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AN

INTRODUCTION

TO

Natural Philoſophy:

OR,

Philoſophical Lectures

Read in the

University of *Oxford,*

Anno Dom. 1700.

To which are Added,

The DEMONSTRATIONS of Monſieur HUYGENS's *Theorems*, concerning the Centrifugal Force and Circular Motion.

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THE PREFACE.

ALTHOUGH now-a-days the Mechanical Philoſophy is in great Repute, and in this Age has met with many who cultivate it; yet in moſt of the Writings of the Philoſophers, there is ſcarce any thing Mechanical to be found beſides the Name. Inſtead whereof, the Philoſophers ſubſtitute the Figures, Ways, Pores, and Interſtices of Corpuſcles, which they never ſaw; the inteſtine Motion of Particles, the Colluctations and Conflicts of Acids and Alkalis, and the Events that thence ariſe, they relate ſo exactly, that there is nothing but a Belief wanting in the Hiſtory of Nature, as often as they ſet forth the Miracles of their ſubtile Matter: I ſay, Miracles, for certainly that muſt be a ſort of a Miracle, which happens contrary to the well-known Laws of Nature, and the eſtabliſhed Principles of

THIS, *however specious it may seem at first appearance, if it be brought to an Examination, will be found contrary to almost all the Laws of Nature. For in the first place the* Cartesians *require their Ethereal Matter to be carried round the Earth in Circles; but from what Cause that Motion arises, or by what Means it is continued, is as difficult to explain, as to give a Reason for Gravity itself. They therefore who contend that Gravity proceeds from thence, undertake to explain what is unknown by what is still more unknown; especially since there may be produced divers Arguments, whereby that Rotation is wholly destroyed. But let us grant to the* Cartesians *their Postulatum, and then let us see, whether this Phenomenon does thence follow. Since it is necessary that the Velocity of the Vortex revolving about the Earth, is, at the Superficies of the Earth, equal to the Velocity of the Earth's Rotation, (for if it was greater, some part of the Motion would be communicated to the Earth; whence it would happen, that its Velocity would be always diminished, and that of the Earth increased, till they arrived at an Equality) so that from the Magnitude of the Earth, and the Time of its Rotation being known, there will be given the Space, which a Body, urged by the Centrifugal Force of the Celestial Matter, can run over in a given Time; namely, that which is equal to the Square of the Arch described in the mean while, applied to the Diameter of the Circle: By* Lemma 2. *in the Demonstrations of Mons.* Huygens's *Theorems of the Centrifugal Force and Circular Motion. If a Calculation be made from this Principle, it will be found that the Space which will be run over in a Second of Time, by a Body agitated by a Centrifugal Force of the Ether, will not ex-*

ceed half a Foot: If therefore the Effect of Gravity was produced mechanically, heavy Bodies in a Second of Time would not descend above half a Foot; but heavy Bodies in their Motion downwards will pass over in the same Time fifteen Feet. So that if after this manner the Ether was the Cause of Gravity, it would act contrary to the Laws of Mechanicks, by making a Body descend through fifteen Feet in one Second of Time.

TO *avoid the Force of this Objection, the* Cartesians *suppose the Rotation of the ethereal Matter to be much swifter than the Rotation of the Earth. Which, though it is impossible, yet if we also concede to them this, there will not thence proceed the Mechanical Action of Gravity. For since the Matter of the Vortex is always carried in Circles parallel to the Equator, and the Directions of the Centrifugal Forces are always made in Lines lying in the Planes of these Circles; it follows, that all Bodies must descend in these Planes, and tend perpendicularly to the Axis, and not to the Surface of the Earth. If therefore the subtile Matter acted mechanically, it would force Bodies perpendicularly to the Axis; whence since, according to these Theorists, it compels them to tend to the Center of the Earth, it produces an Effect contrary to the true Laws of Mechanicks.*

THAT *they may remove this Difficulty, they farther suppose the ethereal Matter to be carried not in Circles parallel to the Equator, but in great Circles of a Sphere. But how it is possible to conceive this, I am at a loss to know; for since every great Circle twice intersects all the other Circles that are infinite in number, it is necessary that the Motion of any Particle should be hindered by infinite*

others

others moving different ways, and at length its Motion ſhould ceaſe, if at firſt an equal Quantity of Motion was impreſſed on all the Parts; or that, laſtly, it ſhould be all carried in parallel Circles, if at the beginning the Motion was greater towards one Part than the other. Moreover, it may be asked, whence it comes to paſs that the ethereal Matter does move in the Superficies of the outermoſt Sphere, ſince it has a Centrifugal Force, it ſeems as if it ſhould recede from thence; what then is it that hinders it from doing ſo? They are wont to anſwer, That the ambient Bodies compreſs the Matter in the exteriour Sphere, and prevent its eſcaping. But ſince this Matter muſt preſs other Bodies encompaſſing it, it is neceſſary that it communicates a Motion to them, and theſe Bodies in like manner will impreſs a Motion on others encompaſſing them, and ſo the Motion of the ſubtile Matter will be propagated in infinitum; *whence of neceſſity its Celerity muſt languiſh by degrees.*

THERE *are many other Difficulties which accompany theſe Mechanical Explications of Gravity; one whereof we ſhall here propoſe, and which extends to all the Theories of this ſort. And it is this: If a Body is after any manner driven downwards by a ſubtile Matter, the Force whereby it is driven, will be neceſſarily as the Number of Particles, by the acting of which together, the Body is conſtrained to tend towards the Earth: but the Number of Particles is as the Superficies of the Body; wherefore the Force whereby the Body is preſſed downwards, will be as its Superficies, and not as the Quantity of Matter in the Body, which is contrary to Experience. Nor will all the reſt of the Hypotheſes, which they frame to account for the Appearance of* *other things, be found less repugnant to the Laws of Nature, if they are brought to an Examination.*

ALL *these Errors seem to spring from hence, that Men ignorant of Geometry presume to Philosophize, and to give the Causes of Natural Things. For what can we expect but Mistakes, from such, as having neglected Geometry, the Foundation of all Philosophy, and being unacquainted with the Forces of Nature, which can only be estimated by the means of Geometry, do yet attempt to explain its Operations, by a Method not at all agreeing with the Rules of Mechanicks?*

AMONGST *the Philosophers of this sort,* Cartes *leads the Van; who though he was a famous Geometer, yet that he might accommodate himself to the idle and common Herd of Philosophers, made no use of Geometry in his Philosophy; and although he pretended to explain all things mechanically by Matter and Motion, yet he introduced a Philosophy, which was as contrary to the true Laws of Mechanicks, as was possible. Those may be reckoned of his Sect, who refuse the Labour of Philosophizing aright, that is, Geometrically: And such are by much the greatest number, being scattered almost over the whole Earth.*

BUT *although so great a part of the Philosophers have embraced the Shadow of Philosophy, and not its Substance; yet there have not been wanting (nor, I hope, ever will) such as have employed real Pains in discovering the true Laws of Nature, and in investigating from thence the Causes of Things by Mechanical Principles.*

AMONGST *the antient Philosophers, the Divine* Archimedes *is of the greatest Eminence; who besides his Geometrical Labours, has left us the Principles of Mechanicks*

nicks and Staticks demonſtrated in two Books de Æquiponderantibus & de Humido Inſidentibus. *After him through a long Series of Years the Mechanical Philoſophy lay in obſcurity, nor was it cultivated except by a very few Perſons of a more refined Underſtanding. Amongſt whom we may juſtly reckon* Roger Bacon *of* Oxford, *and* Cardan. *But at length, about the beginning of the laſt Age, that noble* Lyncean *Philoſopher* Galileo, *having by the means of Geometry penetrated into the Secrets of Nature, framed a new Science of Motion, and ſhewed a Method whereby the Mechanical Cauſes of Things might be diſcovered. Then the Famous* Torricelli *and* Paſchal *treading in his very Footſteps, improved Philoſophy by new Speculations. And laſtly, the Societies of* London *and* Paris *were inſtituted by two powerful Monarchs, for the Advancement of Philoſophy; whereby the Knowledge of Nature has been increaſed with wonderful Inventions, not only ſuch as terminate in bare Speculation, but with many others which are of common Uſe to Men. It would be a difficult Task to reckon up the innumerable Benefits that accrue to Mankind from the Labours of both Societies: nor is it an eaſy matter to ſhew, how much ſucceeding Ages will be obliged to the Geometrical Demonſtrations of the Illuſtrious Monſieur* Huygens, *concerning the Motion of Pendulums; or to the curious Experiments of the Honourable Mr.* Boyle, *whereby he has diſcloſed many wonderful Secrets of Nature. Our late Poſterity will peruſe with a grateful Mind Dr.* Wallis's *Treatiſe of Motion; a Work the moſt perfect of its kind. The Philoſophers will now no longer be troubled to find out the Cauſes of Rivers and Winds, ſince they*

they have been delivered by that acute Geometer Dr. Halley, *tho before him they have been ſought after in vain.*

I SHOULD *proceed in enumerating the Merits of others towards the real Philoſophy, if I did not find myſelf obliged to ſtop, to mention the great Inventions of Sir* Iſaac Newton, *whoſe prodigious Genius has laid open more and abſtruſer Myſteries of Nature, than Men could ever have hoped for: but ſince it is impoſſible to comprehend his Diſcoveries within the narrow Limits of this Preface, we ſhall only undertake to ſay thus much, That what all our Predeceſſors from Time immemorial have handed down to us concerning the Mechanical Philoſophy, does not amount to the tenth part of thoſe Things, which Sir* Iſaac Newton *alone, through his vaſt Skill in Geometry, has found out by his own Sagacity. But how the Mechanical Principles may be eaſily applied to explain the Affections of ſuch Bodies as are at a diſtance from us, namely, the Motions and Inequalities of the Planets; the Learned World has been lately informed by the Philoſophical and Geometrical Elements of Aſtronomy publiſhed by Dr.* Gregory, *the* Savilian *Profeſſor of Aſtronomy: A Work that will laſt as long as the Sun and Moon endure.*

SINCE *therefore ſuch is the State of the Mechanical Philoſophy, that there is no Admittance to it but by the means of Geometry; ſome of my Friends have requeſted of me, that I would undertake to explain to the Youth of this Univerſity its eaſieſt Principles, and ſuch as only depend on the firſt Elements of Geometry; which thing was alſo asked of me in the moſt obliging manner in the world, by the Gentleman who ſubſtituted me in the publick Schools for the under-*

undertaking this Work. And this was Sir Thomas Millington *M. D.* Sedleian *Professor of Natural Philosophy in this University, and President of the College of Physicians in* London; *a Person adorned with Learning of every kind. By his Advice it was, that I read the following Lectures in this University: And in them my principal Care was, that Beginners might have clear and distinct Conceptions of the general Affections of Bodies; for all the Errors in Philosophical Matters have their Origin from the obscure and false Ideas of Things: so that I have endeavoured, as much as I was able, to expound clearly the Extension, Solidity, and Divisibility of Body, which have been delivered by others very obscurely. And then I have explained the Nature and Properties of Motion, which may be easily conceived by all, unless some sort of Philosophers; and I have thence deduced the Laws of Nature, and have shewn that the Force of Gravity, or the Weight of Bodies, is proportionable to the Quantity of Matter in those Bodies, and the Principle whereby, thro' the means of Engines, great Weights are raised. Next, I have made manifest the Laws of Motion, and the Cause of the Acceleration of heavy Bodies depending on them; and by what Proportion the Spaces passed over by heavy Bodies, increase or decrease according to the different Intervals of the Times. To these succeed the Rules of Congress, as well in hard as elastick Bodies, and the Manner whereby the Magnitude of a Stroke may be estimated: to which I have adjoined the Compositions and Resolutions of Motions, and some other Theorems, of no small use in Philosophy. And that the Philosophers may further perceive, how far even the Use of the Elementary Geometry extends*

extends itself in the Knowledge of Natural Things, I have demonstrated from the Elements the very beautiful Theorems of Monf. Huygens, *concerning the Centrifugal Force and Circular Motion.*

LEC-

LECTURE I.

Of the Method of Philoſophizing.

AS it is our Purpoſe in theſe Lectures to endeavour the Explication of the Properties and Affections of Bodies, we have thought it not improper, in the firſt place, to ſay ſomething concerning the Sects, and Principles, and Methods of the Philoſophers; and at the ſame time, to declare our own Manner of Proceeding, in the inveſtigating the Cauſes of Natural Things.

AMONGST the various Sects of Philoſophers that have wrote on Phyſical Subjects, there ſeems to have been four of the greateſt Eminence. The firſt were thoſe who attempted to illuſtrate, (ſhall I ſay?) or rather conceal, the Natures of Things, by the Properties of Numbers and Geometrical Figures. Such were the *Pythagoreans* and the *Platoniſts*; for theſe not being willing that their Opinions ſhould be expos'd to the common People, did therefore, as it were, caſt a Vail over them, by the means of Images and

Hieroglyphicks, that they borrowed from Geometry and Arithmetick: nor indeed did they admit their own Difciples to a thorough Acquaintance with their real Opinions and the greateft Depths of their Philofophy, until they had firft undergone many Years of Probation and Trial. Tho this Method feems extremely well calculated to keep up the Dignity of their Profeffion, yet it has had a very ill effect in refpect to us, the Succeffors of thefe Philofophers. For by this means their Opinions have been handed down to us with fo much Obfcurity and Darknefs, that it is almoft impoffible to difcern, thro the Difguifes that they wrapped them up in, what were their true Sentiments concerning the Natures of Things. But however, notwithftanding the Philofophy of this Sect appears very myfterious to us, yet we may be certain, from their Manner of delivering it, that they looked upon Geometry and Arithmetick to be abfolutely neceffary to folve the Phenomena of Nature, and that they ufed them to that very purpofe.

The fecond fort were the *Peripateticks*. This Sect explained their Philofophy by Matter and Forms, Privations, Elementary Virtues, occult Qualities, Sympathies and Antipathies, Faculties, Attractions, and the like. But thefe Philofophers, in my opinion, feem not fo much to have difcovered the Caufes of Things, as to have given proper Names to the Things themfelves, and to have invented fuch Terms, as are very fit to exprefs natural Actions.

The third Order of Philofophers are fuch as proceed upon *Experiments*: and thefe make it their fole bufinefs, that the Properties and Actions of all Bodies may be manifefted to us, by the means of our Senfes. And indeed Philofophy has received very confiderable Advantages from their Labours; and it might thereby have been ftill farther improved, if the

Expe-

Experimenters themselves had not too often distorted their Experiments and Observations, in order to favour some darling Theories they had espoused.

THE last sort of Philosophers are such as are wont to be called *Mechanical.* They that lift themselves under this Banner, imagine they can explain all the Phenomena of Nature by Matter and Motion, by the Figure and Texture of the Parts, by subtle Particles, and the Actions of Effluvia; and they likewise contend, that these Operations are brought about by the known and establish'd Laws of Mechanicks.

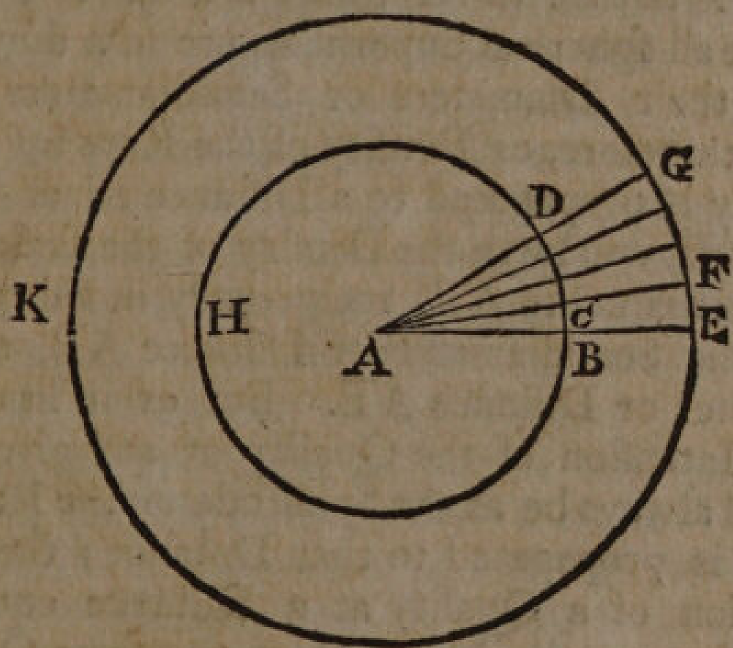
AMONGST these various ways of Philosophizing, as there is no particular one, wherein we do intirely acquiesce; so in each, there are some things which we can approve of. Wherefore we shall chuse out of all of them what may be thought useful, and thence compose the Method we shall here follow.

AND first of all, in imitation of the antient *Pythagoreans* and *Platonists*, we shall call in to our assistance Arithmetick and Geometry, as Arts very necessary to Philosophy, and without which, but little of Certainty can be ever discovered in natural Causes. For since every Physical Action depends upon Motion, or at least it cannot be performed without Motion; the Quantity and Proportion of Motion, the Magnitude, Figures, Number, and Collisions of Bodies in motion, and their Forces necessary to move other Bodies, ought to be investigated. But it is impossible to determine all these, without being instructed in the Nature of Quantity and Proportion: and therefore there is occasion for those Arts, that demonstrate their Properties; and consequently Geometry and Arithmetick must be thought necessary to any one, that would make any progress in Natural Philosophy.

SECONDLY, We ſhall not be aſhamed to uſe, with the Peripateticks, the Terms *Quality*, *Faculty*, *Attraction*, and the like; not that by theſe words we pretend to define the true and phyſical Cauſe and Modus of Action, but as theſe Actions may be augmented and diminiſhed, and therefore ſince they have the Properties of Qualities, the ſame Name may not unfitly be applied to them, ſo that we thereby only mean to expreſs the Ratios of the Forces or their Augmentation and Diminution. For example, we may ſay that Gravity is a Quality, whereby all Bodies are carried downwards, whether its Cauſe ariſes from the Virtue of the central Body, or is innate to Matter itſelf; or whether it proceeds from the Action of the Æther agitated by a centrifugal Force, and ſo tending upwards; or laſtly, whether it is produced after any other manner whatever. So likewiſe we may call the Endeavour of Bodies to approach one another, Attraction; by which word we do not mean to determine the Cauſe of that Action, whether it proceeds from the Action of the Bodies tending mutually to one another, or from their being agitated by Effluvia emitted, or from the Action of the Æther, or Air, or any Medium whatever, that impels the Bodies, which float in it, towards one another: ſuch Actions, I ſay, we may expreſs by theſe Words. And if the true Cauſes are hid from us, why may we not call them occult Qualities? Certainly by the ſame right, as in Algebraical Equations we denote the unknown Quantities by the Letter x or y; and not by a very unlike Method we may inveſtigate the Intenſions and Remiſſions of theſe Qualities, which follow from ſome certain ſuppoſed Conditions. We will illuſtrate this by an Example.

HOWEVER ignorant we are of the Nature of Qualities, and how much ſoever the Modus of Operation is concealed from us, we are able notwithſtanding to demonſtrate the following Theorem concerning their Intenſion and Remiſſion. Every Quality or Virtue, that is propagated every way in right Lines from a Center, is diminiſhed in a duplicate Proportion of the Diſtance from that Center.

LET A be a Point or Center, whence every way the Quality is diffuſed, in the right Lines A B, A C, A D, and innumerable others ſpread indefinitely through the whole Space. I ſay, that the Intenſion of the Quality decreaſes in a Ratio duplicate of that, whereby the Diſtances increaſe; or, which is the ſame thing, its Intenſion at a Diſtance equal to A B is to its Intenſion at a Diſtance equal to the right Line A E, reciprocally in a duplicate Ratio of the Diſtance A B, to the Diſtance A E; that is, directly as the Square of A E to the Square of A B.



Since, from the Hypotheſis, the Quality is propagated in a Sphere every way by right Lines, its Intenſion at any diſtance from the Center, will be proportionable to the Spiſſitude or Denſity of the Rays at the ſame diſtance. By Rays I here mean the rectilineal Ways by which the Quality is diffuſed: Now the Rays that at the Diſtance A B are diffuſed thro

Intensions of Light, Heat, Cold, Perfumes, and the like Qualities, will be reciprocally as the Squares of their Distances from the Point whence they proceed. Hence also may be compared amongst themselves, the Actions of the Sun on different Planets: but this is not the Business of our present Design.

After the Ratios of the Forces are discovered in given Conditions or Suppositions, then those Ratios are to be compared with the Phenomena of Nature, that there may be found out, what Conditions of the Forces belong to every sort of Bodies. But in order to obtain this, many of the Experiments that the third Sect of Philosophers have delivered down to us, must be made use of: tho this ought not to be done without great Caution; for we are well apprised how fond these Gentlemen are of their Theories, how willing they are that they should be true, and how easily they deceive both others and themselves, in trying their Experiments. Such therefore as are produced by all, and which succeed upon every trial, we receive as undoubted Principles or Axioms: as likewise we ought sooner to give credit to those Experiments that are more simple and easy to be shewn, than to those that are more compounded, and difficult to be performed.

Lastly, We ought to inquire with the antient Atomists, and the Followers of the new Philosophy, what are the Phenomena that may be explained by Matter and Motion, and the known and established Laws of Mechanicks.

But that we may proceed in this Affair with the greater safety, and, as much as possible, avoid all Errors; we shall endeavour to observe the following Rules.

First, We shall, after the Method of the Geometers, premise such Definitions, as are necessary to arrive at the Knowledge of Things. It is not to be expected that these ſhould be Logical Definitions, which conſiſt of the *Genus* and *Difference*; or ſuch as diſcover the intimate Eſſence and ultimate Cauſe of the thing defined: theſe Matters I leave to be diſputed by others; for ingenuouſly to confeſs my own Ignorance, the intimate Natures and Cauſes of Things are not known to me. And whatever acquaintance I have with Bodies or their Actions, I obtained it either by the help of my Senſes, or elſe deduced it from ſome of their Properties, which Properties were diſcovered by the ſame means. I ſhall therefore, inſtead of ſuch a Definition as the Logicians are wont to give, exhibite a Deſcription; whereby the thing deſcribed may be clearly and diſtinctly conceived, and likewiſe be diſtinguiſhed from every thing elſe. Wherefore we ſhall define all things by their Properties, chuſing out one or more of the ſimpleſt, which by experience we are certain do really belong to the things themſelves; and then from theſe, we ſhall after a Geometrical manner deduce other Properties of the ſame things. Many modern Philoſophers have offended againſt this Rule, who define things not by any Properties which do certainly belong to them, but by Eſſences and Natures which they ſuppoſe to be in the things themſelves. They ſuppoſe it indeed, but at the ſame time it does not at all appear, whether ſuch Natures as they define, are really in the things themſelves. The *Carteſians*, for example, ſay that a Fluid has its Parts in continual motion, but it does not appear from our Senſes, or Experience, or Reaſon, that ſuch is the Nature of a Fluid: nay, that very Argument which they produce to confirm this their Hypotheſis, we ſhall overthrow by a Geometrical Demonſtration. For they will have it, that the Reſiſtance of a Body moving in a Fluid, is leſs, if the Parts of that Fluid are agitated by an inteſtine Motion, than if there

was

was no ſuch Motion in the Fluid; the contrary to which, we ſhall demonſtrate, when we come to treat of the Reſiſtance of Fluids.

But the Writers of the Mathematical Philoſophy have followed a much better courſe, who take their Definition of a Fluid from its moſt obvious Property. A Fluid, they ſay, is a Body, whoſe Parts yield to the ſmalleſt Force or Impreſſion, and by receding, are eaſily moved in reſpect to each other. From which Definition they deduce many beautiful Theorems that are of ſervice in the Occaſions of Life; whereas the *Carteſian* Philoſophers from theirs have produced nothing either certain or ſolid, much leſs uſeful.

Secondly, While we are inveſtigating natural Truths, it will be of advantage to conſider only the Conditions that were ſuppoſed at firſt, abſtracting for a time from all other Conſiderations whatever. For as the human Mind is finite, ſo if it be diſturbed by too great a number of things at once, it will by that means be rendered incapable of making any Diſcoveries. The Mechanical Writers have obſerved this Rule, in their comparing the Spaces paſſed over by two Bodies in motion: for they look upon the Bodies in this caſe as Points, abſtracting from the Conſideration of their Magnitude, Figure, and Colour, which make no alteration at all in reſpect to the Length they have moved.

Thirdly, It is neceſſary to begin with the moſt ſimple Caſes firſt; and having once ſettled them, we may thence advance to ſuch as are more compounded. So the ſame Mechanical Philoſophers at firſt ſuppoſe the Motion of Bodies to be *in vacuo*, or in a Medium that has no Reſiſtance; and having determined the Laws of Motion in that caſe, they thence proceed to inveſtigate the Laws of Reſiſtance, and laſtly to diſcover what Changes are thereby likely to ariſe to

Bodies

Bodies in motion. For the lefs the Bodies are refifted by the Medium they move in, the lefs will the Laws of the Motion of Bodies in that Medium differ from the Laws before found out. So likewife in Hydroftaticks, we confider the Fluid as having no Tenacity or Coherence of its Parts, but that they are feparable by the leaft Force whatfoever: and it is on this fuppofition, that we determine the Pofitions of Bodies immerfed in fuch a Fluid, and the Preffures of the Fluid on fuch Bodies. Tho perhaps there is not in Nature fuch a Fluid, as has no Cohefion of its Parts; wherefore the Variation or Difagreement in the Laws of a Fluid whofe Parts do cohere, from thofe of a Fluid which we fuppofe to have no Cohefion in its Parts, ought to be inveftigated; and if that Tenacity or Coherence of Parts in the Fluid be but fmall, the Difagreement from the Laws firft difcovered, will be but fmall, and fcarce difcernible.

Most of the Theorifts offend againft this Rule; who neglecting, or not thoroughly underftanding the firft and more fimple Principles of the mechanical Philofophy, at the very firft ftroke attempt the moft difficult Problems, and rafhly enough endeavour to fhew how a World, a Planet, or an Animal might be formed. Thefe are not unlike fome young Beginners in Geometry, who before they are well acquainted with the Elements of that Science, prefently fet upon the Quadrature of the Circle, the Trifection of an Angle by the means of right Lines and Circles, the Duplication of the Cube, and the like: fo our Theorifts, not having laid a good Foundation, raife up but a weak Superftructure; whence it is not to be wondered, if their great Building immediately finks, not without the difgrace of the Architects. But thofe who philofophize aright, ought to take another courfe, and proceed in a quite different Method; and tho they do not pretend to

form

form a World, an Earth, or a Planet, yet they may be able to lay down a ſure Foundation, and firm Principles of the Mechanical Philoſophy, and explain the Phenomena, which are thence to be deduced.

LECTURE II.

Of the Solidity and Extenſion of Bodies.

WE ſhall not here give a Definition of Body, taken from its intimate Nature or Eſſence, wherewith we are not perfectly acquainted, and perhaps never ſhall be: but according to a Rule laid down in the former Lecture, we ſhall define it by ſome of its Proprieties, that diſtinguiſh it from every other Being whatever. We therefore ſay, That a Body is, *what is extended, ſolid, and capable of Motion.*

THERE is no one, I ſuppoſe, ſo ſlow of Apprehenſion, as not readily to perceive, that every finite Body has Bounds or Terminations, which Terminations we call Superficies, and of theſe ſome one is diſtant from its Oppoſite; and again this Superficies (ſince it is not infinite) has Extremes, which we call Lines, and betwixt which there muſt of neceſſity be ſome diſtance. And moreover there are Terminations of theſe Lines, which we name Points, betwixt which alſo there is an Interval. From all theſe Diſtances taken together, we acquire a diſtinct Idea of Extenſion into its triple Dimenſion. For the Diſtance betwixt two oppoſite Superficies of the ſame Body, is called its Thickneſs or Depth; the Diſtance betwixt two oppoſite Lines of the ſame Superficies, is ſaid to be its Breadth; and the Diſtance betwixt both the

the Extremes of a Line, is named its Length. There is no Body that does not partake of this triple Dimenſion, and however ſmall we ſuppoſe it to be, it is neceſſary that it ſhould have Length, Breadth, and Thickneſs: and that which is in a Body, and is deſtitute of all theſe, is not a Body, but a Point; neither is it a Magnitude, but the Beginning or Termination of a Magnitude.

Solidity is that Property of a Body, whereby it reſiſts all other Bodies that preſs it on every ſide; and whereby, whilſt it poſſeſſes any place, it hinders all other Bodies from entring into that place, tho they are forced againſt it ever ſo violently. So, for example, if a Body is held betwixt one's hands, tho it is preſſed with ever ſo great a force, it prevents one's hands from coming to a mutual Contact.

This is that Property, which moſt of the Peripateticks are wont to call Impenetrability; that is, whereby two Bodies are not ſuffered to be in the ſame place at once, or to penetrate each other: tho I rather chuſe, with an illuſtrious Philoſopher of our Age, to call it Solidity. This Property ſeems to be ſo eſſential to all Bodies, that there is no other Being in Nature to which it can belong: for tho there are other ſorts of Magnitudes, corporeal Magnitude alone admits of Solidity; all other Quantities, or indeed ſuch as are no Quantities, as Points, can penetrate, unite, and be in the ſame place. For if two Spheres meet each other, in the place of Concurſe there will be a Point of one united with a Point of the other, or they will agree, or will be in the very ſame Point of Space: So if two Cubes are equal, the one may be ſo placed upon the other, that two of their ſquare Superficies may agree; namely, the Sides of one of the Squares may coincide with the Sides of the other Square; and the Angles of one be united with the Angles of the other,

whence

whence these Quantities mutually penetrate each other, and are in the same place, which is impossible to happen in Bodies.

You may easily perceive, that we use the word *Solidity* in a sense very different from that of the Geometers, who suppose that Solids may mutually penetrate each other. As, for example, when *Euclid*, in the eleventh Book of his Elements, demonstrates the Equality of two Parallelepipedons, placed upon the same Base, and betwixt the same parallel Planes; it follows, that two different Parallelepipedons so placed must necessarily penetrate each other: whence it is manifest, the Geometers suppose their Solids penetrable. The Geometers therefore take the word *Solidity* in a sense different from that of the Philosophers, neither do they oppose their Solids to penetrable Magnitudes, but to Planes, or Superficies, to plane Angles, and Lines; for every thing is reckoned solid by them, that has three Dimensions.

But the Solidity of Bodies is of another sort; which as we have said that it belongs to Bodies alone, so it is common to every Species of Bodies, whether they are fluid, or hard and fix'd, or easily moved and yielding to any Force, or very heavy, or whether they have but little weight, or are absolutely light, if such Bodies can be in Nature; for a Drop of Water or a Particle of Air remaining fixed betwixt any two Bodies, does not less hinder the Contact of those two Bodies, than the hardest Metal or a Diamond would do.

Lastly, By this Property, Body is distinguished from another kind of Extension, which we conceive to be penetrable, which we call Space, and wherein we behold all Bodies to be placed and moved, at the same time regarding that as immoveable.

The *Cartesians*, who define Body by its Nature, which they make to consist in Extension alone, allow of of no Space or Extenſion that is not corporeal; but ſince we have, or at leaſt imagine we have, an Idea of Space diſtinct from the Idea of Body; they certainly offend againſt the Laws of good Method, who place the Nature or intimate Eſſence of Body in any of its Attributes, which, we are not ſure, do only belong to it.

But the *Carteſians* ſay, that the Nature of Body cannot conſiſt in any other of its Attributes, ſince neither Hardneſs, or Colour, or Weight, or Figure, or Taſte, or any of its ſenſible Qualities, can conſtitute its Eſſence. For all theſe Attributes may be taken from the Body, and yet the Nature of Body will remain; but take away Extenſion, and the corporeal Being is immediately deſtroyed: therefore it is neceſſary that the Nature of Body ſhould be placed in Extenſion alone.

This is the Argument of *Cartes* himſelf, tho it is unworthy a Philoſopher; for nothing can thence be deduced, unleſs this, that thoſe ſenſible Qualities that he mentions, are not of the Eſſence of Body, and that Extenſion is an Attribute neceſſary and eſſential to it. But what then? Cannot the ſame univerſal Attribute agree to two different Species of things? Is it neceſſary that all things that have the ſame Attribute, muſt have the ſame Nature, and Eſſence? If this be ſo, there will be no Diſtinction, no Diverſity in things. Tho therefore Space and Body have one and the ſame eſſential Attribute common to them both, yet they are very different things; and there are other eſſential Attributes, peculiar to each, whereby they are ſufficiently diſtinguiſhed.

In the firſt place, the above-deſcribed Solidity is proper to Bodies only, and ſo eſſential to all of them, that you cannot ſo much as ſeparate it from them in your Imagination, but at the very ſame time you deſtroy that very Idea which you had formed of Body:

Body: wherefore if the Essence and intimate Nature of Body is to be placed in some one Attribute, Solidity certainly has a much better pretence to be that Attribute than Extension; especially when there seems to be a Being different from Body, which we call Space, to which likewise Extension appertains: at least, the contrary does not yet appear.

Besides that we have the Idea of this Space altogether distinct from the Idea of Body, each of them seems to be endued with Attributes, not only distinct and proper to themselves, but so contrary, that it is impossible to imagine they could be inherent in the same Subject. For we conceive Body as solid or impenetrable, divisible and capable of Motion, whose Parts may be easily disunited, separated and removed from one another to any distance whatever: one Body may oppose another Body that is moving, it may stop, or at least diminish its Motion; a Body likewise may communicate its Motion to another Body that is at rest, or moves with a less force towards the same or opposite Parts, and carry it along with it.

On the contrary, we conceive Space to be *that*, wherein all Bodies are placed, or, to speak with the Schools, have their *Ubi*; that it is altogether penetrable, receiving all Bodies into itself, and refusing an Ingress to nothing whatsoever; that it is immoveably fixed, capable of no Action, Form or Quality; whose Parts it is impossible to separate from each other, by any Force however great; but the Space itself remaining immoveable, receives the Successions of things in motion, determines the Velocities of their Motions, and measures the Distances of the things themselves. These so disagreeing and repugnant Attributes of Space and Body, it is impossible should belong to the same Subject.

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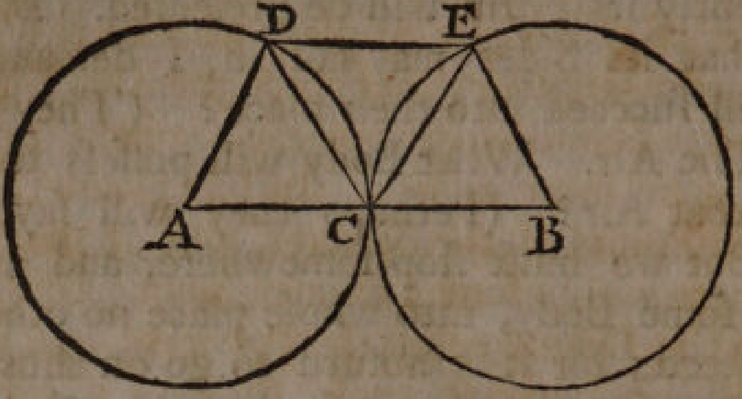
The *Cartesians* perhaps may reply, That this Idea, which we have here given of Space diſtinct from Body, is altogether imaginary and chimerical, the like to which, may they ſay, cannot exiſt in the Nature of things, by any Power whatſoever. But we are ready to demonſtrate againſt the *Cartesians*, that in truth there is given a Space diſtinct from Body; or, in other words, that Space and Body are not the ſame things: but we ſhall firſt of all obſerve, that we do not now intend to prove the real Exiſtence of Space void of all Body, for that we ſhall do in another Lecture; we ſhall at preſent be content, only to ſhew the Poſſibility of it.

Let us ſuppoſe then any Veſſel, and let it at firſt be filled with Air; then let the Air contained in this Veſſel be exhauſted, or, if you will, annihilated by a Divine Power, and let all other Bodies whatever be hindred from entring into its place: I would now ask, if in this caſe there is not given a Space void of all Bodies? All that Body which was within the Veſſel is deſtroyed, the Ingreſs of any other Body is prevented, and the Form of the Veſſel is ſuppoſed to be preſerved; it ſeems therefore to be neceſſary that a Vacuum, or a Space not replete with Body, is given. The *Cartesians* may anſwer, That on theſe Suppoſitions the Sides of the Veſſel muſt fall in, and neceſſarily approach one another. But ſince, according to the *Cartesians* themſelves, no Body can move of itſelf; and ſince, by Hypotheſis, there is no other Body that forces the Sides of the Veſſel towards one another; it therefore follows, that the Sides will not approach one another. They will ſay, perhaps, that the Air diffuſed every way, and preſſing on all hands the Sides of the Veſſel, will cauſe that Motion. But ſince the Preſſure of the Air is of a finite Force, the Strength of the Veſſel may be ſufficient to balance that Preſſure, and conſe-

consequently its Form will be preserved. But let us grant that its Sides may fall in, I demand, what Body will succeed into their place? (They will answer,) The Air. What Body will possess the Space left by that Air? (Perhaps they will say,) More Air? But we must stop somewhere, and at length come to some Body, into whose place no other Body will succeed; for it is absurd to go on thus *ad infinitum:* therefore in that case, it is necessary to suppose a *Vacuum.*

But we shall also shew, by an invincible Demonstration taken from Geometry, at least the possible Existence of a Space void of all Body: towards which Proof, we shall premise the two following Assertions, or Axioms, whose Evidence no Philosopher will call in question. The first is, That no Body or Portion of Matter stands in need of the Existence of another Body, in order to its own Existence. As, for example, a Sphere may exist, whether any other Body exists or not: this evidently follows from the Nature of Substance. Secondly, Any Body, especially if it be hard, can preserve its Figure, if there are no external Bodies or no Agents, which endeavour to induce an Alteration in it. Certainly it must be confessed, that God can preserve any Body in the same State and Position; and whatever may happen externally, he can notwithstanding continue the Figure of that Body immutable.

Since therefore one or two Spheres may exist, without the Existence of any other Bodies; let us suppose all other Bodies, except two Spheres, to be annihilated by God; or rather let us suppose all the Matter in the Universe to be amassed into two Spheres, which may be represented by two Circles, whose Centers let be A and B: and since no other Body is supposed to exist, those spherical Bodies may preserve their Figure, there being no other external



Body either to deſtroy or alter it. Theſe two Spheres then are either contiguous or diſunited; if they are disjoined, there will be an intermediate Space replete with no Body, and therefore all Space is not Body: But if theſe Spheres touch one another, it is neceſſary that they touch one another in one Point only, by the Elements of Geometry: therefore betwixt the other Points of the Spheres there is ſome Diſtance, that is, ſome Space will lie between. For out of the Point of Contact let there be taken any two Points, as D and E; if betwixt theſe Points there is no Space, that is, no Diſtance, theſe Spheres will touch one another in theſe Points, which is impoſſible.

Again, That there is a Space void of all Body, may be thus ſhewn. Let us ſuppoſe two Spheres, in which all the Matter of the Univerſe is accumulated, and that theſe two Spheres are equal; in both which let be accommodated the right Lines CD, CE, each equal to the Semidiameter of either Sphere, join DE; this right Line will be equal to the Semidiameter of one of the Spheres. For, draw AD, BE, and becauſe in the equilateral Triangles ACD, BCE, the Angles ACD, BCE, are each the third part of two right Angles, the Angle DCE will be alſo equal to the third part of two right Angles; for all the Angles at the Point C, make two right ones: whence ſince DC, CE, are equal, the Angles CDE and CED will be likewiſe equal; but taken together, they make two Thirds of two right Angles: wherefore each is one third

third part of two right Angles, whence the Triangle DCE is equi-angular; and consequently, DE will be equal to the Semidiameter of each Sphere, nor in this case can it be more or less. In like manner, betwixt any other Points of the Spheres, out of that of Contact C, there will be some Distance, having a determinate Ratio to the Diameter of the Spheres; and therefore there will be betwixt these Spheres a certain and determinate Space, not replete with Matter: but into that Space may be admitted a Body, whose Dimensions agree with these Distances; but what has greater Dimensions, cannot by any Power whatever be placed within that Space. Whence, since these Properties demonstratively agree with the Space here spoken of, and that Space may really exist, tho no one should think at all, it evidently follows against the *Cartesians*, that the Idea we have of Space, is not chimerical or imaginary; for what is chimerical, can have no Existence out of the Intellect.

We conclude therefore, that there is in reality a Space distinct from all Body; which is as a universal Receptacle, wherein all Bodies are contained and moved. But what is the Nature of this Space, whether it is any thing positive, actually extended in itself, and endued with real Dimensions, or whether its Extension arises from the Relation of Bodies existing in it, so that it may be a mere Capacity, Ponibility, or Interponibility, as some love to express themselves, and to be reckoned in the same Class of Being with Mobility and Contiguity; or whether this our Space is the divine Immensity itself, which is through all and in all, or whether it is created or uncreated, finite or infinite, dependent or independent on God; these things we do not here inquire into, but leave them to be disputed by the Metaphysicians. It serves our turn to be able to explain some of its Properties, and to establish and demonstrate its Diſtinction and Nature to be different from that of Body: They who would have more, may conſult the Philoſophers.

LECTURE III.

Of the Diviſibility of Magnitude.

ALTHOUGH it may be demonſtrated by many Arguments, that Space is really diſtinct from Body, and we have already produced ſome, that ſeem to be unanſwerable; yet both theſe things agree in this, that Extenſion is a univerſal Attribute, neceſſarily and eſſentially belonging to both of them. Before therefore we proceed any farther, it may not be foreign to the purpoſe to explain ſome general Affection of Extenſion; as, for example, its Diviſibility.

THIS Property of Extenſion appertains and neceſſarily adheres to all Species of Magnitude, as well to Lines as Superficies, and as well to Space as to Body. By Diviſibility we would not here be underſtood to mean an actual Separation of Parts from one another, which ſuppoſes Motion, which indeed the Nature of Space does not admit, nor do the Demonſtrations borrowed from Geometry prove ſuch a Separation; but the Diviſibility that we here endeavour to evince, is only the Reſolution of any Magnitude into its Parts, or their Diſtinction and Aſſignment. As, for example, when *Euclid*, in the ninth Propoſition of his third Book, teaches how to cut a rectilineal Angle into two equal Parts, he does not in that Method undertake to ſhew, how one of the equal Parts being ſeparated from the other, recedes

cedes and is placed from it at a given distance; but only delivers a Method, whereby a Line may be drawn, dividing the Angle in such a manner into two other Angles, that the Angle which lies on one side of this Line, shall be equal to the Angle that lies on the other side the same Line. So likewise when, in the following Proposition, he teaches how to bisect any right Line, he only shews how to assign a middle Point, dividing the given right Line into two equal Parts, which Point is the common Termination of both the Parts; namely, where one of the equal Parts ends, and the other begins. This Resolution of Magnitude into its Parts, is so intimate and essential to it, as that which has no Parts, as, for instance, a Point, is not said to be a Magnitude, but the Beginning or End of Magnitude: nor can any Magnitude be produced by any Number of Points, tho infinite; for every Magnitude is not compounded of Points, but Parts, that is, other Magnitudes of the same kind, whereof every one is constituted of other Parts, and each of these is still made up of others, and so on *in infinitum*: nor can we ever arrive at a Magnitude so small, but it may be yet farther divided into Parts; nor is there given, in any Species of Magnitude, an absolute *Minimum*, but whatever is divided, is still farther divisible into Parts. This constant farther Resolution of Matter into Parts, is by the Philosophers called its *Divisibility in infinitum*; and that very truly, since there cannot be assigned any Quantity of Matter so minute, and any finite Number so great, but that the Number of Parts composing that Magnitude, that is, into which it may be resolved, shall be greater than that Number, how large soever it be: *for we call that Infinite, which exceeds any Finite.*

But because this infinite Divisibility of Matter can be demonstrated by Arguments taken from Geometry, and ſince there are now-a-days ſome Philoſophers who attempt to baniſh Geometry out of Phyſicks, by reaſon they are ignorant of that divine Science; and as theſe Gentlemen would be reckoned amongſt the moſt Learned, they leave no means untried, whereby, tho in vain, they may overturn the Force of theſe Demonſtrations: It will be therefore neceſſary, before we produce our Geometrical Arguments, to eſtabliſh their Strength, and to anſwer ſome Objections.

As amongſt the Philoſophers of this Claſs, the famous *John Baptiſt du Hamel*, the Author of the *Burgundian* Philoſophy, is of the greateſt Eminence; we ſhall produce his Opinion on this Subject. He ſays then, that Geometrical Hypotheſes are neither true nor poſſible, ſince neither Points, nor Lines, nor Superficies, as the Geometers conceive them, do truly exiſt in the Nature of Things; and therefore that the Demonſtrations that are produced from theſe, cannot be applied to things actually exiſting, when none of theſe exiſt any where but in our Ideas. He deſires therefore the Geometers to keep their Demonſtrations to themſelves, and not to make uſe of them in Philoſophy, becauſe, according to him, they ſpread over this Science not Light, but Darkneſs. I admire at the Unſkilfulneſs of this otherwiſe moſt Learned Perſon, in this Affair: he might certainly with the ſame Juſtice take away all Phyſical Suppoſitions whatſoever, ſince Geometrical Hypotheſes are equally certain and equally poſſible and real, as are thoſe, which he calls Phyſical. Certainly if Body exiſts, there muſt of neceſſity exiſt real Points, real Lines, and real Superficies, even ſuch as are conceived by the Geometers; as we can eaſily make appear. For if Body be given, that, ſince it is not infinite, has its Terminations; but the Terminations of Body are Superficies, and thoſe Terminations have no Depth: for if they had, they

they would thereby be Bodies, which Bodies would have ſtill other Terminations, which would be Superficies, and therefore there would be a Superficies of a Superficies. Either then this Superficies is deſtitute of all Depth, or not: if the firſt, we have what we require; if the latter, we come again to another Superficies, and ſo we ſhould proceed *in infinitum*, which is abſurd. Wherefore we muſt conclude, that thoſe Terminations are deprived of all Depth, and are therefore true Superficies, and as they are conceived by the Geometers without any Depth, or ſuch as have only Length and Breadth to conſtitute their Eſſence.

AGAIN, Since this Superficies is not infinite, it is likewiſe bounded by its Terminations; but thoſe Terminations are called Lines, which have really no Breadth: for otherwiſe they would be Superficies, and would have alſo their Terminations, which we ought to conceive at leaſt deſtitute of all Breadth; for, as we ſaid before, there cannot be given a Progreſſion *in infinitum*; whence there are really given Lines, which are only extended in Length, without any Breadth. After the ſame manner, Lines alſo have their Terminations, which are called Points, to which belong neither Length, nor Breadth, nor Depth. Wherefore if Body may be ſuppoſed to exiſt, it neceſſarily follows, that Geometrical Superficies, Lines, and Points, may be ſaid not only as poſſible, but alſo to be actually exiſting.

BUT it will be anſwered, That theſe Points, Lines, and Superficies are not material. What then? Who ever aſſerted that a Mathematical Point was Matter? Who ever fancied a material Superficies? If it was material, it would have its Superficies or Termination: but the Superficies of a Superficies, who ever imagined? However, tho neither Superficies, nor Lines, nor Points are real Matter; yet they exiſt or may exiſt in it, as its Modes, Terminations, or Accidents: juſt after the ſame manner, as Figure is not Body itſelf, but only its Affection, whereby it is contained under given Terminations, and this has real Properties wholly diſtinct from thoſe of Body.

AND again our ungeometrical Philoſophers may object, That there is not in Nature a perfectly plane Superficies, a Body perfectly ſpherical, ſuch as the Geometers feign, nor any Curve perfectly circular. But how come they to know all this? Have they ſeen all the Bodies that are in the Univerſe, and view'd them thro a Microſcope? Perhaps they will ſay, that a Superficies cannot be plane or ſpherical, becauſe in theſe Figures there is a Contradiction and an Impoſſibility. But we deſire they would be pleaſed to make out this Contradiction: All Bodies of neceſſity are terminated by ſome Figure, plane or ſpherical Superficies are the moſt ſimple and eaſieſt to be conceived of any; where then lies the Repugnancy, that it ſhould be impoſſible for a Body to be contained under theſe Superficies? I believe there is no one who is acquainted with the Elements of Geometry, but perceives more the Nature and Properties of theſe Figures, and knows more of their Affections, than all the Philoſophers of this ſort put together: but none of thoſe ever found any Repugnancy in theſe Figures; no Geometer ever ſuſpected theſe Contradictions in the Nature of Figures: on the contrary, ſo many beautiful Properties of thoſe Figures diſcovered and demonſtrated by the Geometers, evince their Poſſibility, for of an impoſſible thing there can be no real Property, no Demonſtration. It remains therefore, that they acknowledge theſe Figures as poſſible; and if they are poſſible, it is in the power of God to form out of Matter, Bodies having ſuch Superficies. Let us therefore ſuppoſe two

two Bodies, whereof one is terminated by Planes, the other by a ſpherical Superficies; if therefore the ſpherical Superficies is placed on the Plane, it will touch it; but it will touch it only in one, and that an indiviſible Point, or in a Point that has no Parts, by *Cor. Prop.* 2. *El.* 3. and therefore in that caſe there will be given a true Point. But farther, let us ſuppoſe this ſpherical Body to be moved on the plane Superficies, or to be carried along without any Rotation about its Axis, inſomuch that the Point touching the plane Superficies may be always found in the ſame Plane; then the Tract which that Point deſcribes by its Motion, will be a Mathematical Line, without any Breadth: and if the Diſtance betwixt any two Points in that Plane is the ſhorteſt, there will be generated by that Motion a right Line; but if otherwiſe, either a Curve, or one compounded of right Lines, or partly of theſe and partly of thoſe will be deſcribed. Points therefore, Lines, and Superficies, even ſuch as the Geometers conceive or feign, are poſſible; which was to be ſhewn. And their Poſſibility may be demonſtrated after innumerable other ways, but we are weary of dwelling on theſe Trifles. We ſhall only farther obſerve, that the Diſtance betwixt any two Points of two Bodies, will be their given and determinate Diſtance: As, for example, the determinate Diſtance betwixt the Centers of the Sun and a fixed Star, is that which is meaſured by a right Line lying betwixt thoſe two Points; which, of all the Lines that can be drawn from one Point to the other, will be the ſhorteſt, and paſſed over in the leaſt time with a given Velocity: this Diſtance, I ſay, will remain the ſame, whatever ſhall happen to be the Figure of the intermediate Body, whether it is bounded by Planes, or contained under ſpherical Superficies, or laſtly if there ſhould be no intermediate Body, and nothing but Space lie betwixt;

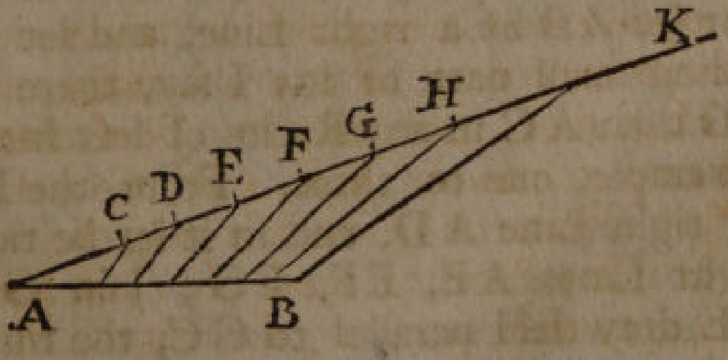
the Line will remain the ſame in Magnitude and Poſition, as long as the Centers of the Bodies remain unmoved.

Having now ſettled theſe Principles, we return to our Purpoſe; which was to demonſtrate that all Extenſion, whether corporeal or incorporeal, was diviſible *in infinitum*, or had an infinite Number of Parts; which we ſhall endeavour to prove by many invincible Arguments. Of which, this ſhall be the firſt: Let AB repreſent a right Line, I ſay it is diviſible into Parts exceeding any finite Number whatever.

Through A let be drawn any right Line AC, and parallel to it let be drawn through B the right Line BD, and in AC let there be taken any Point, as C: if therefore the right Line AB is not diviſible into an infinite Number of Parts, let it be diviſible only into a finite Number of Parts; and let that Number, for example, be ſix. In the Line BD on the ſide oppoſite to C, let there be taken any Number of Points exceeding ſix; for example, the Points E, F, G, H, I, K, L. and let there be drawn by the firſt Poſtulate of *Euclid*, CE, CF, CG, CH, CI, CK, CL. Theſe thus drawn, divide the right Line AB into as many Parts as there are right Lines; for if they do not, then ſome of the right Lines interſect AB in one and the ſame Point: but all of them interſect one another in the common Point C, whence ſome

ſome two right Lines will cut one another twice, or will have the ſame common Segment; both which is contrary to an Axiom in the *Elements*. A B is therefore divided into as many different Parts, as there are right Lines; but there are as many right Lines, as there were Points taken in the right Line B D: wherefore ſince there were taken more Points than ſix, the right Line A B is diviſible into more Parts than ſix. After the ſame manner, how great ſoever the Number aſſumed ſhall be, it may be ſhewn that the Line A B is diviſible into a Number of Parts greater than that Number; namely, by taking in the right Line B D a greater Number of Points, (which may be eaſily done, ſince no finite Number is ſo great, but a greater may be aſſumed, and that in any given Ratio of a greater Inequality) and by drawing right Lines from the Point C to the Points taken in the right Line B D: for theſe right Lines will divide the right Line A B into as many Parts, as there are right Lines, and therefore into more Parts than the Number firſt aſſumed (how great ſoever it was) contains Units; and conſequently the right Line A B is diviſible into more Parts than can be expreſſed by any finite Number, and therefore it is diviſible *in infinitum. Q. E. D.*

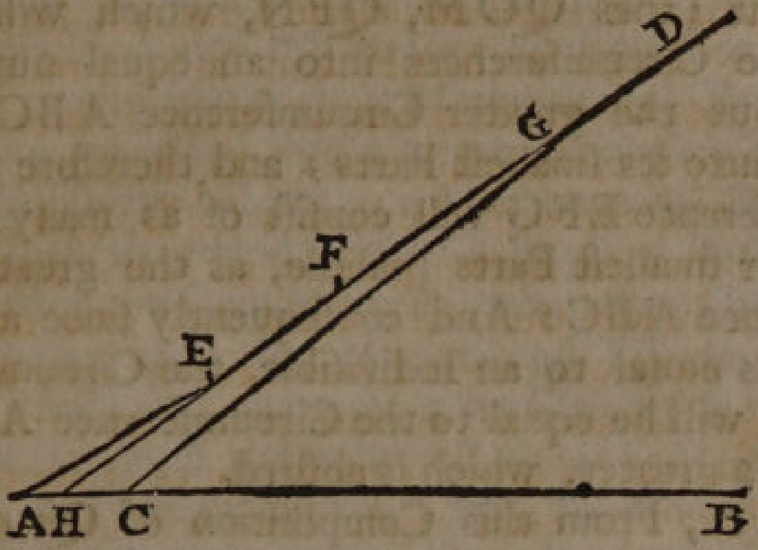
THE ſecond Argument. Let A B repreſent any right Line, I ſay it is diviſible into an infinite



Number of Parts: for if it is not diviſible into an infinite Number of Parts, let it be diviſible into a finite

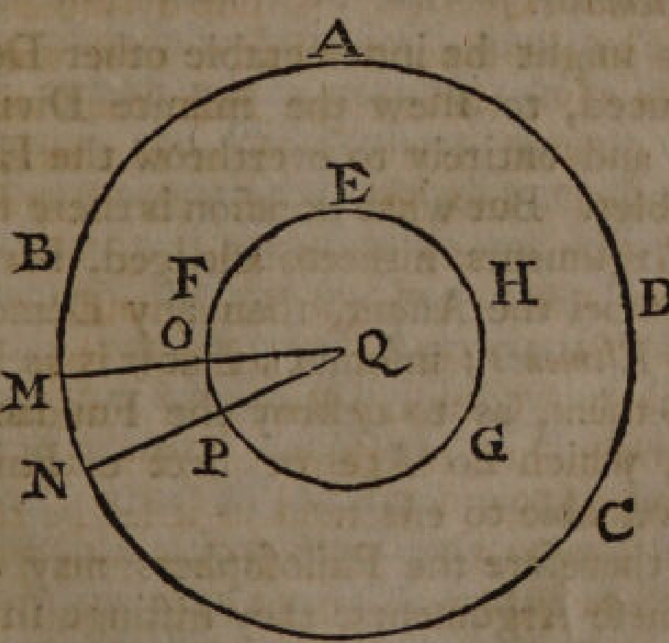
finite Number of Parts; and let that Number be, for example, five. Let any right Line AK be drawn, making any Angle with AB, and in it, produced if neceſſary, let there be taken as many Points as you pleaſe above five, which let be C, D, E, F, G, H, K; join KB, and through the Points C, D, E, F, G, H, let right Lines be drawn parallel to KB: theſe will neceſſarily divide the right Line AB into as many Parts as there are right Lines; for if they do not, more right Lines muſt concur in one and the ſame Point: but they cannot concur, ſince they are parallel; wherefore each right Line will interſect the right Line AB in a different Point, and all will divide the right Line AB into as many Parts, as there were right Lines drawn parallel. But there were more drawn than five, therefore the right Line AB will be divided into more Parts than five. And the ſame may be affirmed of any other Number. Wherefore no Number is ſo great, but the Number of Parts the right Line is diviſible into, is ſtill greater; and conſequently the right Line AB is diviſible *in infinitum.*

THIRDLY, If Quantity is not diviſible *in infinitum*, it muſt be diviſible into Parts, that are not farther diviſible; but there is no Part that cannot be ſtill farther divided, becauſe there can be given no Quantity ſo ſmall, but there may be ſtill taken a ſmaller, and that in any given Ratio of leſſer Inequality. For let AB be a right Line, and let AC be an exceeding ſmall part of it; I ſay, there may be a Line leſs than AC in any Ratio of leſs Inequality, as, for example, one to three. From the Point A draw any right Line AD, and in it let be taken the equal right Lines AE, EF, FG; join GC, and through E draw EH parallel to GC, the right Line AH will be a third part of AC: The Demonſtration thereof is manifeſt from *the ninth Propoſition of the ſixth*



sixth Book of the Elements. And therefore the right Line AC will not be the least that can be taken. The same may be demonstrated of any right Line whatsoever; consequently there is not in Nature an absolute *Minimum.*

Again, If Quantity was composed of Indivisibles, many Absurdities would thence follow: for let there be, for example, two concentrick Circles, ABCD, EFGH, and let the Circumference of the



greater be divided into its indivisible Parts, and let be drawn from the Center Q to each of these Parts, the

the right Lines QOM, QPN, which will divide both the Circumferences into an equal number of Parts, but the greater Circumference ABCD was divided into its ſmalleſt Parts; and therefore the leſs Circumference EFG will conſiſt of as many Indiviſibles, or ſmalleſt Parts poſſible, as the greater Circumference ABC: And conſequently ſince an Indiviſible is equal to an Indiviſible, the Circumference EFGH will be equal to the Circumference ABCD, a leſs to a greater, which is abſurd.

Lastly, From this Compoſition of Quantity of Indiviſibles, there can be no incommenſurate Magnitudes; which is contrary to what the Geometers frequently demonſtrate. For if all Magnitude conſiſted of Indiviſibles, an Indiviſible would be an adequate and common Meaſure of all Magnitudes of the ſame kind; for it would be exactly contained ſome number of times in all, and therefore all Magnitudes would have a common Meaſure, and the Side of a Square would be commenſurate to its Diagonal; contrary to *the laſt Propoſition of the tenth Book of* Euclid's *Elements.*

There might be innumerable other Demonſtrations produced, to ſhew the infinite Diviſibility of Quantity, and entirely to overthrow the Hypotheſis of Indiviſibles. But what occaſion is there for more? ſince the Arguments hitherto alledged, have not leſs force to compel the Aſſent, than any Demonſtration in *Euclid's Elements*; inſomuch that it is as impoſſible to weaken them, as to deſtroy the Fundamentals of Geometry, which no Age or Sect of Philoſophers has been ever able to effect.

That therefore the Philoſophers may avoid the Force of theſe Arguments, they diſtinguiſh betwixt a Mathematical and a Phyſical Body. Being compelled by the Force of Demonſtration, they readily allow a Mathematical Body may be diviſible *in infi-*

nitum;

nitum; but they deny that a Physical Body can be always resolved into still farther divisible Parts. But what, I would know, is a Mathematical Body, but something extended into a triple Dimension? Does not Indivisibility belong to a Mathematical Body, by reason it is extended? But a Physical Body is extended after the same manner: wherefore since Divisibility depends on the Nature and Essence of Extension itself, and owes to it its Origin, it is necessary that it must agree to all Extensions, whether Physical or Mathematical. For, to use a Logical Expression, whatever is predicated of any *Genus*, is predicated of all the *Species* contained under that *Genus*.

There is yet another Distinction amongst the Philosophers, not unlike the former; whereby they own that every Body is mathematically divisible *in infinitum*, but they deny that it is physically so. If these Words have any Meaning, it is certainly this: They acknowledge that a Body is mathematically, that is, really and demonstratively, divisible *in infinitum*; but they deny it to be so physically, or according to their false *Hypothesis*: and so they have a Distinction, against which nothing can be replied.

But because the Philosophers, against whom we dispute, are not acquainted enough with Geometrical Demonstrations, and therefore do not easily perceive their Evidence; before we end this Lecture, we shall produce one Physical Argument taken from Motion, for the infinite Divisibility of Quantity: namely, If Quantity consisted of Indivisibles, it would follow, that all Motion would be equally swift, nor would a slow Snail pass over a less Space in the same Time than the swift-footed *Achilles*. For let us suppose *Achilles* to run very swiftly, and the Snail to creep sluggishly along; if Extension consisted of Indivisibles, the Snail could not in any given

Time

Time pafs over lefs Space than *Achilles*: for if in a Moment's Time *Achilles* paffes over an indivifible Space, the Snail cannot in the fame Moment of Time pafs over lefs Space; by reafon, from the Hypothefis, there cannot be a lefs. For one Indivifible cannot be lefs than another, therefore it will pafs over an equal Space. The fame may be faid of any other Moment of Time: therefore the Spaces paffed over by them both will be equal; and confequently the fwift-footed *Achilles* cannot pafs over more Space than the floweft Snail: which is abfurd. Other Abfurdities of the like fort may be deduced from the fame Hypothefis of Indivifibles; but what we have already faid, is fufficient.

LECTURE IV.

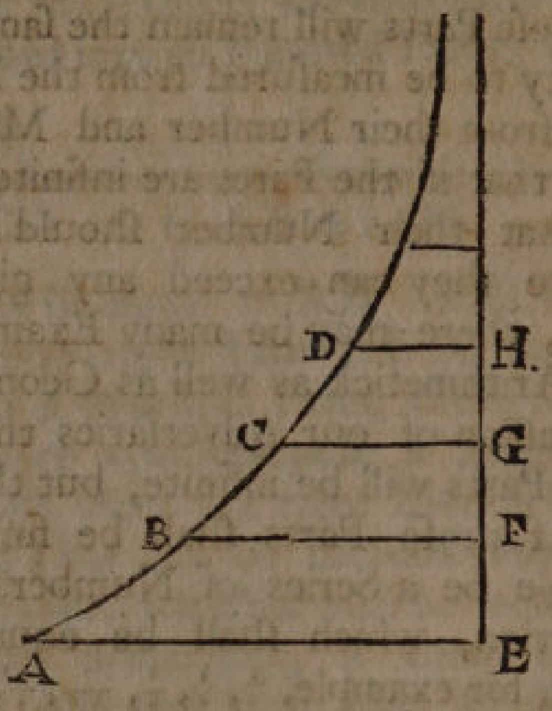
Wherein the Objections ufually brought againft the Divifibility of Matter, are anfwered.

WE have hitherto produced fuch Arguments, as by their help we have fufficiently demonftrated the continual Divifion of Matter into an infinite Number of Parts: it now remains, that we anfwer to the Objections or Cavils of the Philofophers. For there are not a few of them, who labouring under I know not what Obfcurity of Ideas, and not clearly enough perceiving the Evidence of the Arguments we have here made ufe of, produce Arguments of their own againft a thing fo manifeftly true, and to which they are alfo pleafed to give the Title of Demonftrations. But as I have perufed perused many of their Books, I never lit on any thing in all their Writings upon this head, that had so much as the very Appearance of Reason: they were so far from coming up to Demonstrations, that no Geometer, however clear-sighted he was, could ever perceive in them the least Shadow of a Demonstration. I acknowledge there is something in the Nature of Infinites, that seems not to be adequately comprehended by the human Intellect; and therefore it is no wonder, if some things follow from it, which the Mind of Man, involved in thick Darkness, is not able to conceive: and especially in our present Question, there are many things that may seem as Paradoxes, and incredible, to such Philosophers, who are less conversant in these Matters. However, nothing thence follows, that implies either a Contradiction, or is repugnant to any Axiom or Demonstration. But let us consider the Cavils produced by the Atomical Philosophers. The first is that of *Epicurus:* If Quantity was divisible *in infinitum*, it would contain an infinite Number of Parts, and so a Finite would contain an Infinite, which is absurd. But I desire they would explain their Terms, and tell us what they mean by these words, *An Infinite cannot be contained by a Finite.* If they say an infinite Magnitude cannot be contained in a finite Magnitude, I acknowledge it; but the contrary to this does not follow from our Doctrine, nor can they thence ever deduce it by a necessary Consequence. If they say that an infinite Number of Parts, and those infinitely small, cannot be contained in a finite Magnitude, this is the very thing they ought to make out. They would not, I suppose, have us believe their Assertions without any Proof; nor would they place amongst the Axioms, as a self-evident Proposition, such a one, as we have demonstrated by so many strong Reasons the contrary to be true. They may therefore urge, that an infinite Number of Parts muſt compoſe an infinite Magnitude: but this is again to beg the queſtion; for that is the thing itſelf about which we diſpute, *viz.* Whether a finite Magnitude can have an infinite Number of Parts? It is certain, whatever Number of Parts it has, whether finite or infinite, they are equal to their Whole; for as ten tenth Parts of an Unit make an Unit, an hundred hundredth Parts of an Unit taken together do likewiſe compoſe an Unit, and a thouſand thouſandth Parts collected in one Sum cannot be greater than the Whole; ſo likewiſe the infinite infiniteſimal Parts of any Magnitude are equal to that Magnitude. Or thus: Let the right Line AB be divided into an hundred Parts; all theſe taken

A B

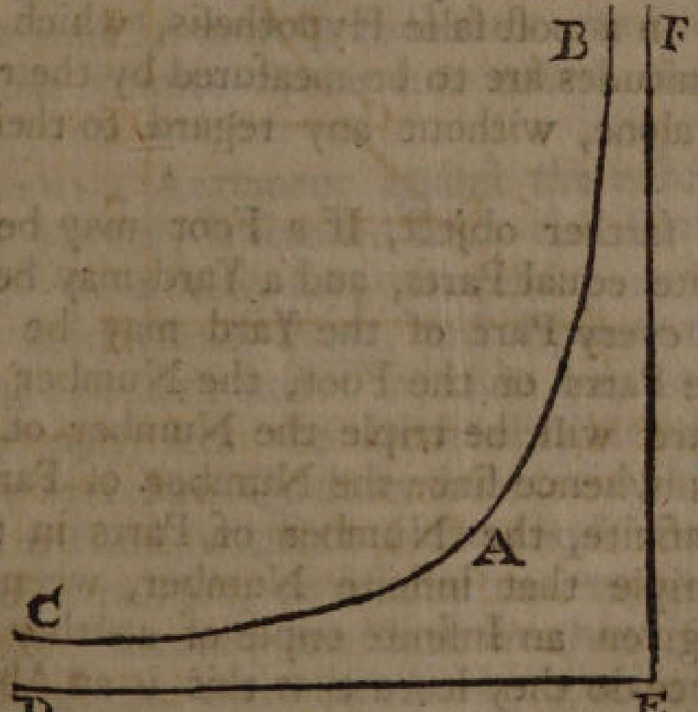
C— C:AB::1:N

together will be equal to AB. And after the ſame manner, if the right Line AB be ſuppoſed to be divided into a thouſand Parts, theſe thouſand Parts taken together will compoſe a Magnitude neither greater or leſs than the right Line AB. Or again, if the right Line AB be divided into a million of Parts, theſe taken together will be equal to AB the Whole. And univerſally, if there be taken two Magnitudes AB and C, ſo that C ſhall have the ſame Ratio to AB, as a Unit has to any Number N, the Quantity C multiplied by the Number N will be equal to AB. For ſince the Quantities C, AB, Unity and the Number N are Proportionals, the Extremes and the Means multiplied into each other will be equal; but ſince AB multiplied by a Unit is equal to AB, (for a Unit neither increaſes by Multiplication, or diminiſhes by Diviſion) the Quantity C multiplied by the Number N, will be equal to AB: therefore however great or ſmall the Number N is, is, this multiplying the Quantity C will always make a Product equal to AB, if the Quantity C be so taken, that it has the same proportion to AB, as Unity has to the Number N. And therefore if N is an infinite Number, and C the infinitesimal Part of the right Line AB, that is, the Quantity C has the same Ratio to AB, as a Unit has to an infinite Number N; also the Quantity C multiplied by an infinite Number N, that is, taken an infinite number of times, will be equal to the Quantity AB, nor as it can be greater, neither can it be less. If therefore the Magnitude of the Parts is diminished in the same Ratio as their Number is increased, the Whole made up of all these Parts will remain the same: nor ought any Quantity to be measured from the Number of its Parts, but from their Number and Magnitude conjointly; so that if the Parts are infinitely small, it is necessary that their Number should be infinitely great, before they can exceed any given Quantity. But besides, there may be many Examples brought both from Arithmetick as well as Geometry, where, by the confession of our Adversaries themselves, the Number of Parts will be infinite, but the Magnitude composed of those Parts shall be finite. Let the first Example be a Series of Numbers decreasing in any proportion, which shall be equal to a finite Number, as, for example, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, &c. the Sum of this Series continued *in infinitum* will be equal to a Unit; but since the Series is continued *in infinitum*, its Terms will be infinite in Number: wherefore in this case the Parts of a Quantity, that are infinite in Number, will make a finite Quantity. And in like manner the Sum of this Series, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, &c. when continued *in infinitum*, will be equal to half an Unit, as may be demonstrated by Arithmetick. But nobody will deny, that this Series being continued *in infinitum*, has an infinite Number of Parts; wherefore there may be an infinite Number of Parts of a Quantity, which however ſhall not exceed one half of an Unit. And in Geometry it is known that there may be given a Space infinitely long, which however ſhall be exactly equal to a finite Space; and this in an infinite number of Examples has been demonſtrated by the famous Geometers, *Torricellius*, *Walis*, *Barrow*, and others; from whom we ſhall produce a few Inſtances. And firſt of all, let the Curve ABCD be of ſuch a nature, that if there are taken in the Aſymptote EH the equal right Lines EF, FG, GH, or the right Lines



EF, EG, EH, are ſuppoſed in Arithmetical Proportion, and to the Points E, F, G, H, are applied the Ordinates AE, BF, CG, DH, which Ordinates let be in a Geometrical Proportion: the Curve ABCD is call'd the Logarithmetical Curve, and the endleſs Space contained betwixt the Aſymptote and the Curve infinitely produced, will be equal to a finite Space, as is demonſtrated by the Great Dr. *Barrow* in his *Geometrical Lectures*. Whence may be ſhewn the abovementioned Property of Numbers decreaſing in any Geo-

Geometrical Proportion. But that we may apply this to our present Purpose; nobody will deny that there are, in the boundless Space HGFE ABCD, which is infinitely long, an infinite Number of Parts, but the Geometers demonstrate all those Parts to be equal to a finite Space; wherefore there are some Parts of Space, tho infinite in Number, yet they do not make an infinite, but a finite Space. After the same manner, in all Hyperbolas, except that of *Apollonius*, the Area contained betwixt the Curve and the Asymptote infinitely produced, will be perfectly quadrable, and equal to a finite Area; but in all these Areas there are an infinite Number of Parts, wherefore an infinite Number of Parts will be equal to a finite Quantity. Besides, in the *Apollonian* Hyperbola CAB, although the indeterminable Area contained betwixt the Curve AB and its Asymptote EF produced *in infinitum*, is an infinite Area, or



greater than any finite one; yet if that infinite Area revolves about its Asymptote, it will thereby generate a Solid, or Body truly infinitely long, which however will be equal to a finite Solid or Body; as

 has

has been moſt elegantly demonſtrated by *Torricellius*, who called this Solid *Hyperbolicum acutum*: but in this Solid there are an infinite Number of Parts, ſince it is infinitely long; therefore the Parts of a Body infinite in Number, may compoſe a finite Body. We might produce innumerable other Examples of this, but we have perhaps beſtowed too much time in anſwering this Objection.

SECONDLY, The Atomiſts object, If all Quantity is diviſible *in infinitum*, any the leaſt Magnitude will be equal to the greateſt, ſince the leaſt has as many Parts as the greateſt. But what Conſequence, I pray, is this? Becauſe a Yard may be divided into an hundred Parts, and a Foot may be likewiſe divided into the ſame number of Parts, therefore does it follow that a Foot is equal to a Yard? But there can be nothing found more alike, than this Way of Argumentation and the preſent Objection; which is grounded on a moſt falſe Hypotheſis, which ſuppoſes that Magnitudes are to be meaſured by their number of Parts alone, without any regard to their Quantity.

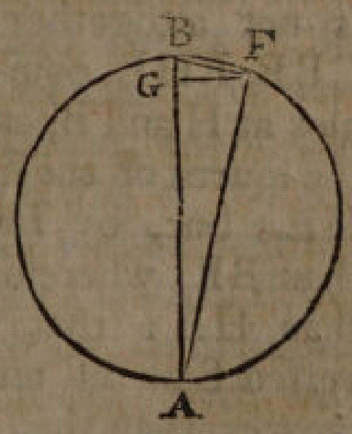
THEY farther object, If a Foot may be divided into infinite equal Parts, and a Yard may be ſo divided, that every Part of the Yard may be equal to one of the Parts of the Foot, the Number of Parts in the Yard will be triple the Number of Parts in the Foot; whence ſince the Number of Parts in the Foot is infinite, the Number of Parts in the Yard will be triple that infinite Number, whence there will be given an Infinite triple of another Infinite. But whence do they learn that this is an Abſurdity? Does it contradict any Axiom commonly received? Not at all, for there is no Axiom that ſuppoſes all Infinites equal. Nor is it contrary to the Nature of an Infinite, that there ſhould be another Infinite ſtill greater; for if there be given an Infinite, as, for exam-

the Divine Power extended thus far, then God could do ſomething that involved a Contradiction, or that was repugnant to his immutable Eſſence. But they farther urge, If all Quantity is diviſible *in infinitum*, and the Parts are actual in the Extenſion, there will be actually given a Part infinitely ſmall, and conſequently not farther diviſible. In the firſt place, I anſwer, It may be denied with *Ariſtotle* that the Parts are actually in the Extenſion, and then their Argument falls to the ground, which they boaſted of as an unanſwerable Demonſtration. Secondly, We grant them that theſe Parts are actually in the Extenſion; we grant that there are Parts infinitely ſmall and indiviſible; laſtly, we grant the Argument: and yet nothing thence follows againſt the continual and infinite Diviſibility of a Quantity that is not infinitely ſmall. This is ſuppoſed indeed in the Argument, but without any Proof. Does it, becauſe an infinite ſmall Part of any Extenſion is not farther diviſible, therefore follow, that a given Part, or one not infinitely ſmall, is alſo not farther diviſible? If any thing thence follows, it is, that all continued Quantity may be reſolved into infinitely ſmall Parts, and therefore it is infinitely diviſible. But the third and true Anſwer is, by denying that the Parts in the Extenſion are ſo minute or ſmall, that they can be farther diviſible; and tho there are given Parts infinitely ſmall, or ſuch as have the ſame proportion to their Whole, as a finite to an infinite Number, or a finite Space to an infinite one; yet we deny that theſe Parts are not farther diviſible: but ſince they are extended, they will be alſo diviſible, not only in two or three, or more Parts, but likewiſe every one may be divided *in infinitum*. The infinite Number of Parts of an infinitely ſmall Quantity, are wont to be called by the Geometers, Infiniteſimals of Infiniteſimals, or Fluxions of Fluxions; and theſe are uſed

by

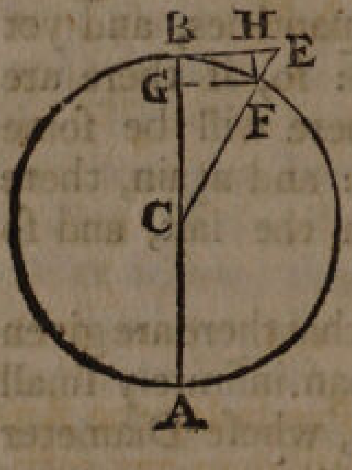
by them in the solving of many intricate Problems. Besides, there are given other Fluxions of these Fluxions, or Parts that are infinitely less than their Wholes; and again, there are other Parts of these Parts, and so on at pleasure. I do not deny, but, from the Weakness of the Human Understanding, this is very difficult to conceive: however, the Truth that is supported by such powerful Arguments is not to be deserted, especially since there are some things, which we do most certainly know, which yet are very difficultly received by our weak Understanding. We might produce many Arguments, but we shall only bring such as serve to illustrate our present Purpose; whereby we shall shew, that there are Quantities infinitely less than some Quantities, and yet also infinitely greater than others: so, if there are given infinitely small Quantities, there will be some Quantities infinitely less than these: and again, there may be others infinitely less than the last, and so always on *in infinitum.*

First then, we shall thus prove, that there are given Quantities that are infinitely less than infinitely small Quantities. Let ABF be a Circle, whose Diameter is AB; and let BF be an infinitely small Part of its Circumference, whose Chord therefore shall be also infinitely small, that is, the Chord BF shall have to any determinate Magnitude, as, for example, to AB the Diameter of the Circle, the same proportion, as any finite Magnitude has to an infinite one. Let be fallen from the Point F, on AB, the Perpendicular FG; BG will be infinitely less than the right Line BF. For let be drawn AF, and the Angle AFB in the Semicircle will be a right one. And therefore in the



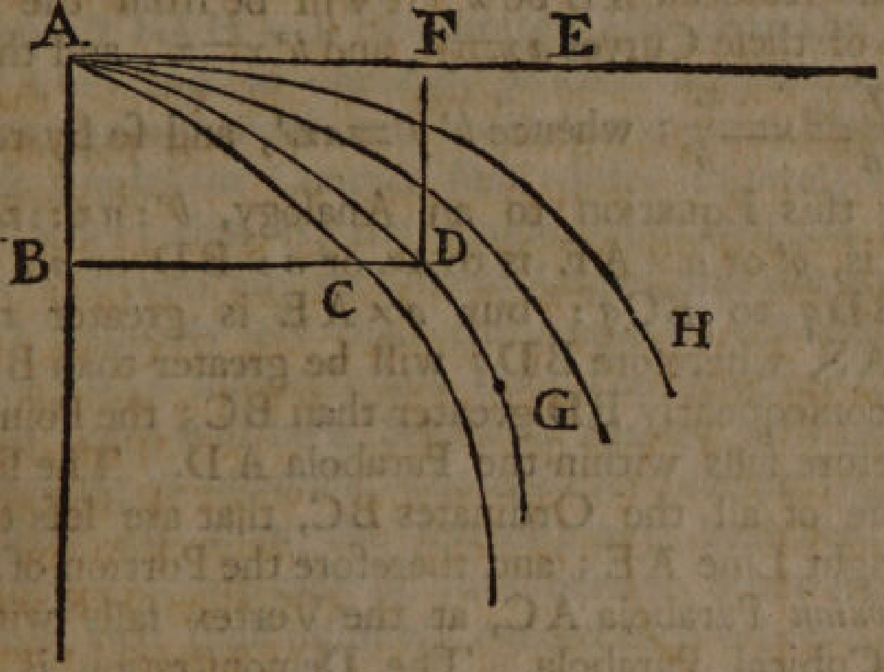
the Triangle AFB rectangular at F, from the Perpendicular FG being let fall on the Base AB, it will be, by the eighth of the sixth Book of *Euclid's* Elements, AB to BF as BF to BG; but, by Hypothesis, AB is infinitely greater than BF, wherefore BF will be infinitely greater than BG. There may therefore be a Quantity, which, although it is infinitely less than a given Quantity, it will be infinitely greater than another Quantity.

So likewise, it is known in the Circle, that as the Sine of any Arch is less, so the Tangent is greater than its correspondent Arch; and therefore the Tangent is greater than the Sine of the same Arch. Let therefore in the Circle, whose Center is C, and Diameter AB, BF be an infinitely small Arch, whose Tangent let be BE, right Sine GF, and versed one GB; through F draw FH parallel to AB, then HE will be equal to the Difference of the right Sine FG and the Tangent BE, which, from what we have shewn, is not altogether nothing.



Now in the Triangles CBE, FHE, equi-angular, by reason of the right Angles at H and B, and the common one E, it will be, by the fourth of the sixth of *Euclid*, CB to BE as FH to HE; but, by Hypothesis, CB is infinitely greater than BE, wherefore FH will be infinitely greater than HE: that is, in the present Case, BG the versed Sine of an infinitely small Arch, is infinitely greater than the Difference betwixt the right Sine and the Tangent of the same Arch. Since therefore CB is infinitely greater than BE, and BE, as we have before demonstrated, is infinitely greater than BG; and again, by what we have now shewn, BG is infinitely greater than HE; the Proposition is manifest.

To the farther Illustration of this Doctrine, we shall produce another Example, borrowed from the Scholium of the first Section of the Great Sir *Isaac Newton's Principia.* Let the Curve AC be the common Parabola, its Axis AB, and AE a Tangent at the Vertex A. The Writers of Conicks demonstrate,



that as in the Circle, so in the Parabola, the Angle of Contact FAC is infinitely less than any rectilineal Angle. To the Axis AB and Vertex A, let there be supposed to be described a Parabola of another kind, as, for example, the Cubical, whose Ordinates increase in a subtriplicate Ratio of the Abscessæ; the Angle of Contact FAD will be infinitely less than the Angle of Contact of the Parabola FAC; or, which is the same thing, there can be described no *Apollonian* Parabolas, or no Circles, with how great soever a Parameter, that will pass betwixt the Cubical Parabola and its Tangent at the Vertex: which may be thus easily demonstrated. Let the Parameter of the *Apollonian* Parabola AC be called a, the Parameter of the Cubical Parabola AD let be b; let there be taken in the Tangent the Point E in such a manner, that

that AE may be a third proportional to the right Lines a and b, that is, that $a \times AE = b^2$; through any Point F betwixt A and E let be drawn FD parallel to the Axis, meeting the Curve AD in D, and let BCD be drawn parallel to the Tangent, and let BD, an Ordinate to the Parabola AD, be called z; but BC, an Ordinate to the Parabola AC, y; and let the Abſceſſa AB be x: it will be from the Nature of theſe Curves $ax = y^2$, and $b^2x = z^3$, and therefore $\frac{y^2}{a} = x = \frac{z^3}{b^2}$; whence $b^2y^2 = az^3$, and ſo by reducing this Equation to an Analogy, $b^2 : az :: z^2 : y^2$ that is, b^2 or $a \times AE$ is to az or $a \times BD$ or $a \times AF$ as BDq to BCq: but $a \times AE$ is greater than $a \times AF$, wherefore BDq will be greater than BCq, and conſequently BD greater than BC; the Point C therefore falls within the Parabola AD. The ſame is true of all the Ordinates BC, that are leſs than the right Line AE; and therefore the Portion of the *Apollonian* Parabola AC, at the Vertex falls within the Cubical Parabola. The Demonſtration is the ſame for any other *Apollonian* Parabola; conſequently there can be drawn no Parabola, and therefore no Circle, (which is always of the ſame degree of Curvity with ſome Parabola) betwixt the Cubical Parabola and its Tangent at the Vertex.

How much ſoever therefore the parabolical or circular Angle of Contact is diminiſhed, it will be yet greater than the Angle of Contact at the Vertex of the cubical Parabola; and therefore any given circular or parabolical Angle of Contact, will be infinitely greater than the Angle of Contact at the Vertex of the cubical Parabola: for that Quantity is infinitely greater than another, which how much ſoever diminiſhed, it ſhall always exceed that other.

Again, to the ſame Axis and Vertex let be deſcribed another parabolical Curve AG, whoſe Ordi-

nates

nates always increase in a subquadruplicate Ratio of the Abscessæ; the Angle of Contact FAG will be infinitely less than the Angle FAD: which is easily demonstrated by a way of Argumentation not unlike the former. After the like manner, to the same Axis and Vertex another parabolical Curve AH may be described, whose Ordinates increase in a subquintuplicate Ratio of the Abscessæ, in which the Angle of Contact FAH will be infinitely less than the Angle FAG: and so one may proceed *in infinitum*, always assigning more and more parabolical Figures, whose Angles of Contact shall differ infinitely from each other; that is, the Angle FAC will be infinitely less than any rectilineal Angle, and the Angle FAD infinitely less than the Angle FAC, and the Angle FAG infinitely less than the Angle FAD: and so there will be a Series of Angles of Contact proceeding *in infinitum*, whereof every latter Angle is infinitely less than the former; nay, between any two Angles there may be innumerable other Angles inserted, that shall infinitely exceed each other. And also between any two of these Angles there may be inserted an infinite Series of intermediate Angles, whereof each following Angle shall be infinitely less than the foregoing. Moreover, there may be innumerable Angles infinitely greater than the circular Angle of Contact, which yet shall be infinitely less than any rectilineal Angle; and so you may proceed *in infinitum*, nor does Nature know any limits.

I Have exhibited these Examples, that our Adversaries may perceive, how prodigiously different their Speculations, concerning the Natures of Things, are from the Things themselves.

LEC-

LECTURE V.

Of the Subtility of Matter.

HAVING, as we believe, proved by undeniable Arguments, the infinite Divisibility of Matter; and sufficiently answered and refuted the Objections that seemed to carry any weight along with them: it remains, that we consider a little the wonderful Subtility of Nature, and those minute Particles into which Matter is actually divided, or of which it is compounded. It would be very easy, by an abundance of Examples, to place these, as it were, before your Eyes, to expose them to your Senses, and even to shew their Smallness by a Calculation: but we shall produce only a few Instances.

AND first of all, from the great Ductility of Gold, several learned Men, as Monf. *Rohault*, in his *Physicks*; our Countryman, the Honourable Mr. *Boyle*, in his Book of *Effluvia*; and lately, the famous Dr. *Halley*, in the *Philosophical Transactions*, N° 194. have made a Computation of the extreme Smallness of its Parts. Dr. *Halley* has shewn, that one Grain of Gold may be cut into 10000 visible Parts; and therefore since a Grain of Gold is nearly equal to $\frac{21}{100000}$ of a cubick Inch, it follows that a cubick Inch of Gold may be divided into 47 619 047 Parts, which shall be all discernible by the naked Eye.

BESIDES, Dr. *Halley* has computed the Thickness of that very fine Skin of Gold, with which in gilding the Artificers cover Silver Wire; and has found it not

not exceed the $\frac{1}{124500}$ of an Inch: that is, if an Inch in Length was divided into 124500 Parts, the Thickness of the gilding would scarce exceed one of those Parts; and therefore the Cube of the hundredth Part of an Inch, or which is the same thing, the $\frac{1}{1\,000\,000}$ Part of a cubick Inch would contain 243 000 000 such Parts.

That noble Philosopher Mr. *Robert Boyle*, has given, in his Book *of the Nature and Subtilty of Effluvia*, many Experiments concerning this Subject; whence we shall here borrow two or three. And, first of all, he dissolved one Grain of Copper in Spirit of Salt *Armoniack*, and that Solution being mixed with distilled Water, gave a very deep and conspicuous blue Tincture to 28534 Grains of Water: whence, since the Quantity of Water, whose Weight is one Grain, is equal to $\frac{37}{10\,000}$ of a cubick Inch, 28534 Grains of Water will be equal in Magnitude to 105,57 cubick Inches. Since therefore one Grain of Copper can give a blue Colour to so great a quantity of Water, it is necessary that there must be some Part of this Copper in every visible part of that quantity of Water; and therefore as many as are the Parts of that Water visible to the Eye, into so many Parts at least was that one Grain of Copper divided. But a Line is sensible to the Sight whose Length is the hundredth part of an Inch, and consequently the Square or the Cube of that Line, will be much more discernible by the Sight: wherefore since the Cube, whose Side is $\frac{1}{100}$ of an Inch, is the $\frac{1}{1000000}$ part of an Inch, it follows, that at the least in 105,57 cubick Inches of Water, there are 105 570 000 Parts distinguishable by the Sight: and therefore by this Solu-

tion

tion one Grain of Copper was at the leaſt divided into as many Parts. But the Magnitude of a Grain of Copper is equal to about $\frac{55}{100000}$ Parts of an Inch; and therefore, ſince a cubick Inch contains almoſt 20000 ſuch Parts, it hence follows, that a cubick Inch of Copper may be actually reſolved into 2111400000000 Parts: and if there be taken the leaſt Grain of Sand, ſuch a one, for example, as its Diameter may be the hundredth part of an Inch, or, which is the ſame thing, the Grain of Sand itſelf be the million part of an Inch, this will contain 2114000 ſuch Parts, into which the Copper was divided.

The ſecond Example that we propoſe, ſhall be drawn from the following Principles.

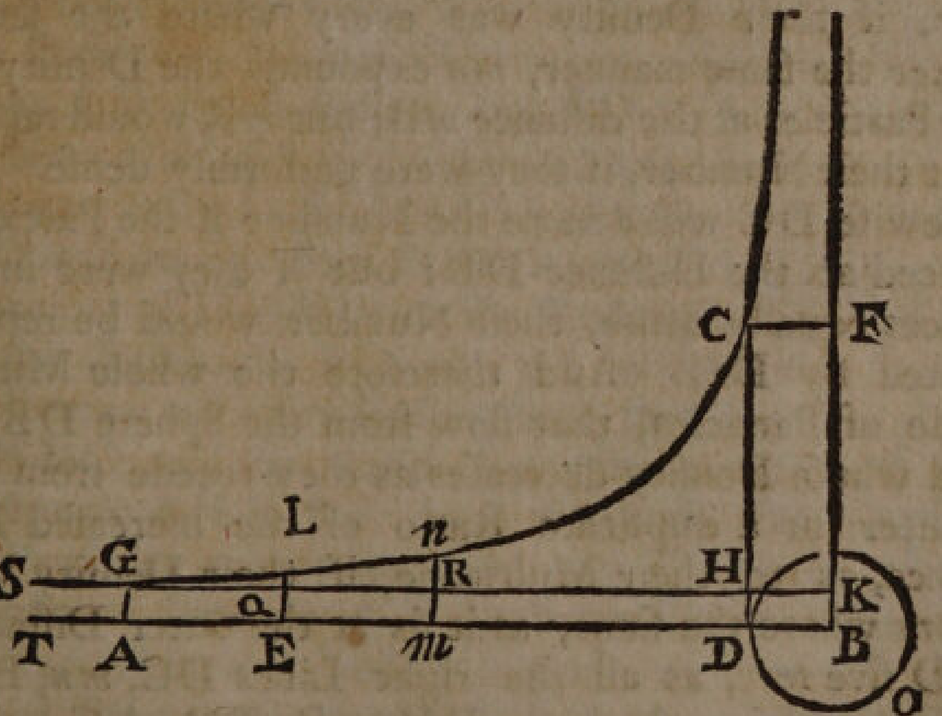
All the modern Philoſophers agree that Odours ariſe from Effluvia, that proceeding from the odoriferous Bodies, are diſperſed on every ſide into the Medium, and which, by means of the Air we draw up our Noſtrils, do ruſh upon the olfactory Nerves, and ſo affect the Senſorium; whence it follows, that in whatever place the Odour of any Body is ſenſible, there muſt be in that very place ſome Particles of the odoriferous Body affecting the Senſe. But there are many odorous Bodies which are eaſily ſmelt at the diſtance of five feet, and which do there affect the olfactory Senſe; there muſt therefore be ſome Particles of the odorous Body diffuſed through all that Space, inſomuch that whereever our Noſtrils ſhall be placed in that Space, there, it is neceſſary, that there be ſome Effluvia of the odoriferous Body; at leaſt there muſt be ſome in that quantity of Air, that we draw through our Noſtrils in Inſpiration. Let us ſuppoſe then, that there is but one Particle of the odorous Body in every Part of that Space, which Part ſhall be equal to the fourth Part of a cubick Inch; and tho it is probable,

Effluvia

Effluvia ſo rare will ſcarce affect the Senſe, yet we ſhall at preſent aſſume no more: ſo many therefore at the leaſt will be the Particles producing the Odour, as there are Spaces, each of which are equal to the fourth part of an Inch, in a Sphere, whoſe Semi-diameter ſhall be five feet; but in that Sphere there will be *57839616* ſuch Spaces: ſo many therefore will be the Particles producing the Odour in that Space.

Having in ſome meaſure determined the Number of Effluvia, we ſhall proceed to diſcover their Magnitude. Since it is neceſſary, that a Body muſt be diminiſhed in Weight, in proportion to the Effluvia that flow from it; the Weight of all the Effluvia proceeding from an odoriferous Body in a given time, will equal the Weight of the Part of the Body that is loſt in that time. Now Mr. *Boyle* has proved by an Experiment, that a certain Maſs of *Aſſa fœtida* expoſed to the open Air, has loſt in ſix days time the eighth part of a Grain in Weight: but ſince the Flux of Effluvia from an odoriferous Body is continual, it is manifeſt that it ought to be proportionable to the Time; and therefore in one Minute's Time the Weight of the Effluvia flowing from the *Aſſa fœtida*, will be equal to $\frac{1}{69120}$ of a Grain. But the Magnitude of a Particle of Water, whoſe Weight is one Grain, is equal to $\frac{369}{100000}$ Parts of an Inch; and therefore a Particle of the ſame Water, whoſe Weight is $\frac{1}{69120}$ of a Grain, will be equal in Magnitude to $\frac{533}{10000000000}$ of a cubick Inch: but the Gravity of *Aſſa fœtida* to that of Water is (as I myſelf have experienced) as 8 to 7, and therefore the Magnitude of a Quantity of *Aſſa fœtida*, whoſe Weight

we ought to know the Quantity of *Assa fœtida* that Mr. *Boyle* exposed to the Air; but we cannot learn this from his Writings. It is therefore necessary we assume some Quantity; but the less we make it, the greater will be, *cæteris paribus*, the Proportion of the Number of Particles proceeding from it, to the Number before found out. That therefore the Number we shall discover may not exceed the Truth, we ought to assume such a Quantity, as probably will be greater than that which Mr. *Boyle* exposed to the Air: and let it be equal to a Sphere, whose Diameter is six Inches, here represented by the Circle DHO; and let the right Line AD be five feet or sixty Inches; AB will be 63 Inches. To the Point A on AB let be erected the Perpendicular AG,



which let represent the Density or Number of Particles in a given Space at the distance AB; and if at all distances the Density of the Particles was the same, their Number might be expounded by the innumerable right Lines EQ, *m*R, DH, &c. compleating the Parallelogram AH, that is, by the Parallelogram

rallelogram AH itself. But since the Number of Particles, in approaching to the Center, is supposed to increase in a duplicate Ratio of the diminished Distance; at the Points E, *m*, D, and innumerable others taken in the right Line AB, let be erected the Perpendiculars EL, *m n*, DC, which let be to AG as the Square of the right Line AB to the Squares of the right Lines EB, *m*B, DB, *&c.* respectively; and through the Points G, L, *n*, C, and innumerable others determined after the same manner, let be drawn a curve Line. Now if AG represents the Number of Particles at the Distance AB, EL will represent their Number at the Distance EB, on a Supposition that the Density of the Particles are reciprocally in a duplicate Ratio of their Distances from the Center; but EQ would denote their Number, if their Density was every where the same. After the same manner, *mn* expounds the Density of the Particles at the distance *m*B; but *m*R would represent their Number, if they were uniformly dense. So likewise DC will denote the Number of the Particles placed at the Distance DB; but if they were in all places equally dense, their Number would be represented by DH. And therefore the whole Multitude of Particles, that flow from the Sphere DBO, and whose Density decreases as they recede from the Center in a duplicate Ratio of the increased Distance, is to their Multitude, if their Density was every where the same, as it is at the outer Distance AD five feet, as all the right Lines DC, *mn*, EL, AG, to the right Lines DH, *m*R, EQ, AG; that is, as the mixtilineal Area ADCG to the Area of the Rectangle GADH.

THE matter therefore is brought to this, that we inquire out the Proportion which the Area GADC has to the Area of the Rectangle AH. But since the Curve GL*n*C is of such a nature, that the right Lines

Lines AG, EL, *mn*, DC, ordinately apply'd to the Asymptote AB, are reciprocally as the Squares of the Distances from the Center; the Curve will be of the hyperbolick kind, and the indeterminable Space CFBTS compounded of Elements that are *reciprocal Secondanes*: and therefore that Space, altho indeterminable, will be perfectly quadrable, and equal to twice the Rectangle CB, by what is demonstrated by Dr. *Wallis* in his *Arithmetick of Infinites*. And consequently the indeterminable or indefinitely extended Area CDTS will be equal to the Rectangle CB; and in like manner the indefinitely extended Area GATS will be equal to the Rectangle GB. The Excess therefore, whereby the Area CDTS exceeds the Area GATS, will be equal to the Excess, whereby the Parellelogram CB exceeds the Parallelogram GB. Let us therefore investigate the Difference of these Rectangles: Since by Hypothesis AD is 60 Inches, and BD 3, AB will be 63 Inches; and let AG be Unity, and since it is as DB*q* to AB*q* so AG to CD, that is, as 9 to 3969; CD will be 441 such Parts, whereof AG is 1: and therefore CD × DB, or the Rectangle CB will be to the Rectangle BG, as 1323 to 63; and so the Difference of the Rectangles, that is, the Area GHDC will be 1260 such Parts, whereof the Rectangle AH is 60. And therefore the Number of Particles proceeding from the *Assa fœtida*, whose Densities decrease in a duplicate Ratio of the increased Distances, and contained within a Sphere whose Diameter is five feet, is to their Number, (if their Density was every where equal to what it is at the distance of five feet) as 1260 to 60; that is, as 21 to 1. If therefore the above-discovered Number 57839616 be multiplied by 21, the Product will give the Number of Particles proceeding from the *Assa fœtida*, viz. 1214631936. Besides, if the Fraction

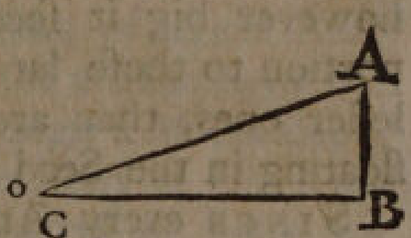
$\frac{8}{10\,000\,000\,000\,000\,000}$, which expressed the Magnitude of the Particles in the former case, be divided by 21, the Quotient $\frac{8}{210\,000\,000\,000\,000\,000}$ or $\frac{38}{1\,000\,000\,000\,000\,000\,000000}$ will exhibite the true Magnitude of each Particle in this second Case.

All these things follow from hence, that we suppose a Man able to smell *Assa fœtida* at the distance of five feet; but there are other Animals, whose Sense of Smelling far exceeds that of a Man, as Hounds, who perceive the Effluvia of wild Beasts remaining on the ground, a great while after the Beast has left the place; and some Birds, that get the Scent of Gun-powder at a great distance. The Subtility of these Effluvia must certainly be much greater than that, which we have discovered by the foregoing Calculation; but for want of Experiments we cannot easily reduce it to Numbers.

The Philosophers that they may still farther shew the Subtility of Matter, produce the Example of those *Animalcula*, that are observed floating in various Liquors, and the Seed of Animals; these indeed are so small in some Fluids, that they appear like Points, through Microscopes that magnify the Object very much. And that curious Observer of Nature, Mr. *Lewenhoeck* has discovered more Animalcules in the Milt of one Cod, even than there are Men at present living on the face of the Earth. But let us discover the Magnitude of these Animalcules: towards which, we shall borrow from Opticks what follow. First, the Image of any Object appears under the same Angle, at the Vertex of Emersion of the Lens, as the Object itself appears under at the Vertex of Incidence; this is demonstrated by Dr. *Gregory*, in the 18th Proposition of his *Elements of Dioptricks*. Secondly, it is found by experience, that those Objects

that

that appear as Points, that is, whoſe Parts the Sight is not able to diſtinguiſh, are ſeen under an Angle not exceeding a Minute. Thirdly, it is certain by frequent Obſervation, that moſt of thoſe Animalcules are of ſo ſmall a Magnitude, that they appear thro a Lens, whoſe focal Diſtance is the tenth part of an Inch, as ſo many Points; that is, their Parts cannot be diſtinguiſhed; and therefore they appear from the Vertex of that Lens, under an Angle not exceeding a Minute. We are now to inveſtigate the Magnitude of an Object that appears under a given Angle at a given Diſtance: In the preſent Caſe, let C be the Vertex of the Lens, AB the Length of the Animalcule, BC its Diſtance from the Lens equal to $\frac{1}{10}$ of an Inch, and the Angle BCA under which it is ſeen at that Diſtance equal to one Minute: from BC and the Angle BCA being given, we are to find AB the Length of the Object. Now in the rectangular Triangle ABC, there being given (beſides the right Angle at B) the Angle BCA one Minute, and the Side BC equal to the tenth part of an Inch, by Trigonometry the Side AB will be found nearly equal to $\frac{3}{100000}$ of an Inch. If therefore theſe Animalcules were of a cubick Figure, that is, of the ſame Length, Breadth, and Thickneſs; their Magnitude would be expreſſed by the Cube of the Fraction $\frac{3}{100000}$; that is, by the Number $\frac{27}{1000\,000\,000\,000\,000}$; that is, to ſo many parts of an Inch is each Animalcule equal.



Hence, what ſome Philoſophers have dreamed concerning Angels, is true of theſe Animalcules, *viz.* that

that many thouſand of them may dance on the Point of a ſmall Needle.

Hence alſo we may gather, how vaſt the Diſtance is betwixt the ſmalleſt floating Animals and the largeſt ſort, how little Proportion the firſt have to the huge Whales, that appear in the Ocean like Mountains, as oft as they raiſe their Heads above the Water. For there are in ſome Liquors Animalcules ſo ſmall, as upon a Calculation, the whole Magnitude of the Earth is not large enough to be a third proportional to thoſe minute floating Animals and the vaſt Whales of the Ocean ; ſo that the Earth itſelf, however big it ſeems, is found to bear a leſs proportion to theſe large Fiſhes, than they do to thoſe leſſer ones, that are beheld through a Microſcope, floating in the Seed of Animals.

Since every Animalcule is an organized Body, let us a little conſider, how delicate and ſubtile ought the Parts to be, that are neceſſary to conſtitute it, and to preſerve its vital Actions. Certainly it is not eaſy to conceive, how it is poſſible there ſhould be contained in ſo narrow a compaſs, the Heart that is the Fountain of its Life, the Muſcles neceſſary to its Motions, the Glands for the Secretion of its Fluids, the Stomach and Bowels to digeſt its Food, and other innumerable Members, without which it is impoſſible an Animal ſhould ſubſiſt. But ſince every one of theſe Members is alſo an organical Body, they muſt have likewiſe Parts neceſſary to their Actions. For they conſiſt of Fibres, Membranes, Coats, Veins, Arteries, Nerves, and an almoſt infinite Number of fine Tubes like to theſe, whoſe Smallneſs ſeems to exceed the very Force of the Imagination. But there are ſome Parts that ought to be almoſt infinitely leſs than theſe, as the Fluids that flow along theſe fine Tubes, and ſuch are the Blood, Lymph, and Animal Spirits, whoſe Subtility even in large Animals is incredible.

LET us then conſider the moſt groſs Parts of the Blood in theſe Animalcules, to wit, the Globules that ſwim in the Blood, and let us endeavour to reduce their Magnitude to a Calculation.

TOWARDS the effecting of which, we ſhall make uſe of the following Hypotheſis; *viz.* That the ſimilar ſolid Parts of different Animals, that is, the ſimilar corporeal Particles, or the Parts conſiſting of three Dimenſions, are as the Magnitudes of the reſpective Animals. Whence it follows, that the ſimilar linear Dimenſions of different Animals are in a ſubtriplicate Ratio of the Magnitudes of the Animals; that is, as the cubick Roots of theſe Animals: as, for example, the Heart of a Man is to the Heart of any Animalcule ſeen through a Microſcope, as the Body of that Man is to the Body of the Animalcule; and therefore, if the Hearts of both are ſimilar Bodies, the Diameter of the one will be to the Diameter of the other, as the cubick Root of the Magnitude of one, to the cubick Root of the other's Magnitude. So likewiſe the ſmalleſt Blood-Veſſels in a Man, are to the like ſmalleſt Veſſels in an Animalcule, as the Magnitude of the Man to the Magnitude of that Animalcule; and the Diameter of a capillary Veſſel in the Body of a Man, is to the Diameter of the like capillary Veſſel in the Body of an Animalcule, as the cubick Root of the Magnitude of the Man to the cubick Root of the Magnitude of the Animalcule.

LET us now ſuppoſe the mean Magnitude of a Man to be three cubick Feet, or 5184 Inches; as therefore the Magnitude of a middle-ſized Man, or 5184 cubick Inches, is to the Magnitude of the Animalcule, found out above, *viz.* $\frac{27}{1\,000\,000\,000\,000\,000}$ Parts of a cubick Inch, ſo are the capillary Veſſels in a human Body to the like capillary Veſſels in an Ani-

Animalcule; and as the cubick Root of the Magnitude of a Man, or the cubick Root of the Number 5184 to the cubick Root of the Magnitude of an Animal, or to the cubick Root of the Number $\frac{27}{1\,000\,000\,000\,000\,000\,000}$, that is, nearly as 17 to $\frac{3}{100\,000}$, so is the Diameter of a capillary Vessel in a human Body to the Diameter of a capillary Vessel in an Animalcule. But Mr. *Lewenhoeck* has by the help of a Microscope discovered Vessels in a human Body so small, that if the Diameter of a Grain of Sand be supposed equal to $\frac{1}{30}$ of an Inch, it will contain 2640 Diameters of the Vessels which he found out in the Body of a Man; and therefore the Diameter of one of these small Vessels will be equal to $\frac{1}{2640} \times \frac{1}{30}$ of an Inch, that is, equal to $\frac{1}{79\,200}$ Parts of an Inch. And although it is certain these Vessels were not the least of all that are in a human Body, for that there must be others much less than these, is easy to be shewn; but however we shall suppose these to be the least of all. Let it be therefore as 17 to $\frac{3}{100\,000}$, so $\frac{1}{79200}$ to another Number; which Number will express in the Parts of an Inch, the Diameter of the smallest Vessel in an Animalcule: which, working by the Rule of Three, is found to be $\frac{3}{134\,640\,000\,000}$, this Fraction reduced to a Decimal, will be nearly $\frac{22}{1\,000\,000\,000\,000\,000}$, or (in round Numbers) $\frac{2}{100\,000\,000\,000\,000}$. But since it is necessary that the Diameter of the Globule, or the fluid Particle, that is contained in any Vessel, be not greater than the Diameter of that Vessel; the Diameter of a Globule of Blood, that flows through

through these smallest Vessels, will not be greater than $\frac{2}{100\,000\,000\,000}$ Parts of an Inch. And therefore the Solidity or Magnitude of these Globules will be less than the Cube of that Diameter, that is, less than $\frac{8}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$ Parts of a cubick Inch: that is, the Magnitude of a Globule is less than that part of a cubick Inch, which is expressed by a Fraction, whose Numerator is eight, but its Denominator a Number consisting of a Unit with thirty and three Cyphers after it.

Since the Fraction, that expresses the Magnitude of these Globules, consists of so many Cyphers, that their true Quantity cannot thence be readily conceived; we shall proceed farther, and compare these Globules with such other small Bodies, as are visible to the naked Eye, *viz.* with the smallest Grains of Sand, such, for example, as their Diameters do not exceed the hundredth part of an Inch: and lastly, we shall compare these small Grains of Sand with other great Bodies of the Earth, as, for example, huge Mountains; that we may perceive what proportion they bear to one another: and so we shall the more easily comprehend the Smallness of Particles. But why do I make use of this word? since I should rather say, by this Comparison their Subtility will appear incomprehensible. For it may be gathered from thence, that Ten thousand two hundred and fifty six of the highest Mountains in the whole Earth do not contain as many Grains of Sand, as one Grain of Sand can of the Blood-Globules of these Animalcules. It is no wonder, if you here stand amazed, and being struck with so prodigious a thing, should call in question the infinite Divisibility of Matter, although it is supported by uncontroulable Demonstrations. But however incredible this

Asser-

Aſſertion may appear at firſt ſight, we ſhall notwithſtanding deduce it from eaſy and evident Principles.

THAT our Calculation may proceed the eaſier, we ſhall call the tenth part of a Foot an Inch, and ſuppoſe that if an hundred Grains of Sand were placed by one another, they would occupy the Space of an Inch in length; or, which is the ſame thing, let a thouſand contiguous Grains of Sand be ſuppoſed to be extended through the length of one Foot, there will therefore be in one cubick Inch 1 000 000 Grains of Sand, and in a cubick Foot there will be 1 000 000 000 Grains. Let a Mile or 1000 Paces be equal to 5000 Feet, there will be then 125 000 000 000 cubick Feet in a cubick Mile: ſo that the Number of Grains of Sand, that can be contained in a cubick Mile, will be 125 000 000 000 000 000 000.

NOW that we may have the Dimenſions of the Mountains, we will make choice of the higheſt, as it is ſuppoſed, of the whole Earth, *viz.* that which is in the Iſland of *Teneriff*, and called *El Pico de Terrario*, whoſe perpendicular Altitude is vulgarly eſteemed three *Italian* Miles. We will ſuppoſe the Figure of this Mountain to be a Cone, and its Circumference at the Baſe to be five and thirty Miles, the Area of the Baſe will be about 97,5 Miles; for as 314 to 100, that is, as the Circumference of a Circle to its Diameter, ſo is 35 to 11,14 the Diameter or Thickneſs of the Mountain at its Baſe, 27,85 the fourth of which being multiplied by the Circumference 35, gives the Area of the Baſe, *viz.* 97,5 ſquare Miles. Since therefore the Mountain is, by Hypotheſis, of a conick Figure, if its Baſe be multiplied into a third part of its Height, the Product will give the ſolid Content of the Mountain; and the third part of the Height is, by Suppoſition, equal to one Mile, which multiplying the Number 97,5, the Product or Solidity of the

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Mountain will be equal to 97,5 cubick Miles: which Number, if it be again multiplied by 125 000 000 000 000 000 000, the Product or Number 12 187 500 000 000 000 000 000 will exhibite the Number of Grains of Sand, of which the Mountain of the Island *Teneriff* might be composed.

These being found out, let us see how many Blood-Globules may be contained in one Grain of Sand. From what has been shewn before, the Magnitude of each Globule is less than the $\frac{8}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$ Parts of an Inch; and the Magnitude of a Grain of Sand is equal to the $\frac{1}{1\,000\,000}$ Part of an Inch: so that if the latter Number is divided by the former, the Quotient $\frac{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}{8\,000\,000}$, or $\frac{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}{8}$; that is,

125 000 000 000 000 000 000 000 000 is less than the Number of Blood-Globules that may be contained within the Magnitude of a Grain of Sand: but this Number 125 000 000 000 000 000 000 000 000 divided by 12 187 500 000 000 000 000 000, the Number of Sands that may be contained in the Mountain of the Island *Teneriff*, the Quotient will be greater than the Number 10 256. So that one Grain of Sand may contain ten thousand two hundred and fifty six times more Blood-Globules in it, than the highest Mountain of the whole Earth does Grains of Sand; or, which is the same thing, ten thousand two hundred and fifty six Mountains, each of which shall be equal to the highest Mountain on the whole Earth, cannot contain in them as many Grains of Sand, as one Grain of Sand may contain in itself of sanguineous Particles of Animalcules, that are by a Microscope seen

ſeen floating in ſome Fluids: which was to be ſhewn. Since therefore theſe Globules are of ſo ſmall a Magnitude, what muſt we think of the Particles compoſing the Fluid in which the Globules are carried, and of the Subtility of the Animal Spirits? This, no doubt, is ſo great, as to exceed all Calculation, and even the Force of the Imagination.

This Subtility of Nature is wonderful beyond meaſure; but there are other Particles of Matter ſtill more ſubtile than theſe, to which if the above-mentioned Globules were compared, they would not only appear as Mountains, but as vaſt Earths. I mean the Particles of Light, which are darted all around from the lucid Body, with an inexpreſſible Celerity, whoſe Subtility the human Mind will perhaps be never, unleſs when it ſhall be made perfect in the Heavens, able thoroughly to comprehend. That it is immenſe, may hence be gathered; that the Light of the ſmalleſt Candle, in a time altogether inſenſible, and without any diſcernible Diminution of the Candle, may be perceived by the Eye, at the diſtance of two Miles; whence it is neceſſary, that in every aſſignable Part of the Sphere of Activity of that Candle, the Diameter of which Sphere is greater than four Miles, and in every aſſignable Portion of Time, there are ſome Particles of that Light, which enter, or are ready to enter the Eye; which will be different in different Parts of Time. And from this ineffable Subtility of Light it is, that the Sun, although from the Beginning of its Creation, it has continually emitted Light, and that with great ſwiftneſs; yet in all that time has not loſt any thing ſenſible of its Magnitude, notwithſtanding it daily, tho inconſiderably decreaſes in Quantity: whence, tho after ſix thouſand Years, its Diminution is not yet become remarkable, notwithſtanding after a finite Series of Years, altho a very protracted one, it will be wholly diſſi-

dissipated. Whence it follows, that this World can neither exist to Eternity, nor could it have existed from Eternity.

FROM *the Demonstration of the infinite Divisibility of Matter are derived the following Theorems, relating to its Rarity, and the Tenuity of its Composition.*

LEMMA.

ANY Quantity of Matter being given, of it, or any part of it, a concave Sphere may be formed, whose Semidiameter shall be equal to any given right Line.

LET the Particle of Matter be a^3, and the given right Line be b. The Ratio of the Circumference of a Circle to its Radius let be as p to r. Let the Semidiameter of the Concavity be called x, the Thickness of the Shell encompassing the spherical Concavity, will be $b-x$, and the Cylinder circumscribed about the Sphere, whose Radius is b, will be $\frac{p\times b^3}{r}$, whence the Sphere inscribed within the Cylinder will be $\frac{2\times pb^3}{3r}$; by the same reason, the Sphere whose Radius is x, will be $\frac{2\times px^3}{3r}$, whose Difference $\frac{2p}{3\times r}\times\overline{b^3-x^3}$ is to be made equal to the spherical Shell, or given Particle of Matter: that is, it will be $\frac{2p}{3r}\overline{b^3-x^3}=a^3$ or $b^3-x^3=\frac{3ra^3}{2p}$ whence $x^3=b^3-\frac{3ra^3}{2p}$ and $x=\sqrt[3]{b^3-\frac{3ra^3}{2p}}$. So that the Thickness of the spherical Shell, or $b-x$, will be $=b-\sqrt[3]{b^3-\frac{3ra^3}{2p}}$.

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the Particle b^3 be supposed to be divided into Parts, whose Number let be n^3, and in each cubick Space let be placed one of those Particles; and by this means the Matter b^3 will be diffused through all that Space. Besides, each Particle of the Matter b^3 being placed, as it were, in its Cell, may be formed into a concave Sphere, whose Diameter may be equal to the given right Line D; whence it will follow, that each Sphere will touch that which is next to it; and the given Particle of Matter, however small b^3, will so fill the given Space, that there will be no Pore in it, whose Diameter will exceed the given right Line D. *Q. E. D.*

Cor. THERE may be given a Body, whose Matter if it be reduced into a Space absolutely full, that Space may be any given Part of the former Magnitude.

The Second THEOREM.

THERE *may be two Bodies equal in Bulk, whose Quantities of Matter may be very unequal, and tho they have any given Ratio to one another, yet the Sums of the Pores or empty Spaces in the Bodies, may almost approach to a Ratio of Equality.* Or, in the *Cartesian* Stile; *All the Space, that is possessed by a subtile Matter within the Pores of one Body, may be almost equal to the Space that is occupied by the like Matter within another Body; although the proper Matter of one Body exceeds ten thousand or an hundred thousand times the proper Matter of the other Body, and the Bodies may be equal in Bulk.*

LET there be, for example, a cubick Inch of Gold, and a cubick Inch of Air not condensed. It is certain the Quantity of Matter in the Gold will exceed twenty thousand times the Matter of Air; yet it may be, that the Spaces in the Gold, either absolutely empty, or replete with a subtile Matter, ſhall be almoſt equal to the Spaces in the Air, either empty, or replete only with a ſubtile Matter.

Let A and B be two Bodies equal in Magnitude; and let both, for example, be a Cube of one Inch. And let the Body A be ten thouſand times heavier than the Body B, whence the Body A will exceed the Body B in Quantity of Matter ten thouſand times. Let us now ſuppoſe the Quantity of Matter in A to be reduced into a Space abſolutely full, which let be the hundred thouſandth part of a cubick Inch; (which is poſſible from the Corollary of the precedent Theorem.) Whence ſince the Matter in A exceeds the Matter in B ten thouſand times, that Matter in B, if it be reduced into a Space abſolutely full, will poſſeſs only the $\frac{1}{1000000000}$ Part of a cubick Inch; ſo that the reſt of the Parts *999999999* will be either abſolutely empty, or replete only with ſome ſubtile Matter, ſuch as the *Carteſians* ſuppoſe. Again, ſince the Quantity of Matter in A only fills the hundred thouſandth Part of an Inch, there will be in the Body A *99999* hundred thouſand Parts, either empty or replete with a ſubtile Matter; that is, by reducing the Fraction to the Denominator of the former Fraction, there will be in A *999990*000 empty Parts. And therefore the Vacuities in A will be to the Vacuities in B, as the Number *999990*000 to the Number *999999999*; which Numbers are almoſt to one another in the Ratio of Equality, for their Difference bears but a ſmall Proportion to the Numbers themſelves. And therefore the empty Spaces, or thoſe only replete with a ſubtile Matter, which are in the two Bodies A and B, obtaining to one

one another the same Ratio as those Numbers, are also almost in a Ratio of Equality. *Q.E.D.*

But that all Bodies are very rare, that is, contain but a very small Quantity of Matter in respect to their Bulk, is most certain from the Properties of diaphanous Bodies; for the *Rays of Light* within Glass or Water are diffused in right Lines, as well as in Air, whatever Side of the diaphanous Body is exposed to the Light: And therefore from any the least assignable Part of the diaphanous Body, to any other Part of it, there is always extended in these Bodies a rectilineal Pore, through which the Light may pass; and this cannot be, unless the Matter of the diaphanous Body obtains but a very little Proportion to its Bulk; and perhaps the Quantity of Matter in Glass has not a greater proportion to its Magnitude, than a Grain of Sand to the whole Bulk of the Globe of the Earth: but that this is not impossible, we have shewn above. Whence, since Gold is not eight times denser than Glass; its Matter also, to its proper Bulk, bears but a very small proportion.

Hence may be gathered the Reason why the magnetick *Effluvia* pervade with the same facility both dense Gold and the more rare Air.

From these Propositions likewise, and from the great Swiftness of Light, may be gathered the Reason, why the Rays of *Light* proceeding from many Objects, and transmitted through a small hole, do not mutually hinder one another, but their Motion is continued in a right Line; which can scarce be explained by the Motion or Impulse of a Fluid, constituting a *Plenum*: for any Body impelled by many Powers at the same time, in different Directions, will receive only one, and a determinate Direction, compounded of them all.

LECTURE VI.

Of Motion, Place, and Time.

SINCE we have hitherto ſufficiently treated of the Solidity, Extenſion, Diviſibility and Subtility of Bodies; we now come to explain Motion, the nobleſt Affection that Body is endued with: by the mediation whereof, Nature diſcovers herſelf acting in that Variety of Things, which ought not to be beheld without Wonder and Aſtoniſhment; and without which, all the Ornament and Beauty of the World would periſh, and a horrid Darkneſs and an infinite Numbneſs would poſſeſs every thing. On this depend the Viciſſitudes of Days and Nights, and the ſo great Variety of Cold and Heat, Snow, Rain, and Sun-ſhine ſucceeding each other, and all the Seaſons of the Year. By Motion Plants grow, Trees are nouriſhed, and Animals live; ſince Life itſelf conſiſts only in Motion, that is, the Circulation of the Blood. But why do I ſpend time in enumerating Particulars? ſince all things owe their birth to Motion.

THE Science therefore of Motion is ſo neceſſary to philoſophize aright, that not the leaſt Operation of Nature can be inveſtigated without it. Hence the famous and moſt true Saying of the Philoſopher, Αναγκαῖον ἀγνοουμένης αὐτῆς κινήσεως ἀγνοεῖσθαι κ̀ τὴν φύσιν, Motion being unknown, Nature muſt of neceſſity be ſo likewiſe.

THE Philoſophers, or rather the Metaphyſicians, have had various Diſputes concerning the Nature, Cauſes, and Communication of Motion; and the Con-

Confusion of Ideas, and the Darkness that thence arose, appear to have been so great, that amongst the Follies of Disputation, the natural and simple Notion that they had of it, seems to have been lost amongst them. For there can scarce be found one of the common People, or the meanest Artificer, but he knows more of the true Nature and Cause of Motion than all these wrangling Philosophers together, some of which were arrived to that height of Folly, as to deny all Motion, as a thing impossible, to Bodies; and they proposed some Cavils, whereby they flattered themselves that they had demonstrated its Impossibility.

We will produce here some of their strongest Arguments: and the first shall be that of *Diodorus Cronus*; namely, If a Body moves, it must either move in the place where it is, or in the place where it is not, both which are impossible: for if it moves in the place where it is, it will never depart from that place, and so there will be no Motion; in like manner, it cannot move in the place where it is not, because nothing acts in a place where it is not: therefore a Body will not move at all. I answer, that a Body neither moves in the place where it is, nor in the place where it is not, but it moves from one place to another.

The second Argument shall be that of *Zeno*, to which he gave the Name of *Achilles*: and thereby he endeavours to prove, that if there be such a thing as Motion, *Achilles*, though ever so swift, could never overtake a Tortoise, though the slowest of Animals. The Argument is this: Suppose *Achilles* to be distant from the Tortoise a certain finite Space, as, for example, a Mile, and let us suppose he moves an hundred times faster than the Tortoise; therefore whilst *Achilles* runs one Mile, the Tortoise has moved forwards the hundredth part of a Mile; so that *Achilles* has not yet overtaken the Tortoiſe: And again, whilſt *Achilles* paſſes over that hundredth part of a Mile, the Tortoiſe in the mean while will have creeped through the ten thouſandth part of a Mile; inſomuch that neither has *Achilles* yet overtaken the Tortoiſe. After the ſame manner, whilſt *Achilles* runs that ten thouſandth part of a Mile, the Tortoiſe will have advanced forwards the million part of a Mile; ſo that *Achilles* has not yet come up with the Tortoiſe. And ſo he may go on *in infinitum*, nor will he ever be able to overtake the Tortoiſe, but there will be always ſome diſtance betwixt *Achilles* and the Tortoiſe.

THIS is the famous Argument of *Zeno*; to anſwer which, ſome have wrote whole Treatiſes: but we ſhall eaſily diſſolve the Knot, by ſaying, That a Mile, together with the hundredth part of a Mile, together with the ten thouſandth part of a Mile, with the million part of a Mile, and ſo on *in infinitum*, will be equal to a finite Quantity. For it is demonſtrated by the Arithmeticians, that the Sum of any Series of Quantities decreaſing in any Geometrical Proportion *in infinitum*, will be equal to a finite Quantity; but the $\frac{1}{100}$ part of a Mile, together with the $\frac{1}{10000}$ part, together with the $\frac{1}{1000000}$ part, together with the $\frac{1}{100000000}$, or the hundred million part, and ſo on *in infinitum*, is a Series of Quantities decreaſing in Geometrical Proportion *in infinitum*: ſo that its Sum, ſince it is equal to a finite Quantity, may be run over by a Body moving with a given Velocity, in a finite time. For let us ſuppoſe *Achilles* in the ſpace of an Hour to run a Mile, and therefore he will paſs over the hundredth part of a Mile in the hundredth part of an Hour, and the ten thouſandth part of a Mile in the ten thou-

thousandth part of an Hour; and after the same manner he will move through the million part of a Mile in the million part of an Hour, and so of the rest. If therefore one Hour, together with the hundredth part of an Hour, together with the ten thousandth part of an Hour, together with the million part of an Hour, $+\frac{1}{100\,000000}$, *&c. in infinitum*; if, I say, the Sum of this Series continued *in infinitum* is equal to an infinite Space of Time, it is certain *Achilles* would never overtake the Tortoise in any finite Time: but since, as we said before, the $\frac{1}{100}+\frac{1}{10\,000}+\frac{1}{1\,000\,000}$, *&c.* Parts of an Hour, is a Series of Quantities decreasing in Geometrical Proportion *in infinitum*; its Sum will be equal to a finite Quantity, *viz.* the ninety-ninth Part of an Hour, as might be easily demonstrated. And within that Space of Time, all the Particles of Time, however infinite in Number, will be passed by. We say therefore, that *Achilles* will overtake the Tortoise, after one Hour, and all those Particles of Time contained in the foregoing Series, and which are infinite in Number, are expired; that is, he will arrive at the Tortoise, after one Hour and the ninety-ninth Part of an Hour: And so the Force of this Argument is destroyed, tho its Patrons have so often boasted of it as unanswerable.

This Argument is also wont to be brought against Motion: Let B and C be two contiguous Points, and the Body A be moved from B to C, in an Instant D: when A is moved, it is supposed to be in B, and therefore in that Instant it cannot arrive at C, namely, because it is supposed to be in B; and it cannot be in both in the same Instant, because nothing can be at once in two places, that is, in the same Instant: and therefore in the Instant in which it is in B, it cannot arrive at C, becauſe it is yet ſuppoſed to be in B; and conſequently, according to the Authors of this Argument, it will never come to C.

It is eaſy to anſwer to this Argument, by ſaying that at the beginning of the Inſtant D, indeed H is in the Point B, but at the end of it, in C; for that Time wherein any finite Motion is performed, muſt have a beginning and an end.

But beſides, in this Argument there are not a few things aſſumed, which are falſe and impoſſible; as, for example, when two Points are ſuppoſed contiguous. If by a Point is meant an indiviſible Part, or the leaſt Quantity, we have before demonſtrated, that there are not given ſuch Points; and therefore if the Argument reſts on this Hypotheſis, it will be impoſſible it ſhould have any force on the human Mind, againſt the Exiſtence of Motion. But if by Points are meant mathematical Points, ſuch, for example, as are the Terminations, Interſections, and Contacts of Lines, theſe indeed we acknowledge as poſſible, however it is impoſſible that any thing ſhould move in them: for whatever moves, it muſt move through a Space; but a mathematical Point contiguous to another Point, cannot make a Space, but a Point only. For as in Arithmetick a thouſand Cyphers, or Nothing taken a thouſand times, ſtill is equivalent to Nothing; ſo in Geometry a thouſand Points, or indeed an infinite Number of them together, do not compoſe any Quantity, but are equivalent to a Point, or no Quantity. Whence, ſince two contiguous Points are but equal to a Point, I readily grant that Motion cannot be through them: but nothing abſurd follows from thence, for Motion through a Space is not taken away, but Motion through a Point; and it would be abſurd indeed to grant ſuch a Motion to be.

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What we have ſaid of Points, the ſame may be accommodated to Inſtants or Moments of Time, by ſhewing that as all Magnitudes, ſo likewiſe Time is diviſible *in infinitum* ; ſo that there is no Particle of Time that can be properly called an Inſtant or Point of Time: as no Part of a Line coincides with a Geometrical Point, and as infinite Points do not compoſe a Line, but a Point, ſo likewiſe infinite Moments, or Points of Time, are equal to no Time. An Interval indeed of Time betwixt different Moments, may be equal to a given Time, but the Moments themſelves will be equal to no Time; for Time is not compounded of Moments, but of Parts, which are alſo Times, nor is Motion performed in an Inſtant, but in Time.

But leaving theſe Trifles, we return to our Deſign.

Since the Motion, of which we are to treat, is local Motion, our Purpoſe requires, that we firſt deliver ſome things concerning Place and Time. Place is wont to be diſtinguiſhed into internal and external. The internal Place is the Space that is filled by the Body which is ſaid to be in that Place ; but the external Place is that alone, which by *Ariſtotle* is defined, and called the concave Superficies of the ambient Body, and which contains the Body that is ſaid to be there placed.

But perhaps Place, as well as Space, is more clearly diſtinguiſhed into abſolute and relative. Abſolute or primary Place is that part of the immoveable, permanent, and every way expanded Space, which is taken up by the Body there placed. Relative or ſecondary Place is that apparent and ſenſible one, which is diſcerned by our Senſes from its Situation in reſpect of other Bodies. For ſince Space itſelf is a ſimilar and uniform Being, whoſe Parts cannot be ſeen, or diſtinguiſhed from one another; therefore it is fitting that the Places of Bodies be

referred

referred to other Bodies, and determined by their Diſtances from, and Poſitions to, other Bodies: as, for example, let us ſuppoſe any one to ſit in the Corner of ſome Houſe; his Place will be defined by the Diſtance, Reſpect, and Poſition, that he has to other Corners, Walls, and ſurrounding Bodies, that are looked upon as immoveable: and as long as he keeps the ſame Diſtance and Situation from theſe Bodies, ſo long is he ſuppoſed to remain in the ſame Place. So likewiſe if any one ſits in a Ship, whether the Ship moves or not, as long as he keeps at the ſame Diſtance from all the Parts of the Ship, which are looked upon as being at reſt, and his Poſition to them all remains the ſame, his relative Place will alſo remain the ſame.

WHAT we have ſaid of Place, may be in like manner applied to Space; for that may be diſtinguiſhed into abſolute and relative. We call that abſolute, which of its own nature, and without relation to any thing beſides, always remains ſimilar and immoveable. But that is relative which is referred to ſome Bodies, by which it is determined and meaſured; whoſe Parts, to wit, always keep the ſame Poſition and Situation to thoſe Bodies, and whoſe Diſtance from them (*viz.* Bodies) always remains unchanged and the ſame.

RELATIVE Space is always the ſame with abſolute Space in Magnitude and Figure, but however it is not neceſſary that it ſhould be always numerically the ſame Space: for if we ſuppoſe the Ship to be moved, the abſolute Space contained within the Cavity of the Ship, will be different in different Places; but ſince the Cavity and Figure of the Ship remains the ſame, the Magnitude of the Space contained in it, and its Figure will be the ſame, and its Parts alike ſituated will always have the ſame Poſition and Diſtance in reſpect to the ſame Parts of the Ship: and there-

therefore it ought to be called the ſame relative Space.

So likewiſe in the Hypotheſis of the Earth's Motion, the Space that is contained within the Walls of any Building, altho by conſidering it as abſolute, always is changed; yet ſince the Cavity of the Building remains the ſame, there is the ſame Figure, and all the ſimilar Parts of the contained Space always keep the ſame Situation to the ſame Parts of the Building; and ſince they always obſerve the ſame Poſition to the relative Space of our Air, as alſo to all the Parts of the Earth, that Space may be called relative.

And after the ſame manner may Time be diſtinguiſhed into abſolute and relative. Abſolute Time flows equally, that is, it never proceeds faſter or ſlower, but without any relation to the Motion of Bodies, it glides along with an equal Tenour. Relative or apparent Time is the ſenſible Meaſure of any Duration by the means of Motion; for ſince the equal Flux of Time itſelf does not affect the Senſes, there muſt be called in the help of an equable Motion, as ſome ſenſible Meaſure, which may determine its Quantity, whoſe Parts may always correſpond and be proportionable to the Parts of Time. But that uniform Motion, which is to be applied to the Meaſure of Time, ought to be the moſt remarkable, obvious to every one, and affecting the Senſes of all, ſuch as are commonly thought to be the Revolutions of the Sun, and Moon, and Stars; by which we divide Time into Hours, Days, Months, and Years. And as we judge thoſe Times to be equal, that paſs whilſt a moving Body, carried by an equable Velocity, runs over equal Spaces; ſo likewiſe thoſe Times are ſaid to be equal, which flow whilſt the Sun or Moon perform their Revolutions, that to our Senſes are equal.

But since, as we said before, the Flux of Time cannot be accelerated or retarded, but all Bodies are moved sometimes faster and sometimes slower, nor perhaps is there given in nature a perfectly equable Motion; it is necessary that absolute Time should be something truly and really distinct from Motion, nor does its Nature more depend on the Motion of Bodies than on their Rest. For let us suppose the Heavens and the Stars to have remained without Motion from the very Creation of the World, it does not thence follow that the Course of Time would have been at a stand, but the Duration of that quiescent State would be equal to the very Time that is now elapsed. Besides, since we learn from the Scriptures, that in the days of *Joshua* the Sun remained for some time immoveable in the same part of the Heavens, yet absolute Time was not therefore at a stand, and began again its Course with the Sun, but flowed in the same Tenour as before, altho all the Sun-Dials shewed the same Hour of the Day, during all the Time of that Station: and so indeed the apparent Time, *viz.* that related to the Motion of the Sun, stood still, when in the mean while the absolute Time proceeded uniformly on.

So likewise since at present the apparent Motion of the Sun is not uniform, neither will its diurnal Revolution be equable, as all the Astronomers acknowledge, but sometimes it proceeds in a swifter, sometimes in a slower degree; and therefore the natural Day, *νυχθήμερον*, or the Space of Time elapsed in one diurnal Revolution, will be sometimes less, sometimes greater: so that the apparent Time does not proceed in the same Tenour as the absolute, whence it ought to be distinguished from it.

Since absolute Time is a Quantity uniformly extended, and in its own nature most simple, it may be aptly represented and proposed to our Imagina-

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tion by the most simple Magnitudes: such, in the first place, seem to be right Lines and Circles, with which and Time there is a certain Analogy. For all the Parts as well of Time as of right Lines and Circles, are every where similar and uniform; and as a Line is generated by the Motion or Flux of a Point, whose Quantity depends on one Length determinated by Motion; so likewise Time may in some measure be looked upon as the Path of an Instant continually gliding along, whose Quantity flows from one Succession, as it were stretched out in Length, and which is demonstrated by the Length of the Space passed over; and therefore may be best represented by the Flux of a Point or a right Line, which will be often done in what follows.

But it must be observed, that we understand by the word Time, that Space of Time wherein Motion is performed; so that when we treat of Philosophical Matters and of Motion, it (*viz.* Time) may be fitly defined with *Aristotle*, *Mensura Motus secundum prius & posterius*; that is, *The Measure of Motion according to what is past and what is to come:* not indeed as considering the absolute Nature of Time, but that Connection which Motion has with it; namely, as no Space can be passed over in an Instant by a Body in Motion, but all Motion is performed successively, and according to the Flux of Time, which Motion therefore may be compared with the Quantity of Time, and be measured by its Flux.

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LECTURE VII.

Definitions.

I. M*OTION is a continual and successive Change of Place.*

II. Celerity *is an Affection of Motion, whereby a Body in Motion passes over a given Space in a given Time.*

III. But *Rest is the Permanence of any Body in the same Place.*

Hence it follows, that Rest, Motion, and Celerity are twofold, according to the double Distinction of Place, *viz.* absolute and relative.

IV. Absolute *Motion is the Change of absolute Place, and its Celerity is measured by absolute Space.*

V. Absolute *Rest is the Permanence of a Body in the same absolute Place.*

VI. Relative *Motion is the Change of relative Place, whose Celerity is measured by relative Space.*

VII. But *relative Rest is the Permanence of a Body in the same relative Place.*

From these it follows, First, That a Person may relatively be at rest, who yet, in respect to absolute Space, may truly and absolutely be in motion: As, for example, if any one should be seated in a Ship, since he keeps the same relative Place, and retains the same Situation and Distance in respect to the other Parts of the Ship, which are looked upon as quiescent, he is relatively at rest; though in the mean while

while he is carried by the ſame Motion, with the ſame Celerity, and according to the ſame Courſe, as the Ship is borne by the Winds; in which caſe, all the Parts of the Ship keeping the ſame Situation amongſt themſelves, will appear to the Spectator placed within the Ship, as if they were at reſt: on the contrary, whilſt the Ship is in motion, the Shore and other circumjacent Bodies out of the Ship, will appear to a Spectator in the Ship, to be moved with the ſame Celerity, but towards contrary Parts, as the Ship, or it will recede from them. The Reaſon of this Appearance may be eaſily ſhewn from Opticks: for thoſe Bodies ſeem to be at reſt, which keep always the ſame Poſitions and Diſtances in reſpect to the Eye itſelf; but thoſe Bodies which we ſee to be moved, we find them change their Diſtance and Poſitions in reſpect to our Eyes. But we will conſider this matter a little farther.

Since Opticks teach us, that every Body, which is viſible, has, by means of the Rays which proceed from that Object, its Image painted on the bottom of the Eye, or Retina; it follows, that thoſe Objects will ſeem to be moved, whoſe Images are moved on the Retina, that is, which paſs over ſucceſſively the different Parts of the Retina, whilſt the Eye is ſuppoſed to be at reſt: but thoſe Objects will be looked upon as being at reſt, whoſe Images always occupy the ſame part of the Retina, that is, when the Motion of thoſe Images are not perceived in the bottom of the Eye. And hence it is, that they who are ſeated in a Ship, do not perceive the Motion of the Ship: for all the Parts of the Ship being relatively at reſt amongſt themſelves, keeping the ſame Poſition and Diſtance in reſpect to the Eye, will have their Images always painted on the ſame Parts of the Retina; therefore their Motion will not be ſeen. But when the Spectator turns his Eyes towards

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the Shore, whilst the Ship is moved, it is necessary that any external Object should change its Situation in respect to the Eye, and therefore its Image will successively occupy different Parts of the Retina; that is, the external Object will seem to move. For the same reason, if the Earth is moved about the Sun, or its own Axis, its Motion will not be perceived by the Inhabitants of the Earth; *viz.* since the Edifices and all visible Objects on the Earth remaining in the same Parts of the Earth, will always keep the same Position amongst themselves and to the Eye; but if the Stars, and all other Bodies not fixed to the Earth, are beheld, those, for the same cause as the Shore before, will be seen to move: that is, if the Earth is turned about its Axis from West to East, the Sun and the other Stars will be beheld to move from the East to the West.

But leaving for a while the Motion of the Earth, let us return to the Example of a Ship: If the Ship is carried in any Direction whatever, as, for example, towards the East; and any one sitting in the Prow, should cast a Stone towards the West, with the same Velocity as the Ship itself has towards the East; the Stone in this case would seem to the Spectator within the Ship, to be moved towards the West, and its relative Velocity would be equal to the absolute Celerity of the Ship itself: yet in truth the Stone would be at rest in the absolute Space, abstracting from the Motion of the Earth, and all that which may arise from Gravity. And if we suppose any Body to be placed out of the Ship hanging in the Air, he would behold the Stone at rest; but since the Stone is heavy, he would see it move only perpendicularly downwards, not tending more towards the East than towards the West: for the Force impressed on the Stone by the Caster, does nothing else but destroy the equal Force of Motion, that was commu-

communicated to it from the Ship moving in a contrary Direction; for any Body or Space being in motion, likewise all Bodies or Particles of Bodies relatively at rest within that Body or Space, will be moved with the same Celerity, and in the same Direction.

But some may object, That the Stone thrown from the Hand of the Caster, lights on the Poup, and gives it a blow; and since the Stone strikes against the Poup itself, it cannot but be moved. I answer, it is true indeed that those who are within the Ship behold the Stone lighting on and striking against the Poup; but if any one is supposed hanging in the Air out of the Ship, he would not see the Stone moving towards the Poup, but the Poup towards the Stone, and striking it; and the Magnitude of the Stroke, that is received on each Body, will be altogether the same as if the Ship was at rest, and the Stone really impelled towards the Poup, with the same Celerity wherewith the Poup arrives at the Stone. For if there are two Bodies, A and B, however equal or unequal; the Force of Percussion will be the same, whether B with a given Celerity impinges on the Body A at rest, or if B be at rest, and A with the same Celerity rushes on it: or if each Body should be moved towards the same part, and the subsequent Body A moving the fastest, should impinge on B; the Quantity of the Stroke will be the same, as if B was altogether at rest, and A was only borne along with the Difference of Celerities, that is, whereby its Celerity exceeds the Celerity of the Body B: or lastly, if A and B were carried towards different Parts, the Magnitude of the Stroke would be the same, as if one was at rest, and the other moved, with a Celerity that is equal

to the Sum of the former Velocities. I ſay, in ſhort, the relative Velocity of Bodies always remaining the ſame, as that wherewith they arrive at one another, the Quantity of Percuſſion will be likewiſe the ſame, however the true Velocities are ſhared, as will be demonſtrated hereafter. But let us return to our Example of the Ship.

If the Force with which a Stone is thrown by the Caſter, is leſs than that which it receives in this caſe from the Motion of the Ship, that Stone is truly carried by its abſolute Motion towards the ſame Parts, as the Ship itſelf: that is, it will be ſeen to move by the Spectator, whom we have ſuppoſed to be placed out of the Ship in the Air, towards the Eaſt, with a Celerity, whereby the Celerity of the Ship exceeds the Celerity of the Motion impreſſed by the Hand of the Caſter; but to thoſe placed in the Ship, the Stone will appear to move towards the Weſt, with the very ſame Celerity that it received from the Hand of the Caſter, with which alſo it will ſeem to ſtrike on the Poup.

But if any one ſetting in the Poup ſhould throw a Stone towards the Prow, its true and abſolute Motion would be towards the Prow or Eaſt, and it would be beheld by our Spectator placed out of the Ship, to be carried with a Celerity that is equal to the Sum of two Celerities, whereof one is that which the Stone received from the Caſter, the other that which was communicated to it from the Motion of the Ship.

All theſe things may be apply'd to the Hypotheſis of the Earth's Motion. For if the Earth only revolved about its Axis from the Weſt towards the Eaſt, and a Stone or Bullet ſhould be thrown by a Cannon to the Weſt, with that Velocity whereby the Earth is turned about its Axis; the Impetus, that the Bullet receives from the Cannon, would deſtroy

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the contrary Impetus, that was impressed on it by the Earth: so that the Bullet would be at rest in the absolute Space, if we have no regard to the Motion arising from its Gravity. Nevertheless those that live on the Earth, and revolve together with it, would behold the Stone or Bullet swiftly carried towards the West; and if any Wall was oppos'd to its apparent Motion, they would see the Bullet striking against the Wall with the same force as if the Wall was really at rest, and the Bullet impinged against it with the Celerity, which in that case it had received from the Explosion: for, as we said before, the Quantity of the Stroke would be the same, whether the Bullet was thrown with a determinate Celerity against the Wall at rest, or the Wall met the Bullet at rest with the same Celerity.

If the Force that is impressed on the Bullet by the Explosion of the Gun, be less than that which is communicated to it by the diurnal Motion of the Earth, the Bullet will be really carried towards the East; but because its Velocity is less than that whereby we are revolved towards the East, the Bullet will appear to us to tend toward the West: and any Object that opposes its apparent Motion, will seem to be struck by it with the same force, as if really the Obstacle had remained in the same absolute Space, and the Bullet had impinged on it with the Force it received from the Gun. If, lastly, the Bullet be disploded towards the East, its absolute Motion will be towards the East, and its Velocity will as much exceed the Velocity wherewith the Earth is moved, as is that which is impressed on the Bullet by the Gun; so that it rushes on and strikes against any Obstacle with that only Difference of Velocity.

And universally, of all Bodies included in a given Space, their Motions amongst themselves will be the same, their Congress the same, the same Force of their Percussion, whether that Space is at rest, or moves uniformly in a right Line.

Having, and that prolixly enough, explained Motion, Rest, Celerity, as well absolute as relative, we now come to define other Terms.

VIII. The *Space passed over is the Way that by a Body is run through in its Motion.*

IX. Its *Length is the right Line that is described by the Center of the moved Body.*

X. The *Direction of Motion is the right Line whither the moving Body tends.*

XI. Equable *Motion is when the moving Body describes all the Parts of the Length or Space passed over always with the same Celerity.*

XII. Accelerated *Motion is that, whose Velocity increases continually.*

XIII. Retarded *Motion is that, whose Velocity is continually diminished.*

XIV. Motion *equably accelerated is that, to which in equal Times there is always added equal Increments of Velocity.*

XV. Motion *equably retarded is that, whose Velocity in equal Times equally decreases, even to Rest.*

XVI. A Momentum *(which is often called the Quantity of Motion, and also simply Motion) is that Power or Force incident to moving Bodies, whereby they continually tend from their present Places.*

XVII. But *an Impediment is that, which obstructs or resists Motion, and destroys, or at least diminishes it.*

XVIII. The *moving Force is the Power of an Agent to cause Motion.*

XIX. An *impressed Force is an Action exerted on a Body, to change its State either of Motion or Rest.*

If a Body A at rest is put in motion with a given Celerity, the Force that is impressed on it, and by receiving

receiving which it begins to move with a given Velocity, is called the impreſſed Force; in which caſe it only differs from the moving Force in the Manner of conceiving it: for the ſame Force, as it proceeds from the Agent, is called the moving Force, and as it is received by the Patient, the impreſſed Force. So likewiſe if a Body B is moving, a certain determinate Force is required to diminiſh its Motion, and alſo a certain determinate Force is neceſſary to put a thorough ſtop to its Motion; which, when it is exerted on the Body B, it is called the impreſſed Force.

I AM not ignorant that there are ſome Philoſophers who do not diſtinguiſh the Quantity of Motion from its Celerity; but ſay thoſe Bodies have equal Motions, that are moved with an equal Celerity, whether the Bodies themſelves are equal or unequal, whether one is very ſmall, and the other ever ſo great, ſo that each Body is carried with the ſame Velocity, they contend that the Quantity of Motion in each will always remain the ſame. But not only Reaſon, but Experience, ſhews that Motion is not only increaſed in the Ratio of the Velocity, but alſo in that of the Bulk or Magnitude, the Bodies being ſuppoſed homogeneal, or of the ſame Species: As, for example, let A and B be two Bodies, A the greater, and B the leſs, and the Momentum or Quantity of Motion of A will be not only greater

than the Momentum of B, if A is moved ſwifter than B; but if both are carried with an equal Celerity, the Force or Energy whereby the greater Body A is carried, will be greater than that which the Body B has to change its place; becauſe there is required a greater contrary Force of an Obſtacle or Impediment to ſtop the Motion of the greater Body A, than that which is neceſſary to deſtroy the Motion of the leſs Body B. For if the Body A is an hundred Pounds weight, but the Body B one Pound only, and if in both Bodies the Celerities are equal; the Force that the Body A exerts, and whereby it endeavours to remove an Obſtacle, (and therefore the Force of the Impediment reſiſting and deſtroying its Motion, will be much greater than the Force of the Motion of the Body B) *viz.* whereby it endeavours to remove the Impediment, and the Force of that Impediment, which is neceſſarily required to deſtroy the Motion of the Body B, will be leſs than the Force of the Impediment that ſhall be ſufficient to take away the Motion of the moving Body A. But we ſhall hereafter give Theorems, whereby may be eſtimated the Quantity of Motion, and its Meaſure be determined.

XX. Those *moving Forces are equal, which acting alike, produce equal Quantities of Motion in a given Time.*

XXI. Those *Forces are contrary, whoſe Lines of Direction are contrary.*

XXII. Gravity *is a Force bearing downwards, whereby Bodies tend to the Earth in right Lines.*

XXIII. *A* Centripetal *Force is that Force whereby a Body continually tends to ſome Point as a Center; and hence it follows, that Gravity is a certain centripetal Force.*

XXIV. But *by a Centrifugal Force we mean a Force whereby any Body is continually ſollicited to recede from a Center.*

But theſe Forces are always eſtimated by the contrary Forces, that are able to keep the Bodies in the ſame ſtate. So if any Body fixed to a String is revolved about an immoveable Center, the Force whereby it endeavours to recede from the Center, is the

the centrifugal Force; but the Action of the String resisting and continually drawing back the Body towards the Center, whereby it comes to pass that the Body is always retained in the same Circle, will be as a centripetal Force equal to the centrifugal one, so that one of these Forces may be aptly estimated by the other. So likewise the Force of Gravity of any Body is known by a Force contrary and equal to it, whereby its Descent may be hindered. But that Force may be either the Weight of another Body acting contrarily by means of a mechanical Instrument, as, for example, a Balance, or a centrifugal Force that will arise, if that Body revolves in a Circle about the Center of the Earth with some certain and determinate Velocity; or lastly, it may be the Firmness and Resistance of another Body on which the pressing Weight rests.

XXV. The *Accelerating Quantity of any Force, is the Measure of the Velocity which that Force generates in a given Time.*

At the same Distance from the Earth, all Bodies, of how unequal weight soever they are, do descend equally swift, and therefore their accelerating Forces are equal; but at unequal Distances they are unequally accelerated, *viz.* at a greater, less; at a less Distance, more.

LECTURE VIII.

HAVING finiſhed the Definitions, which were neceſſary to explain ſuch things as are not ſufficiently clear, or thoſe Terms that are leſs uſual, we now come to treat of the Philoſophical Axioms. But ſince the Object of Natural Philoſophy are Bodies, and their Actions on one another, which are not ſo eaſily and diſtinctly conceived, as thoſe ſimple Species of Magnitudes which are the Subjects of Geometry; I would not have any one, in phyſical Matters, inſiſt ſo much on a rigid Method of Demonſtration, as to expect the Principles of Demonſtration, that is, Axioms ſo clear and evident in themſelves, as thoſe that are delivered in the Elements of Geometry: for the Nature of the thing will not admit of ſuch. But we think it ſufficient, if we deliver ſuch as we apprehend are congruous to Reaſon and Experience, whoſe Truth ſhines out, as it were, at firſt view, which procure the Belief of ſuch as are not obſtinate, and to which nobody can deny his Aſſent, unleſs he profeſſes himſelf to be altogether a Sceptick.

But alſo in demonſtrating, it is neceſſary to make uſe of a more lax ſort of Reaſoning, and to exhibite Propoſitions that are not abſolutely true, but nearly approaching to the Truth. As, for example, when it is demonſtrated that all the Vibrations of the ſame Pendulum made in the ſmall Arches of a Circle, are of equal Duration; it is here ſuppoſed, that the ſmall Arch of a Circle and its Chord are of the ſame Declivity, and of the ſame Length: which however, if we regard the rigid Truth, is not to be admitted; but

but in Physicks, this Hypothesis varies so little from the Truth, that the Difference ought justly to be neglected, and the Disagreement of the Vibrations arising from that Difference, is altogether insensible, as is proved by Experience. So likewise that eminent Philosopher and Geometer, Dr. *Gregory*, in his *Elements of Catoptricks and Dioptricks*, makes use of a more lax Geometry, by assuming Lines and Angles as equal, that in reality are unequal, tho they accede nearly to an Equality: And so he solves many beautiful Physical Problems, which otherwise would prove very intricate. And also this Method seems to be approved of sometimes by Sir *Isaac Newton* himself, as may be seen in *Prop.* 3. *Lib.* 2. of his *Philosophiæ Naturalis Princip. Math.*

But if there are any who harden their Minds against such Principles and Demonstrations, and will not suffer themselves to be convinced by Propositions sufficiently manifest; we leave such to enjoy their supine Ignorance, nor do we think them worthy to be admitted to the Knowledge of the true Philosophy.

AXIOMS.

I. There *are no Properties or Affections of a Non-entity or Nothing.*

II. No *Body can be naturally annihilated.*

III. Every *Mutation induced in a natural Body, proceeds from an external Agent; for every Body is a listless Heap of Matter, and it cannot induce any Mutation in itself.*

IV. Effects *are proportionable to their adequate Causes.*

V. The *Causes of Natural Things are such, as are the most simple, and are sufficient to explain the Phenomena: for Nature always proceeds in the simplest and most expeditious Method; because by this Manner of operating the Divine Wisdom displays itself the more.*

VI. NATURAL *Effects of the same kind have the same Causes: as the Descent of a Stone and a Piece of Wood proceeds from the same Cause; and there is also the same Cause of Light and Heat in the Sun and in a Kitchen-Fire, of the Reflection of Light in the Earth and in the Planets.*

VII. IF *two things are so connected together, that they perpetually accompany each other, that is, if one of them is changed or removed, the other likewise will be in the same manner changed or removed; either one of these is the Cause of the other, or they both proceed from the same common Cause.*

SO if there be a magnetick Needle moveable on an Axis, upon a Loadstone being brought near it, and moved round it, the Needle will immediately be moved with the same Tenour: and if the Motion of the Loadstone be stopped, the Circulation of the Needle likewise stops; and it will again begin to revolve, as soon as the Loadstone is again moved: whence nobody can doubt, but that the circular Motion of the Needle depends on the Motion of the Loadstone. So likewise since the Flux and Reflux of the Sea happens at the same place when the Moon comes to the same Hour-Circle, and constantly observes its Motion; insomuch that a Period of the Tides so precisely answers to the Period of the Moon's Motions, that there has been found no Aberration for so many Ages; for it is retarded 48 Minutes every day, and at the Conjunction of the Sun and Moon, the Tides are always at the highest, and in their Quadratures at the lowest: Whence it must be granted, that the Flux of the Sea depends on the Motion of the Moon, and its Situation in respect to the Sun.

VIII.

VIII. Any *Body being moved in any Direction, all its Particles which are relatively at rest in it, proceed together in the same Direction with the same Velocity; that is, a relative Place being moved, that which is placed therein will be also moved.*

IX. Equal *Quantities of Matter carried along with the same Velocity, their Momenta or Quantities of Motion will be equal.*

For the Momentum of any Body is the Sum of the Momenta of all the Particles composing that Body; and therefore where the Magnitudes and Numbers of the Particles are equal, the Momenta will be equal.

X. Equal *and contrary Forces acting on the same Body, destroy their mutual Effects.*

XI. But *from unequal and contrary Forces there is produced a Motion equivalent to the Excess of the greater Force.*

XII. *A* Motion *produced from conspiring Forces, that is, acting in the same Direction, is equivalent to their Sum.*

XIII. If *what is equivalent be either augmented, or its contrary diminished, then it becomes the greater.*

They who would philosophize mechanically, exhibite the two following.

XIV. All *Matter is every where of the same Nature, and has the same essential Attributes, whether it is in the Heavens or on the Earth, whether it appears under the form of a fluid Body, or a hard, or of any other whatever; that is, the Matter of any Body, for example of Wood, does not differ essentially from the Matter of any other Body whatever.*

XV. But *the different Forms of Bodies are nothing but the different Modifications of the same Matter; and depend on the various Magnitude, Figure, Texture, Position,*

ſition, and other Modes of the Particles compoſing Bodies.

XVI. So *likewiſe the Qualities, or Actions, or Powers of ſome Bodies on other Bodies, ariſe only from the former Affections and Motion conjointly.*

But the Philoſophers ſuppoſe Matter to be the common Subject or *Subſtratum* of all Forms and Qualities; that it is indifferent to them all, ſince it is capable of them all, and remains the ſame under whatever Form it may appear; and whence it is called by the Peripateticks *Materia prima*, or the firſt Matter.

But although Forms and Qualities are altogether accidental to Matter, yet they neceſſarily and eſſentially belong to Body, which is made up of Form and Matter together: as, for example, although the Matter of Wood is altogether indifferent to this or that Form or Figure and Texture of Particles, which being varied infinite ways, remains the ſame; yet the Wood cannot ſubſiſt without that determinate Modification of Particles, which conſtitute the Form of a wooden Body; which being removed, the Wood periſhes, and the ſame Matter paſſes into a Body of another kind. But that the Form of a wooden Body conſiſts in the Modification of Particles, is manifeſt upon putting the Wood into the Fire, whereby its Matter is deprived of its Form; for by the Force of the Fire is diſſolved the Connection and Texture of Particles, and ſome part of them paſſes into Smoke and Vapours, and the other is reduced into Aſhes.

Many Examples are alledged by the Philoſophers, to ſhew that the various Magnitudes, Figures, and Textures of the Particles of the ſame Matter may produce various Forms of Bodies, and that various Qualities may ariſe from their various Motion and Poſition; of which we ſhall here give ſome Inſtances.

First

First of all, when the Particles of Water are rarefied by the Heat of the Sun, they are rais'd out of the Sea aloft into the Air under the form of Vapours; but this new Form is owing to nothing but the changed Situation of the Parts: for by Rarefaction it happens, that the Particles of Water contain in themselves more, and perhaps larger Spaces, either altogether empty, or only replete with a very rare Æther; whence their Matter takes up a larger Space than an equal Quantity of aerial Matter, and it is rendered less intensively dense than the Air, and therefore it is forced upwards after the same manner as Cork is, when it is immersed in Water: Nor do these Vapours ever stop, till they arrive at Air of the same Gravity, where they rest relatively, and compose Clouds that are of a thousand shapes.

Now when by the Course of the Winds the Air is rendered less heavy, the Vapours retaining the same Gravity, must necessarily subside, and in their fall being condensed by the Resistance of the Air, and driven into a less space, they lose their Form, and falling to the Earth, assume the Species of Rain.

Much the greatest part of this is carried to the Sea in Rivers, to be changed again into Vapours; but some part of it mixes with the Earth, and being there deposited, enters into the Roots and Seeds of Trees and Herbs, whence it arises into other and new Species of Bodies. And the same Rain-Water composes different Bodies, as it enters into the different Seeds of things; some passes into Plantanes, some into Grasses, others into Flowers, others into Oaks, Ash, Beech, and other innumerable sorts of Trees and Plants.

Nor in the same Plants does the Rain-Water remain altogether similar, since all Plants consist of innumerable heterogeneous Parts: so in Hemp, for example, the Root is of one form, the Stalk of another,

ther, the slender Fibres of another, the Flowers of another, the Seed of another, the Cells containing the Seed of another.

The Structure of Vessels in the same Hemp is also very various; (for, as in an animated Body, every Plant has its Vessels serving for the Circulation of the Humours) and these Parts are of very different Properties: the Stalk, for example, is a ligneous Body, and after being dryed, becomes very friable; whilst the Rind, or the Membrane covering the Stalk, consists of oblong, very slender and folding Fibres variously connected together.

The Hemp-dressers separate this Membrane from its Stalk, and after they have handled it a thousand different ways, they spin its Fibres into oblong Threds; and the Position and Situation of the Particles being changed, these Fibres take another and a very different Form from that which they had in the green Plant.

Then these Threds being wound up, their smallest Particles continuing the same, appear in the shapes of Bottoms. The Weavers variously connect and weave these Threds, and by their Art make out of them Webs, that yield Garments for Men. These, lastly, being reduced to Linen Rags, are soaked in Water, and by wooden Hammers reduced, as it were, into a soft Pulp; which at length, the aqueous Moisture being dryed up, is transmuted into Paper; which if it be put into the fire, is turned partly into a very fine Powder, and partly vanishes in Smoke.

But all these multifarious Forms under which the same Matter appears, arise from nothing but the changed Figure, Magnitude, and Texture of Particles, and only depend on them.

So when Metals are melted, the Coherence of their Parts is dissolved by the Force of the Fire, and the

the metallick Particles separated from one another, are hurried about with a most rapid Motion, whence they assume the Form of a fluid Body.

Hence also (as it seems) arises the Solution of Salts and Metals in Menstruums: for their Parts are separated from one another by Fermentation, and being resolved into their least Particles, they are agitated by the Motion of the Fluid itself, whence they will appear as fluid Bodies. From these Figures, and the other Modifications of Bodies and their Parts, there arise many Effects, many Qualities proper to each sort of Bodies, which must necessarily be destroyed if the Constitution of Parts is changed. So of the same Matter, as, for example, Iron, may be formed Keys, Plow-shares, Files, Saws, and innumerable other Instruments accommodated to various Uses, whose Qualities and Effects solely depend on their Figures: for whence have Keys the power of opening Doors, but from their Figure, Magnitude, and Congruity of Parts with the Parts of the Locks into which they are put? Whence have Wedges and Plow-shares their power of splitting and dividing Bodies? Have not the Writers of Mechanicks demonstrated, that they derive it from their Figure alone? Whence are those so regular Motions performed in Clocks or Watches, but from the Wheels being disposed amongst themselves, adapted to one another, and joined together? Whence, lastly, is it, that such great Effects are produced by artificial Machines? Certainly the Reason is to be deduced only from their Fabrick.

Nor are natural Bodies less indebted to the Constitution and Modification of their Parts than artificial ones; for all their Operations proceed only from the Motion, Situation, Order, Figure, and Position of their Corpuscles; which being changed in any Body, the Qualities of that Body are also actually changed.

If the Superficies of a Body is uneven and rough, it reflects every way the Light incident on it; because the superficial Parts receiving and remitting the Light, are not all in one and the same regular Superficies, but are placed in almost infinite, and those different Planes: whence the Light falling on these various Planes, must necessarily be reflected every way. Hence Ice, which when it is whole and smooth is almost of no colour, yet broken into pieces, and having rough and angular Parts, it appears white, *viz.* when it reflects Light copiously and on all sides. The Reason is also the same for Water's growing white, upon its being turned into Froth.

But such is the Structure of most visible Bodies, that their Superficies suffocate part of the Rays that fall upon them, and remit part. If their Superficies are so constituted, as to reflect or suffocate equally all sorts of Rays, their Colour would be white or black, or brownish, betwixt black and white; for the white Colour does no other ways differ from black, than as white Bodies reflect abundance of Rays of all sorts, but the black only a few. This appears from the Shade of an opake Body, that whilst the Sun shines, is cast on a white Wall; for that part of the Wall where the Shadow is, as it receives much fewer Rays than the other Parts, it also reflects much fewer Rays, so that in respect to the other Parts it appears black. But if those other Parts of the Wall had not received more Rays than that Part where the Shadow fell, it would be all of the same colour, *viz.* white.

If the Texture of the Superficies is such, that it reflects one sort of Rays more plentifully, and all the rest more sparingly, the Colour of the Superficies will approach to that, which arises from the Rays that are reflected in greater number. This may be hence demonstrated, that the Colour of the same Object

Object will be different, as it receives different ſorts of Rays, the reſt being intercepted; which was firſt diſcovered by the Great Sir *Iſaac Newton.* So if through a Glaſs Priſm red Rays (for ſo we call thoſe that produce a red Colour) are caſt on a blue Object, the Object will change its blue Colour, and put on a red one; but if it receives only yellow Rays, then its Colour will be changed into yellow: if the incident Rays are blue, it will appear blue; and that Colour will be more vivid than all the other Colours, becauſe it reflects many more of theſe Rays, and ſuffocates fewer of them than of the reſt.

If the Superficies of a Body is exactly ſmooth, that is, without any Roughneſs or Unevenneſs; and it reflects from it Rays in great plenty, it will reflect the Rays that proceed from any Object in ſuch a manner, as to afford a viſible Image of that Object. And for that reaſon, the Bodies having ſuch Superficies are called *Speculums.* If the Speculum is a Plane, the Image will be equal to the Object itſelf, and its place will be behind the Speculum at a Diſtance equal to that of the Object itſelf from the Speculum; if the Speculum be a ſpherical Concave, and the radiant Object is farther diſtant from it than the fourth part of the Diameter of the Sphere, the Image will appear hanging in the Air between the Object and the Speculum, and will be leſs than the Object; if the Object be placed in the Center, the Image will be alſo there, and equal to it; if the Object proceeds beyond the Center towards the Speculum, ſo that its Diſtance from it ſhall be a fourth part of the Diameter of the Sphere, the Image from the Speculum will paſs beyond the Center, and be greater than the Object: but when the Object ſhall have arrived at the Diſtance of one fourth of the Diameter of the Sphere, then the Diſtance of the Image will become infinite; but if it approaches but a very little nearer to the Speculum, the Image will be behind the Speculum, and greater than the Object itſelf. All theſe Phenomena that are ſo very different, proceed only from the Alteration of Diſtances, every thing beſides remaining the ſame.

Let us now conſider thoſe various and altogether contrary Effects, which ariſe from an Alteration only in Situation and Poſition, every thing elſe being in the ſame ſtate, except ſuch things as depend on the Mutation of the Situation.

All Philoſophers now acknowledge the Sun to be at reſt in the Center of his Syſtem, but the Earth, like the other Planets, to be carried about him in the ſpace of a Year; yet the Earth moves about the Sun in ſuch a manner, that its Axis remains always parallel to itſelf, not indeed perpendicular to the Plane of the Earth's Orbit, but inclined to it in an Angle of *66°. 30′.* And by reaſon of this Paralleliſm and Inclination, it follows that the Earth muſt ſometimes turn one of its Poles towards the Sun, ſometimes the other; and therefore all the Parts of the Earth will have different Situations in reſpect of the Sun. From this Change in its Situation, ariſe all thoſe Viciſſitudes of Seaſon that yearly happen, *viz.* Summer, Winter, Spring, and Autumn: for if the Axis of the Earth were perpendicular to the Plane of its Orbit, there would be no Alterations of Seaſons, no Differences in the Days and Nights, but every Part of the Earth would always receive, after the ſame manner, the equal Forces of the Sun's Rays.

But ſince every part of the Earth changes its Situation in reſpect of the Sun, and receives its Rays now more, at another time leſs oblique, now for a ſhorter duration, then for a longer; there will thence ariſe different and altogether contrary Appearances. As in the Autumn the Corn becomes ripe, and the Fruits mellow; yet the Fields by degrees

grees lose their green, and pleasant Colour, and the Leaves fall from the Trees. Afterwards the Winter approaching, all things become cold and horrid, the Snow covers the highest Mountains, and the Trees labour with its weight; and what is wonderful, the Waters of the Sea become stable and firm, and what before was only passable by Ships, now bears Tents and Armies.

But the Earth continually revolving in its Orb, each Part thereof changes its Situation in respect of the Sun, and what before was turned from, begins now to respect the Sun: and in the mean while the Snows disappear, the Fields are clothed with Grass, and the Trees adorned with Leaves, the Horse now no longer abides in the Stable, nor the Plowman at the Fire-side; but a new and joyful Face of Things appears, and the Year returns through the Summer to the Autumn.

Now since so many different, so many contrary Effects happen from the sole Alteration of the Situation, and so various Phenomena follow from it, all the other Causes remaining the same; certainly from the Position, Distance, Magnitude, Figure and Structure of the Parts composing Bodies, from the Motion and Subtility of Effluvia, from the Congruity of Bodies, and their Respect to other Bodies; I say, from all these joined and combined together variously, and almost after infinite manners, may proceed almost infinite different Forms, Affections, and Operations amongst themselves of Bodies, nor is any thing to be found in Nature, that does not depend on these. For if these are changed, the Forms, Qualities, and Operations of Bodies will be also changed. For example, it is certain that the magnetick Attractions and Directions arise from the Structure of Parts; for if a Loadstone be struck hard enough, so that the Position of its internal

Parts be changed, the Pole of that Loadſtone will alſo be thereby changed. And if a Loadſtone be put into the Fire, inſomuch that the internal Structure of its Parts be changed or wholly deſtroyed, then it will loſe all its former Virtue, and will ſcarce differ from other Stones.

But although it is in the general ſhewn, that the magnetick Operations do in ſome ſort proceed from the internal Conſtitution of Parts, yet the Modus of Operation, deduced from mechanical Principles, and ſuch as are eaſy to be underſtood, is not yet diſcovered. And what ſome generally boaſt of, concerning Effluvia, a ſubtile Matter, Particles adapted to the Pores of the Loadſtone, *&c.* does not in the leaſt lead us to a clear and diſtinct Explication of theſe Operations: but notwithſtanding all theſe things, the magnetick Virtues muſt be ſtill reckoned amongſt the occult Qualities.

From what has been ſaid, it follows, that the Qualities of Bodies which do not depend on their Forms, and which, the Quantity of Matter remaining the ſame, are not increaſed or remitted, but are in all ſorts of Bodies upon which we can make any Experiments; are the univerſal Qualities of all Bodies. For ſince they do not proceed from the Form or Modifications of Bodies, they muſt depend on the Matter itſelf; but ſince the Nature of all Matter is the ſame, and any part of it differs from another only by ſome Modes, the Qualities not produced by theſe Modes will be the ſame in all Matter.

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the Body B, is double that which is applied to move A, the Momentum of that will be alſo double of the Momentum of this; if the Force is triple, the Motion of the Body B will be likewiſe triple the Motion of the Body A; if but half the Force is impreſſed on the Body B, its Momentum will be half the Momentum of A: that is, ſince the Velocity of the Body A is univerſally to the Velocity of B, as the Force impreſſed on the Body A to the Force impreſſed on B, and as the Force impreſſed on the moveable Body A to the Force impreſſed on the Body B; ſo is the Momentum or Quantity of Motion in A, to the Momentum or Quantity of Motion in B: the Velocity of the Body in motion A will be to the Velocity of the Body in motion B, as the Motion of A to the Motion of the moveable Body B. *Q.E.D.*

Cor. If the Momenta are as the Velocities, the Quantities of Matter in the moveable Bodies will be equal.

THEOR. II.

In *comparing Motions, if the Celerities are equal, the Momenta or Quantities of Motion of Bodies will be as the Quantities of Matter in thoſe Bodies; or if the moveable Bodies are homogeneal, as their Magnitudes.*

Let A and B be two Bodies in motion, and each of them carried with the ſame Celerity C; I ſay, the Momentum of the Body A, is to the Momentum of the Body B, as the Quantity of Matter in A to the Quantity of Matter in B. For if the Quantity of Matter in A is double that in B, A may be divided into two parts, each whereof will have as much Matter, and conſequently [by Axiom 9.] as much Motion as B has: *viz.*

viz. since each Body is moved with the same Velocity, so that the Momentum of the Body A will be double the Momentum of the Body B. If the Quantity of Matter in A is triple of that in B, A may be divided into three parts, each whereof will have a Quantity of Motion equal to that which is in B; and universally, whatever proportion the Matter in A has to the Matter in B, the same proportion will have the Momentum of A to the Momentum of B, if each Body is carried along with the same Velocity.

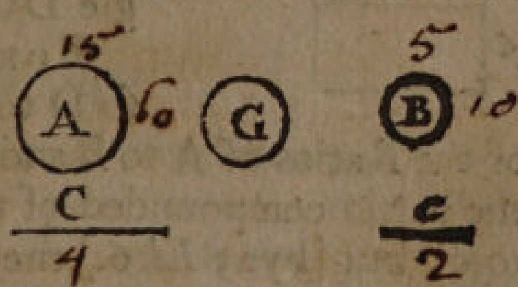
If the Bodies are homogeneal, their Quantities of Matter will be as their Magnitudes or Bulks; and consequently their Motion will be also in the same Ratio of their Magnitudes.

Cor. If the Momenta are as the Quantities of Matter, the Celerities of the Bodies will be equal.

THEOR. III.

In *comparing the Motions of any Bodies, the Ratio of the Momenta is composed of the Ratios of the Quantities of Matter, and of the Celerities.*

Let A and B be any two moveable Bodies, and let A be moved with the Celerity C, but B with the Celerity *c*; I say, the Momentum of A is to the Momentum of B, in a Ratio compounded of the Ratio of the Quantity of Matter in A, to the Quantity of Matter in B, and of the Ratio of the

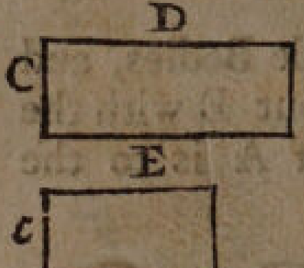


Celerity of the Body A, to the Celerity of the Body B. Let there be a third Body G, whose Matter is equal to that which is in A, but let it be moved with the Celerity of the Body B. From the Elements the Ratio of the Momentum of the Body A, to the Momentum of the Body B, is compounded of the Ratio of the Momentum of the Body A to the

Momentum of the Body G, and of the Ratio of the Momentum of the Body G to the Momentum of the Body B: but [by Theor. 1.] the Momentum of the Body A is to the Momentum of the Body G, as the Celerity C is to the Celerity *c*; and ſince G and B are moved with the ſame Celerity, the Momentum of the Body G will be to the Momentum of the Body B, as the Quantity of Matter in G or A to the Quantity of Matter in B. And therefore the Momentum of the Body A will be to the Momentum of the Body B, in a Ratio compounded of the Celerity C to the Celerity *c*, and of the Quantity of Matter in A or G, to the Quantity of Matter in B. *Q. E. D.*

Cor. 1. If the Bodies are homogeneal, the Ratio of the Momenta will be compounded of the Ratio of the Magnitudes and of the Celerities.

Cor. 2. If it be made as A to B, that is, as the Quantity of Matter in A to the Quantity of Matter in B, ſo is the right Line D to the right Line E; and let be compleated the Rectangles under D and C, and under E and *c*, the Momentum of the moveable Body A will be to the Momentum of the moveable Body B, as the Rectangle DC to the Rectangle E*c*.



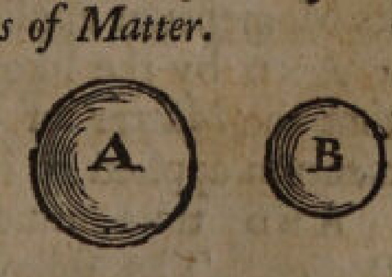
For becauſe it is as A to B, ſo D to E, the Ratio compounded of the Ratios of A to B and of C to *c*, will be equal to the Ratio compounded of the Ratios D to E and of C to *c*; but [by 23 *El.* 6.] the Ratio compounded of the Ratios of D to E and of C to *c*, is equal to the Ratio of the Rectangle DC to the Rectangle E*c*, and [by this third Theor.] the Ratio of the Momentum of the moveable Body A to the Momentum of the moveable Body B, is equal to the Ratio compounded of the Ratios of A to B, or of D to E, and of C to *c*; wherefore it will be as the Rectangle DC to the Rectan-

Rectangle E *c*, so is the Momentum of the moveable Body A to the Momentum of the moveable Body B. The Momentum therefore of any Body may be considered as a Rectangle produced from the Multiplication of the Bulk or Quantity of Matter contained in that Body into its Celerity.

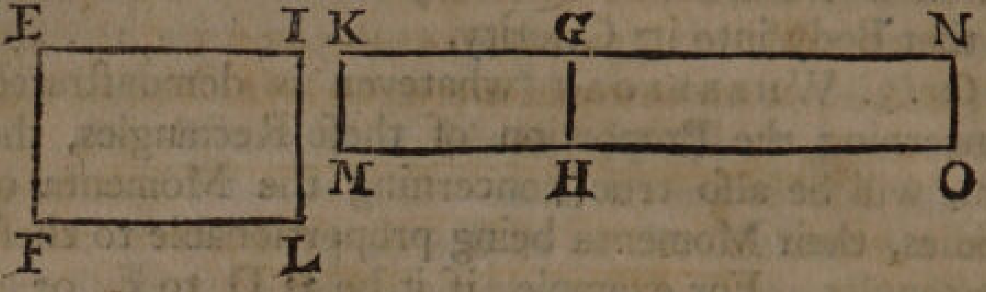
Cor. 3. Wherefore whatever is demonstrated concerning the Proportion of these Rectangles, the same will be also true concerning the Momenta of Bodies, their Momenta being proportionable to these Rectangles. For example, if it be as D to E, or as A to B, so is *c* to C, in this case the Momenta of the moveable Bodies will be equal: for rectangular Parallelograms having their Sides reciprocally proportionable, are [by 14 *El.* 6.] equal; and on the contrary, if the Rectangles are equal, their Sides will be reciprocally proportionable: that is, if the Quantities of Matter, or in Bodies of the same kind, their Magnitudes are reciprocally proportionable to their Celerities, their Momenta will be equal; and conversly, if the Momenta are equal, it will be as the Quantity of Matter in one to the Quantity of Matter in the other, so reciprocally the Celerity of this to the Celerity of that: hence also may be demonstrated the following.

THEOR. IV.

In *compared Motions, the Ratio of the Celerities is compounded of the direct Ratio of the Momenta, and of the reciprocal Ratio of the Quantities of Matter.*



Let A and B be two moveable Bodies, and let A be carried along with the Celerity C, but B with the Celerity *c*: I say, C is to *c*, that is, the Celerity of one of the Bodies A to the Celerity of the other B, is in a direct Ratio of of the Momentum of the Body A to the Momentum of the Body B, and a reciprocal Ratio of the Matter in A to the Matter in B, that is, directly as the Matter in B to the Matter in A. Let it be as A to



B, so the right Line EI to the right Line KG; and let IL be equal to C, but GH equal to *c*; and let be compleated the Rectangles EL, KH. By what has been said above, the Rectangles EL, KH, will represent the Momenta of the moving Bodies A and B respectively: to GH let be applied the Rectangle HN equal to the Rectangle EL. Since therefore HN is equal to EL, it will be [by 16 *El.* 6.] as IL to GH, so is GN to EI; but the Ratio of GN to EI is equal to the Ratio of GN to GK, and of GK to EI; that is, equal to the Ratios of the Rectangle HN or EL to the Rectangle KH, and of GK to EI. Wherefore the Celerity C or IL will be to the Celerity *c* or GH, in a Ratio compounded of the Ratio of the Momentum EL to the Momentum KH, and of the Matter GK to the Matter EI: that is, the Velocity of any Body is always as its Momentum applied to its Matter. *Q. E. D.*

And by the like manner of Reasoning it is found, that the Matter of any Body is always as the Momentum applied to its Velocity.

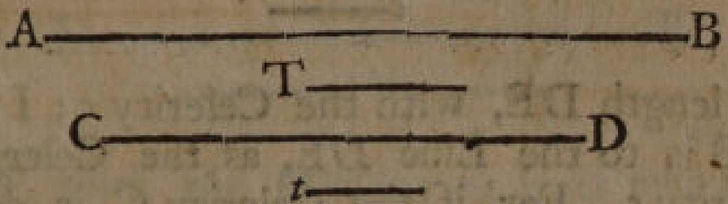
And thus much concerning the Momenta of Bodies, The following Theorems concerning the Proportion of the Spaces passed over by Bodies in motion, are also commonly demonstrated.

THEOR.

THEOR. V.

In *compared Motions, if the Celerities of the moving Bodies are equal, the Spaces run over by them, will be directly as the Times wherein the Motions are performed.*

Let a Body in motion run the length AB, in the Time T, with an equable and uniform Motion; also



let the same or another Body in motion, carried along with the same Velocity, run the length CD, in the Time *t*: I say, the Line AB is to the Line CD, as the Time T to the Time *t*. For if the Time T is double that of *t*, it may be divided into two Parts, each whereof will be equal to *t*; so that each Space run over with the same Velocity in these equal Parts of Time, will be equal to the Space run over in the Time *t*, and the two Spaces taken together will be double the Space run over in the Time *t*. In the same manner, if T is triple of *t*, T may be divided into three equal Parts, and the Spaces run over in each of these Times, will be equal to the Space run over in the Time *t*; and therefore the three Spaces taken together, will be triple the Space run over in the Time *t*. The same may be shewn of other Multiples and Submultiples: wherefore universally, whatever proportion T has to *t*, the same will the Space run over AB have to the Space run over CD. *Q. E. D.*

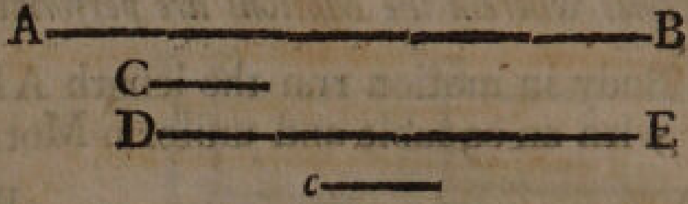
Cor. If the Times are as the Spaces run over, the Celerities will be equal.

THEOR. VI.

In *compared Motions, if the Times of the Motions are equal, the Spaces run over will be as the Celerities.*

Let

Let any moving Body run in a given Time the length AB, with the Celerity C; and in the ſame, or an equal Time, let the ſame or another moving Body



run the length DE, with the Celerity *c*; I ſay, the Line AB is to the Line DE, as the Celerity C to the Celerity *c*. For if the Celerity C is double of *c*, the Space AB run over with the Celerity C, will be double the Space DE run over with the Celerity *c*; if the Celerity C is triple of *c*, the Length AB will be alſo triple the Length DE; if C is the half of *c*, AB will be the half of DE: and univerſally, ſince equal Times are ſpent in running over the Lines, whatever proportion the Celerity C has to the Celerity *c*, the ſame will the Length run over AB have to the Length run over DE. *Q. E. D.*

Cor. If the Celerities are as the Spaces run over, the Times will be equal.

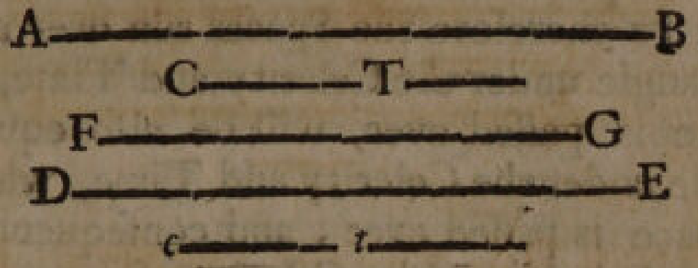
The two firſt Theorems, as alſo the fifth and this ſixth, might be demonſtrated univerſally by Equimultiples, after the manner of *Euclid*; but ſince they are ſo evident, that they may be looked upon as Axioms, they ſcarce need ſo great an Apparatus of Demonſtration.

THEOR. VII.

The *Lengths run over are in a Ratio compounded of the Ratios of the Times and of the Celerities.*

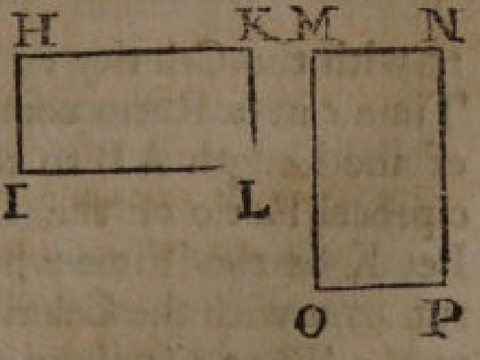
Let the Line AB be run over with the Celerity C, in the Time T; and the Line DE with the Celerity *c*, in the Time *t*: I ſay, the Ratio of AB to DE

DE is compounded of the Ratio of the Celerity C to the Celerity *c*, and the Ratio of the Time T to the Time *t*. Let the Line FG be run over in the



Time T, with the Celerity *c*; it is manifest, that AB is to DE in a Ratio compounded of the Ratios of AB to FG, and of FG to DE. But because AB and FG are run over in the same time, it will be as AB to FG, so the Celerity C to the Celerity *c*; but since the moving Bodies describe the Lines FG and DE with the same Celerity, it will be [by Theor. 6.] FG to DE, as the Time T to the Time *t*. Wherefore since the Ratio of AB to DE is compounded of the Ratios of AB to FG, and of FG to DE, it will likewise be compounded of the Ratios that are equal to these Ratios, *viz.* of the Ratio of the Celerity C to the Celerity *c*, and of the Time T, to the Time *t*.

Cor. 1. If HK be made equal to C, HI equal to T, also MN equal to *c*, and MO equal to *t*, and let be compleated the rectangular Parallelograms HL, MP; it will be AB to DE, as the Rectangle HL to the Rectangle MP: for [by 23 *El.* 6.] the Rectangle HL is to the Rectangle MP, in a Ratio compounded of the



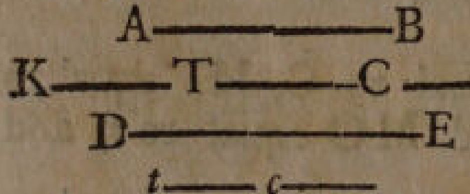
Ratios of HK to MN, and of HI to MO; but [by the preceding Theorem] the Space run over AB is is to the Space run over DE, in a Ratio compounded of theſe Ratios: whence theſe Spaces run over may be conſidered as Rectangles produced from the Times multiplied into the Celerities.

Cor. 2. If therefore the Spaces run over are equal, the Rectangle under the Celerity and Time, wherein one Space is paſſed over, will be alſo equal to the Rectangle under the Celerity and Time, wherein the other Space is paſſed over; and conſequently it will be as the Celerity to the Celerity, ſo reciprocally is the Time to the Time, [by 14 *El.* 6.] that is, if the Spaces run over are equal, the Times will be reciprocally as the Celerities.

THEOR. VIII.

In *compared Motions, the Ratio of the Times is compounded of the direct Ratio of the Length, and of the reciprocal one of the Celerities.*

This Theorem may be demonſtrated from the former, after the ſame manner as the fourth follows the third: but for the ſake of Perſpicuity, we ſhall thus briefly evince it.



Let the Length AB be run over in the Time T, with the Celerity C; alſo let the Length DE be run over in the Time *t*, with the Celerity *c*; I ſay, the Time T is to the Time *t* in a Ratio compounded of the direct Ratio of the Length AB to the Length DE, and of the reciprocal Ratio of the Celerity C to the Celerity *c*. Let K be the Time wherein the Length AB may be run over with the Celerity *c*, the Ratio of the Time T to the Time *t*, will be compounded of the Ratio of T to K, and of K to *t*; but [by the Corol. of the preceding Prop.] it is as T to K, ſo *c* to C (ſince the

the ſame Space is run over in each of the Times) and as K to *t*, ſo is [by the Cor. of Theor. 5.] the Length AB to the Length DE. Wherefore T will be to *t* in a Ratio compounded of the Celerity *c*, to the Celerity C, and of the Length AB to the Length DE; that is, the Times are in a Ratio compounded of the reciprocal Ratio of the Celerities, and the direct one of the Lengths. *Q. E. D.*

After the ſame manner may be ſhewn, that the Celerities are in a direct Ratio of the Lengths, and a reciprocal one of the Times.

Cor. 1. And hence it follows, that the Time is as the Space run over applied to the Celerity.

Cor. 2. The Celerity likewiſe is as the Space run over applied to the Time.

The third and ſeventh Theorem may be demonſtrated from this univerſal Theorem, *viz.*

If any Effects depend at the ſame time on ſeveral Cauſes, in ſuch a manner, that as they are augmented or diminiſhed in the ſame Ratio wherein any of the Cauſes are augmented or diminiſhed, thoſe Effects will be in a Ratio compounded of all the Cauſes: that is, if the Cauſes A, B, C, acting together do produce the Effect E, which, all the reſt remaining the ſame, is as ſome of the Cauſes; and if other Cauſes *a*, *b*, *c*, reſpectively like the former, and acting after the ſame manner, do produce the Effect *e*; it will be as E to *e*, ſo $A \times B \times C$ to $a \times b \times c$. Which may be eaſily ſhewn, by almoſt the ſame Method that we uſed in the preceding Demonſtrations.

After the like manner, if the ſame Effect depends on ſeveral things at once, whereof ſome aſſiſt or augment the ſame, in that Ratio whereby they are augmented; but others impede or diminiſh it, in the ſame Ratio whereby they are augmented: the Effect will always be directly as the aſſiſting Cauſes, and reciprocally as the impeding or diminiſhing Agents.

The seventh Theorem is thus demonstrated in the Style of Sir *Isaac Newton*:

The Celerity being given, the Space run over is as the Time; and the Time being given, the Space run over is as the Celerity: wherefore neither of them being given, it is as the Celerity and Time conjointly.

So likewise may the eighth Theorem be shewn.

The Celerity being given, the Time is directly as the Space run over; and the Space being given, the Time is reciprocally as the Celerity: wherefore neither of them being given, the Time will be directly as the Space and reciprocally as the Celerity.

In like manner may the third and fourth Theorem be explained, and we shall sometimes make use of this Method for the sake of Brevity.

LECTURE X.

In the Demonstrations of the preceding Lecture we have shewn a Method, whereby Physical Matters may be first reduced to Geometry, and afterwards to Arithmetick: for since it is there demonstrated, that the Motions of Bodies are as Rectangles under their Celerity and Quantity of Matter; from the Celerity and Quantity of Matter of any Body being given, there will be given its Momentum, *viz.* equal to a *Factum* or Product made of the Celerity of the Body into its Quantity of Matter. As, for example, let the Body A be 8 parts, but B be 6 parts, the Celerity of A as 5, and the Celerity of the Body B as 3; the Motion of of the Body A will be 40, and the Motion of the Body B but 18 Parts.

So from the given Momentum and Quantity of Matter of any Body, may be known its Celerity; *viz.* if its Momentum be divided by its Quantity of Matter, the Quotient will give its Velocity. For let the Motion in the Body A be 40 parts, and its Quantity of Matter 8 parts; let alſo the Motion in the Body B be 18 parts, and its Quantity of Matter 6 parts: divide 40 by 8, the Quotient will give 5, *viz.* the Velocity of the moving Body A; and by dividing 18 by 6, the Quotient will give 3, the Velocity of the moving Body B.

Since by Examples things become more plain, and Numbers are to be made uſe of in Practice, that the young Beginners may accuſtom themſelves the better to theſe, let us illuſtrate the Science of Motion by Numbers, and make uſe of both ſpecious and numerical Arithmetick; for from ſpecious Arithmetick are diſcovered general Canons, which afterwards may be applied to particular Numbers.

So let A denote the Quantity of Matter in any given Body A, but C ſtand for its Celerity, and let its Momentum be called M; or rather let theſe Letters be put for the Numbers that are proportionable to thoſe Quantities: then it will be $C \times A = M$ and $C = \frac{M}{A}$ and $A = \frac{M}{C}$.

In like manner, ſince the Space run over is always proportionable to the Rectangle under the Celerity and the Time; if the Space be called S, the Time T, and Celerity C, it will be $S = C \times T$; and $C = \frac{S}{T}$; and $T = \frac{S}{C}$; and therefore ſince it is $M = A \times C$, it will alſo be $M = \frac{A \times S}{T}$: or if T is given, it will be $M = A \times S$; that is, the Momentum of any Body is as its Matter drawn into the Space run over by it in a given Time. Many other Conclusions likewise, which some deliver as the Laws of Motion, may be deduced from what has hitherto been demonstrated; but since any Tyro may easily of himself discover them all, it is not worth while to produce them here.

From what has been demonstrated, it is manifest, that the Momentum of any Body arises from the Motion of all its Parts: for in each Particle of the Body there is an Impetus or Force of moving, and from the Sum of these Forces is compounded the Impetus or Quantity of Motion of the whole Body.

Hence also may be gathered, that by how much the greater is the Quantity of Matter in Bodies, by so much the greater ought the Force to be, in order to move those Bodies with a given Velocity, and therefore their Momenta will be greater in the same Ratio. If therefore there are two Bodies carried along with the same Velocity, the Quantities of Matter in them will be always as their Momenta; so that if Bodies which are equal in Bulk, and moved with the same Velocity, have unequal Momenta, it is necessary that the Quantities of Matter in them are unequal: and that which has the less Momentum, will have more Pores or Spaces either altogether empty, or replete with some Matter which does not partake of the Motion of the whole Body, whose Pores it is supposed to fill. So, for example, if there are two Globes, one of Cork, the other of Lead, both of the same Magnitude, and moved with the same Velocity, since it is known by Experience, that the Momentum of one is much greater than the Momentum of the other, it must follow that there are many more Pores in one than in the other; which, it must be allowed, are either altogether empty, or replete with some most subtile Matter, which may so freely pass through its Pores, as not to partake of the Motion of the Body whose Pores it possesses.

But that this Matter may pervade the Pores of other Bodies, without partaking of their Motion, it is necessary that all Bodies should have all their Pores extended in right Lines parallel to the Direction of Motion; *viz.* that there may be no Reflections of the subtile Matter against the Sides of the Pores; otherwise the Matter, although ever so subtile, will be moved together with the Body, whose Pores it is supposed to fill. The subtile Matter therefore cannot but partake of the Motion of the Body, unless the Body moved be so disposed, that it has its Pores parallel to the Direction of Motion; but since its Situation may be varied infinite other ways, that is, the Lengths of the Pores may be inclined to the Line of Direction in infinite Angles; and therefore all these things being supposed, on the Body being moved, the subtile Matter placed in its Pores will be moved along with it: therefore a subtile Matter cannot so freely pervade the Pores of Bodies, but it must partake of their Motion; consequently the Body being moved, the Matter also contained in it will be moved, however subtile it be. If therefore a Body of Cork is moved, it carries along with it the Matter contained in its Pores; so that since it has a less Momentum than a Globe of Lead of the same Magnitude, and moved with the same Velocity, the Quantity of Matter in the Body of Cork will be less, and consequently there will be more Pores or Spaces absolutely empty.

From what has been demonstrated, may be deduced the following Theorem.

THEOR. IX.

THE *Weights of all ſenſible Bodies near the Superficies of the Earth, are proportionable to the Quantities of Matter contained in them.*

FOR, as is manifeſt from many Experiments of Pendulums, all Bodies falling perpendicularly by the Force of Gravity (abſtracting the Reſiſtance of the Air) deſcribe equal Spaces in the ſame Time. For *in Vacuo* or a Medium that has no Reſiſtance, the ſmalleſt Feather takes up no more time in falling than a heavy Piece of Lead; ſo that the Velocities of all Bodies falling in a given Time, are equal; their Momenta therefore will be proportionable to the Quantities of Matter contained in them: but the Forces generating Motion are always proportionable to the Motions or Momenta generated, and therefore in this caſe they will be as the Quantities of Matter in the Bodies moved; but the Forces that generate theſe Motions, are the Gravitations of Bodies, that is, their Weights. Therefore the Weights of all Bodies are proportionable to the Quantities of Matter contained in thoſe Bodies. *Q. E. D.*

Cor. 1. THE Weight therefore of any Body, will be augmented or diminiſhed, from the Quantity of its Matter being only augmented or diminiſhed.

Cor. 2. WHEREFORE the Quantity of Matter continuing the ſame in any given Body, its Weight will alſo continue the ſame; and however its Figure, or the Texture of the Particles compoſing that Body be varied, yet its Weight will not be changed: ſo that the Weight of no Body depends on its Form or Texture.

SINCE [by Axiom 14.] the Nature of all Matter is the ſame, neither does one Body differ from another, but in its Modes, as by the Figure of its Parts,

Parts, Situation, and the like Forms; the Affections of Bodies, which do not depend on their Forms, will be the fame in all Bodies: fo that fince (as has been faid) the Weights of Bodies do not arife from their Forms, but depend on their Quantity of Matter; in equal Quantities of Matter, at the fame diftance from the Earth, the Gravitations towards the Earth will be equal: but if the Weights of two Bodies are unequal, the Quantities of Matter contained in them will be alfo unequal.

Let us fuppofe now two Globes of equal Magnitudes, the one of Lead, the other of Cork; if the Quantity of Matter in both was the fame, (by what has been fhewn) both Bodies would equally ponderate: for the fubtileft Matter occupying the Pores of the Cork, would ponderate equally with the Matter of Lead that is equal to it. But fince there is a great difference in the Weights of thefe two Bodies, there will be alfo a great difference in the Quantity of their Matter; and if Lead is thrice heavier than Cork, the Matter contained in the Lead will be triple of that in the Cork: fo that there will be more Pores or Spaces abfolutely empty in the Cork, than in the Lead. A *Vacuum* therefore is not only poffible, but actually given; which was to be proved. And hence it follows, that the Quantity of Matter in any Body may be properly eftimated by its Gravity.

Since the Momentum may be augmented, as well by increafing the Quantity of Matter, the Velocity remaining the fame, as by increafing the Velocity, the Quantity of Matter remaining the fame; the Antients (who were ignorant of the Force of Gunpowder to give a fwift Motion to Bodies) to beat down their Enemies Walls, made ufe of Machines fo framed, that a huge Mafs of Matter might batter the Walls, though not with a great Velocity, yet with a violent Impetus: but now-a-days fmall Bullets are hurled out of Cannon with a great Velocity, by the Explosion of Gunpowder. But although the warlike Engines of the Antients were much inferiour to ours, yet their Force to overturn Walls was almoſt incredible: for their Battering-Rams were compoſed of vaſt Beams joined together, whoſe Weight may be gattered from hence, that ſome of them required ſix thouſand Men (*viz.* that ſome might ſucceed others) to direct them, and to give them their Motion. That part, whereby the Wall was battered, was made of ſolid Iron, and they were ſo ſlung by Ropes, (I mean the compounded Rams) that their Lengths were parallel to the Horizon; whence being drawn backwards by a great number of Men, and immediately driven forwards by their Weight, and the Forces of the Men acting together, they ſtruck the Walls with their Iron End: and, according to the Teſtimony of *Joſephus*, there were no Towers ſo ſtrong, or Walls ſo thick, as to ſuſtain their conſtant Shocks.

In Machines that raiſe Weights by the Circumgyrations of Wheels, ſometimes the Wheels are rendered heavier by the addition of Lead, *viz.* that the greater Quantity of Matter may receive the greater Impetus or Quantity of Motion; whereby it may the better oppoſe the Reſiſtance, ariſing as well from the Air as from the Friction of the Matter, and the longer preſerve their Motion; which therefore once begun, will be eaſily continued.

From the ſame Principle alſo it is, that Spinners by putting on heavy Wharls on their Spindles, their Gyrations are continued longer: ſince the Part of the Motion loſt by the Reſiſtance of the Air, bears a leſs proportion to the Motion increaſed by the addition of Matter, than that which it had to the Motion not thus increaſed.

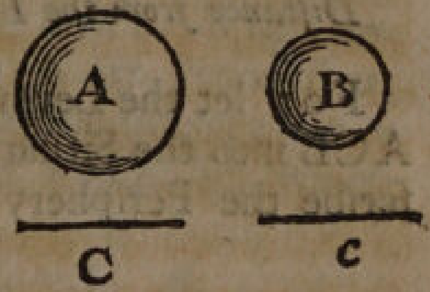
From what has been ſaid, may be ſolved the following Problem.

PROBL.

PROBL. I.

To *find the Velocity, wherewith a given Body must be moved, so that it may have a Momentum equal to any given Momentum.*

Let the given Body be A, whose Momentum ought to be equal to the Momentum of the Body B, moved with the Celerity *c*; let it be as A to B, so the Celerity *c* to another Quantity C; this will be the Velocity sought, wherewith if the Body A is moved, its Momentum will be equal to the Momentum of the Body B, as is manifest from the third Corollary of the third Theorem.



For the Momenta of Bodies are equal, if their Celerities are reciprocally proportionable to those Bodies; but, by Hypothesis, the Celerity of the Body B is to the Celerity of the Body A, as the Body A to the Body B: whence the Momentum of the Body A will be equal to the Momentum of the Body B. *Q. E. I.*

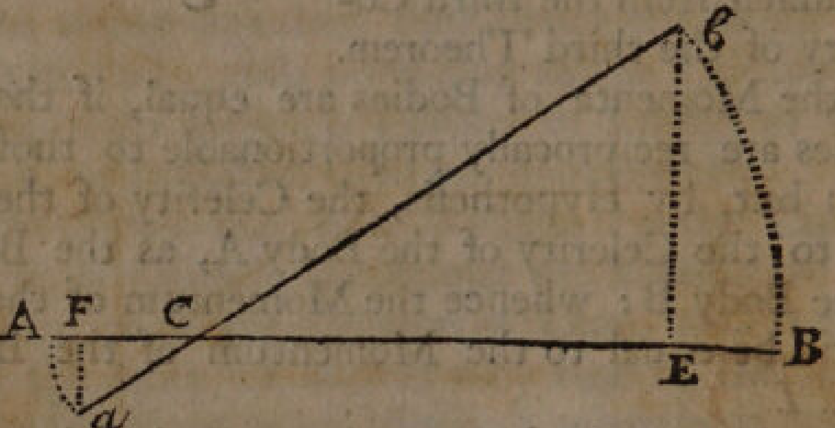
And hence it follows, that a Body, tho ever so small, may have a Momentum equal to the Momentum of a Body, tho ever so great, that is moved with a given Velocity. On this Principle depend all the Engines which are contrived to draw or raise Bodies: for if the Engines are so disposed, that the Velocity of the Power shall be to the Velocity of the Weight, as that Weight is to the Power; I say, in that case the Power will sustain the Weight. We will illustrate this in the five simple mechanick Machines. And first of all in the Lever, which we shall here consider as an inflexible Line, whether a right one or a curve, or composed of many right Lines, that is moveable about a fixed Point, of no Gravity indeed of itself, yet accommodated to sustain and raise Weights.

The fixed Point by which the Lever is sustained, and on which it moves round, is called its *Fulcrum* or Prop.

THEOR. X.

Let AB *be a Lever only moveable round the Fulcrum* C, *the Space described by each of its Points, will be as its Distance from the Fulcrum.*

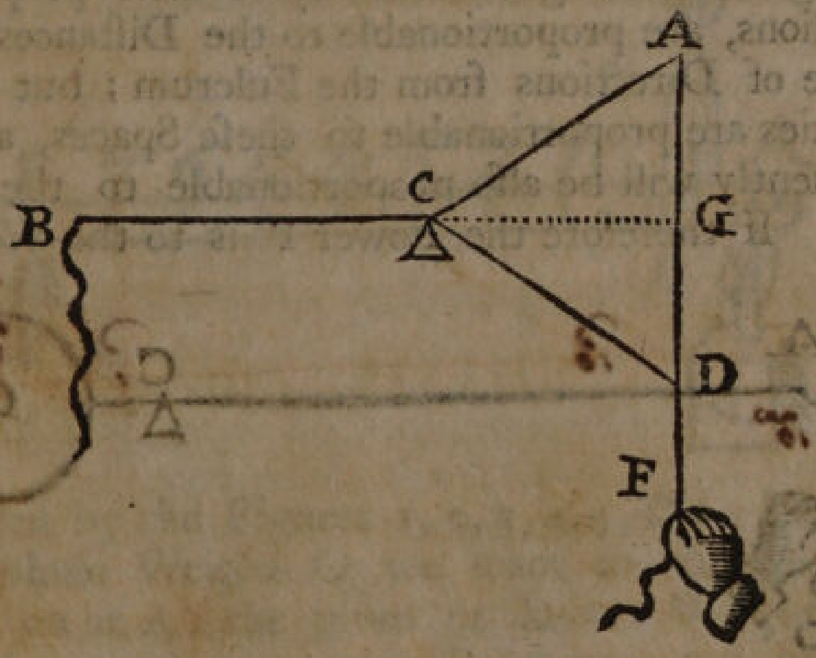
For let the Lever be moved out of the Situation ACB into the Situation αCβ, the Point A will describe the Periphery Aα, but B will pass over the



Periphery Bβ. Now by reason of the similar Sectors ACα, BCβ, Aα is to Bβ, as AC to BC; that is, the Spaces described by the Points A and B, are as their Distances from the Fulcrum. If to the Points A and B be applied Powers drawing the *Brachia* of the Lever perpendicularly; the Spaces that are described by them according or contrary to their Propensions, are not the Peripheries Aα, Bβ, but the Perpendiculars αF, βE, let fall on the Brachia of the Lever. For the Power in A is moved, according to its proper Direction or Propension, through the Space αF only, and no farther; as, for the same cause,

cause, the Way passed through by the Power B, according to its proper Direction, is to be estimated by βE. But by reason of the equi-angular Triangles αCF, βCE, αF is to βE as αC or AC to βC or BC; that is, the Spaces run over by Powers according to their proper Directions, will be as their Distances from the Fulcrum.

BUT if the Direction of the Power is not a right Line, perpendicular to the Brachium of the Lever AC, let from the Fulcrum to the Line of Direction be drawn the Perpendicular CG, and the Space described by the Power according to its Propension, will be proportionable to that Perpendicular: for it matters not, whether the Thred FGA, by which



the Power acts, is affixed to the Point G or A, or indeed to the Point D; for the Line of Direction remaining the same, its Force to move round the Plane ADCB will be the same, as if the Thred was fixed to the Point G, and the Way described by it in a given Time, according to its proper Direction, will be proportionable to the right Line CG. Wherefore it is manifest in every case, that the Way described

described by any Force according to its proper Direction, is proportionable to the Distance of the Line of Direction from the Fulcrum.

THEOR. XI.

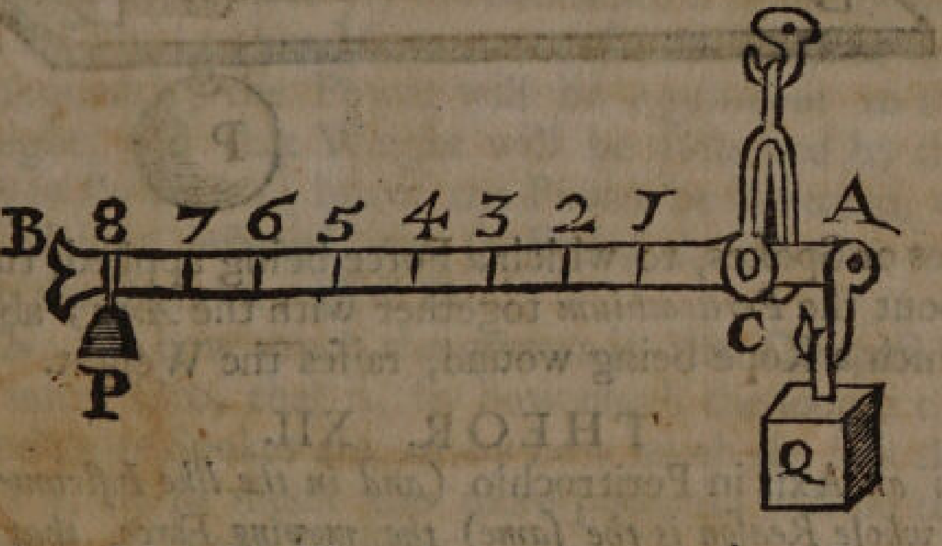
In *a Lever, the moving Force or Power that has to the Weight the same Ratio, which the Distance of the Line of Direction of the Weight from the Fulcrum, has to the Distance of the Direction of the Power from the Fulcrum, will sustain the Weight; and therefore if it be ever so little increased, it will raise the Weight.*

It is manifest from the preceding Theorem, that the Spaces which are described by a Power and Weight according or contrary to their proper Propensions, are proportionable to the Distances of the Line of Directions from the Fulcrum; but the Velocities are proportionable to these Spaces, and consequently will be also proportionable to the Distances. If therefore the Power P is to the Weight Q

as CQ, the Distance of the Direction of the Weight from the Fulcrum to CA, the Distance of the Direction of the Power from the Fulcrum, the Power will be to the Weight, as the Velocity of the Weight to the Velocity of the Power; the Momentum therefore of the Power [by Cor. 3. Theor. 3.] will be equal to the Momentum of the Weight: and consequently, the

the Power will be equivalent to the Weight; which if it be ever ſo little increaſed, it will raiſe the Weight. *Q. E. D.*

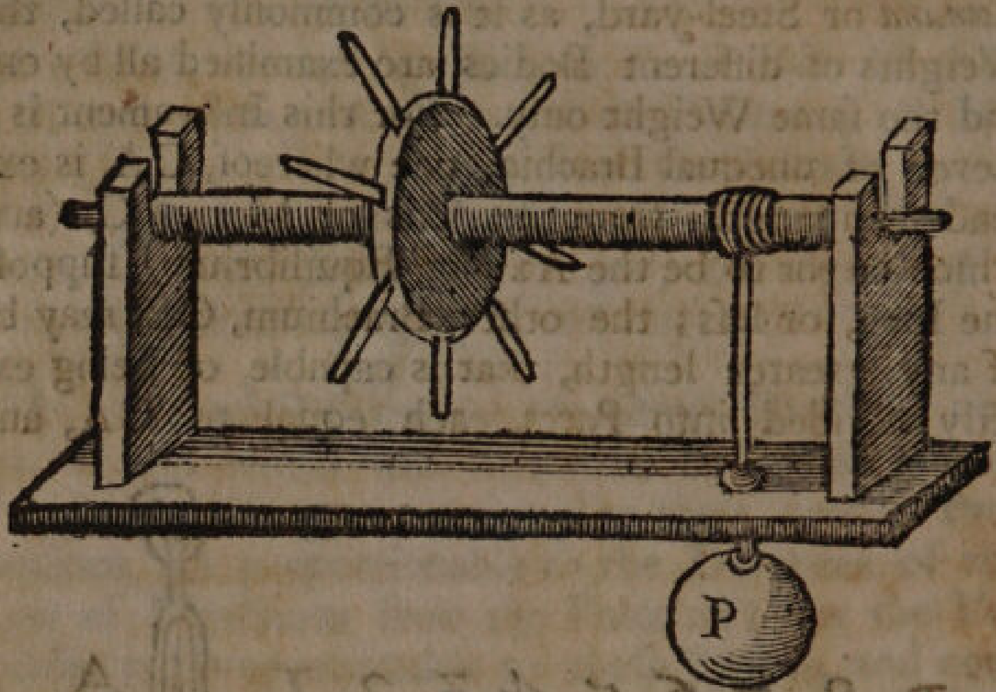
HENCE appears the Reaſon, why by the *Statera Romana* or Steel-yard, as it is commonly called, the Weights of different Bodies are examined all by one and the ſame Weight only. For this Inſtrument is a Lever of unequal Brachia, one whereof, CA, is extended in length, from the Axis of Motion C, (and which ought to be the Axis of Equilibrium) ſuppoſe one Inch, or leſs; the other Brachium, CB, may be of any greater length, that is capable of being exactly divided into Parts, each equal to CA, and



numbered by the Figures 1, 2, 3, 4, 5, &c. Let the Body whoſe Weight Q we want to diſcover, be hanged on at A; the given or known Weight P is moveable on the contrary Brachium, and by removing it from or bringing it nearer the Center C, is diſcovered the Diſtance where is an exact Equilibrium: as, for example, if the Weight P at the Diſtance 8 equiponderates the Weight Q in A, it is thence concluded (by reaſon the Weights are reciprocally proportionable to their Diſtances) that the Weight Q is eight times the known Weight P.

Defin. THERE is a mechanical Inſtrument proper to raiſe Weights, called an *Axis in Peritrochio.* It is compoſed

composed of a Cylinder, that is named the *Axis*, supported at each end by *Fulcra*. About this Cylinder is fixed a Wheel, (which they call *Peritrochium*) and in its Circumference are fastened several Han-



dles or Spokes, to which a Force being applied, turns about the *Peritrochium* together with the *Axis*; about which a Rope being wound, raises the Weight.

THEOR. XII.

In *an* Axis in Peritrochio (*and in the like Instruments, whose Reason is the same*) *the moving Force, that has to the Weight to be sustained the same Ratio, as the Circumference of the Axis to which that Weight is applied, has to the Circumference of the outer Orb to which the Power is applied, will be equivalent to the Weight; which therefore being ever so little increased, will raise the Weight.*

It is manifest from the Make of the Instrument, that in one Conversion the affixed Weight P will be raised to an height that is equal in length to as much of the drawing Rope, as will once encompass the Axis; which therefore is supposed to be equal to its Circumference: and in the mean while the Power applied

applied to the Extremity of one of the Spokes, will have proceeded through the Circumference of the outer Orb, described by the Power in the same Revolution of the Instrument; (that is, the Space passed over by the Power in the same Time, will be equal to the Circumference of the outer Orb) so that the Velocities of the Power and of the Weight, which are as the Spaces passed over in the same Time, will be as the Circumference of the outer Orb, and the Circumference of the Axis. Wherefore if the Weight is to the Power, as the Circumference of the outer Orb is to the Circumference of the Axis, the Velocity of the Power will be reciprocally to the Velocity of the Weight, as the Power to the Weight. Therefore [by Corol. 3. Theor. 3.] the Momentum of the Power will be equal to the Momentum of the Weight; and consequently, the Power will be equivalent to the Weight, and that Weight will be sustained by the *Axis in Peritrochio*: but if the Power be increased, or the Weight lessened ever so little, the Power will raise the Weight. *Q.E.D.*

Cor. By how much the greater is the Compass of the outer Orb, that is, by how much the longer the Handles or Spokes are, or by how much the less the Axis is, by so much the more powerful will be the Force to raise the Weight.

Defin. An Instrument composed of one or more Wheels aptly disposed, moveable about their Axes, and about which Wheels a Rope being put, draws up the Weight, is called a Pulley.

THEOR. XIII.

In *a moveable Pulley, from the Position of the Wheels may be computed what the Force ought to be, which will be equivalent to an affixed Weight; namely, that Force, which is to the Weight, as* 1 *to the Number of Ropes, by which the Weight is suspended, will be able to sustain that Weight: which Force being therefore ever so little increased, will raise the same Weight.*

Fig.

Fig. 1. Let one of the Ends of a Rope be faſtened to a Hook B, and in its Duplicature let be hung a moveable Pulley, to whoſe Block let be fixed the Weight Q; it is manifeſt, that in order to raiſe the

Fig. 1.

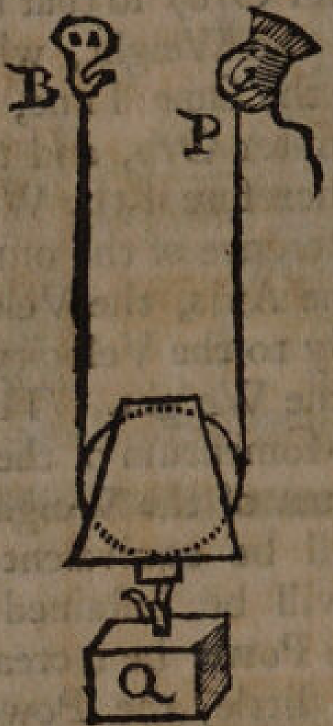


Fig. 2.

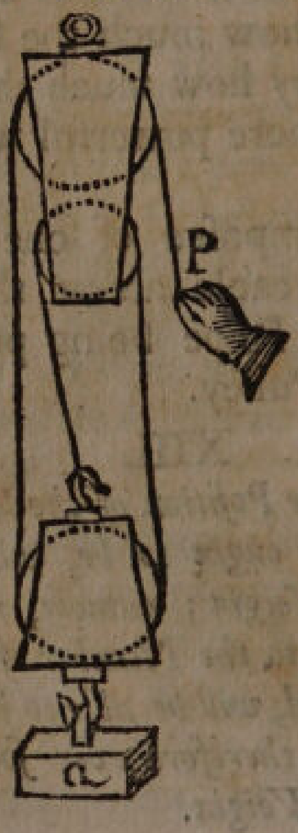


Fig. 3.

Weight

Weight Q one Foot, each Rope sustaining the Block together with the Weight, (reckoning downwards from the Hook) ought to be shortned also one Foot: that is, to raise the Weight one Foot, the Power must move through two Feet; whence in this Instrument, the Way of the Power will be double that of the Weight; and therefore the Celerity of the Power will be likewise double that of the Weight: so that if the Power is to the Weight as 1 to 2, its Momentum will be equivalent to the Momentum of the Weight, and it will sustain the Weight.

Fig. 2. If the Wheels are so disposed, that the Weight Q hangs from three Ropes; and that the Weight may ascend one Foot, all the three Ropes (we call them three, tho indeed they are but the Continuation of one uninterrupted Rope) must be shortened one Foot each, which can be done no otherwise, than by the Power P being moved through the Space of three Feet. Whence since in this Instrument the Way of the Power is triple that of the Weight, its Celerity will be triple the Celerity of the Weight; so that if the Power is to the Weight as 1 to 3, its Momentum will be equivalent to the Momentum of the Weight.

Fig. 3. By the same way of Reasoning, from the third Figure it is manifest, the Power P, which is but the fourth of the Weight Q, will be equivalent to it. In all Cases, the Power that is equivalent to the Weight, if it be ever so little increased, or the Weight diminished, the Power will then raise the Weight. *Q.E.D.*

Defin. A Right Cylinder cut in equally in a helical Form, is called a *Skrew*, whereof there are two sorts, *Male* and *Female*. It is named a male Skrew, when the convex Surface of the Cylinder is cut in; and a female Skrew, when the concave. The first is also called the outside Skrew, as the other is called

the

the inside one. But the female Skrew ought to be so fitted to the male, that their Parts may answer to each other, (*viz.* that the Eminences of the one do agree with the Cavities of the other) whereby, if the

female Skrew is fixed, the male may be moved quite through it; or if the male Skrew is fixed, the female may be freely moved over it. Skrews are chiefly employed to remove Obstacles, to break in pieces or to compress any thing, and to effect other Motions that are to be produced by Pulsion: the Power is applied to an Handle or Handspike.

THEOR.

THEOR. XIV.

In *the Skrew, if it be as the Compass, which the Force or Power applied, passes over in one Revolution of the Skrew, to the Distance between two helical Threds that are contiguous, (reckoned according to the Length of the Skrew) so is the Weight or Resistance to the Power; then the Power will be equivalent to the Weight, and that Power ever so little increased, will move the Impediment.*

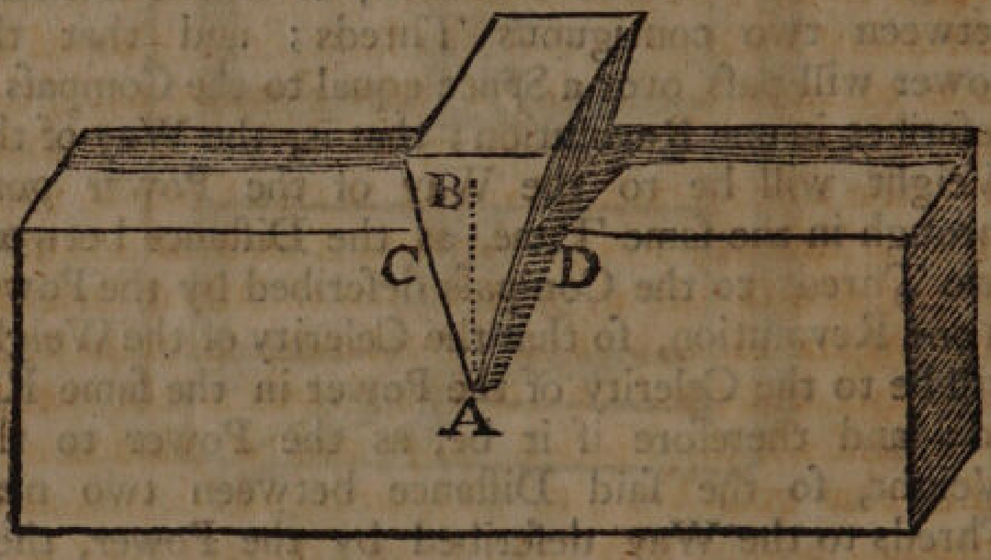
Let CA be a male Skrew, which being turned round by means of the Handle CB, may be passed through a female Skrew, and at the same time raise a Weight P. It is manifest by inspecting the Instrument, that in one Revolution of the Skrew, the Weight will be raised as much, as is the Distance between two contiguous Threds; and that the Power will pass over a Space equal to the Compass it describes in one Revolution; that is, the Way of the Weight will be to the Way of the Power gone through in the same Time, as the Distance between two Threds to the Compass described by the Power in one Revolution, so that the Celerity of the Weight will be to the Celerity of the Power in the same Ratio: and therefore if it be, as the Power to the Weight, so the said Distance between two near Threds to the Way described by the Power, then the Power will be equivalent to the Weight; which being ever so little increased, it will overcome the Resistance. *Q. E. D.*

Defin. A Wedge made of Iron, or some hard Substance, is frequently in use. Its Form is that of a Prism of a small Height, whose opposite Bases are Isosceles Triangles; the Height of these Triangles is the Height of the Wedge, the Base of the Triangle is also called the Thickness of the Wedge, and the right Line which joins the Vertices of the Triangles, is the Edge of the Wedge; and the Parallelogram that joins the Bafes of the Triangles, is called the Back of the Wedge.

THEOR. XV.

A Power *applied directly to the Back of the Wedge, which is to the Refiftance that is to be overcome by that Wedge, as the Thicknefs of half the Wedge to its Height, will be equivalent to the Refiftance; and therefore if it be increafed, it will overcome that Refiftance.*

Let the Refiftance to be overcome by a Wedge, be, for example, the Tenacity or Toughnefs of Wood, or any other Obftacle that is to be cleaved by a Wedge; it is manifeft, that when the Wedge is driven in as far as it appears in the Figure, the



Way of the Power, or the Length run through according to its Propenfity is B A; for fo far, and no farther, is it gone. And after the fame manner, the Way of each Impediment is half C D, and when the Wedge is drove through its whole Height, the Obftacle is divided by the whole Breadth of the Wedge; and confequently each part of the Obftacle is moved half the Breadth of the Wedge; and proportionably in its whole Progrefs, as appears from the Nature of a Triangle. Whence, if the Power be to the Resistance on each side the Wedge, as half its Thickness to its Height, the Resistances will counterballance the Power, so that if the Power be increased, it will overcome the Resistance.

SCHOLIUM.

HENCE it appears, that the Force of the Power is not increased by the means of mechanical Instruments, which is impossible to be done; but by the application of the Instrument, the Velocity of the Weight that is either to be raised or drawn along, is diminished in such a manner, that the Momentum of the Weight becomes less than the Momentum of the Power. So, for example, if any acting Force is able to raise a given Weight of one Pound with a given Velocity, it is impossible, by the means of any Instrument whatever, that the same Force should be made to raise a Weight of two Pounds with the same Velocity; yet it may, by the help of an Instrument, be able to raise a Weight of two Pounds with half that Velocity: nay, the same Power will raise a Weight of a thousand, or even ten thousand Pounds, with the thousandth or ten thousandth part of that Velocity; but the Force of the Power is not thereby increased: the Motion only that it produces in raising that great Weight, is equal to the Motion that is produced when a Weight of one Pound is raised.

FROM what has been said, the Reason is plain, why in Canals of different Capacities, which communicate with one another, the Equilibrium of Liquors is preserved. For let ABCD be a large Canal communicating at C with a smaller one MNKH; if Water be poured in, it will rise in both Canals to the same height, and the Endeavour to descend, or the Force that the Water in the Canal FH has to flow through the Orifice C, is equal to the Force of the

Water in the Canal AC to descend through the same Orifice. For if we suppose, that the Water has descended in the Canal AC through the Height AI; it is necessary, that the Water in the Canal FH do rise to the Height HN, namely, to such a Height, that the Cylinder of Water MFGN be equal to the Cylinder AILD, *viz.* the Cylinder of Water, which descends in the Canal AC: but in equal Cylinders, the Bases and Heights are reciprocal [by 15 *El.* 12.] that is, it will be FM to AI, as the Orifice AD to the Orifice MN or FG; but it is FM to AI, as the Velocity of the Ascent of the Water in the Canal FN, to the Velocity of the Descent of the Water in the Canal AC; and the Orifice AD is to the Orifice MN, as the Water in AC to the Water in the Canal FH, for Cylinders of the same height are as their Bases. Whence the Velocity of the Water ascending in the Canal FH, will be to the Velocity of the Water descending in the Canal AC, as the Water in the Canal AC to the Water in FH; that is, the Velocities of the Waters are reciprocally proportionable to the Quantity of the Waters themselves, and therefore the Momenta of the Waters will be equal: but they are contrary, whence there will follow no Motion.

Hence, by the bye, appears the Reason why Water, or indeed any Fluid, flowing out of a larger into a narrower Channel, is moved with a greater Celerity.

Hence in an Animal Body, if the ſmall Branches of the Arteries, or the Capillary Arteries, have the Sum of their Orifices, or rather of their tranſverſe Sections, greater than the tranſverſe Section of the great Artery or Aorta, from which they all ariſe; the Velocity of the Blood will be leſs in the Extremities of the Body, than in the Aorta: but if this Sum be equal to the tranſverſe Section of the Aorta, the Velocity of the Blood in the Extremities will be equal to the Velocity of the Blood in the Aorta; if the Sum is leſs, the Velocity of the Blood circulating through the extreme Arteries, will be leſs than in the Aorta.

LECTURE XI.

Of the Laws of Nature.

We have hitherto given the Demonſtrations of Theorems relating to the Quantity of Motion, to Spaces paſſed over by moving Bodies, as likewiſe the Corollaries that might be deduced from them; now then we ſhall proceed to the Laws of Nature: namely, ſuch Laws, as it is neceſſary that all natural Bodies do obey. And we ſhall deliver theſe in the ſame Order, and in the very ſame Words, as they are laid down by the Illuſtrious Sir *Iſaac Newton*; the firſt whereof is this:

LAW I.

Every *Body will continue in its State of Reſt, or of moving uniformly in a right Line, except ſo far as it is compelled to change that State by Forces impreſſed.*

SINCE natural Bodies confift of a Mafs of Matter, that of itfelf is not able to induce any Change in its State; if Bodies were once at Reft, it is neceffary that they fhould always remain in that State of Reft, unlefs there is applied a new Force to produce Motion in them: but if they were in Motion, the fame Energy or Force would always preferve the Motion; and therefore Bodies would always retain their Motion, and would always proceed forward in the fame right Line with the fame Tenour, fince they cannot of themfelves acquire either Reft, or a Retardation, or a Change of their Direction to turn on one fide or the other. There are fome Philofophers, who readily enough acknowledge that no Body can move of itfelf, that is, pafs from Reft to Motion of itfelf; but then they are not as willing to grant, that Bodies once moved cannot of themfelves arrive at Reft, by reafon they fee the Motions of Projectiles languifh by degrees, and at laft the moving Bodies themfelves come to Reft.

BUT as no Mode, or Accident, can of its own accord or by itfelf be deftroyed, and as all Effects produced by tranfient Caufes do remain always, unlefs there be fome new and extraneous Caufe that deftroys them; fo likewife Motion once commenced will be continued always, unlefs it is hindered by fome external Caufe: nor is it more in the power of a Body once moved, to lay afide its Motion or Energy to move, and return of itfelf to Reft, than it can put off the Figure that it has been once formed into, and acquire a new one, without fome extrinfick Caufe.

THERE is befides in all Bodies a certain Force, or rather Inactivity, whereby they oppofe every Change; whence it is, that they are very difficultly put out of their State, whatever it is: but that Force is the fame in moving Bodies, as in thofe at Reft, Reſt, nor do Bodies leſs reſiſt the Action, whereby they are brought from Motion to Reſt, than that whereby they paſs from Reſt to Motion; that is, there is not required a leſs Force to put a ſtop to the Motion of any Body, than was before neceſſary to impreſs that Motion on the ſame Body. Whence, ſince the *Vis Inertiæ* or Inactivity of Matter always equally reſiſts equal Changes, it will not be leſs powerful to continue a Body in Motion, which has begun to move, than to preſerve a quieſcent Body in the ſame State of Reſt.

THERE are ſome Philoſophers who ſuppoſe Body of its own nature to be as indifferent to Motion as to Reſt; but by this Indifference they do not, I believe, mean ſuch a Diſpoſition in Bodies, whereby they do not in the leaſt reſiſt Reſt or Motion: for on this Suppoſition it would follow, that any Body, however great, and moved with the ſwifteſt Velocity, might be ſtopped by any the ſmalleſt Force; or if the great Body was at reſt, it might be moved by any Body, however ſmall, without the leaſt Loſs of Velocity in the impelling Body: that is, any ſmall Body impinging on a greater one, would carry that greater Body along with it, without the leaſt Loſs of its Motion; and each Body after the Impulſe would be jointly carried along with that Celerity, that the ſmall Body had at firſt, which, we all know, is abſurd. This Indifference therefore is not placed in a Non-reſiſtance to Motion from a State of Reſt, or to Reſt from a State of Motion; but in this only, that a Body of its own nature is not more propenſe to Motion than to Reſt, nor more reſiſts to paſs from a State of Reſt to Motion, than to return again from that Motion to the ſame State of Reſt: beſides, any quieſcent Body may be moved by any Force, and an equal Force acting in a contrary Direction, will be able to deſtroy that Motion; and in this, they would have this Indifference placed.

SINCE, according to this Law of Nature, a Body once in Motion always continues in that Motion, the Philoſophers ask why all Projectiles loſe by degrees their Motion? (which Motion they are pleaſed to call violent.) Why do they not proceed *in infinitum?* If Motion did not of its own nature decay, a Stone thrown at the beginning of the World would by this time have gone through an immenſe and almoſt infinite Space. And ſo indeed it would, if its Motion had been *in vacuo* or in free Spaces, and without any Gravity. But ſince all Projectiles are carried either through the Air, or on the rough Superficies of other Bodies, they muſt be neceſſarily retarded: for ſince all Bodies in motion muſt drive and thruſt out of its place the reſiſting Air, or overcome the Roughneſs of the Superficies upon which they are moved; they will loſe all that Force and Motion, that is conſtantly employed in overcoming theſe Obſtacles, and conſequently the Motion of Projectiles will be continually diminiſhed: but if there was no Reſiſtance in the Medium, no Roughneſs in the Superficies on which they were moved, no Gravity that continually forces the Bodies towards the Earth, Motion would always continue the ſame, without any Retardation at all. So in the Heavens, where the Medium is exceedingly rare, the Planets do continue in their Motions for a very long time; and upon Ice, or any other very ſmooth Surface without any Roughneſs, heavy Bodies in motion are not ſoon brought to reſt.

LET the Philoſophers at length leave off inquiring into the Cauſe of the Continuation of Motion; for there is no other beſides that firſt Cauſe, which does not only preſerve Motion, but every thing elſe in its *Being*, namely, the All-wiſe and Great God. Nor is Motion continued any other way, than that whereby by is preserved the Figure, the Colour, and any other the like Affections of Body, which always remain the same, unless they are altered by any external Force.

These Philosophers would have done much better, and more suitable to the Rules of good Method, if they had inquired out the Reasons of the Decay or Loss of Motion: but there are some so blind in this Affair, that they suppose that very thing to be the Cause of the Continuation of Motion, from which indeed proceeds its Decay.

Let the Philosophers also leave off disputing concerning the Communication of Motion; for from what has been said, may be understood why a Stone goes with so great an Impetus out of the Hand of the Caster; namely, when the Stone is held in the Hand, it is necessary that it partakes of the Motion of that Hand [by Axiom 8.] so that it will be carried in the same Direction with the Hand: but every natural Body once moved, continues in the same Motion [by the above-mentioned Law] until it is hindered by some external Agent; whence, when the Caster draws back his Hand, but not drawing back the Stone, it will proceed right forwards. After the same manner, if a Ship or Boat is swiftly driven by the Winds or Oars, those that sit in her, have the same swift Motion communicated to them; but if the Ship stop of a sudden, every thing that is placed in that Ship endeavours to continue its Motion, and such things as are not firmly fixed to the Ship, do after the Ship is at rest, leaving their places, still move on; and hence there is danger, lest Men relatively at rest in a Ship, after so sudden and as it were violent Change of its State, are not thrown along, namely, since the Motion which they before received from the Ship, is not yet destroyed.

If a Stone is ſwiftly whirled about in a Sling, it deſcribes a Circle with the ſame Velocity as the bottom of the Sling has, wherein the Stone is placed; but ſince every Body affects to proceed in a right Line, the Stone in every Point of its Orb would go out in a right Line, touching that Orb in the Point wherein it is placed, if it was not hindered by the String: ſo that if the String be let go, broke, or any other ways deſiſts from detaining the Stone, the Stone would no longer move in a Circle, but in a right Line, excluding the Motion ariſing from its Gravity.

That Endeavour, which the Stone thus whirled about has in any Point of its Orb, of running out in the Tangent, ſtretches the String by which it is detained in that Orb; and that Force, whereby the String is ſtretched, ariſes from the centrifugal Force, whereby, namely, it endeavours to recede from the Periphery. This Tenſion any one may eaſily experience in a Sling; and it is found by Obſervation, that the ſwifter the Stone is whirled about, or the greater the Weight is that is put into the Sling, the greater will be the Tenſion of the String, or it will be ſtretched the more.

For this reaſon, ſome Philoſophers will have this centrifugal Force to proceed from Gravity alone, but neither Reaſon nor Experience favours this Opinion; for in a Sling, the String is not only ſtretched when the Stone moves through the loweſt part of its Orb, but likewiſe when it deſcribes the upper part: which cannot ariſe from Gravity, ſince Gravity in the upper part of its Orb can only draw the Stone towards the Center, which is directly contrary to the centrifugal Force, that compels it to recede from the Center. Beſides, if the Stone is whirled about in a Circle parallel to the Plane of the Horizon, the String is then alſo ſtretched; but Gravity can no ways

ways produce that Tension, since the Stone is neither carried upwards or downwards, whose Motion can therefore be neither increased or diminished by this Gravity: the centrifugal Force consequently does not arise from Gravity, but only from that Endeavour, which all Bodies have of proceeding in the same right Line.

If we suppose the Earth to revolve about its Axis, all of us who live upon its Surface must revolve together with it; so that if its Motion should suddenly cease, all things not firmly fixed to it, would be thrown from it with a violent Motion. So likewise if it is carried with an annual Motion about the Sun, and that Revolution should be suddenly stopped, all things would be thrown from it, and, like Planets, would revolve about the Sun, by the same Cause, as the Earth itself was moved before about the Sun.

Since the Earth turns about its Axis, and all things on it describe Circles parallel to the Plane of the Equator, the Philosophers demand how it comes to pass, that all Bodies are not thrown off from its Surface, when by the Law of Nature all Bodies affect a Motion in a right Line? So indeed they would be thrown off, if they were not detained on the Earth by another Force, which is Gravity, and which is much more potent than the centrifugal Force.

If a Vessel full of Water is placed on any horizontal Plane, and suddenly pushed forward by any great Force, the Water in the Vessel will at the beginning be seen to tend towards the Parts that are contrary to the Motion of the Vessel: not that in reality such a Motion is impressed on the Water, but since it endeavours to continue in the same State of Rest, the Vessel cannot immediately communicate its Motion to the Water contained in it; and therefore the Water deserted by the Vessel, and truly at rest, will

seem

ſeem to change its relative place. But at length, after the Motion of the Veſſel is communicated to the Water, and that together with the Veſſel begins to move uniformly and with the ſame Celerity, if then the Veſſel be ſuddenly ſtopped, yet the Water will endeavour to continue in the ſame Motion, and riſing up the Sides of the Veſſel, part of it will even run over.

If a Ship is toſſed by a Storm and a troubled Sea, the Men that are ſitting in her, and relatively at reſt, will be ſeized with Pains, Sickneſs, a Nauſea and Vomiting, eſpecially if they have not been uſed to the Sea; ſince the Liquids contained in their Stomachs, Inteſtines, Blood-Veſſels and other Ducts, do not immediately obſerve the Toſſings of the Ship, whence the Motion of the Fluids in the human Body is diſturbed, and Diſeaſes do ariſe.

LAW II.

The *Change of Motion is always proportionable to the moving Force impreſſed, and is always made according to the right Line, in which that Force is impreſſed.*

This follows from the fourth Axiom; for if any Force generates a certain Motion, a double Force will generate a double Motion, and a triple one a triple Motion: and this Motion, becauſe it is determined always into the ſame Direction with the moving Force, (for it ſolely ariſes from it) it will be always made according to the ſame Direction [by the firſt Law,] nor can the Body decline according to any other Direction, unleſs ſome new Force oppoſes the former: ſo that if the Body was moving before, the Motion produced from the impreſſed Force will either be added to the conſpiring Motion, or ſubducted from the oppoſing Motion, or obliquely joined to the oblique Motion, and will be compounded with it according to the Determination of both.

If any Force produces a Motion in a given Body, [by the first Law] that Body will always continue in its Motion: but if afterwards the same or an equal Force again acts on that same Body according to the same Direction, the Motion thence arising will be equal to the former, and therefore the Sum of the Motions will be double that former; if the same Force a third time acts in like manner on the same Body, the Motion hence occasioned will be also equal to the first, and therefore the Sum of the Motions will be triple the Motion first impressed: and if the same Force should act again on the same Body as before, the Sum of all the Motions will be quadruple the Motion first impressed, and so on continually.

Hence if this new Force should act continually and equally, in equal Intervals of Times, the Motion thence arising would be as the Sum of the Times in which it was generated: so that, since by reason the Body is given, the Motion is as the Velocity, the Velocities so generated will be as the Times from the beginning of the Motion, and the Motion will be equally accelerated. Hence the following Theorems may be easily demonstrated.

THEOR. XVI.

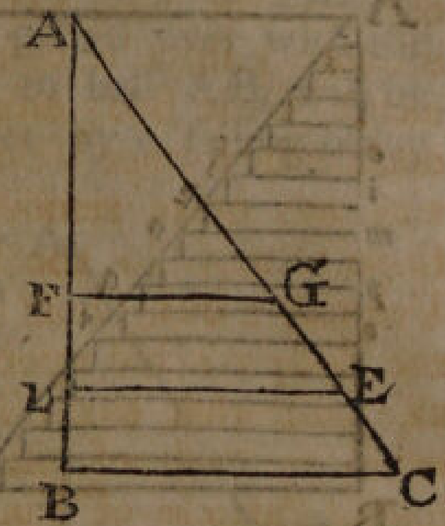
If *Bodies at all Distances from the Earth gravitate equally, the Motion of Bodies falling by their Gravity in the same right Line, will be an equably accelerated Motion.*

Let the Time in which the Body is falling, be supposed to be divided into equal and very small Particles, and let the Gravity acting in the first Particle of Time draw the Body towards the Center; if now after that first Time the Action of Gravity should cease, and the Body be no longer heavy, nevertheless the Motion received from the first Impulse would be always continued, and the Body would equally descend to the Earth, [by the first Law;] but since the Body is constantly heavy, and Gravity acts without intermission, also in the second Particle of Time the same Gravitation will communicate to it another Impulse equal to the former, and the Velocity of the Body after these two Impulses will be double the former Velocity; and if the Force of Gravity was quite removed, yet the Body would continue to move in the same right Line with the same Velocity: but since in the third Particle of Time the Body is acted upon by the same Gravity, it will also after that third Time acquire another Motion equal to each of the former; so likewise in the fourth Particle of Time the Gravitation will add to the heavy Body a fourth Impetus equal to each of the former: and so of the rest. The Impetus therefore or Motions of a given Body acquired by its Gravity, are as the Particles of the Time elapsed from the beginning: so that since the Action of Gravitation is continual, if those Particles are taken infinitely small, the Motion of the falling Body acquired by its Gravity, will be as the Time elapsed from the beginning of the Fall; and since the Body is given, its Motion will be as its Velocity, therefore the Velocity will be always as the Time in which it was acquired. Insomuch that in equal Intervals of Time, a falling Body will receive an equal Increase in its Velocity, and consequently its Motion will be uniformly accelerated. *Q. E. D.*

In like manner, from the same Principles may be demonstrated, that the Motion of Bodies thrown upwards is equably retarded; since the Force of Gravity, acting constantly and equally against the Motion commenced, in equal Times will equally diminish its Motion, even till all the Motion upwards is quite destroyed.

Cor.

Cor. Let the right Line AB represent the Time in which a Body is falling, and BC making with AB a right Angle, represent the Velocity acquired at the end of the Fall; join AC, and through any Point D let be drawn DE parallel to BC: this will be as the Velocity acquired at the end of the Time AD. For (by reason of the similar Triangles ABC, ADE) AB is to AD as BC to DE; but BC represents the Velocity acquired in the Time AB, whence (since the Velocities are as the Times) DE will represent the Velocity acquired at the end of the Time AD: in like manner, FG will represent the Velocity in the Point of Time F, and in every Point of Time the Velocities will be as the right Lines drawn within the Triangle through it, and parallel to the Base BC.



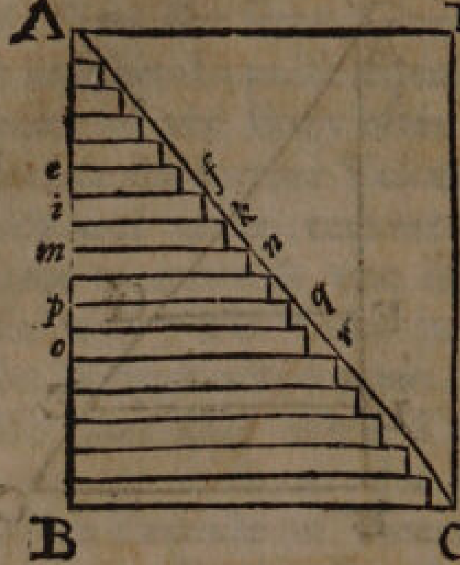
THEOR. XVII.

If *a heavy Body descends from Rest with a Motion uniformly accelerated, the Space, that from the beginning of its Motion is run over by the Body in a given Time, will be half the Space that may be uniformly run over in that Time, with a Velocity equal to that which the heavy falling Body will have acquired at the end of that Time.*

Let AB be the Time in which the heavy Body is a falling, and let BC be the Velocity acquired at the end of the Fall; compleat the Triangle ABC, and Rectangle ABCD; moreover, let the Time AB be distinguished into innumerable Particles, *ei*, *im*, *mp*, &c. draw *ef*, *ik*, *mn*, *pq*, &c. parallel to the

Base;

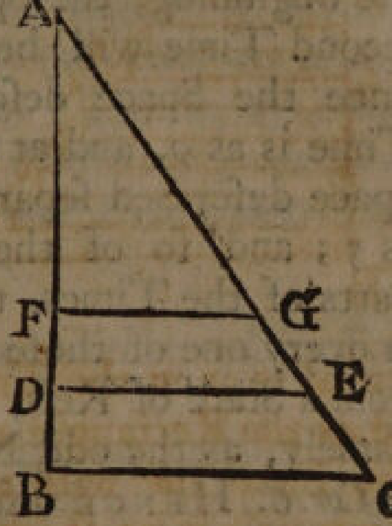
Base; [by the preceding Corol.] ef will be as the Velocity of the heavy Body in the infinitely small Particle of Time ei; and ik will be its Velocity in the Particle of Time im; also mn will be its Velocity at the Point of Time mp; and so pq will be the Velocity in the Particle of Time po: but [by Cor. Theor. 7] the Space passed over in any Time, and with any Velocity, is as a Rectangle under that Time and Velocity: whence the Space



passed over in the Time ei with the Velocity ef, will be as the Rectangle if; so the Space passed over in the Time im with the Velocity ik, will be as the Rectangle mk; so likewise the Space passed over in the Time mp with the Celerity mn, will be as the Rectangle pn: and so of the rest. Wherefore the Space passed over in all these Times, will be as all these Rectangles, or as the Sum of all these Rectangles; but since the Particles of Time are infinitely small, the Sum of all the Rectangles will be equal to the Triangle ABC: but [by the Corol. of Theor. 7.] the Space passed over by a moving Body in the Time AB with an uniform Celerity BC, is as the Rectangle ABCD. Whence the Space passed over in a given Time by a heavy Body falling from Rest, will be to the Space passed over in the same Time with an uniform Velocity, equal to that which is acquired by the heavy Body at the end of its Fall, as the Triangle ABC to the Rectangle ABCD; but the Triangle ABC is half the Rectangle ABCD, whence the Space that is passed over in a given Time by a falling Body from the beginning of its Fall, will be be half that Space which it would have passed over in the same Time, with an uniform Velocity equal to that which it has acquired at the end of its Fall. *Q.E.D.*

Cor. 1. The Space which is passed over with the Velocity CB in a Time equal to half AB, will be equal to the Space passed over by a falling heavy Body in the Time AB.

Cor. 2. From the Demonstration itself it follows, that as the Space passed over in the Time AB may be represented by the Triangle ABC, so the Space described by the heavy Body in the Time AF may be represented by the Triangle AFG; also the Space gone through in the Time AD, may be expounded by the Triangle ADE.



Cor. 3. The Spaces passed over, reckoning from the beginning of the Fall, are in a duplicate Ratio of the Times: for the Space passed over in the Time AB, is to the Space passed over in the Time AF, as the Triangle ABC to the Triangle AFG; but (by reason of the similar Triangles ABC, AFG) the Triangle ABC is to the Triangle AFG in a duplicate Ratio of the Side AB to the Side AF: so that the Space passed over in the Time AB will be to the Space passed over in the Time AF in a duplicate Ratio of the Time AB to the Time AF. The Spaces therefore passed over by a heavy Body from the beginning of its Fall, are as the Squares of the Times in which they are run thorough.

Cor. 4. Hence if a heavy Body falling from a State of Rest, passes over a certain Space in a given Time, the Space passed over in a double Time will be quadruple the former Space; and the Space run over in a triple Time will be nine times greater than the Space first run over, &c. that is, if the Times are taken as 1, 2, 3, 4, 5, &c. the Spaces described in these Times, reckoning from the beginning of Motion, will be as 1, 4, 9, 16, 25, &c.

*Cor.*5. SINCE the Space passed over in the first Time is as 1, in the second as 4, computing from the beginning, the Space described separately in the second Time will be as 3. After the same manner, since the Space described at the end of the third Time is as 9, and at the end of the second as 4, the Space described separately in the third Time will be as 5; and so of the rest: taking therefore equal Parts of the Times, the Spaces described separately in every one of those Times by a heavy Body falling from a State of Rest, will be as 1, 3, 5, 7, 9, 11, &c. namely, as the odd Numbers.

*Cor.*6. HENCE also, since the Velocities acquired by a falling Body are as the Times, the Spaces passed over will be likewise as the Squares of the Velocities: and the Velocities, as well as the Times, will be in a subduplicate Ratio of the Spaces through which the heavy Body falls, computing from the beginning of the Motion.

LEC-

LECTURE XII.

LAW III.

RE-ACTION *is always equal, and contrary to Action; or the Actions of two Bodies upon each other are always equal, and in contrary Directions. That is, by Action and Re-action equal Changes of Motion are produced in Bodies acting upon each other, and these Changes are impressed towards contrary Parts.*

THIS Law will be better illustrated by Examples.

1. IF one Body impinges on another Body at rest, whatever Motion is impressed on the Body at rest, just the same Quantity of Motion is substracted from the impinging Body. For example, if the Body A, whose Motion is twelve Parts, is carried towards the Body B, and if after

the Impulse, it shall have communicated to B five Parts of its Motion, then will there remain to A but seven Parts of Motion, so that the Changes which happen to both Bodies will be equal; and the Effect will be exactly the same, as if a Force equivalent to five Parts of Motion should push the Body B towards C, and another Force equal to the former should act on the Body A, and should press it the contrary way towards D.

2. IF the Body B should not be at rest, but tend towards C, and the Body A moving faster, should impinge on it; the Body A would lose just the same

Quantity of Motion as the Body B would gain, and the Changes of Motion produced by the Impulſe on both the Bodies (that is, the Increaſe of Motion in one, and the Decreaſe in the other) will be equal.

3. If the Bodies A and B meet each other, A being carried towards C with twelve Parts of Motion, but B towards D with three Parts of Motion; whatever Change of Motion happens to the Body B, the very ſame will befall the Body A: for example, if after the Concourſe B is carried towards C with two Parts of Motion, the Change of Motion that is made in it will be five Parts; namely, equal to the Sum of the two Motions, *viz.* of that whereby it was before carried towards D, and which was deſtroyed by the Impulſe of the Body A, and of that which it then received, and wherewith it tends towards C; and the Motion loſt in the Body A, will be exactly equal to theſe five Parts of Motion: ſo that (as in the firſt Example) the Effect will follow exactly the ſame, as it would be, if a Force with five Parts of Motion ſhould puſh B towards C, and another Force equal to the former ſhould be impreſſed on the Body A, and preſs it towards the Parts D.

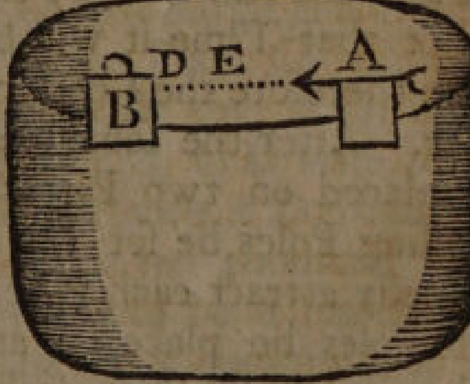
And univerſally, the Magnitude of a Stroke which ariſes from the Concourſe of two Bodies, is always equally received by both the Bodies; whence the Changes of Motion produced by the Stroke in both the Bodies, will be always equal.

So if an Iron Hammer ſtrikes againſt a Glaſs, the Blow is received equally by the Hammer as by the Glaſs, and the Glaſs is broke, when at the ſame time the Hammer remains whole, not becauſe the Force of Percuſſion impreſſed on the Glaſs is greater than that is which is received by the Hammer, but becauſe the Parts of the Iron being harder, and cohering more firmly together, do more ſtrongly reſiſt the ſame Force of Percuſſion, than the brittle and leſs cohering

cohering Parts of the Glass can do. Just after the same manner, if a Body is fastened to a Wall by a fine Thred, a small Force will be sufficient to pull it from thence; but if the same Body is fastened by a strong Rope, the former Force equally applied, will little avail to draw it away.

4. IF a Horse draws forwards a Stone tied to a Rope, the Horse also will be equally drawn backwards towards the same Stone; for the Rope stretched both ways, by the same Endeavour of relaxing itself, will equally strain the Stone towards the Horse, and the Horse towards the Stone: whence the Forces of Attraction, as well in the Horse as in the Stone, will be equal; but since the Firmness and Force of the Horse standing upon the ground is so great, that he is able to resist the drawing of the Rope, he will not in the least yield to the drawing Rope, nor by its Force be removed out of his place; but the Stone, wherein there is not so great a resisting Force, will be moved towards the Horse.

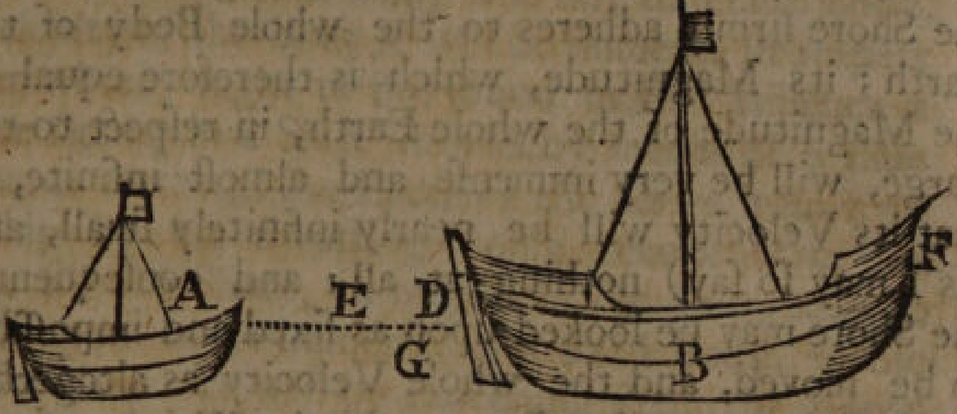
5. WE find by Experience, that in magnetick Attractions the Loadstone does not only draw the Iron, but is in like manner equally drawn by the Iron: For let a Loadstone be placed on a piece of Cork B, and likewise the Iron A on another piece of Cork, and let them both swim in Water; then let the Loadstone be held by any one's Hand, and we shall see the Iron advance to the Loadstone; and if the Iron is kept immoveable, we shall find the Loadstone will come to it: but if both the Bodies are permitted to float freely on the Water, we shall behold both the Loadstone and the Iron mutually



meet each other, and the Force of Attraction acts equally upon them both, producing equal Motions in each: I ſay, the Motions will be equal, but not the Celerities, unleſs the Iron and the Loadſtone weighed alike; for if they are of different Weights, that which weighs the moſt, will have the leaſt Celerity. For example, if the Loadſtone is ten times heavier than the Iron, the Iron reciprocally will have ten times a greater Velocity; namely, that there may be generated equal Quantities of Motion in each Body: ſo that the Loadſtone and the Iron will not meet in the middle Point E, but in the Point D, which will ſo divide the Diſtance BA, as BD will be to DA as the Weight A to the Weight B. So in the preſent Example, if BD is the eleventh part of the whole Diſtance, the Point D will be where the Loadſtone and the Iron will meet each other: for ſince BD is the eleventh part of the Diſtance BA, BD will be to DA as 1 to 10; but as 1 to 10, ſo (by what was ſaid before) will be the Velocity of the Body B to the Velocity of the Body A. Wherefore ſince the Spaces paſſed over in a given Time are proportionable to the Velocities, during the time the Body A is running over the Space AD, the Body B carried with the tenth part of the other's Velocity, will have paſſed over a Space equal to the tenth part of the Space run over by the Body A: ſo that at the end of that Time it will be found in D, in which Point therefore the Loadſtone and Iron will meet each other. After the ſame manner, two Loadſtones being placed on two Pieces of Cork, if their correſponding Poles be ſet towards each other, they will mutually attract each other equally; but if their adverſe Poles be placed near one another, theſe Poles will mutually fly from each other, and the Quantities of Motion produced by the Force wherewith they avoid each other, will be equal in both.

6. THE

6. The ſame thing may be ſhewn in other kinds of Attractions. For let A and B be two Barges upon the Water, and let a Man placed in one of them, as, for example, in A, draw by the means of a Rope the other Barge B towards



himſelf; by this Action of the Man, not only the Barge B will approach towards A, but A alſo will be equally drawn towards B; and the Quantities of the Motions produced by the Attraction, will be equal in each Barge: whence if the Barges are of the ſame weight, they will have, *cæteris paribus*, equal Velocities, and they will meet at the middle Point E, But if one is greater than the other, that is, has in itſelf a greater Quantity of Matter, or a greater Weight, then that which is the biggeſt will have the leaſt Velocity. For example, if the Barge B is ten times greater than the Barge A, the Velocity of A will exceed the Velocity of the Barge B ten times, and the Barges will meet in the Point G, which ſo divides their firſt Diſtance AD, that AG is ten times greater than GD; that is, GD will be the eleventh part of the whole Diſtance AD: but if B ſhould be a Ship a thouſand or ten thouſand times bigger than A, its Velocity would be a thouſand or ten thouſand times leſs than the Velocity of A, and conſequently it would be ſcarcely diſcernible. Now if B ſhould be another Body infinitely great, its Ve-

 locity

locity would be infinitely ſmall, that is, juſt nothing in reſpect of the Velocity of A. Hence if the Rope ſhould be faſtened to the Shore, and the Man in the Barge ſhould draw by the Rope the Shore towards himſelf, the Barge would approach towards the Shore, and the Shore towards the Barge: but ſince the Shore firmly adheres to the whole Body of the Earth; its Magnitude, which is therefore equal to the Magnitude of the whole Earth, in reſpect to the Barge, will be very immenſe and almoſt infinite, ſo that its Velocity will be nearly infinitely ſmall, and (as I may ſo ſay) nothing at all: and conſequently the Shore may be looked upon as fixed and impoſſible to be moved, and the whole Velocity as altogether appertaining to the Barge. If the Weight of the Barge B is a thouſand Talents, and is carried towards F with an hundred Degrees of Motion, the Momentum of that Barge will be [by Theor. 3.] an hundred thouſand parts. Now if to the Barge B is faſtened the Boat A, whoſe Weight is ten Talents, whatever Motion by this means is communicated to the Boat A, the very ſame Quantity of Motion will be taken from the Barge B. So that,

7. If any one in the Boat A draws the Rope AE, by which the Boat is faſtened to the Barge B, in ſuch a manner that the Boat moves forward with five hundred parts of Velocity, the Motion thence ariſing will be five thouſand parts; and ſo much of its Motion the Barge B will loſe, and conſequently there will remain to it ninety-five thouſand Parts of Motion, whence the Velocity of the Barge B will be ninety-five Parts.

8. If any one ſitting in the Boat A, ſhould by a Spreat, or the like Inſtrument, puſh or thruſt the Barge B towards the Parts F, by that puſhing the Boat A would recede towards the contrary Parts; ſo that the Quantity of Motion in both the Veſſels, ariſing

arising from the Force of the Man's pushing, would be equal: whence if the Barge B is ten times greater than the Boat A, it will have ten times a less Velocity; if it is an hundred times bigger, it will likewise have the hundredth part of the Velocity of the Boat A: so that if the Body B is any immense Body, the Velocity of the Boat A will be immense in respect of that which ought to be found in B. Whence if any one sitting in a Ship, should by means of a Pole endeavour to thrust the Earth or the Shore from him, the Ship by his thrusting would recede from the Shore; for the Shore, in respect of the Ship, may be looked upon as an immense and fixed Body, whose Velocity therefore will be exceeding small, or indeed none at all, in respect to that which is found in the Ship.

If the Boat EDG is rowed by Oars, since the Water is by the broad Ends of the Oars A, B, driven backwards towards the Parts C, that will again equally re-act upon the Oars, and force them, together with the Boat to

which they are fixed, towards the Parts H: for which reason alone it is, that the Boat is moved forwards; for if there was no Re-action, and the Water impressed no Motion on the Oars towards the Parts H, when it was thrust by the Oars the contrary way, the Boat would stand still, since there would be nothing that could force it towards the Parts H. But since the Water by its Re-action impresses as great a Motion on the Boat ED, as it receives itself from the Oars, it hence follows, the

greater

greater the broad Ends of the Oars are, or the more they are in number, *cæteris paribus*, or alſo the ſwifter they are moved within the Water, the Boat will be carried with a greater Impetus.

Hence ſince ſwimming is nothing elſe than a rowing with the Hands and Feet, it may be eaſily underſtood, why by ſwimming within the Water we are moved forwards; namely, ſince the Water is impelled backwards by the Palms of our Hands and the Soles of our Feet, that by its re-acting forces thoſe that ſwim towards the contrary parts: ſo that the Motion generated in the Water, is equal to the Motion wherewith the Swimmers proceed forwards. It is juſt the ſame thing in the flying of Birds; for as the Birds by their Wings bear the Air downwards, the Air by its Re-action raiſes them upwards; and if they drive the Air towards the Eaſt, for example, the Re-action of the Air obliges them to tend towards the Weſt. So Gunpowder within a Cannon being fired rarefies, and by its Force acts equally upon the Bullet and the Cannon out of which that Bullet is to be hurled: for the rarefied Air endeavouring to expand itſelf every way, will equally preſs the Cannon backwards, as the Bullet forwards, whence its elaſtick Force will produce equal Quantities of Motion in both; and by dividing theſe Quantities of Motion, as well by the Weight of the Engine as by the Weight of the Bullet, the Velocities thence ariſing will be reciprocally proportionable to the Weights.

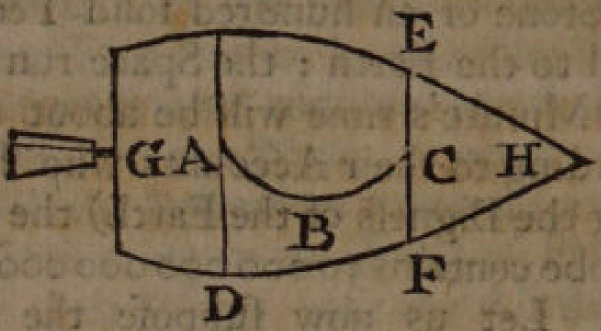
Since all Bodies near the Superficies of the Earth gravitate towards the Earth, in like manner the Earth will gravitate towards each Body, and is attracted towards them; and the Motions generated by this Attraction will be equal, as well in the Earth, as in the heavy Bodies falling to the Earth: ſo if a Stone by the Force of its Gravity falls downwards

to

to the Earth, the Earth in like manner will rise towards the Stone; but since the Quantity of Matter in the Earth exceeds immensely the Quantity of Matter in the Stone, the Velocity of the Stone will exceed immensely the Velocity wherewith the Earth tends towards the Stone, so that (to speak physically) the Velocity of the Earth will be nothing: which will thus appear upon a Calculation. Let us suppose a Stone of an hundred solid Feet in magnitude to fall to the Earth: the Space run over by the Stone in a Minute's time will be about fifteen Feet; but (according to their Accounts who have written concerning the Bigness of the Earth) the whole terraqueous Globe contains 30 000 000 000 000 000 000 000 solid Feet. Let us now suppose the Earth to be every where of the same density with common Stones, (tho it is probable it is much more dense) whence the Quantity of Matter in the Earth will be to the Quantity of Matter in the Stone of a hundred feet, as 300 000 000 000 000 000 000 to 1; and therefore whilst the Stone by its Weight falls through the Space of fifteen feet, the Earth will be drawn towards the Stone the $\frac{15}{300\,000\,000\,000\,000\,000}$ Parts of a Foot: which is so very small a quantity, as not to be conceived by the Imagination, and consequently may be neglected and esteemed as nothing in Philosophy, altho speaking geometrically, and according to Truth, we ought to say that the Earth advanced towards the Stone, and that both the Bodies mutually attracted each other.

If the Moon by its Gravity is retained in its Orb, so that it does not leave the Earth; that is, if the Moon gravitates towards the Earth, in like manner the Earth and all its Parts will gravitate towards the Moon: and hence arises the continual Flux and Reflux of the Sea. But this by the bye, for we shall in in another place explain more fully the Motions of the Sea.

Let ſuch a Ship be at reſt in the Water, as may be eaſily moved by any external Impulſe, yet no Force acting within that Ship, and ſolely contained in it, can communicate any Motion to it at all: for let GH be the Ship, and let any Machine, for ex-



ample, the elaſtick or ſpringing Body ABC, which being violently ſtrained, will endeavour to unbend itſelf, be placed within the Ship: moreover, the Machine being compreſſed, the Side BC will approach the Side AB; the Spring by its natural Energy, or reſtoring Force, endeavouring equally to unbend itſelf on both ſides, will equally impel the Obſtacle DA towards G, and the Obſtacle EF towards H: and therefore the Ship being impelled by theſe contrary and equal Motions, will not be moved at all. Juſt in the ſame manner, as if one ſtanding at the head of the Ship H, ſhould by a Rope draw to himſelf the Stern G; now the Rope being diſtended both ways, by its Endeavour to relax itſelf, it will equally urge the Stern towards the Man that draws, and the Man towards the Stern; and ſince he is ſeated at the Head of the Ship, in like manner the Head will be equally drawn towards the Stern: whence the two contrary and equal Motions will deſtroy each other, and conſequently no Motion at all will follow.

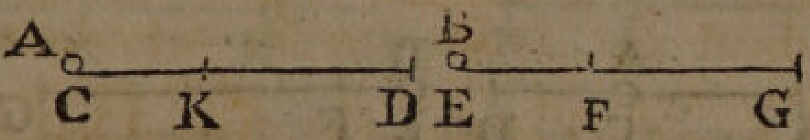
From

From this Law may be demonstrated the following Theorems.

THEOR. XVIII.

If *one Body strikes against another that is either at rest, or moving in the same Direction, but slower; the Sum of the Motions in both the Bodies towards the same Parts, will remain the same after the Stroke, as it was before that Stroke.*

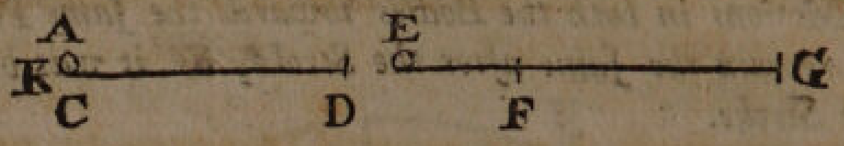
Let the Body A move in the Direction CD from C towards D, and let it strike against another Body B, that is either at rest, or moves slower in the same



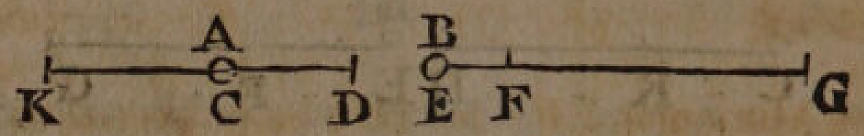
Direction: I say, the Sum of the Motions in both Bodies towards the same Parts, namely, from C towards D, before and after the Stroke, will remain the same. Let CD express the Motion of the Body A, and if the Body B moves, let the right Line EF represent its Motion towards the same Parts, and therefore the Sum of the Motions may be expressed by the Sum of the right Lines CD, EF. Now since Action and Re-action are always equal and contrary, the equal Forces impressed towards contrary Parts, will produce in both the Bodies equal Changes of Motions towards the contrary Parts: if therefore the Motion impressed on B by the Stroke of the Body A shall be represented by FG, the contrary and equal Force acting on the Body A will take as much from its Motion made towards the same Parts; so that by putting DK equal to FG, CK will be as the Motion of the Body A, and EG as the Motion of the Body B after the Stroke or Concourse; and therefore the Sum of the Motions

will

will be as the Sum of the right Lines CK, EG. But ſince FG is equal to KD, if to both there be added EF and CK, then EG and CK will be equal to CD, EF: whence the Sum of the Motions towards the ſame Parts, will remain the ſame, both before and after the Stroke. If FG is equal to CD,



the Point K will coincide with C, and CK will become equal to nothing; whence after the Stroke the Body A will be at reſt. But if FG is greater than



CD, the Point K will fall beyond C, and the Motion of A will become negative, or towards the contrary Parts, *viz.* from C towards K; and the Sum of the Motions towards the Parts G, will be as EG leſſened by CK: for the Sum of two Quantities, whereof one is poſitive, the other negative, is their difference. But becauſe $FG = KD$, let $EF - CK$ be added to them both, and it will be $EF + FG - CK$: that is, $EG - CK = KD + EF - CK$, that is, $EF + CD$: whence the Sum of the Motions towards the ſame Parts, which is here the Difference of the Motions towards the contrary Parts, before and after the Stroke, remains the ſame. *Q. E. D.*

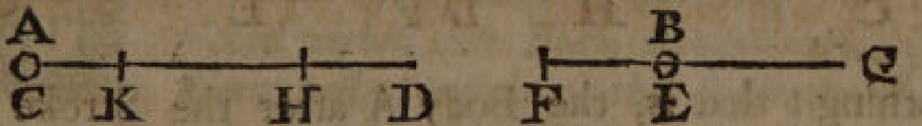
Cor. After the ſame manner, if more Bodies moving towards the ſame Parts ſtrike againſt one another, the Sum of their Motions will not be altered.

THEOR.

THEOR. XIX.

If two Bodies moving towards the contrary Parts, do directly strike against each other; the Sum of their Motions towards the same Parts (which is the Difference of their Motions towards contrary Parts) before and after the Stroke, will always remain the same towards the same Parts.

Let the Body A move from C towards D, whose Motion let be expressed by CD; but let B be moved towards the contrary Part, namely, from E to F,

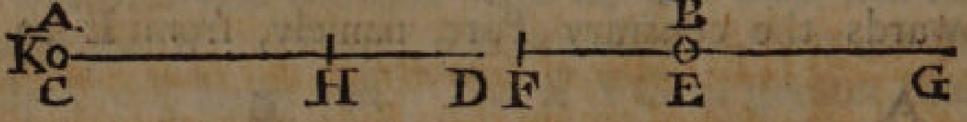


with a Motion as EF: let DH be put equal to EF; and CH, which is the Difference of the Motions towards the contrary Parts, will be as the Sum of the Motions towards G: I say, the same CH will be as the Sum of the Motions towards the same Part G after the Stroke. For let the Motion of the Body B after the Stroke be towards the Part G, and let it be represented by the right Line EG; the Force therefore of Impulse impressed on the Body B towards the Part G, will be equivalent to the Sum of the Motions EF, EG, and will be represented by the right Line FG: for by that Force is destroyed the Motion as EF, towards the Part F, and a new one as EG is impressed towards the contrary Part G. But since the Force of Impulse acts equally on both Bodies towards the contrary Parts, if DK is made equal to FG, it will represent the Force exercised on the Body A towards the Part contrary to its Motion; so that if the Motion as DK is substracted from the Motion as CD, there will remain CK as the true Motion of the Body A towards the Part G.

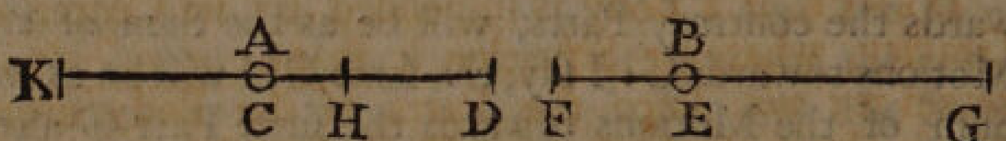
Now

Now since DK is equal to FG, and DH equal to FE, it will be DK, lessened by DH, that is, KH equal to FG, lessened by FE, that is, EG: and therefore since KH is equal to EG, KH will be as the Motion of the Body B after the Stroke; but CK is as the Motion of the Body A, so that CK, KH, that is, CH will be the Sum of the Motions in both Bodies towards the Part G.. *Q.E.D.*

If FG is equal to CD, the Point K will fall on C, and then the Motion of A will become equal to



nothing; that is, the Body A after the Stroke will be at rest, and CH will be equal to EG. But if FG is greater than CD, the Point K will fall beyond



yond C towards the other Part, and the Motion of the Body A will be from C towards K; but (by reason FG is equal to DK, and FE equal to DH) KH is equal to EG: and therefore if from both there be taken CK, it will be CH equal to the right Line EG, lessened by CK. But CH was as the Sum of the Motions towards the Part G before the Stroke, and it is EG lessened by CK, as the Sum of the Motions towards the same Part, namely, as the Difference of the Motions towards the contrary Parts after the Stroke. Wherefore the Sum of the Motions towards the same Part will remain the same before and after the Stroke.

These two last Theorems are both at once thus well expressed by Sir *Isaac Newton*:

The

THE *Quantity of Motion, that is gathered by taking the Sum of the Motions made towards the same Part, and the Difference of those made towards the contrary Parts, is not altered by the Action of Bodies amongst themselves.*

LECTURE XIII.

Second Definitions.

I. THE *Center of Gravity of any Body is a Point placed within that Body, through which if a Plane passes any how, the Segments of the heavy Body that are on each side, as it were, ballanced upon that Plane, will equiponderate.*

HENCE, if a Body is suspended by its Center of Gravity, whatever Situation it is put into, it will retain it: *viz.* since the Parts of the Body every where about the Center consist of equal Momenta, or have equal Propensities to Motion.

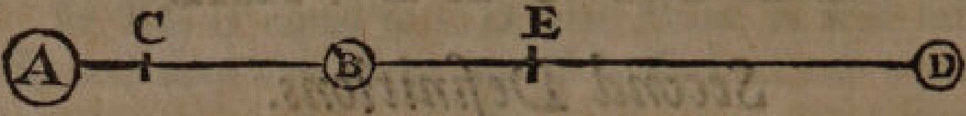
II. WE *call the common Center of Gravity of two Bodies, a Point so situated in the right Line joining their Centers, that the Distances of the Bodies from that Point, are in a reciprocal Ratio of the Bodies.*

LET there be two Bodies A, B, whose Centers of Gravity let be joined by the right Line AB; which let be so divided in C, that AC be to BC as the

Body B, that is, the Matter of B is to the Body A, or Matter in A; that Point C is called the common

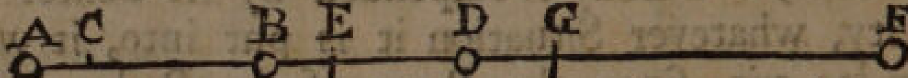
Center of Gravity of the Bodies A and B: namely, becauſe if thoſe Bodies are turned about that Point, at the ſame Diſtances from it, they will retain any given Situation, (as was demonſtrated in Theor. 11.)

III. In *like manner, if there are three Bodies* A, B, D, *and* C *be the Center of Gravity of two of them,* A *and* B, *and the right Line* C D *be ſo divided in* E, *that* C E



is to D E *as the Weight of the Body* D *to the Weight of the two Bodies* A *and* B *together, that Point* E *is called the common Center of Gravity of theſe three Bodies; about which alſo if the Bodies are turned, they will retain any given Situation.*

IV. After *the ſame manner, if there are four Bodies* A, B, D, F, *and* E *is the common Center of Gravity of the three* A, B, D; *the Point* G, *which ſo divides the right Line* E F, *that* E G *is to* G F *as the Weight of the Body* F *to the Weight of the Bodies* A, B, D, *taken together, is called the common Center of Gravity of theſe four Bodies.*



And after the ſame manner the common Center of Gravity of five or more Bodies may be determined.

V. One *Body is ſaid to ſtrike or impinge directly on another, when the right Line, in which it moves, drawn through the Center of Gravity of the impinging Body and the Point of Contact, is perpendicular to the Superficies of the Body that receives the Stroke; or, if the Bodies do not touch one another in a Point, but in a Line or a Superficies, when that Line is perpendicular to this Line or Superficies.*

VI.

VI. But *it is said to strike or impinge obliquely or indirectly on the other, when the before-mentioned right Line is not perpendicular to the Superficies of the Body that receives the Stroke.*

VII. I Call *that a perfectly hard Body, which does not yield in the least to a Stroke; that is, which does not lose its Figure for a moment.*

VIII. That *is a soft Body, which so yields to a Stroke, as to lose its first Figure, and never to endeavour to recover it again.*

IX. That *is an elastick Body, which yields indeed for a little while to a Stroke, yet of its own accord does recover its first Figure.*

X. An *elastick Force is that Force, whereby a Body obliged to quit its Figure, recovers it again.*

XI. A Perfectly *elastick Body is such a one, as recovers its first Figure with the same Force, whereby it was obliged to quit it.*

THEOR. XX.

If *two or more Bodies are carried by an equable Motion towards the same or contrary Parts, their common Center of Gravity before their mutual Concourse will either be at rest, or will move uniformly in a right Line.*

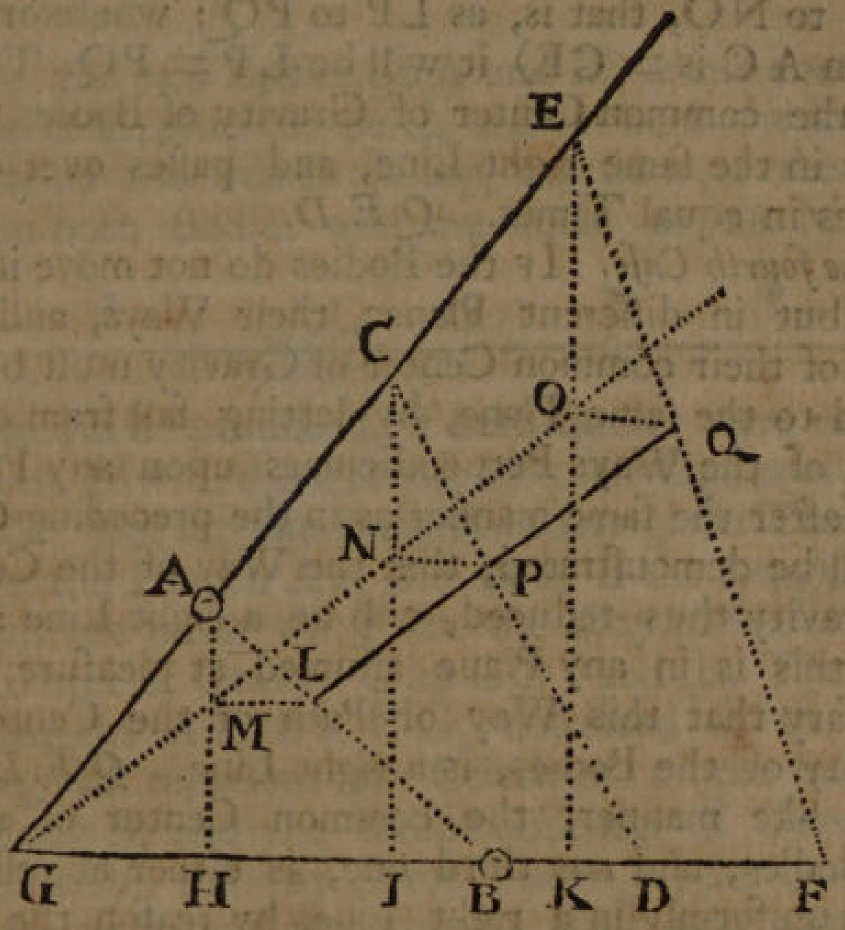
The first Case. Let the Bodies A and B tend towards contrary Parts, whose common Center of Gra-

vity let be C. By reason of the equal Quantity of Motion in both Bodies, the Velocity of the Body A will be to the Velocity of the Body B, as the Body B to the Body A; that is, (from the Nature of the Center of Gravity) as AC to BC: whence, the Spaces passed over in the same Time being proportionable to the Velocities, whilst the moving Body A runs through the Length AC, the Length BC will be run through by the moving Body B; ſo that the Bodies will meet in the Point C, and in that Point will be their Center of Gravity at the time of their Concourſe: but it was in the ſame Point before the Concourſe, ſo that it remained in the ſame place.

It may be ſhewn, after the ſame manner, that if the Bodies receded from the Point C with equal Motions, their Center of Gravity would have been at reſt.

The ſecond Caſe. If Bodies are carried in the ſame right Line towards the ſame Part, or with unequal Motions towards contrary Parts, their common Center of Gravity will be always found in the ſame right Line. For ſince the Bodies do uniformly and directly recede from one another, or approach one another, their Diſtance from one another will be uniformly increaſed or diminiſhed; and therefore the Bodies will uniformly recede from, or approach to any Point, dividing the aforeſaid Diſtance in a given Ratio. The Diſtance therefore of the Bodies from their common Center of Gravity will be uniformly increaſed or diminiſhed; which cannot be, in the aforeſaid Caſes, unleſs that Center is either at reſt, (as in the firſt Caſe) or moves uniformly, as in the preſent Caſe.

The third Caſe. Let the Bodies A and B move in the right Lines AC, BD; and let the Spaces AC, CE, paſſed over by the Body A in equal Times, be equal; and the Spaces BD, DF, paſſed over by the Body B in the ſame Times, be alſo equal. Let the right Lines AC, BD, meet in G; and let it be as AC to BD ſo is AG to GK, and join AH; to which through C and E let CI, EK, be drawn parallel: it will be AC to HI as AG to GH, that is, as AC to BD; wherefore HI is $=$ BD, and therefore HB$=$ID. In like manner, CE is to IK, as AG to to GH, or AC to BD, that is, as CE to DF; wherefore IK is = DF, whence KF = ID = HB. Let L be the common Center of Gravity of the Bodies, when they are placed in the Points A and B; draw LM parallel to BD; the right Lines AB, AH, will be cut similarly: join GM, and let it be produced; this will cut the Parallels to AH in the



Points N and O; *viz.* in the same Ratio as AH or AB is cut. Draw through N and O, parallel to BD, the right Lines NP, OQ; these will cut CD, EF, in the same Ratio as CI, EK, are cut, that is, in the same Ratio as AB is cut in L: but L is the common Center of Gravity of the Bodies when they are found in A and B; wherefore P will be their Center, when they are in the Points C and D, and Q when in E and F. Besides, ML is to HB as AM to AH, or as CN to CI, or as NP to ID: but

but HB and ID are equal; wherefore ML and NP will be equal: in like manner, NP and OQ will be equal. Since therefore the right Lines ML, NP, OQ, are equal and parallel, the right Line drawn through L, and parallel to MO, will paſs through the Points P and Q, and therefore the Center of Gravity will be always placed in the right Line LQ. Beſides, (by reaſon of the Parallels) AC is to CE as MN to NO, that is, as LP to PQ; wherefore (by reaſon AC is = CE) it will be LP = PQ. Therefore the common Center of Gravity of Bodies is always in the ſame right Line, and paſſes over equal Spaces in equal Times. *Q.E.D.*

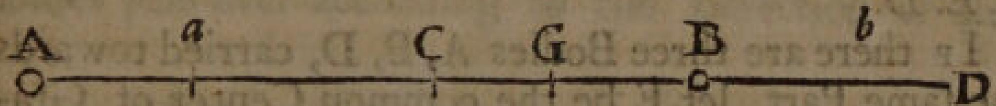
The fourth Caſe. If the Bodies do not move in any one, but in different Planes, their Ways, and the Way of their common Center of Gravity muſt be reduced to the ſame Plane, by letting fall from every Point of the Ways Perpendiculars upon any Plane; and (after the ſame manner as in the preceding Caſe) it will be demonſtrated, that the Way of the Center of Gravity thus reduced, will be a right Line: and ſince this is in any Plane aſſumed at pleaſure, it is neceſſary that this Way or Path of the Center of Gravity of the Bodies, is a right Line. *Q.E.D.*

In like manner, the common Center of theſe two Bodies, and any third one, is either at reſt, or moves uniformly in a right Line, by reaſon the Diſtance of the common Center of Gravity of the two Bodies, and the Center of the third, is divided by it in a given Ratio. And after the ſame manner the common Center of theſe three Bodies, and a fourth, either is at reſt, or moves in a right Line, by reaſon the Diſtance betwixt the common Center of the three, and the Center of the fourth Body, is divided by it always in the ſame Ratio; and ſo of any other number of Bodies. *Q. E. D.*

THEOR. XXI.

If *two Bodies, whether equal or unequal, are carried towards the same Part, with Celerities whether equal or unequal, the Sum of the Motions in both Bodies will be equal to the Motion that would arise, if both the Bodies were carried with the Celerity of their common Center of Gravity.*

Let the two Bodies be A and B, whose common Center of Gravity let be C, and let both the Bodies be carried towards D; I say, the Sum of the Motions in both Bodies will be equal to the Motion



that would be produced, if both Bodies were carried towards D with the Celerity of their Center of Gravity C. For let the Body A in any given Time describe the Length A*a*, the Body B the Length B*b*, and in the mean while let the Way passed over by the Center of Gravity C be CG; and [by Theor. 6.] the Lengths A*a*, B*b*, CG, described in the same Time, will represent the Celerities of the Body A, the Body B, and the common Center of Gravity C respectively; but [by Corol. Theor. 3.] the Quantity of Motion in any Body is as a Rectangle contained under the Quantity of Matter and the Celerity, so that the Motion in the Body A will be as A × A*a*; and in the Body B as B × B*b*; and the Sum of the Motions will be as the Sums of these Rectangles, *viz.* as A × A*a* + B × B*b*. But [by the Def. of the Center of Gravity of Bodies] BC is to AC as A to B; and as A to B, so also [by the same Definition] is *b*G to *a*G: wherefore BC will be to AC as *b*G to *a*G; whence [by 19 *El.* 5.] BC is to AC, that is, A to B, as BC—*b*G to AC—*a*G; that is, as CG

 —B*b*

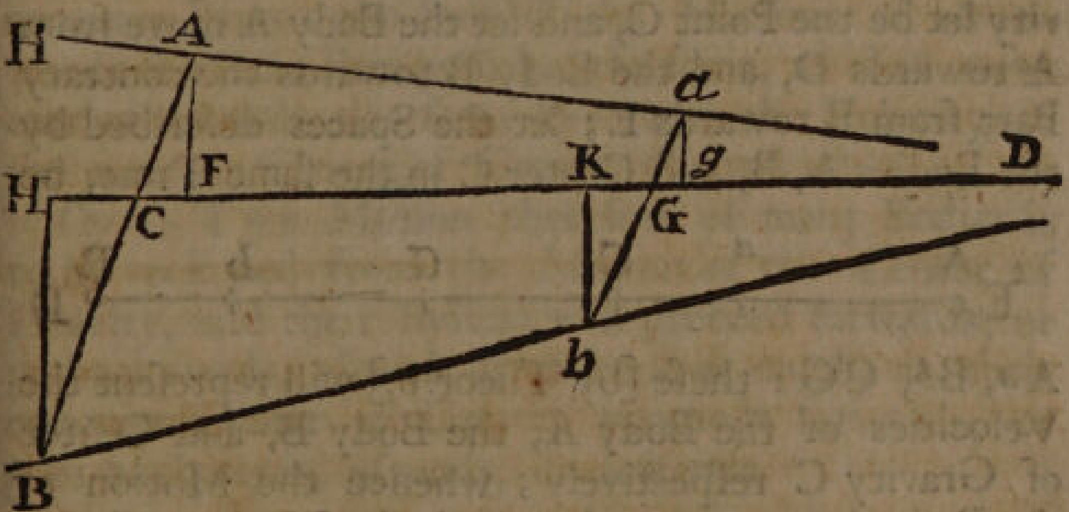
—Bb to Aa—CG. So that [by 16 *El.* 6.] A × Aa —A × CG will be equal to B × CG—B × Bb; and therefore A × Aa + B × Bb will be equal to A × CG + B × CG: but the two Rectangles A × Aa and B × Bb are (as has been ſaid) as the Sums of the Motions in both the Bodies; and the two Rectangles under A and CG, and under B and CG, will be as the Sum of the Motions that would ariſe, if both the Bodies were carried with the Celerity CG of the Center of Gravity: whence the Sum of the Motions in both Bodies is equal to the Motion that would be produced, if both Bodies were carried with the Celerity of their common Center of Gravity. *Q. E. D.*

If there are three Bodies A, B, D, carried towards the ſame Part, let E be the common Center of Gravity of theſe three Bodies; the Sum of the Motions in the three Bodies will be equal to the Motion ariſing from the ſame Bodies carried with the Velocity

of the Point E. For let C be the common Center of Gravity of any two Bodies A and B, the Motion in theſe two Bodies (by what has been demonſtrated before) will be equal to the Motion that would ariſe, if the Bodies coaleſced in one were carried with the Velocity of the Point C; but alſo the Sum of the Motions (*viz.* the Motion of the Bodies ſo coaleſcing, and the Motion of the third Body D) will be equal to the Motion which would happen, if the Body coaleſcing out of the two was moved together with the third Body D, with the Celerity of the Point E; whence the Theorem holds alſo in this Caſe.

The Demonſtration is the ſame, if the Bodies do not move in the ſame right Line, but in Parallels, or alſo

also in Lines any how inclined. But in this Case it must be observed, that the Celerity of the Bodies, wherewith they are carried towards the same Part with the Center of Gravity, is not to be reckoned by the Way which they really pass over, but only by the Way wherein they move according to the Direction of the Center of Gravity. For example, if two Bodies A and B are carried in the right Lines A*a*, B*b*, and CG is the Line described by the common Center of Gravity, whilst the Bodies pass over the Lengths A*a*, B*b*, and from the Points A, *a*, B, *b*, let fall the Perpendiculars AF, *ag*, BH, *b*K upon the right Line CG. Now the Spaces which the Bodies pass over according to the Direction of the



Point C, are not A*a*, B*b*, which are the absolute Spaces described by them; but the Space according to which the Body A is moved towards the Part D, must be computed in the right Line FD, by the Length F*g*; for it moves so much, and no more, according to the Direction of the Point C. In like manner, the Space according to which the Body B is moved towards the Part D, is HK; and by that Space, its Progress in the right Line HD is to be reckoned: so that the Celerities of the Bodies wherewith they are carried towards the same Part, are as the

the right Lines F g, HK; besides, A is to B as BC to AC, or (by reason of the equi-angular Triangles ACF, BCH) as HC to FC: whence the Demonstration will proceed as in the first Case.

THEOR. XXII.

If *two Bodies are carried towards contrary Parts, the Difference of the Motions towards the contrary Parts, or, which is the same thing, the Sum of the Motions towards the same Part, will be equal to the Motion that would be produced, if both the Bodies were carried towards the same Part with the Celerity of the common Center of Gravity.*

Let the Bodies be A and B, whose Center of Gravity let be the Point C, and let the Body A move from A towards D, and the Body B towards the contrary Part from B towards E; let the Spaces described by the Bodies A, B, and Center C in the same Time, be

Aa, Bb, CG: these [by Theor. 6.] will represent the Velocities of the Body A, the Body B, and Center of Gravity C respectively; whence the Motion of the Body A is as $A \times Aa$, and the Motion of the Body B as $B \times Bb$, so that the Difference of the Motions will be $A \times Aa - B \times Bb$. Moreover, from the Nature of the Center of Gravity, BC is to AC as A to B; and as A to B, so will be bG to aG: wherefore it will be as BC to AC, so bG to aG. So that [by 19 *El.* 5.] BC will be to AC, that is, A to B as $BC - bG$ to $AC - aG$; that is, A will be to B as $Bb + CG$ to $Aa - CG$: wherefore [by 16 *El.* 6.] the Rectangle under A and $Aa - CG$ will be equal to the Rectangle under B and $Bb + CG$; that is, $A \times Aa - A \times CG = B \times Bb + B \times CG$: whence

whence it will be $A \times Aa - B \times Bb = A \times CG + B \times CG$; but $A \times Aa - B \times Bb$ is (as was said before) the Difference of the Motions towards the contrary Parts, or the Sum of the Motions towards the same, and $A \times CG + B \times CG$ is the Motion emerging, if both the Bodies should be carried with the Velocity of their common Center of Gravity, whence the Proposition is manifest.

Cor. 1. If the Difference of the Motions towards the contrary Parts is equal to nothing, that is, if in both the Bodies the Quantities of Motion are equal, the common Center of Gravity in this Case will be at Rest.

Cor. 2. If there are several Bodies, either all carried towards the same Parts, or some towards the contrary Parts; the Sum of the Motions of all towards the same Part will be the same, as if all were carried towards the same Part with the Velocity of the common Center of Gravity of them all.

Cor. 3. The Motion therefore of many Bodies is to be reckoned from the Motion of the Center of Gravity, and their System will proceed forwards, or go backwards, ascend or descend as much, as their common Center of Gravity proceeds forwards, or goes backwards, ascends, or descends.

THEOR. XXIII.

If *Bodies impinge on one another, or do act any way upon each other, the State of their common Center of Gravity, of being at rest, or moving uniformly in a right Line, will not be thereby changed.*

If the Bodies impinge on one another, [by Theor. 19.] the Sum of the Motions towards the same Part will be the same before and after the Impulse; but [by Theor. 21, and 22.] the Sum of the Motions before and after the Impulse is the same, as if all the Bodies

Bodies were carried with the Velocity of the common Center of Gravity, and towards the ſame Part with it. Wherefore, ſince the ſame Bodies have the Sums of their Motions before and after the Impulſe equal to each other, and alſo equal to the Motion ariſing from all carried together with the Velocity of the common Center of Gravity; it is manifeſt, that the Velocity of the common Center of Gravity before and after the Impulſe will be the ſame. *Q. E. D.*

We have hitherto delivered ſome general Laws, ſerving to determine the Motions of Bodies of all kinds, we ſhall now proceed to give ſome particular Rules relating to their meeting each other, *viz.* whereby each Body, after the Concourſe, and mutually ſtriking againſt one another, do continue their Motions, and towards what Parts they tend, and with what Velocity. But by reaſon of the different Structure of Bodies, *viz.* as ſome are endued with an elaſtick Force, whilſt others want that Force, the Rules of their Concourſe will be different according to the various Sorts of Bodies: and tho perhaps there is no Body that is either perfectly hard, or perfectly ſoft, or perfectly elaſtick, (for haply all Bodies contain ſomething of all theſe in themſelves) yet it does not hinder, but that we may, by an Abſtraction of the Mind, ſeparate thoſe Qualities, and conſider a Body as only endued with one of theſe Qualities: and the Motions of Bodies will come the nearer to the Rules hereafter given, the more the Bodies themſelves partake of thoſe Qualities and Conditions.

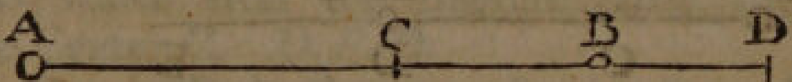
We here ſuppoſe the Bodies to be ſo ſeparated from all others, that their Motions are neither hindered or promoted by ſuch as may lie round about them.

THEOR.

THEOR. XXIV.

If *a hard or soft Body directly impinges or strikes upon another hard or soft Body, whether the Body that receives the Stroke, is either at rest, or moves slower towards the same Part with the other, or lastly towards the contrary Parts, and the Motions are unequal, both Bodies after the Stroke will move together conjointly with the common Center of Gravity.*

Let the Body A impinge on the Body B, which is either at Rest, or moves slower towards the same Part, or is carried towards the contrary Parts with a less



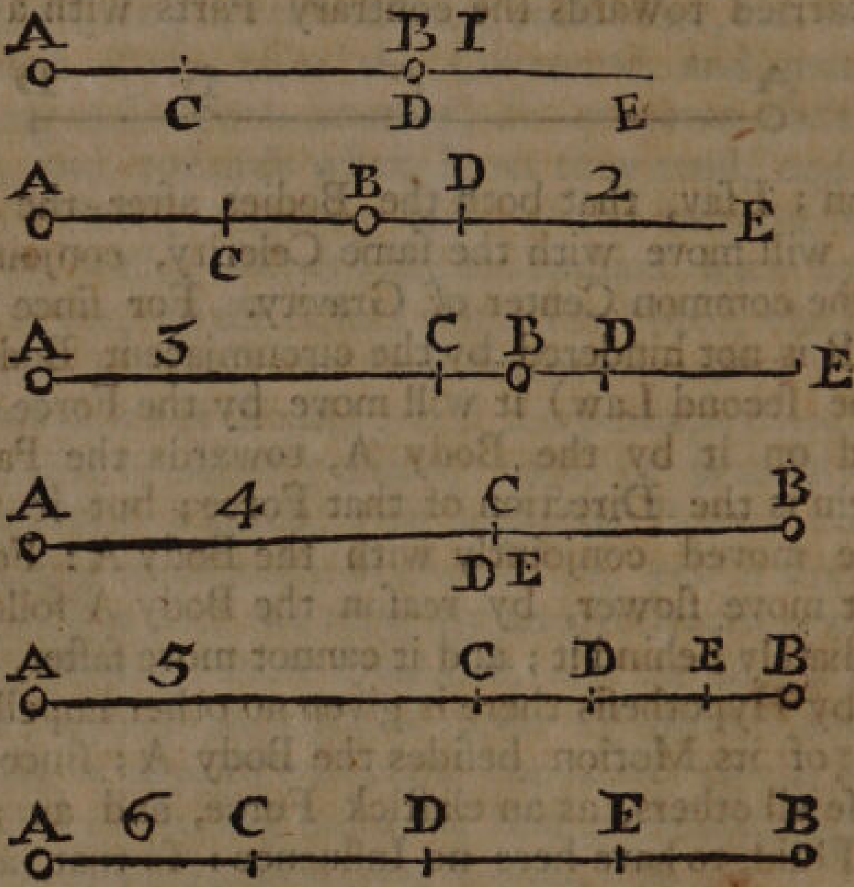
Motion; I say, that both the Bodies after the Impulse, will move with the same Celerity, conjointly with the common Center of Gravity. For since the Body B is not hindered by the circumjacent Bodies, (by the second Law) it will move by the Force impressed on it by the Body A, towards the Parts, wherein is the Direction of that Force; but it will also be moved conjointly with the Body A: For it cannot move slower, by reason the Body A follows immediately behind it; and it cannot move faster, because by Hypothesis there is given no other impelling Cause of its Motion besides the Body A; since we suppose all others, as an elastick Force, and an ambient Fluid, to have here no Influence: so that, after the Concourse, both the Bodies and the common Center of Gravity will move conjointly together. *Q.E.D.*

Cor. If the Bodies are supposed to concur in D, since the Velocities of moving Bodies are as the Spaces described in the same Time, the Velocities of the Body A, the Body B, and the Center of Gravity C, C, will be before the Concourse, as the right Lines AD, BD, CD respectively; for these Lengths are passed over in the same Time.

PROBL. II.

To *determine the Motions of hard or soft Bodies after their direct Impact or Stroke.*

We shall construct all the Cases of this Problem at once. Let therefore A and B be two Bodies, whose Center of Gravity let be C, and let us sup-



pose the Bodies to meet in D, the Celerities of the Body A, of the Body B, and of the common Center of Gravity C, before the Stroke, will be [by the preceding Corol.] as the right Lines AD, BD, and CD respectively. Let now DE be made equal to DC, this will represent the Velocity of the Bodies after

after their Concourse; that is, the Velocity of the Body A before the Impulse will be to its Velocity after the Impulse, as AD to DE; and the Velocity of the Body B before the Stroke will be to its Velocity after the Stroke, as BD to DE: for [by Theor.19.] the Bodies A and B after the Impulse move on together with the Center of Gravity; but [by Theor.18.] the Celerity of the Center of Gravity will be the same both before and after the Impulse, and always towards the same Part. Wherefore if CD represents its Velocity before the Impulse, DE equal to CD will express its Velocity after the Impulse; so that DE will also represent the Celerity of the Bodies A and B, which after the Impulse move together with the Center C. *Q. E. D.*

Cor. 1. If the Body B is at rest, the Point D coincides with B, as in the first Figure; and because B is to A, as AC to BC, or DE, by Composition it will be A + B to A as AB or AD to DE; that is, the Velocity of the Body A before the Stroke is to its Velocity after the Stroke, as the Sum of the Bodies to the impinging Body A.

Example 1. If A is equal to the quiescent Body B, it will be A + B to A as 2 to 1; so that the Velocity of the impinging Body will be double its Velocity after the Concourse.

Example 2. If A is to B as 1 to 9, it will be A + B to A as 10 to 1; so that the Velocity after the Impulse will be only the tenth part of the Velocity before the Impulse.

Example 3. If B is a Body that exceeds A infinitely, the Velocity of the Body A, after the Impulse, will be infinitely small, that is, none at all; for in that Case, A in respect of A + B vanishes, and therefore the Velocity of the Body A after the Concourse likewise vanishes: that is, if the Body A impinges on a firm and immoveable Obstacle, after the Stroke it will be at rest.

Ex-

Example 4. If the Body B is equal to A, and moves ſlower in the ſame Direction, D E or C D will be $= \frac{AB}{2} + BD = \frac{AB + 2BD}{2} = \frac{AD + BD}{2}$; that is, the Velocity after the Impulſe will be half the Sum of the firſt Velocities.

Example 5. If the Bodies tend towards the ſame Parts with equal Motions, the Point D will coincide with C, as was demonſtrated in Theor. 20. and CD, DE, will be equal to nothing, that is, both the Bodies will be at reſt, after their Concourſe.

Cor. 2. Hence the Law of the *Carteſians* is demonſtrated to be falſe, whereby they contend that there is always preſerved the ſame Quantity of Motion in the Univerſe: for Bodies which are not elaſtick, meeting each other from contrary Parts with the ſame Motions, mutually deſtroy each other's Motions.

Example 6. If equal Bodies tend towards contrary Parts with unequal Motions, it will be D E or C D $= CB - BD = \frac{AB}{2} - BD = \frac{AB - 2BD}{2} = \frac{AD - BD}{2}$; that is, the Velocity after the Impulſe will be half the Difference of the firſt Velocities.

All theſe things eaſily flow from the Conſtruction above; but ſince in Practice there is occaſion for a Calculation, a general Solution of this Problem may be thus diſcovered by Calculation.

Let the Velocity of the Body A be called C, and the Velocity of the Body B be c; and if the Bodies are moved according to the ſame Direction, the Sum of the Motions in both towards the ſame Part will be $AC + Bc$: but if they are moved towards contrary Parts, the Sum of their Motions towards the ſame Part will be $AC - Bc$; but [by Theor. 19] in in all Bodies, the Sum of their Motions towards the same Part before and after the Impulse is the same: wherefore the Motion of the Bodies after the Impulse will be either AC + B*c* or AC — B*c*, accordingly as the Bodies did tend towards the same or contrary Parts. There is therefore given the Momentum of the Bodies carried with the same Velocity, whence [by what has been said in *Lect.* X.] their Velocity will be likewise known; namely, if their Momentum is divided by the Bodies themselves, the Quotient will exhibit their Velocities, *viz.* $\frac{AC + Bc}{A + B}$ or $\frac{AC - Bc}{A + B}$; and if B is at rest, that is, if *c* be put equal to nothing, the Velocity of the Bodies will be $\frac{AC}{A + B}$.

Cor. 3. Since the Velocity of the Body A before the Impulse was as AD, and after the Impulse its Velocity was as CD, the Velocity lost will be AC, and therefore the Motion lost by the Stroke $A \times AC$.

THEOR. XXV.

If *a moving Body strikes directly against another either in motion or at rest; the Magnitude of the Stroke is proportionable to the Momentum that is lost at the Concourse by the more powerful Body, whichever of them it is.*

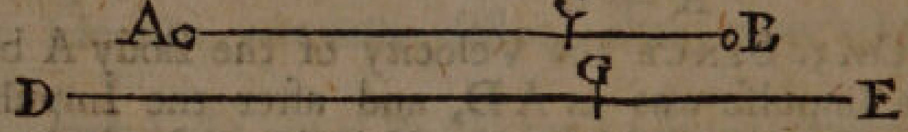
For if we suppose the more powerful Body, (whichever it is) or if they have equal Momenta, either of the Bodies, to be that which gives the Stroke, and the other that which receives it; the Magnitude of the Stroke will be equivalent to the Force impressed by the first on the other: wherefore that Force impressed departs from the percutient Body, [by the third Law] so that the Motion lost in the percutient Body will be proportionable to the Force impressed on the other Body, and consequently to the Magnitude of the Stroke. *Q. E. D.*

Cor. WHERE the Momenta are equal that depart from the percutient Bodies, there the Magnitudes of the Strokes will be equal.

THEOR. XXVI.

IF *a given Body directly impinges on another given Body at rest, the Magnitude of the Stroke will be always proportionable to the Velocity of the impinging Body.*

LET the given Body A impinge on another given Body B that is at rest, with a Velocity which may be expressed by AB. Afterwards let the same Body A impinge on the same quiescent Body B, with ano-



ther Velocity DE; that is, let AB be to DE as the former Velocity to the latter, and let the Distances of the Bodies be AB, DE: for whatever is the Distance at the beginning of Motion, it is the same thing in respect to the Magnitude of the Stroke; and let the common Center of Gravity in the first Situation be C, in the second G. Since the Body A is moved with the Velocity AB, its Velocity after the Concourse will be CB; and since its Motion before the Impulse was A × AB, its Motion after the Impulse will be A × CB; and the Motion lost will be A × AC. After the same manner, if the Body is moved with the Velocity DE, the Motion lost will be A × DG; and consequently, the Magnitude of the Stroke with the Velocity AB will be to the Magnitude of the Stroke with the Velocity DE, as A × AC to A × DG, or as AC to DG: but because it is AC to BC as B to A, it will be

AC

AC to AC + BC, that is, AB, as B to A + B; and in like manner, it will be B to A + B as DG to DE. Wherefore it will be AC to AB, as DG to DE, whence by permutation it will be AC to DG as AB to DE; that is, the Magnitude of the Stroke with the Velocity AB, will be to the Magnitude of the Stroke with the Velocity DE, as the Velocity AB to the Velocity DE. *Q. E. D.*

Cor. If the Body A should rush on B, the Motion lost would be as A × AC; but if B should impinge on A with the same Velocity, the Motion lost would

be as B × BC; but because A is to B as BC to AC, it will be A × AC = B × BC: so that the Quantity of Motion lost by the Stroke, whether B impinges on A with a given Celerity, or A with the same Velocity strikes against B, so that in both Cases the Magnitude of the Stroke will be the same.

THEOR. XXVII.

If *one Body should impinge on another Body moving slower, but in the same right Line and towards the same Part, the Magnitude of the Stroke would be the same, as if the antecedent Body was at rest, and the following Body was carried towards it with a Velocity equal to the Difference of their Velocities.*

Let the two Bodies carried towards the same Part be A and B, whose common Center of Gravity let be C, and let the Point of their Concourse be D;

from what has been delivered above, the Velocities of the Bodies before the Impulse will be as the right

Cor. 1. If therefore the Sum of the Velocities remains the same; that is, the respective Velocity of the Bodies A and B, wherewith they approach each other, whatever is the Difference of Velocities, or after what manner soever that Velocity is shared amongst the meeting Bodies, the Magnitude of the Stroke will be always the same.

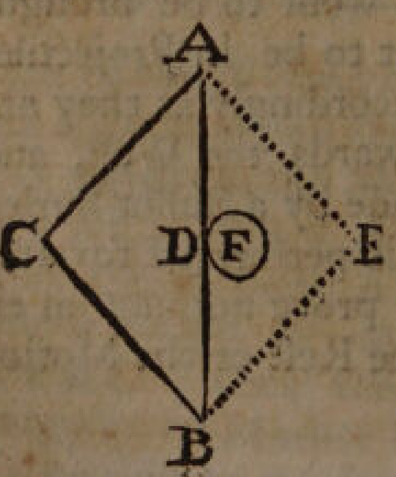
Cor. 2. The Magnitude therefore of the Stroke in given Bodies is always proportionable to their respective Velocities.

Cor. 3. The Motions of Bodies inclosed in a Space are the same amongst one another, whether that Space is at rest, or moves uniformly and directly on; for the Differences of Velocities, wherewith Bodies tend towards the same Part, and the Sums, wherewith they tend towards the contrary Parts, are the same, whether the Space, in which they are inclosed, is at rest, or moves uniformly forwards: so that the Magnitudes of the Strokes being always proportionable to these, will be the same in both Cases. Hence all the Motions within a Ship are performed in the same manner, whether it is at rest, or is moved uniformly forwards. So also the Phenomena of Projectiles and Percussions do all happen just the same amongst us placed upon the Earth, whether they are all together with the Earth in one common Motion, or there is no such common Motion, and the Earth is at rest; so that the Objections that are wont to be brought from the Inequality that ought to be in Projectiles not mov'd by the same Force, accordingly as they are made towards the East or towards the West, and from the unequal Percussions made by a Bullet hurled by a Cannon, as the Explosion is performed towards this or that part, and the like, prove nothing on either side, whether to establish the Rest or the Motion of the Earth.

LECTURE XIV.

IF there were no ſuch thing as *Elaſticity*, the Laws, which we have propoſed in the preceding Lecture, relating to the Percuſſions of hard Bodies, would perfectly agree to all Bodies, and all Bodies after the Impulſe would tend conjointly towards the ſame Parts, towards which, before the Percuſſion, the more powerful Body did tend, that is, the Body whoſe Momentum was the greateſt; and with the Velocity that we determined in the beforementioned Laws. But ſince there are indeed very few Bodies, wherein there is not ſome degree of Elaſticity, (for ſoft Clay, Wax, and the like Bodies, contain in themſelves ſome Particles of Air, which gives them ſome Elaſtick Virtue) it happens by that Elaſtick Force, that Bodies after their Impulſe do not move conjointly together, but fly from one another, and are moved with a different Velocity, ſometimes towards the ſame, and ſometimes towards the contrary Parts. But in order to underſtand the Modus and Cauſe of this Rebounding in Bodies, we will illuſtrate the thing by an Example.

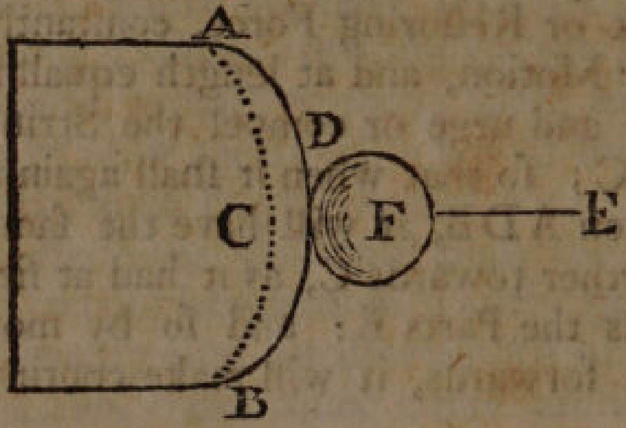
Let AB be a String extended over a Plane, but at ſome Diſtance from it, whoſe Extremities AB let be firmly fixed, and the String itſelf very much ſtretched: Now if the String be drawn by its middle Point D, its Extremities remaining immoveable, into the Situation ACB, ſo that its Point D ſhall be in C, and then be let go, the String will not remain



remain in the Situation ACB, but with a great Force will endeavour to restore itself to its former Situation: and since by the continual Action of the Elastick Force a sufficiently swift Motion is raised in the String, it will happen that when it shall arrive into its Situation ADB, it will in its Motion persevere towards the same Part, till the Elastick or Restoring Force, constantly resisting this farther Motion, and at length equalling it, shall destroy it, and urge or compel the String towards the Parts C; so that when it shall again arrive into the Situation ADB, it will have the same Force of moving farther towards C, as it had at first of tending towards the Parts E: and so by moving backwards and forwards, it will make continual Vibrations.

Let us now suppose the Body F to strike against the String AB, the String will be then compelled to quit its Situation through the Force exerted on it by the Body F; and its Point D, on which the Body F gave the Stroke, will be moved, together with F, towards C: which Motion will continue even till the restoring Power of the String, which is contrary to the Motion of F, shall be equal to that Motion; which as soon as it is, all the Motion towards C will be destroyed. But this elastick Force acting farther, will bring back the String, which therefore will compel the Body F, and move it along with it with the same Velocity. But (by reason we suppose the Tension of the String to be very great) the String will restore itself with the same Force, wherewith it was first inflected: now the Force wherewith it was inflected, was equivalent to the Momentum of the impinging Body, (for that was all spent in bending the String) so that the String by acting on the Body F with such a Force, will restore to it the same Quantity of Motion, as was spent in bending it; so that the Body F will be ſent back with the ſame Velocity that it came forwards with, and ſo a Reflection will be occaſioned.

Let us now put, inſtead of a String, any elaſtick Body A B, which at firſt we ſhall ſuppoſe fixed and immoveable, and let its Superficies A D B be preſſed



inwards by the Force of the Body F ſtriking againſt it. As ſoon as the compreſſing Force, that is, the Motion of the Body F ſhall ceaſe, the ſpringing Body by its innate Force will reſtore itſelf to its former Figure, and by that Force will compel the Body F towards E; and if both the Bodies are perfectly elaſtick, the reſtoring Force of the Spring will be equivalent to the Force that compreſſes it, that is, to the Momentum of the Body F: ſo that it acting on the Body F with this Force, it will compel that Body to return back with the ſame Velocity which it had at firſt. But if the Body ADBC is not fixed, but in ſuch a ſtate as its Motion is not hindered by any other Body, then the elaſtick Force will equally exert itſelf on both the Bodies, and will produce equal Changes of Motions: for if the Body A D B compels the Body F towards the Part E, that again will be equally compelled by the Body F towards the contrary Part; and therefore the Bodies will rebound from each other. And ſo we have demonſtrated after what manner it happens, that Bodies after the Im-

pulſe

pulſe do not either reſt, or move conjointly together, but by rebounding from one another, they move with different Velocities ſometimes towards the contrary Parts, and ſometimes towards the ſame.

THE *Carteſians*, who were ignorant of the Force of Elaſticity to reflect Bodies, have given a quite different Cauſe of Reflection: for they ſay, that Motion is not contrary to Motion, but its Direction to its Direction; ſo that one Body ſtriking againſt another, is reflected, becauſe the Motion of the Body that gives the Stroke cannot be deſtroyed, ſince according to them nothing can be contrary to Motion: but ſince the Direction of one oppoſes the Direction of another, they will have it, that the ſtriking Body after the Impulſe muſt be reflected towards the contrary Parts, the Quantity of Motion remaining the ſame in the Body that gives the ſtroke, and in that which received it.

BUT it is eaſy to ſhew, that this Notion is neither agreeable to Reaſon or Experience: for ſince the Momentum or Quantity of Motion conſtitutes that Force or Energy wherewith the moving Body tends according to its Direction, if two Bodies directly meet each other, the Forces impreſſed according to their contrary Courſes, will be contrary: ſo that if they are equal, they will mutually deſtroy each other; if unequal, the Motion that is of the leſs efficacy, will be deſtroyed. Beſides, one Body ſtriking againſt a greater Body at reſt, or moving ſlower towards the ſame Parts, is reflected; but this cannot be done only by reaſon the Direction is contrary to the Direction: for if the Body B ſtrikes againſt another greater Body A, that is either at reſt, or moves ſlower towards the ſame Parts; ſince all the Force that is found in both the

the Bodies, tends towards C, that Force can never direct the Motion in both the Bodies towards the contrary Parts. For [by the ſecond Law] all Motion is performed according to the right Line, wherein the Force is impreſſed; but [by Hypotheſis] all the Force is impreſſed according to the right Line BC, from B towards C: wherefore if the Reflection of Motion was only made by the Force that is innate in Bodies, without a new Force, there would be cauſed a Motion in a Courſe that is contrary to that wherein the Force is impreſſed; which cannot be. Therefore that Reflection does not ariſe from the Force firſt impreſſed, but from an Elaſtick Force, with which both the Bodies are endued, and which equally acting both ways, compels the Bodies to rebound from one another.

Besides, if Motion was not contrary to Motion, it would be much eaſier to direct a Body that is once in motion, towards the contrary Parts, than to ſtop it quite: for in the firſt Caſe, the Motion of the Body reflecting on one's Hand is not received, but only turned towards the contrary Parts; but in the latter Caſe, all that Motion is employed on the reſiſting Body, which yet is contrary to manifeſt Experience. Laſtly, if nothing was contrary to Motion, where-ever any Body hit againſt any Obſtacle, there will always be a Reflection, which is nevertheleſs contrary to Obſervation: for Lead, Clay, Wax, and other Bodies that have ſcarce any Elaſticity in them, if they fall on the Pavement, are not reflected; yet when Balls made of Wool or Feathers, Ivory, Marble, or Glaſs, and other the like Bodies, which are very elaſtick, are thrown againſt the ſame Pavement, they ſtrongly rebound: That Reflection therefore does not proceed from the Motion that is common to all Bodies, but from an Elaſticity which is only peculiar to reflecting Bodies. Which was to be ſhewn.

But perhaps the *Cartesians* may ask, how we come to know that Balls of Ivory, Glaſs, and Marble, and other reflecting Bodies, which ſeem to be exceeding hard, are endued with an Elaſticity: I anſwer, That they are elaſtick, may be hence concluded, That when they are ſtruck, they yield a Sound, which ariſes from the Vibrations of the Body that receives the Stroke, after the ſame manner as a ſtretched String, by its Vibrations cauſes Undulations in the Air; and therefore it is not in the leaſt to be doubted, but that theſe Bodies are endued with ſome Spring. And this Argument indeed makes it probable that theſe Bodies have an elaſtick Force, but there is another Argument, whereby this matter may be demonſtratively proved.

For let there be two Balls either of Ivory or Glaſs, and if their Figures were perfectly ſpherical, they would touch one another in one only, and that an indiviſible Point; but this cannot be done by any human Art: yet they may be made ſo nearly ſpherical, that they ſhall touch one another in a phyſical Point, that is, in the leaſt viſible part. Now if the Superficies of one of the Balls is ſtained with Ink (or with any Colour that is eaſily wiped off) and another Body impinges on it at reſt, it will appear from the Experiment, that not only a phyſical Point of the Ball that ſtrikes againſt the other, will, after the Impulſe, be ſtained with the Colour of that other Ball, but a ſufficiently large Part thereof; and this could not be, unleſs their Superficies were changed by the Force of the Stroke: but after the Reflection, we find that both the Balls have recovered their firſt Figure. Wherefore the Balls have an elaſtick Force, whereby they are able to recover their former Figure that was alter'd by the Stroke. *Q. E. D.*

Now follow the Rules of Motion obſerved by Elaſtick Bodies.

THEOR.

THEOR. XXIX.

If *two perfectly Elastick Bodies impinge on one another, their relative Velocity will be the same before and after the Impulse; that is, the perfectly Elastick Bodies will recede from each other after the Stroke, with the same Celerity, wherewith they first approached one another.*

For [by Cor. Theor. 27.] the compressing Force, or the Magnitude of the Stroke in given Bodies, arises from the relative Velocity of the Bodies, and is proportionable to it; and [by Def. 11.] perfectly Elastick Bodies restore themselves to their former Figure by the same Force, whereby they were compressed: that is, the restoring Force is equal to the compressing Force, and therefore is equivalent to the Force wherewith the Bodies approach each other before the Impulse. But Bodies are compelled to separate from one another by this restoring Force, whence this Force acting on the same Bodies, will produce a relative Velocity equal to that which they had at first, or it will make the Bodies to recede from one another with the same Velocity wherewith they at first approached each other. *Q. E. D.*

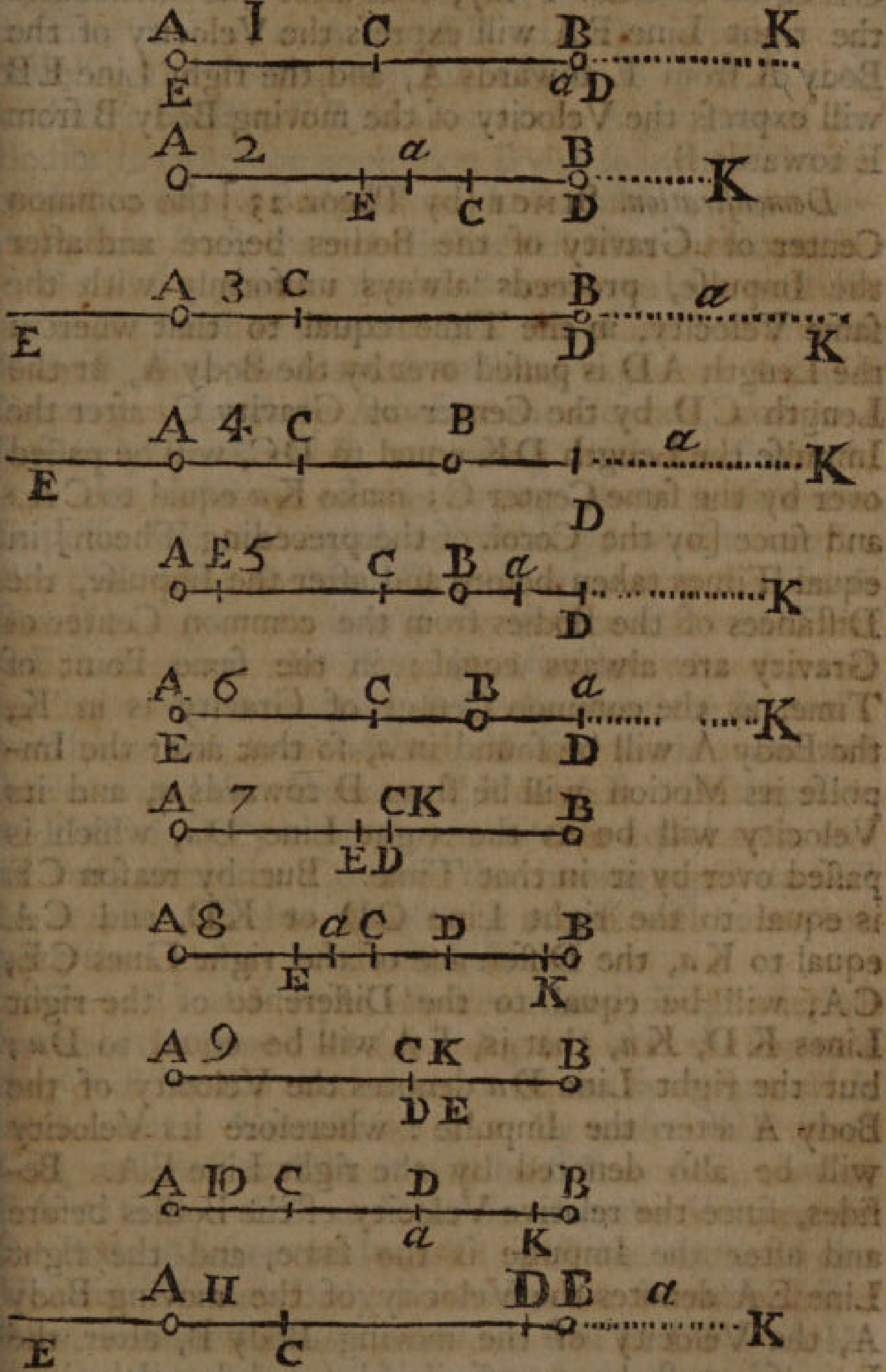
Cor. Therefore in equal Times taken before and after the Impulse, the Distance of the Bodies from one another will be equal; and consequently in the same Times, the Distances of the Bodies from the common Center of Gravity will be also equal.

From this Corollary, the Rules of Motion in the Congress of perfectly Elastick Bodies may be easily discovered; which we shall therefore do in the following Problem.

PROBL. III.

To *determine the Rules of Congress in Bodies that are perfectly Elastick, and which do impinge directly on one another.*

We shall give at once the Construction of all the Cases of this Problem. Let A and B be two perfectly


A 1 C B K
E a D
A 2 a B
K
E C D
A 3 C B a
E D K
A 4 C B a
K
E D
A E 5 C B a
K
D
A 6 C B a
K
E D
A 7 CK B
ED
A 8 a C D B
E K
A 9 CK B
DE
A 10 C D B
a K
A 11 DB a
K
E C

fectly Elastick Bodies, whose common Center of Gravity let be C, and let the Bodies meet in D; make CE equal to CD: I say, that after the Concourse the right Line EA will express the Velocity of the Body A from E towards A, and the right Line EB will express the Velocity of the moving Body B from E towards B.

Demonstration. SINCE [by Theor. 23.] the common Center of Gravity of the Bodies before and after the Impulse, proceeds always uniformly with the same Velocity, in the Time equal to that wherein the Length AD is passed over by the Body A, or the Length CD by the Center of Gravity C, after the Impulse the Length DK equal to DC, will be passed over by the same Center C; make K*a* equal to CA: and since [by the Corol. of the preceding Theor.] in equal Times taken before and after the Impulse, the Distances of the Bodies from the common Center of Gravity are always equal; in the same Point of Time, as the common Center of Gravity is in K, the Body A will be found in *a*, so that after the Impulse its Motion will be from D towards *a*, and its Velocity will be as the right Line D*a*, which is passed over by it in that Time. But by reason CE is equal to the right Line CD or KD, and CA equal to K*a*, the Difference of the right Lines CE, CA, will be equal to the Difference of the right Lines KD, K*a*, that is, EA will be equal to D*a*; but the right Line D*a* denotes the Velocity of the Body A after the Impulse: wherefore its Velocity will be also denoted by the right Line EA. Besides, since the relative Velocity of the Bodies before and after the Impulse is the same, and the right Line EA denotes the Velocity of the moving Body A, the Velocity of the moving Body B, after the Impulse must be necessarily denoted by the right Line EB; namely, from E towards B. *Q. E. D.*

Cor.

Cor. 1. If the Body B is at rest, the Point D will coincide with B, as in the first three Figures: and because B is to A as AC to CB, it will be by Composition B + A to A as AB to CB: whence by doubling the Consequents, it will be B + A to 2 A, as AB to 2 CB or EB; that is, as the Aggregate of the Bodies to twice the impinging Body, so is the Celerity of the impinging Body before the Impulse, to the Celerity of the Body that was first at rest after the Contact.

Cor. 2. So that if A and B are equal, it will be A + B = 2 A; whence EB, the Celerity of the Body B, after the Contact will be equal to AB the Celerity of the Body A before the Contact, and consequently the Point E coinciding with the Point A, AE the Velocity of the moving Body A after the Impulse will be equal to nothing; which may be also thus easily shewn: By reason of the Bodies A and B being equal, it will be AC = CB = CD = CE, wherefore the Point E coincides with A, and consequently the moving Body A, after the Impulse, will be at rest, and the Body B after the Impulse will move with the Celerity EB or AB. If therefore an Elastick Body impinges on another equal to it and at rest, after the Contact the impinging Body will be at rest, and that which was at rest will be moved with the Celerity of the first Body.

Cor. 3. If the equal Bodies A and B are carried towards the same Part (as in Fig. 4.) after the Contact they will be also carried towards the same Part with interchanged Velocities. For by reason CE is = CD, and AC = CB, it will be CE — AC, that is, EA = CD — CB or BD; so that the Velocity of the Body A after the Impulse will be equal to the Velocity of the Body B before the Impulse. Besides, because EA is = BD, it will be EB = AD; and consequently the Velocity of the Body B after the Contact will be equal to the Velocity that the Body A had before the Concourse. *Cor.*

Cor. 4. If the equal Bodies A and B are carried towards the contrary Parts, (as in Fig. 8.) they will after the Impulse recede towards the contrary Parts, with interchanged Velocities. For by reason A C is = C B and C E = C D, it will be A C — C E, that is, A E = C B — C D or B D; so that the Velocity of the Body A, after the Impulse, will be equal to the Velocity the Body B had before the Impulse. Besides, by reason E A is = B D, it will be A D = E B; but A D was the Velocity of the Body A before the Concourse, and E B is the Velocity of the Body B after the Concourse, whence the Corollary is evident.

But since Calculation is always used in Practice, we shall here deliver a Method, whereby the Celerities of Elastick Bodies after the Impulse may be investigated, and brought to Numbers; and it would indeed be easy, after the manner of the foregoing Corollaries, to reduce to Numbers all the particular Cases from the general Construction; but a general Calculation may be thus more readily discovered.

Let us in the first place suppose the Bodies A and B to be moved towards the same Part; and let C be the Velocity of the following Body A, but c the Velocity of the preceding Body B: whence the relative Velocity of the Bodies will be $C - c$, and the Sum of the Motions towards the same Part $AC + Bc$.

Let the Velocity of the Body A after the Impulse towards the same Part, as before, be called x; and because the relative Velocity of the Bodies both before and after the Concourse is the same, the Velocity of the Body B will be $x + C - c$: for the relative Velocity of the Bodies is equal to the Excess of the Velocity, whereby the Velocity of the swifter Body exceeds that of the

the ſlower, ſo that the Exceſs ought to be $C - c$; but ſince the Velocity of the Body A is x, its Motion towards the Part D will be Ax; and ſince the Velocity of the Body B is $x + C - c$, its Motion towards the ſame Part will be $Bx + BC - Bc$; and the Sum of theſe Motions will be equal to the Sum of the former Motions, that is, it will be $Ax + Bx + BC - Bc = AC + Bc$: whence by reducing this Equation, it will be $Ax + Bx = AC - BC + 2Bc$; and $x = \frac{AC - BC + 2Bc}{A + B}$ = to the Velocity of the Body A. Moreover, the Velocity of the Body B is $= x + C - c = \frac{AC - BC + 2Bc}{A + B}$ $+ C - c = \frac{AC - BC + 2Bc + AC + BC - Ac - Bc}{A + B}$ $= \frac{2AC - Ac + Bc}{A + B}$.

If BC is greater than $AC + 2Bc$, x or $\frac{AC - BC + 2Bc}{A + B}$ will be a negative Quantity; ſo that the Velocity of the Body A in that caſe will be towards the contrary part, and its Motion towards D will be negative. If the Body B is at reſt, that is, if $c = 0$, the Velocity of the Body A after the Impulſe will be $+ \frac{AC - BC}{A + B}$ forwards or backwards, as the Sign $+$ or $-$ ſhall prevail.

If the Bodies A and B carried towards the contrary Parts with the Celerities C and c, do directly impinge on each other, their Motion towards the ſame Part will be $AC - Bc$, and the relative Velocities of the Bodies will be $C + c$. Now let x be the Velocity of the Body A after the Impulſe; its Motion towards the ſame Part as before, will be Ax, and the Velocity of the Body B will be $x + C + c$, (for the relative Velocity of Bodies is not altered by the Stroke) and the Motion in the Body B towards D will be $Bx + BC + Bc$; whence the Sum of the Motions towards the same Part will be $Ax + Bx + BC + Bc$, which [by Theor. 14.] will be $= AC - Bc$: so that it will be $Ax + Bx = AC - BC - 2Bc$, and $x = \frac{AC - BC - 2Bc}{A + B}$, and the Velocity of the Body B will be $\frac{AC - BC - 2Bc}{A + B} + C + c$ $= \frac{AC - BC - 2Bc + AC + Ac + BC + Bc}{A + B} = \frac{2AC + Ac - Bc}{A + B}$.

If $BC + 2Bc$ is greater than AC, the Motion of the Body A will be backwards; namely, towards the contrary Part: in which case x or $\frac{AC - BC - 2BC}{A + B}$ will be a negative Quantity.

The first, as I know of, who gave the true Laws of Motion in hard Bodies, was Dr. *John Wallis*, the famous *Savilian* Professor of Geometry in this University, which he did in the *Philosophical Transactions* N° 43. where he also shew'd the true Cause of Reflections in other Bodies, and proved that they proceeded from their Elasticity. Not long after, the famous Sir *Christopher Wren*, and Mons. *Huygens*, imparted to our *Royal Society* the Laws that are observed by perfectly Elastick Bodies, and gave exactly the same Construction, tho each was ignorant of what the other had done. But since they published in the *Philosophical Transactions*, the Constructions and Laws of Motion without any Demonstration; we have thought fit to take thence their very elegant Construction, and to demonstrate it.

After a not unlike Method is constructed the Problem in such Bodies as are indeed Elastick, but which do not restore themselves with a Force equal to that whereby they were compressed. For let there be any two Bodies A and B, whose common Center

Center of Gravity let be C; let AC, BC, be so cut in *a* and *b*, that AC is to *a*C and BC to *b*C, as the

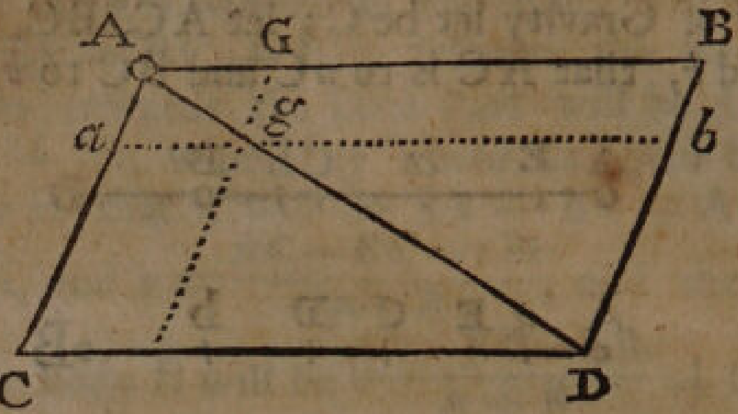
Force compressing the Elastick Body, to the Force whereby it restores itself: and let CE be equal to CD, E*a* will be the Velocity of the Body A after the Impulse from E towards *a*, and E*b* will be the Velocity of the Body B from E towards D.

But if the restoring Force is equal to the compressing Force, the Point *a* will coincide with A, and the Construction returns to the former. The Demonstration is easy to any one that understands the preceding, nor is it necessary to add it here.

THEOR. XXX.

If *a Body* A *is moved uniformly in the right Line* AB, *and in the mean while that right Line* AB *is carried always parallel to itself, as also with an equable Motion according to a Direction that is parallel to* AC; *and let the Velocity of the Body* A *be to the Velocity of the Line* AB *as* AB *to* AC, *and let be compleated the Parallelogram* ABDC, *whose Diagonal let be* AD: *this will be the true Line described by the Body* A *in its Motion.*

When the Line AB shall come to the Situation *ab*, let *g* be the Place of the Body A; and because [by Theor. 6.] the Spaces described in the same Time are as the Velocities, *ag* the Length passed over by the moving Body A, will be to A*a* the Length passed over by the Line AB, as the Velocity of A to the Velocity of the right Line AB, that is [by Hypothe-



ſis] as AB to AC: whence the Parallelogram *a*G will be ſimilar to the Parallelogram CB, and conſequently [by 24 *El.* 6.] the Point *g* will be placed in the Diagonal AD; that is, the Body A will be always found in the right Line AD: ſo that this Line will be paſſed over by it. *Q. E. D.*

Cor. 1. THE Line AD will be deſcribed by the moving Body A in the ſame Time, wherein, without the Motion according to AC, it would paſs over the Line AB; or wherein, without the Motion according to AB, it would deſcribe the right Line AC.

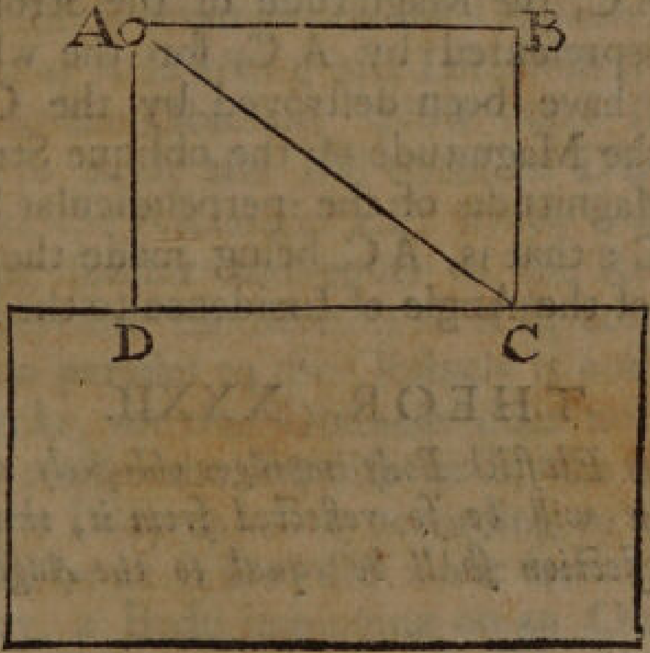
Cor. 2. SINCE the moving Body is therefore carried in the right Line AD, becauſe beſide its own proper Motion, it likewiſe partakes of the Motion of its Place, or of the right Line AB, and its Motion is compounded of both theſe; if any moveable Body has two Motions impreſſed on it at the ſame time, according to the Directions AB, AC; and thoſe Motions, or the Forces whereby they are produced, are as the right Lines AB, AC: AD will be the Line deſcribed by the moving Body, that receives the Motions impreſſed by theſe two Forces, and its Force, wherewith it is carried in the right Line AD, will be to the Forces according to AB, AC, as the Diagonal AD to the Sides of the Parallelogram AB, AC.

Cor. 3. HENCE converſly, if the moving Body with a Force as AD paſſes over the right Line AD, the

the Motion will be the same, and according to the same Direction, as if at the beginning of the Motion it had been impelled by two Forces at once, each proportionable to the right Lines AB, AC, according to the Directions from A to B, and from A to C. And hence any Motion whatever, tho simple in itself, may be considered as compounded of more Motions; and any Forces may be resolved into others, that are more in number, and act according to different Directions.

THEOR. XXXI.

IF *the Body* A *impinges obliquely on a firm Obstacle* DC, *the Energy of the Percussion, or Magnitude of the oblique Stroke, will be to the Magnitude of the Stroke that*



the same Body would have produced, if it had impinged perpendicularly with the same Celerity, as the Sine of the Angle of Incidence ACD *to the Radius.*

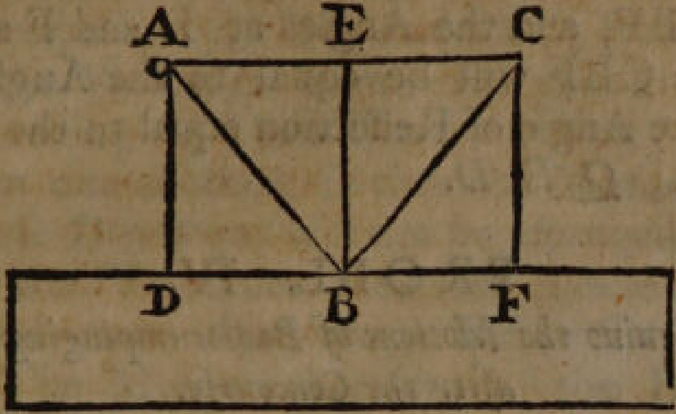
FROM A let fall on the Obstacle the Perpendicular AD, if the Superficies of the Obstacle is a Plane; or if a Curve, let the Perpendicular be let fall on the

Plane touching the Obſtacle in the Point of Incidence C, and compleat the Rectangle DB. Now [by Cor. 3. of the preceding Theor.] the Motion of the Body A being as A C in the right Line A C, will be equivalent to two Motions impreſſed at the ſame time according to the Directions AB, A D, which are to the Motions in AC as the right Lines AB, A D, to A C: but the Obſtacle does no ways reſiſt the Motion in the right Line A B; for ſince A B is parallel to D C, the Body moved in the right Line A B will never impinge on the Obſtacle D C. The Force therefore wherewith it impinges on the Obſtacle, is as the right Line A D: the Force therefore of the Body A in the right Line AC, is to the Force wherewith it impinges on the Obſtacle, as A C to A D. But if it had impinged perpendicularly on the ſame with a Force as A C, the Magnitude of the Stroke would have been repreſented by A C, for the whole Motion would have been deſtroyed by the Obſtacle: wherefore the Magnitude of the oblique Stroke will be to the Magnitude of the perpendicular Stroke as A D to AC; that is, AC being made the Radius, as the Sine of the Angle of Incidence to the Radius.

THEOR. XXXII.

IF *a perfectly Elaſtick Body impinges obliquely on a firm Obſtacle, it will be ſo reflected from it, that the Angle of Reflection ſhall be equal to the Angle of Incidence.*

LET the perfectly Elaſtick Body A fall obliquely on the firm Obſtacle according to the Line AB; I ſay, that will with the ſame Celerity be ſo reflected in the right Line BC, that the Angle of Reflection CBF ſhall be equal to the Angle of Incidence ABD. Let the right Line A B expreſs the Motion of the Body A in the Direction A B; [by Corol. 3. Theor.



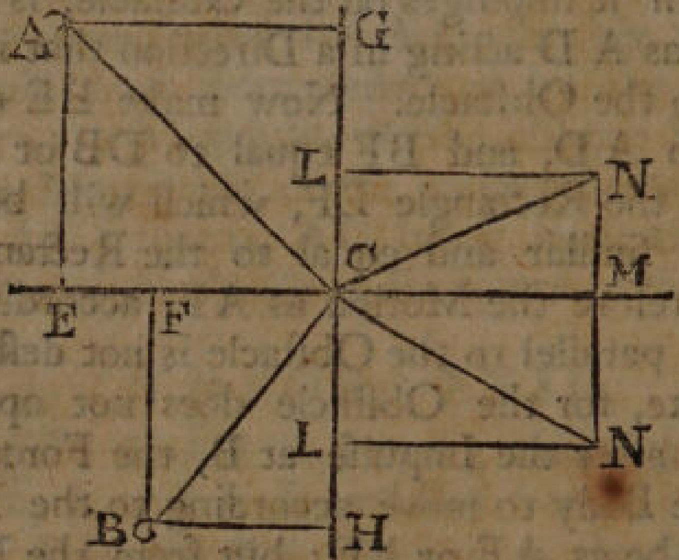
Theor. 30.] let this Motion be resolved into two others, according to the Directions AE, AD, to which the Motion in AB is as AB to AE, AD: but since AE is parallel to the Superficies of the Obstacle, and AD perpendicular to it, or at least to the Plane touching the Obstacle in B; that Force wherewith it impinges on the Obstacle, is only that which is as AD acting in a Direction that is perpendicular to the Obstacle. Now make BE equal and parallel to AD, and BF equal to DB or AE, and compleat the Rectangle EF, which will be in every thing similar and equal to the Rectangle DE. Since therefore the Motion as AE, according to the Direction parallel to the Obstacle is not destroyed by the Stroke, for the Obstacle does not oppose this Motion; after the Impulse at B, the Force remaining in the Body to move according to the Direction BF, will be as AE or BF: but from the Nature of Elasticity, a Body impinging on an Obstacle with a Force as EB according to the Direction EB, is reflected with the same Force according to the same Direction; the Motion therefore of the Body at the Point of Incidence B is compounded of the Motion as BF according to the Direction BF, and the Motion as BE according to the Direction BF. Wherefore [by Corol. 2. Theor. 30.] the Body will be moved in the right Line BC with a Force as BC: but by

reason AD, CF, are equal and parallel, and by reason DB, BF, and the Angles at D and F are equal, the Angle CBF will be equal to the Angle ABD, that is, the Angle of Reflection equal to the Angle of Incidence. *Q. E. D.*

PROBL. IV.

TO *determine the Motions of Bodies impinging obliquely after the Concourse.*

LET any Bodies A and B be moved in the Lines AC, BC, inclined to one another, whose Lengths let respectively express the Velocities of the Bodies A, B: let EFC represent the Plane whereby the Bodies are touch'd in the Point of Concourse; on which from A and B let fall the Perpendiculars AE,



BF, which may express the Velocities wherewith the Bodies approach each other. Compleat the Rectangles EG, FH. [By Cor. 3. Theor. 30.] the Motion of the Body A is resolved into two others, according to the Directions AG, AE; to which the Motion in AC is as AC to AG, AE, respectively: in like manner, the Motion of the Body B is resolved into two other, according to the Directions BF, BH;

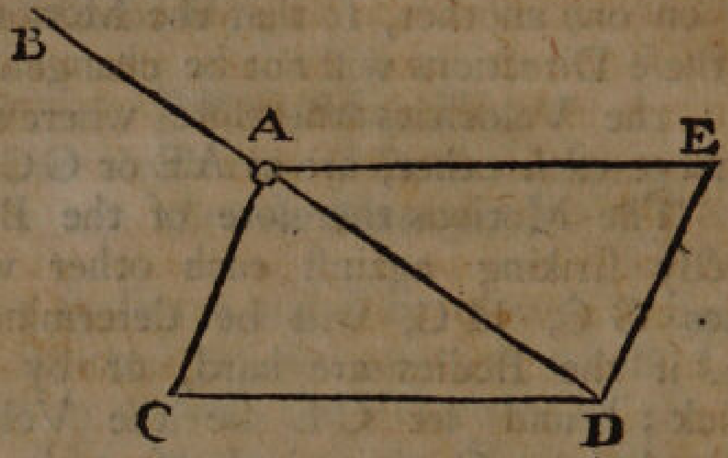
BH; to which the Motion in BC is as BC to BF, BH, respectively. But since AG, BH, are parallel, by the Velocities wherewith the Bodies are moved according to these Directions, they will not impinge on one another, so that the Motion according to these Directions will not be changed by the Impulse; the Velocities therefore wherewith the Bodies meet each other, are as AE or GC and BF or HC. The Motions therefore of the Bodies A, B, directly striking against each other with the Velocities GC, HC, will be determined, [by Probl. 2. if the Bodies are hard, or by Probl. 3. if elastick;] and let CL be the Velocity of the Body A from C towards L after the Impulse, which arises from the Velocities GC, HC. And since, as has been shewn, there remains in the Body a Force of moving according to the Direction parallel to AG with a Velocity as AG, make CM equal to AG, and compleat the Rectangle LM: the Body A, after the Impulse, will move in its Diagonal CN with a Velocity as CN, as appears by Corol. 2. Theor. 30. and after the same manner will be determined the Motion of the Body B after the Impulse. *Q. E. F.*

THEOR. XXXIII.

IF *a moveable Body* A *is drawn by three Powers by the means of three Strings, or by any other manner whatever is sollicited according to the Directions* AB, AE, AC, *so that these three Powers are equivalent to each other, that is, if any two of them destroy the Effect of the other, and the Body is moved by none of them; these Powers will have amongst themselves the same Ratio, as have the right Lines parallel to their Directions, and terminated by their mutual Concourse.*

LET AD express the Power or Force wherewith the moveable Body A is sollicited from A towards B; the

the Force equivalent or equal to this, and ſolliciting the Body A the contrary way from A towards D, may be alſo expreſſed by AD: but [by Cor. 3.

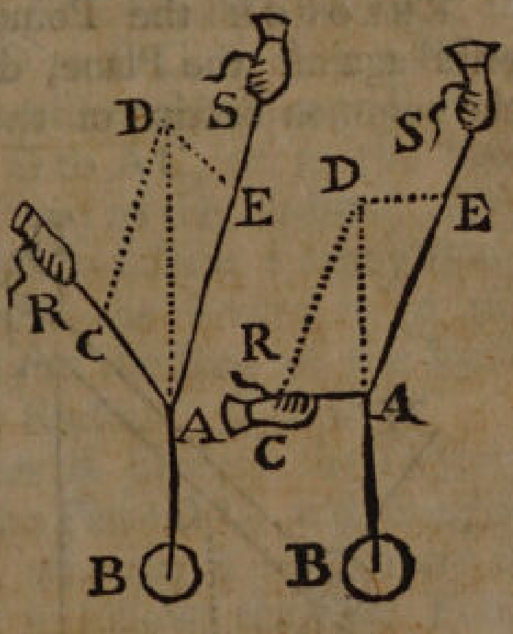


Theor. 30.] the Force impelling the Body from A towards D, will be equivalent to two Forces acting according to the Directions AC, AE, to which the former Force acting from A towards D, is as AD to AC, AE, or to AC, CD, reſpectively; and inverſely the Forces acting according to the right Lines AC, AE, and equivalent together to the Force ſolliciting the Body from A towards D, ought to be to the ſame Force according to AD as AC and AE or CD to AD: wherefore likewiſe the Forces acting according to the right Lines AC, AE, and equivalent to the Force wherewith the Body is ſollicited from A towards B, and deſtroying its Effect, ought to be to the ſame, as AC, CD, to AD; that is, if the ſame moveable Body is ſollicited by three Powers equivalent to each other, according to the Directions AB, AC, AE, theſe three Powers will be as the right Lines AD, AC, AB, reſpectively. *Q. E. D.*

Cor. 1. SINCE in every Triangle the Sides are as the Sines of the oppoſite Angles, AC will be to CD as the Sine of the Angle ADC or DAE to the Sine of the Angle DAC; whence any two Powers will

will be amongst themselves reciprocally as the Sines of the Angles, which the Lines of their Directions contain with the Line of Direction of the third Power. Besides, AD is to AC as the Sine of the Angle C or AED to the Sine of the Angle CDA or DAE; and in like manner, the Power acting according to AB, is to the Power according to AE as the Sine of the Angle AED to the Sine of the Angle ADE or CAD.

Cor. 2. If two Powers R, S, sustain a Weight B, by the means of Strings drawing in the right Lines AR, AS, the Point A will be sollicited by three Powers, whereof two act according to the Direction AR, AS, and the other is the Force of the Gravity of the Weight B, acting according to the right Line AB perpendicular to the Earth: whence the Power R will be to the Force of Gravity as AC to AD, or as the Sine of the Angle DAE to the Sine of the Angle DEA or CAE; and the Power S will be to the Force of Gravity as EA to AD, or the Sine of the Angle CAD to the Sine of the Angle DEA or CAE; and the Power R will be to the Power S as the Sine of the Angle EAD to the Sine of the Angle CAD.



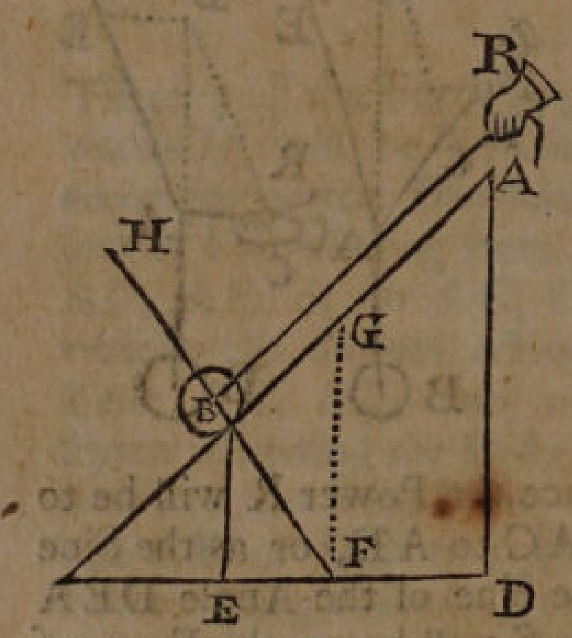
This Theorem, with its Corollaries, is the Foundation of the *New Mechanicks* published by M. *Varignion*; and from it likewise may immediately be deduced most of the mechanick Theorems contained in the famous Book of *Borelli, de Motu Animalium*, for by its help may the Force of the Muscles be determined.

THEOR.

THEOR. XXXIV.

If *an heavy Body* B *is incumbent on an inclined Plane, and is supported by a Power* R, *acting according to a Direction that is parallel to the Plane, so that it does not descend along that Plane; the Power* R *will be to the Weight of the Body* B *as the Sine of the Angle of Inclination to the Radius.*

Through the Point where the heavy Body rests against the Plane, draw AC perpendicular to the common Section of the Plane and the Horizon; from any Point of which A, let fall on the Plane of the Horizon the Perpendicular AD, and join CD; ACD will be [by Def. 6. El. 11.] the Angle of Inclination of the Plane and the Horizon, whose Sine is AD, CA being made the Radius. Now I say, AC is to AD as the Weight of the Body B to the Power R. For the Body B is sollicited by three Powers acting according to different Directions, and placed *in æquilibrio*; whereof the first is the Force of Gravity acting according to the Direction BE perpendicular to CD; the second is the Power R, drawing the Body according to the Direction BR parallel to AC; but the place of the third Power is supplied by the Resistance of the Plane acting according to the Line BH perpendicular to it; for Re-action is always equal to Action, and is made towards the contrary Part: and since the Plane is pressed perpendicularly by the moveable Body,



Body, according to the Direction BH, and that contrary Endeavour is equivalent to the Power sollicting the Body according to BH; and since these three Powers are *in æquilibrio*, and the moveable Body is sustained by them, if FG is drawn parallel to EB, meeting the right Line AG in G, the Power R will be to the Force of Gravity as BG to FG, [by the preceding Theor.] but by reason the Triangle CFG is rectangular, and FB a Perpendicular to its Base CG, it is [by 8 *El.6.*] as BG to FG, so is FG to GC, and as FG to GC, so [by 4 *El.6.*] will be AD to AC: wherefore the Power R is to the Force of Gravity as AD to AC, or as the Sine of the Inclination of the Plane to the Radius. Any Power therefore may be able to sustain a heavy Body on an inclined Plane, so that the Power is to the Weight of the heavy Body as the Sine of the Inclination of the Plane to the Radius. *Q.E.D.*

Cor. 1. Since the Power R hinders the Descent of the heavy Body down the Plane AC, and is equivalent to its Momentum, whereby it endeavours to descend down the Plane; it follows that the Force of any heavy Body descending down an inclined Plane, is to the Force wherewith it endeavours to descend in the Perpendicular, as the Sine of Inclination of the Plane to the Radius.

Cor. 2. Hence there may be assigned such an Inclination of a Plane, that any, the smallest Power, may be able to sustain or even raise upon that Plane any Weight, how great soever.

LEC-

LECTURE XV.

Of the Defcent of heavy Bodies on inclined Planes, and of the Motion of Pendulums.

HAVING finifhed what belongs to Motion in general, we fhall now proceed to treat of thofe Motions that arife from given Forces; in explaining of which, and reviewing the Phenomena thence arifing, is the Bufinefs of the true Philofophy. That therefore we may begin with what is moft fimple, we fhall in the firft place confider that Force, which is always directed toward the fame Part uniformly, that is, every where with the fame Tenour; and fuch is commonly fuppofed the Force of Gravity. For though it is certain that the Force of Gravity is not every where the fame, but at different Diftances from the Center of the Earth, is reciprocally as the Squares of fuchDiftances; yet fince the differentHeights to which heavy Bodies can be thrown by us, are very fmall, in comparifon of the vaft Diftance from the Center of the Earth, in fo very little Difference of Heights, we may fafely, and without any fenfible Error, fuppofe the Force of Gravity to be every where the fame.

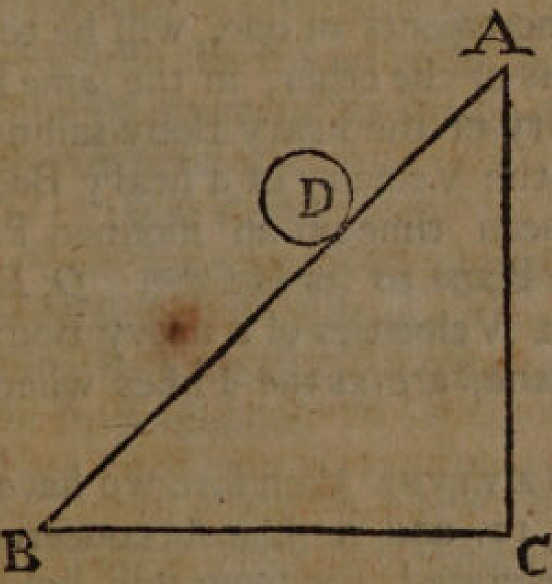
We fhall therefore in this place treat of the Motion of heavy Bodies: but we fuppofe this Motion to be performed, either in Planes inclining to the Horizon, or in curve Superficies, fuch as are fpherical and cycloidal ones; or laftly, in free and unrefifting Spaces: concerning all which we fhall give the following Theorems.

THE-

THEOR. XXXV.

THE *Descent of a heavy Body, upon any inclined Plane, is a Motion equably accelerated. And the Velocity that a heavy Body falling from Rest along an inclined Plane, acquires in any given Time, is to the Velocity acquired in the same Time, by a heavy Body falling perpendicularly, as the Height of the Plane to its Length.*

LET AB be the inclined Plane on which the heavy Body D descends. [By Corol. 1. Theor. 34.] the Force wherewith a heavy Body endeavours to de-



scend on an inclined Plane, is to the absolute Force of Gravity, namely that whereby it would descend in the Perpendicular, in a constant Ratio, which is that of the Sine of the Inclination of the Plane to the Radius, or as the Height of the Plane to its Length; so that since the absolute Force of Gravity of the Body D remains the same, the Force likewise wherewith it endeavours to descend on the Plane AB, will remain the same. That Force therefore will always act on the heavy Body D in the same Tenour; so that being applied in the same manner, it will, by the second Law, always add equal Increments of Veloci-

ties,

ties, not otherwise than is done in heavy Bodies falling perpendicularly. The Descent therefore of heavy Bodies on an inclined Plane, is a Motion uniformly accelerated. *Q. E. D.*

MOREOVER, the Increments of the Velocities of heavy Bodies falling in the Perpendicular, and on an inclined Plane, which are produced in the same indefinitely small Time, are to one another as the Forces wherewith they are produced: but the Forces are in a constant Ratio, namely, as the Length of the Plane AB to its Height AC; wherefore the Increments of the Velocities thence arising, will be in the same Ratio. And consequently, [by 12 *El.* 5.] the Sum of the Increments of one, will be to the Sum of the Increments of the other, in the same Ratio; that is, the Velocity of the heavy Body falling perpendicularly, is to the Velocity of a heavy Body descending in the mean time on an inclined Plane, as the Length of the Plane to its Height. *Q. E. D.*

Cor. 1. THE Velocities of a heavy Body falling on an inclined Plane, are as the Times wherein they are acquired.

Cor. 2. WHATEVER therefore we have demonstrated in Theor. 12. and its Corol. concerning Motion uniformly accelerated, the same will hold true in the Descent of heavy Bodies on inclined Planes; namely, the Space passed over in a given Time by a heavy Body falling on an inclined Plane, computing from the beginning of Motion, will be the half of that, which in the same Time may be passed over by a Body moving uniformly, with the Velocity acquired at the last. Also the Spaces passed over, computing from the beginning of the Motion, are in a duplicate Ratio of the Times or Celerities; and the Celerities and Times are in a subduplicate Ratio of the Spaces passed over.

Cor.

Cor. 3. Hence also the Ascent of a Body along any inclined Plane, is a Motion uniformly retarded, as it is in the Ascent of a Body in the Perpendicular, and it has just the same Properties.

SCHOLIUM.

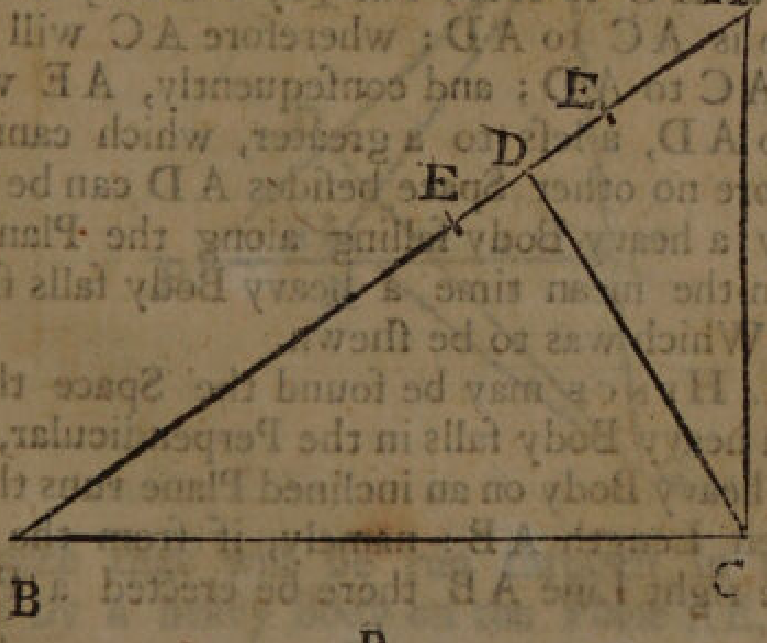
If we have recourse to Experiments, we shall find all things answer to our Reasonings; and in Planes that are not too much inclined, it will be easy to make an Experiment, since Motions that are not very swift may be exactly measured: But it is the contrary in the perpendicular Descent, where the Swiftness of the Motion gives not time to make accurate Observations.

We here suppose the Planes to be exactly smooth, and the Motion on them to be hindered by no Ruggedness or Unevenness.

PROBL. V.

An *inclined Plane being given, to assign the Part of it which a heavy Body will pass over, in the Time whilst another heavy Body shall have fallen through a given Space in the Perpendicular.*

Let AB be the inclined Plane on which the heavy Body descends from A; the Length is to be

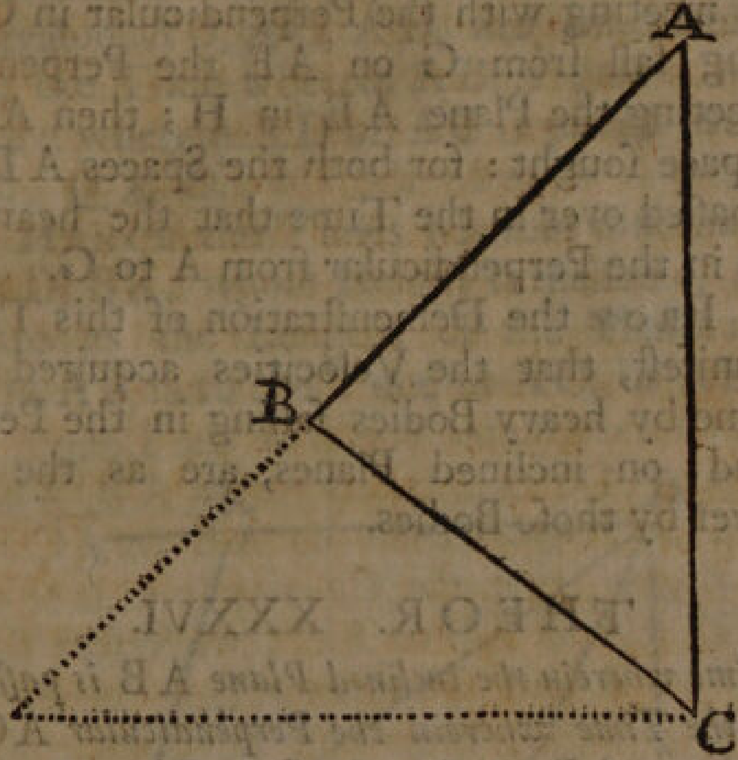


P

assigned

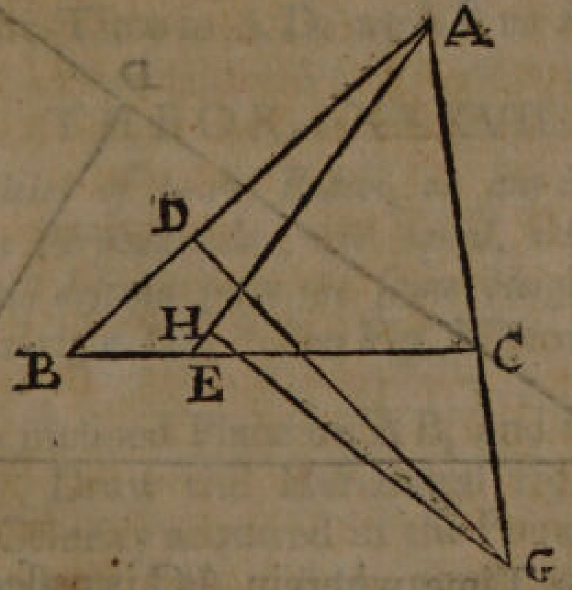
assigned in it, which will be passed over by the heavy Body falling along the inclined Plane: whilst in the mean while another heavy Body falling perpendicularly, shall have run over the Space AC. From the Point C let fall the Perpendicular CD on AB meeting the Plane in D; then AD will be the Space passed over along the inclined Plane, in the Time wherein the heavy Body falls in the Perpendicular from A to C. For if AD is not, let AE be the Space described in the same Time that the heavy Body falls from A to C, which let be greater or less than AD. Draw the horizontal right Line CB. And because by Theor. 12. in the Time that the heavy Body falls from A to C, or from A to E, it may run over twice the Length of AC, with an uniform Velocity, and equal to that which it acquires from falling to C, [as by the Cor. of the preceding Theor.] in the same Time a Length double of AE may be run over, with the Velocity that is acquired in E; the Velocity in C, by Theor. 6. will be to the Velocity acquired in E as twice AC to twice AE, or as AC to AE. But since AC, AE, are run over in the same Time, the Velocity in C [by the preceding Theor.] will be to the Velocity in E as AB to AC: wherefore AB will be to AC as AC to AE; but [by 8 *El.* 6.] as AB to AC, so is AC to AD: wherefore AC will be to AE as AC to AD; and consequently, AE will be equal to AD, a less to a greater, which cannot be. Therefore no other Space besides AD can be passed over by a heavy Body falling along the Plane AB, whilst in the mean time a heavy Body falls from A to C. Which was to be shewn.

Cor. 1. HENCE may be found the Space through which a heavy Body falls in the Perpendicular, whilst another heavy Body on an inclined Plane runs through any given Length AB: namely, if from the Point B to the right Line AB there be erected a Perpendicular



dicular BC, meeting the Perpendicular in C, then AC will be the Space fought.

Cor. 2. If there are two or more inclined Planes AB, AE: and there is given the Space AD in the Plane AB, passed over by a heavy Body in any Time;

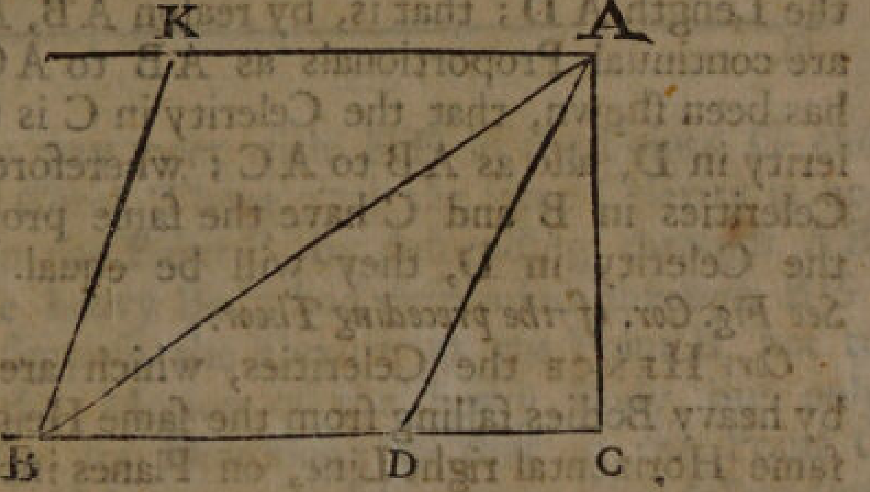


the Space that will be run through in the same Time by a heavy Body on the Plane AE, will be

 found

Ratio of AB to AD, [by Corol. 2. Theor. 35.] that is, by reason AB, AC, AD, are continual Proportionals, the Time wherein AB is passed over is to the Time wherein AD or AC is passed over, as AB to AC. *Q. E. D.*

Cor. HENCE the Times wherein different Planes, AB, AD, BK, whose Height is the same, are passed over, are as the Lengths of the Planes: for the Time in AB is to the Time in AC, as AB to AC;



and the Time in AC to the Time in AD, as AC to AD: wherefore by Equality the Time in AB will be to the Time in AD, as AB to AD.

THEOR. XXXVII.

THE *Celerities of heavy Bodies, on any inclined Plane, and in the Perpendicular, are equal, where the heavy Bodies shall descend from the same Height to the same Horizontal right Line.* See Fig. Theor. 36.

LET the inclined Plane be AB, and the Perpendicular AC. Draw the Horizontal right Line BC. I say, the Celerity acquired in the Point B, after the Descent through AB, will be equal to the Celerity acquired in the Point C, after the Fall through AC. From the Point C let fall the Perpendicular CD on

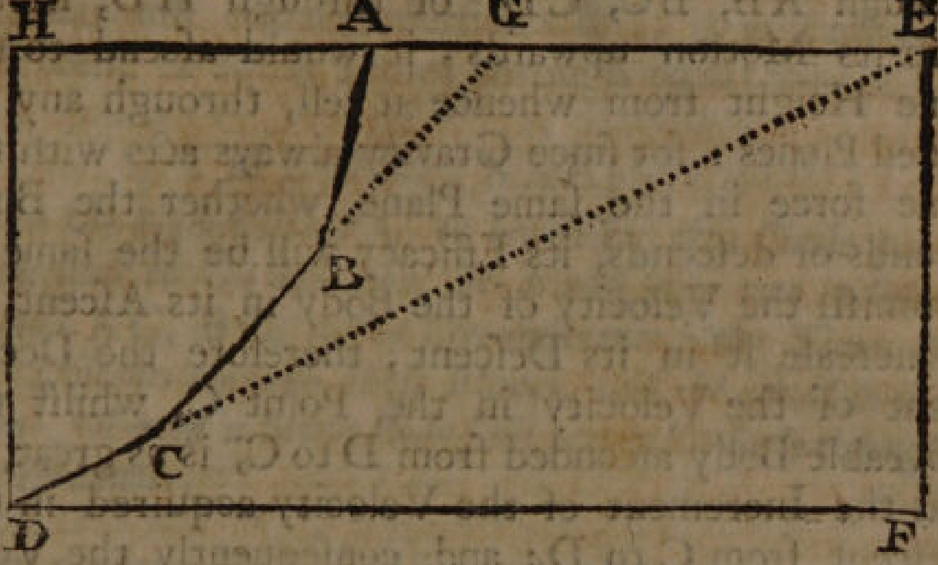
AB. Then AD will be the Space that is run over by the heavy Body falling in the Plane AB, in the Time that another heavy Body deſcends in the Perpendicular through AC: and [by Cor. 3. Probl. 5.] the Celerity in C is to the Celerity in D as AC to AD, or as AB to AC. But becauſe the Celerities acquired in falling along the ſame Plane are in a ſubduplicate Ratio of the Lengths run over by the heavy Body, the Celerity in B will be to the Celerity in D, in a ſubduplicate Ratio of the Length AB to the Length AD; that is, by reaſon AB, AC, AD, are continual Proportionals as AB to AC: but it has been ſhewn, that the Celerity in C is to the Celerity in D, alſo as AB to AC; wherefore ſince the Celerities in B and C have the ſame proportion to the Celerity in D, they will be equal. *Q. E. D.* *See Fig. Cor. of the preceding Theor.*

Cor. Hence the Celerities, which are acquired by heavy Bodies falling from the ſame Height to the ſame Horizontal right Line, on Planes inclined after any manner, are equal amongſt themſelves; for both the Celerities, namely, that which is acquired in the Point B, after the Deſcent through AB, or BK; and that which is acquired in the Point D, after the Deſcent through AD; is equal to the Celerity acquired in the Deſcent of the heavy Body from A to C.

THEOR. XXXVIII.

If *from the ſame Height a moveable Body deſcends with an uninterrupted Motion through any Number of contiguous Planes* AB, BC, CD, *it will always at the concluſion acquire the ſame Velocity; namely, that which is equal to what is acquired by falling perpendicularly from the like Height.*

Through A and D draw the Horizontal right Lines HE, DF, and produce the Planes BC, CD, that

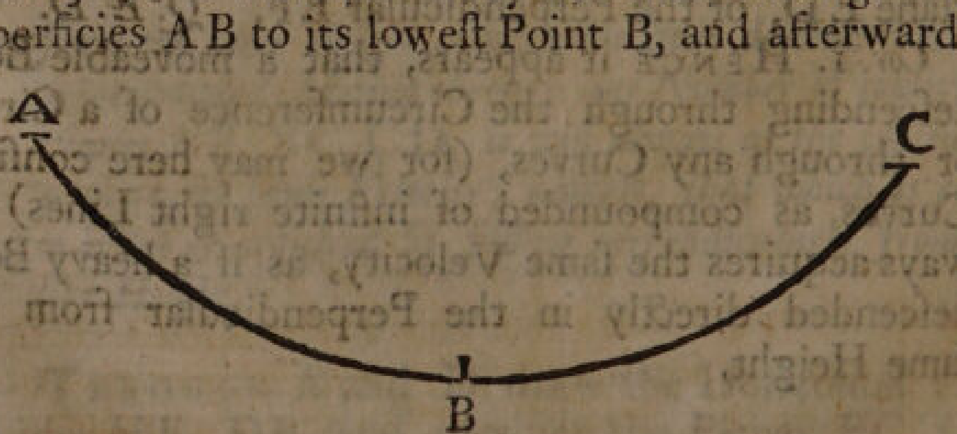


that they may meet with HE in the Points G and E. [By Corol. Theor. 37.] the ſame Celerity is acquired in the Point B, by deſcending through AB, as if the heavy Body had deſcended through BG: but we ſuppoſe the Flexure at the Point B, not to hinder the Motion of the heavy Body, but only to change its Direction; ſo that in the Point C the ſame Velocity will be acquired by deſcending through AB, BC, as if it had deſcended through GC. But by deſcending through GC, the ſame Celerity is acquired, as the heavy Body would have gotten by falling through EC; ſo that ſince the Flexure at C is not ſuppoſed to diminiſh the Velocity of the heavy Body, it will have the ſame Velocity in D, as if it had deſcended through the Plane ED, or the Perpendicular EF. *Q. E. D.*

Cor. 1. Hence it appears, that a moveable Body deſcending through the Circumference of a Circle, or through any Curves, (for we may here conſider Curves as compounded of infinite right Lines) always acquires the ſame Velocity, as if a heavy Body deſcended directly in the Perpendicular from the ſame Height.

Cor. 2. But if a heavy Body, after its Descent through AB, BC, CD, or through HD, should turn its Motion upwards; it would ascend to the same Height from whence it fell, through any inclined Planes: for since Gravity always acts with the same force in the same Plane, whether the Body ascends or descends, its Efficacy will be the same to diminish the Velocity of the Body in its Ascent, as to increase it in its Descent; therefore the Decrement of the Velocity in the Point C, whilst the moveable Body ascended from D to C, is as great, as was the Increment of the Velocity acquired in the Descent from C to D: and consequently the Velocity in C will be the same after the Ascent through CD, as it was before in the same Point, after the Descent through AB, BC. In like manner, the Velocity in B after the Ascent through CB is the same with the Velocity acquired in the Descent through AB or BG. So also Gravity will take as much from the Velocity of the Body in ascending through BA, as was acquired in the Descent through AB; and in the Points that are of an equal height, the Velocity of the Body will be always the same, but the Velocity in the beginning of the Descent, namely, in the Point A, was nothing: so that by ascending, at that Point A all the Velocity is taken away; which Point therefore will be the Boundary to which the moveable Body by ascending will arrive.

Cor. 3. If a moveable Body descends through any Superficies AB to its lowest Point B, and afterwards



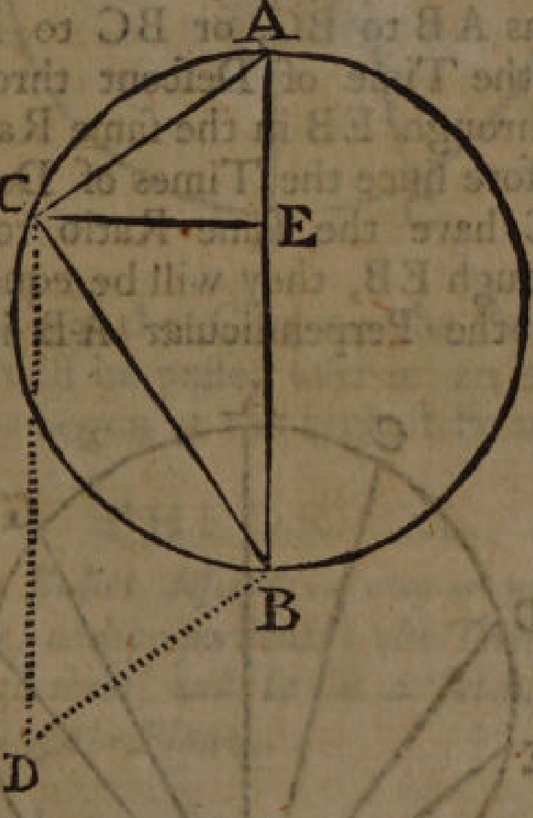
ascends

ascends by the Velocity acquired in the Fall, through the similar and equal Superficies BC; it will ascend and descend in equal Times through equal Spaces.

THEOR. XXXIX.

If from the highest Point A, *or lowest* B, *of a Circle perpendicular to the Horizon, be drawn any two inclined Planes* AC, BC, *to the Circumference; the Times of the Descents through those Planes, will be equal to the Time, wherein a heavy Body falls perpendicularly through the Diameter.*

Let a heavy Body fall from A to C, on the Plane AC; I say, the Time of Descent through AC is equal to the Time of Descent through the Diameter AB. For the Angle ACB in the Semicircle is a right one, [by 31 *El.* 3.] whence since from the



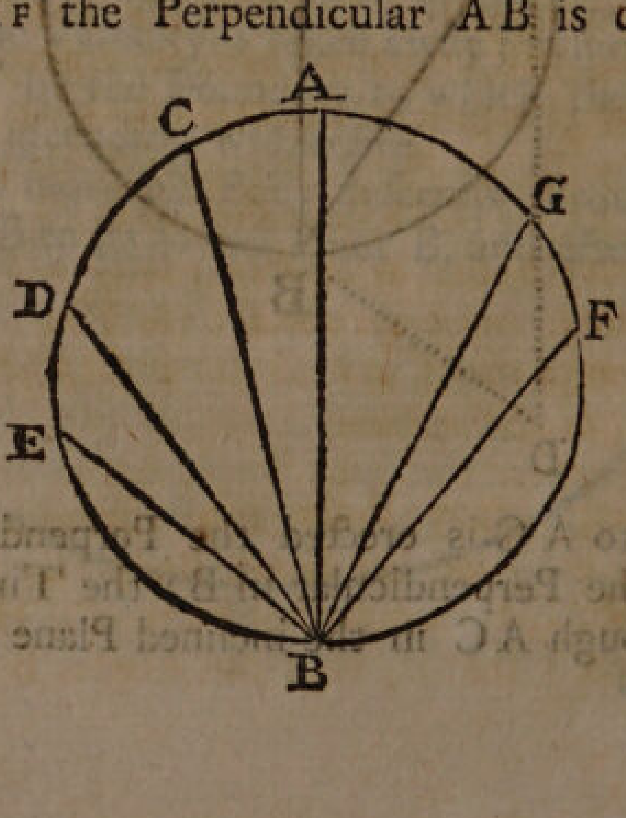
Point C to AC is erected the Perpendicular CB, meeting the Perpendicular in B; the Time of Descent through AC in the inclined Plane will be [by

Corol,

Corol. 1. Probl. 5.] equal to the Time of the Fall through AB in the Perpendicular. I ſay alſo, the Time of Deſcent through CB will be equal to the ſame Time through AB. Draw CD parallel to AB, and DB to AC; and [by 34 *El.* 1.] CD will be equal to AB: and by reaſon of the Angle ACB being right in the Semicircle, the Angle CBD will be a right one. Wherefore ſince from the Point B, upon CB, is erected at right Angles BD meeting with the Perpendicular in D; the Time of Deſcent through CB will be [by Corol. Probl. 5.] equal to the Time of Deſcent through CD: but CD is equal to AB, whence the Time of Deſcent through CB will be equal to the Time of Deſcent through AB.

The ſame thing may be otherwiſe ſhewn thus. The Time of Deſcent through AB is to the Time through EB, in a ſubduplicate Ratio of AB to EB, that is, (by reaſon AB, BC, EB, are continual Proportionals) as AB to BC, or BC to BE; but [by Theor 36.] the Time of Deſcent through BC is to the Time through EB in the ſame Ratio of BC to EB: wherefore ſince the Times of Deſcent through AB and BC have the ſame Ratio to the Time of Deſcent through EB, they will be equal. *Q. E. D.*

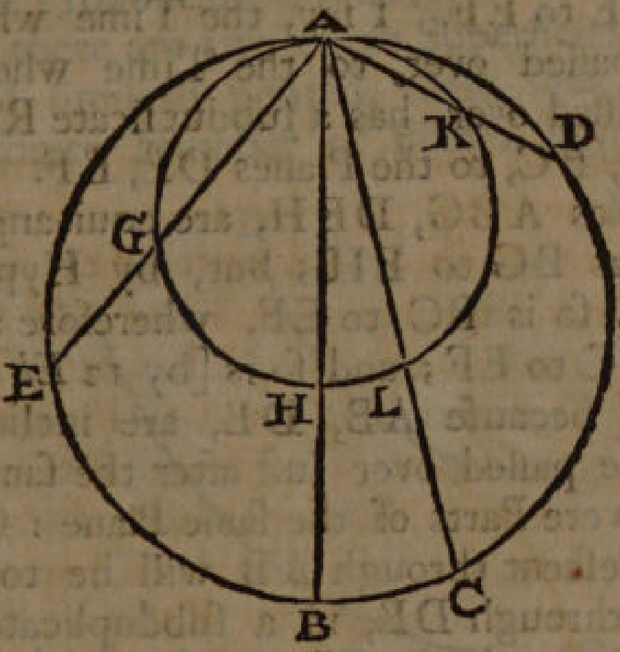
Cor. 1. If the Perpendicular AB is drawn, and



upon

upon the Diameter AB is described a Circle, all the Planes drawn from the Point B, or from the Point A to the Circumference of the Circle, will be run over in the same Time: namely, the Planes AB, CB, DB, EB, FB, GB, will be passed over in the same Time.

Cor. 2. If in the highest Point A, any number of Circles, ABD, AGK, touch each other; and any number of Planes proceed from A, as AB, AC, AD,



AE, intersecting the Circles: the Parts GE, HB, LC, KD, will be passed over in an equal Time, if the Motion is begun at the highest Point A.

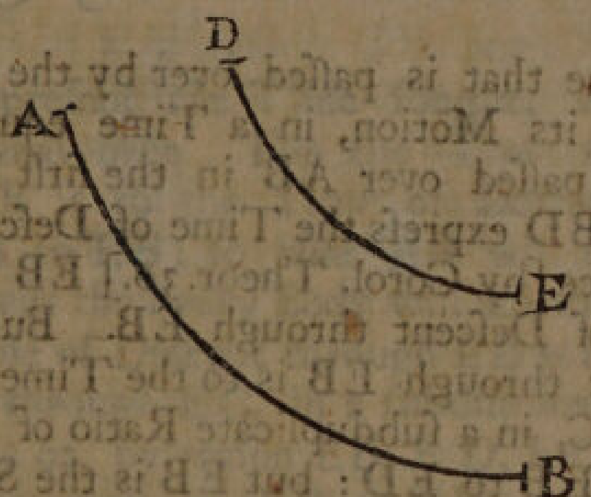
THEOR. XL.

If *two heavy Bodies descend in two or more Planes, inclined alike, and proportional; the Times spent in running through them, will be in a subduplicate Ratio of the Lengths of the Planes.*

Let any heavy Body run through the Planes AB, BC, but another heavy Body run through the Planes DE, EF, inclined alike to the Horizon, and proportional; that is, that the Angles BAG, EDH, are

are equal, as are alſo BGA, EHD; and AB is to BC as DE to EF. I ſay, the Time wherein AB, BC, are paſſed over, to the Time wherein DE, EF, are paſſed over, has a ſubduplicate Ratio of the Planes AB, BC, to the Planes DE, EF. By reaſon the Triangles ABG, DEH, are equi-angular, AB is to DE as BG to EH; but, by Hypotheſis, as AB to DE ſo is BC to EF, wherefore as BG to EH ſo is BC to EF; and ſo is [by 12 *El.* 5.] GC to HF. But becauſe AB, DE, are inclined alike, they will be paſſed over juſt after the ſame manner, as if they were Parts of the ſame Plane: ſo that the Time of Deſcent through AB will be to the Time of Deſcent through DE, in a ſubduplicate Ratio of AB to DE; and the Time of Deſcent through GC is to the Time of Deſcent through HF, in a ſubduplicate Ratio of GC to HF, or in a ſubduplicate Ratio of AB to DE. But the Time of Deſcent through GB is to the Time of Deſcent through HE, in a ſubduplicate Ratio of GB to HE, or AB to DE; ſo that [by 19 *El.* 5.] the Time of Deſcent through BC, after the Deſcent from G or A, is to the Time of Deſcent through EF after the Deſcent from H or D, in a ſubduplicate Ratio of AB to DE; that is, as the Time of Deſcent through AB to the Time of Deſcent through DE. So that [by 12 *El.* 5.] the Time of Deſcent through AB, BC, will be to the Time of Deſcent through DE, EF, as as the Time of Descent through AB, to the Time of Descent through DE; or in a subduplicate Ratio of AB to DE: but by reason AB is to DE as BC to EF, AB will be to DE as AB, BC to DE, EF. So that the Time of Descent through AB, BC, will be to the Time of Descent through DE, EF, in a subduplicate Ratio of AB, BC, to DE, EF. *Q. E. D.* The same thing, in like manner, might be shewn, if there were more Planes on both Parts inclined and proportional; whence the Proposition is manifest.

Cor. If there are two curve Superficies AB, DE, similar and similarly posited, these very little differ from an infinite Number of Planes, infinitely small,



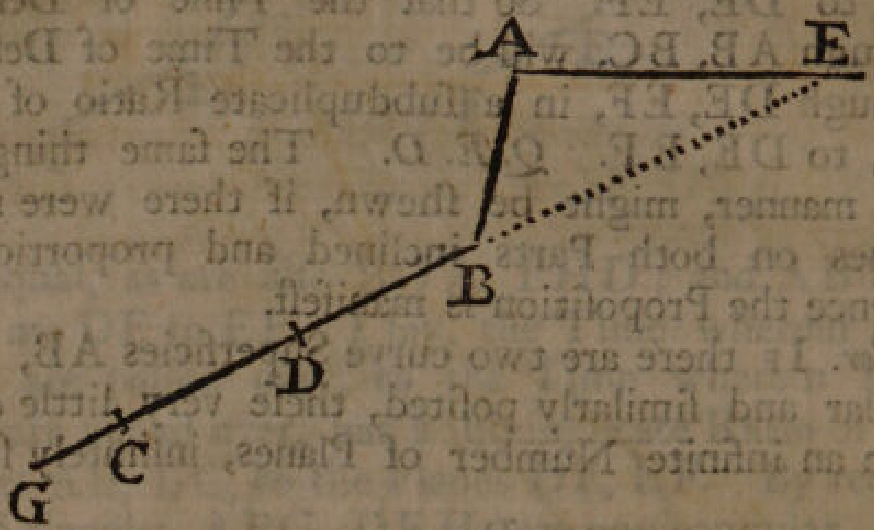
and proportional, and inclined alike to one another; so that the Time of Descent through the Superficies AB, will be to the Time of Descent through the Superficies DE in a subduplicate Ratio of AB to DE.

PROBL. VI.

The *Space* AB *being given in any inclined Plane, and which is passed over in a given Time by a heavy Body falling from rest; to find a Space in another contiguous Plane* BG, *that will be passed over in an equal Time, by the heavy Body continuing its Motion along this second Plane.*

Through

Through A draw the Horizontal right Line AE, and produce BG to E; make BD equal to AB, and take EC a third Proportional to the right Lines EB, ED. Then BC will be the Space in the



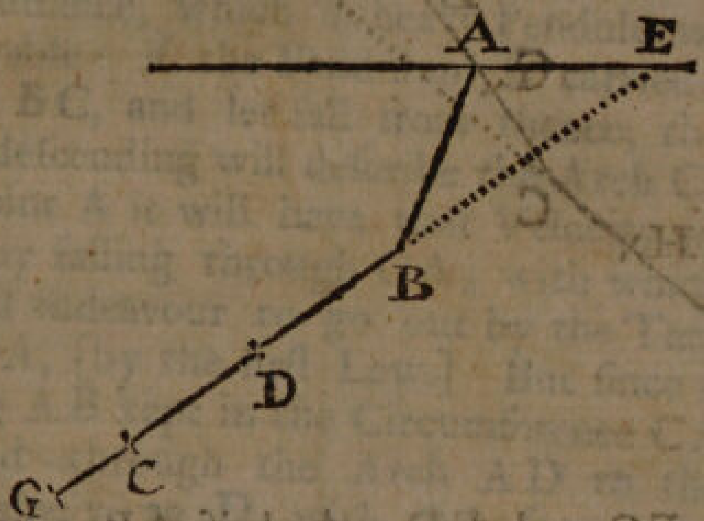
ſecond Plane that is paſſed over by the heavy Body continuing its Motion, in a Time equal to that, wherein it paſſed over AB in the firſt Plane. For let AB or BD expreſs the Time of Deſcent through AB, whence [by Corol. Theor. 36.] EB will expreſs the Time of Deſcent through EB. But the Time of Deſcent through EB is to the Time of Deſcent through EC, in a ſubduplicate Ratio of EB to EC, that is, as EB to ED: but EB is the Space that is paſſed over in the Time as EB; ſo that EC will be the Space that is paſſed over in the Time as ED, and conſequently BC is the Space that is paſſed over in the Time as DB or AB, after the Fall from E or A. Which was to be found out.

PROBL. VII.

The *Space* AB *being given in an inclined Plane, which is paſſed over in a given Time by a heavy Body falling from reſt; as alſo the Space* BC *in another contiguous Plane, wherein the heavy Body continues its Motion: to find the Time wherein that given Space* BC *will be paſſed over.*

Through

Through A draw the Horizontal Line AE, which let BC meet produced in E: betwixt EB, EC, let ED be a mean Proportional. And if AB represents the Time wherein AB is passed over,



BD will represent the Time sought, wherein BC will be passed over. For the Time of Descent through AB is to the Time of Descent through EB, as AB to EB; so that EB will express the Time wherein the heavy Body will fall through EB: but the Time of Descent through EB is to the Time of Descent through EC, in a subduplicate Ratio of EB to EC; or, by reason EB, ED, EC, are continual Proportionals, as EB to ED: but EB is as the Time of Descent through EB, whence DB will be as the Time of Descent through BC. And consequently the Time of Descent through AB, will be to the Time of Descent through BC, as AB to BD. *Q. E. I.*

Cor. Hence if a heavy Body is carried successively through many inclined Planes AB, BC, CD, the Time may be assigned, wherein it will be moving through each of them: for produce BC, CD, that they may meet with the Horizontal right Line drawn through A, in the Points E and F, and let EG be a mean Proportional betwixt EB and EC; as FH

betwixt

vity along the spherical Superficies CAD, if that Superficies was perfectly hard and smooth. For we suppose the Motion about the Point B to be intirely free, and we abstract from the Consideration of the Air's Resistance, which in heavy Pendulums is very inconsiderable: if the Pendulum is carried to the Situation BC, and let fall from thence, the heavy Body in descending will describe the Arch CA, and in the Point A it will have that Velocity which is acquired by falling through CA; with which Velocity it will endeavour to go out by the Tangent in the Point A, [by the first Law.] But since it is by the String AB kept in the Circumference CAD, it will ascend through the Arch AD to the same Height, namely, to D, with that from whence it fell, [by Corol. Theor. 37.] where having lost all its Velocity, it will again begin to descend by its own Gravity; and in the Point A it will acquire the former Velocity, wherewith it will ascend to C: and so by ascending and descending, it will perform continual Vibrations in the Circumference CAD. And if the Motions of Pendulums met with no Resistance from the Air, and if there was no Friction at the Center of Rotation B, the Vibrations of Pendulums would never cease; but from these Causes, the Velocity of the Pendulum in the Point A is a little, tho very insensibly diminished in every Vibration; whence it falls out, that the Ball of the Pendulum does not return precisely to the same Point, but the Arches which it describes constantly become shorter, till at length they grow insensible.

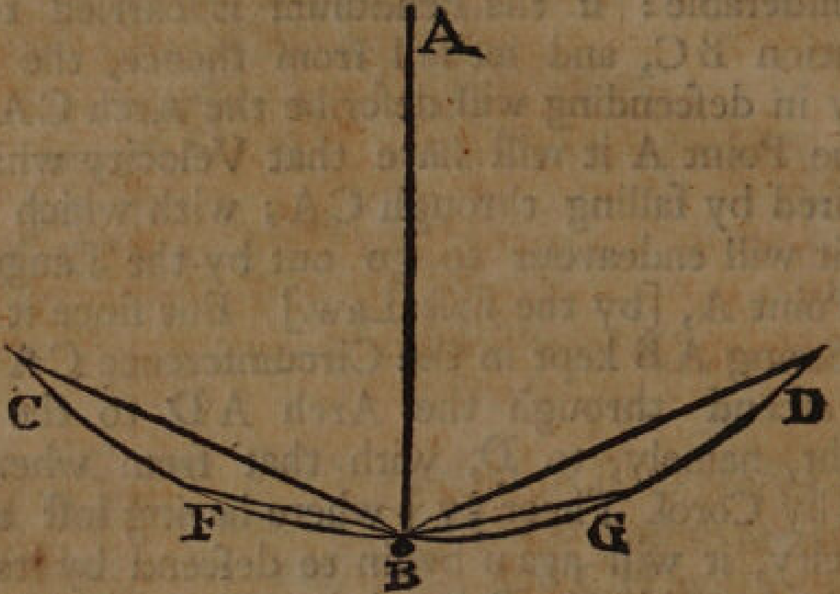
THEOR. XLI.

THE *small Vibrations of the same Pendulum, tho they should be unequal, are almost and to Sense performed in the same Time.*

LET AB be a Pendulum, which by vibrating describes the unequal Arches CBD, FBG; I say, the

 Times

Times taken up in describing them are nearly equal; or the Vibration in the Arch CBD will be nearly performed in a Time equal to that, wherein the Vibration in the Arch FBG will be performed, if the



Arches CB, FB, are not too large. Draw the Subtenses CB, FB, DB, GB, and because the Arches are supposed very small, they differ not much either in Length or Declivity from their Subtenses or Chords: and consequently the heavy Body takes up equal Times, whether it is carried along the Arches CB, FB, or the Subtenses of those Arches; but the Times of Descent through the Subtenses of Arches are equal, [by Theor. 37.] wherefore the Times of Descent through the Arches CB, FB, will be nearly equal; and therefore the Doubles of these Times, namely, wherein the unequal Arches CBD, FBG, are described by vibrating, will be also equal. Wherefore the Vibrations of the same Pendulum, though running out into unequal Arches, are to Sense at least of equal Duration. *Q. E. D.*

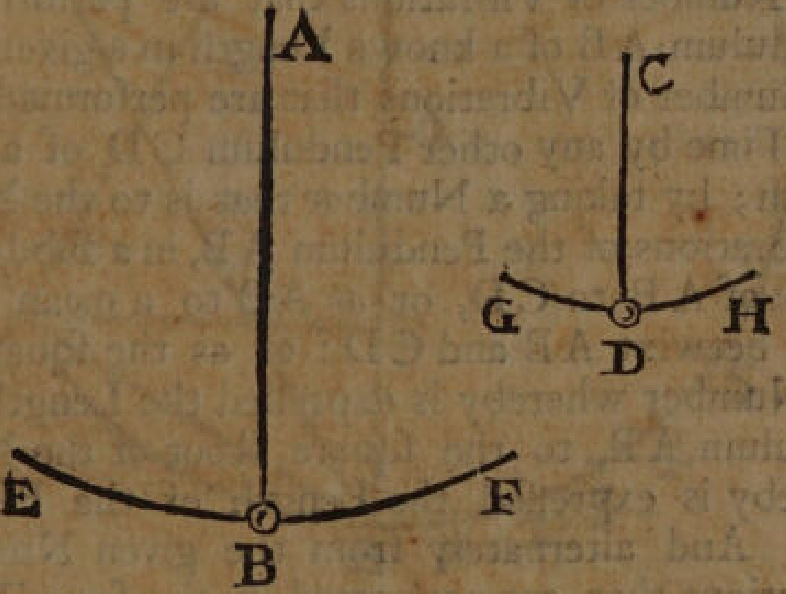
Experience agrees with this Theorem; for two Pendulums of equal Lengths being put in motion, and whereof one describes much greater Arches than

than the other, they will nearly obſerve equal Times in their Vibrations; ſo that in a hundred Vibrations there will ſcarce be the Difference in Time of one Vibration.

THEOR. XLII.

THE *Durations of the Vibrations of two Pendulums, deſcribing ſimilar Arches, are in a ſubduplicate Ratio of the Lengths of the Pendulums.*

LET there be two Pendulums AB, CD, vibrating in ſimilar Arches EBF, GDH; the Time of a Vibration of the Pendulum AB will be to the



Time of a Vibration of the Pendulum CD, in a ſubduplicate Ratio of the Length AB to the Length CD. For becauſe the Arches EB, GD, are ſimilar, and poſited alike, the Time of Deſcent through EB will be [by Theor. 39.] to the Time of Deſcent through GD, in a ſubduplicate Ratio of EB to GD; but the Time of Deſcent through EB is half a whole Vibration in the Arch EBF, as the Time of Deſcent through GD is half a Vibration in the Arch GDH: ſo that the Time of Vibration of the

Pendulum in the Arch EBF will be to the Time of Vibration of the Pendulum in the Arch GDH, in a ſubduplicate Ratio of EB to GD; that is, by reaſon the Arches EB, GD, are ſimilar, in a ſubduplicate Ratio of the Semidiameter AB to the Semidiameter CD, or in a ſubduplicate Ratio of the Length of the Pendulum AB to the Length of the Pendulum CD. *Q. E. D.*

Cor. The Lengths of Pendulums are in a duplicate Ratio of the Times wherein the Vibrations are performed.

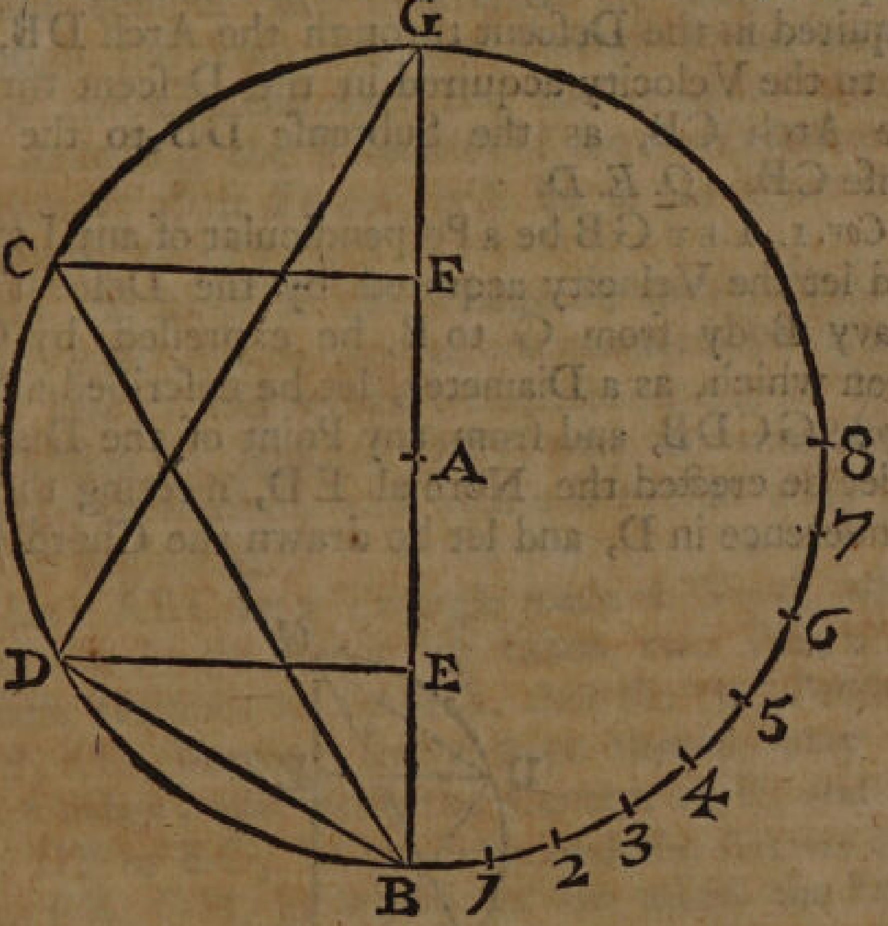
Since the Durations of the Vibrations are reciprocally as the Number of Vibrations performed in the ſame Time, there may be eaſily found from a given Number of Vibrations that are performed by a Pendulum AB of a known Length in a given Time, the Number of Vibrations that are performed in the ſame Time by any other Pendulum CD of a known Length; by taking a Number that is to the Number of Vibrations of the Pendulum AB, in a ſubduplicate Ratio of AB to CD, or as AB to a mean Proportional betwixt AB and CD; or as the ſquare Root of a Number whereby is expreſſed the Length of the Pendulum AB, to the ſquare Root of the Number whereby is expreſſed the Length of the Pendulum CD. And alternately from the given Number of Vibrations that are performed in the ſame Time by two Pendulums AB, CD, and the Length of one, namely, AB being given; there will be given the Length of the other CD, *viz.* by making the Square of the Number of Vibrations of the Pendulum CD to the Square of the Number of Vibrations of the Pendulum AB, ſo the Length of AB to CD the Length ſought.

THEOR. XLIII.

The *Velocity of a Pendulum, in its loweſt Point, is as the Subtenſe of the Arch which it has deſcribed in its Deſcent.*

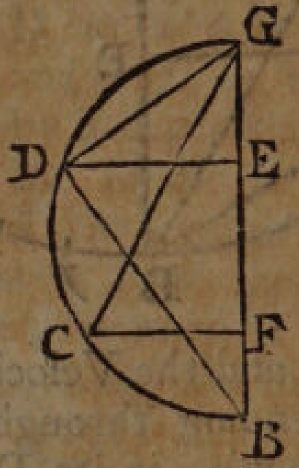
Let

LET AB be a Pendulum, which by its Motion describes the Circle BDCG; I say, the Velocity acquired by falling from D to B, is to the Velocity acquired in B by falling from C to B, as the Chord of the Arch BD, to the Chord of the Arch BC. Through the Points D, C, draw the Horizontal right



Lines DE, CF; and the Velocity the heavy Body acquires by descending through EB will be to the Velocity it will acquire in its Descent through GB, in a subduplicate Ratio of EB to GB, that is, by reason EB, DB, GB, are continual Proportionals, as DB to GB. By the same reason, the Velocity acquired by the moveable Body, in falling through GB, is to the Velocity acquired in the Fall through FB, as GB to CB. Wherefore by Equality, the Velocity acquired, in the Descent of the heavy Body through EB, will be to the Velocity acquired in the Deſcent through FB, as DB to CB; but the Velocity acquired in the Deſcent through the Arch DB, is the ſame with the Velocity acquired in the Perpendicular through EB; and the Velocity acquired in the Deſcent through the Arch CB, is the ſame with the Velocity acquired by the Deſcent in the Perpendicular through FB. Wherefore the Velocity acquired in the Deſcent through the Arch DB, will be to the Velocity acquired in the Deſcent through the Arch CB, as the Subtenſe DB to the Subtenſe CB. *Q. E. D.*

Cor. 1. LET GB be a Perpendicular of any Length, and let the Velocity acquired by the Deſcent of a heavy Body from G to B, be expreſſed by GB; upon which, as a Diameter, let be deſcribed a Semicircle GCDB, and from any Point of the Diameter E let be erected the Normal ED, meeting the Circumference in D, and let be drawn the Chord GD;



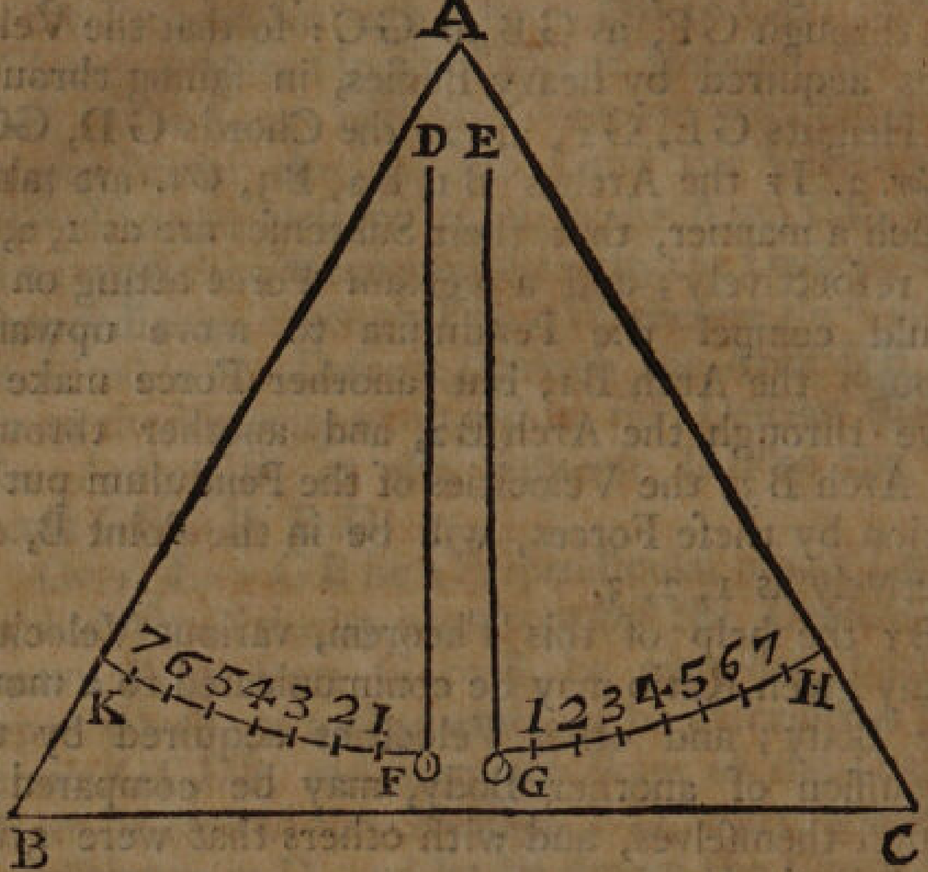
this will be as the Velocity acquired by the heavy Body in falling from the Height GE: For by reaſon BG, GD, GE, are continual Proportionals, the Ratio of BG to GD will be ſubduplicate of the Ratio of BG to GE, ſo that BG will be to GD, as the Velocity acquired by falling from the Height GB, to the Velocity acquired by falling through GE. In like manner, the Velocity acquired by falling

ling through GB, is to the Velocity acquired by the Fall through GF, as GB to GC: so that the Velocities acquired by heavy Bodies, in falling through the Heights GE, GF, are as the Chords GD, GC.

Cor. 2. If the Arches B 1, B 2, B 3, *&c.* are taken in such a manner, that their Subtenses are as 1, 2, 3, *&c.* respectively; and a certain Force acting on it, should compel the Pendulum to move upwards through the Arch B 1, but another Force make it move through the Arch B 2, and another through the Arch B 3, the Velocities of the Pendulum put in motion by these Forces, will be in the Point B, respectively as 1, 2, 3.

By the help of this Theorem, various Velocities in any given Ratio may be communicated to a moveable Body; and other Velocities acquired by the Percussion of another Body, may be compared amongst themselves, and with others that were given at the beginning.

Let ABC be a Triangle made of Wood, wherein near the Angle A let be taken two Points D, E, whose Distance let be such, that the two Pendulums DF, EG, hanging freely from thence, may touch each other; and with the Centers D, E, and Interval DF or EG, let be described the Arches of Circles FK, GH, in which let be taken the Portions F 1, G 1, F 2, G 2, F 3, G 3, F 4, G 4, *&c.* in such a manner, that their Subtenses may be as 1, 2, 3, 4, *&c.* respectively: and if the heavy Body F be raised up to the Point 5 in the Arch KF, but G to the Point 3 in the Arch GH, and are both let down at once, [by Theor. 41.] they will arrive in the same Time at the lowest Points, and the Velocities wherewith they will strike one another, will be as 5 and 3; but if after the Stroke the moveable Body G ascends to 5 in the Arch GH, and the moveable Body F ascends to 3 in the Arch FK, the Velocities of the moveable



able Bodies F and G will be reſpectively as 3 and 5, and towards the contrary Parts. After this manner it will be eaſy to try by Experiments the Rules of Motion as well in hard as elaſtick Bodies, which we have demonſtrated in the 13th and 14th Lectures.

Since the very ſmall Vibrations of theſe Pendulums are nearly of equal Duration, altho the Arches they deſcribe are unequal; hence the great Monſ. *Huygens* has ſhewn the vaſt Uſe of Pendulums to regulate the Motions of Clocks. For although *Galileo*, the Author of this Science, firſt made uſe of Pendulums in Aſtronomical and Philoſophical Obſervations, which require an accurate Meaſure of Time; yet *Huygens* firſt adapted Pendulums to Clocks, and proved by Experience that ſuch Clocks were far more correct than the former, which were regulated by Horizontal Ballances. From that time theſe Pendulum Clocks

Clocks have been in common uſe, whereof ſome have been ſo elaborately made, that they give the Meaſure of Time much more correct than the Sun, which ſhews only the apparent or relative Time, but not the true and abſolute: whence it happens, that Pendulum Clocks ſhew at Times the Hour different from the apparent one, and ſometimes exceeding the Time ſhewn by a Sun-Dial by fifteen or ſixteen Minutes, and ſometimes falling as much ſhort of it in Time: nor does the Sun and Pendulum Clocks ſhew the ſame Point of Time, except four times a Year.

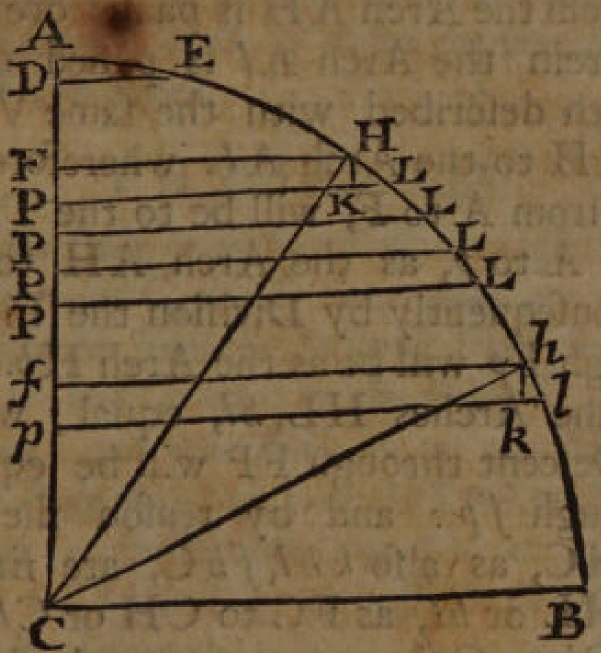
Although the Vibrations of the ſame Pendulum (notwithſtanding it deſcribes unequal Arches) are nearly and to Senſe of equal Duration; yet ſince they are not altogether and geometrically ſuch, but the greater are a ſomewhat longer time in deſcribing than the leſs, and the Vibrations differ from each other a ſmall Quantity in Time; from many of theſe very ſmall Differences at length there do ariſe a ſufficient large Difference, and this is found to be ſo by Experience: For if, as it ſometimes happens in cold Weather, the Wheels are clogged, ſo that they communicate a leſs Force to the Pendulum, the Vibrations become leſs, and the Time ſhewn by the Clock is too faſt; if the Wheels are too glibe, and force the Pendulum to deſcribe larger Arches, then the Times are pointed out too ſlowly. And from ſome late Experiments mentioned in the *Philoſophical Tranſactions*, it appears that the Pendulum of a Clock placed *in vacuo*, the Reſiſtance of the Air being thus removed, performed its Vibrations in larger Arches, and took up more Time in each Vibration. Wherefore that the Vibrations of a Pendulum might be reduced to an exact Equality, and that its Reciprocations, whether performed in greater or leſs Arches, might be made in equal Times; Monſ. *Huygens* found out a Method, whereby the Ball of a Pendulum may be be always carried along the Arch of a Cycloid. We ſhall now demonſtrate in the following Theorems, that the Times of Deſcent through any Arches of the ſame Cycloid to its loweſt Point, which is ſuppoſed to be its Vertex, are equal to one another; ſo that if the Ball of a Pendulum is always moved in the Arch of a Cycloid, the Times of its Vibrations will be exactly equal amongſt themſelves, whether the Pendulum deſcribes greater or leſs Arches.

THEOR. XLIV.

IF *with the Center* C, *and any Interval* C A, *there is deſcribed the Quadrant of a Circle* A H B, *and in the right Line* A C *a moveable Body deſcends with this Law, that its Velocity in any Point* P *ſhall be always as* P L, *the Sine of the Arch* A L; *the Time wherein the Body deſcends from* A *to* C, *will be equal to the Time wherein it might have run through the Circumference* A H B, *with a uniform Velocity as* C B, *which is acquired at the laſt by the falling Body. Beſides, the Time of the Fall through any Space* A F, *will be to the Time of the Fall through the Space* A *p*, *as the Arch* A H *to the Arch* A *l*; *and the Force wherewith the Body is accelerated in any Place* F, *will be as* F C, *the Diſtance of that Place from the Center.*

LET the Circumference A B be diſtinguiſhed into innumerable infinite ſmall Parts L, L, L, L, and let be drawn F H, P L, P *l*, perpendicular to A C; join H C, and let H K be perpendicular to P L. Becauſe the Triangles F H C, K H L, are equi-angular, (for beſides the Angles at F and K being right ones, the Angle F H C is equal to the Angle K H L, the Angle K H C being the Complement of both to a right Angle) F H will be to H C as K H or F P to H L: but [by Hypotheſis] F H is as the Velocity of the moveable Body in the Point F, namely, wherewith

with the infinitely ſmall Line FP is run over, and CH or CB is as the Velocity which is at laſt acquired in falling, when the Body ſhall have come to C, ſo that it will be as the Velocity wherewith the



Arch HL will be deſcribed. The Velocity therefore of the Body deſcending through the infinitely ſmall Line FP, will be to the Velocity of the Body that is moved through the Arch HL, as FP to the Arch HL. Wherefore ſince the Velocities are proportionable to the Spaces paſſed over, the Times wherein thoſe Spaces are paſſed over, will be equal. After the ſame manner it may be demonſtrated, that any other Particle LL of the Circumference may be deſcribed with the Velocity CB, in the ſame Time wherein the correſponding infinitely ſmall Line PP in the Perpendicular will be deſcribed with the correſponding Velocity PL; and conſequently by Compoſition, the moveable Body will deſcend through all the infinitely ſmall Lines PP, that is, through the whole AC, in the ſame Time wherein all the Arches LL, or the whole Circumference AHB is paſſed over, with a uniform Velocity as CB. *Q. E. D.*

Be-

Besides the Time wherein the moveable Body deſcends from A to F, is equal to the Time wherein the Arch AH is paſſed over; and the Time wherein it deſcends from A to *p*, is equal to the Time wherein the Arch A*l* is deſcribed; but the Time wherein the Arch AH is paſſed over, is to the Time wherein the Arch A*l* is paſſed over, (ſince they are both deſcribed with the ſame Velocity) as the Arch AH to the Arch A*l*: wherefore the Time of Deſcent from A to F, will be to the Time of Deſcent from A to *p*, as the Arch AH to the Arch A*l*; and conſequently by Diviſion the Time of Deſcent through F*p* will be as the Arch H*h*. *Q.E.D.*

Make the Arches HL, *hl*, equal; whence the Time of Deſcent through FP will be equal to the Time through *fp*: and by reaſon the Triangles KHL, FHC, as alſo *khl*, *fh*C, are ſimilar; KL will be to HL or *hl*, as FC to CH or C*h*: Alſo *hl* is to *kl* as C*h* to C*f*; and conſequently, by Equality, KL will be to *kl* as CF to C*f*: but KL is as the Increment of the Velocity acquired whilſt the Body paſſed over FP, and *kl* is as the Increment of the Body's Velocity, whilſt it paſſed over in an equal Time the infinite ſmall Line *fp*; but the Forces wherewith the Body is accelerated in the Places F and *f*, are as the Increments of the Velocities generated in equal Times, the accelerating Forces therefore of the moving Body in the Places F and *f* will be as the right Lines KL, *kl*; that is, the Force wherewith a Body is urged in F, is to the Force wherewith a Body is urged in *f*, as the Diſtance GF to the Diſtance CP. The accelerating Forces therefore are in any Places as their Diſtances from the Center. *Q. E. D.*

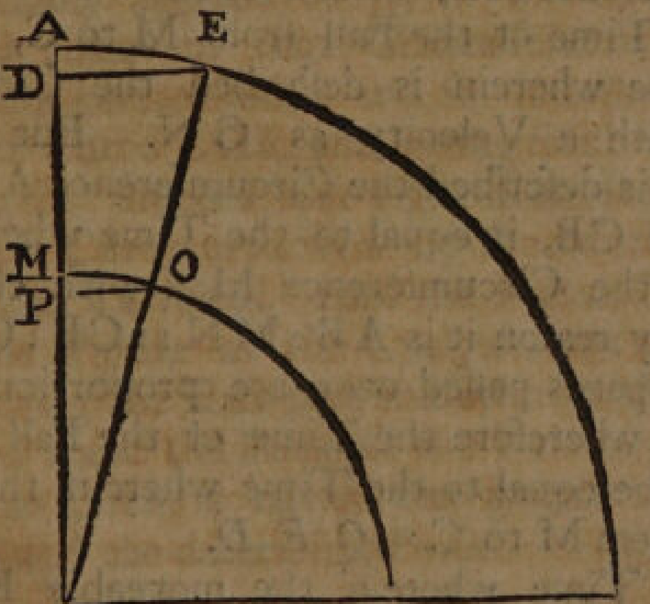
Cor. Hence converſly if a moveable Body in deſcending from A to C is urged by a Force which is as its Diſtance from the Center; and that Force at the

the beginning of Motion is expressed by the right Line DE, the Arch AE being taken infinitely small, the Velocities of the same Body in any Places F, *f*, will be expressed by the Sines FH, *fh*, and the Times by the Arches AH, A*h*, and the Increments of the Velocities, or if the Arches increase equally, the accelerating Forces, will be expressed by the Increments of the Sines.

THEOR. XLV.

IF *a moveable Body be sollicited in the right Line* AC *towards the Point* C, *with Forces that are proportionable to the Distances from the Point* C, *from whatever Height it falls, it will arrive at the Point* C *always in the same Time; and that Time is to the Time wherein a moveable Body may pass over the same way, with a Velocity that is uniform and equal to that which is acquired at last by falling, as half the Circumference of a Circle to its Diameter.*

LET two moveable Bodies be let fall at once from the Points A and M, and let both be sollicited by



Forces that are proportionable to their Distances from the Point C; I say, both the Bodies will arrive

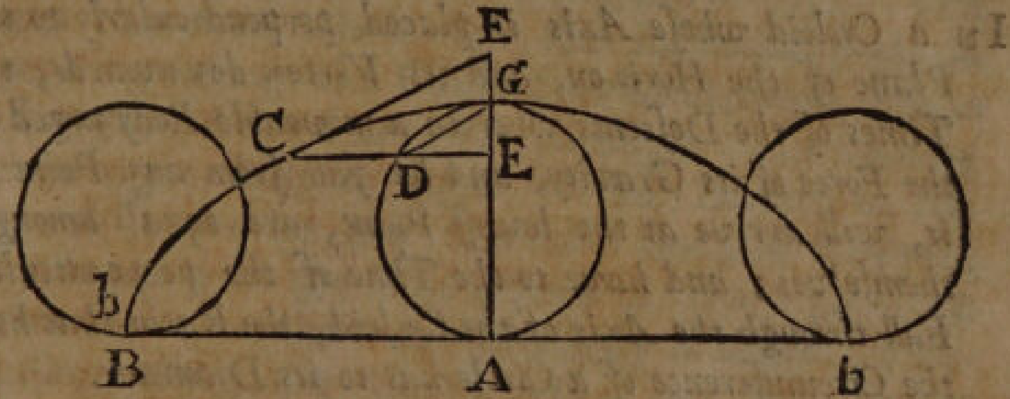
at

at the Point C in the same Time. With the Center C, and Intervals CA, CM, let be described two Quadrants of Circles AB, MN; and let the Force whereby the Body A is urged, or, which is the same thing, its Velocity at the beginning of Motion be represented by DE, the Sine of the infinitely small Arch AE; it is manifest from the Corol. of the preceding Theor. that its Velocity, after the Fall to C, will be represented by the right Line CB. But, by Hypothesis, the Force wherewith the moveable Body is accelerated in A, is to the Force wherewith the moveable Body is accelerated in M, as CA to CM, or as DE to PO, by reason the Arches AE, MO, are similar: wherefore if DE expresses the Velocity of the moveable Body at the beginning of the Fall from A, PO will express the Velocity of the moveable Body at the beginning of the Fall from M; and consequently [by the same Corol.] CN will express the Velocity of the Body in C after its Fall through MC. Besides, the Time of the Fall from A to C, is equal to the Time wherein may be described the Circumference AB, with a uniform Velocity as CB; and the Time of the Fall from M to C, is equal to the Time wherein is described the Circumference MN with a Velocity as CN. But the Time wherein is described the Circumference AB with the Velocity CB, is equal to the Time wherein is described the Circumference MN with the Velocity CN, (by reason it is AB : MN : : CB : CN, namely, the Spaces passed over are proportionable to the Times) wherefore the Time of the Fall from A to C, will be equal to the Time wherein the Body descends from M to C. *Q. E. D.*

The Time wherein the moveable Body passes over the right Line AC, with the Velocity CB, is to the Time wherein it runs through the Arch AB with the same Velocity, as the right Line AC to the

the Arch AB, or as its Double to the Double of this; that is, as the Diameter of the Circle to half its Circumference: but the Time of Descent through the Arch AB is equal to the Time of Descent to C; whence the Time wherein a Body is carried through the right Line AC with a Velocity as CB, will be to the Time of the Fall to C, as the Diameter of the Circle to half the Circumference. *Q. E. D.*

Defin. If a Circle insisting on the right Line B*b*, and touching it in its Point *b*, is supposed to revolve upon that right Line, and by its Circumference be-



ing continually applied to it, to measure a right Line BA*b* equal to that Circumference, till the Point *b* being carried aloft, and so describing in its Motion the Curve BG*b*, and having finished its Circuit, it again touches the right Line BA*b* in the Point *b*; the Curve described by the Motion of the Point is called a Cycloid. And the Figure BGDAB is called the Figure of the Cycloid; the right Line GA, bisecting the Base perpendicularly, the Axis of the Cycloid; and the Point G the Vertex of the Cycloid. The Circle is named the generating Circle, and its Point *b* the describing Point.

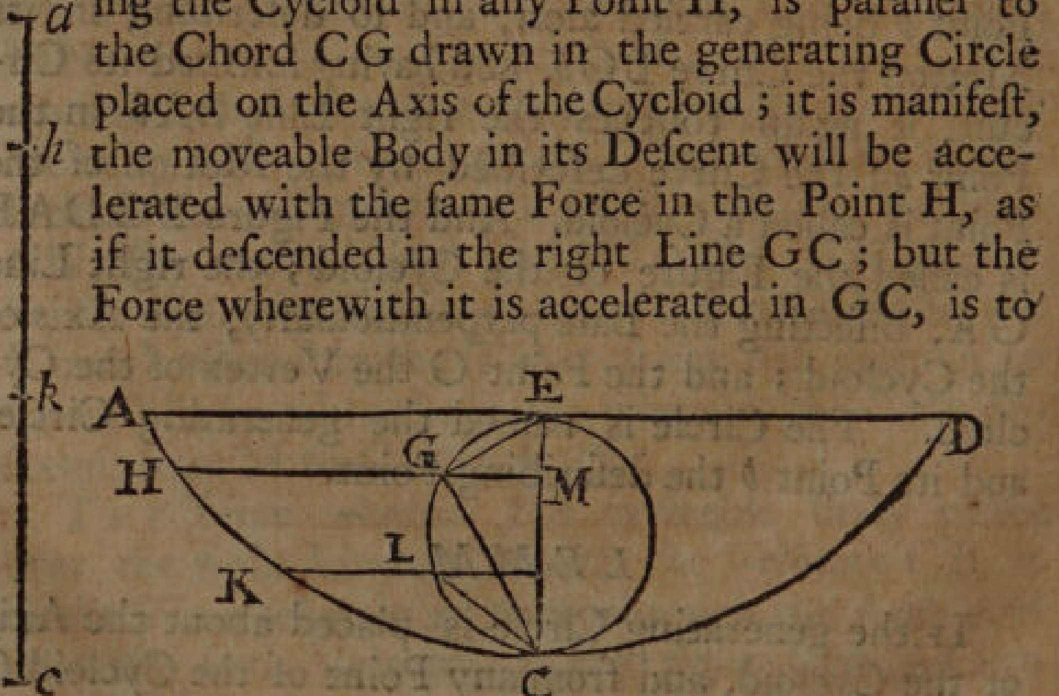
LEMMA.

If the generating Circle is placed about the Axis of the Cycloid, and from any Point of the Cycloid C is is drawn ordinately to the Axis the right Line CE; meeting with the Circumference of the Circle in D; the right Line CD will be equal to the circular Arch GD, the Arch of the Cycloid GC will be equal to double the Chord GD; and the Semi-cycloid BCG will be equal to twice the Diameter AG: but the right Line CF touching the Cycloid in C, will be parallel to the Chord DG. All these things have been demonstrated by Dr. *Wallis*, and others who have wrote concerning the Cycloid.

THEOR. XLVI.

IN *a Cycloid whose Axis is placed perpendicularly to the Plane of the Horizon, and its Vertex downwards, the Times of the Descents wherein a moveable Body urged by the Force of its Gravity, and let fall from any Point in it, will arrive at the lowest Point, are equal amongst themselves; and have to the Time of the perpendicular Fall through the Axis of the Cycloid, the Ratio that half the Circumference of a Circle has to its Diameter.*

LET ACD be a Cycloid, whose Axis is CE, generating Circle ECG. Since the right Line touching the Cycloid in any Point H, is parallel to the Chord CG drawn in the generating Circle placed on the Axis of the Cycloid; it is manifest, the moveable Body in its Descent will be accelerated with the same Force in the Point H, as if it descended in the right Line GC; but the Force wherewith it is accelerated in GC, is to



the

the Force of Gravity as MC to GC: but as MC to GC, so is GC to CE, [by Cor. 8. Prop. El. 6.] wherefore the Force wherewith the moveable Body is accelerated in the Point H, is to the Force of Gravity as GC to CE. By the same reason, the Force of Gravity is to the Force wherewith the moveable Body is accelerated in any other Point K, as CE to CL; wherefore by Equality the Force wherewith the moveable Body is accelerated in H, is to the Force wherewith it is accelerated in K, as GC to LC, or as twice GC to twice LC, that is, as the Curve of the Cycloid HC to the Curve KC. The Forces therefore wherewith the moveable Body in descending along the Cycloid is accelerated, are as the Lengths of the Curve run through. Let us now put the right Line *ac* equal to the Length of the Curve AC, and let any moveable Body be supposed to be urged with the same Forces in the right Line *ac* towards *c*, as the moveable Body is urged, in descending along the Curve AC; but the Forces wherewith the moveable Body is urged, in any Points H and K of the Cycloid, are as the Lengths HC, KC, or *hc*, *kc*: that is, the Forces in any Places are as the Distances of those Places from the Point *c*, and consequently [by the preceding Theor.] the Times of Descent from any Height will be equal. Because therefore in the corresponding Points of the Cycloid and right Line *ac*, the accelerating Forces are equal, the Increments of the Velocities will be also equal: as, for example, putting AH = *ah*, the Accelerations in the Points H and *h* will be equal, as also in the Points K and *k*, so that AK is made = *ak*; and in like manner, in all the other Points of both Lines, which mutually answer to one another, the Increments of the Velocities will be equal: so that if the Bodies begin to descend from corresponding Points, the Sums of the Increments, or the Ve-

 locities

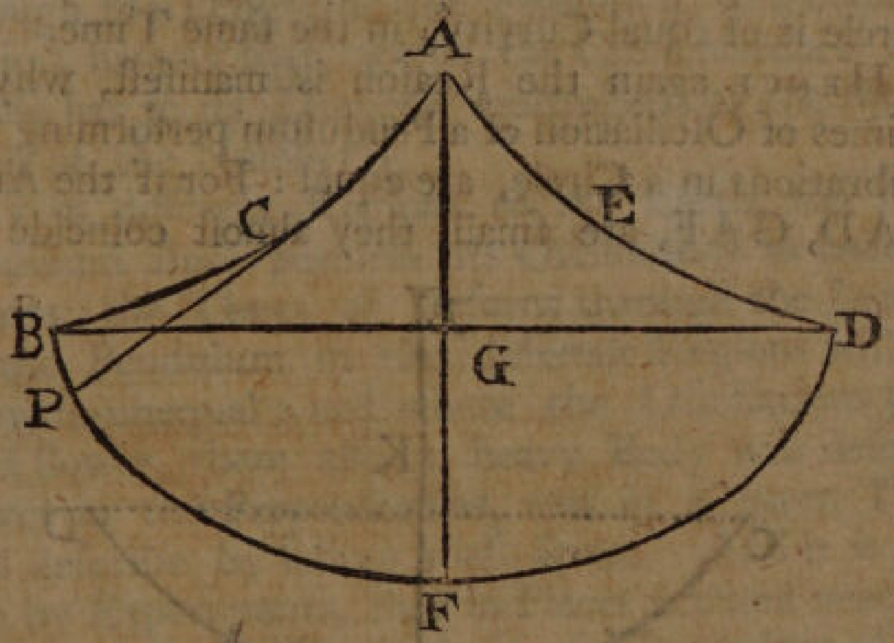
locities acquired in describing equal Spaces, will be equal; and consequently the Times wherein these equal Spaces are described with equal Velocities, will be equal. Therefore the Time of Descent from *a* to *c* in the right Line *ac*, is equal to the Time of Descent from A to C in the Cycloid; and the Time of Descent from *h* to *c* in the right Line *hc*, is equal to the Time of Descent from H to C in the Cycloid; and in like manner, the Time of Descent through KC, is equal to the Time through *kc*, if the beginning of the Fall is from the Points *k*, K: and so of the rest. But the Time of the Fall from *a* to *c*, is equal to the Time of the Fall from *h* to *c*, or from *k* to *c*; wherefore the Time of Descent in the Cycloid from A to C, will be equal to the Time of Descent from H to C, or from K to C. The Times therefore of Descent, wherein a moveable Body, let fall from any Point in the Cycloid, will arrive at the lowest Point, are equal amongst themselves. *Q. E. D.*

Moreover, the Time of the Fall from *a* to *c*, is to the Time wherein *ac*, or 2 EC, is passed over with the Velocity acquired at the last, as the Circumference of a Circle to its Diameter; but the Time wherein 2 EC is passed over with the same Velocity, is equal to the Time wherein the moveable Body, falling by its Gravity, descends through EC, the Axis of the Cycloid: whence the Time of Descent through *ac* or AC, will be to the Time wherein the heavy Body descends through the Axis of the Cycloid, as the Semi-Circumference of a Circle to its Diameter.

Cor. The Time wherein a heavy Body descends in the Cycloid through the Arch AC, and ascends through CD, that is, the Time of the Motion in a Cycloid ACD, is to the Time of the perpendicular Fall through the Axis of the Cycloid, as the whole Circumference of a Circle to its Diameter.

Hence

Hence if the Ball of a Pendulum performs its Vibrations in a Cycloid, whether it describes greater or less Arches, each Oscillation will be always finished in equal Times. Now M. *Huygens*, in the third Part of his *Horologium Oscillatorium*, has delivered a Method whereby the Ball of a Pendulum may vibrate in a Cycloid or any other Curve: namely, by finding out a Curve, by whose Evolution a given Curve will be described; then two Plates must be bent into the same Curviture, betwixt which the Ball being suspended by a String of a determined Length, will describe, not a Circle, but some other Curve. Let there be two Plates ACB, AED, curved into similar and equal Figures; and from the Point A let be



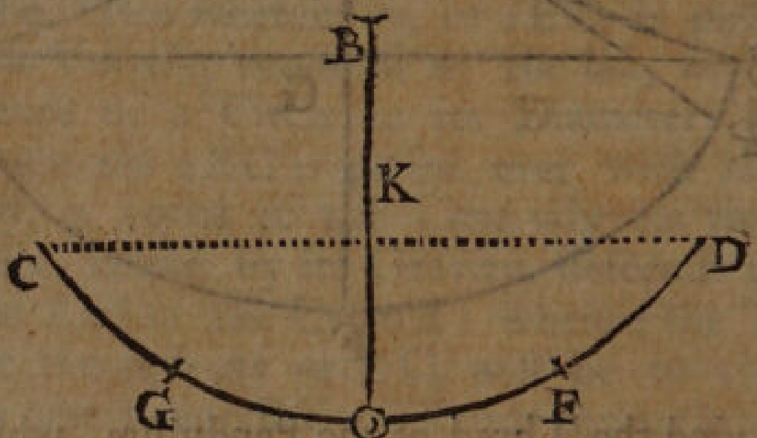
suspended the Thred of the Pendulum, which whilst the Pendulum oscillates, may be applied to the Plates ACB, AED, which it constantly touches: by the Application of the Thred to the Plates, the Motion of the Pendulum in a Circle is continually prevented, and its Ball is carried thro' the Curve BPFD. The Curve ACB or AED is called the Evolute, and the Curve BPFD is said to be described by Evolution. Now if the Curves ACB or AED are two Semi-

 Cycloids,

Cycloids, whose Axes or Diameters of the generating Circles are equal to FG or AG, namely, to half the Length of the Pendulum; the Curve BPFD through which the Ball descends, becomes a whole Cycloid, whose Axis FG is half the Length of the Pendulum, as has been demonstrated by *Huygens* and others.

SINCE the Portion of the Cycloid near the Vertex F is described by the Motion of a String whose Length is AF, and a Circle whose Center is A, Semidiameter AF is described by the same String; that Circle passing through F, almost coincides with the Portion of the Cycloid near the Vertex F, and is of equal Curvity with it. A heavy Body therefore is carried to F, through a small Arch of a Circle, and through the Arch of a Cycloid, to which that Circle is of equal Curvity, in the same Time.

HENCE again the Reason is manifest, why the Times of Oscillation of a Pendulum performing small Vibrations in a Circle, are equal: For if the Arches CAD, GAF, are small, they almost coincide with



the Portion of a Cycloid near its Vertex; the Axis of the Cycloid being AK, half the Length of the Pendulum. So that the heavy Body descends through the Arches of a Circle CA or GA, almost in the same Time, as it would descend through the Arches of a Cycloid nearly coinciding with them: but it descends in equal Times through any Arches of a Cycloid;

cloid; wherefore the heavy Body alſo will fall in equal Times through the ſmall circular Arches CA, GA; and conſequently the whole Oſcillations through the Arches CAD, GAF, are perform'd in equal Times.

The Time therefore wherein a Pendulum performs a very ſmall Oſcillation in a Circle, is equal to the Time wherein an Oſcillation is performed through the Arch of a Cycloid, whoſe Axis is half the Length of the Pendulum. But the Time wherein an Oſcillation is performed in a Cycloid, is to the Time of the perpendicular Fall through the Axis of that Cycloid, that is, through half the Length of the Pendulum, as the Circumference of a Circle to its Diameter. And it hence follows, that the Time of any the ſmalleſt Oſcillation is to the Time of the Fall through the Length of the Pendulum, in a conſtant Ratio; namely that, which the Circumference of a Circle has to its Diameter multiplied by the ſquare Root of the Number 2.

If in different Regions of the Earth, the ſame Pendulum ſhall perform its Oſcillations in unequal Times, the Times of Deſcent through the Length of the Pendulum in the different Regions will be likewiſe unequal; and where the Oſcillations proceed ſlower, there alſo a heavy Body will deſcend ſlower in the Perpendicular, and in a given Time will deſcribe by falling a leſs Space. And it is certain by Experiments, that in Places ſituated near the Equinoctial, the Oſcillations of a Pendulum of the ſame Length are performed in a longer Time, than in Places that are more remote from the Equinoctial: ſo that heavy Bodies in thoſe Regions deſcribe in a given Time by falling a leſs Space, and are accelerated in their Motion by a leſs Force than in our Regions, which have a greater Latitude: ſo that it is confirm'd by Experiments, that the Action of Gravity is leſs in ſuch Places as have leſs Latitude, than in thoſe that are nearer the Pole.

THIS Decreaſe of Gravity ariſes from the centrifugal Force: For ſince from the Rotation of the Earth about its Axis, every Body endeavours to recede from the Center of the Circle it deſcribes, by how much the greater thoſe Circles are, by ſo much the greater is the centrifugal Force of the Bodies deſcribing theſe Circles; which Force therefore is always as the Sine of the Diſtance of the Place from the Pole, which as it is greateſt under the Equinoctial, is nothing at all under the Pole itſelf: ſo that the Force of Gravity is the leaſt under the Equinoctial, but at the Pole the greateſt.

BEFORE we finiſh this matter, we ſhall here give a Solution of a famous Problem, firſt ſought out by *Galileo*, after propoſed by *John Bernouilli* to the Geometers, namely, in the beginning of the Year *1696*; and ſolved by the famous Geometers Sir *Iſaac Newton*, Mr. *Leibnitz*, *James Bernouilli*, the Marquiſs *de l'Hoſpital*, and others. The Problem itſelf was thus propoſed.

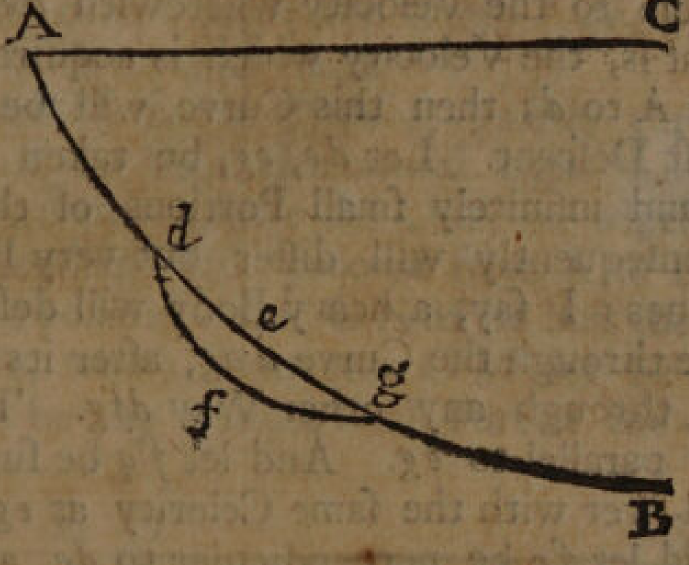
GIVEN *in a vertical Plane two Points* A *and* B, *to aſſign the Way of a moveable Body, through which it deſcending by its Gravity, and beginning its Motion at the Point* A, *it may arrive in the ſhorteſt Time poſſible at the other Point* B.

THE above-mentioned Geometers found out, that this Line would be a Cycloid paſſing through the Points A, B, whoſe Baſe lay in the Horizontal Line drawn through A: to demonſtrate which, we ſhall premiſe the following.

LEMMA.

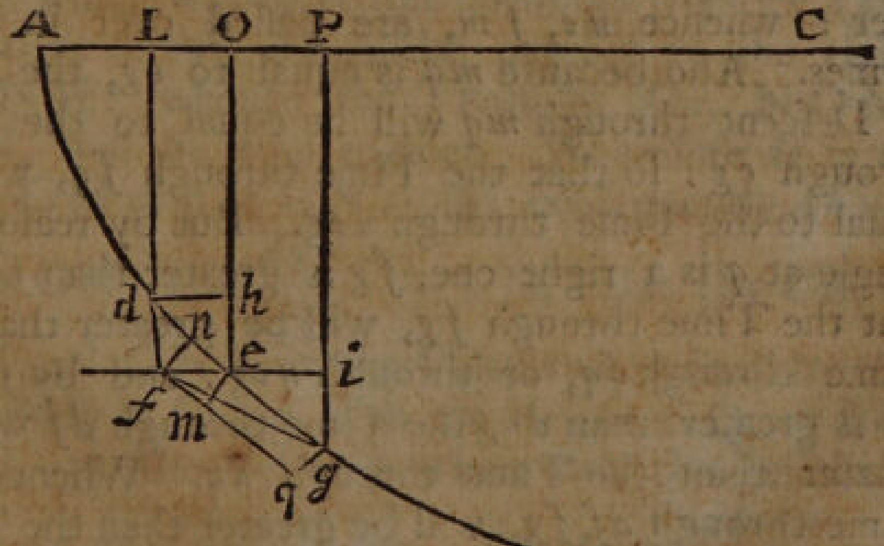
IF AdgB *is the Line of ſwifteſt Deſcent, a heavy Body will deſcend quicker from any Point of it* d, *to any other Point of it* g, *after the Fall from* A, *through the Curve itſelf* deg, *than through any other way whatever.*

FOR



For if you say the heavy Body may descend quicker through dfg, therefore the Way $AdfgB$ will be passed over in a shorter Time than $Adeg$; and consequently the Curve $AdegB$ will not be the Curve of swiftest Descent: which is contrary to the Hypothesis.

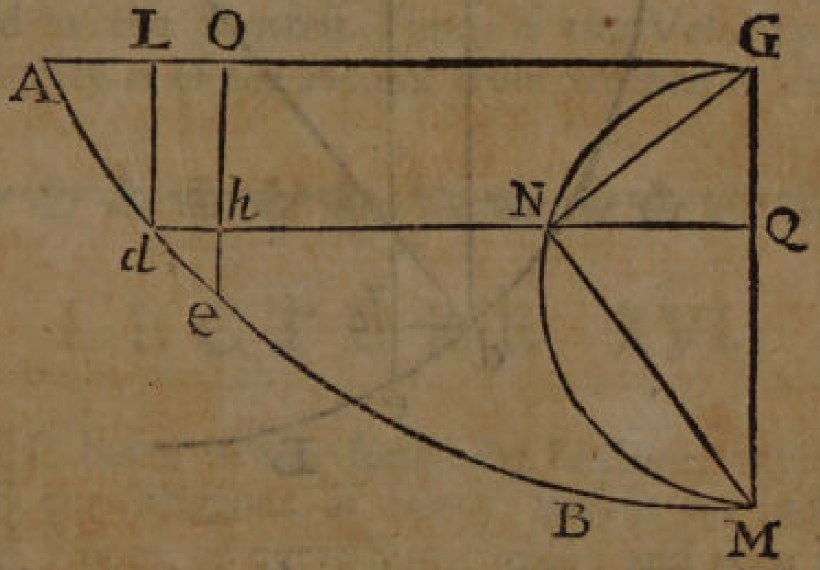
Now let $AdegB$ be the Curve, its Axis A C, and Ordinate applicate dL; the Fluxion or momentaneous Increment of the Axis, let be $LO = dh$; but the Fluxion of the Curve let be de: and let the Rectangle under a given right Line, which we will



call a, and dh or LO, applied to de, be always proportionable to the Velocity wherewith de is paſſed over, that is, the Velocity which is acquired by falling from A to d; then this Curve will be the Line of ſwifteſt Deſcent. Let de, eg, be taken two contiguous and infinitely ſmall Portions of the Curve; which conſequently will differ but very little from right Lines: I ſay, a heavy Body will deſcend in a leſs Time through the Curve deg, after its Fall from A, than through any other Way dfg. Through f draw fq parallel to eg. And let fq be ſuppoſed to be paſſed over with the ſame Celerity as eg is paſſed over; and let fn be perpendicular to de, as alſo me, gq to fq. And by reaſon the Triangles fne, deh, are equi-angular, as likewiſe fme, gei; de is to dh as fe to ne: ſo that it will be $ne = \frac{dh \times fe}{de}$; alſo by reaſon ge is to ei as fe to fm, it will be $fm = \frac{ei \times fe}{ge}$, but it is $\frac{dh \times fe}{de} : \frac{ei \times fe}{ge} :: \frac{dh}{de} : \frac{ei}{ge} :: \frac{dh \times a}{de} : \frac{ei \times a}{ge}$; that is, ne is to fm, as the Velocity whereby ne is paſſed over to the Velocity whereby fm is paſſed over: whence ne, fm, are paſſed over in equal Times. And becauſe mq is equal to eg, the Time of Deſcent through mq will be equal to the Time through eg; ſo that the Time through fq, will be equal to the Time through neg. But by reaſon the Angle at q is a right one, fg is greater than fq; ſo that the Time through fg, will be greater than the Time through fq, or through neg: and by reaſon df is greater than dn, the Time through df will be greater than the Time through dn. Whence the Time through df, fg, will be greater than the Time through dn, ng. A heavy Body therefore will deſcend from d to g, after its Fall from A, through the

Curve *deg*, in a less Time than through any other Way; and consequently the Curve A*deg*B will be the Way of swiftest Descent.

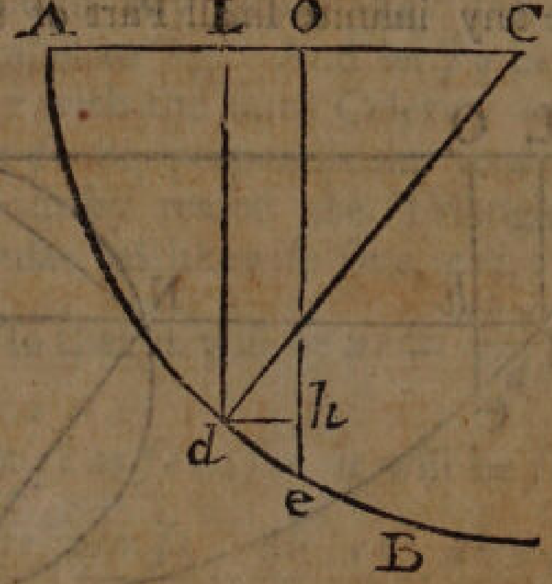
Let ABM be a Cycloid, passing through B, whose Base let be a Horizontal right Line drawn through A; that will be the Line, on which a heavy Body descending, will proceed from A to B, in the least Time possible. Let GNM be half the generating Circle, whose Diameter GM let be called *a*, and let *de* be any infinite small Part of the cycloidal



Curve, which will differ but insensibly from its Tangent in *d*; so that it will be parallel to the right Line NM: whence the Triangles *dhe*, NQM, GMN, will be equi-angular. Wherefore *de* is to *dh*, as GM or *a* to GN; and consequently $dh \times a = de \times GN$: and $\frac{dh \times a}{de} = GN$. But [by Cor. of Theor. 43.] GN is as the Velocity acquired by the heavy Body in falling from the Height GQ, or L*d*; that is, as the Velocity wherewith the infinitely small Line *de* is passed over. Wherefore $\frac{dh \times a}{de}$ will be proportionable to the Velocity wherewith *de* is passed

passed over. The Cycloid A*de*B therefore is the Line of swiftest Descent. *Q. E. D.*

If the Velocity is supposed to be as the Height from whence the heavy Body falls, the Line of swiftest Descent will be the Portion of the Circumference of a Circle, whose Center is in the Horizontal Line drawn through A. For by reason the Triangles



dhe, *d*LC, are equi-angular, *dh* is to *de* as *d*L to *d*C; and consequently, it will be $dh \times dC = de \times dL$, and $\frac{dh \times dC}{de} = dL$. But by Hypothesis *d*L is proportionable to the Velocity: wherefore if *d*C is called *a*, $\frac{dh \times a}{de}$ will be proportionable to the Velocity. Therefore in this Hypothesis the Portion of the Circumference A*de*B will be the Way of swiftest Descent.

If the Velocity in any Point be as the Dignity *m* of the Height whence the Body fell, and let AL be called *x*, *d*L be *y*; it will be $dh = \dot{x}$, $he = \dot{y}$, and $de = \sqrt{\dot{x}^2 + \dot{y}^2}$. Wherefore from the Nature of the

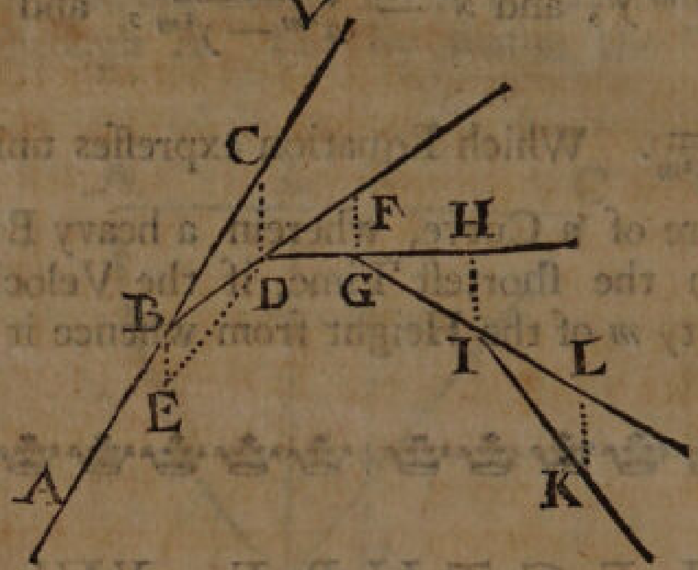
Curve

Curve, it will be $\frac{a^m \dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = y^m$; whence $\frac{a^{2m} \dot{x}^2}{\dot{x}^2 + \dot{y}^2} = y^{2m}$, and $a^{2m} \dot{x}^2 = y^{2m} \dot{x}^2 + y^{2m} \dot{y}^2$, and $a^{2m} \dot{x}^2 - y^{m^2} \dot{x}^2 = y^{2m} \dot{y}^2$, and $\dot{x}^2 = \frac{y^{2m} \dot{y}^2}{a^{2m} - y^{2m}}$, and $\dot{x} = \frac{y^m \dot{y}}{\sqrt{a^{2m} - y^{2m}}}$. Which Equation expresses universally the Nature of a Curve, wherein a heavy Body will descend in the shortest Time, if the Velocity is as any Dignity *m* of the Height from whence it fell.

LECTURE XVI.

WE have, in the foregoing Lecture, explained the Motions of heavy Bodies on inclined Planes, or curve Superficies; and their principal Properties, as far as was consistent with our intended Brevity. It now remains that we speak of the Phenomena of Projectiles: and first of all, we ought to find out the Nature of that Line, which a moveable Body projected in free and unresisting Spaces, will describe by the Force of its Gravity. And indeed if the heavy Body is projected directly upwards or downwards, it will be moved in a right Line; and it is manifest from the former Lectures, that its Motion will be an uniformly-retarded or accelerated Motion, as it is projected either upwards or downwards. But if it is projected in a horizontal Direction, or in any other that is oblique to the Horizon, it will be carried in a curve Line.

For let the Body be projected or cast from A, in the Direction A V; by the first Law of Nature, if no other Force acted on it, it would proceed always in the same right Line, with the same Velocity: so that it



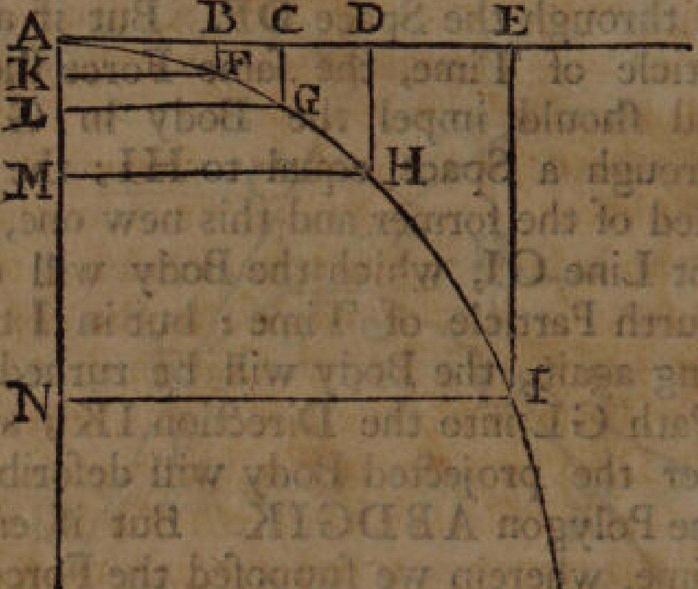
would describe equal Spaces, A B, B C, in equal Times. Let us therefore distinguish the Time into equal Particles; and if after the first Particle of Time, when the moveable Body shall arrive at B, some Force is supposed to act on it, by one Impulse only; and to communicate to it a Motion, whereby it would be carried (the other Motion being taken away) in a Direction perpendicular to the Horizon, through the right Line BE, in the Time that it would describe the right Line BC; compleat the Parallelogram CBED: it is manifest from Cor. 2. Theor. 30. that the Body, by the Motion compounded of both the others, will be moved through the Diagonal BD, and afterwards the projected Body would always proceed in this right Line, if no new Force turned it out of its Path; and it would in an equal Time describe a Space DF equal to BD. But if in the Point D, the same Force should act again with a like Impulse, whereby the Body would be be carried downwards in that Time through a Space equal to FG: the Motion of the Body, compounded of both Motions, would be through the right Line DG, which the Body will describe in the same Time, as it would, without this new Impulse, have proceeded through the Space DF. But if after the third Particle of Time, the same Force should act again, and should impel the Body in G downwards, through a Space equal to HI; the Motion compounded of the former and this new one, will be in the right Line GI, which the Body will describe in the fourth Particle of Time: but in I the same Force acting again, the Body will be turned out of its right Path GL into the Direction IK; and after this manner the projected Body will describe by its Motion the Polygon ABDGIK. But if each Particle of Time, wherein we supposed the Force to act, is diminished *in infinitum*, and their Number is augmented; the Sides of the Polygon will be diminished *in infinitum*, and their Number will be increased *in infinitum*, and consequently the Polygon will be turned into a Curve: that is, if the Force impelling downwards is such, as that it should act always and without ceasing, of which nature is the Force of Gravity, the projected Body, this Force acting, will be caried along a Curve.

THEOR. XLVII.

A PROJECTED *Body, whose Line of Direction is parallel to the Plane of the Horizon, describes by its Motion a Parabola.*

IF the heavy Body is thrown by any extrinsecal Force, as that of a Gun, or the like, from the Point A, so that the Direction of its Projection is the Horizontal Line AD; I say, the Path of this heavy Body will be a Semi-Parabola. For if the Air did not

not resist it, nor was it acted on by its Gravity, the Projectile would proceed with an equable Motion, always in the same Direction; and the Times wherein the Parts of Space AB, AC, AD, AE, were pas-



sed over, would be as the Spaces AB, AC, AD, AE, respectively. Now if the Force of Gravity is supposed to take place, and to act in the same Tenour, as if the heavy Body were not impelled by any extrinsecal Force; that Body would constantly decline from the right Line AE, and the Spaces of Descent or the Deviations from the Horizontal Line AE, will be the same as if it had fallen perpendicularly. Wherefore if the Body falling perpendicularly by the Force of its Gravity, passed over the Space AK in the Time AB, descended through AL in the Time AC, and through AM in the Time AD; the Spaces AK, AL, AM, will be as the Squares of the Times, that is, as the Squares of the right Lines AB, AC, AD, or of KF, LG, MH. But since the Impetus in the Direction parallel to the Horizon always remains the same; for the Force of Gravity, that only sollicites the Body downwards, is not in the least contrary to it: the Body will be equally promoted

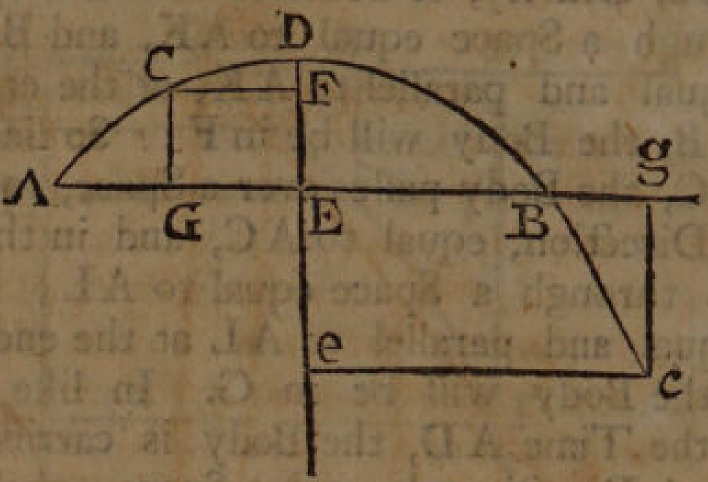
promoted forwards in the Direction parallel to the Plane of the Horizon, as if there was no Gravity at all. Wherefore since in the Time AB, the Body passes over a Space equal to AB; but being compelled by the Force of Gravity, it declines from the right Line AB through a Space equal to AK, and BF being made equal and parallel to AK, at the end of the Time AB, the Body will be in F: So since in the Time AC, the Body passes over a Space, in the Horizontal Direction, equal to AC, and in that Time descends through a Space equal to AL; if CG is made equal and parallel to AL at the end of that Time, the Body will be in G. In like manner, since in the Time AD, the Body is carried in the Horizontal Direction through a Space equal to AD, by the Action of Gravity, it would in the mean while descend through a Space equal to AM; and DH being put equal to AM, at the end of the Time AD, the Body will be in H. And the Path of the Projectile will be in the Curve AFGH; but because the Squares of the right Lines KF, LG, MH, are proportionable to the Abscessæ AK, AL, AM, the Curve AFGH will be a Semi-Parabola. The Path therefore of a heavy Body projected according to the Direction AE, will be a semi-parabolical Curve. *Q. E. D.*

LEMMA.

LET ADB *be such a Curve, as that the Perpendicular* CG *being let fall from any of its Points* C *to* AB, *the Rectangle under* AG, GB, *is equal to the Rectangle under* CG, *and a given right Line* L; *that Curve will be a Parabola.*

LET AB be bisected in E, and the Perpendicular DE be erected; by Hypothesis, the Rectangle under DE and L, will be equal to the Rectangle under

der AE, EB, or AEq. = (by 5 El. 2.) AG × GB + GEq. = CG × L + GEq. = EF × L + CFq. Wherefore the Rectangle under DF and L will be equal to the Square of CF, which is the



Property of the Parabola. If the Point *g* falls in AB produced, which happens when the Curve defcends below AB, the fame Parabola will be the *Locus* of the Point *c*; for (by 6 El. 2.) it is E*g*q. = (*ec*q. =) A*g* × *g*B + EBq. = L × *cg* + L × DE = L × D*e*: which is the Property of the Parabola.

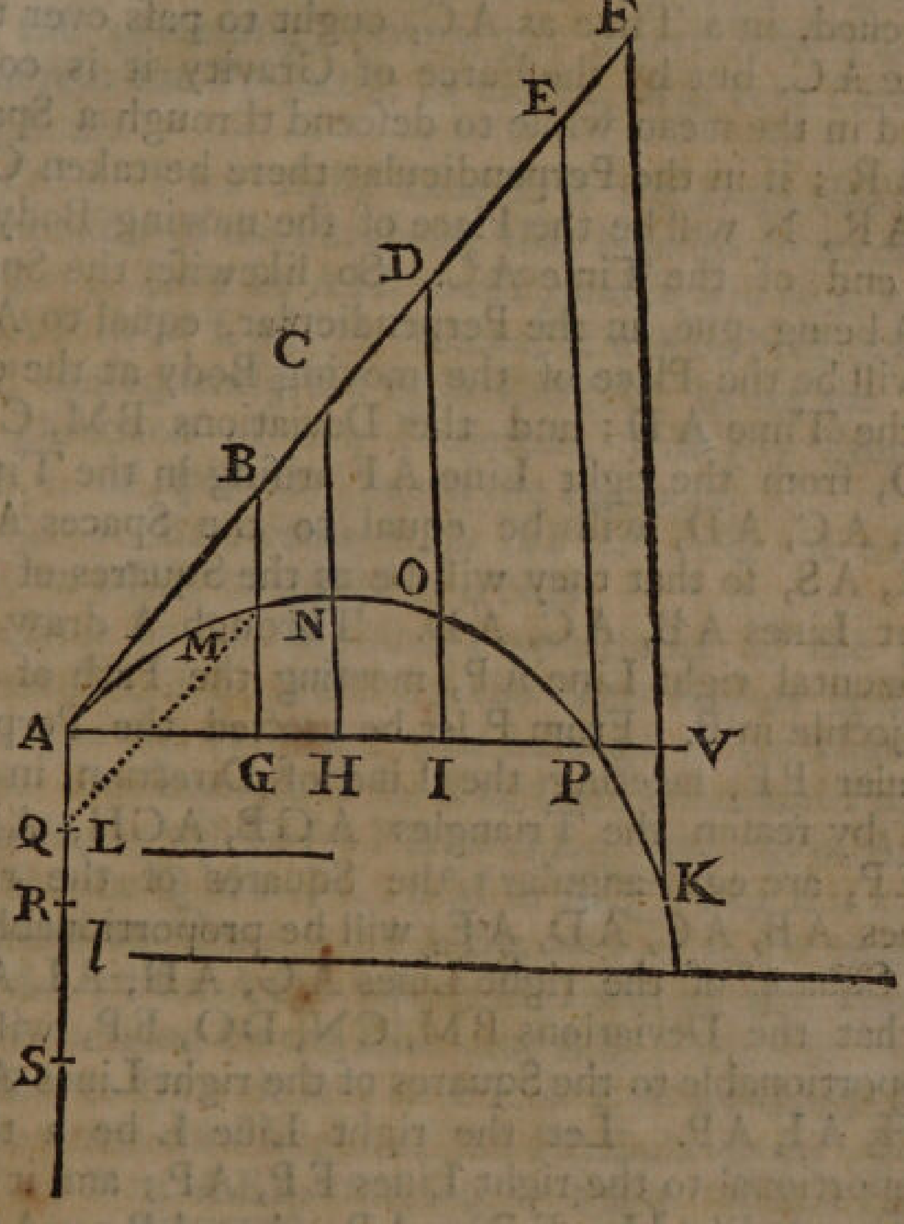
Cor. The right Line L is the *Latus Rectum*, or *Parameter* of the Parabola.

THEOR. XLVIII.

The *curve Line, that is defcribed by a heavy Body projected obliquely and upwards, according to any Direction, is a Parabola.*

Let AF be the Direction of Projection, any ways inclined to the Horizon. Gravity being fuppofed not to act, the moving Body would always continue its Motion in the fame right Line, by the firft Law of Nature, and would defcribe the Spaces AB, AC, AD, proportionable to the Times. But by the Action of Gravity it is compelled continually to decline

cline from the Path AF, and to move in a Curve; I say, this Curve will be a Parabola. Let us suppose the heavy Body falling perpendicularly in the Time AB, through the Space AQ, but in the Time AC,



through the Space AR, and in the Time AD, thro' the Space AS; the Spaces AQ, AR, AS, will be as the Squares of the Times, or as the Squares of the right Lines AB, AC, AD. But because the moving Body not being acted upon by Gravity, in the Time AB would pass over the Space AB, and by the Action of Gravity would in the mean while descend through a Space equal to AQ, it is manifest, if in the Perpendicular BG, there is taken BM = AQ, the Place of the heavy Body, at the end of the Time AB, would be M. In like manner, since the moving Body, by the Impetus first impressed, in a Time as AC, ought to pass over the Space AC, but by the Force of Gravity it is compelled in the mean while to descend through a Space = AR; if in the Perpendicular there be taken CN = AR, N will be the Place of the moving Body at the end of the Time AC. So likewise the Space DO being put, in the Perpendicular, equal to AS, O will be the Place of the moving Body at the end of the Time AD; and the Deviations BM, CN, DO, from the right Line AF arising in the Times AB, AC, AD, will be equal to the Spaces AQ, AR, AS, so that they will be as the Squares of the right Lines AB, AC, AD. Through A draw the horizontal right Line AP, meeting the Path of the Projectile in P. From P let be erected the Perpendicular PE, meeting the Line of Direction in E; and by reason the Triangles AGB, ACH, ADI, AEP, are equi-angular; the Squares of the right Lines AB, AC, AD, AE, will be proportionable to the Squares of the right Lines AG, AH, AI, AP: so that the Deviations BM, CN, DO, EP, will be proportionable to the Squares of the right Lines AG, AH, AI, AP. Let the right Line L be a third Proportional to the right Lines EP, AP; and it will be [by 17 *El.6.*] L × EP = APq. but APq. : AGq. :: EP : BM :: L × EP : L × BM; whence since it is L × EP = APq. it will be L × BM = AGq. In like manner, it will be L × CN = AHq. and L × DO = AIq. But because it is BG : AG :: (EP : AP :: by Hypothesis) AP : L; it will be L × BG = AG × AP = AG × AG + AG × GP = AGq. + AG × GP. But it has been shewn, that

that it is $L \times BM = AGq.$ wherefore it will be $L \times BG - L \times BM = AG \times GP$, that is, $L \times MG = AG \times GP$. By the fame way of Reafoning it will be $L \times NH = AH \times HP$, and $L \times OI = AI \times IP$, as alfo $L \times VK = AV \times VP$. Wherefore by the preceding Lemma, the Curve AMNOPK, wherein the Projectile is moved, will be a Parabola. *Q. E. D.*

Cor. 1. THE right Line L is the *Latus Rectum* or Parameter of the Parabola, that belongs to its Axis.

Cor. 2. LET be $AH = HP$, and it will be $L \times CN = AHq. = L \times NH$, whence it will be $NH = CN$; and confequently the right Line AF being the Line of Direction of the Projectile, will [by Prop. 33. Lib. 1. of *Apollonius*] touch the Parabola.

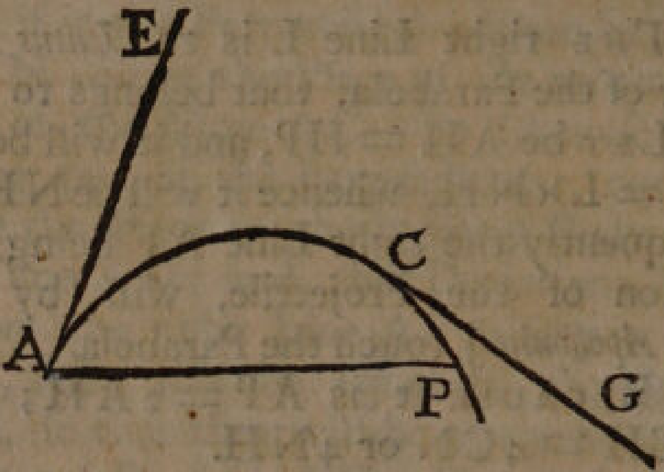
Cor. 3. BECAUSE it is $AP = 2AH$; it will be $PE = 2CH = 4CN$ or $4NH$.

Cor. 4. IF l is a third Proportional to the right Lines PE, AE, l will be the *Latus Rectum* or Parameter of the Parabola, that belongs to the Diameter AS. For becaufe PE, AE, l, are continual Proportionals; it will be $l \times PE = AEq.$ but it is $AEq. : ABq.$ or $QM :: PE : BM$ or $AQ :: l \times PE : l \times AQ$. Wherefore fince it is $AEq. = l \times PE$, it will be $QMq. = l \times AQ$: wherefore l will be the Parameter belonging to the Diameter AS.

Cor. 5. BUT it is $l = PE + L = 4NH + L =$ four times the Height of the Parabola $+ L$. For it is $l \times PE = AEq. = APq. + PEq. = L \times PE + PEq. = \overline{L + PE} \times PE$. Wherefore it will be $l = L + PE = L + 4NH$.

Cor. 6. IF the Times AB, BC, CD, are made equal; the Horizontal Spaces AG, GH, HI, will be equal: that is, if a heavy Body, in its Motion, defcribes a Parabola, in equal Times it will advance equally, according to a Direction that is parallel to the Horizon; and in every Point of the Parabola, the Horizontal Impetus will remain the fame as it was at the beginning of Motion.

Cor. 7. If a Body projected from A in the Direction AE, defcribes the Parabola ACP; in any Point C it will endeavour, by the firft Law of Nature, to run out along the Tangent CG, with all the Velocity that it has in the Point C, and is folely



retained in the Parabolical Curve by the Force of Gravity. But if another heavy Body fhould be projected from C in the Direction CG, with the Velocity that the Body projected from A, has in the fame Point C; that other Body will defcribe the fame Parabola CP. For in the Point C, the Direction, Velocity, and Force of Gravity of both Bodies are the fame; wherefore the Way of both will be the fame.

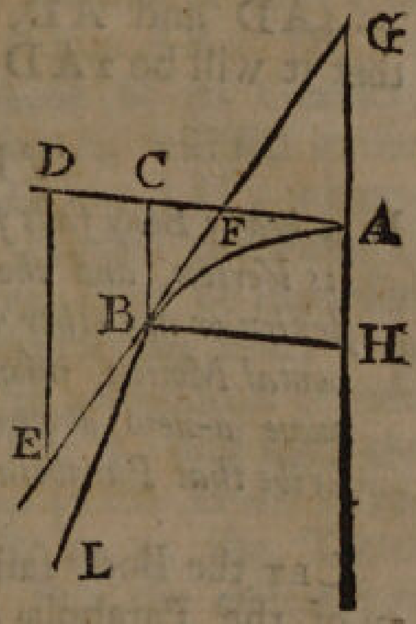
Cor. 8. Hence if a heavy Body is projected downwards, in a Direction oblique to the Horizon; the Path of the Projectile will be a Parabola.

THEOR. XLIX.

The *Impetus of a projected Body in different Points of the Parabola, are as the Portions of the Tangents intercepted betwixt two right Lines parallel to the Axis.*

Let a heavy Body defcribe the Parabola ABL, which let the right Lines AD, BE, touch in the Points

Points A and B. The Impetus of the Body in the Points A and B, will be as CD, EB, the Portions of the Tangents intercepted betwixt two right Lines parallel to the Axis. For if the Body should be deprived of its Gravity in the Point A, it would run out in the Tangent AC, with the same Impetus that it has in the Point A. So likewise the moving Body in B having lost its Gravity, would proceed in the Tangent BE with all the Velocity that it has in the Point B. But in the Points A and B, the Horizontal Impetus remains the same, as appears by Cor. 6. of the preceding Theor. so that the Body in A going out into the Tangent AD, and in B into the Tangent BE, in equal Times it will proceed through equal Spaces according to the Horizontal Direction. Therefore CD in the Tangent AD, and BE in the Tangent BE, will be passed over in equal Times; but the Velocities, or Impetus of the moving Bodies, are as the Spaces passed over in equal Times: wherefore the Impetus of the moving Body in A, is to the Impetus of the same Body in B, as CD to BE. *Q. E. D.*



Cor. If A is the Vertex of the Parabola, and let the Tangent be produced, till it meets the Axis in G; the Impetus in A will be to the Impetus in B, as the Ordinate BH to the Tangent BG; for it is CD : BE :: CF : BF (by reason the Triangles CBF, BHG, are similar) :: BH : BG.

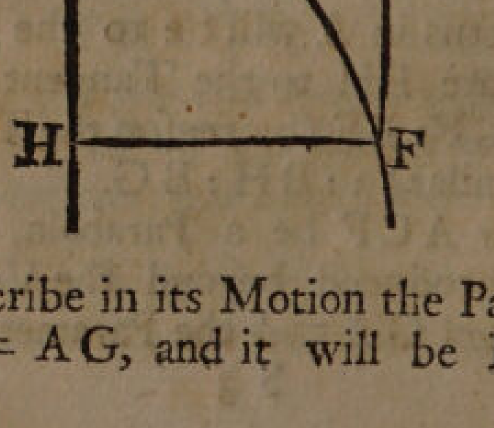
Defin. Let ACF be a Parabola, in whose Axis produced beyond the Vertex let be taken GA = $\frac{1}{4}$ the Parameter. *See the Fig. of the following Theorem.*

The Line G A is called the *Sublimity* of the Parabola, and if beneath the Vertex be taken AD = AG, and DC be an Ordinate to the Axis, it will be DC = 2 AD or 2 AG; for from the Nature of the Parabola, the Rectangle under the *Latus Rectum* = 4AD and AD, that is, 4ADq. = DCq. so that it will be 2AD = DC.

THEOR. L.

If *a heavy Body falls from the Sublimity of a Parabola to its Vertex, and the Motion acquired in falling, by Reflexion or any other way should be changed into a Horizontal Motion, insomuch that the Body may begin to move a-new downwards; the projected Body will describe that Parabola.*

Let the Body fall from the Point G, the Sublimity of the Parabola ACF; and in A by Reflection, or any other way, let the Motion acquired in falling be changed into the Horizontal one in ABE; or, which is the same thing, let the Body be projected in the Direction AE, with the Velocity that is acquired by falling through GA: I say, that heavy



Body will describe in its Motion the Parabola ACF. Let be AD = AG, and it will be DC = 2AG.

Draw

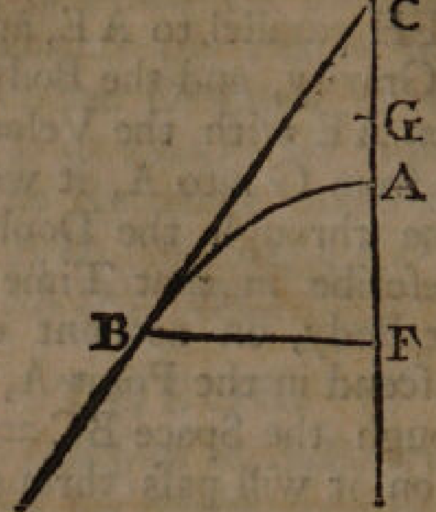
Draw CB parallel to AD; and from any Point F of the Parabola, draw FH parallel to AE, and FE to HA: If there was no Gravity, and the Body was projected in the Direction AE with the Velocity that is acquired in falling from G into A, it would be carried in the same Time through the Double of GA; so that it would describe in that Time AB = DC = 2GA. But the Body, on account of its Gravity, beginning to descend in the Point A, will fall in the same Time through the Space BC = AG; wherefore in its Motion it will pass through the Point C in the Parabola. Again, let the Body be supposed by its Horizontal Motion (abstracting from that which arises from its Gravity) to arrive in a certain Time to E, beyond or on this side B; and since the Motion in the Direction parallel to the Horizon remains equable, AB, AE, will be as the Times wherein they are passed over. But the Descents or Deviations of the moving Body from the right Line AE, are as the Squares of the Times wherein they are made: wherefore by reason BC, EF, are proportionable to the Squares of the right Lines AB, AE; since C is the Place of the heavy Body at the end of the Time AB, F will be the Place of the same Body at the end of the Time AE: and so the heavy Body will be always found in the Parabola ACF. *Q.E.D.*

Cor. Hence the Velocity of a heavy Body in the Vertex of the Parabola which it describes, is that which is acquired by falling from the Sublimity of that Parabola.

LEMMA.

Let BA *be a Parabola, whose Axis is* AF, *Sublimity* AG, *and Tangent* BC, *Ordinate* BF; *it will be* BFq. : BCq. : : GA : GF.

For it is [by 33 Lib. 1. of *Apollonius's* Conicks] CF = 2AF, and from the Nature of the Parabola



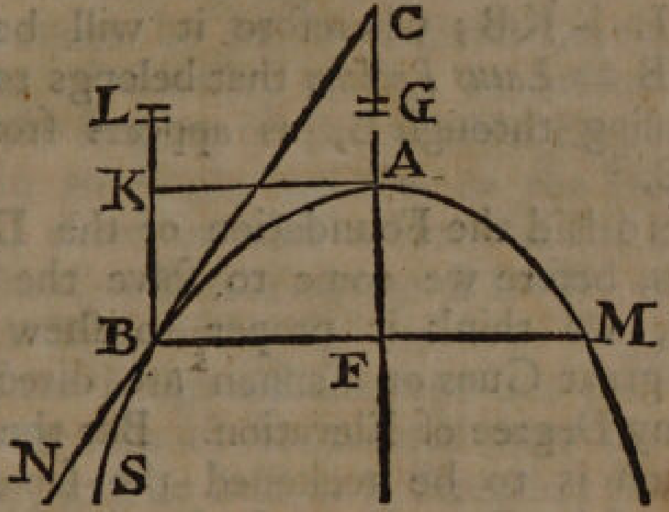
$4GA \times AF = BFq$. Wherefore it will be $BFq : BCq :: 4GA \times AF : 4GA \times AF + CFq :: 4GA \times AF : 4GA \times AF + 4AFq :: GA : GA + AF$ or GF. *Q. E. D.*

THEOR. LI.

A Heavy *Body being projected directly upwards, with the same Impetus wherewith another heavy Body is projected obliquely, will ascend to a Height equal to the Height and Sublimity, taken together, of the Parabola, which the oblique Projectile will describe in its Motion.*

Let from B be projected a heavy Body in the Direction BC, describing in its Motion the Parabola BAM, whose Axis is AF, Vertex A, and Sublimity GA: I say, if the same or another heavy Body be projected from B directly upwards, with the same Impetus, it will ascend to L; so that BL will be equal to FG, the Height and Sublimity of the Parabola taken together. By Cor. of Theor. 49. the Impetus of the heavy Body in B, is to the Impetus of the same Body in A, as BC to BF; but the Impetus acquired by falling from G to F, is to the Impetus acquired by falling from G to A, in a subduplicate Ratio of GF to GA, that is, (by reason it is BCq.

BCq. : BFq. :: GF : GA) as BC to BF. Wherefore the Impetus in B, will be to the Impetus in A, as the Impetus acquired by falling from G to F,



to the Impetus acquired by falling from G to A: but the Impetus of the heavy Body in the Vertex A, is that which is acquired by falling from G to A; wherefore the Impetus or Velocity of the ſame Body in B, is that which is acquired by falling from G to F, or from L to B, which Height is equal to the Height and Sublimity of the Parabola taken together. But the heavy Body projected directly upwards with the ſame Impetus, will aſcend to L; wherefore if a heavy Body is projected directly upwards with the Impetus that another heavy Body, deſcribing the Parabola BAM, has in the ſame Point B, it will aſcend to a Height equal to the Height and Sublimity of the Parabola taken together. *Q. E. D.*

Cor. 1. If a heavy Body falls from L to B, and the Impetus acquired in the Fall remaining the ſame, the Direction of Motion ſhould be changed, by Reflection, or the like manner, into the right Line BC, or BN, ſo that the heavy Body begins to deſcend again; it will in its Motion deſcribe the Parabola SBAM.

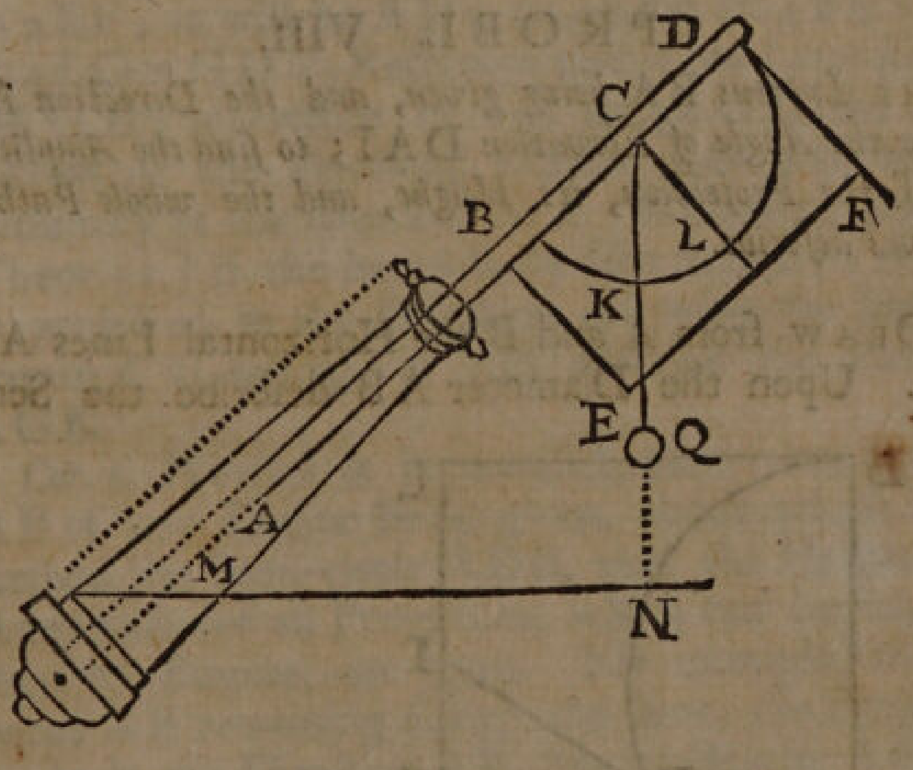
Cor.

Cor. 2. THE Impetus in any Point B of a Parabola, is that which is acquired, by falling through the fourth part of the *Latus Rectum* belonging to the Diameter that passes through that Point. For it is $LB = \frac{1}{4}L + KB$; wherefore it will be $4LB = L + 4KB =$ *Latus Rectum* that belongs to the Diameter passing through B, as appears from Cor. 5. Theor. 48.

HAVING laid the Foundation of the Doctrine of Projectiles, before we come to solve the following Problems, we think it proper to shew the way, whereby great Guns or Cannon are directed according to any Degree of Elevation. But the Direction of a Cannon is to be reckoned the same with the Direction of its Concavity: for by the firing of the Gun-powder, the Bullet is emitted according to the Concavity of the Cannon or Mortar; and if it was not for Gravity, it would proceed in that right Line produced: so that that right Line is the Direction of the Ordnance.

WHEREFORE that the Ordnance may be aimed at the Mark, it ought not to be directed according to its outward Metal, since Ordnance are thicker towards their Breech than near their Orifice, because their Resistance ought to be greatest in that part, which suffers most from the Gun-powder: whence, that a Cannon may be the more readily directed, something is added to the Orifice, (which they call *Dispart*) that its Thickness may equal the Thickness of its Breech, and then it is levelled by a Line parallel to the Concavity of the Piece; and after the above-mention'd manner the Ordnance is aimed directly at the Mark. When Walls are to be battered, or any thing to be done, where a great Impetus is required, and the Mark is not above 200 Paces off, and the Engine sufficiently large; in such Shots, besides what has been said, and the Experience

rience of making proper Charges of Powder, and ſuitable Balls, there is no other Artifice required. But ſince moſt commonly the Citadels, or Enemy that is to be fired upon, by reaſon of the too great diſtance, cannot be hit by a direct Level; or when the Houſes of the City are to be broken down or burnt by the flinging of Bombs; the Engine muſt be elevated in an Angle inclined to the Horizon: to which end, there is uſed a Ruler A B C D, whereunto is fixed a Parallelogram B E F D, on which is inſcribed a Semi-circle divided into Degrees, from whoſe Center hangs a Plummet; but the Extremity of the Ruler A muſt be inſerted into the mouth of the Engine, and be held in a Situation that is paral-



lel to its Axis. And ſo the Ordnance is to be raiſed or depreſſed, till the Perpendicular C Q touches, in the Limb of the Semi-circle, the Point K, namely, the Degree of the Elevation required, reckoning from L towards B; for it is manifeſt, that the Angle L C K is equal to the Angle C M N, the Elevation of the Engine, becauſe the Angle M C N is the Com-

Complement of both to a right Angle. The Parallelogram BEFD is frequently ufed alone without the Ruler, by applying its Side BE to the Mouth of the Engine, whereby the Perpendicular CQ will fhew the Degree of Elevation.

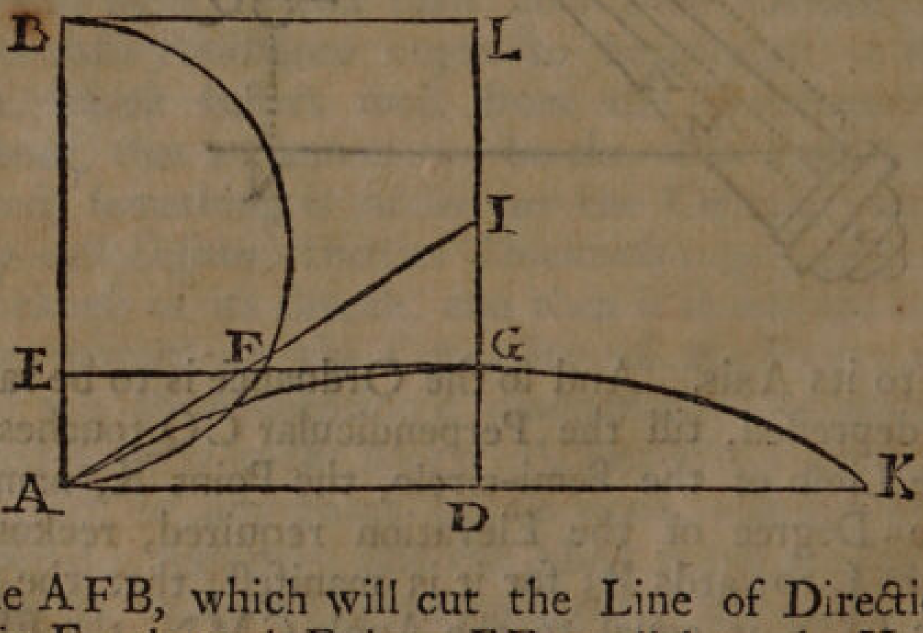
See the Fig. of the next Problem.

Defin. By the Impetus defigned by any Perpendicular AB, we underftand an Impetus fufficient to hurl the propofed heavy Body from A to the higheft Point B of the Perpendicular AB; or, which is the fame thing, the Impetus acquired by falling from B to A: for by no other way can an Impetus be reduced under any certain and univerfal Rule, than by determining it after this manner by Spaces.

PROBL. VIII.

The *Impetus* BA *being given, and the Direction* AI, *or the Angle of Elevation* DAI; *to find the Amplitude of the Projection, its Height, and the whole Path it will defcribe.*

Draw from A and B the Horizontal Lines AD, BL. Upon the Diameter AB defcribe the Semi-

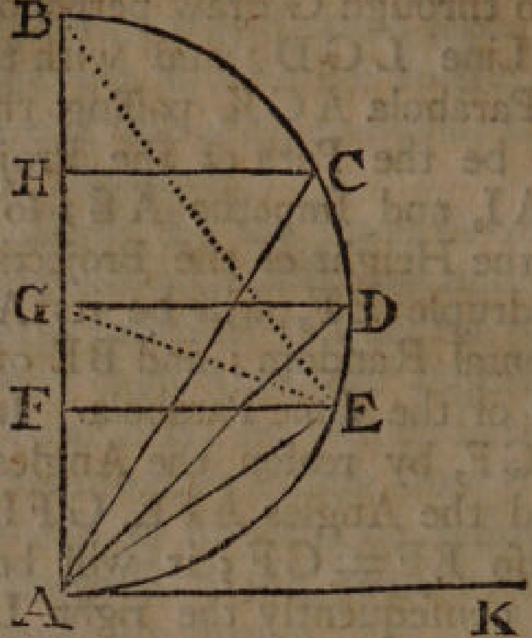


circle AFB, which will cut the Line of Direction AI in F: through F draw EF parallel to the Horizon,

zon, and produce it to G, so that it may be GF = EF; and also through G draw perpendicular to the Horizon the Line LGD; and with the Vertex G describe the Parabola AGK passing through A: I say, this will be the Path of the Projectile, whose Direction is AI, and Impetus AB; so that DG or AE will be the Height of the Projection. Double AD, or quadruple EF, will be its Amplitude or whole Horizontal Random; and BE or LG will be the Sublimity of the same Parabola. In the Triangles AEF, IGF, by reason the Angles at E and G are right, and the Angles AFE, GFI, at the Vertex equal, also EF = GF; it will be IG = AE = DG, and consequently the right Line AI will touch the Parabola. And because it is AD = EG = 2EF; it will be ADq. = 4EFq. = 4BE × EA = 4LG × GD = Rectangle under the *Latus Rectum* and GD: wherefore it will be 4LG = the *Latus Rectum* of the Parabola, whence LG will be the Sublimity of the same Parabola. Wherefore [by Cor. Theor. 51.] if the heavy Body falls from B to A, and is projected in the Direction AI with the Impetus acquired in falling, it will describe the Parabola AGK.

Cor. 1. HENCE it is manifest, from the Impetus AB of any Engine being given, about which is described the Semi-circle ADB, that the Heights and Amplitudes of all Projectiles, which can be made by the same Engine, are given. For example, the Impetus AB remaining always the same, the Projection made in the Direction AE, has its Height AF, and its Amplitude the Quadruple of EF: in like manner, it being made in the Direction AD, its Height will be AG, and its Amplitude the Quadruple of GD; and so of the rest. Whence if the Angle of Elevation DAK is half a right one, the Quadruple of GD will be the greatest of all the Amplitudes, which

can



can be made with the fame Impetus; and the Amplitudes of Projections, which are equally diftant from the Projection whofe Elevation is half a right Angle, are equal: as, for example, the Projection in the right Lines AE, AC, (the Angles DAE, DAC, being made equal) have their Amplitudes quadruple of EF, and quadruple of HC, which are equal. Befides, the Amplitude of the Projection whofe Elevation is half a right Angle, is $= 4GD = 4GB =$ *Latus Rectum* of the Parabola. But the perpendicular Projection upwards, that is, the Impetus of the Projection, will be equal to half the Amplitude of the Projection, elevated to half a right Angle, and made with the fame Impetus. Laftly, to make equal Randoms in a Horizontal Plane, there is required a lefs Impetus in the Projection whofe Elevation is half a right Angle: for if it is not lefs than the Impetus of another Projection, made in another Direction; the Amplitude of the Projection whofe Elevation is half a right Angle, will be greater than the Amplitude of that other Projection.

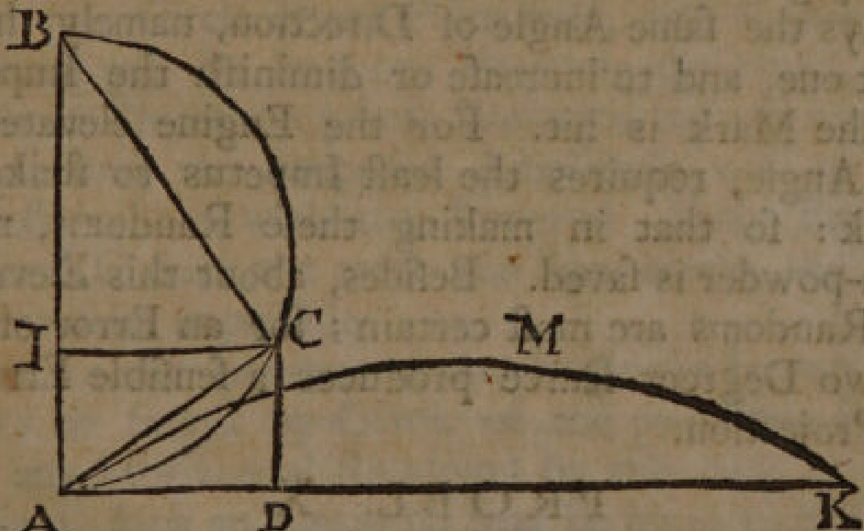
Cor.

*Cor.*2. Because AK touches the Circle, it will be [by 32 *El.* 3.] the Angle ABE = EAK the Angle of Elevation; and consequently, the Angle AGE is double EAK. Wherefore GA, half the Impetus, being made Radius, EF the fourth part of the Amplitude, will be the Sine of double the Angle of Elevation; and AF, the Height of the Projection, will be the versed Sine of the Arch AE, or of double the Angle of Elevation; and FB, the Sublimity of the Parabola, will be the versed Sine of the Arch BE, or of the Complement of double the Angle of Elevation to two right Angles.

PROBL. IX.

The *Amplitude* AK, *and the Angle of Direction* CAK, *being given; to find the Impetus of the Projection, and the Height* AI.

Take AD the fourth part of the Amplitude; and erect the Perpendiculars DC, AB; and make the Angle ACB a right one: I say, AB will be the



Impetus of the Projection, and DC will be its Height. For because the Angle ACB is a right one, the Semi-circle described on the Diameter AB, will pass through C: whence [by Corol. 1. of the preceding

ceding Problem] the Projection whofe Direction is AC, Impetus AB, will defcribe in its Motion the Parabola AMK, whofe Height is DC or AI, and the fourth part of its Amplitude is AD; wherefore converfly the Projectile whofe Direction is AC, fourth part of its Amplitude AD, will have the Impetus AB, and Height DC. *Q. E. D.*

Cor. 1. Hence from any Horizontal Random of any Engine being given, and made at a given Elevation; may be found the Height to which a Ball will rife, being fhot directly upwards: to wit, the Impetus of the Engine, which in great Ordnance exceeds any perpendicular Height to which any Man can come at. But the Impetus being given, the Amplitude and the Height of a Random made from any Elevation, will be given: whence may be difcovered, whether a Mark, whofe Diftance is known, can be hit by a given Engine.

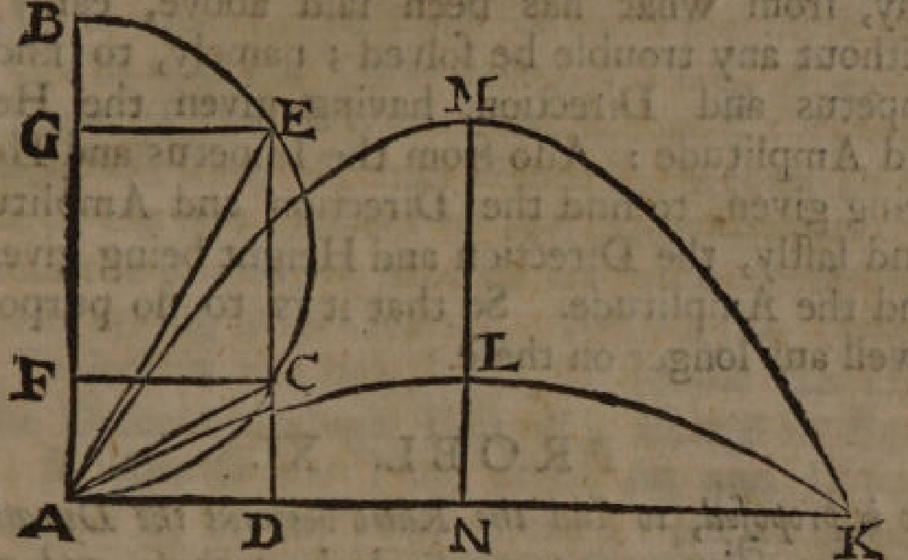
Cor. 2. If AD, the fourth part of the Amplitude, is made Radius, the Height DC will be the Tangent of the Angle of Elevation. To ftrike a Mark at any given Horizontal Diftance, it is beft to keep always the fame Angle of Direction, namely, half a right one, and to increafe or diminifh the Impetus, till the Mark is hit. For the Engine elevated to this Angle, requires the leaft Impetus to ftrike the Mark: fo that in making thefe Randoms, much Gun-powder is faved. Befides, about this Elevation the Randoms are moft certain; for an Error of one or two Degrees fcarce produces a fenfible Error in the Projection.

PROBL. X.

The *Impetus and Amplitude being given, to find the Direction and Height of the Random.*

Let AB be the Impetus, and AD the fourth part of the given Amplitude; upon the Diameter AB

AB describe a Semi-circle ACEB, and erect the Normal DCE, cutting the Semi-circle in the Points C and E: I say, both the Directions, whether AC



or AE, will describe a Parabola, whose Amplitude will be AK quadruple of AD. For the Projections made with the Impetus AB, in the Directions AC or AE, have the Amplitude AK quadruple of FC or GE, [by Probl. 8.] but the Height may be either AF or AG, as is manifest. But if the Normal DC meets the Circle in one Point only, that is, touches it; there will be but one Parabola described, and that by a Projection whose Elevation is half a right Angle: and the proposed Amplitude will be the greatest possible with that Impetus. If the Perpendicular DC does not meet the Semi-circle, the Problem will be impossible.

Cor. If the Impetus of any Engine is known, (as it may be found by Corol. of the preceding Probl. from any Horizontal Random) by the help of this Problem, such an Elevation may be given to the Engine, that the Mark placed at any given Horizontal Distance may be struck; and of two Directions fit for the purpose, equally remote from the Direction whose Elevation is half a right Angle, the more proper may be made choice of.

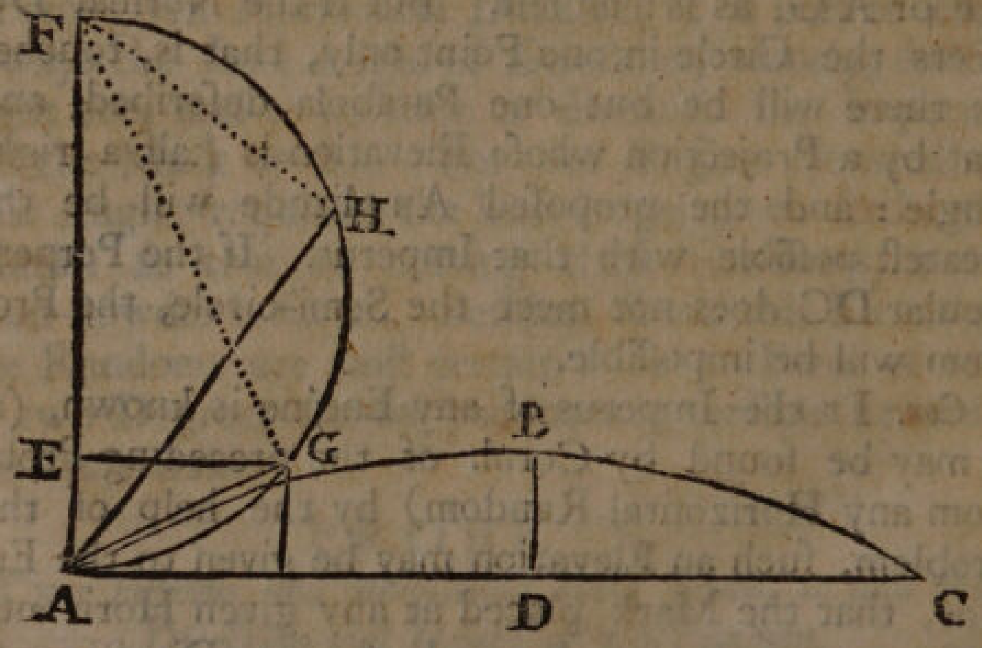
SCHOLIUM.

THE Converse of the three preceding Problems, may, from what has been said above, easily and without any trouble be solved; namely, to find the Impetus and Direction, having given the Height and Amplitude: Also from the Impetus and Height being given, to find the Direction and Amplitude: And lastly, the Direction and Height being given, to find the Amplitude. So that it is to no purpose to dwell any longer on these.

PROBL. XI.

It is proposed, to find the Ratio betwixt the Duration of a Projection made perpendicularly upwards, and of any other, whose Impetus is the same.

LET AF be the Impetus of the Projectile, or the Projection made upwards, and ABC the Projection from any other Elevation AG; about the Diameter



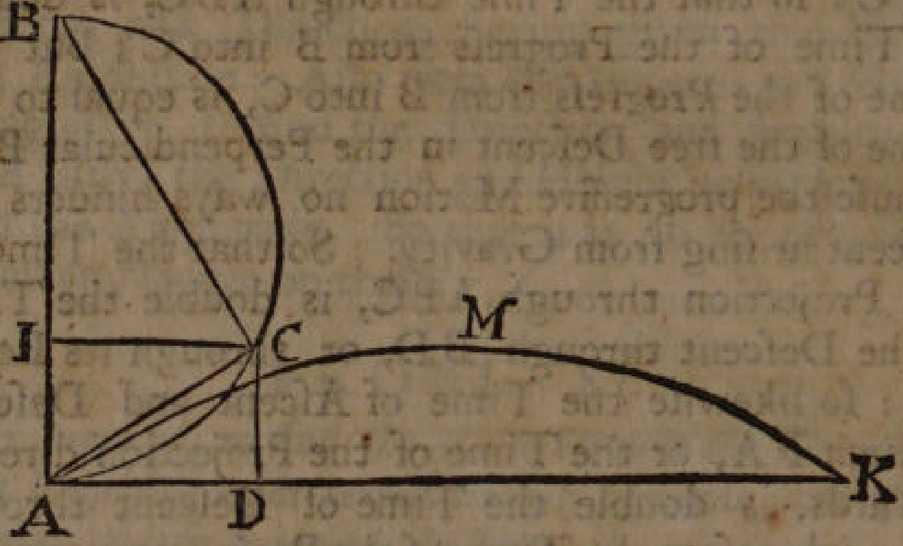
AF describe a Semi-circle, cutting the Direction AG in G: I say, the Duration of the Projection directly upwards, or the Time of Ascent through AF, AF, and of Descent through the same, is to the Duration of the Projection in the Parabola ABC, as AF to AG. The Time of the Progress from A into B, is equal to the Time of the Progress from B into C: so that the Time through ABC, is double the Time of the Progress from B into C; but the Time of the Progress from B into C, is equal to the Time of the free Descent in the Perpendicular BD, because the progressive Motion no ways hinders the Descent arising from Gravity. So that the Time of the Projection through ABC, is double the Time of the Descent through BD, or through its Equal EA: so likewise the Time of Ascent and Descent through FA, or the Time of the Projection directly upwards, is double the Time of Descent through FA; wherefore the Time of the Projection upwards, will be to the Time of the Projection in the Parabola ABC, as the Time of Descent through FA, to the Time of Descent through EA; that is, in a subduplicate Ratio of FA to EA; or, by reason FA, AG, EA, are continual Proportionals, as FA to AG. *Q. E. D.*

Cor. The Durations of Projections made with an equal Impetus in different Directions AG, AH, are in the Ratio of the Chords AG, AH. And if AF is made Radius, AG will be the Sine of the Angle AFG, which is equal to the Angle of Elevation of the Engine: so that the Time of the Projection directly upwards, is to the Time of Projection in a Parabola, as Radius to the Sine of the Angle of Direction.

SCHOLIUM.

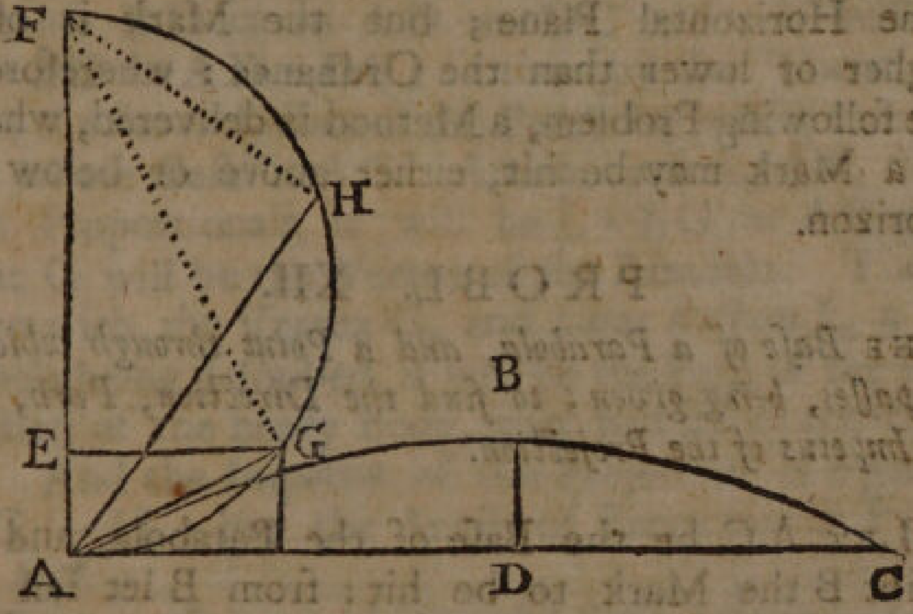
All the Problems about the Projections of heavy Bodies made in a Horizontal Plane, are easily resolved by the means of a Table of Sines and Tangents.

LET AK be the Horizontal Amplitude of any large Piece of Ordnance, elevated to the given Angle CAK, there is required the Height of the Projection, and the Impetus of the Engine. In the



Triangle ADC let it be as the Radius to the Tangent of the Angle of Elevation; so is AD, the fourth part of the given Amplitude, to the Height DC. Also make as the Sine of the Angle of Elevation to Radius, so the Height found DC to AC, which therefore will be given; and in the rectangle Triangle BCA, make as the Sine of the Angle ABC (which is equal to the Angle of Elevation) to Radius, so AC to AB the Impetus, which consequently is known. But the Impetus being given, the Time of the perpendicular Projection will be likewise given. But the Time of the perpendicular Projection is to the Time of the Projection in AC, as AB to AC, or as Radius to the Sine of the Angle of Elevation; and consequently the Time of Projection in AC may be known from a Table of Sines. Hence also from the Time of any Projection being given, made according to a given Elevation, will be given the Time of any other Projection, made with the same Impetus. For as the Sine of Elevation of a Projection,

jection, whose Time is known, is to the Sine of another Elevation; so is the Time known of one Projection, to the Time of the other Projection, which will therefore be also known. But from the Amplitude of one Projection being given, made in a given Direction, will be given the Amplitude of a Projection made in any other Direction. For making half the Impetus Radius, the fourth part of the Amplitude will be the Sine of double the Angle of Elevation; and consequently the Amplitudes are as the



Sines of these Angles. Wherefore if the Amplitude in the Direction AG is known, the Amplitude in the Direction AH will be given: for make as the Sine of double the Angle CAG, to the Sine of double the Angle HAC, so the Amplitude of the Projection according to AG, to the Amplitude of the Projection according to the Direction AH; and if from the Impetus and Horizontal Amplitude being given, there is sought the corresponding Elevation, that may be easily discovered from the same Principle. For it is manifest from Cor. 2. Probl. 8. that double the Impetus is the Amplitude of the Projection whose Elevation is half a right Angle. But the Sines of the Elevations doubled are as the Am-

 plitudes;

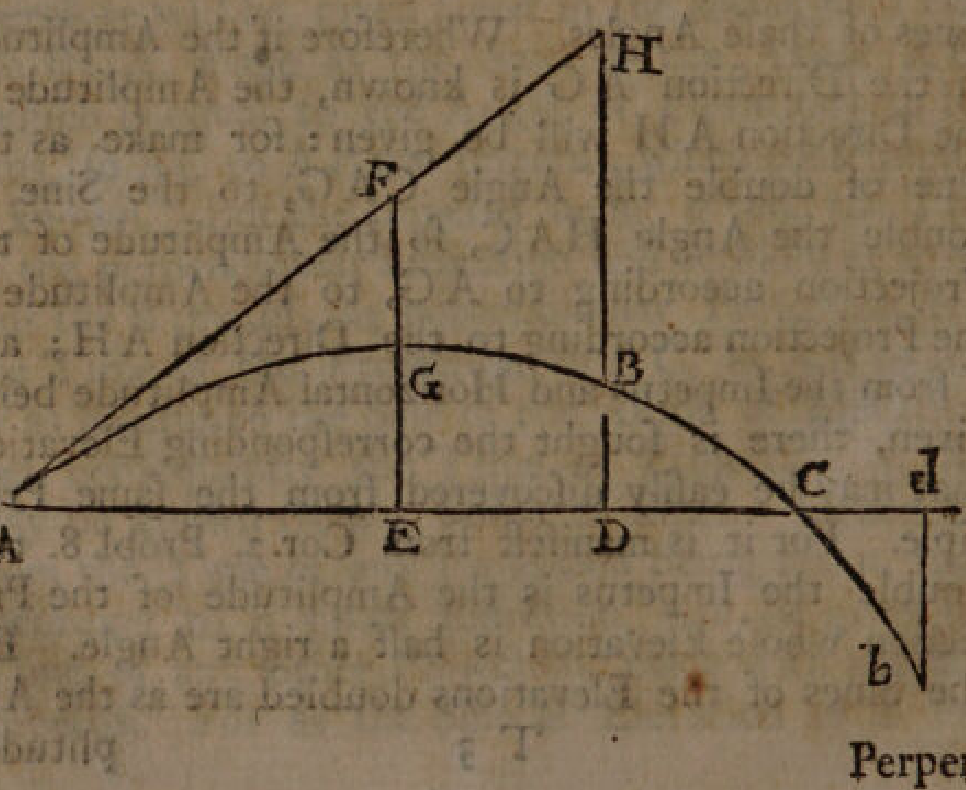
plitudes; wherefore make as double the Impetus to the given Amplitude, so the Sine of double half a right Angle, that is, the Sine of ninety Degrees, or Radius, to another; which will be the Sine of two Arches, whereof one is the Complement of the other to a Semi-circle: and these two Arches halved, will give two Elevations, wherewith the given Amplitude may be reached.

The warlike Engines are not always to be so exploded, as the Bullet shall fall precisely in the same Horizontal Plane; but the Mark is often higher or lower than the Ordnance: wherefore in the following Problem, a Method is delivered, whereby a Mark may be hit, either above or below the Horizon.

PROBL. XII.

The *Base of a Parabola, and a Point through which it passes, being given; to find the Direction, Path, and Impetus of the Projection.*

Let AC be the Base of the Parabola, and the Point B the Mark to be hit: from B let fall the



Perpen-

Perpendicular BD on AC; take L a fourth Proportional to the right Lines BD, AD, DC; L will be the *Latus Rectum* of the Parabola. Bisect AC in E, and from E erect the Perpendicular EF; let EG be a third Proportional to the right Lines L and AE; G will be the Vertex of the Parabola: And if EG is produced, so that it is GF = GE, and AE be drawn, FAE will be the Angle of Direction of the Engine. And the Impetus wherewith the heavy Body is to be projected, will be equal to $EG + \frac{1}{4}L$. Because BD is to AD as DC to L; it will be $L \times BD = AD \times DC$; so that [by Cor. Theor. 48.] L is the *Latus Rectum* of the Parabola, passing through B, whose Base is AC. And because L, AE, EG, are Proportionals, it will be $L \times EG = AE$ q. so that G will be the Vertex of the Parabola. Therefore with the Vertex G, and *Latus Rectum* L, a Parabola being described, it will be the Path of the Projection of the heavy Body, that will strike the Point B. And the Impetus of the Projection is equal to $EG + \frac{1}{4}L$; but the Angle of Elevation is FAE. *Q. E. I.*

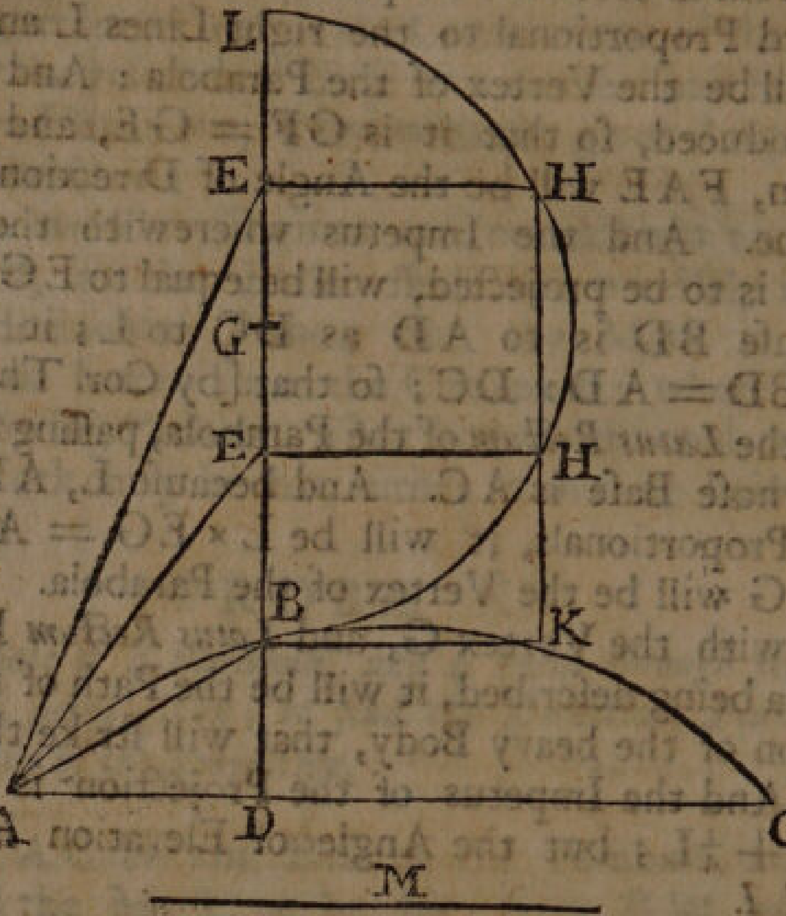
You must proceed after the same manner, if the Point *b* is below the Horizon; for if from *b* be let fall the Perpendicular *bd* on AC produced, and L is taken a fourth Proportional to *bd*, A*d*, *d*C, L will be the *Latus Rectum* of the Parabola passing through *b*.

Cor. AE being made Radius, EF, or the Double of EG, will be the Tangent of the Angle of Elevation; so that if it be made as AE, which is given, to EF also given, so Radius to the Tangent of the Angle FAE, the Angle of Elevation will be given.

PROBL. XIII.

THE *Impetus being given, to find the Direction according to which a heavy Body being projected, that Body may hit a given Point.*

Let M be the given Impetus, and B the Point through which the Projectile ought to pass, whose Distance AB from the Point A is given: from B on

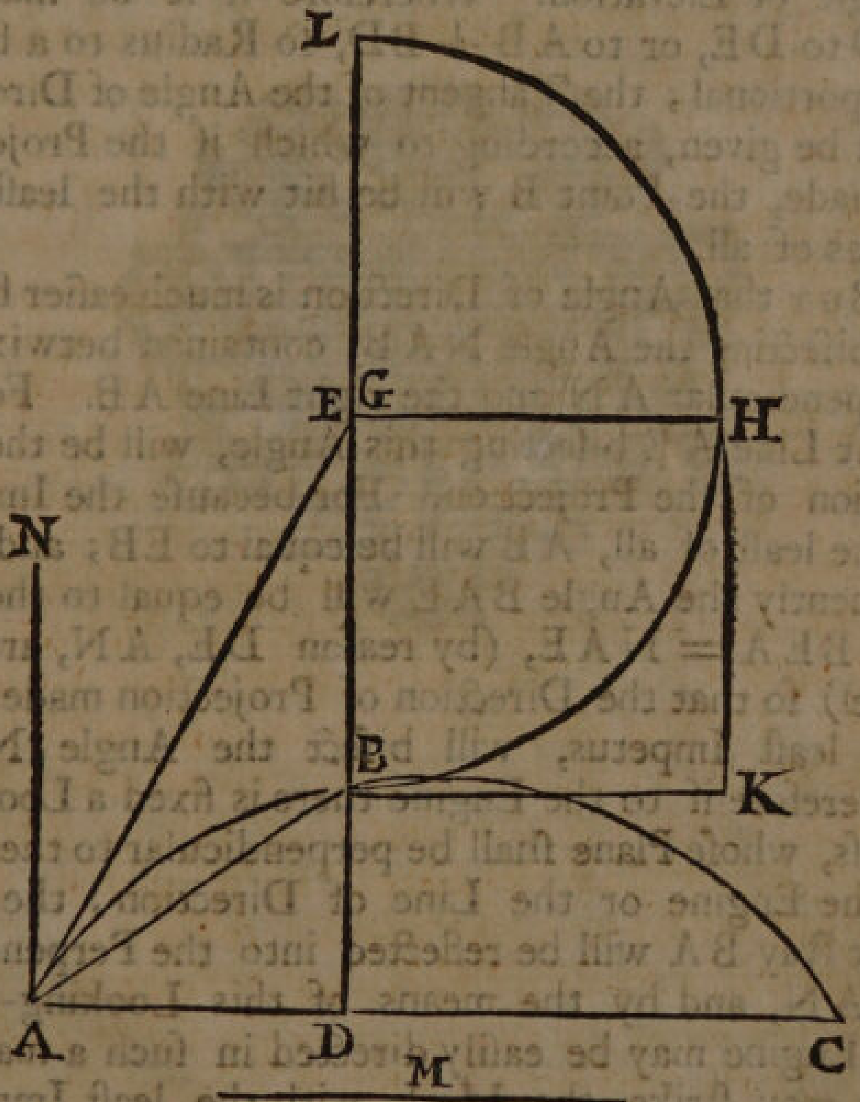


the Horizontal Line AC let fall the Perpendicular BD, in which produced take DG = 2M, and with the Center G and Interval GB describe a Circle, which let the right Line BK = AB touch in B. From K upon BK erect the Perpendicular KH, meeting the Circle in two Points H, H; from which on the Diameter LB let fall the Perpendiculars HE, HE, and draw the right Lines AE, AE, which will be the two Directions sought: that is, a Projectile emitted in the Direction AE with the Impetus M, will pass through the Point B. For it is ADq. + BDq = ABq. = BKq. = EHq. = (from the Nature of a Circle) LE × EB = LB × EB — EBq. = 4M × EB — 2DB × EB — EBq. Wherefore

 it

it will be $4M \times EB = (ADq. + BDq. + 2BD \times EB + EBq. = ADq. + DEq. =) AEq.$ But the Parabola described by a heavy Body, projected in the Direction AE, with the Impetus M, will so cut the right Line DE, that it is $4M \times EB = AEq.$ (as is manifest from Cor. 2. Theor. 51.) Wherefore the Point B is in the same Parabola; and the heavy Body projected with the Impetus M, in the Direction AE, will pass through B. *Q. E. D.*

Cor. If HK meets the Circle in one Point only, that is, if it touches the Circle, there will be only



one Direction that will satisfy the Condition. But if it does not meet the Circle at all, the Problem will

will be impossible; that is, the Point B cannot be hit with the given Impetus. And if KH touches the Circle, that Impetus will be the least of all, wherewith the given Point can be hit. And in that case it will be BK or AB = BE or BG = 2 M — DB; so that it will be BE + BD or DE = 2 M. The least Impetus therefore, wherewith a given Point may be hit, will be equal to half DE = $\frac{AB + BD}{2}$; and DA being made Radius, DE will be the Tangent of the Angle EAD, that is, of the Angle of Elevation. Wherefore if it be made as AD to DE, or to AB + BD, so Radius to a fourth Proportional; the Tangent of the Angle of Direction will be given, according to which if the Projection is made, the Point B will be hit with the least Impetus of all.

But that Angle of Direction is much easier found, by bisecting the Angle NAB, contained betwixt the Perpendicular AN and the right Line AB. For the right Line AE bisecting this Angle, will be the Direction of the Projection. For because the Impetus is the least of all, AB will be equal to EB; and consequently the Angle BAE will be equal to the Angle BEA = NAE, (by reason DE, AN, are parallel) so that the Direction of Projection made with the least Impetus, will bisect the Angle NAB. Wherefore if to the Engine there is fixed a Looking-Glass, whose Plane shall be perpendicular to the Axis of the Engine or the Line of Direction; the incident Ray BA will be reflected into the Perpendicular AN, and by the means of this Looking-Glass the Engine may be easily directed in such a manner, as it may strike the Mark with the least Impetus. For the Engine is to be elevated or depressed, till the Image of the Point B, made on the Plane of the Looking-Glass, may be seen in the Perpendicular

NA:

NA: for by reaſon the Angle of Incidence BAE is equal to the reflected Angle NAE, the Angle NAB will be biſected, and AE will be the Direction of the Engine, when the Point B may be hit with the leaſt Impetus.

THE

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THEOREMS

Of the Famous

Monſ. *HUYGENS*,

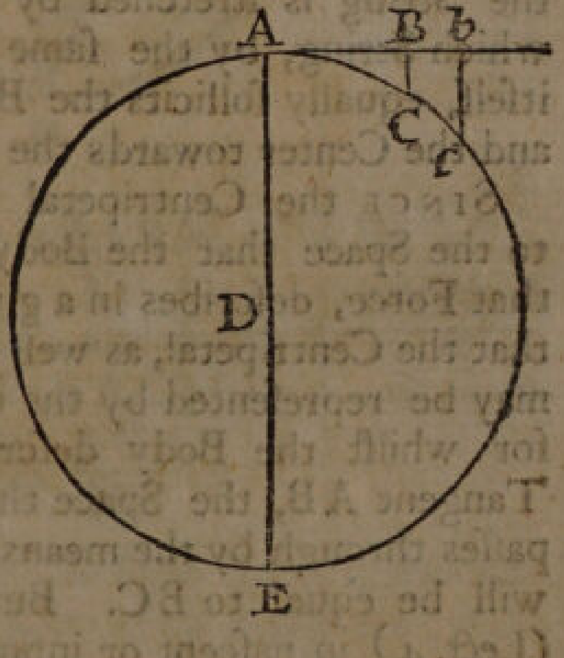
CONCERNING

The Centrifugal Force, and Circular Motion,

DEMONSTRATED.

I *FIRST communicated the Demonſtrations of the following Theorems to the Learned World; for their Author publiſhed them without any Demonſtration. But afterwards the ſame Theorems were demonſtrated by ſome* French-men, *but not in the ſame Order; and now at length the Author's own Demonſtrations are publiſhed in his Poſthumous Pieces, which though they are indeed very ſhort, yet they are more prolix than ours. But ſince theſe Theorems make no mean Part of the Science of Motion,* *we* *we have thought fit to join their Demonstrations to this Work; that the Commonwealth of Learning may see how much the Mechanical Philosophy is capable of being improved by Geometry.*

Defin. 1. A CENTRIPETAL Force is that Force, whereby any moving Body is continually drawn from its rectilineal Motion, and is perpetually sollicited towards some Center. For since, according to a well-known Law of Nature, every Body once in Motion endeavours to proceed always uniformly in the same right Line, it is manifest no moving Body can describe an Orbit in its Motion, unless it is detained in that Orbit by some Force. For example, let a moving Body revolve with a uniform Motion in the Circumference of the Circle ACE, which when it comes to A, if the Force that kept it in the Orbit is taken away, it will proceed along the Tangent AB, and run out *in infinitum*: that it therefore may be de-



tained in the Circumference, it is necessary that some Force continually acts, and which is equivalent to the Force in A sollicting the Body towards D through a Space equal to BC, in the mean time whilst the moving Body by its innate Force proceeds through the infinitely small Space AB: For after this manner, by these Forces joined together, the moving Body will describe the Line AC [by Theor. 30.] This Force, whether it is the Action of a String

String holding the Body, or its Coherence with another revolving Body, or whether it arises from Gravity or any Attraction whatsoever, it may be called a Centripetal Force.

2. A Centrifugal Force is the Re-action or Resistance which a moving Body exerts to prevent its being turned out of its way, and whereby it endeavours to continue its Motion in the same Direction: and as Re-action is always equal, and contrary to Action, so in like manner is the Centrifugal to the Centripetal Force. This Centrifugal Force arises from the *Vis Inertiæ* of Matter, and when a Body revolving in the Circumference of a Circle, is detained by the means of a String from running out; the String is stretched by this Centrifugal Force: which String, by the same Endeavour of relaxing itself, equally sollicits the Body towards the Center, and the Center towards the Body.

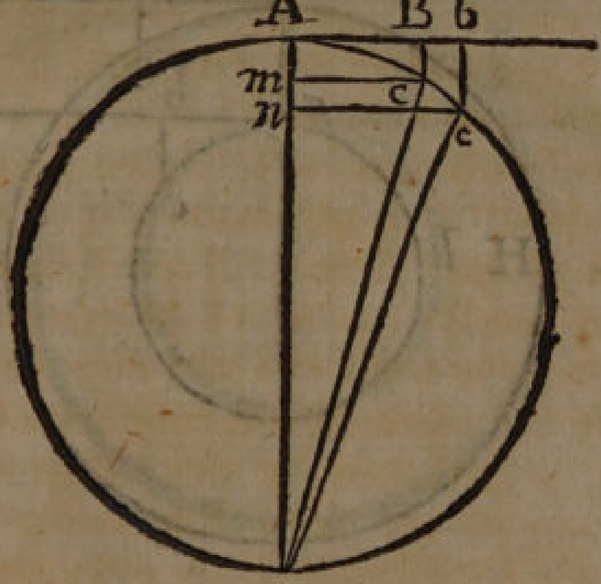
Since the Centripetal Force is proportionable to the Space that the Body, by the Sollicitation of that Force, describes in a given Time; it is manifest, that the Centripetal, as well as the Centrifugal Force, may be represented by the nascent Lines BC or *bc*: for whilst the Body describes the infinitely small Tangent AB, the Space that in the mean time it passes through by the means of the Centripetal Force, will be equal to BC. But we have demonstrated (Lect. 4) in nascent or infinite small Lines AB, AC, that BC is infinitely less than AB or AC. Whence the Centripetal or Centrifugal Force will be infinitely less than the communicated Force AB.

LEMMA I.

In *a Circle the evanescent or infinitely small Subtenses of the Angle of Contact, are in a duplicate Ratio of the conterminal Arches.*

Let

Let the Arches be AC, Ac, the Subtenses perpendicular to the Tangent BC, Bc; draw the Diameter AD, and *Cm*, *cn*, Perpendiculars to the Diameter; and it will be $BC : bc :: Am : An :: Am \times$



$AD : An \times AD$. But it is [by 8 *El.* 6.] $AD : AC :: AC : Am$, and $AD : Ac :: Ac : An$; wherefore it will be $AD \times Am = ACq$, and $AD \times An = Acq$; wherefore it is also $BC : bc :: ACq : Acq$. *Q. E. D.*

Cor. Hence it is $BC = \frac{ACq}{DA}$.

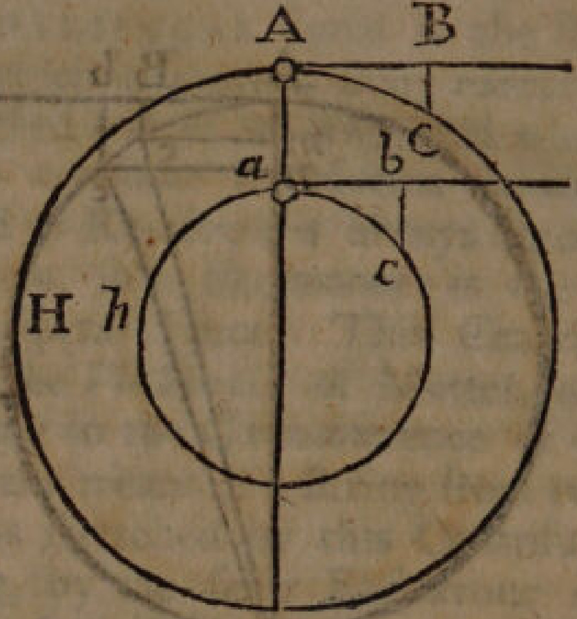
This *Lemma the Great Sir* Isaac Newton *has universally demonstrated in all Curves of the first Order.*

THEOR. I.

If *two equal Bodies in Motion run through unequal Circumferences in equal Times; the Centrifugal Force in the greater Circumference, will be to that in the less, as the Circumferences amongst themselves, or their Diameters.*

Let the Body in motion A run through the Circumference ACH, and in the same Time let the Body

Body *a* run through the Circumference *ach*; and let AC, *ac*, be the infinitely ſmall Arches deſcribed in the ſame Time. Becauſe both Circumferences are



run through in an equal Time, theſe Arches will be ſimilar, and conſequently the Figure ABC will be ſimilar to the Figure *abc*; wherefore BC : *bc* :: AC : *ac* :: Circumference ACH : Circumference *ach*. But it is manifeſt from the foregoing Definition, that the Centrifugal Force of the Body A is to the Centrifugal Force of the Body *a*, as BC to *bc*. Wherefore the Centrifugal Force of the Body A will be to the Centrifugal Force of the Body *a*, as the Circumference ACH to the Circumference *ach*, or as the Diameter of that to the Diameter of this. *Q.E.D.*

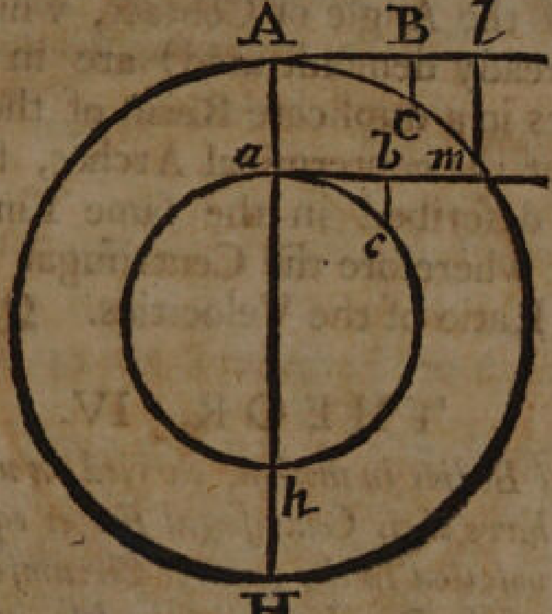
Cor. Hence converſly, if the Centrifugal Forces are as their Diameters, the Periodical Times will be equal.

THEOR. II.

If *two equal Bodies in motion are carried in unequal Circumferences with an equal Celerity, their Centrifugal Forces will be in a reciprocal Ratio of the Diameters.*

Let

LET AC, *ac*, be the leaſt Arches deſcribed in the ſame Time, which, by reaſon the Velocity in both the moving Bodies is equal, will be equal. Make



the Arch A*m* ſimilar to the Arch *ac*, and draw *lm* parallel to BC; and the Centrifugal Force in the greater Circumference will be to that which is in the leſs, as the naſcent Line BC to the naſcent Line *bc*: but BC is to *bc* in a Ratio compounded of BC to *lm* and *lm* to *bc*; and from the preceding Lemma, BC is to *lm*, as AC*q* to A*mq*, and *lm* is to *bc* as A*m* to A*c* or AC. Wherefore it will be BC : *bc* :: AC*q* : A*mq* + A*m* : *ac* :: AC*q* : A*mq* + A*mq* : A*m* × *ac* :: AC*q* or *acq* : A*m* × *ac* :: *ac* : A*m*; that is, as the whole Circumference *ach* to the whole Circumference ACH, or as the Diameter *ah* to the Diameter AH. *Q. E. D.*

THEOR. III.

IF *two equal moving Bodies are carried in equal Circumferences, but both with an equable Motion, (as we would be underſtood to mean in all) the Centrifugal Force of*

 the

the swifter will be to the Force of the slower in a duplicate Ratio of the Celerities.

For the Centrifugal Forces are as the evanescent Subtenses of the Angle of Contact, which (by what we have already demonstrated) are in the same or equal Circles in a duplicate Ratio of the conterminal Arches; but the conterminal Arches, since they are the Spaces described in the same Time, are as the Velocities: wherefore the Centrifugal Forces are in a duplicate Ratio of the Velocities. *Q. E. D.*

THEOR. IV.

If *two equal Bodies in motion, carried in unequal Circumferences, have their Centrifugal Forces equal; the Time of the Revolution in the greater Circumference, will be to the Time of the Revolution in the less, in a subduplicate Ratio of the Diameters.*

Let AC, *ac*, be the least Arches described in the same Time; (*see* Fig. Theor. 2.) because the Centrifugal Forces are equal, it will be $BC = bc$. Let the Time wherein the Circumference ACH is described, be called T, and the Time wherein the Circumference *ach* is described, be *t*: make the Arch A*m* similar to the Arch *ac*, and let us suppose any moving Body to run through the Circumference ACHA in the same Time, as it would run through the Circumference *acha*; and in that case the Arches described in the same Time in both Circumferences will be A*m*, *ac*. But the Velocity of a moving Body passing through the Arch A*m* in any given Time, is to the Velocity of a moving Body passing through the Arch AC in the same Time, as the Arch A*m* to the Arch AC: so that since the Time wherein the same Circumference is run over, is always reciprocally as the Velocity, it will be $T : t :: Am : AC$,

and

and $T^2:t^2::Amq:ACq::ml:BC::ml:bc$: that is, by reason the Arch A*m* is similar to the Arch *ac*, as the Diameter AH to the Diameter *ab*; whence it is manifest, that $T:t::\surd AH:\surd ab$. *Q. E. D.*

Schol. SINCE in every Case the Centrifugal Force is to the Centrifugal Force as BC to *bc*, but it is $BC=\frac{ACq}{AH}$ and $bc=\frac{acq}{ab}$; the Centrifugal Force will be to the other Centrifugal Force as $\frac{ACq}{AH}$ to $\frac{acq}{ab}$: that is, as the Squares of the Arches described in the same Time applied to the Diameters of the Circles. And since those Arches are as the Velocities, the Centrifugal Forces will be as the Squares of the Velocities applied to the Diameters of the Circles.

LEMMA II.

IF *a moving Body revolves in the Circumference of a Circle, the Space, which the moving Body proceeding forwards, and only the Centrifugal Force arising from that circular Motion taking place, would run over in a given Time, will be a third Proportional to the Diameter of the Circle and that Arch, which if it was carried in the Circumference of the Circle it would describe in the same Time.*

LET AC be any Arch described in any the least Particle of Time, and let *n* denote any Time or any Number of those Particles; $n\times AC$ will be the Arch which the moving Body carried in the Circumference will describe in the given Time *n*, and BC the Space which it would run over in the first Particle of that Time, the Centrifugal Force taking place. But since every Body in motion, by the same Force continued always in the fame Direction, defcribes Spaces in a duplicate Ratio of the Times, [by Cor. 3. Theor. 12. Lect. 11. for whatever is demonftrated of Gravity, may be applied to any other Force acting uniformly] the Space defcribed, the Centrifugal Force acting, in the Time n, will be $= n^2 \times BC$. But (as is manifeft from the firft Lemma) it is $AH : AC :: AC : BC$, and as AC to BC, fo $n \times AC$ to $n \times BC$. Wherefore AH is to AE as $n \times AC$ to $n \times BC$; and by multiplying the Confequents by n, AH will be to $n \times AC$ as $n \times AC$ to $n^2 \times BC$: that is, the Diameter of the Circle, the Arch defcribed in a given Time, and the Space which, the Centrifugal Force acting, is run over in the fame Time, are continual Proportionals. *Q. E. D.*

Cor. If the Diameter of the Circle is called D, and the Arch defcribed by the moving Body in any Time A, the Space which the moving Body, the Centrifugal Force acting, proceeding forwards would defcribe in the fame Time, will be $\frac{A^2}{D}$; for D, A, $\frac{A^2}{D}$, are continual Proportionals.

THEOR. V.

If *a moving Body is carried in the Circumference of a Circle, with the Celerity which it has acquired by falling from a Height that is equal to a fourth Part of the Diameter, it will have a Centrifugal Force equal to its Gravity; that is, it will ftretch the String, whereby it is kept to the Center, juft as much as if it was fufpended by it.*

Let the Diameter of the Circle be called D, and the Circumference P: and fince, by Hypothefis, the Velocity of the moving Body carried in the Circumference is uniform, and equal to that which it acquires

acquires by falling through $\frac{1}{4}$ D, it is certain that the moving Body carried in the Circumference will describe in an equal Time an Arch equal to twice that [by Theor. 12. Lect. 11.] that is, $=\frac{1}{2}$ D; whence [by Lemm. 2.] the Space passed over in the mean while, by the acting of the Centrifugal Force, will be $=\frac{1}{4}$ D. For D is to $\frac{1}{2}$ D, as $\frac{1}{2}$ D to $\frac{1}{4}$ D; but by Hypothesis the Space that the moving Body by the Force of Gravity describes in the same Time, is also $\frac{1}{4}$ D. Wherefore since the Spaces run over by these two Forces in the same Time are equal, the Forces themselves will be likewise equal.

Cor. 1. HENCE conversly, if a moving Body carried in the Circumference of a Circle has a Centrifugal Force equal to its Gravity, its Velocity is that which is acquired by falling through $\frac{1}{4}$ D.

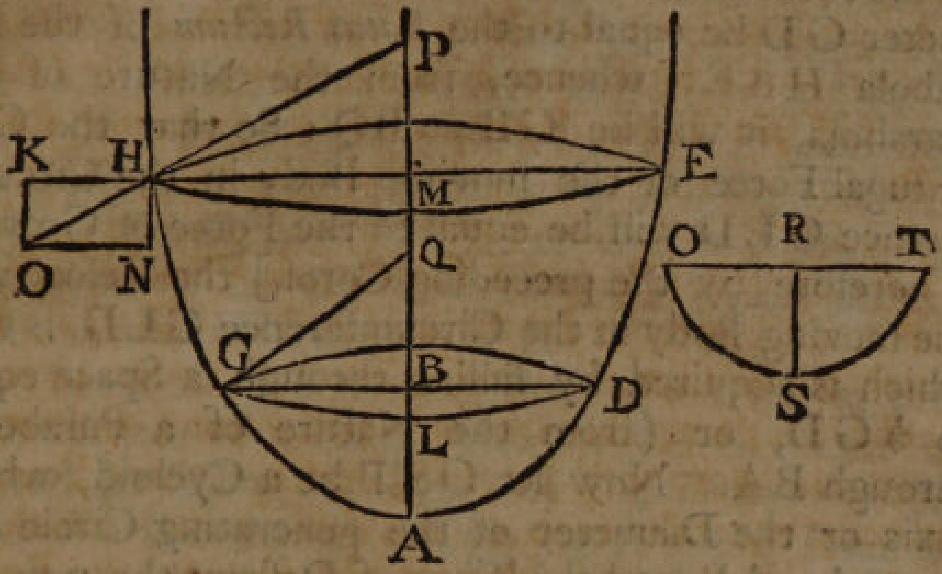
Cor. 2. HENCE the Time of a Revolution is to the Time of Descent through $\frac{1}{4}$ D, as P to $\frac{1}{2}$ D, or as 2 P to D. For in the Time a moving Body, with an accelerated Velocity, runs over $\frac{1}{4}$ D, it will, moved uniformly with the Velocity acquired at the last, run through $\frac{1}{2}$ D: and consequently since the Velocities are equal, the Times will be as the Spaces run over; that is, the Time wherein the moving Body runs through the Circumference, is to the Time wherein it describes $\frac{1}{2}$ D, as P to $\frac{1}{2}$ D, or as 2 P to D; but the Time wherein $\frac{1}{2}$ D is described, is $=$ Time of the Fall through $\frac{1}{4}$ D: whence the Time of the Revolution will be to the Time of the perpendicular Fall through $\frac{1}{4}$ D, as 2 P to D.

THEOR. VI.

IN *the concave Superficies of a Parabolick Conoid, which has its Axis erected perpendicularly, all the Revolutions of a moving Body running through the Circumferences parallel to the Horizon, whether they are small or great,* *are performed in equal Times; each of which Times are equal to two Oscillations of a Pendulum, whose Length is half the* Latus Rectum *of the generating Parabola.*

Let HGADE be a Parabolick Conoid, whose Axis erected perpendicularly is AP; GD, HE, the Diameters of Circles, whose Circumferences parallel to the Horizon the moving Body runs through; which therefore will be sollicited by three Powers mutually equivalent to each other, acting in three different Directions: whereof the first is the Force of Gravity impelling the moving Body in the right Line HN perpendicular to the Plane of the Horizon; the second is the Centrifugal Force arising from the circular Motion, sollicting the moving Body from H towards K; but the Place of the third Power is supplied by the Resistance or contrary Endeavour of the Parabolick Superficies, according to the Line HP acting perpendicularly to it, for Re-action is always equal to Action, and towards the contrary Part. Whence since the Superficies is pressed perpendicularly by the moving Body, it re-acts on the Body according to the Direction HP, and that contrary Endeavour is equivalent to the Power sollicting the moving Body according to the Direction HP. Wherefore since the moving Body is sustained by these three Powers, they must be necessarily *in æquilibrio*, that is, any two destroy the Effect of the other. Whence ON being drawn parallel to HK meeting HN in N, if OH represents the Re-action of the Parabolick Superficies, the right Line ON will expound the Centrifugal Force, and HN the Force of Gravity of the moving Body: But by reason the Triangles HON, HMP, are equi-angular, ON is to HN as HM to MP; that is, the Centri-

fugal



fugal Force of the moving Body defcribing the Circumference of the Circle HME, will be to the Force of its Gravity as HM the Radius of the Circle, to MP the Subnormal. In like manner, in any other Circumference GLD, in the Superficies of the Conoid, the Centrifugal Force of the moving Body defcribing it, is to the Force of Gravity as GB the Radius, to BQ the Subnormal. Moreover, becaufe the Centrifugal Force of the moving Body running over the Circumference HME, is to the Force of Gravity as HM to MP; and the Force of Gravity of the fame moving Body, is to its Centrifugal Force, when it runs over the Circumference GLD, as BQ to BG, or (from the Nature of the Parabola) as MP to BG: by Equality, the Centrifugal Force of the moving Body running over the Circumference HME, will be to its Centrifugal Force when it runs over the Circumference GLD, as HM to BG; that is, the Centrifugal Forces are as the Semi-diameters or Diameters of the Circles, whence [by Cor. Theor. 1.] the Periodical Times are equal. Which was the firft thing to be demonftrated.

Let the Circle GLD be ſo taken, that its Diameter GD be equal to the *Latus Rectum* of the Parabola HAE; whence, from the Nature of the Parabola, it will be GB = BQ: So that the Centrifugal Force of the moving Body in the Circumference GLD will be equal to the Force of Gravity. Therefore [by the preceding Corol.] the Velocity of the moving Body in the Circumference GLD, is that which is acquired by falling through a Space equal to $\frac{1}{4}$ GD, or (from the Nature of a Parabola) through BA. Now let OST be a Cycloid, whoſe Axis or the Diameter of the generating Circle SR is equal to AB, and the Time of Deſcent through the Cycloid OS will be to the Time of the perpendicular Fall through the Axis RS, or through BA, as $\frac{1}{2}$ P to D, by Theor. 46. Lect. 15. But [by the preceding Corol.] the Time of Deſcent through AB is to the Time of the Revolution in the Circumference GLD as D to 2 P; wherefore, by Equality, the Time of Deſcent through the Cycloid OS, is to the Time of a Revolution in the Circumference GLD as $\frac{1}{2}$ P to 2 P, or as 1 to 4: whence the Time of four Deſcents in a Cycloid, or the Time of two Oſcillations in a Cycloid, is equal to the Time of a Revolution in the Circumference GLD. But the Time of two Oſcillations in a Cycloid, is equal to the Time of two the leaſt Oſcillations in a Circle, which is of the ſame curvity with the Cycloid at the Vertex S; by reaſon a Portion of ſuch a Circle and a Portion of the Cycloid nearly coincide at the Vertex S, and conſequently in Phyſical Matters perform the ſame Effect, as is now well known. But the Radius of a Circle of the ſame degree of curvity with the Cycloid at the Vertex S, is equal to twice RS, or twice AB, (as eaſily follows from Corol. Theor. 46. Lect. 15.) So that the Length of a Pendulum

dulum oscillating in that Circle, is equal to twice AB, or to half the *Latus Rectum* of the generating Parabola. Whence the Time of two the least Oscillations of a Pendulum, whose Length is half the *Latus Rectum*, is equal to the Time of two Oscillations in the Cycloid OST, or to the Time of a Revolution in the Circumference GLD, or in the Circumference HME. *Q. E. D.*

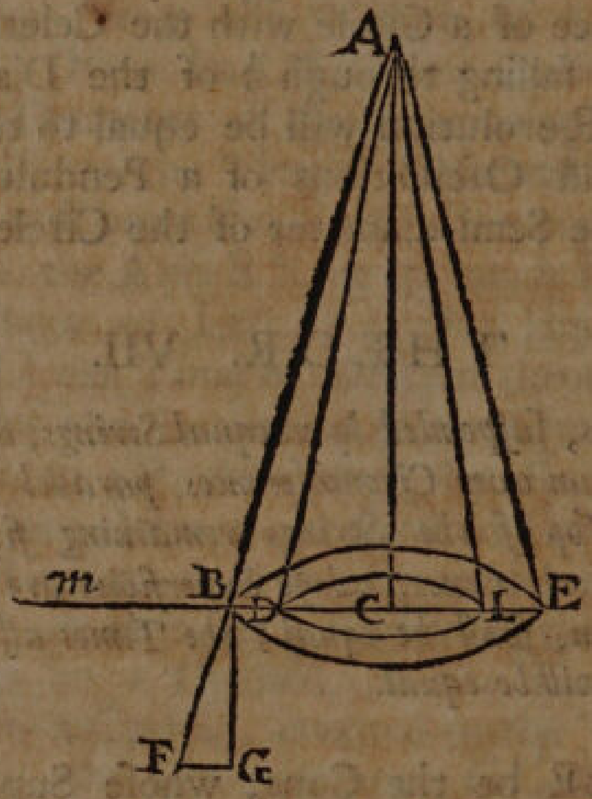
Cor. Hence if a moving Body is carried in the Circumference of a Circle with the Celerity that is acquired by falling through $\frac{1}{4}$ of the Diameter, the Time of a Revolution will be equal to the Time of two the least Oscillations of a Pendulum, whose Length is the Semi-diameter of the Circle.

THEOR. VII.

If *two Bodies, suspended by unequal Strings, do so revolve, that they run over Circumferences parallel to the Horizon, the Top of the Strings remaining fixed, but the Heights of the Cones, whose Superficies are described by this Motion, shall be equal; the Times also of the Revolutions will be equal.*

Let ABE be the Cone, whose Superficies the String AB describes; also ADL the Cone whose Superficies is described by the String AD: and let C be the Center of the Base of both Cones, and AC their common Height. Now let the moving Body B be considered as drawn by three Powers mutually equivalent to each other; whereof one, which is the Force of Gravity, draws the moving Body in the right Line BG perpendicular to the Plane of the Horizon; the other acting according to the Direction B*m*, is the Centrifugal Force whereby the moving Body endeavours to recede from the Center

Center C of its Orbit; but the third, which is equivalent to and refifts the other two, is the contrary Endeavour of the String acting according to the Direction AB; for the Tenfion of a String is inftead of a contrary Power, and in this cafe performs the fame Effect. If therefore BF reprefents the Action of the String, the Centrifugal Force of the moving



Body, and its Force of Gravity, will be expounded by the right Lines FG and BG, [by Theor. 28. Lect. 14.] that is, the Centrifugal Force of the moving Body B, will be to the Force of Gravity as FG to BG, or (by reafon the Triangles FBG, ABC, are equi-angular) as BC to CA. After the fame manner, the Force of Gravity will be to the Centrifugal Force of the moving Body D, as AC to DC: wherefore, by Equality, the Centrifugal Force of the moving Body B, will be to the Centri-

fugal

fugal Force of the moving Body D, as BC to DC; that is, the Centrifugal Forces are as the Semi-diameters of the Circles, whose Circumferences the moving Bodies describe, and consequently [by Cor. Theor. 1.] the Times of the Revolutions are equal. *Q. E. D.*

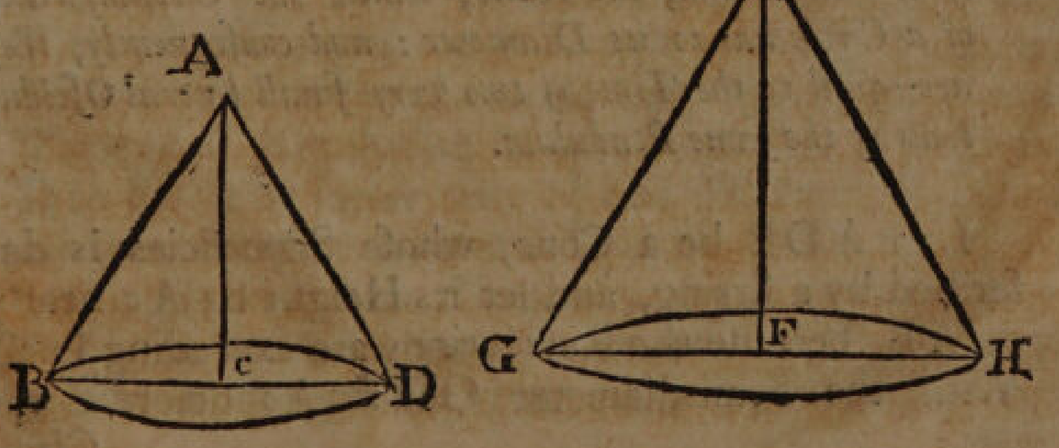
Cor. HENCE the Centrifugal Force is to the Force of Gravity as the Semi-diameter of the Base of the Cone to the Height of the Cone.

Note, BY the Force of Gravity and the Centrifugal Force in this Demonstration, we understand the accelerating Forces of the moving Bodies, unless those Bodies are supposed equal; in which case may be also taken the absolute Forces.

THEOR. VIII.

IF *two Bodies, as before, revolve with a Conical Motion, suspended by equal or unequal Strings, and the Heights of the Cones are unequal, the Times of the Revolutions will be in a subduplicate Ratio of their Heights.*

LET B and G be the two moving Bodies, and let first of all the Cones ABD, EGH, whose Super-



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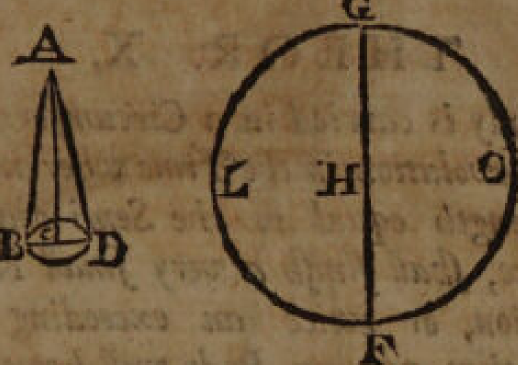
ficies are described by the Strings, be similar; [by Corol. Theor. 7.] the Centrifugal Force of the moving Body B will be to the Force of Gravity as BC to AC; and the Centrifugal Force of the moving Body G will be to the same Force of Gravity as GF to FE. But by reason the Triangles ABC, GEF, are equi-angular, BC is to AC as GF to FE: wherefore the Centrifugal Force of the moving Body B will be to the Force of Gravity, as the Centrifugal Force of the moving Body G to the same Force of Gravity, and consequently these Centrifugal Forces will be equal. Therefore [by Theor. 4.] the Times of the Revolutions of the moving Bodies will be in a subduplicate Ratio of the Semi-diameters; that is, by reason the Triangles ABC, EGF, are equi-angular, in a subduplicate Ratio of the Heights AC and EF. But whatever the Cones are, which the Strings describe, so that their Heights remain unaltered, the Times of the Revolutions will remain unchanged: wherefore in every Case the Truth of this Theorem is manifest. *Q. E. D.*

THEOR. IX.

If *a Pendulum carried in a Conical Motion makes very small Revolutions; the Times of each of them have to the Time of the perpendicular Fall from twice the Height of the Pendulum, the Ratio, which the Circumference of a Circle has to its Diameter: and consequently, they are equal to the Time of two very small lateral Oscillations of the same Pendulum.*

Let ADB be a Cone, whose Superficies is described by a String, and let its Height be A*c* nearly = AB, because the Revolutions are exceeding small. With the Semi-diameter GH = A*c* describe the Circle



Circle GLFO, and let a Body be ſuppoſed to revolve in its Circumference, with the Celerity that is acquired by falling through $\frac{1}{4}$ of its Diameter or $\frac{1}{4}$ D. [By Theor. 5.] its Centrifugal Force will be equal to its Gravity; but the Centrifugal Force of the moving Body B is to the Force of Gravity, and conſequently to the Centrifugal Force of the moving Body carried in the Circumference GLF, as B*c* to A*c* or GH: wherefore the moving Bodies B and G, ſince their Centrifugal Forces are as the *Radii*, will have equal Times of Revolutions, [by Cor. Theor. 1.] But the Time of Deſcent through GF or D, is to the Time of Deſcent through $\frac{1}{4}$ D as D to $\frac{1}{2}$ D, [by Cor. Theor. 12. Lect. 11.] and the Time of Deſcent through $\frac{1}{4}$ D is to the Time of a Revolution in the Circumference GLG, as $\frac{1}{2}$ D to P. Wherefore, by Equality, the Time of Deſcent through D will be to the Time of a Revolution in the Circumference GLF, or to the Time of a Revolution of the Pendulum AB*c*D as D to P. The latter part of this Theorem is manifeſt from the Corollary of Theor. 6.

Cor. Hence ſince the Time of the perpendicular Fall is in a ſubduplicate Ratio of the Space paſſed over by a falling heavy Body, the Time of Deſcent from

from the Height of a Pendulum will be to the Time of an exceeding ſmall Revolution as $D \times \sqrt{\frac{1}{2}}$ to P.

THEOR. X.

IF *a moving Body is carried in a Circumference, and performs each Revolution in the Time wherein a Pendulum, having its Length equal to the Semi-diameter of that Circumference, ſhall finiſh a very ſmall Revolution in a Conical Motion, or twice an exceeding ſmall lateral Oſcillation; that moving Body will have a Centrifugal Force equal to its Gravity.*

BECAUSE the moving Bodies B, G, (by Hypotheſis) perform their Revolutions in equal Times, [ſee Fig. of the preceding Theor.] the Centrifugal Force of the moving Body B will be to the Centrifugal Force of the moving Body G, as BC to GH, or BC to AC; but it is as BC to AC, ſo the Centrifugal Force of the moving Body B to the Force of Gravity, [by Cor. Theor. 7.] wherefore [by 9 *El.* 5.] the Centrifugal Force of the moving Body G will be equal to the Force of Gravity. *Q. E. D.*

THEOR. XI.

THE *Times of the Revolutions of any Pendulum carried with a Conical Motion, will be equal to the Time of the perpendicular Fall from a Height equal to the String of the Pendulum; when the Angle of Inclination of the String to the Plane of the Horizon ſhall be nearly 2 degr. 54 min. But exactly, if the Sine of that Angle ſhall be to Radius as a Square inſcribed in a Circle to the Square of its Circumference.*

LET the Pendulum, whoſe String deſcribes the Conical Superficies CAD, be ſuch, as the Sine of the

the Angle ACE ſhall be to Radius (that is, AE to AC) as $\frac{1}{2}$ D^2 to P^2. Let alſo AFG be the Superficies of a Cone, which the String of the Pendulum deſcribes carried with an exceeding ſmall Motion, whoſe Height therefore is AB = AF = AC. The Time of a Revolution of a moving Body F, will be [by Theor. 8.] to the Time of a Revolution of the moving Body C, in a ſubduplicate Ratio of AB, or AC to AE: but it is as AC to AE, ſo (by Hypotheſis) P^2 to $\frac{1}{2}$ D^2; wherefore the Time of a Revolution of the moving Body F, will be to the Time of a Revolution of the moving Body C, in a ſubduplicate Ratio of P^2 to $\frac{1}{2}$ D^2, that is, in the Ratio of P to $D \times \sqrt{\frac{1}{2}}$. But it is as P to $D \times \sqrt{\frac{1}{2}}$, ſo [by Cor. Theor. 9.] the Time of the leaſt Revolution, that is, the Time of a Revolution of the Body F, to the perpendicular Fall from the Height of the Pendulum: wherefore the Time of the Revolution of the moving Body F, has the ſame Proportion to the Time of the Revolution of the moving Body C, as it has to the Time of the perpendicular Fall from a Height equal to the Length of the Pendulum; and con-

consequently [by 9 *El.* 4.] the Time of the Revolution of the moving Body C, will be equal to the Time of the perpendicular Fall from a Height equal to the Length of the Pendulum. *Q. E. D.*

But since P is to D nearly as 314 to 100, P^2 will be to $\frac{1}{2} D^2$ as *98596* to 5000. But AC is to AE, from the foregoing Demonstration, as P^2 to $\frac{1}{2}D^2$; wherefore *98596* is to 5000 as AC to AE, and as AC to AE, so is [by Trigonometry] the Sine of the Angle AEC, or Radius 100000 to the Sine of the Angle ACE: but *98596* is to 5000 as 100000 to 5070, which therefore is the Sine of the Angle ACE, to which nearly answer 2 Degrees 54 Minutes.

THEOR. XII.

If *two Pendulums equal in Weight, but having their Strings unequal, do revolve in a Conical Motion, and the Heights of the Cones are equal, the Forces wherewith they stretch their Strings, will be in the same Ratio, as is that of the Length of the Strings.*

This is manifest from Theor. 7. For the Force of Gravity in both the Cones, is to the Tension of the String, as the Height of the Cone to the Length of the String; and since the Height of the Cones is the same, it is evident that the Tensions of the Strings are proportionable to their Lengths. *Q. E. D.*

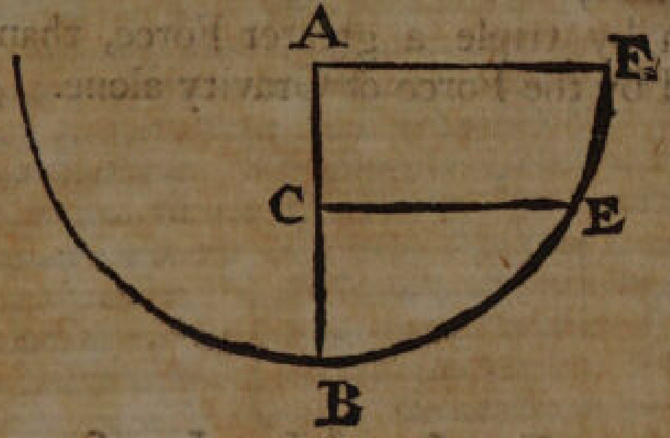
THEOR. XIII.

If *a simple Pendulum is moved with the greatest lateral Oscillation, that is, if it descends through the whole Quadrant of a Circle, when it shall arrive at the lowest Point of the Circumference, it will draw its*

String

String with triple a greater Force, than if it was only ſimply ſuſpended by it.

Let AB be a Pendulum moved through the Quadrant FB, biſect AB in C, through which draw CE perpendicular to AB, and meeting the Circumference in E. If the Pendulum deſcended



only through the Arch EB, it would acquire in the Point B the ſame Velocity, as if it had deſcended through CB = $\frac{1}{4}$ of the Diameter [by Corol. 1. Prop. 38. of the 15th Lect.] ſo that [by Theor. 5.] it will have in the Point B a Centrifugal Force equal to its Gravity: and conſequently the Gravity and the Centrifugal Force joined together, will draw the String with twice a greater Force, than it would be drawn by the Gravity alone. But if the Pendulum is raiſed to F, after its Deſcent to B, it will acquire the ſame Velocity, as if it had fallen through AB. But AB is to BC in a duplicate Ratio of the Velocity acquired in the Deſcent through AB, to the Velocity acquired in the Deſcent through BC; wherefore alſo AB will be to BC [by Theor. 3.] as the Centrifugal Force of the moving Body in the Point B after the Deſcent through FB, to the Centrifugal Force in the Point B, after the Deſcent only through EB. So that the Centrifugal Force of the moving Body after the Descent through F B, will be double the Centrifugal Force after the Descent through E B; that is, the Centrifugal Force in the Point B, after the Fall through F B, will be double the Force of Gravity. Wherefore the String, by the Centrifugal Force and the Force of Gravity acting together, and according to the same Direction, is drawn by triple a greater Force, than if it was stretched by the Force of Gravity alone. *Q. E. D.*

F I N I S.

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