

A compleat treatise on perspective, in theory and practice; on the true principles of Dr. Brook Taylor. Made clear, in theory, by various moveable schemes, and diagrams; and reduced to practice ... Containing diagrams, views, and original designs, in architecture, &c.; ... All originals; / invented, delineated, and, great part, engraved by the author, Thomas Malton.

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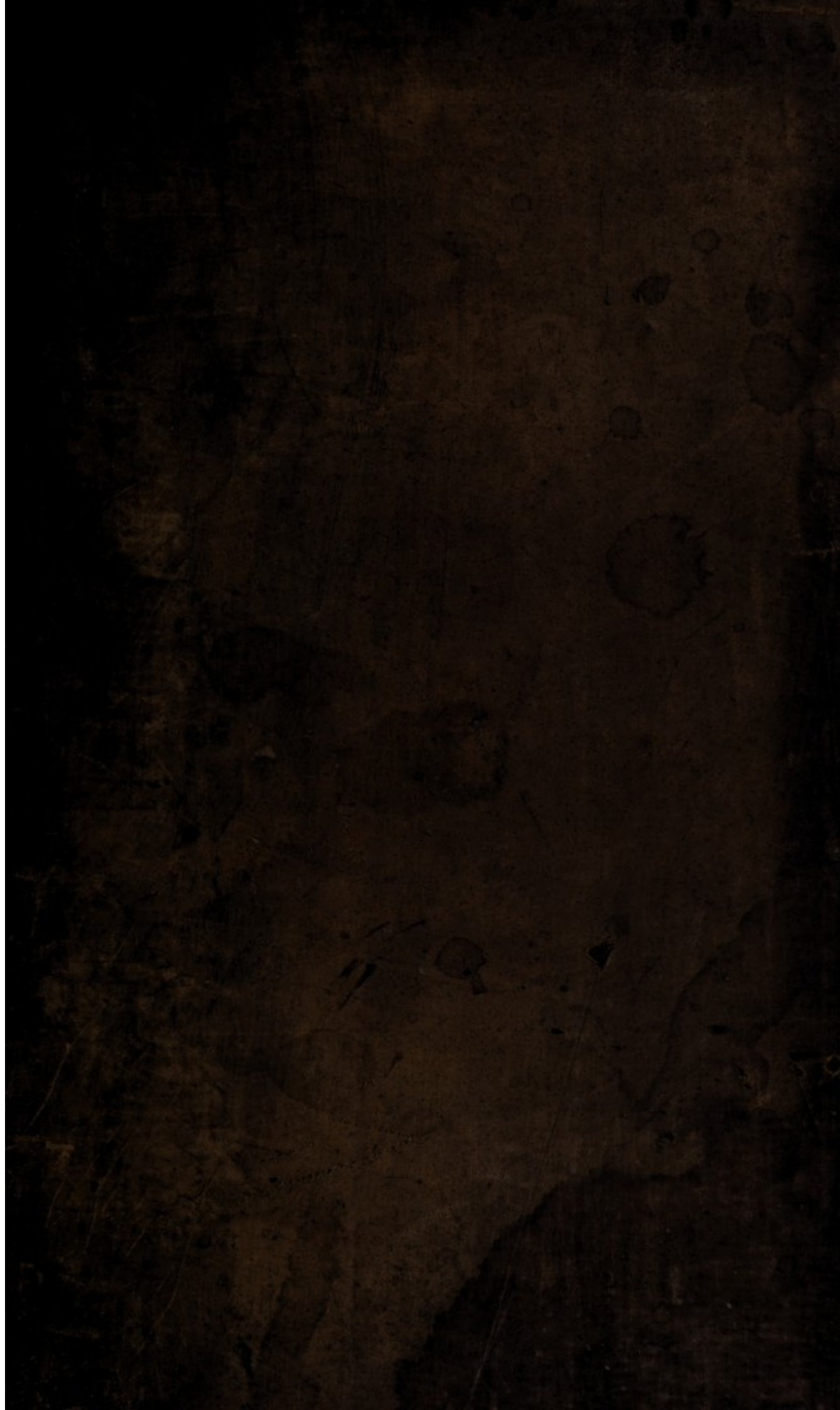
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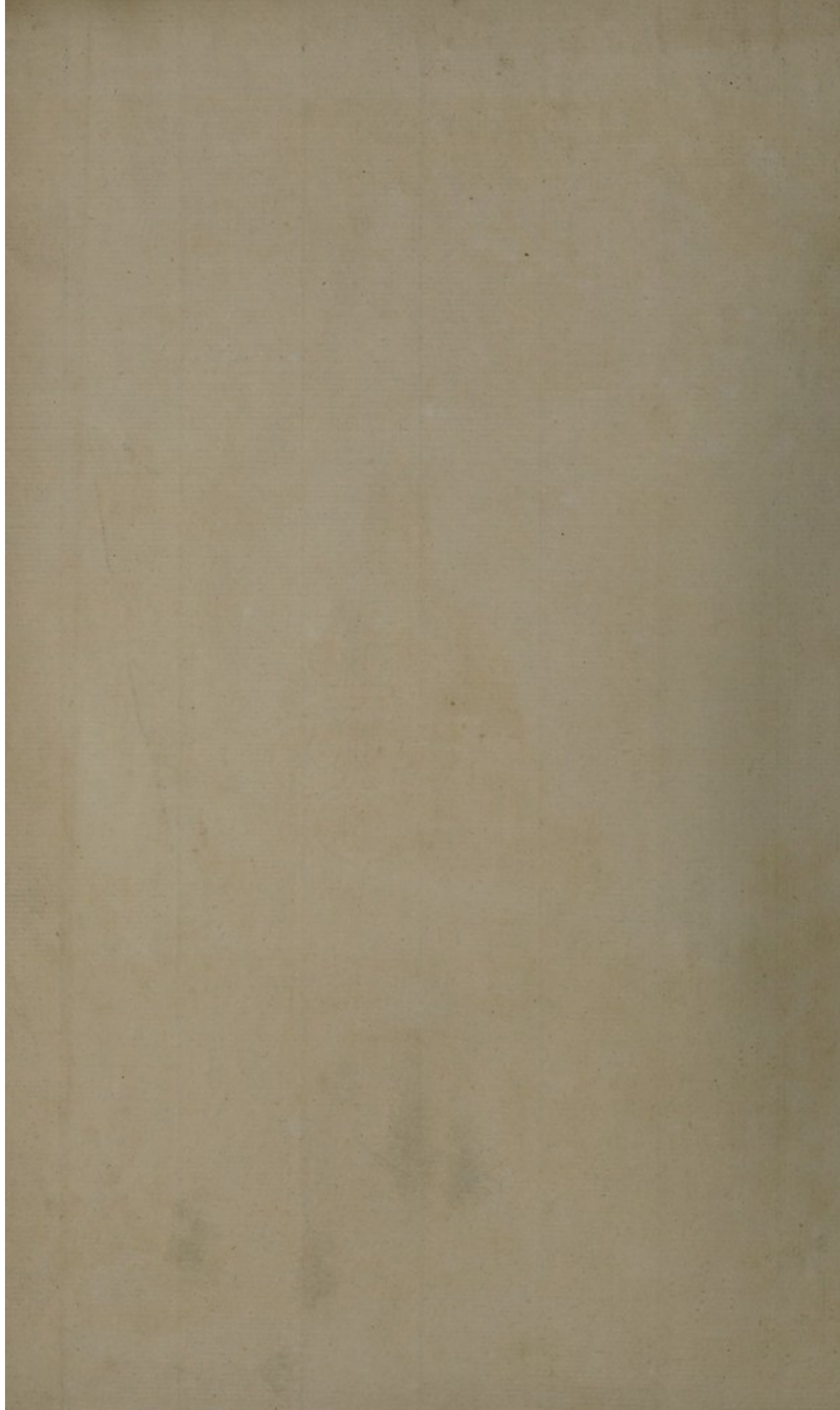
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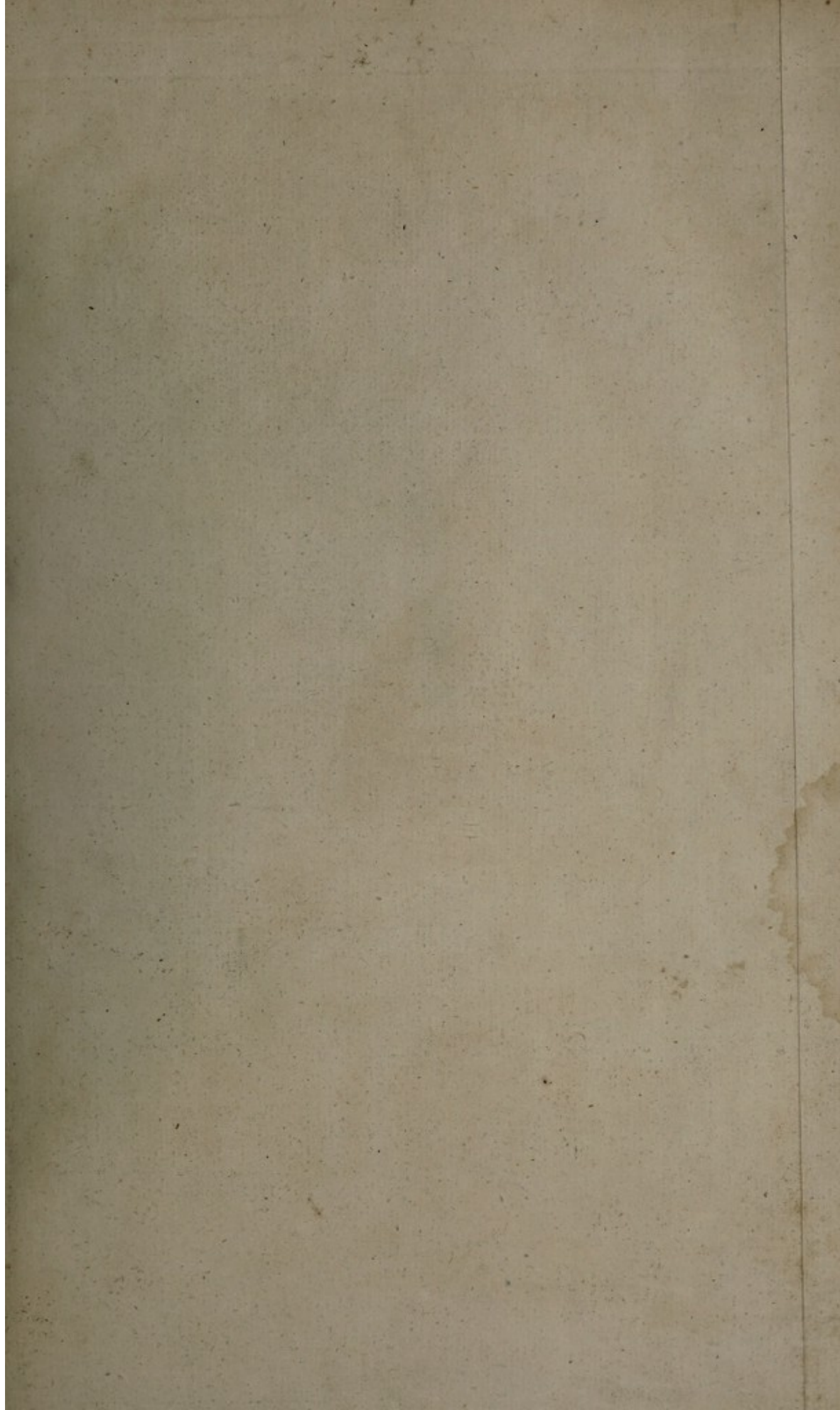
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Mellie's
Life

A
COMPLEAT TREATISE
ON
PERSPECTIVE,
IN

THEORY AND PRACTICE;
ON THE TRUE PRINCIPLES OF
DR. BROOK TAYLOR.

MADE CLEAR, IN THEORY, BY
VARIOUS MOVEABLE SCHEMES, AND DIAGRAMS;
AND
REDUCED TO PRACTICE,
IN THE
MOST FAMILIAR AND INTELLIGENT MANNER;

SHEWING
HOW TO DELINEATE ALL KINDS OF REGULAR OBJECTS, BY RULE.
THE THEORY AND PROJECTION OF SHADOWS, BY SUN-SHINE, AND BY CANDLE-LIGHT.
THE EFFECTS OF REFLECTED LIGHT, ON OBJECTS;
THEIR REFLECTED IMAGES, ON THE SURFACE OF WATER,
AND ON POLISHED, PLANE SURFACES,

IN ALL POSITIONS.
THE WHOLE EXPLICITLY TREATED;
AND ILLUSTRATED, IN A GREAT VARIETY OF FAMILIAR EXAMPLES;
IN FOUR BOOKS.

EMBELLISHED WITH
AN ELEGANT FRONTISPIECE, AND FORTY-EIGHT PLATES.
CONTAINING
DIAGRAMS, VIEWS, AND ORIGINAL DESIGNS, IN ARCHITECTURE, &c. NEATLY ENGRAVED.
ALL ORIGINALS;
INVENTED, DELINEATED, AND, GREAT PART, ENGRAVED BY THE AUTHOR,

T H O M A S M A L T O N.

THE SECOND EDITION, CORRECTED AND IMPROVED; WITH LARGE ADDITIONS.

L O N D O N:

Printed for the Author; and sold by Messrs. ROBSON, in Bond-street; BECKET, Adelphi, Strand; TAYLOR,
near Great Turn-stile, Holborn; DILLY, in the Poultry; and by the AUTHOR, No. 56,
Poland-street, Oxford Road, near the Pantheon.

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T O T H E
P R E S I D E N T A N D M E M B E R S,
O F T H E
R O Y A L A C A D E M Y,
For PAINTING, SCULPTURE, and ARCHITECTURE;
INSTITUTED AT LONDON,
By, and under the Auspices of, his most gracious MAJESTY
G E O R G E T H E T H I R D. .

Gentlemen,

IF the unremitted labour and assiduity, with which I have prosecuted the study of the Science and Art of Perspective, have thrown any new Light on that most necessary branch of the Polite Arts, so, as to render its Principles clearer and better understood, in Theory, more easily applicable, in Practice, more generally useful and subservient to the Arts, of which, it must be allowed to be the foundation, 'tis what I have chiefly aimed at; and presume, this Work may not be wholly undeserving your Patronage and Encouragement, although it may not merit your entire Approbation. Such as it is, Gentlemen, I submit to your Candour, and claim your Protection.

The propriety of dedicating such a Work to Gentlemen, who are, undoubtedly, the most competent Judges of it, will plead an excuse for my presumption; if, on an accurate examination of its Contents, it be found, that I have rendered an apparently intricate Science more familiar, and better adapted to the capacities of young Students, in so essential a part of their Studies; the neglect of which, amongst the rising Artists, is much to be lamented. Perspective seems to be looked on as an Appendage, only, which may be dispensed with, instead of the first requisite; in which, the Student, who would be a Candidate for Fame, should be well grounded.

It

DEDICATION.

It is with reluctance I add, but it is too obvious, that, in many fine Pieces, in respect of Design, Composition, Drawing, or Colouring, there seems to be a want of a just Idea of PERSPECTIVE.

Pardon me, Gentlemen, I do not mean to depreciate the Merits of such experienced Artists, in their several Performances; for, in my opinion, there only wants a thorough knowledge of Perspective, to render the present Age as famous as any former Æra; which, now, is established on the most permanent and infallible Principles; whereby, the trouble of projecting Objects, perspectively, is, in the process, greatly abridged and facilitated.

I am,

Gentlemen,

with great respect,

Your most obedient,

and obliged humble Servant,

THOMAS MALTON.

P R E F A C E.

IN every Age, and in every Climate, where the Polite Arts are cultivated and encouraged, Emulation, and a desire of Fame, inspire the Breasts of the rising Generations; the ingenious Mechanic catches the ardent and laudable Flame; Commerce is extended, the People are necessarily enriched, and the State becomes potent and formidable. Architecture rears her stately Domes and lofty Turrets to the Skies; the sumptuous Edifices, raised by the hand of the able Mechanic, and embellished by the ingenious Sculptor, with breathing Statues and almost animated Marble, proclaim abroad the Magnificence of the Founder, and justly immortalize the skilful Architect; whose Works may vie with the most celebrated relicks of Antiquity, which seem to be of more than human Composition. Whilst Painting, their Sister Art, assists and unites in decorating the interior Domes, Cielings, &c. or, in well-chosen Pieces and Subjects, and masterly Performances, adorns the Stair-case, the Gallery, the Drawing-room, and the Cabinet.

Perspective is allowed by all, who are well acquainted with it, to be the Basis of all the Polite Arts which have their foundation in Drawing; particularly Painting, or Delineating; for, Colouring does not come within its Rules. The Sculptor and Architect may receive great assistance from Perspective. I am sensible it is not absolutely necessary to the Art of Designing, but it is essentially so to see the effect of the Design. An Architect may, doubtless, be famous in his Art without Perspective, but more so if he had that accomplishment; as he would be better able to judge of the effect it would produce, and convey a just Idea of it to others, before the Design is executed. Drawings, in Perspective, from particular Stations, where the Building will be most conspicuous, would answer in that respect, the same purpose as a Model, and at a much less expence.

At a Time when the Arts are in such a Degree of perfection, a Treatise on Perspective may be thought an unnecessary Publication; seeing there are, already extant, many valuable Books on the Subject, containing all the Rules, for practice, which are either necessary, or useful; and consequently, unless some new Principles are proposed, by which, the trouble of projecting Objects, perspectiveally, may be lessened, it is useless to increase the number. For some, with truth, alledge, that, unless Perspective be comprized in a small compass, it will never be studied by those, for whose use it is chiefly intended. On Perspective, as on other Subjects, there are, indeed, a sufficient number of Authors; and yet, I may venture to affirm, that, no Subject, whatever, has been worse handled, in general.

I do not pretend to have found out new Principles, nor do I think, there can or need be any other; those given, by Dr. Brook Taylor, being sufficient for any purpose, whatever; and that, the Principles, on which he has founded his System, are the most simple and perfect that can possibly be conceived. Notwithstanding what many, who have not a true Idea of Perspective, imagine, that there is imperfection in it; that the Rules, prescribed, do not always produce a true or pleasing Representation of Nature; I maintain, that all, which can be done by Rule, is performed on the most perfect and infallible Principles that can be devised; that Perspective is, absolutely, at its *ne plus ultra*; where then, it may reasonably be asked, is the necessity for, or use of this Treatise?

There are, generally, two Motives, which induce every Person to publish to the World, his Inventions or Improvements, or Studies of any kind. The first is Vanity; or, to give it a milder Appellation, a laudable thirst of Fame; (I believe, I shall not err wide if I affirm, that, Vanity is the first, and chief spring of all that is great and laudable) the other is the lucrative Emolument, if not always the Consequence, at least, it may reasonably be expected an Attendant on Fame; it is scarce determinable, whether Vanity or Interest stimulates most to action. To pretend that we are not actuated by either of these Motives is the height of folly; for though our Station, in Life, may set us above the sordid views of Gain, yet no Station, whatever, is exempt from Vanity; which, most wisely, for the best of purposes (the good of the whole) is made a necessary ingredient in our Composition.

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Now, although I have not the least pretence to the invention of new Principles, yet I am firmly persuaded, that I have made use of those we have to the best advantage; that, from the irregular and imperfect Order, as they are given by Dr. Brook Taylor, I have digested it into an useful and practical System; not involved in a labyrinth of mathematical Demonstration, of things which are to little purpose in the Art of Delineating; as the voluminous Work of Mr. Hamilton, published in the Year 1738; which, though a most elaborate and valuable Production, has not been of the least use, to the Arts it was intended to promote. In the mathematical part, I have not entered further than is really necessary, to evince and enforce the Principles, on which the Practice is founded; and, in Practice, I have shewn its immediate and absolute dependance on the Theory. I have by means of an Apparatus, contrived for the purpose, united the Theory and Practice so together, that he must have very little penetration, and a shallow Capacity, who does not in a short Time, and with little Study, conceive a clear and comprehensive Idea of the rationale of Perspective. I have, every where, made such remarks, and thrown such light on the Subject, as, I flatter myself, will make it justly deserve the Title I have given it: my Design, throughout, having been to make the study and practice of Perspective, at the same time, easy and entertaining. The Subjects I have chosen to display, and embellish it with, are such as are common and familiar; for, to what purpose is an intricate and puzzling delineation of the Dodecahedron, &c. except, to shew that the Author understood it? I am of opinion, that the Reader will be much better pleased, with the description of such Objects as are frequently before his Eyes, and such as are fit Subjects to introduce into a Picture; or to form a Picture on. I have not been satisfied with the common Lessons for plane Figures, and Solids composed of or bounded by Planes, and left the Student to decorate his Buildings with the embellishments of Columns, Entablatures, &c. of the several Orders; but I have, minutely, shewn, how to delineate them, singly, and to compose a regular Building of the several Materials, described and treated separately of before.

There are, at this time, but few Gentlemen who are not pleased with a fine Picture, which truly represents Nature, on Canvas; the Deception being so well managed, that, we can almost imagine we see the real Objects, themselves, through the aperture of the Frame. For, it is possible, in perspective Delineations, in which there is a true gradation of Light and Shade, and judicious Colouring (on Cielings, &c. which we cannot come near) to deceive the Eye and Judgment, so, as to imagine, what we see are real, and solid Existences.

Although Perspective is not considered as a necessary part of the Education of a Gentleman, it must be looked on as a genteel and polite Accomplishment, a Qualification for a Prince. For, has not our most gracious Monarch cultivated and particularly encouraged it? nay, descended to learn its Rules, and to delineate, with his own Hand, pieces of Architecture? Is it possible, for a Gentleman to see, as he ought, much less to judge of a Picture, with true taste, and with the discernment of a Connoisseur, without having some notion of Perspective? it expands the Ideas, and makes us see Objects as they really appear, to a judicious Observer; it makes a Person a judge of the disposition, and proportion of the several parts of a Picture, to each other; the Symetry and Harmony of the whole are perceived; if there be any discordance, or unseemly distortion of the Parts, owing to an injudicious choice of the Artist, either in the Distance or Situation of the Object, or in the Position of the Picture, they will be quickly seen; especially, when there is introduced regular Architecture, in Buildings of any kind.

The manner, in which Perspective has hitherto been treated, scientifically, is too rigidly and mathematically so, to be entertaining, to a Person not conversant with the Elements of Euclid; which, though a branch of University Education, has been treated, in those seminaries of Learning, in a manner, which deters numbers from the study of that most useful and necessary Science, the foundation of all mathematical researches. Every Science, and necessary Art, should be so treated as to allure a Person to the pursuit; by making him (in a familiar way) relish

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relish a study, for which he has not a natural propensity. Many Gentlemen imagine, if they neither paint nor draw, that, Perspective is wholly useless, and unnecessary to them; they are greatly mistaken who think so, if they desire to be accomplished, and would bestow a little time and pains to be so.

Of all the mathematical Sciences, the study of Perspective is perhaps the most entertaining; the pleasure and satisfaction which result from it, in delineating, can only be felt, not described. Astronomy (to which Perspective greatly conduces) is the most sublime of all, and next to it I should rank Perspective. The entertainment, resulting from the former, is of a higher kind; we are, as it were, lifted from the Earth, in contemplation of the superior Works of the great Creator, in such wise, as to look on all below not worthy of our notice. The advantages to Society, which are deduced from this most exalted Science, are many and great; and, being well known, would be impertinent to enumerate. On the other hand, the study of Perspective, scientifically, is highly entertaining to a rational Mind; to those who would pursue it, I recommend the Works of Mr. Hamilton. I have not chosen to go further into it, than is really useful; and yet, to some Persons, I shall be thought prolix enough; who want to know it, without the labour of Study; to which, Artists, in general, have great antipathy; I mean, to the study of any thing mathematical.

It is usual, with most writers on Perspective, to introduce so much of Geometry into their Work, as they judge necessary for the knowledge of Perspective. There would be, in my opinion, as much propriety in prefacing every Book, on Literature, with Grammar; lest the Reader should not be acquainted with Syntax, and the Idiom of the Language. I fairly own it above my skill, to draw a line between the necessary and unnecessary, for all is more or less so. Some Theorems, in Geometry, are only necessary for the attaining of others; which, if they cannot be obtained without them, are necessary. In short, if the Student be quite unacquainted with Geometry, he is very unfit to study Perspective; let him first become intimate with Euclid, and then he may safely pursue Perspective; the practical part of one is sufficient for the other. Wherefore, seeing I have compiled a System of Geometry, to which this Work is subservient, I have wholly omitted the Definitions of geometrical Terms, here used; because, it is reasonable to suppose the Student already acquainted with them; otherwise, he must first study that Grammar, of the Science of Perspective.

Perspective being a branch of the Science of Optics (as it is founded on direct Vision) it becomes necessary to consider, in the first place, the structure of the human Eye, and the nature of Vision; to conceive (as far as our Intellects can trace) an Idea, how that extraordinary and astonishing Sense is performed; which being, in some degree, understood, we shall be well prepared for a clear understanding of Perspective, and enabled to distinguish between the Representation of an Object on a Plane, and its real Appearance; a circumstance, which is a great stumbling-block to many, who have not rightly considered the difference.

I have prefaced this Work with a Section on Light and Colour; and as a more essential requisite, to Perspective, the second contains a brief description of the Eye, and of Vision. In the next is contained the foundation of Perspective; it treats on direct Vision, comprised in two Theorems, of universal application; which are the very Essence of Perspective. The two remaining Sections, of the first Book, are wholly digressive; they not being at all necessary, to the understanding of what is contained in the following Work; the Reader may therefore pass them over, if he be so disposed, without breaking the thread of the Subject. They contain matter of mere Opinion; respecting the materiality of Light, Reflection, Refraction, &c.

Book the second contains the whole useful Theory of Perspective, rectilinear and curvilinear; which is somewhat copious, by reason of the Examples given for illustration; and Corollaries, deducible from the Theorems.

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The first Section is a general Introduction, containing many preliminaries, necessary to be known, previous to what follows.

The second is also introductory, and contains a full explanation of all the various kinds of Projection, ichnographic, orthographic and stereographic; with a circumstantial and comprehensive Definition of Perspective, and other introductory matters.

The third is more elementary; it contains full, yet brief Definitions of all the Terms made use of in the Theory.

The fourth Section contains the whole Theory of right-lined Perspective, in fourteen Theorems; from which are deduced several useful and practical Lessons, in Corollaries, Scholia, &c.

In the first seven Theorems, is contained all the necessary knowledge of Perspective, relative to Intersections and Vanishing Lines, in general; the first and most essential requisite in Practice.

The eighth Theorem, though self-evident, is intended as a refutation of an absurd opinion which many entertain, that continued Right Lines, seen direct, cannot be represented by right Lines; seeing that, parallel right Lines always appear to incline towards each other. The ninth teaches all that relates to Planes and Lines parallel to the Picture. The tenth shews where the Intersecting and Vanishing Points of all other Lines are to be found; and the eleventh determines how to find them. The twelfth and thirteenth contain all that relates to Lines not parallel to the Picture, in the most concise, yet full and clear manner that I can conceive. Perhaps the Demonstrations of the last may deter those, who are not Geometricians, from examining it with that attention it requires; let such remember, that, in order to practise Perspective, it is not absolutely necessary to be a Geometrician; because, I practised it long before I understood Geometry; excepting a little practical, and, a superficial knowledge of Lines, in general; which are certainly requisite; as the whole of practical Perspective consists in it.

For which end I have selected, all that is necessary, not only for Perspective, but, also for various mechanical uses, into one Book (the first part of the Royal Road to Geometry) lately published; perhaps, in the most easy and intelligent manner, ever yet done; to which I refer the Reader, for the geometrical Construction of all kinds of Figures here used; as well as to the Elements, for Demonstration, in the Theory.

If those Readers, who have neither time nor inclination to become acquainted with the Elements of Geometry, at least acquire so much, as that Tract contains; I will vouch for the great advantage he will find to his study of Perspective. Indeed it is impossible to understand or practise Perspective without it; for, being in itself, wholly geometrical, the very Language, and Grammar, of Perspective is Geometry; let them take my word, for once, they will not find their time mispent; but, on the contrary, so much knowledge, as it teaches, will be purchased at a very easy rate.

They may, then, boldly venture on the Theory; omitting the Demonstration where they find they cannot comprehend it; which, in general, they will not find enveloped in mystery; having treated that part in the most familiar manner possible, by plain reasoning only, without other reference than to the Elements, for proof of what is advanced. Let them, at least, by the most attentive perusal of the Theorem, endeavour, to conceive the Premises of it; which, in some of them, perhaps, may not be perceived, clearly, at the first reading; the second will open some Ideas, and the third perhaps compleat it; which, if it require a fourth, sometimes, will not be lost labour; for, when the Premises of a Theorem are clearly understood, the proof of it is not difficult, in many cases. However, whether the proof of what is advanced be perceived or not, it is not essential to the Practice of Perspective; the Reader may give me credit for it, I will be responsible for the truth of it; he may be fully satisfied that the thing is certainly so, and make use of that knowledge accordingly.

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The Theorem being understood, let him, at least, go through one Example in each, as he proceeds; his Ideas will not only be strengthened, but he will, possibly, see the truth of the Premises. Then, the Corollaries, Scholia, &c. are but so many easy, useful, and practical Lessons, deduced from the Theorems, which will be found of the greatest use in Practice; and, if the Theorem be clearly conceived, the Corollaries will be so too.

The fourteenth Theorem has no affinity with, or dependance on the foregoing; it contains the whole Theory of practising without Vanishing Lines; an ingenious method, but seldom practised; nor, indeed, is it so generally applicable to practice.

The fifth Section contains so much of the Theory of curvilinear Perspective, as is really useful in Practice; or necessary to be known, by any Artist, whatever.

The sixth is a refutation of several capital Errors and absurd Opinions, which many Persons entertain of Perspective; and which are, there, clearly and fairly stated, and shewn to have no real existence.

The third Book is a copious Treatise on useful practical Perspective, containing Lessons and Examples in every necessary Case, that can occur in Practice; shewing also its dependance on the Theory, in brief Demonstrations, where it is necessary.

The Method, in which I have treated this useful part of Perspective, is such, as I presume will sufficiently recommend it. The Book is divided into twelve Sections, each treating on a distinct subject from the foregoing; beginning with the most simple and first Principles or Elements, and going on, from the most simple, to the most complex Objects, in a regular, progressive succession.

After the necessary Preliminaries and Elements, in the three first Sections; the third shewing also how to find Vanishing Lines, and Vanishing Points, in general, the first requisite in Practice; in the fourth is contained all the practical elementary Problems in Brook Taylor's Essay, which are, of themselves, a complete System of practical Perspective. These Elements, of the following Work, being well understood, will be found of great Utility; which has induced me to perfect such as the Author of them had left very imperfect; and some, particularly his 21st and 22d Figures, are greatly defective; we are nevertheless infinitely obliged to him, for giving them to the World, such as they are.

To understand this Section clearly, and be enabled to apply the Problems it contains, in all Cases, practically (which, it must be observed, is the foundation of the whole) a tolerable share of geometrical knowledge is requisite; without which, it is impossible to turn them to that general use, in Practice, in which they are so universally applicable; inasmuch that, it may be truly said, he who can comprehend them, perfectly, and apply them in all Cases, generally, understands Perspective, thoroughly; but, being superficially looked over and not well digested, they will be found of little use.

This consideration, perhaps, has induced some Persons to imagine, and affirm, that the whole of Perspective may be comprised in a very small Compass. But, I must make free to tell them, they are under a mistake; for, notwithstanding the Principles are few and general, yet, to know how to apply the Rules, in all Cases that may occur in Practice, cannot be so soon acquired; if so, how comes it, that they are no greater Proficients? Or, can they imagine, that, amongst the number of able Men, who have wrote on the Subject, none should hit on that much wished-for Expedient? Either they must acknowledge that their own faculties are not so acute as others (which is paying a sorry Compliment to their Understandings) or they must suppose that there is more in it than they at first imagined. 'Tis true, the whole of Practical Perspective is comprised in this and the foregoing Sections; yet, how few would be able to apply those Problems, though clearly understood, in delineating a regular piece of Architecture, is but too obvious to enlarge on; without previous, practical Lessons, diversified variously, to familiarize them. For which reason, it is necessary to shew their application, in various Examples, which I have not been

sparing of. I have not, as others have done, studied the most easy positions of Objects; but I have endeavoured to give the most pleasing, picturesque, and natural Representations, and render the most difficult Positions easy.

Some Persons know a great deal (in their own opinion) because they know, perhaps, that all Right Lines, which are parallel, have the same Vanishing Point; who yet, know not how to fix and ascertain any one, except the Center of the Picture, and that often very absurdly (indeed they seldom, if ever, use any other) nor how to proportion a Line perspectively; and further, to shew their consummate knowledge in it, if you shew them a piece of Perspective, in which there is not one Line tending to the Center, or Point of View, they will call that Point on the Picture, in which the principal Lines converge, the Point of View; so that, there may, by this criterion, be several Points of View in the same Picture; a circumstance too palpably absurd to need a refutation.

If this be to know Perspective, it may indeed be very soon acquired; it may be taught to a Child. But, when they have drawn a Line to its proper Vanishing Point, how that indefinite Line is to be proportioned, in order to represent, certain finite parts of the Original, they are wholly ignorant, though the chief requisite in Practice, and the principal business of the fourth Section of the third Book.

However, not to discourage them, from attempting to acquire so necessary a part of their Studies, they may be assured that a fixed resolution and perseverance, will soon surmount every apparent difficulty. Some are alarmed, and even frightened, at the sight of so many Lines in a Diagram, for a Lesson in Perspective, imagining they can never comprehend them all. Certainly, they must, to a person wholly ignorant of the use and meaning of them all, appear a general confusion; but when they have set about it, with a resolution to know, and have well digested the Definitions of them, they will, at one glance, comprehend the use of several; and the rest, by analyzing them regularly, will soon become familiar to the Eye, and to the Understanding.

This, I believe, has induced several Persons, who have wrote on Perspective, to deal very unfairly in it, in not giving all the necessary Lines in their Diagrams. I have, in mine, given all, and more than are necessary to be drawn, at once; for let it be observed, that the Radials of Lines, for determining Vanishing Points, seldom need be drawn; and several of the operative Lines, or Visual Rays, which are only used for cutting others, need not be drawn, at all; a Ruler being applied to the two Points (the Eye and the original Point) the indefinite Line may be cut, without drawing the whole Line, which could not answer, in a Diagram, so well as drawing the whole, in order to shew the direction of the Line; besides, in Practice, one part is effected and the Lines rubbed out before others are drawn, which cannot be done in a Diagram, all must remain together.

Others, again, there are, who have almost determined to make an effort, only want to know so much of the Practice as to be able to project a plain Building, &c. and seem quite afraid of attempting the Theory; because there is occasion for Geometry, which, to many, is a terrible Bugbear; so that, rather than exert themselves, they do without it, though the first essential, and chief requisite perhaps of their Profession. They want to know Perspective, but are afraid to venture on the Theory. They deceive themselves if they imagine it an unnecessary Appendage; can it be unnecessary to know how, and why, before we attempt any thing, practically? is not one, the means of effecting the other? I do aver, that the shortest way to acquire Perspective is to understand it first, in Theory; the Practice of it will readily follow.

How different are the dispositions of mankind. After I had (from the Jesuits) acquired (as many would suppose) a proficiency in Practical Perspective, I was far from being satisfied with all that could be obtained from that Author; and although I was not a Geometrician, at that time, and he not giving any Theory, I set about contriving means to have conviction of the truth of his Rules; by which means, I

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fell insensibly into the method used by Vignola and Sirigatti; than which, nothing can be more convictive; and I was accordingly satisfied, that the Rules, given, might be depended on.

But, having acquired such a proficiency in Geometry, as to enable me to comprehend Brook Taylor's Principles, how trifling, how limited appeared all I had learned of it before; I was even ashamed of my own productions, some of which had, undeservedly, been admired: I found, indeed, I scarce knew what was meant by Perspective, till then. The greater progress I made in Geometry, the clearer, and more comprehensive Idea I had of his Principles; which, though founded on common observation of Objects (supported by Geometry) are beyond every thing ever thought on, by any who had wrote on the Subject; and yet, it is so simple, when understood, that 'tis strange it should not have been brought to light sooner.

Having, in the fourth Section of the third Book, displayed the whole Elements of Practical Perspective, the fifth begins the application of them to real use; the whole of which, consists in finding the representation of a Line or Point, in any Plane, any how situated: for, Points are the extremes of Lines, and Lines of Surfaces; and, Solids, or Objects, are bounded by their Surfaces. A Plane, however situated, it is manifest, is the same, in all its properties; its situation, in respect to the Horizon, is, therefore, of no consideration, in Perspective; and, to delineate Plane Figures is all that can be done by Rule, with absolute certainty; the apparent Contours of some round Objects are somewhat difficult to determine, yet they may be done sufficiently correct. In this Section, the Construction of all kinds of plane Figures, perspective, in Planes any how situated, is comprised; and, in the sixth is shewn how to construct Solids, of Planes only; by which, any Plane Building may be projected.

The seventh Section teaches, fully, how to represent Mouldings in general, and to break the same at the Angles of a Building, &c. internal or external, right-angled or otherwise, with other necessary, and decorative, parts of Architecture, &c. as Triglyphs, Consoles, Modillions, &c. also, how to form a Pediment.

The eighth is wholly adapted to circular and round Objects of all kinds; as Arches, Columns, Tuscan and Doric Base and Capital, round Steps, Wheels, Vases, and circular Mouldings; including also the Ionic and Corinthian Capital.

Being now furnished with all the materials of a Building, the ninth Section shews how to compound them, and to form a Building; from the most plain and simple to the most elegant and rich, decorated with the various Orders; also detached Buildings, Views, &c.

The tenth is for internal Subjects, as Rooms, inside views of Churches, Arcades, Staircases, and Ceiling-pieces, representing Domes, and Cupolas; which, though somewhat particular, is founded on the same invariable Principles.

The eleventh is adapted for household Furniture; as Tables, Chairs, Cabinets, Bookcases, Beds, &c. also, for Machines, Coaches, &c.

The twelfth treats of inclined Pictures and Planes, in general, containing Lessons in the most difficult cases, from known data; in which, the excellency of the new Principles is exemplified.

Book the fourth treats on the Perspective of Shadows, which is indeed a copious Subject, and much more might be said of it; but, as I am sensible, that very few take the necessary pains to project their Shadows by Rule (general Effects being all that is studied, or regarded) it would therefore be to little purpose to give Rules, which will seldom, if ever, be followed. Nor is it at all necessary, or even practical, to project every Shadow, mathematically; it would be attended with great loss of time, and perhaps, in some Cases, produce a bad effect, though truly projected. In this branch of Perspective, licence may therefore be taken, justifiably, provided they do not run into gross and palpable absurdities; such as projecting the Shadows of Objects, already immersed in Shade; or, as I have seen in the works of eminent Masters, Shadows cast both ways, in the same Picture, to the right, and to the left.

I have

P R E F A C E.

I have also seen, the Shadow of a curve Line represented by a Right Line, when the Luminary was not nearly in the Plane of the Curve; which, with others, less obvious, ought carefully to be avoided, and guarded against.

Nevertheless, there are certain general Rules may be given, which ought strictly to be adhered to; and never, on any account, departed from. I have, therefore, in this Work, given so much as I conceive necessary to be known; for, in order to be conversant in Shadows, it is absolutely necessary to be acquainted with the invariable Law of Nature, in the projection of Shadows; although it may not always be necessary to follow her dictates, implicitly.

The effect of reflected Light on Objects, from other Bodies, in vicinity with them, is likewise treated of in this Book; also the effect of Distance, usually understood by the term Keeping, properly, aerial Perspective; both which, contribute greatly to the perfection of a Picture; insomuch that, without due regard being had to both, a Picture, ever so well designed and delineated, or coloured, will be but a flat and spiritless performance.

And lastly, the reflected Images of Objects on the surface of still Water, or on polished Mirrours, in any Position, are, in the last Section of this Book, treated on.

Having, now, given a brief account of the Work, I shall just make one more observation, in respect of the notions of some Artists; who being always accustomed to sketch by sight, only, and having not the least conception of projecting Objects by any Rule, they vainly imagine that Perspective will teach them how to represent Objects, exactly as they appear to the Eye, without knowing any thing of their Proportions or Situations, in respect of each other, or of the Picture, which is impossible; so that, when they find there is a necessity for taking and applying their real Measures, or Proportions, being so foreign to what they conceived of it, they imagine so many obstacles to lie in the way, that, they soon desist from the attempt; choosing rather, to remain in ignorance, and proceed with uncertainty, than take the necessary pains to acquire, what, being acquired, they would deem invaluable.

He, therefore, who can hit on such an Expedient, as to teach them how to delineate Objects truly, by Rule, without applying any Rule at all, will be likely to meet with great encouragement from those indolent Artists. I acknowledge my incapacity for the task; and therefore, I shall abide by the Rules, as they are given in the following Work.

A T R U E

C A S E,

BETWEEN THE AUTHOR,

OF A LATE PUBLICATION,

His PRINTERS, and PAPER MERCHANTS.

TRUTH WILL EVER PREVAIL.

OF all the calamitous accidents, which, of late years, have happened in this Metropolis, none has attracted the attention of the Public, more than the Fire, at Messrs. Cox and Bigg's, Printers, in the Savoy, on the evening of March the 2d, 1776; on account of the many valuable literary Productions which were destroyed there; several of which were completed, and ready for Publication. Amongst the rest, was the Compleat Treatise on Perspective, in folio, publishing by Subscription, by Thomas Malton, and intended to be published in March. The following is a true Case between that Author and Messrs. Cox and Bigg; also, between him and Messrs. Wright and Gill, Paper-Merchants, in Abchurch-Lane, Lombard-street.

Those Gentlemen were also sufferers, by that Fire, to the amount of 140 Pounds, for Paper, credited Messrs. Cox and Bigg. Who being lately removed from the Strand, into the Savoy; and having taken in another Partner,* they had been making Conveniences, for carrying on a more extensive Business. In consequence of which, they had neglected to insure, till all was completed; their former Policy being forfeited, or of little value, now. They likewise intended to insure in another Office (the Union) every thing (if I am truly informed) was either ready to be executed or ordered to be ready. Perhaps the Money might not be ready; their neglect of so important and necessary an affair may, on that score, admit of some vindication, in their favour; nevertheless, it was an unpardonable negligence, having so much of others Property on the Premises, and in their possession.

Although the Author of the Treatise on Perspective was, by no means the greatest sufferer, by that accident; respecting the quantity or value of what he lost; yet, all circumstances considered, his Case was singularly hard; and, the treatment, he afterwards experienced, from them, and others, fellow-sufferers,

* A Mr. Nettleton; who, if I am not misinformed, was but nominally so, in trust for Mr. Wells, Mercer, on Ludgate-hill. Be that as it may, it is certain, Mr. Wells was the person who advanced the Money for Nettleton; and, there is not the least doubt, but that he was to be a sharer in the profits of the Business, as a joint Partner.

cruel and inhuman; such as no Person, who had not taken some pains to dissolve every social tie, nay, to divest himself of humanity, could have determined to carry into execution at the time; it seems as if they were determined to crush him, if possible, by their premeditated, and, in some measure, their united efforts. As the disinterested Public are often misled, in forming a judgement of such matters, as they are no way concerned in, I was induced to lay the real and true state of the Case before them, in the most candid and impartial manner.

This Treatise on Perspective was the produce of many years study, the result of my leisure hours, when I followed the profession of a Cabinet-maker, with little success; and which I quitted seven Years ago, encumbered with as many Children, since increased one. Let any one picture to himself, the situation of a Person, who had quitted a Business, not worth following, burthened with such a Family; having no other means to provide for their support, than the produce of his Studies, advanced almost to the meridian of Life.

I began printing the Work about three years ago; but, as all such Works are usually longer in hand than is consistent with the Author's convenience what between the Engravers, Printers, Paper-merchants, and the encumbrance of eight Children, I found myself, when in the middle of my work, not in the most eligible situation; the encouragement, at first, not being adequate to such an arduous and expensive Undertaking. Notwithstanding, I kept jogging on, with resolution and perseverance, though but slowly; for Engravers are a hungry sort of Gentlemen, who will not dig without eating, and well too; when, or before, they have done their Work, they immediately want their Money, and frequently much more than they deserve; many of them being as mere Mechanics as any whatever.

Being thus circumstanced, it is no wonder the Work went so slowly on; the Engravers work'd for me rather than be idle, expecting to be paid some time; the Printers had a very convenient jobb to set their Men on, when they had nothing else to do; and thus, we went gently on for some time, each laying the blame on the other. As time and perseverance will surmount great obstacles, the Royal Road to Geometry was first completed and published, in December 1774, which was well received by the Public. When both were finished, I desired to have them sent Home, altogether, to save the trouble and expence in sending often; but, as there are reasons for every thing, they alledged some plausible excuse, to let them abide in their Warehouse, and send for them as I wanted; which, judging the true reason to be security for Payment, I supinely acquiesced in.

When they moved from the Strand into the Savoy, they moved my Work, without giving me the least intimation of it, although I had requested them to send it Home long before; and surely, as it was to be moved, it was as easy to have sent it home to me, as to carry it farther from me. Not long after (for reasons best known to them) they refused to deliver any more to my Orders, repeated sundry times, to my great hindrance in delivering the Work to the Subscribers, and consequently to my loss (as it happened) in not delivering so many as I might have done, or in making them ready for sale; near a hundred Subscribers wanted Books when the Fire happened. At the same time, they required Security for payment of the remainder of the expence in Printing, Paper, &c. which they had supplied. *Query*, If a person retains my Property, against my Will, as security for payment, is he not accountable for it, if the Property be destroyed? At length, by continual duning, obtaining Notes or accepting Drafts, they were over-paid, before the Fire. *Query*, 2d. Whether a person has a lawful right to be paid for Work done, which was never delivered, although

although requested? and whether, the Property being destroyed, the Money paid should not be refunded, in proportion to the Property lost? There requires, I presume, no great sagacity, or deep knowledge in Law, or, at least, in Equity, to determine.

A few weeks before the Fire, Mr. Bigg, as usual, came to me, and desired me to give him a Note of hand, or two, for the balance of their Account, as they were in great want of Money, that day; having in his hand a slip of paper, on which he had taken (as he said) the sum total from the Book; I think, about twenty seven Pounds. Now, as I never yet had a regular account of the whole, but only for the book of Geometry completed, and part of the Perspective; and as there had been some dispute in respect of the Money paid, which is perfectly clear by the Receipts given; by which, and the accounts I had, I was fully satisfied that very little was due to them; and consequently, I objected to the settling an Account I had never seen; for which, every one who knows any thing of the World, must have condemned me for a Fool. But, he being very urgent and importunate, I agreed, merely to serve their present emergency, as I thought, to give Notes for twenty pounds fifteen shillings, provided they kept back another of ten pounds, due the week after, for two months longer; for which, I gave him another, then, to raise money, which was sent me to pay the first. Mr. Wells, at that time, being concerned with them, those Notes were deposited in his hands; and the first was paid, after the Fire, as it was properly due before; the other, I was fully determined not to pay, unless compelled by Law; as they were scarce legally obtained, and for which I never received value in any shape; but, at the time I gave them, I was then going on with an Appendix to the book of Perspective, which induced me to it; as I concluded, that before the Notes became due, I should be as much in their Accounts; nor, did I doubt their probity, when we should come to settle them.

Being thus situated with the Printers, I shall take my leave of them for a while, and introduce to the Readers acquaintance, two very worthy Gentlemen, Messrs. Wright and Gill; with whom I had contracted for Paper, to the amount of 85*l.* of which I paid them 32*l.* with interest, and gave two Notes for 28*l.* each, bearing Interest. One of these Notes was paid, with Interest, and fifteen Guineas of the other; so that there remained, due to them, twelve pounds and five shillings when the Fire happened.

It is much easier to conceive than describe an Author's situation; who, having gone through the fatigue and expence of such a laborious Work, and completed it, ready for publication, expecting to reap the harvest of his labours; when, lo! a fatal Bar to his hopes, deprives him, in one Hour, of that resource he had so long and so ardently panted after. No redress, no remedy but Patience; the labour, and expence of printing it must be repeated, before he sits down to enjoy the fruits of his past labours, in quiet; ungrateful task; Patience, surely, never had a more ample subject, for exerting her dominion and influence over the Mind.

Such was my situation, on the third day of March, 1776. It is not easy to paint the Prospect which at first presented itself; the Harvest of my labours destroyed, when just ready for the Sickle. But, calling Reason to my aid, I formed the plan of my future proceeding; and comforted myself with the reflection, that the Engravers, Printers, and Paper Merchants were so near paid; there remaining, to the last, the trifling sum of 12*l.* five shillings only, as above; which, considering that they were very eminent, that is, very rich Men, and great traders, I never imagined they would be urgent for such a trifle, to them, at such a time, being reputed to be worth 40 or 50 thousand Pounds, each. What
pitiful

pitiful Ideas must those have, who never felt the sublime pleasure of heaping up Money. There is not, perhaps, a pleasure the World affords that can be compared with it; so exquisitely sublime, that all inferior pleasures are wholly absorbed in it. Those gentlemen having lost 140 Pounds, by the aforesaid Fire, it was a terrible blow to their repose; but they, with others of the same cast, took care to secure, to themselves, the fragments of all that was left on the Estate, in Debts, &c. wholly excluding all such claimants as myself, from having any share in the spoil. Consequently, having lost my present dependance, it was judged a proper time to require payment, of my Debt to them, having stood long in their Books; which single circumstance is, to a lover of Money, a sufficient plea for levying it by distress; the solid argument, that the culprit is not in a situation to pay, with convenience to himself, is an argument that has no weight; the most solid and convincing is, you must and shall pay. Sound, counting-house, reasoning; such as, I make no doubt, every retailer or manufacturer of Goods, for which he has obtained credit of some hungry, rapacious, over-bearing, wholesale Dealer, has experienced one time or other.

In consequence of the above reasoning, I received a Letter from Wright and Gill, requesting immediate payment, as they were really in want of Money. I doubt not the truth of their wanting Money, for that they will to the end of their days. Such penurious Money-rakers make no distinction between the Wants of others, who want the necessaries of Life, and theirs; which is, merely to accumulate, and heap money on money. This Letter was duly answered, requesting them to add a little more to their common stock of Patience, on account of the misfortune which had happened, in which we were common sufferers; and that I would wait on them in a few Days. Accordingly I attended their Levee, when I saw both Wright and Gill: the following is the substance of what passed between us, on that Interview.

Well Mr. Malton (says Mr. Wright) I hope you are come to settle that account, now; it has stood a long time in our Books, and it is very disagreeable to us, to keep Accounts so long open. Yes Sir, says I, if you please we will settle it, but it is not convenient to pay you; as you know the loss I have sustained, you cannot imagine that I am in a situation to pay money yet, having lost the means I had of obtaining it. Why, we are informed that your loss is trifling, a mere shuffling pretence to delay payment. Sir, I scorn the appellation of a Shuffler; * my loss Sir, is greater than yours, which I can justify; but, let that be as it may, I have lost near half the Paper I had from you, with the additional expence of printing, annexed to it. Well, but I knew it had stood long, and ought to have been paid before the Fire happened; that he knew I had got a valuable Work, and that I might pay them if I would. What sound, logical reasoning; how futile must such arguments appear, to an impartial and disinterested person. That it should have been paid long ago was, now, but little to the purpose; it was not paid; and they knew I had lost that, with which I hoped to have paid them and others, ere now; and the very identical Property I had of them, instead of making Money of it; which, had I done, they would be justifiable in the step they took to obtain it. All the arguments I could make use of availed little or nothing; they had lost too much already, by Cox and Bigg, who, he (Mr. Wright) said, deserved to be hanged; and, I make no doubt, they were his real Sentiments. Good God! is it possible any human Being could so far disgrace human Nature? as to say a person deserved to be hanged; for a piece of negligence, in which, they had not lost,

* An opprobrious term, much used by wholesale Dealers; when an honest industrious Manufacturer, cannot make his Payments in due time.

on the whole, above a hundred pounds (who were reputed worth a hundred thousand) the other had lost their Whole, divested of all they had, in the World. And I am also of opinion, that if, in order to pay them the paltry sum of 12 or 13 pounds, I had sold my Copy-right for half, or a third part of its value, they would have highly applauded me, for my Honesty. I should look on such a person in the same light, as one who had rob'd me of fifty pounds value, when he knew he could not make ten shillings, to himself, by his villainy. No, had they never been paid, I know the value of the Work; I have grappled with every difficulty to preserve it my own Property, entire, and it must be some more dire Calamity, to me, that shall wrest it from me.

When all I could say availed nothing, I proposed, that, if they would deduct the Interest I had paid (as surely, in such a case, they could not be hurt if they got the whole Debt, without Interest) I would endeavour to pay them in a certain, short time, mentioned. I must allow them to think and judge for themselves, they said; and generously proposed, if I paid them in the time, to charge no Interest from that time; but they wondered how I could expect what I proposed, after so long credit. I then proposed to give them a Book for the Interest, and put them down as Subscribers; of which, to sell it, they might make two pounds, at least. No, the Book was useless to them. Wretches, who had subscribed ten Guineas, just before, to make a sound in print, or for some other sinister view, yet refuse to encourage a Work, for which they had furnished Paper, to the amount of 85 pounds; and the Subscription to be paid by the interest of the money; and, after I had lost, by fire, near half the Paper I had of them. What pity, that any, who either write or print, should contribute to enrich such selfish mercenaries, by consuming their Paper. I then told them, that they were the best judges what they had to do, and must pursue such measures as appeared to them the most eligible; I could not pay them yet. They, then, assured me they would not wait much longer; in which they spoke the truth; as all such like religiously observe their Word, in paying or in being paid; who frequently make their boast, that they are always punctual in their payments; by way of reproach, to those who are not in a situation to pay them. As if there was any merit in being punctual, to those who have it always in their power; or a reproach to others, who have not; provided they do not unnecessarily consume and embezzle their getings, and are industrious to get.

Being, as I thought, in perfect security, on their account, notwithstanding what had passed; thinking, as being Men, they could not, and as Tradesmen (for their credit) they durst not put their menaces into execution. But I reckoned without my Host; for, on Saturday the 8th of June, a gentleman Officer (one Mr. Armstrong, of Carey-street, Lincoln's-Inn) waited on me, from them, with a scrap of Parchment, and slyly told me I was his Prisoner. On whose account, Sir, says I? let me see your Warrant. I read its contents, desired him to walk up stairs with me, and I would settle the Account; having, fortunately, received as much that morning, which I had then in my pocket. What the consequence would have been, had I not been so lucky, I will not say; but, as it was, it supplied me with courage to support it, with becoming fortitude.

Those who never had the honour to be acquainted with such like Gentlemen, executors of the Law, cannot have the least idea of the delicacy, the humanity, the tender feelings of some of those Harpies in office; who are dead to every sense of feeling, where their Interest is concerned. My Wife (ever anticipating woe) suspecting his business, followed close in the rear; when there followed, perhaps, the most entertaining Scene that can be imagined; of which, I will endeavour to give a true Picture.

There were, in my Study, three of my Children, Boys, who were just come home, from a School in Yorkshire, the oldest just turned of twelve years. Hav-

ing been rather a heavy Tax on me, for three years and a half, of thirty Guineas a year, I had now got them Home; not merely to save that expence, * or some part, but rather to instruct them myself, now; having, as I thought, finished the work I had in hand. As soon as my Wife entered the room, as if she followed a Spectre; I hope my dear (says she) you need not go with the Gentleman? No, my dear, I don't intend it, says I; indeed but you must, says he; Sir, I shall not go out of the House (replied I) I shall pay the Demand you have on me. On which, he started up, and collared me; and putting on one of his fiercest magisterial looks, Sir, says he, do you know who I am? Do you know, that I represent the Sheriff of Middlesex? He has a worthy Representative, says I; but if you represent the Devil † I care not; I can pay your demand on me, and therefore, will not go with you. Immediately he calls up his Hell-hound; who, ere this, had gained entrance, and, like a Russian, attempted to drag me down stairs. The whole House was alarmed; my Wife, striving to screen me from their violence, was handled somewhat roughly; the Boys set up their shrillest Pipes, to a fine old tune; up runs a Daughter and the Maid; a Lady, on the first floor, was frightened almost to death; in short, it was the most diverting scene that can be imagined; what pity, the Authors of it were not present, to enjoy it, and glut themselves with their just Vengeance §. Resistance being vain, I enquired where his habitation was; being informed, in Carey-street, a Coach was called, and away we drove, taking a Son with me, who happened to come in. He had to trudge into Abchurch-Lane, to the Attorney (Mr. Constable) to get a Bill of debt and costs. If he had happened to be gone to his Country house, I perhaps, have taken up my residence in Carey-street, till Monday; it being a custom with those Gentlemen, to seize their prey on a Saturday, in order to detain them the next Day, if matters cannot be settled soon. The Bill was making out, when a messenger was sent, to order twenty shillings to be tacked to it, for Interest; which, with the expences of one pound sixteen shillings, brought it up to fifteen pounds, and one shilling. That business being done, the Office searched, and the Money paid, I regained my Liberty, and returned home, to comfort my disconsolate Family.

I should be glad (for the sake of those, whose misfortunes may subject them to the insults of such Wretches) that some humane Person, of the Law, would, in public print, let such know, how far those Gentlemen have authority to tyrannize over those, who are already too much oppressed; and how they may be punished, for over acting their part. What! is an unfortunate Man, in credit, to be treated, by such Reptiles, like a Cutpurse, or a Felon? If I was under a mistake in point of Law, he should have set me right, in a proper manner. But why should a Person be obliged to go with them (the idea of which is terrible to a Person in Credit) when he can answer the demand against him? Is it to squeeze the last Shilling from him, by all means possible? A Shilling or two for a Coach; a Shilling for being lock'd up; a Bottle or two of Wine; and what they can extort for searching the Office, (though they have no right to demand any thing), 'tis their own Business; and at their peril they detain a Person, if there be nothing more against him. It would be a real Blessing to the unfortunate, if every Sheriff, entering on his Office, would make it his first business to enquire, strictly, how those, to whom they delegate such Power, have acted, in the execution of their Office; and either suspend for awhile, or expel them entirely, if they be found, as too many might be found, to deserve it.

* Let none be surprized at the mention of saving. Having three, at once, it was agreed for ten Guineas each, otherwise it is twelve Pounds a Year; out of which, I will maintain it, they clear five or six, for teaching; then where three are so many in a Family, with oeconomy, there may be a saving.

† Query, Whether it was not a more just, and striking Representation?

§ To give them a better Idea, they may see a faint resemblance, delineat'd in a Print, in the first Number of a Publication, five or six Years ago, called the Gentleman's Museum.

I shall, now, take my leave of those generous and truly humane Gentlemen, those fair and upright Traders; who, by means of their great wealth, have the power to monopolize a most necessary Article, for many miles around the Metropolis; who could have the barbarity to arrest an industrious Person, for the paltry Sum of twelve Pounds five Shillings, the remains of near a Hundred; after having lost near half the Paper he had of them (as related above) although he paid them Interest for their Credit; amounting, on the whole, to four Pounds, twelve Shillings, and ten Pence.

Would it not have redounded more to their Reputation, to have sent for me, and addressed me thus? Well, Mr. Malton, we are sorry that you are, also, a sufferer with us, in that unfortunate Accident; which, to us, is trifling, because we are able to bear it. But for you, who having just completed such a truly laborious Work; burthened, as you are, with so large a Family, to be thus frustrated in reaping the emolument of your indefatigable Labour and Industry, we are really concerned. Having seen and experienced your Probity, we freely offer to credit you Paper, for reprinting the Work; which, we are glad to hear, merits the approbation of the Public; as we shall be happy in being instrumental to the rewarding of so much Merit. There was a small Balance left unpaid; had not our loss been already considerable, we would strike it out of our Books; we cannot suppose you are in a situation now to pay; but if you approve of it, we will add it to the next Account; you may, hereafter, be in a better situation; we will endeavour to contribute towards it; we will each of us be a Subscriber to your Work: as for the Interest you have paid, it shall be remitted, 'tis enough that we get the Principal now. I must observe, here, that when Mr. Wright said, he knew I had got a valuable Work, I asked him what better I was for it, at present? would they let me have Paper, for reprinting it? The answer was, yes, if I would give them security for it. You asked for none before, said I, nor will I attempt to give you any now; surely, now the merits of the Work is established, I am not in a worse situation than when I began it. But I desire not your credit, I can have it elsewhere, and be served at a lower price.

Having thus discharged a duty to the Public, in respect of such Blood suckers; I have only further to add, that I am fully persuaded, they were instrumental to the following affair. Being fortunate, in getting their Money, I must, in course have it in my power to pay the other, also; but if I had, I was far from having the inclination, seeing I never had value for the Money, delivered to me; but, on the contrary, detained from me, by their Caprice, till it was destroyed. With what confidence could they think of compelling me to pay, for Work done, when they were conscious of being the sole cause of my losing it? I shall find means (doubt not) to do myself justice.

On Wednesday the 26th of June, I received a Letter from an Attorney on Mr. Wells's account; giving me to know, that Mr. Wells had put into their hands, the two Notes, for twenty Pounds fifteen Shillings, with order to arrest me, if they were not paid the next Day, or on Friday. I waited on Mr. Wells the next Day, and let him know how I was situated; and that, if I must pay the Notes, they must allow me Time; but that, Cox and Bigg were the Persons properly accountable for them. I knew not, then, that he was a party concerned, otherwise, than having advanced, as he said, a thousand Pounds, which he had lost; he deserved to lose it, for his negligence, in not seeing them insured; by which, others suffered so much. But, as such, generally, are better acquainted with the affairs of those who are indebted to them, than themselves, he knew I could pay them, then, if I would. Perhaps I could; but I had twenty more important, and necessary demands for all the Money I could get.

I wrote

I wrote to him, the next Day, and stated the Case in the most candid and impartial manner; laid the true situation of my affairs open to him; with the burthen of such a Family, as I had to provide for; and that, in three Weeks time, I would use my utmost endeavours to pay one of the Notes. But, what is most extraordinary, although I had requested it, over and over, of Mr. Cox, and he had promised I should, yet, I could never be indulged with an Account of what there was against me; to have, at least, the satisfaction that I ever was so much indebted to them.

Notwithstanding all I had said or wrote to Mr. Wells, I was arrested in a fortnight after, on Saturday (as usual) the 13th of July. As I never imagined that they durst, or that any Person, who was not a Jew or a Barbarian, could have proceeded to compulsion, in such an affair, I was determined not to pay it, till I had advice concerning it; and, having the long Vacation before me, I consequently gave Bail. But, finding I had no Plea to support a Trial, as I could not give direct and positive proof, that he was a Partner in the Business; nor, indeed, did I know it then; and, being at a distance from Home, I sent Orders for paying it, before the Term commenced. I must, however, do Mr. Wells this justice; he took no more than the Principal; I paid no expences, nor did he charge Interest on the Notes; like the other hungry, rapacious Wretches, Wright and Gill, who charged to the last Shilling of Interest.

Thus the affair stands, at present, between me and the Printers. I have waited till now, in hopes they might be in a situation, ere this, to reprint the Work; and, by that means, make me some reparation, without inconvenience to themselves. But, I am never the nearer; the Partnership is dissolved, and therefore, I have but little to expect on that score; nor could I longer delay putting the Work to Press. The Public may now be assured, that it will be regularly published in Numbers, or Parcels, every Month.

I cannot suppose that Messrs. Cox and Bigg are insensible to the peculiar hardship of my Case, as they are conscious that I suffered, entirely, through their caprice; but I think, had they represented the Case properly, and as it really was, they might have alleviated it; and, at least, have acquitted me of those Notes, I had, imprudently, but with good intention to them, last given; especially, as they were not gainers by the payment. I have only, now, to wait with Patience, to see what will be the result of the Brief now impending, and collecting from House to House, throughout every Parish in England, &c. obtained through the Policy and Interest of Mr. Wells; who, although he would disdain to have his Name appear in it, will not, I presume refuse to accept the share of it, which may be allotted to Mr. Nettleton; for that, the World may be assured, is the sole motive that induced him to interest himself so much in it.

I hope for my own, as well as for their Emolument, that the Public will be liberal; they never had a more interesting occasion. Although they have endeavoured to make my loss appear trivial (as it certainly is, compared, in bulk, with some other sufferers) I do here affirm, that I lost 350 Books of Geometry, entirely, and near two hundred (out of five) of the Perspective, which entirely frustrated all my hopes of Emolument, from the publication of that Work for the present.

B O O K I.

OPTICS or VISION.

S E C T I O N I.

OF LIGHT and COLOUR.

TO treat on any Science, properly, it is necessary to begin with the Elements of it, or first Principles; which is the reason, as I suppose, that some writers, on Perspective, have prefaced their Works with a chapter on Optics and Vision. Some of whom, not daring to advance any thing of their own, on the subject, have favoured the World with extracts from Sir Isaac Newton's and Smith's Optics; which, indeed, is very little to the purpose, as the much greater part is no way subservient to Perspective; considering it not merely optical, but purely a mathematical Science, supported on the firm Basis of Geometry.

The ingenious Mr. Hamilton, in his Stereography (a work which does great credit to the Author) to which I acknowledge myself indebted, has given some judicious observations on the subject of Vision; and some, which are rather exceptionable. But, the Theory of Colours (if that can be called a Theory, which is so little known, and nothing in it demonstrable) he, very wisely, declines enlarging on; as not being subservient to Perspective, nor indeed to Painting: I mean the Theory of the Prism, which, Sir Isaac Newton and others have so copiously expatiated on.

I would by no means have it thought that I intend, or have the least desire, to derogate from the character of so great a Man, whose Name I highly revere; but I am persuaded, that the pursuits of the greatest Men are, sometimes, in themselves, trivial and merely amusing; when the character of a great Man is established, it gives a sanction of consequence to his most insignificant amusements; for, why may not the most profound reasoners relax from their intense studies, and amuse themselves, sometimes, with trifling matters? by which means, things of great utility have been brought to light; as must have been the case in respect of the Prism: nor do I find that he has perfected what he had in view, but left his observations for others to improve on, having (in his own words) not inclination to take it up again, and pursue it further, as he had intended.

Had the Theory of Colours, as deduced from the Prism, been amongst the first and chief of that great Man's pursuits, I am much in doubt, if the reputation he has acquired had ever been established, at least on that Basis; things of infinitely more importance to the Community, fixed his credit (most deservedly) on the highest pinnacle of Fame; for, what useful and necessary knowledge has been communicated to mankind, by this acquisition to the Science of Optics? which (with such, apparently wondrous, sagacity and penetration) he has explored and given to the World.

To define, with any degree of precision and perspicuity, what Light is, is not possible; seeing we cannot comprehend, and enter into an accurate disquisition of Fire; which is the only cause we can conceive of Light. From the different and obstinate refrangibility of the different Colours produced by a Prism, when applied to the Sun's Beams, it is concluded, that Light is, in its nature, heterogeneous; that it is composed of Particles, of different qualities, which produce very different effects on Objects. I cannot, from any thing I have yet seen or read on the Subject, give my assent to that opinion; for, I believe, that what we call Light (the vivid glow occasioned by any luminous body) is perfectly homogenous; and that it is not composed of Particles; which implies that it is material, a Body; for, how else can it be a composition of Particles? which, by means of the Prism, are supposed to be dissevered, and separated from each other.

But, what reason can be assigned, that a Prism should have this most extraordinary power of separating them; owing to its form only*, not the matter of which it is composed: But it is not owing to either, as it is manifest by Experiment. Allowing the rays of Light to be thus sifted or separated, what use is made of it, or what further knowledge is deduced from it?

'Tis obvious, that the same Object, having its Surfaces differently disposed, or situated, exhibits different Colours, according as its Surfaces are situated to the Light; that is, to that quarter, from which any thing luminous causes an illumination of them; and those parts, which are most directly opposed to it, are most intense in Colour; whilst the opposite Faces (the Light being obstructed by the Object) are almost deprived of Colour; which they would be entirely, but for other Objects in vicinity with and opposed to them: which other Objects being strongly illumined, are said to reflect Light to them. Hence, a conclusion is drawn, that Objects have, in themselves, no inherent Colour; but that, all the sense we have of different Colours, in the same or in different Objects, is entirely owing to their different qualities and modifications (or, rather, to the construction of their Surfaces, simply) by which, they are fitted and disposed to reflect different coloured Rays, some more copiously than others. But, it does not necessarily follow, that they have no inherent Colour; for although Objects, when opposed to the Sun (that is, when nothing opaque intervenes) exhibit very different, and more brilliant colours than otherwise; yet, if its Rays are obstructed, instantaneously (the Light around them being then supposed stagnated), they still exhibit Colour, and of the same kind, or hue, though less vivid, and greatly inferior in the degree of it.

The chief Argument advanced in favour, and the only reason, of weight, assigned, in support of their Hypothesis, is, that Objects exhibit no Colour, when they are entirely deprived of Light; which argument cannot be denied; because, no Object, in such Case, is perceptible by means of Vision; without Light our Optics are of no use at all. It is the same if we close our Eyes, when Objects are fully illumined; but, certainly, the Colour, as well as the Object, remains, whether, by shutting our Eyes, we perceive it or not. I look on this as a specious, and subtle, not to call it a sophistical way of reasoning; since, without Light, there can be no Vision; and consequently, the sense of Colour, more than of the Figure, Magnitude, or Situation of Objects, cannot be communicated. But, if Light be considered as a Medium existing of itself, without the Luminary, how can there be a privation of it, if it fills the whole surrounding space, as Air does? which, seeing that, Air is also considered as a material Body, is somewhat repugnant to the established and universally received Maxim, that, two Bodies cannot fill or occupy the same Space, at the same Time. And, if Light be a Medium, of what use is it, without the Luminary? or, how can there be a perpetual emission, from the Luminary, the whole Space being, every where, filled with Light? and also, if it be material, what becomes of it? To suppose that it is absorbed, by the Earth and other Planets, would be ridiculous in the highest degree; because, the whole Surfaces of all the Planets bear no sensible proportion to the immense Space between

* A Prism, as defined by Euclid (Def. 13. 11.) is a Solid, contained by Planes, of which, two opposite are equal, similar, and parallel; all the rest are Parallelograms; so that, every Parallelopiped is a Prism, by which, being right-angled, no effect of Colour is produced. Now, as it depends, solely, on the inclination of one Plane to another, an acute angled Pyramid (in which all the Planes are Triangles) or frustum of a Pyramid, is the same, in respect of Colour, as a Prism.

them, in which there is no obstruction to the progression of Light, flowing from the Luminary in all directions.

That Light should exist in darkness (notwithstanding the Scriptures say, that Light was created before the Sun) is not only a direct contradiction in terms, but to reason and common sense, and the nature of things. Also, that Objects exhibit no Colour, when deprived of Light, is certain: because they are no longer visible; but, since every other quality remains, it is most probable (Colour being perceptible by sight only) were they objects of Vision, in total Darkness, Colour would remain likewise: For, although Colour cannot be perceived without Light, yet it is not the efficient cause of different Colours, in Objects composed of different materials.

In treating on Colours, the learned Boyle gives us a wonderful account of a blind Man's distinguishing Colours by the touch; which can only be by the asperity or roughness of the surface of the coloured Body, and which is construed as favouring their Opinion. It may be possible, that a difference in Colours, artificially laid on the surfaces of Bodies, may be felt; or, that the qualities of Dyes or Stains may also affect Bodies so, as to make them sensible to the touch. But, will any person venture to say, that if Bodies, stained or otherwise coloured, were polished or made equally smooth, the difference, in Colour, could be perceived by feeling; or, that the natural Colours of Wood, Metals, or Stones, nearly of the same qualities, except Colour, when equally polished, could be distinguished by that mode of Perception? I affirm they could not; nor can I give any credit to such assertions. Then, since that, Bodies, equally hard and equally polished, exhibit different Colours, I see no reason for supposing that the cause of their differing in Colour is owing to the different texture or construction of their Surfaces; or, that it is possible to distinguish them by their asperity; for, Bodies of equal hardness, and being equally polished, must have Surfaces alike, except in Figure and Colour.

Light, however-propagated, has, I presume, the same properties and influence; and, consequently, acts the same on Bodies. I would ask then, what is the reason, why Blue and Green are scarce distinguishable by the light of Fire? If the action and reaction of Light, on the surfaces of Bodies, be uniform, by a certain law, according as they are fitted and disposed to reflect any particular species of colour-making Rays, whence arises the difference in this case? I should be glad to hear or read an ingenious disquisition of that point; as, I do not doubt, there are Men of sagacity and penetration sufficient to set it in the clearest light, and make it as evident, to the understanding, as any common Case in any Science whatever.

To return to the Prism. It is certainly true (and it is a curious and entertaining Experiment) that, through an acute angled Prism, we perceive Objects very differently coloured from what they really are by Nature; also, by applying the Prism, properly, to the Sun's Beams, there is exhibited a curious and most extraordinary Phenomenon, *viz.* a diversity of Colours, the most lively and beautiful that can be conceived; and, the more uncoloured the Surface is, on which they fall, the more vivid they appear.

Now it is certain, that here is a perception of Colour, on the surfaces of Bodies, which is not inherent; and so is there when the Sun's Beams, only, fall on them: yet, we cannot from thence conclude, that they have no inherent Colour. The Colours which the Prism exhibits are Red, Orange, Yellow, Green, Blue, Indigo, and Violet or Purple. But why is there, here, made a distinction of seven Colours, when, in reality, there are not above three or four simple Colours in Nature? (unless Black and White may be called Colours) *viz.* Red, Yellow, Blue*; and Green; which may also be compounded of Blue and Yellow. Indigo is Blue, Orange and Purple are Compounds, or mixed Colours. Now, if any Philosopher, or Artist whatever, can, from these three or four simple and brilliant Colours, produce all the variety which we see in Nature; or, did the Prism exhibit di-

* It is said in Emerson's Optics (Cor. 3. Prop. 9.). There are as many simple Colours as there are degrees of Refrangibility; and therefore an infinite number. And, hence (in Cor. 5.) it is plain, if the Sun's Light consisted but of one sort of Rays, there would be but one Colour in the World. In one place, he says, Objects have no Colour naturally; and in another, they become coloured, by reflecting the Light of their own Colour, more plentifully than others.

finely, all the simple Colours which are in Nature, only, without mixture, I should then be better disposed to give credit to their Theory: or, when I can have conviction, that a mixture of all these Colours, together, in any ratio, will produce a perfect White (as Snow) I may then be a perfect Profelyte; till then, I am persuaded that I must dissent from their Opinions.

I shall, in the next Place, enquire, how a Prism, particularly, has the wonderful property of separating the different coloured Rays of Light. The Prism is a body of Glass, or it may be of Crystal, or other pellucid, uncoloured Stone; is the property then in Glass, or Stone? No; in the Figure that is made of it, only; amazing! that the disposal of the same Matter into different Forms should produce such very different Effects. Glass, disposed into a portion of a convex Sphere, will make Objects, seen through it, appear magnified; and, being opposed to the Sun's Beams, will collect real Fire; astonishing indeed! If the Surfaces be concave, Objects, seen through them, appear less than they really are; in the forms of Prisms it has various effects; but we do not see the effect of exhibiting Colours through a right-angled Prism; I mean, a four-sided one, whose opposite faces are parallel, and the Angles right ones; neither has it that effect through the right Angle, of a right-angled triangular Prism, but only through the acute Angles. The cause, then, of exhibiting Colour is not in the Matter itself, or Figure, nor in the Surfaces, simply, but in the inclination of the Surfaces to each other; for, being parallel, or at right angles, or nearly so, they do not produce that effect. We may as well ask, why convex Surfaces collect Fire (but, that is well known) rather than plane or other parallel Surfaces, as, why a Prism, whose Planes are inclined to each other, should have the property of exhibiting the different Colours I have mentioned, rather than one that is right-angled; and I am persuaded, that, with all the sagacity and penetration Man is endowed with, he will never be able to account, truly, for either.

Yet, I do not condemn all enquiries into the causes of the various effects which we perceive in Nature, but think the pursuit rational, and truly commendable, when it is founded on certain Data, and real Hypothesis; and the result of our researches productive of real utility, deducible from it, as in some other branches of Optics. It is asserted, by some, that the perfection of Telescopes is owing to the Theory of the prismatic Colours. As I am not conversant in the mechanical construction of Lenses and their application to Telescopes, I cannot affirm that it has not been subservient thereto; but, in my opinion, it has not the least apparent tendency to benefit Mankind accruing from it. Certain I am, that it will never be of use to a Painter, to compound his Colours and form a Theory thereof; by means of which, he may sooner, and with certainty, arrive at perfection, in his Art: and I must needs say, that the attempt made by Brook Taylor, in the Appendix to his second part, does not shew the Doctor's judgment, in that, to be of a piece with the rest of the Work.

The Sun's Beams passing through Glass, whose Surfaces are neither parallel nor perpendicular to each other, or nearly so, exhibit the various Colours, spoken of above, in some degree, according to the inclination of the opposite Surfaces, and purity or clearness of the Glass; through the thick part, near the knob, in the middle of a table of Crown Glass, I have often observed the Phenomenon, in a Window of those Squares: but, it is also observable, that the effect ceases when the Sun ceases to shine on them. It is also certain, that Objects appear coloured otherwise than what Nature assigned them, when we look through Mediums denser than Air; whose Surfaces, through which we look, are inclined to each other in certain Angles; and consequently, there will be the effect of Colour produced, in some degree, which is not natural to Objects, in looking through Object-Glasses, or Lenses, with which Telescopes are constructed; seeing that, their surfaces are inclined, at the edges. The business, therefore, of an Optician, in this respect, is, so to contrive and dispose his Glasses, as to divest the Object of the borrowed Colours, which do not belong to them, naturally; and I could almost affirm, that the perfection, spoken of, has been found out by repeated trials; not from any certain established Law, deduced from the Theory of the prismatic Colours:

Colours: for, if a certain Theory had been established, what hindered the immediate perfection of them, as soon as the cause of the imperfection was known, and certain Laws established, whereby the unnatural effect of Colour might be removed from the Field of View?

From what has been observed, respecting the effect of Colours produced by the Prism (unless it can be proved that the particles of Air, through which, Light must necessarily pass, are Prisms) to infer, that Nature has given no inherent Colour to Objects, is bold and assuming; it is also groundless, seeing that the Colour, which is natural to each, remains when the Sun does not shine on them, though it differs greatly in the degree of it. We may with as much reason conclude, that Objects are much larger than we perceive them to be with the naked Eye, because they appear so, when view'd through Glass whose surfaces are convex; and, we may as well enquire how Vegetables, as Flowers, Fruit, &c. spring out of the Earth, and are adorned with all that beautiful variety of Colours which we see in Nature; by what means they are continually varying in their Colours, from their first formation, in embryo, to maturity; as why, the Prism exhibits the various Colours of the Rainbow, to which they are, in a great measure, similar.

Colours produced by alkaline Salts, &c. and mixtures of different Fluids, with all such like chymical operations, I look on, equally, as entertaining Experiments; but no way productive of any advantage, accruing from it, to the Theory of Colours necessary for a Painter to know, in order to reduce it to real use, in practice.

I shall not, therefore, trespass any longer on the Reader's Time, as it is entirely foreign to the purpose of Perspective; but confine my further observations to that part of Optics relative to Vision, only. In respect of direct Vision, it is certainly of use and subservient to Perspective; and essentially necessary to give a clear Idea of the Principles on which it exists, both in Theory and Practice.

Colour, whether inherent or incident to Bodies, is not material towards a perspective Description or Delineation; 'tis the Figure of the Object, only, it contemplates.

All solid Bodies, whatever, become objects of Perception from their Figure or Colour. Figure is more particularly inseparable from our Ideas of Matter, than Colour; it being impossible, in the nature of things, to have any notion of Extension, abstracted from the Idea of Figure, which limits the Space Bodies occupy or fill. It is the same with or without Light, and may be perceived by the sense of Feeling only. From the known external Figure, and situation of an Object, may be delineated, by mathematical rules, a true perspective Representation, on a Plane; which, by the help of Light and Shade, will raise a perfect Idea in the Mind, of the figure of the Object, and the different forms and positions of its Surfaces, in respect of each other, without the assistance of Colour; but, to exhibit a true and natural Picture of Objects, and create a perfect Idea in every circumstance, Colour is absolutely necessary; and is the last degree of perfection, with which the omniscient Creator has embellished Nature, that can be given to a Picture.

Light, considered as a Medium, by which Vision is convey'd to the Eye, is of too refined a nature for my speculations. But, as the great Author of Nature has given a portion of Reason to every human Being, we have certainly a right to make use of that Reason; and, if we will exert it, properly, it is possible, nay certain, that one Person may penetrate as deeply into the mysteries of Nature as another, though not blest'd with quite so much Learning.

We are told by several great and profound Philosophers (to be a Mathematician is not necessary) that we perceive Objects, only by means of Rays of Light, reflected from every point in their Surfaces to the Eye; which enter there, and form an Image, or Picture of the Objects perceived, on the Retina, or fine Membrane which surrounds all the back part of the Eye, internally. The Retina is said to proceed directly from the Optic Nerve, which dilates or spreads itself as before-mentioned; and, the impression being made thereon, we are further told, is conveyed, by the Optic Nerve, to the Brain, or seat of Perception.

B

But,

But, why do they stop here? I expected, and should be glad to be conveyed into the inmost recesses of the Brain, and be shewn or told, how the Image of the Object, on the Retina, is there perceived; for I must own it is astonishing! that, from what is perceived or felt within, we should have a true Idea of the Figure, Colour, and Magnitude, Situation and Distance of Objects, which are external, or situate without the Eye:

Now, after all this parade, and pompous display of great sagacity and deep penetration, what does the Sum total amount to? Why, that the Object is perceived, i.e. the Mind is sensible of the existence of such Objects as it perceives; and that, the Vision of them is conveyed in right or direct Lines from the Object to the Eye, or from that wondrous Organ to the Object; but how, or in what manner, remains as much a mystery as before. So that, after all which has been said on the Subject, and, allowing the Image, formed on the Retina (inverted or otherwise) as perfect as they please, what nearer are we? where is the Perception of that Image or Picture of the Object? How the Mind, or Soul, perceives the Image on the Retina, any more than the real Object, we are just as much at a loss to account for as ever.

In vain, therefore, does Man torment himself, in endeavouring to explore the hidden mysteries of Nature, which are for ever hid from our researches. Let us then pursue what is within our reach, and not an airy Phantom, which will ever elude the pursuit. In mathematical Sciences we have some certain Data, whereon to frame Hypotheses; and although we must not expect perfection, except in Theory, yet, we can discover so much of Truth as gives us sufficient encouragement to pursue it, and makes ample amends for the imperfection of human nature.

I do not intend, nor shall I attempt to explain the nature of Vision, or enquire into the true cause of it; being fully convinced of the insufficiency of our reasoning faculties for such a disquisition. It is sufficient, for the science of Perspective, to know, or even to suppose that it is conveyed to the Eye, the seat of Sight, in Right Lines. But, whether it be by means of Rays* of Light, reflected from all parts of Objects to the Eye (as is the general Opinion) or whether the Allwise former of the Eye has given a power to that truly wonderful and amazing Organ, to convey to the Mind, by Vision, the perception of Objects by other means, I do not mean to make the subject of my enquiry. Yet, I must acknowledge, that I have strong objections against the general Opinion, as it is now received and almost universally assented to; viz. that the perception we have of external Objects, from Vision, is by means of Rays of Light, reflected from all parts of their Surfaces to the Eye; and that those Rays are material, or composed of Matter.

But, as it is of no consequence, in Perspective, by what means Vision is performed, so it be conveyed in Right Lines, which may pass for an Axiom; seeing that, the refraction of the Rays (if there be any) in the Air which surrounds us, is so very little, in Objects which are at an immense distance beyond the whole Atmosphere, it cannot be sensible at any distance we can perceive Objects, within it, supposing the refraction uniform or regularly curved, between the Object and the Eye. But, I rather suppose the refraction of the Rays to be at the common Surface of two Mediums, only; at entering any other Medium, denser or rarer, than that through which they first pass, and thence proceed again in Right Lines; consequently, there is no Refraction at all, within the Medium; either in Air or Water, I suppose Vision to be convey'd in Right Lines, if the Object and the Eye are both in the same Medium.

At the same time, I would not have it thought, that I suppose any Rays to pass, by means of Reflection, from opaque Bodies; and in luminous ones, the Light, which proceeds from them, filling a concave Sphere, as far as they can be seen, I do not suppose to proceed in Rays, nor in Planes (as a modern Author has supposed); the Hypothesis is absurd; seeing that, the whole Space is illumined in every part, it is impossible, that from every Point in the surface of the luminous Body, Rays can proceed to an immense distance, filling all the Space between.

* I make use of this Term, though I have not the least conception of a Line, or thread of Light, called a Ray; much less of a Pencil, or bundle of Rays.

Suppose every Point in the surface of the Sun to emit a Ray of Light, proceeding in Right Lines from the Center; it is evident, that every Ray, at the distance of the semi-diameter, will fill a Space four-fold of its first dimensions, *viz.* in a duplicate ratio of the Distance; at the distance of the whole Diameter, it must fill a Space nine times as large; i. e. it increases in proportion to the Squares of the Distance; what, then, will be the magnitude of a single Ray of Light, proceeding from the Sun to Jupiter or Saturn, before it reaches those Planets*? or how can the whole of the Space it occupies be filled (as it certainly is) with Light, which proceeded from a single Point, in the Luminary only? Can one individual Ray be shattered into thousands, into millions, lying close to each other? what direction do they proceed in, from the Luminary? not in the direction of a Right Line from the Center, I presume.

I know it may and will be alledged that it expands as it proceeds, still filling the whole Space. I presume then, it cannot proceed in Rays, or Right Lines from the Center of the Luminary, as it is imagined. Air may be rarified and expanded to a very great degree; also, Water and other Fluids are expanded, as in steam, by Fire, or, in Mists and Vapours; by Exhalation, till it floats on Air. But can one drop, or the minutest particle of Water be expanded infinitely? as a Ray of Light, emitted from one single Point in the Luminary, must, in this Case, necessarily be; else, how could we see those Stars, whose distances are, to all sense, infinite?

These things, which may amuse and pass for orthodox with the generality of mankind, cannot be satisfactory to a Person who dare think for himself, and is furnished with the means to do so. Therefore, since all which can be said about it is, at best, but an ingenious conjecture, and since it is in no wise subservient to Perspective, I shall at present take my leave of the Subject, to pursue that part or branch of Optics, which is essentially so, respecting direct Vision; and after that, in a separate Chapter, or Section, I shall give my objections to the received Opinion of the cause of Vision.

* I find by calculation (from the Tables in Harris's Use of the Globes, which I suppose are as much to be depended on as any other) that the distance of the Earth, from the Sun's Center, is 212 semi-diameters of the Sun, nearly. Wherefore, if, instead of a Point, we suppose a certain quantity of Light to flow from one square Inch in the surface of the Sun; each side of that square Inch will, at that distance, be increased to 212 inches, or 17 feet 8 inches; the square of which is 44944 square inches. Consequently, a quadrangular Pyramid, whose Base is 17 feet 8 inches square, and its altitude, the distance of the Sun from the Earth, is filled with Light, from one square Inch, only; and consequently, each single Ray, proceeding from every Point in its Surface, will, at that distance, be multiplied, or magnified 44944 times.

At the distance of Saturn (by the same Tables) a square Inch, and consequently a single Ray, will be increased or multiplied 4,149,369 times; its distance being 2037 semi-diameters of the Sun, nearly; consequently, a quadrangular Pyramid, whose Base is 169 feet 9 inches, on each side, and its altitude 777 millions of Miles, is filled with Light, flowing from every square Inch on the surface of the Sun; an amazing circumstance indeed! if it is supposed, or considered to be a material Body. But, how far Light may proceed, from the Sun, beyond the realms of Saturn, is not yet, I presume, determined.

SECTION II.

Of the structure of the EYE, its parts described, and a short
INTRODUCTION to the nature of VISION.

THE construction of an Eye, that most wonderful organ of Sight, by means of which, we have intelligence of the Objects which surround us, is various; yet has nearly the same Parts, in all the animal Creation. The Parts are few, and apparently simple, but astonishing in their mechanism; as, indeed, are all the animal Functions; but, what is there in Nature which is not so? The Figure of an entire human Eye, within its Socket is globular, of somewhat more than an Inch in Diameter; the Furniture of which are as follow.

Plate I. Let *HIKL* be a vertical Section through the center of the Eye, which, in this Figure, is considerably larger than Life; in order, that the several parts may be described with greater accuracy.

The external part, from *H* to *I*, is called the Cornea, or Horny Coat. It is perfectly transparent, and is somewhat more convex than the part which is within the Socket, by means of a fine, clear, aqueous or watry Humour, between the Cornea and the Iris (the coloured Circle within). Immediately behind the Iris, (*ghik*,) through which there is an Aperture, in the middle, to let in Light (or Vision by means of Light) is a whitish Substance (*MN*) like a strong Jelly, or cold Glue, of a moderate consistence. It is as clear and pellucid as Crystal; from which, it is called the Crystalline Humour. It is in every respect of the same nature and use as a double convex, microscopic Lens; and is said to be more convex on one Side (the inward) than the other. Its use is supposed to contract the Rays of Light, by which Vision is convey'd, to a Focus, beyond this Humour, in the Center of the Eye, at *E*; from whence, they become diverging, and fall on the bottom or back part of the Eye, at *acb*.

The Crystalline Humour (*MN*) is said, by those who are acquainted with and accustomed to dissections of the Eye, to be furnished with muscular Fibres; by means of which, it is made either more or less convex; or, by contracting them, is brought forward or otherwise as occasion requires; in order, to render the Images, or Pictures of Objects perfect, in the bottom of the Eye, as the Objects are nearer to or further from the Eye; all which seems consonant to Reason.

The Aperture or Pupil (*hi*) is also, by some such means, expanded or contracted, as there is occasion for more or less Light to enter. I have observed, in the Eye of a Child, in a Room just light enough to discover it, the Pupil enlarged to three tenths of an Inch in diameter; and immediately, on bringing into a full Light, it was contracted to one tenth, or less; amazing Structure! and this is performed involuntarily, whether we will or no. There can scarce be any Person who has not made the experiment; on going into a darkish Room, he can scarce, at first, distinguish any thing; when presently, he will, after the Aperture in the Iris is open'd to a proper dimension, discern Objects distinctly; and again, on going into a full Light, he cannot bear to look on any illumined Object, till the Pupil is contracted; it is even painful to the Eye, at first, going out of a dark Room into a strong Light, especially if the Sun shines out.

All the back part of the Eye, within, is lined with a fine Membrane of a most curious and delicate texture; called the Retina, from its resemblance of net-work. The large cavity, *VV*, which is bounded by the Retina, backward, and the Crystalline Humour, before (which is contained within it) is filled with a glutinous Fluid like the white of an Egg; being perfectly pellucid and uncoloured, it is called the vitreous or glassy Humour; in which the Rays converge, at *E*, the center of the Eye; and, crossing each other, in that point of contraction, they fall diverging on the Retina, and form the Images of Objects thereon; the sensation, of which, is supposed to be communicated, by the Optick Nerve (which is connected with the Retina) to the Sensorium, in the Brain.

As the manner in which the Rays of Light are refracted, in passing through the various humours of the Eye, is entirely conjectural; seeing that, through a small pin-hole, applied close to the Eye, at *q*, there may be perfect Vision, of an Object of any magnitude; in which Case, the whole System of Rays fall on the Cornea in one Point; how, then, are they refracted? As, this Essay is not intended as a Treatise on Optics, I shall not insist on any thing; but, shall only suppose the Visual Rays (when the Eye is naked) to converge and cross each other in the center of the Eye, at *E* (without considering their Refractions) from whence, I suppose, they diverge in equal Angles, falling on the Retina, at *acb*, in the back part of the Eye.

D E F I N I T I O N S .

1. The AXE of the EYE is a Right Line passing through the Center of the Eye and the Center of the Aperture or Pupil. As $c E C$.

The Point c , where the Axe cuts the Retina, is the place where Vision is perfect; which point is varied as the Eye is moved, or its Axe directed to different Objects, or to different parts of an Object.

2. VISUAL RAY. If AB be supposed an Object of Sight; the Right Lines AE , BE &c. from all parts of the Object to the Eye, or to its Center (E) under which, or by the means of which, the Object is seen, or supposed to be seen, are called Visual Rays.

3. OPTIC ANGLE. By Optic Angle may be understood, either a plane or solid Angle.

If the Object AB be considered as having no dimensions but length, only; i. e. if it be a Right Line, simply; the two Visual Rays EA , EB , from the Eye to each extreme, form a Plane Angle, AEB ; which is the Optic Angle, under which, the Line AB is seen. And, AED or DEB is the Optic Angle of the Segment AD or DB .

4. CONE, or PYRAMID of RAYS. If the Object of Sight be either globular, or circular; as ABG ; the Visual Rays, EA , EB , &c. from the Eye to every part of the extremes of its Surface, being towards the Eye, form a Figure resembling a Cone, which is called the OPTIC CONE.

But, when the Object is either a right lined Solid, or Plane Figure, as CD ; the Visual Rays EF , EC , EH , &c. form a solid Angle, composed of several plane Angles, FEC , CEH , &c. which, being of a pyramidal form, is called a PYRAMID of Rays.

Fig. 2.

The Visual Rays crossing each other, in E , and, from thence, become diverging to the Retina, each opposite plane Angle being equal, † there is formed a similar † 2. 1. El . Pyramid, cEd , which is called the opposite Pyramid; and E is their common Vertex.

The Figure, ab or cd , which is the Base of the opposite Cone or Pyramid of Rays, is the Image or Picture of the Object, AB or CD , on the Retina.

The corresponding Characters shew in what manner the Image is inverted.

Let AB be supposed an Object, direct before the Eye, that is, perpendicular to its Axe (EC). It is supposed to be seen, by means of the Visual Rays EA , EB , EC , &c. from every part or point in the Object, towards the Eye, whether it be Line, Surface, or Solid; which, from the point of contraction (E), proceed diverging, to the back part of the Eye, where they fall on the Retina, and form the Picture, acb . of the Object AB .

It may be alledged, that it is impossible for the Points A and B to be seen, at the same time; seeing, the Rays, from those Points, do not enter the Pupil. I grant they cannot; nor F and G , distinctly; nor indeed, little more than a Point, without moving the Eye; we have but a confused intimation of the surrounding Objects, at a little distance from that Point, to which the Axe of the Eye is directed; of which, every Person's own Experience must soon convince him. But, the quick transition of the Eye, or the direction of its Axe, from one part of an Object to another, is so little attended to, that, many imagine the field of View is much larger than it really is. As for the refraction of the Rays, through the various Humours of the Eye, I shall leave for the speculation of those, who are blest'd with a more fertile imagination than I have any pretension to.

Plate I. The Image of the extreme point A, by means of the Visual Ray A E, falls on the Retina at *a*, and the other extreme, B, is represented at *b*; and C E, the Axe of the Eye, conveys the Image of the point C to *c*, in the center of the Retina. From which it is evident, that the Picture of an Object, formed on the Retina, is necessarily inverted.

Fig. 1.

Nor is there any thing surprizing in this, as some may imagine. For, suppose A the top, and B, the bottom of the Object. It may be imagined, from the inverted position of the Image, *a c b*, that the Object must necessarily appear upside down. But, if we consider, that the sensation of the Point A, being perceived or felt at *a*, is by the direction of the Visual Ray A E, which determines its place in the Object; the Mind, by long experience, having acquired a habit of determining that part of an Object, perceived at *a*, to be above, and that at *b*, below; by the same reasoning it is manifest, that the points of Object on the right hand are pictured on the left, and those to the left, on the right; and which, is no matter of surprize at all; no more than, that, by the same experience, we have acquired an Idea of the distance of any Object perceived.

If a Person, who was born blind, could, when grown to maturity, be made to see, he would have no Idea at all of Distance, or the situations of Objects; which way was up, or which down; and would as soon attempt to lay hold of the most distant Object, which he perceived by the sense of seeing, only, as others which were near him; it being impossible to distinguish, merely by Sight, whether the Point perceived at *c*, on the Retina, be at C or C, or at any distance beyond C, in the direction of E C. But, by Experience and a familiarity with Objects, according to their known magnitudes or distinctness of parts; or, according to the length of Ground, which we imagine to lie between us and the Objects, we judge them to be at such or such a Distance. Whereas, when we look at the Heavens, one Star appears as far off as an other; nor can we form the least Idea of their distance from sight, simply; for, a Star may as well be ten thousand millions of Miles off as ten Miles, its Magnitude being in Proportion to its Distance. (See Fig. 7. A, B and C.)

The Art of Seeing is acquired, regularly and progressively, as all other Arts or Knowledge whatever. Do not we see a Child attempt to catch the Moon or other striking Object, though at an immense Distance? but, growing up in a familiarity with them, by common and frequent experience, they become so intimately connected with our Ideas, that we form a judgement of their Magnitudes and Distances, instantaneously, at first sight; and also of their Situations, in respect of themselves and of each other.

It may, to some Persons, appear strange to call Seeing an Art; but that it is so is certain, although Nature has furnished us with that most wonderful Organ of Sight, the Eye; which, being perfect, is not possible to be open (with sufficient Light for the purpose) but we immediately see what is before the Eye. A Child sees as soon as it is born; but it sees, without judgement, or discerning what it sees. So would a Person born blind, and continued so, till he was mature in Judgment, in every other kind of Perception. Let us suppose such a Person, immediately made to see. What Ideas he would have of what he merely saw, at first opening his Eyes 'tis impossible to say; but, of this I am well assured, that he could not distinguish the most familiar Objects, without Experience; as Figure and Colour are things of which he had formed no conception.

There are not many more opposite, regular Objects, than a Cube and a Sphere, or Globe; nor any, more easily distinguished, by feeling; yet, even these could not, with certainty be known, merely by seeing them, which was one and which the other; because, Experience had not taught, in what manner such Objects affect the Eye. In respect of Colour, 'tis impossible he could have any Ideas, by which he could distinguish them; they would be, to him, as if never existing before. Of Distance, he could have no Conception; for being wholly unacquainted with the real Magnitudes of Objects and the effects which Distance has on them, their relative and apparent Magnitudes could not be discerned, so as to know that one was farther off than another.

This

This may appear strange, to those who have not considered it with attention; but I maintain it to be a truth, founded on sound reason. For, as he knows nothing of the difference between real Magnitudes, and the apparent Magnitudes of Objects, according to their several Distances; he could never imagine that a Man, or a Horse, which he saw at a Distance, was of the same kind as those he saw near him; till Experience had shewn, that the difference in apparent Magnitudes is, merely, the effect of Distance. So, of any other Object, as a House, for instance, into which, as he finds he can freely enter, being near, he could never imagine that to be a House, which by means of its Distance, appeared not so big as his Head. Consequently, since he could not imagine that a Man, or a House, at a Distance, was a Man or a House, of the same Magnitude as others which were near him, he could form no Idea of Distance, by sight; besides, the difference in Effect and Colour, occasioned thereby, are things entirely new; of which, he had not, till now, any Ideas.

Now, the judgement we form of Distance, &c. being acquired early, and grown up with us, is so little attended to, that at first thought of these things, we suppose it to be merely from Sight; than which, no Sense is more delusive and uncertain. The deceptions of Vision are many and frequent; as, when we look at Objects through refracting Mediums, i. e. through any transparent Medium whatever, solid or fluid; according to the different density of the Medium, or to its Figure, &c. we frequently imagine an Object to be nearer or farther off, larger or smaller than it really is; or to be where it really is not; nay, it is possible, by Light and Shade, judiciously disposed, and the assistance of Perspective, to deceive both the Eye and Judgment.

Of all the Senses, that of Sight is by much the noblest; it is, nevertheless, the most deceptive. He who can most deceive the Eye in the art of Delineating and Colouring is esteemed the greatest Genius. It is possible, to describe the representations of many Objects so very accurately on a plane Surface, that (as I have already observed) with the assistance of Colour, and a just distribution of Light and Shade, they would, in the true Point of view, deceive a judicious observer, and appear to be real Objects. As a proof of the deception, in Vision, in respect of Magnitude, simply, I shall give a most striking instance; of which, though it has fallen within the cognizance of Thousands, few, or none, that I have conversed with on the subject, have formed a just Idea, or right judgment. I have asked several Persons, what were their Ideas of the apparent magnitude of the Moon? most have concluded, that it appears, when full, as large as the crown of their Hat; some, as a common Plate, or small Dish. I should be glad to know, by what standard such a judgment could be formed? for, unless some Distance, of the Object to be compared with, be determined, no comparison can be made; as, it may appear as large as the Dome of St. Paul's; but not, I am persuaded, at any distance we can see that Object, by the Light of that Luminary*. Now if we suppose the Plate laid on a Table, before us; or the Hat in one Hand, when we make the comparison, 'tis, perhaps, as random a supposition as can be conceived; and, without some determined Distance, it is vague and undeterminate. I know it will surprize many; but I do affirm, that, at the greatest distance we can hold any thing to compare with the Moon, it does not appear larger than a Silver Penny.

Now, if Seeing be not an Art, to be acquired by Experience, how comes it, that some Persons see with more judgment than others? Since as I am persuaded, that, all human Eyes (being perfect) see alike by Nature; and consequently, all would judge alike, of what they saw; but, it is certain we do not; of which, striking instances might be given. A Picture, or an Object, it is manifest, must strike every Eye alike, in the same Point of view; how is it then, that the most judicious and accurate representation, of a well-known Object, does not communicate, to every beholder, the same Idea of the Original?

* There was a Moon-Light Piece (by a capital Artist) in the Strand Exhibition, last, in which (if I remember well) the Moon appeared near that Object, and almost of equal Magnitude.

It would scarce gain credit, to affirm, that, on asking a Person, who was admiring a large, and well-executed, Print of St. Paul's Cathedral (one who had lived, some Years, near that well-known Object) what it represented? When, after a strict attention to it, I was answered, with an Interrogation, is it Westminster Bridge? 'Tis true, it was a Female; but, she had Eyes, and could distinguish between a Cow and a Horse, at first sight, as well as any other Person.

It is, to me, almost impossible to account for this; as I have known many Children, of three or four Years old, speak with certainty of very paltry Representations. I think it can arise, only, from their inattention to the Objects they see; as it is impossible, that a Child can have had more Experience than a Person grown to maturity. Nevertheless, it is certain, that some have much more Judgment of what they see, with less Experience; or, at least, they acquire it much less time.

That Vision is conveyed in Right Lines to the Eye, I presume no person will attempt to dispute (I would be understood to mean, of such Objects as are situate on or near the Earth; and when no other Medium, but Air, is between the Object and the Eye). I shall, therefore, give it as a general Axiom; which, nevertheless, may be illustrated or ocularly demonstrated, by straining a fine silken thread, to a Right Line, from any Point, as F , of an Object (DF) to the Eye.

Then, if any other Object, as QR , be interposed, as soon as it touches the Thread EF , they will appear to be in contact; and if it be so interposed, as to hide the Point F , the Right Line or Thread, which may represent a Visual Ray, will be refracted, or broken into two Right Lines ER , RF forming an Angle, ERF .

Hence it is evident, that the Points F and F' ; also, the Point C or C' being in the same Right Line with C_2 , will perfectly coincide with each other, to the Eye, at E .

S E C T I O N III.

Containing a brief THEORY of DIRECT VISION.

T H E O R E M I.

OBJECTS appear to have that Proportion to each other, respectively, as the Angles under which they are seen.

Let acb represent a section of the Retina, a portion of a concave Sphere; and, let AB be an object direct before the Eye; divided, in any Ratio, in D and F .

Fig. 1. DEM. It is evident that the Object AB , by means of the Visual Rays EA , EB , is seen under the Angle AEB ; and the several Parts of AB , as AF , FD , and DB , are seen under the several Angles AEF , &c.

But, the opposite Angles aEf , fEd and dEb , are, respectively equal to the Angles AEF , FED , and DEB — — — 2. 1. El.

And, the Images af , fd and db , on the Retina, of the Originals, AF , FD , and DB , are the measures of those Angles, respectively (Th. P. Ang.*) the Retina being supposed a portion of a Sphere; each part is, therefore, equally distant from the Center.

But, the Ark af is to the Angle aEf , as the Ark fd is to the Angle fEd , and as db to dEb . — — — — — 19. 6. El.

* (Th. P. Ang.) refers to the Theory of Plane Angles, in my Treatise of Geometry.

Therefore,

Therefore, the Images or Pictures on the Retina, and consequently, the apparent magnitudes of Objects are in the same ratio, or proportion, as the Angles they subtend at the Eye. Q. E. D.

Nor is there occasion to have recourse to the Images on the Retina; but, allowing the Visual Rays EA, EB, &c. to be Right Lines, and an ark of a Circle, of any radius, being drawn, cutting those Rays; the portions of the Ark, $lmno$, intercepted between the Visual Rays, as lm , mn , &c. measure the several Angles AEF, FED, &c. respectively; and consequently, the parts AF, FD, &c. of the Object AB, will appear to the Eye, at E, in proportion to the Arks lm , mn , &c. which are equally distant from the Eye.

All remote Objects appear equally distant from the Eye; as the Planets, Stars, &c. appear as if they were all in the same concave Sphere; of which every Eye is the Center. So likewise, when we stand in the line of direction of two, three, or more Objects, they appear to touch each other; nor could we judge, merely from sight; which was farthest off, or which the largest.

For, suppose the Visual Rays EF, ED, EB, produced to F , D and B ; any other Objects; as DF and BD , being in the same direction with FD , and DB , and touching the same Visual Rays (EF, EB) will coincide with FD and DB ; the less will hide the greater from Sight, and appear to be of equal magnitude.

2. I am somewhat surprized at what Mr. Hamilton has advanced, in Art. 6, Sect. 1. where he supposes, that Objects do not appear in proportion to the Angles they subtend, but in proportion to the Tangent of half the Angle; and imagines the Retina to be, nearly, a Plane, in the center of the seat of Vision.

How Mr. Hamilton could conceive and endeavour to account for such a strange and unwarrantable Opinion I cannot imagine. If, as he has truly asserted, the space on the Retina, so far as there is distinct Vision, does not exceed an Angle of one or two Degrees, on every side of the Axe; which is as much as to say, it does not exceed an Angle of two or four degrees; and I am fully convinced, that it does not exceed one Degree; what difference can it make, whether that small portion of the Retina be a Plane or a portion of a Sphere? he certainly could not suppose a large portion of the Retina to assume that Form.

Now, if the Retina does assume the form of a Plane, of the space ab , then it is certain that the Images, af , fd , and db , on that Plane, (of which, the Right Line ab may be supposed a Section) are not in proportion to the Angles AEF, FED, and DEB, equal aEf , fEd , and dEb , respectively.

DEM. For, the Tangent cf or cd is not in proportion to ca or cb as the Angle cEf is to cEa ; because, cf is equal fa (AF being equal FC) therefore, the Angle cEf is not equal fEa , or cEa double cEf . — — 3. 6. El.

But, ab is parallel to ab ; therefore, af , fc and cb are in the same Ratio, as a f , f c and c b . — — — — — Cor. to 6. 6. El.

Thus it is clear, if the Retina does take the form of a Plane, that Objects do not appear in proportion to the Angles they subtend; but, as it is the only instance in which I ever knew it contested, so, I am persuaded, it will never be universally adopted, on that Hypothesis. However, as it is merely conjectural, it seems to me more rational, to suppose the Retina to retain its original spherical Form, which the all-wise Creator has given it, than, that it should, continually, at every motion of the Eye, be changing to that of a Plane.

Wherefore, on the presumption that the internal Eye is truly spherical, it is very reasonable to suppose, that, Objects appear, exactly, in proportion to the Angles under which they are seen; and not in proportion to the Tangent of half the Angle, as Mr. Hamilton asserts, and has endeavoured to demonstrate, in 7 and 8 of the same Section: which cannot be as himself has stated it, but on the supposition of the Retina assuming the form of a Plane, which is absurd to suppose. Or

Plate I. granting it really so, the portion of the Retina, which the compass of an Angle of one or two Degrees, at most, takes up, is so very small, that it is impossible to distinguish it, in a Sphere of an inch in Diameter, from a Plane. As the 360th part of the circumference of a Circle, of a small radius, is not distinguishable from a Right Line; consequently, there could be no difference in the apparent magnitude of an Object, in either Case, which is seen under so small an Angle. As, the small portion at *d*, on the Retina, subtending an Angle, at *E*, of about five Degrees, is not distinguishable from a Right Line.

Fig. 1.

3. It is also evident, if the Visual Rays are refracted in passing through the aqueous Humour, before they enter the Pupil (unless that Refraction be rectified again at the Crystalline Humour or elsewhere, and, by that means, proceed in the same direction to the point of convergency, in the center of the Eye) that Objects cannot appear exactly in proportion to the Angles they subtend. Nor have I any conception how a Visual Ray, *AE*, falling remote on the Cornea, can be refracted so as to pass in the same direction through the Aperture, *hi*, to the point of convergency, at *E*, and from thence to the Retina.

For, suppose a Visual Ray, *Ae*, from any Point *A* (seen very oblique) falls perpendicularly on the Cornea, at *e*; if it pass at all through the Pupil, *hi*, whilst the Axe is directed to *C*, it must pass in the direction *eg*; where it falls on the Crystalline, at *g*, and where, it is possible it may again be refracted, in the direction *gb*, to the Retina; making an equal Angle with the Axe, (*EC*) as the original Ray, *Ae*; in which Case, the Image of the Point *A* is seen at *b*, in its proper position.

But, it is manifest, that, whilst the Axe remains in the direction *EC*, the point *A* cannot be seen, at all, nor even the point *F*, distinctly; the intimation we have of Objects, at a less distance from *C*, is so very confused, that, without moving the Eye, we scarce perceive their existence; of which, every Person's own experience will soon convince him. Unless the Axe be directed to any Point (as *E*) no perfect Image of that Point is formed in the Mind, or on the Retina. Consequently, the Rays go direct through the various humours of the Eye; as when we look through a Pinhole; for, how can it be otherwise? seeing, it is manifest they must all pass through the same Point; (*q* is then their common point of convergency) the Image formed on the Retina is, in every respect, the same (except in Hue, for want of sufficient Light) and of the same Magnitude as it appears to the naked Eye. Which single circumstance explodes, in my opinion, the notion of Refraction at the Cornea and through the various Humours, in the passage of the Rays to the Retina. For it is manifest, that, if the Point of convergency be at *q*, without the Eye, the Image will occupy more space on the Retina than at *E*, or any other Point within the Eye. I shall therefore suppose, that, when the Eye is naked, the Point of convergency of the Rays is in its Center; and, on that supposition, do infer, that Objects do appear in proportion to the Angles under which they are seen. Of which I will give an easy and familiar Example.

Fig. 3.

4. Let *AB*, *CD*, and *FG* be three Globes, at different Distances from the Eye, at *E*, and of different Magnitudes; seen under the Angles *AEB*, *CED*, and *FEG* respectively*.

Now, if the Distance of the Globe *CD* be equal to twice the Distance of *AB*, and *FG* be double of *CD*, four times the Distance of *AB*; then will the Angles *AEB*, *CED*, and *FEG*, which they subtend at the Eye, if the Diameters of the Globes are in the same proportion as their Distances, be equal; and consequently, their apparent Magnitudes, to the Eye, at *E*, are equal.

Draw *ES*, to the Center of the Globe *FG*; draw *CD* perpendicular to *ES*, and equal to the diameter of the Globe *CD*; also, draw *BA* perpendicular to *ES*, and equal to the Diameter of the Globe *AB*.

* The full Diameter (*FG*) of a Globe cannot be seen; for, the Visual Rays being Tangents to the Globe, from the same Point *E*, as *Ef*, *Eg*, are all equal (Cor. 2. 16. 3. El.) and, by reason of the convexity, towards the Eye, they cannot touch the two extremes of the same Diameter; unless the Distance be infinite; and then, the Visual Rays are considered as being parallel; *fg* is, therefore, the apparent Diameter of the Globe *FG*.

DEM. It is evident, that the Visual Rays FE and GE, from each extreme of the Diameter of the Globe FG, will pass through the extremes of CD and BA. For, in the similar Triangles FEG, DEC and BEA; $FG : SE :: DC$ (equal CD) : KE, and, as BA (equal AB) : LE. — — 6. 6. El. Then, since they touch the same Visual Rays FE and GE; consequently, they are all seen under the same Angle, FEG. — — by Theorem And, the Diameters, AB, CD and FG, being in proportion to their central Distances, are, therefore, all seen under equal Angles, AEB, CED and FEG. Consequently, the Images, ab, bc, and cd, on the Retina, subtending equal Angles, are equal; and therefore, the appearances of the three Globes AB, CD, and FG are equal. Q.E.D.

If the Globe CD be supposed to be moved to M, its central Distance, EM, being equal ES; and, being, in Diameter, equal half FG; it will subtend an Angle GEH, i. e. CEI, half CED, equal FEG; as the Arks 1 2 3 4, which measure those Angles, exemplify.

Consequently, the Image, (h c) of the Globe GH, at that Distance, is, in magnitude, but one fourth part of the Image (c d) of the Globe CD, of the same Magnitude, and seen at half the Distance; for, they are in the duplicate ratio of the portions of the Ark 30 to 34. — — Cor. 1. 13. 6. El.

5. Yet it is manifest, that two or more unequal Objects, at an equal Distance from the Eye, and seen direct, do not appear exactly of the proportion of their respective Magnitudes; likewise, equal parts of the same Object, on the same side of that Point where it is cut by the Axe of the Eye, do not appear equal.

Let AB be an Object, of length simply, bisected by the Axe EC, to which it is perpendicular, in the Point C; and suppose AC and CB again bisected, in F and G. Fig. 1.

Then, the parts AF, FC, &c. are equal, and AB is double FG; but, they do not appear of that proportion, at E.

DEM. Now because AF is equal to FC, the Angles AEF, FEC, under which they are seen, are unequal — — — — 3. 6. El.

For, if the Angle AEC be bisected, the Right Line EO, bisecting it, will cut AC, in the Point O, in the ratio of EA to EC.

But, EA is greater than EC; for, EC is perpendicular to AC.—Cor. to 12. 1. El.

Wherefore, AO is greater than OC, and the Angles AEO, OEC, under which they are seen, are equal; consequently, the equal parts AF, FC, are seen under unequal Angles; therefore, they appear unequal, by the Theorem.

And, because the Angle CEF is greater than FEA, the equal part CF will appear greater than FA,

After the same manner it may be proved, that the equal part CG appears greater than GB. Consequently, $CG \neq CF$, i. e. FG, appears greater than half AB. Q.E.D.

COR. Hence, it is evident, that if AB be produced, and the equal divisions, CG, GB, BP are continued, they will appear continually less, being seen under less Angles. For the Angle BEP is less than BEG, which, is less than GEC; as the measures, po, or, and rC, of those Angles, sufficiently evince.

N. B. The farther any two or more Objects are from the Eye, the nearer they will appear in the proportion of their respective Magnitudes.

Because they are seen under less Angles; and, the Tangents of small Angles deviate less from the ratio of the Angles, than the Tangents of greater Angles.

6. As Objects of different Magnitudes, at equal Distances, do not appear in the proportion of their respective Magnitudes, so neither do Objects of equal Magnitude, seen at different Distances, appear exactly in the ratio of their several Distances.

Let

Plate I.
Fig. 4.

Let AB be an Object at any Distance (EF) from the Eye, at E ; and, let CD be another Object, parallel to AB , of equal Magnitude, as to length simply; also let CD be at twice the distance of AB (EG double EF). And let them be so situated, that, right lines, AC and BD , joining their extremes, are parallel to EG i. e. perpendicular to AB and CD .

By means of the Visual Rays EA , EB , &c. those Objects are seen under the Angles AEB and CED . I say, the Angle AEB is not double of the Angle CED .

DEM. Because AB is parallel to CD , $EF:EG::Ec:EC$, as $cF:CG$. 6. 6. El.
But EF is equal to $\frac{1}{2}EG$ (Hyp.) wherefore, $cF = \frac{1}{2}CG$, equal AF ; that is, AF is bisected, in the point c .

Now because Ac is equal to cF , the Angle Aec is less than cEF 3. 6. El. for, if they were equal; Ac would be to cF as EA to EF .

But, EF is perpend. to AB , therefore, EF is less than EA . Cor. 3. 12. 1. El.

But Ac and cF appear (to the Eye at E) in proportion to the Angle under which they are seen; i. e. as Aec to cEF ; (by Theo.) and cF appears equal to CG , being seen under the same Angle; consequently CG appears greater than half AF .

By the same reasoning, GD appears greater than half FB .

Therefore, CD appears greater than half AB ; i. e. the Angle CED is greater than half AEB ; although CD is equal to AB , and the distance double.

Let IK be another Object, equal and parallel to AB . and at three times the Distance, from the Eye, at E .

I say, that IK will appear greater than one third of AB .

Produce EG to H , and IK to L , making KL equal to HK ; and draw EK and EL . Then will AF be trisected, in k and l .

DEM. Because AF is parallel HK , $kF:KH::EF:EH$ } — 6. 6. El.
And, because KL is equal to KH , kl is equal to kF . }

But, EF is equal to one third of EH ; consequently, Fk and kl are, each, equal to one third of AF , equal KH .

Then, because Fk , kl , and lA are equal, the Angles FEk , kEl , and lEA are unequal; for, if the Angle FEk , was equal to kEl , Fk would be to kl as EF to El . — — — 3. 6. El.

But EF is perpendicular to AB ; wherefore EF is less than Ek ; consequently, Fk would be less than kl .

And, for the same reason, if the Angles kEl and lEA were equal, kl would be less than lA ; because Ek is less than EA .

But Fk , kl , and lA are equal; therefore, the Angle FEk is greater than kEl , and kEl than lEA .

Therefore, Fk , i. e. HK , appears greater than one third of AF .

By the same reasoning, it is manifest, that HI appears greater than one third of FB , equal HI ;

Therefore, IK , appears greater than one third of AB . Q. E. D.

Thus, I have made it appear, to Demonstration, that Objects do not appear in that proportion to each other, as their respective Distances; in which I have been more particular, because, Mr. Kirby has expressly said, Page 13, that these Pictures (i. e. the appearances of Objects) will be to each other, as their several Distances are to each other*. Rohault says, nearly so; Page 243.

* The Appearances, i. e. the Angles under which Objects are seen, being in proportion to their Distances is, literally, absurd; because, as the distance of an Object is increased, the Angle, under which it is seen, decreases; which kind of reciprocal Proportion is always to be understood in this Case.

7. If the Eye be moved from E to O, making OF equal to FH, and OI, OK be drawn, EB will be cut in the same Points, c and d, as by the Visual Rays EC, ED. But, IK will not appear of the same proportion to AB, at the Point O, as CD, at E, the Objects AB, CD, and IK being equal.

DEM. Because AB, CD, and IK are parallel, the Triangles EcF, ECG, also, OcF, and OKH are similar; wherefore $EF : EG :: cF : CG$, and, $OF : OH :: cF : KH$. — — — 4. 6. El.

But, EF is equal to half EG, and OF is equal to half OH; consequently, cF is equal to half CG, and also to half HK; equal CG.

By the same reasoning, Fd may be proved equal to half GD or HI.

Therefore, AB is cut in the same Points c and d; as it was affirmed.

2. CD appears to the Eye, at E, in proportion to AB, as the Angle CED to AEB; and IK appears, at O, as the Angle IOK to AOB.

Now, the ratio of the Angle CEG to AEG is greater than KOH to AOH.

For the Angle AEG is greater than AOF, equal CEG;

wherefore, the ratio of cEF to AEF is greater than cOF to AOF.

For, let Em be drawn parallel to OK; the Angle FEm is equal FOc consequently, the ratio of FEm to FEc is equal to that, of FOc to FOA, equal FEc.

But Fc is equal to cA (proved) wherefore, Fm is equal to mc.

And, equal Parts of a Right Line subtend less Angles, the farther they are from a perpendicular to the Line, from the Eye; proved.

Therefore, the Angle FEm to FEc, i. e. FOK to FOA, has a less ratio than FEc to FEA, i. e. than GEC to GEA, being nearer equality. Consequently, HK appears less, in proportion to AF, at O, than CD at E.

After the same manner, HI may be proved to appear less, in proportion to FB, at O, than GD at E.

Therefore, IK appears less, in proportion to AB, at O, than CD at E. Q.E.D.

Hence it is manifest, that, notwithstanding the sections of the Visual Rays EC and OK, ED and OI, with AB, are the same; the Distance of the Eye on one side of AB, and the Object on the other being proportional; yet, the appearances of the Objects, to each other, are varied, considerably, as the Distance is greater or less.

For, two Objects of equal height, at a great Distance from each other, and parallel between themselves, and the Eye at the same Distance from one of them, and in a Right Line with both, will appear, in proportion to each other, nearly as their Distances; i. e. the farthest off will appear half the height and half the width of the nearest, being at twice the Distance, and, one third in height and width at three times the Distance, nearly, i. e. somewhat more.

As IK appears, at O, somewhat more than half AB. But, CD (equal IK) appears much greater than half AB, at E. Consequently, if the Eye be at a greater distance from AB, than at O, and the Object at an equal distance on the other side, they would appear nearer in proportion to their Distances, reciprocally; and therefore, when the distance of both is so great, that the Visual Rays, OI, OK, approach nearly to parallelism (which, it is obvious must be infinite, before they can be perfectly so) they will appear nearly as their Distances. Notwithstanding, on a Picture parallel to the Objects, they must ever be represented in the ratio of their respective Distances, and Magnitudes.

THEOREM II.

Parallel Right Lines, however situated, being produced, appear to approach towards each other; and, if produced infinitely, they will appear to meet in a Point, at an infinite Distance*.

Plate I. Let IF and HG be two parallel Lines; and, E, an Eye, situated any how between them. Let K, B, D, and G represent various Distances in the Line HG, and let the Lines IK, AB, &c. be drawn perpendicular to HG, cutting the other Line (also perpendicularly) in the points I, A, C, and F. Also, let the Visual Rays EA, EB, &c. be drawn.

DEM. The space between the Lines, IF and KG, at the several Distances ES, EO, &c. appear, to the Eye at E, under the several Angles IEK, AEB, &c. each of which, as the Distance is farther from the Eye, is less. C. 14. 1. El. For, the Space between the parallel Lines is, every where, equal. Def. 7. Geo.

The Space, at IK, is seen under the Angle IEK; and the space between them, at AB, under the Angle AEB; which is less than IEK, nearly in proportion to the Distances, ES to EO; CD subtends the Angle CED, and FG the Angle FEG; each of which is less than the former; for their Subtenses, AB, CD, &c. are equal; and the Lines EA, EB; EC, ED, &c. forming or containing the Angle, are continually longer; wherefore, the Angle FEG is less than CED, which is less than AEB†, &c. — Cor. to 14. 1. El.

From which it is manifest, that at a greater Distance the Angle will be still less; consequently, it will, at last, become insensible, the interval, between the Lines, is lost to Sight, and, the Lines appear to meet each other. Q. E. D.

Hence, the Lines, IF and HG, are said to vanish; and consequently, the space between them vanishes also.

2. Let ER be drawn, from the Eye, parallel to IF and HG; those Lines will appear to approach, continually, towards ER; and being produced, infinitely, they will all appear to meet in the same Point; however ER may be nearer to one than the other; for, the farther any parallel Line is from ER (which may be called its Radial) the more sudden is its apparent approach to that Radial,

The Space IA, of the one, appears to advance from *b* to *i*; an equal Space KB of the other, advances from *p* to *o*; which is less than the other, in proportion to the Angle KEB to IEA; i. e. as the Ark *po* to *bi*. The Space from A to C appears to advance from *i* to *k*; an equal Space, BD, in the other, from *o* to *n*. From C to F it advances from *k* to *l*, an equal Space, DG, from *n* to *m*; in the proportion of the Arks *kl* to *mn*.

It is evident, that if those Lines were produced longer, they would still appear to approach nearer to the point *e*, as EL, EM, evince. And, being infinitely produced, they would at last appear to terminate in the Point *e*, and be lost to sight.

For, it is manifest, that the farther any Points, L and M (considered as being in the parallel Lines IF and GH) are distant, the nearer the Visual Rays, EL, EM, are to a coincidence with ER; which, it is evident, must be infinite before they can coincide perfectly; in which case, EL, EM, and ER will be the same; for they will all appear to unite.

* It is said in Smith's Optics (Vol. I. Art. 156, Page 52) that parallel Lines, seen obliquely, appear to converge more and more as they are farther extended from the Eye; which, will ever be the case however the Eye is situated, in respect of the parallel Lines. For, being seen obliquely, means nothing more than when we look towards either of their extremes; as they are seen direct, only where a Line, from the Eye, cuts them both at Right Angles; on either side of the Perpendicular they are necessarily seen oblique.

† When the Angle, AEB or CED, under which any Object, as AB, is seen, does not exceed 20 or 30 Degrees, at the most, and that Object is removed to the several Distances, from EO, to EP, EQ &c. the difference between the Tangents OA, OC, &c. deviates but little from the Arks OC, OC, &c. and not still less as the Angle is less; as PC, PF, &c.

3. Hence, it is easy to account for the appearance of a horizontal Plane, or continued level Surface; which, being below the Eye, will gradually appear to ascend; and, being above the Eye, it appears to descend; and, if they were produced infinitely, they would appear to meet in a Right Line, on a level with the Eye.

Let the Right Line HG be a section of a horizontal Plane, below the Eye, at E ; and let IF be a section of one above the Eye. Also, let IK be a section of a Plane direct before the Eye* (at E) perpendicular to the other two.

Now if the Visual Rays, EA , EB , EC , &c. be drawn, they must necessarily cut the Plane of which IK is a Section; in a , b , c , &c.

It is evident that the Space KB will appear to rise, towards S , on the Plane IK , from K , its intersection, to b , where the Visual Ray EB cuts that Plane; the space KD will rise to d , and KG to g .

For the same reason, the space IA will appear to descend from I to a ; IC , from I to c ; and IF , from I to f ; each Plane appearing gradually to approach towards S , where a Perpendicular, ES , to that Plane, cuts it.

And, if the Planes were produced infinitely, they would at last vanish, or be lost to sight, in a Right Line passing through the Point S , parallel to the intersection of the other two Planes, with the Plane IK .

And this is evidently the case however a Plane be situated; whether horizontal, vertical, or inclined, it is the same; for, a Plane is still a Plane, in all positions, and has no properties peculiar to any position, in respect of the Horizon.

Therefore, there may be drawn this Conclusion, that every Plane, in which the Eye is not situate, will appear to approach towards, and at length to meet, another Plane, passing through the Eye, parallel to the former.

4. Let AB , CD , and FG represent three Objects of equal height, at the several Distances, EO , EP , and EQ , from the Eye, at E .

The Visual Rays EA , EB , EC , &c. being drawn, those Objects will appear in proportion to the Angles AEB , CED , and FEG .

For, the portions of an ark of a Circle, whose Center is E , intercepted between the Visual Rays, measure those Angles, respectively, and determine the apparent proportions of the Objects AB , CD , and FG to each other; seeing that, the Angles which they subtend are in the same Ratio. — — — 19. 6. El.

Wherefore, since, in Perspective, the Representation is always on a Plane, and not on a spherical Surface; suppose IK a section of a Plane, in a direct position; the Spaces, $a b$, $c d$, and $f g$, between the Points where the Visual Rays cut and pass through that Plane, considered as a Picture, are the proportions of the Representations of the Objects, AB , CD , and FG ; but their apparent Magnitudes are the portions io , kn , and lm of the Ark bep .

Let HG represent a section of the level Ground, or any other horizontal Plane; and AB , CD , and FG , Objects perpendicular thereto.

If IK be supposed a section of a vertical Plane, the foot or seat B , of the Object AB , will appear, on that Plane, at b ; and consequently, Kb will represent the space of Ground, KB , between the Picture and the Object AB ; i. e. it will appear to rise so high on the Picture; Kd will represent the Space of KD , and Kg of KG . So likewise, a , c , and f shew how much the tops of those Objects appear to descend, on the Picture.

5. Now, if the position of the Picture be inclined to the Horizon, as aK , the Objects and the Eye remaining as before; the Representations ab , cd , and fg , on that Plane, are very different from the proportions of ab , cd , &c. on the Plane IK .

* By being direct before the Eye, I would be understood to mean, in a vertical Position; and when a Right Line, from the Eye, falling in the middle, between its Bounds, cuts the Plane perpendicularly.

For, if the Eye be not in the Plane, or in a continuation of it, the Eye, being considered as a Point, can have but one position, either to a Plane or to a Right Line, being produced; however the Plane may be situated, in respect of the Horizon.

But,

Plate I.
Fig. 5.

But, the apparent proportions, of both, are the same, at E; viz. the portions on the Ark *hep*, which measure the Angles AEB, CED, and FEG.

Nor, is it possible to be otherwise; seeing that, the Objects, AB, CD, &c. remain the same, in respect of each other; and the situation of the Eye, is not varied; consequently, the Angles AEB, CED, &c. are not altered; and the Objects AB, CD, &c. the Subtenses of those Angles, must, necessarily, appear the same, represented on any Surface whatever, and in any Position, cutting the Visual Rays EA, EB, &c.

N. B. The Proportions *CD*, and *FG*, &c. on *AB*, are the same, as *cd* and *fg* on *IK* (*AB* being parallel to *IK*); wherefore, the Triangles AEB, *aEb*, also CED, *CEd*, and *cEd*, &c. are similar; and therefore, as *fg* : *cd*, or to *ab*, :: *FG* : *CD*, or to *AB*. — 4. 6. El.

Consequently, Visual Rays, being cut by parallel Planes, in any position, are not only themselves cut in equal Ratio, but also the Sections, or the Projections on the Planes, by their sections with the Rays. — 1. 8. El.

And, their Proportions are as their Distances ES, EO, EP; i. e. as *fg* : ES :: *FG* : EO as *Fz Gz* : EP, and, as *FG* : EQ; (6. 6. El.) and so of any other.

Hence, it is manifest, that the Sections of any system of Rays, by parallel Planes, are similar.

P R O B L E M.

Fig. 6.

A Right Line (*AB*) being obliquely situated in respect of the Eye, at E, to determine the Point D, to which if the Axe of the Eye be directed, the two Parts, AD and DB, shall be seen under equal Angles.

Without drawing the Visual Rays, or bisecting the Angle made by the Rays.

Make AD to AB, as the distance of the Point A, from the Eye (E) is to the distance of the same Point A, added to the distance of B. — Pr. 32. El.
Then, AD : AB :: EA : EA + EB; and, AD : DB :: EA : EB.

DEM. The Visual Rays EA, ED, EB being drawn, the Angle AEB is bisected by ED; wherefore, AD and DB are seen under equal Angles, at the Point E, For, if the Optic Angle AEB be bisected by the Right Line ED; then is AD : DB :: EA : EB. — 3. 6. El.

2. If the Axe of the Eye be directed to C, the middle Point of AB; then will the Point B be more distinctly seen; because its Image falls nearer to the Image of C than the Point A. And, if the Ark *cf* be made equal *ca*, and *Ef* be drawn, meeting AB produced, in F; AC will then be to CF as EA to EF; and, notwithstanding the difference between AC and CF, their Images on the Retina, and consequently their apparent Proportions, are equal, being seen under equal Angles.

Now, if those Visual Rays are cut by a Plane in the position IK; the part *ac*, the greater portion of *af*, will represent AC, the lesser Segment of AF; and *cf*, the lesser portion, represents CF, the greater Segment; For, since the Optic Angle AEF, is bisected, by EC; *ac* : *cf* :: EA : EF; 3. 6. also, as AC : CF :: EA : EF; consequently, *ac*, *cf* represent AC and CF.

3. If the Right Line AB be produced, at the extreme A, cutting the Plane IK, at I; the whole indefinite representation of that Line, on the Plane IK, is the section IK, of a Plane passing through the Line AB and the Eye, and terminated by a parallel Line from the Eye (EK) to that Plane.

For, because EK is parallel to AB, they will appear to meet in a Point at an infinite Distance; † which is represented by the Point K, on the Plane IK; for, seeing that the Line EK passes through the Eye, its whole appearance is but a Point; consequently, the Point K represents the whole Line EK, produced infinitely; and AB, infinitely produced, will appear to meet EK in that Point.

If the Line AB be produced on the other Side of the Plane IK, and any Point (G) be assumed, between the Eye and the Plane; its representation, on that Plane, is the Point g, in which the Right Line EG, produced, cuts KI produced; the part Ig, beyond the Intersection, I, represents IG, the portion of the Line, A-B, lying on the side of the Plane, IK, towards the Eye; as Ia or Ib represents the part IA or IB, on the other Side; all which is so very obvious, from inspection of the Diagram, it is needless to say more about it.

4. After the same manner as the apparent magnitudes of Objects are determined, so are their apparent Distances, or Bearings, in respect of each other; viz. by the Degrees on the Ark of a Circle, which measure the Angles they make at the Eye.

Let C, D, and F, be three Objects, and E the place of the Eye.

Draw the Visual Rays CE, DE, FE; then, the Angles DEC, CEF are the Optic Angles of their apparent Distances, or Bearings at the Station E.

Notwithstanding their real Distances from each other, CF is nearly double that of CD, yet, the Angle CEF, is much less than the Angle DEC; as it is evident, from the portions, cf and cd, of the Ark df.

If *fd* be supposed the section of a Plane, the appearances, or places of those Objects thereon, are at *f*, *c*, and *d*.

Fig. 7.

5. Suppose C, D, and F to be Stars, in the unbounded Expanse of the Heavens, at an immense distance from each other and from the Eye, at E.

It is impossible to form an Idea of their real Distances or Situations, in respect of each other; for, if the Star C was at B, and F at G, or any where else in the direction EG, their apparent places are still at *c* and *f*, in an imaginary Arch of the Heavens, as it appears to us, equally distant in every part.

Now, since the whole Diameter of the Earth's Orbit is not sufficient to make any sensible difference in their Bearings, and, consequently, of their apparent Places, in the Arch *df*, there cannot be any positive Idea formed of their real Distances; for the portions *fc* and *cd*, of the Arch *fd*, are the measures of their apparent Distances, only; i. e. of the Angles *dec*, *cef*, or the Originals of these Angles DEC, CEF under which they are seen.

Hence is constructed the Celestial Globe or Sphere, which is a true Picture of the Heavens, of Stars, &c. divided into various Figures called Constellations.

If *def* be supposed a portion of a Sphere; or, let HI be an entire Globe.

It is evident, that from its Center, E, which is also the Center of the Arch *df* (E being supposed the Eye of a Spectator) the Star C will appear on the Globe at *o*, and D at *s*, &c. in the same position, situation, and distance, in proportion to the Radius, EH or Eo, to Ec, as in the imaginary Arch of the Heavens, *df*. So that, whether the real Star be at A, B, or C, or any where else in the Right Line EC; its apparent or representative Place, on the Globe HI, will still be at *o*; and consequently, can make no difference, in its true Place thereon; but its apparent Magnitude will be in proportion to its Distance, nearly.

6. Let AB, DF, and GH be three Objects, promiscuously situated in respect of the Eye, at E. It is manifest, that, whilst the Eye remains at E, the three Objects must necessarily appear the same, upon any Surface whatever, cutting the Visual Rays EA, EB, &c.

Fig. 8.

Suppose *ah* a section of the Rays, made by a Plane, and ECa Perpendicular from the Eye to the Plane, cutting it at C. Then, the Visual Rays EA, EB, which are nearest to the Point C, cut that Plane less oblique than EF; and EG, EH are still more oblique, as they are more remote from C. Whereas, on a spherical Surface of which *ahb* may be supposed a Section, being an ark of a Circle, described on the center E, the Rays EA, EF, EH, &c. are all perpendicular

Plate I. Fig. 8. perpendicular to its Surface, being perpendicular to the Ark Cfb . Wherefore, since Ea , Ed , Eg , &c. are equal, for, they are all Radii; consequently, the surface of a Sphere is equidistant from its Center, in every part; and, the Representations, ab , df , and gb , of the Objects AB , DF and GH , on its Surface, are also their true Appearances, from the Point of View E .

But, on the Plane, or its Section, ah , the Object AB , has its Representation, ab , nearly equal to ab on the Curve; whereas, df , it is obvious and demonstrable, is larger than df ; and gh still larger than gb , as it is more remote from the Point C ; nevertheless, its true appearance is gb on the Curve, or spherical Surface; for, they both represent the same Object, from the same Point of View, which cannot vary in its Appearance.

Therefore, the Representations of Objects, on various Surfaces, and in various Situations, although they may differ greatly in Figure, will, if truly represented, appear the same in the true Point of View.

The three Objects AB , DF and GH , though of various dimensions, appear equal; not owing to their Distances, but to their Positions in respect of the Eye, being seen under equal Angles, AEB , DEF and GEH ; as the portions ab , df , and gb , of the Ark Cfb , indicate. Whereas, their Representations on the Plane, of which ah is supposed a Section, are unequal; for, ab , the representation of AB , being nearest to the Center (C) is the least; df is greater than ab , and gh than df .

Yet, it is manifest that the Representations, ab , df , and gh , have the same appearance, to the Eye at E , as their corresponding Images, ab , df and gb , on the spherical Surface. For, the Eye being in the true point of View (at E) they are both seen by the same Visual Rays as the original Objects themselves; and consequently, under the same Optic Angles. Wherefore, their Appearances are the same on either Surface; by Theorem 1st.

From what has been advanced in this Section, it is evident that there is a manifest difference between the Representation of an Object, on a Plane, and its true Appearance; which difference is the greater, as the Eye is nearer to the Object. For, since the Visual Rays must necessarily cut any Plane, on which they fall, more and more oblique, the farther their Intersections are from that Point, in which a perpendicular Line from the Eye would cut the Plane; consequently, the Representations of Objects, on a Plane, cannot be in proportion to their true Appearances, but deviate continually, more and more, as they fall farther from the Point described. Whereas, on a spherical Surface, which is every where equally distant from its Center, every Right Line, and in every direction, which tends to its Center, is perpendicular to its Surface, and none of them can be said to cut it oblique; consequently, the true Representations, and also the true Appearances of Objects, can only be depicted on the Surface of a Sphere; that is, the Representation on its Surface and the true Appearance of the Object, from the Center, as the Point of View, are the same.

S E C T I O N IV.

Containing some objections to the received Opinion of the Cause of VISION.

IN the last Section I have explained, demonstrated, and briefly illustrated the whole Theory of Direct Vision; or so much, at least, as is essential, to give a clear and just Idea of Perspective. To dwell longer on it, in this place, is unnecessary; seeing that, Perspective itself, in Theory, is nothing more than a continuation of the same Science. But, before I enter upon the subject of Perspective, I shall, in this Section, give my objections to the received Opinion of the cause of Vision

It

It is not an easy matter to remove Prejudices, which are deep rooted; especially if they are strengthened with the sanction of Men of known abilities. I am at a loss to conceive why it is, that the generality of mankind so willingly impose on their own judgments, or are so ready to be imposed on by others; eager of being amused with deep and profound Discourses, pretending to penetrate into the mysteries of Nature; which, neither themselves nor their Authors know any thing of. In fact, we see and know nothing further than the surfaces of Bodies; the formation of them, or their constituent parts, we are, and ever shall be, totally ignorant of. We daily see the effects of one Body acting on another, and, by repeated Experiments, we find their full force; that they are regularly and uniformly the same; but, the true causes of those effects are to us unknown; we cannot form an adequate Idea, how, and in what manner they act, and produce such wondrous effects, although so simple in their Nature.

We have not the least Idea how a Solid can become a Fluid and a Fluid become again a Solid, by different degrees of Heat; what Fire is we know not, nor how it acts on Bodies, so as to dissolve some, to change the nature of others, and, in a great measure to annihilate them. We know as little of the Element of the Air we breathe in; or, by what means it is often so violently agitated in one place (as in high Winds) whilst it remains at rest, or has a contrary motion in another. We know not how a Solid is dissolved by a Fluid, and the Solid become perfectly liquidated; how two different Fluids mix and incorporate, so as to be beyond the power or art of Man to separate again, if they have an equal specific Gravity; nay two Solids, as Metals, &c. may be so incorporated as to be quite another kind of Body; and the constituent parts, of each, beyond the power of Art to dissolve, or to distinguish them apart. Some again may be separated, if they have not the same specific gravity, as Gold or Silver, from any other baser Metal. Some Fluids can never incorporate; as Oil or Spirits with Water, &c.

How a Fluid is composed, or what form its minutest particles are of, we are totally ignorant. Is it not ridiculous to suppose Particles, of any Figure? It is the common Opinion that they are globular; because, the least drop of Water or other Fluid assumes that form, than which, nothing is more absurd; because, perfect Globes may be piled into a Pyramid, as firm and permanent, almost, as Cubes, with very little lateral pressure; consequently, a Fluid might be suspended in the same Figure, if its constituent Particles were solid Globes. For, on the supposition of Particles, they must necessarily be solid, or each Particle must, again, be composed of Particles.

To compare a Fluid with fine Sand is truly ridiculous. Sand will support itself in a pyramidal form; and, if the grains were perfectly globular, it would always assume one, of the figure of its Base, terminating in a single grain at the Vertex. It is not so with Fluids; a Fluid cannot rest till it recovers an equilibrium in every part, presently acquiring a perfectly level Surface. Besides, if its Particles were globular, there must be interstices between them filled with Air, which cannot possibly be incorporated with Water, as, in that case, it would be, to all sense. Also, if Metals, &c. when in a fluid state, by means of Fire, were composed of globular Particles, it is reasonable to suppose they would remain so when fixed; consequently, they would be extremely porous, and admit Air to pass freely through, which is repugnant to common experience. No, say those acute reasoners, it is by reason of the Particles changing their Figure that they become fixed. Amazing truly! that the least particles of Matter, which I should suppose (on their Hypothesis) indivisible, consequently impenetrable and immutable, by any means, can be varied in their Shape or Figure. It is also amazing, that, by being globular, they are so easily separated, when in a Fluid state, and, when fixed, are so extremely difficult to dissolve.

If the Particles of solid Bodies are kept together only by cohesion, why does not the same power act on them in their fluid state? or why, when the particles of a
solid

solid Body are, by any means, separated, do they not cohere together again? or, what reason can be given, why they should cohere more strongly one way than another; as in Wood of all kinds? It is downright Sophistry, all; a language without meaning. That Bodies may be separated into Parts or Particles is undoubted; but, that they are composed of Particles, is what I cannot give them credit for. Every Body, whether Solid or Fluid, before any of its parts are separated, is one entire Mass of Matter.

Again, if Fluids are composed of Particles, and allowing them to be transparent in their nature, their globular form would render them, in some degree, opaque; so that, a Fluid would not admit of distinct Vision through it. Now, I am clearly of opinion that all the parts of a Fluid lie perfectly close to each other, without any Cavities interspersed; it is impossible there can be any; consequently, there is not the least particle of Air contained in Fluids. If any dissoluble Body be immersed, as Sugar, &c. in Water; does not the Air, which is contained in its cavities, immediately ascend to the Surface, as soon as it is at liberty, by the dissolution of the immersed Body?

The cause of Transparency, either in Fluids, or, more particularly, in solid Bodies, is, to us, an impenetrable mystery. It is astonishing that there should be distinct and perfect Vision through Glass, or pellucid Stones; and, with all the knowledge and penetration human wisdom can pretend to, we cannot form an Idea tending to comprehend it. Yet, this aspiring, presumptuous, all-sufficient Man, would pretend to account for the cause of Vision, itself, and for all the other Senses; nay (if we will take their Words for it) for every operation in Nature. But I shall make free to tell them, plainly, that they cannot account for one. First, let them account for their own Existence; can they tell where the first spring of self-motion begins? no, not so much as for the motion of a Finger.

As for the generality of Mankind, they neither do nor are capable of judging for themselves, in many cases; and only give their assent, or dissent as they are influenced, or depending on the judgment of others, whom they either know or believe to have Judgment. That this is the case, in general, is certain; and is the reason why the opinions of some great Men are, though ever so arbitrary, adopted as orthodox; because, there are but few who dare venture to contradict them, and think or judge for themselves. And, because those Persons have advanced some received and established truths, these therefore conclude, that all which they presume to advance must necessarily be so.

Those profound Gentlemen knowing their own importance, and the deference which the World pays to their Opinions, are but too apt to be intoxicated with it, and often take too much Consequence to themselves; imagining, that they are really endowed with a penetration, into the mysteries of Nature, beyond the rest of Mankind. Or, from the knowledge they have of Mankind, the great deference they pay to their own superiority over them, and the implicit assent which is given to all their productions, make them forget, that they, as well as the rest, are fallible.

Respecting Vision. It is said in Smith's Optics, Page the first, and that from the Authority of Sir Isaac Newton, "that we shall find it difficult to conceive, "Light to be any thing else but very small and distinct particles of Matter, incessantly thrown out from shining Substances; and every way dispersed by Reflection from all others." I freely own, that I am at a loss to conceive how it can be Matter, and that for several reasons which may be given.

First; it is an indisputable Maxim, universally allowed by all Men of common understandings, that two Bodies cannot occupy or fill the same Space at the same Time; even Air cannot be where there is already any other Body. Steam, or Water expanded, almost, if not totally, excludes Air; consequently, no other Body can occupy the same Space where there is Air. Now, since the whole of Space, within our Atmosphere, at least, is filled with Air, how can it be also filled with Light, if Light be, also, a Body, composed of Matter? for, since it is manifest, to ocular

conviction, that, in full Day Light, there is no part of the Space around us but what is wholly illumined; consequently, being wholly, and in every part, filled with Light, if it be a Body, Air must be totally excluded; agreeable to the general Maxim, that two Bodies cannot fill the same Space. But, I presume, we do not find that to be the Case; for, the Space is filled with Air, and cannot possibly be excluded by Light.

I expect it will be alledged, that Air, being a Fluid, is composed of globular Particles; and that, Light fills up the interstices between them. Of what shape and magnitude, then, are the particles of Light, compared with the particles of Air? If Light be an existing Medium it must also be a Fluid; consequently, its Particles are also globular. And, if it be supposed to proceed from the Luminary, progressively, in Right Lines, in all directions, I own I am at a loss to conceive how that can possibly be effected, through innumerable globular particles of Air; to say nothing of the inconceivable velocity of its motion, being supposed to proceed from the Sun to the Earth in seven minutes and a half. For which extremely nice calculation we are indebted to Roemer; who first observed, that the Eclipses of the Satellites of Jupiter happen, when in Opposition, that is, when the Earth is in a Right Line between the Sun and Jupiter, fifteen minutes sooner than when in Conjunction, or the Sun in a Right Line between the Earth and Jupiter*. The difference of the distance of Jupiter from the Earth, at these two periods, is the whole Diameter of the Earth's Orbit; computed at more than 160 millions of Miles; half of which is the Distance of the Sun from the Earth. From which curious observation it is conjectured, that, Light is projected, from the Sun to the Earth, in seven or eight minutes; that is, the Sun being supposed to be continually emitting Light, it fills a concave Sphere, equal in Diameter to the Earth's Orbit, in that Time. Quere; how is the Sun supplied? and, being Matter, what becomes of it?

Now, admitting this extraordinary observation to be truly calculated, it undoubtedly proves that Vision is not instantaneous, at any distance; which seems reasonable; but I cannot see that it proves the progression of Matter from Jupiter to us. Besides, the Light which proceeds from Jupiter, or his Satellites, is but reflected, at second Hand; perhaps, Light, coming immediately from the Sun, may be much swifter in its motion. I am almost of opinion, that if it were possible to screen the Sun from our sight, by the interposition of an opaque Body, close to its Surface, being removed in an instant, that we should see the Luminary instantaneously; and, the whole Space between be instantly illumined.

The Law of reflected Bodies, that is, the velocity with which an elastic Body rebounds, according to the force by which it is first impelled, I am not acquainted with. But, of this I am well convinced, that the velocity of any impelled Body is not uniformly regular, but is continually varying, and the Body, in motion, tending to rest; consequently, it cannot acquire an additional force by striking against another Body, at rest, so as to rebound with the same velocity by which it was at first impelled; and, if Light, as a Body, acts on the same Principles, it must come, immediately from the Sun to us, with a greater velocity than when reflected from any other Body.

The Distance of Jupiter from the Sun is computed at 424 millions of Miles; and the Earth but 81 millions; which, is not one fifth part of the Distance of Jupiter. Consequently, the velocity, with which Light is first impelled, must be greatly diminished before it reaches Jupiter, and more so in being reflected back from Jupiter to the Earth. I shall endeavour to illustrate what I advance.

* It very rarely, if ever, happens, that they are in a Right Line, in a strict sense; because, the Plane of the Orbit of Jupiter is inclined to the Ecliptic, that is, to the Plane of the Earth's Orbit, in an Angle of one Degree and 20 Minutes. But, the Line of the Nodes, that is, the Line of intersection of the Planes of the two Orbits, is supposed to have a revolution (as the Moon's Orbit in 19 Years) but in what time is not, I believe, as yet determined. Consequently, they can never be perfectly in a Right Line, but only when the Line of the Nodes passes through the Centers of all the three; viz. the Sun, the Earth, and Jupiter. Wherefore, at a Conjunction, Jupiter will appear either above or below the Sun, according as he is in the upper or lower part of this Orbit; and, at an Opposition, the Earth will be either above or below a Right Line passing through the Centers of the Sun and Jupiter.

Plate II. Let S represent the Sun, in the center of our System, of Planets, &c. and
Fig. 9. CDEF the Earth's Orbit, whose semidiameter, SE, is 81,000,000, of Miles.

Let AB be a portion of the Orbit of Jupiter, and J the place of Jupiter therein; whose Distance, SJ, is above five times SE or SC. Let ab be the Orbit of one of Jupiter's Satellites, in which the Satellite is supposed to be, at a, emerging from behind Jupiter, after an Eclipse.

2. It is affirmed that we see Objects, only by means of material Rays of Light, reflected from them to the Eye; which impress on that Organ the perception of Vision.

Suppose the Satellite, at c, to be in a direct Line behind Jupiter; it is plain that it could not be seen at C or E, because it would be hid by the Body of Jupiter; but, as soon as it emerges, that is, as soon as it can appear, as at a, in the Right Line Ca or Ea, it becomes visible; not, merely, because there is no other Body between, but because it becomes illumined from the Sun, in the Right Line Sa; which, whilst behind Jupiter, from a to b was deprived of Light.

Let Sa represent a Ray of Light from the Sun to the Satellite at a; the Earth, at C, being directly between. The same individual particles of Light, which pass by the Sun to Jupiter, pass by the Earth to Jupiter, and back again, before either Jupiter or the Satellite can be seen; according to their Hypothesis.

It is reasonable to suppose, that the velocity with which those particles of Matter first set off, from the Sun, is greater than when they arrive at the Earth; how much, then, must it be decreased before they reach Jupiter? Which Opinion, Rohault (Chap. 27, Art. 34,) endeavours, to confirm, by a comparison with a conical Vessel filled with Water, being raised, at its Surface, by ejecting in more Water at the Vertex. The comparison has indeed some affinity, though a very gross one, because Light is supposed to expand as it spreads; by which means, its direct motion may be equible.

Suppose Light loses one fourth of its velocity before it reaches Jupiter, it can have lost but one twentieth part at the Earth; and it is reasonable to suppose, that the force with which it returns from Jupiter cannot be equal to what it came with. Suppose then it has lost a fourth part more of its force, before it reaches the Earth again, at C; which, considering how languid its Light is, compared with the Light of the Sun, is very moderate. According then to this supposition, Light comes from Jupiter to the Earth, with little more than half the velocity with which it comes from the Sun, directly.

C represents the place of the Earth, at an Opposition and E at the Conjunction; the difference in the Distance of those two situations, from Jupiter to the Earth, is CE, the Diameter of the Earth's Orbit.

Now the motion of a Particle of Light (for a Ray, or continued Right Line of Light, I have no conception of) in its passage from the Sun to the Earth, is, I presume, at its first and greatest velocity, always the same; but, at J, it has lost one fourth of its force; and, from J to C again, it has lost a fourth more; and consequently, in its passage from C to E, which is nearly half JC, it must necessarily have lost one eighth of the velocity it had, at its return to C; at which time the calculation is made.

Wherefore, the motion of Light, from C to E, has, at most, but half the velocity it has from S, to C or E, directly; and, it passes, according to Roemer's calculation, from C to E in 15 Minutes; consequently, it passes from S to E (with double velocity) in a fourth part of the time, that is in 3 minutes and 3 quarters.

But, if the velocity, to and again, be regular and uniform (which I believe is contrary to the laws of motion, of Bodies impelled and reflected again) being 15 minutes in passing from C to E, half of which, SE, must consequently take 7 minutes and a half. For a Satellite, emerging at a, when the Earth is at C, appears 7 minutes and a half sooner, and at E 7 minutes and a half later, than at D or F, where the distance of Jupiter from the Sun and the Earth is equal, according to the calculations made by the ablest Men. But since it is not possible, or, at

least, probable that the velocity of Light can be uniform and equal, at all Distances from the Sun (if it has any motion at all) the greatest velocity must be at its first emission from the Luminary; and consequently, it takes less time in passing from the Sun to the Earth than in its return from Jupiter.

3. It is certain that no opaque Body, I mean, such as are not luminous, can be seen at a great Distance; unless its magnitude be sufficient to obstruct a great quantity of Light, when opposed to a luminous Body; or, when it is so situated, in respect of a luminous Body, that the Light, it receives from the Luminary, is reflected again*; of which, the various appearances of the Moon is sufficient proof. Consequently we could not discern either Jupiter or his Satellites (whose immense Distance is more than seventeen hundred times the Moon's Distance) if they were not illumined by the Sun.

Jupiter being a superior Planet, that is, he moves in an Orbit beyond, yet concentric with the Earth's, he is, consequently, always illumined on that Face towards the Earth; and consequently (being of immense magnitude) may always be seen, when above the Horizon, by the naked Eye; except when in near vicinity with the Sun. But the Satellites are secondary Planets or Moons (of which Jupiter has four) accompanying the primary ones, as the Moon does the Earth; and are so very small, in comparison of their primaries, that they cannot be seen at that distance without a good Telescope.

Now, if those Satellites cannot be seen, after they emerge from behind Jupiter, until the Light (which is always ready, at hand, to illumine them) is reflected, back again, from the Satellite to us; and, suppose this Light to be a material Body, I am firmly persuaded that they would never be seen at all, by us; such an immense Space, from a to E, above 500 millions of Miles, for Matter to be projected in about 45 minutes, is beyond the power of my reasoning faculties to find credit for (with God nothing is impossible.) Besides, they not only reflect the Light directly, but also obliquely, in all Directions, filling a Hemisphere; which, I should suppose, is too gross an improbability (being Matter) to gain credit with a thinking rational Being; let those, who can, find belief and give credit for it.

The whole of this Phenomenon I suppose is this; (for I have never seen the Experiment; as it must require extraordinary Telescopes, which magnify to a great degree, to discern the emission distinctly) either it is impossible to perfect Telescopes sufficient to discern the emission, till after the Satellite is considerably advanced from behind Jupiter; and also, from the near vicinity of the Sun, when in Conjunction with the Earth, at E, whose superior splendor must hinder the Satellite from being seen, for some time; together with the so much greater Distance, than when at C; which, I know from my own observation, renders them scarcely visible, with a common reflector; all which, may concur to render it invisible for 15 minutes, after it might be visible from C. For I make not the least doubt, that their motions and revolutions are as regular and equal as the motion of the Earth itself; which, to us, is the only standard measure of Time.

4. If we see Objects, only by means of material Rays of Light passing from the Object to the Eye; by what means are opaque Objects, which are immersed in Shade, seen at all? as they do not receive Light, immediately, from any luminous Body; nor, perhaps, from any other, opposed to them, by reflection. But, with such reasoners, who can give what properties they please to Matter, there is no arguing; seeing that, they can make Light reflect and rebound from one Object to another, at pleasure. But, are not those ideal properties of Light of their own creating, entirely? That Light is reflected from one opaque Object to another is beyond a doubt; but, that real Matter is projected, to and again, in every direction, I cannot acquiesce in.

* By Light being reflected, I would not be understood to mean, that there is any kind of real Matter projected from the reflecting Body; but only, by being illumined, itself, it becomes, in some degree, luminous, so as to shine with its borrowed lustre, and illumine others.

Plate II. Let AB represent, what is called, a Ray of Light, from some luminous Object, falling on any Surface, as CD .
Fig. 10.

I shall suppose this Ray of Light to be a physical Right Line, the least in its dimensions of breadth and thickness, that can be conceived; consequently, this physical Ray of Light can strike any Surface, on which it falls, in a physical Point only, the smallest that can be conceived.

Now by the laws of Reflection, according to all the writers on Optics, whenever a Ray of Light strikes or impinges on any Body, it is reflected again from that Body, making an equal Angle with a Right Line (BF) perpendicular to the Surface, at the Point in which it strikes the Surface*. Or, if it be a Plane Surface, it makes equal Angles with the Plane ($ABI = EBK$) both, the Original Ray, AB , which is called the incident Ray, and the reflected Ray, BE , being in a Plane, which is vertical to the other Plane.

It is reasonable to conclude, if this Ray of Light, AB , be material, and it is by means of material Rays striking the organs of Sight, that Vision is performed, or the sense of Seeing is effected, that, the Point B could only be seen by an Eye placed, any where, in the direction of BE , as at E ; BE being considered as the same individual Ray, AB , reflected or broken at the Surface, in the Point B †. It is certain, that if the surface CD be Water, or a polished Mirrour, the Image of any Object, at A , will be seen at B , by the Eye at E , or some where in the direction BE , only.

But experience convinces us, that the Point B , or any other Point in the Surface CD , may be seen as perfectly in a thousand other Directions; as at F, G, H , &c. Consequently, if Eyes were placed all around, within the compass of a Hemisphere, of which the Plane CD may be considered as the Base, or being elevated but a few Degrees above the Plane CD , the Point B may be seen by them all, at the same moment of Time.

Now, since the Point B can receive but one Ray of Light from the luminous Body, how can that identical Ray be reflected in ten thousand Directions, in all Directions above the Plane CD , and those Rays to be material? at the same Time, the whole Space is filled with Air in every part. But, the same Eyes can also see every other physical Point in the whole Surface CD , as C, D , &c. consequently, every Point in the Surface must also reflect Rays in all Directions, crossing and cutting the former in every Point; which, unless, not only two, but, an infinite number of Bodies may occupy the same Space, and at the same Time, cannot possibly be, on the supposition that Light is a material Body.

5. Again. Since, as I have observed before, it is asserted that Vision is performed by means of those material Rays of Light, which enter the Aperture of the Eye, and, impinging on the Retina, affects the Optic Nerves with the perception

* It is advanced by Sir Isaac Newton, (Prop. 8, Part 3. of the Second Book of his Optics) that Light is reflected from Bodies before it impinges on their Surfaces. But I must own, that in all the seven reasons he gives for that Opinion, I cannot find one of weight, nor all of them together, sufficient to prove it. Which, Smith, from that authority, and from Sir Isaac's first and fourth Quere, endeavours to prove, is not reflected immediately in an Angle, but makes a regular parabolic kind of curve, at a very small distance from the Body, which he calls the space of activity; and which space, he says, is so extremely small, that, consequently, in physical Experiments, the incurvation of a Ray of Light may be considered as performed in a physical Point. Emerson says (in a Scholium to Prop. 1. of his Catoptrics) that the Curve, described by a Ray of Light, is so extremely small, that it may be looked upon as a single Point.

It is, to me, astonishing, how any Person dare presume to advance such Opinions! and, for what purpose they do advance them, except it be to make the World have a high veneration for their extraordinary sagacity and penetration. Can they, either by experiment or otherwise, prove the truth of their Hypothesis? or do they suppose mankind so credulous as to give them credit for it, from the ridiculous experiment of a Hair, or of the Knives, and fringes of Colours produced by them? And yet it is certain, that some (not to be behind them in penetration) either do or pretend to do; for I have heard the same thing advanced, verbally, by a Person who has a tolerable share of mathematical knowledge.

† See Fig. 10. No. 2. AB is supposed to be an incident Ray, and CD the reflected Ray; making a Curve, BEC ; before it touches the Surface GH ; which Curve is performed in so small a Space, that the Points, B and C , are supposed to coincide. Quere; how is the Curve to be ascertained, and determined? or, how shall we have conviction that it does not touch the Surface, GH ?

of the Objects from which they flow, how shall we account for Vision through Glass, or much harder diaphanous Substances, as Crystal, or Adamant? which is impenetrable, by any tool made of the hardest Steel.

Let any object, as *AB*, be seen, by an Eye at *E*, by means of the Visual Rays *AE*, *BE*, &c. and, let any transparent uncoloured Stone, as Crystal, &c. be interposed directly between the Eye and the Object, as *CD*, of any thickness; having its opposite Surfaces parallel Planes, and well polished. The Object, *AB*, will be seen through it instantaneously, as represented at *a b*, with every circumstance of Colour, &c.

Now, I should be glad to know, how or by what means those Rays of Light, transmitted or reflected from the Object (*AB*) to the Eye (at *E*) pass in an instant through this solid Body of Crystal or Adamant, if they are material? and first, I should be glad to know, what is the cause of Transparency? or, in what the difference consists, between transparent and opaque Bodies?

The reasons given by Sir Isaac Newton, in the third part of the second Book, and which, Smith, in his Optics (Ch. 8, P. 95) has very wisely copied, word for word, concerning the cause of Transparency, Opacity, and Colours, in Bodies, is but little to the purpose; it does not convey, to me, the least Idea of the cause of Transparency, and how Vision is conveyed through transparent Mediums, in every direction, instantaneously.

The learned and reverend Dr. Harris, in his Lexicon, tells us (under the article Transparency) that, a diaphanous or transparent Body is one which, probably, has its Pores all Right or Direct, and nearly perpendicular to its Surface.

By the Pores being all right or direct is meant, I presume, that they go directly through, in Right Lines, without any obstruction; that they are perpendicular, or nearly so, to the Surface, I must needs say, he was extremely sagacious who first found out that extraordinary quality. But how shall we interpret the Word, probably? is it not indeed a tacit acknowledgement, that it is all nothing more than Conjecture? I cannot say that it is even a probable Conjecture.

I am afraid that the Doctor was not very happy in his memory; for, in another place he tells us (under the article Diaphaneity) that the Pores of a diaphanous Body are so ranged and disposed, that the Beams of Light can pass freely through them, every way; which plainly contradicts the former assertion, that they are nearly perpendicular to its Surface; and which, in reality, is saying nothing, because, a Surface may be made at pleasure; and we cannot suppose that Pores can change their Position, as a Surface may be altered, at discretion; or how a Ray of Light, in passing through a triangular Prism, can be nearly perpendicular to both Planes, I am at a loss to devise.

For my part, I look on it as ridiculous and presuming, to tell us that Glass and all transparent Bodies are full of Pores and minute Interstices, through which the Rays of Light have a free passage; because, these Pores must be in every part of it, and in all Directions; neither can there be any solid part, between one Pore and the contiguous or adjoining Pores; for, if there be, it must necessarily impede, or entirely stop the progress of some Rays of Light in their passage to the Eye, and, consequently, will prevent the Vision of the Object from being perfect. Wherefore, since the Pores are so very numerous, have no solid part between them, and that, in all Directions; for it is plain, that they are not confined to one position, but must lie in all directions, which is evident; for, turn the Glass or Crystal (*CD*) as you please, the perception of the Object is as distinct as before; from which it is clear, that the Pores must be the same in all Directions, and then, I would ask, what becomes of the solid Body? for, in reality, it must be all Pores.

It is an insult on our Understandings, on our Reason, and on common Sense, to suppose, or for any person to attempt to persuade us, that Glass, or Adamant, the most compact of Stones, is more pervious, or that the Pores are more direct than in soft Wood, or in any Wood; which every one, who has considered it,

Plate II. knows to be full of Pores (like Veins in the animal Body) through which the Juices pass for its nutriment; and, in many kinds of Wood, they are perceivable by the naked Eye, and lie in direct Lines. Can any Person persuade himself, that the Pores are more numerous, or more direct, or that they are more capacious, so as to admit a freer passage, through Adamant, for the most subtle Fluid whatever (which I will suppose Light to be) than through a thin piece of Wood; in which the Pores are obvious, and clearly visible? Air, which is much denser than Light, or Water, still denser than Air, has a free passage through; but not through Glass, I presume. Yet, if you oppose it to a Candle (being cut very thin across the Pores) you may perceive, indeed, a few scattered Rays of Light pass through, but very far from having distinct Vision of the Candle, or even the out Line of the Flame, only. The reason is very obvious; because, the parts, between the visible Pores, being more condensed and compact, receive the Light which falls on them, and either absorb or reflect it; which, therefore, does not pass through to the Eye; consequently, the parts of the Object, which are in a direct line to the Eye, cannot be seen; therefore, the Vision of the Object is imperfect.

As this is evidently the case when it is opposed to a luminous Body, which is seen by its own Light; what will be the consequence when it is opposed to an opaque Body, which (as we are told) is seen only by means of reflected Light? Why, that there is not the least appearance of it to be seen at all; not even through fine oiled Paper, which, I must needs suppose, is infinitely more porous than any Glass or Stone whatever.

Whereas, through perfectly pellucid Substances, every visible Point, in any Object whether luminous or opaque, is distinctly seen, without the least impediment; consequently, if Light be material, and a Ray is transmitted from every Point, in a visible Surface, to the Eye (for every physical point may be seen) and if those Rays are conveyed through transparent solid Bodies, they must necessarily pass through Pores, direct in all positions.

Now I am fully convinced, that the Pores in Glass, &c. are of those ingenious Gentlemen's own creating; who, when they are at a loss for proof of certain Hypotheses (for want of better) they imagine Bodies to possess such and such qualities as may best answer their purpose. But, are those Chimeras, of their own fertile imaginations, to pass on the World for real existencies? are the Conclusions drawn from such Premises candid? by no means, they are very disingenuous, inasmuch that, I deny it to be in the power of any Man, to give ocular or other demonstrative proof that there are Pores in Glass or transparent Stones; and, I do believe that the most pellucid substances are the freest from Pores; for all porous Bodies are compressible into less compass, which neither Glass nor Stones can possibly be; nor Water, which is perfectly transparent.

Pores are, in my opinion, rather the cause of Opacity than of Transparency in Bodies, seeing that, they absorb the Light in their recesses. Yet, I do not suppose that all Bodies, as Wood or Metals, which are the freest of the kind from Pores, have any degree of transparency, but when they are exceeding thin; as leaf Gold, &c. but, they approach nearer to transparency than the more gross and porous kinds. The real cause of Transparency, and how Vision is conveyed through transparent Bodies, are (I am firmly persuaded) among the hidden mysteries of Nature, which is not given Man to explore.

Fig. 12. 6. It is, I believe, a Paradox, not easily accounted for; that, if two triangular Prisms, of Glass or other diaphanous Substance, having equal Angles, are so placed together (as ABC, BCD) that the outside Faces (AB and CD) are parallel, we have direct Vision through them both (from the Eye, at E, to an Object at F); whereas, if either of them (as BCD) be taken away, the Object is lost to sight from that Point of View, by reason of the supposed refraction of the Rays of Light, in passing through the Prism ABC.

Now,

Now, if the Pores, through which the Rays Ea , Eb , &c. pass, go directly through both Prisms (from a to c , and from b to d) how can they be varied by the removal of one of them? the taking away of one Prism can certainly have no effect on the Pores of the other, so as to alter their Direction; yet (whatever be the Cause) it is certain, if either Prism (as BCD) be removed, that the Vision, of the Object F , is turned aside out of a Right Line, and totally lost to sight, from that Station; for, it will be seen by an Eye at E , and not at E , and appear to be in the direction EF . Or, the Object, at F , being removed to G ; will appear to be at F , in the direction of EF ; and F will be its apparent place.

Again; if the Pores go direct through Glafs, &c. the Rays of Light do not stop at the Surface, and consequently, they cannot suffer either Reflection or Refraction by it. For, I presume, the true definition of a Pore is a small Cavity or Interstice, which admits a free passage for Fluids. If, therefore, they do enter, and pass freely through, how can the surface affect them? or how can the Rays of Light, if Light is a Body, be turned aside, within the Glafs, in any other direction than that of the Pores?

It is affirmed and allowed, that the Angle of Refraction is always to that of Incidence in a certain Ratio or Proportion; and since the Angle of Incidence may be any Angle at pleasure, it necessarily follows, that Light passes freely through in all Directions; which (according to the established Hypothesis, that it is corporeal) implies Pores in all Directions, according to one of the Doctor's Definitions; and, if it were possible for Pores to go right or direct through, in all Directions (which is repugnant to reason and absurd to suppose) the whole Prism must be all Pores; which omniporous quality, being attributed to any kind of bodily Substance, I am persuaded, no Man, in his senses, will acquiesce in.

7. I shall just give one more objection to the materiality of Light, and conclude this Section, and Subject.

Suppose the Eye, at E , viewing an Object, AB . There is supposed to be Rays of Light, AE , BE , &c. transmitted from every point to the Eye, forming an Image of the Object on the Retina, which is generally allowed to be inverted; although a late writer has given some reasons to the contrary. Be that as it may, it is certain, that if those Rays of Light enter the Eye, they must pass through the Aperture or Pupil, and converge to a Point (E) within, before they can diverge again to form an Image of the Object; at a c b . Fig. 13.

Now if the Eye be so situated (in respect of Distance) to the Object, that the extreme Rays, AE and BE , incline to each other in an Angle (AEB) not exceeding 40 Degrees; it is certain, that the Eye is capable of taking in the whole of that Object at one View; although, every part cannot be distinctly seen at once. Every physical Point, in the surface of the Object, as C , D , F , &c. is supposed to transmit a Ray of Light to the Eye, as well as in all other Directions, at the same moment of Time. Consequently, the whole system of Rays generate a Cone or Pyramid of Rays, close wedged together in every part; which, all enter the Eye at the same instant, passing through a Point of the same dimension as one single Ray, at E , the Vertex of the Pyramid or Cone; which Circumstance is so egregiously absurd, that it is sufficient, in my opinion, to refute all that can be alledged concerning the Rays of Light being material.

For, can a great quantity of Air (the rarest Fluid that we are acquainted with) pass through a Pin-hole in an instant? Will it not (like Water) require more or less time, according to the dimension of the Hole, to pass through? can any force drive the same quantity of Air through a Hole of half an inch in Diameter, in the same Time as through one of an Inch? No; the ratio of the Time required, with the same force, will ever be in proportion to the area of the Aperture; that is, the ratio of different Apertures, to each other, is equal to the ratio of the Time required, with equal force.

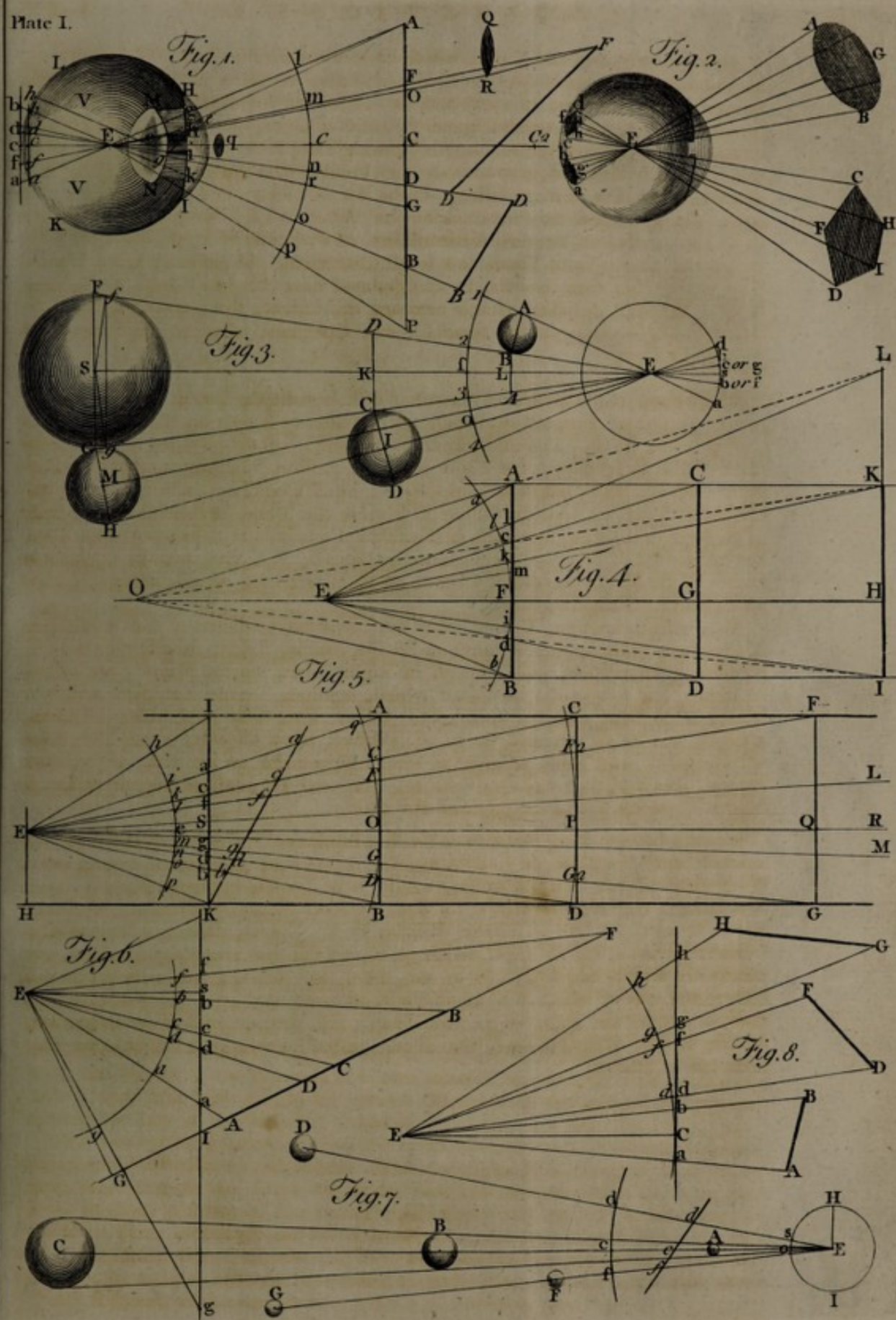
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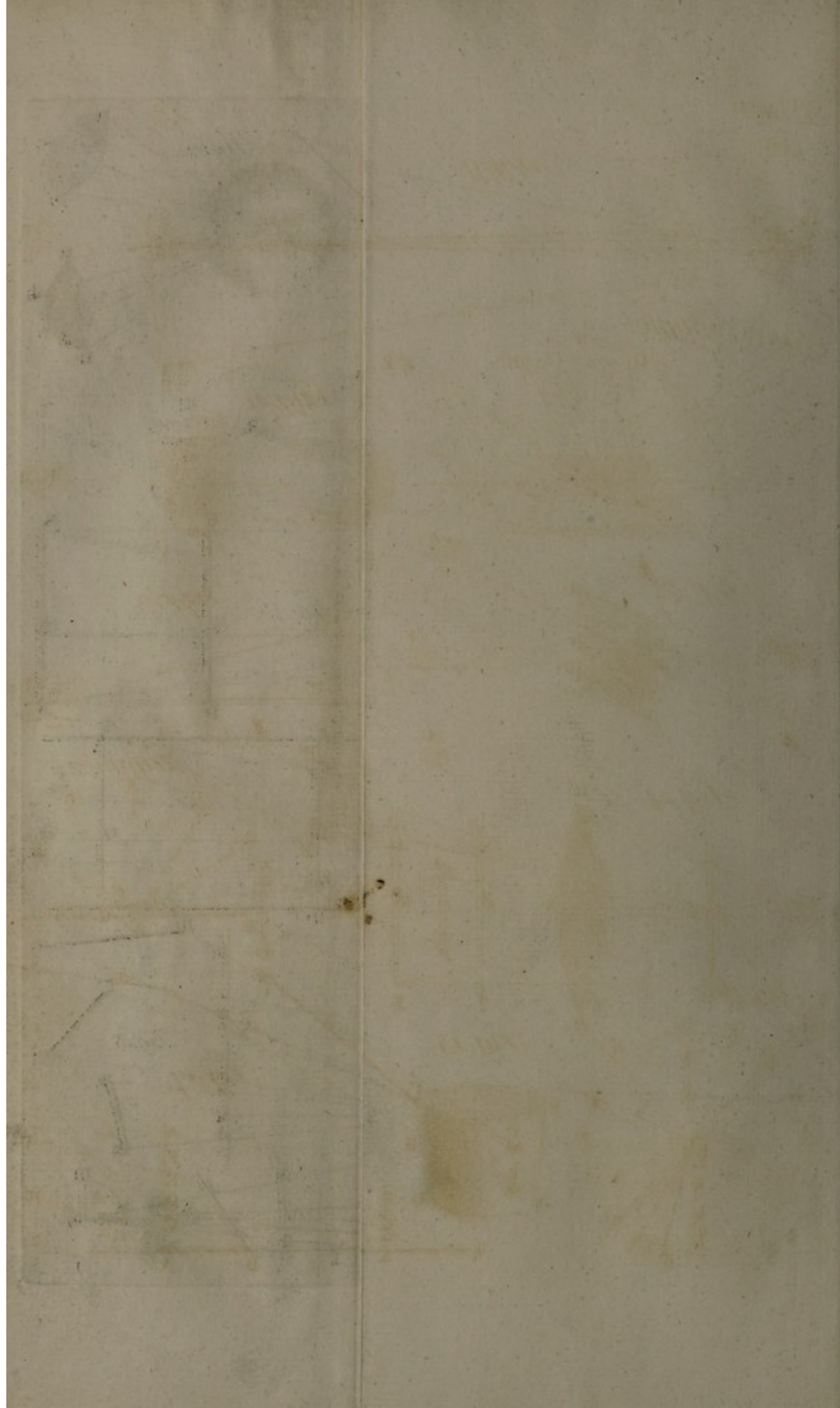
And yet, a Pyramid of Light, whose Base is equal to St. Paul's Cathedral, nay, millions of times greater (for the Eye can take in, not only the Sun, but, a great part of the Hemisphere, at once) and its Altitude of any dimension, as far as the fixed Stars; yet, I say, this prodigious Pyramid of Light, a material Body, a Fluid, can pass (without any known impulse) through ten thousand imaginary Pores in the Cornea, in their passage to an imaginary Point within the Eye; through which, the whole is conveyed in an instant, to the seat of Vision; and what becomes of it after? the Eye must be very capacious to contain it all. Nay, more; the very same Base, or Object, can send forth millions of Pyramids of Light, to other Eyes all around, at the same Time, and in all Directions. Impossible! being Matter. Can any Person form an Idea, to comprehend how the same Body, or Surface, simply, can emit, or reflect what it receives, this instant, from any other Body, in innumerable directions at the same Instant? Let those, who can, find belief and give credit for it.

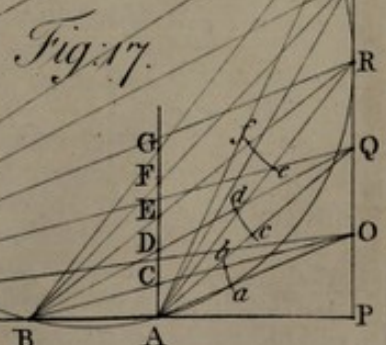
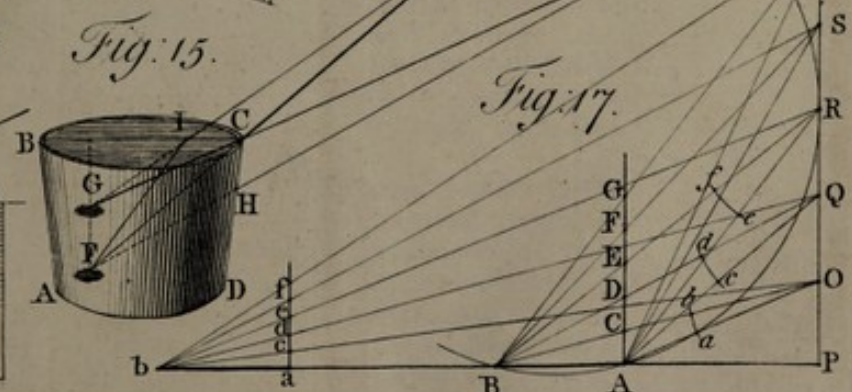
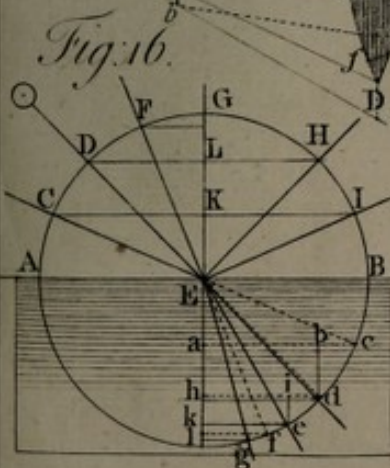
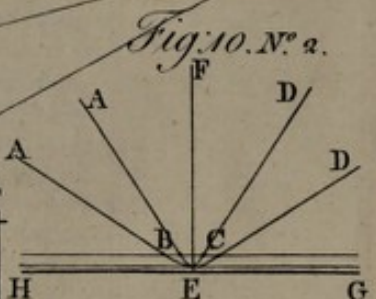
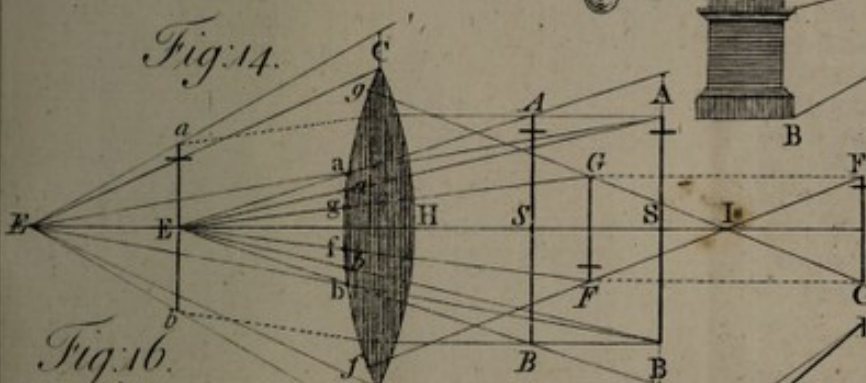
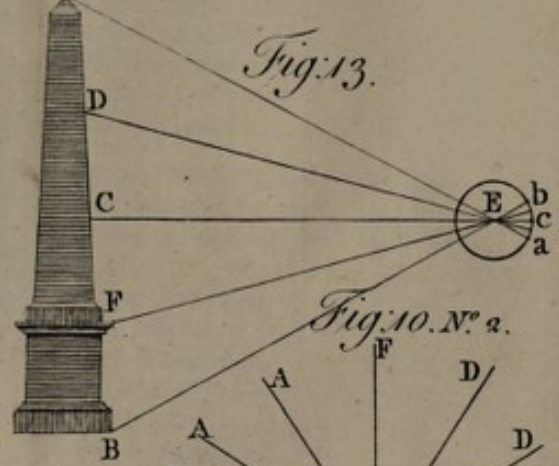
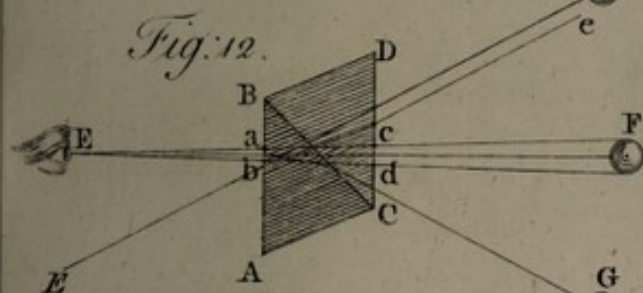
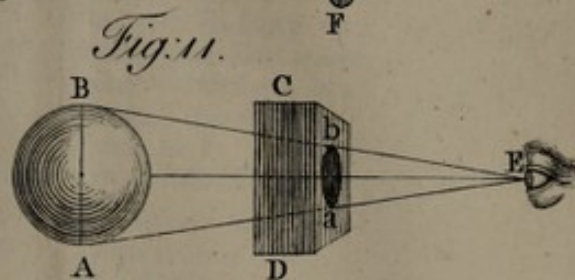
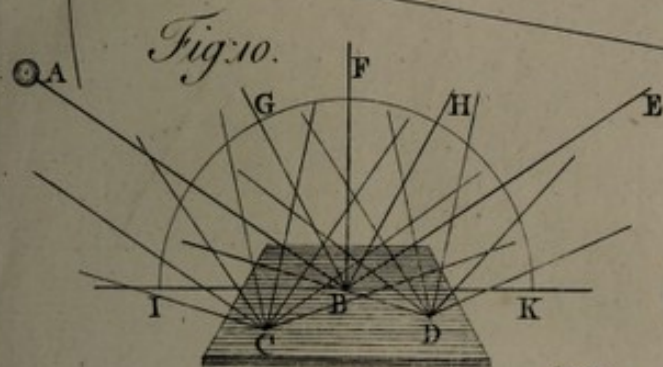
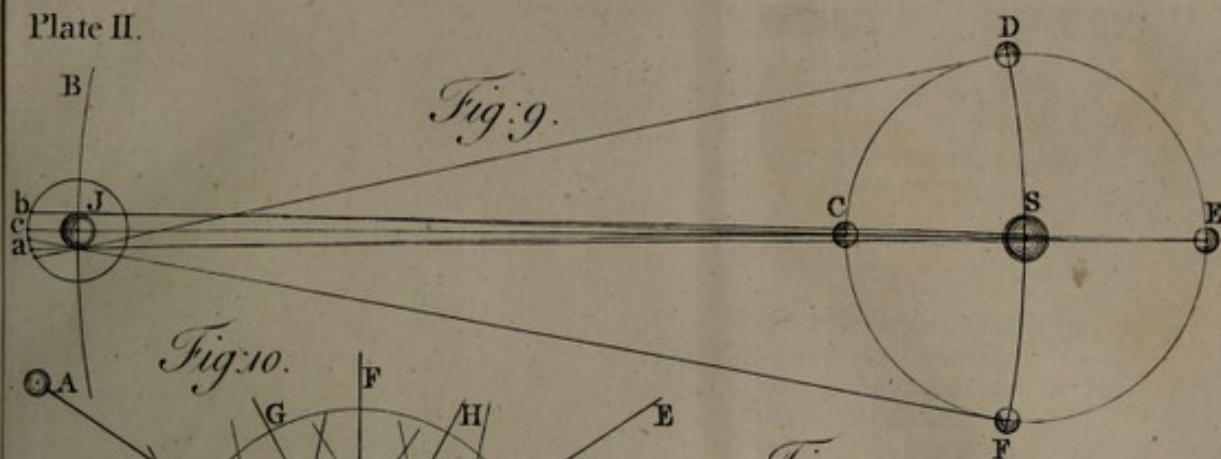
Much more might be said in support of this Argument; but, as it is not directly to the purpose of Perspective, I shall not trespass any longer on the Readers time and patience, in this Digression from the Subject. I shall only beg leave to draw from it this Conclusion; that Light is not an existing Medium composed of Particles; which, being reflected from Objects, in all Directions, and striking on the Organs of Sight, conveys the Vision of them to the Mind, and occasions the sense of Seeing; intimating, by means of the different qualities of its heterogeneous Rays, not merely the existence of the surrounding Objects, but, they are also supposed to excite, in the Mind, the idea of Colours on their Surfaces; which, otherwise, have no existence: strange Doctrine!

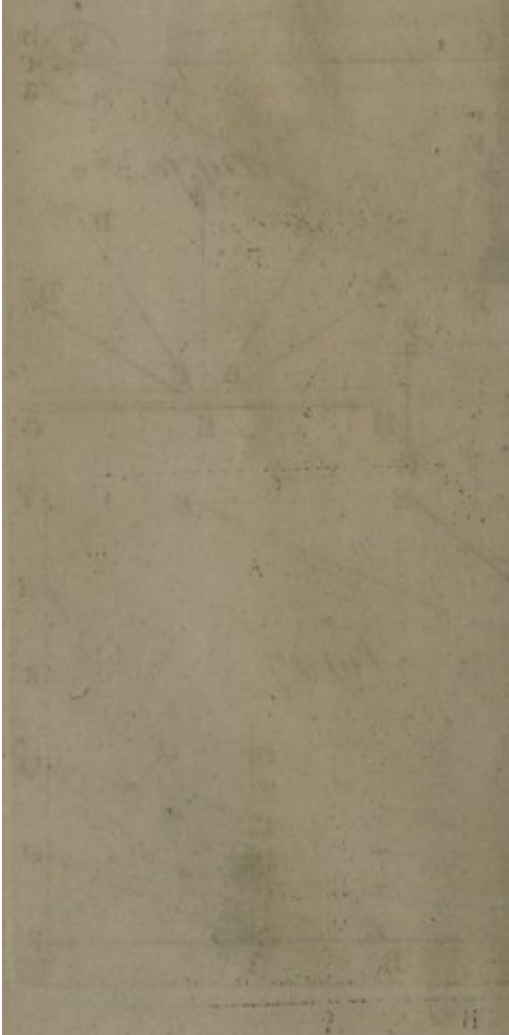
But, with submission, I think, that the perpetual existence of such a Medium is repugnant to the notion of luminous Bodies emitting Light, incessantly; which, proceeding from them, progressively, in Right Lines, excites Vision; not instantaneously, but propagated in Time: which, opinion, is more consistent with the notion of its being reflected from other Bodies. For, how a stagnating Medium, a Fluid, can be so actuated, as to be reflected, from Objects, in all Directions within itself, and consequently, in direct Opposition to its first, or incident motion, and with such amazing velocity, is beyond the reach of human reason to conceive; much less to comprehend and explain.

It is the distinguishing Property of lucid or luminous Bodies to dispense Light all around them; but how, or in what manner, it is not my intention to enquire into; being well assured, that the attempt would be as fruitless as presumptuous. Such Phenomena are, and ever will be, to Man, impenetrable and inscrutable; Mysteries not to be unfolded but by infinite Wisdom itself. To us, there is a large field of knowledge, open and in view, whereon to exercise our reasoning faculties, and which lies within our reach; let us not, then, step aside into intricate Mazes and Labyrinths, out of which it is impossible to extricate ourselves; in which, the farther we wander the more we are bewildered; till, wearied with the vain pursuit, we are, at last, obliged to own, that all our boasted knowledge is but to know how little can be known.









S E C T I O N V.

OF REFRACTED VISION.

THE last Section touched, though very slightly, on that part of Optics called Catoptrics, or reflected Vision. In this I shall briefly touch on Dioptrics or refracted Vision; both which, I look on, in many respects, as Deceptions in Vision. For it is evident, that, in the case of Reflection, on the surface of Water, &c. or polished Mirrours, we do not see the Object, but only its Image or Appearance. So likewise, in the case of Refraction through transparent Mediums, solid or fluid, we do not see the real Object, but its Image or Representation only; which is likewise a Deception; seeing that, the Object appears, in some cases, larger, in others, smaller than it really is, and always appears to be where it really is not; although, we imagine that we are looking directly at the Object.

Perspective Representations, are also manifest Deceptions in Vision. The business of Perspective is to represent Objects on a Plane Surface, by the rules of Optics; which, in the true Point of View, will give the Idea of a real solid Object (being properly and judiciously shaded) having the appearance of projectures and recessings, of one part before or behind another; and having also the same hue or teint of Colour, the Deception becomes stronger; insomuch, that it is possible so to deceive the Eye, as to imagine the Representation to be real and substantial.

In looking at Objects through a Telescope, either reflecting or refracting; although we level it exactly to the Object, in a Right Line, and imagine we look directly at the Object, through the Tube; yet, it is plain that we do not see the Object. For, in a Reflector, we look directly at a concave Mirrour, which is placed direct between the Object and the Eye; consequently, in that case, it is impossible to see the Object, but only its reflected Image on the Mirrour, which is first received on another Mirrour, at the other end through which we look, and reflected again to the former, opposite to the Eye, (a most curious and ingenious invention; which was first constructed by Dr. Gregory, and thence called the Gregorian Telescope.) The Image, of the Object received on the Mirrour, is magnified, by means of convex glasses in the small Tube; so that, we do not see even the reflected Image on the Speculum, but the Image of that, only, magnified, to a great degree. In respect of the refracting Telescope, the Object itself is magnified, in the same manner as the Image on the Mirrour, in the Reflector; so that, we do not, in either case, see the real Object, but only its magnified Image, between the Object Glasses and the Eye. I shall illustrate it by a single magnifying Lens.

1. Suppose an Object, as AB , and a convex Lens, CD , placed between the Object and the Eye, at E . It is manifest, since Vision is conveyed in Right Lines from the Object to the Eye, that, if we saw the real Object, by the Visual Rays AE and BE &c. it would appear, in proportion to the Lens CD , of the magnitude ab , only; but we find, that it appears larger than its real magnitude, in proportion to the Lens.

For, if the Rays Aa and Bb fall perpendicularly on the convex Surface, CHD towards the Object, they will pass directly to the other Surface, which is concave towards the Object, cutting it at a and b , and are thence refracted, to the Eye at E , as aE , bE ; then, if AA and BB be drawn parallel to the Axe of the Lens (EI) and Ea , Eb are produced, cutting AA and BB in A and B , then, is AB the apparent place of AB , or its Image; which, being seen under a larger Angle AEB than the real Object AB , if the Lens was removed, or Plane, it will consequently appear larger; by Theorem 1st.

I

Hence

Plate II. Hence it is manifest, that the real Object, AB , is not seen; for it is not possible
Fig. 14. that the Point, A , can be seen under the refracted Lines Aa , aE ; and if Ea , Eb be produced, it is evident that the Object AB , in that place, must be larger, to appear equal to ab , on the Lens.

This way of determining the Image or the apparent place of the Object, is according to Smith, P. 51. Art. 139. which, in some respects, seems right (He does not indeed determine the Refraction by drawing a right Line, from A or B , to the Center of either Surface; nor does he give any certain Rule to determine the Refraction. It is impossible to ascertain the Point where any incident Ray, from an Object, cuts the Lens; seeing that, the inclination of its Surfaces is continually varying, (from the Center to its Extremes) but I find there are Cases in which it is very exceptionable. For, according to this method of determining the apparent magnitude, or place of the Object, it can never appear larger than at the distance of the Lens. But it is certain, that the Eye and the Object may be so situated, in respect of the Lens, that notwithstanding, the Object, AB , is considerably less than the Lens, it will appear larger; and consequently, its apparent place is on this Side (in the focus of the incident Rays) between the Eye and the Lens, as at aEb the Eye being removed to E ; for ab , the Image of AB , appears, to the Eye at E , under a greater Angle (aEb) than the Lens CD ; and therefore, AB , or its Image ab , appears, to the Eye at E , larger than the Lens CD , which it never could do, if its apparent place was on the other side of the Lens.

In this Case, the Object AB appears erect; but, if another Object, FG , be placed beyond the Focus of the Lens, and Right Lines Ff , Gg , be drawn, through the Center, I , of the opposite Surface, consequently perpendicular to it; and if FG , GF be drawn parallel to the Axe cutting Ff and Gg in G and F ; then is FG the Image, or apparent place of FG , which, in this Case, is inverted; and consequently, the real Object FG is not seen; for the extreme F is seen at f , and G at g , on the surface of the Lens, in a contrary direction to their true places.

The contrary effect is produced through a concave Lens, in which, the Object always appears less than its real magnitude, according to its Distance; which, being but the reverse of the other, 'tis needless to illustrate it.

It is evident, that Vision is conveyed through any transparent Medium whatever, obliquely situated, in refracted or broken Lines; consequently, in such Case, the Object never is where it appears to be. For, the Point A , which is supposed to be seen by the Eye at E , appears to be at A or a , and not at A where it really is; and the Point S , which is seen directly through the Center of the Lens, although it be seen in the Right Line ES (for there is no refraction perpendicularly thro' parallel Surfaces of any kind) yet, its apparent place is at S ; and, if either the Eye or the Lens be removed, the apparent place of the Object is varied.

2. Lenses of all kinds, excepting such as are plane on both Sides, or such Meniscus, (hollow on one Side and convex on the other) as being portions of concentric Spheres, consequently parallel Surfaces, partake of the nature and properties of Prisms, whose Surfaces are inclined to each other. Wherefore, the Rays, in passing through them, are more or less refracted, according as the Surfaces are more or less inclined; that is, the less the radius of the Sphere, to which the Lens is formed, the greater is the magnifying power; because, the refracting Angle is greater; and, consequently, projects the Rays to a greater distance from the Center.

Hence it becomes a Paradox, how a large Object (I mean one, which, according to its Distance, occupies, or appears equal to the whole surface of the Lens) can be regularly magnified; for since it is manifest, that through parallel Surfaces there is no sensible Refraction (except the Surfaces are considerably distant from each other and seen through obliquely, as $Ebce$, Fig. 12.) and the surfaces of every Lens are parallel at the Vertex, and nearly so at a small Distance from it; and, the greater the angle of Inclination, the Refraction is greater; wherefore (the surfaces of a convex Lens, being least inclined at its extremes, i. e. the refracting Angle is the

the greatest; and, seeing it is continually varying, from the Vertex to the Extremes) it seems reasonable, to conclude, that the Object would be more distorted about the edges of the Lens than near the middle; which I do not find to be the Case. Query; by what means is the Object magnified equally?

3. There is another circumstance, which I do not remember to have seen noticed, by those who treat of the properties of Prisms, in respect of Refraction; and which, I do not see a reason for. Right Lines, seen through a triangular Prism, or any two inclined plane Surfaces, being parallel to the intersecting Line of the two Planes, that is, to the refracting Angle of the Prism, do not appear Right Lines, but, curved; and they are more or less curved, as the Angle of Refraction is greater or less; the Curve being concave towards the refracting Angle.

Now it is reasonable to suppose, that the system of Rays from any Right Line to the surface of the Prism, which is a Plane, would also form a Plane; which, by its intersection with the surface of the Prism, must necessarily generate a Right Line, in any position of the Prism; which, would be the Representation of the original Line; and, when the Angle of Refraction is parallel to the original Line, they will also cut the other Surface, towards the Eye, in a Right Line, parallel to the opposite; how is it then that it appears curved? The vertex of the Curve, whatever it be, is where it would be cut by a Plane, passing through the Eye, perpendicular to the Original Line and the Planes of the Prism.

I have already described a Prism, in the last Section, and the nature of Refraction by it, as much as is my design, in respect of the deception of Vision through a Prism. Those who would be more acquainted with the properties of Prisms and Lenses, in general, I refer to Sir Isaac Newton's, or, where they are treated more at large, to Smith's Optics; where, if he has patience to go through it, he will find enough to exercise his patience on. I do not mean to disparage the Work, for I believe it to be the best of the kind, in many respects; yet, I think it deficient, in some Cases; but, in respect of Colour, &c. it is of a piece with the rest, and, being dwelt too long on, becomes tedious, if not trifling.

4. As to the Phenomenon of Colours seen through a Prism it is really surprising; but, it is only the Edges of Objects that are tinged or produce the Colours. A perfectly plane Surface is not at all varied in its Colour, when seen through a Prism; except at the Edges, when it is joined or opposed to a darker Body; and those Edges are most coloured when they are parallel to the refracting Angle; for, when they are perpendicular to it, and seen direct through, they have no Colour at all, but the natural Colour of the Body, whatever it be. Also, the Colours seen through the Prism, do not always follow in the same order as when a Beam of the Sun's Rays passes through; if a small Object be opposed to the Light (as, a Bar in a Sash-Window) being parallel to the Prism, the Object is wholly lost in the Colours, between the Red and Blue; the Red being towards the refracting Angle; an intense Red always follows after any Line or Edge of an Object which is opposed to, or which obstructs the Light in one position of the Prism, the brighter Colours succeeding; but, being reversed, there is a deep Purple, and Blue following; the other Colours are very faint. The Colours are more intense and vivid the brighter the Object, or the greater the opposition and obstruction of the Light by an opaque Object. If a slip of White Paper be laid on any dark Body, being seen through a Prism, its Edges are bordered, one with Red, the other Blue, according to the refracting Angle; but being laid on another, of the same Colour, none is perceived, except it makes a Shade on the other, not lying close, which Shade occasions Colour. Also, if a strong, black Line be drawn thereon, the Red and Blue proceed contraryways from the Line, which is quite lost between them; the Colours being produced, on any Surface, only where there is an opposition with a darker Body. How, then, it is possible to deduce or to draw a conclusion, from such Experiments, that all Objects, which we perceive, are by Nature of the same, or have no Colour at all, save what is effected by the Light reflected from their Surfaces?

5. As,

Plate II. 5. As in Nature, there is necessarily Refraction at the common Surface of any two different Mediums, whether solid or fluid, except when we look perpendicularly through the Surface, or Surfaces, at the Object, although the true reason of it be to us unknown; so it would be superfluous and foreign to my Design to multiply Cases wherein it happens. I shall, therefore, only take notice of one common Case, which is so very common, though but little attended to, as to be obvious to every Person who is blessed with the faculty of Seeing.

Let any Person put a straight Stick into Water, slanting in any direction, and it immediately appears to be broken, at the surface of the Water; the part within, appearing to take a different direction to that which is out of the Water; and, the more it deviates from a Perpendicular, the more it is refracted or broken, till the Stick makes an Angle, with the surface of the Water, of about 45 Degrees; and then, the Refraction is continually less, which is evident; for, when the inclination of the Stick, to the Surface, is such, that the Angle it makes with the Surface is very small, consequently, the Refraction cannot be great; and being immersed perpendicularly, there is no Refraction; which is the same thing as looking directly into the Medium, at an Object in the Water, or through a Body of Glass, very thick; in which Case, the Object appears to be considerably nearer, but, it is in the same Direction as we see it. Also, when we look at the Heavens of Stars, &c. those which are in the Zenith are seen in a Right Line; and, the farther they are from that Point, the more the line of direction is refracted or broken, so that, they are supposed not to be in that place in the Heavens where they appear to be; which Refraction, is said to be owing to the Atmosphere of Air and Vapours, which surrounds the Earth, being denser than the upper Regions.

6. It is a common, though curious and entertaining, Experiment, to immerse a piece of Money or other Object in a Basin full of Water; which will appear to be raised considerably higher than when the Vessel is empty.

Fig. 15. Let ABCD represent a Vessel, suppose of Glass; and suppose E an Eye looking at an Object, at F, at the bottom of the Vessel, being empty. Whilst the Eye remains fixed, at E, so that the Object at F, can just be seen over the edge, at C, let there be Water poured gently into the Vessel; as the Vessel fills, the Object will appear, to rise, gradually, to G, so as to be seen quite clear of the edge of the Vessel, in the direction EG.

Now it is certain, that the Object is at F, which appears to be at G; consequently, if the real Object be seen at all, it is seen in the refracted Lines FI, IE, where the Right Line EG, to its apparent place, cuts the surface of the Water; but, more probably, the Image of the Object, only, is seen at G. Consequently, a straight Stick or Wire being put into the Water, from E to F, will appear broken at the Surface (at C) and appear to go in the Direction CG.

Again. Let the Eye be removed to E, the Water remaining in the Vessel, so that the Object is apparently seen over the edge, at C; and, whilst the Eye remains fixed, let the Water be drawn gently off, by means of a Cock, or otherwise, at the bottom; the Object, apparently at G, will gradually sink lower in the Vessel, and totally disappear from that Station.

Hence it is manifest, that when the Water is in the Vessel, the real Object, at F, is not seen by an Eye at E; for, if it was possible, it must be seen through the Side of the Vessel, at H, which it is plain is not the Case; and, hence it is plain, that the Object being seen, apparently at G, is a manifest Deception in Vision. The Case is the very same, in Objects seen oblique through Glass, or any other pellucid Substance; excepting some small variations in the degree of Refraction.

7. Refraction in Water, according to all writers on Optics, is subject to one invariable Law, without any sensible error, but it is not to be demonstrated mathematically. I have made the Experiment myself, as accurately as it will admit of, and find it to be nearly as follows.

If AB be supposed the surface of Water, and CE, DE or FE an incident Ray of Light falling on it, at E; on which Point, suppose a Circle be described, and the Perpendicular, EG, drawn. Then, if CE be produced to c, and a c be drawn, parallel to AB; take a b equal to three fourths of a c, and draw b d perpendicular to a c, cutting the Circumference in d; the Ray CE is supposed to be refracted, at E, and go in the direction Ed, into the Water. By the same Rule, DE goes in the direction Ec, and FE in Eg. Fig. 16.

But, the same Rays CE, DE, &c. will also be reflected in equal Angles, GEH equal DEG, &c. Can the same Ray be both reflected and refracted? impossible; yet, an Object (☉) may be seen at H, by the reflected Ray EH, as well as at e, by the refracted Ray Ec. It is however certain, that if a Circle be described on the Plane A B G, and Lines are drawn thereon, as in this Figure; being immersed perpendicularly in Water, to the Diameter AB; CED, DEe, and FEg will appear Right Lines; which is very surprising. But yet, I do not see which way this Experiment proves, that Rays of Light go in those directions into the Water; the whole Mass being illumined in every part. If the Eye, being immersed in Water, at e, sees the Object ☉ over the edge of the Vessel, or any other obstacle, at E, on the Surface (which, I presume is the Case) it indicates, that the Object is not seen in a Right Line; or, rather, that its Image is seen at E, on the surface of the Water.

8. Before I conclude this Section and Subject, I cannot help taking notice of an extraordinary passage; which is in the Conclusion drawn from Prop. 8, Part 3, of the second Book of Newton's Optics, Page 69; concerning the extraordinary porosity of Water; which is 19 times lighter, and consequently, he says, 19 times rarer than Gold; and Gold is so rare, that Water may be forced through its Pores. For, as he was informed, by an Eye witness, a Globe of Gold being filled with Water and soldered up (but of what thickness he does not tell us; I have heard say above a quarter of an Inch) and, being pressed with great force, the Water squeezed through its Pores; and stood, all over its Surface, in multitudes of small drops, like Dew. From which, he concludes that Gold has more Pores than solid Parts; but how such a conclusion can possibly be drawn I cannot conceive. I should suppose, that, in such Case, the Water would spout out in streams, rather than stand on the Surface like Dew. Yet Gold will not admit either Air or Light through its Pores, though much rather Fluids than Water; which, he also concludes, from the same Experiment, has above forty times more Pores than Parts; and consequently, Gold (the compactest Metal we know of) according to that Ratio, contains above twice as much empty Space as Matter.

It is no wonder that Water is pellucid, being so extremely porous, and admits Light so freely through its Pores; but, it is somewhat surprising that it is not compressible, which all porous substances must be. And I think it also surprising, that a Man of Sir Isaac's sagacity should advance so much on the credit of any Experiment he never tried himself; which, he certainly had opportunity enough for, being Master of the Mint near thirty years.

If Water be so full of Pores, as Sir Isaac imagines, it might be compressed into less Space; which, the same Experiment absolutely proves that it cannot. If it be composed of globular Particles, they may certainly be squeezed so close together, that (supposing every Globe inscribed in a Cube) the difference between the quantity of Matter and of Space, would be no more than the difference between a Globe and a Cube, whose Diameter and Side are equal, which is nearly double; I mean, that the quantity of Matter contained in Cubes, which admit of no Space between them, is nearly double of that which is contained in the inscribed Globes; and consequently, the Space between Globes, lying in that position, is nearly equal to the Globes; for, the same Ratio must continue *ad infinitum*.

But it is manifest, that Globes will lie considerably closer together; as Circles leave only triangular spaces between them; so Globes of equal Diameters, will lie so together as to leave a kind of triangular pyramidal or prismatic Space between, joining

Plate II. joining each other. The difference between the Globes and Space may be very nearly ascertained, by putting a quantity of equal Globes or Bullets into a hollow triangular Pyramid, which may easily be made, the true figure of a pyramid of Globes (for it is a regular Tetrahedron) and, having placed them regularly, pour in Water just to cover them, filling up the Interstices between; then take out the Globes and fill the Box equally with Water. I have tried the Experiment, and find the difference between the Globes and Space to be, as 2 to 1, as near as can be; in the former Case they are nearly equal. Query; where is the remaining 40 times to be found? are the least particles of Matter, which cannot be conceived of any dimensions, to contain, in each, 40 times more Space than Matter? for, as I have observed before, the ratio, of the Globes to the Space between them, will continue to infinity; and what other regular Figure can there be, to contain more Space between them than Globes?

The Question then is, whether we are to give credit to such Assertions, or are we to credit the evidence of our own Senses and Reason? I shall trust the latter, before the bare assertion of the greatest Man that ever lived.

9. As I have had neither time nor inclination to go through the whole of Smith's Optics; I shall just take notice of one passage in it, in which his geometrical reasoning is somewhat erroneous; but, whether it be of Consequence, in the following part of the Work, I have not enquired; as the Subject is not to my present purpose. I shall, literally, quote his own words, as follow.

Fig. 17. In Vol. 1. Page 59. Art. 157, he says, "The apparent magnitude of a given line, AB, seen very obliquely at a given distance, OA, increases and decreases in proportion to the increase and decrease of OP, the perpendicular distance of the Eye from the line AB produced; provided the distance AO be very large in comparison to AB. For let the Ray BO cut a line AC perpendicular to AB in C; and while the Eye is raised or depressed in the perpendicular OP, the line AC will increase and decrease as OP does; and so will the Angle AOC, subtended by AC, and this Angle measures the apparent magnitude of AB."

First, he says, that the apparent magnitude of AB increases and decreases in proportion, as the Eye is raised or depressed, in the perpendicular OP; and the Angle, it subtends, measures the apparent magnitude of AB.

Let PO be produced to T; in which, take several Divisions, each equal to PO, at Q, R, S, and T; and draw QA, QB and RA, RB, &c.

Now, because PQ is double PO, and PR is triple, the Angle BQA, (according to him) will be double BOA, and BRA triple; for, AG being drawn perpendicular to AB, it is certain, that the Divisions AC, CD, &c. are equal, AG being + Cor. 6. 6. El. parallel to PO; consequently, AC increases as OP does; and so, he says, does the Angle AOC, subtended by AC; wherefore, the Angle at Q is double, and, at R triple, of the Angle AOB. But, the Perpendiculars AC, AD, and AE, are not the measures of those Angles; for they are truly measured by an Ark of a Circle, of the same or an equal Radius, only.

According, then, to his Words, the Angle AOB increases as PO increases; which I deny, and will prove to the contrary.

It is obvious, that the Angle AQB is considerably larger than AOB, but it is not double, although PQ is double PO; but, ARB is very little larger than AQB; and notwithstanding, PR is triple PO, yet the Angle ARB is not double AOB; as the Arks *ab*, *cd*, and *ef*, evince.

Now, by the same reason, the Angles ASB, ATB are continually greater than ARB; but, they are continually less. For, if PR be a mean Proportional between PA and PB, the Angle ARB is the greatest that can be made, by Right Lines drawn from the Extremes of AB, and touching the Perpendicular PT.

DEM. Describe a Circle through the three Points R, A, and B; by Prob. 40 Geo. Then, because PR is a mean Proportional between PA and PB (by Hyp.) the square of PR is equal to a Rectangle under PA and PB; }
 i. e. $PR \times PA = PR^2$ — — — — — } Cor. 9. 6. El.
 Consequently, PR will touch, or be a Tangent to that Circle, drawn through the Points R, A, and B — — — — — P. 16. 3. El.
 But, the Angle ARB is at the Circumference of that Circle; conf. ASB and ATB, which are beyond the Circumference, are less than ARB. — 20. 6.
 Therefore, ARB is the greatest Angle, that can be, touching the Tangent PT, subtended by AB; and consequently, the Angle ASB is not encreased, as the Eye is raised in the Perpendicular OP.

10. Again. He saies, in the following Paragraph, as deduced from the foregoing, that “the apparent magnitudes of equal parts, AB, ab, of a Line, PAa, seen very obliquely, at great distances from the Eye, are, reciprocally, in a duplicate proportion of those distances. For example; let Ob be double of OB, and the angle OBP will be double of ObP; and accordingly, since AB, ab are equal, the perpendicular AC will be double of ac; and, being seen twice as near as ac, will appear four times bigger than ac. Again, if Ob be triple of OB, the line AC will be triple of ac, and being seen three times nearer than ac, will appear nine times bigger than ac; and so on.

Let Ob be double OB; I say, the Angle OBP is greater than twice ObP; Fig. 17. and he expressly saies that OBP is double ObP.

DEM. Because Ob is equal 2 OB, bB is greater than OB. — P. 13. 1. El. wherefore, the Angle bOB is greater than ObP. — — — — — 12. 1.
 But, the Angle OBP, being external, in respect of the Triangle bOB, is equal to bOB + ObP; wherefore, it is greater than double ObP. — 10. 1.
 But, if the Angle abc be equal half ABC, ac will not be half AC. — 3. 6. El.
 Consequently, since the Angle ABC is more than double abc (and AB is equal to ab) AC is still more than double ac.

Therefore, the apparent magnitude of AC to ac is more than duplicate; seeing it is more than half ac, and is also seen at less than half the distance.

Now I am far from supposing, that this Author was so deficient in Geometry as these Examples seem to indicate; nay, I am well convinced he was not, as is evident in the next Book; and his Provisos are some extenuation of the errors; he ought, then, to have told us that they are nearly so, by approximation, and not that they are so, in express Terms. But, in any Case whatever, there is not an equal ratio, or nearly, between the increase and decrease of the Perpendicular AC, and the Angle AOC.

As this and the preceding Article are the only passages which any way tend to advance the Theory of Perspective (having been quoted, by Mr. Kirby, for that, though but little, purpose) I have, therefore, been more particular in my remarks on them. And, as what I have advanced in the two last Sections, and part of the first, is not directly to the purpose of Perspective, it will, I know, by some, be deemed impertinent and foreign to the Design of this Treatise; let them, if they please, pass it over, and proceed immediately to the Subject, which is not vitiated by it. As I do not intend to publish a Treatise on Optics, and as Perspective has a near affinity to that Science, what exceptions I have always had, to sundry passages in the works of optical writers, I thought proper to give here, where I was treating on an essential part of the Theory of Perspective, and a branch of the Science of Optics.

I hope I have not trespassed, too much, on the time and patience of the candid and unprejudiced Reader. I shall now proceed to Book the second, which treats on the Theory of Perspective; where, I shall endeavour to make some amends for his time spent, I hope not lost, in perusing this Digression.

B O O K II.

Of the Theory of Perspective,
rectilinear and curvilinear.

S E C T I O N I,

Containing a general INTRODUCTION to PERSPECTIVE.

TO define the Terms of Art peculiar to any Science, with brevity and perspicuity, I have always looked on as a particular excellence in the Work; but I have frequently found, that an affectation of brevity has left the Term, intended to be explained, rather obscure, at least doubtful; whilst others, endeavouring to render it clear by a multiplicity of words, have, at last, involved it in perplexity.

Although my design is to be as brief as the nature of the Subject will admit of, yet I am afraid I shall rather be thought prolix, than otherwise, in some of the following Definitions or explanation of the Terms; but certain I am, that, the Time spent in acquiring a perfect knowledge of all the Terms I have defined, will not be lost, as they contain many useful Lessons to a young Student. I have endeavoured to explain every Term in the most familiar manner; not saying more than was necessary, yet, enough to be clearly understood. To be too brief is worse than prolixity, so it be not tedious and trifling; the one leaves us doubtful, the other, probably, makes it clear at last.

I may, perhaps, be particular and singular in my opinions; but, I think it better to define each Term separately, than to include several in one Definition, as is very frequent; and I always choose to name the Term, I mean to define, first, rather than end the Definition with it, or name it promiscuously. My reason for which is, that 'tis much easier to find when referred to; or, when there is occasion to look for any particular Definition, without reference, the Number not being known.

The Order in which the Terms are defined is, with me, a material circumstance; beginning at the Foundation, and going gradually on in a regular succession; never using any Term, if possible to avoid it, in the explanation of another, which has not already been defined. In general, I find them promiscuously jumbled together, without Order; going, as it were, from one end of the Science to the other, to and again; too hastily introducing, perhaps, some new or favourite Term, before others which seem necessary to be first known, in order to prepare the way, by removing some impediment. How I have succeeded, must be left to the decision of the candid and impartial Reader.

I am no favourer or admirer of new and uncommon Terms; nor, indeed, do I think that every writer, on any Science, has a right to impose new Terms; unless he has found out some new Principles, on which it was not possible for him to expatiate, by the Terms already known and in use; as Dr. Brook Taylor has done in Perspective; on which, I have spoke more largely in another place. It was impossible for him to convey the Ideas he intended to inculcate by the old Terms, and therefore, he was under the necessity of inventing and enforcing new ones; which are extremely expressive of the thing meant. But, if every writer, on that or any other Subject (who, because he knows something of it, imagines he knows more

than

than any who have wrote before him) was to take the liberty to impose new and unmeaning Terms, of his own, suited only to his own trifling Ideas of the Subject, the Science would, by that means, become perplexed and intricate; each Person, who happened to receive his knowledge of it from different Books, would, consequently, understand and call the same Thing by different Names; than which, especially when they are absurdly or falsely named, nothing tends more to perplex, and involve the Science in obscurity.

I have already, in the Preface, given my reasons for omitting, in this Treatise, geometrical Definitions and Problems; because, I suppose the Reader already tolerably versed in Geometry; if not, I advise him first to study it, at least Practical Geometry; without which, it is useless to attempt, and impossible to succeed in the Study of Perspective: the better he is acquainted with Euclid, the greater progress will he make in Perspective; of which, Geometry is the foundation. To assist him in it, I have compiled and composed a Volume (mentioned in the Preface) which may be called an abridgment; it, nevertheless, contains all that is essential. My chief aim, in that work, was to make it useful, the Study of it pleasant, and attainable by any tolerable Capacity, and applicable to various uses in Life; particularly subservient to this Treatise of Perspective, as I always refer to it for Demonstration: in which case, it may be deemed a part of this Work, and ought always go together. But, although I have, there, fully defined a Plane (Def. 6th.) yet, as it is so very essential in Perspective, it was by no means proper to omit it here; seeing that, on it the whole Theory of Perspective is built.

A PLANE is a perfectly even, straight, and regular Surface, which is neither convex nor concave in any part; but agrees, in every part, with a Right Line or straight Ruler, applied, any how, to the Surface.

2. By the motion of a Right Line, a Plane may be conceived to be generated; either by a direct, lateral motion, or by supposing it whirled around (so as not to generate a Cone) on any Point in it.

3. It is easy to conceive, that, if the Eye of a Person be in a Plane, or in a continuation of it, the nearest extremes or limits, towards the Eye, hide all the rest of the Plane; for, the whole Plane vanishes, and is lost to an Eye in the Plane; which appears but a Right Line, extended in length to the apparent dimensions of the Plane, considered as having only length and breadth: thickness or substance is the property of Solids, a Plane has none; 'tis the Surface, only, that is considered.

N. B. The Picture (in Perspective) is always understood to be a Plane. Therefore, the Board, Canvas, or Paper, on which we draw, is the Plane of the Picture.

N. B. 2. A Plane may be of any Figure; and, in Perspective, it is frequently considered as being infinitely extended, without regard to Figure, or to its limits.

A CUBE is bounded by six Planes, which are all Squares; as A, B, X. Fig. 1. Plate III. Every other Parallelopiped has also six Planes, which are all Parallelograms; either right angled, as Fig. 2, or acute angled, as Fig. 3.

A PRISM is a Solid, whose Base, A, and Top, B, are either similar Triangles*, Quadrangles†, or Polygons‡ of any Number of Sides. The other Planes or Surfaces are all Parallelograms. (As X, X.)

* Fig. 4.
† Fig. 1,
2, & 3.
‡ Fig. 5.

PYRAMIDS have their Bases (A) of any Number of Sides. The other Planes, of the Sides, are all Triangles. (As Y, Y.)

Fig. 6.

N. B. No Solid can be formed of Planes, having less than four; and that must be a Pyramid.

Plate III. Of PLANES and their POSITIONS, in GENERAL.

Fig. 7. 1. HORIZONTAL is the first and most natural position of Planes; such, are all Planes which are parallel to the Horizon; consequently, all horizontal Planes are parallel amongst themselves. As Z, H, H.

2. VERTICAL. All Planes which are perpendicular to, or which cut the Horizon at right Angles, are called vertical Planes. As V. Fig. 7, 8, and 9.

Fig. 8. Vertical Planes may be in all positions, in respect of each other, viz. parallel, perpendicular or inclined; as may be conceived, by revolving a vertical Plane, on a Right Line, AB, perpendicular to the Plane of the Horizon; consequently, they will, if produced, all pass through the Zenith and Nadir * of our Horizon.

3. INCLINED. All Planes whatever, which are neither parallel nor perpendicular to the Horizon, are Inclined Planes.

For, if a Plane cut the Horizon, or would if produced, in an acute Angle, it is not vertical or perpendicular; consequently, it inclines to the Horizon on one side more than the other; which, Inclination, is always measured on that side making the acute Angle; as, for Example.

Fig. 9. The Plane X inclines to the Horizon, in the Angle BCA; and it is also said to incline to a vertical Plane, in the Angle BCE; which Angle, if they have the same Intersection, CD, or parallel Intersections with the Horizon, is the Complement of its inclination to the Horizon. See N. B. Def. 13th. Geo.

4. It may not be improper, here, to observe (for it is necessary to know and understand well) that the Angle of Inclination, of one Plane to another, can be measured, only in a Plane to which both the other are perpendicular; or, which is the same thing, if a Line be drawn in each Plane, from the same Point in their common Intersection, and perpendicular to it, an acute Angle made by these Lines is the Inclination of the Planes; for it is evident, that a Plane passing through those two Lines will be perpendicular to the common Intersection † and consequently to both Planes. To illustrate it.

Suppose the horizontal Plane Z raised up into the Position of X, inclined to the Horizon; the line CD, on which it was supposed to turn, may be considered as the common Intersection of the two Planes, X and Z.

It is evident, that the point A will, in that motion, have described the Ark AB; and, if the Angle ACD be a Right one (as it is supposed) BCD is still a Right Angle; wherefore, AC and CB are both perpendicular to CD; § and the Plane ABC, described by the motion of CB, is also perpendicular to CD, † and consequently, to the two Planes X and Z. ‡ Therefore, the Angle ACB, in the Plane ABC, is the Angle of Inclination of those two Planes.

Draw AD, at pleasure, cutting the common Intersection, CD, in D; and, from the same point D, draw DB, in the inclined Plane X.

DEM. Now, if ACD be a Right Angle, ADC is acute — Cor. 3. 10. 1. El. wherefore, AD is longer than AC; and also, BD than BC — P. 12. 1. El. But, the Chord, or Subtense, AB, subtends both the Angles, ACB and ADB; consequently, the angle ADB is less than ACB. — Cor. to 14. 1. El.

Or, suppose BF perpendicular to the Plane Z; and let FD be drawn.

Then, a Plane DBF, passing through BF, is perpendicular to the Plane Z, but not to X. And, because FD is longer than FC, and BD than BC, it is manifest, that the Angle BCF is greater than BDF.

But, the Plane CBF is perpendicular to both the Planes, X and Z (as before) and, consequently, to their Intersection CD.

* Imaginary Points in the Heavens, diametrically opposite to each other; the one perpendicular over our Heads, the other under our Feet, in the lower Hemisphere.

From which it is clear, that the Angle made by a Plane cutting two other Planes, perpendicular to their common Interfection, is the Angle of Inclination of those two Planes; seeing that, the Angle made by any other Plane, passing through AB or BF , will necessarily be less, the greater the Inclination of AD , or FD , to CD .

To set this matter in the clearest Light possible, it being so very essential in Perspective, as well as in other Sciences and Arts, I have added the following Figure.

Let ABC and CBD be two rectangular Planes, cutting each other in BC , their common Interfection. Let EF and FH be two Right Lines, one in each Plane, perpendicular to their Interfection BC , at the same Point, F . Fig. 10.

Wherefore, the Plane $EIFG$, passing through those Lines, is perpendicular to both Planes, AC and CD ; § and, the Angle EFG , made by that Section, is the largest that can possibly be made by a Plane which is perpendicular to either of them. § 2. 7. El.

For, suppose the Line EH perpendicular to the Plane ACB , only; and, a Plane $EIKL$ to pass through that Line, it will be perpendicular to the Plane AC ;† and because the Plane EFG is perpendicular to both the Planes AC and CD , and passes through the same Point E , it will, also necessarily, pass thro' the Line EH ; wherefore, EH is the common Interfection of those two Planes,* $EIFG$ and $EIKL$ produced. I say, that the Angle EFH , made by the Plane $EIFG$, is greater than EKH . † 9. 7. El.

DEM. Now, EH , the common Interfection of the two Planes IFG and IKL , together with the Interfections EK and KH , of the Plane IKL with the two, Planes AC and CD , form a Triangle; and so does the same line EH with the two Interfections EF and FH , made by the Plane IFG , with the same Planes AC and CD .

But HFH is a Triangle, and, the Angle HFH is presumed to be a Right one; wherefore, HK is longer than HF ;‡ and, for the same reason, EK is longer than EF ; and consequently, the Triangle EFH , having one Side (EH) common with the other Triangle EKH , and, having the other two Sides, EF and FH , less than the two Sides EK and KH , respectively, they, therefore, contain a larger Angle, viz. EFH than EKH . — — Cor. to 14. 1. El.

But, the Plane IFG is perpendicular to both the Planes, AC and CD ; and the Plane IKL is perpendicular to one of them (AC) only.

Therefore, the Angle EFG , made by the section of a Plane which is perpendicular to both the other, is larger than any Angle made by any other Section, of a Plane perpendicular to one of them only; and consequently, the Section EFG measures the true Angle of Inclination of the Planes ABC and CBD . Fig. 10.

N.B. The Angle made by the section of a Plane inclined to both Planes, AC and CD , may be either greater or less than EFG .

5. Although it is not possible to conceive an Idea of a Plane abstracted from one or other of the three Positions I have explained, yet, in the application of Planes, in Perspective, as in Geometry, no particular regard is had to them; for one Plane is said to be perpendicular to another, if it makes right Angles with the other Plane, as H to V : each of which is said to be perpendicular to the other, notwithstanding one of them (H) is horizontal. Fig. 11.

The Planes X and Y are also said to be perpendicular to each other, although both are inclined to the Horizon; and, whatever their Inclination to the Horizon may be, it matters not, if they make right Angles with each other, as at C . For, if the Plane Y was turned up, on AB , its intersection with the Horizon, into the vertical Position W , and, along with it, the Plane X , into the horizontal Position Z ; their Position, in respect of each other, is not altered, if the Angle, at D , be still a Right one, as before, at C .

* EI is the true Interfection, perpendicular to the Plane AC ; and consequently, EH would be IE produced. But, as it would run into the Figure below, I thought it best to dispense with it, as the Demonstration is the same.

Plate III. 6. So likewise, one Plane is said to incline to another, if they do not intersect at right Angles, as H and X, or would not if produced, as X and V.

The Plane X being inclined to both H and V, (Art. 3. of Planes) they are, for the same reason, both inclined to X; yet, one is horizontal and the other vertical; for, the inclination of two Planes is mutual. So that, when it is said that one Plane is perpendicular or inclined to another, it means nothing more, than, that they are at right Angles, or otherwise with each other; no regard being had to the horizontal or vertical Position of either; except the Position of one is previously known or determined, to which, the other is said to be perpendicular or inclined.

Fig. 12. 7. In Perspective, it is also frequently said, that, Lines are perpendicular to certain Planes; whereas, if the Plane be vertical, it is easy to conceive, from what has been said, that all Lines, which are perpendicular to a vertical Plane, are horizontal, and parallel amongst themselves; as AB, EF, and CD, to the Plane GIK. Yet, the Planes, in which these Lines are, may be either horizontal, as ABFE; vertical, as EFCD; or inclined to the Horizon, as ABCD.

If the Plane be horizontal, the Lines perpendicular to it are really perpendicular i. e. to the Horizon; as AJ, ED and FC. But, if the Plane be inclined to the Horizon, then, the Lines, which are perpendicular to it, are also inclined to the Horizon, yet parallel amongst themselves.

Suppose the Plane ILMN vertical, and perpendicular to the inclined Plane GH; the Lines LI and MN, which are at right Angles with its Intersection, IN, are perpendicular to the Plane GH. But the line IP, which is perpendicular to the Horizon, is inclined to the Plane GH, in the Angle LIO. And OI, at right Angles with IP, is horizontal; but, it is also inclined to the Plane GH, in the Angle OIN equal LIP.

DEM. Let NI be produced, towards H. LI is perpendicular to NH.

Then, because LIN is a Right Angle, LIH is, also, a Right one - C. 2. 1. El. Consequently, LIO, equal LIN - OIN, is equal to PIH, or LIH - LIP.

Every Right Line, therefore, which is neither parallel nor perpendicular to a Plane, is consequently, inclined to that Plane; and its Inclination may be known, by drawing a Perpendicular from the extreme, or any other Point, as M, in the Line IM, to the Plane GH; the Complement of the Angle IMN, i. e. IML

* 4. 1. El. equal MIN*, is the Inclination of IM to the Plane GH.

Or, if a Plane (ILMN) be drawn, through the Line IM, perpendicular to the Plane GH; the Angle MIN, which the line IM makes with IN, the intersection of the perpendicular Plane with GH, is the Angle of its inclination to the Plane GH.

N. B. IN is the Seat of the Line IM or IO, and also of ML, or, of the Plane NL, on GH; produced by the Intersection of a perpendicular Plane passing through the Line, as above. Therefore, the Angle which any Line makes with its Seat, on a Plane, is the Angle of its inclination to that Plane.

8. PQRS is a horizontal Plane, cutting the inclined Plane KPQ in the Line PQ. But, PQ is not the Seat of the Plane PR, on KQ, it being inclined to KPQ, in the Angle SPT; for, if ST and RU be drawn, perpendicular to the Plane KPQ, the Lines PT and QU (joining the Points, T and U, where the perpendiculars cut the Plane, with P and Q) are the Seats of the Lines PS and QR; TU is the Seat of the Line RS, or of the Plane TURS; and consequently, PQUT is the Seat of the Plane PQRS, on KPQ. Also, turs is the Seat of that part which is over the Ground Plane.

I would advise the young Student, who is not well versed in these things, to make them familiar to him; for which, the reading over a second time, with due attention to the Figures, will be sufficient.

I have been more particular on this Subject, because I have frequently known Pupils to be mislead, by calling Lines and Planes perpendicular, imagining them to be really so, i. e. to the Horizon; from the common acceptation of the Term, Perpendicular, (to hang down, as a plumb Line) not considering the Position of that Plane or Line, to which, the other Planes or Lines are said to be perpendicular; and to which, due attention must first be given.

9. Suppose the Object AIKC to be, in the lower part, a right angled Parallelopiped (the most general form for Buildings, or the several Parts of a Building) the Planes BEDC, and AIB, of the Front and End, and their opposites, are vertical; for they are perpendicular to the Horizon, or Ground (considered as a Plane) on which it stands. Fig. 13.

Now, the Lines AB and FE, in the Plane AIB, are perpendicular to the Plane BEDC; and the Lines BC, ED, and IK, are all perpendicular to the Plane AIB (i. e. the originals of those Lines are so, in the real Object) yet, they are all parallel to the Horizon, and the three last, parallel between themselves, though in different Planes; § for, a Plane may pass through any two Lines which are parallel. (Ax. 5.) § 4. 7. El.

10. In the Practice of Perspective, it is often necessary to suppose the Object, we are delineating, transparent, as if the whole Object was Glass; and, the Planes or parts of the Building, which are adjacent, are supposed to be seen through the hither Planes; as ABCH, the ground Plane, and FEDG parallel to it; AFGH parallel (or otherwise) to BEDC, and HGDC opposite to AFEB.

These six Planes, forming a right angled Parallelopiped, compose the Body of the Building; every Angle of which, A, B, E, D, &c. is a solid Right Angle; each being composed of three plane Right Angles, as ABC, ABE, and EBC, of the Angle B; consequently, AB, BC, and BE are each perpendicular to the other.

By means of this supposed transparency, the connection of the several parts of an Object are distinctly seen, and delineated with greater accuracy; which, in some Cases, could not, without that expedient, be so certainly ascertained.

11. The Roof, FIKDGE, is a triangular Prism, in its construction. The Planes, EIKD on this side, and FIKG on the other side, are inclined to the Horizon; yet, these Planes may be perpendicular to each other, if the Angles, FIE, GKD, are Right. (See Art. 5.)

They are also inclined to the Front and its opposite, which are vertical Planes; but observe, that, their Inclination is not the internal Angle IEB, or IFA, for those Angles are obtuse; but, if either of the Planes be produced, as BEDC to LM, there is made an acute Angle (IEL) with that Plane, equal to the Complement of IEB to two Right Angles; which, is the Angle of Inclination of those Planes.

The inclined Planes, of the Roof, as well as the horizontal and vertical Planes AC, EC, &c. are perpendicular to the Plane AIB, and to HCK, its opposite.

This, I hope, is sufficient to make all I have said, relative to Planes and Lines (in respect of their positions, and affinity to each other) clearly understood; if so, it is a very material point gained towards understanding Perspective clearly; which will be found a most necessary Introduction to the Theory, and will, also, greatly facilitate the Practice.

Plate IV.

SECTION II.

Of the several kinds of PROJECTION, and other introductory Matters.

PROJECTION is the description or delineation of Objects in Plano, or on a Plane, according to a certain Law; by means of Right Lines, called Rays, supposed to be drawn from every Angle of the Object to some Plane.

Fig. 14. As Aa, Bb, Cc, &c. which, taken altogether, are called the SYSTEM OF RAYS.

When those Rays are all united in a Point (as A O, B O, &c.) it is called a CONE or PYRAMID OF RAYS; and that Point (O) being supposed an Eye, and the Right Lines O A, O B, &c. Visual Rays, the System of Rays is then called, the OPTIC CONE, or PYRAMID (See the 4th Definition, in Optics; Book I. Page 9.)

The Figure described, as a e f b, on the Plane V, a b c e, on the Plane Z, or, a b d e, on the Plane X, is the Projection of the Object A B C.

N. B. When the Object to be delineated is circular or globular, the System of Rays is then properly called a Cone, for it is really so; but when the Object is right lined, it is more properly called a Pyramid, as in Fig. 6. Plate 3; the Base, A, only, being considered as the Object, and a B, b B, &c. as Visual Rays to the Eye, at B, in the Vertex of the Pyramid, a B d, formed by the Rays.

ICHOGRAPHY, or Ichnographic Projection, is that which is described by Right Lines, parallel amongst themselves and perpendicular to the Horizon, from every Angle of the Object, on a Plane parallel to the Horizon.

Fig. 14. No. 1. As Aa, Bb, Cc, &c. from the Angles A, B, C, &c. of the Object, to the Plane Z, which is horizontal. The points, a, b, c, d, &c. where the perpendicular Lines, or Rays, cut that Plane, being joined by Right Lines, a b, b c, c d, &c. and diagonal wise, as a c, b d, &c. is the Ichnographic Projection of the Object, A B C.

The Figure, a e c b, projected on the horizontal Plane Z, is likewise called the PLAN, or SEAT, of that Object on the Ground Plane. The Points a, b, c, &c. are the Seats of the Angles, A, B, C, of the Object. The Line a b is the Seat of the Side A B, and b c of B C, &c. also the Diagonal a c is the Seat of A C; by which means, the Plan of the whole Figure is formed.

ORTHOGRAPHY. If the projecting Rays are parallel to the Horizon, and fall perpendicularly on a Plane (consequently vertical) the Figure described, on that Plane, by the intersection of the Rays, is the Orthographic Projection.

Fig. 14. No. 2. The Lines Aa, Bb, &c. are parallel to the Horizon and to each other; which, falling perpendicular on the vertical Plane V, describe the Orthography of the Object A B C, on that Plane.

Of this kind of Projection there may be infinite variety; for, if either the Plane or the Object be turned, though ever so little, the Figure will be varied on it.

The Figure, a e f b, thus projected, may likewise be called the SEAT of the Object A B C on that Plane; as a e c b on the Ground Plane.

a is the Seat of the Angle A, and b of the Angle B, &c; a b is the Seat of the Side A B, a e of A E; and, the Diagonal a f of the Side A F.

a e may also be considered as the Projection, or Seat, of the Plane A E C, and a b of A B C; for, if they were produced, they would cut the Plane V in those Lines.

N. B. The Seat of a Point, Line, or Plane, may be had on any Plane in whatever Position; by drawing Right Lines from each extreme of the Line, &c. perpendicular to the Plane.

b f on the Plane Z, and b f on the Plane V, are the Seats of the same Line B F, in the Object A B C; and so of the rest.

Ortho-

Orthographic Projection is usually called the **ELEVATION**. But, when it exhibits the End of a Building, or part of the End only, shewing the Projectures of the several parts, from the main Body of the Building, the Contour of the curves of Moulding, &c. it is called a **PROFILE**. And, when the Building, or other Object, is supposed to be cut by a vertical Plane, in any direction through the Building, the hither part being supposed to be removed, and the inside exposed to view, shewing the thickness, &c. of the Walls and Floors, the structure of the Roof and proportion of the Timbers, &c. it is called, a **SECTION**.

All these different kinds of Projection are entirely geometrical, and supposes the Eye of the Spectator at an infinite Distance; the projecting Rays being parallel.

SCENOGRAPHY is the Projection made by a Cone or Pyramid of Rays, Right Lines, from every Angle of the Object, converging to a Point. As $OA, OB, \&c.$

Those Rays being cut by a Plane (X) passing, in any direction, between the Object and the Vertex (O) the Figure projected, by the Rays, on that Plane, is called the Scenographic Projection of the Object ABC .

Fig. 14.
No. 3.

If the Vertex, (O) of this Pyramid of Rays, be considered as the Eye of a Spectator, and the Right Lines $OA, OB, \&c.$ as Visual Rays, the optic Pyramid being cut by the Plane X , considered as a Picture, the Figure, abc , projected thereon, is the **PERSPECTIVE REPRESENTATION** of the Object ABC .

STEREOGRAPHY comprehends the whole Art of representing Solids * on a Plane; which differs from geometrical Projection, in what was explained in the preceding Definition; that, the projecting Rays, supposed to proceed from every part of the Object, terminate in a point, at any distance from the Object; which Point, is always considered as the Eye of a Spectator, and the intersecting Plane as the Picture. The section of the Rays is the Representation.

By this kind of Projection, the several dimensions of Bodies, *viz.* length, breadth and thickness, are all represented at one View, in the greatest degree of perfection that Art is capable of, or Nature exhibits.

As the Stereographic Projection (of Objects) is various, I shall explain it, more fully, under the three several Heads, Perspective, Projected Perspective or Projection, and Transprojection. And first, of **PERSPECTIVE**, which is, more immediately, the Subject of the following Treatise; that part of Stereography being more particularly adapted to the purpose of delineating all kinds of regular Objects, than either of the other; which are, therefore, but seldom practised; and indeed, because they are but little understood, or the difference between them known; yet, they have their several uses in delineating, particularly in projecting shadows.

PERSPECTIVE is a Science founded on Geometry; which teaches how to delineate, or draw, on a plane Superficies (or, simply, on a Plane) the true representations of Objects, according to their Distance and Situation, and Bearings of the Objects to each other; from any Station, at pleasure, real or imaginary.

To give a clear and perfect Idea of the Principles on which the Science of Perspective is built, it is necessary that the Objects, to be represented, are considered as being beyond, or on the other side of the Picture; that is, having fixed on the Station, from which you intend to delineate an Object, imagine a transparent Plane interposed between the Eye and the Object, or Objects; through which you have distinct Vision, of the Objects on the other Side.

It is evident, to all who have considered it, that if, with a steady hand, you trace every Line, of Objects, accurately, as they appear on the transparent Plane (which may be supposed a Window, or a perfectly regular Plate of Glass) keeping the Eye fixed in a Point, there will be true, linear perspective, Representation of all the Objects, on that Plane; which, is considered as the Picture. Now, the performance of this, by geometrical Rules, is what is properly called **PERSPECTIVE**.

* Every Object, having length, breadth, and thickness or depth, comes under the Denomination, geometrically, of a Solid; the concavity, (as of a Box, or House, &c.) not being considered; but only the external Form, consisting of Planes or other Surfaces, variously disposed.

Plate V. N.B. Every Reference, to Figure 15, refers likewise to the Apparatus.

Fig. 15. Let BFIKL be an Object having the three Dimensions, length, breadth, and thickness; which may be supposed a Building, or what you please, to be represented on the Plane, or Picture, MNOP, standing perpendicular on the Ground Plane, SRZ; whose Intersection with it, (MP) is at right Angles with the line of Station, (SL.) The Picture, MNOP, is, therefore, direct, between the Object and the Spectator, ES. EA, EB, EF, &c. may be considered as Visual Rays, in which the vision of the Object, BFIL, is conveyed to the Eye, at E.

2. It is obvious, that if this Plane, or Picture, be transparent, the Representation, aifc, on that Plane, of the Object, BFIL, on the other side, would, by means of the Right Lines EA, EI, EF, &c. from each Angle of the Object to the Eye, exactly coincide, and agree in every part, with the original Object.

For, as it is not possible for Vision to be conveyed but in Right Lines to the Eye (except through dense, refracting Mediums) the Angle A, of the Object, must necessarily appear at a, on the Picture, B will appear at b, and F at f, &c. where the Right Lines, from the Angles of the Object to the Eye, pass through the Picture; and is the supposed reason for the genesis of a Point on the Picture, in Theory. The Right Line ab, or bg, on the Picture, joining the representations of the Angles, or Points, A and B, or B and G, in the Object, will also coincide with, and hide the Original Line, AB, or BG, from the Eye, at E; and is, therefore, its Representation; and so of all the other Lines on the Picture.

This, I think, needs no other Demonstration, for it is ocular, and evident, that the Angle, made by the Pyramid of Rays forming a solid Angle (AEF, FEC) at E, is, not only equal, but, the same, under which, both the Object and its Representation are seen.

So likewise, the representation, fi or fd, on the Picture, of any Line, FI or FD, in the Object, is seen under the same Plane Angle, IEF or FED. Consequently, fi coincides with FI, and fd with FD; the representations, abgh of the Plane ABGH, fghi of FGHI, and bfc of the Plane BFC, coincide with each other, respectively; and consequently, the whole Representation, aifc, or Projection of the Object, AIFC, perfectly coincides with the Original, in the Point of View E, in that Position and Situation of the Picture and Object.

Wherefore, since the Eye is affected, in the same manner, by the Lines and Angles on the Picture, as by the corresponding Lines and Angles of the Object, it is evident, that, if the Picture had the same degree of Light and Shade, and also the true tint of Colour, as the Object, it would be impossible for the Eye, at E, to distinguish whether it was a Picture, delineated on the Plane MNOP, or the real Object, on the others side, that was perceived.

3. Hence it is manifest, that there is, and may be, great deception in Vision; and also, that there may be various Representations, of the same Object, from the same Station or Point of View; which, notwithstanding their difference in figure and dimensions, will have the same Appearance in the true Point of View.

For, if any other Plane, as MNOP, be placed between the Eye and the Object, not parallel to MNOP, the Representation, of the same Object, projected on that Plane, by its intersection with the Visual Rays, will not only be smaller, but, also, very different in Figure and Proportion. For, having drawn the Right Lines SA, SB, SC, from the Foot of the Spectator, at S, to each angle of the Object, towards the Picture, on the Ground Plane, they will determine the extreme width of the representation of that Object on each Picture; and also the proportion, of the representation of the Plane AG to that of GC, which is, as kl to lm, on the Picture MNOP, where the lines SA, SB, and SC cut the bottom edge of the Picture; and, on MNOP, the Proportion is as no to op; but, MP is not parallel to MP; consequently, the proportion, of no to op, is not as kl to lm.

If the Picture was placed on MQ, it is obvious that the Representation would be still less, and also different in its Figure and Proportion; as the parts Qr, rq, intercepted between the lines SA, SB, and SC, sufficiently evince.

Thus, may there be as many different Representations, of the same Object, as you please, and from the same Point of View; from the different Position and Distance of the Picture; all which, will affect the Eye alike, at the Point E, in the Vertex of the Optic Cone or Pyramid of Rays.

PROJECTED PERSPECTIVE, or PROJECTION.

If the Visual Rays EB, EC, ED, &c. are supposed to be produced, or projected beyond the Object, ABCD (a quadrangular Pyramid) and there fall on, or are cut by a Plane (V) the Representation (abcd) of the Object, projected on that Plane, is the Projection of the Object ABCD. Fig. 14.
No. 4.

Projected Perspective is the very same, except in the operation, as common Perspective, i. e. when the Rays are cut by a Plane passing between the Object and the Eye, with only this difference, that, in common Perspective, the Representation is always less than the Object; because the section of the Rays, by the Plane X, is on this side, towards the Eye; in projected Perspective, the Representation must necessarily be larger than the Object, because the Plane of the Section (V) is beyond the Object; but, if the Planes are parallel between themselves, whether the Rays are cut on this or on the other side of the Object, or both, the Representations will be perfectly similar. No. 5.
No. 4.

a, b, c, and d are the projective Representations of the several Angles of the Pyramid, A, B, C, D; which, joined by Right Lines, is the Projection of that Object on the Plane V.

FGHI is the Plan or Seat of the Object ABCD, on the Ground Plane, to which the Base, ACdD, is parallel.

By means of the Seat and the Station Point, S, the Representation, abcd, on the Plane V, may be projected.

The difference between Perspective and Projection is very obvious.

In Perspective, the Object, to be represented, is always supposed to be beyond the Picture; in Projection, the Picture is beyond the Object; which is projected, or supposed to be thrown forward to the Picture, and which is full as rational to suppose. Nor is there occasion, in this Case, to suppose the Picture transparent, as in the former, when the Picture is supposed to be on this side of the Object, and seen through; which indicates the meaning of the term Perspective.

The difference in the operation is very little, and is illustrated by frequent use in the practical part of this Work.

TRANSPROJECTION. If the Visual Rays, from the Object (ABCD) are supposed to pass through the Eye (at E) forming an opposite Pyramid of Rays (aEb) and there fall on, or are cut by a Plane (Y) the Figure (acbd) projected on that plane, by its intersection with the Rays, is the Transprojection of the Object (ABCD). Fig. 14.
No. 6.

It is evident, that, in this kind of Projection, the Representation may be either larger or smaller, or equal to the Object, as the Plane, Y, is removed farther from or nearer to the point E; or, as the point E is removed nearer to, or further from the Object. For, if the point E be in the middle, between the Object and the Plane of the Section, the Representation will be equal to the Object; and, whether it be nearer to, or further from the Object, the Representation will have that Proportion, to the Original, as their Distances from the point E.

It is also evident, that Projections of this kind must necessarily be inverted; as the Rays all pass through one common Point (a is the transprojected place of the Angle A, on the Plane Y, c of C, and b of the Vertex, B, of the Pyramid) in the same manner as optical Philosophers endeavour to account for Vision; by supposing an Image, of every Object perceived, formed in the back part of the Eye,

Plate IV. on the Retina, in the same inverted Position. The Eye being in this respect like a *Camera Obscura*, which is a kind of artificial Eye; in which, the Picture is always inverted. (See Page 10, 1st and 2nd Par. Also, see Fig. 13, Plate II.)

Notwithstanding, if the Plane Y, of the transprojective Picture, be parallel to either the perspective, (X) or the projective, (V) the Representation thereon will be similar to the other, or to both, if they are all parallel amongst themselves.

An ORIGINAL OBJECT is any Object whatever, which is the Subject of the Picture we are delineating.

Fig 15.

BFIL is an original Object, of which, the Projection, a ifc, on the Plane MNOP, or a ifc, on the Plane MNOP, are Representations.

Also, parts of an entire Object, as a Column, a Chimney, &c. are the Originals of their separate Representations.

By ORIGINAL PLANE is meant, not only the Ground, or other Planes upon which Objects are seated, but also, the plane Surfaces of original Objects to be delineated.

ABGH and BFC. &c. in the Original Object BFIL, are Original Planes, as well as the Ground Plane (Z) on which it stands.

It may be necessary to make a distinction between Figures and Objects, although every Plane Figure may be called an Object; but, I think, that Term is more properly, applicable to Solids than to Plane Figures.

By ORIGINAL FIGURE, I shall therefore mean, only, Figures in Original Planes, as Doors, Windows, &c. in Original Objects; or the Figure of the Original Plane itself. As FGH I, or BGFDC, &c.

Any geometrical Plane Figure whatever, in Original Objects, is, therefore, an ORIGINAL FIGURE.

ORIGINAL LINE is any Line in an Original Object, whether Right Line or curved; as the bounds or limits of all Original Planes, or Figures, are Lines.

AB, BG, FG, FD, &c. are Original Lines, in the Original Planes ABGH and BFC, each, of which, is likewise in another Plane; for, each is the common Intersection of two Planes; consequently, each Line is in two Planes.

By Th. 1.7. El. the common Intersection of two Planes is a Right Line, 3. 11. Euc.

AB is the common Intersection of the Ground Plane and the Plane ABGH, and is, therefore, in both Planes; and, BG is, for the same reason, in both the Planes ABGH and BFC; and so of all the rest.

The Intersection of two Lines is a Point; and the extremes of Lines are also Points; wherefore, the Angles F, G, H, &c. of the Object BFIL, are Points; for they are the Intersections, and also the extremes of Lines; and are called ORIGINAL POINTS.

All Objects, whatever, that are formed by Art, as Buildings of all kinds, or other regular Objects, (which are the fittest subject for Perspective) are composed either of Planes or of curved Surfaces, or both. Every Building and parts of Buildings come under the denomination of some one or other geometrical Solid; as Parallelopiped, Prism, Pyramid, Cylinder, Cone, or Sphere; or they are compounded of several, together. For, the Body of the Building is either one Parallelopiped, or it is composed of several, variously disposed, at the discretion of the Architect. The Roofs are, generally, either triangular Prisms, or Pyramids. The Planes, of which Roofs are chiefly composed, are either Triangles, Parallelograms, or Trapezia. The Planes of the Fronts and Ends of a Building are, for the most part, Rectangles; sometimes Pentagons (as BGFDC) or other Polygons; which contain other geometrical Figures, variously disposed, as Doors, Windows, &c. which

which are generally Rectangles, sometimes Circles, Ellipses, or mixed Figures.

Temples, in Gardens, Cupolas, &c. are either cylindrical, or polygonal Prisms; the Roofs, of which, are either pyramidal, or spherical, or mix'd curved Surfaces.

Columns approach nearly to Cylinders; in the lower part they are perfectly so.

Thus, may every part of a Building, or other regular Object, be reduced to some geometrical Solid or other. Solids are composed of Planes or other Surfaces, all which, may again be reduced to their first Principle, Lines; for, as the boundaries of Solids are Planes, or other Surfaces, so the bounds of Planes, &c. are Lines.† Also, the Figures in Planes, whether Doors, Windows, or other Fi-

† Def. 55,
Geo.

gures, are formed of Lines; all which, are ORIGINAL LINES. So likewise, Mouldings, Steps, &c. are represented by Lines; which, in the Originals, are generated either by Planes, only, as streight Steps, or by Planes and curved Surfaces, as in Mouldings and circular Steps. Each Moulding, if it be right lined, is composed of Planes and cylindrical Surfaces; which, by their parallel interfections, generate Right Lines. Circular Mouldings, in Cornices, &c. are composed of Planes, with cylindrical and other curved Surfaces; which, by their regular interfections, generate circular Lines.

The Edges of Columns and other cylindrical Objects, which are represented by Lines, on the Picture, have no real existence in the Originals, but are only apparent; for there is no real Line; as it is but one continued Surface, which returns again into itself, without cutting or intersecting, by which Lines are generated. The Lines which form the representations of the Bases, &c. of Columns are, in the Originals, some of them real and some only apparent; as the Contour of the curve of the Torus, &c. and have various forms, on the Picture, according to the situation of the Eye, or Picture.

Having thus reduced compound Original Objects into their first Principles or Elements, *viz.* Planes and Lines; the next thing, to be considered, is the Position and Situation of those Planes and Lines, in respect of the Picture and of each other; which being premised and well considered, the whole mystery of linear Perspective will be found comprised in a small Compass, both in Theory and Practice. The Principles on which the Theory is built are few, but they are general, and applicable in all positions of the Picture and situation of the Object, or of the Eye. Wherefore, the Distance and Situation of the Object being determined; that is, the Station being fixed, from which an Object is to be delineated, and the Position of the Picture determined, the Representation is also determinable; which, Representation, is in proportion to the Distance of the Picture.

Thus far I have proceeded, by way of Introduction. I have called it an Introduction to Perspective, because, it cannot be called a part or branch of that Science; seeing, all which it contains may be, and is, known to several, who are not acquainted with one Theorem, or any Rules for the practice of Perspective. Nevertheless, as this Work is intended to be a perfect Tutor for young Students, I am well convinced, that, the knowledge they will acquire by this Introduction is by no means to be dispensed with; being as essential to be previously known, as the Definitions of the Terms used in Perspective; which are elementary. It is certainly possible, by Rules laid down, for a Person to practice Perspective without knowing what Perspective means; but that is not compatible with my Design, having entitled this Work a Compleat Treatise, which could not possibly be, without accounting for the Rules given; which I shall do, as briefly as is consistent with the Subject, not dwelling on any unnecessary part of the Science; but certainly, every necessary knowledge, previous thereto (which being, perhaps, no part of any other distinct Science) should first be inculcated.

SECTION

Plate IV.

S E C T I O N III.

Containing the ELEMENTS OF PERSPECTIVE.

IN order to investigate the Theory of Perspective, with clearness and precision, it is necessary to have recourse to certain imaginary Planes, which may be conceived to pass through the Eye of a Spectator, or Point of View, from which an Object is supposed to be delineated, in all Positions as occasions require.

The Picture, as it has already been observed (under the Article Perspective) is supposed to be between the Eye and the Object.

Three of those imaginary Planes, together with the Picture and any Original Plane whatever, are supposed to be constructed as they are represented in Plate IV, Fig. 16, 17, 18, 19, 20 and 21. The Planes, thus constructed, I shall first define; afterwards, the Lines generated by their Intersections; and lastly, the Points produced by the intersections of the Lines; all, which, are so essentially necessary, that, in short, without the assistance of these five fundamental Planes, and the Lines and Points generated by their Intersections with each other, Perspective would be a very intricate and perplexed Study; and, in Theory, a most imperfect Science.

I would particularly advertise young Students, not to pay the least regard to the general positions of the Planes I am about to define, but only their positions in respect of each other; for which reason I have given them in various Positions, and advise the Reader not to give particular attention to the first. For, in the Theory of Perspective, the position they have to the Horizon is not considered, at all; as the Theorems are general, and applicable in all positions and situations of the Picture, whatever; since (as Dr. Brook Taylor, in the Preface to his second Treatise, justly observes) all Planes, simply as Planes, are alike in Geometry, and have the same properties however situated.

D E F I N I T I O N S.

Fig. 16. I. Let ABGH be considered as a part, or a continuation of an ORIGINAL PLANE; being supposed to be produced (if necessary) from any Original Object to the Picture; and, till it cuts a Plane, passing through the Eye of a Spectator, parallel to the Picture.

D E F I N I T I O N II.

The PLANE of the PICTURE is considered as the Board, Paper, or Canvas, on which is to be delineated the representation of some Original Object, or Plane Figure. As ABLM, or the Plane X, is the Picture.

The PICTURE is, generally, and for the most part, vertical, and direct between the Eye and the Object to be delineated; as in Fig. 16, 18 and 19; but may be in any Position whatever; as in Fig. 17, 20 and 21.

D E F I N I T I O N III.

VANISHING PLANE. If a Plane be imagined to pass through the Eye, parallel to any Original Plane, it is called the VANISHING PLANE of that Original Plane; or, simply, the PARALLEL of the Original Plane.

As V, or IKLM, parallel to Z or ABGH, Fig. 16, 17, &c. E is the Eye.

And, a Perpendicular (EC) from the Eye to its Intersection with the Picture, is the RADIAL of the Vanishing Plane.

The DIRECTING PLANE is an imaginary Plane, supposed to pass through the Eye of a Spectator, parallel to the Picture.

GHIK, or the Plane Y, parallel to the Picture (X) is the Directing Plane; the Eye of a Spectator being supposed at E, in the Directing Plane.

N. B. The Distance of the Directing Plane, from the Picture, is always equal to the Distance of the Eye (as E C) and being parallel to the Picture, it makes equal Angles with the Original Plane, as the Picture. (IHA equal MAN).

D E F I N I T I O N V.

The VERTICAL PLANE (ECDF) is supposed to pass through the Eye, perpendicular to the Original Plane and to the Picture. Wherefore, it cuts all the four preceding Planes at right Angles, in all positions of the Picture whatever.

D E F I N I T I O N VI.

RADIAL PLANE is an imaginary Plane, passing through the Eye and any original Right Line, whatever.

As EVID, or ECPD; which, being produced, would pass through the original Line ON, or QP.

Fig. 20.

In Fig. 16. the Original Plane (AG) is supposed horizontal, and the Picture (AL) vertical; consequently, the Vanishing Plane (MK) and Directing Plane (KH) have the same Positions, and cut each other at right Angles.

In Fig. 17. suppose the Picture (ABLM) and the Directing Plane (HIKG) in the same vertical Position; and suppose the four Planes, viz. the Original Plane, the Picture, the Vanishing and the Directing Planes, so fixed together, as to be moveable on their Intersections, AB, GH, IK, and LM, as on hinges; and then, let us suppose, the Picture and Directing Plane (and along with them the Vanishing Plane) pushed into an inclined position, on either side of the original, vertical Position; the Intersections, AB and GH, remaining as they were, unmoved.

It is evident, that the parallelism of the Planes is not destroyed by this motion; for, the Vanishing Plane (IKKM) is still horizontal as before, though removed lower, to iklm; and, the Directing Plane, HIKG or HIKG, is still parallel to the Picture, AmlB or AMB, both being inclined to the Original Plane and its Vanishing Plane, in equal Angles.

In Fig. 18. the Picture (X) and the Directing Plane (Y) are still vertical, but the Original Plane (Z) and its Vanishing Plane (V) are inclined to them, and to the Horizon; making equal Angles with each other, as in the former Case.

Nor is there understood to be any difference, in their Position, between this and the preceding Figure; for, either Plane (AL or HK) may be supposed the Picture, and the other the Directing Plane; by which means, it has both the Positions of the former; only, supposing the Original Plane horizontal, the Picture and Directing Plane are consequently inclined.

In Fig. 19. both the Original Plane and Picture (NBH, and ABLM) are supposed vertical; therefore, the Vanishing and Directing Planes are also vertical; and the vertical Plane (CDFE) in this Case, is consequently horizontal.

In Fig. 20. the Planes are all inclined to the Horizon, and also to each other; excepting the Vertical Plane; which always cuts the other four at right Angles, and is, therefore, perpendicular to them all.

This Figure I recommend, more particularly than any of the other four, to be contemplated by the young Student (although it is the same in all) in order to divest him, entirely, of partiality to any particular Position, respecting the Horizon.

In Fig. 21. The four primary Planes are all moveable, and may be put into all the Positions of the former; either at right Angles, as Fig. 16; or inclined, on either side, as in Fig. 17, in any Angle at pleasure.

Plate IV. As these Planes are all marked with the same Characters, as in the five preceding Figures, it would be superfluous to particularize them here; and, if a Plane be supposed to pass through the four Points, E, C, D, and F, it will be vertical or perpendicular to them all, in every Position.

N. B. Any one of the four Planes may be the Original Plane, the opposite one is, consequently, its Vanishing Plane; either of the other may be the Picture, and the opposite to it is the Directing Plane; E C D F is still the Vertical Plane.

Of LINES, generated by the Intersections of the five elementary Planes.

D E F I N I T I O N VII.

INTERSECTION of the PICTURE, with an Original Plane, is the Line in which any Original Plane cuts the Picture; or, in which an Original Plane, being produced, would cut the Picture.

AB; is the Intersection of the Picture ABLM. with the Original Plane ABGH.

Fig. 16,
17, 18,
19, 20
and 21.

D E F I N I T I O N VIII.

VANISHING LINE is a Line produced by the Intersection of an imaginary Plane, passing through the Eye parallel to any original Plane, with the Picture.

LM, the Intersection of the Vanishing Plane, I K L M or V, with the Picture ABLM or X, is the Vanishing Line of the Original Plane, ABGH or Z.

D E F I N I T I O N IX.

PARALLEL of the EYE is the Line I K, in which the Vanishing Plane (V) and the Directing Plane (Y) intersect each other.

As both these Planes are imagined to pass through the Eye (at E) consequently, their Intersection (I K) passes through the Eye; and, because the Directing Plane is parallel to the Picture, the Parallel of the Eye is parallel to the Picture.

D E F I N I T I O N X.

DIRECTING LINE is the Line G H, in which, an Original Plane (Z) cuts, or would, if produced, cut the Directing Plane (Y).

D E F I N I T I O N XI.

Fig. 16,
&c.

The VERTICAL LINE is the Line C D, in which, the Vertical Plane (E C D F) cuts the Picture; at right Angles with the Vanishing Line and Intersection of the Original Plane.

D E F I N I T I O N XII.

The DIRECTOR of an Original Line. If an Original Line be produced till it cuts the Directing Plane, a Right Line passing through the Eye and that Point is the Director of the Original Line.

E D (Fig. 20 and 21) is the Director of the Line N O, being produced to D.

D E F I N I T I O N XIII.

VISUAL RAY. With optical writers, this Term signifies an imaginary Ray of Light; by which, Vision is supposed to be conveyed from the Object to the Eye; therefore, in Perspective, it is a Right Line imagined to be drawn from any Point, in an Object, to the Eye.

E A, E I, E F, &c. (Fig. 15.) and E N, E O, &c. (20 and 21.) are Visual Rays.

D E F I N I T I O N XIV.

RADIAL LINE is the parallel of any Original Line, producing its Vanishing Point. (See Def. XXII.) As E V (Fig. 20, and 21) parallel to N O.

D E F I -

D E F I N I T I O N XV.

DIRECT RADIAL is a Right Line, from the Eye or Point of View, perpendicular to the Picture. As *EC*.

N. B. If the Original Plane be at right Angles with the Picture (as in Fig. 16.) the Direct Radial (*EC*) is the common Intersection of the Vanishing Plane, (*IKLM*) and the Vertical Plane (*ECDF*); and is, always, in the Vertical Plane.

Of POINTS, and their Distance from the Eye.

D E F I N I T I O N XVI.

The **POINT of SIGHT** is that Point where the Eye, of a Spectator, ought to be placed to look at a Picture; for, in that Point, only, can a perspective Picture be seen perfectly.

E is the place of the Eye, or Point of View, to look on the Picture *ABLM*.

It is the Point where the three imaginary Planes, *viz.* the Vanishing, the Directing, and the Vertical Planes, intersect; consequently, the Eye is in all the three. Or, it is the Point of Intersection between the Parallel of the Eye (*IK*) and the **PRIME DIRECTOR** (*EF*)

N. B. It is the Vertex of the optic Pyramid of Rays (*EA, EB, EF, &c.*) the only Point in which the Images, or Representations (*a i f c.*) on the Planes or Pictures, *MNOP* or *OP*, can exhibit a true Appearance of the Original Object (*BFI L*) on the other side.

Fig. 15.

D E F I N I T I O N XVII.

The **CENTER** of the **PICTURE** is the Point *C*, in which a perpendicular Line from the Eye, or Point of Sight, is cut by the Picture.

The Direct Radial (*EC*) being perpendicular to the Picture, is, therefore, in the Vertical Plane (*ECDF*) consequently, the Center of the Picture, produced by the Perpendicular *EC*, is in the Vertical Line (*CD*) and, when the Original Plane is perpendicular to the Picture, it is the Intersection of the Vertical Line (*CD*) and its Vanishing Line (*LM*).

Fig. 16.

D E F I N I T I O N XVIII.

DISTANCE of the **PICTURE**, or principal Distance, is the Direct Radial, or perpendicular Line (*EC*) from the Eye to the Picture, or to its Center.

N. B. The Center and Distance of the Picture are most essential, and ought to be well understood; for, except the Intersecting and Directing Points, all the rest are dependant on them.

D E F I N I T I O N XIX.

CENTER of a **VANISHING LINE** is the Point where it is cut by a perpendicular Line from the Eye or Point of Sight.

EC (Fig. 15 and 16) or *EC* (Fig. 18, 19 and 20) being a Perpendicular from the Eye (*E*) to the Vanishing Line (*LM*) *C* or *C* is, therefore, its Center.

D E F I N I T I O N XX.

DISTANCE of a **VANISHING LINE**, is the Perpendicular, *EC* or *EC*, from the Eye to its Center; the shortest Line that can be drawn to the Vanishing Line.

D E F I N I T I O N XXI.

POINT of INTERSECTION is that Point in which any Original Line (being produced) cuts the Picture; or the Plane of the Picture produced, if necessary.

I is the Intersecting Point of the Line *NO*, produced to the Picture; *B* and *F* are the Intersecting Points, of the Lines *AB* and *FI*, on the Pictures *MNOP*.

Fig. 20,
and 21.

Fig. 15.

D E F I N I T I O N XXII.

VANISHING POINT. If a Line be drawn from the Eye, parallel to any original Right Line, the Point, where it cuts the Picture, is the Vanishing Point of that Original Line. (See Theo. 2. Sect. 3. Book 1.)

NO

Plate IV. NO is an Original Line, in the Original Plane HNBG; EV is a Line from the Eye, parallel to NO, cutting the Picture; V is, therefore, the Vanishing Point of NO, and EV is its RADIAL, or PARALLEL of the Original Line; consequently it is in the Vanishing Plane, or parallel of that Plane the Original Line is in.

Fig. 15. EC or EV being parallel to the Original Lines AB, GH, FI, &c. C or V, where it cuts the Picture, is, therefore, the Vanishing Point of those Lines.

N. B. The RADIAL EC, or EV, is the Distance of the Vanishing Point, C or V.

DEFINITION XXIII.

DIRECTING POINT is that Point in which any Original Line, being produced, would cut the Directing Plane.

Fig. 20. and 21. D is the Directing Point of the Original Line NO, or PQ, being produced to the Directing Plane (GHIK). It is the Intersection of the Original Line and the Directing Line, of the Plane the Original Line is in.

In Fig. 23. O, P, and Q are the Directing Points of the Lines CB, AB, and AD, where those Lines cut the Directing Plane, GIKH, produced.

DEFINITION XXIV.

STATION POINT is the Point in which a perpendicular Line, from the Eye or Point of Sight, cuts the Ground, considered as a horizontal Plane; or any other horizontal Plane, on which an Object, to be delineated, is seated.

Fig. 15. S, is the real Point of Station, at the foot of a Spectator (E.S.)

In Theory, it is that Point in which a Right Line, from the Eye, cutst he Directing Line, perpendicularly.

Fig. 16, 17, &c. As F, in the Directing Line GH; EF being, supposed, perpendicular to GH.

AXIOMS.

I. One part of a Right Line cannot be in any Plane, and another part of the Line out of that Plane, being produced.

II. Two Planes which are parallel cannot intersect.

III. The common Intersection of two Planes is a Right Line.

IV. The Intersection of two Lines is a Point.

V. Two Right Lines, that are parallel, are in the same Plane: i. e, a Plane may pass through both Lines.

VI. Two Right Lines, meeting in a Point, or which, if produced, would intersect, may be in the same Plane.

VII. Two Right Lines, being parallel, and both cut by another Right Line, are all in the same Plane.

VIII. Three Right Lines, meeting or intersecting each other, are all in the same Plane; consequently, every right lined Triangle is in a Plane.

Several of these Axioms are demonstrable Propositions, in the eleventh Book of Euclid's Elements; but, having a true Idea of a Plane, they are so very evident, that it is but trifling, to little purpose, to attempt to prove them. Nevertheless, as the Student is supposed to understand Geometry, they may, without scruple, pass for Axioms here.

SECTION

SECTION IV.

Containing the THEORY of Rectilinear PERSPECTIVE.

THEOREM I.

AN Original Plane, which is parallel to the Picture, has no Intersection with the Picture, nor Vanishing Line.

DEM. The Original Plane being parallel to the Picture cannot intersect it. Ax. 2.

And, a Plane passing through the Eye parallel to the Original Plane, which should produce the Vanishing Line, by its Intersection with the Picture, is, in this Case, also parallel to the Picture, and the same with the Directing Plane; consequently, it can never cut the Picture, and therefore, cannot produce a Vanishing Line. Q. E. D.

EX. Let $MNOP$ be the Plane of the Picture, parallel to the Plane BFC , of the Original Object ($BFIL$). Fig. 15.

Then, because the Plane BFC is parallel to the Picture, it cannot cut the Picture (though produced infinitely) therefore can have no Intersection with it. Ax. 2. And a Plane ($RSTU$) passing through the Eye (at E) parallel to the Original Plane (BFC) is also parallel to the Picture; therefore, it cannot cut the Picture and produce a Vanishing Line; (agreeable to Def. 8.) for, it is the Directing Plane of that Picture; which is parallel to the Picture. — Def. 4.

COR. Original Lines, which are parallel to the Picture, have neither Vanishing Point, Intersecting, nor Directing Points.

For, an Original Line, parallel to the Picture, is, or may be, in a Plane that is parallel to the Picture; and, a Line passing through the Eye parallel to the Original Line, which should produce its Vanishing Point †, is, in this Case, also parallel to the Picture; and is, therefore, in the Directing Plane, which is parallel to the Picture. Wherefore, it can never cut the Picture and produce a Vanishing Point; or, it may be supposed at an infinite Distance.

Neither can the Original Line, for the same reason, cut the Picture or Directing Plane; therefore, it has neither Intersecting nor Directing Point.

EX. Because the Plane BFC is parallel to the Picture, every Line in that Plane is also parallel to the Picture; therefore, BG , GD , &c.

Consequently, since one part of a Right Line cannot be in any Plane, and another Part of it out of the Plane †, the Line BG , or GD , &c. can never cut the Picture and Directing Plane, and produce an Intersecting and Directing Point; agreeable to Definitions 21 and 23.

For the same reason, a Line passing through the Eye parallel to the Original Line (BG , GF , or GD , &c.) must necessarily be in the Directing Plane ($RSTU$) consequently, it can never cut the Picture and produce a Vanishing Point; agreeable to Definition 22.

† Def. 22.

† Ax. 1.

Plate IV.

THEOREM II.

The Vanishing Line, the Parallel of the Eye, and the Directing Line of any Original Plane, not parallel to the Picture, are parallel to the Intersection of that Plane with the Picture.

Fig. 16. The Construction of the five elementary Planes, with their uses and relation to each other (as explained or defined) 'tis presumed, is well understood.

Suppose, then, the first four, viz. the Original Plane (Z) its Parallel, or Vanishing Plane (V) the Picture (X) and Directing Plane (Y) forming a right angled Parallelopiped; or that part of them, only, which lies between their Intersections (AB, ML, KI, and GH) with each other.

DEM. Now, the Picture and the Directing Plane are parallel, and they are cut by an Original Plane and its Vanishing Plane, which are also parallel. Def. 3 & 4.

But, if two Planes, being parallel, are cut by another Plane, their common Sections are parallel. P. 8. 7. El.

Consequently, if two parallel Planes are cut by two parallel Planes, their Intersections are all parallel amongst themselves.

But, AB, the section of the Original Plane (Z) and the Picture (X) is the Intersection of the Picture. Def. 7.

ML, the section of the Picture (X) and the Vanishing Plane (V) is the Vanishing Line of the Original Plane (Z) Def. 8.

KI, the section of the Vanishing Plane (V) and the Directing Plane (Y) is the Parallel of the Eye. Def. 9.

And, GH, the section of the Directing Plane (Y) and the Original Plane, (Z) is the Directing Line. Def. 10.

Therefore, the Vanishing Line (ML) the Directing Line (GH) and the Parallel of the Eye (KI) are all parallel amongst themselves, and to the Intersection (AB) of the Original Plane with the Picture. Q. E. D. 4. 7. El.

N. B. Whether the Original Plane be at right Angles, or otherwise, with the Picture, the Demonstration holds good, in all positions whatever; since, the Directing Plane is always parallel to the Picture, and the Vanishing Plane to the Original Plane; consequently, their Intersections with each other are still parallel amongst themselves.

Fig. 21.

To assist the Imagination, I have given a Figure with the four principal Planes, moveable; which may be raised up, to a Right Angle or any other, at pleasure.

In every Position, it is evident, that the parallelism of the Planes, and consequently of their Intersections, remains; and the distance of the Intersections between each two, which are in the same Plane, is invariable.

The Vertical Plane may be supposed to pass through the middle, and perpendicular to them all; cutting the Vanishing and Directing Plane, in EC and EF; the Picture and Original Plane, in CD and DF.

COR. 1. If any one of the four Lines be given or determined, and one Point, in any of the other, be also given or found, the whole Line is determined.

For, it is a Right Line, drawn through that Point, parallel to the given Line; seeing, they are all parallel amongst themselves, by the Theorem.

COR. 2. If the Original Plane passes through the Eye, its Intersection and Vanishing Line is the same; and, the Parallel of the Eye is also the Directing Line.

For,

For, seeing that the Original Plane (U, or a b c d, being produced) passes through the Eye (E) there can be no other Plane pass through the Eye parallel to it; but, in this Case, the Original Plane and its Parallel coincide; consequently, its Intersection (L M) with the Picture (A L M B) is also its Vanishing Line. And, for the same reason, the Parallel of the Eye (I K) coincides with the Directing Line; being produced by the same Original Plane. In which case, no Figure in the Original Plane can be represented.

Plate VI.
Fig. 22.

For, as the Original Plane passes through the Eye, the whole Representation of that Plane, and every Figure in it, is its Intersection with the Picture; and is the reason why, Figures, in horizontal Planes, which are on a level with the Eye, cannot be represented; for they are all in the Vanishing Line of horizontal Planes, and, therefore, cannot appear in the Picture.

T H E O R E M III.

All Original Planes, which are parallel amongst themselves, but not to the Picture, have the same Vanishing Line.

DEM. Vanishing Lines, are produced, in Theory, by imaginary Planes, passing through the Eye or Point of View, parallel to the Original Planes. Def. 8.

Wherefore, a Plane passing through the Eye parallel to any Original Plane, producing its Vanishing Line, is also parallel to every Plane which is parallel to the Original Plane; and since this parallel Plane can produce but one Line on the Picture, by its Intersection with it, that Line is, consequently, the Vanishing Line of all the Planes to which it is parallel. Q. E. D.

EX. V, U, and W, are three Original Planes, parallel amongst themselves; A L M B is the Picture, and E the Eye, in the Directing Plane (I K G H.)

Fig. 22.

I K L M passing through the Eye at E, is a continuation of the Plane U; consequently, it is parallel to the Planes V and W; and, seeing that no other Plane can be drawn through the Eye parallel to them, therefore L M is the Vanishing Line of the three parallel Planes V, U and W. Def. 8.

This may be further illustrated by Fig. 15, and by the Apparatus.

Suppose a Plane (R S T U) to pass through the Eye (E) parallel to the Planes B F C and A I L in the Original Object B F I L, cutting the Picture, M N O P, produced, in R U; which is, therefore, the Vanishing Line of those parallel Planes; and, consequently, of all other Planes parallel to them. Plate V.

For the same reason, a Plane (T O P S) passing through the Eye, parallel to the Planes A B G H and C D K L, cutting the Picture M N O P in O P; and the Picture R U O P in O P; O P, or O P, is the Vanishing Line of those Planes, on each Picture, respectively.

SCHOL. Horizontal Planes have horizontal Vanishing Lines; except, the Picture be also horizontal; in which Case they have no Vanishing Line, the Picture being parallel to the Original Plane.

It is also evident, that all vertical Planes have, when the Picture is also vertical, perpendicular or vertical Vanishing Lines; i. e. the Vanishing Lines are, in such case, perpendicular Lines on the Picture, and at right angles with such as are horizontal.

For, 1st. The Intersection of a horizontal Plane, with any other Plane, is necessarily a horizontal Line, seeing it is produced by a horizontal Plane, in which the intersecting Line must always be.

2nd. The common Intersection of two vertical Planes, is a Right Line perpendicular to the Horizon; seeing that, the Line of section, is, in both Planes.

COR.

Plate V. COR. *All Original Lines which are parallel amongst themselves, and not to the Picture, have the same Vanishing Point.*

For, by the same reasoning as in the Theorem, it is evident, since the Vanishing Point of an Original Line is produced, in Theory, by an imaginary Line passing through the Eye parallel to the Original Line †, it necessarily follows, that, a Line passing through the Eye, producing the Vanishing Point of any Original Line, by its Intersection with the Picture, being parallel to the Original Line, is also parallel to all Lines which are parallel to the Original Line; and, since it can produce but one Point on the Picture, it is, consequently, the Vanishing Point of them all. Theor. 2. Book 1st.

Fig. 15. In the same Figure, the Original Lines AB, GH, and FI, are parallel amongst themselves; EC V is a Right Line from the Eye, parallel to them all, cutting the Picture RUOP in V, and MNOP, in C, its Center; the Points C and V, on each Picture respectively, are, therefore, the Vanishing Points of the Original Lines AB, GH, and FI.

The same may be said in respect of the parallel Lines GF and HI, whose Vanishing Point, on the Picture RUOP, produced, is W.

Also, of GD and BC, whose Vanishing Point is Y, on the Picture RUOP; but, being parallel to MNOP, they have, consequently, no Vanishing Point on that Picture, but are parallel to their Representations.

N. B. It may be necessary to observe here, that, in this last, and in most of the Schemes or Diagrams made use of in this Theory, (being drawn perspective) whenever any two, or more, Lines are said to be parallel, if they are not really so, they tend to the same Point, agreeable to Corol. 1. For, I think it unpardonable, in a Treatise on Perspective, to give an Appearance of Objects, or Schemes, not truly and perspective drawn; merely, for the sake of preserving the parallelism of certain Lines, as several Authors have done. Unluckily, for them, they cannot also preserve the true Angles, which are worse represented by that means.

Therefore, whenever Angles are said to be Right, or otherwise; and such or such Lines, to be parallel or perpendicular; if they are not really so, they must be understood to be so; and, I am persuaded, that they will always appear to be so, when truly delineated, perspective.

THEOREM IV.

The Vanishing Lines of all Planes, which are perpendicular to the Picture, pass through the Center of the Picture.

DEM. For, since the Vanishing Line of a Plane is produced by an imaginary Plane passing through the Eye parallel to the Original Plane †, and the Center of the Picture is determined by a Perpendicular, from the Eye to the Picture §, the Original Plane being, in this Case, perpendicular to the Picture, it follows, that its Parallel or Vanishing Plane (producing the Vanishing Line) seeing it passes through the Eye, must, necessarily, pass through the Direct Radial, which is perpendicular to the Picture ‡, and consequently, through the Center of the Picture.

Therefore, the Vanishing Line, produced by the Intersection of every such Plane with the Picture, must pass through the Center of the Picture. Q. E. D.

Fig. 15. EX. 1. All horizontal Planes are perpendicular to a vertical Picture; wherefore, the Plane GHKD, and also the Ground Plane (Z) are perpendicular to both Pictures, MNOP or OP; their Vanishing Plane, therefore, passes through the Direct Radial (EC) of each Picture, and consequently, through C, their Centers.

CX and YV (on each Picture, respectively) parallel to those Planes, drawn through the Centers (C) of both Pictures, are the Vanishing Lines of those Planes. For, by Theo. 3d. the Vanishing Line of every Plane is parallel to its Intersection; and consequently to the Original Plane.

EX. 2. The Planes $AHGB$ and $CDKL$ are perpendicular to $MNOP$, only; their Vanishing Plane ($STOP$) therefore, passes through its Radial (EC) and their Vanishing Line (OP) through the Center (C) of that Picture.

As these two Examples are particular Cases; viz. when the Original Plane is either horizontal or vertical, I shall give another general Case, which will illustrate the Theorem universally. Raise up the Planes of No. 2. $AONB$ is the Picture, GEH the Directing Plane, and E the Eye.

An Original Plane, $IKLM$, is moveable on the Line TU , its common Section with the Ground, or other horizontal Plane, at right Angles with AB , the Intersection of the Picture with the Plane Z . Fig. 15:
No. 2.

It is evident that, as the Original Plane, $IKLM$, is turned (on TU) it is always perpendicular to the Plane or Picture ($AONB$)† for, in every position of the Plane, in that revolution, the Line TU is still perpendicular to the Picture. † 9. 7. El.

Now, if the Plane $IKLM$, revolving, pass through the Eye (at E) and, being all the while parallel to the Original Plane, it must revolve on EC , the Direct Radial, which is parallel to TU ; and, in every Position, its Intersection with the Picture must necessarily pass through C , the Center of the Picture; produced by the Perpendicular EC .

Let the Original Plane ($IKLM$) be raised, making the Angle PQR .

If the Plane $IKLM$, passing through the Eye, makes the same Angle with the Horizon, i. e. if it be parallel to the Plane $IKLM$, it will cut the Picture in ON , the Vanishing Line of that Plane, and pass through C , the Center of the Picture; making the Angle, NCM , with LM , the Vanishing Line of horizontal Planes, equal to the Angle PQR , of the Original Plane with the Ground Plane.

After the same manner, the Vanishing Lines of all Planes, which are perpendicular to the Picture, may be ascertained and drawn on the Picture; knowing the Angle which the Original Plane makes with any horizontal Plane; or with a vertical Plane, which is perpendicular to the Picture.

N. B. The Center of the Picture is the Center of all Vanishing Lines, of Planes perpendicular to the Picture; and they have all the same Radial and Distance (EC) by Def. 19. and 20.

COR. Hence it is evident, that the Center of the Picture is the Vanishing Point of all Lines which are perpendicular to the Picture.

For, EC , a perpendicular Line from the Eye, producing the Center of the Picture, is the common Radial of all Vanishing Planes, that are perpendicular to the Picture; and consequently, parallel to all Lines that are perpendicular to the Picture (for, they are all parallel amongst themselves, and to the Direct Radial, EC) as AB , GH , FI , &c. Fig. 15:

Therefore, since EC is parallel to all such Lines, the Point (C) which it produces on the Picture, is their Vanishing Point; by Def. 22.

But, C is the Center of the Picture; consequently, the Center of the Picture is the Vanishing Point of all Lines that are perpendicular to the Picture.

In this single and particular Vanishing Point, many Artists seem to rest all their knowledge of Perspective (commonly called the Point of Sight) and, whenever a number of Lines tend to the same Point, it is, by them, called a Point of Sight; not considering, or perhaps knowing, that the Center of the Picture is a Vanishing Point, in common with all other, only by virtue of the general Definition (Def. 22.) And, because the knowledge of it is more general, and, as they imagine, the practice much easier, we find it more used than any other; as most regular Objects, particularly Buildings, are right-angled; so that, having one Side or Front parallel to the Picture, (as is usually the case) all the horizontal Lines in the adjoining Sides are, consequently, perpendicular to the Picture, and therefore vanish in its Center; which, from the certainty of the Position of such Lines, is determined and fixed at pleasure, though often very injudiciously.

Plate VI.

THEOREM V.

Original Planes, whose common Interfection is parallel to the Picture, have parallel Interfections with the Picture, and parallel Vanishing Lines.

DEM. The common Interfection of the Original Planes being parallel to the Picture, a Plane may be supposed to pass through that Interfection which is also parallel to the Picture.

Now, the Original Planes being produced, through their common Section, to the Picture, each Plane will cut the Picture in a Line parallel to their common Section (8.7.EL.) Wherefore, since the Interfection of each Original Plane, with the Picture, is parallel to the common Section of both Planes, they are, consequently, parallel between themselves. Q. E. D. 4. 7. EL.

2. But the Vanishing Line of each Original Plane is parallel to its Interfection with the Picture. - - - - - Theorem 3.

And the Interfections are parallel between themselves. Proved.

Therefore the Vanishing Lines are parallel between themselves.

Fig. 22. EX. $abcd$ and $cdef$ are two Original Planes, whose common Section (cd) is parallel to the Picture ($ABLM$); ghi is a Plane, supposed to pass through their common Section, parallel to the Picture.

Now, the Plane $abcd$ being produced to the Picture, cuts it in LM ; and the Plane $cdef$, being produced, cuts it in AB ; wherefore, since the Picture and the Plane ghi are parallel, and are both cut by the Planes $AdcB$ and $LabM$, the Interfections (AB and LM) are, each, parallel to the common Section, cd ; and therefore they are parallel between themselves.

EX. 2. LM is the Vanishing Line of the Plane $cdef$, and LM of the Planes $V, U, \& W$; which are, respectively, parallel to the Interfections $AB, JF, \&c.$ (by Theo. 4.) and therefore they are parallel, also, between themselves.

3. After what has been said, it is manifest that the Directing Lines of Planes, in such Case, are also parallel.

For, since their Interfections with the Picture are parallel, consequently, if they were produced till they cut the Directing Plane, their Interfections with it would likewise be parallel; the Directing Plane being parallel to the Picture.

COR. 1. *The common Interfection of a Plane inclined to the Horizon, with any horizontal Plane whatever, being parallel to the Picture; the Vanishing Line, of that inclined Plane, is a Line parallel to the Horizon.*

For, because the common Interfection is parallel to the Picture, the Vanishing Lines of both Planes are parallel to each other; by Theorem.

But, one of those Planes is horizontal, therefore its Vanishing Line is parallel to the Horizon; by Def. 8.

Consequently, the other Vanishing Line is, also, parallel to the Horizon.

COR. 2. *All Original Planes, that have parallel Vanishing Lines, have the same Vertical Plane; and, consequently, the same Vertical Line; and, also, the same Parallel of the Eye.*

$IKLM$ is the parallel of the Plane W , cutting the Picture in LM , the Vanishing Line of W ; and, $IKLM$, is the Parallel, or Vanishing Plane of the

the Original Plane X, passing through the Eye (at E) and cutting the Picture in LM, the Vanishing Line of that Plane. Also, IKlm being parallel to the Horizon, its Section with the Picture (lm) is the Vanishing Line of horizontal Planes; to which, LM and LM are parallel, by the Theorem.

But, RST is the Vertical Plane of all those Original Planes, and of all other parallel to them; i. e. it cuts them all at right Angles, being perpendicular to their common Sections (cd, fg, &c.) and it cuts the Picture in CD the Vertical Line of them all; for the same Plane cannot cut the Picture in two Lines.

IK is also the Parallel of the Eye of all those Planes; for, it is the common Section of all their Vanishing Planes with the Directing Plane.

This Theorem, and the last Corollary, may also be illustrated by Figure 15.

1. BG, the common Intersection of the two Planes ABGH and BFC, is parallel to both Pictures (MNOP or OP) their Intersections (BE and BK) with MNOP, are therefore parallel; also their Vanishing Lines, RU and OP, on that Picture. The other (MNOP) being parallel to the Plane BFC, has but one Vanishing Line (OP) which is parallel to the Intersections BG and MN.
2. The two Planes, ABGH and BFC, being vertical, a Plane vertical to them, is, consequently, horizontal; for, no other Plane can cut both, or either of them and the Picture, perpendicularly. (See Fig. 19.)

Wherefore, they have the same Vertical Plane; which, in reality, is horizontal; and, consequently, they have also the same Vertical Line; which, in this Case, is the Vanishing Line of horizontal Planes; viz. CX or VY, on either Picture, respectively.

N. B. It is the same in all Positions, whatever, of the Original Planes.

T H E O R E M VI.

The Planes, which produce the Vanishing Lines of two Original Planes, are inclined to each other in the same Angle as the Originals; and have their common Intersection, passing through the Eye, parallel to the common Intersection of the Original Planes.

DEM. For, first, since the Planes, producing the Vanishing Lines of any two Original Planes, are, respectively, parallel to the Originals, they have consequently the same Inclination to each other, as the Originals.

For, if all the Planes are produced (viz. the Original Planes and their Parallels) they will cut each other in parallel Lines, and form a Parallelopiped, between their Intersections. 8. 7. El.

Therefore, they have the same Inclination to each other, respectively. 13. 7.

EX. U, or W, and X, are Original Planes, and KM, KM their Vanishing Planes; being parallel to them, respectively.

The Angle, which the Original Plane (X) makes with W, is dPN (Pd and PN being both perpendicular to their common Section PQ) and it is equal to the Angle LKL which their Vanishing Planes (KM and KM) make with each other

Because KL is parallel to NP, and KZ to dP, the Angle LKL is equal to dPN †. Also, the Angle LKL is equal to LdP, which is equal to dPN ‡.

Therefore, the Planes, extended through those Lines, being respectively parallel, are inclined to each other in the same Angle; i. e. KM has the same inclination to KM, as the Plane X has to U or W.

Fig. 15.

Fig. 22.

† 6. 1. El.
‡ 4. 1. El.

Plate V. Secondly. Because, in this Case, the common Sections (c d, N O, &c.) of the Original Planes are parallel to the Picture, the Intersection (IK) of the Vanishing Planes, passing through the Eye (E) is also parallel to the Picture; and, consequently, to the common Sections of the Original Planes.

For it is in the Directing Plane, which is parallel to the Picture; by Def. 4.

To illustrate this Theorem more familiarly, by moveable Planes.

Fig. 15.
No. 2.

1. Raise up the Plane of the Picture A O N B, perpendicular, or otherwise, to the Horizon. The Directing Plane (G E H) will be parallel to it; and I K L M to the Plane of the Horizon.

No. 3.

Then, if the Plane A D F B (any Original Plane) whose Intersection (AB) with the Horizon, or any horizontal Plane, is parallel to the Picture, be raised up, on AB, making any Angle (P Q R) with the Horizon, and, a Plane, (I K O N) pass through the Eye (at E) parallel to that Original Plane; it is manifest, that this Vanishing Plane (I K O N) makes the same Angle with I K L M, as A D F B with the Horizon; and I K (the parallel of the Eye) the Intersection of the Vanishing Planes, is parallel to AB, the common Intersection of the Original Planes.

No. 2.

2. The Picture, &c. remaining in the same Position, perpendicular to the Ground Plane, and to the common Intersection (T U) of the Original Plane I K L M; making any Angle with the Horizon, at pleasure.

Then, the Vanishing Plane (I K L M) being placed parallel to the Original Plane I K L M, it is evident, that it will make the same Angle with the Horizon, or with a Plane passing through the Eye, parallel to the Horizon, and cutting the Picture in L M (the Vanishing Line of horizontal Planes, as the Original Plane (I K L M) makes with the Horizon.

Also, E C (the Direct Radial) the Intersection of those Vanishing Planes, is parallel to T U, the common Intersection of the Original Planes.

For, they are both perpendicular to the Picture.

In Fig. 23. this Theorem is quite general, i. e. the Position, the Original Planes have to the Horizon, or to the Picture, is not regarded.

Fig. 23.

X and Y are two Original Planes, both inclined to the Horizon, and to the Picture (I T U N) which is also inclined to the Horizon, towards the Eye (at E.) A B is the common Intersection of those two Planes.

Then, if a Plane (I K L M) be supposed to pass through the Eye (E) parallel to the Plane, X; and another Plane (I K L M) parallel to A B D, or Y, cutting the Picture in the Vanishing Lines, L M and M L, of those Planes; they will be inclined to each other in equal Angles, respectively; i. e. the Vanishing Planes, as the Original Planes; and the common Section (E V) of the Vanishing Planes, be parallel to the common Section (A B) of the Original Planes.

COR. 1. *The Radials, producing the Vanishing Points of any two Original Lines, make the same Angle, at the Eye, as the Original Lines make with each other.*

For, they are respectively parallel to their Originals (Def. 22.) and, consequently, they make equal Angles, respectively. - - 5. 7. El.

Fig. 15.

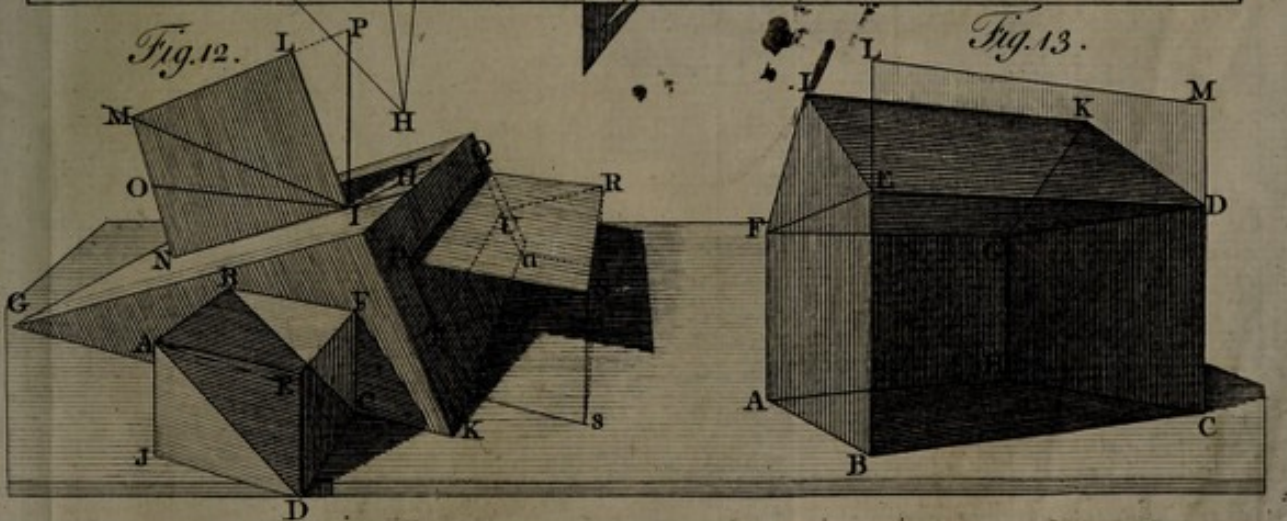
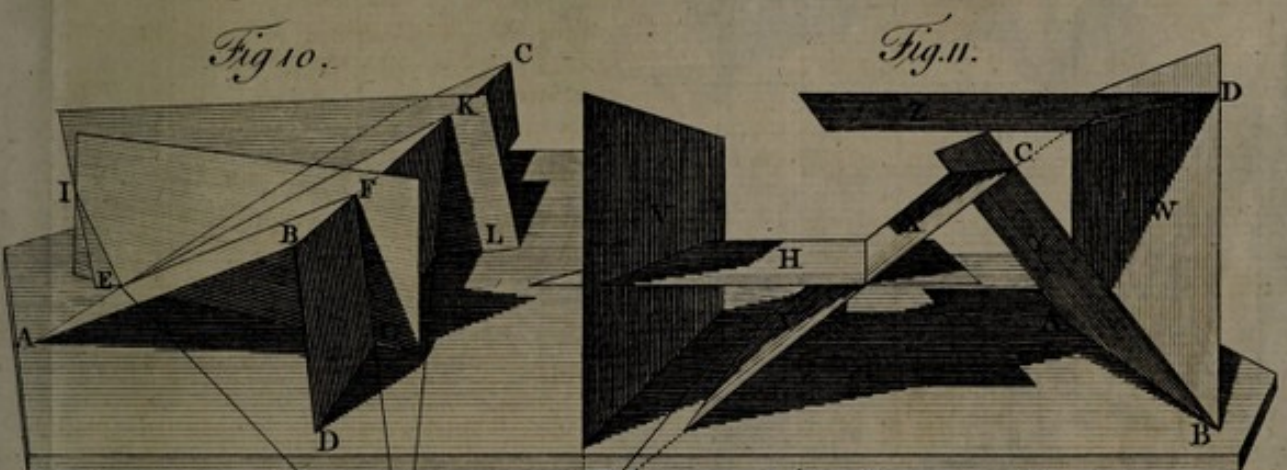
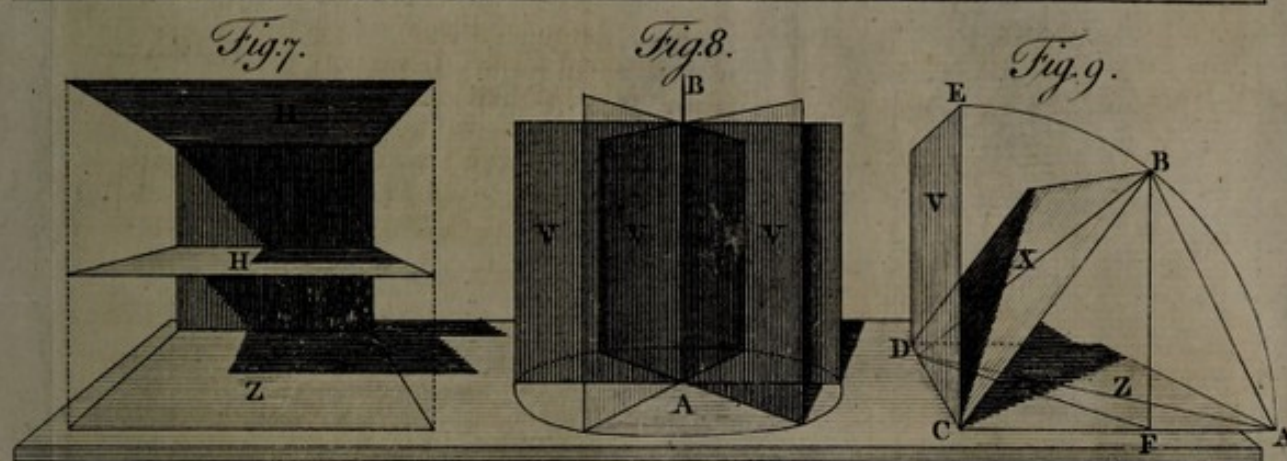
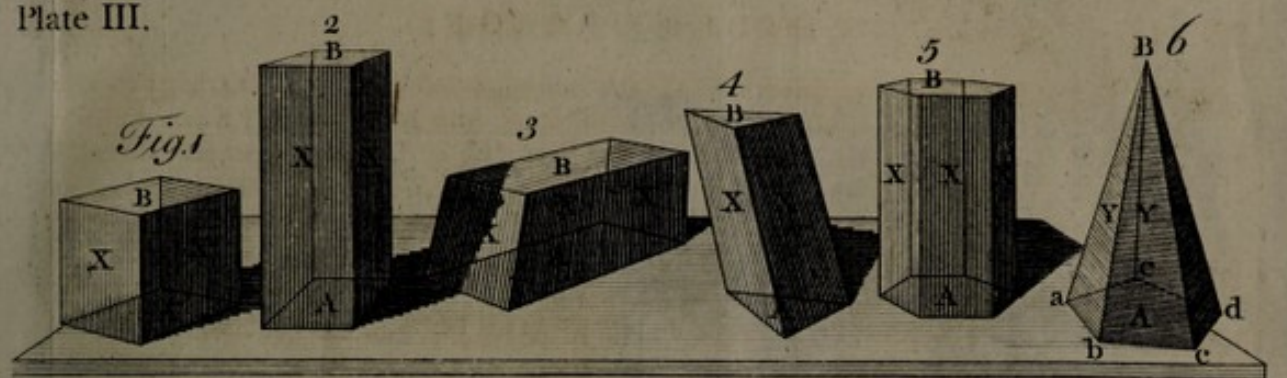
EX. E V is the Radial or Parallel of A B, and E Y of C B.

V and Y are, therefore, the Vanishing Points of A B and C B; the Radials (E V E Y) being parallel, respectively, to the Originals (A B and C B) Def. 22.

Consequently they make the same Angle at the Eye (E) as the Original Lines make with each other; viz. V E Y equal A B C. - - 5. 7. El.

But, A B C is a Right Angle; therefore, V E Y is a Right Angle.

Plate III.



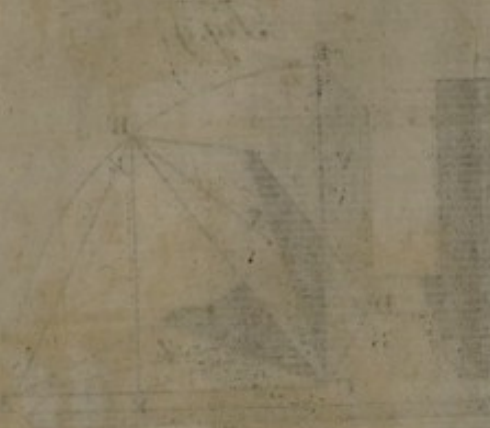


Fig. 14. N^o3.

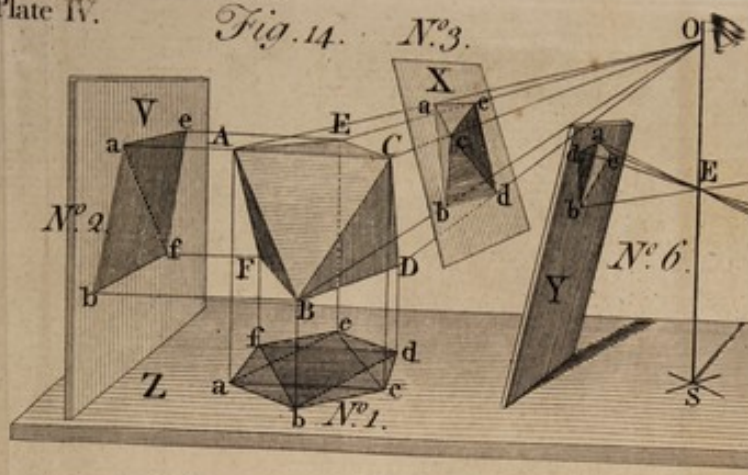


Fig. 14. N^o4.

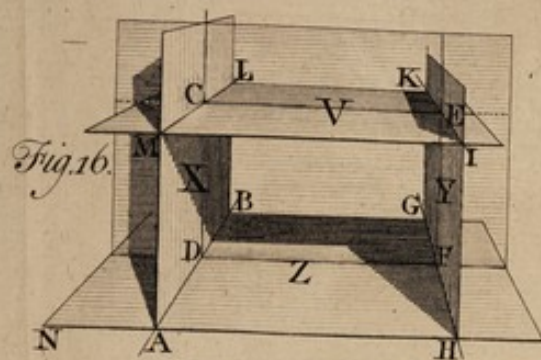
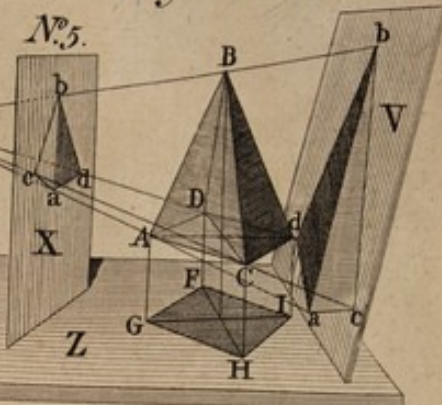


Fig. 17.

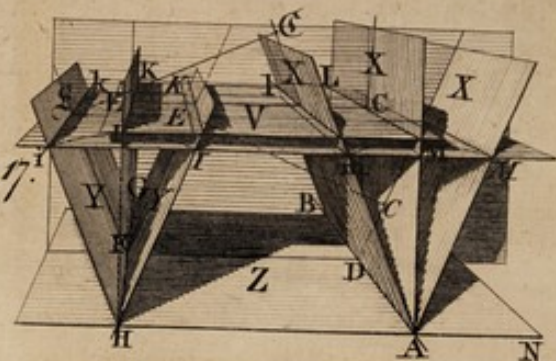


Fig. 19.

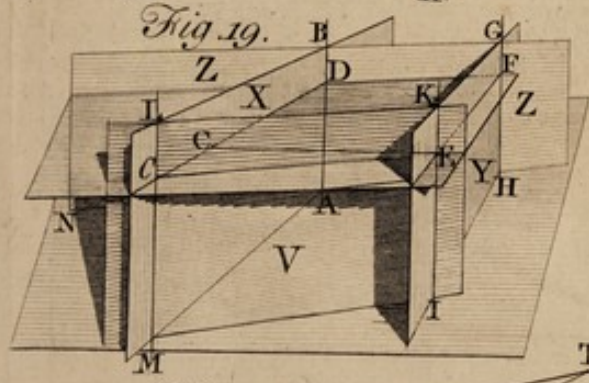


Fig. 18.

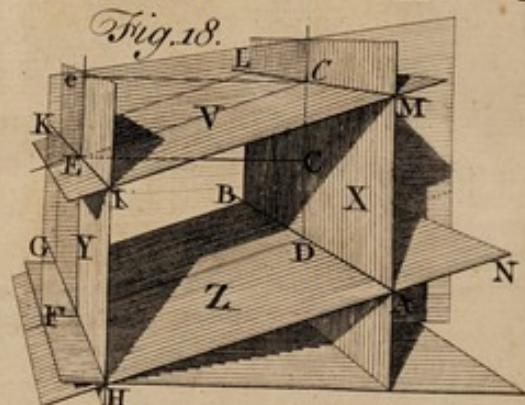


Fig. 20.

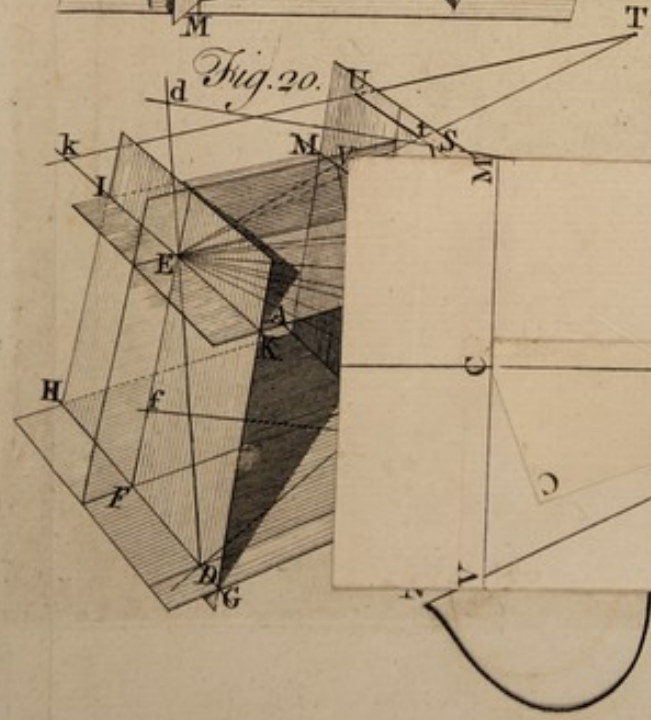
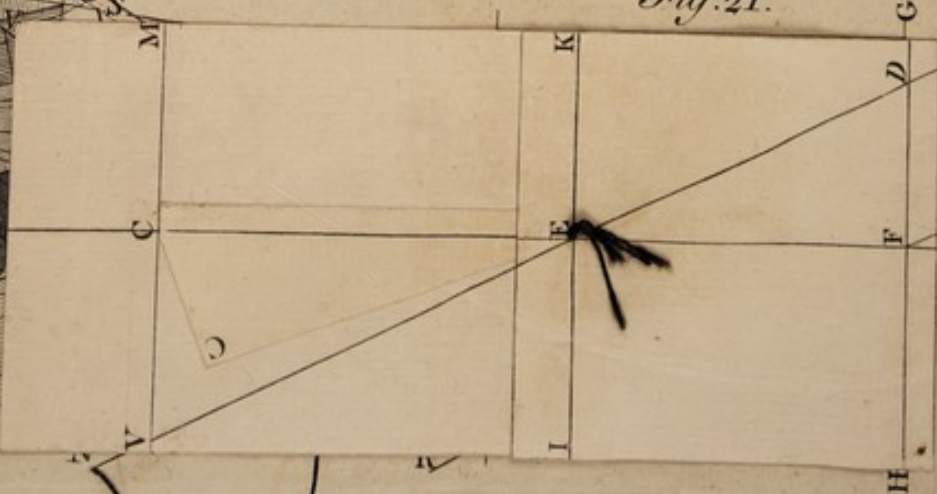


Fig. 21.



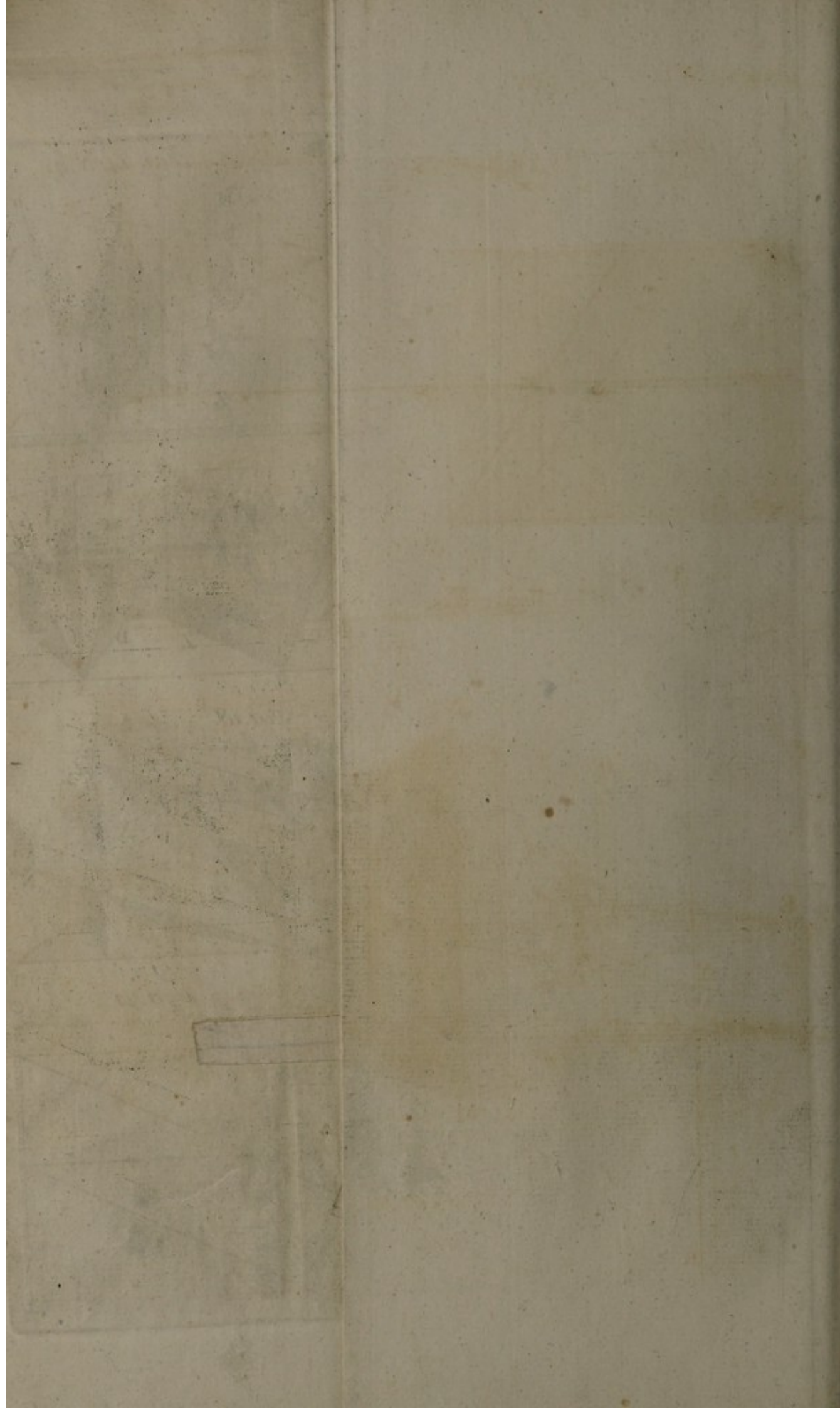


Fig. 15.
N^o 2

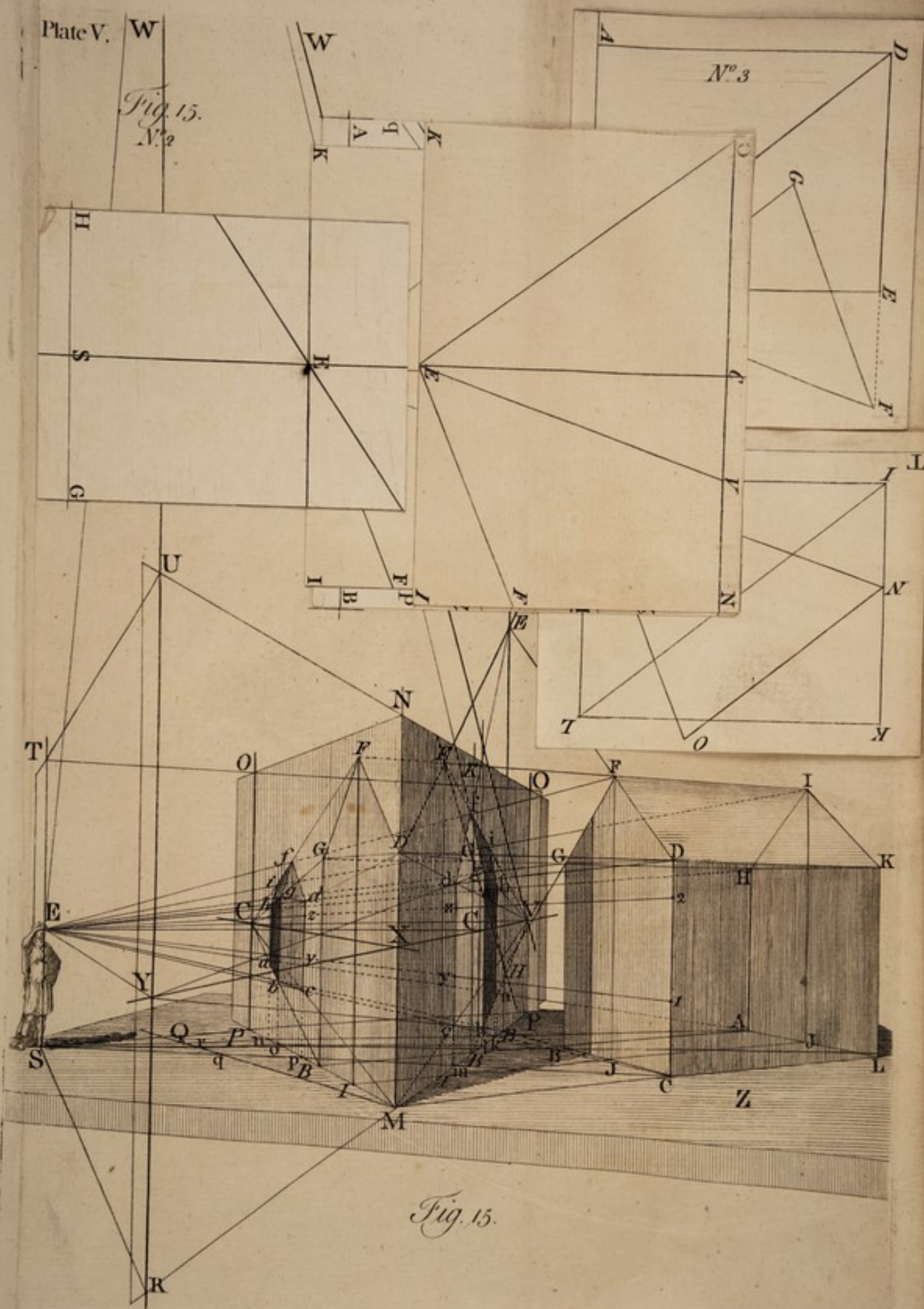


Fig. 15.

This is also illustrated in Fig. 23, and, by changing Y for M, it is described in the foregoing; save only, that ABC is not a Right Angle, consequently VEM is not a Right one, being equal to ABC .

SCHOL. If the Original Lines (not being in the same Plane) are so situated, that they cross, or would cross, each other, without cutting; then, the Angle, which the Radials of such Lines make with each other, is equal to the Angle that would be made between the Originals, if either of them be moved, parallel to itself, in any direction, till it cuts the other.

COR. 2. *The Vanishing Point of the common Intersection, of two Original Planes, is the Point in which their Vanishing Lines intersect each other.*

For, the Intersection of the parallel Planes, producing the Vanishing Lines, passes through the Eye, parallel to the common Intersection of the Original Planes (by the Theorem) and consequently, will cut the Picture in that Point where the Vanishing Lines intersect.

But, the Vanishing Point of a Line is that Point, in which, a parallel Line, from the Eye, cuts the Picture. - - - - - Def. 22.

Therefore, the Point of Intersection of the Vanishing Lines is the Vanishing Point of the common Intersection of the Original Planes.

EX. AB is the common Intersection of the Planes X and Y . ML is the Vanishing Line of the Plane X , and ML of the Plane Y ; their intersecting Point (V) is, therefore, the Vanishing Point of AB ; for EV is parallel to AB .

Fig. 23.

The Visual Rays EA , EB , together with the Line AB , form a plane Triangle, which cuts the Picture in ab (the Representation of AB) whose Vanishing Point is V ; for, it is the Section of a Radial Plane, passing through the Original Line and the Eye, with the Picture; which must necessarily pass through EV , the Radial of AB , and cut the Picture in JV .

† Ax. 5.

Mm is the Vanishing Line of the vertical Plane W , and Mq of horizontal Planes; both which, cut ML , the Vanishing Line of the inclined Plane (X) in M ; therefore, M is the Vanishing Point of CB , the common Intersection of the Planes W and X , and also, of a horizontal Plane ($CB D$.)

EX. 2. Let the triangular piece (in the Apparatus) be fixed to the direct Picture (of which, it is supposed to be a continuation) and draw the thread (at W) through the Eye of the Spectator; also, bring forward the folding part, into the same Plane.

Then, VY is the Vanishing Line of horizontal Planes; RW , of the vertical Planes (BFC and AIL) and VW , of the inclined Plane $FGHI$. V , Y , and W , are, therefore, the Vanishing Points of the common Intersections of those Planes, respectively; viz. V , of GH , the common Intersection of $DGHK$ and $FGHI$; and also of $ABGH$, whose Vanishing Line is OP . Y is the Vanishing Point of the common Intersection of horizontal Planes, and of the vertical Planes, BFC and AIL ; and, W is the Vanishing Point of GF and HI , the common Intersections of those vertical Planes and the inclined Plane $FGHI$.

SCHOL. This Corollary, being so very essential in Practice, cannot be too much enforced; for, having found the Vanishing Point of a Line (in any Plane whatever) which is either the common Intersection of that Plane with some other adjoining Plane, or a Line parallel to it (as V) the Vanishing Line of that other Plane must, necessarily, pass through that Vanishing Point; and, if the Position of the other Vanishing Line be known, its place on the Picture is determined (as OP) otherwise, it is necessary to have the Vanishing Point of some other Line in the same Plane (as W , of the Line FG) in order to determine the Vanishing Line (VW) of the Plane ($FGHI$).

Plate IV.

THEOREM VII.

A Right Line drawn from the Center of the Picture, to the Center of a Vanishing Line, is perpendicular to that Vanishing Line.

DEM. For, because the Direct Radial, producing the Center of the Picture, is perpendicular to the Picture §; and, the Center of every Vanishing Line is produced by a Right Line from the Eye, perpendicular to the Vanishing Line †; consequently, the Radial of the Vanishing Line, producing its Center, and the Direct Radial, together with the Line which joins the two Centers, form a Triangle, and are all in the same Plane † which Plane is perpendicular to the Picture.

For, it passes through the Direct Radial, which is perpendicular to the Picture.

And, it is also perpendicular to the Vanishing Line; because, it is perpendicular to the Picture, and also to the Plane which produced the Vanishing Line.

Wherefore, since the Radial of the Vanishing Line, and the Right Line joining the Center of the Picture and the Center of the Vanishing Line, are both drawn to the same Point in a Plane, at which Point the Vanishing Line is perpendicular to that Plane; consequently, the Line, joining the Center of the Picture and the Center of the Vanishing Line, is perpendicular to the Vanishing Line. Q. E. D.

Fig. 18.
and 19.

EX. LM is the Vanishing Line of the Original Plane Z; C is its Center, and C the Center of the Picture. Draw CC.

I say, that the Line CC is perpendicular to the Vanishing Line LM.

By Def. 17. EC producing the Center of the Picture is perpendicular to the Picture; wherefore, it is in the Vertical Plane (ECDF.)

† Def. 5.

For, it is perpendicular to the Picture (X) and also to the Vanishing Plane (V) †. Consequently, LM, the common Section of the two Planes, V and X, is perpendicular to the Vertical Plane (ECDF.)

Wherefore, EC, the Intersection of that Plane and the Vanishing Plane (V) is perpendicular to LM.

§ Def. 19.

But, EC produced the Center (C) of the Vanishing Line LM §; and consequently, CC, the Intersection of the Vertical Plane with the Picture, is, also, perpendicular to LM; for, EC and CC are two Lines, cutting each other, in the same Plane (ECDF) to which the Vanishing Line (LM) is perpendicular, at the Point (C) of their common Section.

COR. 1. When the Original Plane is perpendicular to the Picture, the Center of the Picture is the Center of the Vanishing Line.

Fig. 16.

For, a perpendicular (EC) from the Eye to the Vanishing Line (ML) producing its Center is the Direct Radial, which is perpendicular to the Picture; and, consequently, produces the Center of the Picture; which is the Center of every Vanishing Line, of Planes perpendicular to the Picture. See Th. 4.

COR. 2. The Distance of every Vanishing Line, which does not pass through the Center of the Picture, is the Hypothenuse of a Right angled Triangle, whose Catheti, or Legs, are the Distance of the Picture, and the Distance between the Center of the Picture and the Center of the Vanishing Line.

Fig. 18.
and 19.

For, EC is the Distance of the Picture, CC is the Distance between the two Centers; and, EC, the Hypothenuse, of the Right angled Triangle ECC, is the Distance of the Vanishing Line LM; being the shortest Line that can be drawn from the Eye to the Vanishing Line.

Cor. 2. 12. 1. El.

To

1

To set this matter in the clearest light possible I have illustrated it by moveable Planes, in Fig. 21.; which, let be raised up, in any Angle at pleasure.

Underneath the Vanishing Plane (IKLM) is a triangular Plane (ECC) which, being turned down, perpendicular to the Vanishing Plane, will represent a Part of the Vertical Plane, and determines the Center of the Picture, in a certain angle of inclination, inclined from the Eye, in the Angle ECC.

The Triangle ECC is right angled, at C; consequently, EC is perpendicular to the Picture, and C is therefore its Center.†

It is evident that CE and CC are both perpendicular to LM, or VM, the Vanishing Line of the Original Plane, NGH.

But, C is the Center of the Vanishing Line, and C is the Center of the Picture; therefore, a Right Line joining the Center of the Picture and the Center of a Vanishing Line is perpendicular to that Vanishing Line.

† Def. 17.

Above the Vanishing Plane is another Right angled Triangle; which being erected, perpendicular to the Vanishing Plane and the Picture, determines its Center, when the Picture is inclined towards the Eye, in the Angle ECC.

In which case, the Center of the Picture necessarily falls above the Vanishing Line; as in the former case it falls below.

Both these Positions are represented in the 17th Figure, and also in the 18th, considering either Plane, X or Y, as the Picture, and E or C the place of the Eye.

But, when the Picture and its Directing Plane are perpendicular to the Original Plane, as in the 16th, and the 17th Figure also shews the same, the Center of the Vanishing Line is the Center of the Picture.

N. B. In this Scheme, and some of the following Diagrams, the original Plane (NGH) must not be considered as necessarily being horizontal; for, the whole Theory, here given, is quite general, and applicable to all positions whatever, either of the Original Plane or of the Picture.

It may perhaps, to some, seem unnecessary to dwell so long on this Proposition, which, at first, may not appear of much consequence; but, I know it to be of the greatest consequence, in the most difficult part of Perspective, *viz.* in the practice on inclined Planes; the Theory, of which, being well digested and understood, the Practice will be found almost as easy as in the most familiar cases, being founded on the same invariable Principles, as will be exemplified in Practice; and therefore, the pains I have been at to enforce it, is by no means to be dispensed with.

The Reader may be well assured, that every Part of the Theory, given in this Work, is of utility, and necessary to be known; and that, the better it is understood, the progress in practical Perspective will be thereby facilitated.

T H E O R E M VIII.

The Perspective Projection of every original Right Line is a Right Line, in every Position or Situation whatever.

DEM. Suppose a radial Plane to pass through the Eye and any original Right Line.

If the Line be situated on the other side of the Picture, the imaginary Plane must, necessarily, cut the Picture; and, if it lie on this side, the Plane will cut the Picture if produced.

But, the Intersection of two Planes is a Right Line - - - Ax. 3.

And, since the Eye is in this imaginary Plane, the whole of that Plane appears but a Line. (See Definition of a Plane, Page 41.)

But, the original Line is supposed to be in that Plane; consequently, the Original Line and every Line in the radial Plane, have their Representations in the Intersection of that Plane with the Picture; seeing, there can be no other Line common to both Planes. Therefore it is a Right Line. † Q. E. D.

† Ax. 3.

Plate V. EX. Let, BFIL be a rectilinear Object, bounded by Planes in various positions.
Fig. 15. Let MNOP be the Picture, placed direct between the Spectator (at E S) and the Object (BFIL).

FI, FG, or FD, is an Original Line (or, take any other Line in the Original Object) E is the Point of Sight, or place of the Eye; and EI, EF, EG, &c. are Visual Rays; in which, the extreme Points, I, F, G, &c. are seen, forming the Optic Angles FEI, FEG, &c.

Now, since the Original Line (FI, FG, &c.) is on the other Side of the Picture, the Visual Rays (EF, EI, &c.) must necessarily cut the Picture.

But, FEI is a Triangle; for, it is three Right Lines touching or cutting each other, and is, therefore, in a Plane. - - - Axiom 8.

The Ray EF cuts and passes through the Picture in the Point f, and EI in the Point i, &c. the Points f, i, g, &c. are, therefore, the Representations of the Original Points F, I, and G; for, they coincide with their respective Originals. Wherefore, the Line fi, or fg, joining the Points f and i, or g, is the Intersection of the Plane Triangle FEI, or FEG, with the Picture.

But, the common Intersection of two Planes is a Right Line. - - - Ax. 3.

And, fi is the Representation, or perspective Projection of the original Line FI, or fg of FG. (See Projection, Scenography, Page 49.)

For, to the Eye, at E, the Point f coincides with the Original Point, F, i with I, and g with G; consequently, the Line fi coincides with its Original FI, and fg with FG.

THEOREM IX.

The Representation, on the Picture, of a Line parallel to the Picture, is parallel to the Original; and, it has that proportion to the Original, as the Distance of the Picture to the Distance of a Plane, passing through the Original Line, parallel to the Picture.

DEM. The Perspective Projection of every Right Line on the Picture, is a Right Line; and it is produced, in Theory, by the Intersection of a Plane, with the Picture, passing through the Eye and the Original Line. - Theo. 8.

Wherefore, since the Original Line is, in this case, supposed parallel to the Picture, a Plane may pass through that Line, also parallel to the Picture.

But, if two Planes being parallel, are both cut by another Plane, their Intersections are parallel. - - - 8. 7. El.

But the Original Line is one of the Sections, and its Projection, on the Picture, is the other (the cutting Plane being supposed to pass through the Original Line and the Eye).

Therefore, the Representation of a Line, which is parallel to the Picture, is parallel to the Original. Q. E. D.

Fig. 22. EX. NO is an Original Line, parallel to the Picture (ALMB) E is the Eye.

Imagine a Plane, passing through the Original Line NO, parallel to the Picture; and, EN, EO, Visual Rays, from the Eye, to each extreme of the Original Line; which, are in the same Plane with NO. - Ax. 8.

Wherefore, NEO is a Radial Plane, passing through the Original Line and the Eye, and cutting the Picture in no, the Representation of NO. - Th. 8.

But, the Plane Y is supposed parallel to the Picture, and NO is in that Plane; consequently, no, its Representation, which is the intersection of the Radial Plane (NEO) with the Picture, is parallel to the Original Line (NO).

For, they are the common Sections of two parallel Planes, by another Plane.

In the same manner, pq, the Representation of PQ, may be proved parallel to PQ.

EX. 2. $AH, BG, CD, \&c.$ are Original Lines, parallel to both Pictures ($MNOP$ or OP) Fig. 15.
 EB and EG are Visual Rays, to the Line BG ; which are all in the same Radial Plane, cutting the Pictures in bg , and bg ; which are, therefore, the Representations of BG , on each Picture, respectively (by Theo. 8th.)
 But, BG , the Original Line, is perpendicular to the Horizon or Ground Plane, (SMZ) and the Pictures are vertical Planes.
 And, the Radial Plane GEB is also vertical, seeing it passes through GB . 9. 7. El.
 Consequently, the common sections of those Planes are Right Lines, perpendicular to the Ground Plane. - - - Cor. to 9. 7. El.
 and, consequently, they are parallel between themselves. - 3. 7.
 Therefore, bg or bg , the Representation of BG , is parallel to BG .
 Also, cd , or cd , is parallel to CD ; and ah , or ah , to AH .

After the same manner, the Representation of any other Line (BC, GD , or $GF, \&c.$) in the Plane BFC , which is parallel to the Picture $MNOP$, only, may be proved parallel to their respective Originals.

DEM. 2. The Representation, no (of NO) is proved parallel to NO , by the 1st Part. Fig. 22.
 Wherefore, the Triangles NEO, nEo , are similar; No. 2.
 and $no : NO :: En : EN$, or as $Eo : EO$. - - - Cor. 3. 2. 6. El.
 But, ECf is perpendicular to the Picture AB , and consequently, to the Plane ONS , which is parallel to the Picture, cutting the Picture in C , its Center, and the Plane ONS in f . Ef is therefore the shortest Line that can be drawn to those Planes, and consequently measures their Distances. 12. 1. El.
 For, EC is the distance of the Picture, and Ef of the Plane ONS .
 But, if two or more Right Lines are cut by parallel Planes, they will be cut proportionally (whether they proceed from one Point or not) - 10. 7. El.
 wherefore, $EC : Ef :: En : EN$, or, as Eo is to EO .
 But, $no : NO :: En : EN$. Th. $no : NO :: EC : Ef$, by equality of Ratios.
 That is, the Representation, no , has that Proportion to NO , its Original, as the Distance of the Picture, EC , has to Ef , the Distance of a Plane passing through the Original Line parallel to the Picture.

N. B. The same Demonstration holds good if NO be considered as the Original Line, on this Side of the Picture, and projected to the Picture. (See Projected Perspective, page 49.)

COR. 1. From the former part, it is evident, that the Projections of any number of Lines, which are parallel amongst themselves and to the Picture, are also parallel amongst themselves and to the Originals.

AH, BG , and CD , are parallel amongst themselves, and to both Pictures; Fig. 15.
 their Representations, on both, are therefore parallel.

Also, BC and GD are parallel between themselves and to the Picture $MNOP$ only; their Representations, on that Picture, are therefore parallel between themselves and to the Originals.

COR. 2. From the second part of this Theorem, may be clearly deduced; that if an Original Line parallel to the Picture, be any how divided, the Representations of the several Parts will have the same Proportion to each other, and to the whole Representation, as the Parts of the Original Line have to each other, and to the whole Line.

If from the divisions, 1 and 2, of the Line CD , the Visual Rays $E1, E2$ be drawn, they will cut the Representations, cd , on both Pictures, to which the Original Line is parallel, in the same Ratio, in the Points y and z .

For, because cd is parallel to CD , the Triangles CED, cEd are similar.
 And, for the same reason, $CE1, CE2$, are similar to $cE1, cE2, \&c.$ C. 3. 2. 6. El.
 Wherefore, $cy : C1 :: cz : C2$; i. e. as $cd : CD$: - 4. 6. El.
 And, consequently, $cy : yz :: C1 : 12$; or, as cd to CD .

S

COR.

Plate VI. COR. 3. Hence it is manifest, that if the Eye be moved, still keeping the same Distance from the Picture; i. e. let the Eye be any where in the Directing Plane; the whole Representation, and each Segment, will still have the same proportion to the Original Line.

For, by the second part of the Theorem, the representation of a Line parallel to the Picture has the same Proportion to the Original, as the Distance of the Picture to the Distance of a Plane, passing through that Line, parallel to the Picture. And the Distance of either is not varied, whilst the Eye is in the Directing Plane; therefore, &c.

COR. 4. The Angle, which the Representations of any two Original Lines, that are parallel to the Picture, make with each other, is equal to the Angle made by the Original Lines. For the Representations are respectively parallel to their Originals.

Fig. 22.
No. 2.

In Fig. 22. No. 2. Let the Original Lines, ON and SR , be produced till they intersect (in P) making the Angle OPS .

Because no is parallel to NO and rs to RS , being produced, they will make the Angle ops equal to OPS , made by the Original Line. 5. 7. El.

Fig. 15.

In Fig. 15. the Angle gfd , on the Picture $MNOP$, to which the Plane BFC is parallel, is equal to the Angle GFD ; and fgd to FGD , &c.

For, fg is parallel to FG , fd to FD , and gd to GD ; by the Theorem.

COR. 5. Original Figures, in Planes which are parallel to the Picture, have their Representations similar to the Originals.

Fig. 22.
No. 2.

The Plane NOS is parallel to the Picture, AB ; wherefore, the Representation op is parallel to the Original Line, OP ; ps is also parallel to PS ; and if os , OS be drawn, os will be parallel to OS . by Theo. Part 1st.

By Cor. 4. the Angle ops is equal to OPS ; and by the second Part of the Theorem, $op:OP::ps:PS$; for, each is as EC to Ef .

Consequently, the Triangles pos , POS are equi-angular; the Angle o is equal O , and s equal S ; and consequently, $os:OS::op:OP$, or as $ps:PS$.

1. 8. El.

Therefore, the Representation, pos , is similar to the Original Triangle, POS .

For, $SEOP$ is a Pyramid, cut by a Plane (AB) parallel to its Base (POS).

Fig. 15.

The Plane BFC , in the Object $BFIL$, is parallel to the Picture $MNOP$.

By Cor. 1. bg and cd , on that Picture, are parallel between themselves, and to their Originals (BE and CD) and so are gf and fd to their Originals.

And by Cor. 4. the Angles bgf and gfd , &c. are equal, respectively, to the Angles BGF and GFD , &c.

But, it is demonstrated, that, $bg:BG::gf:GF$; or, gd to GD , &c. for, each is to its Original, as the Distance of the Picture to the Distance of the Plane they are in; by the Theorem, part 2.

Therefore, since the Sides are directly proportional, and the Angles contained by corresponding Sides, or Diagonals, equal; the Triangle gfd is similar to GFD ; and the whole Representation ($bgfd$) to the Original.

N. B. All Lines, that can be drawn in a Plane, which is parallel to the Picture, are parallel to the Picture; and their Representations have all the same Proportion to their respective Originals.

T H E O.

T H E O R E M X.

All Right Lines, in any Original Plane, not parallel to the Picture, have their Intersecting Points in the Intersecting Line, and their Directing Points in the Directing Line of that Plane.

Also; the Vanishing Points, of Original Lines, are all in the Vanishing Line of the Planes the Original Lines are in.

DEM. If a Right Line be not parallel to the Picture, it will, if produced, cut the Picture, and the Directing Plane, in its Intersecting and Directing Points. § Def. 21. and 23.

Now, since one part of a Right Line cannot be in a Plane and another part of it out of that Plane; † consequently, the Original Line must cut the Picture, somewhere, in the Intersection of the Plane, it is in, with the Picture. † Ax. 1.

And, for the same reason, it will cut the Directing Plane, in the Intersection of the Plane it is in with the Directing Plane, i. e. in the Directing Line of that Plane. Therefore, all Lines, &c. Q. E. D.

Secondly. All Original Lines, in the same Plane, have their Radials in the Vanishing Plane of that Plane; their Intersections with the Picture are, consequently, in the Intersection of the Vanishing Plane with the Picture.

But, the Intersection of the Vanishing Plane, of any Original Plane with the Picture, is the Vanishing Line of that Original Plane. - Def. 8.

Therefore, the Vanishing Point, of every Original Line, is in the Vanishing Line of the Plane that Original Line is in. Q. E. D.

Examples, of this self evident Theorem, are in the Corollaries.

COR. 1. Hence, a Right Line, drawn through any two Intersecting Points, is the Intersecting Line; and, a Right Line drawn through two Directing Points is the Directing Line of the Plane, in which the Original Lines are situated.

It is also manifest, that a Right Line, drawn through any two Vanishing Points of Lines in any Original Plane, is the Vanishing Line of that Plane.

EX. AB, being produced, cuts the Picture in J, and NA cuts it in I; wherefore, a Right Line, drawn through J and I, is the Intersection of the Plane X. Fig. 23.

For, J and I are the Intersecting Points of Right Lines in that Plane; and if two Points in a Right Line are given, the whole Line is determined. (Cor. 1. Th. 2).

CB, produced, being in the same Plane (X) cuts the Picture in d, in the Line JI; and so, it is manifest, would every other Line in that Plane.

AD, produced, cuts the Picture in N; JN is, therefore, the Intersection of the Plane Y; for AB is also in that Plane.

P is the Directing Point of AB, and O of CB; OP is, therefore, the Directing Line of the Plane X; and PQ of the Plane Y*.

The Plane IKLM, passing through the Eye (at E) parallel to the Plane X, produces its Vanishing Line, ML. - Def. 8.

EV is the Radial of AB, and EM of CB; which being parallel, respectively to those Lines, are in the Vanishing Plane IKLM.

And, the Vanishing Points, V and M, are, consequently, in the Line of its section with the Picture, which is the Vanishing Line of that Plane; wherefore, if two Vanishing Points (V and M) of any two Lines are determined, the Vanishing Line (ML) of the Plane they in, is also determined.

For, the Vanishing Point of every Line, in any Plane, is in its Vanishing Line.

* The Point Q should be where AD would cut the Directing Plane, in a Line parallel to JN; i. e. where the Intersection, JN, and the Vanishing Line, ML (which represent parallel Lines) intersect, which Point is out of the Plate. Therefore, PQ is not the true Directing Line, of the Plane ABD.

Plate VI. COR. 2. *All Right Lines, which are parallel to any Original Plane, have their Vanishing Points in the Vanishing Line of that Plane.*

Because, a Plane may be drawn through any Right Line, parallel to any Plane to which the Line is parallel; and two or more Planes, being parallel, have the same Vanishing Line; by Theorem 3d.

COR. 3. *The Intersecting Point of the common Intersection, of two Original Planes, is the Point in which the Intersecting Lines of those Planes cut each other.*

For, the common Section of two Planes is a Right Line, in both Planes.

Wherefore, the Intersecting Point, of that Line, is in the Intersections of both Planes; consequently, it is that Point in which the Intersections cut each other. For, there is no other Point common to both Lines.

Fig. 23. EX. $J I$ is the Intersection of the Plane X , and $J N$ of the Plane Y . J , the Point of their common Intersection, is, therefore, the Intersecting Point of $A B$, the common Intersection of the Planes, X and Y .

$J I$ is the Intersection of the Plane X ; and $i d$ of the Plane W ; d is, therefore, the Intersecting Point of $C B$, the common Intersection of the Planes X and W .

Fig. 15. 2. $F D$ and $F G$ are the Intersections of the vertical Plane $B F C$, and the inclined Planes, $F K$ and $F H$; wherefore, E and H are the intersecting Points of those Lines, on the Picture $M N O P$; where the vertical Intersection ($B E$) of the Plane $B F C$, is cut, by the Intersections, $D F$ and $F G$, of the inclined Planes.

COR. 4. *The Directing Point of the common Intersection of two Planes, is the Intersection of their Directing Lines.*

Fig. 23. EX. $P O$ is the Directing Line of the Plane X , and $P Q$ of the Plane Y ; wherefore, P , the Point of their Intersection, is the Directing Point of $A B$.

Hence, it is evident, may also be deduced the 2nd Corollary to the 6th Theorem, viz. The Vanishing Point of the common Intersection of two Original Planes, is the Intersection of their Vanishing Lines. Which, I advise the Student to be quite clear in; for, by that means, he will be able to determine the Vanishing Lines of contiguous Planes; and so proceed, in the delineation of Objects, from one Plane to another, with the greatest facility.

This Diagram, though apparently intricate, is the completest in the Work, for rendering the knowledge of Vanishing Lines and Points, Intersections, &c. general; and I think, it cannot fail of answering the end I aimed at, viz. to divest the Student of that partiality which most have to the horizontal Vanishing Line; as it is manifest, that the same Principles extend to all, nor is there any difference in the process; all the difficulty, if any, is in finding and determining them, on the Picture; which, when a clear Idea is inculcated, as by this Diagram, will readily be acquired. It is not possible to give, at once, so perfect an Idea, in Representation, as in a Model, as is obvious in Figure 15, of the Apparatus; yet, when regularly analyzed, as in this Theorem, I am persuaded, that all the intricacy will vanish.

THEOREM XI.

The Radial, or Parallel, of an Original Line producing its Vanishing Point, makes the same Angle with the Parallel of the Eye and Vanishing Line, as the Original Line makes with the Intersection and Directing Line, of the Plane that Original Line is in.

DEM. Since the Interfection and Vanishing Line, the Parallel of the Eye and Directing Line, of every Original Plane, are parallel amongst themselves (Th. 2.) and the Radial, producing the Vanishing Point of an Original Line, is parallel to the Original (Def. 22.); it necessarily follows, that, whatever inclination the Original Line has to the Interfection and Directing Line (which are all in the same Plane) the Radial of that Line has the same Inclination to, or makes equal Angles with, the Parallel of the Eye and Vanishing Line, which are all in a parallel Plane. - - - 4. of 1. and 7. El.

EX. NO is an Original Line, which, being produced, cuts the Interfection and Directing Line (AB and GH) in equal Angles. - - - 4. 1. El. Fig. 20. and 21.

EV is the Radial of NO, producing its Vanishing Point (V) in the Vanishing Line (ML) of the Plane NGH, making the Angles EVL, with the Vanishing Line, and VEI, with the Parallel of the Eye, equal.

And, they are also equal to the Angles NIA, NDH, which the Original Line, (NO) makes with AB and GH, the Intersecting and Directing Lines.

Because, the Original Line (NO) and its Radial (EV) are parallel (Def. 22.) and, because they cut parallel Lines in parallel Planes.

2. It is also manifest, that, the Angle (VEC) made by the Radial (EV) of the Original Line (NO) and EC, the Radial of the Vanishing Line (VL) producing its Center (C) is equal to the Angle which the Original Line, (NID) makes with FD, the Interfection of the Vertical Plane (ECDF) with the Plane NGH, in which the Original Line (NO) is situated.

For, these Angles are the Complements of the Angles made by the Original Line and its Radial, with the Interfection of the Picture, and the Parallel of the Eye or Vanishing Line.

3. From the same reasoning, it is evident, that the Angles KED, EDH, made by the Director (ED) of the Line NO, with the Parallel of the Eye and Directing Line, are equal to the Angles LVI, VIA, which the indefinite Representation (IV) of the Original Line (NO) makes with the Interfection (AB) and the Vanishing Line (VL).

For, they are parallel Lines in parallel Planes, and are cut by parallel Lines.

COR. The Center of every Vanishing Line is the Vanishing Point of Lines, which are perpendicular to the Interfection of those Planes of which it is the Vanishing Line.

For, by the Theorem, the Radial of a Line makes the same Angle with the Vanishing Line, as the Original Line makes with the Interfection of the Plane it is in. And in this case, the Original Line makes right Angles with the Interfection.

But, the Center of a Vanishing Line is produced by a perpendicular from the Eye to the Vanishing Line. - - - Def. 19.

Therefore, it is the Radial of all Lines, which are perpendicular to the Interfection of the Plane they are in, &c.

I would recommend it to young Students, to be very clear in this Theorem, as it is most essential in Practice. For, from the knowledge it inculcates, the Vanishing Points of Lines, in any Plane whatever, may be readily found; knowing the Angle which the Original Line makes with the Interfection of the Plane it is in, with the Picture; or with a Line passing through the Station Point, and cutting the Interfection at Right Angles; making an equal Angle, at the Eye, with the Parallel of the Eye; or, with the Radial of the Vanishing Line, producing its Center.

Plate VI.

THEOREM XII. Defin. 25th.

The Indefinite Representation, or Projection on the Picture, of an Original Right Line, not parallel to the Picture, is a Line drawn through its Intersecting and Vanishing Points.

DEM. For, every Right Line, not parallel to the Picture, will, if produced, cut the Picture, somewhere in the Intersecting Line of the Plane it is in.

And the Radial of every Line producing its Vanishing Point must cut the Picture, in the Vanishing Line of the Plane the Original Line is in. Theo. 10.

Wherefore, a Radial Plane, passing through the Original Line and the Eye, must necessarily pass through the Radial of that Original Line, and consequently through its Vanishing Point. Ax. 5.

For, the Radial of every Line is parallel to the Line. Def. 14.

But, the Perspective Projection of every Right Line, is a Right Line, produced by the Intersection of a Radial Plane with the Picture. Theo. 8.

And, the Radial Plane, passes through its intersecting and vanishing Points.

Consequently, the Indefinite Representation of the Original Line, on the Picture, passes through its Intersecting and Vanishing Point. Q. E. D.

Fig. 24. EX. Let HL be a Right Line, cutting the Picture (ACB) in I, and the Directing Plane in D; let E be the Eye; EV is the Radial of HL, cutting the Picture in its Vanishing Point (V) by Def. 22.

Let a Radial Plane (NOPQ) be supposed to pass through the Original Line and the Eye (at E) consequently through the Radial EV (for it is parallel to HL) and also through I, the Intersecting Point of HL.

Now, the Points I and V are in this imaginary Plane, and they are also in the Picture; therefore, a Right Line, drawn through the Points I and V, is the Intersection of the Plane NOPQ with the Picture (ACB).

But, the Plane (NOPQ) passes through the Eye (E) therefore, the Eye is in that Plane; and consequently, the whole of that Plane, and every Line in it, appears but a Right Line.

Wherefore, the Intersection (gVr) of that Plane with the Picture, is the indefinite Representation of the Original Line (HL) on the Picture; seeing that, every Line drawn through E, to any Point (N, M, F, K, &c.) in that Line (except the Point D) is in the Plane NOPQ, and will, if produced, cut the Picture, somewhere in IV, produced both ways, to g and r;

But, the Points n, m, f, &c. where the Visual Rays, or Right Lines, EN, EM, EF, &c. cut the Picture, are the Representations of the Points N, M, F, &c. in the Original Line; consequently, the Representation of every Point in the Line HL, except the Point D, where it cuts the Directing Plane, is in the Intersection gVr, produced indefinitely.

But, the Intersecting Point, I, and V, the Vanishing Point of the Original Line, are in that Intersection.

Therefore, the Indefinite Representation of every Right Line is a Right Line, drawn through its Intersecting and Vanishing Points.

COR. 1. The Representation of an indefinite Right Line, situate on the other Side the Picture, lies between its Intersecting and Vanishing Points.

If that part of the Original Line (IN) which lies beyond its Intersecting Point (I) was produced infinitely, towards H, its whole Representation, on the Picture, is IV.

For, the Point I is its own Representation; Im is the Representation of IM, and mn of the part MN; and, if the Point H be supposed at any finite Distance, its Representation h, must lie between n and V.

For, since the Radial, EV , producing the Vanishing Point (V) is parallel to the Original Line (IN) the Angle VEH will always be equal to EHD .

And, if the Point H be supposed at an infinite Distance, its Representation, (h) will coincide with V ; the Angle VEH will not be sensible, and consequently the Point H will vanish, or be lost to sight; and, therefore, the whole indefinite Representation of IH , infinitely produced, lies between its Intersecting Point (I) and its Vanishing Point (V).

COR. 2. *The Perspective Representation, of a finite Right Line, is part of a Line drawn from its Intersecting to its Vanishing Point.*

For the Visual Rays (EM , EN , EH , &c.) from the several Parts of the Original Line (MN , NH , &c.) to the Eye, determine the representations of those Parts; and, since the whole infinite Line, from I , its Intersecting Point, is represented between its Intersecting and its Vanishing Points (I and V) consequently, the Representation of every finite Part is a part of IV .

Fig. 24.

Im , represents the Part IM , mn represents MN , and, nh represents NH ; the remaining Part, hV , represents all the Line beyond H , infinitely produced.

COR. 3. *The Projective Representation of that Part of an Original Line (ID) which lies between the Picture and the Directing Plane, falls on the other side of its Intersecting Point; and, its whole Representation is infinite.*

EX. The Point F will be projected to f , where the Visual Ray EF , produced, cuts the Picture; and the Point G to g . (See Proj. Perf. P. 49). If is, therefore, the projective Representation of IF , and fg of FG .

If any Point (K) be taken, near D , its Representation will be projected at a great distance from the Intersection I ; and, the nearer the Original Point is to D , the farther will it be projected, from I ; for, EK will always make an Angle with Ig equal to DEK ; but, when the Point K coincides with D , it will be projected to an infinite Distance.

Therefore, the Part, ID is represented by Ig , infinitely produced; and consequently, the Directing Point, D , can have no Representation.

SCHOL. It may be observed, that, as the perspective Part (IV) of the Indefinite Representation, represents all that Part beyond its Intersecting Point, to an infinite Distance; so, the finite part of the Original Line, ID , between the Intersecting and Directing Point, is infinite in its Representation, from I through g .

COR. 4. *All that part of the Original Line, which lies on the other side of the Directing Plane, infinitely produced from its Directing Point, is transprojected to the Picture, beyond its Vanishing Point; and the whole Representation is infinite.*

EX. The representation of the Point Q is at q , and of R at r ; and, the nearer any Point (S) is taken to D , the farther will its Representation be from V ; but, when S coincides with D , its Representation will be at an infinite Distance. (See Transprojection, Page 49).

Also, the farther any Point (L) is taken beyond Q , the nearer its Representation (C) will approach to V ; and, at an infinite Distance, it will coincide with V .

SCHOL. Here, it may be observed, that an indefinite Original Line is represented by an indefinite Line, the Terms being inverted; and that, a Point, on the other Side of D , will be transprojected to an infinite Distance, on one side of the Vanishing Point, V . On this side, it will be projected to an infinite Distance on the other side of the Vanishing Point; whereas, but one Point (D) lies between; and also, that the Representations of two Points, H and L , being considered as the extremes of a Right Line, at infinite Distances, both ways, from the Intersecting Point (I) coincide at V .

Plate VI.

2. The transprojected part of the indefinite Projection, CV , may represent a Line in the same Plane, above the Eye; from the Point C produced indefinitely, through O ; which must also be infinite, if it be parallel to EV , before the Points o and V coincide.

3. If the Original Line TV passes through the Eye, its whole Representation is in its Vanishing Point; for, its Intersecting and Vanishing Point is the same; and, the Point of Sight (E) is its Directing Point.

COR. 5. *The whole indefinite Representation (IV) of an Original Line, from its Intersecting Point (I) is not varied, the Eye being in any Part of its Radial (EV).*

Suppose the Eye, at E , removed to E' , in the Direction VE .

EV , being parallel to IN , the Original Line, the Vanishing Point, V , remains the same; and, I , being its Intersecting Point, is invariable; consequently, the Indefinite Representation (IV) will be the same at any Distance of the Eye.

But, it must be observed, that, it is only so in respect of the whole of IV ; for the finite Parts of the Representation are continually varied, while the Eye moves from V to E ; m being the perspective Representation of the Original Point M , from the Eye at E ; but, removed to E' , it will appear at m' ; consequently, $I m$ is the Representation of the finite Part IM , of the Original Line, from the first Point of View, and $I m'$ from the Point E' . And so, of any other part of the whole Line.

Again, if the Eye move from E , or E' , in the direction of EM , or $E'M$, the Representation m , or m' , remains the same, and the indefinite Representation is varied. For, at E , the indefinite Representation is IV ; but, at E' , it is $I n$; yet $I m'$, the Representation of the finite part IM , is the same, from both Points of View.

COR. 6. *Hence, it is also evident, that, if the Eye be removed to any other Point in ED (the Director of the Original Line, IN) its whole Indefinite Representation, as well as the finite Parts are varied from every Station.*

For, if the Eye be raised to E' , the Radial is $E' C$; consequently, IC is the whole indefinite Representation; but if it be depressed to E'' , then is $E'' n$ the Radial, and $I n$ the whole indefinite Representation; and the finite Parts $I m$, $m n$, &c. are raised higher, or the Points, m , n , fall nearer to the Intersection, I , as the Eye is raised or depressed.

COR. 7. *The Representation of the common Intersection of two Original Lines, is the Point, in which the Indefinite Representations intersect each other.*

Fig. 23.

For, the Point B , of the Intersection of the two Original Lines, AB and BC , is common to both Lines; wherefore, its Representation is in the Indefinite Representations ($J V$ and $d M$) of both Lines; and, consequently, in the Point (b) of their mutual Intersection; for, there can be no other Point common to both.

Also, a , the common Intersection of $J V$ and $N V$, the indefinite Representations of AB and AD is the Representation of A , the Intersection of those Lines.

As this Theorem, and what is deducible from it, with the next, contain the whole essence of practical, rectilinear Perspective, respecting all Lines which are not parallel to the Picture, I would advise the Student to make himself particularly well versed in them before he proceeds to Practice; for if those Theorems be clearly understood and retained, he will find the Practice easy to be acquired, and, at the same time, rationally accounted for.

Plate VI.
Fig. 25.

from the Intersecting Point, to the Distance between the Intersecting and Directing Points, of the Line MN) or, as NA to AC, its Distance from the Intersection, to the Distance between the Intersection and Directing Line, of the Plane the Original Line is in.

DEM. For (having joined IF and DG) because the Directing Plane is parallel to the Picture, IF is parallel to DG - - - 8. 7. El.
(for the Triangle GND is a Plane cutting them both; Ax. 8.)

Wherefore, the Triangles GND, FNI, are similar; and, for the same reason, GNC, FNA, and also, DNC, INA, are similar; consequently $NF : NG :: NI : ND$; or, as $NA : NC$. - - - 4. 6. El.

Wherefore, the Distance of the Original Point from the Picture, and the Distance of the Picture being known; the Distance of its Representation, from the Intersecting Point, is a fourth Proportional, viz. as the Distance of the Original Point from the Picture, added to the Distance of the Picture, i. e. as the Distance NG (of the Original Point, N, from the Directing Plane) is to NF (the Distance of that Point from the Picture) so is the Indefinite Representation, IV (of the Original Line MN) to In; the Distance of n (the Representation of the Point N) from I, the Intersecting Point of the Original Line. Q. E. D. It is, therefore, as $NG : NF :: IV : In$.

For, (by the Theorem) it is, inversely, as $ND : NI :: IV : In$.

Or, from the Vanishing Point, it will be, as $NG : FG :: IV : nV$.

In Numbers, it is thus calculated. The Distance (NF) of the Original Point from the Picture, and the Distance of the Picture (FG) being added (equal NG). The Indefinite Representation, IV, being multiplied by NF, and that Product divided by NG (the Distance of the Original Point from the Directing Plane) gives In; the Distance of n, the Representation of the Original Point, from the Intersecting Point, of the Line it is in. For, $NG : NF :: IV : In$; as above.

Wherefore, the Rectangle, under NG and In, is equal to the Rectangle under NF and IV; and consequently, $NG \times In = NF \times IV$; - 9. 6. El.

It more frequently occurs, in Practice, that the Representation of a certain portion of a Line is required, from some Point already found; the Intersecting Point of the Original Line not being within reach, nor attainable, but only the Vanishing Point. For, the Intersection of every Plane is neither wanted nor can be had, when the Vanishing Line of the Plane is absolutely necessary; it is the same in respect of the Vanishing and Intersecting Points, of Original Lines; as it will be exemplified in the practical part of this Treatise.

COR. 2. *If the whole Indefinite Representation be not given, but only a part, from some determinate Point, in the Original Line; any other portion, of that Line, (from the Point determined) may be perspectively proportioned, geometrically.*

The Representation, m, of the Point M, in the Original Line (MN) being given or found, and the Indefinite Representation (mV) from that Point drawn (V being the Vanishing Point) the Representation, n, of any other Point, N, in the Original Line, will be; as $NM : MD :: mn : nV$; or, as $NM : ND :: mn : mV$.

DEM. For, as $NI : ND :: In : IV$; and, as $MI : MD :: Im : IV$; by Theo. But, - $ND : ID :: IV : nV$; and, - $MD : ID :: IV : mV$; 4. 6. El. for, the Triangles NED, VEN are similar; DE is equal to IV, and ID to VE (Because NDEV is a Radial Plane, passing through the Original Line (NM) and the Eye (E) cutting the Picture and Directing Plane; in IV and ED; and, EV, producing the Vanishing Point (V) is parallel to ND; by Def. 22.)

Now,

Now, $ND:ID::IV:nV$; also $MD:ID::IV:mV$;
 inverfely, $ID:MD::mV:IV$.
 Wherefore, by inordinate equality, $ND:MD::mV:nV$.
 Consequently, $ND-MD (=NM):MD::mV \times nV. (=mn):nV$.
 that is, $NM:MD::mn:nV$, as it was affirmed.
 Wherefore, by addition, $NM:ND::mn:mV$;
 and consequently, by inverfion, $ND:NM::mV:mn$.

Wherefore, the Distance of M (whose Representation, m , is given) and the Distance of any other Point, N, in the Original Line, from the Picture, being known; the difference being Nm , it will, consequently, be, as $NG:Nm::mV:mn$; Q. E. D. For, $NG:Nm::ND:NM$.

Thus may the perspective Proportion, of the known proportions of any Original Line, from any Point in it, be ascertained, geometrically; having the Representation of that Point, in the Indefinite Representation given, and the Distance of that Point, with any other Point in the Original Line, from the Picture, known; seeing, that the Distances of the several Points, from the Picture, are in the same Ratio, as their Distances from the Intersecting Point; and the Distance of those Points, from the Picture, may be had, when, often, their Distances from the Intersecting Point of that Line cannot; for several reasons.

The Distances of the Points M and N, from the Picture, being known, and the Distance of any other Point, in that Line (as O) required; it will be, as $MN:MO$ or $NO::mN:mo$ or $N o$; ($O o$ being supposed parallel to Mm).

So that, if mF , the Distance of the Point M from the Picture, be added, we have the Distance of the Point O, from the Picture, equal oF .

And the Distance of the Picture (FG) being known, the Representation, o , of the Point O, is determinable; for, it will be, as $om:mG::mo:oV$;
 and, by addition, $om:mG::mo:mV$.

By which means, the perspective Representations of several Divisions in a Right Line may be had, geometrically, their Distance from each other being known.

Let ABC be an Original Line, and $CDEG$ a Radial Plane, passing through the Eye (E) and also through the Original Line; cutting the Picture in IV , their common Section.

EV , parallel to AC , is its Radial; IV is, therefore, the whole Indefinite Representation of ABC ; I is its Intersecting, D its Directing, and V its Vanishing Point.

Now, if the Representation (a) of any Point (A) in the Original Line, be determined, all other Divisions ($B, C, \&c.$) in that Line, are also determinable, without the Intersection I ; the Distance of A and B , from the Picture, and the Distance of the Picture being known.

For, $ab:aV::AB:BD$; $ac:aV::AC:CD$; and also, $bc:bV::BC:CD$.

DEM. For, (having drawn AF and BG , parallel to IV) because aV is parallel to AF , $ab:aV::Ab:AF$; also, $ac:aV::Ac:AF$; } Cor. to 6. 6. El.
 and, for the same reason, $bc:bV::bc:BF$; and, as $BC:BG$.

But, the Triangles GEC, CCB are similar, for EG is parallel to CB ; wherefore, $BC:CG::BC:EG$ (equal BD) 4. 6. El.
 Consequently, by compounding, $BC:BG::BC:CD$, equal $CB+GE$.
 But, as $BC:BG::bc:bV$; therefore, $bc:bV::BC:CD$.

c is, therefore, the Representation of the Original Point C , and b of B ; which are determined from the given Representation of A ; the Distance of A and any other Original Point from the Picture, being known; no regard being had to the Distance of the Point A , from the Intersecting Point (I) or from D ,
 the

Fig. 25.

Fig. 26.

Plate VI.

the Directing Point of the Original Line; seeing that, the Distance of the Original Points, from the Picture and Directing Plane, are in the same Ratio, as their Distances from the Intersecting and Directing Points of the Original Line.

In this Theorem is the Perfection of Practical perspective. It is, at the same time, the most mathematical; and the Demonstrations the most perfect, elegant, and convictive, of the whole Theory. By it, we not only know that there is Analogy of Ratios, between the several Distances of Points, in an Indefinite Representation, from the Intersecting Point, and the whole Indefinite Representation; to the several Distances of the Original Points from the Intersecting and Directing Points, or from the Picture and Directing Plane; but, by its means, the Representations of the several portions of an Original Line, in the Indefinite Representation, are determined, with the greatest facility, accuracy, and expedition, geometrically; or they may be determined numerically, by a Scale of equal Parts; but that is seldom practiced, the method of doing it geometrically, being much readier and more accurate than it is possible to calculate, by Numbers.

T H E O R E M. XIV.

The Perspective Projection, or Representation, of every Right Line, is parallel to its Director.

DEM. The Radial of an Original Line, producing its Vanishing Point, is parallel to the Original Line; — — — — — Def. 22.

wherefore, a Radial Plane may pass through the Original Line and its Radial; consequently the Eye is in that Radial Plane. — — — — — Ax. 5.

Now, since one part of a Right Line cannot be in a Plane, and another part of the Line out of that Plane, the Radial Plane will cut the intersecting Line (of the Plane the Original Line is in) in the intersecting Point; and the Directing Line, in the Directing Point, of the Original Line. — — — — — Ax. 1.

But, the Radial Plane passes through the Eye; — — — — — Def. 6. and, consequently, through the Vanishing Point; seeing it passes through the Radial Line, producing the Vanishing Point.

Wherefore, the Section of this Radial Plane, with the Picture, is the Indefinite Representation of the Original Line; seeing, it passes through its Intersecting and Vanishing Points. — — — — — Theorem 12.

But, it also passes through the Eye, and Directing Point, of the Original Line; consequently, it cuts the Directing Plane in the Director of the Original Line. — — — — — Def. 12.

But, the Directing Plane is parallel to the Picture. — — — — — Def. 4.

Wherefore, the Sections of the Radial Plane, with the Picture and Directing Plane, are parallel to one another. — — — — — 8. 7. El.

But, its Section with the Picture is the Indefinite Representation; and, its Section with the Directing Plane is the Director of the Original Line; as above.

Therefore, the Perspective Projection of an Original Line, being a part of its indefinite Representation, is parallel to its Director. Q. E. D.

Fig. 20.

EX. Imagine a Plate (NDEV) passing through the Eye and the Original Line, NO. This Radial Plane must necessarily cut the Picture and Directing Plane, in IV and ED; for EV is parallel to DIN, i. e. to NO; wherefore, EVID is a part of the Radial Plane; which, being produced, would pass through NO, the Original Line.

But, I is the Intersecting, and, V is the Vanishing Point of NO; Def. 21. and 22. wherefore, IV is the Indefinite Representation of NO; — — — — — Def. 25. Th. 12. and, since the Radial Plane passes also through the Eye and Directing Point,

(E and D) its Section with the Directing Plane (ED) is the Director of NO†. † Def. 12.
Consequently, IV, or no, a part of IV, is parallel to ED. 8. 7. El

In Fig. 21. this is more perfectly illustrated, by means of Visual Rays, from the Extremes (N and O) of the Original Line, NO.

For, NO being produced to the Directing Plane, D is its Directing Point; and ED, being drawn, is the Director of NO, as before.

But END is a Triangle, consequently it is a Plane; Ax. 8.
which cuts the Picture in no, or IV, the Indefinite Representation of NO;
and, consequently, it is parallel to ED, its Director.

COR. 1. *If two, or more, Right Lines cut the Directing Plane in the same Point, they will have parallel Representations.*

For they have the same Director, to which they are all parallel, by Theorem.

EX. PQ cuts the Picture in P, and the Directing Plane in D, the Directing Point of NO; and, because it is perpendicular to AB (the Intersection of the Picture) C is, therefore, its Vanishing Point. (EC being perpendicular to LM, is therefore parallel to PQ, being in parallel Planes.)

Consequently, PC is the indefinite Representation of PQ; which is parallel to IV, the indefinite Representation of NO; by Theorem, and 4. 7. El.

And also, by Theo. 5; for, ED, the Director of both (NO and PQ) is the common Intersection of the Radial Planes, DEVI and DECP; passing through both Lines. Therefore, their Intersections, IV and PC, are parallel.

N. B. This Corollary is true if the Lines, NO and PQ, are not in the same Plane.

COR. 2. *All Lines, not having the same Directing Point, but, which cut the same Director, indefinitely produced (through the Eye) both ways, have parallel Representations.*

For, if two Original Lines, NO, or PQ, and RS, cut the Directing Plane in the two Points D and d, in such wise, that a Right Line drawn through the Eye and the Directing Point of one, will also pass through the Directing Point of the other, their Representations, no and pq, are parallel.
Because they have the same Director, DEd.

EV is the Radial or Parallel of NO, V is therefore its Vanishing Point; and Ev is parallel to RS; therefore v is its Vanishing Point; and, Sv its Indefinite Representation.

COR. 3. *All Right Lines cutting the Directing Plane in EF, the Intersection of the Vertical and Directing Planes, will have their Representations parallel to the Vertical Line, (CD.)*

Because, they all have the same Director (EF) to which the Vertical Line (CD) is parallel; being produced by the section of the Vertical Plane (ECDP) with the Picture and Directing Plane. 8. 7. El.

As PY, parallel to RS, cutting EF, the Prime Director, in f.
Ex is its Radial and Pv its indefinite Representation.

COR. 4. *All Lines which cut the Parallel of the Eye of any Original Plane, have their Representations parallel to the Vanishing Line of that Plane.*

Because, the Parallel of the Eye is the Director of all such Lines, and it is parallel to the Vanishing Line; by Theorem 2nd.

TU is an Original Line, cutting IK, the Parallel of the Eye, of the Original Plane NBH, in k; Ex is its Radial, x is therefore its Vanishing Point, and Ux its indefinite Representation; which is parallel to LM, the Vanishing Line of the Plane NBH.

† Theo. 2.

For, IK is parallel to LM †, and it is the common Section of the Plane, $IKLM$ and $EKUX$; therefore, UX is parallel to LM , by Theorem 5th.

COR. 5. *The Representation of any Original Line makes equal Angles with the Intersection and Vanishing Line, of the Plane it is in, as the Director, of that Line, makes with the Parallel of the Eye and the Directing Line of that Plane.*

Because the Representation is parallel to its Director, by the Theorem; and because they cut parallel Lines in parallel Planes.

COR. 6. *If the Representations of any two Lines are parallel, the Originals are either parallel between themselves and to the Picture, or they have the same Director.*

For, if the Original Lines are parallel between themselves and to the Picture, their Representations will be parallel (Theo. 9.); but, if they are not parallel to the Picture, they must have the same Director; seeing, there can be but one Line drawn in the Directing Plane, parallel to both Representations.

N. B. Lines, which are parallel to the Picture, have no Directing Point, but the Director of every such Line, is a Line drawn through the Eye parallel to the Original Line.
For, the Representations are parallel to the Original; by Theorem 9th.

This last Theorem (and the Corollaries deducible from it) contains the whole Theory of the Directing Plane; as, in it and the thirteen preceding Theorems is contained the whole knowledge of rectilinear Perspective; or, at least, all that I conceive to be really useful, in delineating. It has been my aim, not, merely, to amuse or to shew my knowledge in it, but to give useful Instruction; and, I dare venture to affirm, that if the whole of this Theory be clearly understood, the Student will seldom be at a loss in Practice.

It is a mistaken notion which many entertain of Perspective, that, the Theory is unnecessary to a Practitioner. It is certainly possible to practice Perspective, in all common Cases, without being able to account, or give a reason for any rule that is followed; for, the Rules, being deduced from the Theory, will, undoubtedly, if strictly followed, produce certain effects though we know not how to account for it: as there are many Persons very acute in Mensuration, Gauging, Surveying, &c. who know nothing of Geometry, the foundation of the whole. The case is very different in Perspective; for I am well convinced that it is of great use to understand the Theory well, in the first place; and that, the Practice will, by that means, be sooner acquired, and more securely retained. For want of Theory, the Pupil is frequently bewildered, and knows not what he is about; every different Example appearing difficult and strange, though, perhaps, founded on the same invariable Principles. In short, the nearest and most certain road, to Perspective, is to go through the Theory to Practice: and, I will venture to stake all my knowledge in it, that when acquired, the loss of Time (if it be any) will never be regretted.

I shall give three more Theorems, on Circles and spherical Bodies, and then proceed to Practice. If any Person require further knowledge, or a more extensive Theory, I refer him to the elaborate Work of Mr. Hamilton, which is deserving of the highest encomiums, if it was as useful as it is ingenious and learned in the Science; for, he has certainly said all that can be said of it, in Theory: and, I am persuaded, more than any other Person would ever have thought on, and much more than is of real use; for, I think I have omitted nothing that can be useful or necessary to be known, by any Practitioner, whatever.

SECTION V.

Of the THEORY of CURVILINEAR PERSPECTIVE.

IN this Section, I shall chiefly consider the Theory of Perspective relative to circular Objects; which are the most common, and most useful of all curve lined Figures. Other Curves cannot be comprised in any certain Theory, by which their perspective Representations can, with certainty, be ascertained; or if they could, it would answer no purpose to an Artist, seeing that, irregular curved Figures or Objects but seldom occur, in Practice. I would not be understood to mean the curves of the apparent Contours of human or other Figures (endowed with Life or not) which occur in almost every Picture, but which, can never be reduced to Rules, for Practice, from an established Theory, but, irregular curved Figures, in Planes or other Surfaces; winding or serpentine Rivers, Rocks, Mountains, Trees, &c. which are bounded by irregular curved Lines and Surfaces, cannot be reduced to Practice, in delineating them, by any Theory in the Science of Perspective. Notwithstanding they may be delineated with great accuracy, by any Person, who is a little accustomed to sketch by sight only, by means of an Apparatus, which I shall describe in an Appendix to this Work.

To treat, at large, of the various Curves which the Representation of a Circle may take, such as the Parabola or Hyperbola, is foreign to my Design; as it so rarely assumes those forms. Nor is the knowledge thereof of any real use in delineating; seeing that, the small part of the Representations of such Circles as are or can be represented, when they do assume either, could not readily be distinguished from a portion of an Ellipsis; which Curve, as it is the most general and useful, so it is the easiest to describe, and the only one of real use, in Perspective.

In order to a clear understanding of the nature of an Ellipsis and its Properties, it is necessary to be acquainted with the Conic Sections; since every Representation of a Circle or Sphere, in Perspective, is either one or other of the Sections of a Cone. But, as a thorough investigation of it is not necessary, here, I shall refer the Reader, who desires to be perfectly acquainted with the Conic Sections, to Mr. Steel's, or to a later Work, by Mr. Emerson.

Nevertheless, I find it impossible to treat the Theory of the Circle, in Perspective, without having recourse to them, in some degree; therefore I shall, in the first place, define what is a Cone, and, the difference between a Right Cone and a scalene or oblique Cone; for, without that knowledge, all that can be said of it would be to little or no purpose.

The methods of describing an Ellipsis, and all which appertains to it, are treated fully, yet briefly, in six Problems in an Appendix to the practical Part (Book 1st) of my Treatise on Geometry; together with a concise Theory, of its most essential Properties; to which I refer the Reader; I shall consider it, here, only as being the perspective Representation of a Circle or Sphere.

DEFINITIONS.

A **CONE** is a geometrical Solid, whose Base is a Circle, which terminates in a Point, called its **VERTEX**; and, a Right Line, passing through the Vertex and the center of its Base, is called its **AXE** or **AXIS**.*

* The Axis of any thing is either a real or imaginary Right Line passing through its middle, in a certain and determined Position.

If the Object be a Plane Figure, its Axe may be either perpendicular to, or in the Plane of the Figure. If the Axe be in the Plane of the Figure, it is divided, by the Axe, into two equal and similar Figures. Any Diameter of a Circle may be its Axe, an Ellipsis has but two Diameters which are Axes, viz. the Transverse and its Conjugate, at Right Angles with each other.

Plate VII. It may be considered as a Pyramid whose Base is a Polygon of an infinite number of Sides; every Section of which, by a Plane parallel to its Base, will, consequently, be a similar Figure; wherefore, the section of a Cone, parallel to its Base, is a Circle.

2. A **RIGHT CONE** is that which is formed, or supposed to be generated, by the revolution of a right angled Triangle, on one of its Legs.

Fig. 27.
No. 1.

ABC is a right angled Triangle; C is the Right Angle; BC is, therefore, perpendicular to AC, its Base.

If you suppose the Triangle ABC to be revolved quite around, on BC, as an Axis, its Base (AC) will describe the Circle AEDF, which is the Base of the Cone ABD; and the Hypothenuse (AB) will have described the Surface of the Cone; which is every where various, from the Vertex (B) to the Periphery of its Base; its Sides are equal every where, as BA, BE, BD, &c. The Perpendicular (BC) which is the Axe of the Cone, remains at rest; one extreme in B, the Vertex, the other in C, the Center of its Base.

No. 2.

3. AN **OBLIQUE CONE** has, also, a Circle for its Base, but its Axe is inclined to its Base, as BC to AD.

As if the Vertex (B) of a Right Cone, was drag'd on one side, out of its perpendicular position; consequently, the real Axe, of such a Cone, does not pass through the center of its Base, yet BC is called its Axe.

A Right Line bisecting any Angle of a Triangle is called its Axe; wherefore, BE, bisecting the Angle ABD, is the Axe of the Triangle ABD, and consequently of the Cone; for it passes through the middle of such a Solid, whose roundness is elliptical, seeing, its Dimensions, perpendicular to its Axe, are unequal; and, the more the Axe is inclined to the Base, the more its Dimensions differ (its Base remaining the same) and the real Axe is removed farther from the Center (C) towards A.

If a Section of an oblique Cone be made, by a Plane, perpendicular to its real Axe (BE) it is considered as an oblique Section of it; which Section is an Ellipsis, and BE would pass through its Center; consequently, the Solid would revolve regularly on BE, its real Axe. Whereas, on BC it would revolve very irregularly and unequal; but the Cone, ABD, would be equally poised on BC, in a horizontal Position; wherefore, every Section of a Cone, through the Axe BC, bisects the Cone; for the Triangles, ABC, CBD, are equal, in every Section, through BC; by Prop. 18. 1. El.

THEOREM I.

The Representation, or perspective Projection, of every Circle, in a Plane to which the Picture is parallel, is a Circle.

The scenographic Projection, or perspective Representation of an Object, is the Section of the Optic Cone, or Pyramid of Rays, by a Plane, passing between the Eye and the Object. (See Scenography, P. 47.)

Fig. 28.

Let ADGH be a Circle, in any Original Plane (Z) E is the Eye, and EA, EB, &c. Visual Rays, from every Point in the Circumference, to the Eye, forming an oblique Cone (AEF).

DEM. Now, if the Cone of Rays be cut by a Plane (X) parallel to the Plane Z, in which is the Original Circle; adgh, the Section of the Rays, by that Plane, is a Circle.

For, it is the section of a Cone, by a Plane, parallel to its Base; and, whether it be a Right or Oblique Cone, every such Section is a Circle; seeing that, the Cone, aEf , cut off by the Plane X , is similar to the larger Cone (AEF).

This is otherwise demonstrable, from Theorem 9, and Corollaries; in which it is demonstrated, that the Representation of every Plane Figure, parallel to the Picture, is similar to the Original.

Every Diameter in the Original Circle (AF , DH , &c.) being equal, the Representations (af , dh , &c.) have that proportion to their Originals, as the Distance of the Picture to the Distance of the Plane of the Original Circle. (Theo. 9.) consequently they are also equal; therefore, the Representation, $adgh$, is a Circle. Q. E. D. For, all Circles are similar Figures.

COR. Hence it is manifest, that the Representations, in Perspective, of various Circles, in the same Plane, parallel to the Picture, have all the same Ratio amongst themselves, and to each other, as the Original Circles.

THEOREM II.

The Representation of a Circle, in a Plane not parallel to the Picture, is an Ellipsis; except in one certain Point of View, in which, its Representation is, also, a Circle.

Let ADG be an Original Circle in the Plane Z ; E is the Eye, and EA , EB , &c. Visual Rays forming (as before) an Oblique Cone. C is the Center of the Circle. Fig. 28.

DEM. If the Cone of Rays be cut by a Plane (Y) which is considered as the Picture, passing through both sides of the Cone, not parallel to the Base, it is an oblique Section; and, consequently, every such Section, of a Cone, is an Ellipsis; except, as above. Prop. 90. P. 73. Em. Con. Sec.

Wherefore, the Representation, adg , on the Plane Y , (which is an oblique Section of the Cone of Rays EA , EB , ED , &c.) is an Ellipsis. Q. E. D.

Every Circle, which is visible, appears an Ellipsis, except when the Axe of the Eye is perpendicular to the Plane of the Circle, and passes through its Center; for, in that Case, only, the Visual Rays, from the Eye to every Point in the Circumference, are equal, and consequently they generate a Right Cone, the Axe of the Eye being the Axe of the Cone. In every other position, whatever, they must necessarily form an Oblique Cone, seeing that, the Axe of the Eye (which is always the Axe of the Cone) must be inclined to the Plane of the Circle, if it be not perpendicular; and consequently, the Visual Rays, forming the surface of the Cone, have various Inclinations to the Plane of the Circle, and therefore they are unequal.

Wherefore, since the section of an Oblique Cone, perpendicular to its central Axe, is an Ellipsis, and the more the Axe is inclined to the Base, the more excentric is the Section; seeing that, if the Eye be nearly in the Plane of the Circle, the Section approaches nearly to a Right Line; and, the Section, made by a Plane in that position, is the only true Appearance of the Circle; or rather, by a spherical Surface, perpendicular to the Axe, which truly measures the Optic Angle (the Radius being equal to the Distance) under which, the solid dimensions of the Cone, every way, are seen.

COR. If the height of the Eye, or its distance from the Directing Line, be taken a mean Proportional, between the distance of the nearest convex part of the Circumference and the distance of the farthest concave part, from the Directing Line, the Representation will then be a Circle.

Plate VII. Let AB be a Diameter of the Circle, ADG, perpendicular to the Directing Line (KL) BS is the distance of the Circle from it.

Fig. 29.

Make $SE : SA :: SB : SE$; i. e. let $SB : SE :: SE : SA$, by Pr. 30. Geo.

At E, as the Point of View, the Representation of the Circle ADG, on any Plane (X) parallel to the Plane KEL, will be a Circle, and in no other Point whatever, on that Plane, and at the distance BS.

DEM. Draw BI parallel to SE; and, suppose AEB a section, by a Plane passing through CE, the Axe of the Cone, and the Diameter AB, of its Base; and also through S.

Then, because BI is parallel to SE, the Triangles AIB, AES are similar;

Wherefore, $AB : BI :: AS : SE$.

But, by Construction, $AS : SE :: SE : SB$.

Wherefore, $SE : SB :: AB : BI$;

+ 5. 6. El.
|| 4. 1.

and therefore, the Triangles AIB, BES are similar†, for the Angle ABI is equal to ESB ; ESB is equal AIB, and IAB equal BES; seeing that, the Sides which subtend those Angles are proportional.

† 4. 1. El.

But, the Angle BES is equal EBI ‡, for they are alternate; wherefore, EBI is equal IAB (of the Cone AEB) and consequently, the lesser Cone (aEb) cut off by the Plane X (parallel to BI, and the Plane KEL) is similar to the Cone BEA; for, the Angle E (at the Vertex) is common to both; the Angle Eba (equal EBI) is equal EAB, consequently, Eab is equal EBA§, and therefore, the Cone (AEB) is cut sub-contrary, by the Plane X.

§ C. 5. 10. 1.

But, if an Oblique Cone be cut sub-contrary, the Section is a Circle.

Therefore, the Representation, ahbg, on the Plane X, of the Original Circle ADBF, is a Circle. Q. E. D. Prop. 89. P. 73. Em. Con. Sec.

N. B. The Center (s) of the Representation, is not the Representation of the Center (C) of the Original Circle; for it is at c, where the Visual Ray EC cuts ab, the Representation of the Diameter AB. Wherefore, CE is not the real Axe; seeing that, it cannot be the Axe of the Cone AEB, and also, of the lesser Cone (aEb) which is cut sub-contrary, and is therefore similar to AEB; consequently, the true Axe is common to both Cones.

So likewise, df, the Representation of the Diameter DF, in the Original Circle, is not a Diameter of the Representation; but, gh, the Representation of the Chord GH, (which bisects ab) is its Diameter perpendicular to ab.

Wherefore, the Representation of the Segment GBH is the lower Semicircle, gbh; and, gah, the upper Semicircle, represents the large Segment (GAH).

§ Cor. 3.
Theo. 9.

It is evident, that, if the Eye be raised, as at E^1 , the Diameter ab, in the Representation, will be lengthened; seeing, that the Visual Rays, E^1A , E^1B , cut the Picture more oblique; whilst the other Diameter, gh, remains of the same length§; and consequently, the Representation of the Circle, from that Point of View, will be an Ellipsis, and ab its Transverse Diameter, or Axe.

But, if the Eye be lowered, to E^2 , the Diameter ab (now ce) will be shorter than the Diameter gh (which, being parallel to the Intersection of the Picture, will have the same length, seen from any Point in the Directing Plane, as above) and it will then be the Transverse Axe; and ce (the Conjugate to it) is the shortest Diameter of the Ellipsis.

If the Eye be removed on either Side of the Point E, the Representation will be an Ellipsis; for, the Section will not, then, be sub-contrary (the Picture remaining as before). The Representation (ab) of the Diameter AB, will still be a Diameter of the Ellipsis, because it will pass through its Center; but it will be neither the transverse nor the conjugate Axe, for they are always at right angles with each other.

Hence, the Point of View, from which the Representation of a given Circle, on any Picture, however situated, will be a Circle, is easily determined.

The Original Plane (KAL) in which the Circle is situated, being produced if necessary, cuts the Picture in MN, its Intersection.

Having fixed on the Distance, or Station Point, S (at pleasure) in the Diameter AB, produced, which is perpendicular to the Intersection of the Picture; let KL be drawn, through S, parallel to MN; KL is the Directing Line. - Def. 10.

Draw SE perpendicular to KL, and parallel to the Picture.

Make SE a Mean Proportional, between SA and BS; - - Pr. 30. Geo.

E is the place of the Eye, or the Point required; from which, a Circle, agbh, (on the Plane X) whose Center is c, and Diameter ab, will truly represent the Original Circle ADBF, on the Plane AKL.

N. B. The distance of the Picture, in this Case, is not material; for, the Point E being fixed (as above) every Section of the Cone of Rays EA, EF, EB, &c. parallel to the Plane X, or to the Directing Plane, KEL, being cut sub-contrary, will, consequently, be a Circle.

From what has been advanced, it is evident, that the Representation of every Circle, in Perspective, is some one or other of the Conic Sections; which, being a distinct Science, would not be proper to enter on here; but, from that consideration and the preceding Theorems, the two following Corollaries may be deduced.

COR. 1. *If the Circumference of the Original Circle touches the Directing Line, in a Point only, and is not cut by it, the Representation of that Circle will be a Parabola.*

For, whether the Original Circle (ADBF) touches the Directing Line, in S, (the Station Point) or in any other Point (B) it is the same; since that Point, and also the Eye (E) which is the Vertex of the optic Cone of Rays, are both in the Directing Plane (Y) it is evident, that the Directing Plane touches the Cone in the Right Line EB, from its Vertex (E) to its Base (at B). Fig. 30.

Wherefore, since the Picture (X) is always parallel to the Directing Plane †, † Def. 4. the Cone, AEB, is cut by a Plane parallel to its Side (EB) the Curve produced by every such Section is a Parabola. Prop. 76. P. 223. Em. Con. Sec.

That part of the Original Circle (MGAN) which lies beyond the Picture, is represented perspectively, and falls above the Intersection MN (as M a N); the remainder of the Circle, lying between the Picture and the Directing Line, is projected below the Intersection; as l is the representation of the Original Point L. Every other Point (as K) is projected further from the Intersection as it lies nearer to the Directing Line (BS) and, except the Point B, in which it touches the Directing Line, may be supposed to have a Representation, on the Picture, though at an immense distance; but, the Point B can have none; because, the Line (EB) which should produce it, is parallel to the Picture, seeing, it lies in the Directing Plane, and therefore can never cut the Picture. The Representation of which Point, only, is wanting to compleat the Figure and form an Ellipsis; but, for want of that Point, in the Representation, it is kept open, and falls off in Right Lines, nearly, at a Distance, to all sense, infinite.

Hence it is plain, that, on account of the Distance, but a small portion of that part of the Curve which falls below the Intersection, can be represented, in a Picture; and the part which can, together with the part which lies beyond the Picture, differs so little, in its Representation, from an elliptic Curve, that, in delineating, it would be needless to deviate from it; and, I question that ever a distinction was made, in Practice. Nor can it ever be of use, but in delineating the inside of a large Rotunda, Circus, or circular Area; when the Spectator may be supposed to stand on the hither part of the Circumference, or in a Line which is a Tangent to it, and parallel to the Picture; in which case, only the farther, concave part of the Curve can, properly, be represented.

Plate VII. COR. 2. *If the Directing Line (KL) cuts the Original Circle in two Parts; that part (KFAHL) which is on the same side with the Picture, will, in its Representation, form an Hyperbola, below the Vanishing Line (OP) of the Plane of the Circle; the other part (KBL) of the Circle (if it be represented on the same Picture) will be transprojected, and form an opposite Hyperbola, above the Vanishing Line, which is equal and similar to the other.*

For, if the Directing Line passes through the Center of the Original Circle, and the Eye, which is the Vertex of the Cone, is in the Directing Plane; in which Case, the Section, made by the Picture, is parallel to the Axe of the Cone, and the opposite Hyperbolas are equal, and equally distant from the Vanishing Line (OP).

But, when the Circle is cut unequally, by the Directing Line (KL) the Cone (BEAF) is also unequally cut by the Directing Plane (KEL) the Section of which, with the Cone, is the Triangle KEL; the Picture (NOPM) being parallel to the Directing Plane, cuts the larger Segment of the Circle, in MN, and the Cone, in the Curve fg a N, which is an Hyperbola. Prop. 104. P. 160. Em. C. Sec.

If the Sides (AE, FE, BE, &c.) of the Cone BEA, are produced through the Vertex (E) forming an opposite Cone (aEbd) the Picture being produced, will also cut that Cone, produced; its Section, with it, is the opposite Hyperbola, or transprojected Representation of that part of the Circumference of the Circle, (KBL) which lies on the other Side of the Directing Line, by means of the Rays BEb, QEq, &c. produced to the Picture; which, notwithstanding it is the Representation of the lesser Segment of the Circle, is generated by a similar Cone, (aEb) and, although the Section is made at a greater distance from their common Vertex, it is equal and similar to the perspective and projective Representation (fMgaNd) of the larger Segment.

This Curve, like the Parabola, can never be generated, or of use, but when the Area of the Circle is so large, that the Spectator is supposed to stand within it; and that Segment (MGAHN) which lies beyond the Picture, only, is required; and which differs so little from an elliptic Curve, in its Representation, (Mga h N) that the distinction is not very obvious, and seldom, if ever, regarded.

These are all the variety of Curves which a Circle in Perspective can assume, however situated in respect of the Picture, or of the Eye; the chief of which is the Ellipsis. Every Circle, which the Eye is capable of taking, in at one View, it is manifest, has the Appearance of an Ellipsis; except, when the Axe of the Eye is perpendicular to the Plane of the Circle, and passes through its Center; in which position, only, it can appear, to the Eye, a true Circle; although all Representations of Circles, in Planes parallel to the Picture, are Circles†; but, being seen oblique, when the Eye is in the true Point of View, they have the Appearance of Ellipses.

† Theo. 1.

THEOREM III.

The Representation, in Perspective, of a Globe, or Sphere, is an Ellipsis*; except when the Center of the Sphere coincides with the Center of the Picture; in which Case, only, it is a Circle.

The Cone of Rays from the Eye, as its Vertex, to the apparent Circumference of a Sphere, is always a Right Cone. For, a Sphere can have but one Position, either to a Plane, Line or Point; and consequently, the Diameter, every way, always presents itself to the Eye, and always forms a Right Cone.

* I have found it very difficult, nay almost impossible, to convince some Persons of this plain and well known truth; because, as they truly observe, the Diameters of a Sphere always appear equal; which is as much as to say, that the Visual Rays, under which a Globe is supposed to be seen, always form a Right Cone, i. e. whose Axe is perpendicular to its Base; which is not so with Circles, but in one position only. But, such Persons seem to forget, that the Representation is on a Plane, and that Plane is considered as the Plane of the Section; which must, consequently, cut every Cone oblique, but that which has its Axe perpendicular to the Picture.

Now,

DEM. It is evident, that, when a Sphere is so situated, in respect of the Picture and the Eye, that, the Direct Radial coincides with the Axe of the Cone, (i. e. when it passes through the Center of the Sphere) the Section of the Cone, made by the Picture, is parallel to every apparent Diameter of the Sphere, which is the Base of the Cone; and, being equal every way, it is consequently a Circle.

But, the Direct Radial is the Axe of the Cone, which passes through the Center of its Base, and also through the Center of the Picture.

Consequently, the Center of the Picture coincides with the Center of the Sphere, in this Section; and therefore, the Representation is a Circle. Theo. 1st.

For it is the Section of a Cone, parallel to its Base.

2. But, if a Sphere be so situated that the Direct Radial does not pass through its Center, the Axe of the Cone must be inclined to the Picture; which, being the Plane of the Section, the Cone of Rays are cut obliquely by the Picture.

But the oblique Section of a Cone, through both its Sides, is an Ellipsis.

Therefore, the Representation of a Sphere, whose Center is not the Center of the Picture, is an Ellipsis. Q. E. D. For it is an oblique Section of a Cone.

Pr. 90 Em.

EX. Let AB be a Sphere and C its Center (supposed to be seen). Let E be supposed the Eye of a Spectator, and EA, EB, Visual Rays, from the Eye, to the apparent Diameter, AB*. AEB may, therefore, be supposed a Section of a Right Cone, through its Axe, EC, which is an Isosceles Triangle; the Sides, EA, EB, &c. of a Right Cone being equal †.

Fig. 32d

Now, if FG be the Picture (which is a Plane) parallel to AHBI (the Base of the Cone) the Line ab, in which the Plane AEB cuts that Plane, is a Diameter of that Section; which Section, being every way equal, is, consequently, a Circle.

† Def. 2.

But, EC, the Axe of the Cone, being perpendicular to its Base, is perpendicular to the Picture §, and to its Section, FG, with the Triangle AEB; the Point, c, where the Picture is cut by the Axe of the Cone, EC, is the Representation of C, the Center of the Sphere.

§ 6. 7. El.

And, because AB is a Diameter of the Base of the Cone, and C its Center, AB is bisected in C; consequently, ab is also bisected in c; for, the Triangles CE A, CEB are congruous ||; and, because ab is parallel to AB, the Triangles aEc, AEC, and cEb, CEB, are all similar ‡.

|| 7. 1. El.

‡ 2. 6. El.

But, E is supposed the Eye of a Spectator, and, Ec, being perpendicular to FG (the Picture) is the Direct Radial ¶; and the Point c, where it cuts the Picture, is the Center of the Picture §, which is also the Center of the Representation of the Sphere, on that Picture; therefore it is a Circle. Theo. 1.

¶ Def. 15.

§ Def. 17.

Now, if the Cone AEB be supposed to be cut by any other Plane, passing through the Diameter hi, and consequently through c, the Center of the Picture and of the Representation; that Section will be an Ellipsis.

Let SD be a Section of another Plane or Picture, with the Triangle AEB, cutting the Rays EA, EB, in e and d; then is ed, in that Section, the representation of the apparent Diameter AB, of the Sphere, and c is the representation of its Center, as before, in the Section ab; but, ed is not bisected in c.

DEM. Let ace be an Isosceles Triangle; ac=ec; and, ac=cb; th. ec=cb: Ax. 3. El.

But, because b c E, in the Triangle bEc, is a Right Angle, cbE is acute †, and, the Angle cbd, in the Triangle dcb, is, consequently, obtuse. C. 2. 1. 1. Wherefore, cd, subtending the obtuse Angle, is greater than cb; 12. 1. El. And consequently, ec+cd, equal ed, is greater than ac+cb, equal ab.

+ C. 3d

10. 1. El.

But, the Section HEI, which is vertical to AEB, cuts both Pictures in the same Line, hi, equal to ab; as above.

* See the Note to Art. 4. Page 14, on Direct Vision.

Plate VII. Therefore, the Section, $ehdi$, is an Ellipsis; for, the Diameter ed is larger than any other, in that Section.

|| Def. 17. But, ES , perpendicular to SD , is the Direct Radial; wherefore, S is the Center of that Picture; and it does not coincide with the Center of the Sphere (the Eye being at E) consequently, the Representation of a Sphere cannot be a Circle, except when the Center of the Picture coincides with the Center of the Sphere, in the Center of its Representation.

Fig. 33: EX. 2. To illustrate this further. Let X , Y , and Z be three Globes, whose Centers are all in the same Right Line, parallel to ah , the Intersection of the Picture; whose Center is C , and EC its Distance.

Draw the Tangents EA , EB , ED , &c. to the three Globes, the Chords, AB , DF , and GH , of those Tangents, are the apparent Diameters of each; which, it may be observed, are still turned towards the Eye; and are considered as the Diameters of the Bases of the three Cones, AEB , DEF , and GEH , which are all Right Cones.

Now, since they are all cut by the same Plane (of which ah is a Section) each Cone, except the first, AEB (whose Axe, EX , coincides with the Direct Radial, EC) is cut oblique; and, consequently, one Diameter, of each Section, is larger, as the Globe is farther from the perpendicular EC ; as df , and gh , the Representations of a Diameter of each Globe (Y , and Z) df being larger than ab , and gh still larger than df .

§ Cor. 3.
Th. 9.

But, the other Diameters, perpendicular to these, are equal, in all §, for they are supposed parallel to the Picture, and equally distant from it; consequently, the Representation of the Globe X , only, which is in the middle of the Picture, or, the Center, C , of its Representation, in the Center of the Picture, is a Circle; because, the Diameters perpendicular to each other are equal: all others (as of Y and Z) are Ellipses, because they are oblique Sections of Cones, whose Diameters are proved to be unequal.

N. B. It is the same however the Globes are situated; whether above, below, or sideways of the Center of the Picture. If they are equally removed from the Perpendicular EC , and equally distant from the Picture, the Globes being equal, their Representations are equal and perfectly similar, though differently situated; and the farther they are remote, from the Perpendicular, the more excentric is the Ellipsis.

It is obvious and demonstrable, that ab , the representation of the Diameter AB , is bisected in C , the Center of the Picture, and the representation of the Center of the Globe; for, the Triangle aEb is Isosceles. But, df , and gh are not bisected, in i and k , the representations of the Centers of the Globes Y and Z .

For, because the Right Lines EY and EZ (from the Eye to the Centers of the Globes) are perpendicular to the Chords, DF and GH , the Angles DEF , GEH are bisected by those Lines; because the Chords are bisected. C. 9. 1. El.

But, in the Triangles dEf and gEh , because the Angles, at E , are bisected, by the Right Lines Ei , Ek ; df and gh are cut, by those Lines, in the Ratio of the other sides of the Triangles; 3. 6. El. and consequently, $di:if::Ed:Ef$; and $gk:kh::Eg:Eh$.

† 12. 1. El.

But Eg is less than Eh ; because, the Angle gHE is acute, and Egh is obtuse; wherefore, gk is less than kh ; and also, di than if .

Therefore, df and gh are not bisected, by the Lines EY and EZ . And consequently, since they represent Diameters of the Spheres, Y and Z ; their Representations are not Circles; consequently they are Ellipses.

S E C T I O N VI

Containing a full refutation of several Errors and absurd Opinions, which many Artists entertain of Perspective; and, therefore, look on it as an imperfect and fallacious Science.

I Shall, in this Section, in the first place, explain the reason why the Representations of the Diameters of Columns, on a Picture which is parallel, or nearly so, to the Columns, are continually larger the farther they are removed from the Center of the Picture, and consequently from the Eye.

As this is a particular circumstance, which many Persons seem inclined to dispute, or, if it be admitted, they look on it as an imperfection in Perspective, I shall endeavour, and doubt not, to make it appear consonant to reason and Perspective, to their entire satisfaction. It has so near affinity to what has been said, in respect of a Sphere, that the same Diagram might have done for both; but, in order to avoid mistakes, and to keep the Ideas distinct and separate, I have given another.

Let V, X, Y, and Z be the Sections of four Columns, by a horizontal Plane, in which is the Eye, at E. Let LM be a Section of the Picture, parallel to the Columns, and EI, EK, EA, EB, &c. Visual Rays, from the Eye to the apparent Diameters of the Columns, which are still turned towards the Eye, as Globes.

Fig. 34.

It is evident, that the Visual Rays cut the Picture more oblique, the farther the Columns are from the Perpendicular EC; and, notwithstanding the optic Angles DEF, GEH are less, as the Columns are continued, their representative Diameters, df, and gh, intercepted between the Visual Rays, continually increase; and would, if the Columns were continued, till the Interval between them was lost; the Representations of their Diameters still increasing till they touch and cut each other. For, the Space from Center to Center, of the Columns, are equal, and are, consequently, represented so on a parallel Picture, if continued infinitely; what, then, can become of the Space between the Columns, if it be not added to their Diameters, in the Representations on the Picture?

I presume, no Person will say that there is any imperfection in Perspective, in this Case; I do affirm there is none in Perspective; the business, of which, is to represent Objects, truly, on a Plane; according to their Magnitudes, Distances, and Situations in respect of each other, of the Eye, and of the Picture; where, then, is the imperfection in this?

If a Person, not knowing how to choose a proper Distance, take, into the Picture, more than the Eye is capable of taking in at one View; or if, through ignorance, the Picture be absurdly situated, in respect of the Object, is the fault in Perspective, or in his Judgment? In Perspective there is not, nor can be, on the Principles here laid down, any, the least error, if the Elements of Euclid are to be depended on, upon which the whole Fabric is erected; if one falls, the other falls with it.

The Art of Perspective is to represent Objects on a Plane, by geometrical Rules, according to their Situations, &c. (See Perspective, Page 47.)

It is well known (or ought to be) that no Perspective Representation can appear perfect, i. e. it cannot truly represent the Original Object, though ever so accurately delineated, but when the Eye is in the true Point of View.

Suppose, then, E to be that Point; and AEB, DEF, &c. the Optic Angles under which the Columns X, Y, Z (being equal) are seen.

It is also, I presume, allowed, that Objects appear to have the same proportion to each other, respectively, as the Angles under which they are seen. Th. 1. S. 3. D. Vision.

But,

Plate VII. But, the Angle DEF is less than the Angle AEB; because their Subtenses are equal, $AB=DF$ (supposing the full Diameter of a Column to be seen) and the Visual Rays EB, ED, EF, &c. the Sides of those Angles, still longer, the farther they are from the Perpendicular, EC+. How comes it then, that the Diameter of that Column (on the Picture) is the largest, which is seen under the least Angle? The reason is obvious; because, the Picture, LM, cuts those Rays most oblique, where the Angle is the least; in the Points g and h, &c.

4C.14-1 El.

DEM. If EF and EH are produced till they cut a Right Line drawn through the Centers of the Columns (which is parallel to the Picture) in F and H; ED and EG cuts that Line in D and G; all which are beyond the Circumferences of the Circles; wherefore, DF and GH are, each larger than a Diameter; and, if it was not sufficiently obvious, it would be easy to prove that GH is larger than DF. But, AB is less than a Diameter; consequently, ab is less than df, and df than gh; for ah is parallel to AH. - - - 4. 6. El.

Now, if the Eye (at E) be turned towards the Column V; or, if the situation of the Picture be changed to NM, then is S, where ES cuts NM, the Center of that Picture; on which, it is evident, that, ik, the representative Diameter of the Column V, being nearest the Center, is the least; which, on the Picture LM is equal to that of Y; for, they are equally distant from the Center of that Picture; and gh, on the Picture NM, the representation of the Diameter of the Column Z, is considerably larger than gh on the other. Yet, to the Eye, at E, both these Pictures truly represent the Diameters of the four Columns V, X, Y, and Z; V and Y appear equal on both, they being equally distant from the Eye; X, the nearest, will appear the largest, and Z, the farthest, from the Eye, will appear the least.

DEM. With any radius, as ES, on E as a Center, describe an Ark of a Circle, cutting all the Visual Rays EI, EK, &c. from the four Columns.

The parts, ab, df, &c. intercepted between the Rays, are the true proportions of the apparent Diameters of the Columns, and consequently, of their Representations, on both Pictures.

But the Ark ab is the greatest, ik is equal to df, and gh is the least.

Wherefore, the Angle aEb is greater than dEf (equal IEk) and gEh is the least; and consequently, the apparent Magnitudes of the Columns, X, Y, and Z, are in the same Ratio. - - - Theo. 1. Direct Vision.

It is unnecessary to enforce this, by dwelling longer on it; as it is certain, if either of these Pictures be viewed from any other Station, they could not represent the four Columns V, X, Y, and Z, in the Position and Situation they are in.

Suppose the Eye removed to E' (the Point of View ought always to be opposite to the middle of the Picture) and, from that Station, to view the Picture NM.

Draw the Visual Rays E'a, E'b, &c. and on E' describe an Ark of a Circle, cutting them, in 1, 2, 3, &c. The slightest glance of the disproportion of their Appearance, from that Station, is sufficient conviction, that ab (which appears the largest from the true Point of View) whose apparent Magnitude is the Ark 3 4, does not appear half so large as df; and gh appears larger than df (which ought to appear the least) as the Arks 3 4, 5 6, and 7 8, sufficiently evince.

That the true, apparent magnitudes of the Columns V, X, Y, and Z, from the Point of View, E, are the portions ab, df, &c. of the Ark idh, is manifest, when we consider, that the Picture, on which the Columns are represented, is a Plane; but that, their true Appearances can only be represented on a spherical Surface; i. e. on the Surface of a Sphere, the Representation and the Appearance are the same, the Eye being in its Center.

Does any one imagine that there is a real Arch in the Heavens, which has that Appearance? in which, the Stars, &c. appear, equally distant from the Eye in its Center. The Celestial Globe, for instance, is a Picture of the Heavens, Planets,

Stars,

Stars, &c. each Representation of a Star, on its Surface, if they are truly depicted, would, to an Eye in the Center of the Sphere, exactly coincide, and be in the same Right Line with its Original in the Heavens; and, their apparent Distances, from each other, are measured by an Ark of the Sphere; whereas, their real Distances from the Eye and from each other, respectively, have not the least affinity to their Distances, as represented on the Surface of the Globe or from its Center. Wherefore; to an Eye at E, the true Point of View for either Picture (LM or MN) each Diameter, being seen under its true Angle, and the same as its Original, will appear less and less, the farther they are distant from the Eye, or from the Center of the Picture; although their Representations are continually larger and larger, on those Pictures, as the original Columns recede.

And this will ever be the case; on a Picture parallel to the Columns, in some degree, at any distance of the Picture; but, at a proper Distance, for taking in the whole, the difference is so little, and that still less as the Distance is increased, that, it is and ought to be despened with, by making them equal: but, I must observe, it is not, then; true Perspective.

2. Methinks I hear some carping Critic say, I must allow, then, that Perspective is somewhat defective; by no means; I have not yet given up the Point in debate. I say, that, although, at any Distance, there must and will be a difference, though scarce perceptible at a proper Distance, yet I would never advise a Person, who would represent a row of Columns, in full Front, to make the least difference in their Diameters; for, since they support an Entablature, which is represented perfectly horizontal, their Pedestals or Bases the same, consequently parallel, it would be very improper to make the Columns differ in width when they are equal in height, for this reason; because it is impossible to confine the Eye to the true Point of View, always; from which if you deviate, ever so little, the whole Representation is distorted and imperfect.

But, if it were possible to confine the Eye, I would not step the least aside from Perspective, on any account; let the Representation be ever so distorted and preposterous, it will, and must, if truly represented (by the Rules hereafter prescribed) appear to the Eye, in the Point of View, as the Original. If we may be allowed to take liberties, in any Case, where shall we draw the line between the perfect and imperfect Representation? for the whole is more or less distorted; consequently, the Rules of Perspective are not to be depended on, at all. To what, then, must we have recourse? the Eye is not, in many Cases, a competent judge; we should, if we follow its dictates, implicitly, have as many Points of View, in a Picture, as Objects; because, if every Object be represented exactly as it appears to the Eye, on the same Plane or Picture, there can be no point of View for the whole; and consequently, in a long, connected, and continued Object, composed of Planes and Right Lines (as in Buildings of any kind, of regular Architecture) it would be beyond the power of Art to connect the several detached pieces or projectures so, together, as to compose one entire and uniform Picture of the whole.

Notwithstanding an ingenious Author has treated it ludicrously, in a supposed Dialogue between a Lady and an Artist, who was determined to abide by the Rules of Perspective; his Argument has not the least weight, and must be imputed to him, as not having a right notion of Perspective. The Eye is considered as a Point; therefore, whether we suppose it confined to a Pin-hole or not 'tis the same; for he must allow (or he was very unfit to write on Perspective) that there can be but one Point of View for a Picture, in which it can be perfectly seen, as intended by the Artist; consequently, when the Eye is not in that Point, the Objects must necessarily appear more or less distorted, according as they are situated nearer to or farther from the Center of the Picture, or that Point which is opposite to the Eye.

3. I shall, however, for the sake of the Argument, allow each Object to be truly represented, as they appear to the Eye, on a Plane Picture of a tolerable length, as a h: Globes are the fittest Subject to expatiate on, because they are every way the same.

A 2.

Let

Fig. 33.

Plate VII. Let *C* be the Center of the Picture, the Eye being at *E*; and therefore, the Picture may be supposed to be extended equally towards *a* as to *h*.

Now it is certain, that the three Globes appear round, let the Eye be situated where it may; but they cannot, or ought not to be so represented, on the Plane *ah*, to be viewed by the Eye at *E*; for if they were, they would not appear round, but gibbous, or Egg like, standing erect, all but the Globe *X*, in the Center; and the more so, as they are farther removed from it; the Eye must be removed opposite to each, and consequently, there would be as many Points of View as there are Objects, which is an absurd Hypothesis, in one Picture.

Suppose, from the Point of View *E*, I would represent the three Globes, *X*, *Y*, and *Z*, as they appear; that is, the outline of each to be a Circle; the Eye, and its Axe, *EC*, must be turned towards *Y* and *Z*, as *EY*, *EZ*; and consequently, the Picture is turned with it, into the Position *bl*, and *lm*. For, the Picture must be perpendicular to the Axe of the Eye, if the Object be represented as it appears; in which Case, there are three distinct Pictures, viz. *ab*, *bl*, and *lm*; each having a distinct Center, and the same Distance, *EC* or *Ec*; on which Pictures, each Globe is represented by a Circle, and the farthest from the Eye (*Z*) is the least in its Representation, as they really appear.

It must be obvious, if the Globes were represented so on one Plane (of which *ah* is supposed a Section, and *C* its Center) that they would be under a worse predicament than those represented strictly perspectively: such a Picture could not appear true in any Point of View, whatever, every Object having a distinct and separate one.

Suppose them represented on the Plane *ah*, as they appear to the Eye, at *E*, i. e. round; the Representation of the Globe *Y* less than that of *X*, and, of *Z*, less than *Y*, as they are represented on *bl* and *lm*; would they appear in the proportion they are represented, at *E*, or in any other Point? No, certainly; for, at *E*, the Representations would appear less than the Original Objects, and not round (except *X*, only) *Y* would appear more round than *Z*, and *Z* would also appear rounder than any other, more remote from *C*; but they would, all, except *X*, appear elliptical Spheroids; and not Globes (which is obvious, to any Person tolerably acquainted with Optics or Direct Vision) the farthest, from *X*, still more so than the last.

Now, let the Eye be removed to *E*, or opposite to *Z*; *Y*, or *Z*, would then appear round, but not of the same proportion, as from the Point of View *E*; but, at *E*, neither *X* nor *Z* would appear round; and, although equally distant from the Eye, *X* would appear much the largest; where, then, in this Case, must the Eye be placed, to see those Representations such as the real Objects appear? There cannot be a Point determined, for each Representation has a separate Point of View. Can, then, the Picture *ah*, in this Case, be a true perspective Representation of the three Globes, *X*, *Y*, and *Z*, as they appear to the Eye? certainly no, but each Representation, on the Plane *ah*, is as much a distinct and separate Picture, as the three Pictures, *ab*, *bl*, and *lm*; the difference is only, that the three Pictures have but one Point of View, and the Picture *ah* has three, equally distant from it; by supposing the three Pictures placed in a Right Line, *ah*.

4. If what I have here advanced be not sufficient, to divest those Artists of their absurd notions of Perspective, I shall give them one observation more, which they have not, perhaps, considered with that attention it requires; and then leave them to pursue their own way, if it appears to them more eligible and reasonable, and will produce a better and more agreeable Representation, of any Object whatever.

They quarrel and find fault with Perspective, but without reason; because, it is an infallible and most perfect Science. They would have all Objects represented, in Perspective, exactly as they appear to the Eye; there is no such thing to be done; 'tis not in the power either of Art or Science to represent, on a Plane, any single Object, except a Sphere, or a plane Circle, having the Eye opposite to its Center, and the Picture parallel to it, as it appears; and yet, Perspective will give a true and just Representation of every regular Object.

The

The real, and the only cause of all their errors, and false notions of Perspective, is their not rightly distinguishing between the Representation of an Object, on a Plane, and the true Appearance of it; two distinct things, which can never be united, on a Plane Surface or Picture.

The perspective Representation of any Object is the section of the Cone or Pyramid of Rays, by a Plane; and the Appearance of an Object is the section of the Rays by the surface of a Sphere, only; to which, every Visual Ray, from the Eye to the Object, must be Perpendicular; consequently, the portions of the Arks, intercepted between the Visual Rays, measure the Optic Angles, under which, every part of the Representation is seen (the Ark *a*, or *a, db*, Fig. 33 and 34, may be supposed a section of a Sphere, cutting the Visual Rays *EA*, *EB*, &c). It is therefore manifest, that the true Representation, and, at the same time, the true Appearance, can only be represented on a spherical Surface, the Eye being in its Center; but that is not a perspective Representation.

5. I expect it will, again, by some Persons, be alledged, here, that Perspective, then, is not sufficient, to represent Objects as they appear to the Eye. I affirm that it is. Let us, therefore, once more enquire, candidly, what is meant by Perspective, and what effects it is expected to produce.

Perspective is a representation, on a Plane, of an Object, or Objects, in a fixed and determined Position and Point of View. (See Perspective, Page 47.)

I have already shewn the bad effect of viewing a perspective Picture, out of the true Point of View; from which, if we deviate, we cannot expect that the several parts of a Picture can vary their Bearings and Proportions, to each other, as the real Objects; no, certainly, that must be a reality, not a Perspective; which is but a Deception, a Representation of a real Object, on a Plane; and, which, can never represent the Object, truly, from any other Point of View, but that for which it was delineated.

I have also shewn, to ocular conviction (by the Apparatus) that there may be as many various Representations of the same Object, from the same Station, or Point of View, as there can be Positions of the Picture; all which, are true Perspective, and will affect the Eye, alike, in the true Point of View; which, is the Vertex of the Pyramid of Rays; seeing that, every corresponding Line, on each Picture, is seen under the same plane Angle as the Original, and every Surface, as well as the whole Object, is seen under the same solid Angle, or Pyramid of Rays, as the corresponding original Surfaces, or as the original Object.

What, then, is it we are caviling about? would ye have a real, solid Object on a Plane, or a Representation of it only, in a certain Position? Certainly then, if the Eye be not in the true Point of View, the Picture does not truly exhibit an appearance of the intended Object, at the fixed Station; and, although it may be a just, perspective Representation, it may, nevertheless, be a very distorted and disagreeable Picture; not owing to any fault, or imperfection in Perspective, but to the choice of the Situation of the Picture, or the Distance and Position of the Object and the Picture.

Does not almost every Object, except a Sphere, appear different from every different Point of View? and can any Person be so unreasonable as to expect, that a Representation of a solid Object, on a Plane Surface, can appear truly to represent the Original in any other Point of View, but in the Vertex of the Pyramid of Visual Rays, under which the Object itself is supposed to be seen? the very supposition is absurd to the last degree; because, no two parts of the Representation can, at the same time, be seen under the true Optic Angle, in any other Point; consequently, the Representation, must appear erroneous.

6. Now, although the true Representation, and also the true Appearance are depicted on a spherical Surface, yet, I affirm that such a Picture is subject to much greater imperfection than a true Perspective, on a Plane Surface; because, if the Eye be the least removed out of the Center, the whole Appearance and Effect is destroyed,

Plate VII. destroyed, and exhibits a much worse Image of the Object; than a perspective Representation, on a plane Picture, can possibly exhibit, in any Point of View; which is so very obvious, that it is needless to point out the reason: For, suppose the Eye, at *E*, viewing the spherical Picture, *adb*; the Visual Rays, *Ea*, *Ed*, &c. shew, at first sight, how much worse such a Picture must appear, than that on the Plane, *ah*, from the same, though false Point of View; not one of them will appear round, and the Appearance, of all, is preposterous:

From the circumstance I have mentioned, in respect of the true Representation and Appearance being depicted, at once, on a spherical Surface, some Artists imagine, that the Representation on a Plane ought to be so delineated; it cannot be; 'tis impossible, in the nature of things: Suppose a true Representation of a long Building, in full front, delineated on a spherical Surface, and it were possible, afterwards, to reduce the spherical Surface to a Plane; is any Person so weak as to suppose, that such a Representation would appear like the Original, in any Point of View? he must be weak, indeed, and have strange mistaken notions of Perspective, who can; and yet I have heard this Point strenuously supported, or rather argued for, supported it could not be; for, any thinking Person (who can think with propriety about it) must be sensible, that, what should represent Right Lines will be curved, and, the whole, will give the Idea of a Rotunda, or externally round Building; seeing that, the extremes would fall off, not in Right Lines but curved, and they would appear less than the real Object; to say nothing of the almost impossibility of producing or delineating such a Picture, at all, or by any means; I should be glad to be informed, how, or by what Rules.

7. There may be alledged another circumstance, which, I think, must convince the most obstinate, in Perspective. I am persuaded, that, if the Eye be fixed in a Point, at a proper Distance from a transparent Plane, placed between the Eye and an Object, whilst the Hand traced, accurately, every Line of the Object, as it appeared on the transparent Plane; such a delineation, all must allow, would be a true one. Let those, who are not otherwise to be convinced, try the Experiment. I will stake all my knowledge in Perspective, that every Representation of a Right Line is a Right Line, on the Plane; that Columns or Cylinders of equal magnitude, and parallel to the Plane, will be larger as they are more remote from that Point, on the Plane, to which the Eye is opposite; that the Representation of a Circle or Sphere, seen oblique, is an Ellipsis; that Objects of equal magnitude, and equally distant from the Picture parallel to them, however otherwise situated or elevated will be represented equal; with various other circumstances; all which may be fully proved to ocular conviction, which will not admit of the least doubt. Surely then, if by the rules of Perspective the same thing be effected, which certainly will, in every respect, it must exhibit true Representations of Objects.

It has been observed, that the business of Perspective is to produce the Figure of a section of the Cone or Pyramid of Rays, by a Plane, in any determined Position; which, if the Rules it prescribes be truly followed, it will most certainly effect, without any sensible error. For, wherever any Point, or Angle of an Object, appears on a Plane, or other Surface, between the Eye and the Object, there the Visual Ray would cut and pass through the Plane; but when the distance of the Eye is such, that the Visual Rays cut the Plane very oblique, or in Angles, nearly, or perhaps less than, half Right ones, the Representation will consequently be distorted and preposterous, and, in other Points of View, will have a disagreeable and unnatural Appearance.

Here, then, lies the mistake, which, through ignorance or inadvertency, is attributed to Perspective, and supposed to be a deficiency or imperfection in it, 'Tis, generally, in the Point of View, the Situation, &c. of the Picture or Object; which, by being too near the Eye, occasions that Distortion and preposterous Representation we perceive, in several Pictures; for, if the Optic Angle, under which the whole Picture is seen, exceeds 50, or at the most, 60 Degrees, the Distance is not sufficient; as, the Visual Rays will cut the Picture very oblique, near its extremes,

and occasion a disagreeable distortion of the Objects on the extreme parts of it. Yet, as I have observed, at any Distance, the representations of the Diameters of Columns, or other cylindrical Objects, on a Picture parallel to them, will, in true Perspective, ever be the least which are nearest to the Center.

8. I shall now take notice of another great difficulty, which seems to be a stumbling Block to many Artists; who, one would imagine, would not hesitate one moment, to determine about it with propriety; which is, to represent, on a vertical Picture, the appearance of a direct Descent; which, Mr. Kirby has affirmed impossible, in the nature of things, to be done; that it is a strong instance of the insufficiency of Perspective, and that, we must have recourse to Experience, only, in such Cases; intimating, that it is not possible, by the rules of Perspective, to give the Representation of a descending Plane; which is so ridiculous an assertion, that, any Person, who understands Geometry tolerably, will easily be convinced of the contrary.

For, first, we are to consider whether the Descent (which I shall suppose a Plane) is perceivable or not. If this descending Plane can be seen, at all, from any fixed Station, it may, undoubtedly, be represented on the Picture, from that Station, by the strict Rules of Perspective, or there is no truth in Perspective; either it is a perfect and infallible Rule, or it is no Rule at all. If the Plane can be seen, it is a subject of Perspective; if it cannot be seen, it is no subject for a Picture; which needs no Demonstration. To tell us, what descends, and we actually know to go down-hill in Nature, will, if ever so correctly drawn, appear to rise, upwards, on the Picture, is saying nothing to the purpose, the expression is vague and nugatory; for, if the Plane descended so much as not to appear to rise on the Picture, it could have no place or representation thereon; but if it can be seen at all, it must, necessarily and unavoidably, appear to rise; or rather, it must, really, rise on the Picture, for, the Appearance is to descend.

And yet, he says we must have recourse to Experience. What kind of Experience can teach us to do, what is impossible (as he says) in the nature of things, to be done? Experience will teach us all that can be done; Experience, in Perspective, will teach us to delineate regular Objects which we never saw, yea, which never existed; but no Experience whatever, can make us perform impossibilities; to represent Objects which are lost to sight in the Originals, from the determined Station. But, how can any Person be so unreasonable, as to require that to be done, in a Picture, which is so liable to false constructions, or determinations, in Nature; for, a level, or horizontal Plane will appear either to ascend or descend, by the affinity and concurrence of Lines and Objects connected with them; as for Example. When we are going down a regular descent, towards a level Plane, of Land or Water, the greater the descent, the more the level before us will appear to rise; yea, so strong is the Deception, that we are not easily convinced of the contrary; except it be Water, before us, which, we are certain, cannot ascend. Likewise, when we are going up a gentle ascent, towards a level Plane, which is in sight, it will appear to descend; more especially when we see the boundary Lines of both, as in regular Walks, in a Garden, &c. Nor is there any thing surprising in it, if we consider, that, in the former Case, it is owing to the much greater breadth which we see, of the level Plane, and having the same appearance as an Ascent, before us, when we stand or move on a level. In the other Case, the contrary effect, in appearance, is the cause of the Deception.

To draw two parallel and horizontal Lines across the Picture, and to give an Idea, that the space, between them, represents a descending Plane (of a certain length) without shape, bounds or limits, sideways, or any Object situated on the inclination, to bias the judgment, is indeed impossible; but that is giving too great latitude to the meaning of the Expression. I say, it is absolutely impossible, in this Case, to give, by Art, an Idea of a descending Plane, simply as such. For, although a proper gradation of Light and Colour may make a small space, on the Picture, appear to represent a great length, yet, 'tis not possible to say whether it descends,

Plate VII. or represents a level Surface; because, the same space, on the Picture, may represent an equal length, on a declining Plane, to a Spectator standing, as is requisite to represent a level Ground, to a Person laid along, or sitting; consequently, it is not possible to say, with absolute certainty, whether it represents one or the other.

And yet, I question if a skilful and ingenious Painter, in aerial Perspective (who had critically observed and copied Nature) might not, by the effect of Colour, simply, even in this Case, deceive the Eye, and give the appearance of a Descent. But, if there are Objects situated on the inclined Plane, or, if the shape or figure of the Plane, itself, is to be described; whatever can be seen of such Objects (whether Tops or Bottoms it matters not) they may and can be represented, truly and exactly, by the infallible Rules of Perspective; and that, on the same invariable Principles, as the most common and ordinary Cases, whatever, are subject to.

To illustrate what I have advanced, by a simple geometrical Scheme, will not be very difficult; which may be considered as a vertical Section through the whole. Each Plane is, therefore, represented by a Right Line, making the true Angle of the Inclination of the Planes (as I shall call them) with each other.

Fig. 35.

Let AF represent the Plane of the Horizon, and AB a Descent, as a slope Bank, or any other Declivity; making the Angle ABG with the Horizon. Let CD be a Section of the Picture, E the place of the Eye, and EF its Altitude above the Horizon, at the Distance EC from the Picture, whose Center is at C .

Now, if the Eye (at E) be so situated, that a continuation of the inclined Plane would pass through the Eye (as BAE) it is evident that the Plane (AB) from that Station, is totally lost to sight; its whole Representation being the Line of its Intersection with the Picture†.

† Cor. 2.
Theo. 2.

Consequently, if the Eye be moved further back (as at E') its appearance, or place on the Picture, would descend; which cannot be, unless the Plane be considered simply, a Plane, without substance, and seen on the under side; its apparent width from that Station is a D ; but if the Eye be removed, nearer to the Picture, in any direction above BAE the Representation of the Plane rises on the Picture. At E' , its width on the Picture is ab , as the Rays, $E'A$, $E'B$, evince. If the Eye be moved back again, in the direction BE' , to E'' , it is plain that, b , the apparent height of B , remains the same, wherever the Eye is situated in that Line; but the representative width, ab , of AB , will be increased, towards D , as it recedes, if the Picture be at any distance from the Descent. Suppose the Eye brought forward again to E' ; the width of AB from that Station is ab ; and, if the Plane was continued infinitely, a V is its whole apparent breadth, $E'V$ being parallel to AB .

I presume, this is intelligible and clear, hitherto. I will, next, shew that 'tis impossible for the Eye to judge, merely from the width of its Representation on the Picture, whether the Plane be descending or ascending, or perfectly on a level.

From the situation of the Eye at E' , the representative width, on the Picture, of the Slope, AB , is ab ; whereas, if the Plane was more inclined to the Horizon, i. e. if the Descent was more gradual, as AI , it would represent a less space; if it was horizontal, the same ab represents the length AH , only; but, if the Plane ascended from the Picture, as AK , it still represents a shorter length (AK) on that Plane (the Arks li , Hh , &c. shew how much each is shorter than the other) and AL , making right Angles, BLA , $AL E'$, with the Visual Ray $E'B$ is the shortest length, from the Point A , that can be represented, by ab , from that Station, situation, and position of the Picture.

Now, if any Object (as X) be situated on the Inclination AB ; it is evident, if a continuation of the Plane of its Top (MN) passes above or through the Eye, at E' , it cannot be seen from that Point; whereas, if the same, or any equal and similar Object be situated on the Horizontal Plane AH , the top, OP , may be seen from that Station, notwithstanding the Object is more elevated; because, a continuation of the Plane of the Top, OPQ , falls below the Eye, at E' .

The figure of the inclined Plane, itself, or the figure of any Object situated upon it, is described, perspective, in the same manner, and on the same Principles as on a horizontal Plane; which is exemplified in the practical part of this Work.

10. In the last place, I shall demonstrate, that the representations of Objects, which are elevated perpendicularly, above the Horizon, have the same proportion*, on a vertical Picture, as those of the same Magnitude, situated on or near the Horizon; the Object being parallel to the Picture.

It is a mistaken notion which several Persons entertain, that the parts of a Building, which are elevated high above the Horizon, appear to diminish in a greater ratio than those which are extended horizontally; such an opinion may be easily refuted. If the second Part of the 9th Theorem be well considered, it is sufficient refutation.

Let A G be supposed a high Obelisk, and A B, B D, &c. several equal Divisions thereon. Let E S be a Spectator, E the Eye, and a g, or a g, a Section of the Picture; which, being vertical, is parallel to the Object A G.

Fig. 36.

Now, if the Visual Rays E A, E B, &c. be drawn, they will cut the Picture in a, b, d, &c. then is f g, the representation of F G, equal to a b, the representation of A B, or to b d, the representation of B D†.

† 4. 6. El.

For, since the parts A B, B D, &c. of the Original Object, A G, are equal, the representations of them, on the Picture, being parallel to the Object, are also equal. But, A B is an Object, direct before the Eye, at E, on the Horizon A S; and, F G is one of equal length, elevated greatly above it; therefore, the Representations of equal Objects at an equal Distance from the Picture, and parallel to the Picture, are equal. Q. E. D.

The same Object may be considered as a Building, extended horizontally, and A E G a horizontal Plane, in which is the Eye, and also the Visual Rays E A, E B, &c. a g, or a g, is a Section of the Picture, as before. The intersections of the Visual Rays with the Picture, it is evident, are the same.

11. There is yet another point of controversy I have some times been entertained with; which is, that, when we stand, opposite the middle, near a long Building, or range of Buildings in a Right Line, the horizontal Lines, in the Cornice, &c. appear to decline towards either End, yet make no Angle; and therefore, they imagine that the representation of those Lines, in such Case, will be curved. What a poor Idea must a Person have of Perspective, who advances and is really prepossessed of such an Opinion.

I shall say very little on this Point, because, the eighth Theorem is full and perfect Demonstration, that the Representation (on the Picture) of every Right Line, is a Right Line; and, it is so if extended infinitely. Because, a Plane may be supposed to pass through any Right Line and the Eye; and, the Intersection of this imaginary Plane, with the Picture, is the indefinite Representation of every Line it passes through; for, the Eye being in a continuation of it, the whole Plane is lost to sight†. Therefore, the Representation of every Right Line is a Right Line.

† Art. 3.
of a Plane.

The truth, of which, any Person may soon be convinced of, by applying a perfectly straight Ruler, before his Eye, parallel, or otherwise, to a Right Line, in an Object, of any length, and imagine the edge of the Ruler to cut the Picture, which is a Plane; then certainly, if the Ruler coincides with the Original Line, from one end to the other (which it undoubtedly will) the Right Line, in which it is supposed to cut the Picture, is the Representation of the Original Line. It is the same as if a transparent Plane was placed between the Eye and the Object, and any Right Line, in the Object, traced on it, exactly, whilst the Eye is fixed in a Point; the Line, so described, will be a Right Line.

Those Persons never consider (but 'tis plain they are not furnished with the means) that the Picture, being placed in the true Point of View (consequently at its proper Distance) will appear the same as the Original; for the ratio of the Parts is always in proportion to the Distance. Consequently, every part of the Picture, being seen under the same Angle as the Original, will have the same Appearance, in every respect; and consequently, the Right Lines, on it, will appear to decline either way, or both ways, the same as in the Object.

* What I would signify by the proportion of Objects, in this place, is, simply, length and breadth.

12. Respecting the appearance of parallel Right Lines direct before the Eye, there is something paradoxical, which is not easily reconciled to reason; for it is manifest, from the foregoing, and the eighth Theorem, that every Right Line appears a Right Line, when the Eye is directed to it; and yet it is certain, that parallel Right Lines, however situated, appear to converge, and consequently to approach each other.

Suppose two, parallel, horizontal Lines, direct before the Eye. It is manifest, that their Representations, on a Picture parallel to them, are also parallel Right Lines. Notwithstanding which, it is certain, that the nearest Distance subtends a greater Angle, at the Eye, than any other; and that, the Angle subtended, by their perpendicular Distance, is continually less, the farther they are extended on either side. How, then, can they appear Right Lines? seeing that, they appear wider asunder, direct before the Eye, than towards their extremes; nevertheless, since a Plane may pass through each Line and the Eye, they must necessarily appear Right Lines, and consequently parallel; seeing they subtend equal Angles, at equal Distances from those parts which are nearest to the Eye.

This very extraordinary Phenomion may be better conceived, by imagining the Eye in the Center of a concave Sphere; and, imagine two great Circles, which are Meridians, or longitudinal Circles, making an Angle of 15 or 20 Degrees, or more. Now, the Eye being in the common Center of both Circles, each appears a Right Line, which way soever the Eye be turned, towards the Circumference. But, the Meridians intersect and cross each other, in the Poles; which, being diametrically opposite, cannot both be seen at the same time; but, in looking towards either, the Circumferences of the Circles will appear like two Right Lines, converging to a Point; the Pole, in which the Circumferences intersect.

Now, suppose the Eye directed to the Equator of those Meridians; and imagine two equal Chord Lines drawn, one in each Circle, at equal Distances from the Eye, and parallel between themselves. Those Chord Lines, it is manifest, will appear to coincide with the Circumferences of the Circles, the Eye being in their common Center; and would, if they were continued beyond the Circles, infinitely. But, it is certain, that the Circumferences, being meridian Circles, cross each other twice; wherefore, the parallel Lines, one in each Plane of the Circles, will appear to converge to a Point towards each extreme, which are their Vanishing Points; and, which, must be infinite, before they can appear to coincide with the Poles of the Circles. Thus, it is evident, that Right Lines do always appear Right Lines, and are represented by Right Lines; notwithstanding which, parallel Right Lines appear, when direct before the Eye, to converge to a Point, towards each extreme.

I have now, I hope, fully refuted those truly ridiculous and absurd Opinions, of Perspective, which many have imbibed, and are not easily divested of; they, rather, obstinately persist in them, without being able to give a solid reason for their Opinions, and are determined not to give up their Prejudices, at any rate, right or wrong. What strange infatuation must possess that Person, who, having no argument of weight, to support his false notions of Things, has recourse only to Sophistry; and, because he cannot come at sterling Truth, himself, imagines there is no such thing to be found. In points of natural Philosophy, &c. where no certain Criterion can be obtained, to fix our assent, it is no wonder that we meet with so many, widely different, Opinions; and, though few of the arguments advanced have the least foundation in Reason, 'tis amazing with what eagerness and warmth each Assailant attacks his Opponent. But, in Sciences, purely mathematical, all must agree; when Truth appears, there is no resistance can be made; we cannot withhold our assent so prevalent is her influence.

To such as are open to Conviction, and are desirous of coming at Truth, I think I have said enough for their conviction, on the Points debated; but if they are not, all that can be said is to no purpose, 'tis waste of Time and Words. I shall, therefore, leave them to enjoy their Opinions, and please themselves with their great Sagacity; and proceed to lay down certain and infallible Rules for the Practice of Perspective, deduced from a perfect and well founded Theory; which, if truly followed, and proper attention be given to the Lessons, contained in the Introduction, will most certainly produce Harmony and true Effect, of any regular Object, so far as comes within the province of linear Perspective.

Fig. 22.

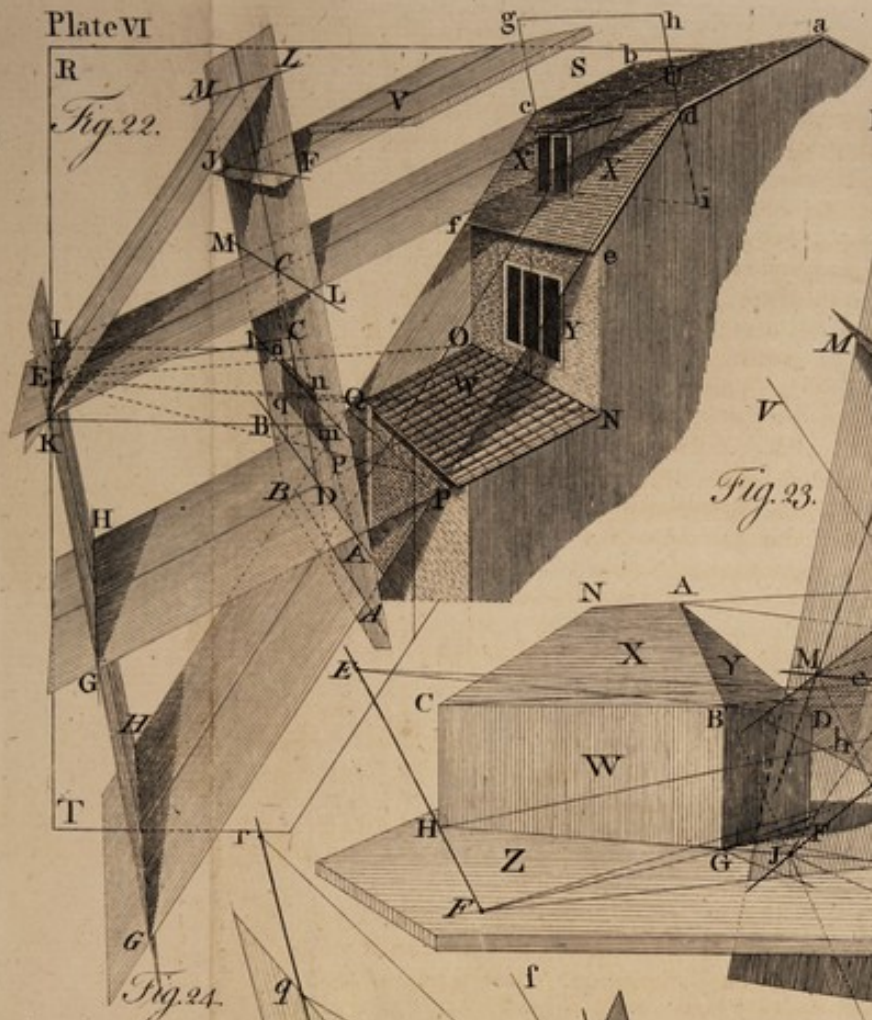


Fig. 22. o

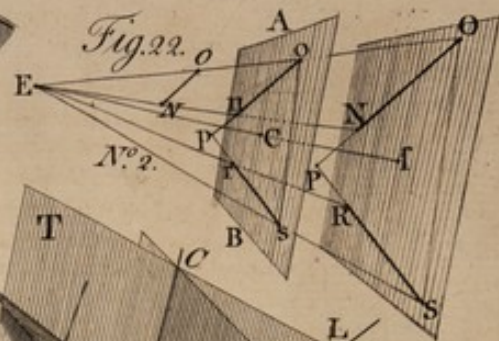


Fig. 23.

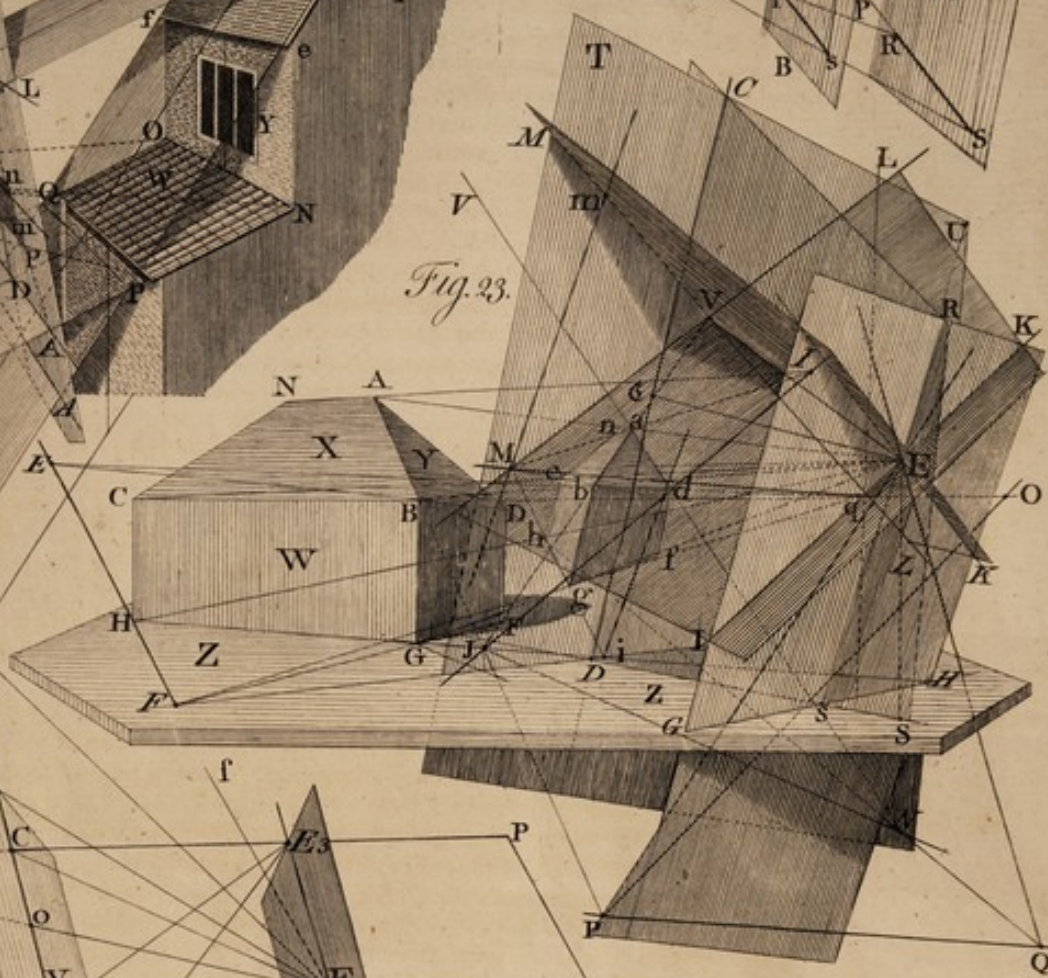


Fig. 24.

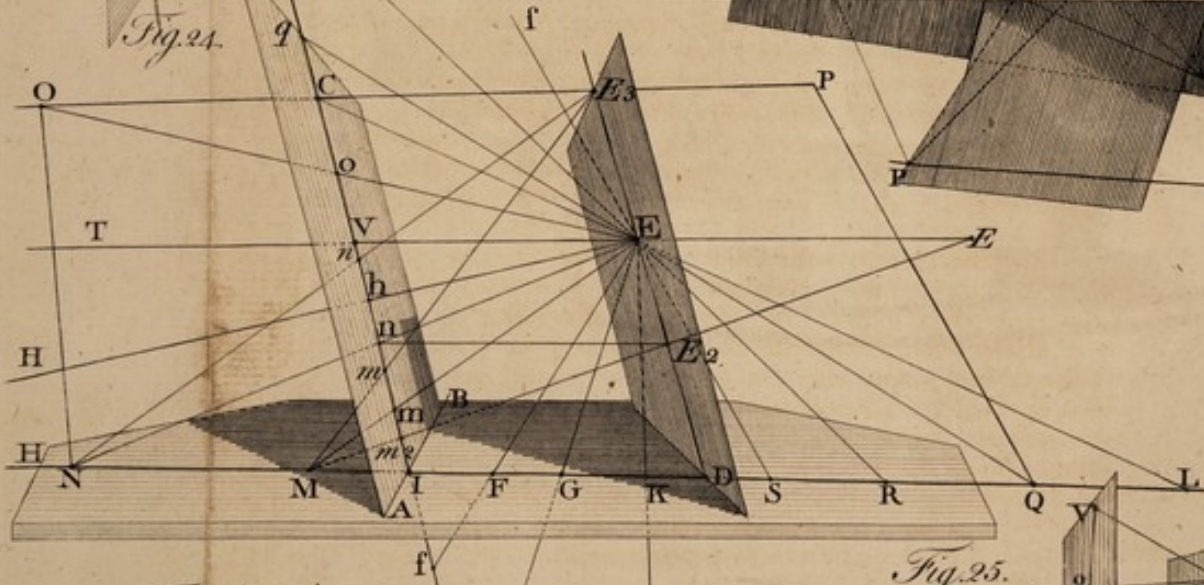


Fig. 26.

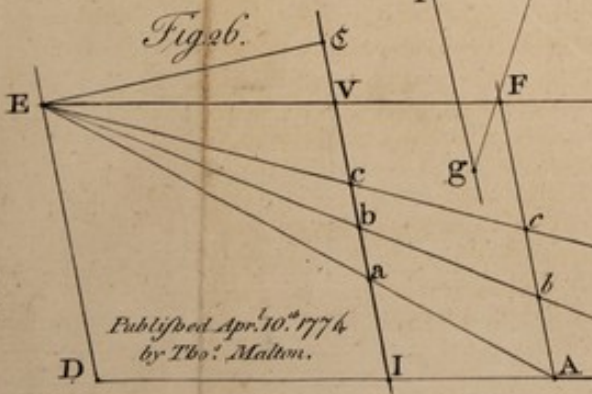
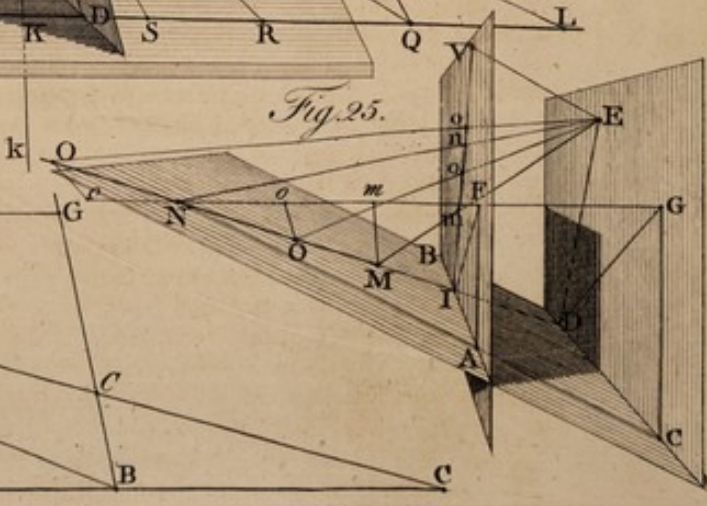


Fig. 25.



*Published Apr. 10. 1774
by Tho. Malton.*



Fig. 27.



Fig. 28.

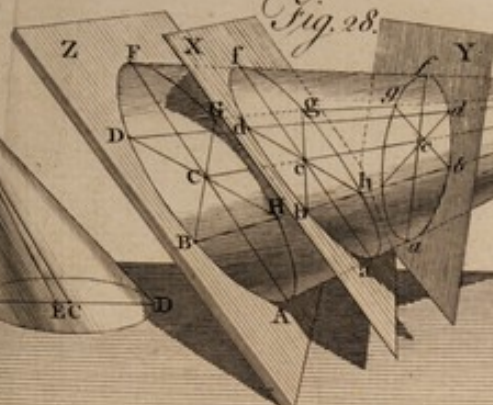


Fig. 29.

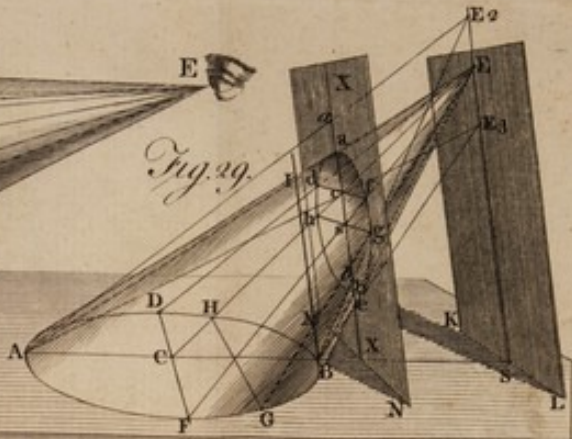


Fig. 30.

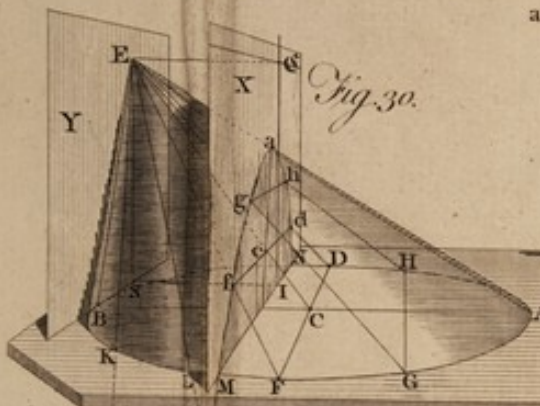


Fig. 31.

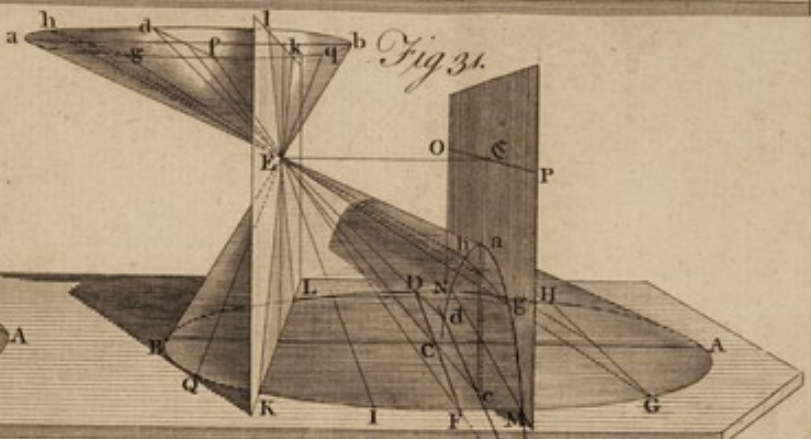


Fig. 32.

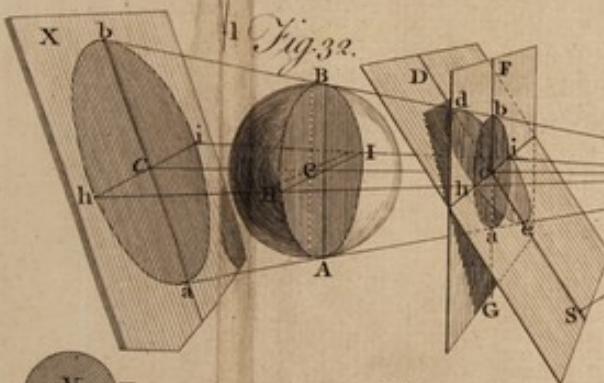


Fig. 33.

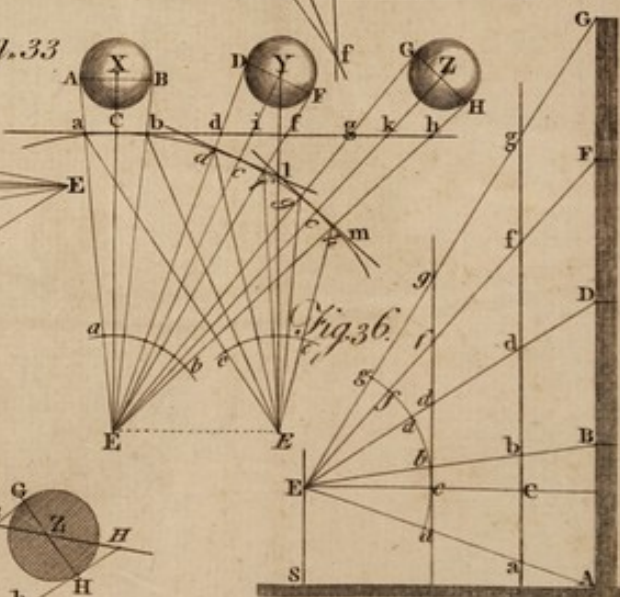


Fig. 34.

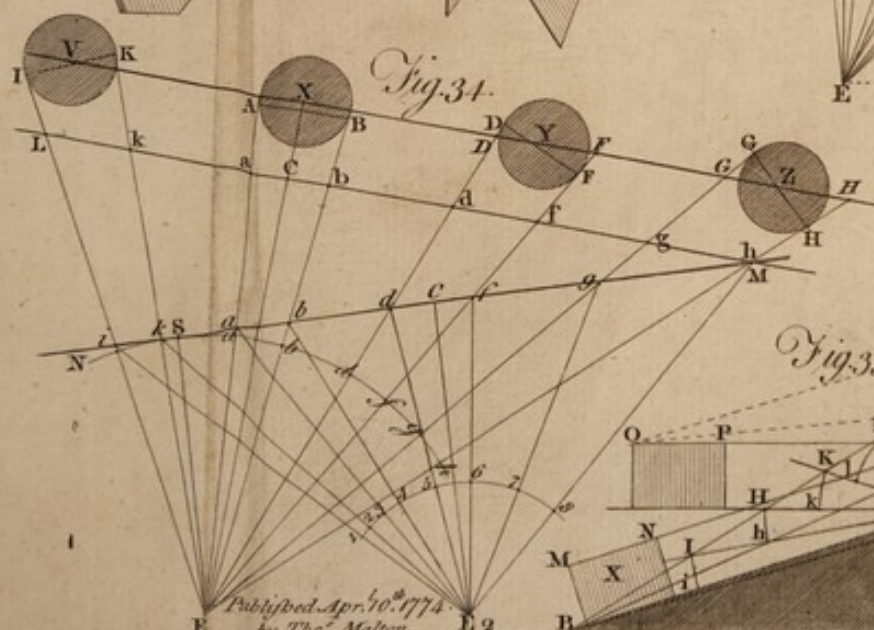


Fig. 36.

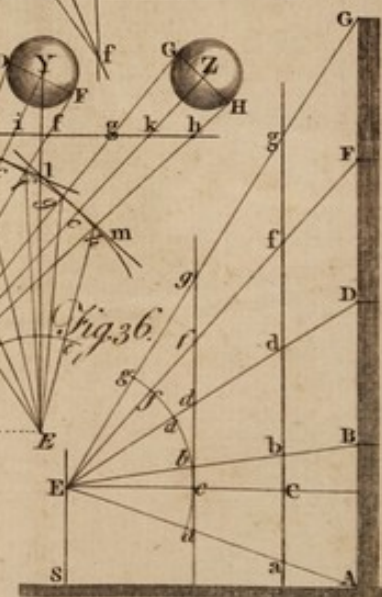
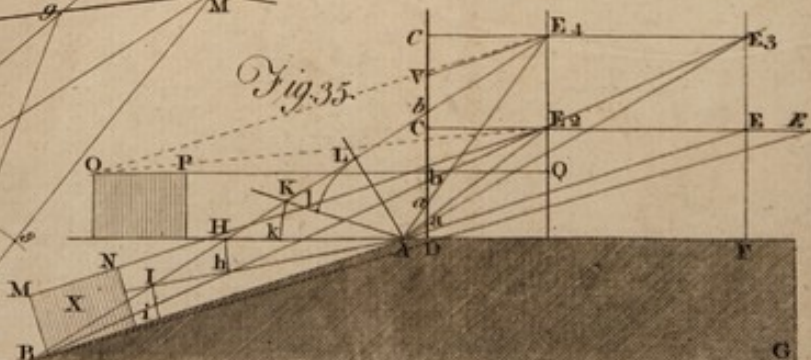
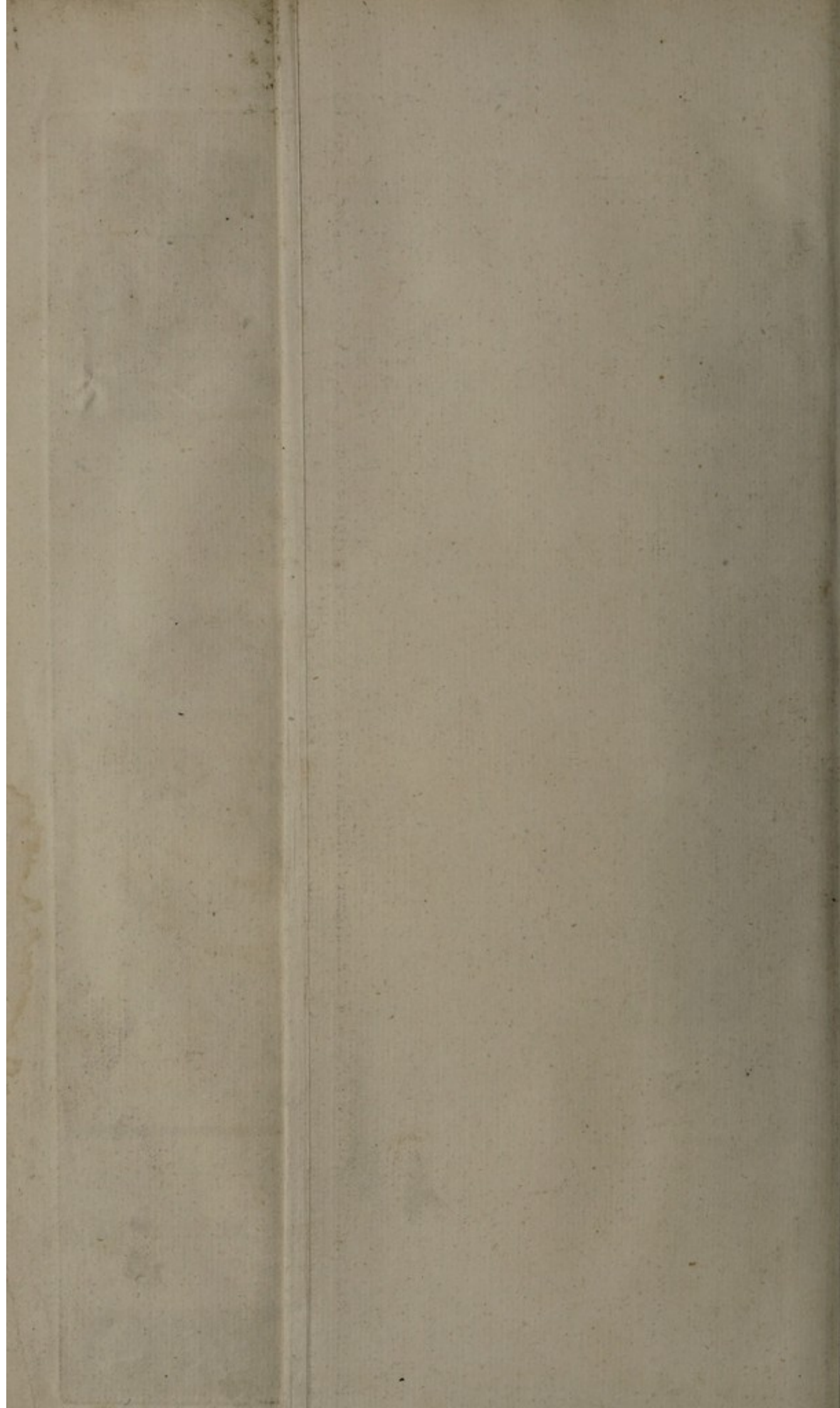


Fig. 35.





B. O O K III.

Of the Practice of PERSPECTIVE.

S E C T I O N I.

An INTRODUCTORY PREFACE.

I Come, now, to the practical, and, in that, the useful part of Perspective; to which, the foregoing Book is an Introduction, only, but a very necessary Introduction; inasmuch, that, without the knowledge inculcated by it, we should proceed in ignorance and uncertainty. Nevertheless, those Persons who have not studied Geometry, and have not, now, perhaps, either leisure or inclination to study it (though, in my Treatise of Geometry, they will find it neither so abstruse, tedious, or dry a Study, as many look on it to be, having treated that most useful branch of Science in a more familiar and intelligible manner, than has been done heretofore) I say, that, without a sufficient fund of Geometry, to perceive and be clear in the Demonstrations, if they will but treasure up, in memory, the Theorems and Corollaries, and take all for granted (as they may depend on the Truths contained in them) they will find the great advantage of it, in Practice.

Every branch of Science is in two parts, *viz.* theoretic and practical. Theory teaches the knowledge of all that is necessary for Practice, in Speculation; the other applies that Knowledge to real use. It is necessary, first, to know how, before we can perform any thing; but notwithstanding, many Persons may be said to know Perspective (as a mathematical Science) yet know not how to apply it to Practice, with success. So, every Art, dependant on Science, may be acquired without the Theory of it, by custom or habit; as in mechanic Trades, derived from Geometry and Mechanics; all which, require time and application, to become familiar to us. So likewise, Perspective may be practiced, without being said properly, that we understand it; seeing, we may not be able to give a sufficient or satisfactory reason for the effects of it, in any instance. And, being well versed in the Theory, we shall nevertheless find, that it will require time and assiduity to apply it, in all Cases that may occur, in Practice, although founded on the same invariable Principles, without familiar Lessons, in every Case, being given. But we shall, certainly, with the assistance of Theory, be able to comprehend, and to understand the various methods of applying the Rules, better than without it; as very little reflection, when we are at a loss, will set us right again, and enforce the universality of its Principles.

It is not, to be wondered at, that the best of Treatises on Perspective has been the least understood; *viz.* that, by Dr. Brook Taylor; because it is very obscure. I call it the best, on account of his general Principles, not from its real utility to Artists, in respect of practicing from its Rules. It can scarce be said, with propriety, that Perspective existed before; it certainly was not properly understood, as a Science. How insipid and imperfect are all the Books on the Subject before his, except Gravesande's and Ditton's; both which seem to touch on the same Principles, but are far short of a perfect, general System.

There are, in that small Treatise, Rules sufficient, in almost all Cases, for Plane Objects; but, various Examples requiring various ways of applying them, he has not made it so useful as it would have been, had he, instead of referring to former Problems, shewn how to apply it there. As, in Example 3rd, Page 24, Book the first, Fig. 21. We are told to make CB represent a Line, equal to that which is

C c

repres-

represented by CA (by Prop. 15.) Now, I question that one Person in fifty (who understands the whole) ever saw the least Affinity between the two Examples; the different situations of the Lines makes it a very different Operation, though built on the same Proposition of Euclid.

Respecting plane Objects, only, Perspective is soon acquired; knowing how to delineate Figures in horizontal Planes, it is the same in any other; having found their Vanishing Lines, with their Centers and Distances; which I have first shewn, in Practice (as in Theory) how to find, in all useful Cases whatever. But, without the embellishments of Mouldings and other architectural Ornaments, in Buildings, &c. all the rest would be to little purpose. That Treatise comprehends only Perspective in Plano; but I have, in this Book, shewn how to apply it in every necessary Case that can be devised; and doubt not, to render it, by that means, the most useful work, of the kind, yet published in any Language, that we know of.

Brook Taylor has, indeed, to his immortal Fame (in Perspective) furnished us with new and extensive Principles; but his Work, at best, is imperfect, and greatly deficient. His Theory is too concise, and is not regularly digested. It may perhaps, by some, be objected, that I have made too much of the Theory. I could have said much more, but know not where to curtail it; the Examples, given for Illustration rather than Demonstration, I am persuaded, will not be found unnecessary, to some, or trifling to any. One Example, in each Case, to some Persons, would be sufficient; to others there can scarce be too many, so they are various, and not a repetition of the former. I shall be guilty of the same fault (if it be a fault) in Practice; I had rather say too much than not say enough, yet I would not be tedious; because, all that can be said, to some, will be too little, or rather too much, seeing it will be all to no purpose. To steer the middle course is a difficulty not easily obviated; but, it is my fixed design to aim at it; others must determine how I have succeeded.

It is the opinion of many Artists, that the whole of useful Perspective may be comprised in a little Compass; that nothing is a greater discouragement to the study of it, than to see a voluminous Work on the Subject. 'Tis certain, that the Principles, on which the Theory of it is founded, are contained in a very small Compass; and, I would recommend Brook Taylor's Epitome for that very reason, which contains *multum in parvo*. Yet, notwithstanding that valuable Treatise has been so long published, it is, at this time, but little known, and less understood; which is a sufficient reason, with me, to suppose that he has not said enough on the Subject. The elaborate Work of Mr. Hamilton is spun out to an immoderate length; yet to as little use as the other. 'Tis my Design, to comprise the whole of useful Perspective in this Book. Neither of these Authors, I am persuaded, had either taught or practised the Art of delineating; and consequently, they were not qualified for treating it in an easy and familiar manner; one great requisite in a Work of this kind. I have had experience in both, and am well convinced, that, to make it useful, it cannot be comprised in so little compass as many imagine; that it requires frequent repetitions of the same Lesson, somewhat diversified, to familiarize the Rules, in various Cases; without which, not one in twenty will ever be a Proficient. Dr. Taylor truly says, it is much better for the Student to devise Examples, himself, in particular Cases, than to go through those of others; but how few are capable of doing so? nay, I find, many are not able to comprehend them at any rate, nor by any means; and therefore, to make such a Work really useful, variety of Cases and Examples must be devised, for Practice and Experience.

In Practice, our Author has given some Problems, containing the most elegant and general Rules that can be, notwithstanding they are but seldom practised; because he has not shewn, properly, how to apply them: that shall be my care to do, where they can be applied usefully. His Diagrams are, in general, very imperfect, and badly devised; 'tis evident he was no practitioner or delineator, himself, even in Plane Objects (for he has given us no other). But, he has departed from his

own Principles, in Example V. Fig. 19. Part II. having projected the Dodecahedron, in Perspective, by means of the Ichnography and Orthography (as by the old Authors) instead of Vanishing Lines and Vanishing Points; which is much more masterly, elegant, and perfect; and is what the difference chiefly consists in. Such Subjects are indeed of little use, except to familiarize us to find Vanishing Lines and Points in all positions of Planes and Lines, to the Picture, and situations of the Object, or Picture.

Some Persons are so bigoted to the old Authors, that they cannot be reconciled to the new Terms, by Brook Taylor; nor indeed to his new Principles, till they find their Excellence by experience. It is not to be wondered at. 'Tis not easy to divest any Person of old habits and methods of practice, though ever so absurd; because it is impossible that they can see the difference, at first, and consequently cannot judge of it; but it is surprizing that they are not to be prevailed on to try; and if they do, it is with seeming reluctance, and with a fixed resolution to prefer, and persevere in their old Prejudices. I am as much against capricious innovations in Science as any Person; but, if there be an appearance of any acquisition to it, we ought, candidly and unprejudiced, to make a fair trial of their merits; without which we cannot judge of their Excellence.

In respect of the new Terms given us by Dr. Brook Taylor, such as Intersection of the Picture, Center of the Picture, Vanishing Lines and Points, &c. (together with Directing Plane and Line, which are most essential, in Theory, though but of little use, in Practice) I am of opinion that no better Terms could possibly be devised; nor any other so expressive of what is meant by them. How narrow, how limited is the Base Line and Horizontal Line (the only Vanishing Line known to the old writers on Perspective) when compared with them! What difference is there, either in Theory or Practice, between the horizontal Vanishing Line and any other, of Planes perpendicular to the Picture? None at all, seeing, they have the same Center and Distance (Th. 4.); nor indeed in any other, except in finding them; the Practice, in all, is the same, in every respect.

It was impossible for him to make the Principles of Perspective general, but by general Terms; which does not regard any Position either of the Picture or of the Original Plane, since all Planes (simply as Planes) are the same. The Intersection of the Picture includes every Intersection whatever, as well as the Base Line, and they are all of the same use. Vanishing Line is not only general, but is, at the same time, so simple and expressive, that it conveys its utility at once to the Mind.

In the second Theorem of the first Book, it is proved, that parallel Right Lines, however situated, appear to approach towards each other; and, consequently, if they are produced, infinitely, they will appear to meet, and vanish, in a Point at an infinite Distance. So likewise, parallel Planes appear to meet each other, and to vanish in a Right Line, (supposed to be infinite) or, properly, in a Point.

Now, if this Theorem be well considered and understood (together with the foregoing) it will be found to be the foundation of the new Principles of Perspective. For if a Plane be supposed to pass through the Eye, parallel to any Plane, whatever, or any number of parallel Planes, and being produced, or continued till they are lost to sight, they will all appear to unite, and to meet the Plane passing through the Eye, at an infinite Distance.

But, if the Eye be in the continuation of a Plane, the whole of that Plane is lost to sight, and appears but a Right Line (Art. 3. of a Plane, P. 41.) And, the Intersection of two Planes is a Right Line, (Ax. 3.) Wherefore, if a Plane (which may be considered as the Picture) be situated any how; the Line, in which this imaginary Plane would cut the Picture, is that in which the parallel Planes unite, and vanish; consequently it represents an infinite Distance; and consequently, the Line, so produced, is their Vanishing Line; for they cannot, if continued infinitely, appear to go beyond it.

Hence,

Hence, those Planes are, very aptly, said to vanish, they being lost to sight. Therefore, all Parallel Planes have the same Vanishing Line. (Theo. 3rd.)

Also, if a Right Line be supposed to pass through the Eye parallel to any number of Lines, they will appear to converge towards that Line, and to meet it in one Point, at an infinite Distance (Theo. II. Direct Vision.)

Wherefore, if this Right Line from the Eye cuts a Plane, any how situated, it will cut the Plane in a Point only; which represents a Point at any Distance whatever, in that direction, and consequently, it represents the Point in which the Lines, parallel to it, converge; which is infinite. Therefore, it is their Vanishing Point; for, they are lost to sight before they appear to reach it, seeing it is infinite.

And, since the Line, producing that Point, passes through the Eye, the whole Line is lost to sight, seeing that one Extreme is at the Eye; and the Extremes of Lines are Points. Therefore, the Point, in which it cuts the Picture, is its whole Representation; and consequently, all Lines, parallel to it, tend to that Point.

Now I must own, that I cannot conceive any Term so fit to express that Line, or Point, in which parallel Planes, or Lines, meet each other, as Vanishing Line, and Vanishing Point; because they are truly said to vanish in them. For the same reason, perhaps, Mr. Noble, the last writer on Perspective, (except Fergusson) has made use of the Terms Entering Line, and Entering Point; seeing that, the Plane, or Line, begins at the one and vanishes in the other. Had this Author been the inventor of those Principles, I should not have found fault with the Names he has given them; but, since there were Names already given, by the Author, which are more significant, I must blame even an attempt to alter them; because, a multiplicity of Names, for the same Thing, occasions a confusion of Ideas, in the Mind, of their signification and use; and cannot possibly be of advantage to the Science.

In the process of this Book, after some necessary observations, on the proportion of the Picture, the Height and Distance of the Eye, &c. in the third Section, I have, first, shewn how to determine on the Position of the Picture, in respect of the Object and the Eye, the Station being previously determined; then, how to prepare the Picture, for Practice, according to the Principles contained in the foregoing Theory.

The Picture being prepared, the Situation and Distance of the Object, and the Position of the Picture being determined, the following Problems, in that Section, shew how to find the Intersections and Vanishing Lines of Planes, in all Positions to the Picture, if they are not parallel to it (for all such have no Vanishing Line. Theo. 1st.) with their Centers and Distances. Then, to fix the Vanishing Points of certain Lines in those Planes, and determine their Distances; by which, the Lines, vanishing in them, are proportioned.

The Position, Situation, and Distance of the Picture (in respect of the Object and the Eye) being determined, the Original Lines, in the Objects, are produced (if necessary, and not parallel to the Picture) to their Intersecting Points; which are always found in the Intersection of the Plane they are in (Theo. 10th) or being much inclined to the Picture they do not, perhaps, fall within the compass of the Picture; then, other expedients are used, to find the representation of some principal Point in the Line, from which Point, the indefinite Representation is drawn (Theo. 12th.) And lastly, the finite Parts, which represent certain portions of Lines in the Original Object, are determined; by Theorem 13th; by which means, the Object is completed; proceeding from one Plane, or Face of the Object, to another; drawing all the Figures, in each, by means of their respective Vanishing Lines. Each two adjoining Faces, having one Line common to both, the Vanishing Point, of that Line, is in both Vanishing Lines (Theo. 10th) consequently, it is in their common Intersection (Cor. 2nd. Theo. 7th.) by the help of which, the Vanishing Lines of contiguous Faces are determined.

In Section 4th, I have briefly illustrated all the remaining, practical, Problems in Brook Taylor's Essay, respecting the proportioning of Right Lines, perspective, and shewn their great and extensive utility; each of which, founded on the most solid and permanent Principles, is of immense value. For, without knowing the whole of Perspective, or practicing by its Rules, rigidly, an Artist, who is accustomed to sketch, by sight, whatever he sees before him, with seeming accuracy, may, by these Problems, rectify any errors, in right lined, or circular, Plane Objects, from the known proportion of one part to another; the affinity of the Planes and Lines to each other, being known, and the ratio of one Part to another; which, may frequently be obtained, when the true measures of those Parts cannot.

These Problems contain all the Rules necessary for Practice. They may be compared to the five fundamental Rules in Arithmetic, by which all others are worked; and, a Person might, with as much propriety, imagine that he had given Arithmetic enough, for every Occasion, in those Rules, as Brook Taylor had of Perspective, in his first Essay; whereas, the Rules, he has there given, are no more than the Elements of practical Perspective.

This Section contains, also, various Expedients; viz. for determining Vanishing Points, when they fall beyond the limits of the Picture, geometrically and arithmetically, i. e. to determine their Distance from the Center of the Vanishing Line and from the Eye, both which are necessary to be known; how to draw Lines to a Vanishing Point which is beyond the bounds of the Picture, &c.

In Section 5th, those Rules are applied to real use, in delineating all kinds of Plane Figures; and, on Planes in various Positions. First, by means of the Figure being geometrically drawn, in the Original Plane. Secondly, without it, by their known Proportions, their position to the Picture, and the properties of the Figure, being regular.

In the 6th Section, from Plane Figures, I have proceeded to Solids, composed of Planes, of various Figures; and in various Positions.

In the 7th, I have applied them to streight Mouldings, composed of Planes and cylindrical Surfaces, in Cornices, Entablatures, &c.

Section the 8th treats of curve lined Objects, in general.

The 9th shews how to apply the whole to compound Objects, in regular pieces of Architecture, and Buildings of various kinds.

The 10th is for internal Views, and horizontal Pictures, in Ceiling Pieces, &c.

The 11th is adapted for the particular Professions of Cabinet-makers, Coach-makers, &c. and for Machines, in general.

The 12th, and last, is on inclined Pictures and Planes, in general; and, applied to Fortification, or military Architecture.

N. B. A Scale of equal Parts is always adapted, or determined on, of the Proportion we intend to delineate the Objects, on the Picture.

* * The methods of making Scales, for various purposes, are given in the Appendix to my Treatise of Geometry.

S E C T I O N II.

A preparatory and elementary INTRODUCTION, to the PRACTICE of PERSPECTIVE.

AS this Book is intended for a compleat practical Treatise, I have treated it in such a manner (in this Section) as if no Theory, or Elements, had been given. For which reason, I have defined a few more Terms, which are suited to Practice only; as there are several Terms in the theoretic List which may be omitted here; and, there are also, in Practice, several which are not useful, in Theory. I have deduced from the Definitions such useful Lessons, which, if carefully attended to, will contain all the necessary Theory for a Practitioner.

PERSPECTIVE, is the Art of delineating the true Representations of Objects, on a Plane Surface, by geometrical Rules; according to the Position, and Distance of the Objects, in respect of the Picture and of the Eye. (See the Apparatus.)

Fig. 15.

The PERSPECTIVE REPRESENTATION of an OBJECT, is the Section of the Pyramid of Rays, *AEI*, by a Plane, in any Position; which, is the Subject of this Third Book. For, the Picture of every Object, truly delineated in Perspective, is supposed to be so situated, in respect of the Object and the Eye, that if Visual Rays, or Threads, i. e. if Right Lines were drawn from each Angle, or other Point in the Object, to the Eye, they would pass through the corresponding Points in the Representation, on the Picture.

As *EA*, *EB*, *EF*, &c. cut the Pictures in *a*, *b*, *f*; *a*, *b*, *f*, &c. which are, therefore, the Perspective Representations, of the Points, *A*, *B*, *F*, &c. on each Picture.

It is evident, that the Perspective Representation of an Object is nothing more than the Figure projected on a Plane, by its Intersection with the Visual Rays from the Eye to the Object; wherefore, the whole business, of practical Perspective, is to find the true Figure of the Section of the Rays, in all Positions, whatever, of the Picture (which is considered as the intersecting Plane) and in every situation of the Object, or of the Eye.

In order to which, it is necessary to reconsider, well, the construction of the elementary Planes; as in Plate IV; or, to make them more familiar, I have given another, which is better suited for Practice; as they are the foundation of the whole, and the origin of all the Lines and Points used in Practice. Also the general Introduction to this Work should be attentively perused, and tolerably well understood, by every one who would be a Proficient in the Practice of Perspective.

Let *BFIL* be an Original Object, representing a Building, and, *ES* a Spectator, viewing that Object, from the Point of View *E* (the Eye.)

It is manifest, that whilst the Eye remains fixed, in that Point, the Object cannot vary in its Appearance. But, it is also obvious and demonstrable, that every different Section of the Pyramid of Rays, by a Plane in different Positions, will exhibit a different Picture; so very different, indeed, they may be, that they can scarcely be supposed to be Representations of the same Object, much less from the same Point of View and Position of the Object, when viewed direct.

Compare the Pictures, *MNOP* and *OP*; how different are the Images of the same Object, to each other. The one, on *MNOP*, has the Representation, *bgfde*, of the end of the Building, *BFC*, similar to the Original, in every respect; the Representation, *abgb*, of the Front, *AHGB*, is distorted, and drag'd out to a preposterous length, when the Eye is opposite to it; whilst the other Picture exhibits a pleasing and natural Appearance, in almost any Point of View, but most so in the true one. Yet, both Pictures affect the Eye alike, in the true Point of View, and appear the same as the Original Object, itself; each Line, Plane,

or Figure (it is evident) being seen under the same Optic Angle as in the Object. Consequently, the Picture being a true Perspective Representation of the Original, if it had likewise the same degree of Colour, Light and Shade, it would not be possible for the Eye, at E, to distinguish whether the Image (*aifc*, or *aifc*) delineated on a Plane, or the Object itself (*AIFC*) was presented to view.

Having, thus clearly, explained what Perspective is, I shall next define all the practical Terms, by means of which, the whole process is performed, and the Representations effected,

Let the Plane *AIKB* (which may be supposed the Picture) be raised up into a vertical Position, i. e. perpendicular to the Plane it is fixed to; which is the most natural, most general, and convenient Position.

Also, raise up the Plane *GKIH*, parallel to the Picture. This Plane is called, the Directing Plane (Def. 4.) and is very useful, in Theory. In it, the Eye is always supposed to be, as at E; and if the Plane *NIKL* be turned down, parallel to *ABGH*, the Line *IK* meeting *IK*, in the Directing Plane; then, *EC* is the Distance of the Eye from the Picture (equal *FD*); and *EF*, (equal *CD*) is its height above the Plane *ABGH*, on which, the Spectator (*EF*) is supposed to stand. Therefore, at whatever Distance the Eye is from the Picture (as *EC*) a Plane *GKIH* is supposed to pass through the Eye, parallel to the Picture.

The Plane *V* being turned up, perpendicular to the Picture, is considered as a part of the prime or chief Vertical Plane; and, *NIKL* is called the Horizontal Plane; both which are supposed to pass through the Eye, at E.

N. B. This Construction of the elementary Planes, so essential in Theory, is, also, very necessary to be well considered by every Practitioner, who may not have an inclination to go through the Theory; he will, most certainly, find his account in it.

DEFINITIONS.

DEF. A. The GEOMETRICAL, or GROUND PLANE is that Plane on which an Original Object, intended to be delineated, is seated.

As *GHZZ*; on which the Plans, *ACDF* and *XYZ* (supposed to be the Seats of some Objects) are geometrically drawn, which are to be represented on the Picture, *AIKB*.

On this Plane is always drawn, or supposed to be drawn, the Figures of the Seats or Plans of Objects situated on it, in their true geometrical Proportion, according as they are situated, in respect of each other and of the Picture; and they are (in common Perspective) always understood to be beyond the Picture.

In the Apparatus, the Plane *ZSZ* is the Ground Plane, representing the real Ground, on which the Object, *BFIL*, stands. The Rectangle *ABCL* is its Plan, or Seat on the Ground Plane.

Note. If the Object stands on a Floor or Table, &c. it is considered as the Ground Plane.

DEF. B. The GROUND LINE, or BASE LINE, is a Right Line on the Picture, drawn parallel to its Top and Bottom, i. e. parallel to the Horizon.

As *AB*, Fig. 37. No. 1 and 3. (See Intersection. Def. 7. in the Theory.)

The Plane *AIKB*, being erected, represents the Picture, and *ABZZ*, being horizontal, is considered as the Ground Plane. Therefore, the Line *AB* is the Intersection of the Picture with the Ground Plane. And, since all Objects which are to be represented, on the Picture, are supposed to be beyond it, they must necessarily appear above the Line of Intersection; for which reason, it is, very properly, called the Ground Line.

Note. On the Ground Line is applied all the measures, geometrically, of all Figures (as Plans, &c.) of Objects on the Ground Plane, to be delineated; and frequently, for other horizontal Planes, whose Intersection with the Picture is not given or drawn.

N. B. Of the same use is the Intersection of any other Original Plane with the Picture, viz. for proportioning Lines and Figures drawn on that Plane.

DEF. C. The HORIZONTAL LINE, is a Right Line drawn parallel to and above the Ground Line; as *ML*. (See Vanishing Line Def. 8; or G.)

The Horizontal Line is called so by way of eminence, or for distinction from all other horizontal Lines, being the first, or principal Vanishing Line; from its fixed and determined Position and Distance, all other Vanishing Lines, whatever, are determined; the Vertical Line is determinable without it.

N. B.

Pl. VIII.
Fig. 37.
No. 1.

Pl. VIII.

Fig. 37.

N. B. Its Distance from the Ground Line is always equal to the height of the Eye from the Ground, being considered as a Plane, on which the Object, to be delineated, is seated. So that, whether we be sitting, standing, or elevated, the height of the Eye, from the Ground Plane, being determined, and AE , or DC , being made equal to it (by the Scale of Proportion) NL drawn through the Point E , or C , parallel to AB , is the Horizontal Line; or Vanishing Line of horizontal Planes.

Note. It is supposed (in Theory) to be produced by a horizontal Plane (N/KL) passing through the Eye and cutting the Picture, in NL , their mutual Intersection.

DEF. D. The VERTICAL LINE, is a Right Line drawn at right angles with the Horizontal and Ground Line; cutting the Picture into two equal Parts, perpendicularly. As ED . (See Def. 11.)

DEF. E. CENTER of the PICTURE (or POINT of VIEW) is the Point C , in which the Horizontal and Vertical Lines cut each other. (See Def. 16 and 17.)

N. B. The real Point of View is the place of the Eye, where it ought to be fixed when viewing a perspective Picture. The Point C , on the Picture, opposite to it, where a Perpendicular from the Eye would cut the Picture, is, therefore, its Center; and is, generally, understood to be the Point of Sight; i. e. to which the Eye must be opposite.

Hence, if the Center of the Picture, C , be first determined (as it frequently is) then a Right Line drawn through C , parallel to the Horizon, as LM , is the Horizontal Line; and, another Line, drawn through C , perpendicular to it, as ED , is the Vertical Line.

DEF. F. PARALLEL of the EYE. If EC be taken, in the Vertical Line, equal to the Distance of the Picture; and, through the Point E (which is considered as the Eye) if a Right Line be drawn, parallel to the Horizontal Line, as IK , it is called the Parallel of the Eye, of horizontal Planes. (See Def. 9.)

DEF. G. VANISHING LINE, of any Original Plane, is a Right Line on the Picture, supposed to be produced by an imaginary Plane passing through the Eye, or Point of View, parallel to the Original Plane; the Line, in which such a Plane would cut the Picture, is the Vanishing Line of that Original Plane.

COR. Hence, the Horizontal Line is the Vanishing Line of the Ground Plane, and all other horizontal Planes.

For, it is produced by a horizontal Plane (N/KL) passing through the Eye, and cutting the Picture.

† Theo. 3.

COR. 2. And hence, all Planes, which are parallel between themselves, (but, not to the Picture) have the same Vanishing Line†.

For, there can be but one Plane passing through the Eye parallel to them all; and since it can produce but one Line on the Picture, by its intersection with it, that Line is, consequently, the Vanishing Line of them all.

† Theo. 2.

N. B. The Vanishing Line, and the Parallel of the Eye, of any Original Plane, are parallel to the Intersection of that Plane with the Picture†.

Therefore, LM , the Horizontal Line, and IK , the Parallel of the Eye, of horizontal Planes, are both parallel to AB , the Ground Line; which is the Intersection of the Ground Plane, with the Picture.

§ Theo. 1.

N. B. 2. Original Planes which are parallel to the Picture have no Vanishing Line, nor Intersection§.

DEF. H. VISUAL RAY is a Right Line supposed to be drawn from the Eye to any Point in an Original Object.

† Def. 4.

Let the Picture, $AIKB$, be turned up into a vertical Position; and, let the Plane W be turned over, till C coincides with the Center of the Picture; then will E coincide with the Eye, or Point of View (E) if the Directing Plane, GEH † be turned up parallel to the Picture.

Let the Plane $AFDC$, on the other Side of the Picture, be also turned over, till it falls into the same Plane with W . Then, the Right Lines EA , EB , EC , are Visual Rays from the Eye (E) to the Original Points, A , B , and C , on the other side of the Picture; which, by their Intersections with the Picture, give the Representations a , b , and c , of the Original Points, A , B , and C , on the Picture.

See the Apparatus; in which, the Threads EA , EB , EF , EG , &c. are Visual Rays; producing, by their Intersections with the Picture, the Representations, a , b , f , g , and a , b , f , g , &c. of the Original Points A , B , F , G , &c. in the Original Object.

PM and PM are the Ground Lines of those Pictures; VY , and CX are the Horizontal Lines; and, CL , and OP are the Vertical Lines. C , their Intersection, is the Center, or Point of View of each Picture; for, the Eye, considered as a Point, is opposite to either.

DEF.

DEF. I. DISTANCE of the PICTURE is the length of the Perpendicular, or the shortest Line that can be drawn from the Eye of a Spectator, in the true Point of View, to the Picture; the Distance between the Eye and the Center of the Picture. As *EC*. (See Def. 15 and 18.)

DEF. K. INTERSECTING POINT, of an Original Line, is that, in which any Original Line (being produced) would cut the Picture.

B, *P*, and *S*, are the intersecting Points of *XY*, *ZY*, and *ZX* of the Three Sides of the Triangle *XYZ*.

In the Apparatus, *F*, *G*, and *H*, are the Intersecting Points of the Original Lines, *IF*, *HG*, and *FG*.

N. B. The Intersecting Point, of every Line, is in the Intersection of the Plane the Line is in, with the Picture (Theo. 10.) Therefore, *FGH*, drawn through the Intersecting Points *F*, *G*, and *H*, is the Intersection of the Plane *FGHI*. Also, *BG*, drawn through the intersecting Points, *B* and *G*, of the Lines *AB* and *HG*, is the Intersection of the Plane *ABGH*.

DEF. L. VANISHING POINT, of an Original Line, is that Point, on the Picture, in which, a Right Line, from the Eye or Point of View, parallel to any Original Line, would cut the Picture.

N, *O*, and *L*, are the Vanishing Points of the three sides of the Triangle *XYZ*. *C* is the Vanishing Point of the Lines *AC* and *FD*; and, *M* and *L* of the Diagonals *AE*, *BF*, &c.

N. B. The Vanishing Point, of every Line, is in the Vanishing Line of the Plane it is in. (Th. 10.)

Therefore, a Right Line drawn through *V* and *W*, the Vanishing Points of *IF* and *GF*, is the Vanishing Line of the Plane *FGHI*.

Also, *RW*, drawn through the Vanishing Points, *Y* and *W*, is the Vanishing Line of the Plane *BFC*; in which, the Lines *BC*, or *GD*, and *GF*, are situated.

Fig. 15.

COR. All Lines which are parallel amongst themselves have the same Vanishing Point †; except they are parallel to the Picture; in which Case they have no Vanishing Point (Cor. to Theo. 1.)

† Cor. 1.
Theo. 3.

For, *EV*, producing the Vanishing Points, *C* and *V*, on both Pictures, being parallel to *AB*, is consequently parallel to *GH*, *FI*, &c. and, since it can produce but one Point on each Picture, that Point is, consequently, the Vanishing Point of them all, by the Definition.

§ 4. 7. EL.

Also, *EW*, being parallel to *FG* and *HI*, produces their Vanishing Point, *W*.

In the Apparatus, *V* is the Vanishing Point of all the Lines *AB*, *GH*, and *FI*: *Y* is the Vanishing Point of *BC*, and *GD*, &c. and *W*, of the Lines *GF* and *HI*; on the Picture *MNOP*.

The Vanishing Point of *FD* is out of the Picture, below the Ground Line.

In the Picture *MNOP*, *C*, its Center, is the only Vanishing Point; viz. of the Lines *AB*, *GH*, &c. to which *EC* is parallel. For they are perpendicular to that Picture.

All the other Lines in the Original Object, in the Plane *BFC*, are parallel to that Picture; and therefore, they have no Vanishing Point.

Also, *BG*, *AH*, &c. being parallel to both Pictures, have no Vanishing Point, on either.

COR. 2. Hence, the Center of the Picture is the Vanishing Point of all Lines which are perpendicular to the Picture.

For *EC*, the Direct Radial, produces the Center of the Picture (Def. 17.) and, because it is perpendicular to the Picture, it is parallel to all Lines that are perpendicular to the Picture, and consequently it produces their Vanishing Point; by this Definition.

DEF. M. STATION POINT is at the foot of the Spectator; or, it is that Point, in which a perpendicular from the Eye would cut the Ground Plane.

If the Directing Plane, *GHIK*, be turned up, on *GH* the Directing Line, perpendicular to the Ground Plane; *E* being the Eye, *EF* perpendicular to *GH*, cuts it in *F*, the Station Point; and *EF* is the height of the Eye above the Ground Plane.

Fig. 37.

DEF. N. STATION LINE is a Right Line drawn from the Station Point, in the Ground Plane, perpendicular to the Ground Line; or, it is the Intersection of the Vertical Plane †, with the Ground Plane. As *FD*, produced.

† Def. 5.

S E C T I O N III.

Containing some preliminary observations, concerning the Proportion and Position of the Picture; of the Height and Distance of the Eye, &c. Secondly, How to prepare the Picture, for Practice. Thirdly, How to find Vanishing Lines, in all common Cases; and to fix the Vanishing Points, of Lines in all Positions, in respect of each other and of the Picture.

IT may appear somewhat strange, and unaccountable to many, of what use are all the imaginary and elementary Planes in delineating Objects; or how so many Planes, cutting each other, can possibly be applied, in Practice; seeing that, the Plane or Picture, on which the Object is delineated, is the only real Plane made use of. It may also be a matter of wonder, how a Visual Ray can be drawn from an Object, which is beyond the Picture, to the Eye, on this Side; by means of which, it is very obvious (from the Apparatus) the Representations of the several Angles, &c. in the Object, are projected on the Picture; but how this can be effected in delineating, must appear strange to a Novice in Perspective. All which, are, I presume, in the following Pages, accounted for, very satisfactorily.

The Art of drawing in Perspective has this advantage of all other. It is no a random sketch, depending on the Hand and Eye; but, every Line (Right Line at least) may be drawn to the utmost exactness; and the Points, where the Visual Rays would pass through the Picture, are determined, mathematically true; yet may be done by a Person entirely unacquainted with Mathematics, who shall adhere to the Rules contained in the following Sections.

It is evident, seeing that Vision is conveyed (from the Object to the Eye) in Right Lines, that every Section of the Pyramid of Rays will appear the same, to an Eye in the Vertex. Consequently, whether the Section be made nearer to the Eye or to the Object, whether by a Plane or other Surface, whether it be direct or an oblique Section, the effect, to the Eye, is the same; but it is obvious, that it can be so only in that Point of View; for, every different Section has a different Representation; but, all parallel Sections are similar Representations. So that, whether the Picture be drawn by a larger or a smaller Scale of Proportion, the Representation is the very same, except in Dimensions; and supposes the Section to be made farther from, or nearer to the Eye, the Vertex of the Pyramid of Rays.

It may also be observed, that, if the Visual Rays were produced or continued, beyond the Model, they would diverge, so, as to take in a real Building, situated exactly as the Model, of any Dimensions. Which being premised, it is manifest, that a Picture, delineated, truly, by a scale of equal Parts, of any Proportion, from a real or imaginary Model, of a Building, &c. will as truly represent the Original, as if the full measures of it were applied; which could not possibly be done.

This consideration may account for the Measures applied in Practice; which must always be in the same Ratio or Proportion, to each other, as the real Measures of the corresponding Parts in the Original Object.

P R E L I M I N A R Y O B S E R V A T I O N S.

IN respect of the Shape and dimensions of the Picture, no Rules can be prescribed, it is always at the discretion of the Artist; unless it be proportioned to some particular Place which determines its Figure and Dimensions. The oblong Rectangle is, in general, a more agreeable and convenient shape than a Square; about the Proportion of 3 to 2, i. e. if the length be three feet, the width may be two, or thereabout; as conveniency, for taking in the Objects, may require. Some Objects requiring it upright; others, and more generally, length-ways.

2. Neither

2. Neither can a certain and invariable Rule be given, for fixing the height of the Eye, and, consequently, of the Horizontal Line. To fix it to half or a third part of the height, absolutely, would be ridiculous; it must ever be at Discretion, in proportion to the Scale of the Drawing; a Landscape View, from an Eminence, may raise the Horizon to the middle of the Picture, or higher, yet may be very natural. In general, five feet, or five feet six inches, the natural height of the Eye, is the most agreeable, being most accustomed to see Objects at that height; altho' it may not be, perhaps, above one fourth or fifth part of the height of the Picture. Too low, in a general View, is not agreeable, because the recedings of the parts of Objects, on the Ground Plane, are not so distinguishable, as, they approach nearly to Right Lines, with each other.

3. Respecting the Distance, something may be ascertained. The Distance of the Picture is a material Circumstance which ought to be well attended to; otherwise the whole performance may be a disagreeable Distortion, instead of a pleasing and natural Representation. The Distance ought always to be considerably more than the height of the Eye; although, Brook Taylor and others, in their Diagrams, have made it much less, which produces a very bad effect; the Representations of the receding parts of the Object, on the Ground Plane, are, by that means, drag'd out to an immoderate length. To illustrate it.

EX. Let AFEB be supposed the prime Vertical Plane, in which the Eye is situated, at E. Let EC, be its Distance from the Picture; of which, CD may be supposed a Section.

Fig. 37.
No. 2.

This, I presume, is easy to conceive; for, suppose two Planes cutting each other at right angles, one of which is considered as the Picture; the other, a Plane passing through the Eye, perpendicular to it. The Right Line CD being considered as their common Section; and the Planes turned around on it, as an Axis (see Fig. 8. Pl. 3. in the general Introduction) in which revolution, either Plane will become direct before the Eye, and the other pass through the Eye, cutting the former in CD.

Now, E is supposed the Eye of a Spectator, and, C the Center of the Picture; EC is, therefore, its Distance. Let an Object be supposed beyond the Picture, whose Seat on the Ground is AD.

If a Right Line, EA, be drawn, cutting CD, the common Section of the Picture and the Vertical Plane, in a; it is obvious that, to the Eye at E, the representation of the Point A will be at a, on the Picture; and the length of Ground, between the Picture and the Point A (equal DA) is represented by Da. For, EA is a Visual Ray, from the Eye to the Point A; in which Direction, that Point is seen; and consequently, a is the Point in which it would pass through the Picture.

As it may be observed in the Apparatus, by placing the Eye in a continuation of either Picture and the Ground Plane; i. e. in the Line of their common Section, PM.

Then (considering the Eye of the Spectator as the Point E; and, A, or any other Point, on the Ground Plane, answering to A, in the Figure) EA represents the Visual Ray, and a, the Point where it passes through the Picture; which is represented by the Right Line CD.

Now, Da represents the length of Ground between the Picture and the Point A, the Eye being at E; but, being moved to E, the Appearance of it is at a; and, the Point a represents, at that Station, the length DG only, which is preposterous.

For, since DC is the whole indefinite Representation of DA, produced (Theo. 12.) and Da, a represent the finite, small Portions DG, GA, only; consequently, the remainder, aC, of the indefinite Representation, DC, represents the whole of DA, produced infinitely beyond A.

It is evident, that the Point, a or a, from either Station, is mathematically determined; agreeable to Theorem 13th. For, DC being the whole indefinite Representation of DA (Th. 12.) the representation of any Point, A, is found; by making Da (or Da) to DC, as AD to AB (or AD added to CE) i. e. Da : DC :: AD : AB, as it is demonstrated in that Theorem.

The Triangles AaD, AEB, and CEa being similar; Da : aC :: AD : CE; consequently, Da : Da + aC (i. e. DC, equal BE) :: AD : AD + CE, equal DB (i. e. AB.)

N. B. While the Eye moves in the Direction EC, the Indefinite Representation DC remains the same, and the finite Parts (Da or Da) of AD is continually varied †.

Also, whilst it moves in the Direction EA the finite part Da remains the same, and the indefinite Representation is varied. As, at e, the whole indefinite Representation is De, and the finite part is still Da.

† Cor. 5.
Theo. 12.

Thus, it is evident, that the Height of the Eye is productive of as great Distortion as the Distance; but, to determine, absolutely, in what proportion one shall be to the other, is not possible, as various circumstances may render all such Rules exceptionable. In general, the Distance ought not to be less than twice the Height of the Eye (as at E) or, at the least, as three to two (as at E') but there may be a necessity, in some Cases, to make it equal, or perhaps higher, for the conveniency of shewing some particular parts of the Object.

Pl. VIII. Mr. Kirby makes it a general Rule, for the Distance not to be greater than the Perpendicular of an equilateral Triangle, whose Side is the length of the Picture; nor less than half the Diagonal; which (when the Center of the Picture is in the middle) if rightly understood and applied, will not produce great Distortion, in the Representations of Objects thereon.

No. 3. For, if S be the Center, or Point of View, of the Picture A N O B, the Distance S N, or S O, may be sufficient, if there be no Objects near the Extremes, at A or B; but if there are, they will be distorted; because, it is evident that the Optic Angle, under which they are seen, is a Right Angle, or 90 Degrees.

But, this Rule, for fixing the Distance, is sometimes injudiciously adhered to; when the Center, or Point of View, is near either Extreme, as at M or L; in which Case, especially if the View be internal, or have Objects situated near the other Extremes A or B, they will be greatly distorted.

Let A S or S N be taken for Radius, and, on M, describe an Ark of a Circle, from E to F. It is manifest, that all the part beyond that Circle, towards L, being seen under an obtuse Angle (which the Eye cannot possibly take in) will be preposterous.

Now, this is so very obvious, to a Person of any discernment, or a small share of knowledge in Geometry, that 'tis needless to expatiate longer on it; yet, this is one principal reason, why many Artists quarrel with Perspective, and pronounce it deficient. For, not being acquainted with the Theory, they would have, on every part of the Picture, Objects represented as they appear; which cannot possibly be, if they are remote from the Center; for the reason given above.

Wherefore, if the Center of the Picture be judiciously fixed (as at C) in the middle of the Picture (respecting its length, only, not height) as it ought always, except in particular Cases, to be, and C N or C O be taken for the Distance, there can no great inconvenience accrue, especially in external Views; as there will, very probably, be nothing but Clouds represented, at N, or O.

The reason of all this is evident. For, wherever the Center of the Picture is fixed, every part of the Picture, ought, at the most, to come within a Circle, whose Radius, or Semi-diameter, is its Distance.

I shall however give one general Rule; which is, to make the Distance, at least, equal to the length of the Picture, E F, the Center, C, being in the middle; but, if it be on either Side, as at M, or L, it must be equal twice M F, or E L, inclusive; seeing that, the Eye is always supposed to take into the Optic Angle, as much on one side of the Center as the other, every way.

4. The Position of the Picture, in respect of the Object and the Eye, is another essential Point to be well considered, and determined on. Without due regard to that, the other Preliminaries are to little purpose; as all the imagined caution, in the Height and Distance, may be rendered abortive, by that means.

The Distance of the Picture ought always to be regulated and governed by the real Distance of the Object, if it be a single one; or, if there are a multiplicity of Objects, it must be calculated from the nearest, intended to be represented.

Fig. 38. Let A B C, be the Seat or Plan of a Building, which is intended to be delineated from the Station E; the Distance from the nearest part is E B.

Let D F be the Intersection of the Picture with the Ground Plane, or Horizontal Plane; and E the Station Point, or the Eye; then is E S the Distance of the Picture, applied close to the nearest part of the Object, A C, which is seen under the Optic Angle A E C.

The Distance E S (equal D F) is sufficient for that Picture, i. e. for the Object A C. But, if there are more Objects, as X and Z, extended to the full length of the Picture, on either side, it is little enough; they being seen under the Angle D E F, which the whole length of the Picture subtends, viz. 55 Degrees, on the Ark d f; and is the least Distance I would ever use, in such Case, when the Objects extend the full width of the Picture; although it is more than Mr. Kirby's greatest.

ES is therefore the Distance of the Picture DF, and also of the Objects, or of a Plane passing through the nearest Planes of X and Z; in which Case, it is evident that the Picture must be as large as the Objects themselves, being applied close to them.

If they are supposed real Buildings, and the Scale of proportion be determined on, viz. a 10th or 12th part, or any other; take Ef (one third part of ES) for the Distance, and draw df parallel to DF; then is Ef equal df; for Ef:df::ES:DF; i. e. they are equal, and consequently, the Objects x, y, and z, being reduced to the same Proportion will all subtend the same Angles, respectively, as the Originals. By which means the Picture may be delineated of any Proportion.

Now, if the Station (E) be determined, from which the Object AC is to be drawn, the Position of the Picture is also determined. For, ES, the Station Line, bisecting the Optic Angle, AEC or DEF, ought always to be perpendicular to the Picture; consequently, the Position of the Picture, DF or df, is determined; i. e. it must be perpendicular to ES.

For, when the Station, or Point of View, is fixed, from which we are determined to delineate the Object, is it not most rational to suppose the Picture, on which the Object is to be represented, placed direct before the Eye? (as MNOP in the Apparatus) or is it more eligible to place it parallel to either Plane, ABGH or BFC (as MNOP) i. e. to AB or BC, as DC or BG, whose Center, or Point of View, is at C or G. The very supposition of it is absurd to the last degree; and yet, this absurdity is committed by every Artist who places the Point of View at either Extreme, or perhaps entirely off the Picture, as is frequent.

Fig. 15.

Fig. 38.

The difference must be obvious to every Person who considers it. For, the Picture being placed direct, according to DF, the Optic Angle is no more than AEC, or a Ec, about 23 or 4 Degrees; but, being placed in the Position DC, the Optic Angle is, a Eg, 102 or 3 Degrees; for, EC, perpendicular to DC, bisects the Optic Angle, on that Picture; as ES bisects the Angle AEC, on the Picture DF; notwithstanding, the Object, ABC, does not occupy one fourth part of the Angle a Eg.

In the Apparatus, SP, on the Ground Plane, bisects the Optic Angle, p St, under which the Object is seen, according to the Position of the Picture MNOP; for, the Angle, at the Station Point (S) is the same as in the Horizontal Plane, at the Eye.

How to prepare the PICTURE for PRACTICE.

Let AIKB (when raised perpendicular to the Ground Plane) be supposed the Picture; also, let the Directing Plane, GHIK, and Horizontal Plane, IKLM, be placed parallel to the Picture, and the Ground Plane.

 Fig. 37.
No. 1.

Suppose the Figures AD, and XYZ, on the other side, are to be represented on the Picture; and the Points a, b, c, &c. where the Visual Rays EA, EB, &c. would pass through the Picture, to be geometrically determined thereon.

In order to which, several preparatory Lines are necessary to be drawn on the Picture; the Center and Distance must always be known or determined, together with the position or situation of the Object, in respect of the Picture and of the Eye, and also its height, above the Ground Plane.

Now, E is the place of the Eye, EC is the Distance of the Picture; and EF is the height of the Eye; the Point C, where the Perpendicular EC cuts the Picture, is its Center. ML, the Line in which the Horizontal Plane cuts the Picture, is the Horizontal Line (Def. C) or the Vanishing Line of horizontal Planes; and AB, in which the Picture cuts the Ground Plane, is the Ground Line (Def. B.) consequently parallel to the Horizontal Line. These two are the first and most useful Lines; the Center of the Picture (C) in this Case, is in the Horizontal Line, the Picture being vertical.

Pl. VIII.

The next Line to be considered, and of use, is the Vertical Line, ED (Def. D.) If the Plane V be turned up, perpendicular to the Picture, it will pass through the Eye, at E, and consequently through EC, cutting the Picture in EC, the Vertical Line; which, being produced will pass through D.

This Line always passes through the Center of the Picture, and is of the same use for all vertical Planes, which are perpendicular to the Picture, as the Horizontal Line for horizontal Planes; i. e. it is their Vanishing Line, which has, consequently, the same Center (C) and Distance (EC).

The construction of these five Planes should be well attended to, and the Lines they generate, by their Intersections; three of which, viz. the Ground Line, the Horizontal Line, and the Vertical Line, are already produced on the Picture; and the Parallel of the Eye, IK (whose real place is out of the Picture) is transposed to the Picture, by turning up the Horizontal Plane on ML, its Intersection, till it coincides with, or falls into the Picture; and with it the Direct Radial, or Distance, (EC) together with the Eye (E) which falls into the Vertical Line, at E.

When the perpendicular position of the Planes has been considered attentively, let them be pushed either from or towards you, keeping the Directing and Horizontal Planes, joined in IK; the Vertical Plane (V) may still be supposed to cut them all at Right Angles, and generates still the same Line on each. In all Positions, i. e. let the Angles they make with each other be what it may, the Planes are still parallel to each other respectively; and their Intersections are still parallel amongst themselves. (Theo. 2nd.)

Now, let the Picture be turned down, on AB, its Intersection with the Ground Plane, till they coincide; and let the Horizontal Plane, with the Eye (E) and the Parallel of the Eye (IK) be also turned down into the Picture (as it is represented on the Picture); also, let the Vertical Plane (V) be turned down on either side, into the Picture, and with it the Direct Radial (EC) i. e. the Distance of the Picture, falling into the Horizontal Line, with the Eye at E. And, lastly, let that part of the Ground Plane, which lies beyond the Picture (on which the Seats of Objects are geometrically drawn out) be supposed to be turned, on AB (its Intersection with the Picture) quite over to the other side; and imagine it, simply, a Plane, without thickness; so that, all the Figures, described on it, are seen on the other Side, inverted; as ABEF, and XYZ; in which Case, it is obvious, that they have the same Position, or Situation to the Picture, as before; and the Lines (which are not parallel to the Picture) being produced, cut the Picture, in the Intersection AB, in the same Points, as before.

Thus, is the Picture prepared for Practice, in common Perspective; and all the elementary Planes are reduced to one Plane, viz. the Picture*.

All the same Lines and Points, may be seen in No. 3. which is divested of that apparent intricacy of Planes on Planes, consequently it is more simple and intelligible; having only the preparatory Lines, answering to the Intersections of the elementary Planes with the Picture, on it. The Figures below AB, the Ground Line, are Plans of Objects, on the Ground Plane, intended to be delineated.

Fig. 37.
No. 3.

To prepare which, let the Ground Line (AB) be first drawn, at such a convenient Distance from the bottom of the Picture, and parallel to it, as to allow room, below it, for drawing the Plans of Objects (as X, Y, Z) intended to be delineated, in their true geometrical Proportion, Place, and determined Position to the Picture, if necessary. The space, below AB (as AFB) is not considered as a part of the Picture, but of the Ground Plane, whose real place is beyond the Picture.

* The Directing Plane (GH IK) not being of use in common Perspective is supposed to be turned down, or removed out of the way.

Through the middle of the Picture, draw ED, perpendicular to the Ground Line, dividing the Picture into two equal Parts. This Line is the Prime vertical Vanishing Line, in which, the Center of the Picture, or Point of View, is always (though not always in the Horizontal Vanishing Line, but when the Picture is vertical, as it is now supposed to be); make CD equal to the determined height of the Eye, C is the Center; through which, draw ML, the Horizontal Line, parallel to AB, the Ground Line.

Then, with the Radius EC (the determined Distance) describe a Semicircle, cutting the Horizontal and Vertical Lines in E, E₁, and F; i. e. make CE, &c. equal to the Distance of the Picture, which are considered as the Eye, transposed to the Picture* (generally understood by the Points of Distance) and, through E, draw IK, the Parallel of the Eye, parallel to the Horizontal Line; then is the Picture prepared, having all the fixed Lines and Points determined thereon. It only remains, now, to find other Intersections, Vanishing Lines, and Vanishing Points, necessary for delineating the Objects intended.

The Ground Line, or Intersection of the Picture with the Ground Plane, is the first and principal Intersection; and the only one, in general, made use of as such. But, the Intersections of other Planes are often wanted, and particularly vertical Intersections; on which, the measures of the heights of Objects are applied; although they are frequently made use of, yet very few consider them as Intersections; and consequently, they do not see the generality of the Principles and Rules by which the Work is performed; as they look on the operation in Planes which are vertical or inclined, in a different light from such as are horizontal; whereas, if it be well considered, they will find it is the very same; for, wherever the geometrical measures are applied, in proportioning any Line, it is considered as an Intersection of some Plane, in which that Line is or may be situated. And, since the Intersection of any Plane determines the Position of its Vanishing Line, I have shewn how to find the Intersections of Original Planes, in all necessary and generally useful Cases.

Inclined Planes, in general, I have reserved for the last Section; but, as various Planes, which are perpendicular to the Picture, are inclined to the Horizon, and vertical Planes are frequently inclined to the Picture, I shall not consider them, in either Case, as inclined Planes, but such only as are inclined to both.

As the Horizontal and Vertical Lines both pass through the Center of the Picture, they are, therefore, the Vanishing Lines of Planes perpendicular to the Picture† (being vertical) which, from their fixed and invariable Position, determine all other Vanishing Lines whatever.

† Theo. 4.

Therefore, in the following Problems, I have, in the first place, shewn how, by them, to determine the Vanishing Lines of all Planes that are perpendicular to the Picture, either with or without the Intersection; which are subject to one general Rule.

Secondly, how to find the Vanishing Lines of vertical Planes, in all Positions to the Picture (except parallel) and also, of certain Planes inclined to the Horizon, which are subject to the same invariable Rule, as vertical Planes.

The fifth Problem shews how to find the Vanishing Lines of Planes any how inclined, both to the Horizon and to the Picture (being vertical) from their known inclination to the Horizon, &c. as specified in the Problem. All which, are frequently necessary in common Practice.

* The Eye, or Point of Distance, may be any where in the Circumference of a Circle, whose Radius is EC, as occasion requires; or, according to the Position of original Planes, which are perpendicular to the Picture, and their Vanishing Lines; all which, have the same Center and Distance (Theo. 4th.)

Plate IX.

P R O B L E M I.

The Intersecting Point of any Line, in a Plane which is perpendicular to the Picture, being given, together with the Angle of its Inclination to the Horizon; to determine its Intersection and Vanishing Line; the Center of the Picture being given.

Fig. 30. Let I be the Intersecting Point of some Line, given; C is the Center of the Picture; and X is the Angle of Inclination of the Plane to the Horizon.

Through C , draw AB , the Horizontal Line, parallel to the Bottom of the Picture; and, through I , draw ID parallel to AB .

Make the Angle DIF equal to the given Angle X ; and, IF is the Intersection required.

Through C , the Center, draw GH parallel to IF ; GH is the Vanishing Line.

Note. The Vanishing Line may be determined, by making the Angle ACG or BCH equal to the Inclination known; without the Intersection.

DEM. Let ID be supposed the Ground Line, and I the Intersecting Point of the common Intersection of the Ground Plane and the other Plane; which, seeing both Planes are perpendicular to the Picture, is also perpendicular to the Picture \dagger .

\dagger Cor. 1.

9. 7. El.

\dagger Th. 10.

\parallel 4. 1. El.

And, since I is the Intersecting Point, of a Line in the Plane, the Intersection of the Plane must necessarily pass through that Point; and consequently, it will make the same Angle with the Ground Line and Horizontal Line, as the Planes make with each other (DIF , equal BLF) equal to the given Angle X . (See Article 4th, of Planes and their Positions. Page 42.)

Therefore, IF is the Intersecting Line of that Plane.

\S Theo. 4.

But, the Vanishing Lines of all Planes, which are perpendicular to the Picture, pass through the Center of the Picture C . And, the Vanishing Line of every Plane is parallel to its Intersection. (Theo. 2.)

Therefore, GH is the Vanishing Line of a Plane perpendicular to the Picture, and inclined to the Horizon in the Angle HCB equal FID , equal X .

N. B. If F be the Intersecting Point of any other Line in that Plane, and EF be drawn parallel to the Horizontal Line; the Angle EFI , equal FID (equal X) produces the same Intersection.

Also, if K be the Intersecting Point of any Line in another Plane, parallel to the former; then, AK , parallel to IF , is its Intersection.

For, all parallel Planes have parallel Intersections. But they have the same Vanishing Line (GH).

If EC , or EC , be equal to the Distance of the Picture; AK , or EF , may be considered as the parallel of the Eye; and EC , or EC , (perpendicular to GH , or AB) is the Vertical Line.

SCHOL. If the Angle of Inclination be greater or less, the process is the same. But, if the Angle be increased or enlarged to a Right one, the Plane is no longer inclined, but vertical; and, if the Intersecting Point, J , be in a Line, passing through C , perpendicular to AB , the Intersection and Vanishing Line are the same, viz. the Vertical Line EJ ; which is the Vanishing Line of all vertical Planes that are perpendicular to the Picture. See this Problem illustrated by moveable Planes; Fig. 15. No. 2.

Fig. 15.

No. 2.

$IKLM$ is the Original Plane, TU is its Intersection with the Ground Plane, or other horizontal Plane; I is its Intersecting Point, and IN is the Intersection of that Plane with the Picture; making the Angle NIB equal to the Angle PQR , of the Original Plane with the Ground Plane.

And, if $IKLM$ (the Vanishing Plane of all Planes which are perpendicular to the Picture) be placed parallel to the Original Plane, $IKLM$, it will cut the Picture in ON , the Vanishing Line of $IKLM$, in that Position.

\dagger 9. 7. El.

For, it passes through the Eye (at E) and the Direct Radial (EC); consequently it is perpendicular to the Picture \dagger , and passes through C , its Center.

Hence; all Vanishing Lines, which pass through the Center of the Picture, have the same Center and Distance, viz. the Center and Distance of the Picture.

Note. If the Plane inclined on the other Side of its Intersection TU , the Angle of its Inclination is made on the other side of the Vertical Line; to which, particular regard must always be had.

P R O B L E M II.

The Vanishing Line of a Plane being given, with its Center and Distance; and the Angle of inclination which any Original Line, in that Plane, makes with the Intersection, of the Plane it is in, with the Picture; to find the Vanishing Point of that Line, and to determine its Distance.

AB is the given Vanishing Line, and C is its Center.

Fig. 40.

Draw CE perpendicular to AB, and equal to the Distance of the Vanishing Line given or found.

Through E draw GH (the Parallel of the Eye) parallel to the Vanishing Line. Make the Angle GEA equal to the Angle which the Original Line makes with the Intersection, cutting the Vanishing Line in A, the Vanishing Point sought; and EA is its Distance, E being considered as the Eye.

Note. Regard must always be had, on which side the Original Line inclines, and the Angle GEA, or HEB must be made accordingly.

DEM. Imagine the Plane AGHB turned over, on AB, till E coincides with the Eye; then is GH in its true Place (still parallel to the Picture); and EA will be parallel to the Original Line, producing its Vanishing Point; and making the same Angle with the Vanishing Line (AB) and Parallel of the Eye, as the Original Line makes with the Intersection and Directing Line, of the Plane it is in ||.

+ Def. 22.
|| Theo. 11.

EX. The Picture, AIKB, being erected, turn over the Vanishing Plane, IKNL, parallel to the Original Plane.

Fig. 37.
No. 1.

Then, the Radials EN, EO, EL, &c. are respectively parallel to their Originals, XY, YZ, and XZ, in the Triangle XYZ; and consequently, they make equal Angles, respectively, with NL, and IK, as the Originals make with AB, the Intersection of the Plane they are in.

P R O B L E M III.

The Intersecting Point of any Line, in a vertical Plane, being given, and the Angle of Inclination of the Plane to the Picture, to find its Intersection and Vanishing Line; the Center and Distance of the Picture being given, and the Picture supposed vertical.

Let I be the intersecting Point given, and C the Center of the Picture.

Fig. 40.

Through C, draw AB, the Horizontal Line, and ED the Vertical Line at right angles; and through I, the Intersecting Point given, draw IH, parallel to ED; which is the Intersection required.

Make CE equal to the Distance of the Picture; and make the Angle CEA equal to the Complement of the given Angle of Inclination; cutting AB in A.

Or (having drawn EG parallel to AB) make GEA equal to the given Angle; and through A, draw FG parallel to IH, which is the Vanishing Line required.

DEM. Because the Picture and the Original Plane are both vertical, their Intersection is perpendicular to the Horizon; and consequently, since I is the Intersecting Point of some Line in that Plane, IH perpendicular to FI, the Ground Line, is the Intersection; by Theorem 10th.

† Cor. to
9. 7. El.

And, because EAC (equal AEG) is equal to the Angle of Inclination, EA is the Radial, or Parallel of the common Intersection of the vertical Plane with horizontal Planes, producing its Vanishing Point. (By Prob. 2.) Therefore, FG, drawn through A, parallel to IH, or ED (i. e. perpendicular to FI) is the Vanishing Line (Theo. 2nd).

G g

For,

Plate IX.

For, imagine the Picture $FGHI$ turned up, vertical, and the Triangle AEC vertical to it, i. e. horizontal; representing a part of the Horizontal Plane; E is the Eye.

Then, a vertical Plane, passing through the Eye and the Line EA , will cut the Picture in FG , the Vanishing Line, which is parallel to IH , the Intersection; by Theorem 2nd.

For, they are the Sections of parallel Planes with the Picture (Def. 8).

N. B. If the Intersecting Point of any other Line, in the Plane, had been given, as B or H , the Intersection IH would be the same.

And consequently, if the Vanishing Point of that Line, or any other, as F , or G , in any vertical Plane, be given or found, the Vanishing Line is determined.

For, it passes through that Point (Th. 10.) perpendicular to the Ground Line, from its Position.

EXAMPLE, by the Apparatus.

Fig. 15.

If the Planes $ABGH$ and BFC were produced, they would cut the Picture $MNOP$, in BK and BE , the Intersections of those Planes; which are parallel between themselves, because their common Section, BG , is parallel to the Picture $+$; and, because BG is vertical, they are, also consequently, vertical, being parallel to BG respectively \dagger .

† Theo. 5.

‡ 4. 7. El.

And, if a Plane ($RSTU$) be supposed to pass through E (the Eye of the Spectator) parallel to BFC , cutting the Picture $MNOP$, produced, in RU , it is the Vanishing Line of that Plane, parallel to its Intersection, BE .

Also, a Plane ($STOP$) parallel to $ABGH$, cuts both Pictures, in OP and OP , the Vanishing Lines of that Plane, on both; parallel to RU or W , i. e. to BK .

When the Original Plane is inclined both to the Horizon and the Picture, having its common Section with horizontal Planes, parallel to the Picture; it is a similar Case to this; and by turning the Picture sideways, on IH , (Fig. 40.) 'tis the very same in every respect; as it is fully illustrated in Fig. 15. No. 3.

No. 3.

EX. $ADFB$ is the Original Plane; AB , its Intersection with the Ground Plane, is parallel to the Picture; i. e. to AB . Let it be raised up, making the Angle PQR , with the Horizon. $AONB$ is the Picture, which being raised up vertical, the Plane $KONI$, passing through the Eye, at E , will be parallel to the original Plane, $ADFB$; and cut the Picture in ON , its Vanishing Line, parallel to the Horizontal Line, LM .

And, if the Original Plane was produced, it would also cut the Picture in a Line parallel to LM , or AB (i. e. to ON) below AB , the Ground Line.

But, if the Original Plane inclined towards the Picture, on this side of AB ; then, its Vanishing Line would fall below the Horizontal Line, and the Intersection above it.

SCHOL. If the Intersection of an Original Plane, in any position whatever, be given, and the Inclination of that Plane to the Picture known, its Vanishing Line is determined as by this Problem; seeing that, the Vanishing Line of a Plane is always parallel to its Intersection with the Picture. Consequently, if the Intersection, be parallel either to the Horizontal or Vertical Line, the Vanishing Line sought is also parallel to them.

But, when the Intersection of the Original Plane is not given; and which, by reason of the great Distance of the Plane, from the Picture, or Inclination to it, cannot be had, nor its Position ascertained; then other Expedients are used, to find the Vanishing Lines of such Planes; viz. by finding the Vanishing Points of two Lines in the Original Plane. (See Prob. 5.)

N. B. The Center of every vertical Vanishing Line is the Point in which it is cut by the Horizontal Line; and the Center of every Vanishing Line, which is horizontal, is the Point in which it is cut by the Vertical Line (by the 7th Theorem) the Picture being vertical.

The Distance of every Vanishing Line which does not pass through the Center of the Picture, is the Hypotenuse of a Right angled Triangle (as AEC) whose Base and Perpendicular are the Distance of the Picture, and the Distance, AC or EC , between the Center of the Picture and the Vanishing Line.

For, in respect of the Vanishing Line, FG , of vertical Planes, if CE be the Distance of the Picture, EA is the Distance of the Vanishing Line, and A is its Center.

But, if GH be a Vanishing Line of a Plane inclined to the Horizon, in the Angle GEA , and to the Picture, in AEC , having the same Distance, AE , E is its Center, and AC is the Distance of the Picture.

Fig. 40.

P R O B L E M IV.

The Vanishing Line of a Plane, its Center and Distance, being given, and the Vanishing Point of some Line in that Plane; to find the Vanishing Point of other Lines, making a given Angle with that Line, whose Vanishing Point is given.

AB is the Vanishing Line, C is its Center, and A the given Vanishing Point. Fig. 41.

Draw CE perpendicular to AB, and equal to the Distance of the Vanishing Line, given or found.

Join AE; and make AEB, or AED, equal to the Angle, which the Original Lines make with each other, cutting the Vanishing Line in B or D, the Vanishing Point sought.

DEM. Imagine the Triangle AEB turned up, on AB, perpendicular to the Picture (if AB be considered as the Vanishing Line of a Plane perpendicular to the Picture) or, making the same Angle with the Picture, as the Plane, of which, AB is the Vanishing Line.

Then is AEB the Parallel of whatever Plane the Original Lines are in, producing its Vanishing Line AB; E coincides with the Eye, and EA, EB, &c. are respectively parallel to the Original Lines, seeing they pass through the Eye and the Vanishing Points of those Lines (Def. 22.) ‡ Def. 8.

But, the Radials of two Lines producing their Vanishing Points, make the same Angle, at the Eye, as the Original Lines make with each other†. Therefore, B, or D is the Vanishing Point required. † Theo. 11.

N. B. If the Angle AEB be obtuse, it is not the Angle of Inclination of the Lines. In which Case, regard ought particularly to be had to the Position of the Original Lines, in respect of each other, and of the Picture.

For, suppose FG to be the Representation of a Line, whose Vanishing Point is A; then, if FB be drawn, AFB represents an Angle equal to AEB; consequently, the obtuse Angle is towards the Picture. In which Case, let the Angle of Inclination be made on the other Side; that is, produce AE to I, and make IEB equal to the given Angle; for if the Angle AEB (i. e. AED) was made equal to the Angle of Inclination, the Point D (in that Case) would not be the Vanishing Point required.

P R O B L E M V.

The Angle of the Inclination of a Plane to the Horizon, together with the Angle which its Intersection, with any horizontal Plane, makes with the Picture, being given, to find its Vanishing Line; the Intersection of the Plane not being given, nor its Position known. The Center and Distance of the Picture are given.

Let AB be the Horizontal Line, and C the Center of the Picture.

Fig. 42.

Draw CE perpendicular to AB, and equal to the Distance of the Picture; and, through E, draw DE parallel to AB.

Draw EA, making the Angle DEA equal to the Angle which the Intersection of the inclined Plane (with horizontal Planes) makes with the Picture, cutting the Horizontal Line in A, the Vanishing Point of the common Intersection.

Draw EB, perpendicular to AE, cutting AB in B; and, through B, draw BG perpendicular to AB, indefinite.

Make BF equal BE; and make the Angle BFG equal to the Inclination of the Original Plane to the Horizon, cutting BG in G, and draw AG, the Vanishing Line sought.

This Problem, for finding the Vanishing Lines of Planes, casually inclined to the Horizon and to the Picture, is universal, and applicable in all Cases, when the Angles are determinable; as in the Roofs of Buildings, Pediments, &c. which, being frequently necessary, in common Subjects, could not be dispensed with

Plate IX. with here; otherwise, I should have omitted it till the last Section, which treats more fully on such Subjects. I shall therefore reserve the Demonstration of it till then; where, every Case and Circumstance, respecting inclined Planes, are fully demonstrated; and exemplified by moveable Planes.

Fig. 42.

† Th. 7.

N. B. The Center of the Vanishing Line, AG, is determined by drawing CH perpendicular to the Vanishing Line, cutting it in H, its Center†, nor does it differ, in that respect, from any other, except in Position; for, CH is a part of the Vertical Line of the Original Plane.

And, if CI be drawn, parallel to the Vanishing Line, and equal to the Distance of the Picture, the Line IH, to the Center, is the Distance of the Vanishing Line AG; which, Distance, is as applicable to that Vanishing Line, as the Distance of the Picture (IC) to all Vanishing Lines which pass through its Center, i. e. of Planes perpendicular to the Picture.

Fig. 15.

EX. In the Apparatus, VW, the Vanishing Line of the Plane of the Roof, HIFG, is determined by this Problem; V being the Vanishing Point of the common Section, GH, with a Horizontal Plane, DGHK; and EW, i. e. the Radial or Right Line from the Eye, parallel to GF determines W, in RU produced, the Vanishing Point of GF and HI, on the Picture MNOP.

For, E, in the Horizontal Line, represents the Eye transposed to the Picture; EY being equal to the Distance of the Eye from the Point Y; and YE W is equal to the Angle DGF of the Inclination of the Roof to horizontal Planes.

Wherefore, since V is the Vanishing Point of one Line (GH or IF) in the inclined Plane, and W is the Vanishing Point of another Line (GF or HI) in the same Plane; consequently, VW (a Right Line drawn through those Points) is the Vanishing Line of the Plane GF IH (Theo. 10th. Cor 1.)

For, a Plane passing through the Eye, and those Vanishing Points, would be parallel to GF IH, and would cut the Picture in the Line VW; which is, therefore, the Vanishing Line of that Plane. Def. G.

S E C T I O N IV.

C O N T A I N I N G

The ELEMENTS of PRACTICAL PERSPECTIVE.

HAVING, in the foregoing Section, shewn how to find and determine the Intersecting and Vanishing Lines, of Planes, in all common Cases; and also the Vanishing Points of Lines, in any Plane whose Vanishing Line is given or found; by means of which Vanishing Points, all original Right Lines (not parallel to the Picture) have their indefinite Representations, on the Picture, truly and accurately described. In this Section I have shewn how to cut off certain portions, from the indefinite Representation, which are the perspective Representations of certain portions, or segments, of Lines in the Original Object.

Having well considered, that most regular Objects are bounded by Planes, and the bounds of Planes are Lines*; it is evident, that to find the Representations of Lines, in all Positions, is to find the Representation of the Figure, or Object, bounded or circumscribed by those Lines. And, since the extremes of Lines are Points, it follows, that, if the two Extremes, of a Right Line, be found, the whole Line is determined; and, by finding sundry Points in curved Lines, the Representation of the Curve is determinable. Wherefore, the whole, of practical Perspective, consists in finding the Representation of a Point, any how situated.

But, since Points are the intersections of Lines, and, to find the Representation of a Point, in Perspective, it must, necessarily, be supposed in some Line; hence it follows, that, to determine the Representations of Lines, in all Cases, is the whole sum and substance of Practical Perspective.

* See Page 50 and 51. Book II. Section II.

Now, Right Lines can have but three Positions, in respect of themselves and of the Picture, *viz.* they must be either parallel, perpendicular, or inclined; and, having learnt how to manage Lines in all these Cases, by the following Problems, there remains little more to be done; for, by constructing a number of Lines together, properly, an Object is formed.

By Theorem 12. the Indefinite Representation of a Right Line, not parallel to the Picture, is a Line drawn through its Intersecting and Vanishing Points. But, since the Intersecting Point is not always wanted (nor is it always attainable) if any Point in the indefinite Representation be determined, a Line drawn through that Point, to the Vanishing Point, is the same; for, it would, if produced, pass through the Intersecting Point of the Original Line.

In this Section, which contains the whole Substance of practical, rectilinear Perspective, I have shewn how the indefinite Representations of Lines, in all Positions, are determined; and then, how to proportion them, in any given or known ratio to the Original; and afterwards, how to manage them when the Vanishing Point is not within the limits of the Picture, by various Expedients.

Let the Reader take particular notice, that I shall, always (to save repetition) in the following Problems, suppose the Center of the Picture to be given, and its Distance known; except, in particular Cases, when it is otherwise expressed.

The Distance of the Picture (being determined) is applied, in Practice, by the same Scale of Proportion to which the Picture is delineated.

Let it also be observed, that I shall always (in the Diagrams) make use of the initial Letters of the following Terms, *viz.* C for the Center of the Picture, E for the Eye, in its first or principal place on the Picture, and *E* for its first transposed place, in any Vanishing Line, &c. and *E'* for the next transposition, &c. and VL for any particular Vanishing Line. But, seeing that the Ground Line, the Horizontal, and Vertical Lines never vary their Places, and are always stronger drawn than the operative Lines, I think it needless to particularize them otherwise.

P R O B L E M VI.

How to find the Representation of a Point whose Seat on the Picture is given, and its Distance from the Picture known.

Let C be the Center of the Picture, and S the Seat of the Original Point.

Fig. 43.

Draw a Right Line CS, through the Center of the Picture, and the given Seat, indefinitely beyond S. Draw CA, at pleasure; and SB parallel to CA.

Make AC equal to the distance of the Picture, and SB equal to the distance of the Original Point from its Seat. Draw AB, which will cut CS in b, the Point sought.

Or, if the Original Point be between the Eye and the Picture, make SB equal to its distance as before, and draw AB, which produce to the Picture, cutting it in *b*.

Then is *b* the projected Representation, of the Point B, on the Picture.

Compleat the Parallelogram ACSD.

DEM. AC is equal to the Distance of the Picture, SB to the Distance of the Original Point from its Seat, and AC is parallel to SB (Con.) Consequently, the Triangles ACb, and bBS are similar. Wherefore, Sb:bC::SB:AC; and, consequently, Sb:Sb+bC (i.e. SC)::SB:SB+AC, equal SD (i.e. BD.) that is, Sb:SC::SB:BD. Also Sb:SC::SB:BD. - Theo. 13.

Now, because SB is the Distance of the Original Point, from the Picture, and AC is equal to the Distance of the Picture; draw CE and SF, both perpendicular to CS, consequently parallel; make CE equal CA, and SF to SB, and draw EF; which will cut SC in the same Point, b.

For, the Triangles CEB, bFS are similar. Wherefore, Sb:bC::SF:EC; i.e. as SB:AC.

H h

Hence

Plate IX. Hence it is evident, that the projection of the Point *b* does not, in the least, depend on the situation of the Lines *CA* and *SB*, in respect of *CS*, but on their parallelism and proportion to each other; wherefore, if the true Distances are not known, but only their Ratio, the Point *b*, will be projected the same. e. g.

Fig. 43.

Take *CI* at pleasure; and make *SG* to *CI*, as *SB* to *AC*, and draw *IG*; the Point *b* will be projected the same, on the Picture.

For, since $SG : CI :: SB : CA$, and $Sb : bC :: SB : CA$; consequently, $Sb : bC :: SG : CI$, and consequently, the Point *b* is the same.

Hence may be seen the universality of the 13th Theorem. For, conceiving *E* to be the Eye, and *EC* the Direct Radial, i. e. the Distance of the Picture, and *SF* the Distance of the Original Point from its Seat, i. e. from the Picture, imagine the Triangle *ECb* to be turned up, on *bC*, till *EC* is perpendicular to the Picture, and suppose *bFS* turned back, on *Sb*, till *SF* is also perpendicular to the Picture, on the other Side; then is *E* in the true place of the Eye, and *F* is in the Place of the Original Point, and consequently, *EF* is a Visual Ray from the Eye to the Point; which it is evident will pass through the Picture, in the Point *b*; and, since Vision is conveyed in Right Lines to the Eye, the Point *F* will appear, on the Picture, at *b*; which is, therefore, the perspective Representation of the Point *F*. (See App.)

Again, because *EC* is parallel to *SF* and cuts the Picture in *C*, its Center, *C* is the Vanishing Point of *SF*; (Cor. 2. Def. L.) for, the Line *SF* is perpendicular to the Picture; and *S* is its intersecting Point; (Def. K.) wherefore, *SC* is its indefinite Representation; and *EF* is a Visual Ray from the Eye to the Original Point; (Def. H.)

† Theo. 12.
Def. 25.

† Ax. 7.

And, because *EC* is parallel to *SF*, and *EF* cuts them both, they are, therefore, all in the same Plane; consequently, the Visual Ray, *EF*, will cut the Picture somewhere in *SC*, the Intersection of that Radial Plane with the Picture, and consequently in *b*; making *Sb* to *bC*, as *SF* (or *SB*) to *EC* (or *AC*) or, *Sb* to *SC*, as *SF* or *SB*, added to *EC*, or *AC*; i. e. to *BD*, or *FH* §; for, *D* is the Directing Point of *BS*, and *H* of *FS*.

§ Theo. 13.
and Th. 11.
8. of El.

SCHOL. Thus, the whole business of practical Perspective will be found (when well understood) to consist in finding the representation or projection of a Point on the Picture, any how situated; i. e. to determine that Point, in which a Visual Ray, from the Original Point to the Eye, would cut and pass through the Picture.

For, if the two Extremes of a Right Line are found, the whole Line is determined; and curved Lines can only be represented by finding the representations of several Points in the Original Curve, and joining them carefully, by hand. I would therefore advise the young Student, to bestow the utmost attention on this and the following Problems, as they really contain the whole essence of Practical Perspective.

P R O B L E M VII.

The Seats, on the Picture, of any two Points in an Original Line being given, and the Distance of the Points from the Picture; to find the Inclination of the Line to the Picture, its Intersecting and Vanishing Points; to draw its indefinite Representation, and to find the Representation of each Point.

Fig. 44.

A and *B* are the Seats of the Points given, and *C* is the Center of the Picture.

Draw *AB*, indefinite; draw *AF* and *BG* perpendicular to *AB*, and equal to the Distance of the Original Points from their Seats, respectively; and draw *FG*, meeting *AB* in the Point *I*.

Then is *I* the Intersecting Point of the Line, in which the Original Points are situated; and *AIF* is the Angle of its Inclination to the Picture.

Through *C*, draw *CV* parallel to *AI*, indefinite; draw *CE* perpendicular to *CV*, and equal to the Distance of the Picture; and, draw *EV* parallel to *IF*, cutting *CV* in *V*, the Vanishing Point.

Draw *IV*, the indefinite Representation of the Original Line; and lastly, draw *EF* and *EG*, or *AC* and *BC*, cutting *IV* in *a* and *b*, the Representations of the Original Points, *F* and *G*.

DEM. Because A and B are the Seats of the Original Points, AB is the Seat of the Line they are in, and consequently, it is the Intersection of a Plane passing through the Line perpendicular to the Picture. (See N. B. Art. 7. General Introduction, Page 44.)

Now, since AF and BG are perpendicular to AB, i. e. to the Picture, and measure the Distance of each Point, respectively, from its Seat; if the Triangle AFI was turned back, behind the Picture and perpendicular to it, F and G would be in the true places of the Original Points, in respect of the Picture, and FG would represent the Original Line; which, being produced to the Picture, would cut it in I, in the Line AB, produced.

For, AF and BG are parallel Lines, and consequently, all other Right Lines which cut them both are in the same Plane †.

Therefore, I is the Intersecting Point of the Line FG, and AIF is the Angle of its Inclination to the Picture, i. e. to its Seat, AB.

Again. Suppose the Triangle CEV turned up, perpendicular to the Picture, it will be parallel to AFI; for CV is parallel to IA, and AFI and CEV are both perpendicular to the same Plane.

Now CE is perpendicular to the Picture, and, since it is equal to the Distance, E is the true place of the Eye; and because EV is parallel to IF, and cuts the Picture in V, V is the Vanishing Point of IF*; seeing they make equal Angles with the Picture; (EVC equal AIF; by Theorem 11th.)

But, I is the Intersecting Point of FG, and V is its Vanishing Point; wherefore, IV is its Indefinite Representation; by Theorem 12th.

And, EF, EG, are Visual Rays, from the Eye (E) on this Side, to the Points F and G, on the other side of the Picture, which would cut and pass through the Picture at a and b.

For, EV is parallel to IF, and they are both cut by IV and EF; wherefore, they are all in the same Plane †; and IV is the Section of that Plane with the Picture, because, the Points I and V are both in the Picture; the Line FG cutting it in I, and EV in V, (Def. K and L.)

Now, since EF and EG are also in the same Plane, they must cut the Picture, somewhere, in the Intersection (IV) of the Plane they are in, consequently in a and b.

For, the Triangles VEa and aIF, VEb and bIG are similar; and therefore, Ia : aV :: IF : EV as Fa : aE; and, Ib : bV :: IG : EV; as by Theorem 13th.

Secondly. AC and BC also determine the same Points, a and b.

For, AF and BG are Lines perpendicular to the Picture; therefore, the Center of the Picture is their Vanishing Point §; EC being parallel to them ||; also, A and B are their Intersecting Points; therefore, AC and BC are the indefinite Representations of AF and BG; by Theorem 12th.

But, the Representation of the common Section of two Lines, is the Point in which the Indefinite Representations cut each other. (Cor. 7. Theo. 12.)

Therefore, a, the Point in which IV and AC cut each other, is the Representation of F; and b of G, in which the Lines BC and IV cut each other.

Also, because CE is parallel to AF, the Triangles CEa, aFA are similar; wherefore, Aa : aC :: Fa : aE :: AF : EC, i. e. :: Ia or IF : aV, or EV. Q. E. D.

Or, if the Distance of the Points F and G, from the Intersecting Point, are transfered to D and H; and the Distance of the Vanishing Point, EV, be set off, from V to E; ED and EH, being drawn, will give the same Points; that is, they will cut IV the same, in a and b.

For, the Ratio of Ia or Ib, to aV or bV, is still as IF or IG, to EV; i. e. as ID or IH, to EV.

Also, if AC, or BC, be considered as the indefinite Representation of a Line (BG) in which the Point G is situated; make BK equal BG, and CE* (on the contrary side) equal CE; KE* will determine the same Point, b, &c.

N. B. The Seat of a Point, on any Plane, is the Point where a Perpendicular from the Original Point cuts the Plane, and consequently measures its Distance. See Ichnog. and Orthog. Sect. II. Page 46.

SCHOL. The Distance of any Point, in an Original Object, from the Picture, may be obtained when the Intersecting Point, of the Line it is in, and its Distance from the Intersecting Point, cannot, for various reasons; and, since it is demonstrated (Theo. 13.) that the Distances of several Points in a Line, respectively, from the Picture, are in the same Ratio as their Distances from the Intersecting Point of that Line; consequently, having their Seats on the Picture, and Distance from their Seats, respectively, their Representations, on the Picture, may be determined without the Intersecting Point. (Cor. 1. Th. 13.)

SCHOL. 2. If the Original Points are in a Plane which is perpendicular to the Picture, the Seats of those Points are in the Intersection of that Plane with the Picture.

Wherefore;

† Ax. 7.

* Def. 22.

† Ax. 7 & 8.

§ Cor. to Theo. 4.
|| Def. 22.

Plate IX.
Fig. 44.

Wherefore, if F and G are two Points in a Line on the Ground Plane, the Seats of those Points are in the Ground Line (ID) where the Perpendiculars FA and GB cut it. A and B are the Seats; and their Distances AF to BG , are as the Distance of the Point F is to the Distance of G , from the Intersecting Point of the Line FG , where it meets AB produced.

COR. ID being the Interfection of a Plane in which the Line FG is or may be situated, VC is its Vanishing Line (Theo. 2. and 10.) and, because it passes through C , the Center of the Picture, it is, consequently, the Vanishing Line of a Plane perpendicular to the Picture (Theo. 4.) whose Distance is EC .

COR. 2. If the Interfection, (ID) and Vanishing Line (CV) of any Plane, with its Center, (C) and Distance, (EC) be given, and any Line (FG) in the Original Plane; its Intersecting Point (I) is found by producing the Line; and its Vanishing Point (V) by drawing EV parallel to FG ; or, by making the Angle $EV C$ equal to AIF . (Theorem 11.)

P R O B L E M VIII.

The Representation of a Line and its Vanishing Point being given or found, to divide the Line, perspectively, in any known Proportion.

Fig. 45.

AB is the Line given, and V is its Vanishing Point.

Draw AH , at pleasure, and EV parallel to AH .

Make AF , FG , and GH in the ratio, or known proportion, of the Original.

Draw BH , and produce it, through B , cutting VE , in E ; and draw EF , and EG , cutting AB in C and D .

Then is AB divided, in C and D , perspectively, as AH is divided; in F and G .

† Cor. to
Theo. 3.

DEM. If VE be considered as the Vanishing Line of a Plane in which the Original of AB is situated, AH may be considered as its Interfection; and EF , EG and EH represent parallel Lines†; for they are all in the same Plane, whose Vanishing Line is EV , and E is their Vanishing Point.

2. 6. El.

Wherefore, since ABH is a Triangle, and CF and DG are supposed parallel to BH ; consequently, AB and AH are divided in the same Ratio, in the Points C , D , and F , G †.

N. B. If aB had been drawn through the other Extreme (B) instead of AH , and aB divided, in the same Ratio as AH , that is, in the given Proportion; Aa being drawn, cutting EV in E , and Ef , Ed , produced, will cut AB in the same Points, C and D .

Any other Line (AH) being divided in the same Ratio as AH , and EV being drawn parallel to AH ; EH drawn, through B , and EF , EG also drawn, will give the same Points C and D , which may be demonstrated in the same manner.

SCHOL. By this Problem, any Right Line in any Object, sketched by hand and depending on the Eye, may be divided, with certainty, in the proportion of the Original, being known, no regard being had to what Plane the Line is in, nor its Vanishing Line.

By the same means, a Line drawn in Perspective may be bisected, as AD , or CB , by making AF , FG , or FG , GH equal; consequently, if the Parts AF , FG , and GH , are all equal, AB is trisected, perspectively.

COR. If AV be the indefinite Representation of a Line, and AC represents some certain portion of the Original; and it is required to cut off another Portion, as CD or CB , in a known Ratio to that represented by AC .

Draw AH , and VE parallel to it, at discretion; assume any Point (E) in EV , and draw EF , through C , cutting AH in F .

Make FG , or FH , to AF in the known proportion of the Original, and draw EG , or EH ; which will cut off CD , or CB , in the proportion required.

COR. 2. If AC represents a certain portion, and it is required to cut off, from any other Point, as D , a part (DB) which shall be to AC in a certain Ratio.

Proceed as before, and, from any Point (E) in EV , draw EF , EG through C and D . Make GH to AF in the known proportion, and draw EH cutting AV in B , the Point sought.

PROBLEM IX.

The Vanishing Line of a Plane being drawn, and the Representations of two Lines in that Plane; if one of the Lines be divided any how, how to divide the other in the same Ratio.

Let AB and CD be the given Representations, of which, let AB be divided, any how, in E ; it is required to divide CD in the same Proportion.

Fig. 46.

GH is the Vanishing Line of the Plane they are in.

At any Distance, at discretion, draw ad parallel to GH ; and, from any Point (G) in the Vanishing Line, draw GA , GE , and GB , and produce them till they cut ad , in a , e , and b .

From the same Point (G) or any other, in the Vanishing Line, as H , draw Hc , Hd , through C and D .

Divide cd , in f , as a b is divided, i. e. make cf to cd , as a e to ab .

Draw Hf , which will divide CD , in F , in the same Ratio as AB is divided.

† Prob. 32.
Geometry.

This Problem may be performed as well by one Point (O) as by two; if the Line, CD , be so situated, as to come within compass. From which construction it is evident, that AB and CD represent equal Lines, as well as being equally divided, ab and cd being equal, and the two Lines having the same Vanishing Point.

For, they represent parallel Lines, and they are both cut by parallel Lines, aO , bO , cO , &c. at equal Distances, ab equal cd , &c.

Or, it may be done by drawing two Lines, AI and CK , which are parallel to GH , through either Extremes (A and C) of the given Lines; by which Construction, the Demonstration evidently appears from the last; seeing that, AI , and ab , or ab ; CK and cd are all divided in the same Ratio.

After the same manner, CD may be divided, perspectively, into as many parts as AB is divided into, and in the same ratio to each other, respectively.

The general utility of this Problem is much the same as the other; the former being, how to divide a Line, perspectively, in the proportion of the Original; and this shews, how to find the Proportion of the Original from the perspective Representation; by which means, any other Line may be divided the same.

In the former, the Vanishing Line was not necessary, only the Vanishing Point of the Line, which may be in any Plane whose Vanishing Line would pass through V , its Vanishing Point, as VE or VE . But, in this Case (the Lines having different Vanishing Points) the Vanishing Line, of the Plane they are in, must be had; and, it must also be observed, that both Lines are in the same Plane, or the Operation cannot be performed by one Vanishing Line.

PROBLEM X.

The Vanishing Line of a Plane being given, and the Representations of two Lines in that Plane, having different Vanishing Points, to cut off, from any Point in one of them, a portion equal to that represented by the other, or in any known Ratio.

The different positions or situations of the Lines, to each other, may make this Operation appear very different; for which reason I shall give it variously, of which, the first is according to Brook Taylor.

I i

Let

Plate X.
Fig. 47.

Let AB be a given Representation of a Line, and FG the indefinite Representation of another Line. NM is the given Vanishing Line, and C its Center.

It is required to cut off (from the Point F) a portion, which shall be to that represented by AB in a known Ratio.

Produce AB to its Vanishing Point, D ; and, through F , draw AH , to the Vanishing Line, cutting it in H .

Draw CE perpendicular to the Vanishing Line and equal to its Distance, and join ED and EG , by Right Lines.

Make EK and EL in the Ratio required; i. e. make EK to EL as the Original Line, represented by AB , is to the other; and draw EM , or EN , parallel to KL .

Draw BH and FD , intersecting in I ; draw IM , or NI , cutting FG , in O or P .

Then, FO , or FP , represents a Line having that Proportion, to the Line which AB represents, as EL to EK .

DEM. If NM be considered as the Vanishing Line of a Plane perpendicular to the Picture; imagine the Triangle NEM turned up perpendicular to the Picture; or, if the Plane in which the Original Lines are situated, be inclined to the Picture, let NEM be supposed parallel to it; then, E is in the true place of the Eye.

† Def. 22.

Now, ED and EG are the Radials of AD and FG , producing their Vanishing Points, consequently they are parallel to them, respectively; † and make the same Angle (DEG) at the Eye, as the Original Lines make with each other. (Cor. 1. Theo. 6.)

And, because EM is parallel to KL , D , G , and M are the Vanishing Points of the three Sides of a Triangle (EKL) the Radials, ED , EG , and EM being parallel to them, respectively.

But, the three Lines, FI , FO , and IO , vanish in those Points, respectively; consequently, FO represents a Triangle similar to EKL ; the Angle DFG represents the Angle DEG (Prob. 4.) and consequently, IM represents a Line parallel to EM ; i. e. to KL .

Therefore, the other two sides, FI and FO , or FP , have that Proportion to each other, respectively, as EK to EL .

† 15. 1. El.

But, because AB and FI , AF and BI have the same Vanishing Points, D and H , respectively, $ABIF$ represents a Parallelogram (Cor. 1. Theo. 3.) consequently, FI represents a Line equal to the Original of AB ; † and therefore, FO , or FP , represents a Line which has that proportion to the Original of AB , as EK to EL .

N. B. If the situation of the Line AB was such, that a Right Line, joining A and F , was parallel to the Vanishing Line NM , then BI must be drawn parallel also.

But, if ab , the given Line, be so situated, that the Line aF is so much inclined to the Vanishing Line as not to reach it within the compass of the Picture, take any Point (J) in the Vanishing Line, and draw Ja , Jb , indefinite; draw AD , at discretion, cutting them in A and B , and proceed as before.

For, AB and ab having the same Vanishing Point (D) also, Aa and Bb have the same Vanishing Point (J) $abBA$ represents a Parallelogram; and consequently, AB , aB , represent equal Lines.

Fig. 48.

Case 2nd. Let the given Lines (AB and FP) be so situated as to cross each other; and, it is required to cut off, from the Point (I) of their Intersection (or any other) a part (IF or IP) equal, or in any Ratio to the Original of AB .

§ Prob. 8.

Draw Ac parallel to the Vanishing Line (MD) indefinite; and, from the Point H , or any other, at discretion, in the Vanishing Line, draw HI and HB , cutting Ac in c and b ; then, Ab is to bc in the Ratio of the Original of AB to BI §.

Make ac equal to Ab and draw aH , cutting AB in d ; dI represents the same measure as AB , viz. ac equal to Ab .

Produce AB to its Vanishing Point (D) and FI , to H ; if it be not already done.

C being the Center of the Vanishing Line, draw CE perpendicular to HD , and equal to its Distance; and draw ED , EH .

Make EK , EL , equal, or in the Ratio required, and join KL ; to which, draw EG parallel, giving the Vanishing Point G .

Draw Gd , and produce it, cutting IF in F ; then, if EK be equal EL , IF represents a Line equal to the Original of dI , that is, of AB , as it was required; or, whatever Ratio EK has to EL , FI represents the same to AB .

Again. Draw mn , through I , parallel to Ac , cutting aH in m .

Make In equal Im ; draw nH cutting AD in O ; and draw OG , cutting FH in P .

IP represents a Line equal to IF , equal AB ; i. e. to their Originals; or in the Ratio of EK to EL , whether equal or otherwise.

DEM.

DEM. Because a m and F I have the same Vanishing Point (H) they represent parallel Lines; and, because m n is parallel to a c. and to the Vanishing Line H D, a m l c represents a Parallelogram; consequently, the Originals of m l and a c are equal.

But, I n was made equal I m, and n H represents a Line parallel to a H; consequently, I O and I d represent equal Lines. (See Problem 8th.)

And, since O P and F d have the same Vanishing Point (G) they also represent parallel Lines; and consequently, I O P, I d F represent equal Angles \dagger ; wherefore, they are similar Triangles, for the Angles P I O, d I F, are equal \dagger .

But, they also represent congruous Triangles, for I P : I F :: I O : I d :: P O : d F; and I O represents a Line equal to I d; consequently, I P represents a Line equal to I F, and P O to d F.

§ C. i. T. 3:
† 4. 1. El.
† 2. 1. El.

If it had been required to cut off, from the Point J, in F H, a portion, in a certain Ratio to A B; draw B J.

If B J be parallel to H D, draw D J cutting A b in a; then J a represents an equal Line as A B.

But, if B J be not parallel to the Vanishing Line, produce it to its Vanishing Point, and proceed as by Fig. 47, using J a for A B, as I d for A B, before.

N. B. If K L was parallel to the Vanishing Line, then P O and d F would also be parallel; for E G would be parallel, and consequently, could not produce a Vanishing Point.

Suppose it was required to cut off, from the Point a (Fig. 47.) in the Line a F, a part, equal, or in any known Ratio, to the Original of A B.

Draw A a till it cuts the Vanishing Line in J, and draw J B; also draw a D to the Vanishing Point of A B. Then a b represents an equal measure as A B, and a F may be cut in any Ratio to a b, as in the next Case; which is the 3rd Example, Prop. 16, of Brook Taylor, mentioned in the Preface to this Book.

Case 3rd, Let A C or C B be the given Line. It is required to cut off from the Point C, a part, which represents an equal measure as the other.

Fig. 49.

D and G are the Vanishing Points of the two Lines, and E is the Eye.

Draw E D and E G, and bisect the Angle D E G, by the Line E H.

Draw A H, which will cut C G in the Point B, as required; or C A in A.

This Case, or this position of the Lines, is but another Example of this Problem, which Brook Taylor calls, finding the Representation of a Circle from the representation of one Radius given. It is certain that the whole Circle may be completed, from a Radius given, though not by this Case only, but by both, as I shall exemplify.

Let A D and F G be drawn indefinite; C, their Intersecting Point, is the representation of the Center, and C B is the Radius given, in the Position by Brook Taylor.

He says, "make C A to represent a Line equal to that represented by C B (by the 15th)" viz. by the first Case of this; "i. e. bisect the Angle E O D;" and, to make it still less intelligible, in his Diagram, the Angle D E G (i. e. E O D, in his Treatise) is not bisected, nor trisected, but is cut, very nearly, in the Ratio of 2 to 5; which negligence, in a Person of his sagacity, is to me surprizing.

It is demonstrated in the 3rd of the 6th of El. that, if any Side of a Triangle be divided in the Ratio of the other two Sides, a Right Line joining the opposite Angle and the point of Section will bisect that Angle. But, there may, very justly, be an exception to this; for, it is necessary that the greater Segment be contiguous to the greater Side, and consequently the lesser Segment to the least Side.

In the Triangle D E G, if D G be divided, in H, in the Ratio of D E to E G, then, E H bisects the Angle D E G; because G H, the greater Segment, is contiguous to the greater Side, E G; which, it is obvious, could not be otherwise.

The Construction, for proof of the Assertion, is to produce either Side, as G E, and make E J equal to E D, and join J D; which is proved to be parallel to E H.

For, because E D = E J the Angle E J D = J D E (9. 1. El.) and, because D E G is equal to them both (10. 1.) and D E H = H E G (Con.) J D E = D E H (Ax. 4.) therefore, E H is parallel to J D (4. 1.)

But, J E is the Radial of C B, produced, and E D of A C; and E J is made equal to E D; also, E H is a Right Line from the Eye, parallel to J D (i. e. to K L, as in the former Case) producing the Vanishing Point of the Line A B (i. e. of I O, and F d, in the two former Cases); by means of which Vanishing Point (H) C A and C B are cut, representing equal portions of those Lines.

Hence may be seen the affinity between the two Cases, in bisecting the Angle D E G.

For

Plate X.
Fig. 49.

† 1. 1. El.

For $EJ = ED$, i.e. EK to EL , and EH is parallel to KL ; consequently, ACB represents a Triangle similar to KEL , or JED ; and consequently, D , G , and H are the Vanishing Points of the three Sides AC , CB and AB ; which is Isosceles, by Construction.

And, since DCG represents an Angle equal to DEG (Prob. 4.) consequently, ACG represents an equal Angle to DEJ ; therefore, the Sides, containing the external Angle DEJ , are, in this Case, made in the ratio (EK to EL) of the Originals, which AC and CB represent; and since they are equal, consequently, AC and CB represent equal Lines.

Now, CA and CB represent equal Lines, from the same Point C ; wherefore, if C be considered as the Center of a Circle, in Perspective, CA and CB are Radii of the Circle; and this is all that Brook Taylor has done towards finding the whole Representation, which is far from being sufficient.

In BC produced, make CF represent a part equal to what CB represents, and CI to CA , or CB ; then, BF , and AI represent Diameters of the Circle.

Proceed as in Case 1st. by making EK equal to EL , and draw EN parallel to KL ; draw AN and BN , cutting BF and AD in F and I , the Points sought.

Or, having drawn AN , only, cutting BF in F ; draw FH , cutting AD in I .

The four Points, A , B , I , and F , are all in the Circumference of a Circle, whose Center is C ; for, CA , CB , &c. represent equal Lines.

But, these four Points are not sufficient, for completing a Circle in Perspective.

Draw HC and NC indefinite, and cut off, from the Center (C) Ca , Cb , Ci , and Cf , representing also equal measures, to those represented by CA , &c. e.g.

Make EM equal to EL ; join ML , and draw EO parallel to ML .

Also, make EL equal EK ; and draw EP parallel to KL .

Draw AO , OI , BP , and PF , cutting the indefinite Lines in a , i , b , and f .

The Angle NEH being bisected, by EQ , shews the Affinity to the first Case.

Through the eight Points, A , a , B , b , I , i , F , and f , an Ellipsis may be described, which will be the representation of a Circle in Perspective; from the given representation of one Radius, AC or CB .

P R O B L E M XI.

From three Points given, in the Circumference of a Circle, to find the Representation of the Circle; having the Vanishing Line of the Plane the Circle is in, and the place of the Eye.

Fig. 50.

A , B , and C are the three given Points, LM is the Vanishing Line, and E is the place of the Eye. ES is the Distance of the Vanishing Line.

Produce AB and CB , cutting the Vanishing Line in I and K ; draw EI and EK . Make the Angles KEL and IEM each equal IEK , producing the Points L and M , in the Vanishing Line.

Draw AK and AM , CL and CI , intersecting in D and F ; the Points D and F are in the Circumference, which passes through A , B , and C .

DEM. Because the Angles KEL , KEI , and IEM are equal (Con.) and E is the Eye, the Angles LDK , KBI , and IFM represent equal Angles (Prob. 4.) and consequently ADC , ABC , and AFC also represent equal Angles †.

Therefore, those Angles touch the Circumference §; for they are in the same Segment, or stand on the same Ark, AbC .

Now, here are five Points, A , D , B , F , and C , in the Circumference; but they are not sufficient for describing the true Curve of the Ellipsis.

Therefore, draw AH at pleasure, cutting the Vanishing Line, in H .

Make the Angle HEJ equal IEK , and draw CJ , cutting AH in G , which is another Point in the Circumference of a Circle.

† 2. 1. El.
§ 10. 3. El.

Thus may as many Points be found in the Circumference as are necessary to describe the whole Curve; which, passing through all the Points, A, G, D, B, &c. will be an Ellipsis; for, it is the Representation of a Circle in Perspective, seeing that those Points are all in its Circumference.

If there be too much Space between any two Points, to describe the Curve with greater accuracy, another Point may readily be found; as a, or b, between A and C.

Draw a B, or b B, at pleasure, cutting the Vanishing Line in S, or O. Join SE, or OE, and make the Angles SEH and SEN, or OEP, equal to IEK, cutting the Vanishing Line in H, N, or P, respectively.

Draw HD, or NF, cutting Sa in a; or PF, cutting Ob in b; which Points, a and b, are also in the Circumference.

For, because AD and CB have the same Vanishing Point (K) also, AB and CF having the same Vanishing Point I, the Originals of the Arks, DB and BF, are each equal AC†; for AD and CB represent parallel Lines‡; consequently, BaD or F, and BAD or F, or BbF, represent equal Angles§, each being equal to the Original of ABC; i. e. IEK.

† Cor. to
10. 3. El.
‡ Cor. 1.
Theo. 3.
§ Cor. 2.
9. 3. El.

Othwise; draw MC, indefinite; produce ME, and make the Angle QER equal to IEK; ER, being produced, will cut the Vanishing Line, somewhere, if it be not parallel to it.

If ER be parallel to the Vanishing Line, draw Aa also parallel; or to the Point R, in which ER would cut the Vanishing Line, cutting MC in a.

For, the Angle QER + REM = two Right Angles†. And, the Angle MaR (i. e. AaC) represents the obtuse Angle MER; and, A BC represents an Angle equal to QER.

† 1. 1. El.

But, the opposite Angles of every Quadrilateral, inscribed in a Circle, are equal to two Right Angles‡.

‡ 11. 3.

Consequently, AaC represents an Angle in the opposite Segment; for, AaC and ABC are opposite Angles, in the Quadrilateral ABCa.

There is not, perhaps, in the whole Book, a more elegant Problem than this, which induced me to give it a place, and to perfect it. Its utility is not so great; yet it may frequently be applied, by those who do not care to be confined strictly to the Rules of Perspective, in every respect. For, having, by any means, obtained the representations of three or more Points, which are known to be in the Circumference of a Circle, the whole may, readily and accurately, be determined; if they know the Distance of the Picture, and the Vanishing Line of the Plane of the Circle.

Brook Taylor has made a lame affair of this elegant Problem, notwithstanding his Principles are the same; by reason of the short Distance he has taken for the Eye, and the prodigious Dimensions of the Circle, it is the most distorted and preposterous Diagram in the whole Book. He finds no more than one Point, on each side, and leaves his Readers to find out the rest; nor does he go about it in a proper manner, there being no occasion (if the Distance was sufficient) to draw another Line (dO) at pleasure, in order to make another Angle, equal to DOE; but, to make equal Angles with EO and DO, on either side, continually. One thing I am much surprized at; he says, "make the Angle dOe equal to DOE; or, having made an Instrument, containing the Angle DEO, turn it round the Center, O, till it comes into the Position dOe," &c. which, is so ungeometrical, that I could scarce conceive it to be the expression of so great a Man.

Having had occasion, in this Problem, for a Vanishing Point (J, or R) which was not within the compass of the Picture, the next Problem shews how to determine the Distance of such Vanishing Points from the Center of the Picture, or Vanishing Line, and also from the Eye.

P R O B L E M XII.

The Vanishing Line of a Plane being given, and the place of the Eye, with a Line, from the Eye, inclining to the Vanishing Line; to find the Center of the Vanishing Line, and to determine the Distance of that Point, in which the inclined Line would cut the Vanishing Line, from the Center, and from the Eye.

K k

AB

Plate X.
Fig. 51.

A B is the Vanishing Line given, and E D is the inclined Line from the Eye, at E. D b is considered as the bounds, or limits of the Picture, and B is the Point in which E D, produced, would cut the Vanishing Line, A F, produced.

Draw E C perpendicular to the Vanishing Line, cutting it in C, its Center.

Take C G any part of C E, a half, a third, or fourth, as E D is less or more inclined to A B, and draw G F parallel to E D.

† 2. and 4.
of 6. El.

Then, as $CG : CE :: CF : CB$ †; that is, if C G be a third part of C E then C F is a third part of C B, and G F of E B; or, in whatever Ratio C G is of C E,

Or, drawn E A perpendicular to E D, cutting the Vanishing Line in A.

Then, find a third Proportional to A C and C E (Prob. 31. Geo.) it will be C B.

For, A E B is a Right Angle (by Construction) and E C is perpendicular to A B;

† 7. 6. El.

therefore, as $AC : CE :: CE : CB$ †;

Also as $AC : AE :: CE : EB$; i. e. E B is a fourth Proportional to A C, C E, & A E.

In Numbers, they are thus determined, by a Scale of equal Parts.

§ 9. 6. El.

First. Take A C and C E by the Scale; square C E, and divide the Product by A C, which, will give C B †; for A C, C E, and C B are three Proportionals.

† same.

Or, if the Triangle C F G be used, multiply C F by C E, and divide the Product by C G; the Quotient will be C B †. For, C G, C F, C E, and C B are four Proportionals.

Secondly. Since $AC : CE :: AE : EB$, consequently they are four Proportionals. Wherefore, multiply C E into A E, and divide the Product by A C, the Quotient will be E B, the Distance of the Vanishing Point, B, from the Eye.

If the Distance of the Picture be known (equal C E) and the Angle of the inclination to the Picture, of one Side of a right angled Object, be determined; the Vanishing Points, and their Distances, are determinable.

Let A B be the Horizontal vanishing Line, and C the Center of the Picture.

Draw C E perpendicular to A B, and equal to the Distance given, or determined.

† Theo. 11.

Make the Angle C E A equal to the Angle given (or to its Complement, according to which side the Line inclines) cutting the Vanishing Line in A, the Vanishing Point of one side †; from which all the rest are determinable.

Or, if the Distance, C E, be so great, that it cannot be laid down on the Picture (as is frequently the Case) take C G half, a third, a fourth, or any portion of C E, and proceed as before; making the Angle C G H equal to C E A, the given Angle. Then will C H be half, a third, &c. of A C, or whatever portion C G was taken of C E †; by which means, the distance of the Vanishing Point A, from the Center, C, is ascertained.

§ 4. 6. El.

Make H G F a Right Angle, i. e. draw G F perpendicular to G H, cutting the Vanishing Line in F; then will C F be also half, a third, &c. of C B, the distance of the other Vanishing Point (B) from the Center (C.) Also, G F will be the same portion of E B.

Thus may the real Distances, be found, and the place of the Eye transposed to the Picture, as it will be exemplified in Practice, when their real places cannot be had thereon; all which may be found arithmetically, as follows.

Let the Distance of the Picture, C E, be given, equal 6, 5 (Feet, or what you please) and let the Vanishing Point A be determined as above, or at discretion, on the Picture; viz. A C equal 2, 6, from which all the rest may be determined.

First; to find the Distance of the Vanishing Point, A, from the Eye.

† Prob. 3.
7. 6. El.

Square A C and C E, which being added together, the square Root of that Product will be A E †; for A E square is equal to A C added to C E square †.

† 20. 1.
§ Prob. 3.
7. 6. El.

Secondly; to find the Distance of the other Vanishing Point B, from the Center.

Square C E, and divide that Product by A C, the Quotient will be C B †.

For, $AC : CE :: CE : CB$. Also, $AC : AE :: AE : AB$.

Thirdly; to find the Distance of the Point B from the Eye.

Having obtained A E, by the first, multiply C E by A E, and divide that Product by A C, the Quotient will be E B; for $AC : CE :: AE : EB$.

The Distance of each Vanishing Point, A and B, from the Eye (E) are as necessary to be had as their places on the Picture, the use of which I shall exemplify and explain in its proper place.

First. AC and CE being given, to find AE; the Distance of A.

Square AC, viz. 2,6;	also, square CE, viz. 6,5	AE=7
multiplied by 2,6	multiplied by 6,5	
156	325	squared 49
52	390	
6,76	42,25	
Square of AC added 6,76		

Sum of both 49,01 (7, sq. Root; = AE.

Secondly. To find CB; AC and CE being given Thirdly. To find EB.

Square CE; 6,5	Multiply CE=6,5
6,5	by AE=7

divide by AC=2,6) 42,25 (16,25=CB divide by AC=2,6) 45,5 (17,5=EB.

Or; having found CB (by the 2nd) add the two Squares, of CE and CB, into one Sum; the square Root, of which, will be EB. (By 20. 1. El.)

When the two Vanishing Points, A and B, are known, to find the Distance of the Picture, CE; the Center being given.

Multiply CB by AC, and extract the square Root of that Product, it will give CE the Distance of the Picture; by Prob. 4. 7. 6. El.

To find the Distance, CE, geometrically; A and B being given, the vanishing Points of Lines at right Angles with each other, and C, the Center.

Draw CE, perpendicular to AB; and, having bisected AB, on F (the point of bisection) with the Radius AF, describe the Ark AE, cutting CE in E. CE is the Distance required, of the Picture, or of the Vanishing Line, AB.

For, AC:CE::CE:CB; consequently, AEB is a Right Angle; by 7. 6. El.

P R O B L E M XIII.

To draw a Line to a Vanishing Point which is not on the Picture,* its Distance from the Center being given, or some Line tending to the Point; the Vanishing Line of the Plane it is in, being also given.

Let AB be the given Vanishing Line, and C its Center.

Let I be the Intersecting Point of some Line; or the Representation of any other Point in the Line, given or found.

Fig. 51.
No. 1.

It is required to draw a Right Line, from the Point I, tending to B, which is out of the Picture; by the known Distance of the Vanishing Point B, from C, the Center of the Vanishing Line. Db is the limits of the Picture.

* By the Vanishing Point not being on the Picture is meant, only, that it does not fall within its prescribed Bounds or Limits. But the Picture may be imagined to be produced, as occasion may require, so that, the Vanishing Points are always supposed to be on the Plane of the Picture.

Take

Plate X.
Fig. 51.
No. 1.

Take CF equal half, a third, or any other equal part of CB .
Draw CI , and FJ parallel to CI . Make $FJ:CI::FB:CB$; i. e. make JF equal half, two thirds, or the same Complement of IC , as FB is of CB .
Draw IJ which will tend to the Point B .

Or; make CK to CI , as CF is to CB ; i. e. if CF be a third part of CB , make CK one third of CI . Join KF , and draw IJ parallel to KF .

DEM. First. In the Triangle CBI , because JF is parallel to IC , $CF:FB::IJ:JB$. - 2. 6. El.
Consequently, $CF:CF+FB$ (equal CB):: $IJ:IJ+IB$ (equal IB) i. e. as JF is to IC . 2. and 4. 6.
Therefore, IJB is a Right Line; for CBI is a right lined Triangle.
2nd. In the Triangle CBI , because $CK:KI::CF:FB$ (Con.) KF is parallel to IB . Q. E. D.

2ndly; by a Right Line tending to the Point B ; the Distance not being known.

Let IJ be a Line tending to the Point B , in the Vanishing Line, AB .
It is required to draw, from the Point a , a Line tending to the same Point.

Draw aC and aI , at pleasure, making an Angle IaC , cutting the Vanishing Line in any Point (C) and the given Line at I . Draw IC .

Take any Point (K) in IC , and draw KF parallel to IJ , cutting the Vanishing Line in F . Draw KL parallel to Ia , and join LF .

A Right Line, ab , drawn through a , parallel to LF , will tend to the Point B .

Note. The Point is given at a and σ , both without and between the given Lines; and the process would be the same if the Point was given on the other Side of the Vanishing Line; or any other Line, instead of the Vanishing Line, which tended to the same Vanishing Point.

DEM. In the Triangles IaC , $I\sigma C$, because KL is parallel to Ia , $CL:La::CK:KI$. - 2. 6. El.
But, $CK:KI::CF:FB$; for KF is parallel to IB , by Construction.
Consequently, $CL:La::CF:FB$; therefore, LF is parallel to aB . Q. E. D.

Or, the same thing may be done in another manner.

No. 2. Let AB and CD be two given Lines; E is a Point given, through which it is required to draw a Line tending to that Point, in which AB and CD would intersect.

Draw AC , at pleasure (through E) and BD , parallel to AC , at discretion.

Draw either Diagonal, as AD ; draw EF parallel to CD , and FG to AB , cutting BD in G ; a Right Line, drawn through E and G , will tend to the same Point with AB and CD .

DEM. In the Triangle ADC , because EF is parallel to CD , the Sides AC and AD , are cut proportionally, in E and F ; i. e. $AE:EC::AF:FD$; and because FG is par. to AB , $AF:FD::BG:GD$; wherefore, $BG:GD::AE:EC$; and consequently, AB , EG , and CD will tend to the same Point.
2ndly. When E falls without the two Lines. Because EF is parallel to CD , $AC:CE::AD:DE$; and because FG is parallel to AB , $BD:DG::AD:DE$, i. e. as AC is to CE ; therefore, &c.

Hence it appears, that nothing more is required than to find a fourth Proportional. For, having drawn a Right Line (AC) at pleasure, through the given Point, E , cutting any two Lines which tend to the same Vanishing Point; and, at a proper Distance, another (BD) parallel to the former (the farther off the better) BD (or LM) being divided, in G , as AC is divided in E ; then, if a Right Line be drawn through the Points E and G (or g) it must necessarily tend to the same Point; for Right Lines, proceeding from the same Point, cut parallel Lines proportionally. This method is the most eligible, because it is the readiest.

§ Cor. to
6. 6. El.

There are various ways of performing this; of which, those by Brook Taylor are very ingenious, and do not require any parallel Lines to be drawn.

No. 3 & 4. Let AB and CD be two given Lines, tending to one Vanishing Point (P)

It is required to draw a Line, through E , which shall represent a Line parallel to the Originals of AB and CD ; and consequently, it must tend to the same Vanishing Point; seeing they are not parallel.

First, when the Point (E) is between the two Lines.

Through the Point E draw two Lines, at pleasure, cutting the two given Lines, **No. 3.**
in A, G, C, and F; and, through A, C, and F, G, draw two Lines, meeting at a.

Draw a H and a B, at pleasure, cutting the given Lines in I, H, B, and D; join BI and DH, intersecting at K.

A Right Line drawn through E and K will tend to the same Point, with AB & CD.

2nd. When the Point E is situated without the two given Lines, AB and CD. **No. 4.**

Draw EA and EF at pleasure, cutting the two Lines, BF and CI, in C, A, F, and G; join AG and CF, intersecting at a.

Draw any other Line, BK, indefinite; and, at D, where it cuts CI, draw DH, through a; also, draw BI, through a.

Lastly, through H and I, draw HK, cutting BD produced, at K.

A Right Line, drawn through E and K, will represent a Line parallel to the Originals of AF and CI; and consequently it will tend to the same Van. Point, P.

Although Dr. Taylor has given Demonstration of all the Problems, previous to this, he has not favoured us with a Demonstration of it. But a little consideration will make it very obvious, on inspection of the Figure.

In No. 3, because AB and CD have the same Vanishing Point, they represent parallel Lines; and if a be considered as the Vanishing Point of AC, FG, &c. they, consequently, represent parallel Lines.

Wherefore, AFGC and IHBD represent Parallelograms; and consequently, E and K, the Intersections of their Diagonals AG, GF, &c. represent the Centers of those Parallelograms †.

And since the Parallelograms are between the same Parallels, their Centers are, consequently, equally distant from each; therefore, EK represents a Line parallel to the Originals of AB and CD.

In No. 4; because BF and CI represent parallel Lines, the Lines AG, CF, &c. which pass through the Point a are all cut proportionally in that Point; wherefore, the Originals of BH and ID, of AF and CG are in the same Ratio respectively, i. e. the Originals of BH:DI::AF:CG; consequently, EA:EC::EF:EG, i. e. as KH:KI; for, KH:KI::HB:ID; i. e. as AF:CG; viz. as EA is to EC.

Wherefore, since EA:EC::KH:KI, consequently, EC:CA::KI:IH; or, as any other Line drawn from K, whose Original is parallel to EA, would be cut, by BF and CI.

But, if KH be supposed parallel to EA, and AH to CI, CAHI represents a Parallelogram; wherefore IH is equal CA; consequently, IK represents a Line equal to EC; and consequently, EK represents a Line parallel to CI. For, ECIK also represents a Parallelogram.

† 16. 1. E.

N.B. Cor. 1.
2. 6. El.

5. If it be required to draw several Lines to a Vanishing Point which is out of the Picture, from various Points, in a given Line.

Let A, B, and C, be three Points in the Line, AC; let DE and FG be two **No. 5.**
Lines which tend to the same Vanishing Point, P.

At any Distance, at discretion, draw HL parallel to AC, and draw AH, DI, BK, and CL parallel to either Line (as FG) cutting HL, in the Points H, I, K, & L.

Assume any Point (G) in the same Line, FG, and draw GI, cutting DE in E; and, through E, draw a c parallel to AC and HL; lastly, draw GH, GK, and GL, cutting a c, in the Points a, b, and c.

Then, if Right Lines Aa, Bb, and Cc are drawn, they will tend to the same Point, P.

For, considering the Originals of AC and HL to be parallel to the Picture, and FG as the Vanishing Line of some Plane; because AH, DI, &c. are parallel to FG, and Ha, Ie, &c. have the same Vanishing Point (G) they are all in parallel Planes, of which FG is the Vanishing Line; and, because ac is parallel to AC and HL, the Original of ac is parallel to their Originals, and consequently may be in the same Plane with either (Ax. 5.) wherefore, Aa, DE, &c. represent parallel Lines, in the same Plane, AacC, or in parallel Planes AHa, DIE, &c. of which FG is considered as the Vanishing Line; and consequently, they vanish in the same Point in that Line.

There are various other Expedients might be given in this Section; but, I shall omit them, till they occasionally occur in the Work; when they will be more intelligible and better understood; and, being immediately applicable, in Practice, they will, at the same time that we acquire them, shew their use; by which means, they will be deeper rooted in the Mind, and the application of them, in future Examples, will be more familiar.

S E C T I O N V.

Of the PRACTICE of PERSPECTIVE, respecting
P L A N E F I G U R E S.

IN the last Section, I have exemplified and illustrated the Elements of Practical Perspective, according to Brook Taylor; which, notwithstanding they are so excellent in themselves, and of great utility in Practice, yet they do not lay a foundation whereon to begin; but teach, only, how to proportion one Part by another, either given or found, on the Picture; so that, a Novice, in these matters, cannot possibly apply them to real use. Nor, in my Opinion, would any Person ever be made a Practitioner, from that Treatise, unless he was endowed with an extraordinary Talent, and a very comprehensive Capacity; being quite conversant in Geometry, and particularly acquainted with the Doctrine of Proportion; having a clear Idea of Planes and their Intersections with each other, and of Right Lines cut by Planes.

Before I proceed to Figures, I have shewn in three Problems, how to find the perspective Representations of Right Lines, in the three Positions, parallel, perpendicular, and inclined to the Picture; having the Intersection and Vanishing Line of the Plane they are in (their Distance from the Picture being known and the Position of the inclined Line; that is, the Angle of its inclination to the Picture, or to the Intersection, and its intersecting Point, or the seat of some Point in it; or the Representation of some Point in the Line.) How to proportion them, that is, to cut off such Portions, or Parts, as are the true perspective Representations of certain Parts in the Originals, (in which the whole foundation of Practical Perspective consists) is contained in the last Section (Prob. 8th, 9th and 10th) and are exemplified in this.

Prob. 6th shews how to find the representation of a Point on the Picture, any how situated; its Seat on the Picture and its Distance from its Seat being given; which, in reality, contains the whole; as it will be found hereafter. I shall, in the next Problem, give a more familiar and introductory Lesson, how to find the representation of a Point, situated on the Ground Plane; which is, undoubtedly, the first Plane to be considered. At the same time, let it be observed, that there is not the least difference, in the Operation, between the Ground Plane and any other, whose Intersection and Vanishing Line is given, and its Distance known; as it will be shewn.

Having, therefore, in the preceding, and in the next four, Problems, given the Elements of the whole, and fully demonstrated it; I shall not trouble the Reader with the Demonstration of every Operation, in the following Work; but only in particular Cases, which may not readily be deduced from the foregoing; as it would only swell the Work to an enormous bulk, and would be of no use to the Practitioner. Therefore, where I see occasion, I shall refer to the elementary Problems, or Theorems, for Demonstration, and to shew how each particular Problem is applicable in Practice, in various Cases, in the course of the Work.

Let it, here, again, be observed, that whenever any Point, Line, or Figure, is given in the Original Plane (the Intersection of the Plane being also given) it is supposed to be so situated on the other side of the Intersection, as it would be, if the Original Plane was turned over, on the Intersection, to the other Side of the Picture, making the same Angle with it; and (if it be not perpendicular) inclined towards the same Part.

See Fig. 37; the Triangle XYZ , on the other Side of the Picture, is inverted, on this Side, for conveniency in Practice.

PROBLEM

P R O B L E M XIV.

To find the Representation of a Point, situate on the Ground Plane, or other horizontal Plane*, its real Place being given thereon; the Intersection of the Plane it is in being given.

There are several ways to find the representation of a Point.

Let *A* be an Original Point in the Geometrical Plane, of which, *BD* is the Intersection; *C* is the Center of the Picture, and *CE* its Distance; *E* is the Eye.

Plate XI.
Fig. 52.
No. 1.

Through *C*, draw *EF* parallel to *BD*, the Intersection; then is *EF* the Vanishing Line of the Original Plane; which being horizontal, consequently, *EF* is the Horizontal Line, or Vanishing-Line of the Ground Plane.

Now, since *BD* is the Intersection of the Original Plane, and *EF* is its Vanishing Line, if the Original Plane was produced infinitely, beyond the Picture, its whole perspective Representation would be between its Intersection and Vanishing Line; consequently, the Representation of any Point, Line, or Figure, in that Plane (beyond the Picture) must be somewhere between those Lines; to find which observe the following Rules. (See Introductory Preface; Page 104.)

Find *S*, the Seat of the given Point; i. e. draw *AS* perpendicular to the Ground Line or Intersection, cutting it in *S*; which is the Seat of the Point *A*; because it is in a Plane perpendicular to the Picture.

Draw *SC*, and *AE* cutting it, in *a*, the Point sought. (See Problem 6.)

For, *EA* is a Visual Ray from the Eye, at *E*, to the Point *A*, which would pass through the Picture at *a*, making *SA* to *aC*, as *AS* to *CE*†; therefore, *a* is the Representation of the Point *A*.

† Theo. 13.

METHOD 2ND. On *C* describe a Semicircle, with the Radius *CE*, cutting the Horizontal Line in *E* and *F*; which are the transposed places of the Eye (*E*) to the Vanishing Line. (See Fig. 37, No. 1, the Vertical Plane (*V*) being turned down, on either Side, into the Picture.)

Therefore, having made *CE* or *CF*, on either Side, equal to the Distance of the Picture, make *SB*, or *SD*, equal to *SA*, and draw *DE* or *BF*, both which will cut *SC* in the same Point, *a*, as before.

For *CE* is parallel to *SB* or *D*, and, *CE* is to *SB* or *SD*, as *CE* to *SA*, viz. as the Distance of the Picture, to the Distance of the Original Point from its Seat, i. e. from the Picture. (See Prob. 6.)

If *CE**, and *SF* be drawn perpendicular to *SC*, or any how, parallel between themselves, it is still the same. For *Fa* is to *aE**, as *SA* is to *aC*, as before.

Note. This Method is the most eligible of all other; for, the Distance of the Picture being known (as it is always supposed to be) and being placed on either Side of the Center, and *SB*, or *SD*, equal to the Distance of the Point, on the other Side, *ED* or *BF*, is a Visual Ray from the Eye to the original Point, as *EA* before.

N.B. If the Original Point be situated in the Station Line, as at *A*, the first method cannot be applied; because its Seat, *D*, being also in the same Line, *DC* and *EA* are one Line, and cannot intersect; in which Case, the Eye being transposed to *E*, on either Side, and *A* to *B*, then is *EB* a Visual Ray, from the Eye, on one side of the Picture, to the Point *A* (i. e. *B*) on the other Side (supposing the Vertical Plane, in which the Visual Ray must be, turned on *CD*, its Intersection with the Picture, direct before the Eye) in its true, geometrical proportion, or length.

* If the Intersection and Vanishing Line, with its Center and Distance, be given or found, of any Plane whatever, whether perpendicular or inclined to the Picture, the Process is the same, in every respect.

Plate XI.
Fig. 52.
No. 1.

If f be the Seat, on the Picture, of a Point in any other Plane, (suppose Vertical) then fG being drawn, parallel to CE , is its Intersection; and, having drawn fC , make fG equal to its Distance, and draw GE , cutting fC in g , the Representation of G ; i. e. of a Point situated beyond the Picture at the Distance fG . Thus the Representation of any Point may be obtained, knowing its situation and Distance from the Picture.

No. 2.

METHOD 3rd. Through the given Point, A , draw AB and AD , at pleasure, cutting the Intersection in B and D .

From E , the Eye, draw EF and EG , parallel, respectively, to AB and AD , cutting the Vanishing Line in F and G , the Vanishing Points of AB and AD . (Def. 1.)

Draw BF and DG , intersecting in a , the Representation of A .

† Cor. 7.
Theo. 12.

For, BF is the indefinite Representation of BA , and DG of DA ; consequently, the Point, in which they intersect, is the Representation of the Point in which the Lines BA and AD intersect†.

N. B. The 2nd Method will be found, on inspection, to be the same as this; for, if EE and AD be drawn, they will be parallel, and EC is parallel to AS ; also CE is parallel to SD ; consequently, C and E are the Vanishing Points of AS and AD respectively. So that, to find the Representation of a Point is to find the Representation of a Line; seeing, it must be supposed in some Line.

No. 3.

METHOD 4th is rather a matter of curiosity than real use, being seldom, if ever, practised; it may be performed without either Intersection or Vanishing Line; nothing more being required than the Place of the Eye, and the Original Point; their Distances from the Picture being known, or the Ratio of their Distances.

On A the given Point, with the Radius AS , its Distance from the Intersection, or from the Picture, describe an Ark, BSD ; and, on E , with the Radius EC , the Distance of the Vanishing Line, or of the Picture, describe the Ark FCG .

Draw the Tangents, BF and DG , intersecting in a , the Representation of A .

Or, if the Radius be taken, in the ratio of the Distances, respectively, it will be the same, i. e. the Tangents will cut each other in the same Point, a .

DEM. Draw AB and AD , EF and EG perpendicular to the Tangents, BF and DG .

Then, the Trapezium $ABaD$ is similar to $aFEG$; for $aB=aD$ and $aF=aG$; (C. 2. 16. 3. El.) also $AB=AD$, and $EF=EG$; the Angles at B , D , F , and G are Right, and $BaD=F a G$ (2. 1. El.) consequently, $BAD=FEG$. (See Th. 1. to 1. El.)

Wherefore, $Ba:aF::AB:EF$; i. e. as $AS:EC$; or, as Aa is to aE (which are corresponding Diagonals) as in the first Method (No. 1.) for, AE is a visual Ray, in both, and a is the Point in which it cuts the Picture.

P R O B L E M XV.

To find the Representation of a Line perpendicular to the Picture; its Place being given in the Original Plane, the Intersection and Vanishing Line of the Plane it is in, and the Place of the Eye.

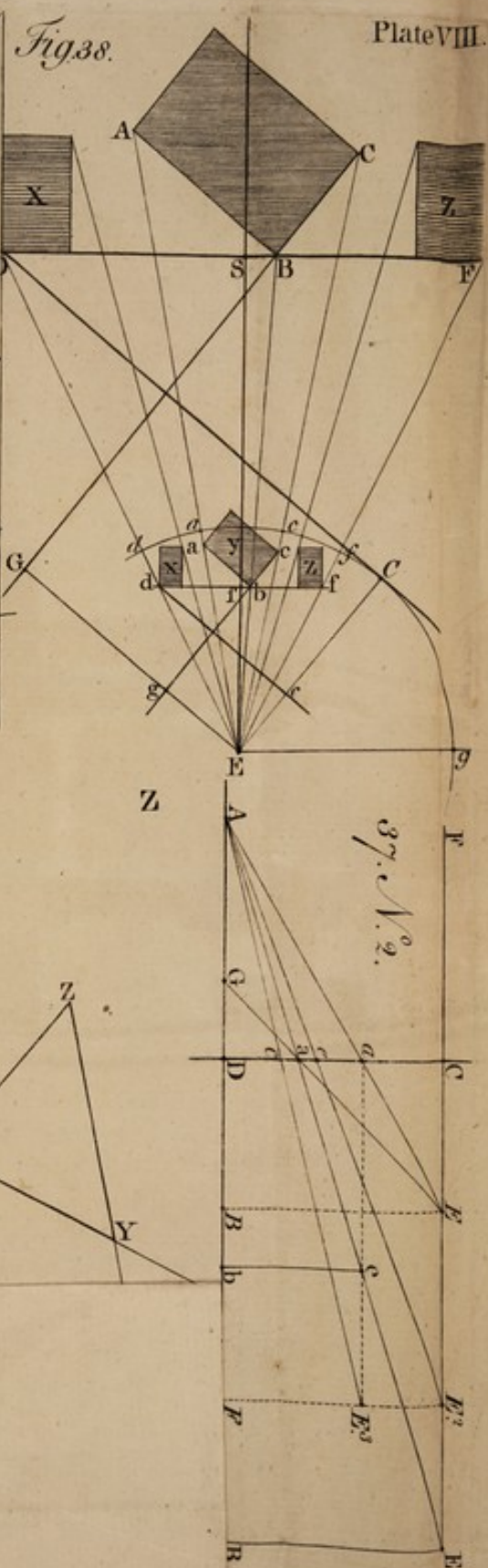
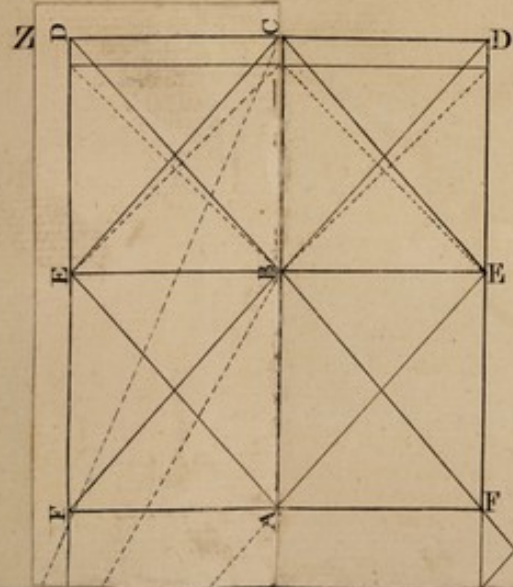
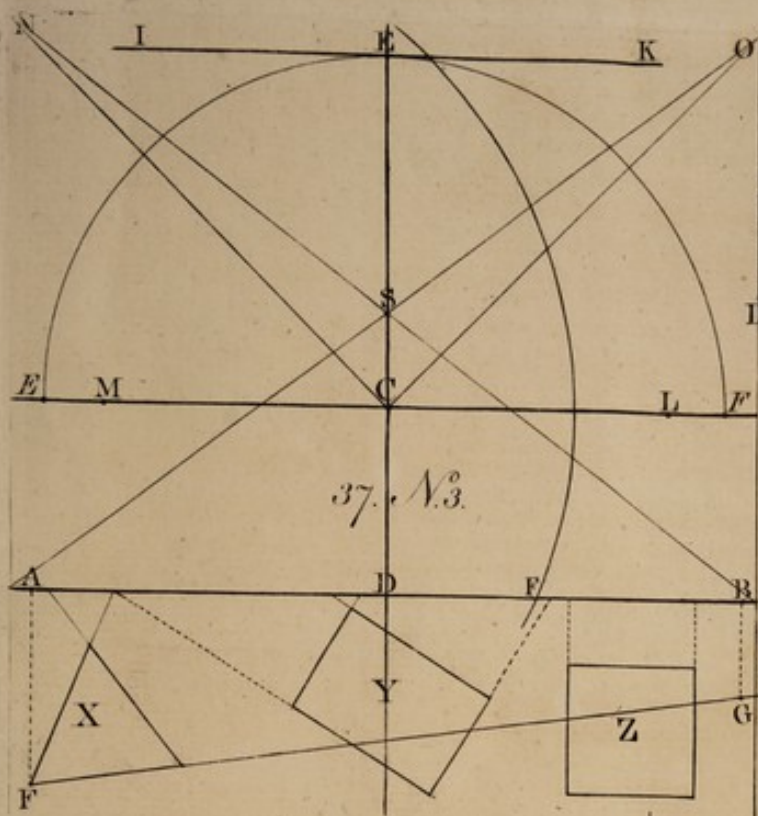
Fig. 53.

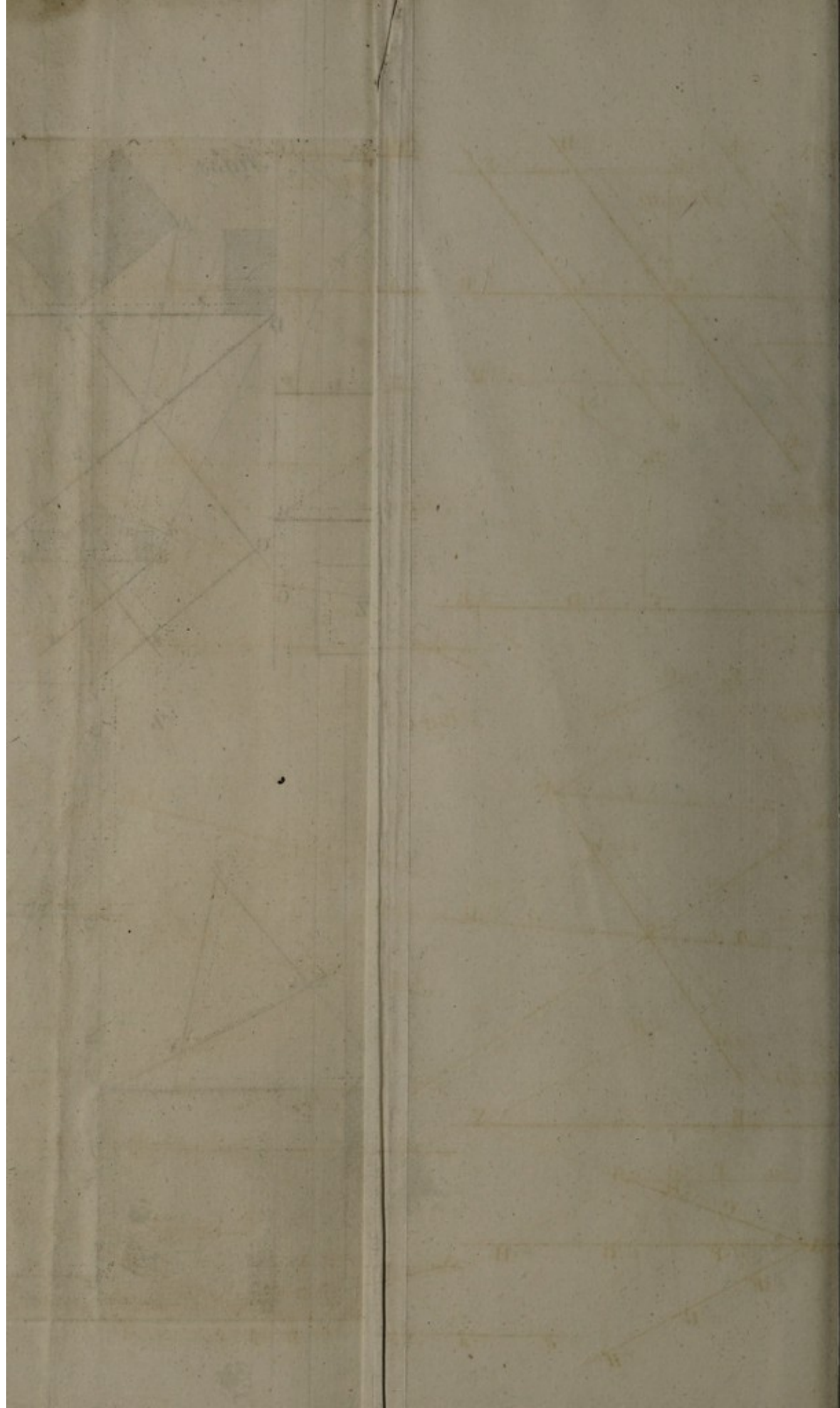
Let AB be the Original Line; AD is the Intersection of the Plane it is in, and ECE is the Vanishing Line; E is the place of the Eye.

Draw EC perpendicular to the Vanishing Line, cutting it in C , the Center; and, because AB cuts the Intersection, A is its Intersecting Point.

Draw AC , and BE cutting it in b ; then is Ab the Representation of AB .

Or (having drawn AC), make CE equal to the Distance of the Picture, and AB equal to the Original Line (AB) and draw BE , cutting AC , in the same Point, b . In which process, the measures, it is obvious, must always be placed on contrary Sides of the indefinite Representation, or the Visual Ray cannot cut it.





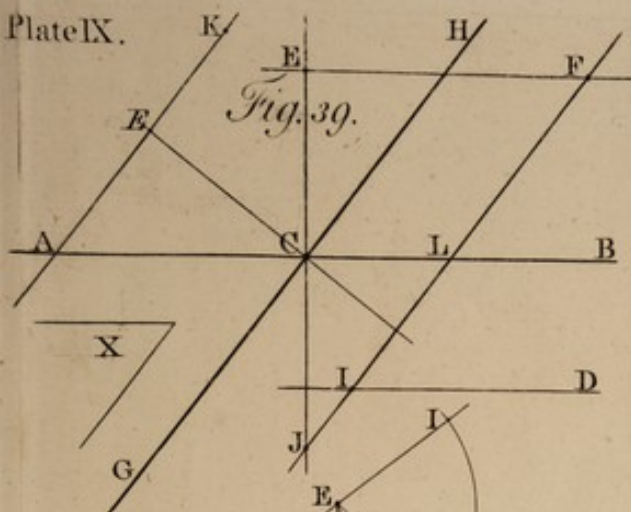


Fig. 39.

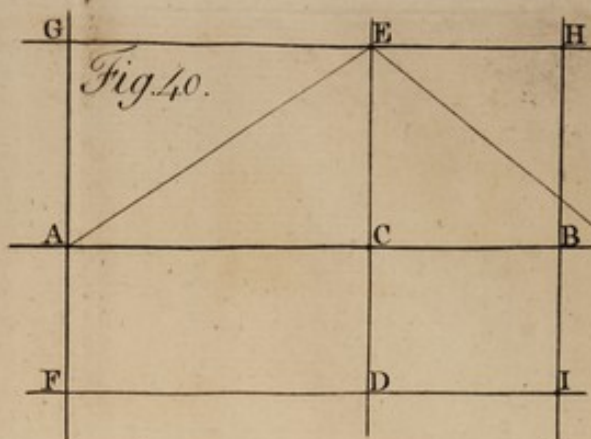


Fig. 40.

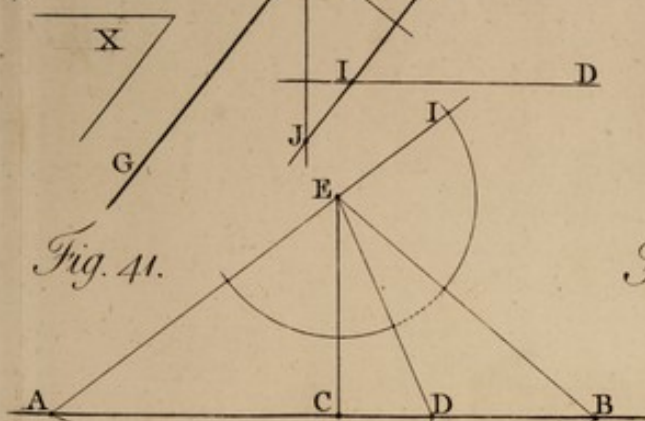


Fig. 41.

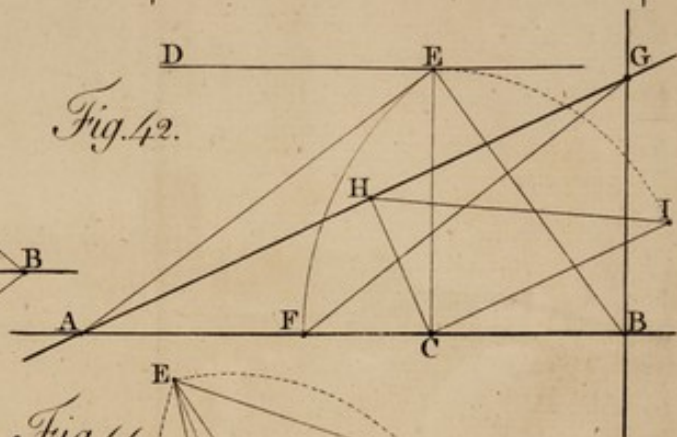


Fig. 42.

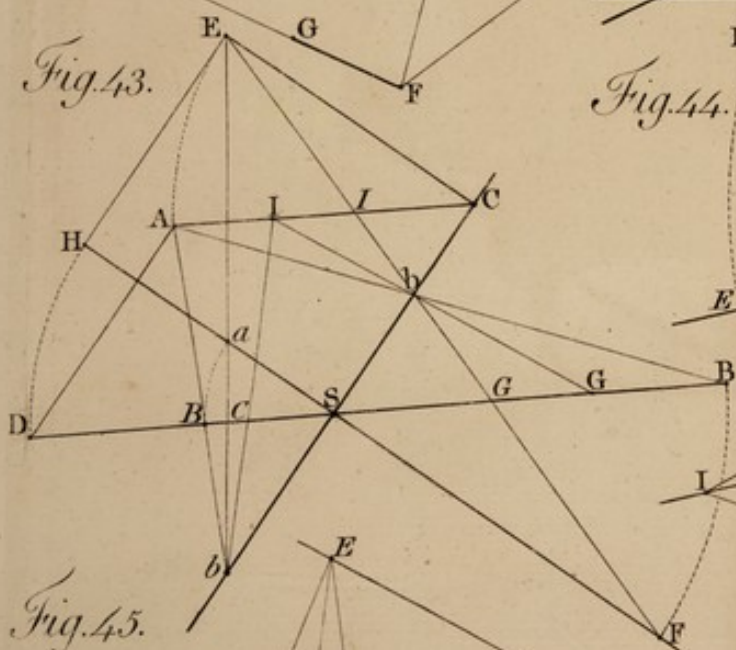


Fig. 43.

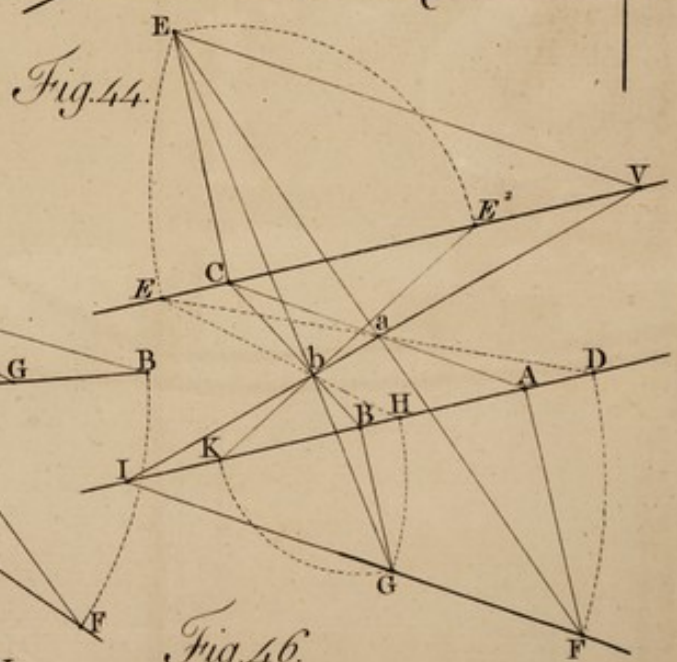


Fig. 44.

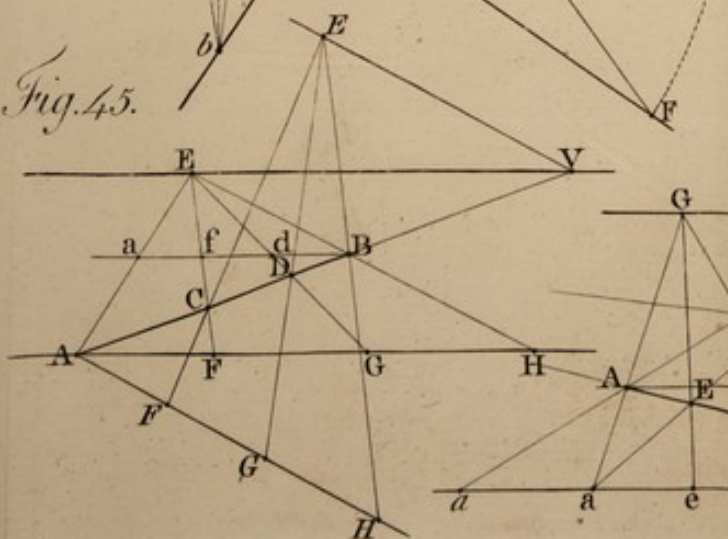


Fig. 45.

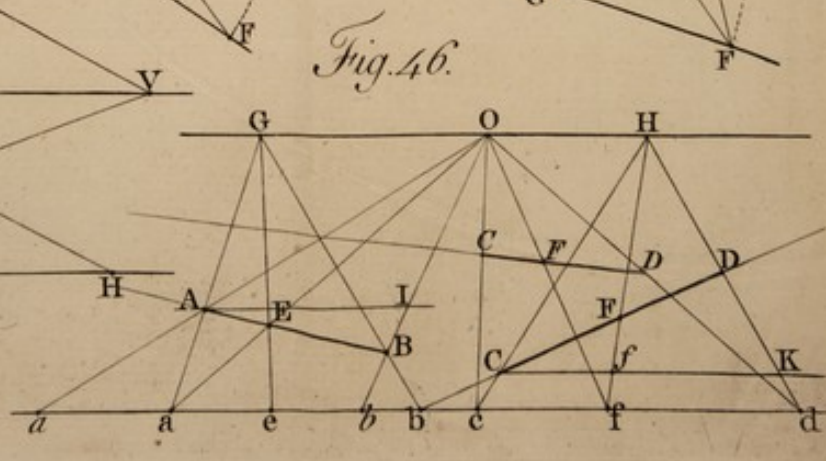


Fig. 46.

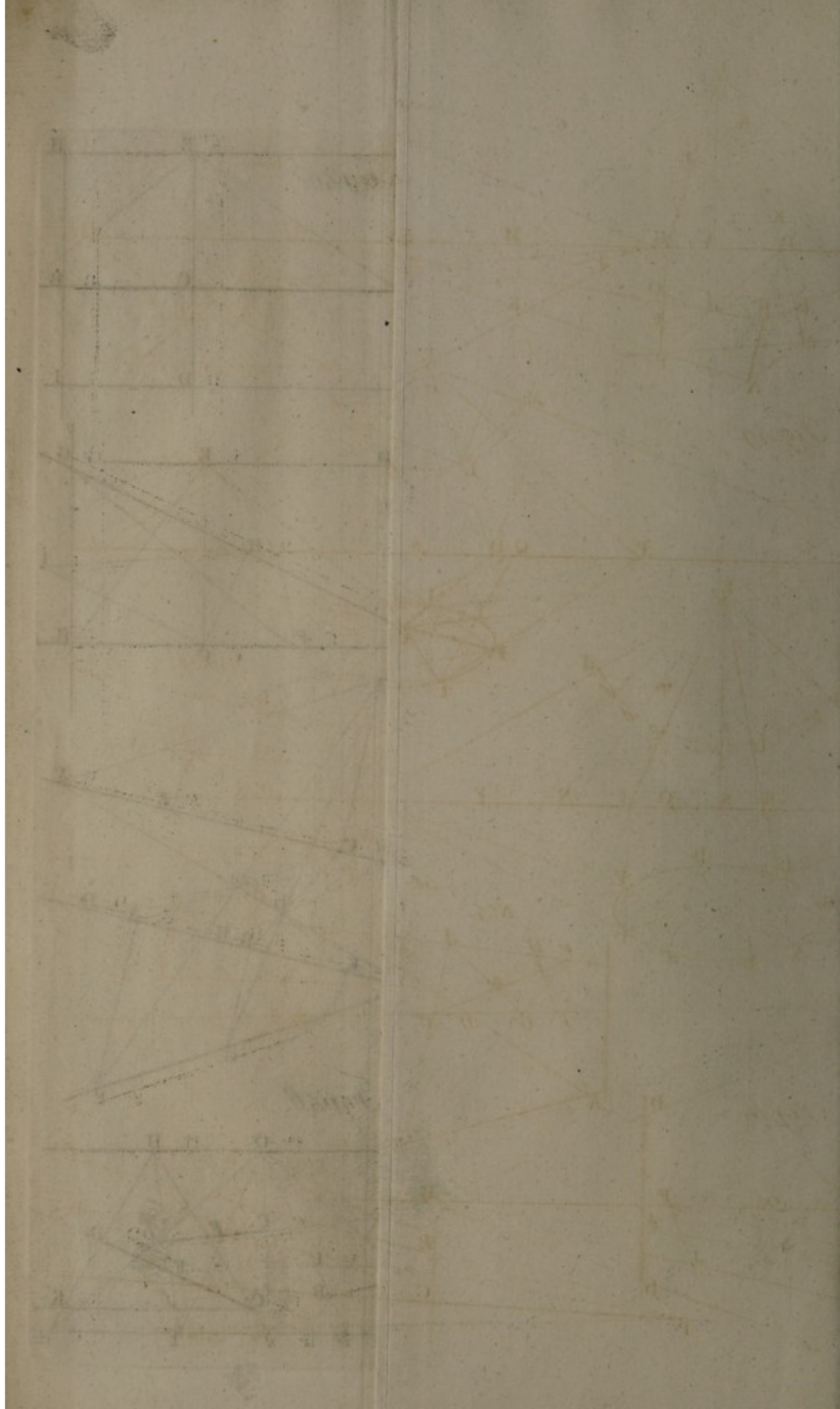


Fig. 47.

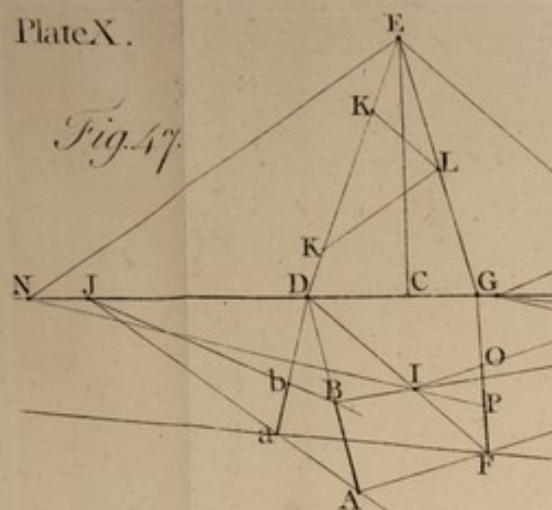


Fig. 48.

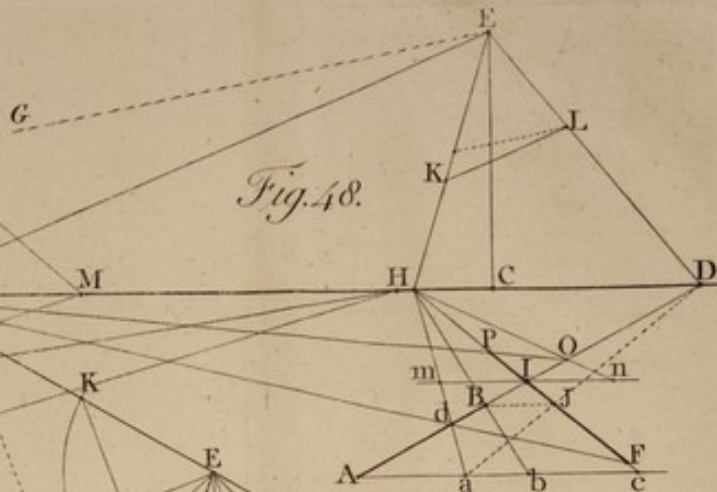


Fig. 49.

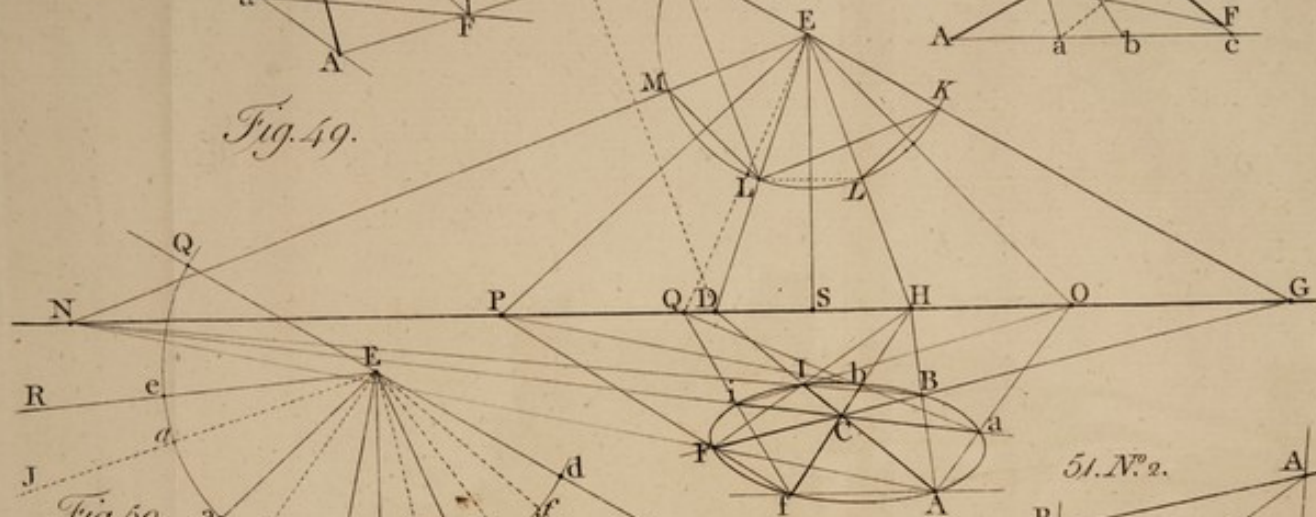
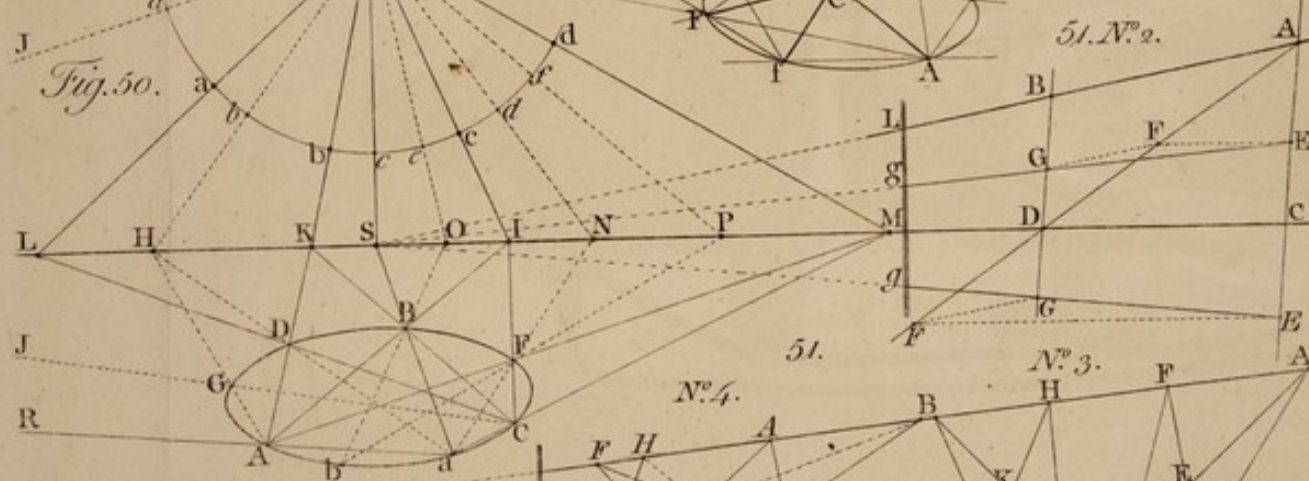


Fig. 50.



51. N° 2.

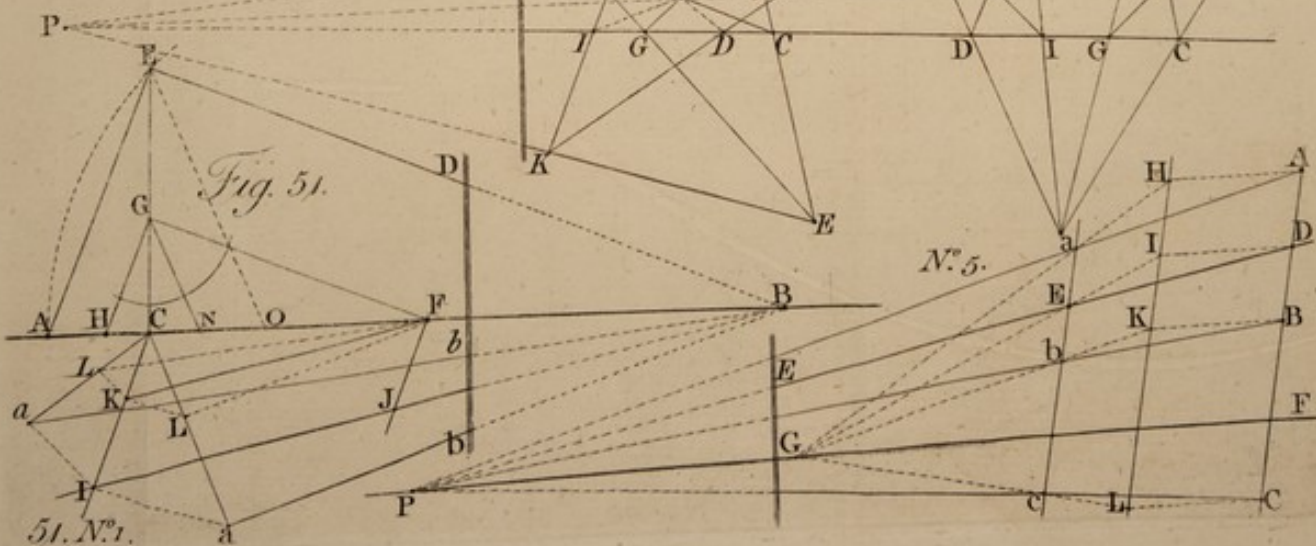
51.

N° 4.

N° 3.

N° 5.

Fig. 51.



51. N° 1.

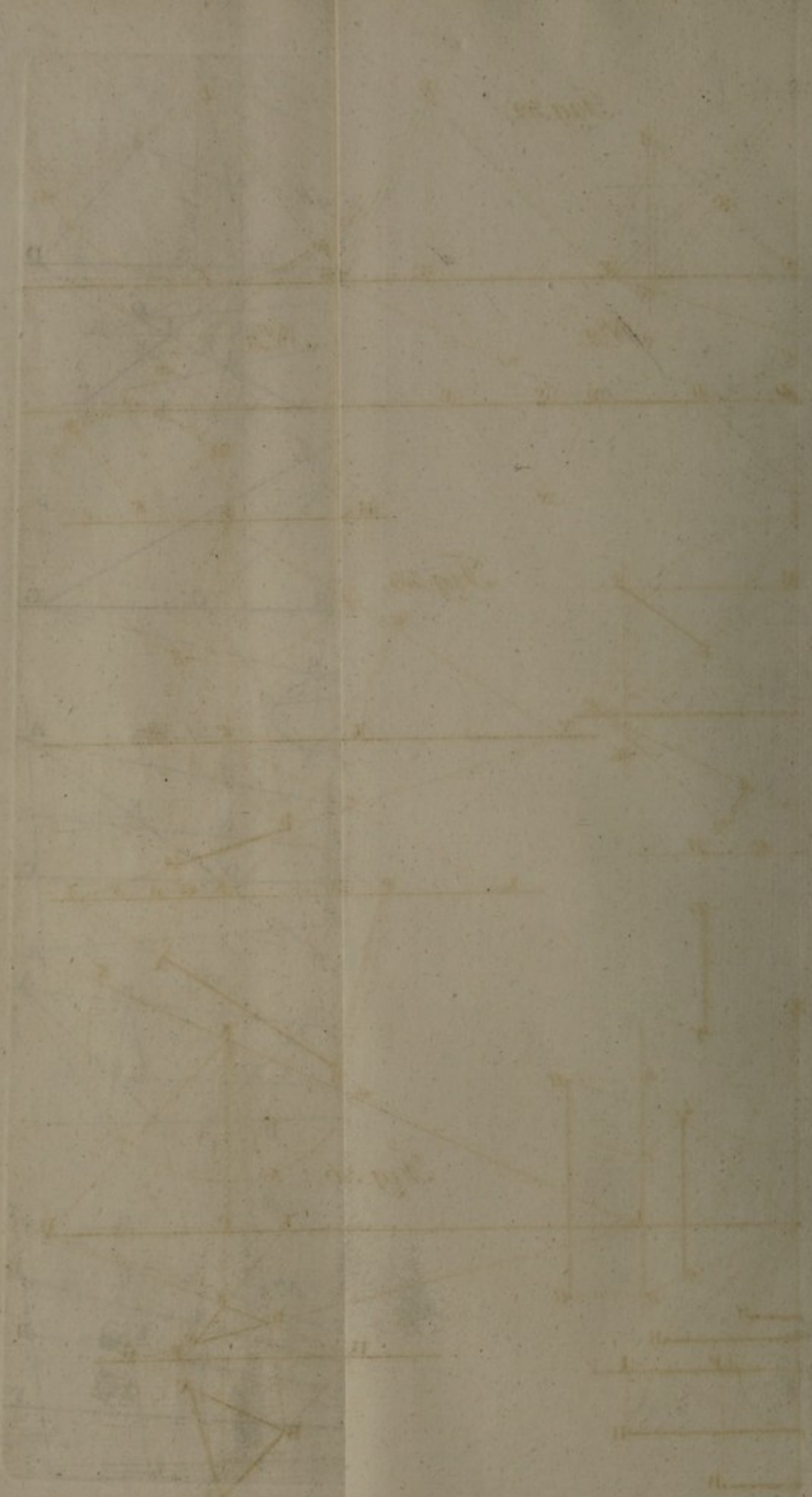


Fig. 52.

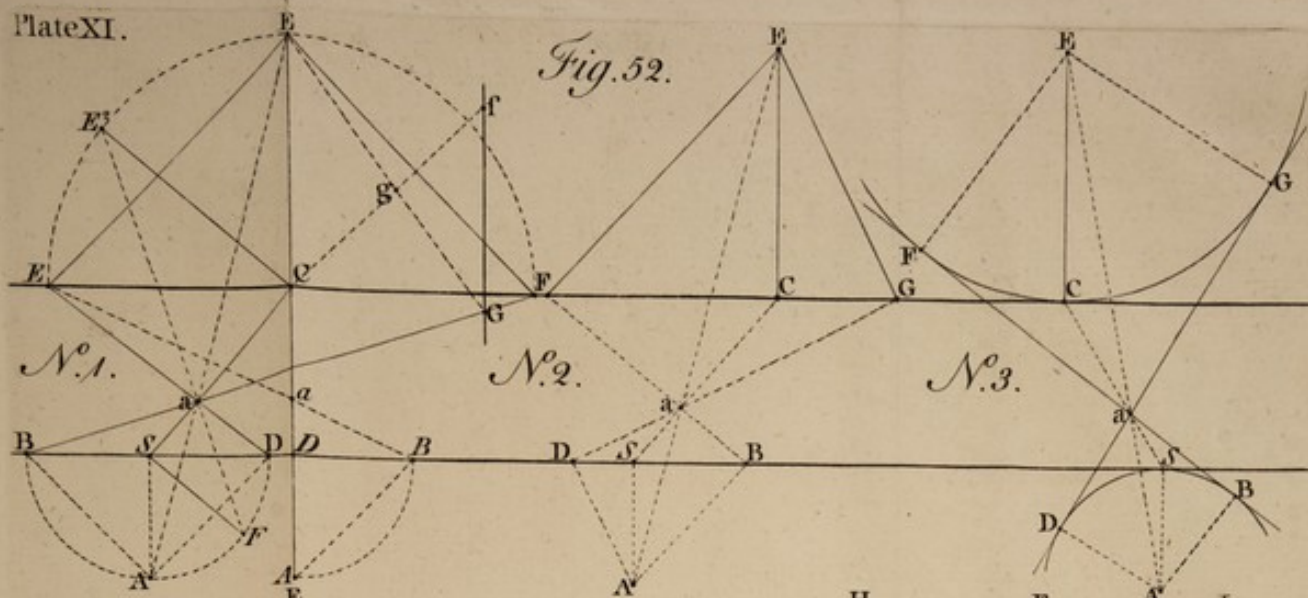


Fig. 53.

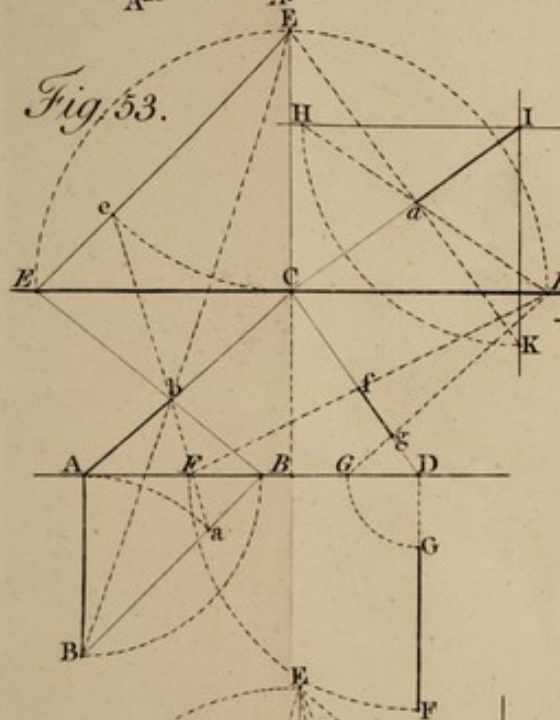


Fig. 55.

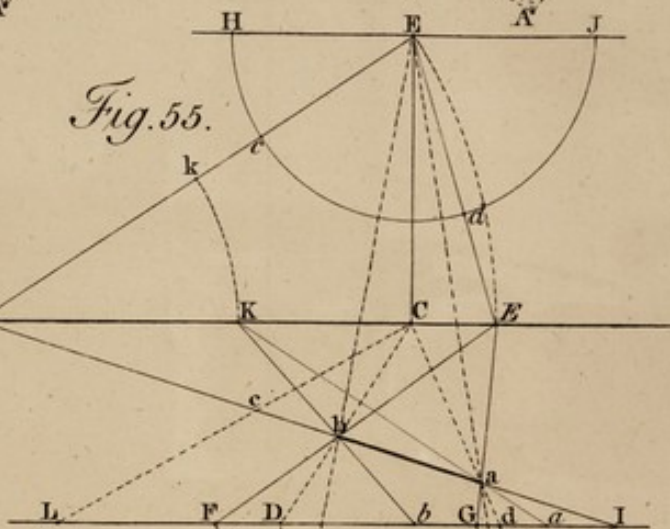


Fig. 54.

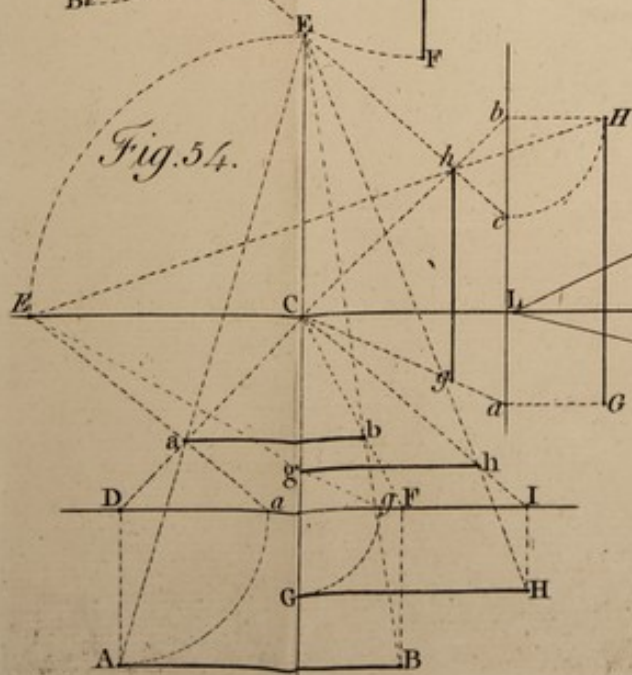
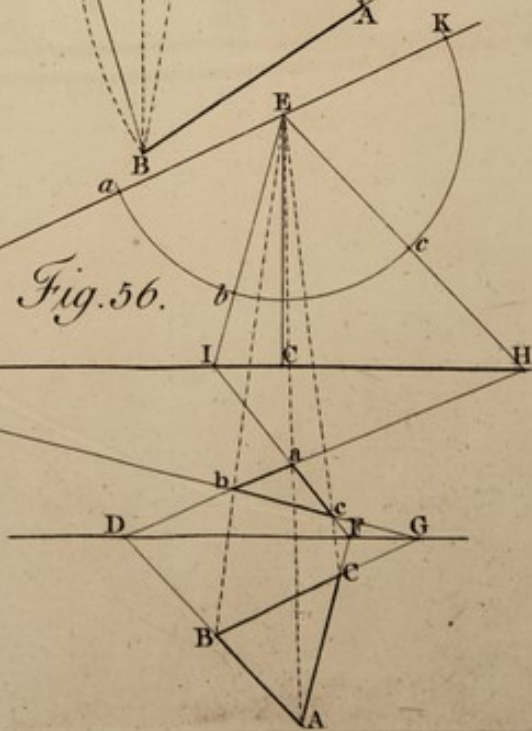


Fig. 56.





DEM. Because AB, the Original Line, is perpendicular to the Picture. EC, the Direct Radial, is its Radial; wherefore, C, the Center of the Picture, is its Vanishing Point †; and, because A is its Intersecting Point, AC is the indefinite Representation of AB, infinitely produced (by Theorem 12th.) But, EB (or EB) is a Visual Ray, from the Eye (E) to some Point (B) in the Line; which must cut the Picture somewhere in the Line AC; which is the Section of a Radial Plane, passing through the Original Line and the Eye †; and consequently it will cut it in b; making $Ab:BC::AB:CE$.
 † Therefore, since A, one extreme of the Line AB, is in the Picture, and b is the Representation of B, another Point in the Line, Ab is the perspective Representation of AB. Q. E. D.

† Def. 22.

† Theo. 8.

To illustrate this, ocularly (See Fig. 37, No. 1.) turn up the Picture AIKB, vertical; also turn over the Plane W, till C coincides with the Center of the Picture, then E will be in the true place of the Eye, and EC is perpendicular to the Picture, consequently parallel to all other Lines, which are perpendicular to the Picture; therefore, to AC and FD, &c. on the other side of the Picture; consequently, C is their Vanishing Point. (Def. L.)

Turn over the Plane AFDC, or X, till it coincides with W; then, the Plane WX is a Radial Plane, passing through the Original Line, AC, and the Direct Radial, EC, its Parallel, consequently through the Eye; producing, by its Section with the Picture, the Right Line RC; which is the indefinite Representation of AC; (Theo. 12) for, it passes through R, the intersecting Point of AB, and also through C, its Vanishing Point.

Now, EA, EB, and EC, are Visual Rays, from the Eye to the Points A, B, and C; and since they are Right Lines (Def. H) and the two Extremes, E, A, &c. are in the Radial Plane, WX, consequently, the whole Line, EA, &c. are in that Plane (Ax. 1.) and consequently, they will cut the Picture, somewhere in RC, the Intersection of the Plane they are in.

Therefore, in a, b, and c; as by Theorem 13th.

N. B. Any Point (B) in the Line AB, may be found, by drawing BB, at pleasure, and EE parallel to it; cutting the Intersection and Vanishing Line, in B and E, and drawing EB; as in Prob. 14.

Or, having drawn BB, at pleasure, and EE parallel to BB; make Ba to Ee, as AB to EC (equal or otherwise) and draw ae; which will cut AC in the same Point, b.

If the given Line does not cut the Intersection, as FG, it must be produced till it does, as at D; and then proceed the same as before; making DG equal DG, and DF equal DF; draw DC; and EF, EG, cutting DC in f and g; fg is the Representation of FG.

If I be the Intersecting Point of a Line, perpendicular to the Picture, whose Representation is required; draw IC; and IH, or IK, parallel to either the Horizontal or Vertical Line; make IH, or IK, equal to the Distance of any Point, in the Original Line, from its intersecting Point, and draw HE or KE, cutting IC in a. Ia is the Representation of a Line, perpendicular to the Picture; from its intersecting Point, I, equal to IH or IK.

P R O B L E M XVI.

To find the Representation of a Line, parallel to the Picture; in a Plane which is perpendicular to the Picture.

Let AB be an Original Line, in the Ground Plane, ECL is the Vanishing Line of the Ground Plane, and DI is its Intersection with the Picture. Fig. 54.

If the Original Line was applied close to the Picture (as DF) it would have its full Dimensions delineated thereon, and be its own Representation.

But, if AB be situated at some Distance from the Picture (as AD) draw AD and BF, perpendicular to the Intersection, cutting it in D and F, the Seats of the extreme Points, A and B. Draw DC and FC.

M m

Having

Plate XI. Having made CE , or CE , equal to the Distance of the Eye, and Da equal
Fig. 54. DA , draw AE or aE , cutting DC in a , the representation of A .

A Right Line, ab , being drawn, parallel to the Interfection, or Vanishing Line, till it cuts FC , is the Representation of AB .

DEM. Because AD and BF are Lines perpendicular to the Picture, C , the Center, is their Vanishing Point (Prob. 15.) and, D and F are their Intersecting Points†; consequently, DC and FC are the indefinite Representations of AD and BF ‡.

† Theo. 10.
‡ Theo. 12. But EA or Ea is a Visual Ray, from the Eye to the Original Point A ; which, it has been proved, would cut the Picture, somewhere, in the Indefinite Representations, DC , making $Da::aC::AD::CE$, &c. by the thirteenth Theorem.

§ Theo. 9. But, the Representation of a Line, parallel to the Picture, is parallel to its Original§; and consequently to the Vanishing Line of the Plane it is in; and has that proportion to the Original, as the Distance of the Picture, to the Distance of a Plane passing through the Original Line parallel to the Picture; which shall be proved here.

|| 15. 1. EL. Now, $DF=AB$ ||; and, in the Triangle DCF , because ab is parallel to DF , the Triangle aCb is similar to DCF ; wherefore, $Ca:CD::ab:Df$, i. e. as ab is to AB , equal DF . (2 & 4. 6. EL.)

But $Ca:aD::CE:AD$; consequently, $Ca:CD$ (i. e. $Ca+aD$):: $CE:CE+AD$.

But, CE is equal to the Distance of the Picture, and AD is equal to the Distance of the Original Line, beyond the Picture; wherefore, $AD+EC$ is equal to the Distance of a Plane, passing through the Original Line, parallel to the Picture.

Therefore, ab , the Representation, is to AB , the Original Line, as the Distance of the Picture, CE , is to $CE+AD$. Q. E. D.

N. B. The Representations, a and b , of the Extremes, A and B , of any Line, may be found, by any of the Methods for finding the Representation of a Point, in Problem 14th.

Any other Line, as GH , parallel to the Picture, is found by the same means.

Or, being situated in any other Plane, as GH , whose Interfection, ab , and Vanishing Line, ECG , is given, or found; whether the Plane it is in be perpendicular to the Picture, or inclined, having the Center and Distance of the Vanishing Line, or of the Picture, the Process is the same.

a and b are the Seats of G and H , and EH or Ec are Visual Rays; the rest is obvious, on inspection of the Figure. But, if the Original Plane be inclined to the Picture, a and b are not the Seats of G and H .

P R O B L E M XVII.

To find the Representation of a Line, any how inclined to the Picture, situated in a Plane perpendicular to the Picture; having the Interfection and Vanishing Line given.

Fig. 55. Let AB be an Original Line in the Ground Plane, whose Interfection is LI , and Vanishing Line, VC ; C is the Center of the Picture.

Draw CE , perpendicular to the Vanishing Line, and equal to the Distance of the Picture. Produce BA , to its intersecting Point, I ; and draw EV , parallel to AB , cutting the Vanishing Line in V , the Vanishing Point of AB .

Draw IV , the indefinite Representation of AB ; and the Visual Rays, AE , BE , being drawn, will cut it in a and b , the Representations of the two Extremes, A and B . ab is the finite perspective Representation of AB .

Or, the Seats of the extreme Points being found, by drawing Ad and BD , perpendicular to the Interfection; dC and DC , being drawn, project the same Points a and b , as before.

Or, make VE equal to VE , IG equal IA , and IF equal to IB .

Draw GE and FE , cutting IV in the same Points, a and b .

DEM.

DEM. Because EV is a Right Line, from the Eye, parallel to the Original Line, and cuts the Picture in V , V is the Vanishing Point of the Line AB ; and I being its intersecting point, IV is its indefinite Representation; a and b are the Intersections of the Visual Rays, with the Picture, as proved in the former Problems. Therefore, ab is the Representation of AB .

† D. f. 22.

† Theo. 12.

GE and FE also project the same Points, a and b .

For VE is parallel to FI , and equal to EV , the Distance of the Vanishing Point; also, IG is equal to IA , and IF to IB ; by Construction.

Wherefore, if EE and BF be drawn, they will be parallel; for the Triangles VEE and BIF , are similar Isosceles; and since IF , IB are respectively parallel to EV and VE , conf. BF is parallel to EE .

Therefore, E is the Vanishing Point of BF ; and conf. $Fb:bE::Bb:bE$, i. e. as $1b:bV$, &c.

The last Method of proportioning inclined Lines will be found the most convenient, in Practice, of all other; and it may also be observed, that if the Representation, a , of any Point, A , in the Original Line, be given or found, and the Intersection of the Plane it is in; the Representation of any other Point (B) in that Line, may be determined, its Distance from A being known, without the Intersecting Point I .

Also, the Inclination of AB being known (equal BIF) the Vanishing Point, V , may be found; by making the Angle HEV , with the Parallel of the Eye (HJ) equal BIF ; or CEV equal to its Complement; or, if EE be the Radial of any other Line whose Vanishing Point is E , make the Angle EEV equal to the Angle which the two Lines make with each other; all, which, produce the same Point V .

Then, having made VE equal VE , the Distance of the Vanishing Point from the Eye; and, if a be the known Representation of some Point A , in the Original Line (AB) and FI the Intersection of the Plane it is in, draw the indefinite Representation, aV , from that Point, and draw Ea cutting the Intersection, in G ; make FG equal to the Distance of the Point B from A , and draw EF , cutting the indefinite Representation, aV , in b , the Representation of B .

If the Vanishing Point, V , be out of the Picture, having found its Distance from the Center, C , and from the Eye, E (by Prob. 12) CE will be equal to the Difference; i. e. make CE equal VE , less by VC .

Or, when the Vanishing Point is in the Picture, and the Point E cannot be had thereon (as is frequently the Case, when the Vanishing Point falls near the Center of the Picture, as E) take any Portion of VE , viz. VK half VE ; draw Ka , cutting the Intersection in a ; make ab equal half AB ; or the same Portion VK was taken of VE , draw Kb , which will project the same Point, b , for the Representation of B , as before. This Expedient will be found, frequently, necessary in Practice.

If the Representations, a and b , of any two Points, A and B , in any Line, be found; their Seats, d and D , are ascertained by drawing Ca and Cb , from the Center, to the Intersection (FI) of a Plane perpendicular to the Picture, in which the Original Line is situated, cutting it in d and D .

By which means, the Representation of any other Point, in that Line, may be found; making DL to Dd , as the Distance of the Point from B , is to AB , and draw CL , cutting aV in c , the Point sought. (See Prob. 8.)

SCHOL. After some one or other of these Methods, all Lines inclined to the Picture are drawn on the Picture, and divided, perspective, as the Original is divided; either by the real measures, being applied, or by the known ratio of the Originals. Various Expedients for performing the same thing will be found necessary, because the same Method cannot always be applied, in all Cases; for which reason, I have given all the Methods, I would devise, which are really useful.

These Methods of proportioning inclined Lines are general and applicable in all Cases whatever; and, on inspection of the Figures, they will be found the very same as for Lines perpendicular to the Picture. For, the Center of the Picture being the Vanishing Point of all such Lines, its Distance is laid down, on either Side as is most convenient, on the Vanishing Line, from the Center; and the Lines, which vanish in the Center, are proportioned by the same means. (See Prob. 15, Fig. 53.)

When

Plate XI. When the Vanishing Point, V (being remote) cannot be had, on the Picture, the transposed place of the Eye, E , may be thus found.

Fig. 55.

The inclination of the Original Line, to the Intersection, being known, subtract it from 180 Degrees, or two Right Angles; take half the sum of the remainder, and make the Angle VEE equal to it.

Or, through E , draw HEJ , the parallel of the Eye, parallel to the Vanishing Line; make HEk equal to the Angle of Inclination of the Original Line; bisect the remainder, kEJ , by the Line EE , cutting the Vanishing Line, at E , the transposed place of the Eye, for the Vanishing Point V .

§ 9. 1. El.

For, the Triangle EEV is Isosceles (EV being equal VE) consequently, the Angle $VEE = VEE$ & equal EEJ , by construction, and by *ar. 1. of Elem. mts.*

Hence it is evident, that Ib is the perspective Representation of IB .

† Cor. to

Theo. 1.

‡ Def. 22.

For, suppose BIF to be the Original of the Triangle bIf ; one, Side (IF) being in the Picture, therefore, it has, no Vanishing Point. The Triangle BIF is Isosceles, and similar to EEV .

Now, EV is parallel to the Side IB , and EE to BF ; wherefore, V and E are the Vanishing Points of those two Sides IB and IF , FE their indefinite Representations; consequently, Ib and If represent equal Lines. IF , being in the Picture, retains its full dimensions, and is its own Representation on the Picture; and, Fb represents the other Side, FB ; therefore, Ib represents IB ; for, b represents B .

† C. 7. T. 12.

P R O B L E M XVIII.

How to find the perspective Representation of a Triangle, having the Original given, in any Plane, whose Intersection and Vanishing Line is also given, and the Place of the Eye.

Fig. 56.

ABC is the Original Triangle, DG is the Intersection, and LH the Vanishing Line; E is the Eye.

Produce the three Sides, AB , AC , and BC , to the Intersection, cutting it in D , F , and G , the intersecting Points of the Sides, respectively.

Draw EH , EL , and EL , respectively parallel to AB , AC , and BC , cutting the Vanishing Line in the Points H , I , and L , their Vanishing Points.

Draw the indefinite Representations, DH , FI , and GL , cutting each other in the Points a , b , and c , and giving the representation of the Triangle by their Intersections.

This is manifest, seeing that the Sides of the Triangle, are in the Lines AD , AF , and BG ; consequently, their Representations must be in the indefinite Representations of those Lines, and consequently, between their Intersections (by Theorem 8. and 12.)

† Cor. 3.

1. 1. El.

‡ 10. 1. El.

N. B. The Angles LEI , IEH , and HEK , at the Eye, are equal, respectively, to ACB , BAC , and ABC ; and they are also equal to two Right Angles; and so are the three Angles of the Triangle.

Hence, if any one Side of the Triangle, as ab , be given, on the Picture, the Vanishing Points, I and L , of the other two Sides, are determined by Problem 4th; and, by drawing Ia and Lb , intersecting at c , the Triangle is completed.

See this illustrated, by moveable Planes, in Fig. 37.

§ Def. 14.

‡ Def. 22.

Raise up the Picture $AIKB$ perpendicular to the Ground Plane, and let the Horizontal Plane be placed parallel to the Ground Plane, i. e. perpendicular to the Picture; or, if the Picture be inclined to the Ground Plane, the Horizontal Plane making an equal Angle with the Picture, as that makes with the Ground Plane, will be parallel to the Ground Plane. In which Case, the Lines EN , EO , and EL are the Radials of the three Sides of the Triangle XYZ , on the other Side of the Picture; being respectively parallel to them. Consequently, the Points N , O , and L , are their Vanishing Points; B , P , and S are their intersecting Points, and BN , PO , and SL are their indefinite Representations, producing the perspective Representation, xyz , by their Intersections.

Let the Picture be turned down, and imagine the Geometrical Plane turned over on its Intersection, AB , to the other Side; in which Case, the Triangle is inverted. Turn down the Horizontal Plane into the Picture; the Radials EN , EO , and EL , are still parallel to the three Sides of the Triangle, and produce the same Vanishing Points, N , O , and L ; also, the Visual Rays EX , &c. being drawn on the Picture, pass through their respective representations, x , y , z ; as it is obvious they would pass through the Picture, in those Points, to the Original Points X , Y , and Z , on the other side.

N. B. The Angle NEO, which the Radials EN and EO make, at the Eye, is equal to the Angle Y, of the Triangle; for, they are respectively parallel to them. EO and EL, the Radials of YZ and XZ, make the Angle OEL equal to the Angle Z; and if NE or LE was produced beyond the Eye (E) they would make an Angle equal to X; which is consonant to Cor. 1st, Theorem 6th.

P R O B L E M XIX.

How to find the Representation of a Square or other Rectangle, given in the Geometrical Plane; one Side being parallel to the Picture.

If the 15th and 16th Problems are well understood, there will be little occasion to explain the Method of proceeding in this; for, if any Rectangle have one Side parallel to the Picture, the other two adjoining Sides must, necessarily, be perpendicular to the Interfection of the Plane it is in, if not to the Picture.

Let ABFD be a Square; one Side, AB, lying close to the Picture, AD and BF are therefore perpendicular to it. C is the Center of the Picture, or of the Vanishing Line, EE, of the Plane of the Square, AB, is its Interfection. Plate XII.
Fig. 57.

Now, AD and BF are perpendicular to the Interfection, therefore they vanish in the Center of the Vanishing Line. (See Problem 15.)

Draw AC and BC, their indefinite Representations; make CE, on either Side, equal to the Distance of the Picture, or Vanishing Line, and draw BE (or AE) diagonal Ways, cutting AC in d, the Representation of D, (or BC in f.)

Draw df parallel to AB. AdfB is the representation of the Square ADFB.

For, BE is the indefinite Representation of the Diagonal, BD, E being its Vanishing Point; for EE is parallel to BD, (CE being equal to CE, and AD to AB) and DF being parallel to the Picture, its Representation, df, is consequently parallel to the Interfection. (Prob. 16.)

2. If another square be required, draw fE, cutting AC in g, and draw gh parallel to df. By which Expedient any length of AC may be obtained.

If the length AG had been required, equal twice AD; AG on the Interfection being made equal twice AB, and GE drawn, gives the same Point g; as before.

Or, if CE be bisected, at K, BK is the indefinite representation of the Diagonal BG; for EK is parallel to BG; the Angle CEK is equal GBH, and, K is the Vanishing Point of that Diagonal; draw BK, cutting AC in g, as before.

3. If the Square be at some Distance from the Picture, as ABFD, produce the two perpendicular Sides to the Interfection, cutting it in a and b; also, produce the Diagonal FA to I, its intersecting Point; or, make aI equal aA, and proceed as in the former Case. The Figure explains the rest.

COR. Hence, a Pavement of Squares, having their Sides parallel to the Picture, may be delineated, with great facility.

Let AB be the Ground Line, and ECD the Horizontal Line.

No. 1.

Take the geometrical measure of a Square, and apply it, on the Interfection, AB, as often as it is required, from A to B; as a, b, &c.

From each Division, draw Right Lines to the Center, AC, aC, &c. and draw a Diagonal, from A or B, to the Eye, at E or D, cutting each indefinite Representation, in the Points a, b, c, &c. through which, draw the Lines FG, HI, &c. parallel to the Interfection, or Vanishing Line, and the several Figures X, Y, Z, &c. are the Representations of Squares on the Picture; which may be repeated, by drawing other Diagonals to any length or width.

N n

Now,

Plate XII. Now, if one of these Squares (as Y or Z) be considered singly, it has scarce the appearance of a Square, having no other, contiguous to it, to bias the judgment; but, by the affinity of the whole, the Eye (being accustomed to see Objects as they appear, in all situations) is not offended, and readily gives the assent; although it is certainly capable of determining, that the several Representations of Squares, are not Squares, but have the appearance, only, of Squares, in certain Positions and Situations.

P R O B L E M XX.

To find the Representation of a Square, or other Rectangle, whose Sides are all equally inclined to the Picture.

First, by the Original Figure being geometrically drawn, in its determined Position to the Picture; the Intersection of the Plane it is in being given.

Fig. 58.

Let ABCD be a Square, the Sides of which are equally inclined to the Picture. C is the Center of the Vanishing Line EE, or of the Picture; E, E, are the transposed places of the Eye, to the Vanishing Line, viz. CE equal CE.

As one Angle of the Square (A) touches the Intersection, it is, consequently, the Intersecting Point of the two Sides AB and AD.

Let the other two Sides be produced to the Intersection, at F and G.

Draw AE, both ways; also, draw FE and GE, diagonal ways, cutting each other. Abcd is the Representation of ABCD.

For the Point A being in the Picture, is its own Representation, AE, AE are indefinite Representations of AB and AD; and FE and GE, of FC and CG (Theo. 12) consequently, they cut each other in the representations of the several Angles, B, C, and D. (Cor. 7. Th. 12.)

If the Rectangle HIKL be at some Distance from the Picture, the Sides being produced to their intersecting Points, a, b, c, d, and the indefinite Representations, aE, bE, &c. being drawn (as in the Figure) give the Representation h i k l of that Rectangle; its Sides being parallel to the Sides of the Square.

EV, parallel to the Diagonal, IL, produces its Vanishing Point.

Note. In this Case, it may be observed, that there is no necessity for the Eye, i. e. the Distance of the Picture or Vanishing Line, being placed above it, but placed equally on either Side of its Center, C, in the Vanishing Line, as CE. Squares, or other Rectangles (wherever they are situated in the Original Plane) having the same Position to the Picture, have their Sides parallel, and consequently, they have the same Vanishing Points.

N. B. The Diagonals of the Square are, in this Case, the one (BD) parallel, and the other (AC) perpendicular to the Picture; consequently, their Representations are either parallel, as b d, or vanish in its Center, as A c.

Let it be, here also, particularly noticed, that E, being the place of the Eye, in the Vanishing Line (commonly called the Point of Distance) is the Vanishing Point of the Diagonal of a Square, whose Sides are parallel and perpendicular to the Picture. Consequently, if the Diagonals are parallel and perpendicular to the Picture, they are the Vanishing Points of its Sides.

By which Points, all Lines perpendicular to the Picture are proportioned, perspectively; as it may be observed in the preceding Problems.

2nd. How to find the Representation of a Square, in this Position, not having the Figure drawn out geometrically, only its measure and place known.

Let A be the Intersecting Point of an Angle of the Square, situate on the left side of the Station Line; at the Distance A J.

Make AF and AG each equal to the Diagonal, and proceed as before.

Or, by the measure of its Sides.

Make

Make EF equal to the Diagonal of a Square, whose Side is CE ; i. e. make EF equal to the Distance of the Vanishing Point E , from the Eye, equal EE .

Make Af equal to AB and draw fF , cutting the indefinite Representation AE in b , the Representation of the Angle B .

Draw bd parallel to FG , the Intersection, cutting AE in d ; and, lastly, draw bE and dE , diagonal ways, cutting each other, which compleats the Figure.

COR. Hence, a Pavement of Squares, diagonal-ways, may be delineated.

Let AB be the Ground Line and ED the Horizontal vanishing Line.

Fig. 58.
No. 1.

Make Aa , ab , &c. equal to the Diagonal of the Square; make CE , and CD each equal to the Distance of the Picture (C being the Center.)

Draw AE , aE , &c. and AD , aD , &c. cutting each other in the representations of Squares, placed diagonal-ways.

Through d , where AD and BE intersect, draw ef parallel to AB ; and where it cuts the several Lines, drawn from a , b , c , &c. viz. in e , a , b , &c. draw aD , bD , and Ea , &c. contrary ways; by which means they may be continued at pleasure.

P R O B L E M XXI.

To find the Representation of any Rectangle, obliquely situated to the Picture.

First, by having the Original drawn in the Geometrical Plane, in its true place and position, in respect of the Picture and of the Eye.

Fig. 59.

$ABCD$ is the Rectangle to be delineated; FI is the Intersection of the Plane it is in; KL is the Vanishing Line, and C its Center. The Distance is known.

Make CE , equal to the Distance, and perpendicular to KL .

Draw EK and EL , parallel to the Sides of the Rectangle, AB and BC , respectively, producing their Vanishing Points, K and L .

The Original being at some Distance from the Picture, produce every Side, DA , CB , &c. to the Intersection, cutting it in F , G , H , and I , their Intersecting Points.

Draw the indefinite Representations FL , GL , HK , and IK ; which, by their mutual Intersections, give the Representation, $abcd$, of $ABCD$.

METHOD 2ND. By the Seats of every Angle, on the Picture, and their Distances from their Seats, respectively.

If the Original be in a Plane perpendicular to the Picture, the Seats of all its Angles are in the Intersection of the Plane they are in.

Let a , b , c , and d , be the Seats of the Angles A , B , C , and D , respectively.

Draw aC , bC , &c. to the Center; make ab equal to the Distance of the Point A , from its Seat, be equal to the Distance of B , cd of C , and df of the Angle D .

Make CE and CF each equal to the Distance of the Picture, and draw bE , aF , dF , and fE , cutting aC , bC , &c. in the Points a , b , c , and d , respectively; which are the representations of the several Angles A , B , C , and D . (Pr. 14. Meth. 1.)

Join the Points a and b , b and c , &c. as in the Figure; and the Quadrilateral, $abcd$, is the representation of the Rectangle, $ABCD$, situated as in the Figure.

Note. This Method, is used by all the old writers on Perspective; particularly in the Jesuits. In which process it may be observed, that there is no occasion for the Vanishing Points, K and L ; for if they made use of Vanishing Points, at all, they were found by producing the Sides ba , bc , &c. to the Vanishing Line; i. e. to the Horizontal Line, for they knew no other Vanishing Line. Which Vanishing Points, so produced, they called accidental Points.

METHOD

Plate XII. METHOD 3rd. By the known Proportion of the Original; its Position, Situation, and Distance being determined.

The Original Figure, ABCD, is drawn out in the Geometrical Plane, to shew how the measures, &c. are applied, but it is, otherwise, of no use in the operation; as the careful and accurate observer will perceive.

IN, the Intersection; KL, the Vanishing Line; C, its Center; and CE its Distance, are given, the same as before.

Through E, draw HI, parallel to the Vanishing Line; draw EK and EL, making the Angles, HEK, and IEL, equal to the known Inclination of the Sides of the Rectangle (AB and BC) to the Intersection, respectively; i. e. make HEK equal to ABF, and IEL equal to CBG; FG being parallel to the Intersection, the Angles ABF and CBG are equal to their inclination to it†.

† 4. 1. El.

Make DB equal to JB; i. e. to the distance of the nearest Angle (B) to the Station Line, and draw BC; also, make Bd equal to its distance from the Intersection (equal BB) and, CE being made equal to CE, draw dE, cutting BC in b, the Representation of the Angle B. (Prob. 14th.)

Draw the indefinite Representations, bK and bL, from the Point b.

It remains, now, to cut off (from b) ba and bc, in the perspective Representations of AB and BC, the Originals.

Make KE' equal to KE, and LE' equal to LE; which Points are used for proportioning all Lines which vanish in K and L, respectively. (Prob. 17.)

Draw E'b, cutting the Intersection in b; make ab equal AB, and draw aE', cutting bK in a, the Representation of A.

Also draw E'b cutting the Intersection in b' and make b'c equal BC (the other Side, whose vanishing Point is L) and draw cE' cutting bL in c, the Representation of C, another Angle of the Figure.

From the Points a and c, draw aL and cK, intersecting at d, which compleats the Figure; for, the Originals of those Sides, being parallel to the Originals of ab and bc, they have, consequently, the same vanishing Points, respectively.

SCHOL. This Method, which is perfectly consonant to the new Principles by Brook Taylor, may appear to some Persons more difficult than either of the other, particularly the first, which is performed, also, entirely on the same Principles; the difference is very obvious, notwithstanding the effect is the same, as it may be seen, by comparing the Figures; or by comparing both, with the 17th Problem, which contains all the various Methods of proportioning inclined Lines, in general.

In the first, there is a necessity for having the Original Figure drawn geometrically, either on the Picture or somewhere apart; for which there is not, frequently, room to spare on the Picture; and we are liable to errors, in transferring the intersecting Points; but, without the Original being placed in the very Position, we cannot draw the Radials, or Parallels from the Eye, producing the Vanishing Points; as by that Method.

Whereas, by the last Method, the Angles, being known, are made equal to the Originals; in which Case, the Radials would be parallel to the original Lines being in their true places.

Or, if there be not room on the Picture, or interfere with the Objects, the Vanishing Points and their Distances, from the Center and from the Eye, are all determinable, by Problem 12th; the Vanishing Points, K and L, and their Distances KE' and LE' being ascertained by it.

There is, likewise, no need for the Original Figure on the Picture, but being any where drawn out, geometrically, or its measures being known, and the Position determined, they are applied to the Picture with the greatest facility, for which, a little Practice will render it quite familiar; regarding always, carefully, to make use of the proper Points for particular Lines, and to distinguish between the Vanishing Points and those which are used, for Visual Rays, i. e. for cutting indefinite Lines in the Ratio required.

P R O B L E M XXII.

To find the Representation of a regular Pentagon, one Side being parallel to the Picture.

Pl. XIII.
Fig. 61.

Let ABCDE be a regular Pentagon, having one Side (AB) parallel to the Intersection (IK) consequently parallel to the Picture.

LM is the Vanishing Line, and E the Eye.

Produce the inclined Sides, FD, CD, AF, and BC, to the Interfection, cutting it in G, H, I, and K.

Draw EL, EM, &c. respectively parallel to them, producing the Vanishing Points, L, M, N, and O, of each Side.

Draw the indefinite Representations GL, HM, IN, and KO; producing, by their Intersections, the Figure, fjcd, of the Original FJCD, (No. 1.) and *IfdcK* of IFDCK (No. 2) so that, the representation of one Side (AB) is wanting, in each; which, on account of its parallelism to the Picture, has neither Intersecting nor Vanishing Point†; and must be found by Problem 16; or, by drawing a Visual Ray, EA, cutting the indefinite Representation, IN, of the Side AF, in a; or, make NP equal NE, and Ia equal IA, and draw aP. (Prob. 17.) Or, by producing either Diagonal, AD or BD; being parallel, respectively, to the Sides BC, and AF, O and N are their Vanishing Points.

† Cor, to Theo. 1.

Having, by any of these Methods, found a, the Representation of A, draw ab, parallel to the Interfection, cutting KO in b; which compleats the Figure.

Secondly. How to determine all the Vanishing Points in this Position, without the Original Figure on the Picture.

Through E, draw RS parallel to the Vanishing Line; and make the Angles REL, LEO, &c. each of 36 Degrees; i. e. divide the Ark of the Semicircle, R 2 3 S, into five equal Parts, and produce the Lines E 1, E 2, &c. to the Vanishing Line, cutting it in L, O, &c. the Vanishing Points of the Sides and Diagonals of a Pentagon, having one Side parallel to the Picture.

SCHOL. The reason of this is obvious; for three of those Angles, viz. REN, is equal to the Angle of a Pentagon; and since one Side is parallel to the Picture, RE is parallel to it; and consequently, EN will be parallel to AF, another Side; the Angle REN being equal BAF.

Also, the Angle which any two Sides, not contiguous (as AF and CB) make with each other, viz. FJC, is equal to one of those Angles; wherefore, EN being parallel to one of those Sides, EO is consequently parallel to the other. (By 6. 1. El.) Also to the Diagonals AD and BD.

This is fully explained, after Problem 11th, in the 4th Book of Elements.

To find the Representation when every Side is inclined to the Picture (the Original being in its true Place) has nothing particular; for, if every Side be produced to its intersecting Point, and their Vanishing Points are found by drawing parallels to every Side, from the Eye, the indefinite Representations being drawn, produce the Representation of the Original; as in the last Problem, or Prob. 18.

If the Vanishing Point of any Side be found, all the rest are determined as before; by producing the Radial of that Side and describing a Semicircle; and dividing it into five equal Parts, as in this Figure.

P R O B L E M XXIII.

How to find the Representation of a Pentagon, having only one Side given, in its true Place and Position, in the Geometrical Plane.

Let AB be the given Side.

It is required to find the Representation of a regular Pentagon, whose Side is equal to AB; and inclined to the Picture in the Angle BDG, or to the Interfection GD. KM is the Vanishing Line, C is its Center, and CE its Distance.

Fig. 6a.

Through E, draw EH parallel to the Vanishing Line, and make the Angle HEN equal to BDG, the given Angle, producing the Vanishing Point I.

Produce BA to D; draw the Indefinite Representation DI; and find the finite part ab, the representation of AB (by Prob. 17) i. e. draw the Visual Rays AE and BE; or, make IP equal IE; and, DF, DG, equal to DA, and DB, respectively; draw FP and GP, cutting DI in a and b.

Produce IE, and describe a Semicircle NacO; divide the Ark into five equal Parts, at a, b, c, and d; draw Ea, Eb, &c. to the Vanishing Line, producing the Vanishing Points, K, L, &c. of the remaining Sides of the Polygon.

Having obtained ab, the Representation of AB, as above; draw aM and bL the indefinite Representations of two Sides, from those Points; for, IaM represents an Angle (IEM) equal to the Angle of a Pentagon; and IbL represents an Angle equal to IEL; which is equal to an external Angle of a Pentagon.

O o.

Draw

Pl. XIII. Draw aK, cutting bL in c; bc is another Side of the Pentagon; for, ac represents a Diagonal (which, in a Pentagon, is parallel to the opposite Side) whose Vanishing Point is K.

Fig. 62.

Draw lf, through c, cutting aM in f; cf represents a Diagonal parallel to the Side a b; and consequently, a f represents another Side.

Lastly; draw fK, and bM cutting it in d, and draw cd, which compleats the Figure, abcdf, required.

The Vanishing Point Q (which is out of the Picture) of the Side cd, and its parallel Diagonal bf, has not been wanted in this process.

Nc and Ob are Diagonals, EN, Ec, or EO and Eb, being supposed Sides of a Pentagon; and EQ, EK are Parallels to them, from the Eye, producing their Vanishing Points. (See Prob. 10.)

Note. If ab or bc, or any Side in the Representation had been given, the whole Figure may be completed, as above, without the Interfection.

P R O B L E M XXIV.

To find the Representation of a regular Hexagon, situated any how to the Picture.

There is no Figure whatever, except a Square, easier to describe than a Hexagon.

To have the Original Figure drawn in the Geometrical Plane, it is evident, is the same as has already been explained, in the foregoing; I shall, therefore, beg leave to pass over that description.

Fig. 63.

Let AB be a Side, given, of a regular Hexagon; the Original of which, is parallel to the Picture. IK is the Vanishing Line.

Make IEK an equilateral Triangle, whose Perpendicular is CE; as follows.

Through E draw HL parallel to the Vanishing Line, and, on E, describe a Semicircle; which, divide into three equal Arks, at a and b.

Draw Ea and Eb, and produce them to the Vanishing Line, cutting it in I and K; which, are all the Vanishing Points required for this Position.

Draw AI and BK; make Ab and Ba each equal to AB, in AB produced, both ways; and draw aI and bK.

Draw BI and AK, or AC and BC, cutting them in F and G, and join FG, which is parallel to AB, and compleats the Figure. ADFGCB.

SCHOL. A regular Hexagon having an even number of Sides, the opposite ones are consequently parallel; and the Diagonal, which passes through the Center, is also parallel to them; and because AB is parallel to the Vanishing Line, its Original is parallel to the Picture; and consequently, the opposite Side FG is also parallel to AB; and, because they are equal, AF and BG are parallel, and consequently their Originals were perpendicular to the Interfection; therefore they vanish in the Center of the Vanishing Line. (Cor. to Theorem 11.)

The two Sides, AD and CG, and the Diagonal, BF, represent parallel Lines; consequently they have the same Vanishing Point, I; also BC, AG, and DF; all which may be seen in the Hexagon ABCGFD, which may be supposed the Original Figure, one Side AB, being in the Picture.

By drawing the Diagonals AG, BF, and CD, the Hexagon is divided into six equilateral Triangles, AOB, BOC, &c. and IEK has its three Sides respectively parallel to them; consequently, since one Side in each Triangle is parallel to the Picture, and has no Vanishing Point (Theo. 1.) EI and EK are parallel to the other two Sides of each Triangle (which are equally inclined) and consequently, to the Sides AD, CG, and BC, DF, of the Hexagon; I and K are, therefore, their Vanishing Points.

COR. Hence, a Pavement of regular Hexagons, or equilateral Triangles, may be easily delineated.

No. 2.

Let AB be the Interfection of the Picture, or the Ground Line of the Pavement; and let Aa be the determined measure of one Side of a Hexagon.

Make ab, bc, &c. equal to Aa, as often as there is occasion.

If C be the Center of the Picture (its Distance being known) find the Vanishing Points, D and E, as I and K above.

Draw

Draw AD, aD, bD, &c. and aE, bE, &c. and, where they intersect, draw Lines parallel to the Intersection, as fg, hi, &c. which divide the whole into the representations of equilateral Triangles; six of which, around the same Point o (as acdefb) form a Hexagon; and thus, as many may be described as are required; for which, the inspection of the Figure is sufficient.

Case second. When the given Side is inclined to the Picture, in any Angle.

Let AB be the Side given, DF is the Intersection, and GL the Vanishing Line Fig. 64. of the Plane it is in; E is the Eye.

Find ab, the representation of AB, by any of the former Problems, its Vanishing Point is G; draw GE, and produce it; on E describe a Semicircle, abcd.

Make ab and bc each equal aE, and draw Eb, Ec, cutting the Vanishing Line, in H and I the Vanishing Points of the other Sides.

Draw aI and bH, the indefinite Representations of two Sides.

For, baf represents the Angle GEI, which is the Angle of a Hexagon; equal $aEb + bEc$, twice 60 Degrees; and abc represents the Angle HEI, equal aEc.

Make af and bc represent Lines equal AB (by Prob. 10, Case 3rd) i. e. bisect the Angle GEH, by the Line EK; or, Ed being equal Eb, having drawn bd, draw EK parallel to db; i. e. perpendicular to EI.

For, EI is the Radial of af; consequently parallel to its Original; and, EK is the Radial of a Diagonal of two contiguous Sides, which is perpendicular to the Original of af; parallel to ab (for abcd is half a regular Hexagon in the Position of the Original) consequently, EK is parallel to a Diagonal whose Vanishing Point is K.

Draw aK cutting bH in c, and draw Gc till it cuts aI in f; for, the Original of the Diagonal cf is parallel to the Original of a b. As ad is parallel to bc.

Draw el; aH, and fH; and GD, cutting fH in e, which compleats the Figure.

Note. If the Vanishing Point H had coincided with C, the Center of the Picture, G and I would be equally distant from C; and the Original of the Hexagon would be regularly posited; having two Sides and a Diagonal perpendicular to the Picture; or to the Intersection of the Plane it is in.

By which means, a Pavement of regular Hexagons may be drawn in that Position.

Let AB be the Ground Line of the Pavement, C the Center of the Picture, No. 21. and DF the Horizontal vanishing Line.

On AB, take as many equal divisions Aa, ab, &c. as you please, each equal to half the width of the Hexagon required.

Make CE equal to the Distance of the Picture, and make the Angles CED, CEF, each of 60 Degrees, i. e. having described the Ark abc, on the Center E, make ba and bc each equal Eb, and draw Ea and Ec, to D and F; C, D, and F, are the Vanishing Points of all the Sides.

From all the Divisions A, a, b, &c. draw Lines to the Center, C; and from every other Division a, c, e, draw aD and aF, &c. to both the other Vanishing Points, D and F; and, where they cut AC and BC, viz. in f, g, &c. draw fF, gD, &c. as in the Figure; by which means they may be continued at pleasure. The rest is obvious, on inspection.

P R O B L E M XXV.

To find the Representation of an Octagon and of a Circle inscribed, having only one Side given.

An Octagon, having four of its Sides in a Square, is very readily delineated in Perspective, seeing that, if one Side be parallel to the Picture, two are necessarily perpendicular to the Intersection, and four are inclined to it, equally, viz. in an Angle of 45 Degrees, half a Right Angle; consequently, the Vanishing Points are the Center, and Distance, of the Vanishing Line of the Plane it is in.

Pl. XIV. Let AB be the given Side of an Octagon, parallel to the Picture; EF is the Vanishing Line of the Plane it is in, and C its Center.

Make CE and CF each equal to the Distance; also make FJ equal FE .
Draw AE and BF , which are two Sides, indefinite.

For, since the given Side, AB , is parallel to the Picture, the adjoining Sides are inclined in half a Right Angle to the Interfection; therefore, E and F are their Vanishing Points. (Pr. 19 & 20.)

Produce AB , both ways; make BD equal AB , and draw DJ , cutting BF in G ; and draw FG parallel to AB , cutting AE in F .

Draw CF and CG , and produce them, till they cut AB , produced, in H and I ; draw IE and HF , cutting HC and IC , in K and L , and draw KL .

Draw AC and BC , cutting KL in M and N ; and draw EN and FM , and produce them, cutting IC and HC in O and P , which compleats the Figure $AFPMNOGB$, of an Octagon.

Or, without drawing AC and BC , make KM equal NL , where DJ cuts KL , and draw EN and FM , as before.

Or, from a and b , where the Diagonals HL and IK cut AF and BG , draw aC and bC , cutting the Diagonals, again, in d and c ; and through them, draw Ec and Fd .

A Curve described, carefully, by hand, touching every Side of the Octagon, passing through the Points a , b , c , and d , will be an Ellipsis; for it is the representation of a Circle, in Perspective.

Or, for greater exactness, through S , where the Diagonals HL and IK intersect, draw ef parallel to AB ; and CS to g , bisecting AB and KL , in g and h ; by which means there are eight Points obtained in the Circumference.

Note. The Figure below HD (which may be considered as the Interfection of the Plane of the Original) is the Original, geometrically drawn. The Italics, corresponding with the Roman Letters in the Representation, shew the Original of each Point, or Line; S represents S , the Center.

To draw an Octagon inclined to the Picture has nothing singular or particular in it; as every Line, which is inclined to the Picture, is projected, on the Picture, by the same general Rule (Prob. 17) which, having been so often exemplified, in the preceding Problems, it would be quite superfluous to repeat.

Of all the various Methods for finding the Representation of a Circle, in Perspective, there is none preferable to this; which I shall exemplify more at large in the 8th Section. For, if any regular Polygon, whatever, be drawn in Perspective, a Circle may be described through all the Angles, or touching every Side. Six Points are not sufficient for describing the Curve with accuracy; besides, an Octagon is readier to describe than a Hexagon, seeing that, the Vanishing Points, required, are only the Center of the Vanishing Line and its Distance. The Octagon being inscribed in a Square, renders it, of all others, the most expeditious for finding the Representation of a Circle.

P R O B L E M XXVI.

The Interfection of a vertical Plane being given, and its Inclination to the Picture, to find the Representation of any Figure in the Plane.

Fig. 66. Let AB be the given Interfection, and C the Center of the Picture.

Through C , draw SD perpendicular to the Interfection, and CE perpendicular to SD , i. e. parallel to AB , and equal to the Distance of the Picture.

Make CES equal to the Complement of the Angle of Inclination, cutting CS in S ; and, through S , draw MN parallel to AB ; MN is the Vanishing Line, S is its Center, and ES is its Distance. (See Prob. 3rd.)

If it be required to find the Representation of a Square, whose Side is parallel to the Picture, and equal to AB ; at the distance Ab from the Interfection.

Draw AS and BS ; make SE' & SE'' equal SE , and draw bE' , cutting AS in F . Draw FE' , cutting BS in H ; and draw FG and HI , parallel to AB .

Then, $FGHI$ represents a Square, in that Plane, at the distance Ab from the Interfection of the Plane of the Original.

If another Square be required, draw another Diagonal HE' or IE' .

Or, if a greater length than a Square be required, make SO to SE (equal SE) as one Side of the Rectangle, required, is to the other, and draw HO , cutting AS in K , and draw KL . For EO , being drawn, is parallel to the Original of HK .

See this illustrated by moveable Planes. (Fig. 37, No. 1.)

Turn up the Plane $AFDC$, (on the other Side of the Picture) at right angles with the Ground Plane, RS is its intersection with the Picture, and AB , with the Ground Plane.

AB and FD , being perpendicular to the Picture, vanish in its Center, C ; and, if the Vertical Plane, V , be turned up, perpendicular to the Picture, EQ will be parallel to the Diagonals, AE and BD , and produce their Vanishing Point; and, Q , on the other Side of the Center, C (in the Vertical Line) equally distant from C , is the Vanishing Point of the other Diagonals, FB and EC .

The rest is obvious; if the Rectangles $AFEB$ and $BEDC$ were Squares, their Diagonals would vanish in E ; CE being equal to the Distance of the Picture.

Second. Let ab be the given Side of a regular Heptagon (in the Plane $AIHB$) parallel to the Horizon; its Vanishing Point is S ; it is required to describe the Heptagon. SE is perpendicular to the Vanishing Line MN .

Having made SE equal SE , make the Angles SEM , SEN each equal to the external Angle of a Heptagon, and bisect them, by the line EO and EP ; also, make the Angles MEQ , NER , equal MEO , &c. i. e. having described a semi-circle, on E , divide the Semi-circumference into seven equal parts, and make ab , bc , and cd , each equal one seventh part.

EQ and ER , being produced to the Vanishing Line, MN , will generate two Vanishing Points, equally distant from S ; which, with M , O , S , P , and N are the Vanishing Points of the Sides and Diagonals; none of them being, in this Case, parallel to the Picture.

Draw aM ; and Nc , through b , indefinite; draw Pc , through a , cutting Nc in c , and draw cS , cutting aM at d . bc and ad are Sides of the Heptagon, whose Vanishing Points are M and N .

Draw cQ , and Rd indefinite (by Prob. 13.) and, through a , draw Ne , cutting cQ at e ; and draw eS , cutting the opposite Side, at f ; and, lastly, draw eO and Pg , through f , cutting eO at g , which compleats the Figure $afecb$, the representation of a regular Heptagon, whose Sides are all inclined to the Picture.

P R O B L E M XXVII.

To find the Representation of any irregular Figure (entirely on the Principles of Brook Taylor) from the known dimensions of the Figure, and the Angles which every Side makes with the adjoining Side or Diagonal; the Place, Position, Situation, and Distance of the Figure, in respect of the Picture and of the Station Line, being given; in a Plane which is inclined to the Picture, in a given Angle, the Center and Distance of the Picture being given, and the Intersection of the Plane of the Figure.

Note. The Original Figure ($ABDFG$) being geometrically drawn, in the Original Plane, is of no other use (in the following Operation) than to determine its Situation and Position; otherwise it is not necessary. The Place of the nearest Angle (A) and the Inclination of a contiguous Side (AB , or AG) in respect of the Intersection, being all that is wanted; the rest are determinable, in respect of themselves; as it will be exemplified.

Fig. 67.

C is the Center of the Picture, HI is the Intersection of the Plane of the Original Figure, and X is the Angle of its Inclination to the Picture.

Pp

A is

Pl. XIV. A is the place of the nearest Angle of the Figure, and JAB is the Angle of the inclination of the Side AB , to the Interfection; AB is the Distance of the Angle A from the Interfection, and Ae is its Distance from the Station Line.

Fig. 67.

Through C , draw EC perpendicular to the Interfection, which is the Vertical Line of the Original Plane of the Figure. (Def. D.)

Draw CE' perpendicular to EC , and equal to the Distance of the Picture.

Make the Angle $CE'C$ equal to Z (the Complement of X) cutting EC in C ; and, through C , draw DG , the Vanishing Line of the Plane of the Figure; C is its Center, and CE' its Distance. (Def. 19 and 20.)

CE , on the Vertical Line, being made equal to CE' ; through E , draw ER parallel to the Vanishing Line; the Parallel of the Eye of that Plane.

† Prob. 2.

‡ Prob. 4.

Draw ED ; making the Angle RED , equal JAB (i. e. to BKH , the Inclination of the Side AB to the Interfection) cutting the Vanishing Line in D ; the Vanishing Point of AB †, and make the Angle DEG equal BAG , cutting the Vanishing Line in G , the Vanishing Point of AG ‡.

Also, make the Angle DEH , equal to ABH , the Supplement of ABD ; and H is the Vanishing Point of BD ; by the same.

Make the Angle GEI equal AGI , the Supplement of AGF ; cutting the Vanishing Line in I , the Vanishing Point of FG .

Lastly, make the Angle IEF equal to DFG ; EF is the Radial of FD ; which, if produced, would cut the Vanishing Line, in the Vanishing Point of FD .

Thus, are the Vanishing Points D , G , H , and I , produced, by the known Inclination of one Side to the Interfection, and the Angles which each Side makes with the adjoining Side, (by Prob. 4.)

The place of the Angle A being given in the Geometrical Plane (which is inclined to the Picture, in the Angle $E'CC$; and to the Horizon, in the Angle $CE'C$) find its representation on the Picture (Prob. 14.)

Draw AB , perpendicular to HI , and draw BC . Make BL equal to BA , and CE equal to CE' , and draw LE , cutting BC in a , the representation of the original Point A .

Or, draw BA parallel to CE' , and make BA equal BA ; produce AB , and draw AS parallel to HI , or perpendicular to BS ; S is the Seat of the point A , on the Picture; and SA its distance from its Seat.

§ Prob. 6.

Draw SC , and AE' cutting it in a , the Representation of A §, as before.

Draw aD and aG the indefinite Representations of AB and AG , from the Point a ; which, if produced, would pass through their Intersecting Points, K & L .

Make DE' equal DE ; and, through a , draw $E'a$ to the Interfection, cutting it in a .

Make ab equal AB , and draw bE' , cutting aD in b ; ab represents a Line equal to AB (Prob. 7, or 17.) Draw bH .

Produce DE , and make EK and EL respectively equal to AB and BD ; or in the ratio of AB to BD ; and draw EM parallel to KL , cutting the Vanishing Line, in M , the Vanishing Point of the Diagonal AD .

Draw aM , cutting bH in d . bd represents a Line in the ratio to that which a represents, as EK to EL , i. e. as AB to BD . (Prob. 10*.)

¶ Def. 22.

For, EK is parallel to AB , and EL to BD ; wherefore KL , consequently EM , is parallel to the Diagonal AD ; therefore, M is its vanishing Point ¶; and abd represents the Triangle ABD . (Pr. 18.)

Now, F , the Vanishing Point of FD (being much inclined) is not in the Picture.

Draw df at pleasure, cutting the Vanishing Line, and EF , in e and f ; and draw gi parallel to df (at any distance, at discretion) and make bi to gb , as ed is to ef .

* From the Position of AB and BD , the measure of AB , or its Ratio to BD , is taken beyond the Eye, in the Radial of AB , produced; as in the third Case, of that Problem.

e. g.

e. g. Draw the Diagonal fb ; and draw dk parallel to the Vanishing Line, cutting fb , produced, in k ; draw ki , parallel to FE , cutting gi in i ; bi is a fourth Proportional to fe , ed , and gb ; i. e. $bi:gb::de:ef$. Draw di ; which will tend to the Vanishing Point F ; (by Prob. 13, No. 2.)

Produce GE , and make EN equal AG ; and, on EI , take EO equal GF (or, make EN and EO in the ratio of AG to GF) and draw EP , parallel to NO , cutting the Vanishing Line in P ; the Vanishing Point of the Diagonal AF .

Draw aP , cutting di in f ; and, lastly, draw If , and produce it till it cuts aG , in g , which compleats the Figure.

N. B. If the Sides of the Original Figure be produced to their intersecting Points, H, I, K , and L , the truth of the process may be perceived, by its affinity and agreement with the former methods, viz. when the Original Figure is drawn in the Geometrical Plane, its Vanishing Line, Center, and Distance being given; which, in this Problem, are given only by Position and Distance; the Original Figure being useless in the Operation, otherwise than to shew the meaning of every step; and that, if the Original Figure be supposed in its true Place, and Position, each Line, from the Eye, producing a Vanishing Point, would be parallel to the corresponding Side, or Diagonal of the Figure.

In this Problem, I have summed up most of the Rules given in the two last Sections; which are called elementary, because, by them, the whole of practical Perspective is performed.

It will, I know, appear, to young practitioners in Perspective, somewhat difficult; yet, if they do but see the Principles on which it is performed, and will be at the trouble to go through the Process a second time, and compare each Operation with the Problem it refers to, that apparent difficulty will vanish; and they will find it of great advantage to them, in applying all the elementary Problems to real use, in delineating Plane Figures.

This Process, difficult and operose as it may appear, at first, is infinitely preferable to any other Method of performing it; nay, it is scarce possible to perform it at all, by any Rules given by the old writers on Perspective, or indeed by any; and, considering that I have given the whole procedure, from the beginning to the end, it is not a long one. I have, not only, referred to the foregoing Problems for each particular step in the Operation, and (as is customary) left the Reader to apply it; but, I have gone on with him, socially, hand in hand; and, like a trusty Guide, pointed out every step he should take.

I have, also, drawn every Line in the whole Operation, which remains in the Plate; yet there is not the least confusion, the use of each being obvious, on inspection; but, being taken regularly, in the course of the Work, every apparent intricacy is unravelled.

At the same time, let it be observed, that there is no need for half the Lines to remain at once; e. g. after having found the Vanishing Line, DG , its Center and Distance, and the Vanishing Points of all the Sides of a Figure, all the operative Lines may be rubbed out before we begin to draw the Figure; also, finding the Seat of A , is not absolutely necessary, but is only another Method to find the representation of the Angle A ; but they must all remain in the Diagram; which circumstance frightens many, with apparent intricacy; which, in reality, is but apparent.

In order to compare the difference, I will shew, how it may be performed otherwise. But, as a Figure of so many Sides would be somewhat confused, I have made choice of a Triangle, in which, the Process is more distinct, and will be better understood.

Let ABD be the given Figure, in the Geometrical Plane; of which, GI is the intersection, and JIM is its inclination to the Horizon; let NI be considered as a vertical Section of the Picture; NIM is the angle of Inclination of the Plane of the Figure with the Picture.

Fig. 68.

Produce NI ; draw AK , BL , and DM , parallel to GI , cutting IM in K , L , and M . Make IK , IL , and IM respectively, equal to IK , IL , and IM ; and draw MN , LO , and KP perpendicular to the Picture; NI being perpendicular to JI .

Draw AF , BG , and DH , perpendicular to GI , and produce them, till they cut KP , LO , and MN , produced, in A , B , and D , the Seats, on the Picture, of the Angles of the Triangle. ABD is its orthographic Projection.

Having thus found the Seats of the three Points A , B , and D , the Representation of each may be found by Prob. 6. as a , the Representation of A , in the preceding Figure; by the second method.

C being

- Pl. XIV. C being the Center of the Picture, draw CE at pleasure, and equal to the Distance of the Picture; and, having drawn AC, BC, and DC, draw Ak, Bl, (or Bl) and Dm (or Dm) parallel to CE (or CE) and equal to the Distance of each Point from its Seat, respectively; viz. Ak equal PK, Bl (or Bl) equal OL, and Dm (or Dm) equal NM.

Draw kE, cutting AC in a; lE (or lE) cutting BC in b, and mE (or mE) cutting DC in d.

The three Points a, b, and d, are the representations of the Angles A, B, and D; which, being joined by Right Lines, compleat the Figure.

In this Process, it is obvious, there is neither Vanishing Line nor Vanishing Point required, the whole being performed by the 6th Problem. If the Intersection, GI, be parallel to the Horizon, ECE may be considered as the Horizontal Line. But, if the Intersection of the Plane of the Figure, be given, it is immaterial what Position it has to the Horizon; the Center of the Picture and Distance, the Place and Position of the Object, and the Inclination of the Plane it is in, being also given.

Although this method is perfectly consonant to the Principles of Brook Taylor, yet it is by no means so masterly or correct, that is, it is more liable to error, than the other, by means of Vanishing Points; which, in complex Objects, having various Lines in various Planes, is preferable to all other. In short, it is truly Perspective, which cannot be said of the old Authors; having few or no elementary Principles, to support or demonstrate the Rules they prescribed, the Operation is, merely, a mechanical Process.

As the old Authors had no Idea of any other Vanishing Line than the Horizontal, nor of placing the Eye any where but in the Point of Distance, therein; their method of finding the representation of each Angle of the Figure, in a Plane which is inclined to the Horizon, is by means of their Seats on the Ground Plane, and their heights above it; as J of M, &c.

As the Process would, in this Place, be confused, by mixing it with the other, I shall defer it to a more proper Time and Place, in an Appendix to this Work, in which the difference will be clearly and fairly stated, and exemplified.

P R O B L E M XXVIII.

To find the Representation of any Line, or Plane Figure, by means of the Directing Plane. (See the 14th Theorem.)

- Fig. 69. Let AB be a Line perpendicular to the Picture, or to the Intersection, IK, of the Plane it is in. Let E be the Eye, in its true Place, as in the foregoing Problems.

At the Distance EP, equal to the Prime Director*, draw ON, the Directing Line, parallel to the Intersection, IK †.

The Space between ON and IK represents all that Space which lies between the Directing Line and the Intersection of the Original Plane with the Picture.

In which Construction, the whole of that Space, together with the Directing Plane, NEO, is supposed to be turned up, on the Intersection IK, till it falls into the Picture; the Original Figures being turned along with it, into the same Position as usual.

- ‡ Def. 12. Produce BA, cutting the Intersection in I, its Intersecting Point; and the Directing Line in D, its Directing Point. Draw ED, which is the Director of AB ‡; to which, draw IC, parallel.

For, C being the Center of the Picture, and I the Intersecting Point of a Line perpendicular to the Picture, IC is its indefinite Representation; and EC, the Direct Radial, is parallel to AB, and equal to ID, a part of AB, produced; consequently, IC is parallel to ED. (15. 1. El.)

As EC (Fig. 37) is equal to BR, a part of AC, intercepted between the Intersecting and Directing Lines; also, EO is equal to GP, and EL to HS; and being parallel, respectively, the Lines RC and EB, EG and OP, &c. are also equal and parallel.

* In Theory, the Prime Director is a Right Line joining the Eye, or drawn through the Eye, and the Station Point of the Original Plane. In common Cases, EP is equal to the height of the Eye above the Ground Plane, always equal to the distance between the Intersection and Vanishing Line.

If the Sides of the Triangle, FGH, be produced, till they cut the Directing Line, in the Points M*, N, and O, their Directing Points, EM, EN, and EO, are their Directors; to which, their several Representations are parallel; J, K, and L, are the intersecting Points of those Lines.

Draw Lh parallel to EO, Kh parallel to EN, and Jf parallel to EM; which, by their Intersections, produce the Figure fgh, the Representation of FGH.

Through C, parallel to IK, draw RS, the Vanishing Line of the Plane of the Triangle; and, having produced the Indefinite Representations to their Vanishing Points, R, S, Q, the Lines ER, ES, and EQ, are the Radials of the Sides of the Triangle, and consequently parallel to them, respectively.

The agreement between these two Methods of producing the Representation, fgh, is so very obvious, as not to need any further explanation.

See this further illustrated, in Fig. 37. Turn up the Picture and Directing Plane.

The Sides of the Triangle ZY and ZX, on the other Side of the Picture, being produced, cut the Intersection (AB) in P and S; and, the Directing Line (HG) in G and H; EG and EH, in the Directing Plane, are respectively parallel to the Representations of those Lines, PO and SI, the Directing Plane being parallel to the Picture. For, they are the Intersections of Radial Planes with the Picture and Directing Plane; which are parallel, by Def. 4.

The Planes W and X being turned over, till E coincides with E; the Section with the Picture, RC, is the indefinite Representation of AC (Theo. 8 and 12) and EB, its Section with the Directing Plane, is the Director of the Line AC (Def. 12) which is parallel to RC (by Theo. 14) being the sections of parallel Planes, by another Plane.

S E C T I O N VI.

Of the PRACTICE of PERSPECTIVE, applied to PLANE SOLIDS.

THE last Section comprehends the whole Art of practical Perspective, in respect of Plane Figures; illustrated with variety of Cases and Examples; in which, I have studied more to render it easy and familiar, than to embellish the Subject with what is no way useful. Every Figure, which I have given, are such as are common, and applicable to Buildings and other Objects; the Pentagon and Heptagon are not so useful in themselves, as for the sake of managing inclined Lines, in all Positions; so that the Lessons deduced from them are excellent. By varying their Positions and Situations, and diversifying the Cases as much as is consistent with utility, I am persuaded that no Person, who has a tolerable Capacity, will be at any loss to know how to describe any Plane Figure, and in any Position whatever.

Although some Persons, of keener Talents, may imagine, that the preceding Section contained Instructions sufficient for every occasion (it certainly contains the Elements of the whole) yet, I presume, if no more had been given, it would be found of but little use to many; when, by applying those valuable Lessons to familiar Subjects, what, before, might appear somewhat mysterious will, hereafter, be applied with ease, as occasion may require.

* Suppose FG produced till it cuts ON, produced, and EM a Line tending to that Point; then EM is the Director of FG.

Plate XV. It may, with equal propriety be alledged, that there is no need for shewing how plane Solids are formed; seeing they are only composed of plane Figures, in various Positions, all which have, already, been copiously treated of. Yet, a few Lessons, and familiar Examples, will not be found superfluous or unnecessary. I shall not dwell on any thing not really useful, but hasten to apply the whole to proper Objects; for which, all the foregoing is but preparatory.

In this Section, I shall always consider the Object as situated on a horizontal Plane, and the Picture as vertical; most of the foregoing Problems, being general, are applicable in all Positions whatever.

In the following part of this Work, I would advise the young Student, as he proceeds, to draw every Object, or a similar one, at his own discretion. Or, if that be not necessary, in some Cases, let him carefully notice what things are given, in the Proposition, and what is required to be done; and then proceed, step by step; not supposing any thing as done, but what he has passed over; by which means, he will see, clearly, how every Line is produced, by which the Objects are formed; but to do it, himself, and varying the Position, a little, is better.

This Precaution would have been necessary in the third or fourth Section, but more particularly so here.

P R O B L E M XXIX.

How to describe a Cube, perspectively, or any other right angled Parallelopiped*, having one Side, given, parallel to the Picture; at any Distance beyond the Picture, its Situation and Dimensions being known.

No. 70. ECE is the Horizontal Line, and PM the Interfection or Ground Line.

C being the Center of the Picture, make CF, on either Side, or on both Sides, equal to its Distance; and let this be understood in future Examples, that CE (E being the Place of the Eye) either on the Horizontal or Vertical Line (the Picture being vertical) always denotes the Distance of the Picture; sometimes CE.

First; let the Picture be supposed applied close to a Face of the Cube; in which Case, being parallel to the Picture, it coincides with the Picture.

This is generally the Case, when there is but one Object; as it answers no purpose to suppose it at any Distance, from the Picture, as in Figures 59, 60, &c. or, when there are various Objects to be delineated, it is, most commonly, supposed to be applied close to the nearest, as in this Example (see Fig. 38.) so that, the Distance of the Picture is the Distance of a Plane direct before the Eye, and touching the Object, or the nearest Object, if there are more than one.

Let AB, in the Ground Line, be the measure of a Cube, to be represented; having one Face in the Picture.

On AB, describe a Square, ABDF, which, being in the Picture, has the full dimensions of the Side given; wherefore, ABDF is the representation of a Face of the Cube required. (Cor. 5. Theo. 9.)

Draw DC and FC; and DE or FE, diagonal-ways, cutting FC in G, or DC in H; draw GH, parallel to FD, which compleats the Figure AGHB, the representation of a Cube, in that Position, and Situation; having but two Faces seen.

Note. Each Side (AF, ED, and DB) of the Square AFDB, representing the front Face of the Cube, is, in this Case, an Interfection of the Picture, as well as AB; viz. of the adjoining Face.

* See the Definition of a Cube, &c. in the General Introduction, Section 1, Page 41.

If another, of equal dimensions, be required, situated on one side of the Station Line, at the Distance LJ (as *ABEH*) and distant from the Picture *La*, equal *La*. Make *LM* equal *AB*; and, on *LM*, describe the Square *LMNO*, as before. Draw *OC*, *NC*, &c. and draw *aE*, cutting *LC* in *a*, and *MC* in *e*. Draw *ab* parallel to *LM*; and *ad*, *bc*, to *MN*; also, draw *cd* parallel to *ab*. i. e. having drawn *ab* parallel to *LM*, describe a Square on *ab*; and, at *e*, describe the Square *efgh*, parallel to *abcd*, which compleats the Figure.

Or, having drawn *LC*, *MC*, and *aE*, compleat the Plan *afeb*. (Prob. 19.)

Describe a Square on *ab*, and draw *cC* and *dC*; draw *fg* parallel to *ad*, and *gh* parallel to *cd*, which compleats the Figure.

DEM. *abcd* being parallel to the Picture (by Hypothesis) is similar to the Original, *LMNO*†; and its Sides have that proportion to the Originals, as the Distance of the Picture (*CE*) to the Distance of the Plane *abcd*; viz. as *EC+La*; i. e. *a* or *d*: *LM*:: *EC*: *EC+La*. (Theo. 9.)

But, *LMNO* is a Square; therefore, *abcd* is a Square.

And, since it is parallel to the Picture, the Originals of the Sides *af*, *dg*, &c. are perpendicular to the Picture; consequently they vanish in its Center, *C*‡.

Also, *afgd* represents a Square in a vertical Plane, perpendicular to the Picture; for *LO* is the Intersection of that Plane§, and *ECE* its Vanishing Line||; wherefore, *CE*, being made equal to *CE*, *E* is the Vanishing Point of Lines (in that Plane) inclined to the Intersection in half a Right Angle; consequently, if one Side (*ad*) be parallel to the Intersection (*LO*) and *OP* being made equal *La*, *PE* passes through a Diagonal of that Side†, therefore, *afgd* represents a Square; and *Ob* being equal *La*, *bE* is a Diagonal of the Top, *dghb*, which also represents a Square, parallel to *afeb*, and *ON* is the Intersection of that Plane, with the Picture.

Therefore, *agc* represents a Cube, having three Faces seen, each of which, is the representation of a Square. Q. E. D. for the opposite Faces are also Squares.

PQ is the Intersection of a Diagonal Plane *cdfe*, whose Vanishing Line would pass through *E*.

The Cube *AGHB* being situated on the Station Line (which, on the Picture, coincides with the Vertical Line) has but two Faces seen; the Front *AFDB*, and Top, *FGHD*, the Eye being above it.

The other Faces, *AFGK* and *BDHI*, seeing they apparently incline towards the Vertical Line, are consequently lost to sight in this Position; as is obvious, supposing the Solid transparent.

† Cor. 5.
Theo. 9.

‡ Cor. to
Theo. 4.
† Theo. 10.
|| Th. 2 &
10.
† See N.B.
Prob. 20.

EXAMPLE I.

To draw the Representation of a high Wall, the End being parallel to the Picture, and at some Distance from it.

This Object, being a right angled Parallelopiped, is delineated as the Cube.

Fig. 71.

Let *PJ* be equal to the Distance of the Wall from the Station Line, on the Left.

Make *PQ* equal to its thickness, and draw *PC* and *QC*; make *Pd* equal to the distance of the end of the Wall, from the Picture; draw *dE* cutting *PC* in *R*, and draw *RS* parallel to *PQ*.

Now, since *P* is the intersecting Point of *RF*, the common Intersection of the Plane of the Wall (which is supposed vertical) with the Ground; *PT*, perpendicular to the Ground Line, or parallel to the Vertical, is the Intersection of that Plane†.

† Cor. 1.
Theo. 2.

And, since the Wall is perpendicular to the Horizon, consequently, its Angles or Corners, *RU* and *SV*, are parallel to *PT* or *QX* the Intersections of the Planes those Lines are in (for they are parallel to the Picture, being vertical) therefore, draw *RU* and *SV* parallel to *PT*, i. e. to the Vertical Line.

Make *PT* equal to the known height, and draw *TC*, cutting *RU* in *U*, and draw *VU* parallel to *RS*; then *RUVS*, represents the End of the Wall, which is parallel to the Picture (on the Ground Line).

Lastly. Make *PM* (on the Ground Line) equal to the Distance of the farther end of the Wall from the Picture, or *dM* equal to its known length.

Draw

Plate XV. Draw EM, cutting PC, the indefinite Representation of its common Section with the Ground Plane, in F; and draw FY, parallel to RU, which compleats the Wall.

Or, if CF (on the Vertical Line) be made, to CE or E, as the height of the Wall to its Length, F will be the Vanishing Point of the Diagonal, RY, which cuts TC, in the Point Y, for its length; draw FY parallel to RU.

Let it be observed, here, that if there be not room enough, on the Interfection, to set off the whole length of the Wall, equal dM , take half the Distance, PJ (equal Pf, in the Plan below) make Ce equal half CE (the Distance of the Picture) and draw Jc, projecting the same Point F.
If the half measure be too much, take a third, or fourth, or any equal portion, of PM (equal Pm) the same part being taken of CE will answer the same purpose (see Prob. 17.)

E X A M P L E II.

How to represent several Parallelopipeds, as Blocks of Stone, &c. for the Basement of a Building, ranged in a Right Line, perpendicular to the Picture.

Fig. 71. W, X, Y, and Z, are the Plans, or Seats of the Blocks on the Ground Plane.

Let ABD, the Front of the first, be close to the Picture, i. e. in the same Plane; consequently, in its geometrical Proportion, of height and width.

Draw AC, BC, and DC. Make Aa, on the Interfection, equal to the length of the first, ab equal to the space between them, and bc equal to the second.

Draw aE, bE, and cE, cutting AC in F, G, and H; draw FI, GK, and HL perpendicular, cutting BC; and IM, KN, and LO, parallel to BD, cutting DC, in M, N, and O, and compleat them, as in the Figure; having regard to the first hiding part of the next, and that of the third, &c.

Now, in order to continue them further, to any length, this Expedient may be used, when the measures exceed the limits of the Picture, and, consequently, cannot be applied to the Interfection, or Ground Line.

Draw Hg, from the farthest corner of the second Parallelopiped parallel to the Interfection, and draw aC and bC, cutting it in e and g.

If it be required to continue them of the same dimensions, and space, make Hf equal eg; or, Ad being made equal to ab, draw dC, cutting gH, at f, and draw fE and gE, cutting AC in a and b. Compleat the Parallelopiped abcd as in the Figure.

After the same manner they may be repeated as often as you please by drawing bg parallel to gH, cutting dC and bC in f and g, and then proceed as before.

This Expedient is, in many cases, better than applying the half-measures, or other portion, on the Ground Line; as they may be continued infinitely, within the compass of Ab (one Block and Space) by drawing Gh (instead of gH) which, being cut by aC, at i, needs not to be transposed, as e to f.

It must be obvious, that Gh (or gH) is cut in the same proportion to the second Block (at the hither end, or the farther) as Ab is to the first; and consequently, iE, hE, &c. will cut AC in the same Points, as if the full measures were applied on the Ground Line; and so of gb, to the next.

E X A M P L E III.

To draw several long Parallelopipeds, parallel amongst themselves, and perpendicular to the Picture; representing large Joists, supporting a rough Floor (over Head).

Let W, X, Y, and Z be the ends of the Joists, of equal dimensions, and equally spaced; and, since they are supposed to be parallel to the Picture, therefore, they have their geometrical proportion on the Picture. (Cor. 5, Th. 9.)

Fig. 72.

Let them be drawn accordingly, by a Scale of equal Parts.

Then, because the Joists are perpendicular to the Picture, in respect of their length; (i. e. they are horizontal, and at right angles with the Picture) draw a C and b C, &c. from every Angle, necessary, as in the Figure; and determine their length, by Prob. 15, i. e. make AB, or AB, equal to the required length, and draw BE, cutting AC in D; or AC in D.

Through D, draw FG parallel to the Vanishing Line (CE) which determines the under Sides, Fc, &c. and the perpendicular Lines, de, &c. compleat them.

The divisions of the Boards are determined by Prob. 8th; i. e. take any measure, Aa, and repeat it, on AB, as often as there is occasion, if the Boards are supposed equal; otherwise Aa, ab, &c. must be made in the same ratio as the Boards, to each other; and at the last division, e, draw e D, cutting the Vanishing Line in H; and draw a H, b H, &c. cutting AD in 1, 2, 3, &c. through which, draw Right Lines parallel to AB, i. e. to the Vanishing Line.

Or, divide AB into the same number of Parts (as A1, 12) and draw Lines to E, which will produce the same, as is obvious; seeing that it does not depend on the dimensions of the Parts, but on their number and ratio to each other. (See Prob. 8.)

Note. A1, 12, on AB, being each equal to a fifth Part of AB, is the geometrical width of the Boards, in proportion to the ends of the Joists, and to Ae, the space between them.

P R O B L E M XXX.

To draw the Representation of a Cube, or other right angled Parallelopiped, any how inclined to the Picture.

First. Let the Square ABCD be the Plan of a Cube, in the Geometrical Plane, equally inclined to the Picture, and to the Intersection, at the Angle A.

Fig. 73.

Lie down the Distance of the Picture, CE', from the Center, as usual; E and E are the Vanishing Points of all the Lines of its Sides. (Prob. 20.)

Draw AE, both ways, A being the intersecting Point of two Sides, AB and AD.

Find the perspective Plan ABCD (by Prob. 10) or of the two Sides, AB, AD, only.

Draw AF perpendicular to the Ground Line, and equal to AB, a Side of the Cube, and draw FE, both ways.

Draw BG and DH, parallel to AF; and draw GE and HE diagonal-ways, cutting each other, at I, which compleats the Cube, AGH.

Otherwise, without drawing the perspective Plan, by means of the Vanishing Lines of its Faces.

Draw AF perpendicular to the Ground Line. Then, AF is the Intersection of two Faces of the Cube. Make AF equal to a Side, AB.

Draw AE and FE, both ways; and through E, on either side, draw the Vanishing Line of the contiguous Face of the Cube. (Prob. 3.) EE, or EF.

Make EE or EF, on either Side, equal EE', and draw AE or AF, cutting the indefinite Representation FE, in G or H, and draw BG, or DH, perpendicular.

Draw BD parallel to the Intersection, and DH parallel to AF; and lastly, draw GE and HE, intersecting at I, as before.

R r

ABGHD

Plate XV. *ABGHD* is the Representation of a Cube, in the position required; situated on a horizontal Plane, and its vertical Faces equally inclined to the Picture.

Fig. 73.

DEM. For, *ABCD* the given Plan is a Square, wherefore *ABCD* represents a Square (Prob. 20) and *FGIH* also represents a Square, in a Plane parallel to *ABCD*. (Th. 3. and Cor. 1.)

And, they are between the same Parallels, *AF*, *BG*, &c. therefore equal - - - (18. 1. El.)

But, *AF* is the Intersection of the vertical Planes, *ABGF* and *ADHF*; and *EE* is the Vanishing Line of *ABGF* (Theo. 2 and 10) *E* is its Center (Def. 19) and *EE* its Distance (Cor. 2. 7.)

Wherefore, *E* is the Vanishing Point of Lines inclined to the Intersection of that Plane, in half a Right Angle or 45 Degrees; consequently, it is the Vanishing Point of the Diagonal of a Square, in that Plane, whose Sides *AF* and *BG* are parallel to the Picture (Prob. 19;) and, *AB* and *FG*, perpendicular to them, vanish in *E*, the Center of the Vanishing Line, *EE*. - - - (Cor. to Th. 11.)

Therefore, *AFGB* represents a Square, at right Angles with *ABCD* and *AFHD*, which (after the same manner) may also be proved to be the representation of a Square; and their opposites, *DHIC* and *CIGB*, being between the same parallel Planes, are also representations of Squares; consequently, *ABGHD* represents a Cube. (See Defin. Page 41.) Q. E. D.

a c f g represents another Cube of the same dimensions, and in the same position to the Picture; whose situation in respect of the former, is as the Plan, *W*, to *BD*; which, on account of the short Distance (*CE*) has a distorted appearance, being remote from the Station Line, though wholly within the Radius of the Distance.

By means of the Intersection (*AF*) of the Face *a d f g*, or *a f* of the Face *a b c d* (*AF*, or *a f*, being made equal to the Side of the Cube) the proportion of the parallel Lines (*a d* and *f g*, or *b c*) in those Planes are determined. For, *AF*, *a d*, and *f g*; also *a f*, *a d*, and *b c* are in the same Plane, and are all parallel to the Picture; consequently, they are parallel between themselves, and to their Originals†.

† Theo. 9.
Cor. 1.

E X A M P L E IV.

How to draw Parallelopipeds ranged in a Right Line, and inclined to the Picture obliquely.

Fig. 74.

Let *X* and *Z* be the Plans of right angled Parallelopipeds; their inclination to the Picture is the Angle *AIA*, which they make with the Ground Line, *CK*.

C being the Center of the Picture, and *CE* its Distance, find the Vanishing Points, *G* and *H*, of the Sides and Ends of the Parallelograms, *X* and *Z*, (Prob. 2, and 4.) and lie down their Distances on the Horizontal Line (Pr. 12.) at *E'* and *E''*.

Transfer all the measures, *IA*, *IB*, &c. of the Objects, and the spaces between them, to the Intersection, at *A*, *B*, and *C*; and, having drawn the indefinite Representation, *IG*, of the whole length, draw *AE'*, *BE'*, &c. cutting it in *a*, *b*, and *c*.

Because the Angle *I* touches the Picture, *IF*, perpendicular to the Ground Line, is the Intersection of the Plane of the Fronts, and also of the End, *IK*.

Make *ID* equal to the height of the Objects, and draw *DG*, *DH*, and *IH*.

Make *IK* equal *IK*, and draw *KE'* cutting *IH*; *Ik* represents *IK*. (Pr. 17.)

Draw *a d*, *b e*, and *c f*, parallel to *ID*; and, where they cut *DG*, in *d*, *e*, and *f*, draw *dH*, *eH*, and *fH*; and from *i*, where *ki* cuts *DH*, draw *iG*, cutting them in *j*, *h*, and *g*, which compleats the upper faces.

h l, parallel to *ID*, &c. cutting *kG*, compleats the End *b e h l*.

If another Object be required, in the same Line, seeing that the measures on the Ground Line, would exceed the limits of that Picture, take *GE'* half *GE'*, and draw *E'c* till it cuts the Intersection, at *a*.

Make *aB* equal to half the space between the Objects, and *Bc* equal half the width of the next Object, and draw *BE'*, and *cE'*, cutting *IG* in *b* and *c*.

If it be required higher than the rest, make *IF* equal to its known height, and draw *FG*. Draw perpendiculars from *b* and *c*, cutting *FG* in *g* and *h*.

Draw

Draw bH cutting kG at d , and draw gH ; and di parallel to bg , which compleats that Object.

Or, if cn be drawn parallel to the Ground Line, and BG cutting it in n ; divide cn , at m , as AC (equal AC) is divided in B ; and, from m and n , draw to E^* cutting IG in the same Points, b and c , as before.

By either of these Expedients they may be continued to any length.

Note. G , the Vanishing Point of the front Line IC , and the parallels to it (being much inclined) may be supposed to be beyond the limits of the Picture; consequently, ic , Df , &c. are tending to a Vanishing Point which is not on the Picture, and must, therefore, be drawn by means of the Expedients, in the 13th Problem; which are very convenient when but few Lines are wanting.

But, a better Expedient than them all is (when the Point is not very remote) to fix a Lath to the Drawing Board, or straining Frame of the Picture, and continue the Vanishing Line as far as is requisite, on the Lath; fix a Pin in the Vanishing Point, and, with a long Ruler, draw the Lines IG , DG , &c. for no other Expedient, whatever, can be so true, i. e. it cannot be performed so accurately as to have the Vanishing Point itself. For large Work (as in Scenery) a fine, smooth Cord, fixed in the Vanishing Point, is a good Expedient.

Prisms of all kinds may be represented after the manner of the foregoing, which are right angled Prisms, having found the Plans of their Bases, on the Ground Plane or other Plane, whether they be triangular, quadrangular, or multangular; of which one or two Examples will be sufficient.

EXAMPLE V.

Pl. XVI.

To represent a hexagonal Prism, perpendicular to the Ground Plane.

The Situation, Distance, and Position being determined, or AB being a Side, given or found, in Perspective, whose Radial is EV , find the Vanishing Points, I and K , of the other Sides, and compleat the Plan, $ABCDEF$, of its Base (by Pr. 24.)

Fig. 75-

Draw Aa , Bb , &c. perpendicular to the Ground Plane, i. e. to its Vanishing Line (IK) or Intersection (GD .)

Produce any Side, as AB , to its intersecting Point, (G) and draw GH , perpendicular to the Ground Line.

GH is the Intersection of the Plane of the Face $AabB$, or gh of $BbcC$ (Pr. 3) for it is vertical; and G , or g , is the Intersecting Point of one Line in the Plane.

§ Th. 2. C. 2.
† Prob. 13.

Make GH equal to the known height of the Prism, and draw HV cutting Aa and Bb in a and b ; the Original of ab being parallel to the Original of AB .

Then, because Aa and Bb are the common sections of the contiguous Faces with $AabB$, draw aI and bK , cutting the other Perpendiculars Ff and Cc , in f and c ; which compleat as many Faces as can be seen.

The other Faces, which are all Parallelograms (See Def. Page 41) Fc , Ed , and dC are supposed to be seen through, the Object being imagined transparent; which, in some Cases, is a necessary Expedient; by which means, the connection of the several Parts are more accurately determined.

EXAMPLE VI.

To represent a pentangular Prism, laid along on the Ground Plane; the Intersection and the Seat of the Object, on the Ground Plane, being given.

AB is the given Side of the Pentagon, and BD the Seat of one Face of the Prism. Fig. 76.

C being the Center of the Picture, and CE its Distance, find the Vanishing Points, I and K , of AB and AD (Prob. 2) by making the Angles JEI and OEK equal,

Pl. XVI. equal, respectively, to BAB and DAD ; or, having determined one, find the other, by Prob. 4. making the Angle IEK equal BAD , a Right Angle.

Fig. 76.

Make ab to represent AB (Prob. 17.) and, on ab , construct a Pentagon, in a vertical Plane, on the same Principles as in Prob. 23.

I being the Vanishing Point of ab ; FG , passing through I , is the Vanishing Line of the Plane of the end of the Prism (being perpendicular to the Ground Plane) I is its Center, and EI its Distance. (Prob. 3.)

As there is not room, on the Picture, beyond the Vanishing Line, FG , to find the Vanishing Points of the Sides of the Pentagon, take IE on this Side equal to IE , through which draw MN (the parallel of the Eye, of that Plane) parallel to FG , and, on E describe a Semicircle; which, divide into five equal parts.

Make ab , bc , and ad , de , each equal to a fifth part, and draw Eb , Ec , &c. which, produce to the Vanishing Line, producing the Vanishing Points, $F, G, \&c.$ by which, the Pentagon $abcde$ is completed, as in the Figure (by Prob. 23.)

Draw aK indefinite, of the Side AD ; and make af to represent a length equal to AD (Prob. 17) also, draw eK and dK .

Draw Hg , through f , and gF , cutting dK in h , which compleats the Figure.

For, $afge$ and $eghd$ represent Parallelograms, which are Faces of the Prism $afhdb$; and $abcde$ represents an End which is a regular Pentagon; to which the other Faces are perpendicular, IEK being a Right Angle (which IaK represents, Prob. 4) equal BAD .

The Expedient of turning over the Vanishing Plane, FEH , to find the Vanishing Points of the Side of the Pentagon, is a very useful and necessary Expedient; because it often happens, that there is not room for turning it over on the other Side of the Vanishing Line, if it be remote from the Center. And, notwithstanding it interferes with the Object in this Diagram, yet being done first, and the Vanishing Points found (as in the Figure) all the operative Lines (for finding them) are rubbed out before we begin to draw the Object; or rather, they are never drawn, for, the Vanishing Points are determined (as F .) by applying a straight Ruler to E and b , &c.

In Fig. 37, the horizontal Vanishing Plane, $NIKL$, or the vertical Plane V , being turned over on their Intersections with the Picture (which are the Vanishing Lines they produce) it is obvious that the Angles, formed by the Radials, EN , EO , &c. are the same on both Sides, and consequently, they produce the same Vanishing Points, N, M , &c. whether the Vanishing Plane be turned up or down, to the Right or to the Left.

EXAMPLE VII.

To represent a quadrangular or hexagonal Pyramid; or any other.

Fig. 77.

Let W and Z be the Plans of the Bases of two Pyramids, whose Altitudes are known, the one a square Base, the other a Hexagon; two Sides, of which, are parallel to two Sides of the Square.

Find the perspective Plan $ABCD$, of the Base, W (by Prob. 21.)

If the Original Figure be drawn in the Ground Plane, the shortest way is by producing its Sides to the Intersection, AK ; and having found the Vanishing Points, M and N , of the Sides, draw their indefinite Representations (Method 1st.)

Draw the Diagonals, AC and BD , intersecting at H , the Center of the Base.

Draw HI perpendicular; and where the Diagonal AC cuts the Intersection, at A , draw AF , also perpendicular to the Ground Line.

Make AF equal to the height of the Pyramid, and draw FG , cutting the Perpendicular HI at I , and draw AI , BI , and DI , which compleat the Figure.

If (as in this Example) on account of the Diagonal, AC , having very little inclination to the Ground Line, the Altitude cannot be determined with accuracy; and the other Diagonal, BD , is so much inclined as not to cut the Intersection within the Picture; it may be thus determined.

From

From either Vanishing Point, M or N, draw a Right Line through the Center of the Base, till it cuts the Ground Line, at a; draw a f parallel and equal to AF, and draw fN, cutting HI in the same Point, I, for the Vertex of the Pyramid, as before. Or, as in the following.

Second. Having found the perspective Plan ABCDFG, of the hexagonal Base, Z (Prob. 24) M and O being two Vanishing Points; the other is out of the Picture. From any Point, P, in the Vanishing Line, draw PH, cutting the Ground Line, in K.

Draw KL perpendicular to the Ground Line, and HI parallel to KL.

Make KL equal to the known Altitude of that Pyramid, and draw LP, cutting HI in I; and draw AL, BI, &c. as before, which compleat that Object.

Prisms and Pyramids are very useful Subjects for practical Lessons. By the first, we learn to form Pavillions, Temples, Cupolas, Alcoves, &c. which are, generally, of some prismatic Form; by the other we may form a Spire, Obelisk, or Pyramid, of any Figure.

As those Objects admit of infinite variety (as many as there may be various Polygons) it would be to little purpose to give more Examples of them, seeing that, if a Polygon of any number of Sides be drawn in Perspective, for the Base (by the various Problems of the last Section) the rest is the same as in the Examples given.

It may, I presume, have been observed, how inconvenient it would be to have the Ground Plans of all the Objects, to be delineated, geometrically drawn, in the Geometrical Plane; besides, the impossibility of having room for them, when the Objects extend to any considerable Distance; as the Plans, W, X, Y, and Z, of the Parallelipeds in the second Example (Plate XV.) evince.

It may also be observed, that they are of no other use than to know how the Objects are situated, in respect both of the Picture and of each other; but since it is obvious, from what has been done, that they are not absolutely necessary to be drawn, seeing that, if their measures are applied to the Intersection of the Plane they are in, it will answer the purpose; therefore I shall, in future Examples, where they are not absolutely necessary, do without them. Yet it must not be understood, that we can do entirely without, which is impossible; but, in many Cases, their Measures and Distances, only, are requisite. In others, the Situation and Position of one Object to another, being irregular, require a true and correct Plan to be first drawn (but not on the Picture) from which all the Vanishing Points may be ascertained; as every Line, in Perspective, is determined from its geometrical Proportion, and the position of the Object to the Picture; consequently, they must be known or imagined, otherwise we have no foundation to build on.

EXAMPLE VIII.

To represent Steps with Kirbs at the ends; as at the entrance into a House, &c.

First, when they are parallel to the Picture; and the Picture close to them.

Fig. 78 is a Section of the Steps, in their geometrical proportion, with the Inclination of the Kirbs to the Horizon, and to the Picture.

Take AB, on the Ground Line, equal to the width, or more properly, to the length of the Steps; and, AD and BJ to the thickness of the Kirbs.

Fig. 79-

Make AF equal to their height, and describe the Rectangles AFGD, and BHIJ, geometrically; which are supposed to be in the Picture.

C is the Center of the Picture, and CE the Distance. Draw AC and BC.

Make Ba equal to the Distance of the first Step from the Front of the Kirb; and a b, b c, and c d, each equal to the breadth of a Step, and draw a E, b E, &c. cutting BC in a, b, c, &c. the perspective breadths of the Steps, on the Ground.

Make Be, e f, &c. each equal to the height of a Step, and draw e C, f C, &c. cutting the Perpendiculars, from a, b, c, &c. at l, m, &c. and transfer them, by the parallel Lines a f, b g, &c. on the Ground, to the other end, cutting AC at f, g, h, &c. from which Points, draw the Perpendiculars fr, gs, &c. and, from l, m, &c. draw Lines parallel to AB, cutting them at r, s, &c. draw r C, s C, &c. cutting them again, at t, &c. and the parallel Lines l r, m s, &c. at the internal Angles, being drawn, compleat the Steps.

S s

The

Pl. XVI.
Fig. 79.

The upper Step having a greater breadth than the other, make dD equal to its breadth, and draw DE cutting BC at e ; the Perpendiculars do and ep cut bC in o and p ; and ou , pv , being drawn parallel to the Ground Line, cut the corresponding Perpendiculars iu and kv , which determine it; *viz.* $opvu$.

If the Kirbs are square, they are right angled Parallelopipeds, and are delineated, as described in Problem the 29th. $AFGD$ and $BHIJ$ are the Fronts, $AFvk$ and $BHp e$ are the Sides, which inclose the Steps; and, $FvwG$ and $HpqL$ are their upper Faces.

If they incline with the Steps (as is common) make the Rectangles AK and BL equal to the Ends; suppose, equal to the height of a Step.

Draw CC perpendicular to the Horizontal Line; and, CE being equal to the distance of the Picture, make the Angle CEC equal to the inclination of the Steps, cutting CC at C , which is the vanishing Point of their Inclination.

Draw KC , LC , &c. cutting GC , IC , &c. in xy , &c. which complete the inclined Faces of the Kirbs, as in the Figure; by drawing xu and oy .

In this Example, the front of each Step, being parallel to the Picture, are similar to each other; and have that Proportion to each other, respectively, as their several Distances, i. e. the second Step, is, in length and height, to the first, as CE is to $CE + ab$; and, the second is to the first, as CE is to $CE + ac$, &c. †

† Theo. 9.

VL , passing through C parallel to the Horizontal Line, is the Vanishing Line of a Plane inclined to the Horizon, in the Angle CEC , equal ECL , and C is its Center (see Fig. 15, No. 3, P. 72.)

For CC is perpendicular to VL ; and EC producing the point C , is the Radial of such Lines in the inclined Plane as are perpendicular to its Intersection KL ; therefore, C is their Vanishing Point. (Cor. Th. 11.)

CASE 2nd. EXAMPLE IX.

When the Steps are inclined to the Picture; the Inclination being determined, also their Situation, in respect of the Station Line.

In this Example I shall suppose, that there is not room on the Picture for ascertaining the Vanishing Points and their Distances, as usual; in order to apply the 12th Problem.

Fig. 80.

C being the Center of the Picture, and KCL the Horizontal Line, draw CE perpendicular, and take CE any equal part (suppose one third part) of the determined Distance of the Picture.

Make the Angle CED equal to the Inclination known; and CD will be one third part of the distance of the Vanishing Point L , of one Side; which being determined, all the rest are determinable, arithmetically (by Prob. 12.)

Or, make DEF a Right Angle; CF will be a third part of CK ; and FE will be a third part of its Distance, by the same, &c.

Having thus obtained the Vanishing Points, K and L , and their Distances E' and E'' being laid down (CE' being equal to three times the difference between FC and FE ; and CE'' to three times the difference between CD and DE) then, proceed as usual; AE being the Intersection of the inward Angle of the Kirb.

Draw AK , the indefinite Representation of the Front, and AL of the End.

Let AJ be the length of the Steps (eq. AB , Fig 79) draw JE' cutting AK at A .

Make Aa equal to the distance of the first Step; ab , and bc , &c. each equal to the breadth of a Step, and draw aE' , bE' , &c. cutting it in a , b , &c. and, seeing there is not room on the Intersection Ac , to apply the measure of the top Step, which, suppose equal to Ac , draw df parallel to Ab , and cL cutting it in f ; and draw fE' cutting AL at B .

Draw the Perpendiculars aj , bf , &c. indefinite.

On AE , take AB , BC , &c. each equal to the height of a Step, and draw BL , CL , &c. cutting the Perpendiculars from a , b , &c. as in the Figure.

Draw

Draw aK, and AL cutting it in k; draw kl perpendicular, and jK cutting it in l; draw lL, and eK cutting it in m, and fK cutting the Perpendicular, mn at n, and proceed, after the same manner, throughout.

Or, having drawn all the Lines eK, fK, &c. indefinite, draw lL; and mn perpendicular to the Ground Line; then nL, and op, &c. by proceeding after that manner the Steps are completed.

The Kirb being equal to BJ (as in the former Figure) draw BE', cutting AK at D; AD represents the thickness of the Kirb, at that end.

The Original of the Point, G, being supposed on this side of the Picture, and being in the Ground Plane, the Representation (G) is consequently below the Ground Line, AB. (See N.B. Prob. 8. Fig. 45. The measure being applied on this side, i.e. below AB, as AH, supposes AB proportioned perspectively; but, being at the other Extreme, as aB, it is projected; and is, consequently, larger than the Original.)

Make Ad' equal to BJ, and draw E'd, cutting KA produced, in G; AG is the projective representation of the end of the Kirb, the Original, of which, is equal to A d, equal BJ.

Draw the Perpendiculars GH, AF, and DG, indefinite; and, AE being made equal to the height of the Kirb, draw EK, cutting them in F, and G; and, being produced, it cuts GH at H.

EL cuts a Perpendicular from B, at I; and, if FL, GL, and HL be drawn, KI, cutting them, compleats the upper Faces, FJ and EIBH. Draw gb perpendicular, cutting GL, at g, which compleats the End.

If the Kirbs are inclined with the Steps, as in the former Case, draw LV, perpendicular to KL; which is the Vanishing Line of the Planes AEIB, AFJ, &c.

Make LE' equal to its Distance, and draw E'V, making the Angle LE'V equal to the inclination of the Steps, cutting LV in V, the Vanishing Point of the inclined Sides*. KV is the Vanishing Line of the Inclination. (Theo. 10. Cor. 1.)

AB being made equal to the height of the Front, draw KB, cutting DG, AF, and GH, in K, H, B, and L; from which Points, draw KV, HV, &c. cutting the horizontal Lines GL, HL, &c. in s, r, t, and u; through which, draw Ku, or join the Points rs, tu, only.

Note. The Picture might, with equal propriety, have been supposed on this side of the Kirb, entirely; but, as it frequently happens, in Practice (the Picture being fixed inadvertently) that some parts of the Object would project on this side; I thought it necessary to give an Example how to proceed in that Case.

SCHOL. In this Example, it is obvious that, the Plane AEIB, in which, the Steps are perspectively proportioned, being farther removed from the Vanishing Line, VH, is fitter for the purpose than GHL; and, if the Original of GH was in the Picture, the measures on AE would be less than the full measure of the Steps, seeing it would be beyond the Picture.

ABob is the Section made by the Picture, with the inclined Kirb, and shews how much is supposed to project on this side. Ab is the section of the Bottom, consequently, parallel to the Horizontal Line; ob is the section of the End, which is vertical, therefore parallel to VL its Vanishing Line; and Bo is the Section with the inclined part, consequently, parallel to the Vanishing Line, KV, of the inclination of the Steps. (Theorem 2nd.)

* This will be clearly demonstrated and made manifest, in the 12th and last Section of this Book.

N.B. The Vanishing Line (VW) of the Roof of the Object in the Apparatus, is determined in this manner (as KV) and is in a similar Position, on the direct Picture, MNOP.

V, the Vanishing Point of HG, answers to K, in this Figure; Y answers to L, and W to V.

Plate
XVII.

S E C T I O N VII.

Of the application of Perspective, to MOULDINGS, &c.

THE foregoing Section contains many useful Lessons in plane Solids, which may be considered as parts of Buildings, &c. In this, I intend to shew how the more decorative parts are formed, such as Pedestals for Columns, Cornices and Entablatures, &c. of the various Orders; without which embellishments, a Building seems naked and destitute of Ornament.

These Lessons, of Steps, are very necessary to the art of delineating Mouldings; which, when we know how to manage well, particularly such Steps as return at the Ends, or on four Sides, forming mitre Angles, the greatest difficulty is surmounted. For Mouldings, breaking round a Pedestal, or the internal and external Angles of a Cornice, &c. properly considered, are but so many Steps, one above or below another, of different dimensions; formed by the Fillets, between the cylindrical parts, which are, properly, the Mouldings; and are effected only by Light and Shade. The Fillets, between them, are narrow Planes, cutting the curved Surfaces in parallel Lines; which being described, by the Rules given, and the mitre Angles of the Mouldings drawn, the business of the linear part is done, and nothing remains but to give the appearance of solidity, convexity, and concavity, by a proper disposition of Light and Shade.

E X A M P L E XI.

How to delineate square Steps, returning on every Side.

Fig. 81. First, when they are parallel to the Picture. AB is their length, ABCD is half the geometrical Plan of the first Step; FGHI of the second, and KLMN of the third; and let OP be the measure of a right angled Block, on the upper Step.

C being the Center, draw AC and BC; and AE, which is a Diagonal, represents AQ, in which are the Seats of G, L, and O, the Corners of the Steps.

Compleat the Square ADCB (Prob. 19) and draw the other Diagonal BD; which, it is evident, tends to a Point on the left hand of the Center, equal to CE, the Distance of the Picture.

Produce FG, KL, &c. to the Ground Line, cutting it in g, l, &c. i. e. make Ag, gl, Bb, &c. equal to the breadth of the Steps, and draw gC, lC, &c. cutting the Diagonals at G, L, H, M, &c. the perspective Seats, on the Ground Plane, of the several corners of the Steps.

Draw AF perpendicular to AB, and make Aa, ab, and bc, equal to the height of the Steps, as in the former Examples; and, having drawn the Perpendiculars from G, L, &c. draw aE, bE, and cE, cutting them at e and e, g and i.

Draw ab, cd, ef, &c. parallel to AB, cutting Perpendiculars from B, H, &c. at b, d, f, &c. which give the representations of the Fronts, corresponding with GH and LM. AabB, being in the Picture, has its full dimensions, in height and length; the other two are in proportion to their Distances.

Having obtained the Fronts (by means of an imaginary Plane passing through the Diagonal AC, which AcFC represents) finish the square of the Top, iklm†.

Then, draw fC and bC, till they cut hk and df, which represent the return of the Steps on that Side, so far as they can be seen; on the other Side they are seen to their full extent, the Point of view being on that Side.

Draw aC and eC , &c. cutting Perpendiculars from D , I , &c. at f , j , and m ; and, having compleated the Face $gm i$, which is somewhat seen, from j and f , draw Lines parallel to the Ground Line, cutting the adjacent Step, which terminate their appearance, and compleat the Steps.

To represent the Block on the upper Step; draw the Perpendiculars Or , and Pq indefinite; make cd (on the vertical Intersection, AF) equal to its height, and draw dE , cutting Or , in r ; eE cuts it at o .

Draw op and rq , parallel to the Steps, which cutting a Perpendicular (Pq) compleat the Front, or qp . OC and rC , cutting Ns , finish the End, or sn , as in the Figure.

I presume that the Reader will, ere this, have discovered, how inconvenient it is to have the perspective Plans, of complex Objects, in the very place where the Object itself is drawn; which, I am sensible, must render it, to young practitioners, perplexing and intricate, to distinguish one part from another. That difficulty shall be obviated in the next Example; but I must observe, that the connection of the several parts, and the agreement between the Plan and Elevation, would not have been so distinctly seen, without the Plan in its true place; hereafter we shall place it either above or below, as is most convenient.

Let it be observed, notwithstanding, that this apparent intricacy will vanish at a second process; and let it be noticed, that, as the Side BC , of the ground Plan, passes through the Angles, d and h , of the Steps, it is not so necessarily; as it must be obvious, if the Center of View (C) was moved, on either hand, it would not. The Line Bp , being considered as a diagonal Line, passing through the Angles of the Steps, d and h , cannot be in the Ground Plane, as BC , a Diagonal of the Square, but making an Angle of elevation with the Diagonal BD ; and consequently, its Vanishing Point cannot be at C , in the Horizontal Line, but above it, in the Vanishing Line of a Diagonal Plane, passing through BQ .

Above the Vanishing Line, let AB be drawn parallel to it, which let be equal to AB ; the Point A perpendicularly over A , and B over B .

Then, if AC and BC be drawn, and the Diagonal AE , cutting BC in C ; and, CD being drawn parallel to AB , compleats the Plan of a Square, $ABCD$, corresponding with $ABCD$ below.

And, if the measure of the Steps be set off, from A and B , at a , b , c , and d , e , f ; aC , bC , &c. being drawn, cut the Diagonals AC and BD , in g , h , &c. which correspond with G , H , &c. below; as may be seen by the dotted Lines gG , hH , mM , pP , &c.

Wherefore, if there be room above, and not below, the Plan may be formed above; by means of which, the Object may be completed; drawing the Perpendiculars gc , lg , &c. cutting aE , bE , and cE ; and bd , mh , &c. cutting the parallel Lines ef , &c. which terminate the other ends of the Steps.

E X A M P L E XII.

Is an Expedient, by means of which, Mouldings, &c. may be delineated, without having the perspective Plan in the Work.

Let $ABCD$ be a perspective Plan of a Square, whose measure is AI , on the Intersection (described by Prob. 20 and 21) F and G are the Vanishing Points of the Sides of the Square, and H of the Diagonal; consequently, FG is the Vanishing Line of the Plane of the Figure; S is its Center.

The Distance of the Vanishing Line, FG , is a mean Proportional between SF and SG (Prob. 12) the Point E , which is the Distance of the Vanishing Point F , from the Eye, is determined by the same; and, the Vanishing Point, H , of the Diagonal, by bisecting the Angle, which the Radials of AB , and AD , make at the Eye; or, by making FH to HG , as the Radial of AB is to the other (3. 6. El.).

See Prob. 10, Case 3rd, Fig. 49. EH , bisecting the Angle DEG , made between the Radials of A and C , gives the Vanishing Point H ; by which, the Lines AC and CB are cut, perspective, representing equal Lines. As, in this Example, by drawing AH (H being obtained, as above) AB being made to represent a measure equal to AI (Prob. 17) and BG drawn; BG is cut, in C , representing an equal Line as AB represents, i. e. equal to AI (Prob. 10.) Then, draw AG ; and FC , till it cuts AG , at D ; $ABCD$ represents a Square, whose Side is equal AI .

It is required to draw, within the Square $ABCD$, the representations of several less Squares, of certain dimensions in proportion to the other, about the same Diagonals (AC and BD) and consequently, having the same Center; i. e. to draw the representations of Borders or Margins within it.

T t

Make

Fig. 81.
No. 2.

Fig. 82.

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Make Aa and ab respectively equal to the breadths of the Margins.

Draw aE and bE , cutting AB in e and d ; then, draw cG and dG , cutting the Diagonals in e and h , i and m ; and draw eF , iF , &c. cutting the Diagonals again, at f , k , &c. and join fg and kl ; or, draw fG and kG .

$efgh$ and $iklm$ are the representations of Squares, having the same Center, o ; and are about the same common Diameters, AC and BD .

Now, because the Intersection (AI) of the Plane those Squares are in, is so near its Vanishing Line (FG) the Sides of the Squares, AB , ef , &c. cut the Diagonal BD very oblique; for which reason, the Points of section, f and k , cannot be ascertained with accuracy; nor can the true breadths of the Margins be determined. Therefore, either below or above the Vanishing Line, draw AB , or a B parallel to AI ; in which, take A or a , in a perpendicular Line from A , and make AB , or a B equal AI ; and make $A1$, or a 1 , and 12 , respectively equal to Aa and ab .

Draw AF and AG , or a F and a G ; and BE , $1E$, $2E$, cutting AF in B , p , and q .

Compleat the Square $ABCD$, or $abcd$, by the same Vanishing Points, F , G , and H , and draw the Diagonal BD , or bd . Draw pG , and qG , cutting the Diagonals, in E , H , i and m ; from which Points draw EF , iF , &c. and finish the interior Squares, $EFGH$, and $iklm$, as in the Figure.

If Perpendiculars are drawn from the Points E , F , or e , f , i , k , &c. they will cut the Diagonals AC and BD in the Points, e , f , i , k , &c. as it is obvious from inspection of the Figure.

+ Theo. 3.
† Cor. to
Theo. 3.

For, if AI be the Intersection of the Plane of $ABCD$, with the Picture, AB , a B may be considered as the Intersections of other Planes, parallel to the former, and therefore, they have the same Vanishing Line, FG ; and consequently, AB , AB , and ab ; also AD , AD , and ad , and all other Lines, whose Originals are parallel to their Originals, have the same Vanishing Points, F and G ; notwithstanding, the Lines are in different Planes; and, H is the Vanishing Point of the Diagonals, AC , AC , and a c , seeing that, they represent parallel Lines, and in parallel Planes.

§ See Def.
Page 41.

Therefore, AB b a d D represents a right angled Parallelopiped or Prism, whose Base, $ABCD$, and Top, $abcd$, are Squares; consequently, they are equal and parallel to each other; and similarly posited. $ABCD$ may be considered as a Section of that Prism, by a Plane parallel to its Base; and consequently, it is also a Square, equal, similar, and parallel to the other.

The same may be said of all the interior Squares; which, with the Perpendiculars Ee , Ff , &c. also form Prisms. The Border, around, represents the edge of a hollow Prism.

In Example 5, the hexagonal Prism has its Base and Top similar Figures, and similarly posited; and being in parallel Planes, their Sides are parallel, and consequently, they have the same Vanishing Points.

Hence it is manifest, that, either the extra Plan $ABCD$, or $abcd$, may be used, for the purpose of drawing perpendicular Lines, in order to determine the extremes of Steps, &c. instead of the real Plan $ABCD$; in which, on account of their being farther from their Vanishing Line, the Intersections are determined with greater exactness, and without incommoding the Object.

E X A M P L E XIII.

How to represent square Steps, obliquely situated to the Picture and having a Pedestal situated on them.

Fig. 83.

Let AB be the Ground Line, and DF the Horizontal vanishing Line, C is its Center; A is the intersecting Point of the nearest Angle of the Ground Plan.

The Distance of the Picture being known, and the Inclination of the Sides of the Object determined, find the Vanishing Points, D and F , of the Sides (by Prob. 12) and G of the Diagonal (as O , Fig. 51; EO bisecting the Angle AEB) also, find E , making DE equal to the Distance of the Vanishing Point D .

Draw AD and AF , indefinite; make AB equal to the Side of the Square of the Steps, and draw BE , cutting AD in d .

Draw AB parallel to $A B$, at any distance at discretion.

Draw

Draw AA perpendicular to AB ; and at the Point A , describe the representation of a Square $ABCD$ (by means of the Vanishing Points D and F ; and O , of the Diagonal) whose Sides AD , &c. represent the measure AB ; as Ad , Ab , above.

Make Aa , ab , each equal to the width of the Steps; and draw aE , bE , &c., cutting AD in g and h .

Draw the Diagonals AC and BD ; and draw gF and hF , cutting them in F , G , &c. and compleat the interior Squares, as in the last Figure.

Draw AH , perpendicular to AB . Take Aa and ac equal to the height of the Steps, and draw aD and aF , terminating at b and d , where Perpendiculars from B and D cut them.

Draw aG and cG , cutting a Perpendicular from F , at e and f .

Draw eD and fD , eF and fF , cutting Perpendiculars from G and H , at g and h , which terminate that Step; bD and dF cutting the next Step, at g and h , compleat the first; and, by the same means, the next is compleated.

It remains, now, to delineate a Pedestal upon the upper Step.

Set off the geometrical projectures of the Mouldings, from b to c , on the Intersection; and transfer them to AD , by means of the Point E ; and draw iF , cutting the Diagonals at L and A ; and draw LD , cutting the Diagonal DB at γ .

Then, on the Intersection AH , make cH equal to the height of the Pedestal; make ci equal to the Plinth, ik to the Base Moulding, kl to the Dado, lH to its Cornice or Sirbase, and Hj to the Plinth of the Base of the Column; which, notwithstanding it is a part of the Base, always goes with the Pedestal.

Fig. 83.

At X and Z , on the right hand, the Profile of the Mouldings and Steps are described, geometrically; from which they are transferred by means of parallel Lines, to the vertical Intersection AH .

Draw cG , iG , kG , &c. cutting Perpendiculars from K and L , in m , n , o , p , and q ; the representations of the corresponding Angles in the Profile.

Draw mD , nD , mF and nF , &c. cutting Perpendiculars from γ and I , in r , s , t , m , n , o , &c. and having divided ik into the smaller divisions of the Mouldings (as at Z) draw to G , cutting Perpendiculars from the corresponding parts in the Ground Plan, at JK , by which means, the whole is compleated; as in the Figure.

Now, when this Object is well considered, what are the several Parts which compose the Pedestal; but so many Steps of different heights and breadths. The Plinth, mnm , is in the place of the next Step, about twice the height; and the Filler, which follows it, is another of a much less height; the Dado, after that, is another still higher, equal kl ; and the Cornice, vx , is but Steps reversed.

The Curve Lines st , no , and no , joining the Angles of the several Fillers, are all that have any linear curvature; although $noon$ represents a cylindrical Surface, yet the appearance of it is effected by Light and Shade only; which, according as it is supposed to be situated, in respect of the Light, will appear either convex or concave; to which, the curve Lines, as its extremes, do not at all contribute.

And now I might take leave of the Reader, and of this Section, and endeavour (as others have done) to persuade him, that I have, in this Lesson, given him sufficient instructions for delineating all kinds of rectilinear Mouldings; but, I have not quite so good an opinion of his Capacity (if it be his first essay) as to suppose any such thing; my intent in this, is only to give him some little Idea of it, by comparing the Process with that of delineating Plane Steps. There remains yet somewhat more to be said on the Subject; therefore I shall not trespass on his time, but give some other Examples for his further Instruction.

E X A M P L E XIV.

How to represent Mouldings around the Basement of a Building, being a regular Pedestal, with an internal and external Angles; whose Planes are parallel and perpendicular to the Picture, and also one to another.

Let AB be the Ground Line, and AD the Intersection of the Picture with the vertical Plane of the Dado, $bcge$; on which, set off all the measures of the heights of the Mouldings, and their distances from each other.

Fig. 84.

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Aa is the height of the Plinth, ab of the Base moulding; bc of the Dado, and cD of the Sirbate, or Cornice of the Pedestal.

Let each Moulding be drawn, geometrical, cdD the Cornice, and abc the Base; draw the Perpendicular ac, and draw the diagonal Lines bc and cd.

C being the Center of the Picture, and EC the Distance, draw AC indefinite.

Make AF the measure of the first break, and draw EF, cutting AC at G.

Draw GI parallel to AB; make AF equal to the projecture of the Pedestal, and draw FC, cutting GI, at I.

Make IJ to GI as the front of the Pedestal to its projecture; that is, as FB to AF; and draw JE, cutting FC, at K; or BE, cutting AC, at H, and draw HK parallel to AB, cutting FC.

From the several Angles, G, I, and K, thus obtained, draw Gr, Is, and Kt, perpendicular to the Ground Line; which constitute the several Planes, from which the Mouldings are supposed to project.

Draw bC, cC, and DC, cutting Gr, at e, g, and c; draw ef, gm, and ek, parallel to the Ground Line, and where they cut Is, at f, m, and k, draw Lines to the Center, cutting Kt, at g, n, and o.

Now, Degc, ekmg, and konm, are the Seats of the Cornice on those Planes; cgeb, &c. is the Dado or plane part between; and AbeG, GefI, and IfgK are the Grounds of the whole Base, thereon.

Draw aC, cutting EF at b; draw bb parallel to AB, cutting EI produced, at b; and draw bC, indefinite.

The Angle k is determined, by drawing a Line from K to the Eye, on the other Side of the Center, equal CE; for Kk is a Diagonal of a Square; whose Vanishing Points are, in this Case, the Points of Distance. (Prob. 19.)

Or, if the Vanishing Point of the Diagonal Kk is not on the Picture, draw Kd parallel to the Ground Line; on which describe the Square Kikd (by Pr. 19) which will give the Angle, k, required; i. e. draw dE cutting KC; and ik parallel to Kd, cutting bC at k, the Angle sought.

From the several Angles, b, h, and k, draw the Perpendiculars bf, bl, and kp; and, through e and k, draw Ec, and Ek, cutting the two first, at f and l.

Or, from c and d, draw Lines to the Center, cutting the first at d and f; from which, draw dB and fl, cutting the second, at B and l; and from them draw Lines to the Center, cutting kp, at l and p; which give all the extreme Angles of the Mouldings.

Draw the diagonal Lines ed, fB, and fg, &c. and observe how the parallel Lines in the geometrical Mouldings, cut their Diagonals; as, at h, the great projecture of the Facia goes beyond it.

Draw the lower Line of the Cimma or Ogee, at h; and, where it cuts the Diagonals, at i and j, draw the perpendicular lines of the Facia, which will terminate it, at the several Angles; by drawing hi, tending to the Center, and ij parallel to the Vanishing Line, &c. The rest is obvious on inspection of the Figure.

The Fillet at z between the Ovolo and Cavetto, touches the diagonal Line, cd; by which, it is easily described, as above.

As the Fillets of the Base Moulding touch the diagonal Line bc, they are very readily returned at the internal Angle, ed, to the external, at fB, and from thence to the other, at gI.

It may perhaps be imagined, that bc, and cd are mitre Angles, at the Corner AD, which they are not. For, suppose ADvu to be a returning Plane, parallel to the Picture, and at right angles with ADcG. Then, because AD was considered as the Section of that Plane, ADvu is in the Picture; and consequently, Abca and cdD are geometrical sections of the Mouldings, by the Picture; and if they were returned on the Plane ADvu, they would project on this Side of the Picture.

Draw EA, meeting ba, produced, at n; and draw nx parallel to AD.

Produce

Produce fd to x ; also produce dc , till it cuts the Perpendicular nx at o ; and draw op and nq parallel to the Ground Line; a diagonal line, ob , is the mitre Angle of that Moulding.

It may be observed, that, if AD passed through E , the mitre Angles would be in that Line entirely; indicating the Eye to be in the diagonal Plane of that Angle; if AD was beyond E , the mitre Angle would be on the other hand of it, when it becomes greatly distorted.

AB being the Intersection of the Ground Plane with the Picture, all the Mouldings on this side project through the Picture, and they are considered as projected to the Picture.

Dq is the height of the Plinth of the Base, which determines it, at r , s , t , on the Pedestal.

E X A M P L E XV.

Is a general and universal Rule for delineating Mouldings, parallel to the Picture; whether they are above or below the Eye.

At either extreme of the Picture, if there be room (or it may be done on another drawing Board, and the Scale Line transfered, carefully, to the Picture) describe the Cornice, &c. geometrically; as at No. 1.

If the Student be not acquainted with Architecture, or has not a knowledge of the geometrical proportions of Mouldings, let him not attempt the delineation of them in Perspective, before he has studied, and practised himself a little in that kind of Projection; as he will but lose his time; it being impossible to succeed in one without the other; seeing that, the projection of Mouldings, in Perspective, depends wholly on the geometrical Profile.

Let AB be the height, and BD the projecture of a Cornice, intended to be delineated; to which let there be added an Architrave, FG ; between which, the plane part, AF , is called the Frize; the whole together composes an Entablature.

Let the several Mouldings be geometrically described, according to their heights and projectures; at a , b , c , &c. and, CD being drawn perpendicular, may be considered as a vertical Section of the Picture, applied close to the greatest projecture of the Cornice, at D .

According to the height which the Cornice is supposed to be above the Eye, take the Point C , and draw CE perpendicular to CD , i. e. horizontal; and equal to the Distance of the Picture.

Fig. 85.

Now, E being considered as the Eye of a Spectator, viewing the Cornice, AD , &c. EA , ED , EF , &c. are Visual Rays, in which direction the several parts are seen; and, CD is supposed to be a Plane, interposed, cutting those Rays; consequently, the several Points, A , a , b , &c. projected by them, are the apparent places of the several edges, or parallel Lines of the Mouldings, on the Picture.

C is the Center of the Picture, and EC is its Distance; by which, the perspective representation of the Mouldings must be projected; provided that, the drawing be required to the Scale CD , of the Profile, ABD .

But, it may, from the same Profile, be made either greater or less. e. g.

If the Rays ED , &c. be continued, beyond D to H , and HI be considered as the Plane of the Picture, the projection of the Cornice is JH , which is greater than AD , and so of the rest; EI is the Distance of that Picture.

Or, if any other Plane be interposed, as at Kd , or CH ; the proportions of the several Parts on each, viz. GF , FA , and AD ; gf , fa , ad , or gf , fa , ad , are, to each other, as their several Distances, EC , to EK , or EC . (6. 6. El.

Hence it is evident, that, from parallel Sections of the Rays, EA , ED , &c. any where, the Mouldings will have the same proportion to each other.

The several Sections, CD , Kd , and CH , are Scales of the perspective Proportions for each respective Distance.

It is required to delineate the Entablature (No. 1.) by the Scale CH .

U u

Let

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Let E be considered as the Center of the Picture or Point of View, and EC the Distance of the Picture; the Terms being inverted.

To the farthest Corner (B) of the given Cornice (i. e. to its height, or Seat of the extreme projecture, D , on the Perpendicular AB) draw EB , cutting CH at H . Take any length, required, as CI , for the Ground Plane of the Cornice.

Draw IK parallel to CH , i. e. perpendicular to EC , the Horizontal vanishing Line; and from H , where EB cut CH , draw HB parallel to the Horizon, cutting IK at B . Draw dD also parallel; and CB , cutting it, at D , gives the Angle D .

Draw Eh and Ei ; and, from their Intersections, with CH , draw Lines parallel to Dd , till they cut BD ; and, from b and i draw Perpendiculars, cutting the Parallels from c and b , at k , which compleats the Corona. ekb represents the Planceer, $b c$, in the Profile. The rest is obvious, for the parallel Cornice.

The projecture of the Architrave, at F , is obtained, by drawing CF , after the same manner as the Cornice, at D .

If the length of the returning Plane exceeds the limits of the Picture, take any equal part of the whole length; as a B , half, in HB , produced; make EE equal to half EC , and draw a E , cutting EB , at b .

To ascertain the mitre Angle, e , seeing that the Vanishing Point of the other Diagonal is not in the Picture, draw bd parallel to aB ; and DE , cutting it in c ; make cd equal bc , and draw dC , cutting DE in e , the Point sought.

Or, draw bL parallel to aB , cutting the Diagonal, DC , at L ; bisect BL perspectively, at f ; draw fb , and produce it, till it cuts DE , at e . Draw ef .

For, because bL is parallel to the Picture, and, C is the Vanishing Point of one Diagonal of a Square, in that position, Bb represents a Line equal to bL ; and BbL representing a Right Angle, is bisected by fb , half the other Diagonal; which, being produced, cuts DE at e , the extreme Angle of the Cornice.

Draw Eb and Ei , cutting eb at 1 and 2; from which, draw Perpendiculars; and draw Lines to E , from the Corona, at k , and all the Fillets, cutting ef ; observing, that the extreme Angle of the Corona, goes beyond the Diagonal.

This Method of representing Mouldings is deduced from the most natural and simple Ideas of it, and cannot, I think, fail of being intelligible to the meanest capacity, who has any emulation to attain to the art of drawing in Perspective.

As there are no Modillions, or other Enrichments, of that kind, in this Cornice, there is no apparent distortion; if there were such, they would be distorted towards the extreme CH , seeing that, the Mouldings extend to the full Distance (EC) by which they are projected.

E X A M P L E XVI.

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To represent a Doric Entablature, with an internal and external Angles.

On account of the variety of Parts, and the many breakings of the Fillets, around the Triglyphs, in the Doric Frize, it is necessary to have it described, geometrically; so that, the Parts may be clearly understood. Indeed every Moulding, delineated in Perspective, ought first to be geometrically drawn, to the Scale of the Drawing; although the measures may be applied from a Scale only.

I have shewn, in the 12th Example, how a perspective Plan may be drawn either above or below the Work; by which the projectures of the Mouldings, &c. may be determined at the Angles, as exemplified in the 13th. But, that process (though the most to be depended on, for accuracy) is attended with extraordinary trouble, which may be lessened greatly, when we are tolerably acquainted with Mouldings. A Specimen of both will be shewn, in the following Example.

Fig. 86.
No. 1.

Let AB be the height of the Entablature, according to the Scale of the Drawing. Let it be bisected at C ; and AC , or CB , is a Module, or Diameter of the Order. AC divided into 60 Minutes, is the Scale; by which the whole is proportioned.

QM , the projecture of the whole Cornice, is a Diameter; the rest is proportioned as by the Scale AB .

DF is the Architrave, FG is the Frize, with its Triglyphs, and GMQ is the Corona. HI is the projecture of a Mutule or Modillion, and K of the whole Planceer; which, being seen in Perspective (the Eye being below it) has a fine effect, and adds greatly to its august appearance.

No. 2 is a geometrical Plan of the Cornice, with an internal Angle at S, and an external, at T; shewing the Planeeer of the Mutules, and panneling between them; for, unless we know the true geometrical form, it is not possible to describe any thing, perspectively. Each return of the Cornice is the same, having two Mutules each way, except the last, on the Right, which is supposed unlimited; and, on the Left, it is limited by the bounds of the Picture; which, on account of its distance from the Center, would be distorted, if it was continued much farther.

No. 2.

BAFG is a perspective Plan of the whole; which is the best method of proceeding, if accuracy be required in the several parts, and we do not grudge the time spent in doing it. It is formed as follows.

No. 3.

At any Distance from the Horizontal Line, *ECE*, either above or below it, draw *AB* parallel to it; one Side, in this Example, being parallel to the Picture.

Having determined on the Place of the Angle *a*, in the Design, take *A* perpendicular over it, and draw *AC*, indefinite; *C* being the Center of the Picture.

Take *AD* equal *ST*, in the geometrical Plan; and, having made *CE* in the Horizontal Line, equal to the Distance of the Picture (equal *ES*) draw *DE*, cutting *AC* in *F*; then, *AF* represents the length of *AD* (equal *ST*, No. 2) and *F* is the internal Angle of the extreme Moulding in the Cornice.

Draw *FG* parallel to *AD*, and *DC* cutting it, in *G*, the external Angle; for *FG* also represents an equal length as *AF*†, each equal to *ST*.

† Prob. 19.

Make *AC* equal *MQ* (No. 1) the projecture of the whole Cornice, and draw *CC* and *AE* cutting it, at *V*. The Diagonal *FX* (i. e. *DE*) also cuts *CC* at *X*; from which, draw *XY* parallel to *AB*.

The Angles *K*, *L*, and *M*, of the Corona (*K*, in the Profile) are determined by making *Ad* equal to *MN*, and drawing *dC*, cutting the Diagonals *AV* and *FX*; by means of which, they are carried around, to *L* and *M*, and, by the same means, the whole Plan is completed, as in the Figure.

To describe every Step, by which the whole perspective Plan is formed, would be as tedious as it would be useless; seeing that, the various Lessons, already given, are sufficient for any right-lined Figure whatever.

Fig. 86.

The Plans of the Mutules, *Z*, *Z*, are the representations of Squares having one Side parallel to the Picture (found by Prob. 19) their places are determined as follows.

Having made *Ae* equal *MO* (in the Profile) draw *eC* cutting the Diagonals *AV* and *FX* at *S* and *W*, and draw *WM* parallel to *AB*.

Make *Aa* equal to *JM*, in the Profile (equal *Ti* in the geometrical Plan) make *ab*, *bc*, &c. equal to the width of the Mutules and the spaces between them (12, 23, &c. in the Plan) and from each Point, *a*, *b*, &c. draw Lines to *C*, the Center, cutting a parallel Line from *S*, in *R*, *P*, &c.

For, the returning Side, *AF*, make *Af*, *fg*, &c. equal *Aa*, *ab*, &c. and draw *fE*, *gE*, &c. cutting *AF*, at 1, 2, 3, from which, draw Lines parallel to *AB*, cutting *SC* at *T*, *V*, &c. and *HC* at *m*, *n*, &c.

For the other parallel Side, *FG*, make *Dm*, *ml*, *lk*, also equal to *Aa*, *ab*, &c. from which draw Lines to the Center; cutting *WM*, at *X*, *Y*, *Z*.

For the apparent width of each Moulding, &c. in Front, draw *MS* perpendicular to the Horizontal Line; take *SE* equal to *EC*, on the Horizontal Line; and, from every Angle, *G*, *H*, *K*, &c. in the Profile, draw *EG*, *EH*, &c. cutting *MS* in the several Points, *g*, *h*, *k*, &c. as in the last Example.

Being thus prepared, we now proceed with the Representation.

a is the determined Angle of the extreme Moulding; draw the Fillet *az* of the upper Moulding (which is in the Picture) geometrical.

Fig. 86.

From *V*, *X*, and *Y* (in the perspective Plan) draw perpendicular Lines indefinite; which are the Angles of the Ground Planes* of the whole.

Then, from the several Angles *H*, *K*, &c. in the perspective Plan, draw Lines perpendicular; and, from the corresponding parts on the vertical Section, *MS*, draw Lines parallel to the Horizon, cutting them. *c. g.*

* By Ground Planes, here, is not meant the real Ground or Floor, as defined, but the vertical Planes, from which the whole Cornice, &c. project; called so by Workmen.

Plate
XVIII.

From *K* draw a Perpendicular, *Kc*, and from *k* (where *KE* cut *MS*) draw horizontal Lines cutting it, at *c*, the representation of the external Angle of the Corona.

Draw *cC*, cutting the Perpendicular from *L* at *l*, the internal Angle of the Corona; and from *l*, draw parallel Lines, cutting the Perpendicular from *M* at *m*, the other external Angle of the same.

Then, from *h* (where *HE* cuts *CM*) draw horizontal Lines cutting the Perpendicular from *H* (in the Plan) at *h*, the representation of the Angle *H*.

Draw *hC* cutting a Perpendicular from *I*, at *j*, and the horizontal Line *j n* gives the other external Angle, *n*.

For the Mutules, draw a Perpendicular from *S*, in the Plan (where they would meet if they were continued to the Diagonal *AV*) and from *i* in the Scale Line, *MS*, draw parallel Lines cutting it, at *f*; which must be returned at all the Angles, as if it was a continued Facia.

From *O*, *P*, &c. draw perpendicular Lines cutting them, in the several Points, *o*, *p*, &c. which are the front Faces of the Mutules. Draw *rC*, *qC*, &c. cutting the horizontal Line *hh*; where they fall against that Facia.

In the returning Side, draw Perpendiculars from *TV*, &c. till they cut *fC*, the returning Facia, and so of all the rest; which are best described by the Figure; observing that, if from the several parts, in the first or nearest front of the perspective Plan (*BAV*) perpendicular lines be drawn, and from the corresponding parts in the Section, or Scale Line, *MS*, parallel Lines cutting them; and, by carefully remarking the mitre Angles of each Member, on *AV*, the true perspective proportions of them are carried round as many Breaks, in the Object, as are required; by means of the Vanishing Point *C* (the Center) for one return, the other parallel; the Originals of them being, in this Example, parallel to the Picture, and the Object right-angled.

The Mouldings around the Mutules are determined, at their projectures, from the Plan, above (at *O*, *P*, &c.) and the width, in Front, from the Section of the Picture, *MS* (at *m*) by means of which, and the Vanishing Point *C*, they are very easily continued around the Mutules, from one Angle to the next, adjoining. First, drawing parallel Lines, from *m*, till they cut Perpendiculars from *O*, *P*, &c. at *e*, *f*, &c. and *fC*, *bC*, &c. cutting Perpendiculars from *J*, at *i* and *j*; through which they are drawn, parallel, to the Angle at *k*, and from *k* they are continued, after the same manner, around the whole.

Every thing being correct, and very particular in the Figure, makes it unnecessary to give a further description of it. To describe every Line in the whole Process, would take several Pages, be extremely tedious and apparently prolix.

For if, as I have before observed, the Student be not acquainted with Architecture, nor understands the several Parts of the Order, it will be impossible for him to succeed in this Example; but if he is, it will be found sufficiently intelligible.

The Architrave, of this Order, it is needless to say any thing about; seeing it is composed of plane Facias, only; projecting, one over another, like Steps, of different heights and projectures, and is managed in the same manner; which, from inspection of the Figure, may readily be described.

The Frize and Triglyphs have nothing difficult in them, especially in this Position. In the parallel Faces they are geometrically proportioned and spaced (Theo. 9. Cor. 5) and in the returning Side, their places are determined from the Plan above, or by the 10th Problem; dividing the upper Line of the Frize, *st*, in the same Ratio, perspectively, as the front Line, *ns*, is divided, at *o*, *p*, *q*. (Prob. 8.)

At No. 4, I have given two Triglyphs, more at large; in order, that the Parts may be more distinctly made out; in which, after dividing the front one, as in the Profile, geometrically, for the flutings (which are right angled) the indented Angles may be obtained, as in the Plan *ABD*, above; by drawing a Line from each Angle *a* and *b*, to the two Distance Points, if they are within reach, cutting at *c*.

Or,

Fig. 57.

Fig. 58.

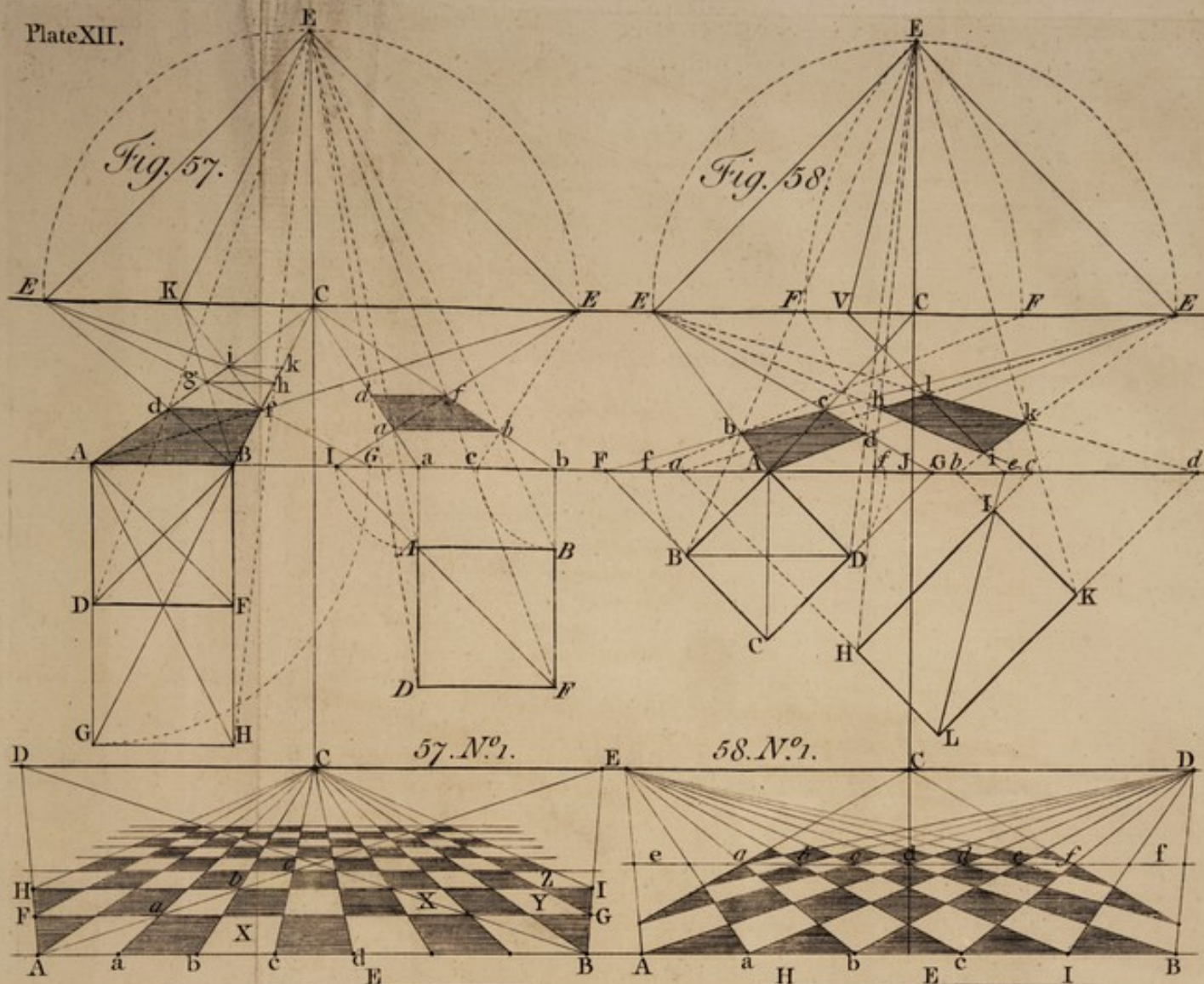
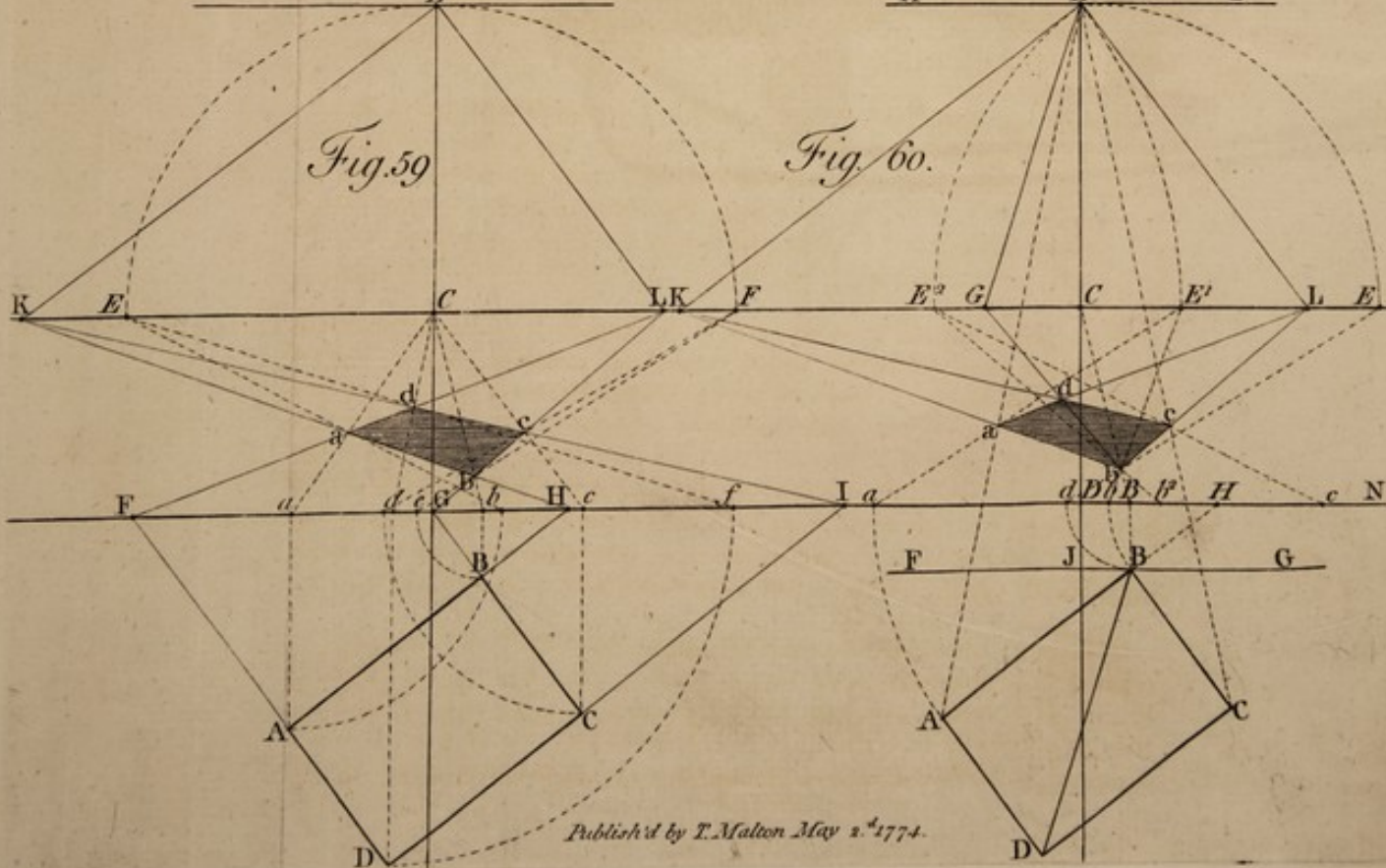


Fig. 59.

Fig. 60.



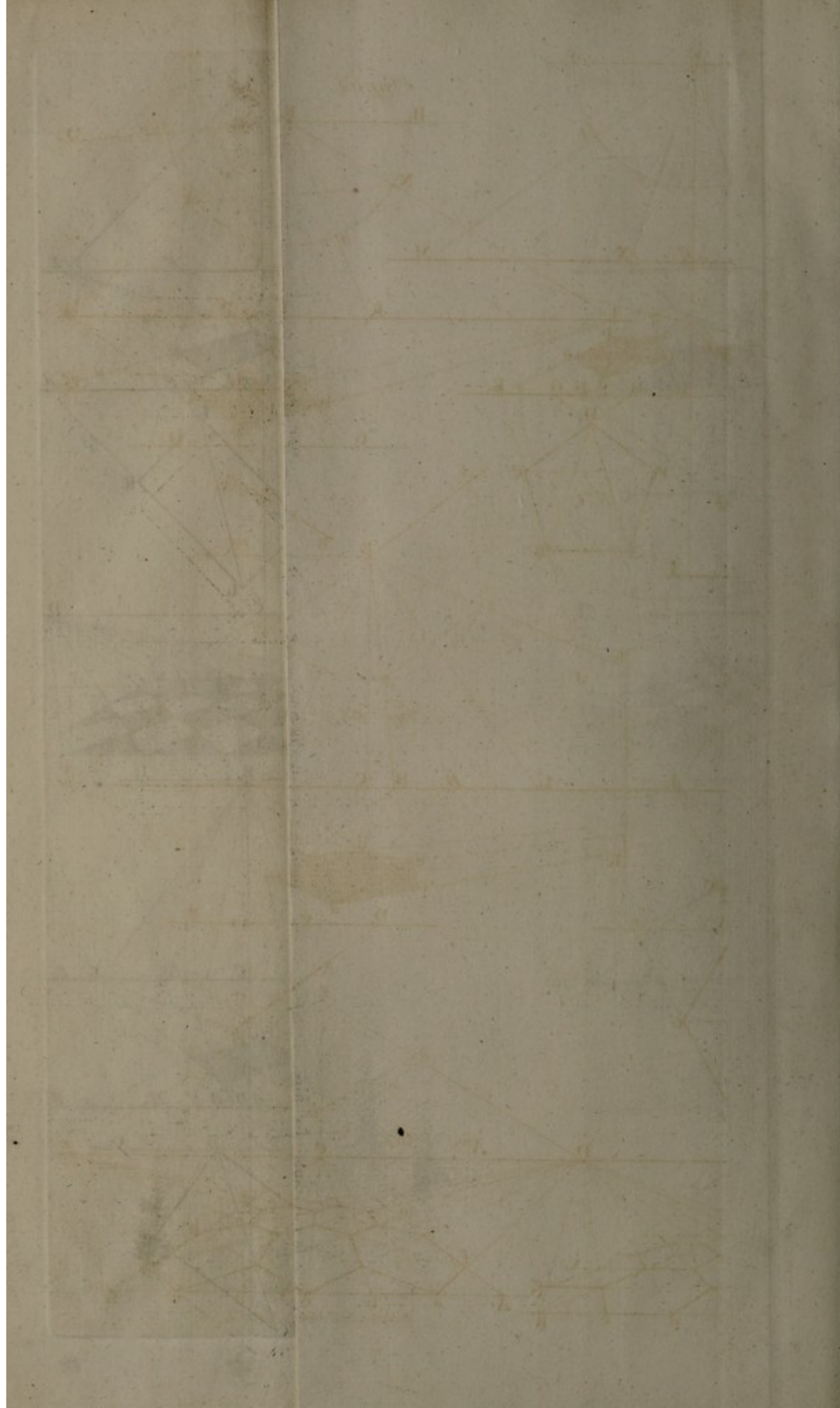
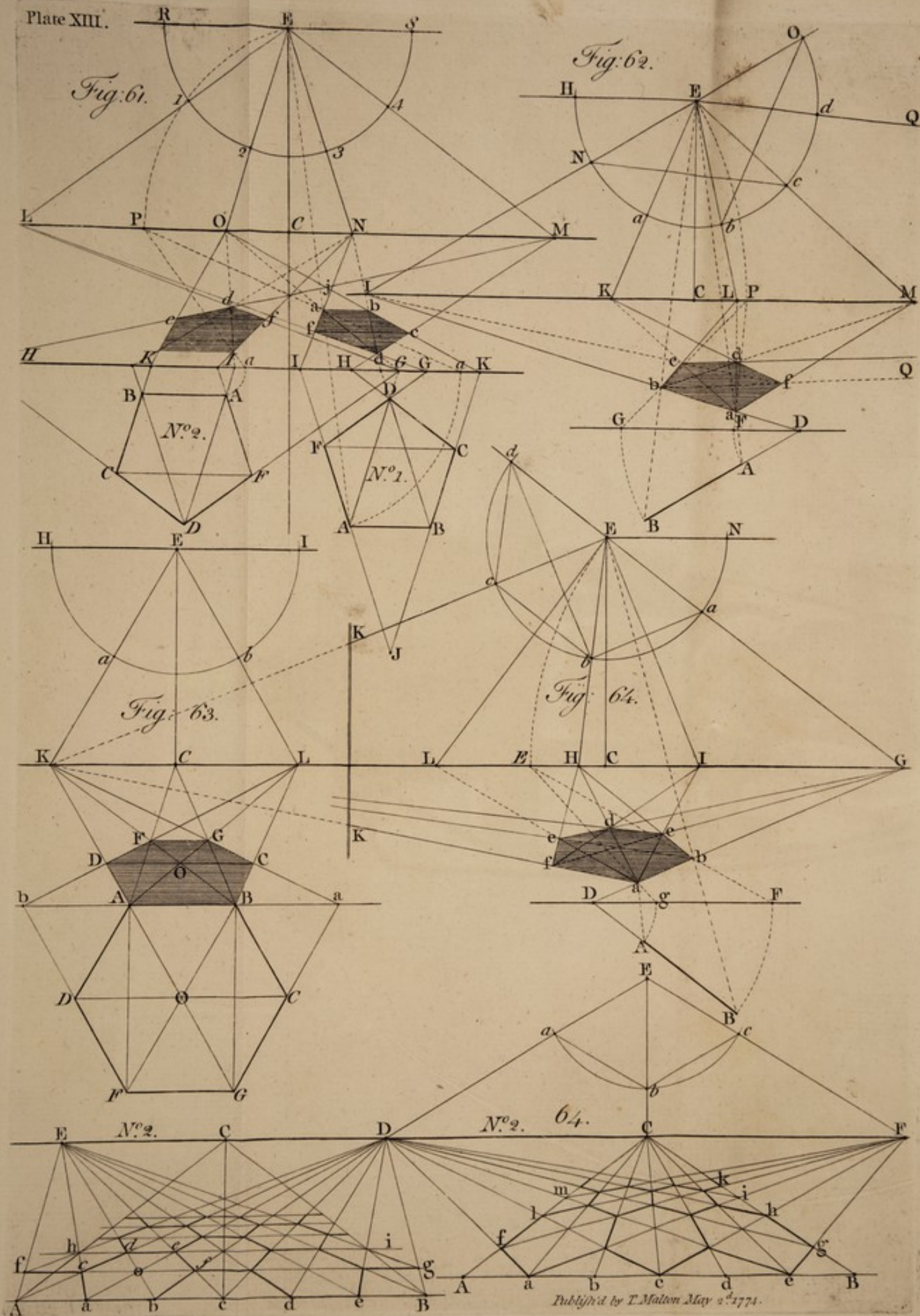


Fig. 61.

Fig. 62.



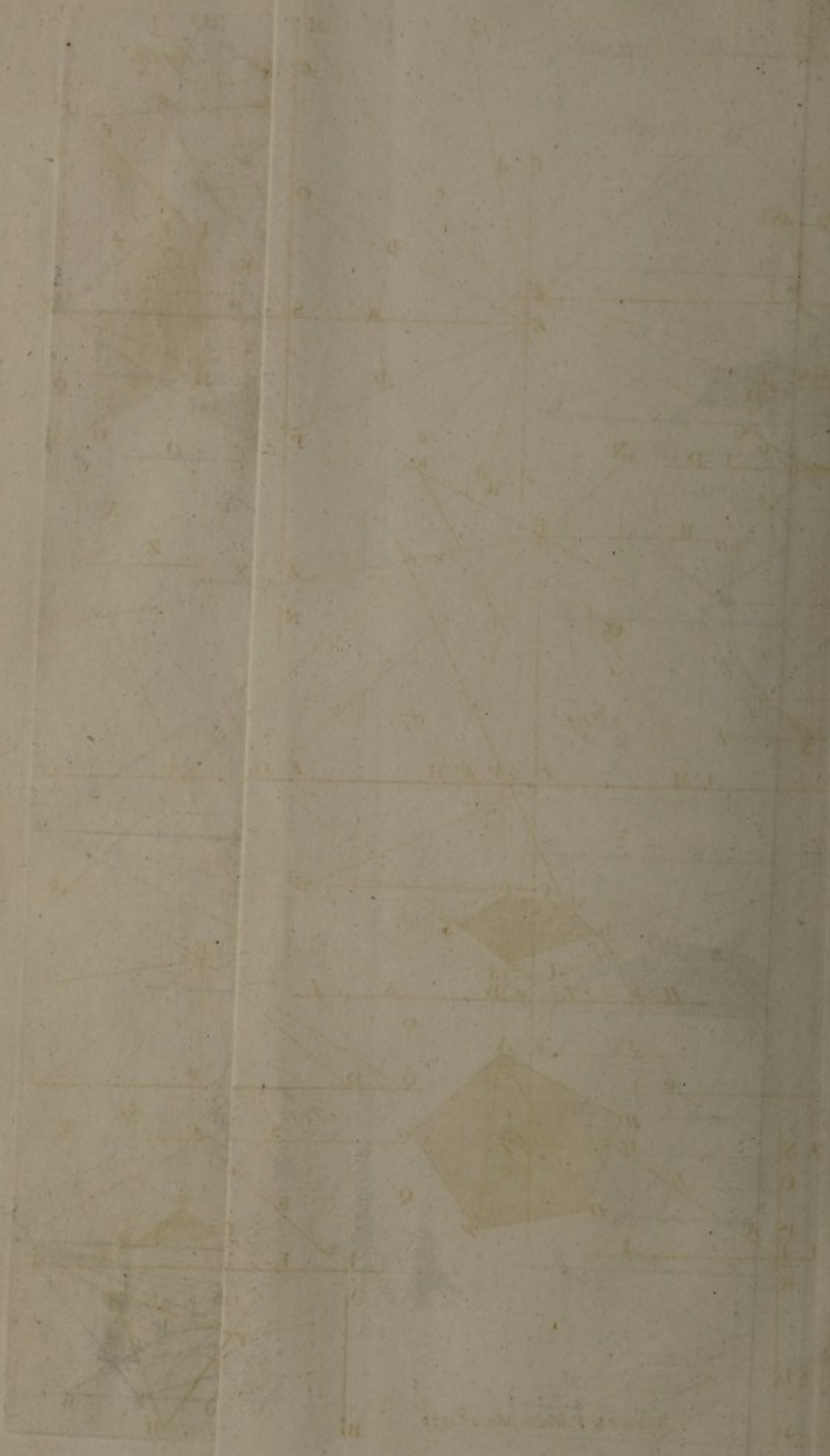


Fig: 65.

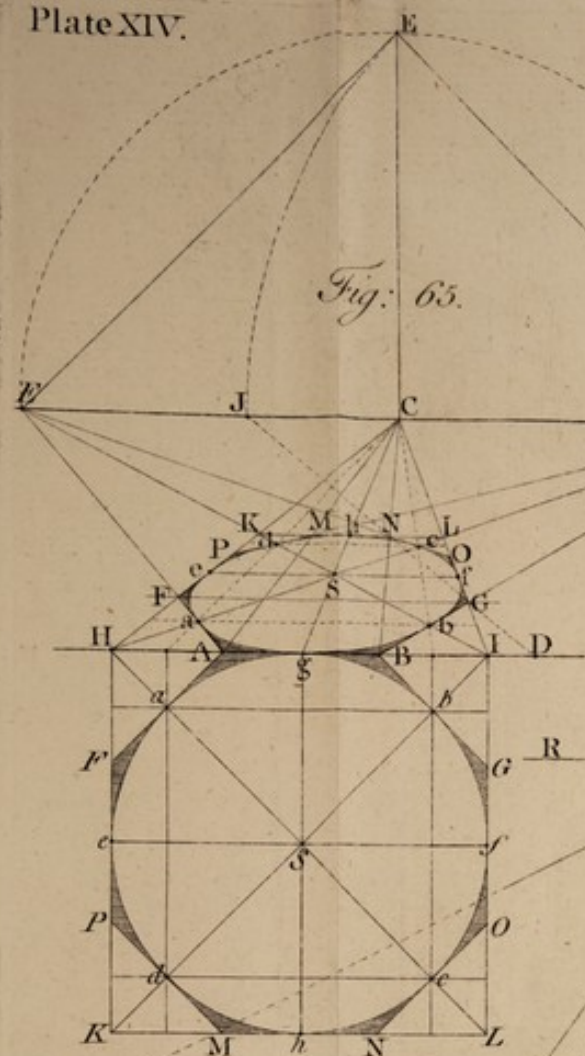


Fig: 66

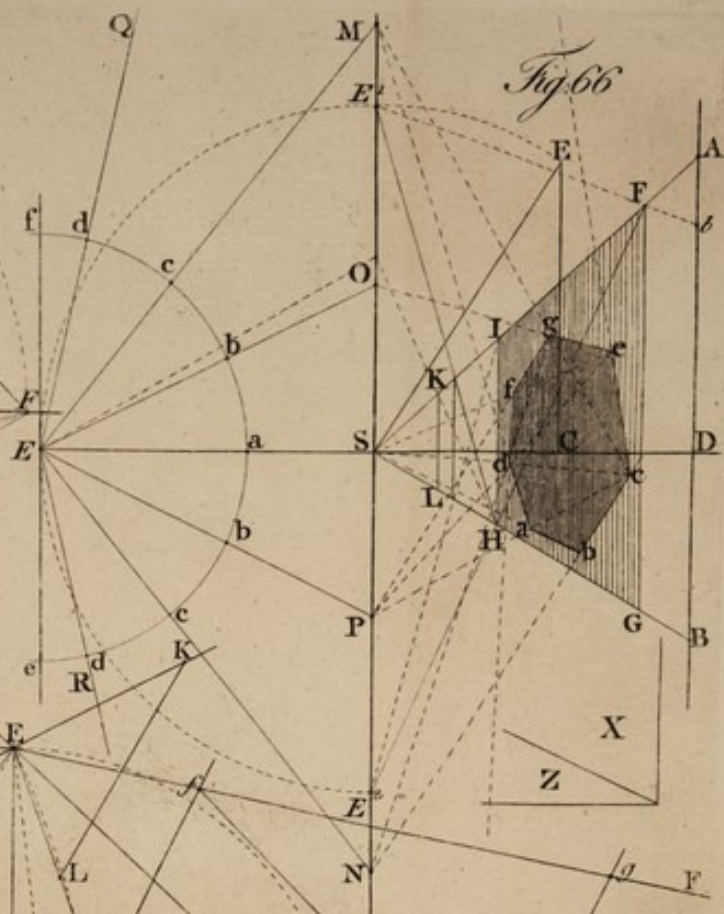


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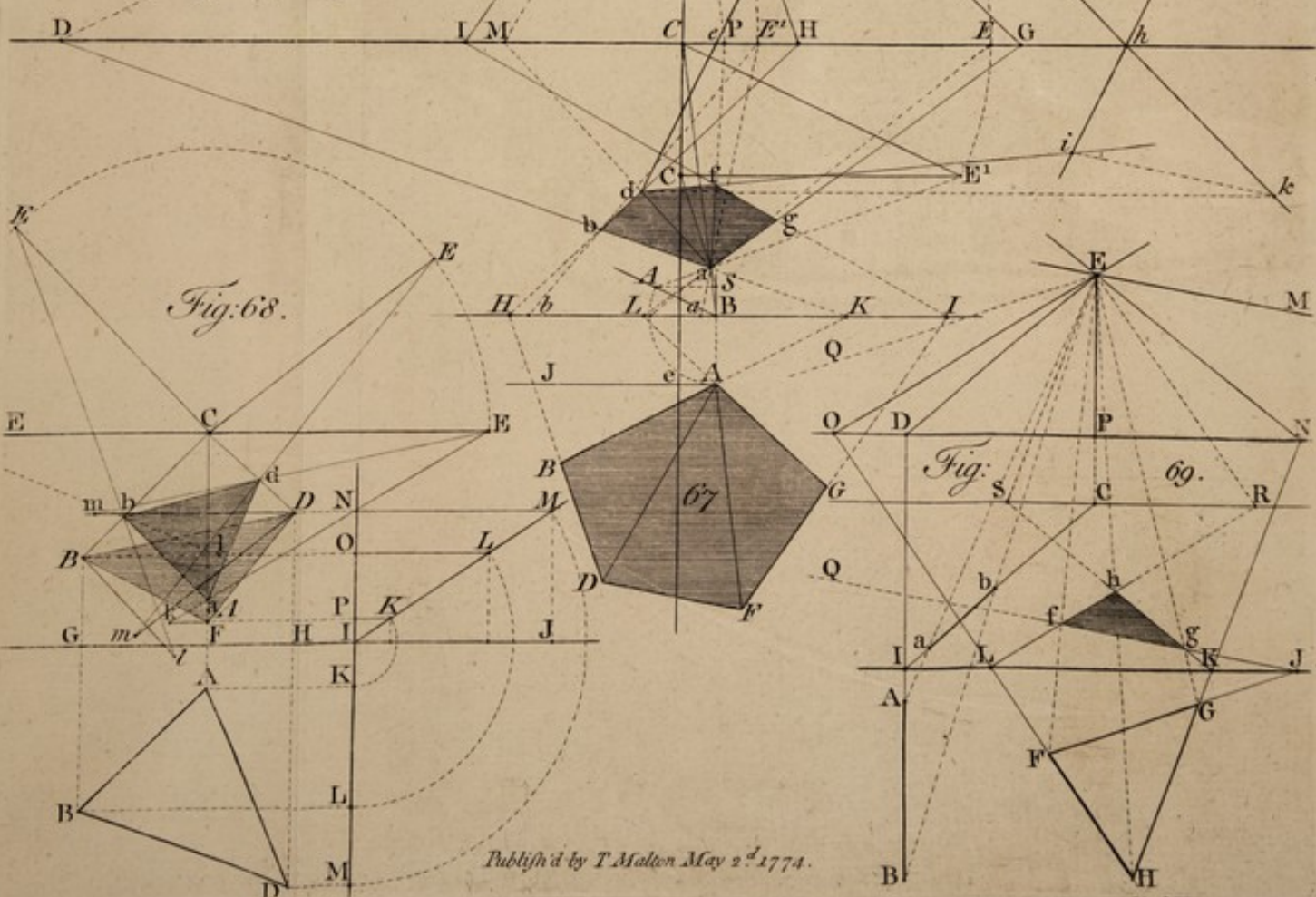


Fig: 68.

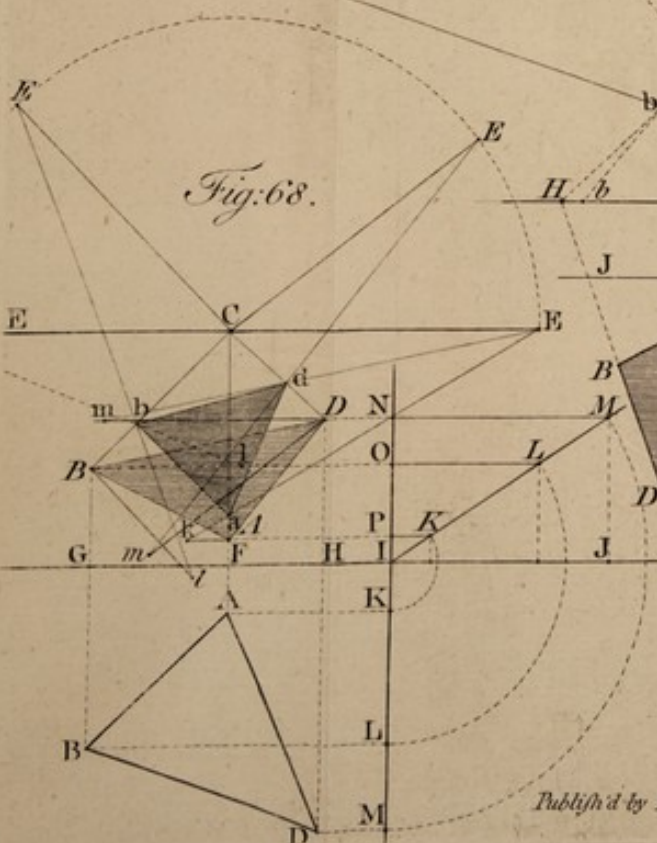
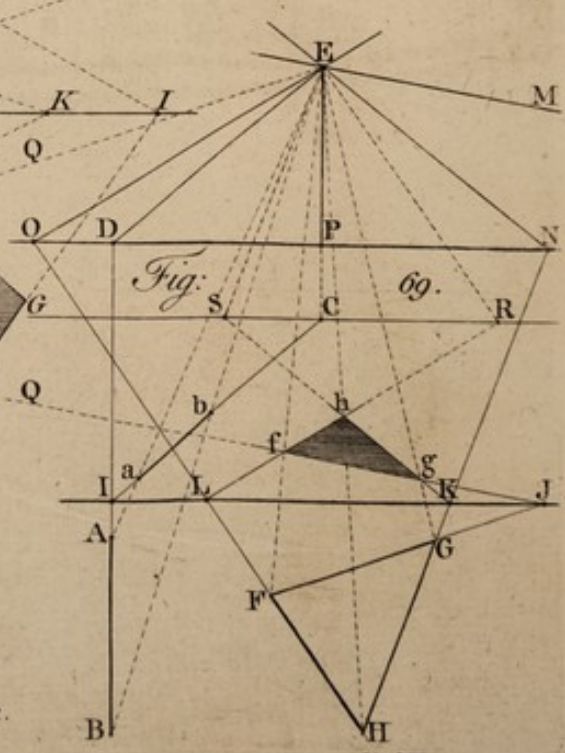
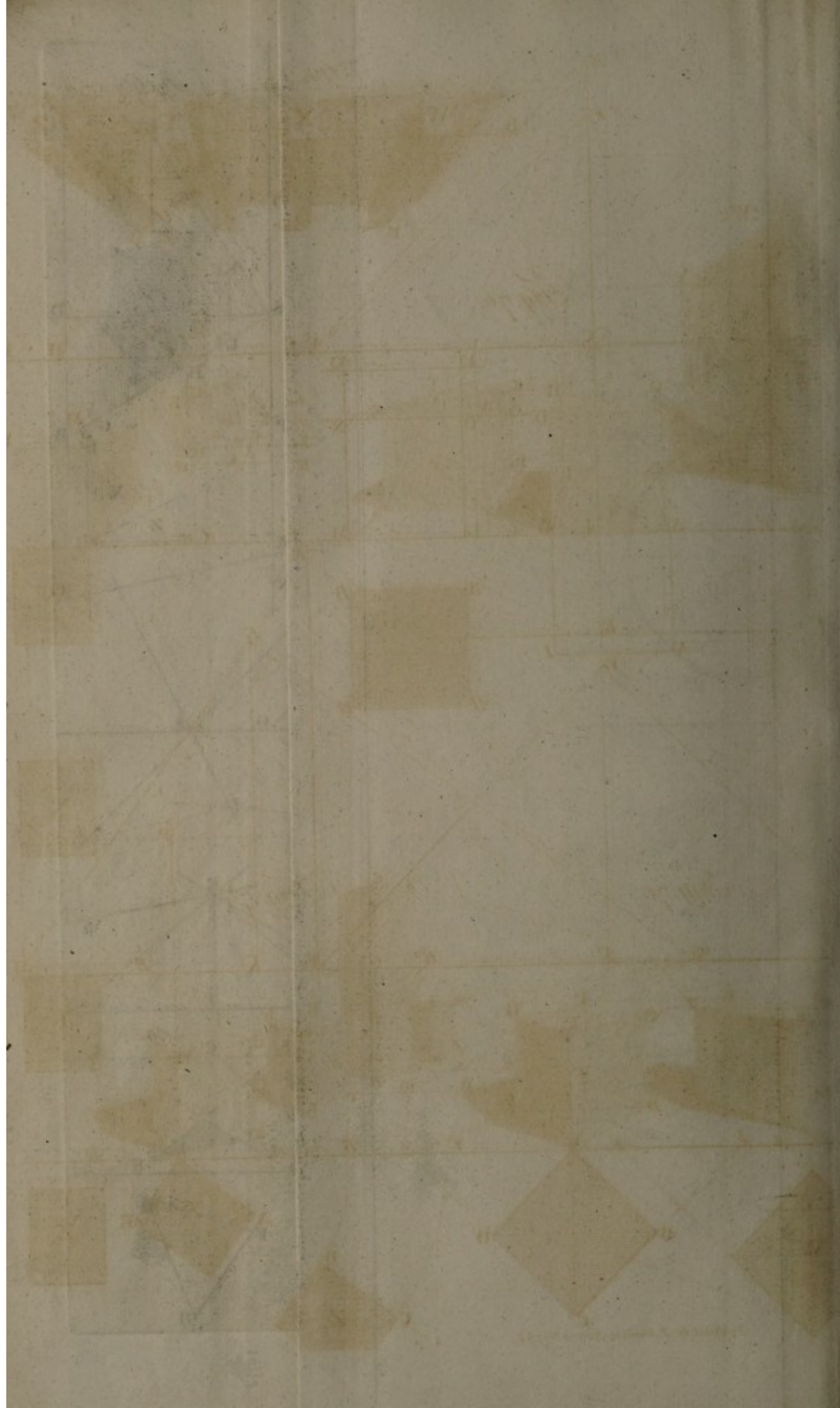
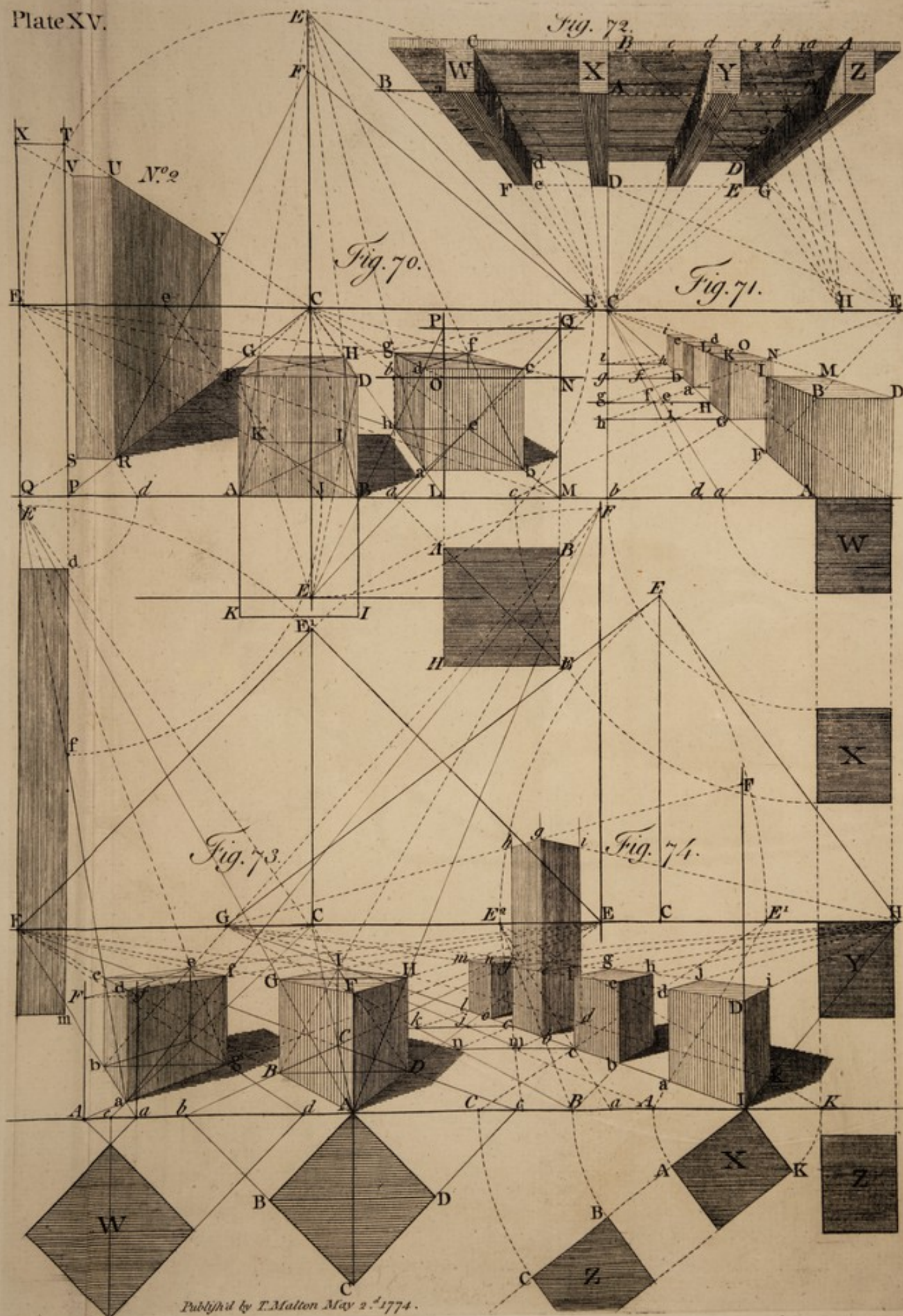


Fig: 69.







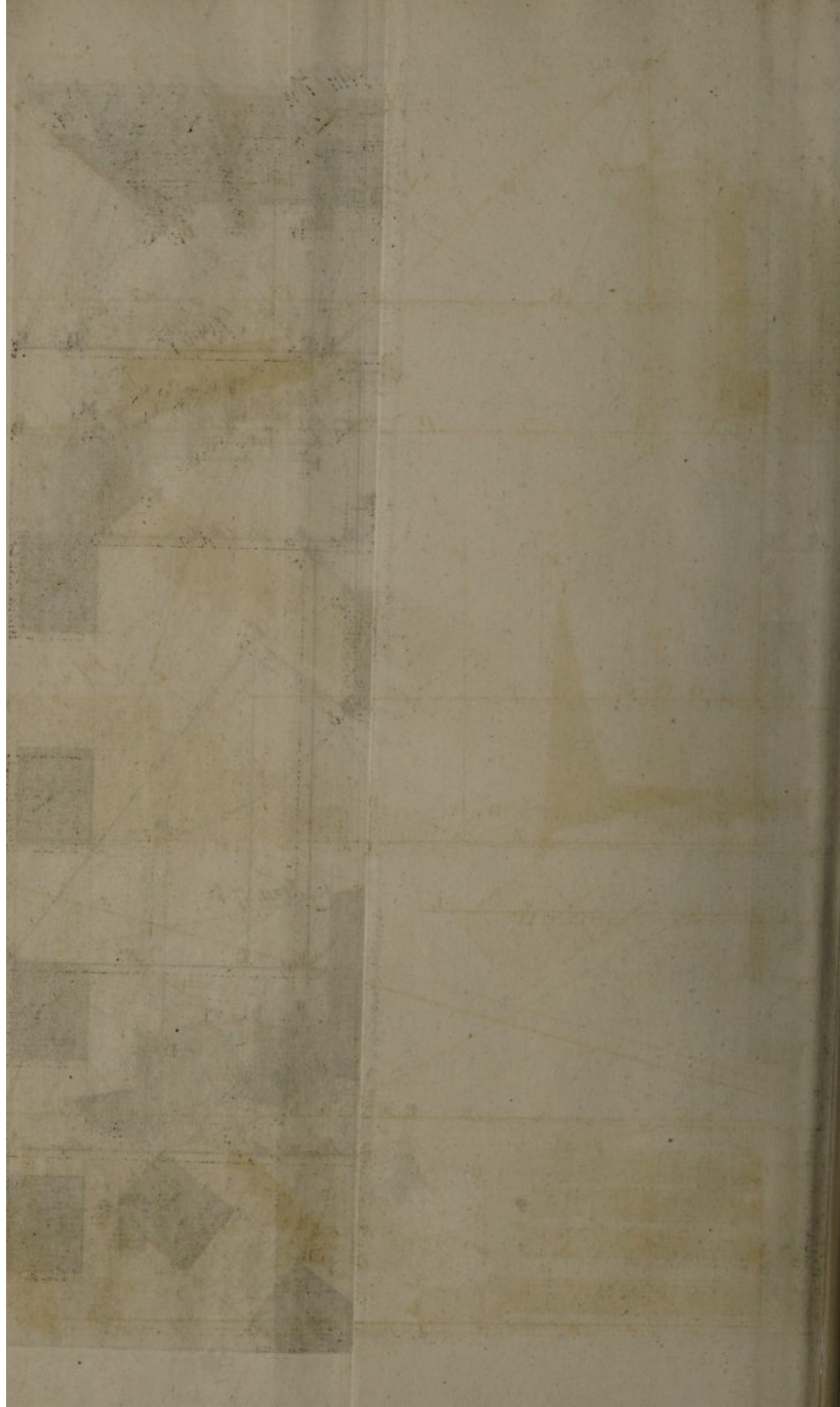


Fig. 75.

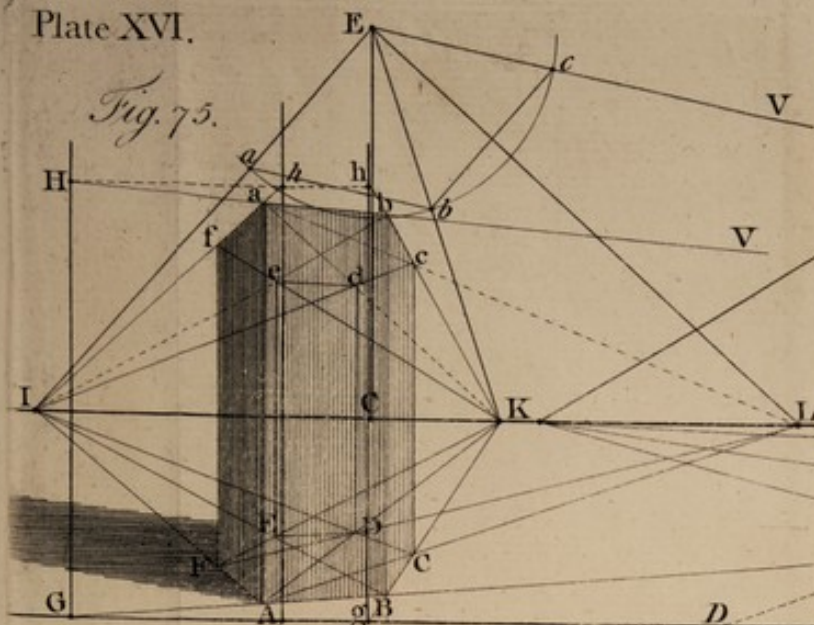


Fig. 76.

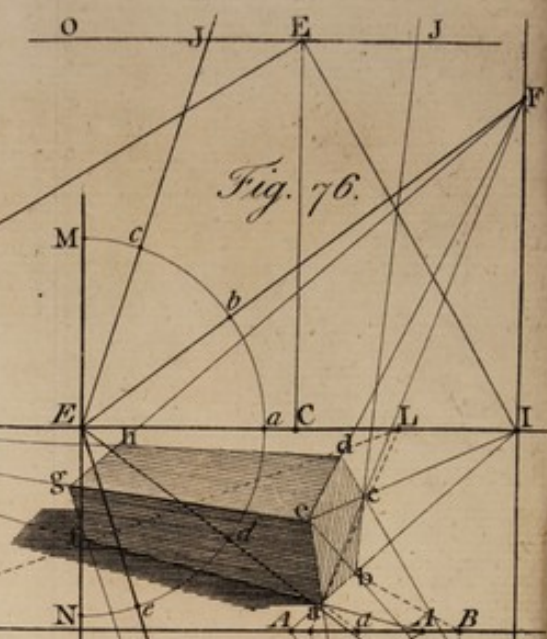


Fig. 77.

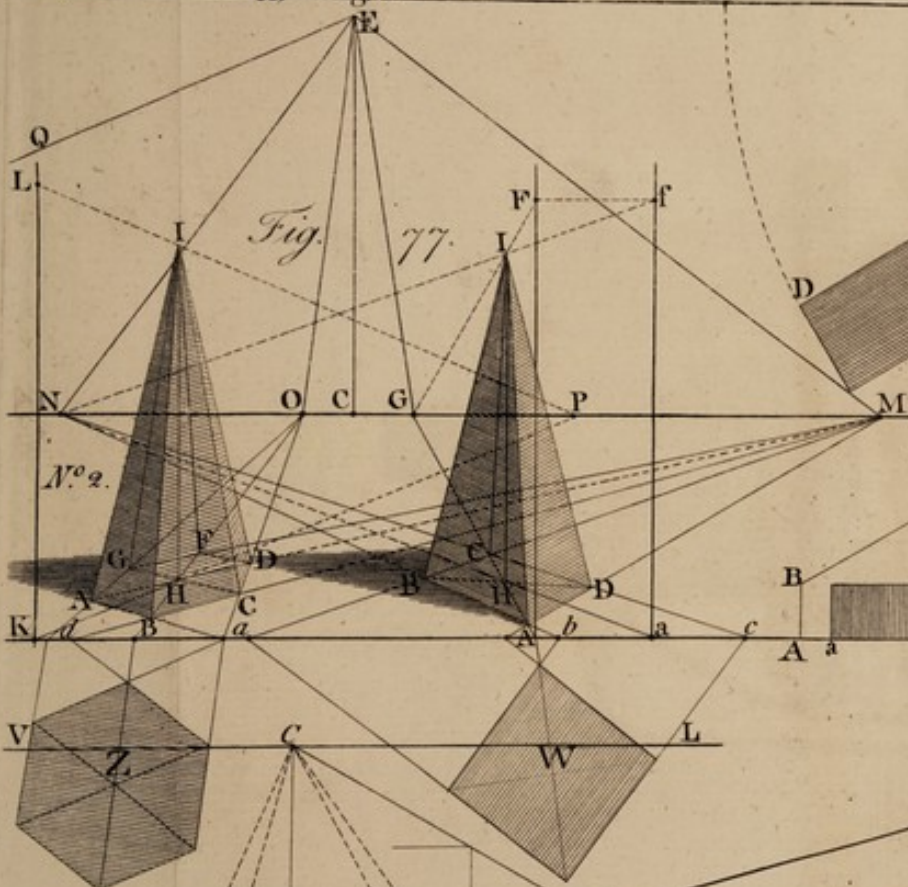


Fig. 78.

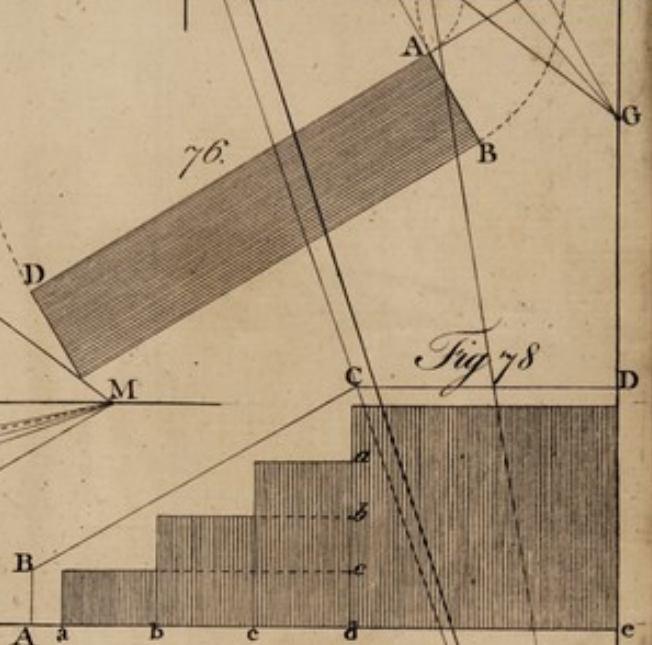


Fig. 79.

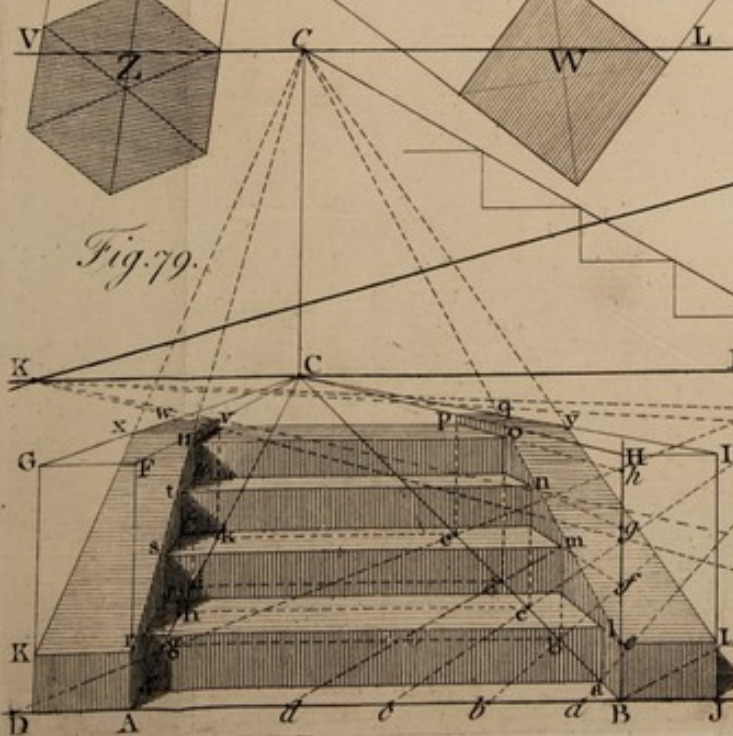
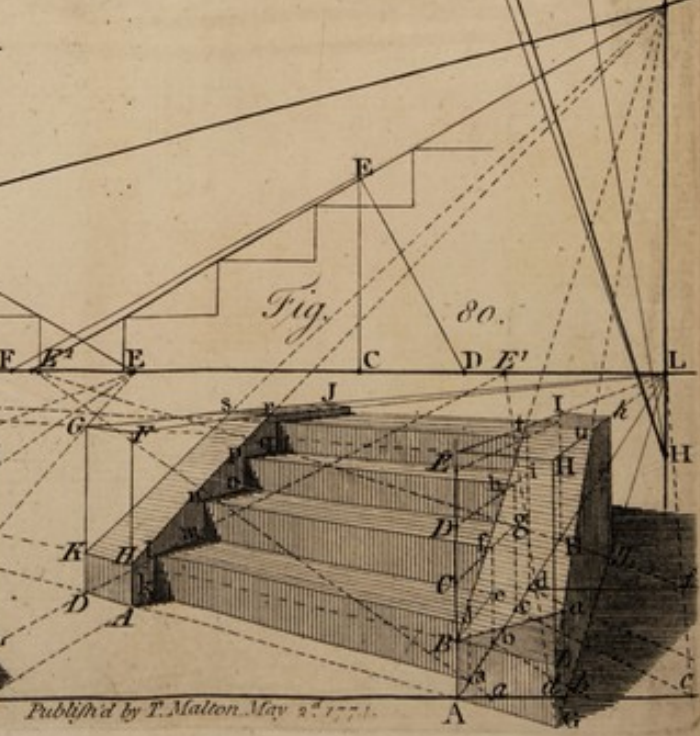
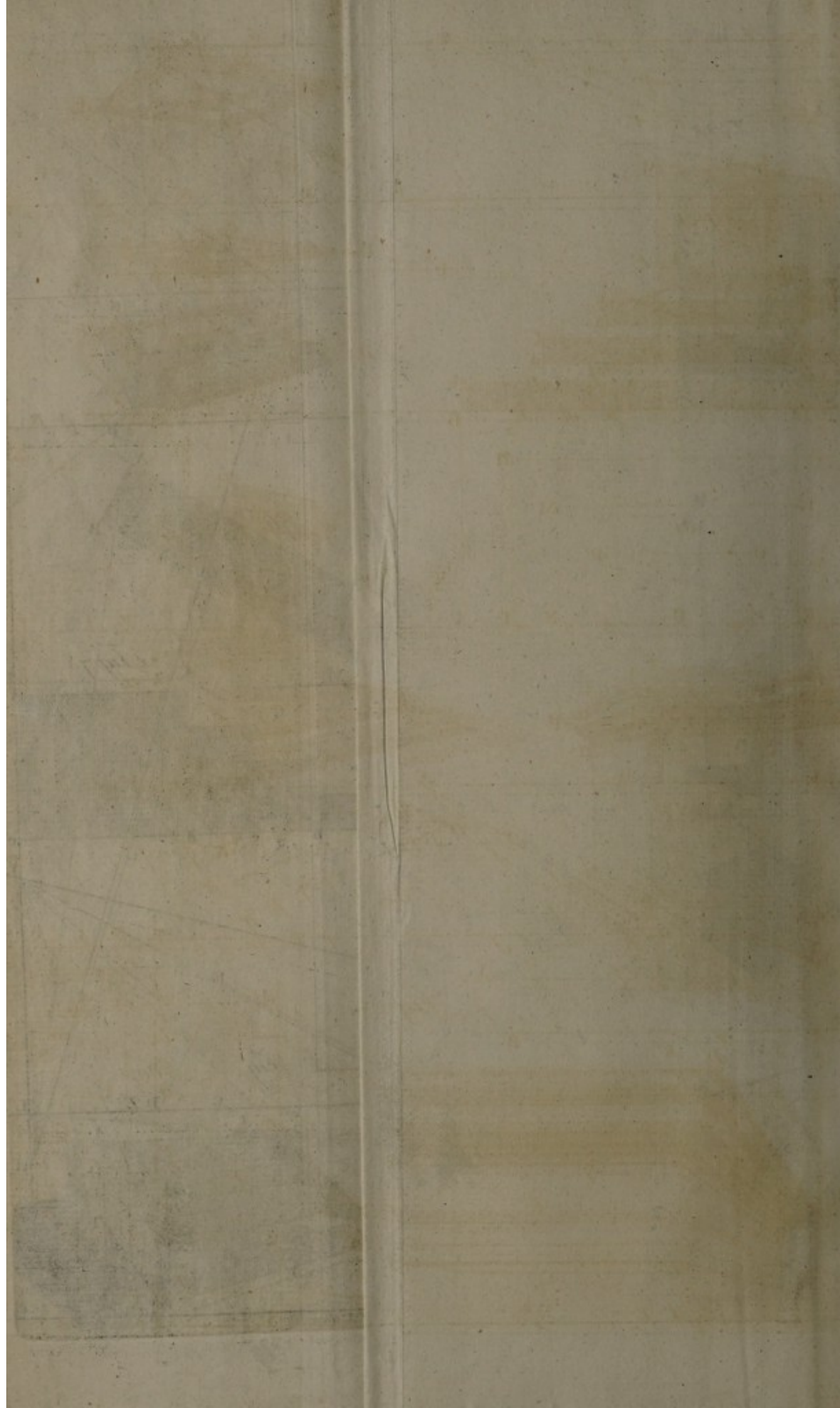
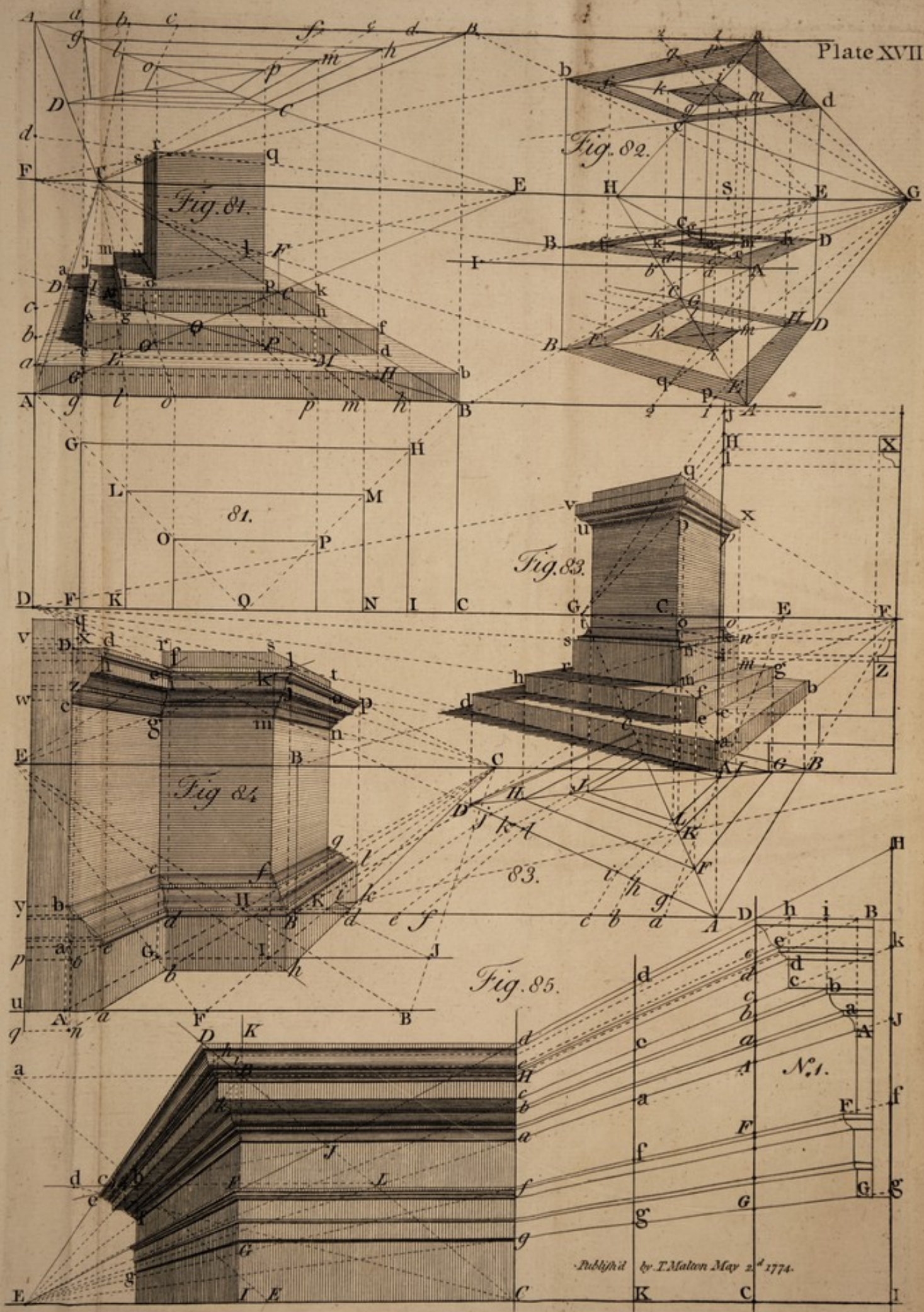


Fig. 80.









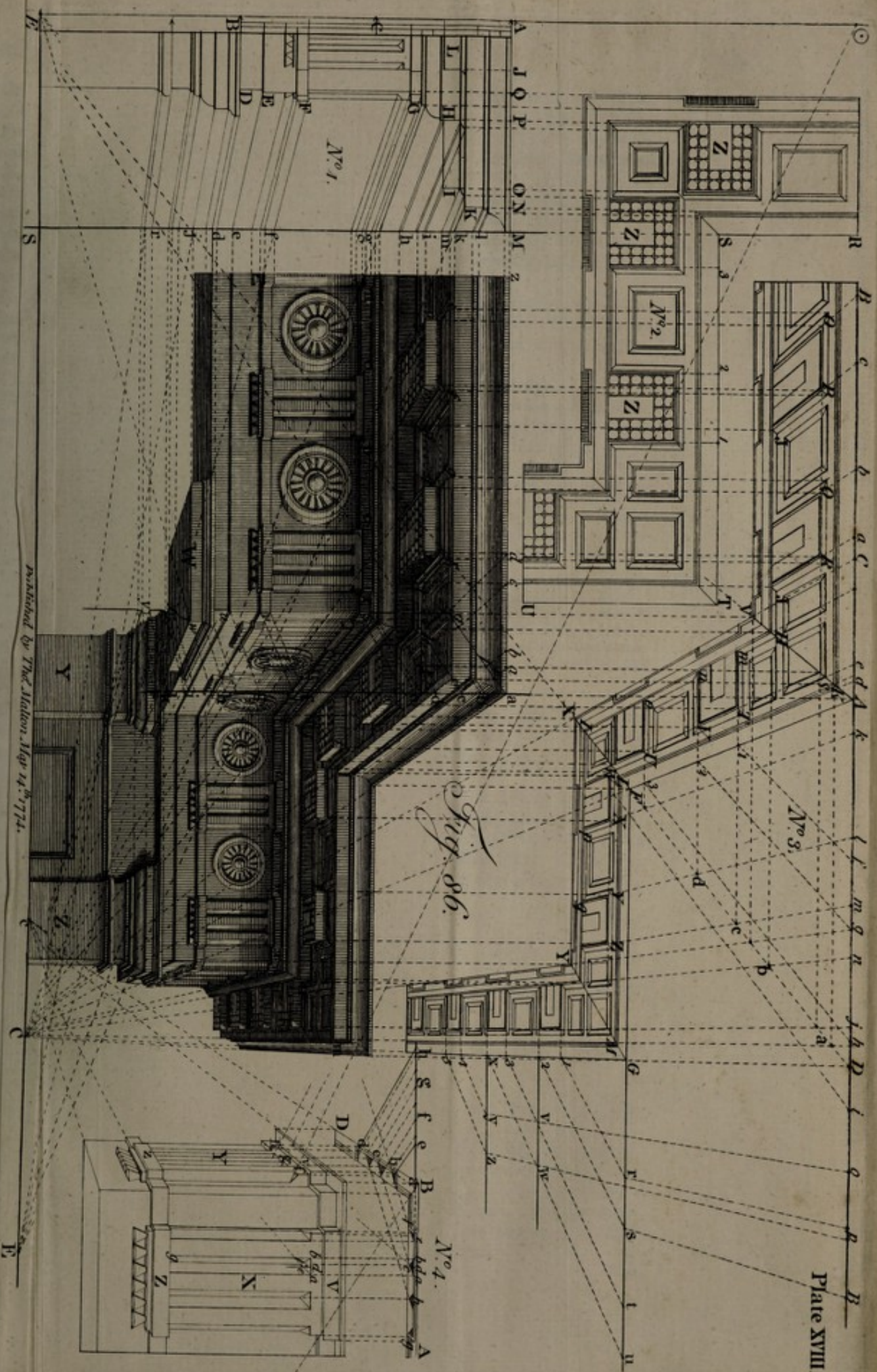


Plate XVIII.

Fig. 86.

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Or, if one Distance (*E*) only; bisect the width of the Flute, at *d*, and draw *dc*, to the Center, cutting *aE* at *c*, as before; *ef*, &c. drawn, to *E*, give the rest.

For the Triglyph in the returning Side; having drawn the front Line, *ad*, to its vanishing Point, *c*; from each angle, *a*, *b*, &c. in the front Triglyph, draw lines to *E*, cutting *ad*, at *b*, *c*, and *d*.

Or, if it be convenient to have the Distance of the Eye on the other Side, it would be better; draw *ah* parallel to *AB*, and take the divisions, from *a*, at *e*, *f*, *g* and *h*, equal to the geometrical measures; and, from them, draw Lines to the other Distance, as in the Figure (supposing it within reach) which give the same Points, more accurately defined. (See Prob. 8.)

From the several divisions, on each Side, draw Lines perpendicular, as in the Figure; by which means, the Triglyphs, *X* and *Y*, are completed. The Perpendiculars, from *c* and *b*, (in the Plan, above) give the indented Angles in the front Triglyph; in the other they are not seen.

This operation is supposed either at the top or bottom of the flutings.

The inclined Lines, at the Top, are determined, truly, by drawing a Perpendicular at the middle point, *d*, making *de* equal to half its width, and drawing a line to *c*, the Center, cutting the Perpendicular from *c*, at *f*; join *af* and *bf*.

It is unnecessary to be more particular, or their vanishing Points may be easily determined; by making the same Angle at the Eye, as the inclined Lines make, either with the horizontal or vertical Line (Prob. 4.) and where it cuts a Line, passing through *E*, perpendicular (the Vanishing Line of an inclined Face) is the Vanishing Point of *af*. Either being found, determines the other; and, the inclined Planes of *Y* being parallel to those of *X*, have the same Vanishing Lines, respectively, passing through the Eye in the Points of Distance.

The Vanishing Points of the inclined Lines, in the front Triglyph, vanish below the Horizon, and those in the returning one (*Y*) which are seen (at *f*, *g*, *h*) vanish at the same Distance above it, at *O*.

Although this method will seldom be practised, yet I affirm it to be the best, being by much the readiest and the most correct. Nor, is there any occasion to be at the trouble of forming the Plan, to get the indented Angle; for, the Flutes being proportioned at *ab* (as above) and the Vanishing Points determined, the Lines, *af* and *bf*, being drawn to their respective vanishing Points, determine at the same time, the internal Angle.

The Tenia (*V* and *Z*) both above and below, breaks regularly around; as it may be seen, and would be needless to explain; similar Subjects having so frequently been repeated. Such minutias are best described by the Figure, only, being accurately drawn, and well defined.

The Abacus (*W*) which may be supposed over a Column, is a Square; and the entire Capitals, over the Pilasters, *Y* and *Z*, at the internal and external Angles, although they are detached Mouldings, are managed (from the Profile) after the same manner as if they were continued; their perspective Plans being delineated, as above; from which, the dotted Lines, corresponding with the Profile Section, (*MS*) determine their several Angles.

METHOD 2ND. EXAMPLE XVII.

How to represent the same thing without having the Profile drawn, or a perspective Plan.

Let *AB* be the given height, which divide into two Modules, and into Minutes, as before (the rest being supposed not drawn.) The Scale (*AM*) of the projecture of the Mouldings, &c. at the top, is also necessary.

From *a*, the determined Angle, draw *ak* perpendicular, and transfer all the measures of the several Mouldings, from the Scale, to *ak*; at *b*, *c*, *d*, &c.

Make *aa*, *ab*, &c. equal to the several projectures *MN*, *NO*, &c. and proceed as in the Plan, above; drawing *aE* for the diagonal Line of the top; and, from the several divisions *a*, *b*, *c*, &c. draw Lines to the Center (*C*) cutting the Diagonal, at *e*, *f*, *g*, &c. from which, draw perpendicular Lines; and, from the measures on *ak*, draw diagonal Lines, to *E*; which give the same Angles *c*, *h*, &c. as before; from which they are carried around the several Faces.

Plate
XIX.

The perspective Plan, in this, is supposed to be formed on the Top of the Cornice, the same as above; and since ak is the Intersection of a Plane, passing thro' the Diagonal or mitre Angle of the Cornice (one Side, in this Case, being parallel to the Picture) E , the Distance Point, is the vanishing Point of that Diagonal, consequently, all horizontal Lines, in that Plane (being parallel) vanish in E .

Wherefore, aE , bE , &c. are each, the indefinite Representation of a Diagonal of a Square, in different horizontal Planes, whose Intersecting Points are a , b , &c. and, the distance of each Angle from the Picture, is aa , ab , &c. which being transferred, perspective, to the Diagonal ag , each Angle, in its proper place, will be perpendicularly opposite to the Points e , f , g , &c. as in the Example.

And thus may the whole be completed, when we are a little versed in Mouldings, by transferring the measures, perspective, from one Diagonal to another, which will be further illustrated in the following Example.

E X A M P L E XVIII.

How to represent a Cornice when it is inclined to the Picture, on both Sides.

If the last Example be tolerably well understood, in the last Process, this will be found easy, being performed by the same means. It must be observed, that the first Method, respecting the Profile, cannot be applied here, nor in any Case; but when the Mouldings are parallel to the Picture. The extra Plan may be used in all Positions; but I shall in the following Examples, do without; as it is only supposing the Plan formed at the Top, from which the Perpendiculars are drawn, and the several Angles of the Facias and Fillets determined.

Fig. 87.

Let AEB be a Profile of the Cornice, to be drawn, and AC the Side of one Plane, on which it is to be projected.

ML is the Horizontal Line, S is the Center of the Picture, and L the Vanishing Point of AC ; which is supposed the upper edge of the Plane from which the Cornice is to project. Let the Top be supposed a Square.

S being the Center of the Picture, and L the Vanishing Point of one Side of a Right Angle (the Distance of the Picture being known) find M the Vanishing Point of the other Side; by Prob. 12. SM is a third Proportional to SL and the Distance of the Picture (found by Prob. 32, Geo.)

Also, find N the Vanishing Point of the Diagonal; by making MN to NL as one Side of the Triangle (whose Perpendicular, at S , is the Distance of the Picture) to the other. As N , or O (Fig. 51.) GN , or EO bisecting the Angle FGH , or AEB .

Draw AM and CM , and AN cutting CM at K ; draw LK , till it cuts AM at D . $ACKD$ is the Top, of a square Prism, from which the Mouldings project.

From each Angle, A , C , and D , draw perpendicular Lines, which represent the Corners of the Prism; the Profile being proportioned to the nearest, AB .

From B (AB being equal to AB) draw BL and BM , cutting the Perpendiculars, from C and D , at G and H . $ACGB$ and $ADHB$ are the Grounds or Seats of the Cornice on the two Planes, which are seen.

The Projecture of the Cornice, around the Prism, is next to be determined.

Draw the Diagonal DC , and produce it both ways; also, produce NA indefinite, and draw Ak parallel to the Horizon; on which, take the diagonal projectures of the Cornice, as on AK , in the Profile.

[†] EK being made equal and perpendicular to AE ; consequently, AK , the Diagonal of the Square $AJKE$, of the projecture AE , is the mitre Angle of a Right Angle, for the Cornice AEB .

From the several Projectures, C , D , &c. draw CG and DF , parallel to AB , and produce them to the Diagonal, cutting it in H and I .

Make Ah , hi , and ik respectively equal to AH , HI and IK ; and having made NO equal to the Distance of the Vanishing Point N , of the Diagonal, draw Oh , Oi , and Ok , till they cut NA , produced, at b , i , and E ; from all which, draw

draw Lines to both Vanishing Points, L and M, cutting the other Diagonal, *CD*, produced both ways, at *F*, *I*, *k*, *l*, &c. which form the perspective Plan of the principal Mouldings. *EB*, *FG*, and *HI*, being drawn, are diagonal Lines of each Angle, from the lower edge of the Fillet, at the top, to the bottom of the Cornice.

Let it be observed, that *N*, being the Vanishing Point of the Diagonal of a Square, i. e. of a Line bisecting the Right Angle, is the same whether the Top be a Square or other Rectangle; but, if it be not a Square, *CD*, produced, would not be the Mitre of the other Angles; because it would not bisect them; neither would *EN* pass through the opposite Angle, or Mitre *KL*; yet *N* would be the Vanishing Point of both, because they would still be parallel. The others are also parallel, and consequently, they have the same Vanishing Point; which, being much inclined, is at a great distance.

Having shewn how the Mouldings are projected forward, on the Diagonal *AE*, and transfered to the other Diagonal; suppose No. 2 the same thing, prepared in the same manner; being divested of several preparatory Lines, which, altogether, render the Work confused, and are supposed to be rubbed out.

Fig. 87.
No. 2.

On *AB*, take *a*, *b*, and *c*, equal to the several heights of the Mouldings (as at *a*, *b*, and *c*, in the Profile) and, from *N*, project them (as *Na*, &c.) till they cut Perpendiculars, from *i* and *b*, in *d*, *e*, *f*, and *g*; and, from them, draw Lines to both Vanishing Points, L and M, cutting Perpendiculars, from *k*, *l*, and *m*, *n*, &c. at *o*, *p*, *q*, &c. which compleat the Facias and small Fillets; and there remains only, to join them by curved Lines, of the same kind as in the Profile.

If the Mouldings are very large, as many Points may be found, in the Curves, as are necessary to describe them with accuracy; by Perpendiculars from *AE*, and measures transfered to *AB*, from corresponding parts in the Profile. But, they may be as well performed by a careful Hand; for after all attempts at exactness, in such minutias, a judicious Person, in Mouldings, and Perspective, would describe them as perfectly, regarding the Position in which they are seen.

Let it be observed, that the Curves are always flatter, in mitre Angles, than in the Profile, and more so, the less the Angle; likewise, in the diagonal Section, *AEB*, they are flatter than in *CFG*; because, the Plane of that Diagonal is nearer to a coincidence with the Eye; for if it be in the Plane (however situated) they are Right Lines.

The Mouldings, in the diagonal Plane *AEB*, seeing that, the full measures are applied, on *AB*, and projected forward to *Ed*, are larger than the Originals; that is, than the given measure; as *Eabc*, than *Aabc*, on *AB*; equal *Aab*, &c. in the Profile. The Picture cuts the plane of the top in *ef*.

In which Case, if *AB* be supposed in the Picture, *AecB* on one Side, and *AfigB* on the other, are sections of the Mouldings by the Picture; and, the whole, *efgB*, of that Section, being on this side of the Picture, is projected, to the Picture. *AEeB*, *CFpG*, and *Hic*, are diagonal Sections through the mitre Angles; and, *jkIm* is a Section perpendicular to the Plane *ACGB*.

If what I have advanced be well understood, all that follows, respecting right-lined Mouldings, will be easy and intelligible, almost from inspection of the Figures.

E X A M P L E XIX.

How to represent an Entablature with Modillions in the Cornice, obliquely situated to the Picture, having internal and external Angles.

ABC is the geometrical Profile of the Cornice; *F* is the front of a Modilion, and *FG* is its Profile. *CD* is the Frize, and *DE* the Architrave; with two Facias.

Fig. 88;

In this Example, I shall suppose the whole Cornice to be beyond the Picture, and touching it at the Angle *A*.

APG is the Intersection of the plane of the top, *HL* is the horizontal Vanishing Line, *C* is its Center, and *CE* the Distance of the Picture. *H* and *L* are the Vanishing Points of the Sides; i. e. of the horizontal Lines of the Mouldings; *M* and *N* are the Distances of those Vanishing Points, for proportioning Lines which vanish in them, respectively; and *O* is the Vanishing Point of the Diagonal, bisecting the Angle, *HEL*, made by the Radials of the Sides.

Draw the vertical Intersection, *AE*, of the diagonal Plane, passing through the mitre Angle; and, transfer all the measures of the heights of the Mouldings, from the Profile, to *c*, *d*, &c. equal *Bc*, *cd*, &c. also, their projectures, to *Aa*, *ab*, *bB*. equal *Aa*, *a b*, and *bB*.

Plate XIX. Draw AL , indefinite; and from all the Points a, b , &c. draw Lines to N , cutting AL at 1, 2, 3; from which, draw Lines to H , cutting a Diagonal, AO , at f, g , &c. the perspective Seats of the projectures; i. e. of the mitre Angles of the Corona, K , and the Facia at F , in the Profile.

From the several Angles, k, l , &c. obtained, as in the foregoing Examples, draw Lines to both Vanishing Points, H and L , indefinite; and proportion them to their respective lengths, by means of the Distances, M , and N , respectively, of those Vanishing Points.

The length being determined, according to the number of Modillions, contained, or otherwise; make AD , equal to the first break, and draw DN , cutting AL at f , the internal Angle; and draw FO , the Diagonal Line of that Angle, in the plane of the Top; indefinite.

Draw bL , cutting that Diagonal, at g ; from which, draw a Perpendicular; and cL cutting it at e , and join ef ; the Diagonal of the Projecture.

Draw He, Hf , and Hg , indefinite; and, make fF to represent a Line in the proportion of the Original, of fF to Af ; (by Prob. 10) or, draw Mf , and produce it to the Intersection, at B ; make BF to AD in that proportion, and draw FM , cutting Hf produced, at F . (Prob. 17.)

Draw FO , cutting Hg , produced, at h ; from which, draw a Perpendicular, cutting He , produced, at i ; and draw FL, hL , and iL , indefinite.

Produce NF to the Intersection, and make FG equal to the length of the front Line of the Cornice; draw GN , cutting FL at m ; and, $G6$ being made equal to the projecture of the Cornice (AB , in the Profile) draw $6N$, cutting FL at n ; and nH , cutting hL , at p ; from which, draw a Perpendicular, cutting iL at q .

Fig. 88.

Thus, are all the Angles at the top and bottom of the Cornice obtained; and, having set off the measure of the upper Fillet, at A , let it be continued around, by means of the Vanishing Points H and L ; then draw diagonal Lines, Fi , and $m q$, by which the Mouldings are all determined (as at AC in the Profile) having, before, obtained their places on the diagonal Line, Ac , at the first Angle.

The perpendicular Lines of the Corona fall below it, from the Point where they are cut by the upper Line of the same, at k .

On the returning Side, on the left hand, the Cornice is supposed to fall against, or is cut off by a Plane, parallel to the front Planes, and consequently has not a mitre Angle.

Draw AH , indefinite; make Aa represent a length equal to what the whole projects from the Plane (by means of Af , Prob. 10; or, by its measure, Pr. 17.)

Draw aL and bH cutting at r ; from which, draw a Perpendicular.

Draw cH cutting it at d , and draw ad , which will terminate all the Mouldings; as the diagonal Lines determine the mitre Angles; by drawing Lines from every Angle, on Ac , to the Vanishing Point H .

The Architrave, E, H, I, K , being composed of plane Facias, with one Moulding, only, on each Face, needs no particular description.

The Modillions are determined, in the same manner, as the Mutules, in Ex. 16.

Make Ag , on the Intersection, AF , equal to Ag , in the Profile, and draw gN , cutting AD , at o , and draw oH cutting the Diagonal AO , at f ; and, from f , draw a Perpendicular, giving the mitre Angle, S , of the front Planes, of the Modillions. Let it be continued around the several Breaks (as the Corona) in Pencil-Lines.

Draw fL , cutting the next Diagonal, at l . fl is the Seat, on the top, of the front of the Modillions on that Side; which may be continued around, from one Diagonal to the other, by means of the Vanishing Points H and L .

Make Ab equal to Ab , in the Profile, and draw bN , cutting AD at 2; and draw $2H$, cutting fl at b . From N , project the Point b to the Intersection, at i ; and make ij, ji, jk , &c. respectively equal to the width of the Modillions, and to the space between them, alternately, as often as is requisite; from all which, draw Lines to the Point N , cutting the Seat, fl , in their perspective widths, at m, n , &c. from which, draw Perpendiculars, giving the several Fronts, x, x , as in the Figure.

The

The Modillions yy and zz , in the returning Side and Front, are determined after the same manner, *viz.* those in the Side, y, y , and also v, v , by means of the Point M , and the front ones, z, z , by the Point N ; their measures being set off at 1, 2, 3, &c. on the Intersection of the Top, as those at x .

Having thus obtained the Fronts, draw Lines to the respective Vanishing Points H and L , till they cut the Plane from which they project; and, having drawn Right Lines for the edge underneath, first, the curved Lines may be drawn by hand, as in the Figure, regarding the different appearance of each, as they recede.

If the Parts are large, and require to be accurately projected, the Moulding, around the Modillions, may be managed the same as in the last Example, by a perspective Plan, above the Cornice. But, a judicious Person will save that unnecessary trouble; for, having obtained their mitre Angles, as at s, t, u , &c. the front Moulding being drawn, as any other continued Moulding, they may be returned at the sides, and meet the inner Moulding, sufficiently correct without the trouble of planing them.

In this Example, there is all the variety that is requisite for such Cornices; but, as the Corinthian Modillion may, to some, appear more difficult than the Ionic, I shall give a specimen how it may be delineated, as briefly as possible.

In the following Lesson, I shall not describe, over again, how the Mouldings are to be drawn; and, I have (in order to have the Modillions larger) omitted the upper Mouldings of the Cornice, and the Corona; having retained only the Planceer; which I shall consider as the Plane, on which the Modillions are seated, and AB the Intersecting Line of that Plane.

The Vanishing Points, H and L , are also the Vanishing Points of this Figure; the Center or Point of view is at S , and the Distance is ES .

Draw the vertical Intersection of the Diagonal, AF ; and, by a geometrical Scale of the Proportions, set off the heights of the Modillions and Mouldings at a, b , &c. also, make ab, bc , &c. equal to the width of the Modillions and spaces between; and transfer them, by means of the Point P (the distance of the Vanishing Point H) to AB (the first dotted Line) which is the Seat of the fronts of the Modillions; and finish the square Blocks $efgh$, &c. which enclose the Modillions, as in the former Example; by which means, they are truly proportioned, perspective, as in the Figure, which would otherwise, be somewhat difficult.

In the side, of each Block, must be drawn, by hand, the Profile of the Modillion, perspective, as it is represented at X (the geometrical Profile) according as they are contracted, or obliquely situated to the Eye.

In the Front, of each, is also described the end of the Scroll, and of the Leaf. In short, having first obtained the Blocks or Cases which contain them, the rest must be delineated by a judicious hand and Eye; for such Figures will baffle all Rules, nor is it possible to subject them to any, by which they may with greater certainty be described; except the Profile, in the side of each, as the Truss in the next Example (No. 2) which is a similar kind of Figure.

The Moulding breaking around the Modillions, is first described as other continued Mouldings, at the front, on which their mitre Angles are obtained, from the Intersection AB ; the measures, on which, not being equal, on both sides, is owing to the front Lines of the Fillets, being in another Plane, standing forwarder than the front of the Modillions.

The Dentils have nothing particular in them; the proportion being known, of their width and spaces between; a Right Line drawn through either Corner, as fg , parallel to the Intersection, being divided in the Ratio, and Lines drawn to any Point, G , in the Vanishing Line, according as the measures are taken, greater or less, will give their measures perspective (as by Prob. 8) but, their true measures can only be applied, at the Intersection of the Plane they are in, with the Picture (drawn through C) and projected, by means of Visual Rays, to the Eye, at P .

Fig. 88.
No. 2.

Plate XIX.

E X A M P L E XX.

How to represent a Door Head, with a Pediment, supported by Trusses or Consols.

Fig. 89. Let A be the determined Angle of the Cornice of the Object, in the Picture.

At No. 1. is the true geometrical proportion of the whole, in Front.

† Prob. 3. Through A , draw AB perpendicular, the vertical Section of a diagonal Plane, passing through the mitre Angle, whose Vanishing Point is G †. H and L are the Vanishing Points of the front and side Lines, which are horizontal; S is the Center of the Picture, and SE its Distance; I and K are the Distance Points of each, respectively, for proportioning Lines which vanish in them‡; EG bisects the Angle HEL ; conf. G is the Vanishing Point of horizontal, diagonal Lines. (Pr. 21.)

‡ Prob. 17.

These things, here premised or given, are supposed to be found and determined, from the known position and situation of the Object, as in the former Examples. (See Prob. 12.)

All the measures, from the Profile, are produced, or applied to the Intersection, AB ; and, by means of the Diagonal AG , the mitre Angle of each Moulding is obtained, as in the foregoing Examples.

To determine the breakings of the Mouldings around the Trusses, draw a D , the Intersection of the Plane they are in, above the Cimma reversa; and, by means of the Point I , project the Point a to that Intersection, at a .

Make ab , bc , and cd , respectively, equal to the Breaks, and to the opening between them, geometrical (as at v u. No. 1.) and draw bI , cI , &c. cutting aL , at b , c , and d , their perspective Proportions.

Draw cH ; indefinite; and by means of the Point K (the Distance Point of H) project the Point c to the Intersection, at d .

Make de equal to the returning Side, and draw eK , cutting cH , at e ; and, through e , draw Le , cutting bH ; which determine all the mitre Angles.

The Mouldings are described, or delineated, as in the last Examples.

To determine the proportion of the Truss, or Console; of which X is the Profile, and Z the Front (No. 1.) Through D , draw a horizontal Line, DF , and produce Hf (from H) to that Intersection, cutting it, at h .

Draw ho , perpendicular; and, from h , set off, geometrically, all the proportions of the Truss, at i , k , &c. in respect of its height (as in the Profile at X) from which, draw Lines to H , cutting fn , at k , l , m , &c. from which they are transferred to the other side, by the Vanishing Point L , at v , u , x ; and, by the Point H , projected forward; which proportions the heights in the other Truss.

The geometrical projectures are perspective proportioned, in the same manner as the returning Moulding, at ce , by projecting the Point f (from K) to the Intersection DF , at i , and making 1 , 2 , 3 , equal to the projectures in the Profile; from which, draw Lines to K , cutting hH at g , b , &c. and from them, draw perpendicular Lines; by which means, finding as many Points as are necessary, the true perspective form may be determined.

At No. 2, is shewn (more at large) how the Truss is described, having all the same Letters of reference; and, in proportion to the other, as 3 to 2.

If the width be not already determined, by the Mouldings, draw Ib , till it cuts the Intersection, DF , at f . H , I , K , and L (No. 2) are supposed to be the Vanishing Points, in the Horizontal Line, answering to those below.

Make ih equal to the width of the Truss, and draw hI , cutting bL at i .

In the Profile (No. 1) take as many Points, in the Curve (1, 2, 3, 4) as are necessary; from which, draw lines parallel to the Horizon, cutting ho , at i , k , l , and m ; and also perpendicular. Let those measures be transferred to ho (No. 2)

from

from which draw Lines to the Vanishing Point H; and, having proportioned fb perspectively, as fh (No. 12) by the Intersection DF (where their geometrical proportions are transferred, at 1, 2, 3) draw perpendicular Lines from the divisions on fb , cutting the Lines from i , k , &c. respectively, at 1, 2, 3, and 4, through which, a Curve being described will represent the out-line of the Console.

From 1, 2, &c. thus obtained, draw Lines to L; by which means, the exterior Curve may be described, as in the Figure; and, also, transferred to the other Console, which must be described after the same manner.

To determine the Pediment. Having obtained the Angle B, its perspective length (Prob. 17) equal twice AA, geometrical; at I (the distance of the Eye from the Vanishing Point L) make the Angle, LIV, equal to the Angle of the Pediment, cutting VL (the Vanishing Line of the front Planes, Prob. 3) at V, and set off an equal Distance on the other side L; from which Points, Lines drawn through A and B, determine the middle, at C; all the Mouldings vanish in those Points.

The height and true pitch, or inclination, may be thus determined.

Make BE equal AB; i. e. make AE equal to twice the height of the Pediment. Draw BF also perpendicular, and draw EL, cutting it at F.

Draw AF and BE, intersecting at C; ACB is the true perspective outline.

Or, having bisected AB, perspectively at G (by Prob. 8) draw a Perpendicular; and, from B, draw BL, cutting it, at C as before; and draw AC and BC.

From all the Mouldings at BC, draw Lines to the Vanishing Point L, cutting CG; and the projectures of the Mouldings being also perspectively proportioned, on CH, at r, s, t; perpendicular Lines, from them, will cut others corresponding, from CG, drawn to H, in the true mitre Angles; and, from those Angles, draw Lines to both Vanishing Points (V) of the Pediment, which will give all the Mouldings, with accuracy, beyond any other method whatever.

The horizontal width of the Frize and Architrave, are proportioned to the Trués and Mouldings, on fn ; by drawing Lines to L; the perpendicular widths of the Architrave (being parallel to the Picture) and the opening of the Door, by the Vanishing Points of the Diagonals of a Square (as at Y) in the Vanishing Line VL of that Plane; regard being had to the projecture of the Mouldings (See Pr. 26.)

The recess of the Door, pq, is determined (by Prob. 10) by its width, mp.

Produce the Radial LE; make EM equal to the width, and EN to the recess; or, make EM and EN, in the ratio of one to the other.

Join MN, and draw EO parallel to MN. O is the Vanishing Point of a Diagonal of the Soffit, or head of the Door Case; and mO being drawn, cuts pH at q, as required. The rest is obvious, from the Figure.

This Method, when it is applicable, is preferable to any other; for, having obtained the true proportion of one Line (mp) in any Plane, any other Line (pq) in that Plane is easily determined, by the 10th Problem. Indeed, it may always be applicable (provided, the Diagonal Line mq (i. e. MN) be not very oblique to the Picture; and consequently, its Vanishing Point (O) very remote; for, if the whole Distance (SE) cannot be used, half, or any other portion, may be taken, with equal propriety; and the Point, O, ascertained the very same (by Prob. 12.)

Otherwise, if no Line, in the same Plane, be found or determined, there is but one general Method; which was applied, to proportion the return of the Moulding ce , or the front Line at b , c , and d , viz. by the true measure applied to the Intersection of the Plane it is in (Prob. 17) which being frequently used, would be needless to repeat; particular regard being had to the true Intersection of either Plane, mpq, of the Soffit (which is horizontal) or of the vertical Plane, W, of the Door Jamb.

For, since pq is the common section of both Planes, it is consequently in both; and therefore, either Intersection will answer the same purpose.

PQ is the Intersection of the horizontal Plane mpq; its true Distance, from any other, being known, equal DQ; a Line drawn through Q, parallel to the Horizon, is its Intersection with the Picture; for, all parallel Planes have parallel Intersections (8. 7. El.) And, if Hp be produced, till it cuts PQ; a Line drawn through P, perpendicular, is the Intersection of the vertical Plane W.

For, pq cuts the Picture at B; therefore, P is its intersecting Point (Def. K) consequently, PR is the Intersection of the vertical Plane, W, that Line is in (Prob. 3,) and, PQ of the horizontal Plane, mpq.

EXAMPLE

Plate XIX.

Fig. 90.

EXAMPLE XXI.

How to delineate a Block-Cornice, and to break the same, or any other, around a Bow Window; which is half a hexagonal Prism.

HL is the Horizontal vanishing Line, C the Center of the Picture, and CE is its Distance; H is the vanishing Point of the Front of the Bow, and L of the Side of the Building, at right angles with it.

Draw EH and EL making a Right Angle (HEL) and, with any Radius, on E, describe an Ark, x, y, z , cutting EH, at x .

Make xy and yz each equal to Ex ; and draw Ey , cutting the Horizontal Line, at M; and Ez , which, would cut it, if produced; M, and N (the supposed Point where Ez would cut HL, produced) are the vanishing Points of the Sides and Diagonals of the Hexagon.

Bisect the Angle $v Ex$; (i. e. HEL) at u , and draw Eu , to P; which is the Vanishing Point of the Diagonal of the Right Angle.

Let AD be the Intersection, of the nearest Angle of the Object, with the Picture; and A the determined height of that Corner; let Aa be the height of a Plinth, above the Cornice; and ab the height of the Cornice, of which, X is the Profile.

The Dimensions of the Bow window, and other parts of the Building, being known, it would almost be superfluous to describe how it is to be determined in Perspective; sufficient instructions for that purpose are contained in the 4th and 5th Examples. However, as the application of them to real Objects, may not, to some, be familiar, I shall, briefly, describe it.

At No. 1. is a Plan of the Window, to a Scale of half the measures applied in delineating; and about one sixtieth part of the real Object.

ak is the position of the Picture, applied close to the Angle a, and cutting the Building at the Angle c; wherefore, great part of the Cornice is projected to the Picture, seeing it is on this side, ak.

Draw AF the Intersection of the Plane of the Top, parallel to the Horizontal Line; make HD equal to EH (the Distance of the Vanishing Point H) and draw AH and AL indefinite.

Make AB equal to the short returning Plane (V) which is at right angles with W, the long side of the Building (ah, continued) i. e. make AB equal twice ab in the Plan, and draw BD, cutting AH, at B.

Draw NB indefinite (Prob. 13) and having found the point P, the distance of N (by Pr. 12) draw PB to the Intersection AF, cutting it at G.

Make Ge, on the Intersection AF, equal to twice bc (No. 1) and draw Pe, cutting NB at C*. Draw CH, indefinite; and draw CD cutting AF at d; make dF equal to Ge (i. e. to the Front) and draw FD, cutting CH at D.

From all the Angles, B, C, and D, draw Perpendiculars, which give all the Faces of the Prism, that can be seen (Y, and Z) i. e. bc, and cd (No. 1.)

As, there is not room, on the Intersection, to set off the whole measure of the Plane W (equal four times ah, No. 1) take AG a third part of it, and LE one third of EL, and draw GE, cutting AL at H; from which draw a Perpendicular.

Thus, having determined the Building, we now proceed to the Cornice.

From X, the Profile, transfer all the measures, to AD, of the heights of the Mouldings, at a b. Draw the Diagonal Pa; and determine the mitre Angle of the Mouldings (as in Example 17th) at aabb.

* Because the point C (in BC) is beyond the Intersection, AF, it is considered as projected to the Picture, its place being on this side, in the Original; as ak (No. 1) the Picture, cuts the Angle c.

Draw aH and bH, cutting Bh, at c and h. Draw Nc and Nh, till they cut Cg, at d and g; and draw dH and gH, cutting Df, at e and f. Also, from i, draw iL, representing a Facia, below the Cornice; and iH, cutting Bh, and continue it around each Face of the Hexagon.

The mitre Angles of the Hexagon are thus obtained.

In every regular Poligon, Right Lines drawn through the Center, bisect the Angles of the Poligon; and in a Hexagon, they divide it into six equilateral Triangles, each Diagonal being parallel to two opposite Sides. (See No. 1.)

xEy and yEz are two such Triangles; consequently, Ex, Ey, and Ez, are parallel to all the Sides and Diagonals of the Hexagon†, producing the Vanishing Points, H, M, and N, of the Diagonals or mitre Angles.

† Cor. 1.
Th. 6.

But, the Angle, at b (No. 1.) where the Hexagon joins with the Building is internal, and its Mitre is gb, produced; consequently parallel to fc; therefore, they have the same Vanishing Point (M)

a b is the Angle of the Building (which is a Right one) where the Mouldings are first projected; ch is the internal, and dg, ef, two external Angles of the Hexagon.

Produce Se, Sd, and Mc, indefinite; and draw Ha, cutting Mc, produced, at c.

Draw Ne, cutting Sd at e, and eH cutting Se, produced, at f; a, c, e, and f, are the extreme Angles of the Cornice; which being obtained, the rest is managed as in the former Examples; by transferring all the measures, of the heights of the Mouldings, from ab to ch, and from ch to dg, &c. also, the projectures of the Mouldings, from the Diagonal aa to cc, from cc to de, and from de to ef; by means of the vanishing Points H and N; from which Diagonals, perpendiculars being drawn, give the Corona, at b, which is carried around in the same manner; and also the small Moulding at b, h, g, f.

The Blocks, are obtained after the same manner as the Modillions, in Ex. 19th.

The internal Angle at q being obtained, as above, and the hither face of the first Block described (by means of the Vanishing Point L) through the outer Angle draw pq parallel to the Horizon, and set off geometrically, the number of Blocks and spaces between them, at 1, 2, 3, &c. from which, draw Lines to some point (P) in the Horizontal Line, cutting pq, at g, b, &c. the perspective proportions of the fronts of the Blocks, on the Face Z.

On the other Sides, they may be determined after the same manner, or otherways; as on the Face Y, by their Seat, BC, at the top, projected to it from the Intersection AF; how they are finished is best explained by inspection of the Figure.

On the Side W, they are determined from their true measures, by drawing mn, through n, parallel to AF; mn is the Intersection of the Plane they are in with the Picture (which pq is not) therefore, having (from the point Q) projected the first to mn, at r, set off the true geometrical measures, from r towards m, at r, s, &c. and, by means of the same point Q, project them to rL; i. e. draw rQ, sQ, &c. cutting rL, which give their true places, on that Side.

Fig. 90.

When the full measures, on nm, exceed the bounds of the Picture, make use of the half measures and distance, &c. as in former Examples.

The Blocks, in the returning Side, V, are determined, after the same manner, from the true measures; by means of the point D, the Distance Point of the vanishing Point, H, of that Side.

The Windows, and the Mouldings around them, are determined as the Door, &c. in the foregoing Example; by setting off the true space and measure of the Moulding from l to k, and transferring them from one Plane to the other, as in the Figure.

Ge and dF, on the Intersection AF, being the true geometrical measure of each Plane; divide them in the true measures of Piers, Mouldings, and Sash Squares, at i, k, l, and m; from which draw Lines to the Points D and O, respectively, cutting BC and CD, at n, o, p, q; and, from them, Perpendiculars being drawn, determine their perspective proportions on those Planes, at r, s, &c. and the proportions of the Sash Squares are projected to their true places, from t and u, by means of the Vanishing Points L and Q, respectively.

Z z

I cannot

I cannot help remarking, here, that the apparent intricacy of representing Mouldings, in Perspective, with accuracy, has led some Persons to imagine that the whole Work is intricate, and the Diagrams complicated; imagining they must necessarily go through the whole, to be acquainted with it. No such thing is absolutely necessary. If a Person be well acquainted with the Principles of the Theory, and the Elements of Practice, contained in the 3d, 4th, and 5th Sections of this third Book, he will find no mystery in their application to any Object whatever; the different Examples, here given, shew how they may be applied, with success, in various cases, and under various circumstances, which might not readily occur, to a Person not thoroughly conversant in it. They must not imagine, because there are a multiplicity of Lines in a Plate, in which there are such variety of Parts in the Object, that the whole is incomprehensible; let them first make themselves Masters of the simple Lessons and Examples, before they attempt a complex one, or it will be impossible for them to succeed in it. Can they imagine, that in such a Subject, as the 86th Figure, where every essential part is minutely described, it can be done without Lines, by contemplating the Figure only? Or, more particularly, the three last Figures, in which, every horizontal Line is inclined to the Picture; every one of which must be perspectively proportioned, to its place and position. Although the description is full and explicit (the preceding Lessons being understood) yet, to give a minute investigation of the 89th Figure, would fill, at least, half a dozen Pages, and would appear the most prolix and tedious description imaginable.

Notwithstanding the operative Lines in them necessarily cross each other, yet, if they are traced to their proper References, that intricacy (which some object to) will no longer exist. It is obvious, that, to have avoided it, wholly, the Figures must have been much smaller, the Plates much larger, or their number greatly augmented, which would have enhanced the price, unnecessarily. If they imagine that others have managed those matters more simply, let them compare, and see what they can make of Pozzo, the only Author, amongst the Italians, who has attempted it; nor has he attempted the description, but the delineation only, and that, in the most easy positions; wherefore, his elegant Designs, convey no better instructions, than any other good Print, by inspection. Mr. Kirby, in his first Essay, has run away from the Subject before he has well begun it; and, in his pompous Work, what has he done? nothing, to any purpose. Mr. Highmore, in his 37th Plate, has delineated a Cornice, at No. 3. tolerably, parallel to the Picture; but No. 4, being inclined, is intolerable, and his description of it more so. In short, a complicated Subject cannot be described without Words, nor can the operation be performed without Lines; although, as I have observed, in the Preface, not half the Lines, which appear in these Diagrams, are necessary to be drawn at all, in the operation.

S E C T I O N VIII.

OF CURVILINEAR OBJECTS.

THE Perspective of curve-lined Objects, the Subject of this eighth Section, is the most difficult, of all other; seeing that, Curve Lines cannot be projected as Right Lines, by means of intersecting and vanishing Points, indefinite; neither can any portion be taken, or cut off, perspectively, otherwise than by drawing a Chord Line from one Point to another in the Original, and finding its Vanishing Point; or, by any means, finding the representation of the extreme Points, from their corresponding Points in the Original.

There are various ways of projecting the representation of a Circle, in Perspective; all which, do no more than find the representations of various Points in the Circumference. For, by the Theory of curvilinear Perspective, it is supposed, that the description or delineation of a Circle, or Sphere, in Perspective, is some one or other of the Sections of a Cone; as in Fig. 28, Plate 7; it is obvious, that if Right Lines (EA, EB, &c.) from the Eye to the several Points A, B, C, &c. in the Curve (which is supposed to be a Circle) in the Plane Z; be cut by another Plane (X or Y) the Points a, b, c, &c. being joined, carefully, by a steady hand will generate a Curve (adg) which, to the Eye at E, will (as it is obvious it must) exactly coincide with the original Curve; seeing that, it is in the surface of the same Cone, of which, the Eye is its Vertex.

Hence it is manifest, that, the more Points there are found in the Representation, the more exactly may the Curve be described; but after all, it depends greatly on the Hand and Eye; insomuch that, without great nicety in both, the representations

representations of curve-lined Objects will have a lame, and very disagreeable appearance. It is, therefore, no wonder to see such bad Representations of round Objects, as are to be met with; but it is a matter of surprize, that any Person, who attempts it, should have so little judgement as to turn the Curves the contrary way; or to make them flatter in those parts, where, it is obvious that, they would have a greater curvature.

Without having the least notion how to project Curves, perspectively, is it possible for a Person, who has been used to sketch at all, to place a cylindrical Object, of any kind, or a Vase, &c. before him, and not see, immediately, how the curves are to be described? or, what part of the Object appears more or less curved than others? is it not obvious to a common Eye? yet may we frequently see Vases represented, whose greatest swell, at the top, is a Right Line, whilst the Curves of the lower part take a contrary direction, to each other; indicating, that the Eye is between them; in which Case, the top would be the most curved. Would not such attempts at Perspective be better let alone? and content themselves with a perfectly geometrical Representation? in which they would all be Right Lines; save only, the external Figure of a vertical Section through the middle.

But, what can be said for the performances of those Artists, whose Works are an honour to their Country and to the Age they lived in, to see them, often, greatly deficient in those particulars. I could wish to see the Works of the present Age more perfect; which, in other respects, seem to vie with the most celebrated amongst the Antients; yet do not pay sufficient attention to those necessary Appendages, which are essentially requisite, to a perfect Picture.

As a Circle is the first and principal of curve Lines, so it is the only one that can be reduced to any certain Rules, in delineating it perspectively. And, of all the various ways to project the representation of a Circle, in Perspective, the best, and most practicable, is to suppose a regular Octagon to be inscribed, or the Circumference divided into eight equal Parts; or, if very large, into sixteen.

Irregular curved Objects are not Subjects for Perspective; all attempts at a Spiral, or twisted Column, &c. by Perspective Rules, would be in vain. Various other Objects, as Rocks, Mountains, Rivers, Trees, &c. are not fit Subjects for Perspective; and consequently Landscape Views cannot be taken or delineated by its Rules; because it is impossible to have the true geometrical Figures and Proportions of such Objects, as before mentioned. I shall however, in an Appendix, describe the use and application of an Apparatus for taking Views, with ease and great exactness; the best calculated, for the purpose, of any I know or have ever heard of. For, notwithstanding what many imagine and affirm, of the possibility of taking Landscape Views with accuracy, by Sight only, I know it is impossible to be done; and cannot conceive it to be any way derogatory to the abilities of the most eminent Artist, to make use of any expedient; by means of which, he may be enabled to make a more correct Portrait, and Picture. I do not mean that he should, rigidly, describe every minutia of the Objects, as in Trees, &c. by it; but I must affirm, that he would take the apparent Magnitudes of the several Objects, their Figures, and their Bearings in respect of each other, with much greater accuracy than it is possible to do by Sight.

P R O B L E M I.

To find the Representation of a Circle; the Original being given, in any Plane, whose Vanishing Line, its Center and Distance are given; according to Brook Taylor.

First; by means of the Vanishing Line, and one Vanishing Point only.

Let AFG be a Circle, in the Geometrical Plane; of which GK is the Intersection, and VE the Vanishing Line; C is its Center, and CE its Distance.

Fig. 91.
No. 1

Draw

Plate XX. Draw AB, at pleasure, cutting the Interfection at *a*; and through C, D, G, &c. Fig. 91. (Points assumed at pleasure, in the Circumference) draw Lines parallel to AB, cutting the Interfection, at *a*, *c*, *d*, &c.

†Cor.Th.3. Draw EV parallel to AB, producing the Vanishing Point, V, of those Lines†. Draw a V, c V, &c. and, to E, draw Visual Rays from every Point, A, D, G, &c. cutting the indefinite Representations of the Lines passing through those Points, respectively, at *a*, *d*, *g*, &c.

Having thus obtained as many Points as are necessary, a Curve (*adifb*) described carefully through those Points, will be an Ellipsis, and it is the true Representation of the Circle AFG.

Or, having drawn the parallel Lines AB, &c. and their indefinite Representations, as before; make VE equal to VE; also, make ab equal to aB, ge equal to gH, &c. and draw bE, cE, &c. which will give the same Points as before.

This needs no Demonstration; seeing that, the Points *a*, *b*, *c*, *d*, &c. are projected the same, as in Prob. 6th, where it is fully demonstrated. And, that the Curve is an Ellipsis, is demonstrated in Th. 2nd. Sect. 5; of Curvilinear Perspective; for it is evident, and manifest, that the Visual Rays EA, ED, &c. from the Eye to every Point in the Circumference, would cut the Picture in the corresponding Points *a*, *c*, *d*, &c. therefore the Curve *adif*, described through those Points, is the true section of the Cone of Rays, and consequently it is an Ellipsis.

Secondly; by the Directing Line and one Director. No. 2.

Fig. 91. Let No. 2. be a Circle, nearly in the same Position to the Eye (at E) as before; No. 2. having the same Interfection, GKl, and the same distance of the Eye (SE.)

Draw CD parallel to the Interfection, and distant from the Eye equal to the distance of the Vanishing Line (VL) from the Interfection (Kl.) CD is the Directing Line of the plane of the Circle. (Def. 10.)

Draw AD, at pleasure, cutting the Directing Line at D; and, from several Points, B, F, &c. in the Circumference, draw Right Lines to D, cutting the Interfection at *c*, *d*, *e*, &c. the Intersecting Points of those Lines.

Draw ED; and, from the Intersecting Points, K, c, &c. draw Right Lines parallel to ED, which are the indefinite Representations of those Lines, respectively.

Draw the Visual Rays EA, EB, &c. as before, cutting those Lines at *a*, *b*, *f*, &c. and describe a Curve through them, which is the true representation of the Circle AFH. Draw VL, the Vanishing Line of the plane of the Circle.

DEM. Because the Lines AD, BD, &c. have the same Directing Point (D) their Representations are parallel between themselves (Cor. 1. Theo. 14.)

Consequently, seeing that Ka, cb, &c. pass through the Intersecting Points, K, c, d, &c. parallel to ED, they are the indefinite Representations of those Lines; and consequently, the Visual Rays EA, &c. will cut them, in the same Points as before, which is manifest; for, if Ea, Eb, &c. be drawn, parallel to AD, BD, &c. respectively, their Vanishing Points, a, b, &c. are produced; by which, the affinity between the different Methods of producing the same thing is accounted for.

P R O B L E M II.

To describe the Representation of a Circle, having the Representation of one Diameter given.

Fig. 92. Let AB be the given Diameter; let ECE be the Vanishing Line of the Plane it is in; C is its Center, and CE its Distance.

Produce AB to its Vanishing Point, V; and bisect AB, perspectively (Prob. 8) e. g. Draw *Abc* parallel to the Vanishing Line, and take two equal Divisions *Ab*, *bc*, at pleasure. Through B, draw cB, and produce it to the Vanishing Line at G, and draw bG cutting AB, at C, the Center of the Circle; through which, draw DF, parallel to the Vanishing Line.

Make VG equal to VE , and draw AG , cutting DF at D ; and, make CF equal to CD ; or, through B , draw GF . DF is a Diameter of the Circle.

Make CE , on both Sides, equal to CE ; draw CC indefinite; and, through D or F , draw ED or EF , cutting CC produced, at H ; through which, draw ab parallel to DF ; and, through D and F , draw CD , CF , cutting it at a and b .

Draw the Diagonals aE and bE ; which will pass through C , the Center of the Circle, and cut aC , and bC , at d and c ; draw cd , cutting CH at I ; IH is a Diameter, and $abcd$ is the representation of a Square, circumscribing the Circle.

Make EE^2 , and EE^3 , each equal to EE ; from both which Points, draw Lines through D and F , cutting the Diagonals at e , f , g , and h ; which Points are also in the Circumference.

Through the Points, A , H , f , F , g , I , h , D , and e , if a Curve be described, it will be the true Representation of a Circle, whose Diameter is equal to ab .

DEM. For, suppose ab the Intersection of the Plane the Original Circle is in; and ECE its Vanishing Line. E , E are the Vanishing Points of the Diagonals of a Square \dagger , and the two Diagonals, ac and bd , cut each other in its Center, C ; which is, consequently, the Center of a Circle, inscribed.

\dagger Prob. 19.
 \dagger 16. 1. El.

And, because HI passes through C the Center of the Circle, HI represents a Diameter perpendicular to ab , seeing it vanishes in C ; and DF , passing through C , is also a Diameter, parallel to the Picture.

Also, because CD is equal to CF , and EE^2 is equal to EE ; E^2 and E^3 are the Distance Points of the Diagonals; and consequently, seeing CD is equal to CF , the diagonal Diameters, eg and fh , represent Lines equal to DF . (See Prob. 8th, and 10th, Case 3rd, for a further Demonstration.)

P R O B L E M III.

How to describe the Representation of a Circle, in any Plane whose Vanishing Line, Center and Distance are given; and Intersection.

Let AB be the Intersection of a vertical Plane, and ECE its Vanishing Line; Fig. 93. its Distance is CE . Let AB be the Diameter given.

Bisect AB , at D ; and draw CD perpendicular to AB .

Make DC equal to AD , and describe the Ark DEF , a fourth part of the Circumference. Draw AC , cutting the Circumference, at E ; from which, draw a Perpendicular, Ec , to the Intersection.

Make Bc equal to Ae , and draw AC , BC , and DC ; also cC and eC ; and draw the Diagonals AE and BE , cutting AC and BC , at I and K ; draw IK ; and FH , through S , parallel to AB .

Through the Points, D , F and H , also through a , e , b , c , and d , where the Diagonals, and IK , are cut by cC , eC , and DC , if a Curve be described, accurately, it will be an Ellipsis, and the true representation of a Circle, viewed oblique.

If the Circle be large, and eight Points are not sufficient; bisect the Arks, DE and EF , at f and g ; from which, draw Perpendiculars to AB ; and make b and d , equally distant, as a and f , from D ; or from A and B .

Draw aC , fC , dC , and bC . From A and G , &c. where the Diagonals are cut, draw Af and Gg , &c. parallel to AB , cutting fC and aC at f and g ; and the others at b , i , k , l , m , and n , through which Points, the Representation will also pass, and may be described with more accuracy.

The affinity between the Original and the Representation, which the corresponding Characters particularize, is sufficient Demonstration. S represents the Center of the Circle, C .

This Method of projecting the Representation of a Circle is, of all others, the best and most practicable. It is nearly the same thing as Prob. 25th; for, the eight Points a , D , e , F , b , c , d , and H being joined by Right Lines, will be the Representation of an Octagon, which is circumscribed by the Circle; as, in the other, the Circle is inscribed, i. e. touches every Side, as this passes through its Angles

Plate XX.

Fig. 93.

Notwithstanding, the Methods, in Prob. 1st, by Brook Taylor, are facile and simple, yet I believe, they are scarce ever used in Practice. If the Circle be large, and Distance adequate thereto, they are utterly impracticable; because there is a necessity for having the whole Circle and Distance, at once, in their true places. Whereas, by the last, the Distance is applied on either, or on both sides of the Center, as in all other Cases whatever. Nor is AB necessarily the Intersection of the Plane of the Original Circle. For, if the place of the Circle be determined, on the Picture (either its Center, or the nearest part of the Circumference, at D) a Line drawn through S or D, parallel to the Intersection, or Vanishing Line, answers the same purpose. AB or FH being made equal to the known Diameter, in that place, and a quarter of a Circle described, as DEF, of that proportion (which, it must be obvious, is as sufficient as the whole) the rest is as already described. Other Points, if requisite, may be taken, as f and g, and more, if necessary.

In common Practice (in a Circle not very large) eight Points being sufficient, there is no real necessity for describing an Ark, geometrically; for e (or c) the Point where a Perpendicular, from E, cuts the Diameter, is distant from A or B somewhat more than one seventh part of the whole Diameter, AB; so that, in all common Cases, it may be ascertained near enough.

The 2nd Problem is useful in many Cases. Having obtained a Line, AB, by any means, on the Picture; which being known to be the Diameter of a Circle, the whole Circle may be projected by the means there described: or by Prob. 10th, Case 3rd, with the greatest exactness. Having obtained the parallel Diameter, DF, the affinity with this last Problem is discernable.

EXAMPLE XXII.

How to represent a plain, circular Arcade, casually inclined to the Picture.

Fig. 94.

Let VM be the Vanishing Line of the Horizon, C the Center of the Picture, CE its Distance, and A the Intersecting Point of the hither Angle of the Steps.

Through A, draw AJ parallel to the Horizon; which may be considered as the Ground Line; also, draw AG perpendicular, the vertical Intersection of a Diagonal Plane; on which, set off, from A, the several heights and proportions of the Object, at D, G, &c.

† Ex. 13.

The near Angles of the Steps being first obtained, as at A, a, b†; their length may be acquired by Prob. 7th. If they exceed the bounds of the Picture, take any equal portion of their length, as AJ, one third part; also, V being the Vanishing Point of that Side (see Prob. 21, Meth. 3rd) and VE its Distance, make VD one third of VE, and draw JD, cutting the indefinite Representation AV, at N. AN represents a length equal to thrice AJ. (Prob. 17.)

The Steps are finished as in the 13th Example.

Next, the proportion of the Plinth, ec f, is determined by the same; and the others, gh, ik, and l, by Example 4; which, with the Piers, are so many Parallelograms, of equal magnitude and equally spaced. (See Example 4.)

Their measures are set off, at 1, 2, 3, on the Ground Line; and projected to their Seats, on the Ground, at 3, 4, 5, 6, &c. by means of the Point E, on one Side; the Distance, VE, of the other Vanishing Point (V) falls out of the Picture.

Their height, and the upper Border at H, are obtained, by their geometrical height, AD (as the Pedestal in Example 13th.) Those above are perpendicularly over the Plinths below; the Band, or Capping on the Top, is determined from G; and, by drawing Lines to both Vanishing Points, V and Y.

The last is not in the Picture, and must be drawn by Prob. 13, or the Point must be ascertained; where, EY, produced, would cut the Vanishing Line VM. E is the Distance Point, of the Eye, for the Vanishing Point (Y) of the Lines AY, &c.

To represent the Arches; which, in this Example, are inclined to the Picture (and consequently, their Representations are Ellipses) the last Figure is adapted.

D, on the vertical Intersection AG, is the height of the spring of the Arch.

Make DE and EF equal to the measures on AB (Fig. 93) i. e. make DF equal to the height of the Arch (half the space between the Piers, equal CF) and DE equal DE (equal LE) and, by means of the Vanishing Point of the Diagonal (M) transfer them to the Angle of the Building, at H, I, K; or, if HK be the determinate height of the Arch, at that Angle, the rest was unnecessary.

From

From H, draw Lines to both Vanishing Points, V and Y, cutting the Piers, at *a*, *f*, *m*, *n*, &c. Bisect *m n*, perpendicularly, at S †, the center of the Arch.

† Prob. 8.

Draw Sp perpendicular, cutting a Line, from K to the Vanishing Point, Y, at *p*, the crown of the Arch; also, draw *no* and *mq* perpendicular; *m q o n* is half a Square, circumscribing the Arch.

Draw the Diagonals So, and Sq, cutting a Line from I, at *r* and *s*.

A Curve described through *m*, *r*, *p*, *s*, and *n*, is the front of the Arch, the Representation of a Semicircle. If more Points are requisite, see the last Problem.

The inner Curve is described as the outer; *n n* being the thickness of the Pier, obtained below; from C and d in the corner Pier.

From all the Points in the outer Curve, draw Lines to the Vanishing Point V; and, *m n*, the inner Diameter, answering to *m n*, being drawn, SV cuts it at *f* the Center; from which, the Diagonals *f o*, *f q*, being drawn, in the Rectangle *m q o n*, give the Points *r*, and *s*; and a Perpendicular from *f* gives *p*; then, a Curve drawn through *m*, *r*, *p*, *s*, and *n*, determines as much of the Soffit of the Arch as can be seen.

After the same manner, the Arches *a d f*, &c. are described, in the returning Side; which would be superfluous to repeat over again; as the Lines themselves shew their use and application the same.

If *V E* be made equal to *V E*, on the Vanishing Line of the front Plane, *E* is the Vanishing Point of one Diagonal (as *ce*) in each front Arch.

E X A M P L E XXIII.

To represent an upright Cylinder, of any given Dimensions.

Let AB be its Diameter. The height of the Cylinder is supposed to be known. Let V be the Center, and VC the Distance of the Picture.

Fig. 95.

Describe a Square ABDg (Prob. 19) and inscribe a Circle aceg (by Pr. 2 or 3.)

At the two Extremes of the Ellipsis, *a* and *d*, draw Perpendiculars, which represent Sides of the Cylinder. Their height must be determined by a Perpendicular either from A or b, or any Point in the Circumference.

Make A A and b b, each equal to the determined height of the Cylinder; the Point *b* is in the Circumference at the Top; and, if AC be drawn, it will cut Perpendiculars from *a* and *e*, at *a* and *e*, which are in the same Circumference. bV will cut a Perpendicular from *f* at *f*; and thus, as many Points as you please may be obtained, as *a*, *b*, *c*, *d*, *e*, *f*, *g*, and *b*, corresponding with *a*, *b*, &c. below; through which, the upper Curve may be described.

Or, having obtained any three Points, the Curve *a*, *b*, *c*, *d*, may be determined, by Prob. 11, with the greatest accuracy.

The Plinth, ABD is parallel to the Picture; and the Abacus, BFG, is equally inclined on both sides, each of which represents a Square (see Prob. 29 and 30) the Curves are still the same; and if they are equally distant from the Vanishing Line (VC) they are equal and similar Ellipsis.

It is obvious that, as the Diameter of the Cylinder depends on the Ellipsis, at either Base, it is very liable to error; as the Ellipsis is contracted or lengthened, it will be smaller or larger in Diameter. To obtain the true Diameters of several Columns, with accuracy, they must be drawn in their true geometrical proportions, as in the next Figure.

Let AB, DF, and GH be the Sections of three Columns by a horizontal Plane, in which the Eye may be supposed to be, at E; and, let AH be a Section of the Picture, i. e. suppose a Plane standing upright on AH parallel to the Columns; EC perpendicular to AH gives its Center, and Distance.

Fig. 96.

The

PlateXXI. The Visual Rays EA , EB , &c. being drawn, are in the same Plane; and the Points A , B , D , &c. where they cut the Picture, it is evident, are the places, where the apparent edges of the Columns will appear, and consequently, AB is the apparent width of the Column AB , DF of DF , and GH of GH ; which, on account of the Rays, EG , EH , intersecting the Picture more oblique than those from DF , has a larger Representation; notwithstanding it is really further from the Eye, and from the Center of the Picture, at C .

If the Eye be moved to E , the dotted Lines shew the difference in their Proportions, from the two Stations or Points of View; where, the difference between the Representations of DF and GH are much larger than at E .

If the Picture be at ah , parallel to AH (the Eye being at E) the Proportions of the Representations are in the same Ratio, as their Distances, Ec to EC .

The true position of the Picture from that Station, is ab , whose Center is at c , Ec perpendicular to ab , bisects the Optic Angle, aEb ; which Optic Angle, of the other Pictures, is iEh ; ci being equal to ch .

And thus the Diameters may be obtained on any Picture, situated to the Columns in any Angle, at pleasure as (HAK) the apparent Diameters on AK , are where the Visual Rays cut it, at A , i , k , l , m , K .

See this matter more fully treated, in the sixth Section, of the Theory, Fig. 34.

E X A M P L E XXIV.

How to represent several Cylinders, in the same Right Line; inclined to the Picture, at pleasure.

Fig. 97.

Let X , Y , and Z , be the Seats of three Columns, on the Ground Plane, between the parallel Lines, AB and CD ; inclined to the Picture in the Angle BAF , at discretion. AF is the Intersection; and HL , the Horizontal Vanishing Line; C is the Center, and CE the Distance of the Picture.

Find the Vanishing Point, H , of the inclination of the Line of the Columns, AB (Prob. 17.) and make HEL a Right Angle; which bisect by the Right Line EO .

L is the Vanishing Point of the other Sides of the Squares; in which the Columns, i.e. their Plans are inscribed; and O of the Diagonals.

It would be superfluous, after so many Examples of the same kind given, to shew how the Representations of those Squares are obtained; either by applying their several Distances from A to a , b , c , &c. (as in Ex. 4) or, as the Square $ABFD$ is found (Prob. 19) severally, at discretion.

The dotted Lines shew how the last is managed; and it is the best, when there are Circles to be inscribed; as they are represented at af , cg , and eh .

Which being obtained, severally, by the last Example, draw AK , perpendicular, and equal to the height of the Columns.

Find the corresponding Squares of the tops of the Cylinders, in which inscribe Circles perspectively†, and draw perpendicular Lines, ai , cl , em , &c. representing the apparent edges of the Cylinders.

† Prob. 3.

If they were to represent Columns, diminishing at the upper ends, the Squares containing the Circles must be represented less, in proportion to their respective Diameters; and the Lines, which are, in this Case, perpendicular, will be gently curved; having obtained the Diameters above and below, a curved Ruler is the best expedient, for drawing the apparent Lines of the sides of the Columns.

Now, although this method of obtaining the apparent Diameters, if it be accurately performed, is strictly true; for, the representations of their Bases and Tops being true Ellipses, there cannot possibly be any error; yet, the performance of it is liable to great error, because it is impossible to describe the Ellipsis accurately; and therefore, the Diameters cannot, by this means, be truly obtained; especially at the bottom, being so near the Vanishing Line; but, much more accurately, by the last Example.

Sect. VIII:

APPLIED TO ROUND OBJECTS.

Then, since EC is the Distance of the Picture, produce it to the Intersection, at G; make GS equal to EC, and conceive the Picture to stand erect, on its Intersection AF, and S to be the Station Point, at the foot of the Spectator; for SG the Distance of the Picture is equal to EC, from the Eye to its Center; the space between the Intersection and the Horizontal Line not being a part of the Distance, seeing they are both on the Picture. (See Fig. 37, No. 1.)

It is equally the same, if S be considered as the Eye, and AF the Horizontal Line; and suppose X, Y, and Z, sections of the Columns by the Horizontal Plane.

Right Lines being drawn to S, as the Station Point; or being, considered as the Eye, in the Horizontal Plane, are Visual Rays to the extreme edges of the Columns; which, by their Intersections with the Picture, at 1, 2, 3, 4, 5, and 6, give the apparent proportions of the Columns, on the Picture; from which, perpendiculars being drawn, give their true places, and representative Diameters, beyond any other means whatever.

E X A M P L E XXV.

How to represent the Tuscan Base, its Plinth parallel to the Picture, and seen obliquely.

C is the Center, nearly as in the last; but the Distance is greater, equal CH: Fig. 9 the Horizontal Line is the same; and the Intersection, or Ground Line, is AB.

At X is a Profile of the Base, consisting of one Torus and Plinth.

Being parallel to the Picture, the Plinth, AabB, in Front, is geometrical; compleat the square of the Base, perspectively†; and, within that, another Square (abcd) equal, in width, to the Circle at the bottom of the Torus, in which describe an Ellipsis‡; the nearer part, efg, only, will be seen.

Produce the corner of the Plinth, from A; AJ is considered as the Intersection of the Diagonal Plane. Make AJ equal to the height of the Fillet above the Torus; and having made a Perspective Plan below (the geometrical proportions, from the Profile, being set off, from A and B, as Aa, Bb) from the Angles A, B, and D, draw Perpendiculars AI, BK, and DL.

From J, on the vertical Intersection, draw JH, cutting AI at I; also draw IK parallel to the Ground Line (AB) and compleat the Square KL, of the Fillet; in which, describe the Ellipsis def, and another parallel to it, perspectively, equal to its width.

Make AG equal to BE; or, through E, the middle of the Torus, draw EG, and compleat the Square EF; in which describe an Ellipsis, as at ghik, which gives the greatest swell of the Torus; and, if greater accuracy be required, describe another Ellipsis, above that; and thus, as many representations of Circles as you please being described, the Curves, dge, and fkg, described over their extremes, falling into the Ellipsis on the Plinth, at e and g, will be the true Contours of the Torus, in Perspective.

Although I can scarce suppose that any Person would go through this process, who would delineate a Base; yet is there no better or readier way of doing it, with certainty. Nevertheless, this dissection of it will greatly assist the imagination; and is the best method of acquiring a true knowledge of the nature of curvilinear practical Perspective.

hi k, at the bottom of the Plinth, is an Ellipsis, which is the perspective Seat of the Shaft of the Column; from which may be obtained the edges of the Column, at MN; but it is of no other use, seeing the Curve of the Ellipsis, at MN, is very different from that below, on account of its being nearer to

Fig. 9

† Prob.

‡ Prob.

Plate XXI.

Fig. 99.

E X A M P L E XXVI.

How to represent a Doric Capital, inclined to the Picture, casually, or at discretion.

S is the Center of the Picture, and C is the Vanishing Point of one side of the Abacus; the other is out of the Picture; the Distance is equal to S L.

E is the Eye, i. e. the Distance of the Vanishing Point of the other Side (found by Prob. 12) and F is the Vanishing Point of a Diagonal, bisecting the Angle.

Let A be the place of the nearest Angle, in the Picture; and let Z be the geometrical Profile. AB is the Intersection of the plane of the top; and it is the measure of the square of the Abacus.

AB being drawn indefinite, draw BE cutting it at B, and draw AC, and BC, and a Diagonal AF, cutting BC; compleat the square of the Abacus (Prob. 21.)

Draw AD the Intersection of the mitre Diagonal, and transfer all the measures from the Profile to AD; from which, draw Lines to F; and having got the Points *c* and *e*, on the Diagonal AF, the Seats of the Projectures of the Facia, Z, and the Fillet at *x* (obtained as usual, by setting off the measures at *a* and *b*) from *c* and *e* draw Perpendiculars, giving the Points *a* and *e*.

Compleat the Squares *cd* and *fg*, in which describe Ellipses; *ab* being drawn parallel to AB, may be considered as the Section of the under Face of the Abacus.

From E, or any other Point in the Horizontal Line, draw Ec to *b*; bisect *ab*, at *d*, and make *a1*, *b2*, each somewhat more than one seventh part of *ab*; or with the Radius *ad* describe an Ark of a Circle, and get the Points 1 and 2, as by Prob. 3rd, by which means, the Ellipsis, i. e. the Points *e*, *f*, *g*, *b*, *i*, by which it is described, are obtained.

After the same manner, in the Square, *efg*, of the fillet at *x*, describe the Ellipsis *j*, *k*, *l*, and another parallel to it, equal to the width of the fillet; and also, another above that, for the thickness of the small Bead, at *x*; which, being so little distant from each other, will not vary much in their Curves; otherwise, every different representation of a Circle must be severally described, as the foregoing.

As it is somewhat more difficult to get the necessary Points, when the Square is inclined than parallel; let the remainder of the Curves be thus obtained.

Through *y* draw a Line parallel to AB; and (because this Object is viewed centrally) where it cuts the Vertical Line, at *i*, set off *if* and *ik* equal; making *fk* equal to the Diameter of the fillet, at *y*, geometrical; draw *fS* and *kS*.

From the Point *j*, where *fd* cuts AD, draw *jF*, cutting *iS* at *o*, the representation of the Center of that Circle; and, through *o*, draw *Lo*, cutting *fS* and *kS* at *h* and *m*; through which, draw *gh* and *lm* parallel to *fd*; *ghlm* is the Square of that Circle, in which an Ellipsis must be described, as before; with the Fillet and Astragal, as was done above; particular care being had, to make them lead true, at the extremes, as if they were continued around.

The Square of the section of the Shaft (*pq*) is obtained in the same manner; and, an Ellipsis being described in it, from the Extremes of which, perpendicular Lines are drawn, for the Shaft or apparent edges of the Column. How they fall into the Cavetto is impossible to describe, seeing, the Line is lost insensibly.

The large Ovolo can only be described, truly, as the Torus, by means of various representations of Circles, and describing, carefully, a Curve over the extremes of them all; for it must be obvious, that the contour of such Mouldings, though circular, is not in a Plane, but is continually changing its place, from the upper Circle to the lower.

Let it be observed, that the Curves are the same, whether the Squares, in which they are described, be parallel to the Picture or oblique, in any inclination whatever.

E X A M P L E

E X A M P L E XXVII.

How to represent a large Crane, or Water Wheel, vertical.

Let ABCD (No. 1.) be the geometrical figure, and proportion of the Wheel; and S its Center, or Axis. Its Position is accidental, as the vertical Line, bg passing through the Center, indicates. Fig. 100.

Let C be the Center of the Picture, and CE its Distance, in the Horizontal Line.

Let AB be the Intersection, of the Plane of the Wheel, with the Picture; which, the Wheel is supposed close to; and let BD be the Ground Line.

Transfer the diameter of the Wheel to the Intersection, AB, at FG.

Draw CV perpendicular to the Ground Line, which is the Vanishing Line of the Face of the Wheel. Make CE equal to CE; draw FC and GC; and FE, cutting GC at H, and draw HI, parallel to AB.

FGHI represents a Square, containing the Wheel; in which, describe the Ellipsis, abcd; and within it another, representing the Rim of the Wheel.

Make BK equal to its determined width, and draw KL parallel to AB, by which describe the Curve OPQ, representing the cylindrical Surface of its convexity; and there may be two other Curves described, equal in width, representing the breadth of the Rims, and another, shewing the thickness of one.

The Levers, or Vanes, a, b, c, d, &c. (No. 1.) may be thus obtained.

Make AG and BF each equal to their width; and draw AC and BC; produce EF, cutting CB, at C, and draw CD parallel to AB, cutting CA, at D.

On CD describe, perspectively, the exterior Square, CDHI, in which describe an Ellipsis, by a Pencil Line; which, with the interior Curves, represent concentric Circles (as ABCD, abcd, No. 1) this last will limit all the extremes of the Levers. Their places are found, by drawing perpendiculars, from each, to the Ground Line; as ip, hq, &c.

Draw a Perpendicular from the Rim, at J (No. 1) cutting the Ground Line at P; from which transfer all the measures Pp, Pq, &c. (where perpendiculars from the Levers, at i, h, &c. cut it) to Bp, Bq, Bg, &c. from which, draw Lines to E, cutting BC at 1, 2, 3, &c. from which draw Perpendiculars, cutting the outer Ellipsis at i, h, &c. as in the Figure, where the corresponding Characters particularize the several Levers.

Any one (as k) being obtained, a Line drawn through S, which represents the Center of the Wheel, gives its opposite, at d.

By the same Method the cross Timbers, &c. may be obtained.

This Method, though the same as the old Authors practised, is not repugnant to the new Principles, by Brook Taylor; and may do, accurately enough, in common Cases, and where room is wanting; but, as a contrast, I will also shew, how much more simple, masterly, and correct, they may be obtained, by means of their respective Vanishing Points; and, when they are not very remote, Intersecting Points.

AB was considered as the Intersection of one Plane of the Wheel, with the Picture. A Right Line, CV, drawn through C, the Center of the Picture, parallel to AB, is the Vanishing Line of them both, and C is its Center.

The Horizontal Line is of no use, in this operation; no regard being had to any other Vanishing Line or Intersection, whatever, but that of the Plane of the Figure.

CE, perpendicular to CV, the Distance of the Picture, is also the Distance of this Vanishing Line, which, is the Vertical Line of the Picture; and E is the true Place of the Eye, for that Plane. (Theo. 4.)

Find the Vanishing Points, D and V, of the Inclination of the Timbers, parallel to AC, and BD (No. 1) i. e. make the Angles CED, CEV, respectively equal to the known or determined Inclination of AC, and BD, with the Horizon; cutting the Vanishing Line at D and V, the Vanishing Points of those parallel Timbers.

Or,

† Prob. 2.
Sect. 3.

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Or, having found either, make DEV a Right Angle, it will give the other; because the Timbers are at right angles with each other. (Prob. 4, Sect. 3.)

Bisect the Angle DEV by the Right Line EF ; F is the Vanishing Point of the Timbers, E , G , &c. (No. 1.) which make half right angles with the other. In the same manner, i. e. by bisecting the Angles all the other Vanishing Points, G and H , &c. of the Levers are acquired; which, seeing they tend to the Center of the Wheel, may be drawn without more preparation; as GS and HS , &c. which gives the Levers, at i , j , and a , k , &c. and FS gives the Timbers at E and G .

The Timbers at I are parallel to them, and have the same Vanishing Point; the others, at O , F , and H , are at right angles with them. Make FEY a Right Angle, and they will tend to that Point, where EY would cut the Vanishing Line.

As the cross Timbers, do not pass through the center of the Wheel, they may be thus obtained; with the greatest accuracy.

Let PJ , at the farther extreme of the Wheel (No. 1) be produced, and produce the Timbers, &c. till they cut it, at M , and N , &c. PQ may be considered at the Intersection of the Picture, coinciding with AB , or KL .

Transfer the intersecting Points M , N , &c. to AB and KL , at v , x , y , and z .

Draw vD and xD , giving the representations of the Timbers, parallel to AC ; whose thickness are obtained at v 1, x 2. yD and zD give those on the other side of the Wheel, as they are seen between the hither Timbers.

From A draw a Line through the Center S , which gives the two Vanes, at u and e , whose Vanishing Point is not in the Picture, nor necessary.

The Intersecting Points of the other cross Timbers, parallel to BD , are below the Intersection, BD , which, if they were remote, could not be acquired; wherefore, from or through either of the extremes of those which are obtained, draw rF and sF cutting the inner Curve of the Rim at t and u ; and, through them, draw Vt , and Vu , which give those Timbers.

The Levers, it is obvious, are all between the two Ellipses, as their Originals are between the Circles, at No. 1.

The Points a , k , i , &c. in the hither Front, being obtained, draw aa , ii , &c. parallel to the Horizon; for the Front is, at right angles with the Picture.

The Supporters, or Stays behind them, are best described by the Figure; to particularize every minutia, would take above another Page to little purpose. The square of the Axle being obtained, at S , the Sides are parallel; and the Timbers at I and O form a Square, whose Vanishing Points are F and Y .

Or, those at I may be produced to the Intersection KL , cutting it at J and K ; and one of the other, at Q . The rest is obvious, on inspection.

E X A M P L E XXVIII.

How to represent circular Steps, in Perspective; with a round Pedestal on the Steps.

Fig. 101.

Let CE be the Horizontal Vanishing Line; C the Center, and CE the Distance; and let AB be the measure of the first Step.

Describe a Square $ABDF$ (Prob. 19th) in which inscribe the Ellipsis $acbd$ $efgh$. Make cG equal to the height of a Step, as at X (No. 1) and by drawing Perpendiculars, $a1$, $b2$, &c. and, from A and B , Diagonals cutting them, the lower Curve may be described.

Draw AI or GH , perpendicular; and on them, from A or c , set off all the measures of as many Steps as are required, viz. AA , AC , or Gc , ci , equal (as in the Profile, for two Steps) and from them draw to E , or C , as in the Figure.

Find the Square of the next Step $JKLM$ (Ex. 11.) in which describe the Ellipsis $abcd$, &c. as before; and the lower Curve, $ghikl$, may be obtained, likewise, in the same manner. And thus, as many Steps may be represented as are required.

N. B. AB is not necessarily the Intersection of the Picture, but parallel to the Intersection; and AB the known or determined measure of the first Step, in that place.

To represent a circular Pedestal, on the Steps.

At No. 1 is the Profile of the Pedestal; from which, transfer all the measures of the Plinth, Mouldings, &c. to the vertical Intersection, GH or AI, or both; the one considered as the section of a vertical Plane perpendicular to the Picture, thro' the Axis of the Pedestal, and the other a diagonal Section.

Fig. 101.

Make CD equal to the Plinth of the Base; CI to the height of the Cornice, and at 1, 2, 3, &c. set off the proportions of the Mouldings.

The square of the Plinth being obtained, at NO, and an equal one at PQ, in both which describe Ellipses; as, for the Steps below.

The other Curves must all be obtained after the same manner, by inscribing Ellipses in Squares; as pq is a Side of the square of the Facia, rs of the bottom line of the Cornice, and tu, of the upper Line of the Base Moulding.

It is obvious, that, as the plane of the Curve falls nearer to the Vanishing Line, the Curve is more excentric, i. e. flatter; as in right lined Objects, every section, parallel to the Base, becomes more obtuse and acute angled, the Figure being more contracted, as it approaches nearer to its Vanishing Line; in which it is wholly lost to sight, seeing that, the Eye is in the Plane of the Figure, and consequently, its whole representation is in the Vanishing Line. Yet, there is no possibility of describing them in another Plane either above or below, so as to be of any service, in respect of the Curve, as every Curve, the farther it is removed from the Vanishing Line of the Plane it is in, becomes more curved, i. e. it is more convex; or, as workmen phrase it, quicker, or more acute; which implies, that it falls off from a Tangent, more suddenly.

From what has been described above, in delineating round Objects, it appears nothing more than drawing a square one, of the same dimensions, and inscribing an Ellipsis in every representation of a Square.

It must be obvious to every Person of discernment, that, wherever the Eye is situated, in respect of a convex Object, as a Vase, &c. (Fig. 102) the Vanishing Line of the Plane of the Circles (CE) cutting the Object, the Curves abc, def, &c. or abc, def, &c. on either side of the Vanishing Line, are always concave towards it; and consequently, if a concave Object be represented (as the Abacus, of the Ionic or Corinthian Capital, Ex. 30 and 32) the Curves will be convex, towards the Vanishing Line. It is also manifest, that if the Curves, that is, if the Planes they are in, be equally distant from a Plane passing through the Eye, parallel to them; being equal and similar Curves, of any kind, and perpendicularly one opposite the other, their Representations will also be equal and similar.

Fig. 102.

How is it possible then, for any Person, who has any pretensions to the Art of delineating, to run into such gross absurdities as are exhibited in No. 2. I have seen a Vase so represented, on the Pannel of a Coach Door, which was very well painted, and, in other respects passable.

The Plinth, AB, was parallel, and the Side, BCD, was seen; but, instead of the Lines, in that Face, tending to a Vanishing Point, they inclined to each other towards the Eye; and consequently, gave the Plane, ABC, the appearance of inclining, or ascending. The Mouldings, at E, where curved contrary to the Base, which plainly indicates the Horizontal Line, HI, to pass between them; and, the Curves, being regular, also evinces the Eye to be at E, and consequently the Face, BCD, of the Plinth, could not be seen.

No. 2.

The Lines at FG, the greatest swell of the Vase, were Right Lines, which also evinces the Eye to be on a level with them; whilst the nulled, or fluted part, abc, was a parabolic Curve, rising as in the Figure; and the Top, at K, gently curved.

Is it not surprizing, that any Artist should be so regardless of his Reputation, as to suffer such a medley of inconsistencies to go out of his hands; and to palliate it by saying, in such things, it is not worth the while to stand considering about it. I grant it would not be worth the while, to delineate the whole by the Rules of

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Perspective; but surely, the Lines in the Plinth are as easily made to incline the right way, as wrong; and if, by curving the horizontal Lines of the Vase, it was intended to give an appearance of roundness; would not that be better effected by curving them properly? it would take no more time; and, if the Delineator was not inclined to think much about it, I would advise all such, to place an Object before them, and their Eye in the Point of View, as they intend the Object to appear; they will then assuredly see, how unpardonable such misrepresentations are, because they are as readily, without loss of time, done right as wrong.

I have seen full as gross absurdities on the Stage; which ought to be regulated by the strictest Rules. In Theatres, Perspective might be displayed to great advantage; and certainly, its Rules ought not to be dispensed with, on any account whatever. I do not mean in every minutia, but in the whole Design; for, if liberties be taken, in order to appear better in one Point of View, it will appear worse in another; particularly, in Scenes representing a grand Palace, &c. or other regular piece of Architecture, there is no possibility of departing from the Rules; in detached Planes, as in Scenery, they cannot represent one entire Building, in any Point of View, without adhering strictly to Perspective.

To the best of my remembrance, in that Master-piece, by Servandoni, in the Pandemonium (though I think, the grotesque Style, of that Design, is not altogether consonant to the grand description of it, in Milton's Paradise lost) the Eye, in that Scene, is on a level with the Bases; and consequently, the Rustics, on the Columns, appear curved. It is a long time since I saw it; but I believe, the first, from the Base, is as much curved as the upper one. The whole has a grand and striking effect, chiefly occasioned by the magnificent display of Lights and Colours; which being left out, I am persuaded that this Design would lose much of the applause it has obtained.

E X A M P L E XXIX.

How to represent an Ionic Capital, perspectively, having one Face parallel to the Picture.

This Example, and more particularly the 31 ft. of the Corinthian Capital, is a matter of real difficulty. To represent such Objects as the Volutes (and other decorative parts, of which the latter is composed) will indeed baffle all Rules. I shall nevertheless attempt to lay down some, which will give their true places, with certainty; and, if the Practitioner will carefully observe how such Objects appear, according to the position in which they are seen, and intended to be delineated, he will not be much at a loss; if he has any hand at Ornament, in Foliage, &c. and carefully adheres to the Rules I shall prescribe.

Fig. 103.

At Fig. 103, No. 1, is the true geometrical Plan and Elevation of the Ionic Capital; not according to the Antique which is not so difficult, seeing that, the Volutes or Scrolls are in one Plane. On the left (at X) is the Plan of the Volute shewing its position in respect of the Abacus and Ovolo, &c. On the right (at Y) is the Elevation; which appears elliptical, on account of its Position, which is known from the Plan.

If *ab* be the Line of its direction (which is somewhat curved) to the front, *AB*; it is obvious, that it makes a considerable Angle with it; to which, perpendiculars being drawn give *ec*, its apparent width in front; which measures, being transposed to the other side, at *d* and *g*, and perpendiculars drawn, give its apparent geometrical Front; of which *Z* is the true.

Having described the geometrical Plan and Elevation, I now proceed to shew how it may be delineated, perspectively.

Let *AB* be considered as the Intersection of the Picture, with the Plane of the top of the Abacus. Let *VL* be the Horizontal vanishing Line, and *C* the Center of the Picture. The Distance is greater than can be applied on the Horizontal Line, take *CE* half the Distance, and describe the Square *ABDG* (Prob. 19) which contains the whole Abacus.

The Curve *efg*, which is the sixth part of the Circumference of a Circle, may be easily obtained; as thus.

From *c*, *F*, and *d*, any Points at pleasure, in *AB*, draw Perpendiculars, *c 1*, *d 2*, &c. to the Curve; and draw *cC*, *F C*, and *dC*, which represent those Lines, indefinite. Make *cc*, *Ff*, &c. represent the finite Parts (Prob. 15) through the Points *c*, *c*, *f*, *d*, and *g*, describe a Curve, which will be a portion of an Ellipsis, and represents the Curve *efg*.

The returning Side, *BD*, is obtained by means of Ordinates, at 1, 2, 3, after the same manner, which are parallel on that Side. The interior Curves may be projected by the same means, and the Plans *X*, *Y*, and *Z* of the Scrolls, as any other geometrical Figure (in Sect. 5) from which, their Representations are projected, as below; the returning Angles at *A* and *B*, being perpendicular and parallel to the Diagonal *AC*, tend to the Distance Points.

This preparation is sufficient for an extra Plan; I shall next shew how far it is useful in finishing the Representation below. This Plan may be supposed to be the top of the Abacus, as a Plane simply; but I shall suppose it to be above, as in Example 12th, and that the Representation is below it.

Let *AB*, No. 3, be now considered as the Intersection of the top, instead of *AB*; to which Line transfer all the measures from *AB*; and form the out Line of the Plane *ABD*, as before; by means of the same Vanishing Points.

To complet the Abacus; bisect *AB*, at *F*, and draw a Perpendicular, *FG*.

From *F*, set off all the measures of the heights of the Mouldings, and draw Lines from them to *C*; supposing *FG* a vertical Section of a Plane through the middle of the Capital.

The Center (*S*) being obtained, draw a perpendicular, from *S*, cutting those Lines; through which, draw the parallel Lines *ce*, *df*, &c. till they cut Perpendiculars from the corresponding Mouldings, in the Plan above. Or, for greater accuracy, make *BC* equal to *Ff*, the receding of the Curve from the Picture; where, make a true Profile of the Moulding, of the Abacus; from which, draw Lines to the Center of the Picture, cutting *ce*, *df*, &c. from the Axis, at *e* and *f*.

Having obtained the returning Moulding, at the Corner *B*, by means of the perspective Plan, above, and a perspective Section, at *FG* (observing, that it is a mitre Angle, touching the Picture, and consequently, recedes more than the true Moulding, as may be seen by the geometrical Plan, No. 1, at *A* or *B*) from which, Lines tending to the Point of Distance, on the right hand, cutting Perpendiculars from the Plan, above, give the other Mitre, *gi*.

From *g*, *b*, and *i*, draw Lines to the Center (*C*) cutting Perpendiculars from the Angle *D*, in the perspective Plan; by which means, the several Angles at *D* are obtained; and the curve Lines may be drawn by a careful hand, as in the Figure, nearly parallel amongst themselves; otherwise, if great accuracy be required, each Curve must be got, severally, in different Planes.

The Abacus being completed, proceed to the Scrolls or Volutes.

Having carefully obtained the perspective Plans (*X*, *Y*, and *Z*) of the Scrolls, which are regular Trapezia (as at *X*, No. 1) not regarding the hollowing.

In this Diagram, it is too near the Vanishing Line; but may, for greater exactness, be at a greater Distance from it (as in Ex. 13.)

Draw *ab* parallel to *AB*, the Intersection of a Plane passing through the middle of the Scrolls (as at *ab*, No. 1) its Distance from *AB*, is equal to the distance of *ab* from the top of the Capital, geometrical.

By means of the Intersection *ab*, the representations of those Trapezia, *x*, *y*, and *z*, are easily obtained; transferring the geometrical measures to that Intersection, first; Perpendiculars, from No. 2, will cut Lines tending to *C*, the Center of the Picture, and give all their Angles, sufficiently correct; but, if greater accuracy be required, their Vanishing and Intersecting Points may be had, as in other Figures, which are in the Horizontal Line; but, on account of their Inclination to the Picture, they are beyond its limits.

Now,

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Now, the use of these Figures is very obvious; by which means, the several apparent Faces, with their several divisions, are determined, passing through the Eye of the Volute, &c. But, if an extra Plan be drawn, as above, Perpendiculars from *X*, *r*, and *Z*, give the same, in respect of their widths.

Hence may be seen the unavoidable distortion of the Volute on the left; which, on account of its oblique position to the Picture, and the Eye, is dragged out beyond its geometrical proportion; and consequently, cannot please the Eye but in the true Point of View. It is true, the Distance is too short; nevertheless, every oblique parallel Representation, of such Objects, must be more or less distorted, according to the distance, and oblique situation of the Eye.

Having got their dimensions, of height, by means of a perpendicular, *hi*, from the middle of their geometrical Plan (*X*, No. 1) and the place where they fall on the Ovolo (which, must be drawn as in the Doric Capital, without its Abacus, Ex. 26) the rest must be delineated by a steady hand, guided by a nice Eye; for, without great nicety in both, it is not possible, by the Rules of Perspective, to project such Objects; seeing that, the Curve is not only continually varying from the Center to its utmost extent, but it is also continually growing forward, from the utmost extreme to its Center, and is not described on the same Plane, or other Surface, which circumstance renders it utterly impossible to be described perspectively, by rule; nevertheless, by the Rules prescribed and judgment in drawing, it may be projected so as to please the most critical Eye,

Below this outline is a finished Capital, divested of the necessary Lines for projecting it. It is somewhat farther removed from the Horizontal vanishing Line, but, in other respects, it is the same; excepting the decorative parts, of foliage, &c. which, according to the position in which the Capital is viewed, will differ in Figure; they were omitted, above, for the sake of continuing the Lines of the Abacus, which they hide, in this. No Rule can possibly be of any use in delineating such Ornaments; a real Object, before the Eye, is the best and only means to effect it.

E X A M P L E XXX.

Is an Ionic Capital, according to the Antients, which is inclined to the Picture.

Fig. 104.

This Capital is delineated with much more ease than the other, for several reasons; because its Abacus is a Square, with only a Cima reversa; and, the Volutes are both in one Plane, which is vertical.

The Abacus, is supposed to cut the Picture, at *A*, through which, draw *AB*, parallel to the Horizon; *V* is the Vanishing Point of the front Lines, its distance is *VD*; the other Vanishing Point is out of the Picture.

Make *AB* equal to a Side of the square of the Abacus, and project the Square *ABC*, perspectively (Prob. 21) and finish the Abacus, as in Ex. 13. of the Pedestal.

The hither Scroll projects through the Picture; because the corner of the Abacus, at *A*, cuts the Picture, and the Scroll projects beyond it, in this Capital.

The Intersection of the Plane of the Volutes may be determined, truly, from a geometrical Plan; as at *X*, an Angle of the Abacus, inclined to *AB* as the Object is supposed inclined to the Picture.

The whole Moulding projects before the plane of the Scrolls; therefore, produce *cb* to *a*, cutting *AB*. Make *Aa* equal *Aa*, draw *ad* perpendicular, and produce *ab*, cutting *ad* at *c*; all that part of the Scroll, on the right hand of *ad*, projects through the Picture.

Make *cd* equal to the height of the Scrolls, or *ad* of the Capital, and draw *dV*.

Describe a Rectangle, inclosing the Scroll, at each extreme, as *cdef*, *cdef*; and, within these, others may be described, as in the Figure (dividing *cd* geometrically, at 1, 2, 3, 4, and drawing Lines to *V*; also, dividing *bf*, perspectively, at 1, 2, 3, and drawing perpendiculars) or, several points may be determined, by means of which, the Scrolls may be very accurately delineated, perspectively, but must own it is a laborious process, and may be drawn accurately enough, within the large Rectangles.

The

The other Front, or rather the End of this Capital, is different, and is not easily delineated, truly. Having obtained the outline of the opposite Scroll; at *g b i*, bisect the bottom Line of the Abacus, at *k*; where, describe the representation of a Semicircle, *kl*, in a Plane parallel to the Front, but different in dimensions to the Scrolls. The proportion, and figure of it, may be had in Books of Architecture.

This Face represents a kind of Balluster, spreading, at its extremes, to the width of the Scroll; but the Axis of it does not pass through their Centers.

The Ovolo is the same in this as in the modern Capital; save only, that it is not seen on the sides, at all, being wholly hid by the Ballusters.

E X A M P L E XXXI.

To represent a Corinthian Capital inclined to the Picture.

This Object is supposed to be direct before the Eye; and consequently, the Axis of the Capital passes through the Center of the Picture, at C.

Fig. 105.

CE (perpendicular) is the Distance, VL the Horizontal Line, and V the Vanishing Point of one side of the Capital, or rather of its Position; for, the Object not being right lined, save the short returning Angles, it has indeed no Vanishing Point; yet they are necessary, in the Delineation.

Let *a* be the Point in which a Square, inclosing the Abacus, touches the Picture; through which, the Intersection of the top may be drawn; but it will be more eligible to take another, above, at BD, parallel to the Horizon.

The Abacus of this Capital is the same, in proportion, and every other respect, as the modern Ionic; therefore, the measures may be taken from Fig. 103, No. 1, being delineated by the same Scale; and excepting its supposed position, being different, the delineation of it would be the same.

Take A perpendicularly over *a*, and draw AV; and AT to the Point in which EY would cut the Horizontal Line (Prob. 13) indefinite.

Bisect the Angle VEY, by the Line EL, L is the Vanishing Point of a Diagonal.

Find F, the distance of the Vanishing Point Y (Prob. 12) and, because the distance of V cannot be in the Picture, take VG three fourths of VE (or any other part, at discretion.)

Take AB three fourths of AB (No. 1) which bisect at *c*, and take all the other measures, at *a*, *b*, &c. on that side, each three fourths of the real measures on AB (No. 1) but, on the other, take *d*, *e*, *f*, &c. the full measures.

From all which Points, draw Lines to F and G, respectively, as in the Figure; cutting AV, and AY, at *g*, *b*, *i*, &c. from which, draw Lines to both Vanishing Points, V and Y, indefinite.

Take A 1, and A 2, equal c 1, Ff, &c. (No. 1) and draw 1 F, 2 F, cutting AY; from which Points draw Lines to V, cutting the Lines from *g*, *b*, &c. to Y, at *n*, *o*, *p*; and from *j*, where 1 V, 2 V, cut the Diagonal AL, draw Lines to Y, cutting *k* V, &c. at *r*, *s*, *t*; through which Points, on both sides, draw the Curves of the Abacus.

At the Corner A, describe an Equilateral Triangle (x) perspectively, and the same at each Corner, B and C; giving the mitre Angles of the Moulding, as at A and B; (No. 1.) which, in this case, are not bisected, having a greater projection at the Corners, than in front.

This operation done, draw a V, and a Y indefinite; to which, draw Perpendiculars from *g*, *b*, *i*, *k*, &c. also, from *n*, *o*, *p*, &c. and draw gY, hY, &c. cutting them at *n*, *o*, *p*, &c. through which Points, describe the Curves bnopq, and arstu.

In respect of these Curves, the operation above is not absolutely necessary.

After the same manner, the Abacus may be compleated; viz. by projecting the curves of the Fillet, in the middle, and at the bottom, in their respective Planes.

The places of the Leaves may be obtained by describing an Ellipsis in the representation of a Square, inclined to the Picture, as the Abacus.

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This may also be done above, at v, u, x, y, z. The Diagonals and the two Diameters, whose Originals are perpendicular to the Fronts, give the apparent width of each Leaf, in the first row, allowing a little space between them, and they also give the middle of the upper Leaves; between each, is a kind of Balluster, from which the Caulicola and Scrolls spring.

The true places and middle of the Heads may be obtained, by describing representations of larger Circles, concentric with the other. At Z is a Leaf, in front.

Describe two Ellipses (cef and ghi) giving the greatest projecture of the Leaves, in their true places; their heights above the Astragal (FG) are taken, from the Profiles (at X) to the greatest projecture of the Heads (at a and b) which projectures (ac, and bd) respectively, are added to the diameter of the Column at the Astragal, for the diameters of those Circles, represented by cf and gi.

These Ellipses being obtained, in their true places, inscribed in Squares (as HI) inclined to the Picture as the Abacus, for the upper one; the Diagonals and parallel Diameters, give the true middle of each head of a Leaf, at c, d, e, &c. Those of the lower row fall between them; as described, in the Plan above.

The Volutes, at the corners of the Abacus, are, except in dimensions, the same as the Ionic, and must be obtained the same, by perspective Plans, at x, y, z.

How they fall into the Caulicola, with the other decorative parts of the Capital, the Figure only can describe; for, I fairly own that it is not in words to describe it, so as to be of any service in delineating. Being well acquainted with the several parts of the Capital, the Rules I have given are sufficient; the rest is best described by a careful and attentive perusal of the Figure.

Those who are not versed in it, would do well to draw from a real Capital, first, in all positions; after which, they will be able to delineate it by Rule.

S E C T I O N IX.

Shewing the application of the whole to entire BUILDINGS,
and regular pieces of ARCHITECTURE.

IN this Section, I have applied the whole of what has been already done to compleat Objects, and particularly to Architecture; as being, of all other, the fittest Subject, and gives the greatest lustre to Perspective; on account of the regular disposition, and arrangement of the several parts of a Building, and being composed, chiefly, of plane Surfaces, which generate Right Lines.

It is, to me, surprizing, that Artists, in general, have no better notions of Perspective; if they would give themselves time to think about it, seriously, they could not commit such palpable mistakes and gross absurdities; even suppose they were not acquainted with the Rules, having a true Idea of the meaning and intent of Perspective; which cannot but be obtained, from the Apparatus, to this Work.

Whatever we are about to delineate, it is supposed that the Object, itself, is on the other side of the Picture, generally (it may be on this side.) Is it not, then, rational to consider, and thence to conclude, that the Picture we are delineating should be directly between the Eye and the Object; or what reason can possibly be assigned that it should not? and yet it is, almost generally, the case. Some, not considering it properly, would tell us, that it is in order to see the End or Side of an Object, as well as the Front; as if that could not be effected otherwise; true, it cannot, on their Hypothesis, that one Face must necessarily be parallel to the Picture.

I shall illustrate the whole of this matter in few words.

Figure 106 exhibits the Plan and Elevation of a plain Building, geometrical, which is to be delineated; nearly the same as the Object, in the Apparatus; in the same Position, and at the same Distance, nearly, by a Scale of the proportion.

In the first place, I shall shew how to determine the true Position of the Picture, the Station being previously determined.

Let S be the Station fixed on, from which it is intended to delineate the Building; X is the geometrical Elevation of the Front, Z is the Plan, or Seat of the Building on the Ground; in the Position, and at the Distance, the Building is to be represented, as seen from the Station S.

The Station, and consequently the Point of View being determined, it is evident, that the Building cannot vary in its appearance at that Station; but it is nevertheless manifest, that there may be a great variety of Pictures, or Representations of the Object; seeing that, every different position of the Picture, or section of the Pyramid of Rays (forming a solid Angle, under which the Object is supposed to be seen) will produce a different Picture. All which, Representations, will affect the Eye alike, in the true Point of View; nor is it possible it should be otherwise, seeing that, when the Eye is in the true Point of View, every Line, on the Picture, is seen under the same plane Angle, and consequently, the whole Representation under the same solid Angle, as the Original.

Hence, it is easy to account for the many distorted and preposterous Representations which are frequently to be seen; and which, is mostly owing to the absurd position of the Picture, in respect of the Object and of the Eye; or frequently to the distance of the Object, itself; a circumstance which ought to be particularly considered; for, if it be too near, it is impossible, in such case, to produce an agreeable Picture.

Then, since every different section of the Pyramid of Rays produces a different Picture (for all parallel Sections produce similar Representations†) certainly, that Section which is perpendicular to the Axe of the Eye, or the Station Line, SL, will be the most natural and agreeable Representation; seeing that, the parts of the Representation will differ but very little (if the Optic Angle ASC be not large) from the true Appearance, i. e. from a Section made by the surface of a Sphere, of which S is the Center.

† 1. 8. El.

Consequently, then, if the Station be fixed, from which it is determined to delineate an Object, the Position of the Picture is also determined, as follows.

ABC is the place of the Object, its Seat on the Ground Plane; and S is the Station determined on. Draw the Right Lines AS and CS, which determine the Optic Angle (ASC) of the Object, AC; for, S may as well be the Eye, and AS, CS Visual Rays, the Eye being perpendicularly over S.

Bisect the Angle ASC, by the Right Line SL; SL is the Station Line for that Object; to which, the Picture must (if placed properly) be perpendicular; all such Sections are similar, and are the most just Representations.

VY, perpendicular to SL, is the true Position for a Picture of that Object, delineated from the Station S; and if Right Lines are drawn, from S, to the several parts of the Plan, as a, b, c, &c. their sections with the Picture, a, b, c, &c. determine the apparent width of each part on the Picture, perhaps with greater accuracy than by any other means.

Here, the Picture is supposed close to the nearest Angle of the Object, at B; and consequently, the perspective section of the Rays, by VY, is the largest that can be made on this side the Object. Now, if an Ark (ABC) be drawn, on the Center S (Radius SB*) it is obvious that the section of the Rays by that Ark (or a Sphere of that Radius) differs but little from the section by the Plane, on VY; because the Optic Angle, ASC, is but small, and therefore the extreme Rays, AS and CS, cut the Plane not very oblique; which, being increased, occasions Distortion.

SV, parallel to AB, and SY, to BC, determine the Vanishing Points of horizontal Lines, in the Building. 1, 2, 3, &c. may be considered as Intersecting Points of several Lines in the Object.

* BS perpendicular to the Picture does not necessarily fall on the Angle B (in the Apparatus it is somewhat on the right hand) for, if the Station was ever so little on either side, it would fall differently, as must be obvious, from the method given of determining it.

Having

Plate
XXIII.

Having thus determined the true position of the Picture from the Station S, answering to the Picture, MNOP, in the Apparatus; which is directly between the Eye and the Object; I shall, next, shew the great absurdity of placing it otherwise.

Let every thing remain in the same Position; S is the determined Station for the same Object; and consequently it must appear the same, whatever Position the Picture is supposed to have, in respect of the Object. But let it be observed, again, that the Representation may differ very widely; as will be shewn.

It is usual with all (who know a little of common Perspective only) to make one Face of the Object parallel to the Picture; in which case (the Object being right angled) the Center of the Picture is, necessarily, the Vanishing Point of horizontal Lines in the other Side. Consequently, if two Faces of the Object are seen, the Picture must necessarily be obliquely situated, in respect of the Object and the Eye.

Now, can any Person conceive it rational, or eligible, to place the Picture in the Position ED, parallel to the End of the Building, to be seen from the Station S; and yet, this is the very supposition, in the Representation exhibited by Fig. 107; whose Center is C, and Distance CE (equal twice SE, 106) which Position answers to the Picture MNOP, in the Apparatus.

How egregiously absurd is the Idea of viewing a Picture in that Position; and yet, in any other Point of View it cannot appear like the Object represented. The distortion of the Front of the Building is obvious, but much more so, is the Face X, of the Bow Window; whilst the other (Z) is geometrical, as well as the end of the Building.

Fig. 107.

The Position being determined, and the Center (C) fixed; and also, the Intersection BG, of the front Plane (its distance from the Center, equal twice EF, 106) the Representation is delineated by Prob. 19 (as by Ex. 1 and 2) in which case, C, the Center of the Picture, is the only Vanishing Point.

Now, those Artists, who make a kind of necessity of delineating their Objects after this method, cannot, I conceive, give a reason, why, in a general View, such Objects must necessarily be so situated to the Picture, as to have some Face parallel to it; or, why the Picture should not be placed parallel to the Front (AB) if it must be so to either; excepting that, they rather choose the one to be parallel than the other; for there is as much propriety in one, as in the other, but the Representation will be much more distorted; because the Distance (SG) of the Picture standing on AG, is less than SE in the former; and, the much greater length of the Front (AB) occasions a greater obliquity of the Rays, with that Picture. For, SG being the Direct Radial of that Picture, ASG is but half the Optic Angle under which it is seen; but it is manifest, that the Eye cannot take in as much on the right-hand, in which case, the whole Angle will be very obtuse.

C A S E T H E S E C O N D .

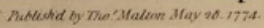
Fig. 108.

Fig. 108 exhibits a Representation of the same Object and from the same Station, in that position of the Picture; in which, the Front is geometrical; but the projectures of the Cornice, Steps, &c. are dragged out preposterously; and the Face X, of the Bow Window, is worse distorted than before.

C is the Center of the Picture, and CE its Distance, equal to twice SG (106.) The whole of the Object ought by no means to exceed the Point E, from the Center; whereas, it extends above twice CE, which is preposterous.

KL is the Vertical Line; and the Angles, CEK, CEL, being made each equal to the Inclination of the Roof, determine the Vanishing Points of the Lines FG, and FD in the Roof; K is also the Vanishing Point of HI parallel to FG; the rest is determined as the former, in which, those Lines are parallel to the Picture.

In both these Pictures, the horizontal Lines, in the Face, X, of the Bow Window, vanish in the Point of Distance, in the Horizontal Line; equal CE, on the other Side of C, because the Figure of it is a regular Octagon. (See Prob. 25.)





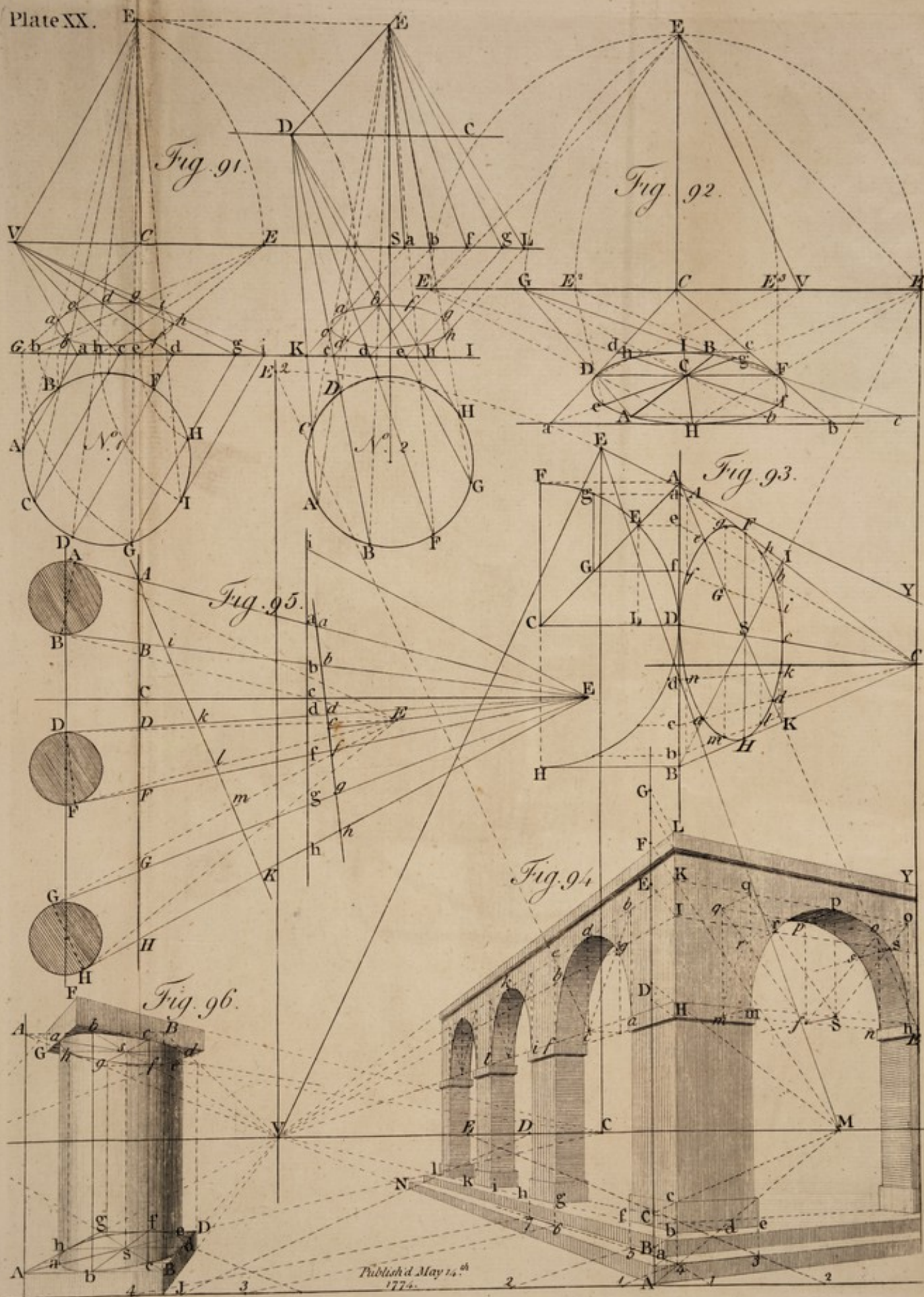




Fig. 97.

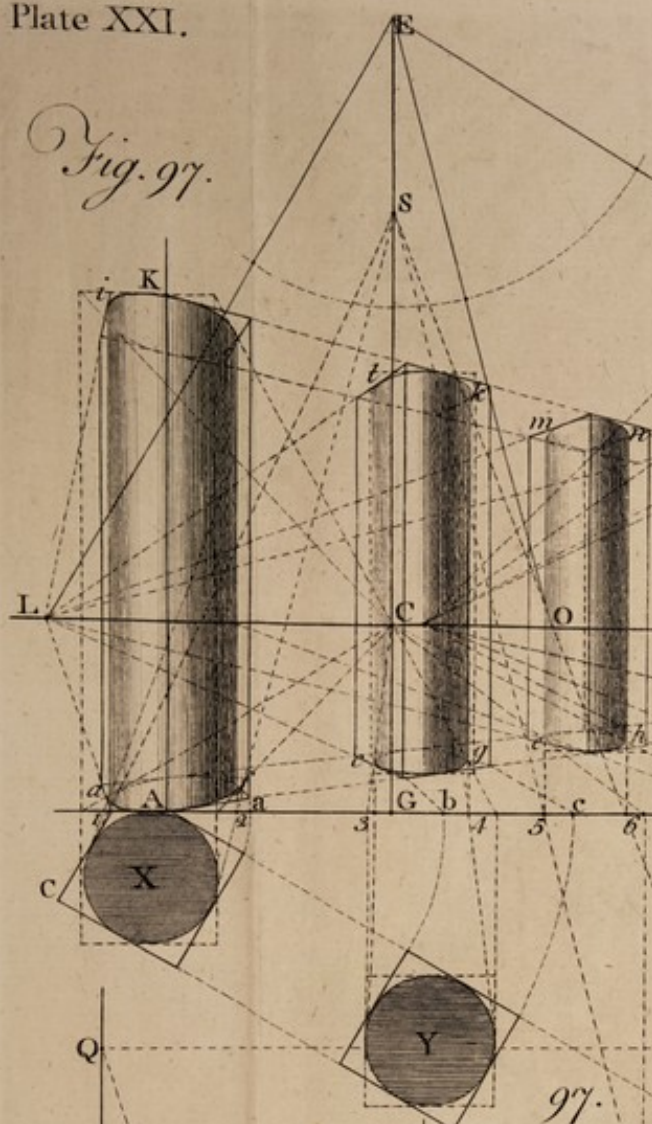


Fig. 99.

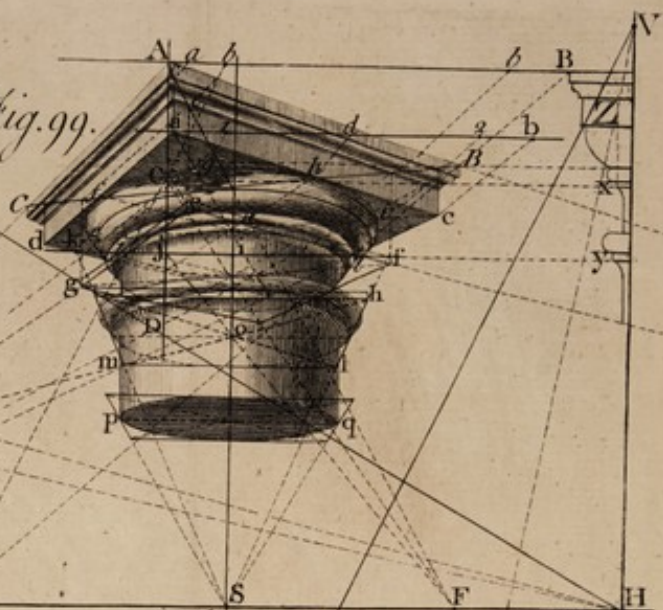
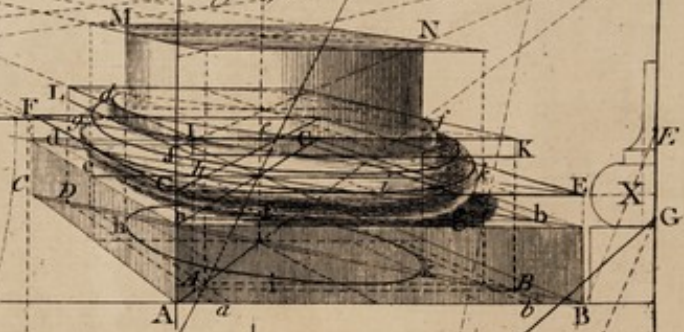
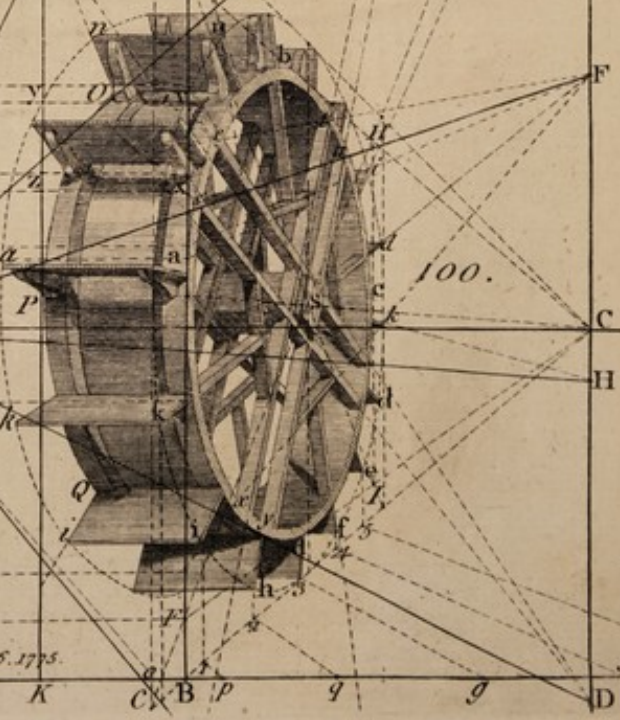
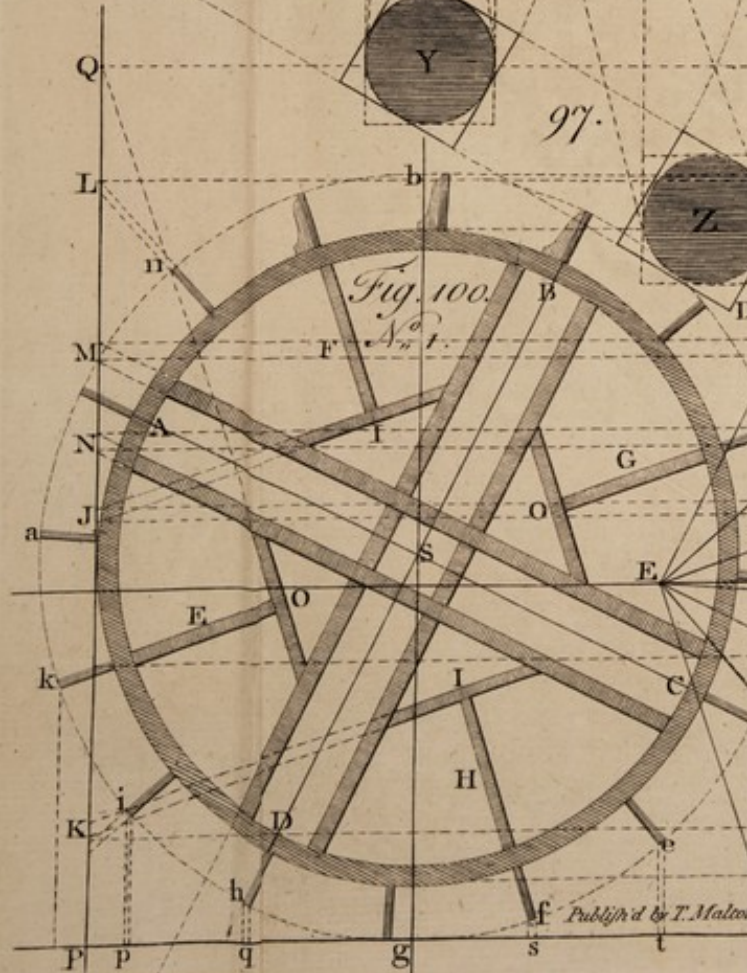


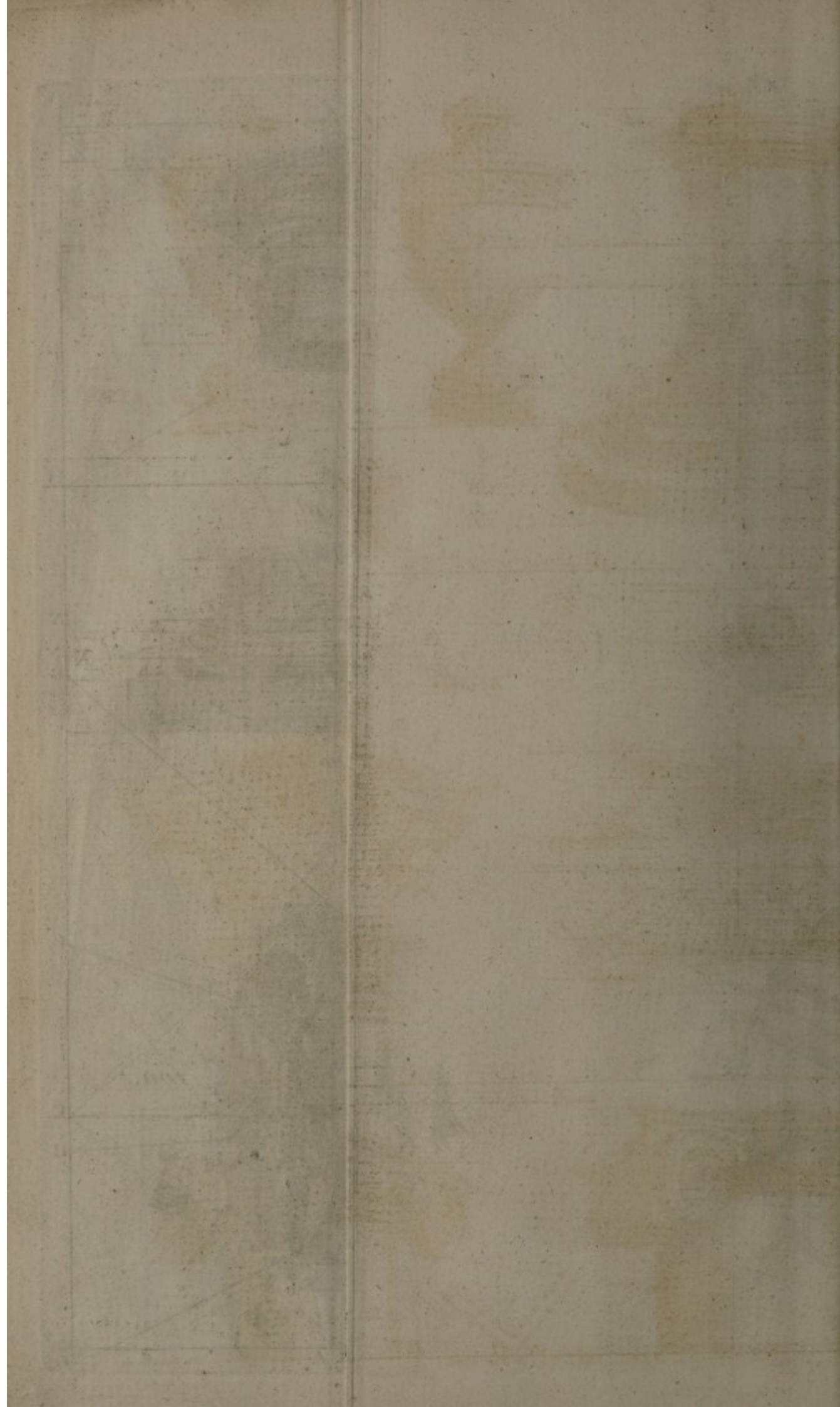
Fig. 98.

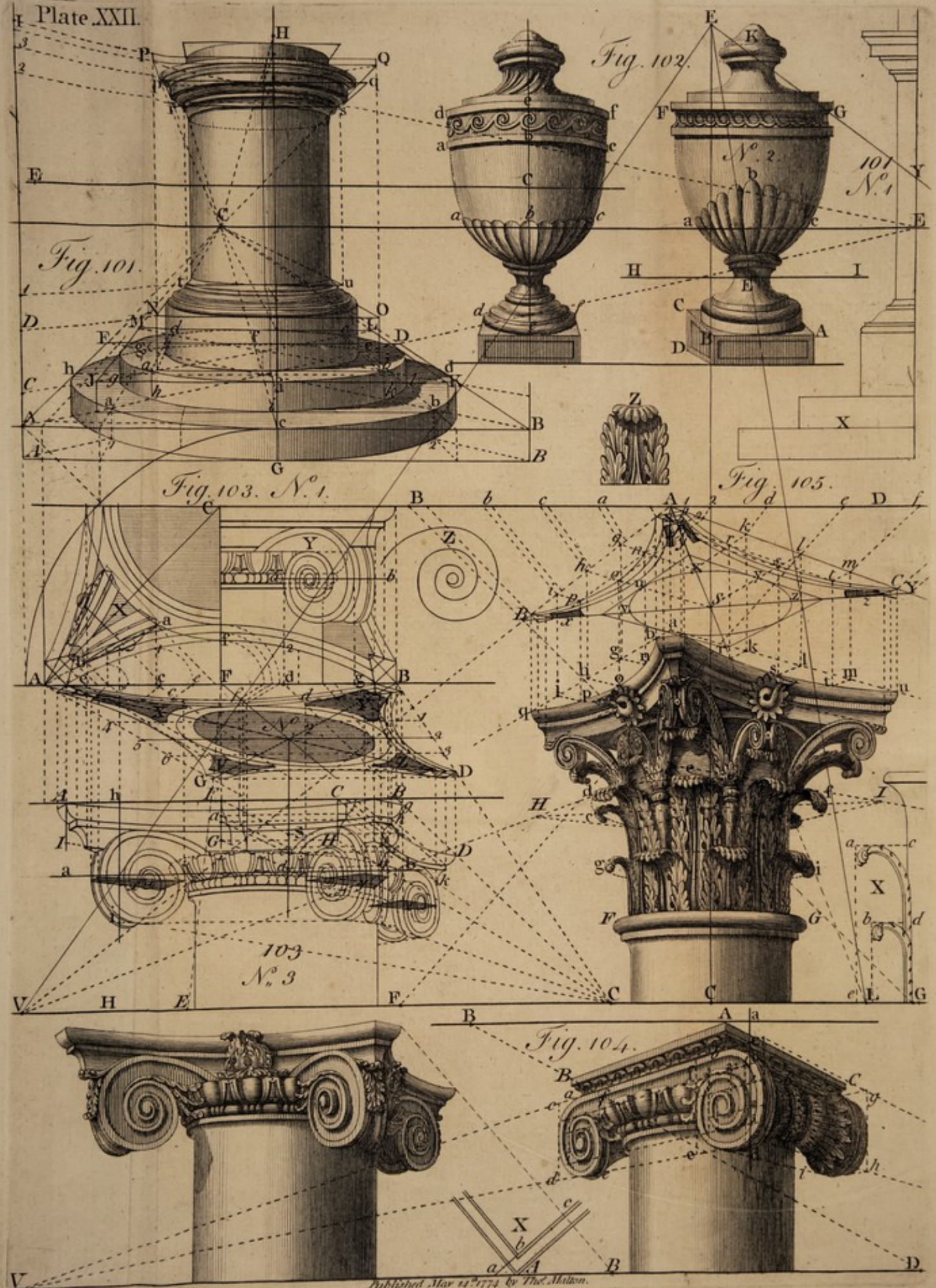


97.

Fig. 100.







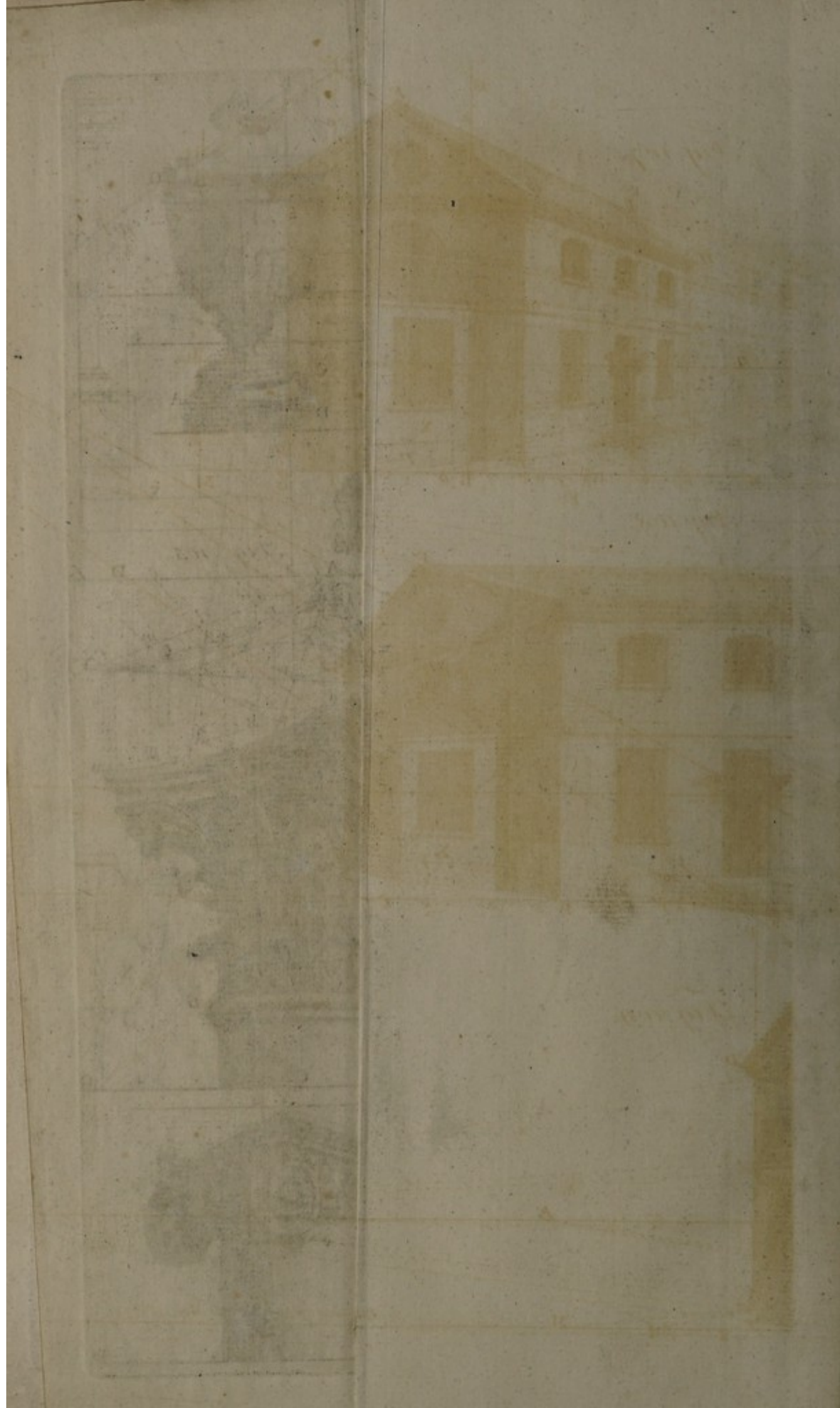


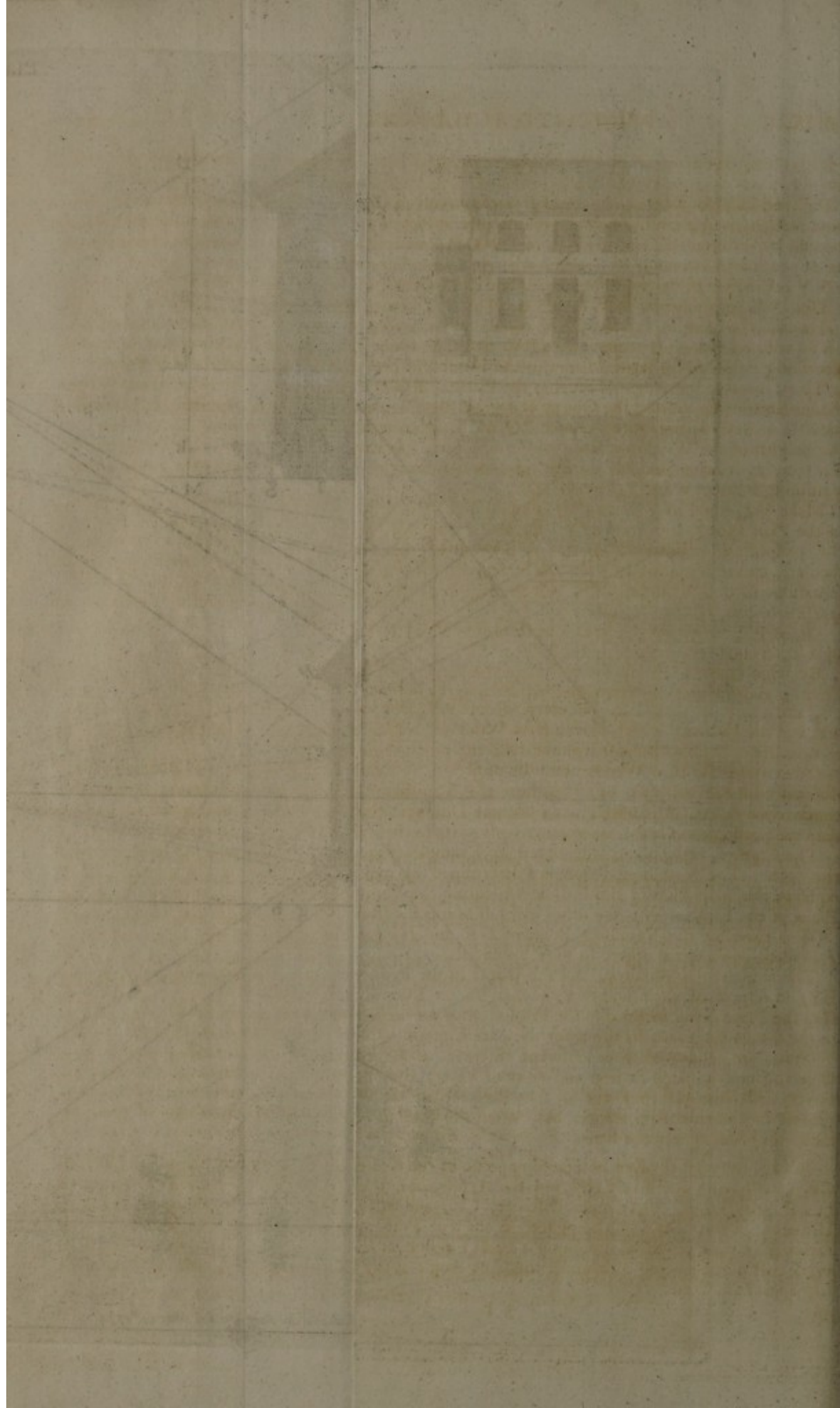
Fig. 106.

Fig. 107.

Fig. 108.

Fig. 109.

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CASE THE THIRD. EXAMPLE XXXII.

Fig. 109 exhibits a true and natural Representation of the same Object from the same Station, in the most judicious position of the Picture; and, in order to shew the affinity between this and the Picture MNOP, in the Apparatus, I have made use of the same Letters for Reference, where they can be properly applied.

Fig. 109.

VY is the Horizontal vanishing Line, and C the Center of the Picture.

Draw CE perpendicular to VY, and equal to the Distance (equal twice SB, 106) and make the Angles CEV, CEY equal, respectively, to the Angles BSV, BSY (106) or, make CV equal twice BV, and CY twice BY; V and Y are the Vanishing Points of horizontal Lines in the Front and End of the Building†.

† Th. 10.
and 11.

Then, because the Angle B touches the Picture, BG is the Intersection of both Planes (in the Apparatus, the Object being at some Distance from the Picture, each Plane has a separate Intersection, BG and BG) on which all the measures, of the heights of the Windows, &c. are set off, geometrically (as in the Elevation, X, 106; each being double) as Bm, mn, &c. from which, draw Lines to both Vanishing Points, V and Y, indefinite.

Make VE equal to VE, and YE 2 equal YE. From B, set off Ba, ab, &c. as in the Plan (106) and draw aE, bE, &c. cutting BV, in the Points a, b, &c. the perspective proportions of the Windows, &c. from which, draw Perpendiculars, cutting Lines drawn from the several heights, on BG to V, in the representations of the Windows, &c. AB the length of the Front, gives BA its perspective length.

Or, if B 1, 2, 3, &c. be set off, as from B to V (106) from which, Lines drawn to Y determine certain Points; on BV, or a V.

By which means, they are projected in the Apparatus, viz. by the Intersecting Points; and the Steps by the same, which may be finished by Example the 13th. Or, the place of each may be taken, from VY, 106, where the Lines, AS, &c. cut the Picture; which for the Bow Window, will be the readiest; observing to take them double, because it is but half the proportion.

Otherwise; the Bow Window may be projected, by Example 5th, observing the difference in Figure; and, being too near the Vanishing Line, another Intersection may be taken, at pleasure, at a sufficient Distance (Ex. 12.) as it has been exemplified in various Cases.

The Cornice, being plain, is managed by Example 18th, and the Pediment, by the 20th. The Vanishing Points are determined by making the Angle YE'W equal to the inclination of the Pediment; cutting the Vanishing Line YW, of the Plane of the End, at W; the other will fall as much below the Horizon.

W is therefore the Vanishing Point of GF and HI (as in the Apparatus) and FD tends down to the other, in WY produced.

The semicircular Window, and the Circle in the Pediment, are determined on BY, in respect of their widths; the heights on BG; as the Arches in Ex. 22; for, the Plane being inclined to the Picture, they are consequently Ellipses; as also in the former position; in the other they are Circles. (Theo. 9, Cor. 5.)

Thus, are delineated three different Pictures, or Representations of the same Object, from the same Station and Point of View; and, notwithstanding they are so very different, yet they are all true Perspective. It is obvious, which is the nearest to the true Appearance, but, when the Eye is in the Point of View, of each, they will appear the same.

In Fig. 107; let the Eye be placed opposite to the Center, at C, at the Distance CE; and having attentively observed the Appearance, then let the Eye be moved to 108, viewing it, at its proper Distance, CE; and lastly, to 109, at its respective Distance. I say, that, each being seen in the true Point of View, exactly (which can only be done perfectly, by fixing a small Pin hole at the true place of the Eye) every part of each being seen under the same Angle as the Original, must necessarily appear the same.

Now, because the Eye is perpendicularly on the Picture, at 109, and at the proper Distance, the Angle it subtends is equal to ASC (106) and, because the Angle does not exceed 25 Degrees, the portion of a Sphere, of that Radius, will not differ much from the Plane of the Picture; and consequently, this Representation is the nearest to the true Appearance.

SCHOL. This Lesson, properly digested, will give the Student a clear Idea, of the cause of Distortion, in the Representation, and teach him how to avoid it; for notwithstanding the Distance may be sufficient for the Object (the Picture being judiciously placed) yet if the Picture be placed according to the two first, it must necessarily be distorted. Although the real Optic Angle, respecting that single Object, is not varied; yet the Angle under which the whole Picture is seen, may be varied and increased infinitely, from 25 Degrees (the true Angle) to any quantity, less than 180, two Right Angles.

A brief and cursory retrospect of the Theory; shewing the immediate and absolute dependance of Practice thereon.

Observe, that No. 15 also refers to the Apparatus.

The first Theorem is exemplified in the Picture MNOP, which is parallel to the Plane BGFDC. This is applied to Practice, by the 9th Theorem.

The second demonstrates, that the Vanishing Line, and Parallel of the Eye, &c. are always parallel to the Intersection of the Original Plane.

This has been exemplified, in practice, throughout the whole; but chiefly in horizontal and vertical Planes. It is illustrated generally, by the Diagrams.

Fig. 15 (No. 2 and 3) illustrates the Theory for all Planes perpendicular to the Picture; or inclined in a certain position, respecting the Horizon and the Picture; and if a Plane passes through the Spectator's Eye (at E) parallel to the Original Plane HIFG, its Vanishing Line, VW, is parallel to the Intersection, FH. (8. 7. El.)

The great use of this knowledge is obvious, for, if either be determined or found, as FH, and any one Point, as V) the Vanishing Point of AB, &c.) the Vanishing Line, VW, is determined; and consequently, the Parallel of the Eye is also determined, the Distance of the Vanishing Line being known; by Theo. 7.

Theorem 3rd has been frequently exemplified, and bespeaks its use and application sufficiently; particularly in Example 12. The Corollaries deduced from it, especially the first, is derived from the second general Theorem, Book the first.

The 4th is a generally useful Theorem; in common Practice, as by the old writers on Perspective, more Planes are perpendicular to the Picture than in any other position. The Horizontal Line (the Vanishing Line of the Ground, and of all other horizontal Planes) also the Vertical Line (of all which are perpendicular to the Picture and to the Horizon) both pass through the Center of the Picture.

The Planes FIHG and FIKD, are perpendicular to the Picture MNOP; their Vanishing Lines also, pass through the Center (C) of that Picture.

The 5th, that Planes, whose common Intersection is parallel to the Picture, have parallel Intersections and Vanishing Lines, is exemplified in the Apparatus.

The Line BG, in the Object, is parallel to both Pictures, and it is the common Intersection of the two Planes, AHGB and BFC.

If those Planes are produced they will cut the Picture in BG and BG; which are their Intersections; the Planes being vertical and the Picture vertical, their Intersections are perpendicular to the Horizon, and consequently parallel. For the same reason, their Vanishing Lines (OP and RW) are parallel; and, because they are parallel to their Intersections, respectively, by Theorem 2nd.

The use of this knowledge is evident; having obtained, by any means, one Intersection or Vanishing Line; and also, by any means, the Intersecting or Vanishing Point of a Line in any other Plane (whose common Section, with it, is parallel to the Picture) the whole Intersection or Vanishing Line is consequently determined.

In the last Example the common Intersection (BG) of the contiguous Planes is in the Picture; consequently, it is the Intersection of both.

By the 6th Theorem, Vanishing Lines, of contiguous Faces of Objects, are determined according to the Inclination of the Original Planes, to one another.

In the last Example they were at right angles; but, being inclined in any Angle whatever, their Vanishing Planes being parallel to them, consequently they make the same Angle with each other, at the Eye; and their common Intersection, which passes through the Eye, is parallel to the Intersection of the Originals; and, when it is not parallel to the Picture, produces its Vanishing Point (Def. 22) which, Vanishing Point, is the Intersection of the Vanishing Lines.

How the Vanishing Lines are found, in particular Cases, is determined by Fig. 15, No. 2 and 3; viz. when the common Intersection is either parallel or perpendicular to the Picture. When it is perpendicular by Problem the first; and when parallel, by the third; and, when it is inclined to the Picture, in general, by the fifth. In all which Cases, it is observable that the Intersection of the Vanishing Lines, is the Vanishing Point, and the Intersection, of the Intersecting Lines, is the Intersecting Point, of the common Intersection of the Original Planes; which is an essential thing to be well understood; for Practice.

Theorem 7th. A Line drawn through the Center of the Picture and the Center of a Vanishing Line is perpendicular to the Vanishing Line.

The use of this Theorem is to find the Center and Distance of every Vanishing Line, the Center of the Picture being determined, and Distance known.

To do which, there is only to draw a Perpendicular, as CS (109) from the Center of the Picture, to the Vanishing Line; and, forming a right angled Triangle, of the Direct Radial, or Distance of the Picture, CE, and the Distance between the two Centers; the Hypothenuse is the Distance of the Vanishing Line. Def. 20.

In Theorem the 8th it is asserted, and proved, that the Representation of every Right Line, in Perspective, is a Right Line.

Any two Visual Rays. EF, EI, or EF, ED, together with the Original Line, form a Triangle, which are all in the same Plane, passing through the Original Line and the Eye; which, by its Intersection with the Picture, generates the Right Line fi, or fd, its Representation; and if the Original Line was extended, infinitely, its representation will still be a Right Line. (1. 7. El.)

The 9th Theorem contains the necessary knowledge for Practice, relative to Planes which are parallel to the Picture.

First; all Lines in such Planes, are parallel to the Picture; consequently they have no Vanishing Point (Cor. 1. Th. 1.) but their Representations are (by Theorem) parallel to the Originals; that is, to the Vanishing Line and Intersection of whatever Plane they are in; and their proportions to the Originals are also there determined; viz. as the Distance of the Picture, to that, of the Plane they are in.

The Corollaries, of this Theorem, are most useful Lessons; which need only to be read over, with attention, to apply them to Practice. The fifth sums up the whole; that the Representation of every Figure, in such position of the original Planes, is similar to its Original; that is, it has the same geometrical Figure.

Theorem the 10th is self-evident, being read attentively; and the Corollaries are useful practical Lessons deduced from it, which are fully explicit in themselves.

Theorem 11th is of the greatest use; insomuch that, without it, we cannot draw the representation of one Line, inclined to the Picture, except by means of expedients; making use of Lines, otherwise, unnecessary in themselves.

Notwithstanding this is the 11th Theorem, yet its application is prior to several others, respecting Vanishing Lines; seeing by its means, only, they can be determined; as in Prob. 2nd. It being impossible to determine any one Vanishing Line, before the Vanishing Point of some Line in the Plane be determined, through which the Vanishing Line must necessarily pass.

For first, the Center of the Picture must be determined, before the horizontal and vertical Vanishing Lines can be drawn; or that of any other Plane, perpendicular to the Picture; or indeed of any other whatever; because it is by means of
Lines,

Lines, in horizontal Planes, others are determined; whose Vanishing Line is always determinable from its known position, the Center of the Picture being fixed; or, which is the same thing, the height of the Eye being determined. Notwithstanding the Center of the Picture (being the Vanishing Point of Lines perpendicular to the Picture) is fixed arbitrarily, yet it is as much subject to the 11th Theorem as any other; and is only determinable from the fixed and invariable position of the Lines, which vanish in it, to the Picture.

As this Theorem has been applied in almost every Example, or Problem, where Vanishing Points of Lines, inclined to the Picture, are required; I shall only illustrate it by an Example or two, from the Apparatus.

The Parallel of the Eye must be imagined, passing through the Eye of the Spectator parallel to the Vanishing Line. Def. 9.

For the parallel Picture; EC cuts them both perpendicularly; therefore it makes equal Angles (i. e. Right Angles) with the Parallel of the Eye and the Horizontal Line; as the Original Lines, AB, HG, &c. make with the Intersection of a horizontal Plane passing through them; or any other, whatever.

In the direct Picture; EV, producing the Vanishing Point of the same Lines, also makes equal Angles with the Parallel of the Eye and the Vanishing Line, of either Plane, AHGB, or FGHI, as the Original Lines, make with the Intersection.

And lastly; EW*, makes equal Angles with the Parallel of the Eye of the Plane FGHI, and the Vanishing Line, VW, of that Plane, as FG, or HI, makes with FH, the Intersection of that Plane with the Picture, or, with WY, of the Plane BFC, as FG makes with BG, its Intersection.

This is practically exemplified, in Problems 5, and 21, Method 3; 23rd, &c.

By Theorem 12th, the Indefinite Representation, of a Right Line, is a Line drawn through its Intersecting and Vanishing Points.

This Theorem has been exemplified frequently; but I shall in this place illustrate it further, it being most essential in Practice.

Whatever Plane any Right Line is in, if it be not parallel to the Picture, it will cut it, somewhere, in the Intersection of the Plane it is in. IF, HG, and FG, are all in one Plane; which being produced, the two first cut both Pictures, at F and G; the other (FG) is parallel to one Picture, and therefore cannot cut it; but it will cut MNOP at H; F, G, and H are, therefore, the Intersecting Points of those Lines; and consequently, the Plane FIHG, being produced, would cut that Picture, in the Line FGH; and the other in FG; as by the 10th.

But, EV and EW being parallel, respectively, to those Lines, the Points V and W are their Vanishing Points; wherefore, the Lines FV and GV, also HW, are the indefinite Representations of those Lines; by this Theorem.

The same thing is applicable in all Planes whatever; as BV, GV, &c. in the Front and End, which are vertical; and a V, BV, MV, &c. on the Ground Plane.

By changing V for C, it is all applicable to the parallel Picture (MNOP) save only, FG is parallel to it, and therefore has no Vanishing Point; for the Indefinite Representations of all Lines, in such case, are infinite, seeing they have neither Intersecting nor Vanishing Point; and are, consequently, parallel to the Vanishing Line of the Plane they are in.

In Theorem 13th, is contained the whole knowledge of proportioning Right Lines, indefinitely drawn on the Picture (by the 12th.) It may, to some, appear too mathematical for Practice; but, notwithstanding that Theorem is more strictly

* Let the triangular Piece be fixed, by the two Pins, to the Picture MNOP, of which it is a continuation; and bring the piece, which folds back, to the same Plane; the continuation of the Vanishing Lines, &c. on them, show their affinity with Figure 109. If the thread, at W, be drawn through the Eye of the Spectator, EW will be parallel to the Lines, FG and HI, in the Roof of the Object; and consequently, W is their Vanishing Point. (Def. 22.)

By the same, the thread EV determines the Vanishing Point of AB, GH, &c. on both Pictures; to the first, it is perpendicular; therefore C is its Center, or Point of View; whilst another, EC, perpendicular to the other Picture, gives its Center (Def. 17) directly in the middle of the Representation, as it ought always, when there is but one Object.

and rigidly demonstrated than any other, it is nevertheless very applicable to practice. For, whether it be considered mathematically or not, 'tis certain that it is put in practice, by every one, in proportioning Lines; even without its ever entering into his Head what Proportion means; on the contrary, if he really understands what is there demonstrated, with how much more pleasure, satisfaction, and certainty, would he proceed in the application of it to Practice.

The Theorem tells us, that the Distance of the Representation of any Point (in the indefinite Representation) from the Intersecting Point (of the Original Line) is in that proportion to the whole Indefinite Representation; as the Distance, between the Original Point, and the Intersecting Point, is to its Distance from the Directing Point of the same Line.

Now when the Premises of this Theorem are clearly understood (for which purpose, the words are adapted, for brevity and perspicuity, as much as may be) what is there of mystery in it? For the first, in respect of the Proportion, simply. If the Distance of the Original Point, from its Intersecting Point, be to its Distance from the Directing Point, as 2 to 3, 5, &c. or as 3, to 5, 7, 8, &c. in any Ratio, whatever; then, the Distance, of the Representation of that Point from the Intersecting Point, will be in the same ratio to the whole Indefinite Representation*.

Can any thing be more simple and easy? nor does it depend on the real measures, but on the ratio of one to the other. Wherefore, since the Distance of some Points, in original Lines, cannot easily be obtained, from their Intersecting and Directing Points; I have shewn, further (which Dr. Brook Taylor has not) that the Distance of the Original Point, either from the Intersection of the Plane it is in, or from the Picture, is in the same ratio to its Distance from the Directing Line or Plane, a circumstance of great utility; because we may, with ease, have the measures of one, when we cannot, without applying Geometry, or by Calculation, have the other. For, the Distance of any Point in the Object, from the Picture, may be had; and the Distance of the Directing Plane, from the Picture, is nothing more than the Distance of the Eye from it; hence it is, with the greatest facility imaginable, reduced to Practice, in any position of the Lines whatever. It is most frequently done by the last; and commonly, by its Distance from the Intersection of the Plane the Original Line is in.

And further; the representation of any Point in a Line being obtained, the representation of any other Point in the same Line, or any other, cutting the former in that Point, may be obtained by the same Theorem; when the Intersecting Point cannot be had, nor is in the least degree necessary.

The 14th Theorem, which is more theoretic than practical, or really useful, is fully explicit in itself; and the Corollaries, deduced from it, need no further Comment, in order to a clearer elucidation.

Thus I have, briefly, taken a transient retrospect of the whole Theory: and I think it cannot fail of answering the End I aimed at, which is most obvious. As the Reader, who has advanced thus far, must now be perfectly acquainted with every Rule given: yet, being eager in his pursuit of what he deemed really and only useful, he may not have given that attention to the Theory (which is frequently referred to) as I could wish, for his more perfect knowledge in it.

He cannot now be insensible of the great advantage resulting from a well-founded Theory, the very essence of all useful Knowledge; for, without it, like a blundering Workman in any mechanic Art, he may continually be running into error; which, for want of a just knowledge of the Theory, he must, unavoidably; nor can he, with ease and certainty, extricate himself from the difficulties he will frequently be immersed in.

* See this fully and practically illustrated in Prob. 6. Sect. 4. and indeed in almost every Problem and Example, throughout the whole Work.

Plate
XXIV.

E X A M P L E XXXIII.

Is a Tuscan Arcade, with Pilasters; having one Face parallel to the Picture.

Fig. 110.

The scale of Proportion being determined, and a Profile, of the Order, proportioned to that Scale (at AD) as in the previous Examples, let AB, the Intersection of the upper face of the Step, be considered as the Ground Line; and, equal to the height of the Eye (in proportion to the Object) let CE be the Horizontal vanishing Line, C its Center, and CE the Distance.

† Cor. to
Theo. 4.

Then, the Object being right angled, the horizontal Lines in the returning Side vanish in the Center of the Picture†; in the other Face, every part of it, which are in Planes parallel to the Picture, are represented by Figures similar to their Originals, according to the 9th Theorem, and its Corollaries.

The place of the Sub-plinth, at X, being determined, by its distance from A, the Step (AB) is first represented; then, the Plinths, X, Y, and Z, are equal, and have equal spaces between them; their places are obtained by setting off their true measures from A towards B, as a, b, c, &c. on the first Step; and after the same manner, the Piers are obtained, and the Plane of the Arches, whose Centers are got by bisecting bc and de, at a and b; and ac being made equal to the Distance of the Plane they are in, from the Picture. Draw cE, cutting aC at f; and a Line fg (parallel to AB) cuts bC, at g; f and g are the Seats of the Centers of the front Arches, on the Plane of the first Step.

Draw FG parallel to AB, and equal to the height of the foot of the Arches; from a and b draw perpendiculars, aF and bG, and from F and G, draw to the Center, C; then, Perpendiculars, from f and g, cutting FC and GC, at h and i, give the Centers of those Arches; which, being parallel to the Picture, are Semicircles. (Cor. 5, Theo. 9.) The Centers of the inner Curves, are in the same Lines, FC, and GC, at k and l, and determined as the other.

The Mouldings in the Cornice, Pilasters, &c. may be proportioned by the general method (Ex. 15) as the Doric Entablature in the 16th.

The true Profile of the whole Order being described geometrical, at AD, and the Picture being supposed close to the Plinth, at A, draw AD perpendicular; which is considered as a vertical Section of the Picture, and CE its Distance.

Then, E being the Eye, and EA, ED, &c. Visual Rays from every Part to the Eye, it is obvious, that the parallel Mouldings have their places and proportions transferred from the Profile, to AD (as in Ex. 15 and 16) and, because the Cornice projects beyond the Picture, as Db, it is projected to the Picture, at D; the difference is only in the proportion of the whole; for if the Picture was at the greatest extreme of the Cornice, as ab; it is manifest, that the proportion of all the Mouldings, &c. would be less than those on AD. (See Example 15th.)

The returning Face (AHIK) being perpendicular to the Picture, the Vertical Line (AD) is the Vanishing Line of all Planes in that Front. (Theo. 5.)

The measures of the Piers, Arches, &c. being set off, at d, e, &c. are proportioned, by drawing Visual Rays dE, eE, &c. as in the foregoing Examples; and the Arches, are managed as in the 22nd Example, their Representations are semi-Ellipses; to particularize the whole would be superfluous.

The Mouldings, in all such Cases, viz. when a true Section of them is parallel to the Picture, may be proportioned, by making a geometrical Section or Profile at S, where the Picture is supposed to cut the returning Mouldings, if they were continued beyond the Picture.

In this Example, the Distance is considerably too little, for taking in so much as is represented; in this position of the Picture. The design, of which, is to shew how distorted the returning Mouldings of the Pilasters, &c. are, beyond the first Arch; which is necessarily the case in all such Views, being parallel, except a sufficient Distance be taken; in which Case, the returning Front would be greatly contracted. On the contrary, being both inclined, they may be shewn to much greater advantage, at a less Distance, without any sensible distortion; as may be seen in the next Example.

E X A M P L E

E X A M P L E XXXIV.

Is the Representation of a Doric Arcade, with Columns, inclined to the Picture.

If the Problems and preceding Examples are understood, there is nothing more requisite to the delineation of this Object; the Situation, Distance, and Position to the Picture being previously determined. Fig. 111.

In Prob. 21, the Rectangle may be considered as the Plan of such an Object, promiscuously situated; or the Plan of the Building in Fig. 106, inclined to the Picture, on VY. The foregoing Example supposes the Picture in the position DE; the Representation, of which, is as Fig. 107.

The height of the Eye being determined, and the Center of the Picture (C) draw the Horizontal Line; and, at a convenient Distance, take AB for the Intersection of the Ground Plane, on which set off the several measures, at a, b, c, &c. and, having obtained the Vanishing Points of the Sides (both which are, in this Case, off the Picture) and their Distances, D and E, being fixed (by Prob. 12) compleat the perspective Plan; as in Example 16, of the Doric Entablature.

The direction of the Lines, in the Figure, shew how it is effected, a repetition of it is almost unnecessary; seeing that, the rectilinear part is managed as inclined Lines, in general; and, the plans of the Columns are representations of Circles inscribed in Squares, by Prob. 3rd.

Let the Reader observe, once for all, that, either a true geometrical Plan of the Building, must be drawn out, or its measures known; which are applied to the Ground Line, or other Intersection, as there is found Occasion. The Profile, and measures of Elevation, must also be known; for it must be obvious, that every Object delineated by the rules of Perspective, are projected by the true and real proportion of one part to another, which must be known, or conceived.

Aa is the width of the Step, ab, bc, &c. being made equal to the Sub-plinths of two Columns, draw aD, bD, &c. cutting AL (drawn to the Vanishing Point of the front Lines, indefinite) at H, K, L, &c. bh and kc are each equal to the break at the Pier, and hk to the opening between the Piers. Being first projected to AL, by means of the point D; from which they are drawn to the Vanishing Point of the Lines in the End, till they meet the returning Lines of the Front (as at I) which are obtained on FQ, drawn to the Vanishing Point. Ad is the measure of the break at the corner of the Step; and, by drawing dE, the internal Angles of the Step and Sub-plinth are obtained, at G. From a and b (making Aa equal to the receding of the Arches, and ab to the thickness of the Wall) draw Lines to D, cutting AL, and from thence to the Vanishing Point of the End; where they cut the Diagonal GE, they are returned at the Front; the rest is obvious, on inspection. The Columns at MN recede from the Front, being in a Line with the first returning Column at the End, at G.

The perspective Plan being compleated, or so much as is necessary (for those Parts which cannot be seen, it would be useless to particularize, except for the affinity of those which are) take A, at the proper Distance from the Vanishing Line (perpendicular over A) equal to the determined height of the Eye; and from it draw indefinite Lines to the Vanishing Points (as AB) and, from the Plan below, cut off such portions as are the representations of the several Parts, and proceed as in the foregoing Examples.

Here, the necessity of a Plan being formed, or, which is the same thing, the Lines proportioned below, by an imaginary intersection, AB, is evident; on account of the great Distance, and the Eye not being far out of the Ground Plane; otherwise, that operation would be unnecessary; for, if the height of the Eye was equal to the Distance of AB, from the Vanishing Line, then, the Plan formed, by means of that Intersection, would be the real Base, or Seat of the representation, of the Object, and no other would be wanted; in which, the expediency of the 12th Example is remarkably obvious. Being farther removed from the Vanishing Line, the several parts are more distinct, and may be done more correct.

The

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The Step being first drawn, the Sub-Plinths, next, are but so many equal Parallelopipeds of a certain proportion and distance from each other (as in Example 4.) The places and proportions of the Columns are determined from their Plans below, by drawing Perpendiculars from them (or by Example 22) the Bases and Capitals, being inclined, by Example 26; and the Entablature, by Ex. 17, 18, and 9.

The Ballustrade, may be planed on the top of the Cornice, or at some distance from it; the Pedestals, or plane parts, are directly over the Columns, and equal to their Diameters at the top; and the places of the Ballusters are got by dividing the Line *AC*, perspectively (by Prob. 8) into the number of Ballusters and spaces between them (equal or otherwise.) The Plinths, at their tops, are perpendicular over the Bases; and central Lines (*hi*) being drawn, through each, the Ballusters are easily formed, from the known geometrical figure of them.

The Doric Entablature is described at large in Example 16 (but parallel to the Picture) how it may be inclined is shewn in the 18th and 19th Examples.

The Plane of the Arches being obtained, the Arches are represented by Example 22, and the Consoles by the 20th; the rest, it would be needless to describe, as the previous Examples contain every necessary Lesson for complex Objects.

E X A M P L E XXXV.

Is the Representation of a compleat Building, with a Portico in the Ionic Order; having a spherical Dome, and Cupola.

Fig. 112.

At No. 1 is half the geometrical Plan, and, at No. 2 is the Elevation; in proportion to the Representation, as one to four.

The Design is intended as an exterior Building, adapted to St. Stephen's, Wallbrook, but of a different Order; having a ~~Front~~ towards the Mansion House.

This Example sums up all the foregoing, in one entire Building.

The Horizontal Line being drawn, at discretion, and the Center of the Picture fixed, as usual, let *AB* be the Ground Line, and *D* the Place of the nearest Angle of the Building. The Distance of the Picture is 15 Inches; *C* is its Center.

The Inclination of the Front and End, to the Picture, is determined by the Line *ab* (No. 1) on which the Picture is supposed to stand.

The Distance of the real Ground Line from the Horizontal, being equal to the height of the Eye, which is elevated above the common height, yet is too low to form a correct perspective Plan, in its true Place; therefore, at any convenient Distance take another Line *AB*, parallel to *AB*, and proceed as in the last Example.

Having laid down the distance of each Vanishing Point, at *E* and *C*, as usual; take the known measures of the several parts of the Building and apply them on each side, from the Angle at *D*. Compleat the exterior Plan, *CDEF*, as in the last Example; from which, the several parts of the Building, above, are proportioned; as the perpendicular dotted Lines shew. *F* is the Vanishing Point of Diagonals; i. e. of a Line bisecting the Angle of the Building.

It must be obvious, that any Line (*AB* or *AB*) being considered as the Ground Line, will give the same proportions; which, being properly considered, there is nothing more in it, than a supposition of the Eye being more elevated above the Ground Plane; and consequently it is more seen and better defined.

The exterior Plan being compleated, from which the corners of the Steps are determined, and drawn in their true places, by Example 13. Also the Sub-plinths of the Columns, *v*, *u*, *x*, &c. from their corresponding Plans below; the Diagonals of which, being drawn, give a central Line, the Axis of each Column; the heights of the several parts are determined on the diagonal Intersection, at *D*, *F* being the Vanishing Point.

The Columns with their Bases are drawn and finished as in the last Example; and the Capital, being the antique Ionic, by Ex. 31st. The interior part of the Portico has a Pilaster opposite each Column, which are also determined from the Plan, below; some of which are hid behind the Columns; perpendicular Lines from the Plan, shew how much, and where they are seen entire.

The Plane of the Front, of the Pediment, over the Portico, being much inclined to the Picture, its Vanishing Line is very remote; the Distance of the Vanishing Point of horizontal Lines, in that Face, may nevertheless be ascertained, (as E) by Ex. 12, and the Lines drawn by the Expedients, in the 13th.

As the Distance of the Vanishing Point of horizontal Lines on the left Hand, would fall off the Picture, it is taken half, at C; and consequently, the measures applied on the Intersection or Ground Line (AB) for that side, are also, each half of the real measure, agreeable to the 13th Theorem, and Prob. 6 and 7.

The front Pediment is thus determined. If the Cornice projected equal to the Steps, which the Picture is supposed to touch, a Right Line drawn through D, perpendicular to the Ground Line, would be the Intersection of the Plane of the Pediment; but, as its projecture is less, it will fall to the left hand of it; as the Line *ab* (No. 1) which is a continuation of the front Line of the Cornice, cutting *ab*, at *d*, indicates sufficiently.

Or, having determined the perspective Plan of the Cornice, at *a*, *b*, *c*, produce *ab*, till it cuts the Ground Line at *d*, and draw *de* perpendicular, which is the Intersection, of the Plane of the Cornice in the Pediment.

Then, having made *Je* equal to the known height, and determined the front Line of the upper Moulding, as *fg*, set up the height of the Pediment, from *e* to *i*, and draw *ik*, tending to the Vanishing Point (by Prob. 13.)

Bisect *fg* (perspectively) at *h*, and draw *hl* perpendicular, cutting *ik*, at *l*, the true pitch, or middle of the Pediment, which may be compleated, by Ex. 20.

The Pediment in the other Front, on the left hand, may be projected by means of its Vanishing Points, by the same Example.

To this Front is added a Pedestal which is continued around the Building; more for the sake of diversifying the Lesson, than propriety in the Building; the heights of the Pedestal, Windows, &c. are proportioned on the Intersection, DK, of that Front, at *a*, *b*, *c*, &c. for the Niches and Windows over them.

The Body of the Building being compleated, the several parts being determined from the Plan below, with the breakings or returnings of the Cornice, and other Mouldings (as in former Examples) let the Circle of the Dome be planed, at GH, perpendicular Lines from the extremes of that Ellipsis, give the extreme apparent edges of the cylindrical part, IK, below the Dome; of which Cylinder, the Ellipsis, on the Ground Plane, may be supposed its Base.

The curve Lines, in the Cornice, around the Dome, are described in their respective Planes; which are thus determined.

From the Vanishing Point on the left, draw through the Center, S, of the Plan below, cutting the Ground Line at J; which represents a Line passing through the middle of the Plan (No. 1) supposing a Section made by a vertical Plane through the middle of the Building.

Draw JL, perpendicular, the Intersection of such a Plane with the Picture (Prob. 3) on which, all the measures of the heights of the Dome, &c. are applied, in the same ratio as on *de*, in the Elevation (No. 2) at M, N, &c.

From M, the height of the Cornice, draw a Line parallel to the Ground Line, which is the Intersection of the Plane of the Cornice, with the Picture. A Right Line, drawn from M to the Vanishing Point on the left, will cut a perpendicular Line from S at *f*, the representation of the Center of the Circle of the Cornice.

From C, the Center of the Picture, draw CS, through *f*; and through *f*, draw *mn*, parallel to the Intersection; make SR and ST each equal to half the Diameter of the extreme Circle of the Cornice; and draw RC and TC, cutting *mn*, at *m* and *n*. *mn* is a Diameter of the Representation, parallel to the Picture; the whole Circle may be compleated, by Prob. 2, Sect. 8th.

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After the same manner, all the other Circles, of the Corona, &c. may be projected, in their respective Planes; as the several Circles in parallel Planes, by Ex. 28.

The Cornice, &c. being compleated, set up, from N, the height of the Dome, and take several Divisions *a, b, &c.* answering to the same ratio on *de*, in No. 2.

Through these divisions, parallel Lines being drawn, as *rs*, each may be considered as the Interfection of a Plane, cutting the Dome (as *rs*, No. 2) in each of which, an Ellipsis being described, representing a Circle of the true diameter, made by that Section; a Curve described over all their extremes (as in Ex. 25 and 26) is the true exterior contour of the Dome; which may be considered as an Ovolo, inverted; as in the Doric Capital.

By any other means (the Dome being plain) I cannot conceive it possible to obtain the true perspective Figure of a Dome; which is generally higher than a Hemisphere; especially in a lofty Building, which would otherwise appear flat, below; besides, it is a more graceful Figure; as the magnificent Dome of St. Paul's sufficiently evinces, to a judicious Eye. But, being a Hemisphere, or other Segment of a Sphere, the Eye being below its Center, the Contour of the exterior Curve, on the Picture, will be elliptical. (Th. 3rd, of curvilinear Perspective.)

By this method, it may be truly projected, let the Figure be what it may, whether a Hemisphere or lesser Segment. Nevertheless, a judicious Eye, and judgment, guided by a thorough knowledge of Perspective, may direct a steady Hand to trace a Curve, sufficiently correct, in common Cases; the height and Vertex being truly ascertained.

The Dome in the Frontispiece is octagonal; and, by means of several Octagons, represented perspectively, in various parallel Planes, the true Figure of it may be projected. Or, by means of several Points, in an Angle; for, if the true Curve of an Angle be obtained (as at Y, No. 3) from the Curve, X, of a vertical Section through the Dome, perpendicular to two opposite Faces (by means of perpendicular and horizontal Ordinates, *ab, cd, &c.* having made *hA* equal to the angular projecture, and divided it at *a, c, e*, as *ah* is divided, at *a, c, e*; perpendiculars, *ab, &c.* being drawn, from the points *b, d, &c.* in the curve at X, draw lines, parallel to *AA*, cutting them at *b, d, &c.* through which the angular Curve *Adg* may be described) then, having drawn any one, perspectively, they may be transfered from one to the other, by the Vanishing Points of horizontal Lines in the Cornice around it, at the foot of the Dome.

By the same means, the Ribs, in this Dome and Cupola, may be described.

After this description of the Dome it would be loss of time to describe particularly the projection of the Cupola. From the Plan below, its place and dimensions are truly ascertained, together with the place of each Pilaster and Window; from which perpendicular Lines are drawn; and, the height of each part set up, on the Vertical Interfection, *HN*, as the Dome, and other Parts, below; of which, the Cupola is a similar Figure, nearly.

E X A M P L E XXXVI.

Is a Representation of that much famed Building, by Inigo Jones, of St. Paul's, Covent Garden; and the Buildings adjacent.

Plate
XXVI.

As the last Example was a Lesson for an entire Building, so this is intended for a Lesson in detached Buildings, or distinct and separate Objects; which may be considered as a Street-View.

Having, in the last, given a compleat perspective Plan of the whole Building, it would be multiplying Examples to little purpose to do the same thing here; let it suffice, therefore, to suppose, that the Plan of the whole, in this piece, is truly described on the Ground Plane, on which the Objects stand; their measures being applied to the Interfection of the Picture or Ground Line, as in the preceding Lessons, which it would be quite superfluous to repeat; I shall, therefore, only make some general Remarks.

It may be observed, that the Church is nearly in the same Position as the last Object, but a much simpler, and consequently an easier Figure to represent.

The Portico is of a Tuscan Order; which, for its simplicity and singular Stile, is much esteemed by the admirers of the antique Grecian Temples, of which, this is esteemed a perfect Model. The great projecture of its Cornice (if it may be so called) has a bold and striking appearance; owing to the great effect of Shade it occasions. The attached Parts, at Y and Z, which is the staircase into the Gallery, seem not to belong to the Body, but are adapted merely for conveniency; I am of opinion that it would make a better Figure without them. The other, at V, is the Vestry Room; which, with a similar Object, on the other Side, adds to the appearance of the back Front.

How these several parts are projected, may be seen in the last Figure; having formed a Ground Plan of the whole; their several heights being set off, on the vertical Intersection of the front Plane, or proportioned to the Pilaster, at the corner.

The two Gates, or entrances into the Church-Yard (X) are a great ornament to the Church, and graceful in themselves. After what has been done, there is nothing singular in their Construction, the geometrical Figure being known.

The Houses, one on each side, are equally distant from the Church; 'tis pity they are not both in the same stile, as that on the Left, for the sake of uniformity. The Hut, adjoining to the House on the Left, at W, is the Parish, Watch-House. I could wish some other place had been destined for it, as it blocks up an Avenue, and breaks the regular Order.

The Fronts of the Houses and the Gates are in a Right Line, much inclined to the Picture; the Portico projects before them. There is no occasion for a Ground Plan of the whole, for the measure of each Part being known, the Line may be proportioned, by the Problems in the 4th and 5th Sections; and, the heights are determined on the Line AB, which may be used as an Intersection of the whole, considered as one Plane.

The Pediment has been frequently described; and, the Bell-Turret may be determined by the length of the Roof; or, more correctly, from a Plan formed below; as the Cupola in the last Example.

On the Left is seen the opening into Henrietta-Street, at P; and one of the Green-Sheds, at Q; the place, of which, is obtained by its known distance from the Church, and the line of Direction in which it stands, respecting the Church, projected if necessary. Or, the Ground Line may be supposed on this side of the Shed, parallel to the bottom edge of the Picture. The Horizon is about the natural height of the Eye; the Center of View is at C, in the middle of the Picture, and the Distance is about 16 Inches,

E X A M P L E XXXVII.

Is the Representation of another well-known Building, viz. The Royal Hospital, of Invalids, at Chelsea.

The Stile, of this Building, is somewhat similar to that of St. Paul's, Covent-Garden, in respect of its simplicity. There is, in the Gusto of the whole, something remarkably pleasing; the Symetry of the several parts, in respect of each other and of the whole, is finely preserved, in a regular and gradual subordination, from the Principal to the several Offices; the agreeable arrangement, of which, is uniform, and perfectly harmonious. Here, the attention is not attracted by the richness of the dress, or masterly execution; the mind is not captivated by grandeur and magnificence, but most agreeably entertained, with an elegance of Design; simple, yet majestic; and comely, though unattired; perfectly agreeable in its native simplicity, and almost naked Beauty, devoid of every luxuriant Ornament.

In the last Example, the Line of the Building was much inclined to the Picture; in this, it deviates little from a Perpendicular; which is far more agreeable than to be perfectly so.

Plate
XXVII.

In

Plate
XXVI.

In this View, many, who pretend to know Perspective, would immediately fix on the Point V, in which all the horizontal Lines, of the Front, converge, to be the Center of View; but it is not so; for, according to my general maxim, it is at C, in the middle of the length; and, notwithstanding there is not one Line vanishes in it, yet, it governs the whole.

If V had been the Center, or Point of View, the Lines BD and HI, &c. would be parallel amongst themselves, and to the Horizon; but, they converge to a Point, at a great Distance, on the left Hand.

The Distance of the Picture is about 19 Inches; at which Distance (the Eye being perpendicularly opposite to C) the whole will appear as the Original, at the station intended; and, I am persuaded no Person will suppose that it would have a better effect, or so good, if V had been the Center of View; for, the Optic Angle would then be almost double of what it now is; and consequently, the Representation would differ more from the true Appearance; which, now, is inconsiderable, the Optic Angle not exceeding 34 Degrees.

The Inclination of the Building to the Picture, and the Distance, being determined, the Vanishing Point, V, is fixed by Prob. 12; and the indefinite Line, GV, being drawn, it is proportioned as usual, by setting off the real measures, or their ratio, from G, at K, L, &c. and drawing Lines, which represent Visual Rays, to E, in the Horizontal Line.

The measures are taken half, because EV is but half the Distance of the Vanishing Point, V; as the whole Distance exceeds the bounds of the Picture.

The horizontal Lines on the returning Sides, whose Vanishing Point is at a great Distance, may be drawn by the Expedient, No. 5, Prob. 13. Or; having proportioned the Line of the Angle, at AB, into the several Divisions, at a, b, c, &c. for the Windows, &c. and having drawn one Line to its Vanishing Point, as BD (by the 12th) the Distance of the Vanishing Point being determined*; produce BD, and, at a convenient Distance, draw FG parallel to AB, and proportion FA in the same Ratio as AB (by Prob. 33, Geom.) draw aa, bb, &c. which will tend to the same Point.

But, when the Inclination is not very great, the best Expedient is to fix a Line^b to the Board and make use of a long Ruler; or, in large work, fix a smooth String to the Vanishing Point, and strain it, in a Right Line, to the several Points a, b, c, &c. and then apply a Ruler to the direction of the Cord.

The Hip of the Roof, at H, may be determined from a Plan, on the Ground Plane; or by the known Inclination of BH to the Picture, and its Vanishing Point; and, the Line HI by the same, as in the 13th Problem.

The Window Frames, in the Roof, being drawn, as the other Windows, and the straight Cornice, fh is obtained, by which, the Pediments over them may be completed, as in Example 20; by finding the Vanishing Points of fg and gh, in the Vanishing Line of the Front, passing through V, perpendicular; the best method for the front Pediments. The Vanishing Line of the side Planes is too remote to use for those Pediments; which, in such case, differ but very little from their geometrical form and proportion.

The Cupola is best done by a Ground Plan, as in the 35th Example.

The Columns and Pediment over them have nothing particular in the delineation; the middle Pediment is more contracted than the hither one, so that the Mouldings are seen on one side only; but is done after the same manner; and, if they have the same inclination to the Horizon, the Lines in them have the same Vanishing Points as those over the Windows; because the Pediments are all in parallel Planes, and consequently they have the same Vanishing Line†.

The Trees (on the right hand) excepting their distances from each other, are not subject to the Rules of Perspective; because no proportion of the Boughs can be taken. The Rails and Posts have nothing singular in them; as the Rails are

* This Vanishing Point is about seven Feet Distance, from the Center of the Picture.

† Theo. 5.
and Cor. 1.

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parallel to the front of the Building, they have consequently the same Vanishing Point (V) as the horizontal Lines, in the Front; and the Posts, being equally or otherwise spaced, are determined as in all other similar Cases.

The heights of the Figures, in the Walk, are thus determined.

Take any Point, L or M, in the Ground Line, and make MN equal to the height of a Figure (by the Scale of proportion, of the measures of the Building) and draw MO and NO, to any Point in the Horizontal Line; then, wherever you intend a Figure, as at *a*, *b*, or *c*, draw *am*, *bm*, &c. parallel to the Ground Line cutting MO, at *m*, or *m*; and draw *mn*, or, *mn*, parallel to MN, which is the height of the Figure in that place, representing an equal height to MN.

Or, wherever you intend a Figure of any kind, draw a Line through the place of the Feet, cutting the Ground Line and Vanishing Line, obliquely; and, where it cuts the Ground Line, draw a perpendicular, and set up the height of the intended Figure; draw another Line to the same Point in the Vanishing Line, between which, a perpendicular being drawn is the height of the Figure.

By the same means, the proportion of any other Object may be determined.

E X A M P L E XXXVIII.

Is a View of the Queen's Palace in St. James's Park, and adjacent Buildings.

This neat and elegant Building was formerly the Town Residence of the Dukes of Buckingham; which Title becoming extinct, it had for some Years been occupied by a distant branch of that ancient Family, without the Title, who made no Figure, suitable to such a noble Mansion; insomuch, that it was gone greatly out of repair when purchased by his Majesty. It has, since, been much enlarged, repaired, and beautified, and made the Winter Residence of the Royal Family. The octagon Building, on the left hand, was added at the same time, and is the Library.

Plate
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Since which, there has been great improvements made; the semicircular Area, in front, has been enlarged; but I think it would have been more advantageous, had the Pallisades stood on a dwarf Wall; for want of which, they are not distinguishable, at some Distance from it. The Gate leading to Chelsea, and the Lodge are also new; and, several old Trees were cut down, to make a more spacious and open View to the Canal, &c. in the Park.

This Building would, with propriety, admit of a full Front View; but such a one is by no means picturesque; as, one Side, in such case, is but a duplicate of the other, which is not the case in this View; the Center, of which, is in the middle of the Picture, as usual.

The Front is inclined, and consequently, the horizontal Lines vanish, but at a great distance, because the Inclination is very great; the Distance of which Vanishing Point is determined by the 12th Prob. The Vanishing Point of horizontal Lines in the other Faces is at V, in the Pavilion on the left hand; the Horizon is high, for the conveniency of seeing the Area, more commodiously.

The Distance of the Picture being determined*, and the Vanishing Point V, the Distance of the other is a third Proportional, to CV and the Distance of the Picture, found by squaring the Distance, and dividing by CV. (Pr. 12.) (Pr. 31. Geo.)

As the octagon Building has two Faces parallel to the Fronts of the House, the adjacent Faces vanish in the Vanishing Points of Diagonals; i. e. all the horizontal Lines, in one Face of the Octagon, vanish in that Point where a Line bisecting the Angle, made by the Radials of horizontal Lines, in the Front and End of the Building, cut the Horizontal Line; and the other, on the Left, where a Line, making a Right Angle with the Line of bisection, cuts it; as it has been exemplified in various cases, in Mouldings, &c. both which fall out of the Picture.

* In this Picture it is about 12 Inches.

Plate
XXIX.

On account of the great Distance, and the Horizon being high (between the Mouldings in the upper part) their inclination is not perceptible; as is frequently the case, in Views taken at a tolerable distance, and perhaps from an eminence, which is generally made choice of in Landscape Views; otherwise, the Ground would be too much contracted, unless diversified with rising Grounds, &c. The distant Buildings, in such case, are represented nearly geometrical; for, unless they are situated considerably above or below the Eye, in proportion to the Distance, the inclination of the Lines is not distinguishable; and, if they are obliquely situated to the Picture, their Faces are contracted, almost geometrically; so that, distant Objects, of all kinds, are not cognizable to the Rules of Perspective, otherwise.

It is unnecessary to say more, in respect of this Object, as Rules have been given for projecting all kinds of Figures of which it is composed; and finding the Vanishing Points, &c. at any given Distance and position of the Picture.

Figure 106 exhibits a general Method for contracting the Faces, &c. of an Object, and for obtaining the true place of each part, in respect of its bearing with others (being detached) from any determined Station, which, in many cases, is the best method of proceeding, and the least liable to error.

E X A M P L E XXXIX.

How to represent Doors and Window Shutters, open, in any given Angle; or at pleasure,

Fig. 113.

Let AV be the indefinite Representation of the Side of a Building in any horizontal or vertical Plane; which, from the point A, is required to be perspectively divided into certain finite parts, representing Windows, Piers, &c.

By the 8th Problem, Ac and VE are drawn parallel, between themselves (VE may be considered as the Vanishing Line, and Ac, the Intersection, of whatever Plane the Original Line is in) make VE equal to the Distance of the Vanishing Point (V) of the Line AV; and, on Ac, take Aa, ab, &c. equal to the known proportions of the Piers and Windows, or Apertures of any kind.

Draw aE, bE, &c. cutting AV in the Points a, b, c, &c. which are the representations of the Original Points, a, b, &c. Wherefore, if ab be supposed the geometrical proportion of a Window, then, ab represents the same perspectively; and so of any other division, as bc of a Pier, &c.

N. B. Let it be observed, that it does not depend on the real measures being applied on Ac, but, that they are in the same ratio, respectively, as EV is of the distance of the Vanishing Point; agreeable to Theorem 13th, which is frequently exemplified in the preceding Work; and, whether AV represents a Line perpendicular or inclined to the Picture, there is no difference in the process.

Fig. 114.

Let AB represent the aperture of the opening of a Door, in the side of a Room, &c. which is required to be seen open, in any position, at pleasure.

Now, AB, the width of the Door, is the radius of a Circle, which it would describe if the Door revolved quite around; and consequently, the Door itself (being a Rectangle) would describe a right Cylinder, of which, BC is its Axis.

By Prob. 10th, Sect. 4th. or, Pr. 2, Sect. 8th, describe the representation of a Semicircle, AaaF, whose Radius (given or found) is AB.

It is manifest, that AB the perspective width of the Door, would, in its semi-revolution (from AB to BF) describe the semi-Ellipsis AaaF; that is, the point A (B being fixed) would describe the semi-circumference of a Circle: and consequently, in whatever position the Door is open, the point A will be somewhere in that Semicircumference, as at a or a; wherefore, aB, aB, &c. are also representations of Radii, of the same Circle.

Take the Point a or a, in the Circumference, at pleasure (according as you require the Door more or less open) and draw aB or aB, which produce to its Vanishing Point, H or H. Draw the Perpendicular ad, or ad, and, through C, draw HQ, or HC till it cuts ad or ad, at d or d; then is aBCd, or aBCd, the representation of the Door, open in the position required,

Or,

Or, if the Angle of the opening, on either side, be known or determined; produce AB to its Vanishing Point (V); S being the Center of the Picture, draw SE perpendicular, equal to its Distance, and join EV ; make the Angle VEH , or VEH , equal to the Angle determined; which will give the Vanishing Point H , or H , of the top and bottom Lines of the Door, in the position required.

If the Door be required more open, as AB , making an acute Angle, ABF , on the other side; then, the Vanishing Point will be on the other side of V .

The process is the same, in every Position.

After the very same manner, Window Shutters, being vertical, are determined; the Radius, of each, being half the width of the Window, Ad or dD . Fig. 115.

$ABCD$ is the aperture of a Window, to which there is required the representations of folding Shutters, open in any position or angle, at pleasure.

Produce AD and BC , both ways, and bisect either, perspectively (as BC) and, make Ba and Cg each represent half BC . On ad and dg describe representations of Semicircles, $abcd$, and $defg$; in which, according to the determined angle or opening of the Shutter, take b or c , e , f , or g ; from which, draw perpendicular, and draw bB , or cC to its Vanishing Point, at S or V ; from which points, draw SA , or VD , and produce them cutting the Perpendiculars, at b or c , which compleats the Shutter $ABbb$, or $CeeD$.

$ABcc$ represents another, less open; and $CffD$ is one parallel to the Picture, consequently, it has the true figure and proportion of half the Window.

$ABaa$, and $CggD$ represent others, quite open, against the Wall.

The Shutters Y and Z describe Semicircles in vertical Planes, as $abcd$, or $abcd$; or, more properly, they describe half horizontal Cylinders. Fig. 116.

The first, being parallel to the Picture, are Semicircles[†]; the others are perpendicular to the Picture; or they may be inclined, in any Angle whatever; the Vanishing Points of the describing Lines are in the vertical Vanishing Line (VL) of the Plane of the Circle, as at V and L , &c. the front Lines are parallel.

In the former they are parallel to the Picture; the front Lines vanish in the Center.

[†] Theo, 9,
Cor, 5.

S E C T I O N X.

OF INSIDE VIEWS, IN GENERAL.

THE application of the Rules of Perspective, whether to the interior or exterior parts of Objects, is the same, for, Planes and Lines, of which Objects are composed, are still the same however situated; whether they form internal or external Angles, of Objects seen externally; or, forming Rooms or Concavities of any kind. Nevertheless, at first Sight, there appears a difference; for I believe that, many who have practised Perspective with tolerable success, in exterior Objects, are somewhat puzzled, at the first attempting an Inside View; not knowing, rightly, where to begin or where to leave off; how to place their Picture, or determine its Distance.

I have seen an attempt to represent an Octagon Building, internally, in which Picture were introduced seven Sides or Faces, out of the eight; it was a misfortune that the other could not be seen also, quite around, and then it would have been a master-piece, indeed. However, from what was seen, and the tendency of the Lines, it was manifest, that the Distance of the Picture was not more than one fourth part of its width, or length; and consequently, the Optic Angle was above 120 Degrees, which ought not on any account (except when the Eye is confined to the true Point of View, always) to exceed 60.

Plate
XXIX.

'Tis the same thing, if, in order to see the whole of a Dome and Cupola, internally, we advance so far into the Building, that the representations of such Subjects are only fit for horizontal Pictures, or Cieling-Pieces; for, when they are represented on a vertical Picture, they appear (at a proper Distance to take in the whole) as if falling, or not upright. Yet are these things to be met with in Pictures and Prints, by Men of some distinction in the Arts.

That the Perspective Representation of the inside of a fine Building is more difficult to manage than the exterior is certain, and is manifested in theatrical performances; which, being represented on several detached Planes, it is impossible, by the Rules of Art, to make them correspond in every Point of View; but, they are very rarely connected in any one Point. There are, undoubtedly, many fine Performances of the kind; yet, without attempting to disparage their Authors, I am confident, that, were they better acquainted with Perspective, they would produce better and more natural Representations; even that famed Scene in Cymon is a jumble of inconsistencies, both in point of Design and Execution; although it was a bold attempt out of the common mode of Representations.

Few Artists have made Perspective so much their Study, to know how to proportion one part to another, on detached Scenes, so, as to make the whole unite in the proper Point of View, whether the Representation be internal or external; indeed, from the construction of the Theatres, it is scarce possible to be done; nor are the Rules of Perspective sufficient, for the purpose, without a tolerable knowledge of Lines, geometrically. It is the least qualification of a Scene Painter to be excellent in Landscape, in which a small knowledge of Perspective is requisite; but, in order to execute Designs in Architecture with correctness, and a just proportion of the several Parts, requires a thorough knowledge of Perspective. Is it not surprizing, that all who are concerned, or any way engaged in Scene Painting, do not make Perspective their immediate study; being the basis, the very soul and ~~evidence~~ ^{essence} of their Profession? yet, to my certain knowledge, several Artists, employed in it, are not only totally ignorant of it, in Theory, but they are, almost, wholly unacquainted with its Rules, which, to me, is most unaccountable.

In inside Views, the bounds of the Picture limit the whole, every way, which renders the operation, in some cases, more difficult than external Views.

In order to shew the inside of a Room, Temple, &c. properly, a Section is supposed to be made; that is, the hither Wall (or inclosure of any kind) is supposed to be removed, and the whole Inside laid open to View; so that, a proper Station and Distance may be taken, from which, the whole, or as much as is required, may be seen under an Angle not too large; for, to suppose that we can be within a Room and exhibit the whole, or nearly, is truly absurd; yet it has been attempted, by those who have not a just notion of Perspective.

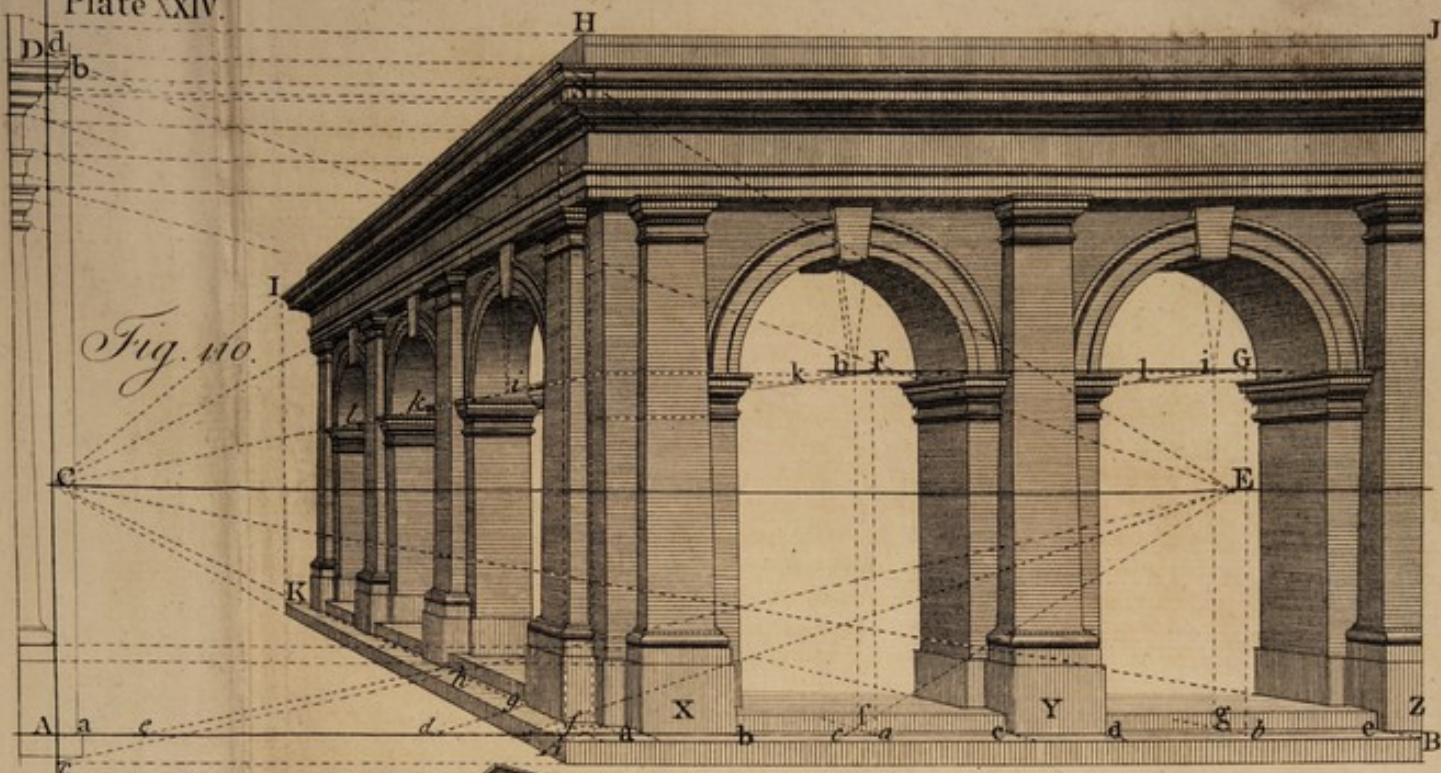
In respect of determining the places and proportions of Doors, Windows, &c. it must be obvious, there can be no difference whether they are interior or exterior; the whole of which is contained in Prob. 8th, and has been universally applied throughout the Work; particularly in Ex. 2nd and 4th, also in the 16th, 19th, and 21st, in finding the places of the Mutules, Modillions, Blocks, &c.

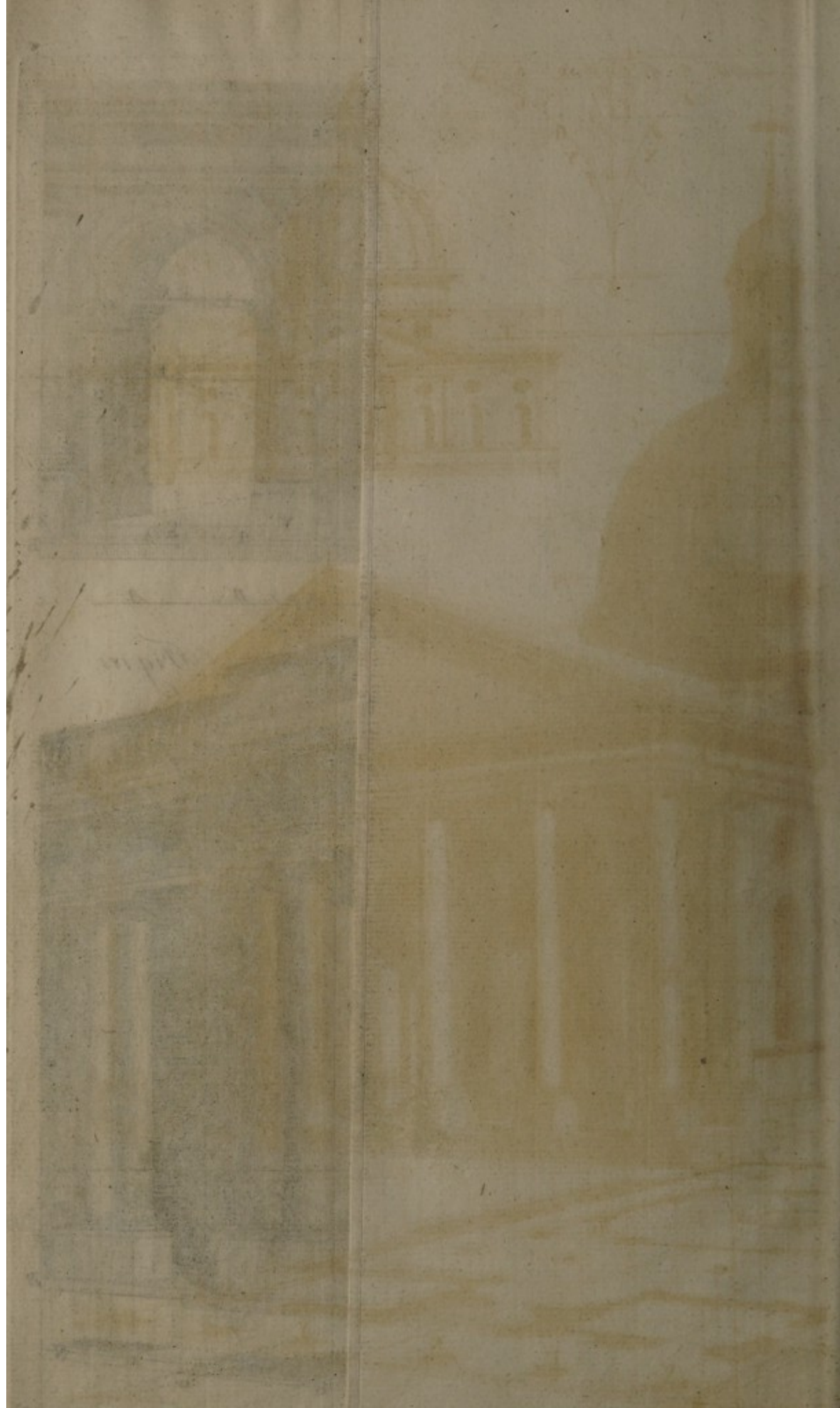
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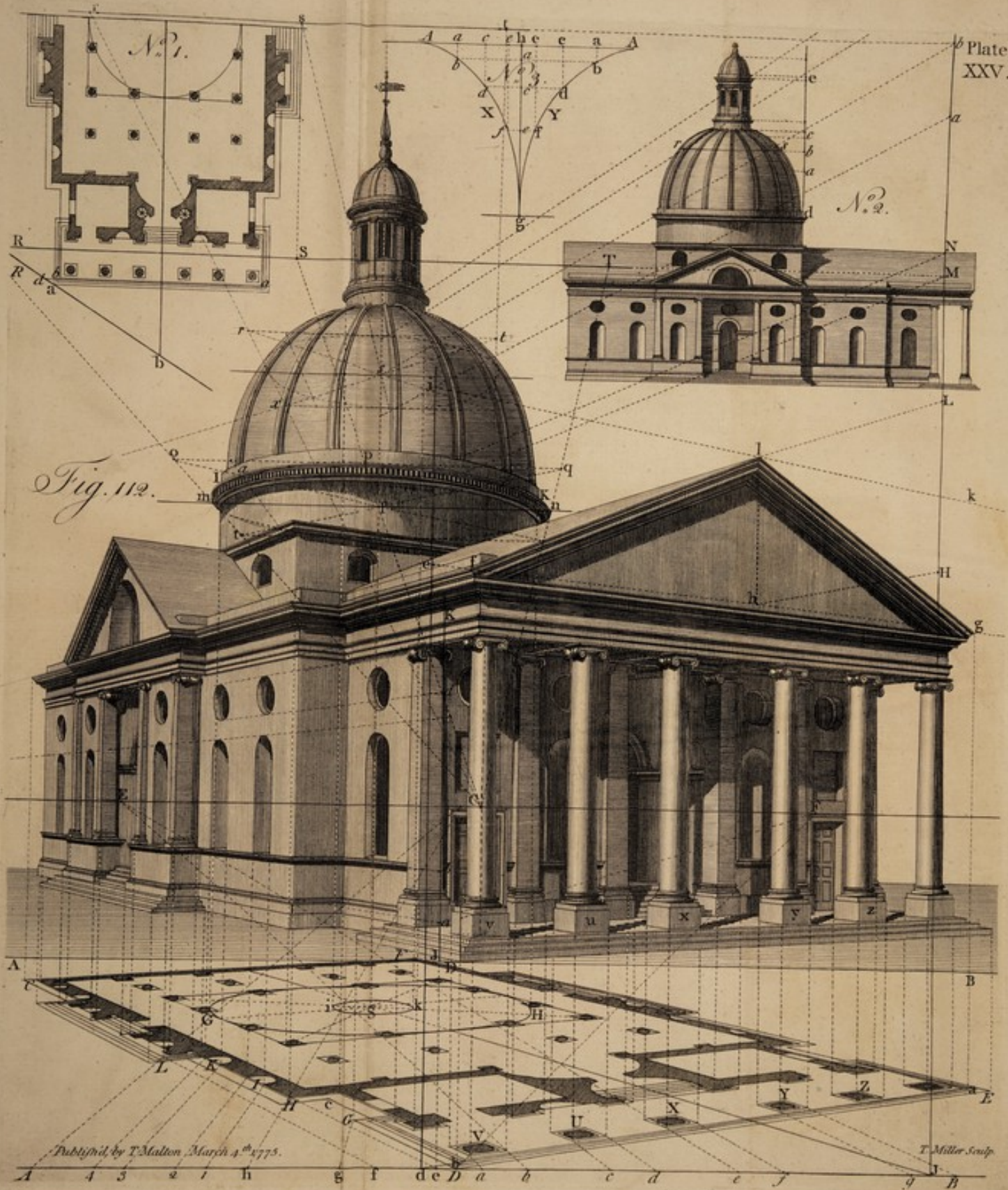
How to represent the inside of a plain Building; the Section, by the Picture, being parallel, to the Ends of the Buildings.

In representing the Inside of any Room, Church, &c. it is usual to take the Station in a Line drawn through the middle of the Building, which indeed appears the most rational; but in such case, in a regular Building (one side being a duplicate of the other) it is not so picturesque, as when the Station is towards either side, as at S; from which, the inside of the Room is to be viewed; by which means, either Side may be shewn to greater advantage, being more opposed to the Eye than the other.

Fig. 1.

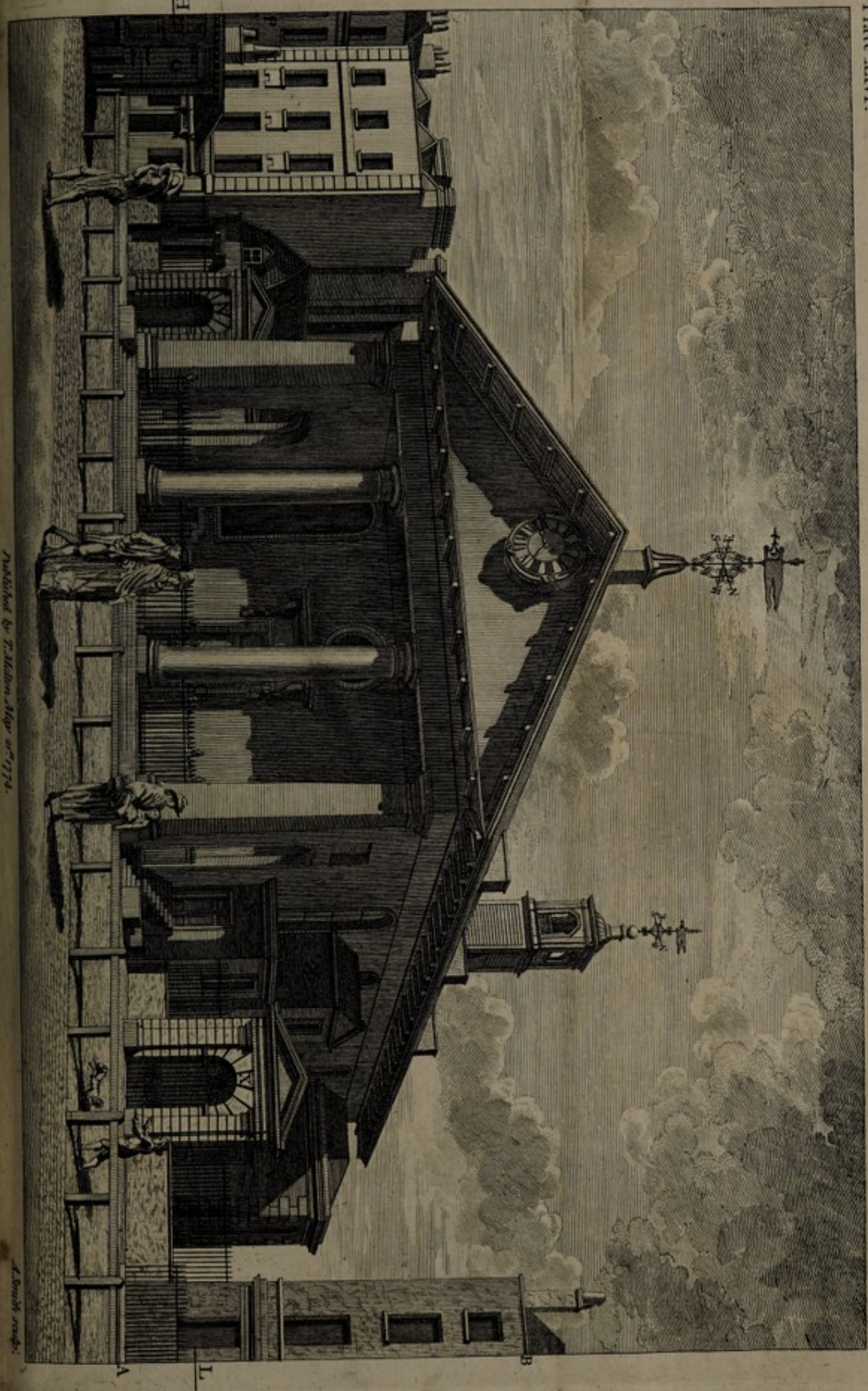






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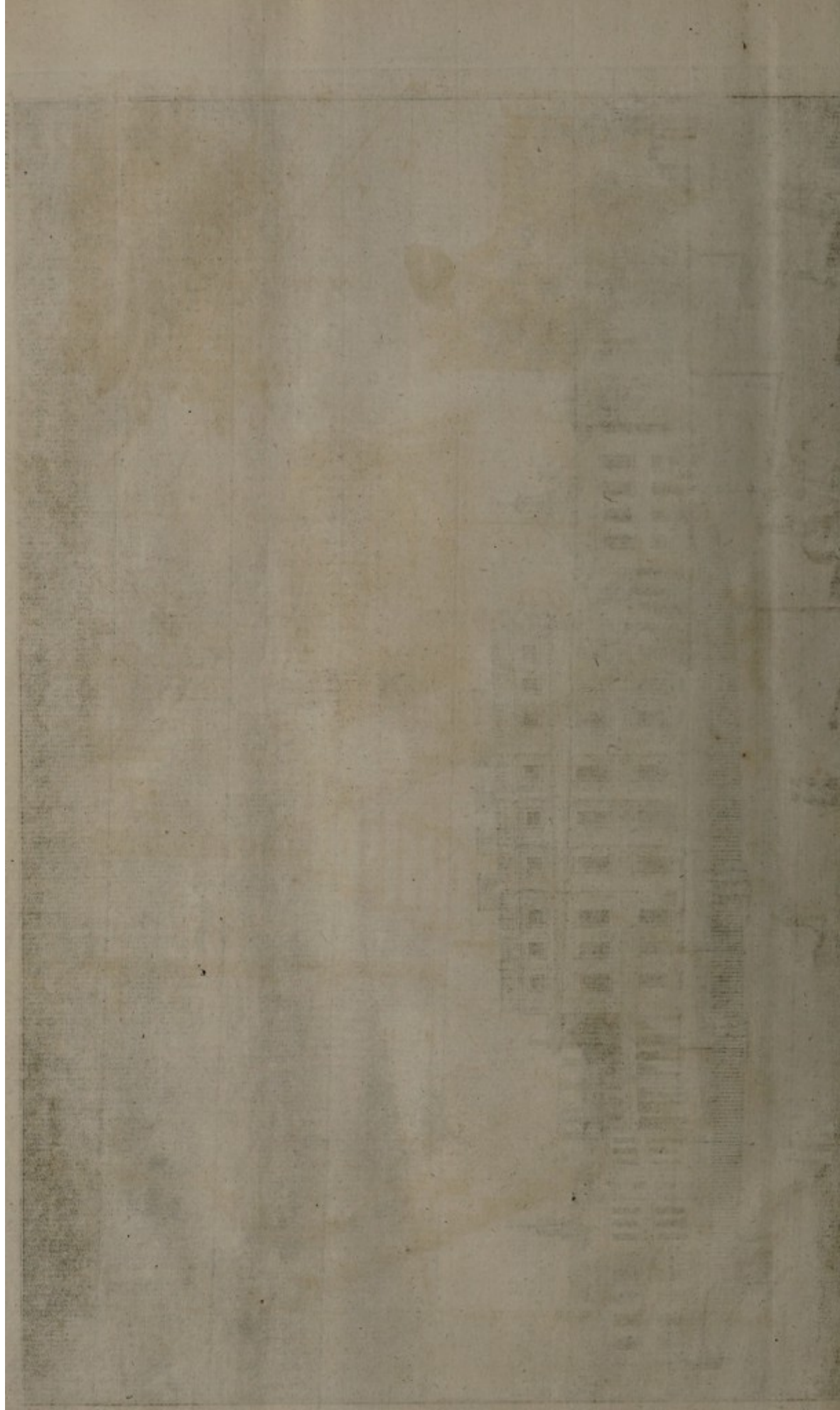
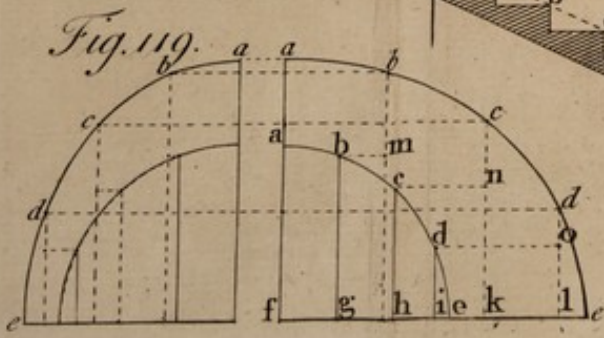
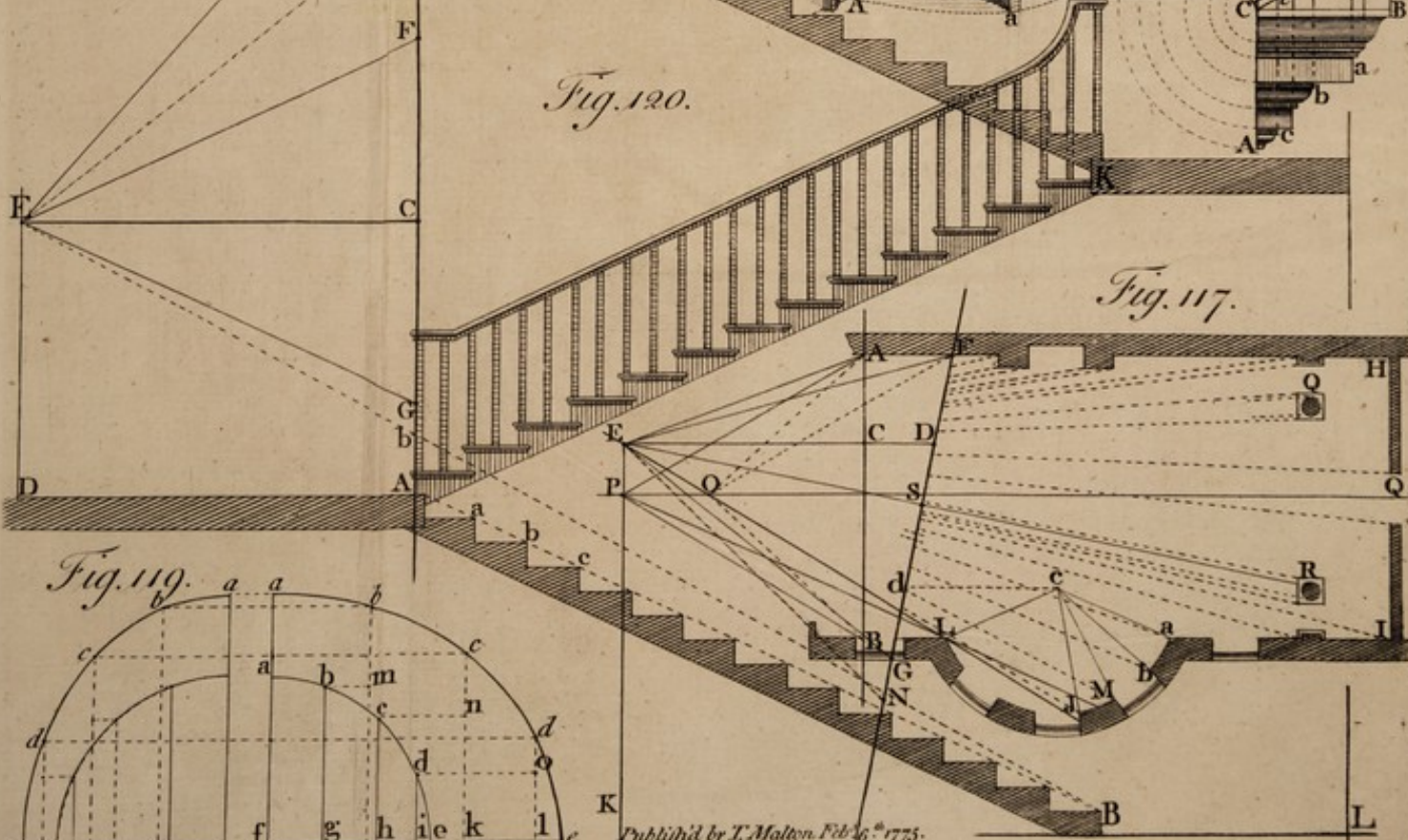
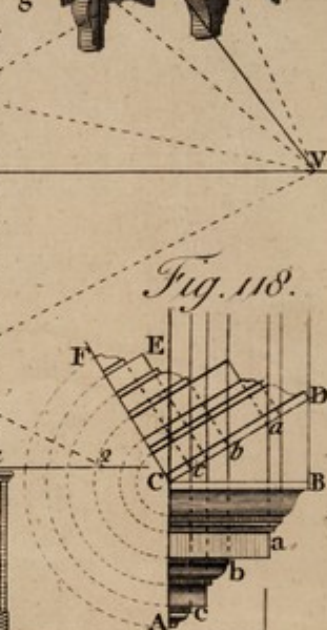
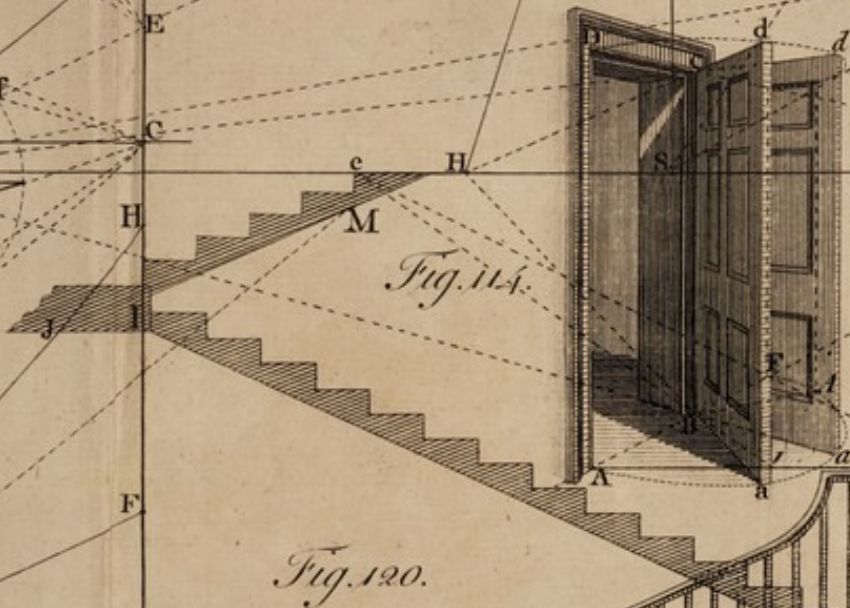
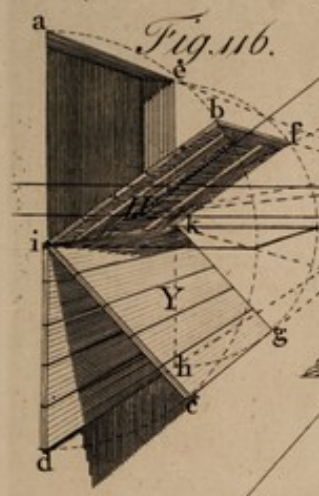
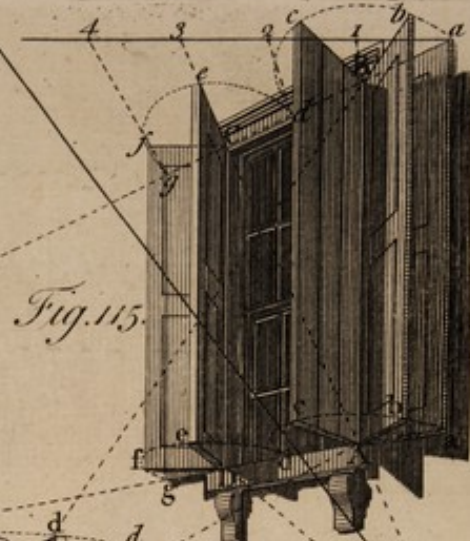
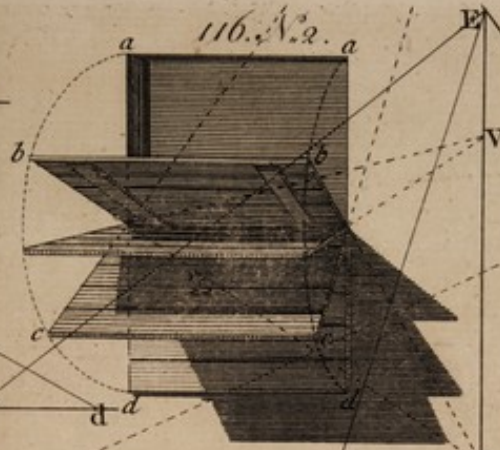
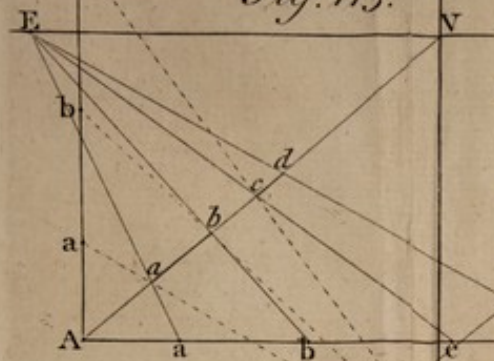
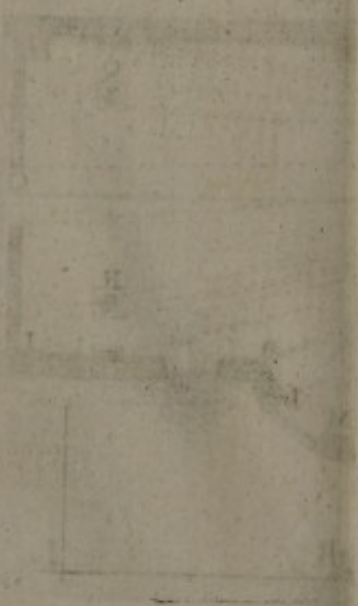


Plate XXIX.
Fig. 113.





No. 1. is the Plan of a Prison; on the left hand are Doors into several Cells, &c. and on the right are Doors and a Window, into the Area, or open Yard; at the farther end is a passage, at R, leading to the Keeper's Apartments, and, at the End is a Stair-case; a few Steps of which lead, through a Door, to the Courts of Justice. V, U, &c. are Piers supporting the Roof. The Elevation, and Sections, are supposed to be understood.

Plate
XXX.
Fig. 1.

Let AB be the place of the Picture; by which, a section of the Building is supposed to be made, and the hither Wall removed; so that, the whole of the Inside, beyond it, being open to view, is intended to be represented on the Picture AB, from some determined Station, as at S; as seen through the Picture, being supposed a transparent Plane. SP, SQ, &c. may be supposed Visual Rays cutting the Picture, which give the true place of each part of the Building thereon; indicating also, what Parts can be seen and which not, being obstructed by the Piers.

Let AB be produced to b, ab is the Ground Line of the Picture; but for the conveniency of making a correct perspective Plan, take AB at a proper distance, below; in which, let AB be taken equal AB (No. 1.) or in any other proportion to AB; it is, here, taken double, or twice.

Let the Horizontal Line (ECG) be drawn, at a proper distance from ab, equal to the height of the Eye above it; in which, take DC double AC (No. 1. where a perpendicular, from S, cuts AB) C is the Center of the Picture; also, make CE double SC (No. 1.) E is the Eye, or Point of Distance, by which the several parts of the Picture are proportioned; CD and CF are each half the Distance. HI, drawn through C, is the Vertical Line of the Picture; which determines the situation of the Eye, in respect of the Building.

Let the perspective Plan be drawn, making IA, IB, &c. each double the measures in the Plan, for the thickness of the Piers, and their distance from the Station Line. How it is perfected, it would be impertinent to explain; the Point E being used, where the measures Cb, Cc, &c. are the full measures (that is, double those at ab, bc, &c. in the Plan, No. 1.) and, when it is more convenient to apply the half measures, make use of the Point D, or F, on either side, as occasion requires; as AD, for the Door at P; the full measure being applied from A to b, gives the same Point p, as the half measure, at D.

AQ is equal to AQ (No. 1.) which, by drawing a Line to D, cuts AC at Q, the place of that Corner, in the perspective Plan; making AQ represent twice AQ (No. 1.) seen at the Distance CE; or equal AQ, at the Distance CD.

AC, or CC, being proportioned at b, c, d, &c. perspective, for the several Piers, &c. Lines parallel to AB, being drawn, determine those on the other side, at g, k, &c. and against the Wall, at n, o, &c. by which means the Plan is completed, in Perspective, from which the Picture is delineated, as follows.

The measure, on AB, being transfered to ab (that is, respecting the space between the Wall and the Piers, the thickness of the Piers, and distance between them, in front) draw aC, cC, &c. and, at A, draw AB perpendicular, the vertical Section of the plane of the Piers, on the right hand, with the Picture; which, being farthest from the Vertical Line, is best, for proportioning the heights of the Piers, Arches, &c. which are set off, from A, at a, b, c, &c. according to known, or determined, measures of their heights, in proportion to the Plan.

Draw AC, aC, bC, &c. and, from the Plan below, draw perpendiculars from g, i, &c. cutting them, at g, i, &c. Perpendiculars being drawn from all the other corresponding parts in the Plan, as l, m, n, o, &c. parallel Lines drawn through g, f, &c. determine the true places of all the Plinths, and Borders, at the tops of the Piers, from which the Arches spring.

The Arches, on each side the Avenue, in front, are Semicircles; wherefore, their Centers being obtained, at S, by bisecting the Diameter RT, they are readily described; and, by drawing Lines, from S, on each side, to C, the Centers of all the receding Arches are determined, in those Lines.

Plate
XXX.

In the returning Fronts, the Arches are semi-Ellipses; their heights are the same as the other, determined on AB , at d ; Cd being produced to the crown of the Arch, at D , obtained from below, at N , projected. For, seeing that the Picture, on AB (No. 1.) is nearer to the Piers than the Wall, consequently, it does not cut the Arches in the middle; wherefore, all the Wall, with the elliptical Window, on this side of AB , projects through the Picture and is projected again to it. C is the Center of that Arch; the Curve, Def , is determined after the same manner as if they were circular, it being obvious, that the same lengths, from i to k , or from f to l , between the Piers, may represent any length of the Indefinite Representation, AC , according to the distance of the Eye (Cor. 5. Th. 12.) AC remaining the same. Hence it is undeterminable whether those Arches represent Circles or Ellipses, of any proportion, the Distance of the Picture being unknown; for the two middle Arches are more excentric than the other two, seeing they have the same height, and the Piers are wider asunder*.

The elliptical Windows, over the Arches, are described after the same manner; as the first on the right hand, half a Window, inscribed in the Rectangle $abgb$, indicates sufficiently.

The groined Arches, over the side Avenues, have an appearance of difficulty without the reality. If a Line be drawn through the crown of the Arch, at c , to C , and, diagonal Lines being drawn between the corners of opposite Piers, in the perspective Plan, as rt and ms , intersecting at u , from which, a perpendicular, being drawn, cuts cC at e , the center of the Groin. Or those Diagonals may be drawn, at the tops of the Piers, and having drawn perpendicular Ordinates to the circular Arch, at A, B , they may be transferred, to the Diagonals; where, perpendiculars being drawn, and, to C , Lines drawn through the points a, b, c , &c. in the front Arch, cutting the diagonal Ordinates at a, b, c , &c. through which the Curve may be accurately described, by encreasing the number of the Ordinates at discretion.

The Arches in the middle Avenue are also semi-Ellipses, geometrically described, being parallel to the Picture; the transverse Diameter of the first is FG , and CD half the Conjugate. Having determined several Points, 1, 2, 3, at equal distances, or otherwise draw the Ordinates o_1, A_2, B_3 parallel to CD ; from which Points, A, B, C , &c. draw Lines to the Center, giving the Diameters of the other Arches, HI and KL , cut in the several Points a, b, c ; a, b, c ; at which Points, Ordinates being drawn, and, from 1, 2, 3, and D , Lines drawn to C cutting them at 1, 2, 3, through which those Curves may also be described.

The Apertures through these Arches, being Circles parallel to the Picture; their Centers being determined, and their Diameters, are only necessary.

The large Beam, at O , is in the middle of the opening, at MN , and tends to the Center, on which the Joists rest, which decline to the Wall on each side, geometrically; their measures and distance from each other being known, are readily determined, by Prob. 10, drawing a Line through B , parallel to MN .

The Stairs, at the far end, are managed by Example 8, in front; the returning flight being parallel to the Picture are inclined, geometrically, to the Horizon. The front Steps may be proportioned to the opening between the Piers; as at Y and Z , in the geometrical Plan; the Hand-rails, of which, tend to a Vanishing Point, in the Vertical Line, at H ; as the Kirbs in Example 8th. The Door is geometrically proportioned, on the Steps.

The middle Avenue of this Building, being loftier than the Sides, the Representation is limited by a Section of the Walls, and Timbers of the Roof, over the Sides, geo-

* At No. 2. is half a circular Arch, that is, a Quadrant, or fourth part of the circumference of a Circle, in a Square $abcd$; c is the Center. $abcd$ is a Rectangle circumscribing a fourth part of an Ellipsis, of the same proportion as the middle Arches to the circular ones at each side; in which, it is obvious that, the Diagonals, ac and ac , cut the Curves at the same height, at e and e ; and if the circular Curve be divided into more parts, at 1 and 2, those being transferred by parallel and perpendicular Ordinates, cut the Diagonals of both in the same proportion; and consequently they give corresponding Points in the Curve.

metrically

metrically drawn; the lower part is bounded by the limits of the Picture in length which, on account of the Station not being in a central Line, has its Center towards the left hand, out of the middle; by which means, it becomes distorted, on the right, being extended too far from the Center; by which the optic Angle is enlarged, and the Visual Rays cut the Picture too oblique, as SB in the Plan (No. 1.) in which case, no more should, properly, be taken into the Picture than the Piers, the optic Angle ASf, being about 50 Degrees.

E X A M P L E XLI.

Is the Inside of a Church, in the same position, as the foregoing Example.

At Figure 2, No. 1, is a half Plan of the Building, as far as the Section by the Picture, on AB. The Piers support the Roof as in the last Example, having Columns on them, of the Ionic Order, with an Entablature all around the middle part of the Building, and a coved Ceiling. At the far End is a semi-octagonal recess, in which is the Altar and communion Table.

It would be wholly superfluous to form a perspective Plan, as in the preceding Example, or to fix the Station, with all the other Preliminaries to the Ground Plan; suffice it to say, that the situation of the Eye is nearly the same in respect of the Building and Picture. C is the Center, and CE half the Distance by which the whole is proportioned; the full Distance cannot be contained within the Picture, on either side.

The places of the Piers, the Windows, &c. it is obvious, are obtained here, as in the foregoing, by applying the half measures (that is, the full measures of the Plan) to the Ground Line, AB, sufficiently correct, inasmuch that, an extra Plan would be unnecessary. The Piers are square, having a Sub-plinth with a Base Moulding, and Impost at the top; on which is a regular Pedestal, which forms the front of the Gallery, receding somewhat from the Pedestal; all which need no other description. The Picture is supposed to cut the Galleries, by which, the Seats, &c. are laid open to view; the Section is drawn geometrically, from which, all the Seats and Backs tend to the Center of the Picture.

The Columns, are supposed to be planed at the bottom of each Pier; but, if a central Line be drawn (as KL) from the middle point of each, and the height of the Column be there ascertained, all the rest are determined by Lines drawn to the Center, giving each its proper Diameter, in proportion to its height.

The Bases of the Columns, being above the Eye, are curved contrary to the usual order when below it; the Curves are still concave towards the Vanishing Line. The Capitals are according to the modern Ionic, in Example 29.

The recesses of the lower Windows, on the Floor, being less than the window Frames, cannot be a difficulty; the sloping part makes up the difference. The Windows above are perpendicular over the other; the heights of which are determined, on the vertical Intersections of the Walls, AD and BG; also the Gallery, and other parts of the Building.

The Nich at the far End, with the Doors and circular Windows over them, are all parallel to the Picture, consequently they are all geometrically proportioned, in front. The receding of the Nich, that is, the perspective widths of the Faces within, are best determined from a perspective Plan, formed below, the figure being determined; or, if the whole be drawn geometrically, and the Station fixed, as in the foregoing, Visual Rays may be drawn, cutting the Picture and giving their proportions thereon.

The Mouldings in the Impost, running around it, have their Vanishing Points, for two Faces, in the Points of Distance. The middle Face is parallel, and in one, seen on the right hand the Lines tend to the Center. The Compartments in the Sides and in the Head are best described by inspection; the Lines, forming the Head, are drawn after the same manner as in an octagonal Dome, seen externally, as described in the 35th Example.

The

Plate
XXX.

The semicircular Heads of the upper Windows have been frequently described; those below have a flat Arch, which is but little seen, by reason of the spherical recesses in the Cieling of the Gallery, which are represented by semi-Ellipses, in the Plane of the Cieling.

The Entablature, supported by the Columns, is formed by means of geometrical Sections, at X and Z; the true place being acquired from a Plan below; or from the Ground Line and vertical Intersections, at H and I.

The Profile of the Mouldings, in the Entablature, on one side, with the large Cove, and a smaller on the other side, being drawn, it would be impertinent to describe the delineation of every Line from each Angle to the Center of the Picture, and how to determine the mitre Angles at the End, where they return, over the Arch; for which, see Example 16, or 19.

For the large Cove, draw Lines from M and N, the internal Angles of the Building to C, cutting Perpendiculars from the Angles of the Frize, or the Pilasters, below the Capital, at O and P; from which, draw Lines to the Eye, at the full Distance on each side, and produce them; then, Lines drawn from Q and R, to C, will cut those Diagonals at S and T, and determine the returning Line or Moulding, which bounds the plane Cieling.

The mitre Angles of the Cove may be truly projected by means of Ordinates from various Points (as *a* and *b*) in the geometrical Curve; but, as it comes so small, at the place, it may be done correctly enough, without that process; taking care that it does not curve too much, but leads truly into the Perpendicular and Diagonal, both which are Tangents to the Curve, as RN and NZ, in the geometrical Section, and must never cut it.

The Compartments, in the Cieling, are but simple Figures, as Rectangles, with Ellipses inscribed, (see No. 1.) which are managed the same as Circles, as it has been shewn in the last Example, in respect of the Arches.

The recesses in the Cieling, over the Galleries, are no more than a repetition of the same thing by a less Scale, having only an Architrave below the Cove, which returns at every Beam from the Columns to the Wall; whose mitre Angles are determined by the same means, from a Profile of the Cove, at G.

Thus, without the real process, I have described every necessary step to be taken in the delineation of this Object; for, as the last is, in respect of its Plan, the same kind of Figure, the same kind of Process will answer for both, in the general parts; and, for particular ones, the Examples referred to give particular Instructions.

E X A M P L E XLII.

Is the representation of an elegant Room, having a large, circular, Bow Window, in the Side, and a Cove-Cieling, inclined to the Picture.

In the two former Examples, the center of View, not being in the middle of the Picture, it may be imagined is wholly owing to the Station being towards one Side more than the other; but I shall shew, here, that it is not; for, let the Station be taken where you please, the Point of View, or rather the Center of the Picture may still be in the middle, and yet, take into the Picture as much of the Object; a circumstance extremely simple in itself, though to some it may appear a mystery.

In Plate 29, Fig. 117, is a Plan of the Room intended to be delineated. Let E be the Station determined on; from which, if Right Lines are drawn to F and B, the extreme on each Side of the Room, intended to be delineated, FEB is the Optic Angle under which it is seen; and, if AB be the Intersection of the Picture, EC, perpendicular to AB, determines its Center, at C, and EC is its Distance; in which case, the Center of View is not in the middle of the Picture. But, if the Optic Angle be bisected by the Right Line ER, then, FG, perpendicular to ER, is the true Position of the Picture, S is its Center, and ES its Distance; and the farther End (HI) is consequently inclined to the Picture; as it is represented in the 31st Plate. See Figure the first.

Let AB be the Ground Line, which is not necessarily the bottom of the Picture; but, whatever falls on this side is supposed projected to the Picture.

Draw AD and BK, perpendicular (the vertical Intersections of the Sides) on which, set up all the measures of the heights (by the Scale of Proportion) for the Dado and Mouldings, the Windows, Chimney Piece, the Entablature, Cove, &c.

On AD, describe, geometrically, the true Profile-Section of all the Mouldings; as DEF for the Cornice, &c. the Lines of which (in this Case) not being perpendicular to the Picture, the Section of it, with the Picture, is not the true Profile; the deviation, in this, is inconsiderable. But, when the Lines are more inclined to the Picture, an Expedient for truly proportioning the Mouldings may be necessary.

Let ABC (Fig. 118, Pl. 29) be the Profile of the Cornice to be represented in the Picture.

The section with the Picture, being vertical, makes no variation in the heights of the Mouldings; but, according to the inclination of the Picture, to the Building, their projectures are varied considerably.

From each projecture a, b, c, &c. draw Lines perpendicular to BC, consequently parallel amongst themselves; make the Angle BCD equal to the complement of the inclination of the Cornice to the Picture; CD will be the extreme projecture of the section of the Cornice, whose Inclination to the Picture is ECD; and, FDC is the true Section, in that position of the Picture; the projecture of each Moulding being taken from CD, where the parallel Lines cut it, at a, b, c, &c.

On the other side, the heights of the Windows, &c. are set up from B to I.

Having drawn the Horizontal Line (VL) and fixed the Center of the Picture, the Vanishing Point V being determined as usual; which, on account of the inclination of the Sides of the Room, is not in the Center, as is customary in Inside Views; the Room, or Building of any kind, being right angled.

In this Case, these Preliminaries are best determined by the geometrical Plan.

If the Ground Plan of any Building which we intend to delineate be drawn, truly geometrical, the Station may be determined, so, as to see such parts of the Objects as we require, whether internal or external.

If it be required to see part of the second window in the Bow; draw a Right Line, from J, through the Angle at L, allowing so much of the recess of the Window, &c. as you wish to represent. It is manifest, that the Station must be somewhere in that Right Line, produced; but if the View was to be central, it would be too near, as at O, for, the Optic Angle, AOB, is too large for the Eye to take in, at one View.

Fig. 117.

At a further Distance, in the same central Line, at P, if PL be drawn, it is plain; that the Window cannot be seen at all, as it cuts the Pier at M; whereas, any where in the direction of JL, as at E, if the Station be fixed so, that, drawing EF and EB, the Optic Angle does not exceed 50 Degrees, it may be represented from that Station, without much distortion. Then, making EG equal EF, FG is the position of the Picture; and ES, bisecting the Angle FEG, is its Distance; wherefore S is its Center (Def. E) for ES is perpendicular to FG†, and SG is equal to SF, consequently it is in the middle of the Picture.

† 9. 2. El.

Now, E, is the Station, or Point of View, and FG is the position of the Picture.

Draw ED parallel to AH and BI; consequently, D, and not S, is the Vanishing Point of those Sides. (Def. L.)

The End (HI) vanishes in a Point where EK, parallel to HI, would cut the Picture, FG, produced; its Distance, from the Center, S, will be to SE, as SE is to SD†, (Prob. 12.) And, if the Angle DEK be bisected, by the Line EN, N is the Vanishing Point of a Diagonal, or mitre Angle.

† 7. 6. El.

Then, if AB (Fig. 1, Pl. 31) be equal to FG, make VC equal to DS; or, in whatever ratio AB (Fig. 1) is to FG, so make VC to DS, and V will be the Vanishing Point of the Sides of the Room; that is, of all the horizontal Lines in those Sides, and of all other Lines parallel to them, whether in the Ceiling, or on the Floor, or elsewhere. (Cor. 1, Theorem 3.)

Being thus prepared, we now proceed to the delineation.

Plate
XXXI.
Fig. 1.

Draw the indefinite Lines AV and BV, and proportion them, by setting off the measures of the several parts from A or B, and drawing Lines to the Distance Point of the Eye, placed on either Side (at E, or E') EV being half, and VE' one third part of the Distance of the Vanishing Point V, there not being room, on the Picture, for the full measure.

Make Aa, ab, &c. each one third of the measures of the Originals, and draw aE', &c. cutting AV at a, &c. which give the Angle of the Chimney, &c.

For its projecture, make Aa equal to, or somewhat more than the real measure (because, AB being inclined, it cannot be equal, but is the Diagonal of the Inclination; i. e. as CD to CB, Fig. 118) and draw aV, cutting a Line drawn from a to the other Vanishing Point, at b. Make ad in the ratio of one third part of the front of the Chimney (as CD to CB, 118) and draw dV, cutting a parallel Line from b at c. Divide bc in the proportion of the Trusses, &c. and draw Lines to E', cutting bV in their perspective Proportions.

Draw perpendiculars from a, b, and c; and, from all the Angles of the Cornice, &c. draw Lines to V; and return the Mouldings, at the several Angles of the Chimney, internally, at gb, and externally, at ik and lm, as described in Example 19; also, at no, the length being obtained by a Perpendicular from f.

To describe every particular would be superfluous and unnecessary, previous Lessons having been frequently and particularly explained.

Having obtained the mitre Angle at no, return the Mouldings of the Entablature, at the End, by means of the Vanishing Point of the End; or by Prob. 13.

The Angle of the Cove may be obtained, with accuracy, having made a geometrical Section, as in the last Example, setting up its height from D, and making its projecture as CD to CB (Fig. 118) in respect of the true projecture; then, draw a Line from the internal Angle to V, cutting the Perpendicular fn at p, and draw the Diagonal (pq) of the Angle, indefinite; and, from the extreme projecture of the Cove, draw a Line to V, cutting it at q.

But, because the Vanishing Point of that Diagonal is not determined, and is out of the Picture; from p, draw a Line tending to the Vanishing Point of the End; and, where it is cut, at r, make rs equal pr, and draw a Line from the Vanishing Point of the other Diagonal; which will give the Point q nearly*.

The other Angle, x, is determined by the Vanishing Point of the Diagonal xy†.

After the same manner as in the foregoing, viz. by drawing several Ordinates to the Curve, at t, u, &c. the perspective Curve at the Angles may be described, and transfered from one Angle to the other.

On the other Side, VE being half the distance of the Eye, the measures on the Ground Line, AB, are applied half the real measures, for proportioning the Piers, and the opening of the Bow Window, &c. from B, to e, f, &c. BI is the height of the Windows. Draw Ih tending to the Vanishing Point of the End of the Room (as for the projecture of the Chimney) and Bd being made equal to the recess of the Window, draw dh; regarding the projectures of the Mouldings.

Having obtained the opening of the Bow Window at ik, which is a Segment of a Circle (by drawing eE and fE, cutting BV) find S the representation of its Center (bisecting ik perspective, at z, and drawing Szn to the Vanishing Point of the End, and zV, cutting it at S; Bz being somewhat more than its Distance from the Wall, viz. as CD to CB, Fig. 118) and draw Si, and Sk, which represent Radii; by means of which, other Radii may be obtained, as Sl, Sm, &c. making certain Angles with Sk perspective (Prob. 10, Case 3rd.) and thus, not only the Curve (klmn) but also, the true place of each Window, &c. is acquired.

The Curve at the Top may be described by the same means.

If the Lines, on the Floor, in the receding of the Windows, tend to the Center S, draw SR perpendicular, and divide it, geometrically, into the several di-

* This is not strictly true, prs not being parallel to the Vanishing Line; but, being so little inclined, the deviation is inconsiderable. rs is less than pr; they may be truly proportioned by Prob. 8, Cor. 1.

† This Vanishing Point is also out of the Picture, being distant from the Center as SN to SE (Fig. 117.) the Distance of the Picture.

vifions for the Mouldings, &c. and producing Sl, &c. to the Vanishing Line; then, Lines drawn from each division, on RS, to the respective Vanishing Point of each Jamb, will divide each Window into its feveral divisions of Mouldings, Saff Squares, &c. and, by the fame means, all the Curves, in the Mouldings, &c. at the top, may be described with the greateft accuracy.

If the Jambs of each Window be parallel between themfelves, then, a Right Line, So, bifecting the Angle lSm, &c. perfpetively, will produce the Vanishing Point of each refpectively, and each Jamb muft, in that Cafe, be divided geometrically, on the Angle, into the feveral divisions required.

The Entablature may alfo be proportioned on BK, the fame as on AD, on the other Side; which will be more accurate, than to depend entirely on the proportions being carried around, from the other Side; which, as they diminish fo much, at the far End, are liable to error.

The places and proportion of the Columns, may be obtained by Prob. 8th, thus.

The Angle f at the foot of the Pilafter being obtained, draw fj parallel to the Ground Line, which divide, geometrically, in the Proportion required; that is, $f1$, equal $j4$, is the diftance of the Column, at the Plinth, from the Wall; and 12 , equal 34 , is the width of the Plinth of the Column; from all which, draw Lines to V , cutting the inclined Line, in which the Columns ftand, in the perfpetive proportions of the Columns and Spaces; which may be compleated from the known proportions of the Order, as in former Examples.

If rs be taken, geometrically, the width of the Door, &c. in proportion to the Columns and fpaces, Lines drawn to V will give the place of the Door, at the farther End, which may be compleated, by Example 20.

The Cieling has nothing of difficulty in it, fave the Ornament, being compofed of Right Lines, regularly difpofed; which, from the geometrical Figure, may eafily be determined. GH , may be confidered and ufed as the Interfection of the Picture with the Cieling, on which, the divisions are geometrically difpofed; from which, draw Lines to V ; and the feveral divisions, in the length, being found perfpetively, on GV or HV , Lines drawn to the other Vanishing Point will cut each Line, drawn to V , in the ratio required; by which, the perfpetive Figure is formed on the Picture.

E X A M P L E XLIII.

Is the Representation of the infide of the Piazza, Covent Garden, from the farther corner of the entrance into the Playhoufe.

The Pofition of the Picture being determined, and confequently, the Inclination of the Lines, in which the Piers ftand, is known.

Let C be the Center of the Picture, EV is the Horizontal Line, and V is the Vanishing Point of one Face of the Piers, found, or determined at pleafure; the other is out of the Picture, on the Left, found as ufual; the Difftance of the Picture is fix Inches and a half, nearly*.

Let AB be the Interfection of the Picture, i.e. the Ground Line, and, let S be the determined Seat (on the Picture) of the corner of the Pier, on the Ground, its Difftance from the Picture being known.

Because there is not room, on the Picture, to fet off its whole Difftance, take CE half the Difftance; and make SD half the difftance of the corner of the Pier; draw DE cutting SC at a , the true place of that Corner, on the Picture.

Draw a V , the indefinite Representation of one Side of the Piers, and a Y , to the other Vanishing Point, whole Difftance, from C , is nine inches and three fourths,

Fig. 2.

* Let it be obferved, that the Difftance, here ufed, is too little; but, being a true Portrait of fo public a Place, I thought proper to difpenfe with it; by representing it as it appears, from the Station determined above. See Preliminary Obfervations, Page 111.

Draw

Plate
XXXI.

Draw ad parallel to AB ; and having transfered the full measure of the Piers and Arches from AB , the Ground Line, to ad (by means of the Point C , or any other in the Vanishing Line) at b , c , and d , draw Lines to E , the Distance Point, of V , giving their places, at ab , and cd , &c. the rest may be obtained to any length, required, by Example 4th. Note, ad may be the Ground Line, by a less Scale.

On the returning Angle, they are obtained by the same measures, applied on the other Side, by the Point F the distance Point of Y .

Produce Va to the Ground Line, cutting it at A , and draw AH perpendicular, on which, set up, from A , the measures of the heights of the Piers, &c. at G and I ; from which, draw Lines to V . Perpendiculars, from a , b , c , &c. cutting them at e , f , g , &c. give the Piers, on that side.

From, ae , the common Interfection, they are returned on the other Side, to the Left.

The circular Arches are all constructed by Exam. 21, as the one on the Left; and the Curves over them, by means of Ordinates ab , cd , &c. their heights being set off from e to i and k , &c. the true Curve being determined (see Fig. 119) and as many Ordinates drawn as are requisite, they are returned on both Sides.

The elliptic Arches and Borders, from the Piers to the Wall may be thus described.

Produce Ya and Ye , and having found the Point J (Prob. 8.) in the middle of the whole width; from J draw a Perpendicular, cutting Ye and Yk , produced, at l and m ; m is the middle Point in the Crown of the Arch.

Draw lV and mV ; and, from the Vanishing Point Y , produce Lines through g , j , &c. from each Pier at the foot of the Arches, cutting lV at n and p , &c. from which, draw Perpendiculars, cutting mV at o and q , &c. the height of each Arch, respectively; as lm , no , and pq .

Then, by means of several Ordinates, as ab , cd , &c. perspectively found, on the Base Line of the Arch, at a , c , e , &c. (see Fig. 119, Pl. 29, for the geometrical construction) their several heights are set off on ek , and projected, by the Vanishing Point Y , cutting Perpendiculars from a , c , e ; by which means, half the Curve of the first Arch is described, as $ebdfm$. The other half is determined by the same means, and all the other Arches after the same manner, by drawing Lines from the Seats, a , c , e , &c. of the Ordinates, to V , cutting the Base Line of each Arch, as gn , at 1, 2, 3, &c.; from which, Ordinates being drawn, and Right Lines from b , d , f , &c. to V , cutting them at 4, 5, 6, &c. then, through their Intersections, the Curve $gsovu$ is described.

Having obtained all the Arches, or Borders, the Groins are next to be determined. The middle Points, r , s , t , &c. are in the Line mV (allowing somewhat for the thickness of the Border) and by means of horizontal Lines, whose Vanishing Point is V , drawn from b , d , f , &c. their lengths, from those Points, as bg , db , fi , &c. may be determined, by the Point E , as in all other similar Cases whatever. Or, perhaps more readily, by perpendicular Lines from the Diagonals, fu and gx , (at the foot of the Arch) where they are cut by Lines drawn from a , c , and e , to V , cutting the former at g , b , and i , respectively.

The Plan of the Plinth of the Column, being determined, from its known measure and place (by its Seat on the Ground Line, or otherwise) there is nothing singular in its construction; save the Blocks or Rustics (W) which are each equal to a Diameter, in height; the space between is the same; they project equal to the Plinth of the Base. Y and V are the Vanishing Points of the horizontal Lines, in the Cornice, &c. The Picture is supposed to cut the Column, consequently its full measures are applied, on BL or somewhat more, seeing it is projected.

The Piazza, on the right-hand, is seen to the End; and, through the Arch, at K , is seen the front of the next, on the other side of James Street.

The distant view of the Church and adjacent Buildings, being so little seen, are best sketched from the place; as it would be attended with unnecessary trouble to find their places, &c. perspectively, from their true geometrical proportions and positions. The Rustics, in the Piers, are a Scale for proportioning them.

E X A M P L E XLIV.

Is the Representation of a Staircase, internal; shewing the descending Stairs, direct.

Plate
XXXII.

This Lesson, is intended, not merely as an Inside View, but in order to shew that a descent may be represented by the Rules of Perspective, on the same Principles as any other Subject, whatever; in which I have reduced to practice, what was heretofore treated on (Art. 8. Sect. 6. B. 1. P. 97) respecting a down-hill Representation.

In this Example the Window Side is parallel to the Picture, and consequently, the Sides of the Staircase are perpendicular to it; therefore, the Center of the Picture is the Vanishing Point of horizontal Lines, in the Sides.

C is the Center, and HL the Horizontal Vanishing Line; and VL, the Vertical Line, is the Vanishing Line of the Sides of the Staircase.

The Vanishing Lines of the ascending and descending Planes, i. e. of the Stairs, are determined by Prob. 2, making CE equal to the Distance of the Picture, and the Angle CEF equal to the inclination of the Stairs to the Horizon. Make CG equal CF, and through F and G, draw Lines parallel to the Horizon; one is the Vanishing Line of the ascent, the other of the descent; and because they are, in this Case, parallel, the Vertical Line, VL, is common to them both, and consequently, F and G are the Centers of those Vanishing Lines, respectively, and their Distance is EF, equal EG. (Theo. 7, and Def. 20.)

† Theo. 7.
Cor. 2.

As this is a circumstance which has occasioned some disputes amongst Artists, I shall display it in the best light I can, and doubt not I shall do it satisfactorily.

In order to which, let Fig. 120 represent a section of the Staircase, geometrical, by half the Scale of the Picture; AB is the descent, from the Landing, at AD, on which the Spectator is supposed to stand, at ED; E is the Eye or Point of View, and EC is the Distance of the Picture*, of which AH is a section. AK is the flight of Stairs, immediately ascending, as AB is descending, and KI is the under side of the next flight, over them, parallel to AB.

Fig. 120†

Now, because the Vanishing Point of every original Right Line is where a parallel from the Eye cuts the Picture‡, and EC the Direct Radial, produces the Center, C is the Vanishing Point of the sides of the Half pace BL, and of the extremes of the Steps, as may be seen in the Picture; also, EF, and EG, being parallel respectively to the Ascent, AK, and Descent, AB, consequently, F and G are the Vanishing Points of all Lines, on the Picture, parallel to them (as the Hand-Rails, the Wainscoting at the Sides, &c.) and consequently, Planes passing through the Eye and those Lines, respectively, parallel to the Stairs, must cut the Picture in the Lines IK and MN passing through F and G, respectively, and therefore they are their Vanishing Line.. (Def. G.)

† Def. 22‡

This I presume is intelligible and clear; and, if Visual Rays are drawn to the several parts of the Staircase, they will determine what can be seen and what cannot; as EB determines how much the Descent rises on the Picture, which it cuts at b; Ab is, therefore, its whole apparent width. The ascending Flight, parallel to AK, it is evident, is seen on the under side, as the Visual Ray EM evinces; and consequently, it descends on the Picture.

Let AB be the Intersection of the Picture with the landing of the Stairs, which, in this Case, is the Ground Line, the Picture being supposed close to the Stairs; all that lies on this side is projected to the Picture; as the Door on the Landing, and Ceiling, as well as the Floor; and it is obvious, that, whatever appears to descend, below the Landing, has its place on the Picture above the Ground Line; consequently, it rises on the Picture.

Make BD equal to the known width of the Stairs, the width of the whole, being represented by AB; the Station is determined by the Vertical Line, VL.

* The Distance of the Picture, it must be observed, is too little, for taking in so much as is here represented; but it is manifest, that, if the Distance was half as much more, or one third lower, we should not see the descending Stairs at all, as the Eye would be either in the Plane of the Stairs, or on the other Side.

Plate
XXXII.

Draw BG and DG, to the center of the Vanishing Line MN; the Originals of those Lines being perpendicular to the Intersection, AB; in which Lines are all the upper edges, or nosings of the Steps; whose places are obtained by Prob. 8th.

Make GE equal EG,* the Distance of the Vanishing Line MN, or IK (Def. 20) Make Ba, ab, &c. double Aa, ab, &c. (Fig. 120) and draw aE, bE, &c. cutting BG at a, b, c, &c. giving their places on the Picture; and parallel Lines, ad, &c. being drawn, give the edges of the Steps; the Ends, being at right angles with the Pictures, vanish in its Center; therefore, draw Ca, Cb, &c. and produce them to the next Step; and thus the descending Stairs are completed.

The Half paces, below, at X, and above, at Y, being horizontal; their Sides, consequently, vanish in the Center; also, the ends of all the Steps, every where. The width of the Half pace being equal to the Steps, draw eC and fE, cutting eC at g, which gives the Angle gh, at the farther end of the Staircase.

The height may be determined on a Perpendicular from B, as BJ, and drawing JC.

The ascending Flight is managed by Example 9th in every respect. The Vanishing Point of the Hand Rails, Wainscoting, &c. of that Flight is at F, as well as of the Lines de and fg, of the Plane defg, being parallel to those below; the other, with the Lines in the Plane Z, is at G.

The height of the Hand Rails, &c. are set up on the Intersection BJ, as at h; or from D, as Di, allowing for the ramping Curve, equal ik.

The panneling of the Wainscot is divided in the ratio required, as in any other Case, by Prob. 8th, making use of the Distance EG (not EC) for the inclined part.

The Windows may be proportioned on gh, at the farther end, geometrical.

The Cornices, and other Mouldings, are managed by the Lessons in Sect. 7.

The Architrave around the Door, being projected, is thus done.

Make Bl, lm, equal, respectively, to its distance from the Steps, and width of the Architrave, and draw El, Em, projecting those measures to CB, produced, cutting it at i and k, and giving ik for the Architrave.

The Cieling is projected in the same manner; and all beyond BP.

AOPB is the Section of the whole by the Picture: OP is the Intersection with the Cieling, and AO and BP with the sides of the Staircase; on which Intersections, all the measures are applied, for each Plane respectively.

OF HORIZONTAL PICTURES.

Horizontal Pieces are considered, by many, as performed by Rules different from vertical Pictures; whereas, they are the same in every respect, being founded on an universal Theory, which regards no particular position of the Picture; but only, the positions of the Planes and Lines in Objects, to the Picture, and to the Eye. For, however the Picture be situated, the Eye being considered as a Point, it must either be in the Plane or out of it, and cannot vary in its position, otherwise; consequently, its Center and Distance, being determined by a perpendicular from the Eye to the Plane, are the same in all positions of the Plane, respecting the Horizon, to which we are naturally partial; but, being properly considered, we shall find it of no consequence in the Theory of Perspective.

Let the Picture be situated as it may, all Original Lines perpendicular to the Picture vanish in its Center (Cor. to Th. 4.) Planes, or Lines, being parallel to the Picture, have no Vanishing Line, or Vanishing Point; but, in such Case, the representations of all Figures, in Objects so situated, are similar to the Originals (Theo. 9, Cor. 5) also, all Original Lines, however inclined to the Picture, vanish in that Point, where a parallel Line from the Eye would cut the Picture; and consequently, all Lines, which are parallel, amongst themselves, have the same Vanishing Point (Th. 3, Cor. 1.) with every other circumstance in the Theory given, which is quite general, and cannot possibly regard any particular position of the Picture, to the Horizon.

* Suppose GN produced and made equal to GE.

The Representations of most Objects (in common Cases) on a vertical Picture, are managed by means of the Horizontal Vanishing Line, only, but, in many Cases, other Vanishing Lines are requisite.

A horizontal Plane, passing through the Eye, which, in other Cases, produces the horizontal Vanishing Line, is, in this Case, parallel to the Picture, and consequently cannot cut it; for it is the Directing Plane, of Pictures in such position. (Def. 4.) The theoretic construction of the elementary Planes is the same, however the Picture be situated; all the difficulty seems to be in fixing the Vanishing Lines, in this Case, having no particular bias in respect of the Horizon.

As, in all common Cases, the Horizontal Line and Vertical Line cut each other, at right angles, in the Center; so, in this, two Lines cutting each other, perpendicularly, in the Center, answer the same purpose, either of which may be considered as the Horizontal Vanishing Line; for, if the Picture be changed to a vertical position, and, the Objects being supposed changed with it, keeping their position to the Picture, it is then a common Case, in every circumstance; the placing of those Lines, most conveniently, is therefore the principal business.

Now, if the Piece be viewed centrally, it is immaterial how they are drawn, for representing a Dome only; but, if we would represent the Inside of any prismatic Object, whatever, it is most eligible to draw the Vanishing Lines parallel to its Sides, as AB and DE; because they are, then, the Vanishing Lines of those Planes, respectively. But, if the View be not central, then, one of the Lines (DE) generally passes through the center (C) of its Base and divides the Dome equally; which may be considered as the Vertical Line of the Picture; the other (FG) as the Horizontal Line; S being the Center of the Picture, whose Distance is known.

Fig. 121.

The chief difficulty, in Cieling pieces, is the representing Objects in such uncommon Positions, they being seen on the under sides, instead of looking direct at them, as standing before us, upright, in common Cases; and are represented as if lying horizontal, in respect of a vertical or upright Picture; in which, the difference consists, wholly. The Cielings, Soffits of Doors and Windows, Planes of Cornices, &c. are, in this Case, parallel to the Picture, as, in other Cases, they are perpendicular to it; being thus reversed, and seldom practised, makes the difficulty greater.

There is one particular circumstance attending Cieling pieces; the Center of the Picture is the only Vanishing Point, generally; not owing to the position of the Picture, but of the Objects to be represented. For the Base of the Dome, or whatever else is represented, is always parallel to the Picture; and consequently, all Lines perpendicular to the Base (being also perpendicular to the Picture) vanish in its Center; and, since the Objects to be represented are always upright, there is no occasion for any other Vanishing Point; as, all Right Lines, in the Objects represented, are either parallel or perpendicular to the Picture. Nor can it be otherwise, unless represented on an inclined Cieling; in which Case, there may be as much variety of Vanishing Lines and Points, as in any other.

These Preliminaries being well considered and digested, the difficulty will vanish, and leave the whole, subject to the same common Rules already laid down; for there are no other used, or necessary.

E X A M P L E XLV.

How to represent on a Cieling, or horizontal Picture, the representation of a Gallery of Communication; with a Dome and Cupola, as seen from below.

ABCD is a geometrical half Plan of the Gallery, &c. to be represented, which is a Square, suppose of 30 feet; and BFHC is a Section of the same. The height, BF, to the Cove, is 18 feet, the Cove 3 feet, and the Base of the Dome (which is a Hemisphere) is 24 feet. Through the Arches, in the middle of three Sides, Galleries are supposed to communicate to various Apartments, above.

Fig. 122.

The

Plate
XXII.
Fig. 123.

The Base of the Gallery being a Square, and the Picture horizontal, consequently parallel to its Base, its representation is therefore a Square. Describe a Square, AHIB, of the dimensions you intend the Picture, which is here, double that of the Plan.

The Station being determined, and the Distance of the Picture, which, in this Case, cannot admit of much variation, from the true, to any Eye which views it. For, I suppose, every Cieling Piece is calculated to be seen from some particular Station, below; and every Person who would see the Piece, truly, must stand in the same place; consequently, the difference, in Distance, is equal to the different heights of the Spectators.

Suppose the Hall, or Saloon, to be a Cube of 30 feet; then, allowing 5 feet for the height of the Eye, its Distance from the Cieling is 25 feet, which is the Distance of the Picture; upon which, is to be represented, a continuation of the Walls, &c. with the Entablature, from B to F, near 20 feet, with the Cove, Dome, and Cupola, covering the whole.

Now, the Distance of the Picture is 25 feet, which is not equal to its width, but cannot, here, be more, unless the Spectator be supposed to sit or lie down. Then, as, in common Cafes, the height of the Eye is next to be determined, so, in this, the place of the Eye in respect of the Picture; which, I would never advise to be in the middle of the Piece, as it cannot produce an agreeable Representation; seeing that, the longitudinal Lines in the Dome (which in that Case are represented by Right Lines) will be but so many Radii of a Circle tending to its Center, as aC, bC, &c. (Fig. 124) and, the latitudinal Circles will be so many concentric Circles, as abc, abc, &c. also, every Side of the Room will advance equally towards the Center.

Therefore, let the Station be determined somewhat towards one Side, where the Cieling may be seen most advantageously; nor is it necessary to be situated centrally, in that Side; although it is a deviation from my general Maxim, of fixing the Center of View in the middle of the Picture; but, since the position of the Picture cannot, in this Case, be varied, I had rather dispense with it than have duplicate Representations.

Let C, then, be the Center of View; somewhat to the right hand. Let AB be considered as the Ground Line; and if, through C, a Right Line, EF, be drawn, parallel to AB, it may be considered as the Horizontal Line of the Picture; and DG, perpendicular to AB, also passing through C, is the Vertical Line, i. e. they are the Vanishing Lines of the Sides of the square Room respectively.

The Galleries, on three Sides, which are supported by Trusses from the Walls, are first to be represented; which I have made to project but three feet, as they would hide the Walls too much if they projected more.

Now, since the Picture is parallel to the Trusses, and they are spaced equally, divide the three Sides AH, HI, and BI, geometrically, into as many spaces and Trusses as are to be represented, at X, X, and describe their Seats on the Picture, truly geometrical. Then, the Lines which measure their thickness or height, being perpendicular to their Bases, and to the Picture, vanish in its Center, as in all common Cafes whatever.

Make CE equal to the Distance of the Picture, in the Vanishing Line EF; draw the indefinite Representations, aC, bC, &c. and proportion them in height, making aa equal to the height, and draw aE, cutting aC at b; which, by drawing parallel Lines, to AH, HI, &c. will determine them all around.

The Cieling of the Gallery, between each Truss, is consequently determined. The under side of the Truss is like an Ionic Modillion, having, also, a Cimma reversa, around the top; which, the Figure sufficiently describes.

The Cornice of the Gallery is described as any other; save only, that the Fascias, or upright Fillets, are contracted, in this Case, as the under Sides, or Plancheers, in common; the difference is owing to the position of the Picture only, the appearance is the same; as it is illustrated in Fig. 125.

Let

Let AB be the Profile of a Cornice, seen from the Point E , by means of the Visual Rays $AE, BE, \&c.$ If the Rays are cut by a vertical Plane (Ab) the representations of the several Angles, at $C, D, \&c.$ are where the Rays are cut by the Plane, at $c, d, \&c.$ and a horizontal Plane, Ba , cuts the same Rays at $a, b, d, \&c.$ It is manifest, that the proportions of the Members, on those Planes or Pictures, differ considerably, yet it is obvious, that each appears the same, in the Point of View, E ; the difference, in the Representations, arising from the different Sections of the Rays, as in any other Case.

For the iron railing which is in the plane of the Facia of the Cornice, from each internal Angle, k , draw kC . Make Ad equal to its width, and de equal to its known height, from the Cieling (equal twice de , Fig. 122) and draw eE , cutting dC at f ; draw fg parallel to AH , and gh parallel to HI ; also fi , parallel to AB , cutting eC , where it joins the Wall, on the other Side.

The divisions and proportions of the Railing are determined, geometrically, on dk^* , &c. at $c, d, \&c.$ from which, Lines are drawn to C .

The Gallery, with the Railing, being compleated, we next proceed with the decorations of the Walls, seen beyond or over it.

Having drawn, geometrically, the Plans, or Seats of the Pilasters, with all the projectures of the mouldings of the Pedestal, &c. at Y, Y ; from each exterior Angle, draw Lines to C , as $jC, lC, \&c.$ and from l , set off, first, the height to the Plinth; then, the measures of the Plinth, Mouldings, Dado, &c. of the Pedestal, at $m, n, \&c.$ and draw $mE, nE, \&c.$ cutting lC in their several perspective Proportions, at $e, f, \&c.$ from each of which draw Lines parallel to their Seat, jl , cutting jC , and return them at right angles. The Diagonals, or mitre Angles are, in this Case, parallel to the Picture, i. e. to AI and BH ; by which means the Mouldings are determined, every where.

The Base Moulding is hid by the projecture of the Plinth, and also the Bases of the Pilasters, on this Side, being greatly foreshortened; which, where they can be seen, are managed after the same manner.

From $o, p, \&c.$ draw Lines to C ; make pq equal to the height of the Capital, from the Cieling, and draw qE , cutting pC , at n , and compleat the square of the Abacus; which being parallel to the Picture, the curviture of it is geometrical.

The Mouldings of the Capital are managed as the other (except in the Abacus, which are all circular curves, being parallel to the Picture) their measures of height, being obtained, from pq , at $r, \&c.$ and their projectures from the Plans below, drawing Lines from each, to C .

The Volute may be determined, from its Plan, below, in respect of its projectures; but to describe it, particularly, would baffle all description.

The Windows, in this Side, and Niches, &c. in the others, are determined geometrically, on AB , &c. the perpendicular Lines of which, vanish in C , and the circular heads are semi-Ellipses, described, by Prob. 3, Sect. 8.

The other Sides are managed after the same manner, from Plans below.

For the Entablature. Let M, N, O , and P , be the Seats of its projecture, on the Picture, in the Diagonals AI and BH . Draw $MC, NC, \&c.$ Make MQ equal to its height from the Picture, and draw QE , cutting MC at n ; draw $no, op, \&c.$ parallel, respectively, to the Sides of the Picture, $nopq$ is the extreme projecture of the Cornice, around.

Make $QR, \&c.$ equal to the measures of the Cornice, Frieze, and Architrave, with their several Mouldings; and draw $RE, \&c.$ giving them perspective, on MC . Their projectures are determined on the Diagonal, AM , geometrical; or, on Diagonals from $n, o, \&c.$ parallel to AM , &c. $mn, \&c.$ being divided in the same ratio, as $A-M$, (Cor. 2. Th. 9.)

* dk is the Intersection of the Plane $dfgk$ of the Railing, distant from AH , equal Ad , i. e. equal to its known distance from the Walls; consequently, d is the Intersecting Point of that Angle.

For, because the Picture is parallel to the Base of the whole, the section of every Plane with it, being perpendicular, is geometrically determined; therefore, the Intersecting Point of every Perpendicular Line in the Objects, as d or k of the Angles of the Railing, is readily found, its distance from each Wall being known, or determined; the whole Base being a true geometrical Plan.

Plate
XXXIII.

The Square of the Cieling above the Cove is thus determined.

From T and U, the Seats of the projecture of the Cove (in the Diagonals AI and BH) draw TC, and UC; and, TV being made equal to its height, draw VE, cutting TC at r; draw rs parallel to the Cornice, and compleat the Square rstu; in which, inscribe a Circle; and, concentric with it, another, leaving a border around the Dome; and also, compleat the same within the Square rstu.

For, because the Cieling is parallel to the Picture, its perspective form is truly geometrical; consequently, one Side being perspectively found, the rest are also determined, in the ratio of rs to the Original.

The Angles of the Cove are determined by Ordinates, on the Diagonals (if it were necessary) geometrically proportioned, alike on each; and others, on AC, &c. perspectively; (See the geometrical construction, Fig. 126, which determines three Points, a, b, and c, in the Curve, equally spaced.) but, as they approach so nearly to right Lines, here, it is an unnecessary process.

To represent the circular Cornice, at the foot of the Dome.

The Frieze or Facia, below the Cornice, is perfectly cylindrical, its height is determined on AC, which represents a Perpendicular from that Angle; on which, every measure of height may be transferred from AB, geometrical, as s, t, u, &c. of the several divisions in the Dome.

From S, the Center of the Base, draw SC, in which Line are the Centers of every Circle in the Dome, &c. for, the representations of all Circles, in Planes which are parallel to the Picture, are Circles; and, because SC represents a Line perpendicular to the Base, in its Center, SC is the Axis of the Dome, and passes through the Center of every Circle in it; which being determined, on SC, or transferred from AC, by lines parallel to the Diagonal AI, as s1, t2, &c. draw lines through 1, 2, &c. parallel to AB.

Now vx is the Diameter of the Base of the Dome, cutting the Axis at o; and 1, 2, &c. are the representations of the several Centers, of Circles in the Cornice.

From v, set off the projectures of the Cornice, in proportion to the Radius vo, at a, &c. from which draw Lines to C, cutting parallel Lines from 1, 2, &c. which give the Radius of each Circle in the Cornice, as 1k, 2e, &c. by which the Cornice may be compleated; being composed, wholly, of Circles.

The parallel Circles in the Dome are determined after the same manner; which are necessary, for describing the longitudinal Borders.

No. 2.

In Fig. 122, draw as many Ordinates parallel to its Base, GI, as are necessary, from the several points a, b, c, equally spaced in the Curve, to the Axis, or central line CH, cutting it at d, e, f. Then, make oa, ob, and oc in the same ratio to ov, as ad, &c. to the Radius GI; from which draw lines to C, as aC, bC, cC; and, from the perspective Centers, 2, 3, 4, and 5, obtained as above, draw parallel lines cutting them at e, f, g, and h, which give the Radius of each Circle in its true place, viz. e2, f3, g4, and h5, which, last, is the Base of the Cupola.

Divide the Base of the Dome into eight equal parts, at a, b, c, &c. so that, ah and de are parallel to AB, &c. and set off the widths of the Borders regularly from those Points (half on each Side) draw oa, ob, &c. and, from the Centers 2, 3, 4, draw lines parallel to oa, ob, &c. cutting their respective Circumferences at i, k, l, m, through which, Curves being traced, they are the central Lines of each Border*. Their widths may also be set off at each Circle, in proportion to its radius, as at the Base, by which they are compleated.

The Compartments are done after the same manner, from those central Lines, respecting the Margins and Mouldings which are longitudinal; the parallel Mouldings are in Circles parallel to the Base.

* The Eye being in the Plane of those passing through d and h, they are, therefore, represented by Right Lines; those passing through b and f, being farthest removed from the Center, are the most curved; for, being viewed centrally, they all appear Right Lines (as in Fig. 124) seeing the Eye is in the Plane of each Curve.

The femicircular Windows, at the Base of the Dome, are but little seen, and may be truly determined by Ordinates, tending to the Center, geometrically divided at the Base, and parallel Circles cutting them, as at *j, i, k, l, m*. The Margins about them may be done by the same means.

The Base of the Cupola being obtained, as above, the Windows in it, are geometrically divided at the Base, and Lines drawn to the Center.

Their apparent height may be obtained on SC or AC, as the other parts.

Thus have I described the whole process of this Cieling Piece, from the beginning to the End; to say more concerning it would be superfluous, as it must be obvious, that the Rules made use of are the same as have been applied in all other Cases, and Subjects whatever.

S E C T I O N XI.

Is applied to Household Furniture, Wheel Carriages, Machines, &c.

THE foregoing Sections contain the Elements and Rules for the practice of Perspective, in all common Cases; I have also shewn how to apply them, in familiar Lessons and Subjects, adapted to any capacity, in the 6th, 7th, 8th, and 9th Sections; and, in the last, to particular Subjects, and in a particular situation of the Picture; so that, there remains nothing more to be done in respect of the application of it, to all useful Subjects, almost whatever.

In this Section, which relates to Objects in particular Professions, yet useful to Artists, I shall only make some cursory remarks, in respect of the representations of such Objects as are here treated on. The former Lessons, being well understood, will be found applicable to whatever Subject the Artist chooses, or has occasion for. To multiply Cases and Subjects, in which the application of the Rules of Perspective may have the appearance of something particular or singular, would be endless, as, in almost every different Object, there is occasion for some variety in the application of its Rules; which, nevertheless, it is obvious, are all founded on the same universal Principles, though the Rules deduced from them are variously applied.

It is a necessary requisite, in many Professions, to be able to give a Design, to a Gentleman, of whatever may be wanted, out of the common run of things; and although a perfect Drawing may not be required, yet to give a slight sketch, with propriety, is certainly an Accomplishment which cannot well be dispensed with. In an Architect it is absolutely necessary, to give a correct Design, but they are generally contented with geometrical Representations, which does not give the true appearance and effect; that can only be done perspectively. Geometrical Drawings to a Workman are necessary; but, to give a true Idea how the Object will appear, when executed, from any particular Station, requires somewhat more; to give such an Idea, without knowing something of Perspective, requires a much greater share of genius than falls to the lot of many.

Next to the Architect, the Cabinet-maker and Upholder will find his account in a knowledge of Perspective; nay, I am of opinion, that it is full as necessary to the Furnisher as to the Builder; having almost as large a field, and as various. There is nothing influences a Gentleman more in the favour of his Workman, when he is pleased to want something whimsical and out of the way, than to take his Pencil and sketch out the Idea the Gentleman had conceived, and was big with, yet could not bring it forth without assistance. He who can do that, and
at

Plate
XXXIV.

at the same time, display a little modern taste, in Ornament, being known, is certain of success, or of employ, at least. I have, therefore, given several pieces of Furniture, of various kinds, for his practice, which will also, frequently, be found useful to the Artist, to furnish his Family, or Conversation Piece.

The Organ Builder, and Coach-maker, may sometimes find it necessary to give their Designs in Perspective; a specimen is given for each Profession. And, lastly, the Mathematical Instrument-maker, Mill-wright, or Engine-builder of any kind, by Perspective, may be able to shew to much greater advantage, the several movements, and mechanism of his Inventions.

There are but very few Persons, who have not had much practice, in Perspective, know any thing more of it, than the application to Objects, whose Planes are parallel and perpendicular to the Picture, and consequently, have seldom occasion for any other Vanishing Point than the Center of the Picture; they always, therefore, adapt every thing to that position, without thinking about the propriety of it. Some cannot think a Drawing is Perspective, unless an End of the Object is represented; for which reason, we seldom see any Object delineated in Perspective without having an End seen, which, in a Building, or other Object of a tolerable length, is absurd to represent, the Front being parallel.

Notwithstanding I have endeavoured to explode the absurdity of the Picture, being necessarily parallel to the front, or some other face of the Object, as not being picturesque; yet, in the following Cases, I would rather prefer it; such as giving a Design of a piece of Furniture, &c. to a Gentleman; because, the Gentleman, not having judgment in Perspective, might be greatly deceived in his Ideas of the proportion of the several parts to each other; whereas, the front of the Object being parallel to the Picture, the true geometrical proportion is preserved, which is most essentially requisite, in order to be clearly understood.

Fig. 129.

Figure 129 exhibits a representation of a Pedestal and Vase direct in front; the Eye is directly opposite to the Ball at the top, being about the height of a Person sitting; for, to stand, when we view such Objects as are wholly below the Eye, although we see no imperfection or distortion in the Object, itself; yet, unless the Distance be greater than we usually stand at, to see such Objects, it will be greatly distorted in the Representation, particularly in the lower parts, seeing that the Distance, in such cases, is frequently less than the height of the Eye.

C is the Center of view, for this Object, the Distance is CV, which is less than the height of the Eye; but, as the far feet, are not seen, and as there are no receding parts seen, below, no distortion is obvious; the Object standing in little compass, the inside of the feet are scarce seen at all, owing likewise to their tapering downwards; otherwise they would be much distorted, on the returning sides.

AE is a Scale of Proportion, on which the heights of the several parts are set up, from A, to B, D, and E, for the Pedestal. EH is the geometrical width, by the same Scale. From E and H, if Lines be drawn to the Center, and FH be made equal to its depth, in proportion to the front, FV being drawn, or a Ruler applied to F and V, cuts HC at I; from which a parallel Line is drawn, cutting EC, and determines the width of the top.

The Mouldings in the front of this Object are the chief difficulty; but the geometrical figure being known, the place of the parallel straight part is easily determined, on EH; and the curved parts, whether convex or concave, are flatter or quicker, as they are nearer to or farther from the Vanishing Line, and may be done sufficiently correct by hand; indeed all attempts at accuracy, by rule, in such minutias, being so small, would be fruitless and lost labour; experience and judgment are the best and only guides.

The place of the foot of the Vase being got, on the Pedestal, at FG, let the height of the greatest swell be set up, from E to J; where, draw a Line parallel to the Horizon; which, if the swell be equal to the straight part of the Pedestal, may

may be used for the side of a Square, enclosing the Circle, making KL equal to its diameter. If the Vase swell more, it will project on this side KL, if less it will fall within it; the real measure, in either case, is still the same, on KL, whether it be projected larger, on this side, or be diminished on the other.

This curve being determined, all the others will be more or less curved, as they are higher or lower; each of which must be done by the same means, if accuracy be required; finding the true place and proportion of each, by drawing a parallel Line, as NO, according to its height, above or below the other. The height of the top is best determined by a central Line, and adjusting the scale of proportion to its distance by means of the Vanishing Point (V) of Diagonals, and a perpendicular Line drawn through K; but, as it is, here, on a level with the Eye, it is determined by the Horizontal Line; and thus, this Object may be completed.

Figure 130 exhibits the representation of a round Pier Table according to the present taste, with tapering, term Feet. Fig. 130.

No. 1, is half the Plan of the bottom, for the place of the Feet, and No. 2 is the figure of the Top, geometrical, in inlaid work; both together form a Semicircle; but, if it has not a folding Top, which, when open, form, together, a complete Circle, I would rather it should be somewhat broader, than half the length; equal to the narrow margin, around; because, the figure within it should be an entire Semicircle, which is not so here; consequently the Figure is not so regular.

Let the Center of view be determined, at C, and take FG equal to the diameter of the Top; as much below C as the Eye is supposed above the Table. The Eye is, here, situated somewhat to the left hand, so that this Object is not viewed direct in front, that is, not centrally, as the foregoing.

Bisect FG, at E, and draw EC, FC, and GC, and from E or F draw a Line to the distance Point (which is not here fixed) cutting GC or EC, at K or D; through which, draw IK parallel to the front Line. In short, complete a Semicircle (IEK) representing a Semicircle, within the Rectangle FIKG; and, within it others, representing the several Borders, &c. in the Top of the figure; which may be obtained by drawing perpendiculars from a, b, and c to FG, and, from A, B, C, draw Lines to C, cutting the inner curve of the border at a, b, c, from which draw Lines to D. On account of the inner Curve, abc, not being a complete Semicircle, bD does not fall in the Diagonal GD.

The Top being completed, take FG parallel and equal to FG, at the distance of the breadth of the Frame, and within it, take a less Diameter, allowing for the projecture of the Top; on which, describe the curve at the bottom of the Frame, and the small Astragal; from the extremes of the inner curve, draw perpendicular, to the lower curve Line expressing the thickness of the Top.

For the place of the Feet, take fg below, at the distance Ff, equal to the height of the Table, in proportion to its width (FG) and having described the representation of a Semicircle, of the diameter of the Frame, the place of the Feet, at i and k, are in the Angles (but let it be noticed, that, the Top not being more than a Semicircle, the curve ixxk is less, because the Top hangs over, behind the Table; so that, d represents the Center)

From x (No. 1.) draw perpendiculars to HE, cutting it at Z; which measures transfer to fg, and draw Lines to C, cutting the curve, at x, which give the place of the front Feet. The Plans, at x, x, being completed by means of their proper Vanishing Points, in the Horizontal Line, draw perpendiculars from each corner, to the Frame; and, within the square of the Foot, describe another, answering to the thickness at the bottom, from all the corners of which, draw Lines to the Frame, where their full dimensions are obtained. The Plinth, at a little height from the bottom, is equal to the thickness of the Foot, at the Frame, the Lines of which tend to proper Vanishing Points, as the Plans, below.

The Compartments in the Frame, and the margins of the Feet, &c. it would be trifling to describe, being divided in the middle, and at each Foot; or otherwise, at pleasure, from the Plan, below; the Figure must supply the rest.

Plate
XXXIV.
Fig. 131.

Figure 131 represents a Chair, direct in front; C is the Center of view, the Distance is CE. GH is the Ground Line, in which, take G and H for the places of the fore Feet; where, make their Plans, and describe the front, GABH, truly geometrical.

Find V and X, the Vanishing Points of the Sides, AF and BD, respectively (Prob. 2.) and having set off, from V, the distance of that Vanishing Point, on the Horizontal Line, make Ba equal to the depth of the Seat, and draw a Line from a to the Eye, cutting BV at D, and draw FD parallel to AB, cutting AX which gives the Figure of the Seat, in the naked Frame; but being stuffed, they must be gently curved at the sides and behind, as on the left side.

The place of the back Feet, on the Floor, must be obtained by the same means, observing that they splay, backward, from the Seat, and consequently, are farther distant from the front; also the line of direction is thereby somewhat varied.

To determine the height of the Back, and the Elbow, draw GB perpendicular, and equal to its height. Draw BC or BX, cutting a perpendicular Line from K or k, at a or c; through which, a parallel Line, ab, determines its height. As to the Figure, 'tis at each Person's option, and depends wholly on the Eye.

The height of the Elbow is set up from A to d, and by drawing a Line to X, its direction, and where it is joined to the Back are ascertained; also, its place on the Rail may be determined, as the whole length, at D. The rest depends entirely on the Hand and Eye; for, no Rules can possibly determine its apparent Figure, perspective.

Fig. 132.

Figure 132 is the representation of a Lady's dressing Table.

In this Picture, the Center of view is taken somewhat to the left hand, at C, the Distance is CE. The height of the Eye supposes the Person sitting.

The measure of its length, in front, may be set off, on the same Ground Line, GH, in proportion to the Chair; or on another, at MN, being supposed to stand farther from the Picture; on which account, its geometrical Figure, in front, is by a less Scale, as MABN.

AB, the front Line of the Top, being determined, let it be bisected, and, AD and BF made equal to half the measure, in any position required; from all which Points, draw to C; make Ab equal to the breadth of the Top, and draw bE, cutting AC at b; draw bf parallel to AD. From b it is transferred, by a dotted Line, to the other side. The hollowing and thickness of the Covers, in front, are determined geometrically, by perpendiculars to AD, or BF, at f and D, or F.

Make Aa equal to the distance of the nest of Drawers, which rise, by a Spring, out of the Frame, and draw aE, cutting AC, at a; draw ac parallel to AB, cutting BC; observing, in this operation, that there is a margin, of above half an Inch allowed on each side, less than AB. Draw the perpendiculars ae and cd, and proportion them to the length ac; or set up the full measure on a Perpendicular, the breadth of the margin within, at B; on which, the several Drawers may be proportioned, at 1, 2, 3, geometrical. The front of the Drawers, being parallel to the Picture, they are geometrically divided into as many Drawers and in what order you please.

The dressing Drawer, being drawn out, it consequently projects on this side the Picture. Having drawn the fronts of the two Drawers, over it, draw CG, and produce it, indefinite; produce HG, and make GI equal to the measure you intend the Drawer to project, and draw E'I, till it cuts CG, produced, at K; draw KL parallel to GH, cutting CH produced, at L; KGH L represents the upper face of the Drawer, projected towards the Eye. HO (the ring) being made equal to its depth, draw CO, till it cuts a perpendicular from L, at J, which determines the front.

The divisions, for the Glass, in the middle, the Boxes, &c. are determined, by their measures on GH, or by a larger Scale, on KL, as Kc being made equal to the first division, cE determines its apparent breadth, at d. After the same manner the rest are determined.

The

The recess, below the Drawers, being an elliptic curve, is thus determined.

The height being determined, draw ag , the transverse Diameter of the Ellipse, describe the geometrical curve $a35g$; ag being bisected at d , $d4$ perpendicular to ag , is half the conjugate Diameter. Take as many Points, 1, 2, 3, &c. on each side, as are necessary, from which, draw Ordinates to the Transverse at a , b , c , &c. observing to make g , f , and e , respectively, the same distances from that extreme, as a , b , and c from the other; from all which, draw Lines to C , and determine the length of each perspective, and respectively, by means of the Point of Distance E ; making bb represent $b2$, and, cc represent $c3$, &c. parallel Lines, bg , &c. will determine those on the other side, giving the Points a , b , c , &c. through which the perspective curve may be described, as in the Figure.

What remains is sufficiently explicit by inspection.

Figure 133 represents a Lady's Secretary and Library; of which there needs no particular description. It is viewed directly central, so that, the Vertical Line cuts it into two similar Figures.

Fig. 133.

Being parallel to the Picture the whole front, of Drawers, &c. is geometrically drawn; the middle part, with the Desk Drawer, may be projected, as the last Figure; or the Picture may be supposed to be wholly on this side, so that, only the Fall of the Desk is projected.

The Shelves, with the Books, recede from the front, it would be impertinent to shew how it is managed here; the Books are disposed at discretion, and, the Door which is open is determined by Example 39; V is the Vanishing Point.

For the rest, the Figure is more explicit than the most elaborate description.

Figure 134 exhibits the representation of a Bed with a Canopy-Teaster, viewed direct in front; one half shews the naked Frame, the other half is furnished.

Fig. 134.

The Front being parallel to the Picture, it is geometrically proportioned. C is the Center, and CV the Distance of the Picture, by which the rest is proportioned.

The Column, on the left hand, being seen entire, is represented as in former Examples; the returning Lines, in the Base and Capital tend to the Center; also the inside of the Foot.

The length of the side (GH) being determined, and the height of the Post, at A , draw AC , cutting a perpendicular from H , at B , the apparent height of the head Post. The geometrical curve of the Laths being determined, the perspective curve, of the side Lath, is determined by means of Ordinates, $a1$, $b2$, &c. the several heights being set up from the corner, at A , and Lines drawn to the Center.

The Laths forming the Canopy, diagonal wise, are determined as groined Arches, by Ordinates from the Diagonals, DE and EF , E being the Center.

The hither one being determined from its known figure, dividing DE perspective, at a , b , c , &c. as the Base of the Original is divided by Ordinates, at discretion; by which means, the several heights aa , bb , &c. are determined, and the Curve described through the Points a , b , c , &c. Then, Lines drawn to the Center, from the Points a , b , c , &c. cut the other Diagonal (FE) at k , i , h , &c. at which Points, the Ordinates kk , ii , &c. being drawn, and from a , b , &c. Lines drawn to the Center, cutting them, at k , i , h , &c. through which, the curve of the other diagonal Lath is drawn.

In respect of the Drapery, the Furniture of the other half, I shall only observe, that it is proportioned as the other; but as for the outline figure and ornament, I will not attempt to give any Rules; such Subjects are always best drawn from a real Object, or they can only be done by a Person of experience and judgment. The figures of the Vallens are geometrical, in front, and no other are seen. The Canopy is but little seen, though it rises considerably; a Right Line, from b , shews how much, save the middle, which appears over it; on which a Vase, or other ornament, at discretion, may be drawn, crowning the whole.

Figure

Plate

XXXV.

Fig. 135.

Figure 135 is a Design for a large Library Bookcase.

In order to give an Idea of the real proportion and figure of such an Object, to a Person who, perhaps, knows not what Perspective means, and who has no other Idea of its proportion and form, than what the Figure really exhibits, it is most proper to give it parallel, as it is here represented, but by no means to shew an End. The Station is central, and consequently, the Point of View is in the middle. The receding parts, and breaking of the Mouldings around them, sufficiently indicate that it is perspectively delineated; and, by reason of its regular position, I am of opinion that it has a much more natural appearance, than when viewed oblique, from either end, which always occasions distortion; as may be seen in Plate 23, Fig. 108, especially when the Object is long.

In respect of the delineation, let it be observed, that the Plinths of the middle part and the two Ends are in one Plane, which are geometrically proportioned, by the Scale. The parts which recede are determined as usual.

At either Corner, as A, of the Plinth, draw the perpendicular AG; on which, set up all the measures of its height, as AB for the Dado part, with its Base and Sirbase; BD the height of the upper Doors, &c. and DF, of the Cornice, &c.

To the Center, C, draw Lines from each division; make Bc equal to the receding of the upper part; or, because the whole Distance cannot be on the Picture, take CE half its Distance, and Ba half Bc, and draw aE, cutting BC at b.

Draw bd perpendicular, cutting the Lines drawn from B, D, &c. to C, which gives the height of the Doors, &c. perspectively, according to their distance from the Front. For their widths, make Ba, equal to the projecture of the Mouldings, and ab to the width of the Door, and draw aC and bC cutting a Line drawn from b, parallel to the Base.

The other End and middle Doors are obtained by the same means. Or, being all in the same Plane, if either the height or width of one be obtained in its place, the whole is determined; for, being parallel to the Picture, the Doors are all represented similar to their Originals; that is, geometrical, in respect of their outward figure, and also of the ornamental part, for Glafs, &c. or as those below.

For the Cornice; if CE, on each Side, be made equal to the Distance of the Picture; they are the Vanishing Points of the Diagonals, which are drawn through the Angles at the top, obtained from the Perpendicular at F, transfered to f. The general method, in Example 15, for Mouldings parallel to the Picture, is applicable here; X, being the Profile, and E, the Point of View.

The figure of the outline of the Pediment is geometrical; which certainly gives the best Idea of its true Figure; and so is all the Ornament, every where.

Fig. 136.

If it be required to see an End of any Object of which a Design is to be made, the Front should then be inclined to the Picture, as in Fig. 136; which exhibits a representation of a Library Table, by a larger Scale than the foregoing Figure. For however prepossessed many Artists are, in the parallel position of the Picture, I am confident, that, were they practised a little in inclined positions, and saw clearly into Perspective, they would not retain their prejudice long; for we cannot possibly see two Faces of a right angled Object, and the horizontal Lines, in either, appear parallel. Why, then, are they so represented? Whenever a single Object is the subject of a Picture, being right angled, and seen oblique, it is, therefore, most palpably absurd to be placed parallel to either Face; or, in any common Case, to have the Center of View out of the middle of the Picture.

The delineating this Object has nothing in it particular. The Inclination of it to the Picture and the Distance being determined, the Vanishing Points are determined, as usual, both which are, in this Piece, out of the Picture.

S is the Center, and d the Distance Point, by which the Front is proportioned; the measures being applied on the Ground Line, AB, as in former Lessons, at a and b, for the Front; which give a and b, on AB, the line of the Plinth, and on FB, of the front Plane, on the Floor, tending to their Vanishing Point.

As there is not room on the Ground Line to set off the full measure of the End, take AD a third part, or Ae half; and let f or S, be in proportion, to the Distance of the Vanishing Point of horizontal Lines in the End; then, Df, or eS, cuts AD in the perspective length required.

From each Angle, A, B, and D, Perpendiculars are drawn, and the real measures of the heights of the Drawers, &c. being applied on EF, where the Seat of the Plane they are in, on the Floor, cuts the Intersection, lines drawn to the Vanishing Point of the front Lines, give their perspective heights.

Perpendiculars from a and b determine the opening; and if, from b, a line be drawn to the Vanishing Point of the End, and another from D, to the other, of the Front, cutting the former, at c, the perspective width of the inside End is determined. The Mouldings are done as usual.

E X A M P L E XLVI.

Is the representation of a large Chamber-Organ, seen oblique.

Fig. 137.

The method of proceeding, in obtaining the general proportions, which is subject to one general Rule, it is unnecessary to repeat; the measures, or their ratio to each other in proportion to the Distance used, being always applied on the Intersection of whatever Plane the Lines are in, or on a Line parallel to it; as on AB, for the Piers, &c. I shall, therefore, pass over that description, and only explain the Parts, where there is any apparent difficulty.

The Finger-Board (U) is usually made to draw out, the length of the Keys, about 6 Inches. Its place being acquired, at a, and V being the Vanishing Point of the End, draw Va, and produce it; draw ae parallel to the Ground Line, make ae represent 6 Inches, in proportion to the length, and draw Ce, (from the Distance Point of V) till it cuts Va produced, at b; and, from b, draw bc to the other Vanishing Point, of the Front, by the Expedients, Problem 13.

Draw bd parallel, on which, set off the number of Keys, as at U; then (having made hc represent the length required) join dc and produce it to the Vanishing Line cutting it at D; to which Point, draw Lines from all the divisions 1, 2, &c. which give the Keys.

The process may be done, more commodiously, below, on the Ground Plane, by dividing bc, below, as b c above; from which, by Perpendiculars, it may be determined, above.

The Mouldings, breaking about the Piers, &c. have nothing singular in them; their heights, on the Pedestal, are set off on AD, the Intersection of the front Plane, with the Picture; and on AF (for the upper part) the Intersection of the Plane of the Pipes, &c. above, at B, C, &c.

At Z (No. 2) is a geometrical Plan of the Breaks.

The Column of Pipes, at each End, are Segments of Circles containing five Pipes each, which are unequal; let their perspective Plans be formed above, at X and Y; from their extreme edges, a, b, &c. draw perpendiculars, as shewn by the dotted Lines, giving the places, below, of the cylindrical Part.

The Mouth of each Pipe, at f, &c. is at about one third part of their height, from which it is conical, inverted. The Ornaments, at the tops and bottoms, are regular, from the middle Pipe, which it would be impossible to describe, being circular; their Figure being known, such minutias are drawn by the Eye.

The Pediment, and middle compartments of Pipes, being inclined, it may be necessary to shew how they are described perspective.

Suppose the geometrical Figure of the Pediment the same as in the Book Case (Fig. 135) which, inscribe in a Rectangle, and draw Ordinates parallel to both Sides, from various points in the Curve, as in that Figure.

The Cornice, at the End, being described, as usual, and the Angle F determined, draw FI to the Vanishing Point of the Front; and make FI represent the distance of the two extreme Angles; at each of which, draw perpendiculars.

Plate
XXXV.

fg is the Intersection of the Plane of the front of the Pediment; from *AF* transfer the height of the top of the Cornice, to *f*, and make *fg* equal to the known height, which, divide in the same ratio as *HI* (Fig. 135) from all which divisions, draw lines to the Vanishing Point of the Front; and having divided *GH* perspective, as *IK* (Fig. 135) perpendiculars drawn from each division, cutting the other, give the Points *a, b, c, &c.* through which the perspective curve is described.

The middle Column of three Pipes is projected as the other.

If the geometrical figure of the two Sides be also determined; then, on *ag*, take the divisions *c, d, f, &c.* geometrical, and draw lines to the Vanishing Point; let the horizontal Line *gk* be divided perspective, at *b, i, &c.* from which, perpendiculars being drawn, cutting the other at *m, n, &c.* give several Points in the Figure. The rest may be drawn, accurately enough, by hand.

The proportion of the Pipes may be determined on any horizontal Line crossing them, as *gk*, either above or below, as is most convenient; dividing it perspective, in the ratio of the Pipes (by Prob. 8) as they are all in the same Plane.

The mitre Angle, of the straight Mouldings with the circular, must be determined by a geometrical Plan, at *W*, which varies according to what portion of the Circle is taken; also, where it falls against the Plane, the projectures may vary considerably; in a Semicircle they have their true projecture.

It is unnecessary to give more examples of Furniture, seeing that, if the Rules of Perspective, here used, be clearly understood, the Examples, given, are sufficient for any Person, who is clear in the Principles on which they are founded; and may, by such, be applied in any Case, whatever, in right lined Objects. Nevertheless, as Chairs, from the various directions of the Lines in them, are somewhat difficult to delineate, I shall give another specimen, how far the Rules of Perspective are applicable to such Objects, any how situated to the Picture.

E X A M P L E XLVII.

Fig. 138.

How to represent a Chair, perspective, casually situated.

The position, and inclination of its Front to the Picture being determined, and the representation of the hither Angle, *ab*, of its Foot, given at discretion, to the scale of the Design you intend.

Let *V* be the Vanishing Point of horizontal Lines, in the front of the Chair. *C* is the Center of the Picture, and *CE* its Distance.

At No. 1. is a half Plan of the Seat, shewing the splay of the back Feet, at *X*, with the inclination of the side Rails to the Front.

Draw *EV*, and make the angle *VEY* equal to the inclination of the Side to the Front, giving *Y* for the Vanishing Point of the Rails, in that Side; also, make the Angle *YEX*, equal to the inclination of one Side to the other; giving the Vanishing Point of the other side Rails, &c.

Then, draw *aV*, and *bV*, *aY*, and *bY*; and *cd*, through *b*, parallel to the Vanishing Line.

Make *bc*, equal to the width of the Front, in proportion to the height (*ab*.)

Make *VD* equal *VE*, and draw *cD*, giving the place of the other front foot, at *i*; also, the breadth of each are set off at *ba* and *cb*.

Let *bd* be equal to the side of the Chair, at its Seat, and *de* to the splay of the back foot; make *YG* equal *YE*, and draw *dG* and *eG*, cutting *bY* at *f* and *g*; *g* is the place of the foot on the Floor; and if a perpendicular be drawn, at *f*, it will cut *aY* in the true width at the Seat.

Make *ac* equal to the width of the seat Rails, and *bd* to the height and width of the low Rail; draw *cV*; and *cY*, cutting the Perpendicular from *f*; also, draw *dY*, for the low Rail, to the back foot, which splays back, from the Seat.

From *g* and *h* draw Lines to *V*, and from *i* and *k* to *X*, cutting them at *l* and *m*, which give the place of the other back foot, and compleats the Seat.

Make

Make be , equal to the recess of the inner Rail from the Front; draw eG cutting bY at f ; from which, draw a perpendicular, giving its place on the low side Rail, at g ; and from g , draw to V , cutting the other Side Rail, the measure of which was transferred from d to the other foot, at h .

The height of the low back Rail is obtained on the front Foot; and transferred to the back foot, by means of the Vanishing Point V .

Thus much is practicable, with the greatest accuracy; but beyond this, little can be done, the whole Back being curved every way.

The height may be truly determined, by means of perpendiculars, from the floor. Having obtained the Line gt , tending to V , over which, a Line, rs , through the corners of the top Rail, is perpendicular, through the Point where it is cut by bY , draw De to the Ground line; make $e1$ equal to its projecture over the Seat, and 12 to its known measure. Draw $1D$ and $2D$, cutting gt at g and t , from which Points, draw perpendiculars. From any Point, as G , in the Horizontal Line, draw Ge , through g ; draw eq perpendicular, make eq equal to the height of the Chair, and draw qG cutting gr in the true place of that corner of the top Rail; rV gives the other, at s .

The various proportions of the Banister, may, in some degree, be obtained, by dividing a line (no) drawn through its middle, into the several heights, 1, 2, 3, &c. through which, Lines drawn to the Vanishing Point, V , determine the raking of the several parts, as in the Figure. For, whether the Back of the Chair be straight or curved, it deviates so little from a Right Line, and from a Perpendicular, that the measures may be applied, geometrically, without any sensible error.

In respect of the modern Chairs, now in use, there is no such thing as applying the rules of Perspective to them, except to ascertain the place of the Feet, on the Floor; also, by inscribing the Seat in a Square, or other Rectangle, its Figure may be nearly determined in its true place.

The height of the Back, and of the Elbows may also be had, but, as the whole Chair is composed of irregular curved Lines, it is absolutely impossible to give its true figure by the Rules of Perspective; such Objects, must therefore depend on the Hand and Eye, for a representation of them; in which, a Person who has judgment in Drawing can only succeed; but, being well versed in Perspective, he will find great assistance from it.

WHEEL CARRIAGES are the next Subject to be handled; on which, as they are frequently necessary to introduce into a Picture, or for the sake of giving a Design to a Gentleman, by those whose profession it is to make them, I shall make a few Observations; but, like many other Objects, not composed of Planes and Right Lines, little can be done by the rules of Perspective, further than to give the proportion of one part to another; also the Wheels may be drawn with tolerable accuracy. Nevertheless, I am conscious, that few Persons who have occasion to delineate them (except the makers) will take the necessary pains to project them by rule; for such as will, I shall lay down some which are practical.

N. B. It must have been observed, that to delineate any Object by the Rules of Perspective, its true geometrical form and proportion of one part to another must be known, or determined on; consequently, the more complicated the Object is, being composed of irregular curved Surfaces, the greater is the difficulty in applying the measures on the Picture; which, as the several parts to be delineated are not Planes, or plane Figures, their places can only be obtained, on the Picture, from their Seats on some Plane or other, to which they are contiguous. By which means, a sufficient number of Points, in any curved Line in the Object, being perspectively found, and joined carefully by hand, the perspective Figure is determined.

E X A M P L E XLVIII.

How to represent a Coach in perspective.

Let $ABCD$ be a geometrical Plan of the Carriage of a Coach; AB is the Axle of the fore Wheels, and CD of the hind Wheels, whose Diameters are ab and dc . $EFGH$ is the Plan of the Top of the Coach, all the other Parts are in proportion to them; as in the Figure.

Fig. 140.

Plate
XXXVI.

S is the Station, from which it is intended to be delineated, and IK the true position of the Picture from that Station. Sv being drawn, parallel to the Axles and front Lines of the Coach, gives their Vanishing Point; Sc, perpendicular to IK, gives its Center; and, Visual Rays from various parts of the Coach, being drawn to S (answering the same as the Eye, in the Horizontal Plane) give their places on the Picture.

The Vanishing Point, of the Line of its direction on the Ground (with which, the Axles, &c. are at right angles) is out of the Picture; its distance from c (the Center) is to cS, as cS to cv. (Prob. 12.)

Fig. 142.

These Preliminaries being settled, as usual, let C be the Center of the Picture, and V, the Vanishing Point, by a Scale of 2 to 1; AJ is the Ground Line; and A, the intersecting Point of the line of direction of the Wheels, on the Ground, which are, here, supposed to be in the same Plane.

The Plane of the Wheels not being vertical, but somewhat inclined, from the Carriage, let DG be the Intersection of the hither Wheels, with the Picture*.

Draw AD perpendicular, make AB equal to the diameter of the fore Wheel, and AD of the hind Wheel; draw BF and DG parallel to the Horizon, cutting AG at F and G. Draw Ae and Gg, also from F, tending to the Vanishing Point of the direction of the Coach.

To obtain the true place and representation of the hind Wheel, bisect AG, and draw sH perpendicular. Draw He, to the Vanishing Point; make HJ equal to the distance of the Seat, on the Ground, of the center of the Wheel, from the intersecting Point H, and draw JE, cutting He at e, (or, if S be its Seat on the Ground Line, draw SC) at which Point, draw a Perpendicular, and from s (in AG) draw a Line to the Vanishing Point, cutting it at s, the representation of the Center of the rim of the Wheel.

Draw sG perpendicular to the Intersection AG; from which Point, set off the true radius of the Wheel, on each side, and draw Lines to the center of the Vanishing Line of the Wheels, cutting a Line drawn through s, parallel to AD, at e and g; eg is a Diameter of the Wheel, in its true place; from which, the circumference of the Wheel, efgh, may be described (by Prob. 3, Sect. 8.)

The fore Wheel (abcd) may be obtained by the same means.

The end of the Nave is not in the plane of the Wheel, but somewhat beyond it, at i and k; also, the Spokes dish inwardly, from the front of the Wheel. Find the circumference of the Naves, abcd and efgh, in their true places, perspectively, at the part where the spokes are fixed in it, and the place of each; also let each Rim be divided, perspectively, into equal parts, the number of Spokes, at 1, 2, 3, &c. (by inscribing a Polygon) from which they are drawn to their corresponding parts in the Nave; or to their common Center, cutting the Nave, in their true places; for, as they are not in the plane of the Rim, their Vanishing Points are not easily determined, nor are they necessary.

The hither Wheels being compleated, the Body of the Coach is next to be determined; the Sides of which not being in one Plane each, their Intersections are not easily determined. Therefore, from the Seat, on the Ground, of the hither Angle of the Coach, perspectively determined, at i (from the Plan) draw ik perpendicular, in which, by means of the Vanishing Point, V, of horizontal Lines in the Front, determine the Angle k; the real height being IK, in the Intersection of the Front. The other Angles, m and n, may also be determined from their Seats; or, by their known distances from k, as in other common Cases.

If the corners of the Coach are perpendicular (as in some Coaches) the measures may be determined on ik, geometrical; but if they are inclined, the point l may be obtained as k, from its Seat; and, from l, lines are drawn to the Vanishing Points, and proportioned as usual.

The Door is not in the same Plane, with the Line kn, but may be determined from the Seat of the hither Line, op, where it opens; and, on any perpendicular

* This Intersection may be truly obtained, by Prob. 6. Sect. 12.

line, cutting the Ground Line, as IK, set up from I, its true measure, of height, at *a* and *b*; then, from I, draw a Line through its Seat (*s*) to the Vanishing Line, cutting it at L; draw *aL*, and *bL*, cutting the Perpendicular, from *s*, at *o*, and *p*; from which, draw Lines to the Vanishing Point of the Side.

The curve of the Bottom being geometrically determined, it may be done perspective, by means of Ordinates, as in foregoing Examples. From the Vanishing Point V, draw a Tangent to the hither Curve, at *r*, which also touches the other.

The contour of the Top may be determined, by several vertical Sections described, perspective (as *dep*, and *fgb* at the Door) then, a Curve described over them all would be the true Contour.

The Axles being parallel to the front of the Coach, their Vanishing Point is V; their thickness may be proportioned to the diameter of the Wheel, and their Shape by means of Ordinates, at 1, 2, 3, &c.

The geometrical Figure of the Crane Neck being described (Fig. 141.) then, by means of squares (the whole being inscribed in a Rectangle, ABCD) it may be accurately drawn in Perspective; with the other appendages of Springs, &c. by the same means. Also the Circle, at X, on which the fore Wheels turn, is determined, at Y, in its true place perspective, as any other Circle, parallel to the Horizon.

The height and true place of the Seat may be easily determined from its Plan on the Ground Plane, perpendicular over the fore Axle. The foot-board, at M, will be best obtained by the same means; or, by its Intersection and Vanishing Line, being a plane Figure. The place of the Bar, to which the Traces are fixed, is easily determined, at E, which tends to the Vanishing Point V.

The other smaller Appendages, such as Iron Stays to the Seat and Springs, Rings, Straps and Buckles, &c. it would be trifling to describe.

The true Figure and proportion of the Carriage being obtained, the Ornaments of carving, &c. will depend on the Hand and Eye, wholly.

And, last of all, the off Wheels must be drawn, after the same manner as the hither ones, nearly; observing that they incline the other way, from the Picture. But, as they are so much hid, by the other parts of the Carriage, their Circumferences and the places of the Spokes therein (with the Point to which they tend) may be obtained from the hither Wheels, by means of the Vanishing Points.

Figure 143 represents a Phaeton, more foreshortened, and going from the Picture; by which the hind part is more seen, as the fore part in the former Figure. Fig. 143.

A description of the method of delineating it, it is obvious, would be, in a great measure, a repetition, consequently tedious. The Position to the Picture being fixed and its Proportions known, the method of proceeding is as in the foregoing Figure, in respect of the Wheels and Carriage.

AB is the Ground Line, on which the Picture, standing perpendicular, is supposed to touch the hither hind Wheel, of which, BD is the Intersection. Bd, and Bb, tending to their respective Vanishing Points, are proportioned, at *a*, *b*, *c*, &c. as usual, for the places and diameters of the Wheels, &c. which are described as the foregoing; and are transferred to the other side, by the Vanishing Point of the Axles, the true figures of which, being in vertical Planes, are determined as other Figures, from their known geometrical form.

On *ef* is set off, perpendicularly, the height of the hinder Axle, and the horizontal movement of the fore Wheels, determined at *f* by a perpendicular from *e*; being cut by a Line drawn from *f*, to the Vanishing Point of the Line of direction of the Carriage, on the Ground. *eg*, on the Ground, is a diameter of that Circle, which being described, it may be determined in its proper place by Perpendiculars; after which, the rest of the Carriage may be drawn by a Person who has knowledge of the several parts.

Plate
XXXVI.

The corners of the Chair, *m, n, o*, and of the Foot-Board, at *l*, may be ascertained by perpendiculars, from their Seats on the Ground, perspectively determined at *m, n*, &c. and certain other Points which are in a right lined direction, as *p* and *q*. For the rest, it would be to as much purpose to give Rules for determining the figure and position of the Man in the Chair, and of the Horses; seeing that, the curve Lines which bound the Figure, save those at the top, are so variable, that no positive determination of their apparent Figures can be described; but must ever depend on the Eye, and judgment in drawing.

E X A M P L E XLIX.

To represent a Machine, for driving Piles; which is in the Repository of the Society, for the encouragement of Arts, Manufactures and Commerce, in the Adelphi, Strand.

Fig. 144.

Objects of this kind, I have frequently observed, are usually drawn with one Face parallel to the Picture; not, I am persuaded, with a view to make it better understood, but to save trouble; or, as it is most probable, for another reason, a want of judgment or knowledge in the Art, to do it otherwise; for, I am opinion, no Person would think that this Object would convey a more adequate Idea of its structure and parts, by being parallel, seeing that, all the same parts are or may be seen, in one as in the other, without any apparent distortion; and which, by being parallel, is unavoidable, being seen oblique. I therefore think it unnecessary to give a Specimen, in that position to the Picture.

C is the Center of the Picture, whose Distance is about 8 Inches; the Vanishing Points are determined, as usual, according to the Inclination given.

Let AD be the Intersection of the Picture, with the plane of the Frame at the Bottom; and A, the Intersecting Point of the nearest Angle of one of the Timbers; also, let E and F be the Distance Points, for the Vanishing Point of each Side, respectively.

The Vanishing Points being ascertained, draw AD and AH, tending to them.

Make Ab equal to the length of that End beyond the other cross Timber; and bd equal to the width between the Timbers, allowing the thickness of one, and draw bE and dE, giving the Points c and d; through which, the other Timbers are drawn.

Make Ac equal to the diagonal of the Inclination, and draw cV, to the Vanishing Point; through c and d, draw Lines to the Vanishing Point of the Side, and produce ec, to the Intersection, AB, cutting it at J.

Make Ab equal to their thickness, and compleat the Ends (v and x); from the two upper Angles of each, draw lines to the Vanishing Points, on the left hand.

How that Frame is finished, the Figure describes sufficiently.

The places of the other Timbers are obtained by setting the Distances, &c. on the Intersection AB, at e and f, and thence transferred to AD; or, from J, to JI.

From J, draw a Perpendicular (JL) the Intersection of the long Timbers; on which, set up all the measures of the Frame for the Wheels, at j, i, k, and l, from each of which, draw lines to the Vanishing Point of the Side, cutting Perpendiculars from c and f, giving the height of the Platform, &c. the length of which, (mn) may be obtained from the Timber, below; and the breadth (no) by the Vanishing Point of a Diagonal (mo.) at e.

The feet of the Supporters (r and f) being got, and the Bearer, or Head piece (X) drawn to the Vanishing Point of the Front (giving what thickness is necessary, at l) draw the Supporters, as in the Figure; also, the Bearers (y) and Stays (z) of the Platform. Their Vanishing Points may easily be obtained, knowing their inclination, in the vertical Vanishing Line of the End; by Prob. 4.

The top of the cross Braces being got, by a line from k, the Figure shews how they are drawn, so as to shew their thickness, properly; first drawing the Front.

Produce the inside of the long Timber to the Ground Line, and somewhat beyond it, at I, draw IK, perpendicular, the Intersection of the vertical toothed Wheel, which begins the motion.

Fig. 142.

Make Ip equal to the height of its Center, and, let pq be its Radius, and qr its Diameter. Draw from p to the Vanishing Point of the Side; and, having bisected the head of the Frame, at g , draw, from g , to the Vanishing Point of the End, which will pass through the Center of the Wheel, at s , through which, draw tu its Diameter, and complete the Rim $hikl$ (by Prob. 3, Sect. 8.) Its Vanishing Line passes through the Vanishing Point of the side Timbers, perpendicular.

The cross Bars (hk and il) have their Vanishing Points in it; making right angles, respectively, with each other, at discretion.

The Teeth, where they are seen, may be done by hand; for although the Rim may be divided into equal parts, yet, to divide it into so many would be unnecessary trouble; observe, as the Wheel is contracted so are the Teeth.

The other vertical Wheel (Z) of a larger Diameter, on the same Axle, is managed by the same means; having the same Vanishing Line, and Vanishing Points. Its use is to accelerate the motion, by its weight.

In such Objects, great regard should be had to describe the hither parts first; taking particular care that they do not cross each other, improperly.

The middle, upright piece, mn , and the Bearer, nC , being drawn, the place of the long Axle is opposite the middle of the vertical Wheel; its place may be found in a Line drawn, on the Ground, through the foot of the upright Supporter, at r .

The small horizontal Wheel (pq) being nearly on a level with the Eye, appears almost in Right Lines. Its Diameter is, in proportion to the vertical Wheel, nearly geometrical.

The height of the upper horizontal Wheel may be had from any perpendicular Line, cutting the Ground Line, as JL ; drawing a Line through r , the Seat of its Center, on the Ground, to the Horizontal Line, cutting it at E .

Make IL , equal to its height, and draw LE , cutting a perpendicular from r at s , and gives s for the Center of the Wheel.

Draw LM parallel to the Ground Line, make LM equal to the radius of the Wheel, and draw ME , cutting a Line drawn through s parallel to LM , at t ; st is the Radius, in its true place; by which, the Circumference, $tuvw$, may be described. The thickness of the Rim, and height of the Teeth, are set up from L ; and the cross Bars tend to the Vanishing Points of the Timbers; or at discretion.

The place of the Stay at u is also determined on IL , at N .

The large upright Timbers may be determined, in height, by the same means.

Produce the bottom Line of the hither cross Timber (at its foot) to the Ground Line, at O ; draw OP perpendicular, and equal to the height. Then, having obtained the place of the upright Timbers, in the middle of the Rails, below, at R and S , and the perpendicular Lines being drawn, a line drawn from P to the Vanishing Point, on the left-hand, determines the height of the hither one, at T ; and, from T to the other Vanishing Point, they may be completed.

The Center and Diameter of the Wheel V may also be obtained on OP , at p and q , and transfered to its place, giving s for its Center, and rs for its Radius; by which it may be completed. (Prob. 3, Sect. 8.)

Observe, it is the circle of the back part of this Wheel that is first got.

The place of the Weight, or Ram, W , is at pleasure; its height is in proportion to the upright Timbers; also the Stays between them, above.

The Roller (Y) for winding up the Ram, is a Cylinder, on the Axis of the vertical Wheel; and it is so contrived, as to revolve on the Axis freely, while the Ram descends, without retrograding the motion of the Wheels. When the Ram is down, it catches, and when the Ram arrives at the Top, the Cylinder is set at liberty again; so that, the motion of the Wheels is continual, the same way.

Behind the Frame is another heavy vertical Wheel (X) on the Axis of the Cylinder, intended to accelerate the motion, whilst the Ram descends.

EXAMPLE

Plate

XXXVI.

EXAMPLE I.

Is the representation of a Crane, from the same Repository.

Fig. 145.

Having been minutely particular in the description of the foregoing Machine, less will suffice in this. It is worked after the same manner, by means of Winches; the Model has three, here is but one; which, is sufficient for the purpose of describing the manner of delineating it.

A geometrical Plan, of the place of each upright piece of Timber, &c. In the Frame, being drawn, to any Scale, and the position of the Picture determined, let Perpendiculars be drawn, from each, to the Picture, giving their Seats. Or, being so prepared, let the Lines, of some of the principal, be produced to the Picture; which, where it can be conveniently done, facilitates the process.

The Center of the Picture is at C, in the same Horizontal Line; the Distance is about 6 Inches and a half. V is the Vanishing Point of horizontal Lines at the Ends, determined at discretion; the other runs out, on the right-hand, distant from the Center almost 14 Inches (a third Proportional to CV and the Distance of the Picture, always, when the Lines are at right angles with each other) BK, on the Floor, is the Ground Line of the Picture, on which it stands.

From A, the intersecting Point of the end Line, at the foot of the higher Supporter of the Frame, draw AV; and in it, find a, the nearest Angle (Prob. 7) from which draw a Line to the other Vanishing Point, and produce it to the Ground Line, at B; and draw BF, perpendicular; on which, the several measures, of the heights, are set up, from B, and Lines drawn to the other Vanishing Point cutting Perpendiculars from a and b.

The Axle (RS) of the large Wheel between the Supporters, is determined by a line drawn from G, the Seat of S on the Ground Line, to the Center, giving its place, at S (in the central Line LS) its Diameter is got by the same means. Or thus.

The height of the Wheel from the Floor being known, draw its Intersection HI, parallel to the Ground Line; and from G, draw Gg, perpendicular, and gC, cutting the Axle in the Center of the Wheel. Make gH and gI each equal to the Radius of the Wheel, and draw HC and IC, cutting a line drawn through the Center (J) parallel to HI, at h and i; by which Diameter (hi) the Circumference may be completed. (Prob. 3. Sect. 8.)

Then, if V be made the Vanishing Point of one of the Spokes (kl) the Vanishing Points of the other are found by making Angles at the Eye, with the Radial of that, equal to the Angles they make with each other; which, in this, are equilateral, having six Spokes, forming a regular Hexagon. (Prob. 24.)

The Winch Wheel (U) touches the Picture; its place being obtained, at K, from the Plan, draw its Intersection Kt, and set up the height of its Center Ks; make rs, and st, each equal to its Radius; and, on the Diameter rt, describe the Circumference, and finish the Wheel from its known Figure.

On the Intersection Kt, of the Face of the Wheel, the height of the Supporters, and also the place of the cross piece (T) may be determined.

The other vertical Wheel, at Y, is got after the same manner, on PQ. Its use is to stop the Crane from going backward, by means of the Catch, at z.

At Z is a Lever, which, by a Cord and Pulley, raises the Catch.

The upper Gudgeon-piece (X) being drawn; find the foot of the Crane, at L, in the central Line; its dimensions being obtained, draw the upright Piece, LM, and set up the measures of the Brace and Headpiece, at d, e, and f.

On the Floor, describe a portion of the circumference of a Circle, perspectively, on the Center L (Radius, the projecture of the Crane) and, according to the direction of the head of the Crane, its Seat being somewhere in that Curve, as at N, draw NO perpendicular, and make NO represent its height from the Floor.

The Rollers, which direct the Cord, are Cylinders; their upper Bases are determined at W, from which, perpendicular Lines are drawn to the head Gudgeon-piece at X, which is level with the Eye, so that, its Plane is not seen.

The Cord, the Pullies, Teeth of the Wheels, the canting of the Timbers, and all such minutias, I shall pass over, as the Figure describes those parts better than the Pen. For those who would be particular, in large Drawings, various Lessons may be found in the Work, for truly projecting every part.

Fig. 1.

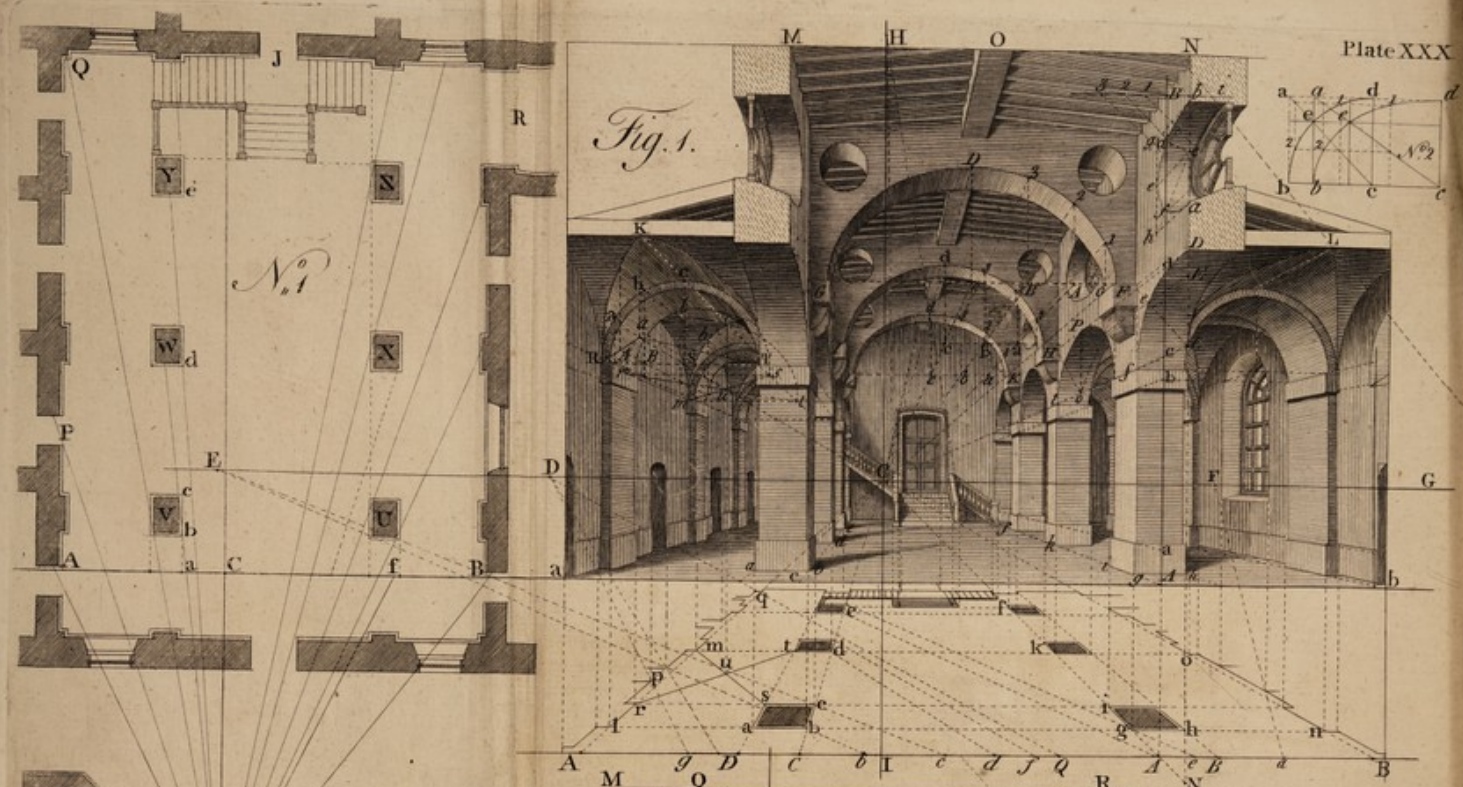


Fig. 2.

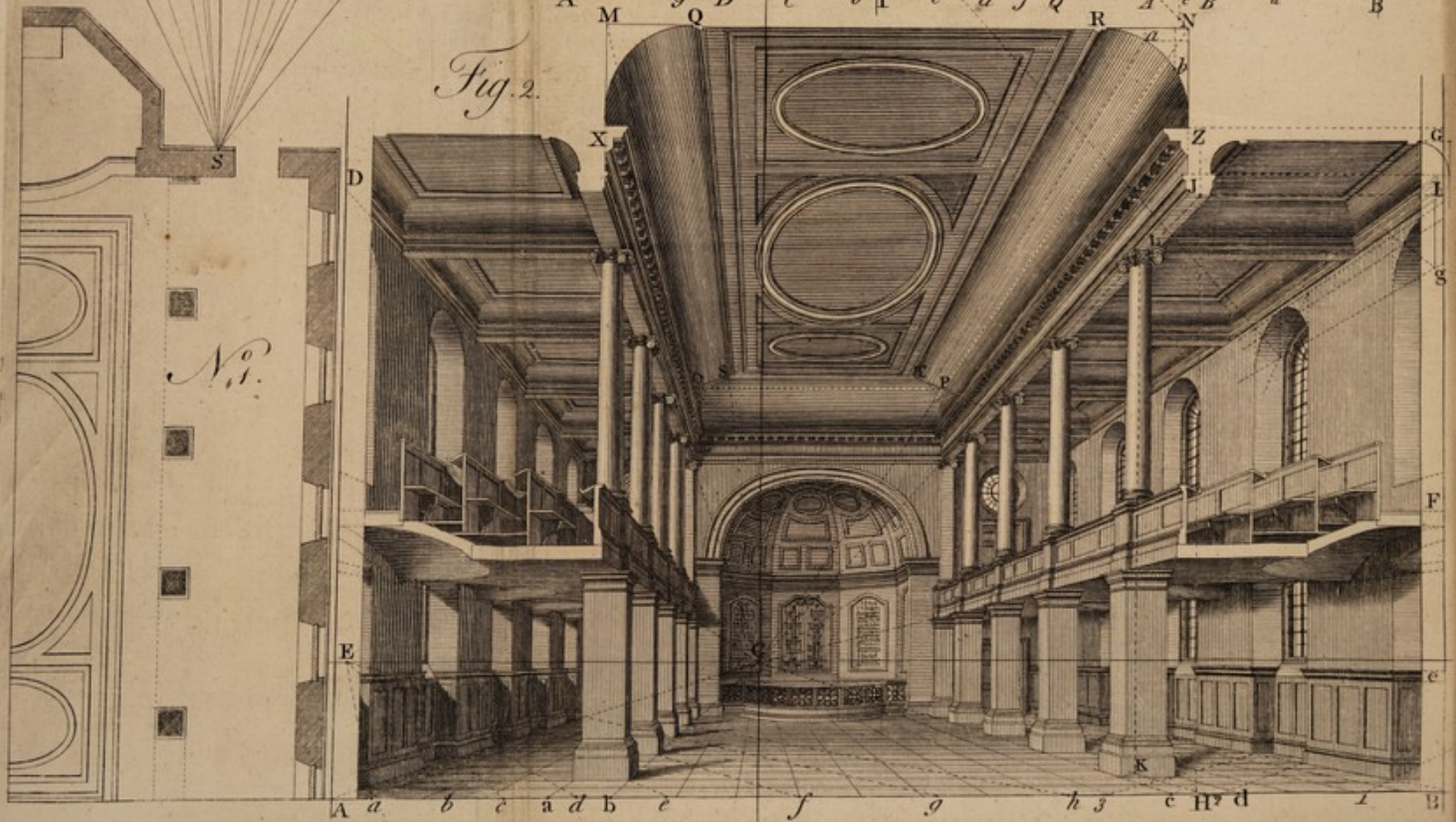


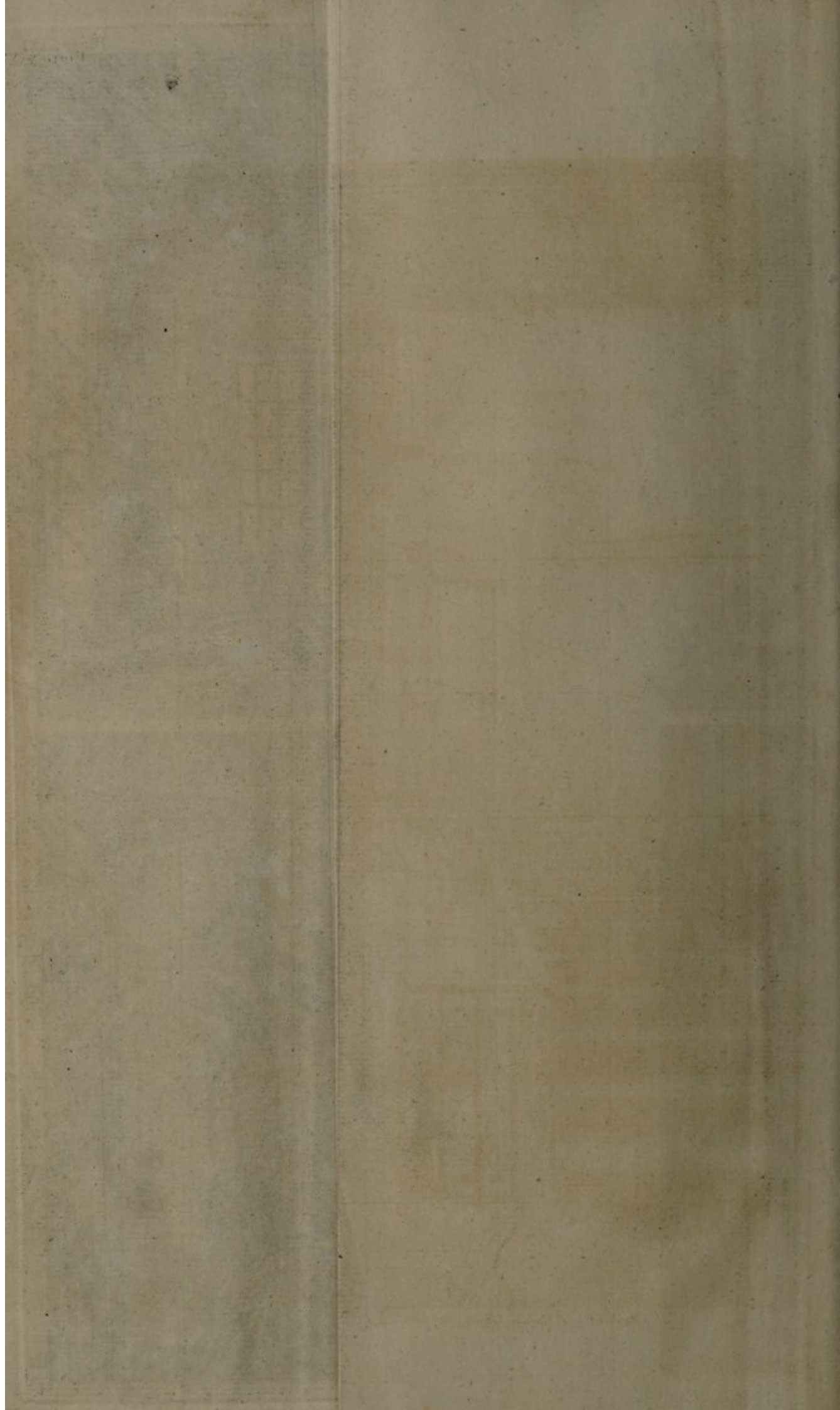


Fig. 1.



Fig. 2.







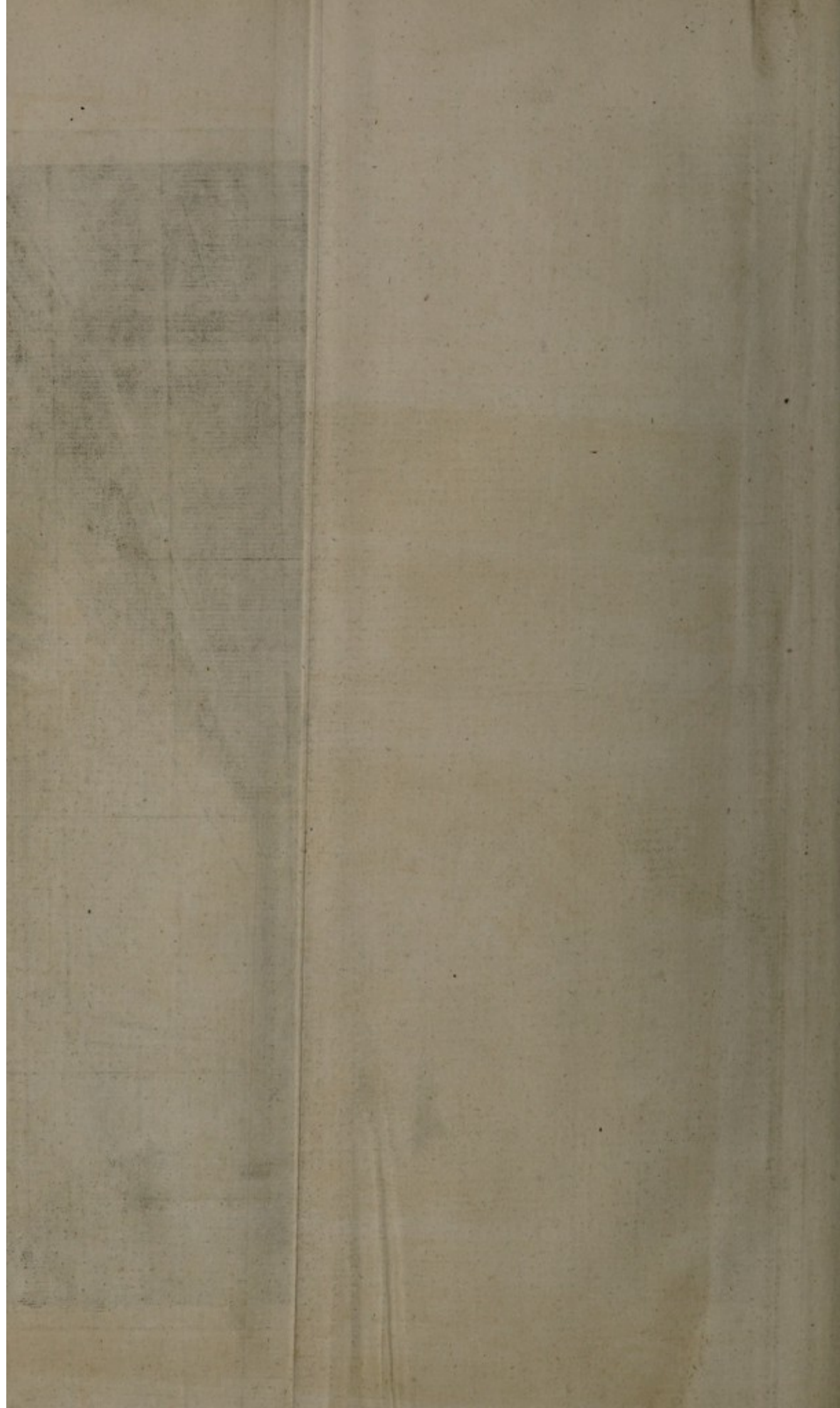


Fig. 125.

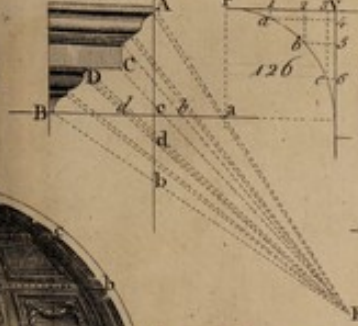


Fig. 126.

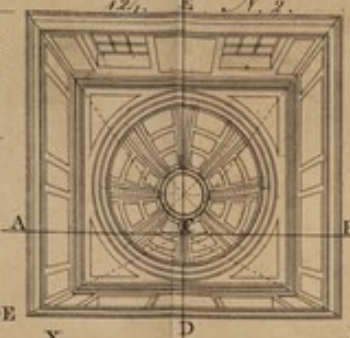


Fig. 124.

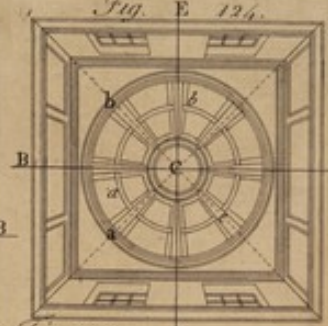


Fig. 123.

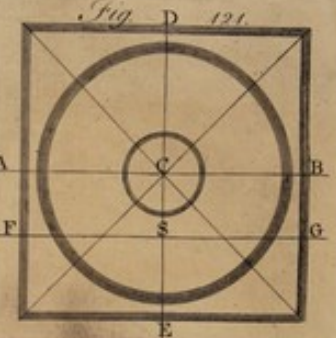


Fig. 122.

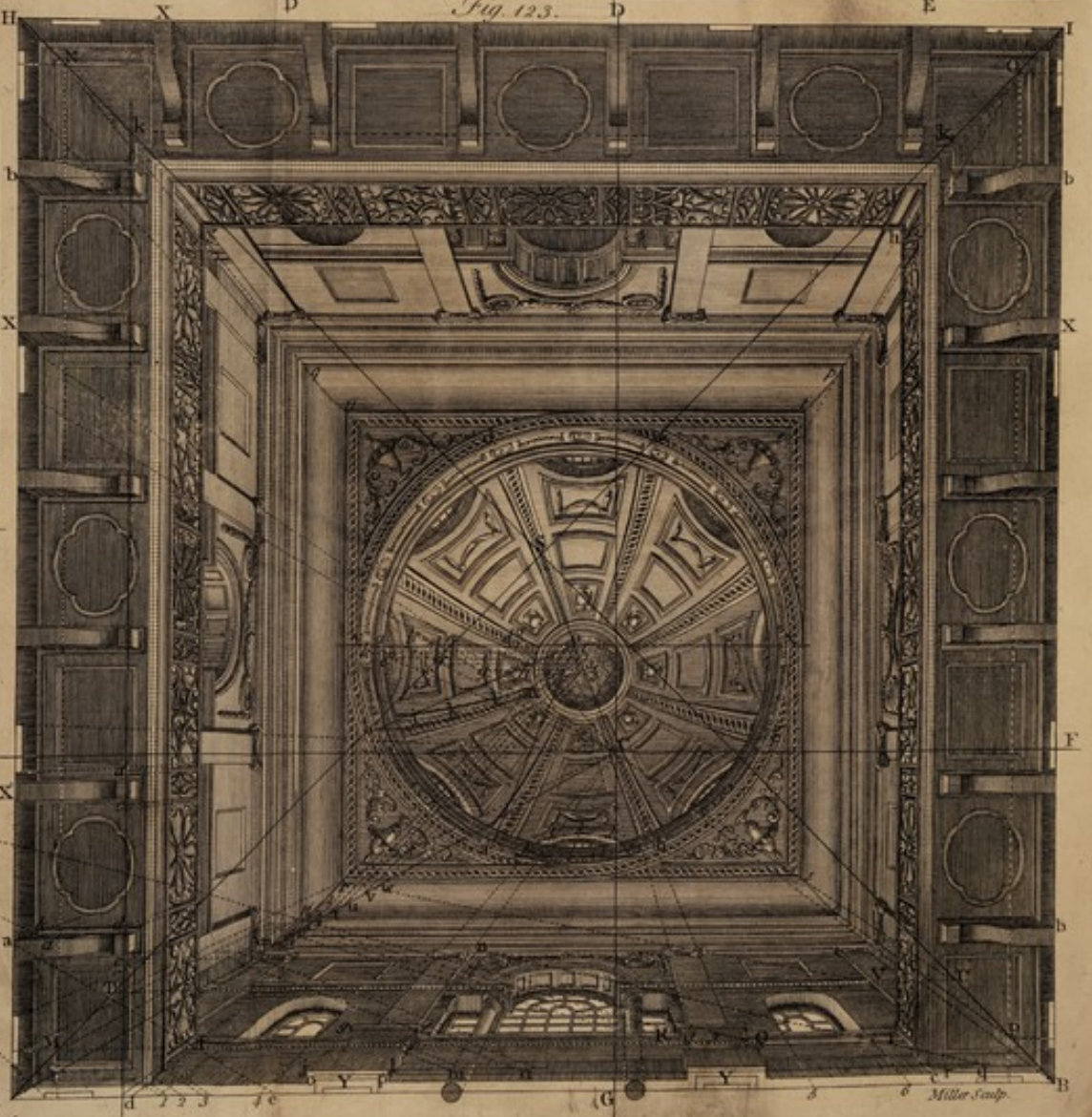
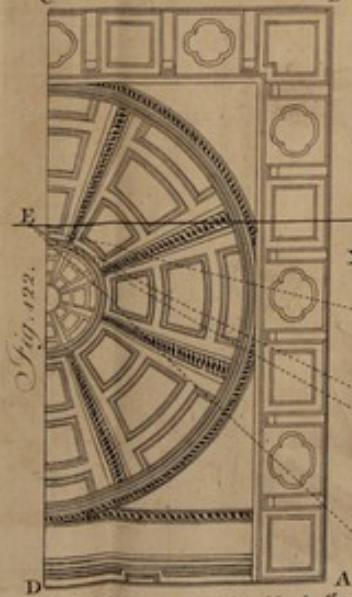
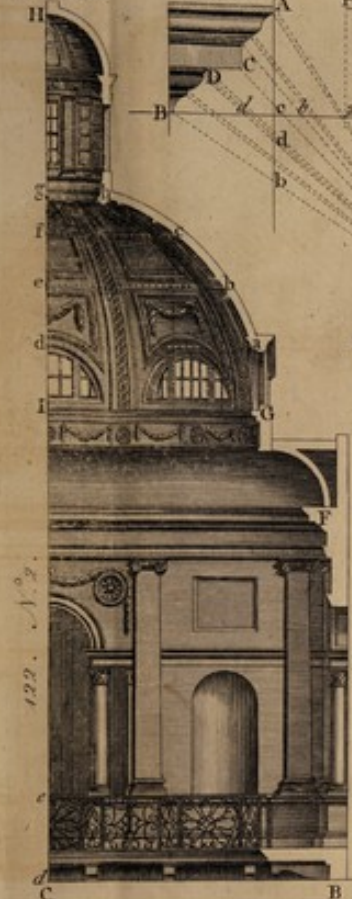




Plate
XXXIV.





Fig. 135.

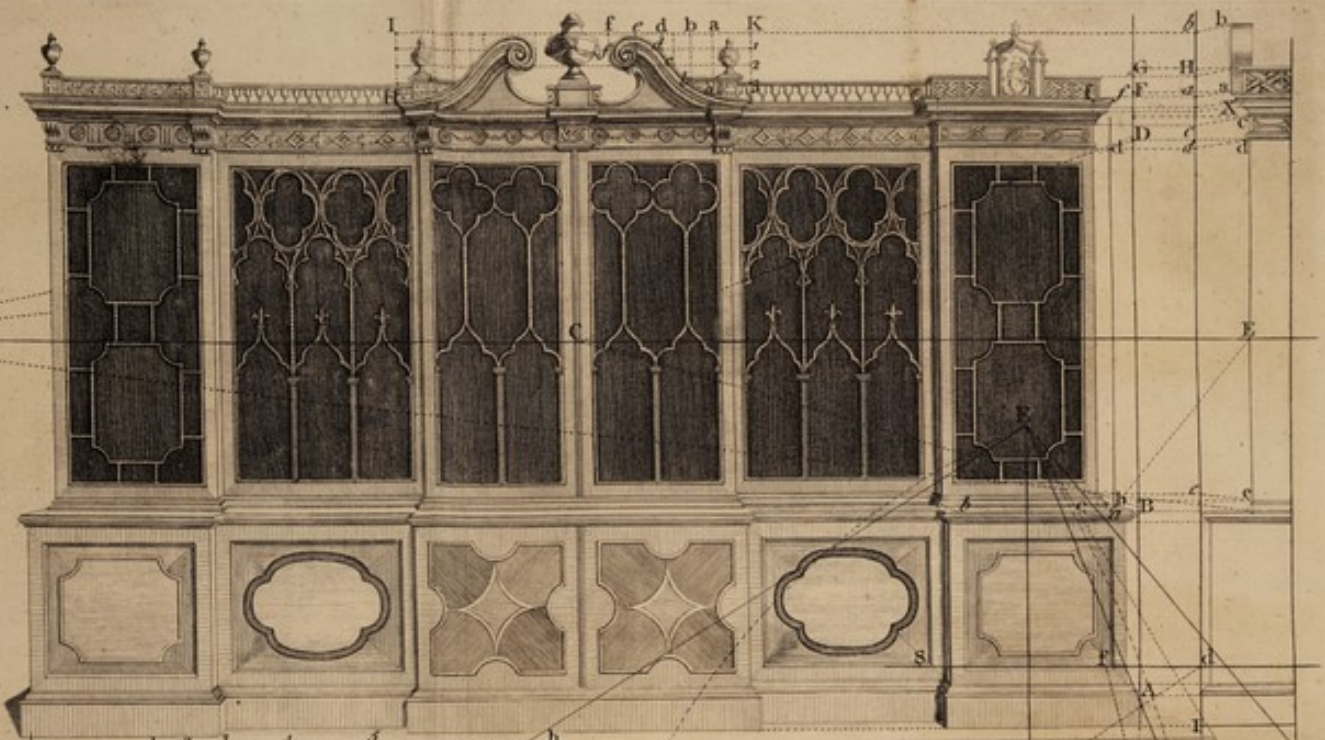


Fig. 136.



Fig. 137.

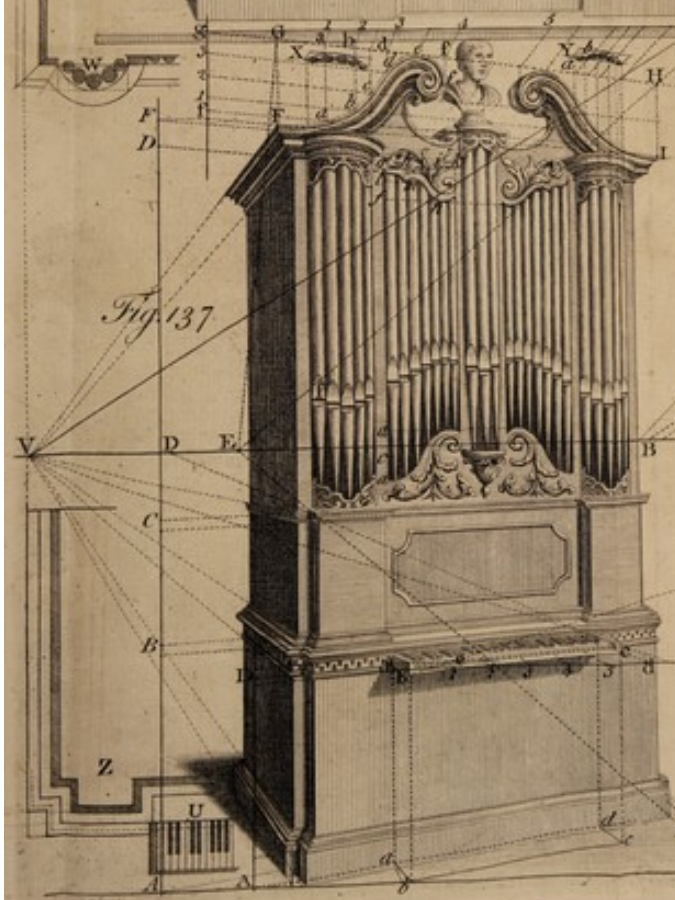
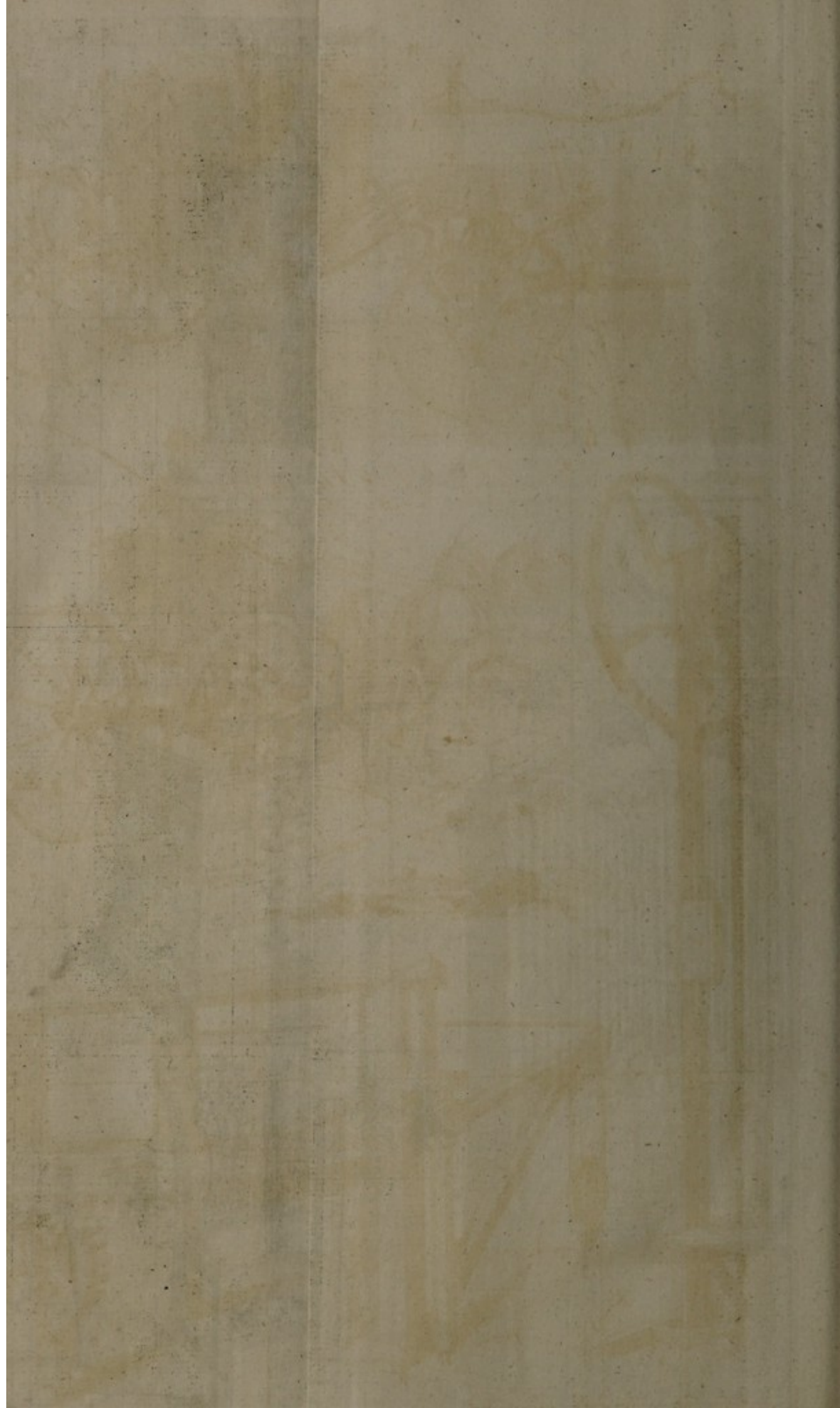


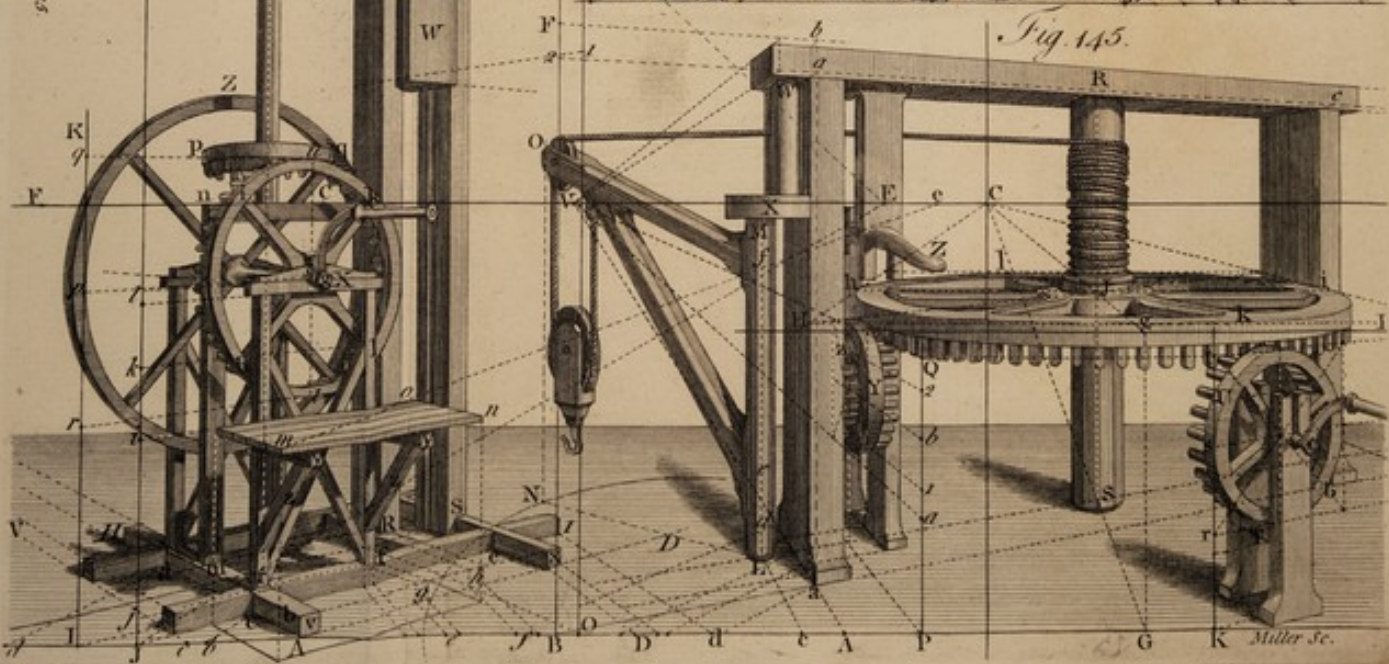
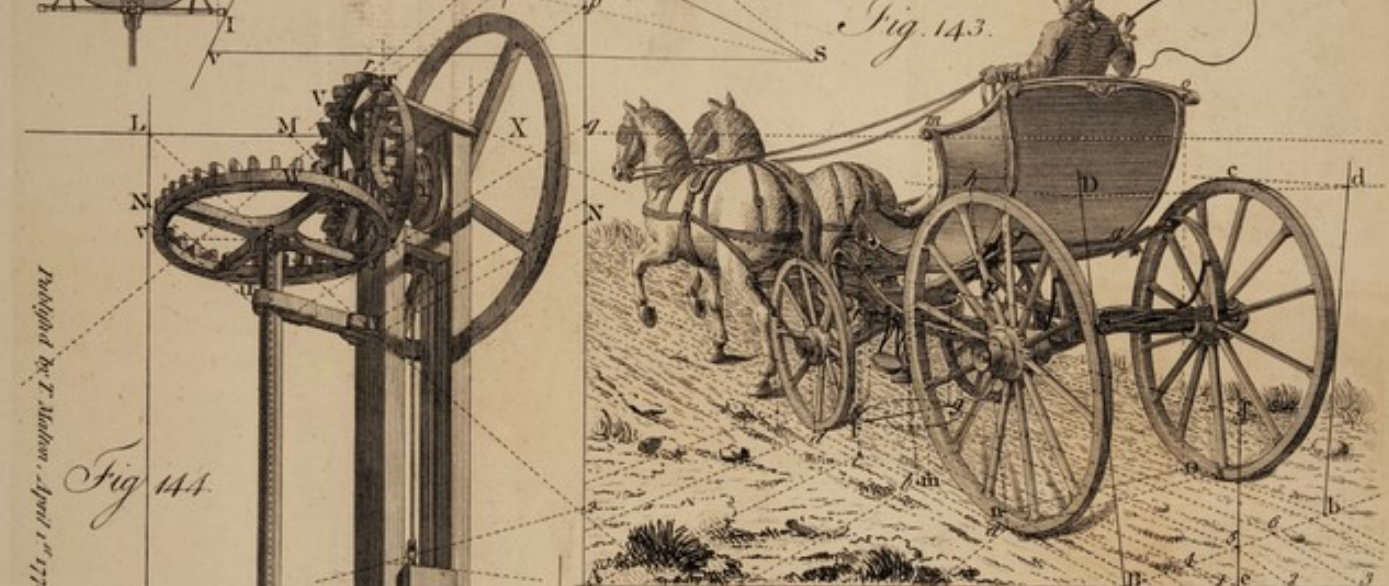
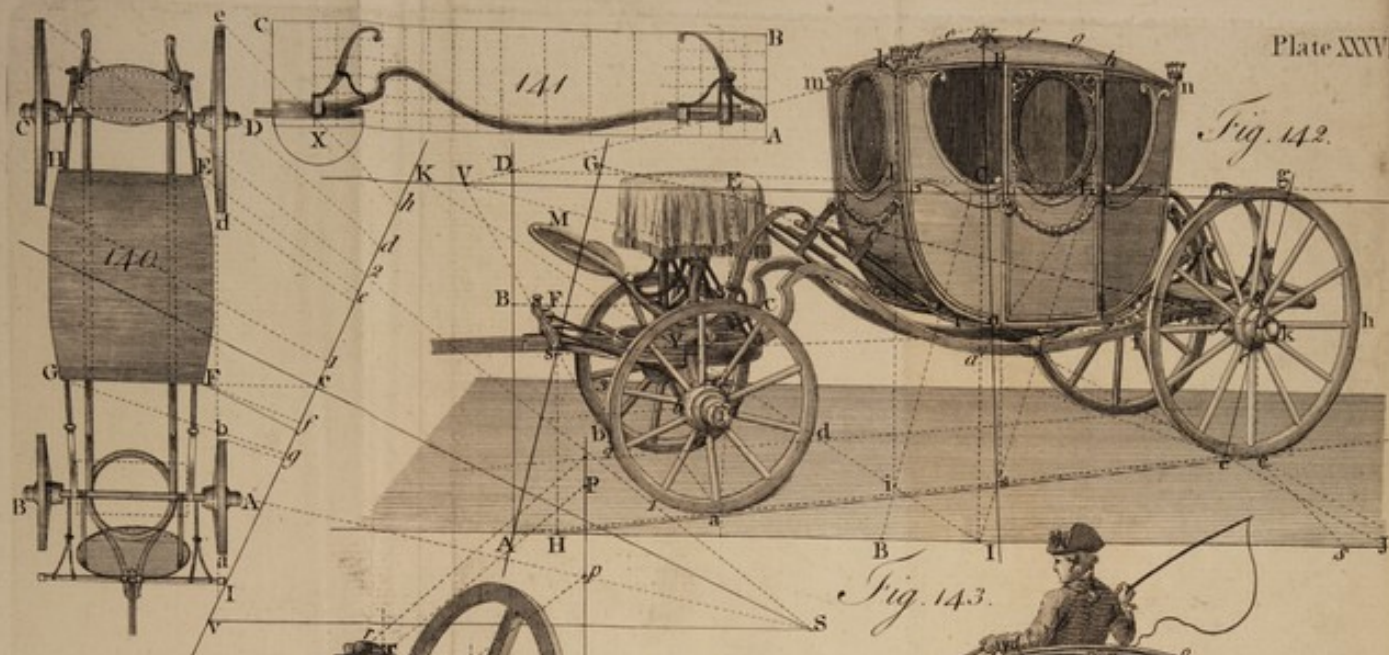
Fig. 139.



Fig. 138.







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K. Muller Sc.



S E C T I O N XII.

On INCLINED PICTURES and PLANES, in general.

AS the Plan which I have hitherto pursued, has been solely confined to the useful study of Perspective, so, this Section is adapted more for the curious Artist than the useful; yet, the inclined Picture may, in some Cases, be necessary. However the universality of the Principles are, here, more fully evinced; it being demonstrated that the position of the Picture to the Horizon, is not of the least consequence, nor has any meaning in the Theory of Perspective, as it has already been shewn, in respect of horizontal Pictures, and is, in this Section, made general.

For, whether the Picture be vertical or horizontal, and the Planes or Lines, in Objects represented on it, be inclined to the Horizon or to the Picture, or both, 'tis the same thing if the Picture be inclined, and the Planes or Lines be either parallel or perpendicular to the Horizon; seeing they are, or may be, nevertheless, inclined to the Picture. Wherefore, however either the one or the other be situated to the Horizon, it matters not, if the inclination of one to the other be known, and the Intersecting Points of any two Lines in either, with the other, be determined; the situation of the Eye being known, in respect of either, as shall be made manifest, to conviction.

P R O B L E M I.

The Intersecting Points of any two Lines in a Plane being given, and the Inclination of the Plane, to the Picture, known; to find the Vanishing Line of that Plane, its Center and Distance; the Center and Distance of the Picture being given.

The Intersecting Points of two Lines, in any Plane with another, being given, the common Intersection of those Planes is determined. (Theo. 10.) Therefore, if A and B are the intersecting Points given, or found, AB is the Intersection of the Plane they are in, with the Picture. (Cor. 3.)

Let A and B be the Intersecting Points of two Lines, in some Plane; and C the Center of the Picture, whose Distance is known.

Draw AB, and CE parallel to AB, equal to the Distance of the Eye; through C, draw DF perpendicular to AB; make the Angle CEF equal to the complement of the inclination of the Plane to the Picture, cutting DC, produced, at F. Through F draw GH, parallel to AB, which is the Vanishing Line required; its Center is F, and its Distance is EF.

Turn up the Triangle CEF, on CF, till EC be perpendicular to the Picture; then, E, is the true Place of the Eye, and EC is the Direct Radial.

DEM. Now, because DF is perpendicular to AB (the common Intersection) and CEF is a Plane, perpendicular to the Picture, passing through DF, it is perpendicular to the other Plane, for, it is perpendicular to their common Section; therefore, DF is the Vertical Line of that Plane, (Def. 11.)

Then, because CEF is the Complement of the angle of Inclination of the Plane to the Picture, EFD is the real Angle, (Cor. 3. 10. 1. El.) for, ECF is a Right Angle, by Construction.

Consequently, since the Vanishing Line of any Plane is parallel to the Intersection of that Plane, (Th. 2) if a Plane be supposed to pass through the Eye, at E, parallel to the Original Plane, it is inclined to the Picture in the Angle, EFD; and the Line GH, in which it must cut the Picture, is the Vanishing Line of that Plane, (Def. 8.) for, the Sections of parallel Planes, by another Plane, are parallel between themselves. 8. 7. El.

But, EF is perpendicular to GH; therefore F is its Center (Def. 19.) and consequently, EF is its Distance (Def. 20.) Also, CF is perpendicular to GH, (Th. 7.) for, it is a Line drawn in the Plane CEF, from the Point F, in which a perpendicular (GH) to that Plane, cuts it. 2. 7. El.

Plate
XXXVII.

COR. Hence the Corollary to the 11th Theorem is manifest; as the Center of the Picture is the Vanishing Point of all Lines which are perpendicular to the Picture; so, the Center of every Vanishing Line is the Vanishing Point, of all Lines which are perpendicular to the Intersection of any Plane, of which it is the Vanishing Line, and of all Lines parallel to them.

Because, EF, producing the Center, (Def. 19.) is parallel to all such Lines.

It must be obvious that this Problem is universal, and cannot possibly regard any position either of the Picture, or of the Original Plane to the Horizon; for, let them be any how situated in that respect, every thing remains the same. But, nevertheless, in many Cases, the position they have to the Horizon is necessary, in practice, to be known; seeing that, all plane Objects, as Buildings, &c. are determined from the position of their Planes to the Horizon; as well as to the Picture; nor is it possible to determine any thing, in respect of their Projections on the Picture, without the previous determination of some Plane or other in respect of them and the Picture, and which is best done, by means of a Plane perpendicular to the Picture, either horizontal or vertical.

PROBLEM II.

The Center and Distance of the Picture being given, and the Vanishing Line of some Plane which is inclined to the Picture; to determine the Inclination of the Plane, and to find the Vanishing Point of Lines perpendicular to that Plane.

Fig. 147. Let AB be the Vanishing Line given, and C the Center of the Picture.

Through C, draw DF perpendicular to AB, indefinite; draw CE perpendicular to DF, i. e. parallel to AB; make CE equal to the Distance of the Picture, and draw ED; also, EF, perpendicular to ED.

EDC is the angle of the Inclination of the Original Plane, to the Picture; and F is the Vanishing Point, of Lines perpendicular to the Plane.

Turn up the Triangle DEF perpendicular, also turn over the Plane X on AB.

DEM. Because CE is perpendicular to the Picture, and equal to its Distance, EC is the direct Radial, and E is the Point of View (Def. 15 and 16) and, because the Plane AEB passes through the Eye, and the Vanishing Line (AB) it is parallel to the Original Plane, (Def. 8.) and its Inclination to the Picture is the same, equal EDC.

But, the Plane DEF is also perpendicular to AEB, and consequently to the original Plane; therefore, it is the Vertical Plane, (Def. 5.) and DF is the Vertical Line, (Def. 11.) which makes equal Angles with them both; and, because it is the Intersection of a Plane cutting two parallel Planes, perpendicularly; therefore, the Angle EDF is equal to the inclination of the Original Planes.

But, EF is in the Vertical Plane, and it is perpendicular to ED (by construction) consequently, EF is perpendicular to the Plane AEB, and consequently it is parallel to all Lines perpendicular to the Original Plane; and therefore, F is their Vanishing Point. (Def. 22.) Q. E. D.

PROBLEM III.

The Center and Distance of the Picture being given, and the Vanishing Point of Lines, which are perpendicular to some Plane, to find the Vanishing Line of that Plane.

Fig. 147. C is the Center of the Picture, and F is the Vanishing Point given.

Join FC, and produce it; draw CE perpendicular to DF, and equal to the Distance given; draw EF, and ED perpendicular to EF, cutting FC, produced, at D; and through D, draw AB perpendicular to DF.

AB is the Vanishing Line required; which is manifest, from the foregoing.

PROBLEM IV.

The Vanishing Line of a Plane being given, and the Vanishing Point of the Intersection, of that Plane, with another Plane, perpendicular to each other; with the Center and Distance of the Picture; to find the Vanishing Line of the other Plane.

Fig. 147. Every thing remaining as in the last Figure; let AB be the Vanishing Line given, and A the given Vanishing Point.

Find the Vanishing Point, F, of Lines perpendicular to any Plane, whose Vanishing Line is AB (by Prob. 2.) and draw AF the Vanishing Line required.

DEM.

DEM. For, since the Plane, whose Vanishing Line is required, is perpendicular to that Plane whose Vanishing Line is AB, F will be the Vanishing Point of Lines perpendicular to the other Plane; and A is the Vanishing Point of some Line in it, by Supposition; consequently, AF is the Vanishing Line sought.

Th. 10.

N. B. If the Vanishing Line given was of a Plane perpendicular to the Picture, seeing it would pass through the Center of the Picture. (Theo. 4.) wherefore, Lines perpendicular to it, would have no Vanishing Point; for EG, perpendicular to EC, being parallel to DF (a Line in the Picture) can never cut the Picture and produce a Vanishing Point; consequently, AF, the Vanishing Line (in that Case) would be perpendicular to AB.

P R O B L E M V.

The Vanishing Line of a Plane being given, and the Vanishing Point of the common Intersection of another Plane with that Plane, whose Inclination to the former is known; to determine the Vanishing Line of the other Plane; the Center and Distance of the Picture being given.

Let AF be the given Vanishing Line; and F the given Vanishing Point.

Fig. 148.

Find AB, the Vanishing Line of a Plane, to which, Lines whose Vanishing Point is F are perpendicular (by the third.)

Make DG equal to DE (the Distance of AB) and draw AG; also draw GB, making the Angle AGB equal to the inclination of the two Planes, cutting AB at B, and draw BF, the Vanishing Line required.

DEM. If the Triangle DEF be turned up, as before, and the Plane AGB be also turned over on AB, DG will coincide with DE. Then, if the Plane AHF be turned over, on AF, AH will coincide with AG, and AHF will be parallel to the Original Plane, whose Vanishing Line is AF (Def. 8.) seeing it passes through the Eye, E, and the Vanishing Line.

But, AGB is perpendicular to AHF (9. 7. El.) for EF is perpendicular to AGB; consequently the Angle of inclination of any other Plane, with that Plane, may be determined in the Plane AGB.

But, AGB is the Angle given, of the Inclination of the Planes; wherefore, if BIF be turned over on BF, till BI coincides with BG; then BIF, passing through EF, is also perpendicular to AGB; consequently, it inclines to AHF in the Angle AGB.

But, Planes which produce the Vanishing Lines of two Original Planes are inclined to each other as the Originals; and, their common Intersection passes through the Eye, parallel to the common Intersection of the Original Planes.

Theo. 6.

Wherefore, EF is parallel to the common Intersection of the Original Planes; consequently, F is its Vanishing Point, (Cor. 2.) for, it is the Intersection of the Vanishing Lines; and BF, the Line in which BIF cuts the Picture, is the Vanishing Line sought; (Def. 8.) for, BIF is parallel to the Original Plane, and passes through the Eye, at E. Q. E. D.

N. B. If AGB was a Right Angle; then, AF, AB, and BF, would be the Vanishing Lines of a solid Right Angle; each Plane, of which, being inclined to the Picture, respectively, as the Planes AGB, AHF, and BIF; all which pass through the Eye, at E.

The Center of each Vanishing Line is where a Perpendicular from C cuts it; as D, K, and L, (Theo. 7.) and DG, KH, and IL are their Distances respectively; G, H and I, are considered as the Eye for each Vanishing Line, in the application of them to practice; each Distance being the Hypothenuse of a right angled Triangle on CE; with CD, CK, and CL, respectively. (Def. 20.)

Note. When the given Vanishing Line passes through the Center of the Picture, it is the Vanishing Line of a Plane perpendicular to the Picture, which also passes through F, as DF; in which Case, A and D coincide; and, the Angle is made with DG, as DGA or DGB; according to which side of that Plane the other Plane inclines.

P R O B L E M VI.

The Intersection of any Plane, which is perpendicular to the Picture, being given, and the Intersecting Point of the common Intersection of that Plane with another Plane, whose Inclination to it is known, and, the Angle which their common Intersection makes with the Picture; to determine its Intersection and Inclination to the Picture.

Turn over the Plane Y out of the way, it being of no use in this Problem.

Let AB be the Intersection given, and A the Intersecting Point.

Make Fig. 149.

Plate
XXXVII.

Make BAE equal the Angle which the common Intersection of the two Planes makes with the Picture; and, at any point (E) in AE , make a right Angle, AEB . Make BED equal to the angle of Inclination of the two Planes, and draw BD perpendicular to BE , cutting ED .

Draw BG perpendicular to AB ; make BG equal to BD , and draw AG .

AG is the Intersection of the inclined Plane, with the Picture.

Turn up the Picture (X) on AB , perpendicular, and the Triangle BDE , on BE , also perpendicular; then, BD will coincide with BG ; also, turn over the Triangle AIE ; they will form a solid Angle, at the Point E .

DEM. BAE is the given Angle, which the common Intersection, AE , makes with the Picture; and, the inclination of the Plane AIE to AEB (which is perpendicular to the Picture, cutting it in AB) is BED ; consequently, AG is its Intersection with the Picture.

For, if the Plane AIE be supposed continued; beyond the Picture, it will cut the perpendicular Plane in a continuation of EA , making equal Angles on the other Side; wherefore, A is the Intersecting Point of their common Intersection, and the Intersection AG remains the same.

Secondly. From any Point (E) in AE , draw EC perpendicular to AB , and CH perpendicular to AG , cutting it in H ; make CF equal CH and draw EF . CFE is the Angle of inclination of the inclined Plane to the Picture.

DEM. For, if the Picture be turned up, and the Plane AIE , meeting it in AG ; then EH is the Hypotenuse of a right angled Triangle CHE , congruous with CFE ; consequently, those Angles are equal.

But, CH and HE are both perpendicular to the common Intersection of the Plane AIE with the Picture, therefore, CHE (equal CFE) is the Angle of its Inclination to the Picture.

This Problem, is general; for, the Intersection, AB , given, is not, necessarily, either horizontal or vertical; nor is the Picture necessarily either, all that's required is, that the Plane be perpendicular to the Picture, whose Intersection is given.

But if it was inclined to the Picture, that inclination being known, and which Side it inclined on, the rest is determinable; whereas, the given Intersection being of a Plane perpendicular to the Picture, it cannot possibly be misunderstood and applied.

In Prob. 5. Sect. 3. is shewn how to find the Vanishing Line of a Plane which is inclined to the Horizon, and to the Picture, when its Intersection with the Horizon is also inclined to the Picture, as in the last Problem. The horizontal Plane, being considered, simply, as a Plane perpendicular to the Picture, the Problem becomes general, and universally applicable; however that Plane or the Picture be situated in respect of the Horizon.

I shall here, as it is there proposed, give a brief Demonstration of that Problem, which will now, I presume, be better understood, the foregoing being previously necessary; and the assistance of moveable Planes will render it far more intelligible.

Let the Picture be turned up perpendicular, and the Plane Y perpendicular to it; also, turn over the Plane V , till FB coincides with EB ; and W being turned over, AE will coincide with AE , &c.

The Planes being thus constructed, let the former be also placed as was directed.

Then, if the Original Plane, Z , be horizontal, Y , i.e. AEB being parallel to it, is the Horizontal Plane; or, Z being considered, simply, as a Plane perpendicular to the Picture, Y is its Vanishing Plane; for AB the Vanishing Line, produced by it, passes through C , the Center of the Picture (Th. 4.)

The Plane W passes through the Eye, parallel to AGE , and therefore produces its Vanishing Line, AG ; and, EB being parallel to EB , also, the Angle BEG being equal to BEG , EG is parallel to EG ; and consequently, G is the Vanishing Point of EG (Def. 22.) also, EA being parallel to EA , A is the Vanishing Point of EA . Wherefore, since A and G are the Vanishing Points of two Lines in the Plane, EA , and EG , consequently, AG is the Vanishing Line of the Plane those Lines are in. (Theo. 10.)

Also, because E is the Eye, and EA , EG , are parallel to EA and EG , respectively; therefore, the Plane AEG is parallel to AEG (7. 7. El.) and it passes through the Eye, at E ; therefore, AG is the Vanishing Line of the Plane. (Def. 8.)

Now, if the Plane V , be turned on BG into the Plane of the Picture, on either Side, BF being equal to BE , and the Angle BFG equal BEG ; consequently, the Point G is the same, however the Plane V be situated to the Picture, in its revolution on BG ; therefore, the Vanishing Point G , is truly ascertained; and consequently, the Vanishing Line AG ; being parallel to the Intersection AG of the Original Plane, also BG to BG . (Theo. 2.)

For, the Original Planes, being situated any where on the other side of the Picture, being parallel, respectively, to their present Situation, their Vanishing Lines are the same. (Theo. 3.)

P R O B L E M VII.

The Inclination of a Line to the Picture being given, and the Angle of inclination of any Plane that Line is in, to the Picture; to determine the inclination of the Line, to the Intersection of the Plane it is in, with the Picture.

Let AB be the given Line (in the Plane AEB) inclined to the Picture in a given Angle. Fig. 150.

Make ABD equal to the Angle known, and, from any Point in AB, draw AC perpendicular to BD; make the Angle CAD equal to the Complement of the inclination of the Plane to the Picture, and ADC, is the real Angle.

On AB describe a Semicircle; and, with the Radius AD, describe the Ark DE, cutting the other at E; i. e. make AE equal AD; draw EB, and ABE is the Angle required.

Draw AE and produce it; make EF equal to CD, and draw BF.

DEM. Turn up the Triangle ADC, on AC, and ACB, on AB, till AD coincides with AE; then, turn up the Plane X, till BC coincides with BF.

Now, X being considered as the Plane of the Picture, and ACB as a Plane passing through the Line AB perpendicular to the Picture, ABC is the angle of its inclination to the Picture, given. But, ADC is the inclination of the Plane it is in, to the Picture, and EB is the Intersection of that Plane with the Picture; consequently, ABE is the inclination of the given Line, AB, to the Intersection, EB.

From these Problems it must be obvious, that the position of the Picture, to the Horizon, is of no consequence in the Theory of Perspective; but is very much so in common Practice; because, all perpendicular Lines, in Objects, are parallel to the Picture, being vertical; and horizontal Lines are easily determined, whether perpendicular or inclined, their inclination being known. To an inclined Picture, their inclination is the Angle they make with their Seats, on the Picture.

N. B. The Vanishing Line or Intersection of some Plane in the Object, must be given, to determine others, if necessary; for which end, the Center and Distance of the Picture are absolutely necessary. The Vanishing Line of horizontal Planes, being perpendicular to the Picture, is therefore first determined, and the Intersection of the Ground Plane; which of all other are fittest. Without them, there would be great difficulty in proceeding. For want of that consideration, the work of Dr. Brook Taylor is almost useless to a Practitioner; he not giving, properly, one specimen, how to find the representation of a Line, from its known length, situation, and place, in respect of the Picture; but only, by means of the Intersecting Points of other Lines, and drawing Visual Rays, from the Eye to the Original Points, in their true places, which is not practical in many cases; or, to determine the Figure from some Line given in it, on the Picture; so that, the Student knows not how or where to begin the process, from the geometrical proportions of the Object, and its known or determined place and position to the Picture, and to the Horizon.

E X A M P L E I.

To find the representation of a right angled Parallelopiped, whose Sides are known, situated at some distance from the Picture; whose Faces are all inclined to the Picture, the inclination of one being determined, and the inclination of one Side in that Face, to the Picture. The Center and Distance of the Picture, with the Seat, and distance of the nearest Angle of the Object, to the Picture, being given.

Let C be the Center of the Picture, and S the given Seat of the hither Angle.

Find a, the representation of that Angle. (Prob. 6. Sect. 4.) Through C, draw DF, at pleasure, and CE, perpendicular to DF; make CE equal to the Distance of the Picture, and the angle CED equal to the Complement of the given Angle of inclination of a Face to the Picture.

Through D, draw AB perpendicular to DF (the Vanishing Line of that Face, Prob. 1.) and, perpendicular to DE draw EF; F is the Vanishing Point of Lines perpendicular to that Plane. (Prob. 2.)

Find the representation of that Face whose Vanishing Line is AB, D is its Center, and DE its Distance. (Prob. 21.) as follows.

Produce FD; make DG equal to DE, and draw GH parallel to AB.

Make

Plate

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Make the Angle HGA equal to the inclination of the given Line to the Intersection of that Face, whose Vanishing Line is AB (Prob. 7.) and, make AGB a right Angle; A and B are the Vanishing Points of its Sides; and, F being the Vanishing Point of Lines perpendicular to that Face, AF, and BF are the Vanishing Lines of the other Faces; as it is manifest by Problem 4.

Draw aA, aB, and aF, the indefinite Representations of three Sides, forming the hither Angle; how to proportion them, I shall shew, as follows.

The measures of those Sides are known, and the distance of the Angle a, from the Picture; which was found by its given Seat, S, and its Distance Sa.

Now, A, B, and F, are the Vanishing Points of the Sides of the Object, any one of which may be in the same Plane with SC (Ax. 6.) wherefore, ACS is the Vanishing Line of such a Plane. (Theo. 10.)

Make ab to represent a Line in proportion to that which Sa represents, as the Original of ab is to the distance of the Angle a (Prob. 10.) thus.

Make CE equal to CE, and perpendicular to AC; draw AE, and produce it; make EM, to EN, in the ratio of the Side, to the distance of the Angle a; join MN, draw EJ parallel to MN, and draw SJ, cutting aA at b; so shall ab represent a Side of the Object, whose length and inclination to the Picture was given.

Draw bB and bF; and, by means of the Radials AG and GB, make ad, or bc, to represent a Line, in proportion to the Original of ab, as one Side of that Face is to the other (Prob. 10.) viz. as GI to GK; and draw Ad (through c) and dF.

The Face abcd, whose Inclination was given, being completed; find e or g, so, that de, or bg, shall represent the proportion of the other given Side, by means of ab, or ad, as before; the Center and Distance of either Vanishing Line being determined, (by Prob. 1.) as S, the Center of BF, by a Perpendicular from C (Th. 4.) and SO the Distance, in AS produced; making BO equal BG, and joining OF, BO and OF are the Radials of the Sides in that Face, forming a right Angle BOF, at the Eye.

Make OP to OQ in the ratio of the Originals of ad to de; draw PQ and OR parallel to PQ. Draw aR which will cut dF at e; and, through e, draw bF, cutting aF at f; and lastly, draw fA, cutting bF at g, which compleats the Parallelopiped, a b d f, required.

For, because of the Vanishing Points A, B, and F, the Sides ab, ad, and af, form a solid Right Angle, at a; AGB, ATF, and BOF being Right Angles, which bad, baf, and daf represent, respectively; and the other Sides vanish in those Points, respectively; as dc and de, &c. which represent Parallels, to ab, af, &c. (Cor. to Th. 3.) Therefore b d f represents a right angled Parallelopiped, whose proportion was known, and position to the Picture determined.

ac, ae, and ag, represent Diagonals, in each Face, respectively, which are in proportion to the Sides, as IK, PQ and TV, respectively, to GI and GK, OP and OQ, TX and TY, respectively.

The Parallelopiped, b d f, is truly determined, according to its position given, in respect of the Picture, its place in respect of the Eye, and its proportion in respect of its Distance; no regard being had to its position respecting the Horizon. Wherefore it is obvious, that, to determine its Position, in that respect, the horizontal Vanishing Line is essentially necessary, and the Vanishing Lines of its Faces are determined in respect of their position to the Horizon, as well as to the Picture, as shall be exemplified in the next.

E X A M P L E II.

To represent an Octaedron, perspective, situated on a Plane inclined to the Horizon, and to the Picture; the angle of Inclination to the Horizon, being given, and the Angle which its Intersection with horizontal Planes, makes with the Picture; together with the Seat of the Object on the inclined Plane; and, its situation in respect of the Picture.*

Fig. 152.
No. 2.

Let ABC (No. 2.) be the Seat of that Face on which the Object rests, on the Plane Z, which is inclined to the Horizon in the Angle X; DF is its Intersection with a horizontal Plane, and DFG the Angle that Intersection makes with the Picture; FG is the Intersection of the Picture with the horizontal Plane. The Station Point is S, the Distance is SG, and SE the Height of the Eye.

* An Octaedron is a regular Solid, one of the five Platonic Bodies, having eight Faces, which are equal, equilateral Triangles; about which Solid, if a Sphere was circumscribed, every Angle of the Solid would be in the Surface of the Sphere.

No. 1.

Its geometrical Construction is necessary to be understood, before it be possible to describe it perspective. Its geometrical Plan, on the Plane on which it rests, is a regular Hexagon, AEBFCD (No. 1.) ABC is the Face, on which it rests. DEF is the upper Face, EAD, EBF, and FCD are inclined Faces, above; and AEB, ADC, and BFC, below, out of sight; IGHC is its geometrical Elevation, or Section through EC, shewing the Inclination of its Faces, to each other; viz. the Angle GIK equal IGH.

These preliminaries being determined, let AB be the Intersection of the horizontal Plane, or Ground Line; and, at the height of the Eye (SE, No. 2.) draw the Horizontal Line, DF, parallel to AB; let C be the Center of the Picture.

Draw CE perpendicular, equal to the Distance of the Picture; and find the Vanishing Point, D, of the inclination given. (Prob. 2. Sect. 3.) Make DEF a right Angle, and through F, draw FG perpendicular to DF. FG is the Vanishing Line of a Plane, to which, Lines vanishing in D are perpendicular. (Prob. 3.)

Make FE equal EF, and the Angle FEG equal X, (No. 2.) and draw DG, the Vanishing Line of the inclined Plane, (as by Prob. 5. Sect. 3.) indefinite.

Produce EC, cutting AB at S; make SB equal GF (No. 2.) draw BD, the indefinite Representation of DF, (No. 2.) and find the Points a and b, representing a and b, in which, BA and CA (No. 2.) cut DF. (Prob. 8.)

Find the Center (C) and Distance of the Vanishing Line DK (Prob. 1.) make CE* (in CC produced) equal to its Distance, and draw DE*.

Make the Angle DEH equal BaD (No. 2.) H is the Vanishing Point of AB (No. 2.)

Make the Angles HEI and IEK each of 60 Degrees; H, I, and K are the Vanishing Points, of the Sides of the two Faces ABC and DEF (No. 1.) i. e. of AB, AC; and BC, (No. 2.) and consequently of the Sides of the opposite Face.

Find the representation of that Face, a b c (Prob. 18) having obtained the Points, a, b, and c (representing a, b, and D, No. 2.) by means of the Intersection AB; making DE*, equal DE; or, by the Intersection BJ, of the Face, a b c.

Find the Vanishing Line HM, of the contrigine Face, whose common Intersection, with that found is a b, and its Vanishing Point H. (by Prob. 5.) Its Inclination is the acute Angle GIK (No. 1.) determined by a perpendicular from E; and IG being made equal IC; which Angle is 70 Degrees.

Find the Center (O) and Distance (OE*) of the Vanishing Line HM, (Prob. 1.)

Draw E*H, and find the Vanishing Points P and Q; as I and K, above; or make equilateral Triangles, X and Y, at the Eye (E*) and produce their Sides to the Vanishing Line H M, cutting it at P and Q.

Through a and b, draw Qa, and Pb, cutting at d; giving a b d for that Face.

The Point d is in the upper Face, and the Sides of opposite Faces are parallel, respectively, two and two; therefore, draw d I, and d K; and Pc, cutting d K at e; draw e H and join a e, which compleats the Figure.

N. B. This Figure having eight Faces, two and two of which are parallel, there are consequently (if none are parallel to the Picture) four Vanishing Lines; but here are only two required, DL; and HM; a third will pass through I and P, and the fourth through K and Q, which, being produced, would meet in the Vanishing Point of a e.

It may be observed, that the three Sides of each Face, vanish in its respective Vanishing Line, (Theo. 10.) and the Vanishing Point of the common Intersection, of any two Faces, is the Intersection of the Vanishing Lines of those Faces, (Cor. 2. Theo. 6.) as H, of a b and e f, the Intersection of the Vanishing Lines DL and HM; and K, of DK and QK, the Vanishing Point of d e, and b e.

The next Figure exhibits two of the same Objects, on a level Plane (or any Plane perpendicular to the Picture) situate alike to the Picture, but on different sides of the Station Line; the Sides, in both, have consequently the same Vanishing Points, and, their Faces have the same Vanishing Lines, respectively.

AB is the Vanishing Line of the Faces which are perpendicular to the Picture, and C its Center; a g is the Intersection of one of those Faces. The Angle a, of one Object, touches the Picture, the other is at some distance beyond it.

FG being the Vanishing Line of a Plane perpendicular to the common Intersection a b, and FE (equal EF) its Distance; the Angle FEG is made equal to that of a diagonal Plane with the Faces, (equal HIC, No. 1.) viz. 55-Degrees. AG is the Vanishing Line of the diagonal Plane a b c d, which is a Square.

The Vanishing Points, A, H, and B, of the Sides a b, a f, and b f, or c d, being found (as above) and EK drawn, perpendicular to EG, cutting GF, produced, at K, the whole is determined. BK and GH intersect at I, the Vanishing Point of e b and d f; and BG, KH, and I A, being produced, would meet in the Vanishing Point of a e and f c.

Fig. 142.

Fig. 143.

Plate
XXXIX.

AB, AI, BG, and GI, are the Vanishing Lines of the Faces; and AG, BI, and KL of the three Diagonal Planes. The rest the Figure describes.

The Vanishing Lines of the Faces produce, by their Intersections, the Vanishing Points of every Side of the Figure, viz. six, A, B, G, H, I, and, if BG and IA were produced, they will meet in the Vanishing Point of a e and f c.

These Examples are sufficient, for finding the Vanishing Lines, and applying them to use, in projecting Figures in inclined Planes, in such Objects. I do not intend to lead the Reader through all the Mazes, attending the projection of the Dodecaedron and Icosaedron, because I know no end it can answer to any Person; such Objects never coming in the course of their several Studies. But, if any Person be inclined to amuse himself with them, he may find Rules in this Section and the 5th. for his purpose, in every position he can devise. As they require a geometrical construction, previous to the projecting them perspectively, I recommend the Reader, who has curiosity, to the Work of Mr. Hamilton; or, to the ingenious Mr. Highmore, they being foreign to my Plan.

E X A M P L E III.

How to represent a piece of regular Fortification, in Perspective.

The foregoing Examples are calculated more for Lessons than real use; in this, the knowledge and application of inclined Planes will be found necessary.

In Fortification or military Architecture, as the Walls of the Polygons and other Outworks are all inclined to the Horizon, and mostly to each other, it seems to be the fittest Subject of any I know. Indeed, the vertical Planes of other Buildings are, frequently, as much inclined to the Picture, but, their position to the Horizon familiarizes them, on a vertical Picture; and yet, in reality, there is no difference, in Theory, and very little in Practice.

Fig. 154.

The figure, and position of the Polygon (No. 1.) to the Picture being determined, find the perspective Plan of its Seat on the Ground Plane (Sect. 5.) Then, BC being the Ground Line, and A the intersecting Point of one Line in the Face X, find the Intersection (AD) of that Plane (Prob. 6.) (V is a section of the Wall, shewing its inclination to the Horizon) also, F being the Vanishing Point of a b, draw FG parallel to AD; FG is the Vanishing Line of that Face (Th. 2.) S is its Center, ES its Distance, and ESC the inclination of the Plane X, to the Picture.

Now, a b is determined, and if the inclination of the Sides a c and b d, to the horizontal Lines, a b and c d, which are parallel, be known, their Vanishing Points are determinable (Prob. 4. Sect. 3.)

Produce SC, and make SE' equal SE. Draw FE', and make the Angles FE'G and FE'I, equal respectively, to the known Angles, producing the Vanishing Point G, of a c; and, if E'I be produced till it meets FG, it will give the Vanishing Point of b d; which does not fall within the compass of the Picture, yet, by means of E'I, b d may be drawn, by Prob. 13.

Then, J being the distance of the Vanishing Point G, draw Ja to the Intersection AD, cutting it at a; make a b equal to the length of the Side a e, and draw bJ, which determines the Angle e; and eF gives the Angle d, b d being drawn indefinite. Or, if the Intersecting Points, D and b be obtained, by making AK equal to the perpendicular height of a e, (as at V) and drawing KD parallel to AB, the rest is unnecessary.

By the same means, any other Face (as Y) may be obtained, and continued around. If the inclination of the Faces, X and Y, to each other be known, the Vanishing Line GH is determined, by Problem 5; or, H, the Vanishing Point of a h, being found, draw GH; by which, the Face Y is described; as X, by means of the Vanishing Line FG.

The method used by the old Authors, for delineating such an Object, is, by forming a perspective Plan of the whole on the Ground, and drawing perpendiculars from the Seats of the interior Angles, as c d, &c.

OF

Of the INCLINED PICTURE.

I presume, the Reader will, ere now, be fully convinced of the universality of the Principles on which the Theory of Perspective is founded; and, that it is the same thing, whether the original Plane or the Picture be inclined, or both, in respect of the Horizon; seeing, it is the position of the original Planes and Lines to the Picture, only, that is considered, in projecting them.

In Example 1st. if the Parallelopiped, being right angled, had any of its Faces parallel to the Picture; or, if they were parallel to the Horizon, and the Picture vertical; or, which is the same thing, if any of its Faces are perpendicular to the Picture; then, the Sides which are perpendicular to them, are parallel to the Picture, and consequently, have parallel representations (Theo. 9.) But, the Picture being inclined to the Horizon, whilst some Face of the Object is parallel to it, the Sides may then be all inclined to the Picture; and, the perpendicular Lines, when the Picture is vertical, are parallel to it, and have no Vanishing Point; but, when the Picture is inclined to the Horizon, they are, consequently, inclined to the Picture, and vanish either upwards or downwards, according as the Picture is inclined, towards the top or bottom of the Object; of which I shall give some Examples.

E X A M P L E IV.

How to represent, an upright Object, on an inclined Picture.

Let AB be an object perpendicular to the Horizon, and let DF be a Section of the Picture, inclined towards the top of the Object; E is the Eye of a Spectator, EC, perpendicular to the Picture, produces its Center and measures its Distance; ED, parallel to the Horizon, determines the Horizontal Line, and EF, being perpendicular, that is parallel to AB, determines the Vanishing Point (F) of Lines perpendicular to the Horizon (Prob. 2.)

Fig. 155.
No. 1.

Let the Picture be prepared accordingly, drawing the Horizontal Line as usual, and the Vertical Line, at right Angles, cutting it at D.

No. 2.

Take DC and CF in proportion to CE the Distance taken (as in No. 1.)

In this Case, it is obvious, that the Center of the Picture, is below the Horizontal Line; as, if the Picture inclined towards B, it would be above it, at S (No. 1.) but, seeing that the Vertical Plane is perpendicular to the Picture, its Center is always in the Vertical Line (DF.)

At No. 3, is shewn the position which the Object has to the Picture (ab) laterally; which Inclination being known, find the Vanishing Points, H, and I, of horizontal Lines in the Object (Prob. 2. and 4, Sect. 3.) DE (equal DE) is the Distance of the Vanishing Line; proceed, in every respect, as in Example, 1st.

F, being the Vanishing Point of Lines perpendicular to the Horizon, and, H and I, the Vanishing Points, of horizontal Lines in the Object, at right angles with each other; the Object being right angled; consequently, HF and IF, are the Vanishing Lines of the upright Planes in the Object (Prob. 4.)

The Seat (*) of any Angle (the nearest to the Picture is the most convenient) being determined; or, the intersecting Point (A) of AB (No. 3.) draw AB, the Intersection of any Plane that Line is in†. Find a, the representation of the Angle A, its distance from the intersecting Point (aA) being known. (Pr. 6, or 7. Sect. 4.) or by its distance, sd, (equal AS) from its Seat, i. e. from the Picture.

† Theo. 2.

By means of the Intersection, AB, of the upper Plane of the cross Arms, is got a, b, &c. the real measures being applied on AB, as usual; or, on AL, the Intersection of the upright Plane (parallel to IF, Th. 2.) in which is the same line, ab; on which Intersection is also applied the measures of height (Ab, ac, &c.) and projected by means of the Point Q, the Distance of the Vanishing Point of those Lines; or, the intersecting Points, K, J, and L, may be found, thus.

Plate
XXXIX.
Fig. 155.

A being determined (by its height above the Eye, the Seat of A , (No. 3.) and distance from its Seat, with the inclination of the Line it is in, to the Picture (dac) or to its Seat) draw Ab through A , perpendicular to HI ; make Aa , Ac , and cb , in the ratio of Ab , bd , and dB ; as CE , to CE , No. 1; draw aK , cJ , and bL , parallel to HI , cutting the Intersection AL , at K , J , and L ; from which, draw lines to the Vanishing Point I , and through a , b , &c. to F , cutting them at f , d , b , &c. From f , and through b , d , &c. draw to H .

Find the Center and Distance of the Vanishing Line HF , and the place of the Eye at O (Prob. 1.) make OM , and ON , in the ratio Ad to Ae (No. 3.) draw OP parallel to NM ; and, through d , draw Pe , and complete the End ek , by means of the Vanishing Point R (in IF) of a diagonal of a Square.

The rest is obvious, from inspection of the Figure.

No. 4. is the same Object on a vertical Picture, whose Distance is ED . (No. 1.)

Here, C is the Center of the Picture, answering to D (No. 1.) AB is the Intersection of the upper Plane of the Cross; and, AD and BE of the upright Faces.

The Intersecting Point of every horizontal Line is geometrically determined; the Intersections of the Planes cutting each other in the Intersecting Points, A , B , D , and E , and the rest are set off geometrically from them; or the Lines are determined, and proportioned, otherwise, as usual.

No. 5. exhibits the same Object, when the Picture is inclined towards its bottom, (as GH , No. 1.) its Center is at S , above the Object, as in No. 2, it is below.

SE is the Distance of the Picture; and, if the Angle SED be made equal to SED (No. 1.) the Horizontal Line DI is determined; also, EF , perpendicular to DE , cuts the Vertical Line, DF , at F , the Vanishing Point of perpendicular Lines, in the Object.

In this Example, the Ground Line (GH) may be used, conveniently; whose distance from the Horizontal, is DB , not EK (No. 1.) i.e. it is in proportion to the Distance of the Picture, ES .

AB is the Intersection of the Plane of the cross Arms, in which the Intersecting Points, A and B , &c. may be determined, as in No. 2. and, BF is the Intersection of a vertical Plane of the Object. The rest it is needless to particularize; the Vanishing Points of horizontal Lines being determined as usual; observing, that the Distance of the Horizontal Line is not the Distance of the Picture, as in a vertical Picture; seeing that, horizontal Planes are not perpendicular to the Picture, the Picture being inclined to the Horizon. Wherefore ED , in both Pictures, is the Distance of the horizontal Vanishing Line; being considered, in every respect, as an inclined Plane, in respect of a vertical Picture.

In these Examples may be seen the universality, and the superiority of Brook Taylor's Principles to all other; by which is shewn (in this last Section) that any Plane Object, whatever, may be projected, from the known position of one Plane to another, in the Object, and the proportion of the Lines in those Planes, together with their position to each other; without regarding their position to any other Plane, whatever, except the Picture. Whereas, by the Old Authors, it was almost impossible to project them at all; or, with the utmost difficulty, by the Seat of each Angle on the Ground Plane, and its height above it, a troublesome and laborious process; by which was obtained the several Angles, and then joined by Right Lines; without Vanishing Points, of which they had not the least conception, in any other Lines but horizontal; and those they called accidental Points, for they had no certain method of producing them. Having found the two extremes of an inclined Line, then, drawing the Line, and producing it to the Horizon, they found its Vanishing Point (properly called accidental) which is now fixed with absolute certainty, in all Positions of Lines to the Picture, and directs the certain position and place of each Line on the Picture, indefinite. And, by means of Visual Rays, drawn on the Picture, certain portions are cut off (truly mathematical) which represent the Originals, as they appear to the Eye, in the true Point of View.

B O O K IV.

OF SHADOWS, in general; of LIGHT and SHADE, REFLECTION, KEEPING, &c.

S E C T I O N I.

An introductory PREFACE, concerning LIGHT and SHADE.

IN the foregoing Work is contained the full knowledge and practice of linear Perspective; the last Book containing Rules, necessary for the projection of any regular Object, almost, whatever; with Examples, varying the application of the Rules, frequently, as occasion requires. The linear part being projected, strictly by the Rules there prescribed, will convey a just Idea of the figure of the Object, and the proportion of its parts to each other (the Eye being in the true Point of View.) Nevertheless, there requires somewhat more to be done, in order to give an appearance of solidity, and preceding of one part behind another; which indeed is effected, in some measure, by their perspective proportions; and perfected, by a just gradation of Light and Shade, properly distributed to each part of the Object; and which, with the projection of Shadows, &c. is the Subject of this fourth Book,

It must not be expected that I should define what Light is, having already given my Opinion on that Subject (Sect. 1. Book 1.) yet, it may be proper to give some general Idea of what is understood and meant by Light, as it is used by Artists in general, in the application of it to a Picture.

LIGHT, in that respect, means nothing more than the bright parts of Objects; which differs greatly in degree, according as their Surfaces are situated in respect of some luminous Body; which is effected either, directly, from the Luminary, or reflected, from some illumined Object.

SHADE is a privation of Light, the dark parts of Objects; occasioned either by the Object itself, on those parts which are not towards the luminous Body, or, by some other opake Substance, interposed between the luminous Body and the Object; depriving it, either wholly or partly, of Light.

To give or prescribe Rules, absolutely, for perfecting a Picture, in respect of Light and Shade, is as impossible as in respect of Colour; yet, by adhering to Reason, and carefully observing Nature, we may arrive at a tolerable degree of perfection. In the first place, it is necessary to consider how the Object is supposed to be situated to the luminous Body, from which it is to receive the Effect.

It has been almost a general Rule, amongst Artists, to suppose Light to flow from the left hand to the right; but that is entirely arbitrary, and can have no foundation in the nature of things, but merely an habitual custom; it is, however, proper to imagine it to flow from one hand or the other; for, to suppose the Luminary, or luminous Body, directly opposed, either on this or on the other side of the Object, can never produce a pleasing effect; seeing that, in the first case, it is almost wholly illumined, in the other, it is wholly deprived of Light; an agreeable mixture of both, judiciously disposed, is what contrasts one Object, or part of an Object from the other, and renders the whole agreeable to the Eye, as a pleasing imitation of Nature.

Plate XL. There is, moreover, an inconveniency arising from the custom of shading Pictures, on the left hand, generally, seeing they cannot suit both sides of a Room; for, being placed, in that respect, properly, adds greatly to the effect of the Piece.

Let it be carefully observed, that, that part of an Object, to which any luminous Body is most directly opposed, will be the brightest; but, when, from the situation of the Eye, it is much contracted, or foreshortened, it may appear as dark, or perhaps darker than other parts, on which, the Light is not so direct.

Fig. 1.

For Example. Let Z be the Plan or Base of some prismatic Object, and AB, &c. Rays of Light, falling thereon. Because those Rays fall more perpendicularly on the Face BC, than on BD, that Face is, consequently, more illumined than BD, and BD than DF, whilst FG is wholly deprived of Light; and GH, being farther removed from the Light, will be darker than FG, from the effect of Light simply, or direct. Now, if a Spectator be so situated, at E, that the Face BC, which is most illumined, is much contracted, it may in some situations of the Object to the Light, appear darker than BD, which is more opposite to the Eye; whereas, if the Station be at E, where both appear equally contracted, the Face BC will appear the brightest; but, if the Light was in the direction of EB they would be, and also appear equally bright, without distinction.

Any direction to the Rays of Light may be given, at discretion, as AB; and, by drawing others, parallel to AB, it is easy to know which Face should be the brightest, and which the next, &c. but to determine in what degree, positively, is not possible; a careful observation of Nature is the best criterion by which to judge of that; and, even in that case, it is not easy to determine, without experience and sound judgment.

After the same manner, the inclination of the Rays of Light to the Horizon, being taken into consideration, may be determined, whether horizontal Faces of Objects, below the Eye, or inclined Faces, are more illumined than the vertical. As Light flows from above, the whole Hemisphere being illumined (when the Luminary itself is not much elevated) the horizontal faces of Objects, or such as are much inclined, are, generally, the brightest; provided they are opposite to that quarter from which the principal Light flows. Notwithstanding it is usual to shade the Roofs of Buildings, Pediments, &c. more than the vertical Planes, it can only be from a supposition of their being of darker materials, as Lead, Slate, &c. for, if they were composed of the same, that is, if they are of one uniform Colour (and otherwise, no positive determination can be made, in respect of Light, from observation) inclined Planes, will be brighter than either vertical or horizontal, being, generally, more opposed to the Light.

In respect of curved Surfaces, from the effect of Light, simply or directly, being convex, those parts which are most towards the luminous Body, are the brightest; and consequently, those which are farthest removed from the Light will (without reflection) be the darkest. On concave Surfaces, the effect is reverse.

Fig. 2.

On cylindrical Surfaces, as Columns, straight Mouldings, &c. the Light is always parallel to the sides, or edges of the Mouldings; the greatest Light being on that part which is most perpendicularly opposed to the luminous Body (as AB) from which it gradates regularly, on each side; consequently, that side which is farthest from the Light will be the darkest, supposing the Luminary to be situated on this side of the Object, on either hand. But, if it be supposed situate on the other side, the Edge, AB, towards the luminous Body, will be the brightest; and it will be gradated from that Edge, not to the other, as in No. 2. (which has not the appearance of an entire Column or Cylinder, but of a Segment, cut off parallel to its Axe) but, it will be darkest somewhat from the other Edge (as No. 3.) not owing to reflection from other Objects, but from the luminous Body, being on the other Side; which, as it is more or less direct, will occasion the darkest part to be more or less removed from the middle.

Convex

Convex spherical Surfaces are brightest on that part to which the luminous Body is perpendicular, from which it gradates every way equally; insensibly varying, to the extremes, provided no other Object interfered, to reflect Light. If a Hemisphere, or lesser Segment, be properly shaded, it will represent equally as well a concave as a convex Surface, by supposing the Light on one side or on the other; that is, when the Eye is so situated as to see the whole circumference of its Base, or nearly; as Fig. 3.

Fig. 3.

Respecting straight mouldings, which are composed wholly of Planes and cylindrical Surfaces, their different situations, towards the flow of Light, occasion variety of Shades. As for example; the Cima-recta (composed of a convex and a concave cylindrical Surface) in its proper position, in a Cornice, &c. has two strong Shades and one Light, in the middle; whereas the very same Moulding, reversed, as for a Base, has two Lights, and one Shade, in the middle, with a faint one at each Edge. The Cima-reversa, in its proper position, has two Lights and one strong Shade, in the middle; the same Moulding reversed, has two faint Shades and one Light, in the middle.

Figure 4 is the Profile of a Cornice; suppose the Light to flow in the direction AB, at discretion. It is easy to determine, what parts of the Cornice will be light, and which shaded. For, drawing several Lines, touching the projectures and prominances, parallel to AB, it is obvious that the edge a, obstructing the Light, must necessarily occasion a strong Shade, below; which gradually dies away into the Light, at b, in the middle, where the Rays fall oblique on it, and where, the prominence or swell of the convex part occasions another Shade, at c; begining faint, and gradually strengthening, reversely.

Fig. 4.

The same reasoning accounts for the Shade in the middle of the Cima-reversa.

The great projecture of the Corona, (X) throws all below in Shade, which gives great expression and force to the whole, making the upper part appear to stand off, from the Canvas or Paper. The whole, below, being immersed in Shade, would be totally lost to sight, was it not, in some degree, illumined by reflection; on which I shall speak in its place; let it suffice, here, to observe, that the effects are almost wholly reversed, and but faintly expressed.

Let No. 2. represent the upper Mouldings reversed, for a Base Moulding.

No. 2.

Here, it is obvious, that the Rays, falling on it in the same direction, illumines the whole; inasmuch that, no part can be said, with propriety, to be in Shade; nevertheless, the parts a, b, &c. which are most directly opposed, will be brighter than the other, which are faintly shaded, as the Surface falls off.

I shall just make an observation on the prevalent Custom of shading Mouldings, in architectural Designs, as strongly illumined by the Sun; which entirely destroys their proper effect.

Suppose AB (No. 3.) to represent a Ray from the Sun. It is obvious, that all the part, from B to b, will be in Shade; and, being a Shadow, projected by the upper edge (BC) strongly defined by a Right Line (bc) has not a very agreeable effect. The Shade, below, is also more sudden and hard.

No. 3.

Now, if by means of Light and Shade, it is intended to give an Idea of Mouldings, I would ask, seriously, which has the most natural appearance and effect? Suppose the Profile cut off, or covered; would any Person conceive what this Moulding is intended to represent? But, where is the necessity to suppose the Sun to shine on them? as there are strong Shades when it does not; nor are they intended as Pictures, but Designs, which ought to exhibit what is intended, in the most expressive manner possible.

If my opinion might be allowed to have any weight, I should suppose that a Section shewing the inside of a Building, geometrical, would be more expressive, by means of penumbral, instead of strong edgy Shades, which are by no means natural, in such Cases. Nay, so fond are some Architects of forced Effects, that we frequently see a strong right line of Light introduced into a Section of a Dome,

through

Plate XL. through an opening at the Top (AB) as strongly defined on the part at C, which is nearly horizontal (where it is absolutely impossible for Light to come) as elsewhere, at D; where, an elliptical figure of the opening would, only, be described by the concave Cylinder, below, being shaded without any edge at all.

Fig. 5.

Similar to this are the forced and unnatural Effects, which may frequently be seen, in Prints and Paintings, of a strong Light being introduced, striking in a Right Line, from the upper edge of the Cornice, across the Side of a Room, as if it was open at the top. Also, on exterior Objects, a strong diagonal Shade frequently crosses a Building, without the least apparent Cause for such an Effect.

There is (to me) another great impropriety; which, though it does not come immediately within my cognizance, I shall beg leave just to mention; which is, the introducing Landscape, and some appearance of Perspective, in a geometrical Design. I am well assured that it need but be pointed out, to convince any sensible Person of its inconsistency, as a Picture; exhibiting a geometrical Elevation of a Building, placed upright in a Garden, &c. where every thing else appears real.

When I see a number of Drawings, geometrical Elevations of Buildings, at an Exhibition, vying with each other, which shall make the best figure, as a Picture, more than as a Design (for which, the Architect is obliged to some Landscape Painter, frequently) I am apt to imagine myself in Vauxhall Gardens, where, the luxuries of Art eclipse the beauties of Nature; and, instead of a Garden, we, in reality, find ourselves in some spacious Building; the several Walks and Avenues of which, resembling Galleries of communication to various detached Apartments.

S E C T I O N II.

The THEORY of SHADOWS, projected by the Sun.

IN treating on Shadows, it is necessary, first, to consider the Situation, Distance, and Magnitude of the luminous Body, by which the Shadows are projected. Secondly, the Situation of the Object to the Luminary, and, of the Lines bounding Planes, &c. whose Shadows are to be projected; their Position in respect of some other Plane, on which the Shadow is to be projected; and lastly, the Position of the Luminary, in respect of the Picture.

The Sun, which is the grand fountain of Light, being placed in the wide Expanse, dispenses its Light, equally, all around, and illumines a concave Sphere, as far as its influence extends; which differs, in degree of Splendor, according to the Distance from the luminous Body. If an opaque Body be interposed, that Body is illumined on its Surface towards the Sun; and consequently, must deprive some part of the Expanse of Light; and, if another opaque Body intervene, the Shade, or Shadow of one, is thrown upon, or projected on the other.

A SHADOW, therefore, is the outline projection of one Object on another, Plane or other Superficies, in Right Lines, from some luminous Body to the PLANE of PROJECTION; i.e. the Plane on which the Shadow is projected.

By RAYS of LIGHT is to be understood Right Lines, from the Luminary, passing by the extremes of an Object, and projecting its Shadow.

Fig. 6.

To illustrate what has been advanced, suppose the Body, a Globe, at S, to represent the Sun, and CD a portion of a concave Sphere, illumined by its Light; let E be supposed an opaque Body, illumined on the Surface towards the Sun.

Now, if this opaque Body be also a Globe, representing the Earth or any other Planet; the Sun being, in proportion to it, immensely great, the Space deprived of Light, in the Expanse, will form a Cone; as ACB.

If a Plane, or any other opaque Surface, be placed at FG, within the extent of the Cone, it will receive, thereon, the Shadow of the opaque Body, at H; which Projection, if the Plane be direct to the Sun, will be a Circle; but, if the Plane be inclined, as DL, it will form an Ellipsis, at I. These Projections, or Shadows, are less than the Object; because the luminous Body is greater. But, if the luminous Body be equal to the Object, or at an infinite Distance; the Shadow, on the direct Plane, will be equal to the Object, in diameter, and may be considered as its orthographic Projection.

Hence it is evident, that, from the magnitude of the Sun in proportion to the Earth, the Rays of Light, from its extremes to the Earth, are converging; but, when its immense Distance is considered, they are, to all sense, parallel. Then, seeing that the largest Object, on the Earth, can scarce be said to bear any proportion to the whole, we may consider them as being perfectly parallel.

Next, the Altitude of the Luminary is to be considered, so as to give the best and most pleasing effect to the Picture; this is generally at the discretion of the Artist. It may, to some, be necessary to explain what is meant by its Altitude.

Let the Semicircle AFB be supposed an Arch in the Heavens, in which is the Sun's *apparent* diurnal motion; and let E be the Earth, *supposed* at rest. Fig. 7.

If the Sun be at C, the Ark CB is the measure of its Altitude; and, the Angle CEB is the Angle of Elevation; EB, being considered as the Horizon, and CE the direction of the Rays of Light. If it be at D, then, DEB is the Angle of the Inclination of the Rays of Light to the Horizon, and the ark DB the measure of its Altitude; but, when its place is at F, in the middle of the Arch (90 Degrees, each way, from A and B) its Rays have, then, no Inclination to the Horizon, AB; and consequently, a perpendicular Line, GE, will project no Shadow.

This can only happen to those parts which lie between the Tropics; for although, at the Equinoxes, the Sun appears to describe a Semicircle, at all parts of the Earth, yet, according to the distance from the Equator, the Plane of its apparent motion is more or less inclined to the Horizon of the place; consequently, its Altitude, when in the Meridian of any place, is more or less elevated above the Horizon, from the Equator to the Poles, where it describes an entire Circle, around the Horizon, having no Elevation, or Meridian.

Lastly; the Angle of Inclination which a vertical Plane, passing through the center of the Luminary, makes with the Picture, is necessary to be known, when it is not in the Plane of the Picture.

Now, although the Rays of Light, proceeding from the Sun, are parallel amongst themselves, in respect of the Earth, yet, their inclination to its Surface varies infinitely.

Suppose a Globe illuminated by the Sun, at S, whose Center only is considered*. Fig. 8.

* In projecting Shadows by the Sun, in order to describe the true Contour, or outline of the Shadow, it is necessary to consider it as a luminous Point, only, at an infinite Distance; for, if its magnitude be taken into consideration (since every part of its Surface emits Light) the Shadow of a Point, would always be, in proportion to the Sun, as the distance of the same Point, from the Plane of Projection, is to its distance from the Sun. Let S be the Sun, and A, a Point whose Shadow is projected, on the Plane X.

Now, the whole extent of the Shadow of the Point A is a Circle, whose Diameter is ab ; for, since it is obvious, that every part of the Sun must emit Light; consequently, a Ray of Light emitted from a , and passing through the Point A, will project its Shadow to a ; and a Ray from b will project the same Point to b , &c. while the Center (S) only, being considered, will project it to B, which is the center of the Shade, and its real Shadow. For (supposing the Point to have substance) every other part of the Shadow, is more languid, the farther it is from the Center, B; consequently, at its extremes, it cannot be distinguished from the surrounding Light; seeing that, every Point, in the Sun's surface, emits light. Wherefore, since the Triangles, aAb , aAb , are similar, it will consequently be, as, Aa is to Aa , so is ab to ab , that is, as AB to AS. (6. 6. El.)

Hence, it is easy to account for the Penumbra of Shadows; which, at a distance, appear distinctly defined, but on approaching near, we find it otherwise; inasmuch that, except where the Lines, in any Object occasioning the Shade, cut the Surface on which the Shadow is projected, we cannot trace a line at all; and, the farther the Shadow is from that Point, the more penumbral it becomes; that is, the less distinctly defined; till, at a considerable distance, it becomes insensibly mixed with the Light.

Let

Fig. 10.

Plate XL.
Fig. 8.

Let AB be the Horizon of the part at D, to which, the Sun is directly opposite. The Ray SC is perpendicular to the Horizon, AB; and consequently, a perpendicular Line, as CD, to the Horizon of that place, can have no Shadow, but will be projected towards the Center. But, if any other Tangent, as EF, be drawn; the direct Ray SC, to the Globe, is inclined to it, in the Angle SDE; and therefore, Lines perpendicular to EF will project Shadows. As CE to D.

Fig. 9.

To illustrate this more clearly; suppose the Plane AB, horizontal, a portion of the Earth's Surface, to which the Rays are inclined, and SE the direction of a Ray of Light. The Angle of Inclination, to the Plane AB, is SED; and, the length of the Shadow of the Perpendicular CD, is DE.

Hence it is manifest, that the Shadow of a Right Line, on a Plane, is always a Right Line (1. 7. El.) for it is projected by a *Plane of Shade*, occasioned by the Line; as CED, which cuts another Plane, whose common Section is the Shadow.

And, it is evident, that, the greater the angle of Elevation is, the shorter is the Shadow; for, FC projects the same Point, C, to G; consequently, being elevated perpendicularly over C, the Line CD will project no Shadow.

In projecting Shadows by the Sun, there are three CASES to be considered, or situations of the Luminary; viz. it must be either on this side, or on the other side, or in the Plane of the Picture.

In order to determine the Shadows of Objects, delineated on the Picture; having first considered and determined on the situation and altitude of the Luminary, the next thing requisite, if the luminous Body, or Point (for it is always considered as such) be not in the Picture, is to find the Vanishing Point of the Rays of Light; but, when it is in the Plane of the Picture, the Rays are parallel on the Picture, and consequently they have no Vanishing Point; the Angle of Elevation is, then, only necessary to be considered.

Fig. 11.

Let FGHI be the Picture, and EC the Distance of the Picture. Also, let SE be supposed a Ray of Light.

Imagine a Plane (ESB) to pass through the Luminary, and through the Eye, at E, cutting the Picture in FG; in which Line, whether the Sun be on this, or on the other side of the Picture, as at S, its apparent, or transprojected place must necessarily be; as at F or G.

For, being on the other side, at S (supposed at an immense Distance) and being in the Plane ESB, consequently ES is in that Plane, and will cut the Picture in their common Intersection (FG) at F. Wherefore, F represents the Sun on the Picture, and is, consequently, the Vanishing Point of the Rays of Light.

When the Sun is on this side, at S', or S'', being still in the same Plane, produced, consequently S', or S'E, being produced, will also cut the Picture in FG, as at G; and, because all the Rays are parallel amongst themselves, and all Lines which are parallel have the same Vanishing Point, consequently, G is their Vanishing Point. (Def. 22.) for S'G is a Ray of Light, to which all other Rays are parallel; or, it is a Right Line, passing through the Eye, parallel to them.

N. B. In the former Case (when the Sun is on the other side of the Picture) it is obvious, that its place, in the Picture, must necessarily be above the Horizontal Line, as it cannot be seen till it is above the Horizon; so, in this Case (seeing it is transprojected) being above the Horizon, its transprojected place must necessarily be below the Horizontal Line; from which (in either Case) it is farther removed, as the Luminary is more elevated above the Horizon. Also, whether its real place be on the right hand, or on the left, its apparent place is the same, in the former Case, but reversed in the other, (See TRANSPJECTION, in the Introduction; Page 52.)

Hitherto I have proceeded introductoryly, which, I have called the Theory of Shadows; seeing that, all which has been said is theoretic. I shall now proceed to practice, and lie down such Rules as are necessary, for projecting the Shadows of all regular Objects, particularly such as are right lined; which may be described on plane Surfaces, with facility and certainty, by adhering to the following Rules.

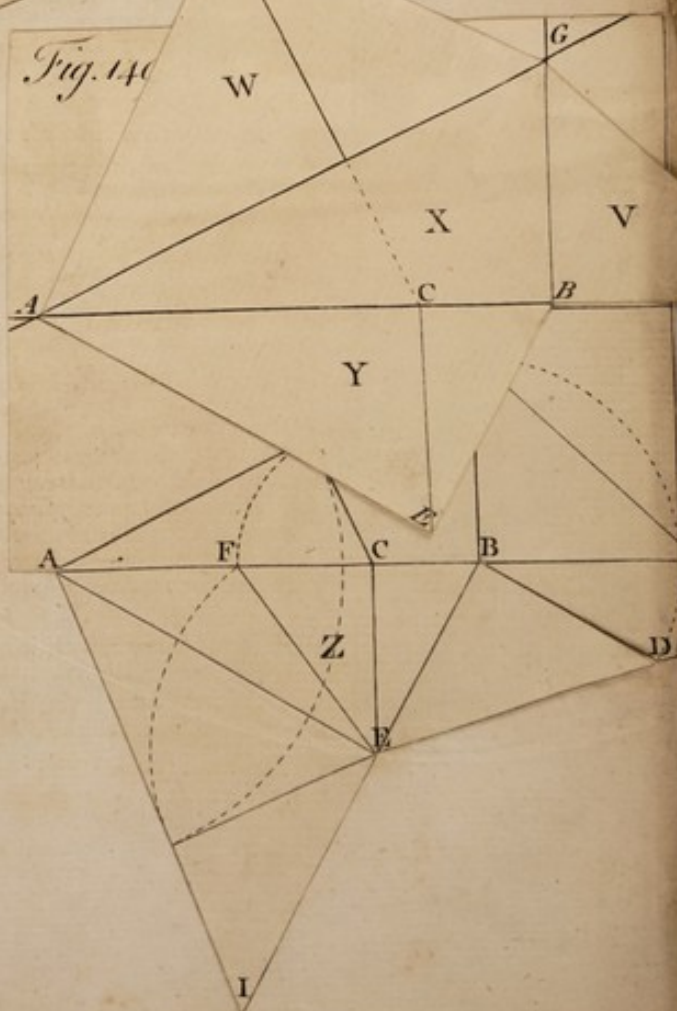
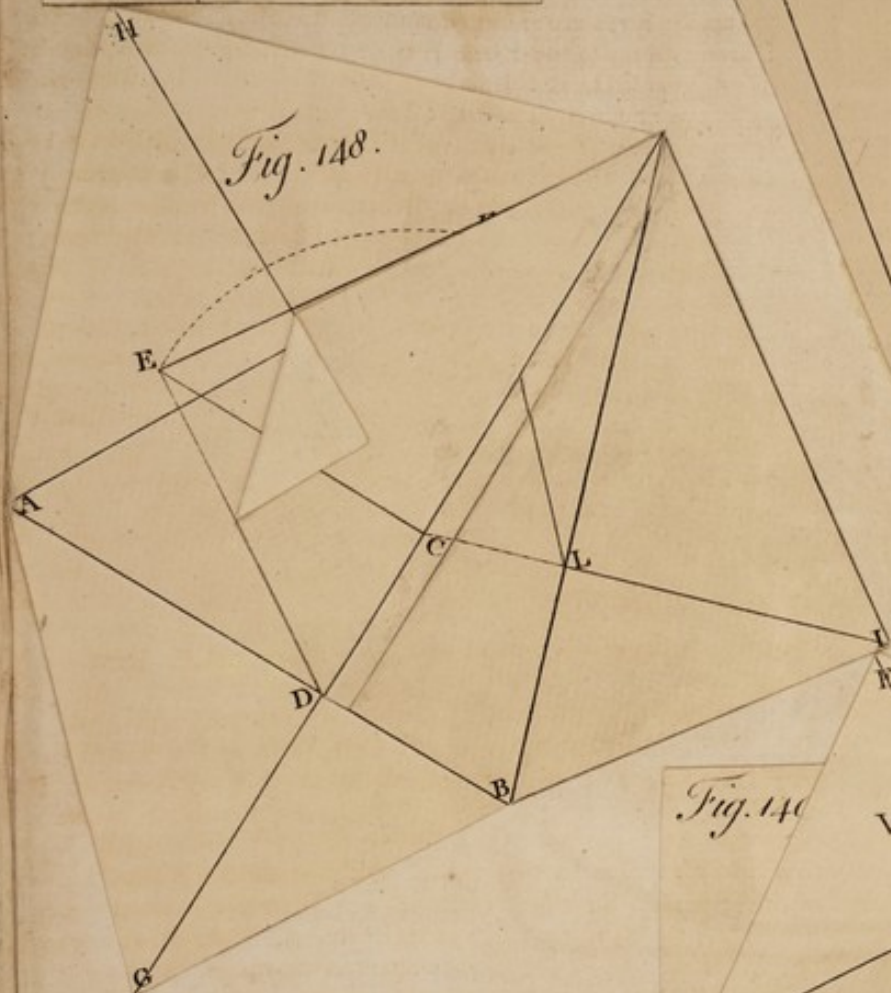
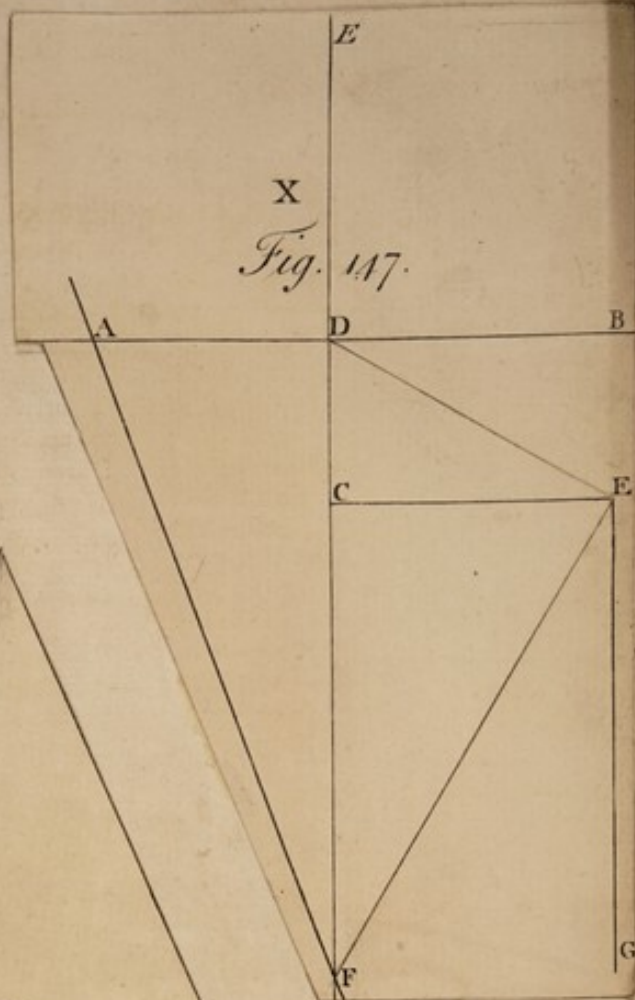
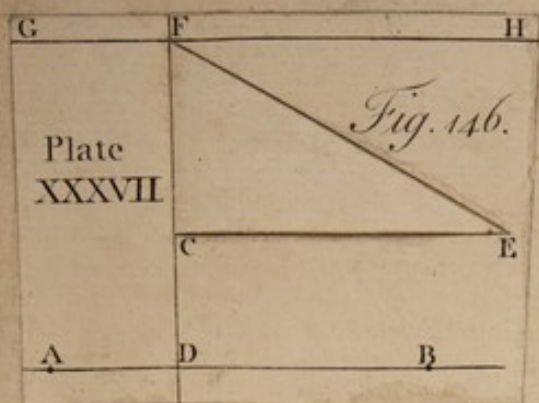




Fig. 152.

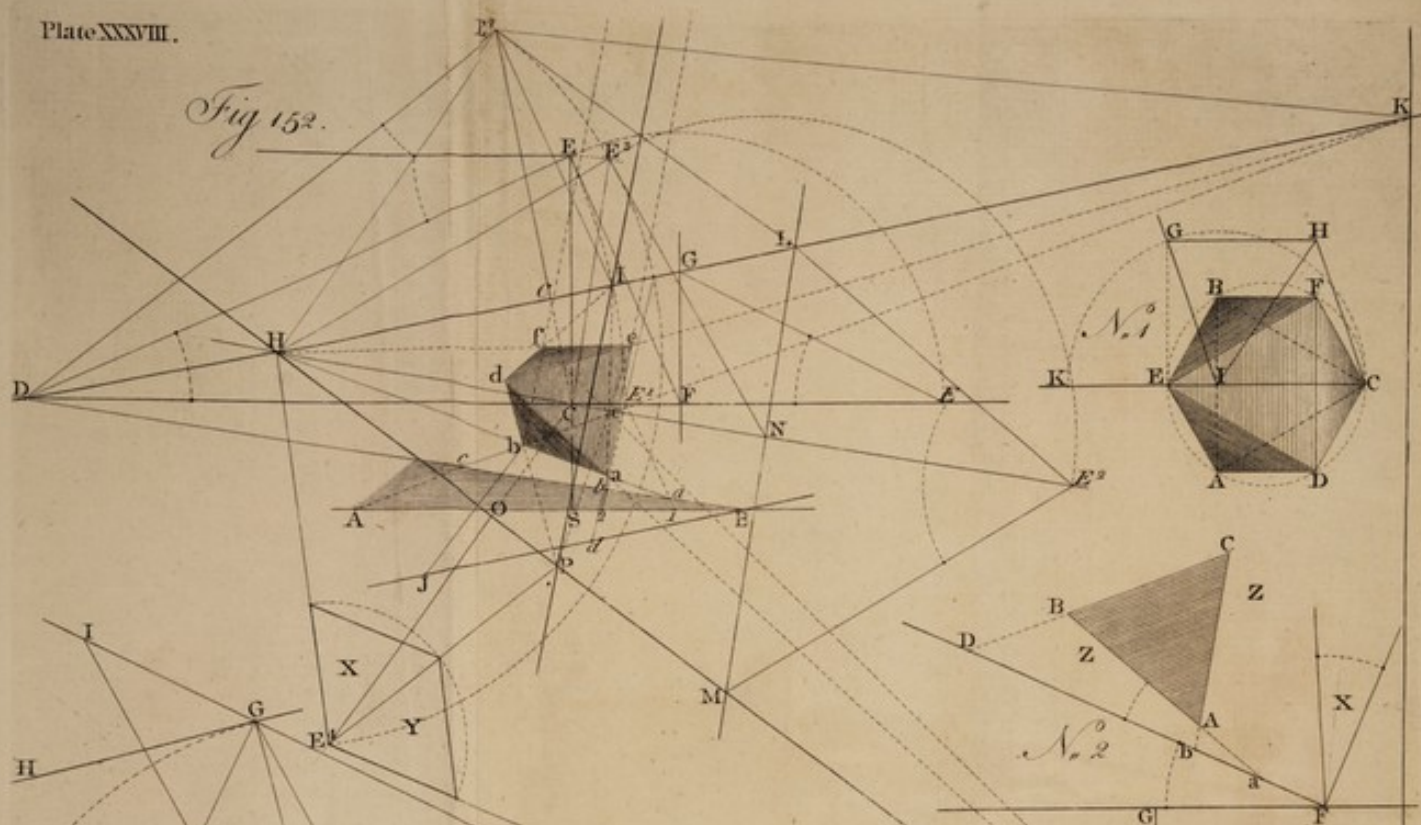


Fig. 151.

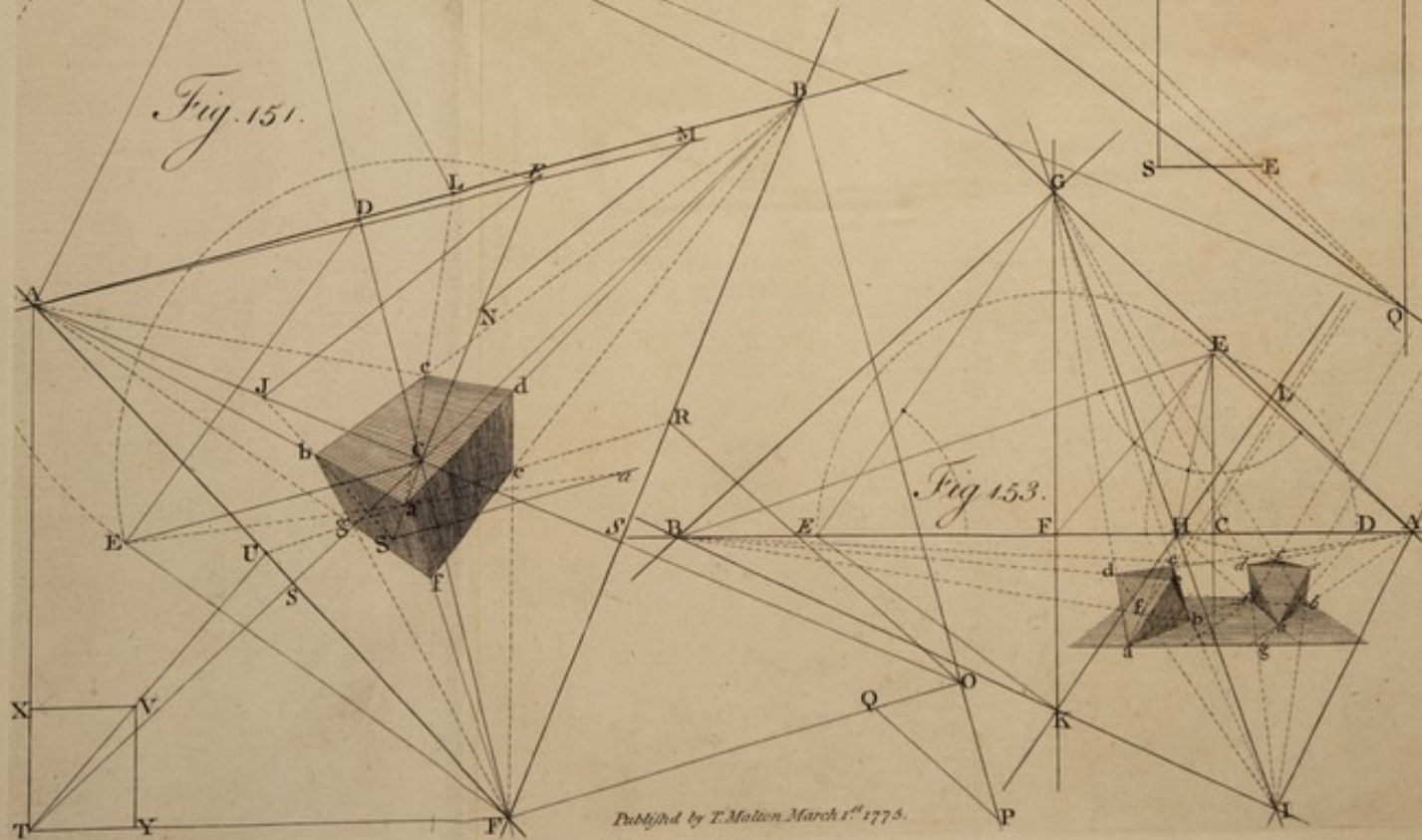
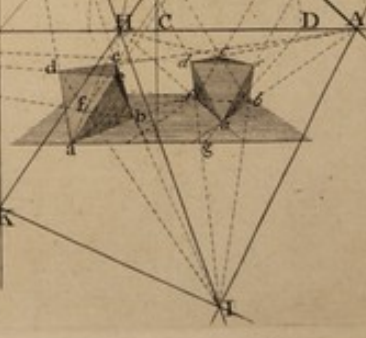


Fig. 153.



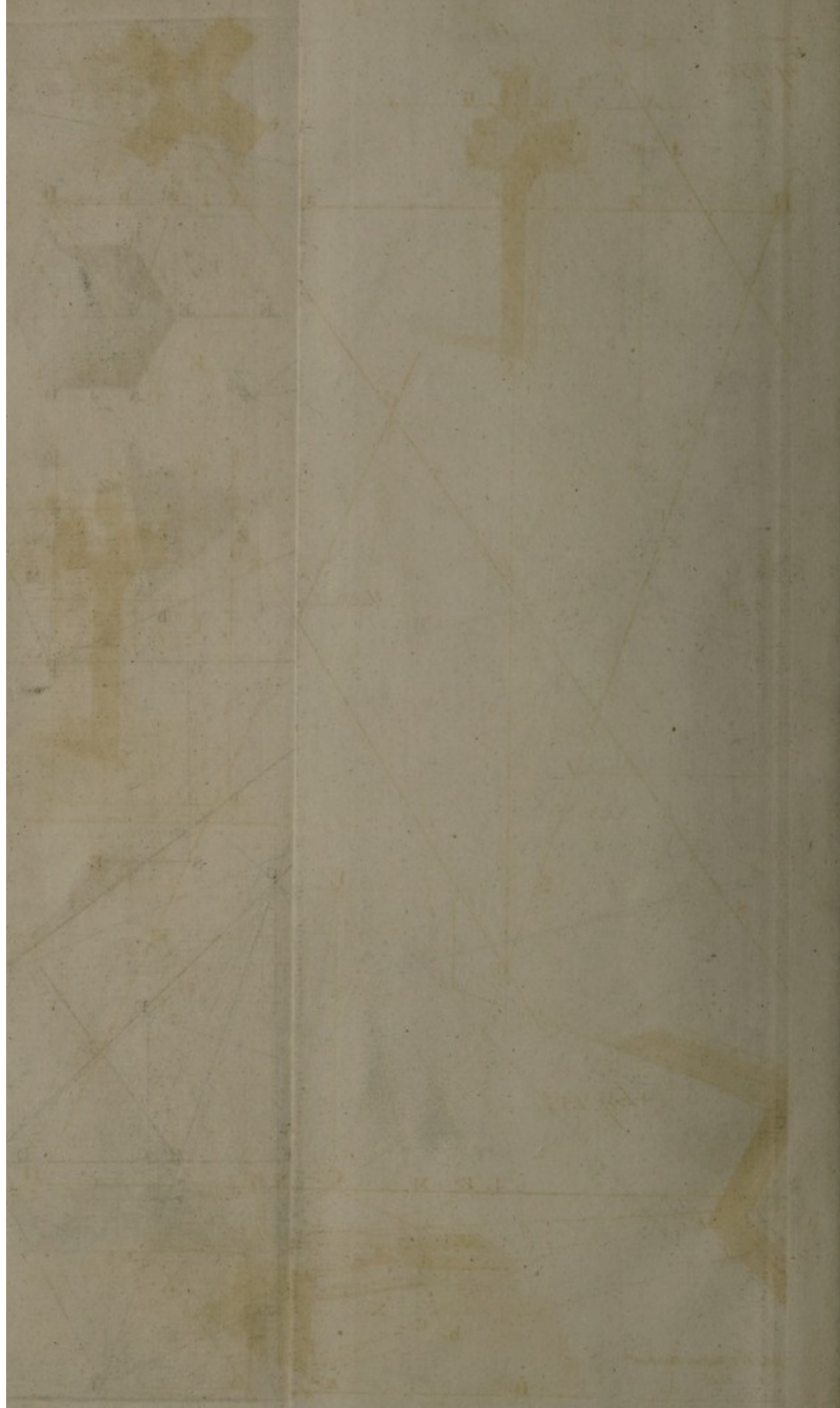


Fig. 153.
N^o 2.

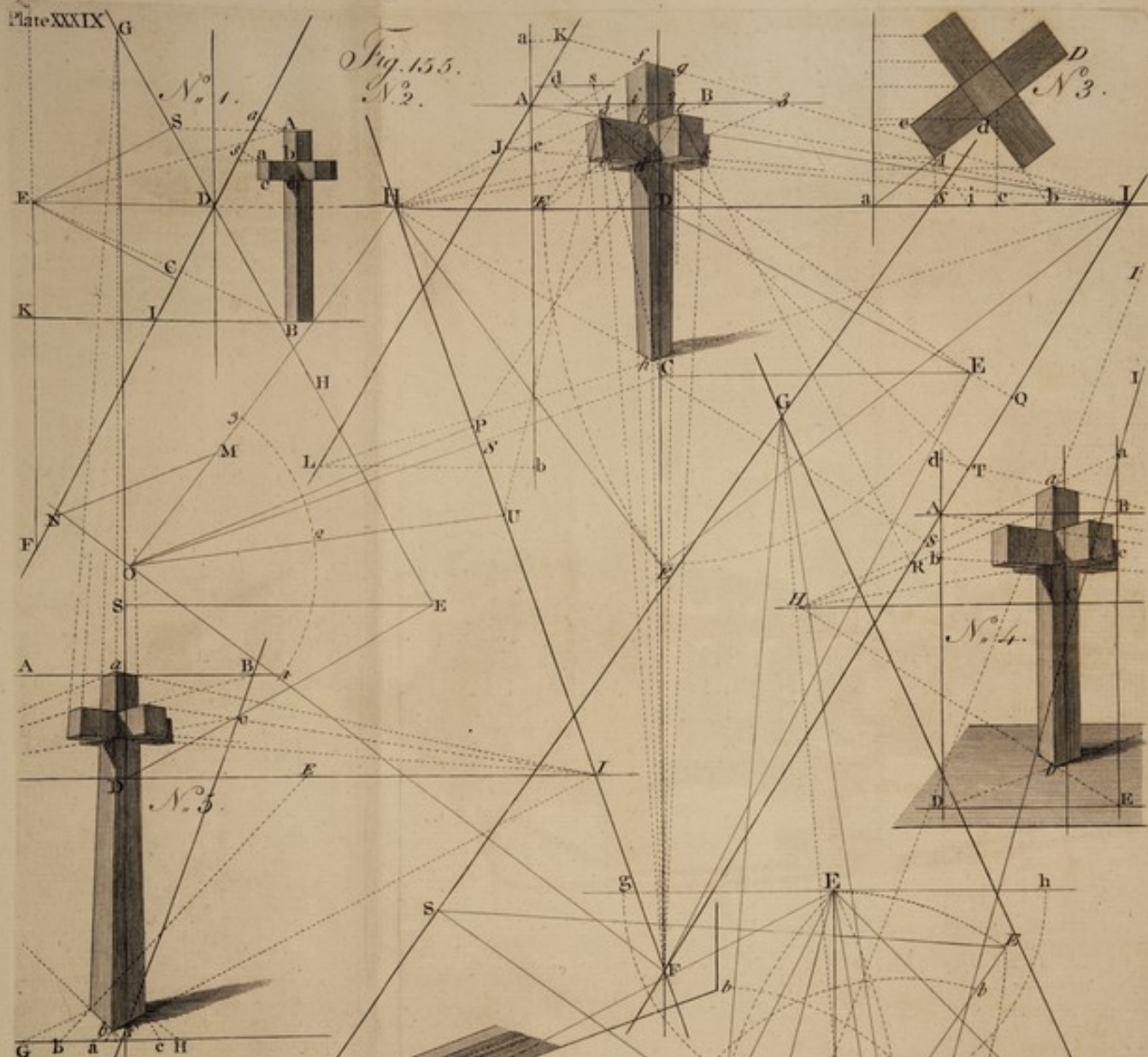
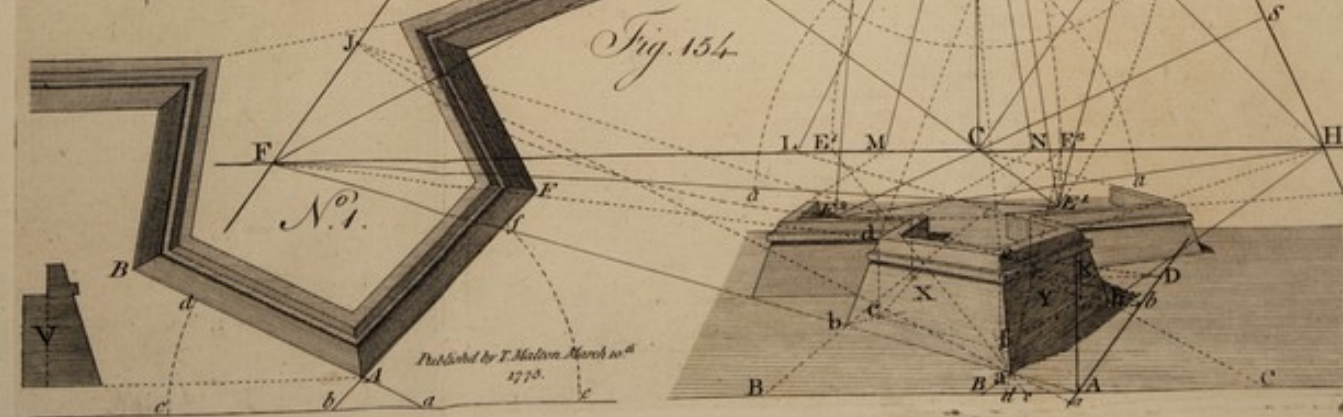
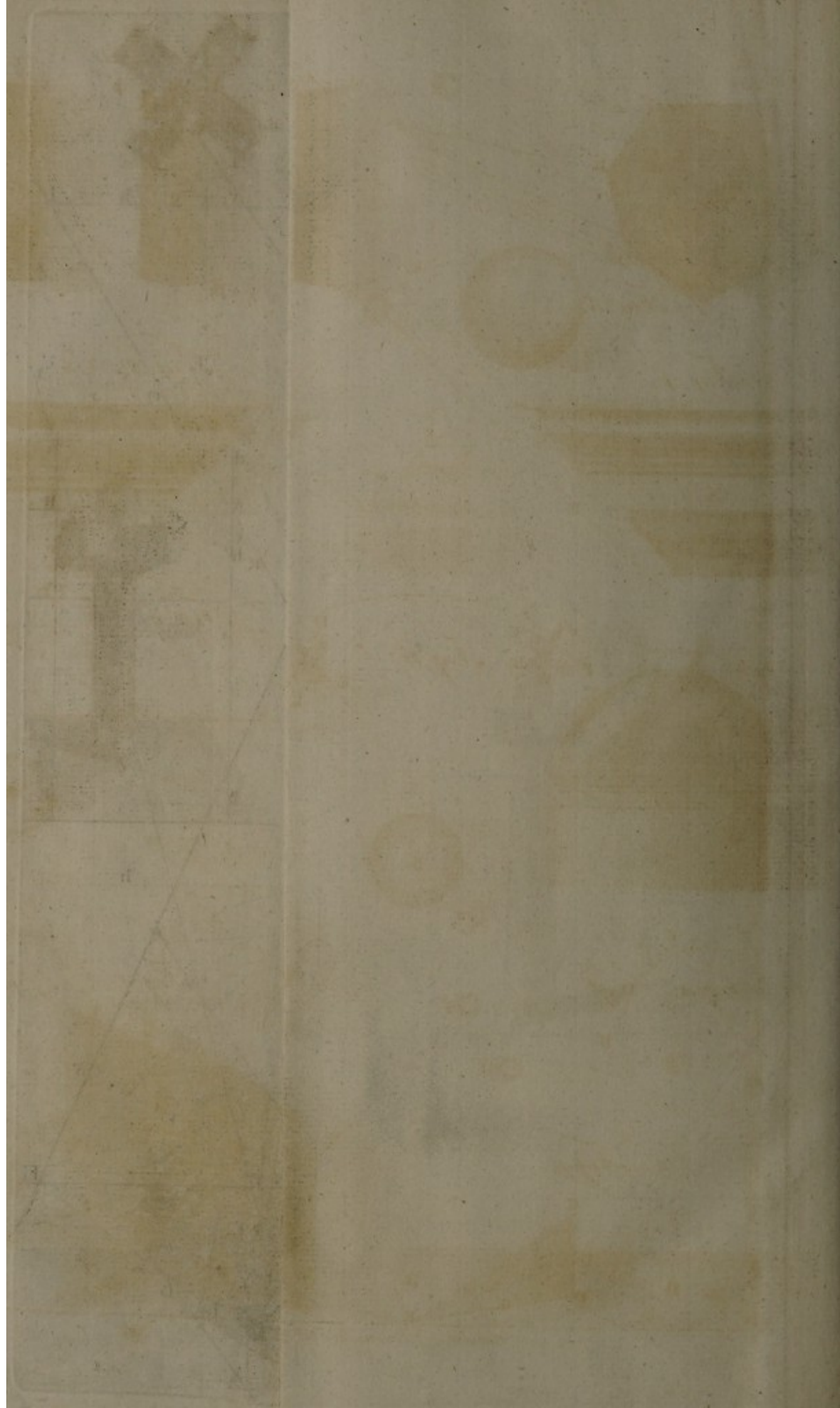
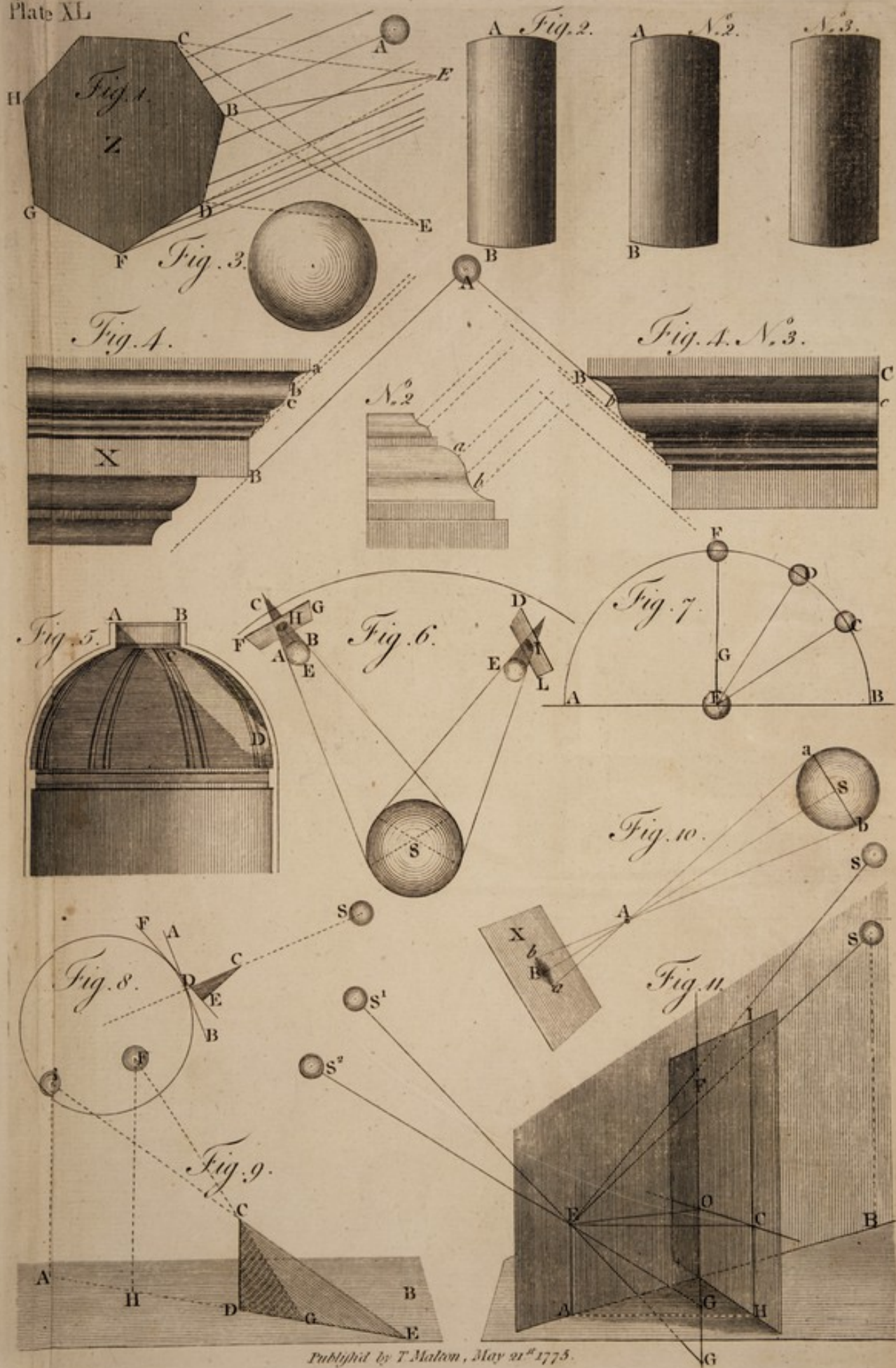


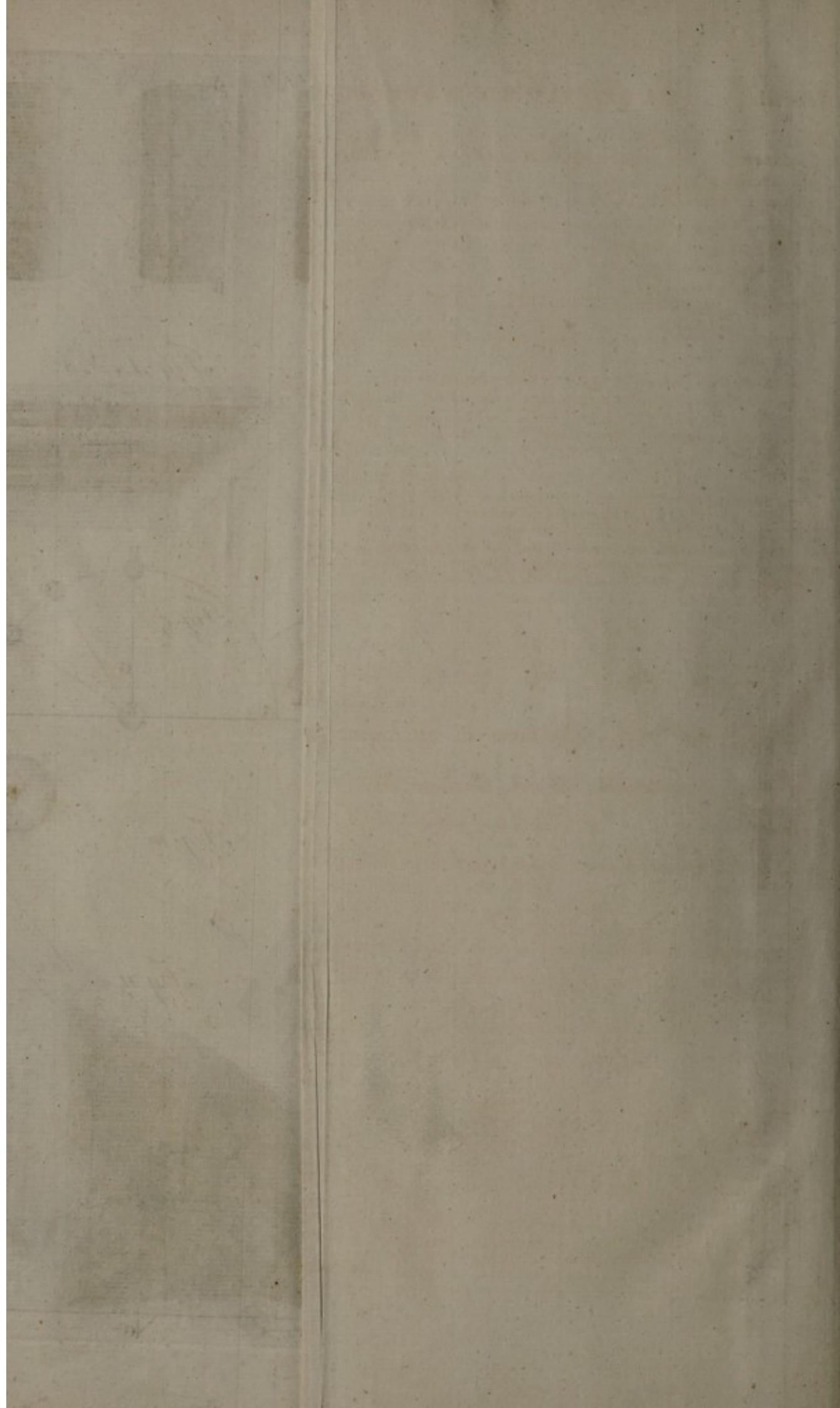
Fig. 154.



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FIRST. The indefinite projection of the Shadow of a Right Line, on any Surface whatever, is the Intersection of that Surface by a Plane, passing through the Luminous Point and the Line; which, for distinction sake, I shall call the **PLANE OF SHADE**. This is obvious in itself.

SECOND. The Vanishing Line of the *Plane of Shade*, projecting the Shadow of any Right Line, is a Right Line drawn through the Vanishing Point of the Rays of Light, and the Vanishing Point of the Line whose Shadow is required (Th. 10.)

Because, the Vanishing Point of the Line, whose Shadow is projected, and the Vanishing Point of the Rays; projecting the Shadow, are in the Plane of Shade.

THIRD. The Vanishing Point of the Shadow of any Right Line, on a Plane, is the intersecting Point of the Vanishing Line of that Plane, and the Vanishing Line of the **PLANE OF SHADE**. (Cor. 2. Theo. 6.)

Because, the common Intersection of those Planes is the Shadow required.

N. B. When the Sun is in the Plane of the Picture, there being no Vanishing Point of its Rays, a Right Line, drawn through the Vanishing Point of any Line whose Shadow is required, parallel to the given Ray, cuts the Vanishing Line of the Plane of projection, in the Vanishing Point of the Shadow.

Also, if the Original Line be parallel to the Picture, and consequently has no Vanishing Point; (the Sun being on either side of the Picture) then, a Right Line drawn through the Vanishing Point of the Rays, parallel to the Line whose Shadow is required, cuts the Vanishing Line of the Plane of projection, in the Vanishing Point of the Shadow.

But, when the Luminary is in the Plane of the Picture, and the Line, whose Shadow is required, parallel to it, the Shadow has no Vanishing Point; for it is, in such Case, necessarily parallel to the Picture; therefore, parallel to the Vanishing Line of the Plane of projection.

S E C T I O N III.

Of the PROJECTION of rectilinear SHADOWS.

P R O B L E M I.

Having the Angle of the Sun's Elevation (X) and the Inclination, to the Picture, of a vertical Plane passing through its Center (equal Z) with the Center and Distance of the Picture given; to find the Sun's place on the Picture, or the Vanishing Point of the Rays of Light.

Let CE be the Horizontal Vanishing Line, and C the Center of the Picture.

Fig. 12.

Make CB equal to the Distance, perpendicular to CE, and draw AB parallel to CE.

Draw BD, making the Angle ABD equal to Z, cutting CE, at D; and through D, draw FG, perpendicular to CE. FG is the Intersection of a vertical Plane passing through the Sun, with the Picture, whose inclination to it is equal Z.

Make DE equal to DB, and the Angle DEF, or DEG, equal to X, the Angle of the Sun's Altitude; F or G is the Vanishing Point sought.

DEM. Turn up the Triangle CBD, perpendicular, and turn over the Plane FEG, or FG, till DE coincides with DB.

Then, if the Sun be on the other side of the Picture, EF is parallel to the Sun's Rays; because the Luminary is in the Plane EFG produced, and, the Angle DEF is equal to its Elevation; consequently, EF produced would pass through the Center of the Luminary, wherefore, F is the place of the Sun on the Picture; i.e. F represents the Sun; consequently all the Rays center there, and therefore it is their Vanishing Point. For its Distance is infinite, to all sense.

CASE 2d. When the Sun is on this side the Picture, being behind the Spectator, it cannot appear in the Picture, but its place is transprojected to G; for, DEG is the Angle of its Elevation, and it is in a continuation of the Plane FEG; wherefore, EG is parallel to the Rays of Light; consequently, G, the transprojected place of the Luminary, is their Vanishing Point.

Or, as Brook Taylor, very pertinently, calls it, the Shadow of the Spectator's Eye, on the Picture; for E is the Eye, wherefore GE, being produced, would pass through the Sun's Center.

Plate XLI.

P R O B L E M II.

The Vanishing Point of the Sun's Rays being given, and the representation of a Line perpendicular to some Plane, whose Vanishing Line is given; together with the Center and Distance of the Picture; to find the representation of the Shadow of that Line, on the Plane, and its Vanishing Point.

Fig. 13. Let S be the Vanishing Point of the Rays, VL the Vanishing Line of the Plane, and AB the given representation of a Line perpendicular to that Plane. C is the Center of the Picture, whose Distance is known.

Find F , the Vanishing Point of Lines perpendicular to the Plane, whose Vanishing Line is VL (Prob. 2. Sect. 12. B. 3.) consequently, AB vanishes in F ; draw SF , cutting VL at V . V is the Vanishing Point of the Shadow.

Draw VB and SA , intersecting at D ; BD is the Shadow of AB , required.

DEM. Whether S be considered as the Image of the Sun, on the Picture, or S its transprojected place, SF is the Vanishing Line of the Plane of Shade, for all Lines perpendicular to that Plane; seeing that, F is the Vanishing Point of all such Lines; wherefore, V , its intersection with VL , is the Vanishing Point of the Shadow, of all Lines perpendicular to the Plane whose Vanishing Line is VL .

For, SV and AB represent parallel Lines (Cor. to Theo. 3.) wherefore, a Plane may pass through both Lines (Ax. 5.) and consequently, SA , VB , AF , and SF are all in that Plane.

But, F is the Vanishing Point of Lines perpendicular to the Plane; and, because S represents a Point at an infinite Distance in that Plane, SF is the Vanishing Line of a Plane passing through AB .

But, S represents the Sun, the Vanishing Point of the Rays of Light, and V is its Seat on the Plane (whose Distance is supposed infinite) consequently, V is the Vanishing Point of the Shadow, BD ; for, the Plane of Shade, SAF , projecting its Shadow, is parallel to a Plane passing through the Eye and the Points S and F , seeing they are at an infinite distance; and the Shadow, BD , is the intersection of that Plane with the Plane of Projection; therefore its Vanishing Point is V , the intersection of their Vanishing Lines, VL , and SF . (Cor. 2. Theo. 6.)

This Problem is universal; I shall, next, apply it to Planes which are perpendicular to the Picture.

No. 2. SECONDLY. Let AB be a Line perpendicular to the Horizon, whose Shadow is required, and S the representation of the Sun, on the Picture; or S its transprojected Image, the Sun being supposed on either Side.

Draw SV perpendicular to the Vanishing Line; then, V is the Seat of the Luminary, its Distance being supposed infinite.

Draw VB and SA , as before, intersecting at D ; BD is the Shadow required.

For, because AB , whose Shadow is to be projected, is perpendicular to the Horizon, and the Picture is supposed vertical; it is parallel to the Picture, consequently SV , parallel to AB , is the Vanishing Line of the Plane of Shade; wherefore, V is the Seat of the Luminary, the Vanishing Point of the Shadow; and, SD is a Ray of Light, projecting the Shadow of the Point A , which determines its length, BD .

This is universally applicable to all Planes which are perpendicular to the Picture, whether they be horizontal, vertical, or inclined to the Horizon.

THIRDLY. When the Luminary is in the Plane of the Picture.

In this CASE, the Rays having no Vanishing Point, the inclination of the Rays to the Plane of projection, being determined, they are all parallel on the Picture; seeing, the distance of the Vanishing Point is infinite.

No. 3. Draw BD parallel to the Vanishing Line, and SA , cutting BD at D , making the Angle SDB equal to the Angle of the Sun's Elevation; BD is the Shadow required.

For, because the Luminary is supposed in the Plane of the Picture, it is consequently in every Plane parallel to the Picture; wherefore, the Plane of Shade, SDB , is parallel to the Picture, seeing the Line AB is parallel; and consequently, BD , its Shadow, is parallel to the Vanishing Line of the Plane of Projection, seeing it is parallel to the Picture.

When

When the Plane of projection is parallel to the Picture, there can be no Shadow projected on it, but when the Luminary is on this side.

Let AB represent a Line perpendicular to the Plane X; and, let S be the trans-
projected Image of the Sun, on the Picture; C is its Center. Join CS.

No. 4

Draw BD, parallel to CS, and AS, cutting BD, at D; BD is the Shadow required

Because the Plane (X) is parallel to the Picture, it has no Vanishing Line; wherefore, seeing the distance of the Luminary is supposed infinite, its Seat, on that Plane, is also at an infinite Distance.

But CS, produced, is the Seat of a Ray of Light, projecting its Vanishing Point, S; for it passes through the Eye, which is perpendicularly opposite to C; wherefore, the Seat of the Luminary is at an infinite distance, in the Line SC, produced. Consequently, the Shadows BD, *BD*, &c. are all parallel to CS; and S is the Vanishing Point of the Rays, which determine the length of the Shadow.

P R O B L E M III.

To project the Shadows of Right Lines, on a Plane to which they are parallel; and, in all positions to the Picture.

FIRST, when the Lines are parallel to the Picture, and the Luminary in the Plane of the Picture.

The Shadows of Lines, on a Plane to which they are parallel, cannot be determined, conveniently, without having their Seats on that Plane; and is the same as finding the Shadows of Lines perpendicular to the Plane; or, the Shadow of one extreme of the Line being found, the other is easily determined.

Let AB be a Line parallel to the Picture, and parallel to the Ground Plane, on which the Shadow is to be projected. Its Seat is *ab*.

Fig. 14

If the Sun be in the Plane of the Picture, and if it was in the Zenith of that place, the Shadow of AB is *ab*. But, if the altitude of the Sun be the Angle SCD; then, drawing AD, parallel to SC, the Shadow of AB is CD; for, the Shadow of a Perpendicular, *Aa*, is *aD*; and CD is equal AB, seeing, ABCD, the Plane of the Shadow, is a Parallelogram.

After the same manner, the Shadow of the inclined Line, EF (being parallel to the Picture) is projected; by producing the Line till it cuts the Plane, at G; then, drawing GI, parallel to the Vanishing Line, and SH, SI, through F and E, parallel to SC, cutting GI, at H and I; HI is its Shadow.

No. 2

SECONDLY. If the Line AB be either perpendicular or inclined to the Picture, and parallel to the Horizon, its Seat, at least of one extreme (B) must be determined; and as the Point B may be in some vertical Plane, (as X) its Seat on the Ground is in the Intersection of that Plane, at *b*; and supposing the Plane X parallel to the Picture, the Luminary is in that Plane; consequently, the Shadow of the extreme B will be somewhere in the Intersection of the Plane.

No. 3

Draw SD, through B, making the Angle SDb equal to the altitude of the Sun, giving the Point D, in the Intersection of the Plane X, for the Shadow of B.

Then, because AB is perpendicular to the Picture, C, is its Vanishing Point; and, because the Shadow is projected on a Plane, to which AB is parallel, the Shadow is necessarily parallel to AB, and consequently it has the same Vanishing Point, C.

Wherefore, draw CD and produce it; draw AE parallel to SD, (the given Ray of Light) cutting CD, produced, at E, the Shadow of A.

DE is the Shadow of AB on the Ground; to which it is parallel and equal.

For the Plane of Shade ABDE is a right angled Parallelogram in Perspective.

FG is its Shadow on an inclined Plane, to which AB is parallel; projected by the same Rays of Light, or Plane of Shade.

SCHOL. Because the Sun is in the Plane X, it cannot be said to be illumined; consequently, no Shadow can be projected on it; otherwise, if the Luminary was ever so little on this side, the Shadow of AB will be first projected on it, from B to D, and then projected to E; in which Case, BD would not be a Ray of Light, but a Shadow, and consequently, not parallel to AE.

CASE

Plate XLI.

CASE the Second and Third.

Fig. 15. When the Luminary is on this side, or on the other side of the Picture.

FIRST. Let AB be parallel to the Picture, and to the Plane, on which its Shadow is to be projected. Let S be the Image of the Sun, on the Picture; or \odot its transprojected Image. Let VL be the Vanishing Line of the Plane of Projection.

Now, the Line AB being parallel to the Picture, has no Vanishing Point; consequently, its Shadow, being on a Plane to which it is parallel, is parallel to the Line. Let D be the Seat of A, on the Plane; C is the Center of the Picture.

Because the Plane, on which the Shadow is to be projected, is inclined to the Picture, through C, draw its Vertical Line EF; and because D is the Seat of A, on the Plane, draw AD and produce it, cutting the Vertical Line at F, the Vanishing Point of Lines perpendicular to the Plane.

For the Seat of a Point on any Plane is produced by a perpendicular to that Plane.

Draw SF, or \odot F, cutting the Vanishing Line at V, or L, the Seat of the Luminary on the Plane, at an infinite Distance.

Draw VD, or LD; and SA, or \odot A, cutting VD or LD, at a or a , the Shadow of the extreme A, on the Plane (Prob. 2.) Draw ab, or ab , parallel to the Van. Line, that is to AB; and, through B, draw SB, or \odot B, cutting ab, or ab , at b, or b .

ab is the Shadow of AB, by means of S, the Image of the Sun on the other side of the Picture; or, ab is its Shadow, projected by the Vanishing Point \odot , the transprojected Image of the Luminary, on this Side; both which, are Vanishing Points of the Rays of Light. aD, or aD , is the Shadow of the Perpendicular AD.

No. 2. SECONDLY. When the Line AB is perpendicular to the Picture, and parallel to the Plane of Projection; C is the Center.

Because the Line whose Shadow is to be projected is perpendicular to the Picture, and parallel to the Plane of Projection, that Plane is consequently perpendicular to the Picture, and its Vanishing Line VC passes through the Center. (Theo. 4.)

Let D be the Seat of the extreme A, on the Plane, and let S, or \odot , be the Image of the Luminary; projected or transprojected.

Draw SV, or \odot V, perpendicular to the Vanishing Line; and, through D, draw VD indefinite. Draw SA, or \odot A, till it cuts VD, at a, or a , the Shadow of the extreme A, on the Plane, in either Case.

For, AD is perpendicular to the Plane, and Da, or Da , is its Shadow. (Pr. 2.)

Draw Ca, or Ca , indefinite, and SB, or \odot B, cutting it at b, or b .

ab is the Shadow of AB, the Sun being beyond the Picture; or, ab is the Shadow of AB, the Sun being on this side, by means of its transprojected Image, \odot .

For, AB being parallel to the Plane of Projection, its Shadow is parallel to AB; consequently, because AB is perpendicular to the Picture, the Center, C, is their common Vanishing Point. And, because the Rays of Light are parallel amongst themselves, they have the same Vanishing Point, S or \odot .

N. B. When the Line, AB, is inclined to the Picture, and the Plane of Projection is perpendicular to it, there is no difference, in the process, but only in its Vanishing Point.

No. 3. THIRDLY. When the Line AB is inclined to the Picture, and the Plane of Projection also inclined to the Picture, let VL be its Vanishing Line.

Find the Vanishing Point (F) of Lines perpendicular to the Plane, (Pr. 2. S. 12.) and draw SF, or \odot F, cutting the Vanishing Line at E, the Seat of the Luminary.

Thro' D, the Seat of A, draw ED indefinite, and SA, or \odot A cutting ED, at a or a .

Then, V being the Vanishing Point of the Line AB, and because the Shadow is parallel to it, draw Va, or Va , indefinite, and SB, or \odot B, cutting it, at b, or b ; ab, or ab , is the Shadow of AB, projected on the Plane whose Vanishing Line is VL.

PROBLEM

P R O B L E M IV.

To determine the Shadows of Lines inclined to the Plane of Projection and to the Picture; in any Angle, whatever.

Let AB be inclined to the Plane Z, on which the Shadow is to be projected; and, Fig. 16.
let ST be a given Ray of Light, the Luminary being supposed in the Picture.

Through V, the Vanishing Point of AB, draw VC parallel to the given Ray, cutting the Vanishing Line of the Plane Z, at C, the Van. Point of the Shadow.

For, the Luminary being in the Plane of the Picture, its Rays have no Vanishing Point; consequently, a Right Line drawn through V, the Vanishing Point of any Line, parallel to the Rays, is the Vanishing Line of the Plane of Shade, projecting the Shadow of that Line.

Therefore, draw AC, the indefinite Shadow of AB; and BD parallel to the given Ray; that is, parallel to VC, the Vanishing Line of the Plane of Shade. AD is the Shadow of AB on the Plane Z.

If the Line EF, whose Shadow is required, does not cut the Plane, on which it is to be projected, let it be produced till it cuts the Plane, at G, and draw GC.

Or, if the Seat of either extreme be given, or found, as *a* of the Point E, or *b* of the extreme F; then, having obtained the Shadow of either, *e* or *f* (Prob. 2.) draw Ce, or Cf; and Rays, through the extremes, E and F, parallel to the given Ray, ST; by which means the Shadow, *ef*, of EF is projected.

CASE the Second and Third.

When the Luminary is beyond, or on this side of the Plane of the Picture.

Let AB be a Line, whose Shadow is to be projected. V is its Vanishing Point. No. 2.
Let S and \odot be the projected and transprojected Images of the Sun.

If the Line does not cut the Plane of Projection, let it be produced, to C.

Draw SV or \odot V, cutting the Vanishing Line EF, at F, the Van. Point of the Shadow. Draw CF; and SA, SB, or \odot A, \odot B, cutting it at *a* and *b*, or *a* and *b*.
ab is the Shadow of AB when the Luminary is beyond the Picture, and *ab*, when it is on this Side; according to its determined place.

Or the Shadow may be found by means of the Seat of either extreme, given or found, on the Plane of Projection; as in the first Case.

SCHOL. Any Line, whose Shadow is to be projected, cutting the Plane of projection (whose Vanishing Line is given or found) the process is the same, whether the Plane of projection be perpendicular or inclined to the Picture; for, if the Vanishing Point of that Line be obtained, the Vanishing Line of the Plane of Shade is also; whose Intersection with the Vanishing Line of the Plane of projection is the Vanishing Point of the Shadow; which, it must be obvious, always passes through that Point in which the Line cuts or would cut the Plane.

2. When the Plane of projection is parallel to the Picture, there can be no Shadow projected on it, but when the Luminary is on this side; in which Case, whether the Line be perpendicular or inclined to the Plane, it is the same to the Picture, and, its Vanishing Point being determined, the process is the same. In this Case, the Plane of projection having no Vanishing Line, the Shadow, of any Line thereon, is parallel to the Vanishing Line of the Plane of Shade.
3. All Lines which are parallel to such Planes are also parallel to the Picture; and their Shadows, being parallel to the Lines, are always so represented; which is not the Case on other Planes, but only, when the Lines are parallel to the Picture, as well as to the Plane of projection.
4. The Seats of all such Lines, on the Plane, must be had, or the Shadow cannot be determined; in which Case, it is but finding the Shadows of two equal Lines, perpendicular to the Plane (as in Fig. 14.) and, joining their extremes, the Shadow of the parallel Line is obtained, as in the second Problem.

Having, in these Problems, given every Rule which I conceive necessary for the projection of right lined Shadows, I shall, next, give some Examples of their utility, in the application of them to the Shadows of Objects.

Plate
XLII.

As every Object is composed either of plane or curved Surfaces, or both, so likewise, those Surfaces are bounded by right Lines or curved; and, as the outline of the Shadow only is required, we should carefully observe on what part of the Object the Light falls, that we do not give the Shadows of such Lines as can (from their situation to the Light) have no Shadow; for, according to the altitude of the Luminary, or its situation to the Object, different Lines in the Object define its Shadow. Wherefore, since it is from the linear perspective representation of the Object, that the Shadow is projected, it is sometimes difficult to determine the Line which casts the Shadow, because they are frequently out of sight; wherefore, they ought to be determined with caution, before we begin the process.

The consideration of the Luminary being beyond, or on this side, or in the Plane of the Picture, is also a circumstance which should be maturely considered. The best effects, are (in my opinion) produced, by considering the Sun to be on this side; because it is obvious, that, when it is beyond the Picture, though ever so little, that is, though ever so much inclined, it is also beyond every Object represented on the Picture; but, if the Sun itself be represented, that is, if it be so little elevated and inclined to the Picture, that the Sun's apparent place is within its limits, it must then be entirely behind the Objects; consequently, the whole of every Object, on that side, is immersed in Shade; and the Shadow, of each, is projected towards the Eye, greatly distorted, and larger than the Object. Whereas, the Sun being on this side, towards either hand, as is most suitable to the Objects (according to what Faces we would have illumined) the Shadows are projected forwards, from the Eye, diminished, and great part of it hid by the Object; which, with the various effects of Light and Shade, on the Objects, cannot fail rendering the whole agreeable, and most pleasing to the Eye.

E X A M P L E I.

To represent the Shadow of a Plane Figure, on a Plane.

Fig. 17. FIRST. Let the Plane ABCDE be perpendicular to the Ground Plane, on which the Shadow is to be projected. Let S and \odot be the Image and transprojected Image of the Sun; VL is the Vanishing Line of the Plane ABCDE, and FG of the Plane of projection.

Draw SF, or $\odot F$, perpendicular to the Horizontal Line, cutting it at F; the Vanishing Point of Lines perpendicular to the Horizon. (Prob. 2.)

Through A, draw Fb, indefinite; and, if the Shadow be projected by the Image, S, through B, draw Sb, cutting the former, at b. Or, draw B \odot , cutting it at b.

Then, for the Shadow of BC (on this side) join SV; if it be parallel to the Horizon, draw bc also parallel; if it be not parallel, then, SV produced will give its Vanishing Point. For the same Line on the other Side, draw V \odot , cutting the Horizontal Line at H, and draw bH. SC, and C \odot , cuts them at c and c, respectively.

CD is parallel to the Plane of projection, and G its Vanishing Point; therefore, draw cG, or cG, and SDd, or D \odot , giving d, or d, for the Shadow of D.

Lastly; for the Line DE (on this side) draw SL, cutting the Horizontal Line at l, and draw ld. For the other Side, draw $\odot L$; which being produced to the Horizontal Line, will give the Vanishing Point of dE, and compleats that Shadow. Or, join dE, or dE, only.

AbcdE is the Shadow of ABCDE projected on the Ground Plane, by the Image (S) of the Sun, on the other side of the Picture; and, AbcdE is its Shadow, by the transprojected Image, \odot , being considered on this Side.

Fig. 18. SECONDLY. Let the Plane ABCDE be either perpendicular or inclined to a vertical Plane, whose Vanishing Line is VL, on which the Shadow is projected.

Whether the Plane, whose Shadow is required, be perpendicular or inclined to the Picture, the process is the same. HF is the Van. Line of the Plane ABCDE.

FIRST. Let S be the Image of the Sun, on the other side of the Picture.

The Line, AB being parallel to the Picture, draw SV parallel to it; V is the Vanishing Point, of its Shadow; wherefore, through A draw Vb, indefinite, and, through B, draw Sb, cutting it at b, which determines its length, Ab.

H being the Vanishing Point of the contiguous Line BC, draw SH, and produce it to I, the Vanishing Point of its Shadow.

Draw bI, and Sc, through C, cutting it at c. bc is the Shadow of BC.

Then, because CD is parallel to the Plane of projection, and G is its Vanishing Point, draw cG, and SD, cutting it at d, the Shadow of D.

Lastly, draw SF, giving the Vanishing Point J if it was necessary; but it is obvious, that, dE, being joined, compleats the Shadow, and tends to J.

If the Shadow was obstructed by another Plane, cutting it at MN, the Vanishing Line of which is FN; SV, being produced, would cut that Vanishing Line in the Vanishing Point of the Shadow of AB, on that Plane; from which, draw Mb. SH produced, cuts the Vanishing Line at P; draw bP, cutting the Ray SCc at e; and, SG produced cuts it at O; draw eO; or join ef only, which compleats it.

SECOND. Let \odot be the transprojected Image of the Sun, on this side.

AB being parallel to the Picture, draw \odot K parallel to it, cutting the Vanishing Line, VK, at K, the Vanishing Point of its Shadow.

Draw AK, and B \odot , cutting it at b, the Shadow of B. \odot H cuts the Vanishing Line at L; draw bL, and C \odot , cutting it at c. Then, draw cG, and D \odot , cutting it at d. cd is the Shadow of CD, being parallel to the Plane of projection.

If \odot F be parallel to the Vanishing Line, VK; then, dE is also parallel to it. Or, it will tend to that Point, in which \odot F, being produced, would cut VK.

SCHOL. If the Plane of projection be much inclined to the Picture, there can be no Shadow cast on it, the Sun being beyond the Picture; or they will be drag'd out to an immoderate length. For, if the Image of the Sun appear in the Picture, it will be behind the Plane of projection, being much inclined.

By observing the Rules, given above, the Shadow of any plane Object may be projected on any Plane, or on various Planes, of which I shall give some Examples.

E X A M P L E II.

To project the Shadows of right angled Parallelopipeds, on the Ground Plane.

FIRST. When a Face of the Parallelopiped is parallel to the Picture, and, the Luminary in the Plane of the Picture.

X is the Object whose Shadow is to be projected.

In this CASE, the Luminary being in the Picture, the Shadow of the whole Solid is no more than of the Plane ABDE. For, the Plane X being parallel to the Picture, cannot be illumined; seeing that the Sun is in the Picture, it is, consequently, in every other Plane parallel to the Picture. The whole Shadow of that Plane is, therefore, but a Line, generated by producing the Plane.

Draw Ab, indefinite, parallel to the Vanishing Line, HL, and, if SB be a given Ray of Light, produce it, till it cuts Ab, at b, the Shadow of B.

Then because BD is perpendicular to the Picture, C, the Center, is its anishing Point; and, being parallel to the Plane of projection, its Shadow consequently parallel, and has, therefore, the same Vanishing Point.

Draw bC; and through the angle D, draw Dd parallel to Sb, cutting bC at d; or, draw Ed parallel to Ab, which compleats the Shadow, AbdE, required.

SCHOL. If a Line (FG) projected from the Plane X, its whole Shadow will be projected on the Ground. For, since the Luminary is in the Plane X, no part of its Shadow can be projected on that Plane; wherefore, fg, its Shadow, is wholly on the Ground Plane.

Fig. 19.

Plate SECONDLY. When its Planes are inclined to the Picture.

XLII. Let AD be a right angled Parallelopiped, the Vanishing Points of its Sides, are
Fig. 20. H and L; and let ST be a given Ray of Light.

Draw Ab parallel to the Horizon; and through B, draw Bb parallel to ST.

Then, L being the Vanishing Point of BC, draw bL; and through C, draw Cc, parallel to ST, cutting it at c. Lastly, draw cH; and Dd parallel to ST; and, through d parallel to Ab. Abcd is the Shadow required.

For the Rays, projecting the Shadow, are all parallel amongst themselves; and the Perpendicular AB, being parallel to the Picture, has its Shadow (Ab) parallel to the Vanishing Line; also, the Shadows of BC and CD have the same Vanishing Points, respectively; because, BC and CD are parallel to the Plane of projection.

CASE 2d. When the Luminary is on the other side of the Picture.

Fig. 21. First; let the Plane AD be parallel to the Picture, and, let S be the Image of the Sun on the Picture, being situated beyond it.

Draw SV, perpendicular to the Horizon, V is the Vanishing Point of the Shadows of Perpendiculars, on the Ground Plane. (Prob. 1.) Draw VA and SB, and produce them, cutting at b, the Shadow of B. Then, draw bd parallel to BD; and, through D, draw Sd, cutting it at d; bd is the Shadow of BD.

Lastly; because DE is perpendicular to the Picture, and C is its Center, draw dC; and, through F, draw VF, or SE, cutting it at e, and compleats the Shadow, AbdeF; which, as it is projected back, towards the Eye, is larger than the Object.

SCHOL. When the Object is inclined to the Picture, as in No. 2, the process is the same in every respect, having regard to the Vanishing Points of the Lines BC and CD, to which their Shadows, bc and cd, are respectively parallel, and consequently have the same Vanishing Points, G and F.

No. 2. CASE 3d. When the Luminary is situated on this side the Picture.

Let AD be a Parallelopiped, right angled, as before; and let \odot be the trans-projected place of the Luminary.

Draw $\odot V$ perpendicular, and, draw AV and B \odot , cutting at b, the Shadow of B. Then, draw bG and C \odot , cutting at c; and lastly, draw cF, which compleats the Shadow Abcd. The Shadow of the Angle D is not seen.

E X A M P L E III.

Plate
XLIII.

To project the Shadow of a plain Building, on the Ground.

Fig. 22. Let ADF represent a Building, obliquely situated to the Picture; and let the Sun be in the Plane of the Picture. The direction of its Rays is SC.

Draw Ab parallel to the Vanishing Line, and Bb to the given Ray, cutting Ab, and determining the Shadow of AB.

Through V, the Vanishing Point of BC, draw VG parallel to the given Ray of Light; and draw bG; SC produced, cuts it at c, the Shadow of C.

Because the Vanishing Point of CD is below the Horizontal Line, equal VH, the Vanishing Point (I) of its Shadow, will be on the other side VH, equal GH.

Make HI equal GH, and draw cI; and Dd parallel to SC, cutting it at d.

Because of the Sun's altitude, and the situation of the Building, the other side of the Roof is illumined; and consequently, the Shadow of CF (the Ridge) is not projected, which is evident; because, being parallel to the Plane of projection, its Shadow is parallel to itself, and consequently has the same Vanishing Point, I. Wherefore, if from c, the Shadow of C, a Line be drawn to L, it will fall on this Side cd, the Shadow of CD; consequently it has no Shadow, here.

Therefore, draw dL, cutting DE; de is the Shadow of a Line from D, parallel to CF (the Eaves of the Roof) on the other side of the Building.

SECONDLY.

SECONDLY. Let the Sun be supposed beyond the Picture, and let S be its Image.

Draw ST, perpendicular; and, through A, draw Tb indefinite; through B, draw Sb, giving Ab, for the Shadow of AB. Fig. 23.

Then, BC being parallel to the Horizon, and L its Vanishing Point, through b, draw Lc, and through C draw Sc, giving bc, the Shadow of BC.

V being the Vanishing Point of CD, draw SV. If SV, be parallel to the Horizon, draw cd, also parallel; if SV, be not parallel, produce it, till it cuts the Horizontal Line, on one side or the other, and draw cd, tending to that Point; also, through D, draw Sd, cutting it at d, the Shadow of D.

Again. Y being the Vanishing Point of DE, draw SY, cutting the Horizontal Line at I, the Vanishing Point of the Shadow of DE.

Draw dI, and SE till it cuts dI, at e; and lastly, join eF, which will tend to its Vanishing Point T, and compleats the Shadow, AbcdeF.

In this Case, it is obvious that the whole Building, save the Plane DB, is in Shade.

CASE 3d. When the Luminary is on this side of the Picture.

Let \odot be its transprojected Image. $\odot K$ being drawn, perpendicular, K is the Vanishing Point of the Shadows of perpendicular Lines. (Prob. 2.)

Draw AK and B \odot , intersecting at b, the Shadow of B.

Then draw $\odot V$, cutting the Van. Line at I, the Van. Point, of the Shadow of BG.

Draw bI, and G \odot , cutting it at g, the Shadow of G.

Join $\odot Y$; and if it be not parallel, produce it to the Vanishing Line.

Through g, draw parallel, or tending to the Point in which $\odot Y$, cuts the Horizontal Line; which compleats the Shadow, as much as can be seen.

I presume, it is obvious, from these Examples, that, the most judicious choice is to suppose the Luminary on this side of the Picture, as in this last Case. First, because the object is more advantageously situated to the Luminary, and consequently, will have a more agreeable effect of Light and Shade. Also, because the Shadow is projected from the Eye, so that, it does not appear distorted and preposterous, as in the second Case; and it is far from being pleasing to have the Shadows of perpendicular Lines all parallel amongst themselves; as when the Luminary is in the Plane of the Picture. Therefore, as I have given Rules and Examples in each, I shall, hereafter, consider the Luminary only on this side of the Picture, unless it be otherwise determined.

E X A M P L E IV.

To project the Shadow of an upright prismatic Object, whose upper Face is inclined to the Horizon, part on the Ground, and part on a vertical Plane, inclined to the Picture, whose Intersection with the Ground is given.

Let X be the Object, whose Shadow is to be projected; let \odot be the transprojected Image of the Sun, and JI the Intersection of a vertical Plane with the Ground; KL is its Vanishing Line. (Prob. 3. Sect. 3. Book III.) Fig. 24.

Draw $\odot H$ perpendicular to the Horizontal Line. Draw AH, and B \odot cutting it at b, the Shadow of the Angle B on the Ground.

But, Ab cuts the Intersection JI, at a; wherefore, because the Plane Y is vertical, AB is parallel to the Plane; and, being also parallel to the Picture, its Shadow is consequently parallel; draw ab, parallel to AB, till it cuts the Ray B \odot , at b, the Shadow of B on the vertical Plane; as b is, on the Ground.

The Vanishing Point of BC is F; which, being inclined to the Horizon is not in the Horizontal Line. Draw $\odot F$, and produce it, till it cuts KL, the vertical Vanishing Line, at K, the Vanishing Point of its Shadow; then, draw bK, and C \odot cutting it at c, the Shadow of C on the vertical Plane.

Plate
XLIII.

Then, G being the Vanishing Point of CD, a Line on the other side, parallel to BE, which projects a Shadow, draw $\odot G$, cutting the vertical Vanishing Line at L. Draw cL and D \odot intersecting at d.

If DM be a perpendicular Line, draw de parallel to it, cutting the Intersection at e, and join eM, which tends to the Vanishing Point, H, of Perpendiculars, and compleats the Shadow; Aaef on the Ground, and abcde on the vertical Plane, as it was proposed.

If the vertical Plane was out of the way, its Shadow on the Ground is *Abcdf*; in which, the Shadow of BC tends to N, and CD to O, in the Horizontal Line.

E X A M P L E V.

To project the Shadow of a Cross, perpendicular to the Horizon, part on the Ground; also, on a Plane inclined to the Horizon, and to the Picture, whose Vanishing Line, and Intersection with the Ground Plane are given.

Fig. 25.

Let AFBK be the Object, inclined casually to the Picture; whose Vanishing Points are M and N, in the Horizontal Line. Let QR be the Intersection of the inclined Plane with the Ground, whose Vanishing Line is OP. ST is a given Ray of Light, the Luminary being in the Picture.

Because the Object is inclined to the Picture, and the Luminary is in the Picture, the Side AB and its opposite, IJ, will project Shadows.

Draw Aa, and Jb parallel to the Horizon, cutting the Intersection, QR, at a and b. Draw ab, and bi, parallel to the Vanishing Line OP, indefinite; and draw Bb, parallel to ST, cutting ab, at b, the Shadow of B.

For, the Line AB being parallel to the Picture, the Plane of Shade occasioned by that Line is, in this Case, parallel to the Picture; consequently, its section with every Surface, on which the Shadow of AB falls, is parallel to the Picture; therefore, Aa is parallel to the Vanishing Line of the Ground Plane (MN) and ab to OP, of the inclined Plane.

Draw NO and MP, parallel to the Ray of Light, ST; O is the Vanishing Point of the Shadows, of all Lines parallel to CD; and P, of all that are parallel to EF, on the inclined Plane.

Draw Cc parallel to ST, cutting the Shadow of AB at c. Draw Oc indefinite, and Dd parallel to the Rays, cutting it at d, the Shadow of D.

DE being parallel to the Picture, draw de, its Shadow, parallel to the Vanishing Line, OP, meeting the Ray of Light from E, at e.

P being the Vanishing Point of the Shadow of EF, draw eP, meeting the Ray from F, at f; and draw fO, till it cuts the Intersection QR, at j; from which it tends to N, the Vanishing Point of FG; being parallel to the Ground.

dO cuts the Intersection at a, from which Point it also tends to N, till it meets the Ray, from K, at k. Draw kM, cutting the Ray from L; and LG being parallel to the Picture, draw lg, parallel to the Horizon, which compleats the Shadow of that Arm, on the Ground.

The Shadow, bbi, of the Top, having the same Vanishing Points, respectively, it would be unnecessary to describe, being obvious, from the Figure.

Secondly. *When the Luminary is on this side of the Picture.*

In the foregoing Case (as in other preceding) the Vanishing Point of the Shadow of every Right Line, on the inclined Plane, was produced, by drawing a Right Line, parallel to the given Ray, from the Vanishing Point of the Line whose Shadow was required, to the Vanishing Line of the inclined Plane; because the Luminary being supposed in the Plane of the Picture, all its Rays are consequently parallel amongst themselves.

In this, the Rays of Light having a Vanishing Point, a Right Line joining the Vanishing Point of the Line, and the Vanishing Point of the Rays, cuts the Vanishing Line, or being produced, would (if it be not parallel) cut the Vanishing Line of the Plane of projection, in the Vanishing Point of the Shadow (Prob. 4.) or, the Rays, instead of being parallel amongst themselves (as in the former Case) converge in a Point, in which consists the difference.

○ is the Vanishing Point of the Rays, MN the Vanishing Line of the Ground, and OP of the inclined Plane, of which, QR, is its Intersection with the Ground. M and N are the Van. Points of hor. Lines in the Object, as before. Fig. 26.

S being the Seat of the Luminary, on the Ground, draw AS, cutting the Intersection, QR, at a. Then, because AB is parallel to the Picture, produce ○S, (being parallel to AB) till it cuts the Vanishing Line of the inclined Plane, at T.

Draw aT; B○, a Ray of Light, cuts aT at b, the Shadow of B; and C○ cuts it at c; through which, a Line drawn from O gives the Shadow of DK, on the inclined Plane; from the Intersection QR it tends to N. Rays from D and K, tending to ○, determine the extremes of the Shadow.

It would be useless to describe the whole, the same Letters of reference, as in the foregoing Case, shew how the Shadow of every Line is produced, by adhering to the former Lessons, in this Case.

EXAMPLE VI.

To project the Shadow of a Right Line, inclined to the Horizon and to the Picture, on several Planes, variously inclined to both. Plate XLIV.

AC represents a Ladder, leaning against a Building, on which its Shadow is to be projected. ○ is the transprojected place of the Luminary; H and I are the Vanishing Points of horizontal Lines in the Building, and E of the Ladder. FI is the Vanishing Line of the Roof of the Building; and GI of the adjoining Shed; on which, the Shadow is also projected. Fig. 27.

Draw E○, and produce it to the vertical Vanishing Line (KI) of the Sides of the Building, cutting it at K; the horizontal Vanishing Line is cut at L, and the inclined Vanishing Lines, at M and N.

K, L, M, N, and E, are all Vanishing Points of the Shadow. (Prob. 4.)

From the foot of the Ladder, draw Lines to the Vanishing Point L, cutting the Intersection of the Plane V, with the Ground, at a and b; AabB, is its Shadow on the Ground.

K being the Vanishing Point of the Shadow, on the vertical Planes, draw Lines from K, through a and b, till they cut the common Intersection of the Planes V and U, at c; from which, draw Lines to E, the Vanishing Point of the Ladder; because it is parallel to the Plane U.

M being the Vanishing Point of the Shadow, on the Plane X, draw dM, cutting the Intersection of that Plane with the next, at e; from which Points draw Lines to K; for the Shadow on the Plane Y, being parallel to V, has the same Van. Point.

From the Points ff (where the Ladder touches the Eaves of the Roof) draw to N; and, where they cut the Intersection of the Chimney with the Roof, draw to K, and E; from C and D the ends of the Ladder, draw to ○, cutting them, which compleats the Shadow of the sides of the Ladder.

If from each Step, on either Side, Lines are drawn to ○, cutting the Shadow of that Side, at a, b, c, &c. and from those Points, Lines are drawn to I, the Vanishing Point of the Steps, their Shadows will be compleated, on the Planes V, X, Y, and Z, as well as on the Ground; because the Steps are parallel to all those Planes; being parallel to the Horizon.

On the vertical Plane U, their Vanishing Point is where a Right Line drawn from I, through ○, would cut the vertical Vanishing Line EH, of that End.

The Chimney being perpendicular, ○S, drawn perpendicular, cutting the Vanishing Line (GI) of the Roof, at R, gives its Vanishing Point; J○ determines its length.

OPQ represents a Sign Post, whose Shadow, Oo, on the Ground, and opQ, on the Wall, is projected by the same Point (○) whose Seat is S; which, being so near the Vanishing Point I, consequently, the Luminary is nearly in the Plane OPQ; wherefore, the Shadow of both, on the Wall, are almost in one Line.

Plate
XLIV.

In this situation of this Object, the Shadow is more pleasing supposing the Sun on the other side of the Picture; as O_s , on the Ground, op of the Post, on the Wall, and pQ of the Bearer. The Sun being on the left hand, V (at a proper distance) may be the Vanishing Point, of the Shadow pQ ; and, the Seat of the Luminary, is the Vanishing Point of O_s , on the Ground (Prob. 2.) OP being perpendicular. The Rays of Light, Pp , &c. indicate the rest.

E X A M P L E VII.

To project the Shadow of a Building, and the several parts of it on the other; the Luminary being on this side of the Picture, having its Altitude and Inclination given.

Fig. 28.

Let the Angle of the Sun's Elevation be equal X ; and the Inclination to the Picture, or a vertical Plane passing through its Center, equal Z .

Find \odot , the transprojected place of the Luminary; by Prob. I.

Draw $\odot S$ perpendicular, giving S , the Seat of the Luminary, on the Ground.

Then, draw AS , cutting the Intersection of the Plane of the Arches, on the Ground, at a ; from which, draw aa , perpend. the Shadow of AB on that Plane.

Produce $\odot S$, till it cuts VO , the Vanishing Line of the inclined Plane X , at O ; the Vanishing Point of the Shadow of AB , on that Plane.

Draw aO indefinite; and, because of the projecture of the Cornice at C , casting a Shadow (JB) on the Plane W , draw $B\odot$, cutting aO at b , the Shadow of B ; from which, draw bc , to the Vanishing Point V ; because, bC , projecting that Shadow is parallel to the Plane X .

Then, M being the Vanishing Point of CD , draw $\odot M$, and produce it, till it cuts the Vanishing Line of the Planes V and W ; and, VO being also cut at P , draw cP , cutting the next plane at e . and draw eQ , to the Vanishing Point, in VE .

L being the Vanishing Point of DE , draw $\odot L$, and produce it to the Vanishing Line VM , of the Roof, cutting it at N . Draw NE , and $D\odot$ cutting it at d , and draw Md , to f , which compleats the Shadow of that part. CD , projecting the Shadow df , is parallel to the Plane Y , and has, therefore, the same Vanishing Point, M .

As the Luminary is not much elevated, the Shadow of the angle of the Cornice falls on the Wall, at h . F being its Seat on the Ground, draw FS , cutting the intersection of the Wall with the Ground at f ; from which draw fh , perpendicular, and $H\odot$, cutting it at h .

RT is the Vanishing Line of the Wall Z , and M being the Vanishing Point of HI , $\odot M$, cutting RT gives the Vanishing Point T , of hi , the Shadow of HI , on the Plane Z . From i , the Shadow of IK tends to K , in RT ; $K\odot$ cuts it at k ; from which, the Shadow of KE (the Ridge, which is not seen) tends to R , till it cuts the Intersection of the Wall with the Ground, whence it tends to V . $E\odot$ cuts it at e ; from which Point, ed is the Shadow of the Hip, at the farther corner.

To determine the Shadow of the projecture of the Cornice. Let $m n$ be the seat of the first Truss, on the top of the Cornice, and m its utmost projecture, which projects the Shadow; or, take any other Point, in mY .

L being the Vanishing Point of the Truss, i. e. of $m n$, draw from \odot through L , till it cuts the Vanishing Line of the Planes W ; from which Point, draw no , and $m\odot$, cutting it at o , the Shadow of m , on the Plane W , produced.

Draw oV , cutting both Planes W , in Gr , and BJ , the Shadow of the extreme projecture of the Cornice, on those Planes.

Having, by the same means, obtained the Shadow of the lower edge of the fronts of the Trusses, their places are obtained by drawing Lines, from each to \odot . The first falls on the Wall, at p , and the first on the other side, at q , on the low Roof.

The Shadows of the Piers of the Arcade, against the Wall, within, are projected by means of the Vanishing Point S , till they meet the wall, where they are upright. Lines drawn to \odot , determine their height. (See Sect. IV. for the Arches.)

Thus, the Shadow of the whole Building is compleated; in which are all the variety of Examples, necessary for projecting the Shadows of right lined Objects.

S E C T I O N IV.

Of the Shadows of curved Lines and curve-lined Objects, on plane and curved Surfaces.

AS the Perspective of curve-lined Objects is more difficult and liable to error than right-lined, on account of the continual bending, and varying in the direction of curve Lines; so, their Shadows are with more difficulty projected. As there are no vanishing Points of the Original Lines, so neither can there be vanishing Points of the Shadows; also, when the Shadows, either of right or curved lines, are projected on a curved Surface, there being no Vanishing Line, the figure of the Shadow cannot, so certainly, be described thereon. Nevertheless, Rules may be prescribed, for projecting Shadows, in such cases; which, if followed, will produce the true projection of as many Points, in the Curve, as are necessary, for describing the contour of the Shadow, with tolerable accuracy. The principal Shadow being determined, the minutias, of Mouldings, &c. may, by a Person who has judgment, be done by Hand, and in many cases they must; for, I fairly own, that it is more than human patience can bear, nor indeed is it possible, to project Shadows, in all Cases, with mathematical exactness.

The Shadow of a Circle, projected upon any Plane, to which the Circle is parallel, is a Circle, of equal dimensions; in all situations of the Luminary, whatever.

For, whether its situation be on this, or on the other side, or in the Plane of the Picture; whether it be more or less elevated, 'tis still the same; the projecting Rays, being parallel amongst themselves, generate a Cylinder, for the most part oblique; consequently, being cut by a Plane parallel to the Circle, its projection must also be a Circle, seeing it is the opposite Base of the Cylinder; but, being cut oblique it is an Ellipsis. So that, in Arches, &c. parallel to the Picture, the Luminary being on this side, and projecting the Shadow on a vertical Plane, parallel to the Arches, the Center of each being determined, the representation of the Shadow of each Arch, may be described with Compasses; which will be in proportion to the Arches, as the distance of the Plane of the Arches, to the distance of the Plane of projection. But, being projected on the Floor, or any other Plane, not parallel to the Arches, their Representations are Ellipses; except when the Cylinder, projecting the Shadow, is cut subcontrary*; in which Case, the Shadow will be a Circle.

In all other positions of the Arches, to the Picture, and in all situations of the Luminary, being projected on a parallel Plane, the Shadows being Circles, their Representations are Ellipses, described as the Arches, themselves; and being projected on any other Plane, the Shadows being Ellipses, their representations are also Ellipses, more excentric, generally, though not always so.

The Shadow of a Sphere or Globe (as well as its perspective Projection) on a Plane, is an Ellipsis, in all Cases, whatever; except when a Right Line passing through the Center of the Luminary, and of the Sphere, is perpendicular to the Plane of Projection.

* An oblique Cylinder or Cone is cut subcontrary, when the Plane of section is so situated, that the Axis of the Cylinder or Cone has the same inclination to it, contrarywise. As, if the Sun be elevated 45 Degrees, and be situated so, that a vertical Plane, passing through its Center, is perpendicular to the Plane of the Arches, i. e. to a vertical Circle; then, if the shadow be projected on the Floor, or any horizontal Plane, it will be a Circle; for the Rays cut the Floor, in the same Angle, viz. 45 Degrees.

Plate
XLV.

EXAMPLE VIII.

To project the Shadows of Cylinders, on a horizontal Plane, lying along; another upright, so situated, that its Shadow crosses one of the former.

Fig. 29. The Bases of the Cylinder (X and Y) being inclined to the Picture, their representations are Ellipses, whose Shadows, may be thus projected.

No. 2. Let ABCD be the representation of a Square, circumscribing a Circle, whose sides are parallel and perpendicular to the Horizon. Draw the Diagonals AC and BD, also the two Diameters ab and cd, parallel to the sides of the Square; and, from the Points e and f, draw perpendiculars to the Base, AD.

Find the Shadow of the Square ABCD (by Prob. 2. and 3.) and draw the Diagonals, AC and DB, draw dS, also gS and hS, cutting the Diagonals; and through their Intersection, draw ba, to the Vanishing Point of AD, &c.

Then, if a Curve be described through the eight Points, a, e, d, f, b, g, c, and d, it will be the true representation of the Shadow of the Circle, a c b d.

Or, because there is but half its Shadow required, proceed thus.

Draw ab perpendicular, through its Center; also, cd and ef parallel to a b.

Find the Shadows of a, c, e, &c. (Prob. 2. Case 3.) and describe the Curve a e b.

Then, the Sides of the Cylinder being parallel to the Plane of projection, draw cV, a Tangent to the Curve; and, the Shadow of the other End being described by the same means, compleats the Shadow of that Cylinder.

Fig. 30. SECOND. The Shadow of the Cylinder Z, on account of its situation, is but little seen. Its Base is turned from the Light, in the other it was illumined, so that, 'tis the other edge of the Circle which projects the Shadow.

THIRD. The Sides of the upright Cylinder being perpendicular to the Horizon, draw AS, a Tangent to its Base, at A, cutting aV, the seat of the Side be, on the Ground, of the Cylinder Z, at A; from which Point, draw Ab perpendicular, cutting be (the greatest projecture of the Cylinder Z) at b.

But, on account of the inclination of the Rays of Light, the Shadow of the Side AE will be continued on the Ground, till it falls into that of the other, at a; and it is continued down the side, from b towards a, till it is lost, insensibly, in the shade of the other Cylinder.

Take as many Points, b, c, d, in the curve of the Base, of the Cylinder Z, as are necessary; and others, answering to them, that is, of equal height, perspectively, at B, C, D, on the Side, AE, of the upright Cylinder; from all which, draw Lines to S, cutting others, drawn from the corresponding Points b, c, d, to the Vanishing Point V, at b, c, d, &c. through which, a Curve being described, will be the Shadow of the Side AE, on the Cylinder Z.

For, the Side of the Cylinder (AE) projecting the Shadow, is perpendicular to the Plane of projection; wherefore, the Plane of Shade, occasioned by that Line, vanishes in S; and, all horizontal Lines (in that Plane) from the Points A, B, C, D, &c. vanish at S; and, consequently, they must cut others, bb, &c. drawn in the Side of the Cylinder Z, being of equal height. Therefore, abcd is the section of the Plane of Shade, with the surface of the Cylinder Z, made by the Side, AE, of the upright Cylinder; and consequently, abcd is its Shadow on that Surface.

The upper Base (EF) of the upright Cylinder, is above the Eye, and, being parallel to the Plane of projection, its Shadow is a Circle; wherefore, having obtained the Shadow of any Diameter, as ef, of EF, by drawing E \odot and F \odot ; the former cutting AS, at e, and drawing ef to the Vanishing Point of EF; then, the Shadow being a Circle, and projected on a Plane, not parallel to the Picture, its representation is an Ellipsis (Theo. 2. Sect. 5. Book I.)

Describe the representation of a Circle, whose Diameter found is ef (Prob. 2. Sect. 8.) and from G, draw a Tangent, tending to S, the Shadow of the other Side of the upright Cylinder, which also cuts the other in the same manner, and compleats the Shadow, as much as can be seen. Let it be observed, that the Shadow, on the Cylinder Z, is darkeft, where the Light is strongest.

E X A M P L E IX.

To project the Shadows of Right Lines, on a convex cylindrical Surface.

Let AK be a Right Line, any how situated, in respect of the Cylinder X; its Vanishing Point is V. Its Shadow on the Cylinder is required; \odot being the transprojected Image of the Luminary. Fig. 31.

Draw V \odot , cutting the Horizontal Line at J, the Vanishing Point of the Shadow on the Ground. Draw AJ, cutting the Base of the Cylinder at a; and, through a, draw $\odot A$, cutting AK at A.

Aa, on the Ground, is the Shadow of AA, part of the Line AK; beyond which, its Shadow is projected on the Cylinder.

Having obtained S, the Seat of the Luminary, (Prob. 2.) also, R, the Vanishing Point of the Seat of AK, on the Ground, draw AR; and, from various Points, B, C, &c. (in AK) at discretion, beyond A, draw BB, CC, &c. perpendicular; from all which Points, B, C, &c. so obtained, draw Lines to S, cutting the Base of the Cylinder, at b, c, &c. from which, draw perpendicular up the Cylinder; and from the several Points, B, C, &c. draw lines to \odot , cutting the corresponding Perpendiculars, at b, c, d, &c. through all which, a Curve, abcdef, being described is the Shadow of AF, part of AK, on the cylindrical Surface.

The remaining part of its Shadow, falls on the Ground, behind the Cylinder.

SECOND. FH, on the top of the Cylinder, is a Square, which may represent the Abacus of the Tuscan Capital; to project its Shadow on the Cylinder and Floor.

Take as many Points f, g, h, &c. as are necessary, in the lower Lines, which cast their Shadows on the Cylinder; from all which, draw to S; also, from the Angle G, cutting the upper Base, or Curve, at 1, 2, 3, &c.

Draw perpendiculars from each Point, down the Cylinder, which are cut, by Rays from each Point, f, g, &c. to \odot , at f, g, &c. the Angle G is projected to G. The Ray k \odot , touches the Cylinder, at k, whence, it is projected to K, on the Floor.

Curve Lines, drawn through Gfgh, and Gik, are the Shadows of the Lines Gk and Gl on the Cylinder; kF is projected on the Floor.

Draw Sa, a Tangent at the Point a, cutting the Ray, from k at K; a K is the Shadow of the side of the Cylinder on the Floor.

Draw KF to the Vanishing Point of GF, and F \odot , cutting it at F, the Shadow of the Angle F; draw FL, to the Vanishing Point of GH, and FI, the upper Line, on the other side, which casts the Shadow; I \odot cuts it at I; through which, draw IM, tending to the vanishing Point of HI, and compleats the Shadow on the Floor.

THIRD. NO is a Line projecting from the Cylinder, at pleafute; let P be its Vanishing Point; to project its Shadow, on the Cylinder.

Draw PQ, perpendicular, cutting the horizontal vanishing Line, in the vanishing Point of the Seat of NO, on any horizontal Plane, imagined at discretion (above or below the Vanishing Line) cutting the Cylinder. Let Nu be a section of such a Plane with the Cylinder.

Draw Oo, perpendicular, cutting Nu at o, and, through o, draw Qn, indefinite; from the extreme N, and from various other Points, as r, s, in NO, draw perpendicular, cutting Qn, at n, t, and u, from which, draw to S, cutting Nu, at N, t, and u. Draw Nn, tr, and us, perpendicular, and N \odot , &c. cutting them at n, r, and s; through which Points, the Shadow of NO is described. The rest is obvious, from the Figure.

EXAMPLE

Plate

XLV.

Fig. 32.

EXAMPLE X.

To project the Shadow of a Tuscan Base on the Floor, casually situated to the Picture.

○ is the transprojected Image, and S the Seat of the Luminary.

Draw AS and B○, cutting at *b*, and draw *bV*; C○, cutting *bV*, determines the Shadow of the edge of the Plinth, BC, provided nothing else interfered. But, the Torus, &c. and Shaft of the Column unite their Shadows with it.

For the Column, draw SB and SF tangents to its Base, on the Ground; from the points of contact, draw BD and FE, perpendicular; which determine where the total Shade, on the Shaft, begins. BD, and EF, are the Sides of the Column which project the Shadow, on the Ground.

Draw D○, and E○, cutting BS, and FS, at *d* and *e*, the Shadows of D and E.

DE, is a Diameter of the Column; and *de* being drawn, tending to the same Vanishing Point, may be considered as its Shadow; on which, as a Diameter found, describe the representation of a Circle (Prob 3. Sect. 8.) or, of a Semicircle, *dfe*, only, which terminates the Shadow. The Column being supposed cut, by a horizontal Plane, through DE, its Shadow is, consequently, a Circle; being projected on a Plane to which it is parallel.

The Shadow of the Fillet *gh*, may be projected after the same manner; but, as so little of it is seen, it would be unnecessary trouble.

Draw a Tangent to its Base, at *b*, from S, and draw *bg* perpendicular, and *g○*, cutting *bS*, at *g*, the Shadow of the Point *g*. By the same means the Shadows of other Points, may be obtained.

The Shadow of the Torus is not easy to describe; seeing its Surface is continually varying, it is scarce possible to determine what part of it projects the Shadow; except the Point *d*, of the greatest swell, determined, as the Fillet, by a perpendicular from *c*. No other Point in that Circle projects a Shadow, on this side; to determine, absolutely, what other Points do, is what I shall not attempt, being persuaded, my Readers will excuse me that trouble.

N.B. The Shadow of the Shaft falls, first, on the Cavetto, before it reaches the Ground; the Fillet on the Torus, and the Torus on the upper Plane of the Plinth; which the Figure describes.

The Shade on the Torus, and on the Column, &c. is more hard and edgy when the Sun shines on them, than otherwise; yet there is no edge defined; also, the Light is broader, and gradates more suddenly into the Shade; and the Reflections are much stronger; but that, I shall reserve to another Section.

EXAMPLE XI.

To project the Shadow of a Doric Capital, on a vertical Plane, whose Vanishing Line (VY) and Intersection (AB) with the Ground Plane are given.

Fig. 33.

CD cuts the Plane at *a*; find its Shadow, *cd* (Prob. 4.) ○* being the Vanishing Point of the Rays, and G, of the Line; which, being joined and produced, cuts the Vanishing Line, VY, at V, the Vanishing Point of the Shadow.

After the same manner, the Shadow *de*, of DE, &c. may be obtained; and the whole Shadow, *cdef*, of the square of the top of the Abacus, in which draw the Diagonals; and let the lesser Trapezium *abcd*, be the supposed Shadow of the square of the Column, parallel to the former, and found by the same means; in which, the Ellipsis, inscribed, may be the Shadow of the top of the Column, in the Plane of the Abacus, from which draw Perpendiculars; or, rather (because of the swelling of the Column) somewhat diverging, and gently curved.

The Shadows of the Angles, *g* and *h*, are obtained as the Square of the top; and, if great accuracy was requisite, the square of the Astragal, *ik* may be also projected, in its true place. But, as nothing more of its Shadow is defined than what projects beyond the Column, it may be dispensed with, the Points, *i* and *k*, being determined, are sufficient.

Rays

Rays of Light, passing by the Ovolo to \odot^s , may be some guide for describing its Shadow; but, on account of the continual variation of its Surface, every way, 'tis not easy to determine at what Points they touch it.

Neither can the curve of the Shadow of fg, be described on the Ovolo, with certainty, as on a Cylinder, yet somewhat may be done towards it; for example; the Shadow of the point b is required.

In this process, it is necessary to have the perspective curve (acf) of the seat of the small Fillet, from which the Ovolo projects, on the under Plane (gh) of the Abacus.

Then, draw bS, cutting it at a; draw ab, perpendicular, and describe the curve bc, a section of the Ovolo, by the Plane of Shade; $b\odot^s$ being drawn, cutting that Curve, determines the Shadow of b, on the Ovolo, at c; which, because it touches the Curve only, is the limit of the Shadow on the Ovolo.

The rest of the Shadow of that Line, from b to g, falls on the vertical Plane.

After the same manner as many Points may be determined as the Student has patience for; and when done, 'tis a great chance if it be more exact than a judicious Person would describe by guess; nevertheless it is useful, to give an Idea how such Shadows may be projected, and what kind of curves they describe.

E X A M P L E XII.

A vertical, concave, cylindrical Surface being so situated, that the Shadow of one part is projected on the other; how to determine that Shadow; also, of Right Lines any how situated to the Surface, and to the Luminary.

Let ABX be a Wall, inclosing a circular Area. \odot^s is the Vanishing Point, of the Rays of Light, and S, of the Shadows of Perpendiculars. Fig. 344

Draw SC, a tangent to the Curve, which determines the point where the Shadow begins, whatever be the Altitude of the Luminary.

Draw AS, cutting the Curve on the Ground, at a; draw ab perpendicular, and $B\odot^s$, cutting it at b, the Shadow of B.

By the same means, the Shadows of as many Points (D, E, &c.) as are requisite, may be found; through which, the Curve, Cdeb, may be described; which is the true Shadow of the Curve CDEB, on the concave Surface.

SECOND. Let FG be a piece of Timber, projecting perpendicularly from the Wall, whose Shadow is required; its Van. Point is where GF would cut SR produced.

Draw Ef perpendicular, either to the top or bottom of the Wall; and, through f, draw a Line to the Vanishing Point of FG, cutting a perpendicular from G. fg is the Seat of FG, on the Plane of the top, being horizontal.

From several Points, h, i, &c. draw perpendicular, cutting fg, at k, l, &c. from g, k, and l, draw lines to S, cutting the Curve of the Top, at m, n, o, from which, draw Perpendiculars, indefinite; and from the Points g, h, i, &c. draw to \odot^s , cutting them, respectively, in the Points g, h, i, which determine the Shadows of those Points on the Wall; through which, a Curve, being described, is the Shadow of the lower edge of the Timber. The rest is evident.

HI is another piece, also perpendicular to the Wall, its Vanishing Point is Y. Its Shadow is described after the same manner; or by means of a section of the Wall (Hi) parallel to the Horizon. Draw IS, cutting it at h; from which, draw perpendicular, and $I\odot^s$ intersecting at K, which determines its length; the Shadow being so nearly perpendicular, it deviates but little from a Right Line.

If its Vanishing Point was at S, the Shadow would be perpendicular.

THIRD. LM is a piece of Timber, inclined to the Wall, its Vanishing Point is Q.

Draw $Q\odot^s$, cutting the Horizontal Line at P, the Vanishing Point of the Shadow on the Ground. Draw LP, cutting the Curve at the bottom, at p; through p, draw $\odot O$, cutting LM, at O. Lp is the Shadow of LO, on the Ground.

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Draw MN, perpendicular; and LP, being drawn, LN is the Seat of LM, on the Ground. From several Points a, b, c , (between O and M) draw perpendicular to the Seat LN, at d, e, f ; from which, draw to S, cutting the Curve at the bottom, at g, h, i , and from them draw perpendiculars up the Wall; also, from a, b, c , draw to O, cutting them, respectively, at a, b, c ; through which, a Curve being described is the Shadow of OM, on the concave Wall.

For the Shadow of each Perpendicular ad , &c. is projected first, from d to g ; and ga , &c. is parallel to ad , &c. ad being parallel to the Wall.

E X A M P L E XIII.

To project the Shadows of Arches; or, the Shadow of the circular outline, on the inside, cylindrical, and plane Surface.

This Example is the same, in every respect, as the first part of the last, except in the position of the Cylinder to the Picture; their position to the Horizon is of no signification.

Fig. 35.

FIRST; the Arch (X) is parallel to the Picture; C is the Center.

The Luminary being on this side, and O its Image; draw CO, indefinite.

Then, because the plane of the Circle is parallel to the Picture, draw a Tangent to the Curve, at A, parallel to CO; also, from various Points B, D, &c. at discretion, draw parallel Ordinates, cutting the opposite side of the Arch, at b, d , &c. from all which, draw to C; and lastly, draw BO, DO, &c. cutting bC, dC, &c. respectively, at b, d , &c. through which, a Curve being described, is the Shadow of all that part of the Circle, from A (where the Shadow begins) to G, where it falls into the perpendicular.

If from A, to b , there be too much space, for, describing the curve accurately, take another point (a) and project its Shadow to a , and more if necessary.

SCHOL. In this, the affinity to the former Example is obvious. The Right Lines, BB, DD, &c. tending to C, (the Center of the Picture) represent perpendiculars to the Bases of the Cylinder; as, in the former they are perpendicular, being parallel to the Picture; also, the Lines Bb, Dd, &c. which are, in this Case, parallel to CO, in the former they vanish in S; because they represent parallel Lines, in a Plane not parallel to the Picture; as, in this Case, they are parallel, being in a Plane parallel to the Picture. The Rays of Light, in both Cases, tend to their Vanishing Point.

2. The Shadow, from A to e , notwithstanding its apparent convexity towards the Front, is really concave, the same as the other; the convexity arises only from its position to the Eye, owing to the exterior Curve Abe; which, in the former, is concave, the Eye being between the two Bases, nearly opposite to that part of the Surface on which the Shadow falls; in which position of the Eye, the true curve of the Shadow is seen.

Fig. 36.

Fig. 36. represents the same thing, inclined to the Picture; in which, the perpendiculars BB, &c. vanish in H, in the Horizontal Line HL, as before in the Center; consequently, bb, dd , &c. vanish in the same Point.

Also Bb, Dd, &c. vanish at S, in the Vanishing Line of the Arch, where HO cuts it; H being the Vanishing Point of Right Lines perpendicular to the Plane of the Arch (Prob. 2.) The Rays of Light vanish at O; consequently, HS is the Vanishing Line of the Plane of Shade, for Lines perpendicular to the Plane of the Arch, and S is its Seat on that Plane.

Fig. 37.

Fig. 37. represents another Arch, with the Light flowing inwardly (as in Example 42. Book 3. of the Piazza.) In which Case, it is the exterior Curve, from A to F, which projects the Shadow, the Sun being on the other side of the Picture (in this Case) at S; and, G is the Vanishing Point of Perpendiculars, as before.

The Figure, having the same Letters of reference, explains the rest.

Here, the curve of the Shadow is seen properly, that is, concave; and it is the same in both the other, in the Object, though represented convex.

In Example 42, of the Piazza, the Luminary is on this side of the Picture, although it may to some appear absurd; but, the direction of the Shadows indicate it. It is indeed very much inclined, nearly approaching to parallel; that is, the Luminary is nearly in the Plane of the Picture.

EXAMPLE XIV.

To project the Shadow of the circular outline of a Nich, on the interior, spherical, and cylindrical Surface.

FIRST; let the Front of the Nich be parallel to the Picture.

C being the Center, draw $C\odot$, indefinite, the Intersection of a Plane passing through the Luminary, perpendicular to the Picture. Fig. 38.

Draw a Tangent to the Curve at A, parallel to $C\odot$, and several parallel Ordinates, BB, DD, &c. at discretion. On the Diameters BB, DD, and EE, describe the representations of Semicircles, in Planes whose Vanishing Line is $C\odot$ (Prob. 2. Sect. 8.) (CI is its Distance) which are sections of the Head, parallel amongst themselves, and to their Plane of Shade.

As the Section, through FF, cuts the cylindrical part of the Nich, also, and that through GG entirely, those Sections are not Semicircles; the former, where it cuts the Head, is circular, but where it cuts the Cylinder (which only is wanted) it is elliptical; and that through GG is a semi-Ellipse, whose transverse Diameter is GG, and its Conjugate, the Diameter of the Nich.

Describe the representations of such Ellipses; which being done, from B, D, &c. draw Rays to \odot , cutting the Curves, respectively, at b, d, &c. through which, describe the Curve A b d e f g, the Shadow of the exterior Curve (from A to G.)

Draw gh, perpendicular, the Shadow of the edge GH, to which it is parallel, and join Hh; which compleats the whole Shadow, in that situation of the Sun.

If the Luminary be less inclined to the Plane of the Nich, but in the same Plane of which CI is the Vanishing Line, its place being \odot^2 ; A a b d e f g b is the Shadow, which has but little convexity, being farther removed from the Curve of the Head, ABE.

SCHOL. This process, though apparently troublesome, is the shortest of any that has yet been published, and much more simple than by vertical Sections, as exhibited in Fig. 39;* in which, the several Points, B, D, &c. in the Arch, by means of perpendiculars to the right Line IK, at the bottom, have corresponding Points, B, D, &c.

Then (S, being the Seat of the Luminary) lines are drawn from B, D, &c. to S, cutting the curve of the bottom of the Nich, at b, d, &c. from which, perpendiculars are drawn to the Curve FGH, where the spherical Surface begins; through which, the representations of parallel Sections are made (as in the Figure) a most difficult process, as; each Curve is a different portion of a Circle. Right Lines from each Point, B, C, &c. tending to \odot^2 , (as in the former Figure) give the Shadow of each Point in its respective Section; through which, the contour of the Shadow is described.

Fig. 39.

Sir William Chambers (if Fame reports true) was the first who took notice, in any publication in England, of this seeming paradox, viz. a convex Shadow of a concave Line; which, since, has been most servilely copied, by almost every Architect, or other Artist, who had occasion to represent a Nich, either in external or internal Designs. Yea, to such an extravagant excess it is carried, that, in geometrical Sections, and where it is impossible that the Sun should ever shine, almost every Nich is so shaded; which, I do maintain, is, in such Case, most unnatural; nor is the Shadow really convex, but only apparently so; yet it is frequently exaggerated, preposterously.

But, I presume that, in Sun-shine, it has not always the appearance of convexity; for, I have shewn (Fig. 38.) that, the less the Inclination of the Sun, to the Plane of the Front, the less convex is the Curve. There is, also, another circumstance which occasions a different appearance. Those Authors, who have favoured the World with the method of describing it, have always given it direct, with the Center of the Picture in the middle of the Nich, as in Fig. 38. But, certainly, in the Front of a Building, perfectly delineated, the Eye cannot be

* In this Example, Fournier is most egregiously mistaken; for, he makes all these vertical Sections pass through B, the Vertex of the Nich, when it is evident, that they are parallel amongst themselves, and to the Plane of Shade of perpendicular Lines. By which means, the contour of his Shadow is more convex than it can ever possibly appear, in any situation of the Luminary, and of the Spectator.

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opposite to Niches at each extreme. Suppose, then, the Eye obliquely situated to a Nich, in a Plane parallel to the Picture, on the same side with the Sun; I presume, the convexity will not be so great, as in direct opposition; yet, it is customary to shade all alike, which is most palpably absurd.

Fig. 39. represents a Nich so situated and shaded; the Luminary being on the left hand, its transprojected Image is, consequently, on the right, at \odot , the Center of the Picture is at S, and the Distance is SO.

Now, although the altitude of the Sun, in this, is less, and its Inclination greater, than in Fig. 38. yet, the contour of the Shadow is concave, where the other is convex; and this is solely owing to the situation of the Eye, in respect of the Nich; for, being seen directly opposite, it would appear more convex than the other.

I think, none have ventured to describe this Shadow, when the Nich is in a Plane inclined to the Picture; which, in delineating, more frequently happens so than otherwise, if the Picture be properly situated. Yet, in such Case, many, not thinking properly about it, would give the contour of the Shadow the same, which can never possibly happen, except when the Rays of Light are very much inclined to the Plane of the Nich.

Fig. 40.

Fig. 40. is the representation of a Nich, in a Plane inclined to the Picture; C is the Center, the Distance is CH, and \odot the Image of the Luminary.

The same Letters of reference indicate the process the same, as already described, with only this difference; that, as, in the former, the Ordinates were parallel (being parallel to the Picture) so, in this, they vanish where C \odot cuts the Vanishing Line of the Plane of the Nich.

The outline of the Nich being delineated, and several Points, in the hither side of the Arch, taken at discretion, draw the Ordinates, tending to their Vanishing Point, from which Point, draw the Tangent, at A, where the Shadow begins.

Then, representations of Semicircles being described on those Diameters (Prob. 3. Sect. 8.) and Rays drawn from the several Points, to \odot , cutting them respectively, give the contour of the Shadow, as before; which, in this Case (as in the last Example) the Eye being opposite to the Surface where the Shadow is defined, is really concave*; and which, in other Points of view, would appear convex.

SCHOL. It must have been observed, that the Center and Distance of the Picture are not necessary in the projection of Shadows, except in determining the Vanishing Point of the Rays; but, in this Example, as the Vanishing Line C \odot is of Planes perpendicular to the Picture, it necessarily passes through the Center (Theo. 4.) and, the representations of the Sections, through the head of the Nich, could not be described without the Distance.

I have now, I presume, furnished the Reader with Rules and Examples, for the projection of Shadows, by the Sun, sufficient for any purpose almost whatever. The Shadows of streight Mouldings, on Planes, are Right Lines, save the contour of the Profile; which, in Mouldings above the Horizon, in Cornices, &c. are almost wholly shaded by the projecting Fillets, &c. and for circular Mouldings, or others below the Eye, the projection of the Shadows of Circles, on plane or other Surfaces (as in this last Section) contain all that can be done in such Subjects.

He who has well digested what is there advanced, will never be much at a loss, in the most difficult Cases (where it is possible to project the Shadow, at all, by Rule) in complex, finished Subjects; of which, Examples may be seen, throughout the Work, particularly in the Frontispiece; that being intended as a general Lesson. Yet, in that, I have designedly deviated from the rigid observance of mathematical exactness; which, as I have observed in the Preface, does not always produce the best effect of Light and Shade. For, Example.

According to the general direction of the Rays of Light given, in that Picture, and the Shadows of perpendicular Lines, it must be obvious, that, as the Columns,

* As the Line which projects the Shadow is concave, the System of Rays, projecting it, form a cylindrical Surface; consequently, the Shadow is concave, towards the Eye, on whatever Surface it falls.

in the Front, stand off, detached, their Shadows would cover more than the whole space between them, on the Wall; which, I am persuaded, would make that part appear heavy, and too hollow, as receding more than it really does. A little Light is also introduced on the circular Colonade, on the right-hand, in order to relieve it from the Building; although it would be wholly in Shade from the Building, according to the situation and altitude of the Luminary, the height, magnitude, and distance of the Object, occasioning the Shade. Such liberties may freely be taken, where it manifestly produces a better general Effect; and, when it does not clash with, and evidently contradict the invariable Law of Nature.

In that Piece, as in most other in this Work, the Luminary is on this side of the Picture; and though, in general, it is productive of the best effects, yet there may sometimes be reasons for supposing it on the other side, or in the Plane of the Picture, as in Plate 27, of Chelsea College. If that Piece had been shaded on the supposition of the Sun being on this side, either it must be much inclined, and consequently the Vanishing Point at a great Distance, or every Face, in that Object, would be illumined, being on the right-hand; or, being on the left, the whole Front would be immersed in Shade; with very little Light, in the Picture; but on the Roof, and end of the Building. Besides, from the situation of that Building, the Sun can never shine on both, the Front and hither End; and I think that circumstance should always be attended to, when a real Object is represented, which is the sole reason why the Queen's Palace (Plate 28.) is shaded from the left; for I am of opinion, that it would have a better Effect being shaded from the other hand; but, as the Sun cannot possibly be on the right-hand, it has certainly the most natural Effect.

Although there be no Face of that Object, as there represented, but what receives Light, immediately from the Sun, save one Face of the Library; yet, as some Faces are more opposed to the Luminary than others, there are various Tints, which distinguish one Plane from another, though not so strongly. For, according as the Rays of Light are less or more inclined to the Face of any Object, it may have all the variety of Tints, from the strongest Light to total Shade; and till the inclination is such, that they are nearly parallel to the Plane, it is still illumined, though in a very small degree; and, when they become parallel, that is, when the Sun is in the continuation of any Plane, it is as much deprived of Light, as if it was, in reality, on the other side; and perhaps more so, as it then receives the least advantage from Reflection.

S E C T I O N V.

Of SHADOWS projected by a Torch or Candle.

IF the Theory of Shadows, in general, (Sect. 2nd.) and the Practice by Sunshine be well understood, little need be said respecting Candle-light Shadows, for in reality there is no difference, except in the distance of the luminous Body, or Point; which, on account of the short Distance, and being situated amidst various Objects, its Light is diffused in all directions; as it is from the Sun, respecting the whole System of Planets: also, in respect of any single Object, of the same magnitude, or nearly, as the Flame, the Rays may be considered as parallel.

In projecting Shadows by Candle-light, there is but one Case, or situation of the luminous Point, in respect of the Picture; for, it is always supposed on the other side. In reality, it is an Object delineated in the Picture; and, when deter-

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mined, it is the Center of the Rays of Light; from which, the Shadows of other Objects are projected all around, in every respect the same as by the Sun, in that Case. Therefore, the Theory being the same, and the Practice having but little variation, few Examples will suffice.

I shall not suppose it necessary to shew how to find the representation of the Flame of the Candle, the common Center of the Rays of Light, which is the same as finding any other Point, in any Object whatever.

The Shadows of Right Lines, perpendiculars, to any Plane, tend to the Seat of the luminous Point on that Plane; which, in this Case, is finite. In respect of the Sun, being supposed at an infinite Distance, it is, consequently, in the Vanishing Line of the Plane; but in this Case, it must be found on the Plane of projection; which being found, the Shadows are readily determined.

Shadows projected by a luminous Point, at a short Distance, it is manifest must necessarily be larger than the Object; seeing that, all the Rays diverge from that Point, the same as from the Sun, when it appears in the Picture; which, on account of its immense Distance, the Rays have not so much divergency. But, the Objects whose Shadows are projected by them, are generally of much greater magnitude; the one being confined to small Objects only, as Chairs, Tables, &c. within a Room, the other is of Buildings, &c. external.

P R O B L E M I.

The representation of a luminous Point, being given, and its Seat on the Ground, or other horizontal Plane, together with the Intersections of other Planes, with that Plane, whether they be perpendicular, parallel, or inclined to the Picture, or to the Horizon, or to both; to determine its Seat on the other Planes, and the Shadows of Right Lines perpendicular to them.

Fig. 41.

Let AB be the Intersection of a vertical Plane with the Ground Plane, parallel to the Picture, and BD of a vertical Plane perpendicular to the Picture. Also, let BD be the Intersection of the two vertical Planes; and DE, DH, EI, EF, &c. Intersections of various Planes with each other.

In short, let LABG, represent a Floor, GBDH a vertical Plane, HDEI the Ceiling, and IEFK an inclined Ceiling, as in a Garret; AFEDB is the farther End of the Room, parallel to the Picture, all the other Planes are perpendicular to it. O is the representation of the luminous Point (as the Flame of a Candle) and S is its Seat on the Floor.

FIRST. To find the Seat of the luminous Point, on any of the other Planes.

OS, being perpendicular to the Floor, consequently parallel to the Picture, imagine a Plane passing through OS, parallel to the Picture, cutting all the Planes (save AFEDB) perpendicularly; the Section of which, with them, is GHIKLG.

Now, because the luminous Point is in that Plane, and it is perpendicular to the Planes BDHG, &c. a Perpendicular from O, to each Plane, must necessarily be in that imaginary Plane; and consequently, must cut each Plane, in its Intersection with the Plane; wherefore, Perpendiculars (Of) to each Intersection, GH, HI, IK, &c. give the Seat (f) of the luminous Point, on each Plane.

The process is so very obvious, that I shall avoid a further description.

To find its Seat, on the Plane AFEDB; draw SC, cutting the Intersection, AB, at r; draw rs, perpendicular, and OC, cutting it at s, the Seat of O on that Plane. For, OC represents a perpendicular to the Plane,† cutting it at s.

† Cor. to
Theo. 4.

SECONDLY. To find the Shadows of Right Lines perpendicular to those Planes.

Through the Point B, or b, where the Perpendicular AB, or ab, cuts the Plane, draw SB, indefinite; and, through the extreme A, or a, draw OC, or Oc, cutting it at C, or c, the Shadow of the Point A, or a.

Consequently, BC , or bc , is the Shadow of AB , or ab .

This process, it is obvious, is the same in each Plane; and, changing S for f , the description, given, serves for all alike.

The Parallelopiped, on which the Candle stands, is right angled; wherefore, AB , &c. are perpendicular to the Floor; by which means, the Shadows, C and c of the Angles, A , &c. being obtained, Cc and Cc are the Shadows of Lines parallel to the Floor, as in the 3rd Problem.

The Seat of the Light, on the upper Face of the Parallelopiped, is in the middle of the foot of the Candlestick, which directs the Shadow of ab .

In the other Block, the side AB , perpendicular to the Floor, projects a Shadow on the Floor, to D , in the Intersection BD ; from which it is cast on the Wall, parallel to itself. The other Side, which is perpendicular to the Wall, is produced till it cuts the Wall, at B , through which Point, draw fC , and OC through A , intersecting at C ; or, till it cuts the perpendicular Shadow, DC .

The Shadow of the piece of Timber (X) projecting perpendicularly from the Wall, is managed as the Lines, simply; in which it is obvious, that, as the Shadows of the Sides tend to f , they are diverging from that Point. The Shadows of ad and de are parallel to them, respectively; wherefore, de has the same Vanishing Point as the Original Line (de) projecting the Shadow.

Perpendiculars to the horizontal Ceiling are in the same position as on the Floor; to the vertical Planes they are horizontal Lines; and to the inclined Ceiling, $IEFK$, they are (as in all other) perpendicular to any Right Line, drawn in the Plane, through the Point, in which the Perpendicular cuts the Plane.

The Perpendicular ab , to that Plane, is so situated, that the Eye is in the Plane of Shade; wherefore, its Shadow cannot be determined as the rest, being in a continuation of the Line, but may be thus found.

Through the Seat, f , of the Light on that Plane, draw fB , at pleasure; and, through b , where ab cuts the Plane, draw bB , perpendicular to ab , cutting fB , at B , and draw Bd , parallel to ab , and ad to bB . Then, through d , draw Oe , cutting fB produced, at e , and draw ce parallel to bB ; which determines, bc , for the Shadow of ab , which could not be determined by the Ray Oc .

SECONDLY. When the Plane is vertical, and inclined to the Picture.

Fig. 42.

Let AB , be the Intersection of a vertical Plane, with the Floor, and D its Vanishing Point. The Center of the Picture is C ; O is the luminous Point, and S , its Seat on the Floor.

To determine its Seat on the vertical Plane, draw CE , perpendicular, and equal to the Distance of the Picture. Join DE , and draw EF , perpendicular to DE , cutting the Horizontal Line at F ; the Van. Point of perpendiculars to the Plane.

Then, draw SF , cutting AB , at r , and rf , perpendicular, cutting OF at f , the Seat of O , on that Plane; for, Of , represents a Perpendicular to the Plane.

Let ab , be a Line, perpendicular to the vertical Plane, its Vanishing Point is F .

To project its Shadow on the Plane; draw fb , through the Point b , where it cuts the Plane, indefinite; and through a , draw Oc , cutting fb , produced, at c . bc , is the Shadow of ab , on the vertical Plane.

THIRDLY. When the Plane is inclined to the Horizon, and to the Picture.

Let AB , be the Intersection of a Plane, inclined to the Horizon and also to the Picture; C is the Center, and D the Vanishing Point of AB . O is the luminous Point, and S its Seat, on the Floor.

Fig. 43.

Find the Vanishing Point, F , of Lines perpendicular to the Intersection AB , (Prob. 4.) and draw SF , cutting it at r , as in the last.

Then, the Inclination of the Plane to the Horizon being known, find its Vanishing Line, DG , (by 5th of the same) and the Vanishing Point H , of Lines perpendicular to the Plane, (Prob. 2. Sect. 12.) which will be that Point, where a

Perpendicular

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Perpendicular to the Vanishing Line, passing through C, the Center of the Picture, is cut by a Perpendicular from F. Also, find the Vanishing Point G, of Lines, in the Plane, perpendicular to the Intersection AB.

Draw rG, and OH, cutting it at f, the Seat of O, (the luminous Point) on that Plane. For, H is the Vanishing Point of Perpendiculars to the Plane, and rG represents a perpendicular to AB; wherefore OH cuts the Plane at f.

Because, OS being perpendicular to the Horizon, and F is the Vanishing Point of Sr, perpendicular to AB; consequently, GH, passing through F, also perpendicular (that is, parallel to OS) is the Vanishing Line of a Plane OfrS, passing through OS, perpendicular to the inclined Plane; and H, being the Vanishing Point of Perpendiculars to the Plane, OH must cut it in rG, the common Section of the two Planes; therefore, f, where OH, cuts rG, is the Seat of the Point O, on the inclined Plane.

The Seat, f, of the luminous Point being found, and ab, a given Line perpendicular to the Plane, its Shadow is determined, by drawing fc, through b, indefinite, and Oc, through a, intersecting at c; giving bc, the Shadow of ab, as in all the foregoing.

In this Problem is contained the whole Theory of projecting Shadows by Candle Light, in which there is not any difference to that of projecting them by the Sun, the Seat being obtained; in respect of Lines parallel or inclined to the Plane of projection, there is very little difference in the process.

The chief difficulty in projecting Shadows, by Candle Light, is to find the Seat of the luminous Point; for which, the Rules given will answer in all Cases, and in all positions of Planes, whatever, having the Intersection of the Plane, with some other, on which, the Seat is already determined, and the Inclination of the Planes, to each other, known.

P R O B L E M II.

To project the Shadows of Right Lines, parallel to the Plane of projection, however situated to the Picture.

The Shadows of Right Lines, on Planes to which they are parallel, whether they are projected by the Sun or by Candle Light, it is obvious, are always parallel to the Lines projecting the Shadows.

Fig. 44.
† 2. 6. El.

For, however the luminous Point be situated, or at whatever Distance, the Rays OA, OB, &c. generating the Plane of Shade, are all cut proportionally by the Line and the Plane of projection†, consequently, seeing the extreme Rays, OA, OB, or any other, are so cut, as OA is to AC, so is OB to BD, it necessarily follows, that CD is parallel to AB (Case 2. 2. 6. El.)

Wherefore, if the Line be parallel to the Picture, the Shadow is parallel on the Picture; that is, to the Vanishing Line of the Plane of projection; and consequently, however otherwise the Line be situated to the Picture, the Shadow, being parallel to the Line, has necessarily the same Vanishing Point (Cor. Th. 3.)

Fig. 45.
No. 1.

Let ACD be a right angled Paralleliped, having the Face AC parallel to the Picture; the Sides, CD, &c. are consequently perpendicular to the Picture, and parallel to the Floor, on which the Shadow is projected.

Let O, be the luminous Point, and S its Seat, on the Floor.

Through A, draw Sb, indefinite, and OB cutting it at b; and, because BC is parallel to the Floor, and to the Picture, its Shadow is parallel to the Line.

Wherefore, draw bc, parallel to BC, and Oc (through C) cutting it at c.

Then, because CD is perpendicular to the Picture, and parallel to the Plane of projection, C (the Center) is the Vanishing Point of its Shadow; wherefore, draw cC, and OC, thro' D, which compleats the Shadow, as much as can be seen.

No 2.

2. EH is another Paralleliped, obliquely situated to the Picture; having one Face so situated, that the luminous Point, is in its Plane, produced.

Produce SE, and, through F, draw Of, cutting it at f, the Shadow of f.

Then, K being the Vanishing Point of FG, draw fK, and Og (through G) cutting it at g; fg is the Shadow of FG, to which it is parallel, having the same Vanishing Point. And, because GH vanishes at I, draw gI, and Oh (through H) cutting it; and lastly, draw hS, which compleats the Shadow E fgh.

To project the Shadows of Right Lines parallel to any other Plane, however situated to the Picture, has nothing more of variety in it; the seat of the Luminary being obtained on the Plane.

3. The Shadows of the Shelves, LM, &c. at the farther End of the Room, may be thus obtained, being parallel to the Picture. No. 3.

Through the feet of the upright pieces, R, and T, draw right Lines, from S, cutting the Intersection of the Wall with the Floor, at r, and the Door, at t; from which, draw Lines parallel to LR, and MT, indefinite. From O, draw through the extremes of the Shelves, L, N, &c. cutting the Wall, in the Shadow rl, at l, n, &c. from which Points, draw parallel to the Shelves, cutting the edge of the upright piece, at l, n, &c. and join, Nn, Oo, &c. as in the Figure.

4. The Shelves, against the side Wall, may have their Shadows projected after the same manner, with only this difference; that, the Shadows, instead of being drawn parallel to the edges of the Shelves, tend to the same Vanishing Point. No. 4.

As ab, the Shadow of AB, and cd of CD, tending to the Center.

5. For the Shadow of the single Shelf on the other side, find the Seat, f, of the Light on the Wall (Prob. 1.) then, ab being perpendicular to the Plane, draw fa, indefinite, and Ob, being produced, cuts it at b. Draw bC, cutting the Angle of the Room, at c, and join cd. No. 5.

After the same manner, the breadth of the Shadows of the Shelves on the other Side are determined.

6. For, the hanging Frame and Shelves, above, proceed thus.

Fig. 45.
No. 6.

Find S, the Seat of the Light on the Ceiling; AB and CD being perpendicular, draw SA, SC, indefinite; and, through B, and D, draw OB, OD, cutting them at b, and d. Draw bd, the Shadow of BD; and, from the Center of the Picture (being the Vanishing Point of BE and DF) draw be, and df, the Shadows of BE and DF, on the Ceiling.

Lastly; through G or H, draw Og, or Ob, cutting be, or df, at g or b; through which, draw gb, parallel to GH (being parallel to the Picture) and, through the extremes of the Shelves, draw Lines from O, cutting gb at a and c.

The rest, for that Shelf, is too obvious to need further explanation.

The other, notwithstanding its inclination, is performed by the same means; only, instead of being parallel, the Shadow tends to the same vanishing Point, as the Shelf; or, both extremes being determined, the vanishing Point is useless. Their widths are determined after the same manner.

7. The Shadow of the Stool, on the Counter and Floor, is thus projected.

No. 7.

From S, the Seat of the luminous Point, draw Right Lines through the feet of the Frame, a, b, &c. till they cut the Intersection, AC, of the Counter with the Floor, at a, b, &c. from which Points, draw Perpendiculars; which are the indefinite Shadows of the Frame, on the front of the Counter; and if, through the Angles, e, f, &c. Lines are drawn from O, cutting the Perpendiculars, at e, f, &c. respectively, their lengths are determined.

The Shadow of the cross Frame is determined, by drawing Lines, from O, through their extremes, cutting the Shadows of the Legs, at i, k, &c.

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No. 8.

8. The Box on which the Candle stands has its upper Face, only, illumined, and consequently its Shadow, only, is projected, on the Floor, &c. by drawing Lines, from S, through the Seat of each Angle, on the Floor; and Om, O, being produced, till they cut them, at *m* and *n*. Op, cuts the other Box, in the perpendicular *rp*, and if *no* be drawn, to the Vanishing Point of *np*, till it cuts the Box, at *o*; and the point *q* being obtained, by the same means, *op* and *pq*, being joined, compleat its Shadow.

P R O B L E M III.

To project the Shadows of Right Lines, any how inclined to the Plane of projection, and to the Picture.

This Problem I shall resolve also, by Examples, as the foregoing.

The common Center of the Rays of Light, that is, the place of the luminous Point being determined, the process is the same as by Sun-shine, having found the Vanishing Line of the Plane of Shade; which passes through the Vanishing Point of the Line, but not through the luminous Point.

Fig. 46.

ABCD represents a high pair of Steps, and U, X, Y, Z, a large folding Screen, so situated, in respect of the Light, at O, that the Shadow of the inclined Sides, (AB and CD) of the Steps, fall on the Screen.

Their situation, in respect of the Picture, and also their position to each other, together with the Seat of the Light, on the Floor, are determined as in No. 2.

No. 2.

Let U, X, &c. be the Seats of the Leaves of the Screen; A, D, E, F, of the feet of the Steps, and, S the Seat of the Light; BC, is the Seat of the top of the Steps, which being contracted, AB and CD, are the Seats of the inclination of the Sides, on the Floor, which cast their Shadows on the Screen; let their inclination to the Horizon be the Angle CDG, equal BAG.

Let V be the Vanishing Point of the Side, AB, of the Steps.

Having found S, the Seat of the Light, also F, the Vanishing Point of the Seat of AB, on the Floor, draw FS, indefinite; VO, being produced, cuts it at G.

Draw GA, and produce it to the Horizontal Line, cutting it at L, and draw VL, the Vanishing Line of the Plane of Shade, of AB.

AL cuts the Intersection, of the Leaf U with the Floor, at *a*; through H, the Vanishing Point of that Intersection, HI being drawn, perpendicular, is its Vanishing Line, which cuts VL at *i*; draw *ai*, cutting the Intersection of the Leaves, U and X, at *b*.

Then, where the Vanishing Line of the Leaf, X, cuts VL, at J, draw *bj*, cutting the Intersection of the Leaves X and Y, at *c*.

The Leaf Y, is parallel to the Picture; wherefore, draw *cd* parallel to the Vanishing Line, VL, cutting the next Intersection at *d*; and, lastly, draw *dK*, to the Point, K, where the Vanishing Line of the Leaf Z, cuts VL, which compleats the Shadow of AB, on the Screen.

The remainder of its Shadow falls behind the Screen, out of sight.

The Shadow of the other Side, CD, whose Vanishing Point is R; falls only on the two Leaves, Y, and Z; and on the Wall, to *g*, mostly out of sight.

RO cuts the Floor at G, and GD determines the Vanishing Point, U, of the Shadow of CD, on the Floor; wherefore, RU is the Vanishing Line of the Plane of Shade, of CD; to which, the Shadow of that Line, on the Leaf Y is parallel; and, on the Leaf Z it tends to X in the Vanishing Line of the Leaf, as the Shadow of AB, on the same Leaf, to K; the Points where the Vanishing Line of the Leaf is cut, by the Vanishing Lines of the Plane of Shade of each Line (AB and CD) respectively.

The Shadow of the Supporter, CE, is projected first on the Floor, to *f*, where it cuts the Wall; and, if the Shadow of any other Point, in the Line, be projected on the Wall, as at *g*, then, *fg* is its Shadow on the Wall,

For the Steps; from *O*, Lines are drawn through the extremes *A*, *B*, *C*, &c. on either Side, till they cut the Floor, or leaves of the Screen, at *a*, *b*, *c*, &c. and from, *a*, *b*, &c. to the Vanishing Point of the Steps, on the Floor, and on the Leaves, being parallel to them. Otherwise, they are drawn to that Point in the Vanishing Line, of each Leaf, in which, the Vanishing Line of the Plane of Shade of the Step cuts it. Or, drawing through both extremes, of each Step, as *B* and *D*, &c. cutting the Shadows of both Sides, on the same Plane, the Vanishing Point is unnecessary; but when it falls within bounds, it is the most correct.

In this Case (viz. of Lines inclined to the Plane of Projection) the Vanishing Line of the Plane of Shade is necessary, when the Shadow is projected on various Planes, but it is not so easily determined, as for Shadows projected by the Sun; nothing more being required than to draw a Right Line through the Vanishing Points, of the Line and of the Rays, because its distance is supposed infinite; and consequently, the same Vanishing Line of any Plane of Shade, serves for all Lines which are parallel. But, here, the Light being at a short distance, it cannot possibly be in the Vanishing Line, which is at an infinite distance; except apparently so, when it happens to be so situated, in respect of the Eye.

That *VL* is the Vanishing Line of the Plane of Shade of *AB*, is manifest; and is determined either by means of the horizontal Vanishing Line (*HL*) or the vertical Vanishing Line (*MN*) of the Wall, *W*, on which the Shadow would be projected, the Screen being out of the way. For, *AB*, cuts the Wall, being produced, at *P*, as the Floor at *A*; and *VO* represents a Line parallel to *AB*; which, is a Ray of Light in the same Plane with *AB*, cutting the Floor, at *G*, and the Wall, at *Q*.

Consequently, *GA*, produced, is the Shadow of *AB*, on the Floor; and, *PQ* produced, is its Shadow on the Wall; the former cuts the Vanishing Line of the Floor, at *L*, the latter cuts the vertical Vanishing Line at *M*; both which, are in the same Right Line passing through *V*; and, since *L* and *M* are Vanishing Points of the Shadow, *VL* is, consequently, the Vanishing Line of the Plane of Shade, occasioned by the Line *AB* (Theorem 10.)

2. The Shadow of the Screen, on the Wall and Ceiling, may be thus projected.

Through, *A*, *B*, &c. at the bottom of the Screen, draw *SA*, *SB*, &c. and produce them to the Intersection of the Wall (*W*) with the Floor, cutting it at *r*, *2*, &c. from which Points, draw Perpendiculars up the Wall; and, through the Angles *F*, *G*, &c. at the top, draw *OF*, *OG*, &c. cutting the Perpendiculars, corresponding with *AF*, &c. at *f*, *g*, &c. and draw *fg*, *gb*, &c.

But the Ray *OI*, cuts the Ceiling; wherefore, having found *i*, the Seat of the Light on the Ceiling, draw *fi*, to that Point where the Perpendicular, from *4*, cuts the Intersection of the Wall with the Ceiling; and *OI*, produced, limits the Shadow, at *i*; let it be produced, also, till it cuts the Perpendicular at *k*, and join *bk*, cutting the Intersection at *j*, and join *ji*.

This process would be indispensibly necessary, provided the Lines, *HI*, &c. were not parallel to the Ceiling; but being so, they are drawn either parallel to the Line (as for *HI*) or to the same Vanishing Point (as for *IK*). *Aifgbjkl*, are the extremes of the Shadow of the Screen, on the Floor, Wall, and Ceiling.

By the same means, the Shadow of *AB*, may be projected on the Screen, thus.

Draw *SA*, *SB*, &c. cutting *AF*, the Seat of *AB* on the Floor, at *n*, *o*, *p*, and *q*; from which, draw Perpendiculars, cutting *AB*, at *A*, *B*, &c. through which Points, draw *Oa*, &c. cutting the corresponding angles of the Screen, at *a*, *b*, *c*, &c. which, being joined by Right Lines, give the same Shadow as before.

This process, though shorter, is by no means so correct and masterly; but, as it is performed in less room, it may be applied when the inclination is such, that the Vanishing Points are very remote.

3. The Shadow of the Table, on which the Candle stands may be thus determined.

Having obtained the Seat (*s*) of the Light on the Table, and (*S*) on the Floor, through *s*, draw *ab*, parallel to the Horizon, also *cd*, and *ef* at pleasure, cutting the Horizontal Line, at *H*, and, *F*; and through *S*, on the Floor, draw *Hc*, and *Fe*, indefinite; also *ab*, parallel to *ab*.

Then, Rays drawn, from *O*, through the extremes of those Lines, viz. *Oa*, *Oc*, &c. (being produced) will cut the corresponding Lines in the Shadow of each, respectively, by which means as many points may be obtained as are necessary, for obtaining the true curve of the Shadow, which is a Circle, the Table being circular.

The

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The Candle standing towards one edge of the Table, the Rays proceeding from it, around its Circumference, form a scalene, or oblique Cone; which being projected to the Floor (to which the Table is supposed parallel) the Section, thereon (which is its Shadow) is consequently a Circle; and, being seen oblique, its Representation is an Ellipsis (Theo. 2. Sect. 5.)

- No. 4. 4. The Shadow of the concave edge of the hollow Cylinder (X) on the interior Surface, may be projected, in the following manner.

Find *s*, the seat of the Light on the Plane of its Base, and draw *sa*, *sb*, &c. at pleasure, cutting the circumference on both sides; from the Points, *d*, *e*, &c. draw Right Lines to the Vanishing Point of the sides of the Cylinder, and draw *Oa*, *Ob*, &c. cutting them, respectively, at *a*, *b*, &c. through which, the contour of the Shadow may be described.

- No. 5. 5. For the Shadow of the edge, of the conical Vessel (W) on the interior Surface.

If Right Lines be drawn from *S*, the Seat of the Light, on the Plane of its upper Base, cutting it on both sides, and making vertical Sections through them, down the Sides, within; then, draw *Oa*, &c. cutting the opposite, corresponding lines, at *a*, *b*, &c. through which, the Shadow may be described, on the Side.

Where they fall on the Bottom, join *cd*, &c. *Ob*, *Oc*, &c. cuts those Lines, at *f*, *g*, &c. through which, the curve of the Shadow is described, thereon.

By taking several Points (*g*, *h*, &c.) in the exterior Curve, and finding their Seats on the Floor, its Shadow (*ghi*) may be described on the Floor.

- No. 6. 6. The Shadows of Globes, it is not easy to describe, with certainty.

If a Tangent be drawn, from the Light (*O*) to any part of its Surface (as *a*) and, through that Point, a Section, by a Plane, be described, perpendicular to the Axis, (*OC*), (which, I freely own, is not easy to do, being obliquely situated*) then, take as many Points in its Circumference as are necessary (*a*, *b*, *d*, &c.) and find their Seats, (*A*, *B*, *D*, &c.) on the Floor, or Ceiling; and, through *S*, or *s*, the Seat of the Light, draw *SA*, *SB*, &c. and *Oa*, *Ob*, &c. cutting them, respectively, at *a*, *b*, *d*, &c. a Curve described through *a*, *b*, &c. will be an Ellipsis, the true Shadow of the Sphere; being an oblique section of the Cone of Rays.

- No. 7. 7. The Shadow in the Nich is described as by Sunshine, with little variation.

Find the Seat (*s*) of the Light on its Plane, (Prob. 1.) from which, draw the Ordinates, *ab*, *cd*, &c. on which Ordinates describe Sections through the head of the Nich, perpendicular to the Plane, and draw *Oa*, *Oc*, &c. giving the Points *a*, *c*, &c. a Curve described through those Points is the true contour of the Shadow, in the Head†. Draw *ef* perpendicular, and join *fg*.

The Shadow of the edge of the Vessel, *def*, falls within the Nich; the rest, with the Stool, falls on the Wall, the Legs being first projected on the Floor, through the Feet, by means of *S*, the Seat of the Light thereon.

* The Shadow of a Globe or Sphere, it is obvious, is no more than the Shadow of a plane Circle, which is not the full diameter of the Globe, because, Tangents to a Sphere, from the same Point cannot touch both extremes of the same Diameter, seeing they would be parallel. The Circle projecting the Shadow is that, whose Circumference is the Base of the Cone of Rays.

† In this Example, the Light is so situated as to cast very little Shadow into the Nich; but, if the Candle was more oblique to it, the contour of the Shadow would be nearly the same figure, the Light being below the head of the Nich; as a tangent to the curve of the Head would always touch it on that side of the Vertex towards the Light. Whereas, by Sunshine, the Light being elevated above the Nich, a Tangent touches it on the other side, and gives a different figure of the Shadow. And, if the Candle be elevated above it, the contour of the Shadow would be nearly the same; the difference arising only from the seat of the Light, on the Plane of the Nich, which, in this is finite, in the other, at an infinite distance; and also from the Rays of Light, which in this are diverging, in the other they are converging, the Light being on this side of the Picture.

S E C T I O N VI.

Of reflected Light; and of the reflected Images of Objects, on Water, and polished, plane Surfaces; Keeping, &c.

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IN treating on this Subject, it may be necessary to consider, in the first place, what is meant by Reflection, simply, or considered abstractedly.

REFLECTION, in a physical sense, signifies a rebounding of Matter, by Percussion; i. e. when an elastic Body, * in motion, strikes another Body, also elastic; it rebounds, from that other Body, in a different direction from that in which it was at first impelled; making an Angle, with its first direction, greater or less, according to the obliquity in which it strikes the Surface of the other Body.

If a Globe strikes a Plane, or the Surface of another Globe (or any Surface whatever) perpendicularly, it will rebound from the other Surface, perpendicularly, in direct opposition to its first, or incident motion; as if A falls, perpendicularly, to the Plane EF, striking it at B, it will rebound again, from B, towards A. But, it is found, from experiment, that if the incident motion be from C to B, oblique to the Plane, or other Surface, it will rebound, or be reflected from B towards D, also oblique; making the Angle ABD, with a perpendicular, at that Point (called the Angle of Reflection) equal to ABC (called the Angle of Incidence) or, which is the same thing, the Angle CBE, in which it inclines to the Plane, in its incident motion, is equal to DBF, in which it reflects from the Plane. And this is the same, in all positions of the Plane, whether horizontal, vertical, or inclined. Hence, Light is said to be reflected, from one Surface to another; and, probably, from that consideration it is imagined to be material.

Fig. 47.

Without taking Matter into consideration, it is certain, that any Surface (not wholly opaque) being opposed to a luminous Body, becomes illumined itself; and, in an inferior degree, illumines other Objects, in vicinity with it.

The first and grand instance, of which, is the Moon and other Planets; as they are more or less illumined, by the Sun, or rather, the more their illumined Surfaces are towards the Earth, the more the Earth is illumined; even to such a degree (though but reflected) as to project Shadows, strongly defined. The Case is perfectly similar in respect of other Bodies, on the Earth. For, however any Surface be situated to the Sun, being illumined by its Light, that Surface illumines others, which are near it, more or less, according as that Surface is situated to the Sun, and as they are situated in respect of each other. Without which, Bodies, or the Surfaces of Bodies, which are not illumined, directly from the Sun or other luminous Body, would be so totally immersed in Shade as not to be visible, excepting their exterior Figure or Outline.

Now, admitting Rays of Light to be emitted from any luminous Body, at S, and falling, directly, on another Body, or plane Surface, at A, they are said to be reflected, directly, again, towards S; but, falling on it oblique, as at B, they are reflected towards D, making the Angle DBE equal SBA.

Fig. 48.

Hence arises an objection to the Newtonian System, respecting the reflection of Light, from one Body to another, and from that other Body to the Eye; by which means, only, it is conjectured, that Objects, not illumined, directly from the Sun, or other Light, become visible.

* By Elastic, in this place, is meant, simply, hard Bodies only, as Stones, Metals, &c. of which, some are more elastic than others. Lead, or pure Gold, yields to the stroke, and therefore, does not rebound like Tin, or Tin like Copper, nor that like Iron, or hard Steel. So Clay will not rebound like Free Stone, nor that like Marble; because, after Percussion, the more remote parts of Matter are still in motion, till the Body is compressed by the Stroke; which deprives it, either wholly or in part, of Motion; and consequently, it rebounds with less force, or not at all.

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Fig. 48.

Let AB be a Plane Surface, directly opposed to the Sun; and, suppose AB the utmost limits of the Plane. Let X be an Object, having one plane Face (CD) parallel to AB; and suppose no other Object, or Body, near.

Now, the distance of the Sun being supposed infinite, its Rays, consequently, fall perpendicularly on AB, in every part.

Let SA, SB, &c. represent Rays of Light from the Sun. Then, according to the Maxim laid down, they are reflected, directly back again, towards S (which, being material, is somewhat repugnant to our Ideas of Matter; seeing that, it is continually flowing, with equal and unremitted velocity, from the Sun to AB, how can it return back, in direct opposition?) consequently, the Object X being situated so, that none of the reflected Rays can possibly fall on the Surface CD, that Surface is wholly invisible, to any Eye at E, or *E*. Query; whether it would be visible or not?

I am of opinion that it would be clearly visible, to any Eye, on this Side CD; not only in respect of its Figure, but that, the Surface, CD, would be illumined, by means of Reflection, from AB; when, according to the general Maxim, it could not, seeing no Rays, from AB, are reflected to CD. Yet, I am fully persuaded that it may be seen; not only, directly, at E, but also oblique, in any direction, as at *E*; although it is manifest, that the Rays, from AB, must be reflected oblique, to CD, and again to E, or *E*, in various Angles and Directions; and yet, the original Rays all fall perpendicularly on AB.

Now, if CD be seen, at all, it is manifest it must be illumined; and it is evident, that it cannot be so from the Sun, directly, consequently, it must be from Reflection; and since no other Body is near, it must be from the Surface AB. Hence, then, it is manifest, that, Light is reflected in other directions, from one Object to another. How or by what means it is reflected, I will not attempt to enquire; but shall only make a few Observations, respecting the effects it produces, on Objects, so essential to the perfecting a Picture, or true Portrait of Nature.

It had, formerly, been customary, with many, to represent Objects, immersed in Shade, so very obscurely, as scarce to be distinguishable; whereas, it is not so in Nature. In clear Day-light, when the Sun does not shine out, we see Objects in their true Colours, and every part is distinct; but, when the Sun breaks out, and darts its Rays on those parts which are opposite, the Colours are more intensely vivid, occasioned by the refulgent lustre of the Sun-Beams. Nevertheless, those parts which are prevented from receiving that additional Light, and appear to be in Shade, cannot possibly be deprived of what Light they had before, but must rather receive additional Light on them; the difference, then, can only arise, from the splendor of the surrounding Light, which dazzles the Eye, and renders those parts obscure, which, before, were distinctly seen. But, when out of the full glow of Sun-shine, we perceive every Object or parts of Objects, which are in Shade, as distinctly as before; and in the same Colours, though greatly different from those on which the Sun shines.

Some, again, of late Years, run into the opposite extreme, and make too little distinction, between the fullest Light and the strongest Shade; by which means, their Pieces look flat, and do not produce, on the Mind, a just Idea of what is intended. For, although it is certain, that, in Sunshine, every part receives additional Light; yet, the contrast is so strong, from the brilliancy of the Colours, where the Rays fall, that, the other Parts appear as if deprived of Light, in comparison with these; and, since that vivid lustre cannot be given by Art, and Colour, the difference must be made by keeping the other parts somewhat under.

Respecting the degree of reflected Light on Objects, it is not easy to determine; that being more or less, according to the situation of the Object, in respect of others; also, according as that other Object is situated to the Light.

If one Object be so situated to another, that the Surface of one, being directly opposed to the Light, is also much opposed to the shaded side of the other Object; the

the reflected Light, on the other, will be stronger than when they are more obliquely situated; either the one, in respect of the Light, or the other, in respect of the illumined Surface.

For example. The Cylinder, AB, being situated near the Wall X, on which the Light is direct; the shaded Edge (AB) has a much stronger reflection, than if the Plane (X) was more inclined to the Light; or, being removed, and no other reflecting Surface near, there would be no Reflection on the Column, in this position of the Light, save what is received obliquely, from the Ground. Fig. 49.

Respecting plane Surfaces. We frequently see (in Prints and Drawings) the shaded face of a Pedestal, &c. very dark, at the hither edge, and gradated towards the other, so, as to give it the appearance of concavity; which cannot be, when no other Object is near. But, when one part of an Object projects from the other, as the Plane X from Z, the Plane Z, being much opposed to a strong Light, and consequently the Shadow of the Plane X, on Z, is very narrow; then, the Light, reflected from Z to X, will be strongest where it joins to Z, and gradually lessened, towards the other edge; but not in so extravagant a degree, as may be seen in various Prints; particularly in Mr. Kirby's Perspective of Architecture. Fig. 50.

Now, the reason for this is extremely obvious; because of the vicinity of the part which joins to Z, the Light reflected on it from Z is stronger than on those parts which are more remote; and consequently, the reflected Light is more languid on the remote parts. But, if the Light was very oblique on Z, so as to cast a great breadth of Shadow, on the Plane; in that Case, the Light, reflected to the Plane X, will be more faint; insomuch that, there is barely a distinction between the Shadow, and the Object occasioning that Shadow; which there always should be, in some degree, the Shadow being darkest; because, the Shadow cannot receive any advantage from Reflection, which the Object does.

In respect of Mouldings; the edge of the Facia, AB, being in full Light, casts its Shadow on the Cavetto below; consequently, the Mouldings cannot have their effect from the direct Light, but from Reflection, only. On the returning Moulding; the under Facia or Planceer (BC) by means of Reflection, from below, is brighter than the vertical Facia, over it; also the vertical Face of each Fillet, is darker than the horizontal; which in other cases, having the Light directly on them, is consequently brighter.

In Figure 4, Plate 40, the great projecture of the Corona (X) deprives the Mouldings, below, of Light, entirely, from above, which are distinguishable only by means of Reflection, from below; consequently, the Ovolo is brightest towards the lower edge, and the Cavetto, towards the upper; and, they are more or less so, according as they are opposed to some illumined Body, reflecting Light on them. It may, therefore, be made a general Rule, with very little exception, that, the effect of reflected Light on Objects is reverse to the direct Light; particular regard being had to the situation of the Object which occasions the reflection. For, certainly, in whatever degree an Object is illumined, or from whatever cause, whether directly or reflected, the effect of it, on the Object, is the same, in proportion to the degree of Light. But when (as is frequent) various Objects reflect Light on the same Object, no one being particularly predominant; the Light by that means is scattered and confused, so that, there is little or no Shade on the Object; insomuch that, the parts can scarce be distinguished, one from another. The same effect is produced from various luminous Bodies, variously situated to an Object, or from various inlets of Light, on Objects, each destroying the effect occasioned by the other.

In a Nich (see Plate 43.) the Shadow is strongest at the edge or outline, and gradually softened towards that side of the Nich, which occasions the Shadow; owing to the strong Reflection from the other Side, on which the Light falls, direct.

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The Shade (without Sun-shine) on a convex or concave segment of a Cylinder, or Sphere, is the same; each may be the other, by supposing the Light on either Hand.

More might be said in respect of reflected Light; but, if the Observations I have made be well attended to, it will be found sufficient, for any purpose whatever.

Of the reflected Images of Objects on Water, and polished Surfaces.

The Reflections of Objects on the Surface of still Water gives (where Water is represented) transparency to the Water, and perfection to the Picture; which, if well managed, has a pleasing effect.

Reflected Images of Objects on Mirrours, except when seen direct, is a real Anamorphosis, or Distortion. On a horizontal Mirrour, the Object standing upright, the Image appears the same, inverted; yet, the Picture is really on the Surface of the Water; for, an Object on the Water, or being cut by its Surface, always appears double, to an Eye at any distance from it, horizontally. For instance, the Eye, being at E, sees the Image of the Object AB inverted, apparently in the Water; but it is manifest, that 'tis not so, in reality, for, the Representation is on the surface of the Water; the part AC being represented by Ac, which is drag'd out, to a preposterous length, whilst bc represents the upper part, the Cone BC, which is much less than the Object; nevertheless, I shall prove, that the Image is represented equal to the Object; but, being seen under unequal Angles, they do not appear exactly equal.

Fig. 51.

It is obvious, whether by means of Rays of Light, reflected to and again, from the Object to the Water, and from thence to the Eye, or otherwise, that, if AD be the Surface of Water (or a polished Mirrour) and AB, an Object, standing erect, we shall see the Image of the Point B, on the Water, at b (the Eye being at E) where Right Lines, drawn from B and E to the Water, at b, make equal Angles with its Surface. Consequently, if Eb be produced; meeting BA, produced, at B, AB, and, AB, equal will be to AB;* is the apparent place of the Object, which appears on the Surface of the Water. Hence it is manifest, that the Image of the Object AB, represented on the Surface of the Water (and which, in real length, is Ab) is represented equal to the Object itself. For, the Point A is common to both, and because B, appears at B, it appears just as much within the Water, as its real place is from the Water, viz. AB equal AB, and AC equal AC, &c.

Hence, we have an unerring Rule, for representing the reflected Images of Objects on polished Surfaces. But it must be observed, that, if the Object be at some distance, beyond the Water, the measure of the Object must be applied from its Seat, on the Plane of the surface of the Water, not from the Water edge, as when it is in the Water or close to it. Also, that the Image represented on the Water, is not similar to the Representation of the Object, on the Picture, which I shall shew; except when a plane Figure only is represented, parallel to the Picture.

E X A M P L E S.

Fig. 52.

Let X be an Object in the Water, having one Face (X) parallel to the Picture.

No. 1.

Produce the Sides, BA and CD, making AB and CD respectively equal to them; also, make the perpendicular EF equal to EF, and join BE and CE; i. e. make the Figure AED similar to AED, inverted, from the Line AD where the Surface of the Water cuts the Object. The Image of the Door, &c. is also similar.

* For, the Angle BbA = EbF (the Angle of incidence, to the Angle of reflection) and, AbB = EbF (2. 1. El.) wherefore BbA = AbB (Ax. 3. El.) then, because the Angles at A are right (AB being supposed perpendicular to the Surface of the Water) and Ab is common to the two Triangles, ABb and AbB, consequently, AB is equal to AB; for the Triangles are congruous (11. 1. El.)

For

For the Face AG, being perpendicular to the Picture, the horizontal Line, BG, vanishes in the Center; wherefore, BG being parallel to the Water, its Image will be parallel to the Line, and consequently, it has the same Vanishing Point.

Draw BC, meeting GH, produced at G, which compleats the Image of that Face, and which, it is obvious, is not similar to the Representation ABGH; seeing that, the Angles which are acute, in one, are obtuse in the other. Likewise, a side of the Roof is represented in the Object, which, it is manifest cannot be seen in the Water, by Reflection.

The Piles, at U, which are perpendicular, have their Images also equal, each to the Representation; likewise, the Pile W is reflected equal, and equally inclined, being parallel to the Picture. But, being otherwise inclined, as Y and Z, one hanging towards, the other from the Picture, the Seats of their tops, on the surface of the Water, being determined, at a, produce the Perpendicular ba, making a \bar{b} equal to ab, and draw from b, to the Timbers, where they are cut by the Surface.

No. 2.

The Vanishing Point of the reflected Image is at the same distance on the other side of the horizontal Line, as that of the Object, whether it be above or below it.

The Image of the Bridge, on the Water, may be obtained after the same manner, particularly the Piers. The reflected Images of all horizontal Lines tend to the same Point, in the horizontal Vanishing Line, as the Representations; for, Right Lines, being parallel to the Surface of the Water, their Images are either parallel to them, or they have the same Vanishing Points, respectively.

No. 3.

The horizontal Line, HI, of the Bridge, vanishes at O; wherefore, PQ, drawn perpendicular through O, is the Vanishing Line of the Plane of the face of the Bridge; IK vanishes at P, and the inclined Line at the hither end at Q, equally distant from O, being equally inclined. Wherefore, kI being made equal to Ik, the horizontal Line has the same Vanishing Point (O) as its original, and the inclined Line IK, tends to Q, representing a parallel to GH, at the hither end.

For the Arches; draw as many perpendicular Lines, ab, cd, &c. as are necessary, and produce them in the Water; making each equal to its corresponding one, a \bar{b} equal ab, &c. by which means, as many Points, in the Curve, as you please, may be obtained, and the image of the Arch described through them; each being a Semi-Ellipsis, of which, ik is a Diameter.

The Crane, &c. on the Wharf, hanging over the Water, has its Image reflected, by means of perpendicular Lines from each part, and finding their Seats on the surface of the Water (as m, of M) repeating the same measure downwards.

No. 4.

For the Hoghead; draw as many Lines (representing lines parallel to the Horizon) as are necessary, tending to the Vanishing Point V, of the horizontal Diameter ab; then, by means of perpendiculars from each extreme of those Lines, their Images are acquired, and the curve of the Head described through them. If other Lines are drawn, from one head to the other, to the Vanishing Point X, of its Axe, the whole Image may be described, as in the Figure.

The Warehouses, &c. standing at some distance from the Water, have their reflected Images described as the other Objects; by supposing the surface of the Water produced, and the Seat of each Angle determined thereon; as of A, B, C, the Angles of the Warehouse W; which are made equal, downward, from their Seats (a, b, c,) respectively, and the horizontal Lines have the same Vanishing Points.

No. 5.

The Spire of the Church, at T, is reflected on the Water; because, its Seat (S) on the level of the Water, being obtained, the height (ST) being applied downward reaches the Water, otherwise it would not.

No. 6.

Some Persons have expressed surprize, that, Objects, at a great distance, should have their Images reflected on Water which is near. The most distant Objects, which can be seen, will be reflected; provided, that the Eye be properly situated, near the Surface, and there be no other Objects intervening, i. e. when Right Lines

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from the Object and from the Eye, making equal Angles with the surface of the Water, meet on the Water; otherwise, the Objects cannot be reflected. It seems indeed rather surprizing, that a Surface, very much inclined to the Horizon, and nearly approaching to perpendicular with the Picture, as a gentle Assent can be reflected; but, the Eye being so situated, as above, and the surface of the Water being on a level with the foot of the hill, its inclined Surface will be reflected thereon.

It has been objected, that the Reflections are too strong, in the Plate; I grant they are, as a Picture; but my design is to shew the figure of the Image on the Water, rather than the effect, considering the Water not simply as Water, but as a Mirrour. Nevertheless, in clear Water, perfectly at rest, the Images of Objects on its Surface will be found as distinct as the Objects themselves appear, according to their Distance; somewhat differing in Colour, rather darker, in the clearest Water.

Thus much may suffice, for Reflection on a horizontal Surface; which is the same, whether it be a clear Fluid (as Water) or other plane Mirrour.

SECONDLY. OF REFLECTION ON MIRROURS, vertical and inclined.

Fig. 53.

Let W be a Mirrour, parallel to the Picture, whose Center is S ; and X , an Object, whose reflected Image is required.

In this Case, it must be observed (as in the former) that the Image, of any Point in an Object, is reflected to the same distance from its Seat on the Mirrour, as the original Point from its Seat; so, that distance is, here, represented, and its Seat obtained, perspectively (in the former it is geometrical) and, because the reflecting Surface is parallel to the Picture, the Center is, consequently, the Vanishing Point of Lines producing its Seat, being perpendicular thereto.

Draw Lines, from the Feet, A, D , &c. of the Chair to S , the Center; and, where they cut the Intersection of the Mirrour with the Floor, at a, d , &c. make aa represent a length equal Aa , &c. (Prob. 8.) and draw ab and dc perpendicular; then, BS and CS , cutting those Perpendiculars, respectively, give their reflected Images. For, as every Object appears to be as far on the other side, as it really is on this side, from the Mirrour; consequently, in this Case, it must be so perspectively.

The reflected Images of horizontal Lines, as the rails of the Chair, &c. (in this Case) tend to a Vanishing Point in the Horizontal Line, at the same distance from the Center of the Picture, on the contrary side, as the Vanishing Point of the Representation. If V be the Vanishing Point of BC , the front Rail; then, in SG , a distance being taken equal to SV , is the Vanishing Point of bc , the reflected Image of BC . I is the Vanishing Point of the side Rail CE ; SK being made equal SI , K is the Vanishing Point of its reflected Image. After the same manner, the middle Point of the top Rail may be obtained, by a Perpendicular, ef , to the Floor, and as many other Points as are necessary.

On a vertical Mirrour, inclined to the Picture, there is no difference in the process, but only in the Vanishing Point of Perpendiculars to the Mirrour; and, if the Vanishing Point of Perpendiculars be determined, 'tis the same in all inclinations.

Fig. 54.

Let Z be a Mirrour inclined to the Horizon, and casually inclined to the Picture. It is required to find the reflected Image of the Chair, on the Mirrour Z .

V being the Vanishing Point of the bottom of the Mirrour, which is horizontal, find the Vanishing Line (VL) of the Mirrour, its inclination to the Horizon being known (Prob. 5. Sect. 3.) and also, the Vanishing Point (F) of Lines perpendicular to the Mirrour (Prob. 2. Sect. 12.)

Then, having found the Intersection (CD) of the Mirrour with the Floor, produced (its Vanishing Point is V) and G , the Vanishing Point of Perpendiculars thereto, draw AG, DG , &c. cutting CD , at A, D , &c. and through them, draw Ab and Dc , tending to the Vanishing Point of the sides of the Mirrour*, indefinite.

* This Vanishing Point is the Intersection of the Vanishing Line (VL) of the Mirrour, and the vertical Vanishing Line RG , produced.

Draw *AF*, *DF*, &c. cutting them, respectively, in the corresponding Points, *a*, *d*, &c. (their Seats on the Mirrour*) beyond which, their Images are represented equal, in Perspective; by making *a*, represent an equal measure as *Aa*, &c. (Prob. 8.)

Find the representation of the Intersection of the plane of the Front, with the Mirrour (which will pass through *V*) and the Intersecting Points of *AB* and *DC*, therein, by producing them; from which Points, draw Lines through *a* and *d*, indefinite, and *BF*, *CF*, &c. cutting them in their respective Images on the Mirrour.

Then, having found the Intersection *EB*, of the Plane of the side of the Chair, with the Mirrour, and the intersecting Points, *E* and *B*, of *GA* and *FB*, draw *Ea* and *Bb* indefinite; and, from *F* and *G*, draw lines to *F*, the Vanishing Point of Lines perpendicular to the Mirrour, cutting them at *f* and *g*.

Thus, as many Points (as *e*, at the top) may be determined as occasion requires, from their Seats on the Floor; by which means, the reflected Image of any Object, whatever, may be represented on the Mirrour.

BC is parallel to the Mirrour, wherefore, the Image (*bc*) of *BC*, has the same Vanishing Point, in the Horizontal Line; otherwise, it would not; for, although all horizontal Lines, whatever, vanish in the Horizon of the Picture; in this Case, the Mirrour is a Picture, and the representation of the Chair is considered as the original Object, in respect of that Picture, each Plane having its peculiar Vanishing Line thereon.

PQ is the Intersection of the Plane of the Mirrour with the Ceiling; by means of which, the Cage, hanging from the Ceiling, at *N*, is reflected. If its Seat, on the Ceiling be determined, the Image of each Angle on the Mirrour is represented, indefinite, and their representative lengths are determined, by drawing lines from each extreme to *F*, the Vanishing Point of perpendiculars to the Mirrour; as *no* is the Seat of *NO*, on the Mirrour, and *no* is its indefinite Image. After the same manner, the corners of the Cage may be represented on the Mirrour.

Of KEEPING; and the effect of DISTANCE.

The Term, **KEEPING**, in the Art of Painting, in general, is used to signify a just and proper subordination of all the parts of a Picture to the principal Object; in respect of Magnitude, Colour, and distinctness of Parts.

The magnitudes of Objects, in respect of each other, perspectively, are various, according to the Station from which they are viewed; and consequently, the subordination of Colour, &c. is not in proportion to the Objects, but to their Distance from the Eye. It is absolutely impossible to lie down Rules, by which the Artist may, with certainty, produce the desired effect; seeing that, in hazy or foggy weather, or in a misty morning, &c. Objects are less distinct, at a short Distance, than, in a clear Day, at a much greater; wherefore, no proportion or degree can be determined. If Objects, of known magnitude, appear not to be far distant, in the Picture, and yet, their parts not distinctly defined, in comparison with others, in the Fore-ground; it implies, that the Air is more gross and hazy, than if the parts were more perfect.

KEEPING is, in a great measure, synonymous with Aerial Perspective; which signifies a diminution of Light, Colour, and distinctness of the parts of Objects, in a regular gradation, as linear Perspective of Magnitude; owing to the effect of Air, between the Eye and the Object; which, being a Medium, obstructs the sight, in some degree, at any Distance; consequently, as there must be a greater quantity of Air between the Eye and distant Objects than near ones, so their parts are less discernable; the Lights and Shades are insensibly mixed, and, at a great Distance, it is all a confused jumble, of Light, Shade, and Colour, without distinction.

It is customary with many, in delineating, and may frequently be seen in Architectural Designs, &c. to make a considerable difference, in Teint, between one plane Surface and another; when the one, in the Original Object, recedes but a

* *Ab* and *Dc* are the Seats of *AB* and *CD* on the Plane of the Mirrour, produced.

Plate
XLVIII.

Fig. 50.

No. 2.

few Inches, which is absurd; for, if that sudden gradation of effect was continued to the distance of a few Yards, it would become totally black, before it was possible for the Air to have any apparent effect on it. Now it is far from being so in Nature, which is obvious to any Eye; for, if the materials are clean, and of an even and uniform Colour (without which no judgment can be made) I will venture to affirm, that two Plane Surfaces (V and Y) parallel between themselves, at several Feet distance from each other, and having a full Light on them, cannot be distinguished one from the other, nor so much as the Line or Edge (AD) seen, at a proper distance for delineating; provided the Light be not (on this side) so very oblique (as at S, No. 3.) as to shade the other, beyond where the Line OC cuts it. I say, the Eye being so situated (at O) that the Angle C, appears to cut the receding Plane, at B, they will appear as one continued Surface. Nevertheless, it may be necessary, in delineating, to make a distinction, in proportion to the Distance; but, if the Plane V has Mouldings, &c. on it, being cut, apparently, by the edge of the other, the Line is sufficiently defined without it.

As it must be obvious, that no positive Rule can be prescribed, I shall just make a few observations, and conclude the Subject; and, with it, the Book.

In respect of Magnitude; it is evident, that Objects may appear, in Perspective, to have any proportion to each other, although not greatly distant. For Example; in Plate 27, of Chelsea College, the farthest Building, with a Pediment, is not above two eleventh parts of the hither one (to which it is equal) in height; and yet it does not offend the Eye, nor appear at too great a Distance. But, if the gradation of Light was in proportion, it would be too great; because, not merely the height is to be considered, but the square of the height, which would reduce it to the proportion of thirty to one; but where we shall fix the standard for unity, I am at a loss to devise.

Now, if we were at twice the Distance from the hither Building (the distance between the two Objects being somewhat more than twice that) the proportion of one to the other, would not be much greater than one to three; the gradation of Light, in that case, would be nearly as one to ten, but one third part of the former. Yet, I presume, that in two Pictures so delineated, it would not be advisable to make that difference in the effect of Aerial Perspective.

In short, as it is absolutely impossible to fix any criterion to determine the Ratio of the gradation of Light and the effect of Distance, but must ever be at the discretion of the Artist, I shall only observe, that the Objects in the Fore-ground, as the hither end of the Bridge (Plate 48) being supposed near the Picture, must have its parts distinct and perfect, with strong Lights and Shades, which graduate to the other End. The Buildings on the Wharf, &c. should be less perfect, as they recede from the Eye. The Church, at a Distance, the Trees and Hills, one beyond another, must consequently be less and less distinct, according to their Distance; till at last they are scarce distinguishable from the Sky. For all which, the Artist can have no other Rule than, carefully and judiciously, to copy Nature (the best Mistress) which, in that Case, is not always uniform, but infinitely variable.

F I N I S.

Fig. 12.

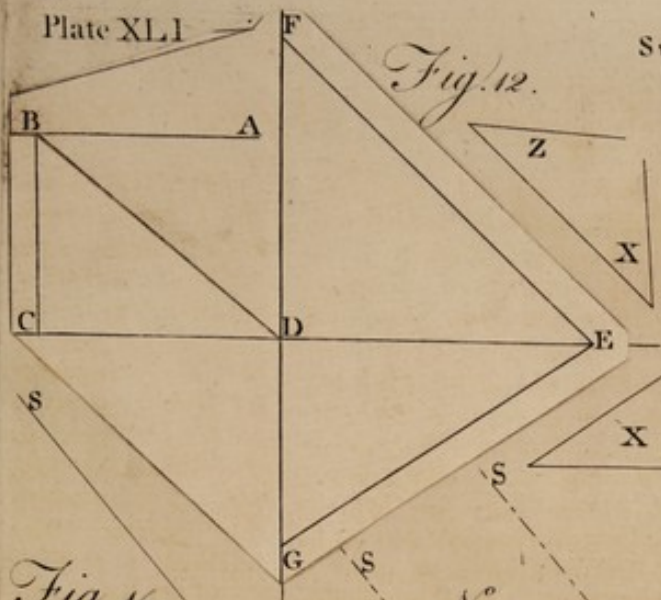


Fig. 13.

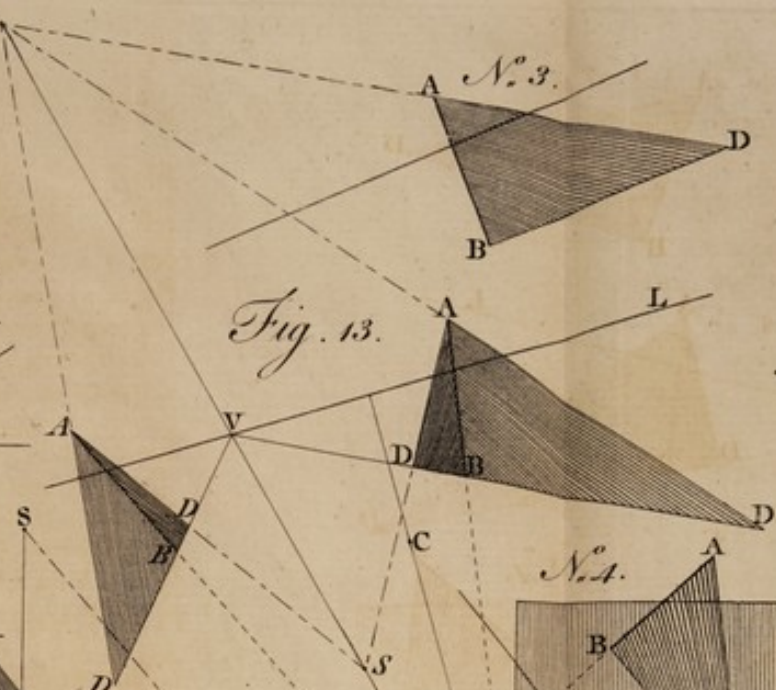


Fig. 14.

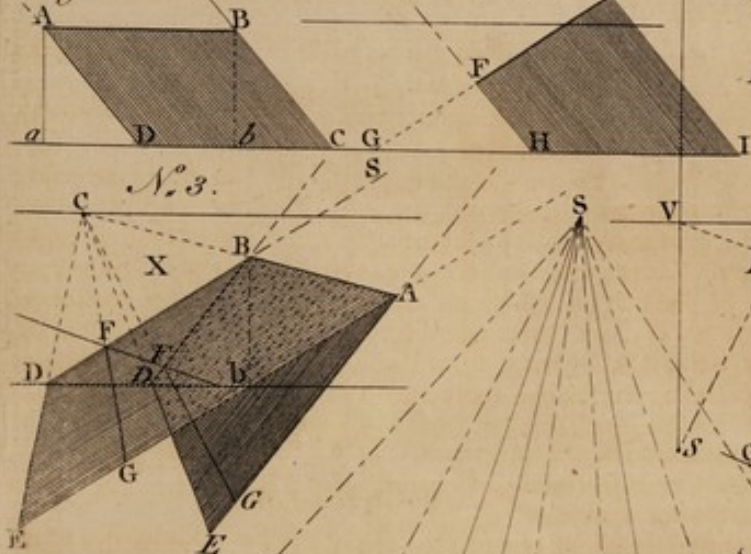


Fig. 16.

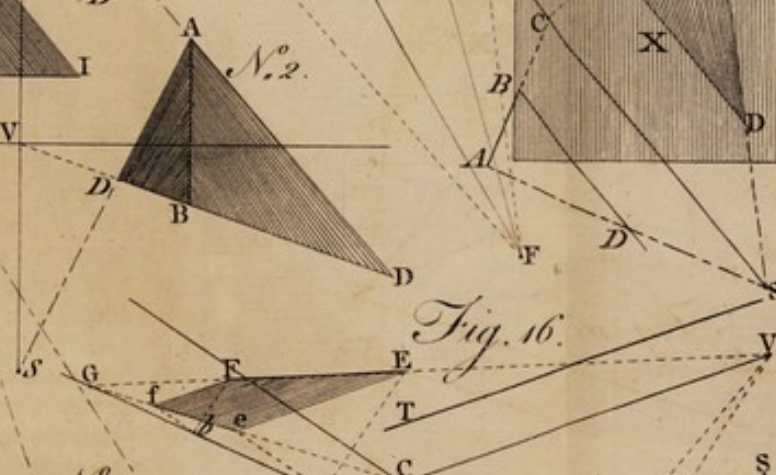


Fig. 15.

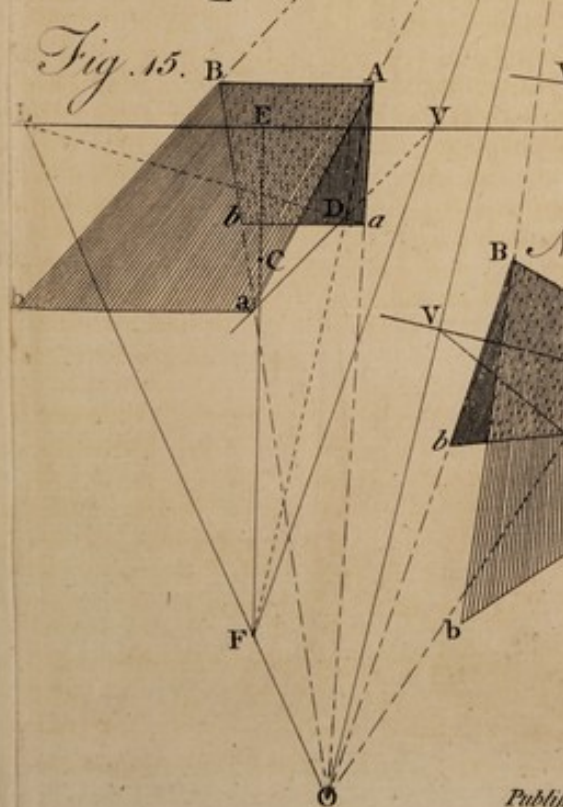
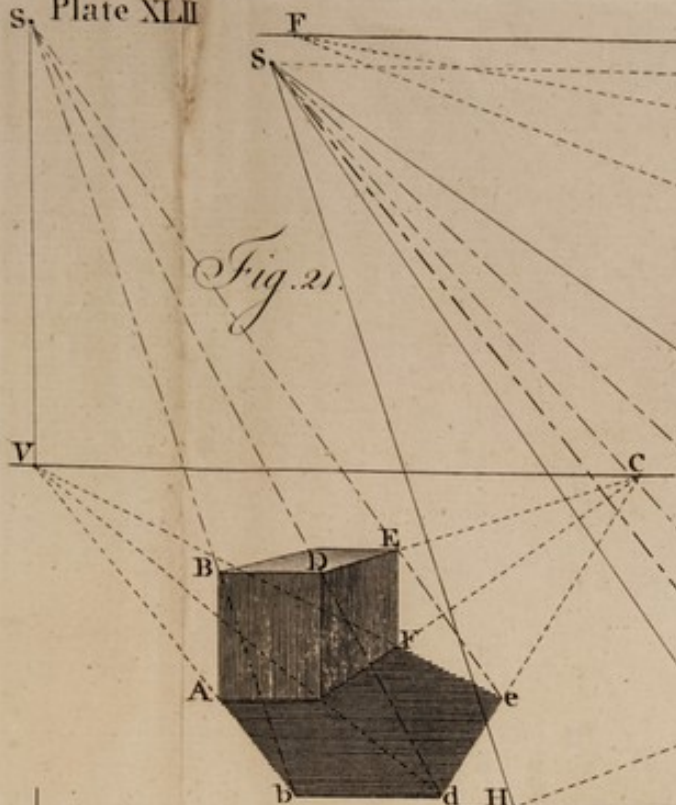




Fig. 21.



21. N^o 2.

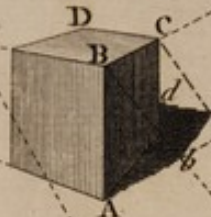


Fig. 18.

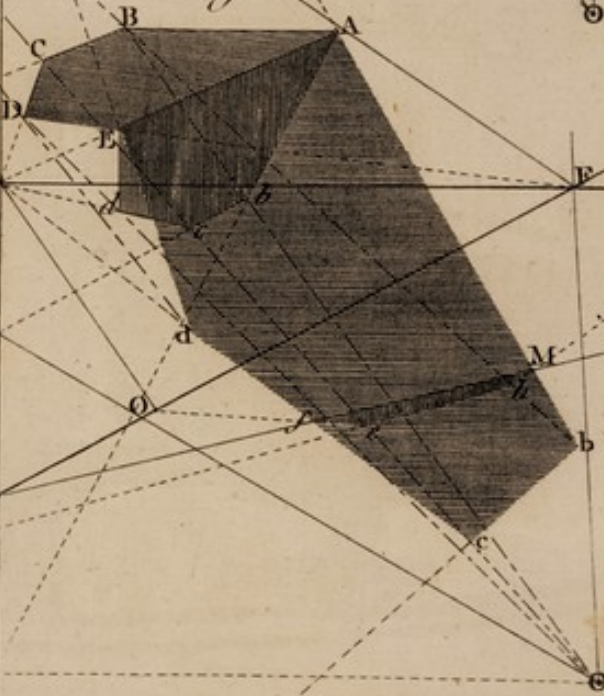


Fig. 17.

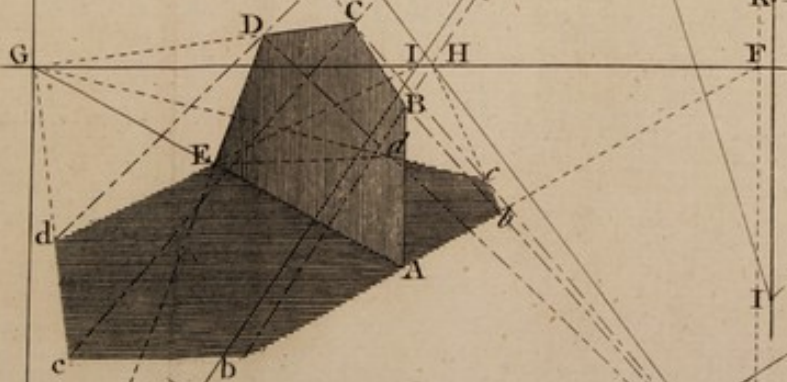


Fig. 19.

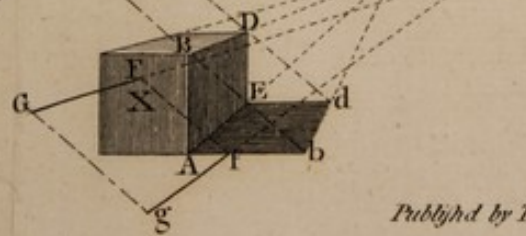


Fig. 20.

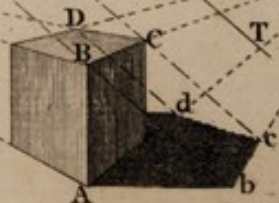




Fig. 22.

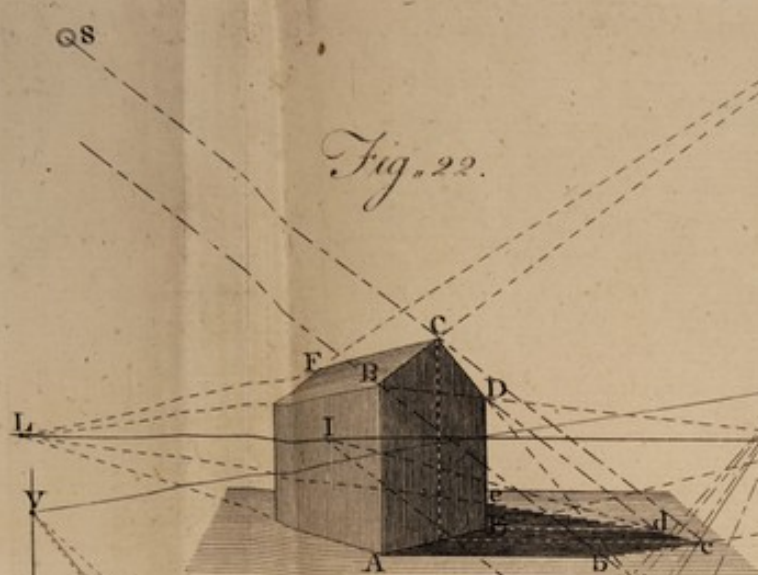


Fig. 24.

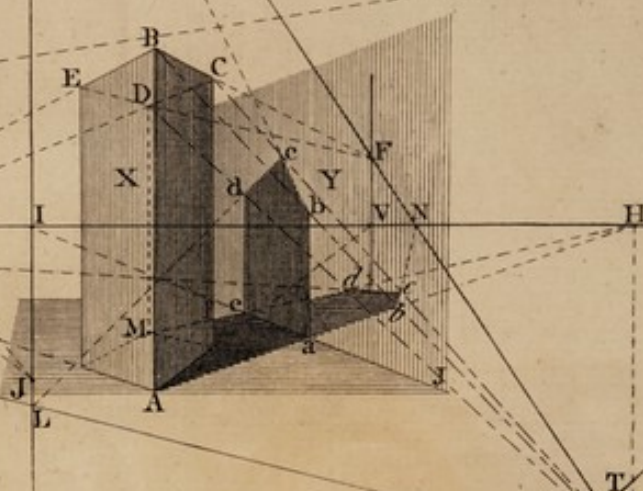


Fig. 23.

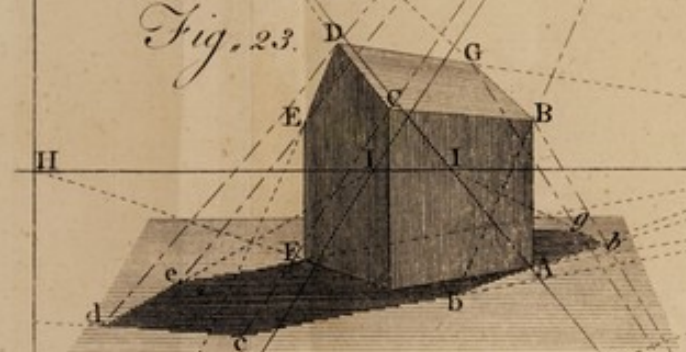


Fig. 26.

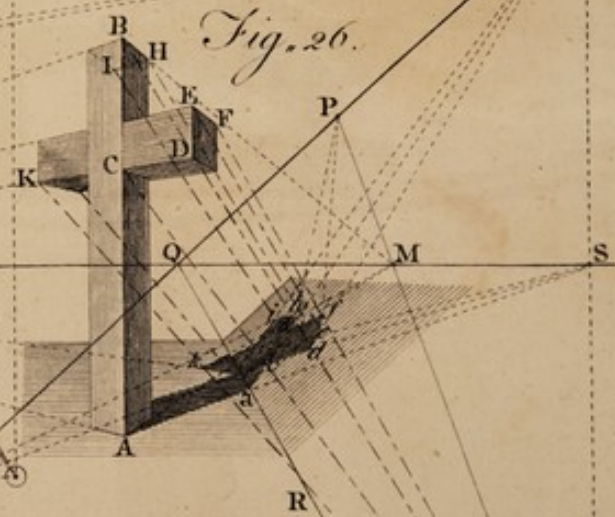
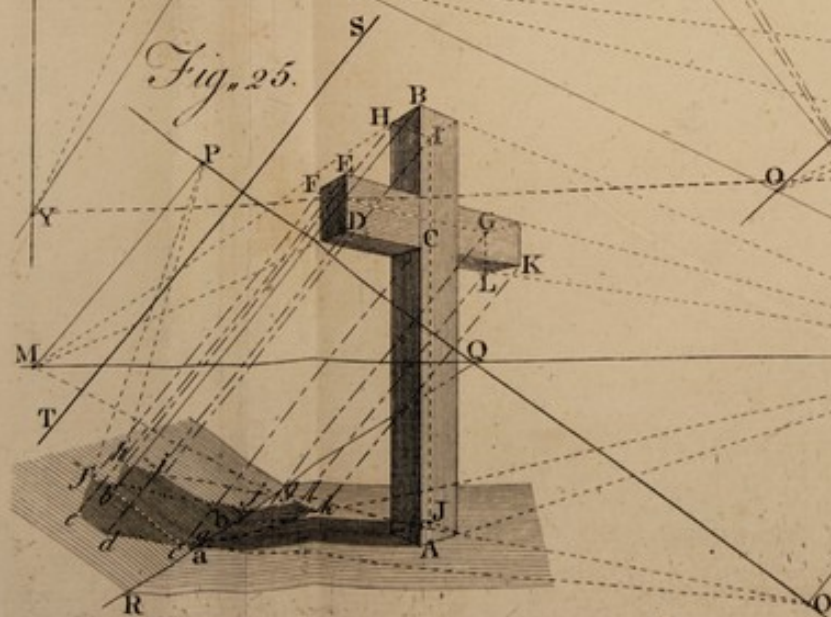


Fig. 25.



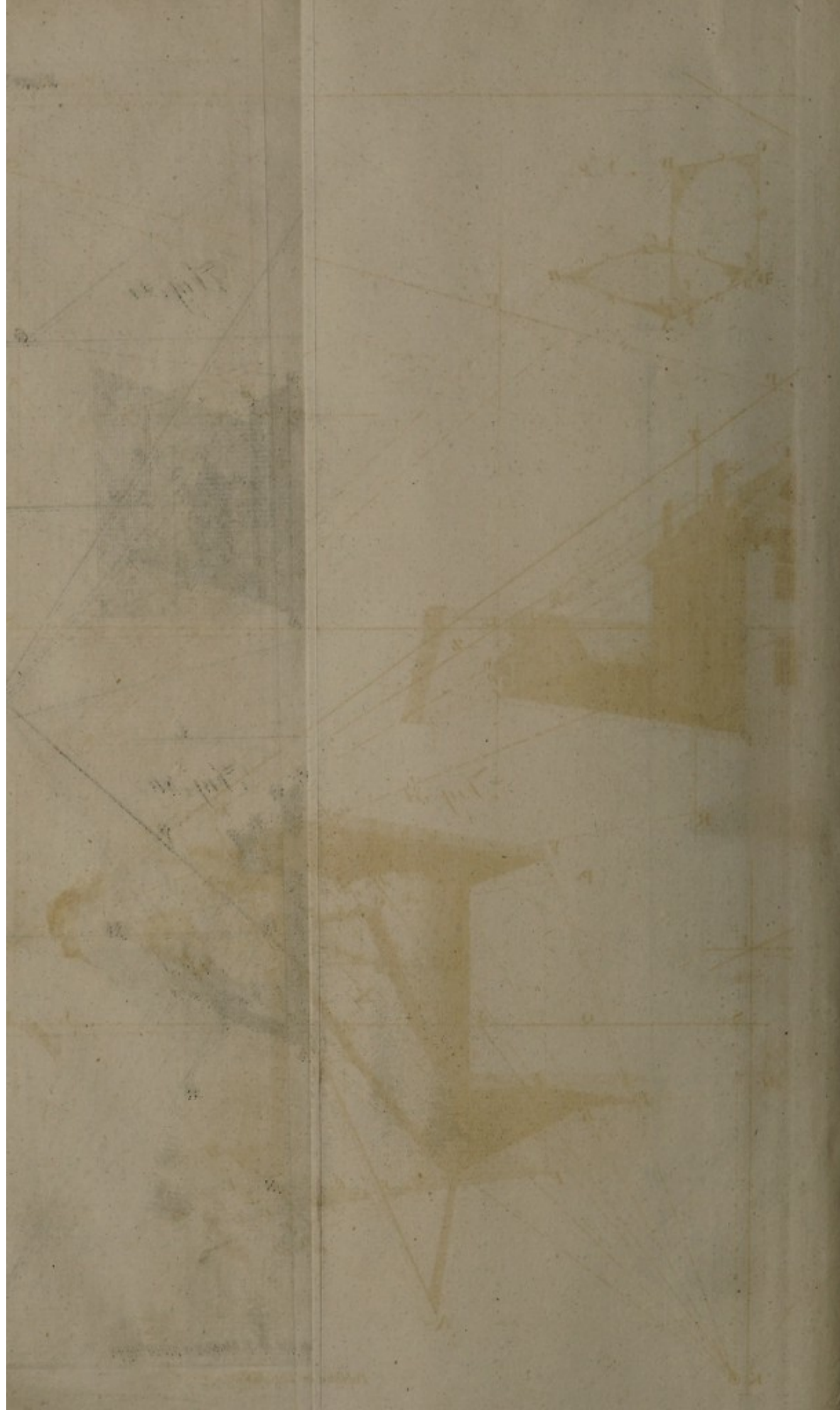


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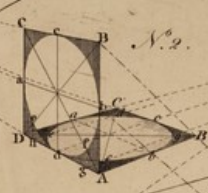
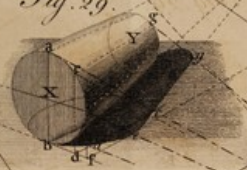


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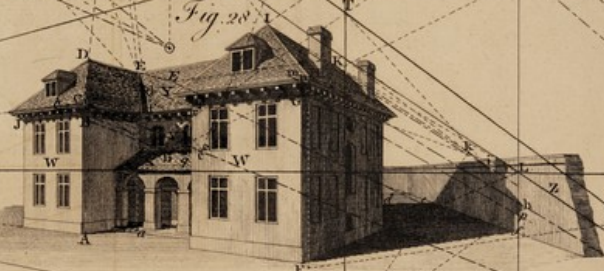


Fig. 27.

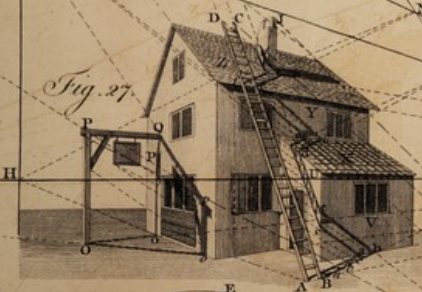


Fig. 30.

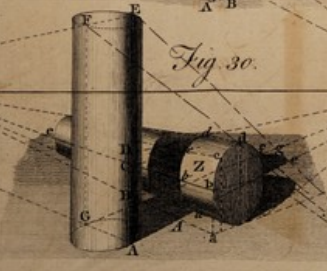
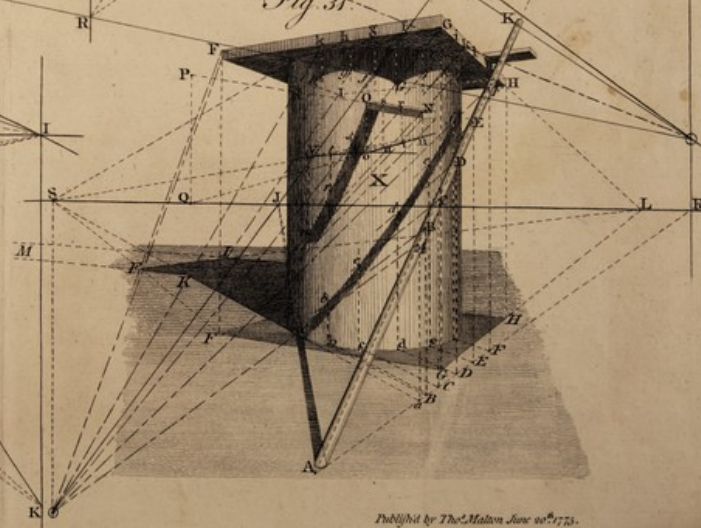


Fig. 31.



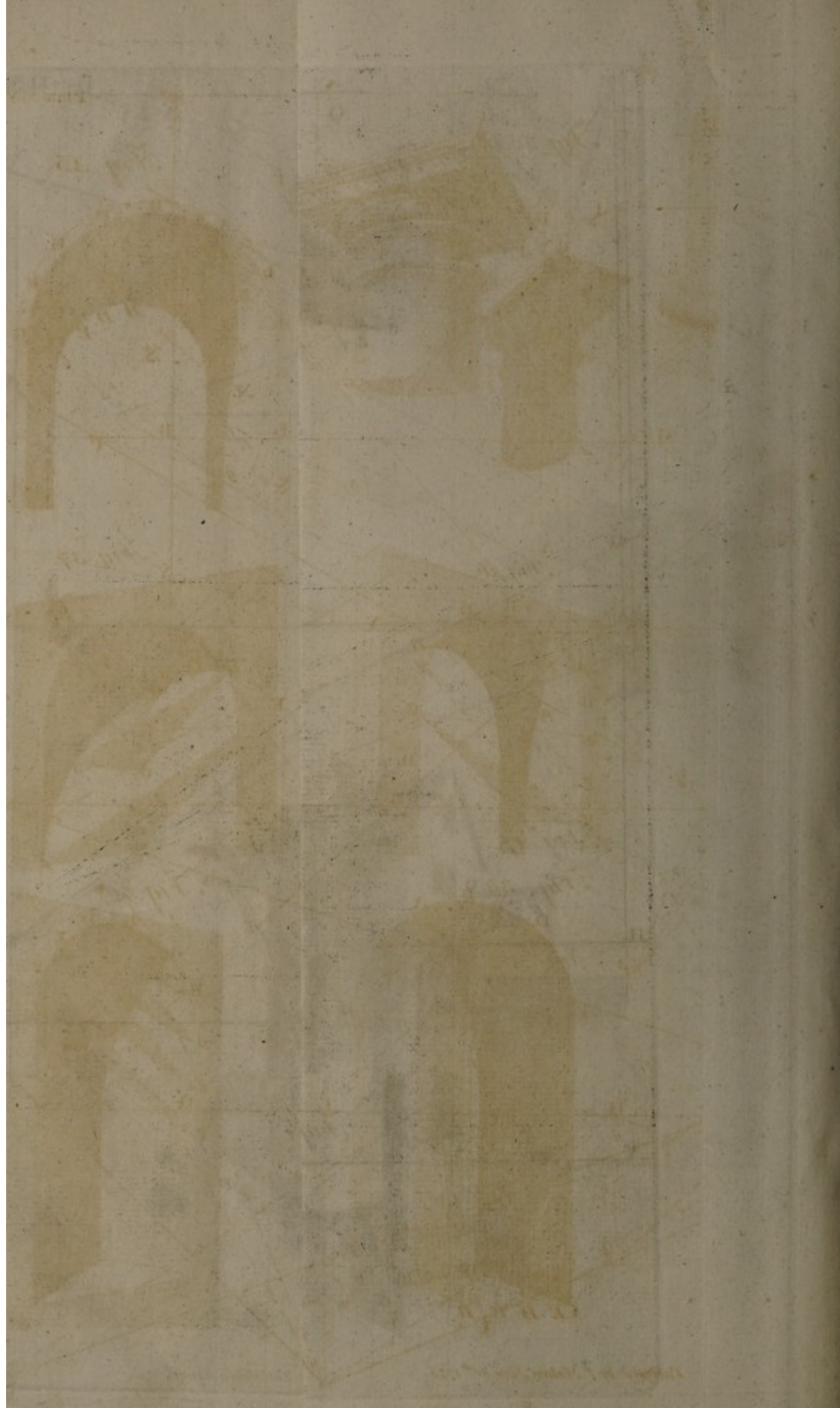


Fig. 32.

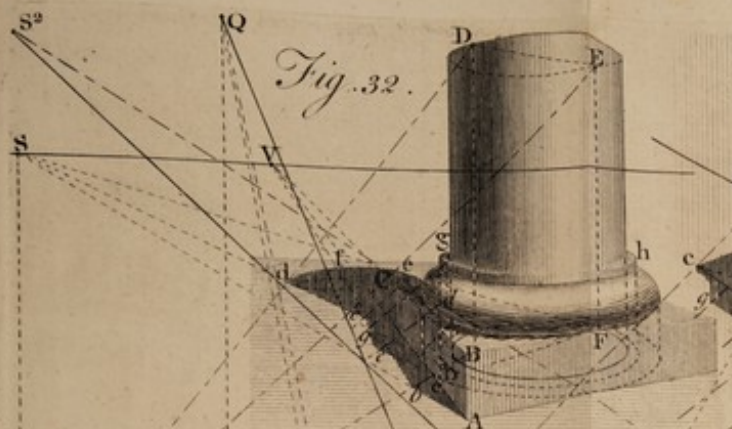


Fig. 33.

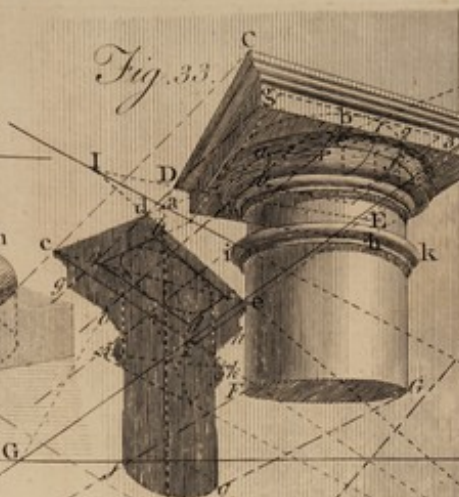


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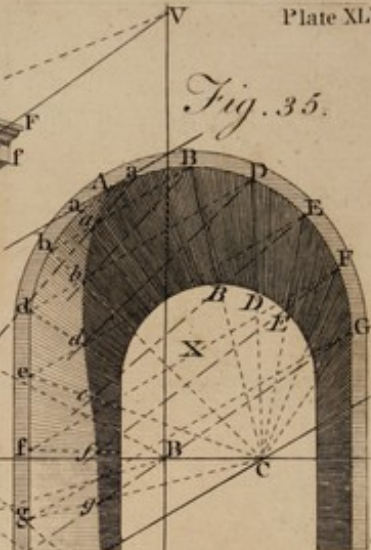


Fig. 34.

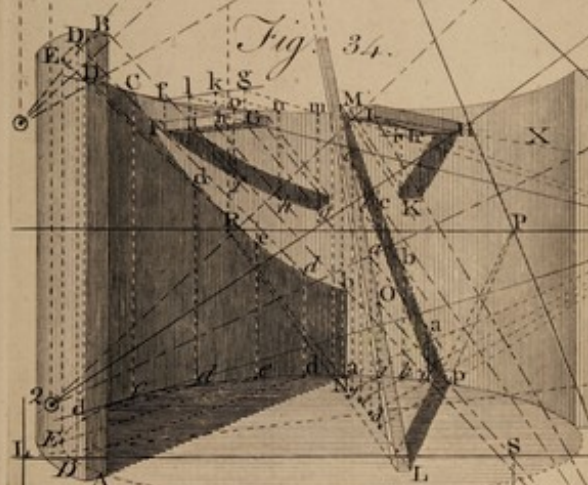


Fig. 36.

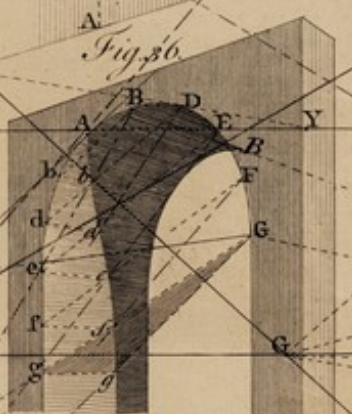


Fig. 37.

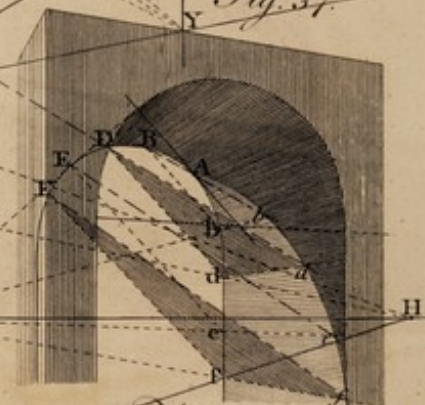


Fig. 38.

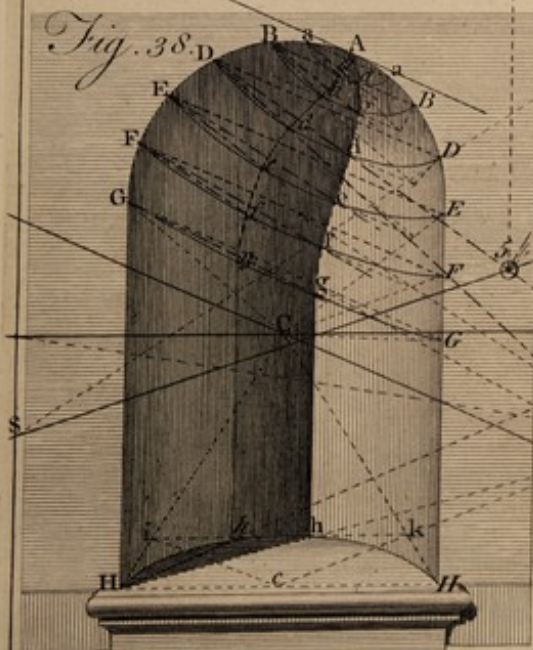


Fig. 39.

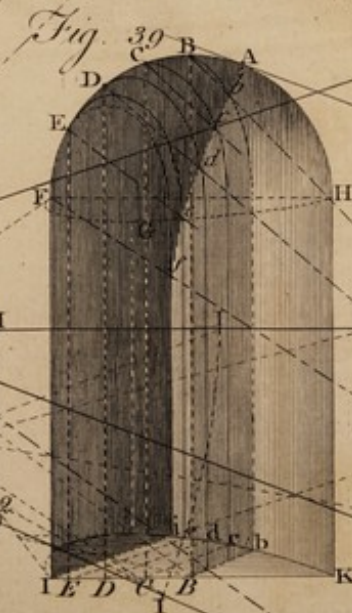
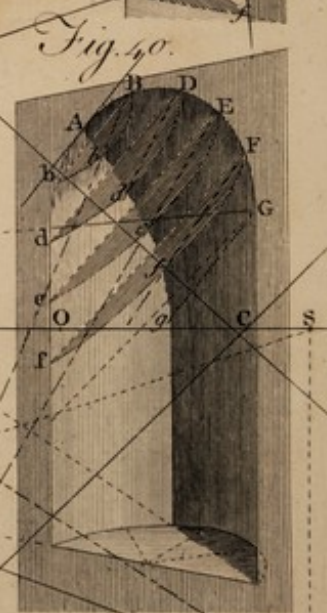


Fig. 40.



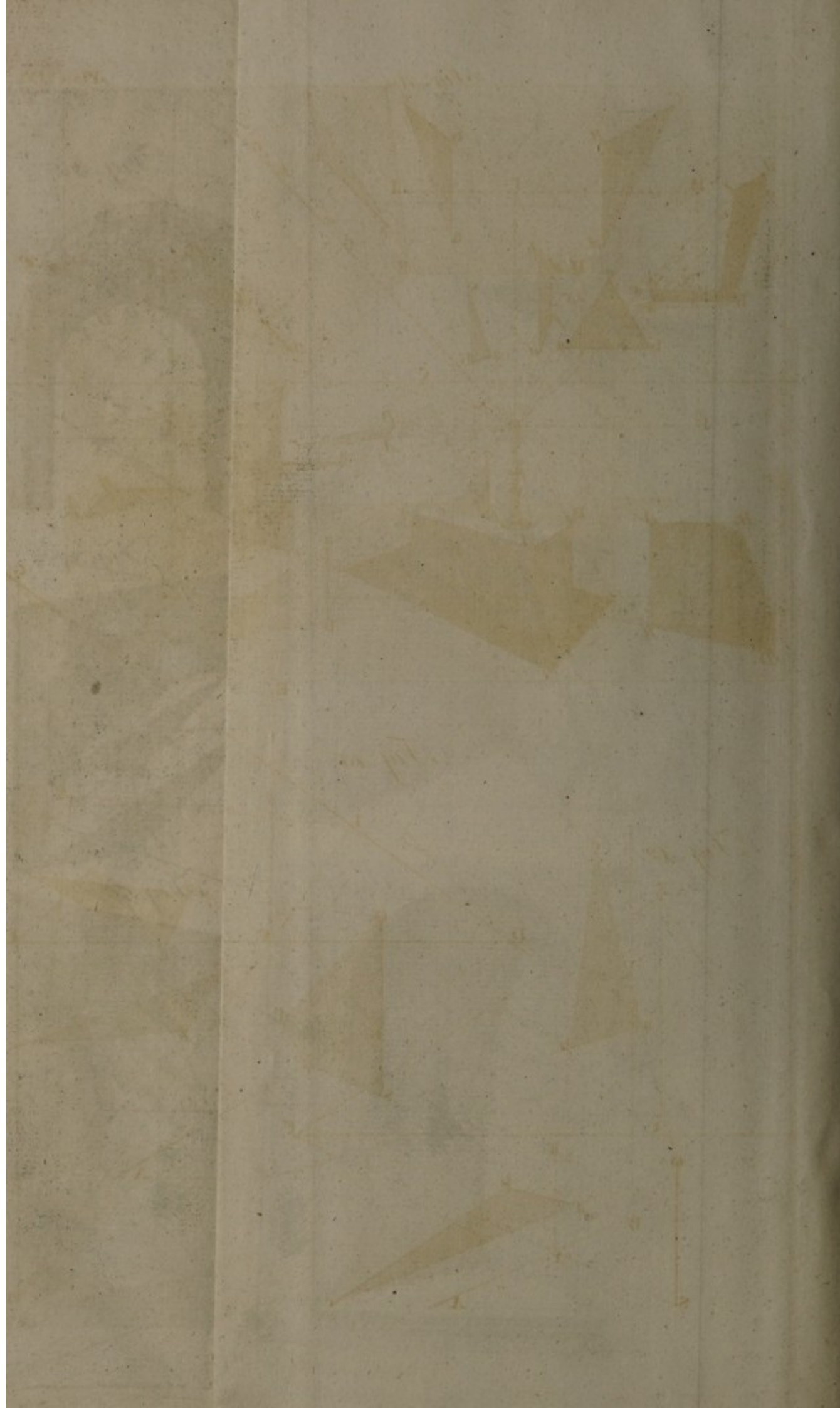


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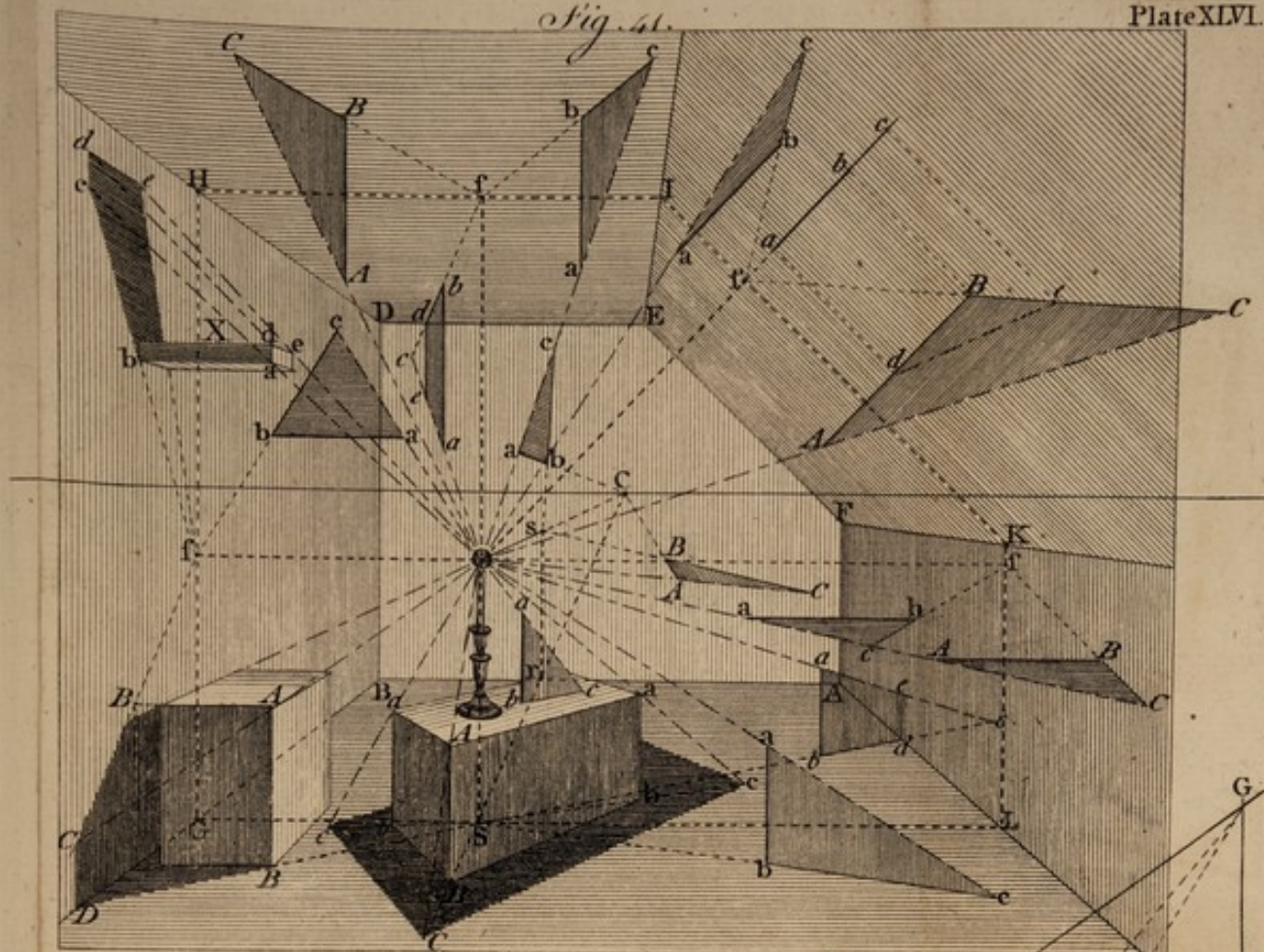


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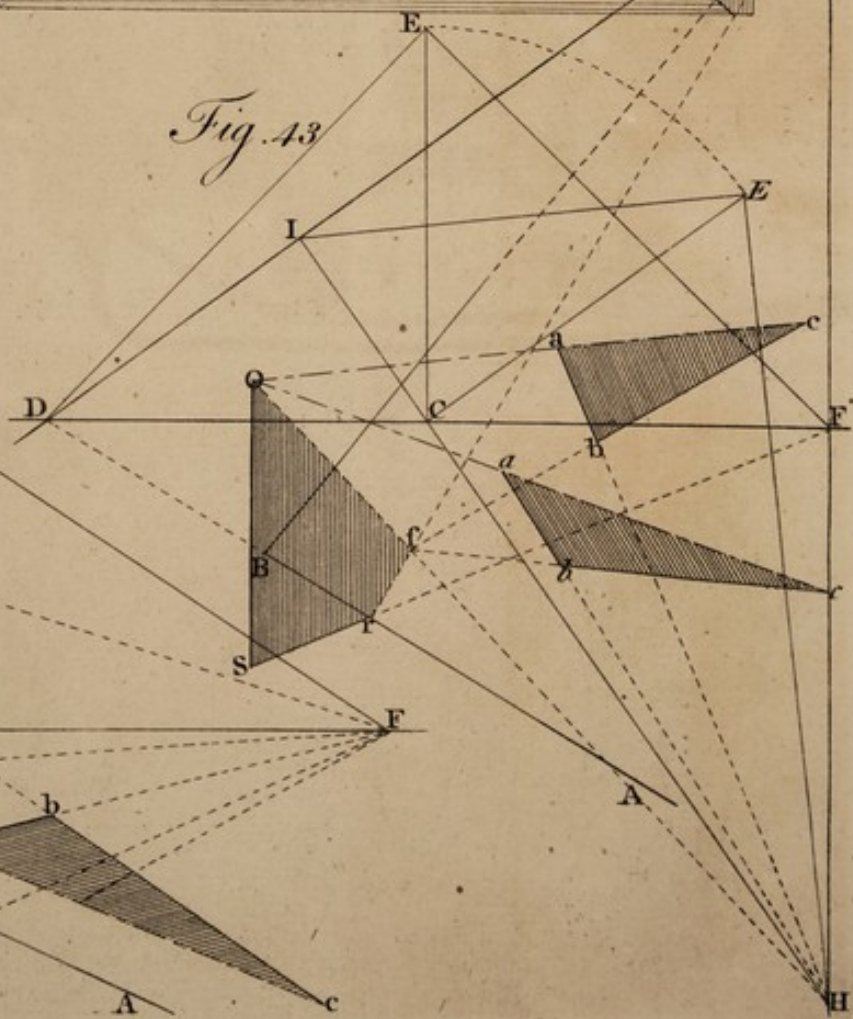
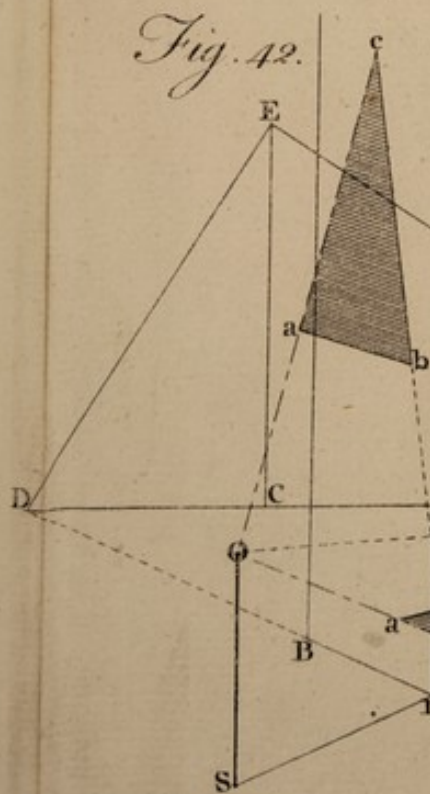


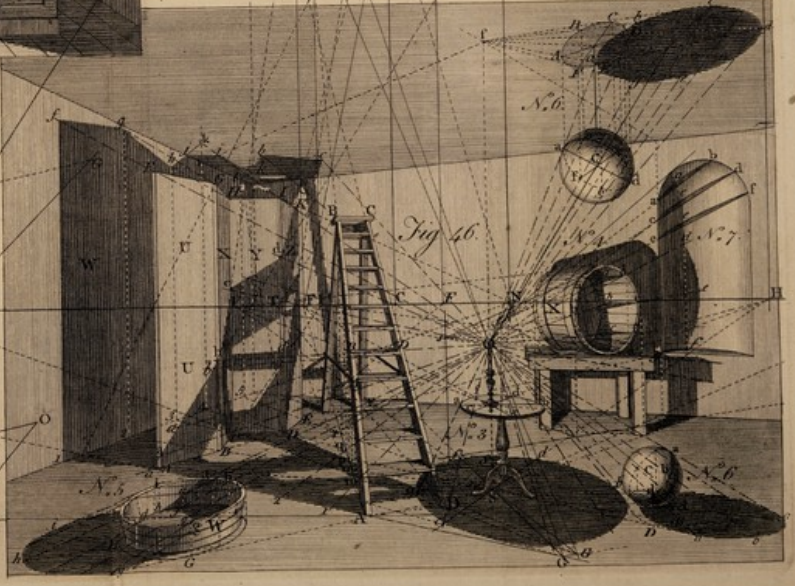
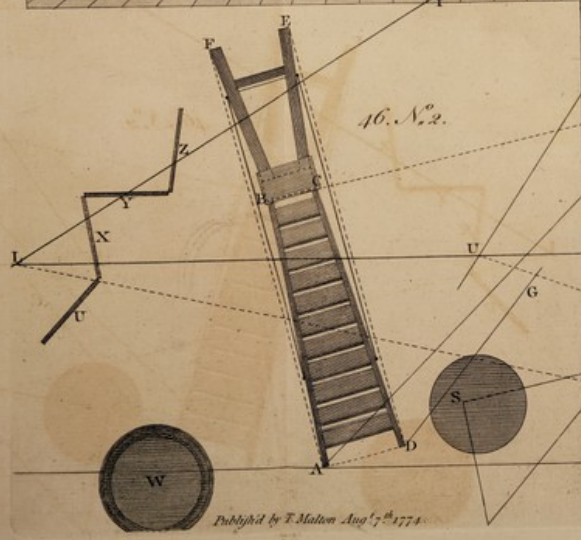
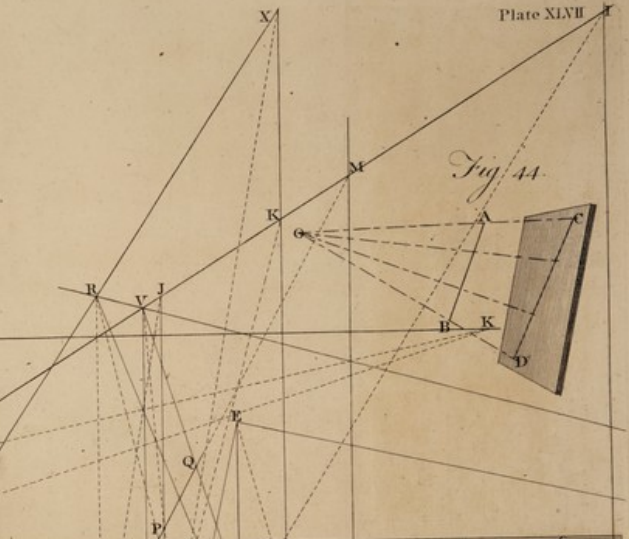
Fig. 42.



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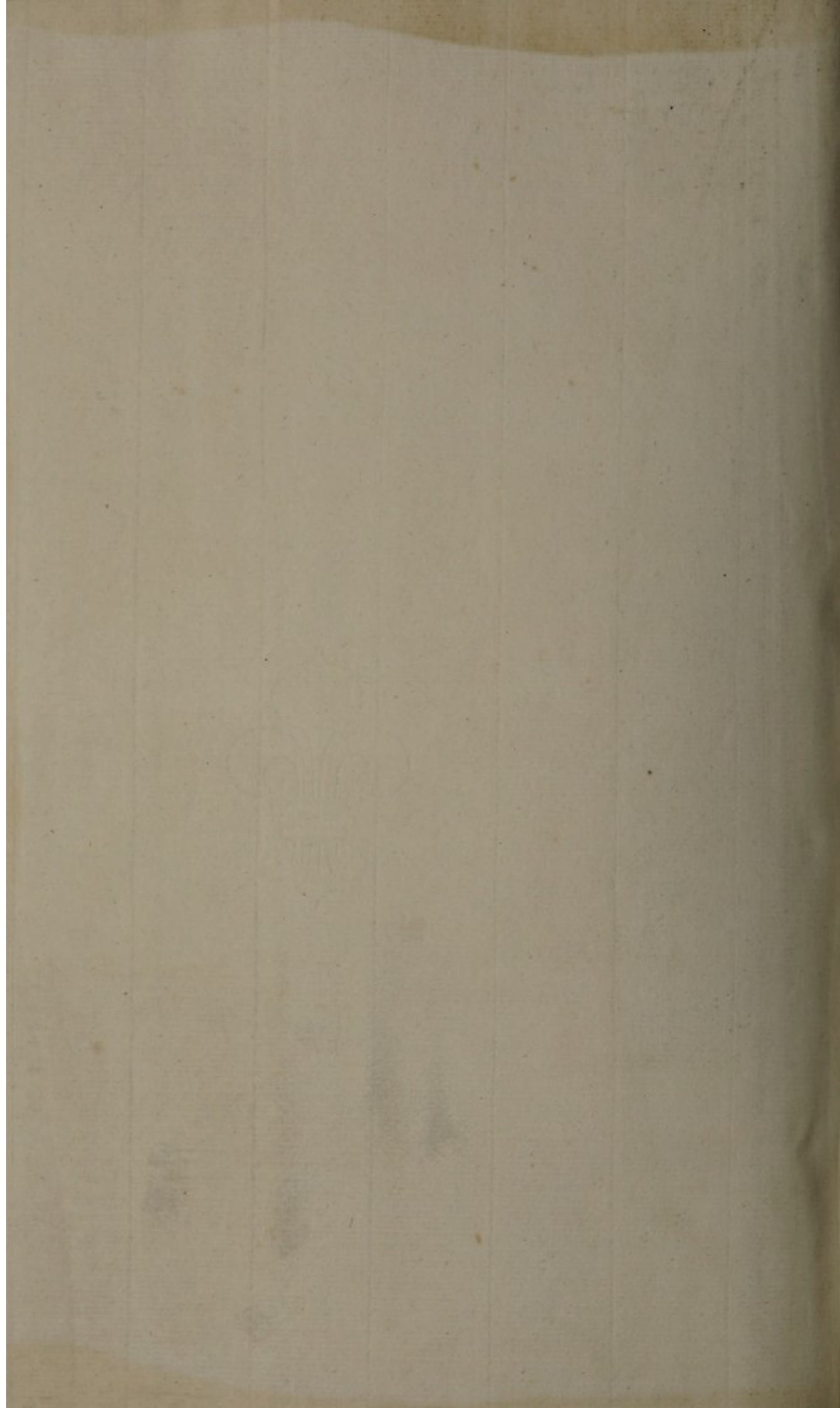
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