. 96, 6, and this will be the length of AC.

SOLUTION of CASE II.

THE solution of this case depends on a well known theorem, viz.

As the sum of the given sides

Is to the difference of those sides,

So is the tangent of the half sum of the unknown angles

To the tangent of the half difference of those angles.

And the angles are readily found by their half sum and half difference being known.

Ex. III. In the triangle ABC, Pl. VI. Fig. 28.

Given $BC = 74$

$BA = 52$

$\angle B = 68^{\circ} 0'$

Required $\angle A$; $\angle C$; & AC.

Preparation.

TAKE the given angle $68^{\circ} 0'$ from 180° , and half the remainder, viz. $56^{\circ} 0'$ is the half sum of the unknown angles which call z ; and let x stand for the half difference of those angles.

ALSO find the given sum of the sides, viz. $BC + BA = 126$.

AND take the difference of those sides, viz. $BC - BA = 22$.

Then the proportions are

As $BC + BA : BC - BA :: \tan. z : \tan. x$.

THEN the sum of z and x gives the greater angle A.

THE difference of z and x gives the lesser angle c ,
 AND as $\text{fine } \angle c : \text{fine } \angle B :: \text{fine } BA : \text{side } AC.$

First by the Logarithm Scales.

To find the tangent of N .

TAKE the extent from 126 (= sum of the given sides) to 22 (= diff. of those sides) on the scale of logarithm numbers; lay this extent from $45^\circ 0'$ to the left on the logarithm tangents; stay the lowest point, and bring that which rested on 45 degrees, to $56^\circ 0'$; remove the compass, and this extent laid from $45^\circ 0'$ towards the left, gives $14^\circ 31'$ equal N .

THEN the sum of $56^\circ 0'$ and $14^\circ 31'$ or $70^\circ 31'$ is the angle A .

AND $14^\circ 31'$ taken from $56^\circ 0'$ leaves $41^\circ 29'$ for the angle c .

To find AC .

THE extent from $41^\circ 29'$ (= $\angle c$) to $68^\circ 0'$ (= $\angle B$) on the logarithm lines, will reach from 52 (= BA) to 72, 75 (= AC) on the scale of logarithm numbers.

IN finding the tangent of (N , or) the half difference of the unknown angles, there were two applications of the compasses to the scale of tangents: Now this happens because the upper tangents which should have been continued beyond 45° , or to the right hand, are laid down backwards, or to the left hand, among the lower tangents (the logarithmic tangents ascending and descending by like spaces at equal distances on both sides of 45°), and thereby the length of the scale is kept within half the length necessary to lay down all the tangents in order, from the left towards the right. But supposing they were so laid down, then the point of $56^\circ 0'$ will reach as far to the right of 45° as it does now to the left, and the

the extent on the numbers from 126 to 22 would reach from the point 56° taken on the right of 45° , to $14^\circ 31'$ at one application; the said extent being applied from 45° downwards, will reach as far beyond $14^\circ 31'$, as is the distance from 45° to 56° ; therefore the legs of the compasses being brought as much closer as is that interval, will reach from 45° to the degrees wanted.

INDEED when the half sum is less than 45° , then the extent from the sum of the sides to their difference, will reach from the tangent of the half sum, downward, to the tangent of the half difference, at once.

AND when the half sum of the unknown angles, and their half difference, are both greater than 45° , then the extent from the sum of the sides to their difference, will reach from the tangent of the half sum of the angles, upwards (or to the right) to the tangent of the half difference of those angles, at once.

Secondly by the double Scales.

Because 126 the sum of the sides will be longer than the scales of lines, therefore take 63, the half of 126, and 11, the half of 22, the difference of the sides; for the ratio of 63 to 11, is the same as that of 126 to 22. Then

1. TAKE the lateral distance 63 on the scales of lines.

2. MAKE this extent a transverse distance to 56° degrees, on the upper tangents.

3. TAKE the transverse distance of 45° on the upper tangents, and make this extent a transverse distance to 45° on the other tangents.

4. TAKE the lateral distance 11, on the lines:

5. To this extent, find the transverse distance on the tangents, and this will be $14^\circ 31' = N$.

AND this is the manner of operation, when M is greater than 45 degrees, and N is less.

But when M } greater } than 45
 & N are each } less } degrees.

Then the third article of the foregoing operation is omitted.

Now having found the angles A and C , the side AC may be found as in the first or second examples.

BUT in this case, the third side AC may be found without knowing the angles. Thus,

1. TAKE the lateral distance of (34 deg.) the half of (68,) the given angle, from the lines.
2. MAKE this extent a transverse distance, to 30 on the lines.
3. WITH the sector thus opened, take the distance from 74 on one leg, to 52 on the other leg, each reckon'd on the lines.
4. The lateral distance, on the lines, of this extent, gives the side $AC = 72, 75$.

FROM the two first articles of this operation, is learn'd how to set the double scales to any given angle.

WHEN the included angle B is 90 degrees, the angles A and C are more readily found, as in the following example, whose solution depends on this principle. That one of the given sides has the same proportion to radius, as the other given side has to the tangent of its opposite angle.

Ex. IV. In the triangle ABC : Fig. 29.

Given $AB = 45$

$BC = 65$

$\angle B = 90$

Required $\angle A$; $\angle C$; & AC .

The proportions are,

For the Angle A.

As side AB : side BC :: radius : $\tan. \angle A$.

AND

AND the $\angle A$ taken from 90° leaves the $\angle C$, then AC may be found as directed in the last example.

First by the logarithmic Scales.

THE extent from 45 ($= AB$) to 65 ($= BC$) on the numbers, will reach from 45 degrees to $55^\circ 18'$ ($= \angle A$) on the tangents.

HERE the angle A is taken equal to $55^\circ 18'$, because the second term BC is greater than the first term AB : But if the terms were changed, and it was made BC to AB , then the degrees found would be $34^\circ 42' = \angle C$.

Secondly by the double Scales.

1. TAKE the lateral distance of the first term, from the lines.

2. MAKE this a transverse distance to 45 deg. on the tangents.

3. Take the lateral distance of the second term, from the lines.

4. THE transverse distance of this extent, found on the tangents, gives the degrees in the angle sought.

IF the first term is greater than the second, then the lateral distance of the first term, must be set to 45 degrees on the lower tangents, and the lateral distance of the second term, must be reckon'd on the same tangents.

BUT if the first term is less than the second, then the lateral extent of the first term must be set to 45° on the upper tangents, and the lateral extent of the second term must be reckon'd on the same tangents.

SOLUTION of CASE III. Fig. 30.

IN the triangle ABC :

Given $BC = 926$.

$BA = 558$.

$AC = 702$.

Requir'd $\angle B$, $\angle C$. $\angle A$.

H 4

THERE

THERE are usually given for the solution of this case by the logarithmic scales two methods; the one best when all the angles are to be found, the other best when one angle only is wanted; both methods will be here delivered.

FIRST. *When all the angles are wanted.*

SUPPOSE a perpendicular AD (Pl. VI. Fig. 30.) drawn to the greatest side BC, from the angle A opposite thereto; then AD divides the triangle ABC into two right angled triangles BDA, CDA; in which if CD and DB were known, the angles would be found, as in the solution of Case I.

TAKE the sum of the sides AC and AB, which is 1260.

ALSO their difference, which is 144.

THEN on the scale of numbers, the extent from 926 (= BC) to 1260, will reach from 144 to 196.

AND the half sum of 926 and 196, is 561 = DC.

AND the half difference of 926 and 196 is 365 = DB.

THE extent from 558 (= BA) to 365 (= BD) on the numbers, will reach on the log. lines from 90° (= $\angle BDA$) to $40^{\circ} 52'$ (= $\angle BAD$.)

THEN $40^{\circ} 52'$ taken from 90° , leaves $49^{\circ} 8'$ for $\angle B$.

AND the extent from 702 (= CA) to 561 (= CD) on the numbers, will reach from 90° (= $\angle CDA$) to $53^{\circ} 04'$ (= $\angle CAD$) on the scale of log. lines.

THEN $53^{\circ} 4'$ taken from 90° , leaves $36^{\circ} 56'$ for the $\angle C$.

ALSO the sum of $40^{\circ} 52'$ and $53^{\circ} 4'$ gives $93^{\circ} 56'$ for the $\angle CAB$.

SECONDLY,

SECONDLY, To find either angle; suppose B.

Preparation.

TAKE the difference between BC and BA, the sides including the angles sought, and call it D = 368.

FIND the half sum of AC and D, call it z = 535

AND the half diff. of AC and D, call it x = 167

THEN as 1 : $\sqrt{\frac{z \times x}{AB \times BC}}$:: radius : sine $\frac{1}{2}$ \angle B.

1. THE extent on the log. numbers from 1 to 535 (= z), will reach from 167 (= x) to a 4th point; mark it and call it G.

2. THE extent from 1 to 558 (= AB), will reach from 926 (= BC) to a 4th point; mark it and call it H.

3. THE extent from the point H to the point G, will reach from 1, downward to a 4th point, mark it and call it K.

4. THE extent from K, to the middle point between it and the 1 next above K, taken on the log. numbers, will reach on the log. sines from 90° to $24^\circ 34'$, which doubled gives $49^\circ 8'$ for the angle B.

BUT the scale of log. versed sines being used, the work will be considerably shortened. Thus,

1. ON the log. numbers take the extent from 535 (= z) to 926 (= BC), this will reach from 558 (= BA) to a 4th point, where let the foot of the compasses rest.

2. THEN the extent from that 4th point to 167 (= x), will reach on the line of versed sines from 0 degrees (at the end) to $130^\circ 52'$, which taken from 180° leaves $49^\circ 8'$ for the angle B.

By

By the double, or sectoral, Scales.

To find the angle B.

1. TAKE the lateral distance 702 ($= AC$, the side opposite to the angle B) from the lines.

2. OPEN the legs of the sector until this extent will reach from 926 ($= CB$) on one scale of lines, to 558 ($= AB$) on the other scale of lines.

3. THE sector being thus opened, take the transverse distance between 30° and 30° on the fines, this distance measured laterally on the fines, one foot being on the centre, will give $24^\circ 34'$ for half the angle B.

THE other angles may be found as $\angle B$ was, or according to the directions in some of the preceding cases.

ALTHOUGH in these examples, oblique triangles were taken as being the most general; yet it may be readily seen, that those concerning right-angled triangles are only particular cases, and may be, for the general, more easily solved.

VARIETY of other examples, shewing the uses of these scales, might be given in various parts of the mathematics, which the reader may of himself supply: However here will be subjoined a few in spherical trigonometry, as they will include some operations not only curious, but perhaps not to be met with elsewhere.

S E C T. XVIII.

The Construction of the several cases of Spherical Triangles by the Scales on the Sector.

TH E cases of spherical triangles are six.

CASE I. Given two sides, and an angle opposite to one of them.

CASE II. Given two angles, and a side opposite to one of them.

CASE III. Given two sides, and the included angle.

CASE IV. Given two angles, and the included side.

CASE V. Given the three sides.

CASE VI. Given the three angles.

These six cases include all the variety that can arise in spherical triangles.

IN the following solutions, are given three constructions to every case, whereby each side is laid on the plane of projection, or (as it is commonly called, the) primitive circle.

To abbreviate the directions given in the following constructions, it is to be understood, that the primitive circle is always first described, and two diameters drawn at right angles.

THE sector is also supposed to be set to the radius wanted, on the scale used ; and the tranverse distance of the degrees proposed is to be taken from the chords, or secants, or tangents, &c. according to the name mentioned in the construction.

So-

SOLUTION of CASE I.

EXAM. In the spherical triangle ABD.

Given $AB = 29^{\circ} 50'$

$DB = 63^{\circ} 59'$

$\angle D = 25^{\circ} 55'$

Required the triangle.

I. To put DB on the primitive circle. Fig. 1. 1. Pl. VII.

1st. Make $DB =$ chord of $63^{\circ} 59'$, and draw the diameter BE.

2d. From D, with the secant of the $\angle D, 25^{\circ} 55'$, cut the diameter $\odot I$ in c: on c as a center, with that radius, describe the circumference DA, and the angle BDA will be $25^{\circ} 55'$.

3d. Make Bd equal to AB, with the chord of $29^{\circ} 50'$.

4th. With the tangent of AB, $29^{\circ} 30'$, from d, cut $\odot B$ produced in b; and from b, with that radius, cut DA in A or a.

5th. Through B, A, E, describe a circumference, and the triangle BDA will be that required; whose parts DA, $\angle B$, and $\angle A$ may be thus measured.

To measure DA.

6th. Make $\odot P$ equal to the tangent of half the angle BDA, viz. $12^{\circ} 57\frac{1}{2}'$; then a ruler on P and A, gives e; and D e measured on the chords, gives the degrees in DA, viz. $42^{\circ} 9'$.

To measure $\angle B$.

7th. Draw the diameter FG at right angles to BE, cutting the circumference BAE in s; a ruler by B & s gives f; make fg equal to the chord of 90 deg.

a ruler on g and B , gives p in the diameter FG . Then Eg on the chords gives the angle $B = 36^\circ 9'$.

To measure $\angle A$.

8th. A ruler on A and p , gives n ; and on A and p , gives m ; and nm measured on the chords, gives $52^\circ 9'$, for the supplement of the angle DAB , which is $127^\circ 51'$.

II. *To put DA on the primitive circle, Fig. 2. I.*

1st. With the secant of the angle D , $25^\circ 55'$, from D , cut the diameter in c ; and on c , with the same radius, describe the arc DB , and the angle BDA will be $25^\circ 55'$.

2d. Make $\odot p$, equal to the tangent of half the angle D ; viz. $12^\circ 57' \frac{1}{2}$.

3d. On the primitive circle, make Dd equal to the given side DB , with the chord of $63^\circ 59'$.

4th. A ruler on B and d , gives B ; then will $BD = 63^\circ 59'$.

5th. Draw $\odot B r$, cutting the primitive circle in r .

6th. Make $r x =$ the chord of 90° ; or twice the chord of 45° .

7th. A ruler on x and B , gives m on the primitive circle.

8th. Make $m q = mp =$ chord of $29^\circ 50'$.

9th. A right line through x & p , x & q , gives f & e in $\odot r$.

10th. On fe as a diameter, describe a circumference, cutting the primitive circle in A , a .

11th. A ruler on A & \odot , gives F .

12th. Through A , B , F , describe a circumference, and the triangle ABD is constructed with DA on the primitive circle as required.

III. *To put AB on the primitive circle. Fig. 3. I.*

1st. Make $AB =$ the chord of $29^\circ 50'$; and draw the diameter BF .

2d. In

2d. In $A b$ drawn perpendicular to AG , take $A b = \text{fine of } AB \ 29^\circ 50'$.

3d. Make the angle $b A g, = \angle D \ 25^\circ 55'$; from A draw $A e$ at right angles to $A g$, and from d , the middle of $A b$, draw de perpendicular to $A b$, cutting Ae , in e ; from e , with the radius $e A$, describe a circumference $A f b$.

4th. From b , with the fine of $BD, 63^\circ 59'$, cut the circumference Afb in f ; and draw $A f$.

5th. From A , draw Ac at right angles to $f A$, meeting $E \odot$ (perpendicular to $A \odot$,) continued, in c ; and on c , with the radius cA , describe a circumference ADG .

6th. Make $Bm = BD$, with the chord of $63^\circ 59'$; from m , with the tangent of $63^\circ 59'$ cut $\odot B$ produced, in n ; on n , with the same radius, cut ADG in D .

7th. Through B, D, F , describe a circumference, and the triangle ABD will be that which was required.

Computation by the logarithmic scales.

To find the angle A.

The fines of the angles of spheric triangles are as the fines of their opposite sides.

Then the extent of the compasses on the line of fines from $29^\circ 50' (= AB)$ to $25^\circ 55' (= \angle C)$; will reach from the fine of $63^\circ 59' (= CB)$ to the fine of $52^\circ 9' (= \angle A)$.

But by construction the $\angle A$ is obtuse; therefore $127^\circ 51'$ (the supplement of $52^\circ 9'$) is to be taken for the angle A .

To find the angle B.

Say, as radius, to the cosine of CB .

So tang. $\angle C$, to the cotang of a fourth arc.

And as tang. AB , to the tang. of CB .

So cosine of the 4th arc, to the cosine of a 5th arc.

Then

Then the difference between the 4th and 5th arcs gives $\angle B$.

The extent from the sine of 90° to the sine of $26^\circ 1'$ ($=$ comp. of $63^\circ 59'$), will, on the tangents, reach from $25^\circ 55'$ to $12^\circ 2'$: But the 4th arc is to be a cotangent; therefore $77^\circ 58'$ (the comp. $12^\circ 2'$) is the 4th arc.

The extent from the tangent of $29^\circ 50'$ to the tangent of $63^\circ 59'$, will reach on the line of sines from $12^\circ 2'$ ($=$ comp. of $77^\circ 58'$) to $48^\circ 9'$.

But the 5th arc is to be a cosine; therefore $41^\circ 51'$ (the comp. of $48^\circ 9'$) is the fifth arc.

And $41^\circ 51'$ taken from $77^\circ 58'$ leaves $36^\circ 7'$ for the angle B.

The extent from the tangent of $29^\circ 50'$ to the tangent $63^\circ 59'$ is thus taken. Set one foot on the tangent $29^\circ 50'$, and extend the other to the tangent of 45° : Apply this extent on the tangents from $63^\circ 59'$ towards the left; rest the left hand foot, and extend the other to 45° , and the compasses will then have the required extent.

To find AC.

Say, as radius, to the cosine of the angle c.

So is the tangent of CB, to the tangent of a 4th arc.

And as cosine of CB, to the cosine of AB.

So is the cosine of the 4th arc, to the cosine of the 5th arc.

Then the difference between the 4th and 5th arcs will give the side AC.

The extent on the sines from 90° to $64^\circ 5'$ (the comp. of $25^\circ 55'$) will reach on the tangents from $63^\circ 59'$ towards the right to $61^\circ 31'$ the 4th arc.

Also the extent on the sines from $26^\circ 1'$ ($=$ comp. of $63^\circ 59'$) to $60^\circ 10'$ ($=$ comp. of $29^\circ 50'$) will reach from the sine of $28^\circ 29'$ (the complement of $61^\circ 31'$) to the sine of $70^\circ 37'$.

But

But the 5th arc is to be a cosine, therefore $19^{\circ} 23'$ is the 5th arc.

And $19^{\circ} 23'$ taken from $61^{\circ} 31'$ leaves $42^{\circ} 8'$ for the side AC.

SOLUTION of C A S E II.

EXAM. In the spherical triangle ABD.

Given $AD = 42^{\circ} 9'$

$\angle A = 127^{\circ} 50'$

$\angle B = 36^{\circ} 8'$

Required the triangle.

I. To put DB on the primitive circle. Fig. 1. 2. Pl. VII.

1st. From B, with the secant of $\angle B$, $36^{\circ} 8'$, cut the diameter $\odot E$ in c; on c, with the same radius, describe the circumference BAF: then the angle DBF = the given $\angle B$.

2d. Make the angle naq equal to $37^{\circ} 50'$, the difference between $127^{\circ} 50'$ and 90° .

3d. Make $aq =$ tangent of DA, $42^{\circ} 9'$; on \odot with the secant of $42^{\circ} 9'$ describe an arc qQ : on c with c q, cut the arc qQ in Q.

4th. Draw $Q\odot G$ cutting the primitive circle in D, and BD will be a side of the triangle.

5th. From Q with Qa, cut BAF in A; and through D, A, G, describe a circumference, and the triangle BAD is that required. Whose parts BD, BA and $\angle D$ are thus measured.

6th. BD measured on the chords, gives 64 degrees.

7th. Make $\odot P =$ tangent of half $\angle B$, viz. $18^{\circ} 4'$; a ruler on P and A gives x; then Bx measured on the chords gives $29^{\circ} 50'$, for BA.

8th. Draw a diameter perpendicular to GD, cutting the circumference DAG in s; a ruler on D and s gives m; make mn 90 degrees, then G n measured on the chords, gives $25^{\circ} 55'$ for the $\angle D$.

II. To put AB on the primitive circle. Fig. 2. 2.

1st. From A, with the secant of the supplement of the $\angle A$, viz. $52^\circ 10'$, cut the diameter $\odot F$ continued in c; on c, with the same radius, describe a circumference ADE.

2d. Make $\odot P \equiv$ the tangent of half the supplement of $\angle A$, viz. $26^\circ 5'$; and make $Ax \equiv$ chord of AD, $42^\circ 9'$: a ruler on P and x, gives D; then is AD equal to $42^\circ 9'$.

3d. On \odot , with the tangent of the angle B, $36^\circ 8'$, describe an arc mc; on D, with the secant of $\angle B$, $36^\circ 8'$, cut the arc mc in c; on c, with the same radius, describe a circumference DB, then the triangle ADB, will be that required.

III. To put DA on the primitive circle. Fig. 3. 2.

1st. Lay down AD with the chord of $42^\circ 9'$: Draw the diameter DF; and another $\odot H$, perpendicular to DF.

2d. On A, with the secant of the supplement of $\angle A$, viz. $52^\circ 10'$, cut the diameter E \odot in c; and on c, with the same radius, describe the circumference ABG.

3d. Make $\odot P$ equal to the tangent of half the supplement $\angle A$, viz. $26^\circ 5'$, a ruler by G and P gives x.

4th. Make $xm \equiv xn$ with the chord of $\angle B$, $36^\circ 8'$; a ruler by G and n gives r, by G and m gives s; on b the middle of rs, with the radius bs, cut $\odot H$ in p.

5th. A ruler on F and p, gives b; make $bk \equiv bD$; a ruler on F and k gives c; with the radius c D, describe the circumference DBF; and the triangle ABD, is that sought.

*Computation by the Logarithmic Scales.**To find the side BD.*Say, as the sine of $\angle B$, is to the side AD.So is the sine of $\angle A$, to the side BD.

Then the extent from the sine of $36^\circ 8'$ to the sine of $42^\circ 9'$, will reach from the sine of $52^\circ 10'$ (the supplement of $127^\circ 50'$) to the sine of $63^\circ 59' =$ side BD.

*To find the side AB.*Say, as radius, is to the cosine of the $\angle A$.

So is the tangent of AD, to the tangent of a 4th arc.

And, as tangent of $\angle B$, to the tangent of the $\angle A$.

So is the sine of the 4th arc, to the sine of a 5th arc.

Then the difference between the 4th and 5th arcs will be equal to the side AB.

The extent from the radius, or the sine of 90° to the sine of $37^\circ 50'$ (the complement of $52^\circ 10'$), will reach on the tangents from $42^\circ 9'$ to $29^\circ 02' =$ 4th arc.

And the extent from the tangent of $36^\circ 8'$ to the tangent of $52^\circ 10'$, will reach on the sines from $29^\circ 02'$ to $58^\circ 54' =$ 5th arc.

Then the difference between $58^\circ 54'$ and $29^\circ 02'$ gives $29^\circ 52'$ for the side AB.

The extent from the tangent of $36^\circ 8'$ to the tangent of $52^\circ 10'$ is taken as shewed in the second operation of the first case.

To find the $\angle D$.

Say, as radius, is to the cosine of AD.

So is the tangent of $\angle A$, to the tangent of a 4th arc.

And as the cosine of $\angle A$, to the cosine of $\angle B$;

So is the sine of the 4th arc, to the sine of the 5th arc.

Then

Then the difference between the 4th and 5th arcs will give the $\angle D$.

Now the extent from the sine of 90° to the sine of $47^\circ 51'$ (the complement of $42^\circ 09'$), will reach from the tangent of $52^\circ 10'$ to the tangent of $43^\circ 40'$. But the 4th arc being a cotangent will be $46^\circ 20'$, the complement of $43^\circ 40'$.

Also the extent from the sine of $37^\circ 50'$ (the complement of $52^\circ 10'$) to the sine of $53^\circ 52'$ (the complement of $36^\circ 08'$), will reach from the sine of $46^\circ 20'$ to the sine of $72^\circ 15'$ the 5th arc.

Then the difference between $72^\circ 15'$ and $46^\circ 20'$ viz. $25^\circ 55'$ will be the angle D .

In applying the first extent, viz. from the sine of 90° to sine of $47^\circ 51'$, to the tangents; set one foot on the tangent of 45° and let the other foot rest where it falls; move the foot from 45° to $52^\circ 10'$; then this extent will reach from 45° to $43^\circ 40'$.

SOLUTION of CASE III.

Ex. In the spherical triangle ABD .

Given $AB = 29^\circ 50'$.

$BD = 63^\circ 59'$

$\angle B = 36^\circ 8'$

Required the triangle.

I. To put AB on the primitive circle. Fig 1. 3. Pl. VII.

1st. Make $AB =$ chord of $29^\circ 50'$; draw the diameter EF , and another $\odot E$ perpendicular thereto.

2d. From B , with the secant of $\angle B$, $36^\circ 8'$ cut $\odot E$ in c , the center of BDF .

3d. From \odot , with the tangent of half $\angle B$, viz. $18^\circ 4'$, cut $\odot E$ in P , the pole of BDF .

4th. Make $Bx = BD$, $63^\circ 59'$; a ruler on P and x , gives D . Through A , D , G , describe a circumference, and the triangle ADB is that required, whose parts AD , $\angle A$, and $\angle D$ may be thus measured.

5th. A ruler on A and s gives z , make $z y =$ chord of 90° ; a ruler on A and y gives p the pole of $A s G$; a ruler on p and D , gives n , and $A n$ measured on the chords gives $42^\circ 8'$ for AD .

6th. $G y$ measured on the chords, gives $52^\circ 11'$ for the supplement of $\angle A$; therefore $\angle A = 127^\circ 49'$.

7th. A ruler on D and p gives r , on D and P , gives m ; and rm , measured on the chords gives $25^\circ 56'$ for the angle BDA .

II. *To put DB on the primitive circle. Fig. 2. 3.*

1st. Make $DB =$ chord of $63^\circ 59'$: draw the diameter BF and perpendicular thereto, the diameter $\odot G$.

2d. From B , with the secant of $\angle B$, $36^\circ 8'$, cut $\odot G$ in c ; on c with CB , describe the circumference BAF .

3d. Make $\odot P =$ tangent of half $\angle B$, $18^\circ 4'$, and $D x =$ chord of AB $29^\circ 50'$, a ruler on P and x gives A ; through D , A , E , describe a circumference, and the triangle ABD is that required.

III. *To put AD on the primitive circle. Fig. 3. 3.*

1. In a right line ed , touching the primitive circle in any point b , take $bd =$ tangent of BD , $63^\circ 59'$; and $be =$ tangent of AB , $29^\circ 50'$.

2. Make the angle $dba = \angle B$, $36^\circ 8'$, and make $ba = be$.

3. From d , \odot , with da , $\odot e$, describe arcs crossing in x ; from x , d , draw the diameters AE , DF ; and others OG , OH , perpendicular to AE , FD .

4. From d , x , with bd , eb , describe arcs crossing in B ; and draw dB , xB .

5. From B draw BC , perpendicular to xB , and meeting $\odot G$ produced in c ; also draw BC perpendicular to dB , and meeting $\odot H$ in c ; then c is the center of a circumference through A , B , E ; and c the center of that through D , B , F ; and the triangle ABD is that required.

Computation by the Logarithmic Scales.

To find the angles A and C.

Say, as the sine of half the sum of the given sides
To the sign of half their difference;
So is the cotangent of half the given angle
To the tangent of half the difference of the re-
quired angles.

And, as the cosine of half the sum of the given sides
To the cosine of half their difference;
So is the cotangent of half the given angle
To the tangent of half the sum of the required angles.
Then the half difference of the required angles add-
ed to their half sum will give the greater angle A.

And the half difference of those angles taken from
their half sum will give the lesser angle D.

Now the sum of the given sides $63^{\circ} 59'$ and $29^{\circ} 50'$ is $93^{\circ} 49'$, their difference is $34^{\circ} 09'$; the half sum = $46^{\circ} 54\frac{1}{2}'$, and the half difference is $17^{\circ} 04\frac{1}{2}'$.

Also half the given angle B is $18^{\circ} 04'$.

Then the extent from the sine of $46^{\circ} 54'$ to the sine of $17^{\circ} 4'$, will reach from the tangent of $71^{\circ} 56'$ (the complement of $18^{\circ} 4'$) to the tangent of $50^{\circ} 57'$ the half difference of the required angles.

Here the extent on the sines is from right to left or decreasing; so the extent on the tangents must be from left to right, which in this case is decreasing.

Also the extent from the sine of $43^{\circ} 6'$ (the complement of $46^{\circ} 54'$) to the sine of $72^{\circ} 56'$ (the complement of $17^{\circ} 04'$), will on the scale of tangents reach from $71^{\circ} 56'$ (the complement of $18^{\circ} 4'$) to $76^{\circ} 53'$ the half sum of the required angles.

Then the sum of $76^{\circ} 53'$ and $50^{\circ} 57' = 127^{\circ} 50' = \angle A$.

And the difference of $76^{\circ} 53'$ and $50^{\circ} 57' = 25^{\circ} 56' = \angle C$.

The angles being known, the other side may be found by opposite sides and angles, and is $42^{\circ} 08'$.

Or the other side may be found without knowing the angles.

Say, as radius is to the cosine of the given angle;

So is the tangent of either given side, to the tangent of a 4th arc.

Which 4th arc will be like the side used when the given angle is acute, otherwise it will be of a contrary kind with the side used.

Then take the difference between the 4th arc and the other given side, call the remainder a 5th arc.

And as the cosine of the 4th arc is to the cosine of a 5th arc;

So is the cosine of the side used in the former proportion

To the cosine of the side required.

Now the extent from the sign of 90° to the sine of $53^\circ 5'$ (= complement of $36^\circ 08'$) will reach from the tangent of $29^\circ 50'$ to the tangent of $24^\circ 51'$ the 4th arc.

And $24^\circ 51'$ taken from $63^\circ 59'$ leaves $39^\circ 8'$ for the 5th arc.

Then the extent from the sine of $65^\circ 09'$ (the complement of $24^\circ 51'$, to the sine of $50^\circ 52'$ (the complement of $39^\circ 08'$) will reach from the sine of $60^\circ 10'$ (the complement of $29^\circ 50'$) to the sine of $47^\circ 51'$; whose complement, *viz.* $42^\circ 09'$ is the side required.

SOLUTION of CASE IV.

Ex. In the spherical triangle ABD:

Given $\angle D = 25^\circ 55'$.

$\angle B = 36^\circ 08'$.

$DB = 63^\circ 59'$,

Required, The triangle.

I. To put DB on the primitive circle. Fig. 1. 4. Pl. VII.

1. Make DB = chord of $63^\circ 59'$; draw the diameter BF, and draw CG perpendicular to BF.

2. From

2 From B, with the secant of $\angle B$, $36^\circ 8'$, cut $\odot c$ in c ; and c will be the center of BAF.

3. From D, with the secant of $\angle D$, $25^\circ 55'$; cut $\odot H$ in c , and c will be the center of DAE; and the triangle DAB is that which was required; whose parts DA, BA, and $\angle A$, are thus measured.

4. Make $\odot p =$ tangent of $\frac{1}{2} \angle D$, $12^\circ 57\frac{1}{2}'$, a ruler on p and A gives x ; then DX measured on the chords gives $42^\circ 10'$ for AD.

5. Make $\odot P =$ tangent of $\frac{1}{2} \angle B$ $18^\circ 4'$, a ruler on P and A, gives z ; then Bz measured on the chords, gives $29^\circ 54'$ for AB.

6. A ruler on A and p , gives n , on A and P, gives m ; and nm measured on the chords gives $52^\circ 10'$ the supplement of the angle A. Therefore $\angle A = 127^\circ 50'$.

II. To put DA on the primitive circle. Fig. 2. 4.

1st. From D, with the secant of $\angle D$, $25^\circ 55'$; cut $\odot F$ in c ; and c is the center of the circumference DBE.

2d. Make $\odot P =$ tangent of $\frac{1}{2} \angle D$, $12^\circ 57\frac{1}{2}'$; and make $DX =$ chord of BD, $63^\circ 59'$; a ruler on P, x , gives B; and DB is $63^\circ 59'$.

3d. Make the angle $CBC = \angle B$, $36^\circ 8'$; through c , draw mc perpendicular to $B\odot$, cutting Bc in c ; on c , with the radius cB , describe the circumference ABG; and the triangle ABD, is that which was required.

III. To put AB on the primitive circle. Fig. 3. 4.

1st. From B, with the secant of $\angle B$, $36^\circ 8'$ cut $\odot F$ in c ; and c is the center of the circumference of BDE.

2d. Make $Bx =$ chord of BD, $63^\circ 59'$; and $\odot P =$ tangent of $\frac{1}{2} \angle B$, $18^\circ 4'$; a ruler on P and x gives D; then is $BD = 63^\circ 59'$.

3d. Make the angle $CDc = \angle D$, $25^\circ 55'$; then mc drawn perpendicular to $\odot D$, meeting Dc in c , gives c the center of the circumference ADG; and the triangle ABD will be that required.

*Computation by the Logarithmic Scales.**To find the angle A.*

Say, as radius is to the cosine of the given side;
So is the tangent of either given angle to the cotangent of a 4th arc,

Call the difference between the other given angle and the 4th arc, the 5th arc.

And, as the sine of the 4th arc, is to the sine of the 5th arc;

So is the cosine of the angle used in the former proportion

To the cosine of the required angle.

The 4th arc will be of the same kind with the angle first used if the given side is less than 90° ; but of a contrary kind if that side is greater than 90° .

Arcs are said to be of the same kind, when both are less, or both greater, than 90 degrees.

The required angle will be of the same kind with the angle used in the proportions, if the 4th arc is less than the other angle; but of an unlike kind when the 4th arc is greater than the other angle.

Now the extent from the sine of 90° to the sine of $26^\circ 01'$ (the complement of $63^\circ 59'$) will reach from the tangent of $25^\circ 55'$ to the tangent of $12^\circ 02'$: But this is the complement of the 4th arc, which is $77^\circ 58'$.

And $36^\circ 08'$ taken from $77^\circ 58'$ leaves $41^\circ 50'$ for the 5th arc.

Then the extent from the sine of $77^\circ 58'$ to the sine of $41^\circ 50'$, will reach from the sine of $64^\circ 5'$ (the complement of $25^\circ 55'$) to the sine of $37^\circ 50'$, which is the complement to $52^\circ 10'$.

But as the 4th arc was greater than $36^\circ 08'$, the angle sought is to be of a contrary kind to $25^\circ 55'$ ($= \angle D$), that is, that A is to be obtuse; so $127^\circ 50'$ (the supplement of $52^\circ 10'$) is to be taken for the angle A.

Now

Now all the angles and one side being known, the other sides may be found by the proportion subsisting between the sines of angles, and the sines of their opposite sides,

Or say,

As the sine of half the sum of the given angles
Is to the sine of half the difference of those angles;
So is the tangent of half the given side
To the tangent of half the difference of the required sides.

And

As the cosine of half the sum of the given angles
Is to the cosine of half the difference of those angles;
So is the tangent of half the given side
To the tangent of half the sum of the required sides.
Then the half difference added to the half sum gives the greater of the sought sides.

And the half difference subtracted from the half sum gives the lesser of the sought sides.

Now the half sum of the given angles, viz.
 $\frac{1}{2}\angle D + \frac{1}{2}\angle B = 31^{\circ} 01\frac{1}{2}'$,

And the half difference of those angles, viz.
 $\frac{1}{2}\angle B - \frac{1}{2}\angle D = 5^{\circ} 6\frac{1}{2}'$,

Also the half of the given side DB, is $31^{\circ} 59\frac{1}{2}'$.

Then the extent from the sine of $31^{\circ} 1'$, to the sine of $5^{\circ} 6'$;

Will reach from the tangent of $31^{\circ} 59'$, to the tangent of $6^{\circ} 3'$.

And the extent from the sine of $58^{\circ} 59'$ (= complement of $31^{\circ} 01'$) to the sine of $84^{\circ} 54'$ (the complement of $5^{\circ} 6'$), will reach from the tangent of $31^{\circ} 59'$ to the tangent of $35^{\circ} 58'$.

Then the sum of $35^{\circ} 58'$ and $6^{\circ} 3'$, viz. $42^{\circ} 01' = AD$.

And the difference of $35^{\circ} 58'$ and $6^{\circ} 3'$, viz. $29^{\circ} 55' = AB$.

SOLUTION of CASE V.

Ex. In the spherical triangle ABD.

Given $AB = 29^{\circ} 50'$

$AD = 42^{\circ} 9'$

$BD = 63^{\circ} 59'$

Required, The triangle.

I. To put AB on the primitive circle. Fig. 1. 5. Pl. VII.

1st. Make $AB =$ chord of $29^{\circ} 50'$; draw the diameter BF.

2. Make $an =$ chord of AD , $42^{\circ} 9'$; and $Bm =$ chord of BD , $63^{\circ} 59'$.

3d. From n , with the tangent of AD , $42^{\circ} 9'$, cut EA produced in c ; and from c , with that radius, describe the arc nn ; from m , with the tangent of BD , $63^{\circ} 59'$, cut FB produced in c ; and from c , with the radius cm , cut the arc nn in D .

4th. Through A, D, E ; B, D, F , describe circumferences, and the triangle ADB is that which was required; whose angles A, B, D , are thus measured.

5th. A ruler on A and a , gives x ; on B and b , gives z ; make xy, zv , each 90° ; a ruler on A and y gives p , in a radius perpendicular to AE ; and a ruler on B and v gives p , in a radius perpendicular to BF .

6th. Ey measured on the chords, gives $52^{\circ} 12'$ for the supplement of the $\angle A$; therefore $\angle A = 127^{\circ} 48'$.

7th. Fv measured on the chords, gives $36^{\circ} 10'$ for the angle B .

8th. A ruler on D and p gives t , and on D and p gives s ; then ts measured on the chords, gives $25^{\circ} 58'$ for the angle D .

The sides AD, DB , are put on the primitive circle, by a construction so like the foregoing one, that it is needless to repeat it. See figures 2. 5. and 3. 5.

Computation by the Logarithmic Scales.

To find the angle A.

The sides including the angle A are $AD = 42^{\circ} 09'$

And $AB = 29^{\circ} 50'$

Their difference call x $= 12^{\circ} 19'$

The side opposite the $\angle A$ is $BD = 63^{\circ} 59'$

Then the sum of BD and x is $76^{\circ} 18'$; the half sum is $38^{\circ} 09'$;

And the difference of BD and x is $51^{\circ} 40'$; the half difference is $25^{\circ} 50'$.

Now take the extent on the line of fines, from the half sum $38^{\circ} 9'$ to either of the containing sides, as to $29^{\circ} 50'$; apply this extent from the other containing side $42^{\circ} 09'$ towards the left, there let the foot rest, and extend the other point (*viz.* that which was set on $42^{\circ} 09'$) to the half difference $25^{\circ} 50'$; then this extent applied to the line of versed fines, will reach from 0 degrees (at the beginning) to $52^{\circ} 12'$; the supplement of which, or $127^{\circ} 48'$ will be the degrees in the angle A.

Again. To find the angle D.

The sides including the angle A, are $BD = 63^{\circ} 59'$

And $AD = 42^{\circ} 09'$

Their difference call x $= 21^{\circ} 50'$

The side opposite to the $\angle D$ is $AB = 29^{\circ} 50'$

Then the sum of AB and x is $51^{\circ} 40'$; the half sum is $25^{\circ} 50'$;

And the difference of BA and x is $8^{\circ} 00'$; the half difference is $4^{\circ} 00'$.

Then the extent on the fines from $25^{\circ} 50'$ to $63^{\circ} 59'$ will reach from the fine of $42^{\circ} 09'$ to some point beyond 90° ; therefore apply the extent between $25^{\circ} 50'$ and $63^{\circ} 59'$ from the fine of 90° downwards, let the point rest where it falls, and bring that point which was set on 90° to $42^{\circ} 09'$; then will the distance between

between the feet shew how far the first extent would reach beyond 90° : Now apply this extent on the lines from the point opposite to the middle 1 on the line of numbers, the other foot falling upwards to the right, let it rest there, and extend the other foot to the half difference $4^\circ 0'$: Then this extent applied to the versed lines, one foot being set on the point opposite the middle 1 on the line of numbers, the other foot will fall on $154^\circ 5'$; the supplement whereof, viz. $25^\circ 55'$ will be the angle D.

SOLUTION of CASE VI.

Ex. In the spherical triangle ABD:

$$\text{Given } \angle A = 127^\circ 50'.$$

$$\angle B = 36^\circ 8'.$$

$$\angle D = 25^\circ 55'.$$

Required, The triangle.

I. To put AB on the primitive circle. Fig. 1. 6. Pl. VII.

1st. From B, with the secant of $\angle B$, $36^\circ 8'$, cut $\odot F$ in c, and c will be the center of the circumference through B, D, E.

2d. From \odot , with the tangent of $52^\circ 10'$ the supplement of $\angle A$, describe an arc xc.

3d. Make an angle $caq = \angle D$, $25^\circ 55'$; make aq equal Bx . (= secant of $52^\circ 10'$.)

4th. From c, with the radius cq , cut xc in ϵ ; From ϵ , with the radius qa , describe a circumference ADG; and the triangle ABD, is that which was required: whose sides AB, BD, DA, are measured as follows.

5th. A ruler on B and a gives d, and on A and b, gives f; make dg , fb , each 90 degrees; a ruler on g and B gives p, and on b and A, gives p, in $\odot F$, $\odot H$, drawn perpendicular to BE, AG.

6th. A ruler on p and D gives n, and on p and D, gives m.

7th.

7th. Then BA , Bn , Am , measured on the chords, gives $29^{\circ} 50'$; $63^{\circ} 59'$; $42^{\circ} 9'$; for the respective measures of BA , BD , AD .

The directions for this construction, may be easily applied to the putting either of the other sides on the primitive circle. Fig. 2. 6. and 3. 6. Pl. VII.

Computation by the Logarithmic Scales.

To find the side BD:

The angles including the side BD , are $\angle B = 36^{\circ} 08'$
And $\angle D = 25 \ 55$

Their difference call $x = 10 \ 13$

The supplement of the \angle opposite to BD is $52 \ 10$

The sum of the supplement of $\angle A$ and x is $62^{\circ} 23'$; the half sum is $31^{\circ} 11\frac{1}{2}'$.

The difference of the supplement of $\angle A$ and x is $41^{\circ} 57'$; the half difference is $20^{\circ} 58\frac{1}{2}'$.

Now on the sines, the extent from the half sum $31^{\circ} 11\frac{1}{2}'$ to $25^{\circ} 55'$ will reach from $36^{\circ} 08'$ to a fourth sine; and the extent from that fourth sine to the sine of the half difference $20^{\circ} 58\frac{1}{2}'$ will reach on the versed sines from the beginning to about 64° the side sought.

And in like manner may the other sides be found.



S E C T. XIX.

Of the proportional Compasses.

TH O S E compasses are called proportional, whose joint lies between the points terminating each leg; in such a manner, that when the compasses are opened, the legs form a cross.

SUCH compasses are either simple or compound.

SIMPLE

SIMPLE proportional compasses, are such, whose center is fixed: One pair of these, serve only for one proportion.

THUS, if a right line is to be divided into 2, 3, 4, 5, &c. equal parts; or the cord of $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, &c. part of a circumference is to be taken; there must be as many of such compasses, as there are distinct operations to be performed.

IN each case, take the length of the right line, or of the radius of the circle, between the longer points of the legs; and the distance of the shorter points will be the part required.

COMPOUND proportional compasses, are those wherein the center is moveable; so that one pair of these will perform the office of several pairs of the simple sort.

IN the flanks of these compasses are grooves, wherein slides the center, which is made fast by a nut and screw.

ON each side of these grooves, scales are placed; which may be of various sorts, according to the fancy of the buyer: But the scales which the instrument-makers commonly put on these compasses, are only two, *viz.* lines and circles.

BY the scale of lines, a right line may be divided into a number of equal parts, not exceeding the greatest number on the scale; which is generally 12.

EXAM. I. To divide a given right line, (suppose of $7\frac{1}{2}$ inches long,) into a proposed number of equal parts. (as 11.)

OPERATION. Shut the compasses; unscrew the button; move the slider until the line across it, coincides with the 11th division on the scale of lines; screw the button fast; open the compasses, until the given line can be received between the longer points of the legs; then will the distance of the shorter points

points be the 11th part of the given line, as required.

By the scale of circles, a regular polygon may be inscribed in a given circle; provided the number of sides in the polygon, do not exceed the numbers on the scale, which commonly proceed to 24.

EXAM. II. To inscribe in a circle of a known radius, (suppose 6 Inches) a regular polygon of 12 sides?

OPERATION. Shut the compasses; unscrew the button; slide the center until its mark coincides with the 12th division on the scale of circles; screw the button fast; take the given radius between the longer points of the legs; then will the distance of the shorter points, be the side of the polygon required.

THESE scales are applicable to several other uses beside the foregoing ones, in the same manner, as the like lines on the sector are.

FROM these operations it is evident, that the lengths of the longer and shorter legs, (reckoned from the center,) must always be proportional to the distance of their extremities.

THEREFORE, to divide a right line into 2, 3, 4, 5, 6, 7, 8, &c. equal parts; the lengths of each leg, from the center, will be expressed by the following series, the whole length of the instrument being taken for unity.

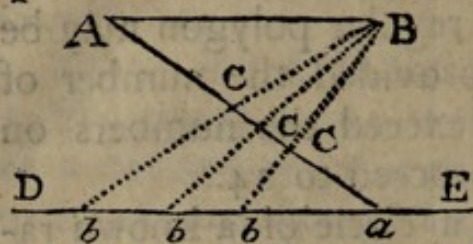
Longer leg $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \&c.$

Shorter leg $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \&c.$

THESE divisions may be very accurately laid on the legs of the compasses by the help of a good sector. (See Prob. 14.)

OR, the divisions of this scale of lines may be found by the following construction.

DRAW the indefinite right line DE; and from any point A, without DE, draw Aa, equal to the shank of the compasses, making any angle at a, with DE. Through A draw the right line AB, parallel to DE, and equal to the given line from whose parts the proportions are taken.



LET Aa contain N parts.

Now that ab may be the *nth* part of AB, or, that AB may be *n* times ab.

LET $ac = \frac{1}{n+1} N$, or $ac = \frac{1}{n+1} N$;

then the point c is the center of the screw pin. And through c, drawing BC, meeting DE in b; then is $ab = \frac{1}{n}$ of AB, or AB = *n* times ab.

$$\text{For } \frac{ab}{AB} = \frac{ac}{AC} = \frac{n}{1}.$$

If the center of the screw-pin be distant from the mark in the slider, the $\frac{1}{m}$ part of N.

Then $ac = \frac{m+s}{s} \times \frac{N}{m}$ (putting $s = n + 1$.)

Ex. If $N = 10000$, $m = 400$, and $n = 1$, or 2, or 3, &c.

Then $ac = 5000$, or 3333, or 2500, &c. when the divisions on the shank respect the center pin.

$$\text{And } ac = \left\{ \begin{array}{l} 5025 \text{ or } 3358 \text{ or } 2525, \text{ \&c.} \\ 4975 \text{ or } 3308 \text{ or } 2475, \text{ \&c.} \end{array} \right\}$$

when the divisions respect a mark in the slider, distant from the center pin, $\frac{1}{25}$ of the length of the Instrument.

THE scale for dividing of the circle, or the divisions for regular polygons may be found thus.

FIND the angles at the center, of as many regular polygons as are to be described on the compasses.

SEEK the sines belonging to the half of each angle, to the radius 1.

To

To each of these fines doubled, add the radius 1.

THEN will the reciprocal of these numbers, be the lengths of the polygonal divisions, on the legs of the compasses, reckoned from the longer point; the length of the instrument being accounted unity.

FOR the longer and shorter legs, (or points) are in the same ratio, as are the radius and chord of the angle at the center.

AND as the sum of the radius and chord, is to the radius; so is the sum of the longer and shorter legs, (or points) to the length of the longer point.

AND hence was the following table composed, which shews the decimal parts on the leg, from the longer point to the center.

N ^o Sides.	Length on the Leg.	N ^o Sides.	Length on the Leg.	N ^o Sides.	Length on the Leg.
3	0,3333	11	0,6396	19	0,7523
4	0,4142	12	0,6589	20	0,7617
5	0,4597	13	0,6763	21	0,7706
6	0,5000	14	0,6921	22	0,7785
7	0,5354	15	0,7063	23	0,7860
8	0,5665	16	0,7193	24	0,7931
9	0,5940	17	0,7313		
10	0,6180	18	0,7423		

THESE divisions may be truly laid off by the help of a good sector; making the whole length of the proportional compasses, a transverse distance to 10 and 10, on the line of lines.

THE complements, to unity, of the numbers in the table, will give the distances of the divisions from the other point of the instrument.

If the mark in the slider, is at some distance from the center, as it commonly is, then this distance, which is always known, must be added to, or subtracted from, the foregoing numbers, according to that side of the center the mark is on; and the sums, or remainders, will give the distances of the divisions from one of the points.

ABOUT *Michaelmas*, 1746, was finished a pair of proportional compasses, with the addition of a very curious and useful contrivance; (see the plate fronting the title page) *viz.* into one of the legs (A) at a small distance from the end of the groove, was screwed a little pillar (*a*) of about $\frac{1}{3}$ of an inch high, and perpendicular to the said-leg; through this pillar, and parallel to the leg, went a screw pin (*bb*); to one end of this screw, was foldered a small beam (*cc*) nearly of the length of the groove in the compasses; the beam was slit down the middle lengthwise, which received a nut (*f*) that slid along the slit (*dd*); this nut could be screwed to the beam fast enough to prevent sliding; one end (*e*) of the screw of the nut (*f*) falls into a hole in the bottom of the screw to the great nut (*g*) of the compasses; the screw pin (*bb*) passed through an adjuster (*b*): To use this instrument, shut the legs close, slacken the screws of the nuts *g* and *f*; move the nuts and slider *k* to the division wanted, as near as can be readily done by the hand; screw fast the nut *f*; then by turning the adjuster *b*, the mark on the slider *k*, may be brought exactly to the division; screw fast the nut *g*; open the compasses; gently lift the end *e*, of the screw of the nut *f*, out of the hole in the bottom of the nut *g*; move the beam round its pillar *a*, and slip the point *e*, into the hole in the pin *n*; slacken the screw of the nut *f*; take the given line between the longer points of the compasses, and screw fast the nut *f*: Then may the shorter points of the compasses be used without any danger of the legs changing their position; this

this being one of the inconveniencies that attended the proportional compasses before this ingenious contrivance ; which was made by Mr. *Thomas Heath*, Mathematical Instrument-maker in the *Strand, London*.

THE proportional compasses had not been long invented before there were several learned and ingenious persons who contrived a great variety of scales to be put thereon ; but these are here omitted, because the credit of the proportional compasses is greatly fallen, since the invention of the sector, the latter being a much more useful instrument than the former, and not so subject to be put out of order ; for if one of the points of these compasses should be blunted or broke, the instrument cannot be used, until the damaged point be replaced by a new one. However, those who are desirous of knowing the construction and use of such scales on the proportional compasses, may be amply satisfied in consulting *Hulsius*, *Horscher*, *Galgemaire*, *Bion*, and others mentioned in the preface to this book.





A P P E N D I X.

CONTAINING

The DESCRIPTION and Use of the GUNNERS CALLIPERS.

✱✱✱ PAIR of Callipers is an instrument used to
✱ A ✱ take the diameters of convex and concave
✱✱✱ bodies.

THE instrument called the Gunners Callipers, consists of two thin rulers or plates, which are moveable quite round a joint, by the plates folding one over the other.

THE length of each ruler or plate is usually between the limits of six inches and a foot, reckoned from the centre of the joint; and from one to two inches broad: But the most convenient useful size is about nine inches long. The figure is best seen in the plate.

ON these rulers are a variety of scales, tables, proportions, &c. such as are esteemed useful to be known by gunners and other persons employed about artillery: But except the taking of the calibre of shot and cannon, and the measuring of the magnitude of *saliant* and *entring angles*, there are none of the articles with which the callipers are usually filled, essential to this instrument; the scales are, or may be, put on the sector; and the tables, precepts, &c. may be put into a pocket book, where they will not need so much contraction: However, for the sake of those who are desirous

desirous of having a single instrument perform many things, the following articles and their disposition on the callipers are here offered : Some of which were proposed many years ago by my much esteemed friend Mr. *Charles Labelye*, engineer to the works of *Westminster-bridge*.

Articles proposed to be put on the Gunners Calipers.

I. THE measures of convex diameters in inches.

II. THE measures of concave diameters in inches.

III. THE weights of iron shot from given diameters.

IV. THE weight of iron shot proper to given gun bores.

V. THE degrees of a semicircle.

VI. THE proportion of Troy and Averdupoise weight.

VII. THE proportion of English and French feet and pounds.

VIII. FACTORS useful in circular and spherical figures.

IX. TABLES of the specific gravity and weights of bodies.

X. TABLES of the quantity of powder necessary for proof and service of brass and iron guns.

XI. RULES for computing the number of shot or shells in a finished pile.

XII. RULES concerning the fall of heavy bodies.

XIII. RULES for the raising of water.

XIV. THE rules for shooting with cannon or mortars.

XV. A LINE of inches.

XVI. LOGARITHMIC scales of numbers, sines, versed sines and tangents.

XVII. A SECTORAL line of equal parts, or the line of lines.

XVIII. A SECTORAL line of plans or superficies.

XIX. A SECTORAL line of solids.

THE Callipers proposed for the reception of the foregoing articles is nine inches long, and each leg two inches broad at the head, and at the points; part of the breadth between the ends is hollowed away in a curve, in order to contain the curvature of the ball, whose diameter is taken between the points; one of ten inches diameter is the largest that can conveniently be taken with a nine inch Calliper; those of six inches cannot well be applied to a shot of more than seven inches diameter.

FOR the ease of reference; it will be convenient to distinguish the four faces of the Callipers by the letters A, B, C, D: Each of the faces A and D consist of a circular head and a leg; the other faces B and C consist only of a leg.

ARTICLE I.

Of the measures of convex diameters.

ON part of the circular head joining to the leg of the face A, are divisions distinguished by the title of *shot diameters*: These are to shew the distance in inches, and tenth parts of an inch, of the points of the Callipers when they are opened.

THE USE.

OPEN the points of the Callipers so, that they may take in the greatest diameter of the ball; then will the bevil edge marked E shew among the foresaid divisions, the diameter of that ball in inches and tenth parts, not exceeding ten inches.

These divisions may be thus laid down by the sector.

OPEN the sector until the radius of the circle, whereon is marked the scale of divisions on the head of the Callipers, taken with the compasses, falls transversely in the scales of lines, on the divisions shewing the distance between the centre of the Callipers and its points: Then the transverse distances of the several divisions

divisions on the scales of lines, being applied like chords to the circle of divisions on the head of the Callipers will give the divisions required.

Thus in the nine inch Callipers, the radius of the head, or circle of divisions being one inch, and the breadth at the points two inches; the distance between the centre and points will be $(\sqrt{82} =) 9,055385$: Then one inch being made a transverse distance on the scales of lines to $9 \frac{5}{1000}$; the transverse distances of 10, 9, 8, 7, 6, &c. being applied to the circle on the head of the Callipers appropriated for the scale, from the mark where the divisions commence, will give the several points, which being cut by the bevil edge E will shew how far the points of the Callipers are distant.

THE workmen generally lay these divisions down by trial.

ARTICLE II.

Of the weights of iron shot.

On the circular bevil part E of the face B, is a scale of divisions denominated by lb *weight of shot*. These are to shew the weights of iron shot when the diameter is taken between the points of the Callipers: For then the number cut by the inner edge of the leg A, shews the weight of that iron shot in pounds averdupoise, when the weight is among the following ones, viz.

lb . $\frac{1}{2}$. 1. $1\frac{1}{2}$. 2. 3. 4. $5\frac{1}{4}$. 6. 8. 9. 12. 16. 18. 24. 26. 32. 36. 42.

OBSERVING that the figures nearest the bevil edge answer to the short lines; and those figures behind them answer to the divisions marked with the longer strokes or lines.

THESE divisions are to be laid down from a table shewing the diameters of iron shots to given weights. Such a table is computed by knowing the weight of one shot of a given diameter: Thus an iron shot of

four inches diameter is found to weigh nine pounds : Then the weights of spheres being to one another as the cubes of their diameters, Say, As 9 lb is to 64, the cube of 4.

So is any other weight, to the cube of its diameter.

THEN the cube root being taken will give the diameter.

Now setting the points of the Callipers to touch one another, make a mark on the bevil edge e opposite to the inner edge of the leg A ; and this mark will be the beginning of this scale of weights : The other divisions will be obtained by opening the points of the Callipers to the distances respecting the weights to be introduced, as shewn by the table, and marking the division opposite to the inner edge of the leg A .

ARTICLE III.

Of the measures of concave diameters.

ON the lower part of the circular head of the face A , and to the right hand of the divisions for the diameters of shot, is another scale of divisions, against which stands the words *Bores of Guns*.

THE USE.

To find the calibre, or the diameter of the bore of a cannon.

SLIP the legs of the Callipers across each other, until the steel points touch the concave surface of the gun in its greatest breadth ; then will the bevil edge F , of the face B , cut a division in the scale shewing the diameter of that bore in inches and tenth parts.

IN the nine inch Callipers these divisions may be extended to 18 inches diameter, but 14 inches is sufficient for both cannon and mortars : And in the six inch Callipers a diameter greater than 10 inches cannot be conveniently introduced.

These

These divisions may be thus laid down by the sector.

SET one inch the radius of the circle on which the divisions are to be put, as a transverse distance to the division $9\frac{5}{100}$ on the scale of lines on the sector : Set the points of the Callipers together, and make a mark on the circular head where it is then cut by the bevil edge F : Then the several transverse distances taken from the sector, and applied on the circumference of the circular head of the Callipers, from the said mark, the several divisions shewing the distance of the points of the Calliper are thereby obtained.

WORKMEN find these divisions by actually setting the points to the distance.

ARTICLE IV.

Of the weights of shots to given gun bores.

WITHIN the scales of shot and bore diameters on the circular part of the face A are divisions marked *Pounders*.

THE USE.

WHEN the bore of a gun is taken between the points of the Callipers, the bevil edge F will cut one of these divisions, or be very near one of them : Then the number standing against it will shew the weight of iron shot proper for that gun ; not exceeding 42 pounds.

THE inner figures $\frac{1}{2}$, $1\frac{1}{2}$, 3, $5\frac{1}{4}$, 8, 12, 18, 26, 36, belong to the longest strokes or lines ; and the figures 1, 2, 4, 6, 9, 16, 24, 32, 42 belong to the short strokes.

THE diameters given by these pounders are larger than those given for the same weights of shot ; because there is an allowance made, called *Windage*, that the shot may roll easily along the chace.

ARTICLE V.

Of the degrees in the semicircular head.

THESE degrees are placed on the upper half of the circular head of the face A, where are three concentric scales of degrees : The outward scale has 180 degrees numbered from the right to left, with 10. 20. 30. 40. &c. to 180 : The middle scale is numbered in the same manner, but the contrary way : And the inmost scale begins in the middle with 0, and is numbered from thence both ways with 10. 20. 30. &c. to 90 degrees.

THE USE.

First to measure an entring, or internal, angle.

APPLY the legs of the Callipers so that its outside edges coincide with the legs of the given angle ; then will the bevil edge F cut the degrees shewing the measure of that angle in the outside scale.

Secondly. To measure a saliant, or external, angle.

SLIP the legs of the Callipers across each other, so as their outside edges may coincide with the legs of the given angle ; then will the bevil edge E cut the degrees shewing the measure of that angle : These degrees are to be counted on the middle scale.

HENCE an angle of any number of degrees may be readily laid down by the Callipers, either on paper, or in the field.

Thus. OPEN the Callipers, the legs being crossed, until the edge E cuts the degrees on the middle scale ; the crossing edges of the instrument will then form the sides of that required angle : The Callipers then laid flat on the paper or ground, lines drawn by the strait sides will express that angle.

Thirdly.

Thirdly. To find the elevation of cannon and mortars, or of any other oblique plane or line.

PASS one end of a fine thread into the notch on the plate B, and to the other end tie a bullet, or other weight: Apply the strait side of the plate A to the side of the body whose inclination is wanted; hold the plate A in this position, and move the plate B until the thread falls upon the line near the centre marked *Perp.* Then will the bevil edge F cut the degrees, counted on the inner scale, shewing the inclination which that body makes with the horizon.

Note. When the edge F cuts o on the inner scale; and the string cuts the *Perp.* mark, then the strait side of the leg A is horizontal: If the head of the Callipers is elevated above the other end, then the edge F must slide downwards towards the strait side of the leg A: But if the head of the Callipers is held lower than the other end, then must the edge F slide the contrary way.

As the outside of a cannon or mortar is not parallel to its chase; therefore a strait stick must be applied to the bottom or top of the bore, touching the chase; and the side of the Callipers be laid on that stick.

A R T I C L E VI.

Of the proportion of Troy and Averdupoise Weights.

ON the face c near the point of the Callipers is a little table shewing the number of pounds that are contained in an equal weight expressed in pounds Troy; and the contrary.

THESE numbers are taken from very accurate experiments made in the year 1744 by the late *Martin Folkes*, Esq; President of the Royal Society, assisted by several other gentlemen of that learned Body.

THE TABLE.

℔ Troy	℔ Averd.	oz. Troy	oz. Averd.
576,00000=700		82	=90
1,00000=0,82274		1,00000=	1,09707
1,21545=1,00000		0,91152=	1,00000

THE USE.

EXAMPLE I. *What weight in pounds Troy is equal to a brass gun weighing 18 C wt.*

Now 18 C wt. is equal to 2016 ℔ ($=18 \times 112$).

THEN 1 : 1,21545 :: 2016 : 2450 ℔ Troy.

OR, 0,82274 : 1 :: 2016 : 2450 ℔ Troy.

OR, 576 : 700 :: 2016 : 2450 ℔ Troy.

EITHER of these methods may be used as the operator pleases.

EXAMPLE II. *What is the worth of a ton of gold ; supposing 1 ℔ Troy makes $44\frac{1}{2}$ guineas.*

Now 1 Ton = 2240 ℔ Averd. ($=20 \times 112$).

AND 1 : 1,21545 :: 2240 : 2722,6 ℔ Troy.

ALSO $44\frac{1}{2}$ Guineas, makes 46,725 £ sterling.

THEN 1 : 2722,6 :: 46,725 : 127213,485 £.

OR, 127213 l. 9s. 8d. $\frac{1}{2}$.

BUT if Troy pounds were given to be converted into Averdupoise pounds, then the numbers in the Troy column must be the first terms of the proportions.

EXAMPLE III. *If a brass gun weighs 2450 ℔ Troy ; What is its weight in Averdupoise ?*

THEN 1 : 0,82274 :: 2450 : 2015,7 ℔ Ave.

OR, 1,21545 : 1,00000 :: 2450 : 2015,7

OR, 700 : 576 :: 2450 : 2016.

ALTHOUGH the Averdupoise pound is heavier than the Troy pound, yet the Troy ounce is heavier than the Averdupoise ounce, nearly in the proportion of 90 to 82.

Ex-

EXAMPLE IV. *In a chest of silver containing 4380 pieces of eight, each piece weighing $\frac{4}{5}$ of an ounce Troy: How many ounces Averdupoise.*

THEN 82 : 90 :: $4380 \times \frac{4}{5}$: 3845,88

Or, 1 : 1,09707 :: $4380 \times \frac{4}{5}$: 3844,13

Or, 0,91152 : 1 :: $4380 \times \frac{4}{5}$: 3844,13

CLOSE to the former table is another, shewing the number of cubic inches in a gallon, both in wine and beer measures; and consequently their proportions: One use is shewn by the following Example.

How long will 33 butts of beer serve a crew of 324 men, allowing to each man 3 wine quarts a day?

Now 33 butts contain 3564 beer gall. (= 108×33)

AND 231 : 282 :: 3564 : $4350\frac{6}{7}$ wine gallons,

AND $4350\frac{6}{7}$ gallons makes $17403\frac{3}{7}$ quarts,

THEN 17403 divided by 324 gives very near 54.

CONSEQUENTLY $\frac{1}{3}$ of 54, or 18 days, is the time that the beer will serve.

If wine gallons were to be converted into beer gallons,

SAY 282 : 231 :: wine gallons : beer gallons.

OR 94 : 77 :: W. G. : B. G.

ARTICLE VII.

Of the proportion of the English and French feet and pounds.

NEAR the point of the face D of the Callipers are two tables shewing the proportion between the pound weights of *London* and *Paris*, and also between the lengths of the foot measure of *England* and *France*. These are according to the accurate standards settled between the Royal Societies of *London* and *Paris* about the year 1743.

THE TABLES.

Eng. lb.	F. lb.	Eng. Ft.	Fr. Ft.
1,08 =	1,00	114 =	107
1,00 =	0,926	1,000 =	0,9386
108 =	100.	1,0654 =	1,0000

THE USE.

EXAMPLE I. *Suppose a crew of 54 English sailors were to attack a French fort, and carry off 6 pieces of brass cannon weighing one with another 980 lb French: How much would each John's share come to, supposing they could sell the cannon at 8 l. a hundred weight English?*

lb F. lb E. lb F.
 Now 100 : 108 :: 980 × 6 : 6350,4, lb Engl.
 lb £. £.
 AND 112 : 8 :: 6350,4 : 453,6 £. sterling.
 M. £. M.
 THEN 54 : 453,6 :: 1 : 8,4 £.
 So that the share of each will be 8 guineas.

EXAMPLE II. *How many English yards are equal to 180 French toises or fathoms?*

Now 1 : 1,0654 :: 180 : 191,672 Eng. Fa.
 THEN 180 French Fathoms are equal to about 383 yards 1 foot.

ARTICLE VIII.

Factors useful in circular and spherical figures.

NEAR the point of the Callipers on the face A is a table containing four rules of the circle and sphere.

THE TABLE.

Diam.	$\times 3,1416$	=circumf.	} of a circle.
Sq. Diam.	$\times 0,7854$	=area	
Sq. Diam.	$\times 3,1416$	=surface	} of a sphere.
Cube Diam.	$\times 0,5236$	=solidity	

THE USE.

EXAMPLE I. *What is the circumference of a circle whose diameter is 12 inches ?*

THEN $(3,1416 \times 12 =)$ 37,6992 is the circumfer.

EXAMPLE II. *What is the area of a circle whose diameter is 12 inches ?*

Now the square of 12 is 144.

THEN $(0,7854 \times 144 =)$ 113,0976 is the area.

EXAMPLE III. *What is the superficies of a sphere whose diameter is 12 inches ?*

Now the square of 12 is 144.

THEN $(3,1416 \times 144 =)$ 452,3904 the superficies of the sphere.

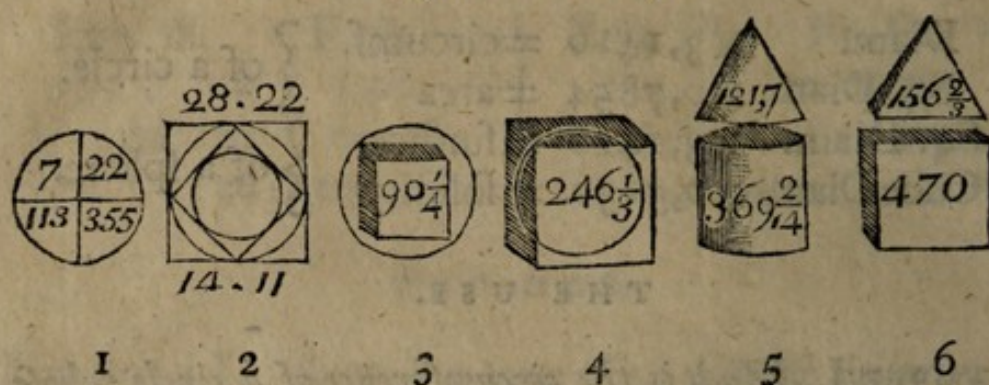
EXAMPLE IV. *Required the solidity of a sphere whose diameter is 12 inches ?*

Now the cube of 12 is 1728.

THEN $(0,5236 \times 1728 =)$ 904,7808 is the solidity.

Upon the circular heads of Callipers are usually placed certain mathematical figures with numbers set to them ; which figures and their numbers may be placed near the points of the Callipers here described, the circular head being appropriated for another use.

The figures are these.



THE numbers in figure 1, are useful for finding the circumference of a circle by knowing its diameter; or to find the diameter by knowing the circumference. Thus

SAY As 7 : 22 :: any given diam: its circum.

AND As 22 : 7 :: any given circum: its diam.

OR As 113 : 355 :: any given diam: its circum.

AND As 355 : 113 :: any given circum: its diam.

FIG. 2. There is a circle inscribed in a square; a square within that circle, and a circle within the inner square: To this figure are set the numbers 28. 22. 14. 11. These numbers signify, that if the area of the outward square is 28, the area of the inscribed circle is 22; the area of the square inscribed in that circle is 14, and the area of its inscribed circle is 11.

THE USE.

EXAMPLE. *What is the area of a circle whose diameter is 12?*

Now the square of 12 is 144.

THEN As 28 : 22 :: 144 : 113,1 the area.

Or As 14 : 11 :: 144 : 113,1.

It may be observed, that the squares are in proportion to one another as 2 to 1; and the two circles are also in the same proportion.

Figure 3. Represents a cube inscribed in a sphere; the number $90\frac{1}{4}$ fixed to it shews, that a cube of iron, inscribed

inscribed in a sphere of 12 inches in diameter, weighs $90\frac{1}{4}$ pounds weight.

Figure 4. Is to express a sphere inscribed in a cube : Now this figure with its number $246\frac{1}{4}$ is to shew the weight in pounds of an iron sphere of 12 inches diameter ; or of a sphere inscribed in a cube whose side is 12 inches.

Figure 5. Represents a cylinder and cone, whose diameters and heights are each one foot : To the cylinder is annexed the number $369\frac{3}{4}$ shewing the weight in pounds of an iron cylinder of 12 inches diameter and 12 inches in height : And the number $121\frac{7}{100}$ joined to the cone shews that an iron cone the diameter of whose base is 12 inches, and the height 12 inches, weighs $121\frac{7}{100}$ pounds.

Figure 6. Shews that an iron cube, whose side is 12 inches, weighs 470 pounds ; and that a square pyramid of iron, whose base is a square foot, and its height 12 inches, weighs $156\frac{2}{3}$ pounds.

THESE numbers which have hitherto been fixed to the four last figures are not strictly true.

FOR by experiment an iron shot of four inches diameter weighs 9 pounds.

AND the weights of spheres being to one another as the cubes of their diameters :

THEREFORE $64 (=4 \times 4 \times 4) : 9 :: 1728 (=12 \times 12 \times 12) : 243$ pounds, for the weight of a sphere of iron which is 12 inches in diameter : Consequently the number 243 should be used instead of $246\frac{1}{4}$ in the 4th figure.

AGAIN. The solidity of a cube inscribed in a sphere of 12 inches in diameter, is 332,55 cubic inches.

AND the weights of bodies of a like matter being in the proportion of their solidities.

THEREFORE, As $904,7808 : 243 :: 332,55 : 89,315$ pounds.

CONSEQUENTLY the number $90\frac{1}{4}$ used at figure 3, should be $89\frac{1}{7}$.

HERE 904,7808 is the solidity of a sphere of 12 inches diameter.

AT figure 5. the weight of the iron cylinder should be 364,5 instead of $369\frac{3}{4}$, and the weight of the cone should be 121,5.

FOR the solidity of a cylinder of 12 inches diameter, and 12 inches high, is 1357,1712 cubic inches.

THEN 904,7808 : 243 : : 1357,1712 : 364,5 pounds.

AND cylinders and cones having equal bases and heights are in proportion as 3 to 1.

THEREFORE the $\frac{1}{3}$ of 364,5, or 121,5 pounds is the weight of the cone.

THE numbers at figure 6 annexed to the cube should be 464 pounds.

AND that fixed to the pyramid should be $154\frac{2}{3}$ pounds.

FOR the cube inches in a foot cube are 1728.

THEN 904,7808 : 243 : : 1728 : 464.

AND a pyramid is $\frac{1}{3}$ of a cube, the bases and height being equal.

THEREFORE the $\frac{1}{3}$ of 464 is $154\frac{2}{3}$ pounds for the weight of the pyramid.

ALTHOUGH it is usually reckoned that a four inch iron shot weighs nine pounds ; and from thence it is deduced that the cubic foot weighs 464 pounds ; yet by the table of specific gravity on the callipers, which is framed from the most accurate experiments, a cubic foot of cast iron weighs almost 446 pounds ; which is 18 pound less than the weight derived from the 4 inch shot, and 24 pound less than that heretofore graved on the callipers ; therefore all the weights found from the said 4 inch shot, should be diminished in the proportion of 464 to 446.

FOR the numbers at figures 3, 4, 5, 6.

As 464 : 446 : : 89,315 : 85,85.

As 464 : 446 : : 243 : 233,5.

As 464 : 446 : : 364,5 : 350,3.

So

So $85\frac{4}{5}$ lb is the weight of an iron cube inscribed in a sphere of 12 inches in diameter.

AND $233\frac{1}{2}$ lb is the weight of an iron sphere of 12 inches diameter.

Also $350\frac{1}{3}$ lb is the weight of an iron cylinder of a foot in diameter and height.

AND $116\frac{2}{3}$ lb is the weight of an iron cone of a foot in diameter and height.

AGAIN 446 lb is the weight of a cubic foot of iron.

AND $148\frac{2}{3}$ lb is the weight of an iron pyramid, having its base a square foot, and its height equal to 12 inches.

ARTICLE IX.

Of the specific gravities and weights of bodies.

ON the leg B of the callipers is a table shewing the weights of a cubic inch or foot of various bodies in pounds averdupoise. To the table here annexed is joined the specific gravities of those bodies, which are omitted on the callipers for want of room.

A Table shewing the weights of bodies and their specific gravities.

Bodies.	Weights.	Spe. Gravity.
Fine Gold. Inch	0,710359	19,640
Standard Gold. Inch	0,706018	19,520
Quicksilver. Inch.	0,497657	13,762
Lead. { Foot	707,0458	11,313
{ Inch	0,409170	
Fine Silver. Inch	0,401150	11,091
Standard Silver. Inch	0,384440	10,629
Copper { Foot	548,0628	8,769
{ Inch	0,317166	
Brass. F.	506,2746	8,104
Steel. F.	490,6241	7,850
Bar Iron. F.	485,2500	7,764
Block Tin. F.	452,3731	7,238
Cast Iron. F.	445,9363	7,135
White Marble. F.	168,8757	2,702
Glass. F.	162,4994	2,600
Stone {	Flint. F.	2,582
	Portland. F.	2,570
	Free. F.	2,352
Brick. F.	125,0000	2,000
Brimstone. F.	112,5000	1,800
Clay. F.	112,0000	1,792
River Sand. F.	110,0000	1,760
Sea Water. F.	64,3732	1,030
Rain { Cubic F.	62,5000	1,000
Water {	Cubic Inch	
	Cylindric F.	
	Cylindric Inch	
Port Wine. F.	61,8000	0,988
Brandy. F.	58,0000	0,928
Olive Oil. F.	57,0624	0,913
Dry Oak. F.	57,1875	0,915
Lime. F.	52,0000	0,832
Elm and Ash. F.	50,0000	0,800
Wheat. F.	50,0000	0,800
Yellow Fir. F.	41,0625	0,657
White Deal. F.	35,5624	0,569
Gun { F.	69,1200	1,106
Powder { In.	0,0400	

IN the foregoing table is contained such bodies as practical engineers and others may have occasion to know their respective weights; there are indeed a great number of other bodies whose specific gravity have been determined by ingenious men: But those only which were apprehended to be the most useful were selected for this subject.

EVERY one will readily conceive how the column of weights may be obtained, namely by procuring masses of a cubic inch or foot of the solids, and carefully weighing them in nice scales to the smallest degree of averdupoise weight: And for the fluids, their weights may be determined by having cubical or cylindrical vessels made to hold a known quantity of cubical inches, and in them to weigh the fluids.

THE specific gravity of a body being the relation which that body has to some other body fixed upon as a standard to compare by; and rain water being found to be alike, or very nearly so, in all places; and therefore chosen by philosophers as the proper standard; consequently by the word specific gravity of a body is meant no more, than that it is so many times heavier or lighter than water, when compared together in equal barks.

THUS fine silver is something more than 11; that is, a mass of fine silver will weigh something above eleven times the weight of an equal mass of water: And, so a common brick weighs twice as much as the rain water that would fill a mould fitted to the brick.

Now the weights of equal masses of several bodies being determined, their specific gravities may be readily found, they being in the same proportion to one another as their weights: And as the comparison is made to rain water, of which, by repeated experiments, it has been found that a cubic foot weighed $62\frac{1}{2}$ pounds averdupoise; therefore *dividing the weight of a cubic foot of any body, by $62\frac{1}{2}$; the quotient will be*

the specific gravity of that body, relative to rain water whose specific gravity is represented by unity.

THE difficulty of procuring masses of metals and other bodies in all parts homogenous, and of having both them and the vessels of capacity constructed to a mathematical exactness, has rendered this method of estimating the specific gravities from the weights of equal bulks, liable to exception : And therefore another method has been contrived to come at these specific gravities, hydrostatically.

IT is a well known thing that any body will weigh less when it is immersed in water than when it is weighed in the open air ; and from a very little reflection, it will be seen that the difference between the weights of any body when weighed in air and in water, will be equal to the weight of so much water as is equal in bulk to the body immersed : But the difference between the weights of a body in air and in water, will shew the weight of a bulk of water equal to the body so weighed : *Therefore to find the specific gravity of any body, find its weight in air and in rain water, and take the difference of those weights ; then the weight in air divided by that difference, will give the specific gravity required.*

IF the solid whose specific gravity is wanted, be lighter than water, so that it cannot sink by its own weight, let it be joined to another so weighty that the compound may sink : But first let the loss of weight which the heavy body alone sustains in water be found as before ; and then let the loss of weight which the compound body sustains be discovered ; from which take the loss of weight of the heavier, and the remainder is the loss of weight sustained by the lighter ; by which dividing the weight in air of the lighter body, and the quotient will shew the specific gravity.

WHEN the specific gravity of fluids are to be compared to each other ; take a solid of any matter and shape, suppose a glass ball, hung by a horse hair, and

immerse this solid in each fluid, and find the loss of weight of the solid in each fluid, the weight of the body in air being first known; then will these losses express the specific gravities of those fluids: For since the loss of weight in each liquor is equal to the weight of as much of the liquor as is equal in bulk to the body weighed; therefore by taking the losses of weight sustained by the same body in the several liquors, the absolute weights are obtained of such portions as are equal in bulk, and consequently the specific gravities of those liquors.

IN this method of finding the specific gravity of solids, it is not necessary that they should be reduced to any regular shape; neither is there wanted a vessel of a known figure and capacity to contain the fluids; and consequently the specific gravities of bodies, whether solids or fluids, may be very easily come at: But from the specific gravities to find the absolute weights of any assigned mass of several bodies, there must be another experiment made, which is to find the loss of weight in water, of a body of a known magnitude; suppose of a cylinder of a homogenous metal, the solidity of that cylinder being most accurately calculated; then will the absolute weight of an equal mass of water be known; and consequently the weight of a cubit foot of water may be accurately obtained, *from whence the absolute weight of a cubic foot of any other body whose specific gravity is known, may be found by multiplying the specific gravity of that body by the weight of a cubic foot of water.*

SOME USES OF THE TABLE.

THE weights of bodies answering to a given solidity are of a twofold use.

FIRST, *To find the weight of a body of a given dimensions, or solidity.*

SECONDLY, *To find the solidity of a body by knowing its weight.*

EXAM. I. *What is the weight of a block of marble 7 feet long, 3 feet broad, and 2 feet thick?*

NOW $7 \times 3 \times 2 = 42$ feet for the solidity.

A CUBIC foot of marble weigh 168,8757 pounds.

THEN $168,8757 \times 42$ gives 7092,7794 pounds.

C qrs. lb

OR, $63 : 1 : 8\frac{3}{4}$ is the weight of that marble.

EXAM. II. *What is the weight of a 13 inch iron bomb shell, the metal being two inches thick on a mean?*

HERE the solidity of two spheres must be found, one of 13 inches diameter, and the other of 9 inches diameter; then their difference being taken will give the solidity of the shell.

NOW the cube of 13 is 2197.

AND the cube of 9 is 729.

ALSO $2197 \times 0,5236$ gives 1150,3492 solidity.

AND $729 \times 0,5236$ gives 381,7044 solidity.

THEIR difference is 768,6448 cubic inches.

AND 768,6448 divided by 1728 gives 0,4448 parts of a cubic foot.

NOW a cubic foot of cast iron weighs 445,9363 pounds.

THEN $445,9363 \times 0,4448$ gives 198,363 pounds for the weight of the shell.

EXAM. III. *How many pigs each of 12 inches long, 6 wide and 4 thick, may be cast out of 10 ton of melted lead?*

NOW 10 ton $= 10 \times 20 = 200$ C. wt.

AND $112 \times 200 = 22400$ pounds in 10 ton.

By the table, 707,0458 pound makes a cubic foot of lead.

AND 22400 divided by 707,0458, gives 31,681 cubic feet, which the 10 ton will make.

Now

Now the solidity of each pig is $\frac{1}{6}$ of a foot.

THEREFORE 31,681 feet solid will make 190 pigs.

FROM several experiments it appears that middling sized men, or those between 5 feet 6 inches and 5 feet 9 inches in height, weigh about 150 pounds, and are in bulk equal to about $2\frac{3}{4}$ solid feet; and the small sized men, or those between 5 feet 3 inches, and 5 feet 6 inches in height, weigh about 135 pounds, and are in bulk equal to about $2\frac{1}{2}$ solid feet: And from those experiments it also appears, that most men are specifically lighter than common water, and much more so than sea water. Consequently could persons who fall into water, have presence of mind enough to avoid the fright usual on such occasions, many might be preserved from drowning: And a very small piece of wood, such as an oar, would buoy a man above water while he had spirits to keep his hold.

A GENTLEMAN who had been on board of a Maltese ship of war, observed hanging to the taffarel, a block of wood almost like a buoy, and so ballanced that one end swam upright, carrying a little flagstaff with a small vane; the person who was on duty on the poop had orders to cut the rope by which the buoy hung, upon any cry of a person's falling over board; and as the block would be in the ship's wake by the time the person floated therein, he was sure of having something at hand to sustain him, till the boat could come to his assistance; and should that take so long time to do, as that the distance from the ship to the man rendered him invisible, yet the boat would have a mark to row towards, shewn them by the vane.

EXAM. IV. *How many spars of white fir, each of 20 feet long and a foot square, are to be lashed together, till the raft is sufficient to float, in common water, 100 barrels of gunpowder conducted by four middling sized men, so as to keep the barrels three inches clear of the water?*

A BARREL of gunpowder, barrel and all, weighs about 120 lb.

So 100 barrels will weigh 12000 lb.

AND 4 men, at 150 lb each, weigh 600 lb.

So that the raft must sustain a weight of 12600 lb.

Now the deal will of it self sink in the water, until the weight of the water displaced is equal to the weight of the wood.

IN each spar there is 20 feet of timber.

A CUBIC foot of white deal weighs 35,5624 pounds.

So $35,5624 \times 20 = 711,248$ lb. the weight of one spar.

AND is also equal to the weight of the water displaced.

A CUBIC foot of common water weighs 62,5 lb.

THEN $62,5 : 1 :: 711,248 : 11,38$ the number of cubic feet which each spar will have immerfed by its own weight.

As the barrels are to be 3 inches clear of the water, therefore the spar must be sunk 9 inches; and consequently 15 feet solid of each spar must be immerfed:

THEN $15 - 11,38 = 3,62$ the additional cubic feet of water to be displaced by each spar, by its incumbent weight.

AND $1 : 3,62 :: 62,5 : 226,25$ lb. the weight which each spar is to sustain.

THEN $226,25 : 12600 :: 1 : 55,6$, &c.

CONSEQUENTLY 56 such spars lashed together will make a float sufficient for to sustain the given weight in the manner proposed.

ARTICLE X.

Of the quantity of powder used in firing of cannon.

ON the circular head of the callipers, on the face D is a table contained between five concentric segments of circular rings; the inner one markt GUNS, shews the nature of the gun, or the weight of ball it carries:

carries: The two next rings contain the quantity of powder used for proof and service to brass guns; and the two outermost rings shew the quantity for proof and service, used in iron cannon.

THE numbers in this table express the English usage, which for the most part, allows the weight of the shot for proof, half its weight in service, and one fourth of its weight of shot for salutes.

THE French allowance of powder, for the charge of the piece for service, used to be two thirds of the weight of the shot; twice as much for proof, and one fourth of the weight of shot for salutes.

THE TABLE.

Nature of guns	Brass		Iron		Salutes	Scaling
	Proof	Service	Proof	Service		
Pounders	lb. oz.	lb. oz.	lb. oz.	lb. oz.	lb. oz.	lb. oz.
1	1.0	0.8	1.0	0.8	0.8	0.1 $\frac{1}{2}$
1 $\frac{1}{2}$	1.8	0.12	1.8	0.12	0.12	0.2
2	2.0	1.0	2.0	1.8	1.0	0.3
3	3.0	1.8	2.0	1.8	1.8	0.4
4	4.0	2.0	4.0	2.0	2.0	0.6
5 $\frac{1}{4}$	5.4	2.10	5.4	2.10	2.10	0.8
6	6.0	3.0	6.0	3.0	3.0	0.8
8	8.0	4.0	8.0	4.0	3.12	0.10
9	9.0	4.8	9.0	4.8	4.0	0.12
12	12.0	6.0	12.0	6.0	4.12	1.0
18	18.0	9.0	15.0	9.0	6.0	1.8
24	21.0	12.0	18.0	11.0	7.0	2.0
26	22.0	13.0	19.0	12.0	7.12	2.4
32	26.12	16.0	21.8	14.0	9.4	2.12
36	28.0	18.0	22.0	15.0	10.0	3.0
42	31.8	21.0	25.0	17.0	11.4	3.4

GUNS carrying shot of the weight 1 lb. $1\frac{1}{2}$ lb. 2 lb. 4 lb. $5\frac{1}{4}$ lb. 8 lb. 26 lb. 36 lb. are now out of use in the British navy.

THE use of this table is obvious: For seek the name of the gun in the inner ring, and the weights of powder for proof and service will be found between the same two strait lines, like radii; and in one of the other rings, according as it is titled at the end.

THUS to a brass 9 pounder, there is allowed 9 lb. of powder for to prove, or try the goodness of the gun when it is first cast; and 4 lb. 8 oz. of powder for each charge in common service: But an iron 9 pounder has 9 lb. for proof, and 6 lb. for service.

WHEN cannon are proved they are usually loaded with two shot.

ON ship board, after there are five or six rounds fired on warm service, the allowance of powder is to be proportionally lessened each time the gun is loaded, until the charge is reduced to one third of the weight of the shot: And the guns as they grow warm in firing, are not to be wetted lest the gun be in danger of splitting by checking the metal with cold water.

THE ingenious Mr. *Robins*, from some hints he gathered from a manuscript lent him by the Right Honourable Lord *Anson*, advises to lessen considerably the common charges allowed to cannon in service: For from those papers it appeared that in service, where 24 pounders have been used to batter in breach, the charge was only 8 pounds of powder: Indeed the velocity of the ball could not be quite so great with 8 pounds of powder as with 12, and consequently the shot would not be drove so far into the rampart, and the breach not made altogether so soon; notwithstanding which, the advantages attending the smaller charges, greatly overbalanced the difference of a few hours in making a sufficient breach.

IN sea service it would perhaps be found of greater use to begin with one third of the weight of shot in

powder, and to diminish that to one fourth or one fifth as the gun waxed warm; for by some experiments it has appeared, that such small charges of powder has produced greater ravage in timber, than has been found with the usual charges: From whence it may be reasonably concluded, that if a shot has just force enough to go through one side of a ship, there will be a greater quantity of splinters rent out of the plank, and consequently do more mischief, than if the shot went with a velocity sufficient to drive it through both sides of the ship.

A R T I C L E X I.

Of the number of shot or shells in a finished pile.

IRON shot and shells are usually piled up by horizontal courses into a pyramidal form, the base being either an equilateral triangle, or a square; or a rectangle; in the triangle and square, the pile finishes in a single ball; but in the rectangle, the finishing is a single row of balls.

IN the triangular and square piles, the number of horizontal rows, or the number counted on one of the angles from the bottom to the top, is always equal to the number counted on one side, in the bottom row.

IN triangular piles, each horizontal course is a triangular number, produced by taking the successive sums of the numbers 1 and 2; 1, 2 and 3; 1, 2, 3 and 4; 1, 2, 3, 4 and 5, &c. Thus.

Numbers in order 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 . 10 . 11, &c.
Triangular numb. 1 . 3 . 6 . 10 . 15 . 21 . 28 . 36 . 45 . 55 . 66, &c.

AND the number of shot in a triangular pile is the sum of all the triangular numbers taken as far, or to as many terms, as the number in one side of the bottom course.

A rule to find the number of shot in a triangular pile.

COUNT the number in the bottom row, and multiply
that

that number more two by that number more one :
Then the product multiplied by one sixth of the said
number, the product will be the sum of all the shot
in the pile.

EXAM. I. *How many shot are in a finished triangular
pile, in one side of whose bottom course are 20 shot ?*

Now the number more two is 22 ; and the number
more one is 21.

AND 22×21 gives 462.

THEN $462 \times \frac{20}{6} = 1540$, the number of shot in that
pile.

EXAM. II. *Required the number of shot in a finished
pile ; there being in one side of the triangular base 40
shot ?*

HERE the number more two is 42 ; and the number
more one is 41.

AND 42×41 gives 1722.

THEN $1722 \times \frac{40}{6} = 11480$ shot in that pile.

IN square piles, each horizontal course is a square
number, produced by taking the square of the number
in its side.

Number in the side 1. 2. 3. 4. 5. 6. 7. 8. 9. 10, &c.
Squares, or horiz. courses 1. 4. 9. 16. 25. 36. 49. 64. 81. 100, &c.

AND the number of shot in a square pile is the sum
of all the squares, taken from one, as far as the num-
ber in the sides of the bottom course.

A rule to find the number of shot in a square pile.

COUNT the number in one side of the bottom course ;
to that number add one, and to its double add one ;
multiply the two sums together ; then their product
being multiplied by one sixth of the said number, the
product will give the number of shot contained in that
pile.

EXAM.

EXAM. III. *How many shot are in a square finished pile, one side of its base containing 20 shot?*

HERE the number is 20.

THE number more one is 21; and its double, more one is 41.

THE product of these numbers is 861 ($=21 \times 41$)

THEN $861 \times \frac{2}{3} = 2870$, the number of shot in that pile.

EXAM. IV. *Required the number of shot in a square finished pile, one side of the lower course, or tier, having 40 shot in it?*

HERE the number counted is 40.

THAT number more one is 41; its double, more one is 81.

AND $81 \times 41 = 3321$ the product.

THEN $3321 \times \frac{4}{5} = 22140$ the number in that pile.

FROM these examples it may be observed, that where room is wanted, 'tis most convenient to have the shot stowed in triangular piles: For on the equilateral triangle, which is less than half the area of a square on one of its sides, there can be piled a greater number than half of what can be raised on the square: Indeed the height of a square pile is somewhat less than a triangular one, as a shot will sink lower in the space between 4 others, than in the space between 3 others, all the shot being of equal diameter; they being so reckoned in every pile.

IN rectangular piles, each horizontal course is a rectangle, the upper one being one row of balls: Now every such oblong pile may be considered as consisting of two parts, one a square pyramid, and the other a triangular prism.

To find the number of shot in a rectangular pile.

1st. TAKE the difference between the number in length and breadth in the bottom course.

2^d.

2d. MULTIPLY the number in breadth, more one, by half the breadth; the product multiplied by the said difference, will give the number in the prismatic pile.

3d. UPON the square of the breadth, find (by the last rule) the number in a pyramidal pile.

4th. THEN the sum of these two piles will shew the number in the rectangular pile.

N. B. The number of horizontal courses, or rows, is equal to the number in breadth at bottom: And the number less one, in the top row, is the difference between the number in length and breadth at bottom.

EXAM. V. *How many shot are in a finished pile of 20 courses, the number in the top row being 40?*

HERE 39 is the difference between the length and breadth.

AND 20 is the breadth.

NOW $20 + 1 = 21$; and $2 \times 20 + 1 = 41$.

THEN $21 \times 41 \times \frac{20}{6} = 2870$, are the shot in the pyramidal pile.

Again. THE breadth more one is 21; and 10 is the half breadth.

AND $21 \times 10 = 210$.

THEN $210 \times 39 = 8190$, are the shot in the prismatic pile.

CONSEQUENTLY the sum of 2870 and 8190, or 11060 shot will be the number contained in that rectangular pile.

IF any of these piles are broken, by having the upper part taken off, and the remaining number of shot are required; it may be obtained by computing what the whole finished pile would contain; and also what the pile wanting, or taken away contained; for then their difference will shew the number remaining.

THE foregoing rules are thus expressed on the Calipers.

NUMBER of shot or shells in a pile.

LET

LET $n = N^\circ$ in an angular row
 $m = N^\circ$ less one in the top row } of a Pile.

THEN $\overline{n+2} \times \overline{n+1} \times \frac{n}{6} = N^\circ$ in a Δ
 AND $\overline{n+1} \times \overline{2n+1} \times \frac{n}{6} = N^\circ$ in a \square } Pile.
 ALSO $\overline{2n+1+3m} \times \overline{n+1} \times \frac{n}{6} = N^\circ$ in a \square

IN Examples I & III. The letter n stands for 20.

AND Examples II & IV. The letter n stands for 40.

IN Example V. The letter n stands for 20.

AND the letter m stands for 39.

THEN $2n+1 = 2 \times 20 + 1 = 41.$

AND $3m = 117.$

So $2n+1+3m = 158.$

ALSO $n+1 = 21.$

AND $\frac{n}{6} = \frac{20}{6}$

THEN $\overline{2n+1+3m} \times \overline{n+1} \times \frac{n}{6} = 158 \times 21 \times \frac{20}{6} = 11060.$

ARTICLE XII.

Concerning the fall of heavy bodies.

WHEN heavy bodies are suffered to fall, it is well known they fall in lines perpendicular to the surface of the earth.

THE force with which any body in motion strikes an obstacle, depends on the weight of that body, and on the velocity or swiftness with which it moves.

THUS a man by throwing, with the same strength, a pound of iron and a pound of cork, will hit a much harder stroke with the iron than with the cork.

ALSO a man and a boy each throwing a pound of iron against the same object, the stroke given by the man will be stronger than that given by the boy, on account of the man's weight flying the swiftest.

M

THE

THE same heavy body by falling from different heights, will strike blows of different strength, that being the strongest where the height is greatest. Consequently heavy bodies by falling acquire velocities greater and greater according to the length of their fall.

THE three following propositions in falling bodies have been proved many ways.

1st. *That the velocities acquired are directly proportional to the times.*

2^d. *That the spaces fallen through are as the squares of the times, or as the squares of the velocities.*

3^d. *That a body moving uniformly with the velocity obtained by falling through any height, will fall twice as far in the same time it was passing through that height.*

EXPERIMENTS shew that heavy bodies fall about 16 feet in one second of time: Consequently at the end of the first second of time, a falling body has acquired a velocity that would carry it down 32 feet in the next second of time.

THEN from the foregoing three propositions may be derived the following rules.

1st. THAT the square root of the feet in the space fallen through, will ever be equal to one eighth of the velocity acquired at the end of the fall.

2^d. THAT the square root of the feet in the space fallen through, will ever be equal to four times the number of seconds of time the body has been falling.

3^d. AND that four times the number of seconds of time in which the body has been falling, is equal to one eighth of the velocity in feet per second, acquired at the end of the fall.

FROM these three rules most of the questions relative to the fall of bodies may be readily solved.

As these rules cannot, for want of room, be put in words at length on the callipers, they are, on the face A of one of the legs, expressed in an algebraic manner. Thus,

FALL OF BODIES.

LET s = space run in feet.

t = time in seconds.

v = velocity in feet per second.

THEN $\sqrt{s} = 4t = \frac{1}{8}v$.

BODIES fall 16 feet in 1st sec.

Note. The character $\sqrt{}$, signifies the square root of the letter joined to it.

SOME USES.

EXAM. I. *How many feet will a bullet fall in 5 seconds of time?*

HERE the time $t = 5$;

THEN $4t$, makes $4 \times 5 = 20$.

NOW $\sqrt{s} = (4t =) 20$.

AND $s = (20 \times 20 =) 400$.

EXAM. II. *From what height must a bullet fall to acquire a velocity of 160 feet per second?*

THE rule is $\sqrt{s} = \frac{1}{8}v$.

HERE v is 160 feet.

AND $\frac{1}{8}v = (\frac{160}{8} =) 20$.

THEN $s = (20 \times 20 =) 400$ feet.

EXAM. III. *How long must a bullet be in falling to acquire a velocity of 160 feet per second.*

THE rule is $4t = \frac{1}{8}v$.

HERE $v = 160$ feet.

AND $\frac{1}{8}v = (\frac{160}{8} =) 20$.

So $4t = 20$.

THEN $t = (\frac{20}{4} =) 5$ seconds of time.

EXAM. IV. *How many seconds will it require for a heavy body to fall through a space equal to 3375 yards?*

THE rule is $4T = \sqrt{s}$.

THEREFORE $T = \frac{1}{4}\sqrt{s}$.

HERE $s = 3375$ yards, or 10125 feet.

AND the square root of 10125 is 100,6.

THEN 100,6 divided by 4 gives 25,15.

So that it will require 25". 9''' of time for the body to fall through 3375 yards.

A R T I C L E XIII.

Rules for the raising of water.

EXPERIMENTS have shewn, that taking horses and men of a moderate strength, one horse will do as much work in raising of water, and such like labour, as five men can.

It has been also found, that one man in a minute, can raise a hoghead of water 12 feet high upon a mean : For a stout man, well plied with strong liquor, will raise a hoghead of water 15 feet high in a minute : Now as the quantity of liquor equal to a hoghead was raised to these heights only by way of experiment for a few minutes, such numbers ought not to be esteemed as the common labour of a man who is to work 4 or 5 hours on a stretch : But it may be reckoned, that of common labouring men, taken one with another, one of them will raise a hoghead of water to 8 feet in height in one minute, and work at that rate for some hours.

It is quite indifferent in what manner the man is supposed to apply his force ; whether by carrying the water in manageable parcels up a stair-case, or raising it by means of some machine : For the advantages gained by using of engines arises chiefly from the ease with which the power can be applied.

ON the face A of the callipers, are the rules, thus denoted.

To raise water.

THE power = P men.

OR to $\frac{1}{3}$ P horses

CAN raise to 8P feet high = F.

THE quantity H, hhd's. in T min.

OR G, gallons in 60 T seconds.

OR, $H \times F = P \times 8 \times T$ minutes.

N. B. CUBIC feet $\times 6,1277$ gives gall.

HERE hogsheds are reckoned at 60 gallons, this estimate being nice enough for any computations on water engines.

SOME USES.

EXAMP. I. *How many hogsheds can six horses raise, by an engine, to 25 feet high in 3 hours?*

Now 6 horses, at 5 men to a horse, is equal to 30 men.

AND the time 3 hours is equal to 180 minutes.

THE height to be raised is 25 feet.

THE general rule is $H \times F = P \times 8 \times T$.

HERE $F=25$; $P=30$; $T=180$.

AND H is required.

$$\text{THEN } H = \frac{P \times 8 \times T}{F}$$

$$\text{OR } H = \left(\frac{30 \times 8 \times 180}{25} \right) = 1728 \text{ hogsheds.}$$

Hence this rule. MULTIPLY eight times the power by the time, the product divided by the height, gives the hogsheds.

EXAMP. II. *It is proposed to throw out of a pond, by an engine, 432 tuns of water in 3 hours by six horses; to what height can the water be raised?*

As 4 Hhds make one tun ; so 432 tuns make 1728 Hhds.

AND 3 hours, or 180 minutes is the time.

Also the power of six horses, is equal to that of 30 men.

THE general rule is $H \times F = P \times 8 \times T$.

HERE $H=1728$; $P=30$; $T=180$.

AND F is required.

$$\text{THEN } F = \frac{P \times 8 \times T}{H}$$

$$\text{OR } F = \left(\frac{30 \times 8 \times 180}{1728} = \right) 25 \text{ feet high.}$$

Hence this rule. MULTIPLY eight times the power by the time ; the product divided by the hogsheds, gives the height in feet.

EXAMP. III. *How long will it require six horses to raise with an engine 1728 hogsheds of water to the height of 25 feet ?*

Now the power 6 horses, is equal to that of 30 men.

THE hogsheds to be raised are 1728.

THE height raised to is 25 feet.

THE general rule is $H \times F = P \times 8 \times T$.

HERE $H=1728$; $F=25$; $P=30$.

AND T is required.

$$\text{THEN } T = \frac{H \times F}{P \times 8}$$

$$\text{OR } T = \left(\frac{1728 \times 25}{30 \times 8} = \right) 180 \text{ minutes, or 3 hours.}$$

Hence this rule. MULTIPLY the hogsheds, by the height in feet ; the product divided by 8 times, the power will give the time in minutes.

EXAMP. IV. *How many horses will it require to work an engine, to raise 1728 hogsheads to the height of 25 feet, in 3 hours?*

Now the hogsheads to be raised are 1728.

THE height to be raised is 25 feet.

THE time to be done in is 3 hours, or 180 minutes.

THE general rule is $H \times F = P \times 8 \times T$.

HERE $H = 1728$; $F = 25$; $T = 180$.

AND P is required.

$$\text{THEN } P = \frac{H \times F}{8 \times T}$$

$$\text{OR } P = \left(\frac{1728 \times 25}{8 \times 180} = \right) 30 \text{ men, or 6 horses.}$$

Hence this rule. MULTIPLY the number of hog-sheads, by the height in feet; the product divided by 8 times the number of minutes, gives the number of men.

A R T I C L E XIV.

Of the shooting in cannon and mortars.

IT has been proved by many writers, that the flight of shot, or the track they describe in the air, is a curve line called a parabola: But then they suppose that the resistance made by the air is so inconsiderable as scarcely to affect the motion of heavy bodies.

UPON this supposition then, which is very far from being true; there have been collected the following observations and rules.

I. ALL bodies projected by any force; are urged with two motions, viz. one in the direction of the power exerted by the engine, and the other in a perpendicular direction to the earth, by the force of gravity; and the track or path described by the body with these two forces is a curve called the parabola.

- II. THE axis of the curve will be at right angles to the horizon ; and the part in which the body descends will be alike to that in which it ascended.
- III. If the point to which the body arrives in its descent, be on the same level with the point from which it was projected, those points are equally distant from the vertex, or highest point of the curve.
- IV. IF a body be projected oblique to the horizon, it will fall there again in the same obliquity, and with the same velocity it was projected withal.
- V. THE horizontal ranges of equal bodies, when projected with the same velocity, at different elevations, will be in proportion to one another ; as the right sines of twice the angles of elevation.
- VI. AMONG equal bodies, projected with equal velocities, the heights to which they will rise in the air, are in the same proportion to one another as the versed sines of twice the angles of elevation.
- VII. WHEN equal bodies are projected with equal velocities, the times of their continuance in the air will be in proportion to one another as the right sines of the angles of elevation.
- VIII. IN the same piece, different charges of equally good gunpowder will produce velocities, nearly in the same proportion as the square roots of the weights of the charges.
- IX. IF equal bodies be projected at the same elevation, but with different velocities, the horizontal ranges will be in proportion to one another, as the squares of the velocities given to the shot, or as the weights of the charges of powder nearly.
- X. THE greatest horizontal range is double to the height from which the body should fall to acquire that force or velocity which would project it to that horizontal range.
- XI. THE greatest horizontal range, or distance to which a body can be thrown, will be obtained when

when it is projected at an angle of 45 degrees of elevation.

XII. THE greatest height to which a projected body can rise, at an elevation of 45 degrees, is equal to one fourth part of its horizontal range.

XIII. To hit an object that lies above or below the horizon of the piece, the best elevation, is equal to the complement of half the angular distance between the object and the zenith.

XIV. AT elevations equally distant from 45 degrees, both above and below it, the horizontal ranges will be equal.

XV. THE time which a heavy body, projected at an elevation of 45 degrees, will continue in the air, before it arrives at the horizon, will be equal to the time that body would take to descend, by the force of gravity, through a space equal to the horizontal range.

It has been found that a 24 pounder at an elevation of 45 degrees, and charged with 16 pound of powder, has ranged its shot upon the horizontal plane about 6750 yards.

THEREFORE 3375 is the impetus, or perpendicular space which a 24 pounder must fall through to acquire such a velocity, as, at an elevation of 45 degrees, would project or throw that shot on the horizon to the distance of 6750 yards.

Now a heavy body falling by the force of gravity through a space equal to 3375 yards or 10125 feet, will, at the end of the fall, acquire a velocity of 804,8 or about 805 feet per second (as shewn at Art. XII.)

AND to fall through a space of 805 feet, it would require 25". 9''' of time.

THE chief of the above principles are shortly expressed on the face B of the callipers in the following manner.

RULES FOR SHOOTING.

Hor. ranges, as right }
 Heights, are as versed } fines of twice } angles of
 Time in air, as right } lines of----- } elevation.
 Impetus = $\frac{1}{2}$ }
 Height = $\frac{1}{4}$ } Hor. range, at 45 deg. of elevation.
 IN ascents or descents, for the best elevation.

TAKE the complement of $\frac{1}{2}$ the angular distance from object to zenith.

To apply these rules to the practice of shooting, it is to be understood that the gunner should make an experiment with every gun he has the care of at some elevation, suppose at 45 degrees, and with the usual charge of powder, and then knowing how far the piece has ranged the shot on the horizontal plane; he may apply the result of those experiments to other elevations and quantities of powder.

EXAMP. I. *Suppose the greatest horizontal range to be 6750 yards: How far will the same piece, and with an equal charge of powder, range a shot at an elevation of 25 degrees?*

WITH equal charges the horizontal ranges are as the right lines of twice the angles of elevation.

THEN, As radius, or the sine of twice 45°

Is to the sine of 50°, or the sine of twice 25°;

So is the greatest horizontal range 6750 yds

To the horizontal range required. 5170 yds.

THAT is, The extent on the line of sines from 90° to 50°.

Will on the line of numbers reach from 6750 to 5170.

EXAMP. II. *The greatest horizontal range of a 24 pounder being 6750 yards: To what height will that shot rise at an elevation of 25 degrees?*

AT an elevation of 45° , the shot will rise $1687\frac{1}{2}$ yards, $=\frac{1}{4}$ of 6750.

AND the heights are as the versed sines of twice the angles of elevation.

THEN, As the versed sine of 90 degrees, or of twice 45° .

Is to the versed sine of 50 degrees, or of twice 25° .

So is the height of an elevation of 45° , viz. $1687\frac{1}{2}$.

To the height at an elevation of 25° . 602,8 yards.

THE logarithm versed sines on the callipers are the supplements of the real versed sines; therefore in the using of this line the supplements of the double angles are to be used.

THEN the extent from the versed sine of 90° to the versed sine of 130° (the supplement of 50°) will on the line of numbers reach from $1687\frac{1}{2}$ to 603.

Or thus. TAKE $1292\frac{1}{2} = \frac{1}{4}$ of 5170, the horizontal range on an elevation of 25° .

THEN. The extent on the log, tangents from radius to 25° , will on the line of numbers reach from $1292\frac{1}{2}$ to about 603 yards.

EXAMP. III. *At an elevation of 25 degrees, how many seconds will a 24 pounder continue in the air before it arrives at the horizon?*

AT 45° elevation the shot takes $35\frac{1}{2}$ seconds in the air *.

And the times in air are as the right sines of the elevations.

THEN As the sine of the elevation 45 degrees

Is to the sine of the elevation 25 degrees

So is the time in air at 45° , viz. $35\frac{1}{2}$ seconds

To the time in air at 25° , viz. 21. seconds.

ON the line of log. sines take the extent from 45 degrees to 25 degrees; then will this extent, applied to the scale of log. numbers, reach from $35\frac{1}{2}$ to 21 seconds.

* This time of $35\frac{1}{2}$ seconds is derived from Rule XV. Then working by the rules belonging to article XII. it will be found that a heavy body will require $35\frac{1}{2}$ seconds to fall through the space of 6750 yards.

AND

AND hence may be estimated the lengths of fuses proper for shells to be fired at given elevations and ranges.

EXAMP. IV. *Required the elevation necessary to strike an object on the horizon at 5170 yards distance, the greatest random of that piece being 6750 yards ?*

SAY. As the greatest random, 6750 yards
 To a proposed random, 5170 yards,
 So is radius, or twice the sine of 45 degrees,
 To double the elevation required, viz. 50 deg.

THE half of which, or 25 degrees, is the elevation necessary to be given to the piece.

THIS elevation is called the lower one.

AND the upper elevation, is at 65 degrees.

FOR 25 degrees and 65 degrees are equally distant from 45 degrees.

EXAMP. V. *At an elevation of 45 degrees, 16 lb. of powder will throw a 24 pounder 6750 yards: How much powder will throw the same shot 5170 yards at the same elevation ?*

By rule IX. THE charges of powder are nearly as the horizontal ranges.

THEN As the horizontal range 6750
 To the horizontal range 5170,
 So is the given charge 16 lb.
 To the required charge 12,26 lb.

THIS proportion may be accurately enough worked by the line of numbers.

FOR the extent from 6750 to 5170, will reach from 16 to $12\frac{1}{4}$.

EXAMP. VI. *At an elevation of 25 degrees, a 24 pounder was ranged on the horizon 5170 yards: Required the impetus that would have given an equal velocity to that shot ?*

WITH an equal charge of powder used at 45 degrees
 of

of elevation, as was used at 25 degrees, the shot would have the greatest horizontal range.

AND with equal charges in the same piece, the impetus is the same at any elevation.

CONSEQUENTLY, to solve this question nothing more is required than to find the greatest horizontal range, which is double to the impetus.

THEN from rule V, by inversion

As the sine of 50 deg. twice the given elevation,

Is to radius, or the sine of twice 45°,

So is the given horizontal range 5170

To the greatest horizontal range 6750,

THE half, or 3375 is the impetus required.

THAT is, the extent on the line of sines from 50° to 90°

WILL on the line of numb. reach from 5170 to 6750.

EXAMP. VII. *Suppose the horizontal range of a piece to be 6750 yards: Required the angle of elevation proper to strike an object 12° above the level of the piece, the horizontal distance of that object being 4680 yards?*

SAY, As the greatest horizontal range 6750

Is to the given horizontal distance 4680

So is the cosine of the object's elevation 78° 00'.

To another sine — — — — 42° 42'.

THUS, the extent on the line of numbers from 6750 to 4680

WILL on the line of log. sines reach from 78° to about 42 $\frac{3}{4}$.

Now on the natural sines, take the extent of 42 $\frac{3}{4}$ deg.

THEN this extent applied from the natural sine of the elevation 12°

WILL give the natural sine of about 62 $\frac{1}{2}$ degrees, whose cosine is about 27 $\frac{1}{2}$.

OR rather 27°. 37'. its half is 13°. 48'.

THE sum of 90° and the given elevation 12° is 102; the half is 51°.

THEN the sum of these halves (51° + 13°. 48' =) 64° 48', is the greater elevation.

AND

AND the difference of these halves ($51^{\circ}-13^{\circ}.48''=$) $37^{\circ}.12'$. is the lesser elevation.

So that the piece pointed at either of these elevations, with the charge of powder that gave the horizontal range, the object will be struck.

BUT in all shooting on ascents or descents, it is best to take the angle between the object and zenith, and get the complement of the half of that angle; then the piece being elevated to that complement, find by trials what charge will reach the object: For on this elevation, a less charge of powder will do the business than on any other elevation.

So in the foregoing example the distance of the object from the zenith is 78° ,

THE half of 78 is 39, and the complement of 39 is 51° , for the best elevation.

ARTICLE XV.

Of the line of inches.

THIS line, the use of which is well known, is placed on the edge of the callipers, or on the strait borders of the faces c, d.

ARTICLE XVI.

Of the logarithmic scales of numbers, sines, versed sines and tangents.

THESE scales are placed along the faces c, d of the callipers, near the strait edges, and are marked and numbered as is shewn in section X; some of the uses of these scales are also shewn in the XV and following sections.

A R-

ARTICLE XVII.

Of the line of lines.

THE line of lines is placed on the callipers on the faces c, d, in an angular position, tending towards the centre of the instrument; its construction and uses are the same as described in treating of the sector; the reader will find sufficient instructions in the sections XI and XII.

ARTICLE XVIII.

Of the lines of plans or superficies.

THESE lines lie on the faces c, d, of the callipers, and like the line of lines tend towards the centre of the instrument: They are marked near the ends of the callipers with the word *Plan*, and have the figures 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 1, running towards the centre: Each of these primary divisions is subdivided into ten parts; and each of the subdivisions is also divided into two, or more parts, according to the length of the callipers.

THESE divisions reckoned from the centre along either leg, are as the square roots of all the whole numbers under 100; and also, of the half numbers: That is, the distance from the centre to the first 1, is as the square root of 1: From the centre to the next division is as the square root of $1\frac{1}{2}$: To the next as 2, the next as $2\frac{1}{2}$, the next as 3, &c.

AND the distance from the centre to the second 1, is as the square root of 10; from the centre to the next division is as the square root of $10\frac{1}{2}$; to the next as 11; to the next as $11\frac{1}{2}$, &c. So that the distances from the centre to 2, to 3, to 4, and so on to 10, are as the square roots of 20, 30, 40, and so on to 100; and the intermediate divisions and subdivisions are estimated as before shewn between 1 and 10.

THIS

THIS line is easily constructed from a table of the square roots of all the units and half units under 100; together with a scale of the intended length of the line of plans, divided into 500 or 1000 equal parts; and such a scale is the line of lines.

In the following solutions, the lengths of lines are supposed to be taken between the points of a pair of compasses: And when the callipers are said to be opened to any line; it means, to the distance of the points of the compass between which that line was taken; the points being applied transversely to the legs of the callipers, as shewn for the sector at section XII.

SOME USES OF THE SCALES OF PLANS.

EXAMP. I. *To find the square root of a given number.*

1st. ON the line of plans seek the division representing the given number: Observing, that numbers of an odd number of places are best found between the divisions 1 and 1; and those of an even number of places, between the 2d 1 and the 10 at the end.

2^d. TAKE, with the compasses, the distance between that division and the centre of the callipers; and this extent being applied, from the centre laterally along the *line of lines*, will give the square root of the number proposed.

THUS the square root of	9	is	3
	of 900	is	30
	of 90000	is	300
	&c.		&c.

THE given numbers being reckoned between the two divisions marked 1 and 1.

AGAIN the square root of	36	is	6
	of 360	is	18,9
	of 3600	is	60
	of 36000	is	189,7.

IF the integer places in the given number are even, the root will consist of half as many places: But if the number

number of integers be odd, increase it by one, and the integer places in the root will be half that number of places.

THUS numbers of two, four, six, eight integer places, will have roots consisting of one, two, three, four, &c. places : And numbers consisting of one, three, five, &c. places, have roots of one, two, three, &c. places.

EXAMP. II. *Between two given numbers (suppose 4 and 9) to find a mean proportional.*

1st. TAKE the greater of the given numbers (9) laterally from the line of lines, and make this extent a transverse distance to (9 and 9) the same number on the lines of plans.

2^d. TAKE the transverse distance between (4 and 4) the lesser given number on the lines of plans, and this extent applied laterally on the line of lines, will give (6 for) the mean proportional sought.

FOR $4 : 6 :: 6 : 9$.

BY this example it is easy to see how to find the side of a square equal to a superficies whose length and breadth are given.

EXAMP. III. *Two similar, or like, superficies being given ; to find what proportion they have to one another.*

1st. TAKE one side of the greater superficies between the points of the compasses, and make this extent a transverse distance on the line of plans between 10 and 10 ; or 100 and 100 : or on any other number.

2^d. APPLY a like side of the less superficies transversely to the line of plans, and the divisions it falls on will shew the number, that to the former number (taken transversely for the side of the greater superficies) bears the same proportion of the lesser superficies to the greater.

THIS proposition may be wrought laterally on either of the legs, reckoning from the centre : For like sides of similar plans being laid from the centre on either

leg, will give numbers shewing the proportion of those plans.

EXAMP. IV. *To find the sides, or other lines, of a superficies A, which shall be similar to a given superficies B, and in a given proportion to B, suppose as 3 to 7?*

1st. To the scales of plans, apply transversely, any given line of B to the consequent of the given ratio, as from 7 to 7.

2^d. TAKE the transverse distance, on the plans, of the given antecedent, as from 3 to 3, and this extent will be a like line of the figure A.

3^d. As many lines being thus found as is necessary, the figure A may be constructed.

EXAMP. V. *To find the sides, or other lines, of a superficies D, which shall be like to either of two given plane figures A and B; and also be equal to the sum or difference of A and B.*

1st. FIND (by Ex. 3.) two numbers expressing the proportion of the given figures A and B; and take the sum and difference of those numbers.

SUPPOSE the proportion of A to B, to be as 3 to 7. THEIR sum is 10, and their difference is 4.

THEN if D is to be like A.

FOR the sum, it will be $3 : 10 :: A : D$.

FOR the diff. it will be $3 : 4 :: A : D$.

BUT if D is to be like B.

THEN, for the sum, it will be $7 : 10 :: B : D$.

AND for the diff. it will be $7 : 4 :: B : D$.

2^d. FIND (by Ex. 4.) the sides of a superficies D, similar to A, so that A may be to D as 3 to 10 for the sum, or as 3 to 4 for the difference; or if like to B, so that B may be to D as 7 to 10 for the sum, or as 7 to 4 for the difference.

AND thus, a sufficient number of lines being found the figure D may be constructed.

EXAMP. VI. *Three numbers being given to find a fourth in a duplicate proportion : Or, the like sides a, b, of two similar figures A, B, being known, and also the area A, of one, to find the area B, of the other.*

ON the scale of plans, take the given superficies A laterally ; and on the scale of lines, apply this distance transversely to the given side *a* of that superficies : Take the transverse distance of the given side *b* of the other superficies, from the scale of lines ; then this distance applied laterally on the scale of plans, will shew the area of B.

THUS. *If 40 poles be the side of a square whose area is 10 acres ; what is the area of that square whose side is 60 poles ?*

TAKE the lateral distance 10 on the scale of plans ; apply this distance transversely to 40 and 40 on the line of lines : Then the transverse distance of 60 and 60 on the lines, applied laterally to the scale of plans, will give $22\frac{1}{2}$ acres the area required.

AGAIN. *How many acres of woodland measure, of 18 feet to the pole, is in that field which contains 288 acres, at $16\frac{1}{2}$ feet to the pole ?*

APPLY the lateral distance of 288, taken from the scale of plans, to the line of lines, transversely from 18 to 18 ; then the transverse distance of $16\frac{1}{2}$ and $16\frac{1}{2}$ on the lines, will, on the scale of plans, give 242 the area in woodland acres.

EXAMP. VII. *To open the callipers, so that the lines of plans make with one another a right angle ?*

ON the line of plans take the lateral extent of any number thereon.

THEN set the callipers so, that this extent shall be a transverse distance to the halves of the former number,

and the lines of plans will then stand at right angles to one another.

THUS : The lateral extent of 60 on the plans, put transversely to 30 and 30 on the plans, will set those lines at right angles to one another.

ARTICLE XIX.

Of the line of solids.

THESE lines are laid on the faces of D, C, of callipers, like sectoral lines tending to the centre, and are distinguished by the letters SOL placed at their ends.

THERE are twelve primary divisions on these lines marked 1, 1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; each of the eleven spaces or intervals is divided into ten other parts; and each of these parts is divided into two or more parts, according to the length of the instrument.

THESE divisions are best taken from a scale of equal parts, such as the line of lines, and thence transferred to the scales of solids, reckoning from the centre; from whence the several distances of the divisions are, as the cube roots of such numbers under 100 as are intended to be introduced.

THUS, the distance of the first 1 from the centre is as the cube root of $\frac{1}{10}$, and the greater divisions following to the second 1, express the cube roots of $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, &c. to the number 1, which the second 1 stands for; and if these spaces are subdivided, their distances from the centre are as the cube roots of $\frac{15}{100}$, $\frac{25}{100}$, $\frac{35}{100}$, $\frac{45}{100}$, &c.

THE distance from the centre to the second 1 is as the cube root of 1, and the greater divisions between the second 1 and the third 1, are as the cube roots of the whole numbers 2, 3, 4, 5, 6, 7, 8, 9; the intermediate smaller divisions are as the cube roots of the mixed numbers to which they belong: Thus if the space between the divisions representing the roots of 1 and 2 is parted into 4; then those subdivisions will be

as

as the cube roots of $1 \frac{25}{100}$, $1 \frac{5}{10}$, $1 \frac{75}{100}$; and the like for other subdivisions.

THE distance between the centre and the third 1 is as the cube root of 10; and so the following divisions marked with 2, 3, 4, &c. to 10, are as the cube roots of 20, 30, 40, &c. to 100: each of these spaces are divided into 10 parts, which are as the cube roots of the intermediate whole numbers; and if these subdivisions are again divided, these latter divisions will be as the cube roots of the mixed numbers to which they belong.

ON the French instruments, the divisions of this line is usually extended to 64; and consequently only the cube roots of all the integer numbers under 64 are thereon expressed: Now whether the divisions proceed only to 64 or to 100, the best way of laying them down is from a table of cube roots ready computed, reckoning the length of the greatest root, or the length of the scale of solids, to be equal to the length of the line of lines, taken from the centre.

THE cube roots of $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, $\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, $\frac{9}{10}$, are, 0,464. 0,585. 0,669. 0,737. 0,794. 0,843. 0,888. 0,928. 0,965.

THE following table contains the cube roots of all the whole numbers from 1 to 100.

TABLE of cube numbers and their roots.

Cubes	Roots	Cubes	Roots	Cubes	Roots	Cubes	Roots	Cubes	Roots
1	1,000	21	2,759	41	3,448	61	3,936	81	4,327
2	1,260	22	2,802	42	3,476	62	3,958	82	4,344
3	1,442	23	2,844	43	3,503	63	3,979	83	4,362
4	1,587	24	2,884	44	3,530	64	4,000	84	4,379
5	1,710	25	2,924	45	3,557	65	4,021	85	4,397
6	1,817	26	2,962	46	3,583	66	4,041	86	4,414
7	1,913	27	3,000	47	3,609	67	4,061	87	4,431
8	2,000	28	3,036	48	3,634	68	4,082	88	4,448
9	2,080	29	3,072	49	3,659	69	4,102	89	4,465
10	2,154	30	3,107	50	3,684	70	4,121	90	4,481
11	2,224	31	3,141	51	3,708	71	4,141	91	4,498
12	2,285	32	3,175	52	3,732	72	4,160	92	4,514
13	2,351	33	3,207	53	3,756	73	4,179	93	4,531
14	2,410	34	3,240	54	3,780	74	4,198	94	4,547
15	2,466	35	3,271	55	3,803	75	4,217	95	4,563
16	2,520	36	3,302	56	3,826	76	4,236	96	4,579
17	2,571	37	3,332	57	3,848	77	4,254	97	4,595
18	2,621	38	3,362	58	3,871	78	4,273	98	4,610
19	2,668	39	3,391	59	3,893	79	4,291	99	4,626
20	2,714	40	3,420	60	3,915	80	4,309	100	4,642

THE numbers in the foregoing table may be laid on the line of solids in the following manner.

MAKE the length of the line of solids equal to the length of the line of lines, apply this extent transversely to 4,642 on the line of lines; then the other numbers in the table taken transversely from the line of lines, are to be laid laterally, from the centre, on the line of solids.

Some uses of the lines of solids.

EXAMP. I. *To find the cube root of a given number.*

SEEK the given number on the line of solids, and take its extent from the centre.

THEN this extent applied laterally to the line of lines will give the cube root sought. IT

It should be remarked, that a given number of
 of 1, 2 or 3 places, has a root of one place.
 of 4, 5 or 6 places, has a root of two places.
 of 7, 8 or 9 places, has a root of three places.

AND when a given number is sought for on the line of solids,

THE primary divisions from 1 to 10 may be reckoned either as so many hundreds, or as so many hundred thousands, or as so many hundred millions.

THUS the division marked 5 may either represent 500, or 500000, or 500000000.

AND the like of the other primary divisions and their intermediates.

AND hence the divisions between the centre and the first of the primary ones, are to be estimated for numbers of one, two, four, five, seven and eight places.

EXAMP. II. *To a number given, to find another in a triplicate ratio of two given numbers.*

THUS. Suppose a shot of 4 inches diameter to weigh 9 $\frac{1}{2}$; required the weight of that shot which is 8 inches in diameter?

HERE a number is to be found, that to 9 shall be in the triplicate ratio of 4 to 8.

THAT is, as the cube of 4 is to the cube of 8, so is 9 to the number sought.

Now from any scale of equal parts, suppose inches, take 4; and make it a transverse distance to 9 and 9 on the line of solids (reckoning the 10 at the end, as 100): Then will the extent of 8 inches, applied transversely to the line of solids, give 72 for the number sought, which is the pounds weight of a shot of 8 inches diameter.

AGAIN.

AGAIN. Suppose a ship of 2000 tons burthen is 144 feet 6 inches on the keel, and 51 feet by the beam: Required the length and breadth of another similar ship that shall be of 1415 tons burthen?

FROM any scale of equal parts take $144\frac{1}{2}$ and make this extent a transverse distance to 2000 on the line of solids; then will the transverse distance of 1415 taken on the line of solids give the length of the keel, which applied to the said scale of equal parts will give about $128\frac{3}{4}$ feet.

ALSO the extent in equal parts of 51 being made a transverse distance to 2000 on the lines of solids; then the transverse distance on the solids of 1415 will give in equal parts $46\frac{1}{3}$ feet for the breadth by the beam.

EXAMP. III. Between two given numbers or lines to find two mean proportionals.

1st. FROM any scale of equal parts take the measure of the greatest of the given lines or numbers, and apply this extent transversely to that number on the line of solids; then the transverse extent on the solids, of the least of the given numbers, being taken, will be the greater of the required means, whose measure will be found on the said scale of equal parts.

2^d. MAKE the extent of the greater mean, a transverse distance to the greater of the given numbers, on the line of solids; then the transverse distance of the lesser of the given numbers, taken from the line of solids, will give the lesser of the required means.

Suppose two mean proportionals were required between 9 and $41\frac{2}{3}$.

THE lateral extent of $41\frac{2}{3}$, taken from the line of lines, apply transversely to $41\frac{2}{3}$ and $41\frac{2}{3}$ on the line of solids; then the transverse extent of 9 and 9 taken on the solids, and applied laterally to the line of lines will give 25 for the greater of the two means.

APPLY

APPLY the said extent of 25 transversely to $41\frac{2}{3}$ and $41\frac{2}{3}$ on the line of solids; then the transverse extent on the solids from 9 to 9 applied laterally to the line of lines, will give 15 for the lesser mean.

FOR 9, 15, 25 and $41\frac{2}{3}$ are in continual proportion.

EXAMP. IV. *To find the side of a cube equal to a parallelopipedon whose length, breadth and depth are given.*

1st. BETWEEN the breadth and depth find a mean proportional by Ex. 2. Art. 18.

2^d. FIND the measure of the mean proportional on the line of lines, and apply it to the lines of solids transversely, at the numbers expressing that measure: Then the transverse extent of the length being taken from the line of solids and applied laterally to the line of lines, will give the side of a cube equal to that parallelopipedon.

THUS, Suppose a parallelopipedon, whose length is 72, breadth 64, and depth 24.

THE number 64 taken laterally from the line of lines and applied transversely to 64 and 64 on the line of plans; then the transverse distance of 24 and 24 on the plans measured laterally on the line of lines gives about 39,2 for the mean proportional.

APPLY the extent of the mean proportional, to 39,2 transversely on the line of solids; then the transverse extent of 72 and 72 on the solids, being applied to the line of lines laterally, will give 48 for the side of the cube equal in solidity to the given parallelopipedon.

FOR $48 \times 48 \times 48 = 24 \times 64 \times 72 = 110592$.

EXAMP. V. *Two similar solids A and B being given, to find their ratio.*

1st. TAKE any side of the solid A, and apply it transversely

transversely on the line of solids from 10 to 10, or from any other number to its opposite.

2d. APPLY the like side of the solid B transversely to the lines of solids, and observe the number it falls on : Then will the numbers on which those transverse extents fall, shew the ratio of the solids A and B.

EXAMP. VI. *A solid A being given to find the dimensions of a similar solid B, that to A shall have any assigned ratio.*

1st. ON the line of solids seek two numbers expressing the terms of the given ratio.

2d. TAKE the extent of one side of the given solid A, and apply it transversely on the lines of solids to the antecedent of that given ratio ; then the transverse extent of the consequent taken on the lines of solids will be a like side of the solid B.

THUS. *To find the side of a cube B, double to a given cube A.*

HERE the ratio is as 1 to 2.

APPLY the side of the cube A to the lines of solids transversely from 1 to 1 ; that is from 10 to 10 ; then will the transverse distance of the numbers 2 and 2 or 20 and 20 shew the side of the cube B.

AGAIN. *To find the diameter of a sphere B, that to the sphere A, whose diameter is given, shall be in the ratio of 3 to 2.*

MAKE the diameter of the sphere A a transverse distance to 2 and 2 on the lines of solids ; then will the transverse distance of 3 and 3 on the line of solids be the diameter of the sphere B.

EXAMP. VII. *Any number of unequal similar solids being given ; to find the side of a similar solid equal in magnitude to the sum of the magnitudes of the given solids.*

TAKE, in equal parts, a number expressing the side
of

of one of the given solids, and apply this extent to the line of solids transversely, to any number (suppose 10 at the 3d 1).

ALSO take in the same equal parts, the numbers shewing the similar sides of the other solids, and apply these extents to the lines of solids transversely, noting the numbers they fall on.

THEN will the transverse extent on the line of solids of a number equal to the sum of the noted numbers, be the like side of the similar solid required, which applied to the same scale of equal parts the others were taken from will give the measure of that side.

THUS. *What will be the diameter of that iron shot cast from 3 other shot whose diameters were 4 inches, 4, 4 inches, and 5 inches ; supposing no waste in melting ?*

MAKE 4 inches a transverse extent on the line of solids, to any number suppose 10. Then 4,4 inches applied transversely to the solids will give about $13\frac{1}{3}$; and 5 inches also applied transversely to the solids will give about $19\frac{2}{3}$: Now the sum of the noted numbers 10 and $13\frac{1}{3}$ and $19\frac{2}{3}$ will be 43 ; then the transverse extent of 43 on the line of solids will give $6\frac{1}{2}$ inches for the diameter of the new shot.

EXAMP. VIII. *To find the dimensions of a solid which shall be equal to the difference of two given similar solids, and also similar to them.*

APPLY a dimension of one solid transversely to the line of solids at any number ; and also note what number on the line of solids, the like dimension of the other solid falls transversely on ; take the difference of those noted numbers ; and on the line of solids take transversely the extent of the remainder, and that will be a like dimension of the similar solid required.

THUS.

THUS. *With the powder out of a shell of 10 inches concave diameter is filled a shell of 7 inches : What sized shell will the remaining powder fill ?*

THE extent of 10 inches being applied transversely to the lines of solids, at any number suppose 100 ; the extent of 7 inches will fall transversely on the lines of solids, about the number $34\frac{1}{4}$: The difference between 100 and $34\frac{1}{4}$ is $65\frac{3}{4}$: Then the transverse extent at $65\frac{3}{4}$ on the line of solids, will give 8,7 inches for the concavity of that shell which the remaining powder will fill.

F I N I S.



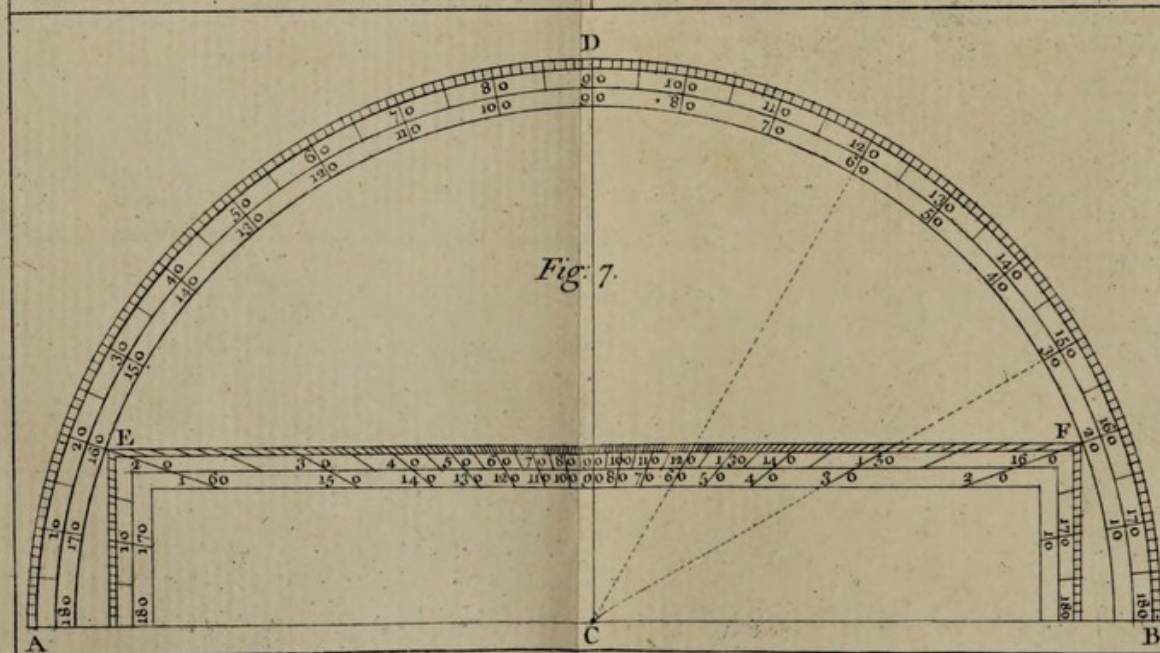
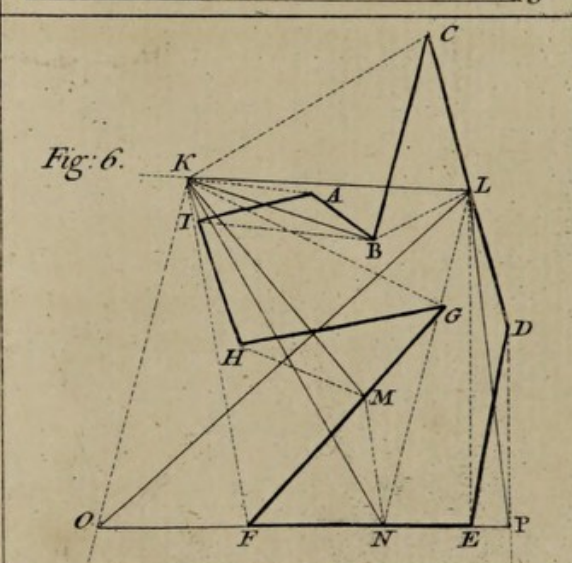
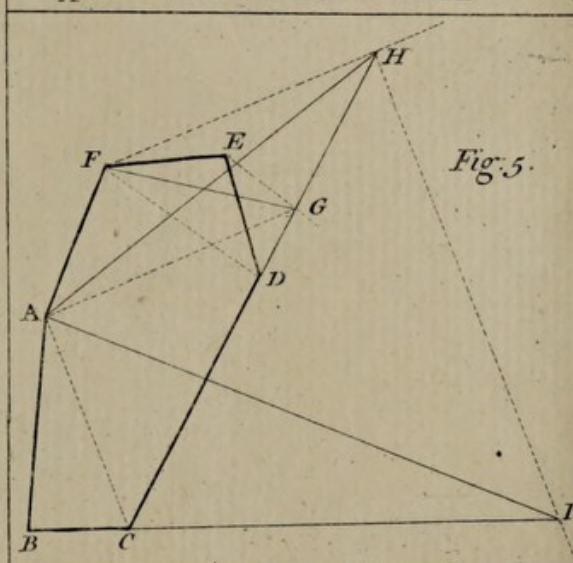
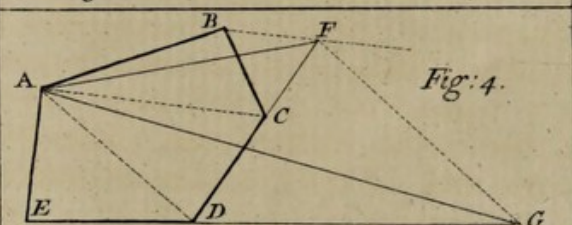
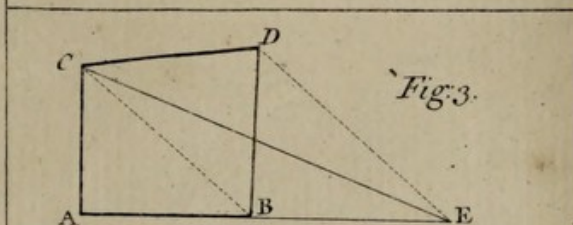
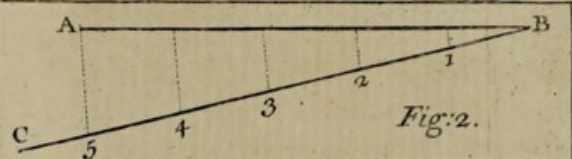
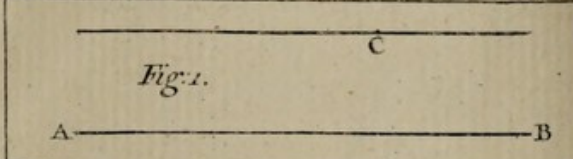




Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5

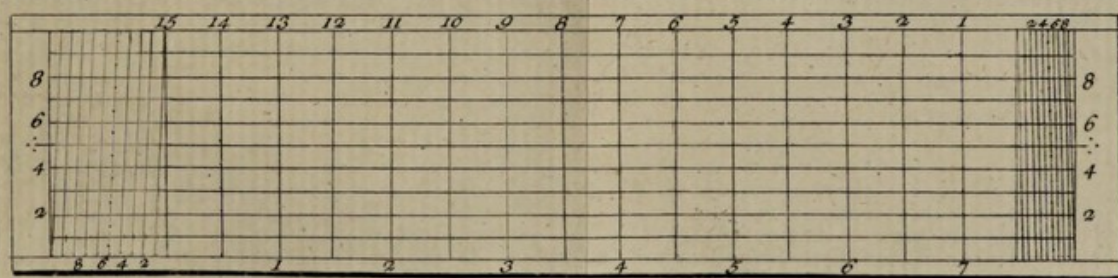
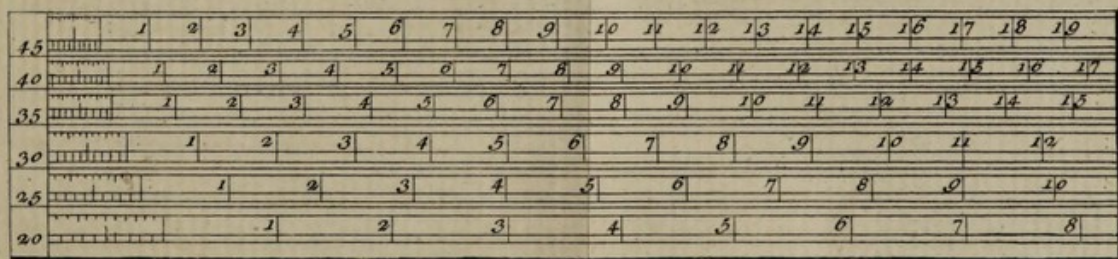
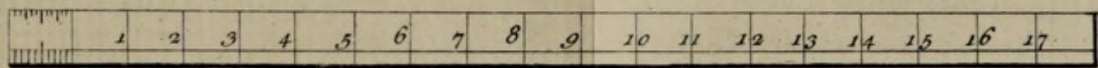
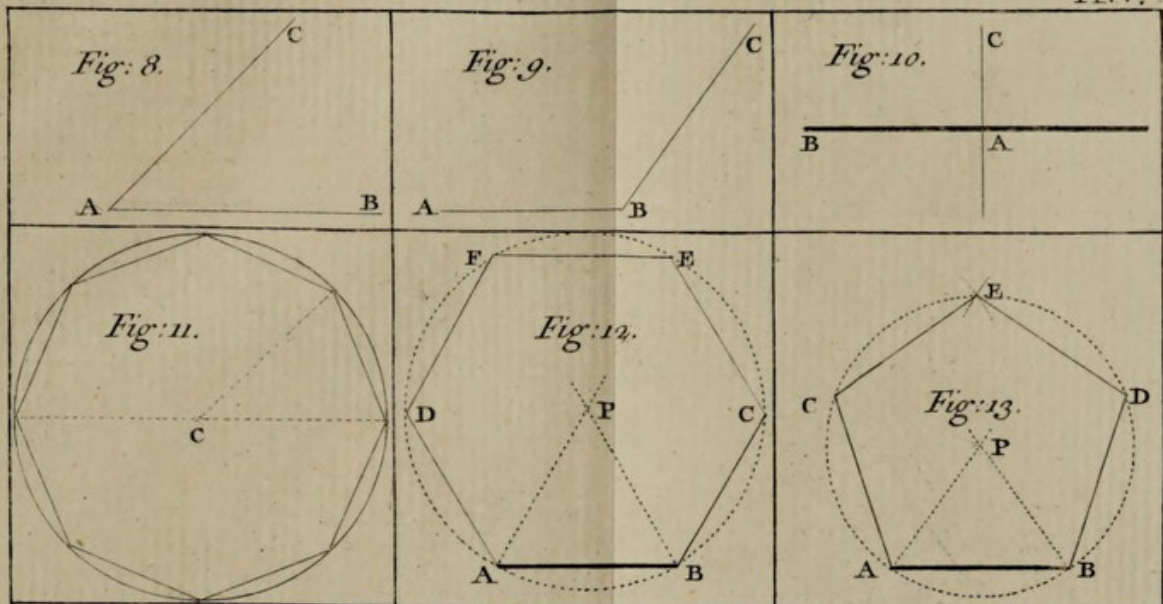
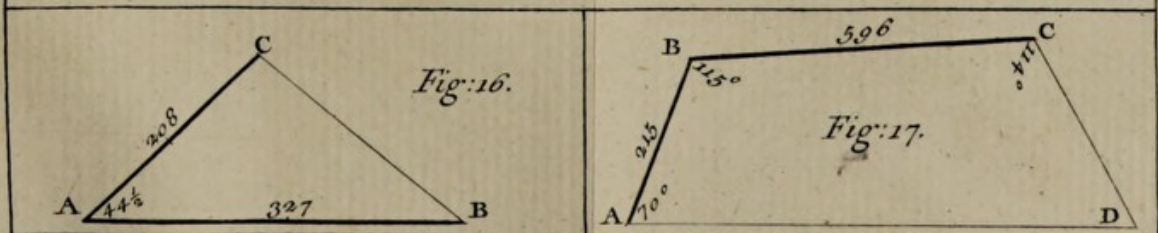
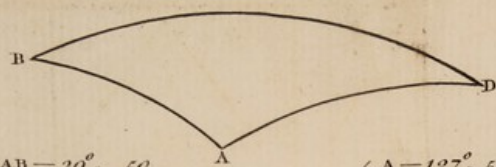


Fig. 15.





$$AB = 29^{\circ} 50'$$

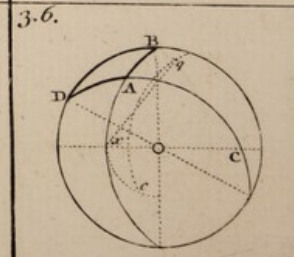
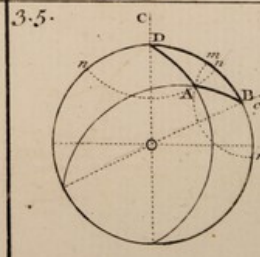
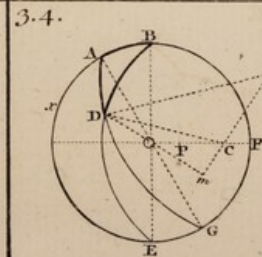
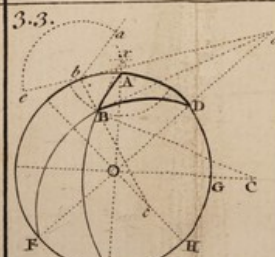
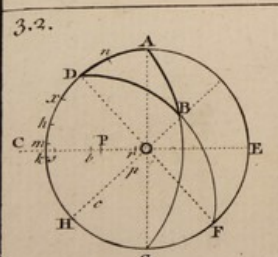
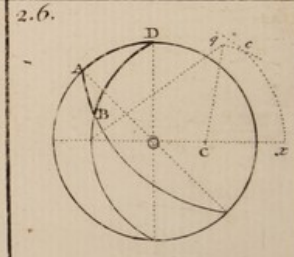
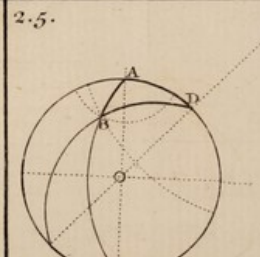
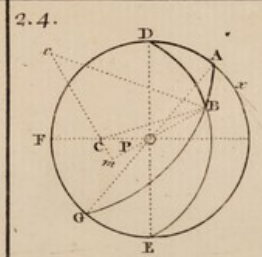
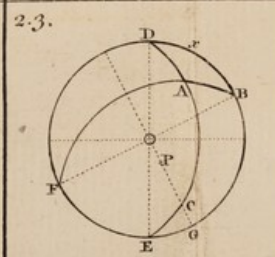
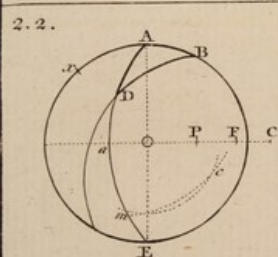
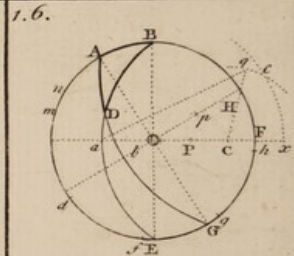
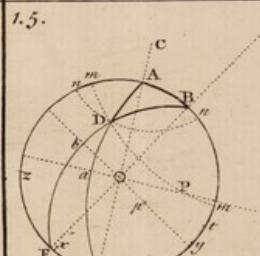
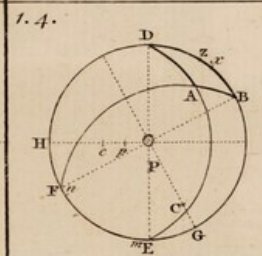
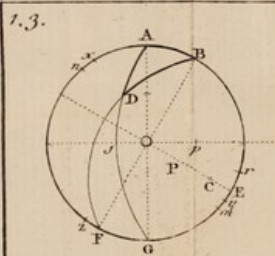
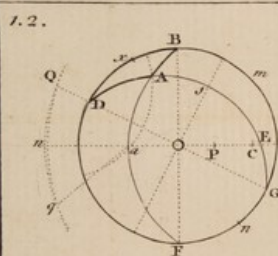
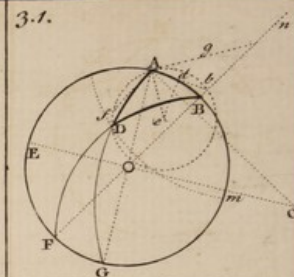
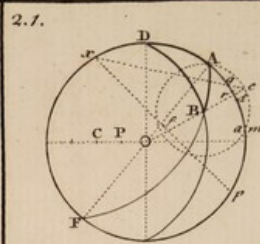
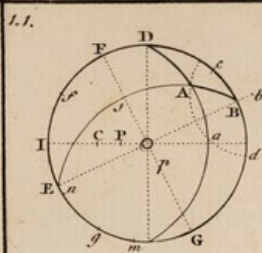
$$AD = 42^{\circ} 09'$$

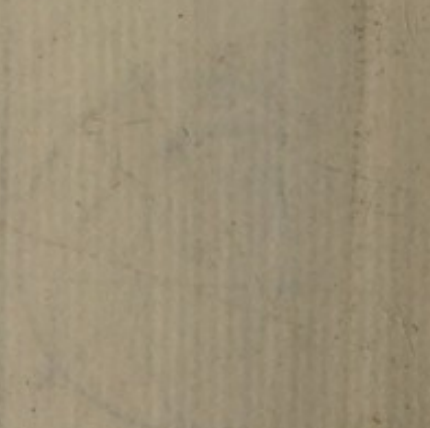
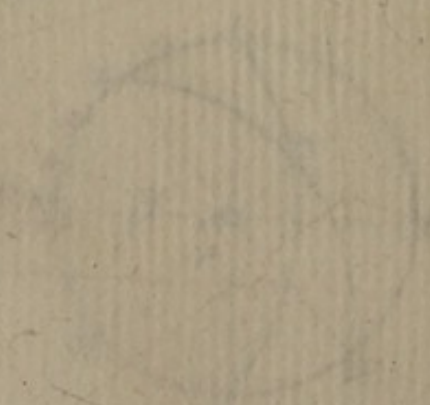
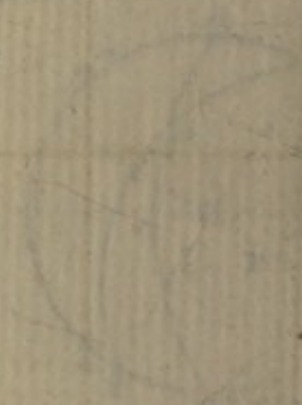
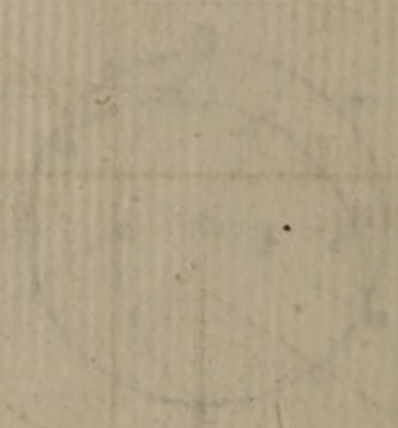
$$BD = 63^{\circ} 59'$$

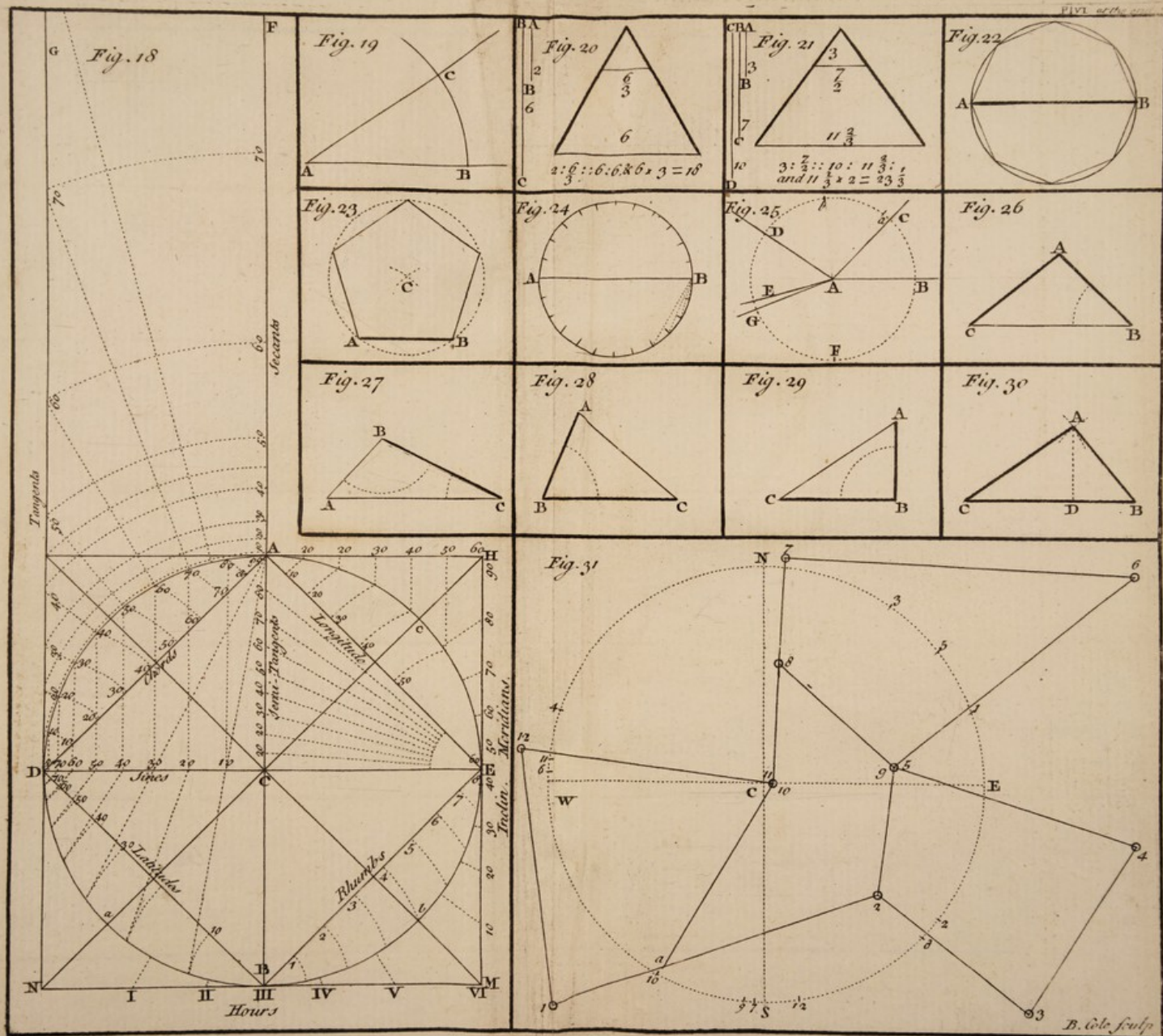
$$\angle A = 127^{\circ} 50'$$

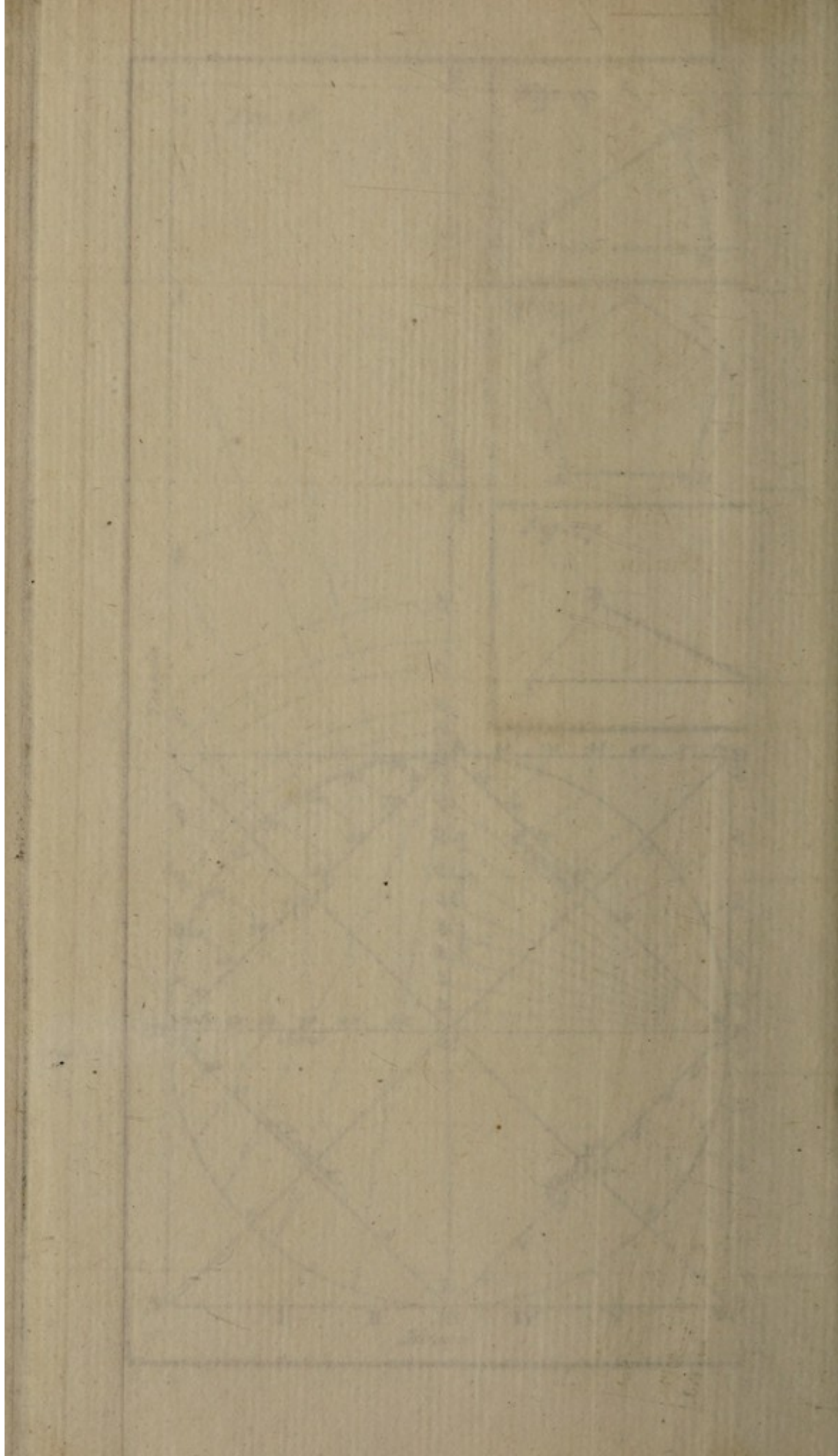
$$\angle B = 36^{\circ} 8'$$

$$\angle D = 25^{\circ} 55'$$

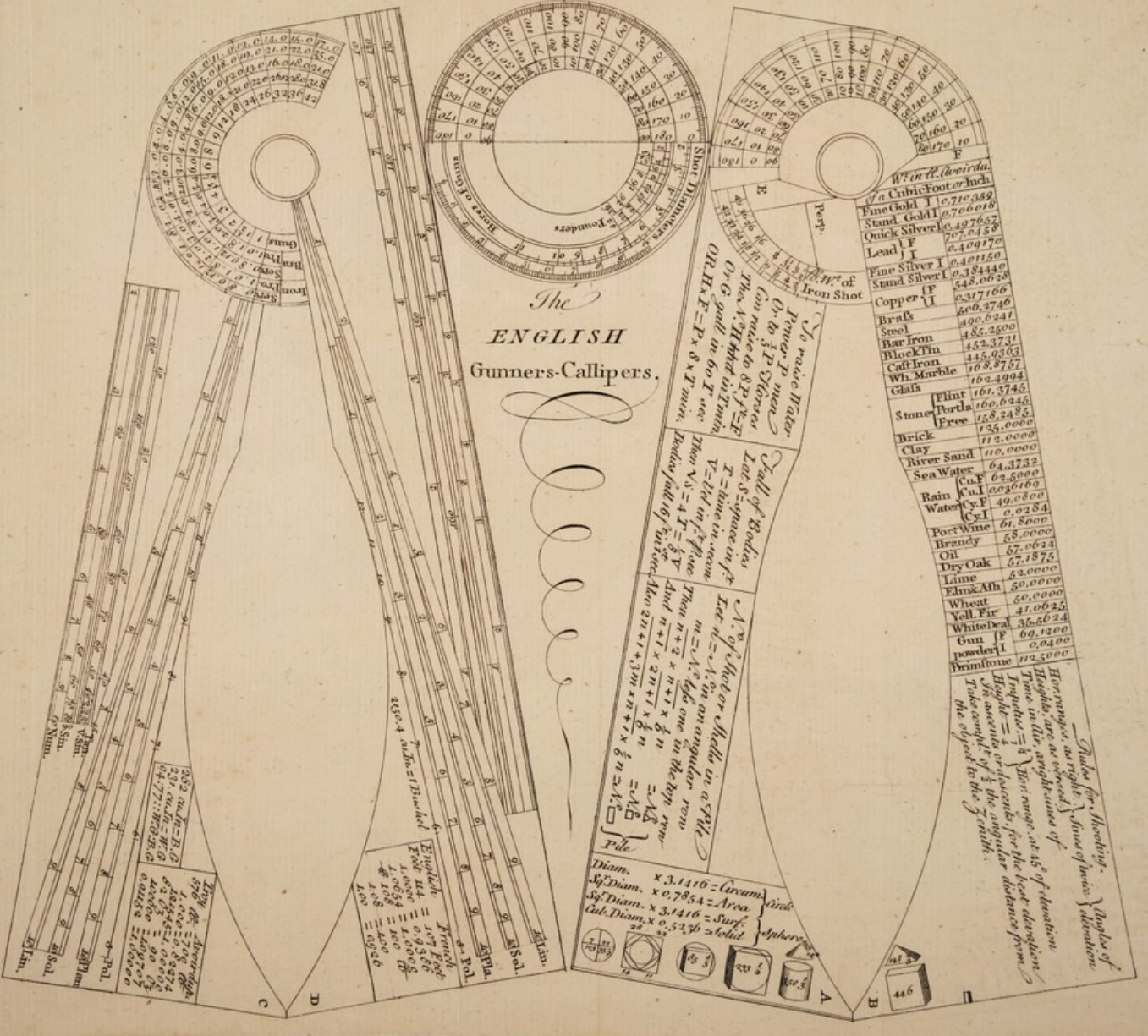






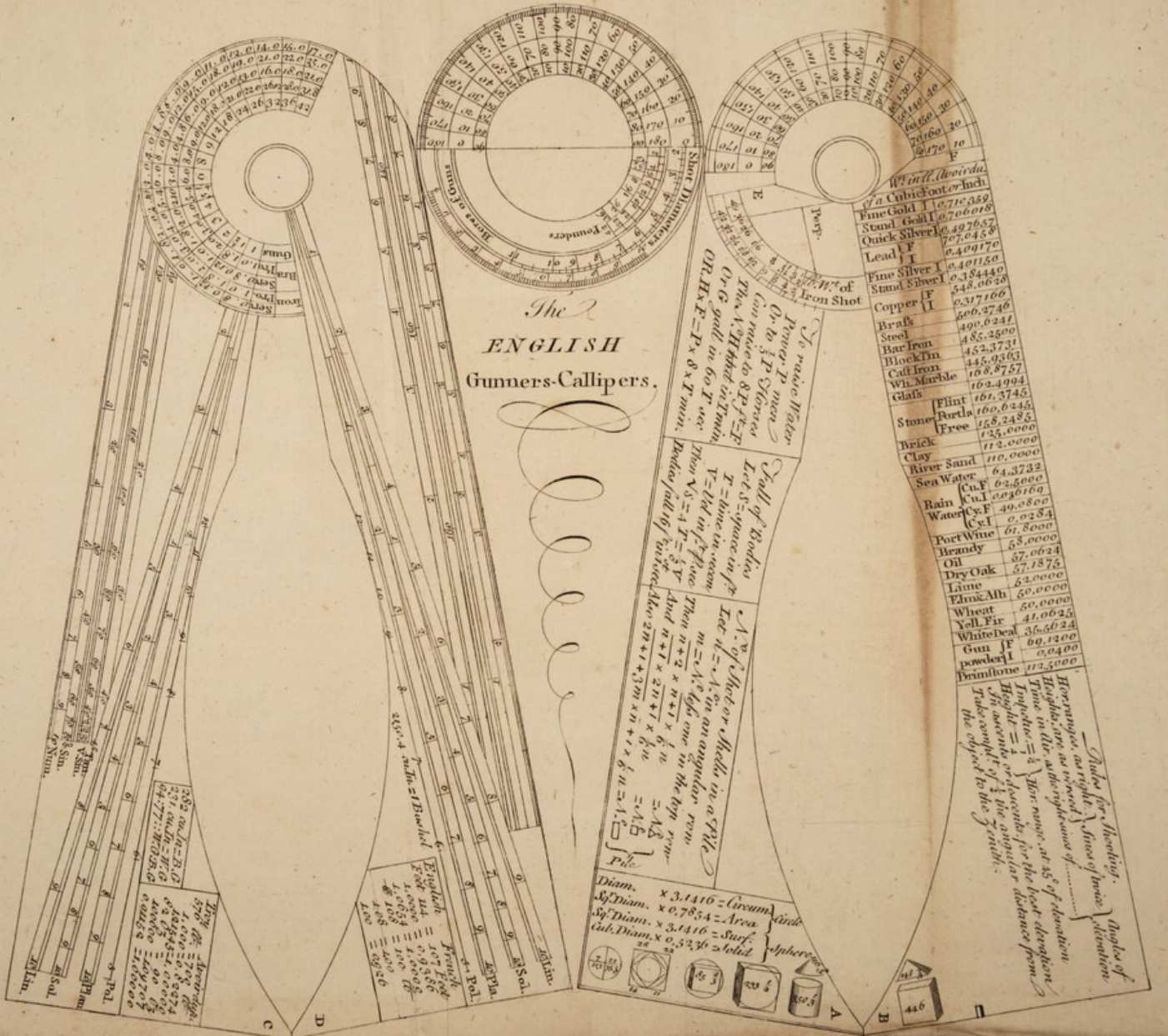


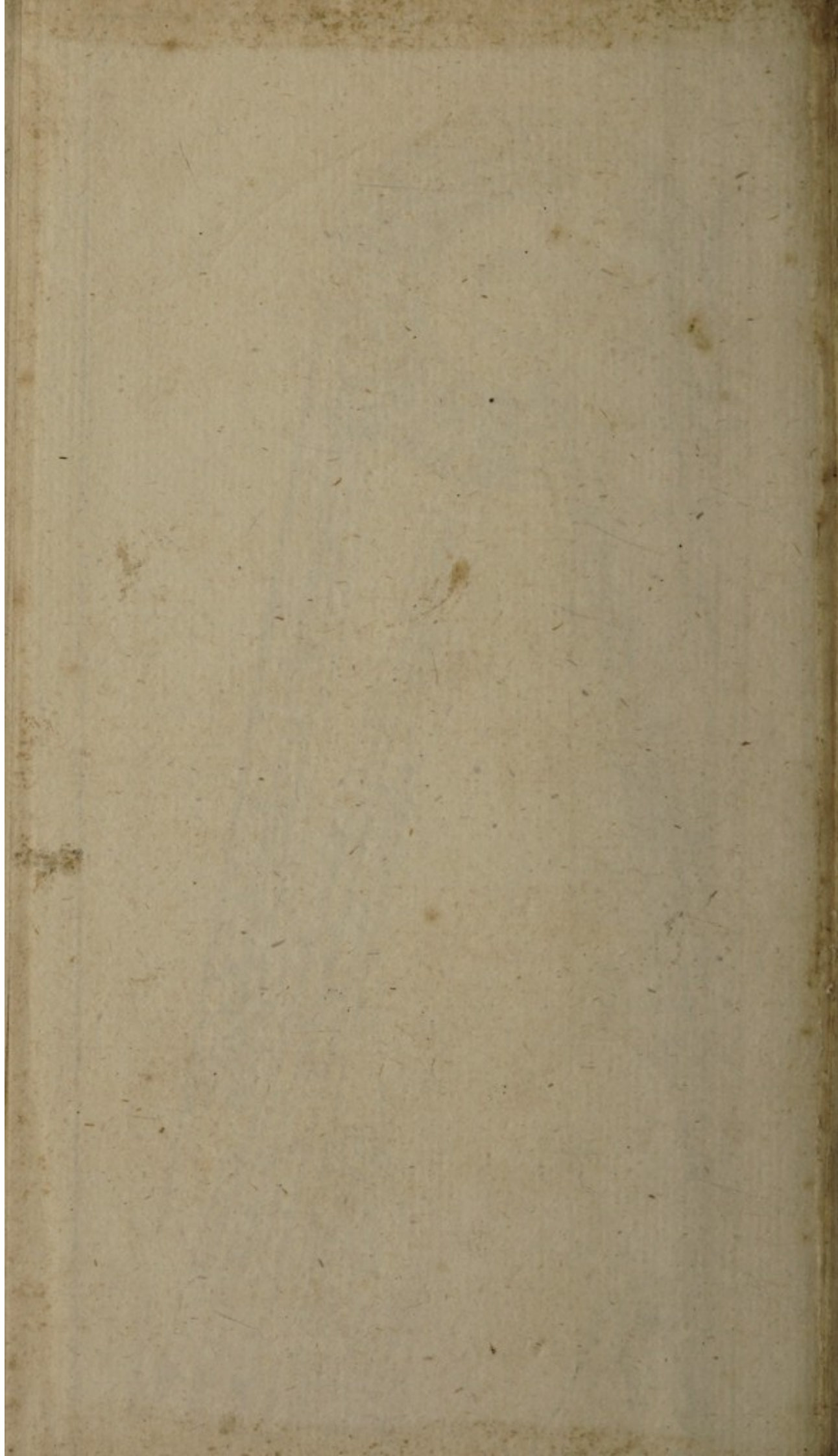
The ENGLISH Gunners-Callipers.

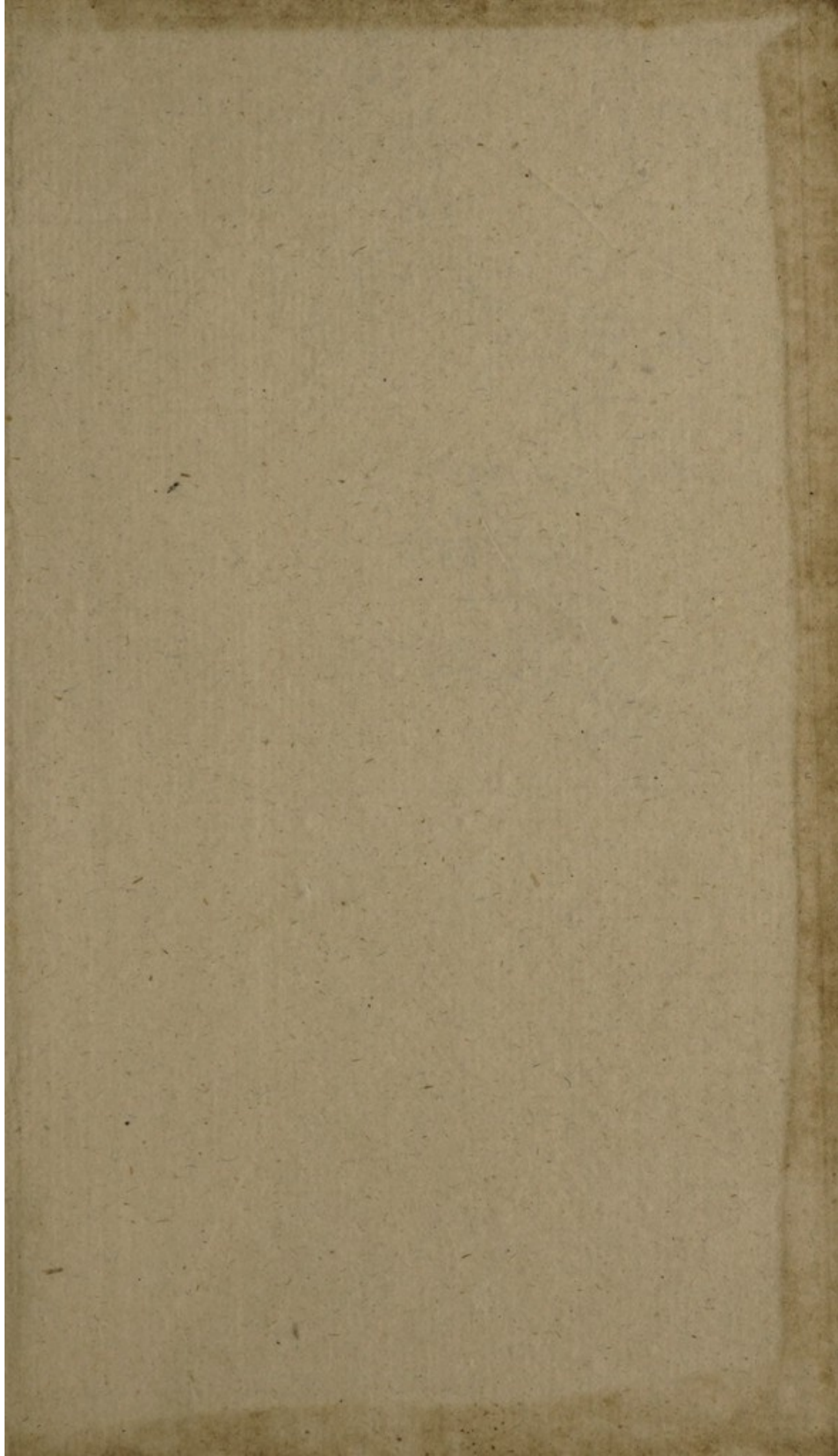




The ENGLISH Gunners-Callipers.







$$64:9::1$$

16

$$64 \overline{) 144} \left(2 \frac{1}{4} \right)$$

128

16

$$64:9::175$$

3/4

9

$$64 \overline{) 675} \left(10 \right)$$

64

35

$$64:144::411875$$

144

1647500

1647500

411875

50310000

99

59.3

75

375

525

15625

75

18125

39375

411875

