

An essay to ascertain the value of leases and annuities for years and lives, and to estimate the chances of the duration of lives. Wherein are many observations on bills of mortality, etc / [Anon].

Contributors

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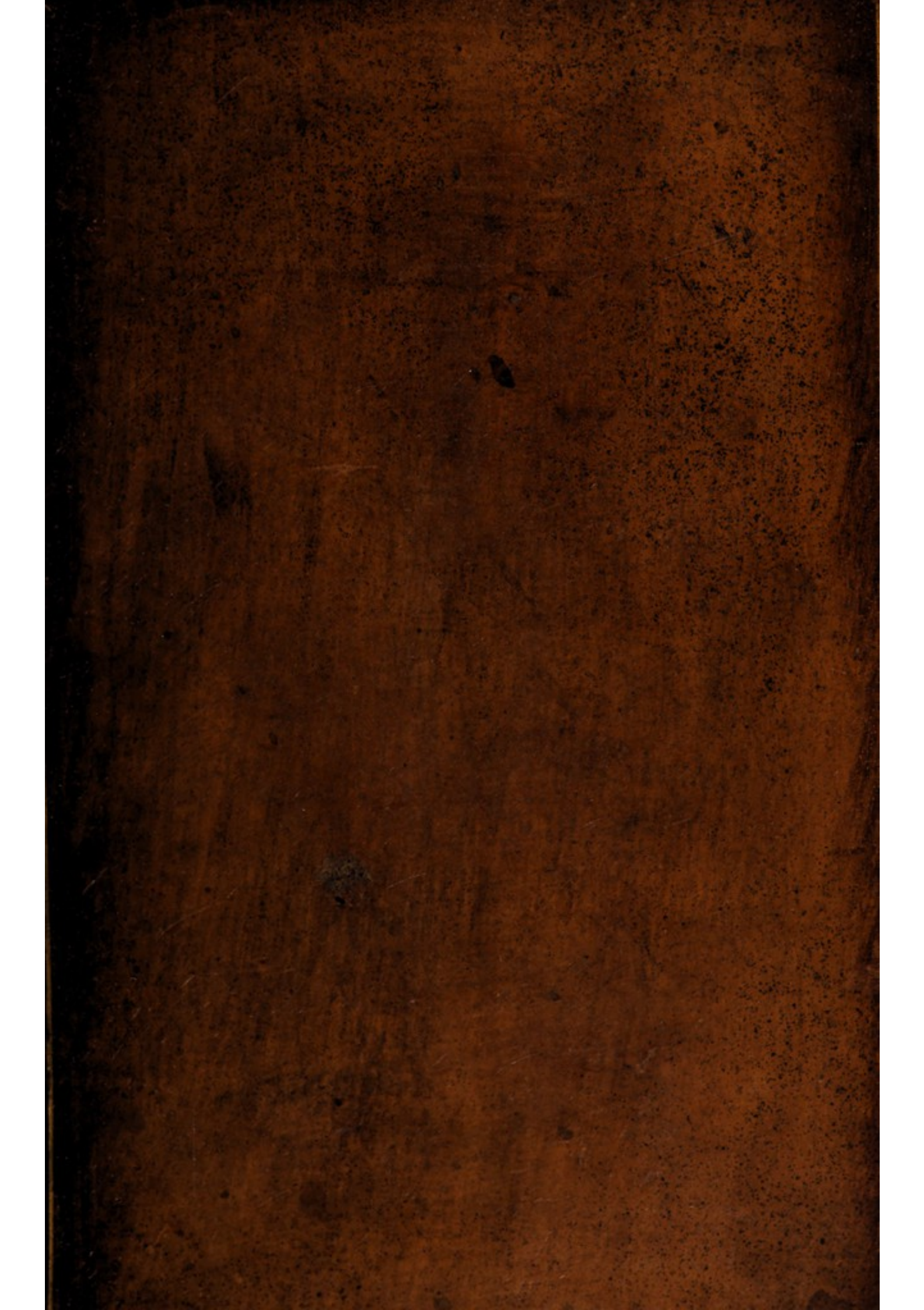
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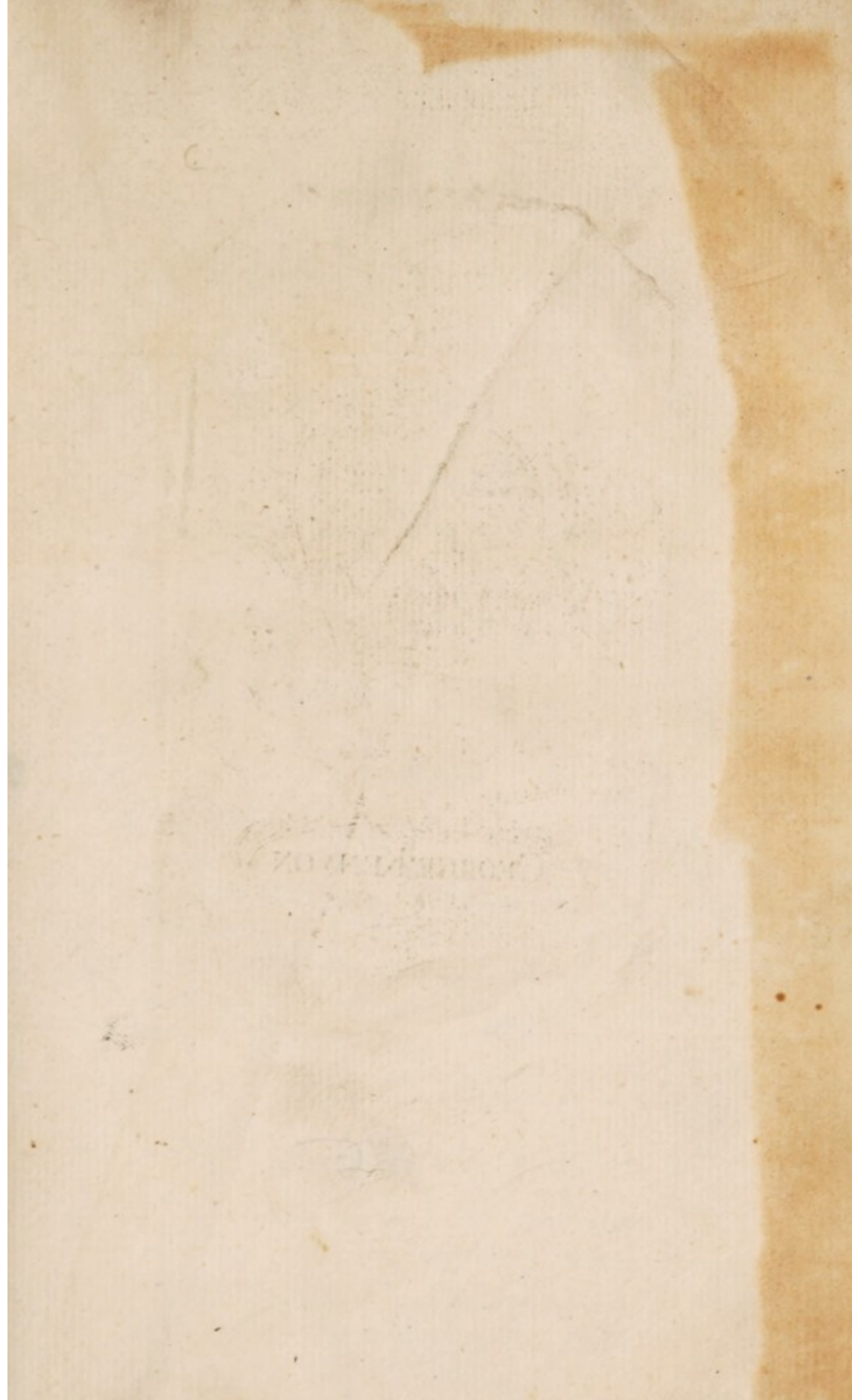
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THE ART and MYSTERY OF
ARITHMETIC AND ANNUITIES
FOR
YEARS and LIVES
AND A TABLE OF
CHANCES
OF THE
DURATION OF LIVES

Many Observations on Bills of Mortality
on the State of the Coin, the Interest of Money
the Price of Wines, in different Ages: And Notes
of several Learned Writers on this Subject, and their
Reasons for the same, particularly for the Duration
of Lives, and for the manner of Living, the most
advantageous and prudent.

L O N D O N,

Printed by S. BARNARD, at the Black Swan in St. Dun-
stons Church-yard; and J. STURGEON, at the
Sign of the Green Dragon in St. Dunstons Church-yard.
MDCCLXXII.



A N
E S S A Y

To ascertain the VALUE of
LEASES and ANNUITIES

F O R
YEARS and LIVES,

And to Estimate the
CHANCES

O F T H E
DURATION of LIVES.

WHEREIN ARE

Many OBSERVATIONS on Bills of Mortality,
on the State of the Coin, the Interest of Money, and
the Price of Things, in different Ages: And Notice
taken of several Writers on this Subject; and their
Rules and Tables, particularly for the Valuation of
ANNUITIES for one or more Lives, are demon-
strated to be erroneous.

L O N D O N,

Printed for S. BIRT, in *Ave-mary-Lane*;
D. BROWNE, at the *Black Swan* without
Temple-Bar; and J. SHUCKBURGH, at the
Star near the *Inner-Temple-Gate, Fleet-Street*.

M,DCC,XXXVII.

E S S A Y

TO ACCOMPANY THE
HISTORICAL
MEDICAL
JOURNAL

FOR
YEARS and LIVES

And to Estimate the

CHANCES

OF THE

DURATION OF LIVES

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strated to be erroneous.

L O N D O N,

Printed for S. Birt, in Ave-mary-Lane;
D. Browne, at the Black-Swan without
Temple-Bar; and J. Sturges, at the
Gate near the Inner-Temple-Gate, Fleet-Street.

MDCCLXXVII.



TO THE
R E A D E R.



I SHALL not need to give any Reasons for writing the following Treatise, or for publishing it now 'tis wrote, because the Subject-Matter is of a very common and general Concern: Nor shall I trouble you with a long Preface, to give an Account of my Undertaking, or to commend my Performance. If my Notions and Calculations are right, and of any Service, their own Weight will and shall bear them out; and if they are wrong, or are useless, I have not so great an Opinion of my own, or so little an one of other People's Understanding, as to expect that the World will have any Regard to what I may say to recommend them.

*But as in writing on this Subject I have travell'd out of my proper Profession, and undertaken a Province which may seem not to belong to me; and in so doing have encounter'd, not vulgar Opinions only, but some which have been received on the Authority of great Men; and since I have carried the Matter so far, as to attack Rules and Computations which are grounded, or supposed to be grounded, on Mathematical Operations; I hope you will, and I doubt not but you will, do me the Favour,
and*

and yourself the Justice, to read, examine, and weigh the Arguments which I bring for my Assertions, before you pass any Judgment on them. When that is done, and if, on due Consideration, they have not that Weight in them which to me they seem to have; you will be at Liberty, and to be sure you will use the Liberty, to withhold your Assent from any Notions which may have been taken up and entertain'd, by me or any Man else, without sufficient Reason and Foundation.

For the rest: If you have any other and better Thoughts, on this Subject, than you meet with here, you will be so good as to give me Part of them; but if not, and until such do appear, you are welcome to these, and to my Pains herein.

Your's,

October 14,
1736.

W. L.

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Addenda

Addenda & Emendanda.

PAGE 14, Line 5, for *or* read *for*. Pag. 62, l. 22, for *Valuation* read *Value*. After Pag. 62, for 61 read 63. Pag. 64, l. 26, for 200 l. read 2000 l. Pag. 77, l. 20, for *they* read *we*. Pag. 92, l. 21, for *Annuity*, solely read *Annuity solely*. Pag. 125, l. 25, for *of, that* read *of that*. Pag. 137, l. 25, for *Merris's* read *Morris's*. Pag. 139, l. 17, for *and in the* read *and the*. Pag. 219, l. 6, for *that there* read *there*. Pag. 257, l. 6, for *County* read *Country*. Pag. 246, l. 29, add, *but not out of the Individuals who were such at his Accession*. Pag. 266, l. 21, for *these* read *the*. Pag. ib. l. 22, for *the* read *these*. Pag. 305, l. 3, for *the* read *and the*. Pag. 306, l. 10, for *Life makes* read *Life, makes*. Pag. 333, l. 20, for 35 read 55. Pag. 337, l. 6, for *coinside* read *coincide*. Pag. 364, cross Lines, for *ten* read *twelve*. Pag. 375, l. 32, for *take* read *takes*. Pag. 379, l. 17, for *above the* read *above, the*. After Page 381, for 373 and 372 read 382, 383. Pag. 373, l. 9, for *Regard of* read *Regard*. Pag. 414, l. 10, for *three Lives* read *three Joint-Lives*. Pag. 426, l. 11, for *at* read *of*. Pag. 428, l. 33, for 47 read 44. Pag. 432, l. 24, for 10 read 11. Pag. 449, l. 30, for *commuicate* read *communicate*.



A N
E S S A Y

To Ascertain the VALUE of
LEASES, &c.



PROPOSE to consider here, whether there be any, and what is, the Rule to compute the Value of Leases for Terms of Years, and for one or more Lives; so far that Persons who have Occasion to transact those Affairs, either to buy, or to sell, or to renew, (and one or other of these Transactions almost daily occurs,) may know whether the Bargain they make is an even and a fair one. That there should be a Want of such a Rule at this Time of Day, after so many Affairs of this Kind have been transacted, may seem a little strange; and yet none has been produced, or at least none has fallen under my Observation, to which some material Objection, as I apprehend, does not lie in
A the

the present State of Things : And I am not so fond of my own Productions as to imagine no Objection can or will be made to the Method I mean to offer : Thus far only I shall go ; I will propose it with my Reasons to support it, and hope it will appear to come nearer the Mark than any now extant, or commonly made Use of ; and that, with some few Additions and such Alterations as may make it suit the particular Estate in View, it will answer the Purposes intended.

The common Methods for computing the Value of Leases for Lives are, in my Judgment, much the most erroneous, and therefore I mean to consider them more specially ; and the Method I propose for that Purpose, is by reducing them to Terms for Years, and by estimating them as such. Now if this can be done within any tolerable Degree of Probability, 'tis certain the Value of them may be so far known ; for the Value of Terms for Years may be computed to the Exactness of a single Shilling, if necessary, at any given Rate of Interest : And the Thing, I think, may be done as near the Truth as in a Matter of so much Uncertainty can be expected ; and is to be done, by supposing such Leases to have a Duration equal to the Term to which the given Life or Lives on an even Chance are estimated to be equal : And then the Value of them likewise may be adjusted, and known to a reasonable Degree of Probability.

But when Leases for Lives are converted into Terms for Years, or may be consider'd as such, there

there seem to me some other Circumstances to be taken into the Account, before we can determine the Value of them : And since these Circumstances are such as do or may attend all Leases, whether for Lives or Years, I shall not need to make any Distinction between such Leases with Regard to these ; but what I say upon Leases for Years under this Head, I think is applicable, and I mean should be applied, to Leases for Lives likewise.

It has been often, and I believe very truly, said, that there is hardly one Cause comes into a Court of Equity, but has some Circumstances attending it, peculiar to itself, which make it vary from any other : And, I think, it may as justly be said that scarcely two Leasehold Estates in the Kingdom, whether for Lives or Years, are in all Instances just so much alike, that any one common Rule for estimating the Value, either in Purchases or Renewals, will quadrate with them : But that some other Particulars, besides those of the Reserv'd and Improv'd Rent, are to be taken into Consideration, and yet such others are rarely or very slightly regarded.

In the North and West of *England* the Tax on Lands is in general much easier in Proportion than in the South and East Parts ; and even in the same Countries Estates of the same Value in Rent are rated in those Assessments in an unequal Manner. In some Countries the Buildings in general are of Stone or Brick, in some of Timber and Lime, and in others of something less durable ; and Estates have some more some

fewer Buildings on them, and those in a better or worse Condition. Now where Estates differ in any of these Circumstances, if the Difference be to any Degree, I confess I cannot see any Reason or Equity to make use of a general Rule of so many Years Value of the improved Rent, after deducting the reserved Rent only, either in taking, buying, or renewing such Estates. And where these Things, as undoubtedly in some Cases they must, and in Fact are, taken into Consideration; I very much doubt whether it be done in a regular and even Way.

I have before observed that the Value of Leases for Years may be computed to a great Exactness, and most certainly it may; but then those Leases must be first reduced to absolute Annuities, by which I mean Sums certain, clear of all Deductions and Outgoings whatsoever; for 'tis on such Supposition only, that their Value can be exactly ascertain'd. Besides this; I think an Allowance should be made to the Tenant for assuring such Estate and the Rent from all casual Losses and Damages, and also for his Trouble in managing and looking after the Estate, and receiving and paying the Rent, and that these ought to be made in Proportion to the improv'd Value of the Estate. And then, that the Surplusage or Neat-Money arising, after such Reduction and Allowances made, be look'd upon as the Lessee's Estate and Interest; for 'tis well known that the Proprietors of absolute Annuities are free from all Engagements which can possibly make them subject to
any

any Deductions, or to any Losses or Charges whatever. This Neat Produce then being stated as the Lessee's Interest, I propose that the Money laid out on a Purchase, or Renewal of it be reckon'd at an Interest of *6 l. per Cent.* or one *per Cent.* above common Interest: By which I would not be understood to mean that such Tenant is to make or will make *6 l. per Cent.* of his Money; for though it should be so reckon'd in Computation, yet in Fact there will not be so much coming to the Tenant for his own Use; as I shall take Occasion to show under the proper Head.

I am very sensible that most Persons, in Transactions of this Kind, consider and allow only the Rent reserved to the Landlord; but surely all other Payments made by the Tenant, whether Fee-Farm, Chief, or Quit-Rent, or whatever other Denomination such Payments come under, if they are certain and perpetual, and are chargeable on and issuing out of the Estate, and are not usually and commonly discharged by an ordinary Tenant at Will, must be and will be admitted to come into the same Class; and as they are determinate Sums, and are or may be certainly known, I apprehend there can't well be any Dispute about them, if they are, and where they are, of such a Value as to deserve Notice.—

As for those three Articles, *viz.* the Land-Tax, Repairs, and an Allowance for Charges and accidental Losses, all which I have proposed and contend ought to be discounted by the Les-

for, before we can truly and properly say the Lessee's Estate is an absolute Annuity; about these there is great Disagreement, as well in Practice, I believe, as in Theory. The Author of a Treatise call'd, *The Value of Church and College Leases consider'd*, who has been said in the common Prints to be a Dignitary or Prelate of the Church, urges, that all these Burdens are to lie on the Tenant: And Mr. *Richards*, in a Tract published some few Years since, which he calls, *The Gentleman's Steward and Tenant of Manors instructed*, admits that a Tenant is to have Compensation for the two last, but not for the first Article; and I am of Opinion, and find I am not singular in Opinion, that the Landlord ought to make an Allowance for every one of these Articles. Before I give my Reasons for my Opinion, I intend to consider what is alledged for the contrary one. The Reverend Author tells us then, That Taxes, Repairs, and Accidents, are not peculiar to Church or College Estates; but Estates of Inheritance are equally liable to the same Inconveniencies: I add, neither are the Estates of the Laity, where let at a small Rent with a Fine taken, exempted from these Burdens; for I mean to make no Difference between them, unless any particular Argument necessarily leads me into it. Now I make no Doubt but that forty Years Experience, and more, has made most People sensible of this great Truth, that Lands of Inheritance are subject to the Land-Tax; and I should be much pleased if I could have the Opportunity of concurring with

with this learned Writer, in ranking it among the Inconveniencies attending Estates in Land which are accidental only, for at present it must be number'd among the certain ones: But let the Taxes and Repairs be certain or accidental Inconveniencies, I confess I don't see the Consequence. Owners of Lands of Inheritance, if they let those Lands at the improved Rent, must pay the Taxes, and do Repairs, and not their Tenants; therefore Owners of Lands of Inheritance, where they let their Lands at a small annual Rent, and take a Fine instead of and equivalent to the rest, must not pay the Taxes, or do Repairs, but their Tenants must: Whereas, if any Inference can be drawn from this Way of Reasoning, the Reverse is the most natural and most obvious one.

If we should ask the Reason why Owners of Lands of Inheritance, if they let those Lands at the improved Rent, must pay the Taxes; we shall undoubtedly be told, because such Owners receive the Rents, or may take the Profits, of such Estates. For the very same Reason, and on the same Grounds, if Landlords letting at a small Rent do, and so far forth as they do, receive the Rents and take the Profits of their Estates, though in a Manner and at a Time differing from that in which the Owners of Estates who let at the improved Rent, receive and take theirs, then so far and for so much of the Rents and Profits of their Estates as comes to their Hands, they must likewise pay the Taxes; they, as well as the rest of the Kingdom, being obliged

to take and hold their Estates with such Burdens as the Laws of the Land lay on them.

I observe that this Author walks over this Head very tenderly, and contents himself with telling us he believes, where the Rent reserved is a third or a fourth Part of the improved Value, that there a Consideration and Allowance is had by such Landlords; which is a tacit Confession that there is some Equity and Reason in the Expectation of Tenants in some Cases. I suppose therefore, when he tells us in general Terms, that Tenants are unreasonable and partial to themselves, if they expect to hold their Leasehold Estates free from these Inconveniencies, he does not mean that there is an Argument to prove that all Tenants ought to be charged with them, but such only as hold at a small Rent: But then he is so good to Landlords as to leave them a great Latitude in judging, or rather makes them sole Judges, what they shall call a small Rent, or what shall be look'd upon as a reasonable Consideration on a great Rent. But if Tenants, whether holding at a great Rent or at a small Rent, should give the same civil Appellations to their Landlords, and call them unreasonable and partial Men, if they expect to be exempted from these Burdens; would not the Argument on their Side have just the same Weight, and full as much Manners, as on the other Side?

We are told farther, that these Charges on Lands of Inheritance, in some Parts of *England* at least, make a Draw-back of 30 *l.* per Cent. which

which we admit may be true; but then 'tis equally true with Regard to Lease-hold Estates, at least in the same Parts of *England*: The necessary Consequence of which is, that the Owners of such Estates, and in those Parts of the Kingdom, can receive for their own Uses no more than 70 *l. per Annum* out of an Estate which is call'd, and at the improved Rent is worth, 100 *l. per Annum*. Now though such Tenants, or the Person whom he dresses up and introduces in a Fool's Coat, that he may have an Opportunity of shewing him away, do not understand, or will not attend to, nice Calculations; yet, I suppose, they may have Sense enough to find out that 70 *l.* is not quite so much as 100 *l.* and that the one is not so good an Interest for the Sum of Money supposed to be laid out in the Purchase of this Estate, as the other is; though, perhaps, they may not be able to discover exactly what Interest they have for such Money, for want of a little of that same which some others are so great Masters of.

And if this Author would have paid a proper Regard to his own Assertions, or had attended here to Calculations, I don't say any nice ones, but one of his own, I think he would not, I am sure he should not have complain'd of the Generality of the World, as unreasonable and partial to themselves, in expecting to make full 6 *l. per Cent.* of their Money, above all Deductions, when they purchase Lease-hold Estates: Or if these Expectations in such Purchasers are really unreasonable, he, of all Men, should not have
given

given them Grounds whereon to raise such Expectations, and yet he has done it in the strongest Terms.

In Page the sixth of his Treatise, having inform'd the Tenant, to whom he addresses his Letter, that on the nicest Calculations, the Rule for renewing seven Years lapsed in a Lease of twenty-one Years, supposing the Interest of Money at 6 *l. per Cent.* is near two Years and an half Value; with an Air of Insult, he asks him, if at this Time he can put out his Money on Land Security, and have 6 *l. per Cent.* for it: And in a Line or two after tells him, if his Landlord, on renewing his Lease, had treated him on that Foot, that he ought to be thankful that his Money was laid out securely upon Land, after the Rate of 6 *l. per Cent.* The Insinuations here, that a Person renewing his Lease at the Rate here mention'd, would have 6 *l. per Cent.* for his Money, are express'd in Terms so strong, or so artful, that from thence, and thence only, every Reader must have concluded that he meant to say such a Tenant would really make 6 *l. per Cent.* of his Money. In this Sense he has been understood by some, and must and will be understood by every Man, who does not attend to his subsequent Declarations; which plainly enough show, that something else was his Meaning.

Immediately after, he gives us what he calls a familiar Instance, to show the great Advantages which a Purchaser will make in laying out his Money at the Rate of 6 *l. per Cent.* in buying Leasehold Estates. He does not indeed assert, that

that a Purchaser of such Estates, in the Instance and on the Terms there mention'd, makes 6 *l. per Cent.* of his Money ; for he uses this wary Expression, that the Purchaser values his Money laid out at 6 *l. per Cent.* but does not say that he makes 6 *l. per Cent.* of his Money so laid out. But though he does not assert, he ought to have asserted and proved, that such Purchaser makes 6 *l. per Cent.* of his Money, if he would have proved any Thing pertinent to the Purpose ; for let the Valuation be what it will, if the effectual Produce does not answer the Computation, the Manner of computing the Interest, where two Persons dispose of Money in two different Ways, will never show which of the two has the greater Interest for his Money, or the better Bargain. We may compute the Interest at 6 *l. per Cent.* and in Fact make but 3 *l.* as I shall take Occasion, in another Place, to show, that a Purchaser, in the Instance here given, will make no more, if these inconvenient Draw-backs, the Land-Tax, Repairs, and casual Losses and Charges, are to come out of his Estate ; and we may compute the Interest at 5 *l.* and in Fact make 4 *l.* or 3 *l. 10 s.* and then there can be no Question who has the larger Interest for his Money, and the most advantageous Bargain.

And in this Case, if it were not necessary to assert and prove that such a Purchaser would make 6 *l. per Cent.* of his Money ; yet I think the Author, out of Regard to the supposed Ignorance of his Lease-hold Tenant, to whom he is giving Advice, ought to have been more explicit :

plicit: For we know all Men will not attend to nice Calculations, and perhaps all Men do not understand Expressions so well guarded. In Fact, so it comes out; for there is a learned Land Surveyor, who says, that a Purchaser of Lease-hold Estates on the Terms mention'd by this Reverend Author makes 7 *l. per Cent.* of his Money, and if he makes 7 *l.* to be sure he makes 6 *l.* and cites this Instance as a Proof and Demonstration of it.

Since we have mention'd this Instance, and it is brought to induce Persons to become Purchasers of Leases, and to prevail on Tenants of such Estates to pay their Fines with a free and a liberal Hand; and if an advanced Fine is paid, possibly it may be no great Matter whether Taxes, Repairs, and other Charges on the Estate, are or are not taken into the Consideration: And since the Author tells us 'tis a plain and familiar Instance and a Demonstration suited to every Man's Capacity, and the Writer just mention'd, I suppose, that he might not be deem'd to want common Capacity, very readily gives his Assent to it, as he does to whatever the other advances, though never so extravagant; and another, and a third Writer, quote it as a ruled Case that is to govern every Thing to be offer'd on this Subject; let us see what this Instance is. and what mighty Matters it proves.

The Reverend Author asserts, that 'tis better Husbandry to buy Leases at twelve Years Value, and to renew from seven Years to seven Years, after the Rate of two Years and an half Value,
or

or thereabouts, than to give twenty or twenty one Years Value for Lands of Inheritance; and to prove the Assertion gives this Case.

The Leasehold to be purchased, he supposes to be 100 *l.* *per Annum*, besides the reserved Rent: That if a Man gives 1200 *l.* for this Estate, he gives 800 *l.* or 900 *l.* less than he must give for an Estate in Fee, and consequently that he has 800 *l.* at least to improve; though he has the same Income as if he had laid out 2000 *l.* in Lands of Inheritance. And as for the 800 *l.* Surplus Money, this, at the End of seven Years produces 280 *l.* at 5 *l.* *per Cent.* Simple Interest; that if the Landlord should take 250 *l.* which is two Years Value and an half, for a Renewal, such Tenant would be a Gainer 30 *l.* besides the Improvement he might make of his Interest; and that if the Landlord take only one Year's Value, as Churches and Colleges now do, the Fine would be less by 150 *l.* (it should be said 180 *l.*) than the Simple Interest of the 800 *l.*; the Tenant consequently is a Gainer to that Value.

This Instance, though it may seem a familiar one, will never be admitted to be a fair one, because the Comparison ought not to have been framed between 2000 *l.* laid out in Lands of Inheritance of one Side, and on the other Side, 1200 *l.* laid out on Leasehold Lands, and 800 *l.* reserved at Interest, but the whole 2000 *l.* on both Sides should have been supposed to have been vested in Lands; and if 'tis good Husbandry to lay out Money on Leases, the more Money is
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laid out in those Estates, so much the greater Advantage will be reap'd from thence. But the Author, I suppose, stated his Case in this Manner, that his Lease-hold Tenant might be sure of a Fund or Payment of his Fine at the End of seven Years; and that being a provident Care for the Landlord as well as the Tenant, the Case shall e'en stand as here put.

But this Case, even as 'tis stated, is so far from being a Demonstration, that the Lease-hold Purchaser has the Advantage of the Fee Simple Purchaser, that it proves just nothing at all; because it asserts, or takes for granted, that the Owner of the Lease-hold Estate has the same Income out of his purchased Lands, as the Owner of the Fee-Simple has out of his; which, in Fact, is not true, and the very Case supposes it not to be true. The Case supposes, and if it did not suppose it, there necessarily must be a Rent reserved; now for this reserved Rent the Land-Tax must be paid; for this a proportionable Allowance for Repairs, and other Outgoings, must be made; and consequently the Income of one must be less than that of the other, just so much as these Charges on the reserved Rent amount to. I observe, the Author is so wary as not to mention what the reserved Rent might be: And if we suppose it to be equal to a third or a fourth Part of the improved Rent, the Deductions on those Heads will considerably abate such Tenant's Income: But be it more, or be it less, something it necessarily must be, and the very Case is so put; and therefore

fore the Demonstration, let it be suited to whose Capacity it will, fails, and comes to nothing.

This Reverend Author having assured us, that this is a plain and familiar Instance; tho' as plain as it is, we see some Persons have made a Shift not to understand it, or to understand it to prove something which it does not, nor, I suppose, was ever intended to prove: I beg Leave to make Use of it to show, as I think I can do, I will not say to a Demonstration, but to some Degree of Probability, that a Purchaser of a Leasehold will have a much worse Bargain than he that purchases a Fee-Simple, on the Terms here proposed. We will suppose then, that one Person purchased a Fee-Simple of the Value of 100 *l. per Annum*, about the Year 1718, the Time when this Treatise was first published; and because I am desirous that he should have a well-condition'd Estate, I will suppose that he gave for it twenty one Years Value, and that another Person about the same Time, being well instructed from this Instance how to buy Bargains, purchased a Leasehold of the same annual Value, over and above the reserved Rent, and gave for it twelve Years Value; and I will suppose this Estate too to be a well-condition'd one, and so I am sure I ought, and I will state the reserved Rent annually payable out of it at no more than ten Groats, if the Author will assure me any such is to be found in the Kingdom; and that this last Person disposed of 900 *l.* which is the Sum that the Leasehold Estate cost less than the Fee-Simple did, in Public, or
any

any other Securities, just as he shall think fit to direct : Let us see then how the present Case of these two Purchasers stands, and examine whether now, in 1736, they are the same ; or whether, and how far, they differ, and on which Side the Advantage, if any, lies.

I have stated the Rent payable out of this Leasehold Estate so low, that the Income of both Estates, arising from the meer Rents, must be admitted to have been the same, within a Trifle, ever since the supposed Purchase ; but 'tis not necessary to admit that the Owner of the Fee-Simple Land has all this Time made no Advantage by breaking up Meadow, or Pasture-Land, by cutting Timber, or by some casual or accidental Profits arising from his Lands, such Advantage as the Owner of the Leasehold has no Right or Power to meddle with. But not to insist on this ; though I must observe, if no such casual Profit has been made, the Estate is become so much the more valuable to the present, or any future Owner : Here I put the whole Difference on that single Point, the Rise of the Value of Lands, in the Purchase, within this Compass of Time.

On this Head, I do not think I make an over Estimate, when I compute the Value of the Fee-Simple Lands, I mean the Inheritance, at 500 *l.* that is at five Years Purchase, more than seventeen or eighteen Years ago ; and I am sure I make a fair Allowance if I compute the Value of the Leasehold Lands, I mean to be sold, at 120 *l.* that is, at one Year and near a Quarter's Purchase,

Purchase, more than in 1718: For the Fall of the Interest of Money from 5*l.* to 4*l.* does not make a Rise of full one Year and a Quarter's Value in a Term for twenty one Years, though it makes a Rise of five Years Value in Lands of Inheritance. This Computation, which is a fair and a plain one, and, I believe, suited to every Man's Capacity, leaves a manifest Difference of 380*l.* clear Gain, on the Side of the Owner of Fee-Simple Land; and when, and where, and out of what the Owner of Leasehold Land can make up this Difference, I believe, must be left to his learned Advocate, for I am sure the Tenant can't make it up out of this his Estate.

As for the 900*l.* supposed to be retain'd in such Purchaser's Hands, or at least not invested in Land, and the Interest arising upon it; the Author, or his Tenant, may dispose of it just as they in their great Wisdom shall think convenient. They are at Liberty to employ it in renewing the Lease at seven Years End, at any Rate they shall fix, or let it alone; they shall put out the whole, or any Part of it, at Interest at 5*l.* or at 10*l. per Cent.* and accumulate Interest upon Interest, if they can; and they shall suppose it, at seven Years End, to be 2000*l.* or at twenty one Years, to be 20,000*l.* just as they please: And yet, if all these fine Things were really something more than meer Speculation, they are nothing to the Purpose; for the Question is only about the Produce of Money laid out in the Purchase of Lands of Inheritance, or

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Lease-

Lease-hold Lands, and not about the Produce of Money which is not invested in Lands, but put out at Interest: And this, I think, plainly justifies my former Observation, that the Author's Comparison in the first Instance was not rightly instituted.

And since this Author so frequently puts us in Mind of the Probability of the Decrease in the Interest of Money, and so strongly inculcates the Necessity of having Regard to the Price paid on the Purchase of Fee-Simple Lands; after so much Provocation, I must do him the Justice to take some farther Notice of it. I have already observed what Influence such Decrease has on the Value of a Fee-Simple, and on a Lease-hold Estate in a Purchase; *viz.* that a Fall from 5 *l.* to 4 *l.* *per Cent.* enhances the Price of one full five Years, and the Price of the other only one Year and near a Quarter, that is, it advances one from twenty to twenty five Years, and the other from twelve and three Quarters, and something over, to fourteen Years Value. To which I add farther, that if the Fall of Interest be from 4 *l.* to 3 *l.* the Rise of Lands of Inheritance is to thirty three Years and eight Months Value, and the Rise of a Term for twenty one Years to fifteen Years one Quarter; and if the Fall be to two and a Half, the Rise, in one Case, is to forty Years Value, and in the other to sixteen Years and one Eighth. If this were pursued farther, and Money supposed to be so low as one *per Cent.* I believe, though I have not computed it as I have done
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in the other Instances, that it would come out, that a Lease for twenty one Years would not be worth twenty Years Value, and consequently the supposed Lease-hold Estate not worth 2000*l.* when 'tis plain and obvious to every Man's Understanding, that the supposed Fee-Simple Estate would be worth an Hundred Years Value, that is, 10,000*l.* I am surprized therefore, that any Person, who had the Decrease of Interest professedly in his View, and supposed the Thing might probably soon take Effect, and the Event has shown the Justness of the Thought, could conceive it to be better Husbandry to lay out Money in Lease-hold Estates at twelve Years Value, than in Fee-Simple at twenty or twenty one : And to urge this View and such Supposition, which he frequently does, as an Argument to prove it, is absurd and ridiculous.

I have taken the more Notice of the different Influence which the Variation of Interest has on the Value in the Purchase of the one Kind of Estates, from what it has in the other ; because I believe many Persons are apt to imagine, that on a Decrease of Interest the Value rises in the same Proportion in the one as in the other Estate ; which the Instances I have already given show manifestly to be a false Notion. And if we look back and examine what Influence the Decrease already incurr'd has had on these different Estates, we shall see the Falsity of the Notion, perhaps, still more clearly. When Money was at 10*l. per Cent.* Lands of Inheritance were worth ten Years Value, and now that

it is at 5*l. per Cent.* the Worth is twenty Years Value; at the one Time the Value of a Term for twenty one Years was about eight Years and an Half, and now 'tis about twelve and three Quarters, that is, in one Case the Increase in Value is fully double, and in the other one Half, or thereabouts.

Since then the Decrease of Interest has an Influence on the Value of both these Sorts of Estates, and where the Decrease is from 5*l.* to 4*l.* the Increase in Value will be in such a Proportion, that in an Estate of 100*l. per Annum*, the Rise, if it be a Fee-Simple Estate, will be 500*l.* and if it be a Leasehold 120*l.* only; and by the common Tables for Annuities this appears plainly to be the Case: It may be asked, What is become of, and who has the Advantage of, the remaining Sum, *viz.* 380*l.*? This is certain; if an Estate in Fee-Simple, which is let at the improved Rent, is advanced in Value 500*l.* by such Variation of Interests; an Estate in Fee-Simple, which is let on Lease at a small Rent, must be advanced in Value the like Sum one Way or other, where we suppose, as we do here, that the real Value and other Circumstances of both Estates are the same, the Rent reserved only excepted. In the Case then where a Term for Years is existing at a small Rent, the Term and Reversion may be and are considered as separate Estates, and as such may have a distinct Valuation set upon them; and the Value of the Term and Reversion put together, is one and the same as the Value of the Fee-

Fee-Simple entire. To the Question then, What is become of this 380*l.* and who has the Advantage of it? The Answer is, That it accrues to the Reversionary Part of the Estate, and belongs to the Landlord. But then with Regard to Church-Men and Colleges, whom this Author had chiefly in View, this Circumstance is of no Service; for the Advance in Value of this Part of the Estate will create no Advantage either to Landlord or Tenant; for it cannot be sold, mortgaged, or run into, any farther than by filling up the usual and ancient Term, or the customary Lives; nor, if the Leases were suffer'd to expire, can the annual Rent be raised meerly upon this Account: So that the whole Advantage coming to the one or the other, or both, from this Advance in the Value of the whole Estate, must arise solely from, and is confined to the Term, for the one cannot sell, or the other buy, more than the usual and accustomed Term.

It may, perhaps, be more material to have an Answer to another Question which may be ask'd, *viz.* Who is to have and will have the Benefit of the advanced Value in the Term here supposed, I mean the 120*l.*? I doubt the Reverend Author will find a Difficulty in answering this Question in such a Manner as shall be consistent both with the Advice to his Tenant, and with his Reasoning on Behalf of Landlords. If he admits that the Tenant is to have the Advantage of this Rise, he directly contradicts his own Principles; for this greater Value of the

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Term,

Term, unless there be a greater Rent, which is out of the Question here, is the sole Foundation on which Landlords can raise any Right or Pretence to advance their Fines, and his Arguments tend all to this Purpose. Nay, this is not only so far true, but even this lower Interest, which produces this proportionably greater Value, is the very Rule which he gives and urges to be the Rule by which the Quantity of such advanced Fine must be adjusted.

If the Author leaves his Tenant to make the best of his Bargain, and insists, as I doubt not but he will, that this Advance in the Value of the Term belongs to the Landlord; and it must belong to him, or he can receive none from this Change of Circumstances; and if he claims and takes this as a Perquisite appertaining to his Interest, as he has Power to do on subsequent Renewals, then the Tenant can have none at all: For we are speaking here solely of the Advantage arising on the Term, and 'tis impossible both the one and the other should have one and the same Thing; so that we shall be at a Loss to find out the Wholesomeness or Disinterestedness of the Advice here given.

A Tenant indeed, who purchased when the higher Rate of Interest prevail'd, and consequently bought at the lower Price, if he takes Time by the Fore-lock, and meets with an unwary Purchaser, may sell as soon as the lower Rate of Interest takes Place, and by that Means will have the Advantage of the Advance in the Value of the Term; but if he misses his Opportunity, and waits a Renewal, the Landlord, according

according to this Doctrine, has a Right, and possibly may advance his Fine, and then the imaginary great Bargain amounts to just nothing.

This then being the Case, that the whole Advantage arising by the Increase of the Value on Purchases accrues to the Purchaser of Lands of Inheritance; and little more than one fifth Part of such Increase comes to the Purchaser of Leasehold Lands, and even this little will be coming to him only on a Supposition that he makes the best of his Bargain that he can possibly make; that is, if he disposes of his Estate, and has the good Fortune to do it at the right Juncture of Time, or, upon keeping his Estate, if his Landlord is so good to him as to renew at the old Fine: If this be the true State of the Case, I suppose Persons will not be much at a Loss to find out where to dispose of their Money with the best Husbandry, if they are minded to vest it in Lands.

I intend to consider elsewhere, whether a Landlord has, and how far he has, a just Pretence to raise his Fine on the Decrease of Interest of Money; or, which is the same Thing, on the Increase of the Value of a Term of Years. Here I have been stating only the Difference between purchasing the one Estate and the other, on a Supposition of such a Decrease: And on the Comparison I think it appears very plainly, that in the one Case there is a very great Advance; and that the Right to it, the whole of it, is indisputably with one Party; and in the other Case, that the Advance, comparatively, is very inconsiderable, and that the

Right to it is at least very precarious with the other Party, since it depends on the good Fortune of the Proprietor, or the Good-nature of his Superior, of whom he holds it.

Hitherto we have been only discussing that Point, which of two Purchasers of an Estate of these different Tenures will have the better Bargain, on a Supposition that both of them buy with a View to part with, or in Fact do part with, their Estates soon after their Purchase; and upon the whole, I think it abundantly apparent on which Side the Advantage lies, and that it lies on the Side of a Purchaser of Fee-Simple Lands to a great Degree, if look'd on in this View. Let us go on now to examine how this Matter will stand, if we suppose both Parties to keep their respective Estates, and to employ the Rents and Profits of them to such Uses in Life as they have Occasion for.

This Author, in stating his Case, kept a Reserve of 800 *l.* or 900 *l.* in Hand, in Order to provide, as I supposed, for the Payment of the Fine for renewing at the End of seven Years. In that Respect he states his Case very judiciously, because he might think, possibly he might know, that such Tenants frequently delay Payment of their Fine, or wrangle and quarrel about the Quantity of it, more for that they have not the Money ready to pay the Fine, than that they have any just and reasonable Objection to make to it; and that when Payment can be no longer shuffled off, the Debate ends in a Mortgage of the Estate, this Mortgage continues
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'till another Renewal comes, and then an additional Mortgage is made, and so, *toties quoties*, 'till several Renewals have eat out the Principal. To prevent this Mischief, which but too frequently happens upon these Sorts of Estates, I must agree that the Provision here made is a very proper and a prudent one; But then I doubt it will be enumerated among the Inconveniencies which attend these Estates; and we are to seek the Remedy for it out of something which does not arise from the Estate itself.

In another Respect, and which is the only one that should have been here regarded, that is, whether it would be Prudence to lay out Money on such Estates, it must have been better to have supposed the whole Sum laid out in a Purchase; because, in Point of Computation at least, such a Purchaser must have had the Advantage, just as much as Money at 6 *l. per Cent.* if it were really to be had, will produce a better Income than the same Sum at 5 *l. per Cent.* and as a larger Estate, when sold, will yield a greater Gain in Proportion than the less Estate, on a Supposition of a Rise in the Value.

But what if this 800 *l.* or 900 *l.* which is so providently retain'd in Hand as a Fund to raise the Fines on Renewals, and out of which such vast Mountains of Gold are to be raised, should be short enough to answer the first Purpose, *viz.* to raise the Fine? The Interest of Money, since the Year 1718, has fallen, though not by Law, yet in Fact and in Practice, so that this Principal reserved and put out at Interest, sup-
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posing it to be 900 *l.* in seven Years Time, at 4 *l. per Cent.* will produce no more than 252 *l.* at Simple Interest; and if the Tenant was so wary and so happy as to accumulate Interest, and he must have a great Share of Sagacity and good Fortune too that can do it, yet the Produce would not be 300 *l.* Now if 'twere right, in 1718, for a Landlord to take, and prudent in a Tenant to give, two Years and an half Value, that is 250 *l.* for a Renewal of this Estate; then, in 1736, it would be Justice in the Landlord to demand, and Prudence in the Tenant to pay, near three Years Value, that is, near 300 *l.* for such Renewal. To which if we add the Charges of Journeys, Attendances, Fees, &c. to buy this Bargain at first; and the Pleasure of repeating it once in seven Years for ever; the Upshot is, that this mighty Fund, out of which such vast Savings were to be made, out of which such Heaps of Riches were to be amass'd, and a Fortune and Family to be raised, is barely sufficient to bear its own Charges.

But if this Fund were more than sufficient to answer the Purpose of raising the Fines, and should even produce a Surplus, as it will do on a Supposition that an Interest of 5 *l. per Cent.* is made of it, yet this Author does not see, or if he does, some of his auxiliary Writers do not see, that the Interest of this Money, or so much of it as is applied for Payment of the Fines, is sunk, and entirely lost to the Owner; so that the Proprietor of this Leasehold Estate, who has laid out 1200 *l.* in the Purchase of it, and
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has placed out 800 *l.* at Interest, has only 70 *l.* *per Annum* in the whole, as the Income of his 2000 *l.* that is no more than 3 *l.* 10 *s.* *per Cent.* for his Money. I say, the whole of the Income is no more than 70 *l.* *per Annum*, for the Interest arising from the Money, or the greatest Part, must be applied to the Payment of the Fine, and out of the 100 *l.* *per Annum* in Land are to be deducted the Taxes on it, the Repairs, and accidental Losses, which this Author states at 30 *l.* *per Cent.* and when these are deducted, as he contends they ought to be, the Neat Money left is only 70 *l.* The Income arising from the Lands of Inheritance being supposed to be the same, and in the present State of Things, the Surplusage coming from the Money at Interest being an inconsiderable Sum, and if Things were to remain on the same Foot that they now do ; yet I do not think, nor do I believe the most fond Admirer of these Estates can think, that this Circumstance only, *viz.* the Identity of Income arising from both Estates sets the two Owners in a Condition equally good.

But admitting it does so, if Interest of Money should fall, which this Author supposed very probable, and since the Publication of his Treatise in Practice it has fallen ; the necessary Consequence of such Fall is, that the Income arising from the Money will sink, and at the same Time, and for the same Reason, the Fine will be raised on Renewals ; and all the while the Rents of the Lands of Inheritance and of the Lease-hold will continue the same ; so that the
Income

Income of one probably may, is subject at least to, be diminish'd, when the other is liable to no such Contingency. And if we state this Case fairly, and as it ought to have been stated, and put both Persons on an equal Foot, we must suppose the Purchaser of Lands of Inheritance to lay out only 1200 *l.* in the Purchase of such Lands, and to place out 800 *l.* at Interest. If we do so, the Lands purchased will be about 60 *l.* *per Annum*; and if we allow for the Land-Tax, Repairs, and accidental Losses, after the Rate of 30 *l.* *per Cent.* the Rent Neat will be 42 *l.* the Interest of the 800 *l.* will be 40 *l.* and both together will be 82 *l.* which is a better Income than the Lessee-Tenant will see from his Land and Money, unless his Landlord be more kind to him in his Fine than some Writers are willing to allow he should.

After all, when the Debate is, What Advantages are to be made by purchasing Leasehold Estates, 'tis nothing to the Purpose to examine what another and a different Estate will produce, or whether a certain Sum of Money, not laid out in Land, but put out at Interest, will make such a Return as shall be sufficient to satisfy this or that, or any other Demand; since the sole Question here is, What is the natural and genuine Product of this Estate in particular? If the Tenant has an Opportunity, by artificial Aids drawn from another Estate, to help himself so far as that the Bargain shall not be an ill one, and, farther than that, this Contrivance will not carry him, yet this is only an accidental Relief,
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and does not arise from the Estate itself. And when we consider the Estate naked as it is, and stripp'd of this foreign Assistance; if we should admit that it might be purchased at a Rate where Interest is computed at 6 *l. per Cent.* and a Fee-Simple where 'tis computed at 5 *l.* only; and though we should suppose that the Land-Tax, Repairs, and accidental Losses, were not to be placed to the Termer's Account; yet even upon these Terms, I am not of Opinion that the Prudence and good Husbandry is on the Side on which this Author has put it. In Theory and meer Computation it must be with him, because, past all Peradventure, 'tis better to have 6 *l. per Cent.* if 'tis to be had, than to have 5 *l.* whether it be for Expence or for Increase, if there were nothing to be regarded but meerly the Income; nay, and if the meer Income is consider'd, I doubt, in Practice and in the Event, even this will not, and in Fact rarely does, answer such Expectations.

The Owner of a Fee-Simple has a fix'd permanent Estate; his Property is an entire Thing, lies together, and is ready to his Hand; and being vested in *Terrâ Firmâ*, is subject to no Hazards, Accidents, or Contingencies, none I mean but what a Lease-hold Property is equally liable to: For I have no Regard to the Insinuation, that a Tenant by Lease has an indisputable Title to his Estate, since there are many more Suits between Landlord and Tenant, either about the Estate granted, or the Terms on which 'tis granted, than there are between the
Buyers

Buyers and Sellers of Lands of Inheritance. But where and whence is the Tenant by Lease to have his Principal, or his Fine, if he has the Favour to renew? He is to have it by saving the Surplusses of the Rents of his Estate as they arise Yearly or Half-Yearly, by putting such Surplusses out at Interest, and accumulating Interest. But if his Under-Tenants, or any of them, omit making regular Payments of their Rents at the Day; if he himself misses an Opportunity once only during the whole Term instantly to dispose of his great Surplusses, and the Interest of such Surplusses; if in any Part of the Time he takes but one bad Security, nay, if a Security be only so far bad as not to make punctual Payment of his Interest; or if the Interest of Money should decrease within the Time: If any of these Contingencies happen, the Principal, or the annual Income, must so far fall short.

Now if this Author will show me one Leasehold Tenant who ever escaped all these Inconveniencies, which are not only possible but probable; I may safely engage to produce him an Hundred, who, instead of accumulating Interest, putting out their Surplusses at Simple Interest, or even of saving those Surplusses, have mortgaged their Estates to raise Money for their Fines on a Renewal. Therefore, however exact and regular this Scheme may be in Numbers and Figures, however fine and well it may look in Notion; yet we are to consider whether it can, or how far it can, be reduced into Practice,
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and what usually and commonly is the Event of such Undertakings : And we are likewise to take Mankind to be such as by Experience and in common Life we find them to be, and not that all Lease-holders are as provident, and as dextrous, *Rebus agundis*, as this Gentleman himself may be, or as he would have us suppose all such Tenants to be.

But then, in stating the Produce of this Lease-hold, if we charge the Land-Tax, Repairs, and accidental Losses on the Tenant, we have already seen that the clear Income from the Land and Money together is no more, or very little more, than 70 *l. per Annum*, or about 3 *l. 10s. per Cent.* which I can hardly persuade myself any one will think to be a sufficient Produce for Money so laid out. Nay, even this 3 *l. 10s. per Cent.* arises by calling in the Assistance of the Money not laid out in Land ; and if that be left out of the Case, as in all Reason it ought to be, it being an Aid entirely foreign to the Estate, we shall see presently, that the Net Income arising from the Estate solely will not be so much as after the Rate of 3 *l. per Cent.* for the Money laid out in the Purchase.

I would first take Notice, that Mr. *Richards*, whom I have mention'd before, admits that such a Tenant is entitled to a Compensation, in some Way or other, for Repairs, and other Out-goings, whether certain or casual ; but he excepts the Land-Tax, for a Reason peculiar to himself, *viz.* because, as he says, Money is chargeable to the Assessments on Land. That
Money

Money is by Act of Parliament made liable to the Aid on Land is undoubtedly true; but 'tis as notoriously true that it very seldom or ever is so charged: And when it becomes the common Practice to assess it, that there may be Time enough to debate that Point. However, I concur in Opinion with this Gentleman, that these Estates are to be look'd upon and treated rather as Money than Land; but it will be very hard on one Side to tax them because they are Land, and on the other Side because they are Money, for that will be placing the poor Tenant between the Hammer and the Anvil, and then he is sure to be crush'd to nothing.

But I say, in whichever Light we look upon the Lessee's Interest, it ought to be exempted from all these Burdens; and to prove it, argue thus. Whenever a Person takes a Lease for a Term of Years, or purchases one already granted, he must have this in his View, *viz.* out of the Rents and Profits of this Estate, to take so much yearly for his own Use as amounts to the Interest of the Money he lays out, computing it at such a Rate as he is contented to take, or might elsewhere have had for such Money; and with the Overplus of such Rents and Profits to re-imburse the Principal during the Term, or to raise Money for a Fine to renew. Supposing then, that the Rate of Interest on which he buys be stated at 6 *l.* and I think in all these Cases it ought to be so stated, that is at one *per Cent.* higher than common Interest, and under another Head shall give my
Reasons

Reasons for it, the Person may give for a Term of twenty one Years is 11 Years $\frac{2}{4}$ Value, as may be seen in the common Tables. Now tho' such Purchaser makes his Computation at 6 *l. per Cent.* and though he should receive the whole of the stated Income in neat Money; yet, in Fact, he will have little more than five and a Half for his own Use, as I shall take Occasion to show elsewhere. But if he must go further, and out of the Rents and Profits of this Estate must discount Taxes, Repairs, and accidental Losses, which this Author has asserted, and I have admitted, may amount to 30 *l. per Cent.* on the full or improved Rent; sure I am, that such a Purchaser must sit down contented with less than 3 *l. per Cent.* as the Interest of his Money to be applied to his own Use, or his Principal at the End of the Term, or his Fine to renew, will come short.

The Calculations, in common Use, for the Value of such Estates on the original Purchase, or on the Renewal of them, let them be framed or approved by Sir *Isaac Newton*, or the most exact Calculator in the Universe, are not applicable to these Estates, unless or until they are reduced to absolute Annuities; for the Truth and Justness of these Computations depend on that Point, that the Estates are clear of all Deductions: And to make Use of one and the same Rule to adjust the Value of Interests subject to Draw-backs, and of Interests subject to no Draw-backs, is absurd, to say no worse. And then to be told, as we are by one with a grave Face,

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how we may dispose of Money where'tis valued at *6 l. per Cent.* Interest, that is, in Purchases of Leases; and afterwards to be told by him, as we are almost in the same Breath, that the Owners are unreasonable and partial Wretches if they expect to make *6 l. per Cent.* of their Money, over and above Taxes, Repairs, and other accidental Losses; that is, if they expect to make *3 l. per Cent.* of their Money; for, in Fact, they will not make so much, if these Deductions are made out of their Estates: This is to suppose that the Generality of Mankind, at least one large Part of 'em, want common Sense; and that the Author had a Right to impose on them.

I have asserted, if these Incumbrances are to be charged on a Tenant, even supposing him to have purchased where Interest is computed at *6 l. per Cent.* that he will not be able to make so much as *3 l. per Cent.* of his Purchase-Money for his own Use; and to prove it, shall make Use of the Instance before mention'd, and, when applied to my Purpose, I hope it will appear as plain and as familiar as when applied to some others.

We will suppose then, the Estate to be taken, or to be purchased, is *100 l. per Annum*, over and above the Rent reserved, and that it is to continue for twenty one Years; and then, if Interest be computed at *6 l. per Cent.* the Price to be paid for it, if we state it at the exact Sum, is *1176 l.* Now if this *100 l. per Annum*, supposed to be the Tenant's Annuity, were a neat Income, free from all Reprizes, I agree that such Tenant will make about five and a Half *per Cent.* of his Money, and something over; but what

what Interest will he make, if the Deductions in Controversy are made out of his Estate? For there is but small Comfort in valuing his Money at 6 *l. per Cent.* in the Purchase, if in the Event it does not produce half the Sum.

We have supposed these Deductions, in common Cases, and in common Years, may amount to 30 *l. per Cent.*; and since we have grounded our Supposition on a very great Authority, and find it confirm'd by much Experience, I apprehend this will not be controverted. If the Rent reserved on the Lease of this Estate be only 10 *l. per Annum*, then the Value at the improved Rent will be 110 *l. per Annum*, and the Deductions out of this 110 *l.* if made after the Rate of 30 *l. per Cent.* will be 33 *l. per Annum*. In this Case a Fund must be provided out of the Estate for re-imburfing the Principal by the Expiration of the Term for twenty one Years; and this Fund, I say, must be 33 *l. per Annum*, for a less annual Sum, though computed at Compound Interest, and at 5 *l. per Cent.* will not produce 1176 *l.* in twenty one Years Time. Now when these three Sums, 10 *l.* 33 *l.* and 33 *l.* making all together 76 *l.* are deducted out of the improved Rent 110 *l. per Annum*, there remains to the Tenant for his Interest no more than the Sum of 34 *l. per Annum*, which is not 3 *l. per Cent.* for his Principal Money of 1176 *l.*

This Instance, I think, is suited to the Capacity of every Man, at least to the Capacity of every one who has common Understanding in Numbers and Figures; and evidently shows,

that in the Purchase and Renewal of Lease-hold Estates, the Articles of Taxes, Repairs, and accidental Losses, are not to be placed to the Tenant's Account, since no Man will pretend that 3 *l. per Cent.* is a reasonable Interest for Money laid out in that Manner: And even this 3 *l. per Cent.* arises on a Supposition that Compound Interest is in Fact made; which is almost as certainly not true in Practice, as in Computation 'tis true.

If we vary this Case, and put it on a Foot of an higher Rent reserved, we shall see still more plainly the Justice and Necessity of placing these Reprizes to the Landlord's Account, and not the Tenant's. If the Estate at the improved Rent be 200 *l. per Annum*, and the Rent reserved be 100 *l.* the Annuity left to the Tenant will be 100 *l.*; and if no Regard is to be had to any Out-goings but that of the Rent reserved, the Value of this Estate in the Purchase will be the same as of the former; the annual Sum to re-imburse the Principal must be the same here as before, *viz.* 33 *l.*; the Deductions, if computed at the former Rate, will be 60 *l.*: And if these three Sums, 100 *l.* 33 *l.* and 60 *l.* making in all 193 *l.* are deducted out of 200 *l.* the Remainder is no more than 7 *l.* as the Annuity or Income for the Principal Sum of 1176 *l.*

I might very well rest the Matter here, since this Instance demonstrates in Numbers and Figures, which are very stubborn Things, that these Reprizes are not to be made out of the Tenant's Estate: But since this Reasoning is
founded

founded on Suppositions, and those may be unfair or precarious ones, tho' this cannot well be objected to me, since I made not the Suppositions here, but assented to those ready made to my Hands ; I will proceed to show, that the Argument drawn from the Reason and Justice of the Thing is with me, as well as that in Numbers and Figures.

If an Estate were out of Lease, and let to a Tenant at Will at the improved Rent ; I would ask, Whether the Landlord then, let him be Church or Lay-Man, must not pay or discount the Land-Tax for the whole Estate ? The Collector of this Tax, or His Majesty's Receiver-General, to be sure will not be satisfy'd with some little Part, with a small Allowance, or with some equitable Considerations had of the Taxes, but must and will have the whole. And if it should so happen that the Dwelling-House, a Barn or two, or the like, belonging to this Estate, should tumble down, or want Repairs ; and such Things may happen in this the most prosperous State of the Nation ; I suppose the Owner would think convenient to rebuild or repair at his own Costs and Charges, rather than suffer the Estate to be untenanted. And if it be the constant and daily Practice for the Landlord to pay or allow the Taxes and Repairs to his Tenant at Will, or to make him a Satisfaction for them by an Easement in the Rent, and the Right and Justice of the Thing is that he should do so, which no Man can controvert ; then it must be right and just for him to pay or

allow the same to his Tenant by Lease: For 'tis impossible to show the Difference between the Cases where a Person receives the whole Produce of his Estate by Yearly or Half Yearly Rents, and at several Instalments; and where he receives such whole Produce, Part by Way of Anticipation, and as a fore-hand Rent, and the remaining Part by Yearly or Half-Yearly Rents, and at several Instalments: Therefore, where the Tenant cannot have an Allowance for the Land-Tax and Repairs out of the Rents paid by Instalments, as on a Lease at a small Rent he cannot have, he must, and in Equity and Justice ought to have a Compensation for them out of the anticipated or fore-hand Rent, that is, out of the Fine, whether paid on an original Grant, or on a Renewal.

To pursue this Matter a little farther: If one of these Estates, supposed to be out of Lease, at the improved Rent were worth an 100 *l. per Annum*, and the Possessor were desirous of a certain Rent, the best he could get, without taking any Fine or Premium; would any Man of common Honesty and Integrity expect to have, or any one of common Understanding agree to give, an absolute Rent of 100 *l. per Annum* for it; that is, charge himself with the Land-Tax and Repairs, without having a Consideration for them, either by Abatement of the Rent to be reserved, or by Discount on it when to be paid? And if a Fine were to be paid on sinking Part of the Rent, could any Proprietor in Conscience ask, or would any Tenant of common Sense pay, a
Fine

Fine for such Part of the improved Rent as was to be abated or discounted for the Land-Tax and Repairs? Or if so much Rent only were sunk as was just equal to these Out-goings, could the one ask, or would the other pay, any Fine at all? It is ridiculous in Notion, and in Practice would be something worse; and if this be the Case in a Grant on an original Lease, as undoubtedly it is, it will be impossible to make out that a Grant on an old Lease renewed differs in the Reason and Justice of the Thing.

For these Reasons I am clear of Opinion, that the Land-Owner is to pay or allow the Taxes and Repairs; so let us now consider how these Articles are to be adjusted; and these, I think, depend on, and must be determined by the particular Circumstances in this Kind, attending every individual Estate to be dealt in. Though there seem to be some Difficulties in the Way, yet I observe they are no other than what occur in the Purchase of Lands of Inheritance; and which, I think, with some Enquiries made, a little Experience and Discretion used, and some Share of Honesty observed, which in all these Cases is necessary, may be conquer'd.

As for the Land-Tax, though the Rate of it in the different Parts of the Kingdom varies very much, to a Degree beyond what could well be imagined, if Matter of Fact did not prove it, and in the same County in different Districts there is great Disproportion, yet the Rate at which any particular Estate is assess'd, by proper Enquiries, may be known; and when a Yearly

Payment is ascertain'd, the Payments for a Number of Years may be settled by a proper Medium: And if we take the Mean between the highest and lowest, and for a Term, I suppose 10 *l. per Cent.* will be a moderate Computation.

The Matter of Repairs has still more Difficulties in it; for, besides that every individual Estate has its particular Circumstances of this Kind, it may not be right, perhaps, to state the Charges for a Number of Years from that of any one Year: And yet even this may be got over; for by looking back some Years, and computing what has been the Charge, and by looking forwards and estimating what the future Charge may be, and upon a View or Enquiry of the Quantity and Quality of the Buildings on the Premises, some Sort of reasonable Measure might be framed by Persons of Skill in these Affairs; and if 10 *l. per Cent.* were fix'd for common and usual Cases, with an Exception to such as have any Particularities in them, I believe we should not be far off the Mark.

The Reverend Author, whom I have so frequently before mention'd, supposes Taxes and Repairs, throwing in accidental Losses, to be 30 *l. per Cent.*; and I would state them at the same Rate, if he would give me Leave to add an Article of Charge, for managing the Estate and receiving and paying the Rent; but, in Truth, unless this Article be included, which, there does not seem to be the same Reason for in Lands of Inheritance as in Lease-holds, I think the Author here stretch'd the Point a little,

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to serve a Turn ; for his Argument led him to depreciate the Value of Lands of Inheritance, and he therefore states these Deductions at the most advanced Rate ; and never considers, that having laid the Load of these Deductions on his Lease-hold Tenants, he does thereby sink the Value of their Estates in the same Proportion.

Though I have here stated 10 *l. per Cent.* on the improved Rent, as a reasonable Allowance for Repairs in common Cases, yet there are Exceptions to be made out of this general Rule : For where an Estate consists in the whole, or the greatest Part, in Meadow or Pasture Land, no Repairs, or very little, can be call'd for, even during a long Term : On the other Side, if the whole, or greatest Part, of the Estate consists in Houses, though they be in the best of Repair and Condition, yet from the natural Decay of Buildings, from Change of Tenants, and other Accidents, which in the Course of some Years must happen, it will become necessary to lay out Money in Repairs, in Alterations, or in Ornaments, or it may be impossible to keep up the Rents of such Houses : To this if we add the Hazard of Want of Tenants during some Part of a Term, for which Time, in the Case of Houses, there is a Loss of the total Profit : If I should state these Charges and Contingencies at 20 *l. per Cent.* I believe it will not be thought extravagant ; and Cases undoubtedly may be put, where, from the ruinous State of the Houses, their unfortunate Situation, or some other bad Qualities accompanying them, it would not

not appear too much to allow one half of the real Rent on this Article.

I know, in this and such like Cases, the common Method and Practice is, to allow a Tenant or a Purchaser an higher Rate of Interest, as a Compensation; but this, in Truth, is a fallacious one; and though it may not lie open to the Eye of a common Observer, for all the Writers on this Subject that ever I met with use it and commend it, yet 'tis greatly to the Prejudice of any Person's Purchasing, and will not answer the Purpose design'd, unless the Rate of Interest be set so high as would appear extravagant to all Mankind.

To show how these two Methods differ from one another, and what are the several Results of each, I will take Leave to put a Case, which, perhaps, may better explain my Meaning, and at the same Time show which of the two is more eligible. We will suppose that an Estate in Houses is to be let, or sold, for a Term of twenty one Years; and that the Rent of it, clear of all Deductions, excepting this one Article of Repairs, is an 100*l. per Annum*; and that it will cost 50*l. per Annum, communibus Annis*, for the Term, to keep it in Repair: In my Method I reckon the Money laid out by the Tenant in the Purchase at 6*l. per Cent.* and then the whole Rent being supposed to be an 100*l. per Annum*, and the annual Sum to be paid, and which I say is to be put to the Landlord's Account for Repairs, being 50*l. per Annum*, if this be deducted out of the former Sum, the Neat Rent remain-

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ing to the Tenant will be 50 *l. per Annum*; and this being the Interest which the Purchaser is to enjoy and to pay for, the Price to be paid, Interest being reckon'd at 6 *l. per Cent.* will be 588 *l.*

In this Way of putting it, the Purchaser plainly sees what he buys, and the Price he pays for it, and 'tis evident that he has an adequate Compensation for taking upon himself the Repairs; for they are supposed to be at the Rate of 50 *l. per Annum*, and for doing this he has 50 *l. per Annum* allotted to him, by Way of Discount; and with the other 50 *l.* the whole being supposed to be 100 *l.* he pays himself Interest for his Money laid out, and re-imburfes the Principal during the Term.

In the other Method, which is to allow a Purchaser an higher Rate of Interest, Who shall say what is such an higher Rate of Interest as is just an Equivalent for these Out-goings in Repairs, and how shall we determine the Price to be given for the Estate? I'll take upon me to say, and no great Undertaking neither, that it cannot be adjusted but by one of these two Ways; either by deducting this Charge as an annual Charge out of the supposed annual Rent, and so valuing the Estate at 50 *l. per Annum* only, which is the Method I propose; or by putting a Value on this Incumbrance in a gross Sum, and then deducting such Sum out of the Purchase-Money to be paid, valuing the Estate at an 100 *l. per Annum*: But then the Value of this Incumbrance, in the Gross and for the whole Term, cannot be known

known but by stating first what will be the annual Charge; and this, in Effect, is my Method still, only we take a longer Course to come at the same End.

If a Person, without Computation in one or other of these Ways, will sit down to Guess-Work, and determine what is the higher Rate of Interest to be reckon'd in this Case, I doubt he would run himself into an Error; for, I believe, most Men would be apt to conclude, that an Allowance of 12 *l. per Cent.* for the Purchase-Money, upon the whole Rent of an 100 *l. per Annum*, would be equal to the Allowance of 6 *l. per Cent.* upon 50 *l. per Annum*; whereas if the common Tables are consulted, it will appear, that for an 100 *l. per Annum* at an Interest of 12 *l. per Cent.* 750 *l.* and above, must be given, when, in my Way of computing, no more must be given for this Estate than 588 *l.*; and 'tis worth no more. But if he will call the Estate here an 100 *l. per Annum*, and will state his Interest at such a Rate as will repay him his Principal, Interest, and the supposed Charges of Repairs, he must then reckon it at 17 *l. per Cent.* as near as can be; which, as I observed before, is so great a Rate of Interest that a Man could hardly think of, or have the Face to demand, and yet at so much he must state it, or he will be a Loser by the Bargain.

Having consider'd, under the Head of Outgoings, what Articles of Deductions are necessary to be made to reduce these Estates to absolute Annuities, and in what Manner, and at what Rate

Rate these Reducements are, in a general Way, to be made, leaving a Latitude for peculiar Circumstances that may attend particular Estates ; we have now to settle the Articles of Allowance to be made to the Tenants, for their Trouble and Charge in managing their Landlord's Estate, and for assuring it and the Rents from accidental Losses ; which I choose to divide into two Heads, and to set each at *5 l. per Cent.* on the improved Value.

I suppose a Landlord, whether Ecclesiastical or Lay, does not expect that his Tenant will be his Bailiff, and set and let his Estate for him, and inspect the State of it from Time to Time ; that he will receive and pay his Rent, and that, perhaps, at a Place many Miles distant from the Place where the Estate lies, at a Time, to be sure, many Months before he can receive it from the Under-Tenants, and that he will be out of Pocket, at least, to the Amount of half a Year's Rent during the whole Term ; all these Things no one can imagine will be coming to a Landlord from a Tenant, out of stark Love and Kindness : No ; the Tenant has, in this Case, undoubtedly, a Right to be consider'd for his Trouble as well, and as much, as any other Person would be who should take these Matters upon himself ; and for the Management of this Estate, where there's added an Engagement to pay at a Day and Place certain, which is more than Stewards or common Agents undertake, the Tenant, to be sure, may well deserve *5 l. per Cent.* at least, on the improved Rent, when

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'tis no more than the customary Allowance, in many the like Cases, to meer Stewards.

As to the Premium to the Tenant, for assuring the Estate against the Casualties of Fire, Water, and accidental Losses; I state that likewise at *5 l. per Cent.* in the like Manner: And these two Allowances, put together, amounting to *10 l. per Cent.* this Sum is the Medium, or near the Medium, at which the Author, so often cited, values these Undertakings and Charges in Estates of Inheritance; for out of such Estates he makes a Draw-back of *30 l. per Cent.* in one gross Sum, for Taxes, Repairs, and accidental Losses. In my Way I have divided these Reprizes into *10 l. per Cent.* for Taxes, *10 l.* for Repairs, and *10 l.* more for accidental Losses, or rather for managing and assuring the Estate and Rent. If the Partition of these Deductions, thus made by me, will not suit with all Estates alike, as undoubtedly they will not, a Deduction in a gross Sum will still less suit; but in this Method the Parties interested, both of one Side and the other, will have a more clear View of the State of their Case, and be able to form a better Judgment, whether on any, and on which Articles an Addition or a Subtraction may be made, than when all are thrown into one Article. Upon the whole, when these Out-goings and Allowances are deducted, the Rent remaining to the Lessee, which, in common Cases, may be about *70 l. per Cent.* on the improved Rent, is the neat Rent, or the absolute Annuity, which the Tenant is to pay for in his Purchase.

Instead

Instead of this Allowance of 10 *l. per Cent.* to the Tenant, for his Trouble and Charge in managing and assuring the Estate of his Landlord, or to make him a Compensation for the Hazards which he runs on his Side, that he shall be a Loser by some Contingencies or other, during his Term, and that his Superior, on the other Side, is made secure in all Events; Mr. *Richards*, in stating the Value of Leases, makes Use of another Method of Computation to adjust this Difference. He reckons the Out-goings, that is, whatever is paid to, or on Account of the Landlord, as an Annuity certain, as it really is, because in all Events it must be paid, and first paid, and values it at a Rate of 4 *l. per Cent.* Interest; and the Annuity belonging to the Tenant, either on Purchase or Renewal, he reckons as an uncertain one, as it really is; because this Annuity is to arise out of the Surplussage of the Rents of the Estate, and those Rents may not be sufficient to answer both Annuities, and values it accordingly at 6 *l. per Cent.*; and this different Valuation of the Annuities he judges to be an Equivalent to the Tenant for his Undertaking, and then subtracting one Value from the other, he states the Remainder as the Value of the Lease.

That a Tenant, or a Purchaser, is entitled to some Compensation for his Undertakings of this Kind, is agreed between us; and on my first Thoughts upon this Scheme for that Purpose, I confess I came into this Notion: But upon a Calculation of the Equivalent which would arise

to a Tenant on such Scheme, upon Supposition of greater or less Out-goings, I have alter'd my Opinion ; for this Reason, because where the Out-goings are large, this Method produces too great a Satisfaction, and where they are small, too little a Satisfaction for this Purpose. My Notion here is, that the Provision for such Purposes ought in all Instances to be equal ; I mean that it ought to be one and the same for an Estate of one and the same Value, be the Out-goings more or less ; for that some very odd Consequences will follow from a different Provision.

This Author observes, that Cases may be so put, that the Value of a Lease of a shorter Continuance may be greater than the Value of a Lease of a longer Continuance on the same Estate ; and this, he says, will be the Case, where the Sum paid out is large in Respect to the yearly improved Rent, and at the same Time the Rate of Interest to be allow'd a Purchaser is very great, on Account of the ruinous Condition of Houses, or the like ; the Consequence of which must be, that it may be worth the while to take a Lease of such Houses for twenty one Years ; but when seven of them are expired, it may not be so to renew the Lease, and to make it up twenty one again.

From this Observation, which is a just one, he raises an Objection against his own Method of allowing a Purchaser a greater or less Rate of Interest for his Money laid out, as the Estate is more or less liable to Casualties, or the Charge
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of Repairs are greater or less. To this Objection, urged thus far and no farther, and to that Purpose he mentions, and no other, *viz.* where the Out-goings for Repairs are very great, his Answer may be right enough, *viz.* that Houses may easily be supposed in such a State as to stand and be kept up at a reasonable Charge for a small Term, but in the Compass of a longer must be rebuilt in Part, or in the whole, so as the shorter Term shall be of greater Value than the longer.

But this Observation may be carried farther, and has more in it than this Author saw, or at least has taken Notice of, *viz.* that this will be the Case, that a Lease for a shorter Term will be of greater Value than a Lease for a longer Term, supposing his Manner of calculating be a right one, not only where the Out-goings for Casualties or Repairs are great, but even in all Cases where the Out-goings are to a certain Value, let these Out-goings be of what Kind soever you will: For Instance; if the real Value or improved Rent of an Estate be an 100 *l. per Annum*, and the Rent reserved to the Landlord be 80 *l. per Annum*, or four Fifths of the whole; though there be no Out-goings for Taxes or Repairs, or if the Landlord takes them upon himself; yet if the Method here proposed for adjusting the Equivalent to a Tenant for undertaking to pay this Rent in all Events, is a right one, it must follow, that a Lease of such an Estate, though it should consist wholly of Meadow or Pasture Ground, for seven or fourteen Years, is of some Value; for twenty one Years,

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of less Value; for thirty one Years, at Par, or thereabouts; and for fifty two Years, worth less than nothing considerably: And this, I think, has so much of the Absurd in it as to be an Objection to his Method, when applied to purchasing, and has more of it, if used, in renewing.

I know few or no such Church or College-Leases can well be supposed; but it may be supposed, and in Fact is true, that Gentlemen of great Estates frequently let them in such a Manner; taking upon themselves the Land-Tax, allowing Timber and Materials for Repairs, and putting Contingencies and Casualties on their Tenants, and then reserving an absolute Rent without a Fine. Now to say, that an Estate of an 100 *l. per Annum*, let at 80 *l. per Annum* Rent on such Conditions for a Term of thirty or fifty Years, is of no Value, or worth, less than nothing, is absurd; and yet 'tis a necessary Consequence of this Method of adjusting this Equivalent. But in my Method I state an annual Premium of 10 *l. per Cent.* to the Tenant for taking upon himself these Contingencies and Trouble, and deduct it out of the improved Rent, and then there remains 10 *l. per Annum* neat Annuity to the Tenant; and in this Way of computing 'tis plain, that a Lease of such an Estate, and upon such Terms, is of some Value, and increases in Value in Proportion to the Number of Years for which 'tis to continue; and I believe all Mankind will judge that it must do so.

I have before asserted, that this Method provides too great an Equivalent where the Outgoings

goings are large, and too small an one where they are little: And, upon a Computation, it comes out, where these Out-goings are 80 *l.* in the 100 *l.* or four Fifths of the whole Rent, the Deduction to be made under this Head is 20 *l.* *per Cent.* on the improved Rent, which surely is too favourable to Lessees: On the other Side, where the Out-goings are low in Proportion to the whole Rent, suppose 20 *l.* in the 100 *l.* or one Fifth of the Whole, then the Deduction to be made on this Head will not amount to so much as 5 *l.* *per Cent.* on the improved Rent; which seems to me to under-do the Matter as much as in the other Case it over-does it.

I must confess I don't see how the *Quantum* of the Out-goings can in any wise be made a Rule to govern these Allowances; for whether they be great or small, the Charge and Trouble of managing the Estate is the same, the casual Losses incident to it are the same, and both depend on the *Quantum* of the improved Rent, and nothing else: But to make the Out-goings a Rule to regulate the Value of this Charge and these Losses, is to make something a Measure of the Value of the Charge and Losses, which is plainly no Measure, either of the Charge or of the Losses. The Charge and Trouble of setting and letting the Estate, of receiving and paying the Rents, and the like, arise from and must be govern'd by the Quantity of the whole Estate, and not by the *Quota Pars* coming to the Landlord and Tenant respectively; and the Casualties and Losses which may happen must

and will happen in, or in Proportion to, the whole, and not to any particular Part that may be going out.

It is, or perhaps may be, urged, that where a Tenant undertakes for so large Out-goings as four Fifths of the full Value of the Estate, he runs a greater Hazard than if he undertakes for one Fifth only : And in one Case, that it may come out, by Losses which may happen, that some Years the Estate will not be sufficient to answer the Demands to be made upon it ; since the Out-goings certain are supposed to be four Fifths of the whole, and consequently that the Tenant must make good such Deficiency out of his own proper Estate : But, on the other Side, that this cannot be supposed ever to happen in an Estate where the Out-goings are only a fifth Part. Admitting that this may happen where the Out-goings are large, as it may, and sometimes does ; and admitting it can never happen, where those Out-goings are small, yet the Loss to the Tenant is the same, whether the Payments on these Articles are one, or four Fifths of the whole Rent. In the first Case, if the Lease Estate falls short of answering the Demands upon it, the Tenant must make them good directly and immediately out of his own separate Estate ; and in the latter Case, though the Lease Estate does not fall short of answering these Demands where they are the largest that well can happen, yet such Estate is the Fund out of which the Tenant is to receive his Annuity or Interest, call it which you will, and out of which he is likewise to re-imburse his Principal ;

Principal ; and upon such Contingencies happening, as we are now speaking of, this Fund must be deficient with Respect either to the Annuity or the Principal, and such Deficiency must be made good out of such Tenant's separate Estate : So that directly or indirectly, and in the Event, this Loss falls upon, and must be supply'd out of, the Tenant's proper Estate, in the one as well as in the other Case.

All that I shall observe farther on this Head is, that if these Out-goings are about two Fifth Parts, or something between that and a Moiety of the whole Rent, then the Value of an Estate for a Term of Years, either in the Purchase or Renewal, will be the same both in this Author's and my Method of making and computing these Allowances : And those who think his to be a reasonable and equitable Provision between Landlord and Tenant, and are so far conversant in Numbers and Figures as to go through his Operation, now they see the different Results of both Methods, are at Liberty to be sure to make Use of his ; though mine seems to me the most equal, and in Practice, I believe, will be found the more easy and more safe one. In the main, we agree that such an Allowance is to be made, and that some Method of obtaining it must be observed, or the Price of such an Estate will not be fairly and evenly adjusted.

There is an Objection remains behind, which goes to all these Articles, the Taxes, the Repairs, and accidental Losses, or to the Allowances for managing and assuring the Estate ; for

which Reason I reserved the Consideration of it to this Place ; and it arises from the usual Covenants in such Leases. The Allegation then is, that the Tenants, by the Covenants in such Leases, have obliged themselves to pay the Land-Tax, to keep their Estates in Repair, and to answer all Losses and Out-goings, whether certain or casual ; and have also thereby undertaken to pay the Rents at the Places and Times fix'd, though it should prove never so expensive or inconvenient. All this is certainly true, but is fond Talk to amuse themselves, or to impose on ignorant or unwary People ; for where such Engagements are enter'd into, as in most Leases belonging to corporate Bodies they are, it must be taken for granted, in Regard to that Justice and Equity which is supposed to be between Landlord and Tenant, and that common Sense which a Tenant, unless carved out on Purpose to serve a Turn, may have, that a Compensation, adequate to these Covenants, is, some Way or other, provided for such Undertakings. The Question then is not, What are the Effects of such Covenants, and what will be the Consequence of them ; for every Man knows that they must be perform'd, if the Party is able to perform them : But, Whether any, and what Compensation, in Reason and Justice, a Man is intitled to, upon or for entering into such Covenants, and taking upon himself these Charges and Incumbrances.

Since this Objection has but little Weight in it, and whatever it has, the same has been, in a
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great Measure, obviated by what I have already alledged under another Head, where I have proved that the Articles of Taxes and Repairs are not to be placed to the Tenant's Account ; I shall only add here : The Case being so, that these Covenants must of Necessity continue to be inserted in Church and College-Leases, or otherwise they would not be valid ; if, by Virtue of such Covenants, the Owners of those Estates are now become burden'd with Incumbrances which were either not at all in Being, or were of small Significance, when those Covenants were first inserted ; and if it be reasonable, as the State of such Incumbrances now stands, to make such Tenants a Compensation for such new Burdens, which were then unforeseen, and therefore not possible to be provided for, the Justice of which, I think, no Man can deny ; and since no Equivalent can be contrived for such Lessees, but from the Fine to be paid on Renewals, out of the Fine it ought and must come, and be discounted.

Having offer'd to Consideration the several Articles of Deductions and Allowances, which I apprehend the Owners of Leasehold Estates may fairly insist on in Order to reduce such their Estates to absolute Annuities, and in what Manner, and at what Rate such Reducements in a general Way may be made and computed ; I shall proceed now to consider and state what may be a reasonable Interest to allow them, for the Money they lay out in purchasing or renewing such Leases.

I think then, as I proposed at first, that for the Principal Money laid out in the original Purchase, or in the Renewal of those Estates, Interest should be computed at 6 *l. per Cent.*; but I do not mean here, that on such Computation the Interest, or Annuity, coming to the Purchaser for his own Use, should be, or will be 6 *l. per Cent.* for his Money so laid out; for I know of Necessity it must be less. To see how this Matter stands, and what Interest will in Fact be coming to one purchasing on such Terms, we will state a plain and obvious Case; and from thence every Man may draw his own Conclusions, and form a Rule for his own Practice, just as he likes best. We will suppose then the Estate to be purchased or renew'd, at the improved Value, is worth 200 *l. per Annum*; the Out-goings and Allowances, which under the former Heads we have stated at 30 *l. per Cent.* must here be 60 *l.* annually, and if the Rent reserved to the Landlord be 40 *l. per Annum*, when these two Sums are deducted, the neat Income remaining to the Tenant will be an 100 *l. per Annum*. For a Term of twenty one Years in this clear 100 *l. per Annum*, I say, a Person may give 11 $\frac{3}{4}$ Value, or in Money 1175 *l.*; and for renewing it at seven Years End may give two Years Value, and near an Half, or in Money 246 *l.*; and this is the Price to be paid for the one and the other, Interest computed at 6 *l. per Cent.*

I don't doubt but most Persons have a Notion, we have seen in Fact some have, that a Purchaser on these Terms makes 6 *l. per Cent.* of his Money,

ney, but this is a great Mistake; for if we enquire what it is that such a Person proposes to himself, and what he is to do, and must do, on such a Purchase; we see most plainly, that his Expectations are, and the Effect of his Undertaking must be, that the Rents and Profits of the Estate purchased shall yield him a reasonable Interest for his Money, to be applied to such Uses of Life as he shall see convenient; and a farther Sum, to be applied to re-imburse the Principal by the End of the Term, or to pay the Fine from Time to Time, as he shall renew the Lease.

The Interest of 1175*l.* the Sum supposed to be laid out in this Purchase, at 6*l. per Cent.* is 70*l. 10 s.*; and if this Sum be deducted out of the 100*l. per Annum*, which is the Neat Income left to the Tenant, the Surplusage will be 29*l. 10 s.* Now this Surplusage will not produce a Principal of 1175*l.* within the Term, but upon a Supposition that you can, and in Fact do, make 6*l. per Cent.* of it, and that you can and do put it out at Compound Interest; and all this at regular and stated Times. But 'tis notorious, that by Law you cannot let it at more than 5*l. per Cent.* and almost as notorious, that in Practice you cannot get so much; the necessary Consequence of which is, that the Purchaser must sink his Annuity, or lose Part of his Principal, in Proportion to such Decrease in common Interest.

The Fund, out of which the Return of the Principal is to be made, and out of which the
Interest

Interest or Annuity is to be paid, being one and the same, *viz.* the neat Rent; in Point of meer Computation, 'tis one and the same Thing whether the Principal or Interest falls short; but since, in Point of Prudence, the Proprietor would chuse to sink his Annuity rather than his Principal, I will suppose this Deficiency in the Fund, which is an unavoidable one, to fall on the Annuity solely. If then the Purchaser, out of the Rents of this Estate, will reserve such a Sum as will be sufficient, at a Rate of 5 *l. per Cent.* to make a Return of the Principal, he must apply annually 33 *l.* to that Purpose, and he must make both Simple and Compound Interest of it likewise; and then there remains to the Owner for his Interest, and as the *Quantum* which he may employ for his own Use, no more than 67 *l. per Annum*; which comes out to be 5 $\frac{3}{4}$ *per Cent.* for his Money, or something less: So that the Law has cut him short $\frac{1}{4}$ *per Cent.* at least, if there were nothing else in the Case.

But then we are to consider farther, that for large Sums the common Practice now is to accept 4 *l.* or 4 $\frac{1}{2}$ *per Cent.*; and for small Sums, such as this Surplusage is, very great Trouble and Difficulty will attend the placing them out at all, and to do it in the still smaller Sums of the growing Interest, to be sure, is impracticable. And if both these were, and where they are, practicable, yet the Proprietor of the Estate must wait for doing it 'till his Tenants pay their Rents, and his Securities pay their Interest; and though none of them should prove so bad

as to be totally insolvent, yet the Delays in such Cases are so notorious, that no prudent Man will or does reckon on those Payments at the stated and limited Times to an Exactness, and yet the Justice of the Computations, both on Simple and Compound Interest, depends on the Punctuality of Payment, even to the Nicety of a Day. These Contingencies, though there are others attending the Affair of Interest, I think, may modestly be computed to sink the Tenant's Annuity $\frac{3}{4}$ *per Cent.* more, and reduce it to 5 *l.*; and I believe the World will judge that he is not only a provident Tenant, but an happy one too, if he can make it hold up to this Rate through all these Difficulties.

If we examine this Matter more closely, and compute what Proportion of this 1175 *l.* Principal Money is severally to arise from these Surpluses, what Part from the Simple Interest, and what from the Compound Interest, of such Surpluses, we shall be able to form a better Judgment what Part of his Income of an 100 *l. per Annum* he may apply to his own Use, safely and consistently with re-imbursing the Principal. We have already seen that he must make a Reserve of the annual Sum of 33 *l.* as a Fund, for the Purpose of re-imbursing the Principal, for that no less annual Sum, even computed at Compound Interest, will produce 1175 *l.* in twenty one Years; and, upon a Computation, it appears, that from the meer Surplusage, *viz.* the 33 *l. per Annum*, there arises in twenty one Years no more than 693 *l.* so that the remaining
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Sum of 482 *l.* must arise from the Interest ; and it appears likewise, that from the Simple Interest only of those Surplusages there arises in the same Time no more than 346 *l.* 10 *s.* so that there is a Deficiency of 136 *l.* 10 *s.* which is left to arise and be gain'd solely by accumulating Interest. If Compound Interest is not taken into the Consideration, and in Truth it hardly deserves it, and the Surplusage of Rents, and Simple Interest of such Rents, be look'd on as the Fund to raise this Principal, the annual Sum to be reserved for this Purpose must be 38 *l.* and above ; and then the Annuity left to the Proprietor of the Lease, for his own Use, will be near 62 *l.* and about 5 *l.* $\frac{1}{4}$ *per Cent.* for his Money : But even this is grounded on something which undoubtedly in Practice is never the Case, that the whole of the Rents is punctually paid to a Day at the Year's End, and the Overplus as regularly placed out. If Simple Interest be likewise left out of the Consideration, as it generally may be, if Consideration is then only to be had of it when and where it is actually made, and the meer Surplusage of the Rents be the only Fund for raising this Money, then the Sum reserved for that Purpose must be the exact Sum of 56 *l.* *per Annum* ; and then 44 *l.* being left as the clear Annuity, does not leave full 4 *l.* *per Cent.* to the Tenant, as Interest for his Principal Money. Having thus shown the Manner of making Computations of this Kind, and fairly stated how and from whence the Money is to arise to make good such Computations, I leave

it with the Reader to determine what Part of his Rents a Purchaser of such an Estate, and on such Terms, may safely apply to his own Use, without Danger of diminishing his Principal.

If any Leasehold Tenant should be so unreasonable and partial to himself, as to think that the Overplus Rent only should be look'd on as the Fund for raising this Principal, or the Fine to renew, and that no Regard ought to be had to the Interest of such Overplus, either Compound or Simple; or if any such Tenant should be so unwary or so unhappy as to content himself meerly with saving such Surplusses to answer these Purposes; and were it not for Fear of incurring some Gentlemen's Displeasure and Censure, I should readily suppose, that some such there were; then such Tenants at least will have no more than 44 *l. per Annum* out of an 100 *l.* which is less than 4 *l. per Cent.* for the Money laid out in the Purchase of these Estates; and this is the Case, though Interest be computed at 6 *l. per Cent.* in the Purchase, and the Estate be a clear neat Income, or an absolute Annuity. But if such Tenant is likewise to be saddled with the Land-Tax, Repairs, and other Charges incident to Estates in Land, and to have no Compensation for such Burdens, which being stated at 30 *l. per Cent.* will, in this Case, amount to 60 *l. per Annum*; if this Load is likewise to be laid on such Tenant, there will be nothing remaining for his own Use, or nothing to reimburse the Principal.

On

On a View of these Calculations I cannot help being of Opinion, that *5 l. per Cent.* is as much as can reasonably be expected to arise to a Tenant purchasing on these Terms; and tho' this be a greater Interest than will be coming to one who purchases Lands of Inheritance, and something more than in some Cases may be coming to one who disposes of his Money in Mortgages, yet 'tis no more than legal Interest; and so much, in Consideration had of the Hazards and Undertakings he engages in, is, I think, no more than what he is well entitled to. What is usually practised in these Cases, I don't pretend to know, nor can any one know without applying it to Particulars; and in different Cases the Circumstances may be so various as to be a Secret to any one but those who are immediately concern'd in buying and selling; and if we consult them, we shall only learn from thence, that the Seller sold dear and the Buyer bought cheap, no Body being willing to be thought to have been over-witted in a Bargain.

I am led to take Notice here what Valuation other Writers on this Subject put upon these Sorts of Estates; and I observe, that the Reverend Author, whom I have so often mention'd before, supposes a Purchaser to buy at twelve Years Value, or where Money is computed at *6 l. per Cent.*; but then he does not say, that a Purchaser on these Terms will make *6 l. per Cent.* of his Money; and to be sure he did not think he makes so much, because he rails most plentifully at the Unreasonableness and Partiality of such

such who expect it: But then this Opinion is not grounded on the Reasons I have given, *viz.* the Decrease of Common Interest, and the Contingencies of not making Compound or even Single Interest on such small Sums, and in a Manner so regular as the Computations suppose; but because he thinks that the Load of Taxes, Repairs, and accidental Charges, are to lie on such Purchaser's Estate. And yet, whatever he thought, or would be understood to have thought, a secondary Writer, who implicitly follows him, or means to follow him, understands him in that Sense; that the Purchaser will make 6 *l.* or even 7 *l. per Cent.* of his Money, and quotes his Authority, and his Instance, as a Demonstration of it. Now whoever will read, and does attend to what he reads, will certainly see, that this Reverend Author does not prove, or undertake to prove, any Thing more by his Instance, than that one who disposes of his Money in such a Manner as he there directs, *viz.* partly in a Lease, and partly in Securities at Interest, will have the same Income out of his Land, besides some Savings over out of his Interest, as another shall have who lays out the like Sum on Lands of Inheritance. And if we enquire how much his Income will be from 2000 *l.* laid out in Lands of Inheritance, his Argument necessarily leads him to say, and he says it in so many Words, that he will have about 70 *l. per Annum*, that is, 3 *l. 10 s. per Cent.* for his Money so laid out, and consequently the Purchaser of Leaseholds is to expect no more from his Land merely;

ly; and indeed, in the Case as stated, 'tis supposed that the total Income of both Estates in Land is one and the same, and both liable to the same Deductions. But then the Interest of the Money on Securities being more than sufficient in all Events to pay the Fine from Time to Time, on the Account of such Overplus he gives the Preference to the Leasehold Tenant. But whatever the first Writer asserts, or whatever he meant or has been understood to have asserted, 'tis demonstrable that the Purchaser of the Leasehold does not make so much as 5*l. per Cent.* of his Money. The neat Produce of the Land which cost 1200*l.* is 70*l. per Annum*; the Interest of his 800*l.* on Securities can be no more than 40*l.*; both together are an 110*l.* the Fine to be paid at seven Years End, if the Superior takes only one Year's Value, will be 100*l.* and to raise that Sum, the Tenant must reserve out of his Income more than the annual Sum of 10*l.* for 10*l. per Annum* for seven Years, at Compound Interest, will not produce an 100*l.* and consequently the Neat Income to the Tenant, both from his Land and his Money, must be less than an 100*l.* that is, less than 5*l. per Cent.* for his 200*l.*

It seems to me, as if this Secondary Writer was determined in all Events, whenever the first advances any Thing extravagant, to assert the same Thing with some Additions; whether he did understand or did not understand the first, or whether he saw any Reason for it, or saw none: And seeing the Estimate which they have
both

both given of the Value of these Estates is a very high one, and greatly exceeds the Value which I have here put upon them, we will examine them both, though the latter is only an ill Copy of an ill Original.

The First then affirms, that a Person might well afford to give fifteen or sixteen Years Purchase for a Lease of twenty one Years, if he could be assured that his Superior would always take one Year's Value for the Renewal of seven: And his Second says, that if a Person gives sixteen Years Purchase for such an Estate, with a View of renewing at two Years Purchase and an half, he will have near *5 l. per Cent.* for his Money. These two Estimates, we see, differ very widely from one another, as well as both do from mine; for one Year's Purchase on a Renewal is very short of two and an half, and a View only is not so good as an Assurance; tho' I confess, a View of renewing at two Years Purchase and an half may amount almost to an Assurance.

If we would know what Interest a Purchaser will have for his Money, where he buys at sixteen Years Value, as this Secondary Writer states it; let us take the same Instance we used before, and suppose the Estate at the full Rent to be an *110 l. per Annum*, the Rent reserved to the Landlord to be *10 l.* and then the Sum to be given for this Estate, at this Computation, will be *1600 l.* The Deductions for the Out-goings, which he admits are usually borne by Tenants, and means that of Right they ought to be so,

E stating

flating them at 30 *l. per Cent.* by a Medium for common Cafes, amount to 33 *l. per Annum*; the annual Sum to be referved to re-imburfe the Principal Sum of 1600 *l.* must be 45 *l.* or very near it, and this must be computed at 5 *l. per Cent.* and with Compound Interest: And when these two Sums, 33 *l.* and 45 *l.* making together 78 *l.* are deducted out of the 100 *l.* the clear Annuity left to the Owner, as the Interest of his Money, will be only 22 *l.* which is less than 1 $\frac{1}{2}$ *per Cent.* for his Principal Money laid out.

If we take into our View, that this Lease may be renew'd at seven Years End, and so from seven Years to seven Years, at 2 $\frac{1}{2}$ Years Value, the Fine on Renewals will be 250 *l.*; and to raise that Sum in seven Years, computing Money at 5 *l. per Cent.* and Compound Interest, there must be referved 31 *l.* annually, or very near it; and when this Sum, and the Sum of 33 *l.* to be fet off for Out-goings, making together 64 *l.* are deducted out of this Estate, the Remainder to the Tenant will be no more than 36 *l. per Annum*, which is just 2 $\frac{1}{4}$ *per Cent.* for his 1600 *l.*

There is a Way, indeed, of making the Computation here, which may seem to produce an Interest almost of 4 $\frac{1}{4}$, though, by the Bye, that is not near 5 *l.* and is thus made. The Rent referved to the Landlord is 10 *l.* and the Out-goings in Taxes, and other Charges, are 33 *l. per Annum*; which two Sums being deducted out of the 110 *l. per Annum*, the Neat Income remaining to the Proprietor is 67 *l.*: Now if the Whole of this 67 *l.* be put to the Account of Interest,

Interest, the Tenant then will have $4\frac{1}{4}$ *per Cent.* for his Money, or near it. But whoever computes in this Manner, and I cannot conceive by what other Method this Writer can produce such an Interest as he talks of, dreams so much and so long upon the Interest, that he forgets that the Principal is by this Way sunk, and the Estate entirely lost. If the Purchaser here takes near $5l.$ *per Cent.* as the Interest of his Money laid out, he must take near $80l.$ *per Annum* of the $100l.$; nay, if he takes only $4\frac{1}{2}$, he must take $68l.$ and if the Tenant must bear the Burden of the Land-Tax, and other Out-goings, as this Writer asserts; and if these Charges amount to $33l.$ *per Annum*, and they are cut and carved out according to the Model framed by his Friend from whom he copies; 'tis obvious there is nothing left to re-imburse the Principal, or to raise the Fine for renewing, and consequently the Principal sunk, and the Estate totally lost.

But to leave this Author to his own Understanding, in which he greatly abounds, let us examine the Assertion, or rather Concession, of the original Author, which is not quite so extravagant; who only says it is a Matter confess'd, that a Person may well afford to give fifteen or sixteen Years Purchase for such an Estate, on an Assurance of renewing always at one Year's Purchase for seven Years. The total Rent, at the full Extent, being an $110l.$ and the Rent reserved to the Landlord $10l.$ the Price to be paid for the $100l.$ *per Annum* resting, at fifteen Years

Value, will be 1500*l.* The Deductions for the Land-Tax, and other Out-goings, will be, as before, 33*l. per Annum*, and the Sum to be annually reserved to raise the Fine of an 100*l.* will be about 13*l.* if we compute at 5*l. per Cent.* Compound Interest. And if these three Sums, 10*l.* 33*l.* and 13*l.* making together 56*l.* are deducted out of an 110*l.* the Remainder will be 54*l.* which is exactly $3\frac{2}{5}$ *per Cent.* for the Purchase Money of 1500*l.*; and I doubt not but this Author did himself see that this, or some such like Sum would be the Interest coming to a Person purchasing on those Terms, whatever his blundering Second saw, or thought he saw.

If indeed the Assurance of renewing once in seven Years for ever, at one Year's Value, were a legal and binding one, such an one I mean as could be justified in *Westminster-Hall*; or if the Lease were a perpetual one on those Terms, *viz.* that the Tenant once in seven Years should pay the whole Rent of his Estate at the extended Value, and no more, I should agree that such a Lease might be worth fifteen or sixteen Years Value; and the Reason of it is obvious. By these Means the Interest is become a certain and permanent one; and though in the Terms of the Law it cannot be call'd a Fee-Simple, yet in common Sense, and common Understanding, 'tis equal to one, it being a Perpetuity; and the Fine or Premium to be paid once in seven Years is ascertain'd and fix'd, and the Value of it may be adjusted with great Exactness. But that a Lease, where the Tenure is so far precarious
that

that the Superior may or may not prolong it, as he judges convenient, and from Time to Time may advance the Fine at his Will and Pleasure ; that such an Estate should be worth fifteen or sixteen Years Value, I can by no Means admit : And I believe this Author, with all his Learning and Abilities, will never be able to make out, to the Satisfaction of Persons who are unprejudiced, and see what is the Produce of it.

The Truth is, that a Purchaser of such an Estate, if it were a neat Income, at sixteen Years Value, would make little more than 2 *l.* 10 *s.* *per Cent.* of his Money ; and a Purchaser at fifteen Years Value, little more than 3 *l.* *per Cent.* of it : For the Value of an absolute Annuity, or of a Lease reduced to an absolute Annuity, for a Term of twenty one Years, is very little more than sixteen Years Purchase, if Money were at 2 $\frac{1}{2}$ *per Cent.* and very little more than fifteen Years Purchase, if it were at 3 *l.* *per Cent.* : Or, which is the same Thing, and the Proposition is convertible, that 2 $\frac{1}{2}$, or thereabouts, is the Interest which is made of such an Annuity, if valued at sixteen Years Purchase, and 3 *l.* *per Cent.* if valued at fifteen Years Purchase. And since these Writers lay the Burden of the Land-Tax, and other Out-goings, on the Tenants, 'tis certain they will have no more than 2 $\frac{1}{4}$ in one Case, and 3 $\frac{1}{5}$ *per Cent.* in the other, as the Interest for their Money laid out at those Rates ; and even thus much will be coming to them, only on a Supposition that the Term is continued by repeated Renewals, the former at the Rate

of $2\frac{1}{2}$ Years Value, and the latter at one Year's Value only ; therefore I cannot but concur with this Reverend Author in his Advice, that Persons who buy these Estates should be upon their Guard, how and at what Rate they purchase upon such Expectations.

However, though these Authors exalt the Value of a Term for twenty one Years to sixteen Years Purchase, when they are in their Altitudes ; yet they seem to set it no higher than twelve Years, where they are more immediately considering how and in what Manner such Value is to be ascertain'd. And I cannot forbear observing the pleasant Method which one of them takes of doing this, where he tells us, that the most agreeable Proportion between Lease-holds for three Lives, and those for twenty one Years, is as sixteen to twelve ; which is neither better nor worse than it would be to tell us, from the Contents of a Circle, which never were known, and perhaps never will be known, you may know the Contents of a Square, which are known, or may be known, to a mathematical Point.

But in the Comparison made of these two Estates, 'tis not the only Fault that 'tis trifling and simple ; for there is another worse Fault, and which indeed induced me to take Notice of it, *viz.* that it is grounded on something which, I doubt, in Fact is not true, *viz.* that Estates for three Lives are worth, and commonly sold for, sixteen Years Value, and that Covenants are frequently enter'd into to give and take at that Price. That such Estates are worth sixteen
Years

Years Value, when Estates for twenty one Years are worth twelve Years, depends on a Supposition that an Estate for three Lives is equal to a Term for sixty Years ; and whether there be any Sense or Reason for making such a Supposition shall be discuss'd elsewhere ; and it will be there demonstrated that there is no Foundation for making any such extravagant Estimate. That Estates for three Lives, where nothing more or farther is granted, are commonly sold for sixteen Years Value, sure enough must be false in Fact ; because this is the Market-Price of Estates for three Lives, to which is annexed a Right of Free-Bench, which is a Right to a Widow's Life, that is, a fourth Life to commence after the Determination of the other three. This I take to be the common Price of these Sorts of Estates, because I have been so inform'd by those who usually do, and have themselves granted many such ; and this Author himself does, in Effect, inform us the same Thing, when he tells us, that Copyholds for Lives, to which are annexed the absurd Custom, as he calls it, of the Widow's Free-Bench, are of equal Value with Leases for three Lives. Now if we look upon such fourth Life, not as a Life concurrent with the other three, but as a Life in Reversion after the other three, as in Fact it is, we may, at a Medium, suppose it to be of the Age between the oldest and youngest, and then it may very well be estimated equal to a Term of twenty Years, or thereabouts ; and I believe those who have dealt in granting such Estates have found

Reason to make the Estimate at this Rate : And such an Estate indeed, in my Way of Computation, may be equal to a Term of sixty Years, and commonly sell at the Price mention'd. But to talk of Estates for three Lives, meerly and simply such, as the same Things as Estates for three Lives, and a fourth in Reversion after the End of the other three, and to put both on the same Foot in Point of Value, has something in it which I cannot account for, so I must leave it with the Author.

I have propos'd, that in the Purchase of these Estates Interest should be computed at 6 *l. per Cent.* which, under a good Management, may produce about 5 *l.* for the proper Use of the Owner, thinking for myself, that such an Interest is not an unreasonable one on these Estates : But my Intention here is not to prescribe what Interest shall be made by a Purchaser ; let every Man judge of that for himself. My Aim is chiefly to point out, if such or such an Interest is expected to be made, such or such a Manner of computing must be used in the Purchase ; which some People either do not themselves see, or are unwilling others should see. But where Persons plainly see and know what Interest they shall have on any particular Computation, if they will accept of $2\frac{1}{4}$, or $3\frac{3}{4}$ *per Cent.* for their Money, I have nothing farther to say ; if they are contented, so am I.

It will be alledged, perhaps, since I look upon the Tenant's Interest here as an Annuity, and have guarded it so strongly against all possible Hazards

Hazards and Contingencies, as indeed I mean it should, as far as the Nature of the Thing will admit, that such an Annuity is, upon the Matter, as certain and as good as any other whatever, and equally valuable; and therefore that 'tis unreasonable for such a Tenant to expect 5 *l.* *per Cent.* for his Money, when a common Annuitant would be well contented with 4 *l.* Upon this I observe; where Persons buy these Estates meerly for their Convenience, suppose an House in *London*, or elsewhere, for their own Dwelling, a Farm or Land for their own Occupation; or because they are contiguous to, or lie intermix'd with, other of the Purchaser's Estates, or in any other Respects are greatly commodious to him: With such Views Persons may be induced, and there may be Reason to be contented with a lower Rate of Interest, or not to stand so strictly to the Allowances for accidental Losses and Charges, because by occupying the Premises they pay themselves their Interest, and the Contingencies of Losses and Charges are by that Means lessen'd. On such Occasions I have only this to say; if in the Main they keep these general Rules in their Eye, they cannot be much amiss in their Purchases.

But in common and ordinary Cases, such where Persons buy these Estates meerly for the Sake of the Rents, and solely as a Manner of disposing their Money on Land Security to receive Interest for it, there I can by no Means admit that such a Tenant's Annuity is as good as any other, or ought to be so valued. Sure enough,

enough, an Annuity certain, one where the Fund is safe, and the Day and Place of Payment is fix'd, must be better than one where the Fund may prove deficient, and where the Day of Payment is uncertain, at the best, is always a long Day, and sometimes may never happen to come at all.

Now since we may consider these Estates as having several Annuities issuing out of them to different Persons, or for different Purposes, and some of them are more and some are less certain, it cannot, with any Reason, be expected, that the Proprietors of them should incumber their Estates with the Payment of the certain Annuities, and themselves be content with a less certain one, unless such less certain one have the Advantage of being larger. And since the Rent payable to the Landlord, the Out-goings for the Land-Tax, and other Charges incident to the Estate; as also the annual Sum to be reserved for re-imbursing the Principal, or raising the Fine to renew, may all be look'd upon as certain and absolute Annuities; and they are really and truly such, and must be estimated as such; for in all Events they must be paid or provided for, whether the Owners can or cannot raise them out of the Lease-hold Estate; for not only that their Estate, but all other their Estate whatever, is charged or chargeable with them; if these Annuities or Payments were valued at a Rate of 4 *l. per Cent.* the Estimate, perhaps, would not be thought too high.

But

But the Annuity coming to the Tenants, which is the Interest they are to receive for their Money, cannot, to be sure, be stated at a Rate so high; because the Fund for Payment of their Annuity is one and the same as is to pay the former Annuities, and those other Annuities, in Point of Charge and Payment, are all precedent to this; so that if the Fund ever falls short of paying all the Annuities, and in the Compass of some Years, 'tis almost impossible but that it should sometimes happen, such Deficiency must all fall upon the Tenant's Annuity. And if the Fund should prove so good as in no one Year to fail, yet the very Possibility on that Side against the Non-Possibility of the other Side, is of some Consideration; and in all Events, such Tenants, as they stand last in Course of Payment, cannot be paid their Annuity 'till all the others are satisfied, and must therefore of Necessity take up with the Remnants of Rent, and such as come last to Hand. On these Accounts I think such Tenants may very well expect and deserve an Annuity of *5 l. per Cent.* as the Interest of their Money thus laid out; and is no greater an Annuity than every Man, I believe, will expect, who buys an Annuity with his Eyes open, where the Fund, out of which 'tis to issue, may with any Possibility be ever deficient, and such Deficiency must be his Loss, and where the Times and Manner of Payment are such as of Course they will be in Cases of this Nature.

These

These Estates, if purchased at the Rate and upon the Terms I have mention'd, or thereabouts, may be desirable Things ; but surely they over-act a Part who set them out so pompously, as if there were hardly any other in the Kingdom worth enquiring after, in Comparison of these ; and who magnify the Advantages of laying out Money in the Purchase of them, as if no Way of disposing of Money in Land could come into any Sort of Competition with this. If we look upon them as Annuities, they are determinable ; if as Mortgages, or Land Securities attended with Interest, they are redeemable ; if the Landlord thinks fit to suffer the Term to expire, or the Lives to fall in. They do indeed differ from common Mortgages, because they cannot be redeem'd absolutely at the Will of the Lessor, but must continue for a Time certain, and in that Respect may have some Advantage of ordinary Mortgages ; yet still they are determinable by Effluxion of the Time : And if in Fact they are not so determined, but are suffer'd to continue by Renewals, there is Trouble, sometimes very unnecessary Trouble, and Disputes, and always some Charge, attending such Renewals.

To this we may add ; that there is much more Care and Caution to be used in managing these Estates, than in absolute Annuities, or ordinary Mortgages, with which some People cannot, and others will not, incumber themselves ; and that they are not so easily and so readily sold and transferr'd as those Securities are. On these Accounts

counts these latter, though attended with an Interest of 4 *l. per Cent.* only, shall be generally preferr'd to the former, though the Interest be 5 *l. per Cent.* or more: For the Certainty of the Income, and the punctual Payment of it at the Times stated, is to some, and the Facility in selling and transferring their Interest is to others, of so great Value and Consideration, that we see in daily Experience it makes a Difference of one *per Cent.* at least.

We are told indeed, that this Manner of disposing of Money is laying it out in the Purchase of Lands; and on that Account frequent Caution is given to the Gentlemen of the Church and Colleges, that in making and renewing their Leases, they are so far selling their Estates; which may be very proper Advice, but comes from this Hand with an ill Grace; yet if we call it buying and selling Lands, for I shall have no Quarrel about the Terms if they agree in the Thing; What do we learn, or what can be inferr'd from thence? Why truly, nothing that I know of, but that the one buys and the other sells the growing Rents of the Estate, and them only, for a Term of Years, or for one or more Lives; and if their Tenants renew their Leases, then there is buying and selling from seven Years to seven Years, and from Life to Life, and so on to the End of the Chapter; and by that Means the Owners of Leases have frequent Opportunities to repeat the Pleasure of laying out their Money in the Purchase of Lands.

Now

Now admitting that the Purchase of a Lease is a Purchase of Land, yet that does not make the Estate in it to be a perpetual or a permanent Interest, nor is it such; but must be esteem'd and is treated only as a Mortgage redeemable, or an Annuity determinable, at the Pleasure of the Lessor under some Limitations. And that 'tis no other, or better, is plain not only from thence, because 'tis entirely in the Lessor's Breast whether he will renew or not, and if he does not renew, there is an End put to the Interest in whatever View we look upon it; but because if he does renew he may raise the Fine, and in these our Days of Prosperity, that is no such wild or imaginary Supposition; and every Advance in the Fine, though never so small, is so far forth a Determination of the old Annuity, and a Substitution of one less in its Room; for every Rise in the Fine does of Necessity make an Abatement of the Annuity.

Nay, we may go farther; for tho' a Church or College, or other Landlord, has taken one and the same Fine, suppose at one Year's Value on the neat Rent, for these one Hundred or two Hundred Years last past, or should continue to do so for as many Years to come, yet the very Change of Times, and the Decrease in the Interest of Money, does necessarily itself determine or diminish these Annuities or annual Income; for every Decrease in the Interest of Money makes it necessary to enlarge the Fund for raising the Principal, or the Fine to renew, and this cannot be made greater, but by making the annual

nual Income less. And since a farther Decrease of Interest may possibly, and probably will come to pass in the Course of some Years, though perhaps not so soon as may be commonly imagined, and this will, and unavoidably must, draw along with it some Diminution of the Annuity; and since the Advancement of Fines is labour'd so strongly, and so warmly pursued, and this must make another and a farther Diminution of such Annuity: These Considerations, added to those already alledged, have induced me to think, that, upon Purchases of these Estates, Interest, at present, should be computed at 6 *l. per Cent.*; upon which Computation a Purchaser may expect to get about 5 *l. per Cent.* for his own Use. If these Calculations of mine, and the Reasonings grounded on them, do not appear to others to have that Weight in them which I have given to them, sure enough, they are at Liberty to think, and will act, for themselves as they see convenient: And there I rest this Matter.

We shall go on now to consider what is the Method of adjusting the Fine to be paid for renewing, or filling up the Years lapsed in an old Term; or for adding any Number of given Years to the Years in Being in any Lease. In general, to be sure, the Rule for renewing a Lease must be the same as for purchasing; for it can never be consonant to Justice or Reason, to assign one Rule for purchasing the Whole, and another for purchasing a Part of the same Whole. To make a different Rule for these Purposes, let the Deviation be on which Side we will, is, in Effect,

Effect, to suppose that seven Years (for Instance) to be added on a Renewal, and which, in Time, will come to be seven Years in Possession, will not be the same Thing when they are become seven Years in Possession, as the like Number of Years now in Possession is.

A Renewal then being only a new, or another Purchase, and to be govern'd by the same Rules, we must reduce the Estate to an absolute Annuity, in the Manner before directed; we must make our Computation at the Rate of 6 *l. per Cent.*; and then, if we subtract the Value of the Lease in Being from the Value of the whole Lease, or subtract the Years Purchase of the Term in Being from the Years Purchase of the whole Term, the Remainder is the Value or the Years Purchase to be given for renewing. If the Term be not a certain one, or not known, as in the Case of a Life or Lives to be added it will not, there this previous Operation will be necessary, *viz.* we must calculate to what Term the Life or Lives in Being, in Conjunction with the Life or Lives to be added, when all put together, are equal; or we must compute the Value of them, when all put together; and the Term to which they are equal, or the Value of them, when put together, is the Term or the Value out of which the Subtraction is to be made. Out of this Term or Value a Subtraction must be made of the Term or Value of the Life or Lives in Being, and the Remainder will be the Term and Value respectively of the Life or Lives to be added, as in the former Case:
And

And in the same Manner we are to proceed, whenever the new Term is to vary from the old one, by fixing first the Number of Years of such new intended Term.

To this Method I see no Objection, except any should arise from that Part of it which directs that these Estates should be first reduced to absolute Annuities; and therefore, tho' the Arguments which I have already offer'd under another Head, where I treated of the Rate of purchasing such Estates, have, in great Measure, obviated any such Objection here; yet, it being a Point of some Moment in the Case, I will take the Liberty to repeat, in Part at least, what was there alledged to remove this Difficulty.

We will suppose then that an Estate is out of Lease, and to be now let to a Tenant at Will, or from three Years to three Years, at a Rent certain, the best that could be got without taking any Fine or Premium; and let us see what Rent a Person would give, where he undertakes to make Payment at a Day and Place certain, where he discharges the Land-Tax, and all other Out-goings whatever incident to the Estate, out of his own Pocket, and engages to leave the Estate in such Repair and Condition as a future Tenant shall approve. To imagine that any Man, for an Estate of an 100 *l. per Annum* at the improved Rent, would give an 100 *l. per Annum* neat Rent, is ridiculous, and to practise it, or attempt it, would be something worse. In my Way of Thinking, and according to my little Experience, I should very much question

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whether

whether any one, I mean any one of Sense enough to know what he engages to do, and of Substance and Honesty enough to make good his Engagements, would give in clear Rent any more than two Thirds, or thereabouts, of the improved Rent, on the Terms I mention'd.

I don't suppose here, that the Taxes, Repairs, and other incidental Charges would amount to the other third Part, and yet in Truth they will not fall very much short of it, because they have been already stated at 30 *l. per Cent.* in common and ordinary Cases where Lands are let at Will on a Rack-Rent: But then 'tis to be consider'd, that it very rarely happens that any Persons will undertake these Things on any Terms, specially the Payment of the Land-Tax; and many are not to be trusted with it; and those who will undertake them, and can be confided in, will not do it, on an uncertain and precarious Tenure, unless they have a considerable Allowance, such an one as, perhaps, upon a long Term, and a more certain Interest, they would not insist on.

If we carry this Matter farther, and suppose this Estate to be let out in future Times by a Lease for seven, fourteen, or twenty one Years, yet these or the like must be the Terms and the Rent; and then 'tis evident that the one Tenant and the other has an Equivalent for taking those Charges and Incumbrances on himself, as in Justice he ought. If we proceed, and suppose such Landlord to sell, and such Tenant to buy off Part of this Rent, 'tis weak, or wicked, to think that one may sell, or the other ought to buy,

buy, any Thing but Part of the Neat Rent; for the other Part is something that does not, and is supposed, not to accrue either to the Landlord or Tenant. If then a Renewal is only a Purchase repeated, or only a new Purchase call'd by another Name, as in Truth 'tis no other, 'tis impossible, in the Right and Justice of the Thing, to distinguish between letting an Estate at Will, on a Rent certain, granting a Term in it without Fine on the like Neat Rent, granting a Term in it on a Fine proportionate to the Abatement of such Rent, and renewing such Term on a Fine answerable to the Number of Years lapsed: And therefore, in all the Cases, Consideration must be had of those Out-goings, in the last Case, equally with any of the former.

The Justice and Reasonableness of the Method proposed by me for adjusting Fines, and wherein, and how far it differs from, or may be preferable to any others; either that in common Use, or a new one recommended by Mr. *Richards*; cannot better be discover'd than by computing the Fine in an Instance or two in each Method: And I will take the Liberty to do it on two different Cases; in one where the Rent reserved to the Landlord shall be supposed a great one, and in the other a small one, in Proportion to the whole Value of the Estate. The common Method is to subtract the Lord's Rent only out of the whole Rent, mine is to subtract the Lord's Rent, and all other Out-goings and Allowances, out of the whole Rent; and in both Cases to take the Rent remaining as the Sum on which to compute the Fine: And Mr. *Rich-*

ards's Method is ; he reckons the whole Rent for the Years left in Being in the Term as an Annuity at 6 *l. per Cent.* and the Out-goings for the same Number of Years as an Annuity at 4 *l. per Cent.* ; he subtracts the second Annuity out of the first, and takes the Remainder as the Value of the Annuity for the Term left in Being ; and then, if the Value of the Annuity for the Term still in Being is subtracted out of the Value of the Term when it was full, such Value of the full Term being first adjusted in the same Manner as the Value of the Term resting is adjusted ; the Sum remaining is the Fine to be given for renewing or filling up the Term.

We will state Interest at 6 *l. per Cent.* at which Computation the Fine will be $2\frac{1}{2}$ Years Value, or near it ; and the Estate we will suppose to be 200 *l. per Annum* at the improved Rent ; and the Rent to the Lord in one Case to be 20 *l. per Annum*, and in the other 120 *l.* ; and let us compute what the Fine will be in these two different Cases, according to these several Ways of adjusting it. On the first Case ; in the common Method, where the Lord's Rent only is deducted, the Rent remaining to the Tenant will be 180 *l.* and the Fine will be 450 *l.* for renewing seven Years lapsed in a Term of twenty one Years : In my Method, where the Lord's Rent of 20 *l.* and the Out-goings in Repairs, in the Land-Tax, and for Contingencies, come to 60 *l.* more, the Rent remaining to the Tenant will be 120 *l.* and the Fine 300 *l.* for a like Renewal : And in Mr. *Richards's* Method, where the Lord's Rent only, and the Out-goings for Re-

pairs

pairs are deducted, and which will come only to 40*l.* the Rent remaining to the Tenant or his Annuity will be 160*l.* and the Fine will be 354*l.*; and his Computation is thus made;

The Value of 200*l.* *per Annum* } *l.*
for twenty one Years, at 6*l.* } 2352.
per Cent. is

The Value of 40*l.* *per Annum*, }
being the Out-goings in Rent } 560.
and Repairs, at 4*l.* *per Cent.* }
is

If this Sum be deducted out of }
the former, the Remainder } 1792 — 1792.
will be the Value of the Lease }
for twenty one Years, and is

The Value of 200*l.* *per Annum* }
for fourteen Years, at 6*l.* *per* } 1860,
Cent. is

The Value of 40*l.* *per Annum*, }
being the Out-goings for four- } 422.
teen Years, at 4*l.* *per Cent.* is

If this Sum be deducted out of }
the former, the Remainder } 1438 — 1438.
will be the Value of the }
Lease for fourteen Years, }
and is

If the Value of the Lease for }
fourteen Years be subfrac- }
ted out of the Value of the }
Lease for twenty one, the } ——— 354.
Remainder will be the Fine, }
and is

On the other Case, in which the reserved Rent is supposed to be 120 *l.* in the common Method, where the Lord's Rent only is deducted, the Rent remaining to the Tenant will be 80 *l.* and the Fine 200 *l.* for seven Years; in my Method, where the Rent and Out-goings will come to 180 *l.* all together, the neat Rent to the Tenant will be 20 *l.* only, and the Fine 50 *l.*; and in Mr. *Richards's* Method, who reckons for Lord's Rent and Repairs 140 *l.* only, the neat Rent, or Annuity to the Tenant, will be 60 *l.* and the Fine only 8 *l.* for seven Years additional Term.

I shall leave these several Computations to the Reader's Reflections, without any of my own, except by remarking only, that if we vary this last Case so far as to state the Repairs on it at 24 *l.* instead of 20 *l.* as in Mr. *Richards's*, and in my Way of Thinking we may do, since we agree that the Repairs are to be allow'd for, at such a Rate as, in Fact and Reality, they come out to be, as near as can be computed; then the Fine, in my Method, would be 40 *l.* and in his nothing at all. The Rent and Out-goings, in my Way of reckoning, would then be 184 *l.* the Rent left to the Tenant no more than 16 *l.* and the Fine for that 40 *l.*: And in Mr. *Richards's* Way, the Rent and Repairs only being Out-goings, and which come to 144 *l.* the Computation will stand thus:

The Value of 200 <i>l.</i> per Annum	} 2352.	<i>l.</i>
for twenty one Years, at 6 <i>l.</i>		
per Cent. is		

The

The Value of 144 *l. per Annum*,
being the Out-goings in
Rent and Repairs, at 4 *l. per*
Cent. is } *l.*
2017.

If this Sum be deducted out of
the former, the Remainder
will be the Value of the
Lease for twenty one Years,
and is } 335 — 335.

The Value of 200 *l. per Annum*
for fourteen Years, at 6 *l.*
per Cent. is } 1860.

The Value of 144 *l. per Annum*,
being the Out-goings for 14
Years, at 4 *l. per Cent.* is } 1519.

If this Sum be deducted out of
the former, the Remainder
will be the Value of the Lease
for fourteen Years, and is } 341 — 341.

This last Remainder, which is the Value of the Lease for fourteen Years, being greater than the first Remainder, which is the Value of the Lease for twenty one Years, there is manifestly nothing left for a Fine ; on the contrary, the Value for fourteen Years being greater than the Value for twenty one Years, by 6 *l.* the Landlord ought to pay the Tenant that 6 *l.* to induce him to take an additional seven Years,

I have before observed, that this Gentleman makes Use of the same Method in settling the Price to be paid on taking or purchasing such an Estate, as he does in renewing it ; and under

that Head I found Fault with it, as producing a very absurd Effect, *viz.* that a Lease for twenty one Years would be of some Value, but a Lease for fifty Years would be worth nothing. The very same Absurdity, or rather one more gross, recurs in renewing; for if this Rule of computing the Fine is a right one, the Tenant ought not to renew, even though no Fine was demanded, in the Case last stated; which is, in Effect, to say, that a Lease of twenty one Years was of Value, if made in 1720, but that a like Lease of the same Estate, if made in 1727, was worth nothing. This indeed may come out to be the Case in some Instances; that is, where the Estate consists mostly in Houses, which in seven Years Time may be much impair'd, and become ruinous; but then the Rule cannot be a general one, and applied in all Cases alike; and to be sure is not applicable to the Case just mention'd, where the Repairs are not of much Consequence. Nay, the Rule will not do in any Case where the Out-goings amount so high as 144 *l.* in 200 *l.* let those Out-goings be in the Lord's Rent only, as in Case of Meadow or Pasture Land, where no Repairs at all may be necessary; for if this Rule be observed, and no Fine is to be paid on renewing the Lease of such an Estate, the necessary Consequence will be, that a Lease for twenty one Years of such an Estate would be a valuable one, but a Lease for fourteen Years and seven Years in Reversion immediately following, that is, a Lease for twenty one Years of one and the same Estate, would

be

be of no Value, but worth less than nothing; which is as much as to say, that one and the same Thing, for one and the same Time, is of some Value, and of no Value.

I assert farther, and if any one will give themselves the Trouble to make the Calculation as I have done, they will find it to be true, that a Fine of one Year's Value, as now frequently taken, does then become a just Fine for a seven Years Renewal, according to this Rule, when the Out-goings in Rent or otherwise are about $\frac{1}{2}$ Parts, or about a Moiety of the whole Value. For which Reason I am a good deal surprized that the Framer of this Rule should on one Side lay a Load on the Clergy and Fellows of Colleges, for accepting one Year's Rent, as a Fine for adding seven Years to a Lease of fourteen in Being, and on the other Side should charge their Tenants with Extortion and Oppression, in forcing their Landlords to submit to such Terms; since undoubtedly there are some, very probably there are many of these public Estates, where the Lord's Rent and the Out-goings, especially if the Land-Tax is brought to that Account, as most certainly it is, or ought to be, must amount to more than a Moiety of the full Value; in which Case, such a Fine cannot be taxed of Partiality of the one Side or the other.

For the same Reason I do as much wonder, that the same Person should assert in general Terms, that seven Years in Reversion after fourteen is worth and sold for $2\frac{1}{2}$ Years Value, if belonging to the Laity; for wherever Estates

are

are under the Circumstances which have been just mention'd, let the Owners be who they will or what they will, if they take more than one Years Value for adding seven, they who take, and not they who pay such a Fine, will be chargeable with Extortion and Oppression.

When the Rent and Out-goings are less, that is, more properly speaking, where the Neat Rent, or Annuity coming to the Tenant, is greater, the Fine undoubtedly should be larger; for most certainly it must bear a Proportion to that which he is to receive, whatever that be, and not to something which he never can receive. And yet, notwithstanding this Writer magisterially pronounces that a reversionary Term of seven Years is worth $2\frac{1}{2}$ Years Value, I very much question whether he can produce a single Instance of a Leasehold Estate in the Kingdom, where such seven Years are worth two Years Value, computing by his own Rule: For if the Out-goings in the Rent to the Lord, and in Repairs, are *15 l. per Cent.* the Fine will be short of two Years Value. To make the Out-goings come to *15 l. per Cent.* I have inserted the Repairs, because he agrees they are to be allow'd, but have omitted the Land-Tax; and if that is likewise to be inserted, as I must insist it ought to be, it will be impossible to find out an Estate where the Circumstances are such, that the Out-goings, under all these Heads, shall be so little as *15 l. per Cent.*

That the Repairs belong to the Landlord he admits, and in his Computations always reckons

on them as such; and surely he must be stupid himself, or must think others to be so who maintains the contrary, if those who are Owners of the Estate are to bear this Burden; and if we may not esteem and call them Owners of the Estate who take the whole Profits and Produce of the Estate, partly in the annual Rents, and partly by a fore-hand Rent; I don't know in what Light we must look on, or under what Denomination we must put, them who take the Whole in annual Rents.

For this Reason, which I have mentioned under another Head, and shall not need to repeat any farther, I assert that the Land-Tax is to be put to the Landlord's Account, as well in stating the Price to be paid on renewing, as on purchasing these Estates. Nay, this Writer himself, though he charges this Tax to the Tenant in all Cases, and without Distinction, yet in some Cases he does, in Effect, and in the Event, make the Tenant an Allowance for them; not indeed by the Name of the Land-Tax, but under the Notion of Contingencies and Casualties: And the Allowance he makes for this Purpose is a very ample one, for it exceeds not only what I crave for such Purpose, but it even exceeds any Rate at which that Tax ever yet was, or, I hope, ever will be assess'd; it being, in some Instances, above 28 *l. per Cent.* where a Renewal, and the Fine to be paid upon it, is under Consideration. This appears plainly in the particular Instance last mentioned, where I
computed

computed the Fine, or rather no Fine to be paid, on supposing the Lord's Rent and Repairs only to be 144 *l. per Annum*: Where we see that the rest of the Rent, *viz.* 56 *l. per Annum*, is sunk and lost to the Landlord under the Article of Contingencies or Casualties; but what those should be, which happen alike in all Estates which are of a like Value, unless the Land-Tax be accounted as one and put under that Cover, is past my Comprehension.

If we make Use of the other Instance, on which I calculated the Fine to be paid, where the Lord's Rent and Repairs are comparatively small, there indeed the Allowance to the Tenant for the Land-Tax is very small, or rather none at all, on a supposed Renewal: But where's the Reason or the Justice of this great Distinction? The neat Rent or Annuity to the Tenant in one Case is stated at 56 *l. per Annum*, and in the other at 160 *l.*; but I suppose the Difference in the Tenant's Annuity, solely will hardly pass for a just Cause of exempting one and charging the other with this Tax. 'Tis true indeed, that the Engagements and Incumbrances lying on one Tenant, if compared with those lying on the other, are very small; but then 'tis true likewise, that he who has the less or the fewer Incumbrances on his Estate has paid a Fine to his Landlord in Proportion to his Exemption, and for such Exemption.

In short, let us take it in which Light we please, whether this Article of Contingencies was or was not intended to make a Provision for
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the Land-Tax, the Method of computing will still come out wrong. If it does include the Land-Tax, then there is no Provision at all made for it in Behalf of a Tenant who has a large Annuity, though the Provision for him who has a small Annuity is so very liberal an one; and yet the one has all the Estate he has paid for, and the other has paid for all he has. If the Land-Tax was not intended, nor is included in this Provision, it will be very difficult to say what those Contingencies or Casualties are, which in such an Estate can amount to so great an annual Sum; and much more difficult to give a tolerable Reason for making so great a Distinction on one and the same Estate, where 'tis supposed to differ in one Circumstance only, that of a greater or a less Rent to the superior Lord.

As to that Matter, what Fine the Laity or private Persons do in Fact take on such Renewals, I shall not pretend to say, not knowing all the Persons in the Kingdom, much less their Estates; and therefore it would be idle in me to contradict those who affirm that they take $2\frac{1}{2}$ Years Value, for some People may know what others do not know; but if those Persons mean to say that this is universally the Practice among the Laity, as they strongly insinuate, if they don't plainly speak out; in this I will take upon me to contradict them, because some Gentlemen, to my Knowledge, never take more than one Year's Value. I will venture to affirm farther, that those who regulate their Fines by deducting the Lord's Rent only out of the total
Rent,

Rent, and take $2\frac{1}{2}$ Years Value after such Deduction, will necessarily be guilty, even in this Writer's Estimation, of extorting more than is justly due to them. Nay, I may add, and I think I have already proved, that if they take two Years Value in the Manner he computes, they will go to the very Extremity which even his Rule, in any Instance whatever, will well admit.

According to my Method of adjusting a Fine for seven Years, the Fine, if computed and regulated by deducting the reserved Rent only, will come out to be one Year's Value, where the reserved Rent is a Moiety of the improved Value, or near it; where the reserved Rent is three Parts in four of the Whole, or thereabouts, no Fine will be due; and where the reserved Rent is a tenth Part only of the Whole, there the Fine will be one Year's Value and an half, and something over; in common and ordinary Cases. By common Cases I mean such where the Land-Tax and Repairs may be stated, each of them at *10 l. per Cent.*: But as these, in particular Estates, will be more or less easy, there is Room left to vary the Fine; and where these, and all other Out-goings, are stated at the lowest Rate that can well be supposed to happen in any Estate whatever, a Fine of two Years Value is the highest that can be given, even by the most willing Purchaser.

Here I make no Distinction between Lay and Ecclesiastical Estates; and whether Churchmen and Colleges will think it proper and prudent
to

to be more easy in their Fines than Lay-Lessors generally are or should be, and how far they ought to carry this Point, that there I shall leave to their Consideration: For I do not intend here to obtrude my Advice on those who are wise enough to think, and able enough to judge, for themselves; nor shall I make any Suggestions, which they will readily enter into and make a proper Use of, without any Interposition of mine.

It will be alledged, perhaps, by those who take two Years or two Years and an half Value for a Fine, if any such there be, and by those who are desirous of getting such a Fine, and possibly there may be many of them; that a Term for twenty one Years, and consequently a Term for seven Years being a Part of it, increases in Value and Proportion to the Decrease of Interest of Money; that, when seven Years, Part of a Lease for twenty one Years, are elapsed, those seven Years in Reversion are become the Property of the Landlord, and therefore that his Interest in such seven Years must be proportionably increased in Value; and if he sells them to a Stranger, or adds them to the Term in Being by renewing to the old Tenant, that the Price on Sale or Renewal ought to be advanced in Proportion.

This Allegation I have already taken some Notice of; and having first premised, that as Interest has decreased on one Side, so as to give a greater Value to such reversionary Years, so on the other Side there may possibly be some

new Charges on Lease-hold Estates, which may have prevented such Increase in Value, in Part, or in the whole; and whether that be the Case, or how far it is the Case, I reserve to be discuss'd hereafter: I say, under such possible Limitations I admit the Allegation to be true, and shall consider it as such.

I cannot say, with any Certainty, when Mr. *Æcrold* lived, and framed his Tables for Fines on Renewals; but I conjecture he might make them about two Hundred Years ago, or towards the latter End of the Reign of *Henry* the VIIIth; and possibly for the Use of the Gentlemen into whose Hands the Church-Lands were dispersed, on the Dissolution of the Monasteries. Be that as it will, it appears pretty plainly from the Construction of these Tables, that the Interest of Money was then about 10*l.* per Cent.; and at that Rate, a Term for twenty one Years was worth eight Years and half, 51 Days Value, and the Fine for renewing seven Years lapsed in such a Lease he states at one Year's Value. Amongst the many Writers on this Subject, some of whom are very full of Complaint, or rather Clamour; for in some of them 'tis meer Clamour without Knowledge, of the present supposed Minuteness of Fines taken by Churchmen and Colleges; I meet with none who finds Fault with this Gentleman for fixing them at the Rate he has done for his own Time, and for that Rate of Interest. At the present Time, the Interest of Money is at 5*l.* per Cent. and at that Rate, a Term for twenty one Years is in Value twelve Years, three Quarters

Quarters, 25 Days; and since the Value of such a Term at the former Rate of Interest was 8-2-51, by this Decrease of Interest the Increase in Value on the whole Term is one half, that being the Proportion as near as can well be fix'd: Therefore the Increase of the Value on the seven Years lapsed must likewise be one half, that is, half a Year; consequently, the Fine to be paid for such reversionary seven Years, if Regard is to be had to these Tables of Mr. *Æcroid*, and a Fine at present is to bear a Proportion to a Fine as formerly, will be one Year and an half's Value, the Landlord's Interest here being manifestly no other or more than the seven Years lapsed.

To this if we add another Consideration, which I reserved to myself a Liberty to add, *viz.* that there may be some Charges now imposed on these Estates to which in former Times they were almost Strangers, and if those Charges are to come entirely out of the Landlord's Interest, which they must do, unless a Difference can be shown between a Payment of one and the same Value where made by annual Instalments and where made by septennial ones; and if this cannot be effected but by an Abatement in the Fine, here will be a farther Reducement of such Fine. The new Imposition I mean will easily be understood to be the Tax on Land, which in *Æcroid's* Time was never laid, and if any other were laid of the same Nature, it was only occasionally and rarely done, whereas it is now annually done; and whoever knows any Thing of the present Crown Revenue, and the Monies

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usually

usually raised for the current Services of each Year, must acknowledge that it will be a constant and certain Tax for twenty one Years at least, if not for ever.

If this be the Case on Mr. *Æcroid's* Tables for Renewals, I suppose we shall hear no more Arguments drawn from thence, or any farther Appeals to them; but then I must insist that it will not be right neither to apply to this Purpose some modern Tables, which are plainly and avowedly calculated on a Supposition that a Lease of Houses or Lands, deducting only the reserved Rent, are the same Thing as absolute Annuities; which no Body will admit but the most strenuous Advocates for advancing Fines, and even these are not all of them willing to go the whole Length. And yet these Gentlemen, as if the Justice of these Tables when applied to these Purposes, was as clear as when applied to absolute Annuities, run Riot against Tenants, charge them with Tyranny, Extortion, Oppression, and other foul Practices; and give their Landlords some Language not much more civil, the one for paying, and the other for accepting, Fines at one Year's Value only.

But pray, who has given them Authority to call me Fool, or my Tenant a Knave, because we can and do agree about a Fine, or I sell and he buys something, without consulting them, or abiding by their Tables? Am not I Master of my own, and may not I do with it what I think proper, without asking their Leave? If I set an higher Value on my Ease, or on my Convenience,

nience, or a lower Value on my Estate, than some others may do, surely I may exchange the one for the other at what Price I please without incurring their Censure; where, to be sure, I oppress no Man. And I would not have them to be so certain and so positive as some of them seem to be, that none of the Laity do now grant or renew Leases on these easy Terms, because this is to assert a Negative, and it happens that I can and do affirm it to be false on my own Knowledge.

But to let this pass; for a Comparison of what is done by the Laity with what is done by the Clergy, is very little to the Purpose, unless the Practice of one would justify or excuse the Practice of another, without Regard had to the Equity or Iniquity of the Action: I will suppose that such Practice of taking low Fines prevails only or mostly amongst Ecclesiastical and Collegiate Persons, and that 'tis really a Grievance; but how is it to be removed, if Tenants are obstinate, and will not advance their Fines? Why, their Landlords must suffer their Leases to expire, and take their Estates into their own Hands; and this is urged upon them not only as a Matter of Prudence and Advantage, but as a Point of Duty necessarily incumbent on them, in whatever Circumstances they may be.

Whether such a Project as this, if it could be and were carried into Execution, would produce any such great Increase in the annual Income of these Bodies of Men as may be commonly expected, is a Thing which some People, perhaps,

will make a Doubt of. Those Persons will then be fully convinced, if it will be any Satisfaction to them to be convinced, that Landlords must bear the Burden of the Land-Tax; and they will find that they must submit to many other Inconveniencies likewise, of which, in their present Situation, they are not sensible, as being wholly exempt from them. I have before made an Estimate, if these Estates were in Hand and let out to Tenants at Will on a Rack-Rent without Fine, that they would hardly yield to the Owners in neat Money more than two Thirds of the improved Value; and I cannot imagine but that, in the present Way of managing them, they must produce more than a Moiety of such improved Value, the annual Rents and Fine being put together; and where they do in Reality produce less, I think the one may reasonably expect a larger Fine, and the other may well comply with it, and I will not suppose any so foolishly obstinate as not to comply with it; and therefore I do not see the Necessity or the great Use of this Device, the Event of which no Man can foresee.

I know very well that this Course has been frequently pursued of late Years amongst some of the Laity, but with what Success it has been attended, or what Service it has done the Owners, I do not pretend to know: But if a Man were to make a Judgment of it from the Multitude of Farms which are now in the Owners Hands for want of Tenants, where to be sure they do not see half Rent, he would not be very
hasty

hasty to run into this Project; but, perhaps, would be apt to suspect rather, that at the Bottom this Advice had no more in it, than that one Fox had lost his Tail amongst the Briars or in a Gin, and would willingly draw in his Brethren to cut off theirs.

But whatever may have been the Success of these Undertakings by private Persons, I can in no Sort agree that this is a good Reason for Persons in a publick Capacity to engage in such an Affair. It may be supposed, and in this Case it must be supposed, where a private Person can and does forbear taking a Fine, that his stated annual Income will supply his common and ordinary Occasions; and 'tis certain that the Benefit of such Forbearance in Time will come to him or his; possibly to himself, in all Events to some of his Family. With Regard to Persons in a public Capacity, the Case is quite otherwise; for if these forego their Fines in their Course, some of them will be almost destitute of a Subsistence, others will be greatly straiten'd by it; and even those whose Circumstances are such as leave Room to forego these accidental Advantages can have no Assurance, perhaps no Probability, that they shall have any Benefit themselves of such Forbearance: And therefore, to deny themselves their Fines, and the Opportunities of providing thereby for their own or their Families Subsistence, would be an Act of downright Folly in some of them, and in none of them would it be an Act of Prudence, if the prudential Part only be regarded.

The Advisers of this Scheme forget surely that, when these Leases expire, the then Proprietor of the Estate, if he be a single Person, or the Community, if it be a Corporation aggregate, as the Law stands now will have Power, and therefore may very legally and rightly grant new Leases at the ancient Rent, and take a Fine proportionate to the rest; so that this admirable Project will produce no more than this, that it will stop the Hands of the present Proprietor, or of the Individuals who now compose a Community, from taking a Fine, to give an Opportunity to a Successor, or to the Individuals who may hereafter be the Community, to take a larger Fine.

A Law indeed may be made, as one of them seems to hint, which shall oblige these public Persons to permit their old Leases to expire, and which shall restrain them from granting any new ones afterwards, on any other Terms but at the most improved Rent which can be got without taking any Fine; and I must acknowledge this would destroy the whole Iniquity of Fine-taking, let it be of which Side we will: But I very much doubt the Parliament will not come into such an Act, unless the Houses should be very thin indeed, because this is to deprive one Man of his Property, in some Cases possibly of his whole Subsistence, merely to enlarge the Property of another; and this is to be done without making any Compensation to the Party who is so deprived, and in a Case where the Interest of the Persons losing and of the Persons

Persons gaining is of equal Concern to the Public.

If the Case be such, that the Revenue arising from these Estates, unless you alter the Manner in which they are now managed, will not maintain the Successors answerable to their Functions, or their Dignities; that there may be a good Reason to make some farther Provision for them; but then that Reason is equally good to make a farther Provision for the now Possessors, since the present Generation is as much concern'd that the now Proprietors of these Offices or Preferments should have an adequate Maintenance, as the succeeding Generation can be that the future Proprietors should have such: And 'tis a very sorry Reason to diminish the Income of the immediate Owners, which the Argument supposes to be full low enough already, only because 'tis proper to augment the Income of their Successors.

As for what is further urged, that 'tis the Duty of the present Owners of such Estates, whatever their Circumstances may be to forego the Advantage of their Fines, for the Benefit, possibly of themselves, and more certainly of their Successors; I see some unfair and not very decent Insinuations in the Allegation, but for Reason or Sense in it I see none. The Laws of the Land have provided that these public Persons may let Leases for twenty one Years, or three Lives, in Possession, and farther than that those Laws have prohibited them to go; a private Person, by Settlement, has reserved to himself a

Power to let Leases under the same Limitations, and farther he cannot go : But will any Man say, that the one or the other is guilty of Injustice if he does go so far ? Surely the Laws of the Land in one Case, and a Law of his own making in the other Case, is the Measure of Justice, or there's none. Or shall we say that a Man in private Life and Concerns may justly go as far as he has reserved to himself a Power to go, and that one in public Station is guilty of Breach of Duty if he goes as far as the Law of the Land gives him Liberty to go ? This is to make a Distinction where there is really no Difference, or if there is any, the Grounds of Complaint from the other Party are much better founded.

Every one who keeps his Leases always full by renewing, whether he be in public or private Life, though he does no more than what he legally may do, and in taking a Fine takes no more than what is rightfully his own, most certainly does so far depreciate the Estate belonging to him ; but the one does a Detriment to his Successor, that is to some Body, he knows not whom, one who is a perfect Stranger to him ; and the other takes to himself something, which would otherwise come, possibly to an eldest Son and Heir ; but in all Events to some Body of his Family to whom he stands nearly related. Notwithstanding which, as much Clamour as is raised against the one Class of Men for renewing their Leases and taking Fines, we hear of none against the other for doing the same Thing.

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Ay, but Persons in a publick Station of Life, Clergymen specially, are to take particular Care of their Successors, and to manage their Estates for their Benefit in a peculiar Manner. If by this no more is meant, than that they ought to make Use of their Estates in such a Way as to do no Injury to their Successors, this most certainly is true; but then 'tis equally true with Respect to all Mankind as well as them, and in all other Instances as well as this; for every Man is obliged so to use his own as not to injure the Property of another. If more be meant here, and that they are in Duty obliged to let their Leases expire, and leave them in that State for the Benefit of their Successors, this Notion I can by no Means come into.

What they ought, or must in Justice do, we have already seen; for the Laws are the Measure of that, or 'tis past my Understanding to find out what is the Measure of it. What they should do as an Act of Charity or Munificence, must be determined by the Circumstances and Abilities of the Proprietors; and even where these are supposed to be sufficient for such Purposes, yet surely we must leave it to their Judgment and Discretion: Or we shall be forced to say in one Case, that one Man must debar himself of the Conveniencies, possibly of the very Necessaries of Life, in order that another may enjoy them in a greater Plenty than himself: And in the other Case, if those whose Condition is such that they are in a Capacity to do this are under Obligation to do it, whether their own Judgment

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concurr in it or not, that very Obligation makes the Act cease to be, what it is intended for, an Act of Charity or Munificence.

Where these Bodies of Men, or any private ones amongst them, are in such Circumstances that they can well spare taking a Fine, as some of them to be sure occasionally may do; if they are disposed to Acts of Charity or Munificence to those of their own Order, I cannot but think, if they will look into their own Estates, and observe what they frequently consist of, that the Vicars and Curates of the Parishes where their Estates lie, and out of which their Estates are derived, will appear more proper Objects of their Benevolence, than their Successors can do. It so happens, not perhaps from the original Donation, but by some subsequent artful Exchanges, that many of the Estates belonging to these Persons are Rectories and impropriate Tythes, and where they are so, the Provision left for the Minister, if any be left, is some wretched Pittance; and where this is the Case, and 'tis much too common a Case, surely the Practice of those Clergymen and Colleges is greatly to be commended, who, on renewing their Leases, sink the Fine in the Whole, or in Part, and instead of it, put their Tenants under an Obligation to make a proportionable Augmentation of the Minister's Stipend: And as this may be done with Ease and Convenience, and to the Satisfaction of all Parties interested, sure enough it will promote the Interest of the Church of *England* more effectually than any Scheme for
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letting Church Leases run out, and for Clergymens taking their Estates into their own Hands. And since the Bounty of Queen *Anne* gives these Gentlemen, and indeed all others, an Opportunity to double their Benefaction; if such a Method for augmenting poor Livings continues to be encouraged by the Clergy of superior Fortunes, and if the Laity are ready to concur in prosecuting so good a Design, the inferior Clergy may hope, in some reasonable Time, to have a competent Maintenance, and the People an able and discreet Minister as a Guide for their Souls.

Before I conclude this Head, it will be expected, perhaps, I should, and therefore I shall, take Notice of another Thing, which is frequently mention'd, and insisted on as a Matter of great Moment, to determine this Controversy about Fines; I mean the low Value of Money, and the high Price of the Necessaries of Life, comparing the present with former Times. If by former Times we understand here only forty or fifty Years ago, or the Times of King *Charles* and King *James* the Second, which one of these Writers has his Eye upon, the Difference of the Price of Money, or of the Necessaries of Life, will make nothing for advancing Fines. The Decrease in the Price or Value of Money has been one *per Cent.* since those Times, but such Decrease is over-balanced by the Increase of the Charges which since then have been laid on Lands; the Article of the Land-Tax alone, not to mention others, being equal at least to the Gain

Gain on the other Side ; so that such Decrease operates nothing towards this Purpose.

As to the Necessaries of Life, it will be admitted that many of them are advanced in their Price within this Period ; but every Body knows, who is not wilfully blind, that such additional Price is to be attributed to the modern Duties of Excise and Customs ; but such of those Things which are the direct and immediate Produce of Land, and which are exempt from those Duties, 'tis not true that their Price is generally enhanced ; and if 'twere true, the necessary Consequence of such Advance should be, and would be, that the Lands which produce them would yield a better annual Rent, and yet this certainly is not the Case, unless the Lands have received some Improvements ; and if that were the Case, and where it is the Case the Lessors have the Benefit of such Addition in the Rent by an Addition to the Fine in Proportion, even on a Supposition that they take no more than one Year's Value.

But for a Landlord to raise his Fine, which is but of the Nature of a fore-hand Rent, meerly because many Things in common Life are grown dearer by new Excises or new Customs, by which Means the Expences of him or his Family are enlarged, has no more Sense in it than there would be if he should pretend to raise the annual Rent of his Estate because there is an annual Tax on it, and his Income thereby lessen'd : For the Diminution of the Income in one Case is just as good a Reason for raising one Kind of Rent, as the Enlargement of the Expence in
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the other Case is for raising the other Kind. But I doubt few Landlords will find their Tenants such Asses, notwithstanding the low Opinion which some of them entertain of their Understanding, as to submit to this Burden, unless they can put them into a Way of throwing it off from their own Shoulders, and laying it on the Consumer of the Produce of the Land, as common Tradesmen in exciseable Goods do, by raising the Price of the Commodities they vend: And yet, in the Event, even this would bring it back on themselves in great Measure. The Truth is, if Land Owners, in this and other Instances, can and do prevent the Load of a Tax from falling directly and immediately on themselves, yet in the last Resort there it will fall, let them shift it seemingly as far off as they will in the first Imposition; and, perhaps, just so much farther off from them as 'tis laid in the first Instant, and according to common View and Estimation, just so much the more heavily it comes upon them at the last.

If by former Times we mean those of two Hundred or three Hundred Years ago, this may require a distinct Consideration. Every one, who has at all look'd into Things of this Nature, knows very well, that in the Times of *Henry* the Fourth, or about three Hundred Years ago, a Pound of Silver Money in Tale was an actual and effective Pound of Silver, wanting a few Shillings, and allowing a small Matter for Allay; a Solid or Shilling, or what was so denominated in Accounts, was a twentieth Part of a Pound, and a Penny

Penny a twelfth Part of a Shilling. From hence, without Doubt, was introduced the Method of reckoning by Pounds, Shillings, and Pence; meaning thereby such Quantities of Silver: And we continue still to make our Accounts by Pounds, Shillings, and Pence, tho' it is now meerly a Fiction, and subsists no where but in Imagination; for a Pound in Money, as now call'd, does certainly contain no more than a third Part of the ancient Pound, which was a Pound in Weight as well as in Tale.

It is known too, equally well, that fifteen Pounds in Silver, whether in Coin or in Bullion, they being both the same or near it, were equal to one Pound Weight in Gold: And a Pound of Silver in Weight being now cut into 3 *l.* 2 *s.* or sixty two Shillings, fifteen such Pounds make now 46 *l.* 10 *s.* in Silver Money in Tale; and a Pound of Gold is now cut into forty four Guineas and an Half, which makes in Gold Money in Tale 46 *l.* 14 *s.* 6 *d.* each Guinea reckon'd at 1 *l.* 1 *s.*: So that Gold and Silver in our Days bears the same, or very near the same, Proportion to one another, as they did in former Days.

How the State of the Silver Coin stood about two Hundred Years ago, or in the Times of *Henry* the Eighth, is not so easy to fix without distinguishing the Times; however, it may be sufficient here to say in general, that in the Beginning of his Reign a Pound of Silver made 2 *l.* 5 *s.* in Tale, and at the latter End 7 *l.* 4 *s.*; so great was the Debasement of the Coin then by the Mixture of Allay: And yet once, in
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the Time of his Son and Successor, *Edward* the Sixth, this Coin was made much worfe, a Pound of Silver fine making 14 *l.* 8 *s.* in Tale.

I cannot forbear observing here, that many Authors, amongst others Bishop *Fleetwood* in his *Chronicon Preciosum*, take Notice of this Debasement of the Coin, and complain of it as a great Grievance to the Nation, as undoubtedly it was; but assert, that an Advancement of Money in the Denomination, where the Standard of which 'tis made remains the same, is no Prejudice to the Publick. Nay, some carry it so far as to contend it would be of Advantage; whereas an Advancement and a Debasement most certainly are in equal Mischief, where made in an equal Degree. If the Silver Species now in Being was all call'd in and re-coin'd half of Silver and half of Alloy, the Grievance of this is readily seen and admitted; but if a Crown Piece were call'd an Angel, and made to go for ten Shillings, and the rest of the Silver Pieces in Proportion; in this, say they, there's no Harm, the Change is meerly nominal.

As to Sales or Bargains to be made in future, I agree such Change in general is meerly nominal; because the Parties selling or contracting will have Regard, not to the Denomination solely, but to the real Quantity of Silver or Gold which is known, or supposed, to be contain'd in the Money in which such Contracts are presumed to be paid and perform'd; and therefore the Owners of Goods would then expect to have double the Quantity of Money in Pounds,
Shillings,

Shillings, and Pence, for the same Commodities which they would now sell for the single Quantity : So that in these Instances there would be neither Prejudice nor Benefit accruing to either Party. But would not the Case be the same, and the Change be merely nominal, if the Money were debased in an equal Proportion? Exactly the very same, and for the same Reason : For there would be the like Right, the like Opportunity, and the like Method, of Persons helping themselves, *viz.* by taking then a double Quantity of Money, in Monies number'd, instead of a single one which they now take.

But, besides those Transactions of buying and selling, and Contracts, in future, there are other Money Matters which are to be regarded, and where this Exchange will produce an Effect which shall be more than Names and meer Words. There are such Things as old and unalterable Rents, Annuities perpetual and certain, Debts subsisting, and Contracts in Being, both of a public and private Nature, where the Sums are fix'd and determinate, to the Value of many Millions, perhaps not much short of a Moiety of the Value of the Estate of the whole Kingdom ; the Proprietors of all which necessarily must lose just as much as the Money is nominally advanced, for that they will receive just so much in Silver less, and in Gold proportionably, as is equal to the supposed Increase in the Denomination of the Money. With Regard therefore to these Persons, an Increase in the nominal Value of Money, and a Decrease in the intrinsic Value, supposing

supposing them to be in one and the same Proportion, will have one and the same mischievous Effect; that is, the one or the other will sink a Moiety of their Substance. Whether such a Project as this might not pass in a neighbouring Kingdom for a reasonably good Way of discharging public Debts, I can't tell; but I believe he who should advise it, and put it in Practice in *England*, in most People's Opinion, would deserve an Ax or a Halter.

And for the Advantages which are surmised would arise from hence to the Public, *viz.* that such a Scheme would make the Species of Silver more plentiful, and prevent its being melted down and exported; they are both of them very vain and very groundless Expectations. If the Silver Coin, supposing there are now six Millions of that Species in the Kingdom, were advanced to double the present nominal Value, we might indeed say then, that we had twelve Millions of that Species; but this same Silver Species, when call'd twelve Millions, will operate in Trade, or any Business of the Nation, just so far, and no farther, than the same Species now does, which is call'd only six Millions: For that, after such Alteration, there can be no Money Transactions in which Silver is made use of, let the Transaction be in a greater or a less Sum, but we must necessarily then make Use of double the Quantity of that Money in Tale, or the same Quantity in Weight, which we now make Use of, for any Purpose whatever, excepting that of paying old Debts, and making

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good previous Contracts. To say otherwise, or that the same Quantity of Silver in Coin, when call'd twelve Millions, would be of more Use than when call'd six Millions, has no more Truth or Sense in it, than there would be in saying; if a Piece of Cloth of five Yards long were cut into ten Pieces, and those Pieces were called Yards, that then it would go farther in making a Suit of Cloaths, than it would do if those Pieces were call'd, as they really are, half Yards only. Just in the same Manner, if a Pound of Silver, which is cut into sixty two Pieces, and which are now call'd Shillings, should hereafter be call'd Solids, or by any other new Name, and made to pass for two Shillings; yet a Pound of Silver, whether the sixty two Pieces, of which 'tis made, are call'd Shillings or Solids, is one and the same Thing, and of one and the same Value; and an old Shilling will go as far at the Market as a new Solid.

On this Account, and for the same Reason, any such Scheme as this, whether it be by increasing the nominal Value of Money by giving it a new Name, or by decreasing its real Value by debasing it, will have no Sort of Effect towards preventing the Exportation of our Coin, either of Gold or Silver. Traders and Merchants, especially those who deal in foreign Commodities, will not have Regard to the Pounds, Shillings, and Pence in Tale, to which so many Pieces of Silver or Gold Coin may amount; but to the Pounds, Ounces, and Grains, which those Pieces will make in Weight at the Scale;

Scale; and therefore, for a Parcel of Goods, the same in Quantity and Quality, they will then expect, and in Fact will receive, Silver or Gold in Exchange, the same in Quantity and Quality which they now do: And consequently may export Silver or Gold with the same Ease and equal Profit, whatever the State of the Coin shall be: And if they can do it to Advantage, I suppose they will do it; and I confess I do not see any Reason why they should be prohibited.

To return from this Digression. What was the Interest of Money, or of Silver and Gold of which it consisted, or the Price given for it on Loan, in the two several Periods I have been speaking of, *viz.* two Hundred and three Hundred Years ago, I confess I have not been able to discover, so as to fix it with any Certainty. Thus far, however, we may safely go, and perhaps that may be sufficient for our Purpose, as to affirm, that about two Hundred Years ago it was at least as high as 10 *l. per Cent. per Annum*, there being an Act of Parliament made at the latter End of the Reign of *Henry* the VIIIth, that it should not exceed that Rate; so that it is a very fair Conjecture, that it had been before that Time higher, and in the Times of *Henry* the IVth, possibly as high as 15 *l. per Cent.*: And every Body knows, that the Interest or Price of it now is no more than 5 *l. per Cent.* setting it at the highest Rate it can be let at.

From these Premises, I mean from the different State of the Silver Coin, which generally governs that of the Gold Coin, and from the

different Rates of Interest, in the two former or in any other Periods, and in the present Period of Time, I make these Deductions : That when a Pound of Money in Tale was an effective Pound of Silver in Weight, and the Interest or Price of Money was at 10 *per Cent.* if we compare the Price of Commodities in *England* now, with the Price of them at those Times, the Price now should be in the Proportion of six to one ; and if the Price or Interest of Money was at 15 *per Cent.* and the like Comparison were made, the Proportion now should be as nine to one. My Notion in this Matter is, that the Price of Things at one Time will bear that Proportion to the Price of them at another Time, which the effective Silver in the nominal Pound at one Time bears to the effective Silver in the nominal Pound at another Time, and the Interest or Price of such Pounds on Loan at the different given Times ; Computation being made upon both those Heads.

From hence I infer, if in the Times of *Henry* the VIIIth a Pound of Silver was cut into forty or forty five Shillings, and now into sixty or sixty two, that the Price of any Commodity which then was two must now be three Pounds on the Account of that Difference only ; and if the Interest or Price of Money was then 10 *per Cent.* which is now only five, the Price on that Account must be farther doubled, and, in all, be six Pounds ; that is, the present Price must be three Times as much as in that Reign. In like Manner, if in the Times of *Henry* the VIth a Pound of Silver was cut into thirty, and
now

now into sixty two Shillings, from thence only the Price of Things now must be double the Price of them then; and if the Interest of Money then was at 15 *per Cent.* as very probably it might, that Difference from the Interest now will make an Addition of a treble Price, and the Whole be as six to one. So in the Times of *Henry* the IVth, when a Pound of Money in Tale was a Pound of Silver in Weight, and the Interest was 15 *per Cent.* which is treble the Value and treble the Price of each now, we must treble the Price on each Account, and the Price now will be as nine to one.

To show this, I will make Use of a familiar Instance. A Farmer or Merchant, in the Times when a Pound of Money in Tale was an effective Pound of Silver in Weight, and the Price of it on Loan was 10 *per Cent.* is possess'd of a Parcel of Sheep, Oxen, Wool, Corn, or other Necessaries of Life, to the Value of an 100 *l.* at the Market Price; when he sells them at that Rate, will receive an 100 *l.* in Silver, in Weight as well as in Tale, or in Gold proportionably. A Farmer or Merchant in these Times therefore must have 300 *l.* in Money for a like Parcel of Goods, or else he will not have the same Quantity of Silver or Gold in Weight, though the Goods sold are in Quantity and Quality the same; that is, the present Owner must have three Times the Price, in Money as now told, as the ancient Owner had, for the same Things. That this is the Fact in the Instance of Gold, as a Commodity, is evident; for that a Pound of Gold in *Henry* the IVth's Time, was sold for 15 *l.* or

thereabouts, in Money in Tale, and now it fells for 45 *l.* and something over, in the like Money in Tale, that is, at three Times the Price.

To go on to the Difference in the Rate of Interest now and formerly. If these Merchants have not present Occasion for their several Sums in their Way of Trade and Business, or are desirous to let them out at Interest: The Merchant of old Times, on the Loan of his 100 *l.* at the End of the Year will receive 10 *l.* in Silver, in Weight as well as in Tale, supposing Interest to have been then at 10 *per Cent.*; but the Merchant of these Times, for the Interest of his 300 *l.* will have no more than 15 *l.* in Tale, which is equal only to 5 *l.* in Weight: Therefore, that the one and the other may have equal Advantage from the same Parcel of Goods, the modern Merchant must sell his Goods at 600 *l.* for no less Sum than that will yield him 10 *l.* of Silver in Weight for Interest by the End of the Year, that is, he must sell them at six Times the Price: or otherwise these two Persons, at the End of such Year, will not be in equal Circumstances, though they were so at the Beginning when they were possess'd of their Goods.

If these Merchants should invest their Money in Lands of Inheritance, instead of putting it out at Interest, the Case will still come out the same. When Money is at 10 *per Cent.* the Price of Estates in Fee-Simple is ten Years Value: So that an 100 *l.* in old Times would have purchased an Estate of 10 *l. per Annum*, and a Rent of 10 *l. per Annum* was a Rent of

so many Pounds Weight in Silver, or near it, and of Gold in Proportion. If a Man were now to purchase an Estate of equal Goodness, he must pay for it 300 *l.* even though Interest were supposed to be the same now as formerly, and there were no Advance in the Year's Purchase; for no Rent less than 30 *l. per Annum* will produce ten Pounds in Silver in Weight, or proportionably in Gold. And when we take into the Account the Difference of Interest, 'tis plain, that the Fall of Interest to one Half makes a Rise of Land in the Purchase to a double Price: and therefore, now Interest is no more than 5 *per Cent.* he must and does pay 600 *l.* for an Estate of 30 *l. per Annum*; that is, for an Estate of equal Goodness, and which shall yield an equal Quantity of real Silver or Gold, which might have been bought 300 Years ago for one 100 *l.* he must now pay just six Times as much.

Or take the Matter in this Light. The Sum of an 100 *l.* in former Ages, if laid out in Lands, or let out at Interest, would, in a Year's Time, produce to the Owner ten Pounds Weight of Silver, or a proportionable Quantity of Gold; the Sum of 200 *l.* now, if laid out in Lands, or let out at Interest, would, in a Year's Time, produce to the Owner ten Pounds in Money, as now counted: But ten Pounds in Money, as formerly counted, had three Times as much Silver or Gold in it, as ten Pounds in Money, as now counted: Therefore, to produce as much Silver or Gold within the Year as formerly, three Times 200 *l.* or 600 *l.* as Money is now

counted, must be laid out in Land, or put out at Interest ; that is, six Times the Money must now be employ'd to produce the same Quantity of Silver or Gold as was produced formerly by one 100 *l.* only. The Consequence of this is, that the Necessaries and Conveniencies of Life, which are the Things out of which Money is to arise, when they come to a Market, must now be sold at six Times the Price, or six Times the Pounds, Shillings, and Pence, which they were formerly sold at ; or the very same Things will not answer the same Purposes of Life now, which they would have done three Hundred Years ago.

It will be needless to repeat here this Reasoning, and show that 'tis applicable to the Case, which I have supposed possible, that the Interest of Money has been in some Times at 15 *l. per Cent.* and that the Price of Goods between those Times and these should be in the Proportion of nine to one. It may suffice to say in general, that it will hold good in that Instance, and in any other which may be supposed ; so that wherever the Quantity or real Silver in a nominal Pound, and the current Interest of Money can be known and determined, there the Price of Things may be known and determined likewise. My Notion is here, that the real and intrinsic Value of the absolute Necessaries of Life, such as Food and Rayment, were always, and always will be, much one and the same, except where an accidental Plenty or Scarcity makes a temporary Variation ; but if we compute the Value of those

those Neceſſaries by a third Thing as a common Measure between them, the Price, or nominal Value ſo meaſured, muſt vary as ſuch Measure itſelf varies. If Silver be made that Medium, as for many Ages it has been in theſe Parts of the World ; and if in former Times a real Pound of Silver was call'd a Pound, a twentieth Part of a Pound was call'd a Shilling, and the twelfth Part of a Shilling a Penny ; if in Times ſubſequent the Measure itſelf be varied, and a third Part of a Pound of Silver be denominated a Pound, and Shillings and Pence in the like Proportion, the Price or nominal Value of the Neceſſaries of Life will and muſt vary with it. A Quarter of Wheat or Malt, a Pound of Beef, or a Yard of Cloth, are now of the ſame real Value as heretofore ; for they will go as far towards the Support of human Life now as they did five Hundred or a Thouſand Years ago, and no farther ; but their Price or nominal Value, as meaſured by the current Coin of the Kingdom, muſt vary as the Coin itſelf varies ; and this we muſt admit to be the Caſe in Silver itſelf, or we muſt be forced to ſay, that one third Part of a Pound of Silver, ſuppoſing no more than a third Part of a Pound of Silver to be in a preſent nominal Pound, is of equal Value with a whole Pound. For if we conſider Silver, not as a Measure of Traffick only, but as a Neceſſary or Convenience of Life, and as a Merchandizable Commodity, as we certainly may and do ; one Pound of Silver is of the ſame Value as another, and of the ſame Value at one Time as at another,

ther, greater or less Plenty excepted, but the Price or nominal Value was formerly one Pound only, but now we see it is three Pounds and above in Silver, as a Measure: And we may rightly say, that it ceases to be one and the same Measure, when it ceases to be one and the same Thing in its Weight and Value.

From hence we plainly discern one Reason why Money now is not of the same Value as it was two Hundred or three Hundred Years ago, *viz.* because the Silver of which it consists in any given Sum in Tale is now only a Moiety, or a third Part of the Quantity which was formerly in the like Sum in Tale; and it being a Convenience of Life and a saleable Commodity, the real Quantity of Silver in the Money is the true Measure of its Value. But besides this Decrease in the Value of Money arising from the Variation in the Species; we find another in the Interest, or annual Premium for it. This, to be sure, has proceeded from another Cause; and has arisen, as I apprehend, from the great Increase of the moveable and easily transferable Estates in the Kingdom, which for some Time has been growing upon us, and of late has been so vastly enlarged by the public Funds, and not from the Increase of Gold and Silver in the Kingdom, either in Coin or Bullion, any otherwise than as they constitute a Part of such moveable Estates. How, and in what Manner, these Decreases of the Value of Money in both Kinds have an Influence on the Price of the Necessaries of Life, we have already taken Notice; we shall proceed, therefore,

therefore, to consider how such Decreases, and the Increase in the Value, or rather Price, of these Necessaries, are or may be applied to the Purpose of advancing Fines since those Days.

What Influence the Decrease in the Interest of Money should have in this Affair, we have already consider'd ; and I shall not need to repeat what has been offer'd to that Purpose under another Head. As to the Increase in the Price of Things, it can be nothing to the Purpose, unless we could suppose that the very numerical Fine was now taken for the same individual Estate, which was taken two Hundred Years ago, which every Body knows to be false. If a Landlord, in his Catalogue of the Necessaries of Life, should insert Wine, Brandy, Coffee, Tea, and Chocolate, and alledge that the Price of these Things is greatly enhanced of late Years ; I doubt his Tenant, who holds by the Year, will hardly be brought to admit, that an Advance, in the Price of those Commodities, or in the Use of them, is a sufficient Reason to raise his Rent which is paid annually, nor his Tenant, who holds by Lease, to raise his fore-hand Rent or his Fine, which is paid at once : because the Produce of his Land, out of which such Rents are to arise, to be sure, is neither made better, nor of greater Value, by any such Advance ; 'tis well if 'tis not made worse, and of less Value.

If the Price of Beef and Mutton, of Wool and Corn, or the like, is advanced threefold suppose, since the Times of *Henry* the VIIIth, or in any other Proportion since any other deter-

minate

minate Period of Time; whether such Rise has proceeded from lowering the Interest of Money, or from diminishing its intrinsic Value, or from both together; such Advancement in one Kind has given Occasion for, and produced, an Advancement in another Kind, *viz.* in the annual Rent: For the Rent of Land and the Price of the Things which that Land produces, always do, and necessarily must rise or fall together: And since the annual Rent is the Measure of the fore-hand Rent, an Enlargement of one has made an Enlargement of the other likewise in Proportion; so that a Lessor, in the present Way of Computation, has already raised his Fine answerable to the Rise which has been in the Price of those Necessaries of Life, in which he can any wise be concern'd, or his Estate be affected by: And where the Price of those Things which the Land produces has not been raised, there's no Sense or Reason in raising the annual or fore-hand Rent.

I shall add here, that the Cheapness of Money and Dearness of the Necessaries of Life, which already have or hereafter may come to pass, have no Influence on Lease-hold Estates which in any Respect are beneficial to the Owners, but directly the contrary: though with Regard to Fee-Simple Lands, the Proprietors of them have a visible and immediate Advantage if they part with their Interest, and if they keep it, in all Probability, in the Course of some Years, must have one in another Kind. The Consequence of this Decrease in Interest on one Side is, that
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the Annuity coming to the Owner of the Lease must sink, as certainly, though not altogether in the same Proportion, as if his Money were invested in *South-Sea*, or other public Annuities, and the Government should lower those Annuities; or if his Money were out on a Mortgage, and the Interest of it were decreased by Law; and the Principal, in either of those Cases, can in no wise be enlarged: And, on the other Side, the Proprietor of inheritable Lands has a great Advance in his Principal, by the Rise in the Value on Sale, and his Rent or Annuity out of all Danger of sinking.

If this Decrease in the Value of Money, and Increase in the Price of the Necessaries of Life, which always go together, or rather are one and the same Thing, should occasion a Rise in the Rent of Lands, as in the Compass of some Time it certainly will, tho' it proceed by Degrees not easily perceptible; the Advantage of such Rise will redound wholly to the Proprietor of Fee-Simple Estates, and the Leaseholder can have no Share in it, for his Landlord has undoubtedly the Right to, and will undoubtedly take the whole Benefit of, that in his subsequent Fines.

Now I suppose a less Income arising to a Tenant, and at the same Time a greater Expence becoming necessary, as they always accompany one another, is no very agreeable Thing in Enjoyment, or comfortable in Prospect: So that I humbly apprehend those many Harangues upon the present prosperous State of the Nation, and the Intimations of the Probability of a farther Decrease

Decrease in the Interest of Money, which so frequently occur in some of the Writers on this Subject, however true they may be, and of Service to Tenants of Lands in Fee-Simple, if applied, and when applied to Tenants by Lease, and Monied Men, are not exceedingly much to their Satisfaction, or to the present Purpose.

Since then, upon the whole, Estates for a Term of twenty one Years, or any other determinate Number of Years, may be ascertain'd to the Exactness of a single Shilling, if there were Occasion for it, provided they be, and when they are, reduced to absolute Annuities: And having offer'd to Consideration the several Articles of Deductions and Allowances to be made, which the Purchasers of such Estates may fairly insist on, in order to reduce them to such Annuities: And having stated what I apprehend to be a reasonable Interest for the Money laid out in purchasing or renewing them, which I think, in a general Way, should be one *per Cent.* higher than the common current Rate: I shall only add, that I have annexed a Table, N^o I., for valuing Annuities for any Term for fifty one Years, at the several Rates of 3, 4, 5, 6, 7, and 8 *per Cent.* Interest; in which, instead of Decimals for Parts of a Year or Parts of a Month, I have inserted Quarters of a Year and Days, and in the following Discourse express myself in that Manner; apprehending such a Form would be more useful and more ready for other People, as I found it to be for myself.

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In one Case indeed, where the annual Rent or Annuity is an even 100*l.* or is easily reduced to such, there a Table by Years and Decimals, or rather Centesimals, is more ready than by Quarters and Days; for there the very Figures, which express the Years and centesimal Parts, express the Value in Money: For Instance, if Enquiry was made, what is the Value of an Annuity, or a Lease reduced to an Annuity, for twenty one Years, Interest computed at 4*l. per Cent.*; the Answer is, 14-01 in Years and Centesimals, which in Money is 1401*l.* For which Reason, and because some Persons may have a better Taste, or have been more accustomed, to these Accounts in the Way of Decimals; and since in the following Discourse I often make Use of, and have Reference to, a Table drawn in that Form; to my Table for the Value of Annuities I have added a third Column, which gives the Decimal, or rather Centesimal, Parts of a Year, corresponding to the Quarters and Days. I must observe here, that Mr. *Richards*, in his Table, and throughout his Discourse, calls those Parts of a Year Decimals; for what Reason I cannot understand; but not intending to have any Dispute about Terms, where the Meaning of them is well enough understood, I have complied with the Use of his Expression; though the Parts intended are really Centesimal Parts of a Year, and ought to be so call'd.

The Use of this Table in purchasing these Estates, whether they are originally and properly Terms for Years, or are Estates for a Life
or

or Lives, and are reduced to Terms for Years, is so obvious and so well known, that I need to add nothing to explain it. In renewing such Estates of either Kind, there is Occasion only for an easy Operation: that is, in Case of Leases for Years, to put together the Number of Years remaining in the Lease and the Number of Years to be added: in Case of Leases for Lives, to put together the Number of Years to which the Life or Lives remaining are equal, and the Number of Years to which the Life or Lives to be added are equal; to state the Value of the whole Term: and then out of such Value to deduct the Value of the Term of Years in Being in the Lease; and the Remainder is the Value of the Number of Years, or of the Life or Lives, to be added; that is to say, is the Fine to be paid for renewing. For this Reason I thought it unnecessary to insert any Table for renewing Leases; but it may, perhaps, be of Service to such who may be concern'd in these Sort of Transactions to give them a Caution, that they first reduce the Estate in View to an absolute Annuity; for that those same Tables, either for purchasing or renewing, are very improperly, I may say very unjustly, applied to Estates held by Lease, unless and until they are so reduced: For the Computations, by which both the one and the other are constructed, are made on this Foot, that the Annuities are absolute. And this Caution appears to me the more necessary, because the common Tables of Renewals may tend, and perhaps were calculated and published with a View,

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to take off Peoples Minds from attending to this Point, viz. what Parts of the annual Value of his Estate a Tenant is to pay for in the Fine on a Renewal; whereas, in my Judgment, this is the only Matter that deserves or requires any Attention, and which I have been here attempting to adjust and establish; and I hope have put into such a Method as will appear to be a fair and equitable one between Landlord and Tenant.

End of the First Book.



BOOK I



B O O K II.

*An ESSAY to estimate the Chance of
the Duration of a Life, and to ascertain
the Value of Leases and Annuities for
Life, &c.*



WE proceed now to consider of, and propose, a Method to ascertain the Value of Leases for one or more Lives, and those, I think, as well as Leases for Years, must first be reduced to absolute Annuities; and this we must do by the same Rules, and in the same Manner, as has been prescribed in common Leases for Years. When this is done, or provided for, Estates, whether held by Lease, or by any other Tenure for a Life, will have one and the same Consideration; and to adjust the Value of such Interests, I apprehend it to be the best, and, indeed, the only Method, to resolve them into Terms for Years certain. To do this, we must compute to what Age there is a Chance or Probability that any Person or Persons named shall live; or, which is the same Thing, to what Term of Years any given Life or Lives are equal; so far, I mean, as the Chance of any Persons living can be estimated: And when this is done, and from hence, as I have before observed, we may see, and readily

dily determine what is the Value of such an Estate, to such a Degree of Exactness and Certainty as the Chance of the Life can be ascertain'd.

We want to know, for Instance, what is the Value or the Sum of Money to be paid for a stated Annuity for a given Life ; or, such a Sum of Money we have to dispose of, such is the Annuity we would have during a Life, and are desirous to know what Sort of Life, or what Aged Life, must be nominated, so as to buy an equal Bargain. If those Cases were put upon a Term of Years, if we are not contented to abide by the Tables in common Use, the Method to calculate this, and by which the common Tables were framed, is this. In the first Case, where the Term or Number of Years is agreed on, we add together the Sum of the Reversions, or the present Value of the Sum to be paid at the End of each Year, at the stated Interest, for the whole Term of Years : and the Total arising from thence is the Value of the Annuity, or the Sum to be paid for it, at such a Rate of Interest. In the other Case, where the Sum of Money to be paid and received is agreed on, the Method is this : Out of the stated annual Income we deduct the Interest which we expect for our Money, and compute in what Number of Years the Surplusage of the Annuity, at Compound Interest, at the current Rate, will produce the Principal ; and the Number of Years which produces the Principal is the Term for which the Purchaser must enjoy his Annuity.

Now let this be applied to an Annuity to be purchased on a given Life ; to adjust the Value of which we must add together the Sum of the Reversions, at the stated Interest, for the given Life ; but before we can effect this we must determine, by a Computation made in the best Manner we can make one, for what Term or Number of Years there is a Chance or Probability that the Person whose Life is given may live ; and then, indeed, we may proceed as in a Term of Years, it being now reduced to such. But unless this be done, and until it is done, I see no Rule or Reason why in adding together Reversions we may not stop at five Years End, or go on to fifty, every one according to his own particular Conceit : And in Truth some of the Computations of this Kind are such, as would tempt a Man to think that they were made without either Rule or Reason.

In the Second Enquiry upon a Life ; that is, if we would know what Sort of Life we must have, or of what Age the Person to be named should be, for a determinate Sum, and at a stated Interest : How are we to proceed there ? By the same Method as is observed in a Term for Years : We are to deduct Interest out of the annual Payments at a stated Rate ; to compute in what Time the Surplusage after such Deduction, at Compound Interest at the common Rate, will produce the Principal ; and the Time which produces the Principal is the Time for which the Purchaser must enjoy the Annuity. But, by knowing the Time for which the Annuitant is to
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enjoy his Estate, we are not come to the Knowledge of the Sort of Life to which such Time is equal, or what Aged Life must be nominated to answer such Purpose: Nor can we ever, as I apprehend, come to such Knowledge, without computing first and determining, by an even Chance or on a reasonable Probability, what Sort of Life, or what Aged Life, may be equal to, or be likely to continue to, such a Time.

Now if the Chance of Life can be adjusted upon any good and probable Grounds; and we must suppose that this is capable of being done to some tolerable Degree, for if the Chance of the Duration of a Life cannot be adjusted with some sort of Probability, all the rest signifies just nothing: I say, if this can be done, and when 'tis done; that is, if we can make, and have made, a reasonable Estimate how long any Person named may live; since the Lease or Annuity is supposed to continue as long as the Life continues, all farther Calculations, whether built on imaginary or on real Foundations, are, in my Judgment, perfectly useless: for they can only discover something which is already sufficiently discover'd, since the only Thing which remains to be known is the Value of the Term to which we have estimated such or such a Person is likely to live, which the common Tables for the Value of Annuities for Terms of Years show us at once, and that with Certainty, and to the Exactness of a single Shilling, and at every Rate of Interest: And to search farther is neither better nor worse, if I may have Leave to use a familiar

Simile, than it would be to seek for a Needle in a Truss of Hay when we have it all the while between our Finger and Thumb.

In Pursuance of, and in Conformity to, this my Way of thinking as to the Method of computing the Value of Annuities for a Life, I have drawn the Scheme in Table N^o II., in the two first Columns of which is exhibited the Term of Years to which any single Life is equal, computing for every five Years of Age of Life, from ten Years to eighty Years of Age, inclusive of both: And such a Scheme for a single Life must be, as I apprehend, the Foundation of any that can be framed for two or more Lives. This I have form'd upon the best Observations and Calculations that I could think of, and those not a few, some of which I shall hereafter take Notice of: And yet I do not expect it will be agreeable to every Man's Taste, and that no Objection can be raised against it; for in Things of this Nature, which are subject to a vague Estimate, every one has his particular Way of Thinking: All that I shall say of it is, that to me it seems liable to fewer Objections than any that has yet appear'd, to me at least, not excepting that which was drawn by the best Hand. On which Account, though there are many Tables already extant for the Valuation of these Estates, yet if the Method of making such Valuation was meerly imaginary, and not known before Dr. Halley's Rules for that Purpose came out, as has been confidently affirm'd, and I cannot contradict; and if the Tables, which since that Time have been

been publish'd were constructed by those Rules, I will venture to affirm, that there are none now extant free from Errors, and, if I may have Leave to say so, Errors apparent and amendable; this will be a sufficient Excuse for offering a new one: specially if it avoids the Errors of former Tables, and keeps clear of new ones in any tolerable Degree: But whether it does, and how far it does this, or exceeds any other Tables, is submitted to the Reader's Judgment.

I am very sensible, that a Man ought to be wary, and go on good Grounds, when he presumes to attack any Calculations made by so great a Master of those Things as Dr. *Halley*; and yet if his Calculations appear to me liable to Exceptions, and those Exceptions can be maintain'd, I doubt not but the World and the Doctor himself will excuse my Attempt: And I verily perswade myself, when the Animadversions I shall make on his Tables and his Rule are thoroughly consider'd, it will appear to others, as it does to me, that they are not fram'd with that Accuracy which usually attends this Gentleman's Performances of this Kind.

The chief Exception which I take to the Doctor's Table is, that where the Value of an Annuity for a Life in one Period of Life is compared with the Value of an Annuity for a Life in another Period of Life, the Term to which such Value in one Period is equal does not differ or vary, by a just and proportional Degree of Decrease, from the Term to which such Value in another Period is equal: By Means, and in Con-

sequence whereof, the Chance of the Duration of the Annuity does not keep equal Pace with, and sink by the same Degrees as the Chance of Vitality on the Life decreases: Nor does the Value of the Annuity itself, if it were consider'd, and could it be consider'd singly and abstractedly from the Life on which it depends, sink and decrease from one Period of Life to another, in a just and proper Proportion.

What I mean by the Decrease in the Value of the Annuity, and by the Difference of the Term to which such Value is equal, from one Period of Life to another, will appear best by an immediate Inspection into the Tables; but to give an Instance of it here, it stands thus. An Annuity for the Life of a Person between the Age of 15 and 20 is in Value 13-33, which is equal to a Term of 27-2-61; and an Annuity for the Life of a Person between the Age of 20 and 25 is in Value 12-78, which is equal to a Term of 25-0-00, according to the Doctor's Table: And if we deduct 12-78 out of 13-33, the Remainder will be 00-55, and this is the Decrease in the Value; and if we deduct 25-0-00 out of 27-2-61, the Remainder will be 2-2-61, and this is the Difference in the Term, in those two Periods.

But this, perhaps, will be still more plain and obvious if express'd in Terms more common and familiar; and so express'd stands thus. A Person of the Age between 15 and 20 has a Chance to live 27 Years, 2 Quarters, and 61 Days; and one of the Age between 20 and 25 has

has a Chance to live 25 Years; that is, less than a Person of the precedent Age 2 Years, 2 Quarters, and 61 Days: And this is what I call the Difference of the Term of Years from one Period of Life to another: And since the Decrease in Value will always correspond to the Difference in the Term, the Value of the Annuity in the second Period is less than the Value of the Annuity in the first Period by 00-55 in Decimals, which is equal to a Term of 2-2-61 in Reversion after a Term of 25-0-00.

I have been the more exact in explaining this Difference in the Term or Decrease in the Value, which are much one and the same Thing or always co-incide, because the Objection I make to the Doctor's Table is, that it does not preserve a just and proper Proportion in this Instance: such a Proportion I mean as can be justified by any Bills of Mortality, or can be accounted for by any Facts that ever happen in Nature. To show this Defect in the most clear and distinct Manner I can, I have inserted the Doctor's Table in N^o III., in the Appendix; and since the same Exception lies against Mr. *Hayes's*, Mr. *Richards's*, and Mr. *Merris's* Tables for the Value of Annuities on a Life, I have inserted them likewise, or Part of them, in the Tables N^o IV, V, and VI: and to each of them I have added three Calculations of my own: The first Column of which contains an Account of the Term of Years and Parts of a Year, to which every Life is equal as it corresponds to the Value in the Tables respectively: the second gives the several Differences

rences in the Term of Years, and the third the Decreases in the Value of the Annuity, from one Period of Life to another, as they come out on the several Tables respectively. In my own Table, N^o. II. I have inserted solely the Term to which a Life is equal, because the Value of the Annuity will on Course attend on and be equal to the Term; and to this I have added a third Column, which gives this Difference in the Term according to such Table, and the Decrease in the Value will be in the very same Proportion: from whence, and by an Inspection into the several Tables, and a Comparison of them together, we may see which of them best preserves that proportional Difference in the Term and Decrease in the Value, which I say, and undoubtedly, ought to be observed in this Case.

Since then these Tables are all of them defective in this Point, some in a greater some in a less Degree, and this appears by a meer Inspection into the Tables themselves; I shall confine my Reflexions here solely to Dr. *Halley's* Table, but intend that my Reasoning in general on this Head should be applied, as it certainly is applicable, to all of them equally.

The particular Periods of Life which I lay my Finger on, and say are Faulty in this Table in this Respect, are 20 and 40: in the first of which the Difference in the Term is much too great, and in the latter too small. In the first Column of this Table, which contains the Ages of Persons, we have 20, which is the Period in-
cluding

cluding the Lives from 20 to 25; and against it, in the fourth Column, which gives the Difference between the Term in that Period and the Term in the preceeding Period, we find the Difference to be 2-2-61: And in the same fourth Column we see the Difference for the preceeding Period to be 00-2-30, and that of the subsequent one to be 2-0-46: and this Difference 2-2-61, compared with such the precedent and subsequent Difference, is such a Proportion, I say, as cannot be maintain'd by any Bills of Mortality, at least any that ever I saw, or by any Reason in the Nature of the Thing.

If we consult the Bills of Mortality for *London*, I mean the modern ones, where the Ages of Persons dying are distinguish'd in Periods from ten Years to ten Years, and in the Extract which I have added in the Appendix, it appears that the Number of Persons dying in the Period between 20 and 30 is less than the Number of those dying in the Period between 30 and 40; and if we should suppose of those dying between 20 and 30, one half were of the Age between 20 and 25, and the other half between 25 and 30, which is plainly a more favourable Supposition than there are just Grounds to make; yet since the Difference in the intermediate Period, that is, in the Period between 20 and 25, ought to bear a Proportion to the precedent and subsequent one, it should therefore be something greater than the precedent and something less than the subsequent one: Whereas in this intermediate Period the Difference is 2-0-31 more than

than in the precedent one, which is an extravagant Decrease; and 'tis also more than the subsequent one by 00-2-15, when the subsequent Difference, if there were any Variation, undoubtedly ought to be the greater; for that fewer Persons out of an equal Number die in the Period between 20 and 25, than in the Period between 25 and 30. And if we should suppose, as we have before supposed, that the Number of Persons dying of the Age between 20 and 25 and between 25 and 30 was the same; yet the Number of Persons co-existing in the first of these two Periods must be greater than in the second, and consequently the Chance of Mortality in the latter Period must be greater than in the former, the same Number dying out of a less Number living: and therefore the Difference in the latter ought to be greater.

I should take Notice, however, that, at the Time when the Doctor composed this Table, the Bills of Mortality for *London* were not published in the Manner they have been of late Years, that is, with the Distinction of the Ages of the several Persons dying; so that these Bills could be no Guide to him, nor his Table be found Fault with merely because it does not quadrate with a Rule which was not then in Being. But then the *Breslaw* Table, which, if I understand him rightly, was in great Measure of his own Formation, was in Being and before him when he wrote; and if this be the Standard, and in Truth 'tis the only proper Standard, whereby to try this Table for the Value of Annuities for a Life,

a Life, my Objection of the Want of a proportionable Difference in the Term and Decrease in the Value is stronger upon an Inspection into that Table than in any other Light whatever.

If then we consult the *Breslaw* Table, which I have inserted in the Appendix, it will appear that the Number of Persons dying in a Year in the Period between 15 and 20 is in the whole 30, the Number dying in the Period between 20 and 25 is 31, and between 25 and 30 is 36. Upon this I observe, tho' the Number of Persons co-existing in the first of these Periods of Life is greater than the Number co-existing in the second; yet that the Number of Persons dying in the first is less than the Number of those dying in the second Period; and the same Observation holds good in comparing the second with the third Period: From whence it necessarily follows that the Difference in the Term ought to be in a Proportion correspondent, or, in other Words, such Difference in the Term ought to bear a Proportion answering to the Increase in the Chance of Mortality: And if the *Breslaw* Table be a Rule for the Increase of the Chance of Mortality, it must be a Rule for the Difference in the Term, and yet for this Difference in the Term we see that the proper Proportion is directly contradicted, or at least apparently unpreserved, in this Table of the Value of Annuities.

Many Persons, I know, have a Notion that the Age of 30 is the best Age of Life on which to have an Annuity depending, for that young
People

People under that Age are exposed to more Hazards and Casualties than those who are arrived to some Maturity of Age and Discretion; and consequently that the Chance of Vitality in the younger Part of Life is less, at least more uncertain, than at the Age of 30, or thereabouts; and that a loose Calculation of the Chance of the Duration of a Life may suffice here. But Dr. *Halley* discover'd no such Thing in the Bills of Mortality for *Breslaw*, nor can any one discover it in the *Breslaw* Tables which he form'd from thence; for there the Number of Persons dying in the Period from 10 to 20 is 63, from 20 to 30 is 67, and from 30 to 40 is 86. If we look into the modern Bills of Mortality for *London*, and my Extract from thence, there we find the Case to be the same in the main, though not in the same Proportion, and that the Number of those dying in the older Period of Life greatly exceeds the Number of those dying in the Younger: and the Account there stands thus. Out of every thousand Persons dying, there die, of the Age between 10 and 20, 30; of the Age between 20 and 30, 72; and of the Age between 30 and 40, 93: And from hence it appears evidently that this Notion of the best Age of Life is not a just one.

But if there was a better Foundation for the Notion than there appears to be, and if there was Room for a Latitude in the Calculation of the Chances of Mortality for the Age between 10 and 30, this can avail nothing in the Age of 40 or thereabouts; and yet in the Period of 40

we have an improper Proportion in the Difference of the Term, for that it is too small, when compared with the Difference in the Periods on both Sides of it. In this Instance, indeed, the Deviation is not great; but if any Difference were made, it should have been greater in this than in the precedent, and less than in the subsequent Period; that is, it should grow greater and greater gradually, as the Age of Life advances: whereas the Difference is less in this Period than in the Periods on either Side. And since all that I have urged against the Want of Proportion by an over Difference is equally applicable to this Case of an under Difference; I shall only add, that I believe no Bills of Mortality whatever or any Thing in Nature do, and that I am very sure the Bills of Mortality for *Breslaw* or for *London* do not, warrant the one or the other.

These are the single Articles in this Table which are most exceptionable; but, in Truth, this Defect goes through the whole Performance, which I crave Leave to show by an Instance, which, in my Apprehension, demonstrates the Defect, and that it really is such. The Doctor, on the Bills of Mortality for *Breslaw*, observes, and I believe all Mankind will agree it to be true in the rest of the World as well as there, that out of a certain Number of Persons in the Decline of Life, more die in a Year, or any determinate Number of Years, than there do out of an equal Number of Persons in the Youth and Vigour of Life: And he has himself given

us the *Breslaw* Table, as a Scheme of the Increase of Mortality, according to the Advance in Age. Now if the Chance of Mortality increases, and the Chance of Vitality decreases, in Proportion to the Advance in Age, as is here asserted, and is undoubtedly true; and if the Value of an Annuity for a Life, or the Number of Years to which a Life is equal, which differ in nothing but the Manner of Expression, does depend on the Chance of the Vitality of the Life and on nothing else, as most certainly it does, and shall be proved beyond Contradiction; it evidently and necessarily follows, that the Value of an Annuity for a Life, or the Number of Years to which a Life is equal, must decrease faster and by greater Degrees in the older than in the younger Stages of Life, and in the same Proportion as the Chance of Vitality decreases.

Let us see now how this Difference or Decrease in the Term will stand, if we put together the whole of the Decreases for the four Periods of younger Life, of middle Life, and of the oldest Life. The whole then for the first Set is a Term of 7-1-46, for the second 7-0-15, and for the third 7-1-00: which, we see, is directly contrary to the Rule laid down, and almost inverts the Proportion. If we would know what is the right and just Proportion to be observed, and might depend on the *Breslaw* Tables as giving us such Proportion, and this Author's Table for the Value of Annuities was framed from those Tables; on a Computation from thence, the Case, as between the Persons of the youngest

youngest and the oldest Period of Life, stands thus. The Number of Persons co-existing above the Age of 10 and under 30, which contains the four Periods of younger Life, consisting of twenty Years, is in all 11875; and the Number of Persons dying out of them in one Year is 130; that is, one in 91, or thereabouts. The Number of Persons existing above the Age of 55 and under 75, which contains the four Periods of older Life, consisting likewise of twenty Years, is in all 3726; and the Number of Persons dying out of them is 204, that is one in 18, or thereabouts. The Proportion here then is plainly five to one: that is, of an equal Number of Persons of each Stage of Life, the Number of the older Stage of Life dying in a Year will be five, and of the younger Stage one only: Or if one Person only be nominated of each Stage of Life, the Chance of Mortality on the Side of the Person of the older Stage of Life against the Person of the younger Stage is five to one. Whether this Proportion of the Increase of Mortality, as it arises from a Computation on the *Breslaw* Table, be a just one, I neither affirm nor deny; but be it right, or be it wrong, this Author's Table for the Value of Annuities for a Life was framed from thence; and therefore the Difference or Decrease in the Term, from one Period of Life to another, and between the Periods of younger and elder Life, should bear such a Proportion to one another on the Table for the Value of Annuities, as the Increase of the Chance of Vitality in

one Period of Life bears to that Increase in another Period, according to the *Breslaw* Table: And I may appeal to the Doctor's own Calculations, whether the Decreases given in one Table are conformable to the Decreases in the other Table, or are agreeable to any Rule which arises from the Nature of the Life of Man.

I might rest the Matter here; but since it may be urged, although the proportional Decrease contended for is not preserved, when we compare it, as we have hitherto done, with the Scheme of the Difference of the Term; yet if we compare it with a Scheme of the Decrease of the Value, and the Table of Annuities is framed by a Computation from the Value and not from the Term, there possibly the proper Proportion may be well enough preserved. For this Reason I made the Calculation inserted in the Fifth Column of the Table N^o. III, which contains an Account of the Decrease of the Value of an Annuity from one Period of Life to another, computed by the Values only; and we find the very same Defects here as we had before, and in the very same Instances: And indeed, it must of Necessity so come out, since the Value and the Term do always reciprocally correspond.

The single Periods of Life which we found Fault with, according to the Scheme in the Terms, were 20 and 40; and in these two Articles the same Error occurs upon this Scheme as we met with on the former. On the Period 20 we objected that the Decrease, when compared with the precedent and subsequent one, was too great; and here the several Decreases of
these

these three Periods being, as stated in Decimals, 11, 55, 51, it appears plainly on the Face of them that the Middle one, *viz.* that for the Period 20, is greater than the precedent one by a Difference much too large, and also greater than the subsequent one, when undoubtedly it ought to be something less. And as for the Period 40, to avoid Repetitions, I would refer you to the Scheme itself; an Inspection into which does plainly enough discover the Defect; for the Decrease in that Period is less than in either of those on each Side, and therefore cannot possibly be in a proper Proportion to both of them.

These, indeed, are not very great Mistakes, or, however, by a small Variation might be rectified; but if these were set right, yet the grand Error will still remain, *viz.* Want of Proportion through the whole Scheme. The Decrease in the four first Stages of Life is indeed in Value, computed by Way of Decimals, 1-72; and in the latter Stages is 3-91; notwithstanding which, in Reality, there is no great Difference in these two Decreases, and what Difference there is lies on the wrong Side, *viz.* on the Side of 1-72, that being the greater Decrease of the two. To say that 1-72 Value in Decimals is something greater than 3-91 Value in Decimals, seems to be a Paradox, and if said of them simply and singly taken, is manifestly a Contradiction: but then they may be so placed with Reference to something else, or may be consider'd as Part of, or taken out of, something else in such a Manner, as that the first shall be a Value greater than the

second. For Instance, 1 simply taken and by itself is certainly not so much as 2 so taken, but 1 consider'd as Part of 10, or as drawn out of 10, is something greater than 2 consider'd as Part of 40, or as drawn out of 40; for the one is a tenth and the other a twentieth Part only of the Thing to which it stands related: And that this is the Case here will appear from hence.

A Term for 90 Years, Interest computed at 6*l. per Cent.* is in Value 16-58; and the Proposition inverted is a true one; 16-58 in Value is equal to a Term of 90 Years: Now if out of 16-58 we draw 41 Decimals, the Remainder will be 16-17, and 16-17 is equal to a Term of 60 Years, and no more; so that a Subtraction of 41 Decimals only lessens the Term here 30 Years. A Term for 21 Years, at the same Rate of Interest, is in Value 11-76; and if we draw out of this 41 Decimals, the Remainder will be 11-35, and 11-35 is equal to a Term of 19-2-30: So that the Subtraction of 41 Decimals out of the shorter Term lessens that Term only one Year and an half, or little more; whereas a Subtraction of 41 Decimals out of the longer Term lessens that Term 30 Years. From hence 'tis plainly seen, that one and the same Value in Decimals, drawn out of a greater given Value in Decimals, and out of the longer Term to which such given Value is equal, lessens the Term out of which 'tis subtracted in a much greater Degree, than the same Value in Decimals, drawn out of a less given Value in Decimals, and out of the shorter Term to which such given Value

Value is equal, lessens the Term out of which that is subtracted. Therefore it may be true, that 1-72 Value in Decimals, though not half so great a Value as 3-91, drawn out of one Value, shall lessen the Term out of which it is subtracted as much, or more, than 3-91 Value in Decimals, though more than double the Value of the other, drawn out of another Value, shall lessen the Term out of which that is subtracted.

If we examine out of what these two Values 1-72, and 3-91, are respectively drawn; it appears that 1-72 is drawn out of the Value 13-44, which is equal to a Term of 28-1-00; that is out of the greater given Value, and the longer Term corresponding: and 3-91 is drawn out of the Value 9-21, which is equal to a Term of 13-3-30, that is, out of the less given Value and the shorter Term corresponding. If we proceed in the Computation, and apply this, it stands thus. The Value 1-72 subtracted from 13-44, leaves a Value 11-72: the Value 11-72 is equal to a Term of 20-3-45: a Term of 20-3-45 subtracted from a Term of 28-1-00 leaves a Term of 7-1-46; and this is the Decrease in the Term for the four Periods of younger Life. The Value 3-91 subtracted from 9-21 leaves a Value 5-30; the Value 5-30 is equal to a Term of 6-2-30; a Term of 6-2-30, subtracted from a Term of 13-3-30 leaves a Term of 7-1-00; and this is the Decrease in the Term for the four Periods of older Life. Therefore, upon the whole, whether the Decrease here be taken immediately from the Term, or be com-

puted from the Value, and so to the Term, this Decrease in the Term comes out to be one and the same; that is, the Decrease of the Term in the Stages of younger Life is greater than in the Stages of older Life: and consequently, the gradual Decrease of the Chance of Vitality, in Proportion to the Increase of Age, is not preserved: which was the Thing to be demonstrated, and I think is fully done.

This Way of Reasoning may seem, perhaps, to have some Difficulty in it, but, I believe, will be render'd more intelligible when I have shown, as I shall have Occasion to do hereafter, the great Difference between adding to, or subtracting from a Term of Years, and adding to or subtracting from the Value corresponding to the Term. It will appear then: If a Term and a Value corresponding are given, where a Subtraction is made of a proportionate Part, (suppose a Quarter) out of the given Value, and a Subtraction of the same proportionate Part out of the given Term, that the Value remaining will not correspond to the Term remaining on such Subtraction. If this be the Case, which I shall in a proper Place demonstrate to be the Case; it may be true, that 1-72 being the Value remaining on a Subtraction made out of one given Value and a Term corresponding, and 3-91 being the Value remaining on a Subtraction made out of another given Value and a Term corresponding, shall be equal to one and the same Term; or 1-72 the less Value shall be equal or correspond to a longer Term, than 3-91, tho' the greater Value.

That

That this is the Fact here, is proved beyond Contradiction by the Way of arguing which follows, which I hope will be clear enough, and I'm sure is right. By the Doctor's Table for the Value of Annuities for Lives, the Life of a Person aged 10 is equal to a Term of 28-1-00; the Life of the same Person, when aged 30, is equal to a Term of 20-3-45: the Difference between these two Terms is evidently 7-1-46: that is, in 20 Years Time the Life of this Person, whilst in the younger Periods of Life, is decreased or grown less in Computation a Term of 7-1-46. The Life of this Person, when aged 30, is equal, as noted before, to a Term of 20-3-45; the Life of the same Person, when aged 50, is equal to a Term of 13-3-30; the Difference between these two Terms is 7-0-15; that is, in these 20 Years Time the Life of this Person, in these middle Periods of Life, is impair'd or grown less upon an Estimate a Term of 7-0-15. The Life of this Person, when aged 50, is equal, as noted before, to a Term of 13-3-30; the Life of the same Person, when aged 70, is equal to a Term of 6-2-30; the Difference between these two Terms is 7-1-00; that is, in 20 Years Time in the oldest Periods of Life, the Life of this Person is grown worse and diminish'd, upon the Chance, a Term of 7-1-00: So that the Differences or Decreases in the Term will stand thus; for the 20 Years in youngest Life 7-1-46, for the 20 Years in middle Life 7-0-15, and in the oldest of all 7-1-00.

Now since this Table for the Value of Annuities is constructed on the Foundation of the Table of Mortality for *Breslaw*, it ought, as I observed before, to agree with and be conformable to that Table, or there is a Defect in the Structure; but on looking into this latter Table, it appears that the Number of Persons dying in the first Period, that is between 10 and 31, is 138; in the second Period, between 30 and 51, the Number is 196; and in the last Period, between 50 and 71, the Number is 215; and yet the Number of Persons co-existing is greater in the younger than in the older Period of Life, and gradually decreases as Age increases: From whence 'tis plainly seen, in this Calculation as well as in a former, that the Table of Annuities has no sort of Conformity with the Table of Mortality, though the one be built, in the main, on the Foundation of the other. And as this Want of Conformity between these two Tables, in a proportionable Decrease, runs through the whole, so I cannot forbear observing the foul Work that it makes in one particular Instance, such as most certainly overthrows the Justice of this Table for Annuities, and of the Rule too, by which it was framed.

The Value of an Annuity for a Life of 10 Years Age is by this Table 13-44, which is equal to a Term of 28-1-00: and the Value of an Annuity for a Life of 70 is 5-32, which is equal to a Term of six Years and two Thirds. If we consult the *Breslaw* Table, the even Chance of the Duration of the Life of 10 Years Age is

41 Years

41 Years and over, and the Chance of the Duration of the Life of 70 is six Years, and about two Thirds of a Year. Now that an Annuity for the Life of a Person aged 10 Years should be in Value 13-44, that is, should be equal only to a Term of 28-1-00, when his Life, on the even Chance, is equal to 41 Years and above; and that an Annuity for the Life of a Person aged 70 should be in Value 5-32, which is equal to a Term of six Years and two Thirds, when his Life, on the even Chance, is equal only to six Years and two Thirds: I say, that the Chance of the Duration of the Annuity in one Case should fall short of the Chance of the Duration of the Life, and so much as thirteen Years; and in the other Case, that the Chance of the Duration of the Annuity and of the Life should be so near an Equality, or the very same; has something in it so much of the absurd, that I cannot reconcile it to my Understanding. And yet if we compute the Value of an Annuity for a Life of 80, and make Use of the Rule which the Doctor prescribes and made Use of in framing his Table for such Value, we shall discover something still more extraordinary: *viz.* that an Annuity for such a Life will be in Value 3-84, which is equal to a Term of four Years and an half, when on the *Breslaw* Table the even Chance of the Duration of such a Life is not full four Years; which I think is so palpable a Contradiction to common Sense, that nothing can maintain the Rule by which it was produced. But this Objection goes to the Rule itself by which
the

the Table was framed, and I meant here to take Notice only of the Defects which arise on the Table, so I shall wave any farther Reflexions on that Point; intending to resume the Consideration of it in a more proper Place: and conclude that I have made good my first Exception, that a proper and proportional Decrease in the Term, or in the Value of these Annuities is not preserved in these Tables.

A second Objection lies against Mr. *Hayes's* and Mr. *Richards's* Tables, viz. that, being computed for the several Rates of 4, 5, 6, 7, and 8 *per Cent.* they give us such a Value of an Annuity for a Life, as that one and the same Life is equal to a different Term for each Rate of Interest. There is no Room for this Defect in Dr. *Halley's* or Mr. *Morris's* Tables, they being computed for a single Rate of Interest only; but in Truth, the Objection does lie against the Doctor's Rule for forming these Tables; for that any Table drawn by his Rule, and for more Rates of Interest than one, will have the same Defect. This appears plainly enough by Mr. *Richards's* Tables, which were constructed by this Rule; and that this of Necessity must be the Case in all Tables for Variety of Interests drawn by the Rule, will be demonstrated when I come to speak to the Rule itself.

Mr. *Morris's* Table stands clear of this Error, and so indeed would any Table he should frame for Variety of Interests, if made in the Manner in which I suppose his to have been made. He does not directly tell us by what Method he
form'd

form'd his Table; but I think I can see that he made Use of Dr. *Halley's* Table as his Model; and his Way of Reasoning upon it was thus. An Annuity for the Life of a Person, for Instance of ten Years Age, by the Doctor's Table, is in Value 13-44; the Value 13-44 at 6 *per Cent.* Interest is equal to a Term of 28-1-00, or thereabouts; every Life of ten Years Age is equal to one and the same Term, at whatever Rate Interest be computed; a Term of 28-1-00 at 4 *l. per Cent.* is in Value 16-72, or thereabouts: Therefore the Value of an Annuity on such a Life, where Interest is computed at 4 *l. per Cent.* must be 16-72: and accordingly this is the Sum given in his Table as the Value of an Annuity on such a Life at that Rate of Interest. That this must have been his Manner of Reasoning and forming his Table, and sure enough 'tis a just one, is evident from hence; that the Term to which the Value given in his Table corresponds at the different Ages of Life varies only a few Days from the Term to which the Value given in the Doctor's Table corresponds at the same Age of Life; and in two Instances the Term corresponding to the Value given in one Table is the same, to a single Day, as in the other.

As to Mr. *Hayes's* and Mr. *Richards's* Tables, that these two Tables for every Rate of Interest give us such a Value of an Annuity for a Life as does in Fact make one and the same Life equal to a different Term of Years, and what that Variance is on each of them, may be best and most plainly seen by the short Schemes in Tables

N^o. VII.

N^o. VII. and N^o. VIII. The first of these gives us the Value of an Annuity for a Life of 30 Years Age, and the Term to which such Value is equal at the several Rates of Interest, as stated by Mr. *Hayes* himself; and 'tis marvellous to me how he could make the Computation, and not see the Absurdity of the Supposition on which it must be grounded: And the other gives the Value of an Annuity for a Life of 12 Years Age, at the like several Rates of Interest, as stated by Mr. *Richards*, and the Term to which such Value is equal, as I compute the same.

Now I say, that the Values stated in these two Tables thus corresponding to and producing a different Term of Years to which one and the same Life is equal, according to the several Rates of Interest at which the Calculations are made, undoubtedly are, and necessarily must be, wrong for that Reason because they produce such different Terms. This being the Case on both these Tables; and since Mr. *Hayes* has not acquainted us what Rule he made Use of in forming his Table, and Mr. *Richards* informs us that he built on Dr. *Halley's* Hypothesis, and 'tis evident he did so: And since one and the same Defect goes through both Tables, I shall apply myself chiefly to Mr. *Richards's* Table, and the Rule by which that was constructed: Only I take Notice here, that my Observations on one of them, in most Instances, and specially in that which I have before mention'd, will equally affect and be applicable to the other, whatever was the Rule by which it was framed.

I suppose

I suppose no one will controvert those Points: That he who has an Annuity for the Life of a Person has an Annuity for such a Term of Years as such Person in Fact shall live; and when he buys it, the Term of Years to which any Person's Life shall be prolonged being uncertain, that he buys it for such a Time as there is a Chance or reasonable Probability that the Person may live whose Life is nominated: And I am sure 'tis possible that five several Annuities, at the five several Rates of Interest in these Tables may be granted on one and the same Life, as well as on different ones. To say then, that in Case of one Annuity there is a Chance or Probability that the Person on whose Life the Annuity depends may live for 30 Years, or near it; in Case of a second Annuity, that he may live for 29 Years, or thereabouts; and on a third, on a fourth, and on a fifth Annuity, on each of them, that he can and probably may live for a different Term of Years; is ridiculous and absurd to the last Degree. And if we take a particular Instance, and suppose two Annuities only, at the two different Rates of 4 *l.* and 8 *l. per Cent.* to be granted to two distinct Persons on the Life of one and the same Nominee; to say, with Regard to the Annuitant who purchases at 8 *l. per Cent.* that the Nominee, on whose Life the Annuity depends, may or has an even Chance to die in 25 Years; but with Regard to the Annuitant who purchases at 4 *l. per Cent.* that the same individual Person may probably live, or has a Chance to live 30 Years, is a flat Contradiction; and is, in Effect,

to

to assert, either that one of the Annuities will continue after the Nominee is dead on whose Life it depends, or that the other will cease whilst the Nominee is living for whose Life 'tis to continue; when the Annuities must both determine at one and the same Time, on the Death of the Nominee.

Now if Mr. *Richards's* Tables were the Result of Dr. *Halley's* Observations, and were calculated by the Method prescribed by him, or by Mr. *Moivre*, as I believe they were; yet if the Error I have mention'd proceeds from the Foundation or the Method, as most certainly it does, I apprehend it will prove that the one or the other of them is wrong, much more strongly, than the Foundation or the Method will prove that the Calculations are right, notwithstanding the great Authority of the Inventors of them: and therefore, since the Operations upon them have produced such absurd Effects, I don't see they are of any Significance, except to have demonstrated that the Instructions are wrong. I must confess I do not know any Method, and very much question whether there be any, to adjust the Value of an Annuity for a Life, but by computing for what Term there is a Chance or Probability such Life shall continue; and when that is done, the common Tables, which give us the Value of any Term of Years at a stated Interest, give us at the same Time the Value of an Annuity for the Life sought after at that Rate of Interest. But since Dr. *Halley* has found out and publish'd another Rule for this Purpose, we will consider it.

The

The Rule then which he prescribes, and which he and Mr. *Richards* made Use of, to form these their Tables for the Value of Annuities on a single Life, is grounded on the *Breslaw* Table, and some Propositions deduced from thence. The *Breslaw* Table exhibits the whole Number of People supposed to be living in that City at a certain Time, and the supposed current Age of them all, from the Birth to the Age of 84 : And at present I will suppose, though I do by no Means grant, that this Table gives a true Account of the Persons living there, and their several Ages. To ascertain the different Degrees of Vitality in all Ages of Life, this Author advances this Proposition. *If the Number of Persons of any Age remaining after one Year be divided by the Difference between that and the Number of the Age proposed, it shows the Odds that there is, that a Person of that Age does not die in one Year. For Instance; a Person of 25 Years of Age has the Odds of 560 to 7, or 80 to 1, that he does not die in a Year: Because that of 567 living, of 25 Years of Age, there do die no more than 7 in a Year, leaving 560 of 26 Years old.*

Here I have given the Author's Proposition and Instance in his own Words; but if I may have Leave to express the Meaning of them in my Way, which, perhaps, may tend to explain and illustrate them, I should say: That the Number of Persons living of any Age proposed was the Number of Chances belonging to a Person of that Age; that the Number of Persons living of any subsequent Age, suppose at one Year's,

Year's, at ten Years, or at twenty Years Distance, was the Number of Chances with him, or the Number of Chances that he had to live to such subsequent Age; and that the Difference between the Number of Persons living of the Age proposed, and the Number of Persons living of the subsequent Age, that is the Number of Persons dying in that Time was the Number of Chances against him, or the Chances that he would die within that Time, or before he arrived at such subsequent Age. In the Instance here given, a Person of the Age of 25 has 567 Chances, so many Persons being living of that Age; of the Age of 26, which is the next Year, there are 560 Persons living, and within the Compass of that Year there have died seven Persons; so that on the Life of a Person of 25, there are 560 Chances with him that he lives to the End of one Year, so many being living at the End of that Year, and there are 7 Chances against him that he dies within that Time, so many being dead in that Time. This Method, if pursued, gives the Chances for him and against him in like Manner for any other subsequent Year. For Instance; the Number of Persons living of the said Age of 25 is 567, the Number of Persons living at ten Years End, or of the Age of 35, is 490; the Difference between these two Numbers, or the Number of Persons dying in that Time, is 77; therefore the Chances with him, or the Chances that the Person lives to ten Years End, are 490; and the Chances against him, or the Chances that he dies within that Time, are 77.

I have

I have here avoided the Use of the Word Odds, because 'tis an equivocal Term, and may mean the Odds in the Number of Chances ; or the Odds in the Value of those Chances, and instead thereof I make Use of the Expression, Chances with him or Chances against him, and shall do so hereafter ; for if this Author does never intangle himself and his Reasoning with the Ambiguity of that Term, I am sure another does : For I have met with an algebraical Calculation of the Length of two Pages, composed by the Honourable *Francis Roberts* a Gentleman very learned in Figures, to prove a Proposition which he calls an arithmetical Paradox, which, in Truth, is no better than a meer Quibble on the double Meaning of the Word Odds ; for, instead of the Word Odds in the Gentleman's Proposition, let us insert the Expression, Chances for and against him, and the Paradox is gone, and the Demonstration unnecessary.

The Proposition, as it stands in a Collection of Tracts, which are taken out of the Philosophical Transactions, is this. *There are two Lotteries, in either of which a Gamester paying a Shilling for each Ticket ; the first Lottery, upon a just Computation of the Odds, has 3 to 1 of the Gamester, the second has but 2 to 1 ; nevertheless, the Gamester has the same (and no more) Disadvantage in playing at the first, as in the second Lottery.* I admit that the Gentleman proves the Truth of his Position by a rational Argument, and I doubt not but he proves it by his algebraical

cal Proceſs likewise; but this, I think, is full as great a Paradox in Arithmetick, *viz.* that 2 is as much as 42; and yet most certainly this is a true Proposition, if by 2 I mean two Pieces of Gold call'd Guineas, and by 42 I mean so many Pieces of Silver call'd Shillings: But I believe there is never a Cobler in the Kingdom who sings over old Shoes, or Ploughman that whistles after his Horses, but could readily clear up this Difficulty. And the seeming Paradox here, to be sure, might have been solv'd without such a vast Profusion of Algebra as is expended on this Occasion; specially when it had been laid down as a Maxim, that the Value of a Ticket in a Lottery is the Sum produced on dividing the total Value of all the Prizes by the Number of Tickets; for this algebraical Proceſs, as long as it is, proves nothing but what was before taken for granted or proved by this Maxim; and therefore some will surmise that this is meerly an Impropriety of Language, and that the Proceſs in Algebra need not to have been quite so long to prove that a Man may talk or write improperly, specially if he studies to do so.

And though this Author has very well proved his main Position, that an Adventurer has an equal Disadvantage in either of these Lotteries; yet he has mis-stated the Case in both of them, or has mis-computed the Odds in them, when he tells us, the Odds in one is 3 to 1, and in the other 2 to 1; for the Odds are 2 to 1 in one Lottery, if 4 Blanks to 2 Prizes makes such Odds, and an even Chance in the other Lottery,
if

if an equal Number of Prizes and Blanks makes an even Chance: For the Number of Tickets in each Lottery is fix, and in the first there are four Blanks and two Prizes, and in the latter three Blanks and three Prizes. But be that as it will, I do not see the mighty Discovery in finding out that he who has only one Chance in three (which I think is a Blunder to call the Odds of 3 to 1 against him) to gain a Prize in one Lottery, has or may have the same Advantage as he that has one in two or an even Chance in another Lottery; for we need only to make the Value of the Prizes in Proportion, and he that has only one Chance in a 1000 *l.* shall be in as good a Condition as he that has one Chance in two.

I do not say Dr. *Halley* has been guilty of the same Fault with this Honourable Gentleman, for the long and elaborate Calculations just mention'd tend to no Purpose but to prove something which is previously supposed or proved by a Maxim; and when a Position is so proved, I don't think a long String of Algebra adds either Weight or Beauty to the Argumentation: But I am afraid, before we come to the End of this Discourse, we shall find that the Doctor, or Mr. *Moirre*, or both, in the Computation of the Value of Annuities for Lives, have, in some Instances, brought to Account more Chances of Vitality on Lives than belong to them, in other Instances, have inserted such Chances as do not and can not belong to them, and in some Cases have assign'd wrong Values to right Chances: And where any of these Mistakes are committed,

all the Algebra in the Universe will never prove that the Sum total of the Values arising on such Computations will be the true Value of the Annuities inquired after.

The Doctor having deduced his Proposition from the *Breslaw* Table, and asserted, that the Valuation of Annuities for Lives depends on it; and having laid it down for a Maxim, that a Purchaser of an Annuity on a Life ought to pay for such Parts only of the Value of the Annuity as he has Chances that his Nominee is living, which is undoubtedly true; and having told us how the present Value of a Sum of Money, payable at any future Time, and at any Rate of Interest, may be known; and given us a Scheme for that Purpose, at the Rate of 6 per Cent. Interest: He assumes, from the foregoing Proposition; *That, as the Number of Persons living after a Year, or after any Number of Years, are to the Number of Persons dead within that Time, so are the Number of Chances with him to the Number of Chances against him, that the Person nominated is then living:* In Consequence of which, the Rule for ascertaining the Value of an Annuity for a given Life will be this. *As the Number of Persons living of an Age proposed is to the Number living after one Year, or after any Number of Years; so the present Value of the Sum payable at the End of one Year, or at the End of the given Number of Years, is to the Sum which ought to be paid for the Chance which the Person has to enjoy his Annuity for that one Year, or the given Number of Years.* And in the Conclusion he adds;

If

If this Operation be repeated for every Year of the Person's Life, that the Sum of all the present Values of those Chances is the true Value of an Annuity for such Person's Life: And that by this Rule he framed the Table he gives us, which is for the Value of Annuities on a single Life at 6 l. per Cent. Interest.

I will not positively say, that the Author, in forming his Table, did not himself observe his own Rule, because he says he used some Compendia in forming it, and in seeking a shorter Way he might possibly miss the right Way; but if I understand his Rule right, as I am sure I do, unless the Expression, for every Year of the Person's Life, has some Meaning in it which I cannot find out; and if my Calculations by that Rule are right, as I am confident they are: Thus much I may say, that the Rule does not in every Instance produce that Value which is exhibited in the Table as the Value of an Annuity for a Life. The Rule, in short, is this: We are, in the first Place, to compute what is the Value of an Annuity for one Year of the Life, beginning at the Age of the Person nominated, looking upon it as an absolute Annuity for one Year, allowing and discounting out of it a Value proportionate to the Chance of Mortality arising in that Year; and this is done by the first Part of the Rule: and then we are to proceed in the same Manner for every Year of such Nominee's Life; and 'tis asserted, that all these Values put together are the Value of the Annuity on such a Life. But then the latter

Part of the Rule, where it directs that this Operation must be repeated for every Year of the Nominee's Life, does not distinctly and expressly say for what Number of Years this Computation must be made: But I cannot find out that 'tis capable of any other Meaning than one of these two, either that it must be made for so many Years as the Nominee has an even Chance to live, or for so many Years as he has any Chance at all, or a Possibility to live. However, let the Computation be made in which Way of the two he pleases, it produces a Value different from what this Table gives; one of them a Value much under, and the other a Value a little over, the Value exhibited in the Table, and this is the Case in more Instances than one.

The Value exhibited in the Table for an Annuity on a Life of ten Years old, Interest computed at 6 *l. per Cent.* is 13-44; and if an Annuity be granted for 10000 *l. per Annum*, suppose on the Life of *A.* of ten Years old, the total Value of it will be 134: 400 *l.* To show that this is not the true Value of such an Annuity, even on a Computation made according to the Rule here laid down, I have made a Calculation pursuant to that Rule, for every Year of such a Person's Life, from the Age of his Life when nominated, that is, from ten Years old, to the Extremity of old Age, that is to an 100 Years old; and have set it forth in Table N^o. IX. In this Table, and in the second Column, we have the present Value of such an Annuity payable at the End of each Year for 90 Years on an absolute Term, transcribed from
this

this Author; and in the sixth and last Column, we have the present Value of such an Annuity payable at the End of each Year for 90 Years, on the Chance of Mortality on the Life of a Person of ten Years Age, form'd exactly, I think, by the Rule given. And because the Chance of Mortality is calculated by the *Breslaw* Table, I have inserted that likewise in Table N^o. X.; with an additional Computation for 16 Years, from the Age 84 to the Age of an 100, omitted in the former Table.

From the Table, N^o. IX. and the sixth Column, and by a short Calculation added at the End, it appears; if we compute the Values on the Chance of Mortality for so many Years only of such Person's Life as he has an even Chance to live, that is for 41 Years, the Value is no more than 13-02; and if we compute such Values to the Extremity of Life, that is, for 90 Years, the Value is 13-50. I admit that the Value given in the Doctor's Table, and the Sum arising here, does not vary greatly; for 13-44 is not much short of 13-50, which in so large a Total is an inconsiderable Sum: But then I observe, and it appears from my Table, and the Calculation at the Foot of it, that 13-44 is equal only to a Term of 58 Years, if a Person of ten Years old so long lives, whereas I suppose we ought to compute here to the Extremity of Life, that is for 90 Years, and the Sum produced ought to be equal to a Term for 90 Years, if the Person so long lives: So that in this Author's Way of computing in this Instance, which has

produced no more than 13-44, there is an Omif-
 fion of no lefs than 32 Years in the Computa-
 tion.

Now I fuppofe it will not be pretended that
 an Annuity for 58 Years, if a Perfon of ten
 Years old fo long lives, is of equal Value with
 an Annuity for 90 Years, if fuch a Perfon fo
 long lives; for though the Difference is fmall,
 fome Difference there is, and there is no more
 Reason for omitting 32 Years in the Computa-
 tion, than there is for omitting 42 Years, or any
 other Number of Years whatever; therefore
 13-44, the Value given in the Table, is not the
 right Value of an Annuity on fuch a Life, even
 though the Calculation be made according to the
 Rule here laid down. If it fhould be alledged,
 that this Variation is no other than what might
 eafily arife from a compendious Way of making
 the Calculation, which the Author intimates he
 made Ufe of, and that 'tis not of Confideration
 enough to overthrow the Rule; yet I affert,
 that neither 13-44, nor 13-50, is the true Value
 of fuch an Annuity, but that its Value is 15-12:
 And if I prove it, this, I fuppofe, will be a fuf-
 ficient Demonftration of the Faultinefs of this
 Table, and of the Rule too by which it was
 framed.

To prove this, and from the Doctor's own
 Pofitions, as I mean to do, and can very readily
 do, I muft have Recourfe to, and transcribe
 hither, another Proposition advanced by this
 Author, and which he calls his third Ufe of the
Breflaw Table. *If it be inquired at what Number*
of

of Years it is an even Lay (or Chance) that a Person of any Age shall die, this Table readily performs it; for if the Number of Persons living of the Age proposed be halved, it will be found by that Table at what Year the said Number is reduced to Half by Mortality; and that is the Age to which 'tis an even Wager (or Chance) that a Person of the Age proposed shall arrive before he dies. As for Instance; a Person of 30 Years of Age is proposed, the Number of that Age is 531, the Half thereof is 265, which Number is found to be between 57 and 58 Years; so that a Man of 30 Years of Age may reasonably expect (or has an even Chance) to live between 27 and 28 Years. This Rule for the Valuation of Annuities upon a Life, founded on the Proposition last mention'd, is a very obvious and a very natural one, easily to be understood, and readily to be practis'd; and, if applied to the Case we put, stands thus. The Number of Persons of the Age of 10, of which Age we have supposed *A.* to be, is 661; the Half of this is 330; this Number we find between 51 and 52, so that *A.* has an even Chance to live 41 Years, and something over: A Term for 41 Years, Interest computed at 6 *l.* per Cent. is in Value 15-12, as may be seen in Table N^o. I.; therefore an Annuity for the Life of *A.* who has an even Chance to live 41 Years and beyond, must be worth 15-12, and above.

If to this Case we apply the Doctor's Maxim, that the Purchaser of an Annuity on the Life of *A.* ought to pay only for such Parts of the Value of the Annuity as he has Chances that *A.* is living,

living, it will stand thus. If the Annuitant has an even Chance that *A.* will live above 41 Years; though we admit there are many Chances against him in that Time, or many Chances that he does not live 41 Years; yet there must be so many Chances with him that he does live to 41, and so many Chances with him that he lives beyond 41, that the Number of all the Chances put together must be such that his Chance upon the whole is an even one that he lives for 41 Years: The Chances with him, so long as they last, are Chances to enjoy the whole and every Part of the Annuity; but those Chances with him are so many, that they must continue 'till it becomes an even Chance that *A.* dies, that is, 'till he has lived 41 Years, and over: And therefore a Purchaser having so many Chances of enjoying the whole and every Part of this Annuity as amount to an even Chance of enjoying it for 41 Years, and upwards, must pay for it as for an absolute Annuity for 41 Years, that is, must pay for it 15-12. Or put it thus: If the Annuitant has an even Lay or Wager, or may reasonably expect, that *A.* his Nominee will live 41 Years, he may for that Reason expect to receive the whole or all the Parts of the Annuity for that Time; for he must receive the whole of it, if his Nominee so long lives, and that he shall so long live is admitted to be a reasonable Expectation; and then I am sure the Seller of the Annuity on his Part has the same Reason to expect to be paid for the whole Value of the Annuity as an absolute Annuity for 41 Years, as the Purchaser has Reason

Reason to expect to receive the whole of the Annuity for that Term of Years.

I cannot conceive, for my Part, on what Account the Doctor deserts this easy, plain, and familiar Method of ascertaining the Value of these Annuities, specially when it has this farther very advantageous Circumstance attending it, *viz.* that it determines the Value of such Annuities for every Rate of Interest at once, for so it manifestly does, by giving the Number of Years to which a Life is equal; whereas of the Method he uses he says himself, and 'tis notoriously true, that it requires a very long and laborious Calculation; and 'tis as true, that at best it gives us the Value of Annuities at one Rate of Interest only, and for every other Rate of Interest the same long and laborious Calculation must be entirely repeated; and most other Persons, all who think as I do, that the Author of it has not himself pursued his own Method with Exactness, will be apt to say of it, that 'tis an intricate and an abstruse one.

I have already observed that this Author has advanced two Propositions, from each of which may be deduced a Rule to compute the Value of this Annuity; and from one of those Rules, which is drawn from the first Proposition, and which the Author himself makes Use of, is produced, as we have seen, the Sum of 13-44, or rather 13-50, as the Sum to be paid for it, on the Contingency of Mortality; and from the other Rule, which is drawn from the second Proposition, is produced the Sum of 15-12 as the
Sum

Sum to be paid for it; in which latter Method nothing is allow'd for the Contingency of Mortality during the Term of 41 Years, but the Life of *A.* being on an even Chance equal to a Term of 41 Years, is stated as equal to an absolute Term for so many Years. Now if both these Propositions are true, one or other of these Rules deduced from thence, producing these two different Sums, must be false; for 'tis impossible that two Rules, producing two Sums so widely different, should both be right. That both Propositions are true will easily be admitted, for they are in Effect one and the same, only cloath'd in different Expressions; at least, the second is a Deduction and a necessary Consequence of the first: And if we can deduce a Rule from a third Proposition which shall give us the same Value of this Annuity as is produced by the Rule drawn from the second Proposition, this will be a strong Confirmation of the Justness of the second Rule, and of the Badness of the first.

The third Proposition, which the Doctor calls the second Use of the *Breslaw* Table, and which is framed to find out the Odds (or the Chances) that any Person nominated does not die before he attains any proposed Age, is this: *That we divide the Number of Persons living of the proposed Age, by the Difference between that Number and the Number of Persons living of the Age of the Person nominated, and the Number produced shows the Odds (or the Chances) that the Person lives to the Age proposed.* *A.* the Person nominated,

nated, is of the Age of 10 Years, and the Question is, What are the Odds (or Chances) that he lives to the Age of 51, that is, that he lives 41 Years? The Number of Persons living of the Age of 51, the Age proposed, is 335; the Number of Persons living of the Age of *A*. is 661; and if out of this Number we subtract 335, the Number left will be 326, which is the Difference between the Number of the Persons living of one Age, and of the Persons living of the other Age; or, which is the same Thing, 326 is the Number of Persons who have died in these 41 Years, which comes out to be something less than a Moiety: The Odds therefore that *A*. lives 41 Years, or to be 51 Years old, are plainly these, as 335 is to 326; or, if I may use my Manner of Expression, there are 335 Chances with him that he lives 41 Years, and 326 Chances against him, that he does not live so many Years: That is, in either Way there is something better than an even Chance that he does live 41 Years.

We have seen already, if we make Use of the second Proposition, and the Rule deduced from thence, and a very plain and easy Rule it is; that 'tis more than an even Chance that *A*. supposed to be ten Years old, will live 41 Years and over, from the Time that the Annuity is granted: The Consequence of which is evidently this, that the Value of the Annuity depending on this Life, by whichever Rule of these two we compute the Term of such Life, will be 15-12: And from these Premises, which are collected from the Doctor's own Positions, I conclude

clude thus. Since the two first Rules for adjusting the Value of this Annuity vary so much in the Quantum which they respectively produce, that one or other of them must necessarily be false, for so one must be, unless they concur in producing the same, or very near the same Quantum; and since we see here a third Rule, drawn from another Proposition of the same Author's, which produces the same Chance of the Duration of the Life, as that Rule which I call his true Rule does; and the Duration of the Life must be the Measure of the Duration of the Annuity; the Presumption will be a very strong one, if it does not amount to a Demonstration, that the other Rule, which is the Rule the Doctor made Use of, is not a true one.

So much for the Proofs of the Falsity of this Rule drawn from the Author's Positions; let us examine now how the Argument stands on the Effects which the Rule has produced in the several Tables which have been form'd from thence for the Valuation of these Annuities. In my Observations on the Doctor's Table for this Purpose, I took Notice that it does not preserve a Decrease either in the Value or in the Term to which an Annuity is made equal in a Proportion answerable to the Decrease in the Chance of Vitality from one Period of Life to another. In particular, that on an even Chance a Life of ten Years Age will have a Duration for a Term of 41 Years and above, and the Annuity depending on that Life for 28 Years only, and about a Quarter over; but that, on the like
even

even Chance, a Life of 70 Years of Age, and an Annuity depending on that Life, have one and the same, or very near the same, Duration. I omitted, but might have taken Notice of the same Defect in Mr. *Richards's* Table for the like Annuities, with this small Difference; that on his Table the Chance of the Duration of the Annuity for the elder Life is made about three Months short of the Chance of the Duration of the Life. The even Chance of the Duration of a Life of 12 Years Age, which is the Age inserted in Mr. *Richards's* Table, is a Term of 40 Years, and the Chance of the Duration of the Annuity for that Life is a little short of 28 Years; but on a Life of 72 the even Chance of its Duration is just 4 Years, and the Annuity depending on it is by his Table made equal to three Years three Quarters. But even this small Difference will be reconciled, and both Tables appear equally and alike defective in this Instance, when an Error in the Doctor's Table is rectified, and the Value of an Annuity for a Life of 70 is made only 5-23, as upon a Computation made I find it should be, instead of 5-32: For then the Difference between the Chance of the Duration of a Life of 70 compared with the Duration of the Annuity on it in one Table, and the Chance of the Duration of a Life of 72 compared with the Duration of the Annuity on it in the other Table, will be much one and the same, *viz.* about a Quarter of a Year. Here again we have another Error in the Doctor's Table, even on a Computation made according to

to his own Rule ; for I have computed the Value of an Annuity for a Life of 70 by that Rule, and 'tis no more than 5-23, when he gives us 5-32. Since then this Rule, where it has been made Use of by Variety of Hands, concurs in producing the same or much the same Effects, I cannot but think that those Effects are necessary ones ; and if such, I leave it to the Reader to make his Conclusions on the Rule which produced them.

Under a former Head I took Notice, that Mr. *Richards* formed his Table for the Value of these Annuities likewise from this same Rule, at least at the Rates of 4*l.* 6*l.* and 8*l.* per Cent. Interest, and at the Rates of 5*l.* and 7*l.* by an Equation ; and I observed there, that for every different Rate of Interest he has stated such a Value, as the Value of an Annuity on one and the same Life, that the Term corresponding will be different as the Rate of Interest is different : And I affirm'd that such Difference gives such a Term to which a Life is equal, that 'tis not in Nature possible that a Life can be equal. I assert then, that the Value of an Annuity for a Life depends upon and is govern'd by the Term to which the given Life is equal, or has an even Chance to continue in Being ; as much and as certainly as the Value of an Annuity for a Term of Years does depend upon and is govern'd by the Number of Years for which that is to continue : And that one and the same Life cannot possibly be equal to two or more different Terms, because and when two or more different Annuities,

ties, granted at two or more different Rates of Interest, are depending on such Life; for 'tis in Effect to say, that a Person does exist and does not exist at one and the same Time; and that there is one and the same Chance that he does live, and that he does not live, to a determinate Time. The Consequence of this must be, if this Difference in the Term to which a Life is equal is a necessary Result from this Rule, that it effectually destroys the Rule; I shall proceed therefore to show that this Difference, or some Difference, if not the identical one we find in these Tables, does necessarily result from the Rule; and that the Value of an Annuity for a Life at the lower Interest must be equal to a longer Term, than the Value of an Annuity for one and the same Life is equal to at the higher Interest.

I affirm then; if a certain Value and Term corresponding are given, and out of that Value a proportionate Part be deducted; first, that the Term corresponding to the Value remaining will not bear the same Proportion to the Value remaining as the original Term does to the original Value: Secondly, that the Term corresponding to the Value remaining after a Deduction, at one Rate of Interest, will greatly differ from the Term corresponding to the Value remaining, after a Deduction, at another Rate of Interest: And Thirdly, that it differs in this Manner; *viz.* at the lowest Interest, that is at 4*l. per Cent.* that the Difference or Decrease in the Term is the least, and at the highest Interest,

that is at 8 *l. per Cent.* the Difference or Decrease in the Term is the greatest. To put this Matter in the clearest Light I can; I will state a Case, and suppose that the Person on whose Life an Annuity is granted is ten Years old, that the Life of such a Person is equal to a Term of 28 Years, and at all Rates of Interest equal to the same Term.

	Value.	Term.	
An Annuity, for the whole Life, is	16-64	28-0-00	Interest at 4 <i>per C.</i>
A Moiety of the Annuity, is	08-32	10-2-00	

An Annuity, for the whole Life, is	11-05	28-0-00	Interest at 8 <i>per C.</i>
A Moiety of the Annuity, is	05-52	07-2-00	

On this Case, in which the proportionate Part of the Value deducted is a Moiety, the Truth of my first Position appears; for that the original Value and Term were equal to one another, but the new Term is only a third Part, or thereabouts, of the old Term, when the new Value is a Moiety of the old one: And the Second and Third Positions are as evident, for that at 4 *l. per Cent.* which is the lowest Interest, the new Term, or the Term corresponding to the Value remaining after the Deduction, is more than one

one third Part of the old Term, or the Term corresponding to the first Value ; for the Term corresponding to the first Value is 28-0-00, and the Term corresponding to the second or remaining Value is 10-2-00 : But at 8 *l. per Cent.* which is the highest Interest, the new Term, or the Term corresponding to the Value remaining, is not one Third (I should rather say, is little more than a Fourth) Part of the old Term, or the Term corresponding to the first Value ; for the Term corresponding to the first Value is 28-0-00, and the Term corresponding to the second or remaining Value is only 7-2-00.

From these Premises it necessarily follows ; if the proportionate Part deducted out of a given Value be a twentieth or an hundredth Part only, yet that the Term corresponding to the Value remaining after the Deduction must be different at different Rates of Interest ; and that at the lowest Interest the Difference or Decrease in the Term will be the least, and at the highest Interest that Difference or Decrease will be the greatest : From whence it follows, if we compute the Value of an Annuity for a Life at the several different Rates of Interest, and compute such Values by deducting a proportionate Part of the Value of an absolute Annuity out of each Year of the Life of the Nominee, as the Rule directs us to do : I say, it necessarily follows, that the Term corresponding to the Value remaining after such Deduction made, must be a shorter Term for every Year of the Life, and consequently a shorter Term for the whole Life, in

those Cases where a proportionate Part of the Value is deducted out of each Year of Life, at the higher Interest, than in those Cases where a proportionate Part of the Value is deducted out of each Year of Life, at the lower Interest: which was the Thing to be proved.

Since then the Matter of Fact, on a Computation, is agreeable to the Reason of the Thing, as from Mr. *Richards's* Tables which were form'd by the Rule it appears to be, I might very well rest the Matter here; for this, as I apprehend, is a Demonstration that the Rule itself must be wrong; since in Fact it does, and necessarily must, produce such Terms, as the Terms to which a Life is equal, to which Terms 'tis in Nature impossible that a Life can be equal, unless a Life can be in Being and not in Being at one and the same Time. However, we shall go on to examine the Directions of the Rule, and we shall discover, perhaps, from thence, not only that 'tis erroneous, but wherein the Error of it lies: And this I choose the rather to do, because the Arguments which tend to disprove the Doctor's Rule will prove and establish mine.

To determine the Value of the Annuity for the whole Life, the Rule directs us to adjust first the Value of it for each Year of the given Life, and to carry on such Computation as long as Life can well be supposed possible to last, and then to put together all these Values: And 'tis affirm'd, that the Aggregate or Sum total is the Value of the Annuity for the given Life. To adjust the Value of the Annuity for each Year
of

of Life, the Rule directs us; of the Value of the Annuity, as an absolute one for a Year, to take such Parts only as correspond to the Chances that the Nominee lives to the End of the Year; or, which comes to the same Thing, out of the Annuity, as an absolute one for a Year, to deduct such Parts of it as correspond to the Chances that the Nominee is dead before the End of the Year; and 'tis affirm'd, that the Parts of the Value which are taken in the first Case, or the Parts of the Value which are remaining in the latter Case, which will be one and the same Sum total, are the Value of the Annuity for each Year of the Life.

'Tis certain, and will be admitted, that in each Year of the Life of any Person there are many Chances arising that he dies within such Year, which we may call Chances of Mortality or losing Chances, as well as there are many Chances that he lives to the End of those Years respectively, and these we may call Chances of Vitality, or gaining Chances: and if the Rule be applied to the Life of *A.* of ten Years Age, and on an Annuity of 10000 *l. per Annum*, the Value corresponding to the gaining Chances arising in the first Year of the Life of *A.* will be the Sum of 9319 *l.* as the Value of the Annuity for such first Year of that Life, as may be seen by my Table N^o. IX.; and the Sum of 115 *l.* will be the Value corresponding to the losing Chances arising in the same Year, and to be deducted out of the present Value of that Annuity taken as an absolute Annuity for one Year:

Those two Sums 9319*l.* and 115*l.* making together 9434*l.* which is the Value of the Annuity as an absolute one for one Year. Now for these losing Chances arising in each Year I assert, that no Value at all is to be deducted in the Year or Years in which they respectively arise; but in another Manner, and in another Place, as I will prove presently.

Supposing here, but not admitting, that on this Account some Value were to be deducted, I do deny that 115*l.* is the right Sum; because that Sum is the Value corresponding to the Chances that may be lost in each Year, and those Chances are proportionate Parts of the Value of the Annuity; whereas the Sum to be deducted, if any, ought to be the Value corresponding to the Term or Number of Days which on the Chance of Mortality may be lost in each Year. Now if these two Sums will be different; that is, if the Sum produced as the Value corresponding to the Chances which may be lost will be one Sum, and the Sum produced as the Value corresponding to the Number of Days in a Year which may be lost will be another Sum, these two different Sums will have a different Term corresponding: And since the Sum produced in the latter Way is the Value corresponding to the Term or Number of Days, such Sum must give us the right Term or Number of Days to be deducted, and the former as giving a different Term must give us a wrong one. This Way of Reasoning will hold equally good, and in the same Manner, in the other Case; I mean if

if we take the Value corresponding to the gaining Chances in each Year as the Value of the Annuity for such Year; when we ought to take the Value corresponding to the Term or Number of Days which may be gain'd in each Year; here likewise, the Sums produced, and the Term or Number of Days corresponding will be different, and the latter only the right Sum. I admit that the Difference in the Term or Number of Days in each Year, arising from these two Ways of Computation, will not be great; but then it is to be considered, that this Operation is to be continued through all the possible Years of Life, that is for 90 Years in some Cases; and if there be some Difference, be it more, or be it less, such Difference, though it were only one Day in each Year, and whether it be too much or too little by that one Day, is fatal to the Rule: For if my Position is right, that the Value of an Annuity for a Life depends on the Number of Years and Days to which such Life is equal, if the Value produced does not correspond to such Number of Years and Days, such is not the Value of the Annuity, nor the Rule a right one. To prove then, that the Sum corresponding to the Value of the Chances of Mortality arising in each Year will be a Sum different from the Sum corresponding to the Value of the Term or Number of Days of Mortality arising in each Year, I argue thus.

A Term of Years given corresponds and is equal to a certain Value, and a Value given corresponds and is equal to a certain Term of Years.

For Instance; a Term of 20 Years, at 4 *l.* per Cent. Interest, is equal to 13-57 in Value: and on the Reverse, 13-57 in Value is equal to a Term of 20 Years. But, where a Value and Term corresponding are given, if we add to, or if we multiply, or if we subtract from, each of them the same proportionate Parts, the Value and Term arising on the Addition or Multiplication, or the Value and Term remaining on the Subtraction, will, in none of the Cases, correspond to one another. Thus; a Term of 20 Years is in Value 13-57, if we add to the Term a quarter Part, the Term will be 25 Years, and if we add to the Value a quarter Part, that is 3-39, the Value will be 16-96: but a Term of 25 Years is in Value no more than 15-60; and the Value 16-96 is equal to a Term of 29 Years, or very near. Thus in Subtraction likewise; if out of that Term and that Value we subtract a Moiety of each, the Term remaining will be 10 Years, and the Value remaining will be 6-78: but a Term of 10 Years is in Value 8-09, and the Value 6-78 is equal only to a Term of 8-1-00, or thereabouts. The Case is the same if we make Use of Multiplication, as Mr. *Moiure* in his Rules frequently does, *viz.* if we multiply a given Term into itself, or different Terms into one another, and do the same by the respective Values corresponding, the Terms and the Values produced will in no sort correspond. For Instance; a Term of 10 Years multiplied into a Term of 10 Years produces an 100; and 8-09, the Value corresponding to that Term, being multiplied

into

into itself produces 65-44: but this Term and this Value produced are out of all Bounds of Proportion to one another.

I affirm farther; if we multiply a longer and a shorter Term, each into itself, and do the same by the Values respectively corresponding, and out of the Produce of the longer Term and of the greater Value respectively subtract the Produce of the shorter Term and the smaller Value, that the Term and Value remaining will not correspond. A Term of 10 Years multiplied into itself produces an 100; and 8-09, the Value corresponding to that Term, so multiplied produces 65-44: A Term of 9 Years multiplied into itself produces 81; and 7-42, the Value corresponding to that Term, multiplied in the same Manner, produces 54-05: If then we subtract 81 out of an 100, the Term remaining will be 19 Years; and if we subtract 54-05 out of 65-44, the Value remaining will be 11-39: But the Term remaining and the Value remaining do not correspond, for a Term of 19 Years is in Value 13-12, and the Value 11-39 is equal only to a Term of 15-2-00, or thereabouts. From hence is evidently seen, that a Term given corresponds to the Value, and a Value given corresponds to the Term; but when we add to that Value, or subtract from it, or multiply it, that the Value arising on the Addition or Multiplication, or remaining on the Subtraction, does not give us a Term corresponding which bears a Proportion to the Increase and Decrease in the Value respectively: And in like Manner,
if

if we add to, or subtract from, or multiply, a Term given, that the Value corresponding to the new Term will bear no Proportion to the Increase and Decrease of the Term respectively. I cannot forbear observing here, though it may be a little out of Time, that these Positions, which are demonstrated in Numbers and Figures, do effectually destroy the Foundations of Dr. *Halley's* and Mr. *Moiré's* Rules for calculating the Value of Annuities on Lives, not only of those for a single Life, but also those for any Number of Lives, in whatever Form and Shape they are combined: For these Rules are evidently grounded or depend upon this Supposition, which is manifestly a false one, that to add to, or subtract, or multiply, or in any other Form to operate on the Value of an Annuity, is one and the same Thing as to perform the same Operation on the Term to which the Annuity is equal: And by this Means they do, in Effect, make the Value of the Annuity to be the Measure of the Term to which the Life or Lives are equal; whereas the Value of the Annuity, as it must attend and depend upon the Life or Lives, must be governed by, and follow, the Term, or the Chance of the Term, of the Life or Lives.

At present to apply these Positions to the Life of *A.* on an Annuity of 10000*l.* and to the Method of calculating the Value of this Annuity on his Life, for the first Year of his Life, as prescribed by this Rule, the matter stands thus.

The number of Persons living of the Age of *A.* supposed to be ten Years old, is 661; the
Number

Number of Persons living of the Age succeeding is 653; the present Value of a 10000*l.* to be paid at the End of one Year absolutely is 9434, and the Sum produced by the Rule, is 9319; that is $\frac{653}{661}$ Parts of 9434*l.* is 9319*l.*; and this is the Value, or the Sum to be paid for the first Year of this Annuity on the Contingency of Mortality. Now if $\frac{1}{2}\%$ Parts, or a Moiety of a Term of 20 Years is one Thing, and produces one Sum for its Value, and $\frac{1}{2}\%$ Parts or a Moiety of the Value of such a Term is another Thing, and produces a different Sum for its Value: Then $\frac{653}{661}$ Parts of a Term for one Year must be one Thing, and produce one Sum for its Value, and $\frac{653}{661}$ Parts of the Value of such a Term must be another Thing, and produce a different Sum for its Value: But $\frac{1}{2}\%$ Parts or a Moiety of a Term of 20 Years does produce a Sum different from the Sum which $\frac{1}{2}\%$ Parts or a Moiety of the Value of such a Term does produce, as we have already shewn: Therefore, $\frac{653}{661}$ Parts of a Term must produce a Sum different from the Sum which $\frac{653}{661}$ Parts of the Value of such a Term produces. The true Value of this Annuity for this one Year, as well as for all the Years of the whole Life, depends upon, and is governed by, the Term or Number of Days of the Year, for which it will continue: But by the foregoing Proposition $\frac{653}{661}$ Parts of 9434*l.* or 9319 must be one Thing, and $\frac{653}{661}$ Parts of a Term of one Year or 365 Days must be another Thing, and produce a different Sum for its Value: Therefore the

Value

Value of the Annuity for this one Year, or the Sum to be paid for it for this one Year, must be another, and a different Sum from 9319*l.*; the necessary Consequence of which, is, that the Rule which produces 9319*l.* as the Value or Sum to be paid for this one Year, is not a right Rule: which was the Thing to be proved.

But since I have asserted that no Sum at all is to be deducted in each Year of Life, in Consideration of those Chances of Mortality, which arise in such Year; 'tis not much material whether the Sum of 115*l.* or any other Sum, might be the more proper Sum to be deducted; and therefore I shall proceed to maintain that Assertion. I observe then, that upon the Foundation of this Rule, there are two Methods of adjusting the Value of these Annuities; one by the Value of the gaining or losing Chances arising in each Year of Life, which is the Method which this Author alledges he made Use of in forming his Table of such Value; the other is, by the Value of the gainable or loseable Days, arising in each Year of his Life; so let us examine how the Computation is made in the one way, and in the other.

In this Author's Method, we compare the Number of Persons living; suppose of the Age of 10, with the Number of Persons living of the Age of 11, and one we find to be 661, and the other 653 and the Difference to be 8, that is 8 have died in that Year; so there are 653 Chances with the Annuitant, that his Nominee lives that Year out; or 653 Chances of Vitality, and there
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are 8 Chances against him, or 8 Chances of Mortality in the same Time; in the second Year there are 646 Chances with him, and 7 Chances against him; in the third Year, 640 with him, and 6 against him; and so we proceed for the whole 90 Years, that is to the very Extremity of possible Life. By a Table given us here, which I have transcribed into my Table, N^o. IX. we find what is the present Value of 10000 *l.* to be paid at the End of each Year of an 100 Years, where 'tis to be paid as an absolute Annuity, and without any Chance of losing any Part; and then we take such Parts of the present Value of the Annuity absolute, as corresponds to the Value of the Chances of Vitality, or gaining Chances, arising in each Year, as the Value of the Life Annuity for that Year: In which way of Computation, we leave out or deduct, in each individual Year, through the whole possible Life, a Sum corresponding to the Value of the several losing Chances, or Chances of Mortality, in each Year as they respectively arise. In the other Method, which differs very little from the former, we compute how many Days in each Year may be gained by the Chance of Vitality, and how many may be lost in the same Time, on the Chance of Mortality; and we take the Number of gainable Days arising in each Year respectively, as the Term to which the Life is equal in that Year, or the Value corresponding to such gainable Days as the Value of the Annuity for that Year; and in this Way as well as in the other, in every Year through the whole possible

possible Life, we leave out or deduct from the whole Number of Days in a Year, the loseable Days arising in that Year, and from the Value of an absolute Annuity for one Year, we deduct the Value corresponding to the loseable Days in the Year: the gaining Chances in one Case answering to the Days gainable on the Chance of Vitality in the other, and the losing Chances to the loseable Days on the Chance of Mortality.

Now I assert, that neither the losing Chances in each Year, if they are taken Simply and without Regard to their Values corresponding, nor the Value of those losing Chances when those Chances are resolved into the Sums corresponding; nor the loseable Days in each Year, nor the Value corresponding to those loseable Days, are to be deducted in those Years in which they respectively arise; but that those Deductions are to be made out of something else, and in another Manner; and herein lies the Error of the Rule.

To begin with the loseable Days in each Year, because the Reasoning on that Head may be the plainest and most clear; I say that those loseable Days are not to be deducted out of each Year as they arise, but all the loseable Days arising in the whole Life are to be accumulated, and the total is to be deducted out of the total Days of 90 Years, or of so many Years as the Life can possibly last; and the Remainder is the Term to which the given Life is equal: Or, which is the same Thing, only the Method is inverted, all the gainable Days in each Year for 90 Years, or for all the possible Years of the given Life
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are to be accumulated, and the Sum total is the Term to which a Life is equal. This Proposition and Method of attaining the Term to which a Life is equal, is so plain and obvious as to need neither Proof nor Illustration: But to try the Truth of the Method, I have made a Calculation on the Life of a Person of 30 Years old, of the Number of gainable and loscable Days arising in each Year of his Life for 70 Years; that is to the Extremity of Life; and the Sum total of the gainable Days comes out to be 10024, that is the Life of a Person of 30 Years old is equal to 10024 Days, or 27 Years and 169 Days; and if we consult this Author's second Proposition, and the Rule for computing the Chance of Vitality deduced from thence, he there uses this Instance, and makes such a Life equal to a Term of 27 Years, and something above; so that these two Rules may very fairly be said to produce one and the same Term, to which a Life is equal.

This then being the Produce in my Method of calculating the Chance of Duration, or the Value of an Annuity, on the Life of a Person of 30 Years old, which concurs with that produced by this Author's second Proposition and Rule; now let us see what is the Produce, if the Value be computed by his first Rule. If we consult his Table, which was form'd by such first Rule, the Value of such an Annuity there given is 11-72, which is equal to a Term for 21 Years, wanting four Decimals, or about half a Quarter: If we inquire how it comes to pass, that the
Value

Value or Number of Years in the last Way of computing, falls so much short of the Value and Number of Years in the former Way of computing, the Reason is very obvious and plain. In the former Method, all the loseable Days arising in the whole Term of 70 Years being accumulated, are deducted out of the whole Term; by which means, all the gainable Days in the Term of 70 Years, are computed as so many Years and Days in immediate Possession, and all the loseable Days, as so many Years in Reversion, after the Term in Possession, or at the end of Life; and in the latter Method, the loseable Days arising in each Year, are deducted in each Year; so that the gainable Days are not connected and all computed, as so many Years and Days in immediate Possession, there being loseable Days intervening in each Year, and those loseable Days are not computed as Days, all of them, in Reversion at the End of Life, but in Reversion at the End of each Year: For Instance, in the Calculation I made, of the gainable and loseable Days arising in each Year, of the Life of a Person of the Age of 30; (and if the Calculation there, were faulty a Day, more or less, 'tis not material here,) I found the loseable Days in the first Year to be 6, in the second Year 11, and in the third 17, and so on; which Days in this Author's Method, are deducted in each Year as they respectively arise, or a Value corresponding to them; and in my Method, the total of the loseable Days are deducted out of the total of 70 Years or the whole Term of possible

possible Life : That is, in one Case we take these fix loseable Days out of the first Year, and in the other Case we take them out of 70 Years, and a Value respectively corresponding : Therefore the Value of the Annuity, and the Number of Years to which a Life is equal, must vary greatly according as the Estimate is made by these different ways of Computation. From hence we discover plainly, what it is in these Methods which gives Occasion for the great Variety we find in the Produces of them; and that very same Thing, I say, is the Error in this Author's Method, *viz.* that it directs that those loseable Days arising in each Year, should be deducted in the Years in which they respectively arise.

That this is an Error in such Method I assert; because 'tis grounded on something which never does, and never can, in Fact, happen; for the Life of Man is made up of a continued Series of so many Years and so many Days, and not of 365 Days wanting 6 Days in the first Year, and of 365 Days wanting eleven Days in the second Year, and of 365 wanting 66 Days, suppose in the eleventh Year; and therefore since a Life cannot be discontinued at the End of 359 Days, and begin again *de novo* at the End of 365 Days; and since the Person has a Chance to live many Days in the subsequent Year, as in the Case stated 'tis suppos'd he has, he must of necessity first enjoy the fix last Days of the precedent Year, before he can enjoy the gainable Days in the succeeding Year; and if he does not there enjoy them, he never can enjoy them; so that if

they are there deducted, we deduct some Days which he has a Right, as having a Chance, to enjoy.

If we make Use of the same Rule, but the inverted Method, of ascertaining the Number of Years and Days to which the given Life is equal; that is, by computing how many gainable Days there are in each Year to the Extremity of Life, and taking those gainable Days accumulated as the Years and Days to which such Life is equal; this Way of Reasoning will be full as strong and perhaps still more clear. The Number of gainable Days, or Days which an Annuitant has a Chance to enjoy, on the Chance of the Vitality of his Nominee, in the first Year of his Life are 359, and in the second Year, 354; and the supernumerary Days, that is the 6 Days, which on the Chance of Mortality, are loseable Days in the first Year; I say, must be supplied from the gainable Days in the second Year, and placed to the Account of the Days in the first Year, so as to make up such first Year a compleat Number of 365 Days; for if they are not brought to the Account as enjoyed in the first Year, they cannot be brought to Account as enjoyed any where; and then you do not allow the Person to have a Chance to enjoy some of the Days, which the Case stated has admitted him to have a Chance to enjoy.

If therefore we will consider the Life of Man, as such a Thing as it really is, that is, that it must continue, so long as it has any Being at all, in an interrupted Series of Years and Days, and of necessity must run the Course of the precedent Year,

Year, before it can enter upon a Subsequent one; we must compute the gainable Days in each Subsequent Year as a Part of the Year precedent, so as to make it up to its full Complement of an intire Year: or, which comes to the same Thing, we must take the gainable Days in each Year for the whole Life, and join them together as in one Link, and the Sum total, which in both Ways must be exactly the same, will be the Years and Days to which a Life is equal. The Truth is, that these loseable Days, or the Days which on the Chance of Mortality may be lost in each Year, in Fact are not lost, unless Life itself is lost, and Life is suppos'd not to be lost; and therefore ought not to be computed as lost, as long as the even Chance is that it continues; and since Life, so far as its Tether goes, must be connected in a continued Chain of Years and Days, we must compute all the Days of a Man's Life, as so many continued Days in immediate Possession.

Having prov'd then that the loseable Days arising in each Year, are not to be deducted out of those Years in which they arise severally; and shown likewise, when, and where they are to be left out, and when and where they are to be brought to Account: I proceed now to make good my other Assertion, that neither the losing Chances arising in each Year, nor the Values corresponding to those Chances, are to be deducted in those Years in which they respectively arise; tho' I think there need not many Words to convince us, that the Reasoning made

Use of under the former Head, will hold good and be applicable here; yet, since this great Author's grand Rule depends on that Point, that these Deductions are to be made in this Manner, and at those Times; I will take the liberty to repeat the force of my Arguments so far, as to shew that they are applicable here; and shall make Use of the Instance on the Life of *A.* having before stated the Chances, and the Values corresponding, on his Life.

The Number of Persons living of that Age, that is the whole Number of Chances, is 661; the Number of Persons living of the Age succeeding is 653; so that the gaining Chances in that Year are 653, and the losing Chances are 8: The Value corresponding to the 661 Chances, or, which is the same Thing, the present Value of that Year's Annuity, taken absolutely and as Subject to no losing Chance, is 9434*l.* and on the same Annuity taken as Subject to the Chance of Mortality, the Value corresponding to the gaining Chances is 9319*l.* and to the losing Chances 115*l.* making together 9434*l.* The Rule then giving for the gaining Chances 653, and for the Value of them 9319*l.* and for the losing Chances 8, and the Value of them 115*l.* asserts that the greater Sum is the Sum gained that Year, and to be accounted as the Value of the Annuity for the first Year; and that the less Sum is the Sum lost, and to be deducted in that Year. My assertion is, that the Sum of 9434*l.* which is the Value of that Year's Annuity, as an absolute one, is the Value of the first Year's Life Annuity;

Annuity; and that the Sum of 115 *l.* supposed to be lost, and which I admitted, on the Chance of Mortality, possibly may be lost, is not to be deducted in that Year, but at the End of Life. I observe here, as I did under the former Head, that 'tis in Nature impossible, that *A.* should enter into a second Year of his Life, unless, and until he has lived all the Days of his first Year; and consequently that he cannot enjoy the gaining Chances of a subsequent Year, until he has enjoyed all the Chances, both gaining and losing Chances, of the precedent Year: And since 'tis supposed in the Case, that he has many gaining Chances in the next and other subsequent Years, it necessarily follows that we must look upon him as enjoying, or having a Right to enjoy, the whole Number of Chances, both gaining and losing Chances in such first Year; and then the Sum to be taken as the Value of that Year's Annuity, must be a Sum equal to the Value of the whole entire Year, that is 9434 *l.*

It is agreed that an Annuity of 10000 *l.* for one Year, and payable at the End of one Year, is worth 9434 *l.* where such Annuity is absolute, and not subject to any Contingency; and I will admit that such an Annuity, if for one Year only, on the Chance that *A.* shall so long live, is in Value no more than 9319; and that the Loss on the Chance of Mortality in that Year, is in Value 115 *l.*; and therefore that a Purchaser of such an Annuity for one Year and no more, must have an Abatement out of 9434 *l.* in proportion to the Loss possible on such Chance of Mor-

tality. But for what Reason and on what Foundation must such an Abatement be made? Most certainly for this, because on the Chance of Mortality within that Year, there is a Possibility of a Want of Value, and such Want of Value is computed at 115*l*.; and unless this Annuitant has an Equivalent for such Want of Value within that Year, he can have it no where else, for the whole of his Interest ends with the Year; but if such an Annuity be purchased for the Life of *A*. absolutely, or for 90 Years, if *A*. shall so long live, which are in Value the same; in this Case the Reason of the Thing don't hold throughout. It is indeed here, as well as in the former Case, supposed that there is some Chance that *A*. may dye in the first Year of the 90; but in this Case there is another Supposition, which destroys or sets aside, all Effect of the former; for 'tis supposed likewise, that there are many Chances of Life to be enjoyed in the second Year, which cannot be had unless, and until, he has lived out the whole of the first Year. To which I shall only add, that there is no Necessity here, though there is in the former Case, for the possible Want of Value, on the Chance of Mortality in the first Year, to make the Abatement for it out of such first Year; for that it may be made elsewhere, and I say ought to be made elsewhere, and the Place where is the very Extremity of Life, and is to be done and readily may be done, by computing the Number of losing Chances arising in 90 Years, and deduct-
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ing them out of the whole Number of Chances arising in that whole Term.

The Fault then of the Rule, in short, lies in this, *viz.* in computing the Value of each Year's Annuity, it supposes that the losing Chances arising in the first Year, and so successively in each Year, are actually lost in such Years respectively, and directs a Value, proportionate to such Loss, to be deducted out of the Value of each Year's Annuity, as not an absolute Annuity: Whereas the Fact is, that those Chances, or any Part of the Value of the Annuity, are not lost, and therefore ought not to be computed as lost; for no Part is lost or can be lost, unless the whole is lost; and the Case itself supposes that neither the one nor the other is lost; for it supposes that there are gaining Chances in the subsequent Year, and more than an even Chance to enjoy the Annuity of the second Year; but those Chances cannot be had, or the Annuity be enjoy'd in such second Year, without a previous Enjoyment of all the Chances, and of the whole Annuity of the first Year; and this holds good in each Year and on every Year's Annuity successively, till we come to that Year in which it is an even Chance, that Life is at an End. The true State of the Case then is, that there are no losing Chances, or any Want of Value in the Annuity, do in Fact, or can arise till Life itself determines, and the supposition is, that Life continues; and if it continues, it must continue without any Breach or Interruption, and so must the Annuity which is attendant on it, till

it ceases to be an even Chance, that Life has any longer Duration.

Or take the Matter in this Light; and possibly in this View we shall see the Error of the Rule more plainly, and what led this great Author into it; and the Error perhaps more properly and strictly may be in this. The Rule then directs, though not in so many Words, yet in Meaning and Effect, that we compute the Number of gaining Chances arising separately in each Year of Life, for so many Years as there is any possible Chance that the given Life may be extended to; then to ascertain the Values corresponding to such Chances, and to take all those Values put together, as the Value of the Annuity for such Life. Now I admit that the Rule gives us all the possible gaining Chances arising in Life, and Values corresponding to such Chances; and if we view the Rule in this Light, it may seem to give us the Whole of the Value of an Annuity for a Life; but I say, that the Values corresponding to those Chances, are not the very Values which the Annuitant has a Chance to enjoy. It must be admitted, and it is allowed by this Author, that on the *Breslaw* Table it is an even Chance, an even Wager, or Lay, call it which you will, that *A.* before named lives 41 Years, and that he lives no longer; and it being admitted, that this Rule does rightly ascertain the Number of gaining Chances upon this Life, it necessarily follows, that the Annuitant must enjoy all these gaining Chances within the Compass of 41 Years, and
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not afterwards, or we run ourselves into two contradictory Suppositions. The one is, that the Annuitant has not a Chance of enjoying this Annuity, which upon the Case stated is to continue for the Life of *A.*, at a Time when we have before supposed that he has a Chance that *A.* is living, which is supposing *A.* to be alive and not alive at the same Time; and the other is, that the Annuitant has several Chances of enjoying this Annuity, which, upon the Case stated, is to determine on the Death of *A.* at and after a Time, when we have already supposed that the even Chance is that *A.* is dead; which is supposing *A.* to be living at, and long after, the Time we have supposed him to be dead and gone.

If then all these gaining Chances upon the Life of *A.* must be taken in the first Part of Life, and in a continued Series, and so they must be taken, or they cannot be taken at all; and if the Values annexed to these Chances so taken, be put together; then those Values put together, the Value of the even Chance on the Life of *A.*, and the Value of an absolute Term for 41 Years, will all be the same, *viz.* 15-12. But if these gaining Chances and the Values annexed to them, are to be taken as they are computed severally to arise, in each Year of Life, and these Values put together are the Value of the Annuity for this Life, as this Rule asserts; Since many of these Chances are supposed to arise after the Expiration of 41 Years, and the Chances which arise in the first Part of Life or before 41 Years expire,

expire, are of much greater Value than those which arise in the latter Part of Life, or after 41 Years ; for Instance, if the Value of all the gaining Chances arising in the last ten Years of possible Life, is not greater than the Value of eight such Chances arising in the first Year of Life; and if we leave out or subtract the Value of eight Chances in the first Year of Life, and in their stead insert, suppose the Value of an 100 Chances in the last ten Years of Life, as this Rule directs, it may very well come out that the Value of the Annuity on such a Life, where the Computation is made in this Manner, shall be no more than 13-44, or 13-50, and if made in the former Manner, shall be 15-12 : But that those Chances or the Values corresponding to them are not then and there to be taken, but when and where they are to be taken, I think has been sufficiently shewn.

To go on therefore to another Position, which is a Consequence of the last ; I affirm that this Sum 13-44 (or rather 13-50) given in these Tables as the Value of an Annuity for the Life of *A.* supposed to be ten Years old, is the Value of another Annuity, or rather of several Annuities joined together, different from an Annuity for the Life of *A.* absolutely, or for 90 Years, if *A.* so long lives. I say then, that this Value is the Aggregate or Sum total of the Value of 90 several Annuities, granted to 90 several Persons, suppose to N^o. 1. to N^o. 2. and so on to N^o. 90. made in this Manner, *viz.* to N^o. 1. for one Year, if he shall live from 10 to 11 Years Age;
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to N^o. 2. for one Year, if he shall live from 11 to 12 Years Age; and so on successively to N^o. 90. for a Year, if he shall live from 99 to be an 100 Years old; each Person to be nominated at the Beginning of that Year for which the Annuity is granted, determinable by his Death within the Year. I affirm then, that this Sum 13-50 is the Value of those 90 several Annuities, separately computed for each Life, and then put together; and that the Interest, suppose of N^o. 100. in an Annuity for the Life of *A*. absolutely, or for 90 Years, if *A*. so long lives, is of a different and much greater Value than the Interest of N^o. 1. N^o. 2. and so on to N^o. 90. all put together: The plain and necessary Consequence of which is, that 13-50 is not the Value of an Annuity for the Life of *A*.

My first Assertion is, that the Sum of 13-50 is the Value of these several 90 Annuities put together. This is a Point that seems to require little Proof, since such Sum appears to be nothing more or other than a Collection of 90 several Annuities, separately computed for each Year of a Life for 90 Years, deducting out of each Year a Value corresponding to the Chance of Mortality arising on the Life of a Person of the Age of each Year respectively. In my Table, N^o. IX. we have a Computation of the Value of an Annuity of 10000 *l.* *per Annum*, calculated on the Chance of Mortality for each Year of the Life of a Person of ten Years old for 90 Years, drawn according to the Direction of the Doctor's Rule; the Sum total of all
which

which Annuities put together, is 135000. Now I say that any one and every one of the Annuities for the Year, if N^o. 1. N^o. 2. and so on to N^o. 90. shall live to the End of that Year for which the Annuity is respectively granted to them, is of the same Value, neither more nor less, as any one and every one of the annual Annuities exhibited in that Table; the Annuity of N^o. 1. being the same as the Annuity for the first Year of the Life there computed; the Annuity of N^o. 2. the same as the Annuity for the second Year; and so on successively and correspondently, the Annuity of N^o. 90. the same as the Annuity for the 90th Year of the Life of the Person intended in such Calculation. This will appear very evidently, on comparing the Interest of one with the Interest of the other Annuity corresponding; and, for Instance, we will take the Interest of N^o. 1. and compare it with the Interest of the Annuity for the first Year of the Life on the Table, and the Interest of N^o. 20. and compare it with the Annuity for the corresponding Year, the 20th Year of the Life on the Table.

The Interest of the Annuity belonging to N^o. 1. is the same as the Interest of the Annuity for the first Year of the Life in the Table, and must be the same; for that both are computed and ascertain'd by one and the same Rule, *viz.* by taking the Value of one Year's Annuity as an absolute one, after deducting out of such Value so much as is equal to the Contingency of Mortality on such Life within that Year. The very
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same is the Case on the Interest of N^o. 20. in his Annuity, and the Interest on the 20th Year of the Life of the Person described in the Table. Both Annuities are in Reversion, commence at one and the same Time, in all Events expire at the End of the Year, and both depend on a Contingency of determining within the Year, and upon two Lives of equal Age; and upon the Foot of such Circumstances and such Contingencies, which are in all Respects the same, the Value of each Annuity is computed.

My second Assertion was; that an Interest of an Annuity for the Life of *A*. absolutely, or for 90 Years, if *A*. so long lives, is of better Value than all these several 90 Annuities put together. This I shall prove by showing, that an Annuity for the first Year of the Life of *A*. where 'tis absolutely for his Life, is more valuable than an Annuity for the Life of N^o. 1., which is to determine with the Year; and in like Manner, that the Annuity for the second, the third, and every other Year of the Life of *A*. successively, 'till we come to that Year of the Life of *A*. in which it ceases to be an even Chance that his Life has any farther Duration, is of more Value than the Annuity for the Life of N^o. 2. N^o. 3. and so on successively, 'till we come to N^o. 42, within which Time it is computed that *A*. may be dead: Comparing the Annuities belonging to each Proprietor on the several Years as they respectively correspond to each other.

For Instance; the Interest of N^o. 1. in his Annuity may determine within the Year by his
Death

Death in that Time, and must determine at the End of the Year by the Effluxion of his Term. The Interest of N^o. 100. in his Annuity for the Life of *A*. may likewise determine within the same Year by the Death of *A*. his Nominee; but does not determine at the End of the Year by the Effluxion of the Term, for 'tis to continue for 89 Years more, if *A*. so long lives: That is, to one of the Annuities are annexed two Limitations; one of which possibly may, and the other necessarily must, put an End to it by the Completion of the Time for which it was granted. The other has one Limitation only, *viz.* the very same Contingency of Mortality which the other has; but has no such Termination at the End of the Year. Therefore N^o. 100. in his Annuity must have a better Chance to the same Thing, or a Chance to something more or better than N^o. 1. has in his Annuity. For this Reason, and on this Foundation, the Circumstances of every other of these 90 Annuitants for a Life determining with the Year, at least 'till we are come to that Year when the Chance is an even one that *A*. is dead, will be the very same; that is, their Annuities respectively will be of less Value than the Annuity for the Life of *A*. in the Year corresponding.

If we enquire what is the Difference, and what makes the Difference, in the Value of these Annuities; 'tis plainly this, *viz.* N^o. 1. N^o. 2. and so on, cannot be look'd upon to have a Chance, and their Annuities are computed upon such Foot as if they had not, and most certainly they have

have not a Chance to enjoy all the Parts of an Annuity for a Year; for within the Year they cannot be said to have a Chance to enjoy that which on the Contingency of Mortality they have a Chance to lose in that Time; and beyond the Year they have no Chance, or Possibility of Chance, because their respective Annuities absolutely determine with the Year. But with Regard to N^o. 100. he may very well be look'd upon as having a Chance to enjoy all the Parts of this Year's Annuity, because his Annuities not determining with the Year, he has many Chances of enjoying the second Year's Annuity, and by that Means he gains in such second Year what there was a Chance might be lost in the first: And this Chance of gaining in a subsequent Year what might be lost in the precedent Year continuing in the Case of N^o. 100. as long as the Chance of the Vitality of *A*. continues to be an even Chance, that is, 'till *A*. arrives to the Age of 51; the Annuitant in the whole has so many and such Chances of enjoying the Annuity for 41 Years as make it equal in Value to an absolute Annuity for that Term of Years.

As this appears to be the true State of the Matter, when we consider the Case of the Annuitant for the Life of *A*. absolutely compared with the Case of the several Annuitants for a Year whose Interests are determinable by their Lives; the same will come out to be the State of the Matter, if we take a View of the Case of the Grantor of such Annuities respectively, or of him who is to pay the same. This is an unquestionable Truth, that the Chance to pay or
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not to pay, and the Chance to receive or not to receive the Annuity, must be reciprocal and one and the same; so that if the Grantor of the Annuity has a Chance to pay a greater Sum to the Annuitant for the Life of *A.* singly, than he has a Chance to pay to the several 90 Annuitants, it plainly follows, that the single Annuity is of more Value than all the several 90 Annuities put together. With Respect to the Annuitant N^o. 1. I agree that the Grantor has a Chance not to pay such Annuity, and I admit he has just the like Chance not to pay the Annuity for the first Year of the Life of *A.*, there being within that Year an equal Chance of Mortality both against the one and the other. With Respect to the Annuity to N^o. 2., I will admit likewise that the Grantor is on the same Foot as with Respect to the Annuity for the second Year of the Life of *A.*; but then I admit this on a Supposition that *A.* is living at the Beginning of such second Year; for N^o. 2. is supposed to be then living, or rather to be then nominated to take the Annuity; for unless both are supposed to be living, and both Annuities to exist still, there is no Room to make a Comparison of the Annuities for such second Year. If then *A.* is supposed to be living in the second Year, it necessarily follows, that the Grantor must have paid the Annuity for the first Year of the Life of *A.* as an Annuity absolute, and without any Defalcation; and this is the Case in every other Year of the Life of *A.* so long as it continues an even Chance that *A.* is living: that is, we can set no Value on, or make
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any Computation of the Price to be paid for, the Annuity in any subsequent Year of the Life of *A.* without previously supposing that *A.* is then living; the necessary Consequence of which is, that the Annuity for the precedent Year must also be supposed to have been paid, and if paid the Sum paid must have been the full Value of such Year's Annuity, as an absolute one without any Deductions.

The Doctor's Hypothesis, and the Rule form'd upon it, is faulty therefore, in making two Suppositions which are not consistent one with the other, *viz.* it supposes that the Grantor of the Annuity in each Year of the Life of *A.* has such a Chance of the Mortality of *A.* as may possibly exempt him from paying that Year's Annuity; and at the same Time it supposes there is a Chance that *A.* is living in the subsequent Year without paying the Whole of the Annuity for the precedent Year as an absolute one. Now 'tis possible that *A.* may die in the first Year, for without Doubt there is some Chance he may die in each Year, and in every Day and Hour of such Year, and if he does die, the Grantor will save the Payment of the Annuity for that Year, and all the subsequent Years; but such Grantor has no Chance of *A.*'s dying or not dying in a second or other subsequent Year, unless he has, and until he has, lived out the first and precedent Year: Consequently the Grantor can have no Chance of paying or of not paying the Annuity of the second or any subsequent Year, unless he has, and until he has, paid the whole of the An-

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nuity for the first and precedent Year: So that if we make the first Supposition, there is no Room to make the second at all, and if we make the second the first is set aside and made void.

Where 90 several Annuities are granted for 90 Years, if so many several Persons live to the End of those Years respectively for which such Grants are made, here the same Suppositions are made as before, and here are rightly made. 'Tis evident in this Case, that the Grantor of these Annuities has a Chance of Mortality of the Nominees in each of these Years, such a Chance as shall discharge him from Payment of the Annuity of that Year; and if the Nominee does die within the Year, the Grantor will save that Year's Annuity. The second Supposition is likewise true here; for the Grantor will have a Chance of the Death of the Nominee in the second Year, or any other subsequent Year, whether the Nominee in the first, or any other precedent Year, did or did not die within the Year; and he will have the Chance of not paying in the subsequent Years, whether he paid the whole or paid no Part of the Annuity in the precedent Years. Therefore, the Chance of the Grantor of the single Annuity and of the several Annuities, not being one and the same, the Value of the Annuity for the single Life must be a Sum different from the Sum of the Values of the several Annuities collected together, that is, from the Sum of 1350 *l*.

There is still another wrong Supposition made in this Rule, and which, I apprehend, gave Occasion

casion to the two former, *viz.* that the Chances of Mortality are the same on the Annuity for a single Life as in these Annuities for 90 several Lives, such as I have before described; or rather that the Chance of Mortality may as often arise in the one Case as in the other. Where there are 90 several Annuities depending on so many several Lives, 'tis obvious there is a Chance of Mortality on each Life, and such Chance may arise in the Year of the respective Life; but in the Case of an Annuity on a single Life there can be but one Chance of Mortality, at least, such Chance can arise but once, for a Man can die but once only. On this Account it seems absurd to compute how many Times in each Year, or in 90 Years, the Chance of Mortality may come up, since in the whole 90 Years it can come up but once only; and most certainly 'tis unnecessary to make such a Computation, since during all the 90 Years of the possible Life of *A.* the Grantor, in the whole, has but one Chance of being discharged from paying the Annuity, and in every Year, until that one blank Chance does come up, he must pay the Whole of the Annuity, as an absolute one.

With a View of ascertaining the Time to which the Life of a Person may probably be extended, it might not be improper to compute how many Chances of Mortality there are in each Year of such Life, and from thence to collect in how many Years such blank Chance, in all Probability, will come up; and then we may state it, that the Life and the Annuity attending

on it will continue to that Year in which on such Computation the Chances of Vitality and Mortality are even, that is, when the Chances of Vitality of the given Life are sunk to a Moicty; and from and after that Year, that the Life and Annuity with it will cease; but this evidently destroys the Rule.

Since then the Chance of the Grantor of an Annuity for the Life of *A.* is a single one, and more than once it cannot come up; and the only Thing necessary to be computed is, within what Time on the probable Chances of Mortality *A.* may die: And since the Chance of his living for 41 Years and dying before 42 Years is an even Chance, as computed on the *Breslaw* Table, and several Rules deduced from thence; and seeing in any one and every one of those Years in which the Annuity is paid, the Whole of the Annuity must be paid: on this Case, and from these Premises, I infer, where an Annuity is granted for the Life of *A.* absolutely, or for 90 Years, if *A.* so long lives, that the Annuity for any one and every one of the Years of the Life of *A.* until it becomes an even Chance that *A.* does no longer exist, is of more and greater Value than an Annuity for any one and every one of the Years corresponding, where the like Annuity is granted to 90 several Persons for 90 several Years, if such several Nominees shall respectively live to the End of that Year for which the Annuities are respectively granted.

The Conclusion, upon the Whole, is this: Since the Sum of 13-44, (or rather 13-50,) which

which is the Value of an Annuity for the Life of *A.* when such Value as computed by this Rule, is the Aggregate or Sum total of the Value of 90 distinct and separate Annuities for a Year for so many single Years as the several Nominees in them shall respectively live, and which determine at the End of each Year; and since the Value of an Annuity for the Life of *A.* absolutely, or for 90 Years, if *A.* so long lives, is the Aggregate or Sum total of the Value of so many Years of 90 Years as *A.* shall live to the End of, and which do not determine at the End of each Year, but at the End of all the Years, or by the Death of *A.* and then only: And since the Value of the Annuity in each Year, where it is for a Life absolutely, amounts to a greater Sum than the Value of the Annuity, where it determines at the End of each Year, amounts to: The Conclusion is, that this Sum of 13-44 or 13-50, which is the Sum total of the Value of those Annuities which are for Lives, and determine at the End of each Year, is not the Sum total of the Value of the Annuities which are for Life absolutely, and do not determine with the Year, that is, is not the Value of the Annuity for the Life of *A.* Q. E. D.

The Doctor, in framing this Rule, and as a Foundation on which he builds it, lays down this Maxim; that a Purchaser of an Annuity for a Life is to pay for such Parts only of the Value of the Annuity as he has Chances that his Nominee is living, which most certainly is true; but then 'tis as certainly true, that he is to pay

for all the Parts and the very Parts of the Annuity which he has Chances to receive on the Life of his Nominee: But whether this Maxim be rightly and thoroughly complied with and preserved in this Rule, is a great Question. In the Case before stated on the Life of *A.* 'tis pretty plain, that the Rule does not bring to the Annuitant's Account those Parts of the Value of the Annuity which he has a Chance to enjoy, since he has a Chance to enjoy all the Parts of it, as an Annuity absolute and entire for so many Years as he lives, whereas the Rule assigns to him such Parts only as are answerable to the Chances of Vitality arising in each Year. If we vary this Case, and put it, that an Annuity is purchased on the Life of *B.* whom we will suppose to be of such an Age, be that what it will, as that the even Chance is that he lives one Year and no more; I apprehend we shall see more plainly that the Annuitant, purchasing according to the Direction of the Rule, does not pay for all and the very Parts of the Annuity which he has the Chance to enjoy, and must enjoy, if he enjoys any.

We will take then the Age of 80 as the Age in which *B.* has an even Chance to live one Year, and no more; and in stating this to be the Chance of Vitality for such Age, we necessarily suppose, and ground our Case upon this, that the Chances of Vitality and Mortality arising in that Year are even. Now if the Purchaser of the Annuity for the Life of *B.* absolutely pays no greater Sum for his Interest than a Purchaser
pays

pays for his Interest who buys the like Annuity for one Year only, determinable by the Death of *B.*, 'tis obvious, and I suppose it will be admitted, that the Owner of the Life Annuity absolute does not pay for all the Parts of his Annuity which he has Chances that his Nominee is living. The Proprietor of the Annuity for a Year, determinable by Death within the Time, pays only for so many Chances of Vitality as arise within the Year, that is half a Year's Value, and in Justice ought to pay for no more, since he can have no Chances arise beyond the Year; but the Proprietor of the Life-Annuity absolute, if the Price of it were the same and no more, would have many Chances for which he pays nothing. In this Instance, where there are as many Chances of Vitality to arise beyond the Year as there are Chances arise within the Year, he would have just as many Chances of Vitality left at the end of the Year, for which he pays nothing, as he has Chances for which he does pay.

It being admitted then, that the Purchaser of the Annuity for the Life absolute, is to pay more than he who purchases for one Year only, if *B.* so long lives; it is said that these Chances of Vitality left at the End of such first Year, are not totally sunk and unpaid for, by the Purchaser of the Life-Annuity absolute; for though they are not brought to Account and paid for as Chances of Vitality arising in the first Year, yet the Rule directs, that the Chances of Vitality are to be computed likewise for the subsequent Years, as long as Life can be supposed possi-

bly to last; that the Chances arising in those subsequent Years of possible Life, are supposed in the Case to be the same, in Number, as the Chances left at the End of the first Year; and that such a Purchaser must pay for those Chances. I admit, that the Rule does direct that these remaining Chances of Vitality are to be paid for, and by that Means, that all the Chances in some Sense, and at some Rate, are paid for; yet these are not to be paid for as Chances arising in the first Year, but as Chances arising in the several subsequent Years of possible Life; and then the Annuitant does not pay for the very Parts of the Value of the Annuity, which he has a Chance to receive on the Life of the Nominee; for sure enough, the Chances arising in the subsequent Years of possible Life, are not those very Parts of the Value of the Annuity which he has a Chance to receive on such Life.

If we reckon the whole Number of Chances, in the first Year of the Life of this Nominee to be twenty, it being just an even Chance that he lives to the end of that Year and no longer, ten of those Chances in that Year will be Chances of Vitality, and ten of Mortality; so that at the end of that Year, there will be ten Chances of Vitality remaining. As for the ten Chances of Vitality arising in such first Year, there is no Controversy but those are to be paid for as Chances arising in that Year; nor can there be any Dispute but that the Sum to be paid for them is one half of the present Value of the Annuity, as an absolute one. But as for the ten
Chances

Chances of Vitality, supposed to be remaining after the end of the Year, in my Way of stating the Value of this Annuity, these ten Chances must be looked upon as all arising within such first Year; for none can arise in any subsequent Year, until and unless these have first arisen, and that they ought to be paid for, as for Values in present Possession; but in the Way of stating such Value, according to the Rule, these ten Chances are to be look'd upon as arising all after the Expiration of the first Year; some arising in the second, some in the third, and some in the twentieth Year of possible Life; and the Sum total of the Values corresponding to those Chances so arising, is to be taken as the Sum to be paid for these remaining Chances. Now, since the Number of Chances of Vitality remaining unpaid for, in either Way of stating it, is one and the same, *viz.* ten; here where those ten Chances are all in Reversion, and some very distant, and the Values corresponding reversionary ones, the Sum total of those Values must be greatly less than the Sum total of the Values corresponding to the like Number of Chances, in my Way of stating it, where the Chances and Values corresponding, are all in immediate Possession, as supposed to arise in the first Year.

I am afraid then that a Person, who undertakes to pay an Annuity for such a Life, will think he has an ill Bargain, if he is to be paid for such Undertaking, according to the Rate of such reversionary and remote Values; and will be apt
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to surmise, that the Purchaser does not pay for the very Parts of the Value of the Annuity, which he will have the Chances to receive. He says then, and mentions it only, because it has been insisted on already, that on computing the Chances of Life and Death, the even Chance in this Case is, and the Nominee on whose Life the Payments depend, may reasonably expect, that he shall live to the End of one Year, that on the same Grounds he must expect to pay the Annuity for that Year; that if he does so, he must make present and immediate Payment: And therefore that it is unreasonable, that the Annuitant should pay him less than the present Value of the Annuity, payable at the End of the Year, or pay his Purchase-Money, by a Computation on reversionary Values, that will fall greatly short of the present Value of the Annuity for a Year, and yet so much the Annuitant has an even Chance to receive.

He has this farther to offer under this Head; that if the Nominee survives the Year, he shall be obliged to pay the whole and entire Annuity, for that he knows no Contingency in his Case, that will discharge his Engagements by paying Part only; for there is but one Chance in the whole, either that the Nominee does, or that he does not survive the Year; and here the even Chance is, that he does survive the Year, in the second Year of the Nominee's Life; the same Observation recurs, that he has no Contingency of performing his Contract by a partial Payment; for if the Nominee beyond Expectation should

should outlive a second Year, he must again pay the Whole; but here the even Chance is, that the Nominee is dead, and that he has nothing to pay. The Case will be the very same, if the Nominee be young, and in Course of Nature, likely to live many Years, yet there is no Room, that there can be no Chance of dividing the Annuity in any one Year, and paying a Part, and saving the rest in that Year. Here indeed, the Chance of Vitality gives room to make a Division of the whole Annuity, by the Number of Years of the whole, as of 70, 80, or 90 Years, to which the given Life may possibly extend; and we may, on a Computation, estimate, that to so many Years such a Life probably will extend, and beyond so many Years it will not extend; but of the Annuity in each individual Year, no Division can be made. There being then no Medium, on any one Year, between paying the Whole and paying nothing; the Right of the Thing seems to be where the even Chance is, that the Nominee lives to the End of the Year, and so long as that Chance continues, so far and for so many Years, the Purchaser must pay for the whole and every Part of the Annuity, as an absolute one; and where the even Chance is, and from the Time when the Chance begins to be such, that the Nominee is dead, that from thence the Purchaser is to pay nothing.

He admits it may be true, that the Annuitant pays for all the Chances of Vitality on the Life, if by all, is meant all in Number, but he apprehends the Sense of the Maxim laid down

down by the Doctor, to be, that the Chances which the Purchaser is to pay for, are to be equal in Value as well as Number, to the Chances which he has to receive. He has been taught by another Learned Author, that a Man who has one Chance in three only, may have as much Advantage as he that has one in two; and I will add, that he that has the fewer Chances may have ten Times, or a thousand Times a better Chance, than he that has the greater Number of Chances, provided the Sum which he has a Chance to gain, be proportionally greater than the Sum which the other has a Chance to gain. He acknowledges therefore, that it is not enough for him, that for the ten Chances of Vitality, supposed to be left at the end of the first Year, he is to be paid in the Value of so many Chances, which by a fictitious and groundless Supposition, are computed to be so left, and to arise in the subsequent Years, Years long subsequent to the Time, when in all Probability the Nominee is dead, and in his Grave; Since the Value of all those Chances so arising, at the highest Calculation that can justly be made in his Favour, will not exceed a third Part of a Year's Value of the Annuity, and he on the other side has an even Chance to pay, nay, is as sure to pay, as he is sure to pay any Thing for those ten Chances, as if they were the Value of a Moiety of the Year's Annuity; that is, as for so many Chances of Vitality arising in the first Year. In Fact and Reality, these supposed remaining Chances must arise in the first Year, and before any other

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in any subsequent Year do arise, or they can never arise at all, and as such they must be paid to the Annuitant, or none must be paid; and therefore, as such, they ought to be paid for by the Annuitant, or none ought to be paid for: So that in the Event it comes out, if the Purchase-Money to be paid is calculated by this Rule, the Purchaser will not pay for all, and the very Parts of the Value of the Annuity which in Reality he has the Chances to receive, or has the Chance that his Nominee is living.

He adds this farther; that his Case upon the Life of *B.* where the even Chance is that he lives to the End of one Year, and no longer, and where the Chances in all are supposed to be twenty, and ten of them are Chances of Vitality, and ten of Mortality, may very well be resembled to the Case of a Proprietor of a Lottery, and the Life of Man has but too much Resemblance to a Lottery, in which the whole Number of Tickets is twenty, and one Moiety of them is Prizes, and the other Blanks. We will suppose then that the Prizes in this Lottery are each of the Value of 9434*l.* or the present Value of 10000*l.* to be paid at the End of one Year, and which, according to my Computation, is the Value of the Annuity for the Life of *B.*; and the Question will be only this, How many Tickets an Adventurer in such Lottery must be possess'd of, to have an even Chance to a Benefit-Ticket, and what Price he must pay for each Ticket?

I say

I say then ; to have an even Chance for a Benefit-Ticket, he must buy two Tickets, and for each he must pay a Moiety of the Value of a Prize ; that is, for his two Tickets he must pay the Sum of 9434 $\frac{1}{2}$. To prove these Assertions, it will hardly be necessary to resort to a long algebraical Calculation, though they may be so proved, because I believe there is not a Broker in *Exchange-Alley*, even of the lowest Form, but knows, if he cannot demonstrate, the Truth of them. The Case then of him who purchases an Annuity on the Life of *B.*, and of him who adventures in this Lottery, being in all Respects parallel ; for the Chance of gaining is equal, and the Sum to be received is one and the same ; therefore one and the same Sum is to be given on each Adventure.

If it be true, as I admit it is, that the Annuitant may receive no Part of this Annuity, as he will receive none if *B.* dies within the Year, and this undoubtedly is possible ; so it is true, and must be admitted, that the Lottery-Adventurer likewise may receive nothing, as will be his Case, if both his Tickets come up Blank ; and this is possible too, as well as the other. But as such Chance, or Possibility rather, of both Tickets being Blank, in Reason, ought not, and to be sure will not prevail on the Proprietor of the Lottery, who is to pay the Prizes, to sell two of his Tickets, which will give the Adventurer an even Chance to a Prize, at a Rate lower than the full Value of a Prize : So the Grantor of an Annuity on the Life of *B.* absolutely, though there
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is a Possibility that *B.* may die within the Year, and he may have nothing to pay, since he undertakes to pay the Annuity on the even Chance of one Year's Life of *B.* cannot, in Justice to himself, accept of a Sum less than the present Value of one Year's entire Annuity.

If it be said on the other Side, that there is a Contingency that both Tickets may be Prizes, it may also be said that possibly *B.* may live a second or a third Year: But such Possibility of having two Prizes, will never be allowed to be a just Reason for giving more for two such Tickets than the Value of a single Prize; and the Possibility of two or more Years Continuance of the Life of *B.* will hardly prove that more is to be given for the Annuity, than one Year's present Value of it; and to be sure will never prove that less is to be given, as the Rule directs. I may therefore safely leave it to Dr. *Halley*, and those who follow his Rule, to shew in what Instance these Cases are not parallel; or to prove that the Sum of 9434 *l.* is not the Parts of the Value of the Annuity which the Proprietor has a Chance to receive, and the Grantor a Chance to pay, on the Life of this Nominee.

From hence, I apprehend, it will follow; either that the Maxim laid down, that a Purchaser is to pay for such Parts only of the Value of an Annuity, as he has Chances that his Nominee is living, is not in all Senses true; or that in such Sense in which it is true, it is sometimes wrongly applied by the Rule. In this Sense of the Maxim, *viz.* that such Parts of the Value of
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the Annuity are to be paid, for which the Annuitant has the Chances to receive, supposing the Parts to be paid for, and the Parts to be received are in Number the same, whether they are in Value the same or not the same; in that Sense the Maxim is not true. In the Sense in which it is true, *viz.* that such Parts of the Value of the Annuity are to be paid for, as in Value are equal to those Parts which the Annuitant has a Chance to receive; in that Sense the Rule has, in this Instance, made a wrong Application of the Maxim; for those Parts of the Annuity which the Annuitant has the Chances to receive, are of much greater Value, then those Parts of it, which by this Rule he is directed to pay for in his Purchase.

In the former Instance, on an Annuity for the Life of *A.*, supposed to be ten Years old, the Maxim is as ill applied as here, though not in the same Manner. There the Rule applies it to every Year of the Life of *A.*, as separate and distinct Years, and as separate and distinct Annuities for those Years; which is applying it to something in which the Annuitant for the Life of *A.* is in no wise concerned; for his Annuity has no Termination at the End of each Year, but is to continue for 90 Years in an uninterrupted Line, unless Death intervenes and breaks it. The Maxim may indeed be applied to this Annuity, and will be rightly applied thus far, and in this Manner. The Value of an Annuity for a Term of 90 Years absolute, is in Value so much; of this Term and an Annuity attendant
on

on it, the Proprietor of the Life Annuity has an even Chance, or the Chances to enjoy so many Years; and if we enquire in particular, what Parts of the Term or the Annuity the Proprietor has the Chances to enjoy, I say he has so many Chances that *A.* his Nominee, will continue in Life, as that the Chances in the whole shall amount to an even Chance, that he lives for 41 Years from the time of the Annuity granted, and by two Rules of this Authors, which we have before mentioned, that is computed to be his Chance of Vitality: Consequently that for these 41 Years, the Annuitant will receive the whole and all the Parts of the Annuity as an absolute Annuity, and therefore by the Maxim must pay for it as such for that Term; for it is certain, as he ought not to pay for more than all, so he ought to pay for all, and such as he receives, or has an even Chance to receive.

I must observe here, that this Maxim does not prove, nor has any Tendency to prove, what is the Chance of Vitality on a Life, or what are the Parts of the Value of an Annuity, which an Annuitant has the Chance to receive on the Life of his Nominee; but, it being supposed or proved that this or that is the Chance of Vitality, that these or those are the Parts of the Value of the Annuity, which the Annuitant has the Chance to receive, it follows by Virtue of the Maxim, and if the Maxim had never been heard of, it would have followed, that the Value of the Annuity would be so much: But the Chance itself, or the Parts of the Value to be re-

ceived, cannot from thence be collected and ascertained. In the Instance upon the Life of *A.* it is possible that the Annuitant may receive all the Parts of the Annuity for 90 Years, because it is possible *A.* may live for 90 Years, or to be an 100 Years old; it is possible likewise, that he may enjoy no Parts at all of the Annuity, because *A.* may dye within the first Year: But the Parts of the Value of the Annuity, which the Annuitant must receive on the Life of this Nominee, are those Parts which arise and grow due, within the first Years that shall immediately ensue from the Time of the Grant of the Annuity, be the Number of those Years more or less. But what that Number of Years will be, is in no sort determined from hence, but must be collected from the Chance of Vitality on that Life; and it being calculated by a Rule of the Doctor's, which is certainly a right one, so far as the *Breslaw* Table is a just one, that the Chance of Vitality on such a Life, is a Term of 41 Years, it follows, that the Parts of the Value of the Annuity which the Annuitant will, or has a Chance to receive, are those Parts which will arise in 41 Years from the Time of the Grant; because the even Chance is, that *A.* lives for 41 Years from such Time; and that the Parts of the Value of the Annuity which the Annuitant will not, or has not a Chance to receive, are those Parts which might arise and grow due after the first 41 Years, from the Time of the Grant, because the Nominee will not, or the even Chance

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is, that he does not live beyond those 41 Years.

I cannot forbear taking Notice here, that the Sum total of the Parts of the Value of an Annuity on the Life of *A.*, which are computed to arise after the Expiration of the Term of 41 Years, and which Dr. *Halley* says, are to be taken as Parts of the Value of the Annuity on the Life of *A.*, and which I say, are to be omitted and left out of such Value, as arising after the Expiration of the 41 Years, the Sum total of these, I say, is no more than 42, or rather 48. On the other side I observe, that the Sum total of the Parts of the Value of the Annuity on such Life, which arise within the said Term of 41 Years, and which Dr. *Halley* says are to be left out and omitted, as being no Part of the Value of such Annuity, and which I say, are to be accounted and inserted as Parts of such Value, because in Fact, they do arise, and may be received within such 41 Years, the Sum total of these, I say, amounts to 2-09; so that it may very well come out, as in Fact it does, that the Value of this Annuity in one Way of Computation, shall be no more than 13-44 (or rather 13-50) and in the other Way 15-12.

That the even Chance of Vitality on such a Life as this is a Term of 41 Years, is collected from the Doctor's Rule on the *Breslaw* Table; and whether this be the right Chance or no does not affect the present Argument; however, we have hitherto supposed it to be so: And since I have sufficiently proved, that the Parts of the Value of the Annuity which the Annuitant has

a Chance to receive on this Life, are all those Parts of such Value which arise so long as the even Chance of Vitality continues, and no other, or for any longer Time; all I shall add farther, and by Way of Conclusion, on this Head, is this. Since it is not only possible but probable and a reasonable Expectation, that the Annuitant will receive all the Parts of the Value of the Annuity which arise within these 41 Years, because the even Chance is, that the Nominee lives those 41 Years, and in the Nature of the Thing, it is absolutely necessary that he should receive all these Parts before he can receive any other Parts: On the other side, since it is not probable nor a reasonable Expectation, that the Annuitant will receive any Parts of the Value of the Annuity, which may arise after the Expiration of these 41 Years, because the even Chance is, that the Nominee will be dead before any of them can arise; and from the very Being of the Life of Man, it is impossible he should receive any such Parts which arise after 41 Years, unless he does, and before he does receive all those which arise before 41 Years, for the Nominee must live 41 Years before he can live 42 Years. Upon the whole, I leave it with the Reader to determine, what are the Parts of the Value of the Annuity, which the Annuitant has a Chance to receive on the Life of this Nominee.

I have insisted the longer on the Proof of the falsity of this Rule, not only on Account of the Character of its Author, to whom great Defe-
 rence is due; but because the Arguments I have
 alledged to disprove his Rule, are of equal Force
 against

against all other Rules and Calculations whatever, which have not, and I have met with none that have, a proper Regard to the Chance of the Duration of a Life; and they tend likewise to prove and stablish the Rule, which I have offered for the Valuation of these Annuities; for the Justice of mine depends intirely on that Position, that the Chance of the Duration of a Life is the sole Measure of the Duration, and consequently, of the Value of the Annuity depending on it. The Method then, which I have proposed for adjusting the Value of these Annuities is, as appears from my Scheme in Table N^o. II. by Estimating the Duration of every Life, and supposing it equal to a determinate Number of Years; and such an Estimate I have framed for every five Years of Life, from 10 to 80 Years of Age. This I lay down as a general or common Rule, but to be sure many Exceptions are to be made out of it, from the greater or less Salubrity of the Place of Inhabitation, from the remarkable Longevity in some Families, and the contrary in others, and from other particular Circumstances which may attend particular Persons; and for these, Considerations must be had, as they respectively arise, since no one Rule can be framed to suit them all.

This Scheme I have drawn, chiefly from a View and Observations on the Bills of Mortality for *London*; and to make my Reflections on them more plain and conspicuous, I have made an Abstract of them for these six Years last past; wherein a Distinction is made of the Ages of

the several Persons dying, in the several Periods of Life, and have set it forth in Table N^o. XI. In this Table we see at one View, the Number of Persons dying in each Period of Life, in each of these six Years, the Number of Persons dying in each Period, for all the six Years put together, and the Number of them dying in each Period in a Year by a Medium. We have likewise there, the whole Number of Persons dying in each of these six Years, the whole Number of Persons dying in all the six Years put together, and the Number dying in each Year by a Medium. In these Bills we have always two Articles; of Abortives and Stillborn, which do not properly come under the present Consideration, and therefore I have put them under a separate Head, and have subtracted them out of the general Account of Mortality: And in the last Column, I have inserted a Calculation of the Number of Persons dying in each Period of Life, in every 1000 dying.

I have begun my Computation at a Life of ten Years of Age, and Estimated it as the best Life; though I think one of 7, 8, or 9 Years of Age may be equally good, and indeed, better in those Cases, where the Persons have got over the Small-pox, Measles, and such like Ailments incident to Youth, and have attained some firmness of Body. This I was induced to do from these Bills of Mortality, for *London*; from whence it appears, that the Number of Persons dying in the Period, between 5 and 10 Years of Age, exceeds the Number of Persons dying of the Age between 10 and 20, in the Proportion of 38, to 30; and yet the first Period takes up only

only half the Time of the latter. It appears likewise from thence, notwithstanding the Hazards and Casualties which may be attendant on Persons, between the Age of ten and twenty, and the vulgar Opinion, that the Chances of Mortality is greater in young than in middle Life, yet that fewer Persons dye in that Period of Life, than in the Period between 30 and 40, though the Number of Persons coexisting in the younger Period of Life, must be much greater than the Number of those coexisting in the elder. In the next Period of Life, *viz.* between 20 and 30, though the Mortality is considerably increased, yet the same Observation holds good in a great Measure, the Mortality in the two subsequent Periods, being still higher than in this, and in a great Degree so.

Having Estimated a Life of ten Years of Age as the best of Lives, I have stated it as equal to a Term of 28 Years, which will not be thought too high, by those who approve the *Breslaw* Table and Dr. *Halley's* Scheme; nor indeed by any one who will form an Estimate of the Duration of Lives, from the *London* Bills of Mortality. But in my Table, I have made an Allowance at large, for the uncommon Accidents of Pestilences, Famines, and civil Wars, to which Regard must be had, when a Calculation is made for a Number of Years; and being one of those who think the City of *London* to be a more healthful Place, than almost any other in this Kingdom, at least more than the Generality of Places are; I have so far complied with that

Notion, as to fix the Number of Years lower than those Bills seem to require; for those Bills will admit of a Calculation for a Term of 30 Years on such a Life, at the lowest Rate of setting it; and at the highest Rate, and where you will reckon to the Extremity of the Years, they may reach a Term of 35 Years.

On looking into my Abstract of these Bills, we find that more than one half of the Persons dying in *London*, are under ten Years of Age; the Consequence of which must be, that one half of the Persons born alive there, dye before they arrive at that Age, and the other half live beyond that Age. If we proceed and take a View of the other Moiety of Persons dying, it appears that the Number of Persons dying of the Age of 50 and upwards, exceeds the Number of those dying between the Age of 10 and 40; and of those dying between the Age of 40 and 50, if we suppose one half to be between 40 and 45, and the other between 45 and 50; then one half of this second Moiety of Persons dying, is under the Age of 45, and the other half is above 45: And the Consequence in this, will be the same as in the former Case, that of those who live to ten Years of Age, one half dies before the Age of 45, and the other lives beyond it. If these Conclusions are right, it must follow, if we compute the Chance of Vitality of a Person of ten Years old, for the City of *London*, to the utmost Extent, the Life of such a one is equal to a Term of 35 Years, he having an even Chance to live to the Age of 45. When I say,

say, if we compute to the utmost Extremity, I mean, if we begin our Computation precisely at ten Years of Age, for that we must not take a Life of 12, 13, or 14, nor a Medium between 10 and 15; because on a Life of 15, if we make a Calculation of the Chance of its Duration, in the same Manner as I have done on a Life of 10, the Chance is very little above 31 Years; and therefore as a Medium between 10 and 15, we cannot take more than 33 Years.

By the same Abstract, and in like Manner, I have computed the Chance of Vitality for the Period of Life, 30 to 35; and it appears, that the Number of Persons of 50 and upwards, exceeds the Number of those dying between 30 and 50; from whence I conclude as before, that such a Life must be equal to 20 Years and over; since of those who live to the Age of 30, more than one half live to 50 and above, that is above 20 Years. I have made a Computation on such a Life more particularly, and in another Method, which I shall mention presently, because it stands contradicted by an Estimate, made by the Author of a *Treatise*, called *A true Estimate of the Value of Leasehold Estates, &c.* who reckons such a Life as the best, and states it as equal only to 14 Years, which is short of mine some Years. This Gentleman has some very good Thoughts on this Subject, in which I readily concur with him; but I can by no Means subscribe to his Opinion, either that a Life of 30 is the best Life, or that such a Life is equal only to 14 Years. His Notion of the best Age of Life, is directly contradicted

contradicted and overthrown by what has been already offered, and might be farther collected from the *London Bills of Mortality*; and his Observations and Positions do in no sort justify his Conclusion, that such a Life is equal only to a Term of 14 Years; but rather produce another and a different Conclusion, such a one as confirms my Estimate; for which Reason I shall consider them at large.

The Observations he makes, are on the Registers of two Country Parishes, which have been regularly and well kept these 40 Years last past; from which it appears, as he expresses it, *that the whole Number of Inhabitants above two Years old, are buried in 26 Years or something above*; from whence he concludes, that a Life is equal to little more than 13 Years: And then, making an Allowance of a Year more, for the greater Mortality of Children from 2 till about 10 Years old, than afterwards, he fixes the Term of the best Life as equal to 14 Years.

Admitting that the Author's Observations of Facts are true, as there is no sort of Reason to call in Question his Veracity; yet his Position, that the whole Number of Inhabitants of two Years old in these Places, were buried in 26 Years, does not prove that the Life of any one, or every such Inhabitant of the Age of 30, is equal only to 13 or 14 Years.

This Position is capable of being understood in two different Senses; either that every individual Inhabitant of two Years old, who were living suppose in 1700, were all dead and buried
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by or before the End of the Year 1727; or that a Number of Inhabitants of two Years old and upwards, equal to the whole Number of that Age, at any given Time, died and were buried in that Compass of Time. That the first is the Meaning of the Position I can hardly think, because in that Sense, surely it could never be true, in these or in any other Places; and in the other Sense, it may indeed be true: But in either Sense, I conceive that it does not prove his Inference, or rather that it proves something which must have another Conclusion, than what is here made.

To shew the Difference of these two Senses, and that in the first Sense, the Position is very unlikely to be true, though possibly in the second it may; I shall give a known and familiar Instance. The Number of Bishops in *England* is 26, and the Number of Aldermen for the City of *London* is the same; now it may be true, that in 26 Years last past, and to be sure in a much less Time, a Number of Bishops, and likewise of Aldermen, equal to the whole Number, that is 26 Bishops and 26 Aldermen, may have died: But it is not true that the very individual Bishops and Aldermen who were such, 26 Years ago, are all dead; for my Lord of *Worcester*, has been Bishop above forty Years, and his Grace of *Canterbury* about thirty Years; and I believe two or three at least, of the Senior Aldermen, have been of that Body above 26 Years: And if in such Bodies of Men, where rarely any come to be Members of it, till they arrive at some Ma-
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turity of Age, the Assertion is not true, it is much more likely not to be true, where many of the Persons are of a very young Age.

But the Sense of the Author's Proposition here, may be collected with most Certainty from his own Account and Calculations, and if we look into his Computations for the less Parish, he states the Number of Inhabitants above two Years old, to be 119, and the Number of Persons above two Years old dying annually, to be $4\frac{1}{2}$; and then having divided the former by the latter Number, and the Produce being 26, he concludes that the whole Number of Inhabitants are buried in 26 Years. Now this is a Demonstration of the Proposition, if understood in the second Sense; viz. that in 26 Years Time a Number of Inhabitants, equal to the whole Number of Inhabitants, that is, the Number of 119 has died; but is no sort of Proof at all, that the very individual 119 Persons, taken at any one Time, were dead in 26 Years. And in Truth, the Proposition in the first Sense is intirely incredible; for that it must necessarily follow, that there was never a Native of either of the Parishes, living in either of them in 1728, who was then 30 Years old; every individual who was living there in the Year 1700, and then two Years old, being dead and buried.

Our Enquiry in general is, How long it is likely any Person of a given Age may live; and in particular here, to what Time any one of the present Inhabitants of these Parishes, between the Age of 30 and 40, may live. For the So-
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lution of this Question we are directed to make Use of this Method; we must compute in what Time a Moiety of a certain Number of Persons given of that Age have in Fact died in that Place, and take that Time as the Term to which there is an even Chance that any one Person of that Age, and in that Place now living, shall live. Now this Rule for estimating the Duration of a Life means, and is applicable to, Individuals only; and this Author's Assertion, I suppose, is not true, and cannot be understood to speak of Individuals; for it could not be true that the whole Number of the individual Inhabitants of these Parishes did die in 26 Years, or one Half of them in 13 Years: The Consequence is, that this Assertion does not prove that the Life of any one of them of the Age between 30 and 40 is equal to 13 Years, and no more, because the Rule means one Thing, and the Assertion another.

And if the Matter of Fact were such, in these Parishes, or in either of them, that it would maintain the Position in the first Sense, and if the Author could be understood to speak of Individuals; yet I cannot admit that the Position, so understood, proves that the Life of a Person here, of the Age between 30 and 40, is equal only to a Term of 13 or 14 Years; since it will follow, even from thence, that such Life must be equal to a Term much longer.

To make Use of the Instance here produced of the less Parish: The Assertion, if applied here, is this; that of 119 Persons subsisting in
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this Parish in the Year 1700, one Moiety, that is 60 of them, died and were buried by the End of the Year 1713: Now admitting that this Moiety were really a Moiety of the Individuals subsisting in 1700, if the Rule for estimating the Duration of Lives, which we have before mentioned, is a right Rule, and Dr. *Halley* makes Use of it, this Author makes Use of it, and if this be not, I very much doubt there is no such Thing as a right Rule; it necessarily follows, that the Life of every one of these Inhabitants who were in Being in 1700, and then above two Years old, that is of every one of these 119 Persons, and not of those only who were of the Age between 30 and 40, at an Average, was equal to a Term of 13 Years; one Moiety of such Persons living to the End of 13 Years.

If this be the Consequence, and nothing does so naturally and so certainly follow; then, in Order to make every Life, one with another, to come out to be equal to a Term of 13 Years, it will be necessary that we greatly increase the Chance of Vitality in the Lives of that Part of these Inhabitants who are in the Period of Life between 30 and 40, or it will be impossible to make up the Deficiency that will arise in the Chance of Vitality for that Part of the Inhabitants which is in the bad Periods of Life, that is from 2 to 10 Years old, or towards it, and from 40 upwards, which are admitted on all Hands to be among the Periods of Life where the Chance of Mortality is greater than in the Period between 30 and 40.

In enumerating the Periods of Life which are bad, I have omitted the Whole of the Time between 10 and 30, because in my Judgment 'tis not such, but quite the contrary; and yet, according to this Author's Way of Thinking and Reasoning, it ought to be inserted, for if the Chance of Mortality in this Period is not greater than in the Period between 30 and 40, a Life in this Period must necessarily have the Chance of the longer Duration, or be a better Life, since it is so much farther removed from the Decline of Life, than a Life in the other Period is. Leaving therefore this Period out of the Account of bad ones, let us calculate what may be the Increase of Mortality for the two Periods from 2 to 10, and from 40 upwards, which are agreed on all Sides to be bad ones.

This Author takes Notice of, and makes an Allowance for, the greater Mortality in the Period between 2 and 10; but then 'tis only by the Subtraction of one Year from thence, and making an Addition of one Year to the best Period of Life, *viz.* from 30 to 40, and stating such a Life as equal to 14 Years. But this Allowance is surely too small, and will certainly appear so to any one who consults the Bills of Mortality for *London*, and finds there, and whoever does consult them will find, that the Number of Persons dying between 2 and 10 is more than four to one of those dying between 10 and 20, the Proportion being as 126 to 30, and yet the Period not so long neither.

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But if 'twere admitted that this is a sufficient Allowance here ; yet I observe no Notice at all is taken of the Periods from 40 upwards, nor any Deduction made for the greater Chance of Mortality in those Ages of Life ; which yet undoubtedly ought to have been made, and is very considerable. If we would keep up the Average, and make each of the Lives in these two different Periods, that is in the Period between 30 and 40, and the Period of 40 and upwards, to be equal to a Term of 13 Years one with another, the Increase of Vitality in one Period must be just equal in the Whole to the Decrease of Vitality in the other : So that the Question will be, What may be the Decrease of Vitality in this older Period of Life.

I have computed, by Way of Supposition only, that these 119 Inhabitants, in Point of Age, might be thus divided ; that is, between 2 and 10 Years Age the Number might be 30, between 10 and 30 might be 40, between 30 and 40 might be 20, and from 40 upwards 30 ; making in all 120. If this Repartition should not be very exact, though I believe 'tis not far off the Point, yet a small Variation will have no great Influence on my Reasoning, and whatever it be, it will appear on the Computation. From this Author's Scale of the Term of Years to which a Life is equal, I have computed, in several Ways, to what Number of Years every of the Lives in the oldest Period, I mean from 40 upwards, at an Average, must be equal ; and making Use of that Method of Computation
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which produces the longest Term to which such Lives can be equal, it comes out that they can be equal to no more than eight Years at the most, one with another. If then the Lives of every one of these 30 older Persons are at an Average equal only to eight Years, we must from every one of such Lives take off five Years, each of their Lives wanting so much of being equal to 13 Years; and if the Number of Persons of that Period be 30, we must take off in the Whole 5 Times 30, that is 150. And since this whole Number of 150 Years so taken off here must be added to the Lives of the Persons in the Period between 30 and 40, and those are only 20 Persons, 150 divided by 20 producing $7\frac{1}{2}$, we must add seven Years and an half to the Life of every Person, in this younger Period, one with another; that is, we must add these Years to 13, or rather to 14, which we had before; and then their Lives, at an Average, are equal to twenty one Years and an Half.

If the Number of Persons of this Parish in these two Periods of Life were equal, that is, if there were as many Persons living of the Age between 30 and 40, as there were of the Age of 40 and upwards, which is not at all likely to have been the Case, either here or any where else, then the Number of Years to be added to each of the Lives of the younger Set of Persons would be just the same as the Number of Years taken off from each of the Lives of the elder Set, that is an even five Years; but then on that Foot, each of the Lives in the

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younger Set would be equal to a Term of 19 Years.

From these Premises these Conclusions may very fairly and very justly be drawn. If this Author might be understood to speak of Individuals, when he asserts that the whole Number of Inhabitants died in 26 Years, and a Moiety of them in 13 Years Time, it follows in the first Place, that each of their Lives, at an Average, was equal to a Term of 13 Years; and from hence it follows, in the second Place, and as a necessary Consequence of the former Conclusion, that the Lives of those Persons in the bettermost Period of Life, that is in the Age between 30 and 40, must each of them be equal to a longer Term than 13 Years, or otherwise it will be impossible that each Life of the whole Number, which includes those between the Age of 2 and 10 Years, and those of the Age of 40 and upwards, should be equal to a Term of 13 Years: And therefore that the Life of every Person of this Parish between the Age of 30 and 40 may very well be estimated equal to a Term of 20 Years and above, even on a Supposition that the Author speaks of Individuals.

But in Fact this is not the Case, for this Calculation includes not only all the Inhabitants between 2 and 10, and above 40, who were in Being at the Beginning of the supposed 26 Years or 13 Years, but it includes also all those who from Time to Time during the whole Course of these 26 Years or 13 Years grew up and succeeded into the bad Period of Life between 2
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and 10; which is a wide Difference, and a Defect of greater Consequence than the former. The Number of Persons above two Years old living in this Parish, suppose in the Year 1700, was 119; by the End of the Year 1726 an equal Number of Inhabitants above that Age, that is 119, had died and been buried there; but who were the Persons dying there in this Compass of Time? Not the very individual Persons who were subsisting in 1700, for that, as we have already seen, is neither credible, nor can be the Meaning of the Expression: But the Number is made up partly out of the Individuals, and the rest out of such as did afterwards grow up to the Age of two, of which there must be annually and constantly some through the whole Course of the 26 Years. If we would know what the Number of Persons might be thus growing up and succeeding into this List, the Author's Account here gives us an Opportunity of computing it, and it stands thus. The Number of Persons annually born is 6, the Number of Persons under two Years Age annually dying is $1\frac{1}{2}$, therefore the Number of Persons annually added to the List is $4\frac{1}{2}$, and consequently in 26 Years the whole Number added is 117; so that in Fact these 119 Persons dying were neither the Individuals, nor out of the like Number of Individuals, but out of a Number of Individuals almost double.

The Rule, and the only unexceptionable Rule, to adjust the Chance of the Duration of a given Life, is; to compute in what Time a Moiety of

a Number of Persons of the same or the like Age with the given Life, in Fact has lived or commonly does live, and to take the Time to which such Moiety lives as the Term to which 'tis probable the given Life may extend. But then we must make the Cases in every Instance parallel, as we must do wherever we will argue from one Case to another; that is, here we must take a Set of Persons of the same Age or near the same Age as the given Life, and should retain and hold ourselves close to the same Persons; but this Author has not observed this Method in either Instance. The Enquiry is, to what Time 'tis probable that Persons of the Age between 30 and 40 may live; and to determine this he has taken a List of Persons of all Ages from two upwards, and does not keep to them neither solely, but brings in another Set of Persons of an allow'd bad Age of Life, *viz.* between 2 and 10 Years Age; and this he does not for once only but in a continued Succession for the whole Time of 26 Years.

The Truth is, into such a List not only no new crazy Lives, but even none at all, good, bad, or indifferent, ought to be introduced to supply the Vacancies that do and must happen by Mortality; for that in Effect brings us back to the former Point, that the Death of a Number of Inhabitants equal to the whole Number of Inhabitants is, in such a Computation, the same Thing as the Death of the Individuals nominated. That this is not right, is plain enough from the Observations I have made on this Parish; but
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will perhaps appear more evidently and certainly in the Instances of the Bishops of the Kingdom, and of the Aldermen of the City of *London*; for my Lord Bishop of *Worcester* has lived to bury his Brethren more than twice over, and the Senior Alderman of the City his Brethren twice over or near it; that is, a Number of Bishops and Aldermen equal to the whole Number of those on each Bench, *viz.* 26, have died twice over, that is, 52 have died during their Time; that is, twice the Number of the Individuals, but not the very Individuals themselves, have died in a determinate Time.

To apply this to the particular Purpose which we are now considering; if an Annuity had been purchased by one Person, suppose in 1693, when the now Bishop of *Worcester* was first made a Bishop, to continue for the Lives of the individual Persons then on the Bench of Bishops, and by another an Annuity to continue 'till a Number of Bishops equal to the whole Number of Bishops, that is 26, should have died; one of the Annuities would be still subsisting, and the other would have determined above twenty Years ago: For that, on a Computation made, I find that in 17 Years Time or thereabouts there are 26 Bishops die; and the Case among the Aldermen of the City of *London* is much the same. If we turn our Manner of Reasoning another Way, and suppose a Man to buy an Annuity now on the Life of one of the present Aldermen, and that the Purchaser was to take the Life of one Mean betwixt the oldest and the

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youngest of that Body; it might be a tolerable good Rule of estimating the Continuance of such Life, to examine the Time in which 13 Aldermen of any determinate Set of Aldermen have died, and to take that Time as the Term to which such Alderman now to be nominated might live; but it would be a wrong Rule to examine in what Time any 13 Aldermen of the Body at large have died, and take that Time for such Term; because in one Method 13 is certainly a Moiety of the List, and in the other 'tis manifestly not so, the List being enlarged by the continual supplemental Lives. Or take it thus. In one Way the Chance of Mortality by the Addition of such succeeding Lives, and keeping the Number always full, continues much at a Par for the whole Time; but in the other Way such Chance is gradually decreasing as the Number of Individuals decreases by Death; so that the Death of 13 shall happen much sooner in the one Way than in the other; and this always comes out to be the Fact, when Trial is made and applied to particular Cases. And that this is the Fact, with Regard both to the Bishops of the Kingdom, and the Aldermen of *London*, is evident from hence, that from his present Majesty's Accession to the Crown, and before the ninth Year of his Reign expired, there died twelve Bishops and thirteen Aldermen.

This Author estimating the Age between 30 and 40 as the best of Age of Life, and having grounded his Calculations on the whole Number of the Inhabitants of the Parish he mentions, without

without distinguishing their Ages, and not on any certain Set of Individuals of the Parish, I can make no positive and certain Calculation from thence to what Number of Years such a Life may be equal, and shall only say: Since I have shown under a former Head, that from his Premises and in his own Way of computing such a Life must be equal to a Term of 20 Years or above, I may infer, if his Computation had been made from Individuals and not the Inhabitants at large, the Chance of Vitality on such a Life might have come up to the Estimate which I have put upon it in my Tables, or something near it. But this Author has more Observations on this Subject which, I think, tend to confirm my Opinion, that a Life in the best Stage of it, *viz.* of ten Years of Age, may very well be computed equal to a Term of 28 Years; and as I took the first Hint of such Estimation from thence, it is but Justice to acknowledge from whence I received it, and how and by what Means I collected it.

He observes then, that one Half of the Mass of Mankind dies before ten Years of Age; which holds true for the City of *London*, and I believe in most other Places, except in the Tables for the City of *Breslaw*: Now if this Gentleman would give himself the Trouble to take another View of the Inhabitants of this Parish which he knows so well, and examine the Register there once more, which gives an Account of the Ages of the 125 Persons buried in the 20 Years there mention'd, he might readily find the

Term of Years to which any Person, of that Parish at least, has a Chance to live; for the Age of the Person dying, whose Age was the Middle between the oldest and youngest Person dying, always excluding out of the Account Children under ten Years old, the Age of such Person, when we have deducted out of it so many Years as are equal to the Age of the Person whose Life we enquire after, gives us the Term of Years to which the Life enquired after is equal.

The Way of Reasoning is this. One Moiety of the Mass of Mankind dies before or about the Time of ten Years of Age; or, which is the same Thing, a Moiety of the Mass of Mankind lives to the Age of ten Years, or thereabouts; therefore every individual Person born, and when born, has a Chance to live ten Years. Of the Persons who live to ten Years or upwards, one Moiety dies before such a Time, or one Moiety lives to such a Time; therefore every Individual, who arrives to ten Years of Age, has a Chance to live to such Time. To determine what this particular Time is, we must consult Registers and Bills of Mortality which distinguish the Ages of Persons dying; and from thence we must collect, as near as we can, the Age of the Person dying, whose Age was the Middle or Mean between the oldest and youngest dying; and in doing this, the larger the Number of Persons is, and the longer the Period, the more exact the Medium must come out; because some Years and some Seasons produce

duce Distempers which carry off more Persons in Proportion of the younger or middle Age of Life, and other Years and different Seasons shall bear hardest on those in the Decline of Life : and the Age at which the Person of the middle or mean Age died is the Age to which the Person enquired after has a Chance to live.

To show better the Meaning and Use of this Rule, we will apply it to a Person of the Parish so often mention'd, and to one of ten Years of Age ; from whence it will appear, that the Life of such an one may very well be estimated as equal to a Term of 28 Years. The whole Number of Persons, all Children included, dying here in 20 Years Time, was 125, and a Moiety of them, that is 62, died before ten Years of Age, so that the only Enquiry necessary is of the other Moiety. If we begin then at the youngest of the Persons above 10 Years old who was buried during this Period, and count forwards, or begin with the oldest who was buried, and count backwards, the Age of the Person dying when we come to a Moiety of 62, that is to 31 ; this is the Age we enquire after, or the Term we want, deducting ten out of it. I appeal then to this Gentleman's farther Review of this Parish, and for the Period he mentions or any other, whether there were not more Persons buried of the Age of 38 and upwards, than there were of the Age between 10 and 38. If that comes out to be the Case, as I verily believe it will, it follows, that even in this Spot of Ground, which does not seem to be amongst the most healthful,
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a Person of ten Years Age has an even Chance to live to be 38 Years old, and his Life consequently is equal to a Term of 28 Years; that being evidently the Term to which such a Person has a Chance to live.

That this, which I have surmised, would come out to be the Case in this Country Parish, is only a Conjecture, though not an ill grounded one; but that this is the Case in the City of *London* is more than Conjecture. On the Bills of Mortality for that City for these last six Years, I observe, that the Number of Persons dying of the Age of 40 and upwards exceeds the Number of those dying of the Age between 10 and 40; and the Proportion of one to the other is as 202 to 195, as may be collected from my Abstract of those Bills in the Table N^o. XI.: And if we take the Number of those only dying between 10 and 38 on one Side, and on the other Side, those dying from the Age of 38 upwards, the Excess is much greater, and the Proportion is 220 to 177, as near as I can state it. The Truth is, as I have observed before on these Bills, that the Number of Persons dying in that City of the Age of 45 and upwards is full as great as the Number of those dying of the Age between 10 and 45; and consequently, in *London*, a Life of the Age of ten must be estimated as equal to a Term of 30 Years, or above, and in other Places equal to a Term of 28 Years.

On this Abstract of the Bills of Mortality for *London* I observe farther, that the Number of Persons dying there of the Age between 10 and

20, to the Number dying there between 30 and 40, is in the Proportion of 30 to 93: So that whatever might be the State of the Country Parish here mention'd, or whatever Reasons this Author, or any other, might have for stating a Life of the Age between 30 and 40 as the best Life, either in this or any other Place, such an Estimate will by no Means suit with the Chances of Vitality for *London*.

From these Premises, and the Calculations even on the State of this Village, as far as the Knowledge of it is come to our Hands, as well as on the Bills of Mortality for *London*, I think we might safely conclude, that the Period of Life between 30 and 40 is not the best Stage of Life; but whether it be the best or be not the best, yet a Life in that Stage may be equal to a Term of 20 Years, and above; and on the other Side, that a Life of the Age of 10, or thereabouts, is the best Life, and in a general Way may be stated as equal to a Term of 28 Years: But I have something to add farther under both those Heads.

I have look'd into the List of a certain Society which consisted of 70 Persons, and was in Being in 1700, the Persons and Ages of whom I knew, and know who are now living and who are dead, and the Times of their Death; all of them pretty exactly. If the whole Society were divided into two Parts, one Moiety was of the Age from 18 to 30, or thereabouts, and the other Moiety of the Age from 30 to 45, or thereabouts; and upon Examination I find, that one
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Half at least of the elder Moiety were living at the End of 20 Years, and one Half of the younger Moiety at the End of 26 Years; and of the elder Moiety there are now, in 1736, only seven or eight, at most, living; and of the younger Moiety either 15 or 16. The Consequence is, that a Life of any Person of the elder Moiety should be equal to a Term of 20 Years, and one of the younger Moiety to a Term of 26 Years; those being the Terms to which a Moiety at least of each Set respectively lived; and that a Life of the Age between 30 and 40 is not so valuable as one of a younger Age.

To shew what is the Chance of Vitality on Lives, where the Parties interested in them are at Liberty to pick and choose their Lives, and there we are to suppose that they will take the best, and such at least as, in their Judgment, and to common Appearance, are the best; I shall add one Thing more, which has been communicated to me by a Friend since writing this, *viz.* an Account of the Time to which a Moiety of those Persons lived, whose Names were put into the Annuities on Survivorship which were granted by an Act of Parliament of the late King *William* and Queen *Mary*. The Persons for whose Lives those Annuities were to be granted, were nominated at or before *Midsummer* 1693; at *Midsummer* 1728, that is at the End of 35 Years, one Moiety were living, but the Number exceeding a Moiety was then so small, that at *Christmasts* following it was sunk under a Moiety; from whence, upon the Rule formerly laid down, it

it will follow, that the Lives of any one, and every one, of the Persons nominated in the Orders of those Annuities in Pursuance of this Act of Parliament, were equal to a Term of 35 Years.

On this I remark, that the Term of 35 Years is the Term to which I have before asserted, that the Lives of Persons living in *London* of the Age of 10, might be computed to be equal, if we would carry the Computation to the utmost Extent. But then on the City of *London*, I have taken notice that it is more healthful than most other Places, in my Opinion at least: Thus much however is certain, that People in *London* generally live to a greater Age, or more Persons in Proportion die of a great Age there, than almost any where else in this Kingdom: And upon the length of the Term, to which a Moiety of the Persons lived, who were named in the Annuities on Survivorship, every one will readily observe, that those Lives were picked and chosen Lives; for which Reasons, I think that the Duration of the Lives of the Inhabitants of the City of *London*, or of the Nominees in these Annuities, will not be a good Rule in all Cases, to measure the Duration of Lives for that Part of the Mass of Mankind, which is in the best Stage of Life; I mean, of the Age between 10 and 15. Thus much however I may conclude, that these Accounts will sufficiently justify any Estimate, which states such Life as equal to a Term of 28 Years and no more; and all the Doubt

Doubt remaining with me is, whether they don't require that it should be set higher.

As for the other several Periods of Life, I have made the Estimates by a gradual Decrease, in a Way and Proportion, which I believe will appear to be a reasonable and a just one; that is, I have made the Decrease in each subsequent Period, greater than the Decrease in the precedent Period, by the Addition of one Quarter of a Year to each; except only, that in the Periods from 60 to 65, 65 to 70, and 70 to 75; to each I have made an Addition of two Quarters instead of one. I should take Notice too, that in the two last Periods of Life, I have not made the Decrease in any Proportion to the rest; partly because I found it not easily practicable, for that Persons who are arrived to that Age, generally speaking, are past most of the Distempers of Life, except that of old Age, and a decay of Nature, which shows itself and can be judged of, only on Individuals; and partly, because I saw no Necessity of being very exact in the Estimates of those Lives, they being of small Consideration, and inserted only for a particular Purpose.

To the Remarks already made on the Bills of Mortality for *London*, I crave leave to add one or two more; the first, to remove an Objection which is made against them, that they are an improper Rule to determine the Chance of the Duration of a Life; and a second, which tends to shew the Healthfulness of that City, so far as the Longevity of Persons living there, shows it; and with these I shall conclude this Head.

Head. It is alledged then, that the whole Number of Persons dying there, are not included in these Bills, for that none are inserted but those who die in the Communion, or are buried by the Service, of the Church of *England*; and that the Number of Persons who are only casual or temporary Inhabitants, is very great and incertain, and yet, if they dye there, and there are buried, that they are reckoned in these Bills: Therefore such Bills, not being collected for all the Inhabitants, or for the constant Inhabitants, cannot be a good Measure to compute the Chance of Mortality for that City.

Admitting these Allegations to be true, as most certainly they are, I agree the Rule is not quite so certain and exact as if the Numbers were intire, and there were no Accession of Foreigners; but then I say, that Dissenters from the Church of *England*, of whatever Sect or Religion they may be, and whatever Exemptions they may claim from the Law of the Land, they can have none from the common Course of Nature; but are subject to the same Laws of Mortality with other People, and in the same Place, or in the same Country and Climate, dye much at a much, in the same Proportion as other Mortals of the like Age, and of a different Religion; therefore, be the Number of those who on this Account are omitted out of these Bills, greater or less, yet I doubt not but in the like Periods of Life, the Number of those who die and are buried out of the Pale of this Church, and are left out of the Bills, is in the same

same, or near the same Proportion, as the Number of those who live and die within such Pale, and are included in the Bills. The same Way of Reasoning, in great Measure at least, will hold good, with Regard to the great and continual Resort of Strangers to that City, and their Death there; for even these, taking them all together, young and old, sane and insane, are liable to much the same Chance of Mortality, with the natural Inhabitants.

These Allegations will indeed have great Weight, where these Bills of Mortality are made Use of, to compute what is the whole Number of Persons living in that City, or to state what are the several Ages of the Persons living there; and even, if these Bills were in these Respects intire and compleat, I cannot discover that such Bills would be a sure Rule for making such Calculations; though, believe me, they may give us a far better Account than the wild Guesses and vague Computations, which some People give us. To fix the Number of Souls in that City, or in any other Place, otherwise than by a distinct Numeration of the Individuals, is a Matter of much Difficulty, and, notwithstanding the Attempts of some great Persons, remains yet under great Incertainties. To do this; it is not sufficient to know solely the Number of Persons dying in each Period of Life, but we must know, or be able to state, out of what Number of Persons living in each Period, they did so die; but no Bills of Mortality, that I have seen or heard of, have ever yet carried their Computations

tions so far; or, we must fix the Proportion which the whole Number of Persons dying bears to the whole Number living: but the Computures of this Kind, which have been made by different Hands, to compute the Number of Inhabitants of a given Place or County, have been some of them so wide and so distant from one another, that the one Computure shall give you near double the Number, which the other does; for I have seen one that computes one to die annually, out of 21 living; and another, one out of 40 living.

Now, on the other side, to fix the Chances of Mortality on any Person named, or between Persons of one Age and another, I say it is sufficient to know the Proportion, which the Number of Persons dying of one Age bears to the Number dying of another Age: And thus much these Bills for *London*, or the Improvements I have made upon them, have done. For Instance: Of the Persons dying there in a Year, or in several Years collected together, we see that one Moiety or above, is under ten Years old; therefore on any Person or every Person born alive, the even Chance is, that he lives to be ten Years old, or thereabouts: Of the Persons of ten Years old and above, dying there in one Year, or in a Number of Years by a Medium, we may discover from thence, that one Moiety or thereabouts is above 45, and the other between 10 and 45; therefore on a Person of 10 Years old, the even Chance is, that he lives to be 45 Years old; consequently his Life is equal to a Term of 35 Years;

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Again;

Again; of a 1000 Persons dying, 5 are of the Age between 90 and 100, or, which is the same Thing, one in 200 dying, is of the Age between 90 and 100, therefore one in 200, must first live to the Age of 90, or one in 200 could not possibly die, of, or above that Age: Therefore, since these Bills do give us, or are a sufficient Guide to us, to discover these Proportions, I conclude that they are a good Rule, to determine the Chances of Vitality and Mortality, for the Place where.

The other Observation which I make on these Bills, is, that upon a Computation from thence, it appears, that in *London*, one in 2786 lives to the Age of an 100, one in 200 to 90, one in 33 to 80, one in $14\frac{1}{2}$ to 70, one in 8 to 60, one in 5 to 50, and one in 4 to 45: And as a Moiety or more of the Persons dying, is under 10 Years of Age, and a Moiety or less is above 10 Years Age; then out of the Persons of ten Years Age and above who die, one in a Moiety, of the former Numbers respectively, lives to the several and respective Ages before-mentioned. And if we take another Age, suppose 50, and enquire what is the Chance that such an one lives to be 100 Years old: The whole Number of Persons dying in six Years Time, of the Age of 50 and above, being 30242, and the Number of those dying of the Age of an 100 and above, being 54, the former Number being divided by the latter, leaves 560; that is, of the Persons of 50 Years Age and above, dying one in 560, is of the Age of 100; or one in 560 of 50 Years Age,

Age, lives to the Age of 100. Now, I believe, if we were to search all the Registers in the Kingdom, we shall hardly find one where these Propositions are exceeded, or even attained. In the little Country Parish, so often before spoke of, the Gentleman, who has known the Inhabitants for many Years, and has examined the Register there; tells us, that of 125 buried in 20 Years last past, Children included, but 3 have arrived at 70 Years, and 2 at 80; from whence, if there be no Mistake in the Numbers, he concludes very rightly, that in that Place and in that Period of 20 Years, one only in 41, arrived to 70 and upwards; and one only in 62, to 80 and upwards; which very great Difference from the Chance of Vitality for *London*, evidently shows the Longevity of People in that City, at least beyond the Inhabitants of this Gentleman's Parish; and I am very inclinable to think, beyond those of most other Places of this Kingdom.

End of the Second Book.





B O O K III.

*An ESSAY to Estimate the Chance of
the Duration of two or more given Lives,
and to ascertain the Value of Annuities
and Leases for such Lives, &c.*



WE are come now to the last and most difficult Task, to ascertain the Value of Leases and Estates for two or more Lives; and as this is the most difficult Part of the Work, so it has been the most weakly perform'd by the several Authors, who have treated this Subject; and Mr. *Richards's* Performance is no better than any of the rest, notwithstanding he ushers it into the World with great Pomp and Solemnity; and tells us in his Preface, that his Tables, such of them as are new, are built on the Foundation laid by Dr. *Halley*, and formed by the Method prescribed by Mr. *Moiure*; and in his Book gives us to understand, that his Tables, in particular for the Value of Annuities, for two and for three Lives, meaning surviving Lives, are framed by a Rule set out by Mr. *Moiure*, and by him proved Mathematically, to be a true one for that Purpose.

Not

Not to enter into that Point at present, whether the Foundation be wrong, or the Rule be wrong; or whether the Author of the Tables did not understand, or did not pursue the Rule; this I am sure of, and doubt not but I shall be able to make it evidently appear, that his Tables for the Value of Annuities for two Lives and for three Lives, are wrong: And the Errors in them are, I think, so gross and so very extravagant, that they will bear hard upon the Method which produced them, let who will be the Father or Grandfather of it, and whatever Support it may have from pretended Demonstrations.

These Tables being so Solemnly introduced, and alledged to be framed by a Mathematical Rule, gave me great Expectations of finding a right Estimate of the Value of Annuities for these Lives; but upon first looking into them I saw, and I wonder Mr. *Richards*, who seems to understand and to be very well versed in Figures, did not see, that his Value of Annuities for three Lives, of 12 Years Age, and at 5*l.* per Cent. Interest, must certainly be false; since it appears at first View to be very near equal to the Value of the Inheritance. It was this particular Estimate, and its being the Value of Annuities for three Lives, at the best Age of Life, and made at the legal and most common Rate of Interest, which gave Occasion and first induced me to enter into this Matter; and after much Thought and many Calculations on this Subject, made at different and distant Times, as I had leisure, a Method occurred to me of making these Esti-

mates, which I think will answer the Purpose more effectually, and more nearly than any other yet made public, or that I have seen: But this I must submit to the Judgment of the Reader.

Before I give you the Result of my Thoughts and my Pains, it may not be amiss to shew the Defects of former Calculations; for if any of them are right, mine is certainly unnecessary, and possibly wrong. The Calculations of that Author, who has *Sir Isaac Newton's* Name and Approbation prefixed to his Book, so far as they relate to these Estates, are built on such wild and groundless Suppositions, that they do not deserve, and in Truth, they have not the Sanction of, that great Man's Authority, whatever Appearance it might be intended they should have of it. He does indeed approve the Method of making the Calculations, and the Calculations there exhibited; but he says not a word of the Application of them to the Purpose, and in the Manner in which this Author there applies them. And as for the Method of computing the Value of Annuities for two, or for three Lives, by stating the Value of an Annuity for the first Life, as equal to a certain Number of Years, and an Annuity for the Subsequent Lives by decreasing such Term by one Year only, for every additional Life, it is so far from the Truth, whoever was the Maker or the Approver, that it seems to me not to be even guessing what might be the real Value of an Annuity for such additional Life or Lives, but merely studying a Measure
that

that might depreciate the Value of the Annuity for the first Life, or the Life in Possession, and to enhance the Value of the Annuity for the Life or Lives in Reversion, or to be added, purely for the sake of advancing the Fines, on adding or renewing a Life or Lives.

With this View, he states the first Life or Life in Possession, as equal to a Term of 10 Years only; because he supposes two Lives, or three Lives, or any other Number of Lives, to be equal to a Term certain; and if we allot a short Term only for the first Life or Life in Possession, there is better Room to make an Apportionment of a larger Term for the Life or Lives in Reversion, or to be added; and consequently an Appointment of a larger Sum for the Fine; and unless he had this Reason, he had none, to make a Life in Possession, equal only to a Term of 10 Years. If a Life were equal to no longer Term than this, an Annuity for a Life, where Interest were computed at 8*l. per Cent.* or higher, would not have been worth seven Years Value; and they must have been most absurd Computers, who could compute a third Life, or a Life to be added to two in Being, worth seven Years Value in Reversion, and at the same Time compute the first Life or Life in Possession, not worth seven Years Value in Possession: And yet they must have computed the Value of a third Life at this Rate, who stated the Fine for renewing or adding a third Life to two in Being, at one Year's Value. Having fixed the Term for the first Life at ten Years, he computes a

second or additional Life at nine Years, a third Life at eight Years; and so he proceeds to decrease each additional Life by one Year.

Supposing now the first Life to be equal to a Term of 10 Years only; a second and a third Life, whether they be added to the former, by way of original Purchase, or by Renewal, can be estimated equal to no longer a Term than 10 Years, if we consider them singly and separately by themselves, and as Lives in immediate Possession: But, as the first additional Life is a Life in Reversion, to commence after ten Years, it is pretty extraordinary to compute such Life equal to nine Years, but it is more so to compute the second additional Life, which is in Reversion after 19 Years; as equal to eight Years. But the greatest Defect in this Way of computing, lies in this: A Person who purchases an Annuity or Lease by renewing, or adding a second or third Life to another in Being, is to pay no more for such second or third additional and concurrent Life or Lives, than a third Person would do for the same; and they are really of no more Value to the one than to the other: But such third Person ought to buy them and pay for them as Annuities or Leases on a Contingency only, as well as Annuities or Leases in Reversion; since with Regard to him it may so happen, that one of them, or neither of them may ever take Place. Now the Chance between the first and second Life, being one in two, or an even Chance, that the second does not survive the first Life, and between the first and second Life

Life on one side, and the third Life on the other side, the Chances that the third Life does not survive both the other Lives, are two in three; it becomes an even Chance that the first additional Annuity never takes Place, and two Chances in three, that the second never does: And therefore, the first additional Life is equal only to a Moiety of nine Years, and the second only to a third Part of eight Years; the two Terms of nine Years and eight Years, being the Terms to which these two Lives were computed to be respectively equal, considered merely and solely as Lives in Reversion, so that the Term in all the three Lives put together, would be little more than 17 Years. But to this Point of the Chances of Survivorship, between two or more Lives, and how they are to be estimated, I mean to speak at large, under a more proper Head of Discourse, so shall press it no farther here.

Mr. *Haye's* Rule for valuing these Annuities is still more extravagant than the last Author's; for he allows no Decrease at all, in the Term for the second or the third Life; his Direction being, to put together the Term to which each Life, singly taken, is equal, whether two, three, or, I suppose, more Lives, and to take the respective aggregate Terms, as the Terms to which the two Lives, the three Lives, or all the Lives, whatever the Number may be, are equal. Very few Words will show the great Absurdity of this Rule. Where Interest is computed at 4*l. per Cent.* a Life of 30 is equal to a Term of 25 Years,

Years, according to this Author's Computation; a second and a third Life of the same Age, must therefore each of them be equal to a Term of 25 Years; the Sum total of the Term on all the three Lives, will be 75 Years; each of the three Persons are supposed to be 30 Years old, when the Annuity is granted; therefore one of the three, must be supposed to live to be an 105 Years old.

Mr. *Morris's* Rule for this Purpose, is merely an Extract from Mr. *Moirre*, and since Mr. *Richards's* Tables for the Value of these Annuities, were constructed by the same Rule, this will be considered under the following Head; for I mean to examine those Tables more at large; partly because they have the Appearance of Regularity, and of being supported by Mathematical Demonstrations; and partly because such Examination will, in some Measure, let us into the Understanding and the Reason of the Rule, which I mean to offer for these Valuations of the Annuities. The Method which Mr. *Richards* made Use of, for Computing and Forming these Tables, under Mr. *Moirre's* Directions, is this: He states first the Value of Annuities for a single Life, for two Joint-Lives, and for three Joint-Lives; and then adding some farther Calculations, grounded on these several Values, he collects the Value of these Annuities for surviving Lives. These Tables thus framed, I affirm, do not give us the true Value, or any Thing that is near the true Value, of Annuities for two or for three Lives, and the Survivor;
and

and therefore, either Mr. *Richards* does not fairly represent Mr. *Moirre*, which I see no Reason at all to suppose, or he does not pursue his Method, which upon repeated Experiments, I am satisfied, he does pursue; or the Method prescribed, or the Hypothesis on which the Method is founded, or both, are wrong: And this I principally aim to shew, and I think is very fairly deducible, from the Errors in these Tables, if those Errors are the genuine and necessary Effects of the Method.

I have observed under a former Head, that this Author's Table for the Value of Annuities on a single Life are faulty, on Account that they make one and the same Life equal to a different Term of Years, according to the various Rates of Interest, at which the Calculations are made; and this, for the Reasons there given, I have asserted, cannot possibly be the right Value, and yet that such Value is the necessary Produce of the Hypothesis. And since this Table, for the Value of an Annuity for a single Life, is the Foundation of the Table, for the Value of an Annuity for two Joint-Lives, and for three Joint-Lives, and those Tables for the Value of Annuities, for two and for three Joint-Lives, with some farther Calculations and Deductions from them, are the Principles, from which the Tables for the Value of Annuities, for two and for three Lives and the Survivor, are framed; the Consequence is, that these last Tables necessarily must, and in Fact they do, partake of the same Imperfections,

Imperfections, which the precedent Tables, from whence they are derived, labour under.

However, that is not the Error which I chiefly insist on here, though an Error undoubtedly it is, and an indefensible one; but if I should admit, that this Table for the Value of Annuities for a single Life was right, at any one Rate of Interest, and since every Man takes himself to be at Liberty to estimate a Life, to be equal to such a Number of Years as he likes best, the Table may be right at some one Rate of Interest, or at every Rate of Interest, considered singly and uncompar'd with the Table at any other Rate of Interest; and admitting, that the Tables for the Value of Annuities for two Joint-Lives, and for three Joint-Lives are right, and at present, I do neither affirm or deny them to be so; yet I say, that these Tables for the Value of Annuities, for two and for three Lives, and the Survivor, do not give us the true Value; and from thence it will follow, if the Method for constructing those Tables has been rightly pursued, that such Method is a wrong one.

To examine first the Table for the Value of Annuities for two Lives, let us see whether it gives us such Values as in Fact doe, or possibly can belong to such Lives. If we look into this Table for the Value of an Annuity for two Lives, each of 22, and compare it with the Tables for the Value of an Annuity for three Lives, two of 22 each, and a third Life of 82; they stand thus, on this Table.

Interest

	<i>l.</i>	<i>l.</i>	<i>l.</i>	<i>l.</i>	<i>l.</i>
Interest at	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
The two Lives	20-46	17-48	15-08	13-04	11-81
The 3 Lives	20-46	17-46	15-07	13-04	11-79

It appears here evidently, in two of the Instances, that the Value of an Annuity for two Lives and the Survivor, each of 22, is equal to an Annuity for three Lives and the Survivor, two of 22, and one of 82; and in the other three Instances it appears, that an Annuity for two Lives and the Survivor, each of 22, is of greater Value than an Annuity for three Lives and the Survivor, two of 22, and one of 82; which is impossible to be right, unless it be right to say, that two and two make four, and that two and two, after an Addition of something more, make still but four, and sometimes less than four; which I apprehend may be contrary to a received Maxim in the Mathematicks, and I am sure is contrary to common Sense. It will not, in this Case, be to any Purpose to alledge, that a Life of 82 Years of Age is of small Value; for if 'tis not *plus* nothing, I am sure 'tis not *minus* nothing, as this Calculation makes it; or, in other Words, if an Addition of a third Life of 82 to two Lives of 22, makes no Addition to the Value of an Estate held by those Lives, yet 'tis impossible it can make a Subtraction from such Value.

This Error is so apparent that it cannot, I think, be denied; and so gross as not to be defended; and yet it does not arise from any Defect in Mr. *Richards's* Table for the Value of an Annuity on a single Life; for if such Table

at any one Rate of Interest is right, and if his Table for the Value of Annuities for two joint Lives, at a corresponding Interest, is likewise right; and if the Values in the Tables, which I have here transcribed, both for the Annuities on the two Lives and three Lives, and the Survivor, are form'd according to Mr. *Moiivre's* Directions, as I am satisfied they are, having made the Experiment in all the Instances; the Conclusion must be, that this Error is to be imputed to some Defect in the subsequent Calculations, grounded on those previous Tables, and in the Method prescribed.

Another Error which occurs in this Table for Annuities on two Lives and the Survivor, is, that two such Lives, at 5 *l. per Cent.* Interest, are made to be equal to a longer Term than at any other Rate of Interest, even longer than when Interest is computed at 4 *l. per Cent.* This appears plainly in the Instance of two Lives, each of 12 Years Age; where at 4 *l. per Cent.* the Value given in the Tables is 21-39, which is equal to a Term of 49 Years only, or a little above, and at 5 *l. per Cent.* the Value given is 18-41, which is equal to a Term of 51 Years, or near it.

This Error, again, could not arise from Mr. *Richards's* Tables; either that for Annuities on a single Life, or that for Annuities on two joint Lives, and those only are made Use of here to compose this third Table. It could not arise from the Table for the Value of Annuities on a single Life, because the Value of an Annuity for
a single

a single Life is set higher in such Table, and the Term to which Value corresponds is longer, if Interest be computed at the Rate of *4 l. per Cent.* than if computed at the Rate of *5 l.*; and consequently, the Term to which two such Lives and the Survivor are equal, should have been longer, where the Estimate is made at the Rate of *4 l. per Cent.* than where 'tis made at the Rate of *5 l. per Cent.*; and I say such Term would have been longer, if some subsequent wrong Calculations had not intervened and prevented it.

Neither could this Error arise from the Table for the Value of Annuities on two joint Lives; because the Calculation which Mr. *Moivre* directs to be made here, with Regard to the Value of Annuities for two such Lives, must necessarily have produced quite a different Effect. By his Directions, whether Interest be computed at *4 l. per Cent.* or at *5 l.* this same Value of Annuities for two joint Lives is to be deducted out of the Value of the two Lives singly taken and put together; and the Value of each of these Lives singly taken is greater, and the Term to which such Value corresponds is longer, when Interest is computed at *4 l. per Cent.* than when 'tis computed at *5 l. per Cent.*, and consequently the Value of these two Lives, when put together, must be greater, and the Term to which such Value corresponds must be longer, according to the different Rate of Interest. Now if two Sums total are given, the one a greater and the other a less Sum, out of each of which a Subtraction is to be made; if we subtract a greater Sum

Sum out of the less Sum total, and a less Sum out of the greater Sum total, the Sum given which was the less Sum total at first, must still continue the less Sum total after such Substraction.

To apply this to the present Case. The Term which corresponds to the Value of these two Lives, singly taken and put together, is 59-2-0, if Interest is computed at 4*l. per Cent.*; if computed at 5*l.* the Term is 58-2-88; as may be seen by my Table N^o. VII.: And these are the Terms out of which the Substraction is respectively to be made. The Term which corresponds to the Value of those two joint Lives, calculated by Mr. *Richards's* Tables, is 18-3-7, if Interest be computed at 4*l. per Cent.*; and if computed at 5*l.* is 18-3-54: And these are the Terms which are respectively to be subtracted. But 18-3-7, which is the less Term to be subtracted, being subtracted out of 59-2-0, which is the greater Term total, leaves a Remainder, 40-2-84; and 18-3-54, which is the greater Term to be subtracted, being subtracted out of 58-2-88, which is the less Term total, leaves a Remainder 39-3-34; consequently, 40-2-84 should be the Term to which the two Lives and the Survivor are equal, where Interest is computed at 4*l. per Cent.*; and 39-3-34 should be the Term to which the same two Lives are equal, if Interest were computed at 5*l. per Cent.*

From these Tables for the Value of Annuities on two joint Lives it appears; if an Annuity be granted for two joint Lives, and Interest is computed

puted at 4*l. per Cent.* the Chance is that one of the Nominees will die in 18-3-7; but if granted for the same Lives, and Interest be computed at 5*l. per Cent.* then the Chance is that one of the Nominees will not die 'till 18-3-54. Upon which I cannot forbear remarking the great Absurdity of this Calculation; and yet this is still aggravated by something which follows, *viz.* that if an Annuity be granted on any of these Lives singly taken, and Interest were computed at 4*l. per Cent.* then the Term to which such single Life is equal, comes out to be longer than if Interest were computed at 5*l. per Cent.*: And from thence it necessarily follows, that these Tables for the Value of Annuities for joint Lives are erroneous as well as the rest; for it can never be true, that a single Life of a given Age, Interest computed at 4*l. per Cent.*, should be equal to a Term longer than a single Life of the same Age, Interest computed at 5*l. per Cent.*; and at the same Time be true that two joint Lives of the given Age, Interest computed at 5*l. per Cent.*, are equal to a longer Term, than the very same Lives would be equal to if Interest were computed at 4*l. per Cent.*

I will now state a Case, which I think will perfectly clear up this Matter, and show plainly, that the Table, which we are now considering, is false throughout, and in every Instance, unless by Accident; and if this be the Case, as indeed it must necessarily be the Case, if the Method of framing it is wrong, I think this will be a very strong Proof of my Position, that the Method is

wrong. We will suppose then an Annuity of 100*l. per Annum*, and at 4*l. per Cent.*, to be granted to N^o. 1. for the joint Lives of *A.* and *B.* who are both of the Age of 12 Years; a second to be granted to N^o. 2., to commence from the Death of the first of those two who dies, and to continue for the Life of the Survivor of them; and a third to N^o. 3. for the Lives of *A.* and *B.* and the Survivor. 'Tis evident here, that the Interest of N^o. 3. is just equal to the Interests of N^o. 1. and N^o. 2. put together; for their several Interests here commence and determine at one and the same Time. By this Table, the Value of an Annuity for two such Lives, and the Survivor, at 4*l. per Cent.* is 21-39; if the Annuity be 100*l. per Annum*, the Worth in Money will be 2139*l.* and is equal to a Term for 49 Years, or thereabouts; and this is the Interest of N^o. 3. The Interest of N^o. 1., which is in the joint Lives of *A.* and *B.* by the Tables for those Estates, is in Value 13-01; in Money is 1301*l.*, and is equal to a Term of 19 Years, wanting some few Decimals, or near a Quarter of a Year. The Interest of N^o. 2., which is to commence from the Death of any one of the two joint Nominees, and to continue for the Life of the Survivor of them, is a Reversion only, and will come into Possession on the Death of the Nominee who shall die first. And the Chance being, as computed by these Tables, that one or other of the Nominees will die in 19 Years, or something less Time, the Interest of N^o. 2. is a Reversion to take Place at the End of 19 Years;

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at the End of 19 Years, *A*. or *B*. or whoever is the Survivor of them, will then be 31 Years old; so that the Interest, and the whole of the Interest of N^o. 2. is in a Life of 31 to commence after a Term of 19 Years. The Life of this single surviving Person, being of the Age of 31, by this Author's Table for the Value of Annuities for one Life, is in Value 15-18, and is equal to a Term of 24 Years wanting some few Decimals, or near a Quarter of a Year; and a Term for 24 Years in Reversion after a Term for 19 Years, is in Value 07-40, and in Money is worth 740 *l*.: So that the Interest of N^o. 1. is 1301 *l*. and the Interest of N^o. 2. is 740 *l*.; both put together is 2041. Or take it thus; the Interest of N^o. 1. is near equal to a Term of 19 Years, the Interest of N^o. 2. is near equal to a Term of 24 Years, and both put together are near equal to a Term of 43 Years; and a Term for 43 Years in an Annuity of 100 *l*. *per Annum*, at 4 *l*. *per Cent*. Interest, is worth in Money 2041 *l*. All these Computations are made from Mr. *Richards's* Tables; and since his Tables for two Lives and the Survivor give us a Term of 49 Years as the Term to which the Interest of N^o. 3. is equal, and the Sum of 2139 *l*. as the Value of this Annuity in Money; and since his Tables for two joint Lives and a single Life give us only a Term of 43 Years, as the Term to which the Interests of N^o. 1. and N^o. 2. put together are equal, and the Sum of 2041 *l*. only as the Value in Money of both these Annuities put together; if the Interest of N^o. 1. and N^o. 2.

put together is just equal to the Interest of N^o. 3. by itself, as most certainly it is; if the Tables for a single Life and two joint Lives were right, it must unavoidably follow that his Tables for two Lives and the Survivor are false: and if Mr. *Richards* understood Mr. *Moiivre's* Method rightly, and there is no Room to make a Doubt of it, and pursued such Method in framing these Tables, as by repeated Experiments I am satisfied he did; the Conclusion must be, that the Method itself is wrong.

In stating this Case, and making the Computations upon it, one plain Defect has occurred to me; and it lies in this. In my Calculations of the Interest of N^o. 2., which we will suppose to be in the Life of *B.* as the Survivor, and where the Lives are equal 'tis no Matter which is supposed to be the Survivor, I have computed such Interest as a Reversionary one, and to commence from the Death of *A.*; and so far I suppose this Method of computing may fall in with mine: But then I go farther, and compute the Life of *B.* to be equal to such a Term only as the Life of *B.* is equal to at the Time when the Interest of N^o. 2. in such Life commences; and undoubtedly the Life of *B.*, with Regard to N^o. 2. is not equal to a longer Term than this, and *B.* at the Commencement of the Interest of N^o. 2. in his Life is really 31 Years old, 19 Years of his Life having elapsed during the Life of *A.*, and a Life of 31 is equal only to 24 Years. But now, in the Computations made here, according to Mr. *Moiivre's* Directions, the Interest of N^o. 2.
in

in the Life of *B.*, the supposed Survivor, is made equal to such a Term as the Interest would be equal to, in Case no Part of the Life of *B.* were elapsed, and as if *B.* were no more than 12 Years old at the Commencement of the Interest in his Life, and his Life were then equal to a Term of 30 Years. This evidently appears to be the Case in the Instance just now mention'd; for the Interest of N^o. 1. in the joint Lives of *A.* and *B.* is equal only to 19 Years, and since these Computations must make the Interests of N^o. 1. and N^o. 2. put together to be equal to a Term of 49 Years, or otherwise they will not be equal to the same Term to which the two Lives and the Survivor are equal; it necessarily follows, that the Interest of N^o. 2. in the Life of *B.* the Survivor must be here computed as equal to 30 Years: And in other Ages of Life I have made the Experiment, and they all appear to have been computed much on the same Foot. From whence one may very well conclude, that Mr. *Moiivre's* Mathematical Demonstrations, however elaborate and accurate they may be, and whatever it is they do prove, do not, and certainly cannot, prove and establish a Method of Calculation which produces such evident and gross Errors. But whether Mr. *Moiivre* mistook the Point to be proved, or Mr. *Richards* has mistaken his Meaning, that I shall leave at present to them to accommodate between themselves, as they think fit; but as I shall have an Opportunity of resuming the Consideration of

this Matter, I will then attempt to show on which Part the Mistake lies.

From this Case, so far as we have already gone in it, I think it appears pretty plain that this Table does not give us the true Value of Annuities for two Lives and the Survivor; and if we carry it a little farther, I believe we shall see to what Sort of Annuities on Lives, or to what Sort of Lives these Calculations may more justly be applied: I must add in this particular Instance; because I apprehend they cannot rightly be applied to any Lives as an universal Rule; and when they are applied in this Instance, we shall find them to be Lives much different from two concurrent and surviving Lives.

To put the Case then, as before, I will suppose that N^o. 1. N^o. 2. and N^o. 3. have Grants for the Lives of *A.* and *B.*, and on the same Terms as in the former Case; and farther, that N^o. 4. has a Grant for the same Lives and the Survivor of them, with this Addition to his Grant, that on the Death of *A.* whom we will suppose to die first, he has a Liberty to renew his Grant, and to insert another Life instead of *B.* the Survivor. Let us examine now what is the Value of these two Interests belonging to N^o. 4., viz. his original Grant, and his Right to renew and exchange a Life. His original Grant is, in Effect, the same Thing as a Grant for the joint Lives of *A.* and *B.*, and is therefore equal to a Term of 19 Years, or thereabouts; but the Value of his Right of renewing and exchanging a Life depends upon the Terms which are or
may

may be put upon him, as to the Age of the Life whom he has Power to nominate on such Exchange. If he has Liberty to nominate at large, and does nominate a Person of the best Age of Life, that is about 10 or 12, then the Life of such Nominee will be equal to a Term of 30 Years in Reversion after 19; and then the Interests of N^o. 4. taken together, will be equal to a Term of 49 Years.

But as these Cases are severally stated, can any Pretence be made, that the Interest of N^o. 3. is equal to the Interest of N^o. 4.? It is impossible; for at the End of 19 Years, when *A.* is supposed to die, the Interest remaining to N^o. 4. is equal to a Term of 30 Years, that being the Term to which the Life of the Person is equal whom he nominates to succeed to *A.*; whereas in the Case of N^o. 3., at the End of 19 Years, when *A.* is supposed to die, the Interest remaining to N^o. 3. is only in the Life of *B.*; at the Death of *A.* the Age of *B.* will be 31, a Life of the Age of 31 is equal only to 24 Years; therefore the Interest remaining to N^o. 3. is equal only to 24 Years, at the same Time as the Interest remaining to N^o. 4. is equal to 30 Years.

Since then these Tables give us a Term of 49 Years, as the Term to which two Lives and the Survivor are equal; but two such Lives and concurrent Lives as are comprized in the Grant to N^o. 3. are not equal to such a Term, or any Thing near it; and two such Lives and a succedaneous Life as are mention'd in the Grant to N^o. 4. are equal to such a Term, or thereabouts;

we see to what two Lives these Tables happen to be applicable in this particular Instance. The Result of the whole is, that Mr. *Richards* has taken a great deal of Pains, as in Truth he has, to calculate the Value of Annuities for two Lives, intending thereby Annuities for two Lives and the Survivor; but his Tables, if they give any, give us the Value of Annuities for two joint Lives and a single succeeding Life: So that, if Mr. *Moiſre's* Demonstrations prove the latter, which I very much question, yet even then they prove something that is not at all to the present Purpose.

Let us see now what the Rule is which Mr. *Moiſre* has prescribed for adjusting the Value of Annuities for these Lives, and examine it particularly; for perhaps we may discover something in it besides the enormous Errors which it has produced, which will convince us, that the Rule itself must necessarily be a false one, notwithstanding the Allegation that the Truth of it has been demonstrated: For the Demonstrations, whatever they are, may have proceeded on Principles or Suppositions, for which there are no just Foundations in the Reason or Nature of the Thing. The Rule then is this; to put together the Value of the Annuity for the two given Lives singly taken, and from that Value to subtract the Value of those Lives jointly taken, and the Sum remaining, he says, is the Value of the Annuity for the two Lives and the Survivor. I say, on the contrary, that such Remainder is not, and cannot be the Value; and that the Cases which I have

have already produced, and another or two which I crave Leave to state, do put the Matter beyond all Controversy.

To apply this in an Instance or two; we will first make Use of the Lives of *A.* and *B.*; suppose them to be each 12 Years old, that *B.* becomes the Survivor, and that the Interest is 4 *l.* per Cent. as before.

An Annuity for the Life of <i>A.</i> singly	}	17-20
taken, is in Value		
One for the Life of <i>B.</i> singly taken, is	}	17-20
in Value		

These Values put together make 34-40

An Annuity for their joint Lives is in Value 13-01

This last Value, subtracted from the	}	21-39
precedent Value, leaves a Remain-		
der, as the Value of the Annuity for		
the two Lives and the Survivor,		

This Sum 21-39, which comes out to be the Remainder, and to be the Value of an Annuity for these two Lives and the Survivor, is equal to a Term of 49 Years, and above; but that this, which is the Term produced by Mr. *Moiré's* mathematical Operation, is not the true one, I affirm; and I think, what I have already offer'd, and what I shall farther add, will put it past Contradiction.

On the Case thus stated, we will suppose N^o. 1. has the Interest in the Life of *A.* singly taken; N^o. 2. in the Life of *B.* singly taken; N^o. 3. in the Joint-Lives of *A.* and *B.*; and N^o. 4. in the
Lives

Lives of *A.* and *B.* and the Survivor. The Value of the several Interests of N^o. 1. N^o. 2. and N^o. 3. are before stated and agreed upon; and we want to know what is the Value of the Interest of N^o. 4. This Author's Rule asserts, that this Value is adjusted barely by deducting the Interest of N^o. 3. out of the Interest of N^o. 1. and N^o. 2. singly taken and put together: and I assert the contrary. That the Interest of N^o. 4. is not so valuable as the Interests of N^o. 1. and N^o. 2. put together is admitted on both Sides; and it must be admitted likewise, that such Want of Value arises from hence, that the Interest of N^o. 4. in the two Lives, so long as both of them last, is combined, and with Regard to him and his Annuity both the Lives are as one only; so that the Enquiry is, what Prejudice or Loss accrues to him from such Combination; and whether any and what after such Combination ended.

If we admit that we may adjust the Value of an Annuity for these two Lives and the Survivor, by deducting something out of the Value of the two Lives singly taken and put together; such Deduction must be of something and every Thing, whatever that be, which does come to, or may be enjoy'd, either by N^o. 1. or by N^o. 2.; but does not come to, or can in any Sort, or by any Chance, be enjoy'd by N^o. 4. in the Whole or in Part.

I agree then in this Case, that N^o. 1. and N^o. 2. have each of them a Chance to enjoy their Annuities for the Joint-Lives of *A.* and *B.*, that is, for a Term of 18-3-07, or thereabouts; and that

that each of them, during that Time, will be entitled to 13-1, as the Value corresponding to such Term; and that N^o. 4. has a Chance to enjoy that Term and that Value no more than once only during those Joint-Lives: And 'tis from thence urged, that one of those Terms and one of those Values is lost to N^o. 4., since he has not a Chance to enjoy them both, and consequently one of them is to be deducted out of the Interest on the two Lives singly taken and put together. If we grant this to be true, as most certainly it is, we must then compute all the Losses accruing to N^o. 4. as well as this, all I mean that stand on the same Foot with this, whether arising during the Combination of the two Lives, or after it is ended: And if we do so, believe me, before we come to an End of our Computation, we shall meet with so many Losses and Deductions, that we shall leave an Annuity for two concurrent Lives and the Survivor, of no greater Value than an Annuity for any one of the Lives singly taken.

In the Case stated 'tis supposed, that the Interest of N^o. 1. in the Life of *A.* is in Value 17-20, and equal to a Term of 29-3-00; and that the Interest of N^o. 2. in the Life of *B.* is the same; and 'tis supposed likewise, that at the End of 18-3-00 one of the two dies, and that the Interest of N^o. 3, being an Interest for those Joint-Lives, then determines; and it not being material which dies first, we have supposed *B.* the Survivor. From hence it follows, that N^o. 4. has not a Chance to enjoy so much on the Life
of

of *A.* as N^o. 1. has ; for the Chance of N^o. 4. on that Life is only a Term of 18-3-00, in Value 13-01 ; whereas the Chance of N^o. 1. on that Life is a Term of 29-3-00, in Value 17-20 : So that on this Life there is a Loss of a Term of 11 Years, and 4-19 in Value. The Deductions then out of the Value and the Term of the two Lives, singly taken and put together, will stand thus. For the Loss during the Joint-Lives in the Term 18-3-00, for the Loss on the Life of *A.*, he being dead, 11-0-00 in the Term ; in all 29-3-00 : For the Loss during the Joint-Lives in the Value 13-01, for the Loss on the Life of *A.*, he being dead, in the Value 4-19 ; in all 17-20 : That is, just the Term and the Value of one of the Lives singly taken is lost to N^o. 4. by this Way of computing.

That this Term of 11 Years, and this Value 4-19, must be look'd upon as lost to N^o. 4., and be deducted out of the Term and the Value of the two Lives singly taken and put together, might be thus proved. If this be good Reasoning, *viz.* N^o. 1. and N^o. 2. have each of them a Chance to enjoy a Term of 18-3-00, in Value 13-01, during the Joint-Lives of *A.* and *B.* ; but N^o. 4. has a Chance to enjoy one of the Terms and one of the Values only, therefore one of them is to be deducted out of the Term, and the Value of the two Lives singly taken and put together, as something lost to N^o. 4 ; which must be Mr. *Moiivre's* Way of Reasoning : Then this also must be good Reasoning, *viz.* N^o. 1. and N^o. 2., besides the Term 18-3-00, and the Value

Value 13-01, incurred and grown due during the Joint-Lives, have each of them a Chance likewise to a farther reversionary Term of 11 Years, in Value 4-19; for the Chance of each of them is in the Whole a Term of 29-3-00, in Value 17-20; but N^o. 4. has a Chance to enjoy this reversionary Term of 11 Years, in Value 4-19, once only, for the very same Reason that he cannot, during the Joint-Lives, enjoy more than one Term and one Value, for that his Annuity is a single one only, and only one Nominee living: one of these Terms therefore, and the Value corresponding, should be deducted out of the Value and the Term of the two Lives singly taken and put together, as something lost to N^o. 4.

Now it is obvious, that both these Losses must not be computed, and deducted as such; for that this Method makes the Value of an Annuity, for two Lives and the Survivor, to be equal only to an Annuity for one of the two Lives, singly taken: But then I affirm, that a bare Subtraction of the Value of the Annuity for the Joint-Lives, that is the Subtraction of 13-01, which is the Value of the Annuity for these Joint-Lives, out of 34-40, which is the Value of the Annuity; for these two Lives singly taken, and put together, does not and cannot leave us the Value of the Annuity for the two Lives and the Survivor. If out of this total Value 34-40, we make no other Subtraction than the Value 13-01, we manifestly leave in such Sum total 4-19, the reversionary Value on the Life of *A.*;

and

and another 4-19, the reversionary Value on the Life of *B.* and once 13-01, supposed to be due and received during the Joint-Lives: For 4-19, 4-19, and 13-01, put together, make 21-39; and the Rule does plainly enough so direct. Now N^o. 4. has no Right to both these reversionary Values, but upon a Supposition that *A.* lives to the End of 29-3-00, or has a Chance to live so long; and that *B.* also lives, or has a Chance to live, to the same Time; and that there is the same Chance that both of them, when jointly considered, may live to the End of that Term, as there is on each of them, when singly considered; which is not only not true, but is also contrary to the former Supposition, that one or other of them dies at the End of 18-3-00: And we must suppose likewise, that N^o. 4. has two Annuities, when the Case states it as a single one only.

To avoid these Inconsistencies, I mean these on one side, if at the End of 18-3-00, we deduct the Whole of one of these reversionary Values, then we make an Annuity for two Lives equal only to an Annuity for a single Life; on the other side, if we deduct neither of these reversionary Values, we shall make the Chance of the Duration of both Lives, when Joint, to be the same, as when each of them is Single, and equal to 29 Years; and besides out of one Annuity, we shall carve two: To avoid this Difficulty, we need only to take the Fact, and the Chance, to be such as it is; *viz.* that the Annuity is a single one, and the Chance to be, that one or other
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of the Nominees will be dead at the End of 18-3-00; and if we compute the Value of the Annuity for the Life of the Survivor, on this Foot, it will come out to be more than that Value 4-19 once taken, and something less than that Value twice taken: But of this Point, and of the Manner in which the Life of the surviving Nominee, and the Annuity depending on it, is to be estimated, I shall have Opportunity to consider under another Head.

— As this Way of Reasoning, may seem perhaps a little abstruse, and not to shew clearly and distinctly enough, that the Value of the Annuity for the Joint-Lives is not the proper or the sole Deduction to be made; and since this Rule is supported by a Claim to a Mathematical Demonstration, and a Mathematical Demonstration if rightly applied, is an uncontestable Evidence; the Reader will excuse me, if I produce another Instance, in which this Error is still more gross and more palpable; and indeed lies so open, that one would wonder how the Constructor of the Tables could miss discovering it, when he made his Calculations. From hence it will be seen, and when pointed out, it will be acknowledged by every Man, who has common Understanding in Numbers and Figures, evidently to appear; that, in Order to find out the Value of the Annuity for the two Lives, and the Survivor, the Value of the Annuity for the two Joint-Lives, is not the sole Value, or the proper Value, to be deducted out of the Value of the Annuity

Annuity for the two Lives, singly taken and put together.

The Case is upon two Lives, each of the Age of 82, and stands thus.

An Annuity for the Life of *A.* aged 82, } 1-41
is in Value

One for the Life of *B.* aged 82, is in Value } 1-41

Both Values put together, are 2-82

An Annuity for their Joint-Lives, is in Value } 0-55

The last Value deducted out of the precedent one, leaves } 2-27

This remainder then, is the Value of an Annuity for these two Lives, and the Survivor of them, as it comes out upon the Operation, made by Mr. *Moiivre's* Rule, and is the same as is given in Mr. *Richards's* Tables. The Lives of *A.* and *B.* being equal, and the Chance of dying, consequently equal, we will suppose that *A.* dies first, and the Interest of an Annuitant in these two Lives and the Survivor, is plainly this, and no more than this, *viz.* in their Joint-Lives, during the Life of *A.* and after his Death in the Life of *B.* surviving. An Annuity for their Joint-Lives, is in Value 0-55, an Annuity for the Life of *B.* the Survivor, when the Interest on both Lives commences, is only 1-41; therefore, the whole of this Annuitant's Interest, cannot possibly be more than 55 and 1-44, both put together 1-96.

That the Interest in the Joint-Lives of *A.* and *B.*, and the Interest in the single Life of *B.*
the

the Survivor, are all the Interests which this Annuitant has, in the Lives of *A.* and *B.* and the Survivor, is past all Controversy: These two Interests put together are, as before collected, no more than 1-96 in Value; and it is impossible they should be more, unless we suppose the Life of *B.* the Survivor, at the Time he becomes the Survivor; that is, at a Time when he is six or seven Months older, than when the Interest first commenced; is become a better Life now, when it is single, than it was at first, when it was single. But this is a contradiction to common Sense and Reason, and to speak a little in the Mathematical Language, is contrary to the Postulatum at first laid down, that such a Life is in Value 1-41, and no more; therefore the Annuity for these two Lives and the Survivor, cannot be more than 1-96 in Value.

If in this Case, as well as in the former, we make only a single Subtraction, and take 2-27 as the Value of the Annuity on these two Lives and the Survivor, here occurs the same Exception that we took before, and appears in a stronger light; that the Term in the Life of *B.* supposed to be the Survivor, will be much too long, and the Value of the Annuity much too great. There, if we made only a single Subtraction, we left the Term in the Life of the Survivor, the same at the Time when he became the Survivor; that is, when he was 30 or 31 Years old, as the Term of his Life was, when the Interest first commenced, that is, when he was 12 Years old only. Here, if we make only a single

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Subſtraction, the Value of the Annuity for the Life of the Survivor will be greater, and the Term correſponding to ſuch Value, will be longer, at the Time he becomes the Survivor, than the Value and Term of the ſame Life was, even at the Commencement of the Intereſt. At the Commencement of the Intereſt, the Value of the Annuity for the Life of *B.* was 1-41, and no more; and at the Time when *B.* becomes the Survivor, the Value will be 1-72; for if out of 2-27, which is the Value given of the Annuity for the two Lives and the Survivor, we Subſtract 00-55, which is the Value of the Annuity for the two Joint-Lives; the Value remaining, will be 1-72, as the Value of the Annuity for the Life of *B.* when he becomes the Survivor, and the Term in the Life, will correſpond to ſuch Value. Here the Defect is more viſible than before, and from hence it neceſſarily follows, where we are to adjust the Value of the Annuity for theſe two Lives and the Survivor, that the Value of the two Joint-Lives, *viz.* 0-55, is not the ſole Value to be deducted out of the Sum total, of the Value of the two Lives ſingly taken and put together, *viz.* out of 2-82; for ſuch a Value deducted, if nothing more is deducted, leaves 2-27, as the Value of ſuch an Annuity; which is ſo great a one, as we ſee cannot poſſibly be the true Value of this Annuity.

Nay we may go farther, and aſſert that it follows from hence, not only that 2-27 is not the Value of the Annuity for theſe two Lives and
the

the Survivor, but also that it is not the Value of two such Lives of 82, and a succedaneous Life; I mean, where the Annuitant upon the Death of him who dies first, has the Privilege in the room of the Survivor to nominate a Successor of the Age of which the Survivor was, at the Commencement of the Interest, that is, of the Age of 82: For it appears from hence, that an Annuity for two such Lives, is in Value 1-96 and no more, since the Interest on the Joint-Lives, is 0-55, and the Interest in the single succeeding Life, supposed to be of the Age of 82, is 1-41, and those make together 1-96 and no more.

And since it is plain, that an Interest for these two concurrent Lives, and the Survivor of them, is not so valuable as an Interest in these two Lives taken as Joint-Lives, and an Interest in a single Life to succeed and be nominated in the Manner, and at the Time I have mentioned; it follows farther, that we must make a Deduction even out of this 1-96, before we can come at the Value of the Annuity, for these two Lives and the Survivor. Upon the Whole, I very much question, whether Mr. *Moirre* or Mr. *Richards* can assign any certain, or determinate two Lives whatever, of 82, to which an Annuity of the Value of 2-27 will correspond: But whether they can or they cannot, I think this Case gives us a convincing Proof, that a Subtraction solely, of the Value of the Annuity for the two Joint-Lives, out of the Value of the Annuity for the two Lives singly taken and put together, will not leave us the Value of the Annuity for those

two Lives and the Survivor; for that the Value 2-27, which is the Produce of such Operation, is not, nor can possibly be, the Value of an Annuity, for the two Lives and the Survivor.

To make this Matter still more clear, I crave leave to produce one Instance more, where the two Lives are of different Ages; and I mean here, to consider the Rule in another Light, and to examine what are the Parts of the Value of such an Annuity, which by this Rule are directed to be brought to the Annuitant's Account, as belonging to him. And here I assert, that more and other Parts of the Value of such an Annuity, are by the Rule brought to his Account, as belonging to him, or more Parts are left in the Account of the Value of the Annuity, for the two Lives singly taken and put together, than the Annuitant has a Chance to enjoy: And I will attempt to assign, the particular Values or Parts of Values, so wrongly brought to Account.

In an Instance on the Life of *A.* aged 12, and *B.* aged 42; Interest computed at 6l. per Cent., the Account stands thus.

An Annuity for the Life of <i>A.</i> singly	}	13-36
taken, is in Value		
One for the Life of <i>B.</i> singly taken, is	}	10-24
in Value		
Both Values put together, are		23-60

An

An Annuity for their Joint-Lives, is	}	08-84
in Value		
This last Value deducted out of the	}	14-76
precedent Value, leaves		

Which Sum of 14-76, is the Sum produced by an Operation made, according to Mr. *Moirre's* Method, and is the Value given in Mr. *Richards's* Tables, as the Value of an Annuity for these two Lives and the Survivor. We have before observed, that the Reason for deducting the Value of the Joint-Lives out of 23-60, the total Value of both Lives put together, is this; because in that Sum total, the Value of the Annuity on the Life, both of *A.* and of *B.*, is included, as Parts of the Annuity to be received by the Annuitant, whereas during their Joint-Lives, he receives the Value of one only; so that during that Time, one or other of the two Lives, is no better than a dormant and barren Interest, producing no other Value to the Annuitant, than the Life of his Companion produces: And therefore unless 8-84, which is the Value of the Joint-Lives, and is twice brought to Account in the total Value, is once subtracted out of it, the Annuitant will be computed to receive that Value twice, when he can receive it once only.

If this be a good Reason for subtracting the Value of the Joint-Lives once, and not estimating such Value as Part of the Annuity belonging to the Annuitant, as undoubtedly it is; then the same Reason will hold good, to deduct or

leave out of this same total Value, any and every other Value which stands in this Account, as a Sum to be received, if any there be, which in Fact cannot be received. And most certainly in this total, there still rests a Value, or a Part of a Value, which can never be received; for at the Death of the Nominee who dies first, be he which we will, the Deduction of 8-84 already made, does not make a Deduction of the intire Value of the Annuity, either on the Life of *A.* or of *B.*; for the one is 13-36, and the other is 10-24. But after the Death of one of the Nominees, such Part of the Value of the Annuity on the Life of the Person dying, as remains unreceived at his Death, never can be received; since the Annuitant has neither Title nor Possibility to receive it in the Life-time of the Survivor. Title to receive he has none; for the Interest in that Life is now more than dormant, for it is absolutely extinct and gone; and to receive the Remanet of the Annuity on the Life of the Nominee dead, and at the same Time to receive the Remanet of the Annuity on the Life of the Nominee living, is not possible; for here occurs an insuperable Difficulty, that we make two Annuities going on together, and a double Receipt, where we suppose but one, and there can be no more than one.

I have before admitted, that this my Way of computing and deducting, will make an Annuity for these or any other two Lives, to be of no greater Value than an Annuity for one of the same two Lives; and yet the Reason is full as strong

strong for making the one Deduction, as for making the other; it being equally true, that the Annuitant does not and cannot receive his Annuity double, during the Life of the surviving Nominee, as that he cannot receive it double during the Joint-Lives of both the Nominees: And I do not urge this as an Argument to prove that my Way of computing is right, for I know it to be false, and from thence argue, that both of them are false.

To make an Annuity on these two Lives, to be of greater Value than on any one of them singly taken, as most certainly it is; we must reckon the Interest of the Annuitant in the Life of the Survivor, as of more Value than the Remanet on his particular Life, at the Time he becomes the Survivor, though not equal to the Remanet on both Lives. I mean thus: The whole Value of the Annuity for the Life of *A.* is 13-36; the Value on their Joint-Lives 8-84, which being deducted, the Remanet on the Life of *A.* will be 4-52: The whole Value on the Life of *B.* is 10-24; and 8-84, the Value on the Joint-Lives, being deducted, the Remanet on the Life of *B.* will be 1-40. We must admit then, and the Fact is so, that the Interest of the Annuitant in the Life of *A.*, if he becomes, and when he becomes the Survivor, is of greater Value than 4-59, the Value then remanent on his Life; and in like Manner, if *B.* becomes the Survivor, that the Interest in his Life will then be greater than 1-40, the Value then remanent on his Life: But

neither of them will be equal to the Values remanent on both their Lives.

If the Question be asked, in what Manner the Quantity of this greater Value of the Interest in the Life of the Survivor is to be adjusted, which I promised before to consider; my Answer is, that it must be done by looking upon the Life of the Survivor as equal to such a Term, as his Life will be equal to, at the Time he becomes the Survivor, or has the Chance to become such; and as such it may very well be looked upon by him, and so it would be looked upon by any third Person, who should then purchase an Annuity on his Life, since then and from thence it is a single Life; and the Interest of the Annuitant, either the old or the new one, depends now on the Contingency of Mortality on that surviving Life only: And the Term on that Life so computed, must be taken with regard to either of the Annuitants, as a Term in Reversion, to commence from the Determination of the Term, in the two Joint-Lives.

To this Question, the Answer of Mr. *Moivre* by his Rule is, that the remanent Value of the Annuity on the Life of the Nominee dead, must be added to the remanent Value on the Life of the Nominee surviving; and that these two Values put together, give us the Interest of the Annuitant in the Life of the Survivor: To apply this to the Lives of *A.* and *B.*: If *A.* be the Survivor, the remanent Value on his Life is 4-52; to this we must add 1-40, the remanent Value on the Life of *B.*; these two put together will

will be 5-92; and this will be the Value of the Interest of the Annuitant, in the Life of *A.* the Survivor: To this if we add 8-84, the Value received during the Joint-Lives, the whole will be 14-76; the intire Value of the Interest of the Annuitant in these two Lives and the Survivor. In the other possible Case, that *B.* becomes the Survivor; to the remanent Value on his Life, which is 1-40, we must add the remanent Value on the Life of *A.*, which is 4-52; these put together will be 5-92, and is the Interest of the Annuitant in the Life of *B.* the Survivor: And to this, when we have added 8-84, the Value received during the Joint-Lives, the Whole of the Interest in the two Lives, and the Survivor, comes out to be 14-76; the same as was before computed on the Life of *A.*, if he were the Survivor.

This, indeed, is not the Direction of the Rule in so many Words, but 'tis the Sense of it, and in the Event it comes to this: for if we put together the Value of an Annuity for two Lives singly taken, and out of this total Value we deduct the Value of an Annuity for the Joint-Lives once, supposed to be lost to the Annuitant during such Joint-Lives, what are the Values then left in the Account? Most plainly the Values left in the Account are the Value of the Annuity for the Joint-Lives supposed to be taken during the Joint-Lives, for in the Sum total this Value is twice included; and the Surplusage or Remanet on each of the Lives singly taken, not yet supposed either to be taken or lost: For here
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the Value of the Annuity for the Joint-Lives supposed to be taken is 8-84; the Remanet on the Life of *A.* is 4-52, and on the Life of *B.*, 1-40; all which put together make 14-76, the total Value of the Annuity for the two Lives. If this then be the Rule or the Effect of it, that the Remanet on both Lives put together is to be accounted as the Value of the Annuity for the Life of the surviving Nominee, let us take a View of it, as such Value has Correspondence to the Term to which such surviving Life is equal, and we shall see the absurd Consequences. I say then, that the Age of the Survivor, at the Time he becomes the Survivor, is the Measure by which we must judge of the Duration of such a Life, and the Measure of his Life is certainly the Measure of the Annuity, or of the Parts of the Annuity, which will be received during such Life. Mr. *Moiivre* says otherwise, and that we must take the Value of the Annuity on the Life of *A.* and the Value of the Annuity on the Life of *B.*, not received during their Joint-Lives, and these two Values put together are the Value of the Annuity during the surviving Life. Now in this Way of computing such Value, we plainly take a Part of the Value of an Annuity on two Lives put together, as a Measure of the Duration of one of the two Lives; when a Part, or several combined Parts, of the Value of an Annuity are not so much as a Measure of the Duration, even of that very Life on which the Annuity depends; on the contrary, the Duration of the Life is the Measure of the Value of the

the Annuity; much less can a Part of the Value of an Annuity on one Life, and a Part of such Value on a second Life, combined together, give us the right Term to which such second Life is equal, or the proper Parts of the Annuity remaining and to be received during such Life. Nay in this Case the Life of the Survivor is in Effect a third Life: For though the Person surviving must be one and the same with one of the Nominees, yet his Life should be and must be considered as a different Life now from what it was before, since he is supposed to be advanced in Age about thirteen Years. To which may be added, as something more extraordinary, that these remanent Parts of the Value of the Annuity on these two Lives put together are made the Measure of the surviving Life, or, as I may more properly call it, a third Life, let which will of the two be the Survivor, and let the Ages of the two Nominees differ never so widely.

And farther; if we could suppose that the several Parts of an Annuity on a single Life, where such several Parts have been calculated annually, on the Chance of Mortality, thro' the whole possible Term of a given Life, and when put together, do give us the true Value of the Annuity for such Life; and if we could suppose, that a Part of the Value of an Annuity on one Life added to a Part of such Value on a second Life would give us the true Value of the Annuity for the surviving Life of these two, which I call a third Life; and that this will hold good, let the Age of the surviving Life be what it will:

Which

Which Suppositions we must make where we follow the Directions of this Rule; though I confess I can reconcile neither of them to my Understanding: Yet, even upon these Suppositions, as ill grounded as they are, the Values here specified, that is, the Values not received during the Joint-Lives, cannot be the Values resting to be received, it being impossible for the Annuitant to receive those individual Values, as will be obvious when we consider what are those Values unreceived. The Value of the Annuity during the Joint-Lives, and which is received, is 8-84, and is equal to a Term of 13 Years, very little over; so that at the End of 13 Years there remains on the Life of *A.* 4-52, as the Value to be received on his Life, which is equal to a Term of 14-3-00, or thereabouts; and on the Life of *B.* there remains 1-40 as the Value to be received on his Life, which is equal to a Term of 3-1-00, or very near it; both which Terms are Terms in Reversion after the Expiration of the 13 Years incurred during the Joint-Lives, and the Values are correspondent.

Now the Annuitant can't possibly receive, or have a Chance to receive, both these Sums of 4-52 and 1-40, because in the Case stated we have supposed that the Annuity is only a single one, and that neither of these two Values do arise and grow due 'till after the Expiration of 13 Years, and the Determination of the Interest for the Joint-Lives, that is, after the Death of one of the Nominees: So that to put the Annuitant in a Capacity of receiving both these Sums,

we must suppose there are two Annuities, which is contrary to the Case stated; and we must suppose that *A.* and *B.* do both survive the Term of 13 Years, and each of them one another, which not only contradicts the former Supposition that one of them is dead, but common Sense too. 'Tis certain there is a Chance that *A.* may survive *B.*, and a Chance likewise, that *B.* may survive *A.*, but the Annuitant can't have both these Chances come up Benefit-Tickets; for 'tis in Nature impossible, and a Contradiction in Terms, that *A.* does in Fact survive *B.*, and *B.* at the same Time survive *A.*: So that there is a Certainty that the Annuitant will enjoy the Sum attending one of these Chances, but an Impossibility that he should enjoy both: And yet this Method of Mr. *Moiivre's* does necessarily suppose that he is entitled to, and will receive, both Annuities; and I doubt Dr. *Halley's* Hypothesis supposes that both Nominees will become the Survivor.

This Rule may be consider'd in another Light, and express'd in different Terms, and yet, in Effect, remain the same; and in this Light, perhaps, we shall see better what are the Parts of the Value of this Annuity which do not belong to such an Annuitant, and yet are here brought to his Account. The Rule will then stand thus. To take the Value of the Annuity for any one of the Lives, intire as it is; and out of the Value of the Annuity for the other Life to subtract the Value of the Annuity for those two Lives as Joint-Lives, and to add the Remainder

to the Value of the first Life: And to take those two Values put together as the Value of the Annuity for those two Lives and the Survivor. If we state the Rule in this Manner, it is in Substance the same as in the former. There we deduct the Value of the Annuity for the Joint-Lives out of the Value of the Annuity for both Lives singly taken and put together; here we keep the Value of the Annuity for the two Lives separate, and out of the Annuity for one of the Lives separately and singly taken we deduct the Value of the Annuity for the Joint-Lives: And then we put together the Value of one of the Annuities intire, and so much of the Value of the other as remains after the Value of the Annuity for the Joint-Lives is deducted: And therefore we deduct the same Value out of the same Value in both Cases, and the Sum total remaining will be the same.

If the Rule be look'd upon in this View, we see more clearly what Values we have; *viz.* the Value of the first Life intire, and so much of the second Life as remains after deducting the Value of the Annuity for the Joint-Lives. Now here I admit that 'tis right to take the Value of the Annuity for the first Life intire as it is; and the Question is solely, Whether the Annuitant for the two Lives and the Survivor has a Right or a Chance to those Parts of the Value of the Annuity for the second Life as are here assign'd to him, and I conceive not; and for this Reason. The Parts of the Value of the Annuity which the Annuitant has a Right to on the
second

second or additional Life, are the Parts of the Value of an Annuity in Reversion after the Determination of the Annuity on the first Life; but the Parts of the Value of the Annuity here assign'd to him, are the Parts of the Value of an Annuity in Possession; for the Parts of the Value deducted here are deducted out of a Value in Possession, and consequently the Parts of the Value remaining, which are the Parts allotted to the Annuitant, are Parts of a Value in Possession.

An Instance will explain and illustrate this Position. To ascertain the Value of an Annuity for the Lives of *A.* and *B.* both of 12 Years Age and the Survivor, we are to take the Whole Value of the Annuity for the Life of *A.* which is 17-20, and of the Value of the Annuity for the Life of *B.*, which is likewise 17-20, we are to take so much as remains when the Value of the Annuity for their Joint-Lives is deducted. The Value of the Annuity for these Joint-Lives is 13-01, and if we deduct 13-01 out of 17-20, the Remainder is 4-19, which 4-19 being added to 17-20, the Value of the Annuity for the first Life, gives us 21-39; which is the Value of the Annuity for these two Lives and the Survivor, produced by Mr. *Moirre's* Method, and given in the Tables. It appears from hence very plainly, that this Value 4-19 are Parts of a Value of an Annuity in Possession, for they are Parts of the Value 17-20, which is the Value of an Annuity for 29-3-00 in immediate Possession: And 'tis as plain, that the Annuitant has a Right only to the

the Parts of the Value of an Annuity in Reversion, and that on such second Life they are Values in Reversion after the End of 29-3-00.

And if we look upon this Value 4-19 as Parts of a Value in Reversion, as in some Respects it may be look'd upon, and examine what these Parts are; we shall discover that still they are such Parts as the Annuitant has no Right or Possibility to receive. The Rule, in the Manner I have now stated it, directs us to take the whole Value 17-20, which is equal to a Term of 29-3-00 on the Life of *A.*, as the Value of the Annuity for that Life; and for the Annuity for the Life of *B.* the Survivor to take so much of the Value of the Annuity for his Life as remains when the Value of the Annuity for the Joint-Lives is deducted thereout. I admit that the Annuitant has a Right to something over and besides the Value and the Term on the Life of *A.*; but 'tis plainly a Value in Reversion after 17-20, and after a Term of 29-3-00: But the Value taken by Virtue of this Rule is a Value in Reversion after 13-01, and after a Term of 18-3-00; and this evidently makes the Reversion to fall into Possession before in Fact it does, and makes this particular Value to be twice brought to Account; for 'tis already once brought to Account in the Life of *A.*, so that we have two Annuities subsisting for this Space of Time, instead of one: Whereas the farther or additional Value which the Annuitant has a Right to, over and above the Value of the Annuity for the first Life, is a Value and Term in Reversion after
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the Determination of the first Value and Term, that is after 17-20 and the Term of 29-3-00; the Rule itself, as stated by me, does suppose such remaining Interest on the second Life to be no other or greater. From whence 'tis apparent that the Value 4-19, which is here assign'd to the Annuitant on the Life of *B.* the Survivor, is not the right one; for the Value which he is intitled to is the Value of a Term in Reversion after the End of a Term of 29-3-00, whereas the Value here assign'd is the Value of a Term in Reversion after the Determination of the Interest for the two Joint-Lives, that is, after a Term for 18-3-00: And consequently, the Parts of the Value of the Annuity not belonging to this Annuitant, which are here brought to his Account, are the Difference between the Value of an Annuity for 11 Years in Reversion after a Term of 18-3-00, and the Value of an Annuity for 11 Years in Reversion after a Term of 29-3-00; or the Sum by which the former Value exceeds the latter.

By Way of Digression a little; let us try what will be the Effect of this Rule, which was framed for adjusting the Value of this Annuity for the Life of *A.* and *B.* and the Survivor, if we apply it to adjust the Term to which such an Annuity is equal: And if the Rule be a right one to ascertain the Value of this Annuity, I see no Reason why it should not be a right one to ascertain the Term. I should rather say, that I see great Reason why it should be a good Rule to determine the Term in the Annuity, though a very

bad one to determine the Value. The Value of the Annuity for the Life of *A.* being 17-20 is equal to a Term of 29-3-00, and the Annuity for the Life of *B.* is the same; the Value of the Annuity for their Joint-Lives is 13-01, and is equal to a Term of 18-03-00, or thereabouts; and if we deduct this Term out of 29-3-00, the Term of the second Life, the Term left will be 11-00-00; which being added to 29-3-00, the Term on the first Life makes 40-3-00, as the Term to which the Annuity for both the Lives and the Survivor is equal.

Now this, for any Thing I see to the contrary, may be the true Term to which this Annuity is equal, I am sure at least that 'tis very near the true one; for in my Way of computing such Term, if each Life were supposed to be equal to a Term of 29-3-00, as here they are, the Term of such an Annuity would be 40 Years and above; and on a Computation by Mr. *Richards's* Tables, it will come out to be very little more. If we take the Term of this Annuity from the Value of it given in his Tables for those Annuit-ies, there indeed the Term comes out much longer; for the Value in those Tables is 21-39, which is equal to a Term of 49-0-00, or above: But this Value and Term is plainly equal to the Value and Term of an Annuity for two Joint-Lives and a succeedaneous Life, to be nominated when the Annuity for the Joint-Lives is determined and one of the Nominees is dead, and not of an Annuity for two Lives and the Survivor of them; as I have elsewhere shown. But

if

if we suppose the Survivor, at the Time when his Companion dies, that is at the End of 18-3-00, to be, as really he is, of the Age of 30 and above, and if we take the Value and the Term of a Life of 30 Years old, and the Annuitant most certainly has not a Right in a Life of any better Value and Duration; the Value of such a Life is about 14-66 in Value, which is equal to a Term of 22-2-00; which being added to 18-3-00 makes 41-1-00, as the Term to which these two Lives are equal.

This Instance shews the Truth and Justice of my Reasoning and Observations in another Place, where I alledged, that an Addition to, or a Subtraction from, a Value given was not the same Thing as an Addition to, or a Subtraction from, a Term given: For that on a Value and Term correspondent given if an Addition were made to such Value of a Quarter, or any other proportionable Part, and an Addition of the like proportionable Part were made to such Term, the new Value and Term arising on such Addition would not correspond; and the Case is the same, and of Necessity must be the same, on a Subtraction. And the Reason of the Thing is plainly this; that in the Case of the Term, whatever is added or subtracted is a Term wholly in Reversion, but in the Case of a Value, what is added is a Value in Possession, or Part of a Value in Possession. From hence I infer; since the Value of Annuities on Lives depends on the Time or Term for which any given Lives endure, or on an even Chance may be computed to

endure; and since a Part of a Term, with an Annuity attending it, will not be the same Thing as a like proportional Part of the Value of the Annuity; that this Rule, assigning on such second Life a proportionable Part of the Term of the whole Life as the Term to which a Life which depends on the Chance of Survivorship is equal, may very well give us the true Term to which such Life so depending on the Chance of Survivorship is equal; but where it assigns a proportionable Part of the Value of the Annuity for the whole Life as the Value of an Annuity for a Life which depends on the Chance of Survivorship, there the Value so assign'd shall not be the true Value of the Annuity for the Life so depending on the Chance of Survivorship.

To return to the Point. As Dr. *Halley's* Rule for the Valuation of Annuities on a single Life is faulty, in assigning Values in Reversion to the Annuitant on a Life instead of, and where, Values in Possession, if any Values at all, do belong to such Annuitant; and by that Means, the Term to which an Annuity for a Life is equal is sunk and depress'd to a shorter Term than the Life nominated has an even Chance to endure: So the Method here is faulty on the other Side, in assigning Values in Possession, or Values in Reversion, as taking Place in Possession earlier than in Fact they do take Place, when Values in Reversion only, or such as are at a more remote Distance of Time than is named, do belong to the Annuitant; and by that Means
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the Term to which an Annuity for two Lives is equal, is extended beyond the Term to which it can in any Reason be supposed to continue.

That this must be the Case will appear by comparing the Chance of the Duration of a single Life and of two Lives and the Survivor, with the Value and Duration of an Annuity for such single Life, and for such two Lives: For the Doctor, as I have before observed, and those who follow his Rule, make a Difference between the Chance of the Duration of a single Life and the Chance of the Duration of the Annuity depending on the Life; and so they do likewise in the Case of two Lives and the Survivor. If we compute the Chance of Vitality by the *Breslaw* Table on a Life of ten Years of Age, such a Life is equal to a Term of 41 Years, but by the Doctor's Tables for the Value of Annuities on Lives, an Annuity for such a Life, at 6 *l.* per Cent. Interest, is equal only to a Term of 28-1-00, and on Mr. *Richards's* Table much the same. By the same *Breslaw* Table I have made a Calculation of the Chance of Vitality on two Lives of ten Years Age, and of the Survivor; and the Chance of Vitality of one or other of the Lives reaches to 52 Years, or thereabouts; and the Value of the Annuity for the two Lives and the Survivor, at the like Interest on Mr. *Richards's* Table, form'd by this Rule of Mr. *Moiivre's*, is 15-59; answering to a Term of 48 Years, or near it. The Remark which I make upon this is, that on a single Life the Chance of the Duration of the Annuity falls short of the Chance

of the Duration of the Life about 13 Year, but on two Lives such first Chance falls short of the second about 4 Years only, when undoubtedly it ought to exceed the former: And this Disproportion, I say, can no Ways be accounted for, but by supposing that in computing the Value of the Annuity for the two Lives, by Mr. *Moiivre's* Rule, more Values or greater Values are inserted and brought to the Annuitant's Account than do belong to him.

But this Matter may be carried farther; for I am of Opinion, not only that Mr. *Moiivre's* Rule for calculating the Value of an Annuity for two Lives and the Survivor, but even that the Foundation of his Rule, viz. Dr. *Halley's* Hypothesis, for ascertaining the Chance of the Duration of two Lives and the Survivor, is not a right one: And if Mr. *Moiivre's* Method for calculating the Value of Annuities for two Lives and the Survivor depends on the Justice of the Hypothesis, as I take it for granted it does, and I suppose is the Thing which he has undertaken to demonstrate; then, if the Foundation is wrong, the Method and all the Tables constructed thereby, will be wrong also. It may be true, for any Thing I shall at present say to the contrary, that Mr. *Moiivre* has demonstrated that his Method is right, on a Supposition that Dr. *Halley's* Hypothesis, upon which it is grounded, is right; but these Tables of Mr. *Richards* show either that the Hypothesis is wrong, or that the Method is wrong; for that I may be confident that these Tables constructed by the Method are not

not right; and therefore that Mr. *Moiivre's* Demonstrations, if they prove that the Method is agreeable to the Hypothesis, prove that the Hypothesis on which the Method is founded is wrong. My Way of Reasoning hitherto has been principally thus. These Tables are wrong; therefore, either the Method by which they were constructed, or the Hypothesis on which the Method is founded, or both, are wrong. Here I shall invert the Argument, and say; the Hypothesis and the first Foundation is wrong, therefore the Method is wrong, and consequently the Tables which were constructed by it are wrong also.

Before I enter into the Discussion of that Point, I must premise, that the Method which Mr. *Richards* made Use of to form his Tables for the Value of Annuities for two and so for three Lives and the Survivor, is that which Mr. *Moiivre* prescribes, and not that which Dr. *Halley* prescribes, for that Purpose; which may perhaps be thought the same, but are really very different. The Doctor, in calculating the Value of Annuities for two Lives, and so for three Lives, directs the same Method which he directed and made Use of for calculating the Value of Annuities for a single Life. His Way is this; he calculates the Value of the Annuity as an absolute one for each Year of the Life of the Nominee, deducting out of each Year's Value a Sum answering to the Chance of Mortality arising in such Year, and this he repeats for every Year of the possible Life of such Nominee; and all

those Values, so collected and put together, he states as the Value of the Annuity for that Life. The very same Method he prescribes to calculate the Value of Annuities for two or more Lives, by taking the Values arising in each Year of the Life of the Nominees, or of any one of them, through all the possible Years of the Life of all or any one of them; and then putting all these Values together, he states them as the Value of an Annuity for such Lives. In this Way of computing the Value of Annuities for one or more Lives, no Regard is had, or intended to be had, of the even Chance of Vitality on the single Life, or on any of the Lives if more than one; but the Computation is to be made solely on this Foot, *viz.* so long 'tis possible that the single Person, and if more than one, so long 'tis possible both or one of them may live; therefore so long the Computation must be continued.

Let us see now how Mr. *Richards* framed his Tables for the Values of these Annuities. In forming those for the Value of Annuities for a single Life, he made Use of the same Method which Dr. *Halley* prescribed and made Use of to form his Tables for the Life Annuities: But in stating the Value of Annuities for two or more Lives and the Survivor, there Mr. *Richards* deserts the Doctor's Method; and no Wonder, since the Operation would prove so excessively tedious and long, and resorts to Mr. *Moiivre's*. This, as we have before observed, is to put together the Value of the Annuity on each of two Lives singly taken, and out of that Produce to deduct

deduct the Value of the Annuity for those two Lives as Joint-Lives, and to take the Sum remaining as the Value of the Annuity for the two Lives and the Survivor. Now this Method is manifestly founded on Dr. *Halley's* Hypothesis, by which he calculates the Chance of the Duration of two or more Lives and the Survivor, and not that by which he calculates the Value of Annuities for two or more such Lives: But even this Hypothesis for calculating the Chances of the Duration of two or more Lives is, as I said before, in my Apprehension and Judgment, a wrong one.

In a Matter of so much Intricacy and Perplexity as this is, whether I shall be able to express my Sentiments and Reasons in such a Manner as satisfy the Understanding of others, and to prove my Position, I must leave to the Reader's Judgment; however, I shall attempt it. The *Doctor's* Rule for ascertaining the Chance of the Duration of a single Life, is this; to take the Number of Persons living of the Age named, which Number is given in the *Breslaw* Table, as the Chances of that Person's Life; to compute in what Number of Years those Persons are reduced to an half, and the Year wherein those Persons are so reduced, is the Time to which such Person has an even Chance to live, and beyond that Year he has not an even Chance to live: And the Number of Persons living in the first or any subsequent Year, I would call the Chances of Vitality on such Persons Life, and the Number of Persons dead within such Year or Years, the Chances of Mortality. The Number of Persons living, that is the Chances
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of Vitality on the Life of a Person of ten Years of Age is 661; the Half of that is 330; in 41 Years, or thereabouts, the Number of Persons living is reduced to 330; that is, the Chances of Vitality and Mortality are even: Therefore the even Chance is, that a Person of such an Age lives for 41 Years, and not beyond.

The Rule for ascertaining the Chance of Duration of two Lives and the Survivor, is built upon the same Foundation; but with this Difference. On a single Life, the Number of Persons living of the Age given, is the Chances of Vitality of that Person's Life; here on two Lives, we are to multiply the Number of Persons living of the Age given by the same Number, if both Persons named are of the same Age; and if they are of a different Age, we must multiply the Number of Persons living of one Age, by the Number of Persons living of the other Age; and the Produces are the Chances of Vitality on the Lives of those two Persons. In like Manner for the Chances of Mortality; we are to multiply the Number of Persons of the Age given, who have died within any stated Time, by the same Number, if both Persons named are of the same Age; and if they are of a different Age, we must multiply the Number of Persons of the one Age given, who have died within the stated Time, by the Number of Persons of the other Age given, who have died within the same Time; and the Produces are the Chances of Mortality on those two Persons for the Time so stated. And here, if we enquire to what Time

'tis an even Chance that two Persons named, or one of them, shall live, and beyond which they, or either of them, will not live; we state first the Chances of Vitality, on the two Lives as before directed, and then we compute the Chances of Mortality on the same Lives to a stated Time, 'till we come to the Time or Year wherein the Chances of Mortality are a Moiety of the Chances of Vitality as before fix'd, at which Time both Chances, in Effect, are even; and this is the Time, or Year, to which 'tis an even Chance that one or other of the Nominees will live, and beyond which they, or either of them, will not live.

To make the Rule and my Objections more intelligible, I will apply it in an Instance. If we take the Life of *A.* and *B.*, both of ten Years Age, the Number of Persons living of the Age of 10 is 661; 661 multiplied by 661 produces 436921, and that Produce is the Chances of Vitality on those two Lives. The Number of Persons of that Age who have died, suppose in 55 Years, is 469; 469 multiplied by 469 produces 218761, and this Produce is a Moiety of the former Produce, and a little over, that is in 55 Years, the Chances of Vitality and Mortality on these two Lives, are come near to a Par; and therefore the even Chance is, that *A.* and *B.*, or one of them, will live for 55 Years, or near it, and neither of them beyond that Time. I deny then that this Calculation gives us the true Chance of the Duration of these two Lives and the Survivor,

vivor, or that *A.* and *B.*, or either of them, have a Chance to live for 55 Years.

My Objection to the Rule is this. The Number 661 is the Number of all the Chances of Vitality through the whole possible Life of *A.*, and of all the Chances of Vitality through the whole possible Life of *B.*, that is for 90 Years on each of them; and consequently $661 + 661$, or 436921, is the Number of all the Chances of Vitality on the whole possible Lives both of *A.* and of *B.*, that is for 90 Years both on the one Life and the other. But I say, an Annuitant, or other Person, having an Interest in the Duration of the Lives of *A.* and *B.* and the Survivor has not a Right to the Chances of the Duration of the Lives both of *A.* and of *B.* through the whole possible Lives of each of them, that is for 90 Years on each of them. Therefore this Calculation assigning all the Chances of Vitality on both Lives, and through their whole possible Lives, allots to such an Annuitant having Interest in their Lives more Chances on such Lives than belong to them.

The first and third Propositions are so obvious and clear as to need no Proof; and the second, *viz.* that an Annuitant, having an Interest in these Lives, has not a Right to the Chances of the Duration of the Lives both of *A.* and of *B.*, through the whole possible Lives of each of them, that is for 90 Years on each of them, which only has any Difficulty in it, may thus be proved. If we compute the Chance of Vitality on the Lives of *A.* and *B.* singly and separately taken,
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and make Use of the *Breslaw* Table for that Purpose, the even Chance on each of their Lives is equal to a Term of 41 Years, or thereabouts; and if we compute such Chance on those Lives as Joint-Lives, and by the same Tables, in my Way of making such Computation, the even Chance on their Lives as Joint-Lives is equal to a Term of 30 Years, or thereabouts. Now at the End of 30 Years, or whenever, in any Way of Computation, the even Chance is that either *A.* or *B.* are dead, from thence the Annuitant has the Chance of the Duration of one of the Lives only, the other being from that Time to be consider'd as extinct and gone; and consequently, so many of the Chances of Vitality as are computed to arise on the Life of the Nominee so dying, after the Expiration of 30 Years, and when one or other of the Persons is on an even Chance computed to be dead, do, in Fact, not belong to such Nominee, and therefore ought not to be inserted in such first general Produce or Account. I admit, that in calculating the Chances of Mortality on these two Lives, we compute the Chances on both their Lives, and through the whole possible Lives of each, and deduct them out of the Chances of Vitality; but it will not from thence follow, as perhaps may be urged, that the Calculation of the one as well as of the other Chances ought to be made on the same Foot: For the Cases I say differ; and when we observe how they differ, that Difference will show us that the one Calculation

lation ought to be made and is justly made on that Foot, and that the other ought not.

We will suppose then, at present, that 'tis an even Chance that at the End of 30 Years either *A.* or *B.* is dead, yet I say we must continue our Computation of the Chances of Mortality from thence in the same Manner as before, because the Chance of both dying remains the same at least after the Death of one, as whilst both are living. On the other Side, after the End of these 30 Years, and the supposed Death of one, the Chances of Vitality do not remain the same as before; and if we continue our Computation of those Chances through the whole possible Lives of both, that is for 90 Years, as the Rule manifestly does, we compute as if both were possibly living for a Term of 90 Years, when for Part of that Time we have supposed one to be dead, which is, in Effect, to suppose it possible that both are living when one is dead. The Rule here does plainly take it for granted that the Chance of both being living, and the Chance of both being dead, go equally through the whole Course of the Lives of both, or for 90 Years, which is certainly a wrong Presumption; for in any given Number of Years there is always a Chance that both are dead, but not always or for the whole Time a Chance that both are living, because one may be dead, and for great Part of the Time the even Chance is that one is dead, living the other.

For Instance, at the End of 50 Years there is a Chance that *A.* and *B.* are both dead, and the
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Chance that both are dead is not the less, because one of them is computed to have been dead some Time; but at the End of 50 Years there is no Chance that *A.* and *B.* are both living, for the even Chance is that one of them has been dead these twenty Years; and therefore the Chance on the Lives of *A.* and *B.* at the End of 50 Years must be a Chance different from and less than a Chance that both are living, for the Chance is that they are not both living: And yet the Rule does plainly suppose that both are living, or possibly may be living; for the Sum 436921 contains all the Chances through the whole possible Lives of both, and for that Year gives us the Chances of Vitality answerable. And if we carry this Matter the farthest it can be carried, and admit that there is a possible Chance of both being living at the End of these 50 Years; yet the even or probable Chance is that one of them is dead, and we cannot put the possible Chance of both being living on the same Foot with the even or probable Chance that one is dead, without supposing that a possible Chance is as strong and as good as an even or a probable one.

The true State of the Chances here is this; the Chance of Mortality both of *A.* and *B.*, through all the Years of the possible Life of *A.* and *B.* on the one Side, and on the other Side, the Chance of Vitality of *A.* and *B.* for so many Years as *A.* and *B.* have a Chance both of them to live, and after that, the Chance of Vitality through all the remaining possible Years of the
Life

Life either of *A.* or *B.* whichsoever shall become the Survivor. That this is a true Representation of the Chances of Mortality, I suppose will be admitted, because 'tis agreeable to the Direction of the Rule; and that these, and no other or more than these, ought to be taken as the Chances of Vitality, is evident from this Argument. No other or more Chances of Vitality do or can belong to those or any other Lives, than such as arise whilst those Lives are in Being, or possibly in Being, but if we take such and so many as the Rule directs, or any more than I have assign'd, we necessarily suppose that *A.* and *B.* are both in Being or possibly in Being, after the Time when we have supposed one or other of them to be dead. 'Tis indeed possible that *A.* may extend his Life to 50 or to 90 Years, and possible that *B.* may extend his to the same Time, and therefore that we may suppose it possible that both may be in Being at the End of 50 Years: But this Supposition, that 'tis possible that *A.* and *B.* are both living and in Being at the End of 90 or of 50 Years, is at an End and must cease when the other Supposition takes Place, that one of the two is dead: Or else we shall make contradictory Suppositions, that *A.* survives *B.* and *B.* likewise survives *A.*, and that both are living when one is dead.

When we compute the Chances of Vitality on the single Life of *A.*, we suppose it possible that he may live for 90 Years, or to be an 100 Years old; but when on a Computation made we have stated that the even Chance of the Duration of his

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his Life is equal to 41 Years, we have no farther Regard to the first possible Chances on his Life: So if we compute the Chances of Vitality on the Lives of *A.* and *B.* and the Survivor, from the End of 30 Years, when *A.* is supposed to be dead, supposing he dies first, from thence we are to have no Consideration of the original possible Chances on his Life, for they are at an End; and after they are at an End, I see no Sort of Reason why we should make any Account of them in this Case, any more than we do of the possible Chances of Vitality on his Life remaining after the End of 41 Years, where we calculated the Chance of the Duration of his Life on the Foot of a single Life. All that rests to be done after this, is to compute the Chances of Vitality on the Life of *B.* the Survivor as a single Life, and this Computation must be taken from the Time of his becoming the Survivor, and be carried on through all the Years of his possible remaining Life. And that the Doctor's Hypothesis and Rule for ascertaining the Chance of the Duration of these two Lives and the Survivor brings to the Account of the Chances of Vitality on their Lives as well all the Chances possibly remaining to arise on the Life of *A.* after the End of 30 Years, that is after the supposed Death of *A.*, as the Chances possibly remaining on the Life of *B.* supposed to survive *A.* and the Term of 30 Years, is evident from hence; for that in the first general Account of the Chances on both Lives are inserted all the original possible Chances on both Lives singly taken. From

whence I collect and conclude; since in the Number 436921 are included all the Chances which arise on the whole Life of *A.* and the whole Life of *B.*, as well those which arise whilst both are living, as those which may arise on the Life of *B.* the Survivor, and those which are computed to arise on the Life of *A.* after 30 Years, that is after he is dead, or supposed to be dead; from hence I infer, that such and so many Chances are inserted in that Sum total more than ought to be, as are computed to arise on the Life of *A.*, after he is supposed to be extinct and gone.

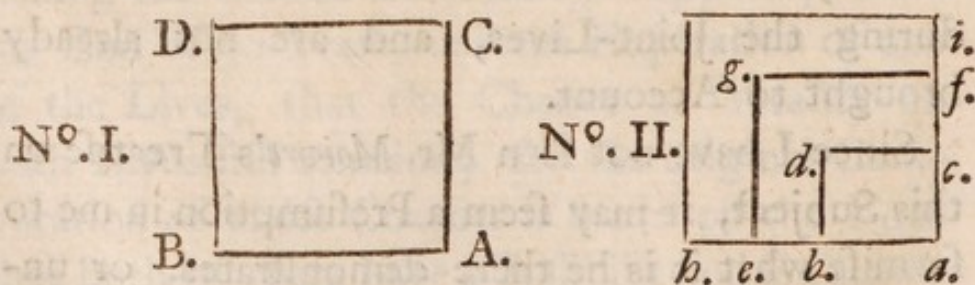
On this Conclusion naturally arises this Observation, that the Doctor's Rule for ascertaining the Chance of the Duration of two Lives and the Survivor, and Mr. *Moiivre's* Method for calculating the Value of an Annuity for two such Lives, have one and the same Defect, grounded on one and the same wrong Supposition, that both the Nominees are living when one is dead, or computed to be dead. The Rule for ascertaining the Chance of Vitality supposes, after the End of 30 Years, and the Expiration of one of the Lives, that the Chance of Vitality on both Lives still remains; for the original Computation of those Chances is made on the Foot that both of them have a Possibility of Existence for 90 Years; and the Method of valuing an Annuity for those Lives does the very same Thing. Agreeably to this, when we compute the Value of an Annuity for these two Lives and the Survivor, the Method is this. We first take
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the Value of the Annuity for the Joint-Lives of *A.* and *B.*, and afterwards so much of the Value of the Annuity on each of the Lives singly computed as remains after we have deducted out of each of them the Value of the Annuity for the Joint-Lives: We put together these three Values, the Value of the Joint-Lives once taken, and such Part of the Value on each of the Lives singly computed as remains after such Deduction of the Value of the Annuity for the Joint-Lives, and the Sum total is the Value of this Annuity for the two Lives and the Survivor. Just in the same Manner, where we compute the Chances of Vitality on these two Lives and the Survivor, we carry to Account first the Chances of Vitality which have arisen during the Lives of both of them, and afterwards when one is supposed to be dead, or ought to be supposed dead, we add to such Account such and so many of the original possible Chances on both these Lives as remain after deducting the Chances arisen during the Joint-Lives; or, which is the same Thing, so many of those first Chances as have not arisen during the Joint-Lives, and are not already brought to Account.

Since I have not seen Mr. *Moiivre's* Treatise on this Subject, it may seem a Presumption in me to surmise what it is he there demonstrates, or undertakes to demonstrate; but I apprehend he only attempts to prove, and it should seem sufficient for his Purpose to prove, that his Method for calculating the Value of Annuities for two or more Lives was consonant to Dr. *Halley's* Hy-

pothesis for ascertaining the Chances of the Duration of such Lives; and I very much question, the Reasons of which we shall see hereafter, whether he has proved even so much as this. However, whether he has or has not demonstrated it, I think, from what I have offer'd, it is evident and certain, that the Method of one is, in some Instances at least, consonant to the Hypothesis of the other: The necessary and unavoidable Consequence of which is; since the Hypothesis itself is false, that those very Demonstrations which were brought to stablish the Method do themselves destroy it.

I am very sensible that Dr. *Halley* endeavours to explain and support this Hypothesis by a mathematical Scheme, as well as by Numbers and Figures; but his Attempt this Way is as fruitless and ineffectual as in the other, since both are grounded on wrong Suppositions, and on something which in Fact is not true, nor can possibly ever happen; which, perhaps, will be seen more plainly on a Scheme than in Numbers.



In the Scheme N°. I. we will suppose the Line A. B. to represent the Chances of Vitality on the Life of *A.* aged ten Years, and the Line A. C. the Chances of Vitality on the Life of *B.*

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of the same Age; and the Line *a. b.* in the Scheme N^o. II. to represent the Chances of Mortality on the Life of *A.*, and the Line *a. i.* the Chances of Mortality on the Life of *B.* If Enquiry is made, what is the Chance of Vitality for the first Year of the Life of *A.* and of *B.*, each singly and separately taken; if we suppose the Lines *a. b.* and *a. c.* to represent the Chances of Mortality for the first Year of these two Lives respectively; the Answer will be, *A. B. minus a. b.* is the Chance on the Life of *A.* and *A. C. minus a. c.* the Chance on the Life of *B.* If we would be inform'd in what Time the even Chance is that *A.* may die, we compute what are the Number of Persons of his Age dying, or the Chances of Mortality arising, in each Year of his Life, beginning at the Age ten, and of the Line *a. b.* we take in each Year a-Part in Proportion to such Chances, 'till we come to that Year in which the Line by such Takings becomes of half the Length of the Line *A. B.*; that is when the Number of Persons of his Age living and the Number of those dead, that is the Chances of Vitality and Mortality, are even; and that is the Year to which 'tis an even Chance that *A.* will live, and not beyond it.

If we carry our Enquiries to the Lives of *A.* and *B.*, and the Survivor of them, and would know what are the Chances that both are, or one of them is, living at the End of a given Number of Years; and what is the Year in which the even Chance is that both are dead; we proceed thus, according to the Doctor's Rule.

The Line *A. B.* represents the Chances of the Life of *A.*, and *A. C.* represents the Chances of the Life of *B.*: *A. B.* + *B. C.* produces the Square *A. B. C. D.*, and this Square represents the Chances of Vitality on both their Lives. The Lines *a. b.* and *a. i.* represent the Chances of Mortality on the Lives of *A.* and *B.* respectively; and if we suppose, as we did before, that the Lines *a. b.* and *a. c.* represent the Chances of Mortality for the first Year of each of their Lives, then *a. b.* + *a. c.* will give us the Square *a. b. c. d.*; and that Square represents the Chances of Mortality on both their Lives for such first Year, I mean the Chances that both are dead: And consequently *A. B. C. D.* — *a. b. c. d.* is the Chance that both are, or one of them is, living at the End of one Year. To fix the Year in which 'tis an even Chance that both are dead, we compute what are the Chances of Mortality arising in each Year of the Life of *A.* beginning at the Age of ten, and of the Line *a. b.*, we take such a Part as bears Proportion to the Chances of Mortality arising in each Year respectively, and the same we do on the Life of *B.*, and take a like proportionable Part of the Line *a. i.*; and this we repeat 'till we come to the Year in which the Parts of the two Lines so taken, being multiplied one into the other, produce a Square which is half the Magnitude of the Square *A. B. C. D.*, and that is the Year in which 'tis an even Chance that *A.* and *B.* are both dead. Now if we suppose the Lines *A. B.* and *A. C.* to be in Length each of them 661 Inches, which 661 is the

the Number of Persons living of the Age of *A.* and *B.*, and of the Chances of Vitality on each of their Lives; $661 + 661$ produces 436921 , and the Square of the two Lines will be *A. B. C. D.* If we suppose the Line *a. e.* to be the Parts of the Line *a. b.* which are taken (suppose in 55 Years) as the Line of Mortality on the Life of *A.*, and the Line *a. f.* to be the Parts of the Line *a. i.*, which are taken as the Line of Mortality on the Life of *B.* (suppose for the same Time,) and that those Lines are in Length each 469 Inches; $469 + 469$ produces 219961 , and the Square of the two Lines will be *a. e. f. g.* The Sum 219961 is a Moiety of 436921 , and something over, and the Square *a. e. f. g.* is half the Magnitude of the Square *A. B. C. D.* and something more: But 55 Years, or near it, must pass before we have so many Chances of Mortality arise on the Life of *A.*, and so many on the Life of *B.*, as shall amount to 469 Chances on each, and before we take so many Parts of the Line *a. b.* and so many Parts of the Line *a. i.* as shall be equal to 469 Inches on each: And therefore 55 Years must pass before we shall have the Number 219961 , and the Square *a. e. f. g.*; therefore 55 Years must pass before 'tis an even Chance that *A.* and *B.* are both dead.

This is the Doctor's Representation of this Matter, as to the Sense and Substance of it, tho' possibly not in his Expressions, his Treatise not being now before me; but is framed, I say, on a Supposition which ought not, and cannot, in this Case, be made. If we examine and state

what these three Squares $A. B. C. D.$, $a. b. c. d.$, and $a. e. f. g.$, are compounded of, we shall see with Certainty what each of them contains. The Square $A. B. C. D.$ is made by multiplying the Line $A. B.$ by the Line $A. C.$; the Line $A. B.$ contains all the possible Chances of Vitality on the whole Life of $A.$, that is for 90 Years; and the Line $A. C.$ all the like Chances on the Life of $B.$, and for the same Term: Therefore the Square $A. B. C. D.$ contains all the possible Chances of Vitality on both their Lives for a Term of 90 Years. The Square $a. b. c. d.$ is made by multiplying the Line $a. b.$ by the Line $a. c.$; the Line $a. b.$ contains the Chances of Mortality arising on the Life of $A.$ in the first Year, and the Line $a. c.$ the like Chances on the Life of $B.$ and for the same Time; therefore the Square $a. b. c. d.$ contains the Chances of Mortality on both their Lives for such first Year. The Square $a. e. f. g.$ is made by multiplying the Line $a. e.$ by the Line $a. f.$; the Line $a. e.$ contains the Chances of Mortality arising on the Life of $A.$ in 55 Years Time, and the Line $a. f.$ the like Chances on the Life of $B.$ for the same Time: Therefore the Square $a. e. f. g.$ contains all the Chances of Mortality on both these Lives for a Term of 55 Years. The Hypothesis then asserts, that as the Square $A. B. C. D.$ is to a smaller Square, made by multiplying such Part of the Line $a. b.$ by such Part of the Line $a. i.$ as answers to the Chances of Mortality arising in any given Year, so are the Chances of Vitality to the Chances of Mortality on these Lives for that

that Year; or, which is the same Thing, if we deduct in any Year the smaller Square belonging to that Year out of the greater Square, that the Square or Area so left represents the Chances of Vitality on these Lives for that Year: For Instance; as $A. B. C. D.$ is to $a. e. f. g.$, the Square answering to the 55th Year on these two Lives, so are the Chances of Vitality to the Chances of Mortality on those Lives for that Year; or $A. B. C. D. - a. e. f. g.$ is the Chances of Vitality for such 55th Year on those two Lives: And therefore $A. B. C. D. - a. e. f. g.$, or the Area left, being in Magnitude rather less than the Square $a. e. f. g.$, 'tis less than an even Chance that either of them are living at the End of 55 Years.

I affirm then, first, that an Annuitant who has an Interest in the Lives of $A. B.$ and the Survivor, has not a Chance to gain the Whole of the greater Square $A. B. C. D.$ in each Year for a Term of 55 Years; much less for a Term of 90 Years: And, secondly, that on the other Side he has a Chance to lose the smallest Square, viz. $a. b. c. d.$, in the first Year of the Lives of $A.$ and $B.$, and in every subsequent Year a larger Square in Proportion to the Chances of Mortality arising in each Year 'till the 55th Year; and in that Year he will have a Chance to lose the Square $a. e. f. g.$: Therefore a Deduction of the smaller Square of each Year out of the greater Square for the Term of 55 Years, will not leave to the Annuitant for each Year for 55 Years such a Square as he has a Chance to; consequently, a Deduction
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of the smaller Square *a. e. f. g.*, which is the Square of Mortality for the 55th Year, out of the greater Square *A. B. C. D.*, will not leave to the Annuitant such a Square or Area as he has a Chance to for such 55th Year.

The Conclusion here is a necessary Consequence of the Premises; the second Position is supposed in the Hypothesis, and if not so, is apparent from hence; for that the Squares of Mortality to be deducted in each Year contain the Chances of Mortality arising in each Year on the two Lives, neither more nor less; and the first Position I have already proved under the former Head, and here shall add only thus much farther to confirm it.

If we admit, that for the first and every other Year of the Life of *A.* and *B.* for 30 Years, or during the Joint-Lives of *A.* and *B.*, a Deduction of the smaller Square, the Square of Mortality, out of the greater Square, may leave to the Annuitant a proper Square for such Year; the Reason of that is, because during those Years the Annuitant may have a Chance to the greater Square; and this Chance he may have, because *A.* and *B.* are supposed on an even Chance both to live so long; but when that Reason ceases, and the Annuitant has not a Chance that both are living, which after the End of 30 Years he has not, from that Time he has not a Chance to the greater Square *A. B. C. D.* From that Time, I say, such Annuitant has not a Chance that both are living, because it contradicts the former Supposition, that on the even and probable

bable Chance one or other of them is dead : And if we should admit that there is a possible Chance, tho' not a probable one, that both may live to be an 100 Years old, such Admission must be grounded on this ; that because when we consider each Life singly and separately we suppose it possible that *A.* may live so long, and possible that *B.* may live so long ; therefore, when we take the Lives of *A.* and *B.* jointly we may suppose that both of them may possibly live to that Age. Now I will not say that 'tis in Nature absolutely impossible that both should live to such an Age ; but if this Way of Reasoning is right in the Case of two Lives, by the same Rule we may proceed to three or any other Number of Lives *in infinitum* ; for where the Possibility ends, or the Impossibility is to begin, cannot be fix'd : And if any one can make such extravagant Suppositions, and will make Account of Chances arising on such remote and imaginary Possibilities, he must go on as far as he pleases, but I desire to leave him. In Truth, this Hypothesis for calculating the Chance of Survivance on two or more given Lives does really suppose that all the Lives conjunctly taken, whatever the Number of them may be, may possibly live to the Extremity of Age, that is to an 100 Years old ; and in the Case of three Lives does in Fact take it for granted that all of them possibly may live to that Age, and on that Foot makes a Computation of the Chance of Survivance of one of them : But there I can precisely and determinately fix the very Chances of Vitality which
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on this groundless Supposition are allotted, but do not belong to those Lives, or any of them; as shall be made appear in its proper Place.

But I confess I don't see why any Regard should be had to the Squares of the one Sort or the other, after the End of the 30 Years, when one of the Lives is extinct, or supposed to be extinct; on the contrary, I see very good Reason why no Regard should be had to either of them, and so will every Man who sees the absurd Consequences of continuing them to any longer Period. From that Time, one of the Lives, suppose *A.*, and one of the Lines, viz. *A. B.* is cut short; so that there is now no Chance left on the Life of *A.*, nor any Line *A. B.* now in Being, whereby to frame a Square; and it is therefore very absurd to carry on the Calculations to a Time, beyond which *A* is not living, and the Line not existing. On the other side, when *A.* is dead and gone, no Chances of Mortality can in strictness arise on the Life of *A.*, for none can happen *de futuro*; and with Regard to the Line *a. b.*, the Line of Mortality on his Life, if we take any Parts of it for the Chances of Mortality, we must now make a Taking once for all and take all, for *A.* by his Death has filled up the Measure of his Life and his Line. And if after the supposed Death of *A.*, we might continue our Computations on the same Foot as if *A.* were living as well as *B.*, and with Regard to the Line *A. B.* and *A. C.* might take the Square of them as the Chances of Vitality on them both, as still subsisting, which is greatly absurd; what must

must we doe with Regard to the Lines *a. b.* and *a. i.* the Lines of Mortality; must we go on to take a Part of each of them, and put such Part on the same Foot likewise, and continue the Squares of them too? most certainly we must; for if we suppose *A.* and *B.* both to be living, we do *eo ipso* suppose neither of them to be dead; and yet, though we do not suppose *A.* to be already dead, sure enough we may suppose he will die, and the Chance of his dying may proceed *pari Passu*, with the Chance of *B.*'s dying; and then, if we go on in every Year subsequent to the 30th, to take a Part of the Line *a. b.* in the same Proportion, as we do of the Line *a. i.*, and from these Parts so taken, we form a Square for the Chance of Mortality for that Year, and if we carry on this Process till the Square so to be formed, is of half the Magnitude of the greater Square, which will not be the Case till 25 Years after the End of 30 Years, or till 35 Years from the Beginning of our Computation: If we proceed thus, and the Hypothesis does manifestly so intend and so practise, we certainly and evidently suppose something, which in Fact, is not true, or can ever be supposed; for then we suppose that neither *A.* nor *B.* is dead till the End of 55 Years, and *eo ipso* we suppose that both are living at and to the Time, to which we have computed that one of them only is living. And since we have supposed *A.* and *B.* to be of the same Age, and possibly born of a Day, let us even suppose too, as they say is sometimes the

the Case of Twins, that they both die of a Day.

If Consideration be had of this Square of Vitality, that is the Square A. B. C. D, after the Expiration of the 30 Years and the Death of *A.*, supposing him to die first, another absurd Consequence following from thence, and which makes the strongest against it, is this; that the Original possible Chances on the Life of *A.*, will be brought to Account twice, once during the continuance of the said Term, and again after it is ended. That they are brought to Account after the Term of 30 Years is ended, is obvious from hence; for that in the Square A. B. C. D. are contained, all the possible Chances arising on the Life of *A.* as well as of *B.*, not only for the Term of 30, but of 90 Years. And that these same Original Chances which possibly may arise on the Life of *A.* after the End of 30 Years, are all brought to Account during the Term of the 30 Years, is proved from hence; for that in computing what is the probable Chance of the Continuance of both Lives as Joint-Lives, we have taken into Consideration all the possible Chances of both Lives through their whole Lives, and for a Term of 90 Years, and consequently all the possible Chances arising after, as well as those arising in the Term of 30 Years; and on comparing the Number of them with the Number of Chances of Mortality, arising in the same Time we have collected, that in 30 Years Time the Number will be equal, and consequently the even Chance is, that both of them
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may live for 30 Years, and one only beyond that Time. As in ascertaining the Chance of the Duration of the single Life of *A.*, we take all the Chances through his whole Life, to the Extremity of old Age, in which are included all the possible Chances arising after the End (suppose of 41 Years) as well as those which arise during that Time; in the same manner on the Joint-Lives of *A.* and *B.*, and on comparing the Chance when one or other will die, we take into our Account all the Chances through their whole possible Lives, and to the Extremity of the Age of both, and therein are included all the possible Chances arising after the End (suppose of 30 Years) as well as those which have arisen during that Time: And from thence, we collect and state the even or probable Chance on the Life of *A.*, to be a Term of 41 Years, and on *A.* and *B.* as Joint-Lives, to be a Term of 30 Years: Therefore the Original possible Chances on the Life of *A.*, are here twice brought to Account.

From hence and upon the Whole: Since the Square of *A. B. C. D.*, contains all the possible Chances of Vitality through the whole Lives both of *A.* and *B.* that is, for 90 Years; and the Hypothesis directs us to take this Square, deducting thereout the Square of the Chances of Mortality, for a Term beyond 30 Years, to a Term of 55 Years, and in so doing, supposes this Square and consequently the possible Chances of both Lives, which compose such Square to have an Existence after the End of 30 Years
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and to 55 Years, and affirms that Square, deducting as aforesaid, taken for a Term of 55 Years, is the Chance of Vitality for the Lives of *A.* and *B.* and the Survivor; And since, after the End of 30 Years, when *A.* is supposed to be dead, this Square is determined or can be no longer continued; and since all the possible Chances on the Life of *A.* through his whole Life, are brought to Account during the Term of 30 Years: From hence I infer that the Rule, in computing the Chances of Vitality for these two Lives and the Survivor, brings to the Account so many more Chances than belong to those Lives, as are supposed, after the Expiration of 30 Years, and one of the Lives, to arise on that one Life so determined.

As this is the Case on Dr. *Halley's* Hypothesis, for ascertaining the Chances of the Duration of two Lives and the Survivor, and on Mr. *Moiivre's* Method for adjusting the Value of an Annuity for two such Lives, which is founded upon, and is pursuant to the Hypothesis; the same we shall find it to be, where the Rule is applied to Particulars, as in Mr. *Richards's* Tables, and that they all agree in one and the same Defect.

That Defect which I mean here, and which I say is agreeable to, and proceeds from, the Rule, is this; that these Tables make the Life of the Survivor of two Lives, after he becomes the Survivor, to be equal to a longer Term than they make the same Life to be equal to, when the Interest on the two Lives first commenced, and even after such Survivor is advanced in Age,
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some seventeen or eighteen Years. We will here make Use of the same Lives of *A.* and *B.* as before, and instead of 4*l.* *per Cent.* which we used before, here we will compute after the Rate of 8*l.* *per Cent.*; because the Defect is most visible and apparent at that Rate of Interest, though at all Rates it continues in some Measure. The Value of an Annuity for those two Lives and the Survivor, is 12-02, which is equal to a Term of 43-3-00; the Value of an Annuity for those two Lives as Joint-Lives, is 9-32, which is equal to a Term of 17-3-00; therefore to make the Term in the two Lives and the Survivor to be equal to 43-3-00 in the whole, we must compute the Life of the Survivor to be equal to a Term of 26-0-00; for no less Term will make 17-3-00, the Term for the Joint-Lives, to be in the whole 43-3-00. But the Value of an Annuity for the Life either of *A.* or *B.* singly taken, and taken when they are 12 Years old, is no more than 10-67, which is equal only to a Term of 25 Years; and the Survivor, at the Time he becomes such, is advanced in Age near 18 Years, and is near 30 Years old; and therefore his Life should then be equal to a Term shorter, and it is impossible it should be equal to a Term longer, than it was at first. This Error is so apparent and so gross, that nothing to be sure can defend; and if the Table in this Instance is rightly constructed according to the Rule, as upon Experiment I find it is, and if this Value is the necessary Produce of the Rule as most certainly it is, the Rule, has the same Fault, and is as indefensible as the Produce of it.

If we carry our Inquiries a little farther, and examine how it comes to pass, that the Term in the Annuity for the Joint-Lives is so short, and the Term in the Annuity for two Lives and the Survivor is so very long, we shall discover what makes this Variance, and that the Defect in the Rule is the same, and no other than that which I have assigned. I have before taken Notice, that if in our Computation we make Use of 4*l.* *per Cent.* as our Interest, then the Term in the Annuity for the two Joint-Lives is equal to 18-3-00, and if 8*l.* *per Cent.*, then the Term will be equal to 17-3-00, according to Mr. *Richards's* Way of stating the Value of such Annuities. And I have asserted, and it will be admitted, that neither 17-3-00, nor 18-3-00, are the Terms of the Duration of these two Joint-Lives; but that the even Chance is, where we compute by the *Breslaw* Table, which makes each of the Lives singly taken equal to a Term of 41 Years, that then both Lives will continue in Being for a Term of 30 Years or thereabouts: The Chance then of the Duration of both these Lives, being equal to a Term of 30 Years; if we inquire why the Chance of the Duration of the Annuity for those Joint-Lives, is a Term so much shorter as to be equal only to 18-3-00 at one Rate of Interest, and at another Rate, no more than 17-3-00; the Reason is obvious. To compute the Value of Annuities either on a single or on two or more Joint-Lives, the Rule directs that in each Year of Life, the Value corresponding to the Chances of Mortality arising

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in such Year, be deducted out of the Value of the Chances of Vitality, and so for every Year of possible Life; and by that Mean the Term to which the Duration of the Life is equal, and the Term to which the Value of the Annuity is equal, cannot co-inside; but the Term in the Annuity is shorter than the Term in the Life. I shall not here dispute the Justice of the Rule, which makes the Term to which a given Life or Lives is equal to be one, and the Term to which the Value of an Annuity which depends on those Lives is equal to be another, but Just or not Just, so the Fact is. Accordingly, in Dr. *Halley's* Table for the Value of Annuities on a single Life, we see the Value of an Annuity on the Life of a Person of ten Years of Age, is equal to a Term only of 28 Years and a little over; and yet in his Way of computing the Chance of Vitality of such a Person, by the *Breslaw* Table, and the Way is undoubtedly a right one, the even Chance is, that he lives for 41 Years and above. The same, as we have seen, is the Case on Mr. *Richards's* Table for the Value of Annuities on a single Life, and at all Rates of Interest; and so it is on his Tables for Annuities on Joint-Lives.

The Difficulty then which arises here, and wants to be accounted for, is this; since in Annuities for a single Life and two Joint-Lives, the Term in the Annuity is much shorter than the Term in the Life or Lives, what it is that gives occasion that in Annuities for two Lives and the Survivor, the Term in the Annuities is

in some Instances longer than the Term in the Lives; at least in the Case of the Life surviving, the Annuity; for that Life is computed to last longer than the Life on which it depends. The Account that must be given of this Matter is, for since the Operator has performed his Part in Mode and Figure, no other Account can be given but this; either that the Hypothesis in Calculating the Chances of Vitality on two Lives and the Survivor, or the Method in settling the Value of Annuities for those two Lives, have inserted more Chances of Vitality, or more or greater Values than do belong to such Lives; or rather, that both of them are respectively guilty of the Error.

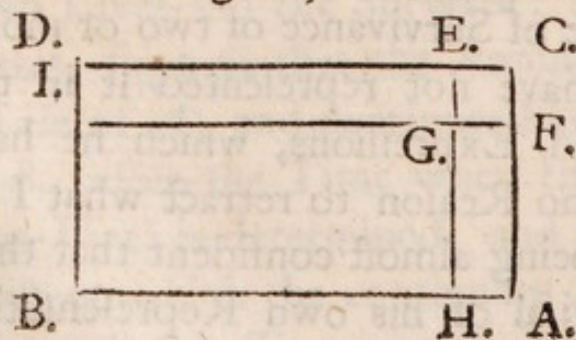
To settle the Value of an Annuity for the Life of *A.* and *B.* and the Survivor, Mr. *Moirre* directs us to take the Value of the Annuity for their Joint-Lives once, and afterwards and besides, that so much of the Value of the Annuity for the Life of *A.* singly taken, and wholly taken, as remains after deducting thereout the Value of the Joint-Lives, and to doe the same on the Life of *B.*, and to state all those three Values put together, as the Value of the Annuity for those two Lives and the Survivor: And do not we, in this, suppose that the Annuity continues on the Life of *A.*, and continues likewise on the Life of *B.*, after the Time when the Interest in the Joint-Lives is determined, and when one of the Lives on which the continuing Annuity depends is extinct? To compute the Chance of the Duration of these two Lives and the Survivor,

vor, Dr. *Halley* directs us to take all the Chances of Vitality on the Life of *A.* for his whole possible Life; that is for 90 Years, and the same on the Life of *B.*, which are found by Multiplying the one by the other, to deduct thereout the Chances of Mortality, which arise on the Life both of *A.* and of *B.*, for a certain limited Time; and to look upon the Chances remaining, as the Chances of the Duration of those Lives, or the Survivor of them, for such limited Time: And do not we here, in the first Instant, and in every Year, through which we carry our Computation, necessarily suppose that *A.* and *B.*, are both then living or possibly living; for the original Sum out of which we make our Deduction, is the same through all the Years of the Computation; and when we compute the Chance of the Duration of those two Lives and the Survivor, do not we continue this Computation for 55 Years, when the even Chance is, that one or other of them is dead in 30 Years, and his Chances all extinct? most certainly, and most evidently this is the Case, both on the Hypothesis and on the Method, and there I rest the Matter, leaving it to the Authors to make them good.

Upon looking into Dr. *Halley's* Scheme for the Chance of Survivance of two or more Lives, I find I have not represented it in the same Manner and Expressions, which he has done; but I see no Reason to retract what I have advanced, being almost confident that the *Doctor*, on a Revival of his own Representation, will

admit that I have represented the Matter, in the Manner in which he should have done, though not in the Manner in which he has done. In the Case stated by me, which is on two Lives of equal Age, the Number of Chances of one Life, multiplied by the Number of Chances on the other Life, produces a Square Number; and a Line representing the Number of Chances on one Life, multiplied by a Line representing the Number of Chances on the other Life, produces a Square: And the Number produced by such Multiplication of the Figures, is the Number of Chances on both Lives, and the Square produced by the Multiplication of the two Lines, is a Square answering to the Chances on both Lives. Upon the Case stated by the *Doctor*, which is on two Lives of unequal Age, the Number of Chances on one Life, multiplied by the Number of Chances on the other Life, produces the Number 298900, which may be called a Parallelogram Number; and a Line representing the Number of Chances on one Life, multiplied by a Line representing the Number of Chances on the other Life, produces an Area, which is a Parallelogram.

To make my Reasoning on the *Doctor's* Scheme more intelligible, I have here inserted it.



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The *Doctor* in the subsequent Part of his Representation, substitutes *Nn* in the Room of, and as equal to the Number 298900; and supposes the Line *A B* to represent the Number of Chances on one Life, and the Line *A. C.* to represent the Number of Chances on the other Life, and that those two Lines multiplied one into the other, will produce the Parallelogram *A. B. C. D.*, and that *A. B. C. D.* will be *N n*, or the Product of the two Numbers representing the Number of Persons of the two Ages given. Agreeably to this; the Inquiry he makes being, what are the Chances that both are dead in 8 Years, he multiplies the Number of Persons of one Age, dead in eight Years Time, by the Number of Persons of the other Age, dead in the same Time, and the Produce is 3650; and here, as before, in the Room of the Number 3650, he substitutes *Y y* as equal to that Number. He goes on and supposes the Line *C. E.* to represent the Number of Persons of one Age, dead in eight Years, and the Line *C. F.* to represent the Number of Persons of the other Age, dead in the same Time: And I add, though he has omitted it, that these two Lines multiplied one into the other, produce the Area *C. E. F. G.* Thus far the *Doctor's* Representation is undoubtedly right; but the Part which follows, is undoubtedly wrong, and wrong for that very Reason, because the first Part is right, and it is impossible both should be right. Where he says, that the Rectangle *F. E.* is the Product of the deceased, or *Y y* an equal Number of both dead;

he is mistaken: For Yy is not the same or equal to the Rectangle $F. E.$, but to the whole smaller Area $C. E. F. G.$; for Yy represents the Number 3650, and that Number is made up by multiplying the Number of Persons of one Age dead, by the Number of Persons dead of the other Age, in eight Years; whereas $F. E.$ is no more than a meer single Line put into such a Position, as to form a Rectangle, and can represent only the Number of Persons of one Age dead in eight Years, added to the Number of Persons of the other Age, dead in the same Time. And whereas he says, that as the whole Rectangle $A D$ or $N n$ is to the Gnomon $F A B D E G$ or $N n - Y y$, so is the whole Number of Persons or Chances, to the Number of Chances, that one of the two Persons is living; this again is certainly wrong: For $N n$ is not the same, or equal to the Rectangle $A D$, but to the whole Parallelogram $A B C D$, and he himself so states it; and, as before observed, Yy is not the same or equal to the Rectangle $E F$, but to the whole Area $C E F G$. But the worst Part of the Representation is in this Part, *viz.* that the Gnomon $F A B D E G$ is intended to be, and manifestly is an Area, and the Rectangle $A D$ is no other than a meer Line, or two Lines joined, formed in such a Manner as to create a right Angle; and we can make no Comparison, or frame an Idea of any Proportion between a meer Line and an Area, whether a Square or a Parallelogram. In the next Sentence he repeats the same Mistake, when he says; as
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the Rectangle AD or Nn is to (the Rectangle) FE or Yy , so are all the Chances, to the Chances that both are dead.

Now, I apprehend, that in all these Places, and in some others in the same Paragraph, we should read rectangular Parallelograms, instead of Rectangles; and if we read so, then our Representations of this Matter will agree, saving only, that his Instance is of Lives of unequal Age, and the Lines multiplied must produce Parallelograms, and mine is of Lives of equal Age, and the Lines must produce Squares. And that we ought so to read, is past all doubt, by his Representation of the Case on three Lives. There he makes all his Products Parallelopipedes, as well the several Chances of Survivorship, as the Chances of some being living, and all being dead: Now we cannot by a single Multiplication, of the Lines of a Rectangle, immediately form any Solid, but of necessity we must make an intermediate Operation, and first by a Multiplication form an Area, and a Multiplication afterwards, will in this Case produce a Parallelopiped; but here, unless the intermediate Chances on two Lives are supposed to be Area's, we form Solids from meer rectangular Lines. For this Reason on the Case of two Lives, and because the whole Number of Chances is in the first Instant call'd, and rightly call'd, a Parallelogram, I conclude that the several Chances on those Lives, should all have been called rectangular Parallelograms, and not merely Rectangles.

However, be the Representation right or wrong, I have asserted, and do insist on it, that this Rule, whether it be consider'd in the Shape
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of Numbers or of Lines, does not give us the true Chances of the Duration of two Lives and the Survivor, for that the Proportion is not as the whole Number of Chances is to the Chances of both being dead, or as the Area of the whole Square or of the whole Parallelogram is to the Gnomon of the Square or of the Parallelogram, or as the greater Square or as the greater Parallelogram is in Magnitude when the less Square or less Parallelogram is deducted thereout. I have already given my Reasons for this Assertion, and shall not repeat them, but crave Leave to add here a Proof something of a different Kind, to confirm it.

I observe, that the Method here prescribed by the Doctor for computing the Chance of the Duration of two or more Lives differs from that made Use of on the *Breslaw* Table; for I have made a Calculation of the Chance of the Duration of the Lives of two, of three, and of four Persons, all supposed to be of the Age of ten Years, to discover to what Age the even Chance is that one of the two, one of the three, and one of the four, respectively may live, or within what Time the even Chance is that all will be dead. On such Computation it appears that of the two Persons one or other of them has a Chance to live for 55 Years or near it; one or other of the three for 61 Years or near it; and one or other of the four for 65 Years; that is, one or other of them may live to be of the respective Age of 65, 71, and 75. But if we make Use of the *Breslaw* Table as a Rule to adjust these

these Chances on these several Lives, the even Chance will be that one or other of the two may live for 52 Years, one or other of the three for 58 Years, and one or other of the four for 61 Years; that is, one or other of them may live to the respective Ages of 62, 68, and 71. The necessary and obvious Inference from hence is, that the one or the other of these Methods for adjusting such Chances cannot be right.

If we compute, by the same *Breslaw* Table, how many Persons of the Age of ten Years live to 65, 71, and 75, which are the several Ages to which the Survivor of these Persons respectively may live according to the Doctor's Method of calculating the Chances of Survivance, which we are now considering, it appears there are 192 live to 65, 132 live to 71, and 88 to 75. The Number of Persons living of the Age of ten being 661; if we divide that Number severally by 192, by 132, and by 88, the several Quotients will be $3\frac{1}{2}$, 5, and $7\frac{1}{2}$; that is, of 661 Persons of the Age of ten Years one only in $3\frac{1}{2}$ lives to 65, one in 5 lives to 71, and one in $7\frac{1}{2}$ to 75: Therefore $2\frac{1}{2}$ in $3\frac{1}{2}$ die in 55 Years, 4 in 5 in 61 Years, and $6\frac{1}{2}$ in $7\frac{1}{2}$ in 65 Years.

My Way of reasoning and computing on the *Breslaw* Table is this. Two Persons in three of a given Age die within such a Time, three Persons in four within such a Time, 99 in an 100 within such a Time; or, on the Reverse, one Person only in three of a given Age lives to such a Time, one only in four to such a Time, and one

one only in an 100 to such a Time ; therefore if a Person would have an Annuity on Lives adequate to such Terms respectively, he must have such and such and so many Lives. And this is the very Method which Dr. *Halley* prescribes, for adjusting the Chance of the Duration of a single Life, when the *Breslaw* Table is made Use of; viz. one Person in two of a given Age, dies within such a Time, or, on the Reverse, one only in two lives to such a Time ; or of a greater Number of Persons, one Moiety lives to such a Time, and the other Moiety dies within the Time ; therefore the even Chance is, that a single Person of the given Age, may live to such Time ; and not beyond that Time.

If we apply this to the present supposed Persons of ten Years Age ; two in three die in 52 Years, three in four die in 58 Years, and four in five in 61 Years ; and therefore, if a Person would have an Annuity on Lives adequate to such Terms ; for the one he must have two Lives, for the other three Lives, and for the third four Lives, all of ten Years Age : But if he expects an Annuity for the several Terms of 55, 61, and 65 Years, which are the Terms respectively, to which two Lives, three Lives, and four Lives, are equal, in the Doctor's Method of calculating the Chance of the Duration of two or more Lives, now under Consideration ; in the one Case, he must have two Lives and an half, in the second four Lives, and in the last six Lives and a half, otherwise he will not have an even Chance, that his Annuities will continue
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for the several Terms intended; for that any Number of Lives less than these, will not leave the Annuitant an even Chance, that any one of his Nominees shall survive, to the End of the respective Terms.

That this Way of computing the Chances of Survivance on the *Breslaw* Table, is a right one, so far as the Table itself is a just one, I make no doubt: But I think the Computation here made, is a strong Indication that this Table is not a just one, or at least that it will not suit any other Place, if possibly it may that City. It is very improbable, and I believe contrary to all Experience elsewhere, that of the Persons who live to ten Years of Age, one in three lives to the Age of 62, one in four to the Age of 68, and one in five to the Age of 71; or, which is pretty much the same Thing, that of the Persons born alive, one in six arrives to the Age of 62, one in eight to 68, and one in ten to 71: And more nearly, one in $5\frac{1}{2}$, one in $7\frac{1}{2}$, and one in $9\frac{1}{2}$ to those Ages respectively. I cannot to be sure affirm, that this is never the Case in any individual Place, in this or any other Kingdom; but whether it be so or no, I appeal and must leave it to every Man's own Observation, and any Man may make the Observation, and with great ease. If it be true that one Person (suppose in 10) in any given Place, lives to be of the Age of 71, it is grounded upon this, or this must be the Consequence, that one Person in 10 of that Place, at any given Time in general, is of the Age of 71 or upwards; and this is easily known
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and computed in Places not over great. If Inquiry were made for this Purpose, I believe we should more readily find one or more Places, where there are not one in 50 living of that Age, than we should find one Place where there was one in 10 living. And yet, as if this Computation were not sufficiently extravagant, the Doctor's Method for calculating those Chances, gives us the Ages of 65, 71, and 75, as the Ages to which one in three, one in four, and one in five, does respectively live. And since I cannot, in my Understanding, reconcile and make consistent, these different Produces of the Chances of the Duration, of two or more Lives, which arise on these two Methods of computing; otherwise than by supposing, as I before contended, that in the Doctor's Hypothesis, more Chances of Vitality, or fewer Chances of Mortality than belong to those Lives, are somewhere inserted; I think the great excess in the Computation in the Hypothesis, must be accounted for in that Way, and can be accounted for no other Way.

After all, it may very well be made a Question, whether the original Positions about the Chances of Vitality and Mortality, and which are the Grounds of the subsequent ones, are themselves right or no: I mean the Positions, that the Number of Persons living of one Age, multiplied by the Number of Persons living of the same, or of another Age, does give us the Chances of the Lives of two such Persons; and that the Line A B representing the Number of
 Chances

Chances on one Life, and the Line AC representing the Number of Chances on a second Life, that if these Lines are multiplied one into the other, which will produce the Parallelogram $ABCD$, that such Parallelogram will represent the whole Number of Chances on both those Lives. The Case is the very same, with Regard to the Chances of Mortality, so I shall not repeat them. These Positions which in Effect and in Sense, are one and the same, only one is express'd in Numbers, and the other by Lines, the Doctor lays down without Proof, at least I see none that he offers to that Purpose; and an Assertion in the Negative, is just as good as one in the Affirmative, unless it be supported by Argument; which in my humble Opinion, is wanting in this Case. Sure I am, if the Doctor's Representation of this Matter, in the latter Part of it, in that Part I mean, with which I before found Fault, be true and a right one; it can be such on that Foot solely, and upon a Supposition, that the Rectangle AD , which is a meer Line made by adding two Lines together, and not the Parallelogram, which is formed by a Multiplication of the two Lines, does truly represent all the Chances on both those Lives.

However, as I have suppos'd that this Part of the Representation is erroneous and ought to be rectified, I do not insist on that: But say, that there is nothing in the Life of A , or of B , or in any other Number of Lives whatever, or in the Line AB , as representing the Life of A , or in the Line AC , as representing the Life
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of *B.*, which can be a sufficient Ground for supposing those Lines to be, or forming them into, a Plane; whether Square, Parallelogram, or any Area whatever. The Line *A B* is supposed to be, and is a meer straight Line, and the Line *A C* is supposed to be so likewise; and it being ascertained, as far as the even Chance goes, that the Line *AB*, when considered by itself, will be extended to such a Length, and the Line *A C* to such a Length, and there being a Chance that one or other of the two Lines, when considered as combined, will be extended to a greater Length than any one of the Lines singly taken; the Inquiry is solely what such greater Length is. To discover this, we are directed to form these two Lines into Squares, or Parallelograms, as the Case happens, to be of two equal or unequal Lives and Lines; and from thence 'tis suggested, that we may collect, what will be the Length of the Life and the Line of the longer liver. Now here we form these Lines into Squares and Parallelograms, which yet we cannot do, without supposing them to be such Things as they are not, nay such Things as we are not concerned in, for our Inquiry is upon meer Lines; and to what Purpose do we this? When we have introduced these Squares and Parallelograms, we can frame no Proportion between the Length of a Line, and the Magnitude of a Plane of any Sort or Kind, and if we could conceive any such, these Squares and Parallelograms are made to represent something, to which they have no Sort of Resemblance, for the Lives of two Persons can
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have no Resemblance to an Area of any Denomination whatever. The Length of the Life of *A.* and of *B.*, are meerly the Space of Time, through which the one or the other may live; and if such space of Time may be represented by two Lines, as well enough it may, yet we cannot form these Lines into Areas, without supposing the Lines and the Lives to be something which they are not, or ever can be, and that they are to be Measured in Breadth as well as Length. And by the Way, I wish that this were the only Instance, in which Mathematicians form Lines into Areas, without sufficient Reason, or any Foundation in the Nature of the Thing, for so doing.

If these same Lines do not represent the Lives of these Persons, but the Number of Chances belonging to their respective Lives, yet the Difficulty is not removed, only shifted off from one Position to the other; for it wants to be proved and still remains to be proved, that the Chances of one Life, multiplied by the Chances of the other Life, give us the Chances on both Lives. This I say ought to be proved, for it is more natural, and in my Judgment more rational to say; if the Number of Persons living of one Age, are added to the Number of Persons living of the other Age, that the Sum produced, is the Number of Chances on both Lives; than it is to say, if the Number of Persons living of one Age, are multiplied by the Number of Persons living of the other Age, that the Sum produced, is the Number of

Chances on both Lives. The Chances on the Life of one Person named, are the Persons living of the same Age as the Person named; and if the Person named, suppose *A.*, as in the Doctor's Instance, be of the Age of 18, the Number of Persons living, and consequently the Chances on his Life will be 610; to this if we add the Life of a second Person, suppose *B.*, of the Age of 35, the Number of Persons living of that Age are 490, and those are the Chances on the second Life. Admitting then, as we do admit, that 610 are the Chances on the Life of *A.*, and 490 the Chances on the Life of *B.*; if these two Sums are added together, the Produce will be 1100, and if they are multiplied one into another, 610×490 is 298900; and the Consequence is as easy and as just, therefore 1100 are the Chances on both Lives, as is the other Consequence; therefore 298900 are the Chances on both Lives: Nay the Advantage is manifestly on the side of the former, because 1100 is the real Number of the Persons living of both Ages, and 298900 is not so, and the Number of Persons living, is admitted to be the Measure of the Chances of Vitality on each of their Lives, when singly taken.

And as I see no just grounds in the Nature of the Life of Man, to multiply these Numbers or to form these Lines into Areas, so neither do I see any occasion for making such Multiplications; for if these Numbers and these Lines added together only, will ascertain what is intended to be done by the Numbers multiplied, or by the supposed

supposed Areas; that is, will discover what are the Chances of survivorship on these two Lives; these Transformations are unnecessary. But if these Numbers, which are the real Numbers of the Chances on these Lives, and these Lines which truly represent these Lives or their Chances, will not make such discovery; it is very unlikely that these multiplied Numbers, which are not the real Number of Chances, or that these Areas, which do in no sort resemble those Lives, should give us any Assistance towards such discovery: And the Thing perhaps, is not discoverable by either of the Ways.

To pursue this Matter a little farther in the Doctor's Instance and in his Manner; the Lives of *A.* and *B.* being proposed, the Chances on those Lives are required after 8 Years. The Number of Chances for *A.* is 610, and for *B.* 490; and in 8 Years there are 50 dead of the Age of *A.*, and 73 of the Age of *B.* There are in all $610 + 490$, or 298900 Chances; of these there are $50 + 73$, or 3650, that they are both dead: And as 298900 is to $298900 - 3650$, or to 295250, so are the Chances that one or other of them lives 8 Years; now this I deny. In this Matter, where the Chances of Vitality at the End of 8 Years, are to be determined and finally adjusted, by deducting out of such first and original Chances, the Number of Chances of Mortality arising in those 8 Years; it is the very same Thing, and the Error will be as fatal, whether a greater Number of Persons are supposed to be living, than really are living, or a

fewer Number supposed to be dead than really are dead; or if the Number produced on such a Computation, does not give us the right Number of the Chances of Mortality arisen in that Time, or not the right Number of the Chances of Vitality sunk in that Time. The Chances of Vitality on these two Lives, in the first Year of these 8 Years, are 298900; and the Chances of Mortality arising in those 8 Years, are 3650; therefore $298900 - 3650$, or 295250, are the Chances of Vitality left at the End of those 8 Years, as the Doctor states it. Now this Sum of 295250, is not the Number of Chances that both are living, at the Beginning of the 9th Year; for the Number of Persons then living of the one Age will be 560, and of the other Age 417; and $560 + 417$ makes no more than 233520, which are therefore the Chances that both are living. This will easily be admitted to me, since the Doctor's Position states it in that Manner, *viz.* not that 295250 are the Chances of both being living, which may be no more than 233520, but the Chances that one or other of them is living. Now if this Sum of 295250 is solely the Number of Chances, that one or other of them is living at the End of the eighth, and the Beginning of the ninth Year, then some Chances of Vitality have been sunk during these eight Years, of which no Notice is taken, or some Chances of Mortality have arisen within that Time, which are omitted out of this Account; for 3650, is evidently not the whole Number of Chances sunk on these two Lives within

within such Time. During these 8 Years, and indeed in the first Year, and in every other subsequent Year, there are double Chances of Mortality arise, *viz.* that both may die, and also that one or other may die; but 3650 are no other, or more than one or other of these Chances, either that they both die in those 8 Years, or the Chances that one or other dies in that Time, but does not include both Chances. And it is one and the same Thing, as I observed before, whether the Number of Persons living at the Beginning of a determinate Term, give us too many Chances of Vitality, or the Number of Persons dying within such Term, give us too few Chances of Mortality; in either Case, the Sum total of the Chances of Vitality at the Foot of the Account, will be a wrong one.

If it be urged, that this Sum 3650 contains the double Chances of Mortality, I mean as well the Chances that both are dead; as the Chances that one or other is dead, we must of necessity say, that the Sum 298900 contains likewise the double Chances of Vitality, I mean as well the Chances that both are living, as the Chances that the one or other is living; otherwise we set Chances against one another, which are not opposite one to the other. Now if this should be right in the first Instant, and 298900 should contain the double Chances of Vitality, yet in the very next step we take, we are sure to be wrong: For at the End suppose of 8 Years, the Sum 295250 is not the Number of the double Chances of Vitality, for that as we have seen before

is $569 + 417$, which makes no more than 233520, and 295250 is solely the Number of Chances that one or other is living, and is so stated by the Doctor himself; and the Case will be the very same at the End, even of the first Year, for the Sum arising will be the Number of Chances, that one or other is living, and not that both are living.

The Truth is, this Number 3650 can enumerate only, and is intended to enumerate only, the Chances that both are dead, for it is made up by multiplying the Number of Persons dead of one Age, by the Number of Persons dead of the other Age. From hence it appears, that no Notice is taken, or any Account made, of the Chances that one or other is dead in the 8th, or any precedent Year, of these two Lives; and if we should admit, which is making the best of it we can make; that, whilst it continues an even Chance that *A.* and *B.* are both living, for so many Years the Chances that both are living, and the Chances that one or other is living, are well set in Opposition to the Chances that both are dead, and the Chances that one or other is dead; yet from the Time when the even Chance commences, that one or other of the two is dead; from thence undoubtedly we must increase the Chances of Mortality, or decrease the Chances of Vitality, for it is obvious and certain, that from that Time some Chances of Vitality are sunk and gone, and the Chances of Mortality not grown less.

The

The Hypothesis therefore is mistaken, in one Respect or other; either in supposing that the Number of Persons living of one Age, being multiplied by the Number of Persons living of another Age, will give us the Chances of Vitality on two Lives of those Ages, for in so doing it supposes, that the Chances of Vitality continue through all the Years of the possible Life of both Persons, which most certainly is not the Case; or it is mistaken in supposing, that the Number of Persons of one Age dead within a determinate Time, being multiplied by the Number of Persons of the other Age dead within the same Time, will give us all the Chances of Vitality sunk in that Time; for in such Way of computing we have solely the Chances that both are dead, when there is a farther Chance that one or other is dead: Or lastly, it is mistaken in supposing, that the Sums produced by such Multiplications, may properly be set one against the other, whereas the Chances contained in the Sums so produced, are not opposite to one another: And since these Suppositions are made without any Grounds or Foundation, nay, if any one of them only were so made, it is fatal to the Hypothesis; it follows that the Hypothesis is wrong.

Before I conclude this Head, I will make one Observation more; and as it has less Intricacy in it than some others which I have before insisted on, it may perhaps for that Reason more plainly prove, that we shall certainly miscompute the Chances of the Duration of two Lives, if we

proceed upon the Doctor's Hypothesis, taking it in which Sense we will, and supposing it capable of two Senses. He undertakes, as I have already observed, upon the Lives of *A*. and *B*., supposed to be of the Ages of 18 and 35, to state what are the Chances that one or other is living at the end of eight Years, and what are the Chances that the elder is dead in that Time living the younger, and what are the Chances that the younger is dead living the elder; and this he does, both by the Way of Numbers and of Lines. In computing these Chances by Way of Numbers, the Doctor keeps closely enough to his multiplied Numbers, though by the by, his Expression in one Place is very deficient; but when he states the Chances by Way of Lines and Figures, he gives us the Line *A B*, as representing the Number of Persons living of the younger Age, and *A C*, as representing the Number of Persons living of the elder Age, and tells us, that those two Lines multiplied into one another, will produce the Parallelograms *A B C D*, and that such Parallelogram will represent the Number of Chances on the Lives of both Persons. After this he drops his Parallelogram, and we find nothing throughout but Rectangles, or Letters representing Rectangles; and the Proportions afterwards stated, of which there are many, are all between Rectangles of one Kind or other; except in one Instance, and there it is between an Area and a Rectangle. Now a Rectangle, is no more or other than two meer Lines joined together, in such a Position

as to form a right Angle; and if a Rectangle represents the Number of Persons, and the Chances on two Lives, it must be on this Ground and for this Reason, because the Number of Persons living of one Age, being added to the Number of Persons living of the other Age, gives us the Number of Persons living, and the Chances on the Lives of both Ages. Whether or no the Doctor intended here, to prescribe or to make use of two Methods for calculating these Chances, *viz.* by Numbers and Lines added together, and by Numbers and Lines multiplied one into another, I don't know; but this I do know, whether he meant them for two Rules or did not mean them for two, that neither of them is a right one, to ascertain these Chances.

To show this, I have made a Computation of the Time to which 'tis an even Chance that one or other of these two Lives, given by the Doctor may live, and this I have done in both Ways; I mean by Numbers multiplied, or by Lines form'd into Parallelograms, which, in Effect, is one and the same; and by the Numbers singly taken and put together, or by Lines singly taken and form'd into a Rectangle, which also, in Effect, is one and the same. In the first Way of computing, the Term to which one of the two may live, and beyond which neither of them will live, is a Term between 40 and 41 Years, and in the other Way the Term is about 39 Years. Now either of these Terms seems to be near the Mark, and yet the Methods by which the Computations are made are certainly both of them

them wrong; at least, neither of them will fit all Instances. If the Lives given are of great Inequality in Age, the one suppose of 10, and the other of 60, or any Age beyond that, if we use the first Method of Computation, *viz.* by Numbers and Lines multiplied one into another, the Term to which the two Lives will be equal will be no longer than the Term to which the youngest Life by itself is equal: And if the Persons are of equal Age, and we use the latter Method, *viz.* by Numbers and Lines added together, the Case will be the same, and the Term for any one of the Lives will be as long as for both Lives. The Persons living of the Age of 10 are 661, and of the Age of 60 are 242: $661 + 242$ is 159962; in 41 Years, or thereabouts, one Moiety, or 331, of the Persons of the younger Age are dead, and all those of the elder Age: $331 + 242$ is 80082, which is just about a Moiety of the Whole or first Number; that is, in so many Years as the even Chance is that one is dead, it will be an even Chance that both are dead. If the Persons named are both of the Age of 10, the Persons living are 661; and if both Numbers are put together, the Whole will be 1322, in 41 Years, or thereabouts, one Moiety of each will be dead, that is, a Moiety of the Whole will be dead, and the Term on both the Lives the same only as on any one of the single Lives. From whence I conclude, and there I rest the Matter, that neither of the Doctor's Methods, if he really intended them for two, will give us the Term to which two Lives are equal.

Having

Having consider'd Mr. *Richards's* Tables for the Value of Annuities on two Lives and the Survivor, and produced such Errors in them as sufficiently show that they are false; and having demonstrated, as I apprehend, that the Rule by which they were framed must necessarily be wrong: We will now proceed to examine the Tables made for the Value of Annuities on three Lives and the Survivor. Upon View of them it will appear that they have the same Errors as the former Tables, with an Addition of some others more extravagant and more gross: And these too, as far as I can judge, must have their Foundation, in some Measure at least, in the Method prescribed for forming them, or in the Hypothesis on which such Method is founded.

I have before observed that these Tables for the Value of Annuities on three Lives and the Survivor make an Annuity for three such Lives, each of 12 Years Age, and at 5 *l. per Cent.* Interest, equal almost to the Inheritance; for the Value there given is 19-54, and the Inheritance at that Interest is no more than 20: Which Value 19-54 is equal to a Term of 78 Years. To this Observation I add another here, that by these Tables such an Annuity, and on the same Lives, at the Rate of 7 *l. per Cent.* Interest, is made 13-46, which is equal only to a Term of 41-2-00, which is 36 Years less than an Annuity on the same Lives at 5 *l. per Cent.*; which is to say, in plain Language, that if one be the Rate of Interest *A. B.* and *C.* will all die in 41 Years and an half, but if another be the Rate, one or other

other of them will live 78 Years. Now this Value 13-46, and the Term 41-2-00, is certainly as much below *Par* as the Value and the Term at the Rate of 5 *l. per Cent.* is above *Par*. I don't mean to say here, that an Annuity for three such Lives, at 7 *l. per Cent.* or at any other Rate of Interest, is equal to a longer Term than 41 Years and an Half, for by Accident that may be something near the Truth; but when I say this Value and Term are below *Par*, I mean below the State which they should be when compared with the Values and Terms at other Rates of Interest, and according to his Method of Calculation in other Instances.

To make this Error more visible, though 'tis of so enormous a Size that it cannot well miss being seen, I have drawn a short Scheme; giving the several Rates of Interest, the Value which Mr. Richards's Tables put upon an Annuity for three Lives of 12 Years Age, and the Term to which such Values are respectively equal, drawn by me, but from his Tables for the Value of Terms for Years.

The Value and Term of an Annuity for three Lives and the Survivor, all of ten Years Age.	Interest.	Value.	Term.
	4	23-01	65-0
	5	19-54	78-0
	6	16-19	61-0
	7	13-46	41-2
	8	12-29	56-0

N. B. 13-46 should be
read 13-44.

The

The Fault then in this Table, which evidently shows itself in this Scheme, is a Want of that Proportion which the Term in an Annuity for three Lives at one Rate of Interest ought to bear to the Term in such an Annuity at other Rates of Interest, particularly in the Instances given at the two Rates of 5 *l.* and 7 *l. per Cent.*; admitting it at present to be right, though 'tis undoubtedly not right, that the Term in such an Annuity can vary as the Rate of Interest varies. The Term to which such Annuities are equal, at the Rates of 4 *l.* 6 *l.* and 8 *l. per Cent.* are 56, 61, and 65 Years; and this is such a Variation in the Terms as would and must arise meerly from the Table of the Value of Annuities on a single Life, because in that Table on a single Life an Annuity at 4 *l. per Cent.* is stated as equal to a longer Term than one at 5 *l. per Cent.*, and one at 6 *l.* as longer than one at 8 *l.*; and in such a Degree as might produce this proportionable Variation in the Term to which three Lives at these Rates of Interest are respectively made equal. But how comes it to pass, or how can it be right, that an Annuity for three Lives, at 5 *l. per Cent.*, is equal to a Term longer by 13 Years than such an Annuity at 4 *l. per Cent.*, for the one is equal to a Term for 78 Years, and the other only for 65 Years; when an Annuity for one Life at 5 *l. per Cent.* is equal to a shorter Term than such an Annuity at 4 *l. per Cent.*; for the first is equal to a Term of 29-1-44, and the latter to a Term of 29-3-00? On the other Side, How comes it to pass, that an Annuity for

for three Lives, at 7 *l. per Cent.* is equal to a Term shorter by 15 Years than the like Annuity at 8 *l. per Cent.*, when an Annuity for one Life at 7 *l. per Cent.* is equal to a longer Term than the like Annuity at 8 *l. per Cent.*? Whence these particular Errors have arisen may not be very clear, but they are so apparent and so gross that no one can avoid seeing and acknowledging them now they are pointed out, or can in any Sort defend.

But whencesoever these Errors may have arisen, or whoever was the Author or the Cause of these extravagant Estimates of the Value of these Annuities at the Rates of 5 *l.* and 7 *l. per Cent.* Interest, which it may be not very material to know; I have computed what Number of Lives of 12 Years Age an Annuitant must have, whose Annuity for their Lives shall be in Value 19-54, and equal to a Term of 78 Years. To have a Number of Lives of 12 Years Age which shall be equal to 78 Years, 'tis obvious we must have so many Lives at least that the Chance shall be an even one that one of the Number lives to be 90 Years old. If we look into the *Breslaw* Table, it appears, that of 646 Persons of 12 Years Age eight only, that is one in 80, lives to 90 Years Age; and from the Bills of Mortality for *London*, or my Extract, we find, that of the Persons above ten Years old dying in a Year; one in an 100, by a Medium, is of the Age of 90: So that, instead of three Lives of 12 Years Age, an Annuitant must have 80 by one Rule, and by the other he must have an

100 Nominees, to have an even Chance that one of them lives to the Age of 90 Years.

This proves beyond Contradiction, as I apprehend, that this Table for the Value of Annuities for three Lives of 12 Years Age, and at 5 *l. per Cent.*, cannot be right; and I assert farther, that this Table, form'd by Mr. *Moirre's* Method, is wrong in every other Instance, and at every Rate of Interest, unless meerly by Accident; and this, I think, will appear very evident from the Case I shall state, and from this Author's own Tables for the Value of Annuities on a single Life, on two Joint-Lives, and on three Joint-Lives, which are the Foundation on which he computes the Values of Annuities for three Lives and the Survivor.

We will suppose then, that N^o. 1. has an Annuity, purchased after the Rate of 6 *l. per Cent.*, for the Lives of *A. B.* and *C.* and the Survivor, all of 12 Years Age; and the Value of this Interest is the Point of Enquiry. We will suppose also, that N^o. 2. N^o. 3. and N^o. 4. have each of them a separate Annuity in this Manner: N^o. 2. for the Joint-Lives of the same *A. B.* and *C.* to determine on the Death of the Nominee who shall die first; and to avoid Repetitions and Intricacies, we will suppose *A.* to die first, *B.* second, and that *C.* becomes the last Survivor; N^o. 3. for the Joint-Lives of *B.* and *C.* to commence on the Death of *A.*, and to determine on the Death of *B.*; and N^o. 4. for the Life of *C.* the Survivor, to commence on the Determination of the two precedent Estates. We will suppose,

pose, in the third Place, that N^o. 5. has a like Annuity for the Joint-Lives of the same *A. B.* and *C.*, with this Priviledge annexed to his Grant, that on the Death of *A.*, supposed to die first, he shall have a new Grant for two Joint-Lives, and be at Liberty to exchange the Lives of *B.* and *C.*, and in their Stead to nominate two other Persons, suppose *D.* and *E.*, who at the Time of Nomination shall be 12 Years old, and no more; and on the Death of *D.* or *E.* shall have another Grant with the like Liberty to exchange a Life, and instead of the Survivor of them to insert another Person's Life, suppose *F.*, who at the Time of such Insertion shall be 12 Years old, and no more.

To apply this in a particular Instance, and upon Mr. *Richards's* Tables for the Value of these several Annuities; the Matter stands thus. The Interest belonging to N^o. 1. being an Annuity for three Lives and the Survivor, all of twelve Years Age, as stated in these Tables, is in Value 16-19; which is equal to a Term of 61 Years: And this, I say, is not the true Value. The Interest of N^o. 2., being an Annuity for the Joint-Lives of *A. B.* and *C.*, they being 12 Years old, is in Value 9-50, equal to a Term of 14-2-00: The Interest of N^o. 3., being an Annuity for the Joint-Lives of *B.* and *C.*, they being 26 Years old, or thereabouts, at the Death of *A.*, that is, when the Interest in their Lives commences, is in Value 9-60, equal to a Term of 14-3-00: And the Interest of N^o. 4., being an Annuity for the Life of *C.* the Survivor, he
being

being about the Age of 41 at the Death of *B.*, that is, when the Interest in his Life commences, is in Value 10-90, equal to a Term of 18-0-00: Which three Terms 14-2-00, 14-3-00, and 18-0-00, make in all 47-1-00; which is in Value about 15-60. The Interest of N^o. 5. in the Joint-Lives of *A. B.* and *C.*, they being 12 Years old, is in Value 9-50, equal to a Term of 14-2-00; his Interest in the Joint-Lives of *D.* and *E.*, they being 12 Years old, is in Value 11-13, equal to a Term of 18-3-00; and his Interest in the Life of *F.*, he being likewise 12 Years old, is in Value 13-36, equal to a Term of 27-3-00: Which three Terms, 14-2-00, 18-3-00, and 27-3-00, make in all 61-0-00; and are in Value 16-19.

Now 'tis certain and obvious, that the Interest of N^o. 1. is just equal to all the three Interests of N^o. 2., N^o. 3., and N^o. 4., put together; for that the Interests of N^o. 1. and N^o. 2. are supposed to commence at one and the same Time, and the Interest of N^o. 1. will continue just as long as the Interests of N^o. 2., N^o. 3., and N^o. 4., or any of them, and no longer; and they must all determine at one and the same Time, *viz.* upon the Death of *A. B.* and *C.*, and the Survivor of them. 'Tis as certain, and as evident, that all the three Interests of N^o. 2., N^o. 3., and N^o. 4., put together, are not equal to the single Interest of N^o. 5. The Interest of N^o. 2. in the Joint-Lives of *A. B.* and *C.* is plainly the same as the Interest of N^o. 5. in the same Lives; but the Interest of N^o. 3. in the

Joint-Lives of *B.* and *C.* is evidently not so valuable as the Interest of *N*^o. 5. in the Joint-Lives of *D.* and *E.*, nor the Interest of *N*^o. 4. in the Life of *C.* the last Survivor near so valuable as the Interest of *N*^o. 5. in the single Life of *F.*; for that the Interest of *N*^o. 3. in the Joint-Lives of *B.* and *C.* commences at the same Time as the Interest of *N*^o. 5. in the Joint-Lives of *D.* and *E.* and at the Commencement of such Interests *D.* and *E.* are supposed to be 12 Years old, and no more, and *B.* and *C.* are supposed to be then 26 Years old, or, in Fact, are so many Years older than 12 as have incurr'd during the Joint-Lives of *A.* *B.* and *C.*: And since this Way of Reasoning is as strong, and has a greater Effect, when a Comparison is made between the Interest of *N*^o. 4. in the Life of *C.* the Survivor and the Interest of *N*^o. 5. in the single Life of *F.*, there being near 30 Years Difference in their Ages. From hence 'tis plain, as plain as Numbers and Figures, and a Computation made on Facts evidently and notoriously true, can make it, that the Interest of *N*^o. 5. is greater than the Interests of *N*^o. 2. *N*^o. 3. and *N*^o. 4. put together; that the Interest of *N*^o. 1. is the same as the Interests of *N*^o. 2., *N*^o. 3., and *N*^o. 4., put together; consequently the Value 16-19, and the Term 61-0-00, being the Value and the Term belonging to *N*^o. 5. cannot be the Value and the Term belonging to *N*^o. 1; that is, cannot be the Value and the Term of the Annuity for these three Lives and the Survivor; which was the Thing to be proved. From whence, since this Table

for

for the Value of these Annuities was all constructed by the same Rule and in the same Manner, it follows necessarily, that the whole of it is false.

And as this Error visibly runs through the whole Scheme, and therefore must arise from a Defect in the Rule, so does that which I mention'd before, *viz.* the great Deviation of the Value of these Annuities at the Rates of 5 and 7 *per Cent.*, from the Value of them at other Rates. Thus much, at least, is certain, that this does not arise solely from Mr. *Richards's* Tables for the Value of Annuities for three Joint-Lives, for two Joint-Lives, and a single Life; for if he had contented himself with a Method arising from thence, and not gone in Search of one in the Clouds, the Tables might have had an Appearance of Truth, or to be sure had avoided this gross Deviation. I will take Leave then to vary the Case just put, and suppose this Annuity of N^o. 5. to have been granted at the Interests of 5*l.* and 7*l. per Cent.*: And let us examine to what Term such an Annuity would be equal, and what would be the Value of it, if we consult only those three Tables of this Author, and go no farther. The Interest of N^o. 5. in the Joint-Lives of *A. B.* and *C.*, they being 12 Years old, is in Value 10-00, equal to a Term of 14-1-00; his Interest in the Joint-Lives of *D.* and *E.*, they too being 12 Years old, is in Value 12-05, equal to a Term of 18-3-00, and something above; and his Interest in the Life of *F.*, he being of the same Age of 12 likewise, is in

Value 15-23, equal to a Term of 29-1-00, and something above: And these three Terms, 14-1-00, 18-3-00, and 29-1-00, make in all a Term of 62-1-00, in Value 19-05, or thereabouts. If we suppose the Grant to N^o. 5. to have been at 7 *l. per Cent.* and compute his Interest in such Grant in the same Manner as we have before done on the Grant at 5 *l. per Cent.*, it will come out to be in Value 13-97, equal to a Term of 57-01-00. The Interest then of these several Grants to N^o. 5., at these several Rates, standing thus; *viz.* at 5 *l. per Cent.*, the Term 62-1-00, in Value 19-05; at 6 *l. per Cent.* the Term 61, in Value 16-19; and at 7 *l. per Cent.* the Term 57-0-00, in Value 13-97; where Computation is made on Mr. *Richards's* three Tables, without entering into any farther Calculations: And since the Interest for these three Lives, intended to be three concurrent Lives and the Survivor, comes out to be at 5 *l. per Cent.* the Term 78 Years, in Value 19-54; at 6 *l. per Cent.* the Term 61 Years, in Value 16-19; and at 7 *l. per Cent.* the Term 41-2-00, in Value 13-44; where the Calculation is made in the Manner prescribed by Mr. *Moiure's* Table: 'Tis evident from hence, if Mr. *Richards* had relied on his own Tables, and omitted the several Additions, Substractions, and Combinations of Values, which he was led into by pursuing that Rule, that he would have avoided these extravagant Variations, and had given us the Value of Annuities for some Sort of combined Lives, though

though not for three concurrent Lives and the Survivor: And that the Values set down in these Tables as the Values of Annuities for three such Lives, which will correspond with no three Lives in Nature, which Way soever combined, was the Produce of this imaginary mathematical Rule.

I may go farther, and affirm, if these Tables for the Value of Annuities for three Joint-Lives, two Joint-Lives, and a single Life, were rightly form'd, or if any such could be rightly form'd, which is a Matter of much Difficulty, specially in Lives of unequal Ages, that from thence, without any farther Calculations, we might very easily and readily form a Table for the Value of Annuities for three concurrent Lives and the Survivor: For 'tis certain, if a Computation be made in the Manner I have computed the Interest of N^o. 2., N^o. 3., and N^o. 4., the Produce will be the true Value of an Annuity for three such Lives; it being evident that the Interest of N^o. 1., which is an Interest in three concurrent Lives and the Survivor, is equal to the Interest N^o. 2., N^o. 3., and N^o. 4., put together; for they are the Value of an Annuity for three Joint-Lives, for the two joint surviving Lives, and for the single surviving Life: But then such two joint surviving Lives, and such single surviving Life, must be computed equal to such a Term only, as their two Joint-Lives and the single Life, respectively, are equal to at the Time when the Interest in them respectively takes Place.

Besides this Defect in these Tables, on the Value of Annuities for three Lives of 12 Years Age, and at the Rate of 5 *l.* and 7 *l. per Cent.*, the same Fault occurs again, in another Period of Life, and at a different Rate of Interest, *viz.* in three Lives of 32 Years of Age, and at 8 *l. per Cent.* Interest. If we look into these Tables for the Value of an Annuity for three such Lives, it appears, that the Term to which such Lives are equal at the Rates of 4 *l.*, 5 *l.*, 6 *l.*, and 7 *l. per Cent.* Interest, is in some tolerable Proportion, that is in such a Proportion as is answerable to, and is the necessary Consequence of, the Term of a single Life being longest at the lowest Rate of Interest, and being the shortest at the highest Rate of Interest, that being the Manner in which the Value of Annuities for single Lives is calculated; according to which proportional decrease we find that three Lives of the Age of 32, in the Estimate of them given by these Tables, are equal to a Term of 44 Years at 4 *l. per Cent.*, and at 5 *l.*, 6 *l.*, and 7 *l.*, are equal to a Term gradually less: But at 8 *l. per Cent.* they are made equal to a Term of 50 Years, wanting only a Decimal, or about half a Year, so that the Term corresponding to the Value of three Lives, in this Period of Life, and at this Rate of Interest, is extended much beyond the Term to which any such Lives at any other Rate of Interest are equal.

I must add here, that I have made a Calculation of the Value of Annuities for those three Lives at 8 *l. per Cent.*, according to the Method prescribed

prescribed by Mr. *Moiivre*, and find that Mr. *Richards* has pursued it, except only that he has committed a small Mistake, and has made the Value of those Lives 12-18, which should be 12-16, which last Value is equal to a Term of 49 Years and an Half. I have likewise calculated to what Term three such Lives at this Interest would be equal, if Mr. *Richards* had contented himself with his own three Tables, *viz.* those for three Joint-Lives, two Joint-Lives, and a single Life; and the Term to which they would be equal is 41 Years, or thereabouts, which is a Term bearing a tolerable Proportion to the Term to which those Lives, at the other Rates of Interest, are equal: So that the Deviation here in three Lives of 32 at 8 *l. per Cent.*, must be placed to the same Account, as the like Deviation in three Lives of 12 at 5 *l.* and 7 *l. per Cent.* was placed, that is to the Defect in Mr. *Moiivre's* Method of calculating the Values of Annuities for such Lives.

It is unnecessary, and would be too tedious, to enter into all the Errors of this Table for the Value of Annuities for three Lives, but I cannot forbear taking Notice of another; *viz.* a Want of a Decrease in the Value of Annuities for these Lives and in the Term to which such Lives are equal in the different Periods of Life, in a Proportion answerable to the Increase of Age. I choose to mention this Fault, because it runs through, and goes to the whole Performance; and undoubtedly take its Rise from the Hypothesis itself, and the Foundation on which

all the Tables are grounded. I observe then; of Annuities for three Lives of 12 Years Age at 4*l. per Cent.* the Value is 23-1, equal to a Term of 65 Years, and of Annuities for three Lives of 32 Years Age at the like Interest, is 20-63, equal to a Term of 44 Years: Of Annuities for three Lives of 12 Years Age at 8*l. per Cent.*, the Value is 12-29, equal to a Term of 56 Years, and of Annuities for three Lives of 32 Years Age, at the like Interest, is 12-18, read rather 12-16, equal to a Term of 50 Years, or near it. Here then in one Case, where Interest is computed at 4*l. per Cent.* the Value is sunk 2-38 Decimals, and the Term decreased twenty Years by an Advance of 20 Years in the Age, which is very extraordinary, specially in the youngest Part of Life; but 'tis much more so, that in the other Case, where Interest is computed at 8*l. per Cent.*, in the like Advance in Age, and in the same Period of Life, the Value is decreased 11 Decimals only, and the Term no more than six Years. But for the Smalness of the Decrease in this Period of Life, at this last Rate of Interest, the Author has taken Care to make a sufficient Amends in the next ten Years of Life; for he has given the Value of Annuities for three Lives of the Age of 42-11-52, equal to a Term of 33-2-00, that is, in these ten Years he has sunk the Value 64 Decimals, and the Term sixteen Years and an half.

This Excess of the Decrease in the Value and in the Term, in these different Periods of Life,

and

and at this Rate of Interest, the Decrease in one being so much too great, and in the other so much too small, is so gross and so obvious, that I am surprized that Mr. *Richards*, on the meer View of his Tables, did not see it; for there it stands thus; three Lives Age 12, Value 12-29; Age 22, Value 12-27; Age 32, Value 12-18, read 12-16; and Age 42, Value 11-52. But if an Inspection only would not have discover'd these Errors, 'tis impossible he could have miss'd seeing them if he had reduced these Values to the Terms corresponding, which he might easily and readily have done by his own Table; and the Terms stand thus. Value 12-29, the Term 56; Value 12-27, Term 55; Value 12-16, Term 49-2; and Value 11-52, Term 33-2: Where it appears that the Decrease in the first Period of ten Years is in Decimals 2 only, and in the Term one Year; and in the last Period of ten Years the Decrease is in Decimals 64, and in the Term 16 Years.

If any one is not convinced, on the View of the several Values and Terms, that the Decreases in these Instances are wrong; what follows will demonstrate it. From this Author's Table for the Value of Annuities on a single Life at 8 *per Cent.*, it appears that the Decrease in the Term on a single Life answering to the Advance of the Age of such Life from 22 to 42, is only three Years and one Third; for a Life of 42 is equal to a Term of 15 Years and above, and a Life of 32 is not equal to a Term of 18 Years and

and an half; and therefore the Decrease for three such single distinct Lives, when all such Decreases are put together, can amount to no more than ten Years. Now it may easily be true, and most certainly is true, that the decrease of the Term on three combined Lives of that Age, on such Advance in the Age, shall be less than the decreases in three such single distinct Lives, when such decreases are put together, that is shall be less than 10 Years; but on the other side it can never be true, that the decrease of the Term on the three combined Lives on such Advance in the Age, should be greater than the decreases in three such single distinct Lives, when such decreases are put together, that is should be more than 10 Years; unless it should be true that the Whole may be bigger than all the several Parts which make up the Whole; for the Whole in this Case, that is the decrease on the three Lives combined has no other Parts, and can have no other Parts, besides the several decreases on the same three Lives singly and distinctly taken; and in Truth has not the whole even of them; From whence it necessarily follows, that the decrease of the Term, stated here at 16 Years, and the decrease of the Value of the Term, stated here at 64 Decimals, are both false.

The decrease in the other Period of ten Years of Life, is as ill fixed on the other side, and is proved to be so, by an Argument as strong and as plain as in the former Case. This decrease, in this Period between 12 and 22, in the Way
of

of stating by the Value, is two Decimals only ; and in the Way of stating it by the Term, is only one Year ; yet in his Table for the Value of Annuities on single Lives, a Life of 12 is in Value 10-67, equal to a Term of 25 Years exactly ; and one of 22 is in Value 10-17, equal to a Term short of 22 ; so that the decrease on a single Life from 12 to 22, is in Value 50 Decimals, and in the Term three Years and above ; so that this decrease of two Decimals in the Value, and in the Term one Year only in three Lives, is as absurdly and preposterously out of all Bounds of Proportion on one side, as 64 Decimals and a Term of sixteen Years is on the other side ; for if the decrease of the Term upon each Life of three when singly taken, is three Years and above the decrease upon all the same three Lives, when combined, must be at least as much as the decrease of one of the three Lives, that is, must be three Years and above ; but is made here one Year only : That is in short, in one Period of Life, the decrease on the three Lives combined, exceeds the decrease on all the three Lives singly taken and put together ; and in the other, the decrease on all the Lives combined, is not so great as the decrease on one of the three Lives singly taken.

On this Excess in the decrease of the Term and Value of an Annuity, according to the Advance in Age, which in one shape or other, appears almost in every Period of Life, and at every Rate of Interest ; I observe that it labours under two material Defects, neither of which is justified

justified by any Thing in the Life of Man. The one is, that this decrease in the Term, and the Value corresponding, is greater, in some Instances at least, in a younger Period of Life than in an older; and the other, that this decrease, for the same Periods of Life, is not uniform through all the Rates of Interest, but varies as the Rate of Interest varies; so that we have a different decrease of the Term, for every different Rate of Interest, for one and the same Period of Life; and how far this difference is carried in some Cases we shall see presently.

That the Chance of Mortality increases as the Life of Man advances in Age, must be, and is admitted on all Hands; the necessary Consequence of which is, that the Term to which the Lives of three Persons in an advanced Age of Life is equal, must be shorter than the Term to which the Lives of three Persons in a younger Age of Life is equal; and yet in these Tables, no Regard is had to this Circumstance in Life, but on the contrary, as if the Rule were the Reverse, and the Chance of Mortality the greatest in the youngest Periods of Life, the decrease in the Term is there made the largest. An Annuity for three Lives of 12 Years of Age, at *5l. per Cent.* is by these Tables made equal to a Term of 78 Years; one for three Lives of 22 Years of Age, equal to a Term of 58 Years; and one for three Lives of 32, equal to a Term of 43 Years; so that the decrease for the first Period of ten Years, is twenty Years, for the second is fifteen Years,
and

and for both put together 35 Years. An Annuity for three Lives of 52 Years of Age, is made equal to a Term of 28 Years or near it; one for three Lives of 62 Years of Age, equal to a Term of 19 Years, something over, and one for three Lives of 72, equal to a Term of 11 Years and a half, or thereabouts; so that the decrease for the first of these Periods of ten Years, is 9 Years and a little over, and for the second, is near 8 Years; and for both put together about 17 Years. Here we see that this decrease grows gradually less, when it should grow gradually greater, and in the twenty Years of older Life, is not half so much as in the like Number of Years of the youngest Life; and this Want of a proper decrease, a decrease proportionate to the Advance in Age, shows itself in every other Rate of Interest in some Degree, but not always for the same Period of Life; which makes an Additional Irregularity in these Tables.

As to the second Defect which I mentioned before, *viz.* that this decrease in the Term, and in the Value, is different, for every different Rate of Interest; I have affirmed, that it appears in some Periods of Life, but I believe the Fact is so in all Periods, because I think the Foundations on which the whole Scheme is framed, makes it necessary that it should be so: However, we will take the Period of ten Years of youngest Life, and see how this decrease stands there. We have already observed, that at 5 *l.* *per Cent.* the decrease for this Period is twenty Years,

Years, and at 8*l. per Cent.* only one Year; and it appears by the Tables, that at 4*l. per Cent.* it is about six Years, and at 6*l. per Cent.* about four Years: But at 7*l. per Cent.* we have something more extraordinary than any Thing which has as yet occurred. At all other Rates of Interest, there is some decrease in the Value and the Term in each Period of Life, and undoubtedly some there ought to be, in Regard of the Chance of a less Duration of Life, must lye on the side of Lives as they grow older; and in particular, we have seen that at one Rate of Interest, this decrease for the first Period of Life, is no less than a Term of twenty Years; notwithstanding which, at 7*l. per Cent.* Interest, and for the same first Period of Life, we have not only a want of any decrease, but we have instead of it, an increase of the Value and the Term: These Tables giving us for the Value of Annuities for three Lives of 12 Years of Age, 13-46; and for the Value of Annuities for three Lives of 22 Years of Age, 13-58; the one equal to a Term of 41 Years only, the other to 43 Years.

These Tables I say, give us 13-46 as the Value of Annuities for three Lives of 12 Years of Age, and I observe that they give us the very same for the Value of three Lives, where two are 12, and the third is 22 Years of Age; but upon a Computation made in Mr. *Moiivre's* Method, I find that 13-46 is not the true Value for either, but the first is 13-44, equal to a Term short of 41 Years; and the second is

13-60,

13-60, equal to a Term of 44 Years, or very near it. I believe that these Produces for the Value of Annuities for these two Sets of Lives, if the Framers of the Tables found them to be the same as I have done, might a little puzzle him, and perhaps mistrusting his own Calculation, rather than his Instructor's Mathematical Rule, he might give us the Values as they stand in the Tables; but whether he did it voluntarily or did it inadvertently is not material, since he has not perfectly cured the Fault; for these Values, take the one or the other, are both erroneous; since it is as untrue that the Value of both these Annuities are one and the same, as it is that the latter is greater than the former, though the Error in both Cases may not be of equal Magnitude.

We have then here a Set of three Lives, one of which Lives is of a greater Age than 12, and yet the Value of Annuities on such Set of Lives, is greater than the Value of Annuities for a Set of three Lives, where they are all 12 Years old and no more; but besides this, we have another Set of Lives where two are of 12, and the third of 52 Years of Age; the Value of which is set down 13-90, equal to a Term of 53 Years and above; we have a second Set of Lives where one is of the Age of 12, and two of the Age of 22, the Value of which is stated at 13-62, equal to a Term of 44 Years; and a third Set of Lives where all are of the Age of 22, the Value of which is fixed at 13-58, equal to a Term near 44 Years; when in the same Table 13-46, or rather

rather 13-44, is given us as the Value of Annuities for three Lives all of 12 Years of Age, which is not equal to a Term longer than 41 Years. What Conclusion Mr. *Richards* or Mr. *Moiure* will draw from these Premises, I determine not; but some Things there are which are so repugnant to common Sense, that all the Mathematical Demonstrations in the Universe can never prove to be true, at least not to the Satisfaction of a Man of common Understanding. Amongst these somethings I may undoubtedly place this; that an Annuity for three Lives, where some one, two, or all three of the Lives, are of a greater Age than 12, should be of better Value and equal to a longer Term, than an Annuity for three Lives, where all the Persons are of the Age of 12 and no more.

Where are we now to place this Fault? It does not arise from hence, that Mr. *Richards* has not pursued Mr. *Moiure's* Method; for in most Instances he has followed it reasonably well, and where he has fallen into another Method, there a close Pursuit of it would have made the Error still greater; and therefore the Fault must lye in the Method itself, or perhaps in the Foundation on which the Method is built, and possibly they may both be wrong: And this particular Fault, I mean that the decrease in the Term, for one and the same Period of Life, is different according to the different Rates of Interest, arises from the Hypothesis itself. I have already observed on Mr. *Richards's* Table for the Value of Annuities on a single Life, that such Table gives

us such a Value that the Term corresponding is a different one, as the Rate of Interest is different; for Instance, that the Term of an Annuity for one Life, Interest computed at 4*l. per Cent.*, is near thirty Years, at 8*l. per Cent.* only 25 Years. The necessary Consequence of which is, that the Term to which an Annuity for two Lives and three Lives is equal, must likewise vary as the Rate of Interest varies; and these Tables for Annuities for two and three Lives do in some Measure agree with such Variation: The Consequence from thence is, that the Decrease in the Term, for each Period of Life, must differ too as well as the whole Term for each Period differs; and if these several Differences are a necessary Consequence of the original Hypothesis, and such Differences are really a Fault in the Tables, it follows that the Hypothesis itself is wrong.

That these Differences, which are all consequent or dependant one on another, are really Faults in the Tables, both that for a single Life, for two Lives, and for three Lives, is proved very manifestly by the Arguments which I produced in another Place, where I undertook to prove, that a Table for the Value of Annuities on a single Life, could not be a right one which gave us such Values of Annuities on a single Life, as that the Terms corresponding to those Values were different, according as the Rate of Interest was different: And I crave Leave here to repeat the Force of my Argument,

in short, as there applied to the Table for the Value of Annuities on single Lives, and to apply it here to the Tables for the Value of Annuities on two and three Lives, and it stands thus.

If the Notion is right, and the Foundation on which it is built is right, that an Annuity for a single Life, for two Lives, or for three Lives, is of such a Value that the Term corresponding to the Value shall be different, as the Rate of Interest is different; for Instance, if at *4l. per Cent.* the Value of an Annuity on a single Life of 12 Years of Age is such, that the Term corresponding is 30 Years or near it, and at *8l. per Cent.* the Value is such, that the Term corresponding shall be 25 Years only; if at *4l. per Cent.* the Value of an Annuity on two Lives of 12 Years of Age is such that the Term corresponding is 49 Years, and at *8l. per Cent.* the Value of the like Annuity is such, that the Term corresponding is 44 Years only; and if at *4l. per Cent.* on three Lives of 12 Years of Age, the Value of the Annuity be such, that the Term corresponding is 65 Years, and at *8l. per Cent.* the Value of the like Annuity has a Term corresponding of 56 Years only: And this evidently appears to be the Case on Mr. *Richards's* three Tables: and this likewise is the Case on Mr. *Hayes's* Table for the Value of Annuities for a single Life, and the Difference in a much greater Proportion.

If

If these Values, and the Rule by which they were ascertained, is right; then, if two different Persons purchase an Annuity at the two different Rates of 4*l.* and 8*l. per Cent.* Interest, on one and the same single Life, the same two Lives, and the same three Lives; the necessary and evident Consequence of this, is that with Regard to such Purchasers the Chance of Vitality of the Persons on whom their Interest depends, though they are one and the same Persons, must be different: For with Regard to the Purchaser at 8*l. per Cent.* the Chance of Vitality on his single Nominee determines at the End of 25 Years, of his two Nominees at the End of 44 Years, and of his three Nominees at the End of 56 Years; and yet at the same Time, on the very same Persons, with Regard to the Purchaser at 4*l. per Cent.*, the Chance of Vitality continues, to 30 Years, to 49 Years, and 65 Years, respectively: And this is another of those Somethings which, I say, all the Mathematicks in the Universe can never reconcile to common Sense and common Understanding.

Now if the Term, to which the Value of an Annuity on a Life and Lives is equal, varies according as the Rate of Interest varies, and this be the necessary Produce of the original Hypothesis, as I affirm it is, and have already proved in *page* the 177th; such Variety, if nothing else could be offered, would be sufficient to destroy the Hypothesis. I do not mean to say here that this Hypothesis, and the Rule formed upon

it, will produce a Variety in the Term identically the same which Mr. *Richards's* Tables give us, nor is it necessary I should; because any Variation in the Term, if a necessary Consequence of the Hypothesis, is fatal to the Hypothesis: For the Chance of Vitality on the Life of one or more the same individual Persons cannot vary in the least, unless it could be demonstrated that the Life of one and the same Person, and the Lives of the same Persons if more than one, may have both a longer and a shorter Duration; which I suppose has not yet been done.

I put in this Caution with respect to the Variety in the Term, which may possibly arise on a Calculation made by the Doctor's Rule, and the Variety given in Mr. *Richards's* Tables; because the great Variation in the Term which appears in the Tables on the Value of Annuities for three Lives, at 5*l.* and 7*l. per Cent.*, from what the Term is at these Rates of Interest, may have arisen, in some Measure, from the Method which Mr. *Richards* made Use of to compute the Value of Annuities for a single Life at these two particular Rates of Interest; *viz.* not by the Doctor's Rule absolutely and strictly, but by an Equation, for that is an infallible Way, let who will Use it in such Computations as these, to have a wrong Value; at least to have such a Value, as that the Term corresponding to the Value, shall not be a Term between the two Terms, at 4*l.* and 6*l.*, and between 6*l.* and 8*l.*, for a proportionate Value does not
give

give a proportionate Term, nor on the Reverse, does a proportionate Term give a proportionate Value; as I have demonstrated under a former Head, in *page* the 177th.

I observe upon this; since Mr. *Richards* in forming his Tables for the Value of Annuities for a single Life, at 5*l. per Cent.*, made Use of Equation and not Dr. *Halley's* Hypothesis directly, and since these are the Foundation of his Tables of Annuities for three Lives; that the Value 19-54, given in these Tables, which is equal to a Term of 78 Years, might arise, in Part at least, from using such Equation and not from a Defect, either in the Doctor's Hypothesis, or in Mr. *Moiivre's* Method. Admitting this to be the Case at that Rate of Interest, yet it is not the Case at the Rate of 4*l. per Cent.*, for in that Instance he made Use of their Rule throughout; but I affirm that 23-01, which is given as the Value of an Annuity for three Lives of 12 Years of Age, and at the latter Rate of Interest, cannot possibly be the true Value of such an Annuity. This Value 23-01 is equal to a Term of 65 Years, but this Term exceeds the Term to which any one of three such Persons has a Chance of living by some Years, even on Dr. *Halley's* Rule for calculating the Chance of the Duration of such Life; for on such Calculation the Duration of any one of those Lives, hardly comes up to a Term of 59 Years; and it is impossible the Annuity should last longer, or have a Chance to last longer than any one of the

Lives on which it depends, has an Existence or a Chance to an Existence. The Calculation of the Chances of Vitality and Mortality on those three Lives, according to the Doctor's Rule, is an easy and a short one, and stands thus. The Number of Persons living of the Age of 12 is 646: $646 + 646$ is 417316; $417316 + 646$ is 269586136; and this is the Number of Chances of Vitality on these three Lives. The Number of Persons dead of that Age in 59 Years is 515: $515 + 515$ is 265425; $265425 + 515$ is 136693875; and this is the Number of Chances that all three are dead in 59 Years. But this last Sum is more than a Moiety of the Sum of the Chances of Vitality; therefore it is more than an even Chance that all the three Persons are dead in 59 Years.

In the Case of an Annuity on a single Life it may be alledged, and the Allegation is a plausible one; though the even Chance of the Duration of a single Life is 41 Years, yet that there is a Chance of Mortality occurring during that Term, and therefore during that Term there is a Chance of the Termination of the Life and of the Annuity, and that a Value equivalent to such Chance must be deducted out of the Annuity for such Time: but when we proceed farther, and to this one Life we make an Addition of two more Lives of the same Age, and inquire what is the Chance of the Duration of those three Lives or of one of them, and what is the Value of an Annuity for those Lives, and

to

to what Term such Value is equal: To say here that the Value is such that the Term in the Annuity will be longer than the Term in the Lives, not only runs counter to the Case on a single Life, for there the Term in the Annuity is shorter than the Term in the Life, but it is in Effect to say that the Annuity has some Chance of Continuance after the Lives on which it depends have no Chance of Continuance: which is so contradictory to Reason and common Sense, in my Understanding, that I must leave it with the Author to reconcile it if he can. I must here do Dr. *Halley* the Justice to take Notice that this absurd and preposterous Value is not the Produce of his Hypothesis, notwithstanding Mr. *Moiivre*'s Allegation that his Method is founded on that Hypothesis, but is solely and only chargeable on the Method, as we shall see presently when we consider the Method, by which such Value is produced; which we shall proceed now to do.

The Rule then which Mr. *Moiivre* prescribes, and which Mr. *Richards* made Use of, to compute the Value of Annuities for three Lives and the Survivor of them, and by which this extravagant Value of Annuities for three Lives of 12 Years of Age at 4*l. per Cent.*, viz. the Value 23-01, is produced, is this. To put together the Value of all the three Lives singly taken, and to this Sum to add the Value of the three Lives jointly taken once; to compute the Value of these Lives jointly taken, two and two com-

bined and crossed, and to put these three Values together; to subtract the last Sum total out of the first Sum total; and to take the Remainder as the Value of the Annuity for the three Lives and the Survivor.

To treat this Matter distinctly, and to shew clearly the Manner of using this Rule, we will apply it in an Instance on the Lives of *A.*, *B.*, and *C.*, and Survivor, all supposed to be of the Age of 12 Years, and at 4*l.* per Cent. Interest.

An Annuity for the Life of *A.* is } 17-20
in Value

The same for the Life of *B.* is 17-20

The same for the Life of *C.* is 17-20

All these Annuities put together } 51-60
are

An Annuity for these three Joint- } 10-44
Lives is

These Values put together a- } 62-04--62-04
mount to

An Annuity for the Joint-Lives } 13-01
of *A.* and of *B.* is

The same for the Joint-Lives of } 13-01
A. and *C.* is

The same for the Joint-Lives of } 13-01
B. and *C.* is

These three Values put together } 39-03--39-03
amount to

This

This last Sum total, being deduced out of the first Sum total, leaves a Remainder, as the Value of the Annuity for these Lives. ——— 23-01

This Remainder, 23-01, is the Sum given in these Tables, as the Value of this Annuity. We will examine then here, as we did under the Article of Annuities for two Lives, what are the Parts of the Value of the Annuity for these Lives which by this Rule are directed to be brought to Account as Parts of such Value: and by that Means we shall discover whether the true and right Values, or whether more or greater Values are not brought to the Account of an Annuitant on these Lives than do really belong to such Lives: And here I cannot only certainly say that more Values are brought to such Account than ought to be, but I think I can also determinately fix those more Values so brought to this Account.

The first Sum brought to such Annuitant's Account, or left standing in his Account, is 10-44, the Value of the Annuity for the Joint-Lives of *A. B.* and *C.*; for by the Rule this Value is added to the first Sum total, and is nowhere subtracted; and this Sum to be sure ought to be, and is rightly brought to the Account, as a Sum which the Proprietor will receive during the Joint-Lives of *A. B.* and *C.* If we proceed to examine what is farther carried to such Account
or

or left standing on such Account, it will appear to be so much of the Value of the Annuity on the Life of *A. B.* and *C.* each singly taken, as remains when the Value of the Joint-Lives two and two combined is substracted. By the Method in which Mr. *Richards* puts it, we collect together the Value of the several Annuities for the three Lives singly taken, and into such general Account we insert the Value of the three Joint-Lives once, and make one gross Sum, and out of this Sum we substract the Whole of the Value for the Joint-Lives, two and two combined. Now this Sum thus substracted in Effect and in the Event is substracted out of the Value of the Annuities for the three Lives singly taken and put together, and might be previously so substracted, and the Value of the Annuity for the three Joint-Lives be afterwards added to the Sum remaining: or the Value of the three Interests during the two Joint-Lives may be singly and separately substracted out of the Value of the Annuities for each of the three Lives singly taken, and the Value of the Annuity for the three Joint-Lives be afterwards added to the three Sums remaining: And in either Ways of the three the final Produce will be one and the same Sum. From whence 'tis evident that the Parts of the Value of the Annuity here taken are in the whole; the Value for the three Joint-Lives once, and so much of the Value of the three Lives singly taken as remains when the Value of the Joint-Lives two and two combined

bined is subtracted; whether such Subtraction of the three Interests on the Joint-Lives two and two combined be made singly out of the Value of each Life singly taken, or out of the Value of the three Lives singly taken and put together, or out of such last Value when the Value for the three Joint-Lives is added.

This, perhaps, will more clearly appear to be the Case, if we put the Rule into a different Form; as thus. To take the Value of the Annuity for the three Joint-Lives once, and out of the Value of the Annuity on each of the Lives singly taken separately to subtract the Value of the Annuity for one of the Sets of the Joint-Lives two and two combined, by which Means all three Sets will be subtracted, in this Way as well as the other: And to take the three Remainders, together with the Value of the Annuity for the three Joint-Lives, as the Value of the Annuity for the three Lives and the Survivor. In either Way of using the Rule the Values will, and necessarily must, come out the same, for the Difference is only this: In one Case we subtract three Sums out of three Sums separately, and in the other Case we first collect each of the same three Sums into one Sum, and then make the Subtraction collectively. To use the same Instance we had before.

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The Value of an Annuity for the Joint-Lives of <i>A.</i> , <i>B.</i> , and <i>C.</i> , is	}	10-44
The Value of an Annuity for the single Life of <i>A.</i> is	}	17-20
The Value of an Annuity for the Joint-Lives of <i>A.</i> and <i>B.</i> is	}	13-01
The second Sum substracted out of the first leaves	}	04-19 — 4-19
The Value of an Annuity for the single Life of <i>B.</i> is	}	17-20
The Value of an Annuity for the Joint-Lives of <i>A.</i> and <i>C.</i> is	}	13-01
The second Sum substracted out of the first leaves	}	04-19 — 4-19
The Value of an Annuity for the single Life of <i>C.</i> is	}	17-20
The Value of an Annuity for the Joint-Lives of <i>B.</i> and <i>C.</i> is	}	13-01
The second Sum substracted out of the first leaves	}	04-19 — 4-19-00
		Sum total—23-01-00

Now it will readily be admitted that 10-44 the Value of the Annuity for the three Joint-Lives of *A.* *B.* and *C.* is to be brought to Account; and I affirm, that the other Values to be taken into this Account are, the Value of the Annuity for the Joint-Lives of the two Survi-
vora,

vors, and the Value of the Annuity for the Life of the single Survivor; each of those Values to be computed from the Time when the Interest in those Lives respectively commences, or on an even Chance may be estimated to commence. But whatever are the Values to be added, I say, 'tis impossible that the three Values here assign'd, viz. 4-19, 4-19, and 4-19, can be the right Values; for this Assignment, most assuredly, has a double Error in it.

In the first Place, no Value, directly and strictly as such, is brought to Account, or left remaining in the first and general Account; for such Parts of the Value of this Annuity as arise after the Determination of the Interest for the three Joint-Lives, and during the Continuance of the Interest in the two surviving Lives; when, past all Peradventure, the Annuitant for three Lives and the Survivor has a Right to, and must receive, some Parts of the Value of his Annuity during the Joint-Lives of those two who become the Survivors; and therefore the total Omission of this Value can no ways be justified. And that this Omission will create an Error in the total Value of the Annuity for the three Lives, is evident from hence; for that the Annuitant here is intitled to a Value and a Term in immediate Reversion, and the Value and Term assign'd to him by the Rule is a Value and Term in remote Reversion. The Value first assign'd to him is 10-44, which is equal to a Term of 13-3-00; after which 'tis plain the Annuitant has a Title to a Value in immediate Reversion
after

after 10-44, and to a Term which should take Place on the Determination of the former Term of 13-3-00: But the Value here allotted to him, *viz.* 4-19, is a Value in Reversion after 13-01; and the Term allotted to him is a Term which takes Place after the Determination of a Term of 18-3-00; that being the Term to which the Value 13-01 corresponds. From hence 'tis plain that for five Years, that is from the End of the Term 13-3-00 to the End of the Term 18-3-00, no Value or Term at all is allotted to the Annuitant; and if any other is allotted to him as an Equivalent, 'tis impossible that it should be a right one, and we shall see presently what it is, and that 'tis a wrong one.

The second Error committed in the Rule is, that when one of the Nominees in the Annuity is dead, such a Sum is brought to Account as is equal to the Value of an Annuity, as if all three of them were still living, and the Sum to be received by the Annuitant were three distinct Annuities, and each of them equal to the Whole; when 'tis supposed in the Case, that one of the Nominees is dead, and that the Annuity is no more than a single one. 'Tis indeed possible that *A.* may be the Survivor of the three, and possible that *B.* may, and possible that *C.* may; but that every one of them should in Fact become the Survivor, is Nonsense in Terms, and in Nature impossible. And as to the Parts of the Value of the Annuity to be received in the Life of the two Survivors, that is when the Interest in the Estate for the three Joint-Lives is determined, there can
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be no more Reason to bring the Value of the Annuity to Account oftner than once during such surviving Lives, than there is for bringing it to Account oftner than once during the three Joint-Lives; and yet this Rule brings it to Account but once only during the Term of the three Joint-Lives, and for the Rest of the Time brings it to Account three Times.

If we look back and observe what are the Parts of the Value of an Annuity, which by this Rule are brought to Account in the Valuation of an Annuity for two Lives and the Survivor, we shall find, that there the Value of the Annuity for the two Joint-Lives once taken, and so much of the Value of the Annuity on both the Lives singly taken as remains when one only is surviving; that these Values put together are stated as the Value of such an Annuity. Here it appears, that the Value of the Annuity for the three Joint-Lives once taken, and so much of the Value of the Annuity on the three Lives singly taken, as remains when two are surviving; that these Values put together are stated as the Value of the Annuity for three Lives. This being the Case, and the Rule proceeding on the same Foundations here as before, and having shown, I think demonstrated, that other and more Parts of the Value of an Annuity for two Lives are brought to Account there, than do or can possibly belong to such an Annuitant, I might very well refer myself to my Reasoning under the former Head; but since I can determinately say what are the particular Parts of the Value

Value of the Annuity here which by the Rule are brought to the Annuitant's Account, but do not belong to him, I will go on, and try to fix those Parts by the Instance following; on the Lives of *A. B.* and *C.*, each of 12 Years Age.

We will suppose then, that N^o. 20 grants an Annuity to N^o. 1. for those three Lives and the Survivor of them: and that N^o. 40. grants a like Annuity to N^o. 2. for those three Joint-Lives; to N^o. 3. the like Annuity for the Life of *A.* if *A.* survives either *B.* or *C.*, and to commence on the Death of *B.* or *C.*, whichever of those two dies first; to N^o. 4. the like Annuity for the Life of *B.* if *B.* survives either *A.* or *C.*, and to commence on the Death of *A.* or *C.*, whichever of those two dies first; and to N^o. 5. the like Annuity for the Life of *C.* if *C.* survives either *A.* or *B.*, and to commence on the Death of *A.* or *B.*, whichever of those two dies first. The Value of the Annuity granted by N^o. 20. to N^o. 1., as given in the Tables and computed before, is 23-01. Let us compute, in the next Place, what is the Value of the four Annuities granted by N^o. 40. to N^o. 2., 3., 4., and 5. The Value of the Annuity granted to N^o. 2., being an Annuity for the Joint-Lives of *A. B.* and *C.*, is in Value, as by the Tables, 10-44. The Value of the three Annuities granted to N^o. 3., 4., and 5., are evidently the same; because they are all to commence on an equal Contingency, and on an even Chance are to commence and determine at one and the same Time: So that a Computation of
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the Value of one gives the Value of each. The Annuity granted to N^o. 3. is for the Life of *A.* in Reversion after the Death of *B.* or *C.*, and in Case he survives either of them, that is, after the Determination of the Joint-Lives of *B.* and *C.* An Annuity for the Life of *A.*, absolutely and in the Whole, is 17-20, and an Annuity for the Joint-Lives of *B.* and *C.* is 13-01, as given in the Tables and the Computation above; and if 13-01 be deducted out of 17-20, the Sum remaining will be 4-19, which is the Value of the Annuity granted to N^o. 3. The Annuity belonging to N^o. 4. and N^o. 5. being each of the same Value as that belonging to N^o. 3. must each be 4-19. If we put together all these four Annuities granted by N^o. 40., they stand thus.

The Annuity to N ^o . 2.	10-44
That to N ^o . 3.	4-19
That to N ^o . 4.	4-19
That to N ^o . 5.	4-19
Total Value	<u>23-01</u>

This being the total Value of all the four Annuities put together, and the Value of the single Annuity belonging to N^o. 1., as stated in the Tables, being the same, *viz.* 23-01; let us see how the Case stands with Regard to the respective Grantors, N^o. 20. and N^o. 40.: And, to avoid Confusion, we will suppose *A.* to die first, *B.* second, and *C.* to be the Survivor. Now during the Joint-Lives of *A.* *B.* and *C.*, N^o. 20.

and N^o. 40. are on the same Foot, that is, each must pay one Annuity; on the Death of *A.* the Annuities to N^o. 4. and N^o. 5. are both to commence, and N^o. 40. will have two Annuities to pay; but N^o. 20. will have one only to pay; and on the Death of *B.* each of the Grantors will be again on the same Foot, and each of them have one Annuity only to pay. This Instance demonstrates, that the Value of the four separate Annuities is no more than 23-01; that the Value of the single Annuity for the three Lives and the Survivor cannot be so much; that during the Joint-Lives of *B.* and *C.*, the two surviving Lives, two Annuities are in the Computation calculated to be coming to that Annuitant, when in Fact only one belongs to him: Consequently, that one of these two Annuities is brought to his Account more than belongs to him.

To confirm the Justice of this Calculation, if it needs any Confirmation; or rather to show, if Mr. *Moirre's* Method of computing the Value of Annuities for three Lives and the Survivor is pursued, that the Case is and necessarily must be such that this particular Chance of a double Annuity during the Continuance of the two surviving Lives is always inserted and brought to the Annuitant's Account, I crave Leave to make Use of the Instance given by Mr. *Richards*, Page the 83d, &c. He there supposes a Man to give his three Daughters, whom I will call *A.* *B.* and *C.*, an Annuity of 20*l.* per Annum each during their Lives; and to order, that when any of them

dies,

dies, the Annuity of the Sister dying should fall to the other two in equal Shares; and at the Death of another, that the whole should belong to the Survivor for her Life. He goes on thence to compute the Value of this Gift to each of these Women; and gives us the Value of the Share of the eldest Sister thus. First 20*l.* *per Annum* during her own Life. Secondly, 10*l.* *per Annum* in Reversion of the youngest, to continue from thence during the Joint-Lives of the other two. Thirdly, 10*l.* *per Annum* in Reversion of the Second, to continue from thence during the Joint-Lives of the First and Third. Fourthly, 40*l.* *per Annum* in Reversion of the two youngest Lives during her own Life. I have here omitted the Values, because I do not intend to enter into that Matter, whether the Values there given, or the Method by which they are computed, be right or no, though that too may very well be question'd; but the single Point I insist on here, is that more Articles are inserted than do belong to, or can possibly ever be enjoy'd by this Sister.

I will not charge the Author with using some Art in the Expressions here; but he must give me Leave to vary them a little, and I would express the Thing thus. First, 20*l.* *per Annum* to *A.* during her own Life. Secondly, 10*l.* *per Annum* in Reversion after the Death of *C.*, to continue during the Joint-Lives of *A.* and *B.* Thirdly, 10*l.* *per Annum* in Reversion after the Death of *B.*, to continue during the Joint-Lives of *A.* and *C.* If we read the Account thus, it

appears evidently that there are two Articles or two Annuities of 10 *l. per Ann.* each, on the several Joint-Lives of *A.* and *B.*, and of *A.* and *C.*: And a Man may read as he runs, that this necessarily supposes that *B.* survives *C.*, and at the same Time that *C.* survives *B.*: But since 'tis impossible in Nature that both should happen, 'tis impossible in Fact that *A.* should enjoy both those Annuities. She has undoubtedly a Chance to one or other of those two Annuities, and to determine to which of the two she has a Chance, and to compute the Value of it, may have some Difficulty in it, but there is none at all to see she cannot possibly have a Chance to both. Her first reversionary Interest commences on the Death of *C.*, and not before, and from thence during the Joint-Lives of herself and *B.* she will receive 10 *l. per Annum*: But when, or which Way, will the second reversionary Interest arise and commence, or how can she ever receive the 10 *l. per Annum* on the Joint-Lives of herself and *C.*, when *C.* is already dead and gone? We see then in this Case, just in the same Manner as in the Case of Annuities for three Lives, that in both Accounts the Annuity during the Joint-Lives of the two Survivors is twice inserted, when it ought to stand there once only. From hence therefore, as Mr. *Richards* concludes that his Computation of each Sister's Portion is a right one, from its Agreement with the Value of the whole Estate of all the three Sisters on the Survivor of the three Lives; so from the same Agreement I conclude, since his Computation
of

of the Value of each Sister's Portion is a wrong one, that the Computation of the Value of the whole Estate on the Survivor of the three Lives is a wrong one likewise.

If we carry our Enquiries further, and examine whence this Excess in the Valuation of these Annuities arises, we shall see plainly that it arises from the original Hypothesis, either in part or in the whole, for most certainly it assigns more Chances of Vitality on the Lives of three Persons and the Survivor of them than do or can possibly belong to them. The Hypothesis asserts, that an Annuitant on the Lives, suppose of *A. B. and C.* and the Survivor of them, has seven Chances of Vitality, or seven Chances that some or one of them is living; and I affirm on the contrary, that he has six only, and that he cannot possibly have more than six, either at the Commencement, or at any Time during the Continuance, of the Interest in their Lives. The Seven several Chances of Vitality assign'd to the Annuitant upon these three Lives and the Survivor by the Hypothesis stands thus; and may be rank'd in three Classes, as follows.

The first on the Lives of *A. B. and C.* all living.

The second on the Lives of *A. and B.* *C.* dead.

The third on the Lives of *A. and C.* *B.* dead.

The fourth on the Lives of *B. and C.* *A.* dead.

The fifth on the Life of *A.* *B. and C.* dead.

The sixth on the Life of *B.* *A. and C.* dead.

The seventh on the Life of *C.* *A. and B.* dead.

In the Order and Manner in which I have placed these Chances, which does not in Substance vary from that in which the Hypothesis places them, I think it appears on the View, that in the third or last Class there is one Chance inserted which the Annuitant has no Right or Pretence to, there being in Fact no such Chance. At the Commencement of the Interest in the Lives of these three Persons, and whilst they are all living, there is undoubtedly a Chance that *A.* may become the last Survivor, that *B.* may become such, and that *C.* may become such: But the Hypothesis does not state it in this Manner; and if it did so state it, then there must be three such Chances reckon'd, for so many are then subsisting, or there are none, and thus we shall have nine Chances instead of Seven. But the Truth is, and the Hypothesis states it in this Manner, *viz.* when, and if *A.* is dead, that either *B.* or *C.* is the last Survivor; when, and if *B.* is dead, that either *A.* or *C.* is the last Survivor; and when, and if *C.* is dead, that either *A.* or *B.* is the last Survivor. Now these Chances, or any of them, do not commence and accrue to the Annuitant, unless and until either *A.* or *B.* or *C.* is dead; and when any one of them is dead, suppose *A.*, 'tis as plain as Words and Figures can make it, that between the two Survivors *B.* and *C.* there remain to the Annuitant no more than two Chances of Survivorship, *viz.* either that *B.* or that *C.* will survive; for after *A.* is dead there can be no such Chance that *A.* will survive;

survive; it being now, to this Purpose, the same Thing as if *A*. never had any Being at all.

And that the Chances of the third Class, that is the Chances of the last Survivor, do not commence 'till one of the three is dead, is as evident a Position as the former; because whilst all three are living, the Annuitant's Chances are those of the second Class, and no other; and the Hypothesis states it in this Manner. But whether it does so state it, or does not so state it, in the third Class, and after one of the three is dead, there are then but two Chances left; and 'tis impossible to make three Chances of them, but by supposing that they are all three living. Now if we make this Supposition, that all three of them are a second Time living, as in Truth the Hypothesis does; the Consequence must be, and we see the Consequence in the Computation is, that the Chances on the Survivorship of two are a second Time brought to Account; for so long and as often as all three are living, or supposed to be living, so long and as often the Chances of the Survivorship of two must continue and be accounted.

To shew more apparently, that this Hypothesis for calculating the Chance of Vitality on these Lives, does certainly insert more Chances than belong to them, if not the individual one just mentioned; I crave leave to add one Argument more, which in Substance is the same, as I urged under the Head of the Chances of Survivorship on two Lives, and which perhaps may appear more obvious and open than the former.

If we make a Calculation of the Chance of the Duration of three Lives or one of them, all supposed to be of ten Years of Age, according to the Method prescribed by the present Hypothesis; such Duration comes out to be equal to a Term of 61 Years or near it. If we make a Calculation of such Chance and on the same Lives by the *Breslaw* Table, and that Table is by the Doctor recommended, and most certainly is a good Rule, so far as the Table itself is a just one, to calculate the Chance of the Duration of a single Life, and the Reason which proves such Table to be a good Rule, for calculating the Chance of the Duration of a single Life, is the very same and equally good to prove, that the Table is a good Rule to calculate the Chance of the Duration of two, three, or more Lives, and the Survivor of them, at least where the Lives are of equal Age: Now by this Table, the Chance of the Duration of these three Lives or any one of them, is equal only to a Term of 58 Years; the Method of making which Calculation, and the Reasons confirming the Justice of it, may be seen in *page* 346 of this Treatise, to which I crave leave to refer myself.

Since then the Chance of the Duration of these Lives, in one Way of computing, is equal to a Term shorter by three Years, than in the other Way of computing the same, and since it is impossible both should be right, I must leave it with the Favourers of this Hypothesis to make out, that the Error does not lie on the same side
on

on which the Excess lies; and an Error on one side or the other, there must certainly be.

This being the Case, if Mr. *Moiure* has demonstrated that his Rule for calculating the Value of Annuities for three Lives, is consonant to the Doctor's Hypothesis, for calculating the Chance of the Duration of three Lives, which I have Reason to believe he has not done, though I never saw his Treatise; yet, admitting it to be so, I do not see any Thing is gained by his Demonstrations, unless the Satisfaction of seeing that such his Demonstrations have destroyed that very Thing, which they were calculated to establish; for if the Hypothesis is false, the Rule too, if it be mathematically consonant to it, and for that very Reason, must be false too. But I will venture to say, whatever Mr. *Moiure* has demonstrated, or supposed himself to have demonstrated, that his Calculations have not prov'd that his Rule is in all Instances consonant to the Hypothesis, unless they have prov'd, which I may be very confident they have not done or ever can do, that a Term for 59 Years is equal to and one and the same Thing, as a Term for 65 Years. I have made a Calculation, and set it forth in *page 389* above, of the Term to which three Lives of 12 Years of Age or one of them, has an even Chance of Duration, where such Chance is calculated by the Doctor's Hypothesis, and the Term is barely 59 Years: But an Annuity for three such Lives, computation being made at 4l. *per Cent.* Interest, at which Rate of Interest Mr. *Richards* tells us he strictly pursued

perſued Mr. *Moiſvre's* Rule, in forming his Tables, is in thoſe Tables ſtated at 23-01; and the Value 23-01 is equal to a Term of 65 Years. Now this Term of 65 Years, for which the Annuity muſt continue, or the Value of it cannot be 23-01, is not only monſtrouſly abſurd, as making the Annuity to have a Chance of Continuance, longer than any one of the Lives on which it depends has any Chance of Continuance, but it demonſtrates that Mr. *Moiſvre's* Rule for calculating the Value of Annuities on ſuch Lives, is not agreeable to the Doctor's Hypotheſis for calculating the Chance of the Duration of theſe Lives.

And as the Rule for calculating the Value of Annuities on theſe Lives, does not in all Inſtances agree with the Hypotheſis for calculating the Chance of the Duration of ſuch Lives; for we ſee here it varies in the moſt material Point; ſo neither does this Rule in any Sort agree with the Rule which the Doctor preſcribes, for calculating the Value of Annuities on thoſe Lives. The Doctor's Rule for calculating the Value of Annuities on two, three, or more Lives, is the ſame in Subſtance, with that which he uſes for calculating the Value of an Annuity on a ſingle Life, *viz.* by eſtimating the Annuity on a Life or Lives, as an abſolute one for ſo many Years as ſuch ſingle Life, and where more Lives than one, for ſo many Years as any one of the given Lives, can poſſibly have a Continuance; after having deducted thereout a Value correſponding to the Chances of Mortality, which in each Year of ſuch

such Life or Lives may have arisen, or incur'd during such possible Continuance. By this Means, as well in the Case of a single Life, as of two or more Lives, the Chance of the Duration of the Annuity in the Lives, is not adequate to, but is always less, or should be less, than the Chance of the Duration of the Lives themselves; and the Doctor does not alledge or suppose, that his two Rules would produce one and the same Term for the Annuities and for the Lives, though if they had concurred in such Produce, I do not think such Concurrence would have been any Objection to the Rules; but Mr. *Moiivre's* Rule, whatever he may alledge or might mean, does not agree with either of them.

The Doctor's Method for calculating the Value of Annuities on two or more Lives, as he himself very justly complains, requiring a very long and tedious Calculation, and being attended likewise with some Intricacies, Mr. *Moiivre* has found out, or imagines he has found out, another Rule for this Purpose, which is less difficult and more concise, and which is surmised to be built on the Doctor's Foundations, and agreeable thereto. Whether there be any and what Agreement between these two Rules, we shall see presently; but in the Values which they severally produce, as the Values of Annuities on the same Lives, and in the Terms to which those Values correspond, sure I am that they differ very widely, and that Mr. *Moiivre's* is by much the worst. I have by the Doctor's Rule made a Calculation, and a tedious Piece of

Work

Work it was, of the Value of an Annuity for two Lives, and for three Lives, and the Survivor, all supposed to be of ten Years of Age, and at 6 *l. per Cent.* Interest; and the Value of the first Annuity is 15-41, which is equal to a Term of 45-0-00, and the Value of the second is 15-91, which is equal to a Term of 55-0-00. If we consult Mr. *Richards's* Tables for the Value of Annuities on two Lives, and three Lives of 12 Years of Age, which is the Age at which his Computations are made; and he assures us, those Tables were framed by Mr. *Moiivre's* Rule, and upon Tryal, I am satisfied that they are rightly and justly so constructed; we find the Value of the first of these Annuities, *viz.* that for two Lives to be 15-59, which is equal to a Term of 48-0-00, and the Value of the second, *viz.* that for three Lives to be 16-19, which is equal to a Term of 61-0-00. Now the Value of an Annuity must be greater, and the Term corresponding to it longer, where the Annuity depends on two, or on three Lives of 10 Years of Age, than where it depends on the like Lives of 12 Years of Age, and if the Rules which produced these Values and Terms were consonant to each other, would certainly be so; whereas we see here that the Annuities depending on the elder, and consequently the shorter Lives are of a greater Value, and equal to a longer Term, than those which have their Dependance on younger, and consequently longer Lives; the one an even three Years, and the other six Years; which is not only very absurd,

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but evidently demonstrates that the two Rules are not the same in Substance; but that Mr. *Moiivre's* of the two, is the most faulty, as making an Addition to a Value already too great, and to a Term already too long.

Having sufficiently prov'd that Mr. *Moiivre's* Rule, for calculating the Value of Annuities on three Lives, does not, in all Circumstances, and in those which are most Material, agree, either with Dr. *Halley's* Hypothesis for calculating the Chances of Vitality on such Lives, or his Rule for computing the Value of Annuities on such Lives, since his Rule produces such a Value and Term for these Annuities, as varies greatly from the Value and Term which the Doctor's Hypothesis or Rule produces: Let us examine now wherein these two great Authors agree, for in one Particular at least, they certainly do agree; but it is in a very unhappy Particular, *viz.* the most weak Part of the Doctor's Hypothesis. This, as I have before observed upon it, in stating the Chances of Vitality on three Lives, has stated them at seven, whereas there are no more, and I think I have prov'd beyond all Contradiction, that there can be no more than six; and herein the Rule of the one, is consonant to the Hypothesis of the other; for it makes Account of such seventh Chance, and inserts a Value instead of, and as an Equivalent for, such Chance.

There being then this Consonance between them, it may be ask'd, What gives occasion that they produce such a Variety in the Values, and in the Terms of the Annuities on these
Lives

Lives? The Answer to this Question will plainly shew, there is a Fault in Mr. *Moiivre's* Rule, consider'd separately and distinctly by itself, over and besides the Defect which is in it, in that Instance wherein it agrees with the Doctor's Hypothesis. The supernumerary Chance of Vitality, which the Hypothesis brings to Account on three Lives, is a Chance arising between the Determination of the Interest on the three Lives, and the Commencement of the Interest on the single Life, that is during the Interest of the two surviving Lives; for it gives three Chances of Survivance between those two Nominees, when there can be no more than two; by which means the Chance of Duration of one or other of the three Lives, is extended beyond its due Limits, for a certain Number of Years equal to such extraordinary Chance. Mr. *Moiivre* enters into the same Notion, supposes the same supernumerary Chance, and in his Rule inserts a Value corresponding to such Chance, and in the same Place: And very likely he may have demonstrated, for in Fact the Case is such, that his Rule does insert such and no other, or more Values on such Lives, than correspond to the Chances on such Lives, which the Hypothesis has inserted. But the Chance extraordinary induced in one, is a Thing so different from the Value extraordinary induced in the other, that these Things, supposed and seeming to be the same, create the Difference of the Values and Terms on those Lives, according as they are computed by the one or the other of the Rules.

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That this Chance in the Hypothesis, is of a different Nature and Account from the Value in the Rule, will appear from hence. The Hypothesis, by inducing this Chance too much, enlarges the Term, to which all three Lives are equal for a Term, suppose of seven Years, and whether this be the exact Number of Years, by which such Term is enlarged, is not here material; but a Term of seven Years, or any other Number of Years, whether inserted during the three Joint-Lives, or during the two surviving Lives, or during the Life of the single Survivor, or wherever it be inserted, does no more and goes no farther, than to add seven Years too much to the Term to which those Lives are equal. But where a Value corresponding to such supernumerary Chance, or to such additional Term, is to be added to such Account, or is inserted more than belongs to such Account, there is great Difference, whether such a Value, as corresponds to the like Term, be taken during the three Joint-Lives, or during the two surviving Lives, or during the single surviving Life, or in Reversion after them all. A Term of seven Years added, let us suppose it inserted where we will, is only an additional Term in Reversion after the End of a prior Term, and enlarges such Term no farther than the seven Years so added; but a Value equal to seven Years, if taken and inserted in the middle of a longer Term, is not a Value wholly in Reversion of the former Value, but shall increase the original Term,

Term, perhaps twice seven Years, and of necessity must increase such Term more than seven Years; because the Value of a Term in Reversion, suppose after twenty Years, is much greater, and the Term corresponding much longer, than the Value of a Reversion, and the Term corresponding, suppose after a Term of forty Years. Now since Mr. *Moiure's* Rule inserts, as it manifestly does, a Value corresponding to the Term, supposed to arise immediately after the Determination of the Interest on the three Joint-Lives, and during the Lives of the two surviving Nominees; the Value of a Term for seven Years in Reversion, then and there taken and added to the Account, will add such a Value to the Whole, as shall greatly exceed a Term of seven Years.

For Instance, and not that this is exactly the Case; if the true and proper Term of these three Lives was a Term of forty Years, the Value of an Annuity for those Lives at 4 *l. per Cent.* would be 19-78; if to this Term we add seven Years, as we suppose the Hypothesis to do, the Term would then be 47 Years and no more, and the Value of that Term is 21-05: But if to the Value 19-78, we add the Value of a Term for seven Years in Reversion after 20 Years, as we suppose Mr. *Moiure's* Rule to do, and as in Fact it evidently does, the Value of the whole Annuity will stand thus: The Value of those seven Years in Reversion after 20 Years is 2-74, and if this Sum be added to 19-78, the total Value will be 22-52, which is equal to a
Term

Term of 59 Years. From hence 'tis apparent that Dr. *Halley's* Hypothesis and Mr. *Moiivre's* Rule do not agree in all Instances; and it follows likewise, though the Hypothesis were admitted to be right, yet that the Rule must be false throughout, for that one and the same Error runs through the whole Scheme of it. The Error I mean is, that it supposes that to add to or subtract from a given Value a proportional Part of it is one and the same Thing, as to add to or subtract from a given Term a like proportional Part, which we have before proved, and which appears from the Instance just now given to be entirely wrong.

Now this Defect in the Rule has so strong an Operation in all Computations of this Kind, that if we could suppose an Annuity on a single Life, Interest computed at 4*l. per Cent.*, to be in Value 19-78, and to be equal to a Term of 40 Years; and if the *Breslaw* Table be a good Rule to estimate the Duration of a Life, a Person of 10 or 12 Years of Age has an even Chance to live 40 Years; and if the Life be equal to that Term, I make no doubt but the Annuity depending on that Life will be equal to the same Term: Upon these Suppositions, and if we make use of this Rule to calculate the Value of an Annuity for three such Lives, the Value produced will exceed the Value of the Inheritance considerably. But in Truth, to discover the Extravagance of the Estimates of the Value of Annuities on these Lives, when made according to this Rule, and that such Extravagance does

not proceed intirely from Dr. *Halley's* Hypothesis, but from the Rule itself as introducing a Value instead of a Term, we need not to go out of Mr. *Richards's* Tables which were constructed by the Rule. If we consult them, the Value of an Annuity for three Lives, suppose of the Age of 15, as a Medium between 12 and 17, is 23-01, Interest stated at 4*l. per Cent.*, which is equal to a Term of 65 Years; but if we make Use of the *Breslaw* Table to compute the Duration of these Lives or any one of them, and I am sure those Tables extend the Chance of such Duration beyond the Term which can well be justified, yet the Term of those Lives is no more than 53 Years and an half; and if we follow Dr. *Halley's* Hypothesis in making such Computation, the Term is only 56 Years and a Quarter. If we put this Matter into another Light, and compute upon the *Breslaw* Tables how many Persons of the Age of 15 live to be of the Age of 80, and here again the Allowance is sufficiently large, it appears manifestly that not one in 15 arrives at that Age; the Number of Persons living of the Age of 15 being 628, and those of the Age of 80 no more than 41; and if we divide 628 by 41, the Number arising is 15 and something over; which shews, that to have an even Chance that any one of our Nominees shall live to be of the Age of 80, we must nominate more than 15 Persons. But Mr. *Richards's* Tables for the Value of an Annuity for three such Lives, which were formed by Mr. *Moiure's* Rule, suppose there is an even Chance that one Person in

three

three of such Age will live to be 80 Years old, for they compute the Annuity to continue for 65 Years from the Time of the Grant made, at which Time all the three Nominees in the Grant are of the Age of 15. This, I think, is so plain and clear a Case, that I need not to add any Thing farther to prove, what I have undertaken to prove, that in this Value of an Annuity for three such Lives, which makes it an even Chance that one of the Lives continues in Being for 65 Years, more or greater Values, though it does not discover in particular what those more or those greater Values are, have been carried to the Account, than do or can belong to the Proprietor of an Annuity for three such Lives.

Since then, neither Dr. *Halley's* nor Mr. *Moiivre's* Rule, for calculating the Value of Annuities for two, three, or more Lives and the Survivor of them, nor any Tables which have been constructed by those or by any other Rules, do give us the right Value of them; I shall proceed now to propose my own for that Purpose. My Method, and which I believe is the only one to do this within any Degree of Justness, is to compute here, as we did in the Case of Annuities on a single Life, to what Term the given Lives or one of them has an even Chance of Duration; for the Annuity most certainly depends on and must continue as long as any of the Lives continues; and when we know for what Term the even Chance is that any one of the Lives and the Annuity attendant on it will continue, we know on the same even Chance what the Value

of the Annuity is, for 'tis equal to that Term to which the Lives are equal.

To compute the Term to which two, three, or more Lives, meaning concurrent Lives and the Survivor of them, are equal; I propose this Method. Where two Lives are of equal Age and Goodness there can be no Difference which of the two is supposed the Survivor, the Chances of Survivorship between them being equal; here then we may take the Term to which either of the Lives is equal, simply as it is, as Part of such Term. The second or other Life we must look upon as a Life or Term in Reversion, as in reality it is; and when it falls into Possession, we must not account of it as a Life equal to the same Term, as it might be estimated at the Time when the Interest in the two Lives first commences; because so many Years are elapsed before it commences, as incurred during the continuance of the first Life; but we must state such second Life as equal to such a Term only, as such Life will be equal to at the Time of its suppos'd Commencement. This then would be the Term to which such second Life would be equal if it were a Thing certain that it did ever take Place; but there being a Contingency, by the Death of such second Person during the Life of the first, that it may never come into Possession at all, a Deduction is to be made out of such reduced Term equal to that Contingency: And these two Terms, *viz.* the Term to which the first Life is equal taken simply as it is, and the Term to which the second Life is equal when
computed

computed in this Manner on these two Reductions; these two Terms put together are the Term to which the two given Lives and the Survivor of them are equal.

This Method pursued will give us the Term for any three or more given Lives. The Term for two first Lives being known; the third Life as well as the second being only a reversionary Interest, and to have its Continuance from the Time when it commences, and such Commencement depending likewise on a Contingency: Such reversionary Life must be computed to commence from the Expiration of both the precedent Lives, and for the Contingency of Non-commencement a Deduction must be made from the reduced Term, in Proportion to the Number of Years by which the Lives of both the precedent Persons, or the Terms to which their Lives are equal, exceed the Term to which the third or last Life is equal. And these three Terms, *viz.* the Term of the first Life, taken simply as it stands, and the Term in both the other Lives, such double Reductions being first made, give us the Term for the three given Lives and the Survivor.

I do not pretend to prove this Rule right by a Mathematical Demonstration, for I am not Master enough of the common Processes in Algebra to enter into that Sort of Proof, and possibly the Thing may not be capable of it; and even that Proof sometimes fails us for Want of a proper Application; so we must content ourselves with an Instance or two, which may explain

my Meaning and Method; and at the same Time show the Grounds and Foundation of the Rule.

We will suppose then the Estate, whose Value for two or for three Lives we are enquiring after, to be 100*l. per Annum*; the Rate of Interest to be 6*l. per Cent.*; and the Term to which a single Life for the different Stages of Life is equal, to be according to the Estimate exhibited in my Table: Though at every other Rate of Interest, and upon any other fix'd Estimate of the Term to which any single Lives are equal, the Method will be the same, and will produce the Term to which two or three Lives are equal. We will put it then that N^o. 1. has an Interest in such an Estate for a Term of 28 Years in immediate Possession; N^o. 2. for a second Term of 28 Years to commence from the Determination of the first; and N^o. 3. for the like Term to commence from the Determination of the second; that N^o. 4. has an Interest in the like annual Estate for the Lives of *A.* and *B.* and the Survivor; and N^o. 5. for the Lives of *A. B.* and *C.* and the Survivor, all of 12 Years of Age. Now let us examine what is the Value of these several Interests belonging to these different Persons, and compare them together; and to avoid Perplexity, we will consider them separately.

The Value of the Term belonging to N^o. 1., being a Term for 28 Years in immediate Possession, is 1340*l.*: the Value of that belonging to N^o. 2., being a Term for 28 Years in Reversion after 28 Years, is 260*l.*; and the Value of that of N^o. 3., being a farther like reversionary

ary Term, is 54*l.*; and the Value of all the three Terms, being all together a Term of 84 Years, is in all 1654. The Interest of N^o. 4. in the Life of *A.* is equal to the Interest of N^o. 1., for they are both supposed to be Estates in Possession, and by my Table the Life of *A.* is estimated to be equal to a Term of 28 Years; the Value of that Interest therefore is 1340*l.* The Interest of N^o. 4. in the Life of *B.* supposed to be the Survivor is not equal to the Interest of N^o. 2., because the Interest of N^o. 2., though in Reversion, is an absolute Term, and certain both as to its Commencement and Continuance; whereas the Interest of N^o. 4. in the Life of *B.*, if ever it takes Place, cannot be computed to have a Continuance for a Term of 28 Years, but for such a Term only as the Life of *B.* is equal to at the Death of *A.*; and besides such Interest may never take Place at all, because of the Contingency that *B.* may die before *A.* At the Death of *A.*, that is at the End of 28 Years, the Age of *B.* will be 40, a Life of 40 by my estimate is stated to be equal to a Term of 21 Years, omitting the Fraction, and the Value of a Term for 21 Years, in Reversion after 28 Years, is 228: And this would be the Value of the Interest of N^o. 4. in the Life of *B.*, if the Interest were sure to take Place. But, there being a Contingency that *B.* may die before *A.*, a farther Deduction is to be made out of this Term in Proportion to such Contingency; and this Proportion is thus to be adjusted, *viz.* *A.* and *B.* being of the same Age, the Chance is

just an equal one which of the two becomes the Survivor; and an equal Chance to the Whole being the same Thing as an even Chance to a Moiety, one half of this Term of 21 Years must be deducted or withdrawn, and then 10 Years and an half will be left, as the Term to which the Life of *B.* is equal; and the Interest of N^o. 4. in the Life of *B.*, will be in Value 149*l.*; and his Interest in the two Lives of *A.* and *B.* will be equal to a Term of 38 Years and an half, in Value 1489*l.*

If we proceed in this Method and compare the Interest of N^o. 5. in the Life of *C.* with the Interest of N^o. 3. in his reversionary Term of 28 Years, it will appear on the Comparison that the Interest of the former falls much short of the Interest of the latter, for the same Reasons and under the same Heads as in the Case precedent. The Interest of N^o. 5. in the Life of *C.* cannot be computed equal to 28 Years, because at the Commencement of the Interest of N^o. 5. in the Life of *C.*, which is after the Determination of the two precedent Interests in the Lives of *A.* and *B.*, there can be no Pretence that *C.* should have an even Chance to live 28 Years; but his Life must be computed equal to a Term answerable to the Age he will be of at the suppos'd and computed Time of the Expiration of the former Lives; and it being suppos'd that *A.* lives 28 Years, and computed that *B.* will live 10 Years and an half, in all 38 Years and an half; the Age of *C.* at the Commencement of the Interest in his Life will be

50 and above, and a Life of 50 by the Estimate in my Table is stated at 17 Years, and a Term of 17 Years, in Reversion after 38 Years, is in Value 107*l*.: This therefore would be the Value of the Interest in his Life, were it sure to take Place. But this being uncertain, because of the Contingency that *C.* may die before *A.*, or *B.*, or before either of them, this Term must be reduced as before; and the Chance with him that he outlives them both being one in three, and the Chance against him that one or other of the two others outlive him being two in three, and a Chance of one in three to the whole Term of 17 Years being the same Thing, as an even Chance to a third Part of the Term, the Interest of N^o. 5. in the Life of *C.* will be equal to a Term of five Years and two thirds of a Year in Reversion after 38 Years, which is in Value about 42*l*.: The Interest therefore of N^o. 5, in the three Lives all together, will be equal to an absolute Term of 44 Years, and in Value 1531*l*. or thereabouts.

To these Computations of the Term, and Value of the several Annuities of N^o. 4. and N^o. 5. respectively, I see no Objection which can be made; and on the contrary, I apprehend they are confirmed by Dr. *Halley's* Maxim, which with a very little Variation may be applied, and will be rightly applied here. The Purchaser of an Annuity for the Term of a given Life, must pay for such Parts, and only such Parts, of the Term and the Annuity attending on it, as he has Chances, or the even
Chance

Chance, that his Nominee is living, and that he enjoys the Annuity. That the Interest of N^o. 4. in the Life of *A*. is equal to a Term of 28 Years will admit no Dispute; because it is supposed in the Case that the Life of *A*. is equal to a Term of 28 Years: and that his Interest in the Life of *B*. is equal to the very Term which I have assigned, neither more nor less, will I think almost as little admit of a Dispute. Upon the Interest of N^o. 4. in the Life of *B*. there can be no Sort of Pretence that it is at any Value during the Life of *A*.; or when it commences, that it can be computed to continue any longer than for such a Term as the Life of *B*. is equal to, or has a Chance to continue, from the Time of such Commencement: And here again 'tis supposed in the Estimate, that the Life of *B*., when the Interest in his Life commences, is equal to a Term of 21 Years, he being then of the Age of 40. As to the Deduction made out of this Term of 21 Years, by which it is reduced to 10 Years and an half; it must be admitted, that there is a Contingency that *B*. does not survive *A*., and they being of the same Age, 'tis plain that the Chance of Survivorship is just equal; and I suppose it will be admitted without an Algebraical Calculation, though it is capable of being so prov'd, that he who has an equal Chance or one Chance in two to the whole Term of 21 Years, has just an even Chance to a Moiety of it; and since the Interest of N^o. 5. in three Lives, in all respects stands on the same Foot; I conclude that the Terms and the Value

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of the Annuities attending on them, which I have assigned to the respective Annuitants, are the very Terms and Values which they have respectively the Chance to enjoy.

If then these my Computations of the Term and Value of these several Interests are right, the Interest of N^o. 4., which is for the Lives of *A.* and *B.* and the Survivor, is equal to a Term of 38 Years and an half, and is in Value 1489: And the Interest of N^o. 5., which is in the Lives of *A.* *B.* and *C.* and the Survivor of them, is equal to a Term of 44 Years, and in Value 1531, or thereabouts. To which I add by way of Corollary, that the Term to which such two or such three Lives and the Survivor are equal, at every Rate of Interest, whether it be higher or whether it be lower, is and must be the same; for that the Rate of Interest has no Influence, and it is impossible it should have any, on the Chance of the Duration of the Lives.

I cannot forbear observing here the great Difference which there will be in the Value of the Interest of N^o. 4. in the Life of *B.*, if such Value be computed by taking a Moiety of the Term, and what it will be if we take a Moiety of the Value of such Term. Here, where we take a Moiety of the Term of 21 which is 10 Years and an half, the Value corresponding to such 10 Years and an half, which is undoubtedly the Value of the Interest in the Life of *B.*, is 144: But a Term of 21 Years, in Reversion after 28 Years, is in Value 2-28, and if we take a Moiety of such Value, we shall have no more
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than 1-14, as the Value of the Interest in the Life of *B*; and the Value of the Annuity for the two Lives and the Survivor will be no more than 14-54, equal to a Term of 35 Years and something over. And if we compute the Interest of N^o. 5. in the Life of *C*. on the Chance of Survivorship in the same Manner, by taking one third of the Value of the Term when we should have one third of the Term itself, we shall find the like Deficiency; and instead of the Value 42, which is the Value of the third Part of the Term, and which on my Computation is the Value of the Interest in the Life of *C*., we shall have no more than 38, as being a third Part of the Value 1-14: So that, in this Way of computing, the Value of the Annuity for the three Lives would be no more than 14-92, equal only to a Term of 39 Years or less.

To make this Observation applicable to the present Purpose, I must take Notice that Mr. *Richards*, in his Table for the Value of Annuities on a single Life, makes the Term, to which a single Life of 12 Years of Age is equal, to be a Term of 28 Years or near it, Interest computed at 6 *l. per Cent.*; and that an even 28 Years is the Term to which I estimate a single Life of that Age to be equal, at that and at every other Rate of Interest: But in his Table for the Value of Annuities for two such and three such Lives, he makes the one equal to a Term of 48 Years and above, which I make no more than 38 Years and an half; and the other 60 Years and above, which I make only 47 Years. Now,
since

since in forming these Tables he made Use of Mr. *Moiivre's* Rule, and that Rule directs the Use of the Value of the Terms and not the Terms themselves, in making our Computations of the Value of these Annuities; and since, as I observed before, the Way of computing by the Values produces a shorter Term, as the Term to which two and three such Lives shall be equal, than the Way of computing by the Term produces: From hence I infer, if all the Chances of Vitality and the Terms corresponding to them are brought to Account in my Calculations, as I am almost confident they are, that in these Calculations there are inserted, where or by whose Hands I shall not here dispute, more Chances of Vitality, as belonging to two and to three such Lives, than do or can belong to them; otherwise the Term to which two such Lives would be equal, in that Way of Computation, would be little more than a Term of 35 Years; and the Term to which three such Lives would be equal would not only fall short of being a Term of 60 Years, but even of a Term of 39 Years.

What Values too great or too many are by Mr. *Moiivre's* Rule here inserted so as to swell the Values and the Terms in these Annuities to so monstrous a Degree, we have already considered; and have shown, that upon a fictitious Supposition that *A.* has a Chance to survive *B.*, and *B.* also a Chance to survive *A.*, and such a Chance that both may happen and take Effect, a Value answering to such double Chance is brought to the Account; and this appears evidently

dently to be the Case here on these two Lives. Here where the Value of the Chance of Survivorship on the Life of *B.* only is inserted, the Value is no more than 14-54; but if we add to this Sum 1-14, which is the imaginary Value of the Chance of Survivorship on the Life of *A.*, the total Value will be 15-68; which is the Value, within few Decimals, given in Mr. *Richards's* Table as the Value of an Annuity for two such Lives and the Survivor. Now 'tis not only impossible that *A.* should in Fact survive *B.* and *B.* also survive *A.*, and an Annuity cannot subsist unless the Life on which it attends subsists; but there is this farther Error, that we have already inserted in our Account a Term and a Value answerable to the whole Life of *A.*; therefore no Value or Term or any Chance of either can be now remaining. And if on the Life of *A.* there is no Chance left, and when he is dead sure enough there is none, and if the Chance on the Life of *B.* surviving is only a Chance to a Moiety of the Term to which his Life is equal at the Time he becomes the Survivor, which I think is very evident; it follows that my Rule inserts all and the Whole, and Mr. *Moiure's* Rule more than all and the Whole, of the Term belonging to such Lives.

In the Instance already given, the Lives are supposed to be of the same Age, but the more common Cases are that they are of different Ages, but there the same Rule will give us the Term to which two or three Lives are equal. The Calculation is indeed somewhat longer, but stands on the same Reason as in the former Case.

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Here the youngest of the Lives, whether they be two, three, or more Lives, should be placed first; and the Term to which such first Life is equal estimated as it stands by itself, must be taken as Part of the Term. As for the second and third Life, they must be placed in Order as they are in Juniority of Age; and out of the Term to which they are respectively equal at the Time of their Nomination, a double Deduction must be made here as before: One must be made in Regard that the Interest in their Lives is a reversionary one, and commences only from the Expiration of the precedent Estates; and this is regulated by computing those Lives as equal to such a Term as Lives are computed at which are of that Age of which the Nominees are when the Interest in their Lives respectively commences. Another Deduction is to be made on the Account of the Contingency, that by Death intervening such their Interest may be prevented from ever taking Place: And this is adjusted by subtracting out of the Term, when reduced in the Manner before directed, so many Years as are equal to the Chance that there is against such second Person that he does not survive the first, in Case of two Lives; and in Case of three Lives, so many Years as are equal to the Chance that the third Person does not survive the two precedent ones.

In the former Instance, where the Lives were all of the same Age, and where the Chance of Survivorship against the second Life was an equal Chance or one in two, and the Chance against the third Life was two in three; there the Term

to be deducted appear'd readily to be a Moiety of the reduced Term in one Case, and in the other two Thirds of the reduced Term. Here, where we suppose the Lives to be of different Ages, if we take three Lives whose Term is, suppose 28, 21, and 11 Years; the Manner of making the Computation is this. On the two first Lives, all the Chances put together are 49, *viz.* on the first Life 28, and on the second 21; and therefore the Chances of Survivorship with the second Life will be $\frac{21}{49}$ Parts of the reduced Term of such second Life, and the Chances against him $\frac{28}{49}$ Parts of the same Term: So that if we deduct these $\frac{28}{49}$ Parts out of the reduced Term of the second Life, the Term remaining will be the Term to which such second Life is equal; and then the Term of the first Life taken simply as it stands, and the Term on the second Life thus doubly reduced, when both put together, give us the Term to which the two Lives and the Survivor of them is equal. On the three Lives, all the Chances put together are 60, *viz.* on the two first Lives 49, and on the third Life 11; and therefore the Chances of Survivorship with the third Life will be $\frac{11}{60}$ Parts of the reduced Term on such third Life, and the Chances against him $\frac{49}{60}$ Parts of the same Term: So that if we deduct these $\frac{49}{60}$ Parts out of the reduced Term of such third Life, the Term remaining will be the Term to which such third Life is equal; and then this remaining Term, added to the Term for the two Lives, gives us the Term to which all three Lives and the Survivor of them is equal. We may calculate the Term to which this second

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and third Life are equal, on the Chance of Survivorship, in another and more easy Method, which will produce one and the same Term, *viz.* by taking $\frac{2}{4}\frac{1}{9}$ Parts of the reduced Term on the second Life as the Term to which such second Life is equal, and $\frac{1}{6}\frac{1}{5}$ Parts of the reduced Term on the third Life as the Term to which such third Life is equal.

To make my Rule and the Manner of using it more intelligible, we will calculate the Chance of the Duration of two and of three Lives of different Ages. We will suppose then *A.* to be 12 Years old, and his Life to be equal to a Term of 28 Years: *B.* to be 40 Years old, and his Life to be equal to a Term of 21 Years: And *C.* to be 60 Years old, and his Life to be equal to a Term of 11 Years, omitting Fractions. The Life of *A.*, taking that Life first, is equal to 28 Years; and this stands simply as it is, and is the first Term. At the End of 28 Years, when the Life of *A.* is supposed to determine, *B.* will be 68 Years old, and his Life will be then equal to a Term of 8 Years, and this would be the Interest in his Life, if it were sure to take Place. The Chance of its taking Place or not taking Place is $\frac{2}{4}\frac{8}{9}$ against him, and $\frac{2}{4}\frac{1}{9}$ with him; therefore his Interest is equal to $\frac{2}{4}\frac{1}{9}$ Parts of a Term of eight Years. The Term to which such Interest is equal is adjusted by multiplying the first and last Terms, that is 21 by 8, and dividing by the middle Term, that is by 49; and it comes out to be a Term of 3-1-63. This then is the Term to which the Life of *B.* is

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equal;

equal; and if this be added to the former Term on the Life of *A.*, the Whole will be 31-1-63; and the Interest in the Lives of *A.* and *B.* and the Survivor of them, will be equal to that Term. After the End of this Term of 31-1-63, *C.* the third Life will be 90 Years old; the Term in his Life will be equal to one Year only, and is the Interest in his Life, if it were sure to take Place. The Chance of its taking Place is 49 in 60 against him, and 11 in 60 with him; his Interest therefore is equal to $\frac{11}{60}$ Parts of one Year, or 365 Days, which comes out to be 67 Days: And if all these Terms, *viz.* 28-0-00, 3-1-63, and 0-0-67, are put together, the total Term will be 31-2-39; and is the Term to which the Lives of *A.* *B.* and *C.* and the Survivor is equal.

That the Annuitant who has an Estate for these two or for these three Lives and the Survivor, in this Way of computing his Interest, has all the Parts of the Terms of these two or these three Lives for which he has a Chance that his Nominees, or any one of them, are living, and if he has all the Terms he will enjoy all the Parts of the Annuity which he has a Chance to enjoy, since the Annuity will attend on the Term, is proved, if not mathematically, yet by a little common Reason. The Life of *A.* is supposed to be equal to a Term of 28 Years; and a Term of 28 Years is accordingly assign'd to the Annuitant as Part of his Term. During the Life of *A.* the Interest of the Annuitant in the Lives of *B.* and *C.* is, in some Sense, no
other

other than a dormant one, for as to the Receipt and Payment of the Annuity, the Being of one Life avails as much as the Being of all three Lives: But the Persons themselves, during this Time, are living on and advance in Age, and their Lives decay and are impair'd in Proportion to the Duration of the first Life; so that their Lives, at the Commencement of the valuable Interest in them, can be reputed equal to such a Term only as their Lives are respectively equal to at the Commencement of such Interest. In Pursuance of this we have reduced the Term to which the Life of *B.* is equal to a Term of eight Years, and the Life of *C.* to a Term of one Year; those being the Terms respectively to which the Lives of *B.* and *C.* respectively are equal at the Commencement of the profitable Interest in their Lives.

As to the Contingency of Survivorship, I have inserted all the Chances that can possibly arise on each of them, all I mean from which the Annuitant can receive any Benefit; and I may be confident that my Calculation of the Terms answering to such Chances is right, since 'tis instructed, as I apprehend, by the common Rules of Chances, and is the same which *Dr. Halley* makes Use of, with this Difference, that I apply the Rule to ascertain a Term, which he applies to ascertain a Value. In my Way I argue thus. The Number of Years to which the Life of *B.* is equal, when the Chance of Survivorship is to be calculated, is no more than eight Years in the whole, and that is all the Term he can have

if all the Chances of Survivorship were with him, and he were in Fact become the Survivor. The Life of *A.* is supposed to be equal to 28 Years, and the Life of *B.* to be 21 Years; so that the whole Number of Years or Chances between them both is 49; of these 49 21 are with *B.* that he becomes the Survivor, and 28 are against him that he does not: Therefore the Chance of *B.* is a Chance to $\frac{21}{28}$ Parts of a Term of eight Years, which on a Computation proves to be a Term of 3-1-63. Dr. *Halley* reasons thus. The total present Value of an Annuity, suppose of 10000 *l.* for one Year is 9434 *l.*; and no more than that Sum is such an Annuity worth, Interest computed at 6 *l. per Cent.*, where the Annuitant has all the Chances there are to receive it, and the Annuity were an absolute one. The whole Number of Chances, suppose in the first Year of the Life of a Person of 10 Years old, is 661; of these Chances 653 are with the Nominee that he does survive the Year, and 8 are against him that he does not survive the Year: Therefore the Chance of the Annuitant to the Annuity for the first Year is $\frac{653}{661}$ Parts of the Sum of 9434, which is 9319. This is my Way of Reasoning exactly, only differently applied: From whence I conclude, that the Annuitant has all the Terms, with the Annuity attending on them, which he has the Chance that any one of his Nominees is living.

To prevent any Surmise which may be made, that I have myself, under a former Head, disapproved this Way of Reasoning in Dr. *Halley*,

as I may seem to some to have done, I observe, that I do admit the Way of Reasoning to be right in general, but I urge, that 'tis wrongly applied in the Manner and in the Instance in which 'tis there applied. I have asserted, in Opposition to the Doctor, that 9319*l.* is not the Value of the Annuity for the first Year of the Life of a Person of 10 Years old, where the Annuity is for his Life absolutely; but where the Annuity is for one Year, and the first Year only of the Life of a Person of such an Age, if he shall so long live, and then to determine; there I admit that 9319*l.* is the true Value of an Annuity for such one Year, and the Annuity so determinable. The Difference of these two Cases is this: In the first Case I contend that there are no Chances of Mortality arise which are to be deducted in the first Year of the Life of the Nominee, but that such Chances as are computed to arise in such first Year are to be discounted out of the whole Life at large; but in the latter Case, the Chances of Mortality arising in the first Year must be then allow'd and deducted, or they cannot be allow'd and deducted at all, the Interest expiring with the Year. Therefore, where there are Chances of Mortality against an Annuitant, and an Allowance and Deduction ought to be made for them, there the Reasoning is rightly applied; but where there are no Chances against him, or none that are to be then and there deducted, here it will be wrongly applied. But in the Case on the Life of *B.*, and when his Chance of surviving *A.*

comes under Consideration, 'tis obvious, and very certain, that there are many Chances against him that he does not out-live 28 Years and become the Survivor, and that those Chances must then and there be deducted, or there is no Room to deduct them at all, and therefore the Reasoning is justly applied.

I am sensible, that by this Method, so far as we have hitherto gone into it, no Provision is made for the possible Contingency that a different Person or Persons, where they are of unequal Ages, may become the Survivors or Survivor; which yet may produce a Variance in the Term for the two or the three Lives respectively. Now the great Difficulty in adjusting the Value of Annuities for two or for three Lives and the Survivor, is to ascertain what is the Chance, or the Value of the Chance, on the possible Contingency of Survivorship, when two of three Nominees of unequal Ages are to become the Survivors, and when one of two is to be left the sole Survivor. Upon this I observe, that some Persons have been so careful to make a Provision sufficient for this possible Contingency, that in the Case of two Lives they have supposed both the Nominees to be the Survivor; and in the Case of three Lives, all the three to be surviving when there are two only. Now this must certainly be wrong; for though on two Lives there is a Contingency that *A.* may survive *B.*, and a Contingency that *B.* may survive *A.*, yet 'tis impossible it should happen that *A.* does in Fact survive *B.*, and *B.* also survive *A.* :

A. : And on three Lives, though there is a Chance that *A.*, and that *B.*, and that *C.*, whilst all three are living, may be the last Survivor, yet when one of the three is dead, there is no Chance that such one who is dead can be the last Survivor. Notwithstanding which, in some Calculations, and from the best of Hands too, we have seen that on two Lives the Interest in the Lives of both is computed as in Being, when the Interest of the one or the other is extinguish'd by Death; and that on three Lives, a Chance of all three becoming the last Survivor is inserted at a Time when one of the three is supposed to be dead and gone.

On my Method I observe; whether the Calculation be made on two, on three, or any other Number of Lives proposed, if the Persons are all in the same Period of Life, and their Lives of equal Goodness, the Contingency of Survivorship amongst them is entirely out of the Question, because there can be no Difference in the Chance: And with Regard to such Lives, I may venture to say, though I will not undertake to prove by Algebra, that my Method is mathematically right. Where Persons are in different Periods of Life, this Contingency of Survivorship must undoubtedly have some Influence on the Term to which the Lives are equal, or on the Chance of the Term to which they may be computed equal: And yet it will not make such a Variance in the Term, or in the Chance of the Term, as may commonly be apprehend-

ed, or such as perhaps would be thought of Moment.

But be the Influence more or less; the Rule, so far as we have already consider'd it, directs that the youngest of the given Lives be taken as the first, that the rest be placed as they are in Juniority of Age, that we compute the Term to which those Lives are separately and successively equal, that we make this Computation once only, and that we take the several Terms so arising, when all put together, as the Term to which the two Lives or the three Lives and the Survivor are equal. Now 'tis obvious that this Order of placing and computing the Chance of the Term on unequal Lives does not provide for all the several Contingencies of Survivorship which there are on those Lives; but yet I see Reason to think that, in general, it must produce a longer Term, as the Term to which these Lives are equal, than any other Order of placing them will produce. Upon the Calculations I have made for this Purpose, and I have made great Variety of them, I have found that it does produce either one and the same Term as any other Order of placing them produces, in some Instances to the Exactness of a single Day even in the Case of a large Number of Years, but most commonly produces a longer Term than any other Order of placing and computing does.

However, since there are Instances, for some few have fallen under my Observation, where this Method of proceeding will not give us the longest Term as the Term for such Lives; we
must

must vary the Position of the Lives given as oft as those possible Contingencies on the Lives admit of; and then proceed, in the same Method as before, to calculate to what Term each of the given Lives will be equal in all the possible Shapes of Survivorship: And that Term, which, upon repeating the Calculation so often, and which, upon putting together the several Terms so collected, comes out to be the longest Term for the Lives nominated, is the Term to which those Lives are equal: And this, I think, allots to the Annuitant just the Term which he has a Right to, *viz.* the best of the Chances in the Term of the given Lives.

Mr. *Richards*, in his Tables for the Value of Annuities on two and on three Lives, which he form'd by Mr. *Moiivre's* Rule, has assign'd such a Value of Annuities for such Lives as I have just mention'd, *viz.* for two Lives of 12 and 42, and for three Lives of 12, 42, and 62, as that the Terms corresponding to those Values, Interest computed at the Rate of 6 *l. per Cent.* do much exceed the Terms to which I have made such Lives, or Lives rather better, to be equal: For the Value of the Annuity for the two Lives given in the Tables is equal to a Term of 36 Years and above, which in my Way and on my Lives I make no more than 31-1-63; and the Value on the three Lives there given is equal to a Term of 40 Years, or near it, which in my Lives I make no more than 31-2-39. And yet I observe, on his Tables for the Value of Annuities on a single Life, computed at this Rate of Interest,

Interest, and the Observation holds good on Dr. *Halley's* Table for the Value of the like Annuities, and at the same Rate of Interest, that the Term corresponding to the Value, given in these Tables, as the Value of the Annuity for every one of these Lives, when singly taken, is a Term shorter than I assign to those Lives, and the Life of 42 in particular falls short of the Term to which I compute such Life to be equal by five Years, or near it: The Consequence of which must be, that the Term corresponding to the Value of all the three Lives singly taken and put together, in their Way of computing, must fall short likewise of the Term which I assign to the same three Lives singly taken and put together. Therefore, since the Term, to which these three Lives singly taken and put together is equal, in their Way of Estimate, falls so much short of the Term, to which those Lives singly taken and put together is equal, in my Way of Estimate: And since the Term to which these three Lives taken as combined and surviving Lives, in their Way of computing is equal, so greatly exceeds the Term to which these Lives taken as combined and surviving Lives, in my Way of computing is equal: From hence it necessarily follows, either that in my Account of the Term for the combined and surviving Lives I have not inserted all the Parts of the Terms in these Lives which the Annuitant has a Chance to enjoy, or that Mr. *Richards* has inserted more than all the Parts of the Value of the Annuity which he has a Chance to receive.

Upon

Upon either of our Computations I will make no farther Reflections, having insisted very largely on them before ; and shall add only this. Calculations in Algebra may be right, infallibly right ; but then they must be grounded on Principles or Suppositions ; and those Principles, unless they are self evident, and the Suppositions, unless they are such as all Mankind will concur in, must be proved ; and by a Proof which perhaps may arise from something which has no Relation to nor can be measured by a mathematical Rule : And therefore I leave it with the Authors to make good the Suppositions on which their algebraical Calculations are founded.

I should here put an End to this Discourse ; but I cannot forbear first taking Notice of another algebraical Calculation, made by a different Hand, to adjust the Credibility of human Testimony ; where we meet with a most wicked Application of the Mathematicks. It is not indeed directly and immediately to the present Purpose, but it will serve to show us that Mathematicians sometimes make very strange Suppositions, and then build thereon very good Calculations, and from right Calculations sometimes deduce very wrong Consequences : And thus far the Calculations have an Affinity the one to the other, that this Writer makes use of the Value of Money as a Measure to ascertain the Duration of the Credibility of human Testimony much in the same Manner, which Dr. *Halley* and Mr. *Moivre* make Use of it to determine the
Chance

Chance of the Duration of a Life or Lives ;
and I think with equal Success.

I have the rather chosen to make some Remarks on this Calculation, because it has been taken Notice of by the Author of that celebrated Piece, *Christianity as old as the Creation*, and I suppose he would be understood that it was applicable and of Moment to his Purpose. He does not indeed directly make an Application of it ; but whether he did not know how to make it, or meant to leave it to his willing Reader to do it, I can't say ; but I believe no Man makes any Doubt of his Good-Will.

The Proposition is this ; which I shall give you in the Author's own Words, because I think it to be a very extraordinary one, since 'tis not only founded on chimerical Suppositions, but the Conclusion is directly wrong.

At the Rate of 6 l. per Cent. Interest, the present Value of any Sum payable after twelve Years is but half the Sum ; so if the Proportion of Certitude transmitted by each Reporter be $\frac{1}{100}$, the Proportion of Certainty after twelve Transmissions will be but as half ; and it will grow by that Time an equal Lay whether the Report be true or no.

If we admit his first Position, on the present Value of a Sum of Money payable *in futuro*, to be true, as most certainly it is ; yet it is grounded on a Supposition of his own making, to serve a Turn, for we may suppose Money at any other Rate of Interest as well as at 6 l. per Cent. : And if we admit the second Position, on the Proportion of Certainty or Credibility, to be

be true likewise, and here the Suppositions are such that no Man surely but himself could make: Yet the Conclusion, *viz.* it will grow an equal Lay whether the Report be true, has neither Sense nor Truth in it. We do indeed, in a common Way of speaking, say that such a Report is true, or such a Report is false; but the Meaning of that Expression is, that the Fact reported is true or is false: And surely here, where the Proposition is to be demonstrated by a mathematical Proof, it would have been much more proper, at least, to have concluded that it would be an even Lay whether the Fact reported were true, and not whether the Report were true; for most certainly there may be a true Report of a Fact not true.

If then we understand this Proposition in such a Manner as to make Sense of it, yet 'tis neither true nor just, because there is plainly more in the Conclusion than in the Premises; for the Premises and the subsequent Calculations do not show, nor are they intended to show the Duration of the Truth of a Fact reported, for that is perpetual, but the Duration of its Credibility; and the Truth of a Fact and the Credibility of it are very different Things. Matters of Fact may be true, and many are so, which yet are not, nor ever were credible, to some Persons at least, for want of proper Evidence to prove them; and if once they were credible they may cease to be so, and will cease to be so, if and when the Evidence is lost or sunk; but if once they were, they will always continue to be true. Here then
the

the Author manifestly over-does the Matter, when he concludes to the Verity or Reality of a Fact, whereas his Undertaking was, and he could undertake, only to compute its Credibility. Therefore the Conclusion should have been this, and no more than this: After twelve Transmissions it will grow an equal Lay whether the Fact reported be credible or no.

But when the Proposition is stated, as it ought to have been, in this plain Language, and is understood in this Sense, it manifestly appears to be false: And I cannot but think that the Author, if he would have given himself Leave to have express'd his Thoughts in this open Manner, must have seen the Falsity of it. Admitting that by twelve Transmissions of a Fact reported one Half of the Degrees of Certainty or Credibility of the Fact are sunk, does it follow that the other Half are sunk too, and that there are none left; or that a Fact reported ceases wholly to be credible, because or when 'tis but half as credible as it was at first? 'Tis downright palpable Nonsense; and it would be as good Reasoning and as much Sense to say of a Sum of Money, suppose a 1000 *l.* that when 'tis sunk to 500 *l.* it ceases to be any Money at all, or that the Sum remaining, if there could be said to be any, was of no Value.

For my own Part, I must confess, I cannot frame any Idea of the Relation between the Increase and Decrease of Money, or of any other mathematically measurable Quantity, and the Increase or Decrease of the Credibility of a Fact reported;

reported; and most People, I believe, would look upon a Pound or an Ell of Credibility as a very great Rarity: But if any one can conceive a Similitude between them, they must admit this farther Similitude in the Case. According to my little Mathematicks, if out of a Pound of Money in a Purse we take at stated Times, let those Times return never so often, a Part in Proportion to what we find in the Purse; or if from a Line of an Ell long we should cut off once a Year, or once a Day if you will, a Part of such Line in Proportion to what is remaining; I think we shall never, as long as the World lasts, get to the Bottom of the Purse, or to the End of the Line, so as that the one or the other shall be reduced to nothing; and therefore if the Credibility of a Fact is compared to a Pound of Money or to an Ell of Line, so proportionably decreasing, it can never be totally annihilated, or a Fact reported, that was once credible, ever lose the Whole and all its Degrees of Credibility.

Or, if the Credibility of a Fact reported may be resembled to Money, take it in this Light. If a Bag of Money is transmitted through several Hands, on these Terms, that every one into whose Hands it comes is at Liberty to take out a certain proportionable Part of what he finds in it, the Case never will be such that there shall be no Money in the Bag, though it goes through a Million of Hands, unless and until it falls into the Hands of some one who is Knave enough to take more than his Proportion, and take all. Just
in

in the same Manner will it fare with a Fact transmitted by Report through several Hands, supposing it to lose a proportionable Part of its Credibility by every Hand through which it goes; if the Fact was once credible, it will continue eternally credible, unless and until the Report falls into the Hands of a Person whose Character is such that from his Hands, and his Hands are the sole Hands through which it can any farther be transmitted, it will entirely cease to be credible.

From these Premises then, and if we can make the Suppositions and Resemblances here made, the Conclusion will not be, that after twelve Transmissions of a Fact reported it will become an even Lay whether it be true or be credible or not: On the contrary, the Conclusion will be, and it will be a mathematically necessary one, though the Transmissions should be so many as not to be knowable and numerable, which is almost always the Case, yet that it can never become an equal Lay that 'tis not credible. Nay, if we make such Suppositions here as may be made and must be made on a Report which has gone through twelve Transmissions, *viz.* that 'tis a Fact worthy of Notice and of some Consequence, and that 'tis reported as such Facts usually and commonly are reported; the Inference will be, and the Inference here too will be mathematically necessary, that the Fact, whether it be a recent one, or one of former Ages, by this Multiplicity of Transmissions will gather Degrees

Degrees of Credibility, and as it grows older in Age will grow stronger in Credibility.

This Author's Suppositions in his second Position are, that a certain and stated Proportion of Credibility, may be assigned to twelve Reporters of a Fact, and that every one of such twelve Transmissions, is from a single Person to a single Person only; which I say are such Suppositions as no one could make but himself, and most certainly they subsist no where but in the Imagination of a most fruitful Brain. When and where is that Report to be met with, that every Report-Maker has just the very same Degrees and Parts of Degrees of Credibility appertaining to him, which this Calculator has severally carved out for him? What is that Fact, or how can we conceive a Fact to be such, that there shall be twelve Reporters of it who shall successively transmit it from one to the other, and no one of them shall make a Report of the same Fact to more than a single Person? Now if any one of these Transmitters of this Report should happen to have more Degrees of Credibility inherent in him than this Author has been willing to allow him, the Auditor will be cruelly at a Loss, notwithstanding this Mathematical Rule which he may have in his Pocket, when and where he is, and when and where he is not, to give Credit to a Report; and if any one of these Transmitters should communicate the Fact to more than one Person, and if it be not worth relating to more than one Person, he may e'en as well keep it wholly to himself, but if he

so divulges it there's an End of our supposed single Transmissions, and the Report may come to the last Hand by a double, treble, or sextuple Transmission, and then the Evidence is enlarged, as this Author himself admits, in Proportion to the Degrees of Credibility inherent in each Reporter, and to the Number of Persons reporting.

And after all these wild Suppositions, and the Calculations grounded upon them, we remain still in the same State of Uncertainty as to the Credibility of a Fact reported; for all the Disputes that ever were or possibly can be on a Matter of Fact, if Persons know or mind what they are disputing about, though I doubt that is not always the Case, are on that Point solely how far and to what Degree the Person first relating a Fact, and the Persons successively transmitting the Report of it, are credible Persons; for the Credibility of a Fact depends solely and wholly on the Credibility of the Person testifying and reporting it. Now the Credibility of the Person depends on his Character, that is on his Abilities, and the Opportunities which he had to be rightly informed in the Things he saw, or were related to him, and his Honesty and Integrity in faithfully transmitting what he so saw, or was related to him; and his Character in these Instances will be the same, whether he be the first, or the twelfth, or the thousandth Hand: So that the sole Difficulty is, to discover and form a right Judgment what Degrees of Credibility may justly be assigned to each Person.

Now

Now this Gentleman takes it for granted, that this Matter, which is the very Thing and the only Thing, which is or can be in Dispute, is a settled and stablish'd Point; and very gravely states the Degrees of Credibility of Persons, at a Rate and Proportion certain, for every one the same: And then with great Sagacity, and upon a just Calculation, finds out that if 50 Degrees of an 100 are gone, there is no more than one Half left, which I suppose he might have done without the Aid of a Mathematical Rule: But the Discovery which he makes in his Conclusion, that when one Half is left they are all gone, that there is so new and peculiar to himself, that I doubt, even with the Aid of his Mathematical Rule, he will not be able to make it good.

I have asserted before that this Author's first Position on the present Value of a Sum of Money payable at the End of twelve Years, depends on a Supposition of his own making, and one made on purpose to serve a Turn; and most certainly it does depend on this, that the Sum of Money, calculated to be the present Value of a Sum payable at the future Time, is supposed to be put out at 6 *l. per Cent.* Interest, that it is computed at Compound Interest from the End of each Year and no oftner, and that at the stated future Time it will become just double the Sum, and be equal to the Sum total then payable. But what if Persons should differ in their Opinion about the Degrees of Credibility, which may be assigned to the several, or any of the Reporters of a Fact, and one should think that

the Proportion here stated is too great, and another should think it too small? Why, truly for this possible Case the Author has made a Provision tolerable good; for as there are different Degrees of Credibility in the Parties testifying or transmitting the Report of a Fact, so there are different Rates of Interest of Money; and as the Proportion of Credibility runs higher or lower, we must take a higher or lower Rate of Interest; and the Receipe will certainly keep either at Land or at Sea: So that a Person need only to have a common Table of Interest, computed at Variety of Interests, and by a proper Application to the particular Case, may readily calculate how many Degrees of Credibility there must be left before they are all gone, and he may justly withhold his Assent to the Truth of a Fact reported.

But, since this Gentleman supposes that the Interest of Money is computed as due once a Year only, and once a Year only such Interest is accumulated, and supposes that the Transmission of the Report of a Fact from one Hand to another in one Case, must be just as often and correspond to the Times of computing, and compounding Interest in the other; what if a Person, who knows *Res agere*, and understands the Difference of compound Interest, where it is computed once a Year, and where it is computed every half Year, or every Quarter, or every Day, should have a Mind to compute and accumulate his Interest every Day, and if one, who is as credulous as the other is covetous, should claim

claim as many Transmissions of a Report as the moneyed Man takes Times of accumulating his Interest; What shall we do here? This seems to be a Case omitted: but, where Interest is computed and in Fact accumulated every Day, I suppose the moneyed Man in ten Years Time perhaps, or thereabouts, may see his original Sum doubled, or, which is the same Thing, the present Value of the Sum payable at the End of ten Years, Interest so often compounded, will be no more than half the Sum; and then the Person who is measuring the Degrees of Credibility, and has a Title to as many Transmissions as the other takes Times of accumulating Interest, instead of twelve Transmissions will be entitled to ten Times 365, that is 3650 Transmissions, there being so many Days of accumulating Interest in the Space of ten Years, before the Degrees of Credibility will be sunk to one Half. And the latter will have greatly the Advantage too, because the moneyed Man will not see his Principal doubled in ten Years Time, unless in Fact as well as in his Computations he accumulates Interest daily, whereas the Dealer in Credibilities can never meet with a Disappointment; for if his Computation is right, and after such plain Instructions there is no great Difficulty in making it right, the rest is ready cut and dried to his Hands.

Upon the Whole; I think the Representation here given, of the Duration of the Credibility of a Fact reported on human Evidence, is a Piece of meer Pageantry, and that not well

drawn neither: And the Use I would make of it is only this, that where vain or wrong Suppositions are made, and on them just Calculations are founded; and where just Calculations are made, and from them wrong Inferences are deduced, we can make no safe or just Conclusions: and there I rest the Matter.



E L B A T

T A B L E I.

Showing the Value of Annuities, for the Term, and at the Interest, in the Table; in Years, Quarters, Days; and the centesimal Parts of a Year, corresponding to the Quarters and Days.

Terms of Years.	3 <i>l.</i> per Ct.			C. P.	4 <i>l.</i> per Ct.			C. P.	5 <i>l.</i> per C.			C. P.
	Y.	Q.	D.		Y.	Q.	D.		Y.	Q.	D.	
1	00	3	80	97	00	3	77	96	00	3	69	95
2	01	3	58	91	01	3	47	88	01	3	40	86
3	02	3	29	83	02	3	11	78	02	2	80	72
4	03	2	77	71	03	2	44	62	03	2	14	54
5	04	2	29	58	04	1	69	44	04	1	29	33
6	05	1	62	42	05	0	84	23	05	0	25	07
7	06	0	87	24	05	3	87	99	05	3	14	79
8	07	0	11	03	06	2	80	72	06	1	77	46
9	07	3	22	81	07	1	62	42	07	0	40	11
10	08	2	22	56	08	0	36	10	07	2	80	71
11	09	1	14	29	08	3	00	75	08	1	22	31
12	09	3	87	99	09	1	44	37	08	3	44	87
13	10	2	62	67	09	3	80	97	09	1	51	39
14	11	1	29	33	10	2	18	55	09	3	51	89
15	11	3	80	97	11	0	36	10	10	1	47	38
16	12	2	33	59	11	2	51	64	10	3	33	84
17	13	0	69	19	12	0	55	15	11	1	07	27
18	13	3	11	78	12	2	51	64	11	2	66	68
19	14	1	36	35	13	0	44	12	12	0	29	08
20	14	3	51	89	13	2	33	59	12	1	77	46
21	15	1	62	42	14	0	03	01	12	3	25	82
22	15	3	69	94	14	1	69	46	13	0	58	16
23	16	1	69	44	14	3	33	84	13	1	84	48
24	16	3	62	92	15	1	00	25	13	3	14	79
25	17	1	55	40	15	2	40	61	14	0	33	09

TABLE I. continued.

Term Years.	3 l. per C.			C.P.	4 l. per C.			C.P.	5 l. per C.			C.P.
	Y.	Q.	D.		Y.	Q.	D.		Y.	Q.	D.	
26	17	3	40	86	15	3	80	97	14	1	44	37
27	18	1	22	31	16	1	22	31	14	2	51	64
28	18	3	00	75	16	2	51	64	14	3	51	89
29	19	0	62	17	16	3	80	97	15	0	47	13
30	19	2	29	58	17	1	14	29	15	1	40	36
31	19	3	84	98	17	2	25	57	15	2	33	59
32	20	1	44	37	17	3	36	85	15	3	18	80
33	20	3	00	75	18	0	40	11	16	0	00	—
34	21	0	44	12	18	1	44	37	16	0	69	19
35	21	1	84	48	18	2	44	62	16	1	44	37
36	21	3	29	83	18	3	40	86	16	2	14	54
37	22	0	62	17	19	0	36	10	16	2	73	70
38	22	1	87	49	19	1	29	33	16	3	40	86
39	22	3	14	79	19	2	22	56	17	0	03	01
40	23	0	40	11	19	3	11	78	17	0	55	15
41	23	1	62	42	20	0	00	—	17	1	18	30
42	23	2	77	71	20	0	73	20	17	1	66	43
43	24	0	00	—	20	1	51	39	17	2	11	53
44	24	1	07	27	20	2	25	57	17	2	55	65
45	24	2	11	53	20	2	84	73	17	3	07	77
46	24	3	11	78	20	3	55	90	17	3	47	88
47	25	0	07	02	21	0	18	05	17	3	84	98
48	25	1	00	25	21	0	73	20	18	0	29	08
49	25	1	80	47	21	1	33	34	18	0	62	17
50	25	2	62	67	21	1	80	47	18	1	00	25
51	25	3	40	86	21	2	36	60	18	1	29	33
60	27	2	66	68	22	2	40	61	18	3	66	93
70	29	0	44	12	23	1	51	39	19	1	33	34
80	30	0	77	21	23	3	55	90	19	2	33	59
90	31	0	00	—	24	1	03	26	19	3	00	75
100	31	2	36	60	24	2	00	50	19	3	36	85
Fee	33	1	29	33	25	—	—	—	20	—	—	—

TABLE I. continued.

Term of Years.	6l. per C.				C.P.	7l. per C.				C.P.	8l. per C.				C.P.
	Y.	Q.	D.			Y.	Q.	D.			Y.	Q.	D.		
1	00	3	69		94	00	3	66		93	00	3	62		92
2	01	3	29		83	01	3	22		81	01	3	11		78
3	02	2	62		67	02	2	44		62	02	2	29		58
4	03	1	77		46	03	1	51		39	03	1	22		31
5	04	0	77		21	04	0	36		10	03	3	87		99
6	04	3	62		92	04	3	07		77	04	2	44		62
7	05	2	29		58	05	1	51		39	05	0	77		21
8	06	0	77		21	05	3	80		97	05	3	00		75
9	06	3	18		80	06	2	03		51	06	1	00		25
10	07	1	40		36	07	0	07		02	06	2	77		71
11	07	3	47		88	07	2	00		50	07	0	51		14
12	08	1	47		38	07	3	69		94	07	2	14		54
13	08	3	36		85	08	1	40		36	07	3	55		90
14	09	1	14		29	08	2	87		74	08	0	87		24
15	09	2	77		71	09	0	40		11	08	2	22		56
16	10	0	36		10	09	1	73		45	08	3	36		85
17	10	1	80		47	09	3	03		76	09	0	44		12
18	10	3	29		83	10	0	22		06	09	1	44		37
19	11	0	58		16	10	1	29		33	09	2	36		60
20	11	1	80		47	10	2	33		59	09	3	25		82
21	11	3	03		76	10	3	29		83	10	0	07		02
22	12	0	11		03	11	0	22		06	10	0	73		20
23	12	1	18		30	11	1	07		27	10	1	44		37
24	12	2	18		55	11	1	80		47	10	2	11		03
25	12	3	11		78	11	2	55		65	10	2	62		67
26	13	0	00		—	11	3	29		83	10	3	22		81
27	13	0	77		21	11	3	87		99	10	3	66		93
28	13	1	55		40	12	0	51		14	11	0	18		05
29	13	2	29		58	12	1	11		28	11	0	58		16
30	13	3	03		76	12	1	58		51	11	1	03		26

TABLE I. continued.

Term of Years.	6l. per C.			C.P.	7l. per C.			C.P.	8l. per C.			C.P.
	Y.	Q.	D.		Y.	Q.	D.		Y.	Q.	D.	
31	13	3	66	93	12	2	18	55	11	1	36	35
32	14	0	33	09	12	2	66	68	11	1	66	43
33	14	0	84	23	12	3	18	80	11	2	00	50
34	14	1	47	38	12	3	58	91	11	2	22	56
35	14	2	03	51	13	0	03	01	11	2	44	62
36	14	2	47	63	13	0	36	10	11	2	66	68
37	14	2	87	74	13	0	66	18	11	2	87	74
38	14	3	33	84	13	1	00	25	11	3	18	80
39	14	3	69	94	13	1	25	32	11	3	40	86
40	15	0	11	03	13	1	47	38	11	3	62	92
41	15	0	44	12	13	1	66	43	11	3	80	97
42	15	0	73	20	13	1	84	48	12	0	03	01
43	15	1	11	28	13	2	11	53	12	0	18	05
44	15	1	40	36	13	2	29	58	12	0	29	08
45	15	1	66	43	13	2	44	62	12	0	40	11
46	15	2	00	50	13	2	58	66	12	0	47	13
47	15	2	22	56	13	2	73	70	12	0	55	15
48	15	2	44	62	13	2	87	74	12	0	62	17
49	15	2	66	68	13	3	07	77	12	0	69	19
50	15	2	87	74	13	3	18	80	12	0	77	21
51	15	3	14	79	13	3	29	83	12	0	84	23
60	16	0	62	17	14	0	00	—	12	1	40	36
70	16	1	51	39	14	0	40	11	12	1	69	44
80	16	2	00	50	14	0	55	15	12	1	80	47
90	16	2	29	58	14	0	66	18	12	1	84	48
100	16	2	40	61	14	0	73	20	12	1	87	49
Fee	16	2	61	67	14	1	14	29	12	2	—	50

T A B L E II.

Showing in the first Column the Age of Persons, in the second the Term of Years and Quarters to which any given Life, in those Periods, is computed to be equal, according to my Way of making the Estimate; to which I have added, in the third Column, the Difference or Decrease in the Term, from one Period to another.

Ages of Persons from	Term to which such Lives equal.	Decrease in the Term.
10 to 15 —	28 0	—
15 — 20 —	27 2	0 2
20 — 25 —	26 3	0 3
25 — 30 —	25 3	1 0
30 — 35 —	24 2	1 1
35 — 40 —	23 0	1 2
40 — 45 —	21 1	1 3
45 — 50 —	19 1	2 0
50 — 55 —	17 0	2 1
55 — 60 —	14 2	2 2
60 — 65 —	11 2	3 0
65 — 70 —	8 0	3 2
70 — 75 —	4 0	4 0
75 — 80 —	2 —	—
80 &c. —	1 —	—

From this and the precedent Table is readily seen the Value of an Annuity for any Life given at any Rate of Interest: For this Table gives the Term to which the Life is equal, and the precedent Table gives the Value, or Year's Purchase, to which such Term is equal; that is the Value of the Annuity.

T A B L E III.

Shewing, in the first Column, the Age of Persons, in the second the Value of Annuities for every fifth Year of Age, from the 10th to the 70th Year of Man's Life, as stated by Dr. *Halley*, Interest computed at 6*l. per Cent*: To which I have added, in the third Column the Years, Quarters, and Days, to which such Value corresponds; in the fourth Column the Difference or Decrease in the Term, from one Period to another; and in the fifth, the Difference or Decrease in the Value, in the same Manner.

Ages of Persons	Years Value or Purchase.	Term to which such Value equal	Decrease in the Term.	Decrease in the Value.
10	13 44	28 1 00		
15	13 33	27 2 61	0 2 30	00 11
20	12 78	25 0 00	2 2 61	00 55
25	12 27	22 3 45	2 0 46	00 51
30	11 72	20 3 45	2 0 00	00 55
35	11 12	18 3 45	2 0 00	00 60
40	10 57	17 1 10	1 2 35	00 55
45	09 21	15 2 00	1 3 10	00 66
50	09 21	13 3 30	1 2 61	00 70
55	08 51	12 1 12	1 2 18	00 70
60	07 60	10 2 00	1 3 12	00 91
65	06 54	08 2 00	2 0 00	01 06
70	05 30	06 2 30	1 3 61	01 24

TABLE IV.

Shewing, the Value of Annuities for a Life,
at the Ages mentioned, Interest computed
at 4*l.* *per Cent.*, as stated in Mr. *Hayes's*
Tables; with the same Additions as in the
precedent Tables.

Ages of Persons	Years Value or Purchase.	Term to which such Value equal	Decrease in the Term.	Decrease in the Value.
30	15 54	24 3 23		
35	14 51	22 0 55	2 2 68	1 03
40	13 49	19 3 28	2 1 27	1 02
45	12 41	17 2 22	2 1 06	1 08
50	11 28	15 1 45	2 0 77	1 13
55	10 10	13 0 84	2 0 61	1 18
60	08 80	11 0 33	2 0 51	1 30
65	07 26	08 3 07	2 0 26	1 54
70	05 54	06 1 84	2 1 13	1 72
72	0	42 2 1	07 0 21	12 80
73	0	30 2 1	07 1 11	11 80
74	1	07 2 1	00 2 00	80 70
75	1	00 0 2	00 2 70	09 70
76	1	08 2 1	11 2 70	87 40
77	1	00 0 2	11 2 30	82 30
78	1	22 1 2	08 1 10	03 10

TABLE V.

Showing, the Value of Annuities for a Life,
at the Ages mentioned, Interest computed
at 6*l. per Cent.*, as stated by Mr. *Richards*:
with the same Additions as in the precedent
Tables.

Ages of Persons.	Years Value or Purchase.	Term to which such Value equal	Decrease in the Term.	Decrease in the Value.
7	13 42	28 0 36		
12	13 36	27 3 00	0 1 36	0 06
17	12 99	25 3 74	1 3 17	0 37
22	12 54	23 3 76	1 3 89	0 45
27	11 97	21 3 00	2 0 76	0 57
32	11 47	20 0 00	1 3 00	0 50
37	10 90	18 0 77	1 3 14	0 57
42	10 24	16 1 10	1 3 67	0 66
47	09 58	14 2 74	1 2 27	0 66
52	08 91	13 0 50	1 2 24	0 67
57	08 11	11 1 76	1 2 65	0 80
62	07 08	09 2 00	1 3 76	1 03
67	05 90	07 2 00	2 0 00	1 18
72	04 78	05 3 11	1 2 80	1 12
77	03 29	03 3 11	2 0 00	1 49
82	01 36	01 1 80	2 1 22	1 93

T A B L E VI.

Showing the Value of Annuities for a Life, at the Ages mention'd, Interest computed at 4 *l. per Cent.*, as stated by Mr. *Morris*: with the same Additions as in the precedent Tables.

Ages of Persons.	Years Value, or Purchase.	Term to which such Value equal.	Decrease in the Term.	Decrease in the Value.
10	16 72	28 0 68		
15	16 53	27 2 68	0 2 00	0 19
20	15 62	25 0 22	2 2 46	0 91
25	14 80	22 3 68	2 0 45	0 82
30	13 96	20 3 45	2 0 23	0 84
35	13 08	18 3 60	1 3 75	0 88
40	12 29	17 1 15	1 2 45	0 79
45	11 39	15 2 30	1 2 75	0 90
50	10 45	13 3 15	1 3 15	0 94
55	09 55	12 1 18	1 1 88	0 90
60	08 40	10 2 00	1 3 18	1 15
65	07 12	08 2 50	1 3 31	1 28
70	05 70	06 2 45	2 0 05	1 42

T A B L E VII.

Showing, in the first Column, the Rate of Interest, in the second the Value of an Annuity for a Life of 12 Years Age, transcribed from Mr. *Richards's* Tables: To which I have added, in the third Column, the Term of Years to which such Value corresponds, as I calculate the same.

Interest.	Value.	Term of Years.
4	17 20	29 3 00
5	15 23	29 1 44
6	13 36	27 3 00
7	11 72	25 1 23
8	10 67	25 0 00

T A B L E VII.

Showing, in the first Column, the Rate of Interest, in the second, the Value of an Annuity for a Life of 30 Years Age, extracted from Mr. *Hayes's* Tables: To which I have added, in the third Column, the Term of Years to which such Value corresponds, as stated by Mr. *Hayes* himself.

Interest.	Value.	Term of Years.
4	15 54	25
5	13 12	22
6	11 27	20
7	09 83	18
8	08 68	16

Mr. *Hayes* seems to me, in this Table, first to have stated to what Term a Life of 30, or any other Age, is equal, and from thence to have computed the Values: Now, in what Way of Thinking, or on what Grounds, he could in the first Instant suppose that one and the same Life could be equal only to a Term of 16 Years, and at the same Time be equal to a Term of 25 Years, I confess exceeds my Comprehension.

Interest.	Value.	Term of Years.
4	17 20	25
5	15 23	22
6	13 30	20
7	11 72	18
8	10 07	16

T A B L E

T A B L E IX.

Exhibiting a Computus of the Value of an Annuity of 1000*l.* per Annum, for a Life of 10 Years Age, made according to Dr. Halley's Rule, for every Year of Life to an 100 Years of Age.

	multiply'd	by	produces		Integrals.—Fractions
1	9 434	653	6 160 402	This last Sum, divided by 661, leaves a Remainder,	9 319 — 543
2	8 900	646	5 749 400		8 698 — 022
3	8 396	640	5 373 440		8 129 — 171
4	7 921	634	5 021 914		7 597 — 297
5	7 473	628	4 693 044		7 099 — 605
6	7 050	622	4 385 100		6 634 — 026
7	6 650	616	4 096 400		6 197 — 183
8	6 274	610	3 827 140		5 789 — 611
9	5 919	604	3 575 076		5 408 — 388
10	5 584	598	3 339 232		5 051 — 521
11	5 268	592	3 118 656		4 718 — 058
12	4 970	586	2 912 420		4 406 — 054
13	4 688	579	2 714 352		4 106 — 286
14	4 423	573	2 534 379		3 834 — 105
15	4 173	567	2 366 091		3 579 — 372
16	3 936	560	2 204 160		3 334 — 386
17	3 714	553	2 053 842		3 107 — 115
18	3 503	546	912 638		2 893 — 365
19	3 305	539	1 781 395		2 695 — —
20	3 118	531	1 655 658		2 504 — 514
21	2 941	523	1 538 143		2 326 — 657
22	2 775	515	1 429 125		2 162 — 043
23	2 618	507	1 327 326		2 008 — 038
24	2 470	499	1 232 530		1 864 — 426
25	2 330	490	1 141 700		1 727 — 153
26	2 198	481	1 057 238		1 599 — 299
27	2 074	472	978 928		1 480 — 648
28	1 956	463	905 628		1 370 — 058

T A B L E IX. continued.

29	I	845	454	837	630	I	267	—	143
30	I	741	445	774	745	I	017	—	053
31	I	643	436	716	348	I	083	—	485
32	I	550	427	661	850	I	001	—	189
33	I	462	417	609	654		922	—	212
34	I	379	407	561	233		849	—	064
35	I	301	397	516	497		781	—	256
36	I	227	387	474	849		718	—	251
37	I	158	377	436	566		660	—	306
38	I	092	367	400	764		606	—	198
39	I	031	357	368	067		556	—	551
40		972	346	336	312		508	—	524
41		972	335	325	620		492	—	408
42		972	324	314	928		476	—	292
43		972	313	304	236		460	—	176
44		972	302	293	544		444	—	060
45		726	292	211	992		320	—	472
46		726	282	204	732		309	—	483
47		726	272	197	472		298	—	494
48		726	262	190	212		287	—	505
49		726	252	182	952		276	—	516
50		543	242	131	406		198	—	528
51		543	232	125	976		190	—	386
52		543	222	120	546		182	—	244
53		543	212	115	116		174	—	102
54		543	202	109	686		165	—	621
55		406	192	77	952		117	—	615
56		406	182	73	892		111	—	521
57		406	172	69	832		105	—	427
58		406	162	65	772		99	—	333

This last Sum, divided by 661, leaves a Remainder,

TABLE IX. continued.

59	406	152	61 712	93 — 239
60	303	142	43 026	65 — 061
61	303	131	39 693	60 — 033
62	303	120	36 360	55 — 005
63	303	109	33 027	49 — 638
64	303	98	29 694	44 — 610
65	227	88	19 976	30 — 146
66	227	78	17 706	26 — 520
67	227	68	15 436	23 — 233
68	227	58	13 166	19 — 607
69	227	49	11 123	16 — 547
70	169	41	6 929	10 — 319
71	169	34	5 746	8 — 458
72	169	28	4 732	7 — 105
73	169	23	3 887	5 — 582
74	169	20	3 380	5 — 075
75	126	18	2 268	3 — 285
76	126	16	2 016	3 — 033
77	126	14	1 764	2 — 442
78	126	12	1 512	2 — 190
79	126	10	1 260	1 — 599
80	94	8	752	1 — 091
81	94	7	658	— 658
82	94	6	564	— 564
83	94	5	470	— 470
84	94	4	376	— 376
85	71	3	213	— 213
86	71	2	142	— 142
87	71	1	71	— 071
88	71	$\frac{1}{2}$	35	— 035
89	71	$\frac{1}{4}$	17	— 017
90	53	$\frac{1}{4}$	13	— 013

This last Sum, divided by 661, leaves a Remainder,

TABLE IX. continued.

Totals.	Col. 2.	Column 4.	divided by 661 leaves	Column 5.
Page 1.	134 061	79 085 357		119 645—012
Page 2.	28 258	9 810 401		14 841—500
Page 3.	5 409	357 724		541—123
Totals.	167 728	89 253 482		135 027—635

N. B. Where this Computation
 is carried on for 41 Years } 130-265--347
 only, the Produce is
 Where carried on for 58 Years, } 134-486--512
 the Produce is
 Where carried on for 90 Years, } 135-027--635
 the Produce, as above, is

That is, in the first Case, the Value is 13-02
 In the second Case, the Value is 13-44
 In the last Case, the Value is 13-50

From whence 'tis apparent, where the Value
 of such an Annuity is stated at 13-44 only, as
 in Dr. Halley's Table, that there is an Omission
 of 32 Years in making the Calculation.

T A B L E X.

Showing the Number of Persons living in the City of *Breslaw* in *Silesia*, and their Ages current respectively, from 1 to 84 Years Age.

Age current.	Persons living.	A.	P.
1 ---	1000	6 —	710
2 ---	855	7 —	692
3 ---	798	8 —	680
4 ---	760	9 —	670
5 ---	732	10 —	661

The rest of this Table is inserted in the third Column of the precedent Table, and need not be here repeated: And I have there added the Age current and Persons living from 84, where Dr. *Halley's* Table breaks off, to Age an 100: And the Table, with these Additions, makes the total Number of Persons there living to be the same as supposed by the Doctor, *viz.* 34000.

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