

Lectures on select subjects in mechanics, hydrostatics, pneumatics, and optics. With the use of the globes, the art of dialing, and the calculation of the mean times of new and full moons and eclipses / By James Ferguson.

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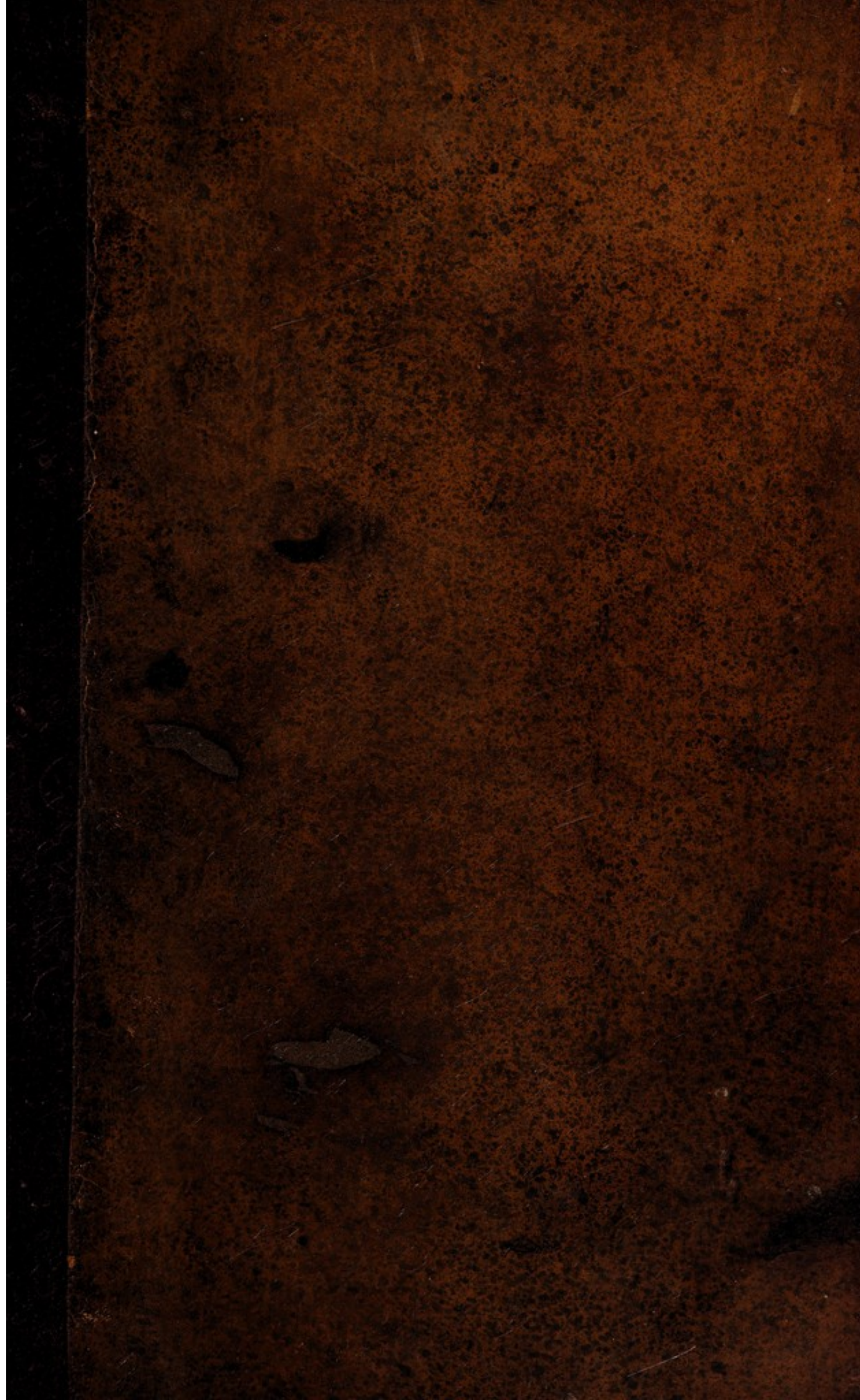
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LECTURES

ON

SELECT SUBJECTS

IN

MECHANICS, || PNEUMATICS,
HYDROSTATICS, || AND OPTICS:

WITH

The USE of the GLOBES,

The ART of DIALING,

AND

The Calculation of the Mean Times of NEW
and FULL MOONS and ECLIPSES.

By JAMES FERGUSON.

Philosophia mater omnium bonarum artium est. CICERO. 1 Tusc.

LONDON,

Printed for A. MILLAR in the Strand.

MDCCLX.

1760

THE FUTURE

ON

SELECTED SUBJECTS

A
T

MECHANICS
HYDROSTATICS AND OPTICS



The Collection of the New
and Full Moore and Bellamy

BY JAMES FERGUSON

LONDON

Printed by A. Milne in the Strand

TO HIS
ROYAL HIGHNESS
Prince EDWARD.

S I R,

AS heaven has inspired your ROYAL
HIGHNESS with such love of in-
genious and useful arts, that you not
only study their theory, but have often
condescended to honour the professors
of mechanical and experimental philo-
sophy with your presence and particular
favour; I am thereby encouraged to
lay myself and the following work at

DEDICATION.

your Royal Highness's feet: and at the
same time beg leave to express that ve-
neration with which I am,

S I R,

Your ROYAL HIGHNESS'S

Most obliged,

And most obedient,

Humble Servant,

James Ferguson.

P R E F A C E.

***E**VER since the days of the LORD CHANCELLOR BACON, natural philosophy hath been more and more cultivated in England. THAT great genius first set out with taking a general survey of all the natural sciences, dividing them into distinct branches, which he enumerated with great exactness; he enquired scrupulously into the degree of knowledge already attained to in each, and drew up a list of what still remained to be discovered: this was the scope of his first undertaking. Afterward he carried his views much farther, and shewed the necessity of an experimental philosophy, a thing never before thought of. As he was a professed enemy to systems, he considered philosophy no otherwise than as that part of knowledge which contributes to make men better and happier: he seems to limit it to the knowledge of things useful, recommending above all the study of nature, and shewing that no progress can be made therein, but by collecting facts, and comparing experiments, of which he points out a great number proper to be made.*

*But notwithstanding the true path to science was thus exactly marked out, the old notions of the schools so strongly possessed people's minds at that time, as not to be eradicated by any new opinions, how rationally soever advanced, until the illustrious Mr. BOYLE, the first who pursued LORD BACON's plan, began to put experiments in practice with an assiduity equal to his great talents. Next, the ROYAL SOCIETY
being*

P R E F A C E.

being established, the true philosophy began to be the reigning taste of the age, and continues so to this day.

The immortal SIR ISAAC NEWTON insisted, even in his early years, that it was high time to banish vague conjectures and hypotheses from natural philosophy, and to bring that science under an entire subjection to experiments and geometry. He frequently called it the experimental philosophy, so as to express significantly the difference between it and the numberless systems which had arisen merely out of the conceits of inventive brains: the one subsisting no longer than the spirit of novelty lasts; the other never failing whilst the nature of things remains unchanged.

The method of teaching and laying the foundation of physics, by exhibiting public courses of experiments, was first undertaken in this kingdom, I believe, by Dr. JOHN KEILL, and since improved and enlarged by Mr. HAUKS BEE, Dr. DESAGULIERS, Mr. WHISTON, Mr. COTES, Mr. WHITESIDE, and Dr. BRADLEY our present Regius and Savilian professor of Astronomy. Nor has the same been neglected in other countries: Dr. JAMES, and Dr. DAVID GREGORY, Sir ROBERT STEWART, and after him Mr. MACLAURIN, in Scotland; Dr. HELSHAM in Ireland, Messieurs s'GRAVESANDE and MUSCHENBROEK in Holland, and the ABBÉ NOLLET in France, have acquired just applause thereby.

The substance of my own attempts in this way of instrumental instruction, the following sheets (exclusive of the astronomical part) will shew: the satisfaction they have generally given, read as lectures

P R E F A C E.

to different audiences, affords me some hope that they may be favourably received in the same form by the public.

I ought to observe, that though the four last lectures cannot be properly said to concern experimental philosophy, I considered however that they were not of so different a class, but that they might, without much impropriety, be subjoined to the preceding ones.

My apparatus (part of which is described here, and the rest in a * former work) is rather simple than magnificent; which is owing to a particular point I had in view at first setting out, namely, to avoid all superfluity, and to render every thing as plain and intelligible as I could.

* *Astronomy explained upon SIR ISAAC NEWTON's principles, and made easy to those who have not studied mathematics.*

E R R A T A.

Page 35, line 8, for *wx* read *wz*. Page 42. l. 20, *dele* only. Page 117, l. 15 from the bottom, for *three* read *six*. Page 147, l. 3 and 5 from the bottom, for *0.785339* read *0.785399*. Page 200, l. 1. for *VIII* read *VII*. Page 211, l. 19, for *ABE* read *aBE*. Page 212, l. 16, for *eb* read *eB*. Page 213, l. 11 from the bottom, for *IkH* read *IkK*. Page 216, l. 5, for *merciscus* read *meniscus*. Page 265, l. 13, for *plane* read *place*.

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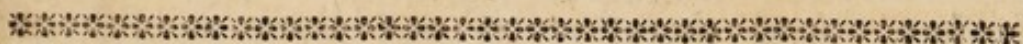
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L E C.

LECTURES

O N

SELECT SUBJECTS.



LECT. I.

Of matter and its properties.

AS the design of the first part of this course is to explain and demonstrate those laws by which the material universe is governed, regulated, and continued; and by which the various appearances in nature are accounted for; it is requisite to begin with explaining the properties of matter.

By the word *matter* is here meant every thing that has length, breadth, and thickness, and resists the touch. Matter,
what?

The inherent properties of matter are solidity, inactivity, mobility, and divisibility. Its properties.

The *solidity* of matter arises from its having length, breadth, thickness, and being impenetrable. Hence it is that all bodies are comprehended under some shape or other, and that every particular body hinders all others from occupying the same part of space which it possesseth. Thus, if a piece of wood or metal be squeez'd ever so hard between two plates, they cannot be brought into

B

contact.

contact. And even water or air has this property ; for if a small quantity of it be fixed between any other bodies, they cannot be brought to touch one another.

Inactivity.

A second property of matter is *inactivity*, or *passiveness* ; by which it always endeavours to continue in the state that it is in, whether of rest or motion. And therefore, if one body consist of twice or thrice as much matter as another body, it will have twice or thrice as much inactivity ; that is, it will require twice or thrice as much force to give it an equal degree of motion, or to stop it after it hath been put into such a motion.

That matter can never put itself into motion is allowed by all men. For they see that a stone, lying on the plane surface of the earth, never removes itself from that place, nor does any one imagine it ever can. But most people are apt to believe that all matter has a propensity to fall from a state of motion into a state of rest ; because they see that if a stone or a cannon-ball be put into ever so violent a motion, it soon stops ; not considering that this stoppage is caused 1. by the gravity or weight of the body, which sinks it to the ground in spite of the impulse ; and 2. by the resistance of the air through which it moves, and by which its velocity is retarded every moment till it falls.

A bowl moves but a short way upon a bowling-green ; because the roughness and unevenness of the grassy surface soon creates friction enough to stop it. But if the green were perfectly level, and covered with polished glass ; and the bowl perfectly hard,

hard, round, and smooth, it would go a great way further ; as it would have nothing but the air to resist it : if then the air were taken away, the bowl would go on without any friction, and consequently without any diminution of the velocity it had at setting out : and therefore, if the green were extended quite round the earth, the bowl would go on for ever.

If the bowl were carried several miles above the earth, and there projected in a horizontal direction, with such a velocity as would make it move more than a semidiameter of the earth, in the time it would take to fall to the earth by gravity ; in that case, and if there were no resisting medium in the way, the bowl would not fall to the earth at all ; but would continue to circulate round it, keeping always in the same tract, and returning to the same point from which it was projected, with the same velocity as at first. In this manner the moon moves round the earth, although she be as unactive and dead as any stone upon it.

The third property of matter is *mobility* ; for we ^{Mobility.} find that all matter is capable of being moved, if a sufficient degree of force be applied to overcome its resistance.

The fourth property of matter is *divisibility*, of ^{Divisibi-} which there can be no end. For, since matter can ^{lity.} never be annihilated by cutting or breaking, we can never imagine it to be cut into such small particles, but that if one of them be laid on a table, the uppermost side of it will be further from the table than the undermost side. Moreover it would be absurd

to say that the greatest mountain on earth has more halves, quarters, or tenth parts, than the smallest particle of matter has.

We have many surprising instances of the smallness to which matter can be divided by art: of which the two following are very remarkable.

1. If a pound of silver be fused with a single grain of gold, the gold will be equally diffused through the whole silver, so that taking one grain from any part of the mass (in which there can be no more than the 5760th part of a grain of gold) and dissolving it in *aqua fortis*, that quantity of gold will fall to the bottom.

2. The gold-beaters can extend a grain of gold into a leaf containing 50 square inches; and this leaf may be divided into 500000 parts. For an inch in length can be divided into 100 parts, every one of which will be visible to the naked eye: consequently a square inch can be divided into 10000 parts, and 50 square inches into 500000. And if one of these parts be viewed with a microscope that magnifies the diameter of an object only 10 times, it will magnify the area 100 times; and then the 100th part of a 500000th part of a grain (that is the 50 millionth part) will be visible. Such leaves are commonly used in gilding; and they are so very thin, that if 124500 of them were laid upon one another, and pressed together, they would not exceed one inch in thickness.

Yet all this is nothing in comparison of the lengths that nature goes in the division of matter. For Mr. *Leeuwenhoek* tells us, that there are more
animals

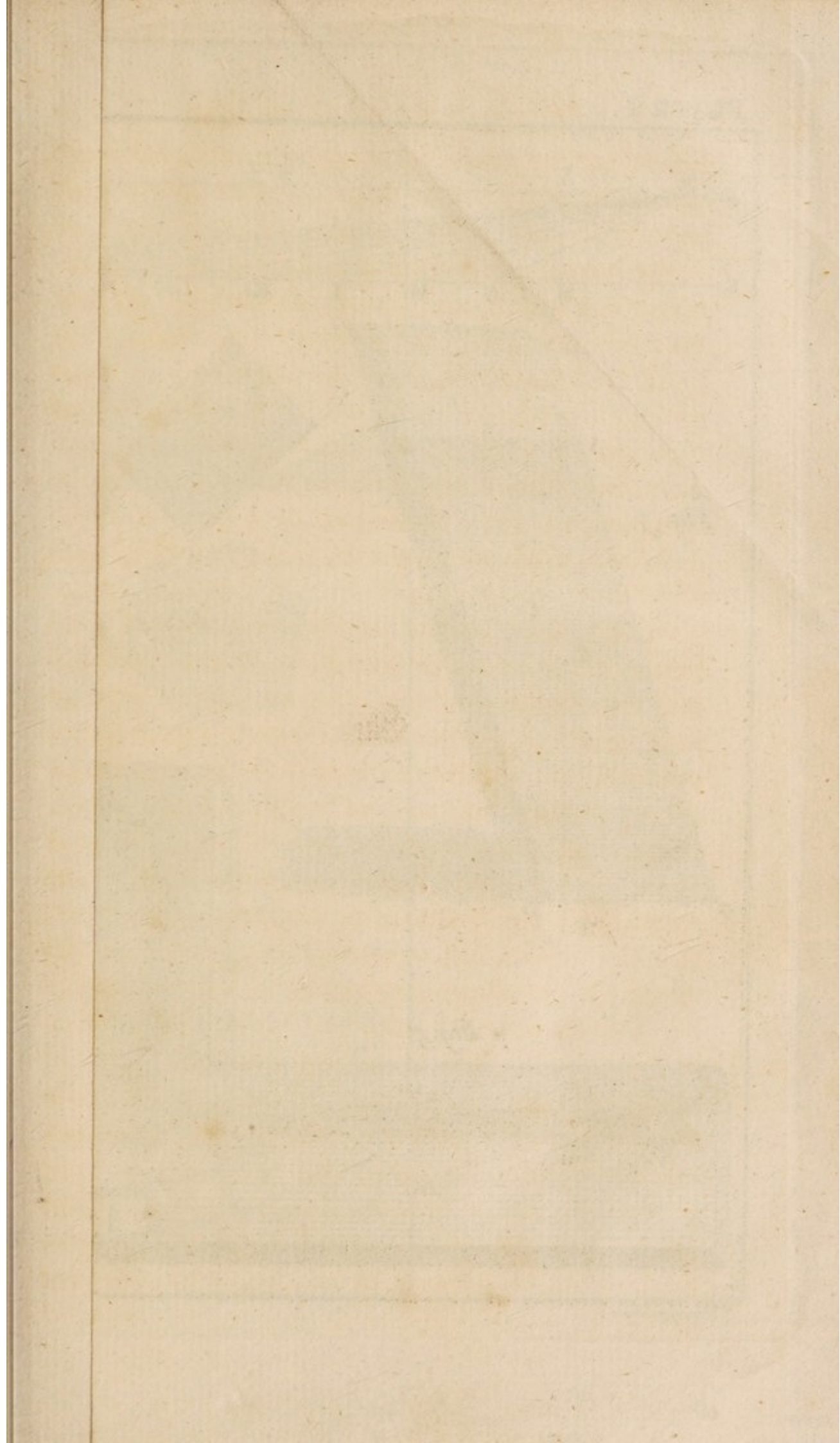
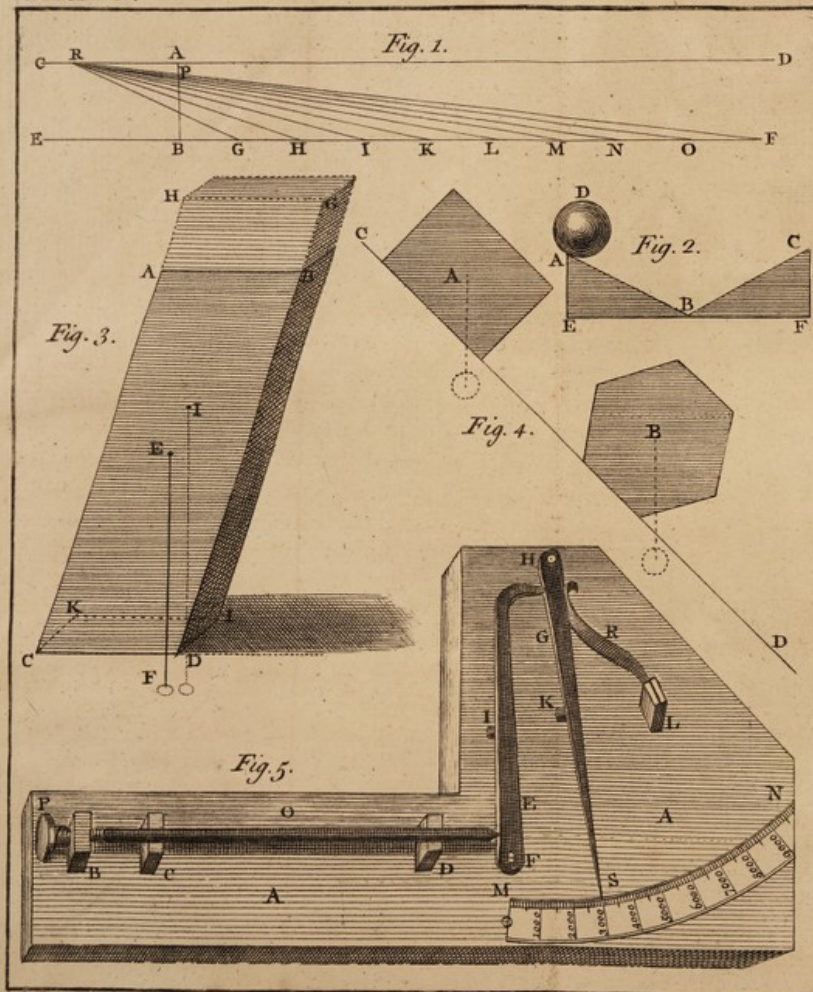


PLATE I.



J. Ferguson delin.

J. Mynde sc.

animals in the milt of a single cod-fish, than there are men upon the whole earth: and that, by comparing these animals in a microscope with grains of common sand, it appeared that one single grain is bigger than four millions of them. Now, each animal must have a heart, arteries, veins, muscles, and nerves, otherwise they could neither live nor move. How inconceivably small then must the particles of their blood be, to circulate through the smallest ramifications and joinings of their arteries and veins? It has been found by calculation, that a particle of their blood must be as much smaller than a globe of the tenth part of an inch in diameter, as that globe is smaller than the whole earth! and yet, if these particles be compared with the particles of light, they will be found to exceed them as much in bulk as mountains do single grains of sand. For, the force of any body striking against an obstacle is directly in proportion to its quantity of matter multiplied into its velocity: and since the velocity of the particles of light is demonstrated to be at least a million of times greater than the velocity of a cannon-ball, it is plain, that if a million of these particles were as big as a single grain of sand, we durst no more open our eyes to the light, than we durst expose them to sand shot point-blank from a cannon.

That matter is infinitely divisible in a mathematical sense, is easy to be demonstrated. For, Plate I.
Fig. 1. let AB be the length of a particle to be divided; and let it be touched at opposite ends by the parallel lines CD and EF , which, suppose to be in-

The in-
finite divi-
sibility of
matter pro-
ved.

finitely extended beyond D and F . Set off the equal divisions BG, GH, HI , &c. on the line EF , towards the right hand from B ; and take a point as at R , any where towards the left hand from A , in the line CD : Then, from this point, draw the right lines RG, RH, RI , &c. each of which will cut off a part from the particle AB . But after any finite number of such lines are drawn, there will still remain a part as AP at the top of the particle, which can never be cut off; because the lines DR and EF being parallel, no line can ever be drawn from the point R to any point of the line EF that will coincide with the line RD . Therefore the particle AB contains more than any finite number of parts.

Attraction. A fifth property of matter is *attraction*, which seems rather to be infused than inherent. Of this there are four kinds, viz. *cohesion*, *gravitation*, *magnetism*, and *electricity*.

Cohesion. The *attraction of cohesion* is that by which the small parts of matter are made to stick and cohere together. Of this we have several instances, some of which follow.

1. If a small glass tube, open at both ends, be dipt in water, the water will rise up in the tube to a considerable height above its level in the basin: which must be owing to the attraction of a ring of particles of the glass all around the tube, immediately above those to which the water at any instant rises. And when it has risen so high that the weight of the column balances the attraction of the tube, it rises no higher. This can be no
ways

ways owing to the pressure of the air upon the water in the basin ; for, as the tube is open at top, it is full of air above the water, which will press as much upon the water in the tube as the neighbouring air does upon any column of an equal diameter in the basin. Besides, if the same experiment be made in the exhausted receiver of an air-pump, there will be found no difference.

2. A piece of loaf sugar will draw up a fluid, and a sponge will suck in water : and on the same principle sap ascends in trees.

3. If two drops of quicksilver be placed near each other, they will run together and become one large drop.

4. If two pieces of lead be scraped clean, and pressed together with a twist, they will attract each other so strongly as to require a force much greater than their own weight to separate them. And this cannot be owing to the pressure of the air, for the same thing will hold in an exhausted receiver.

5. If two polished plates of marble or brass be put together, with a little oil between them to fill up the pores in their surfaces, and prevent the lodgment of any air ; they will cohere so strongly, even if suspended in an exhausted receiver, that the weight of the lower plate will not be able to separate it from the upper one. In putting these plates together, the one should be rubbed upon the other, as a joiner does two pieces of wood when he glues them.

6. If two pieces of cork, equal in weight, be put near each other in a basin of water, they will

move equally fast toward each other with an accelerated motion, until they meet: and then, if either of them be moved, it will draw the other after it. If two corks of unequal weights be placed near each other, they will approach with accelerated velocities inversely proportionate to their weights: that is, the lighter cork will move as much faster than the heavier, as the heavier exceeds the lighter in weight. This shews that the attraction of each cork is in direct proportion to its weight or quantity of matter.

This kind of attraction reaches but to a very small distance; for, if two drops of quicksilver be rolled in dust, they will not run together, because the particles of dust keeps them out of the sphere of each other's attraction.

Repulsion. Where the sphere of attraction ends, a *repulsive force* begins; thus, water repels most bodies till they are wet; and hence it is that a small needle, if dry, swims upon water; and flies walk upon it without wetting their feet.

The repelling force of the particles of a fluid is but small; and therefore, if a fluid be divided, it easily unites again. But if glass, or any other hard substance be broke into small parts, they cannot be made to stick together again without being first wetted: the repulsion being too great to admit of a re-union.

The repelling force between water and oil is so great, that we find it almost impossible to mix them so as not to separate again. If a ball of light wood

wood be dipt in oil, and then put into water, the water will recede so as to form a channel of some depth all around the ball.

The repulsive force of the particles of air is so great, that they can never be brought so near together by condensation as to make them stick or cohere. Hence it is, that when the weight of the incumbent atmosphere is taken off from any small quantity of air, that quantity will diffuse itself so as to occupy (in comparifon) an infinitely greater portion of space than it did before.

Attraction of gravitation is that power by which ^{Gravitation.} distant bodies tend towards one another. Of this we have daily instances in the falling of bodies to the earth. By this power in the earth it is, that bodies, on whatever fide, fall in lines perpendicular to its surface; and consequently, on opposite fides, they fall in opposite directions; all towards the center, where the force of gravity is as it were accumulated: and by this power it is, that bodies on the earth's surface are kept to it on all fides, so that they cannot fall from it. And as it acts upon all bodies in proportion to their respective quantities of matter, without any regard to their bulks or figures, it accordingly constitutes their weight. Hence,

If two bodies which contain equal quantities of matter, were placed at ever so great a distance from one another, and then left at liberty in free space; if there were no other bodies in the universe to affect them, they would fall equally swift towards one another by the power of gravity, with velocities

cities accelerated as they approached each other ; and would meet in a point which was half way between them at first. Or, if two bodies containing unequal quantities of matter, were placed at any distance, and left in the same manner at liberty, they would fall towards one another with velocities which would be in an inverse proportion to their respective quantities of matter ; and moving faster and faster in their mutual approach, would at last meet in a point as much nearer to the place from which the heavier body began to fall, than to the place from which the lighter body began to fall, as the quantity of matter in the former exceeded that in the latter.

All bodies that we know of have gravity or weight. For, that there is no such thing as positive levity, even in smoke, vapours, and fumes, is demonstrable by experiments on the air-pump ; which shews, that although the smoke of a candle ascends to the top of a tall receiver when full of air, yet, upon the air's being exhausted out of the receiver the smoke falls down to the bottom of it. So, if a piece of wood be immersed in a jar of water, the wood will rise to the top of the water, because it has a less degree of weight than its bulk of water has : but if the jar be emptied of water, the wood falls to the bottom.

Gravity demonstrated to be as the quantity of matter in bodies.

As every particle of matter has its proper gravity, the effect of the whole must be in proportion to the number of attracting particles ; that is, as the quantity of matter in the whole body. This is demonstrable by experiments on pendulums ;
for,

for, if they are of equal lengths, whatever their weights be, they vibrate in equal times. Now it is plain, that if one be double or triple the weight of another, it must require a double or triple power of gravity to make it move with the same celerity: just as it would require a double or triple force to project a bullet of twenty or thirty pounds weight with the same degree of swiftness that a bullet of ten pounds would require. Hence, it is evident that the power or force of gravity is always proportional to the quantity of matter in bodies, whatever their bulks or figures are.

Gravity also, like all other virtues or emanations which proceed or issue from a center, decreases as the distance multiplied by itself increases. That is, a body at twice the distance of another attracts with only a fourth part of the force; at thrice the distance, with a ninth part; at four times the distance, with a sixteenth part; and so on. This too is confirmed by comparing the distance which the moon falls in a minute, from a right line touching her orbit, with the distance through which heavy bodies near the earth fall in that time. And also by comparing the forces which retain Jupiter's moons in their orbits, with their respective distances from Jupiter. These forces will be explained in the next lecture.

It decreases
as the
square of
the distance
increases.

The velocity which bodies near the earth acquire in descending freely by the force of gravity, is proportional to the times of their descent. For, as the power of gravity does not consist in a single impulse, but is always operating in a constant and uniform

uniform manner, it must produce equal effects in equal times ; and consequently in a double or triple time, a double or triple effect. And so, by acting uniformly on the body, must accelerate its motion proportionably to the time of its descent.

To be a little more particular on this subject, let us suppose that a body begins to move with a celerity constantly and gradually increasing, in such a manner, as would carry it through a mile in a minute ; at the end of this space it will have acquired such a degree of celerity as is sufficient to carry it two miles the next minute, though it should then receive no new impulse from the cause by which its motion had been accelerated : but if the same accelerating cause continues, it will carry the body a mile farther ; on which account, it will have run through four miles at the end of two minutes ; and then, it will have acquired such a degree of celerity as is sufficient to carry it through a double space in as much more time ; or eight miles in two minutes, even though the accelerating force should act upon it no more. But this force still continuing to operate in an uniform manner, will again in an equal time produce an equal effect ; and so, by carrying it a mile further, cause it to move through five miles the third minute : for, the celerity already acquired, and the celerity still acquiring, will each have its compleat effect. Hence we learn, that if the body should move one mile the first minute, it would move three the second, five the third, seven the fourth, nine the fifth, and so on in proportion.

And

And thus it appears, that the spaces described in successive equal parts of time, by an uniformly accelerated motion, are always as the odd numbers 1, 3, 5, 7, 9, &c. and consequently, the whole spaces are as the squares of the times, or as the velocities. For, the continued addition of the odd numbers yields the squares of all numbers from unity upward. Thus, 1 is the first odd number, and the square of 1 is 1; 3 is the second odd number, and this added to 1 makes 4, the square of 2; 5 is the third odd number, which added to 4 makes 9, the square of 3; and so on for ever. Since therefore, the times and velocities proceed evenly and constantly as 1, 2, 3, 4, &c. but the spaces described in each equal time are as 1, 3, 5, 7, &c. it is evident that the space described

In one minute will be - - - 1 = square of 1
 In 2 minutes - - 1 + 3 = 4 = square of 2
 In 3 minutes - 1 + 3 + 5 = 9 = square of 3
 In 4 minutes 1 + 3 + 5 + 7 = 16 = square of 4 &c.

N. B. The character + signifies *more*, and = *equal*.

As heavy bodies are uniformly accelerated by the power of gravity in their descent, it is plain that they must be uniformly retarded by the same power in their ascent. Therefore, the velocity which a body acquires by falling, is sufficient to carry it up again to the same height from whence it fell: allowance being made for the resistance of the air, or other medium in which the body is moved. Thus, the body *D* in rolling down the inclined plane

The descending velocity will give a power of equal ascent.

Fig. 2.

plane *AB* will acquire such a velocity by the time it arrives at *B*, as will carry it up the inclined plane *BC*, almost to *C*; and would carry it quite up to *C*, if the body and plane were perfectly smooth, and the air gave no resistance.—So, if a pendulum were put into motion in a space quite void of air, and all other resistance, and had no friction on the point of suspension, it would move for ever: for the velocity it had acquired in falling through the descending part of the arc, would be still sufficient to carry it equally high in the ascending.

The center
of gravity.

The *center of gravity* is that point of a body in which the whole force of its gravity or weight is united. Whatever supports that point bears the weight of the whole body: and whilst it is supported, the body cannot fall; because all its parts are in a perfect equilibrium about that point.

And line of
direction.

An imaginary line drawn from the center of gravity of any body to the center of the earth, is called the *line of direction*. In this line all heavy bodies descend, if not obstructed.

Since the whole weight of a body is united in its center of gravity, as that center ascends or descends we must look upon the whole body to do so too. But as it is contrary to the nature of heavy bodies to ascend of their own accord, or not to descend when they are permitted; we may be sure that, unless the center of gravity be supported, the whole body will tumble or fall. Hence it is, that bodies stand upon their bases when the line of direction falls within the base; for in this case the body cannot be made to tumble or fall without first raising the

the center of gravity higher than it was before. Thus, the inclining body $ABCD$, whose center of gravity is E , stands firmly on its base $CDIK$, because the line of direction EF falls within the base. But if a weight as $ABGH$ be laid upon the top of the body, the center of gravity of the whole body and weight together is raised up to I ; and then, as the line of direction ID falls without the base at D , the center of gravity I is not supported; and the whole body and weight tumble down together. Fig. 3.

Hence appears the absurdity of people's rising hastily in a coach or boat when it is likely to overset: for, by that means they raise the center of gravity so far as to endanger throwing it quite out of the base; which if they do, they overset the vehicle effectually. Whereas, had they clapt down to the bottom, they would have brought the line of direction, and consequently the center of gravity, farther within the base, and by that means might have saved themselves.

The broader the base is, and the nearer the line of direction is to the middle or center of it, the more firmly does the body stand. On the contrary, the narrower the base, and the nearer the line of direction is to the side of it, the more easily may the body be overthrown: a less change of position being sufficient to remove the line of direction out of the base in the latter case than in the former. And hence it is, that a sphere is so easily rolled upon a horizontal plane; and that it is so difficult, if not impossible, to make things which
are

Fig. 4.

are sharp pointed to stand upright on the point.— From what hath been said, it clearly appears that if the plane be inclined on which the heavy body is placed, the body will slide down upon the plane whilst the line of direction falls within the base; but it will tumble or roll down when that line falls without the base. Thus, the body *A* will only slide down the inclined plane *CD*, whilst the body *B* rolls down upon it.

When the line of direction falls within the base of our feet, we stand; and most firmly when it is in the middle: but when it is out of that base, we immediately tumble or fall. And it is not only pleasing, but even surprising, to reflect upon the various and unthought of methods and postures which we use to retain this position, or to recover it when it is lost. For this purpose we bend our body forward when we rise from a chair, or when we go up stairs: and for this purpose a man leans forward when he carries a burden on his back, and backward when he carries it on his breast; and to the right or left side as he carries it on the opposite side. A thousand more instances might be added.

The quantity of matter in all bodies is in exact proportion to their weights, bulk for bulk. Therefore, heavy bodies are as much more dense or compact than light bodies of the same bulk, as they exceed them in weight.

All bodies
porous.

All bodies are full of pores, or spaces void of matter: and in gold, which is the heaviest of all known bodies, there is perhaps a much greater quantity

quantity of space than of matter. For the particles of heat and magnetism find an easy passage through the pores of gold; and even water itself has been forced through them. Besides, if we consider how easily the rays of light pass through so solid a body as glass, in all manner of directions, we shall find reason to believe that bodies are much more porous than is generally imagined.

All bodies are some way or other affected by heat; and all metallic bodies are expanded in length, breadth, and thickness thereby.—The proportion of the expansion of several metals, according to the best experiments I have been able to make with my pyrometer, is nearly thus. Iron and glass as 3, steel 4, copper 4 and one eighth, brass 5, tin 6, lead 6 and one eighth. An iron rod 3 feet long is about one 70th part of an inch longer in summer than in winter.

The pyrometer here mentioned being (for aught I know) of a new construction, a description of it may perhaps be agreeable to the reader.

AA is a flat piece of mahogany, in which are fixed four brass studs *B, C, D, L*; and two pins, one at *F* and the other at *H*. On the pin *F* turns the crooked index *E I*, and upon the pin *H* the straight index *G K*, against which a piece of watch-spring *R* bears gently, and so presses it towards the beginning of the scale *MN*, over which the point of that index moves. This scale is divided into inches and tenth parts of an inch: the first inch is marked 1000, the second 2000, and so on. A bar of metal *O* is laid into notches in the top of the

The expansion of metals.

The pyrometer.

Fig. 5.

C

studs

studs *C* and *D*; one end of the bar bearing against the adjusting screw *P*, and the other end against the crooked index *E I* at a 20th part of its length from its center of motion *F*.—Now it is plain, that however much the bar *O* lengthens, it will move that part of the index *E I* against which it bears just as far: but the crooked end of the same index, near *H*, being 20 times as far from the center of motion *F* as the point is against which the bar bears, it will move 20 times as far as the bar lengthens. And as this crooked end bears against the index *G K* at only a 20th part of its whole length *G S* from its center of motion *H*, the point *S* will move through 20 times the space that the point of bearing near *H* does. Hence, as 20 multiplied by 20 produces 400, it is evident that if the bar lengthens but a 400th part of an inch, the point *S* will move a whole inch on the scale; and as every inch is divided into 10 equal parts, if the bar lengthens but the 10th part of the 400th part of an inch, which is only the 4000th part of an inch, the point *S* will move the tenth part of an inch, which is very perceptible.

To find how much a bar lengthens by heat, first lay it cold into the notches of the studs, and turn the adjusting screw *P* until the spring *R* brings the point *S* of the index *G K* to the beginning of the divisions of the scale at *M*: then, without altering the screw any farther, take off the bar and rub it with a dry woollen cloth till it feels warm; and then, laying it on where it was, observe how far it pushes the point *S* upon the scale by means of the crooked

crooked index *EI*: and the point *S* will shew exactly how much the bar has lengthened by the heat of rubbing. As the bar cools, the spring *R* bearing against the index *KG*, will cause its point *S* to move gradually back towards *M* in the scale: and when the bar is quite cold, the index will rest at *M*, where it was before the bar was made warm by rubbing. The indexes have small rollers under them at *I* and *K*; which, by turning round on the smooth wood as the indexes move, make their motions the easier, by taking off a great part of the friction, which would otherwise be on the pins *F* and *H*, and of the points of the indexes themselves on the wood.

Besides the universal properties above-mentioned, there are bodies which have properties peculiar to themselves: such as the loadstone, in which the most remarkable are these, 1. it attracts iron and steel only. 2. It constantly turns one of its sides to the north and another to the south, when suspended by a thread that does not twist. 3. It communicates all its properties to a piece of steel when rubbed upon it, without losing any itself. Magnetism.

According to Dr. *Helfham's* experiments, the attraction of the loadstone decreases as the square of the distance increases. Thus, if a loadstone be suspended at one end of a balance, and counterpoised by weights at the other; and a flat piece of iron placed beneath it, at the distance of four tenths of an inch, the stone will immediately descend and adhere to the iron. But if the stone be again removed to the same distance, and as

many grains be put into the scale at the other end as will exactly counterbalance the attraction, then, if the iron be brought twice as near the stone as before, that is, only two tenth parts of an inch from it, there must be four times as many grains put into the scale as before, in order to be a just counterbalance to the attractive force, or to hinder the stone from descending and adhering to the iron. So, if four grains will do in the former case, there must be sixteen in the latter. But from some later experiments, made with the greatest accuracy, it is found that the force of magnetism decreases in a ratio between the reciprocal of the square and the reciprocal of the cube of the distance; approaching to the one or the other, as the magnitudes of the attracting bodies are varied.

Electricity. Several bodies, particularly amber, glass, jet, sealing-wax, agate, and almost all precious stones, have a peculiar property of attracting and repelling light bodies when heated by rubbing. This is called *electrical attraction*, in which the chief things to be observed are, 1. if a glass tube about an inch and an half in diameter, and two or three feet long, be heated by rubbing, it will alternately attract and repel all light bodies at the distance of 10 or 15 inches. 2. It does not attract by being heated without rubbing. 3. Any light body being once repelled by the tube, will never be attracted again, till it has touched some other body. 4. If the tube be rubbed by a moist hand, or any thing that is wet, it totally destroys the electricity. 5. Any body except air, being interposed, stops
the

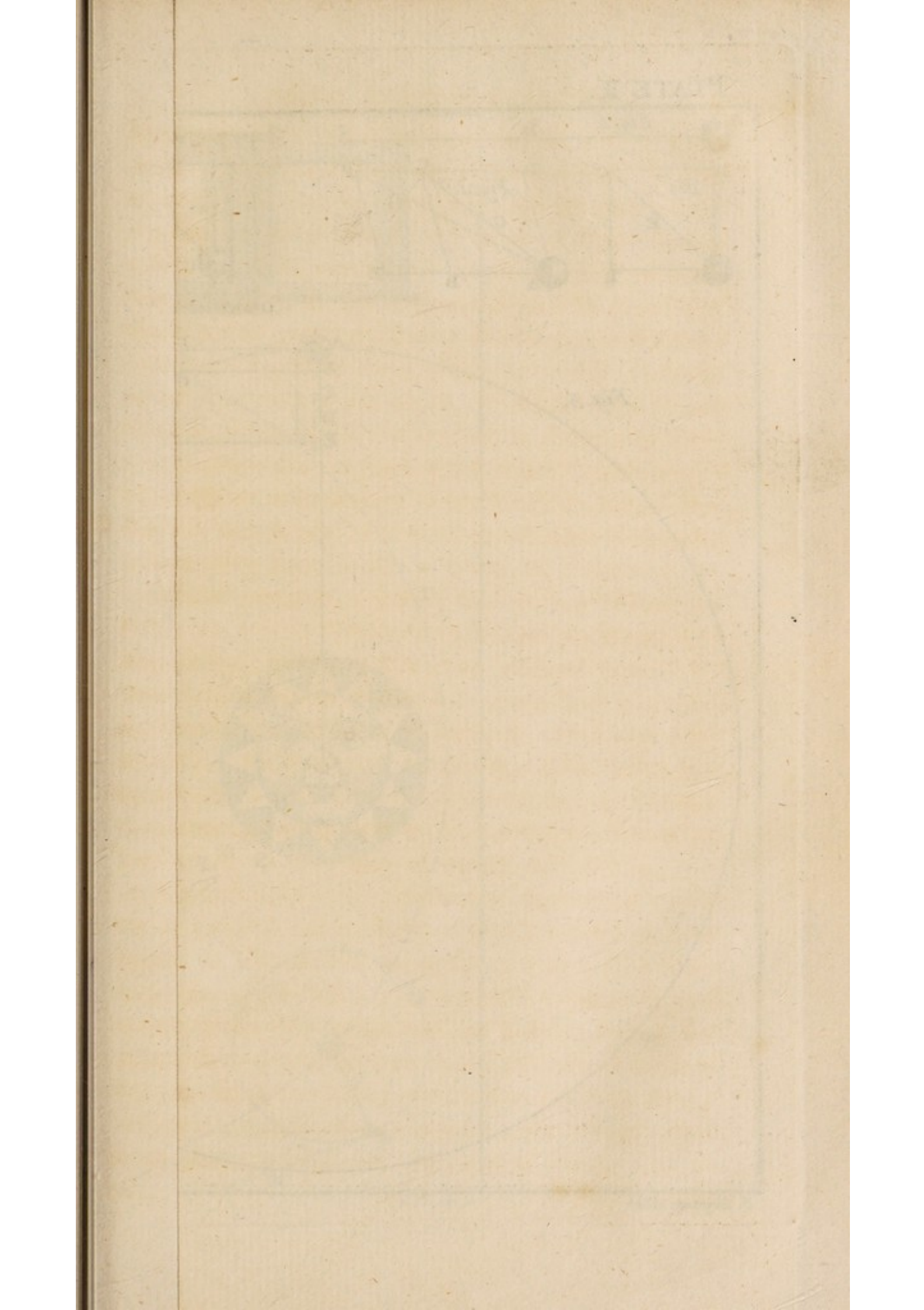
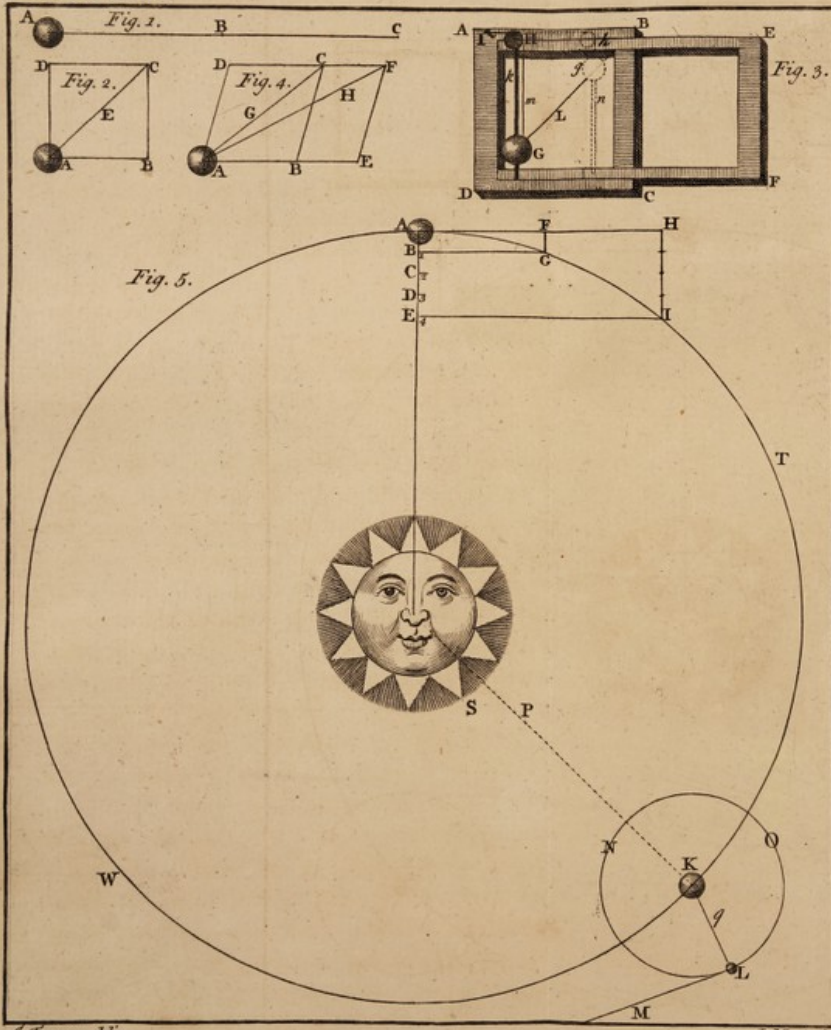


PLATE II.



J. Ferguson delin.

J. Mynde sc.

the electricity. 6. The tube attracts stronger when rubbed over with bees-wax, and then with a dry woollen cloth. 7. When it is well rubbed, if a finger be brought near it, at about the distance of half an inch, the effluvia will snap against the finger, and make a little crackling noise; and if this be performed in a dark place, there will appear a little flash of light.

L E C T. II.

Of central forces.

WE have already mentioned it as a necessary consequence arising from the deadness or inactivity of matter, that all bodies endeavour to continue in the state they are in, whether of rest or motion. If the body *A* were placed in any part of free space, where nothing either draws or impels it any way, it would for ever remain in that part of space, because it could have no tendency of itself to remove any way from thence. If it receive a single impulse any way, as suppose from *A* towards *B*, it will go on in that direction; for, of itself it could never swerve from a right line, nor stop its course.—When it has gone through the space *AB*, and met with no resistance, its velocity will be the same at *B* as it was at *A*: and this velocity, in as much more time, will carry it thro' as much more space, from *B* to *C*; and so on for ever. Therefore, when we see a body in motion, we conclude that some other substance must have

Motion or rest equally indifferent to all bodies.

Plate II.
Fig. 1.

given it that motion; and when we see a body fall from motion to rest, we conclude that some other body or cause stopt it.

All motion
naturally
rectilinear.

As all motion is naturally rectilinear, it appears, that a bullet projected by the hand, or shot from a cannon, would for ever continue to move in the same direction it received at first, if no other power diverted its course. Therefore, when we see a body move in a curve of any kind whatever, we conclude it must be acted upon by two powers at least; one to put it in motion, and another drawing it off from the rectilinear course it would otherwise have continued to move in: and whenever that power which bent the motion of the body from a straight line into a curve, ceases to act, the body will again move on in a straight line touching that point of the curve in which it was when the action of that power ceased. For example, a pebble moved round in a sling ever so long a time, will fly off the moment it is set at liberty, by slipping one end of the sling-cord; and will go on in a line touching the circle it described before: which line would actually be a straight one, if the earth's attraction did not affect the pebble, and bring it down to the ground. This shews that the natural tendency of the pebble, when put into motion, is to continue moving in a straight line, although by the force of the sling it be made to revolve in a circle.

The effects
of combi-
ned forces.

The change of motion produced is in proportion to the force impressed: for the effects of natural causes are always proportionate to the force or power of those causes,

By

By these laws it is easy to prove that a body will describe the diagonal of a square or parallelogram, by two forces conjoined, in the same time that it would describe either of the sides, by one force singly. Thus, suppose the body *A* to represent a ship at sea; and that it is drove by the wind in the right line *AB*, with such a force as would carry it uniformly from *A* to *B* in a minute: then, suppose a stream or current of water running in the direction *AD*, with such a force as would carry the ship through an equal space from *A* to *D* in a minute. By these two forces, acting together at right angles to each other, the ship will describe the line *AEC* in a minute: which line (because the forces are equal) will be the diagonal of an exact square. To confirm this law by an experiment, let there be a wooden square *ABCD* so contrived, as to have the part *BEFC* made to draw out or push into the square at pleasure. To this part let the pulley *H* be joined, so as to turn freely on an axis, which will be at *H* when the piece is pushed in, and at *b* when it is drawn out. To this part let the ends of a straight wire *k* be fixed, so as to move along with it, under the pulley; and let the ball *G* be made to slide easily on the wire. A thread *m* is fixed to this ball, and goes over the pulley to *I*; by this thread the ball may be drawn up on the wire, parallel to the side *AD*, when the part *BEFC* is pushed as far as it will go into the square. But, if this part be drawn out, it will carry the ball along with it, parallel to the bottom of the square *DC*. By this means, the ball *G* may either be drawn

perpendicularly upward by pulling the thread m , or moved horizontally along by pulling out the part $BEFC$, in equal times, and through equal spaces; each power acting equably and separately upon it. But if, when the ball is at G , the upper end of the thread be tied to the pin I , in the corner A of the fixed square, and the moveable part $BEFG$ be drawn out, the ball will then be acted upon by both the powers together: for it will be drawn up by the thread towards the top of the square, and at the same time carried with its wire k towards the right hand BC , moving all the while in the diagonal line L ; and will be found at g when the sliding part is drawn out as far as it was before; which then, will have caused the thread to draw up the ball to the top of the inside of the square, just as high as it was before, when drawn up singly by the thread without moving the sliding part.

Fig. 4.

If the acting forces are equal, but at oblique angles to each other, so will the sides of the parallelogram be: and the diagonal run through by the moving body will be longer or shorter, according as the obliquity is greater or smaller. Thus, if two equal forces act conjointly upon the body A , one having a tendency to move it through the space AB in the same time that the other has a tendency to move it through an equal space AD ; it will describe the diagonal AGC in the same time that either of the single forces would have caused it to describe either of the sides. If one of the forces be greater than the other, then one side of the parallelogram

rallelogram will be so much longer than the other. For, if one force singly would carry the body through the space AE , in the same time that the other would have carried it through the space AD , the joint action of both will carry it in the same time through the space AHF , which is the diagonal of the oblique parallelogram $ADEF$.

If both forces act upon the body in such a manner, as to move it uniformly, the diagonal described will be a straight line; but if one of the forces acts in such a manner as to make the body move faster and faster as it goes forward, then the line described will be a curve. And this is the case of all bodies which are projected in rectilineal directions, and at the same time acted upon by the power of gravity, which has a constant tendency to accelerate their motions in the direction wherein it acts.

From the uniform projectile motion of bodies in straight lines, and the universal power of gravity or attraction, arises the curvilinear motion of all the heavenly bodies. If the body A be projected along the straight line AFH in open space, where it meets with no resistance, and is not drawn aside by any power, it will go on for ever with the same velocity, and in the same direction. But if, at the same moment the projectile force is given it at A , the body S begins to attract it with a force duly adjusted *, and perpendicular to its motion at A , it

The laws of the planetary motions.

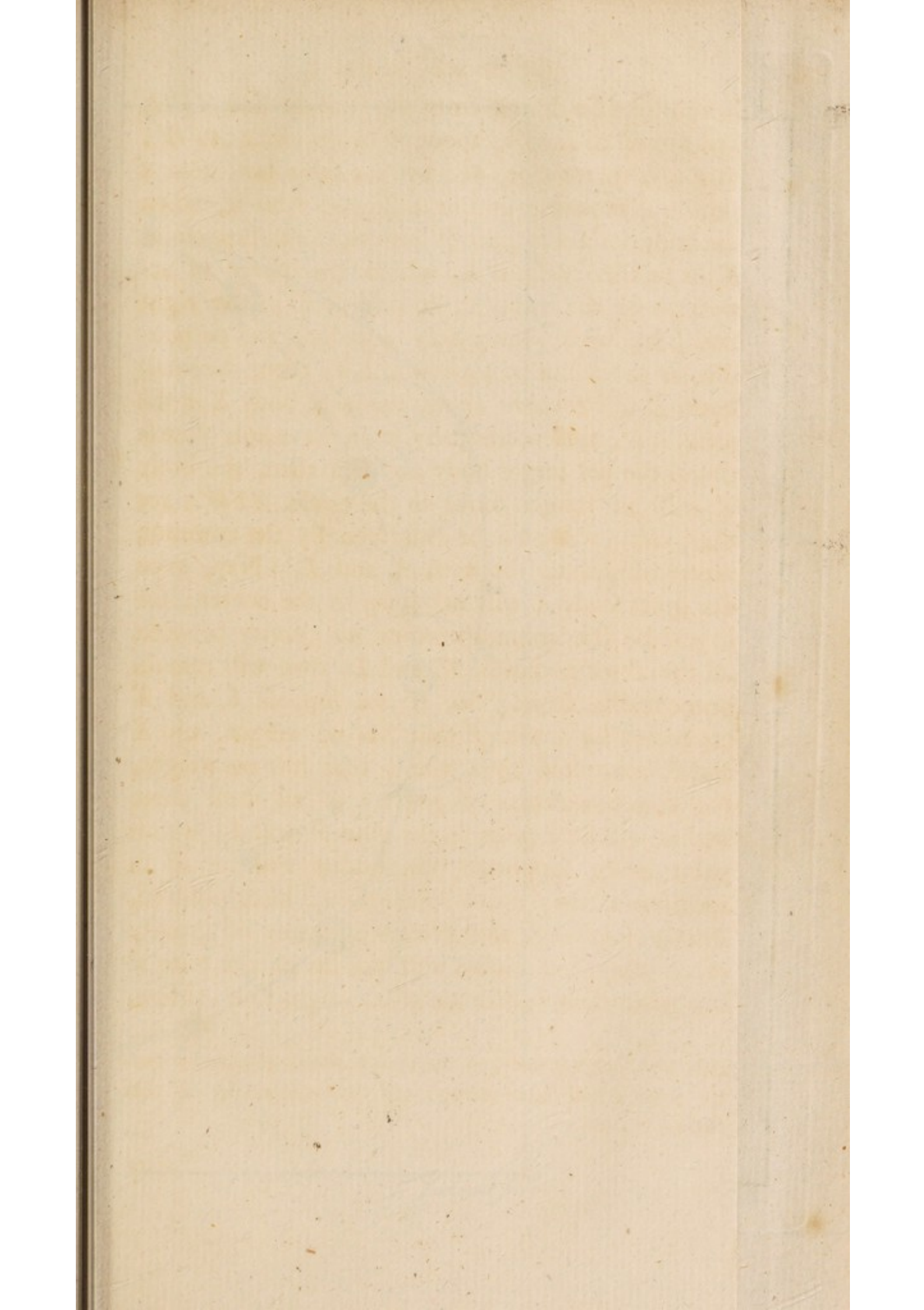
Fig. 5.

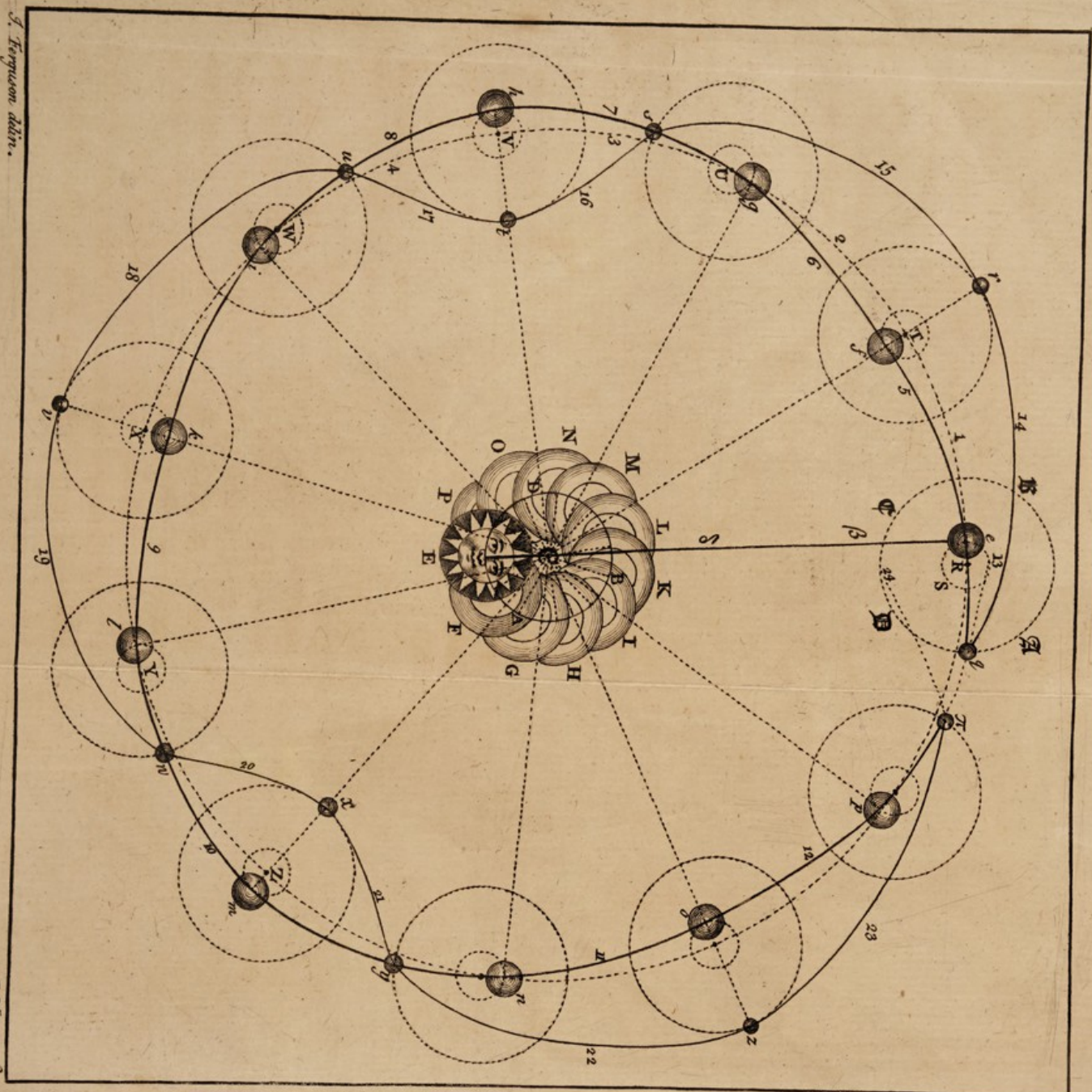
* To make the projectile force a just balance to the gravitating power, so as to keep the planet moving in a circle, it must

it will then be drawn from the straight line AFH , and forced to revolve about S in the circle ATW ; in the same manner, and by the same law, that a pebble is moved round in a sling. And if, when the body is in any part of its orbit, (as suppose at K) a smaller body as L , within the sphere of attraction of the body K , be projected in the right line LM , with a force duly adjusted, and perpendicular to the line of attraction LK ; then, the small body L will revolve about the large body K in the orbit NO , and accompany it in its whole course round the yet larger body S . But then, the body K will no longer move in the circle ATW ; for that circle will now be described by the common center of gravity between K and L . Nay, even the great body S will not keep in the center; for it will be the common center of gravity between all the three bodies S , K , and L , that will remain immoveable there. So, if we suppose S and K connected by a wire P that has no weight, and K and L connected by a wire q that has no weight, the common center of gravity of all these three bodies will be a point in the wire P near S ; which point being supported, the bodies will be all in *equilibrio* as they move round it. Though indeed, strictly speaking, the common center of gravity of all the three bodies will not be in the wire P but when these bodies are all in a right line. Here,

must give such a velocity as the planet would acquire by gravity when it had fallen through half the semidiameter of that circle,

S may





J. Ferguson delin.

J. Mynde sc.

S may represent the sun, K the earth, and L the moon.

In order to form a general idea of the curves described by two bodies revolving about their common center of gravity, whilst they themselves with a third body are in motion round the common center of gravity of all the three; let us first suppose E to be the sun, and e the earth going round him without any moon; and their moving forces regulated as above. In this case, whilst the earth goes round the sun in the dotted circle $RTUWX$, &c. the sun will go round the circle ABD , whose center C is the common center of gravity between the sun and earth: the right line $\beta\delta$ representing the mutual attraction between them, by which they are as firmly connected as if they were fixed at the two ends of an iron bar strong enough to hold them. So, when the earth is at e the sun will be at E ; when the earth is at f the sun will be at F , and when the earth is at g the sun will be at G , &c.

See Plate III.

The curves described by bodies revolving about their common center of gravity.

Now, let us take in the moon q (at the top of the figure) and suppose the earth to have no progressive motion about the sun; in which case, whilst the moon revolves about the earth in her orbit ABC , the earth will revolve in the circle $S 13$, whose center R is the common center of gravity of the earth and moon; they being connected by the mutual attraction between them in the same manner as the earth and sun are.

But the truth is, that whilst the moon revolves about the earth, the earth is in motion about the sun:

sun: and now, the moon will cause the earth to describe an irregular curve, and not a true circle, round the sun. For, it is the common center of gravity of the earth and moon that will describe the same circle which the earth would have moved in, if it had not been attended by a moon. For, supposing the moon to describe a quarter of her progressive orbit about the earth in the time that the earth moves from *e* to *f*; it is plain, that when the earth comes to *f* the moon will be found at *r*; in which time, their common center of gravity will have described the dotted arc *R 1 T*, the earth the curve *R 5 f*, and the moon the curve *q 14 r*. In the time that the moon describes another quarter of her orbit, the center of gravity of the earth and moon will describe the dotted arc *T 2 U*, the earth the curve *f 6 g*, and the moon the curve *r 15 s*, and so on.—And thus, whilst the moon goes once round the earth in her progressive orbit, their common center of gravity describes the regular portion of a circle *R 1 T 2 U 3 V 4 W*, the earth the irregular curve *R 5 f 6 g 7 h 8 i*, and the moon the yet more irregular curve *q 14 r 15 s 16 t 17 u*; and then, the same kind of tracks over again.

The center of gravity of the earth and moon is 6000 miles from the earth's center towards the moon; therefore, the circle *S 13* which the earth describes round that center of gravity (in every course of the moon round her orbit) is 12000 miles in diameter. Consequently, the earth is 12000 miles nearer the sun at the time of full moon than at the time of new. [See the earth at *f* and at *b*.]

To

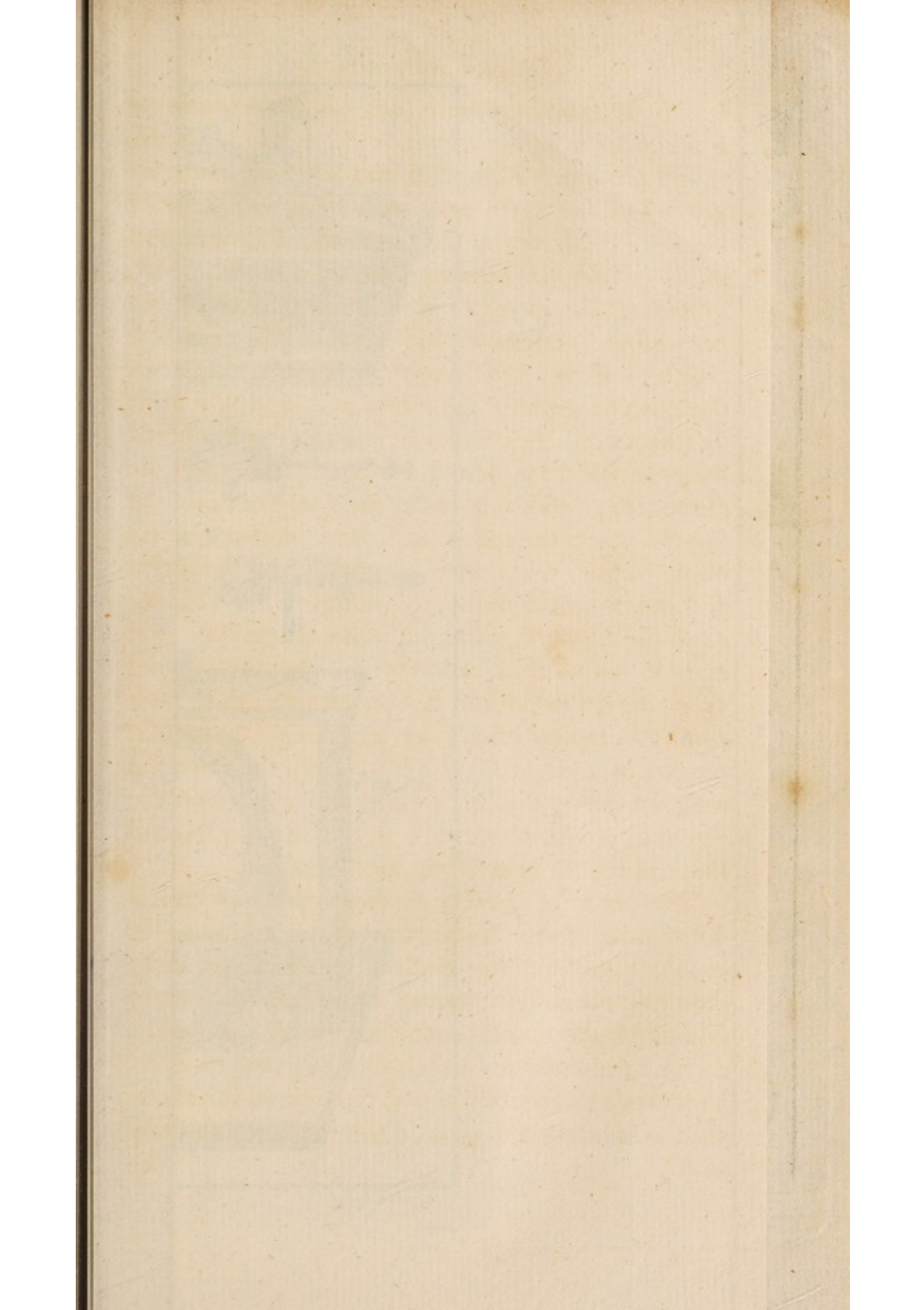


PLATE IV.

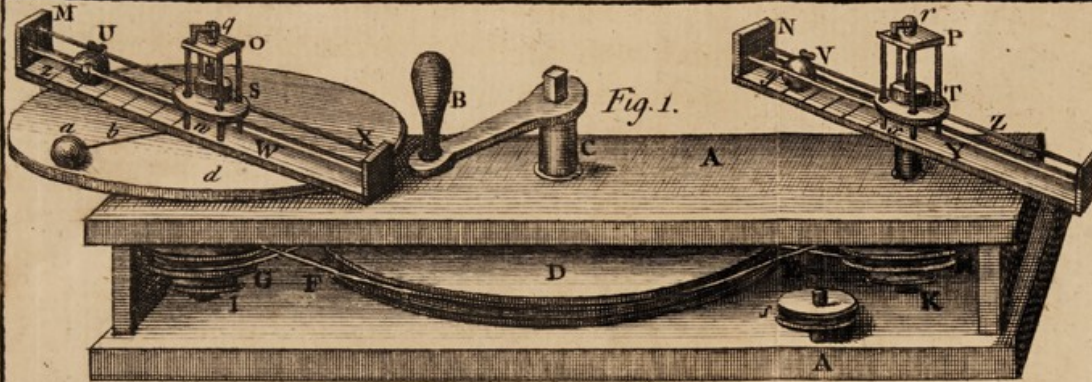


Fig. 1.

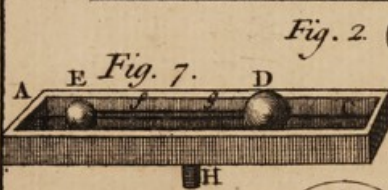


Fig. 2.

Fig. 3.

Fig. 4.

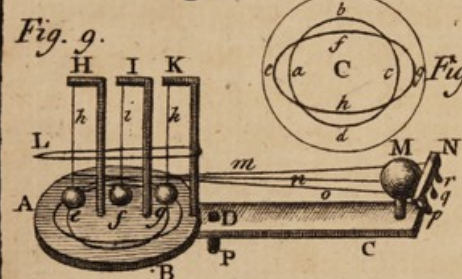


Fig. 5.

Fig. 6.

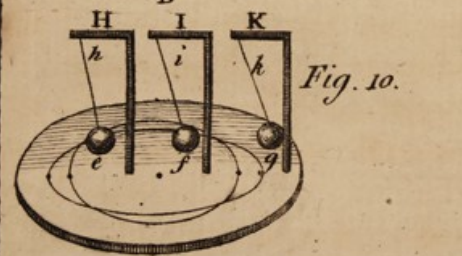


Fig. 7.

Fig. 8.

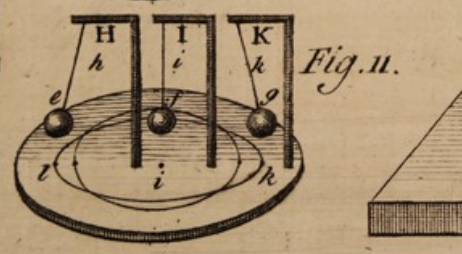


Fig. 9.

Fig. 10.

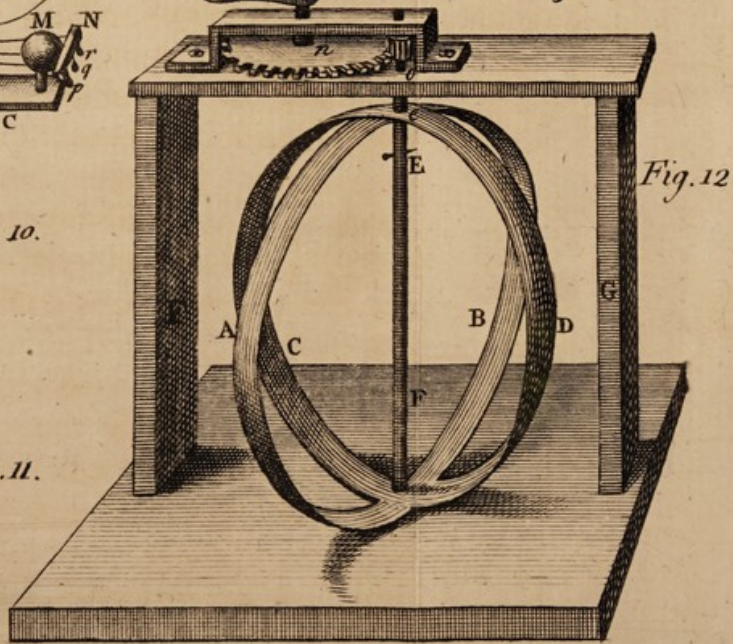


Fig. 11.

To avoid confusion in so small a figure, we have supposed the moon to go only twice and an half round the earth in the time that the earth goes once round the sun: it being impossible to take in all the revolutions which she makes in a year, and to give a true figure of her path, unless we should make the semidiameter of the earth's orbit at least 84 inches; and then, the proportional semidiameter of the moon's orbit would be only a quarter of an inch.—For a true figure of the moon's path I refer the reader to my treatise of astronomy.

If the moon made any compleat number of revolutions about the earth in the time that the earth makes one revolution about the sun, the paths of the sun and moon would return into themselves at the end of every year; and so be the same over again: but they return not into themselves in less than 19 years nearly; in which time the earth makes nearly 19 revolutions about the sun, and the moon 235 about the earth.

If the planet *A* be attracted towards the sun with such a force as would make it fall from *A* to *B* in the time that the projectile impulse would have carried it from *A* to *F*, it will describe the arc *AG* by the combined action of these forces, in the same time that the former would have caused it to fall from *A* to *B*, or the latter have carried it from *A* to *F*. But, if the projectile force had been twice as great, that is, such as would have carried the planet from *A* to *H* in the same time that now, by the supposition, it carries it only from *A* to *F*; the sun's attraction must then have been four times

Plate II.
Fig. 5.

A double
projectile
force ba-
lances a
quadruple
power of
gravity.

as strong as formerly, to have kept the planet in the circle ATW ; that is, it must have been such as would have caused the planet to fall from A to E , which is four times the distance of A from B , in the time that the projectile force singly would have carried it from A to H , which is only twice the distance of A from F^* . Thus, a double projectile force will balance a quadruple power of gravity in the same circle; as appears plain by the figure, and shall soon be confirmed by an experiment.

Plate IV.
Fig. 1.

The whirling-table described.

The whirling-table is a machine contrived for shewing experiments of this nature. AA is a strong frame of wood, B a winch or handle fixed on the axis C of the wheel D , round which is the catgut string EF , which also goes round the small wheels G and H , crossing between them and the great wheel D . On the upper end of the axis of the wheel G , above the frame, is fixed the round board d , to which the bearer MSX may be fastened occasionally, and removed when it is not wanted. On the axis of the wheel H is fixed the bearer NTZ : and it is easy to see that when the winch is turned, the wheels and bearers are put into a whirling motion.

Each bearer hath two wires W, X , and Y, Z , fixed and screwed tight into them at the ends by nuts on the outside. And when these nuts are unscrewed, the wires may be drawn out, in order

* Here the arcs AG, AI must be supposed to be very small; otherwise AE , which is equal to HI , will not be quadruple to AB , which is equal to FG .

to change the balls *U* and *V*, which slide upon the wires by means of loops fixed into the balls, and keep them up from touching the wood below them. A strong silk line goes through each ball, and is fixt to it at any length from the center of the bearer to its end, as occasion requires, by a nut-screw at the top of the ball; the shank of the screw going into the center of the ball and pressing the line against the under side of the hole that it goes through.—The line goes from the ball, and under a small pulley fixt in the middle of the bearer; then up through a socket in the round plate (see *S* and *T*) in the middle of each bearer; then through a slit in the middle of the square top (*O* and *P*) of each tower, and going over a small pulley on the top, comes down again the same way, and is at last fastened to the upper end of the socket fixt in the middle of the above-mentioned round plate. These plates *S* and *T* have each four round holes near their edges for letting them slide up and down upon the wires which make the corners of each tower. The balls and plates being thus connected, each by its particular line, it is plain that if the balls be drawn outward, or towards the ends *M* and *N* of their respective bearers, the round plates *S* and *T* will be drawn up to the top of their respective towers *O* and *P*.

There are several brass weights, some of two ounces, some of three, and some of four, to be occasionally put within the towers *O* and *P*, upon the round plates *S* and *T*: each weight having a round hole in the middle of it, for going upon the
Fig. 2.
 sockets

sockets or axes of the plates, and is slit from the edge to the hole, for allowing it to be slipped over the foresaid line which comes from each ball to its respective plate.

Fig. 1.

The propensity of matter to keep the state it is in.

The experiments to be made by this machine are,
 1. Take away the bearer *MX*, and take the ivory ball *a* to which the line or silk cord *b* is fastened at one end; and having made a loop on the other end of the cord, put the loop over a pin fixt in the center of the board *d*. Then, turning the winch *B* to give the board a whirling motion, you will see that the ball does not immediately begin to move with the board; but, on account of its inactivity, endeavours to continue in the state of rest which it was in before.—Continue turning, until the board communicates an equal degree of motion with its own to the ball, and then turning on, you will perceive that the ball will remain upon one part of the board, keeping the same velocity with it, and having no relative motion upon it: as is the case with every thing that lies loose upon the plane surface of the earth, which having the motion of the earth communicated to it, never endeavours to remove from that place. But stop the board suddenly by hand, and the ball will go on, and continue to revolve upon the board, until the friction thereof stops its motion: which shews, that matter being once put into motion, would continue to move for ever, if it met with no resistance. In like manner, if a person stands upright in a boat before it begins to move, he can stand firm; but the moment the boat sets off,

off, he is in danger of falling towards that place which the boat departs from : because, as matter, he has no natural propensity to move. But, when he acquires the motion of the boat, let it be ever so swift, if it be smooth and uniform, he will stand as upright and firm as if he was on the plane shore : and if the boat strike against any obstacle, he will fall towards that obstacle ; on account of the propensity he has, as matter, to keep the motion which the boat has put him into.

2. Take away this ball, and put a longer cord to it, which may be put down through the hollow axis of the bearer *MX* and wheel *G*, and fix a weight to the end of the cord below the machine ; which weight, if left at liberty, will draw the ball from the edge of the whirling board to its center.

Draw off the ball a little from the center, and turn the winch ; then the ball will go round and round with the board, and will gradually fly off farther and farther from the center, and raise up the weight below the machine : which shews that all bodies revolving in circles have a tendency to fly off from these circles, and must have some power acting upon them from the center of motion, to keep them from so flying off. Stop the machine, and the ball will continue to revolve for some time upon the board ; but as the friction gradually stops its motion, the weight acting upon it will bring it nearer and nearer to the center in every revolution, until it brings it quite thither. This shews, that if the planets met with any resistance in going round the sun, its attractive power would bring

Bodies moving in orbits have a tendency to fly out of these orbits.

D

them

them nearer and nearer to it in every revolution, until they fell into it.

Bodies
move faster
in small or-
bits than in
large ones.

3. Take hold of the cord below the machine with one hand, and with the other throw the ball upon the round board as it were at right angles to the cord, by which means it will go round and round upon the board. Then, observing with what velocity it moves, pull the cord below the machine, which will bring the ball nearer to the center of the board, and you will see that the nearer the ball is drawn to the center, the faster it will revolve; as those planets which are nearest the sun revolve faster than those which are more remote; and not only go round sooner, because they describe smaller circles, but even move faster in every part of their respective circles.

Their cen-
trifugal for-
ces shewn.

4. Take away this ball, and apply the bearer MX , whose center of motion is in its middle at w , directly over the center of the whirling board d . Then, put two balls (U and V) of equal weights upon their bearing wires, and having fixed them at equal distances from their respective centers of motion w and x upon their silk cords, by the screw nuts, put equal weights in the towers O and P . Lastly, put the catgut strings E and F upon the grooves G and H ; which, being of equal diameters, will give equal velocities to the bearers above, when the winch B is turned: and the balls U and V will fly off towards M and N ; and will raise the weights in the towers at the same instant. This shews, that when bodies of equal quantities of

of matter revolve in equal circles with equal velocities, their centrifugal forces are equal.

5. Take away these equal balls, and instead of them, put a ball of six ounces into the bearer MX , at a sixth part of the distance wz from the center; and put a ball of one ounce into the opposite bearer, at the whole distance xy , which is equal to wx , from the center of the bearer; and fix the balls at these distances on their cords, by the screw nuts at top: then the ball U , which is six times as heavy as the ball V , will be at only a sixth part of the distance from its center of motion; and consequently will revolve in a circle of only a sixth part of the circumference of the circle in which V revolves. Now, let any equal weights be put into the towers, and the machine turned by the winch; which, (as the catgut string is on equal wheels below) will cause the balls to revolve in equal times; but V will move six times as fast as U , because it revolves in a circle of six times its radius; and both the weights in the towers will rise at once. This shews, that the centrifugal forces of revolving bodies (or their tendencies to fly off from the circles they describe) are in direct proportion to their quantities of matter multiplied into their respective velocities; or into their distances from the centers of their respective circles. For, supposing U , which weighs 6 ounces, to be two inches from its center of motion w ; the weight multiplied by the distance is 12: and supposing V , which weighs only one ounce, to be 12 inches distant from its center of motion x , the weight 1 ounce multiplied

by the distance 12 inches is 12. And as they revolve in equal times, their velocities are as their distances from the center, namely, as 1 to 6.

If these two balls be fixed at equal distances from their respective centers of motion, they will move with equal velocities; and if the tower *O* has 6 times as much weight put into it as the tower *P* has; the balls will raise their weights exactly at the same moment. This shews that the ball *U* being six times as heavy as the ball *V*, it has six times as much centrifugal force, in describing an equal circle with an equal velocity.

A double velocity in the same circle, is a balance to a quadruple power of gravity.

6. If bodies of equal weights revolve in equal circles with unequal velocities, their centrifugal forces are as the squares of the velocities. To prove this law by an experiment, let two balls *U* and *V* of equal weights be fixed on their cords at equal distances from their respective centers of motion *w* and *x*; and then, let the catgut string *E* be put round the wheel *K*, (whose circumference is only one half of the circumference of the wheel *H* or *G*) and over the pulley *s* to keep it tight: and let four times as much weight be put into the tower *P*, as in the tower *O*. Then, turn the winch *B*, and the ball *V* will revolve twice as fast as the ball *U*, in a circle of the same diameter, because they are equidistant from the centers of the circles in which they revolve; and the weights in the towers will both rise at the same instant, which shews that a double velocity in the same circle will exactly balance a quadruple power of attraction at the center of the circle. For the weights in the towers

towers may be considered as the attractive forces in the centers, acting upon the revolving balls; which, moving in equal circles, is the same thing as if they both moved in one and the same circle. Consequently, if the planets had moved twice as fast round the sun as they do, the sun's attraction must have been four times as great as it now is, to have retained the planets in their orbits. So that the sun must have either been four times as big as he now is, or four times as dense under the same bulk he now has, or four times as far from the common center of gravity of the whole system as he now is, in order to have preserved a just balance between the attractive and projectile forces.

7. If bodies of equal weights revolve in un-^{Kepler's} equal circles, in such a manner that the squares of ^{Problem.} the times of their going round are as the cubes of their distances from the centers of the circles they describe; their centrifugal forces are inversely as the squares of their distances from those centers. For, the catgut string remaining as in the last experiment, let the distance of the ball *V* from the center *x* be made equal to two of the cross divisions on its bearer; and the distance of the ball *U* from the center *w* be three and a sixth part; the balls themselves being of equal weights, and *V* making two revolutions, by turning the winch, in the time that *U* makes one: so that if we suppose the ball *V* to revolve in one moment, the ball *U* will revolve in two moments, the squares of which are one and four: for the square of 1 is only 1, and the square of 2 is 4; therefore, the square of the

D 3 period

period or revolution of the ball V is contained four times in the square of the period of the ball U . But the distance of V is 2, the cube of which is 8, and the distance of U is $3\frac{1}{6}$, the cube of which is 32 very nearly; in which 8 is contained four times: and therefore, the squares of the periods of V and U are to one another as the cubes of their distances from x and w , which are the centers of their respective circles. And if the weight in the tower O be four ounces, equal to the square of 2, the distance of V from the center x ; and the weight in the tower P 10 ounces, nearly equal to the square of $3\frac{1}{6}$, the distance of U from w ; it will be found upon turning the machine by the winch, that the balls U and V will raise their respective weights very nearly at the same instant of time. Which confirms that famous proposition of KEPLER, viz. that the squares of the periodical times of the planets round the sun are in proportion to the cubes of their distances from him; and that the sun's attraction is inversely as the square of the distance from his center: that is, at twice the distance his attraction is four times less; and thrice the distance, nine times less; at four times the distance, sixteen times less; and so on, to the remotest parts of the system.

8. Take off the catgut string E from the great wheel D and the small wheel H , and let the string F remain upon the wheels D and G . Take away also the bearer MX from the whirling-board d , and instead thereof put the machine AB upon it, fixing this machine to the center of the board by the

the pins *c* and *d*, in such a manner, that the end *ef* may rise above the board to an angle of 30 or 40 degrees. In the upper side of this machine there are two glass tubes *a* and *b*, close stoppt at both ends; and each tube is about three quarters full of water. In the tube *a* is a little quicksilver, which naturally falls down to the end *a* in the water, because it is heavier than its bulk of water; and on the tube *b* is a small cork which floats upon the top of the water at *e*, because it is lighter; and is small enough to have liberty to rise or fall in the tube. While the board *b* with this machine upon it continues at rest, the quicksilver lies at the bottom of the tube *a*, and the cork floats on the water near the top of the tube *b*. But, upon turning the winch, and putting the machine in motion, the contents of each tube will fly off towards the uppermost ends (which are farthest from the center of motion) the heaviest with the greatest force. Therefore, the quicksilver in the tube *a* will fly off quite to the end *f*, and occupy its bulk of space there, excluding the water from that place, because it is lighter than quicksilver; but the water in the tube *b* flying off to its higher end *e*, will exclude the cork from that place; and cause the cork to descend towards the lowermost end of the tube, where it will remain upon the lowest end of the water near *b*; for the heavier body having the greater centrifugal force, will therefore possess the uppermost part of the tube; and the lighter body will keep between the heavier and the lowermost part.

The absurdity of the Cartesian vortices.

This demonstrates the absurdity of the *Cartesian* doctrine of the planets moving round the sun in vortexes : for, if the planet be more dense or heavy than its bulk of the vortex, it will fly off therein, farther and farther from the sun ; if less dense, it will come down to the lowest part of the vortex, at the sun : and the whole vortex itself must be surrounded with something like a great wall, otherwise it would fly quite off, planets and all together. — But while gravity exists, there is no occasion for such vortexes ; and when it ceases to exist, a stone thrown upwards will never return to the earth again.

If one body moves round another, both of them must move round their common center of gravity.

9. If a body be so placed upon the whirling-board of the machine (Fig. 1.) that the center of gravity of the body be directly over the center of the board, and the board be put into ever so rapid a motion by the winch *B*, the body will turn round with the board, but will not remove from the middle of it : for, as all parts of the body are in *equilibrium* round its center of gravity, and the center of gravity is at rest in the center of motion, the centrifugal force of all parts of the body will be equal at equal distances from its center of motion ; and therefore the body will remain in its place. But if the center of gravity be placed ever so little out of the center of motion, and the machine be turned swiftly round, the body will fly off towards that side of the board on which its center of gravity lies. Thus, if the wire *C* with its little ball *B* be taken away from the demi-globe *A*, and the flat side *ef* of this demi globe be laid upon the whirling-

Fig. 4.

whirling-board of the machine, so as their centers may coincide; if then the board be turned ever so quick by the winch, the demi-globe will remain where it was placed. But if the wire *C* be screwed into the demi-globe at *d*, the whole becomes one body, whose center of gravity is now at or near *d*. Let the pin *c* be fixed in the center of the whirling-board, and the deep groove *b* cut in the flat side of the demi-globe be put upon the pin, so as the pin may be in the center of *A*, [See Fig. 5. where Fig. 5. this groove is represented at *b*] and let the whirling-board be turned by the winch, which will carry the little ball *B* (Fig. 4.) with its wire *C*, and the demi-globe *A*, all round the center-pin *ci* and then, the centrifugal force of the little ball *B*, which weighs only one ounce, will be so great, as to draw off the demi-globe *A*, which weighs two pounds, until the end of the groove at *e* strikes against the pin *c*, and so prevents *A* from going any farther: otherwise, the centrifugal force of *B* would have been great enough to have carried *A* quite off the whirling-board. Which shews, that if the sun were placed in the very center of the orbits of the planets, it could not possibly remain there; for the centrifugal forces of the planets would carry them quite off, and the sun with them; especially when several of them happened to be in any one quarter of the heavens. For the sun and planets are as much connected by the mutual attraction that subsists between them, as the bodies *A* and *B* are by the wire *C* which is fixed into them both. And even if there were but one single planet

planet in the whole heavens to go round ever so large a sun in the center of its orbit, its centrifugal force would soon carry off both itself and the sun. For, the greatest body placed in any part of free space could be easily moved: because if there were no other body to attract it, it could have no weight or gravity of itself; and consequently, though it could have no tendency of itself to remove from that part of space, yet it might be very easily moved by any other substance.—And perhaps it was this consideration which made the celebrated ARCHIMEDES say, that if he had a proper place at some distance from the earth whereon to fix his machinery, he could move the whole earth.

Fig. 6. 10. As the centrifugal force of the light body *B* will not allow the heavy body *A* to remain in the center of motion, even though it be 24 times heavier than *B*; let us now take the ball *A* (Fig. 6.) which weighs only 6 ounces, and connect it by the wire *C* with the ball *B*, which weighs only one ounce; and let the fork *E* be fixed into the center of the whirling-board: then, hang the balls upon the fork by the wire *C* in such a manner, that they may exactly balance each other; which will be when the center of gravity between them, in the wire at *d*, is supported by the fork. And this center of gravity is as much nearer to the center of the ball *A*, than to the center of the ball *B*, as *A* is heavier than *B*, allowing for the weight of the wire on each side of the fork. This done, let the machine be put into motion by the winch; and the balls *A* and *B* will go round their common center

center of gravity *d*, keeping their balance, because either will not allow the other to fly off with it. For, supposing the ball *B* to be only one ounce in weight, and the ball *A* to be six ounces; then, if the wire *C* were equally heavy on each side of the fork, the center of gravity *d* would be six times as far from the center of the ball *B* as from the center of the ball *A*, and consequently *B* will revolve with a velocity six times as great as *A* does; which will give *B* six times as much centrifugal force as any single ounce of *A* has: but then, as *B* is only one ounce, and *A* six ounces, the whole centrifugal force of *A* will exactly balance the whole centrifugal force of *B*: and therefore, each body will detain the other so as to make it keep in its circle. This shews that the sun and planets must all move round the common center of gravity of the whole system, in order to preserve that just balance which takes place among them. For, the planets being as unactive and dead as the above balls, they could no more have put themselves into motion than these balls can: nor have kept in their orbits without being balanced at first with the greatest degree of exactness upon their common center of gravity, by the Almighty hand that made them and put them in motion.

Perhaps it may be here asked, since these balls cannot go round unless their center of gravity be supported by the prop or fork *E*; what prop it is that supports the center of gravity of the solar system, and consequently bears the weight of all the bodies in it; and by what the prop itself is supported?

supported? The answer is easy and plain; for the center of gravity of our balls must be supported, because they gravitate towards the earth, and would therefore fall to it: but as the sun and planets gravitate only towards one another, they have nothing else to fall to; and therefore have no occasion for any thing to support their common center of gravity: and if they did not move round that center, and consequently acquire a tendency to fly off from it by their motions, their mutual attractions would soon bring them together; and so the whole would become one mass in the sun: which would also be the case if their velocities round the sun were not quick enough to create a centrifugal force equal to the sun's attraction.

But after all this nice adjustment, it appears evident that the Deity cannot withdraw his regulating hand from his works, and leave them to be solely governed by the laws which he has imprest upon them at first. For if he would leave them so, their order would in time come to an end; because the planets must necessarily disturb one another's motions by their mutual attractions, when several of them are in the same quarter of the heavens; as is often the case: and then, as they will attract the sun more towards that quarter than when they are in a manner dispersed equably around him, if he was not at that time made to describe a portion of a larger circle round the common center of gravity, the balance would be destroyed; and as it could never restore itself again, the whole system would begin to fall together, and would in time unite in
a mass

a mass at the sun.—Of this disturbance we have a very remarkable instance in the comet which now appears; and which, in going last up from the sun, went so near to Jupiter, and was so affected by his attraction, as to have the figure of its orbit much changed; and not only so, but to have its period altered, and its course to be different in the heavens from what it was before.

11. Take away the fork and balls from the Fig. 7. whirling-board, and place the trough *AB* thereon, fixing its center to the center of the whirling-board by the pin *H*. In this trough are two balls *D* and *E*, of unequal weights, connected by a wire *f*; and made to slide easily upon the wire *C* stretched from end to end of the trough, and made fast by nut-screws on the outside of the ends. Let these balls be so placed upon the wire *C*, that their common center of gravity *g* may be directly over the center of the whirling-board. Then, turn the machine by the winch, ever so swiftly, and the trough and balls will go round their center of gravity so as neither of them will fly off; because, on account of the equilibrium, each ball detains the other with an equal force acting against it. But if the ball *E* be drawn a little more towards the end of the trough at *A*, it will remove the center of gravity towards that end from the center of motion; and then, upon turning the machine, the little ball *E* will fly off, and strike with a considerable force against the end *A*, and draw the great ball *B* into the middle of the trough. Or, if the great ball *D* be drawn towards the end *B* of the

the trough, so that the center of gravity may be a little towards that end from the center of motion, and the machine be turned by the winch, the great ball *D* will fly off, and strike violently against the end *B* of the trough, and will bring the little ball *E* into the middle of it. If the trough be not made very strong, the ball *D* will break through it.

Of the
tides.

12. The reason why the tides rise at the same absolute time on opposite sides of the earth, and consequently in opposite directions, is made abundantly plain by a new experiment on the whirling-table. The cause of their rising on the side next the moon every one understands to be owing to the moon's attraction: but why they should rise on the opposite side at the same time, where there is no moon to attract them, is perhaps not so generally understood. For it would seem that the moon should rather draw the waters (as it were) closer to that side, than raise them upon it, directly contrary to her attractive force. Let the circle *abcd* represent the earth, with its side *c* turned toward the moon, which will then attract the waters so, as to raise them from *c* to *g*. But the question is, why should they rise as high at that very time on the opposite side, from *a* to *e*? In order to explain this, let there be a plate *AB* fixed upon one end of the flat bar *DC*; with such a circle drawn upon it as *abcd* (in Fig. 8.) to represent the round figure of the earth and sea; and such an ellipsis as *defg* to represent the swelling of the tide at *e* and *g*, occasioned by the influence of the moon. Over this

Fig. 8.

Fig. 9.

this plate *AB* let the three ivory balls *e*, *f*, *g*, be hung by the silk lines *b*, *i*, *k*, fastened to the tops of the crooked wires *H*, *I*, *K*, in such a manner that the ball at *e* may hang freely over the side of the circle *e*, which is farthest from the moon *M*, (at the other end of the bar;) the ball at *f* may hang freely over the center, and the ball at *g* hang over the side of the circle *g*, which is nearest the moon. The ball *f* may represent the center of the earth, the ball *g* some water on the side next the moon, and the ball *e* some water on the opposite side. On the back of the moon *M* is fixt the short bar *N* parallel to the horizon, and there are three holes in it above the little weights *p*, *q*, *r*. A silk thread *o* is tied to the line *k* close above the ball *g*, and passing by one side of the moon *M*, goes through a hole in the bar *N*, and has the weight *p* hung to it. Such another thread *n* is tied to the line *i*, close above the ball *f*, and passing through the center of the moon *M* and middle of the bar *N*, has the weight *q* hung to it, which is lighter than the weight *p*. A third thread *m* is tied to the line *b*, close above the ball *e*, and passing by the other side of the moon *M*, through the bar *N*, has the weight *r* hung to it, which is lighter than the weight *q*.

The use of these three unequal weights is to represent the moon's unequal attraction at different distances from her. With whatever force she attracts the center of the earth, she attracts the side next her with a greater degree of force, and the side farthest from her with a less. So, if the weights

Fig. 10.

weights are left at liberty, they will draw all the three balls towards the moon with different degrees of force ; and cause them to make the appearance shewn in Fig. 10 ; by which means they are evidently farther from each other than they would be if they hung at liberty by the lines *b, i, k* ; because the lines would then hang perpendicularly. This shews, that as the moon attracts the side of the earth which is nearest her with a greater degree of force than she does the center of the earth, she will draw the water on that side more than she draws the center, and so cause it to rise on that side : and as she draws the center more than she draws the opposite side, the center will recede farther from the surface of the water on that opposite side, and so leave it as high there as she raised it on the side next to her. For, as the center will be in the middle between the tops of the opposite elevations, they must of course be equally high on both sides at the same time.

But upon this supposition the earth and moon would soon come together : and to be sure they would, if they had not a motion round their common center of gravity, to create a degree of centrifugal force sufficient to balance their mutual attraction. This motion they have ; for as the moon goes round her orbit every month, at the distance of 240000 miles from the earth's center, and of 234000 miles from the center of gravity of the earth and moon, so does the earth go round the same center of gravity every month at the distance of 6000 miles from it ; that is, from it
to

to the center of the earth. Now as the earth is (in round numbers) 8000 miles in diameter, it is plain that its side next the moon is only 2000 miles from the common center of gravity of the earth and moon; its center 6000 miles distant therefrom, and its farthest side from the moon 10000. Therefore the centrifugal forces of these parts are as 2000, 6000, and 10000, that is, the centrifugal force of any side of the earth when it is turned from the moon is five times as great as when it is turned towards the moon. And as the moon's attraction (expressed by the number 6000) at the earth's center keeps the earth from flying out of this monthly circle, it must be greater than the centrifugal force of the waters next her, and consequently sufficient to raise them; but as her attraction on the opposite side is less than the centrifugal force of the water there, the excess of this force is sufficient to raise the water just as high on the opposite side.—To prove this experiment-
Fig. 9.
ally, let the bar *DC* with its furniture be fixed upon the whirling-board of the machine (Fig. 1.) by pushing the pin *P* into the center of the board; which pin is in the center of gravity of the whole bar with its three balls *e*, *f*, *g*, and moon *M*. Now, if the whirling-board and bar be turned slowly round by the winch, until the ball *f* hangs over the center of the circle, as in Fig. 11. the ball *g* will be kept towards the moon by the heaviest weight *p*, (Fig. 9.) and the ball *e*, on account of its greater centrifugal force, and the lesser weight *r*, will fly off as far to the other side, as in Fig. 11.

E

And

And so, whilst the machine is kept turning, the balls *e* and *g* will hang over the ends of the ellipsis *lfk*. So that the centrifugal force of the ball *e* will exceed the moon's attraction just as much as her attraction exceeds the centrifugal force of the ball *g*, whilst her attraction just balances the centrifugal force of the ball *f* and makes it keep in its circle. And hence it is evident that the tides must rise to equal heights at the same time on opposite sides of the earth. This experiment, to the best of my knowledge, is entirely new.

The earth's
motion de-
monstra-
ted.

From the principles thus established, it is evident that the earth moves round the sun, and not the sun round the earth: for the centrifugal law will never allow a great body to move round a small one in any orbit whatever; especially when we find that if a small body moves round a great one, the great one must also move round the common center of gravity between them two. And it is well known that the quantity of matter in the sun is 227000 times as great as the quantity of matter in the earth. Now, as the sun's distance from the earth is at least 81,000,000 of miles, if we divide that distance by 227000, we shall have only 357 for the number of miles that the center of gravity between the sun and earth is distant from the sun's center. And as the sun's semidiameter is $\frac{1}{4}$ of a degree, which, at so great a distance as that of the sun, must be no less than 381500 miles, if this be divided by 357, the quotient will be $1068\frac{2}{3}$, which shews that the common center of gravity is within the body of the sun; and is only the $1068\frac{2}{3}$ part

part of his semidiameter from his center toward his surface.

All globular bodies which do not turn on their axes must be perfect spheres, because all parts of their surfaces are equally attracted toward their centers. But all globes which do turn on their axes will be oblate spheroids; that is, their surfaces will be higher, or farther from the center, in the equatoreal than in the polar regions. For, as the equatoreal parts move quickest, they must have the greatest centrifugal force; and will therefore recede farthest from the axis of motion. Thus, if two Fig. 12.
circular hoops *AB* and *CD*, made thin and flexible, and crossing one another at right angles, be turned round their axis *EF* by means of the winch *m*, the wheel *n*, and pinion *o*; and the axis be loose in the pole or intersection *e*, the middle parts *A, B, C, D* will swell out so as to strike against the sides of the frame at *F* and *G*, if the pole *e*, in sinking to the pin *E*, be not stopt by it from sinking farther: so that the whole will appear of an oval figure, the equatoreal diameter being considerably longer than the polar. That our earth is of this figure is demonstrable from actual measurement of some degrees on its surface, which are found to be longer in the frigid zones than in the torrid: and the difference is found to be such as proves the earth's equatoreal diameter to be 35 miles longer than its axis.—Since then, the earth is higher at the equator than at the poles, the sea, which like all other fluids naturally runs downward (or towards the places which are nearest the earth's
E 2 center)

center) would run towards the polar regions, and leave the equatoreal parts dry, if the centrifugal force of the water, which carried it to those parts, and so raised them, did not detain and keep it from running back again towards the poles of the earth.

LECT. III.

Of the mechanical powers.

The foundation of
all mechanics.

IF we consider bodies in motion, and compare them together, we may do this either with respect to the quantities of matter they contain, or the velocities with which they are moved. The heavier any body is, the greater is the power required either to move it or to stop its motion: and again, the swifter it moves, the greater is its force. So that the whole *momentum* or quantity of force of a moving body is the result of its quantity of matter multiplied by the velocity with which it is moved. And when the products arising from the multiplication of the particular quantities of matter in any two bodies by their respective velocities are equal, the *momenta* or entire forces are so too. Thus, suppose a body, which we shall call *A*, to weigh 40 pounds, and to move at the rate of 2 miles in a minute; and another body, which we shall call *B*, to weigh only 4 pounds, and to move 20 miles in a minute; the entire forces with which these two bodies would strike against any obstacle would be equal to each other, and therefore it would require equal powers to stop them. For 40 multiplied

multiplied by 2 gives 80, the force of the body *A*; and 20 multiplied by 4 gives 80, the force of the body *B*.

Upon this easy principle depends the whole of mechanics: and it holds universally true, that when two bodies are suspended by any machine, so as to act contrary to each other; if the machine be put into motion, and the perpendicular ascent of one body multiplied into its weight, be equal to the perpendicular descent of the other body multiplied into its weight, these bodies, how unequal soever in their weights, will balance one another in all situations: for, as the whole ascent of one is performed in the same time with the whole descent of the other, their respective velocities must be directly as the spaces they move through; and the excess of weight in one body is compensated by the excess of velocity in the other.—Upon this principle it is easy to compute the power of any mechanical engine, whether simple or compound; How to compute the power of any mechanical engine. for it is but only enquiring how much swifter the power moves than the weight (*i. e.* how much farther in the same time,) and just so much must the power be increased by the help of the engine.

In the theory of this science, we suppose all planes perfectly even, all bodies perfectly smooth, levers to have no weight, cords to be extremely pliable, machines to have no friction; and in short, all imperfections must be set aside until the theory be established, and then proper allowances are to be made.

The mechanic powers, what?

The simple *machines* usually called *mechanical powers* are six in number, viz. the *lever*, the *wheel and axle*, the *pulley*, the *inclined plane*, the *wedge*, and the *screw*.—They are called mechanical powers, because they help us to raise weights, move heavy bodies, and overcome resistances, which we could not effect without them.

The *lever*.

1. A *lever* is a bar of iron or wood, one part of which being supported by a prop, all the other parts turn upon that prop as their center of motion: and the velocity of every part or point is directly as its distance from the prop. Therefore, when the weight to be raised at one end is to the power applied at the other to raise it, as the distance of the power from the prop to the distance of the weight from the prop, the power and weight will exactly balance or counterpoise each other: and as a common lever has but very little friction on its prop, a very little additional power will be sufficient to raise the weight.

There are four kinds of levers. 1. The common sort, where the prop is placed between the weight and the power; but much nearer to the weight than to the power. 2. When the prop is at one end of the lever, the power at the other, and the weight between them. 3. When the prop is at one end, the weight at the other, and the power applied between them. 4. The bended lever, which differs only in form from the first sort, but not in property. Those of the first and second kind are often used in mechanical engines; but there

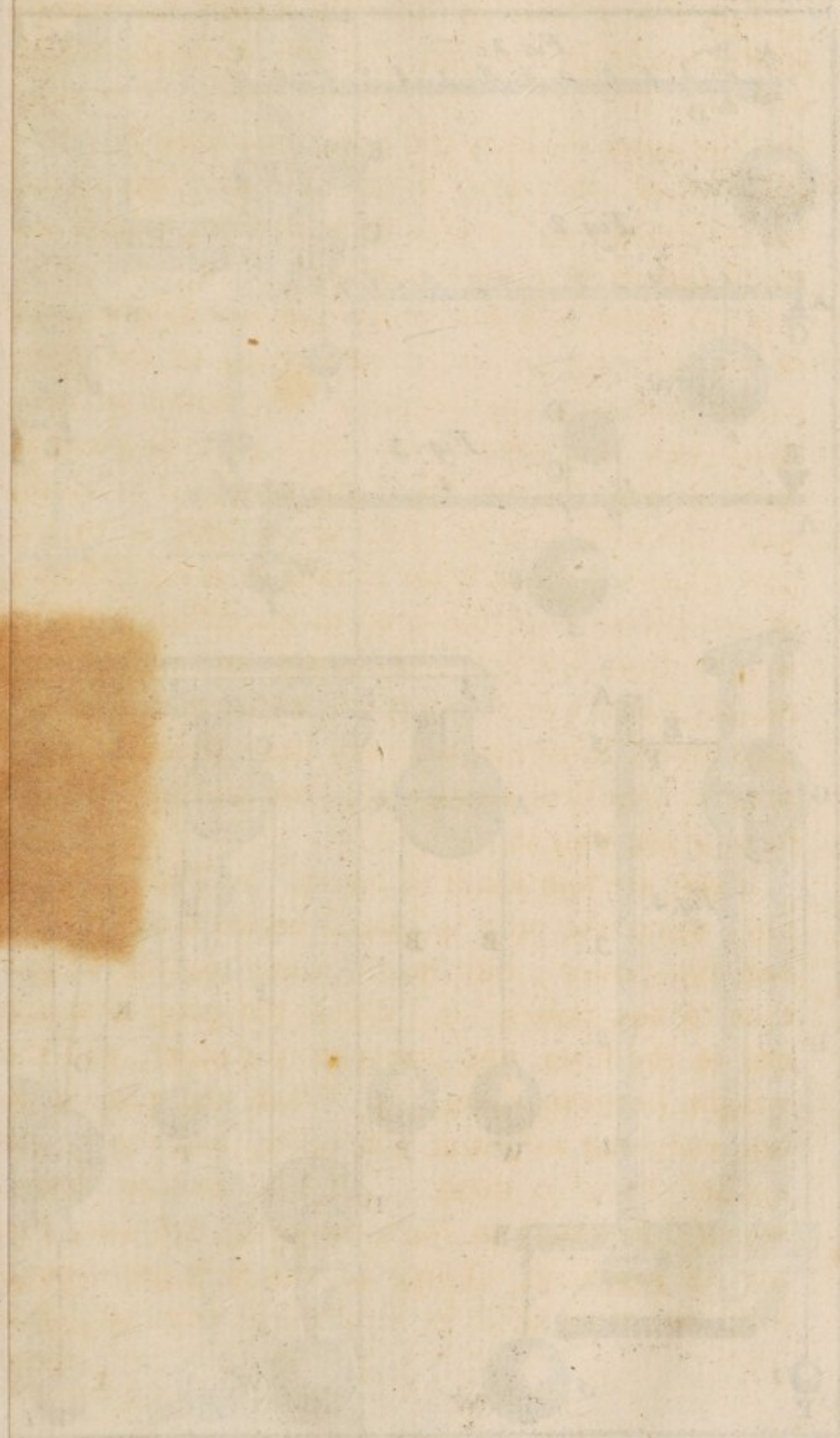
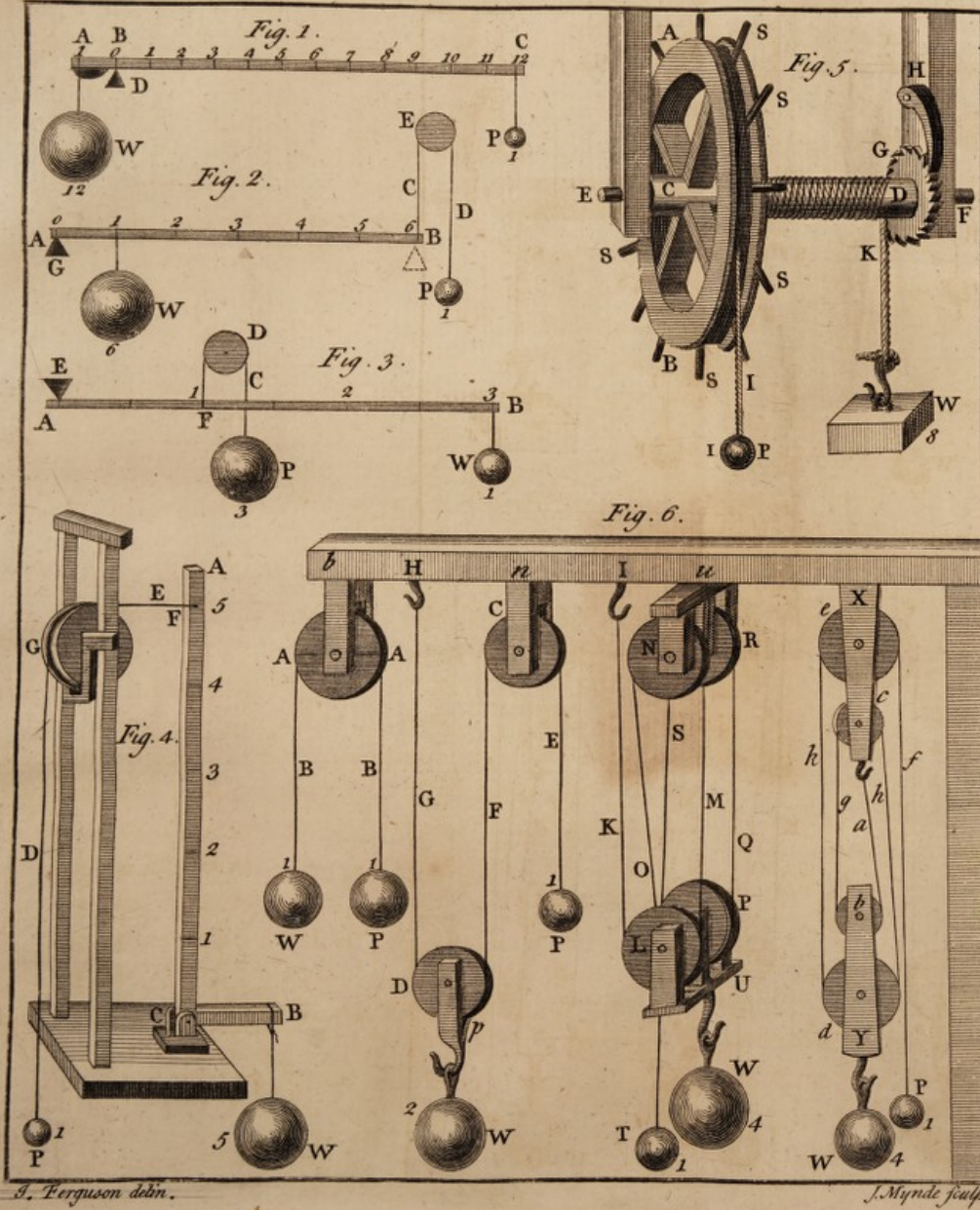


PLATE V.



there are few instances in which the third sort is used.

A *common balance* is a lever of the first kind ; The balance. but as both its ends are at equal distances from its center of motion, they move with equal velocities ; and therefore, as it gives no mechanical advantage, it cannot properly be reckoned among the mechanical powers.

A lever of the first kind is represented by the bar *ABC*, supported by the prop *D*. Its principal use is to loosen large stones in the ground, or raise great weights to small heights, in order to have ropes put under them for raising them higher by other machines. The parts *AB* and *BC*, on different sides of the prop *D*, are called the *arms* of the lever : the end *A* of the shorter arm *AB* being applied to the weight intended to be raised, or to the resistance to be overcome ; and the power applied to the end *C* of the longer arm *BC*. Plate V.
Fig. 1.
The first kind of lever.

In making experiments with this machine, the shorter arm *AB* must be as much thicker than the longer arm *BC*, as will be sufficient to balance it on the prop. This supposed, let *P* represent a power, whose intensity is equal to 1 ounce, and *W* a weight whose intensity is equal to 12 ounces. Then, if the power be 12 times as far from the prop as the weight is, they will exactly counterpoise ; and a small addition to the power *P* will cause it to descend, and raise the weight *W* ; and the velocity with which the power descends will be to the velocity with which the weight rises, as 12 to 1 : that is, directly as their distances from the

E 4

prop ;

prop; and consequently, as the spaces through which they move. Hence, it is plain that a man who by his natural strength, without the help of any machine, could support an hundred weight, will by the help of this lever be enabled to support twelve hundred. If the weight be less, or the power greater, the prop may be placed so much the farther from the weight; and then it can be raised to a proportionably greater height. For universally, if the intensity of the weight multiplied into its distance from the prop be equal to the intensity of the power multiplied into its distance from the prop, the power and weight will exactly balance each other; and a little addition to the power will raise the weight. Thus, in the present instance, the weight W is 12 ounces, and its distance from the prop is 1 inch; and 12 multiplied by 1 is 12; the power P is equal to 1 ounce, and its distance from the prop is 12 inches, which multiplied by 1 is 12 again; and therefore there is an equilibrium between them. So, if a power equal to 2 ounces be applied at the distance of 6 inches from the prop, it will just balance the weight W ; for 6 multiplied by 2 is 12, as before. And a power equal to 3 ounces placed at 4 inches distance from the prop would do the same; for 3 times 4 is 12; and so on, in proportion.

The *steelyard*.

The *statera* or roman *steelyard* is a lever of this kind, contrived for finding the different weights of different sorts of bodies, or of a greater quantity of the same sort, by one single weight placed at different distances from the prop or center of motion

motion *D*. For, if a scale hangs at *A*, the extremity of the shorter arm *AB*, and is of such a weight as will exactly counterpoise the longer arm *BC*; if this arm be divided into as many equal parts as it will contain, each equal to *AB*, the single weight *P* (which we may suppose to be 1 pound) will serve for weighing any thing as heavy as itself, or as many times heavier as there are divisions in the arm *BC*, or any quantity between its own weight and that quantity. As for example, if *P* be 1 pound, and placed at the first division 1 in the arm *BC*, it will balance 1 pound in the scale at *A*: if it be removed to the second division at 2, it will balance 2 pounds in the scale: if to the third, 3 pounds; and so on to the end of the arm *BC*. If each of these integral divisions be subdivided into as many equal parts as a pound contains ounces, and the weight *P* be placed at any of these subdivisions, so as to counterpoise what is in the scale, the pounds and odd ounces therein are by that means ascertained.

To this kind of lever may be reduced several sorts of instruments, such as scissars, pinchers, snuffers; which are made of two levers acting contrary to one another: their prop or center of motion being the pin which keeps them together.

In common practice, the longer arm of this lever exceeds the weight of the shorter; which gains an advantage, because it adds so much more to the power.

A lever of the second kind has the weight between the prop and the power. In this, as well as the

The second
kind of lever.

the former, the advantage gained is as the distance of the power from the prop to the distance of the weight from the prop: for the respective velocities of the power and weight are in that proportion; and they will balance each other when the intensity of the power multiplied by its distance from the prop is equal to the intensity of the weight multiplied by its distance from the prop.

Fig. 2. Thus, if AB be a lever on which the weight W of 6 ounces hangs at the distance of 1 inch from the prop G , and a power P equal to the weight of 1 ounce hangs at the end B , 6 inches from the prop, by the cord CD going over the fixed pulley E , the power will just support the weight: and a small addition to the power will raise the weight, 1 inch for every 6 inches that the power descends.

This lever shews the reason why two men carrying a burden upon a stick between them, bear unequal shares of the burden in the inverse proportion of their distances from it. For it is well known, that the nearer any of them is to the burden, the greater share he bears of it: and if he goes directly under it, he bears the whole. So, if one man be at G and the other at P , having the pole or stick AB resting on their shoulders; if the burden or weight W be placed five times as near the man at G as it is to the man at P , the former will bear five times as much weight as the latter. This is likewise applicable to the case of two horses of unequal strength, to be so yoked as that each horse may draw a part proportionable to his strength; which is done by dividing the beam so, that

that the point of traction may be as much nearer to the stronger horse than to the weaker as the strength of the former is greater.

To this kind of lever may be reduced oars, rudders of ships, doors turning upon hinges, cutting knives which are fixed at the point of the blade, and the like.

If in this lever we suppose the power and weight to change places, so that the power may be between the weight and the prop, it will become a lever of the third kind: in which, that there may be a balance between the power and the weight, the intensity of the power must exceed that of the weight just as much as the distance of the weight from the prop exceeds the distance of the power from it. Thus, let E be the prop of the lever AB , and W a weight of 1 pound placed 3 times as far from the prop as the power P acts at F by the cord C going over the fixed pulley D ; in this case, the power must be equal to three pounds in order to support the weight.

The third
kind of le-
ver.

Fig. 3.

To this sort of lever are generally referred the bones of a man's arm: for when we lift a weight by the hand, the muscle that exerts its force to raise that weight is fixed to the bone about one tenth part as far below the elbow as the hand is. And the elbow being the center round which the arm turns, the muscle must therefore exert a force ten times as great as the weight raised.

As this kind of lever gives no advantage to the moving power, it is never used but in cases of necessity; such as that of a ladder, which being
fixed

fixed at one end, is by the strength of a man's arms reared against a wall. And in clock-work, where all the wheels may be reckoned levers of this kind, because the power that moves every wheel except the first, acts upon it near the center of motion by means of a small pinion, and the resistance it has to overcome acts against the teeth round its circumference.

The fourth
kind of lever.

Fig. 4.

The fourth kind of lever differs nothing from the first, but in being bended for the sake of convenience. ACB is a lever of this sort, bended at C which is its prop or center of motion. P is a power acting upon the longer arm AC at F , by means of the cord DE going over the pulley G ; and W is a weight or resistance acting upon the end B of the shorter arm BC . If the power be to the weight as BC to CF , they are in *equilibrio*. Thus, suppose W to be 5 pounds acting at the distance of one foot from the center of motion C , and P to be 1 pound acting at F five feet from the center C , the power and weight will just balance each other. A hammer drawing a nail is a lever of this sort.

The wheel
and axle.

Fig. 5.

2. The second mechanical power is the *wheel and axle*, in which the power is applied to the circumference of the wheel, and the weight is raised by a rope which coils about the axle as the wheel is turned round. Here it is plain that the velocity of the power must be to the velocity of the weight as the circumference of the wheel is to the circumference of the axle: and consequently, the power and weight will balance each other when the intensity

tenſity of the power is to the intenſity of the weight as the circumference of the axle is to the circumference of the wheel. Let AB be a wheel, CD its axle, and ſuppoſe the circumference of the wheel to be 8 times as great as the circumference of the axle; then, a power P equal to one pound hanging by the cord I , which goes round the wheel, will balance a weight W of 8 pounds hanging by the rope X , which goes round the axle. And as the friction on the pivots or gudgeons of the axle is but ſmall, a ſmall addition to the power will cauſe it to deſcend, and raiſe the weight: but the weight will riſe with only an eighth part of the velocity that the power deſcends, and conſequently, through no more than an eighth part of an equal ſpace, in the ſame time. If the wheel be pulled round by the handles S, S , the power will be increaſed in proportion to their length. And by this means, any weight may be raiſed as high as the operator pleaſes.

To this ſort of engine belong all cranes for raiſing great weights; and in this caſe, the wheel may have cogs all around it inſtead of handles, and a ſmall lantern or trundle may be made to work in the cogs, and be turned by a winch; which will again increaſe the power of the wheel as much as its number of cogs exceeds the number of ſtaves or rounds in the trundle, if the radius of the trundle and that of the winch be equal: but if the radius or length of the winch be double the radius or ſemidiameter of the trundle, it will again give a double power. So that, if the circumference of the wheel be 8
times

times the circumference of the axle, the power of the axle is eight times as great as the power of the wheel; and if the trundle makes 8 revolutions to one of the wheel, the power of the axle is 8 times 8 times, or 64 times, as great as that of the trundle: if the length of the winch be twice the semi-diameter of the trundle, this will again give a double power; and so make it twice 64 or 128 on the axle. Therefore, a man who by his natural strength could lift an hundred weight from the ground in his arms, will, by the help of such an engine as this, be able to raise 128 hundred weight, or 6 tons and almost an half. In this sort of machines it is requisite to have a ratchet-wheel *G* on one end of the axle, with a catch *H* to fall into its teeth; which will at any time support the weight, and keep it from descending, if the workman should through inadvertency or carelessness quit his hold whilst the weight is raising. And by this means, the danger is prevented which might otherwise happen by the running down of the weight when left at liberty.

The pulley. 3. The third mechanical power or engine consists either of one *moveable pulley* or a *system of pullies*, some in a block or case which is fixed, and others in a block which is moveable and rises with the weight. For though a single pulley that only turns on its axis, and rises not with the weight, may serve to change the direction of the power, yet it can give no mechanical advantage thereto: but is only as the beam of a balance whose arms are of equal length and weight. Thus, if the equal weights

weights W and P hang by the cord BB upon the pulley A , whose block b is fixed to the beam HI , they will counterpoise each other, just in the same manner as if the cord were cut in the middle, and its two ends hung upon the hooks fixt in the pulley at A and A , equally distant from its center.

But if a weight W hang at the lower end of the moveable block p of the pulley D , and the cord GF go under the pulley, it is plain that the half G of the cord bears one half of the weight W , and the half F the other; for they bear the whole between them. Therefore, whatever holds the upper end of either rope sustains one half of the weight: and if the cord at F be drawn up so as to raise the pulley D to C , the cord will then be extended to its whole length, all but that part which goes under the pulley: and consequently, the power that draws the cord will have moved twice as far as the pulley D with its weight W rises; on which account, a power whose intensity is equal to one half of the weight will be able to support it, because if the power moves (by means of a small addition) its velocity will be double the velocity of the weight; as may be seen by putting the cord over the fixt pulley C (which only changes the direction of the power without giving any advantage to it) and hanging on the weight P , which is equal only to one half of the weight W ; in which case there will be an equilibrium, and a little addition to P will cause it to descend, and raise W through a space equal to one half of that through which P descends.—Hence, the advantage
gained

gained will be always equal to twice the number of pullies in the moveable or undermost block. So, when the upper or fixt block u contains two pullies, which only turn on their axes, and the lower or moveable block U contains two pullies, which not only turn upon their axes but also rise with the block and weight, the advantage gained by this is as 4 to the power. Thus, if one end of the rope $KMOQ$ be fixed to a hook at I , and the rope pass over the pullies N and R , and under the pullies L and P , and have a weight T of one pound hung to its other end at T , this weight will balance and support a weight W of four pounds hanging by a hook at the moveable block U , allowing the said block as a part of the weight. And if as much more power be added as is sufficient to overcome the friction of the pullies, the power will descend with four times as much velocity as the weight rises; and consequently through four times as much space.

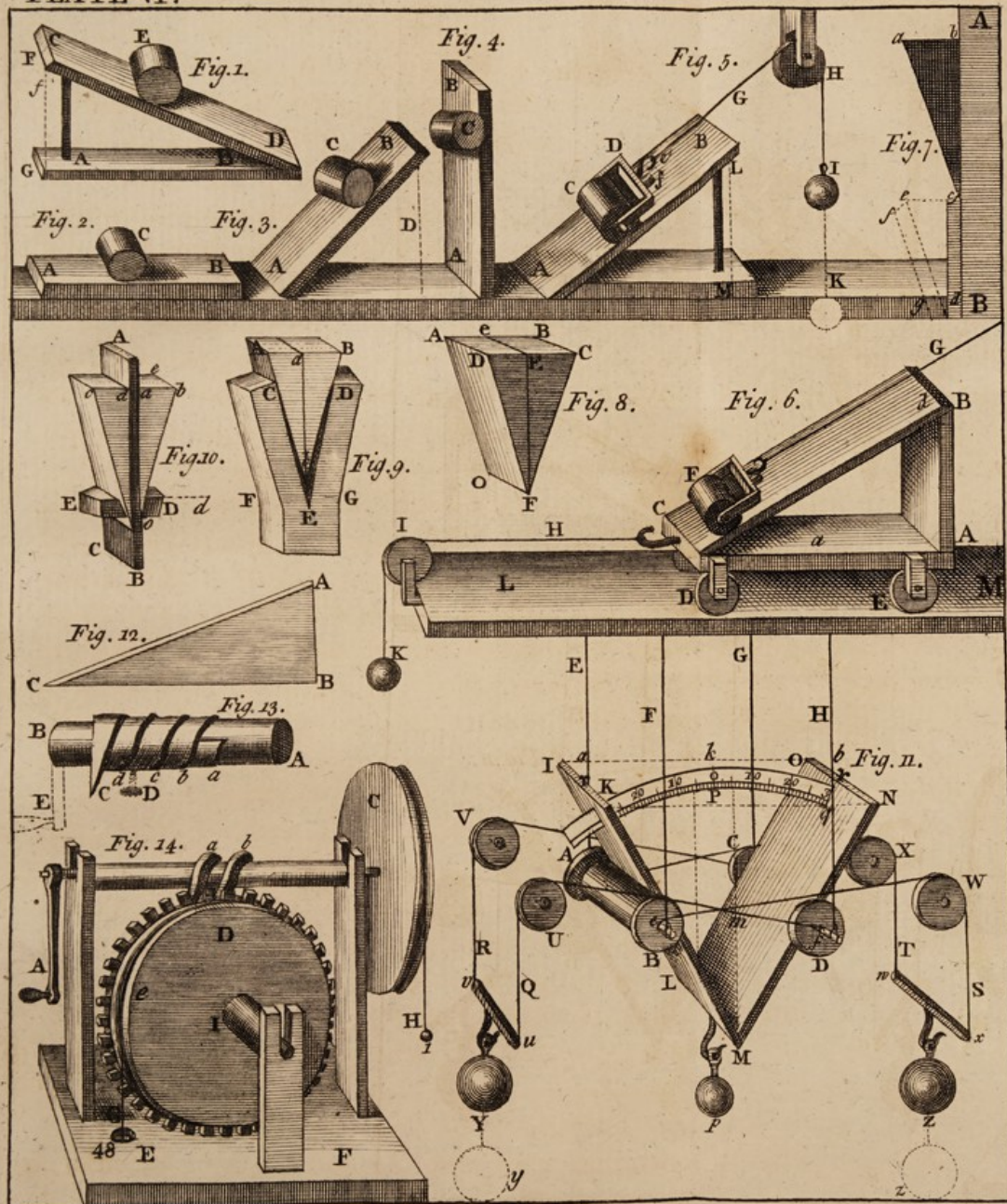
The two pullies in the fixed block X , and the two in the moveable block Y , are in the same case with those last mentioned; and give the same advantage to the power.

As a system of pullies have no great weight, and lie in a small compass, they are easily carried about; and can be applied, in a great many cases, for raising weights, where other engines cannot. But they have a great deal of friction on three accounts:

1. because the diameter of their axis bears a very considerable proportion to their own diameter;
2. because in working they are apt to rub against one



PLATE VI.



J. Ferguson delin.

J. Mynde sc.

one another, or against the sides of the block ;
3. because of the stiffness of the rope that goes round them.

4. The fourth mechanical power is the *inclined plane* ; and the advantage gained by it is as great as its length exceeds its perpendicular height. The inclined plane. Let Plate VI. *AB* be a plane parallel to the horizon, and *CD* a Fig. 1. plane inclined to it ; and suppose the whole length *CD* to be three times as great as the perpendicular height *GfF* : in this case, the cylinder *E* will be supported upon the plane *CD*, and kept from rolling down upon it, by a power equal to a third part of the weight of the cylinder. Therefore, a weight may be rolled up this inclined plane with a third part of the power which would be sufficient to draw it up by the side of an upright wall. If the plane was four times as long as high, a fourth part of the power would be sufficient ; and so on, in proportion. Or, if a pillar is to be raised from a floor to the height *GF*, by means of the engine *ABDC*, (which will then act as a half wedge, where the resistance gives way only on one side) the engine and pillar will be in *equilibrio* when the power applied at *GF* is to the weight of the pillar as *GF* to *CD* ; and if the power be increased, so as to overcome the friction of the engine against the floor and pillar, the engine will be driven, and the pillar raised : and when the engine has moved its whole length upon the floor, the pillar will be raised to the whole height of the engine, from *G* to *F*.

The force wherewith a rolling body descends upon an inclined plane is to the force of its absolute

F

gravity,

- gravity, by which it would descend perpendicularly in a free space, as the height of the plane is to its length. For, suppose the plane AB to be parallel to the horizon, the cylinder C will keep at rest upon any part of the plane where it is laid.
- Fig. 2.
- Fig. 3. If the plane be so elevated, that its perpendicular height D is equal to half its length AB , the cylinder will roll down upon the plane with a force equal to half its weight; for it would require a power equal to half its weight to keep it from rolling. If the plane AB be elevated, so as to be perpendicular to the horizon, the cylinder C will descend with its whole force of gravity, because the plane contributes nothing to its support or hindrance; and therefore, it would require a power equal to its whole weight to keep it from descending.
- Fig. 4.
- Fig. 5. Let the cylinder C be fitted to turn upon slender pivots in the frame D , in which there is a hook e with a line G tied to it: let this line go over the fixed pulley H , and have its other end tied to a hook in the weight I . If the weight of the body I be to the weight of the cylinder C added to that of its frame D , as the perpendicular height of the plane LM is to its length AB , the weight will just support the cylinder upon the plane, and a small touch of a finger will either cause it to ascend or descend with equal ease: then, if a little addition be made to the weight I , it will descend, and draw the cylinder up the plane. In the time that the cylinder moves from A to B , it will rise through the whole height of the plane ML ; and the weight will

will descend from H to K , through a space equal to the whole length of the plane AB .

If the plane be made to move upon rollers, or friction-wheels, and the cylinder be supported upon it; the same power will draw the plane under the cylinder, which before drew the cylinder up the plane, provided the pivots of the axes of the friction-wheels be small, and the wheels themselves be pretty large. For, let the machine ABC (equal Fig. 6, in length and height to ABM , Fig. 5.) be provided with four wheels, whereof two appear at D and E , and the third under C , whilst the fourth is hid from sight by the horizontal board a . Let the cylinder F be laid upon the lower end of the inclined plane CB , and the line G be extended from the frame of the cylinder, about six feet, parallel to the plane CB ; and, in that direction, fixed to a hook in the wall; which will support the cylinder, and keep it from rolling off the plane. Let one end of the line H be tied to a hook at C in the machine, and the other end to a weight K , the same as drew the cylinder up the plane before. If this line be put over the fixed pulley I , the weight K will draw the machine along the horizontal plane L , and under the cylinder F : and when the machine has been drawn its whole length, the cylinder will be raised to d , equal to the perpendicular height AB above the horizontal part a .

To the inclined plane may be reduced all hatchets, chisels, and other edge-tools which are chamfer'd only on one side.

The wedge. 5. The fifth mechanical power or engine is the wedge, which may be considered as two equal inclined planes DEF and CEF , joined together at their bases $e EF$: then, DC is the whole thickness of the wedge at its back $ABCD$, where the power is applied; EF is the depth or height of the wedge; DF the length of one of its sides, equal to CF the length of the other side; and OF is its sharp edge, which is entered into the wood intended to be split by the force of a hammer or mallet, striking perpendicularly on its back. Thus, ABb is a wedge driven into the cleft CDE of the wood FG .

When the wood does not cleave at any distance before the wedge, there will be an equilibrium between the power impelling the wedge downward, and the resistance of the wood acting against the two sides of the wedge, if the power be to the resistance as half the thickness of the wedge at its back is to the length of either of its sides; that is, as Aa to Ab , or Ba to Bb (Fig. 9.) And if the power be increased, so as to overcome the friction of the wedge and the resistance arising from the cohesion or stickage of the wood, the wedge will be drove in, and the wood split asunder.

But, when the wood cleaves at any distance before the wedge (as it generally does) the power impelling the wedge will not be to the resistance of the wood as half the thickness of the wedge is to the length of one of its sides; but as half its thickness is to the length of either side of the cleft, estimated from the top or acting part of the wedge.

For, if we suppose the wedge to be lengthened down from b to the bottom of the cleft at E , the same proportion will hold; namely, that the power will be to the resistance as half the thickness of the wedge is to the length of either of its sides: or, which amounts to the same thing, as the whole thickness of the wedge is to the length of both its sides.

In order to prove what is here advanced concerning the wedge, let us suppose the wedge to be divided lengthwise into two equal parts; and then it will become two equally inclined planes; one of which, as abc , may be made use of as a half wedge Fig. 7. for separating the moulding cd from the wainscot AB . It is evident, that when this half wedge has been driven its whole length ac between the wainscot and moulding, its side ac will be at ed ; and the moulding will be separated to fg from the wainscot. Now, from what has been already proved of the inclined plane, it appears, that to have an equilibrium between the power impelling the half wedge and the resistance of the moulding, the former must be to the latter as ab to ac ; that is, as the thickness of the back which receives the stroke is to the length of the side against which the moulding acts. Therefore, since the power upon the half wedge is to the resistance against its side, as the half back ab is to the whole side ac , it is plain that the power upon the whole wedge (where the whole back is double the half back) must be to the resistance against both its sides, as the thickness of the whole back is to the length of both the

F 3
sides;

sides; supposing the wedge at the bottom of the cleft: or as the thickness of the whole back to the length of both sides of the cleft, when the wood splits at any distance before the wedge. For, when the wedge is driven quite into the wood, and the wood splits at ever so small a distance before its edge, the top of the wedge then becomes the acting part, because the wood does not touch it anywhere else. And since the bottom of the cleft must be considered as that part where the whole stickage or resistance is accumulated, it is plain, from the nature of the lever, that the farther the power acts from the resistance, the greater is the advantage.

Some writers, particularly *Gravesande*, *Worster*, and *Emerson* advance, that the power of the wedge is to the resistance to be overcome, as the thickness of the back of the wedge is to the length only of one of its sides; which seems very strange: for, if we suppose *AB* to be a strong inflexible bar of wood or iron fixt into the ground at *CB*, and *D* and *E* to be two blocks of marble lying on the ground on opposite sides of the bar; it is evident that the block *D* may be separated from the bar to the distance *ab* by driving the inclined plane or half wedge *abo* down between them; and the block *E* may be separated to an equal distance on the other side in like manner by the half wedge *cdo*. But the power impelling each half wedge will be to the resistance of the block against its side, as the thickness of that half wedge is to the length of its acting side. Therefore the power to drive both the
half

half wedges is to both the resistances, as both the half backs is to the length of both the acting sides, or as half the thickness of the whole back is to the length of either side. And, if the bar be taken away, the blocks put close together, and the two half wedges joined to make one; it will require as much force to drive it down between the blocks, as is equal to the sum of the separate powers acting upon the half wedges when the bar was between them.

To confirm this by an experiment, let two cylinders, as *AB* and *CD*, be drawn towards one another by lines running over fixed pulleys, and a weight of 40 ounces hanging at the lines belonging to each cylinder: and let a wedge, of 40 ounces weight, having its back just as thick as either of its sides is long, be put between the cylinders, which will then act against each side with a resistance equal to 40 ounces, whilst its own weight endeavours to bring it down and separate them. And here, the power of the wedge's gravity impelling it downward will be to the resistance of both the cylinders against the wedge, as the thickness of the wedge is to the length of both its sides; for there will then be an equilibrium between the weight of the wedge and the resistance of the cylinders against it, and it will remain at any height between them; requiring just as much power to push it upward as to pull it downward.—If another wedge, of equal weight and depth with this, and only half as thick, be put between the cylinders, it will require twice as much weight to be hung at

Fig. 11.

the ends of the lines which draw them together, to keep the wedge from going down between them. That is, a wedge of 40 ounces, whose back is only equal to half the length of one of its sides, will require 80 ounces to each cylinder, to keep it in an equilibrium between them; and twice 80. is 160, equal to four times 40. So that the power will be always to the resistance, as the thickness of the back of the wedge is to the length (not of its one side but) of both its sides.

The best way, though perhaps not the neatest that I know of, for making a wedge with its appurtenances for such experiments, is as follows.

Fig. 11. Let $IKLM$ and $LMNO$ be two flat pieces of wood, each about fifteen inches long and three or four in breadth, joined together by a hinge at LM ; and let P be a graduated arch of brass, on which the said pieces of wood may be opened to any angle not more than 60 degrees, and then fixt at the given angle by means of the two screws a and b . Then, $IKNO$ will represent the back of the wedge, LM its sharp edge which enters the wood, and the outsides of the pieces $IKLM$ and $LMNO$ the two sides of the wedge against which the wood acts in cleaving. By means of the said arch, the wedge may be opened so, as to adjust the thickness of its back in any proportion to the length of either of its sides, but not to exceed that length: and any weight as p may be hung to the wedge upon the hook M , which weight, together with the weight of the wedge itself, may be considered as the impelling power; which is all the same in experiment,

periment, whether it be laid upon the back of the wedge to push it down, or hung to its edge to pull it down.—Let AB and CD be two wooden cylinders, each about two inches thick where they touch the outsides of the wedge; and let their ends be made like two round flat plates, to keep the wedge from slipping off endwise between them. Let a small cord with a loop on one end of it go over a pivot in the end of each cylinder, and the cords S and T belonging to the cylinder AB go over the fixt pullies W and X , and be fastened at their other ends to the bar wx , on which any weight as Z may be hung at pleasure. In like manner, let the cords Q and R belonging to the cylinder BC go over the fixt pullies U and V to the bar uv , on which a weight Y equal to Z may be hung. These weights, by drawing the cylinders towards one another, may be considered as the resistance of the wood acting equally against opposite sides of the wedge: the cylinders themselves being suspended near and parallel to each other, by their pivots in loops on the lines E, F, G, H ; which lines may be fixed to hooks in the cieling of the room. The longer these lines are, the better; and they should never be less than four feet each. The farther also the pullies W, V , and W, X are from the cylinders, the truer will the experiments be: and they may turn upon pins fixed into the wall.

In this machine, the weights Y and Z , and the weight p , may be varied at pleasure, so as to be adjusted in proportion of the length of the wedge's sides to the thickness of its back; and when they
are

are so adjusted, the wedge will be in *equilibrio* with the resistance of the cylinders.

The wedge is a very great mechanical power, since not only wood but even rocks can be split by it; which would be impossible to effect by the lever, wheel and axle, or pulley.

The screw. 6. The sixth, and last mechanical power is the *screw*; which cannot properly be called a simple machine, because it is never used without the application of the lever acting as a winch or handle in turning it: and then it becomes a compound engine, of a very great force either in pressing the parts of bodies close together, or in raising great weights. It may be conceived to be made by cutting a piece of paper *ABC* (Fig. 12.) into the form of an inclined plane or half wedge, and then coiling it round a cylinder *AB* (Fig. 13.) And here it is evident, that the winch *E* must turn the cylinder once round before the weight or resistance *D* can be moved from one spiral winding to another, as from *d* to *c*: therefore, as much as the circumference of a circle described by the handle of the winch is greater than the interval or distance between the spirals, so much is the force of the screw. That is, supposing the distance between the spirals to be half an inch, and the length of the winch twelve inches; the circle described by the handle where the power acts will be 76 inches nearly; or about 152 half inches, and consequently 152 times the distance between the spirals: and therefore, a power at the handle, whose intensity is equal to no more than a single pound, will balance 152 pounds

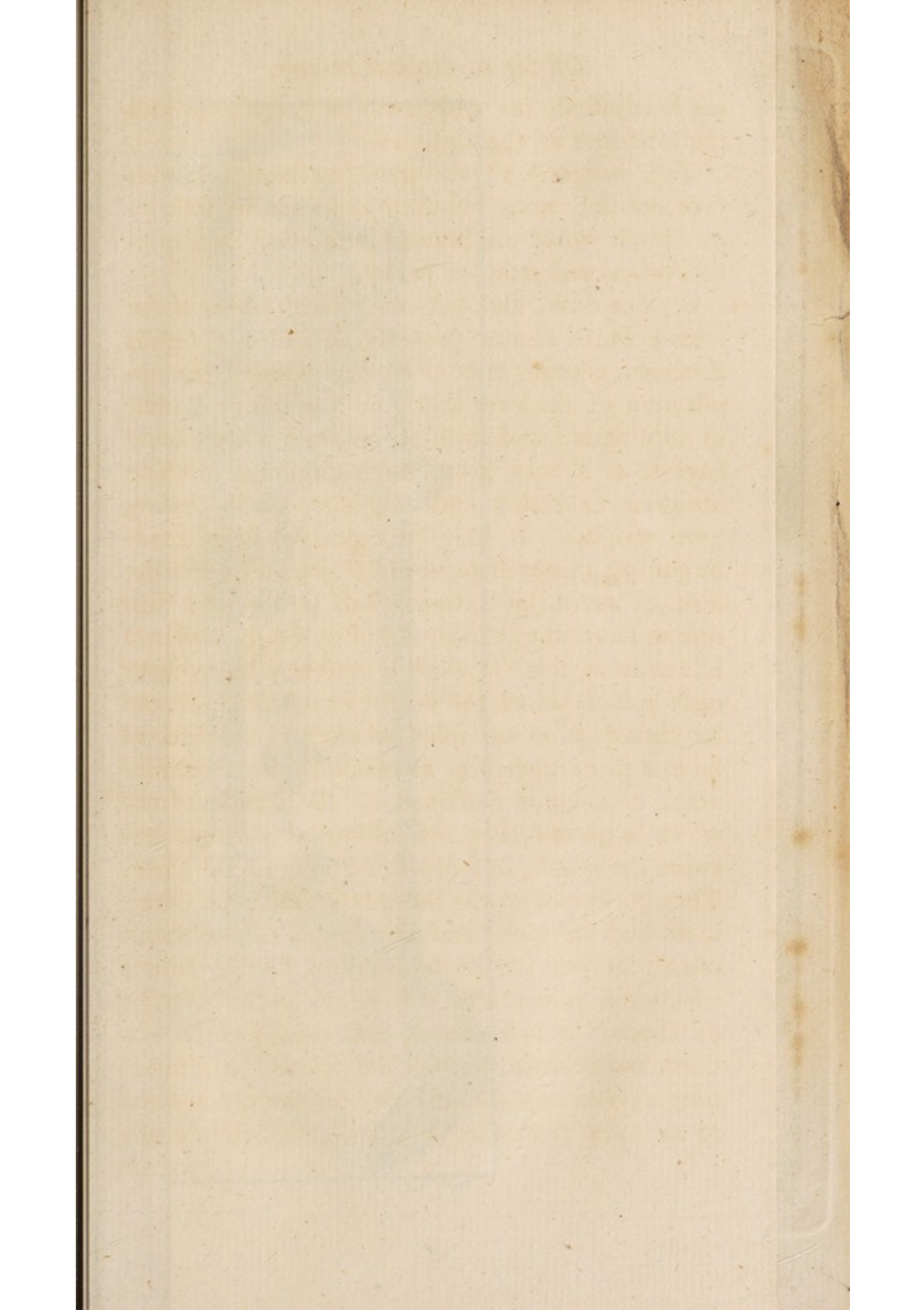
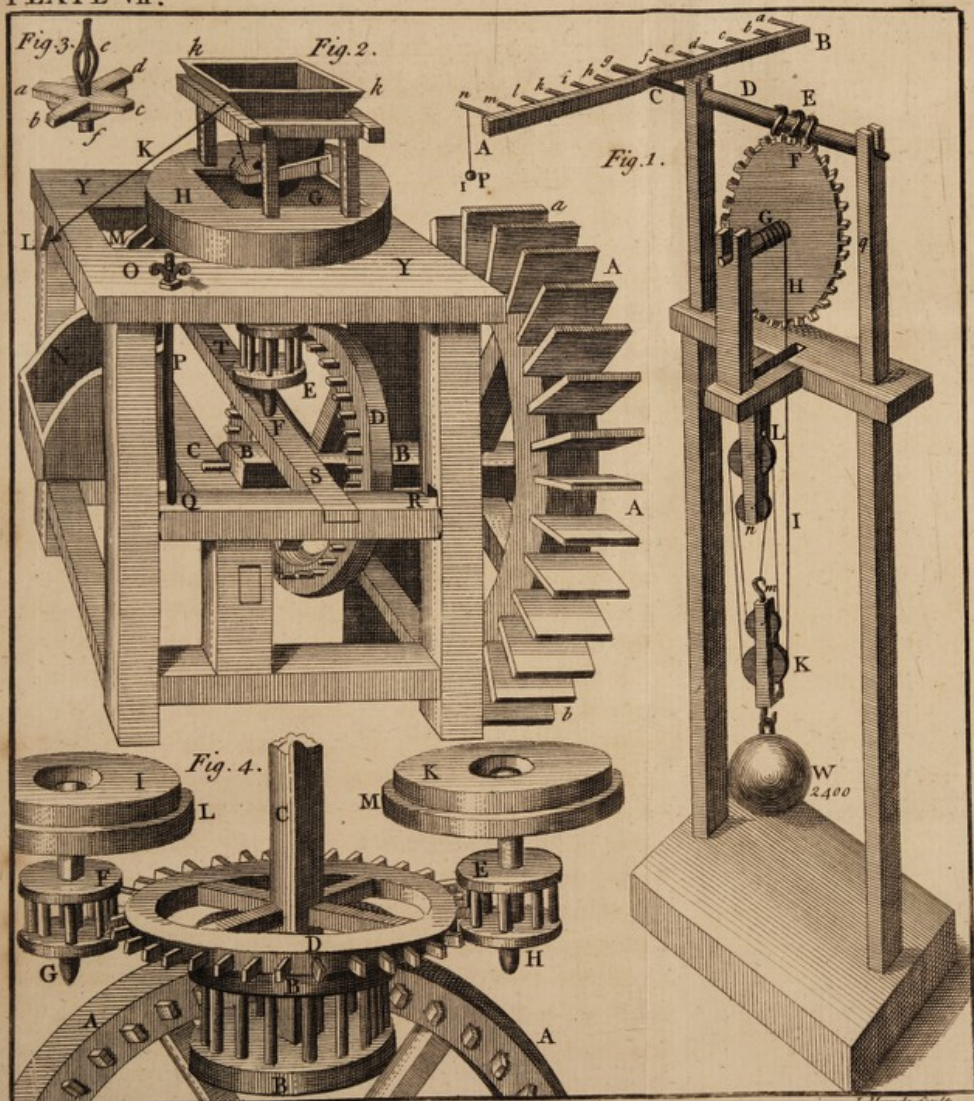


PLATE VII.



J. Ferguson delin.

J. Mynde sculp.

pounds acting against the screw; and as much additional force, as is sufficient to overcome the friction, will raise the 152 pounds; and the velocity of the power will be to the velocity of the weight, as 152 to 1. Hence it appears, that the longer the winch be made, and the nearer the spirals are to one another, so much the greater is the force of the screw.

A machine for shewing the force or power of Fig. 14. the screw may be contrived in the following manner. Let the wheel *C* have a screw *ab* on its axis, working in the teeth of the wheel *D*, which suppose to be 48 in number. It is plain that for every time the wheel *C* and screw *ab* are turned round by the winch *A*, the wheel *D* will be moved one tooth by the screw: and therefore, in 48 revolutions of the winch, the wheel *D* will be turned once round. Then, if the circumference of a circle described by the handle of the winch be equal to the circumference of a groove *e* round the wheel *D*, the velocity of the handle will be 48 times as great as the velocity of any given point in the groove. Consequently, if a line *G* goes round the groove, and has a weight of 48 pounds hung to it below the pedestal *EF*, a power equal to one pound at the handle will balance and support the weight.—To prove this by experiment, let the circumferences of the grooves of the wheels *C* and *D* be equal to one another; and then, if a weight *H* of one pound be suspended by a line going round the groove of the wheel *C*, it will balance a weight of 48 pounds hanging by the line *G*; and
a small

a small addition to the weight *H* will cause it to descend, and so raise up the other weight.

If the line *G*, instead of going round the groove *e* of the wheel *D*, goes round its axle *I*; the power of the machine will be as much increased, as the circumference of the groove *e* exceeds the circumference of the axle: which, supposing it to be 6 times, then one pound at *H* will balance 6 times 48 or 288 pounds hung to the line on the axle: and hence, the power or advantage of this machine will be as 288 to 1. That is to say, a man who by his natural strength could lift an hundred weight, will be able to raise 288 hundred, or $14\frac{8}{10}$ ton weight by this engine.

Plate VII.
Fig. 1.

A combination of
all the mechanical
powers.

But the following engine is still more powerful, on account of its having the addition of four pulleys: and in it we may look upon all the mechanical powers as combined together, even if we take in the balance. For, as the axis *D* of the bar *AB* is in its middle at *C*, it is plain that if equal weights are suspended upon any two pins equidistant from the axis *C*, they will counterpoise. It becomes a lever by hanging a small weight *P* upon the pin *n*, and a weight so much heavier upon either of the pins *b*, *c*, *d*, *e*, or *f*, as is in proportion to the pin's being so much nearer the axis. The wheel-and-axle *FG* is evident; so is the screw *E*, which takes in the inclined plane, and with it the half wedge. Part of a cord goes round the axle, the rest under the lower pulleys *K*, *m*, over the upper pulleys *L*, *n*, and then it is tied to a hook at *m* in
the

the lower or moveable block, on which hangs the weight W .

In this machine, if the wheel F has 30 teeth, it will be turned once round in 30 revolutions of the bar AB , which is fixt on the axis D of the screw E : if the length of the bar is equal to twice the diameter of the wheel, the pins a and n at the ends of the bar will move 60 times as fast as the teeth of the wheel: consequently, one ounce at P will balance 60 ounces hung upon a tooth at q in the horizontal diameter of the wheel. Then, if the diameter of the wheel F is 10 times as great as the diameter of the axle G , the wheel will have 10 times the velocity of the axle; and therefore one ounce P at the end of the lever AC will balance 10 times 60 or 600 ounces hung to the rope H which goes round the axle. Lastly, if four pulleys be added, they will make the velocity of the lower block K , and weight W , four times less than the velocity of the axle: and this being the last power in the machine, which is 4 times as great as that gained by the axle, it makes the whole power of the machine 4 times 600, or 2400. So that a man who could lift one hundred weight in his arms, by his natural strength, would be able to raise 2400 hundred weight by this engine.—But it is here as in all other mechanical cases; for the time lost is always as much as the power gained, because the velocity with which the power moves will ever exceed the velocity with which the weight rises, as much as the intensity of the weight exceeds the intensity of the power.

The

The friction of the screw itself is very considerable: and there are few combined engines, but what, upon account of the friction of the parts against one another, will require a third part of more power to work them when loaded, than what is sufficient to constitute a balance between the weight and the power.

L E C T. IV.

Of mills, cranes, wheel-carriages, and the engine for driving piles.

AS these machines are so universally useful, it would be ridiculous to make any apology for describing them.

Plate VII,
Fig. 2.
A common
mill.

In a common *breast mill*, where the fall of water may be about ten feet, *AA* is the great wheel, which is generally about 17 or 18 feet diameter, reckoned from the outermost edge of any float-board at *a* to that of its opposite float at *b*. To this wheel the water is conveyed through a channel, and so falling upon the wheel, turns it round.

On the axis *BB* of this wheel, and within the mill-house, is a wheel *D*, about 8 or 9 feet diameter, having 61 cogs which turn a trundle *E* containing 10 upright staves or rounds; and when these are the number of cogs and rounds, the trundle will make $6\frac{1}{10}$ revolutions for one revolution of the wheel.

The trundle is fixt upon a strong iron axis call'd the spindle, the lower end of which turns in a
brass

brass foot, fixt at *F*, in the horizontal beam *ST* called the bridge-tree; and the upper part of the spindle turns in a wooden bush fixt into the nether millstone which lies upon beams in the floor *XY*. The top part of the spindle above the bush is square, and goes into a square hole in a strong iron cross *abcd*, (See Fig. 3.) called the rynd; under which, and close to the bush, is a round piece of thick leather upon the spindle, which it turns round at the same time as it does the rynd.

The rynd is let into grooves in the under surface of the running millstone *G* (Fig. 2.) and so turns it round in the same time that the trundle *E* is turned round by the cog wheel *D*. This millstone has a large hole quite through its middle, called the eye of the stone, through which the middle part of the rynd and upper end of the spindle may be seen; whilst the four ends of the rynd lie hid below the stone in their grooves.

The end *T* of the bridge-tree *TS* (which supports the upper millstone *G* upon the spindle) is fixed into a hole in the wall; and the end *S* is let into a beam *QR* called the brayer, whose end *R* remains fixt, and its other end *Q* hangs by a strong iron rod *P* which goes through the floor *XY*, and has a screw-nut on its top at *O*; by the turning of which nut, the end *Q* of the brayer is raised or depressed at pleasure; and consequently the bridge-tree *TS* and upper millstone. By this means, the upper millstone may be set as close to the under one, or raised as high from it, as the miller pleases. The nearer the millstones are to one another, the
finer

finer they grind the corn, and the more remote from one another, the coarser.

The upper millstone *G* is inclosed in a round box *H*, which does not touch it any where; and is about an inch distant from its edge all around. On the top of this box stands a frame for holding the hopper *kk*, to which is hung the shoe *I* by two lines fastened to the hind-part of it, fixed upon hooks in the hopper, and by one end of the crook-string *K* fastened to the fore-part of it at *i*; the other end being twisted round the pin *L*. As the pin is turned one way, the string draws up the shoe closer to the hopper, and so lessens the aperture between them; and as the pin is turned the other way, it lets down the shoe, and enlarges the aperture.

If the shoe be drawn up quite to the hopper, no corn can fall from the hopper into the mill; if it be let a little down, some will fall: and the quantity will be more or less according as the shoe is more or less let down. For the hopper is open at bottom, and there is a hole in the bottom of the shoe, not directly under the bottom of the hopper, but forwarder towards the end *i*, over the middle of the eye of the millstone.

Fig. 3.

There is a square hole in the top of the spindle, in which is put the feeder *e*: this feeder (as the spindle turns round) jogs the shoe three times in each revolution, and so causes the corn to run constantly down from the hopper, through the shoe, into the eye of the millstone, where it falls upon the top of the rynd, and is, by the motion of the
rynd

rynd and the leather under it, thrown below the upper stone, and ground between it and the lower one. The violent motion of the stone creates a centrifugal force in the corn going round with it, by which means it gets farther and farther from the center, as in a spiral, in every revolution, until it be thrown quite out; and, being then ground, it falls through a spout *M* called the mill-eye, into the trough *N*.

When the mill is fed too fast, the corn bears up the stone and is ground too coarse; and besides, it clogs the mill so as to make it go too slow. When the mill is too slowly fed, it goes too fast, and the stones by their attrition strike fire against one another. Both which inconveniencies are avoided by turning the pin *L* backwards or forwards, which draws up or lets down the shoe; and so regulates the feeding as the miller sees convenient.

The heavier the running millstone is, and the greater the quantity of water that falls upon the wheel, so much the faster will the mill bear to be fed; and consequently so much the more it will grind. And on the contrary, the lighter the stone, and the less the quantity of water, so much slower must the feeding be. But when the stone is considerably wore, and become light, the mill must be fed slowly at any rate; otherwise the stone will be too much borne up by the corn under it, which will make the meal coarse.

The quantity of power required to turn a heavy millstone is but very little more than what is sufficient to turn a light one: for as it is supported

ported upon the spindle by the bridge-tree *ST*, and the end of the spindle that turns in the brass foot therein being but small, the odds arising from the weight is but very inconsiderable in its action against the power or force of the water. And besides, a heavy stone has the same advantage as a heavy fly; namely, that it regulates the motion much better than a light one.

In order to cut and grind the corn, both the upper and under millstones have channels or furrows cut into them, proceeding obliquely from the center towards the circumference. And these furrows are each cut perpendicularly on one side and obliquely on the other into the stone; which gives each furrow a sharp edge, and in the two stones they come, as it were, against one another like the edges of a pair of scissars: and so cut the corn, to make it grind the easier when it falls upon the places between the furrows. These are cut the same way in both stones when they lie upon their backs, which makes them run cross ways to each other when the upper stone is inverted by turning its furrowed surface towards that of the lower. For, if the furrows of both stones lay the same way, a great deal of the corn would be drove onward in the lower furrows, and so come out from between the stones without ever being cut.

When the furrows become blunt and shallow by wearing, the running stone must be taken up, and both stones new dressed with a chissel and hammer. And every time the stone is taken up, there must be some tallow put round the spindle upon the bush, which

which will soon be melted by the heat that the spindle acquires from its turning and rubbing against the bush, and so will get in betwixt them: otherwise the bush would take fire in a very little time.

The bush must embrace the spindle quite close, to prevent any shake in the motion, which would make some parts of the stones grate and fire against each other; whilst other parts of them would be too far asunder, and by that means spoil the meal in grinding.

Whenever the spindle wears the bush so as to begin to shake in it, the stone must be taken up, and a chissel drove into several parts of the bush; and when it is taken out, wooden wedges must be drove into the holes; by which means the bush will be made to embrace the spindle close all around it again. In doing this, great care must be taken to drive equal wedges into the bush on opposite sides of the spindle; otherwise it will be thrown out of the perpendicular, and so hinder the upper stone from being set parallel to the under one, which is absolutely necessary for making good work. When any accident of this kind happens, the perpendicular position of the spindle must be restored by adjusting the bridge-tree *ST* by proper wedges put between it and the brayer *QR*.

It often happens, that the rynd is a little wrenched in laying down the upper stone upon it; or is made to sink a little lower upon one side of the spindle than on the other; and this will cause one edge of the upper stone to drag all around upon the other,

whilst the opposite edge will not touch. But this is easily set to rights, by raising the stone a little with a lever, and putting bits of paper, cards, or thin chips betwixt the rynd and the stone.

The diameter of the upper stone is generally about six feet, the lower stone about an inch more : and the upper stone when new contains about $22\frac{1}{2}$ cubic feet, which weighs somewhat more than 1900 pounds. A stone of this diameter ought never to go more than 60 times round in a minute ; for if it turns faster it will heat the meal.

The grinding surface of the under stone is a little convex from the edge to the center, and that of the upper stone a little more concave : so that they are farthest from one another in the middle, and come gradually nearer towards the edges. By this means, the corn at its first entrance between the stones is only bruised ; but as it goes farther on towards the circumference or edge, it is cut smaller and smaller ; and at last finely ground just before it comes out from between the stones.

The water-wheel must not be too large, for if it be, its motion will be too slow ; nor too little, for then it will want power. And for a mill to be in perfection, the floats of the wheel ought to move with a third part of the velocity of the water, and the stone to turn round once in a second of time.

Such a mill as this, with a fall of water about $7\frac{1}{2}$ feet, will require about 32 hogsheads every minute to turn the wheel with a third part of the velocity with which the water falls ; and to overcome
the

the resistance arising from the friction of the geers and attrition of the stones in grinding the corn.

The greater fall the water has, the less quantity of it will serve to turn the mill. The water is kept up in the mill-dam, and let out by a sluice called the penstock, when the mill is to go. When the penstock is drawn up by means of a lever, it opens a passage through which the water flows to the wheel: and when the mill is to be stopt, the penstock is let down, which stops the water from falling upon the wheel.

A less quantity of water will turn an overshot mill (where the wheel has buckets instead of float boards) than a breast mill where the fall of the water seldom exceeds half the height *Ab* of the wheel. So that, where there is but a small quantity of water, and a fall great enough for the wheel to lie under it, the bucket (or overshot) wheel is always used. But where there is a large body of water, with a little fall, the breast or float-board wheel must take place. Where the water runs only upon a little declivity, it can act but slowly upon the under part of the wheel at *b*; in which case, the motion of the wheel will be very slow: and therefore, the floats ought to be very long, though not high, that a large body of water may act upon them; so that what is wanting in velocity may be made up in power: and then the cog wheel may have a greater number of cogs in proportion to the rounds in the trundle, in order to give the millstone a sufficient degree of velocity.

They who have read what is said in the first lecture, concerning the acceleration of bodies falling freely by the power of gravity acting constantly and uniformly upon them, may perhaps ask, why should the motion of the wheel be equable, and not accelerated, since the water acts constantly and uniformly upon it? The plain answer is, that the velocity of the wheel can never be so great as the velocity of the water that turns it; for, if it should become so great, the power of the water would be quite lost upon the wheel, and then there would be no proper force to overcome the friction of the geers and attrition of the stones. Therefore, the velocity with which the wheel begins to move, will increase no longer than till its *momentum* or force is balanced by the resistance of the machine; and then the wheel will go on with an equable motion.

A hand mill. [If the cog wheel *D* be made about 18 inches diameter, with 30 cogs, the trundle as small in proportion with 10 staves, and the millstones be each about two feet in diameter; and the whole work be put into a strong frame of wood, as represented in the figure, the engine will be a hand-mill for grinding corn or malt in private families. And then, it may be turned by a winch instead of the wheel *AA*: the millstone making three revolutions for every one of the winch. If a heavy fly be put upon the axle *B*, near the winch, it will assist greatly in regulating the motion.]

If the cogs of the wheel and rounds of the trundle could be put in as exactly as the teeth are cut in the wheels and pinions of a clock, then the
trundle

trundle might divide the wheel exactly: that is to say, the trundle might make a given number of revolutions for one of the wheel, without a fraction. But as any exact number is not necessary in mill-work, and the cogs and rounds cannot be set in so truly as to make all the intervals between them equal; a skilful mill-wright will always give the wheel what he calls a *hunting cog*; that is, one more than what will answer to an exact division of the wheel by the trundle. And then, as every cog comes to the trundle, it will take the next staff or round behind the one which it took in the former revolution: and by that means, will wear all the parts of the cogs and rounds which work upon one another equally, and to equal distances from one another in a little time; and so make a true uniform motion throughout the whole work. Thus, in the above water mill, the trundle has 10 staves, and the wheel 61 cogs.

Sometimes, where there is a sufficient quantity of water, the cog-wheel *AA* turns a large trundle *BB*, on whose axis *C* is fixed the horizontal wheel *D*, with cogs all around its edge, turning two trundles *E* and *F* at the same time; whose axes or spindles *G* and *H* turn two millstones *I* and *K*, upon the fixed stones *L* and *M*. And when there is not work for them both, either may be made to lie quiet, by taking out one of the staves of its trundle, and turning the vacant place towards the cog wheel *D*. And there may be a wheel fixt on the upper end of the great upright axle *C* for turning a couple of boulding-mills; and other work for

drawing up the sacks, fanning and cleaning the corn, sharpening of tools, &c.

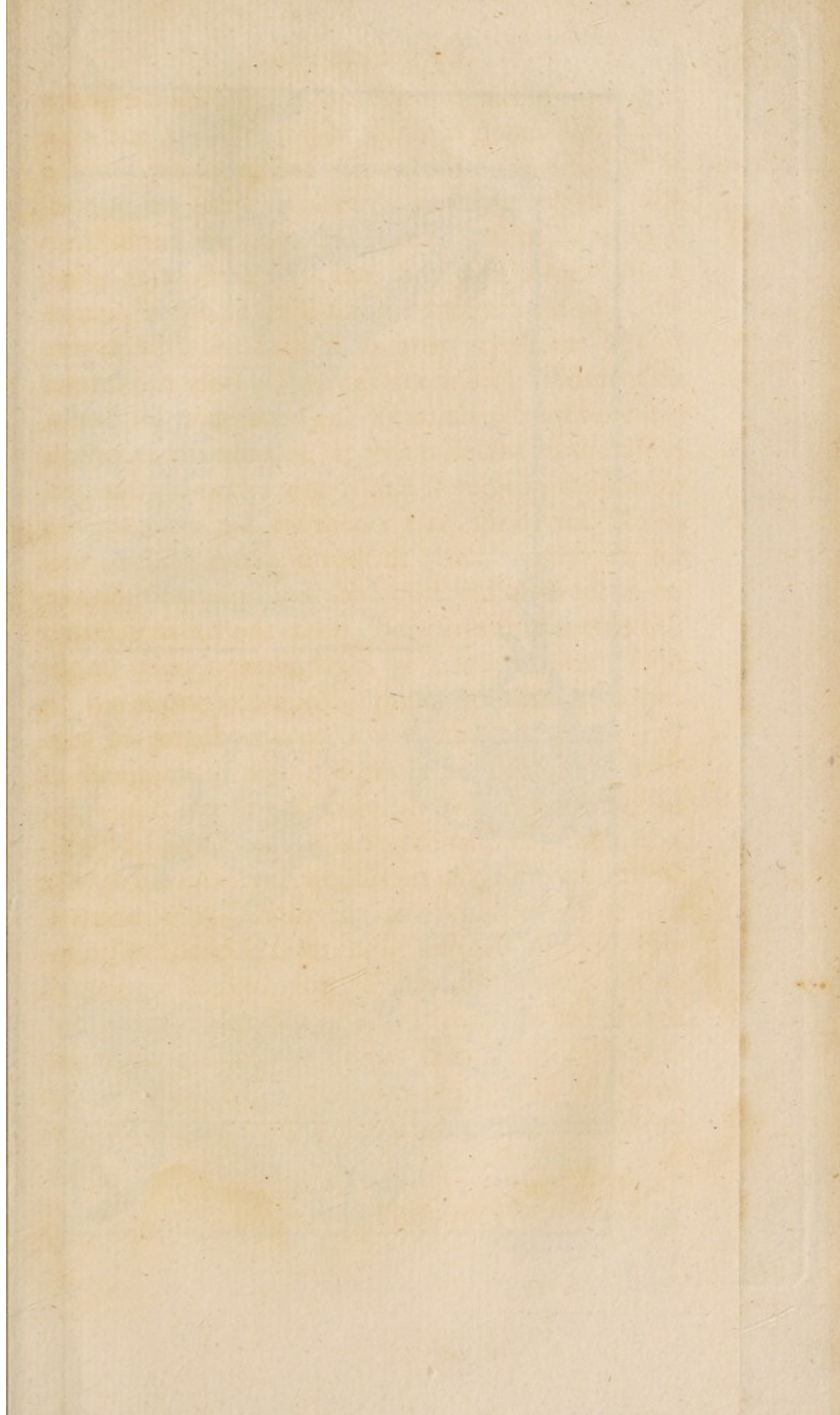
A horse mill. If, instead of the cog wheel *AA* and trundle *BB*, horizontal levers be fixed into the axle *C*, below the wheel *D*; then, horses may be put to these levers for turning the mill: which is often done where water cannot be had for that purpose.

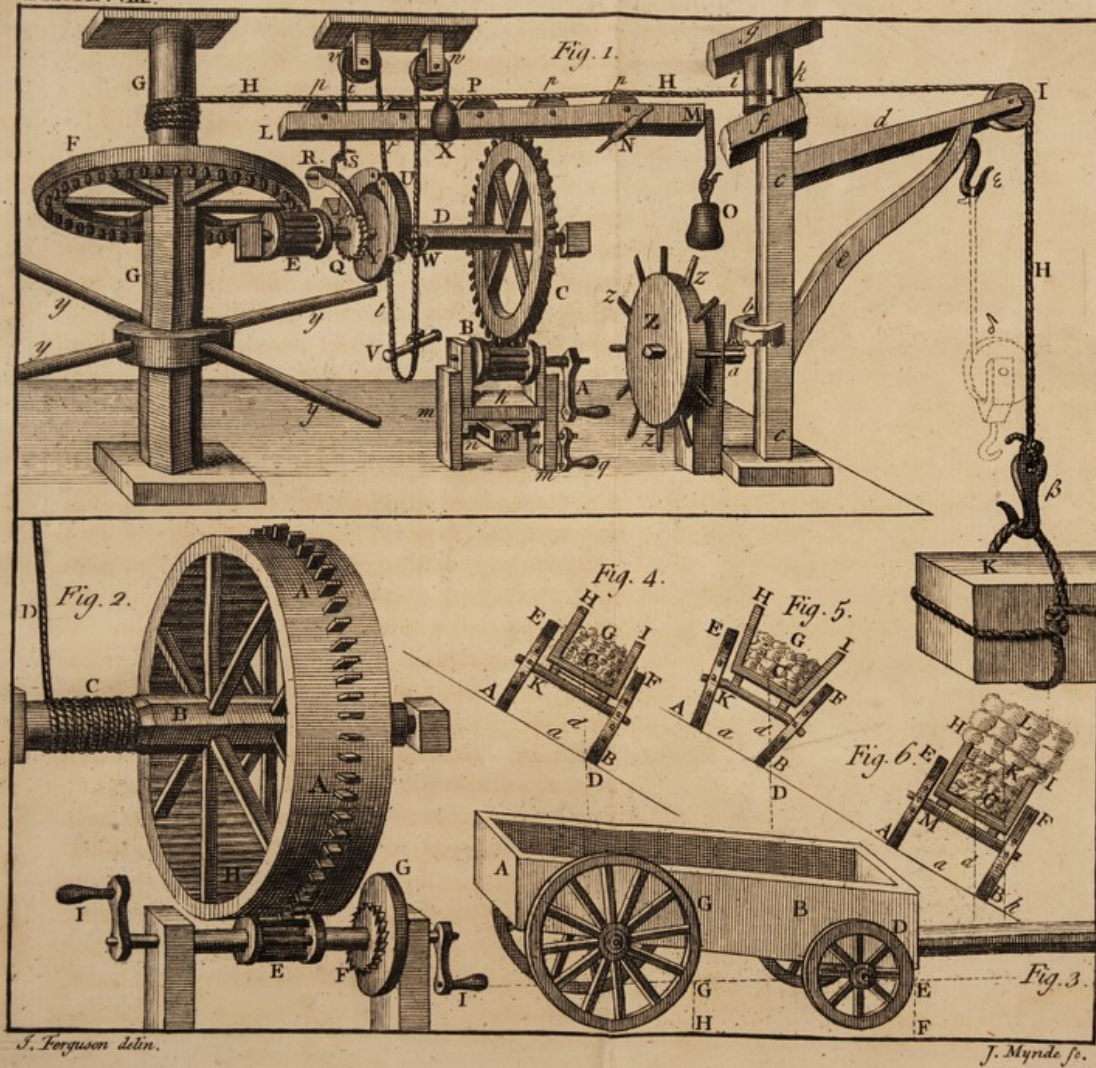
A wind mill. The working parts of a wind mill differ very little from those of a water mill; only the former is turned by the action of the wind upon four sails, every one of which ought (as is generally believed) to make an angle of $54\frac{2}{3}$ degrees with a plane perpendicular to the axis on which the arms are fixt for carrying them. It being demonstrable, that when the sails are set to such an angle, and the axis turned towards the wind, it has the greatest power upon them. But this angle answers only to the case of a vane or sail just beginning to move*: for, when the vane has a certain degree of motion, it yields to the wind; and then that angle must be increased to give the wind its full effect.

Again, the increase of this angle will be different, according to the different velocities from the axis to the extremity of the vane. At the axis it will be $54\frac{2}{3}$ degrees, and thence continually increase, giving the vane a twist, and so causing all the ribs of the vane to lie in different planes.

Lastly, these ribs ought to decrease in length from the axis to the extremity, giving the vane a curvilinear form; so that no part of the force of

* See *MAC LAURIN's* Fluxions, near the end.





any one rib be spent upon the rest, but all move on independent of each other. All this is required to give the sails of a wind mill their true form: and we see both the twist and the diminution of the ribs exemplified in the wings of birds.

It is almost incredible to think with what velocity the tips of the sails move when acted upon by a moderate gale of wind. I have several times counted the number of revolutions made by the sails in ten or fifteen minutes; and from the length of the arms from tip to tip, have computed, that if a hoop of that diameter was to run upon the ground with the same velocity that it would move if put upon the sail-arms, it would go upwards of 30 miles in an hour.

As the ends of the sails nearest the axis cannot move with the same velocity that the tips or farthest ends do, although the wind acts equally strong upon them; perhaps a better position than that of stretching them along the arms directly from the center of motion, might be to have them set perpendicularly across the farther ends of the arms, and there adjusted lengthwise to the proper angle. For, in that case, both ends of the sails would move with the same velocity; and being farther from the center of motion, they would have so much the more power: and then, there would be no occasion for having them so large as they are generally made; which would render them lighter, and consequently, there would be so much the less friction on the thick neck of the axle where it turns in the wall.

A crane

A crane.

Plate VIII.
Fig. 1.

A crane is an engine by which great weights are raised to certain heights, or let down to certain depths. It consists of wheels, axles, pullies, ropes, and a gib or gibbet. When the rope *H* is hooked to the weight *K*, a man turns the winch *A*, on the axis whereof is the trundle *B*, which turns the wheel *C*, on whose axis *D* is the trundle *E*, which turns the wheel *F* with its upright axis *G*, on which the great rope *HH* winds as the wheel turns; and going over a pulley *I* at the end of the arm *d* of the gib *ccde*, it draws up the heavy burthen *K*; which, being raised to a proper height, as from a ship to the quay, is then brought over the quay by pulling the wheel *Z* round by the handles *z, z*, which turns the gib by means of the half wheel *b* fixt on the gib-post *cc*, and the strong pinion *a* fixt on the axis of the wheel *Z*. This wheel gives the man that turns it an absolute command over the gib, so as to prevent it from taking any unlucky swing, such as often happens when it is only guided by a rope tied to its arm *d*; and people are frequently hurt, sometimes killed, by such accidents.

The great rope goes between two upright rollers *i* and *k*, which turn upon gudgeons in the fixed beams *f* and *g*; and as the gib is turned towards either side, the rope bends upon the roller next that side. Were it not for these rollers, the gib would be quite unmanageable; for the moment it were turned ever so little towards any side, the weight *K* would begin to descend, because the rope would be shortened between the pulley *I* and axis *G*; and so the gib would be pulled violently to that side, and
either

either break itself to pieces, or every thing that came in its way. These rollers must be placed so, that the sides of them round which the rope bends, may keep the middle of the bended part directly over the center of the hole in which the upper gudgeon of the gib turns in the beam *f*. The truer these rollers are placed, the easier the gib is managed, and the less apt to swing either way by the force of the weight *K*.

A ratchet-wheel *Q* is fixt upon the axis *D*, near the trundle *E*; and into this wheel falls the catch or click *R*. This hinders the machine from running back by the weight of the burthen *K*, if the man who raises it should happen to be careless, and so leave off working at the winch *A* sooner than he ought to do.

When the burthen *K* is raised to its proper height from the ship, and brought over the quay by turning of the gib, it is let down gently upon the quay, or into a cart standing thereon, in the following manner: A man takes hold of the rope *tt* (which goes over the pulley *v*, and is tied to a hook at *S* in the catch *R*) and so disengages the catch from the ratchet wheel *Q*; and then, the man at the winch *A* turns it backward, and lets down the weight *K*. But if the weight pulls too hard against this man, another lays hold of the handle *V*, and by pulling it downward, draws the gripe *U* close to the wheel *X*, which, by rubbing hard against the gripe, hinders the too quick descent of the weight; and not only so, but even stops it at any time, if required. By this means,
heavy

heavy goods may be either raised or let down at pleasure, without any danger of hurting the men who work the engine.

When part of the goods are craned up, and the rope is to be let down for more, the catch *R* is first disengaged from the ratchet wheel *Q*, by pulling the cord *t*; then the handle *q* is turned half round backward, which, by the crank *nn* in the piece *o*, pulls down the frame *b* between the guides *m* and *m* (in which it slides in a groove) and so disengages the trundle *B* from the wheel *C*: and then, the heavy hook *β* at the end of the rope *H* descends by its own weight, and turns back the great wheel *F* with its trundle *E*, and the wheel *C*; and this last wheel acts like a fly against the wheel *F* and hook *β*; and so hinders it from going down too quick; whilst the weight *X* keeps up the gripe *U* from rubbing against the wheel *Y*, by means of a cord going from the weight, over the pulley *w* to the hook *W* in the gripe: so that the gripe never touches the wheel, unless it be pulled down by the handle *V*.

When the crane is to be set at work again, for drawing up another burthen, the handle *q* is turned half round forwards; which, by the crank *nn*, raises up the frame *b*, and causes the trundle *B* to lay hold of the wheel *C*; and then, by turning the winch *A*, the burthen of goods *K* is drawn up as before.

The crank *nn* turns pretty stiff in the mortise near *o*, and stops against the farther end of it when it has got just a little beyond the perpendicular; so

so that it can never come back of itself: and therefore, the trundle *B* can never come away from the wheel *C*, until the handle *q* be turned half round.

The great rope runs upon rollers in the lever *LM*, which keep it from bending between the axle at *G* and the pulley *I*. This lever turns upon the axis *N* by means of the weight *O*, which is just sufficient to keep its end *L* up to the rope; so that, as the great axle turns, and the rope coils round it, the lever rises with the rope, and prevents the coilings from going over one another.

The power of this crane may be estimated thus, suppose the radius of the winch *A* to be twice the radius of the trundle *B*; this will make the power of the trundle upon the wheel *C* to be as 2: but if the trundle has 8 staves, and the wheel 64 cogs, the trundle will increase the power of the wheel 8 times, which multiplied by 2, the power of the winch, is 16; and if the trundle *E* on the axis of the wheel *C* has 9 rounds, and the wheel *F* 72 cogs, the power is again 8 times increased; which makes the power of the wheel *F* as 8 times 16, or 128: and lastly, if the diameter of the wheel *F* be 8 times the diameter of that part of its axle *G* on which the rope winds, this again increases the power 8 times, and so makes it 8 times 128, or 1024, which is the whole power of the machine; and so much more weight a man can raise by it than he could lift in his arms with an equal degree of strength.

If this power be thought greater than what may be generally wanted, the wheels may be made with
fewer

fewer cogs in proportion to the staves in the trundles; and so the power may be of whatever degree is judged to be requisite. But if the weight be so great as will require yet more power to raise it (suppose a double quantity) then the rope H may be put under a moveable pulley as δ , and the end of it tied to a hook in the gib at ϵ ; which will give a double power to the machine, and so raise a double weight hook'd to the block of the moveable pulley.

When only small burthens are to be raised, this may be quickly done by men pushing the axle G round by the handspikes y, y, y, y ; having first disengaged the trundle B from the wheel C : and then, this wheel will only act as a fly upon the wheel F ; and the catch R will prevent its running back, if the men should inadvertently leave off pushing before the burthen be unhooked from β .

Lastly, when very heavy burthens are to be raised, which might endanger the breaking of the cogs in the wheel F ; their force against these cogs may be much abated by men pushing at the handspikes y, y, y, y , whilst the man at A turns the winch.

I have only shewn the working parts of this crane, without the whole of the beams which support them; knowing that these are easily supposed, and that if they had been drawn, they would have hid a great deal of the working parts from sight, and also confused the figure.

Another
crane.

Another very good *crane* is made in the following manner. AA is a great wheel turned by men walking

walking within it at *H*. On the part *C*, of its axle Fig. 2. *BC*, the great rope *D* is wound as the wheel turns; and this rope draws up goods in the same way as the rope *HH* does in the abovementioned crane, the gib-work here being supposed to be of the same sort. But these cranes are very dangerous to the men in the wheel; for, if any of the men should chance to fall, the burthen will make the wheel run back and throw them all about within it; which often breaks their limbs, and sometimes kills them. The late ingenious Mr. *Padmore* of Bristol (whose contrivance the forementioned crane is, so far as I can remember its construction after seeing it once about twelve years ago) observing this dangerous construction, contrived a method for remedying it, by putting cogs all around the outside of the wheel, and applying a trundle *E* to turn it; which increases the power as much as the number of cogs in the wheel is greater than the number of staves in the trundle: and by putting a ratchet wheel *F* on the axis of the trundle, (as in the abovementioned crane) with a catch to fall into it, the great wheel is stopt from running back by the force of the weight, even if all the men in it should leave off walking. And by one man working at the winch *I*, or two men at the opposite winches when needful, the men in the wheel are much assisted, and much greater weights are raised, than could be by men only within the wheel. Mr. *Padmore* put also a gripe-wheel *G* upon the axis of the trundle, which being pinched in the same manner as described in the former crane, heavy burdens may be

be let down without the least danger. And before this contrivance, the lowering of goods was always attended with the utmost danger to the men in the wheel; as every one must be sensible of, who has seen such engines at work.

And it is surprising that the masters of wharfs and cranes should be so regardless of the limbs or even lives of their workmen, that, excepting Sir *James Creed*, there is scarce an instance of any who uses this safe contrivance.

*Wheel car-
riages.*

The structure of *wheel carriages* is generally so well known, that it would be needless to describe them. And therefore, we shall only point out some inconveniencies attending the common method of placing the wheels, and loading the wag-gons.

In coaches, and all other four wheel'd carriages, the fore-wheels are made of a less size than the hind ones, both on account of turning short, and to avoid cutting the braces: otherwise, the carriage would go much easier if the fore-wheels were as high as the hind ones, and the higher the better, because their motion would be so much the slower on their axles, and consequently the friction proportionably taken off. But carriers and coachmen give another reason for making the fore-wheels much lower than the hind-wheels; namely, that when they are so, the hind-wheels help to push on the fore ones: which is too unphilosophical and absurd to deserve a refutation, and yet for their satisfaction we shall soon shew by experiment that it has no existence but in their own imaginations.

It is plain that the small wheels must turn as much oftener round than the great ones, as their circumferences are less. And therefore, when the carriage is loaded equally heavy on both axles, the fore axle must endure as much more friction, and consequently wear out as much sooner, than the hind axle, as the fore-wheels are less than the hind ones. But the great misfortune is, that all the carriers to a man do obstinately persist, against the clearest reason and demonstration, in putting the heavier part of the load upon the fore axle of the waggon; which not only makes the friction greatest where it ought to be least, but also presseth the fore-wheels deeper into the ground than the hind-wheels, notwithstanding the fore-wheels, being less than the hind ones, are with so much the greater difficulty drawn out of a hole or over an obstacle, even supposing the weights on their axles were equal. For the difficulty, with equal weights, will be as the depth of the hole or height of the obstacle is to the semidiameter of the wheel. Thus, Fig. 1. if we suppose the small wheel *D* of the waggon *AB* to fall into a hole of the depth *EF*, which is equal to the semidiameter of the wheel, and the waggon to be drawn horizontally along; it is evident, that the point *E* of the small wheel will be drawn directly against the top of the hole; and therefore, all the power of horses and men will not be able to draw it out, unless the ground gives way before it. Whereas, if the hind-wheel *C* falls into such a hole, it sinks not near so deep in proportion to its semidiameter; and therefore, the point *G* of the

H large

large wheel will not be drawn directly, but obliquely, against the top of the hole; and so will be easily got out of it. Add to this, that since a small wheel will often sink to the bottom of a hole, in which a great wheel will go but a very little way, the small wheels ought in all reason to be loaded with less weight than the great ones: and then the heavier part of the load would be less jolted upward and downward, and the horses tired so much the less, as their draught raised the load to less heights.

It is true, that when the waggon road is much up-hill, there may be danger in loading the hind part much heavier than the fore part; for then the weight would overhang the hind axle, especially if the load be high, and endanger tilting up the fore-wheels from the ground. In this case, the safest way would be to load it equally heavy on both axles; and then, as much more of the weight would be thrown upon the hind axle than upon the fore one, as the ground rises from a level below the carriage. But as this seldom happens, and when it does, a small temporary weight laid upon the pole between the horses would overbalance the danger; and this weight might be thrown into the waggon when it comes to level ground, it is strange that an advantage so plain and obvious as would arise from loading the hind-wheels heaviest, should not be laid hold of, by complying with this method.

To confirm these reasonings by experiment, let a small model of a waggon be made, with its fore-wheels $2\frac{1}{2}$ inches in diameter, and its hind-wheels

wheels $4\frac{1}{2}$; the whole model weighing about 20 ounces. Let this little carriage be loaded any how with weights, and have a small cord tied to each of its ends, equally high from the ground it rests upon; and let it be drawn along a horizontal board, first by a weight in a scale hung to the cord at the fore part; the cord going over a pulley at the end of the board to facilitate the draught, and the weight just sufficient to draw it along. Then, turn the carriage, and hang the scale and weight to the hind cord, and it will be found to move along with the same velocity as at first: which shews, that the power required to draw the carriage is all the same, whether the great or small wheels are foremost; and therefore the great wheels do not help in the least to push on the small wheels in the road.

Hang the scale to the fore cord, and place the fore-wheels (which are the small ones) in two holes, cut three eighth parts of an inch deep into the board; then put a weight of 32 ounces into the carriage, over the fore axle, and an equal weight over the hind one: this done, put 44 ounces into the scale, which will be just sufficient to draw out the fore-wheels: but if this weight be taken out of the scale, and one of 16 ounces put into its place, if the hind-wheels are placed in the holes, the 16 ounce weight will draw them out; which is little more than a third part of what was necessary to draw out the fore-wheels. This shews, that the larger the wheels are, the less power will draw the carriage, especially on rough ground.

H 2

Put

Put 64 ounces over the axle of the hind-wheels, and 32 over the axle of the fore ones, in the carriage; and place the fore-wheels in the holes : then, put 38 ounces into the scale, which will just draw out the fore-wheels ; and when the hind ones come to the hole, they will find but very little resistance, because they sink but a little way into it.

But shift the weights in the carriage, by putting the 32 ounces upon the hind axle, and the 64 ounces upon the fore one ; and place the fore-wheels in the holes : then, if 76 ounces be put into the scale, it will be found no more than sufficient to draw out these wheels ; which is double the power required to draw them out, when the lighter part of the load was put upon them : which is a plain demonstration of the absurdity of putting the heaviest part of the load in the fore part of the waggon.

Every one knows what an out-cry was made by the generality, if not the whole body, of the carriers, against the broad-wheel act ; and how hard it was to persuade them to comply with it, even though the government allowed them to draw with more horses, and carry greater loads, than usual. Their principal objection was, that as a broad wheel must touch the ground in a great many more points than a narrow wheel, the friction must of course be just so much the greater ; and consequently, there must be so many more horses than usual, to draw the waggon. I believe that the majority of people were of the same opinion, not considering, that if the whole weight of the waggon
and

and load in it bears upon a great many points, each sustains a proportionably less degree of weight and friction, than when it bears only upon a few points ; so that what is wanting in one is made up in the other ; and therefore will be just equal under equal degrees of weight, as may be shewn by the following plain and easy experiment.

Let one end of a piece of packthread be fastened to a brick, and the other end to a common scale for holding weights : then, having laid the brick edgewise on a table, and let the scale hang under the edge of the table, put as much weight into the scale as will just draw the brick along the table. Then, taking back the brick to its former place, lay it flat on the table, and leave it to be acted upon by the same weight in the scale as before, which will draw it along with the same ease as when it lay upon its edge. In the former case, the brick may be considered as a narrow wheel on the ground ; and in the latter, as a broad wheel. And since the brick is drawn along with equal ease, whether its broad side or narrow edge touches the table, it shews that a broad wheel might be drawn along the ground with the same ease as a narrow one, (supposing them equally heavy) even though they should drag, and not roll, as they go along.

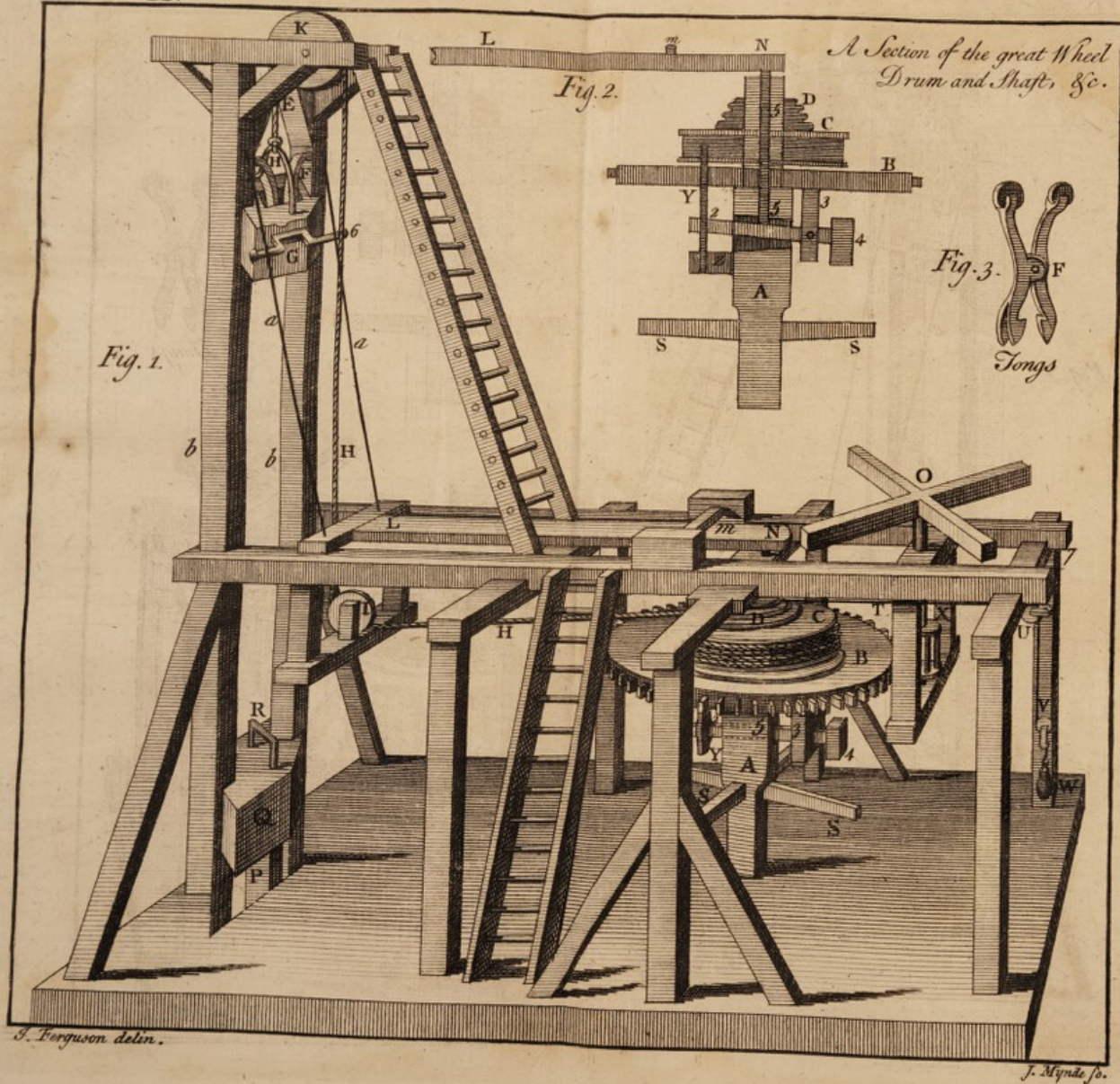
As narrow wheels are always sinking into the ground, especially when the heaviest part of the load lies upon them, they must be considered as going constantly up-hill, even on level ground. And their edges must sustain a great deal of friction by rubbing against the sides of the ruts made by

them. But both these inconveniencies are avoided by broad wheels; which, instead of cutting and ploughing up the roads, roll them smooth and harden them; as experience testifies in places where they have been used, especially either on wetish or sandy ground: though after all it must be confessed, that they will not do in stiff clayey cross roads, because they would soon gather up the weight of an ordinary load of clay.

If the wheels were always to go upon smooth and level ground, the best way would be to make the spokes perpendicular to the naves; that is, to stand at right angles to the axles, because they would then bear the weight of the load perpendicularly, which is the strongest way for wood. But because the ground is generally uneven, one wheel often falls into a cavity or rut when the other does not; and then it bears much more of the weight than the other does: in which case, concave or dishing wheels are best, because when one falls into a rut, and the other keeps upon high ground, the spokes become perpendicular in the rut, and therefore have the greatest strength when the obliquity of the load throws most of its weight upon them; whilst those on high ground have less weight to bear, and therefore need not be at their full strength. So that the usual way of making the wheels concave is by much the best.

The axles of the wheels ought to be perfectly straight, that the rims of the wheels may be parallel to each other; for then they will move easiest, because they will be at liberty to go on straight forwards.

PLATE IX.



forwards. But in the usual way of practice, the axles are bent downward at their ends ; which brings the sides of the wheels next the ground nearer to one another than their opposite or higher sides are : and this not only makes the wheels to drag sidewise as they go along, and gives the load a much greater power of crushing them than when they are parallel to each other, but also endangers the overturning of the carriage when any wheel falls into a hole or rut ; or when the carriage goes in a road which has one side lower than the other, as along the side of a hill. Thus, (in the hind view of a *Fig. 4.* waggon or cart) let AE and BF be the great wheels parallel to each other, on their straight axle K , and HCI the carriage loaded with heavy goods from C to G . Then, as the carriage goes on in the oblique road AaB , the center of gravity of the whole machine and load will be at C^* ; and the line of direction CdD falling within the wheel BF , the carriage will not overset. But if the wheels be inclined to each other at the ground, as AE and *Fig. 5.* BF , and the machine be loaded as before, from C to G , the line of direction CdD falls without the wheel BF , and the whole machine tumbles over. When it is loaded with heavy goods (such as lead or iron) which lie low, it may travel safely upon an oblique road so long as the center of gravity is at C , and the line of direction Cd falls within the wheels ; but if it be loaded high with lighter goods (such as woolpacks) from C to L , the center of *Fig. 6.* gravity is raised from C to K , which throws the line

of direction *Kk* without the wheel *BF*, and then the load overfets the waggon.

If there be some advantage from small fore-wheels, on account of the carriage turning more easily and short than it can be made to do when they are large; there is at least as great a disadvantage attending them, which is, that as their axle is below the level of the horses breasts, the horses not only have the loaded carriage to draw along, but also part of its weight to bear; which tires them sooner, and makes them grow much stiffer in their hams, than they would be if they drew on a level with the fore axle. And for this reason, we find coach horses soon become unfit for riding. So that on all accounts it is plain, that the fore-wheels of all carriages ought to be so high, as to have their axles even with the breasts of the horses; which would not only give the horses a fair draught, but likewise cause the machine to be drawn by a less degree of power.

Plate IX.
Fig. 1, 2.

We shall conclude this lecture with a description of Mr. *Vauloue's* curious engine, which was made use of for driving the piles of Westminster-bridge: and the reader may cast his eyes upon the first and second figures of the plate, in which the same letters of reference are annexed to the same parts, in order to explain those in the second, which are either partly or wholly hid in the first.

The pile engine.

A is the great upright shaft or axle, on which are the great wheel *B* and drum *C*, turned by horses joined to the bars *S, S*. The wheel *B* turns the
trundle

trundle *X*, on the top of whose axis is the fly *O*, which serves to regulate the motion, and also to act against the horses, and keep them from falling when the heavy ram *Q* is discharged to drive the pile *P* down into the mud in the bottom of the river. The drum *C* is loose upon the shaft *A*, but is locked to the wheel *B* by the bolt *Y*. On this drum the great rope *HH* is wound; one end of the rope being fixed to the drum, and the other to the follower *G*, to which it is conveyed over the pullies *I* and *K*. In the follower *G* is contained the tongs *F* (See Fig. 3.) that takes hold of the ram *Q* by the staple *R* for drawing it up. *D* is a spiral or fusee fixt to the drum, on which is wound the small rope *T* that goes over the pulley *U*, under the pulley *V*, and is fastened to the top of the frame at 7. To the pulley-block *V* is hung the counterpoise *W*, which hinders the follower from accelerating as it goes down to take hold of the ram: for, as the follower tends to acquire velocity in its descent, the line *T* winds downwards upon the fusee, on a larger and larger radius, by which means the counterpoise *W* acts stronger and stronger against it; and so allows it to come down with only a moderate and uniform velocity. The bolt *Y* locks the drum to the great wheel, being pushed upward by the small lever 2, which goes through a mortise in the shaft *A*, turns upon a pin in the bar 3 fixt into the great wheel *B*, and has a weight 4 which always tends to push up the bolt *Y* through the wheel into the drum. *L* is the great lever turning on the axis *m*, and resting upon the forcing
bar

bar 5, 5, which goes down through a hollow in the shaft *A*, and bears upon the little lever 2.

By the horses going round, the great rope *H* is wound about the drum *C*, and the ram *Q* is drawn up by the tongs *F* in the follower *G*, until the tongs comes between the inclined planes *E*; which, by shutting the tongs at the top, opens it at the foot, and discharges the ram, which falls down between the guides *bb* upon the pile *P*, and drives it by a few strokes as far into the mud as it can go; after which, the top-part is sawed off close to the mud, by an engine for that purpose. Immediately after the ram is discharged, the piece 6 upon the follower *G* takes hold of the ropes *a, a*, which raise the end of the lever *L*, and cause its end *N* to descend and press down the forcing bar 5 upon the little lever 2, which, by pulling down the bolt *X*, unlocks the drum *C* from the great wheel *B*; and then the follower, being at liberty, comes down by its own weight to the ram; and the lower ends of the tongs slip over the staple *R*, and the weight of their heads causes them to fall outward, and shuts upon it. Then the weight 4 pushes up the bolt *X* into the drum, which locks it to the great wheel, and so the ram is drawn up as before.

As the follower comes down, it causes the drum to turn backward, and unwinds the rope from it, whilst the horses, great wheel, trundle, and fly go on with an uninterrupted motion: and as the drum is turning backward, the counterpoise *W* is drawn up, and its rope *T* wound upon the spiral fusee *D*.

There

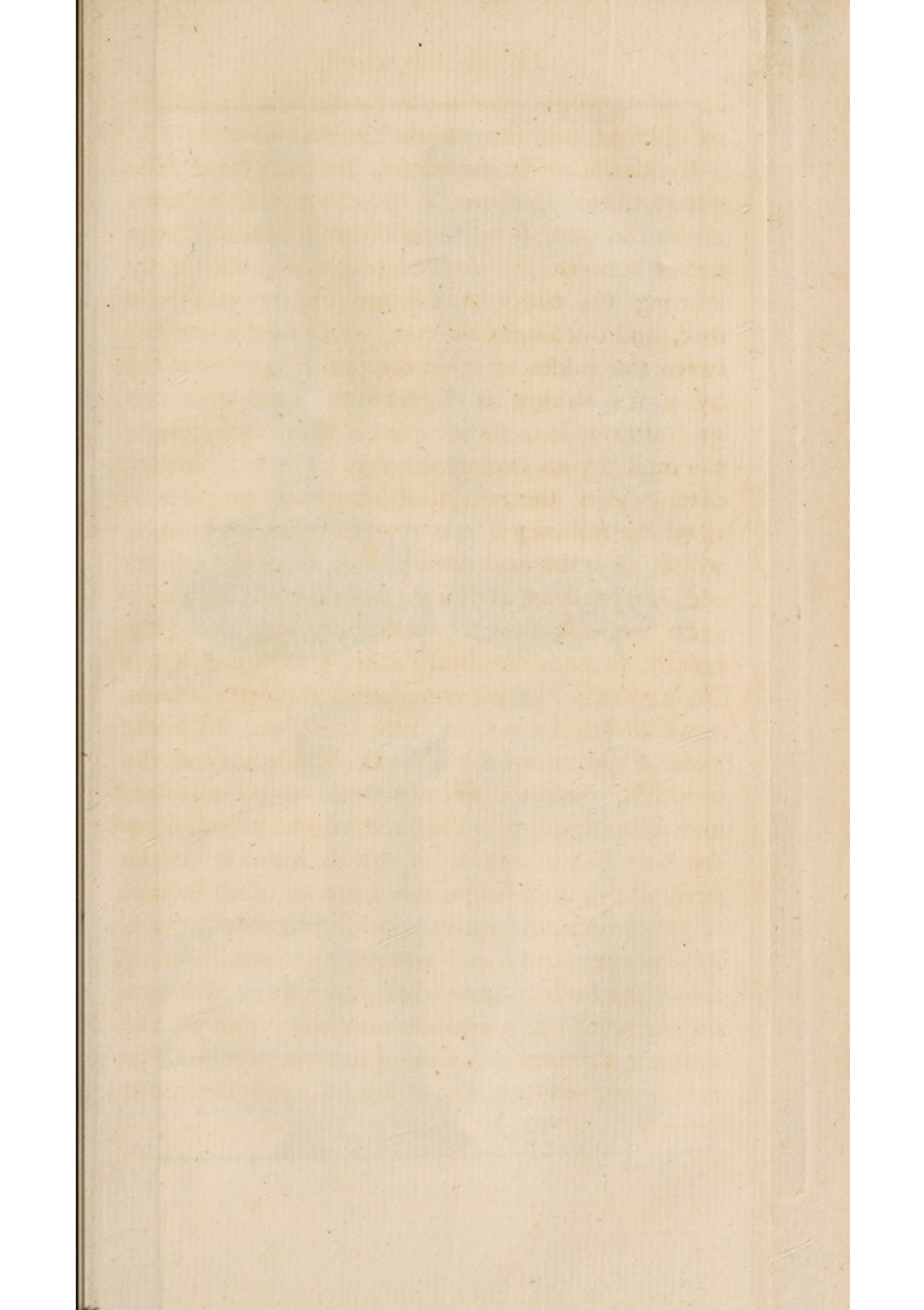
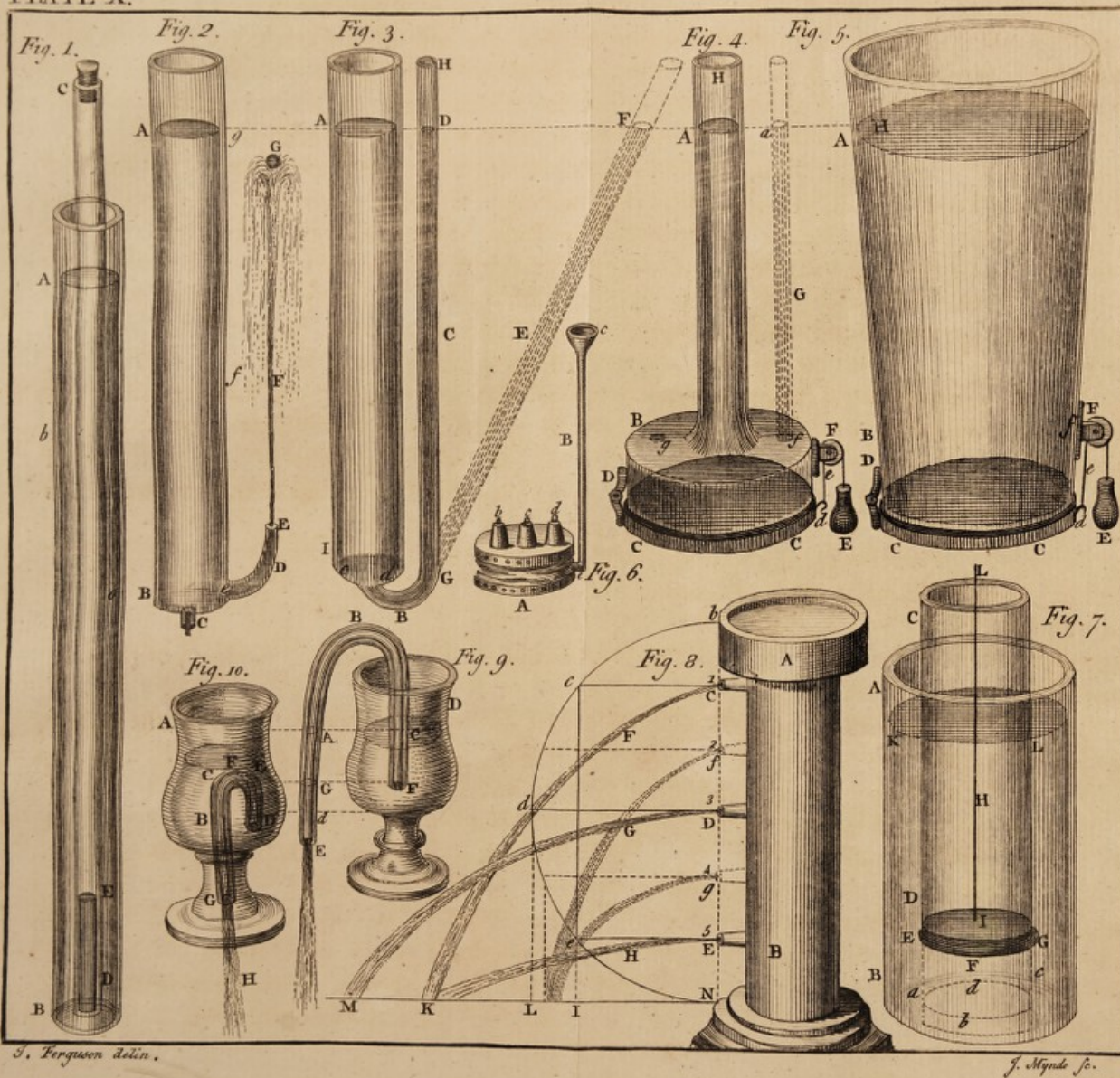


PLATE X.



There are several holes in the under side of the drum, and the bolt *X* always takes the first one that it finds when the drum stops by the falling of the follower upon the ram; until which stoppage, the bolt has not time to slip into any of the holes.

This engine was placed upon a barge on the water, and so was easily conveyed to any place desired.—I never had the good fortune to see it, but drew this figure from a model which I made from a print of it; being not quite satisfied with the view which the print gives. I have been told that the ram was a ton weight, and that the guides *bb*, between which it was drawn up and let fall down, were 30 feet high. I suppose the great wheel may have had 100 cogs, and the trundle 10 staves or rounds; so that the fly would make 10 revolutions for one of the great wheel.

L E C T. V.

Of hydrostatics, and hydraulic machines, in general.

THE science of *hydrostatics* treats of the nature, gravity, pressure, and motion of fluids in general; and of weighing solids in them.

A fluid is a body that yields to the least pressure, or difference of pressures. Its particles must be exceedingly small, because they cannot be discerned by the best of microscopes; they must be hard, because no fluid, except air or steam, can be pressed into a less space than it naturally possesses; they must also be round and smooth, because they are so easily

Definition
of a fluid.

easily moved among one another ; unless it can be shewn that their particles do not actually touch each other, which would indeed seem to be the case with such as are pungent and sharp to the taste.

All bodies, both fluid and solid, press downwards by the force of gravity : but fluids have this wonderful property, that their pressure upwards and sidewise is equal to their pressure downwards ; and this is always in proportion to their perpendicular height, without any regard to their quantity : for, as each particle is quite free to move, it will move towards that part or side on which the pressure is least. And hence, no particle or quantity of a fluid can be at rest till it is every way equally pressed.

Plate X.

Fig. 1.

Fluids press
as much
upward as
downward.

To shew by experiment that fluids press upward as well as downward, let *AB* be a long upright tube, filled with water near to its top ; and *CD* a small tube open at both ends, and immersed into the water in the large one : if the immersion be quick, you will see the water rise in the small tube to the same height that it stands in the great one, or until the surfaces of the water in both are on the same level : which shews that the water is pressed upward into the small tube by the weight of what is in the great one ; otherwise it could never rise therein, contrary to its natural gravity ; unless the diameter of the bore were so small, that the attraction of the tube would raise the water ; which will never happen if the tube be as wide as that in a common barometer. And, as the water rises no higher in the small tube than till its surface be
on

On a level with the surface of the water in the great one, this shews that the pressure is not in proportion to the quantity of water in the great tube, but in proportion to its perpendicular height therein: for there is much more water in the great tube, all around the small one, than what is raised to the same height in the small one, as it stands in the great.

Take out the small tube, and let the water run out of it; then it will be filled with air. Stop its upper end with the cork *C*, and it will be full of air all below the cork: this done, plunge it again to the bottom of the water in the great tube; and you will see the water rise up in it to the height *E*, which shews that the air is a body, otherwise it could not hinder the water from rising up to the same height as it did before, namely to *A*; and in so doing, it drove the air out at the top; but now the air is confined by the cork *C*: and it also shews that the air is a compressible body, for if it were not so, a drop of water could not enter into the tube.

The pressure of fluids being equal in all directions, it follows that the sides of a vessel are as much pressed by a fluid in it, all around in any given ring of points, as the fluid below that ring is pressed by the weight of all that stands above it. Hence, the pressure upon every point in the sides, immediately above the bottom, is equal to the pressure upon every point of the bottom. To shew Fig. 2. this by experiment, let a hole be made at *e* in the side of the tube *AB* close by the bottom; and
another

another hole of the same size in the bottom at C ; then pour water into the tube, keeping it full as long as you choose the holes should run, and have two basons ready to receive the water that runs through the two holes, until you think there is enough in each bason; and you will find by measuring the quantities, that they are equal; which shews that the water run with equal speed through both holes: and this it could not have done, unless it had been equally pressed through them both. For, if a hole of the same size be made in the side of the tube, as about f , and if all three are permitted to run together, you will find that the quantity run through the hole at f is much less than what has run in the same time through either of the holes C or e .

In the same figure, let the tube be re-curved from the bottom at C into the shape DE , and the hole at C be stopt with a cork. Then, pour water into the tube to any height, as Ag , and it will spout up in a jet EFG , nearly as high as it is kept in the tube AB , by continuing to pour in as much there as runs through the hole E ; which will be the case whilst the surface Ag keeps at the same height. And if a little ball of cork G be laid upon the top of the jet, it will be supported thereby, and dance upon it.—The reason why the jet rises not quite so high as the surface of the water Ag , is owing to the resistance it meets with in the open air: for if a tube either great or small was screwed upon the pipe at E , the water would rise in it until the surfaces of the water in both tubes were on the same level; as will be shewn by the next experiment.

Any

Any quantity of a fluid, how small soever, may be made to balance and support any quantity how great soever. This is deservedly termed *the hydrostatic paradox*, which we shall first shew by an experiment, and then account for it upon the principle above mentioned, namely, that *the pressure of fluids is directly as their perpendicular height, without any regard to their quantity.* The hydrostatic paradox.

Let a small glass tube DCG , open at both ends, Fig. 3. and bended at B , be joined to the end of a great one AI at cd , where the great one is also open; so that these tubes may in their openings freely communicate with each other. Then pour water through a small necked funnel into the small tube at H ; this water will run through the joining of the tubes at cd , and rise up into the great tube; and if you continue pouring until the surface of the water comes to any part, as A , in the great tube, and then leave off, you will see that the surface of the water in the small tube will be just as high, at D ; so that the perpendicular altitude of the water will be the same in both tubes, however small the one be in proportion to the other. This shews, that the small column DCG balances and supports the great column Acd ; which it could not do, if their pressures were not equal against one another in the recurved bottom at B .—If the small tube be made longer, and inclined in the situation GEF , the surface of the water in it will stand at E , on the same level with the surface A in the great tube; that is, the water will have the same perpendicular height in both tubes, although the column

column in the small tube is longer than that in the great one ; the former being oblique, and the latter perpendicular.

Since then, the pressure of fluids is directly as their perpendicular heights, without any regard to their quantities, it appears, that whatever the figure or size of vessels be, if they are of equal heights, and if the areas of their bottoms are equal, the pressures of equal heights of water are equal upon the bottoms of these vessels ; even though the one should hold a thousand or ten thousand times as much water as would fill the other. To confirm this part of the hydrostatical paradox by an experiment, let two vessels be prepared, of equal heights, but very unequal contents, such as *AB* in Fig. 4. and *AB* in Fig. 5. Let each vessel be open at both ends, and their bottoms *Dd*, *Dd* be of equal widths. Let a brass bottom *CC* be exactly fitted to each vessel, not to go into it, but for it to stand upon ; and let a piece of wet leather be put between each vessel and its brass bottom, for the sake of closeness. Join each bottom to its vessel by a hinge *D*, so that it may open like the lid of a box ; and let each bottom be kept up to its vessel by equal weights *E* and *E*, hung to lines which go over the pullies *F* and *F* (whose blocks are fixed to the sides of the vessels at *f*) and the lines tied to hooks at *d* and *d*, fixed in the brass bottoms opposite to the hinges *D* and *D*. Things being thus prepared and fitted, hold the vessel *AB* (Fig. 5.) upright in your hands over a basin on a table, and cause water to be poured into the vessel slowly,

slowly, till the pressure of the water on its bottom bears it down from the vessel at the side d , and raises the weight E ; and then the water will run out at d . Mark the height at which the surface H of the water stood in the vessel, when the bottom began to give way at d ; and then, holding up the other vessel AB (Fig. 4.) in the same manner, cause water to be poured into it at H ; and you will see that when the water rises to A in this vessel, just as high as it did in the former, its bottom will also give way at d , and it will lose all the water.

The natural reason of this surprising phenomenon is, that since all parts of a fluid at equal depths below the surface are equally pressed in all manner of directions, the water immediately below the fixed part Bf (Fig. 4.) will be pressed as much upward against its lower surface within the vessel, by the action of the column Ag , as it would be by a column of the same height, and of any diameter whatever; (as was evident by the experiment with the tube Fig. 3.) and therefore, since action and reaction are equal and contrary to one another, the water, immediately below the surface Bf , will be pressed as much downward by it, as if it was immediately touched and pressed by a column of the height gA , and of the diameter Bf : and therefore, the water in the cavity $BDdf$ will be pressed as much downward upon its bottom CC , as the bottom of the other vessel (Fig. 5.) is pressed by all the water above it.

To illustrate this a little farther, let a hole be made at f in the fixed top Bf , and let a tube G
I
be

Fig. 4.

be put into it; then, if water be poured into the tube *A*, it will (after filling the cavity *Bd*) rise up into the tube *G*, until it comes to a level with that in the tube *A*; which is manifestly owing to the pressure of the water in the tube *A*, upon that in the cavity of the vessel below it. Consequently, that part of the top *Bf*, in which the hole is now made, would, if corked up, be pressed upward with a force equal to the weight of all the water which is supported in the tube *G*: and the same thing would hold at *g*, if a hole were made there. And so, if the whole cover or top *Bf* were full of holes, and had tubes put into them, the water in each tube would rise to the same height as it is kept into the tube *A*, by pouring more into it, to make up the deficiency that it sustains by supplying the others, until they were all full, if equally high: and then the water in the tube *A* would support equal heights of water in all the rest of the tubes. Or, if all the tubes except *A*, or any other one, were taken away, and a large tube equal in diameter to the whole top *Bf* were placed upon it, and cemented to it, and then if water were poured into the tube that was left in either of the holes, it would ascend through all the rest of the holes, until it filled the large tube to the same height that it stands in the small one, after a sufficient quantity had been poured into it: which shews, that the top *Bf* was pressed upward by the water under it; and before any hole was made in it, with a force equal to that wherewith it is now pressed downward by the weight of all the water above it in the great tube. And therefore,

therefore, the re-action of the fixed top *Bf* must be as great, in pressing the water downward upon the bottom *CC*, as the whole pressure of the water in the great tube would have been if the top had been taken away, and the water in that tube left to press directly upon the water in the cavity *BDdf*.

Perhaps the best machine in the world for demonstrating the upward pressure of fluids, is the hydrostatic bellows *A*; which consists of two thick oval boards, each about 16 inches broad and 18 inches long, covered with leather, to open and shut like a common bellows, but without valves; only a pipe *B*, three feet high, is fixed into the bellows at *e*. Let some water be poured into the pipe at *c*, which will run into the bellows, and separate the boards a little. Then, lay three weights *b, c, d*, each weighing 100 pounds, upon the upper board; and pour more water into the pipe *B*, which will run into the bellows, and raise up the board with all the weights upon it: and if the pipe be kept full, until the weights are raised as high as the leather which covers the bellows will allow them, the water will remain in the pipe, and support all the weights, even though it should weigh no more than a quarter of a pound, and they 300 pounds: nor will all their force be able to cause them to descend and force the water out at the top of the pipe.

The reason of this will be made evident, by considering what has been already said of the result of the pressure of fluids, from its being as their perpendicular height without any regard to their

Fig. 6.
The hydro-
static bel-
lows.

quantity. For, if a hole be made in the upper board, and a tube be put into it, the water will rise in the tube to the same height that it has in the pipe; and would rise as high (by supplying the pipe) in as many tubes as the board could contain holes. Now, suppose only one hole to be made in any part of the board, of an equal diameter with the bore of the pipe *B*; and that the pipe holds just a quarter of a pound of water: if a person claps his finger upon the hole, and the pipe be filled with water, he will find his finger to be pressed upward with a force equal to a quarter of a pound. And as the same pressure is equal upon all equal parts of the board, each part whose area is equal to the area of the hole will be pressed upward with a force equal to that of a quarter of a pound: the sum of all which pressures against the under side of an oval board 16 inches broad, and 18 inches long, will amount to 300 pounds; and therefore, so much weight will be raised up, and supported, by a quarter of a pound of water in the pipe.

How a man
may raise
himself up-
ward by his
breath.

Hence, if a man stands upon the upper board, and blows into the bellows through the pipe *B*, he will raise himself upward upon the board: and the smaller the bore of the pipe is, the easier he will be able to raise himself. And then, by clapping his finger upon the top of the pipe, he can support himself as long as he pleases; provided the bellows be air-tight, so as not to lose what is blown into it.

This figure, I confess, ought to have been much larger than any other upon the plate; but it was not

not thought of, until all the rest were drawn : and it could not so properly come into any other plate.

Upon this principle of the upward pressure of fluids, a piece of lead may be made to swim in water, by immersing it to a proper depth, and keeping the water from getting above it. Let CD be a glass tube, open at both ends, and EFG a flat piece of lead, exactly fitted to the lower end of the tube, not to go within it, but for it to stand upon ; with a wet leather between the lead and tube to make close work. Let this leaden bottom be half an inch thick, and held close to the tube by pulling the packthread IHL upward at L with one hand, whilst the tube is held in the other by the upper end C . In this situation, let the tube be immersed in water in the glass vessel AB , to the depth of three inches below the surface of the water at K ; and then, the leaden bottom EFG will be plunged to the depth of somewhat more than eleven times its own thickness : holding the tube at that depth, you may let go the thread at L ; and the lead will not fall from the tube, but will be kept to it by the upward pressure of the water below it, occasioned by the height of the water K above the level of the lead. The reason whereof is, that as lead is 11.33 times heavier than its bulk of water ; and is therefore immersed to a depth somewhat more than 11.33 times its thickness, and no water getting into the tube between it and the lead, the column of water $EabcG$ below the lead is pressed upward against it by the water $KDEGL$.

How lead
may be
made to
swim in
water.
Fig. 7.

all around the tube; which being a little more than 11.33 times as high as the lead is thick, is sufficient to balance and support the lead at the depth *KE*. If a little water be poured into the tube upon the lead, it will increase the weight upon the column of water under the lead, and cause the lead to fall from the tube to the bottom of the glass vessel; where it will lie in the situation *bd*. Or, if the tube be raised a little in the water, the lead will fall by its own weight, which will then be too great for the pressure of the water around the tube upon the column of water below it.

How light
wood may
be made to
lie at the
bottom of
water.

Let two pieces of wood be plained quite flat, so as no water may get in between them if they be put together: let one of the pieces as *bd* be cemented to the bottom of the vessel *AB* (Fig. 7.) and the other piece be laid flat and close upon it, and held down to it by a stick, whilst water is poured into the vessel; then remove the stick, and the upper piece of wood will not rise from the lower one: for, as the upper one is pressed down both by its own weight and the weight of all the water over it, whilst the contrary pressure of the water is kept off by the wood under it, it will lie as still as a stone would do in its place. But if it be raised ever so little at any edge, some water will then get under it; which being acted upon by the water above, will immediately press it upward; and as it is lighter than its bulk of water, it will rise, and swim upon the surface of the water.

All fluids weigh just as much in their own element as they do in open air. To prove this by experiment,

experiment, let as much shot be put into a phial, as, when corked, will make it sink in water: and being thus charged, let it be weighed, first in air, and then in water, and the weights in both cases wrote down. Then, as the phial hangs suspended in water, and counterpoised, pull out the cork, that water may run into it, and it will descend, and pull down that end of the beam. This done, put as much weight into the opposite scale as will restore the equipoise; which weight will be found to answer exactly to the additional weight of the phial when it is again weighed in air with the water in it.

The velocity with which water spouts out at a hole in the side or bottom of a vessel, is as the square root of the depth or distance of the hole below the surface of the water. For, in order to make double the quantity of a fluid run through one hole as through another of the same size, it will require four times the pressure of the other, and therefore four times its depth below the surface of the water: and for the same reason, three times the quantity running in an equal time through the same sort of hole, must run with three times the velocity, which will require nine times the pressure; and consequently must be nine times as deep below the surface of the fluid: and so on.— To prove this by an experiment, let two pipes, as

* The square root of any number is that which being multiplied by itself produces the said number. Thus, 2 is the square root of 4, and 3 is the square root of 9: for 2 multiplied by 2 produces 4, and 3 multiplied by 3 produces 9, &c.

I 4

C and

Fig. 8. C and g , of equal siz'd bores, be fixed into the side of the vessel AB ; the pipe g being four times as deep below the surface of the water at b in the vessel as the pipe C is: and whilst these pipes run, let water be constantly poured into the vessel, to keep the surface still at the same height. Then, if a cup that holds a pint be placed at K to receive the water that spouts from the pipe C , and at the same moment a cup that holds a quart be placed near L to receive the water that spouts from the pipe g , both cups will be filled at the same time by their respective pipes.

The horizontal distance to which water will spout from pipes.

The horizontal distance, to which a fluid will spout from a horizontal pipe, in any part of the side of an upright vessel below the surface of the fluid, is equal to twice the length of a perpendicular to the side of the vessel, drawn from the mouth of the pipe to a semicircle described upon the altitude of the fluid: and therefore, the fluid will spout to the greatest distance possible from a pipe, whose mouth is at the center of the semicircle; because a perpendicular to its diameter (supposed parallel to the side of the vessel) drawn from that point, is the greatest that can possibly be drawn from any part of the diameter to the circumference of the semicircle. Thus, if the vessel AB be full of water, the horizontal pipe D be in the middle of its side, and the semicircle $Nedcb$ be described upon D as a center, with the radius or semidiameter DgN , or Dfb , the perpendicular Dd to the diameter NDb is the greatest that can be drawn from any part of the diameter to the circumference $Nedcb$.

Nedcb. And if the vessel be kept full, the jet *G* will spout from the pipe *D*, to the horizontal distance *NM*, which is double the length of the perpendicular *Dd*. If two other pipes, as *C* and *E*, be fixed into the side of the vessel at equal distances above and below the pipe *D*, the perpendiculars *Cc* and *Ee*, from these pipes to the semicircle, will be equal; and the jets *F* and *H* spouting from them will each go to the horizontal distance *NK*; which is double the length of either of the equal perpendiculars *Cc* or *Dd*.

If either of these pipes be elevated 45 degrees above the level or horizontal position, it will spout the water still farther; and at that elevation will spout it to the greatest horizontal distance possible; taking this distance on a level with the pipe. At any greater or less elevation, it will spout to a less distance: and at equal elevations above or below 45 degrees, it will spout to equal distances.

Fluids by their pressure may be conveyed over hills and valleys in bended pipes, to any height not greater than the level of the springs from whence they flow. For it has been already shewn that they will rise to the same level in bended pipes *. How water may be conveyed over hills and valleys. * Page 111.

But when they are designed to be raised higher than the springs, forcing engines must be used; which shall be described when we come to treat of pumps.

A *siphon*, generally used for decanting liquors, is a bended pipe whose legs are of unequal lengths; and the shortest leg must always be put into the liquor intended to be decanted, that the perpendicular altitude of the column of liquor in the other leg

leg may be longer than the column in the immersed leg, especially above the surface of the water. For, if both columns were equally high in that respect, the atmosphere, which presses as much upward as downward, and therefore acts as much upward against the column in the leg that hangs without the vessel, as it acts downward upon the surface of the liquor in the vessel, would hinder the running of the liquor through the syphon, even though it were brought over the bended top by suction. So that there is nothing left to determine the motion of the liquor, but the superior weight of the column in the longer leg, on account of its having the greater perpendicular height.

Fig. 9.

Let D be a cup filled with water to C , and ABC a syphon, whose shorter leg BCF is immersed in the water from C to F . If the end of the other leg were no lower than the line AC , which is level with the surface of the water, the syphon would not work, even though the air should be drawn out of it at the mouth A . For although the suction would draw some water at first, yet the water would stop at the moment the suction ceased; because the air would act as much upward against the water at A , as it acted for it by pressing downward on the surface at C . But if the leg AB comes down to G , and the air be drawn out at G by suction, the water will immediately follow, and continue to run, until the surface of the water in the cup comes down to F ; because, until then, the perpendicular height of the column BAG will be greater than that of the column CB ; and consequently, its weight will

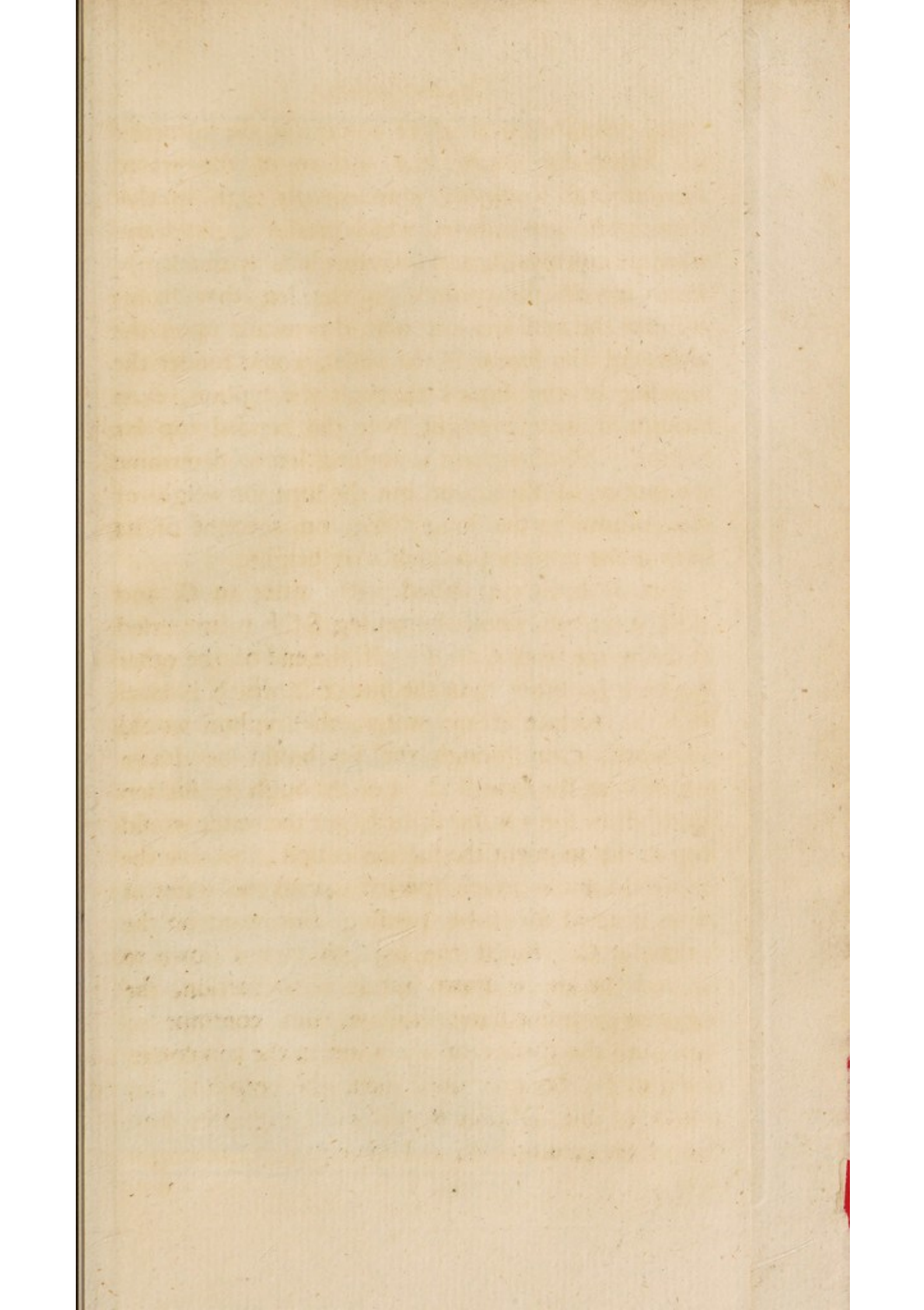
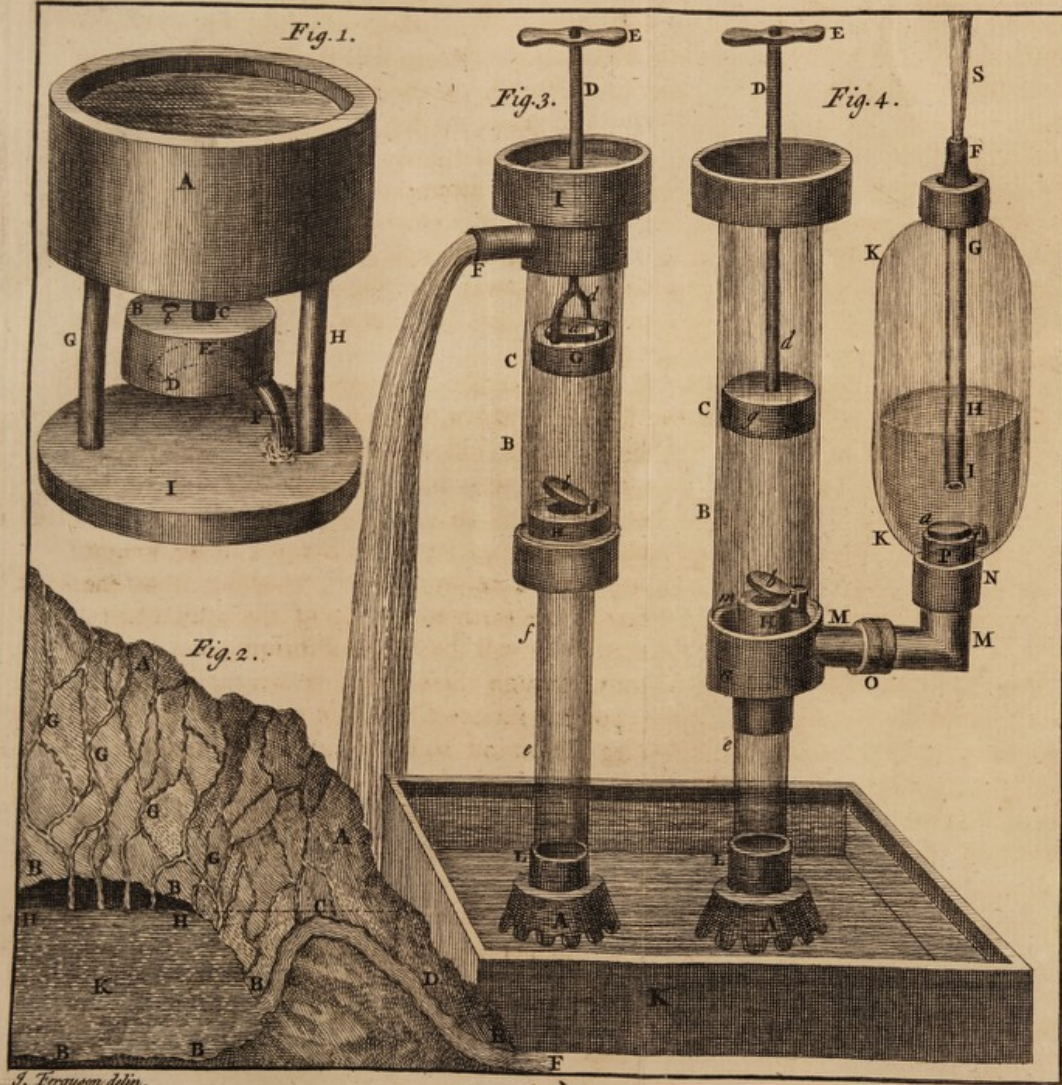


PLATE XI.



J. Ferguson delin.

J. Mynde sc.

will be greater, until the surface comes down to *F*; and then the syphon will stop, though the leg *CF* should reach to the bottom of the cup at *b*. For which reason, the leg that hangs without the cup is always made long enough to reach below the level of its bottom; as from *d* to *E*: and then, when the syphon is emptied of air by suction at *E*, the water immediately follows, and by its continuity brings away the whole from the cup; just as pulling one end of a thread will make the whole clue follow.

If the perpendicular height of a syphon, from the surface of the water to its bend at *B*, be more than 33 feet, it will draw no water, even though the other leg were much longer, and the syphon quite emptied of air; because the weight of a column of water 33 feet high is equal to the weight of as thick a column of air, reaching from the surface of the earth to the top of the atmosphere; so that there will be an equilibrium, and consequently, though there would be weight enough of air upon the surface *C* to make the water ascend in the leg *CB* almost to the height *B*, if the syphon were emptied of air, yet the weight would not be sufficient to force the water over the bend; and therefore, it could never be brought into the leg *BAC*.

Let a hole be made in the bottom of the cup *A*, Fig. 10. and the longer leg of the bended syphon *BCE* Tantalus's cup. *D* be cemented into the hole, so that the end *D* of the shorter leg *DE* may almost touch the bottom of the cup within. Then, if water be poured into this

this cup, it will rise in the shorter leg by its upward pressure, extruding the air all the way before it through the longer leg: and when the cup is filled above the bend of the syphon at *F*, the pressure of the water in the cup will force it over the bend of the syphon; and it will descend in the longer leg *CBG*, and run through the bottom, until the cup be emptied.

This is generally called *Tantalus's* cup, and the legs of the syphon in it are almost close together; and a little hollow statue, or figure of a man, is sometimes put over the syphon to conceal it; the bend *E* being within the neck of the figure as high as the chin. So that poor thirsty *Tantalus* stands up to the chin in water, imagining it will rise a little higher, and he may drink; but instead of that, when the water comes up to his chin, it immediately begins to descend, and so, as he cannot stoop to follow it, he is left as much pained with thirst as ever.

*The fountain
at command.*

Plate XI.
Fig. 1.

The device called *the fountain at command* acts upon the same principle with the syphon in the cup. Let two vessels *A* and *B* be joined together by the pipe *C* which opens into them both. Let *A* be open at top, *B* close both at top and bottom, (save only a small hole at *b* to let the air get out of the vessel *B*;) and *A* be of such a size, as to hold about six times as much water as *B*. Let a syphon *DEF* be foldered to the vessel *B*, so that the part *DEe* may be within the vessel, and *F* without it; the end *D* almost touching the bottom of the vessel, and the end *F* below the level of *D*: the vessel *B*
hanging

hanging at *A* by the pipe *C* (folded into both) and the whole supported by the pillars *G* and *H* upon the stand *I*. The bore of the pipe must be considerably less than the bore of the syphon.

The machine being thus constructed, let the vessel *A* be filled with water, which will run through the pipe *C*, and fill the vessel *B*. When *B* is filled above the top of the syphon at *E*, the water will run through the syphon, and be discharged at *F*. But since the bore of the syphon is larger than the bore of the pipe, the syphon will run faster than the pipe, and will soon empty the vessel *B*; upon which the water will cease from running through the syphon at *F*, until the pipe *C* re-fills the vessel *B*; and then it will begin to run as before. And thus the syphon will continue to run and stop alternately, until all the water in the vessel *A* has run through the pipe *C*.—So that after a few trials, one may easily guess about what time the syphon will stop, and when it will begin to run: and then, to amuse others, he may call out *stop*, or *run*, accordingly.

Upon this principle, we may easily account for *Intermitting springs*. Let *AA* be *intermitting or reciprocating springs*. part of a hill, within which there is a cavity *BB*; Fig. 2. and from this cavity a vein running in the direction *BCDE*. The rain that falls upon the side of the hill will sink and strain through the small pores and crannies *G, G, G, G*; and fill the cavity with water *K*. When the water rises to the level *HHC*, the vein *BCDE* will be filled, and begin to run at *F*, like a syphon; which running will continue until

until the cavity be emptied, and then it will stop until the cavity be filled again.

The common
pump.

The *common sucking pump*, with which we draw water out of wells, is an engine both pneumatic and hydraulic. It consists of a pipe open at both ends, in which is a moveable piston, bucket, or sucker, as big as the bore of the pipe in that part wherein it works; and is leathered round, so as to fit the bore exactly; and may be moved up and down, without suffering any air to come between it and the pipe or pump barrel.

We shall explain the construction both of this and the forcing pump by pictures of glass models, in which, both the action of the pistons and motion of the valves are seen.

Fig. 3.

Hold the model *ABC* upright in the vessel of water *K*, the water being deep enough to rise at least as high as from *A* to *L*. The valve *a* on the moveable bucket *G*, and the valve *b* on the fixed box *H*, (which quite fills the bore of the pipe or barrel at *H*) will each lie close, by its own weight, upon the hole in the bucket and box, until the engine begins to work. The valves are made of brass, and covered underneath with leather for closing the holes the more exactly: and the bucket *G* is raised and depressed alternately by the handle *E* and rod *Dd*, the bucket being supposed at *B* before the working begins.

Take hold of the handle *E*, and thereby draw up the bucket from *B* to *C*, which will make room for the air in the pump all the way below the bucket

to dilate itself, by which its spring is weakened, and then its force is not equivalent to the weight or pressure of the outward air upon the water in the vessel *K*: and therefore, at the first stroke, the outward air will press up the water through the notched foot *A*, into the lower pipe, about as far as *e*: this will condense the rarefied air in the pipe between *e* and *C* to the same state it was in before; and then, as its spring within the pipe is equal to the force or pressure of the outward air, the water will rise no higher by the first stroke; and the valve *b*, which was raised a little by the dilation of the air in the pipe, will fall, and stop the hole in the box *H*; and the surface of the water will stand at *e*. Then, depress the piston or bucket from *C* to *B*, and as the air in the part *B* cannot get back again through the valve *b*, it will (as the bucket descends) raise the valve *a*, and so make its way through the upper part of the barrel *d* into the open air. But upon raising the bucket *G* a second time, the air between it and the water in the lower pipe at *e* will be again left at liberty to fill a larger space; and so its spring being again weakened, the pressure of the outward air on the water in the vessel *K* will force more water up into the lower pipe from *e* to *f*; and when the bucket is at its greatest height *C*, the lower valve *b* will fall, and stop the hole in the box *H* as before. At the next stroke of the bucket or piston, the water will rise through the box *H* towards *B*, and then the valve *b*, which was raised by it, will fall when the bucket *G* is at its greatest height. Upon depressing the bucket again, the
water

water cannot be pushed back through the valve *b*, which keeps close upon the hole whilst the piston descends. And upon raising the piston again, the outward pressure of the air will force the water up through *H*, where it will raise the valve, and follow the bucket to *C*. Upon the next depression of the bucket *G*, it will go down into the water in the barrel *B*; and as the water cannot be driven back through the now close valve *b*, it will raise the valve *a* as the bucket descends, and will be lifted up by the bucket when it is next raised. And now, the whole space below the bucket being full, the water above it cannot sink when it is next depressed; but upon its depression, the valve *a* will rise to let the bucket go down; and when it is quite down, the valve *a* will fall by its weight, and stop the hole in the bucket. When the bucket is next raised, all the water above it will be lifted up, and begin to run off by the pipe *F*. And thus, by raising and depressing the bucket alternately, there is still more water raised by it; which getting over the pipe *F*, into the wide top *I*, will supply the pipe, and make it run with a continued stream.

So, at every time the bucket is raised, the valve *b* rises and the valve *a* falls; and at every time the bucket is depressed, the valve *b* falls and *a* rises.

As it is the pressure of the air or atmosphere which causes the water to rise, and follow the piston or bucket *G* as it is drawn up; and since a column of water 33 feet high is of equal weight with as thick a column of the atmosphere, from the earth to the very top of the air; therefore, the perpendicular

The pump rod is never raised directly by such a handle as *E* at the top, but by means of a lever, K
whose

whose longer arm (at the end of which the power is applied) generally exceeds the length of the shorter arm five or six times; and, by that means, gives five or six times as much advantage to the power. Upon these principles, it will be easy to find the dimensions of a pump that shall work with a given force, and draw water from any given depth. But, as these calculations have been generally neglected by pump-makers (either for want of skill or industry) the following table was calculated by the late ingenious Mr. *Booth* for their benefit. In this calculation, he supposed the handle of the pump to be a lever increasing the power five times; and had often found that a man can work a pump four inches diameter, and 30 feet high above the bucket, and discharge $27\frac{1}{2}$ gallons of water (English wine measure) in a minute. Now, if it be required to find the diameter of a pump, that shall raise water with the same ease from any other height above the bucket; look for that height in the first column, and over-against it in the second you have the diameter or width of the pump; and in the third, you find the quantity of water which a man of ordinary strength can discharge in a minute.

Height

Height of the pump above the bucket.	Diameter of the bore where the bucket works.		Water discharged in a minute, English wine measure.	
Feet.	Inches.	100 parts	Gallons.	Pints.
10	6	.93	81	6
15	5	.65	54	4
20	4	.90	40	8
25	4	.38	32	6
30	4	.00	27	2
35	3	.70	23	3
40	3	.47	20	4
45	3	.26	18	1
50	3	.10	16	3
55	2	.95	14	7
60	2	.83	13	5
65	2	.71	12	4
70	2	.62	11	5
75	2	.53	10	7
80	2	.44	10	2

The *forcing pump* raises water through the box *H* in the same manner as the sucking pump does, when the plunger or piston *g* is lifted up by the rod *Dd*. But this plunger has no hole through it, to let the water in the barrel *BC* get above it, when it is depressed to *B*, and the valve *b* (which rose by the ascent of the water through the box *H* when the plunger *g* was drawn up) falls down and stops the hole in *H*, the moment that the plunger is raised to its greatest height. Therefore, as the water between the plunger *g* and box *H* can neither get through the plunger upon its descent, nor back again into the lower part of the pump *Le*, but has

a free passage by the cavity *mn* into the pipe *MM*, which opens into the air-vessel *KK* at *P*; the water is forced through the pipe *MM* by the descent of the plunger, and driven into the air vessel; and in running up through the pipe at *P*, it opens the valve *a*; which shuts at the moment the plunger begins to be raised, because the action of the water against the under side of the valve then ceases.

The water, being thus forced into the air-vessel *KK*, by repeated strokes of the plunger, gets above the lower end of the pipe *GHI*, and then begins to condense the air in the vessel *KK*. For, as the pipe *GH* is fixed air-tight into the vessel below *F*, and the air has no way to get out of the vessel but through the mouth of the pipe at *I*, and cannot get out when the mouth *I* is covered with water; and is more and more condensed as the water rises upon the pipe, the air then begins to act forcibly by its spring against the surface of the water *H*: and this action drives the water up through the pipe *IHG F*, from whence it spouts in a jet *S* to a great height; and is supplied by alternately raising and depressing of the plunger *g*, which constantly forces the water that it raises, through the valve *H*, along the pipe *MM*, into the air vessel *KK*.

The higher that the surface of the water *H* is raised in the air-vessel, the less space will the air, which before filled that vessel, be condensed into; and therefore the force of its spring will be so much the stronger upon the water, and will drive it with the greater force through the pipe at *F*: and as the spring of the air continues whilst the plunger *g* is rising,

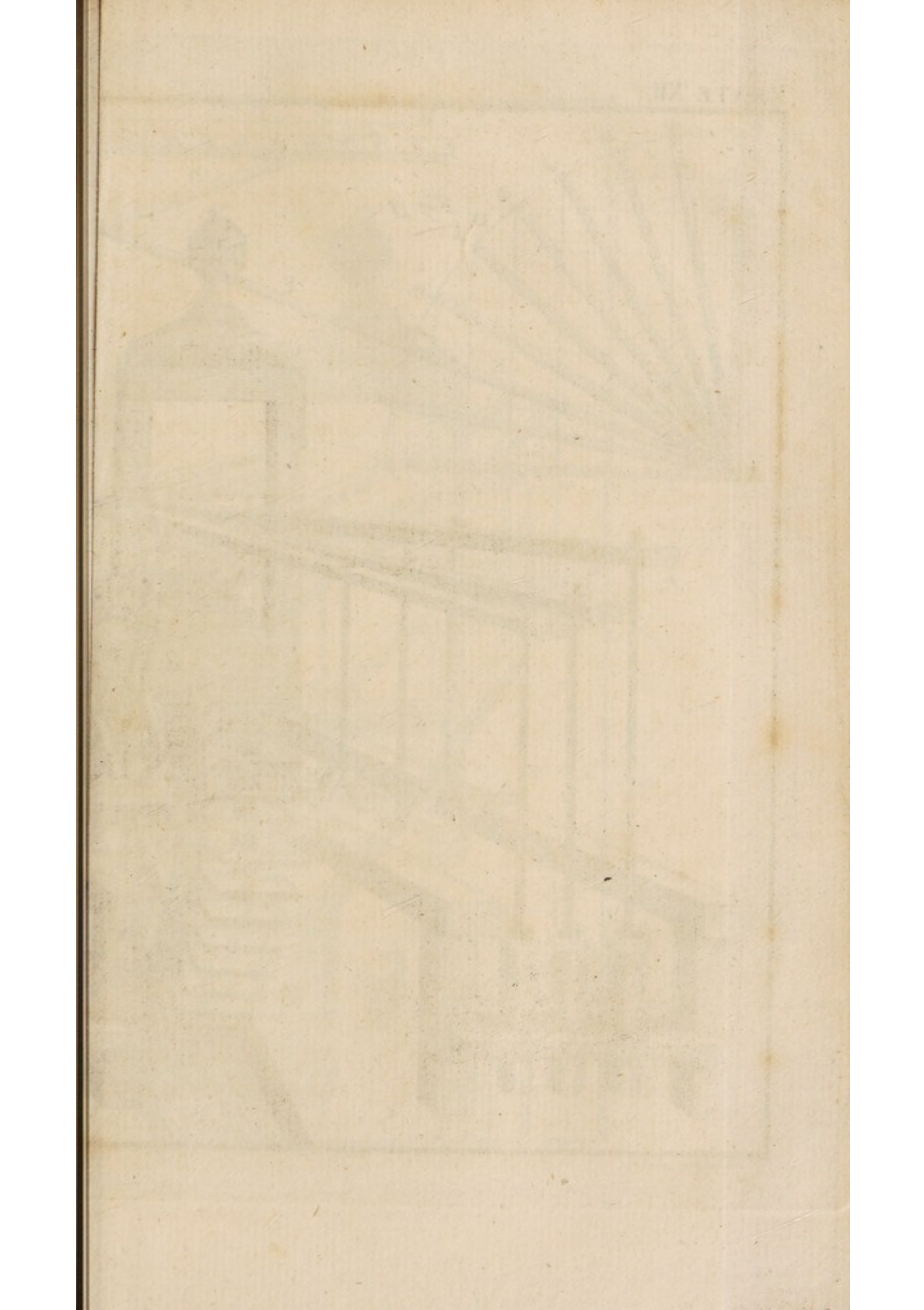
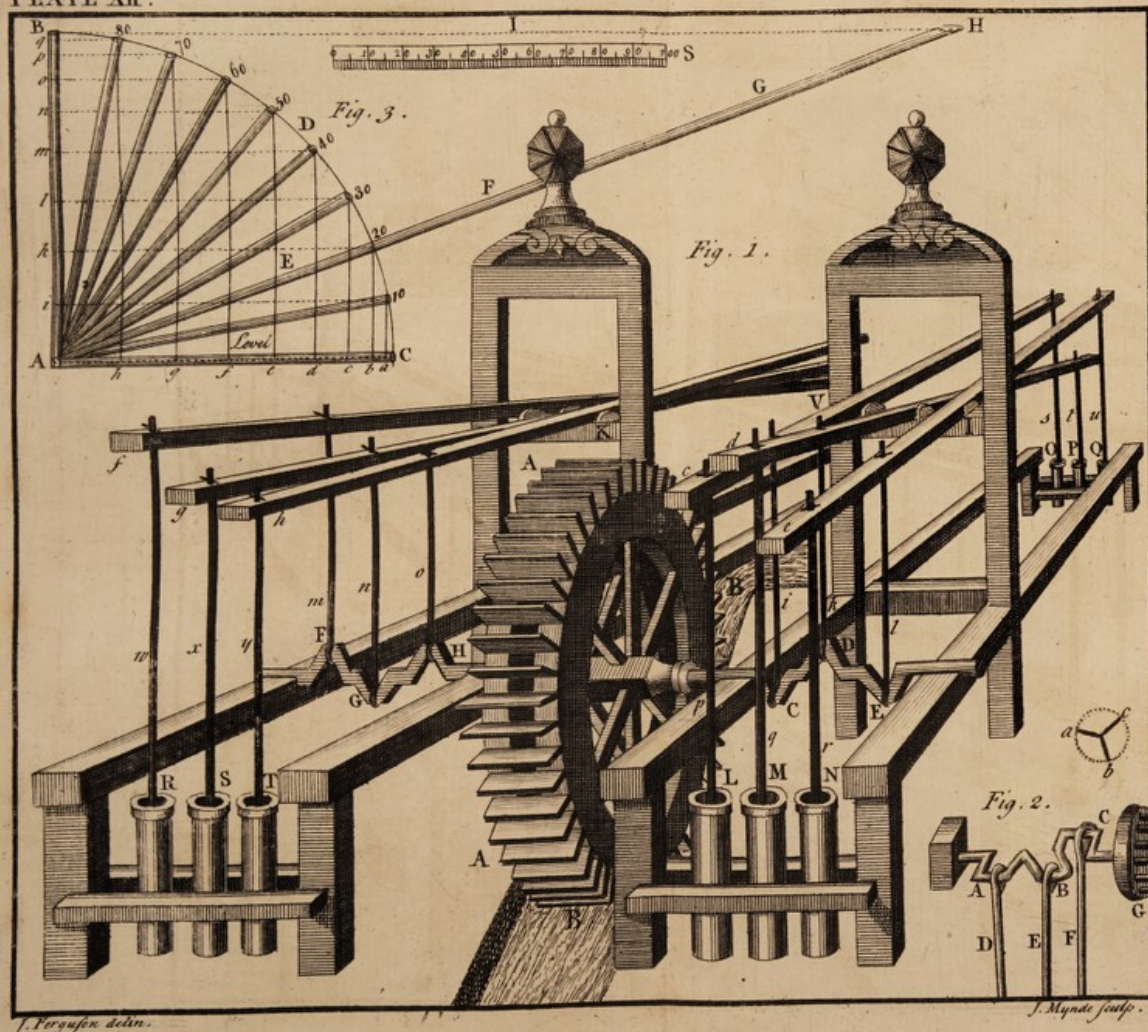


PLATE XII.



rising, the stream or jet S will be uniform as long as the action of the plunger continues: and when the valve b opens, to let the water follow the plunger upward, the valve a shuts, to hinder the water, which is forced into the air vessel, from running back by the pipe MM into the barrel of the pump.

If there were no air-vessel to this engine, the pipe GHI would be joined to the pipe MMN at P ; and then, the jet S would stop every time the plunger is raised, and run only when the plunger is depressed.

Mr. *Newsham's* water-engine, for extinguishing fire, consists of two forcing pumps, which alternately drive water into a close vessel of air; and by forcing the water into that vessel, the air in it is thereby condensed, and compresses the water so strongly, that it rushes out with great impetuosity and force through a pipe that comes down into it; and makes a continued stream by the condensation of the air upon its surface in the vessel.

By means of forcing pumps, water may be raised to any height above the level of a river or spring; and machines may be contrived to work these pumps either by a running stream, a fall of water, or by horses. An instance in each sort will be sufficient to shew the method.

First, by a running stream, or a fall of water. Plate XII.
Let AA be a wheel, turned by the fall of water Fig. 1.
 BB ; and have any number of cranks (suppose six) as C, D, E, F, G, H , on its axis, according to the strength of the fall of water, and the height to which the water is intended to be raised by the

A pump
engine to
go by wa-
ter.

engine. As the wheel turns round, these cranks move the levers *c, d, e, f, g, h*, up and down, by the iron rods *i, k, l, m, n, o*; which alternately raise and depress the pistons by the other iron rods *p, q, r, s, t, u, w, x, y*, in twelve pumps; nine whereof, as *L, M, N, O, P, Q, R, S, T*, appear in the plate; the other three being hid behind the work at *V*. And as pipes may go from all these pumps, to convey the water (drawn up by them to a small height) into a close cistern, from which the main pipe proceeds, the water will be forced into this cistern by the descent of the pistons. And as each pipe, going from its respective pump into the cistern, has a valve at its end in the cistern, these valves will hinder the return of the water by the pipes; and therefore, when the cistern is once full, each piston upon its descent will force the water (conveyed into the cistern by a former stroke) up the main pipe, to the height the engine was intended to raise it: which height depends upon the quantity raised, and the power that turns the wheel. When the power upon the wheel is lessened by any defect of the quantity of water turning it, a proportionable number of the pumps may be laid aside, by disengaging their rods from the vibrating levers.

This figure is a representation of the engine erected at *Blenheim* for the Duke of *Marlborough* by the late ingenious Mr. *Aldersea*. The water wheel is $7\frac{1}{2}$ feet in diameter, according to Mr. *Switzer*'s account in his *Hydraulics*.

When

When such a machine is placed in a stream that runs upon a small declivity, the motion of the levers and action of the pumps will be but flow; since the wheel must go once round for each stroke of the pumps. But, when there is a large body of flow running water, a cog or spur wheel may be placed upon each side of the water wheel *AA*, upon its axis, to turn a trundle upon each side; the cranks being upon the axis of the trundle. And by proportioning the cog wheels to the trundles, the motion of the pumps may be made quicker, according to the quantity and strength of the water upon the first wheel; which may be as great as the workman pleases, according to the length and breadth of the float-boards or wings of the wheel. In this manner, the engine for raising water at *London-Bridge* is constructed; in which, the water wheel is 20 feet diameter, and the floats 14 feet long.

Where a stream, or fall of water cannot be had, ^{A pump engine to go by horses.} and gentlemen want to have water raised, and brought to their houses from a rivulet or spring; this may be effected by a horse-engine, working three forcing pumps which stand in a reservoir filled by the spring or rivulet: the pistons being moved up and down in the pumps by means of a triple crank *ABC*, which, as it is turned round by the trundle *G*, raises and depresses the rods *D, E, F*. The trundle may be turned by such a wheel as *F* ^{Fig. 2.} in Fig. 1. of Plate VIII. having levers *y, y, y, y*, on its upright axle, to which horses may be joined for working the engine. And if the wheel has three

times as many cogs as the trundle has staves or rounds, the trundle and cranks will make three revolutions for every one of the wheel: and as each crank will fetch a stroke in the time it goes round, the three cranks will make nine strokes for every turn of the great wheel.

The cranks should be made of cast iron, because *that* will not bend; and they should each make an angle of 120 with both of the others, as at *a, b, c*; which is (as it were) a view of their *radii*, in looking endwise at the axis: and then, there will be always one or other of them going downward, which will push the water forward with a continued stream into the main pipe. For, when *b* is almost at its lowest situation, and is therefore just beginning to lose its action upon the piston which it moves, *c* is beginning to move downward, which will by its piston continue the propelling force upon the water: and when *c* is come down to the position of *b*, *a* will be in the position of *c*.

The trundle may be either on the upper or under side of the rim of the great wheel, at the discretion of the workman.

The more perpendicularly the piston-rods move up and down in the pumps, the freer and better will their strokes be: but a little deviation from the perpendicular will not be material. Therefore, when the pump-rods *D, E*, and *F* go down into a deep well, they may be moved directly by the cranks, as is done in a very good horse-engine of this sort at Sir *James Creed's*, which forces up wa-
ter

ter about 40 feet from a well under ground, to a reservoir on the top of his house at *Greenwich*. But when the cranks are only at a small height above the pumps, the pistons must be moved by vibrating levers, as in the above engine at *Blenheim*: and the longer the levers are, the nearer will the strokes be to a perpendicular.

Let us suppose, that in such an engine as Sir *James Creed's*, the great wheel is 12 feet diameter, the trundle 4 feet, and the radius or length of each crank to be 9 inches, working a piston in its pump. Let there be three pumps in all, and the bore of each pump be four inches diameter. Then, if the great wheel has three times as many cogs as the trundle has staves, the trundle and cranks will go three times round for each revolution of the horses and wheel, and the three cranks will make nine strokes of the pumps in that time, each stroke being 18 inches (or double the length of the crank) in a 4 inch bore. Let the diameter of the horse walk be 18 feet, and the perpendicular height to which the water is raised above the surface of the well be 64 feet.

A calculation of the quantity of water that may be raised by a horse-engine.

If the horses go at the rate of two miles an hour (which is very moderate walking) they will turn the great wheel 187 times round in an hour.

In each turn of the wheel the pistons make 9 strokes in the pumps, which amount to 1683 in an hour.

Each stroke raises a column of water 18 inches long, and 4 inches thick, in the pump-barrels; which column, upon the descent of the piston, is forced

forced into the main pipe, whose perpendicular altitude above the surface of the well is 64 feet.

Now, since a column of water 18 inches long and 4 inches thick, contains 226.18 cubic inches, this number multiplied by 1683 (the strokes in an hour) gives 380661 for the number of cubic inches of water raised in an hour.

A gallon, in wine measure, contains 231 cubic inches, by which divide 380661, and it quotes 1468 in round numbers, for the number of gallons raised in an hour; which, divided by 63, gives $26\frac{1}{2}$ hogsheads.—If the horses go faster, the quantity raised will be so much greater.

In this calculation it is supposed that no water is wasted by the engine. But as no forcing engine can be supposed to lose less than a fifth part of the calculated quantity of water, between the pistons and barrels, and by the opening and shutting of the valves, the horses ought to walk almost $2\frac{1}{2}$ miles *per* hour, to fetch up this loss.

A column of water 4 inches thick, and 64 feet high, weighs $349\frac{2}{6}$ pounds averdupoise, or $424\frac{5}{2}$ pounds troy; and this weight, together with the friction of the engine, is the resistance that must be overcome by the strength of the horses.

The horse-tackle should be so contrived, that the horses may rather push on than drag the levers after them. For if they draw, in going round the walk, the outside traces will rub against their sides and hams; which will hinder them from drawing at right angles to the levers, and so make them pull at a disadvantage. But if they push the lever before

before their breasts, instead of dragging it behind them, they can always walk at right angles to it.

It is no ways material what the diameter of the main or conduct pipe be: for the whole resistance of the water therein, against the horses, will be according to the height to which it is raised, and the diameter of that part of the pump in which the piston works; as we have already observed. So that by the same pump, an equal quantity of water may be raised in (and consequently made to run from) a pipe of a foot diameter, with the same ease as in a pipe of five or six inches: or rather with more ease, because its velocity in a large pipe will be less than in a small one; and therefore its friction against the sides of the pipe will be less also.

And, the force required to raise water depends not upon the length of the pipe, but upon the perpendicular height to which it is raised therein above the level of the spring. So that the same force, Fig. 3. which would raise water to the height AB in the upright pipe $AiklmnopqB$, will raise it to the same height or level BIH in the oblique pipe $AEFGH$. For the pressure of the water at the end A of the latter, is no more than its pressure against the end A of the former.

The weight or pressure of water at the lower end of a pipe, is always as the sine of the angle to which the pipe is elevated above the level or horizon. For, although the water in the upright pipe AB would require a force applied immediately to the lower end A equal to the weight of all the water in it to support the water, and a little more

more to drive it up, and out of the pipe; yet, if that pipe be inclined from its upright position to an angle of 80 degrees (as in *A 80*) the force required to support or to raise the same cylinder of water will then be as much less, as the sine 80 *b* is less than the radius *AB*; or as the sine of 80 degrees is less than the sine of 90. And so, decreasing as the sine of the angle of elevation lessens, until it arrives at its level *AC* or place of rest, where the force of the water is nothing at either end of the pipe. For, although the absolute weight of the water is the same in all positions, yet its pressure at the lower end decreases, as the sine of the angle of elevation decreases; as will appear plainly by a farther consideration of the figure.

Let two pipes, *AB* and *BC*, of equal lengths and bores, communicate with one another at *A*; and let the pipe *AB* be divided into 100 equal parts, as the scale *S* is; whose length is equal to the length of the pipe.—Upon this length, as a radius, describe the quadrant *BCD*, and divide it into 90 equal parts or degrees.

Let the pipe *AC* be filled with water, and elevated to 10 degrees upon the quadrant; then, part of the water that is in it will rise in the pipe *AB*, and if it be kept full of water, it will raise the water in the pipe *AB* from *A* to *i*; that is, to a level *i 10* with the mouth of the pipe at 10: and the upright line *a 10*, equal to *Ai*, will be the sine of 10 degrees elevation; which, being measured upon the scale *S*, will be about 17.3 of such parts as the pipe contains 100 in length: and therefore, the force

or pressure of the water at A , in the pipe $A 10$, will be to the force or pressure at A in the pipe AB , as 17.3 to 100.

Let the same pipe be elevated to 20 degrees in the quadrant, and if it be kept full of water, part of that water will run into the pipe AB , and rise therein to the height Ak , which is equal to the length of the upright line $b 20$, or to the sine of 20 degrees elevation; which, being measured upon the scale S , will be 34.2 of such parts as the pipe contains 100 in length. And therefore, the pressure of the water at A , in the full pipe $A 20$, will be to its pressure, if that pipe were raised to the perpendicular situation AB , as 34.2 to 100.

Elevate the pipe to the position $A 30$ on the quadrant, and if it be supplied with water, the water will rise from it, into the pipe AB , to the height Al , or to the same level with the mouth of the pipe at 30. The sine of this elevation, or of the angle of 30 degrees, is $c 30$; which is just equal to half the length of the pipe, or to 50 of such parts of the scale, as the length of the pipe contains 100. Therefore, the pressure of the water at A , in a pipe elevated 30 degrees above the horizontal level, will be equal to one half of what it would be, if the same pipe stood upright in the situation AB .

And thus, by elevating the pipe to 40, 50, 60, 70, and 80 degrees on the quadrant, the sines of these elevations will be $d 40$, $e 50$, $f 60$, $g 70$, and $h 80$; which will be equal to the heights Am , An , AO , Ap , and Aq : and these heights measured upon the scale S will be 64.3, 76.6, 86.6, 93.9, and

and 98.5 ; which exprefs the preffure at *A* in all thefe elevations, confidering the preffure in the upright pipe *AB* as 100.

Because it may be of ufe to have the lengths of all the fines of a quadrant from 0 degrees to 90, we fhall here annex a table fhewing the length of the fine of every degree in fuch parts as the whole pipe (equal to the radius of the quadrant) contains 1000. Then the fines will be integral or whole parts in length. But if you fuppofe the length of the pipe to be divided only into 100 equal parts, the laft figure of each part or fine muft be cut off as a decimal ; and then thofe which remain at the left hand of this feparation will be integral or whole parts.

Sine of	Parts	Sine of	Parts	Sine of	Parts	Sine of	Parts	Sine of	Parts
D. 1	17	D. 19	325	D. 37	601	D. 55	819	D. 73	956
2	39	20	342	38	616	56	829	74	961
3	52	21	358	39	629	57	839	75	966
4	70	22	375	40	643	58	848	76	970
5	87	23	391	41	656	59	857	77	974
6	104	24	407	42	669	60	866	78	978
7	122	25	423	43	682	61	875	79	982
8	139	26	438	44	695	62	883	80	985
9	156	27	454	45	707	63	891	81	988
10	173	28	469	46	719	64	898	82	990
11	191	29	485	47	731	65	906	83	992
12	208	30	500	48	743	66	913	84	994
13	225	31	515	49	755	67	920	85	996
14	242	32	530	50	766	68	927	86	997
15	259	33	545	51	777	69	933	87	998
16	275	34	559	52	788	70	939	88	999
17	292	35	573	53	799	71	945	89	999
18	309	36	588	54	809	72	951	90	1000

Thus, if the radius of the quadrant (fuppofed to be equal to the length of the pipe *AC*) be divided

vided into 1000 equal parts, and the elevation be 45 degrees, the sine of that elevation will be equal to 707 of these parts: but if the radius be divided only into 100 equal parts, the same sine will be only 70.7 or $70\frac{7}{10}$ of these parts. For, as 1000 is to 707, so is 100 to 70.7.

As it is of the greatest importance to all engine-makers, to know what quantity and weight of water will be contained in an upright round pipe of a given diameter and height; so as by knowing what weight is to be raised, they may proportion their engines to that force; we shall subjoin tables shewing the number of cubic inches of water contained in an upright pipe of a round bore, of any diameter from one inch to six and an half; and of any height from one foot to two hundred: together with the weight of the said number of cubic inches, both in troy and averdupoise ounces. The number of cubic inches divided by 231 will reduce the water to gallons in wine measure, and divided by 282 will reduce it to the measure of ale gallons. Also, the troy ounces divided by 12 will reduce the weight to troy pounds, and the averdupoise ounces divided by 16 will reduce the weight to averdupoise pounds.

And here I must repeat it again, that the weight or pressure of the water, acting against the power that works the engine, must always be estimated according to the perpendicular height to which it is to be raised, without any regard to the length of the conduct pipe when it has an oblique position; and as if the diameter of that pipe were just equal
to

to the diameter of that part of the pump in which the piston works. Thus, by the following tables, the pressure of the water, against an engine whose pump is of a $4\frac{1}{2}$ inch bore, and the perpendicular height of the water in the conduct pipe is 80 feet, will be equal to 8057.5 troy ounces, and to 8848.2 averdupoise ounces; which makes 671.4 troy pounds, and 553 averdupoise.

One inch diameter.				$1\frac{1}{2}$ inch diameter.			
Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.	Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.
1	9.42	4.97	5.40	1	21.10	11.19	12.29
2	18.85	9.95	10.92	2	42.41	22.38	24.58
3	28.27	14.92	16.38	3	63.62	33.57	36.87
4	37.70	19.81	21.85	4	84.82	44.76	49.16
5	47.12	24.87	27.31	5	106.03	55.95	60.45
6	56.55	29.84	32.77	6	127.23	67.14	73.73
7	65.97	34.82	38.23	7	147.44	78.34	86.02
8	75.40	39.79	43.69	8	169.65	89.53	98.31
9	84.82	44.76	49.16	9	190.85	100.72	110.60
10	94.25	49.7	54.62	10	212.06	111.91	122.89
20	188.49	99.48	109.24	20	424.11	223.82	245.78
30	282.74	149.21	163.86	30	636.17	335.73	368.67
40	376.99	198.95	218.47	40	848.23	447.64	491.57
50	471.24	248.69	273.09	50	1060.29	559.55	614.46
60	565.49	298.43	327.71	60	1272.35	671.46	737.35
70	659.73	348.17	382.33	70	1484.40	783.37	860.24
80	753.98	397.90	436.95	80	1696.46	895.28	983.14
90	848.23	447.64	491.57	90	1908.52	1007.19	1106.03
100	942.48	497.38	546.19	100	2120.58	1119.09	1228.92
200	1884.96	994.76	1092.38	200	4241.15	2238.18	2457.84

Example. Required the number of cubic inches, and the weight of the water, in an upright pipe 278 feet high, and $1\frac{1}{2}$ inch diameter?

Here,

Here, the nearest single decimal figure is only taken into the account; and the whole being reduced by division, amounts

Feet	Cubic inches	Troy oz.	Averd. oz.
200	4241.1	2238.2	2457.8
70	1484.4	783.3	860.2
8	169.6	89.5	98.3
<hr/>			
Anf.	278	5895.1	3111.0
			3416.3

to $25\frac{1}{2}$ wine gallons in measure; to $259\frac{1}{4}$ pounds troy, and to $213\frac{1}{2}$ pounds averdupoise.

2 Inches diameter.				$2\frac{1}{2}$ Inches diameter.			
Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.	Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.
1	37.70	19.89	21.85	1	58.90	31.08	34.14
2	75.40	39.79	43.69	2	117.81	62.17	68.27
3	113.09	59.68	65.54	3	176.71	93.26	102.41
4	150.79	79.58	87.39	4	235.62	124.34	136.55
5	188.49	99.47	109.24	5	294.52	155.43	170.68
6	226.19	119.37	131.08	6	353.43	186.52	204.82
7	263.89	139.26	152.93	7	412.33	217.60	238.96
8	301.59	159.16	174.78	8	471.24	248.69	273.09
9	339.29	179.05	196.63	9	530.14	279.77	307.23
10	376.99	198.95	218.47	10	589.05	310.86	341.37
20	753.98	397.90	436.95	20	1178.10	621.72	682.73
30	1130.97	596.85	665.42	30	1767.15	932.58	1024.10
40	1507.96	795.80	873.90	40	2356.20	1243.44	1365.47
50	1884.96	994.75	1092.37	50	2945.25	1554.30	1706.83
60	2261.95	1193.70	1310.84	60	3534.29	1865.16	2048.20
70	2638.94	1392.65	1529.32	70	4123.34	2176.02	2389.57
80	3015.93	1591.60	1747.80	80	4712.39	2486.88	2730.94
90	3392.92	1790.55	1966.27	90	5301.44	2797.74	3072.30
100	3769.91	1989.50	2184.75	100	5890.49	3108.60	3413.67
200	7539.82	3979.00	4369.50	200	11780.98	6217.20	6827.34

These tables were first calculated to six decimal places for the sake of exactness; but in transcribing them, there are no more than two decimal figures taken into the account, and sometimes but one; because there is no necessity for computing to hun-

3 Inches diameter.				3½ Inches diameter.			
Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In aver-dupoise ounces.	Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In aver-dupoise ounces.
1	84.8	44.76	49.16	1	115.4	60.9	66.9
2	169.6	89.53	98.31	2	230.9	121.8	133.8
3	254.5	134.29	147.47	3	346.4	182.8	200.7
4	339.3	179.05	196.63	4	461.8	243.7	267.6
5	424.1	223.82	245.78	5	577.3	304.6	334.5
6	508.9	268.58	294.94	6	692.7	365.6	401.4
7	593.7	313.35	344.10	7	808.2	426.5	468.3
8	698.6	358.11	393.25	8	923.6	487.4	535.3
9	763.4	402.87	442.41	9	1039.1	548.3	602.2
10	848.2	447.64	491.57	10	1154.5	609.2	669.1
20	1696.5	895.28	983.14	20	2309.1	1218.6	1338.2
30	2544.7	1342.92	1474.70	30	3463.6	1827.8	2007.2
40	3392.9	1790.55	1966.27	40	4618.1	2437.1	2676.3
50	4241.1	2238.19	2457.84	50	5772.7	3046.4	3345.4
60	5089.4	2685.83	2949.41	60	6927.2	3655.7	4014.5
70	5937.6	3133.47	3440.98	70	8081.7	4265.0	4683.5
80	6785.8	3581.11	3932.55	80	9236.3	4874.3	5352.6
90	7634.1	4028.75	4424.12	90	10390.8	5483.6	6021.7
100	8482.3	4476.39	4915.68	100	11545.4	6092.8	6690.8
200	16964.6	8952.78	9821.36	200	23090.7	12185.7	13381.5

dreth parts of an inch or of an ounce in practice. And as they never appeared in print before, it may not be amiss to give the reader an account of the principles upon which they were constructed.

The solidity of cylinders are found by multiplying the areas of their bases by their altitudes. And ARCHIMEDES gives the following proportion

4 Inches diameter.				4½ Inches diameter.			
Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.	Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.
1	150.8	79.6	87.4	1	100.8	100.7	110.6
2	301.6	159.2	174.8	2	381.7	201.4	221.2
3	452.4	238.7	262.2	3	572.6	302.2	331.8
4	603.2	318.3	340.6	4	763.4	402.9	442.4
5	753.9	497.9	436.9	5	954.3	503.6	553.0
6	904.8	477.5	524.3	6	1145.1	604.3	663.6
7	1055.6	557.1	611.7	7	1337.9	705.0	774.2
8	1206.4	636.6	699.1	8	1526.8	805.7	884.8
9	1357.2	716.2	786.5	9	1717.7	906.5	995.4
10	1508.0	795.8	873.9	10	1908.5	1007.2	1106.0
20	3115.9	1591.6	1747.8	20	3817.0	2014.4	2212.1
30	4523.9	2387.4	2621.7	30	5725.6	3021.6	3318.1
40	6031.9	3183.2	3495.6	40	7634.1	4028.7	4424.1
50	7539.8	3997.0	4369.5	50	9542.6	5035.9	5530.1
60	9047.8	4774.8	5243.3	60	11451.1	6043.1	6636.2
70	10555.8	5570.6	6117.3	70	13359.6	7050.3	7742.2
80	12063.7	6366.4	6991.2	80	15268.2	8057.5	8848.2
90	13571.7	7162.2	7865.1	90	17176.7	9064.7	9954.3
100	15079.7	7958.0	8739.0	100	19085.2	10071.9	11060.3
200	30159.3	15916.0	17478.0	200	38170.4	20143.8	22120.6

tion for finding the area of a circle, and the solidity of a cylinder raised upon that circle.

As 1 is to 0.785339, so is the square of the diameter to the area of the circle. And, as 1 is to 0.785339, so is the square of the diameter multiplied by the height to the solidity of the cylinder. By this analogy the solid inches and parts of an

5 Inches diameter.				5½ Inches diameter.			
Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.	Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.
1	235.6	124.3	136.5	1	285.1	150.4	164.3
2	471.2	248.7	273.1	2	570.2	300.9	328.5
3	706.8	373.0	409.6	3	855.2	451.4	492.8
4	492.5	497.4	546.2	4	1140.4	601.8	657.1
5	1178.1	621.8	682.7	5	1425.5	752.3	821.3
6	1413.7	746.1	819.3	6	1710.6	902.7	985.6
7	1649.3	870.4	955.8	7	1995.7	1053.2	1149.9
8	1884.9	994.7	1092.4	8	2280.8	1203.6	1314.1
9	2120.6	1119.1	1228.9	9	2565.9	1354.1	1478.4
10	2356.2	1243.4	1365.5	10	2851.0	1504.6	1642.7
20	4712.4	2486.9	2730.9	20	5702.0	3009.1	3285.4
30	7068.6	3730.3	4096.4	30	8553.0	4513.7	4928.1
40	2424.8	4973.8	5461.9	40	11404.0	6018.2	6570.8
50	11781.0	6217.2	6827.3	50	14255.0	7522.9	8213.5
60	14137.2	7460.6	8192.8	60	17106.0	9027.4	9856.2
70	16493.4	8704.1	9558.3	70	19957.0	10531.9	11498.9
80	18849.6	9947.5	10923.7	80	22808.0	12036.5	13141.6
90	21205.8	11190.9	12289.2	90	25659.0	13541.1	14784.3
100	23561.9	12434.4	13654.7	100	28510.0	15045.6	16426.9
200	47123.9	24868.8	27309.3	200	57020.0	30091.2	32853.8

inch in the tables, are calculated to a cylinder 200 feet high, of any diameter from 1 inch to $6\frac{1}{2}$ inches; and may be continued at pleasure.

The weight
of running
water.

And as to the weight of a cubic foot of running water, it has been often found upon trial, by Dr. Wyberd and others, to be 76 pounds troy, which is equal to 62.5 pounds averdupoise. Therefore, since

6 Inches diameter.				6½ Inches diameter.			
Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.	Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.
1	339.3	179.0	196.6	1	398.2	210.1	230.7
2	678.6	358.1	393.3	2	797.4	420.3	461.4
3	1017.9	537.2	589.9	3	1195.6	630.4	692.1
4	1357.2	716.2	786.5	4	1593.8	840.6	922.8
5	1696.5	895.3	983.1	5	1991.9	1050.8	1153.6
6	2035.7	1074.3	1179.8	6	2390.1	1260.9	1384.3
7	2375.0	1253.4	1376.4	7	2788.3	1471.1	1615.0
8	2714.3	1432.4	1573.0	8	3186.5	1681.2	1845.7
9	3053.6	1611.5	1769.6	9	3584.7	1891.3	2076.4
10	3392.9	1790.6	1966.3	10	3982.9	2101.5	2307.1
20	6785.8	3581.1	3932.5	20	7965.8	4202.9	4614.3
30	10178.8	5371.7	5898.8	30	11948.8	6304.4	6921.4
40	13571.7	7162.2	7865.1	40	15931.7	8405.9	9228.6
50	16964.6	8952.8	9831.4	50	19914.6	10507.4	11535.7
60	20357.5	10743.3	11797.6	60	23897.6	12608.9	13842.9
70	23750.5	12533.9	13763.9	70	27880.5	14710.4	16150.0
80	27143.4	14324.4	15730.2	80	31863.4	16811.8	18457.2
90	30536.3	16115.0	17696.5	90	35846.3	18913.3	20764.3
100	33929.2	17905.6	19662.7	100	39829.3	21014.8	23071.5
200	67858.4	35811.2	39325.4	200	79658.6	42029.6	46143.0

since there are 1728 cubic inches in a cubic foot, a troy ounce of water contains 1.8949 cubic inch; and an averdupoise ounce of water 1.72556 cubic inch. Consequently, if the number of cubic inches contained in any given cylinder, be divided by 1.8949, it will give the weight in troy ounces; and divided by 1.72556, will give the weight in

averdupoise ounces. By this method, the weights shewn in the tables were calculated; and are near enough for any common practice.

The fire-engine.

The *fire-engine* comes next in order to be explained: but as it would be difficult, even by the best plates, to give a particular description of its several parts, so as to make the whole intelligible, I shall only explain the principles upon which it is constructed.

1. Whatever weight of water is to be raised, the pump-rod must be loaded with weights sufficient for that purpose, if it be done by a forcing-pump, as is generally the case: and the power of the engine must be sufficient for the weight of the rod, in order to bring it up.

2. It is known, that the atmosphere presses upon the surface of the earth with a force equal to 15 pounds upon every square inch.

3. When water is heated to a certain degree, the particles thereof repel one another, and constitute an elastic fluid, which is generally called *steam* or *vapour*.

4. Hot steam is very elastic; and when it is cooled by any means, particularly by its being mixed with cold water, its elasticity is destroyed immediately, and it is reduced to water again.

5. If a vessel be filled with hot steam, and then closed so, as to keep out the external air, and all other fluids; when that steam is by any means condensed, cooled, or reduced to water, *that* water will fall to the bottom of the vessel; and the cavity of the vessel will be almost a perfect vacuum.

6. Whenever

6. Whenever a vacuum is made in any vessel, the air by its weight will endeavour to rush into the vessel, or to drive in any other body that will give way to its pressure; as may be easily seen by a common syringe. For, if you stop the bottom of a syringe, and then draw up the piston, if it be so tight as to drive out all the air before it, and leave a vacuum within the syringe, the piston being let go, will be drove down with a great force.

7. The force with which the piston is drove down, when there is a vacuum under it, will be as the square of the diameter of the bore in the syringe. That is to say, it will be driven down with four times as much force in a syringe of a two inch bore, as in a syringe of one inch: for the areas of circles are always as the squares of their diameters.

8. The pressure of the atmosphere being equal to 15 pounds upon every square inch, it will be equal to about 12 pounds upon every circular inch. So that if the bore of the syringe be round, and one inch in diameter, the piston will be prest down into it by a force nearly equal to 12 pounds: but if the bore be 2 inches diameter, the piston will be prest down with 4 times that force.

And hence it is easy to find with what force the atmosphere presses upon any given number either of square or circular inches.

These being the principles upon which this engine is constructed, we shall next describe the chief working parts of it: which are, 1. A boiler. 2. A cylinder and piston. 3. A beam or lever.

The *boiler* is a large vessel, generally made of iron or copper; and commonly so big, as to contain about 2000 gallons.

The *cylinder* is made about 40 inches diameter, bored so smooth, and its piston fitting so close, that little or no water can get between the piston and sides of the cylinder.

Things being thus prepared, the cylinder is placed upright, and the shank of the piston is fixed to one end of the *beam*, which turns on a center like a common balance.

The boiler is placed under the cylinder, with a communication between them, which can be opened and shut occasionally.

The boiler is filled about half full of water, and a strong fire is made under it: then, if the communication between the boiler and the cylinder be opened, the cylinder will be filled with hot steam; which would drive the piston quite out at the top of it. But there is a contrivance by which the piston, when it is near the top of the cylinder, shuts the communication at the top of the boiler within.

This is no sooner shut, than another is opened, by which a little cold water is thrown upwards in a jet into the cylinder, which mixing with the hot steam, condenses it immediately; by which means a vacuum is made in the cylinder, and the piston is pressed down by the weight of the atmosphere; and so lifts up the loaded pump-rod at the other end of the beam.

If

If the cylinder be 42 inches in diameter, the piston will be pressed down with a force greater than 20000 pounds, and will consequently lift up that weight at the opposite end of the beam : and as the pump-rod with its plunger is fixed to that end, if the bore where the plunger works were 10 inches diameter, the water would be forced up through a pipe of 180 yards perpendicular height.

But, as the parts of this engine have a good deal of friction, and must work with a considerable velocity, and there is no such thing as making a perfect vacuum in the cylinder, it is found that no more than 8 pounds of pressure must be allowed for, on every circular inch of the piston in the cylinder, that it may make about 16 strokes in a minute, about 6 feet each.

Where the boiler is very large, the piston will make between 20 and 25 strokes in a minute, and each stroke 7 or 8 feet ; which, in a pump of 9 inches bore, will raise upwards of 300 hogsheads of water in an hour.

It is found by experience that a cylinder, 40 inches diameter, will work a pump 10 inches diameter, and 100 yards long : and hence we can find the diameter and length of a pump, that can be worked by any other cylinder.

For the conveniency of those who would make use of this engine for raising water, we shall subjoin part of a table calculated by Mr. *Beighton*, shewing how any given quantity of water may be raised in an hour, from 48 to 440 hogsheads ; at any given depth, from 15 to 100 yards ; the machine

chine working at the rate of 16 strokes *per* minute, and each stroke being 6 feet long.

One example of the use of this table, will make the whole plain. Suppose it were required to draw 150 hogsheads *per* hour, at 90 yards depth; in the second column from the right hand, I find the nearest number, viz. 149 hogsheads 40 gallons, against which, on the right hand, I find the diameter of the bore of the pump must be 7 inches; and in the same collateral line, under the given depth 90, I find 27 inches, the diameter of the cylinder fit for that purpose.—And so for any other.

A Table

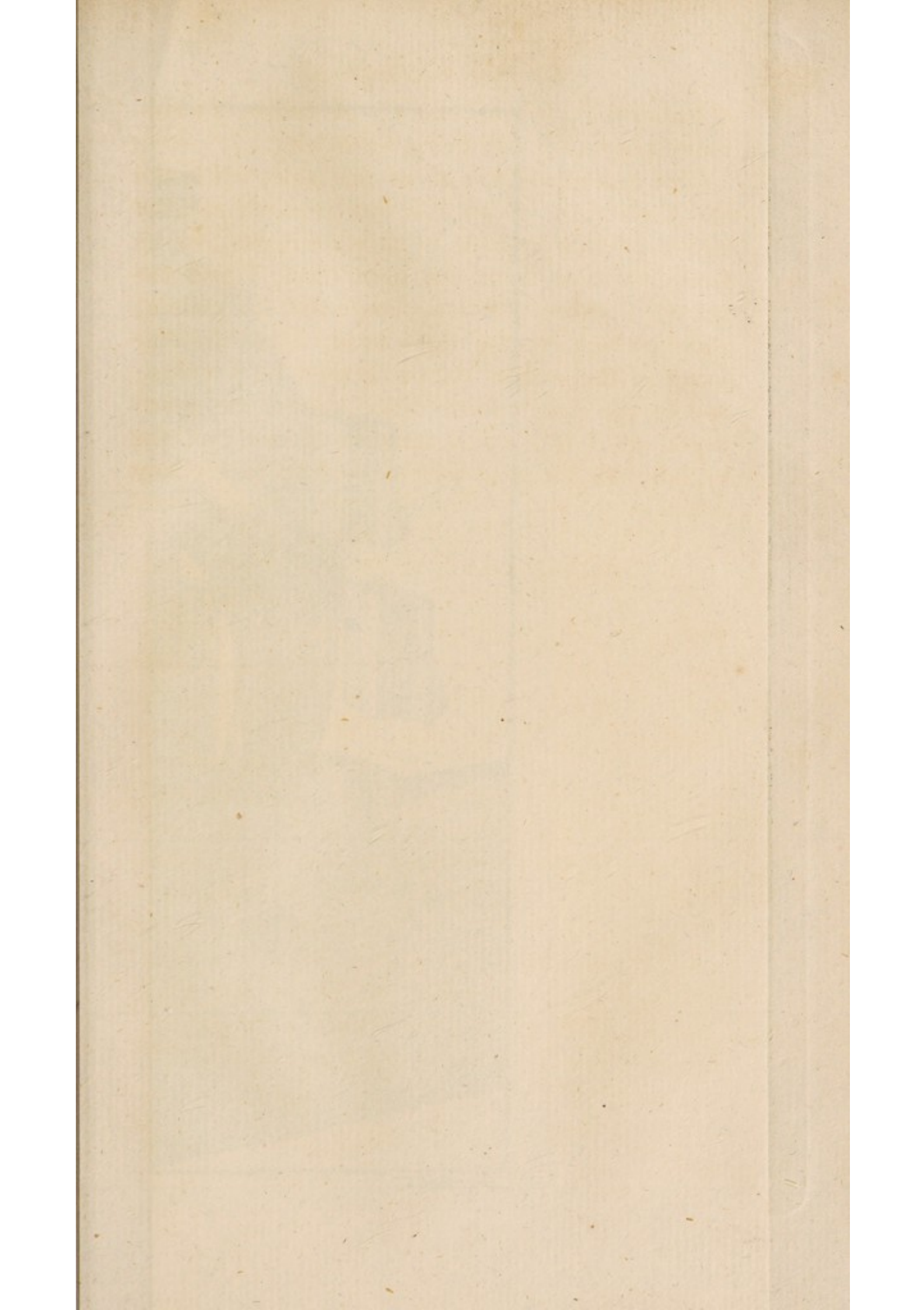
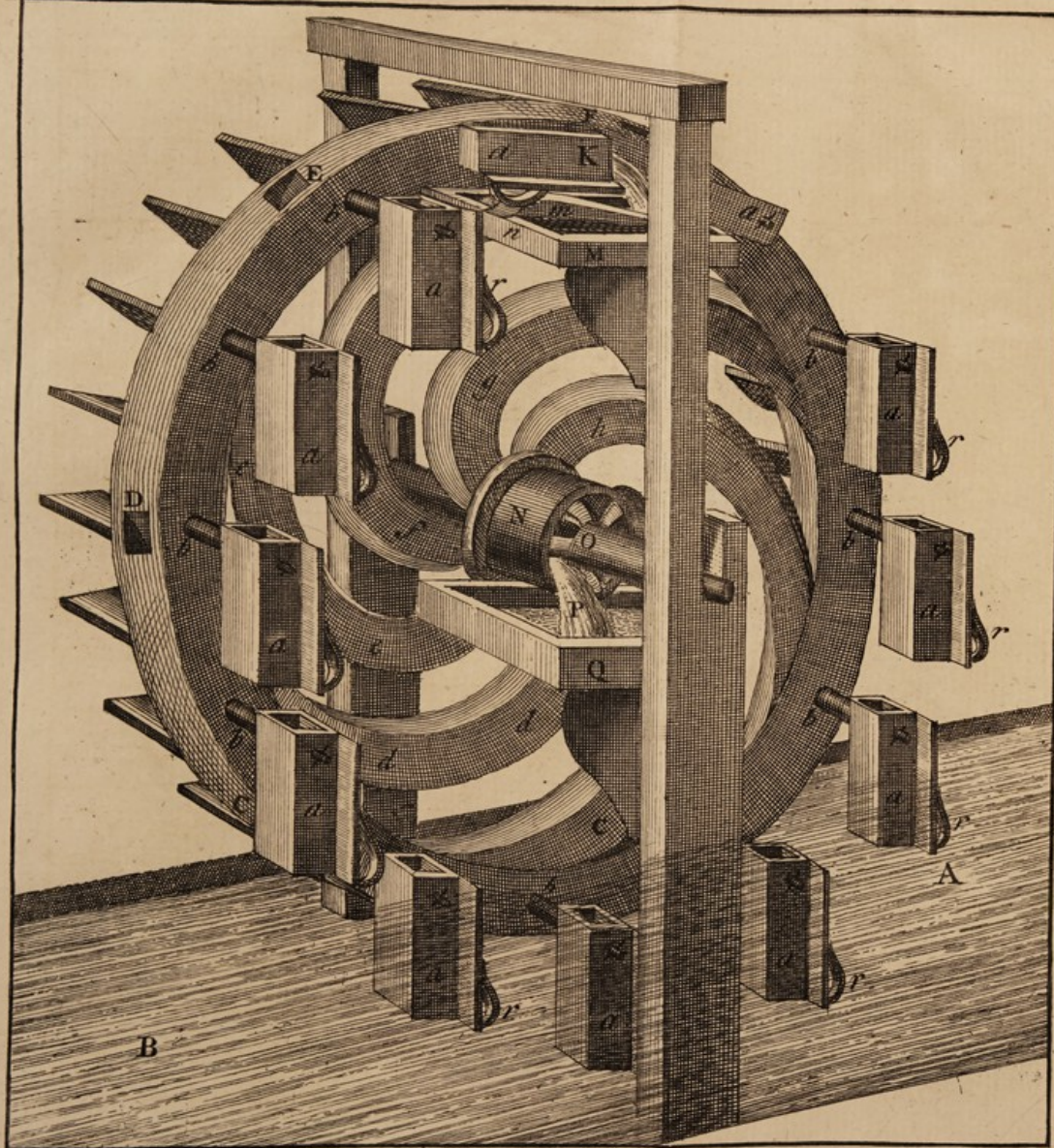


PLATE XIII.



J. Ferguson delin.

J. Mynde sc.

A Table shewing the power of the engine for raising water by fire.

This table is calculated to the measure of ale gallons, at 282 cubic inches *per* gallon.

The depth to be drawn in yards.															In one hour.	Diam. of pump.
															Hogsh.	Inches.
15	18	17	15	14	13	12	11	10	9	8	7	6	5	4	440	12
20	21	19	18	16	15	14	13	12	11	10	9	8	7	6	369	11
25	24	22	20	18	17	16	15	14	13	12	11	10	9	8	304	10
30	26	24	22	20	19	18	17	16	15	14	13	12	11	10	247	9
35	28	26	24	22	21	20	19	18	17	16	15	14	13	12	221	8½
40	30	28	25	23	22	21	20	19	18	17	16	15	14	13	195	8
45	32	29	27	24	23	22	21	20	19	18	17	16	15	14	182	7½
50	34	31	28	25	24	23	22	21	20	19	17	16	15	14	172	7
60	37	34	31	28	26	25	24	23	22	20	19	17	16	15	149	6½
70	40	37	33	30	28	27	26	25	24	22	20	19	18	17	128	6
80	39	36	33	31	29	28	27	25	24	22	20	19	18	17	110	5½
90	38	35	32	30	28	27	26	25	24	22	20	19	18	17	94	5
100	40	36	33	30	28	27	26	25	24	22	20	19	18	17	66	4½
															60	4
															48	
															51	

Diameter of the cylinder in inches.														
12	11	10	9	8½	8	7½	7	6½	6	5½	5	4½	4	3

Water

Plate XIII.
The *Persian*
wheel.

Water may be raised by means of a stream *AB* turning a wheel *CDE*, according to the order of the letters, with buckets *a, a, a, a*, &c. hung upon it by strong pins *b, b, b, b*, &c. fixed in the side of the rim: but the wheel must be made as high as the water is intended to be raised above the level of that part of the stream in which the wheel is placed. As the wheel turns, the buckets on the right hand go down into the water, and are thereby filled, and go up full on the left hand, until they come to the top at *K*; where they strike against the end *n* of the fixed trough *M*, and are thereby overset, and empty the water into the trough; from which it may be conveyed in pipes to the place which it is designed for: and as each bucket gets over the trough, it falls into a perpendicular position again, and goes down empty, until it comes to the water at *A*, where it is filled as before. On each bucket is a piece *r*, which going over the top or crown of the bar *m* (fixed to the trough *M*) raises the bottom of the bucket above the level of its mouth, and so causes it to empty all its water into the trough.

Sometimes this wheel is made to raise water no higher than its axis; and then, instead of buckets hung upon it, its spokes *c, d, e, f, g, b*, are made of a bent form, and hollow within; these hollows opening into the holes *C, D, E, F*, in the outside of the wheel, and also into those at *O* in the box *N* upon the axis. So that, as the holes *CD*, &c. dip into the water, it runs into them; and as the wheel turns, the water rises in the hollow spokes *c, d*, &c. and

and runs out in a stream *P* from the holes at *O*, and falls into the trough *Q*, from whence it is conveyed by pipes. And this is a very easy way of raising water, because the engine requires neither men nor horses to turn it.

The art of weighing different bodies in water, and thereby finding their specific gravities, or weights, bulk for bulk, was invented by ARCHIMEDES; of which, we have the following account.

Hiero, king of *Syracuse*, having employed a goldsmith to make a crown, and given him a mass of pure gold for that purpose, suspected that the workman had kept back part of the gold for his own use, and made up the weight by allaying the crown with copper. But the king not knowing how to find out the truth of that matter, referred it to *Archimedes*; who having studied a long time in vain, found it out at last by chance. For, going into a bathing tub of water, and observing that he thereby raised the water higher in the tub than it was before, he concluded instantly that he had raised it just as high as any thing else could have done, that was exactly of his bulk: and considering that any other body of equal weight, and of less bulk than himself, could not have raised the water so high as he did; he immediately told the king, that he had found a method by which he could discover whether there were any cheat in the crown. For, since gold is the heaviest of all known metals, it must be of less bulk, according to its weight, than any other metal. And therefore, he desired that a mass of pure gold, equally heavy with the crown

when

Of the specific gravities of bodies.

when weighed in air, should be weighed against it in water; and if the crown was not allayed, it would counterpoise the mass of gold when they were both immersed in water, as well as it did when they were weighed in air. But upon making the trial, he found that the mass of gold weighed much heavier in water than the crown did. And not only so, but that, when the mass and crown were immersed separately in one vessel of water, the crown raised the water much higher than the mass did; which shewed it to be allayed with some lighter metal that increased its bulk. And so, by making trials with different metals, all equally heavy with the crown when weighed in air, he found out the quantity of allay in the crown.

The specific gravities of bodies are as their weights, bulk for bulk; thus, a body is said to have two or three times the specific gravity of another, when it contains two or three times as much matter in the same space.

A body immersed in a fluid will sink to the bottom, if it be heavier than its bulk of the fluid. If it be suspended therein, it will lose as much of what it weighed in air, as its bulk of the fluid weighs. Hence, all bodies of equal bulk, which would sink in fluids, lose equal weights when suspended therein. And unequal bodies lose in proportion to their bulks.

The *hydrostatic balance*.

The *hydrostatic balance* differs very little from a common balance that is nicely made: only it has a hook at the bottom of each scale, on which small weights may be hung by horse-hairs, or by silk threads.

threads. So that a body, suspended by the hair or thread, may be immersed in water without wetting the scale from which it hangs.

If the body thus suspended under the scale, at one end of the balance, be first counterpoised in air by weights in the opposite scale, and then immersed in water, the equilibrium will be immediately destroyed. Then, if as much weight be put into the scale from which the body hangs, as will restore the equilibrium (without altering the weights in the opposite scale) that weight which restores the equilibrium, will be equal to the weight of a quantity of water as big as the immersed body. And if the weight of the body in air be divided by what it loses in water, the quotient will shew how much that body is heavier than its bulk of water. Thus, if a guinea suspended in air, be counterbalanced by 129 grains in the opposite scale of the balance; and then, upon its being immersed in water, it becomes so much lighter, as to require $7\frac{1}{4}$ grains put into the scale over it, to restore the equilibrium, it shews that a quantity of water, of equal bulk with the guinea, weighs $7\frac{1}{4}$ grains, or 7.25; by which divide 129, (the aerial weight of the guinea) and the quotient will be 17.793; which shews that the guinea is 17.793 times heavier than its bulk of water. And thus, any piece of gold may be tried, by weighing it first in air and then in water; and if upon dividing the weight in air by the loss in water, the quotient comes out to be 17.793, the gold is good; if the quotient be 18, or between 18 and 19, the gold

How to find
the specific
gravity of
any body.

gold is very fine; but if it be less than $17\frac{1}{2}$, the gold is too much allayed, by being mixed with some other metal.

If silver be tried in this manner, and found to be 11 times heavier than water, it is very fine; if it be $10\frac{1}{2}$ times heavier, it is standard; but if it be of any less weight compared with water, it is mixed with some lighter metal, such as tin.

By this method, the specific gravities of all bodies that will sink in water, may be found. But as to those which are lighter than water, as most sorts of wood are; the following method may be taken, to shew how much lighter they are than their respective bulks of water.

Let an upright stud be fixed into a thick flat piece of brass, and in this stud let a small lever, whose arms are equally long, turn upon a fine pin as an axis. Let the thread which hangs from the scale of the balance be tied to one end of the lever, and a thread from the body to be weighed, tied to the other end. This done, put the brass and lever into a vessel; then pour water into the vessel, and the body will rise and float upon it, and draw down the end of the balance from which it hangs: then, put as much weight in the opposite scale as will raise that end of the balance, so as to pull the body down into the water by means of the lever; and this weight in the scale will shew how much the body is lighter than its bulk of water.

There are some things which cannot be weighed in this manner, such as quicksilver, fragments of diamonds, &c. because they cannot be suspended

in threads; and must therefore be put into a glass bucket, hanging by a thread from the hook of one scale, and counterpoised by weights put into the opposite scale. Thus, suppose you want to know the specific gravity of quicksilver, with respect to that of water; let the empty bucket be first counterpoised in air, and then the quicksilver put into it and weighed. Write down the weight of the bucket, and also of the quicksilver; which done, empty the bucket, and let it be immersed in water as it hangs by the thread, and counterpoised therein by weights in the opposite scale: then, pour the quicksilver into the bucket in the water, which will cause it to preponderate; and, put as much weight into the scale as will restore the balance to an equipoise; and this weight will be the weight of a quantity of water equal in bulk to the quicksilver. Lastly, divide the weight of the quicksilver in air by the weight of its bulk of water, and the quotient will shew how much the quicksilver is heavier than its bulk of water.

If a piece of brass, glass, lead, or silver, be immersed and suspended in different sorts of fluids, its different losses of weight therein will shew how much it is heavier than its bulk of the fluid; *that* fluid being the lightest, in which the immersed body loses least of its aerial weight. A solid bubble of glass is generally used for finding the specific gravities of fluids.

Hence we have an easy method of finding the specific gravities both of solids and fluids, with regard to their respective bulks of common pump
M water,

Of the specific gravities of bodies.

water, which is generally made a standard for comparing all the others by.

In constructing tables of specific gravities with accuracy, the gravity of water must be represented by unity or 1.000, where three cyphers are added, to give room for expressing the ratios of other gravities, as in the following table.

A Table of the specific gravities of several solid and fluid bodies.

A cubic inch of	Troy weight.			Averdup.		Comparative weight.
	oz.	pw.	gr.	oz.	drams.	
Very fine gold — —	10	7	3,83	1	5,80	19,637
Standard gold — —	9	19	6,44	10	14,90	18,888
Guinea gold — — —	9	7	17,18	10	4,76	17,793
Moidore gold — —	9	0	19,84	9	14,71	17,140
Quicksilver — — —	7	7	11,61	8	1,45	14,019
Lead — — — — —	5	19	17,55	6	9,08	11,325
Fine silver — — —	5	16	23,23	6	6,66	11,087
Standard silver — —	5	11	3,36	6	1,54	10,535
Copper — — — — —	4	13	7,04	5	1,89	8,843
Plate brass — — —	4	4	9,60	4	10,09	8,000
Steel — — — — —	4	2	20,12	4	8,70	7,852
Iron — — — — —	4	0	15,20	4	6,77	7,645
Block tin — — — —	3	17	5,68	4	3,79	7,321
Speltar — — — — —	3	14	12,86	4	1,42	7,065
Lead ore — — — —	3	11	17,76	3	14,96	6,800
Glass of antimony —	2	15	16,89	3	0,89	5,280
German antimony — —	2	2	4,80	2	5,04	4,000
Copper ore — — — —	2	1	11,83	2	4,43	3,775
Diamond — — — — —	1	15	20,88	1	15,48	3,400
Clear glass — — — —	1	13	5,58	1	13,16	3,150
Lapis lazuli — — —	1	12	5,27	1	12,27	3,054
Welch asbestos — —	1	10	17,57	1	10,97	2,913

The

The Table concluded.

A cubic inch of	Troy weight.			Averdup.		Compa- rative weight.
	oz.	pw.	gr.	oz.	drams.	
White marble — — —	1	8	13,41	1	9,06	2,707
Black ditto — — —	1	8	12,65	1	9,02	2,704
Rock crystal — — —	1	8	1,00	1	8,61	2,658
Green glass — — —	1	7	15,58	1	8,26	2,620
Cornelian stone — — —	1	7	1,21	1	7,73	2,568
Flint — — —	1	6	19,63	1	7,53	2,542
Hard paving stone — — —	1	5	22,87	1	6,77	2,460
Live sulphur — — —	1	1	2,40	1	2,52	2,000
Nitre — — —	1	0	1,08	1	1,59	1,900
Alabaster — — —	0	19	18,74	1	1,35	1,875
Dry ivory — — —	0	19	6,09	1	0,89	1,825
Brimstone — — —	0	18	23,76	1	0,66	1,800
Allum — — —	0	17	21,92	0	15,72	1,714
Ebony — — —	0	11	18,82	0	10,34	1,117
Human blood — — —	0	11	2,89	0	9,76	1,054
Amber — — —	0	10	20,79	0	9,54	1,030
Cow's milk — — —	0	10	20,79	0	9,54	1,030
Sea water — — —	0	10	20,79	0	9,54	1,030
Pump water — — —	0	10	13,30	0	9,26	1,000
Spring water — — —	0	10	12,94	0	9,25	0,999
Distilled water — — —	0	10	11,42	0	9,20	0,993
Red wine — — —	0	10	11,42	0	9,20	0,993
Oil of amber — — —	0	10	7,63	0	9,06	0,978
Proof spirits — — —	0	9	19,73	0	8,62	0,931
Dry oak — — —	0	9	18,00	0	8,56	0,925
Olive oil — — —	0	9	15,17	0	8,45	0,913
Pure spirits — — —	0	9	3,27	0	8,02	0,866
Spirit of turpentine — — —	0	9	2,76	0	7,99	0,864
Oil of turpentine — — —	0	8	8,53	0	7,33	0,772
Dry crabtree — — —	0	8	1,69	0	7,08	0,765
Sassafras wood — — —	0	5	2,04	0	4,46	0,482
Cork — — —	0	2	12,77	0	2,21	0,240

Take away the comma from the numbers in the right hand column, or (which is the same) multiply

M 2

them

them by 1000, and they will shew how many ounces averdupoise are contained in a cubic foot of each body.

In troy weight, 24 grains make a pennyweight, 20 pennyweight make an ounce, and 12 ounces a pound. In averdupoise weight, 16 drams make an ounce, and 16 ounces a pound. The troy pound contains 5760 grains, and the averdupoise pound 7000; and hence, the averdupoise dram weighs 27.34375 grains, and the averdupoise ounce 437.5.

Because it is often of use to know how much any given quantity of goods in troy weight do make in averdupoise weight, and the reverse; we shall here annex two tables for converting these weights into one another. Those from page 144 to page 149 are near enough for common hydraulic purposes; but the two following are better where accuracy is required in comparing the weights with one another: and by trial I find, that 175 troy ounces are precisely equal to 192 averdupoise ounces, and 175 troy pounds are equal to 144 averdupoise. And although there are several lesser integral numbers, which come very near to agree together, yet I have found none less than the above to agree exactly. Indeed 41 troy ounces are so nearly equal to 45 averdupoise ounces, that the latter contains only $7\frac{1}{2}$ grains more than the former: and 45 troy pounds weigh only $7\frac{3}{10}$ drams more than 37 averdupoise.

A Table

A Table for reducing troy weight into averdupoise weight.

Troy weight.	Averdupoise.			Troy weight.	Aver. Drams.
	lb.	oz	drams.		
Pounds—4000	3291	6	13.68	Penny wt. 19	16.67
300	2528	9	2.26	18	15.79
2000	1645	11	6.84	17	14.92
1000	822	13	11.42	16	14.04
900	740	9	2.28	15	13.16
800	658	4	9.14	14	12.29
700	576	0	0.00	13	11.41
600	493	11	6.85	12	10.53
500	411	6	13.71	11	9.65
400	329	2	4.57	10	8.78
300	246	13	11.42	9	7.90
200	164	9	2.28	8	7.02
100	82	4	9.15	7	6.14
90	74	0	13.62	6	5.27
80	65	13	4.11	5	4.39
70	57	9	9.60	4	3.51
60	49	5	15.08	3	2.63
50	41	2	4.57	2	1.75
40	32	14	10.05	1	0.88
30	24	10	15.54	Grains—23	.84
20	16	7	5.03	22	.80
10	8	3	10.52	21	.77
9	7	6	7.86	20	.73
8	6	9	5.21	19	.69
7	5	12	2.56	18	.66
6	4	14	15.90	17	.62
5	4	1	13.25	16	.58
4	3	4	10.60	15	.55
3	2	7	7.95	14	.51
2	1	10	5.30	13	.47
1	0	13	2.65	12	.44
Ounces—11		12	1.09	11	.40
10		10	15.54	10	.36
9		9	13.99	9	.33
8		8	12.43	8	.29
7		7	10.88	7	.26
6		6	9.32	6	.22
5		5	7.77	5	.18
4		4	6.22	4	.15
3		3	4.66	3	.11
2		2	3.11	2	.07
1		1	1.55	1	.04

A Table for reducing averdupoise weight into troy weight.

Averdupoise weight.	Troy weight.				Averd. weight.	Troy weight.			
	lb.	oz.	pw.	gr.		lb.	oz.	pw.	gr.
Pounds 6000	7291	8	0	0	Ounces 15	1	1	13	10.50
5000	6076	4	13	8	14	1	0	15	5
4000	4861	1	6	16	13	11	16	23.50	
3000	3645	10	0	0	12	10	18	18	
2000	2430	6	13	8	11	10	0	12.50	
1000	1215	3	6	16	10	9	2	7	
900	1093	9	0	0	9	8	4	1.50	
800	972	2	13	8	8	7	5	20	
700	850	8	6	16	7	6	7	14.50	
600	729	2	0	0	6	5	9	9	
500	607	7	13	8	5	4	11	3.50	
400	486	1	6	16	4	3	12	22	
300	364	7	0	0	3	2	14	16.50	
200	243	0	13	8	2	1	16	11	
100	121	6	6	16	1	0	18	5.50	
90	109	4	10	0	Drams 15		17	2.10	
80	97	2	13	8	14		15	22.76	
70	85	0	16	16	13		14	19.42	
60	72	11	0	0	12		13	15.08	
50	60	9	3	8	11		12	12.74	
40	48	7	6	16	10		11	9.40	
30	36	5	10	0	9		10	6.06	
20	24	3	13	8	8		9	2.72	
10	12	1	16	16	7		7	23.38	
9	10	11	5	0	6		6	20.04	
8	9	8	13	8	5		5	16.70	
7	8	6	1	16	4		4	13.36	
6	7	3	10	0	3		3	10.02	
5	6	0	18	8	2		2	6.68	
4	4	10	6	16	1		1	3.34	
3	3	7	15	0	3/4		0	20.51	
2	2	5	3	8	1/2			13.67	
1	1	2	11	16	1/4			6.83	

The two following examples will be sufficient to explain these two tables, and shew their agreement.

Ex. I. In 6835 pounds 6 ounces 9 pennyweights 6 grains troy, *Qu.* How much averdupoise weight? (See page 165.)

		Averdupoise.		
		lb.	oz.	drams.
Pounds troy—	{ 4000	3291	6	13.68
	{ 2000	1645	11	6.84
	{ 800	658	4	9.14
	{ 20	16	7	5.02
	{ 10	8	3	10.51
	{ 5	4	1	13.25
	oz. 6		6	9.32
	pw. 9			7.90
	gr. 6			.22
Answer.		5624	10	11.88

Ex. II. In 5624 pounds 10 ounces 12 drams averdupoise, *Qu.* How much troy weight? (See page 166)

		Troy.			
		lb.	oz.	pw.	gr.
Pounds averd.	{ 5000	6076	4	13	8
	{ 600	729	2	0	0
	{ 20	24	3	13	8
	{ 4	4	10	6	16
	oz. 10		9	2	7
	dr. 12			13	15.08
Answer.		6835	6	9	6.08

How to
find out the
quantity of
adulterati-
on in me-
tals.

The use of the table of specific gravities will best appear by an example. Suppose a body to be compounded of gold and silver, and it is required to find the quantity of each metal in the compound.

First find the specific gravity of the compound, by weighing it in air and in water, and dividing its aerial weight by what it loses thereof in water, the quotient will shew its specific gravity, or how many times it is heavier than its bulk of water. Then, subtract the specific gravity of silver (found in the table) from that of the compound, and the specific gravity of the compound from that of gold; the first remainder shews the bulk of gold, and the latter the bulk of silver, in the whole compound: and if these remainders be multiplied by the respective specific gravities, the products will shew the proportion of weights of each metal in the body. Example.

Suppose the specific gravity of the compounded body be 13; that of standard silver (by the table) is 10.5, and that of gold 19.63: therefore 10.5 from 13, remains 2.5, the proportional bulk of the gold; and 13 from 19.63, remains 6.63 the proportional bulk of silver in the compound. Then, the first remainder 2.5, multiplied by 19.63 the specific gravity of gold, produces 49.075 for the proportional weight of gold; and the last remainder 6.63 multiplied by 10.5, the specific gravity of silver, produces 69.615 for the proportional weight of silver in the whole body. So that, for every

every 49.07 ounces or pounds of gold, there are 69.6 pounds or ounces of silver in the body.

Hence it is easy to know whether any suspected metal be genuine, or allayed, or counterfeit; by finding how much it is heavier than its bulk of water, and comparing the same with the table: if they agree, the metal is good; if they differ, it is allayed or counterfeited.

A cubical inch of good brandy, rum, or other proof spirits, weighs 235.7 grains; therefore, if a true inch cube of any metal weighs 235.7 grains less in spirits than in air, it shews the spirits are proof. If it loses less of its aerial weight in spirits, they are above proof; if it loses more, they are under. For, the better the spirits are, they are the lighter, and the worse, the heavier. All bodies expand with heat and contract with cold, but some more and some less than others. And therefore the specific gravities of bodies are not precisely the same in summer as in winter. It has been found, that a cubic inch of good brandy is 10 grains heavier in winter than in summer; as much spirit of nitre, 20 grains; vinegar 6 grains, and spring water 3. Hence it is most profitable to buy spirits in winter, and sell them in summer, since they are always bought and sold by measure. It has been found, that 32 gallons of spirits in winter will make 33 in summer.

The expansion of all fluids is proportionable to the degree of heat; that is, with a double or triple heat a fluid will expand two or three times as much.

How to try
if spirituous
liquors.

The thermometer.

Upon

Upon these principles depends the construction of the thermometer, in which the globe or bulb, and part of the tube, are filled with a fluid, which, when joined to the barometer, is spirits of wine tinged, that it may be the more easily seen in the tube. But when thermometers are made by themselves, quicksilver is generally used.

In the thermometer, a scale is always fitted to the tube, to shew the expansion of the quicksilver, and consequently the degree of heat. And, as *Fahrenheit's* scale is most in esteem at present, I shall explain the construction and graduation of thermometers according to that scale.

First, let the globe or bulb, and part of the tube, be filled with a fluid; then immerse the bulb in water just freezing, or snow just thawing; and even with that part in the scale where the fluid then stands in the tube, place the number 32, to denote the freezing point: then put the bulb under your arm-pit, when your body is of a moderate degree of heat, so that it may acquire the same degree of heat with your skin; and when the fluid has risen as far as it can by that heat, there place the number 97: then divide the space between these numbers into 65 equal parts, and continue those divisions both above 97 and below 32, and number them accordingly.

This may be done in any part of the world; for it is found that the freezing point is always the same in all places, and the heat of the human body differs but very little: so that the thermometers made in this manner will perfectly agree with one another;

another; and the heat of several bodies will be shewn by them, and expressed by the number upon the scale, thus.

Air, in severe cold weather, in our climate, from 15 to 25. Air in winter, from 26 to 42. Air in spring and autumn, from 43 to 53. Air at midsummer, from 65 to 68. Extreme heat of the summer sun, from 86 to 100. Butter just melting, 95. Alcohol boils with 174 or 175. Brandy with 190. Water 212. Oil of turpentine 550. Tin melts with 408, and lead with 540. Milk freezes about 30, vinegar 28, and blood 27.

A body specifically lighter than a fluid will swim upon its surface, in such a manner, that a quantity of the fluid equal in bulk with the immersed part of the body, will be as heavy as the whole body. Hence, the lighter a fluid is, the deeper a body will sink in it; upon which depends the construction of the *hydrometer* or water-poise.

From this we can easily find the weight of a ship, or any other body that swims in water. For, if we multiply the number of cubic feet which are under the surface, by 62.5, the number of pounds in one foot of fresh water; or by 63, the number of pounds in a foot of salt water; the product will be the weight of the ship and all that is in it. For, since it is the weight of the ship that displaces the water, it must continue to sink until it has removed as much water as is equal to it in weight: and therefore the part immersed must be equal in bulk

How the weight of a ship in water may be estimated.

bulk to such a portion of the water as is equal to the weight of the whole ship.

To prove this by experiment, let a ball of some light wood, such as fir or peartree, be put into water contained in a glass vessel; and let the vessel be put into a scale at one end of a balance, and counterpoised by weights in the opposite scale: then, marking the height of the water in the vessel, take out the ball; and fill up the vessel with water to the same height that it stood at when the ball was in it; and the same weight will counterpoise it as before.

From the vessel's being filled up to the same height at which the water stood when the ball was in it, it is evident that the quantity poured in is equal in magnitude to the immersed part of the ball; and from the same weight counterpoising, it is plain that the water poured in is equal in weight to the whole ball.

LECT. VI.

Of pneumatics.

THIS science treats of the nature, weight, and pressure of the air, and the effects arising from it.

The properties of air.

The air is that thin transparent fluid body in which we live and breathe. It encompasses the whole earth to a considerable height; and, together with the clouds and vapours that float in it,
is

is called the atmosphere. The air is justly reckoned among the number of fluids, because it has all the properties by which a fluid is distinguished. For, it yields to the least force impressed, its parts are easily moved among one another, it presses according to its perpendicular height, and its pressure is every way equal.

That the air is a fluid, consisting of such particles as have no cohesion betwixt them, but easily glide over one another, and yield to the slightest impression, appears from that ease and freedom with which animals breathe in it, and move through it without any difficulty or sensible resistance.

But it differs from all other fluids in the three following particulars. 1. It can be compressed into a less space than what it naturally possesseth, which no other fluid can. 2. It cannot be congealed or fixed, as other fluids may. 3. It is of a different density in every part, upward from the earth's surface, decreasing in its weight, bulk for bulk, the higher it rises; and therefore must also decrease in density. 4. It is of an elastic or springy nature, and the force of its spring is equal to its weight.

That air is a body, is evident from its excluding all other bodies out of the space it possesses: for, if a glass jar be plunged with its mouth downward into a vessel of water, there will but very little water get into the jar, because the air of which it is full keeps the water out.

As air is a body, it must needs have gravity or weight: and that it is weighty, is demonstrated by experiment. For, let the air be taken out of
a vessel

a vessel by means of the air pump, then, having weighed the vessel, let in the air again, and upon weighing it when re-filled with air, it will be found considerably heavier. Thus, a bottle that holds a wine quart, being emptied of air and weighed, is found to be about 17 grains lighter than when the air is let into it again; which shews that a quart of air weighs 17 grains. But a quart of water weighs 14625 grains, this divided by 17 quotes 860 in round numbers; which shews, that water is 860 times heavier than air near the surface of the earth.

As the air rises above the earth's surface, it grows rarer, and consequently lighter, bulk for bulk. For since it is of an elastic or springy nature, and its lowermost parts are pressed with the weight of all that is above them, it is plain that the air must be more dense or compact at the earth's surface, than at any height above it; and gradually rarer the higher up. For, the density of the air will always be as the force that compresses it: and therefore, the air towards the upper part of the atmosphere being less pressed than that which is near the earth, it will expand itself, and thereby become thinner than at the surface of the earth.

Dr. Cotes has demonstrated, that if altitudes in the air be taken in arithmetical proportion, the rarity of the air will be in geometrical proportion. For instance,

At

At the altitude of	7	Miles above the surface of the earth, the air is	- - - - - 4	times thinner and lighter than at the surface.
	14		- - - - - 16	
	21		- - - - - 64	
	28		- - - - - 256	
	35		- - - - - 1024	
	42		- - - - - 4096	
	49		- - - - - 16384	
	56		- - - - - 65536	
	63		- - - - - 262144	
	70		- - - - - 1048576	
	77		- - - - - 4194304	
	84		- - - - - 16777216	
	91		- - - - - 67108864	
	98		- - - - - 268435456	
	105		- - - - - 1073741824	
	112		- - - - - 4294967296	
	119		- - - - - 17179869184	
	126		- - - - - 68719476736	
	133		- - - - - 274877906944	
	140		- - - - - 1099511627776	

And hence it is easy to calculate, that a cubic inch of such air as we breathe, would be so much rarefied at the altitude of 500 miles, that it would fill a sphere equal in diameter to the orbit of Saturn.

The weight or pressure of the air is exactly determined by the following experiment.

Take a glass tube about three feet long, and open at one end; fill it with quicksilver, and putting your finger upon the open end, turn that end downward, and immerse it into a small vessel of quicksilver, without letting in any air: then take away

The Torricellian experiment.

away your finger, and the quicksilver will remain suspended in the tube $29\frac{1}{2}$ inches above its surface in the vessel; sometimes more, and at other times less, as the weight of the air is varied by winds and other causes. That the quicksilver is kept up in the tube by the pressure of the atmosphere upon that in the basin, is evident; for, if the basin and tube be put under a glass, and the air taken out of the glass, all the quicksilver in the tube falls down into the basin; and when the air is let in again, the quicksilver rises to the same height as before. Therefore the air's pressure on the surface of the earth, is equal to the weight of $29\frac{1}{2}$ inches depth of quicksilver all over the earth's surface, at a mean rate.

A square column of quicksilver, $29\frac{1}{2}$ inches high, and one inch thick, weighs just 15 pounds, which is equal to the pressure of air upon every square inch of the earth's surface; and 144 times as much, or 2160 pounds, upon every square foot; because a square foot contains 144 square inches. At this rate, a middle siz'd man, whose surface may be about 14 square feet, sustains a pressure of 30240 pounds, or $13\frac{1}{2}$ tons, when the air is of a mean gravity: a pressure which would be insupportable, and even fatal to us, were it not equal on every part, and counterbalanced by the spring of the air within us, which is diffused through the whole body; and re-acts with an equal force against the outward pressure.

Now, since the earth's surface contains (in round numbers) 200,000,000 square miles, and every
square

square mile 27878400 square feet, there must be 5575680000000000 square feet on the earth's surface; which multiplied by 2160 pounds (the pressure on each square foot) gives 1204346880000000000 pounds for the pressure or weight of the whole atmosphere.

When the air is taken out of a pipe, and the end of the pipe immersed in water, the water will rise in it to the height of 33 feet above the surface of the water in which it is immersed; but will go no higher: for it is found, that a common pump will draw water no higher than 33 feet above the surface of the well: and unless the bucket goes within that distance from the well, the water will never get above it. Now, as it is the pressure of the atmosphere, on the surface of the water in the well, that causes the water to ascend in the pump, and follow the piston or bucket, as all the air above it is lifted up; it is evident, that a column of water 33 feet high, is equal in weight to a column of quicksilver of the same diameter, $29\frac{1}{2}$ inches high; and to as thick a column of air, reaching from the earth's surface to the top of the atmosphere.

In serene calm weather, the air has weight enough to support a column of quicksilver 31 inches high; but in tempestuous stormy weather, not above 28 inches. The quicksilver, thus supported in a glass tube, is found to be a nice counterbalance to the weight or pressure of the air, and to shew its alterations at different times. And being now generally used to denote the changes in the weight of

The barometer.

N

the

the air, and of the weather consequent upon them, it is called the *barometer*, or weather glass.

The pressure of the air being equal on all sides of a body exposed to it, the softest bodies sustain this pressure without suffering any change in their figure; and so do the most brittle bodies without being broke.

The cause
of winds.

The air is rarefied, or made to swell with heat; and of this property, *wind* is a necessary consequence. For, when any part of the air is heated by the sun, or otherwise, it will swell, and thereby affect the adjacent air: and so, by various degrees of heat in different places, there will arise various winds.

When the air is much heated, it will ascend towards the upper part of the atmosphere, and the adjacent air will rush in to supply its place; and therefore, there will be a stream or current of air from all parts towards the place where the heat is. And hence, we see the reason why the air rushes with such force into a glass house; or towards any place where a great fire is made. And also, why smoke is carried up a chimney, and why the air rushes in at the key-hole of the door, or any small chink, when there is a fire in the room. So we may take it in general, that the air will press towards that part of the world where it is most heated.

The trade-
winds.

Upon this principle, we can easily account for the *trade-winds*, which blow constantly from east to west about the equator. For, when the sun shines perpendicularly on any part of the earth, it will

will heat the air very much in that part, which air will therefore rise upward, and when the sun withdraws, the adjacent air will rush in to fill its place; and consequently cause a stream or current of air from all parts towards that which is most heated by the sun. But, as the sun, with respect to the earth, moves from east to west, the common course of the air will be that way too; continually pressing after the sun: and therefore, at the equator, where the sun shines strongly, there will be a continual easterly wind; but, on the north-side, it will incline a little to the north, and on the south-side, to the south.

This general course of the wind about the equator is changed in several places, and upon several accounts; as, 1. By exhalations that rise out of the earth at certain times, and from certain places; in earthquakes, and from volcano's. 2. By the falling of great quantities of rain, causing thereby a sudden condensation or contraction of the air. 3. By burning sands, that often retain the solar heat to a degree incredible to those who have not felt it, causing a more than ordinary rarefaction of the air contiguous to them. 4. By high mountains, that alter the direction of the winds in striking against them. 5. By the declination of the sun towards the north or south, heating the air on the north or south side of the equator.

To these and such like causes is owing, 1. The ^{The mon-}irregularity and uncertainty of winds in climates ^{soons.} distant from the equator, as in most parts of *Europe*. 2. Those periodical winds called *monsoons*,
N 2 which

which in the *Indian* seas blow half a year one way, and the other half another. 3. Those winds which on the coast of *Guinea*, and on the western coasts of *America*, blow always from west to east. 4. The sea-breezes, which, in hot countries, blow generally from sea to land, in the day-time; and the land-breeze, which blows in the night; and, in short, all those storms, hurricanes, whirlwinds, and irregularities, which happen at different times and places.

The *vivifying spirit*
in air.

All common air is impregnated with a *vivifying spirit*, which is necessary to continue the lives of animals: and this, in a gallon of air, is sufficient for one man during the space of a minute, and not much longer.

This spirit in air is destroyed by passing through the lungs of animals: and hence it is, that an animal dies soon, after being put under a vessel which admits no fresh air to come to it. This spirit is also infused into water; for fish die when they are excluded from fresh air, as in a pond that is closely frozen over. And the little eggs of insects, stopped up in a glass, do not produce their young, though assisted by a kindly warmth. The seeds also of plants mixed with good earth, and inclosed in a glass, will not grow. So that fresh air is absolutely necessary, both for the production and continuation of the lives of animals and plants.

This enlivening quality in air is also destroyed by the air's passing through fire; particularly charcoal fire, or the flame of sulphur. Hence, smoaking chimneys must be very unwholesome, especially if the rooms they are in be small and close.

Air

Air is also vitiated, by remaining closely pent up in any place for a considerable time; or perhaps, by being mixed with malignant steams and particles flowing from the neighbouring bodies; or lastly, by the corruption of the vivifying spirit; as in the holds of ships, in oil-cisterns, or wine cellars which have been shut up for a considerable time. The air in any of them is sometimes so much vitiated, as to be immediate death to any animal that comes into it.

Air that has lost its vivifying spirit is called *Damps*, not only because it is filled with humid or moist vapours, but because it deadens fire, extinguishes flame, and destroys life.—The dreadful effects of damps are sufficiently known to such as work in mines.

If part of the vivifying spirit of air in any country begins to putrefy, the inhabitants of that country will be subject to an epidemical disease, which will continue until the putrefaction is over. And as the putrefying spirit occasions the disease, so if the diseased body contributes towards the putrefying of the air, then the disease will not only be epidemical, but pestilential and contagious.

The atmosphere is the common receptacle of all the effluvia or vapours arising from different bodies; of the steams and smoke of things burnt or melted; the fogs or vapours proceeding from damp watery places; and of the effluvia from sulphureous, nitrous, acid and alkaline bodies. In short, whatever may be called volatile, rises in the air to greater or less heights, according to its specific gravity.

*Fermenta-
tions.*

When the effluvia, which arise from acid and alkaline bodies, meet each other in the air, there will be a strong conflict or *fermentation* between them; which will sometimes be so great, as to produce a fire: then, if the effluvia be combustible, the fire will run from one part to another, just as the inflammable matter happens to lie.

Any one may be convinced of this, by mixing an acid and an alkaline fluid together, as the spirit of nitre and oil of cloves; upon the doing of which, a sudden ferment, with a fine flame, will arise; and if the ingredients be very pure and strong, there will be a sudden explosion.

*Thunder
and light-
ning.*

Whoever considers the effects of fermentation, cannot be at a loss to account for the dreadful effects of *thunder* and *lightning*: for the effluvia of sulphureous and nitrous bodies, and others that may rise into the atmosphere, will ferment with each other, and take fire very often of themselves; sometimes by the assistance of the sun's heat.

If the inflammable matter be thin and light, it will rise to the upper part of the atmosphere, where it will flash without doing any harm: but if it be dense, it will lie nearer the surface of the earth, where taking fire, it will explode with a surprising force; and by its heat rarefy and drive away the air, kill men and cattle, split trees, walls, rocks, &c. and be accompanied with terrible claps of thunder.

The heat of lightning appears to be quite different from that of other fires; for it has been known to run through wood, leather, cloth, &c. without

without hurting them; while it has broken and melted iron, steel, silver, gold, and other hard bodies. Thus it has melted or burnt asunder a sword without hurting the scabbard, and money in a man's pocket without hurting his cloaths: the reason of this seems to be, that the particles of the fire are so fine, as to pass through soft loose bodies without dissolving them; whilst they spend their whole force upon the hard ones.

It is remarkable, that knives and forks which have been struck with lightning have a very strong magnetical virtue for several years after; and I have heard that lightning striking upon the mariner's compass will sometimes turn it round; and often make it stand the contrary way; or with the north pole towards the south.

Much of the same kind with lightning are those *Fire damps*, explosions called *fulminating* or *fire damps*, which sometimes happen in mines; and are occasioned by sulphureous and nitrous, or rather oleaginous particles, rising from the mine and mixing with the air, where they will take fire by the lights which the workmen are obliged to make use of. The fire being kindled, will run from one part of the mine to another, like a train of gunpowder, as the combustible matter happens to lie. And as the elasticity of the air is increased by heat, that in the mine will consequently swell very much, and so, for want of room, will explode with a greater or less degree of force, according to the density of the combustible vapours. It is sometimes so strong, as to blow up the mine; and at other times so

N 4

weak,

weak, that when it has taken fire at the flame of a candle, it is easily blown out.

Air that will take fire at the flame of a candle, may be produced thus. Having exhausted a receiver of the air-pump, let the air run into it through the flame of the oil of turpentine; then remove the cover of the receiver, and holding a candle to that air, it will take fire, and burn quicker or slower, according to the density of the oleaginous vapour.

*Earth-
quakes.*

When such combustible matter, as is above mentioned, kindles in the bowels of the earth, where there is little or no vent, it produces *earthquakes*, and violent storms or hurricanes of wind, when it breaks forth into the air.

An artificial earthquake may be made thus. Take 10 or 15 pounds of sulphur, and as much of the filings of iron, and knead them with common water into the consistence of a paste: this being buried in the ground, will, in 8 or 10 hours time, burst out in flames, and cause the earth to tremble all around to a considerable distance.

From this experiment we have a very natural account of the fire of mount *Ætna*, *Vesuvius*, and other volcano's, they being probably set on fire at first by the mixture of such metaline and sulphureous particles.

Plate XIV.
Fig. 1.

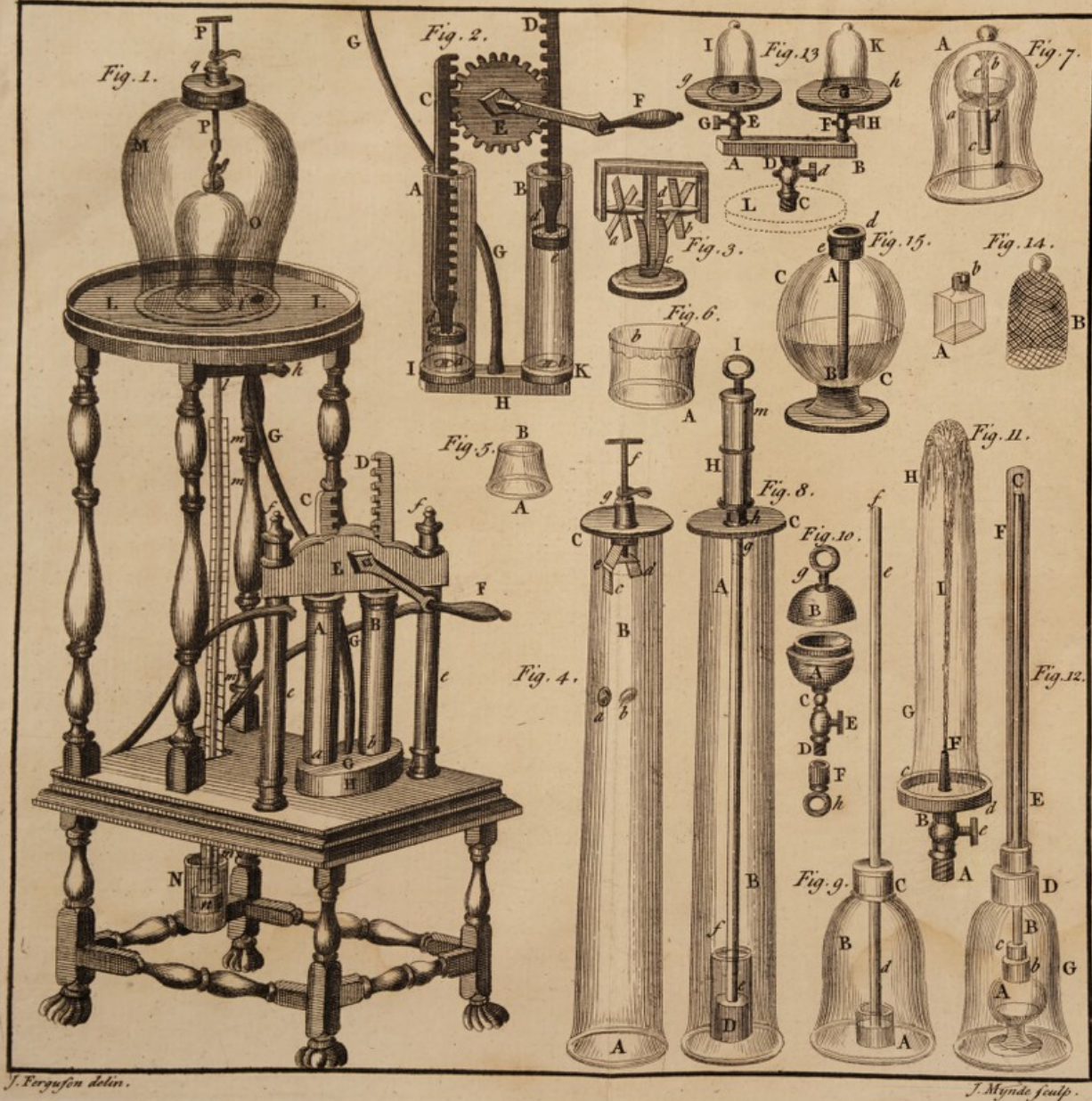
The *air-pump* being in effect the same as the water-pump, whoever understands the one, will be at no loss to understand the other.

The *air-
pump.*

Having put a wet leather on the plate *LL* of the air-pump, place the glass receiver *M* upon the leather,



PLATE XIV.



J. Ferguson delin.

J. Mynde sculp.

leather, so that the hole *i* in the plate may be within the glass. Then, turning the handle *F* backward and forward, the air will be pumped out of the receiver; which will then be held down to the plate by the pressure of the external air, or atmosphere. For, as the handle (Fig. 2.) is turned backwards, it rises the piston *de* in the barrel *BK*, by means of the wheel *F* and rack *Dd*: and as the piston is leathered so tight, as to fit the barrel exactly, no air can get between the piston and barrel; and therefore, all the air above *d* in the barrel is lifted up towards *B*, and a vacuum made in the barrel from *e* to *b*: upon which, part of the air in the receiver *M* (Fig. 1.), by its spring, rushes through the hole *i*, in the brass plate *LL*, along the pipe *GG* (which communicates with both barrels by the hollow trunk *IHK*, Fig. 2.), and pushing up the valve *b*, enters into the vacant place *be* of the barrel *BK*. For, wherever the resistance or pressure is taken off, the air will run to that place, if it can find a passage.—Then, as the handle *F* is turned forward, the piston *de* is depressed in the barrel; and, as the air which had got into the barrel cannot be pushed back through the valve *b*, it ascends through a hole in the piston, and escapes through a valve at *d*; and is hindered by that valve from returning into the barrel, when the piston is again raised. At the next raising of the piston, a vacuum is again made, in the same manner as before, between *b* and *e*; upon which, more of the air which was left in the receiver *M* gets out thence by its spring, and runs into the barrel *BK*, through
the

the valve *b*. The same thing is to be understood with regard to the other barrel *AI*; and as the handle *F* is turned backwards and forwards, it alternately raises and depresses the pistons in their barrels; always raising one whilst it depresses the other. And, as there is a vacuum made in each barrel when its piston is raised, every particle of air in the receiver *M* pushes out another, by its spring or elasticity, through the hole *i* and pipe *GG* into the barrels; until at last the air in the receiver comes to be so much dilated, and its spring so far weakened, that it can no longer get through the valves; and then, no more can be taken out. Hence, there is no such thing as making a perfect vacuum in the receiver; for the quantity of air taken out at any one stroke, will always be as the density thereof in the receiver: and therefore it is impossible to take it all out, because, supposing the receiver and barrels of equal capacity, there will be always as much left as was taken out at the last turn of the handle.

There is a cock *k* below the pump-plate, which being turned, lets the air into the receiver again; and then it becomes loose, and may be taken off the plate. The barrels are fixed into the frame *ee* by two screw-nuts *ff*, which press down the top-piece *E* upon the barrels: and the hollow trunk *H* (in Fig. 2.) is covered by a box, as *GH* in Fig. 1.

There is a glass tube *lmn* open at both ends, and about 34 inches long; the upper end communicating with the hole in the pump-plate, and the lower end immersed in quicksilver at *n* in the vessel

vessel *N*. To this tube is fitted a wooden ruler *mm*, called the *gage*, which is divided into inches and parts of an inch, from the bottom at *n* (where it is even with the surface of the quicksilver) and continued up to the top, a little below *l*, to 30 or 31 inches.

As the air is pumped out of the receiver *M*, it is likewise pumped out of the glass tube *lmn*, because that tube opens into the receiver through the pump-plate : and as the tube is gradually emptied of air, the quicksilver in the vessel *N* is forced up into the tube by the pressure of the atmosphere. And if the receiver could be perfectly exhausted of air, the quicksilver would stand as high in the tube, as it does at that time in the barometer : for it is supported by the same power or weight of the atmosphere in both.

The quantity of air exhausted out of the receiver on each turn of the handle, is always proportionable to the ascent of the quicksilver on that turn : and the quantity of air remaining in the receiver, is proportionable to the defect of the height of the quicksilver in the gage, from what it is at that time in the barometer.

I shall now give an account of the experiments made with the air-pump, in my lectures ; shewing the resistance, weight, and elasticity of the air.

1. To shew the resistance of the air.

1. There is a little machine, consisting of two Fig. 3.
mills, *a* and *b*, which are of equal weights, independent

pendent of each other, and turn equally free on their axes in the frame. Each mill has four thin arms or sails, fixed into the axis: those of the mill *a* have their planes at right angles to its axis, and those of *b* have their planes parallel to it. Therefore, as the mill *a* turns round in common air, it is but little resisted thereby, because its sails cut the air with their thin edges: but the mill *b* is much resisted, because the broad sides of its sails move against the air when it turns round. In each axle is a pin near the middle of the frame, which goes quite through the axle, and stands out a little on each side of it: upon these pins, the slider *d* may be made to bear, and so hinder the mills from going, when the strong spring *c* is set on bend against the opposite ends of the pins.

Having set this machine upon the pump-plate *LL* (Fig. 1.) draw up the slider *d* to the pins on one side, and set the spring *c* at bend upon the opposite ends of the pins: then push down the slider *d*, and the spring acting equally strong on each mill, will set them both a going with equal forces and velocities: but the mill *a* will run much longer than the mill *b*, because the air makes much less resistance against the edges of its sails, than against the sides of the sails of *b*.

Draw up the slider again, and set the spring upon the pins as before; then cover the machine with the receiver *M* upon the pump-plate, and having exhausted the receiver of air, push down the wire *PP* (through the collar of leathers in the neck *q*) upon the slider; which will disengage it from the
pins,

pins, and allow the mills to turn round by the impulse of the spring: and as there is no air in the receiver to make any sensible resistance against them, they will both move a considerable time longer than they did in the open air; and the moment that one stops, the other will do so too.—This shews that air resists bodies in motion, and that equal bodies may meet with different degrees of resistance, according as they present greater or less surfaces to the air, in the planes of their motions.

2. Take off the receiver *M*, and the mills; and having put the guinea *a* and feather *b* upon the brass flap *c*, turn up the flap, and shut it into the notch *d*. Then, putting a wet leather over the top of the tall receiver *AB* (it being open both at top and bottom) cover it with the plate *C*, from which the guinea-and-feather tongs *ed* will then hang within the receiver. This done, pump the air out of the receiver; and then draw up the wire *f* a little, which by a square piece on its lower end will open the tongs *ed*; and the flap falling down, as at *c*, the guinea and feather will descend with equal velocities in the receiver; and both will fall upon the pump-plate at the same instant. *N. B.* In this experiment, the observers ought not to look at the top, but at the bottom of the receiver; in order to see the guinea and feather fall upon the plate: otherwise, on account of the quickness of their motion, they will escape the sight of the beholders.

II. *To shew the weight of the air.*

1. Having fitted a brass cap, with a valve tied over it, to the mouth of a thin bottle, or *Florence* flask, whose contents is exactly known, screw the neck of this cap into the hole *i* of the pump-plate: then, having exhausted the air out of the flask, and taken it off from the pump, let it be suspended at one end of a balance, and nicely counterpoised by weights in the scale at the other end: this done, raise up the valve with a pin, and the air will rush into the flask with an audible noise; during which time, the flask will descend, and pull down that end of the beam. When the noise is over, put as many grains into the scale as will restore the equilibrium; and they will shew exactly the weight of the quantity of air which has got into the flask, and filled it. If the flask holds an exact quart, it will be found, that 17 grains will restore the equipoise of the balance, when the quicksilver stands at $29\frac{1}{2}$ inches in the barometer: which shews, that when the air is at a mean rate of density, a quart of it weighs 17 grains: it weighs more when the quicksilver stands higher; and less when it stands lower.

2. Place the small receiver *O* (Fig. 1.) over the hole *i* in the pump-plate, and upon exhausting the air, the receiver will be fixed down to the plate by the pressure of the air on its outside, which is left to act alone, without any air in the receiver to act against it: and this pressure will be equal to as
many

many times 15 pounds, as there are square inches in that part of the plate which the receiver covers ; which will hold down the receiver so fast, that it cannot be got off, until the air be let into it by turning the cock *k* ; and then it becomes loose.

3. Set the little glass *AB* (which is open at both ends) over the hole *i* upon the pump-plate *LL*, and clap your hand close upon the top of it at *B* : then, upon exhausting the air out of the glass, you will find your hand pressed down with a great weight upon it ; so that you can hardly release it, until the air be re-admitted into the glass by turning the cock *k* ; which air, by acting as strongly upward against the hand as the external air acted in pressing it downward, will release the hand from its confinement.

4. Having tied a piece of wet bladder *b* over the open top of the glass *A* (which is also open at bottom) set it to dry, and then the bladder will be as tight as a drum. Then place the open end *A* upon the pump-plate, over the hole *i*, and begin to exhaust the air out of the glass. As the air is exhausting, its spring in the glass will be weakened, and give way to the pressure of the outward air on the bladder, which, as it is pressed down, will put on a spherical concave figure, which will grow deeper and deeper, until the strength of the bladder be overcome by the weight of the air ; and then it will break with a report as loud as that of a gun. —If a flat piece of glass be laid upon the open top of this receiver, and joined to it by a flat ring of wet leather between them ; upon pumping the
air

air out of the receiver, the pressure of the outward air upon the flat glass will break it all to pieces.

Fig. 7. 5. Immerse the neck *cd* of the hollow glass ball *eb* in water, contained in the phial *aa*; then set it upon the pump-plate, and cover it and the hole *i* with the close receiver *A*; and then begin to pump out the air. As the air goes out of the receiver by its spring, it will also by the same means go out of the hollow ball *eb*, through the neck *dc*, and rise up in bubbles to the surface of the water in the phial; from whence it will make its way, with the rest of the air in the receiver, through the air-pipe *GG* and valves *a* and *b*, into the open air. When it has done bubbling in the phial, the ball is sufficiently exhausted; and then, upon turning the cock *k*, the air will get into the receiver, and press so upon the surface of the water in the phial, as to force the water up into the ball in a jet, through the neck *cd*; and will fill the ball almost full of water. The reason why the ball is not quite filled, is because all the air could not be taken out of it; and the small quantity that was left in, and had expanded itself so as to fill the whole ball, is now condensed into the same state as the outward air, and remains in a small bubble at the top of the ball; and so keeps the water from filling that part of it.

Fig. 8. 6. Pour some quicksilver into the jar *D* and set it on the pump-plate near the hole *i*; then set on the tall open receiver *AB*, so as to be over the jar and hole; and cover the receiver with the brass plate *C*. Screw the open glass tube *fg* (which has a brass

a brass top on it at *b*) into the syringe *H*, and putting the tube through a hole in the middle of the plate, so as to immerse the lower-end of the tube *e* in the quicksilver at *D*, screw the end *b* of the syringe into the plate. This done, draw up the piston in the syringe by the ring *I*, which will make a vacuum in the syringe below the piston; and as the upper end of the tube opens into the syringe, the air will be dilated in the tube, because part of it, by its spring, gets up into the syringe; and the spring of the undilated air in the receiver acting upon the surface of the quicksilver in the jar, will force part of it up into the tube: for the quicksilver will follow the piston in the syringe, in the same way, and for the same reason, that water follows the piston of a common pump when it is raised in the pump barrel; and this, according to some, is done by suction. But to refute that erroneous notion, let the air be pumped out of the receiver *AB*, and then all the quicksilver in the tube will fall down by its own weight into the jar; and cannot be again raised one hair's breadth in the tube by working the syringe: which shews that suction had no hand in raising the quicksilver; and, to prove that it is done by pressure, let the air into the receiver by the cock *k* (Fig. 1.) and its action upon the surface of the quicksilver in the jar will raise it up into the tube, although the piston of the syringe continues motionless.—If the tube be about 32 or 33 inches high, the quicksilver will rise in it very near as high as it stands at

O

that

that time in the barometer. And, if the syringe has a small hole, as *m*, near the top of it, and the piston be drawn up above that hole, the air will rush through the hole into the syringe and tube, and the quicksilver will immediately fall down into the jar. If this part of the apparatus be air-tight, the quicksilver may be pumped up into the tube to the same height that it stands in the barometer; but it will go no higher, because then the weight of the column in the tube is the same as the weight of a column of air of the same thickness with the quicksilver, and reaching from the earth to the top of the atmosphere.

Fig. 9.

7. Having placed the jar *A*, with some quicksilver in it, on the pump-plate, as in the last experiment, cover it with the receiver *B*; then push the open end of the glass tube *de* through the collar of leathers in the brass neck *C* (which it fits so as to be air-tight) almost down to the quicksilver in the jar. Then exhaust the air out of the receiver, and it will also come out of the tube, because the tube is close at top. When the gauge *mm* shews that the receiver is well exhausted, push down the tube, so as to immerse its lower end into the quicksilver in the jar. Now, although the tube be exhausted of air, none of the quicksilver will rise into it, because there is no air left in the receiver to press upon its surface in the jar. But let the air into the receiver by the cock *k*, and the quicksilver will immediately rise in the tube; and stand as high in it, as it was pumped up in the last experiment.

Both

Both these experiments shew, that the quicksilver is supported in the barometer by the pressure of the air on its surface in the box, in which the open end of the tube is placed. And that the more dense and heavy the air is, the higher does the quicksilver rise; and, on the contrary, the thinner and lighter the air is, the more will the quicksilver fall. For, if the handle *F* be turned ever so little, it takes some air out of the receiver, by raising one or other of the pistons in its barrel; and consequently, that which remains in the receiver is so much the rarer, and has so much the less spring and weight; and thereupon, the quicksilver falls a little in the tube: but upon turning the cock, and re-admitting the air into the receiver, it becomes as weighty as before, and the quicksilver rises again to the same height.—Thus we see the reason why the quicksilver in the barometer falls before rain or snow, and rises before fair weather; for, in the former case, the air is too thin and light to bear up the vapours, and in the latter, too dense and heavy to let them fall.

N.B. In all mercurial experiments with the air-pump, a short pipe must be screwed into the hole *i*, so as to rise about an inch above the plate, to prevent the quicksilver from getting into the air-pipe and barrels, in case any of it should be accidentally spilt over the jar: for if it once gets into the pipes or barrels, it spoils them, by loosening the folder, and corroding the brass.

8. Take the tube out of the receiver, and put one end of a bit of dry hazel branch, about an

inch long, tight into the hole, and the other end tight into a hole quite through the bottom of a small wooden cup: then pour some quicksilver into the cup, and exhaust the receiver of air, and the pressure of the outward air, on the surface of the quicksilver, will force it through the pores of the hazel, from whence it will descend in a beautiful shower into a cup placed under the receiver to catch it.

9. Put a wire through the collar of leathers in the top of the receiver, and fix a bit of dry wood on the end of the wire within the receiver; then exhaust the air, and push the wire down, so as to immerse the wood into a jar of quicksilver on the pump-plate; this done, let in the air, and upon taking the wood out of the jar, and splitting it, its pores will be found full of quicksilver, which the force of the air, upon being let into the receiver, drove into the wood.

Fig. 10.

10. Join the two brass hemispherical cups *A* and *B* together, with a wet leather between them, having a hole in the middle of it; then screw the end *D* of the pipe *CD* into the plate of the pump at *i*, and turn the cock *E*, so as the pipe may be open all the way into the cavity of the hemispheres: then exhaust the air out of them, and turn the cock a quarter round, which will shut the pipe *CD*, and keep out the air. This done, unscrew the pipe at *D* from the pump; and screw the piece *Fb* upon it at *D*; and let two strong men try to pull the hemispheres asunder by the rings *g* and *h*, which they will find hard to do: for if the diameter of the hemispheres

hemispheres be four inches, they will be pressed together by the external air with a force equal to 188 pounds. And to shew that it is the pressure of the air that keeps them together, hang them by either of the rings upon the hook *P* of the wire in the receiver *M* (Fig. 1.) and upon exhausting the air out of the receiver, they will fall asunder of themselves.

11. Place a small receiver *O* (Fig. 1.) near the hole *i* on the pump-plate, and cover both it and the hole with the receiver *M*; and turn the wire so by the top *P*, that its hook may take hold of the little receiver by a ring at its top, allowing that receiver to stand with its own weight on the plate. Then, upon working the pump, the air will come out of both receivers; but the large one *M*, will be forcibly held down to the pump by the pressure of the external air; whilst the small one *O*, having no air to press upon it, will continue loose, and may be drawn up and let down at pleasure, by the wire *PP*. But, upon letting it quite down to the plate, and admitting the air into the receiver *M*, by the cock *k*, the air will press so upon the small receiver *O*, as to fix it down to the plate; and at the same time, by counter-balancing the outward pressure on the large receiver *M*, it will become loose. This experiment evidently shews, that the receivers are held down by pressure, and not by suction, for the internal receiver continued loose whilst the operator was pumping, and the external one was held down; but the former became fast immediately by letting in the air upon it.

Fig. 11.

12. Screw the end *A* of the brass pipe *ABF* into the hole of the pump-plate, and turn the cock *e* until the pipe be open; then put a wet leather upon the plate *cd*, which is fixed on the pipe, and cover it with the tall receiver *GH*, which is close at top: then exhaust the air out of the receiver, and turn the cock *e* to keep it out; which done, unscrew the pipe from the pump, and set its end *A* into a bason of water, and turn the cock *e* to open the pipe; on which, as there is no air in the receiver, the pressure of the atmosphere on the water in the bason will drive the water forcibly through the pipe, and make it play up in a jet to the top of the receiver.

13. Set the square phial *A* (Fig. 14.) upon the pump-plate, and having covered it with the wire cage *B*, put a close receiver over it, and exhaust the air out of the receiver; in doing of which, the air will also make its way out of the phial through a small hole in its neck under the valve *b*. When the air is exhausted, turn the cock below the plate, to re-admit the air into the receiver; and as it cannot get into the phial again, because of the valve, the phial will be broke in a thousand pieces by the pressure of the air upon it. Had it been of a round form, it would have sustained this pressure like an arch, without breaking; but as its sides are flat, it cannot.

To

To shew the elasticity or spring of the air.

14. Tie up a very small quantity of air in a bladder, and put it under a receiver; then exhaust the air out of the receiver, and the small quantity which is confined in the bladder (having nothing to act against it) will expand itself so by the force of its spring, as to fill the bladder as full as it could be blown of common air. But upon letting the air into the receiver again, it will overpower the air in the bladder, and press its sides almost close together.

15. If the bladder so tied up be put into a wooden box, and have 20 or 30 pounds weight of lead put upon it in the box, and the box be covered with a close receiver; upon exhausting the air out of the receiver, that air which is confined in the bladder will expand itself so, as to raise up all the lead by the force of its spring.

16. Take the glass ball mentioned in the fifth Fig. 7. experiment, which was left full of water all but a small bubble of air at top, and having set it with its neck downward into the empty phial *aa*, and covered it with a close receiver, exhaust the air out of the receiver, and the small bubble of air in the top of the ball will expand itself, so as to force all the water out of the ball into the phial.

17. Screw the pipe *AB* into the pump-plate, Fig. 11. place the tall receiver *GH* upon the plate *cd*, as in the twelfth experiment, and exhaust the air out of the receiver; then, turn the cock *e* to keep out the
O 4 air,

air, unscrew the pipe from the pump, and screw it into the mouth of the copper vessel *CC* (Fig. 15.) the vessel having first been about half filled with water. Then turn the cock *e* (Fig. 11.) and the spring of the air which is confined in the copper vessel will force the water up through the pipe *AB* in a jet into the exhausted receiver, as strongly as it did by its pressure on the surface of the water in a basin, in the twelfth experiment.

18. If a fowl, a cat, rat, mouse, or bird, be put under a receiver, and the air be exhausted, the animal is at first oppressed as with a great weight, then grows convulsed, and at last expires in all the agonies of a most bitter and cruel death. But as this experiment is too shocking to every spectator who has the least degree of humanity, we substitute a machine called the *lungs-glass* in place of the animal; which, by a bladder within it, shews how the lungs of animals are contracted into a small compass when the air is taken out of them.

19. If a butterfly be suspended in a receiver, by a fine thread tied to one of its horns, it will fly about in the receiver, as long as the receiver continues full of air; but if the air be exhausted, though the animal will not die, and will continue to flutter its wings, it cannot remove itself from the place where it hangs in the middle of the receiver, until the air be let in again, and then the animal will fly about as before.

Fig. 12.

20. Pour some quicksilver into the small bottle *A*, and screw the brass collar *c* of the tube *BC* into the brass neck *b* of the bottle, and the lower end

end of the tube will be immerfed into the quickfilver, fo that the air above the quickfilver in the bottle will be confined there, becaufe it cannot get out about the joinings, nor can it be drawn out through the quickfilver into the tube. This tube is alfo open at top, and is to be covered with the receiver *G* and large tube *EF*, which tube is fixed by brafs collars to the receiver, and is clofe at the top. This preparation being made, exhaust the air both out of the receiver and its tube; and the air will by the fame means be exhausted out of the inner tube *BC*, through its open top at *C*; and as the receiver and tubes are exhausting, the air that is confined in the glafs bottle *A* will prefs fo by its fpring upon the furface of the quickfilver, as to force it up in the inner tube as high as it was raifed in the ninth experiment by the preffure of the atmofphere: which demonftrates that the fpring of the air is equivalent to its weight.

21. Screw the end *C* of the pipe *CD* into the hole of the pump-plate, and turn all the three cocks *d*, *G*, and *H*, fo as to open the communications between all the three pipes *E*, *F*, *DC*, and the hollow trunk *AB*. Then, cover the plates *g* and *b* with wet leathers, which have holes in their middle where the pipes open into the plates; and place the clofe receiver *I* upon the plate *g*: this done, fhut the pipe *F* by turning the cock *H*, and exhaust the air out of the receiver *I*. Then, turn the cock *d* to fhut out the air, unfcrew the machine from the pump, and having fcrewed it to the wooden foot *L*, put the receiver *K* upon the plate

b;

b; this receiver will continue loose on the plate as long as it keeps full of air; which it will do until the cock *H* be turned to open the communication between the pipes *F* and *G*, through the trunk *AB*; and then the air in the receiver *K*, having nothing to act against its spring, will run from *K* into *I*, until it be so divided between these receivers, as to be of equal density in both; and then they will be held down with equal forces to their plates by the pressure of the atmosphere; though each receiver will then be kept down but with one half of pressure upon it, that the receiver *I* had, when it was exhausted of air; because it has now one half of the common air in it which filled the receiver *K* when it was set upon the plate; and therefore, a force equal to half the force of the spring of common air, will act within the receivers against the whole pressure of the common air upon their outside. This is called transferring the air out of one vessel into another.

Fig. 14.

22. Put a cork into the square phial *A*, and fix it in with wax or cement; put the phial upon the pump-plate, with the wire cage *B* over it, and cover the cage with a close receiver. Then, exhaust the air out of the receiver, and the air that was corked up in the phial will, by the force of its spring, break the phial outwards, because there is no air left on the outside of the phial to act against the air within it.

22. Put a shrivelled apple under a close receiver, and exhaust the air; then the spring of the air within the apple will plump it out, so as to cause all

the wrinkles disappear ; but upon letting the air into the receiver again, to press upon the apple, it will instantly return to its former decayed and shrivelled state.

23. Take a fresh egg, and cut off a little of the shell and film from its smallest end, then put the egg under a receiver, and pump out the air ; upon which, all the contents in the egg will be forced out into the receiver, by the expansion of a small bubble of air contained in the great end, between the shell and film.

24. Put some warm beer in a glass, and having set it on the pump, cover it with a close receiver, and then exhaust the air. Whilst this is doing, and thereby the pressure more and more taken off from the beer in the glass, the air therein will expand itself, and rise up in innumerable bubbles to the surface of the beer ; and from thence it is taken away with the other air in the receiver. When the receiver is near exhausted, the air in the beer, which could not disentangle itself quick enough to get off with the rest, will now expand itself so, as to cause the beer to have all the appearance of boiling ; and the greater part of it will go over the glass.

25. Put some warm water in a glass, and put a bit of dry wainscot or other wood into the water. Then, cover the glass with a receiver, and exhaust the air ; upon which, the air in the wood having liberty to expand itself, will come out plentifully, and make all the water to bubble about the wood, especially about the ends, because the pores lie lengthwise.

lengthwise. A cubic inch of dry wainscot has so much air in it, that it will continue bubbling for near half an hour together.

Miscellaneous experiments.

25. Screw the syringe *H* (Fig. 8.) to a piece of lead that weighs one pound at least; and, holding the lead in one hand, pull up the piston in the syringe with the other; then, quitting hold of the lead, the air will push it upward, and drive back the syringe upon the piston. The reason of this is, that the drawing up of the piston made a vacuum in the syringe, and the air, which presses every way equally, having nothing to resist its pressure upward, the lead and syringe were pressed up, contrary to their natural tendency by gravity. If the syringe so loaded, be hung in a receiver, and the air be exhausted, the syringe and lead will descend upon the piston-rod by their natural gravity; and, upon admitting the air into the receiver, they will be drove upward again, until the piston be at the very bottom of the syringe.

26. Let a large piece of cork be suspended by a thread at one end of a balance, and counterpoised by a leaden weight, suspended in the same manner, at the other. Let this balance be hung to the inside of the top of a large receiver; which being set on the pump, and the air exhausted, the cork will preponderate, and shew itself to be heavier than the lead; but upon letting in the air again, the equilibrium will be restored. The reason
of

of this is, that since the air is a fluid, and all bodies lose as much of their absolute weight in it, as is equal to the weight of their bulk of the fluid, the cork being the larger body, loses more of its real weight than the lead does; and therefore must in fact be heavier, to balance it under the disadvantage of losing some of its weight: which disadvantage being taken off by removing the air, the bodies then gravitate according to their real quantities of matter, and the cork, which balanced the lead in air, shews itself to be heavier when in *vacuo*.

27. Set a lighted candle upon the pump, and cover it with a tall receiver. If the receiver holds a gallon, the candle will burn a minute, and then, after having gradually decayed from the first instant, it will go out: which shews, that a constant supply of fresh air is necessary to feed flame; and so it also is for animal life. For a bird kept under a close receiver will soon die, although no air be pumped out; and it is found that a gallon of air is sufficient only for one minute for a man to breathe in.

The moment that the candle goes out, the smoke will be seen to ascend to the top of the receiver, and there it will form a sort of cloud: but upon exhausting the air, the smoke will fall down to the bottom of the receiver, and leave it as clear at top as it was before it was set upon the pump. This shews, that smoke does not ascend on account of its being positively light, but because it is lighter than air; and its falling to the bottom when the
air

air was taken away, shews, that it is not destitute of weight. So most sorts of wood ascend or swim in water; and yet there are none who doubt of the wood's having gravity or weight.

28. Set a receiver, which is open at top, upon the air-pump, and cover it with a brass-plate, and wet leather; and having exhausted it of air, let the air in again at top through an iron pipe, making it pass through a charcoal flame at the end of the pipe; and when the receiver is full of that air, lift up the cover and let down a mouse or bird into the receiver, and the burnt air will immediately kill it. If a candle be let down into the air, it will go out directly, but will purify the air so far as it was let down; and so, by lighting it and letting it down over and over again, all the air in the receiver will be purified.

29. Set a bell upon a cushion on the pump-plate, and cover it with a receiver; then shake the pump to make the clapper strike against the bell, and the sound will be very well heard: but, exhaust the receiver of air, and then, if the clapper be made to strike ever so hard against the bell, it will make no sound at all; which shews, that air is absolutely necessary for the propagation of sound.

30. Let a candle be placed on one side of a receiver, and viewed through the receiver at some distance; then, as soon as the air begins to be exhausted, the receiver will be filled with vapours which rise from the wet leather, by the spring of the air in it; and the light of the candle being refracted

refracted through that medium of vapours, will have the appearance of circles of various colours, such as are seen about the moon in a hazy air at night.

The air-pump was invented by *Otho Guericke* of *Magdeburg*, but was much improved by *Mr. Boyle*, to whom we are indebted for our greatest part of the knowledge of the wonderful properties of the air, demonstrated in the above experiments.

The elastic air which is contained in many bodies, and is kept in them by the weight of the atmosphere, may be got out of them either by boiling, or by the air-pump, as shewn in the 25th experiment: but the fixed air, which is by much the greater quantity, cannot be got out but by distillation, fermentation, or putrefaction.

If fixed air did not come out of bodies with difficulty, and spend some time in extricating itself from them, it would tear them to-pieces. Trees would be rent by the change of air from a fixt to an elastic state, and animals would be burst in pieces by the explosion of air in their food.

Dr. Hales found by experiment, that the air in apples is so much condensed, that if it were let out into the common air, it would fill a space 48 times as great as the bulk of the apples themselves: so that its pressure outwards was equal to 11776 lb. and, in a cubic inch of oak to 19860 lb. against its sides. So that if the air was let loose at once in these substances, they would tear every thing to-
pieces

pieces about them with a force much superior to gunpowder. Hence, in eating apples, it is well that they part with the air by degrees, as they ferment in the stomach, otherwise, an apple would be immediate death to him who eats it.

The mixing of some substances with others will release the air from them, all of a sudden, which may be attended with very great danger. Of this we have a remarkable instance in an experiment made by Dr. *Slare* ; who having put half a dram of oil of carraway-seed into one glass, and a dram of compound spirit of nitre in another, covered them both on the air-pump with a receiver six inches wide, and eight inches deep, and then exhausted the air, and continued pumping until all that could possibly be got both out of the receiver, and out of the two fluids, was extricated : then, by a particular contrivance from the top of the receiver, he mixed the fluids together ; upon which, they produced such a prodigious quantity of air, as instantly blew up the receiver ; although it was pressed down by the atmosphere with upwards of 400 pound weight.

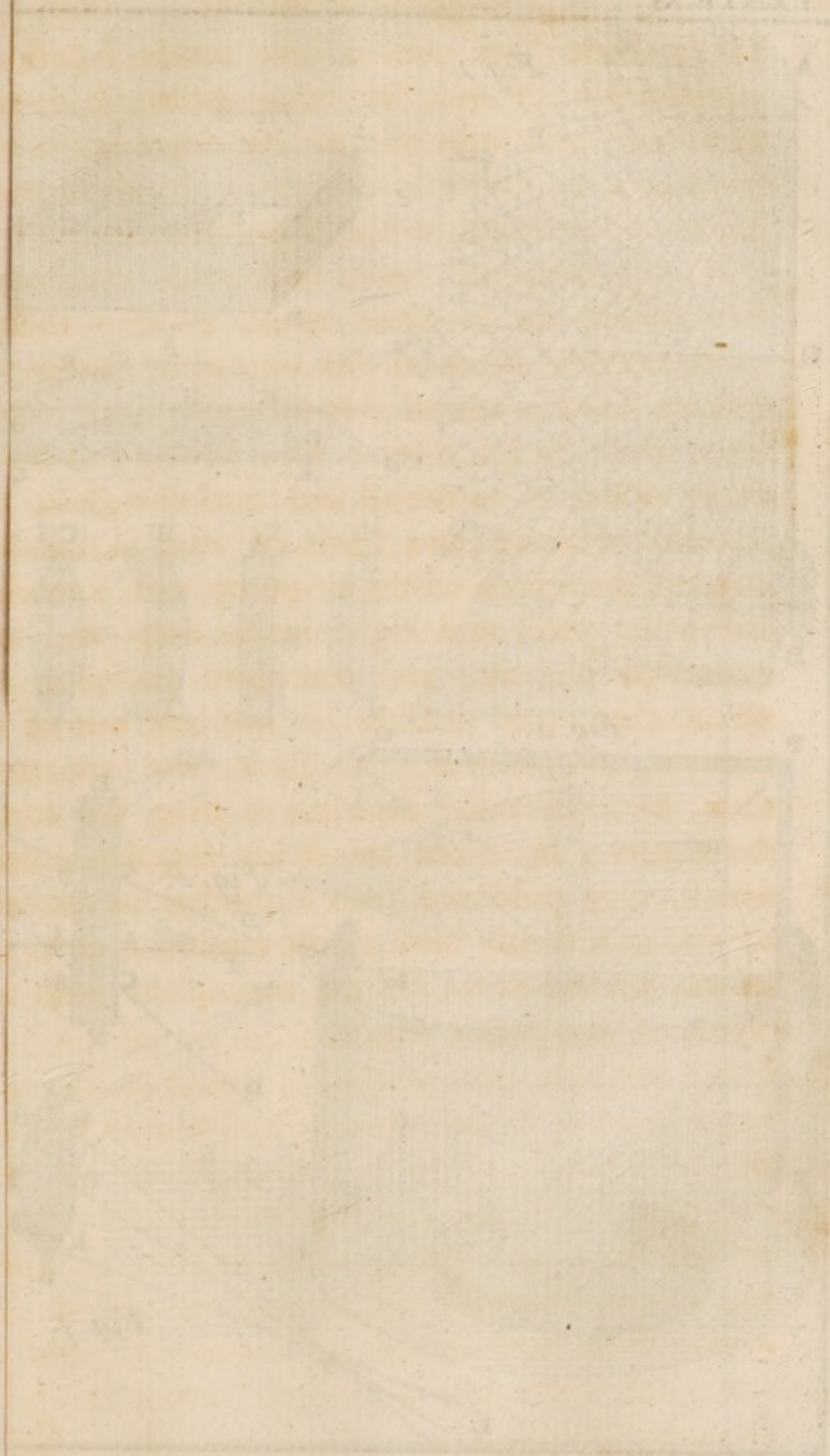
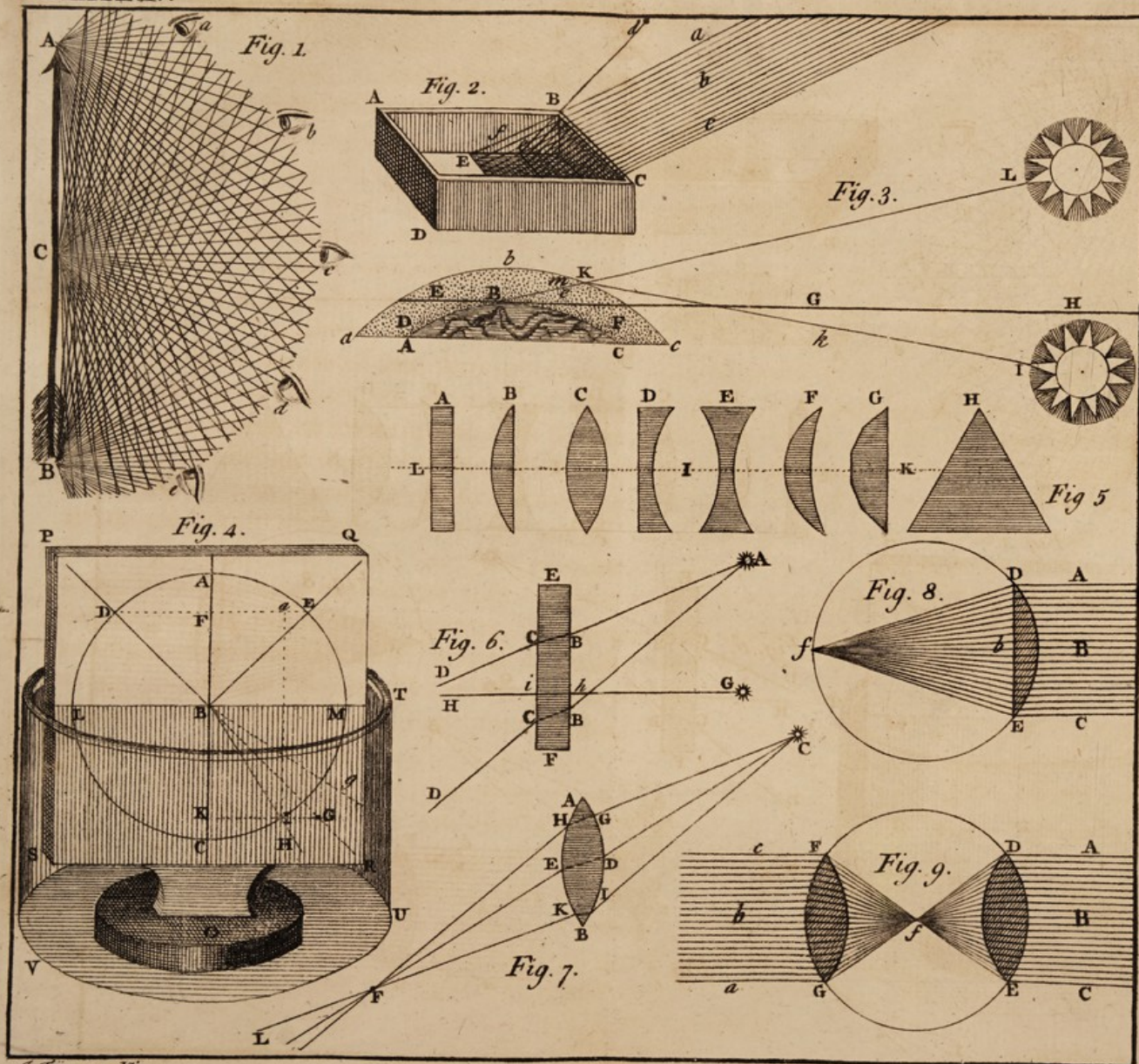


PLATE XV.



L E C T. VIII.

Of optics.

LI G H T consists of an inconceivably great number of particles flowing from a luminous body in all manner of directions ; and these particles are so small as to surpass all human comprehension.

That the number of particles of light is inconceivably great, appears from hence, that if a candle be lighted, and there be no obstacle in the way to obstruct the passage of its rays, it will fill all the space within two miles of it every way with luminous particles, before it has lost the least sensible part of its substance.

A ray of light is a continued stream of these particles, flowing from any visible body in a straight line : and that the particles themselves are incomprehensibly small, appears from the following experiment. Make a small pin-hole in a piece of black paper, and hold the paper upright on a table facing a row of candles standing by one another ; then place a sheet of pasteboard at a little distance behind the paper ; and some of the rays which flow from all the candles through the hole in the paper, will form as many specks of light on the pasteboard as there are candles on the table before the plate : each speck being as distinct and clear, as if there was only one speck from one single candle : which shews, that the particles of light are

The amazing smallness of the particles of light.

exceedingly small, otherwise they could not pass through the hole from so many different candles without confusion.—Dr. *Niewenlyt* has computed, that there flows more than 6,000,000,000,000 times as many particles of light from a candle in one second of time, as there are grains of sand in the whole earth; supposing each cubic inch of it to contain 1,000,000.

These particles, by falling directly upon our eyes, excite in our minds the idea of light. And when they fall upon bodies, and are thereby reflected to our eyes, they excite in us the ideas of these bodies. And as every point of a visible body reflects the rays of light in all manner of directions, every point will be visible in every part to which the light is reflected from it. Thus the object *ABC* is visible to an eye in any part where the rays *Aa, Ab, Ac, Ad, Ae, Ba, Bb, Bc, Bd, Be*, and *Ca, Cb, Cc, Cd, Ce*, come. Here we have shewn the rays as if they were only reflected from the ends *A* and *B*, and the middle point *C* of the object; every other point being supposed to reflect rays in the same manner. So that, wherever a spectator is placed with regard to the body, every point of that part of the surface which is towards him will be visible, when no intervening object stops the passage of the light.

Plate XV.

Fig. 1.

Reflected
light.

Since no object can be seen through the bore of a bended pipe, it is evident that the rays of light move in straight lines, whilst there is nothing to refract or turn them out of their rectilineal course.

Whilst

Whilst the rays of light continue in any * medium of an uniform density, they are straight ; but when they pass obliquely out of one medium into another, which is either more dense or more rare, they are refracted towards the denser medium : and this refraction is more or less, as the rays fall more or less obliquely on the refracting surface of the medium.

To prove this by experiment, set the empty vessel *ABCD* into any place where the sun shines obliquely, and observe the part where the shadow Fig. 2. of the edge *BC* falls on the bottom of the vessel at *E* ; then fill the vessel with water, and the shadow will reach no farther than *e* ; which shews, Refracted light. that the ray *aBE* which came straight in the open air, just over the edge of the vessel at *B* to its bottom at *E*, is refracted by falling obliquely on the surface of the water at *B* ; and instead of going on in the rectilinear direction *ABE*, it is bent downward in the water from *B* to *e* ; the whole bend being at the surface of the water : and so of all the other rays *abc*.

If a stick be laid over the vessel, and the sun's rays be reflected from a glass perpendicularly into the vessel, the shadow of the stick will fall upon the same part of the bottom, whether the vessel be empty or full ; which shews, that the rays of light are not refracted when they fall perpendicularly on the surface of any medium.

* Any thing through which the rays of light can pass is called a medium ; as air, water, glass, diamond, or even a vacuum.

The rays of light are as much refracted by passing out of water into air, as by passing out of air into water. Thus, if a ray of light flows from the point e , under water, in the direction eB ; when it comes to the surface of the water at B , it will not go on thence in the rectilineal course Bd , but will be refracted into the line Ba . Therefore,

To an eye at e looking through a plane glass in the bottom of the empty vessel, the point a cannot be seen, because the side Bc of the vessel interposes; and the point d will just be seen over the edge of the vessel at B . But if the vessel be filled with water, the point a will be seen from e ; and will appear as at d , elevated in the direction of the ray eb *.

The days
are made
longer by
the refraction
of the
sun's rays.

The time of sun-rising or setting, supposing its rays suffered no refraction, is easily found by calculation. But observation proves that the sun rises sooner, and sets later every day than the calculated time; the reason of which is plain from what was said immediately above. For, though the sun's rays do not come part of their way to us through water, yet they do through the air or atmosphere, which being a grosser medium than the free space between the sun and the top of the at-

* Hence a piece of money lying at e , in the bottom of an empty vessel, cannot be seen by an eye at a , because the edge of the vessel intervenes; but let the vessel be filled with water, and the ray eb being then refracted at B , will strike the eye at a , and so render the money visible, which will appear as if it were raised up to f in the line aBf .

mosphere,

mosphere, the rays, by entering obliquely into the atmosphere are there refracted, and thence bent down to the earth. And although there are many places of the earth to which the sun is vertical at noon, and consequently his rays can suffer no refraction at that time, because they come perpendicularly through the atmosphere; yet there is no place to which the sun's rays do not fall obliquely on the top of the atmosphere, at his rising and setting; and consequently, no clear day in which the sun will not be visible before he rises in the horizon, and after he sets in it: and the longer or shorter, as the atmosphere is more or less replete with vapours. For, let ABC be part of the earth's surface, DEF the atmosphere that covers it, and $EBGH$ the horizon of an observer at B . As every point of the sun's surface sends out rays of light in all manner of directions, some of his rays will constantly fall upon, and enlighten, one half of our atmosphere; and therefore, when the sun is at I , below the horizon H , those rays which go on in the free space Ikk preserve a rectilineal course until they fall upon the top of the atmosphere; and those which fall so about K , are refracted at their entrance into the atmosphere, and bent down in the line KeB , to the observer's place at B : and therefore, the sun will appear at L , in the direction of the ray BeK , above the horizon BGH , when he is really below it at I .

The angle contained between a ray of light, and a perpendicular to the refracting surface, is called *the angle of incidence*; and the angle contained

Angle of incidence.

Fig. 4.
Angle of re-
fraction.

tained between the same perpendicular, and the same ray after refraction, is called *the angle of refraction*. Thus, let LBM be the refracting surface of a medium (suppose water) and ABC a perpendicular to that surface; let DB be a ray of light, going out of air into water at B , and therein refracted in the line BH ; the angle ABD , is the angle of incidence, of which DF is the sine; and the angle KBH , is the angle of refraction, whose sine is KI .

When the refracting medium is water, the sine of the angle of incidence is to the sine of the angle of refraction, as 4 to 3; which is confirmed by the following experiment, taken from Doctor SMITH's optics.

Describe the circle $DAEC$ on a plane square board, and cross it at right angles with the straight lines ABC , and LBM ; then, from the intersection A , with any opening of the compasses, set off the equal arcs AD and AE , and draw the right line DFE : then, taking Fa , which is three quarters of the length FE , from the point a , draw aI parallel to ABK , and join KI , parallel to BM : so KI will be equal to three quarters of FE or of DF . This done, fix the board upright upon the leaden pedestal O , and stick three pins perpendicularly into the board, at the points D , B , and I : then set the board upright into the vessel TUV , and fill up the vessel with water to the line LBM . When the water has settled, look along the line DB , so as you may see the head of the pin B over the head of the pin D ; and the pin I will

will appear in the same right line produced to G , for its head will be seen just over the head of the pin at B : which shews, that the ray IB , coming from the pin at I , is so refracted at B , as to proceed from thence in the line BD to the eye of the observer; the same as it would do from any point G in the right line DBG , if there were no water in the vessel: and also shews, that KI , the sine of refraction in water, is to DF , the sine of incidence in air, as 3 to 4*.

Hence, if DBH were a crooked stick put obliquely into the water, it would appear a straight one, as DBG . Therefore, as the line BH appears at BG , so the line BG will appear at Bg ; and consequently, a straight stick DBG put obliquely into water, will seem bent at the surface of the water in B , and crooked as DBg .

When a ray of light passes out of air into glass, the sine of incidence is to the sine of refraction, as 3 to 2; and when out of air into a diamond, as 5 to 2.

Glass may be ground into eight different shapes at least, for optical purposes, viz.

1. A *plane-glass*, which is flat on both sides, Fig. 5. and of equal thickness in all its parts, as A .
2. A *plano-convex*, which is flat on one side and convex on the other, as B .
3. A *double-convex*, which is convex on both sides, as C .

* This is strictly true of the red rays only, for the other coloured rays are differently refracted; but the difference is so small, that it need not be considered in this place.

Lenses.

4. A *plano-concave*, which is flat on one side and concave on the other, as *D*.

5. A *double-concave*, which is concave on both sides, as *E*.

6. A *merciscus*, which is concave on one side and convex on the other, as *F*.

7. A *flat-plano-convex*, whose convex side is ground into several little flat surfaces, as *G*.

8. A *prism*, which has three flat sides, and when viewed endwise, appears like an equilateral triangle, as *H*.

Glasses ground into any of the above shapes *B*, *C*, *D*, *E*, *F*, are generally called *lenses*.

A right line *L I K*, going perpendicularly through the middle of a lens, is called *the axis of the lens*.

Fig. 6. A ray of light *G b*, falling perpendicularly on a plain glass *E F*, will pass through the glass in the same direction *b i*, and go out of it into the air in the same right course *i H*.

A ray of light *A B*, falling obliquely on a plain glass, will go out of the glass in the same direction, but not in the same right line; for in touching the glass, it will be refracted in the line *B C*, and in leaving the glass, it will be refracted in the line *C D*.

Fig. 7. A ray of light *C D*, falling obliquely on the middle of a convex glass, will go forward in the same direction *D E*, as if it had fallen with the same degree of obliquity on a plane glass; and will go out of the glass in the same direction with which it entered: for it will be equally refracted at the points *D* and *E*, as if it had passed through a plane surface. But the rays *C G* and *C I*, will be

be so refracted, as to meet again at the point F . Therefore, all the rays which flow from the point C , so as to touch the glass, will meet again at F ; and if they go farther onward, as to L , they cross at F , and go forward on the opposite sides of the middle ray $CDEF$, to what they were in approaching it in the directions HF and KF .

When parallel rays, as ABC , fall directly upon a plano-convex glass DE , and pass through it, they will be so refracted, as to unite in a point f behind it; and this point is called the *principal focus*; the distance of which, from the middle of the glass, is called the *focal distance*; which is equal to twice the radius of the sphere of the glass's convexity. And,

Fig. 8.
The properties of
different
lenses.

When parallel rays, as ABC , fall directly upon a glass DE , which is equally convex on both sides, and pass through it; they will be so refracted, as to meet in a point or principal focus f , whose distance is equal to the radius or semidiameter of the sphere of the glass's convexity. But, if a glass be more convex on one side than on the other, the rule for finding the focal distance is this; as the sum of the semidiameters of both convexities is to the semidiameter of either, so is double the semidiameter of the other to the distance of the focus. Or, divide the double product of the radii by their sum, and the quotient will be the distance sought.

Fig. 9.

Since all those rays of the sun which pass through a convex glass are collected together in its focus, the force of all their heat is collected into that part;

part; and is in proportion to the common heat of the sun, as the area of the glass is to the area of the focus. Hence we see the reason why a convex glass causes the sun's rays to burn, after passing through it.

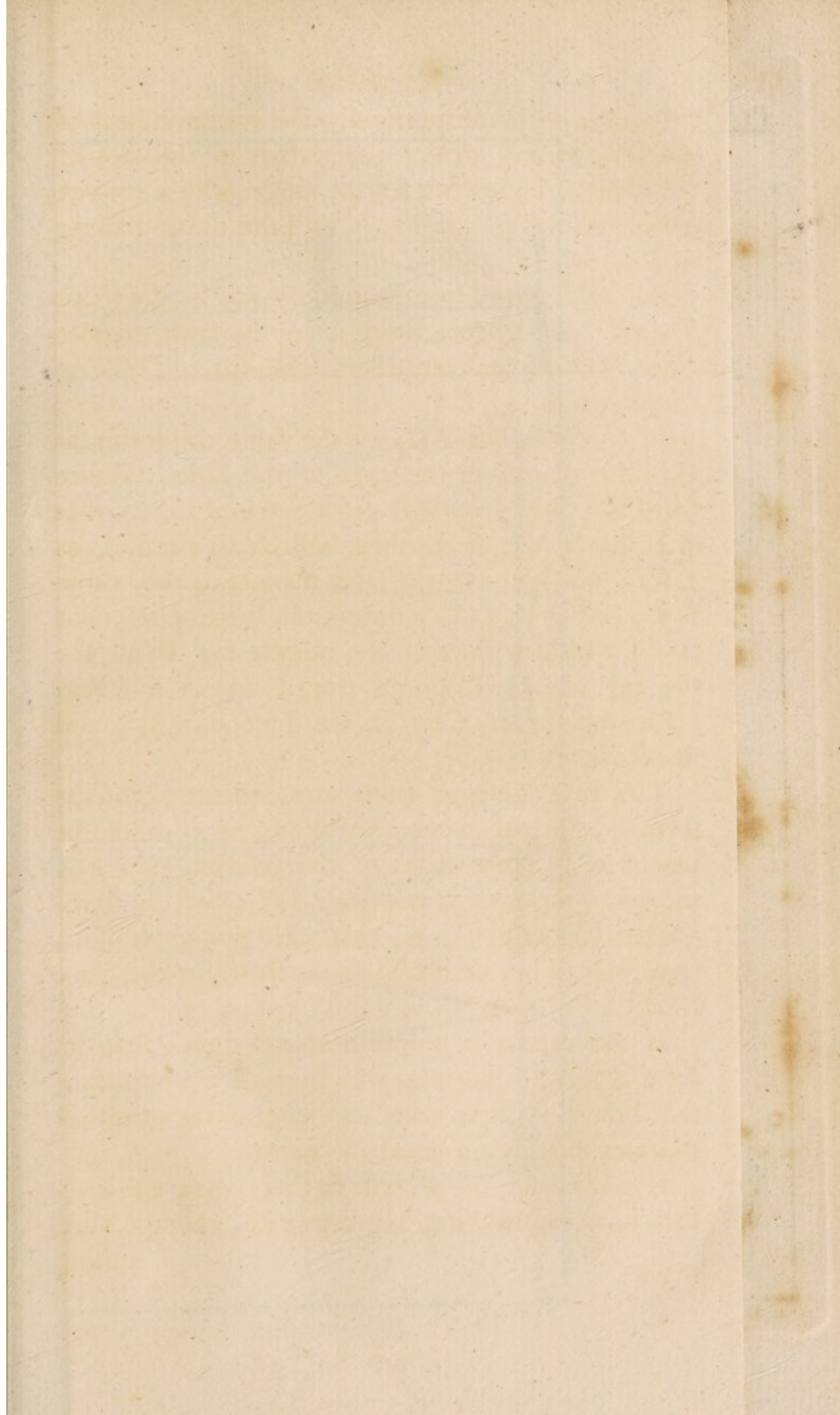
All these rays cross the middle ray in the focus f , and then diverge from it in the same manner FfG , as they converged in the space DfE in coming to it.

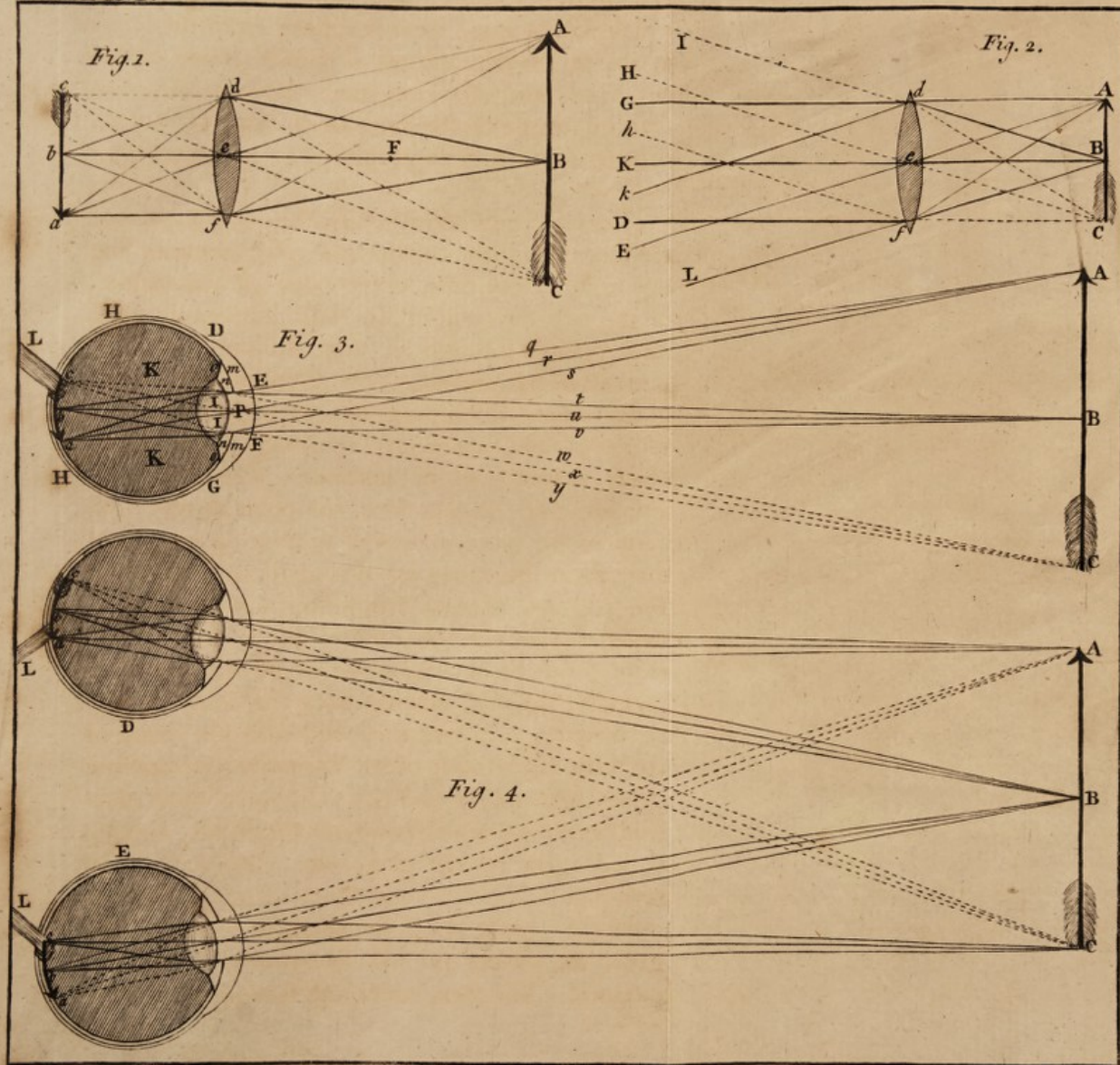
If another glass FG , of the same convexity as DE , be placed in the rays at the same distance from the focus, it will refract them so, as that after going out of it, they will be all parallel, as a, b, c ; and go on in the same manner as they came to the first glass DE , through the space ABC ; but on the contrary sides of the middle ray Bfb : for the ray ADf will go on from f in the direction fGa , and the ray CEf in the direction fEc ; and so of the rest.

The rays diverge from any radiant point, as from a principal focus: therefore, if a candle be placed at f , in the focus of the convex glass FG , the diverging rays in the space FfG , will be so refracted by the glass, as, that after going out of it, they will all become parallel, as shewn in the space cba .

If the candle be placed nearer the glass than its focal distance, the rays will diverge after passing through the glass, more or less, as the candle is more or less distant from the focus.

If the candle be placed farther from the glass than its focal distance, the rays will converge after
passing





passing through the glass, and meet in a point, which will be more or less distant from the glass, as the candle is nearer to, or farther from, its focus: and where the rays meet, they will form an inverted image of the flame of the candle; which may be seen on a paper placed in the meeting of the rays.

Hence, if any object ABc , be placed beyond Plate XVI.
Fig. 1. the focus F , of the convex glass def , some of the rays which flow from every point of the object, on the side next the glass, will fall upon it, and after passing through it, they will be converged into as many points on the opposite side of the glass, where the image of every point will be formed; and consequently, the image of the whole object, which will be inverted. Thus, the rays Ad , Ae , Af , flowing from the point A , will converge in the space daf , and by meeting at a , will there form the image of the point A . The rays Bd , Be , Bf , flowing from the point B , will be united at b by the refraction of the glass, and will there form the image of the point B . And the rays Cd , Ce , Cf , flowing from the point C , will be united at c , where they will form the image of the point C . And so of all the other intermediate points between A and C . The rays which flow from every particular point of the object, and are united again by the glass, are called *a pencil of rays*.

If the object ABC be brought nearer to the glass, the picture abc will be removed to a greater distance. For then, more rays flowing from every
single

single point, will fall more diverging upon the glass; and therefore cannot be so soon collected into the corresponding points behind it. Consequently, if
 Fig. 2. the distance of the object ABC , be equal to the distance eB of the focus of the glass, the rays of each pencil will be so refracted by passing through the glass, that they will go out of it parallel to each other; as I, H, b , from the point C ; G, K, D , from the point B ; and k, E, L , from the point A ; and therefore, there will be no picture formed behind the glass.

If the focal distance of the glass, and the distance of the object from the glass, be known; the distance of the picture from the glass may be found by this rule, *viz.* multiply the distance of the focus by the distance of the object, and divide the product by their difference; the quotient will be the distance of the picture.

Fig. 1. The picture will be as much bigger or less than the object, as its distance from the glass is greater or less than the distance of the object. For, as Be is to eb , so is AC to ac . So that, if ABC be the object, cab will be the picture; or, if cab be the object, ABC will be the picture.

The manner of vision.

Having described how the rays of light, flowing from objects and passing through convex glasses, are collected into points, and form the images of the objects; it will be easy to understand how the rays are effected by passing through the humours of the eye, and are thereby collected into innumerable points on the bottom of the eye, and thereon form the images of the objects which they
 flow

flow from. For, the different humours of the eye, and particularly the chrystalline humour, are to be considered as a convex glass; and the rays in passing through them, to be effected in the same manner as in passing through a convex glass.

The eye is nearly globular. It consists of three Fig. 3. coats and three humours. The part *DHHG* of the outer coat, is called the *sclerotica*, the rest *GFE D* the *cornea*. Next within this coat is that called the *choroides*, which serves as it were for a lining to the other, and joins with the *iris mn, mn*. The *iris* is composed of two sets of muscular fibres; the one of a circular form, which contract the hole in the middle, called the *pupil*, when the light would otherwise be too strong for the eye; and the other of radial fibres, tending every where from the circumference of the iris towards the middle of the pupil; which fibres, by their contraction, dilate and enlarge the pupil when the light is weak, in order to take in the more of its rays. The eye described. The third coat is only a fine expansion of the optic nerve *L*, which spreads like net-work all over the inside of the *choroides*, and is therefore called the *retina*; upon which are painted (as it were) the images of all visible objects, by the rays of light which either flow or are reflected from them.

Under the cornea is a fine transparent fluid like water, which is therefore called the *aqueous humour*. It gives a protuberant figure to the cornea, fills the two cavities *mm* and *nn*, which communicate by the pupil *P*, and has the same limpidity, specific gravity, and refractive power, as water.—At
the

the back of this lies the *chrystalline humour II*, which is shaped like a double convex glass; and is a little more convex on the back than the fore part. It converges the rays, which pass through it from every visible object, to its focus at the bottom of the eye. This humour is transparent like chrystal, is of the consistence of hard jelly, and exceeds the specific gravity of water in the proportion of 11 to 10. It is inclosed in a fine transparent membrane, from which proceed radial fibres *oo*, called the *ligamentum ciliare*, all around its edge; and join to the circumference of the iris. These fibres have a power of contracting and dilating occasionally, by which means they alter the shape or convexity of the chrystalline humour, and also shift it a little backward or forward in the eye, so as to adapt its focal distance at the bottom of the eye to the different distances of objects; without which provision, we could see only these objects distinctly, that were all at one distance from the eye.

At the back of the chrystalline, lies the *vitreous humour K K*, which is transparent like glass, and is largest of all in quantity, filling the whole orb of the eye, and giving it a globular shape. It is much of a consistence with the white of an egg, and very little exceeds the specific gravity and refractive power of water.

As every point of an object *ABC* sends out rays in all directions, some rays from every point on the side next the eye will fall upon the cornea, between *E* and *F*; and by passing on, through the humours
and

and pupil of the eye, they will be converged to as many points on the retina or bottom of the eye, and will thereon form a distinct inverted picture *abc* of the object. Thus, the pencil of rays *qrs*, that flows from the point *A* of the object, will be converged to the point *a* on the retina; those from the point *B* will be converged to the point *b*; those from the point *C* will be converged to the point *c*, and so of all the intermediate points; by which means the whole image *abc* is formed, and the object made visible; although it must be owned, that the method by which this sensation is carried from the eye by the optic nerve to the common sensory in the brain, and there discerned, is above the reach of our comprehension.

But, that vision is effected in this manner, may be demonstrated experimentally. Take a bullock's eye whilst it is fresh, and having cut off the three coats from the back part, quite to the vitreous humour, put a piece of white paper over that part, and hold the eye towards any bright object, and you will see an inverted picture of it upon the paper.

Since the image is inverted, many have wondered why the object appears upright. But we are to consider, 1. that *inverted* is only a relative term; and 2. that there is a very great difference between the real object and the means or image by which we perceive it. When all the parts of a distant prospect are painted upon the retina, they are all right with respect to one another, as well as the parts of the prospect itself; and we can only judge of an object's being inverted, when it is turned reverse to its natural position with respect to other objects which we see
and

and compare it with.—If we lay hold of an upright stick in the dark, we can tell which is the upper or lower part of it, by moving our hand upward or downward; and know very well that we cannot feel the upper end by moving our hand downward. Just so we find by experience, that upon directing our eyes towards a tall object, we cannot see its top by turning our eyes downward, nor its foot by turning our eyes upward; but must trace the object the same way by the eye to see it from head to foot, as we do by the hand to feel it: and as the judgment is informed by the motion of the hand in one case, so it is also by the motion of the eye in the other.

Fig. 4.

In Fig. 4. is exhibited the manner of seeing the same object *ABC*, by both the eyes *D* and *E* at once.

When any part of the image *abc* falls upon the optic nerve *L*, the corresponding part of the object becomes invisible. On which account, nature has wisely placed the optic nerve of each eye not in the middle of the bottom of the eye, but towards the side next the nose: so that, whatever part of the image falls upon the optic nerve of one eye, may not fall upon the optic nerve of the other. Thus, the point *a* of the image *abc*, falls upon the optic nerve of the eye *D*, but not of the eye *E*; and the point *c* falls upon the optic nerve of the eye *E*, but not of the eye *D*: and therefore, to both eyes taken together, the whole object *ABC* is visible.

The nearer that any object is to the eye, the larger is the angle under which it is seen, and the magnitude

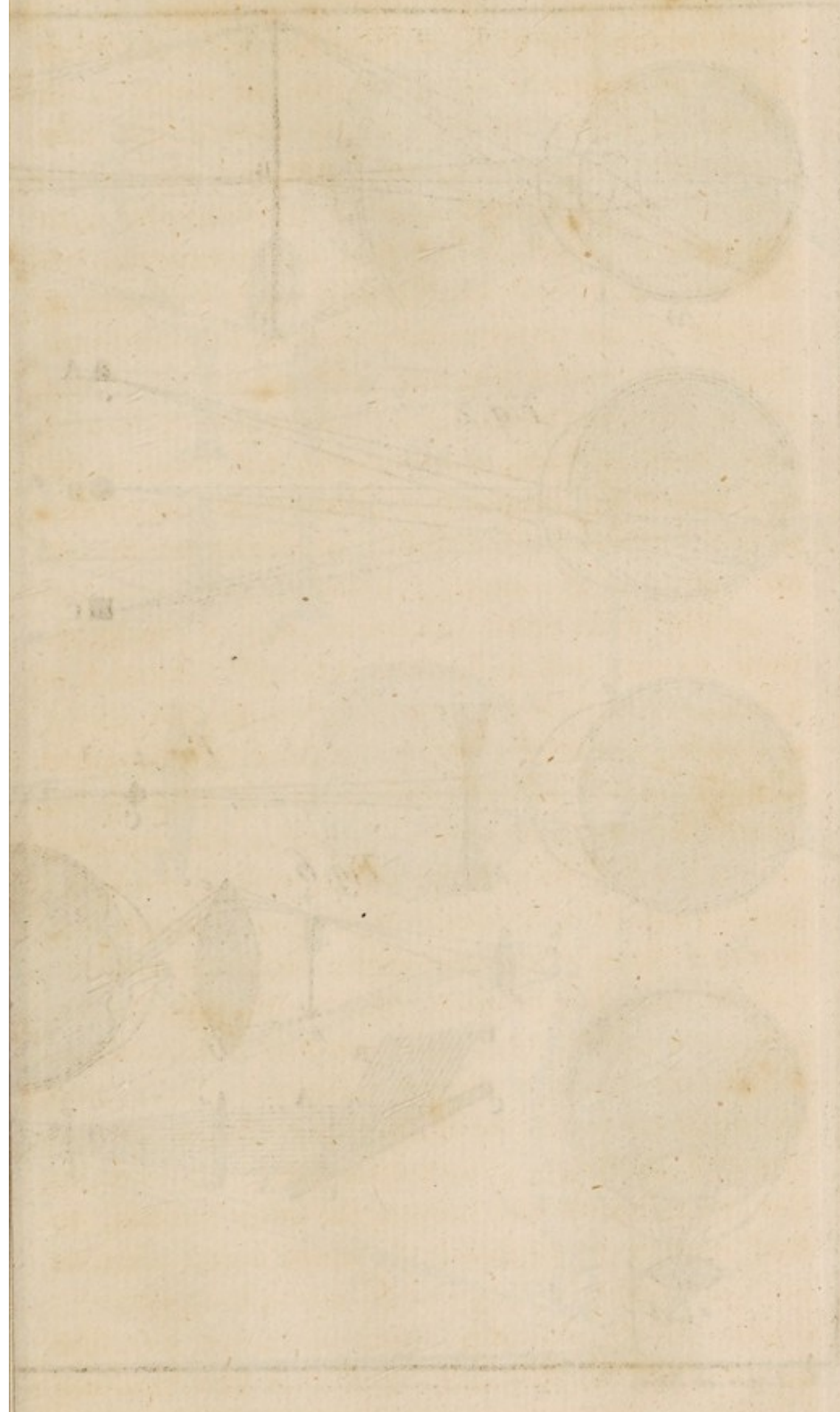
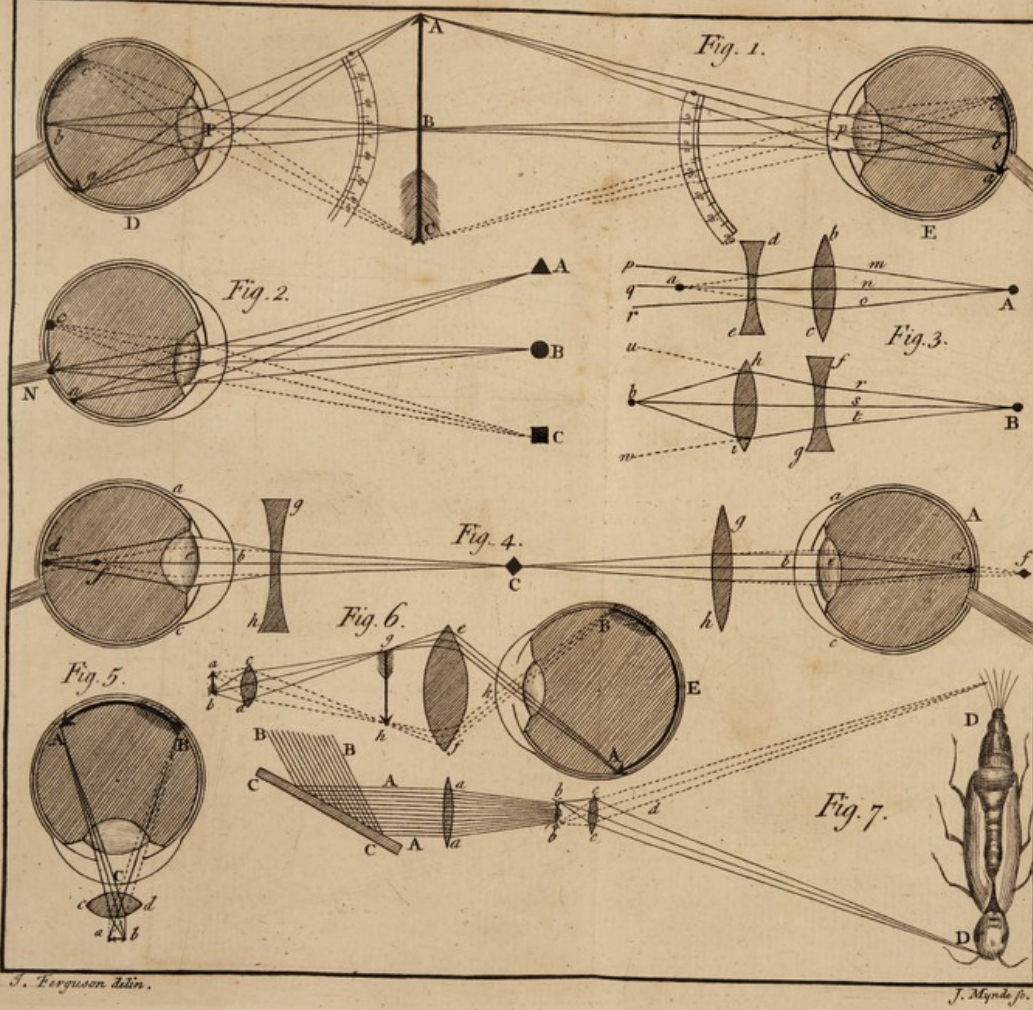


PLATE XVII.



magnitude under which it appears. Thus to the eye *D*, the object *ABC* is seen under the angle *APC*; and its image *abc* is very large upon the retina: But to the eye *E* at a double distance, the same object is seen under the angle *ApC*, which is equal only to half the angle *APC*, as is evident by the figure. The image *abc* is likewise twice as large in the eye *D*, as the image *abc* is in the eye *E*. In both these representations, a part of the image falls on the optic nerve, and the object in the corresponding part is invisible. Plate XVII.
Fig. I.

As the sense of seeing is allowed to be occasioned by the impulse of the rays from the visible object upon the retina of the eye, and forming the image of the object thereon, and that the retina is only the expansion of the optic nerve all over the choroides; it should seem surprising, that the part of the image which falls on the nerve should render the like part of the object invisible; especially, as the nerve is allowed to be the instrument by which the impulse and image are conveyed to the common sensory in the brain. But this difficulty vanishes, when we consider that there is an artery within the trunk of the optic nerve, which entirely obscures the image in that part, and conveys no sensation to the brain.

That the part of the image which falls upon the middle of the optic nerve is lost, and consequently the corresponding part of the object rendered invisible, is plain by experiment. For, if a person fixes three patches *A, B, C*, upon a white wall, at the height of the eye, and the distance of about a foot

Q

from

from each other, and places himself before them, shutting the right eye, and directing the left towards the patch *C*, he will see the patches *A* and *C*, but the middle patch *B* will disappear. Or, if he shuts his left eye, and directs the right towards *A*, he will see both *A* and *C*, but *B* will disappear; and if he directs his eye towards *B*, he will see both *B* and *A*, but not *C*. For whatever patch is directly opposite to the optic nerve *N* vanishes. This requires a little practice, after which he will find it easy to direct his eye so, as to lose the sight of whatever patch he pleases.

We are not commonly sensible of this disappearance, because the motions of the eye are so quick and instantaneous, that we no sooner lose the sight of any part of an object, than we recover it again; much the same as in the twinkling of our eyes, for at each twinkling we are blinded; but it is so soon over, that we are scarce ever sensible of it.

Fig. 4.

Why some eyes require spectacles.

Some eyes require the assistance of convex glasses to make them see objects distinctly, and others of concave. If either the cornea *abc* or chrystalline humour *e*, or both of them, be too flat, as in the eye *A*, their focus will not be on the retina, as at *d*, where it ought to be, in order to render vision distinct; but beyond the eye, as at *f*. And therefore, those rays which flow from the object *C*, and pass through the humours of the eye, are not converged enough to unite at *d*; and therefore the observer can have but a very indistinct view of the object. This is remedied by placing a convex glass

glafs *g b* before the eye, which makes the rays converge sooner, and imprints the image duly on the retina.

If either the cornea, or chryſtalline humour, or both of them, be too convex, as in the eye *B*, the rays that enter it from the object *C*, will be converged to a focus in the vitreous humour, as at *f*; and by diverging from thence to the retina, will form a very confused image thereon: and, ſo of courſe, the obſerver will have as confused a view of the object as if his eye had been too flat. This inconvenience is remedied by placing a concave glaſs *g b* before the eye; which glaſs, by cauſing the rays to diverge between it and the eye, lengthens the focal diſtance ſo; that if the glaſs be properly choſen, the rays will unite at the retina, and form a diſtinct picture of the object upon it.

Such eyes as have their humours of a due convexity, cannot ſee any object diſtinctly at a leſs diſtance than ſix inches; and there are numberleſs objects too ſmall to be ſeen at that diſtance, becauſe they cannot appear under any ſenſible angle. The method of viewing ſuch minute objects is by a *microſcope*, of which there are three forts, *viz.* the *ſingle*, the *double*, and the *ſolar*.

The *ſingle microſcope* is only a ſmall convex glaſs, Fig. 5; as *c d*, having the object *a b* placed in its focus, and the eye at the ſame diſtance on the other ſide; that the rays of each pencil, flowing from every point of the object on the ſide next the glaſs, may go on parallel to the eye, after paſſing through the glaſs; and then, by entering the eye at *C*, they will be ^{The *ſingle* *micro-* *ſcope*.} converged

converged to as many different points on the retina, and form a large inverted picture AB upon it, as in the figure.

To find how much this glass magnifies, divide the least distance (which is about six inches) at which an object can be seen distinctly with the bare eye, by the focal distance of the glass; and the quotient will shew how much the glass magnifies the diameter of the object.

Fig. 6. The *double* or *compound microscope*, consists of an object-glass cd , and an eye-glass ef . The small object ab is placed at a little greater distance from the glass cd than its principal focus, so that the pencils of rays flowing from the different points of the object, and passing through the glass, may be made to converge, and unite in as many points between g and b , where the image of the object will be formed: which image is viewed by the eye through the eye-glass ef . For the eye-glass being so placed, that the image gb may be in its focus, and the eye much about the same distance on the other side, the rays of each pencil will be parallel, after going out of the eye-glass, as at e and f , till they come to the eye at k , where they will begin to converge by the refractive power of the humours; and after having crossed each other in the pupil, and passed through the chrystalline and vitreous humours, they will be collected into points on the retina, and form the large inverted image AB thereon.

The double microscope.

The magnifying power of this microscope is as follows. Suppose the image gb , to be six times the distance of the object ab from the object-glass cd ;
then

then will the image be six times the length of the object : but since the image could not be seen distinctly by the bare eye at a less distance than six inches, if it be viewed by an eye-glass *ef*, of one inch focus, it will thereby be brought six times nearer the eye ; and consequently viewed under an angle six times as large as before ; so that it will be again magnified six times ; that is, six times by the object glass, and six times by the eye-glass ; which multiplied into one another make 36 times : and so much is the object magnified in diameter, more than what it appears to the bare eye ; and consequently 36 times 36, or 1296 times in surface.

But, because the extent or field of view is very small in this microscope, there are generally two eye-glasses, placed sometimes close together, and sometimes an inch asunder ; by which means, although the object appears less magnified, yet the visible area is much enlarged by the interposition of a second eye-glass ; and consequently a much pleasanter view is obtained.

The *solar microscope*, invented by Dr. *Lieberkhn*, Fig. 7. is constructed in the following manner. Having procured a very dark room, let a round hole be made in the window-shutter, about three inches diameter, through which the sun may cast a cylinder of rays *AA* into the room. In this hole, place the end of a tube containing two convex glasses and an object, *viz.* 1. A convex glass *aa*, of about two inches diameter, and three inches focal distance, is to be placed in that end of the tube which is put into

The solar
micro-
scope.

the hole. 2. The object *bb*, being put between two glasses (which must be concave to hold it at liberty) is placed about two inches and an half from the glass *aa*. 3. A little more than a quarter of an inch from the object, is placed the small convex glass *cc*, whose focal distance is a quarter of an inch.

The tube may be so placed, when the sun is low, that his rays *AA* may enter directly into it: but when he is high, his rays *BB* must be reflected into the tube by the plane mirror or looking-glass *CC*.

Things being thus prepared, the rays that enter the tube will be conveyed by the glass *aa* towards the object *bb*, by which means it will be strongly illuminated; and the rays *d* which flow from it, through the convex glass *cc*, will make a large inverted picture of the object at *DD*, which, being received on a white paper, will represent the object magnified in length, in proportion of the distance of the picture from the glass *cc*, to the distance of the object from the same glass. Thus, suppose the distance of the object from the glass to be $\frac{3}{10}$ parts of an inch, and the distance of the distinct picture to be 12 feet or 144 inches, in which there are 1440 tenths of an inch; and this number divided by 3 tenths, gives 480; which is the number of times the picture is longer or broader than the object: and the length multiplied by the breadth shews how much the whole surface is magnified.

Before

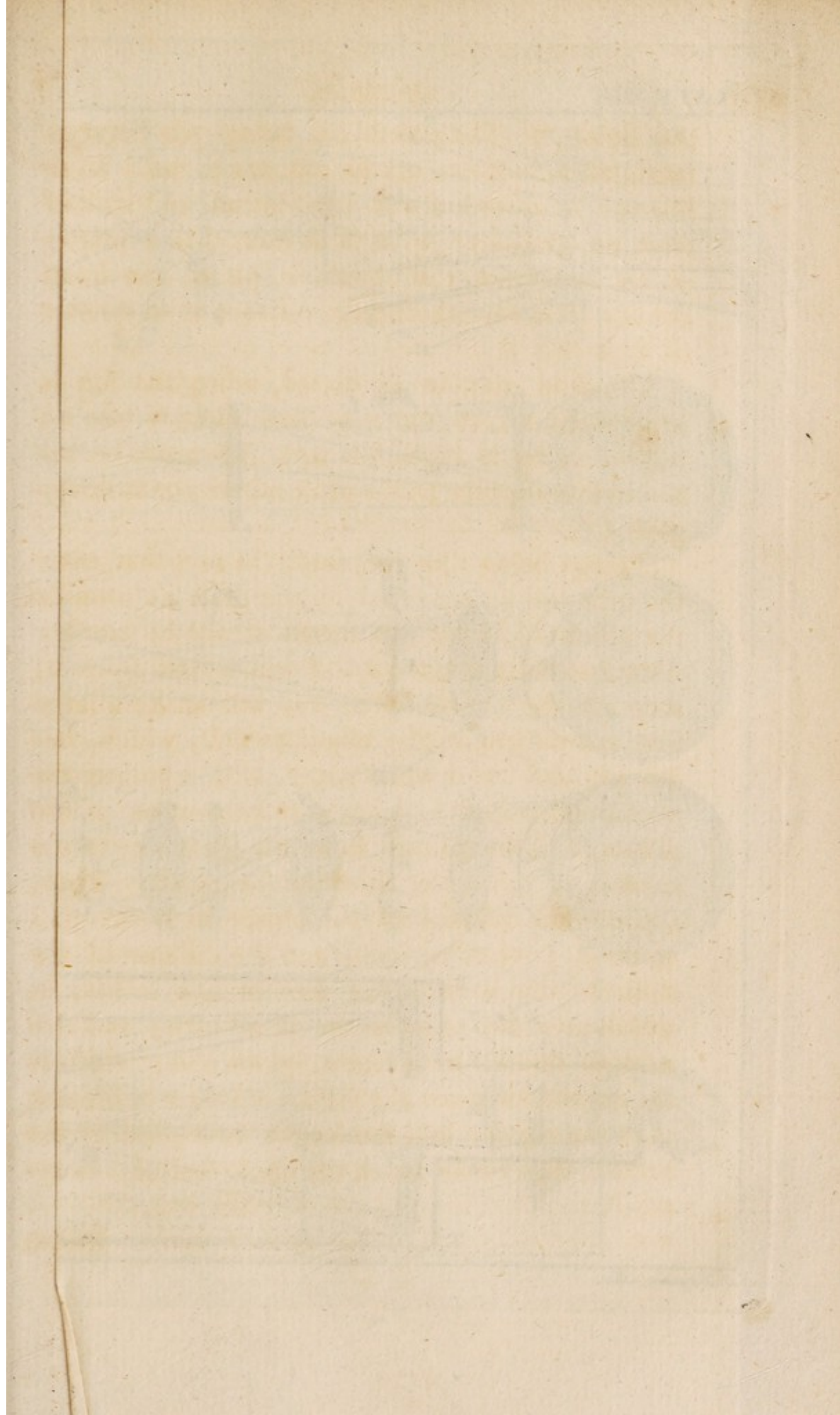
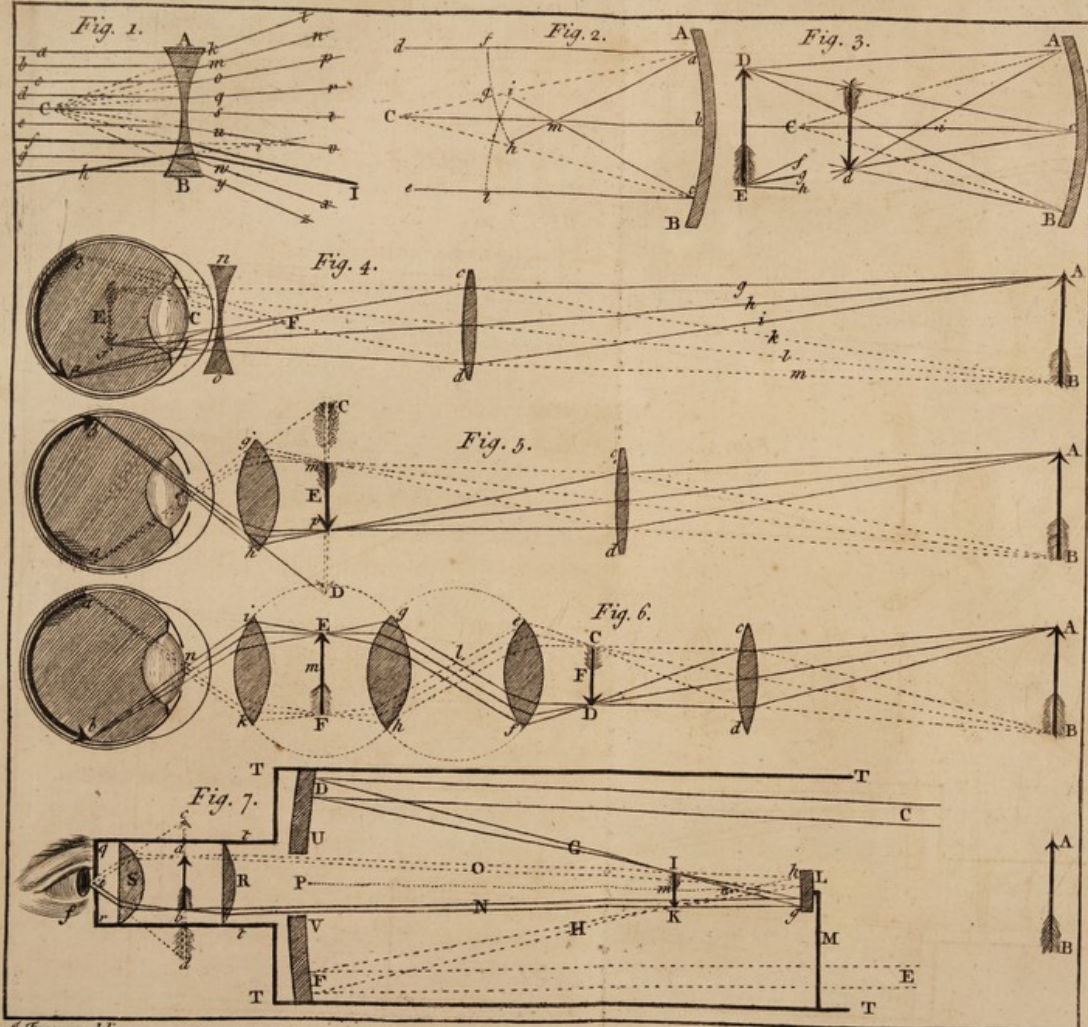


PLATE XVIII.



S. Ferguson delin.

J. Mynde sc.

Before we enter upon the description of tele-^{Telescopes.}scopes, it will be proper to shew how the rays of light are affected by passing through concave glasses, and by falling upon concave mirrors.

When parallel rays, as *abcdefgb*, pass directly ^{Pl. XVIII.} through a glass *AB*, which is equally concave on ^{Fig. 1.} both sides, they will diverge after passing through the glass, as if they had come from a radiant point *C*, in the center of the glass's concavity; which point, is called the negative or virtual focus of the glass. Thus, the ray *a*, after passing through the glass *AB*, will go on in the direction *kl*, as if it had proceeded from the point *C*, and no glass in the way. The ray *b* will go on in the direction *mn*; the ray *c* in the direction *op*, &c.—The ray *C*, that falls directly upon the middle of the glass, suffers no refraction in passing through it; but goes on in the same rectilineal direction, as if no glass had been in its way.

If the glass had been concave only on one side, and the other side quite plane, the rays would have diverged, after passing through it, as if they had come from a radiant point at double the distance of *C* from the glass; that is, as if the radiant had been at the distance of a whole diameter of the glass's concavity.

If rays come more converging to such a glass, than parallel rays diverge after passing through it, they will continue to converge after passing through it; but will not meet so soon as if no glass had been in the way; and will incline towards the same side to which they would have diverged, if they

had come parallel to the glass. Thus, the rays f and b , going in a converging state towards the edge of the glass at B , and converging more in their way to it than the parallel rays diverge after passing through it, they will go on converging after they pass through it; though in a less degree than they did before, and will meet at I : but if no glass had been in their way, they would have met at i .

Fig. 2. When parallel rays, as dfa , Cmb , elc , fall upon a concave mirror AB (which is not transparent, but has only the surface AbB of a clear polish) they will be reflected back from that mirror; and meet in a point m , at half the distance of the surface of the mirror from C , the center of its concavity: for they will be reflected at as great an angle from a perpendicular to the surface of the mirror, as they fell upon it, with regard to that perpendicular; but on the other side thereof. Thus, let C be the center of concavity of the mirror AbB , and let the parallel rays dfa , Cmb , and elc , fall upon it at the points a , b , and c . Draw the lines Cia , Cmb , and Cbc , from the center C to these points; all which will be perpendicular to the surface of the mirror, because they proceed thereto like so many *radii* or spokes from its center. Make the angle Cab equal to the angle dac , and draw the line amb , which will be the direction of the ray dfa , after it is reflected from the point a of the mirror: so that the angle of incidence dac , is equal to the angle of reflection Cab ; the rays making equal angles with the perpendicular Cia on its opposite sides.

Draw

Draw also the perpendicular Cbc to the point c , where the ray elc touches the mirror; and, having made the angle Cci equal to the angle Cce , draw the line cmi , which will be the course of the ray elc , after it is reflected from the mirror.

The ray Cmb passing through the center of concavity of the mirror, and falling upon it at b , is perpendicular to it; and is therefore reflected back from it in the same line bmc .

All these reflected rays meet in the point m ; and in that point the image of the body which emits the parallel rays da , Cb , and ec , will be formed: which point is distant from the mirror equal to half the radius bmc of its concavity.

The rays which proceed from any celestial object may be esteemed parallel at the earth; and therefore, the images of that object will be formed at m , when the reflecting surface of the concave mirror is turned directly towards the object. Hence, the focus m of parallel rays is not in the center of the mirror's concavity, but half way between the mirror and that center.

The rays which proceed from any remote terrestrial object, are nearly parallel at the mirror; not strictly so, but come diverging to it, in separate pencils, or, as it were, bundles of rays, from each point of the side of the object next the mirror: and therefore they will not be converged to a point, at the distance of half the radius of the mirror's concavity from its reflecting surface; but into separate points at a little greater distance from the mirror. And the nearer the object is to the mirror,

rour, the farther these points will be from it; and in them an inverted image of the object will be formed, which will seem to hang pendent in the air; and will be seen by an eye placed beyond it (with regard to the mirrour) in all respects like the object, and as distinct as the object itself.

Fig. 3. Let AcB be the reflecting surface of a mirrour, whose center of concavity is at C ; and let the upright object DE be placed beyond the center C , and send out a conical pencil or bundle of diverging rays from its upper extremity D , to every point of the concave surface of the mirrour AcB . But to avoid confusion, we only draw three rays of that pencil; as DA , Dc , DB .

From the center of concavity C , draw the three right lines CA , Cc , CB , touching the mirrour in the same points where the foresaid rays touch it; and all these lines will be perpendicular to the surface of the mirrour. Make the angle CAd , equal to the angle DAC , and draw the right line Ad for the course of the reflected ray DA : make the angle Ccd equal to the angle DcC , and draw the right line cd for the course of the reflected ray Dc : make also the angle CBd equal to the angle DBC , and draw the right line Bd for the course of the reflected ray DB . All these reflected rays will meet in the point d , where they will form the extremity d of the inverted image ed , similar to the extremity D of the upright object DE .

If the pencil of rays $Efgb$ be also continued to the mirrour, and their angles of reflection from it be made equal to their angles of incidence upon it,

it, as in the former pencil from D , they will all meet at e by reflection, and form the extremity e of the image ed , similar to the extremity E of the object DE .

And as each intermediate point of the object, between D and E , sends out a pencil of rays in like manner to every point of the mirror, the rays of each pencil will be reflected back from it, and meet in as many intermediate points between the extremities e and d of the image; and so the whole image will be formed, not at i , half the distance of the mirror from its center of concavity C ; but at a greater distance, between i and the object DE : and the image will be inverted with respect to the object.

This being well understood, the reader will easily see how the image is formed by the large concave mirror of the reflecting telescope, when he comes to the description of that instrument.

When the object is more remote from the mirror than its center of concavity C , the image will be less than the object, and between the object and mirror: when the object is nearer than the center of concavity, the image will be more remote and bigger than the object: thus, if DE be the object, ed will be its image; but if ed be the object, DE will be its image; for, as the object recedes from the mirror, the image approaches nearer to it; and as the object approaches nearer to the mirror, the image recedes farther from it; on account of the lesser or greater divergency of the pencils of rays which proceed from the object: for, the less

less they diverge, the sooner they are converged to points by reflection; and the more they diverge, the farther they must be reflected before they meet.

If the radius of the mirror's concavity and the distance of the object from it be known, the distance of the image from the mirror is found by this rule; divide the product of the distance and radius, by double the distance made less by the radius, and the quotient is the distance required.

If the object be in the center of the mirror's concavity, the image and object will be coincident, and equal in bulk.

If a man places himself directly before a large concave mirror, but farther from it than its center of concavity, he will see an inverted image of himself in the air, between him and the mirror, of a less size than himself. And if he holds out his hand towards the mirror, the hand of the image will come out towards his hand, and coincide with it, of an equal bulk, when his hand is in the center of concavity; and he will imagine he may shake hands with his image. If he reaches his hand farther, the hand of the image will pass by his hand, and come between his hand and his body: and if he moves his hand towards either side, the hand of the image will move towards the other; so that whatever way the object moves, the image will move the contrary.

All the while a by-stander will see nothing of the image, because none of the reflected rays that form it enter his eyes.

If

If a fire be made in a large room, and a smooth mahogany table be placed at a good distance near the wall, before a large concave mirror, so placed that the light of the fire may be reflected from the mirror to its focus upon the table; if a person stands by the table, he will see nothing upon it but a longish beam of light: but if he stands at a distance towards the fire, not directly between the fire and mirror, he will see an image of the fire upon the table, large and erect. And if another person, who knows nothing of this matter before-hand, should chance to come into the room, and should look from the fire towards the table, he would be startled at the appearance; for the table would seem to be on fire, and by being near the wainscot, to endanger the whole house. In this experiment, there should be no light in the room but what proceeds from the fire; and the mirror ought to be at least fifteen inches in diameter.

If the fire be darkened by a screen, and a large candle be placed at the back of the screen; a person standing by the candle will see the appearance of a fine large star, or rather planet, upon the table as bright as Venus or Jupiter. And if a small wax taper (whose flame is much less than the flame of the candle) be placed near the candle, a satellite to the planet will appear on the table: and if the taper be moved round the candle, the satellite will go round the planet.

For these two pleasing experiments, I am indebted to the reverend Dr. LONG, *Lowndes's* professor of astronomy at Cambridge, who favoured

voured me with the sight of them, and many more of his curious inventions.

The re-
fracting te-
lescope.

In a *refracting telescope*, the glass which is nearest the object in viewing it, is called *the object-glass*; and that which is nearest the eye, is called *the eye-glass*. The object-glass must be convex, but the eye-glass may be either convex or concave: and generally in looking through a telescope, the eye is in the focus of the eye-glass; though that is not very material: for the distance of the eye, as to distinct vision, is indifferent, provided the rays of the pencils fall upon it parallel: only, the nearer the eye is to the end of the telescope, the larger is the scope or area of the field of view.

Fig. 4. Let cd be a convex glass fixed in a long tube, and have its focus at E . Then, a pencil of rays ghi , flowing from the upper extremity A of the remote object AB , will be so refracted by passing through the glass, as to converge and meet in the point f ; whilst the pencil of rays klm , flowing from the lower extremity B , of the same object AB , and passing through the glass, will converge and meet in the point e : and in these points f and e , the images of the points A and B will be formed. And as all the intermediate points of the object, between A and B , send out pencils of rays in the same manner, a sufficient number of these pencils will pass through the object-glass cd , and converge to as many intermediate points between e and f ; and so will form the whole inverted image eEf , of the distinct object. But because this image is small, a concave glass no , is so placed in the end
of

of the tube next the eye, that its virtual focus may be at F . And as the pencils of rays pass converging through the concave glass, but converge less after passing through it than before, they go on further, as to b and a , before they meet; and the pencils themselves being made to diverge by passing through the concave glass, they enter the eye, and form the large picture ab upon the retina, whereon it is magnified under the angle bFa .

But this telescope has one inconveniency which renders it unfit for most purposes, which is, that the pencils of rays being made to diverge by passing through the concave glass *no*, very few of them can enter the pupil of the eye; and therefore the field of view is but very small, as is evident by the figure. For none of the pencils which flow either from the top or bottom of the object AB can enter the pupil of the eye at C , but are all stopt by falling upon the iris above and below the pupil: and therefore, only the middle part of the object can be seen when the telescope lies directly towards it, by means of those rays which proceed from the middle of the object. So that to see the whole of it, the telescope must be moved upwards and downwards, unless the object be very remote; and then it is never seen distinctly.

This inconvenience is remedied by substituting a Fig. 5. convex eye-glass, as gb , in place of the concave one; and fixing it so in the tube, that its focus may be coincident with the focus of the object-glass cd , as at E . For then, the rays of the pencils flowing from the object AB , and passing through

through the object-glass cd , will meet in its focus, and form the inverted image mEp : and as the image is formed in the focus of the eye-glass gb , the rays of each pencil will be parallel, after passing through that glass; but the pencils themselves will cross in its focus on the other side, as at e : and the pupil of the eye being in this focus, the image will be viewed through the glass, under the angle geb ; and being at E , it will appear magnified, so as to fill the whole space $CmepD$.

But, as this telescope inverts the image with respect to the object, it gives an unpleasant view of terrestrial objects; and is only fit for viewing the heavenly bodies, in which we regard not their position, because their being inverted does not appear, on account of their being round. But whatever way the object seems to move, this telescope must be moved the contrary way, in order to keep sight of it; for, since the object is inverted, its motion will be so too.

The magnifying power of this telescope is, as the focal distance of the object-glass to the focal distance of the eye-glass. Therefore, if the former be divided by the latter, the quotient will express the magnifying power.

When we speak of the magnifying of a telescope or microscope, it is only meant with regard to the diameter, not the area nor solidity of the object. But as the instrument magnifies the vertical diameter, as much as it does the horizontal, it is easy to find how much the whole visible area or surface is magnified: for, if the diameters be multiplied into one

one another, the product will express the magnification of the whole visible area. Thus, suppose the focal distance of the object-glass be ten times as great as the focal distance of the eye-glass; then, the object will be magnified ten times, both in length and breadth: and 10 multiplied by 10, produces 100; which shews, that the area of the object will appear 100 times as big when seen through such a telescope, as it does to the bare eye.

Hence it appears, that if the focal distance of the eye-glass were equal to the focal distance of the object-glass, the magnifying power of the telescope would be nothing.

This telescope may be made to magnify in any given degree, provided it be of a sufficient length. For, the greater the focal distance of the object-glass, the less may be the focal distance of the eye-glass; though not directly in proportion. Thus, an object-glass, of 10 feet focal distance, will admit of an eye-glass, whose focal distance is little more than $2\frac{1}{2}$ inches; which will magnify near 48 times: but an object-glass, of 100 feet focus, will require an eye-glass somewhat more than 6 inches; and will therefore magnify almost 200 times.

A telescope for viewing terrestrial objects, should be so constructed, as to shew them in their natural posture. And this is done by one object-glass *cd*, Fig. 6. and three eye-glasses *ef*, *gb*, *ik*, so placed, that the distance between any two, which are nearest to each other, may be equal to the sum of their focal distances; as in the figure, where the focus of the glasses *cd* and *ef* meet at *F*, those of the glasses *ef*
R and

and gb , meet at l , and of gb and ik , at m ; the eye being at n , in or near the focus of the eye-glass ik , on the other side. Then, it is plain, that these pencils of rays, which flow from the object AB , and pass through the object-glass cd , will meet and form an inverted image CFD in the focus of that glass; and the image being also in the focus of the glass ef , the rays of the pencils will become parallel, after passing through that glass, and cross in its opposite focus at l ; from whence they pass on to the next glass gb , and by going through it they are converged to points in its other focus, where they form an erect image EmF , of the object AB : and as this image is also in the focus of the eye-glass ik , and the eye on the opposite side of the same glass; the image is viewed through the eye-glass in this telescope, in the same manner as through the eye-glass in the former one; only in a contrary position, that is, in the same position with the object.

The three glasses next the eye, have all their focal distances equal: and the magnifying power of this telescope, is found the same way as that of the last above; *viz.* by dividing the focal distance of the object-glass cd , by the focal distance of the eye-glass ik , or gb , or ef , since all these three are equal.

Why the
object ap-
pears co-
loured
when seen
through a
telescope.

When the rays of light are separated by refraction, they become coloured, and if they be united again, they will be a perfect white. But those rays which pass through a convex glass, near its edges, are more unequally refracted than those
which

which are nearer the middle of the glafs. And when the rays of any pencil are unequally refracted by the glafs, they do not all meet again in one and the ſame point, but in ſeparate points; which makes the image indiftinct, and coloured, about its edges. The remedy is, to have a plate with a ſmall round hole in its middle, fixed in the tube at *m*, parallel to the glaſſes. For, the wandering rays about the edges of the glaſſes will be ſtopt, by the plate, from coming to the eye; and none admitted but thoſe which come through the middle of the glafs, or at leaſt at a good diſtance from its edges, and paſs through the hole in the middle of the plate. But this circumscribes the image, and leſſens the field of view, which would be much larger if the plate could be diſpenſed with.

The great inconvenience attending the management of long teleſcopes of this kind, has brought them much into diſuſe ever ſince the *reflecting te-* The re-
flecting te-
leſcope. *leſcope* was invented. For one of this ſort, fix feet in length, magnifies as much as one of the other an hundred. It was invented by Sir *Iſaac Newton*, but has received conſiderable improvements ſince his time; and is now generally conſtructed in the following manner, which was firſt propoſed by Dr. *Gregory*.

At the bottom of the great tube *TTTT*, is placed the large concave mirrour *DUVF*, Fig. 7. whoſe principal focus is at *m*; and in its middle is a round hole *P*, oppoſite to which is placed the ſmall mirrour *L*, concave toward the great one; and ſo fixed to a ſtrong wire *M*, that it may be moved

farther from the great mirrour, or nearer to it, by means of a long screw on the outside of the tube, keeping its axis still in the same line Pmn with that of the great one.—Now, since in viewing a very remote object, we can scarce see a point of it but what is at least as broad as the great mirrour, we may consider the rays of each pencil, which flow from every point of the object, to be parallel to each other, and to cover the whole reflecting surface $DUVF$. But to avoid confusion in the figure, we shall only draw two rays of a pencil flowing from each extremity of the object into the great tube, and trace their progress, through all their reflections and refractions, to the eye f , at the end of the small tube tt , which is joined to the great one.

Let us then suppose the object AB to be at such a distance, that the rays C may flow from its lower extremity B , and the rays E from its upper extremity A . Then, the rays C falling parallel upon the great mirrour at D , will be thence reflected converging, in the direction DG ; and by crossing at I in the principal focus of the mirrour, they will form the upper extremity I of the inverted image IK , similar to the lower extremity B of the object AB : and passing on to the concave mirrour L (whose focus is at n) they will fall upon it at g , and be thence reflected converging, in the direction gN , because gm is shorter than gn ; and passing through the hole P in the large mirrour, they would meet somewhere about r , and form the lower extremity b of the erect image ab , similar to the
lower

lower extremity B of the object AB . But by passing through the plano-convex glass R in their way, they form that extremity of the image at b . In like manner, the rays E , which come from the top of the object AB , and fall parallel upon the great mirror at F , are thence reflected converging to its focus, where they form the lower extremity K of the inverted image IK , similar to the upper extremity A of the object AB ; and thence passing on to the small mirror L , and falling upon it at b , they are thence reflected in the converging state bo ; and going on through the hole P of the great mirror, they would meet somewhere about q , and form there the upper extremity a of the erect image ab , similar to the upper extremity A of the object AB : but by passing through the convex-glass R in their way, they meet and cross sooner, as at a , where that point of the erect image is formed.—The like being understood of all those rays which flow from the intermediate points of the object, between A and B , and enter the tube TT ; all the intermediate points of the image between a and b will be formed: and the rays passing on from the image, through the eye-glass S , and through a small hole e in the end of the lesser tube tt , they enter the eye f , which sees the image ab (by means of the eye-glass) under the large angle ced , and magnified in length, under that angle, from c to d .

In the best reflecting telescopes, the focus of the small mirror is never coincident with the focus m of the great one, where the first image IK is formed, but a little beyond it (with respect to the

eye) as at n : the consequence of which is, that the rays of the pencils will not be parallel after reflection from the small mirror, but converge so as to meet in points about q, e, r ; where they would form a larger upright image than ab , if the glass R was not in their way: and this image might be viewed by means of a single eye-glass properly placed between the image and the eye: but then the field of view would be less, and consequently not so pleasant; for which reason, the glass R is still retained, to enlarge the scope or area of the field.

To find the magnifying power of this telescope, multiply the focal distance of the great mirror by the distance of the small mirror from the image next the eye, and multiply the focal distance of the small mirror by the focal distance of the eye-glass: then, divide the product of the former multiplication by the product of the latter, and the quotient will express the magnifying power.

I shall here set down the dimensions of one of Mr. *Short's* reflecting telescopes, as it is mentioned in Dr. *Smith's* optics.

The focal distance of the great mirror 9.6 inches, its breadth 2.3; the focal distance of the small mirror 1.5, its breadth 0.6: the breadth of the hole in the great mirror 0.5; the distance between the small mirror and the next eye-glass 14.2; the distance between the two eye-glasses 2.4; the focal distance of the eye-glass next the metals 3.8;

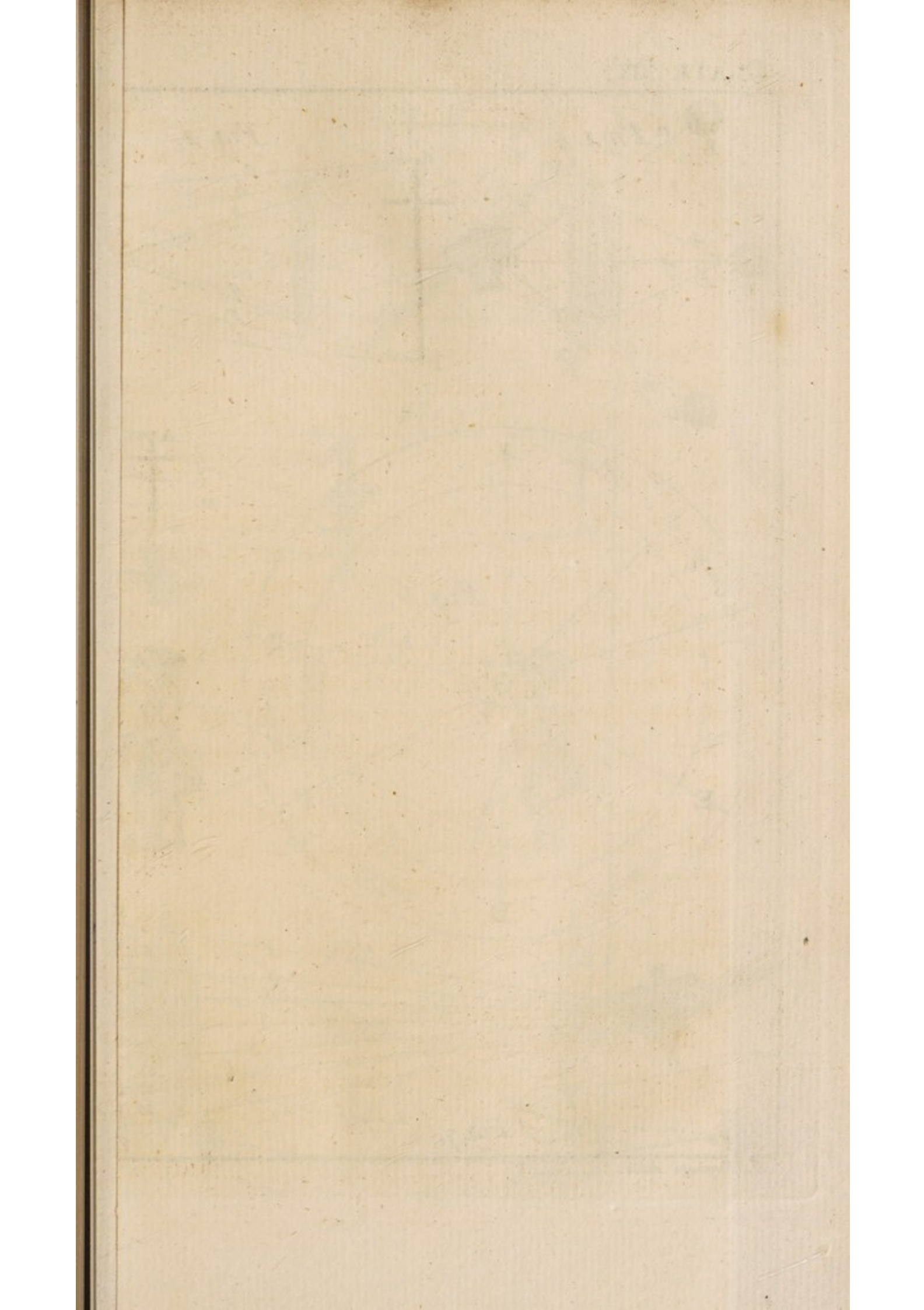
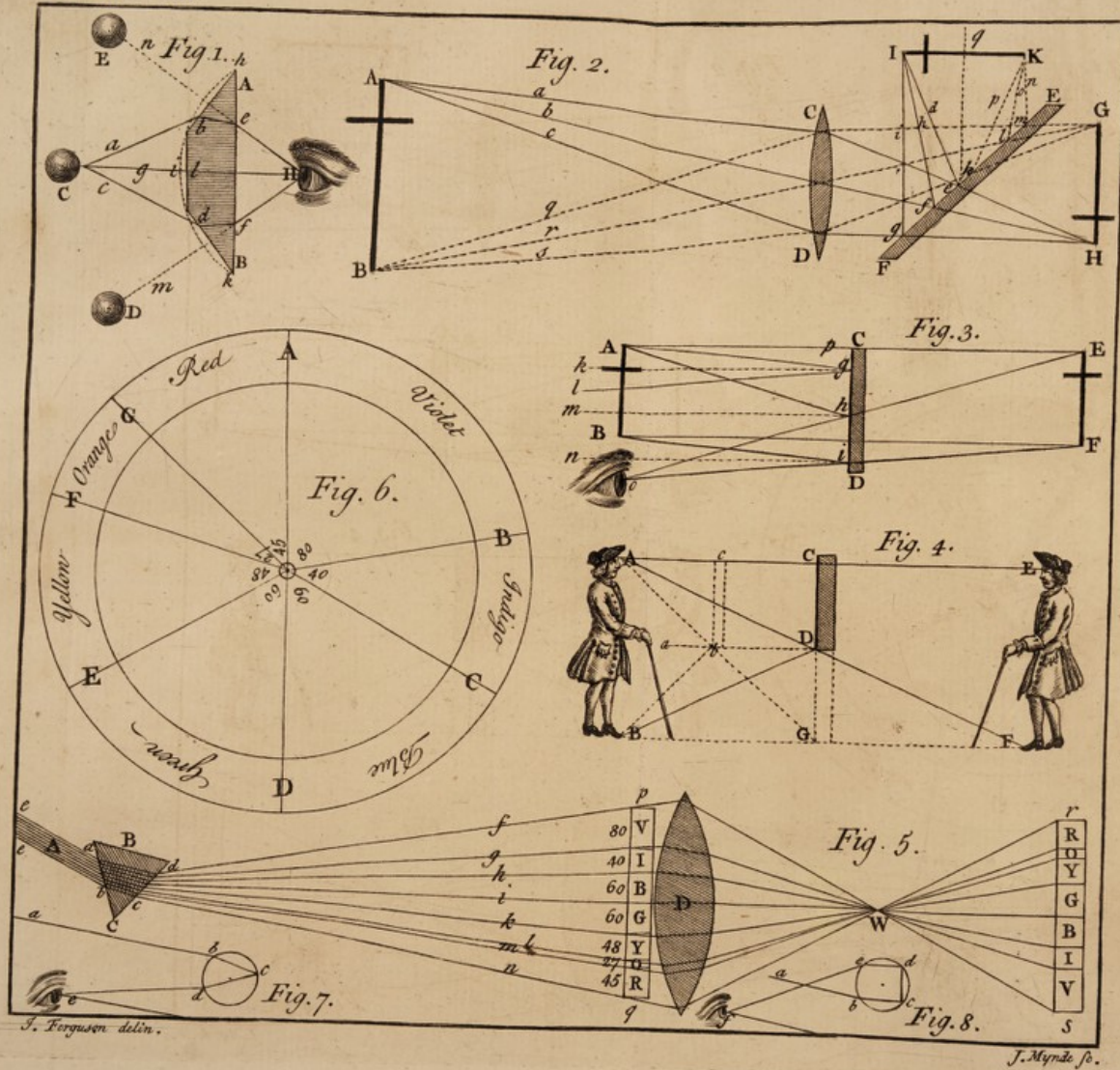
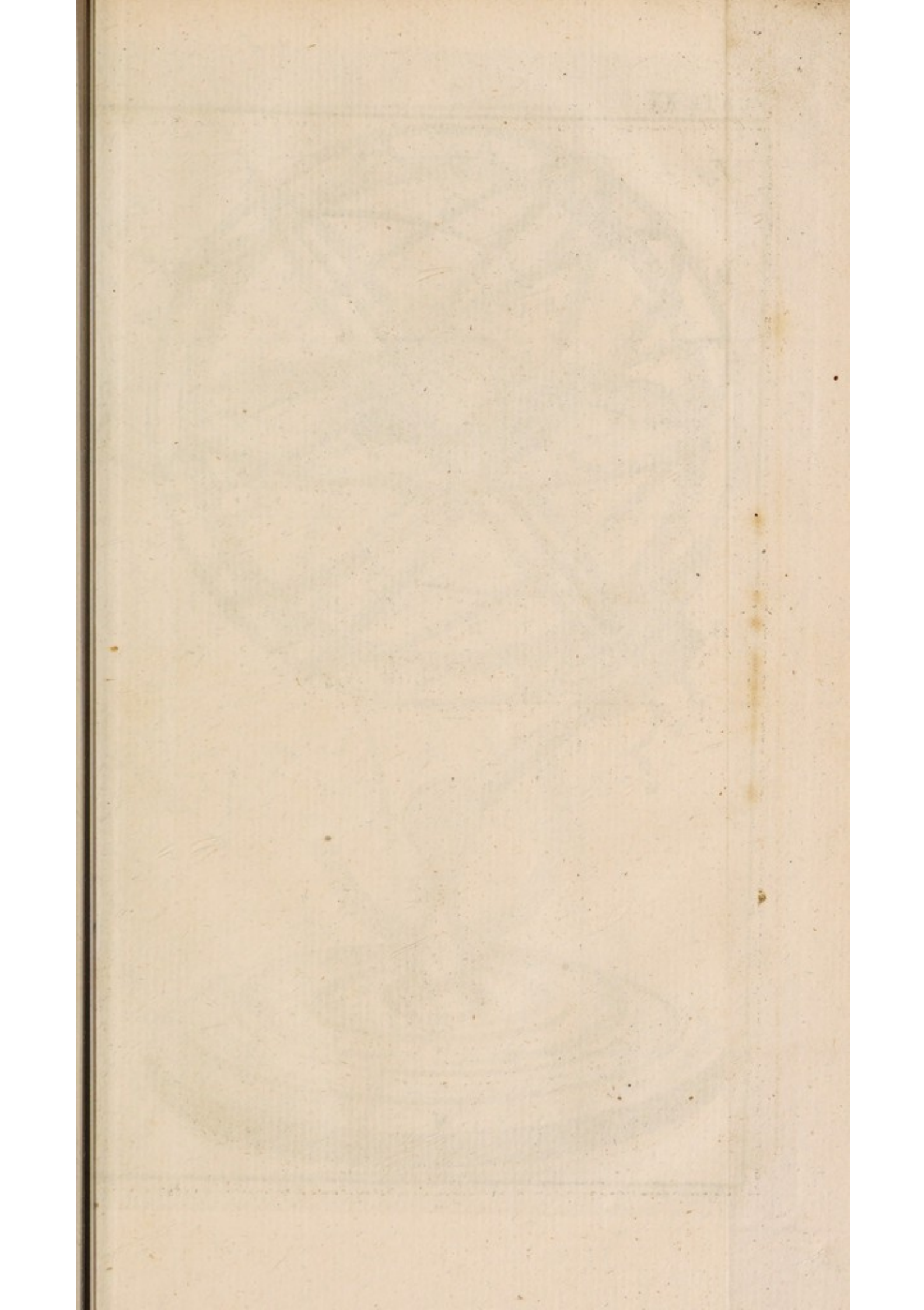
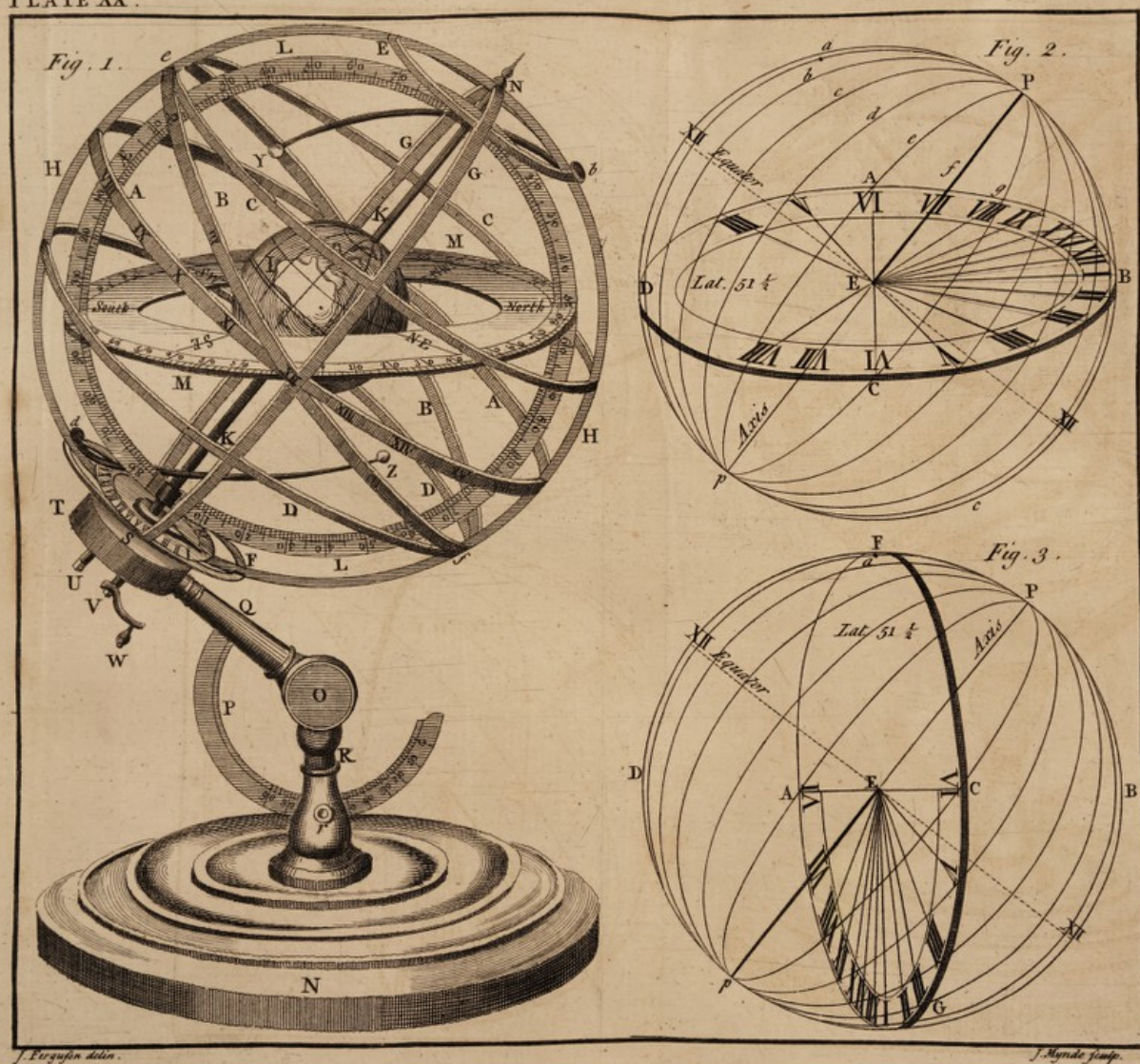


PLATE XIX.







3.8 ; and the focal distance of the eye-glass next the eye 1.1.

One great advantage of the reflecting telescope is, that it will admit of an eye-glass of a much shorter focal distance than a refracting telescope will ; and, consequently, it will magnify so much the more : for the rays are not coloured by reflection from a concave mirror, if it be ground to a true figure, as they are by passing through a convex-glass, let it be ground ever so true.

The adjusting screw on the outside of the great tube fits this telescope to all sorts of eyes, by bringing the small mirror either nearer to the eye, or removing it farther : by which means, the rays are made to diverge a little for short sighted eyes, or to converge for those of a long sight.

The nearer an object is to the telescope, the more its pencils of rays will diverge before they fall upon the great mirror, and therefore they will be the longer of meeting in points after reflection ; so that the first image *IK* will be formed at a greater distance from the large mirror, when the object is near the telescope, than when it is very remote. But as this image must be formed farther from the small mirror than its principal focus *n*, this mirror must be always set at a greater distance from the large one, in viewing near objects, than in viewing remote ones. And this is done by turning the screw on the outside of the tube, until the small mirror be so adjusted, that the object (or rather its image) appears perfect.

In looking through any telescope towards an object, we never see the object itself, but only that image of it which is formed next the eye in the telescope. For, if a man holds his finger or a stick between his bare eye and an object, it will hide part (if not the whole) of the object from his view. But if he ties a stick across the mouth of a telescope, before the object-glass, it will hide no part of the imaginary object he saw through the telescope before, unless it covers the whole mouth of the tube: the only effect it will have, is, to make the object appear dimmer, because it intercepts part of the rays. Whereas, if he puts only a piece of wire across the inside of the tube, between the eye-glass and his eye, it will hide part of the object which he thinks he sees: which proves that he sees not the real object, but its image. This is also confirmed by means of the small mirror *L*, in the reflecting telescope, which is made of opaque metal, and stands directly between the eye and the object towards which the telescope is turned: and will hide the whole object from the eye at *e*, if the two glasses *R* and *S* are taken out of the tube.

Plate XIX. The multiplying glass is made by grinding
 Fig. 1. down the round side *bik* of a convex glass *AB*,
 The multi- into several flat surfaces, as *bb*, *bld*, *dk*. An
 plying glass. object *C* will not appear magnified, when seen through this glass, by the eye at *H*; but it will appear multiplied into as many different objects as the glass contains plane surfaces. For, since rays will flow from the object *C* to all parts of the glass,

glafs, and each plane surface will refract these rays to the eye, the same object will appear to the eye, in the direction of the rays which enter it through each surface. Thus, a ray giH , falling perpendicularly on the middle surface, will go through the glafs to the eye without suffering any refraction; and will therefore shew the object in its true place at C : whilst a ray ab flowing from the same object, and falling obliquely on the plane surface bb , will be refracted in the direction be , by passing through the glafs; and upon leaving it, will go on to the eye in the direction eH ; which will cause the same object C to appear also at E , in the direction of the ray He , produced in the right line Hen . And the ray cd , flowing from the object C , and falling obliquely on the plane surface dk , will be refracted (by passing through the glafs and leaving it at f) to the eye at H ; which will cause the same object to appear at D , in the direction Hfm .—If the glafs be turned round the line glH , as an axis, the object C will keep its place, because the surface bld is not removed; but all the other objects will seem to go round C , because the oblique planes, on which the rays ab , cd fall, will go round by the turning of the glafs.

The *camera obscura* is made by a convex glafs Fig. 2.
 CD , placed in a hole of a window-shutter. Then, The camera
obscura.
 if the room be darkened so, as no light can enter but what comes through the glafs, the pictures of all the objects (as fields, trees, buildings, men, cattle, &c.) on the outside, will be shewn in an inverted order, on a white paper placed at GH in
 the

the focus of the glass: and will afford a most beautiful and perfect piece of perspective or landscape of whatever is before the glass; especially if the sun shines upon the objects.

If the convex glass CD be placed in a tube in the side of a square box, within which is the plane mirror EF , reclining backwards in an angle of 45 degrees from the perpendicular kq , the pencils of rays flowing from the outward objects, and passing through the convex-glass to the plane mirror, will be reflected upwards from it, and meet in points as I and K (at the same distance that they would have met at H and G , if no mirror had been in the way) and will form the aforesaid images on an oiled paper stretched horizontally in the direction IK ; on which paper, the out-lines of the images may be easily drawn with a black lead pencil; and then copied on a clean sheet, and coloured by art, as the objects themselves are by nature.—In this machine, it is usual to place a plane glass unpolished, in the horizontal situation IK , which glass receives the images of the outward objects; and their out-lines may be traced upon it by a black lead pencil.

N. B. The tube in which the convex-glass CD is fixed, must be made to draw out, or push in, so as to adjust the distance of that glass from the plane mirror, in proportion to the distance of the outward objects; which the operator does, until he sees their images distinctly painted on the horizontal glass at IK .

The

The forming a horizontal image, as IK , of an upright object AB , depends upon the angles of incidence of the rays upon the plane mirror EF , being equal to their angles of reflection from it. For, if a perpendicular be supposed to be drawn to the surface of the plane mirror at e , where the ray $AaCe$ falls upon it, that ray will be reflected upwards in an equal angle with the other side of the perpendicular, in the line edI . Again, if a perpendicular be drawn to the mirror from the point f , where the ray Abf falls upon it, that ray will be reflected in an equal angle from the other side of the perpendicular, in the line fbI . And if a perpendicular be drawn from the point g , where the ray Acg falls upon the mirror, that ray will be reflected in an equal angle from the other side of the perpendicular, in the line giI . So that all the rays of the pencil abc , flowing from the upper extremity of the object AB , and passing through the convex-glass CD , to the plane mirror EF , will be reflected from the mirror, and meet at I , where they will form the extremity I of the image IK , similar to the extremity A of the object AB . The like is to be understood of the pencil qrs , flowing from the lower extremity of the object AB , and meeting at K (after reflection from the plane mirror) the rays form the extremity K of the image, similar to the extremity B of the object: and so of all the pencils that flow from the intermediate points of the object to the mirror, through the convex glass.

If

The opera-
glass.

If a convex-glass, of a short focal distance, be placed near the plane mirror, in a short tube, and a convex-glass be placed in a hole in the side of the tube, so as the image may be formed between the last mentioned convex-glass, and the plane mirror; the image being viewed through this glass, will appear magnified.—In this manner, the *opera-glasses* are constructed; with which a gentleman may look at any lady at a distance in the company, and the lady know nothing of it.

The com-
mon looking
glass.

The image of any object that is placed before a plane mirror, appears as big to the eye as the object itself; and is erect, distinct, and seemingly as far behind the mirror, as the object is before it: and that part of the mirror, which reflects the image of the object to the eye, (the eye being supposed equally distant from the glass with the object) is just half as long and half as broad as the object itself. Let AB be an object placed before the reflecting surface gbi of the plane mirror CD ; and let the eye be at o . Let Ab be a ray of light flowing from the top A of the object, and falling upon the mirror at b ; and bm be a perpendicular to the surface of the mirror at b : the ray Ab will be reflected from the mirror to the eye at o , making an angle mbo equal to the angle Abm : then will the top of the image E appear to the eye in the direction of the reflected ray ob produced to E , where the right line ApE , from the top of the object, cuts the right line obE , at E . Let Bi be a ray of light proceeding from the foot of the object at B to the mirror at i ; and

Fig. 3.

ni a perpendicular to the mirrour from the point i , where the ray Bi falls upon it: this ray will be reflected in the line io , making an angle nio , equal to the angle Bin , with that perpendicular, and entering the eye at o : then will the foot F of the image appear in the direction of the reflected ray oi , produced to F , where the right line BF cuts the reflected ray produced to F . All the other rays that flow from the intermediate points of the object AB , and fall upon the mirrour between b and i , will be reflected to the eye at o ; and all the intermediate points of the image EF will appear to the eye in the direction of these reflected rays produced. But all the rays that flow from the object, and fall upon the mirrour above b , will be reflected back above the eye at o ; and all the rays that flow from the object, and fall upon the mirrour below i , will be reflected back below the eye at o : so that none of the rays that fall above b , or below i , can be reflected to the eye at o ; and the distance between b and i is equal to half the length of the object AB .

Hence it appears, that if a man sees his whole image in a plane looking-glass, the part of the glass that reflects his image must be just half as long and half as broad as himself; let him stand at any distance from it whatever: and that his image must appear just as far behind the glass as he is before it. Thus, the man AB viewing himself in the plane mirrour CD , which is just half as long as himself, sees his whole image as at EF , behind the glass, exactly equal to his own size.

A man will see his image in a plane looking-glass, that is but half his height.

Fig. 4.

For,

For, a ray AC , proceeding from his eye at A , and falling perpendicularly upon the surface of the glass at C , is reflected back to his eye in the same line CA ; and the eye of his image will appear at E , in the same line produced to E , beyond the glass. And a ray BD , flowing from his foot, and falling obliquely on the glass at D , will be reflected as obliquely on the other side of the perpendicular abD , in the direction DA ; and the foot of his image will appear at F , in the direction of the reflected ray AD , produced to F , where it is cut by the right line BGF , drawn parallel to the right line ACE . Just the same as if the glass were taken away, and a real man stood at F , equal in size to the man standing at B : for to his eye at A , the eye of the other man at E would be seen in the direction of the line ACE ; and the foot of the man at F would be seen by the eye A , in the direction of the line ADF .

If the glass be brought nearer the man AB , as suppose to cb , he will see his image as at CDG : for the reflected ray CA (being perpendicular to the glass) will shew the eye of the image as at C ; and the incident ray Bb , being reflected in the line bA , will shew the foot of his image as at G ; the angle of reflection abA being always equal to the angle of incidence Bba : and so of all the intermediate rays from A to B . Hence, if the man AB advances towards the glass CD , his image will approach towards it; and if he recedes from the glass, his image will also recede from it.

Having

Having already shewn, that the rays of light are refracted when they pass obliquely through different mediums, we come now to prove that some rays are more refracted than others; and that, as they are differently refracted, they excite in our minds the ideas of different colours. This will account for the colours seen about the edges of the images of those objects which are viewed through telescopes.

Let the sun shine into a dark room through a small hole, as at *ee*, in a window-shutter; and place a triangular prism *BC*, in the beam of rays *A*, in such a manner, that the beam may fall obliquely on one of the sides *abC* of the prism. The rays will suffer different refractions by passing through the prism, so that instead of going all out of it on the side *dcC*, in one direction, they will go on from it in the different directions represented by the lines *f, g, h, i, k, l, m, n*; and falling upon the opposite side of the room, or on a white paper placed as at *pq*, to receive them, they will paint upon it a series of most beautiful lively colours (not to be equalled by art) in this order, *viz.* those which are least refracted by the prism, and will therefore go on between the lines *n* and *m*, will be of a very bright intense red at *n*, degenerating from thence gradually into an orange colour, as they are nearer the line *m*: the next will be of a fine orange colour at *m*, and from thence degenerate into a yellow colour towards *l*: at *l* they will be of a fine yellow, which will incline towards a green, more and more, as they are nearer and nearer

Fig. 5.

The prism.

The colours of the light.

nearer k : at k they will be a pure green, but from thence towards i they will incline gradually to a blue : at i they will be a perfect blue, inclining to an indigo-colour, from thence towards b : at b they will be quite the colour of *indigo*, which will gradually change towards a violet, the nearer they are to g : and at g they will be of a fine violet colour, which will incline gradually to a red, the nearer they are to f , where the coloured image ends.

There is not an equal quantity of rays in each of these colours ; for, if the oblong image pq be divided into 360 equal parts, the red space R will take up 45 of these parts ; the orange O , 27 ; the yellow Y , 48 ; the green G , 60 ; the blue B , 60 ; the indigo I , 40 ; and the violet V , 80 : all which spaces are as nearly proportioned in the figure as the small space qp would admit of.

If all these colours be blended together again, they will make a pure white ; as is proved thus. Take away the paper on which the colours pq fell, and place a large convex-glass D in the rays f, g, b , &c. which will refract them so, as to make them unite and cross each other at W : and if a white paper be placed there to receive them, they will excite the idea of a strong lively white. But if the paper be placed farther from the glass, as at rs , the different colours will appear again upon it, in an inverted order, occasioned by the rays crossing at W .

As white is a composition of all colours, so black is a privation of them all, and, therefore, properly no colour. Let

Let two concentric circles be drawn on a smooth round board *ABCDEFGG*, and the outermost of them divided into 360 equal parts or degrees: then, draw seven right lines, as $\odot A$, $\odot B$, &c. from the center to the outermost circle, making the lines $\odot A$ and $\odot B$ include 80 degrees of that circle; the lines $\odot B$ and $\odot C$ 40 degrees; $\odot C$ and $\odot D$ 60; $\odot D$ and $\odot E$ 60; $\odot E$ and $\odot F$ 48; $\odot F$ and $\odot G$ 27; $\odot G$ and $\odot A$ 45. Then, between these two circles, paint the space *AG* red, inclining to orange near *G*; *GF* orange, inclining to yellow near *F*; *FE* yellow, inclining to green near *E*; *ED* green, inclining to blue near *D*; *DC* blue, inclining to indigo near *C*; *CB* indigo, inclining to violet near *B*; and *BA* violet, inclining to a soft red near *A*. This done, paint all that part of the board black which lies within the inner circle; and putting an axis through the center of the board, let it be turned very swiftly round that axis, so as the rays proceeding from the above colours, may be all blended and mixt together in coming to the eye; and then, the whole coloured part will appear like a white ring, a little greyish; not perfectly white, because no colours prepared by art are perfect.

All the prismatic colours blended together, make a white.

Any of these colours, except red and violet, may be made by mixing together the two contiguous prismatic colours. Thus, yellow is made by mixing together a due proportion of orange and green; and green may be made by a mixture of yellow and blue.

All bodies appear of that colour, whose rays they reflect most; as a body appears red when it reflects most of the red-making rays, and absorbs the rest.

Transpa-
rent co-
lours be-
come opaque
if put to-
gether.

Any two or more colours that are quite transparent by themselves, become opaque when put together. Thus, if water or spirits of wine be tinged red, and put in a phial, every object seen through it will appear red; because it lets only the red rays pass through it, and stops all the rest. If water or spirits be tinged blue, and put in a phial, all objects seen through it will appear blue, because it transmits only the blue rays, and stops all the rest. But if these two phials are held close together, so as both of them may be between the eye and object, the object will no more be seen through them than through a plate of metal; for whatever rays are transmitted through the fluid in the phial next the object, are stopped by that in the phial next the eye. In this experiment, the phials ought not to be round, but square; because nothing but the light itself can be seen through a round transparent body, at any distance.

As the rays of light suffer different degrees of refraction by passing obliquely through a prism, or through a convex-glass, and are thereby separated into all the seven original or primary colours; so they also suffer different degrees of refraction by passing through drops of falling rain; and then, being reflected towards the eye, from the sides of these drops which are farthest from the eye, and again refracted by passing out of these drops into the

the air, in which refracted directions they come to the eye; they make all the colours to appear in the form of a fine arch in the heavens, which is called the *rain-bow*. The rain-bow.

There are always two rain-bows seen together, Fig. 7. the interior of which is formed by the rays ab , which falling upon the upper part b , of the drop bcd , are refracted into the line bc as they enter the drop, and are reflected from the back of it at c , in the line cd , and then, by passing out of the drop into air, they are again refracted at d ; and from thence they pass on to the eye at e : so that to form the interior bow, the rays suffer two refractions, as at b and d ; and one reflection, as at c .

The exterior bow is formed by rays which suffer two reflections, and two refractions; which is the occasion of its being less vivid than the interior, and also of its colours being inverted with respect to those of the interior. For, when a ray ab , falls upon the lower part of the drop $bcd e$, it is refracted into the direction bc by entering the drop; and passing on to the back of the drop at c , it is thence reflected in the line cd , in which direction it is impossible for it to enter the eye at f : but by being again reflected from the point d of the drop, it goes on in the drop to e , where it passes out of the drop into the air, and is there refracted downward to the eye, in the direction ef . Fig. 8.

LECT. VIII.

The description and use of the globes, and armillary sphere.

The terrestrial globe.

IF a map of the world be accurately delineated on a spherical ball, it will be a true representation of the earth: for the highest hills are so inconsiderable with respect to the bulk of the earth, that they take off no more from its roundness, than grains of sand do from the roundness of a common globe; for the diameter of the earth is 8000 miles, in round numbers, and no known hill upon it is three miles in perpendicular height.

Proof of the earth's being globular.

That the earth is spherical, or round like a globe, appears, 1. from its casting a round shadow upon the moon, whatever side be turned towards her when she is eclipsed. 2. From its having been sailed round by several persons. 3. From our seeing the farther, the higher we stand. 4. From our seeing the masts of a ship, whilst the hull is hid by the convexity of the water.

And that it may be peopled on all sides without any one's being in danger of falling away from it.

The attractive power of the earth draws all terrestrial bodies towards its center; as is evident from the descent of heavy bodies in lines perpendicular to the earth's surface, at the places whereon they fall; even when they are thrown off from the earth on opposite sides, and consequently, in opposite directions. So that the earth may be compared to a great magnet rolled in filings of steel, which attracts and keeps them equally fast to its surface

surface on all sides. Hence, as all terrestrial bodies are attracted toward the earth's center, they can be in no danger of falling from the earth on any one part, or side of the earth, more than from any other.

The heaven or sky surrounds the whole earth : *Up* and *down*, we mean only ^{down, what?} with regard to ourselves ; for no point, either in the heaven, or on the surface of the earth, is *above* or *below*, but only with respect to us. And let us be upon what part of the earth we will, we stand with our feet toward its center, and our heads toward the sky : and so we say, it is *up* toward the sky, and *down* toward the center of the earth.

To an observer placed any where in an indefinite space, where there is nothing to limit his view, all objects that are very remote appear equally distant from him ; and seem to be placed in a vast concave sphere, of which his eye is the center. Every astronomer can demonstrate, that the moon is much nearer to us than the sun is ; that some of the planets are sometimes nearer to us, and sometimes farther from us, than the sun ; that others of them never come so near us as the sun always is ; that the remotest planet in our system, is beyond comparison nearer to us than any of the fixed stars are ; and that it is highly probable some stars are, in a manner, infinitely more distant from us than others. And yet, all these celestial objects appear equally distant from us. Therefore, if we imagine a large hollow sphere of glass, to have as many golden studs fixed to its inside, as there are

All objects
in the hea-
ven appear
equally
distant.

The face of
the heaven
and earth
represented
in a ma-
chine.

stars visible in the heavens, and these studs to be of different magnitudes, and placed at the same angular distances from each other as the stars are; it will be a just representation of the starry heavens, to an eye supposed to be in its center, and viewing it all around. And if a small globe, with a map of the earth upon it, be placed on an axis in the center of this starry sphere, and the sphere be made to turn round on this axis, it will represent the apparent motion of the heavens round the earth.

The equi-
noctial.

If a great circle be so drawn upon this sphere, as to divide it into two equal parts, or hemispheres, and the plane of the circle be perpendicular to the axis of the sphere, this will represent the *equinoctial circle*, which divides the heaven into two equal parts, called the *northern* and the *southern hemispheres*; and every point of that circle will be equally distant from the *poles*, or ends of the axis in the sphere. That pole which is in the middle of the northern hemisphere, will be called the *north pole of the sphere*, and that which is in the middle of the southern hemisphere, the *south pole*.

The poles.

The
ecliptic.

If another great circle be drawn upon the sphere, in such a manner as to cut the equinoctial at an angle of $23\frac{1}{2}$ degrees; in two opposite points, it will represent the *ecliptic*, or circle of the sun's apparent annual motion: one half of which is on the north side of the equinoctial, and the other half on the south.

If a large stud be made to move eastward in this ecliptic, in such a manner as to go quite round it,
in

in the time that the sphere is turned round westward, 366 times upon its axis; this stud will represent the *sun*, changing his place every day a 365th The sun, part of the ecliptic; and going round westward, the same way as the stars do; but with a motion so much slower than the motion of the stars, that they will make 366 revolutions about the axis of the sphere, in the time that the sun makes only 365. During one half of these revolutions, the sun will be on the north side of the equinoctial; during the other half, on the south; and at the end of each half, in the equinoctial.

If we suppose the terrestrial globe, in this ma- The earth, chine, to be about one inch in diameter, and the diameter of the starry sphere to be about five or six feet; a small insect on the globe would see only a very little portion of its surface; but would see one half of the starry sphere; the convexity of the globe hiding the other half from his view. If the sphere be turned westward round the globe, and the insect could judge of the appearances which arise from that motion, he would see some stars The appa- rising to his view in the eastern side of the sphere, rent motion whilst others were setting from his view on the of the hea- western: but as all the stars are fixed to the sphere, vens. the same stars would always rise in the same points of his view on the east side, and set in the same points of his view on the west side. With the sun it would be otherwise, because the sun is not fixed to any point of the sphere, but moves slowly along an oblique circle in it. And if the insect should look towards the south, and call that point of the

S 4

globe,

globe, where the equinoctial in the sphere seems to cut it on the left side, the *east point*; and where it cuts the globe on the right side, the *west point*; the little animal would see the sun rise north of the east, and set north of the west, for $182\frac{1}{2}$ revolutions; after which, for as many more, the sun would rise south of the east, and set south of the west. And in the whole 365 revolutions, the sun would rise only twice in the east point, and set twice in the west. All these appearances would be the same, if the starry sphere stood still, (the sun only moving in the ecliptic) and the earthly globe were turned round the axis of the sphere eastward. For, as the insect would be carried round with the globe, he would be quite insensible of its motion; and the sun and stars would appear to move westward.

We are but very small beings compared with our earthly globe, and *it* is but a dimensionless point compared with the magnitude of the starry heavens. Whether the earth be at rest, and the heaven turns round it, or the heaven be at rest, and the earth turns round, the appearance to us will be exactly the same. And, because the heaven is so immensely large, in comparison of the earth, we see one half of the heaven as well from the earth's surface, as we could do from its center.

*Circles of the
sphere.*

We may imagine as many circles described upon the earth as we please; and we may imagine the plane of any circle, described upon the earth, to be continued until it marks a circle in the concave sphere of the heavens.

The

The *horizon* is either *sensible* or *rational*. The *sensible* horizon is that circle which a man, standing upon a large plain, observes to terminate his view all around, where the heaven and earth seem to meet. The plane of our sensible horizon continued to the heaven, divides it into two hemispheres; one visible to us, the other hid by the convexity of the earth. The horizon.

The plane of the *rational horizon*, is supposed parallel to the plane of the sensible; to pass through the center of the earth, and to be continued to the heavens. And although the plane of the sensible horizon touches the earth in the plane of the observer, yet its plane, and that of the rational horizon, will seem to coincide in the heavens, because the whole earth is but a point compared to the sphere of the heavens.

The earth being a spherical body, the horizon, or limit of our view, must change as we change our place.

The *poles of the earth*, are those two points on its surface in which its axis terminates. The one is called the *north pole*, and the other the *south pole*. Poles.

The *poles of the heaven*, are the two points in which the earth's axis produced terminates in the heaven: so that the *north pole* of the heaven is directly over the north pole of the earth; and the *south pole* of the heaven is directly over the south pole of the earth.

The *equator* is a great circle upon the earth, every part of which is equally distant from either of the poles. It divides the earth into two equal parts, Equator.

parts, called the *northern* and *southern hemispheres*. If we suppose the plane of this circle extended to the heaven, it will mark the *equinoctial* therein; and divide the heaven into two equal parts, called the *northern* and *southern hemispheres* of the heavens.

Meridian.

The *meridian* of any place is a great circle passing through that place and the poles of the earth. We may imagine as many such meridians as we please, because any place that is ever so little to the east or west of any other place, must have a different meridian from it: for no one circle can pass through any two such places and the poles of the earth.

The meridian of any place is divided by the poles, into two semicircles: that which passes through the place is called the *geographical*, or *upper meridian*: and that which passes through the opposite place, is called the *lower meridian*.

*Noon, and
midnight.*

When the rotation of the earth brings the plane of the geographical meridian to the sun, it is *noon* or *mid-day* to that place; and when the lower meridian comes to the sun, it is *midnight*.

All places lying under the same geographical meridian, have their noon at the same time, and consequently all the other hours. All those places are said to have the same *longitude*, because no one of them lies either eastward or westward from any of the rest.

Hour circles.

If we imagine 24 semicircles, one of which is the geographical meridian of a given place, to meet at the poles, and to divide the equator into

24 equal parts ; each of these will come round to the sun in 24 hours, by the earth's equable motion round its axis in that time. And, as the equator contains 360 degrees, there will be 15 degrees contained between any two of these semicircles which are nearest to one another : for 24 times 15 is 360. And as the earth's motion is eastward, the sun's apparent motion will be westward, at the rate of 15 degrees each hour. Therefore,

They whose geographical meridian is 15 degrees *Longitude.* eastward from us, have noon, and every other hour, an hour sooner than we have. They whose meridian is 15 degrees westward from us, have noon, and every other hour, an hour later than we have : and so on in proportion, reckoning one hour for every 15 degrees.

As the earth turns round its axis once in 24 hours, and shews itself all round to the sun in that time ; so it goes round the sun once a year, in a great circle called the *ecliptic*, which crosses the *Ecliptic.* equinoctial in two opposite points, making an angle of $23\frac{1}{2}$ degrees with the equinoctial on each side. So that one half of the ecliptic is in the northern hemisphere, and the other in the southern. It contains 360 equal parts, called degrees, (as all other circles do, whether great or small) and as the earth goes once round it every year, the sun will appear to do the same, changing his place almost a degree, at a mean rate, every 24 hours. So that whatever place or degree of the ecliptic the earth is in, the sun will appear in the opposite. And as one half of the ecliptic is on the north side of
the

the equinoctial, and the other half on the south; the sun, as seen from the earth, will be half a year on the south side of the equinoctial, and half a year on the north: and twice a year in the equinoctial itself.

*Signs and
degrees.*

The ecliptic is divided by astronomers into 12 equal parts, called *signs*, each sign into 30 *degrees*, and each degree into 60 *minutes*: but in using the globes, we seldom want the sun's place nearer than half a degree of the truth.

The names and characters of the 12 signs, are as follow; beginning at that point of the ecliptic where it crosses the equinoctial to the northward, and reckoning eastward round to the same point again. And the days of the months on which the sun now enters the signs, are set down below them.

<i>Aries,</i>	<i>Taurus,</i>	<i>Gemini,</i>	<i>Cancer,</i>	<i>Leo,</i>	<i>Virgo,</i>
♈	♉	♊	♋	♌	♍
March,	April,	May,	June,	July,	August,
20	20	21	21	23	23
<i>Libra,</i>	<i>Scorpio,</i>	<i>Sagittarius,</i>	<i>Capricorn,</i>	<i>Aquarius,</i>	<i>Pisces,</i>
♎	♏	♐	♑	♒	♓
Septemb.	Octob.	Novemb.	Decemb.	January,	Feb.
23	23	22	21	20	18

By remembering on what day the sun enters any particular sign, we may easily find his place any day afterward, whilst he is in that sign, by reckoning a degree for a day: which will occasion no error of consequence in using the globes.

When the sun is at the beginning of *aries*, he is in the equinoctial; and from that time he declines northward

northward every day, until he comes to the beginning of *cancer*, which is $23\frac{1}{2}$ degrees from the equinoctial. From thence he recedes southward every day, for half a year; in the middle of which half, he crosses the equinoctial at the beginning of *libra*; and at the end of that half year, he is at his greatest south declination, in the beginning of *capricorn*, which is also $23\frac{1}{2}$ degrees from the equinoctial. Then, he recedes northward from *capricorn* every day, for half a year; in the middle of which half, he crosses the equinoctial at the beginning of *aries*; and at the end of it he arrives at *cancer*.

The sun's motion in the ecliptic is not perfectly equable, for he continues eight days longer in the northern half of the ecliptic, than in the southern: so that the summer half year, in the northern hemisphere, is eight days longer than the winter half year, and the contrary in the southern hemisphere.

The *tropics* are lesser circles in the heavens, parallel to the equinoctial; one on each side of it, touching the ecliptic in the points of its greatest declination; so that each tropic is $23\frac{1}{2}$ degrees from the equinoctial, one on the north side of it, and the other on the south. The northern tropic touches the ecliptic at the beginning of *cancer*, the southern, at the beginning of *capricorn*; for which reason, the former is called the *tropic of cancer*, and the latter the *tropic of capricorn*. *Tropics.*

The *polar circles* in the heavens, are each $23\frac{1}{2}$ degrees from the poles, all around. That which *Polar circles.*
goes

goes round the north pole, is called the *arctic circle*, from *ἄρκτος*, which signifies a *bear*; and there is a constellation or groupe of stars near the north pole, which goes by that name. The south polar circle, is called the *antarctic circle*, from its being opposite to the arctic.

The ecliptic, tropics, and polar circles, are drawn upon the terrestrial globe, as well as upon the celestial. But the ecliptic, being a great fixed circle in the heavens, cannot properly be said to belong to the terrestrial globe; and is laid down upon it only for the conveniency of solving some problems.

In order to form a true idea of the earth's motion round its axis every 24 hours, which is the cause of day and night; and of its motion in the ecliptic, round the sun every year, which is the cause of the different lengths of days and nights, and vicissitude of seasons; take the following method, which will be both easy and pleasant.

An idea of
the seasons.

Let a small terrestrial globe, of about 3 inches diameter, be suspended by a long thread of twisted silk, fixt to its north pole: then, having placed a lighted candle on a table, to represent the sun, in the center of a hoop of a large cask, which may represent the ecliptic, the hoop making an angle of $23\frac{1}{2}$ degrees with the plane of the table; hang the globe within the hoop, near to it; and if the table be level, the equator of the globe will be parallel to the table, and the plane of the hoop will cut the equator at an angle of $23\frac{1}{2}$ degrees; so that one half of the equator will be above the hoop,

hoop, and the other half below it : and the candle will enlighten one half of the globe, as the sun enlightens one half of the earth, whilst the other half is in the dark.

Things being thus prepared, twist the thread towards the left hand, that it may turn the globe the same way by untwisting ; that is from west, by south, to east. As the globe turns round its axis or thread, the different places on its surface will go regularly through the light and dark ; and have, as it were, an alternate return of day and night in each rotation. As the globe continues to turn round, and shew itself all around to the candle, carry it slowly round the hoop by the thread, from west, by south, to east ; which is the way that the earth moves round the sun, once a year, in the ecliptic : and you will see, that whilst the globe continues in the lower part of the hoop, the candle (being then north of the equator) will constantly shine round the north pole ; and all the northern places, which go through any part of the dark, will go through a less portion of it than they do of the light ; and the more so, the farther they are from the equator : consequently, their days are then longer than their nights. When the globe comes to a point in the hoop, mid-way between the highest and lowest points, the candle will be directly over the equator, and enlighten the globe just from pole to pole ; and then, every place on the globe will go through equal portions of light and darkness, as it turns round its axis ; and consequently, the day and night will be of equal

equal length at all places upon it. As the globe advances thence, toward the highest part of the hoop, the candle will be on the south side of the equator, shining farther and farther round the south pole, as the globe rises higher and higher in the hoop; leaving the north pole as much in darkness as the south pole is in the light, and making long days and short nights on the south side of the equator, and the contrary on the north side, whilst the globe continues in the northern or higher side of the hoop: and when it comes to the highest point, the days will be at the longest, and nights at the shortest, in the southern hemisphere; and the reverse in the northern. As the globe advances and descends in the hoop, the light will gradually recede from the south pole, and approach towards the north pole, which will cause the northern days to lengthen, and the southern days to shorten in the same proportion. When the globe comes to the middle point, between the highest and lowest points of the hoop, the candle will be over the equator, enlightening the globe just from pole to pole, when every place of the earth (except the poles) will go through equal portions of light and darkness; and consequently, the day and night will be then equal, all over the globe.

And thus, at a very small expence, one may have a delightful and demonstrative view of the cause of days and nights, with their gradual increase and decrease in length, through the whole year; together with the vicissitudes of spring,
summer,

summer, autumn, and winter, in each annual course of the earth round the sun.

If the hoop be divided into 12 equal parts, and the signs be marked in order upon it, beginning with *cancer* at the highest point of the hoop, and reckoning eastward (or contrary to the apparent motion of the sun) you will see how the sun appears to change his place every day in the ecliptic, as the globe advances eastward along the hoop, and turns round its own axis: and that when the earth is in a low sign, as at *capricorn*, the sun must appear in a high sign, as at *cancer*, opposite to the earth's real place: and that whilst the earth is in the southern half of the ecliptic, the sun appears in the northern half, and *vice versa*: that the farther any place is from the equator, between it and the polar circle, the greater is the difference between the longest and shortest day at that place; and that the poles have but one day and one night in the whole year.

These things premised, we shall proceed to the description and use of the terrestrial globe, and explain the geographical terms as they occur in the problems.

This globe has the boundaries of land and water laid down upon it, the countries and kingdoms divided by dots, and coloured to distinguish them, the islands properly situated, the rivers and principal towns inserted, as truly as they have been ascertained upon the earth by measurement and observation.

The terrestrial globe described.

The equator, ecliptic, tropics, polar circles, and meridians, are laid down upon the globe in the manner already described. The ecliptic is divided into 12 signs, and each sign into 30 degrees, which are generally subdivided into halves, and into quarters if the globe is large. Each tropic is $23\frac{1}{2}$ degrees from the equator, and each polar circle $23\frac{1}{2}$ degrees from its respective pole. Circles are drawn parallel to the equator, at every ten degrees distance from it on each side, to the poles: these circles are called *parallels of latitude*. On large globes there are circles drawn perpendicularly through every tenth degree of the equator, intersecting each other at the poles: but on globes of or under a foot diameter, they are only drawn through every fifteenth degree of the equator: these circles are generally called *meridians*, sometimes *circles of longitude*, and at other times *hour-circles*.

The globe is hung in a brass ring, called the *brass meridian*; and turns upon a wire in each pole, sunk half its thickness into one side of the meridian-ring; by which means, that side of the ring divides the globe into two equal parts, called the *eastern* and *western hemispheres*; as the equator divides it into two equal parts, called the *northern* and *southern hemispheres*. This ring is divided into 360 equal parts or degrees, on the side wherein the axis of the globe turns. One half of these degrees are numbered, and reckoned, from the equator to the poles, where they end at 90: their use is to shew the latitudes of places. The degrees on the other half of the meridian-ring, are numbered
from

from the poles to the equator, where they end at 90 : their use is to shew how to elevate either the north or south pole above the horizon, according to the latitude of any given place, as it is north or south of the equator.

The brasen meridian is let into two notches made in a broad flat ring, called the *wooden horizon*, the upper surface of which divides the globe into two equal parts, called the *upper* and *lower hemispheres*. One notch is in the north point of the horizon, and the other in the south. On this horizon, are several concentric circles, which contain the months and days of the year, the signs and degrees answering to the sun's place for each month and day, and the 32 points of the compass.—The graduated side of the brass meridian lies toward the east side of the horizon, and should be generally kept toward the person who works problems by the globes.

There is a small *horary circle*, so fixed to the north part of the brasen meridian, that the wire in the north pole of the globe is in the center of that circle ; and on the wire is an *index*, which goes over all the 24 hours of the circle, as the globe is turned round its axis. Sometimes there are two horary circles, one between each pole of the globe and the brasen meridian ; which is the contrivance of the ingenious Mr. *Joseph Harris*, master of the assay-office in the Tower of London ; and makes it very convenient for putting the poles of the globe through the horizon, and for elevating the pole to small latitudes and declinations of

the sun; which cannot be done where there is only one horary circle fixed to the outer edge of the brasen meridian.

There is a thin slip of brass, called the *quadrant of altitude*, which is divided into 90 equal parts or degrees, answering exactly to so many degrees of the equator. It is occasionally fixed to the uppermost point of the brasen meridian, by a nut and screw. The divisions end at the nut, and the quadrant is turned round upon it.

The globe being a machine which has been seen by most people, and upon the figure of which, in a plate, neither the circles, nor countries, can be properly expressed, we judge it would signify very little to refer to a figure of it; and shall therefore only give some directions how to choose a globe, and then describe its use.

Directions
for choo-
sing of
globes.

1. See that the papers be well and neatly pasted on the globes, which you may know, if the lines and circles thereon meet exactly, and continue all the way even and whole; the circles not breaking into several arches, nor the papers either coming short, or lapping over one another.

2. See that the colours be transparent, and not laid too thick upon the globe to hide the names of places.

3. See that the globe hang evenly between the brasen meridian and the wooden horizon; not inclining either to one side or to the other.

4. See that the globe be as close to the horizon and meridian, as it conveniently may; otherwise you will be too much puzzled to find against what
part

part of the globe any degree of the meridian or horizon is.

5. See that the equinoctial line be even with the horizon all around, when the north or south pole is elevated 90 degrees above the horizon.

6. See that the equinoctial line cuts the horizon in the east and west points, in all elevations of the pole, from 0 to 90 degrees.

7. See that the degrees of the brazen meridian, marked with 0 and 90, be exactly over the equinoctial line of the globe.

8. See that there be exactly half of the brazen meridian above the horizon; which you may know, if you bring any of the decimal divisions on the meridian to the north point of the horizon, and find their complement to 90 in the south point.

9. See that when the quadrant of altitude is placed at the highest point of the brazen meridian, the beginning of the degrees of the quadrant reaches just to the plane surface of the horizon.

10. See that whilst the index of the hour-circle (by the motion of the globe) passes from one hour to another, 15 degrees of the equator pass under the graduated edge of the brazen meridian.

11. See that the wooden horizon be made substantial and strong: it being generally observed, that in most globes, the horizon is the first part that fails, on account of its having been made too slight.

In using the globes, keep the east side of the horizon towards you (unless your problem requires the turning of it) which side you may know by

*Directions
for using
them.*

the word EAST, upon the horizon : for then you have the graduated side of the meridian towards you, the quadrant of altitude before you, and the globe divided exactly into two equal parts, by the graduated side of the meridian.

In working some problems, it will be necessary to turn the whole globe and horizon about, that you may look on the west side thereof ; which turning will be apt to jog the ball so, as to shift away that degree of the globe which was before set to the horizon or meridian : to avoid which inconvenience, you may thrust the feather-end of a quill in between the ball of the globe and the brasen meridian ; which, without hurting the ball, will keep it from turning in the meridian, whilst you turn the west side of the horizon towards you.

P R O B L E M I.

*To find the * latitude and † longitude of any given place upon the globe.*

Turn the globe upon its axis, until the given place comes exactly under that graduated side of the brasen meridian, on which the degrees are numbered

* The latitude of a place is its distance from the equator, and is north or south, as the place is north or south of the equator. Those who live at the equator have no latitude, because it is there that the latitude begins.

† The longitude of a place is the number of degrees (reckoned upon the equator) that the meridian of the said place

numbered from the equator; and observe what degree of the meridian the place then lies under; which is its latitude, north or south, as the place is north or south of the equator.

The globe remaining in this position, the degree of the equator, which is under the brasen meridian, is the longitude of the place; which is east or west, as the place lies on the east or west side of the first meridian of the globe.—All the *Atlantic Ocean*, and *America*, is on the west side of the meridian of *London*; and the greatest part of *Europe*, and of *Africa*, together with all *Asia*, is on the east side of the meridian of *London*, which is reckoned the *first meridian* of the globe by the *English* geographers and astronomers.

P R O B L E M II.

The longitude and latitude of a place being given, to find that place on the globe.

Look for the given longitude in the equator, (counted eastward or westward from the first meridian, as it is mentioned east or west;) and bring

place is distant from the meridian of any other place from which we reckon, either eastward or westward, for 180 degrees, or half round the globe. The English reckon the longitude from the meridian of London, and the French now reckon it from the meridian of Paris. The meridian of that place, from which the longitude is reckoned, is called the *first meridian*. The places upon this meridian have no longitude, because it is there that the longitude begins.

the point of longitude in the equator to the brazen meridian, on that side which is above the south point of the horizon: then count from the equator, on the brazen meridian, to the degree of latitude given, towards the north or south pole, according as the latitude is north or south; and under that degree of latitude on the meridian, lies the place required.

P R O B L E M III.

To find the difference of longitude, or difference of latitude, between any two given places.

Bring each place to the brazen meridian, and see what its latitude is: the lesser latitude subtracted from the greater, if both places are on the same side of the equator, or both latitudes added together, if they are on different sides of it, is the difference of latitude required. And the number of degrees contained between these places, reckoned on the equator, when they are brought separately under the brazen meridian, is their difference of longitude; if it be less than 180: but if more, let it be subtracted from 360, and the remainder is the difference of longitude required. Or,

Having brought one of the places to the brazen meridian, and set the hour index to XII, turn the globe until the other place comes to the meridian, and the number of hours and parts of an hour, past over by the index, will give the longitude in time; which may be easily reduced to degrees, by
allowing

allowing 15 degrees for every hour, and one degree for every four minutes.

N. B. When we speak of bringing any place to the brazen meridian, the graduated side of the meridian is meant.

P R O B L E M IV.

Any place being given, to find all those places that have the same longitude or latitude with it.

Bring the given place to the brazen meridian, then all those places which lie under that side of the meridian, from pole to pole, have the same longitude with the given place. Turn the globe round its axis, and all those places which pass under the same degree of the meridian that the given place does, have the same latitude with that place.

Since all latitudes are reckoned from the equator, and all longitudes are reckoned from the first meridian, it is evident, that the point of the equator which is cut by the first meridian, has neither latitude nor longitude.—The greatest latitude cannot exceed 90 degrees, because no place is more than 90 degrees from the equator. And the greatest longitude cannot exceed 180 degrees, because no place is more than 180 degrees from the first meridian.

P R O B L E M

P R O B L E M V.

*To find the * antœci, † periœci, and ‡ antipodes, of any given place.*

Bring the given place to the brazen meridian, and having found its latitude, keep the globe in that

* The *antœci* are those people who live on the same meridian, and in equal latitudes, on different sides of the equator. Being on the same meridian, they have the same hours; that is, when it is noon to the one, it is also noon to the other, and when it is midnight to the one, it is also midnight to the other, &c. Being on different sides of the equator, they have different or opposite seasons at the same time; the length of any day to the one, is equal to the length of the night of that day to the other; and they have equal elevations of the different poles.

† The *periœci* are those people who live on the same parallel of latitude, but on opposite meridians: so that though their latitude be the same, their longitude differs 180 degrees. By being in the same latitude, they have equal elevations of the same pole (for the elevation of the pole, is always equal to the latitude of the place), the same length of days or nights, and the same seasons. But being on opposite meridians, when it is noon to the one, it is midnight to the other.

‡ The *antipodes* are those who live diametrically opposite to one another upon the globe, standing with feet towards feet, on opposite meridians and parallels. Being on opposite sides of the equator, they have opposite seasons, winter to one, when it is summer to the other; being equally distant from the equator, they have the contrary poles equally elevated above the horizon; being on opposite meridians, when

that situation, and count the same number of degrees of latitude from the equator towards the contrary pole, and where the reckoning ends, you have the *antæci* of the given place, upon the globe. Those who live at the equator have no *antæci*.

The globe remaining in the same position, set the hour index to the upper XII on the horary circle, and turn the globe until the index comes to the lower XII; then, the place which lies under the meridian, in the same latitude with the given place, is the *periæci* required. Those who live at the poles have no *periæci*.

As the globe now stands, (with the index at the lower XII,) the *antipodes* of the given place will be under the same point of the brasen meridian where its *antæci* stood before. Every place upon the globe has its *antipodes*.

P R O B L E M VI.

To find the distance between any two places on the globe.

Lay the graduated edge of the quadrant of altitude over both the places, and count the number of degrees intercepted between them on the quadrant; then multiply these degrees by 60, and the

when it is noon to the one, it must be midnight to the other; and as the sun recedes from the one when he approaches to the other, the length of the day to one must be equal to the length of the night at the same time to the other.

product

product will give the distance in geographical miles: but to find the distance in English miles, multiply the degrees by $69\frac{1}{2}$, and the product will be the number of miles required. Or, take the distance betwixt any two places with a pair of compasses, and apply that extent to the equator; the number of degrees, intercepted between the points of the compasses, is the distance in degrees of a great circle*; which may be reduced either to geographical miles, or to English miles, as above.

Great circle. * Any circle that divides the globe into two equal parts, is called a *great circle*, as the equator or meridian. Any circle that divides the globe into two unequal parts, (which
Lesser circle. every parallel of latitude does) is called a *lesser circle*. Now, as every circle, whether great or small, contains 360 degrees, and a degree upon the equator or meridian contains 60 geographical miles, it is evident, that a degree of longitude upon the equator, is longer than a degree of longitude upon any parallel of latitude, and must therefore contain a greater number of miles. So that, although all the degrees of latitude are equally long upon an artificial globe, (though not precisely so upon the earth itself) yet the degrees of longitude decrease in length, as the latitude increases, but not in the same proportion. The following table shews the length of a degree of longitude, in geographical miles, and hundredth parts of a mile, for every degree of latitude, from the equator to the poles: a degree on the equator being 60 geographical miles.

The number of miles in a degree of longitude,
in all degrees of latitude.

Deg.	Miles.	Parts.	Deg.	Miles.	Parts.	Deg.	Miles.	Parts.	Deg.	Miles.	Parts.	Deg.	Miles.	Parts.	Deg.	Miles.	Parts.
1	59.99		19	56.73		37	47.92		55	34.41		73	17.54				
2	59.96		20	56.38		38	47.28		56	33.55		74	16.53				
3	59.92		21	56.02		39	46.63		57	32.68		75	15.52				
4	59.85		22	55.63		40	45.97		58	31.79		76	14.31				
5	59.77		23	55.23		41	45.28		59	30.90		77	13.50				
6	59.67		24	54.81		42	44.59		60	30.00		78	12.48				
7	59.56		25	54.38		43	43.88		61	29.09		79	11.45				
8	59.42		26	53.93		44	43.16		62	28.17		80	10.42				
9	59.26		27	53.46		45	42.43		63	27.24		81	9.38				
10	59.09		28	52.96		46	41.68		64	26.30		82	8.35				
11	58.89		29	52.47		47	40.92		65	25.36		83	7.32				
12	58.69		30	51.96		48	40.15		66	24.41		84	6.28				
13	58.46		31	51.43		49	39.36		67	23.44		85	5.24				
14	58.22		32	50.88		50	38.57		68	22.48		86	4.20				
15	58.95		33	50.32		51	37.76		69	21.50		87	3.15				
16	57.67		34	49.74		52	36.94		70	20.52		88	2.10				
17	57.38		35	49.15		53	36.11		71	19.53		89	1.05				
18	57.06		36	48.54		54	35.27		72	18.54		90	0.00				

PROBLEM VII.

A place on the globe being given, and its true distance from any other place, to find all the other places upon the earth which are at the same distance from the given place.

Bring the given place to the brasen meridian, and screw the quadrant of altitude to the meridian, directly over that place; then keeping the globe in that position, turn the quadrant quite round

round upon it, and the degree of the quadrant that touches the second place, will pass over all the other places which are equally distant with it from the given place.

This is the same as if one foot of a pair of compasses was set in the given place, and the other foot extended to the second place, whose distance is known; for if the compasses be then turned round the first place as a center, the moving foot will go over all those places which are at the same distance with the second from it.

P R O B L E M VIII.

The hour of the day at any place being given, to find all those places where it is noon at that time.

Bring the given place to the brazen meridian, and set the index to the given hour; this done, turn the globe until the index points to the upper XII, and then, all the places that lie under the brazen meridian have noon at that time.

N. B. The upper XII always stands for noon; and when the bringing of any place to the brazen meridian is mentioned, the side of that meridian on which the degrees are reckoned from the equator is meant, unless the contrary side be mentioned.

P R O B L E M IX.

The hour of the day at any place being given, to find what o'clock it is at that time at any other place.

Bring the given place to the brazen meridian, and set the index to the given hour; then turn the globe, until the place where the hour is required comes to the meridian, and the index will point out the hour at that place.

P R O B L E M X.

*To find the sun's place in the ecliptic, and his * declination, for any given day of the year.*

Look on the horizon for the given day, and right against it thereon, you have the degree of the sign in which the sun is (or his place) on that day at noon. Find the same degree of that sign in the ecliptic line upon the globe, and having brought it to the brazen meridian, observe what degree of the meridian stands over it; for that is the sun's declination, reckoned from the equator.

* The sun's declination is his distance from the equinoctial in degrees, and is north or south, as the sun is between the equinoctial and the north or south pole.

P R O B L E M

P R O B L E M XI.

The day of the month being given, to find all those places of the earth over which the sun will pass vertically on that day.

Find the sun's place in the ecliptic for the given day, and having brought it to the brazen meridian, observe what degree of the meridian is over it; then, turning the globe round its axis, all those places which pass under that degree of the meridian, are the places required: for as their latitude is equal in degrees to the sun's declination, he must be directly over head to each of them at its respective noon.

P R O B L E M XII.

*A place being given in the * torrid zone, to find those two days of the year, on which the sun shall be vertical to that place.*

Bring the given place to the brazen meridian, and mark the degree of latitude that is exactly over it

* The globe is divided into five zones; one torrid, two temperate, and two frigid. The *torrid zone* lies between the two tropics, and is 47 degrees in breadth, or $23\frac{1}{2}$ on each side of the equator: the *temperate zones* lie between the tropics and polar circles, or from $23\frac{1}{2}$ degrees of latitude,

it on the meridian; then turn the globe round its axis, and observe the two points of the ecliptic which pass exactly under that degree of latitude: lastly, find on the wooden horizon, the two days of the year in which the sun is in those points or degrees of the ecliptic, and they are the days required: for on them, and none else, the sun's declination is equal to the latitude of the given place.

P R O B L E M XIII.

To find all those places of the north frigid zone, where the sun begins to shine constantly without setting, on any given day, from the 21st of March to the 23^d of September.

On these two days, the sun is in the equinoctial, and enlightens the globe exactly from pole to pole: therefore, as the earth turns round its axis, which terminates in the poles, every place upon it will go equally through the light and the dark, and so make equal day and night to all places of the earth. But as the sun declines from the equator, towards either pole, he will shine just as many degrees round that pole, as are equal to his decli-

tude, to $66\frac{1}{2}$, on each side of the equator; and are each 43 degrees in breadth: the *frigid zones* are the spaces included within the polar circles, which being each $23\frac{1}{2}$ degrees from their respective poles, the breadth of each of these zones is 47 degrees. As the sun never goes without the tropics, he must every moment be vertical to some place or other in the torrid zone.

nation from the equator; so that no place within that distance of the pole will then go through any part of the dark, and consequently the sun will not set to it. Now, as the sun's declination is northward, from the 21st of March to the 23^d of September, he must constantly shine round the north pole all that time; and on the day that he is in the northern tropic, he shines upon the whole north frigid zone; so that no place within the north polar circle goes through any part of the dark on that day. Therefore,

Having brought the sun's place for the given day to the brazen meridian, and found his declination, (by Prob. 9.) count as many degrees on the meridian, from the north pole, as are equal to the sun's declination from the equator, and mark that degree from the pole where the reckoning ends: then, turning the globe round its axis, observe what places in the north frigid zone pass directly under that mark; for they are the places required.

The like may be done for the south frigid zone, from the 23^d of September to the 21st of March, during which time the sun shines constantly on the south pole.

P R O B L E M XIV.

To find the place over which the sun is vertical, at any hour of a given day.

Having found the sun's declination for the given day, (by Prob. 9.) mark it with a chalk on the
brazen

brazen meridian: then bring the place where you are (suppose London) to the brazen meridian, and set the index to the given hour; which done, turn the globe on its axis, until the index points to XII at noon; and the place on the globe, which is then under the point of the sun's declination marked upon the meridian, has the sun that moment in the zenith, or directly over head.

P R O B L E M X V .

The day and hour at any place being given, to find all those places where the sun is then rising, or setting, or on the meridian: consequently, all those places which are enlightened at that time, and those which are in the dark.

This problem cannot be solved by any globe fitted up in the common way, with the hour circle fixed upon the brass meridian; unless the sun be on or near some of the tropics on the given day. But with a globe fitted up according to Mr. *Joseph Harris's* invention, (already mentioned, page 275) where the hour circle lies on the surface of the globe, below the meridian, it may be solved for any day of the year, according to his method; which is as follows.

Having found the place to which the sun is vertical at the given hour, if the place be in the northern hemisphere, elevate the north pole as many degrees above the horizon, as are equal to the latitude of that place; if the place be in the southern

hemisphere, elevate the south pole accordingly; and bring the said place to the brasen meridian. Then, all those places which are in the western semicircle of the horizon, have the sun rising to them at that time; and those in the eastern semicircle have it setting: to those under the upper semicircle of the brass meridian, it is noon; and to those under the lower semicircle, it is midnight. All those places which are above the horizon, are enlightened by the sun, and have the sun just as many degrees above them, as they themselves are above the horizon: and this height may be known, by fixing the quadrant of altitude on the brasen meridian over the place to which the sun is vertical; and then, laying it over any other place, observe what number of degrees on the quadrant are intercepted between the said place and the horizon. In all those places that are 18 degrees below the western semicircle of the horizon, the morning twilight is just beginning; in all those places that are 18 degrees below the eastern semicircle of the horizon, the evening twilight is ending; and all those places that are lower than 18 degrees, have dark night.

If any place be brought to the upper semicircle of the brasen meridian, and the hour index be set to the upper XII or noon, and then the globe be turned round its axis; when any place comes to the western semicircle of the horizon, the index will shew the time of sun-rising at that place; and if the place be brought to the eastern semicircle of the horizon, the index will shew the time of sun-set.

To

To those places which do not go under the horizon, the sun sets not on that day ; and to those which do not come above it, he does not rise.

P R O B L E M XVI.

The day and hour of a lunar eclipse being given ; to find all those places of the earth to which it will be visible.

The moon is never eclipsed but when she is full, and so directly opposite to the sun, that the earth's shadow falls upon her. Therefore, whatever place of the earth the sun is vertical to at that time, the moon must be vertical to the antipodes of that place : so that the sun will be then visible to one half of the earth, and the moon to the other.

Find the place to which the sun is vertical at the given hour, (by Prob. 14 ;) elevate the pole to the latitude of that place, and bring the place to the upper part of the brazen meridian, as in the former problem : then, as the sun will be visible to all those parts of the globe which are above the horizon, the moon will be visible to all those parts which are below it, at the time of her greatest obscuration.

But with regard to an eclipse of the sun, there is no such thing as shewing to what places it will be visible, with any degree of certainty, by a common globe ; because the moon's shadow covers but a small portion of the earth's surface, and her latitude, or declination from the ecliptic, throws her

shadow so variously upon the earth, that to determine the places on which it falls, recourse must be had to long calculations.

P R O B L E M XVII.

*To rectify the globe for the latitude, the * zenith, and the sun's place.*

Find the latitude of the place (by Prob. 1.) and if the place be in the northern hemisphere, raise the north pole above the north point of the horizon, as many degrees (counted from the pole upon the brazen meridian) as are equal to the latitude of the place. If the place be in the southern hemisphere, raise the south pole above the south point of the horizon, as many degrees as are equal to the latitude. Then, having brought the place to its latitude on the brazen meridian, fasten the quadrant of altitude so, that the chamfered edge of its nut (which is even with the graduated edge) may be joined to the zenith, or point of latitude. This done, bring the sun's place in the ecliptic for the given day (found by Prob. 10.) to the graduated side of the brazen meridian, and set the hour index to XII at noon, which is the uppermost XII on the hour circle; and the globe will be rectified.

* The *zenith*, in this sense, is the highest point of the brazen meridian above the horizon; but in the proper sense, it is that point of the heavens which is directly vertical to any given place.

The

The latitude of any place, is equal to the elevation Remark. of the nearest pole of the heaven above the horizon of that place ; and the poles of the heaven are directly over the poles of the earth, each 90 degrees from the equinoctial line. Let us be upon what place of the earth we will, if the limits of our view be not intercepted by hills, we see one half of the heaven, or 90 degrees every way round, from that point which is over our heads. Therefore, if we were upon the equator, the poles of the heaven would lie in our horizon, or limit of view : if we go from the equator, towards either pole of the earth, we shall see the corresponding pole of the heaven rising gradually above our horizon, just as many degrees as we have gone from the equator : and if we were at either of our earth's poles, the corresponding pole of the heaven would be directly over our head. Consequently, the elevation or height of the pole in degrees above the horizon, is always equal to the number of degrees that the place is from the equator.

P R O B L E M XVIII.

*The latitude of any place, not exceeding * $66\frac{1}{2}$ degrees, and the day of the month, being given ; to find the time of sun-rising and setting, and consequently the length of the day and night.*

Having rectified the globe for the latitude, and the sun's place on the given day, (as directed in the

* All places whose latitude is more than $66\frac{1}{2}$ degrees, are in the frigid zones : and to those places the sun does not

the preceding problem) bring the sun's place in the ecliptic to the eastern side of the horizon, and the hour index will shew the time of sun-rising; then turn the globe on its axis, until the sun's place comes to the western side of the horizon, and the index will shew the time of sun-setting.

The hour of sun-setting doubled, gives the length of the day; and the hour of sun-rising doubled, gives the length of the night.

P R O B L E M XIX.

The latitude of any place, and the day of the month, being given; to find when the morning twilight begins, and the evening twilight ends, at that place.

This problem is often limited: for, when the sun does not go 18 degrees below the horizon, the twilight continues the whole night; and for several nights together in summer, between 49 and $66\frac{1}{2}$ degrees of latitude: and the nearer to $66\frac{1}{2}$, the greater is the number of these nights. But when it does begin and end, the following method will shew the time for any given day.

Rectify the globe, and bring the sun's place in the ecliptic to the eastern side of the horizon; then mark that point of the ecliptic with a chalk which is in the western side of the horizon, it being the point opposite to the sun's place: this done,

set in summer, for a certain number of diurnal revolutions; which occasions this limitation of latitude.

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lay the quadrant of altitude over the said point, and turn the globe eastward, keeping the quadrant at the chalk-mark, until it be just 18 degrees high on the quadrant; and the index will point out the time that the morning twilight begins: for the sun's place will then be 18 degrees below the eastern side of the horizon. To find the time when the evening twilight ends, bring the sun's place to the western side of the horizon, and the point opposite to it, which was marked with the chalk, will be rising in the east: then, bring the quadrant over that point, and keeping it thereon, turn the globe westward, until the said point be 18 degrees above the horizon on the quadrant, and the index will shew the time when the evening twilight ends; the sun's place being then 18 degrees below the western side of the horizon.

P R O B L E M XX.

To find on what day of the year the sun begins to shine constantly without setting, on any given place in the north frigid zone; and how long he continues to do so.

Rectify the globe to the latitude of the place, and turn it about until some point of the ecliptic, between *aries* and *cancer*, coincides with the north point of the horizon where the brasen meridian cuts it: then find, on the wooden horizon, what day of the year the sun is in that point of the ecliptic; which is the day that the sun begins to shine

shine constantly on the given place, without setting. This done, turn the globe until some point of the ecliptic, between *cancer* and *libra*, coincides with the north point of the horizon, where the brass meridian cuts it; and find, on the wooden horizon, on what day the sun is in that point of the ecliptic; which is the day that the sun leaves off constantly shining on the said place, and rises and sets to it as to other places on the globe. The number of natural days, or compleat revolutions of the sun about the earth, between the two days above found, is the time that the sun keeps constantly above the horizon without setting; for all that portion of the ecliptic, which lies between the two points that intersect the horizon in the very north, never sets below it: and there is just as much of the opposite part of the ecliptic that never rises; therefore, the sun will keep as long constantly below the horizon in winter, as above it in summer.

Whoever considers the globe, will find, that all places of the earth do equally enjoy the benefit of the sun, in respect of time, and are equally deprived of it. For, the days and nights are always equally long at the equator: and in all places that have latitude, the days at one time of the year, are exactly equal to the nights at the opposite season.

P R O B L E M XXI.

*To find in what latitude the sun shines constantly without setting, for any length of time less than * $182\frac{1}{2}$ of our days and nights.*

Find a point in the ecliptic half as many degrees from the beginning of *cancer*, (either toward *aries* or *libra*) as there are † natural days in the time given; and bring that point to the north side of the brasen meridian, on which the degrees are numbered from the pole towards the equator: then, keep the globe from turning on its axis, and slide the meridian up or down, until the foresaid point of the ecliptic comes to the north point of the horizon, and then, the elevation of the pole will be equal to the latitude required.

P R O B L E M XXII.

The latitude of a place, not exceeding $66\frac{1}{2}$ degrees, and the day of the month being given; to find the sun's amplitude, or point of the compass on which he rises or sets.

Rectify the globe, and bring the sun's place to the eastern side of the horizon; then observe what

* The reason of this limitation is, that $182\frac{1}{2}$ of our days and nights make half a year, which is the longest time that the sun shines without setting, even at the poles of the earth.

† A natural day, contains the whole 24 hours: an artificial day, the time that the sun is above the horizon.

point

point of the compass on the horizon stands right against the sun's place, for that is his amplitude at rising. This done, turn the globe westward, until the sun's place comes to the western side of the horizon, and it will cut the point of his amplitude at setting. Or, you may count the rising amplitude in degrees, from the east point of the horizon, to that point where the sun's place cuts it; and the setting amplitude, from the west point of the horizon, to the sun's place at setting.

P R O B L E M XXIII.

*The latitude, the sun's place, and his * altitude, being given; to find the hour of the day, and the sun's azimuth, or number of degrees that he is distant from the meridian.*

Rectify the globe, and bring the sun's place to the given height upon the quadrant of altitude; on the eastern side of the horizon, if the time be in the forenoon; or the western side, if it be in the afternoon: then, the index will shew the hour, and the number of degrees in the horizon, intercepted between the quadrant of altitude and the south point, will be the sun's true azimuth at that time.

N. B. Always when the quadrant of altitude is mentioned in working any problem, the graduated edge of it is meant.

* The sun's altitude, at any time, is his height above the horizon at that time.

If

If this be done at sea, and compared with the sun's azimuth, as shewn by the compass, if they agree, the compass has no variation in that place: but if they differ, the compass does vary; and the variation is equal to this difference.

P R O B L E M XXIV.

The latitude, hour of the day, and the sun's place, being given; to find the sun's altitude and azimuth.

Rectify the globe, and turn it until the index points to the given hour; then lay the quadrant of altitude over the sun's place in the ecliptic, and the degree of the quadrant cut by the sun's place is his altitude at that time above the horizon; and the degree of the horizon cut by the quadrant is the sun's azimuth, reckoned from the south.

P R O B L E M XXV.

The latitude, the sun's altitude, and his azimuth being given; to find his place in the ecliptic, the day of the month, and hour of the day, though they had all been lost.

Rectify the globe for the latitude and * zenith, and set the quadrant of altitude to the given azi-

* By rectifying the globe for the zenith, is meant screwing the quadrant of altitude to the given latitude on the brass meridian.

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muth in the horizon; keeping it there, turn the globe on its axis until the ecliptic cuts the quadrant in the given altitude: that point of the ecliptic which cuts the quadrant there, will be the sun's place; and the day of the month answering there-to, will be found over the like place of the sun on the wooden horizon. Keep the quadrant of altitude in that position, and having brought the sun's place to the brazen meridian, and the hour index to XII at noon, turn back the globe, until the sun's place cuts the quadrant of altitude again, and the index will shew the hour.

Any two points of the ecliptic which are equidistant from the beginning of *cancer* or of *capricorn*, will have the same altitude and azimuth at the same hour, though the months be different; and therefore it requires some care in this problem, not to mistake both the month and the day of the month; to avoid which, observe, that from the 20th of March to the 21st of June, that part of the ecliptic which is between the beginning of *aries* and beginning of *cancer* is to be used: from the 21st of June to the 23d of September, between the beginning of *cancer* and beginning of *libra*: from the 23d of September to the 21st of December, between the beginning of *libra* and the beginning of *capricorn*: and from the 21st of December to the 20th of March, between the beginning of *capricorn* and beginning of *aries*. And as one can never be at a loss to know in what quarter of the year he takes the sun's altitude and azimuth, the above caution, with regard to the quarters of
the

the ecliptic, will keep him right as to the month, and day thereof.

P R O B L E M XXVI.

To find the length of the longest day at any given place.

If the place be on the north side of the equator, find its latitude by Prob. 1.) and elevate the north pole to that latitude; then, bring the beginning of *cancer* ☊ to the brasen meridian, and set the hour index to XII at noon. But if the given place be on the south side of the equator, elevate the south pole to its latitude, and bring the beginning of *capricorn* ☋ to the brass meridian, and the hour index to XII. This done, turn the globe westward, until the beginning of *cancer* or *capricorn* (as the latitude is north or south) comes to the horizon; and the index will then point out the time of sun-setting, for it will have gone over all the afternoon hours, between mid-day and sun-set; which length of time being doubled, will give the whole length of the day, from sun-rising to sun-setting. For, in all latitudes, the sun rises as long before mid-day as he sets after it.

P R O B L E M XXVII.

To find in what latitude the longest day is of any given length, less than 24 hours.

If the latitude be north, bring the beginning of *cancer* to the brasen meridian, and elevate the north pole

pole to about $66\frac{1}{2}$ degrees ; but if the latitude be south, bring the beginning of *capricorn* to the meridian, and elevate the south pole to about $66\frac{1}{2}$ degrees ; because the longest day in north latitude, is when the sun is in the first point of *cancer* ; and in south latitude, when he is in the first point of *capricorn*. Then set the hour index to XII at noon, and turn the globe westward, until the index points at half the number of hours given ; which done, keep the globe from turning on its axis, and slide the meridian down in the notches, until the afore-said point of the ecliptic (viz. *cancer* or *capricorn*) comes to the horizon ; then, the elevation of the pole will be equal to the latitude required.

P R O B L E M XXVIII.

*The latitude of any place, not exceeding $66\frac{1}{2}$ degrees, being given ; to find in what * climate the place is.*

Find the length of the longest day at the given place, by Prob. 26. and whatever be the number of hours whereby it exceedeth twelve, double that

* A *climate*, from the equator to either of the polar circles, is a tract of the earth's surface, included between two such parallels of latitude, that the length of the longest day in the one exceeds that in the other by half an hour : but from the polar circles to the poles, where the sun keeps long above the horizon without setting, each climate differs a whole month from the one next to it. There are 24 climates between the equator and each of the polar circles ; and six from each polar circle to its respective pole.

number,

number, and the sum will give the climate in which the place is.

P R O B L E M XXIX.

The latitude, and the day of the month, being given ; to find the hour of the day when the sun shines.

Set the wooden horizon truly level, and the brazen meridian due north and south by a mariner's compass : then, having rectified the globe, stick a small sewing-needle into the sun's place in the ecliptic, perpendicular to that part of the surface of the globe : this done, turn the globe on its axis, until the needle comes to the brazen meridian, and set the hour index to XII at noon ; then, turn the globe on its axis, until the needle points exactly towards the sun (which it will do when it casts no shadow on the globe) and the index will shew the hour of the day.

P R O B L E M XXX.

A pleasant way of shewing all those places of the earth which are enlightened by the sun, and also the time of the day, when the sun shines.

Take the terrestrial ball out of the wooden horizon, and also out of the brazen meridian ; then set it upon a pedestal in sun-shine, in such a manner that its north pole may point directly towards the north pole of the heaven, and the meridian of the place where you are be directly towards the
X south.

south. Then, the sun will shine upon all the like places of the globe that he does on the real earth, rising to some when he is setting to others; as you may perceive by that part where the enlightened half of the globe is divided from the half in the shade, by the boundary of the light and darkness: all those places on which the sun shines, at any time, having day; and all those on which he does not shine, having night.

If a narrow slip of paper be put round the equator, and divided into 24 equal parts, beginning at the meridian of your place, and the hours set to those divisions in such a manner, that one of the VI's may be upon your meridian; the sun being upon that meridian at noon, will then shine exactly to the two XII's; and at one o'clock to the two I's, &c. So that the place, where the enlightened half of the globe is parted from the shaded half, in this circle of hours, will shew the hour of the day.

The principles of dialing shall be explained in the next lecture, by the terrestrial globe. At present we shall only add the following observations upon it; and then proceed to the use of the celestial globe.

1. *The latitude of any place is always equal to the elevation of the pole above the horizon of that place; and the elevation of the equator is equal to the complement of the latitude, that is, to what the latitude wants of 90 degrees.*

2. *Those places which lie on the equator have no latitude, it being there that the latitude begins; and those places which lie on the first meridian have no longitude,*

gitude, it being there that the longitude begins. Consequently, that particular place of the earth where the first meridian intersects the equator, has neither longitude nor latitude.

3. In all places of the earth except the poles, all the points of the compass may be distinguished in the horizon: but from the north pole, every place is south; and from the south pole, every place is north. Therefore, as the sun is constantly above the horizon of each pole for half a year in its turn, he cannot be said to depart from the meridian of either pole for half a year together. Consequently, at the north pole it may be said to be noon every moment for half a year; and let the winds blow from what part they will, they must always blow from the south; and at the south-pole, from the north.

4. Because one half of the ecliptic is above the horizon of the pole, and the sun, moon and planets, move in (or nearly in) the ecliptic; they will all rise and set to the poles. But, because the stars never change their declinations from the equator, (at least not sensibly in one age) those which are once above the horizon of either pole, never set below it; and those which are once below it never rise.

5. All places of the earth do equally enjoy the benefit of the sun in respect of time, and are equally deprived of it.

6. All places upon the equator have their days and nights equally long, that is, 12 hours each, at all times of the year. For, although the sun declines alternately, from the equator towards the north and towards the south, yet, as the horizon of the equator

cuts all the parallels of latitude and declination in halves, the sun must always continue above the horizon for one half a diurnal revolution about the earth, and for the other half below it.

7. *When the sun's declination is greater than the latitude of any place, upon either side of the equator, the sun will come twice to the same azimuth or point of the compass in the forenoon, at that place; and twice to a like azimuth in the afternoon; that is, he will go twice back every day, whilst his declination continues to be greater than the latitude. Thus, suppose the globe rectified to the latitude of Barbadoes, which is 13 degrees north; and the sun to be any where in the ecliptic, between the middle of taurus and middle of leo; if the quadrant of altitude be set to about * 18 degrees north of the east in the horizon, the sun's place be marked with a chalk upon the ecliptic, and the globe be then turned westward on its axis, the said mark will rise in the horizon a little to the north of the quadrant, and thence ascending, it will cross the quadrant towards the south; but before it arrives at the meridian it will cross the quadrant again, and pass over the meridian northward of Barbadoes. And if the quadrant be set about 18 degrees north of the west, the sun's place will cross it twice, as it descends from the meridian towards the horizon, in the afternoon.*

8. *In all places of the earth between the equator and poles, the days and nights are equally long, viz. 12 hours each, when the sun is in the equinoctial: for, in all elevations of the pole, short of 90 degrees*

* From the middle of gemini to the middle of cancer, the quadrant may be set 20 degrees,

(which

(which is the greatest) one half of the equator or equinoctial will be above the horizon, and the other half below it.

9. The days and nights are never of an equal length at any place between the equator and polar circles, but when the sun enters the signs ♈ aries and ♎ libra. For in every other part of the ecliptic, the circle of the sun's daily motion is divided into two unequal parts by the horizon.

10. The nearer that any place is to the equator, the less is the difference between the length of the days and nights in that place; and the more remote, the contrary. The circles which the sun describes in the heaven every 24 hours, being cut more nearly equal in the former case, and more unequally in the latter.

11. In all places lying upon any given parallel of latitude, however long or short the day or night be at any one of these places, at any time of the year, it is then of equal length at all the rest; for, in turning the globe round its axis (when rectified according to the sun's declination) all these places will keep equally long above or below the horizon.

12. The sun is vertical twice a year to every place between the tropics; to those under the tropics, once a year, but never any where else. For, there can be no place between the tropics, but what there are two points in the ecliptic whose declination from the equator is equal to the latitude of that place; and but one point of the ecliptic which has a declination equal to the latitude of places on the tropic which that point of the ecliptic touches: and as the sun never goes

without the tropics, he can never be vertical to any place that lies without them.

13. To all places in the * torrid zone, the duration of the twilight is least, because the sun's daily motion is the most perpendicular to the horizon. In the frigid † zones, greatest; because the sun's daily motion is nearly parallel to the horizon; and therefore he is the longer of getting 18 degrees below it (till which time the twilight always continues). And in the § temperate zones it is at a medium between the two, because the obliquity of the sun's daily motion is so.

14. In all places lying exactly under the polar circles, the sun, when he is in the nearest tropic, continues 24 hours above the horizon without setting; because no part of that tropic is below their horizon. And when the sun is in the farthest tropic, he is for the same length of time without rising; because no part of that tropic is above their horizon. But, at all other times of the year, he rises and sets there, as in other places; because all the circles that can be drawn parallel to the equator, between the tropics, are more or less cut by the horizon, as they are farther from, or nearer to, that tropic which is all above the horizon: and when the sun is not in either of the tropics, his diurnal course must be in one or other of these circles.

15. To all places in the northern hemisphere, from the equator to the polar circle, the longest day and shortest night is when the sun is in the northern tropic;

* Between the tropics. † Between the polar circles and poles. § Between the tropics and polar circles.

and

and the shortest day and longest night is when the sun is in the southern tropic ; because no circle of the sun's daily motion is so much above the horizon, and so little below it, as the northern tropic ; and none so little above it, and so much below it, as the southern. In the southern hemisphere, the contrary.

16. In all places between the polar circles and poles, the sun appears for some number of days (or rather diurnal revolutions) without setting ; and at the opposite time of the year without rising ; because some part of the ecliptic never sets in the former case, and as much of the opposite part never rises in the latter. And the nearer unto, or the more remote from the pole, these places are, the longer or shorter is the sun's continual presence or absence.

17. If a ship sets out from any port, and sails round the earth eastward to the same port again, let her take what time she will to do it in, the people in that ship, in reckoning their time, will gain one compleat day at their return, or count one day more than those who reside at the same port ; because, by going contrary to the sun's diurnal motion, and being forwarder every evening than they were in the morning, their horizon will get so much the sooner above the setting sun, than if they had kept for a whole day at any particular place. And thus, by cutting off a part proportionable to their own motion, from the length of every day, they will gain a compleat day of that sort at their return ; without gaining one moment of absolute time more than is elapsed during their course, to the people at the port. If they sail westward, they will reckon one day less than the people do who reside

at the said port, because by gradually following the apparent diurnal motion of the sun, they will keep him each particular day so much longer above their horizon, as answers to that day's course: and by that means they cut off a whole day in reckoning, at their return, without losing one moment of absolute time.

Hence, if two ships should set out at the same time from any port, and sail round the globe, one eastward and the other westward, so as to meet at the same port on any day whatever; they will differ two days in reckoning their time, at their return. If they sail twice round the earth, they will differ four days; if thrice, then six, &c.

The celestial globe.

To rectify it.

Having done, for the present, with the terrestrial globe, we shall proceed to the use of the celestial; first premising, that as the equator, ecliptic, tropics, polar circles, horizon, and brazen meridian, are exactly alike on both globes, all the former problems concerning the sun are solved the same way by both, and therefore it is needless to repeat them. The method also of rectifying the celestial globe is the same as rectifying the terrestrial, *viz.* Elevate the pole according to the latitude of your place, then screw the quadrant of altitude to the zenith, on the brass meridian; bring the sun's place in the ecliptic to the graduated edge of the brass meridian, on the side which is above the south point of the wooden horizon, and set the hour index to the uppermost XII, which stands for noon.

N. B. The sun's place for any day of the year stands directly over that day, on the horizon of the celestial globe, as it does on that of the terrestrial.

The

The *latitude* and *longitude* of any *star*, or other celestial phenomenon, is reckoned in a very different manner from the latitude and longitude of any place on the earth: for all terrestrial latitudes are reckoned from the equator; and longitudes from the meridian of some remarkable place, as of London by the English, and Paris by the French; though most of the French maps begin their longitude at the meridian of the island *Ferro*.—But the astronomers of all nations agree in reckoning the *latitudes* of the stars, planets, and comets, from the *ecliptic*; and their *longitudes* from the * *equinoctial colure*, in that semicircle of it which cuts the ecliptic at the beginning of *aries* ♈; and thence eastward, quite round, to the same semicircle again. Consequently, all those stars which lie between the equinoctial and the northern half of the ecliptic, have north declination and south latitude; and all those which lie between the equinoctial and the southern half of the ecliptic, have south declination and north latitude; and all those which lie between the tropics and poles, have their declinations and latitudes of the same denomination.

* The great circle that passes through the *equinoctial points* at the beginning of ♈ and ♎, and through the *poles of the world*, (which are two opposite points, each 90 degrees from the equinoctial) is called the *equinoctial colure*: and the great circle that passes through the beginning of ♈ and ♏, and also through the *poles of the ecliptic*, and poles of the world, is called the *solstitial colure*.

There

There are six great circles on the celestial globe, which cut the ecliptic perpendicularly, and meet in two opposite points in the polar circles; which points are each 90 degrees from the ecliptic, and are called its poles. These points divide the said circles into 12 semicircles; and they cut the ecliptic at the beginnings of the 12 signs. They resemble so many meridians on the terrestrial globe; and as all places which lie under any particular meridian semicircle on that globe, have the same longitude, so all those points of the heaven, through which any one of the above semicircles are drawn, have the same longitude.—And, as the greatest latitudes on the earth are at the north and south poles of the earth, so the greatest latitudes in the heaven, are at the north and south poles of the ecliptic.

Constellations.

In order to distinguish the stars, with regard to their situations and positions in the heaven, the antients divided the whole visible firmament of stars into particular systems, which they called *constellations*; and digested them into the forms of such animals as are delineated upon the celestial globe. And those which lie between the figures of these imaginary animals, and could not be brought within the compass of any of them, were called *unformed stars*.

Because the moon and all the planets were observed to move in circles or orbits which cross the ecliptic (or line of the sun's path) at small angles, and to be on the north side of the ecliptic for one half of their course round the heaven of stars, and
on

on the south side of it for the other half, but never to go quite 8 degrees from it on either side, the ancients distinguished that space by two lesser circles, parallel to the ecliptic, (one on each side) at 8 degrees distance from it. And the space included between these circles, they called the *zodiac*, because most of the 12 constellations placed therein, resemble some living creature.—These constellations are, 1. *Aries* ♈, the ram; 2. *Taurus* ♉, the bull; 3. *Gemini* ♊, the twins; 4. *Cancer* ♋, the crab; 5. *Leo* ♌, the lion; 6. *Virgo* ♍, the virgin; 7. *Libra* ♎, the balance; 8. *Scorpio* ♏, the scorpion; 9. *Sagittarius* ♐, the archer; 10. *Capricornus* ♑, the goat; 11. *Aquarius* ♒, the water bearer; and 12. *Pisces* ♓, the fishes.

Zodiac.

Its signs, or divisions.

It is to be observed, that in the infancy of astronomy, these twelve constellations stood at or near the places of the ecliptic, where the above characteristics are marked upon the globe: but now, each constellation is a whole sign forwarder, on account of the recession of the equinoctial points from their former places. So that the constellation of *aries*, is now in the former place of *taurus*; that of *taurus*, in the former place of *gemini*; and so on.

Remark.

The stars appear of different magnitudes to the eye; probably because they are at different distances from us. Those which appear brightest and largest, are called *stars of the first magnitude*; the next to them in size and lustre, *stars of the second magnitude*; and so on to the *sixth*, which are the smallest that can be discerned by the bare eye.

Some

Some of the most remarkable stars have names given them, as *castor* and *pollux* in the heads of the twins, *sirius* in the mouth of the *great dog*, *procyon* in the side of the *little dog*, *rigel* in the left foot of *orion*, *arcturus* near the right thigh of *bootes*, &c.

These things being premised, which I think are all that the young *Tyro* need be acquainted with, before he begins to work any problem by this globe, we shall now proceed to the most useful of those problems; omitting several which are of little or no consequence.

P R O B L E M I.

*To find the * right ascension and † declination of the sun, or any fixed star.*

Bring the sun's place in the ecliptic to the brazen meridian, then that degree in the equinoctial which is cut by the meridian, is the sun's *right ascension*; and that degree of the meridian which is over the sun's place, is his *declination*. Bring any fixed star to the meridian, and its *right ascension* will be cut by the meridian in the equinoctial; and the degree of the meridian that stands over it, is its *declination*.

* The degree of the equinoctial, reckoned from the beginning of *aries*, that comes to the meridian with the sun or star, is its *right ascension*.

† The distance of the sun or star in degrees from the equinoctial, towards either of the poles, north or south, is its declination, which is north or south accordingly.

So

So that *right ascension* and *declination*, on the celestial globe, are found in the same manner as *longitude* and *latitude* on the terrestrial.

P R O B L E M II.

To find the latitude and longitude of any star.

If the given star be on the north side of the ecliptic, place the 90th degree of the quadrant of altitude on the north pole of the ecliptic, where the twelve semicircles meet; which divide the ecliptic into the 12 signs: but if the star be on the south side of the ecliptic, place the 90th degree of the quadrant on the south pole of the ecliptic: keeping the 90th degree of the quadrant on the proper pole, turn the quadrant about, until its graduated edge cuts the star: then, the number of degrees in the quadrant, between the ecliptic and the star, is its latitude; and the degree of the ecliptic cut by the quadrant, is the star's longitude, reckoned according to the sign in which the quadrant then is.

P R O B L E M III.

To represent the face of the starry firmament, as seen from any given place of the earth, at any hour of the night.

Rectify the celestial globe for the given latitude, the zenith, and sun's place, in every respect, as taught

taught by the 17th problem, for the terrestrial; and turn it about, until the index points to the given hour: then, the upper hemisphere of the globe will represent the visible half of the heaven for that time: all the stars upon the globe being then in such situations, as exactly correspond to those in the heaven. And if the globe be placed duly north and south, by means of a small sea-compass, every star in the globe will point toward the like star in the heaven: by which means, the constellations and remarkable stars may be easily known. All those stars which are in the eastern side of the horizon, are then rising in the eastern side of the heaven; all in the western, are setting in the western side; and all those under the upper part of the brazen meridian, between the south point of the horizon and the north pole, are at their greatest altitude, if the latitude of the place be north: but if the latitude be south, those stars which lie under the upper part of the meridian, between the north point of the horizon and the south pole, are at their greatest altitude.

P R O B L E M IV.

The latitude of the place, and day of the month, being given; to find the time when any known star will rise, or be upon the meridian, or set.

Having rectified the globe, turn it about until the given star comes to the eastern side of the horizon, and the index will shew the time of the star's

star's rising; then turn the globe westward, and when the star comes to the brasen meridian, the index will shew the time of the star's coming to the meridian of your place; lastly, turn on, until the star comes to the western side of the horizon, and the index will shew the time of the star's setting.

N. B. In northern latitudes, those stars which are less distant from the north pole, than the quantity of its elevation above the north point of the horizon, never set; and those which are less distant from the south pole, than the number of degrees that it is depressed below the horizon, never rise: and *vice versa* in southern latitudes.

P R O B L E M V.

To find at what time of the year a given star will be upon the meridian, at a given hour of the night.

Bring the given star to the upper semicircle of the brass meridian, and set the index to the given hour; then turn the globe, until the index points to XII at noon, and the upper semicircle of the meridian will then cut the sun's place, answering to the day of the year sought; which day may be easily found against the like place of the sun, among the signs on the wooden horizon.

P R O B L E M

P R O B L E M VI.

*The latitude, day of the month, and * azimuth of any known star being given; to find the hour of the night.*

Having rectified the globe for the latitude, zenith, and sun's place; lay the quadrant of altitude to the given degree of azimuth, in the horizon; then, turn the globe on its axis, until the star comes to the graduated edge of the quadrant; and when it does so, the index will point out the hour of the night.

P R O B L E M VII.

The latitude of the place, the day of the month, and altitude † of any known star, being given; to find the hour of the night.

Rectify the globe as in the former problem, guess at the hour of the night, and turn the globe until the index points at the supposed hour; then lay the graduated edge of the quadrant of altitude over the known star, and if the degree of the star's

* The number of degrees that the sun, moon, or any star, is from the meridian, either to the east or west, is called its *azimuth*.

† The number of degrees that the star is above the horizon, as observed by means of a common quadrant, is called its *altitude*.

height

height in the quadrant upon the globe, answers exactly to the degree of the star's observed altitude in the heaven, you have guessed exactly: but if the star on the globe is higher or lower than it was observed to be in the heaven, turn the globe backwards or forwards, keeping the edge of the quadrant upon the star, until its center comes to the observed altitude in the quadrant; and then, the index will shew the true time of the night.

P R O B L E M VIII.

An easy method for finding the hour of the night by any two known stars, without knowing either their altitude or azimuth; and then, of finding both their altitude and azimuth, and thereby the true meridian.

Tie one end of a thread to a common musket bullet; and, having rectified the globe as above, hold the other end of the thread in your hand, and carry it slowly round betwixt your eye and the starry heaven, until you find it cuts any two known stars at once. Then, guessing at the hour of the night, turn the globe until the index points to that time in the hour circle; which done, lay the graduated edge of the quadrant over any one of these two stars on the globe, which the thread cut in the heaven. If the said edge of the quadrant cuts the other star also, you have guessed the time exactly; but if it does not, turn the globe slowly backwards or forwards, until the quadrant (kept
Y upon

upon either star) cuts them both through their centers: and then, the index will point out the exact time of the night; the degree of the horizon, cut by the quadrant, will be the true azimuth of both these stars from the south; and the stars themselves will cut their true altitudes in the quadrant. At which moment, if a common azimuth compass be so set upon a floor or level pavement, that these stars in the heaven may have the same bearing upon it (allowing for the variation of the needle) as the quadrant of altitude has in the wooden horizon of the globe, a thread extended over the north and south points of that compass will be directly in the plane of the meridian: and if a line be drawn upon the floor or pavement, along the course of the thread, and an upright wire be placed in the southmost end of the line, the shadow of the wire will fall upon that line, when the sun is on the meridian, and shines upon the pavement.

P R O B L E M IX.

To find the place of the moon, or of any planet; and thereby to shew the time of its rising, southing, and setting.

Seek in *Parker's* or *Weaver's* Ephemeris the *geocentric place of the moon or planet in the ecliptic, for the given day of the month; and, ac-

* The place of the moon or planet, as seen from the earth.

ording

cording to its longitude and latitude, as shewn by the Ephemeris, mark the same with a chalk upon the globe. Then, having rectified the globe, turn it round its axis westward; and, as the said mark comes to the eastern side of the horizon, to the brazen meridian, and to the western side of the horizon, the index will shew at what time the planet rises, comes to the meridian, and sets, in the same manner as for a fixed star.

P R O B L E M X.

To explain the phenomena of the harvest-moon.

In order to do this, we must premise the following things. 1. That as the sun goes only once a year round the ecliptic, he can be but once a year in any particular point of it: and that his motion is almost a degree every 24 hours, at a mean rate. 2. That as the moon goes round the ecliptic once in 27 days and 8 hours, she advances $13\frac{1}{6}$ degrees in it, every day, at a mean rate. 3. That as the sun goes through part of the ecliptic in the time the moon goes round it, the moon cannot at any time, be either in conjunction with the sun, or opposite to him, in that part of the ecliptic where she was so the last time before; but must travel as much forwarder, as the sun has advanced in the said time; which being $29\frac{1}{2}$ days, makes almost a whole sign. Therefore, 4. The moon can be but once a year opposite to the sun, in any particular part of the ecliptic. 5. That the

Y 2 moon

moon is never full but when she is opposite to the sun, because at no other time can we see all that half of her which the sun enlightens. 6. That when any point of the ecliptic rises, the opposite point sets. Therefore, when the moon is opposite to the sun, she must rise at * sun-set. 7. That the different signs of the ecliptic, rise at very different angles or degrees of obliquity with the horizon, especially in considerable latitudes; and that the smaller this angle is, the greater is the portion of the ecliptic that rises in any small part of time; and *vice versa*. 8. That, in northern latitudes, no part of the ecliptic rises at so small an angle with the horizon, as *pisces* and *aries* do; and therefore, a greater portion of the ecliptic rises in one hour, about these signs, than about any of the rest. 9. That the moon can never be full in *pisces* and *aries* but in our autumnal months, for at no other time of the year is the sun in the opposite signs *virgo* and *libra*.

These things premised, take $13\frac{1}{2}$ degrees of the ecliptic in your compasses, and, beginning at *pisces*, carry that extent all round the ecliptic, marking the places with a chalk, where the points of the compasses successively fall. So you will have the moon's daily motion marked out for one compleat revolution in the ecliptic (according to § 2 of the last paragraph.)

* This is not always strictly true, because the moon does not keep in the ecliptic, but crosses it twice every month. However, the difference need not be regarded in a general explanation.

Rectify

Rectify the globe for any considerable northern latitude, (as suppose that of London) and then, turning it round its axis, observe how much of the hour-circle the index has gone over, at the rising of each particular mark on the ecliptic; and you will find that seven of the marks (which take in as much of the ecliptic as the moon moves through in a week) will all rise successively about *pisces* and *aries*, in the time that the index goes over two hours. Therefore, whilst the moon is in *pisces* and *aries*, she will not differ in general above two hours in her rising for a whole week. But if you take notice of the marks on the opposite signs, *virgo* and *libra*, you will find that seven of them take nine hours to rise; which shews, that when the moon is in these two signs, she differs nine hours in her rising within the compass of a week. And so much later as every mark is of rising than the one that rose next before it, so much later will the moon be of rising any day, than she was on the day before, in the corresponding part of the heaven. The marks about *cancer* and *capricorn*, rise with a mean difference of time between those about *aries* and *libra*.

Now, although the moon is in *pisces* and *aries* every month, and therefore must rise in those signs within the space of two hours later for a whole week, or only about 17 minutes later every day than she did on the former; yet she is never full in these signs, but in our autumnal months *August* and *September*, when the sun is in *virgo* and *libra*. Therefore, no full moon in the year will continue

to rise so near the time of sun-set for a week or so, as these two full moons do, which fall in the time of harvest.

In the winter months, the moon is in *pisces* and *aries* about her first quarter; and as these signs rise about noon in winter, the moon's rising in them passes unobserved. In the spring months, the moon changes in these signs, and consequently rises at the same time with the sun; so that it is impossible to see her at that time. In the summer months she falls in these signs about her third quarter, and rises not until midnight, when her rising is but very little taken notice of; especially as she is on the decrease. But in the harvest months she is at the full, when in these signs, and being opposite to the sun, she rises when the sun sets, (or soon after) and shines all the night.

In southern latitudes, *virgo* and *libra* rise at as small angles with the horizon, as *pisces* and *aries* do in the northern; and as our spring is at the time of their harvest, it is plain their harvest full moons must fall in *virgo* and *libra*; and will therefore rise with as little difference of time, as ours do in *pisces* and *aries*.

For a fuller account of this matter, I must refer the reader to my Astronomy, in which it is described at large.

P R O B L E M XI.

To explain the equation of time, or difference of time between well regulated clocks and true sun-dials.

The earth's motion on its axis being perfectly equable, and thereby causing an apparent equable motion of the starry heaven, round the same axis produced to the poles of the heaven; it is plain that equal portions of the celestial equator pass over the meridian in equal parts of time, because the axis of the world is perpendicular to the plane of the equator. And therefore, if the sun kept his annual course in the celestial equator, he would always revolve from the meridian to the meridian again in 24 hours exactly, as shewn by a well regulated clock.

But as the sun moves in the ecliptic, which is oblique both to the plane of the equator and axis of the world, he cannot always revolve from the meridian to the meridian again in 24 equal hours; but sometimes a little sooner, and at other times a little later, because equal portions of the ecliptic pass over the meridian in unequal parts of time, on account of its obliquity. And this difference is the same in all latitudes.

To shew this by a globe, make chalk-marks all around the equator and ecliptic, at equal distances from one another (suppose 10 degrees) beginning at *aries* or at *libra*, where these two circles intersect each other. Then turn the globe round

its axis, and you will see that all the marks in the first quadrant of the ecliptic, or from the beginning of *aries* to the beginning of *cancer*, come sooner to the brazen meridian than their corresponding marks do on the equator : those in the second quadrant, or from the beginning of *cancer* to the beginning of *libra*, come later : those in the third quadrant, from *libra* to *capricorn*, sooner ; and those in the fourth, from *capricorn* to *aries*, later. But those at the beginning of each quadrant come to the meridian at the same time with their corresponding marks on the equator.

Therefore, whilst the sun is in the first and third quadrants of the ecliptic, he comes sooner to the meridian every day than he would do if he kept in the equator ; and consequently he is faster than a well regulated clock, which always keeps equable or equatoreal time : and whilst he is in the second and fourth quadrants, he comes later to the meridian every day than he would do if he kept in the equator ; and is therefore slower than the clock. But at the beginning of each quadrant, the sun and clock are equal.

And thus, if the sun moved equably in the ecliptic, he would be equal with the clock on four days of the year, which would have equal intervals of time between them. But as he moves faster at some times than at others, (for he is eight days longer in the northern half of the ecliptic than in the southern) this will create a second inequality ; which combined with the former, arising from the obliquity of the ecliptic to the equator, makes up that

that difference, which is shewn by the common equation tables to be between good clocks and true sun-dials.

The description and use of the armillary sphere.

Whoever has seen a common *armillary sphere*, Plate XX. Fig. 1. and understands how to use it, must be sensible that the machine here referred to, is of a very different, and much more advantageous construction. And whoever has seen the curious glass sphere invented by Dr. LONG, or the figure of it in his Astronomy, must know that the furniture of the terrestrial globe in this machine, the form of the pedestal, and the manner of turning either the earthly globe, or the circles which surround it, are all copied from the Doctor's glass sphere; and that the only difference is, a parcel of rings instead of a glass celestial globe; and all the additions are a moon within the sphere, and a semicircle upon the pedestal.

The exterior parts of this machine are a com- The armil- lary sphere. pages of brass rings, which represent the principal circles of the heaven: viz. 1. The equinoctial *AA*, which is divided into 360 degrees (beginning at its intersection with the ecliptic in *aries*) for shewing the sun's right ascension in degrees; and also into 24 hours, for shewing his right ascension in time. 2. The ecliptic *BB*, which is divided into 12 signs, and each sign into 30 degrees, and also into the months and days of the year; in such a manner, that the degree or point of the ecliptic

in which the sun is, on any given day, stands over that day in the circle of months. 3. The tropic of *cancer* CC , touching the ecliptic at the beginning of *cancer* in e , and the tropic of *capricorn* DD , touching the ecliptic at the beginning of *capricorn* in f ; each $23\frac{1}{2}$ degrees from the equinoctial circle. 4. The arctic circle E , and the antarctic circle F , each $23\frac{1}{2}$ degrees from its respective pole at N and S . 5. The equinoctial colure GG , passing through the north and south poles of the heaven at N and S , and through the equinoctial points in the ecliptic *aries* and *libra*. 6. The solstitial colure HH , passing through the poles of the heaven, and through the solstitial points *cancer* and *capricorn*, in the ecliptic: each quarter of the former of these colures is divided into 90 degrees, from the equinoctial to the poles of the world, for shewing the declination of the sun, moon, and stars; and each quarter of the latter, from the ecliptic at e and f , to its poles b and d , for shewing the latitudes of the stars.

In the north pole of the ecliptic is a nut b , to which is fixed one end of a quadrantal wire, and to the other end a small sun X , which is carried round the ecliptic BB , by turning the nut: and in the south pole of the ecliptic is a pin d , on which is another quadrantal wire, with a small moon Z upon it, which may be moved round by hand: but there is a particular contrivance for causing the moon to move in an orbit which crosses the ecliptic at an angle of $5\frac{1}{3}$ degrees, in two opposite points called the *moon's nodes*; and also for shifting these points

points backward in the ecliptic, as the *moon's nodes* shift in the heaven.

Within these circular rings is a small terrestrial globe *I*, fixt on an axis *KK*, which extends from the north and south poles of the globe at *n* and *s*, to those of the celestial sphere at *N* and *S*. On this axis is fixt the flat celestial meridian *LL*, which may be set directly over the meridian of any place on the globe, and then turned round with the globe, so as to keep over the same meridian upon it. This flat meridian is graduated the same way as the brass meridian of a common globe, and its use is much the same. To this globe is fitted the moveable horizon *MM*, so as to turn upon two strong wires proceeding from its east and west points to the globe, and entering the globe at opposite points of its equator, which is a moveable brass ring let into the globe in a groove all around its equator. The globe may be turned by hand within this ring, so as to place any given meridian upon it, directly under the celestial meridian *LL*. The horizon is divided into 360 degrees all around its outermost edge, within which are the points of the compass, for shewing the amplitude of the sun and moon, both in degrees and points. The celestial meridian *LL* passes through two notches in the north and south points of the horizon, as in a common globe: but here, if the globe be turned round, the horizon and meridian turn with it. At the south pole of the sphere is a circle of 24 hours, fixt to the rings,
and

and on the axis is an index which goes round that circle, if the globe be turned round its axis.

The whole fabric is supported on a pedestal *N*, and may be elevated or depressed, upon the joint *O*, to any number of degrees from 0 to 90, by means of the arc *P*, which is fixed into the strong brass arm *Q*, and slides in the upright piece *R*, in which is the screw *r*, to fix it at any proper elevation.

In the box *T* are two wheels, and two pinions whose axes come out at *V* and *U*; either of which may be turned by the small winch *W*. When the winch is put upon the axis *V*, and turned backward, the terrestrial globe, with its horizon and celestial meridian, keep at rest; and the whole sphere of circles turns round from east, by south, to west, carrying the sun *X*, and moon *Z*, round the same way, and causing them to rise above and set below the horizon. But when the winch is put upon the axis *U*, and turned forward, the sphere with the sun and moon keep at rest; and the earth with its horizon and meridian turn round from west, by south, to east; and bring the same points of the horizon to the sun and moon, to which these bodies came when the earth kept at rest, and they were carried round it; shewing that they rise and set in the same points of the horizon, and at the same times in the hour circle, whether the motion be in the earth or in the heaven. If the earth be turned, the hour index goes round its circle; but if the sphere be turned, the hour circle goes round below the index.

And

And so, by this construction, the machine is equally fitted to shew either the real motion of the earth, or the apparent motion of the heaven.

To rectify the sphere for use, first slacken the screw *r* in the upright stem *R*, and taking hold of the arm *Q*, move it up or down until the given degree of latitude for any place be cut by the stem *R*; and then the axis of the sphere will be properly elevated, so as to stand parallel to the axis of the world, if the machine be set north and south by a small compass: this done, count the latitude from the north pole, upon the celestial meridian *LL*, down towards the north notch of the horizon, and set the horizon to that latitude; then, turn the nut *b* until the sun *X* comes to the given day of the year in the ecliptic, and the sun will be at its proper place for that day: find the place of the moon's ascending node, and also the place of the moon, by an Ephemeris, and set them right accordingly: lastly, turn the winch *W*, until either the sun comes to the meridian *LL*, or until the meridian comes to the sun, (according as you want the sphere or earth to move) and set the hour index to the XII, marked noon, and the whole machine will be rectified.—Then turn the winch, and observe when the sun or moon rise and set in the horizon, and the hour index will shew the times thereof for the given day.

As those who understand the use of the globes will be at no loss to work many other problems by this sphere, it seems needless to enlarge any farther upon it.

LECT. IX.

The principles of dialing.

Prelimina-
ries.

A Dial is a plane, upon which lines are described in such a manner, that the shadow of a wire, or of the upper edge of another plane, erected perpendicularly on the former, may shew the true time of the day.

The edge of the plane by which the time of the day is found, is called the stile of the dial, which is always parallel to the earth's axis; and the line on which the said plane is erected, is called the substile.

The angle included between the substile and stile, is called the elevation, or height of the stile.

Those dials whose planes are parallel to the plane of the horizon, are called horizontal dials; and those dials whose planes are perpendicular to the plane of the horizon, are called vertical, or erect dials.

Those erect dials, whose planes directly front the north or south, are called direct north or south dials; and all other erect dials are called decliners, because their planes are turned away from the north or south.

Those dials whose planes are neither parallel nor perpendicular to the plane of the horizon, are called inclining, or reclining dials, according as their planes make acute or obtuse angles with the horizon; and if their planes are also turned aside from

from facing the south or north, they are called declining-inclining, or declining-reclining dials.

The intersection of the plane of the dial, with that of the meridian, passing through the stile, is called the meridian of the dial, or the hour-line of XII.

Those meridians, whose planes pass through the stile, and make angles of 15, 30, 45, 60, 75, and 90 degrees with the meridian of the place, (which marks the hour-line of XII) are called hour-circles: and their intersections with the plane of the dial, are called hour-lines.

In all declining dials, the substile makes an angle with the hour-line of XII; and this angle is called the distance of the substile from the meridian.

The declining plane's difference of longitude, is the angle formed at the intersection of the stile and plane of the dial, by two meridians; one of which passes through the hour-line of XII, and the other through the substile.

This much being premised concerning dials in general, we shall now proceed to explain the different methods of their construction.

If the whole earth *aPcp* were transparent, and hollow, like a sphere of glass, and had its equator divided into 24 equal parts by so many meridian semicircles, *a, b, c, d, e, f, g, &c.* one of which is the geographical meridian of any given place, as London, (which is supposed to be at the point *a*) and if the hours of XII were marked at the equator, both upon that meridian and the opposite one,

Plate XX.
Fig. 2.

The universal principle on which dialing depends.

one, and all the rest of the hours in order on the rest of the meridians, these meridians would be the hour-circles of London: then, if the sphere had an opake axis as PEp , terminating in the poles P and p , the shadow of the axis would fall upon every particular meridian and hour, when the sun came to the plane of the opposite meridian, and would consequently shew the time at London, and all other places on the meridian of London.

*Horizontal
dial.*

If this sphere was cut through the middle by a solid plane $ABCD$, in the rational horizon of London, one half of the axis EP will be above the plane, and the other half below it; and if straight lines were drawn from the center of the plane, to those points where its circumference is cut by the hour-circles of the sphere, these lines would be the hour-lines of a horizontal dial for London: for the shadow of the axis would fall upon each particular hour-line of the dial, when it fell upon the like hour-circle of the sphere.

Fig. 3. If the plane which cuts the sphere be upright, as $AFCG$, touching the given place (London) at F , and directly facing the meridian of London, it

*Vertical
dial.*

will then become the plane of an erect direct south dial: and if right lines be drawn from its center E , to those points of its circumference where the hour-circles of the sphere cut it, these will be the hour-lines of a vertical or direct south dial for London, to which the hours are to be set as in the figure, (contrary to those on a horizontal dial) and the lower half Ep of the axis will cast a shadow on the hour of the day in this dial, at the same time

me that it would fall upon the like hour-circle of the sphere, if the dial-plane was not in the way.

If the plane (still facing the meridian) be made ^{*Inclining*} to incline, or recline, any given number of degrees; ^{*and reclining dials.*} the hour-circles of the sphere will still cut the edge of the plane in those points to which the hour-lines must be drawn straight from the center; and the axis of the sphere will cast a shadow on these lines at the respective hours. The like will still hold, if the plane be made to decline any given number ^{*Declining dials.*} of degrees from the meridian, towards the east or west: provided the declination be less than 90 degrees, or the reclination be less than the co-latitude of the place: and the axis of the sphere will be the gnomon, or stile, for the dial. But it cannot, when the declination is quite 90 degrees, nor when the reclination is equal to the co-latitude; because in these two cases, the axis has no elevation above the plane of the dial.

And thus it appears, that the plane of every dial represents the plane of some great circle upon the earth; and the gnomon the earth's axis, whether it be a small wire, as in the above figures, or the edge of a thin plate, as in the common horizontal dials.

The whole earth, as to its bulk, is but a point if compared to its distance from the sun: and therefore, if a small sphere of glass be placed upon any part of the earth's surface, so that its axis be parallel to the axis of the earth, and the sphere have such lines upon it, and such planes within it, as above described; it will shew the

Z

hours

hours of the day as truly as if it were placed in the earth's center, and the shell of the earth were as transparent as glass.

But because it is impossible to have a hollow sphere of glass perfectly true, blown round a solid plane; or if it was, we could not get at the plane within the glass to set it in any given position; we make use of a wire-sphere to explain the principles of dialing, by joining 24 semicircles together at the poles, and putting a thin flat plate of brass within it.

*Dialing by
the com-
mon ter-
restrial globe*

A common globe, of twelve inches diameter, has generally 24 meridian semicircles drawn upon it. If such a globe be elevated to the latitude of any given place, and turned about until any one of these meridians cuts the horizon in the north point, where the hour of XII is supposed to be marked, the rest of the meridians will cut the horizon at the respective distances of all the other hours from XII. Then, if the globe be taken out of the horizon, and a flat board or plate be put into its place, even with the surface of the horizon; and if straight lines be drawn from the center of the board, to those points of the horizon which were cut by the 24 meridian semicircles, they will be the hour-lines of a horizontal dial for that latitude, the edge of whose gnomon must be in the very same situation that the axis of the globe was, before it was taken out of the horizon: that is, the gnomon must make an angle with the plane of the dial, equal to the latitude of the place for which the dial is made.

If

If the pole of the globe be elevated to the * co-latitude of the given place, and any meridian be brought to the north point of the horizon, the rest of the meridians will cut the horizon in the respective distances of all the hours from XII, for a direct south dial; whose gnomon must make an angle with the plane of the dial, equal to the co-latitude of the place: and the hours must be set the contrary way on this dial, to what they are on the horizontal.

But if your globe have more than 24 meridian semicircles upon it, you must take the following method for making *horizontal* and *south dials*.

Elevate the pole to the latitude of your place, and turn the globe until any particular meridian (suppose the first) comes to the north point of the horizon, and the opposite meridian will cut the horizon in the south. Then, set the hour-index to the uppermost XII on its circle; which done, turn the globe westward until 15 degrees of the equator pass under the brazen meridian, and then the hour-index will be at I (for the sun moves 15 degrees every hour) and the first meridian will cut the horizon in the number of degrees from the north point, that I is distant from XII. Turn on, until other 15 degrees of the equator pass under the brazen meridian, and the hour-index will then be at II, and the first meridian will cut the ho-

To construct a horizontal dial.

* If the latitude be subtracted from 90 degrees, the remainder is called the co-latitude, or complement of the latitude.

rizon in the number of degrees that II is distant from XII: and so, by making 15 degrees of the equator pass under the brazen meridian for every hour, the first meridian of the globe will cut the horizon in the distances of all the hours from XII to VI, which is just 90 degrees; and then you need go no farther, for the distances of XI, X, IX, VIII, VII, and VI, in the forenoon, are the same from XII, as the distances of I, II, III, IV, V, and VI, in the afternoon: and these hour-lines continued through the center, will give the opposite hour-lines on the other half of the dial: but no more of these need be drawn, than what answer to the sun's continuance above the horizon of your place on the longest day, which may be easily found by working the 26th problem of the foregoing lecture.

Thus, to make a horizontal dial for the latitude of London, which is $51\frac{1}{2}$ degrees north, I elevate the north pole of the globe $51\frac{1}{2}$ degrees above the north point of the horizon, and then, turn the globe until the first meridian (which is that of London on the English terrestrial globe) cuts the north point of the horizon, and set the hour-index to XII at noon.

Then, turning the globe westward until the index points successively to I, II, III, IIII, V, and VI, in the afternoon; or until 15, 30, 45, 60, 75, and 90 degrees of the equator pass under the brazen meridian; I find the first meridian of the globe cuts the horizon in the following numbers of degrees from the north towards the east, *viz.* $11\frac{2}{3}$,
 $24\frac{1}{4}$,

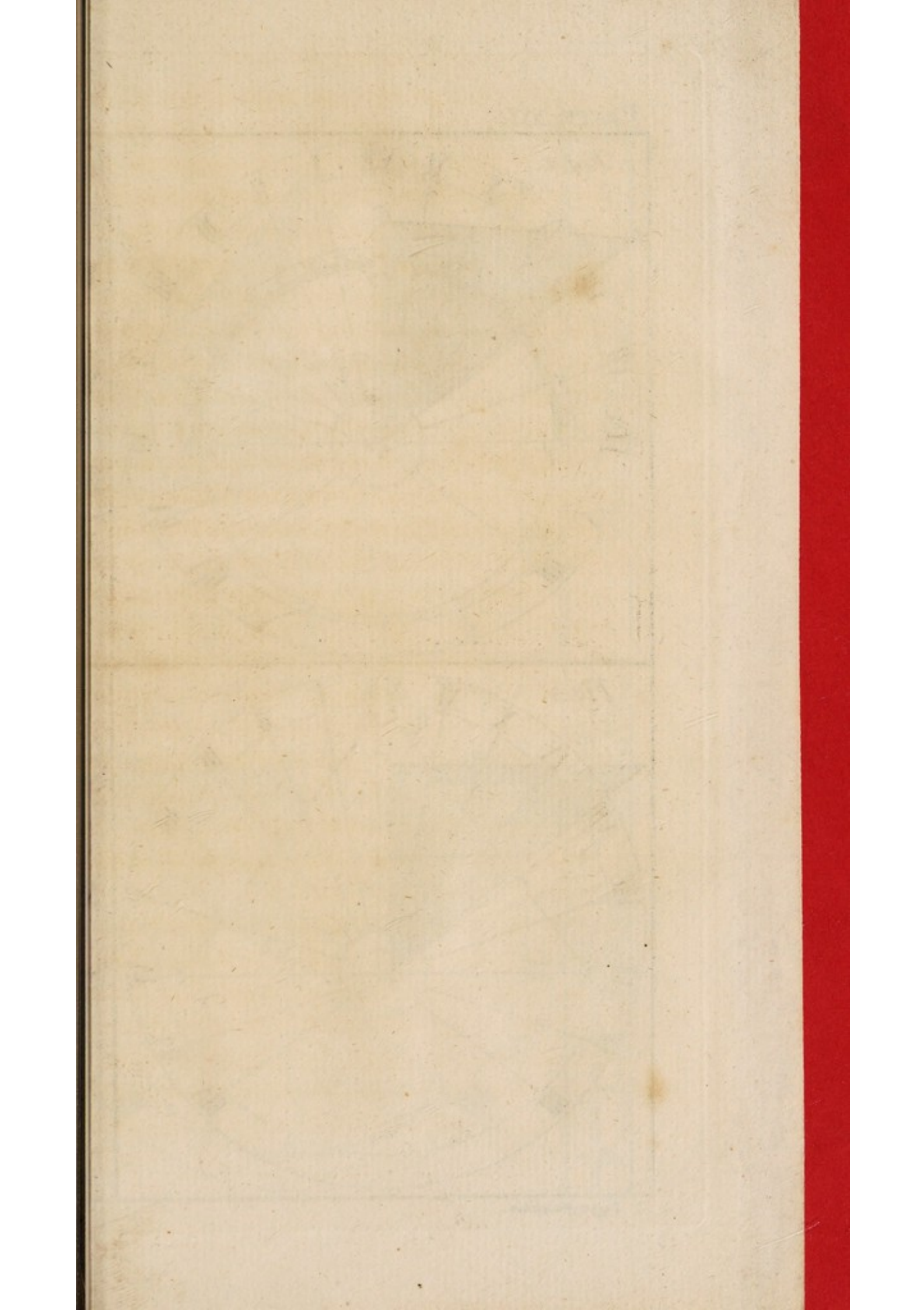
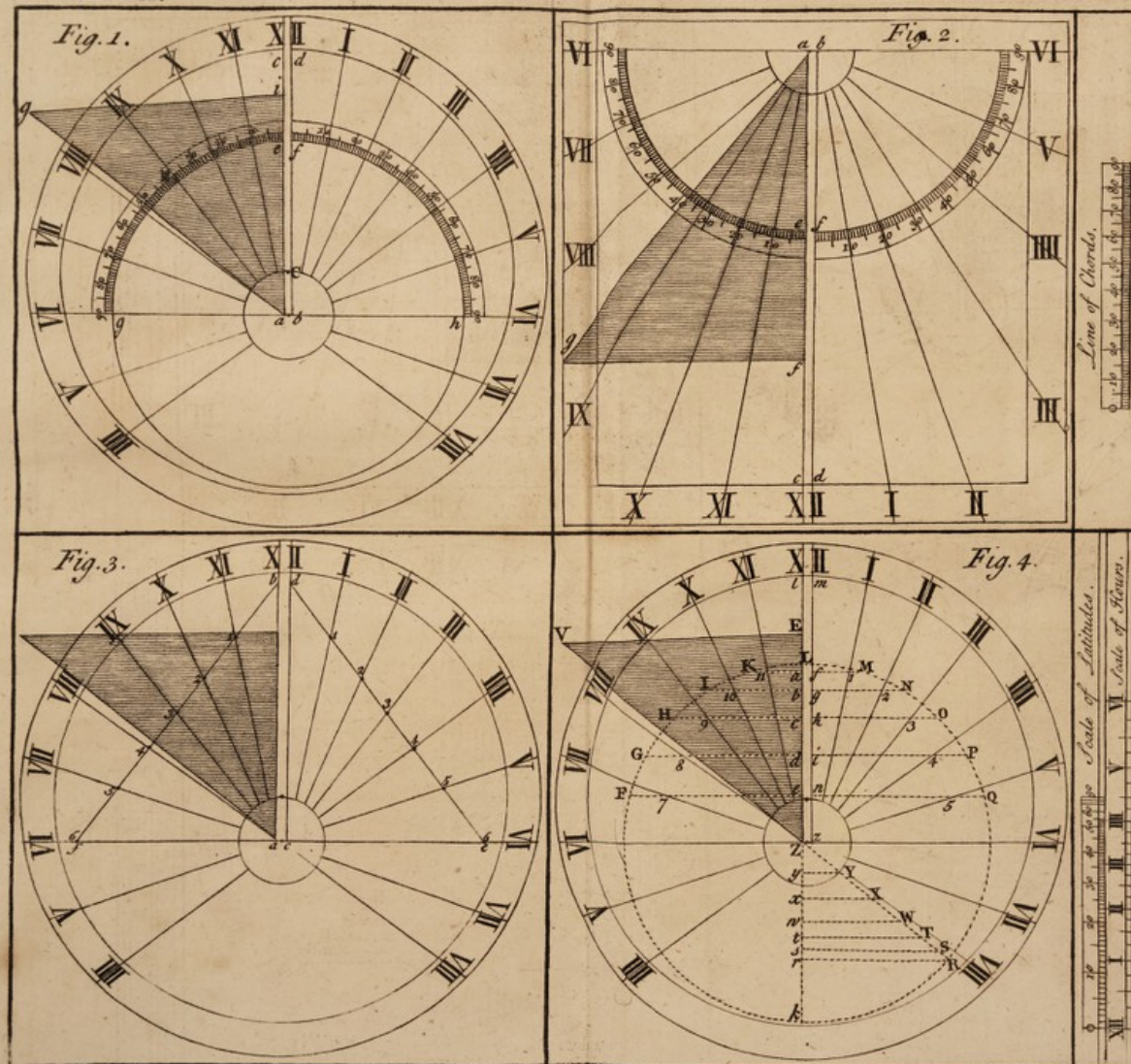


PLATE XXI.



J. Ferguson delin.

J. Mynde sc.

$24\frac{1}{4}$, $38\frac{1}{2}$, $53\frac{1}{2}$, $71\frac{1}{5}$, and 90; which are the respective distances of the above hours from XII upon the plane of the horizon.

To transfer these, and the rest of the hours, to ^{Plate XXI.} a horizontal plane, draw the right lines ac and bd ^{Fig. 1.} upon that plane, as far from each other as is equal to the intended thickness of the gnomon or stile of the dial, and the space included between them will be the meridian or twelve o'clock line on the dial. Cross this meridian at right angles with the six o'clock line gb , and setting one foot of your compasses in the intersection a , as a center, describe the quadrant ge with any convenient radius or opening of the compasses: then, setting one foot in the intersection b , as a center, with the same radius describe the quadrant fb , and divide each quadrant into 90 equal parts or degrees, as in the figure.

Because the hour-lines are less distant from each other about noon, than in any other part of the dial, it is best to have the centers of these quadrants at a little distance from the center of the dial-plane, on the side opposite to XII, in order to enlarge the hour distances thereabouts, under the same angles on the plane. Thus, the center of the plane is at C , but the centers of the quadrants are at a and b .

Laying a ruler over the point b , (and keeping it there for the center of all the afternoon hours in the quadrant fb) draw the hour-line of I through $1\frac{2}{3}$ degrees in the quadrant; the hour-line of II, through $24\frac{1}{4}$ degrees; of III, through $38\frac{1}{2}$ degrees;
Z 3
grees;

grees ; IIII, through $53\frac{1}{2}$, and V through $71\frac{1}{5}$: and because the sun rises about four in the morning, on the longest days at London; continue the hour-lines of IIII and V in the afternoon, through the center *b* to the opposite side of the dial.— This done, lay the ruler to the center *a* of the quadrant *eg*, and through the like divisions or degrees of that quadrant, *viz.* $11\frac{2}{3}$, $24\frac{1}{4}$, $38\frac{1}{2}$, $53\frac{1}{2}$, and $71\frac{1}{5}$, draw the forenoon hour-lines of XI, X, IX, VIII, and VII; and because the sun sets not before eight in the evening on the longest days, continue the hour-lines of VII and VIII in the forenoon, through the center *a*, to VII and VIII in the afternoon; and all the hour-lines will be finished on this dial; to which the hours may be set, as in the figure.

Lastly, through $51\frac{1}{2}$ degrees of either quadrant, and from its center, draw the right line *ag* for the hypotenuse or axis of the gnomon *agi*; and from *g*, let fall the perpendicular *gi*, upon the meridian line *ai*, (called also the substile) and there will be a triangle made, whose sides are *ag*, *gi*, and *ia*. If this triangle be made a plate as thick as the distance between the lines *ac* and *bd*, and set upright between them, touching at *a* and *b*, its hypotenuse *ag* will be parallel to the axis of the world, when the dial is truly set; and will cast a shadow on the hour of the day.

N. B. The trouble of dividing the two quadrants may be saved, if you have a scale with a line of chords upon it, such as that on the right hand of the plate: for if you extend the compasses

passes from 0 to 60 degrees on the line of chords, and with that extent as a radius, describe the two quadrants upon their respective centers, the above distances may be taken with the compasses upon the line, and set off upon the quadrants.

To make an erect direct south dial. Elevate the pole to the co-latitude of your place, and proceed in all respects as above taught for the horizontal dial, from VI in the morning to VI in the afternoon; only the hours must be reversed, as in the figure; and the hypotenuse *ag*, of the gnomon *agf*, must make an angle with the dial-plane equal to the co-latitude of the place. As the sun can shine no longer on this dial, than from six in the morning until six in the evening, there is no occasion for having any more than twelve hours upon it.

Fig. 2.
To construct an erect south dial.

To make an erect dial, declining from the south towards the east or west. Elevate the pole to the latitude of your place, and screw the quadrant of altitude to the zenith. Then, if your dial declines toward the east (which we shall suppose it to do at present) count in the horizon the degrees of declination, from the east point towards the north, and bring the lower end of the quadrant to that degree of declination at which the reckoning ends. This done, bring any particular meridian of your globe (as suppose the first meridian) directly under the graduated edge of the upper part of the brazen meridian, and set the hour-index to XII at noon. Then, keeping the quadrant of altitude at the degree of declination in the horizon, turn the globe

To construct an erect declining dial.

eastward on its axis, and observe the degrees cut by the first meridian in the quadrant of altitude (counted from the zenith) as the hour-index comes to XI, X, IX, &c. in the forenoon, or as 15, 30, 45, &c. degrees of the equator pass under the brazen meridian at these hours respectively; and the degrees then cut in the quadrant by the first meridian, are the respective distances of the forenoon hours from XII on the plane of the dial.—Then, for the afternoon hours, turn the quadrant of altitude round the zenith until it comes to the degree in the horizon opposite to that where it was placed before; namely, as far from the west point of the horizon toward the south, as it was set at first from the east point toward the north; and turn the globe westward on its axis, until the first meridian comes to the brazen meridian again, and the hour-index to XII: then, continue to turn the globe westward, and as the index points to the afternoon hours I, II, III, &c. or as 15, 30, 45, &c. degrees of the equator pass under the brazen meridian, the first meridian will cut the quadrant of altitude in the respective number of degrees from the zenith, that each of these hours is from XII on the dial.—And note, that when the first meridian goes off the quadrant at the horizon, in the forenoon, the hour-index shews the time when the sun will come upon this dial: and when it goes off the quadrant in the afternoon, the index will point to the time when the sun goes off the dial.

Having thus found all the hour-distances from XII, lay them down upon your dial-plane, either
by

by dividing a semicircle into two quadrants of 90 degrees each, (beginning at the hour-line of XII) or by the line of chords, as above directed.

In all declining dials, the line on which the stile or gnomon stands (commonly called the *substile-line*) makes an angle with the twelve o'clock line, and falls among the forenoon hour-lines, if the dial declines towards the east; and among the afternoon hour-lines, when the dial declines towards the west: that is, to the left hand from the twelve o'clock line in the former case, and to the right hand from it in the latter.

To find the distance of the substile from the twelve o'clock line; if your dial declines from the south toward the east, count the degrees of that declination in the horizon from the east point toward the north, and bring the lower end of the quadrant of altitude to that degree of declination where the reckoning ends: then, turn the globe until the first meridian cuts the horizon in the like number of degrees, counted from the south point toward the east; and the quadrant and first meridian will then cross one another at right angles, and the number of degrees of the quadrant, which are intercepted between the first meridian and the zenith, is equal to the distance of the substile-line from the twelve o'clock line; and the number of degrees of the first meridian, which are intercepted between the quadrant and the north pole, is equal to the elevation of the stile above the plane of the dial.

If

If the dial declines westward from the south, count that declination from the east point of the horizon towards the south, and bring the quadrant of altitude to the degree in the horizon at which the reckoning ends; both for finding the forenoon hours, and distance of the substile from the meridian: and for the afternoon hours, bring the quadrant to the opposite degree in the horizon, namely, as far from the west towards the north, and then proceed in all respects as above.

Thus, we have finished our declining dial; and in so doing, we made four dials, *viz.*

1. A north dial, declining eastward the same number of degrees. 2. A north dial, declining west the same number. 3. A south dial, declining east. And, 4, a south dial declining west.

Only, placing the proper number of hours, and the stile or gnomon respectively, upon each plane. For (as above-mentioned) in the south-west plane, the substilar-line falls among the afternoon hours; and in the south-east, of the same declination, among the forenoon hours, at equal distances from XII. And so, all the morning hours on the west decliner will be like the afternoon hours on the east decliner: the south-east decliner will produce the north-west decliner; and the south-west decliner, the north-east decliner, by only extending the hour-lines, stile and substile, quite through the center: the axis of the stile, (or edge that casts the shadow on the hour of the day) being in all dials whatever parallel to the axis of the world, and consequently pointing towards the north pole

pole of the heaven in north latitudes, and towards the south pole, in south latitudes. *See more of this in the following lecture.*

But because every one who would like to make a dial, may perhaps not be provided with a globe to assist him, and may probably not understand the method of doing it by logarithmic calculation; we shall shew how to perform it by the plain dialing lines, or scale of latitudes and hours; such as those on the right hand of Fig. 4. in Plate XXI, or at the top of Plate XXII, and which may be had on scales commonly sold by the mathematical instrument makers.

An easy
method for
construct-
ing of dials.

This is the easiest of all mechanical methods, and by much the best, when the lines are truly divided: and not only the half hours and quarters may be laid down by all of them, but every fifth minute by most, and every single minute by those where the line of hours is a foot in length.

Having drawn your double meridian line ab , Fig. 3. cd , on the plane intended for a horizontal dial, and crossed it at right angles by the fix o'clock line fe , (as in Fig. 1.) take the latitude of your place with the compasses, in the scale of latitudes, and set that extent from c to e , and from a to f , on the fix o'clock line: then, taking the whole fix hours between the points of the compasses in the scale of hours, with that extent set one foot in the point c , and let the other foot fall where it will upon the meridian line cd , as at d . Do the same from f to b , and draw the right lines ed and fb , each of which will be equal in length to the whole

whole scale of hours. This done, setting one foot of the compasses in the beginning of the scale at XII, and extending the other to each hour on the scale, lay off these extents from *d* to *e* for the afternoon hours, and from *b* to *f* for those of the forenoon: this will divide the lines *de* and *bf* in the same manner as the hour-scale is divided, at 1, 2, 3, 4, and 6; on which the quarters may also be laid down, if required. Then, laying a ruler on the point *c*, draw the first five hours in the afternoon, from that point, through the dots at the numeral figures 1, 2, 3, 4, 5, on the line *de*; and continue the lines of IIII and V through the center *c* to the other side of the dial, for the like hours of the morning: which done, lay the ruler on the point *a*, and draw the last five hours in the forenoon through the dots 5, 4, 3, 2, 1, on the line *fb*; continuing the hour-lines of VII and VIII through the center *a* to the other side of the dial, for the like hours of the evening; and set the hours to their respective lines, as in the figure. Lastly, make the gnomon the same way as taught above for the horizontal dial; and the whole will be finished.

To make an erect south dial, take the co-latitude of your place from the scale of latitudes, and then proceed in all respects for the hour-lines, as in the horizontal dial; only reversing the hours, as in Fig. 2; and making the angle of the stile's height equal to the co-latitude.

I have drawn out a set of dialing lines upon the top of the 22d Plate, large enough for making a
dial

dial of 9 inches diameter, or more inches if required; and have drawn them tolerably exact for common practice, to every quarter of an hour. This scale may be cut off from the plate, and pasted on wood, or upon the inside of one of the boards of this book; and then it will be somewhat more exact than as it is on the plate, for, being rightly divided upon the copper-plate, and printed off on wet paper, it shrinks as the paper dries: but when it is wetted again, it stretches to the same size as when newly printed; and if pasted on while wet, it will remain of that size afterward.

But lest the young *tyro* should have neither globe nor wooden scale, and should tear or otherwise spoil the paper one in pasting, we shall now shew him how he may make a dial without any of these helps. Only, if he has not a line of chords, he must divide a quadrant into 90 equal parts or degrees for taking the proper angle of the stile's elevation; which is easily done.

With any opening of the compasses, as ZL , de- Fig. 4.
scribe the two semicircles LFk and LQk , upon the centers Z and z , where the six o'clock line ^{Horizontal} crosses the double meridian line; and divide each ^{dial.} semicircle into 12 equal parts, beginning at L (though, strictly speaking, only the quadrants from L to the six o'clock line need be divided:) then connect the divisions which are equidistant from L , by the parallel lines KM , IN , HO , GP , and FQ . Draw VZ for the hypotenuse of the stile, making the angle VZE equal to the latitude of your place; and continue the line VZ to R . Draw the line Rr parallel

parallel to the fix o'clock line, and set off the distance at aK from Z to γ , the distance bI from Z to X , cH from Z to W , dG from Z to T , and eF from Z to S . Then draw the lines Ss , Tt , Ww , Xx , and γy , each parallel to Rr . Set off the distance $y\gamma$ from a to 11, and from f to 1; the distance xX from b to 10, and from g to 2; wW from c to 9, and from h to 3; tT from d to 8, and from i to 4; sS from e to 7, and from n to 5. Then laying a ruler to the center Z , draw the forenoon hour-lines through the points 11, 10, 9, 8, 7; and laying it to the center z , draw the afternoon lines through the points 1, 2, 3, 4, 5; continuing the forenoon lines of VII and VIII through the center Z , to the opposite side of the dial, for the like afternoon hours; and the afternoon lines IIII and V through the center z to the opposite side for the like morning hours. Set the hours to these lines as in the figure, and then erect the stile or gnomon, and the horizontal dial will be finished.

South dial.

To construct a south dial, draw the line VZ , making an angle with the meridian ZL equal to the co-latitude of your place; and proceed in all respects as in the above horizontal dial for the same latitude, reversing the hours as in Fig. 2, and making the elevation of the gnomon equal to the co-latitude.

Perhaps it may not be unacceptable to explain the method of constructing the dialing lines, and some others; which is as follows.

With

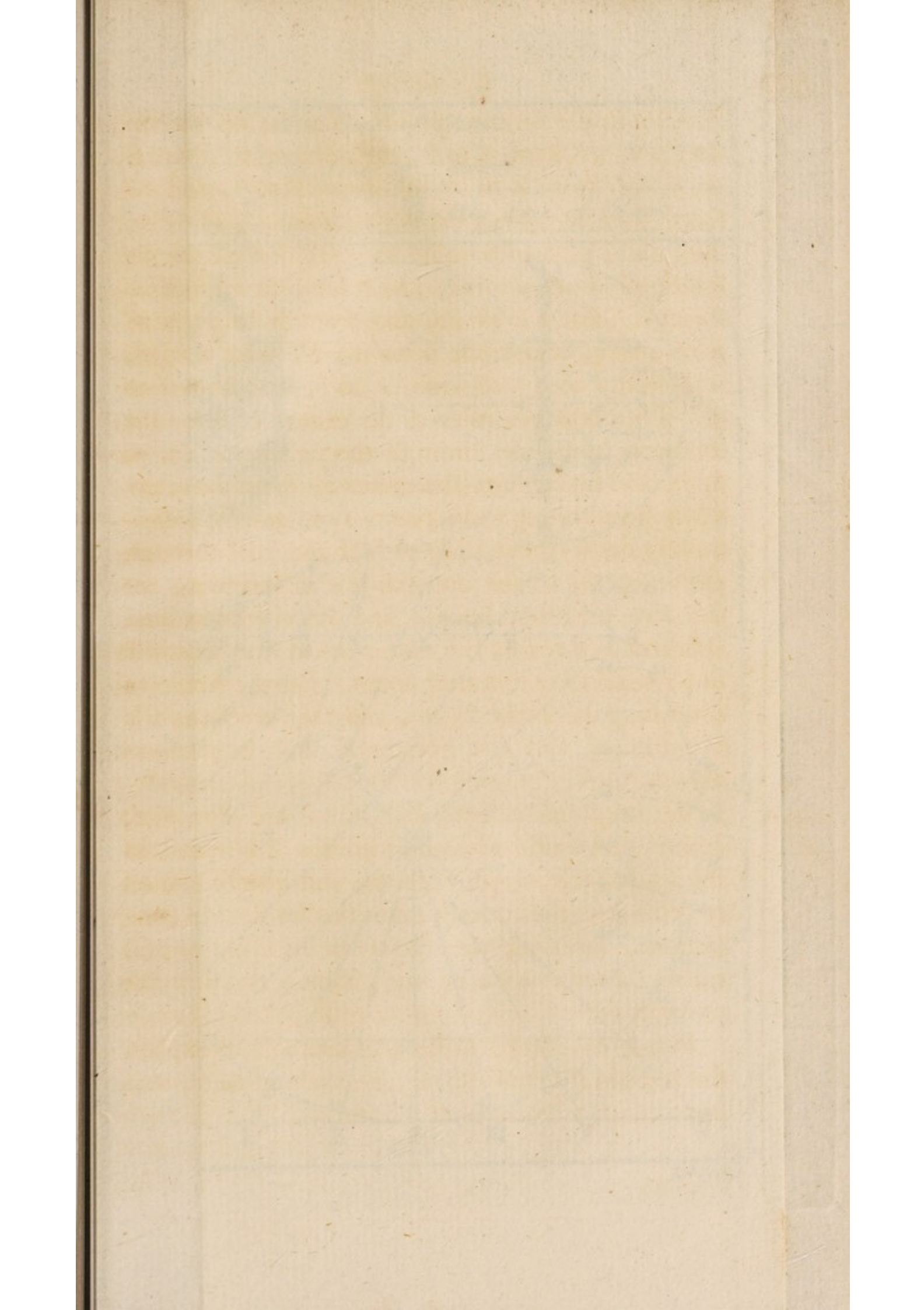
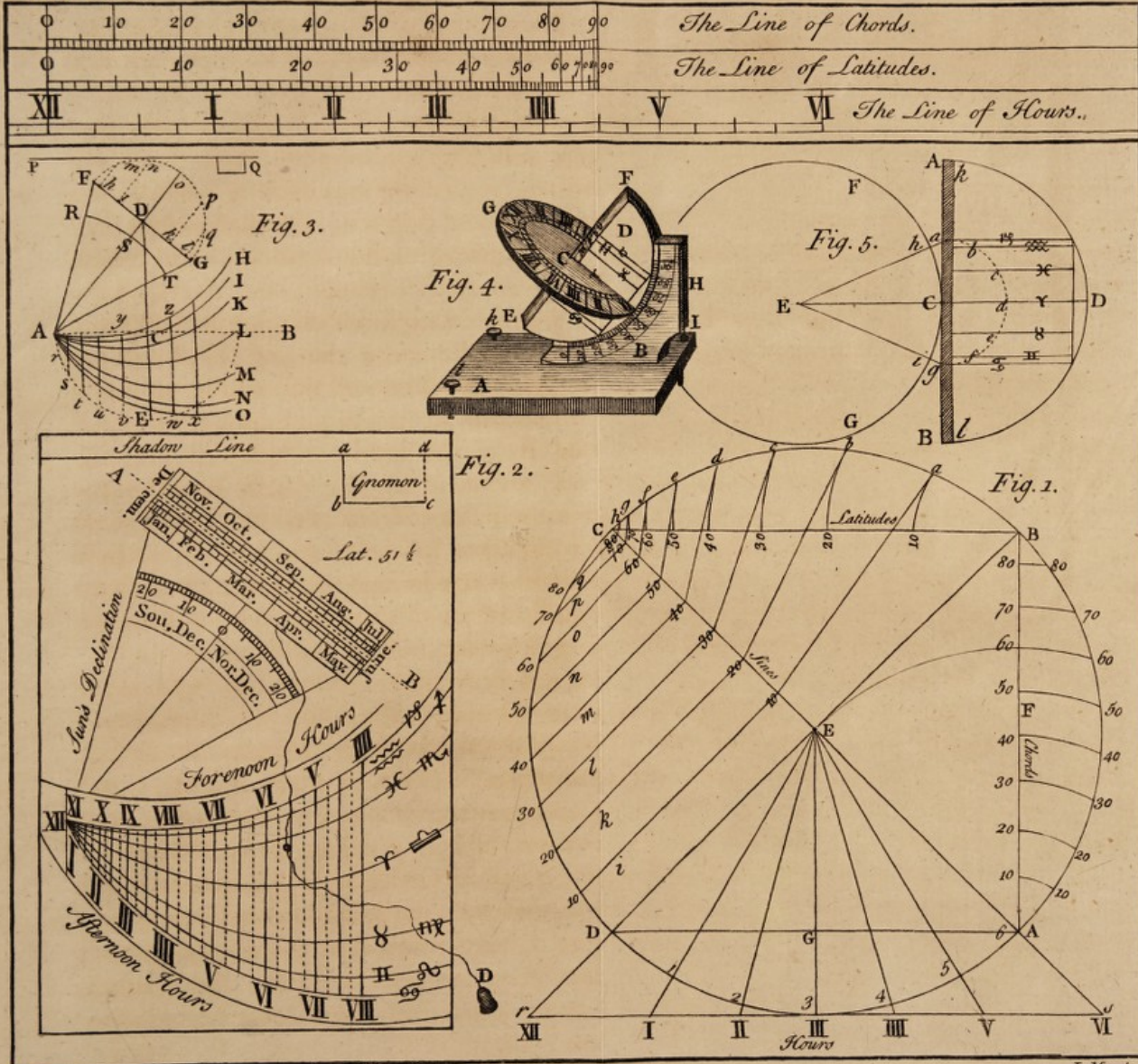


PLATE XII.



J. Ferguson delin.

J. Mynde sc.

With any opening of the compasses, as EA , according to the intended length of the scale, describe the circle $ABCD$, and cross it at right angles by the diameters AEC and BED . Divide the quadrant AB first into 9 equal parts, and then each part into 10; so shall the quadrant be divided into 90 equal parts or degrees. Draw the right line AFB for the chord of this quadrant, and setting one foot of the compasses in the point A , extend the other to the several divisions of the quadrant, and transfer these divisions to the line AFB by the arcs 10 10, 20 20, &c. and this will be a line of chords, divided into 90 unequal parts: which, if transferred from the line back again to the quadrant, will divide it equally. It is plain by the figure that the distance from A to 60 in the line of chords, is just equal to AE , the radius of the circle from which that line is made; for if the arc 60 60 be continued, of which A is the center, it goes exactly through the center E of the arc AB .

*Dialing lines
how constructed.*

And therefore, in laying down any number of degrees on a circle, by the line of chords, you must first open the compasses so, as to take in just 60 degrees upon that line, as from A to 60; and then, with that extent as a radius, describe a circle, which will be exactly of the same size with that from which the line was divided: which done, set one foot of the compasses in the beginning of the chord-line, as at A , and extend the other to the number of degrees you want upon the line; which extent, applied to the circle, will include the like number of degrees upon it.

Divide

Divide the quadrant CD into 90 equal parts, and from each point of division draw right lines, as i , k , l , &c. to the line CE , all perpendicular to that line, and parallel to DE , which will divide EC into a line of fines; and although these are seldom put among the dialing lines on a scale, yet they assist in drawing the line of latitudes. For, if a ruler be laid upon the point D , and over each division in the line of fines, it will divide the quadrant BC into 90 unequal parts, as Ba , Bb , &c. shewn by the right lines $10a$, $20b$, $30c$, &c. drawn along the edge of the ruler. If the right line BC be drawn, subtending this quadrant, and the nearest distances Ba , Bb , Bc , &c. be taken in the compasses from B , and set upon this line in the same manner as directed for the line of chords; we have a line of latitudes BC , equal in length to the line of chords AB , and of an equal number of divisions, but very unequal as to their lengths.

Draw the right line DGA , subtending the quadrant DA , and parallel to it draw the right line rs , touching the quadrant DA at the numeral figure 3. Divide this quadrant into six equal parts, as 1, 2, 3, &c. and through these points of division draw right lines from the center E to the line rs , which will divide it at the points where the six hours are to be placed, as in the figure. If every sixth part of the quadrant be sub-divided into four equal parts, right lines drawn from the center through these points of division, and continued to the line rs , will divide each hour upon it into quarters.

In

In Fig. 2. we have the representation of a portable dial, which may be easily drawn on a card, and carried in a pocket-book. The lines ad , ab , and ac of the gnomon are cut quite through the card; and as the end ab of the gnomon is raised occasionally above the plane of the dial, it turns, as it were, upon the uncut line cd . The line AB is slit quite through the card, and the thread C is put through the slit, and has a knot tied behind, to keep it from being easily drawn out. On the other end of this thread is a small plummet D , and on the middle of it a small bead for shewing the hour of the day.

To rectify this dial, set the thread in the slit, right against the day of the month, and stretch the thread from the day of the month over the angular point where the curve lines meet at XII, then shift the bead to that point on the thread, and the dial will be rectified.

To find the hour of the day, raise the gnomon (no matter how much or how little) and hold the edge of the dial next the gnomon towards the sun, so as the uppermost edge of the shadow of the gnomon may just cover the *shadow-line*; and the bead then playing freely on the face of the dial, by the weight of the plummet, will shew the time of the day among the hour-lines, as it is forenoon or afternoon.

To find the time of sun-rising and setting, move the thread among the hour-lines, until it either covers some one of them, or lies parallel betwixt any two; and then it will cut the time of sun-

A a

rising

rising among the forenoon hours, and of sun-setting among the afternoon-hours, for that day of the year to which the thread is set in the scale of months.

To find the sun's declination, stretch the thread from the day of the month, over the angular point at twelve, and it will cut the sun's declination, as it is north or south, for that day, in the proper scale.

To find on what days the sun enters the signs : when the bead, as above rectified, moves along any of the curve lines which have the signs of the zodiac marked upon them, the sun enters those signs on the days pointed out by the thread in the scale of months.

Fig. 3.

The construction of this dial is very easy, especially if the reader compares it all along with Fig. 3, as he reads the following explanation of that figure.

Draw the occult line AB parallel to the top of the card, and cross it at right angles with the fix o'clock line ECD . Then, upon C , as a center with the radius CA , describe the semicircle AEL , and divide it into 12 equal parts (beginning at A) as Ar , As , &c. and from these points of division, draw the hour-lines r , s , t , u , v , E , w , and x , all parallel to the fix o'clock line EC . If each part of the semicircle be subdivided into four equal parts, they will give the half hour lines and quarters (as in Fig. 2.) Draw the right line $ASDo$, making the angle SAB equal to the latitude of your place. Upon the center A , describe the arch RST , and
set

Set off upon it the arcs SR and ST , each equal to $23\frac{1}{2}$ degrees, for the sun's greatest declination; and divide them accordingly, as in Fig. 2. Through the intersection D of the lines ECD and ADo , draw FDG at right angles to ADo . Lay a ruler on the points A and R , and draw the line ARF through $23\frac{1}{2}$ degrees of south declination in the arc SR ; and then laying the ruler over the points A and T , draw the line ATG through $23\frac{1}{2}$ degrees of north declination in the arc ST : so shall the lines ARF and ATG cut the line FDG in the proper length for the scale of months. Upon the center D , with the radius DF , describe the semicircle FoG ; which divide into six equal parts, Fm , mn , no , &c. and from these points of division, draw the right lines mb , ni , pk , and ql , each parallel to oD . Then setting one foot of the compasses in the point E , extend the other to A , and describe the arc AzH for the tropic of VS : with the same extent, setting one foot in G , describe the arc AEO for the tropic of S . Next, setting one foot in the point b , and extending the other to A , describe the arc ACI for the beginning of $\text{A}\text{r}\text{d}\text{r}$ and J ; and with the same extent, setting one foot in the point l , describe the arc AN for the beginning of I and Q . Set one foot in the point i , and having extended the other to A , describe the arc AK for the beginning of H and M ; and with the same extent, set one foot in k , and describe the arc AM for the beginning of S and M . Then, setting one foot in the point D , and extending the other to A , describe the curve AL for the beginning of P

and ☐; and the signs will be finished. This done, lay a ruler from the point *A* over the sun's declination in the arch *RST* (found by the following table) for every fifth day of the year; and where the ruler cuts the line *FDG*, make marks; and place the days of the months right against these marks, in the manner shewn by Fig. 2. Lastly, draw the shadow-line *PQ* parallel to the occult line *AB*; make the gnomon, and set the hours to their respective lines, as in Fig. 2. and the dial will be finished.

There are several kinds of dials which are called *universal*, because they serve for all latitudes. Of these, the best one that I know, is Mr. *Pardie's*, which consists of three principal parts; the first whereof is called the *horizontal plane* (*A*), because in practice it must be parallel to the horizon. In this plane is fixed an upright pin, which enters into the edge of the second part *BD*, called the *meridional plane*; which is made of two pieces, the lowest whereof (*B*) is called the *quadrant*, because it contains a quarter of a circle, divided into 90 degrees; and it is only into this part, near *B*, that the pin enters. The other piece is a *semicircle* (*D*) adjusted to the quadrant, and turning in it by a groove, for raising or depressing the diameter (*EF*) of the semicircle, which diameter is called the *axis* of the instrument. The third piece is a *circle* (*G*), divided on both sides into 24 equal parts, which are the hours. This circle is put upon the meridional plane so, that the axis (*EF*) may be perpendicular to the circle; and the point

Fig. 4.

An universal dial.

point *C* be the common center of the circle, semicircle, and quadrant. The straight edge of the semicircle is chamfered on both sides to a sharp edge, which passes through the center of the circle. On one side of the chamfered part, the first six months of the year are laid down, according to the sun's declination for their respective days; and on the other side, the last six months. And against the days on which the sun enters the signs, there are straight lines drawn upon the semicircle, with the characters of the signs marked upon them. There is a black line drawn along the middle of the upright edge of the quadrant, over which hangs a thread (*H*), with its plummet (*I*), for leveling the instrument. *N. B.* From the 23d of September to the 20th of March, the upper surface of the circle must touch both the center *C* of the semicircle, and the line of ♈ and ♎; and from the 20th of March to the 23d of September, the lower surface of the circle must touch that center and line.

To find the time of the day by this dial. Having set it on a level place in sun-shine, and adjusted it by the leveling screws *k* and *l*, until the plumb line hangs over the black line upon the edge of the quadrant, and parallel to the said edge; move the semicircle in the quadrant, until the line of ♈ and ♎ (where the circle touches) comes to the latitude of your place in the quadrant: then, turn the whole meridional plane *BD*, with its circle *G*, upon the horizontal plane *A*, until the edge of the shadow of the circle falls

precisely on the day of the month in the semicircle; and then, the meridional plane will be due north and south, the axis EF will be parallel to the axis of the world, and will cast a shadow upon the true time of the day, among the hours on the circle.

N. B. As, when the instrument is thus rectified, the quadrant and semicircle are in the plane of the meridian, so the circle is then in the plane of the equinoctial. Therefore, as the sun is above the equinoctial in summer, (in northern latitudes) and below it in winter; the axis of the semicircle will cast a shadow on the hour of the day, on the upper surface of the circle, from the 20th of March to the 23d of September: and from the 23d of September, to the 20th of March, the hour of the day will be determined by the shadow of the semicircle, upon the lower surface of the circle. In the former case, the shadow of the circle falls upon the day of the month, on the lower part of the diameter of the semicircle; and in the latter case, on the upper part.

Fig. 5. The method of laying down the months and signs upon the semicircle, is as follows. Draw the right line ACB , equal to the diameter of the semicircle ADB , and cross it in the middle at right angles with the line ECD , equal in length to ADB ; then EC will be the radius of the circle FCG , which is the same as that of the semicircle. Upon E , as a center, describe the circle FCG , on which, set off the arcs Cb and Ci , each equal to $23\frac{1}{2}$ degrees, and divide them accordingly into that number, for the sun's declination. Then, laying

laying the edge of a ruler over the center *E*, and also over the sun's declination for every * fifth day of each month, (as in the card dial) mark the points on the diameter *AB* of the semicircle, from *a* to *g*, which are cut by the ruler; and *there*, place the days of the months accordingly, answering to the sun's declination. This done, setting one foot of the compasses in *C*, and extending the other to *a* or *g*, describe the semicircle *abcdefg*; which divide into six equal parts, and through the points of division draw right lines, parallel to *CD*, for the beginning of the fines, (of which one half are on one side of the semicircle, and the other half on the other) and set the characters of the signs to their proper lines, as in the figure.

The following table shews the sun's place and declination, in degrees and minutes, at the noon of every day of the second year after leap-year; which is a mean between those of leap-year itself, and the first and third years after. It is useful for inscribing the months and their days on sun-dials; and also for finding the latitudes of places, according to the methods prescribed at the end of the table.

* The intermediate days may be drawn in by hand, if the space be large enough to contain them.

A Table shewing the sun's place and declination.

January.				February.			
Days	Sun's Pl.		Sun's Dec.	Days	Sun's Pl.		Sun's Dec.
	D.	M.			D.	M.	
1	11	5 ^p	23 S 1	1	12	38 ^m	17 S 2
2	12	6	22 55	2	13	39	16 45
3	13	8	22 49	3	14	40	16 27
4	14	9	22 43	4	15	41	16 10
5	15	10	22 37	5	16	41	15 51
6	16	11	22 29	6	17	42	15 33
7	17	12	22 22	7	18	43	15 14
8	18	13	22 14	8	19	43	14 55
9	19	14	22 5	9	20	44	14 36
10	20	16	21 56	10	21	45	14 17
11	21	17	21 47	11	22	45	13 57
12	22	18	21 37	12	23	46	13 37
13	23	19	21 27	13	24	46	13 17
14	24	20	21 17	14	25	47	12 57
15	25	21	21 6	15	26	47	12 36
16	26	22	20 54	16	27	48	12 15
17	27	24	20 43	17	28	48	11 54
18	28	25	20 30	18	29	48	11 33
19	29	26	20 18	19	0 ^x 49		11 12
20	0 ^m 27		20 5	20	1	49	10 50
21	1	28	19 52	21	2	50	10 29
22	2	29	19 38	22	3	50	10 7
23	3	30	19 24	23	4	50	9 45
24	4	31	19 10	24	5	51	9 23
25	5	32	18 55	25	6	51	9 0
26	6	33	18 40	26	7	51	8 38
27	7	34	18 24	27	8	51	8 16
28	8	35	18 9	28	9	51	7 53
29	9	35	17 53	In these Tables, N signifies north declination, and S south.			
30	10	36	17 36				
31	11	37	17 19				

A Table shewing the sun's place and declination.

March.			April.		
Days	Sun's Pl.		Days	Sun's Pl.	
	D.	M.		D.	M.
1	10	X 52	1	11	✓ 38
2	11	52	2	12	37
3	12	52	3	13	36
4	13	52	4	14	35
5	14	52	5	15	34
6	15	52	6	16	33
7	16	51	7	17	31
8	17	51	8	18	30
9	18	51	9	19	29
10	19	51	10	20	28
11	20	51	11	21	27
12	21	50	12	22	25
13	22	50	13	23	24
14	23	50	14	24	23
15	24	49	15	25	21
16	25	49	16	26	20
17	26	48	17	27	18
18	27	48	18	28	17
19	28	48	19	29	15
20	29	47	20	08	14
21	0	✓ 47	21	1	12
22	1	46	22	2	11
23	2	45	23	3	9
24	3	45	24	4	7
25	4	44	25	5	6
26	5	43	26	6	4
27	6	42	27	7	2
28	7	42	28	8	0
29	8	41	29	8	59
30	9	40	30	9	57
31	10	39			

A Table shewing the sun's place and declination.

May.				June.					
Days	Sun's Pl.		Sun's Dec.		Days	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	10	8 55	15	N 7	1	10	Π 44	22	N 5
2	11	53	15	25	2	11	41	22	13
3	12	51	15	43	3	12	39	22	21
4	13	49	16	0	4	13	36	22	28
5	14	47	16	18	5	14	34	22	35
6	15	45	16	35	6	15	31	22	41
7	16	43	16	51	7	16	28	22	47
8	17	41	17	8	8	17	26	22	53
9	18	39	17	24	9	18	23	22	58
10	19	36	17	40	10	19	20	23	3
11	20	34	17	55	11	20	18	23	7
12	21	32	18	10	12	21	15	23	11
13	22	30	18	25	13	22	12	23	15
14	23	28	18	40	14	23	9	23	18
15	24	25	18	54	15	24	7	23	20
16	25	23	19	8	16	25	4	23	22
17	26	21	19	22	17	26	1	23	24
18	27	19	19	35	18	26	58	23	26
19	28	16	19	48	19	27	56	23	27
20	29	14	20	1	20	28	53	23	28
21	0	Π 11	20	13	21	29	50	23	28
22	1	9	20	25	22	0	⊖ 47	23	28
23	2	7	20	37	23	1	45	23	28
24	3	4	20	48	24	2	42	23	27
25	4	2	20	59	25	3	39	23	26
26	4	59	21	10	26	4	36	23	24
27	5	57	21	20	27	5	33	23	21
28	6	54	21	30	28	6	31	23	19
29	7	52	21	39	29	7	28	23	16
30	8	49	21	48	30	8	25	23	12
31	9	47	21	57					

A Table shewing the sun's place and declination.

July.				August.			
Days	Sun's Pl.		Sun's Dec.	Days	Sun's Pl.		Sun's Dec.
	D.	M.			D.	M.	
1	9	22	23 N 8	1	8	58	18 N 2
2	10	19	23 4	2	9	55	17 47
3	11	16	23 0	3	10	53	17 32
4	12	14	22 55	4	11	50	17 16
5	13	11	22 49	5	12	48	17 0
6	14	8	22 43	6	13	45	16 43
7	15	5	22 37	7	14	43	16 26
8	16	0	22 30	8	15	41	16 9
9	17	2	22 23	9	16	38	15 52
10	17	57	22 16	10	17	36	15 35
11	18	54	22 8	11	18	33	15 17
12	19	51	22 0	12	19	31	14 59
13	20	49	21 52	13	20	29	14 41
14	21	46	21 43	14	21	26	14 23
15	22	43	21 33	15	22	24	14 4
16	23	40	21 22	16	23	22	13 45
17	24	38	21 14	17	24	20	13 26
18	25	35	21 3	18	25	17	13 7
19	26	32	20 52	19	26	15	12 47
20	27	29	20 41	20	27	13	12 27
21	28	27	20 30	21	28	11	12 7
22	29	24	20 18	22	29	9	11 47
23	0	21	20 6	23	0	7	11 27
24	1	19	19 54	24	1	5	11 6
25	2	16	19 41	25	2	3	10 46
26	3	13	19 28	26	3	1	10 25
27	4	11	19 14	27	3	59	10 4
28	5	8	19 1	28	4	57	9 43
29	6	6	18 46	29	5	55	9 21
30	7	3	18 32	30	6	53	9 0
31	8	0	18 17	31	7	51	8 38

A Table shewing the sun's place and declination.

September.				October.			
Days	Sun's Pl.		Sun's Dec.	Days	Sun's Pl.		Sun's Dec.
	D.	M.			D.	M.	
1	8	49	8 N 16	1	8	8	3 S 14
2	9	47	7 55	2	9	7	3 37
3	10	46	7 33	3	10	7	4 1
4	11	44	7 10	4	11	6	4 24
5	12	42	6 48	5	12	5	4 47
6	13	40	6 26	6	13	4	5 10
7	14	39	6 3	7	14	4	5 33
8	15	37	5 41	8	15	3	5 56
9	16	35	5 18	9	16	3	6 19
10	17	34	4 55	10	17	2	6 42
11	18	32	4 32	11	18	1	7 5
12	19	31	4 9	12	19	1	7 27
13	20	29	3 46	13	20	0	7 50
14	21	28	3 23	14	21	0	8 12
15	22	26	3 0	15	22	0	8 35
16	23	25	2 37	16	23	0	8 57
17	24	24	2 14	17	23	59	9 19
18	25	22	1 50	18	24	59	9 41
19	26	21	1 27	19	25	58	10 3
20	27	20	1 4	20	26	58	10 24
21	28	19	0 40	21	27	58	10 46
22	29	17	0 17	22	28	58	11 7
23	0	16	0 S 6	23	29	58	11 28
24	1	15	0 30	24	0	58	11 49
25	2	14	0 53	25	1	58	12 10
26	3	13	1 17	26	2	58	12 31
27	4	12	1 40	27	3	58	12 51
28	5	11	2 4	28	4	58	13 12
29	6	10	2 27	29	5	58	13 32
30	7	9	2 50	30	6	58	13 51
				31	7	58	14 11

A Table shewing the sun's place and declination.

November.			December.		
Days	Sun's Pl.		Days	Sun's Pl.	
	D.	M.		D.	M.
1	8	m 58	1	9	16
2	9	58	2	10	17
3	10	59	3	11	18
4	11	59	4	12	19
5	12	59	5	13	20
6	13	59	6	14	21
7	15	0	7	15	22
8	16	0	8	16	23
9	17	0	9	17	24
10	18	1	10	18	25
11	19	1	11	19	26
12	20	2	12	20	27
13	21	2	13	21	28
14	22	3	14	22	29
15	23	4	15	23	30
16	24	4	16	24	31
17	25	5	17	25	33
18	26	5	18	26	34
19	27	6	19	27	35
20	28	7	20	28	36
21	29	7	21	29	37
22	0	7	22	0	38
23	1	9	23	1	40
24	2	10	24	2	41
25	3	11	25	3	42
26	4	11	26	4	43
27	5	12	27	5	44
28	6	13	28	6	46
29	7	14	29	7	47
30	8	15	30	8	48
			31	9	49
					23 6

To find the latitude of any place by observation.

The latitude of any place, is equal to the elevation of the pole above the horizon of that place. Therefore it is plain, that if there was a star fixt in the pole, there would be nothing required to find the latitude, but to take the height of that star with a good instrument. But although there is no star in the pole, yet the latitude may be found by taking the greatest and least height of any star that never sets: for if half the difference between these heights be added to the least height, or subtracted from the greatest, the sum or remainder, will be equal to the height of the pole at the place of observation.

But because the night must be more than 12 hours in length, in order to have two such observations; the sun's meridian altitude and declination are generally made use of for finding the latitude, by means of its complement, which is equal to the elevation of the equinoctial above the horizon; and if this complement be subtracted from 90 degrees, the remainder is the latitude: concerning which, I think the following rules take in all the various cases.

1. If the sun has north declination, and is on the meridian to the south of your place, subtract the declination from the meridian altitude, (taken by a good quadrant) and the remainder is the height of the equinoctial or complement of the latitude north.

E X A M P L E.

Sup- { The sun's meridian altitude $42^{\circ} 20'$ South
pose { And his declination, (subtr.) $10 \ 15$ North

Rem. the comp. of the latitude $32 \ 5$

Which subtract from — — $90 \ 0$

And the rem. is the latitude $57 \ 55$ North.

2. If the sun has south declination, and is southward of your place at noon, add the declination to the meridian altitude; the sum, if less than 90 degrees, is the complement of the latitude north: but if the sum exceeds 90 degrees, the latitude is south, and if 90 be taken from that sum, the remainder will be the latitude.

E X A M P L E S.

The sun's meridian altitude $65^{\circ} 10'$ South

The sun's declination, (add) $15 \ 30$ South

Comp. of the latitude — $80 \ 40$

Subtract from — — $90 \ 0$

Rem. the latitude — $9 \ 20$ North.

The sun's meridian altitude $80^{\circ} 40'$ South

The sun's declination, (add) $20 \ 10$ South

The sum is - - - - $100 \ 50$

From which subtract - $90 \ 0$

Remains the latitude — $10 \ 50$ South.

3. If

3. If the sun has north declination, and is on the meridian north of your place, add the declination to the north meridian altitude; the sum, if less than 90 degrees, is the complement of the latitude south: but if the sum is more than 90 degrees, subtract 90 from it, and the remainder is the latitude north.

E X A M P L E S.

Sun's meridian altitude	-	60° 30'	North
Sun's declination, add	- -	20 10	North
<hr/>			
Compl. of the latitude	- -	80 40	
Subtract from	- - - -	90 0	
<hr/>			
Remains the latitude	- =	9 20	South.
Sun's meridian altitude	-	70° 20'	North
Sun's declination, add	- -	23 20	North
<hr/>			
The sum is	- - - -	93 40	
From which subtract	- -	90 0	
<hr/>			
Remains the latitude	- -	3 40	North.

4. If the sun has south declination, and is north of your place at noon, subtract the declination from the north meridian altitude, and the remainder is the complement of the latitude south.

E X A M.

E X A M P L E.

Sun's meridian altitude	-	-	52° 30'	North
Sun's declination, subtract	-	-	20 10	South
<hr/>				
Compl. of the latitude	-	-	32 20	
Subtract this from	-	-	90 0	
<hr/>				
And the rem. is the latitude			57 40	South.

5. If the sun has no declination, and is south of your place at noon, the meridian altitude is the complement of the latitude north: but if the sun be then north of your place, his meridian altitude is the complement of the latitude south.

E X A M P L E S.

Sun's meridian altitude	-	-	38° 30'	South
Subtract from	-	-	90 0	
<hr/>				
Remains the latitude	-		51 30	North.

Sun's meridian altitude	-	-	38° 30'	North
Subtract from	-	-	90 0	
<hr/>				
Remains the latitude	-		51 30	South.

6. If you observe the sun beneath the pole, subtract his declination from 90 degrees, and add the remainder to his altitude; and the sum is the latitude.

E X A M P L E.

Sun's declination	-	-	-	20° 30'
Subtract from	-	-	-	90 0
<hr/>				
Remains	-	-	-	69 30
Sun's altitude below the pole				10 20
<hr/>				
The sum is the latitude	-			79 50.

Which is north or south, according as the sun's declination is north or south : for when the sun has south declination, he is never seen below the north pole ; nor is he ever seen below the south pole, when his declination is north.

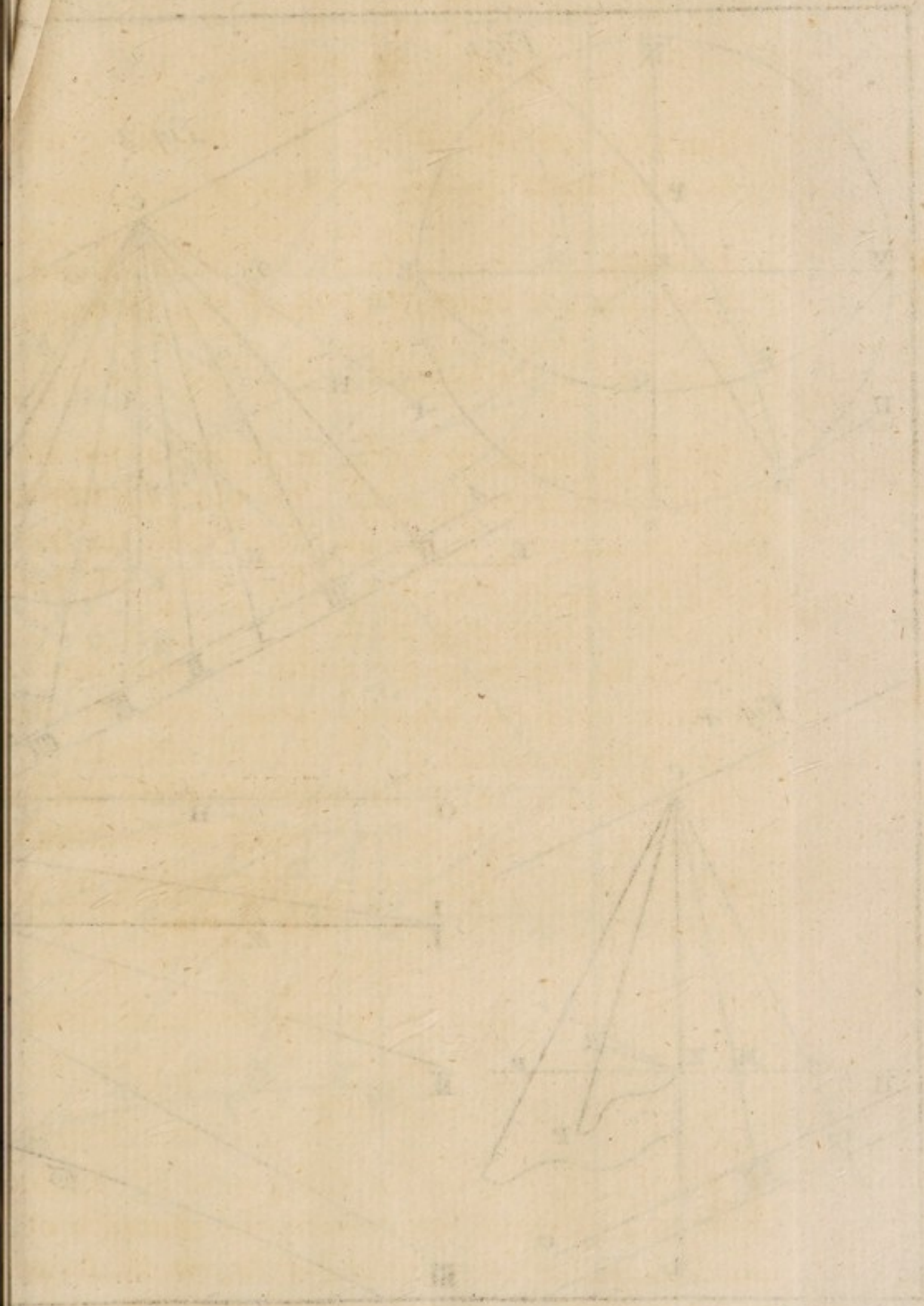
7. If the sun be in the zenith at noon, and at the same time has no declination, you are then under the equinoctial, and so have no latitude.

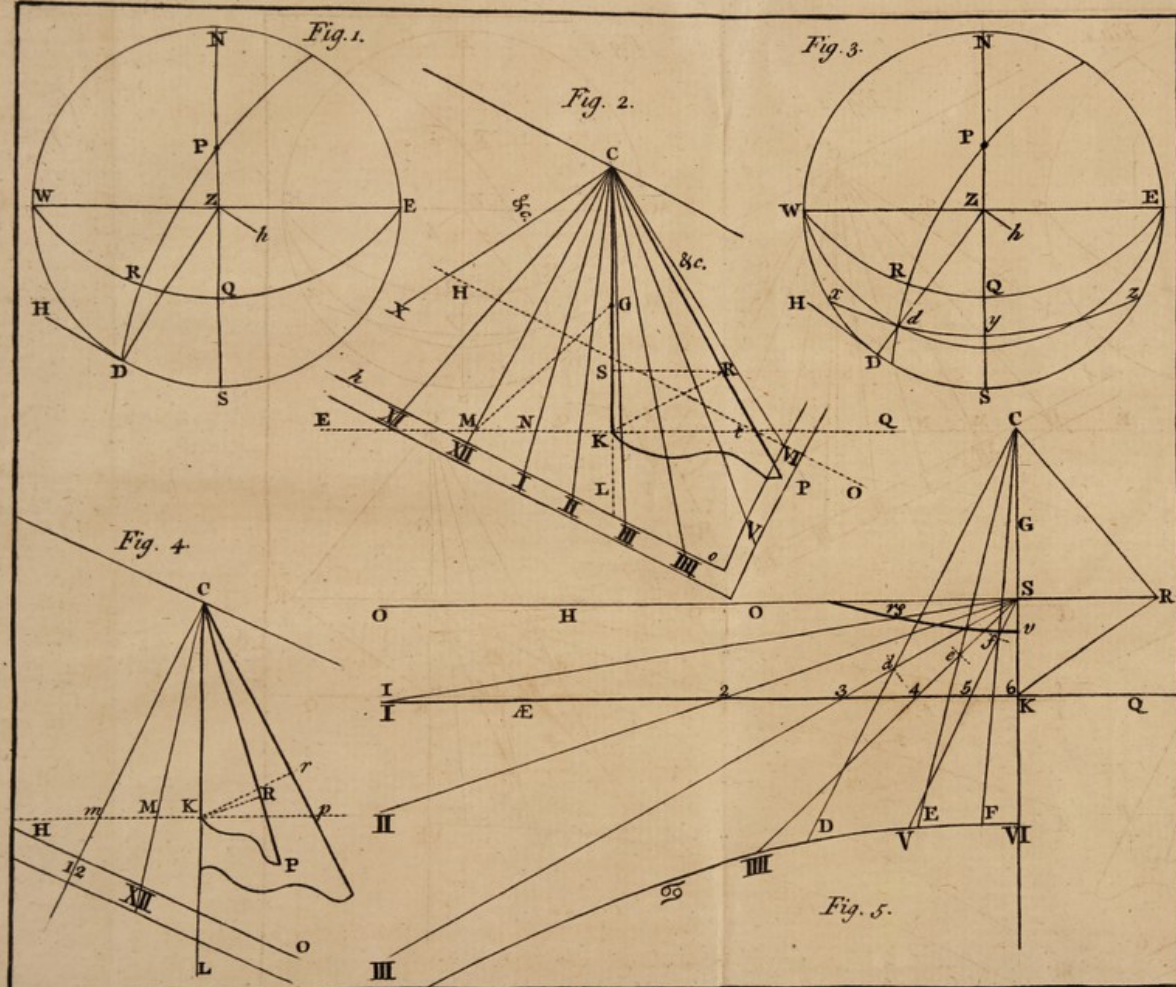
8. If the sun be in the zenith at noon, and has declination, the declination is equal to the latitude, north or south. These two cases require no examples.

L E C T. X.

Of dialing.

HAVING shewn in the preceding Lecture how to make sun-dials by the assistance of a good globe, or of a dialing scale, we shall now proceed to the method of constructing them arithmetically ; which will be more agreeable to those who





who have learnt the elements of trigonometry, because globes and scales can never be so accurate as the logarithms, in finding the angular distances of the hours. Yet, as a globe may be found exact enough for some other requisites in dialing, we shall take it in occasionally.

The construction of sun-dials on all planes whatever, may be included in one general rule: intelligible, if that of a horizontal dial for any given latitude be well understood. For there is no plane, however obliquely situated at any given place, but what is parallel to the horizon of some other place; and therefore, if we can find that other place by a problem on the terrestrial globe, or by a trigonometrical calculation, and construct a horizontal dial for it; that dial, applied to the plane where it is to serve, will be a true dial for that place.— Thus, an erect direct south dial in $51\frac{1}{2}$ degrees north latitude, would be a horizontal dial on the same meridian, 90 degrees southward of $51\frac{1}{2}$ degrees north latitude; which falls in with $38\frac{1}{2}$ degrees of south latitude. But if the upright plane declines from facing the south at the given place, it would still be a horizontal plane 90 degrees from that place; but for a different longitude: which would alter the reckoning of the hours accordingly.

C A S E I.

1. Let us suppose that an upright plane at London, declines 36 degrees westward from facing

B b 2

the

the south; and that it is required to find a place on the globe, to whose horizon the said plane is parallel; and also the difference of longitude between London and that place.

Rectify the globe to the latitude of London, and bring London to the zenith under the brass meridian, then, that point of the globe which lies in the horizon at the given degree of declination, (counted westward from the south point of the horizon) is the place at which the abovementioned plane would be horizontal.—Now, to find the latitude and longitude of that place, keep your eye upon the place, and turn the globe eastward, until it comes under the graduated edge of the brass meridian; then, the degree of the brass meridian that stands directly over the place, is its latitude; and the number of degrees in the equator, that are intercepted between the meridian of London and brass meridian, is the place's difference of longitude.

Thus, as the latitude of London is $51\frac{1}{2}$ degrees north, and the declination of the place is 36 degrees west; I elevate the north pole $51\frac{1}{2}$ degrees above the horizon, and turn the globe until London comes to the zenith, or under the graduated edge of the meridian; then, I count 36 degrees on the horizon, westward from the south point, and make a mark on that place of the globe over which the reckoning ends, and bringing the mark under the graduated edge of the brass meridian, I find it to be under $30\frac{1}{4}$ degrees in south latitude; keeping it there, I count in the
equator

equator the number of degrees between the meridian of London and the brasen meridian, (which now becomes the meridian of the required place) and find it to be $42\frac{3}{4}$. Therefore, an upright plane at London, declining 36 degrees westward from the south, would be a horizontal plane at that place, whose latitude is $30\frac{1}{4}$ degrees south of the equator, and longitude $42\frac{3}{4}$ degrees west of the meridian of London.

Which difference of longitude being converted into time, is 2 hours 51 minutes.

The vertical dial, declining westward 36 degrees at London, is therefore to be drawn in all respects as a horizontal dial for south latitude $30\frac{1}{4}$ degrees; save only, that the reckoning of the hours is to anticipate the reckoning on the horizontal dial, by 2 hours 51 minutes: for so much sooner will the sun come to the meridian of London, than to the meridian of any place whose longitude is $42\frac{3}{4}$ degrees west from London.

2. But, to be more exact than the globe will shew us, we shall use a little trigonometry.

Let *NE SW* be the horizon of London, whose zenith is *Z*, and *P* the north pole of the sphere: and let *Zb* be the position of a vertical plane at *Z*, declining westward from *S*, (the south) at an angle of 36 degrees; on which plane an erect dial for London at *Z* is to be described. Make the semidiameter *ZD* perpendicular to *Zb*, and it will cut the horizon in *D*, 36 degrees west of the south *S*. Then, a plane in the tangent *HD*, touching the sphere in *D*, will be parallel to the plane *Zb*;

and the axis of the sphere will be equally inclined to both these planes.

Let WQE be the equinoctial, whose elevation above the horizon of Z (London) is $38\frac{1}{2}$ degrees; and PRD be the meridian of the place D , cutting the equinoctial in R . Then, it is evident, that the arc RD is the latitude of the place D , (where the plane Zb would be horizontal) and the arc RQ is the difference of longitude of the planes Zb and DH .

In the spherical triangle WDR , the arc WD is given, for it is the complement of the plane's declination from S the south; which complement is 54° (*viz.* $90^\circ - 36^\circ$): the angle at R , in which the meridian of the place D cuts the equator, is a right angle; and the angle RWD measures the elevation of the equinoctial above the horizon of Z , namely $38\frac{1}{2}$ degrees. Say therefore, as radius is to the co-sine of the plane's declination from the south, so is the co-sine of the latitude of Z to the sine of RD the latitude of D : which is of a different denomination from the latitude of Z , because Z and D are on different sides of the equator.

As radius	—	—	10.00000
To co-sine 36°	$0' = RQ$		9.90796
So co-sine 51°	$30' = QZ$		<u>9.79415</u>

To sine $30^\circ 14' = DR$ (9.70211) = the lat. of D , whose horizon is parallel to the vertical plane Zb at Z .

N. B.

N. B. When radius is made the first term, it may be omitted, and then, by subtracting it mentally from the sum of the other two, the operation will be shortened. Thus, in the present case,

To the logarithmic sine of $WR = *54^{\circ} 0' 9.90796$
 Add the logarithmic sine of $RD = \dagger 38^{\circ} 30' 9.79415$

Their sum — radius — — — 9.70211
 gives the same solution as above. And we shall keep to this method in the following part of the work.

To find the difference of longitude of the places D and Z , say, as radius is to the co-sine of $38\frac{1}{2}$ degrees, the height of the equinoctial at Z , so is the co-tangent of 36 degrees, the plane's declination, to the co-tangent of the difference of longitudes. Thus,

To the log. sine of $** 51^{\circ} 30' — 9.89354$
 Add the log. tang. of $\dagger 54^{\circ} 0' — 10.13874$

Their sum — radius — — — 10.03228 is the nearest tangent of $47^{\circ} 8' = WR$; which is the co-tangent of $42^{\circ} 52' = RQ$ the difference of longitude sought. Which difference, being reduced to time, is 2 h. $51\frac{1}{2}$ m.

* The co-sine of $36^{\circ} 0'$, or of RQ .

† The co-sine of $51^{\circ} 30'$, or of QZ .

** The co-sine of $38^{\circ} 30''$, or of WDR .

‡ The co-tangent of 36° , or of DW .

B b 4

3. And

3. And thus having found the exact latitude and longitude of the place *D*, to whose horizon the vertical plane at *Z* is parallel, we shall proceed to the construction of a horizontal dial for the place *D*, whose latitude is $30^{\circ} 14'$ south; but anticipating the time at *D* by 2 hours 51 minutes, (neglecting the $\frac{1}{2}$ minute in practice) because *D* is so far westward in longitude from the meridian of London; and this will be a true vertical dial at London, declining westward 36 degrees.

Fig. 2. Assume any right line *CSL* for the substile of the dial, and make the angle *KCP* equal to the latitude of the place (*viz.* $30^{\circ} 14'$) to whose horizon the plane of the dial is parallel; then *CRP* will be the axis of the stile, or edge that casts the shadow on the hours of the day in the dial. This done, draw the contingent line *EQ*, cutting the substilar line at right angles in *K*; and from *K* make *KR* perpendicular to the axis *CRP*. Then *KG* ($= KR$) being made radius, that is, equal to the chord of 60° or tangent of 45° on a good sector, take $42^{\circ} 52'$ (the difference of longitude of the places *Z* and *D*) from the tangents, and having set it from *K* to *M*, draw *CM* for the hour-line of XII. Take *KN*, equal to the tangent of an angle less by 15 degrees than *KM*; that is, the tangent $27^{\circ} 52'$; and through the point *N* draw *CN* for the hour-line of I. The tangent of $12^{\circ} 52'$, (which is 15° less than $27^{\circ} 52'$) set off the same way, will give a point between *K* and *N*, through which the hour-line of II is to be drawn. The tangent of $2^{\circ} 8'$ (the difference between 45° and

and $42^{\circ} 52'$) placed on the other side of CL , will determine the point through which the hour-line of III is to be drawn: to which $2^{\circ} 8'$, if the tangent of 15° be added, it will make $17^{\circ} 8'$; and this set off from K towards Q , on the line EQ , will give a point for the hour-line of III: and so of the rest.—The forenoon hour-lines are drawn the same way, by the continual addition of the tangents 15° , 30° , 45° , &c. to $42^{\circ} 52'$ (= the tangent of KM) for the hours of XI, X, IX, &c. as far as necessary; that is, until there be five hours on each side of the substile. The sixth hour, accounted from that hour or part of the hour on which the substile falls, will be always in a line perpendicular to the substile, and drawn through the center C .

4. In all erect dials, CM , the hour-line of XII, is perpendicular to the horizon of the place for which the dial is to serve: for that line is the intersection of a vertical plane with the plane of the meridian of the place, both which are perpendicular to the plane of the horizon: and any line HO , or bo , perpendicular to CM , will be a horizontal line on the plane of the dial, along which line the hours may be numbered; and CM being set perpendicular to the horizon, the dial will have its true position.

5. If the plane of the dial had declined by an equal angle toward the east, its description would have differed only in this, that the hour-line of XII would have fallen on the other side of the substile

substile CL , and the line HO would have a subcontrary position to what it has in this figure.

6. And these two dials, with the upper points of their stiles turned toward the north pole, will serve for other two planes parallel to them; the one declining from the north toward the east, and the other from the north toward the west, by the same quantity of angle. The like holds true of all dials in general, whatever be their declination, and obliquity of their planes to the horizon.

C A S E II.

Fig. 3. 7. If the plane of the dial not only *declines*, but also *reclines*, or *inclines*. Suppose its declination from fronting the south S , be equal to the arc SD on the horizon; and its reclination be equal to the arc Dd , of the vertical circle DZ : then it is plain, that if the quadrant of altitude ZdD , on the globe, cuts the point D in the horizon, and the reclination is counted upon the quadrant from D to d ; the intersection of the hour-circle PRd , with the equinoctial WQE , will determine Rd , the latitude of the place d , whose horizon is parallel to the given plane at Z ; and RQ will be the difference in longitude of the planes at d and Z .

Trigonometrically thus: let a great circle pass through the three points, W , d , E ; and in the triangle WDd , right angled at D , the sides WD and Dd are given; and thence the angle DWd is found, and so is the hypotenuse Wd . Again, the difference, or the sum, of DWd , and DWR
the

the elevation of the equinoctial above the horizon of Z , gives the angle dWR ; and the hypothenuse of the triangle WRd , was just now found: whence the sides Rd and WR are found, the former being the latitude of the place d , and the latter the complement of RQ the difference of longitude sought.

Thus, if the latitude of the place Z be $52^{\circ} 10'$ north, the declination SD of the plane Zb (which would be horizontal at d) be 36° , and the reclination be 15° , or equal to the arc Dd ; the south latitude of the place d , that is, the arc Rd , will be $15^{\circ} 9'$; and RQ the difference of longitude $36^{\circ} 2'$. From these data therefore, let the dial (Fig. 4.) be described; as in the former example.

8. Only it is to be observed, that in reclining or inclining dials, the horizontal line will not stand at right angles to the hour-line of XII, as in erect dials; but its position may be found as follows.

To the common substilar line CKL , on which Fig. 4. the dial for the place d was described, draw the dial $Crpm$ 12 for the place D , whose declination is the same as that of d (*viz.* the arc SD): and HO , perpendicular to Cm the hour-line of XII on this dial, will be a horizontal line on the dial $CPRM$ XII. For, the declination of both dials being the same, the horizontal line remains parallel to itself, while the erect position of one dial is reclined, or inclined, with respect to the position of the other.

Or, the position of the dial may be found by applying it to its plane, so as to mark the true hour
of

of the day by the sun, as shewn by another dial; or by a clock, regulated by a true meridian line and equation table.

9. There are several other things necessary in the practice of dialing; the chief of which, I shall give in the form of arithmetical rules, simple and easy to those who have learnt the elements of trigonometry. For, in practical arts of this kind, arithmetic should be used as far as it can go; and scales never trusted to, except in the final construction, where they are absolutely necessary in laying down the calculated hour-distances, on the plane of the dial. And although the inimitable artists of this metropolis have no occasion for such instructions, yet they may be of some use to students, and to private gentlemen who amuse themselves this way.

R U L E I.

To find the angles which the hour-lines on any dial make with the substile.

To the logarithmic sine of the given latitude, or of the stile's elevation above the plane of the dial, add the logarithmic tangent of the * hour-distance from the meridian, or from the † substile;

* That is, of 15, 30, 45, 60, 75°, for the hours of I, II, III, IIII, V, in the afternoon; and XI, X, IX, VIII, VII, in the forenoon.

† In all horizontal dials, and erect north or south dials, the substile and meridian are the same: but in all declining dials, the substile line makes an angle with the meridian.

and

and the sum *minus* radius will be the logarithmic tangent of the angle sought.

For, in Fig. 2. KC is to KM in the ratio compounded of the ratio of KC to KG , ($=KR$) and of KG to KM ; which, making CK the radius 10,000000, or 10,0000, or 10, or 1, are the ratio of 10,000000, or of 10,0000, or of 10, or of 1, to $KG \times KM$.

Thus, in a horizontal dial, for lat. $51^{\circ} 30'$, to find the angular distance of XI in the forenoon, or I in the afternoon, from XII.

To the log. sine of $51^{\circ} 30'$	9.89354*
Add the log. tang. of $15^{\circ} 0'$	9.42805
	<hr/>

The sum — radius is 9.32159 = the logarithmic tangent of $11^{\circ} 50'$, or of the angle which the hour-line of XI or I makes with the hour of XII.

And by computing in this manner, with the sine of the latitude, and the tangents of $30, 45, 60$, and 75° , for the hours of II, III, IIII, and V in the afternoon; or of X, IX, VIII, and VII in the forenoon; you will find their angular distances from XII to be $24^{\circ} 18', 38^{\circ} 3', 53^{\circ} 35',$ and $71^{\circ} 6'$; which are all that there is occasion to compute for.—And these distances may be set off from XII by a line of chords; or rather, by taking 1000 from a scale of equal parts, and setting that

* In which case, the radius CK is supposed to be divided into 1000000 equal parts.

extent

Fig. 2. extent as a radius from C to XII; and then, taking 209 of the same parts, (which, in the tables, are the natural tangent of $11^{\circ} 50'$) and setting them from XII to XI and to I, on the line *bo*, which is perpendicular to C XII: and so for the rest of the hour-lines, which, in the table of natural tangents, against the above distances, are 451, 782, 1355, and 2920, of such equal parts from XII, as the radius C XII contains 1000. And lastly, set off 1257 (the natural tangent of $51^{\circ} 30'$) for the angle of the stile's height, which is equal to the latitude of the place.

The reason why I prefer the use of the tabular numbers, and of a scale decimally divided, to that of the line of chords, is because there is the least chance of mistake and error in this way; and likewise, because in some cases it gives us the advantage of a *nonius*' division.

In the universal ring-dial, for instance, the divisions on the axis are the tangents of the angles of the sun's declination placed on either side of the center. But instead of laying them down from a line of tangents, I would make a scale of equal parts, whereof 1000 should answer exactly to the length of the semi-axis, from the center to the inside of the equinoctial ring; and then lay down 434 of these parts toward each end from the center, which would limit all the divisions on the axis, because 434 are the natural tangent of $23^{\circ} 29'$. And thus, by a *nonius* affixed to the sliding piece, and taking the sun's declination from an ephemeris, and the tangent of that declination
from

from the table of natural tangents, the slider might be always set true to within two minutes of a degree.

And this scale of 434 equal parts might be placed right against the $23\frac{1}{2}$ degrees of the sun's declination, on the axis, instead of the sun's place, which is there of very little use. For then, the slider might be set in the usual way, to the day of the month, for common use; but to the natural tangent of the declination, when great accuracy is required.

The like may be done wherever a scale of sines or tangents is required on any instrument.

R U L E II.

The latitude of the place, the sun's declination, and his hour-distance from the meridian, being given; to find (1.) his altitude, (2.) his azimuth.

1. Let d be the sun's place, dR his declination; Fig. 3. and in the triangle PZd , Pd the sum, or the difference, of dR and the quadrant PR , being given by the supposition, as also the complement of the latitude PZ , and the angle dPZ which measures the horary distance of d from the meridian; we shall (by Case 4. of Keill's oblique spheric. Tr.) find the base Zd , which is the sun's distance from the zenith, or the complement of his altitude.

And (2.) As $\sin. Zd : \sin. Pd :: \sin. dPZ : dZP$, or of its supplement DZS , the azimuthal distance from the south.

Or,

Or, the practical rule may be as follows.

Write A for the sine of the sun's altitude, L and l for the sine and co-sine of the latitude, D and d for the sine and co-sine of the sun's declination, and H for the sine of the horary distance from VI.

Then the relation of H to A will have three varieties.

1. When the declination is toward the elevated pole, and the hour of the day is between XII and VI; it is $A = LD + Hld$, and $H = \frac{A - LD}{ld}$.

2. When the hour is after VI, it is $A = LD - Hld$, and $H = \frac{LD - A}{ld}$.

3. When the declination is toward the depressed pole; we have $A = Hld - LD$, and $H = \frac{A + LD}{ld}$.

Which theorems will be found useful, and expeditious enough, for solving those problems in geography and dialing, which depend on the relation of the sun's altitude to the hour of the day.

EXAMPLE I.

Suppose the latitude of the place to be $51\frac{1}{2}$ degrees north; the time five hours distant from XII, that is, an hour after six in the morning, or before VI in the evening; and the sun's declination 20° north. *Required the sun's altitude.*

Then,

Then, to $\log. L, = \log. \sin. 51^{\circ} 30' - 1.89354^*$
 add $\log. D, = \log. \sin. 20^{\circ} 00' - 1.53405$

Their sum - - - 1.42759

gives $LD, = \log. 0.267664$, in the natural sines.

And, to $\log. H, = \log. \sin. \dagger 15^{\circ} 00' - 1.41300$

add $\left\{ \begin{array}{l} \log. l, = \log. \sin. \ddagger 38^{\circ} 00' - 1.79415 \\ \log. d, = \log. \sin. \parallel 70^{\circ} 00' - 1.97300 \end{array} \right.$

Their sum - - - 1.18015

gives $Hld, = \log. 0.151408$, in the natural sines.

And these two numbers (0.267664 and 0.151408) make $0.419072 = A$; which, in the table, is the nearest natural sine of $24^{\circ} 47'$, the sun's altitude sought.

The same hour-distance being assumed on the other side of VI, then $LD - Hld$ is 0.116256 , the sine of $6^{\circ} 40\frac{1}{2}'$; which is the sun's altitude at V in the morning, or VII in the evening, when his north declination is 20° .

But when the declination is 20° south (or towards the depressed pole) the difference $Hld - LD$ becomes negative, and thereby shews that, an hour before VI in the morning, or past VI in the evening, the sun's center is $6^{\circ} 40\frac{1}{2}'$ below the horizon.

* Here we consider the radius as unity, and not 10.00000 ; by which, instead of the index 9 we have -1 , as above: which is of no farther use, than making the work a little easier.

† The distance of one hour from VI.

‡ The co-latitude of the place.

|| The co-declination of the sun.

E X A M P L E II.

In the same latitude and north declination, from the given altitude to find the hour.

Let the altitude be 48° ; and because, in this case,
 $H = \frac{A - LD}{l \times d}$ and A (the natural sine of 48°) = .743145,
 and $LD = .267664$, $A - LD$ will be .475481,
 whose logarithmic sine is -1.6771331
 from which taking the
 $\log. \text{ sine of } l \times d, = \left. \begin{array}{l} \\ \end{array} \right\} -1.7671354$

Remains -1.9099977 the log. sine
 of the hour-distance sought, viz. of $54^\circ 22'$;
 which, reduced to time, is 3 hours $37\frac{1}{2}$ min. that
 is, IX-h. $37\frac{1}{2}$ m. in the forenoon, or II h. $22\frac{1}{2}$ m.
 in the afternoon.

Put the altitude = 18° , whose natural sine is
 .3090170; and thence $A - LD$ will be = .0491953;
 which divided by $l \times d$, gives .0717179, the sine of
 $4^\circ 6\frac{1}{2}'$; in time $16\frac{1}{2}$ minutes nearly, before VI in
 the morning, or after VI in the evening, when the
 sun's altitude is 18° .

And, if the declination 20° had been towards
 the south pole, the sun would have been depressed
 18° below the horizon at $16\frac{1}{2}$ minutes after VI in
 the evening; at which time, the twilight would
 end: which happens about the 22d of November,
 and 19th of January, in the latitude of $51\frac{1}{2}^\circ$ north.
 The same way may the end of twilight, or be-
 ginning of dawn, be found for any time of the
 year.

NOTE

NOTE 1. If in theorem 2 and 3 (pag. 384) A is put $= 0$, and the value of H is computed, we have the hour of sun-rising and setting, for any latitude, and time of the year. And if we put $H = 0$, and compute A , we have the sun's altitude or depression at the hour of VI. And lastly, if H , A , and D , are given, the latitude may be found by the resolution of a quadratic equation; for $l = \sqrt{1 - L^2}$.

NOTE 2. When A is equal 0, H is equal $\frac{LD}{ld} = T, L \times T, D$, the tangent of the latitude multiplied by the tangent of the declination.

As, if it was required, *what is the greatest length of day in lat. $51^\circ 30'$?*

To the log. tangent of $51^\circ 30'$ - - 0.0993948

Add the log. tangent of $23^\circ 29'$ — 1.6379563

Their sum — 1.7373511
is the log. sine of the hour-distance $33^\circ 7'$; in time 2 h. $12\frac{1}{2}$ m. The longest day therefore is 12 h. + 4 h. 25 m. = 16 h. 25 m. And the shortest day is 12 h. — 4 h. 25 m. = 7 h. 35 m.

And if the longest day is given, the latitude of the place is found; $\frac{H}{T, D}$ being equal to T, L . Thus, if the longest day is $13\frac{1}{2}$ hours = 2×6 h. + 45 m. and 45 minutes in time being equal to $11\frac{1}{4}$ degrees.

From the log. sine of $11^\circ 15'$ — 1.2902357

Take the log. tang. of $23^\circ 29'$ — 1.6379562

Remains — 1.6522795 =
the log. tangent of lat $24^\circ 11'$.

C c 2

And

And the same way, the latitudes where the several geographical *climates* and parallels begin, may be found; and the latitudes of places that are assigned in authors from the length of their days, may be examined and corrected.

NOTE 3. The same rule for finding the longest day in a given latitude, distinguishes the hour-lines that are necessary to be drawn on any dial, from those which would be superfluous.

In lat. $52^{\circ} 10'$, the longest day is 16 h. 32 m. and the hour-lines are to be marked from 44 m. after III in the morning, to 16 m. after VIII in the evening.

In the same latitude, let the dial of Art. 7. Fig. 4. be proposed; and the elevation of its stile (or the latitude of the place d , whose horizon is parallel to the plane of the dial) being $15^{\circ} 9'$; the longest day at d , that is, the longest time that the sun can illuminate the plane of the dial, will (by the rule $H = T, L \times T, D$) be twice 6 hours 27 minutes, = 12 h. 54 m. The difference of longitude of the planes d and Z was found in the same example to be $36^{\circ} 2'$; in time, 2 hours 24 minutes: and the declination of the plane was from the south towards the west. Adding therefore 2 h. 24 m. to 5 h. 33 m. the earliest sun-rising on a horizontal dial at d , the sum 7 h. 57 m. shews that the morning hours, or the parallel dial at Z , ought to begin at 3 min. before VIII. And to the latest sun-setting at d , which is 6 h. 27 m. adding the same 2 h. 24 m. the sum 8 h. 51 m. exceeding 6 h. 16 m. the latest sun-setting at Z , by 35 m. shews that

that none of the afternoon hour-lines are superfluous. And the 4 h. 13 m. from III h. 44 m. the sun-rising at Z to VII h. 57 m. the sun-rising at d , belong to the other face of the dial: that is, to a dial declining 36° from north to east, and inclining 15° .

E X A M P L E III.

From the same *data* to find the sun's *azimuth*.

If H , L , and D are given, then (by Art. 2. of Rule II.) from H having found the altitude, and its complement Zd ; and the arc Pd (the distance from the pole) being given; say, As the co-sine of the altitude is to the sine of the distance from the pole, so is the sine of the hour-distance from the meridian to the sine of the azimuth-distance from the meridian.

Let the latitude be $51^\circ 30'$ north, the declination $15^\circ 9'$ south, and the time II h. 24 m. afternoon, when the sun begins to illuminate a vertical wall: and it is required to find the position of the wall.

Then, by the foregoing theorems, the complement of the altitude will be $81^\circ 32\frac{1}{2}'$, and Pd the distance from the pole being $109^\circ 5'$, and the horary distance from the meridian, or the angle dPZ 36° .

To log. sin. $74^\circ 51'$ - - - 1.98464

Add log. sin. $36^\circ 0'$ - - - 1.76922

And from the sum - - - 1.75386

Take the log. sin. $81^\circ 32\frac{1}{2}'$ - 1.99525

Remains - 1.75861 = log. sin.
 35° , the azimuth distance south.

C c 3

When

When the altitude is given, find from thence the hour, and proceed as above.

This praxis is of singular use on many occasions; in finding the declination of vertical planes more exactly than in the common way, especially if the transits of the sun's center is observed by applying a ruler with sights, either plain or telescopical, to the wall or plane, whose declination is required.—In drawing a meridian line, and finding the magnetic variation.—In finding the bearings of places in terrestrial surveys; the transits of the sun over any place, or his horizontal distance from it being observed, together with the altitude and hour.—And thence determining small differences of longitude.—In observing the variation at sea, &c.

The learned Mr. *Andrew Reid* invented an instrument several years ago, for finding the latitude at sea from two altitudes of the sun, observed on the same day, and the interval of the observations, measured by a common watch. And this instrument, whose only fault was that of its being somewhat expensive, was made by Mr. *Jackson*. Tables have been lately computed for that purpose.

But we may often, from the foregoing rules, resolve the same problem without much trouble; especially if we suppose the master of the ship to know within 2 or 3 degrees what his latitude is. Thus,

Assume the two nearest probable limits of the latitude, and by the theorem $H = \frac{A + LD}{ld}$, compute the hours of observation for both suppositions. If one interval of those computed hours coincides with

with the interval observed, the question is solved. If not, the two distances of the intervals computed, from the true interval, will give a proportional part to be added to, or subtracted from, one of the latitudes assumed. And if more exactness is required, the operation may be repeated with the latitude already found.

But which ever way the question is solved, a proper allowance is to be made for the difference of latitude arising from the ship's course in the time between the two observations.

Of the double horizontal dial; and the Babylonian and Italian dials.

To the *gnomonic* projection, there is sometimes added a *stereographic* projection of the hour-circles, and the parallels of the sun's declination, on the same horizontal plane; the upright side of the gnomon being sloped into an edge, standing perpendicularly over the center of the projection: so that the dial, being in its due position, the shadow of *that* perpendicular edge is a vertical circle passing through the sun, in the stereographic projection.

The months being duly marked on this dial, the sun's declination, and the length of the day at any time, are had by inspection; (as also his altitude, by means of a scale of tangents). But its chief property is, that it may be placed true, whenever the sun shines, without the help of any other instrument.

Let d be the sun's place in the stereographic pro- Fig. 3.
jection, xyz the parallel of the sun's declination,

C c 4

Z d

Zd a vertical circle through the sun's center, *Pd* the hour-circle; and it is evident, that the diameter *NS* of this projection being placed duly north and south, these three circles will pass through the point *d*. And therefore, to give the dial its due position, we have only to turn its gnomon toward the sun, on a horizontal plane, until the hour on the common gnomonic projection coincides with that marked by the hour-circle *Pd*, which passes through the intersection of the shadow *Zd* with the circle of the sun's present declination.

The *Babylonian* and *Italian* dials reckon the hours not from the meridian, as with us, but from the sun's rising and setting. Thus, in *Italy*, an hour before sun-set is reckoned the 23d hour; two hours before sun-set, the 22d hour; and so of the rest. And the shadow that marks them on the hour-lines, is *that* of the point of a stile. This occasions a perpetual variation between their dials and clocks, which they have to correct from time to time, before it arises to any sensible quantity; by setting their clocks so much faster or slower. And in *Italy*, they begin their day, and regulate their clocks, not from sun-set, but from about mid-twilight, when the *ave Maria* is said; which corrects the difference that would otherwise be between the clock and the dial.

The improvements which have been made in all sorts of instruments and machines for measuring time, have rendered such dials of little account. Yet, as the theory of them is ingenious, and they are really, in some respects, the best contrived of
any

any for vulgar use, a general idea of their description may not be unacceptable.

Let Fig. 5. represent an erect direct south wall, Fig. 5. on which a *Babylonian dial* is to be drawn, shewing the hours from sun-rising; the latitude of the place, whose horizon is parallel to the wall, being equal to the angle KCR . Make, as for a common dial, $KG = KR$ (which is perpendicular to CR) the radius of the equinoctial AEQ , and draw RS perpendicular to CK for the stile of the dial; the shadow of whose point R is to mark the hours, when SR is set upright on the plane of the dial.

Then it is evident that, in the contingent line AEQ , the spaces $K1$, $K2$, $K3$, &c. being taken equal to the tangents of the hour-distances from the meridian, to the radius KG , one, two, three, &c. hours after sun-rising, on the equinoctial day; the shadow of the point R will be found, at these times, respectively in the points 1, 2, 3, &c.

Draw, for the like hours after sun-rising, when the sun is in the tropic of capricorn VQV , the like common lines CD , CE , CF , &c. and at those hours the shadow of the point R will be found in those lines respectively. Find the sun's altitudes above the plane of the dial at these hours, and with their co-tangents Sd , Se , Sf , &c. to radius SR , describe arcs intersecting the hour-lines in the points d , e , f , &c. So shall the right lines $1d$, $2e$, $3f$, &c. be the lines of I, II, III, &c. hours after sun-rising.

The construction is the same in every other case, due regard being had to the difference of longitude
of

of the place at which the dial would be horizontal, and the place for which it is to serve. And likewise, taking care to draw no lines but what are necessary; which may be done partly by the rules already given for determining the time that the sun shines on any plane; and partly from this, that on the tropical days, the hyperbola described by the shadow of the point *R*, limits the extent of all the hour-lines.

The most useful however, as well as the simplest of such dials, is that which is described on the two sides of a meridian plane.

That the *Babylonian* and *Italic* hours are truly enough marked by right lines, is easily shewn. Mark the three points on a globe, where the horizon cuts the equinoctial, and the two tropics, toward the east, or west; and turn the globe on its axis 15° , or 1 hour; and it is plain, that the three points which were in a great circle (viz. the horizon) will be in a great circle still; which will be projected geometrically into a straight line. But these three points are universally the sun's places, one hour after sun-set (or one hour before sun-rise) on the equinoctial and solstitial days. The like is true of all other circles of declination, besides the tropics; and therefore, the hours on such dials are truly marked by straight lines limited by the projections of the tropics; and which are rightly drawn, as in the foregoing example.

Note 1. The same dials may be delineated without the hour-lines *CD*, *CE*, *CF*, &c. by setting off the sun's azimuths on the plane of the dial,

dial, from the center S , on either side of the substile CSK , and the corresponding co-tangents of altitude from the same center S , for I, II, III, &c. hours before or after the sun is in the horizon of the place for which the dial is to serve, on the equinoctial and solstitial days.

2. One of these dials has its name from the hours being reckoned from sun-rising, the beginning of the *Babylonian* day. But we are not thence to imagine that the *equal* hours, which it shews, to be those in which the astronomers of that country marked their observations. These, we know with certainty, were unequal, like the *Jewish*, as being twelfth parts of the natural day: and an hour of the night was, in like manner, a twelfth part of the night; longer or shorter according to the season of the year. So that an hour of the day, and an hour of the night, at the same place, would always make $\frac{1}{12}$ of 24, or 2 equinoctial hours. In Palestine, among the Romans, and in several other countries, 3 of these unequal nocturnal hours were a *vigilia* or *watch*. And the reduction of equal and unequal hours into one another, is extremely easy. If, for instance, it is found, by a foregoing rule, that in a certain latitude, at a given time of the year, the length of a day is 14 equinoctial hours, the unequal hour is then $\frac{1}{14}$ or $\frac{7}{6}$ of an hour, that is, 70 minutes; and the nocturnal hour is 50 minutes. The first watch begins at VII (sun-set), the second at three times 50 minutes after, *viz.* IX h. 30 m. the third always at midnight; the morning watch at $\frac{1}{2}$ hour past II.

If

If it were required to draw a dial for shewing these unequal hours, or 12th parts of the day; we must take as many declinations of the sun as are thought necessary, from the equator towards each tropic: and having computed the sun's altitude and azimuth for $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$ th parts, &c. of each of the diurnal arcs belonging to the declinations assumed: by these, the several points in the circles of declination, where the shadow of the stile's point falls, are determined: and curve lines drawn through the points of an homologous division will be the hour-lines required.

Of the right placing of dials, and having a true meridian line for the regulating of clocks and watches.

The plane on which the dial is to rest; being duly prepared, and every thing necessary for fixing it; you may find the hour tolerably exact by a large equinoctial ring-dial, and set your watch to it. And then the dial may be fixed by the watch at your leisure.

If you would be more exact, take the sun's altitude by a good quadrant, noting the precise time of observation by a clock or watch. Then, compute the time for the altitude observed, (by the rule, page 386) and set the watch to agree with that time, according to the sun. A *Hadley's* quadrant is very convenient for this purpose; for, by it you may take the angle between the sun and his image, reflected from a basin of water; the half
of

of which angle, subtracting the refraction, is the altitude required. This is best done in summer, and the nearer the sun is to the prime vertical (the east or west azimuth) when the observation is made, so much the better.

Or, in summer, take two equal altitudes of the sun in the same day; one any time between 7 and 10 o'clock in the morning, the other between 2 and 5 in the afternoon; noting the moments of these two observations by a clock or watch: and if the watch shews the observations to be at equal distances from noon, it agrees exactly with the sun: if not, the watch must be corrected by half the difference of the forenoon and afternoon intervals; and then the dial may be set true by the watch.

Thus, for example, suppose you had taken the sun's altitude when it was 20 minutes past VIII in the morning by the watch; and found by observing in the afternoon, that the sun had the same altitude 10 minutes before III; then it is plain, that the watch was 5 minutes too fast for the sun: for 5 minutes after XII is the middle time between VIII h. 20 m. in the morning, and III h. 50 m. in the afternoon; and therefore, to make the watch agree with the sun, it must be set back five minutes.

A good *meridian line*, for regulating clocks or watches, may be had by the following method. *A meridian line.*

Make a round hole, almost a quarter of an inch diameter, in a thin plate of metal; and fix the plate in the top of a south window, in such a manner,

ner, that it may recline from the zenith at an angle equal to the co-latitude of your place, as nearly as you can guess: for then, the plate will face the sun directly at noon on the equinoctial days. Let the sun shine freely through the hole into the room; and hang a plumb-line to the cieling of the room, at least five or six feet from the window, in such a place as that the sun's rays, transmitted through the hole, may fall upon the line when it is noon by the clock; and having marked the said place on the cieling, take away the line.

Having adjusted a sliding bar to a dove-tail groove, in a piece of wood about 18 inches long, and fixed a hook into the middle of the bar, nail the wood to the abovementioned place on the cieling, parallel to the side of the room in which the window is: the groove and bar being towards the floor. Then, hang the plumb-line upon the hook in the bar, the weight or plummet reaching almost to the floor; and the whole will be prepared for farther and proper adjustment.

This done, find the true solar time by either of the two last methods, and thereby regulate your clock. Then, at the moment of next noon by the clock, when the sun shines, move the sliding bar in the groove until the shadow of the plumb-line bisects the image of the sun (made by his rays transmitted through the hole) on the floor, wall, or on a white screen placed on the north side of the line; the plummet or weight at the end of the line hanging freely in a pale of water placed below it on the floor.—But because this may not be quite
correct

correct for the first time, because the plummet will not settle immediately, even in water; it may be farther corrected on the following days, by the above method, with the sun and clock; and so brought to a very great exactness.

N. B. The rays transmitted through the hole will cast but a faint image of the sun, even on a white screen, unless the room be so darkened that no sun-shine may be allowed to enter, but what comes through the small hole in the plate. And always, for some time before the observation is made, the plummet ought to be immersed in a jar of water, where it may hang freely; by which means the line will soon become steady, which otherwise would be apt to continue swinging.

As this meridian line will not only be sufficient for regulating of clocks and watches to the true mean time by equation tables, but also for most astronomical purposes, I shall say nothing of the magnificent and expensive meridian lines at *Bologna* and *Rome*, nor of the better methods by which astronomers observe precisely the transits of the heavenly bodies on the meridian.

L E C T. XI.

Shewing how to calculate the mean time of any new or full moon, or eclipse, from the creation of the world to the year of CHRIST, 5800.

I N computing the following tables for this purpose, I have kept to the mean motions in Mr. *Meyer's* celebrated tables, published in the *Göttingen* Trans-

Transactions ; but have altered the form, in order to bring them into as short a compass as possible.

P R E C E P T S.

To find the mean time of any new or full moon in any given year and month after the Christian Æra.

1. If the given year be found in the second column of the *Table of the moon's mean motion from the sun*, under the title *years before and after CHRIST* ; write out that year, with the mean motions belonging to it, and thereto join the given month with its mean motions. But, if the given year be not in the table, take out the next lesser one to it that you find, in the same column ; and thereto add as many *complete years*, as will make up the given year : then, join the given month, and all the respective mean motions.

2. Collect these mean motions into one sum of signs, degrees, minutes, and seconds ; remembering, that 60 seconds (") make a minute, 60 minutes (') a degree, 30 degrees (°) a sign, and 12 signs (s) a circle. When the signs exceed 12, or 24, or 36, (which are whole circles) reject them, and set down only the remainder ; which, together with the odd degrees, minutes, and seconds already set down, must be reckoned the whole sum of the collection.

3. Subtract the result or sum of this collection, from 12 signs ; and write down the remainder,

Then,

Then, look in the table, under *days*, for the next less mean motions to this remainder, and subtract them from it, writing down their remainder.

This done, look in the table under *hours*, (marked H.) for the next less mean motions to this last remainder, and subtract them from it, writing down their remainder.

Then, look in the table under *minutes*, (marked M.) for the next less mean motions to this remainder, and subtract them from it, writing down their remainder.

Lastly, look in the table under *seconds*, (marked S.) for the next less mean motions to this remainder, either greater or less; and against it you have the seconds answering thereto.

4. And these times collected, will give the mean time of the *required new moon*; which will be right in common years, and also in January and February in leap-years; but always one day too late in leap-years after February.

E X A M P L E I.

Required the mean time of new moon in September 1764? (a year not inserted in the table.)

Moon from sun.

		•	°	'	''
To the year after <i>Christ's</i> birth 1753		10	9	24	56
Add compleat years ———	11	0	10	14	20
(sum 1764)					
And join September — — —		2	22	21	7
The sum of these mean motions is		1	12	0	23
Which, being sub. from a circle, or		12	0	0	0
Leaves remaining — — —		10	17	59	37
Next less mean mot. for 26 days, sub.		10	16	57	34
And there remains — — —		1	2	3	
Next less mean mot. for 2 hours, sub.		1	0	57	
And the remainder will be —		1	6		
Next less mean mot. for 2 min. sub.		1	1		
Remains the mean mot. of 12 sec.					5

These times, being collected, would shew the mean time of the required new moon in September 1764, to be on the 26th day, at 2 hours 2 min. 12 sec. past noon. But, as it is in a leap-year, and after February, the time is one day too late. So, the true mean time is Sept. the 25th, at 2 m. 12 sec. past II in the afternoon.

N. B. The tables always begin the day at noon, and reckon thence forward, to the noon of the day following.

To

To find the mean time of full moon in any given year and month after the Christian Æra.

Having collected the moon's mean motion from the sun for the beginning of the given year and month, and subtracted their sum from 12 signs, (as in the former example) add 6 signs to the remainder, and then proceed in all respects as above.

E X A M P L E II.

Required the mean time of full moon in September 1764?

Moon from sun.

To the year after Christ's birth. 1753	10 9 24 56
Add compleat years ——— 11	0 10 14 20
(sum 1764)	
And join September ———	2 22 21 7
The sum of these mean motions is	1 12 0 23
Which, being subtr. from a circle, or	12 0 0 0
Leaves remaining ———	10 17 59 37
To which remainder add ———	6 0 0 0
And the sum will be ———	4 17 59 37
Next less mean mot. for 11 days, subtr.	4 14 5 54
And there remains ———	3 53 43
Next less mean mot. for 7 hours, subtr.	3 33 20
And the remainder will be ———	20 23
Next less mean mot. for 40 min. subtr.	20 19
Remains the mean mot. for 8 sec.	4

D d 2

So,

So, the mean time, according to the tables, is the 11th of September, at 7 hours, 40 min. 8 sec. past noon. One day too late, being after February in a leap-year.

And thus may the mean time of any new or full moon be found, in any year after the Christian Æra.

To find the mean time of new or full moon in any given year and month before the Christian Æra.

If the given year before the year of CHRIST be found in the second column of the table, under the title *years before and after CHRIST*, write it out, together with the given month, and join the mean motions. But, if the given year be not in the table, take out the next greater one to it that you find; which being still farther back than the given year, add as many compleat years to it as will bring the time forward to the given year: then join the month, and proceed in all respects as above.

EXAMPLE III.

Required the mean time of new moon in May, the year before Christ 585?

The next greater year in the table is 600; which being 15 years before the given year, add the mean motions for 15 years to those of 600, together with those for the beginning of May.

To

Moon from sun.

	'	°	'	''
To the year before <i>Christ</i> 600	5	11	6	16
Add compleat years mot. 15	6	0	55	24
And the mean motions for May	0	22	53	23
<hr/>				
The whole sum is — — —	0	4	55	3
Which, being subtr. from a circle, or	12	0	0	0
<hr/>				
Leaves remaining — — —	11	25	4	57
Next less mean mot. for 29 days, sub.	11	23	31	54
<hr/>				
And there remains — — —	1	33	3	
Next less mean mot. for 3 hours, sub.	1	31	26	
<hr/>				
And the remainder will be —		1	37	
Next less mean mot. for 3 min. sub.		1	31	
<hr/>				
Rem. the mean mot. of 14 seconds			6	
<hr/>				

So, the mean time, by the tables, was the 29th of May, at 3 hours, 3 min. 14 sec. past noon. A day later than the truth, on account of its being in a leap-year. For, as the year of CHRIST 1 was the first after a leap-year, the year 585 before the year 1 was a leap-year, of course.

If the given year be after the Christian *Æra*, divide its date by 4, and if nothing remains, it is a leap-year in the old style. But if the given year was before the Christian *Æra*, (or year of CHRIST) subtract one from its date, and divide the remainder by 4; then, if nothing remains, it was a leap-year; otherwise, not.

D d 3

Ta

To find whether the sun is eclipsed at the time of any given change, or the moon at any given full.

Of eclipses. From the *Table of the sun's mean motion*, (or distance) from the moon's ascending node, collect the mean motions answering to the given time; and if the result shews the sun to be within 18 degrees of either of the nodes at the time of new moon, the sun will be eclipsed at that time. Or, if the result shews the sun to be within 12 degrees of either of the nodes at the time of full moon, the moon will be eclipsed at that time, near the contrary node: otherwise not.

EXAMPLE IV.

The moon changes on the 26th of September 1764, at 2 h. 2 m. (neglecting the seconds) after noon. (See Example I.) Qu. Whether the sun will be eclipsed at that time?

				Sun from node.			
				s	o	'	"
To the year after <i>Christ's</i> birth 1753				1	28	0	19
Add compleat years ——— 11				7	2	3	56
(sum 1764)							
And {	September	—	—	8	12	22	49
	26 days	—	—	27	0	13	
	2 hours	—	—			5	12
	2 minutes	—	—				5
Sun's distance from the ascend. node				6	9	32	34
				Now,			

Now, as the descending node is just opposite to the ascending, (*viz.* 6 signs distant from it) and the tables shew only how far the sun has gone from the ascending node, which, by this example, appears to be 6 signs, 9 deg. 32 min. 34 sec. it is plain that he must be eclipsed; being then only $9^{\circ} 32' 34''$ short of the descending node.

EXAMPLE V.

The moon will be full on the 11th of September 1764, at 7 hours 40 min. past noon. (See Example II.)

Qu. Whether she will be eclipsed at that time?

Sun from node.

	s	o	'	"
To the year after <i>Christ's</i> birth 1753	1	28	0	19
Add compleat years ——— 11	7	2	3	56
(sum 1764)				
And { September — — —	8	12	22	49
11 days — — — —		11	25	29
7 hours — — — —			18	11
40 minutes — — — —				1 44
Sun's distance from the asc. node	5	24	12	28

Which being subtracted from 6 signs, leaves only $5^{\circ} 47' 32''$ remaining: and this being all the space that the sun is short of the descending node, it is plain that the moon must then be eclipsed, because she is no farther from the contrary node.

EXAMPLE VI.

Qu. Whether the sun was eclipsed in May, the year before CHRIST 585? (See Example III.)

		Sun from node.			
		°	0	'	"
To the year before Christ	600 —	9	9	23	51
Add the mean mot. of	15 compl. years	9	19	27	49
And {	May — — — —	4	4	37	57
	29 days — — — —	1	0	7	10
	3 hours — — —			7	48
	3 min. (neglecting the seconds)				8
Sun's distance from the asc. node		<hr/> 0 3 44 43 <hr/>			

Which being less than 18 degrees, shews that the sun was eclipsed at that time.

*Thales's
eclipse.*

This eclipse was foretold by *Thales*, and appears to be the eclipse which put an end to the war between the Medes and Lydians.

When
eclipses must
happen.

The times of the *sun's conjunction with the nodes*, and consequently the *eclipse-months* of any given year, are easily found by the *Table of the sun's mean motion from the moon's ascending node*; and much in the same way as the mean conjunctions of the sun and moon are found by the table of the moon's mean motion from the sun. For, collect the sun's mean motion (which is the same as his distance gone) from the moon's ascending node, for the beginning of any given year, and subtract it from 12
signs ;

signs ; then, from the remainder, subtract the next less mean motions belonging to whatever *month* you find them in the table ; and from their remainder subtract the next less mean motions for *days*, and so on for *hours* and *minutes* : the result of all which will shew the time of the sun's mean conjunction with the *ascending node* of the moon's orbit.

EXAMPLE VII.

Required the time of the sun's conjunction with the ascending node in the year 1764?

Sun from node.

To the year after <i>Christ's</i> birth 1753	1	28	0	19
Add compleat years 11	7	2	3	56
Mean dist. at begin. of A. D. 1764	9	0	4	15
Subtr. this distance from a circle, or	12	0	0	0
And there remains — — —	2	29	55	45
Next less mean motion for March subtr.	2	1	16	39
And the remainder will be —	0	28	39	6
Next less mean mot. for 27 days, subtr.	0	28	2	32
And there remains — — —			36	34
Next less mean mot. for 14 hours, subtr.			36	21
Rem. (nearly) the mean mot. of 5 min.				13

Hence it appears, that the sun will pass by the moon's *ascending node* on the 27th of March, at 14 hours

hours 5 minutes past noon ; *viz.* on the 28th day, at 5 minutes after II in the morning, according to the tables : but this being in a leap-year, and after February, the time is one day too late. Consequently, the true time is at 5 minutes past II in the morning on the 27th day ; at which time, the descending node will be directly opposite to the sun.

If 6 signs be added to the remainder arising from the first subtraction, (*viz.* from 12 signs) and then the work carried on as in the last example, the result will give the mean time of the sun's conjunction with the descending node. Thus in

E X A M P L E VIII.

Let it be required to find when the sun will be in conjunction with the descending node in the year 1764?

Sun from node.

	°	0	'	"
To the year after <i>Christ's</i> birth 1753	1	28	0	19
Add compleat years — — — 11	7	2	3	56
Mean dist. fr. asc. nod. at beg. of 1764	9	0	4	15
Subtr. this distance from a circle, or	12	0	0	0
And the remainder will be —	2	29	55	45
To which add half a circle, or —	6	0	0	0
And the sum will be — —	8	29	55	45
Next less mean mot. for Septemb. subtr.	8	12	22	49
And there remains — — —	0	17	32	56
Next less mean mot. for 16 days, subtr.	0	16	37	4
And the remainder will be —		55	52	
Next less mean mot. for 21 hours, subtr.		54	32	
Rem. (nearly) the mean mot. of 31 min.			1	20
				So

So that, according to the tables, the sun will be in conjunction with the *descending node* on the 16th of September, at 21 hours 31 minutes past noon: one day later than the truth, on account of the leap-year.

When the moon changes within 18 days before or after the sun's conjunction with either of the nodes, the sun will be eclipsed at that change: and when the moon is full within 12 days before or after the time of the sun's conjunction with either of the nodes, she will be eclipsed at that full: otherwise not.

If to the mean time of any eclipse, either of the sun or moon, we add 557 Julian years, 21 days, 18 hours, 11 minutes, and 51 seconds, (in which there are exactly 6890 mean lunations) we shall have the mean time of the return of that eclipse. For, at the end of that time, the moon will be either new or full, according as we add it to the time of new or full moon; and the sun will be only 45" farther from the same node, at the end of the said time, than he was at the beginning of it; as appears by the following example*.

The

* Dr. HALLEY's period of eclipses contains only 18 years, 11 days, 7 hours, 43 minutes, 15 seconds; in which time, according to his tables, there are just 223 mean lunations: But, as in that time, the sun's mean motion from the node is no more than $11^{\circ} 29' 31'' 49''$, which wants $28' 11''$ of being as nearly in conjunction with the same node at the end of the period as it was at the beginning; this period can be of no long duration for finding eclipses, because it will in time fall quite without their limits. The following tables make this period 31 seconds

The period.		Moon fr. sun.				Sun fr. node.			
		s	o	'	''	s	o	'	''
Compl. years	500 —	3	5	32	47 —	10	14	45	8
	40 —	8	26	50	37 —	1	23	58	49
	17 —	3	2	21	39 —	10	28	40	55
	days 21 —	8	16	0	21 —	21	48	38	
hours	18 —		9	8	35 —		46	44	
minutes	11 —			5	35 —				29
seconds	51 —				26 —				2
Mean motions		<hr/> 0 0 0 0 — <hr/>				<hr/> 0 0 0 45 <hr/>			

And this period is so very near, that in 6000 years it will vary no more from the truth, as to the restitution of eclipses, than $8\frac{1}{4}$ minutes of a degree; which may be reckoned next to nothing. It is the shortest in which, after many trials, I can find so near a co-incidence of the sun, moon, and the same node.

31 seconds shorter, as appears by the following calculation.

The period.		Moon from sun.				Sun from node.			
		s	o	'	''	s	o	'	''
Compl. years	18 —	7	11	59	4 —	11	17	46	18
days	11 —	4	14	5	54 —		11	25	29
hours	7 —		3	33	20 —			18	11
min.	42 —			21	20 —			1	49
sec.	44 —				22 —				2
Mean motions		<hr/> 0 0 0 0 — <hr/>				<hr/> 11 29 31 49 <hr/>			

This

A Table of mean lunations.

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This table is made by the continual addition of a mean lunation, viz. $29^d 12^h 44^m 3^s 6''' 21'' 14'' 24'' 0'''$.

Lun.	Days.	H.	M.	S.	Th.		
1	29	12	44	3	6		
2	59	1	28	6	13		
3	88	14	12	9	19		
4	118	2	56	12	25		
5	147	15	40	15	32		
6	177	4	24	18	38		
7	206	17	8	21	44		
8	236	5	52	24	51		
9	265	18	36	27	57		
10	295	7	20	31	3		
20	590	14	41	2	7		
30	885	22	1	33	11		
40	1181	5	22	4	14		
50	1476	12	42	35	18		
100	2953	1	25	10	35		
200	5906	2	50	21	11		
300	8859	4	15	31	46		
400	11812	5	40	42	22		
500	14765	7	5	52	57		
1000	29530	14	11	45	54		
2000	59061	4	23	31	48		
3000	88591	18	35	17	42		
4000	118122	8	47	3	36		
5000	147652	22	58	49	30		
10000	295305	21	57	39	0		
20000	590611	19	55	18	0		
30000	885917	17	52	57	0		
40000	1181223	15	50	36	0		
50000	1476529	13	48	15	0		
100000	2953059	3	36	30	0		

In 100000 mean lunations, there are 8085 Julian years, 12 days, 21 hours, 36 minutes, 30 seconds = 2953059 days, 3 hours, 36 minutes, 30 seconds.

Proof of the Table.

		Moon fr. sun				
		.	o	'	"	
Jul. years	In					
	{	4000	1	14	22	12
		4000	1	14	22	12
		80	5	23	41	15
		5	10	0	18	28
Days	12	4	26	17	20	
Hours	21		10	40	1	
Min.	36			18	17	
Sec.	30				15	
M. fr. sun		o	o	o	o	

Having by the former precepts computed the mean time of new moon in January, for any given year, it is easy by this Table to find the mean time of new moon in January for any number of

by means of a small table of lunations for 12 or 13 months, to make a general table for finding the mean time of new or full moon in any given year and month whatever.

	D.	H.	M.	S.	Th.
In 11 lunations there are	324	20	4	34	10.
In 12 lunations —	354	8	48	37	16.
In 13 lunations —	383	21	32	40	23.

But then it would be best to begin the year with March, to avoid the inconvenience of losing a day by mistake in leap-year.

A Table of the moon's mean

Julian peri- od.	Years bef. and after CHRIST.	Moon from sun.				Com- pleat years.	Moon from sun.			
		°	'	"	'''		°	'	"	'''
706	4008	5	28	1	17	11	0	10	14	20
714	4000	5	9	23	24	12	5	2	3	11
1714	3000	11	20	28	57	13	9	11	40	35
2714	2000	6	1	34	30	14	1	21	18	0
3714	1000	0	12	40	3	15	6	0	55	24
3814	900	10	19	46	36	16	10	22	44	15
3914	800	8	26	53	9	17	3	2	21	39
4014	700	7	3	59	43	18	7	11	59	4
4114	600	5	11	6	16	19	11	21	36	27
4214	500	3	18	12	49	20	4	13	25	19
4314	400	1	25	19	23	40	8	26	50	37
4414	300	0	2	25	56	60	1	10	15	56
4514	200	10	9	32	29	80	5	23	41	15
4614	100	8	16	39	3	100	10	7	6	33
4714	1	6	23	45	36	200	8	14	13	7
4814	101	5	0	52	9	300	6	21	19	40
4914	201	3	7	58	43	400	4	28	26	13
5014	301	1	15	5	16	500	3	5	32	47
5114	401	11	22	11	49	1000	6	11	5	33
5214	501	9	29	18	23	2000	0	22	11	6
5714	1001	1	4	51	9	3000	7	3	16	39
6414	1701	0	24	37	2	4000	1	14	22	12
6466	1753	10	9	24	56					
6514	1801	6	5	26	15					

The year of the Julian period 706 is supposed to be the year of the creation.	Compleat years.	Moon from sun.				Months	Moon from sun.			
		°	'	"	'''		°	'	"	'''
All these are Julian years, consisting of 365 d. 6 h.	1	4	9	37	24	Jan.	0	0	0	0
	2	8	19	14	48	Feb.	0	17	54	48
	3	0	28	52	13	Mar.	11	29	15	16
	4	5	20	41	4	April	0	17	10	3
	5	10	0	18	28	May	0	22	53	23
	6	2	9	55	52	June	1	10	48	11
	7	6	19	33	17	July	1	16	31	32
	8	11	11	22	7	Aug.	2	4	26	20
	9	3	20	59	32	Sept.	2	22	21	8
	10	8	0	36	55	Oct.	2	28	4	29
						Nov.	3	15	59	17
						Dec.	3	21	42	37

This table agrees with the *old style*, until the year 1753; and after that, with the *new*.

Days.	Moon from sun.				Moon from sun.				Moon from sun.			
	s	o	'	"	H. M. S.	o	'	"	M. S. Th.	'	"	"
1	0	12	11	27								
2	0	24	22	53								
3	1	6	34	20	1	0	30	29	31	15	44	47
4	1	18	45	47	2	1	0	57	32	16	15	16
5	2	0	57	13	3	1	31	26	33	16	45	44
6	2	13	8	40	4	2	1	54	34	17	16	13
7	2	25	20	7	5	2	32	23	35	17	46	42
8	3	7	31	34	6	3	2	52	36	18	17	10
9	3	19	43	0	7	3	33	20	37	18	47	39
10	4	1	54	27	8	4	3	49	38	19	18	7
11	4	14	5	54	9	4	34	18	39	19	48	36
12	4	26	17	20	10	5	4	46	40	20	19	5
13	5	8	28	47	11	5	35	15	41	20	49	33
14	5	20	40	14	12	6	5	43	42	21	20	2
15	6	2	51	40	13	6	36	12	43	21	50	31
16	6	15	3	7	14	7	6	41	44	22	20	59
17	6	27	14	34	15	7	37	9	45	22	51	28
18	7	9	26	0	16	8	7	38	46	23	21	56
19	7	21	37	27	17	8	38	6	47	23	52	25
20	8	3	48	54	18	9	8	35	48	24	22	54
21	8	16	0	21	19	9	39	4	49	24	53	22
22	8	28	11	47	20	10	9	32	50	25	23	51
23	9	10	23	14	21	10	40	1	51	25	54	19
24	9	22	34	41	22	11	10	30	52	26	24	48
25	10	4	46	7	23	11	40	58	53	26	55	17
26	10	16	57	34	24	12	11	27	54	27	25	45
27	10	29	9	1	25	12	41	55	55	27	56	14
28	11	11	20	27	26	13	12	24	56	28	26	43
29	11	23	31	54	27	13	42	53	57	28	57	11
30	0	5	43	21	28	14	13	21	58	29	27	40
31	0	17	54	47	29	14	43	50	59	29	58	8
32	1	0	6	15	30	15	14	18	60	30	28	37

$$1 \text{ Lunation} = 29^d 12^h 44^m 3^s 6^{th} 21^{iv} 14^v 24^{vi} 0^{vii}.$$

In leap-years, after February, a day and its motion must be added to the time for which the moon's mean distance from the sun is given. But, when the mean time of any new or full moon is required in leap-year after February, a day must be subtracted from the mean time thereof, as found by the tables. In common years they give the day right.

A Table of the sun's mean motion from

Years of the world.	Years bef. and after CHRIST.	Sun from node.				Compleat years.	Sun from node.			
		•	•	/	//		•	•	/	//
0	4008	7	6	17	9	11	7	2	3	56
8	4000	0	11	4	55	12	7	22	11	39
1008	3000	9	10	35	11	13	8	11	17	2
2008	2000	6	10	5	28	14	9	0	22	25
3008	1000	3	9	35	44	15	9	19	27	49
3108	900	7	24	32	46	16	10	9	35	31
3208	800	0	9	29	48	17	10	28	40	55
3308	700	4	24	26	49	18	11	17	46	18
3408	600	9	9	23	51	19	0	6	51	43
3508	500	1	24	20	53	20	0	26	59	24
3608	400	6	9	17	54	40	1	23	58	49
3708	300	10	24	14	56	60	2	20	58	13
3808	200	3	9	11	58	80	3	17	57	37
3908	100	7	24	8	59	100	4	14	57	2
4008	1	0	9	6	1	200	8	29	54	3
4108	101	4	24	3	3	300	1	14	51	5
4208	201	9	9	0	4	400	5	29	48	7
4308	301	1	23	57	6	500	10	14	45	8
4408	401	6	8	54	8	1000	8	29	30	17
4508	501	10	23	51	9	2000	5	29	0	33
5008	1001	9	8	36	18	3000	2	28	30	50
5708	1701	4	23	15	30	4000	11	28	1	6
5760	1753	1	28	0	19					
5808	1801	8	25	44	44					

The 4008th year bef. the y. of CHRIST is sup. to be the year of the creation.	All these are Julian years, as in the preceding table.	Compleat years.	Sun from node.				Months	Sun from node.			
		•	•	/	//	•		•	/	//	
	1	0	19	5	23	Jan.	0	0	0	0	
	2	1	8	10	47	Feb.	1	2	11	48	
	3	1	27	16	10	Mar.	2	1	16	39	
	4	2	17	23	53	April	3	3	28	27	
	5	3	6	29	16	May	4	4	37	57	
	6	3	25	34	40	June	5	6	49	45	
	7	4	14	40	3	July	6	7	59	14	
	8	5	4	47	46	Aug.	7	9	11	1	
	9	5	23	53	9	Sept.	8	12	22	49	
	10	6	12	58	33	Oct.	9	13	32	18	
						Nov.	10	15	44	5	
						Dec.	11	16	53	34	

This table agrees with the *old style*, until the year 1753; and after that, with the *new*.

Days.	Sun from node.				Sun from node.				Sun from node.			
	s	o	'	''	H. M. S.	o	'	''	M. S. Th.	'	''	'''
1	0	1	2	19								
2	0	2	4	38								
3	0	3	6	57	1	0	2	36	31	1	20	31
4	0	4	9	16	2	0	5	12	32	1	23	7
5	0	5	11	36	3	0	7	48	33	1	25	43
6	0	6	13	54	4	0	10	23	34	1	28	9
7	0	7	16	13	5	0	12	59	35	1	31	55
8	0	8	18	32	6	0	15	35	36	1	33	31
9	0	9	20	51	7	0	18	11	37	1	36	6
10	0	10	23	10	8	0	20	47	38	1	38	42
11	0	11	25	29	9	0	23	23	39	1	41	18
12	0	12	27	48	10	0	25	58	40	1	43	54
13	0	13	30	7	11	0	28	33	41	1	46	36
14	0	14	32	26	12	0	31	9	42	1	49	5
15	0	15	34	15	13	0	33	45	43	1	51	41
16	0	16	37	4	14	0	36	21	44	1	54	17
17	0	17	39	23	15	0	38	57	45	1	56	53
18	0	18	41	41	16	0	41	32	46	1	59	29
19	0	19	44	0	17	0	44	8	47	2	2	5
20	0	20	46	19	18	0	46	44	48	2	4	41
21	0	21	48	38	19	0	49	20	49	2	7	17
22	0	22	50	57	20	0	51	56	50	2	9	53
23	0	23	53	16	21	0	54	32	51	2	12	29
24	0	24	55	35	22	0	57	8	52	2	15	5
25	0	25	57	54	23	0	59	43	53	2	17	41
26	0	27	0	13	24	1	2	19	54	2	20	17
27	0	28	2	32	25	1	4	55	55	2	22	53
28	0	29	4	51	26	1	7	31	56	2	25	29
29	1	0	7	10	27	1	10	7	57	2	28	4
30	1	1	9	29	28	1	12	43	58	2	30	40
31	1	2	11	48	29	1	15	9	59	2	33	16
32	1	3	14	7	30	1	17	55	60	2	35	52

In leap-years, after February add one day and one day's motion to the time at which the sun's mean distance from the ascending node is required.

FINIS.

Date	Sun from node			Moon from node			Sun from node		
	h	m	s	h	m	s	h	m	s
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0
51	0	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0
57	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0
59	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0
61	0	0	0	0	0	0	0	0	0
62	0	0	0	0	0	0	0	0	0
63	0	0	0	0	0	0	0	0	0
64	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0	0	0
67	0	0	0	0	0	0	0	0	0
68	0	0	0	0	0	0	0	0	0
69	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	0	0
71	0	0	0	0	0	0	0	0	0
72	0	0	0	0	0	0	0	0	0
73	0	0	0	0	0	0	0	0	0
74	0	0	0	0	0	0	0	0	0
75	0	0	0	0	0	0	0	0	0
76	0	0	0	0	0	0	0	0	0
77	0	0	0	0	0	0	0	0	0
78	0	0	0	0	0	0	0	0	0
79	0	0	0	0	0	0	0	0	0
80	0	0	0	0	0	0	0	0	0
81	0	0	0	0	0	0	0	0	0
82	0	0	0	0	0	0	0	0	0
83	0	0	0	0	0	0	0	0	0
84	0	0	0	0	0	0	0	0	0
85	0	0	0	0	0	0	0	0	0
86	0	0	0	0	0	0	0	0	0
87	0	0	0	0	0	0	0	0	0
88	0	0	0	0	0	0	0	0	0
89	0	0	0	0	0	0	0	0	0
90	0	0	0	0	0	0	0	0	0
91	0	0	0	0	0	0	0	0	0
92	0	0	0	0	0	0	0	0	0
93	0	0	0	0	0	0	0	0	0
94	0	0	0	0	0	0	0	0	0
95	0	0	0	0	0	0	0	0	0
96	0	0	0	0	0	0	0	0	0
97	0	0	0	0	0	0	0	0	0
98	0	0	0	0	0	0	0	0	0
99	0	0	0	0	0	0	0	0	0
100	0	0	0	0	0	0	0	0	0

In the above table the Moon's position is given for every day and hour of the month. The Sun's position is also given for every day and hour of the month. The Moon's position is given in the first column, the Sun's position in the second column. The Moon's position is given in the first column, the Sun's position in the second column. The Moon's position is given in the first column, the Sun's position in the second column.

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BOOKS printed for A. MILLAR in the *Strand*.

1. **A**STRONOMY explained upon Sir *Isaac Newton's* Principles, and made easy to those who have not studied Mathematics, by *James Ferguson*. 4to.

2. *Cyclopædia: Or, An Universal Dictionary of Arts and Sciences*. By *E. Chambers*. F. R. S. The Seventh Edition, corrected and amended, with some Additions. In 2 vols.

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6. An Account of Sir *Isaac Newton's Philosophical Discoveries*. In Four Books. By *Colin Mac Laurin*, A. M. late Fellow of the Royal Society. Professor of Mathematics in the University of *Edinburgh*, and Secretary to the Philosophical Society there. Published from the Author's MSS. Papers. To which is added, An Account of the Life and Writings of the Author. By *Patrick Murdoch*, M. A. and F. R. S.

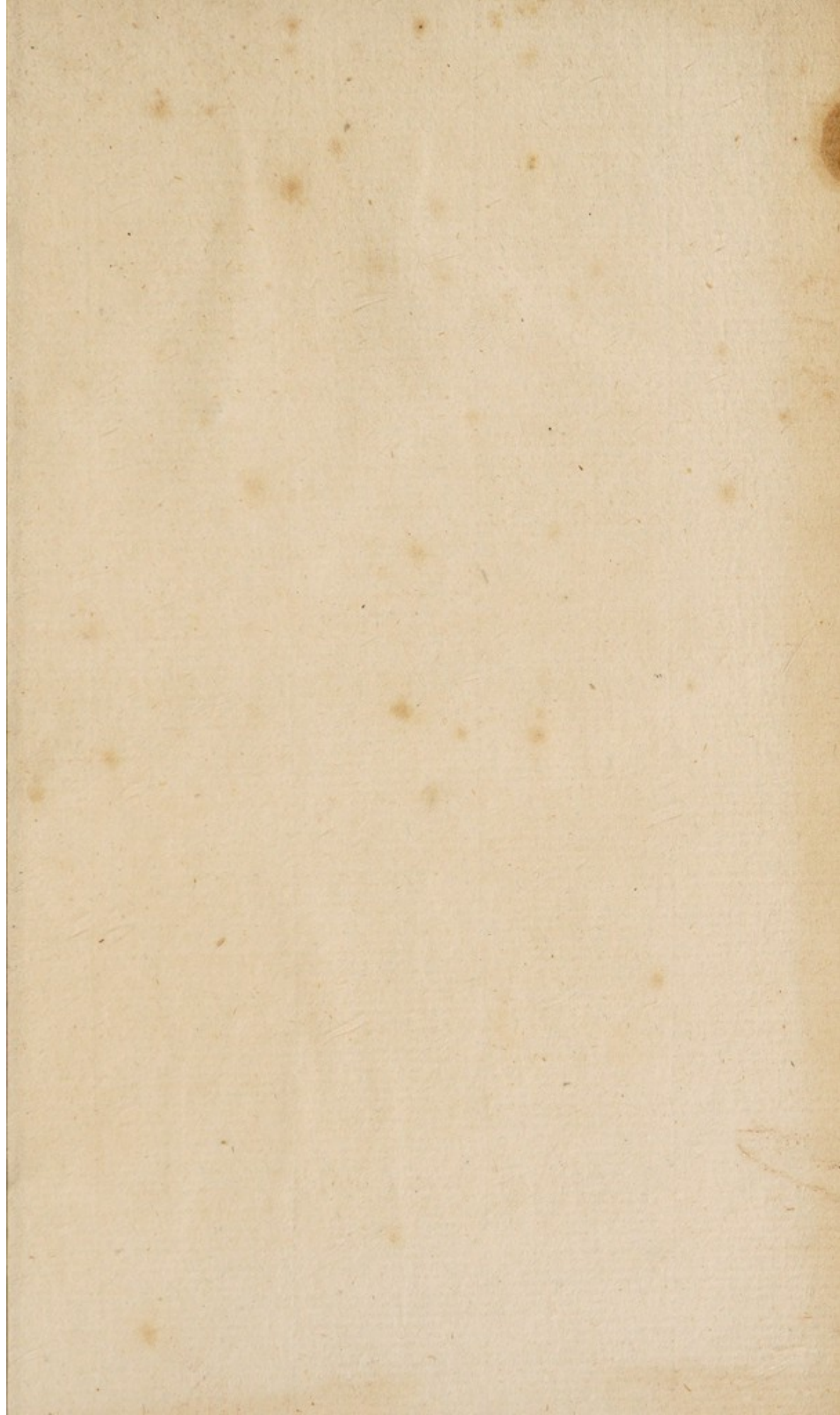
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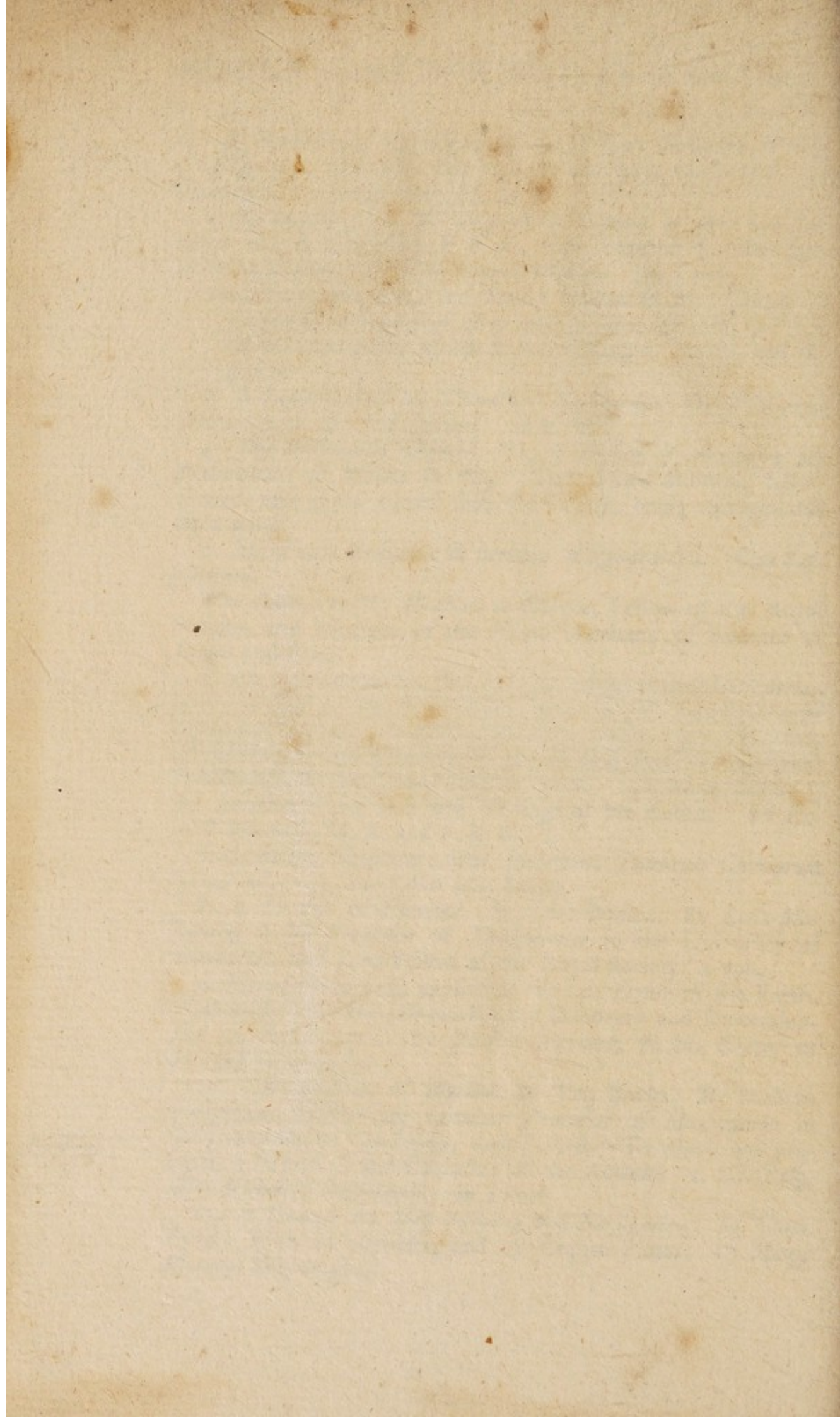
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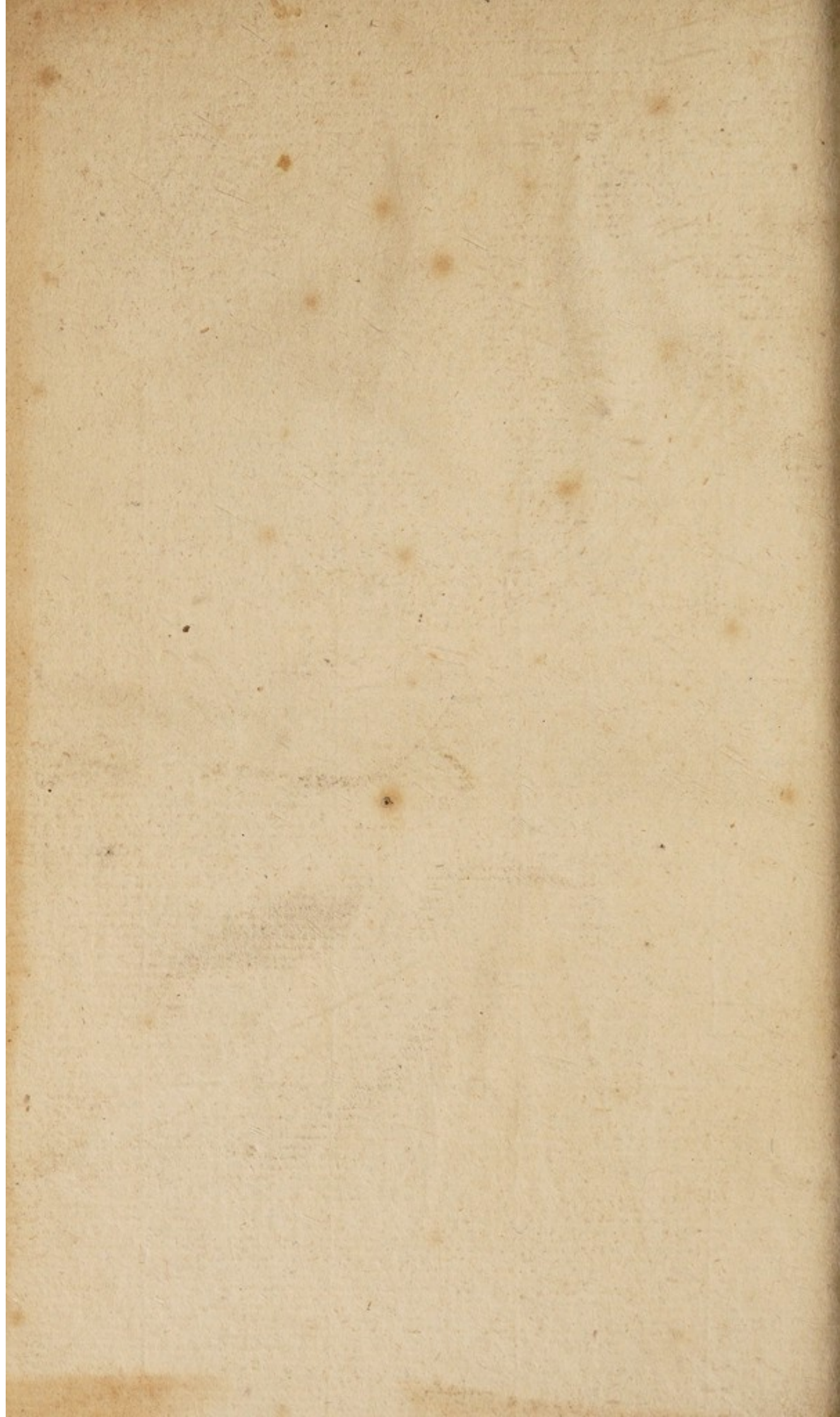
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Science

