

Astronomy, in five books / by Roger Long.

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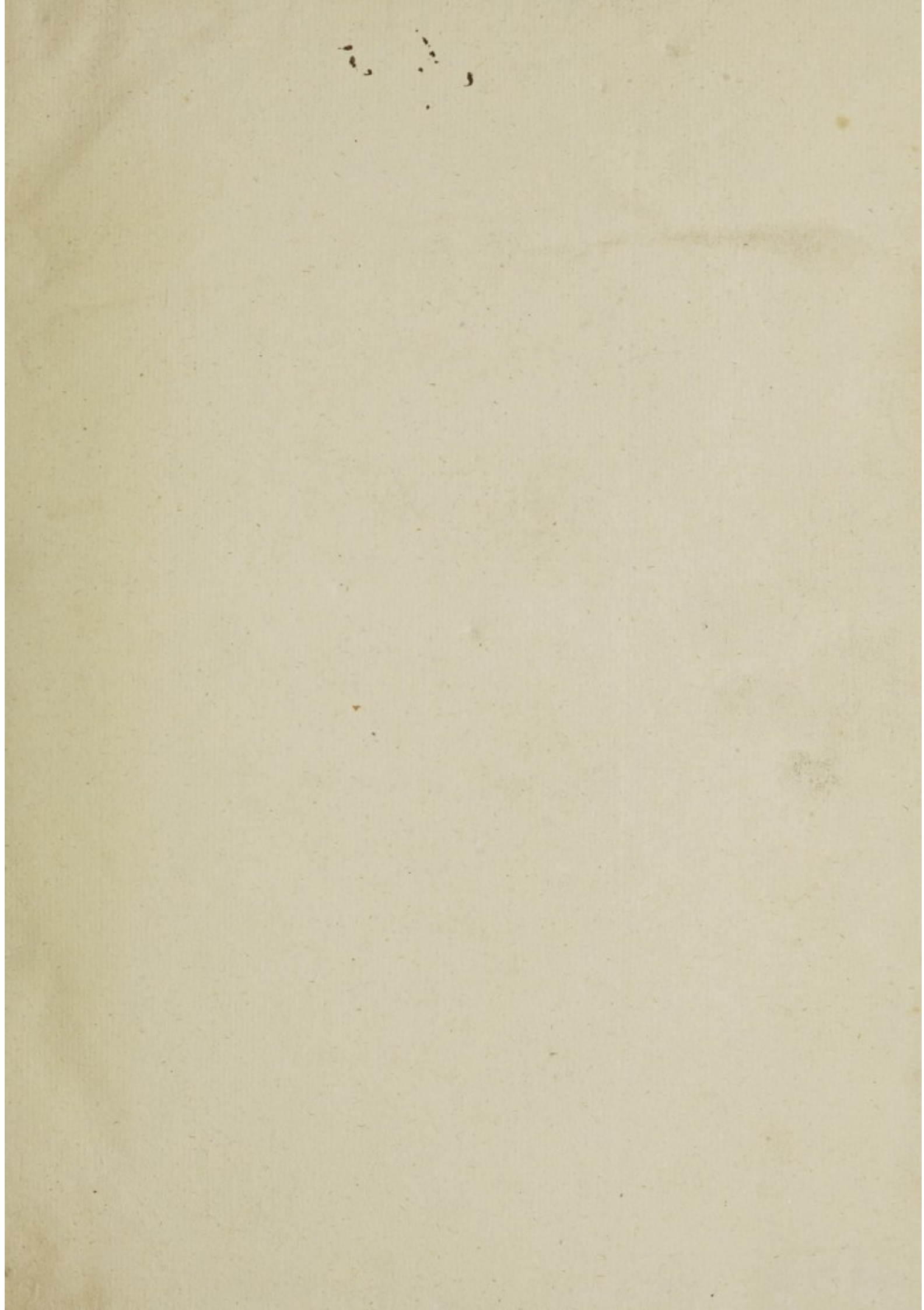
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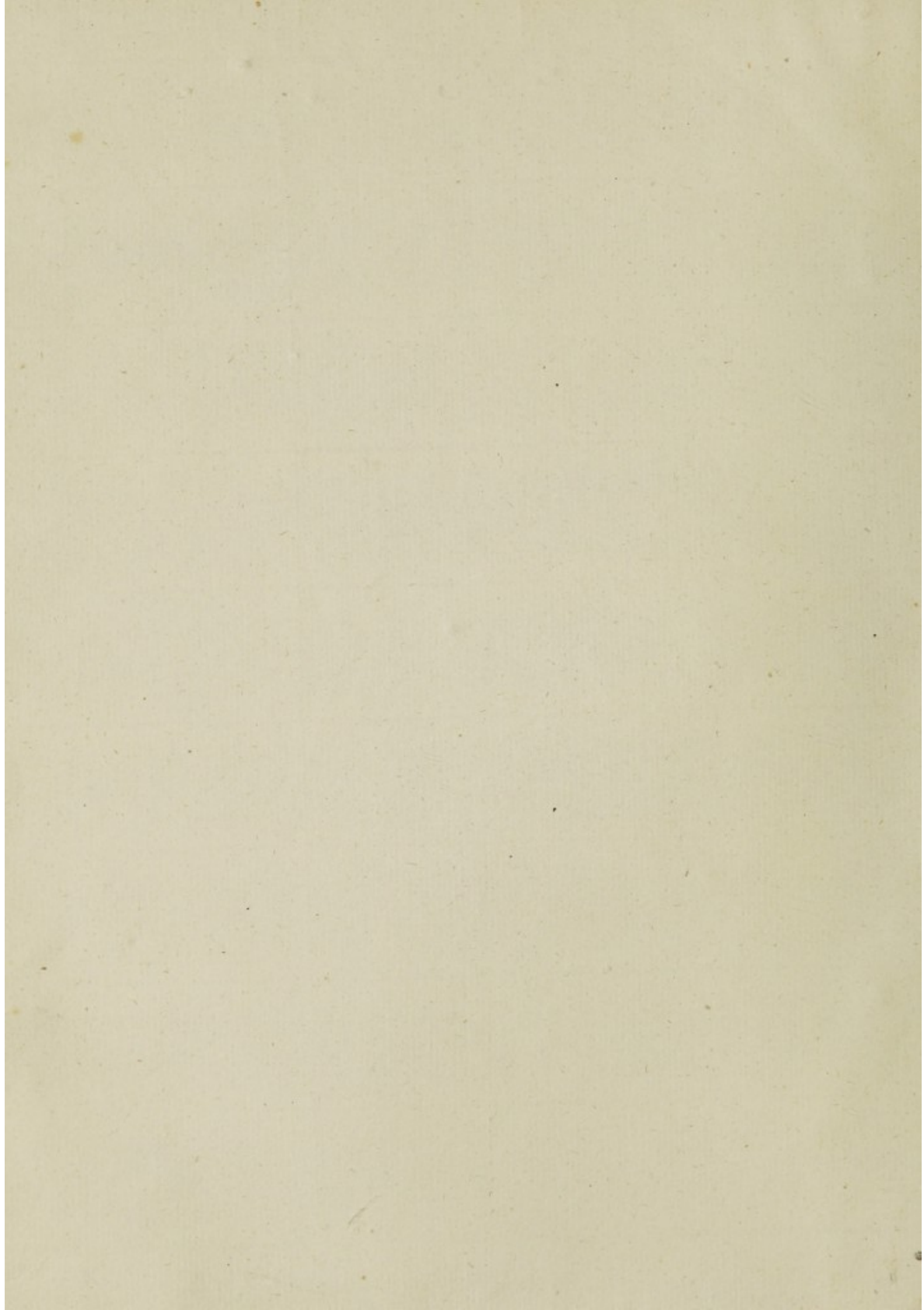


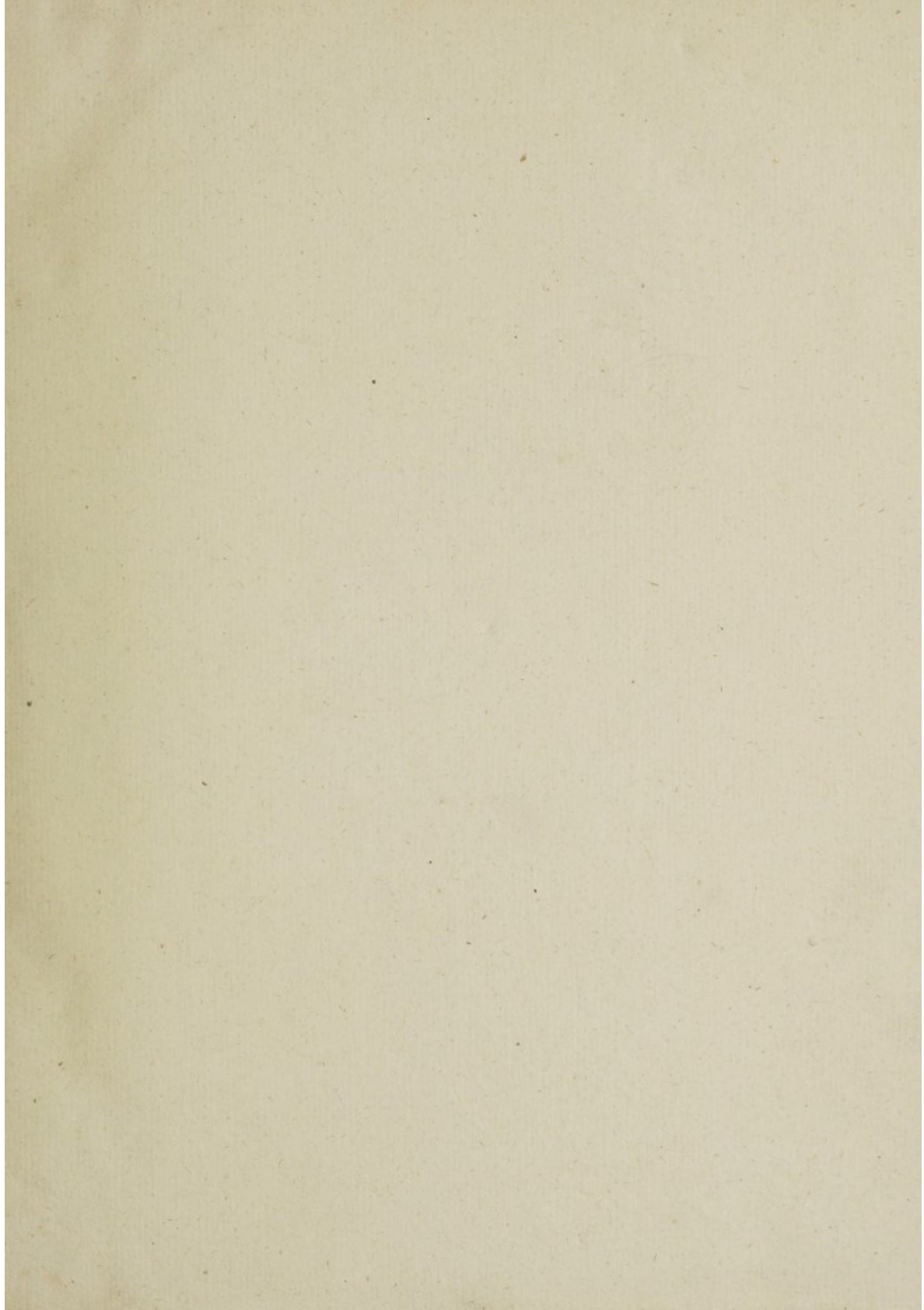
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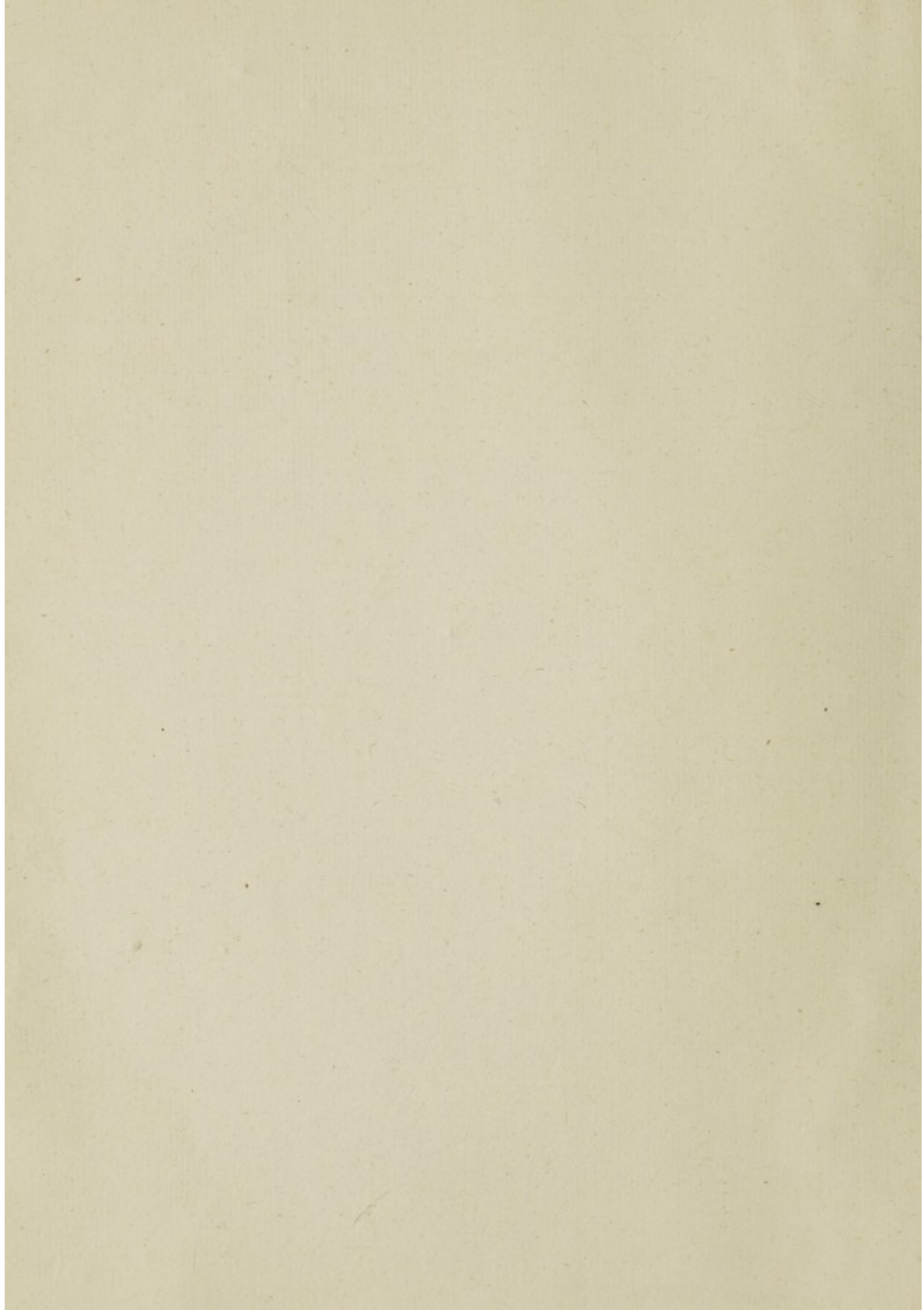
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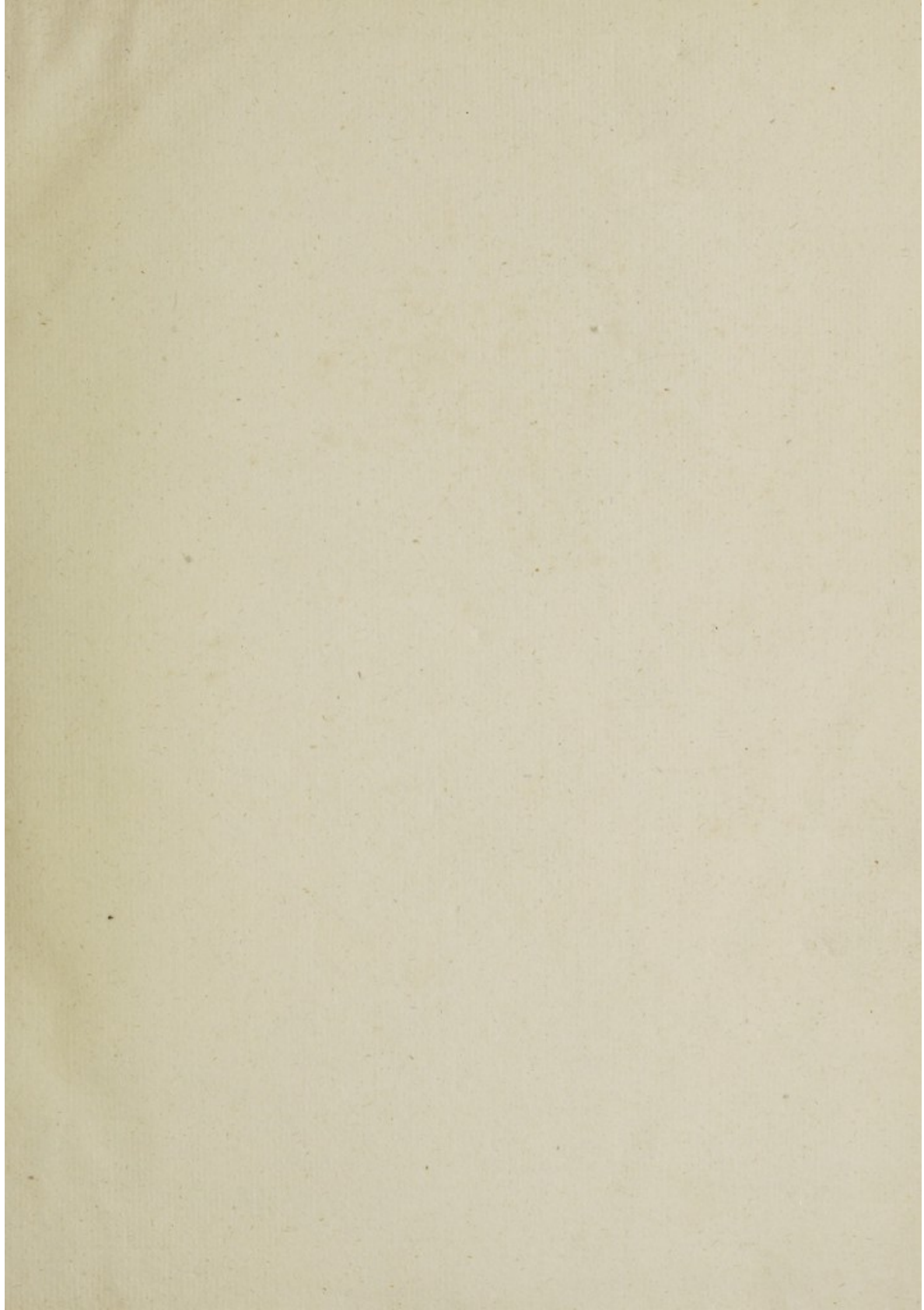
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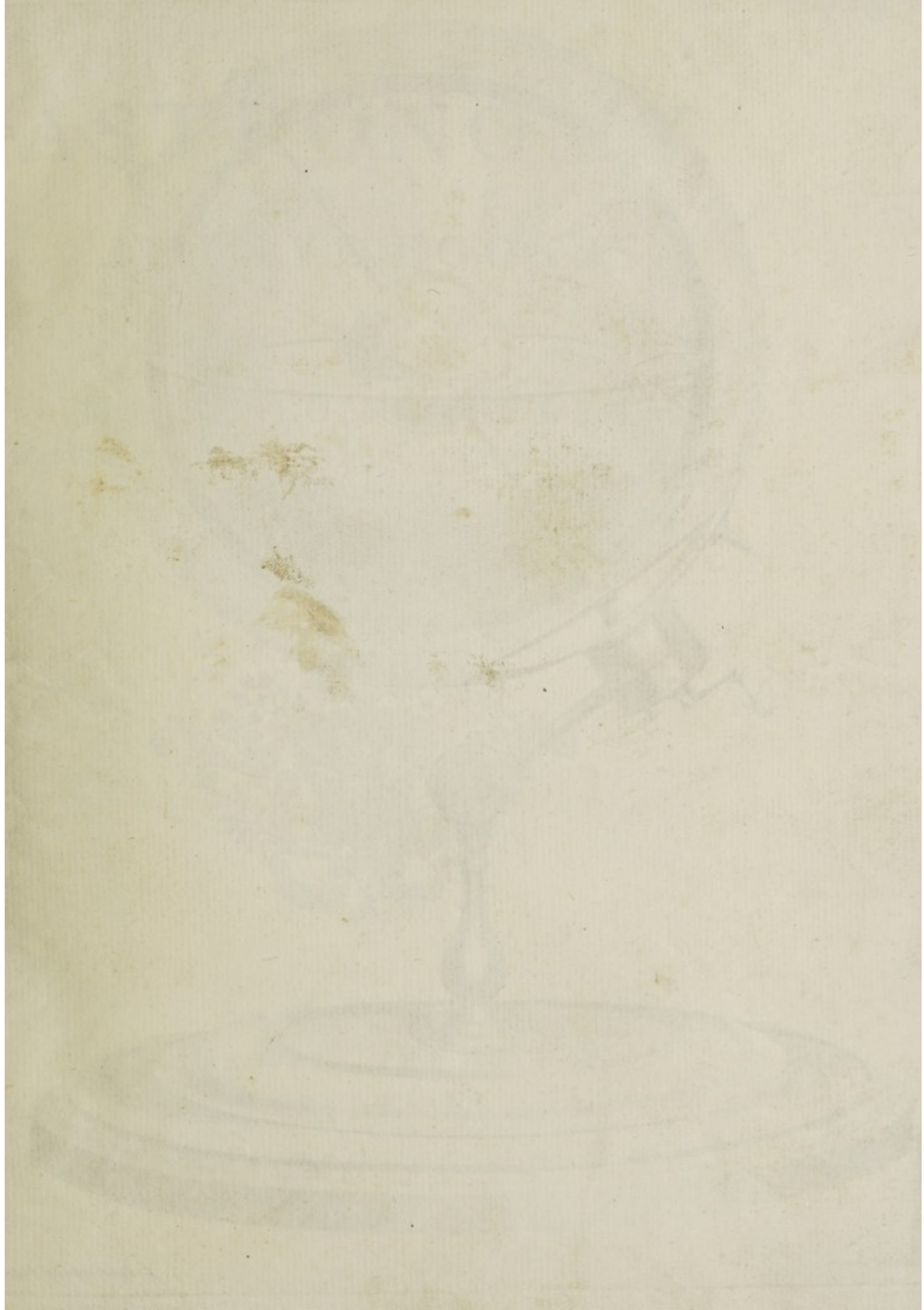


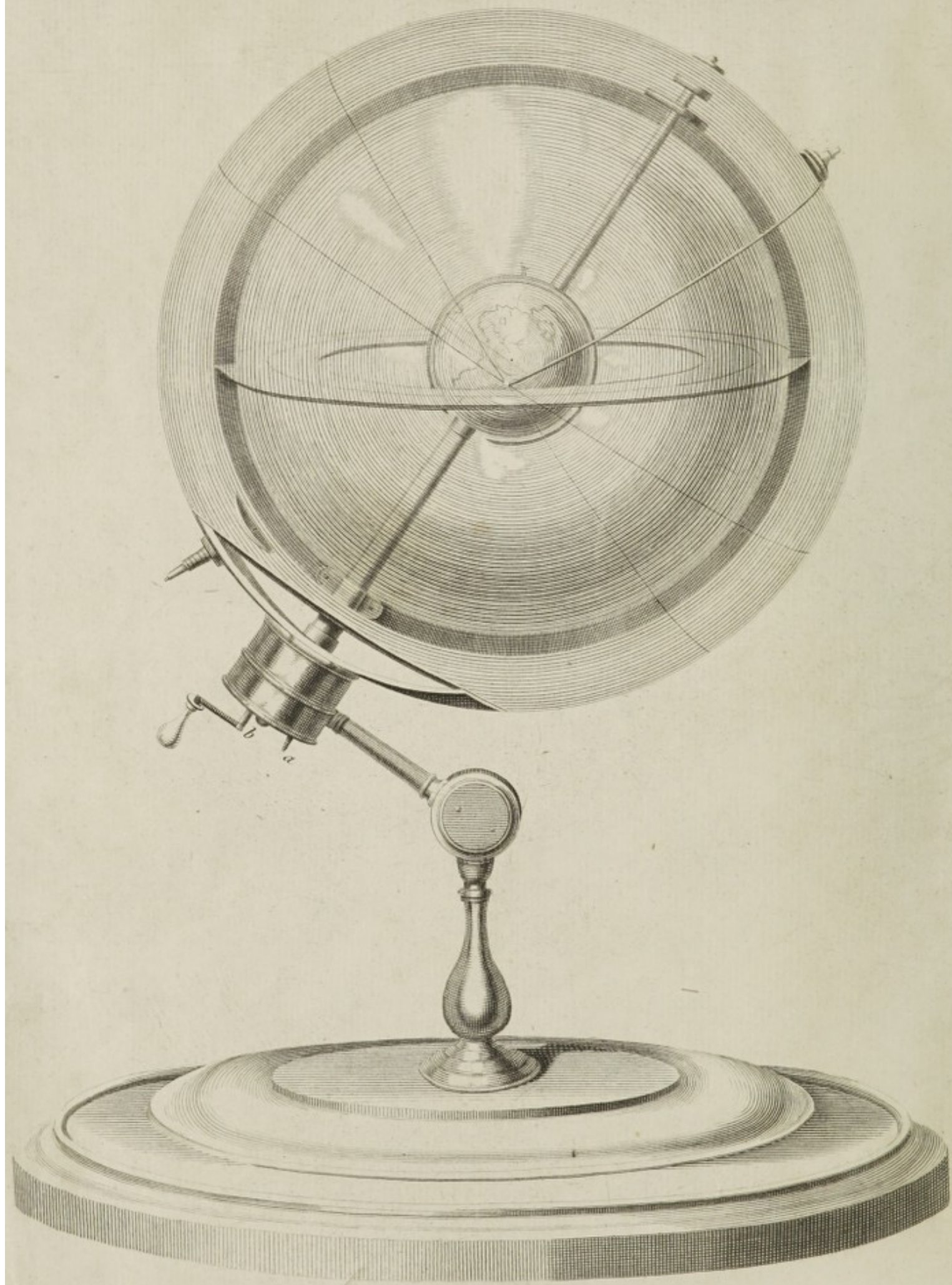




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ASTRONOMY, IN FIVE BOOKS.

B Y

ROGER LONG, D.D. F.R.S.

MASTER OF PEMBROKE HALL IN THE UNIVERSITY
OF CAMBRIDGE.

VOL. I.



CAMBRIDGE,
PRINTED FOR THE AUTHOR,
M. D C C. X L I I.



P R E F A C E.

ASTRONOMY is a science which, in all ages and countries flourishing in arts and politeness, has engaged the attention of the curious: it has not only employed the pens of the most eloquent orators and embellished the writings of poets of the most elevated genius; but has also been cultivated by the greatest princes, the ablest statesmen, and the wisest philosophers; as will appear at large, when we come to look into the history of its rise and progress in several parts of the world.

It should not indeed be dissimbled, that the great inducement to search into the nature and motions of the heavenly bodies with such care and pains as many persons have done was oftentimes the hopes they entertained of being thereby enabled to gratify a darling passion of human nature, the desire of looking into futurity: how mankind came first to give into this opinion, and how groundless a fancy it is, will appear in several places of this work: not that I shall take the pains professedly to shew the vanity of *judicial astrology*; this has already been done sufficiently by several able hands^a: I shall only make some short remarks upon it as occasion offers it self.

It is not much to be wondred at, that among nations abandoned to idolatry and superstition, or immersed in darkness and ignorance, it should be believed that the stars are the dispensers of good and evil to the inhabitants of this lower world, and that by their configurations they point out to mankind which of these two is to be expected from their influences; but it seems strange that so many of those who had the light of revelation to guide them have embraced so absurd opinions.

Idolatry and divination were of great antiquity among the Egyptians^b; the Jews, by their long stay among them when they were a rich and flourishing people, contracted a great fondness for their superstitions: after their going out of Egypt into the promised land they were to be surrounded by nations addicted to various kinds of idolatrous and superstitious practices; there was very good reason therefore why their law should so expressly forbid, not only all idolatrous worship in general, and that of worshipping the host of heaven in particular, but all kinds of divination also, and particularly that of *observing times*^c, or enquiring after the lucky or unlucky moments of entering

^a Cic. *de divinatione* l. 2. Favorinus *ap. Gellium* l. 14. c. 1. Sextus Empiricus *adversus astrologos*.
Augustin. *de civ. l. 5. c. 1. &c.* ^b Exod. 8. 26. Gen. 41. 8. ^c Lev. 19. 26. Deut. 18. 10, 14.

upon any action, which astrologers pretend to find out by the positions of the stars and planets. The sacred historians tell us, that, notwithstanding these prohibitions, the Jews frequently fell into the practice of the former of these abominations, and we have no reason to believe they did at those times keep clear of the latter: it is expressly recorded of *Manasseh* that in the beginning of his reign he was guilty of both, that he *worshipped the host of heaven, and observed times*^a; and we may conclude the same of his people, from what we are told in general terms, that *he made Judah and the inhabitants of Jerusalem to do worse than the heathen*^b. There is such a connection between star-worship and star-divination, that the latter of these has with great probability been thought to owe its origin to the former.

It has been observed by some, that the Jews were never guilty of idolatry after the Babylonish captivity: but it is certain that in after times they gave into the belief that the stars and planets have an influence upon the tempers and actions of men: to this purpose there are several passages in the *talmud*, wherein it is said what will be the disposition and circumstances of life of a man born under such or such a planet^c: and though the wisest of them decry such notions, as indeed wise men of all times and places have done, yet we find one of their Rabbins^d wrote a treatise of judicial astrology, and another^e foretold the coming of their Messiah would be A. D. 1464, because then, as he pretended to find by his calculations, the heavenly bodies would be in the same state as they were at the time of the giving of the law. To mention no more, their attachment to this superstition sufficiently appears by a saying common among them to a proverb, *that every thing depends upon the stars*^f.

As to the *primitive Christians*, judicial astrology is by good interpreters thought to be one of those curious arts whereof some of the first converts shewed their detestation, by burning their books in great numbers, and to a considerable value^g: however that be, it is condemned by some of the most ancient fathers^h: afterwards, when the Christian religion had obtained the support of the secular power, we find astrologers, who had been banished Rome by some heathen Emperorsⁱ, ranked in the same class with apostates, sacrificers to idols, magicians, enchanters and dealers in witchcraft; and together with them threatened by the canons of the church with excommunication^k: and by the imperial laws with death^l. This denunciation, we may suppose, for some time at least, very much discouraged the practice it was levelled at; but in

a 2 Kings 21. b 2 Chron. 33. 9. c Buxtorf. *florileg. Hebraic. tit. Sydera.* d Aben Ezra.
e Abraham dictus Princeps apud Picum disput. in astrolog. l. 5. c. 12. f Buxtorf. *ubi supra.* g Acts 19. 19.
h Tertullian. *de idololatr.* 9. Bardefanes & Origenes ap. Euseb. *prap.* l. 6. i Sueton. in Tiber. & Vitell.
k Concil. Laodic. can. 36. v. Photii *nomocan.* l. 9. tit. 18. *de maleficis et mathematicis.*

the succeeding ages of barbarism and ignorance, and especially after the Europeans became acquainted with the learning of the Arabians, astrology prevailed so much that in the 14th century that great genius *Picus* Earl of Mirandola wrote against it, and that in a very masterly manner; exposing, with great learning, acuteness and strength of reasoning, the vanity of all the rules of that pretended art: the astrologers published answers to *Picus*: some of them boast, as a full evidence of the truth of their art, that the death of that incomparable person happened according to their predictions; one of them^a is said to have foretold that he should dye in the 33d year of his age: another^b tells us three astrologers had pronounced he would not live to six and thirty, and that this provoked him to write against them: the truth is this, *Picus* dyed in his 32d year; now if they really came thus near the fact as is pretended, and those predictions were not trumped up after the event, which is often the case, it did not require any great sagacity to guess that so extraordinary a person as *Picus*, of so much fire and so great application, would be short lived: perhaps, if they had found he was likely to prove them false prophets, he might have been in some danger of having his life shortened by them; this is no more than what the famous *Cardan* is said to have practised upon himself, who having, calculated his own nativity, and found in what year of his age he should die, starved himself to death, to verify the prediction^c.

That astrologer pretends to out do all that went before him in calculating nativities^d; he tells us he bestowed an hundred hours upon that of our *Edward* the 6th, from which he foretold several sicknesses which would attack him in the 34th and 55th year of his age; whereas that hopeful prince did not outlive his sixteenth year. It is pleasant enough to see how *Cardan* endeavours to bring himself off after this event: he says he will not, like some other astrologers, pretend that he knew what would happen, and kept it to himself only out of fear; but that he had omitted something in his calculation, which, if he had gone through it, as he might have done in half an hour more, would have shewed him that the King would be in great danger of death in his 16th year: that he looked upon this omission as a piece of good fortune to himself, on account of the great danger such a prediction might have brought him into, when he was at the English court: and he mentions two astrologers to whom foretelling the untimely death of princes had pro-

^a Bellantius ap. Gassendum de syderum effectibus prope finem.

^b Gauricus, v. *Elogia Pici ejus operib. prefixa.*

^c Gassendus ubi supra.

^d comment. in Ptolem. et in lib. geniturar.

Cardan pretended to calculate the nativity of *Jesus Christ*, and to deduce from thence the nature and duration of the Christian religion: others have been guilty of the like presumption: see *Bayle's dict. art. Cardan.*

ved fatal. This was not the only time *Cardan* was out in his predictions: I believe it would not do him much wrong, to apply to him what he says himself of the generality of astrologers, that *we should be in the right to expect events just contrary to what they foretell*. In the last century judicial astrology was again attackt by the learned *Gassendus*: this was no unnecessary work; for, notwithstanding what *Picus* had written, it continued to be in great vogue, not only in Italy, where it was patronized by Popes, Cardinals, and many of the petty Princes; but in other parts of Europe also.

But, how vain soever their pretensions be who would persuade us they can read future events in the stars, astronomy still deserves to be cultivated with the greatest application, as it is one of the noblest branches of natural philosophy; whether we consider the dignity of the subject, the certainty which we are able to arrive at in many parts of it, or its usefulness in civil life.

As to the first of these, the astronomer has for the subject of his speculations the whole universe of material beings: he considers the nature of matter in general, and enquires by what laws the several parts of matter act upon one another: but his thoughts are more particularly employed about those great bodies that compose the visible system of the world, which are in common speech comprehended under the appellation of *the sun moon and stars*. He finds the magnitude of these to be vastly greater than is commonly imagined: he is able to demonstrate, that very few of them are so small in bulk as the earth we live upon; and that much the greater part of them far exceed it in dimensions: that the sun appears so small as he does, by reason of his great distance from us; whereas in reality he is in magnitude equal to a million of our earths: he discovers that there are several other earths, some of them much larger than ours, which receive heat and light from the sun, are carried round him with prodigious velocities, and may probably be inhabited by various creatures both rational and irrational: he knows that the stars which seem to be so thick sown in the firmament are at immense distances from one another, as well as from us: he has proofs to shew, that, how small soever they appear to us, they are really very large bodies, many of them not inferior to the sun in magnitude; and that there is therefore reason to believe they may serve for the like uses, of giving light and heat to habitable earths with which they are environed: his glasses shew him a prodigious number of stars which by reason of their vast distances from us are invisible to the unassisted eye: and, the better his glasses are, the greater is the number of stars thus discovered: this makes it reasonable to conclude that there is still an innumerable multitude of them diffused through the immensity of space, beyond the reach of any glasses that can ever be made. How magnificent

cent an idea does this give us of the universe, in comparison of what it appears to an ordinary spectator, who looks upon the stars only as groupes of glittering spangles fixed by way of ornament in the vaulted roof of heaven, or as so many little lamps lighted up there, for no other use than to send a faint glimmering upon the earth, of less service to the inhabitants thereof than the light which the moon alone furnishes them with?

The pride of man is apt to make him fancy, that all things were made for his use: he has indeed a dominion given him over those works of God which are within the limits of the earth and sea^a, but it reaches no farther: the Psalmist, when he contemplated the heaven, and the luminaries thereof, the sun, moon, and stars, the handy work of the great Creator of all things, which he has disposed and set in order in so much beauty and perfection, was so far from thinking that all these glorious beings were created for the service of man only; that he is humbled into an admiration of the great condescension of the author of such wonderful works, that he should vouchsafe to cast an eye of regard upon man, and make him the object of his favour^b.

It is an observation of a philosopher^c that mathematical sciences have a tendency to purify the soul: the active principle within us must have some employment; if it be taken up with abstruse speculations, it will be less attacht to sensual pleasures: but, if we go no farther than that, we may fall very short of acting up to the dignity of a reasonable nature: in order to this, we must carry on our contemplations of the frame and constitution of the universe to their proper mark, so as, from beholding the wonders of the creation, to be brought to adore the wisdom power and goodness of the Creator: there is indeed no part of the creation which does not display these attributes to an attentive mind; though *an unwise man doth not well consider this, and a fool doth not understand it: but the heavens, in a more eminent manner, declare the glory of God, and the firmament sheweth his handy-work*^d: in this divine book, the most unlettered may find enough to excite their admiration, though it will display the greatest wonders to those who peruse it with the closest attention: but then we must take care always to approach it, as we should the other book of God, the holy scriptures, with reverence and humility; not having too high an opinion of our own abilities, as if we could with the line of human reason fathom all the depths of the divine counsels; *but thinking soberly as we ought to think*: for want of these dispositions, some men read the scriptures, and become teachers of new doctrines, departing from the simplicity of the primitive faith; others philosophise about the visible creation, till they forget the invisible creator; till they become materi-

^a Psal. 8. 6, 7.^b Psal. 8. 4.^c Hierocles in *aur. carm.* Pythag.^d Psal. 19. 1.

alists or epicureans; and fancy they can account for the formation and preservation of this beautiful frame of the universe, by matter and motion: or, at best, only consider it as a curious machine, without ever lifting up their thoughts to the adorable author and Lord of all things.

As to the *certainty* of this noble science, there are many things therein which, though to an ordinary understanding they seem incredible, or beyond the reach of human skill to know, are capable of strict demonstration: when we are speaking of the vast magnitude, the immense distance of some of the heavenly bodies, and the prodigious velocity of their motions, a man unacquainted with the methods by which we arrive at the knowledge of those matters is apt to think it all conjecture; whereas we go a great way in these speculations by the infallible steps of arithmetic and geometry.

The ignorant vulgar may think it a trifling and unprofitable employment, for a man to spend his time in enquiring into the nature and motions of the stars: but *the usefulness of astronomy* will quickly appear, if we consider what a dependance several other arts and sciences have upon it: without astronomy, *chronology* and *geography* would be more imperfect than they are; and, in consequence of that, *history* would be liable to greater uncertainty than it is. Without astronomy, we should be so far from being able to measure time so accurately as we now do, by the help of sundials, clocks, and watches; that we could not so much as divide the year into months, or know the length of the days and nights, or mark out the seasons exactly enough even for the purposes of agriculture, and other uses of common life. Without astronomy, all intercourse and commerce with distant nations must cease; the sailor could neither know the situation of the place to which he is bound, nor be able to find the way to it. In short, it is to this science that we owe the first attempt to discover America, as well as the success of that attempt: and it is to the same that we are beholden for the means of sailing thither with safety and certainty, to fetch home the riches and rarities of that new world.

It is now time to give the reader *some account of the following treatise*, and in what manner I have handled the subject before me: my proposals promised to make the study of astronomy easy to persons unacquainted with mathematics; this, I hope, it will be thought is performed, if it be made much easier than was ever done before: in order to this, I have in the introduction explained such mathematical terms as must be made use of, and set down several propositions to which reference is sometimes made in the body of the work: some of these are very necessary to be previously known, and are easy enough; others, which are more difficult, and are inserted either to make the introduction more methodical, or for the sake of those who have

have a mind to look farther into the subject, are distinguished from the rest by a full point placed before them, as may be seen page 12; 14, 15, &c, that they may be passed over by such readers as desire only to have a general view of astronomy: my intention at first was to have distinguished the more difficult parts from those easy to be understood, all along throughout the work; but, upon considering the great difference there must needs be in the capacities and attainments of my readers, I thought that improper: only I have in some places thrown the most abstruse mathematical demonstrations into *the remarks*, where they may be passed by without breaking the connection: I must however take notice, that the remarks are not all of this sort; for some of them are philological, as those upon the names and characters of the sun and planets, book 2 chap. 3; and those upon the signs and their characters, book 2 c. 6: others are explanations of or additions to something that went before, as in book 2 c. 14.

As for those readers who are the least prepared for studies of this sort, I would not have them discouraged by seeing schemes of lines and circles, and some of them of an appearance a little complicated; for they will often find them to be no more than pictures or perspective views of the heaven, or of the routes which the heavenly bodies are supposed to take, or of the tracks wherein they appear to us to move. The line which an arrow or a bullet shot through the air describes may be drawn upon paper, though neither of them leaves any visible traces behind it; in the same manner, the astronomer represents the motions of the heavenly bodies: and, in order to this, he is not always confined to the earth, and the prospects which he has from thence; but, one while, he supposes himself to be carried to the sun, and considers in what manner our earth and the planets would from that point of sight appear to go round him: at another time, he imagines himself to be an inhabitant of one of the planets, and from thence to view the sun, our earth and the rest of the planets: and sometimes, when he has beheld the vast concave of the heaven, adorned with numberless stars, his imagination transports him out of the confines of this visible system, to some point at an immense distance from it, from whence he takes a view of the convex of this imaginary sphere: *the common celestial globe* shews the heaven with the stars in their proper situations in this view: *the glass sphere*, of which I have given a picture for the frontispiece, and a description page 71, has a representation of the same sort, on its outside; and the inner concave surface of it shews how the starry heaven by which we are surrounded appears to us. This machine, which was invented by me above 20 years ago, as is well known to a great number of people, has since been imperfectly copied by several persons, and by some

of them, ignorantly enough, called the sphere of *Archimedes*: it seems they had heard of the glass sphere of that great mathematician, but had not met with those authors who give an account of it; if they had, they would have found it very different from mine. I have sometimes considered the heaven as a sphere of glass, that so any circles conceived to be drawn upon it might be exhibited entire; this will be found a great help to the imagination.

It will not be amiss to inform those who have not seen my *astronomical apparatus* at Pembroke Hall in the University of Cambridge, that I have there a large hoop about 20 feet in diameter, and 3 in breadth, whereon are delineated the constellations of the zodiac, with the ecliptic and such other circles as I thought proper: fitting in the midst of this I have the same view of things as if I was looking at so much of the starry heaven it self. If a sphere of these dimensions were made to turn round a stage whereon the spectators were placed, with a *planetarium* or *Orrery* in the center; most of the appearances in astronomy might be exhibited thereby in a more easy and familiar manner than has ever yet been done by schemes or mechanism. I should have been glad to have been enabled to erect such a fabric, for the service of the younger part of the university whereof I have been many years a member; but have no great reason to expect to be assisted and encouraged in such an undertaking: however I intend, if I have time, to leave behind me a model of such a machine; though I think the hint I have here given is enough to direct any ingenious person who understands astronomy and mechanicks, how to execute that design. What I have already done this way has helped to furnish my book with several schemes which will be found entirely new. I have in some of my figures trespassed upon the rules of perspective, by shewing a little more than the eye takes in at one view; this is done but seldom, only when I thought I could thereby convey a more perfect idea of what I intended to represent. I have quoted authors, sometimes as vouchers of facts, sometimes to shew to whom I am beholden, and sometimes for the sake of the youth of the Universities, whom I had often in my thoughts, to let them know where they may find the subjects treated of more at large; though I have generally given the substance of what has been said by former writers, where I thought it worth the while: such of my notes are in latin, as I thought would be of no use to an english reader; for whose sake I have generally quoted those authors in english which I knew were translated into that language, though I had them before me in the originals.

As my chief design was to make the knowledge of the heavenly bodies as easy as the nature of the thing will admit, I have taken all the care I was able to dispose the several parts of my work in such a manner, that what comes

comes first in order prepares the way for what is to follow; the want of which method is often an occasion of great obscurity in books that treat of any science. In the style I have studied conciseness, as far as is consistent with perspicuity. The paragraphs or articles which have numbers prefixt to them I call *sections*, and refer to them by the character §; though that mark as well as the word section is made use of by some authors to signify a larger division. A short view of the particulars treated of may be seen, by running over the heads of the chapters, and the words printed in *italic*, or by the table of *contents* at the end of this volume; I have made it very full, and marked the most difficult parts with an obelisk or dagger.

Since the *addenda* were printed off, I have met with a farther confirmation of the *change of the obliquity of the ecliptic* mentioned p. 274 & 356: it is from the observations of the Chinese astronomers, who are said, in the year 66 before Christ, to have supposed the obliquity of the ecliptic 24° ; and in the year 1280 after Christ, to have found it $23^{\circ} 33' 40'' 17'''$ or $18'''$. v. *Observations mathematiques & astronomiques &c tirees des anciens livres Chinoises par Souciet, Vol. 2. & aët. eruditor. Lips. mens. August. Ann. 1732.*

I ought now to make some apology to my subscribers for my long delay: this, among other reasons which it is not worth while to trouble the public with, has been owing in some measure to my enlarging my first plan: so that I hope I shall have their pardon, when they find I have given them a good deal more than I promised in my proposals; as *the principles of navigation, the use of maps and globes*, and some other particulars which I thought would make the work more entertaining or useful; though no body would have had any reason to complain of me if I had omitted them: but an author that intends to content his readers must endeavour in the first place to please himself, it is in writing as in building; we are often drawn in farther than we at first intended: this has swelled my book into two volumes, and obliges me, contrary to my first design, to publish only the first at present: the public may be assured the second shall follow with the utmost expedition I am able; great part of it being already printed in books I use to put into the hands of those who attended the course of lectures in astronomy, which, for above 20 years I have been constantly reading in this University.



ERRATA.

Page 11. line 2. for AF read AE. p. 12. l. 5. f. DC r. CE. p. 19. l. 15. f. 1008 r. 1034. *ibid.* l. 19. f. 1016064 r. 1069156. f. 1008 r. 1034. p. 19. l. 1. f. twice r. four times. p. 21. l. 20. f. ACE r. ACB. p. 22. l. 22. f. versed r. versed sine. *ibid.* l. 35. f. AC, is the versed sine r. AR, is the versed sine. p. 23. l. 11. f. BT r. BL. *ibid.* l. 15. f. DV r. BF. p. 31. l. 8. f. the center of the quadrant r. his eye. *ibid.* l. 9. f. it r. the quadrant. p. 37. in the margin f. 29 r. 92. p. 42. l. 33. f. 197474980 r. 197819550. p. 45. l. 20. f. DR. L. *ibid.* l. 23. f. 9. r. 1. *ibid.* l. 27. f. whether it be thinner, or more dense than the first r. if it be thinner than the first. *ibid.* l. 30. f. different r. thinner p. 48. in the margin dele 93 & 94. p. 49. l. 25. f. DCE r. FEG. *ibid.* l. 27. f. CR. E. p. 56. l. 26. for ab read na. for bc in one to bc read bc in one to mn. *ibid.* l. 27. for cd to cd read cd to ml. p. 57. l. 27. for g to f read g to b. for f to e read b to i. *ibid.* l. 28. for e to d read i to l. p. 59. l. 34. dele from g to b. *ibid.* l. ult. & in the margin f. 151 r. 152. p. 60. l. 2. dele na & g b. *ibid.* l. 4. insert g b, before h i, & na, after mn. *ibid.* l. 21. f. GR. I. p. 62. l. penult. ult. f. on the east side of Spain r. round the cape of Good-hope. p. 63. l. 24. f. semidiameter r. diameter. *ibid.* l. 25. f. above r. about. *ibid.* l. 34. f. 273 r. 274. p. 64. l. 4. f. 274 r. 275. *ibid.* l. ult. f. 275 r. 276. p. 65. l. 18. f. AR. Z. p. 67. l. 2. for a read A. *ibid.* l. 17. f. over r. in the focus of. p. 72. l. 8. after epq; insert the plane of. p. 75. l. 21. for a read g. p. 76. l. 20. f. Pgs r. Pbs. *ibid.* l. 21. f. Pbs r. Pis. *ibid.* l. 23. f. res r. rfs. p. 81. l. 37. f. answer to 15" r. answer to 1'. f. 15 thirds r. 1 second. p. 84. l. 36. after London: insert the third column shews the same in degrees, minutes, &c: p. 87. l. 21. for eq read ao. p. 88. l. 3. f. rho r. rho. for rho read pio. p. 97. l. penult. f. about one degree r. a little more than one third of a degree. p. 98. l. 8. f. 12th of december r. 11th of june. p. 105. l. ult. f. several r. some. p. 109. l. 7. after vertical insert much. p. 110. l. 7, 8. f. moveable round its center, and with telescope lights r. with a telescope moveable round its center. *ibid.* l. 30. f. 60° 30' r. 62° 30'. *ibid.* l. 32. f. 40° 30' r. 42° 30'. p. 112. l. 34. f. COP r. CPO. *ibid.* l. penult. f. other side r. other sides. p. 114. l. 2. f. 213½ to 600; just the same as r. 600 to 209; a little different from what. p. 115. l. 9. f. elliptic r. ecliptic. *ibid.* l. ult. f. l. 2. r. l. 3. p. 116. l. ult. f. higher r. lower. p. 117. l. 19. f. § 419 r. § 412. p. 120. l. 27. f. 235. r. 435. p. 128. l. 27. f. 900 r. 9000. p. 131. l. 2. f. 5703 r. 57033. *ibid.* in the margin f. 58 r. 57. p. 134. l. 12. dele and then for every 10', from 1°, to 2°: p. 136. l. 18. f. 2094655 r. 20949655. p. 146. l. 37. f. London r. Naples. *ibid.* l. penult. f. Naples r. London. p. 150. l. 13, 16, 22. dele E. p. 151. l. 17. after rhumb insert except the east and west. p. 154. l. 18. f. § 258 r. § 257. p. 157. l. 15. f. chap. 17 r. chap. 16. p. 158. l. 7. & in the margin dele 89. *ibid.* l. 33. f. extream meridians r. bounding lines. *ibid.* l. 37. f. extream parallels r. bounding lines. p. 160. l. 19. f. 180 r. 120. p. 165. l. ult. f. Plin. nat. hist. l. 4. c. 10. r. d Plin. nat. hist. l. 4. c. 12. p. 182. l. penult. f. 368 r. 369. p. 190. l. 12. dele certainly. p. 199. l. 2. after sun, insert some of them. p. 204. l. 12. f. § 339 r. § 340 p. 205. l. 2. f. s r. ©. p. 218. l. 10. f. a planet r. mercury. *ibid.* l. 11. after orbit insert proxime. *ibid.* l. 12. after L: insert the greatest possible elongation of venus is when the visual line is a tangent to her orbit, and the earth is near her perihelion. p. 224. l. 28. f. 1608 r. 1708. p. 229. l. 12. f. Pr. D. p. 231. l. 19. for n read a. p. 232. l. 16. f. § 670 r. § 620. p. 233. l. 13. f. § 706 r. § 711. p. 236. l. 4. f. lemma 3 r. lemma 4. *ibid.* l. 20. f. retrograde to direct r. direct to retrograde. *ibid.* l. 24. f. direct to retrograde r. retrograde to direct. p. 237. l. 37. f. c. 16 r. c. 6. *ibid.* l. ult. f. c. 10 r. c. 14. p. 240. l. 22. f. at A, fig. 47 r. at E, fig. 48. *ibid.* in the margin. f. 47 r. 48. p. 268. l. 3. f. EQ r. AQ. *ibid.* l. 25. f. NAS r. NDS. *ibid.* l. 27. f. E r. Q. p. 276. l. 12. f. § 802 r. § 805. p. 279. l. 17. f. 1700 r. 1692. p. 288. l. 32. f. DA r. PB. *ibid.* l. 33. f. DB r. PA. p. 301. l. 5. after by and by: insert what is said here and in the remarks § 879 is true only of stars that are on the same side of the true pole as the apparent pole is when the star passes through the zenith: by the true pole I mean the center of the circle described by the earth's axis extended, see § 829. p. 312. l. 1. f. quadrature r. conjunction or opposition. *ibid.* l. 2. f. A or C r. B or D. p. 319. l. 7. f. ABCD r. ABCH. p. 321. l. 14. after than insert the north pole is.

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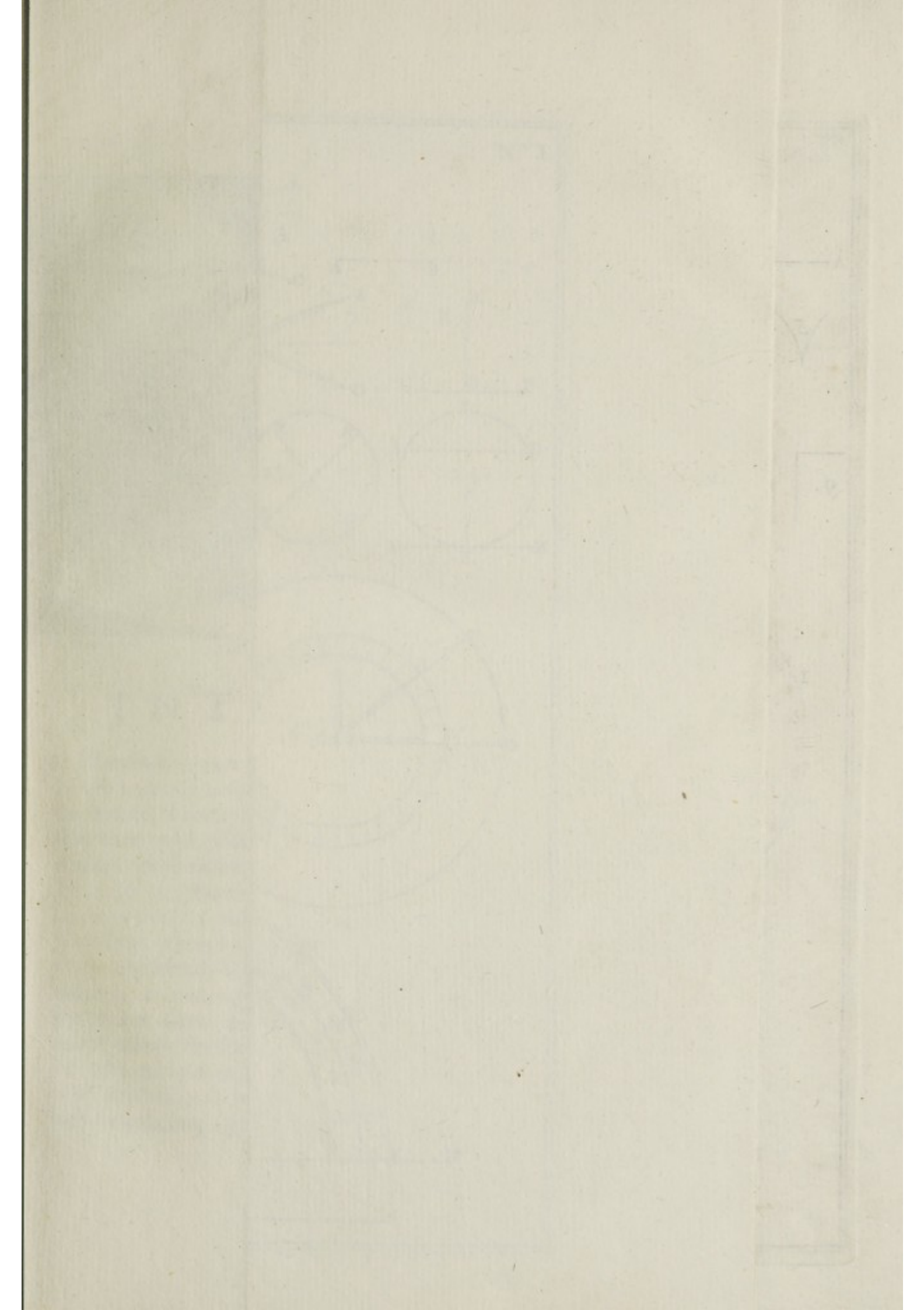
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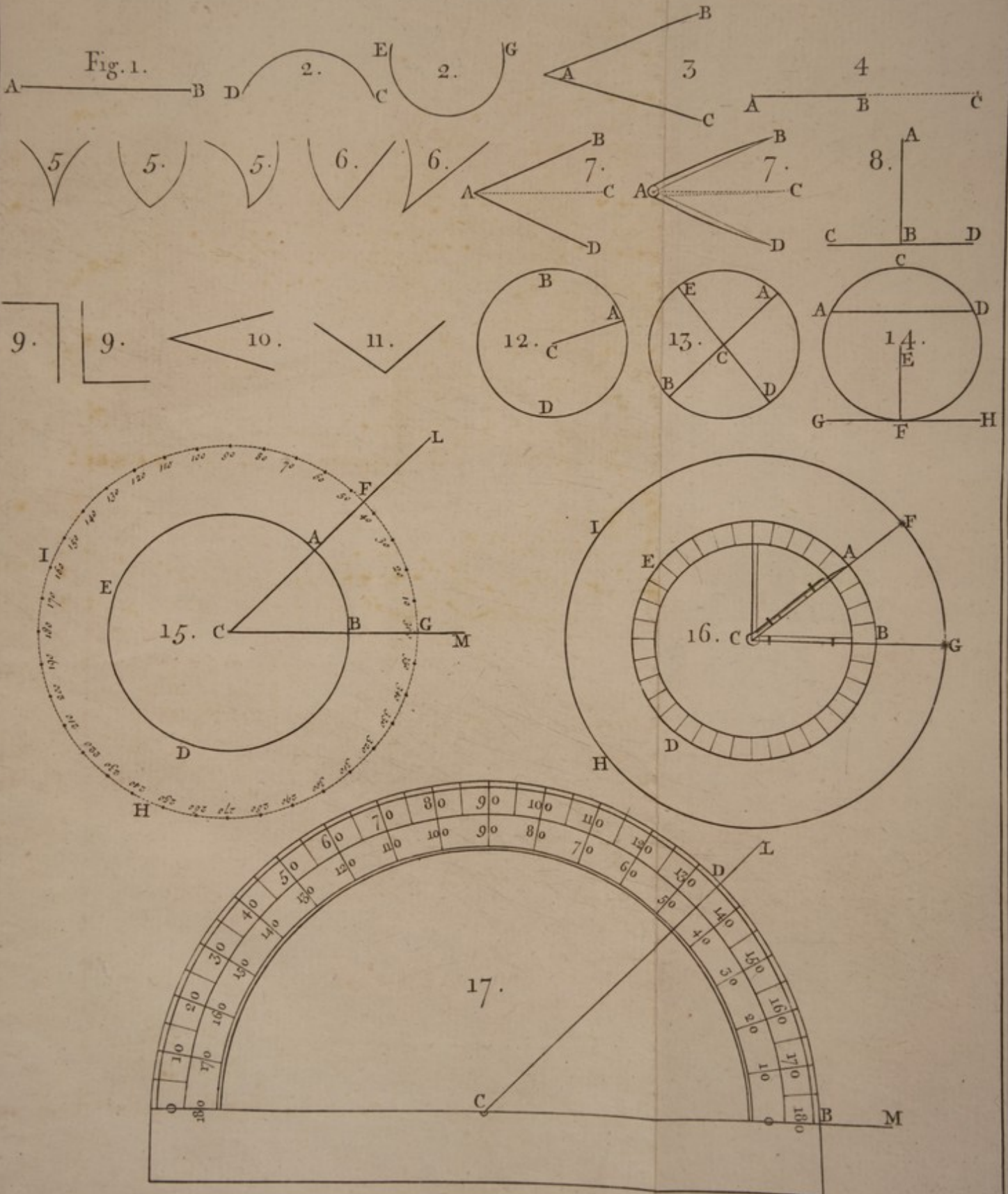
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G. Vander Gucht Sculp.

INTRODUCTION.

§. I. ASTRONOMY IS A SCIENCE WHICH TREATS OF THE HEAVENLY BODIES: by the heavenly bodies are meant the sun, moon, stars, planets, and comets: the nature of these it is the business of an astronomer to search into, to consider their order, distance, magnitude, shape, motion, light, and other particulars which relate to them; their different appearance to the inhabitants of the earth, at different times, and in different places, and to enquire into the causes thereof: in this enquiry, the earth also upon which we live, being the place from whence we view all the appearances in the heaven, must of necessity be considered, with regard to its shape and magnitude, its situation in the universe, its motion, or rest, and many other affections and properties. Among the ancient Greek and Latin writers astronomy and astrology were but different names for the same thing, but in modern language, an astrologer is one who pretends to be acquainted with the influences of the heavenly bodies upon our earth and its inhabitants, and would have it believed that, from their various positions, aspects, and configurations, he can foretell future events.

FIG. 2 It is impossible to treat of astronomy without borrowing, several terms from that part of the mathematics which is called geometry, such as a line, an angle, a circle, &c: these terms I shall, for the sake of those readers that are unacquainted with that science, take care to define and explain, before I make use of them.

3 The object about which mathematical reasoning is employed is *quantity*; that is, every thing concerning which we may enquire how great or how much it is: thus, magnitude, weight, number, duration, motion, and many other affections or properties which may be conceived to belong to material beings, may be considered as quantity.

4 *Magnitude* is that quantity which we conceive extended and divisible into parts. There are three sorts of magnitude, *viz.* a line, a surface, and a solid.

5 *A line* is that magnitude in which we consider extension in length only, without breadth or thickness: thus, if I imagin a line drawn from one star in the firmament to another, I consider nothing but the length of that line, or the distance between the two stars; it is true, if I draw a line upon paper, it must have some breadth to make it visible, and if I form the idea of a visible line in my mind, I must give that some breadth likewise, to make it the object of my imagination; nevertheless, if I would consider a line as mathematicians do, I must consider the length of it only, without any regard to its breadth.

6 *A line* is either *strait*, which is sometimes called a *right line*, or crooked, which is frequently called a *curve*; the line AB, fig. 1, is a strait line; the lines DC and EG, fig. 2, are curves.

7 A strait line may be imagined to be continued or extended both ways as far as we please; when no bounds are set to this extension, the line is said to be *infinite*, or more properly *indefinite*.

8 *A point* is understood to be without any extension at all: to make a point visible upon paper, I must give it some magnitude, and when I form the idea of a visible point in my imagination, I must give that also some magnitude, though the least that is possible, as when I think of the point of a needle; nevertheless mathematicians consider a point as without extension, without parts and indivisible: thus the ends of a line are points, that is, they are considered only as bounds which terminate the line, or as places to which the line is extended; if the ends of a line were conceived as extended, and making parts of the line, they would be then short lines and not points. By this time I suppose it appears that geometrical points and lines are objects of the understanding, not of the sense or imagination; and our ideas of them are formed by abstraction after this manner; from the idea of a visible point the mind takes away every thing but place; from the idea of a visible line the mind takes away every thing but extension in length only.

9 A line may be conceived to be *described* by the motion of a point: thus, I FIG. 1
may imagin the line AB, fig. 1, to be described by the motion of the point A, 1
from A to B.

10 A *surface* or *superficies* is that magnitude in which we consider extension in length and breadth only: thus, a piece of land is measured how long and how broad it is, without any regard to the depth of the earth: the shadow which is cast upon the ground in sunshine by an opaque body gives us a sensible image of a surface, for a shadow has no thickness or depth.

11 A surface is either plane or curve; the surface of a common looking-glass, or the smooth side of a marble slab made exactly flat and even, may represent a *plane surface*: a plane surface is usually called a *plane*.

12 A *curve surface* is, either *convex*, that is, swelling out, as the outside of an egg-shell, or *concave*, that is, hollow, as the inside of an egg-shell.

13 A surface may be conceived to be *described* by the motion of a line, not directly endways; a strait line describes a plane; a curve describes a curve surface: this will be illustrated when we come to consider the formation of a circle, square, or globe: if a line is moved directly endways, it will not describe a surface, but only a line.

14 A plane surface may be imagined to be extended every way as far as we please: when no bounds are set to this extension, the surface is said to be infinite, or rather indefinite.

15 Any surface terminated or enclosed every way is called a *figure*: a plane surface terminated or enclosed every way is called a *plane figure*: a plane figure may be formed by one curve, or by two, or by a curve and a strait line; but a plane surface cannot be enclosed by fewer strait lines than three.

16 The *area* of a plane figure is the quantity of surface contained therein.

17 An *angle* is the opening of two lines which touch one another in a point, and do not lie directly one against the other endways: thus, fig. 3, let the line AB touch the line AC in the point A, the opening between these two lines is called an angle. 3

Some define it thus, an angle is the inclination towards each other of two lines which meet in a point, &c: the difference between this definition and the former is that in this, we consider the relation between the lines, as they come towards the point where they touch, in the other, as they go from it.

It is mentioned in both definitions, that the lines, in order to form an angle, must not lie directly endways, because two strait lines which lie so and touch, will not form an angle, but make one strait line: thus, fig. 4, the lines AB and BC form one strait line, AC: two curves also, which lie directly one against the other endways and touch, will not make an angle, but form one curve; thus, fig. 13, the curves AE and EB make one curve, AEB. A 2 18 An 13

FIG. 18 An angle is rectilinear, curvilinear, or mixt. A *rectilinear angle* is formed by two strait lines, as in the third figure; a *curvilinear angle* by two curves, as in the fifth figure; a *mixt one* by a strait line and a curve, as in the sixth figure. I am at present treating of rectilinear angles.

19 The lines which form an angle, are called its *legs*: the point A, fig. 3, where they touch one another, is the *vertex* of the angle, or *the angular point*.

20 An angle is usually marked with three letters of the alphabet, or other marks, one placed at the vertex, the other two at the other ends of the legs; in expressing an angle, the letter or mark at the vertex is always named in the second place: thus, the angle, fig. 3, may be called BAC or CAB; by this means, several angles which touch in one point may be distinguished from one another; thus, fig. 7, there are three angles touch in the point A, viz. BAC, CAD, and BAD. Sometimes, when it can be done without the danger of occasioning any mistake or ambiguity, an angle is expressed by one letter or mark only: thus I may call the angle which is drawn in the third figure, the angle A.

21 The quantity of an angle does not depend upon the length of the lines which form it, but upon the wideness of their opening: imagin the two lines AB and AC, fig. 7, to be fastened together at A, as with a joint, so as to be moveable like the legs of a pair of compasses; the wider you open them asunder, the larger angle will they form, the nearer you bring them together, the less will the angle be between them: thus, fig. 7, the angle BAD is greater than the angle BAC. How the quantity of an angle is measured, will be shewn when a circle is treated of, § 30.

22 If one strait line AB, fig. 8, stands exactly upright upon another CD, between the ends of it, there will be formed at the point B two angles, viz. ABC and ABD, which will be equal, because the line AB is supposed to stand upright, and not lean one way more than another: such are called *right angles*, and the line AB is said to be *perpendicular* to CD. Two lines may also be perpendicular to each other, and form a right angle, when they touch at one end, as fig. 9.

23 Any angle less than a right one is called an *acute angle*, as fig. 10; any angle greater than a right one is called an *obtuse angle*, as fig. 11: and both acute and obtuse are called *oblique angles*.

24 If a strait line CA, fig. 12, be fixt at one end C, and the other end A be carried round, it will describe a figure called a *circle*: the curve ABD is the *circumference*, the point C at the immoveable end of the line is the *centre* of the circle; any strait line drawn through the centre and terminated at each end by the circumference is a *diameter* of the circle: thus, fig. 13, AB and DE are diameters. Every diameter divides a circle into two equal parts, which are called *semicircles*: any strait line drawn from the centre to the circumference, as CA, CB, CD, CE, is a *semidiameter* or *radius* of a circle.

25 All semidiameters of the same circle are equal; this is evident from the manner of drawing a circle; for the describing line AC, fig. 12, is supposed to continue of the same length, all the while it is carried round; and if a circle be drawn with a pair of compasses, the feet of the compasses are all the while kept at the same distance the one from the other. FIG. 12

26 A circle is properly the whole figure, § 24; however, when we name a circle, we frequently mean the circumference only; sometimes by a circle is meant only the surface contained within the circumference, but this is properly *the area of a circle*: the same is to be observed in speaking of a semicircle.

27 A piece of the circumference, is called *an arch*, or rather *an arc of a circle*, thus, fig. 14, the line AD divides the circle into two arcs, ACD and AFD: a straight line drawn from one end of an arc to the other is called *the chord* of that arc: thus the line AD is the chord of the arc ACD, or of the arc AFD. 14

28 A straight line is said to be *a tangent to a circle* when it only touches it in a point, in such a manner, that, if it were extended both ways from the point of contact, it would not enter into the circumference, or cut any part of it: thus, fig. 14, GH is a tangent to the circle ACDF: geometers demonstrate that a radius of a circle drawn to the point of contact is perpendicular to the tangent: thus, fig. 14, the radius EF is perpendicular to GH: *Eucl. l. 3. prop. 18.* 14

29 Every circle may be imagined to be divided into 360 equal parts, which are called *degrees* of a circle: every degree into 60 equal parts, which are called *minutes*, every minute into 60 *seconds*, every second into 60 *thirds*, &c: the mark for a degree is $^{\circ}$, for a minute $'$, for a second $''$, &c: forty four degrees seven minutes fifty two seconds are thus exprest, $44^{\circ} 7' 52''$.

30 The quantity of an angle is found, by taking its vertex for a centre, drawing a circle which shall cut the legs of the angle, and measuring the arc contained between the legs: thus, to measure the angle LCM, fig. 15, set one foot of a pair of compasses in the vertex C, and open them to any distance at pleasure, so that the other foot of the compasses may draw a circle cutting somewhere the two legs of the angle, as the circle ABDE does at the points A and B, let the circumference be divided into 360° , and observe how many degrees are contained in the arc AB, for of so many degrees is the angle said to be: in the present example the arc AB is 45° , therefore the angle LCM is an angle of 45° . 15

31 The quantity of an angle will be found the same, of what size soever the circle be drawn that measures it: thus, if instead of ABDE, which for distinction, may be called the *black circle*, the angle LCM be measured by a larger one, as the *pointed circle* FGHI, divided into 360° ; I say the arc FG will contain the same number of degrees as the arc AB does, and consequently give you the same measure of the angle LCM: for if you imagin the line LC to be carried round 15

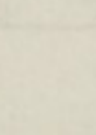
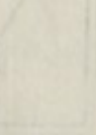
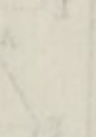
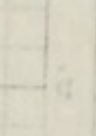
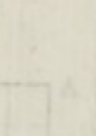
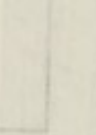
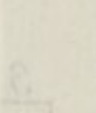
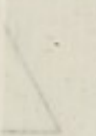
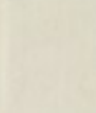
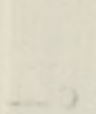
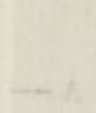
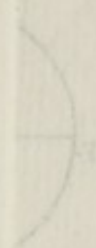
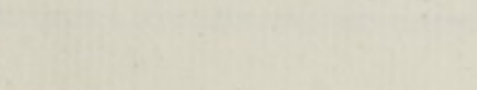
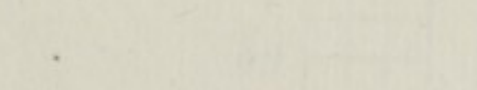
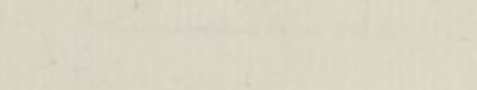
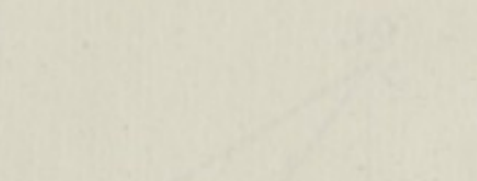
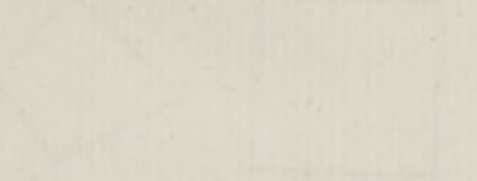
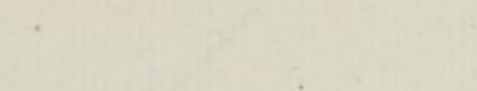
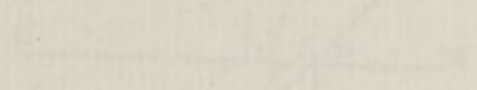
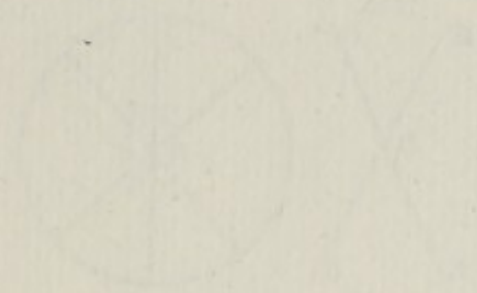
FIG. round the point *c*, according to the order of the letters *FGHI*, it is manifest
 15 that the point *F* will describe the whole pointed circle, in the same time that
 the point *A* describes the whole black circle: therefore when the line *LC* is car-
 ried any part of its round, as into the situation *MC*, the point *F* will have de-
 scribed just as great a part of the pointed circle, as the point *A* will have de-
 scribed of the black circle; that is, the arcs *FG* and *AB* shall each of them bear
 the same proportion to its respective circle: so that if *FG* is an eighth part of
 the pointed circle, *AB* will be an eighth part of the black circle: if one of these
 arcs be 45° , the other shall be likewise of the same number of degrees.

32 It is in consequence of this last proposition, that we are able with a small
 circle, or piece of a circle, of brass or wood, to measure arcs in those vast cir-
 16 cles which we imagin in the starry heaven it self: for let *ABDE*, fig. 16, be a
 circle of brass, whose circumference is divided into 360° , let *F* and *G* be two
 stars, whose distance from one another is to be measured: if the star *G* be view-
 ed through two sights placed in the line *CB*, the eye being at *c*, and at the same
 time the star *F* be viewed through two sights in a moveable ruler, whose edge
 lies upon the line *CA*, then will the number of degrees in the arc *AB* of the brass
 circle, which in the present example is 40° , shew the number of degrees in *FG*,
 an arc of a circle imagined to be drawn in the heaven through the stars *F* and
G; and the distance of those stars is said to be 40° .

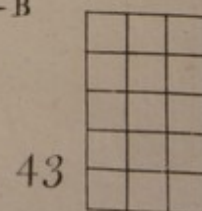
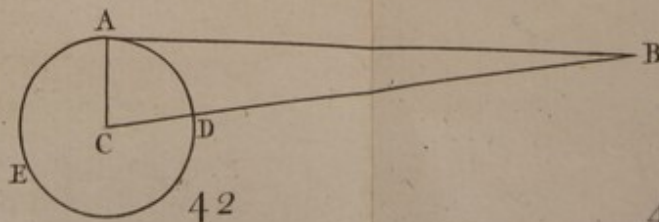
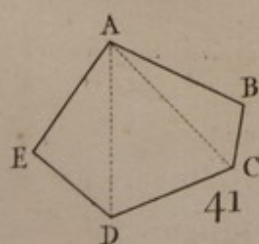
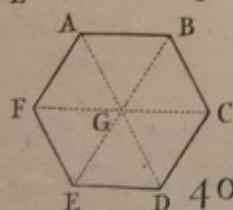
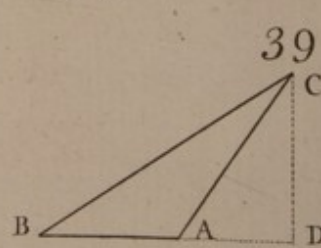
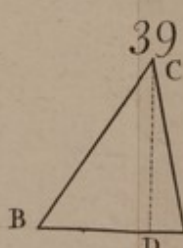
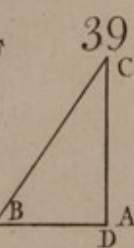
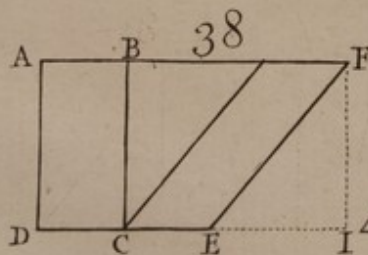
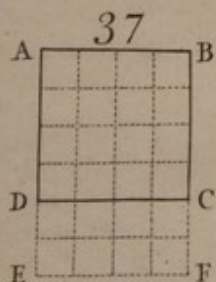
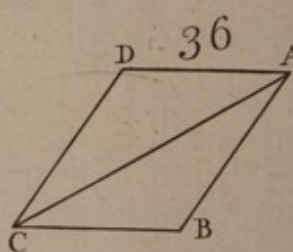
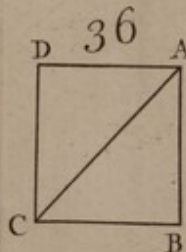
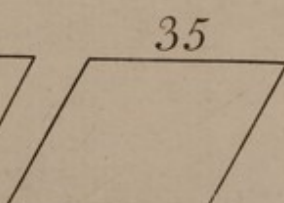
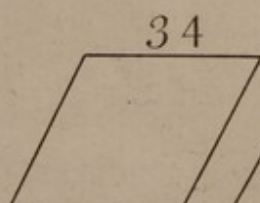
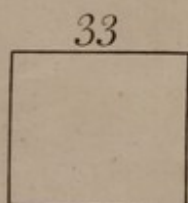
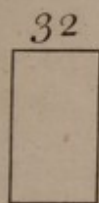
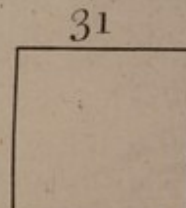
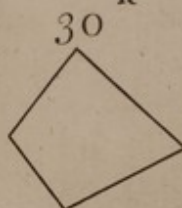
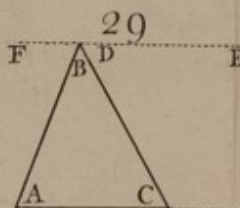
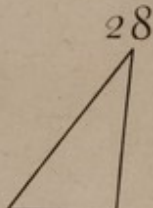
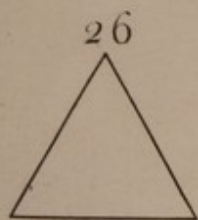
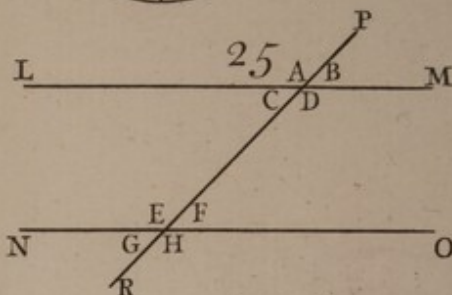
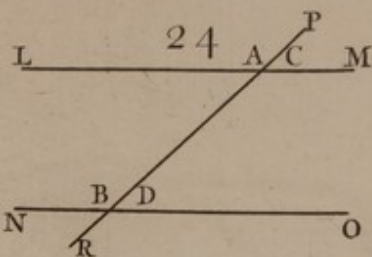
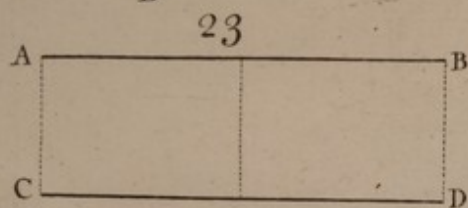
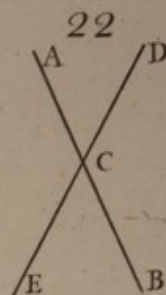
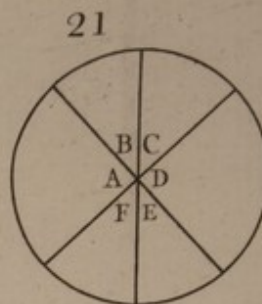
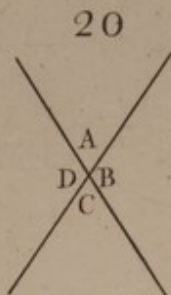
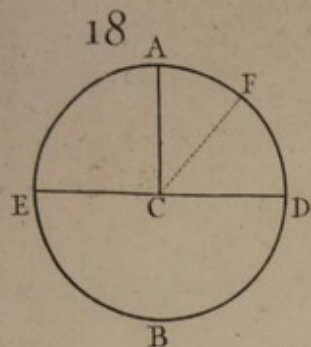
33 To save the trouble of dividing a circle into 360° , in order to measure
 the degrees of an angle, you may make use of an instrument called a *protractor*,
 which is a semicircle of silver or brass divided into degrees, and, if it be large
 enough to admit of such division, into halves and quarters of a degree: the se-
 venteenth figure gives the picture of the protractor, the manner of using it is
 17 as follows: if it be required to measure the angle *LCM*, fig. 17, lay the central
 point of the protractor upon *c*, the vertex of the angle, with the semidiameter
 of the protractor *CB* upon *CM*, one of the legs of the angle; the arc of the pro-
 tractor *DB* contained between the legs of the angle shews the number of de-
 grees the angle *LCM* contains, which in the present example is 45° .

34 By the same instrument may an angle be drawn of any number of de-
 17 grees required: thus, to lay down an angle of 45° ; draw a strait line at pleasure
CM, lay upon it the semidiameter of the protractor *CB*, so that its central point
 may fall upon one end of the line, as *c*, make a mark *D* with a pen, close to the
 circumference of the protractor, at the division of 45° , take off the protractor,
 and draw a line from *c* through *D*, as *CDL*: the angle *LCM* will be of 45° , as
 was required.

35 *An axiom* is a self evident proposition: such are these that follow;
 Axiom 1, The sum of all the parts is equal to the whole. 2 If equal quanti-
 ties



Introduction



ties be added to equal ones, the sums thereby produced will be equal. 3 If FIG.
 from equal quantities equal ones be taken, the remainders will be equal. 4 If
 any two quantities are equal to a third quantity, they are equal to one another.
 5 Those quantities which having a third quantity added to them or taken from
 them are equal, are equal themselves.

36 Those angles which are measured, or as they sometimes speak, *subtended*
 by equal arcs of the same circle, are equal: thus, if the arc AD, fig. 18, be equal 18
 to the arc AE, the angles ACD and ACE are equal; this is evident from the way
 of measuring angles, which is taught, § 30.

37 If one strait line AC falls upon another, ED, any where between the ends
 of it, fig. 18, there will be formed two angles ACD and ACE, which have one 18
 leg AC common to them both: such may be called *consequent angles*.

38 The measure of any two consequent angles ACD and ACE is a semicircle, 18
 or 180° ; for if from their common vertex C you draw a circle ADBE, cutting
 their legs in the points A, D, and E, then DE shall be a diameter, and EAD a se-
 micircle, by § 24; but EAD is the measure of the angles ACD and ACE; § 30.

39 Any two consequent angles ACD and ACE are equal to two right ones: for 18
 if AC be perpendicular to DE, then the angles ACD and ACE are right ones, by
 the definition of a right angle, § 22: if FC be drawn oblique to ED, then I say
 the oblique angles FCD and FCE are equal to the two right angles ACD and ACE
 by the foregoing proposition, because they have the same measure as they have,
viz. the semicircle EAD.

40 If two consequent angles be divided into a greater number of angles, by more
 lines drawn to the vertical point, fig. 19, the sum of all these angles A, B, C, D, is 19
 equal to two right angles; for a semicircle EFG measures them all, and it is still
 the same quantity of two right angles, divided into a greater number of parts.

41 If two strait lines cut one another, fig. 20, the four angles A, B, C, D, round 20
 the point of intersection are equal to four right angles: *demonstration*, A and B are
 equal to two right angles, because they are consequent, § 39; D and C are equal
 to two right angles, for the same reason, being also consequent; therefore the
 sum of A, B, C, D, is equal to four right angles; which was the thing to be proved.

42 Since a semicircle measures two right angles, § 38 and 39, a quarter of a
 circle 90° , or, as it is frequently called, a *quadrant*, is the measure of one right
 angle; a whole circle 360° is the measure of four right angles.

Corollary. If more than two strait lines cross one another in a point, fig. 21, 21
 all the angles A, B, C, D, E, F, formed round the point are equal to four right ones:
 for the measure of them all is a circle, as appears by the figure.

43 If two strait lines AB and DE, fig. 22, intersect one another, there will be 22
 formed round the point of intersection c, four angles, whereof any two oppo-
 site

FIG. fite to each other at the vertex are called vertical: in the figure before us, the
 22 two acute angles ACD and BCE considered together are called *vertical angles*, in like manner the two obtuse angles ACE and BCD are vertical.

22 44 All vertical angles are equal: in the figure last mentioned, the two acute angles are equal, the two obtuse angles are also equal: *demonstration*, ACD and ACE are equal to two right angles, being consequent, § 39; ACE and ECB are equal to two right angles, for the same reason: therefore, § 35 *axiom* 5, ACD and ECB are equal; which was to be proved.

23 45 Two lines as AB and CD , fig. 23: are said to be *parallel*, when they are in every part at equal distances from one another: the distance between parallel lines is measured by other lines drawn perpendicular to them, as the pointed lines are in this figure; if these pointed lines are all equal, the lines AB and CD are parallel: the opposite edges of a flat ruler, which is made throughout of the same breadth, will serve very well to represent parallel lines.

46 Parallel lines, to what length soever they be extended, will never meet, but continue still equidistant from each other.

24 47 If a strait line PR crosses two parallel lines LM and NO , fig. 24, it will have the same inclination to both; if it is perpendicular to one, it will be so to the other; if it crosses one obliquely, it will cross the other with the same obliquity: that is, the angle A will be equal to B , and the angle C to D ; this is evident from considering the intersecting line as inflexible, which therefore cannot bend from or towards one line more than the other, if they are parallel, as is supposed in the proposition.

25 48 If a strait line PR cuts two parallel lines LM and NO , fig. 25, it makes with them eight angles: four of which, A, B, H, G , are *external*; the other four C, D, E, F , are *internal*. The angles D and E considered together, are called *alternate*, as are also c and F . The angles E and A considered together, are called *opposite angles on the same side*, as are also B and F . The angles c and E considered together, are called *internal angles on the same side*, as also are D and F .

25 49 The alternate angles c and F , fig. 25, are equal; *demonstration*, c is equal to B , because they are vertical, § 44, B is equal to F , by § 47, therefore c is equal to F ; which was to be proved. In the same manner, D and E may be proved equal to one another, as being both equal to A .

25 50 The internal angles on the same side D and F are equal to two right ones: *demonstration*, F is equal to c by the preceding proposition, being alternate; c and D are equal to two right angles, being consequent, § 39; therefore by § 35, *axiom* 5, D and F are equal to two right angles; which was to be proved.

51 One proposition is called *the converse* of another, when after a conclusion is drawn from something supposed, in the converse proposition, that conclusion

clusion is first supposed, and then that which was before supposed, is drawn FIG. from it as a conclusion: for example, in this proposition, if two parallel lines are crossed by a third, the alternate angles will be equal; the supposition is, that the lines are parallel, the conclusion, that the angles are equal: the converse of this proposition is, if the alternate angles are equal, the lines are parallel; here the supposition is, the angles are equal, and the conclusion, that the lines are parallel. In geometry wherever the inseparable and incommunicable property is found, there is the thing itself; and every proposition about such properties infers the truth of its converse.

52 A *triangle* is a plane figure enclosed by three lines, which meet and form three angles. The lines which form a triangle are called its *sides*; if the sides are all straight lines, it is a *plane triangle*; if they are all curves, it is a *curvilinear* one; if they are of both kinds, it is a *mixt triangle*: we are at present treating of plane triangles: in every triangle an *angle* may be said to be *opposite* to a side, or a *side opposite* to an angle: thus fig. 29, the angle C is opposite to the side BA, 29 the angle B is opposite to the side CA, and the angle A to the side BC: in the same figure, the side BA is opposite to the angle C, the side BC is opposite to the angle A, and the side CA to the angle B.

53 A triangle may have all its sides and all its angles equal, as fig. 26; or 26 only two sides and two angles equal, as fig. 27; or all the sides and all the an- 27 gles may be different, as fig. 28. A *right-angled triangle* is that which has one 28 of its angles a right one: an *obtuse-angled triangle* is that which has one obtuse angle: an *acute-angled triangle* is that which has all its angles acute.

54 Sometimes we call one side of a triangle *the base*, and then the other two sides are called its *legs*: any side of a triangle may be considered as its base; but we commonly call that so which lies nearest to us, when we view the triangle: thus, fig. 29, AC is the base of the triangle ABC.

55 In every triangle ABC, fig. 29, the three angles A and B and C, taken to- 29 gether, are equal to two right ones: *demonstration*, if through the vertex of the angle B the line FE be drawn parallel to the base AC, the lines BA and BC may be considered as falling upon these two parallel lines; and then the angles A and ABE are equal to two right ones, being internal opposite on the same side, § 50: now the angle ABE contains the two angles B and D; therefore A and B and D are equal to two right ones, but D and C are equal, because alternate, § 49; therefore A and B and C added together are equal to two right angles; which was to be proved. *Eucl. l. 1. p. 32.* This proposition is said to have been invented by *Pythagoras*, and is brought by *Aristotle* and others as an example of a perfect demonstration: many corollaries or conclusions may be drawn from it, among which these that follow are the most useful.

FIG. *Cor.* 1, The sum of the angles of any one triangle is equal to the sum of the angles of any other triangle; *viz.* 180° , or two right angles. 2, The quantity or number of degrees, of any one angle of a triangle being given or known, the quantity of the other two angles may be known: thus, in the preceding figure, if the angle A be 70° , the sum of the angles B and C will be 110° : for 70° added to 110° is 180° . 3, The quantity of two angles of a triangle being given, 29 the quantity of the remaining angle may be known: thus, in fig. 29, if the sum of the angles B and C be 110° , the quantity of the angle A is 70° . 4. If two angles in one triangle are equal to two angles in another triangle, the remaining angles are equal. 5, In every right-angled triangle one acute angle is *the complement* of the other to 90° . 6, If one angle in a triangle be a right one, the other two angles are acute. 7, If any right-angled triangle has one acute angle equal to one of the acute angles of another right-angled triangle, those triangles are equiangular.

56 *Trigonometry* is the art of measuring triangles; to measure a triangle is to find out the quantity of every angle of it, and the length of every side: sometimes, by measuring a triangle we mean finding out the area of it, the way of doing this see below, § 64. There are in every triangle fix things chiefly to be considered, *viz.* three angles and three sides; trigonometry teaches us how we may, if some of these are given, find out all the rest: the corollaries immediately preceding point out to us some of the first steps to be taken in trigonometry; before any further progress therein is attempted, several things are to be premised; and first it is necessary to treat of *quadrilateral figures*, or such as have four sides.

57 If a quadrilateral figure has either its opposite sides or its opposite angles unequal, it is called a *trapezium*, fig. 30. If a quadrilateral figure has its opposite sides parallel, it is called a *parallelogram*, as figg. 31, 32, 33, 34, 35, 36. If a parallelogram has all its angles right ones and all its sides equal, it is called a *square*, fig. 31. If a parallelogram has all its angles right ones and only 31 its opposite sides equal, it is called a *rectangled parallelogram*, or more usually 32 a *rectangle*, figg. 32, 33. If a parallelogram has all its angles oblique and all 33 its sides equal, it is called a *rhombus*, fig. 34. If all the angles of a parallelogram 34 are oblique, and only the opposite sides equal, it is called a *rhomboid* fig. 35.

58 A parallelogram is named by setting down the four letters or other marks placed at the four corners of it, and sometimes by mentioning only two letters 36 or marks placed at two opposite corners; thus the 36th figure may be called the parallelogram ABCD, or the parallelogram AC, or BD.

59 *The generation* or formation of a *parallelogram* may be conceived in the following manner: let it be supposed that a right line AB, fig. 37, which may be

be called a *describent*, is carried or moved, so as to be all the while parallel to it- FIG.
 self in its first situation, along another line *AE*, which will be a *dirigent*; when 37 - AE
 the describent has moved from its situation *AB*, and is come into the situation
EF, the surface it has passed over or described is the parallelogram *ABFE*.

60 The mensuration of squares and rectangles is easily understood, from considering the manner in which they are generated: let it be supposed that the describent *AB* is, before its motion, divided into any number of equal parts, 37
 as 4, and the dirigent, which in order to generate a rectangle or square must stand at right angles to the describent, into as many of the like parts as it contains: for example, let the length of the describent be 4 inches, and the length of the dirigent 6; when the describent has moved one inch upon the dirigent, the surface described by it will be 4 square inches; when it has moved 2 inches upon the dirigent, it will have described 4 more, in all 8 square inches &c; when it has moved upon the dirigent a length equal to its own, viz. 4 inches, it will then have described a square *ABCD*, which contains an area 4 times 4, that is 16 square inches: let this motion of the describent be continued till it has moved upon the dirigent the whole 6 inches, and it will then have described a rectangle *ABFE*, whose area is six times 4, that is 24; square inches.

61 From hence it appears, that the way to find the area of any rectangle *AF*, 37
 is to measure two contiguous sides of it, as *AB* and *AE*, and see what number of feet, inches, or tenths of an inch, or any other known measure, each side contains, and to multiply the number of feet, inches, &c. of one side by the number of feet, inches, &c. of the other side: thus, if *AB* is 4 inches and *AE* 6, multiply those numbers 4 and 6 together, and the product 24 is the number of square inches contained in the rectangle *AF*: since all the sides of a square are equal, the area of a square is found by multiplying any one of its sides by itself: thus if a room is square, and has each side 12 feet long, the area of the floor is 12 times 12, that is 144 square feet.

62 Not only squares, and rectangles, but all other kinds of plane figures, as circles, triangles, &c. have the quantity of their areas expressed, by saying how many square feet, square inches, &c. they contain; the same way of measuring is also applicable to convex or concave surfaces, as well as plane ones; thus, I may find how many square feet &c. are contained in the surface of a mountain or valley, or even of the whole earth itself. I shall now shew the method of measuring the areas of such plane figures, as we may have occasion to consider in the following work; the demonstrations, which are here omitted, may be seen in the common books of geometry.

63 All parallelograms, whether oblique-angled ones or rectangles, having equal bases and equal perpendicular heights are equal: *Eucl.* 1. 35. thus, fig. 38, 38

FIG. the rectangle AC and the oblique parallelogram CF are equal, if their bases DC³⁸ and CE are equal, and if they have the same perpendicular height AD and FI, so that both of them can stand between the same parallel lines AF and DI: from hence it follows, that the way to find the area of any oblique-angled parallelogram as CF, fig. 38, is this; multiply the base ~~DE~~ by the perpendicular height FI, and the product is the area.

64 A strait line drawn from any corner of a parallelogram to the opposite one is called a *diagonal*: thus, fig. 36, AC is a diagonal. Every diagonal divides a parallelogram into two equal triangles: thus, fig. 36, ABC and ADC are equal, as is easy to demonstrate, *Eucl. l. 1. p. 34*: and therefore, every triangle is half a parallelogram which has the same base and the same perpendicular height: therefore, to find the area of any triangle BAC, fig. 39, multiply the base AB by the perpendicular height CD, and half the product is the area.

65 A *polygon* signifies a plane figure with many angles: no figure is called by that name which has not more than four angles: many polygons are called by particular names, expressing the number of angles they contain, which is always the same with the number of their sides; thus if a polygon has five angles or sides, it is called a *pentagon*: if six, an *hexagon*: if seven, an *heptagon*: if eight, an *octagon*: &c. these are all called *regular*, when their angles and sides are all equal, and are understood to be regular when nothing is said to the contrary. Any polygon, fig. 40, may be divided into as many triangles as it has sides, if any where within the polygon, you take a point G, and from thence draw a line to every angle A, B, C, D, E, F: the area of any polygon is the sum of the areas of the several triangles into which it is thus divided.

66 To find the area of any irregular plane figure ABCDE, fig. 41, let it be divided into triangles by drawing lines from one angle to another, as is here done, find the area of each of those triangles, by § 64, the sum of them is the area of the whole figure.

67 To find the area of a circle is the same thing as to find the quantity of its surface in square measure, which is commonly called, to *square the circle*; this may be done, if we can find a square, triangle, or any other rectilinear plane figure, equal to the circle; because the area of any such figure may be found, by § 64 and 66; for this purpose, the following proposition is demonstrated by geometers,^a

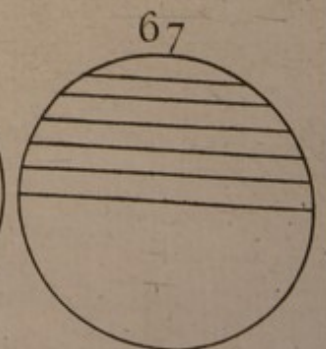
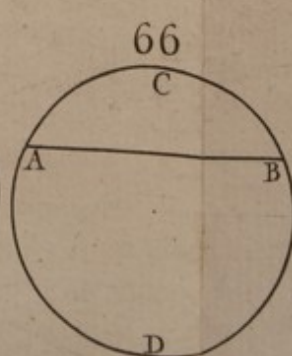
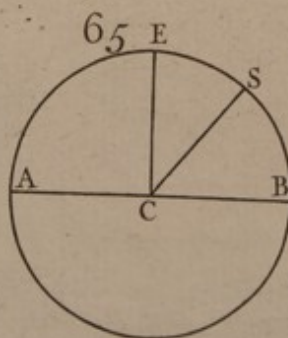
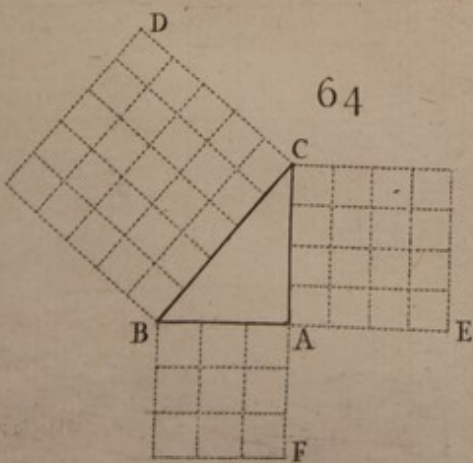
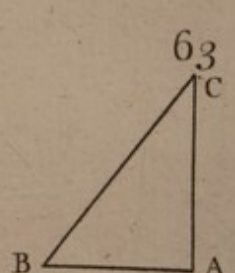
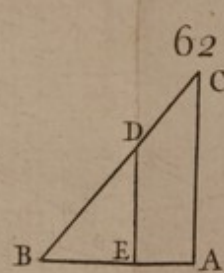
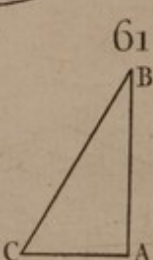
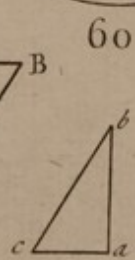
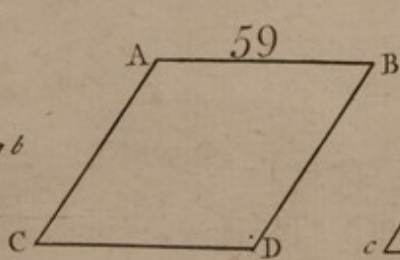
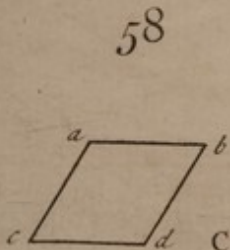
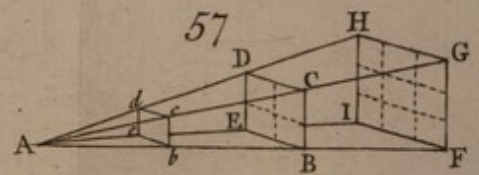
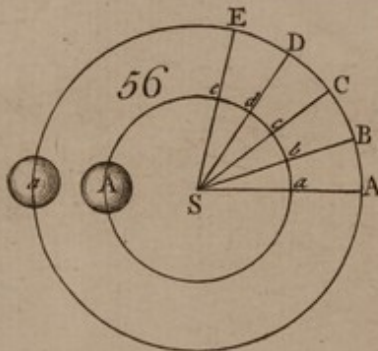
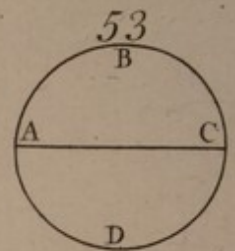
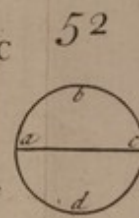
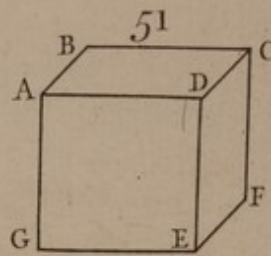
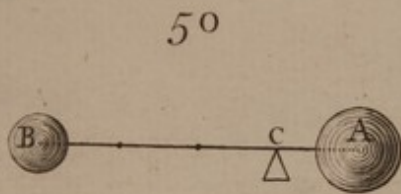
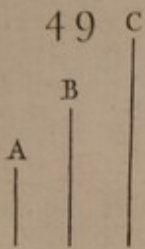
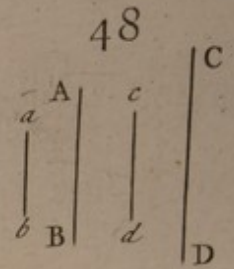
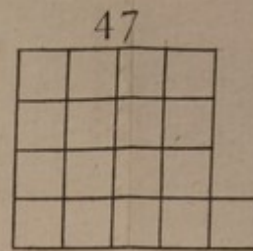
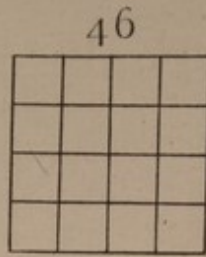
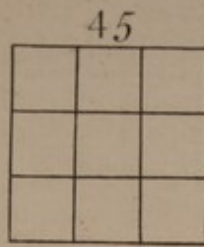
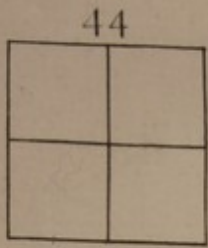
42 The area of any circle ADE, fig. 42, is equal to the area of a right-angled triangle BAC, which has one leg AC equal to the radius, and the other leg AB equal to the circumference of the circle.

Scholium, the only difficulty here is to find a strait line equal to the circumference, the radius being given, or, which is the same thing, to find the ratio between the diameter and circumference; this can never be done exactly, though

^a Tacquet's *Geometria practica* l. 2. cap. 12. Pardie *Geometr. l. 4. p. 31.*



Introduction



we may come as near exactness as we please: we say in gross, and inaccurately, that the diameter of a circle is to its circumference as 1 is to 3; so that if the diameter of a circle is one foot, the circumference of it will be about 3 feet: but these numbers are fit only for such uses as do not require much exactness. *Archimedes* found the diameter of a circle to be to the circumference as 7 to 22; this is exact enough for many uses: *Ludolf van Ceulen*, by a laborious calculation, found the ratio between the diameter of a circle and its circumference, in numbers of 36 places each; this was looked upon to be so considerable a performance, that the numbers stand engraved upon his tombstone, to perpetuate the memory thereof: His numbers are these

if the *diameter* be 100000, 000000, 000000, 000000, 000000, 000000,
the *circumference* will be 314159, 265358, 979323, 846264, 338327, 950288,
in these numbers, that by which the circumference is expressed is too small;
and yet if instead of (8), the first figure upon the right-hand, we were to put
the figure (9), the number expressing the circumference would then be too
great; with either of these figures, it is so near the truth, that the diameter of the
earth being given, we might from thence compute the number of sands equal
to the solid content of the whole earth, so near as not to differ one grain of sand
from the truth. *Van Ceulen's* numbers have been doubled by that industrious
arithmetician *Mr. Abraham Sharp*; this has carried the affair to an accuracy
almost beyond belief; sufficient, if the diameter of the sphere of the fixt stars
were given, in grains of sand to compute the number of grains of sand which
might be contained in the concavity thereof.

68 We shall be exact enough for most purposes, if we take these numbers only as far as the four last places on the left hand, and say the diameter of a circle is to its circumference, as 1000 to 3141: with these numbers, by the help of the golden rule, the diameter of a circle being given, the circumference may be found: or the circumference being given, the diameter may be found, very near the truth. See this whole affair treated of more at large, by *Tacquet*, in his *Geometria practica*, l. 2. c. 12, & *selecta theoremata ex Archimede*, prop. 6: see also *Sherwin's tables* p. 53.

Example 1, If the circumference of a circle be 138204 feet, how many feet is its diameter? the answer is 44000 feet; for by the golden rule,

$$3141 : \text{circumf. } 138204 \text{ feet} :: 1000 : \text{diam. } 44000 \text{ feet.}$$

Example 2, If the diameter of a circle be 2000 feet, how many feet is the circumference of it? the answer is 6282 feet; for

$$1000 : \text{diam. } 2000 \text{ feet} :: 3141 : \text{circumf. } 6282 \text{ feet.}$$

the diameter and circumference of a circle being thus found, by either of them being given, multiply half the diameter by half the circumference, the product is the area of the circle, by § 64 and 67.

FIG. 69 A *plane number* is that which may be produced by the multiplication of one number by another: thus, 6 is a plane number, because it may be produced by the multiplication of 3 by 2, for two times 3 is 6; again, 15 is a plane number, because it is the product of 5 multiplied by 3. Those numbers which multiplied by each other produce a plane number, are called *the sides of the plane*: thus 5 and 3 are the sides of the plane 15. If we suppose the units of which any plane number consists, to be represented by little squares, those squares may be placed in the form of a rectangle; thus, the number 15 may
 43 be represented by a rectangle, one of whose sides is 3, the other 5: fig. 43.

70 A *square number* is a plane number whose sides are equal; the product of any number multiplied by it self is therefore always a square number: thus, 4
 44 is a square number, because it may be produced by multiplying 2 by 2, fig. 44;
 45 again, let 3 be multiplied by 3, the product is 9, a square number, fig. 45; let 4
 46 be multiplied by it self, the product will be the square number 16, fig. 46, &c.
 The side of a square is called *the square root* of that number, thus 4 is the square root of 16. Every square number may be ranged in form of a square; and the converse of this is true, *viz.* that every number which can be ranged in the form of a square is a square number; see the three figures immediately preceding. We are taught in the common books of arithmetic how to find out the sides, or, as it is usually called, to extract the square root of any number given; if a number is a square, its root may be found exactly in whole numbers; thus the square root of 16 is 4: if a number is not exactly square as 17,
 47 fig. 47, its root cannot be found exactly in whole numbers, but instead thereof it is usual to take the root of the next square number below it, as 16, and make use of it with some fraction.

71 By comparing lines and numbers together, we see that a rectangle in geometry is the same to the lines by which it is generated, as the product in arithmetic is to the numbers from the multiplication whereof it arises: the square of a number is the product of that number multiplied by it self, and the square of a line is the square produced by the multiplication of the line into itself, that is, by imagining a describent to be carried along a dirigent equal to itself, as already has been explained § 60.

72 One kind of quantity is frequently expressed by another: numbers will serve to express all kinds of quantity; thus, if I compare two cannon balls together, and find the weight of one double to the weight of the other, I may express the weight of the heaviest ball by the number 8, and the weight of the other by the number 4: in the same manner may different quantities of motion, of heat, light, &c. be expressed by numbers: all kinds of quantity may be also expressed by lines; thus, I may express the time of one hour by a line of
 a deter-

a determinate length, and the time of two hours by a line twice as long as the former: different quantities may be also express'd by rectangles or squares of different magnitude; by this means, geometry as well as arithmetic is useful in computing all sorts of quantity: for when quantities are represented by numbers, lines, or squares, we may proceed to compute with those numbers, lines, or squares, and thereby discover the proportions of those quantities which they represent.

·73 *An aliquot part* of any quantity is that which repeated a certain number of times, (*aliquoties*,) exactly measures that quantity; *an aliquant part* of any quantity is that which does not measure it: thus, in numbers, 2 is an aliquot part of 8, because 2 repeated 4 times is equal to 8: but 3 is an aliquant part of 8, because if repeated twice, it is less than 8, if repeated thrice, it is more than 8: thus, a line 2 inches long is an aliquot part of a foot, a line 5 inches long is an aliquant part of a foot.

·74 Any two quantities are *commensurable*, or, as they are sometimes called, *rational*, when a third quantity can be found which is a common measure to them both: thus 2 is a common measure to 4 and 12: and 3 is a common measure to 9 and 15: all numbers are commensurable; because an unit is their common measure; and all commensurable quantities may be express'd by numbers.

·75 *Incommensurable, irrational, or surd quantities* are such as have no common measure, and cannot be express'd by numbers: thus geometers demonstrate, that the diagonal and side of a square are incommensurable.^a

·76 When we compare two quantities together, as to more or less, and enquire whether they be equal or not, and, if unequal, how much one is greater or less than the other, we are said to consider the *ratio* or *reason* of those quantities, and we say one is in such a ratio to the other: in such comparisons the first quantity is called *the antecedent*, the second *the consequent*: if the antecedent and consequent are equal, they are said to be in a *ratio of equality*; if unequal, in a *ratio of inequality*: when the antecedent is greater than the consequent, as in the numbers [8:4,] it is in a *ratio of majority*: when the antecedent is less than the consequent, as [4:8,] it is in a *ratio of minority*: in ratios of inequality the less number is either an aliquot part of the greater, as in the numbers [12:3,] or an aliquant part of it, as [10:3.] The names of some of those ratios of inequality that most frequently occur, are these that follow.

·77 In a ratio of majority, when the antecedent contains the consequent twice precisely, as [8:4,] the antecedent is said to be in a *duple ratio* to the consequent: when it contains it three times, as [9:3,] it is in a *triple ratio*: when four times, as [12:3,] in a *quadruple ratio* &c: when the antecedent con-

^a Tacquet Euclid. edit. Cantabrig. Schol. in lib. 1. prop. 47.

FIG. tains the consequent once, and a half of the consequent, as in these numbers, $[3:2,]$ it is in a *ratio sesquialtera*: when it contains the consequent once and a third part, as in these numbers, $[4:3,]$ it is in a *ratio sesquitertia* &c.

·78 In ratios of minority, the same terms are used as before, with the addition of *sub*; thus, when the antecedent is contained in the consequent twice, as $[4:8,]$ the *ratio* is *subduple*: when three times, as $[3:9,]$ it is *subtriple*: when four times, as $[3:12,]$ it is *subquadruple*: when once and a half, as $[2:3,]$ it is *subsesquialtera*: when once and a third part, as $[3:4,]$ it is *subsesquitertia*, &c.

·79 When we consider four quantities together, and compare them by pairs, two with two, and find that the ratio between one pair is equal to the ratio between the other pair, we call this equality or similitude of ratios *proportion*; and those four *quantities* are said to be *proportional*. Proportion is either direct or reciprocal; *direct proportion* is where the first antecedent is to the first consequent, as the second antecedent is to the second consequent: thus, in these four numbers, 4, 6, 8, 12, if I compare the pair $[4:6,]$ with the pair $[8:12,]$ I shall find that 4 is in the same ratio to 6, as 8 is to 12; which is usually thus expressed, $4:6::8:12$; thus, in the four lines of fig. 48, so that $ab:AB::cd:CD$: that is, the line ab : is in the same ratio to AB , as cd is in to CD . If the quantities are expressed by numbers, in direct proportion more requires more, or less requires less; the greater the third number is, the greater must the fourth number be; or the less the third number is, the less must the fourth number be: thus, in this example, if one degree of a great circle upon the earth, is equal to 60 Italian miles, to how many such miles will 360° , the whole circumference of the earth, be equal? here more degrees require more miles; the answer is 21600 miles; thus expressed, $1^\circ:60^{\text{miles}}::360^\circ:21600^{\text{miles}}$.
 2^d Example, if the moon goes through a great circle in the heaven, or 360° in 27 days, how many degrees will she go in one day? here less requires less, that is, in less time the motion will be of fewer degrees; the answer is 13° : with a fraction or piece of a degree, thus expressed, $27^{\text{days}}:360^\circ::1^{\text{day}}:13^\circ\frac{2}{7}$. In numbers directly proportional, three numbers being given the fourth is found by the single golden rule direct.

·80 When three quantities are such that the first quantity is in the same ratio to the second as the second is to the third, the second quantity is called a *mean proportional* between the first and the third: thus, in numbers, 12 is a mean proportional between 6 and 24; for $6:12::12:24$: thus, in lines, fig. 49, B is a mean proportional between A and C; for $A:B::B:C$.

·81 *Reciprocal proportion* is where more requires less, or less requires more; that is, the greater the third quantity is, the less will the fourth be, or the less the third quantity is, the greater will the fourth be; thus, if I would poise

two

two unequal heavy bodies A and B, fig. 50, fixed at the two ends of a beam AB, FIG. so as to make them rest in *æquilibrio*, or to be of equal weight, I must place 50 *the fulcrum* or support c under the beam in such a manner, that the distances of the centers of the heavy bodies from it may be reciprocally as their weights; that is, if the weight of A be triple the weight of B, the distance of B from c must be triple the distance of A from c: here greater weight requires proportionally less distance, and less weight requires proportionally greater distance from the fulcrum; the point c, where the fulcrum must be placed to support the bodies A and B in *æquilibrio*, is called *the center of gravity* of those bodies. In numbers reciprocally proportional, three numbers being given, the fourth is found by the single golden rule inverse.

•82 Any quantity considered as capable by multiplication, of producing, or being produced by, another quantity, is called *a power*: the product or *factum* of any quantity multiplied by it self is called the *second power*, or *square* of the simple quantity so multiplied: if that square is multiplied by the first simple quantity, the product is called the *third power*, or *cube*, &c: in all these cases, the simple quantity, to which the rest owe their origin, is called *the first power*, *the root*, and in geometry *the side* of the square, or cube, &c: thus, in numbers, let the root be 4, the square number produced by 4 multiplied by 4, is 16; the square 16 multiplied by the root 4, produces the cube number 64: thus, in lines, fig. 51, the line AB, multiplied by it self gene- 51 rates the square ABCD: this square multiplied by the side AB, or carried parallel to it self, in its first situation, along a line AG, perpendicular to its plane, and equal to the side AB, generates the cube ACEG.

•83 *Quantities* that being multiplied into themselves produce equal squares, equal cubes &c, are said to be *equal in power*: in latin, *æquipossē*.

•84 Ratios may be considered not only between simple quantities, but also between their powers: the ratio between two squares or second powers is called *a duplicate ratio*: the ratio between two cubes or third powers is called *a triplicate ratio*: the ratio between a square and a cube, or between the second and third power is called *a sesquiplicate ratio*.

•85 *First example* of direct proportion of powers; the areas of any two circles *abcd* and ABCD, fig. 52 and 53, are in a duplicate ratio of their diameters, 52 or as the squares of their diameters: thus, let the diameter *ac* be 12, and the 53 diameter AC be 20; the square of 12 is 144, the square of 20 is 400; then I say, the area of *abcd*: area of ABCD::144:400. *Euclid. lib. 12. prop. 2.*

•86 *Second example*; the surfaces of any two spheres A and B, fig. 54 and 55, 54 are as the squares of their diameters^a: thus, let the diameter of A be 6, and 55

^a *Archimedes de sphaera & cylindro, Tacquet selecta ex Archimede theoremata.*

FIG. the diameter of B 8, the square of six is 36, the square of 8 is 64; I say then
 54 that the surface of A is to the surface of B, as 36 is to 64.

55 ·87 *Third example*; the solid contents of two spheres, A and B, of different
 54 magnitude, fig. 54 and 55, are in a triplicate ratio of their diameters, or as the
 55 cubes of their diameters: thus, let the diameter of A be 6, and the diameter of
 B 8; the cube of 6 is 216, the cube of 8 is 512, I say then the solid content of
 A is to the solid content of B, as 216 is to 512. *Eucl.* 12, 18.

·88 *Fourth example*; of reciprocal proportion of powers; the intenseness of
 all corporeal qualities or powers which are diffused every way in strait lines
 from any natural body, as from a center, such as light, heat, smell &c, is re-
 ciprocally as the squares of the distances of the acting body from the bodies acted
 upon. To give an instance in light; it is evident that the intenseness of the
 suns light, falling upon any given surface, is greater or less, according as his
 rays fall upon it in greater or less quantity; or in other words, any surface is
 more or less illuminated, according as the light falls thicker or thinner upon it;
 now that this density of the suns light is reciprocally as the squares of the
 distances of the illuminated bodies from the sun, is thus demonstrated: let s
 56 be the sun sending forth rays of light every way round, in the strait lines SA,
 SB, SC, SD, &c, all these rays, at the distance sa, are spread uniformly over
 the spherical surface abcde; at the distance SA, the same rays are spread uni-
 formly over a larger spherical surface ABCDE; the light then must be as much
 thinner upon this larger spherical surface than it is upon the smaller one, as the
 larger spherical surface is greater than the smaller one: now the surfaces of
 spheres are as the squares of their diameters, by § 86; that is, the spherical sur-
 face ABCDE is to the spherical surface abcde, as the square of SA is to the
 square of sa: therefore the light at A is to the light at a, as the square of sa is
 to the square of SA: which was to be proved. The following experiment, taken
 out of *Smith's Optics* pag. 17, may serve for a further illustration of this matter.

57 Let the light which flows from a point A, and passes through a square
 hole bcde, be received upon a plane BCDE, placed parallel to the plane of the
 hole; if the distance of the plane BCDE from the point A be twice as great
 as the distance of the hole from that point, the light will then be spread over
 a surface which is four times as large as the aperture of the hole, as appears
 by the figure, where BCDE is divided into four squares, each of which is e-
 qual to the square bcde; if the distance of the plane which receives the
 light coming through the hole be three times the distance of the hole from the
 point A, the light will then fall upon a square surface FGHI, nine times as
 large as the square aperture of the hole, as is evident also from a view of the
 figure; so that at double the distance, the plane illuminated by the point A is
 twice

twice as large; at three times the distance, it is nine times as large; at four times the distance, the illuminated surface is sixteen times as large, &c; the surfaces illuminated increasing as the squares of their distances from the illuminating point: thus, let there be two bodies of equal surfaces A and a, fig. 56, at different distances from the sun, let the distance of A from the sun be as 4, and the distance of a as 8; then will the quantity of the suns light falling upon A, compared with his light which falls upon a, be in the same ratio as 64, the square of 8, is to 16, the square of 4: by this method, the several distances of the planets from the sun being given, the force of the suns heat or light upon them respectively may be compared: thus, if the distance of Mercury from the sun be as 4, and the distance of the earth as 10; the heat upon Mercury shall be to the heat upon the earth, as 100, the square of 10, is to 16, the square of 4. By this method also the different force of the suns heat upon the earth, arising from her different distances from the sun, may be found: thus, let the distance of the earth from the sun in summer, be as 1008, her distance in winter, as 1000; then, if no other cause of the difference of the suns heat were to be considered, but the difference of his distance from the earth, summers heat would be to the winters heat, as 1000000 the square of 1000, the winters distance, is to 16064, the square of 1008, the summers distance. 1034

89 *Fifth example*; the gravitation of the several primary planets towards the sun is reciprocally as the squares of their distances from him: *Newt. princ. p. 362. ed. 2.* Thus, let the distance of the earth from the sun be 10, the distance of Mercury 4; the square of 10 is 100, the square of 4 is 16; I say then, Mercury's gravitation is to the earths gravitation in the same ratio as 100 is to 16: the gravitation of Jupiters satellits towards Jupiter, and of Saturns satellits towards Saturn, is in the same proportion, viz. reciprocally as the squares of their distances from their primary; and may be compared by this method. 1-1069156-

90 *Sixth example*, of proportion between different powers; if the periodical times in which the planets go round the sun be compared with their middle distances from the sun, the proportion stands thus; their periods are in a sesquiplicate ratio of their distances, or in other words, *the squares of their periods are as the cubes of their distances*: thus, if we consider Venus and the earth together, we shall find that the square of the period of Venus is to the square of the earths period, as the cube of the distance of Venus is to the cube of the earths distance. The periodical times of the secondary planets motion round their primaries are likewise in the same ratio of their distances from their primaries. *Præfat. in edit. 2. Newtoni Princip. & lib. 3. pag. 259. & 360.*

The six following sections, are useful for the understanding the way of making tables of sines, &c, but may be passed over by those who are not desirous to look into that affair.

FIG. 91 *Similar plane figures* are such as have their corresponding parts proportional.

92 *All circles are similar*: the circumference of any one circle is in the same ratio to its diameter, radius, or arc of a given number of degrees, or chord of a given arc, as any other circle is to its diameter, radius, or arc of a like number of degrees, or chord of a like arc. § 31 and 32.

93 *Similar parallelograms* are such as have their corresponding sides proportional, and their corresponding angles equal: thus, the two parallelograms AD and *a d*, fig. 58 and 59, are similar, if the ratio between the sides is such that, $ab:AB::ac:AC$, and if the angle *a* is equal to A, and the angle *b* to B.

94 Triangles which are equiangular are similar: *Eucl.* 6. 4. thus, fig. 60 and 61, if the angle *a* is equal to A, the angle *b* to B, and *c* to C, those triangles are similar, and their corresponding sides are proportional; that is, $ab:AB::ac:AC$. and $ab:AB::cb:CB$. and $ac:AC::cb:CB$.

95 If any triangle BAC, fig. 62, has a line DE drawn in it parallel to one of its sides CA, the other two sides of it BC and BA will be cut proportionally; and there will be formed another triangle BED, similar to BAC: so that, $BE:BA::BD:BC$. and $BE:BA::ED:AC$, &c. *Eucl.* 6. 2. *this proposition is of great use in making the tables of sines, tangents, and secants.*

96 If one acute angle in one right-angled triangle is equal to one acute angle in another right-angled triangle, those triangles are similar; § 55, *Cor.* 7, and § 94. We may now resume the doctrine of trigonometry, so much of it as we shall have occasion to consider in the present work.

97 In every right-angled triangle BAC, fig. 63, the longest side BC, which is always that opposite to the right angle A, is called the *hypotenuse*; the other sides AB and AC, are called *the legs*.

98 *In every right-angled triangle, the square of the hypotenuse is equal to the squares of the legs.* This proposition is sometimes thus expressed, in every right-angled triangle the hypotenuse is *equal in power* to the legs: thus, take the triangle BAC, fig. 64, upon the hypotenuse BC erect the square BD, upon the leg AC erect the square CE, upon the leg BA, erect the square BF; I say the area of the square BD will be equal to the sum of the areas of the other two squares CE and BF. This proposition, the demonstration of which may be seen in the common books of geometry, *Eucl.* 1, 47, is of great use in trigonometry, for thereby, the measure of any right-angled triangle is easily found, if we have some *data*, or things known to go upon: thus, in the right-angled triangle, fig. 63, the length of the hypotenuse BC and one leg AC being given, the other leg may be found in the following manner; let the length of the hypotenuse be 10 feet, yards, or miles, or any other known measure, in the present example

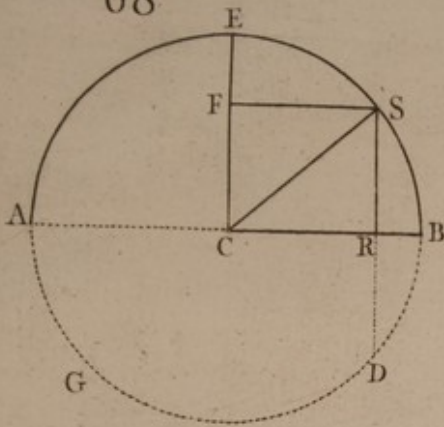
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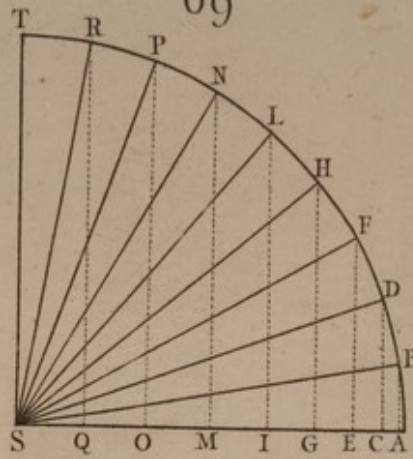
Introduction .

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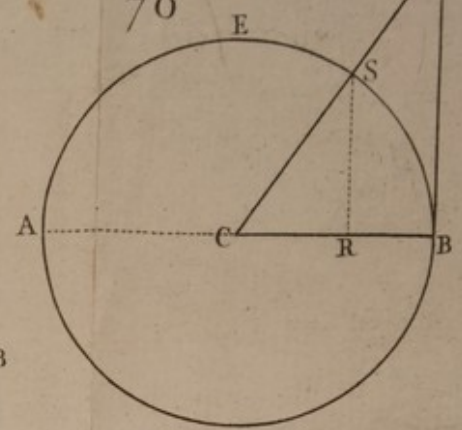
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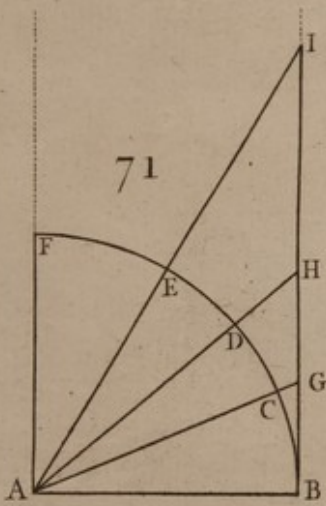
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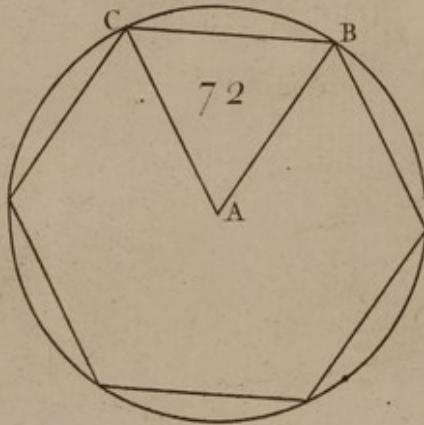
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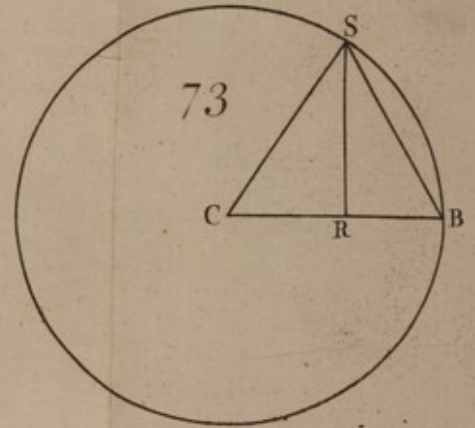
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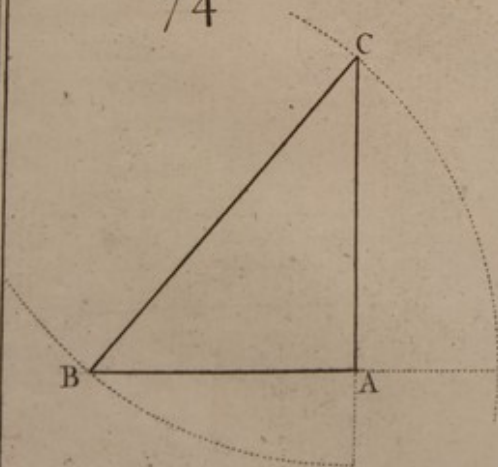
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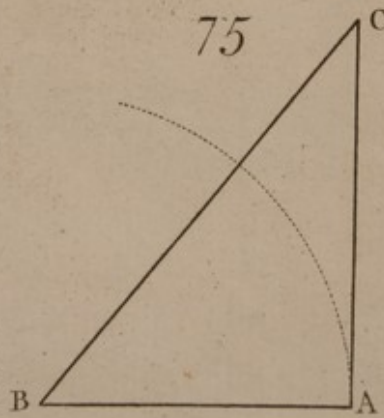
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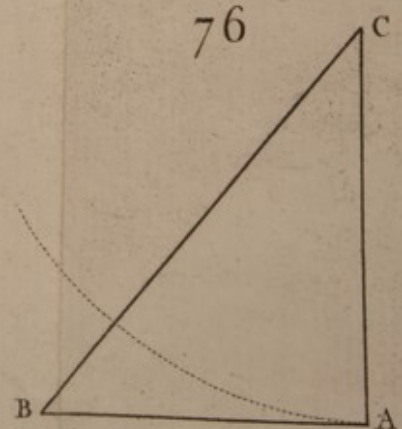
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we will suppose feet; the square of 10 is 100: let the length of the leg AC be 8 FIG. feet, the square of 8 is 64: let the square 64, be taken from 100, the remainder is 36, the square of the other leg AB; find the square root of 36, viz. 6, and you 63 have the length of the leg AB, 6 feet. In like manner the two legs being given, the hypotenuse may be thus known; find the squares of the legs and add them together, the sum is the square of the hypotenuse: *example*, let the leg BA be 6 feet, the leg AC 8 feet, the square of 6 is 36, the square of 8 is 64, the sum of 36 and 64 is 100, the square of the hypotenuse: the square root of 100 is 10, the length of the hypotenuse therefore is 10 feet; which was to be found. *This proposition is another of the sources of the tables of sines, tangents, and secants, which for the sake of some of my readers I shall now explain.*

*99 *The complement of an arc* is so much as it wants of a quarter of a circle: thus, fig. 65, the arc SE is the complement of the arc SB; and the arc SB is 65 the complement of the arc SE.

*100 *The supplement of an arc* is so much as it wants of a semicircle: thus, fig. 65. SEA is the supplement of the arc SB, and SB is the supplement of the 65 arc SEA: some writers call this the complement to a semicircle.

*101 *A chord or subtense*, AB, fig. 66, is a strait line drawn within a circle, 66 terminated at each end by the circumference: every chord as AB divides the circumference of the circle into two arcs, ACB and ADB: the chord is said to be the chord of either of those arcs: a chord is said to subtend its arc, because it is stretched under it, being as a string to a bow. ACB

*102 The larger any arc is, the longer is the chord, till the arc is a semicircle, and the chord a diameter; which is the longest strait line that can be drawn within a circle, as appears by the sixty seventh figure. 67

*103 *A right sine*, or as it is generally called, *a sine of an arc*, is a strait line drawn within a semicircle, from one extreme of an arc, perpendicular to a diameter drawn from the other extreme of that arc: in the black semicircle AEB, fig. 68, let the arc whose sine is to be found be SB, from s one extreme of 68 the arc, draw SR, perpendicular to AB the diameter which is drawn from B, the other extreme of the given arc; SR is the right sine of it: and since the arc SB measures the angle SCB, § 30, the line SR is called the sine also of the angle SCB. Sine is a relative term, and is indifferently referred either to the angle, or to the arc which measures the angle; in trigonometry it is generally called the sine of an angle.

*104 If we compleat the circle, as is done in the preceding figure by the 68 pointed semicircle AGB, and take an arc BD equal to SB, and continue the line SR to D, it is obvious, by *Eucl.* 3. 3, that the sine SR, is half SD, the chord of the arc SBD, which is double to the arc SB; this proposition is commonly thus expressed,

FIG. preft, *the fine is half the chord of a double arc*: thus it appears that a fine is the fame in a femicircle that a chord is in the whole circle.

• 105 As a chord is referred to both parts of the circumference of a circle, as well that which exceeds a femicircle as that which falls fhort of it, § 101; fo a fine is referred to both parts of a femicircle, as well the arc greater than a
68 quadrant as that which is lefs than a quadrant: thus SR the fine of the arc SB , is alfo the fine of the arc SEA , the fupplement of SB : if we confider the angles, inftead of the arcs, we may then fay, that the fine of every acute angle SCB , is alfo the fine of the obtufe angle SCA , the fupplemental angle to SCB , which added to it makes two right angles: for this reafon, tables of fines are never computed for angles that exceed 90° ; but if the fine of any obtufe angle be required, we take for it the fine of the acute angle which added to it makes 180° : thus, for the fine of 160° , we take the fine of 20° ; for the fine of 110° , we take the fine of 70° .

69 • 106 The larger any angle or arc is, the greater will the fine be, till the arc is a quadrant, and the angle a right one; and then the fine is the radius of the circle; which being the largeft fine poffible, and all other fines being taken out of it, is called *the whole fine*: in fig. 69, the pointed lines are fines of angles or arcs increafing from 10° , to 90° , thus, AB is the fine of an angle or arc of 10° , CD the fine of 20° , EF of 30° , &c.

68 • 107 A *verfed fine* of an arc is a ftrait line drawn from one extreme of that arc to the right fine of it, perpendicular to the right fine: thus, RB is the verfed *fine* of the arc SB , drawn from B to R , making the angle BRS a right one: thus alfo, AR is the verfed fine of the arc AEs , drawn from A to R , and making ARS a right angle.

68 • 108 The point R , where the right fine SR falls upon the diameter AB , divides the radius CB into two parts, CR and RB , that part RB next the circumference is the verfed fine, called by fome the *fagitta* of the arc SB ; the other part *viz.* CR next the center is called *the fine of the complement*, becaufe, by *Eucl.* 1, 34, equal to FS , which is in reality the fine of the arc SE , the complement of the arc SB . Verfed fines are feldom inferted in tables^a, becaufe eafily found by knowing the radius and fine complement; thus, for any arc SB lefs than a quadrant, take the fine complement CR from the radius CB , the remainder RB is the verfed fine: for any arc AEs greater than a quadrant, add the fine complement CR to the radius AC , and the fum of both, *viz.* AR , is the verfed fine. *AR*

70 • 109 A *tangent of an arc* is a ftrait line perpendicular to the diameter, drawn out from one extreme of the arc, till it meets with the radius continued through the other extreme of the arc: thus, in fig. 70, BT is the tangent of

^a Verfed fines are in *Sr Jonas Moor's* and *Sherwin's tables*.

the arc SB , perpendicular to the diameter AB , drawn out from the point B , till it meets with the radius CS continued, in T . FIG. 70

• 110 A *secant of an arc* is the radius continued through one end of the arc, till it meets the tangent drawn from the other end: in fig. 70, CT is the secant of the arc SB . The secant or tangent of any arc is, as was said of the sine, the secant and tangent also of the supplement of that arc; thus, TB is the tangent, and TC the secant, of the arc SEA , the supplement of SB .

• 111 The larger any angle or arc is, the longer will the secant and tangent be, till the arc is a quadrant, and the angle a right one; and then they are said to be infinite; because, fig. 71, the secant will be the radius AF continued, as by the pointed line, to which the tangent BT is parallel, and therefore they can never meet, § 46. The seventy first figure shews how the secants and tangents grow longer, as the arcs are larger; thus, of the arc BC , the tangent is BG ; the secant AG : of the arc BD , the tangent is BH ; the secant AH : of the arc BE , the tangent is BI ; the secant AI : of the arc BF the tangent is BI ; the secant AF : both infinitely extended. BI BF

• 112 The sine, tangent, or secant, of the complement of a given arc or angle, is frequently called *the cosine, cotangent, and cosecant* of that arc or angle.

• 113 To find the sine, tangent, or secant, of an arc or angle, is to find what proportion it bears to the radius; that is, suppose the radius were divided into a certain number of equal parts, as 10000000 parts, to find how many of such parts the sine, secant, or tangent, is equal to: for this purpose, tables have been made, wherein an angle or arc being given, we may find the sine, tangent, or secant thereof, or the sine tangent or secant being given, we may find the angle or arc.

• I shall now give the Reader, for a specimen, the first two pages of a table of sines, &c. together with directions for the manner of using it.

• 114 The table is divided into six columns, the first of which is the column of minutes: the second, of sines: the third, of tangents: the fourth, of secants: the fifth, of logarithms of sines: the sixth, of logarithms of tangents.

• 115 At the top of each page, except the first and the third, after the word *grad.* is set down a number which signifies whole degrees: and in the first column under the word minutes are the minutes which added to those degrees express the quantity of any angle whose sine, tangent, &c. may be found in that page. The first and third pages of the table contain only minutes from 1' to 60', and therefore at the top of those two pages is put *grad. 0.*

Minut.	Grad. o.				
	Sinus	Tang.	Secant.	Log. Sin.	Log. Tang.
0	0	0	100000.00	0	0
1	29.09	29.09	100000.00	6.4637261	6.4637261
2	58.18	58.18	100000.02	6.7647561	6.7647562
3	87.27	87.27	100000.04	6.9408473	6.9408475
4	116.36	116.36	100000.07	7.0657860	7.0657863
5	145.44	145.44	100000.11	7.1626960	7.1626964
6	174.53	174.53	100000.16	7.2418771	7.2418778
7	203.62	203.62	100000.21	7.3088239	7.3088248
8	232.71	232.71	100000.27	7.3668157	7.3668169
9	261.80	261.80	100000.34	7.4179681	7.4179696
10	290.89	290.89	100000.42	7.4637255	7.4637273
11	319.98	319.98	100000.51	7.5051181	7.5051203
12	349.06	349.07	100000.61	7.5429065	7.5429091
13	378.15	378.16	100000.72	7.5776684	7.5776715
14	407.24	407.25	100000.83	7.6098530	7.6098566
15	436.33	436.33	100000.95	7.6398160	7.6398201
16	465.42	465.42	100001.08	7.6678445	7.6678492
17	494.51	494.51	100001.22	7.6941733	7.6941786
18	523.60	523.60	100001.37	7.7189966	7.7190026
19	552.68	552.69	100001.53	7.7424775	7.7424841
20	581.77	581.78	100001.70	7.7647537	7.7647610
21	610.86	610.87	100001.87	7.7859427	7.7859508
22	639.95	639.96	100002.05	7.8061458	7.8061547
23	669.04	669.05	100002.24	7.8254507	7.8254604
24	698.13	698.14	100002.44	7.8439338	7.8439444
25	727.21	727.23	100002.65	7.8616623	7.8616738
26	756.30	756.32	100002.86	7.8786953	7.8787077
27	785.39	785.41	100003.08	7.8950854	7.8950988
28	814.48	814.50	100003.31	7.9108793	7.9108938
29	843.57	843.60	100003.55	7.9261190	7.9261344
30	872.65	872.69	100003.80	7.9408419	7.9408584

Minut.	Grad. 89.				
	Sinus	Tang.	Secant.	Log. Sin.	Log. Tang.
60	10000000	Infin.	Infin.	10.0000000	Infin.
59	99999.99	343774667.	343774682.	9.9999999	13.5362739
58	99999.98	171887319.	171887348.	9.9999999	13.2352438
57	99999.96	114591530.	114591574.	9.9999998	13.0591525
56	99999.93	85943630.	85943689.	9.9999997	12.9342137
55	99999.89	68754887.	68754960.	9.9999995	12.8373036
54	99999.84	57295721.	57295809.	9.9999993	12.7581222
53	99999.79	49110600.	49110702.	9.9999991	12.6911752
52	99999.73	42971757.	42971873.	9.9999988	12.6331831
51	99999.66	38197099.	38197230.	9.9999985	12.5820304
50	99999.58	34377371.	34377516.	9.9999982	12.5362727
49	99999.49	31252137.	31252297.	9.9999978	12.4948797
48	99999.39	28647773.	28647948.	9.9999974	12.4570909
47	99999.28	26444080.	26444269.	9.9999969	12.4223285
46	99999.17	24555198.	24555402.	9.9999964	12.3901434
45	99999.05	22918166.	22918385.	9.9999959	12.3601799
44	99998.92	21485762.	21485995.	9.9999953	12.3321508
43	99998.78	20221875.	20222122.	9.9999947	12.3058214
42	99998.63	19098419.	19098680.	9.9999940	12.2809974
41	99998.47	18093220.	18093496.	9.9999934	12.2575159
40	99998.30	17188540.	17188831.	9.9999927	12.2352390
39	99998.13	16370019.	16370325.	9.9999919	12.2140492
38	99997.95	15625908.	15626228.	9.9999911	12.1938453
37	99997.76	14946502.	14946837.	9.9999903	12.1745396
36	99997.56	14323712.	14324061.	9.9999894	12.1560556
35	99997.35	13750745.	13751108.	9.9999885	12.1383262
34	99997.13	13221851.	13222229.	9.9999876	12.1212923
33	99996.91	12732134.	12732526.	9.9999866	12.1049012
32	99996.68	12277396.	12277803.	9.9999850	12.0891062
31	99996.44	11854018.	11854440.	9.9999845	12.0738656
30	99996.19	11458865.	11459301.	9.9999835	12.0591416

• 116 If an angle is given, whose quantity is exprest in degrees and minutes, and the sine, tangent, or secant thereof required; find the number of degrees on the top of the page, and the number of minutes in the column of minutes in the same page, and on the same line with the number of minutes, you have the sine, in the column of sines: the tangent, in the column of tangents: the secant, in the column of secants.

• 117 *Example*, To find the sine, &c of $89^{\circ} 45'$; in the second page of the table, which has on the top *grad.* 89, look in the column of minutes for the number 45; on the same line, you have the sine of $89^{\circ} 45'$, viz. 99999.05: the tangent 22918166: the secant 22918385.

• 118 When an angle is given of a number of minutes only, find that number in the first or third page of the table, in the column of minutes; and on the same line, in its proper column, you have the sine, tangent, and secant of the angle given.

• 119 *Example*, To find the sine, tangent, or secant of an angle of $14'$; look in the first page of the table, in the column of minutes, for the number 14; and on the same line in the column of sines, you have the sine 407.24: in the column of tangents, you have the tangent 407.25: in the column of secants, the secant 10000083.

• 120 If the sine, tangent, or secant be given, and the angle corresponding to it required, proceed thus; find the sine in the column of sines, or the tangent in the column of tangents, &c, and in the column of minutes, on the same line, you will find the minute, which with the number of degrees on the top of the page is the angle required: thus, if the sine 99999.58 be given, find that number in the column of sines, which in the specimen before us is in the second page; and on the same line is the number 50, in the column of minutes: add $50'$ to 89° , the number of degrees at the top of the page, and you have the angle required; viz. $89^{\circ} 50'$.

• 121 The numbers are so placed in these two pages, which in the common books of tables face one another, that the degree and minute in one page, is the complement to the degree and minute on the same line, in the other page; so that sine and cosine, tangent and cotangent, secant and cosecant are seen at one view: thus, on the same line with $10'$, in the left hand page, together with its sine, tangent, and secant, we have in the right hand page its complement, $89^{\circ} 50'$, with its sine, tangent, and secant.

• 122 Sometimes a sine, tangent, or secant is given, of a number which is not to be found in the table; this often happens when the sine, &c given is the fourth number in an operation by the golden rule; in this case, instead of the number given, take the number nearest to it in the table: *example*, if the
fine

fine given whose angle is required be 8699567, instead of this number, which FIG. is not in the table, take the nearest to it viz. 86992.56, and the corresponding angle $60^{\circ} 27'$ is the angle required.

• 123 In the column of lines there is all along placed a point amongst the figures by which every fine is exprest, in such a manner, that every number has two places on the right hand of the point: there is a point also among the figures by which every tangent, or secant is exprest, and for the greatest part of the table, every number by which any tangent or secant is exprest, has two places on the right hand of the point: but when the numbers rise so high as to have more figures than can conveniently stand in the column, it is usual to omit one or two of those figures, which were to be placed on the right hand of the point, whose places must always be supplied with as many cyphers, in order to have the true tangent or secant: thus, the tangent of $89^{\circ} 56'$ is set down in the table 85943630, without any figure or cypher on the right hand of the point; two cyphers therefore are to be added, and then you have the true tangent, viz. 85943630.00.

• 124 In some tables we have the fines, tangents, and secants, for angles of less than $1'$: the table published with *Oughtred's trigonometria* gives them for centesmes of a degree: by *Pitiscus's* or *Sherwin's* tables, they may be found for an angle of any number of seconds from $1''$ to $60''$.

• 125 Though the labour of computing a table of fines &c need not now be undertaken, because it has been already gone through by others with incredible industry and patience; yet some readers may be desirous to see something of the method which they proceeded in, who calculated the tables we have: the eleven following sections are for that purpose.

• 126 Sines are either primary, or secondary; *primary fines* are such as are found out by an immediate calculation; *secondary fines* are such as are deduced from the primary ones.

• 127 The first primary fine is the radius or whole fine, which we may suppose divided into any number of parts; for the ease of calculation, the number assumed for it is an unit with a certain number of cyphers, as 10000000.

• 128 The radius is equal to the side of an hexagon inscribed in a circle; *Eucl.* 4. 15: thus, let an hexagon be inscribed in a circle, fig. 72, I say any 72 side thereof CB is equal to the radius: *demonstration*, draw the radius's AC and AB, these are equal, by § 25, therefore in the triangle ABC the angles C and B are equal, *Eucl.* 1. 5; the sum of the angles A, C, B, is 180, § 55, whereof the angle at A is a third part, viz. 60° , by the work, which supposes CB to be the chord of a sixth part of the whole circumference 360° ; therefore the angle A is equal to B or C: and then the side CB is equal to the radius AC or AB, *Eucl.* 1. 6.

FIG. 129 The radius being given, the sine of 30° may be known; for the radius is the chord of 60° , by § 128, and the sine of 30° is half the chord of 60° , by § 104; thus, the radius being 10000000, the sine of 30° is 5000000.

130 The sine of an arc being given, its cosine may be found by this proposition; *the sine of an arc and its cosine are in power equal to the radius: demonstr.* the sine SR of an arc SB together with its cosine CR and radius CS, fig. 68, form a right-angled triangle, in which the radius CS is the hypotenuse, by the definitions, § 67, 103, 108: but in every right-angled triangle the hypotenuse is in power equal to the legs, § 98: therefore CS^2 : thus the sine of 60° is found in the following method: SR the sine of 30° is 5000,000, § 129, let the square thereof 25000,000,000,000, be subtracted from 100,000,000,000,000 the square of the radius, and the remainder 75000,000,000,000, is the square of the cosine, the square root of which *viz.* 8670254 is the sine of 60° .

131 The right sine and the versed sine of an arc are in power equal to the chord of that arc: *demonstr.* in fig. 73. let the given arc be SB, the sine thereof SR together with the versed sine RB and the chord SB, form a right-angled triangle, whereof the chord SB is the hypotenuse, by the definitions, § 97, 103, and 107: but in every right-angle triangle the legs are equal in power to the hypotenuse, by § 98: therefore CS^2 .

132 *Coroll. 1.* The sine of an arc being given, the chord of that arc may be found, in this manner; find the versed sine of the given arc by § 108, and add the square thereof to the square of the sine; the square root of the sum of them is equal to the chord required.

133 *Cor. 2.* The sine of an arc being given, the sine of half that arc may be found, in this manner; find the chord of the arc by the preceding corollary, and half that chord is the sine of half the given arc, by § 104.

134 This is sufficient to shew something of the method of investigating sines; a primary sine is first found, and the secondary sines are derived from it, by finding the sine of half the given arc, by § 133; and then the complement of that half, by § 130; and then again the half of that complement, by § 133; and thus we may proceed as far as is required: thus, from the sine of 30° , are found the fifteen following sines.

Sine of 30° and its halves		Comple- ments,		Halves of the Complements.		Comple- ments.		Halves of the Comp.		Comple- ments.	
30°	0'	60°	0'								
15°	0'	75°	0'	(37°	$30'$	52°	$30'$	26°	$15'$	63°	$45'$
				(18°	$45'$	71°	$15'$				
7°	$30'$	82°	$30'$	41°	$15'$	48°	$45'$				
3°	$45'$	86°	$15'$								

135 When

135 When the fines are found by the preceding propositions, the tangents and secants are easily found by the golden rule: thus, let the given arc be SB , fig. 70; the fine thereof RS together with the cosine CR and radius CS , form a right-angled triangle CRS , by the definitions § 103 and 108; the radius CB together with the tangent BT and secant CT , form another right-angled triangle, CBT , by the definitions § 109 and 110: these triangles having one acute angle at C common, are similar, by § 96: therefore to find the tangent, say, $CR:RS::CB:BT$. that is, as the cosine CR is to the fine RS , so is the radius CB to the tangent BT .

136 To find the secant, say thus. $CR:CS::CB:CT$. that is, as the cosine CR is to the radius CS , so is the radius CB to the secant CT .

137 What ever number is assumed for the radius, almost all the fines tangents and secants are incommensurable to it, as being found either by extracting the square root, or by the golden rule; in the first of these operations, the root extracted is often a surd number, as being extracted out of a number not exactly square; in the second operation, viz. that by the golden rule, the 4th number is often a whole number with a fraction; for these reasons tables of fines, tangents, and secants cannot be made perfectly accurate: but they may be made so nearly accurate, that no number in them shall differ from the true one so much as one of those parts into which the radius is supposed to be divided in the tables; thus, in the common tables, where the radius is assumed of 10000000 parts, no fine or tangent differs from the true number so much as a 10000000th part of the radius: to make the tables thus near the truth, it was necessary for authors either to consider fractions in their calculations of fines &c, which would have made them exceedingly operose; or else to assume the radius of a much greater number of parts in the calculation, than it is said to consist of in the tables after they are made: the latter method as the more easy has been made use of: thus, *Rheticus* in order to compute a table of fines, &c, to a radius of 1000000,0000, assumed a radius of 100000,00000,00000 parts; and after the calculation, cut off from each number 5 figures on the right hand: *Pitiscus* for some fines at the beginning of his table, assumed the radius of 100000,00000,00000,00000,00000 parts, *Pitisc. trigonometr. l. 2. n. 26*. For the rest, see *Tacquets geometr. pract. l. 1. c. 2.* and the Authors there cited: *Lansberg. triangulorum geometria: Newton's compendium of trigonometry: Sberwin's tables p. 44: Keils trigonometria, &c.*

138 By the definitions § 103 and 108; the fine SR , of any acute angle SCR , fig. 68, together with the cosine CR , and the radius CS form a right-angled triangle, CRS : the converse of this is true, viz. that in every right-angled triangle CRS , the sides are radius, fine, and cosine of one of its acute angles.

139 By

FIG. *139 By the definitions § 109 and 110, the tangent TB, of any acute angle
 70 TCB, together with its secant CT, and the radius CB, form a right-angled triangle TBC: the converse of this is true, *viz.* that in every right-angled triangle TBC, the sides are radius, tangent, and secant of one of its acute angles.

74 *140 In every right-angled triangle, BAC fig. 74, any side may be made radius, and then each of the other sides will be either sine, tangent, or secant of one of the acute angles: *case* 1. If the hypotenuse BC be made radius, the legs will be sines of their opposite angles; that is, CA is the sine of its opposite angle ABC, and BA is the sine of the angle ACB: *case* 2. If one of the legs be made radius, then shall the other leg be tangent of the angle opposite to it, and the
 75 hypotenuse secant of the same: thus, fig. 75, if the leg BA be made radius, the other leg CA will be tangent of its opposite angle B, and the hypotenuse BC
 76 secant of the same: again, fig. 76, if the leg CA be made radius, the other leg BA will be tangent to its opposite angle C, and the hypotenuse BC secant.

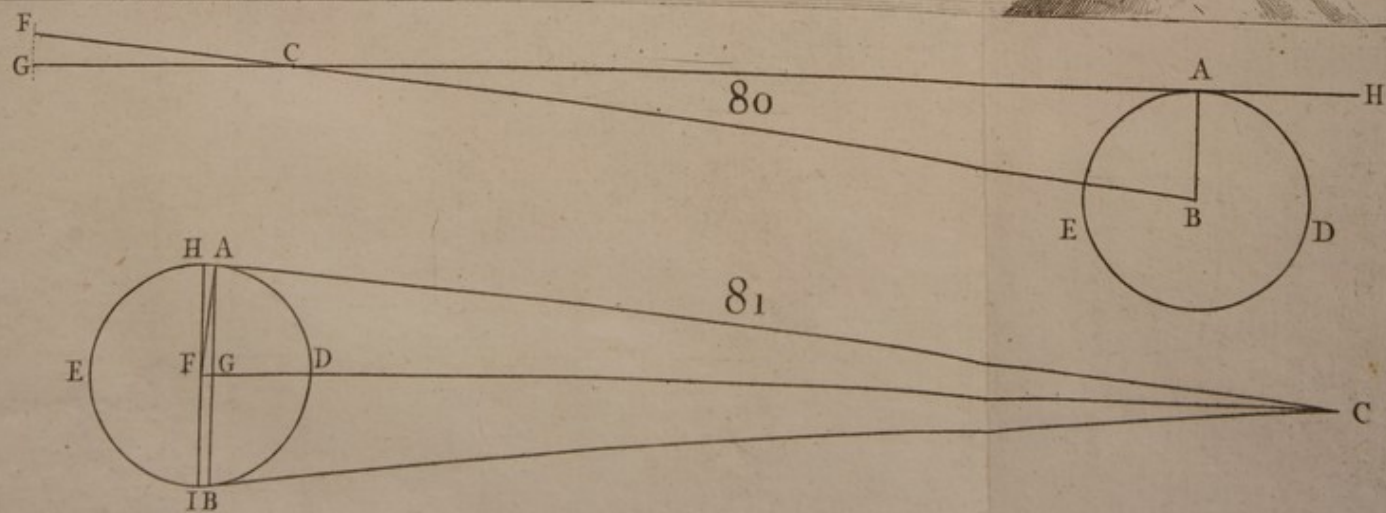
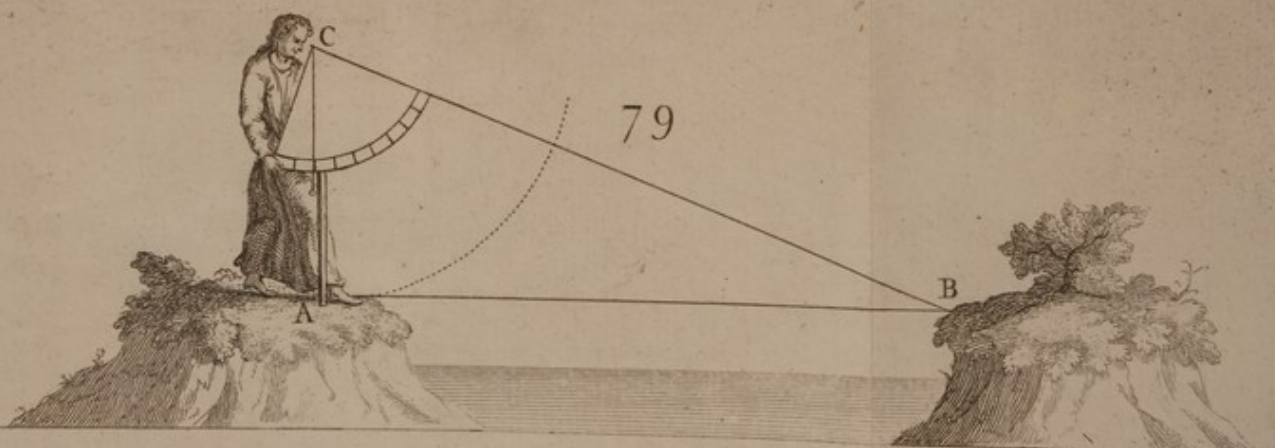
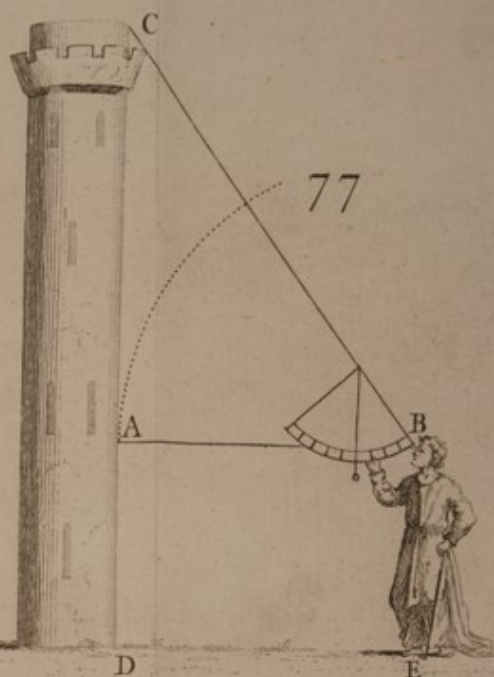
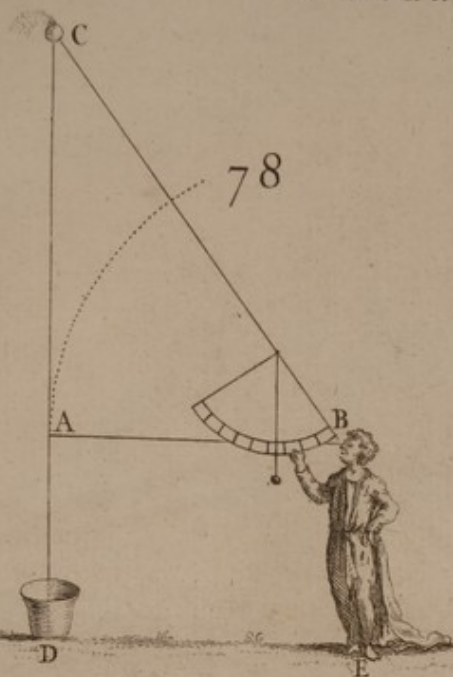
*141 All the angles of a right-angled triangle being given, the ratio between any two of its sides may be found: for every side may be considered either as radius, or sine, or tangent, or secant of a given angle, § 138 and 139; and the
 74 be found by the table, § 113: thus, fig. 74, the side BC being considered as radius, is 10000000; the leg BA as sine of the angle C, suppose of 50° , is found in the table 76604.41: the leg CA as cosine, or sine of the angle B of 40° the complement of 50° , is 64278.76: and the ratios between the sides stand thus;
 $BC : AB :: 10000000 : 76604.41$. and $BC : CA :: 10000000 : 64278.76$.

*142 If the ratio between any two quantities, and the real measure of one of
 74 them be given in numbers, the measure of the other quantity may be found by the golden rule: thus, fig. 74, if in the triangle BAC I find the ratio between the sides BC and BA is such that $BC : BA :: 100 : 76$, and that the actual measure of BC is 200 feet, then I may be assured that BA is 152 feet; for
 $100 : 200^{feet} :: 76 : 152^{feet}$.

*143 This proposition then in trigonometry is universally true, that in every right-angled triangle, if one acute angle and the length of one side be given, the whole triangle may be known: that is, we may find the quantity of every angle, and the length of every side.

*144 This proposition being here once demonstrated, will be referred to as an axiom, in other parts of this work: in the mean time it will not be amiss to give the reader a taste of the usefulness thereof, and of the tables of sines &c, in the following examples; wherein the method will be seen in which we may proceed by trigonometry, not only to measure the heights and distances of such objects upon the earth as are inaccessible, but also to discover the distances and magnitudes of the heavenly bodies.

Introduct.



*145 *Example 1*; suppose DC is an upright tower whose height is required, **FIG.** it may be found in this manner; with a chain or staff of a known length, ⁷⁷ measure a distance at pleasure from D the foot of the tower, to E where the station of the observer is to be, in the present instance let DE be taken 100 feet; let the observer standing at E look first, through the sights of a quadrant held horizontally, at the tower, and where the visual ray terminates make a mark A , which will shew the height of his eye above D the bottom of the tower, then keeping the center of the quadrant in the same place B , let him turn it, till through the sights he can see the top of the tower C : here is a triangle BAC , made by these three lines, *viz.* BA the distance of the observer's eye from the tower, CA the height of the tower above his eye, and BC a line drawn from his eye to the top of the tower; in this triangle, the angle A is a right one by the supposition, the angle at B is found by the quadrant, suppose it 60° , and the measure of one side AB is known, *viz.* 100 feet: from these *data* we may find, first, the ratio between AB and AC , after this manner; make BA radius, and AC , will, by § 140, be tangent of the angle B , which is by observation found 60° ; now in a table where the whole sine or radius is 10000000, the tangent of 60° is 173205.08; therefore $BA : CA :: 10000000 : 173205.08$. Secondly, the real measure of BA being 100 feet, AC will by the golden rule be found 173 feet with a fraction or part of a foot, which after reduction comes to a little more than two tenths of a foot. The operation in numbers stands thus, 10000000 : 100^{feet} :: 173205.08:

X100

$$1 \mid 0000000 \quad 173 \mid 2050800 \quad (173 \frac{2}{10} \mid \frac{050800}{000000})$$

AC being thus found 173 feet $\frac{2}{10}$, add thereto AD , equal to BE the height of the observers eye from the ground, which suppose to be 5 feet, and you have the whole height of the tower CD , 178 feet $\frac{2}{10}$.---I have in this example supposed E the ground upon which the observer stands and D the bottom of the tower to be upon a level; if one of these is higher than the other, allowance must be made for the difference between them, in assuming the height of the observers eye.---By the same method may the utmost height of a bomb or rocket thrown up perpendicularly be found, by an observer standing at E , **fig. 78**, a ⁷⁸ known distance, suppose 100 yards, from D the place where it is discharged, and looking at it through the sights of a quadrant when at its utmost height C : a view of the figure is sufficient, if compared with the last, for the triangle BAC is measured in the same manner as in the last figure; *viz.* the angle A is a right one by the work, the angle B is found by the quadrant, and the distance AB equal to DE is given in real measure, *viz.* 100 yards: so that if BA be made radius, AC will be tangent of the angle at B , &c.

*146 *Ex-*

*his eye
the quad.*

FIG. 146 *Example 2*; let it be required to measure the distance AB, which may
 79 be the breadth of a river or valley fig. 79; set up perpendicularly a stick CA of a known length, suppose it 4 feet two inches, in all 50 inches long; place the center of your quadrant at c the top of the stick, and look through the sights of it till the visual ray points at the object B, whose distance from A the place of the observer is to be measured: here is a right-angled triangle BAC, with one of its acute angles and one of its sides given; for the angle A is a right one by the supposition, the side AC is 50 inches by the same, the angle c is found by the quadrant, suppose it $89^{\circ} 30'$: make CA radius, and AB will be tangent of the angle at c, of $89^{\circ} 30'$, § 140 *case 2*: say then, as radius CA is to 50 inches, so is the tangent of $89^{\circ} 30'$ to the number of inches contained in AB: now in a table where the radius is 10000000, the tangent of $89^{\circ} 30'$ is 11458865.00, so that for an operation by the golden rule we have three numbers given, by means whereof the fourth number required, viz. 5729 inches and 4 tenths, the length of AB, is easily found; the work in natural numbers stands thus,

$$10000000 : 50^{\text{inches}} :: 11458865.00$$

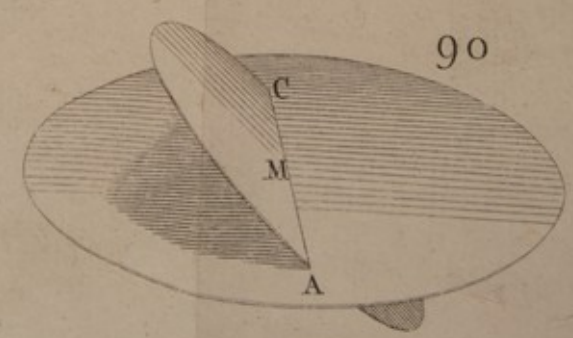
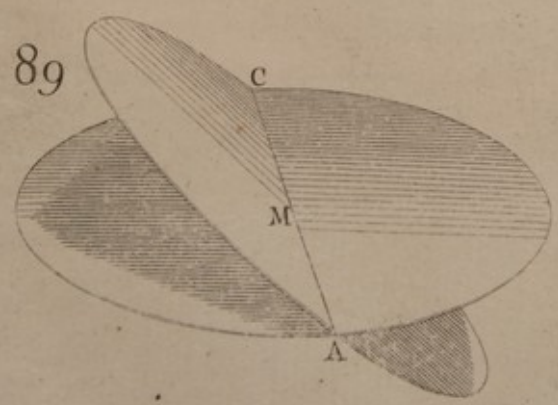
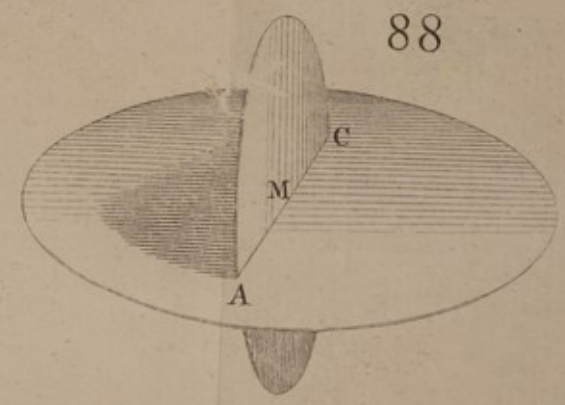
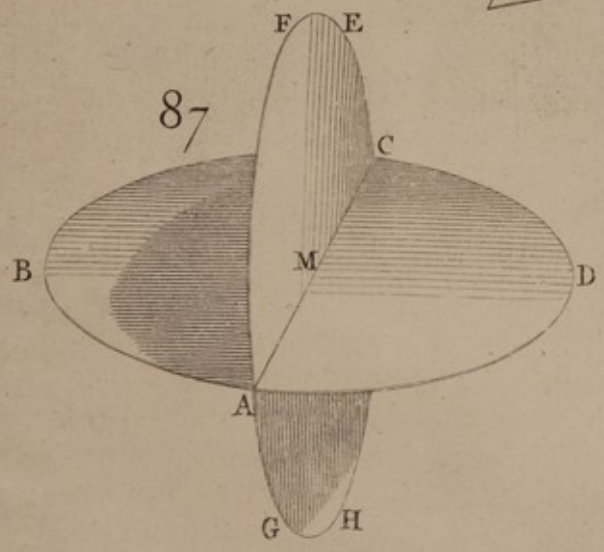
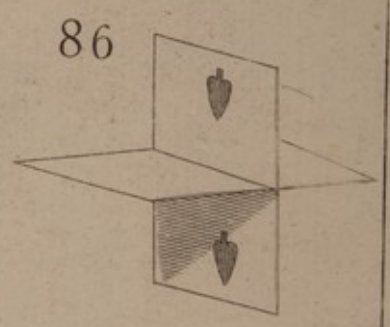
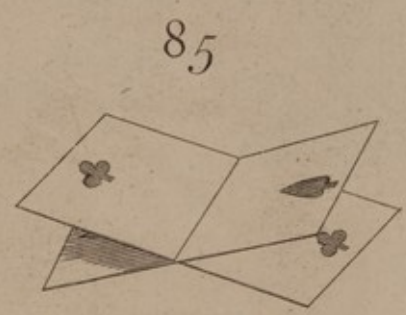
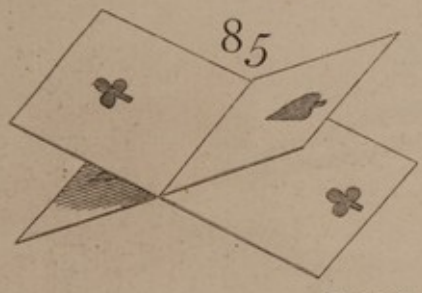
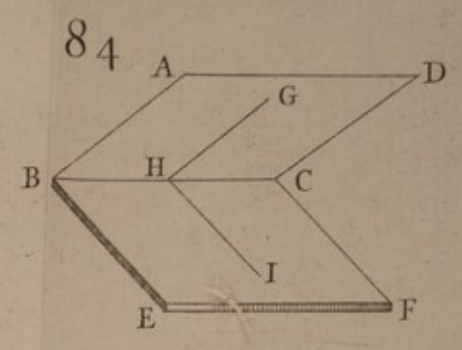
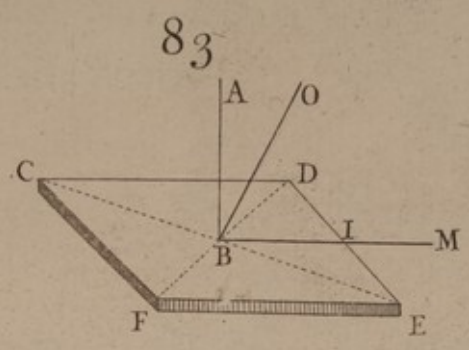
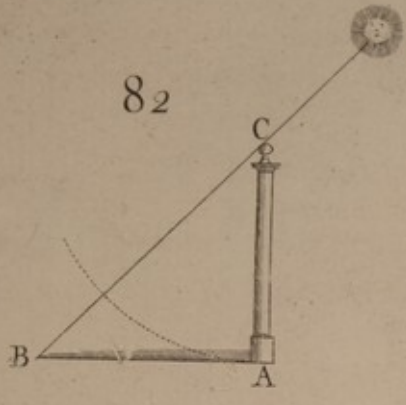
$$1 | 0000000 \quad) \quad \begin{array}{r} 50 \\ 5729 | 4325000 \end{array} \quad (\begin{array}{r} 5729 \frac{4}{10} | 325000 \\ 000000 \end{array}$$

these 5729 inches reduced to feet or yards, will give the distance of B from A 477 feet, or 159 yards, and 5 inches 4 tenths. I have in this example, to make it more easy, supposed the points A and B to be upon a level, as they must be if we would have the angle at A a right one: to shew how to find a distant point upon the same level with one that is near us, would I think be foreign to my present design.

80 . 147 *Example 3*; to find the moons distance from the earth; let ADE fig. 80, represent the earth, c the moon, A the place of the spectator observing the moon in his sensible horizon HC: here is a triangle BAC, made by AB a semidiameter of the earth, BC a line drawn from the center of the earth to the moon, and AC a line drawn from the place of the spectator to the moon; in this triangle the angle BAC is a right one, by § 28, *Eucl.* 3. 18: the angle ACB is the angle of the moons parallax, which, by methods hereafter to be taught, has been found to be $57'$; the side AB, by finding the circumference of the earth, the manner of doing which will also hereafter be shewn, is found to be in round numbers, 4000 miles: make BC radius, and then AB, will be sine of the angle c; say then, as BA sine of $57'$ is to 4000 the number of miles contained in the earths semidiameter, so is BC radius to a 4th number, viz. 241255, the number of miles contained in BC, the moons distance from the center of the earth; the sine of $57'$ is 165799; the proportion then stands thus,

165799

Introduct.



165799^{fine} of $57' : 4000^{\text{miles}} :: 10000000^{\text{rad}} : 241255^{\text{miles}}$

FIG.

in round numbers we may say the distance of the moon from the earth is 240000 miles.

*148 *Example 4*; to find the diameter of the moon; let $ADBE$ be the moon, fig. 81, let her be viewed by a spectator upon the earth at C , her apparent diameter is measured by the angle ACB , which, by methods hereafter to be taught, is found to be $30'$; half of this, *viz.* the angle ACF is therefore $15'$: here then is a triangle CAF whose angle CAF is a right one, *Eucl.* 3. 18, the angle ACF is $15'$ by observation, and the side CF the moons distance from the earth is known by § 147: from these *data* the semidiameter of the moon AF is thus found; in the triangle ACF make the hypotenuse CF radius, and AF will be the sine of the angle ACF , of $15'$: say then, as radius 10000000 to 240000, the number of miles contained in CF the moons distance from the earth, so is 43633 the sine of $15'$ to the number of miles contained in AF , the moons semidiameter: from these three numbers given, the fourth *viz.* 1047 miles is easily found by the golden rule: for $10000000 : 240000^{\text{miles}} :: 43633 : 1047^{\text{miles}}$. 1047 miles is then the moons semidiameter, and the double thereof 2094 miles her diameter.

*149 *Scholium*; by the figure before us it appears that we do not see an entire hemisphere of the moon HDI at one view from C , but only the segment ADB which is a little less than an hemisphere; notwithstanding this the preceeding calculation helps us to the true diameter of the moon HI : for the visual angle ACB and the half of it ACF being found, we do not proceed to find the measure of the chord AB or of its half AG ; what we deduce from thence is the measure of the line AF , which is a semidiameter of the moon, and consequently equal to half the line HI the moon's true diameter. I need not here add that the distance of the moon from us is so great that the segment of the moon seen by us differs not sensibly from an hemisphere as will be observed § 241.

*I have in these examples, to make the computation more easy, made use of round numbers instead of the true ones, *viz.* 4000 miles for the earths semidiameter, to find the moons distance, and 240000 miles for the moons distance, to find her diameter; for this reason the distance and diameter of the moon here found are a little different from the true ones: but that difference is not considerable, and I shall in the proper place give the true distance and diameter of the moon, deduced from the true semidiameter of the earth.

*150 *Example 5*; let it be enquired how far the shadow of an upright pillar will be extended at noon, upon a level pavement, the meridian altitude of the sun and the height of the pillar being given; suppose the meridian altitude of the sun is 50° , and the height of the pillar 100 inches: here is a triangle CAB

E

whose

FIG. whose angle at A is a right one by the supposition; the angle at B is 50° by the
 82 supposition also; therefore the angle C the complement of B is 40° § 55 Cor. 5; in this triangle if CA be made radius, AB will be the tangent of the angle C of 40° , § 140 case 2; we may say then, as radius to tangent of 40° , so CA is to AB; the tangent of 40° is 8390996, therefore $10000000 : 8390996 :: CA : AB$; but the measure of CA is given 100 inches; from these three numbers given, the 4th, viz. the length of AB in inches, is easily found by the golden rule to be 83 inches: for $10000000 : 100^{inches} :: 8390996 : 83^{inches}$.

• 151 In the foregoing examples I have made use of natural sines &c; it remains to shew the use of the other part of the tables; viz. the logarithms of the sines and tangents. Logarithms are numbers so contrived that by making use of them, the difficult operations of arithmetic may be performed by the help of more easy ones: thus, instead of multiplication in natural numbers, we may work by addition in logarithms; instead of division, by subtraction; instead of squaring or cubing a number, or raising other powers, we work by multiplication; instead of extracting the root of a square, cube, or other power, we use division.

• 152 There is usually at the end of the table of sines, another table in which any number within the compass of the table being given, its logarithm may be found; or the logarithm being given, its number may be found.

• If two numbers are to be multiplied together, add together the logarithms of the given numbers; and the sum of them is the logarithm of the product.

• If one number is to be divided by another, subtract the logarithm of the divisor from the logarithm of the dividend; the remainder is the logarithm of the quotient.

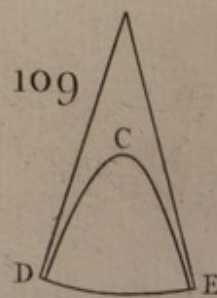
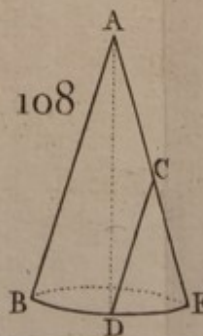
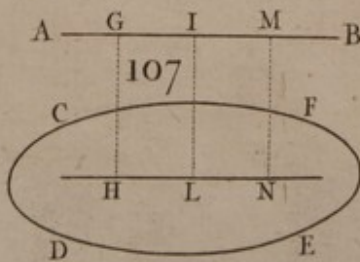
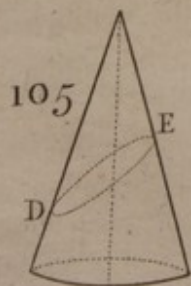
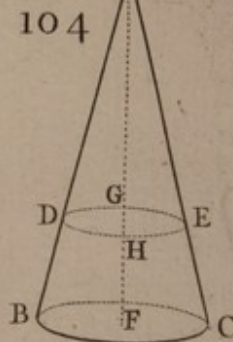
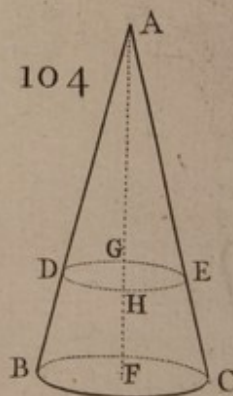
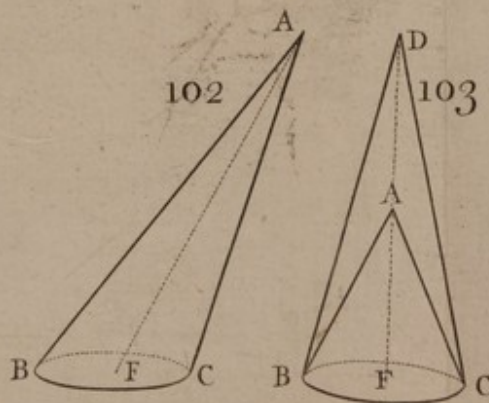
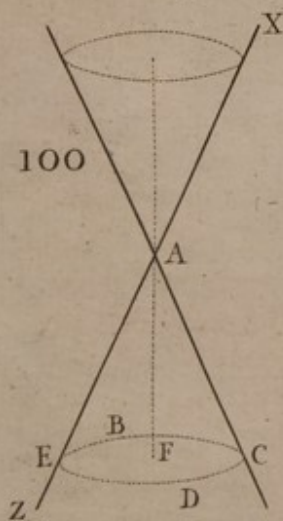
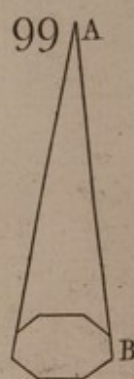
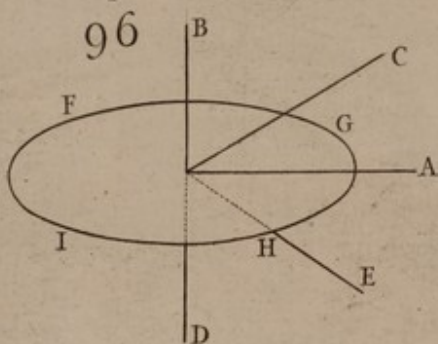
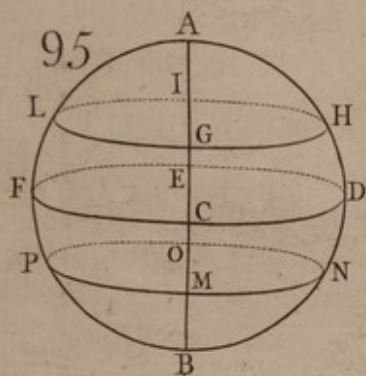
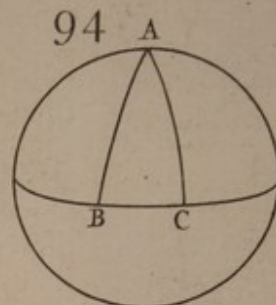
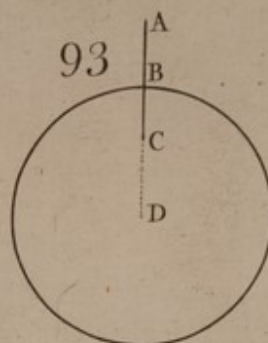
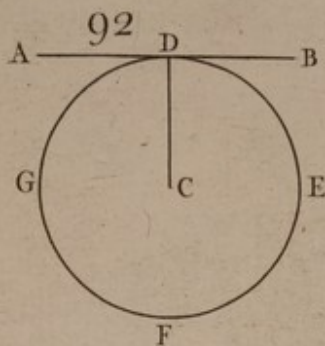
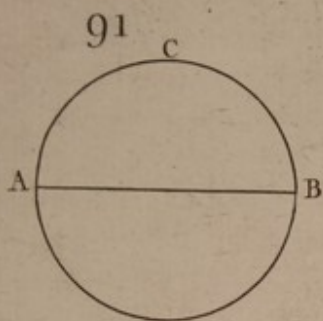
• From whence it follows, that in the direct golden rule, if you add the logarithms of the second and third terms together, and from the sum of them subtract the logarithm of the first, the remainder is the logarithm of the fourth term.

• The calculation of spherical triangles and of some oblique-angled plain triangles is made more easy, if we make use of logarithmic sines, and tangents instead of natural ones; I have said so much of them in this place, to explain the tables of which I gave a specimen: for a fuller account of logarithms see Wallis's *Algebra*, c. 12. Halley *Philos. trans.* N. 216. Pardie's *Geometr.* l. 5. Keil de *Logarithmis*. Sherwin's tables. &c.

153 If a strait line stands exactly upright upon a plane, as an upright pillar does upon a level pavement; that line is said to be *perpendicular to the plane*:
 83 thus, the line AB, fig. 83, is perpendicular to the plane CDEF, if it makes right angles with all the lines, as CE or DE, which can be drawn upon the plane through



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through the point B: a thread with a bullet at the end of it, hung over a table FIG. that stands exactly level, will be perpendicular to the surface of the table.

154 If a strait line stands obliquely upon a plane, as a pillar that leans does upon a level pavement; that *line* is said to be *inclined to the plane*, and the acute angle contained between the plane and the line where they approach nearest to one another is *the angle of the lines inclination*: thus, fig. 83, the line OB ⁸³ is inclined to the plane CDEF, and the angle of its inclination is OBI.

155 A strait line AB is parallel to a plane CDEF, and the plane CDEF is parallel to a strait line AB, fig. 107, when every part of the line is equally distant from the plane: that is, when all the lines which can be let fall from the line, perpendicular to the plane, as GH, IL, MN, are equal. ¹⁰⁷

156 A point is said to be in a plane produced, when the plane continued or extended would pass through that point: a point is said to be elevated above a plane, when a line drawn from the point to the center of the plane, is either perpendicular or inclined to the plane: thus, fig. 83, the point M is in the ⁸³ plane CDEF produced, but the points A and O are elevated above that plane: thus also, fig. 96, let FGHI be a circle viewed obliquely, the point A is in the ⁹⁶ plane of this circle produced, but the points, B, C, D, E, are all elevated above the plane of the circle: viz. B and C above one side of the plane, and D and E above the other side of it.

157 Parallel planes are such as, how far so ever they be extended, will never meet, but continue still equidistant: the distance between parallel planes is measured by strait lines drawn from one plane to the other, so as to be perpendicular to both of them; if these perpendiculars are all equal, the planes are parallel: thus, if a coach glass be throughout of the same thickness, the two surfaces of it are parallel planes; the opposite sides of a die are also parallel planes.

158 If one plane stands exactly upright upon another, as an upright wall does upon a level pavement, those planes are said to be perpendicular to one another, or to intersect each other at right angles. See fig. 86. ⁸⁶

159 If one plane stands obliquely upon another, as a wall that leans does upon a level pavement, it is said to *incline* to it, and the angle contained between those two surfaces of the planes which are nearest to each other, is called ⁸⁴ *the angle of their inclination*. See fig. 84 and 85. ⁸⁵

160 Any two planes ABCD and BEFC, fig. 84, which are not parallel, will, ⁸⁴ if both of them be extended, intersect each other in a right line BC, which is called *their common section*.

161 The inclination of two planes is measured by an angle contained between two right lines which are drawn upon the planes, perpendicular to

FIG. their common section, and meet in a point of it: thus, fig. 84, let there be two
 84 planes ABCD and BEFC, whose common section is BC; from any point thereof
 as H draw upon the planes the lines HG and HI perpendicular to BC; the angle
 GHI is the angle of inclination of those planes.

162 If two planes intersecting one another be imagined to move upon their
 common section, as the lid of a snuff box does upon its hinge, the wider the
 planes are opened asunder, the greater is the angle of their inclination, till they
 are opened so wide that one is perpendicular to the other. If two cards, or
 85 pieces of stiff paper be cut, each half through, and let one into the other, you
 86 may by opening them more or less represent the different inclinations of two
 planes: see the eighty fifth and eighty sixth figures.

163 Any two circles described upon parallel planes with their centers ex-
 actly opposite to one another, so that if a line were drawn perpendicularly to
 the planes through the center of one circle, that line if extended would pass
 through the center of the other circle, are *parallel circles*: circles drawn upon
 the opposite sides of a die with their centers exactly opposite are parallel. Cir-
 cles may be parallel whether they be equal to each other or not.

164 Any two *circles* are said to *incline* to each other which are drawn
 from the same center upon inclining planes, and the angle of their inclination
 is the same as that of the planes upon which they are drawn. Two *circles* are
perpendicular which are drawn from the same center upon perpendicular
 planes: two circles may be perpendicular or inclined, whether they be equal
 87 or not: the 87th figure gives a view of two equal circles, ABCD and EFGH,
 88 having the same center M, drawn upon planes perpendicular to each other: the
 89 88th figure shews two unequal and perpendicular circles: the 89th figure repre-
 90 sents two circles equal and inclined: and the 90th figure two circles unequal and
 inclined: in every one of these figures, AC is the common section of the planes
 of the intersecting circles, and M is their common center. Circles having dif-
 ferent centers may also be drawn upon inclining planes; but these fall not under
 our present consideration.

165 A *solid* or *solid figure* is that magnitude in which we may consider three-
 fold extension, *viz.* length, breadth, and thickness: every piece or particle of
 matter, of what shape soever, is a solid, because it is conceived as having this
 threefold extension, and it may be enquired how long, how broad, and how
 thick or deep it is; but we need at present consider only two or three of the
 regularly shaped solids, as a sphere, a cone, a cylinder, and a cube.

166 When geometers speak of a solid, they do not always mean that it
 should consist of gross material parts, as the word solid may seem to imply;
 pure space it self, though it be conceived perfectly empty and void of all matter,

has

has this threefold extension; and any part of infinite space, when we imagin FIG. it of any determinate largeness and figure, is a solid in geometry: thus, the inside of a vessel, whatever its shape be, may be called a solid, whether the vessel be full or empty; because it may be measured in length, breadth, and thickness, and its capacity found how much it may contain: so that when we speak of a sphere, cone, or cube, we may only consider a figure having threefold extension, of such a shape and size, without regarding whether it consists of matter or of pure space.

167 *A sphere or globe* is a solid every way perfectly round: the ivory balls used at a billiard table, and the marbles children play with are spheres or globes. A sphere and a globe signify originally the same thing, *viz.* a round body, in different languages; but custom has prevailed so, that when we speak of the bodies of the sun, moon, earth, planets, or stars, which we suppose to be round bodies, we call them globes; but when we speak of the heaven, in the concave surface of which the sun, moon, stars, and all the heavenly bodies appear to be placed, we call it *the sphere of the heaven, or of the fixt stars*: geometers also when they consider a round solid abstractedly, and the circles or other lines which may be imagined to be drawn upon the surface of it, whether considered as convex or concave, are said to treat of the sphere. We may imagin as many circles as we please drawn upon the convex surface of the globe of the earth, or upon the concave surface of the sphere of heaven: the circles of both these kinds, are called in books of astronomy *the circles of the sphere*, and an explanation of the nature and use of them in astronomy or geography is called *the doctrine of the sphere*.

168 *The formation of a sphere* is commonly thus explained; imagin a semicircle ACB, fig. 91, to turn round upon its diameter AB, till it comes to its first 91 situation, and it will describe a sphere: the center of this semicircle will also be the center of the sphere thus described by it. Any strait line drawn through the center of a sphere, and terminated at each end by the surface of it, is called *a diameter*. Any strait line drawn from the center to the surface, is *a radius*, or *semidiameter of the sphere*. All radius's or semidiameters of the same or equal spheres, are equal; as are also all their diameters: this is evident from § 25.

169 *A great circle* is that which divides the sphere into two equal parts: *a less circle* is that which divides the sphere into two unequal parts: fig. 95. 95

170 Any strait line which touches a sphere on its convex surface, in such a manner that the line, if extended both ways, would not enter into the sphere, is called *a tangent to the sphere*. Any radius of a sphere drawn to the point of contact, is perpendicular to the tangent: in fig. 92, AB is a tangent to the 92 sphere DEFG; and CD drawn to the point D is perpendicular to AB. Any tangent to a great circle of a sphere is a tangent to that sphere. 171 Any

FIG. 171 Any strait line AB or CB, fig. 93, standing exactly upright either upon the
 93 concave or convex surface of a sphere, is said to be *perpendicular to the sphere*:
 such a line continued through the sphere would pass through the center of it;
 and the converse of this is true, that any strait line drawn from the center to
 the spherical surface, or through it, will be a perpendicular.

172 If the arcs of three great circles be drawn upon the surface of a sphere,
 94 so as to meet in three points, they will form a *spherical triangle*, as ABC, fig. 94.

94 173 The measure of a spherical angle BAC, fig. 94, is the arc BC of a great
 circle described from the angular point A, intercepted between the sides AB
 and AC continued to quadrants.

174 The common *terrestrial globe* represents very well the earth and sea,
 the situation of mountains, rivers &c, with the extent of kingdoms, and states;
 and has besides several circles drawn upon it, to represent such as we may con-
 ceive to be drawn upon the spherical surface of the earth it self.

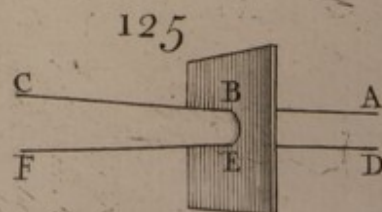
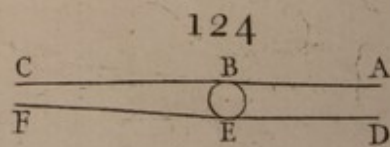
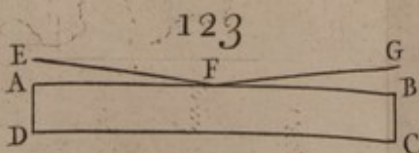
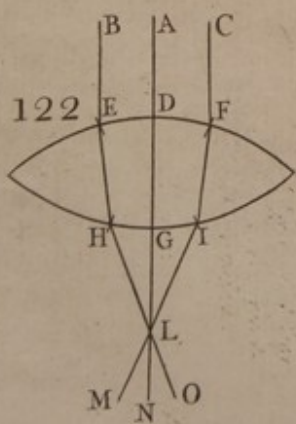
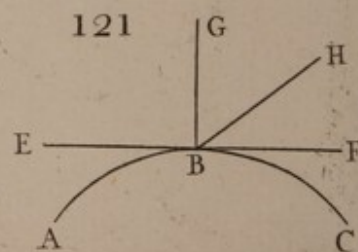
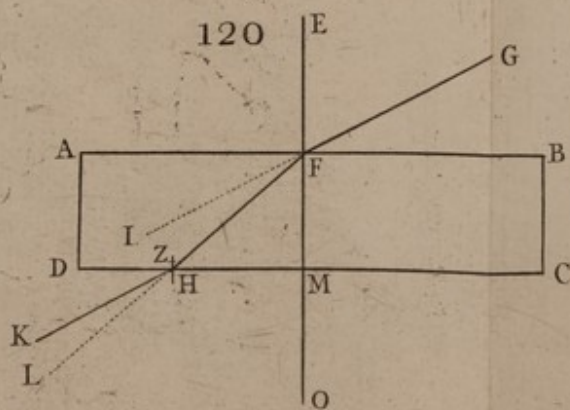
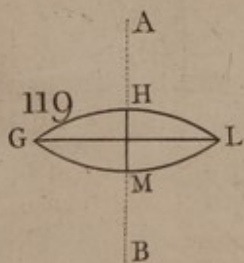
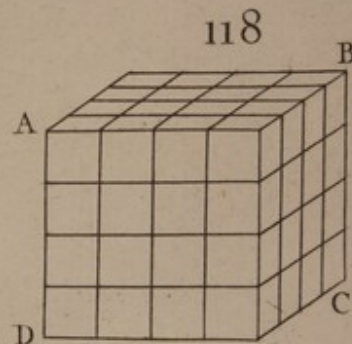
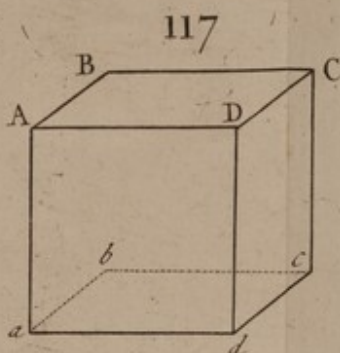
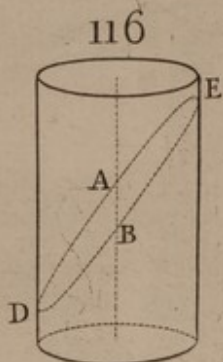
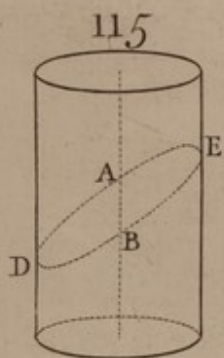
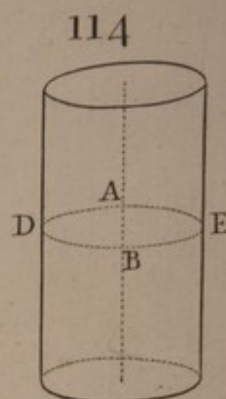
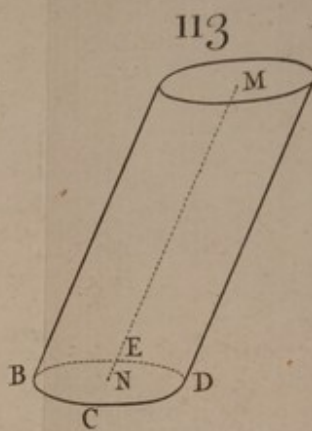
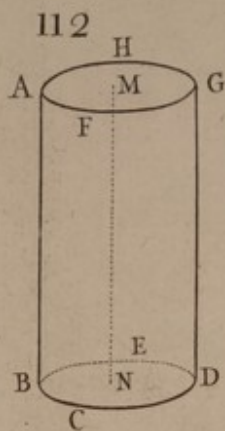
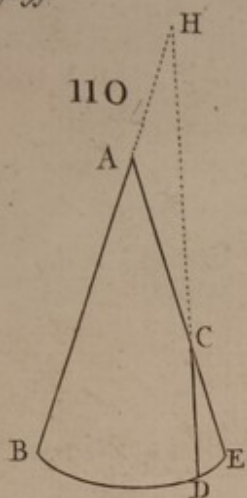
175 The common *celestial globe* represents the convex surface of the sphere
 of the heaven, such as it would appear to us, if we were placed without, at an
 immense distance from it; and has several circles drawn upon it, to represent
 some which may be imagined to be drawn upon the sphere of the heaven. *An*
artificial sphere is a machine which has the principal circles of the sphere
 drawn upon hoops, or represented by rings of brass or other metal. The circles
 of the sphere may also be drawn upon the surface of a hollow sphere made
 of glass: such an one I have for many years made use of in reading lectures.

95 176 A sphere ADBF, fig. 95, may be considered as put into such a whirling
 motion round one of its diameters AB, as boys give a top; this motion is call-
 ed a *rotation*, because it is like the turning of a wheel round its axletree: the
 diameter AB, round which the rotation of the sphere is made, is called *the*
axis: the two extreme points of the axis A and B, terminated by the surface
 of the sphere, are *the poles of the sphere*: by this rotation, every point of the
 spherical surface, except the poles, describes a circle; thus, the point G describes
 the circle GHIL, the point C describes the circle CDEF, the point M describes
 the circle MNOP: the circle CDEF described by a point C equally distant from
 both poles, is a great circle; to this the other circles are all parallel: every pa-
 rallel grows less, the nearer the point describing it is to either of the poles.
 Any circle may be conceived as a great circle drawn upon a sphere, and to
 have its axis and its poles. I have in the figure supposed the sphere to be trans-
 parent, that the circles drawn upon it might be seen entire.

177 Any two great circles of the same sphere which intersect each other,
 will divide one another into two equal parts. *Theodos. Spheric. l. 1. prop. 11.*

178 If a plane be imagined to pass through a sphere, *the section of the sphere*,
 that

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that is, the curve described upon the plane by the surface of the sphere passing through it, will be a circle: if the plane which cuts through a sphere passes through the center of it, the section of it is a great circle of that sphere; if the plane does not pass through the center, the section is a less circle: a plane that cuts through a sphere, and passes through the center, divides it into two equal parts, which are called *hemispheres*, or half spheres: a plane that cuts through a sphere, and does not pass through the center of it, divides it into two unequal parts, either of which, especially the least, is called a *segment of a sphere*: thus, fig. 95, AHGLI is a segment of the sphere ADBF.

179 If we imagin a strait line AB, figg. 97, 98, 99, one end of it A being fixt, to be carried round, all the while touching the sides of a triangle, square, or polygon, above whose plane the point A is elevated; it will describe a solid called a *pyramid*: the *vertex* of the pyramid is the immoveable point A, the *base* of it is the triangle, square, or polygon, round which the line is supposed to be carried.

180 If we take an immoveable point A elevated above the plane of a circle BCDE, and suppose a strait line xz drawn through the point, and extended both ways from it to an indefinite length, to be carried quite round the circle, all the while touching its circumference, and continuing still fixt to the immoveable point, the line by this motion will describe two *conic surfaces*, which are *vertical*, or opposite, having their common vertex at the immoveable point A, fig. 100.

181 The solid contained within the conic surface, between the immoveable point A and the circumference of the circle BCDE, is a *cone*. fig. 100: the immoveable point A is the *vertex*: the circle BCDE, is the *base*: and a strait line AF drawn from the vertex to the center of the base, is the *axis of the cone*: all strait lines drawn from the vertex to the circumference of the base, as AB, AC, AD, AE, &c. are *sides of the cone*.

182 If the axis of a cone be perpendicular to its base, it is called a *right cone*, such as is represented, fig. 101: if the axis be inclined to the base, it is called a *scalenous* or *oblique cone*, such as is drawn, fig. 102: a right cone is always understood when the contrary is not exprest.

183 The formation of a right cone may be conceived in another manner, by the revolution of a right-angled triangle round one of its legs, as upon an axis: thus, fig. 101, let there be a right-angled triangle BFA, let one of its legs AF be fixt, and let the triangle revolve round it; there will then be described the cone ACB, whereof AF the fixt leg of the triangle will be the axis, and the circle BECD described by the other leg BF will be the base.

184 A cone described as in the preceding section will be more or less acute,

FIG. 103. cute, according as the acute angle BAF adjoining to AF the immoveable leg of the describing triangle is more or less acute: thus, fig. 103, the cone BDC described by the revolution of the triangle BFD is more acute than the cone BAC described by the revolution of the triangle BFA ; because the angle BDF is less than the angle BAF .

185 The acuteness of a cone may also be thus estimated; from the extreme points of any diameter of the base, as BC , fig. 101, draw strait lines to the vertex A , as BA and CA , these will form a triangle BAC , by whose angle at the vertex A , the acuteness of the cone is measured; if two cones of equal bases are of different length, the longer is said to be more acute than the other: thus, fig. 103, the cone BDC is more acute than the cone ABC , because the angle BDC is less than the angle BAC .

186 *Similar cones* are such as have their axes and the diameters of their bases proportional: all right cones having the same acuteness are similar: and consequently, if any right cone ABC , fig. 104, be cut through by a plane DE parallel to the base, the segment ADE will be a cone similar to the cone ABC .

187 Vertical cones whose bases are parallel, are similar: this is evident from a view of the 100th figure, and needs not be demonstrated.

188 If a plane be imagined to pass through all or part of the sides of a cone, the curve line described upon the plane by the sides is called a *conic section*.

189 If all the sides of a cone are cut through by a plane, to which the axis is perpendicular, the section of the cone is a *circle*: thus, fig. 104, if the cone BAC be cut through by a plane DE to which the axis AF is perpendicular, the section is the circle $DGEH$.

190 If all the sides of a cone are cut through by a plane to which the axis is inclined, the conic section is called an *ellipsis*. fig. 105, and 106.

An ellipsis made by the section of the same cone, or of two cones that are similar, is more oblong, the less the angle is which the axis of the cone makes with the intersecting plane: thus, the ellipsis DE is more oblong in fig. 106, than the ellipsis DE in fig. 105.

191 If a cone is cut through by a plane to which one of the sides of the cone is parallel, the section is called a *parabola*: thus, fig. 108, let the cone ABE be cut through by a plane DC to which the side AB is parallel, the section will be a parabola, such as is represented, fig. 109, by the curve DCE .

192 If a cone is cut through by a plane to which one of the sides of the cone is inclined in such a manner, that the plane and the side extended both ways from the base of the cone would meet in a point beyond the vertex of it, the section is called an *hyperbola*: thus, fig. 110, let the cone ABE be cut through by the plane DC to which the side BA is so inclined, that both of them extended

extended both ways from the base would meet somewhere beyond the vertex, FIG. in the point H, the section will be an hyperbola, such as is represented, fig. 111, by the curve DCE.

193 If a strait line, which is either perpendicular or inclined to the plane of a circle BCDE, fig. 112, be carried round the circle, all the while continuing parallel to itself in its first situation, and touching with one end B the circumference of the circle, the line will describe a *cylindrical surface*: the circle BCDE is *the base*; to this the point A at the other end of the line will describe another circle equal and parallel, viz. AFGH, which may be also called a base: the solid contained between the cylindrical surface and the planes of these two bases, is called a *cylinder*: a strait line MN drawn from the center of one base to the center of the other, is *the axis of the cylinder*: the describing line AB in every different situation as AB, GD, &c is a *side of the cylinder*.

194 If the axis of a cylinder is perpendicular to the base, it is a *right cylinder*, fig. 112: rolling stones used in gardens are right cylinders. If the axis of a cylinder is inclined to the base, it is a *scalenous* or *oblique cylinder*, fig. 113.

The formation of a right cylinder is by some explained, by imagining a rectangle to revolve round one of its sides: thus, fig. 112, if we imagin the rectangle ABMN to turn round the side MN, that side being all the while fixt, there will be described the cylinder ABDG.

195 If a plane DAEB to which the axis is perpendicular cuts through all the sides of a cylinder, the section arising from thence is a circle, fig. 114.

196 If a plane to which the axis is inclined cuts through all the sides of a cylinder, the section is an ellipsis, figg. 115, 116: this ellipsis is more oblong, the less the angle is which the axis makes with the intersecting plane: thus, the ellipsis DAEB in fig. 116, is more oblong than the ellipsis DAEB in fig. 115.

197 A *Cube* is a solid terminated by six sides, every one of which is an exact square: such is a well made die, called in Greek *cubos*, from whence the word cube is derived.

198 The formation of a cube may be thus conceived; imagin a square ABCD, fig. 117, to be carried parallel to it self, the length of one of its sides, as along the line Aa equal to AD, when this square has passed from its first situation ABCD, and is come into the situation abcd, it will have described the cube ABccda. The side or root of the square by which the cube is generated is called also *the root of the cube*.

199 If every side of a cube is a foot square, the cube is called a cubic foot: if a side is an inch square, it is a cubic inch, &c: the measure of any cube is exprest by saying how many cubic feet, cubic inches, &c it may be divided into.

200 To find how many cubic feet, inches, &c are contained in a cube, find
F how

FIG. how many square feet, inches &c are contained in one of the sides, and multiply that number by the square root; thus, let a side of the cube ABCD, fig. 118, be 16 inches square, the root or side of the square 16 is 4, multiply 16 by 4, the product 64 is the number of cubic inches in the cube ABCD.

201 If a square number be multiplied by its square root, the product is a cube number; which is so called, because if we imagin every unit of which the product consists to be represented by a little cube, the whole product may be represented by a larger cube containing all those little ones: thus, if the number 16 the square of 4 be multiplied by 4 the square root of it, the product 64 may be represented by the cube ABCD, fig. 118, containing 64 little cubes.

202 As a square is the measure of all surfaces, § 62, so a cube is the measure of all solids, the dimensions of which are exprest by saying how many cubic feet, cubic inches, or cubic tenths of an inch, &c they contain: thus, I may consider and exprest how many cubic feet are contained in a mountain, or in the globe of the earth, or in the sun or moon, or in any planet or star.

203 How to measure either the superficial or solid content of a cone, cylinder, or any other regular solid, may be seen in the common books of geometry^a; a sphere or globe is the only solid, besides the cube, whose mensuration we shall have occasion to consider at present: the way to measure the cube has already been shewn, § 200; how to find the dimension of the globe will be seen in the two following sections.

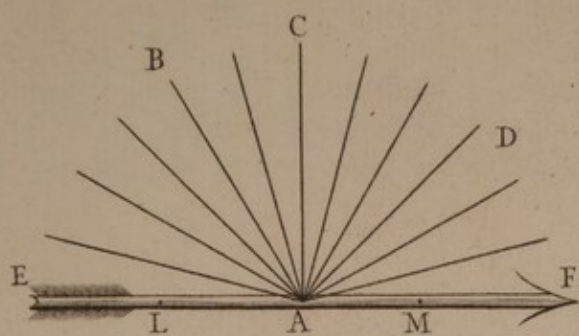
204 To find the superficial content of a globe, the circumference or diameter being given; find the area of a great circle of the globe by § 68, and multiply that area by the number four, the product is the superficial content required: thus, suppose the circumference of the earth to be 24930 english miles, the diameter 7935, § 457; the area of a great circle upon the earth will be 49454887½ miles: the product of this number multiplied by 4 is 197819550, which is the number of square english miles contained in the surface of the earth.

205 To find the solid content of a globe, the superficial content being given; multiply the superficial content by a sixth part of the diameter of the globe, the product is the solid content required: thus, suppose the superficial content of the globe of the earth be found 197474980 square english miles, by the preceding section; multiply this number by 1322½ the sixth part of 7935, the number of english miles the diameter of the earth amounts to, and the product 261616354875, is the number of cubic english miles contained in the globe of the earth.

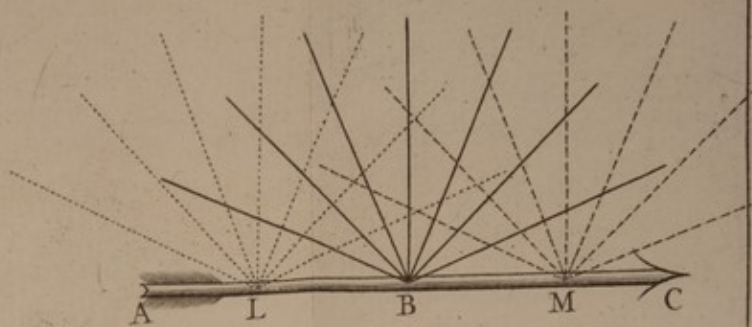
The following propositions about the nature of light, refraction, and vision are so useful in astronomy, that I shall make no apology for inserting them in this place.

^a Tacquet Geometria practica l. 3. c. 18, Pardie Geometr. l. 5.

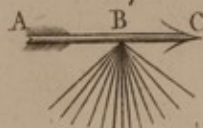
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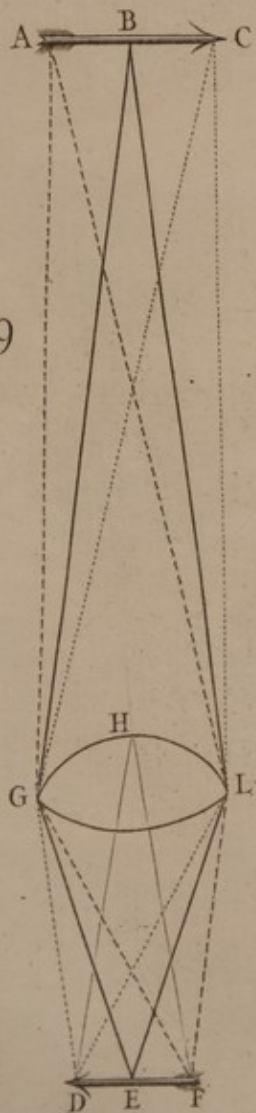
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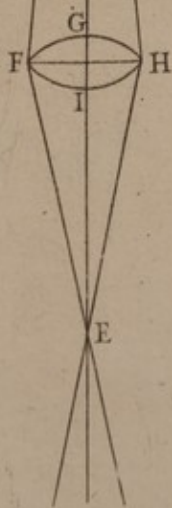
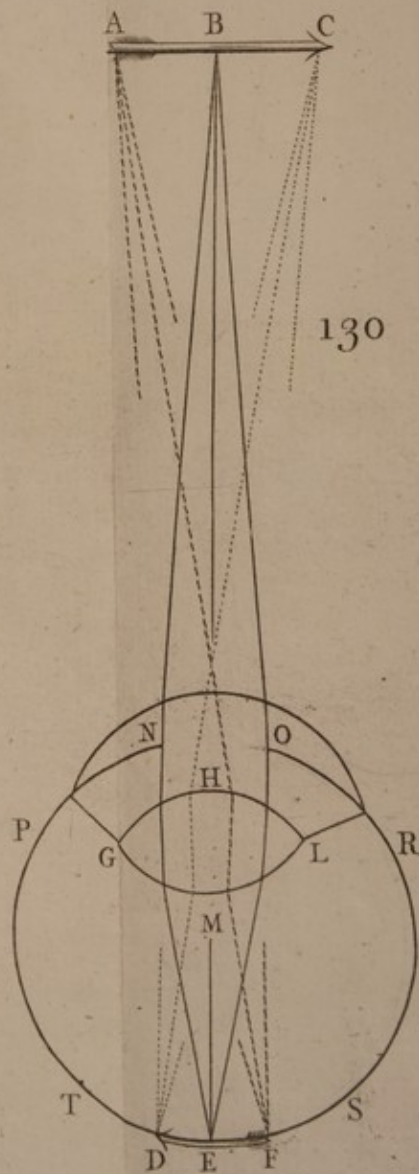
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206 *Light* consists of exceedingly small distinct parts of matter^a, which FIG. moving with a prodigious swiftness enter our eyes, and impress upon the internal parts of them that motion which excites in our minds the sensation we call seeing: light either comes directly from a luminous body, as the sun, a star, fire, &c; or else being emitted from a luminous body, and falling upon an opake one, is from thence reflected, or thrown back, and scattered every way: luminous bodies are incessantly streaming out these particles of light; opake bodies continue to reflect light no longer than they receive it from luminous ones.

207 Though the progressive motion of light be exceedingly swift, yet it is not instantaneous, but it spends some time in passing from one place to another: what the velocity of light is, will be shewn in a proper place.

208 Light may be considered either as contemporary or successive^b: the *light* which falls upon an object all at once, in the same instant, is *contemporary*; the *light* which from time to time continues to fall upon an object, is *successive*, consisting of parts following one another, as the parts of water do that are forced in a continued stream out of the spout of an engine.

209 The least light or part of light which can be conceived to act or be acted upon alone by itself, is called *a ray of light*.

210 Empty space, or any body that is transparent, that is, through which any rays of light can pass, as air, water, oil, &c, is called *a medium*.

211 *A lens* is a solid, convex on both sides, shaped like a lentil, from whence it has its name: each side of a lens is a segment or piece of a sphere: the sides of a lens may be segments of equal spheres, or of unequal ones: when we speak of a lens in optics, we generally mean a piece of glass of this figure; such as a burning-glass, or a spectacle-glass: a lens may be made of water, or any other transparent liquor, by inclosing it between two thin concave pieces of glass: the chryselline humour in the eye is a lens formed of that liquor which constitutes it, inclosed between the coats of the eye. If a circular plane be imagined to divide the segments of spheres which form a lens, that plane may be called *the section of the lens*: *the axis of a lens* is a line drawn from the middle point of one of its convex surfaces to the middle point of the other: the axis of a lens extended both ways would pass through the centers of the spheres whereof the convex surfaces of the lens are segments: a lens is turned directly towards an object, when its axis if extended would fall upon the middle point of the object: in fig. 119, GHLM is a lens; HM the axis: GL the 119 section, A and B the centers of the spheres whereof the surfaces of the lens are segments: in fig. 127, the lens F G H I is turned directly towards the object ABC. 127

^a *Newt. Optics* p. 345.

^b *Newt. Opt.* p. 2.

FIG. 212 The rays of light, in passing through a medium that is throughout of the same density, go in strait lines. If a ray of light goes out of one medium into another of a different density, as out of air into glass, or out of glass into air, if it falls perpendicularly upon the surface of the second medium, it will continue to go on in a strait line, in the same direction as through the first medium: thus, fig. 120, let $ABCD$ be a solid piece of glass, a ray of light EF coming out of air, and falling perpendicularly upon the surface of the glass AB , will go on in a strait line EFM ; in like manner, the ray going out of glass into air at M , in the direction FM , perpendicular to the surface of air CD , will go strait on in the direction FMO .

213 If a ray of light, coming out of one medium into another of a different density, falls obliquely upon the second medium, it will be bent at the point of incidence, and go through the second medium in a different direction: this bending of the rays is called *refraction*. The laws of refraction are these; if a line be drawn perpendicular to the surface of the second medium, a ray going out of a thinner medium into a more dense one, is refracted towards the perpendicular; but a ray going out of a denser into a thinner medium, is refracted from the perpendicular: thus, fig. 120, a ray GF falling obliquely upon AB , will be refracted at F , and since glass is denser than air, it will be bent towards the perpendicular FM , and instead of the strait line FI , will go on in the refracted line FH ; again, at H is its point of incidence out of glass into air, out of a denser into a thinner medium, there it will be bent from the perpendicular HZ , and instead of going on in its former direction FHL it will go in the refracted line HK .

214 In the foregoing example, the surface of the second medium was supposed to be a plane, when it is a curve surface, we consider it in the following manner; a line which, being drawn to the point of contact, is perpendicular to the tangent of a curve, is perpendicular to that curve; a line which, being drawn to the point of contact, makes oblique angles with the tangent, makes the like angles with the curve: thus, fig. 121: let the line EF touch the curve ABC in the point B , if GB is perpendicular to the tangent, it will be also perpendicular to the curve; let the line HB fall obliquely upon the tangent, it will fall upon the curve with the same obliquity; that is, the mixt angle CBH is equal to the rectilinear angle FBN . See *Molyneux dioptr.* p. 6.

215 A ray of light passing out of one medium into another of a different density, when the surface of the second medium is curve, is subject to the same general laws of refraction, as when it is a plane: if it falls perpendicularly upon the surface of the second medium, it will continue to go on in a strait line, in the same direction as through the first medium; if it falls obliquely upon

upon the surface of the second medium, it will be refracted at the point of incidence, so as to be bent towards the perpendicular, when it passes out of a thinner medium into a more dense one; and from the perpendicular, when it passes out of a denser into a thinner medium: thus, fig. 122, let $EFIH$ be a lens of glass, upon the middle point D of its convex surface, let the line AD fall perpendicularly, it will then pass on in a strait line DG coinciding with the axis of the lens, and again at G , where it leaves the glass, falling perpendicularly upon the surface of the air HGI , it will continue to go on in the same direction, in the line GL ; so that the passage of this ray, first out of air into glass, and then out of glass into air, will be in the strait line $ADGL$.

Let other rays BE , CF , parallel to AD , fall upon the same lens at the points E and F , these will fall obliquely upon the convex surface of the glass, the denser medium, and therefore will be bent towards the perpendiculars, which are here express'd by short dotted lines, and instead of going strait on in their first direction, will go through the glass in the directions EH and FI ; at H and I falling obliquely upon the concave surface of the air, they will be again refracted and bent from the perpendicular, so as to go in the directions HL and IL .

216 From hence it appears, that parallel rays, as BE , AD , CF falling upon a lens $EFIH$ turned directly towards them, will be refracted so as to converge or draw nearer to one another, till they meet in a point D , fig. 122: there they will cross, and from thence proceed *diverging*, or spreading further asunder, as is express'd by the lines LM , LN , LO . *Molyn. dioptr. p. 6. For the physical cause of these refractions, See Newt. Optics, p. 248. 320. 324. Smith's Optics, l. 9. c. 7.*

217 When a ray of light, in passing out of one medium into another of a different density, falls very much inclined to the surface of the second medium, that is, makes a very small angle with it, it will not enter the second medium, whether it be thinner, or more dense than the first; but it will be reflected, in such a manner, that the angle of reflection shall be equal to the angle of incidence: thus, fig. 123, let a ray EF fall at the point F upon AB the surface of a different medium, so as to make the angle EFA a very small one, the ray then will not enter the second medium $ABCD$, but will be reflected in the line FG , so that the angle of reflection GFB shall be equal to the angle of incidence EFA . *if it be thinner & first thinner*

218 I here consider refraction and reflection of light at large and in a general view, as if all the rays of light were alike affected in passing through different mediums, which accurately speaking they are not; for with the same angle of incidence some rays are more refracted, that is, bent further from the strait line than others, and when they fall very much inclined to the second medium some are sooner reflected than others, that is, with the same angle of incidence

FIG. incidence some rays shall be reflected and others not, but they will require to have the the angle of incidence still less, to make them reflected; this is the meaning of this proposition, *the light of the sun consists of rays differently refrangible: Newt. Opt. p. 21*: and of these propositions, *the suns light consists of rays differing in reflexivity*; and *those rays are more reflexible than others which are more refrangible*: these and many other curious particulars relating to the nature of light, I shall not dwell upon here, as not pertinent to my subject; it is sufficient just to mention some of the most remarkable ones.

219 Light is not refracted or reflected by impinging upon the solid parts of the refracting or reflecting medium, but by a power uniformly diffused through the medium, which acts upon the particles of light, without immediate contact, in a direction perpendicular to the surface of the medium: *Newt. Opt. p. 237.*

220 Light in passing close by the sides of bodies is bent out of the way, in such a manner, that parallel rays passing on each side any body, converge; and 124 passing between two bodies, diverge: thus, fig. 124, let BE represent an hair viewed endways, parallel rays of light AB and DE passing by the sides of the hair will not go on in strait lines, but will be bent, and made to go on in the 125 converging lines BC and EF: again, fig. 125, let BE be an hole in a plate of brass, parallel rays passing through the hole will not go on in strait lines, but will be bent, and made to go on in the diverging lines BC and EF. *Newt. Opt. p. 293 & seqq. & 313. Qu. 2.*

221 Rays of light of different degrees of refrangibility, reflected from opaque bodies to our eyes, excite in us sensations of different colours: when those rays which are least refrangible are reflected by any body, alone, or in much the greatest quantity, the rest being totally or in a great measure absorbed, that body appears red; when the most refrangible rays are reflected alone, or in the greatest quantity, the other rays being absorbed, the reflecting body appears of a violet colour; we say then, that the red-making rays are least refrangible and the violet-making rays are most refrangible, and the rays which produce intermediate colours, orange, yellow, green, blew, are of intermediate degrees of refrangibility: probably the particles which constitute the red-making rays are the largest, and those which constitute the violet-making ones the smallest, and the intermediate ones are of intermediate magnitudes, whereby different vibrations are excited in the retina, which give us sensations of different colours: *Newt. Opt. p. 319, and 320.*

222 The several colours just now mentioned are called *primary*, and the rays which produce a primary colour are called *homogeneal* or *similar*: all other colours arise out of different mixtures of the primary colours: whiteness is a mixture of all colours; and light the mixture of all the several sorts of rays which

which are capable of producing the different primary colours. *Newt. p. 217.* FIG.

223 Every point of a visible object sends from itself, as from a center, rays of light in strait lines, every way through an hemisphere: thus, fig. 126, the rays of light are diffused every way from the point A in strait lines AB, AC, AD, &c through the hemisphere EBCDF; this is evident from hence, that wherever a spectator is placed within this hemisphere, so that a strait line can be drawn from A to his eye, the point A will be visible to him; it is for this reason the same object may be visible to an hundred people at the same time: we are here to understand by a point of an object, not a mathematical but a *physical point*; that is, the least imaginable part of an object, or the least part that can reflect light enough to make itself visible: the light is in like manner diffused from all other points of the object as L and M. See fig. 128.

224 An object is said to be distant from a lens, when the surface of the lens which is towards the object is small, compared with the distance of the object: an object is distant from the eye, when the aperture of the pupil is small, compared with the distance of the object: as suppose the diameter of the pupil is but about an hundred and fiftieth part of the distance of the object.

225 All the rays which can be drawn from any one point B, fig. 127, of an object ABC, distant from a lens FH, to that surface of it FGH which is towards the object, may be looked upon as *physically parallel*: thus, the rays BF and BH are so near being parallel, that they will be refracted in the same manner as if they were exactly parallel: from hence, and from § 216, it follows, that,

226 The rays which come from the same point of an object, and fall upon the surface of a lens turned directly towards the object, unite in a point beyond the lens: thus, fig. 127, let the object be ABC, the lens FGH the rays which come from the point B, and fall upon the surface of the lens FGH, will be so refracted at their passage out of air into glass, and out of glass into air, that they will be made to converge so as to meet somewhere at a point, as at E.

227 The whole parcel of rays which thus coming from any point B fall upon the surface of the lens, and passing through it are united in a point E, are called a *pencil of rays*: a pencil of rays is a double cone of rays, the common base whereof is the section of the lens FH: the vertex of one cone is B the radiating point of the object, the vertex of the other cone is E *the tip of the pencil*: the axis of a pencil rays is a line BE drawn from one vertex to the other: the axis may be considered as a strait line. *Molyneux dioptr. p. 8.*

228 The rays of light coming from different points of the same object cross each other in the manner represented fig. 128, this they do in innumerable places, without hindring one anothers progress, by reason of the inconceivable smallness

FIG. smallness of the particles of light: this may be looked upon as one of those
 93 many wonders which we meet with, in our searches after natural knowledge.

229 An object is seen when the rays of light coming from all parts of it enter the pupil of the eye, and paint its picture upon *the retina*^a, which is a thin membrane at the bottom of the eye: the following experiments are proofs of this, *exper. 1*, take out the eye of an ox or sheep new killed, and strip off
 94 carefully the skin and fat from the back part of it, till only a thin film is left there; upon this film the picture of any object placed before the pupil of the eye will be painted in little.

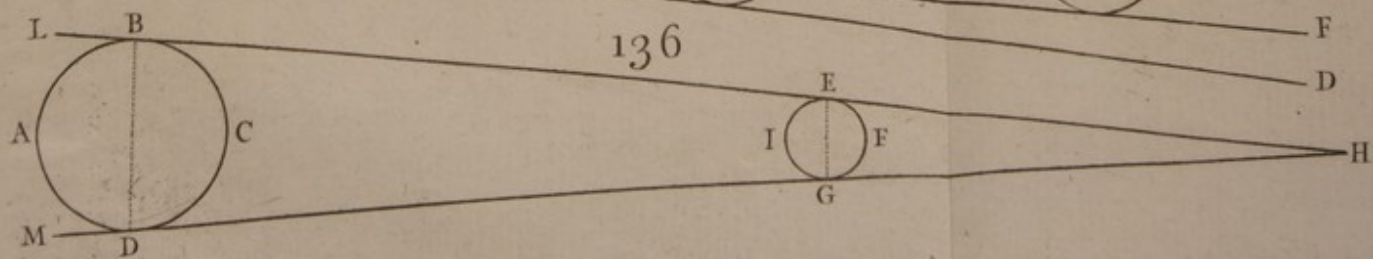
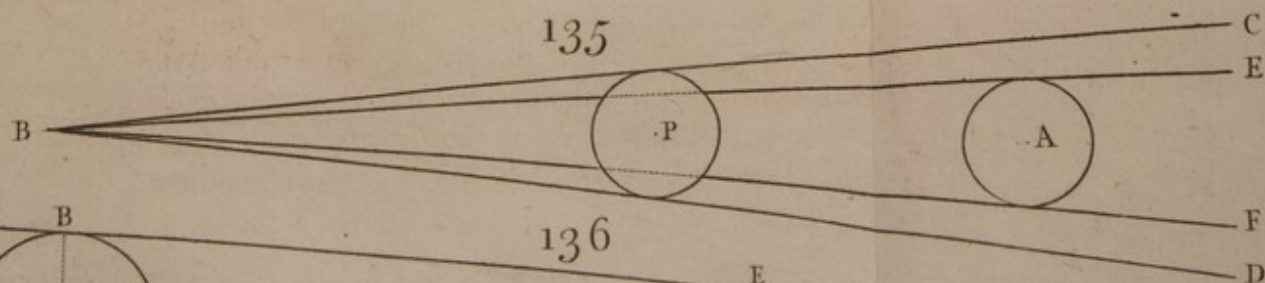
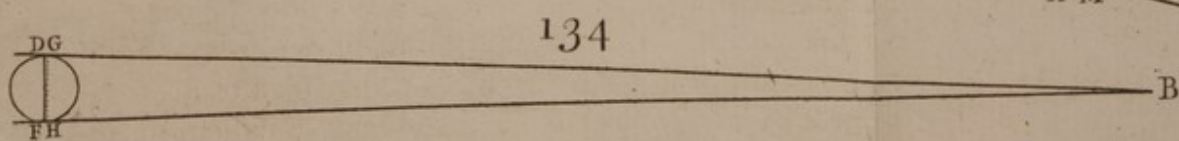
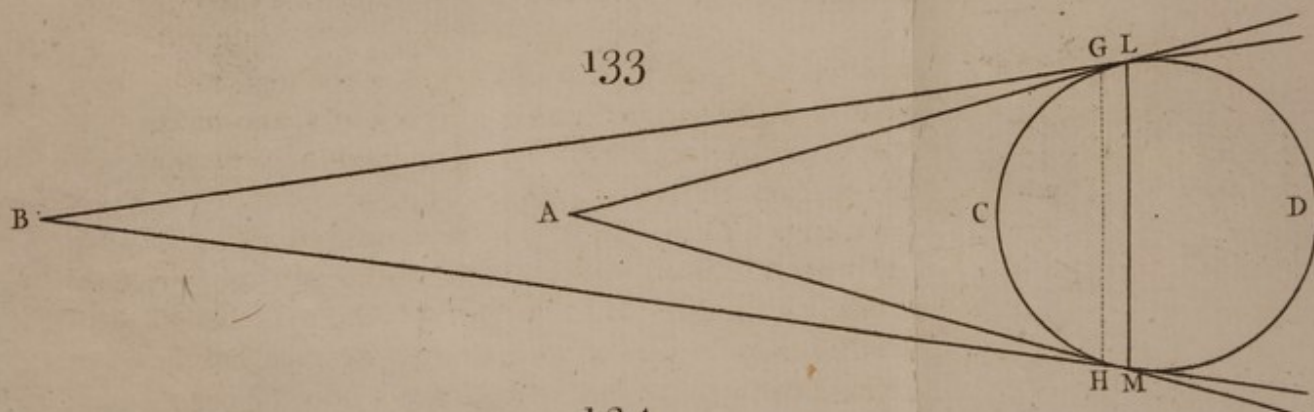
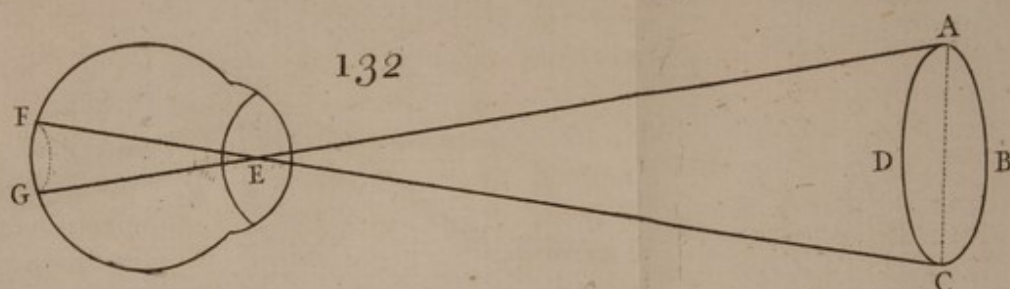
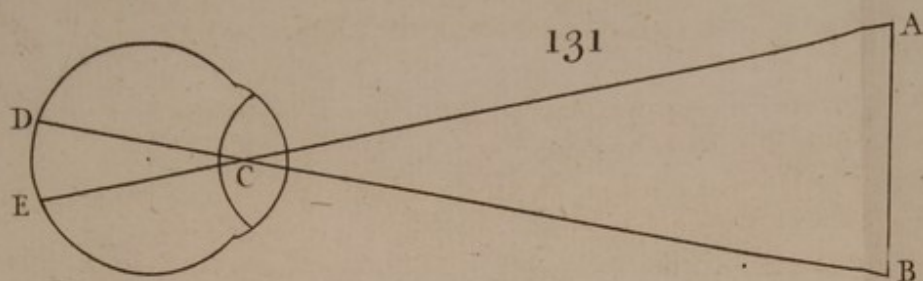
230 *Exper. 2*. Having made a room dark, make a small hole in the door or window-shutter, hold a white paper over against the hole at a proper distance from it, within the room, and there will be painted upon the paper the picture of those objects without doors which are within view of the hole: this experiment succeeds best when the sun shines upon that side of the objects which is towards the hole: the picture will be more strong and lively if a lens be placed in the hole: the proper distance for the paper is found by tryal, by moving it nearer the hole or further from it, till the picture is in its greatest perfection.

231 In both these experiments, the picture of the object is inverted, as it is also in the eye of a living person; this leads us to this enquiry, how does an inverted image make us see the object erect? to which it may be answered, that in vision, as in the perceptions by our other senses, there is no likeness between the impression made by the sensible object upon the bodily organ and the sensation thereby excited in the soul; there is only a connection between these two, so that such an impression shall always excite such a sensation; in the case before us, such an impression upon the retina as is always used to be made by an erect object will always make us perceive the object erect; this impression is always made, when the rays of light coming from that part of the object which is nearest the earth fall upon that part of the retina which is furthest from the earth, and the rays of light coming from that part of the object which is furthest from the earth, fall upon that part of the retina which is nearest to the earth. v. *Molyneux dioptr. p. 105. Smith's Optic. p. 38 and 49.*

129 232 Let *GHL* fig. 129, be a lens placed in a hole of a shutter to a dark room, and turned directly towards an object *ABC*; from each point of the object will come a pencil of rays: thus, from the point *B* comes the pencil *BGEL*, expressed by black lines, terminating in *E*; and from the point *C* comes the pencil *CGDL*, expressed by dotted lines, terminating in *D*; from the point *A* comes the pencil *AGFL*, expressed by dashed lines, terminating in *F*. A plane *DEF*

^a *Newt. Opt. p. 319.*

Introdut.



passing through the tips of the pencils, is called *the distinct base*; because a FIG. paper held there will have painted upon it the distinct picture of the object; if the paper be held nearer the lens, or further from it, the picture will be confused.^a

233 In the same manner *vision* is performed; fig. 130, let $PRST$ be the globe 130 of the eye with its coats and humours, whereof we consider chiefly the chrySTALLINE humour, which forms a lens $GHLM$; by this lens, the rays coming from the several points of the object through the pupil of the eye NO , are so refracted, that the pencils terminate upon the retina DEF , and make there a distinct picture of the object. *Molyneux dioptr. p. 103.*

234 The pencils of rays have here been considered, only to shew how an object comes to appear distinct or confused: in speaking of the apparent magnitude of objects, we consider only the axis of each pencil, and of those pencils in particular which come from the extreme points of the object: we may say then, that the magnitude of the picture upon the retina is determined by strait lines drawn from the extreme points of the object, through the pupil to the retina: thus, fig. 130, the length of the picture DF is determined by the lines 130 AF and CD . See § 227.

235 *The apparent magnitude of a strait line* AB , fig. 131, is the angle form- 131 ed by two strait lines drawn from the extreme points of it A and B , and meeting at a point in the eye at C : *the optic or visual angle*, under which the line AB is seen, is the angle ACB . *Tacquet. optic. l. 1. definit. 4.*

236 Suppose an object were a circular plane $ABCD$, viewed directly, fig. 132, imagin strait lines to be drawn from every point of it through the pupil, these will cross and form two similar cones, one of which, DCE has its base upon the retina; the other, which is called *the optic or visual cone*, has the object for its base; the common vertex C of these two cones is near the center of the chrySTALLINE: the visual cone is formed by strait lines drawn from every point of the object to a point in the eye. It is obvious, that if instead of a circle, the object were a triangle, square, or polygon, the lines drawn from it would have formed a visual pyramid instead of a cone; but a circle is the more simple figure, and therefore more proper for our present consideration.

237 *The apparent magnitude of a circle* is the apparent magnitude of its diameter: thus, fig. 132, the apparent magnitude of the circle $ABCD$ is the 132 angle AEC ; and we say its *apparent diameter* is so many seconds, minutes, or degrees, as that angle contains: to this angle the angle FEG is equal. §. 44.

^a If a lens be held directly to the sun, the place of the distinct base is called the *focus*, or burning point of the lens; because there the pencils of rays from every point of the suns body terminate, paint his image most strongly, and excite a violent heat, so as to inflame combustible bodies.

FIG. 238 If an opaque globe be at a moderate distance from the eye, the picture of it upon the retina is a circle, properly diversified with light and shade, so as to excite in the mind the sensation of a globe: if a globe be at a great distance from the eye, the difference between those lights and shades which form the picture of a globe will not be perceptible, and then the globe will appear like a circular plane: thus, the moon viewed with the naked eye, and the planets viewed through a telescope appear like circular planes. A luminous globe, though at a moderate distance from the eye, appears like a luminous plane: the eye cannot distinguish a globe of iron heated so hot that it is almost ready to run, from a flat circular piece, at the distance of a few yards; much less can the eye distinguish a luminous globe from a plane, when the distance of it is very great, as is that of the sun from us: such a globe appears luminous, because the rays emitted from it are of all sorts mixed together § 222; and it appears a flat circular plane, because the rays of light come from all parts of it, as to sense, with equal fullness and strength, without any shades, or difference of colour: for this reason, the sun, though really a globe, appears to us a circular plane, whether we receive his picture upon a piece of white paper in a dark room, or look at him through a smoked glass.

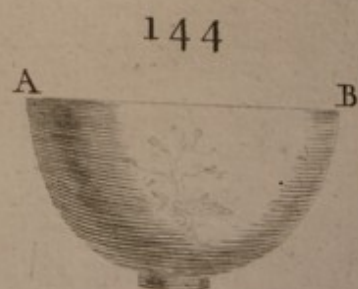
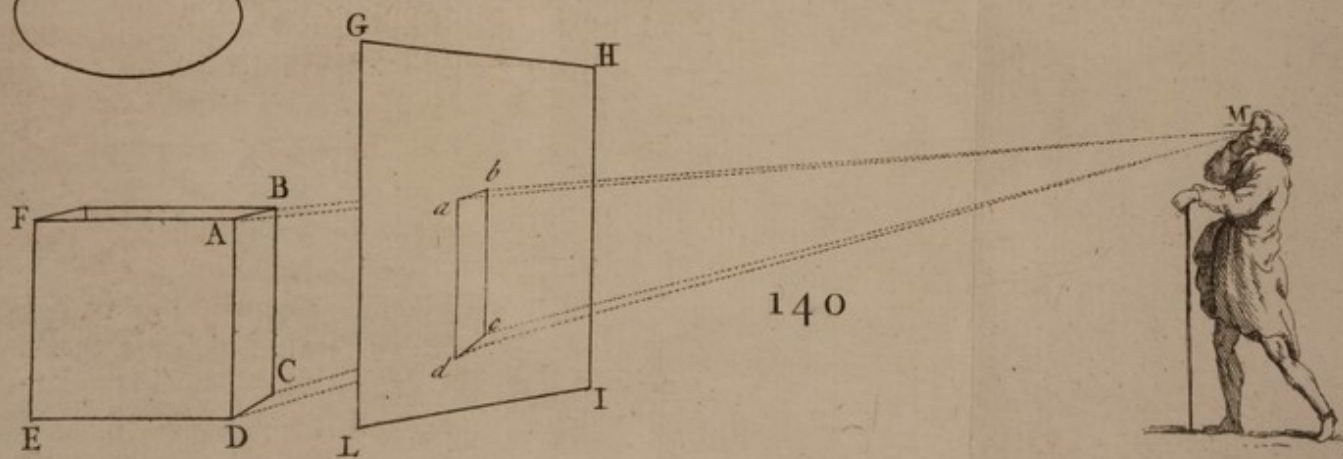
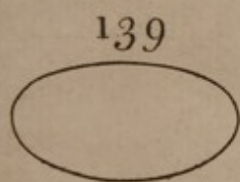
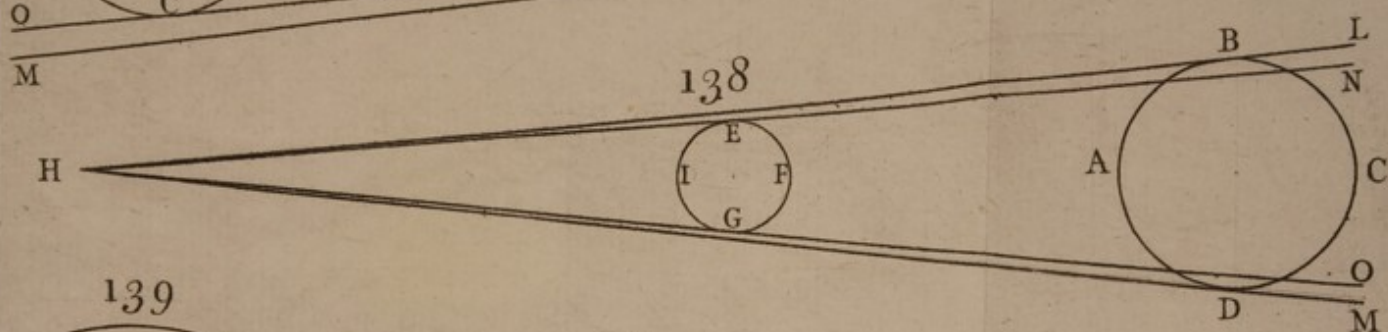
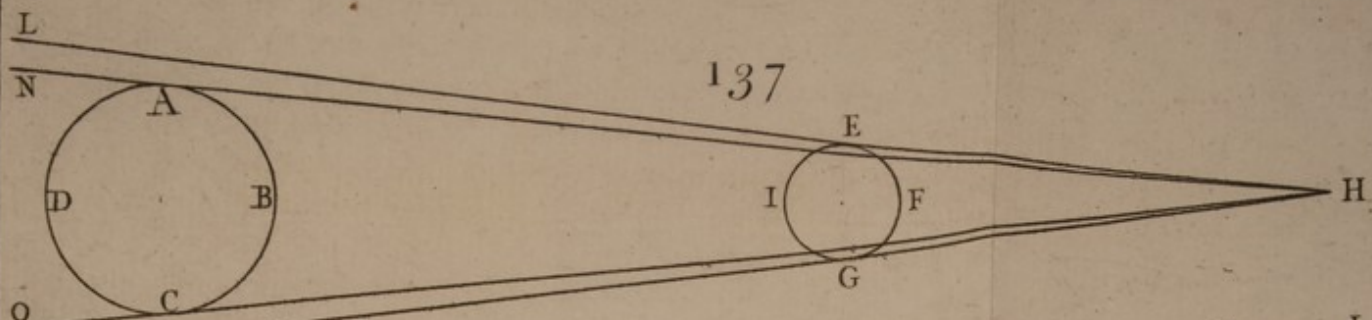
239 When we consider the sun, moon, or planets in this view, we call them the *disks* of the sun, moon, or planets; the apparent diameters of which I shall shew how to measure, in a proper place: the apparent diameter of the disk of the sun differs not sensibly from the apparent diameter of the globe of the sun: § 149: the same may be said of the disk of the moon, or any of the planets.

240 The nearer a globe is to the eye, the less segment of it is visible; the further off it is, the greater is the visible segment: thus, fig. 133, let the globe be $C G D H$, let the eye be at A , from A draw tangents to the globe which will touch it at G and H ; if these lines be imagined to be carried round, still touching the globe, the point A being immovable, they will then include the segment of the globe, $G C H$, which is visible at A : if the eye be further off as at B , from B draw tangents $B L$ and $B M$, these carried round, as before, will include a larger segment of the globe, *viz.* $L C M$, which is visible at B .

241 If a globe is at a very great distance from the eye, so as to appear under a very small angle, which is the case of the sun, moon, stars, and planets, the visible segment of the globe will not be sensibly less than half the globe: and then fig. 134, the line $G H$ which subtends the visual angle, differs not sensibly from $D F$ the diameter of the globe. *Tacquet. Optic. l. 1. prop. 37 & seqq.*

242 The nearer a globe is to the eye, the greater is its apparent diameter; that is, under the greater angle will it appear; the further off a globe is, the less





less is its apparent diameter; that is, the less is the optic angle under which the diameter is viewed: thus, fig. 135, suppose the eye is at B, the moon at P FIG. 135 is seen under the angle CBD; let the moon be further from the eye, as at A, the moon will then be seen under the angle EBF, less than CBD: the converse of this is true, *viz.* the larger the apparent diameter of the same globe is, the nearer is that globe to the eye: from hence we know, the sun, moon, and planets are sometimes nearer to us than they are at other times; because their apparent diameters are sometimes larger than at other times, if they be measured with instruments proper for that purpose.

243 Two globes of different magnitude may appear under equal angles, if their distances are in a direct proportion to their diameters: for this reason, if the sun be as much further from us than the moon, as the sun's diameter is greater than the moon's, their apparent diameters will be equal: thus, fig. 136. 136 let ABCD be the sun, EFGI the moon, let the eye be at H, they will both appear under the same angle LHM: if the larger globe is further off than according to this proportion, the less globe will be seen under the greatest angle: thus, fig. 137, let ABCD be the sun, EFGI the moon, let the eye be at H; the sun 137 will then be seen under the angle NHO, less than the angle LHM, under which the moon is seen.

244 The apparent diameters of the sun and moon are always nearly equal: the distance of each of them is so varied, that sometimes their apparent diameters are exactly equal, as in fig. 136: sometimes the apparent diameter of 136 the sun is a very little less than the apparent diameter of the moon, as in fig. 137: and sometimes the apparent diameter of the moon is a very little less 137 than the apparent diameter of the sun, as in fig. 138: from hence it comes to 138 pass, that in a central eclipse of the sun, the sun is sometimes just covered by the moon, sometimes he is a little more than covered, and sometimes the moon does not quite cover the sun, but a ring of his light appears round the moon.

245 If an object, whose real magnitude is known, is seen alone, without any intermediate objects to compare it with, we judge of its distance by its apparent magnitude: thus, we judge the distance of a crow flying in the air to be greater, the less the crow appears: the greater the apparent magnitude of an object is, the larger is the picture of it upon the retina; pictures of different magnitude excite different sensations in us; it is matter of experience, that when such objects have excited such sensations, they have been at such distances, and therefore whenever the same objects excite the same sensations, we make the same judgment of their distance, as well as of other circumstances of them, which we have often experienced to be true; just as we judge a piece of iron to be hot, when we see it of such a colour, as we have often seen iron to have, which we otherwise knew to be hot.

246 Another way by which we judge of the distance of a known object, is by the distinctness or confuseness of its appearance: we have a thousand times experienced, that we can distinguish the several parts of an object better, the nearer it is to us, and that at such a distance it appears with such a degree of distinctness: colour is another circumstance that suggests to the mind the idea of distance: thus we judge a mountain to be at a great distance, when it appears of a faint bleuish colour; such as painters for that reason use, when they would represent the distant parts of a landscape.

247 As for an object that is seen with other objects, as is usually the case, the distance of it from our eye, besides the circumstances already mentioned, is also suggested to us, by the apparent magnitude of other adjoining objects, obliquely extended either beyond the object in view, or between it and our eye: thus, we judge an house to be further off, the more ground we see between it and the place where we stand to view it: See *Smith's Optics* l. 1. c. 5. It is matter of experience, that in perspective, lines that are really parallel will appear to converge towards a point, which is of the height of the spectators eye, and is called *the point of sight* by writers of perspective: thus, the furthest parts of a cieling seem to descend; the furthest parts of a level pavement appear to rise higher; a walk of trees set in parallel lines, viewed at one end, seems to grow narrower as the trees are more distant from the eye: and conversely, since the same ideas always accompany the same impressions upon the organ, we judge that part of a level pavement to be the furthest off, which appears highest; and we judge that part of the cieling to be furthest off, which appears lowest; and when we look along through a walk of trees set in parallel lines, we judge that part of the walk to be furthest off which appears narrowest. *Tacquet. opt. l. 1. prop. 24 & 25.*

248 Without some of these circumstances, the eye does not enable us to judge of the distance of an object: the most proper for our present consideration are luminous objects: let a number of lamps be placed irregularly, at different distances from the eye, in a dark night; here being no sensible difference of colour, no intermediate objects to be seen, no converging towards the point of sight, the eye perceives no difference in their distance, and they will therefore seem all to be equally distant from the spectator; for the same reason, the sun, and moon, stars and planets, seem all to be at the same distance from us: it is indeed very probable, that some of the stars are many millions of miles further from us than others; the distance of the sun is demonstrated to be much less than the distance of any star; the moon and some of the planets, we know by ocular proof, are nearer to us than the sun, because they sometimes come between the sun and our eye, and hide his whole disk or part of it

it from our view: this the moon does every time the sun is eclipsed: nevertheless the sense perceives no difference, in the distances of the heavenly bodies; to us they appear to be all equally distant, and to be placed in the surface of a sphere whereof our eye is the center: the heaven therefore may very well be represented by the glass sphere, or by the common celestial globe. FIG.

249 The concave surface in which the stars appear to be placed, is called *the sphere of the fixt stars*: this sphere is so immensely large, that our eye would seem to be in the center of it, and we should find no sensible difference in the apparent places of the stars, though we were removed many millions of miles from our present situation; because such a distance, though great in it self, dwindles to nothing, and becomes insensible, when compared with the vastly greater distance of the stars from us: this may be illustrated by this easy experiment; let any person look at two or three objects that are at a pretty large distance from him, but appear near to one another, a small removal of the eye will not alter the prospect of those objects: thus, suppose a man standing in a certain place, looks at two or three churches that are three or four miles distant from him, but appear near to one another, he will see them in a certain position; let him remove two or three feet from his first station, and they will still appear, as to sense, in the same situation as before; because the distance of two or three feet bears an exceedingly small proportion to the distance of three or four miles; in so small a difference of stations, the difference of the pictures of so distant objects upon the retina, is too small to be perceptible.

The following propositions in perspective are very useful to help the imagination, in conceiving the representations of several appearances in the heavenly bodies: they are distinguished by points, that the reader may if he pleases pass them over for the present, and read them hereafter, when they are referred to, as they will be in proper places.

250 *Perspective* is the art of describing upon a plane the representation of any object, or in other words of drawing the picture of it: a picture drawn in the utmost perfection, and placed in a proper position, ought so to appear to the spectator, as not to be distinguishable from the object it represents: in order to produce this effect, the rays of light ought to come from the several parts of the picture to the eye of the spectator, in the same direction, and with the same strength of light, shadow, and colour, as they would do from the corresponding parts of the object it self, situated in the place wherein the picture represents it to be: thus, fig. 140, if we suppose a spectator whose eye is at M to be 140 looking at a picture of a cube *abcd*, and the original cube *ABCDEF* to be actually placed behind the picture, at the same distance from his eye, and in the same situation as the picture represents it to him; the light from any point *a* of
of

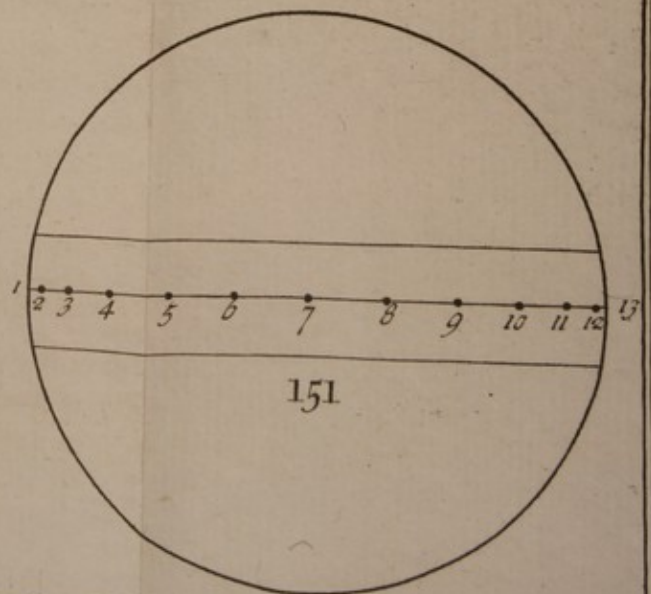
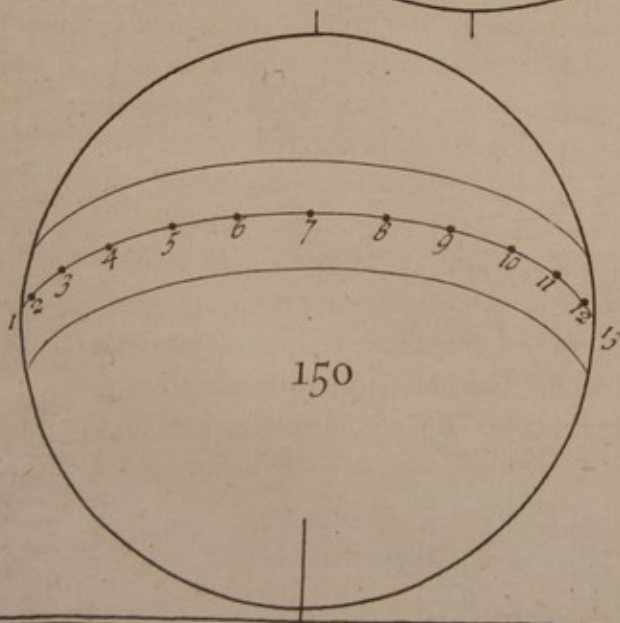
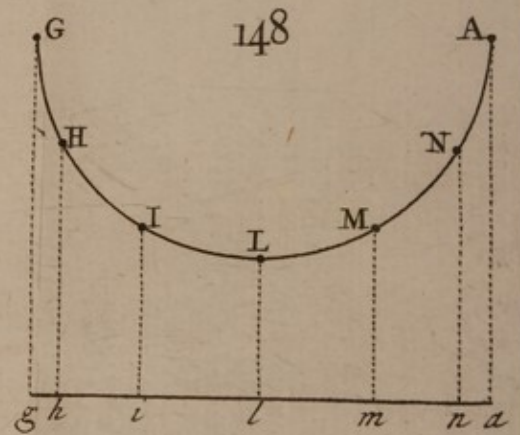
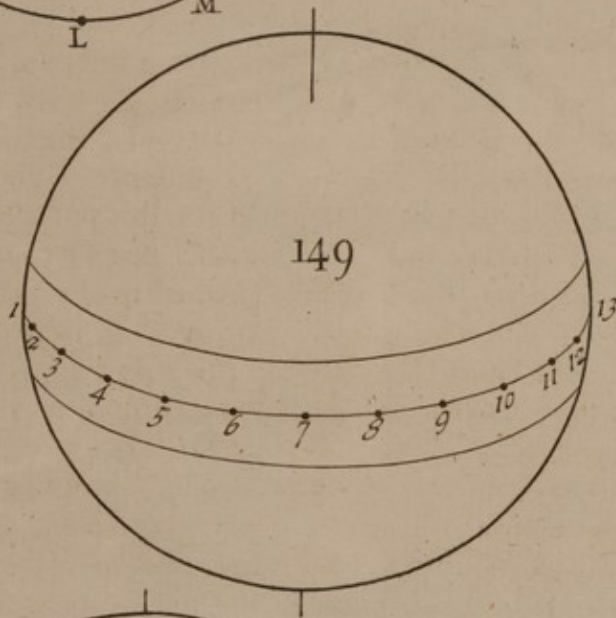
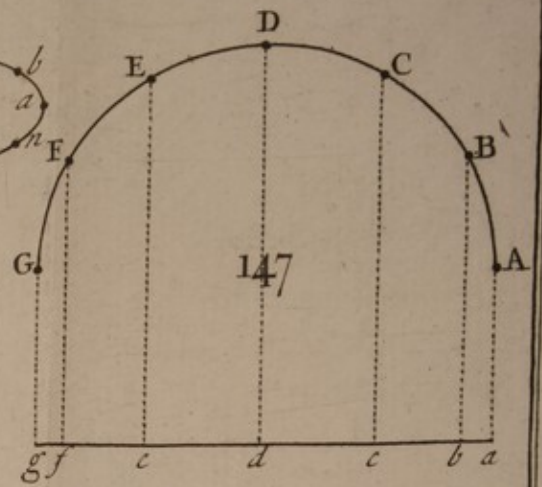
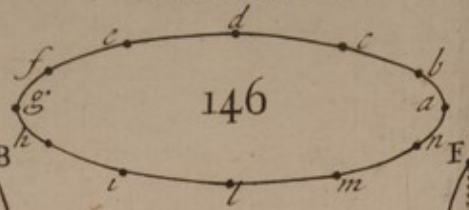
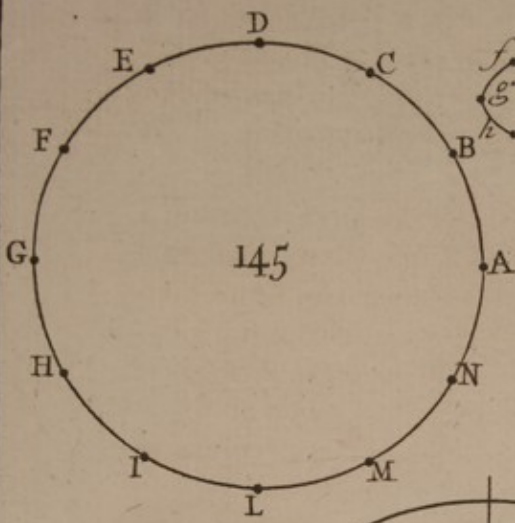
FIG. of the picture, ought to come to the spectators eye M by the ray aM , in the
 140 same direction, with the same colour, and strength of light and shadow, as it
 would do from the corresponding point A of the original cube, by the ray AM .

• 251 The plane upon which the picture is drawn may be called *the glass*, because it may be imagined transparent, as glass is, for the rays of light coming from the object to the eye to pass through.

• 252 If we imagin lines drawn from every point of an object to the eye, through the glass, they will form a plane figure upon the glass, similar to the picture of the object upon the bottom of the eye: the out lines of the figure thus formed upon the glass are called *the projection* of the object.

• 253 When the eye is at a moderate distance from an object, the *projection* of it is called *scenographic*: in this manner the scenes of a theater are designed, as are also pictures of buildings, gardens, &c drawn in perspective: I call that *a moderate distance* where the apparent magnitude of the object bears a sensible proportion to the distance of it from the eye of the spectator.

• 254 When the eye is at an infinite distance from the object, the *projection* of it is call *orthographic*: in this case, all the lines which may be imagined drawn from the several points of the object to the eye, though they really converge and meet in a point in the eye, yet may be considered as physically parallel; because they have the same effect, are projected in the same manner upon the glass, and form the same picture upon the retina, as if they were really parallel: that is commonly called *an infinite distance*, which is immensely great, where the apparent magnitude of the object is exceedingly small compared with the distance of it from the eye of the spectator; where consequently the object appears under a very small angle: this is the case with regard to the sun, which, though really of a great magnitude, by reason of his vast distance from us, appears under a small angle, of no more than about $30'$: for this reason, all the rays of light which come from the sun to our earth may be looked upon as physically parallel; and therefore if any plane or solid figure be held in the sun, the shadow of it, cast upon a plane held perpendicular to the rays of the sun, will be the orthographic projection of that figure: thus, the shadow of a circle held directly to the sun, so that his rays fall upon it perpendicular to the plane of it, will be a true circle, if projected upon a plane perpendicular to the rays of the sun: thus also, a circle held obliquely to the sun, so that his rays fall inclined to the plane of it, will be a true ellipsis, if projected upon a plane perpendicular to the rays of the sun. The orthographic projection is of greatest use in astronomy, and therefore principally to be considered: I shall first explain as much of that as is necessary in the twelve following propositions, and then say something of the scenographic projection.



*255 The projection of a globe is a circle, see § 236 and 238.

*256 The projection of a circle viewed directly, so that a line drawn from the eye to the center of it would be perpendicular to the plane of it, is a circle.

*257 The projection of a circle viewed obliquely, so that a line drawn from the eye to the center of it would make an acute angle with the plane of the circle, is an ellipsis.

*258 A circle viewed in such a position that the plane of the circle continued would pass through the eye, appears a strait line: these three last propositions may be illustrated by the different appearances of the circular rim of a china basin: look directly into the basin, and the rim will appear a circle, as fig. 141: ¹⁴¹ look obliquely upon it, and it will appear an ellipsis, more or less oblong, as it is viewed more or less obliquely: fig. 142 represents the basin viewed oblique- ¹⁴² ly; fig. 143 shews the same viewed more obliquely; and fig. 144 represents ¹⁴³ the basin, as it appears viewed with the eye in the plane of the circular rim of ¹⁴⁴ it produced; in which case the projection or view of it is a strait line AB.

*Scholium, In this experiment, in order to make the projection nearly orthographic, and the curve not sensibly different from a true ellipsis, the basin must be placed at a good distance from the spectator; the distance of about 100 diameters of the circular rim from his eye is sufficient for this purpose.

*259 In all projections of a circle, except the direct one described § 256, the parts which are equal in the original circle will appear unequal in the projection of it: thus, suppose a circle is divided into 12 equal parts, fig. 145, if it ¹⁴⁵ be viewed so obliquely that the projection of it is the ellipsis *adgl*, fig. 146, ¹⁴⁶ the points ABCDEFGHILMN, equally distant in the original circle, fig. 145, ¹⁴⁵ will appear unequally distant in the projection, fig. 146; viz. in the situation ¹⁴⁶ *abcdefghijklmn*.

*260 When a circle is viewed obliquely, or with the eye in the plane of it continued, one half of it is nearer to the eye than the other: thus, if you look obliquely at fig. 145, or with your eye in the plane of the paper continued, ¹⁴⁵ the book being held in its proper situation a little distance from you, the semicircle GHI LMNA will be nearer to your eye than the semicircle ABCDEFG: the nearest of these I call *the convex semicircle*, because it is convex towards the eye: that which is furthest from the eye I call *the concave semicircle*, because it is concave towards the eye.

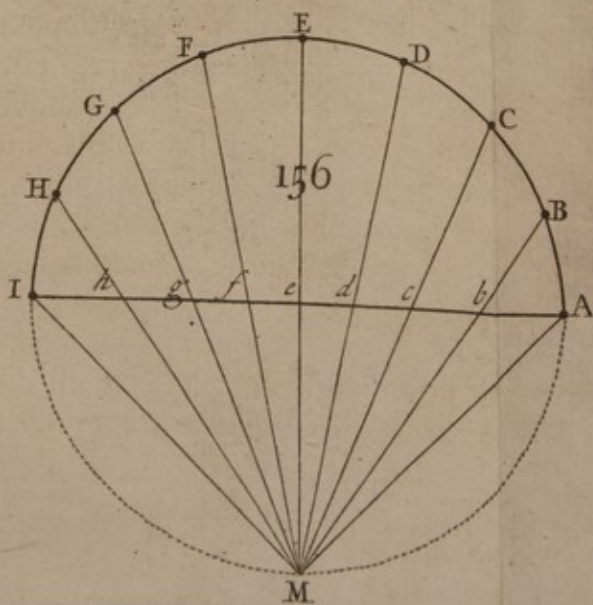
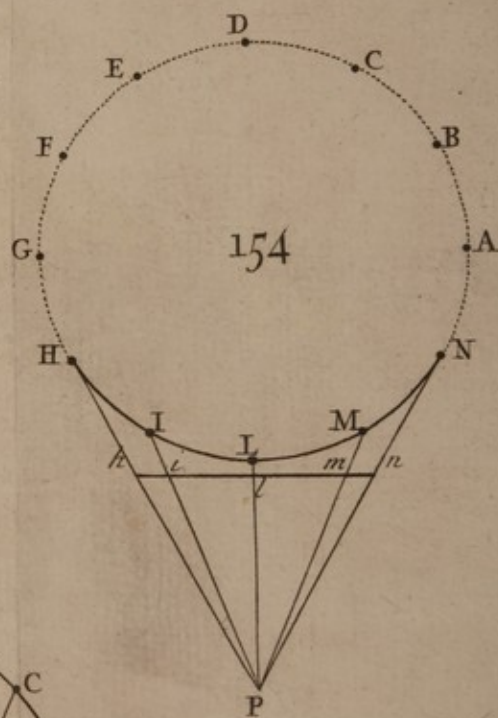
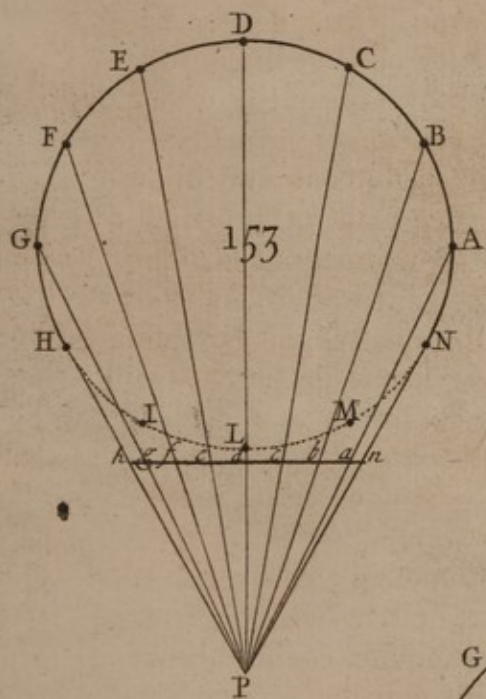
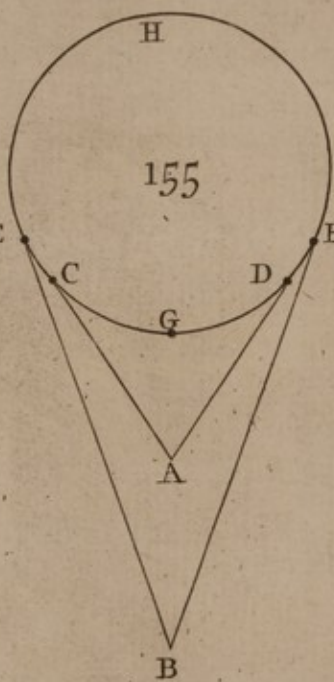
*261 If a concave semicircle divided into any number of equal parts be viewed obliquely, or with the eye in the plane of it continued, the points equally distant in the original semicircle will be so projected, that those nearest the middle of the projection will appear the furthest from one another: thus, if the concave semicircle ABCDEFG, fig. 145, be viewed obliquely, so that ¹⁴⁵ the

FIG. the projection of it is the semiellipsis $abcdefg$, fig. 146, the arcs cd and de
 146 are greater than bc , or ef ; and the arcs bc and ef are greater than ab , or fg ; and
 145 if a concave semicircle $ABCDEF G$, fig. 145, be viewed with the eye in the
 147 plane of it continued, so as to be projected into the strait line ag , fig. 147, then
 cd and de will be longer than bc , or ef ; and bc or ef will be longer than ab ,
 or fg .

262 If a convex semicircle divided into any number of equal parts be viewed
 obliquely, or with the eye in the plane of it continued, the points equally
 distant in the original semicircle will be projected in such a manner, that those
 nearest the middle of the projection will appear to be most distant from one
 145 another: thus, if the convex semicircle $GHILMNA$, fig. 145, be viewed so
 146 obliquely that the projection of it is the semiellipsis $ghilmna$, fig. 146, the
 arcs il and lm are greater than bi or mn , the arcs bi and mn are each of them
 greater than gb or na : again, if the same convex semicircle $GHILMNA$, be
 viewed with the eye in the plane of it produced, so that the projection of it
 148 is the strait line $ghilmna$, fig. 148, the lines il and lm are longer than bi or
 mn , and the lines bi and mn are each of them longer than gb or na .

263 In the orthographic projection of a circle, the projections of the con-
 vex semicircle and the concave one are similar, whether the projection be an
 146 ellipsis or a strait line: thus, fig. 146, the semiellipsis $abcdefg$ is similar to the
 semiellipsis $ghilmna$; and the largest arcs in one are equal to the largest
 arcs in the other; as are also the smallest arcs in one to the smallest in the other,
 and the intermediate arcs in one to the corresponding intermediate ones in the
 other: that is, ab is equal to na , as is also bc to mn , and cd to lm &c; thus
 147 also, the line ag , fig. 147, is similar to the line ag , fig. 148, that is ab in one
 148 is equal to ab in the other of these lines, as is also bc in one to bc in the o-
 ther, and cd to cd &c.

na —
 m l.
 264 Corollary. From hence it follows, that a body moving in a circle regu-
 larly, with the same velocity through all the parts of the circle, will appear to
 move irregularly, to a spectator who views the circle obliquely, or has his eye
 145 in the plane of the circle produced: thus, fig. 145, let A be a body moving
 uniformly, equal arcs in equal times, in the circle $ABCDE$, &c. that is, mov-
 ing from A to B , in the same time that it moves from B to C , or from C to D , or
 from D to E , &c. If this circle be viewed so obliquely, that the projection of it
 is the ellipsis $abcdef$, &c, fig. 146, the apparent motion of the body will be
 unequal; for it will appear to move from a to b , in the same time that it moves
 from b to c , or from d to e &c: this apparent inequality in the motion is such,
 that both in the concave and convex semicircle, the moving body appears to go
 quickest nearest the middle: for in the projection of the concave semicircle, cd
 and



and de are greater than bc or ef ; and bc and ef are greater than ab or fg ; and **FIG.**
 in the projection of the convex semicircle, il and lm are greater than bi or ¹⁴⁶
 mn , and bi and mn are greater than gb or na & c : again, let the projection of
 circle in which the body moves uniformly, be a strait line; equal divisions in
 the concave semicircle will appear as in fig. 147, therefore the moving body ¹⁴⁷
 will appear to move from a to b in the same time as from b to c , & c , that is,
 quickest near the middle of the line: and equal divisions in the convex semi-
 circle will appear as in fig. 148, therefore the moving body will appear to ¹⁴⁸
 describe the line gb in the same time as it does the line bi or il , or lm , & c :
 which apparent motion is also quickest, nearest the middle of the line.

•265 The following examples will serve to illustrate, as well as shew the
 usefulness of the proposition now before us, and are therefore brought in this
 place by way of anticipation; *the first example*; the orbits of the satellits of Ju-
 piter and Saturn are at such a distance from us, that the projection of them is
 orthographic, when our eye is elevated above any one of their planes, the ele-
 vation is so little, that the projection of the orbit is a very oblong ellipsis, such
 as is drawn fig. 146, so that the satellit appears to move unequally, going ¹⁴⁶
 according to the order of the letters unequal arcs in equal times; *viz.* from a
 to b in the same time as from b to c , or from c to d , & c , first in one semiellipsis
 $abcdefg$, from the right hand towards the left, and then in the other semiel-
 lipsis $ghilmna$, from the left hand towards the right: sometimes our eye is in
 the plane of a satellits orbit produced, so that the projection of it is a strait
 line, in which case also the satellit appears to move unequally, going, whilst
 in its concave semicircle, according to the order of the letters, in a strait line
 from a to b , in the same time that it does from b to c , or from c to d , & c , fig.
 147; and whilst in its convex semicircle, going back again with the same un- ¹⁴⁷
 equal motion, moving in the same time from g to f , as it does from f to e , or ^{*h to i*}
 from e to d , & c , fig. 148. ¹⁴⁸

•266 *The second example*; the sun has a rotation round his axis, as will be
 be more at large shewn hereafter; if we imagin a great circle, which may be
 called *the suns equator*, to be drawn upon the globe of the sun, equally distant
 from both his poles, with other circles parallel to his equator, § 176, since
 only half the suns globe can be seen by us at a time, only half of each of these
 circles, *viz.* the convex semicircles, would be visible to us at a time, and the
 projection of these semicircles would appear differently, according as our eye
 is differently situated with regard to the suns equator: thus, if the eye be a
 little elevated above the north side of the plane of the suns equator, it will be
 likewise elevated above the north side of the planes of all the parallels, and the
 projections of them will be such semiellipses as are represented fig. 149: if the ¹⁴⁹

FIG. eye be elevated above the south side of the plane of the suns equator, it will be in like manner elevated above the south side of the planes of all the parallels, and the projections of them will be such semiellipses as are drawn, fig. 150: if the eye be in the plane of the suns equator produced, it will likewise be in the planes produced of all the parallels, § 254, and then the projections of them will be strait lines, as fig. 151, now if we imagin these circles drawn upon the suns globe to be divided into equal parts, by visible points, these points will appear unequally distant in these three several projections; as they are represented in fig. 149, 150 and 151: from whence it follows, that a spot upon the spherical surface of the sun, being visible only whilst that hemisphere upon which it is placed is towards our earth, will, by the really uniform rotation of the sun round his axis, appear to move unequally upon the suns disk, so as to go quicker, the nearer it is to the middle of the disk, and slower the nearer it is to the edges; and this will be the case, whether the eye be elevated above the plane of the suns equator, so as to make the projections of that circle and its parallels very oblong ellipses, as in fig. 149, and 150, or the plane of the suns equator continued, passes through the eye, so as to make the projections of that circle and its parallels strait lines, as in fig. 151: a view of these figures is sufficient to shew the inequality of the apparent motion of a solar spot; the numeral figures shew the apparent place of a spot every day for 13 days, which is about half the time in which a spot appears to us to revolve round the sun: thus, fig. 149, 150, and 151, if any day at noon we observe the apparent place of a spot at 1, the next day at noon it will be at 2; the third day from the first observation it will be at 3; the fourth day it will be at 4; &c.

267 The scenographic projection of the out-lines of a figure may be made by casting the shadow of it upon a wall with a candle; thus, a circle of wire held obliquely between a candle and a wall will cast a shadow of an elliptic form, such as is represented fig. 152; where gla the lower part of the projection is the half of a wider ellipsis than the upper part adg is; but the most accurate as well as the easiest way of taking the projections of all kind of figures, is by the *camera obscura*, or dark room, which is now so commonly known, that I need add nothing to the short description already given of it, § 230, but this, that when a lens is made use of, the projection will be larger, the longer the focus of the lens is; that is, the flatter the lens is, or the larger the spheres are whereof the sides of the lens are segments; and that with the same lens you may project the object of different magnitudes, according as you place it at different distances from the lens.

268 In the scenographic projection of a circle, the projections of the concave and convex semicircles are not similar, whether the projection be an elliptic

liptic curve or a strait line; for though in both these the parts which are equal in the original circle appear larger in the projection, the nearer they are to the middle of it, equal parts in the original semicircles will be more unequal in the projection of the convex, than in that of the concave semicircle: thus, if a circle divided into 12 equal parts, fig. 145, placed at a little distance from the eye, be viewed with the eye a little elevated above the plane of it, so that the projection of it is the elliptic curve, fig. 152, the points $A B C D E \&c$, equally distant in the original will be projected into the points $abcdef\&c$, unequally distant, in such a manner that in the lower and convex semiellipsis $ghilmna$, there will be a greater difference between the smallest and greatest arc, than there is between the smallest and greatest arc in the upper and concave semiellipsis $abcdefg$, again, if the same circle placed at a little distance from the eye, be viewed with the eye in the plane of it continued, the concave semicircle will be projected into the strait line nb , fig. 153, where the points $N, A, B, C, D, \&c$ equally distant in the original, are projected into the points unequally distant $n, a, b, c, d, e, f, g, h$: but the convex semicircle will be projected into the strait line bn fig. 154, in which the points H, I, L, M, N , equally distant in the original, are projected into points still more unequally distant, viz. h, i, l, m, n .

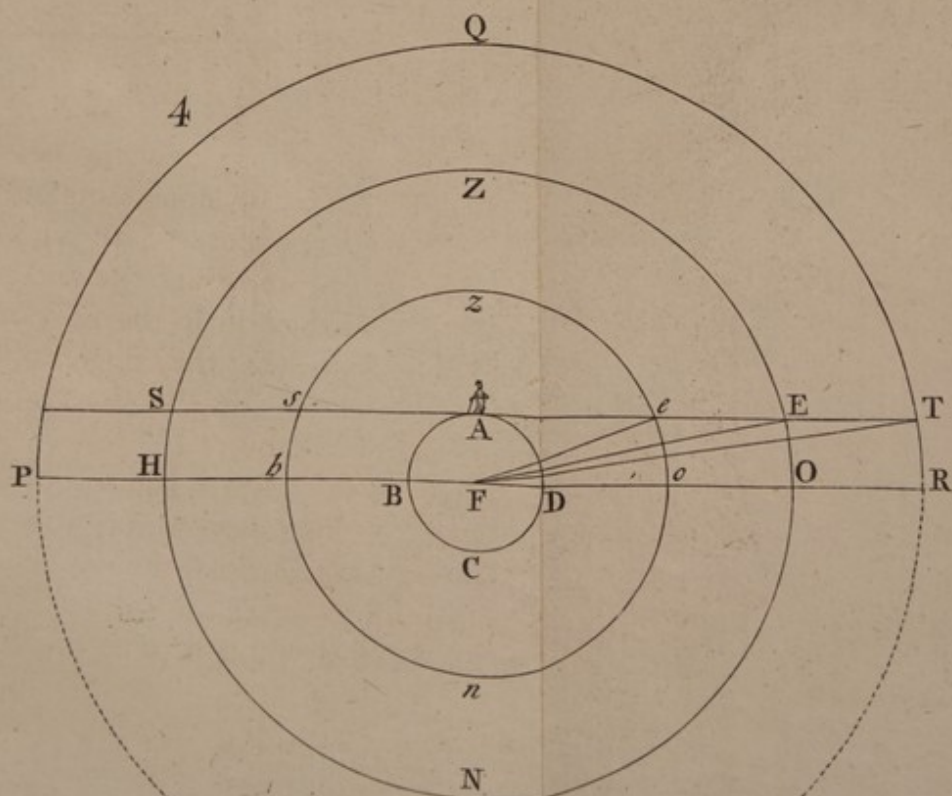
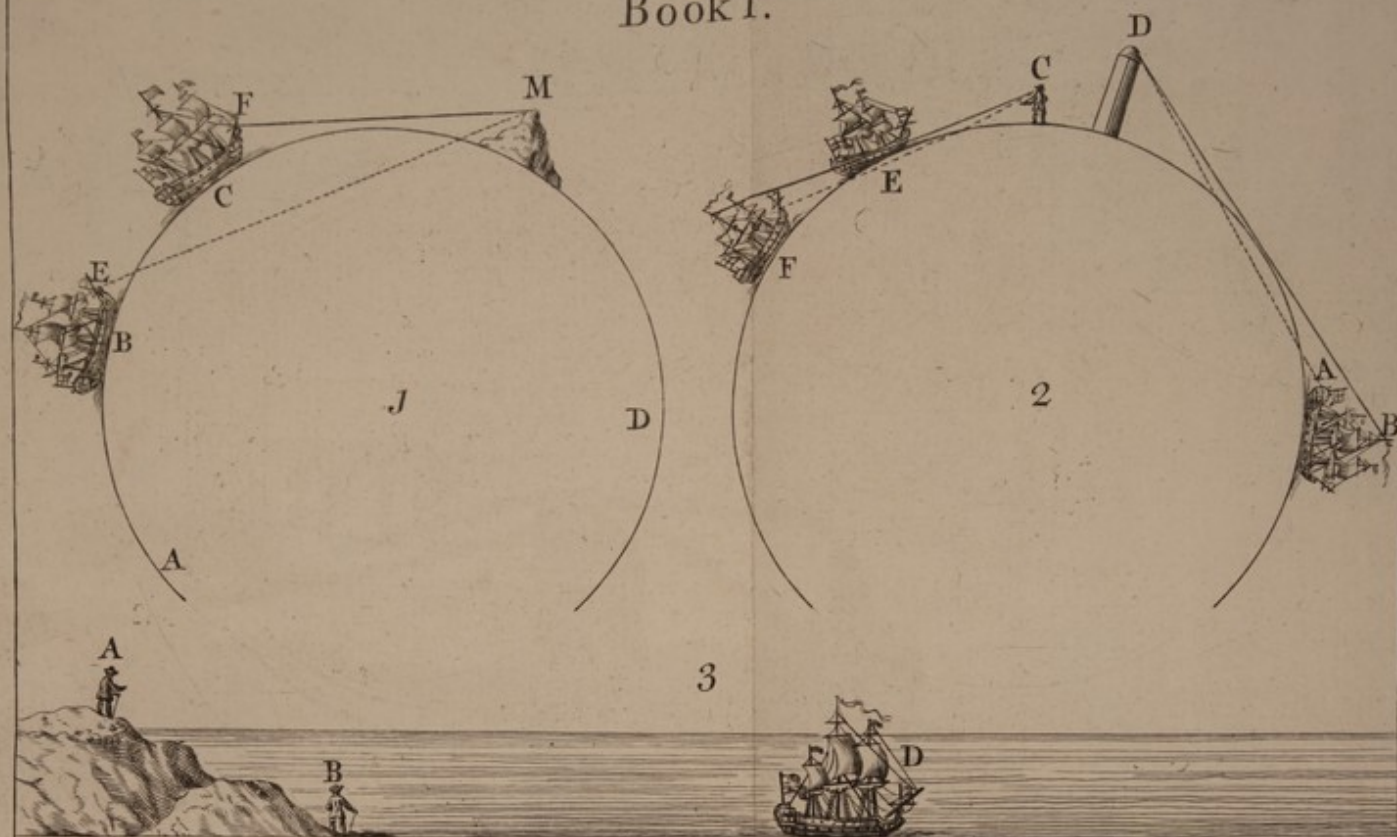
269 Corollary; if a body moving uniformly round in a circle, at a moderate distance from the eye, be viewed with the eye a little elevated above the plane of the circle, it will appear to move unequally throughout the whole circle, but most unequally whilst it is in its convex semicircle: for let fig. 152 be the projection of the circle in which the body moves, it will appear to move in equal times from a to b , from b to c , from c to d , from d to e , from e to f , &c; if the same body moving uniformly round in a circle, at a moderate distance from the eye, be viewed with the eye in the plane of the circle continued, it will appear to move unequally in strait lines, but most unequally whilst in its convex semicircle: thus, fig. 153, let the body A move round in the circle $A B C D \&c$, according to the order of the letters, the eye being in the plane of the circle continued at P ; whilst it is in its concave semicircle, it will appear to move unequally, going in equal times from a to b , from b to c , from c to d , from d to e , &c; but whilst the body is in its convex semicircle, fig. 154, it will appear to move more unequally, going the contrary way to the former, in equal times from g to h , from h to i , from i to l , &c;----- this may be illustrated by the following example, which is also brought in here by way of anticipation, and will be better understood hereafter; suppose the distance of our eye from the orbit of Mercury, and its elevation above the plane of it were such, that the projection of this orbit seen from our earth would be the curve $abcd \&c$, fig. 151; the figure shews what the apparent motion of Mercury round

FIG. round the sun would be, seen from our earth, if the earth were all the while to
 152 continue at rest; namely, in equal times the unequal arcs *na, ab, cd, de, ef, fg, gh,*
 from the right hand towards the left, whilst he is in his concave semicircle,
 and more unequal arcs *bi, il, lm, mn,* in equal times, from the left hand to-
 wards the right, whilst he is in his convex semicircle: in like manner, if the
 153 orbit of Mercury were represented by the circle *ABCD &c*, fig. 153, and 154,
 154 and the eye were at rest in the plane of it continued at *P*, the apparent une-
 qual motion of Mercury whilst in his concave semicircle, would be in equal
 times in the unequal strait lines from *n* to *a*, from *a* to *b*, from *b* to *c*, &c,
 fig. 153, from the right hand towards the left; and whilst Mercury is in his
 convex semicircle, his apparent motion would be in equal times in more un-
 equal strait lines from *b* to *i*, from *i* to *l*, &c, fig. 154, in a direction contrary
 to the former, from the left hand towards the right.

270 *Scholium*, although I have in this and the preceding section used the
 word semicircle, it is to be observed, that when a circle at a moderate distance
 from the eye is viewed, the convex part of it is not quite so much as a semi-
 circle, and the concave part of it is a little more than a semicircle; for if we
 imagin the lines *PH* and *PN* to be drawn from the eye at *P* tangents to the
 153 circle, fig. 153 and 154, they will touch the circle at *H* and *N*; so that the con-
 154 cave part of the circle will be *N A B C D E F G H*, greater than the convex part,
 which is *H G L M N*; and the nearer the eye is to the circle, the greater is the
 difference between the concave and convex part of it: thus, if the circle *E G D H*
 155 fig. 155, be viewed, the eye being at *B*, the concave part of it is *E H F*, the con-
 vex *E G F*; if the eye be nearer the circle as at *A*, the concave part of it is then
C H D, and the convex part is *C G D*.

271 In the scenographic projections hitherto explained, the eye is supposed
 to be placed a greater distance from the center of the circle whose projection
 is to be found, than the semidiameter of the circle: there is another kind of
 projection of a circle into a strait line, in which the eye is supposed to be in the
 plane of the circle, at a distance of a semidiameter from the center of it: in
 this projection, which takes in only the concave semicircle, equal parts in the
 original semicircle will appear smaller in the strait line, the nearer they are to
 156 the middle of it: thus, fig. 156, the eye being at *M*, the equidistant points
A, B, C, D, E, F, G, H, I, will be projected into the unequally distant points
A, b, c, d, e, f, g, h, i, where the lines *de* and *ef* are shorter than *cd* or *fg*; the
 lines *cd* and *fg* are shorter than *bc* or *gh*; the lines *bc* and *gh*
 are shorter than *Ab* or *hi*.

Book I.





ASTRONOMY. BOOK I.

CHAP. I. OF THE SHAPE OF THE EARTH.

§ 272. THE EARTH is the place from whence we view the heavenly bodies, the appearance of these bodies is different to the inhabitants of different parts of the earth, this is evident from the experience of all who have travailed into far distant countries: the sun, moon, and stars rise and set in Greenland, in a manner very different from what they do in the East Indies; and in both these places they appear differently from what they do in England: the cause of this difference is the shape of the earth; this therefore is one of the first things into which an astronomer ought to enquire.

273 *The earth is spherical*: by the earth is meant the earth and sea; now that the earth and sea together constitute a body of a figure every way round, we have several proofs, some of which shall be here produced, the rest being reserved for a more proper place. 1. When we are at sea on board a ship, we may be out of sight of land, when the land is near enough to be visible, if it
were

FIG. were not hid from our eye by the convexity of water; thus, fig. 1, let $ABCD$ represent a piece of the globe of the earth; let M be the top of a mountain, this cannot be seen by a person on board a ship at B , because a line drawn from M to his eye at E is intercepted by the convexity of water: but let the ship come to C , and then the mountain will be visible, because a line may be drawn from M to his eye at F . 2. The higher the eye is, the further will the view be extended: it is very common for sailors from the top of the mast of a ship to discover land or ships at a greater distance, than they can do when they stand upon deck: thus, fig. 2, the top of the tower D may be seen from B the top of the mast, when it cannot be seen from A , by reason of its being hid by the convexity of the water. 3. When we stand upon the shore, the highest part of a ship is visible at the greatest distance: if a ship is going from us out to sea, we shall continue to see the mast after the hull or body of the ship disappears, and the top of the mast will continue to be seen longest; if a ship is coming towards us, the top of the mast comes first into view, and we see more and more of it, till at last the hull appears: thus, fig. 2, let the eye be at C , when a ship is as far off as at F , only the top of the mast is visible, the hull is hid behind the water, when the ship is nearer, as at E , the hull may also then be seen. If the surface of the sea were a flat, fig. 3, a line might be drawn from any object situate upon it, as the ship D , to the eye, whether placed high or low, at A or at B : in this case, any object upon the earth, or sea, would be visible at any distance which was not so great, as to make the appearance of it, too faint, or the angle under which it appears too small, to be seen by us: an object would be visible at the same distance, whether the eye were high or low: not the highest but the largest objects would be visible, to the greatest distance, so that we should be able to see the hulk of a ship further off than the mast: all which is contrary to experience. 4. The convexity of the water may be seen upon any still water, as upon a river which is extended in a strait line, a mile or two in length; a little boat upon the water may be seen at a mile distance, by a man whose eye is any height above the water: but if he stoops down, and lays his eye near the water, he will find the surface thereof rising up in such a manner as to cover the boat, and intercept his view of it. 5. Several navigators have sailed quite round the globe; not in an exact circle, for this the winding of the shores, the land running out in many places into the sea, would not admit of; but going in and out, as the shores happen to lie, they have held on the same course, and come home on a different side from that they first set out from: thus, *Ferdinand Magellan* setting out on the west side of Spain, continued sailing westward till he returned home on the east side of Spain; our *S^t Francis Drake*, *Capt. Dampier* and others have done the

the like; the course they have some of them run is marked upon some maps and globes: these proofs are sufficient to evince the roundness of the water; that the earth is also round is evident, if we consider, that the ocean is diffused all over the globe, so as to divide the earth as it were, into large islands; and that the surface of the earth is no where very much higher than the sea, as is evident from the course of rivers, which are at a moderate estimate computed not to have above one mile fall, in running a thousand miles, and have their shores pretty much of the same height^a; as also from the height of mountains, which are none of them found to be above 3 or 4 miles higher than the surface of the sea: these experiments prove the earth and sea to have a convex surface every where. 6. All the appearances in the heaven, both at land and sea, are the same, as they would be, if the earth were a globe, which proves it to be of that shape; and lastly, in eclipses of the moon, which are caused by the shadow of the earth falling upon the moon, as will appear when eclipses are treated of, this shadow is always circular, whatever situation the earth is in at that time: now a body can be no other than a globe, which in all situations casts a circular shadow: it is true the surface of the earth is not an exact geometrical globe, because it sinks into valleys in some places, and rises into mountains in others, but these inequalities upon its surface are as inconsiderable, when compared with the magnitude of the earth, as the little asperities upon the rind of an orange are to the bulk of the orange; and accordingly we find that mountains and vallies upon the surface of the earth, cause no irregularities in its shadow, in a lunar eclipse, but the circumference thereof is even and regular, as if it were cast by a body exactly globular: the ~~semidiameter~~ diameter of the earth is above 8000 English miles, the highest mountain is not 4 miles, higher than the surface of the sea, which is but a 2000th part of the earth's diameter: thus it appears that the highest mountain bears no greater proportion to the bulk of the earth, than a grain of dust does to a common globe.^b

We may imagin as many circles as we please to be described upon the earth, or upon the sphere of the heaven: we may also, when we have imagined a circle to be described upon the earth, conceive the plane of that circle to be extended every way, till it marks a circle concentric to itself, upon the sphere of the heaven.

273 The horizon is either sensible or rational: *the sensible horizon* is a circle, the plane whereof is supposed to touch the spherical surface of the earth, in the place of the spectator whose horizon it is, and to be continued to the heaven: thus, if I imagin the plane of the floor upon which I stand to be extended every way, till it reaches the starry heaven, this plane is my sensible

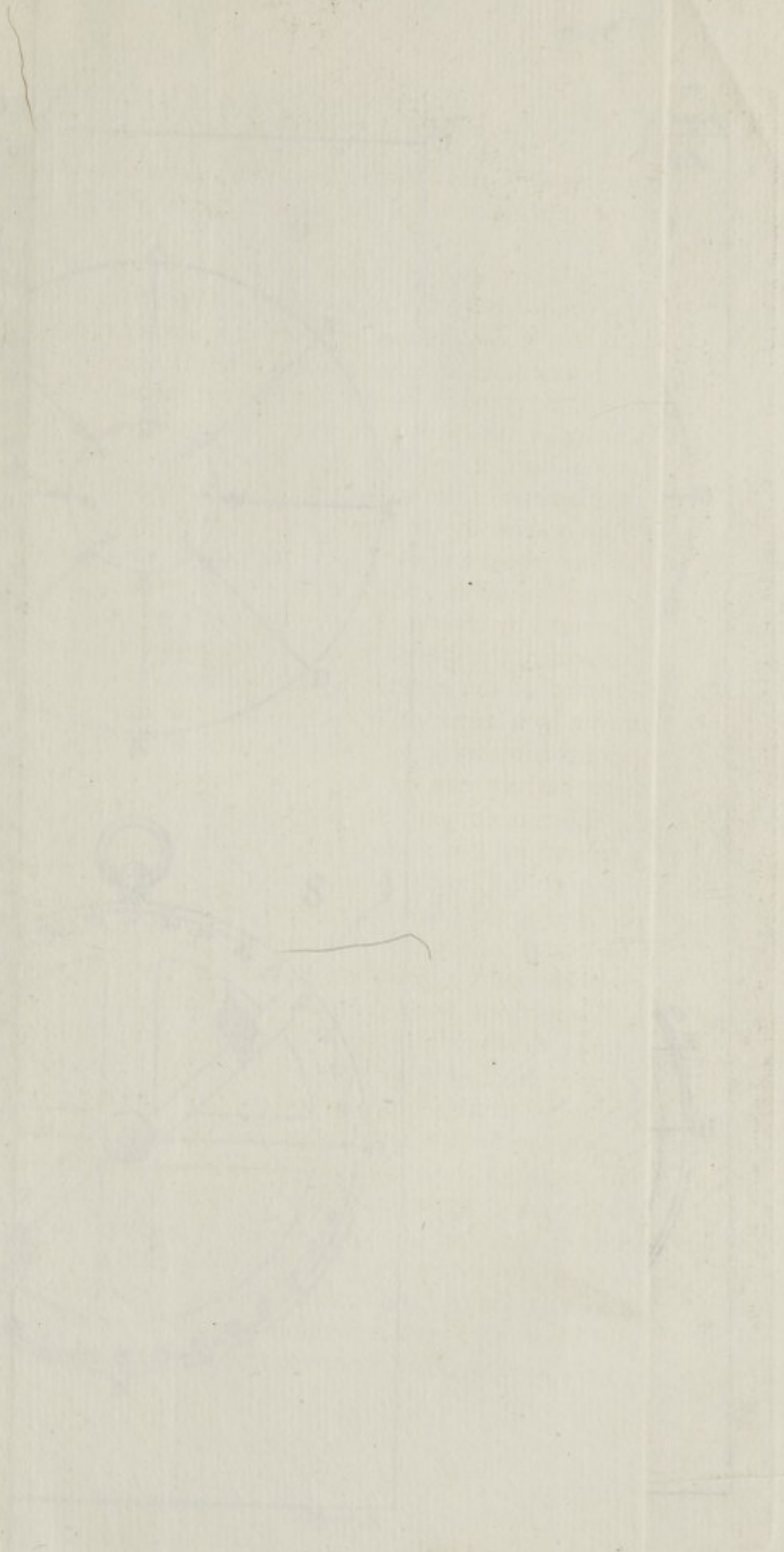
^a Kepler epit. Astronom. l. 1. pag. 21.

^b Varenus Geograph. l. 1. c. 9.

horizon:

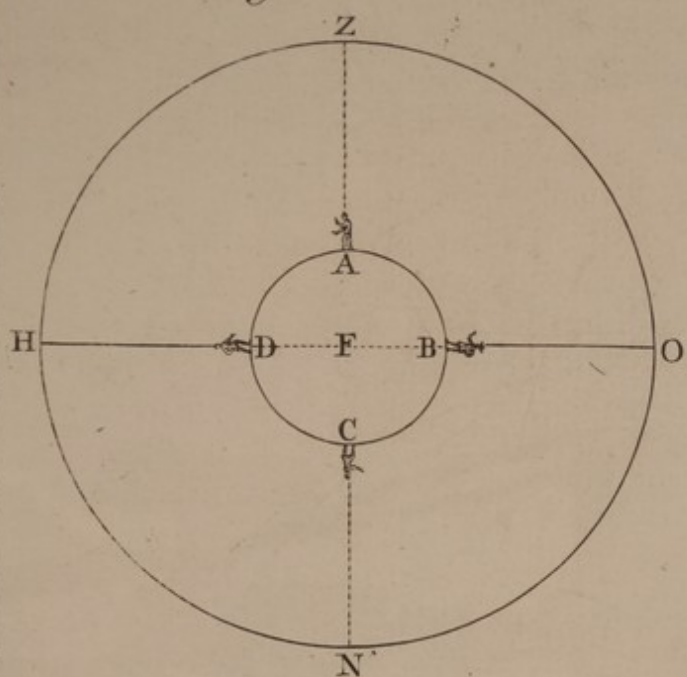
FIG. horizon: *the rational horizon* is a circle whose plane passes through the center of the earth, parallel to the plane of the sensible horizon, and is continued to the heaven.

274 The earth is so small in comparison of the immense largeness of the sphere of the heaven, that the planes of the sensible and rational horizon coincide; that is, the distance between them, when measured in the sphere of heaven, is insensible, not great enough to be discovered by any observation: in fig. 4, let $ABCD$ represent the earth, $zbnO$ the sphere of the starry heaven, if an inhabitant of the earth stands upon the point A his sensible horizon is fe , his rational horizon bo , the distance between the planes of these two is AF the semidiameter of the earth, which is measured in a great circle upon the sphere of the heaven, by the angle efO or by the arc eo , this arc in so small a circle as $zbnO$ would amount to several degrees, and consequently the difference between the sensible and rational horizon would be great enough to be easily discovered by observation: but this circle in the figure is a great deal too small to represent the sphere of the heaven, proportional to such an earth as is here supposed of the magnitude $ABCD$; let us then represent the sphere of the heaven by a larger circle $zHNO$, here the same semidiameter of the earth AF measured in this larger circle amounts to fewer degrees, for the arc EO is of fewer degrees than the arc eo and the angle efO is less than efO but this angle also may amount to several degrees or minutes, and be large enough to be measured by astronomical observation: if we imagin the sphere of the heaven to be still larger in proportion to the earth, so that half of it may be represented by the semicircle PQR , the distance between the planes of the sensible and rational horizon measured by the angle TFR less than the angle efO , or by the arc TR , less than the arc EO , is now of a fewer degrees than in the last supposition, but still may be large enough to be observed with instruments: from a view of this scheme it appears, that the larger the sphere of heaven is, in proportion to the globe of the earth, the less sensible is the difference between the sensible and rational horizon, as being measured by a less angle; we may go on further, and suppose the sphere of the heaven larger; so large, that the angle which should measure the distance of the sensible from the rational horizon does not amount to a few seconds, or perhaps not so much as one second of a minute, and consequently is too small to be measured by any astronomical observation: this is in fact the case, and therefore the difference between the sensible and rational horizon is insensible; just as it would be if the earth were a point of no sensible magnitude; and this is meaning of that common expression, that *the earth is but a point* compared with the starry heaven.

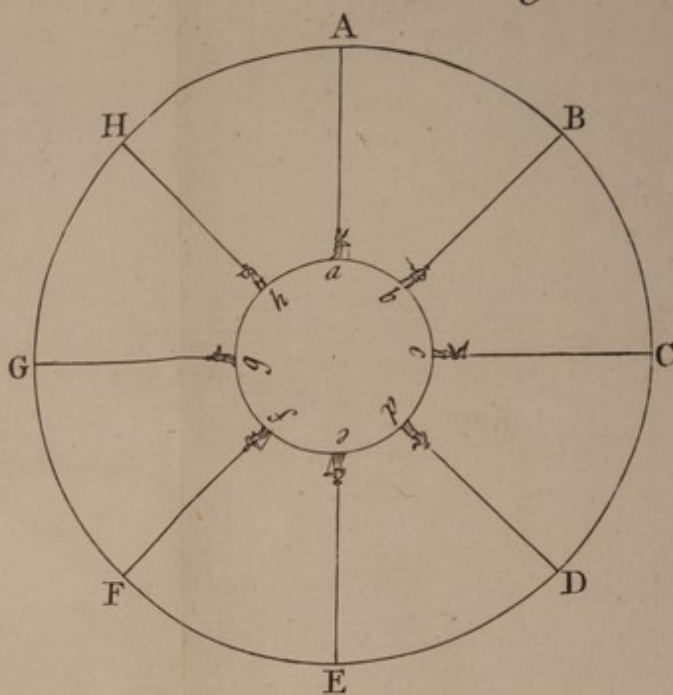


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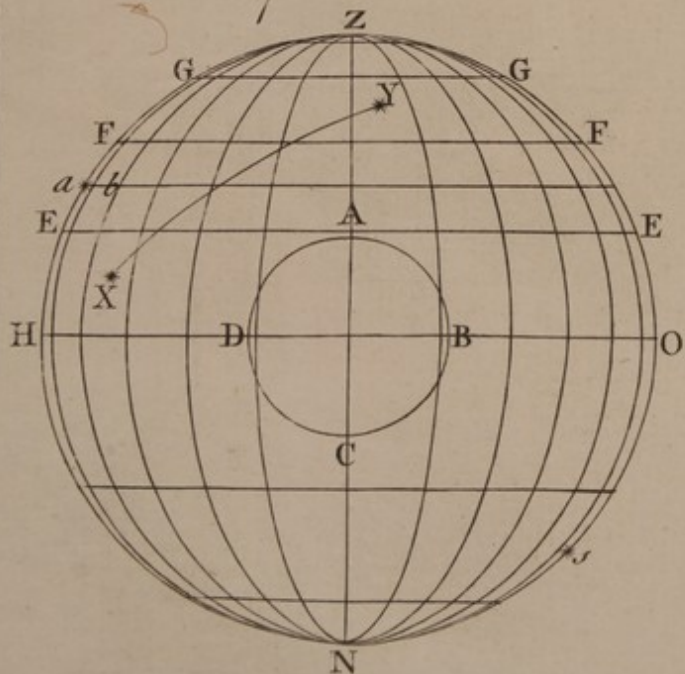
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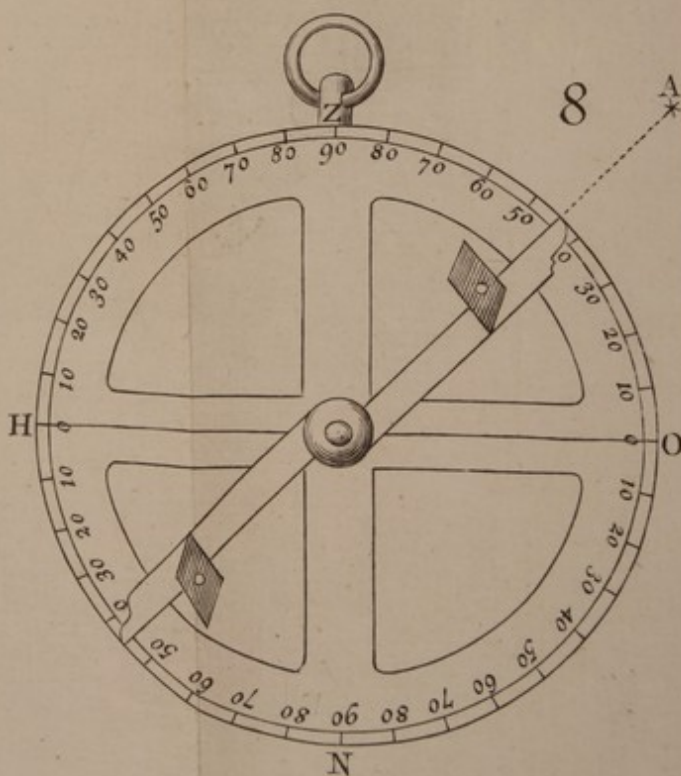
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7



8



276 The plane of our horizon continued to the heaven divides it into two FIG.
hemispheres, one *visible* to us, the other *invisible*; as being hid from us by the
 interposition of the earth: thus, fig. 5, let $ZONH$ represent the heaven, $ABCD$ 5
 the earth, the horizon of an inhabitant of the earth at A , is HO , his visible
 hemisphere is HZO , his invisible hemisphere HNO ; let another inhabitant of
 the earth be placed at C , on a part of the earth opposite to A , his horizon is
 HO , his visible hemisphere HNO , his invisible hemisphere HZO ; the visible
 hemisphere is often called also *the upper hemisphere*; and whatever stars or o-
 ther objects in the heaven are within it, they are said to be *above the horizon*;
 as whatever is in the invisible hemisphere is said to be *below the horizon*, or in
the lower hemisphere: this way of speaking may, without due caution, lead us
 into a vulgar error; to avoid which we are to remember, that no part of the
 heaven is absolutely in itself above or below, but only relatively, and in respect
 of us; for that which is above with regard to one part of the earth, is below
 with regard to another: thus, if a man is placed upon the earth at A , the point 5
 Z in the heaven is over his head, the opposite point N is under his feet; on the
 contrary, if a man be placed upon the earth at C , the point N is over his head,
 and the point A under his feet.

277 What has now been mentioned may be sufficient to satisfy those who
 think it impossible there should be any *antipodes*, or *inhabitants of the earth*
with their feet opposite to ours, and imagin that, if there were any such, they
 would fall downwards, as they call it, into the sky which is under us: but to
 take away this prejudice more effectually, we may consider *the earth as a large*
round loadstone, whose attractive power causes all heavy bodies upon the surface
 of it to tend towards its center, and that the earth is alike on all sides surround-
 ed by the heaven; and then the reason will be easily seen why the inhabitants
 of all parts of the earth stand equally upright upon it, with their feet towards
 F , the center of it; namely, because towards that the weight or tendency of
 every mans body is directed, whether he lives at A or at B , at C or D , fig. 5; and 5
 therefore a man living at A has no more reason to imagin that a man who
 lives at C would fall towards N , than he has to fear that he himself is in dan-
 ger of falling towards Z .

278 The horizon is the first circle which offers itself to the consideration
 of an astronomer, and may be thus defined: *the horizon* is that circle which
 any one upon a large plain observes, terminating his view every way round
 him, where the heaven and earth seem to meet: this definition, though it seems
 chiefly to regard the sensible horizon, is applicable to the rational one, because
 that is coincident with the sensible, as has been already shewn, § 275; in
 I astronomy

FIG. astronomy and geography, whenever the horizon is mentioned, the rational horizon is usually understood, when nothing is said to the contrary.

279 Since the horizon is a great circle of the sphere of the heaven, dividing it into two equal parts, § 276, it must also divide into two equal parts every great circle of that sphere, which is intersected by it, § 177; and since the horizon, as will appear hereafter, divides every such great circle into two equal parts, this is a proof that the horizon is a great circle, § 51.

280 By reason of the earth's roundness, *every place upon the surface of the earth has a different horizon*: this is easily understood by fig. 6; if a man lives at *a*, his horizon is *GC*; if he lives at *b*, his horizon is *HD*; if at *c*, it is *AE*; &c. this difference of horizons in different parts of the earth is another proof that the earth is spherical: if the earth were a flat, all the inhabitants thereof would have the same horizon.

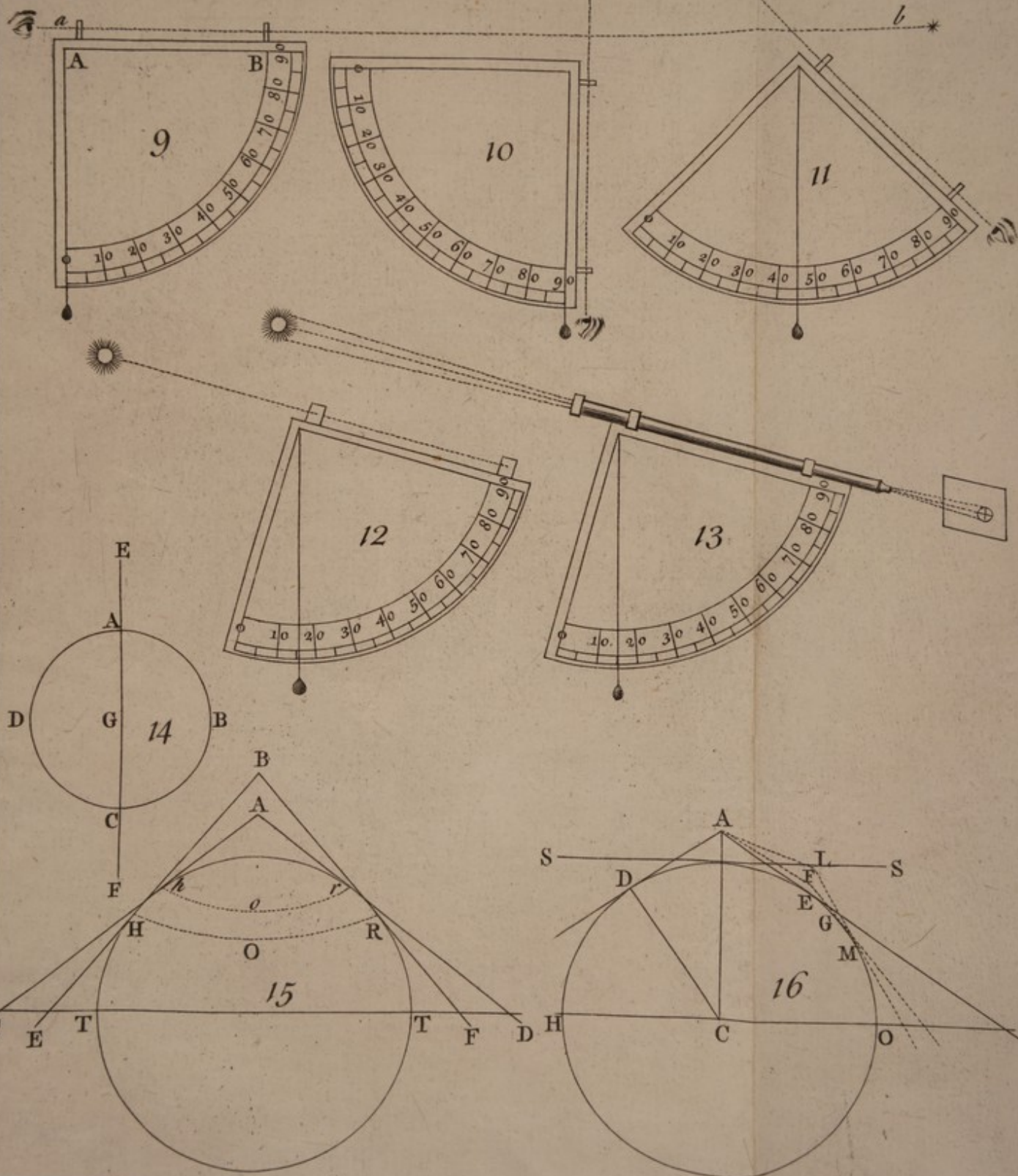
281 *The poles of the horizon*, ^a are called the zenith and nadir; they are called also the *vertical points*; the *zenith* of any inhabitant of the earth is that point in the heaven which is over his head: the *nadir* is the opposite point in the heaven, directly under his feet: thus, fig. 6, if a man lives at *a*, his zenith is *A*, his nadir *E*; if he lives at *b*, his zenith is *B*, his nadir *F*; &c.

282 We may imagin as many great circles as we please to be described upon the sphere of the heaven, so as to intersect each other in the two vertical points; these are called *vertical circles* or *verticals*; they are also called *aximuths*: in fig. 7, let *ABCD* represent the earth, *ZONH* the sphere of the heaven, if an inhabitant of the earth lives at *A*, his horizon is *HO*, his zenith *Z*, his nadir *N*, the semiellipses terminated at *Z* and *N* are projections or pictures of his vertical circles, whereof only one half of each vertical as *zAHN*, or *zbN*, &c. is here drawn, to avoid confusion: verticals are sometimes called *secondaries of the horizon*; and in general, any great circles which are imagined to be drawn through the poles of another great circle, are called secondaries of that circle.

283 *The altitude of any point of the heaven above the horizon*, or its *depression below the horizon*, is the arc of a vertical intercepted between that point and the horizon: thus, if it be enquired what is the height of the star *a* above the horizon *HO*, fig. 7; imagin a vertical circle *ZONH* to be drawn through the star, the arc *aH* contained between the star and the horizon is 30° , the height of the star *a* then is 30° : the same figure shews the star *s* is 45° below the horizon.

284 *The way of measuring the height of any of the heavenly bodies* will be understood, by an explanation of fig. 8; let *ZONH* be a circle of brass divided into 360° , suspended in such a manner, that the plane of it is perpendicular, and the line *HO* parallel to the horizon; then will *HO* always represent the

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the horizon, and whenever the circle is turned so, that the plane of it continued FIG. would pass through a star *a*, the graduated edge will represent a vertical 8 drawn through that star; let there be a ruler with sights, moveable about the center of the circle, turn the ruler about, till you can see the star through the sights, and the number of degrees intercepted between the edge of the ruler and the horizontal line *HO*, will shew the height of the star above the horizon: thus, in the figure before us the height of the star *A* is 40° .^a

285 The zenith, which is the highest point in the heaven, is but 90° above the horizon; we may therefore instead of a whole circle, make use of a quadrant, having a small thread with a plummet at the end of it, to measure the altitude of any star, the sun, moon, &c: a quadrant may be either furnished with *plain sights*, which are two thin pieces of brass pierced through with two very small holes, and placed so, that a line *ab* drawn through the centers of the two holes is parallel to the quadrant's radius *AB*, fig. 9; or else a telescope is 9 fixed to the quadrant, in such a manner, that a line drawn from the center of the eye-glass to the center of the object-glass is parallel to the radius; there 13 are then usually two fine hairs stretched over the eye-glass, so as to cross each other at the center of it, by means whereof the telescope is directed more accurately towards any object: thus, if a star is to be viewed with these *telescopic sights*, in order to take its altitude, the quadrant must be held so that the point of intersection of the cross hairs may cover the star. *The limb* or margin of a quadrant is divided into 90° , every degree into halves or quarters, and sometimes, if the instrument be large enough to admit of it, into minutes: the way of using it will easily be understood by what followeth.

286 If when a star is viewed through the sights of a quadrant, the thread falls upon (o) in the limb of the quadrant, the star has then no altitude, but is in the horizon, fig. 9: if the thread falls upon (90), the star is in the zenith, 9 fig. 10: if a star is in any intermediate place between the zenith and the horizon, when it is viewed through the sights of a quadrant, the thread will lie upon that degree in the limb which shews its altitude: thus, fig. 11, the altitude of the star *A* is 45° : in using the quadrant to observe the height of a star, care must be taken that the thread just touches the limb of the quadrant, so as to play freely without pressing upon it, and then the plane of the graduated face of the instrument will be coincident with the plane of a vertical drawn through that star. 10 11

287 When the sun's altitude is to be observed by a quadrant with plain sights, it is sufficiently exact for common uses to let the sun shine through the hole of one sight, and turn the quadrant, till the little round bright spot made by his rays, falls with its center upon the middle of the hole of the other sight;

^a See § 31 & 32.

FIG. see fig. 12: if *telescopic sights* are made use of, we may receive the suns image
 12 through the telescope upon a piece of white paper, held so that his rays may
 fall perpendicular upon it, and form thereon a luminous circle, with the sha-
 13 dows of the crofs hairs intersecting each other in the center of it; see fig. 13.

288 If we would take *the altitude of the sun more accurately*, it may be done
 by *two observers*, taking at the same time, with different instruments, one, the
 height of the upper edge, the other, the height of the lower edge of the suns
 disk; the middle height between these two is the height of his center: thus,
 14 fig. 14, let $ABCD$ be the sun, if the height of the upper edge A be 40° , the
 height of the lower C $39^\circ 30'$, the difference between them is $30'$, take half
 thereof, $15'$ from 40° , the height of the upper edge, or add it to $39^\circ 30'$, the
 height of the lower edge, and you have the height of the center G , $39^\circ 45'$.

289 *The suns altitude* may also be taken *accurately by one observer*, his ap-
 parent diameter being given, after this manner; the height of the upper edge
 of the suns disk being found by the quadrant, take from it half his diameter,
 the remainder is the height of his center: or the height of the lower edge be-
 ing found, add thereto the suns semidiameter, the sum of both is the altitude
 of his center. Whenever we look at the sun, whether through plain sights or
 a telescope, it is necessary to hold a piece of *glafs smoaked* over a lamp or a
 candle, before the eye, to prevent its being injured by the glare of his beams.

290 Sometimes, that an instrument of a large radius may not be unweildy,
 a *sextant*, or a sixth part of a circle, and sometimes, an *octant*, or eighth part
 of a circle is made use of, instead of a quadrant.

291 We may imagin as many *circles* as we please to be drawn upon the
 sphere of the heaven, *parallel to the horizon*; these are called *almicantarabs*;
 and grow less and less, as they come nearer to either of the poles of the ho-
 7 rizon: in fig. 7, the horizon is represented by HO , the almicantarabs, by the
 lines EE , FF , GG , &c, parallel to HO .

292 If we are to *measure the apparent distance between any two points in the*
sphere of the heaven, we must imagin a great circle to be drawn through them,
 and find how many degrees, minutes, &c are contained in that arc of it which
 is intercepted between those points; this great circle is called *a circle of di-*
 7 *stance*: thus, if I am to measure the distance between two stars x and y , fig. 7,
 I must find how many degrees, minutes, &c are contained in the arc xy^a .

The apparent diameter of the suns disk is measured by a circle of distance
 14 drawn through the center of it: thus, fig. 14, let $ABCD$ be the suns disk, $EACF$
 a piece of a great circle drawn through the center of it, the arc AC contain-
 ed between two opposite points in the circumference of the disk is the mea-
 sure of the suns diameter; and of so many minutes of a degree as that con-

^a See § 32.

tains,

rains, is the suns diameter said to be: the apparent diameter of the sun may FIG. be found by two observers, one of them taking the height of the upper, the other, the height of the lower edge of his disk, at the same time; the difference between the heights of these two is the suns diameter: thus, if the height of the upper edge A be observed 40° , and the height of the lower edge C, 39° 14 30', the difference between these two, viz. 30', is the apparent diameter of the sun: the several methods of measuring the apparent diameters of the heavenly bodies, or the distance between any two points in the heaven, will be shewn more at large in the proper place, it is sufficient for the present to know in general what is meant by measuring them.

293 Sometimes the sensible, or rather visible horizon is considered, only with regard to such objects as are upon the earth, and may be thus defined; *the visible horizon* is a less circle upon the surface of the earth, comprehending all objects upon it, which are within our view: *the higher the eye is, the farther is the visible horizon extended*: thus, fig. 15, let H b r R represent a part of the 15 spherical surface of the earth, if the eye be at A, draw A b and A r tangents of the globe of the earth, imagin one of these A b, the point A continuing immoveable, to be carried round, all the while touching the surface of the earth, the point b will describe the visible horizon, part of which is here shewn by the pointed curve b o r: if the eye be placed higher, as at B, the tangents B H and B R will reach farther off, and the visible horizon will be larger, viz. H O R: what the extent of the visible horizon is, and how it may be found, will be shewn, after the dimension of the earth has been treated of.

294 The visible horizon is most accurately observed upon the sea, and is therefore sometimes called *the horizon of the sea*; it may be observed, by looking through the sights of a quadrant at the most distant part of the sea within view; in making this observation, the visual rays A D and A E, fig. 16, will, by 16 reason of the spherical surface of the sea, always point a little below the true sensible horizon s s, before described, in § 274, and consequently below the rational horizon H O, which is parallel to it, and coincident with it, § 275: how much *the depression of the horizon of the sea* is below the true horizon, is seen by the quadrant, which gives the measure of the angle D A C. It is obvious, from fig. 15, that the higher the eye is, the greater is the depression of the 15 horizon of the sea; for the tangents B E and B F point lower, or are more depressed below the true horizon T T, than the tangents A C and A D.

295 *The depression of the horizon of the sea is variable*^a, so as to be sometimes a little greater than at other times, though the height of the eye be the same, in the several observations; this difference is but small, amounting only to a

^a *Observations astronomiques faites par Cassini en divers endroits du Royaume, pag. 16.*

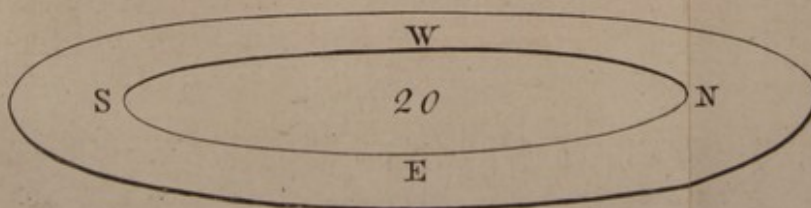
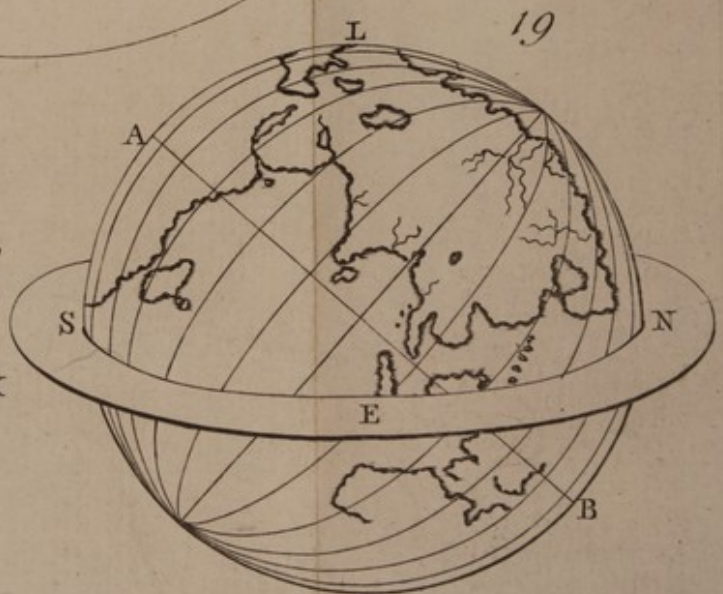
FIG. few seconds, and is owing to the difference in the air, which, at different times, refracts the visual rays more or less, according as it is of a greater or less density: in fig. 16, without refraction, the visual ray is AE , and E is the most distant point which could be seen; but by refraction, the ray FG , coming from the point G may be bent at F , so as to go on from thence in the line FA , and then the view is extended as far as G , and the depression of the horizon of the sea is in the line AF ; which points higher than AE , but extends the view farther: a view of the figure is sufficient to shew that if the refraction were greater, the view would be extended still farther, as to M ; though the depression of the horizon of the sea would then be less; as is shewn by the line ALM : from hence it appears, that our visible horizon is, by the different refraction of the air, sometimes extended farther than at other times.

CHAP. 2. THAT THE EARTH TURNS ROUND ITS AXIS.

296 THE sphere of the fixt stars, together with all the heavenly bodies, which by reason of their distance seem to be placed therein, appears to revolve round us in the space of 24 hours; this is a fact, of the truth of which every body may be convinced, with a very little attention; if the sun's place in the heaven be observed any day at noon, he will appear to go round in a circle, and return to the same place the next day at noon; the same is observable with regard to the stars, though they are not so generally taken notice of: observe the place of any star at any hour of the night, look at it some hours after, and you will find it has changed its place; continue to observe it, and you will find that in 24 hours it describes a circle in the sphere of the heaven, and returns to its place wherein you first observed it to be: this apparent motion of the heavenly bodies is called *the first^a*, and *the diurnal motion*, and can be accounted for, but one of these two ways, either they revolve round in the manner they appear to do, and our earth stands still; or they stand still, and our earth revolves round, the contrary way to the apparent motion of the heaven: which of these opinions is most probable, will be considered in another place; it is enough at present to shew, that the senses alone cannot determine the point, because every thing will appear in the same manner to them, whether it be the earth or the heaven that moves: suppose a man were in a boat, in the middle of a large lake of water, the banks whereof were in some places covered with trees, and in other places with buildings; if his boat were, unknown to him, without any noise or shock, turned evenly round one way,

^a Either because this motion of the heavenly bodies is most obvious, and therefore first taken notice of, or because it is supposed to owe its rise to the motion of the *primum mobile*.

Book I.



by a diver underneath it, or by springs and wheels, the trees and buildings FIG. would appear to him to move round his boat the contrary way; nor would it be possible for him, by his senses alone, to determine whether this apparent motion of the trees and buildings were owing to the real motion of them, or of his boat: in like manner, if the earth, which we consider as always in the center of the sphere of the heaven, has a rotation round its axis, such a motion will cause to all the inhabitants of the earth an apparent revolution of the sphere of the heaven, the contrary way, round the axis of the earth produced.

297 The cut which I have chosen for the frontispiece to this work is a picture of *my glass sphere*, on the surface of which the figures of the constellations, some of the principal stars, and such circles of the sphere as were necessary are engraved with a diamond; so that this glass represents the sphere of the heaven; within, in the middle of it, is placed a little globe of the earth, supported by a steel axis, the ends of which go through the sphere in the places of the celestial poles; *a* and *b* are two square steel arbors, upon either of which a little winch may be put, to turn it round with; if the winch be put upon the arbor *a*, and turned round the same way the key of a watch is usually turned in order to wind it up, it will, by means of some wheel-work enclosed in the brass box *c d*, carry the little globe round within the sphere, the same way we suppose our earth to revolve; if the winch be put upon the arbor *b*, as it is drawn in the cut, and turned round the contrary way to the former, the glass sphere will go round the globe, the same way that the heaven appears to us to move. Thus, by this machine, the real motion of the earth round its axis within the sphere of the heaven, or the apparent motion of the heaven round the earth, may be represented; a view of it is sufficient to take off the prejudice of sense, and to shew that the appearances of the heavenly bodies would be the same to us, whether they moved round our earth, as they appear to do, and our earth stands still; or they stand still, and our earth is carried round the contrary way: I shall explain the other parts of this sphere, as occasion offers, some of the following figures are representations of so much of it as is necessary for our present purpose: *the rotation of the earth having been considered, several astronomical terms occur, which it is now proper to explain; I shall first treat of those which more immediately respect the earth.*

298 *The axis of the earth* is an imaginary line, passing through the center of the earth, round which its revolutions are made: *the poles of the earth* are the two points in the surface thereof, which terminate the axis; one of these, *p*, fig. 17, is called *the north pole*, the other *s*, *the south pole*: *the equator* is a great 17 circle upon the earth, every part of which is equally distant from either of the poles;

FIG. poles; in fig. 17, eq is the equator: by sailors the equator is frequently called
 17 *the line*, and when they sail over it, they are said to cross the line.

299 *The meridian* of any place is a great circle upon the earth drawn through the place, and through the poles of the earth: the meridian of any place cuts the horizon at right angles, and marks upon the plane of it the *north* and *south points*, and divides the globe of the earth into two *hemispheres*, the
 17 *eastern* and the *western*: thus, fig. 17, let the point l be London, the meridian of London is $epqs$; the horizon is HO , the north point of the horizon is o , the south point of it b ; all that part of the globe which is shewn in the figure, is the eastern hemisphere, in respect of London, and all parts of the earth that are in the hemisphere opposite to this, which is not here described, but may be imagined, or may be seen upon a common globe, are in the western hemisphere.

300 The meridian of any place is by the poles of the earth divided into two semicircles; one of which is drawn through the place whose meridian it is, the other passes through the point in the earth which is opposite to that place: by the meridian of any place, writers in geography and astronomy, very often mean the semicircle which passes through the place: this therefore may be called *the geographical meridian*: all places lying under this semicircle are said to have the same meridian; the semicircle opposite to this is then
 17 called *the opposite meridian*, or the opposite part of the meridian: thus, fig. 17, *plebs* is the geographical meridian of London, and also of all the parts of the earth lying under that semicircle; the opposite meridian to this is *poqs*.

301 The meridian of any given place is, by the definition thereof in the two last sections, immoveably fixt to that place, supposing the place to keep its situation upon the earth, and the poles of the earth to keep their places also; the meridian therefore of any place must be carried round along with it, by the rotation of the earth.

302 When the geographical meridian of any place is by the rotation of the earth brought to point at the sun, it is *noon* or *midday* in that place: thus,
 17 in fig. 17, if we suppose the sun to be at M , the geographical meridian of London *plebs* points at the sun, so that if the plane of that semicircle were extended to the sun it would pass through his center; in such a situation of the earth as this, it is noon at London, and at all places which lye under the semicircle *plebs*.

303 The plane of the meridian of any place may be imagined to be extended to the sphere of the fixt stars: when by the rotation of the earth the plane of the meridian comes to any point in the heaven, as the sun, moon, or a star, &c; that point appears to come to the meridian: according to this explanation

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tion, I would always be understood, when I use the common expressions, as FIG. I often shall do, and say the sun, a star, &c comes to, or passes through the meridian.

304 *The rotation of the earth is from west to east*: east and west are relative terms; so that a place may lye east with regard to one place, and west with regard to another: thus, let fig. 18 represent the globe of the earth, with 24 18 geographical meridians drawn upon it, at equal distances from one another; let $PLgs$, be the meridian of London, passing through London at L ; let N be Naples, $PNbs$ is then the meridian of Naples; let Q be Canaria, $PQfs$ is the meridian of Canaria: it is evident, from a view of the figure and from § 299, that the meridian of London, is west from the meridian of Naples, and east from the meridian of Canaria.

If we were to go round the earth upon the equator ao eastward, we should go according to the order of the letters $abcd$ &c from a to b , from b to c , 18 from c to d , &c: the rotation of the earth being eastward is therefore according to the order of the letters $abcdef$ &c; and is the cause that *the heaven and heavenly bodies appear to move the contrary way from east to west*.

The true east and west points in any place are those where the horizon of that place is intersected by the equator: thus, fig. 19, let L be London, AB 19 the equator, SEN part of the plane of the horizon of London; E is the east point for London; and the point opposite to this in the horizon, which cannot be seen in this figure, but may easily be imagined, is the west point: in fig. 20, I have given an oblique view of the horizon of London, similar to that in 20 the preceding figure, only in this the picture of the globe of the earth is omitted, that the horizon may be seen entire, with *the four cardinal points*, as they are called, east, west, north, and south; which are here marked with their initial letters, E, W, N, S : in a looser and more general way of speaking, we may call that half of the horizon, SEN , in the middle of which the east point (E) is, *the east side of the horizon*; because the sun, moon, stars &c, appear to rise on this side^a: in the same way of speaking, we may call the other half of the horizon, SWN , which has the west point (W) in the middle of it, *the west side of the horizon*; because below this the heavenly bodies appear to set^b.

305 The time which passes between the noon of any one day, in a given place, and the noon of the day following, in the same place, is called a *natural day*: thus, if when the earth is in the situation described, § 302, it is noon at 17 London; the time that the earth takes up in turning once round its axis, so as to return into the same situation, and make it noon again at London, is a na-

a, b, The east is, in our own and several other languages, expressed by a word which signifies *rising*; and the west by a word which denotes *setting*: see the words east and west, in the index.

FIG. tural day^a: the natural day comprehends the day and night, and is divided into 24 *hours*, every hour into 60 *minutes*, every minute into 60 *seconds*, every second into 60 *thirds*, &c: minutes and seconds of time are exprest by the same characters as minutes and seconds of a degree are: 9 hours 56 minutes 44 seconds are thus set down, 9^h. 56'. 44":

306 Noon is the beginning of the natural day, and therefore all places which lye under the same meridian, and have noon at the same time, have also every other hour of the natural day at the same time: thus, when it is one in the afternoon at London, it is also one in the afternoon at every place under the meridian of London; when it is two at London, it is two every where under the same meridian; when it is three at London, it is three there; &c.

CHAP. 3. THE LONGITUDE OF PLACES: THE FIRST MERIDIAN.

307 All places under the same meridian are said to have the same *longitude*: all places that lye under different meridians are said to have different longitude: this difference of longitude may be eastward or westward: *the difference of longitude between any two places* is the distance of their meridians, measured in degrees, minutes, and seconds, upon the equator: thus, if the meridian of any place cuts the equator in a point which is distant 15° east from the point where the meridian of London cuts the equator, we say that place differs from London in longitude 15° eastward: if the meridian of any place cuts the equator in a point which is 30° distant westward from the point where the meridian of London cuts the equator, that place differs from London in longitude 30° westward, &c: thus Naples marked with N, fig. 18, differs in longitude from London 15° eastward; and Canaria, marked with Q, differs from London 15° westward; Grand Cairo in Egypt marked with the letter c, differs 30° from London eastward.

308 It is usual for geographers to pitch upon the meridian of some remarkable place for *the first meridian*: and to reckon the difference of longitude of any other place, by the distance of its meridian from the first meridian, measured upon the equator eastward. The ancient Greek geographers^b placed the first or great meridian in Hera or Junonia, one of the Fortunate islands, as they were then called; this is supposed to be the island Teneriff, one of the Canaries, and was the most western part of the earth then discovered: the Arabian geographers,

^a If we would exprest it according to the apparent motion, we may say, the natural day in London is the time which passes between the suns leaving the meridian of London and returning to it again.

^b Ptolem. *Geogr.* l. 1. ex Marino; v. Gregorii *posthuma*, p. 266. a treatise about the use of the globes; Christmanni *Comment. in Alfragan. Astron.* cap. 11. *præcipue vero Ricciol. Geographia reformat.* l. 9. cap. 2,

ambitious

ambitious of having the beginning of the longitude taken from them, chose to FIG. fix the great meridian upon the utmost shore of the western ocean: some later geographers made it to pass through the island Corvo, one of the Açores^a, because on that island the magnetic needle, at that time, pointed directly north, without any variation; and it was not then known that the variation of the needle is it self subject to variation, as has since been discovered: *Bleau* in his mapps and globe replaced the first meridian in Teneriff; and to ascertain the place of it more exactly, made it to pass through the middle of a mountain in that island famous for its great height, and called, from the smallness of its top, *el pico*^b: it is now grown a custom amongst geographers, for every one to make the first meridian pass through the principal city of that country to which he belongs; thus the English, in their mapps and globes, make the first meridian to pass through London; the Dutch theirs through Amsterdam; the French theirs through Paris; and particularly through the royal observatory there. I shall all along take the meridian of London for the first meridian, as *Mr Senex* has done in his globes, the most correct, I believe, that have yet been published.

309 *The longitude of any place* is the number of degrees, minutes and seconds, upon the equator, which can be reckoned eastward from the first meridian to the meridian of that place: thus, fig. 18, if we take the meridian of 18 London *PLgs* for the first, and draw meridians through every 15° of the equator, beginning at that point of it *a*, through which the meridian of London passes, and go round eastward; we shall have the globe divided as it appears in the figure before us, which is a picture of half of it, as much as can be seen at one view: then I say, all places under the meridian *PLgs* are in the same longitude as London is; all places under the meridian marked *P15s* are in 15° of longitude; all places under the meridian *P30s* are in 30° of longitude; all places under the meridian *P45s* are in 45° of longitude; &c so that the numeral figure placed upon the equator at the point where each meridian cuts the equator, shews the longitude of any place which lyes under that meridian: thus fig. 18, *N* Naples is in 15° of longitude, *c* Cairo is in 30° of longitude, *Q* Canaria is in 345° of longitude *c*.

310 If we imagin 12 great circles, one of which is the meridian of a given place, to intersect each other at the poles of the earth, and divide the equator into 24 equal parts, these are *the hour-circles* of that place: these circles are by

^a So called from the Goshawks there: *azor* in spanish is a Goshawk.

^b *El pico* signifies a birds beak.

^c These places, Naples, Cairo, and Canaria, which last is a small island that took its name from the large dogs bred there, *Canariam vocari a multitudine canum iugentis magnitudinis scribit Plin. l. 6. c. 32*, and gave the name to the rest of the Canary islands, are all of them some few minutes and seconds different in longitude from what is here set down, as appears by the table of the longitude and latitude of places; I have however, to make the examples the easier, supposed the meridians drawn upon the 20 figure 15° or one hour distant from one another, to pass through these places, though speaking exactly they do not, but only very near them.

FIG. the poles divided into 24 femicircles; an arc of the equator contained between any two of these *hour-femicircles* which are nearest to one another, is 15° , for 15° is a 24th part of 360° ; by the rotation of the earth, the plane of every hour-femicircle points at the sun, an hour after the femicircle which is next it towards the east has pointed at the sun; and thus they successively point at the sun every hour, so that the planes of all the 24 hour-femicircles extended, pass through the sun in a natural day: fig 18 is a picture of the globe of the earth, upon which the hour-femicircles of London are drawn, as many as can be seen at one view of the earth; let *p g s* be the meridian of London, imagin the sun to be in the place where your eye is when you look directly at the figure, the meridian of London then points at the sun, so that it is noon at London; imagin the earth to revolve round, according to the order of the letters, *abcdef*, and you will easily see, that in one hours time, the next meridian *p f s* will point at the sun, and then it will be one in the afternoon at London; in another hour, the meridian *p e s* will point at the sun, and then it will be two in the afternoon at London; &c, so that at the end of 24 hours, the earth will return into the same situation, and the meridian *p g s* will point at the sun again; and make it noon at London for the next day.

18 311 From a view of the figure thus explained, we may observe, that a meridian *p g s*, 15° east from London, comes to point at the sun, one hour sooner than the meridian of London does; a meridian *p b s*, 30° east from the meridian of London, comes 2 hours to the sun, before the meridian of London does, &c; a meridian *p e s*, 15° west from London, points at the sun, one hour later than the meridian of London does: and so agreeably to this proportion of 15° for an hour, the meridian of any place comes to point at the sun, a longer or shorter time, sooner or later than the meridian of London does, according as that meridian is more or less distant from London, eastward or westward: for this reason, the difference of longitude between any two places is sometimes exprest in time, *viz.* hours, minutes, and seconds; instead of degrees, minutes, and seconds: thus, supposing the meridian of London the first meridian, *the longitude of any place in time* is the number of hours, minutes, and seconds, by which the meridian of that place comes to the sun, sooner or later than the meridian of London does: thus, the meridian of Naples is distant from the meridian of London one hour, the meridian of Cairo two hours, eastward; the meridian of Canaria is distant from the meridian of London one hour, westward.

312 From the two preceding sections it appears, that they whose meridian is 15° east from London have noon, and consequently every other hour of the natural day, an hour sooner than they have it at London; they whose meridian is 15° west from London have noon and every other hour of the natural

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ral day an hour later than at London: the converse of this is true, that they who have noon and every hour of the natural day an hour sooner than they have at London, have their meridian 15° east from the meridian of London; as also that every place which has noon and every other hour of the day an hour later than London, differs from London in longitude 15° towards the west, and so on in the same proportion, so that one hour difference in time is 15° difference in longitude.

313 From hence it follows, that *by any instantaneous appearance in the heaven, seen at two places which lye under different meridians, the difference of their longitude may be found*, if the hour of the day be known at each place: thus, the beginning of a lunar eclipse, when the moon first touches the shadow of the earth, is an instantaneous appearance, as is also the end of a lunar eclipse, when the moon leaves the shadow of the earth; these appearances are visible to all the inhabitants of that half of the globe of the earth which at that time is turned towards the moon; suppose, that when a lunar eclipse begins, I find it is exactly 12 at night at London, and an observer from Naples writes me word, that in that place it was one in the morning, when the same eclipse began; I may from thence conclude, that the meridian of Naples is 15° distant from the meridian of London, eastward; if another person sends me word, that he observed the same eclipse at Canaria, and that there the beginning of it was at 11 at night, I may be assured Canaria differs in longitude from London 15° westward.

314 I have in the preceding examples, to make them the more easy, supposed a whole hour difference in time, which answers to 15° ; but since one minute of time answers to $15'$ of a degree, and one second in time answers to $15''$ of a degree, &c; the difference in longitude between any two places may be found in minutes or seconds of a degree: thus, if at the beginning of an eclipse of the moon, it is 12 at night at London, and at the same moment, it is $1^h. 55'. 55''$. in the morning at Constantinople, then the meridian of Constantinople is $28^{\circ}. 58'. 45''$. east from the meridian of London.

315 There is also another consequence which follows from § 312, proper to be mentioned in this place, which is this; *if a man goes round the earth eastward, he will reckon one day more to have passed during the time of his voyage, than they do who live at the place from whence he set out; and be one day forwarder in the week than they are, at his return thither; so that he will count their monday to be tuesday; their tuesday to be wednesday: &c.* On the other hand, *if a man goes round the earth westward, he will at his return count one day less to have passed, than they do who have continued all the while at the place from whence he began his voyage; and be one day in the week later than they:*

FIG. they; so that it will be tuesday with him, when it is wednesday with them; and
 18 monday with him, when it is tuesday with them; &c. the reason of this will
 easily be seen, if we consider the 18th figure as a picture of the globe of the
 earth, in the following manner; suppose a man begins his tour from L Lon-
 don eastward, on a tuesday, when he is got to the meridian of Naples *Pbs*, 15°
 from the meridian of London, it will be noon with him one hour sooner than
 with those who live at London; when he is got to the meridian of Cairo, *Pis*,
 30° east from the meridian of London, he will have noon two hours sooner
 than they at London; and so on in this proportion, every meridian he arrives
 at 15° further east from London, it will be noon, that is, the beginning of the
 natural day with him, one hour more, sooner than at London: so that when he
 has gone quite round, and is returned to London again, he will have gone 360°
 east from the meridian of London, and it will be noon with him, 24 hours,
 or a whole day, sooner than to those who staid at London: that is, it will be
 tuesday noon with him, when it is but monday noon with them, &c. On the
 other hand, if a man sets out from London westward, when he comes to the
 meridian of Canaria *Pfs*, 15° west from the meridian of London, his noon will
 be an hour later than at London; when he comes to a meridian *Pes* 30° west
 from the meridian of London, it will be noon with him two hours later than
 at London; so that when he is got quite round to the meridian of London again,
 he will have gone through the 24 meridians described § 310, and therefore
 his noon will be 24 hours, or a whole natural day, later than at London; so
 that it will be monday noon in his account, when they who staid all the
 while at London reckon it tuesday noon, &c: in this affair, it is not necessary
 that a man should go round the globe upon the equator, or any of its parallels;
 it is sufficient if he passes through the several meridians, as all navigators must
 do who go round the globe eastward or westward. And thus much may suf-
 fice at present, to give a general view of the nature of the longitude of places;
 the more particular consideration of it is reserved to another part of this work.

316 The two following tables are taken out of one of the almanacks pub-
 lished yearly by the order of the royal academy of sciences at Paris, under
 the title of *connaissance des temps*; one use of these tables is, when the difference
 of longitude of places is exprest in time, as hours, minutes, seconds, &c, to
 reduce it to measure, in parts of the equator, as degrees, minutes, seconds: or
 conversely, when it is exprest in measure, to reduce it to time.

a *Connaissance des temps pour l'annee 1727. au meridiem de Paris, publiee par l'ordre de l'Academie Royale
 des Sciences, & calculee par M. Lieutaud de la meme Academie.*

THE FIRST TABLE

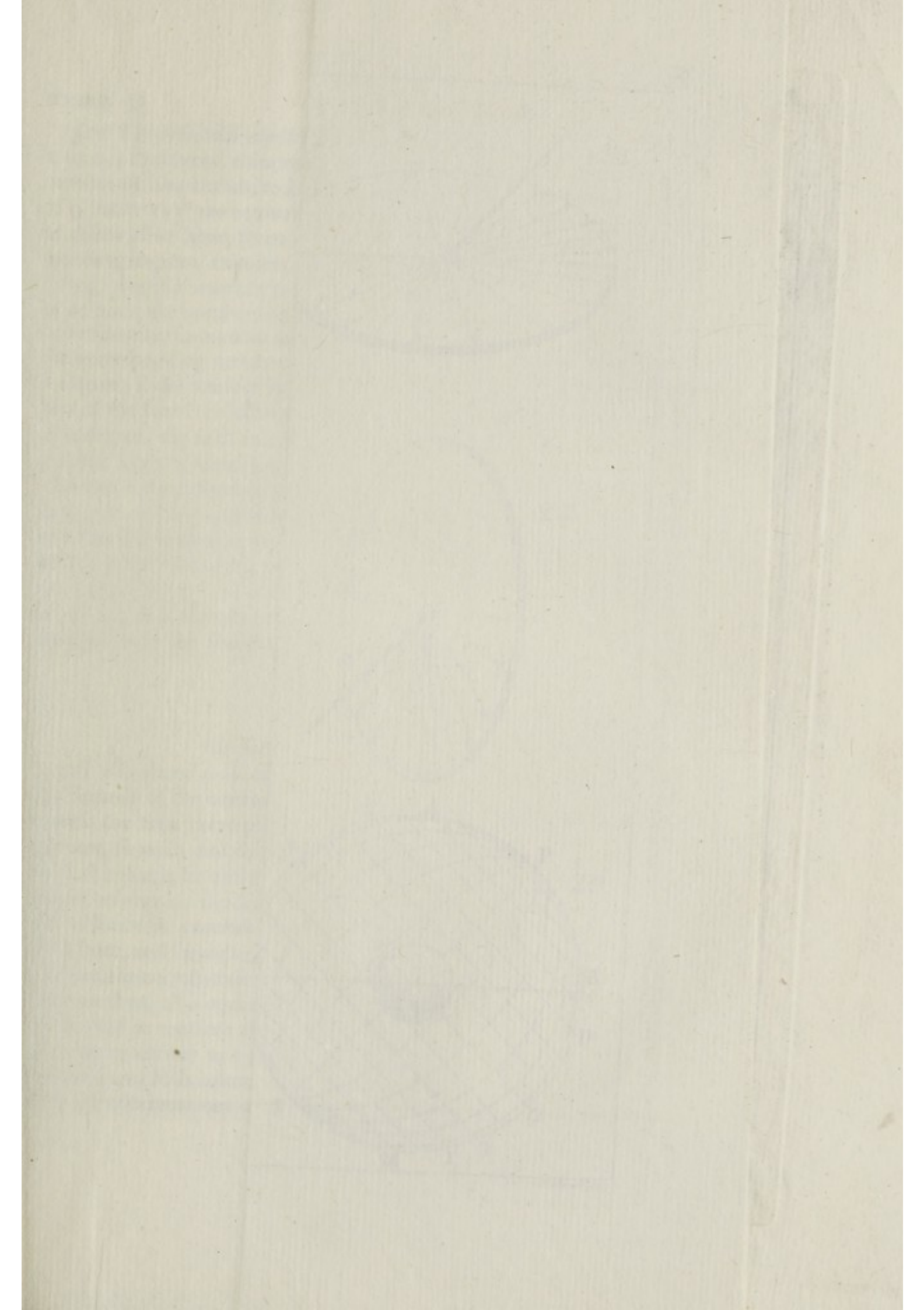
By which the longitude exprest in hours, minutes, and seconds, $\text{\textcircled{c}}$, may be reduced to degrees, minutes, and seconds, $\text{\textcircled{c}}$.

Hours	Degrees	Min.	Deg. Min.	Min.	Deg. Min.
		Sec.	Min. Sec.	Sec.	Min. Sec.
		Thirds	Sec. Th.	Thirds	Sec. Thirds.
1	15	1	0 15	31	7 45
2	30	2	0 30	32	8 0
3	45	3	0 45	33	8 15
4	60	4	1 0	34	8 30
5	75	5	1 15	35	8 45
6	90	6	1 30	36	9 0
7	105	7	1 45	37	9 15
8	120	8	2 0	38	9 30
9	135	9	2 15	39	9 45
10	150	10	2 30	40	10 0
11	165	11	2 45	41	10 15
12	180	12	3 0	42	10 30
13	195	13	3 15	43	10 45
14	210	14	3 30	44	11 0
15	225	15	3 45	45	11 15
16	240	16	4 0	46	11 30
17	255	17	4 15	47	11 45
18	270	18	4 30	48	12 0
19	285	19	4 45	49	12 15
20	300	20	5 0	50	12 30
21	315	21	5 15	51	12 45
22	330	22	5 30	52	13 0
23	345	23	5 45	53	13 15
24	360	24	6 0	54	13 30
		25	6 15	55	13 45
		26	6 30	56	14 0
		27	6 45	57	14 15
		28	7 0	58	14 30
		29	7 15	59	14 45
		30	7 30	60	15 0

THE SECOND TABLE

By which the longitude exprest in degrees, minutes, and seconds, &c, may be reduced to hours, minutes, and seconds.

Deg.	H.	M.	Deg.	H.	M.	Degrees.	Hours.
Min.	M.	S.	Min.	M.	S.		
Sec.	S.	T.	Sec.	S.	T.		
1	0	4	31	2	4	70	4 40
2	0	8	32	2	8	80	5 20
3	0	12	33	2	12	90	6 0
4	0	16	34	2	16	100	6 40
5	0	20	35	2	20	110	7 20
6	0	24	36	2	24	120	8 0
7	0	28	37	2	28	130	8 40
8	0	32	38	2	32	140	9 20
9	0	36	39	2	36	150	10 0
10	0	40	40	2	40	160	10 40
11	0	44	41	2	44	170	11 20
12	0	48	42	2	48	180	12 0
13	0	52	43	2	52	190	12 40
14	0	56	44	2	56	200	13 20
15	1	0	45	3	0	210	14 0
16	1	4	46	3	4	220	14 40
17	1	8	47	3	8	230	15 20
18	1	12	48	3	12	240	16 0
19	1	16	49	3	16	250	16 40
20	1	20	50	3	20	260	17 20
21	1	24	51	3	24	270	18 0
22	1	28	52	3	28	280	18 40
23	1	32	53	3	32	290	19 20
24	1	36	54	3	36	300	20 0
25	1	40	55	3	40	310	20 40
26	1	44	56	3	44	320	21 20
27	1	48	57	3	48	330	22 0
28	1	52	58	3	52	340	22 40
29	1	56	59	3	56	350	23 20
30	2	0	60	4	0	360	24 0



317 *The first table explained*; the first column contains whole hours, from 1 to 24; the second column has the number of degrees which answer to each number of hours: thus, to 4 hours the corresponding number of degrees is 60; to 9 hours 135° are equivalent: in the third column are minutes, or seconds, or thirds of an hour, from 1 to 30; and in the fourth column are the corresponding degrees, minutes, seconds, or thirds of a degree: here it is to be observed, that if a number in the third column be understood to signify minutes of an hour, the numbers on the same line in the fourth column signify degrees, and minutes; if a number in the third column signifies so many seconds of time, the corresponding numbers in the fourth line signify minutes, and seconds of a degree; if the number in the third column signifies thirds of time, the numbers of the fourth column which answer to it are so many seconds, and thirds, of a degree: the fifth column is a continuation of the third, from 31 to 60; and the sixth column is a continuation of the fourth.

Example; the difference in longitude between the meridian of the Royal Observatory at Paris and that of Pekin in China is 7^h 37^m 6^s; if it be enquired what this difference is in parts of the equator, the answer is 114° 16' 30"; which is thus found by the table; to 7^h in the first column, 105° answer in the second; to 37^m in the fifth column, the corresponding numbers in the sixth are 9° 15'; if 6 seconds be found in the third column, the equivalent parts of the equator in the fourth column are 1' 30", add these together,

$$\begin{array}{r}
 105^{\circ} \quad 0' \quad 0'' \\
 9 \quad 15 \quad 0 \\
 \quad \quad 1 \quad 30 \\
 \hline
 \text{the sum is } 114 \quad 16 \quad 30.
 \end{array}$$

318 *The second table explained*; the first column contains degrees, minutes, and seconds of the equator, from 1 to 30; the numbers in the second column express the time corresponding to each number in the first column, in hours, minutes, seconds, and thirds: here it must be observed, that if the number in the first column be understood to mean so many degrees, the numbers in the second column on the same line are hours, and minutes; if the number in the first column be minutes of a degree, the numbers in the second are minutes of an hour, and seconds; if the number in the first column expresses seconds in measure, the corresponding numbers in the second column are seconds, and thirds in time: thus, 1° is equivalent to 0^h. 4^m; one minute of a degree to 4" in time; one second in measure to four thirds in time: thus also, 15° answer to 1^h; 15' of a degree answer to 15" of an hour; 15" in measure to 15 thirds in time: the third and fifth columns are continuations of the first; and the fourth and sixth are continuations of the second.

L

Example;

FIG. *Example*; the meridian of Siam is distant $98^{\circ} 30'$ east, from the meridian of the R. Observatory at Paris: if it be enquired how much this difference is in time, the answer will be $6^h 34^m$; for to 90° in the fifth column, 6^h answer in the sixth; to 8° in the first column, $0^h 32^m$ are set down as equivalent, in the second column; and to $30'$ in the first column, 2^m correspond in the second: add together 6^h and 32^m and 2^m , the sum is $6^h 34^m$,

CHAP. 4. THE GENERAL GROUNDS OF DIALLING.

- 319 If we imagin the hour-circles of any place, as London, to be drawn upon the globe of the earth, and suppose this globe to be transparent, and to revolve round a real axis, which is opaque, and casts a shadow; whenever the plane of any hour-semicircle points at the sun, the shadow of the axis will fall upon the opposite semicircle: thus, fig. 21, if the semicircle *pas* points at the sun, the shadow of the axis *ps* will fall upon the semicircle *pos*.
- 320 If we imagin any plane to pass through the center of this transparent globe, the shadow of half the axis will always fall upon one side or other of this intersecting plane: thus, fig. 21, let *ABCD* be the plane of the horizon of London; so long as the sun is above the horizon, the shadow of the upper half of the axis *pg* will fall somewhere upon the upper side of the plane *ABCD*: when the sun is below the horizon of London, then the shadow of the lower half of the axis *gs* falls upon the lower side of the plane *ABCD*.
- 321 When the plane of any hour-semicircle points at the sun, the shadow of the axis marks the respective hour-line upon the intersecting plane; *the hour-line* therefore is a line drawn from the center of the intersecting plane to the point where the intersecting plane is cut by the semicircle opposite to the hour-semicircle: thus, fig. 21, let *ABCD* the horizon of London be the intersecting plane; when the meridian of London *pas* points at the sun, as in the figure before us, the shadow of the half axis *pg* falls upon the line *gB*, which is drawn from *g* the center of the horizon, to *B*, the point where the horizon is cut by the opposite semicircle *pBs*: therefore *gB* is the line for the hour of 12 at noon.
- 322 By the same method the rest of the hour-lines are found, by drawing for every hour a line from the center of the intersecting plane, to that semicircle which is opposite to the hour-semicircle: the 21 figure shews the hour-lines drawn upon the plane of the horizon of London, so many as are necessary, which are only for those hours during which the sun is above the horizon of London, the longest day in summer: the hour-lines being thus found by the intersecting points of the hour-semicircles, let the semicircles be taken away,

away, as the scaffolding is when the house is built, and what remains, as in FIG. 22 fig. 22, will be an *horizontal dial* for London.

- 323 If instead of 12 hour-circles as above described we take twice that number, we may, by the points where the intersecting plane is cut by them, find the lines for every half hour; if we take four times the number of hour-circles, we may find the lines for every quarter of an hour, &c.
- 324 I have here taken the horizon for the intersecting plane, by which the method is seen of making an horizontal dial; if we take any other plane for the intersecting plane, and find the points where the hour-semicircles pass through it, and draw lines from the center of the plane to those points, we shall have the hour-lines for that plane: the 23^d figure shews how the hour-lines are found upon a south plane perpendicular to the horizon; and the 24th figure shews a *south dial* with its hour-lines, without the semicircles by means whereof they are found. *The gnomon of every sundial represents the axis of the earth*, and is therefore always placed parallel to it, whether it be a wire, as in the figures before us, or the edge of a plate, as in the common brass horizontal dials.

- 325 If a little transparent globe with hour-semicircles and an axis be so placed upon the surface of the earth, that the axis is parallel to the axis of the earth, all things mentioned in the preceding sections will be the same in this globe as in the globe of the earth itself, supposed transparent; or as they would be in this little globe placed in the center of the earth: because, by § 274, the earth may be considered as a point, and consequently in every such little globe, the plane *abcd* being parallel to the plane *ABCD*, fig. 25, may be looked upon as coincident with it, and the axis *ps* being in every such globe parallel to *PS*, may be considered as coincident with it: for this reason, a dial placed upon the surface of the earth shews the hour as truly, as if it were placed at the center of the earth: in fig. 25, let the large dial in the middle represent a dial made upon the plane of the rational horizon, let the little dials round this be pictures of dials made upon horizontal planes, placed upon the surface of the earth; the sun, by reason of his immense distance, shines in the same manner upon all these, all his rays which fall upon these several dials are physically parallel, have the same effect as if they were exactly parallel; and therefore, since by the rotation of the earth the plane *abcd* of every one of these little dials is carried parallel to the plane of the horizon *ABCD*, they will shew the hours in the same manner as the large dial does.

And thus much is enough to shew the general grounds and principles of dialling, which consist in finding where the shadow of a strait wire parallel to the axis of the earth will fall upon a given plane, every hour, every half hour, &c: the hour-lines are found by the hour-semicircles as described above:

FIG. there are several ways of finding the intersecting points of the hour-semicircles, for all sorts of planes, either by calculation, or by mechanical operations, but these are to be learned from those writers who professedly treat of dialling.

CHAP. 5. THE LATITUDE OF PLACES: THE ELEVATION OF THE POLE:
THE COMPLEMENT OF LATITUDE, OR HEIGHT OF THE EQUATOR.

326 *The equator* divides the globe of the earth into two parts, the *northern* and *southern*: all places upon the earth lying under the equator, are said to have no latitude: all other places upon the earth are said to be in *north* or *south* latitude, according as they are situated on the north or south side of the equator, towards the north or south pole.

327 *The latitude of any place* is its distance from the equator, measured in degrees, minutes, and seconds, upon the meridian of that place: thus, fig. 17, let *l* be London, *p* the north pole, *eq* the equator, *ples* is the meridian of London, the latitude of London is the arc *el*, this arc is $51^{\circ} 31'$; London is then in $51^{\circ} 31'$ of north latitude.

328 *The complement of latitude* in any place is the number of degrees, minutes, and seconds which, added to the latitude, make up 90° : thus, the complement of the latitude of London is $39^{\circ} 29'$; for $39^{\circ} 29'$ added to $51^{\circ} 31'$ make 90° . *The complement of latitude shews the height of the equator*, or the angle the plane of it makes with the plane of the horizon: thus, fig. 17, let *l* be London, the latitude of London is the arc *el*; the complement of latitude is the arc *eb*, which measures the height of the equator, or the angle *eib* contained between the planes of the equator *eq* and the horizon of London *bo*.

329 *The elevation of the pole* in any place is the least distance of that pole from the horizon, measured in degrees, minutes, and seconds, upon the meridian^a of the place: thus, fig. 17, let *l* be London, *p* the north pole, *bo* the horizon of London; the elevation of the north pole above the horizon of London is the arc *po*.

330 *The elevation of the pole in any place is equal to the latitude* of that place: thus, fig. 17, the arc *elp* is 90° , by § 298, and the arc *lpo* is 90° , by § 274; take from each of these the arc *lp*, and the remaining arcs *le* and *po* will be equal, by § 35, axiom. 3.

331 Here follows a table of the longitudes and latitudes of some remarkable places; the second column shews the difference of their meridians from the meridian of London, in hours, minutes, and seconds; (E) shews the place to lye east, and (W,) west from London: the last column contains the latitude or elevation of the pole: all the places here set down, except the Cape of good hope, are in north latitude.

^a The meridian is here taken for the whole circle, as defined § 299.

	Difference from the meridian of London						Lat. or height of the pole.			
	H.	M.	S.		D.	M.	S.	D.	M.	S.
*Alexandria in Egypt	2	1	27	E	30	21	45	31	11	20
Araçta	2	59	41	E	44	55	15	36	0	0
Bologna	0	46	46	E	11	42	15	44	30	0
Cairo	2	6	6	E	31	31	30	30	2	30*
Cambridge	0	16	11	E	4	2	45	52	17	0
Cape of G. hope	1	20	39	E	20	9	45	34	15	0
Cayenne	3	32	19	W	53	4	45	4	56	0
Dantzic	1	14	25	E	18	36	15	54	22	0†
Dublin	0	26	0	W	6	30	0	53	12	0
Edinburgh	0	12	0	W	3	0	0	55	57	0
Florence	0	45	39	E	11	24	45	43	46	30
*Goa	4	55	21	E	73	50	15	15	31	0
Jerusalem	2	21	41	E	35	25	15	31	50	0
Ifpahan	3	31	41	E	52	55	15	32	25	0
Lisbon	0	33	19	W	8	19	45	38	45	0*
London	0	0	0		0	0	0	51	31	0*
†Madrid	0	12	19	W	3	4	45	40	26	0†
Marfeilles	0	22	9	E	5	32	15	43	19	30
Mexico	6	54	19	W	103	34	45	20	0	0†
Moscow	2	41	41	E	40	25	15	55	36	0
Naples	0	59	1	E	14	45	15	40	48	0
†Nuremberg	0	44	37	E	11	9	15	49	26	0†
Oxford	0	5	0	W	1	15	0	51	43	0
Paris R. observ.	0	9	41	E	2	25	15	48	50	10
Pekin	7	46	47	E	116	41	45	39	54	0
Petersburg	2	7	41	E	31	55	15	60	0	0
Pic of the Açores	1	52	19	W	28	4	45	38	35	0
Pic of Teneriff	1	2	19	W	18	0	0	28	30	0
Rome	0	51	1	E	12	45	15	41	54	0
Siam	6	43	41	E	100	55	15	14	18	0
Stockholm	1	18	1	E	19	30	15	59	20	0†
Venice	0	51	1	E	12	45	15	45	25	0
Vienna	1	7	51	E	14	57	45	48	14	0*
Uraniburg	0	51	51	E	12	57	45	55	34	5
York	0	4	0	W	1	0	0	54	0	0

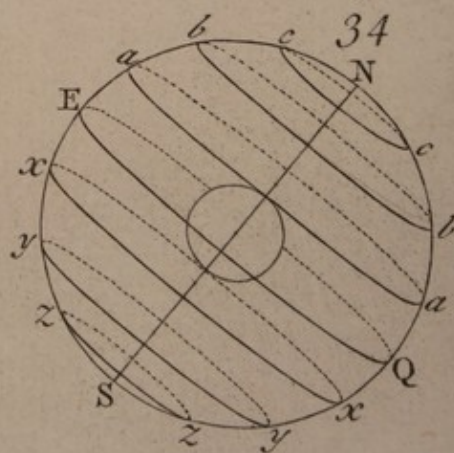
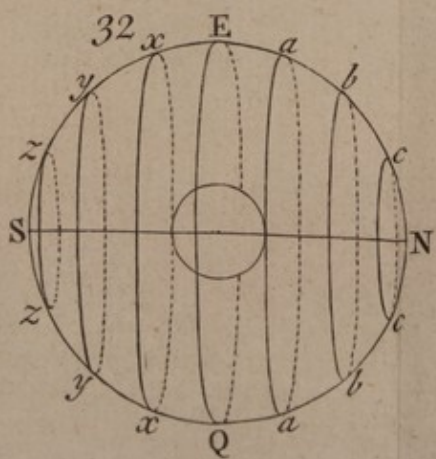
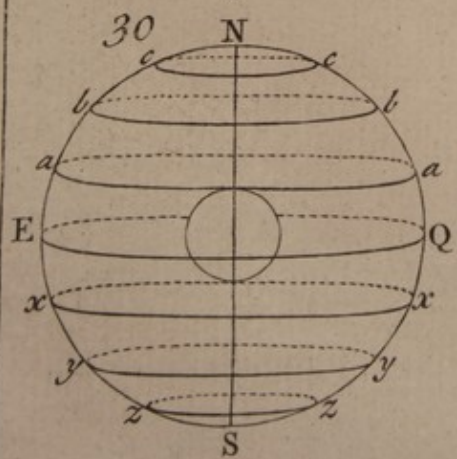
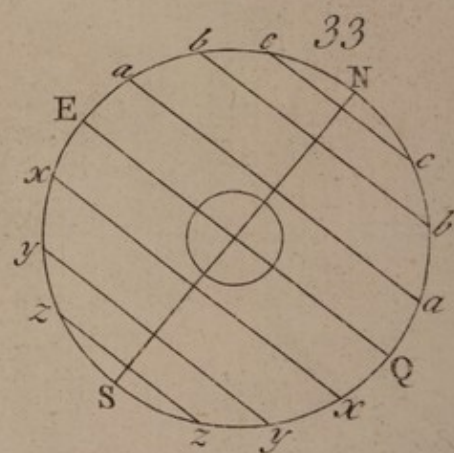
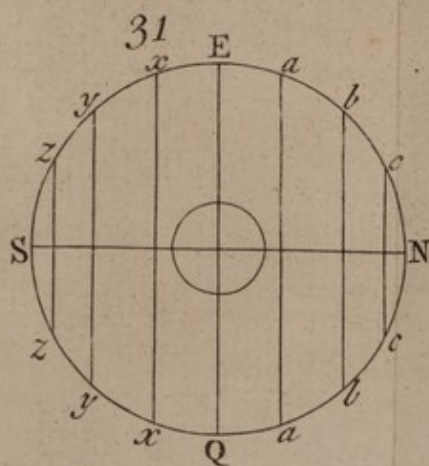
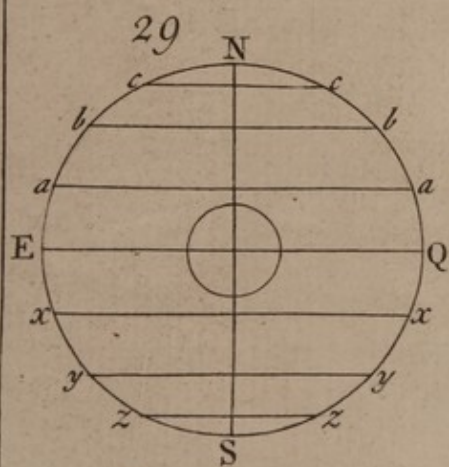
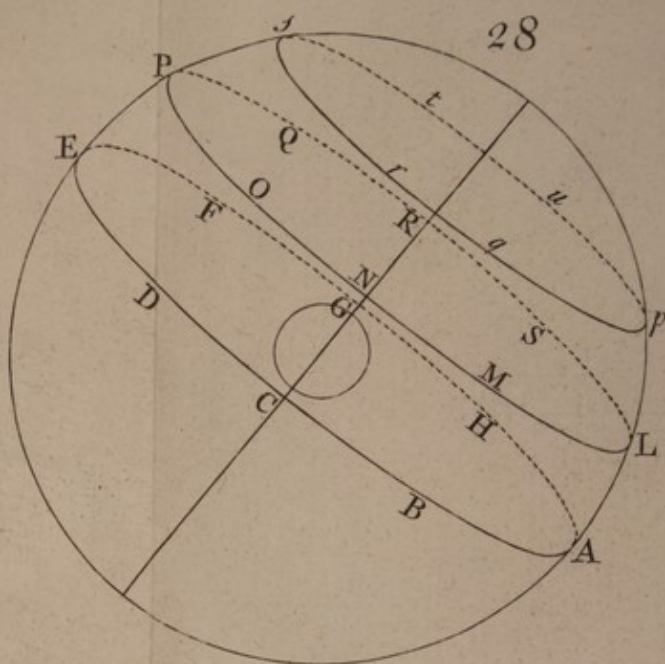
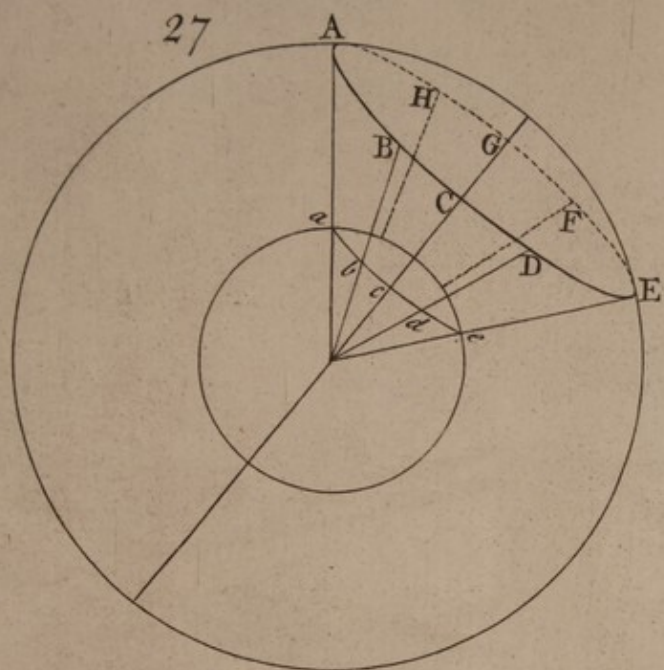
Tables of the longitude and latitude of places are to be found in many books of geography; *Ricciolus* has a very large one, in his *geographia reformat*a; l. 9. c. 4. *Souci*et has another, at the end of his *observations mathematiques, astronomiques, &c, tirees des anciens livres Chinois, ou faites nouvellement aux Indes et a la Chine, par les peres de la C. de J. Lieutaud* gives us one of four pages, in the *connoissance des temps*: him I have chiefly followed in the foregoing specimen; in which those longitudes or latitudes that are marked with a star, are such as have been verified by the astronomical observations of some of the members of the Royal Academy of sciences at Paris; those marked with a cross, are from the observations of other astronomers.

332 *A table of the longitude of places for one meridian being given, we may make a table for any other meridian*, if the difference between those meridians be also given: thus, in *Lieutaud's* table, the longitudes are from the R. Observatory at Paris; those in my specimen were from thence computed for the meridian of London, by the following method. I find in *Lieutaud's* table, that the meridian of London is distant from the meridian of the Royal Observatory at Paris, $0^h 9^m 41^s$ in time, or $2^\circ 25' 15''$ in measure, westward: every place therefore that lyes east from Paris, is so much further east from London; and must have $0^h 9^m 41^s$, or $2^\circ 25' 15''$, added to its longitude from Paris, if we would have its longitude from London: on the other hand, every place that lyes west from Paris, is $0^h 9^m 41^s$, or $2^\circ 25' 15''$, less distant from London, westward; and must have so much taken from its longitude reckoned from the meridian of Paris, if we would have its longitude from the meridian of London: thus, the longitude of Rome from Paris is $0^h 41^m 20^s$ eastward; add to this, $0^h 9^m 41^s$, and you have the longitude of Rome from London, $0^h 51^m 1^s$: thus again, Mexico is $106^\circ 0' 0''$ west from Paris; take from that number of degrees, $2^\circ 25' 15''$, and there remains $103^\circ 34' 45''$, the longitude of Mexico west from London.

333 Another use of the table is this, *if the time of the day be given at one place, the time of the day at any other place, may be known*, if the different longitude of those places be also given: thus, suppose it is now 2 in the afternoon at London, what is it o' clock at Jerusalem? I find by the table, that the meridian of Jerusalem is $2^h 21^m 41^s$ east, so long therefore it comes sooner to the sun than the meridian of London, and consequently so much more of the day is spent at Jerusalem than at London: add then $2^h 21^m 41^s$ to the time of the day at London, and you have the time of the day at Jerusalem, $4^h 21^m 41^s$ in the afternoon.

Another example; suppose, I find by an almanack, that the beginning of an eclipse of the moon will be this present night, 4 minutes after 9 at London; if

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if it be enquired what time of the night it will then be at Lisbon, the answer FIG. is thus found; Lisbon is $0^h 33^m 19^s$ west from London, therefore so much less of the natural day will be then past there, than at London; take $0^h 33^m 19^s$ from $9^h 4^m 0^s$, the time at London, and the remainder $8^h 30^m 41^s$ is the time of the night at Lisbon.

Another example; when it is noon at Ispahan, what o' clock is it at London? I find by the table, Ispahan is $3^h 31^m 41^s$ east; so much further then is the day advanced there than at London: take $3^h 31^m 41^s$ from $12^h 0^m 0^s$ there remains the time of the day at London, $8^h 28^m 19^s$ in the morning: other uses of these tables will be shewn, as occasion offers itself.

334 *The different longitude and latitude of different places upon the earth, is a proof that the earth is spherical*: for that different meridians come successively to point at the sun, as has been described § 310, is a proof that the earth is round from east to west; and the different elevation of the pole in different places under the same meridian, shews it to be round from north to south.

335 We may imagin as many circles as we please to be drawn upon the earth, parallel to the equator; these grow less and less as they are nearer to either of the poles of the earth; such circles are called *parallels*, or parallels of latitude: all places having the same latitude lye under the same parallel; and conversely, all places under the same parallel have the same latitude: in fig. 18, 18 these parallels are express'd by strait lines, parallel to the equator eq ; amongst which RLT is the parallel of London; vcx the parallel of Cairo.

The astronomical terms which arise from considering the rotation of the earth, and relate more immediately to the heaven, are next to be explained.

CHAP. 6. THE CELESTIAL POLES, EQUATOR, PARALLELS: CIRCLES OF DECLINATION: DECLINATION OF THE SUN, STARS, &c.

336 *The celestial poles* are two points in the sphere of the heaven, through which the axis of the earth infinitely extended both ways would pass: one of these, P , fig. 17, through which the north end of the axis would pass, is 17 called *the arctic pole*, from the greek word *arctos*, which signifies a bear, because this pole is near the constellations of the great and little bear; the other celestial pole s , being opposite to P , is called *the antarctic pole*: the celestial poles are often called the north and south pole.

337 The elevation of the pole may be considered in the sphere of the heaven, and then it is the arc of a vertical, contained between the celestial pole and the plane of the horizon extended: thus, the height of P the celestial pole, 17
fig.

FIG. fig. 17, is the arc PO : the height of the pole is of the same number of degrees, minutes, and seconds, whether we consider the terrestrial or celestial pole: for, SP being a strait line, the angle PbO must be equal to the angle pbo ; and the arc PO to the arc po : § 30, and 31.

338 We may imagin the plane of the equator upon the earth to be extended every way, till it reaches the starry heaven, and to mark upon it *the celestial equator*. We may imagin as many *circles* as we please to be drawn in the *sphere of the heaven, parallel to the celestial equator*; these parallels grow less and less, the nearer they are to the celestial poles; in fig. 26, eq is the equator upon the earth, and EQ the celestial equator; the parallels to the celestial equator are $AB, CD, FG, \&c.$

339 We may imagin as many great circles as we please to be drawn upon the sphere of the heaven, intersecting each other in the celestial poles; these will cut the equator at right angles, and are called *circles of declination*, and sometimes, secondaries of the equator, § 282: in fig. 26, some of the circles of declination are represented by the strait line ps , and the curves $PES, pas, pbs, pcs, \&c.$, intersecting each other in the poles P and s .

340 *The declination of any point in the heaven*, as the sun, moon, a star $\&c.$ ^a is its distance from the celestial equator, in degrees, minutes, and seconds, measured upon a circle of declination drawn through that point; *declination* is either north or south: thus, fig. 26, EQ is the celestial equator, P the north pole, s the south; the star A is in north declination, so much as the arc AE amounts to; in the same figure, the south declination of the star H is measured by the arc HE .

341 We may imagin any parallel upon the earth to have a corresponding parallel in the sphere of the heaven; thus, fig. 27, $abcd \&c$ is the parallel of London, and $ABCDEFGH$ the corresponding parallel in the heaven: we may imagin any parallel in the sphere of the heaven to have a corresponding parallel drawn upon the globe of the earth; thus, $ABCDEFGH$ is a parallel in the heaven, the corresponding parallel to which upon the earth is $abcd \&c$: *corresponding parallels* are such where the celestial parallel is the same number of degrees, minutes, and seconds distant from the celestial equator, as the parallel upon the earth is from the equator upon the earth.

342 If we imagin a line to be drawn from the center of the earth through any place, till it reaches the sphere of the heaven, it will mark there the zenith of that place: thus, fig. 27, a line drawn from the center of the earth through London at a , marks the zenith of London A : by the rotation of the earth, the zenith of every place which is not exactly under one of the poles, perpetu-

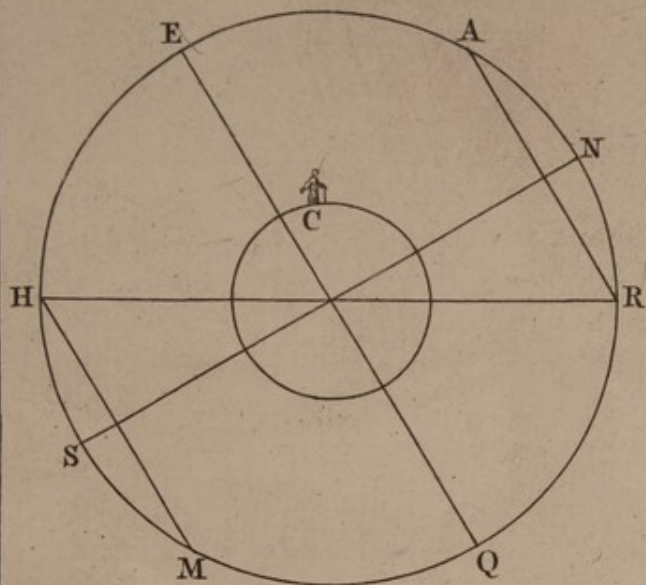
^a By the declination of the sun, moon, $\&c.$ is meant the declination of their centers.



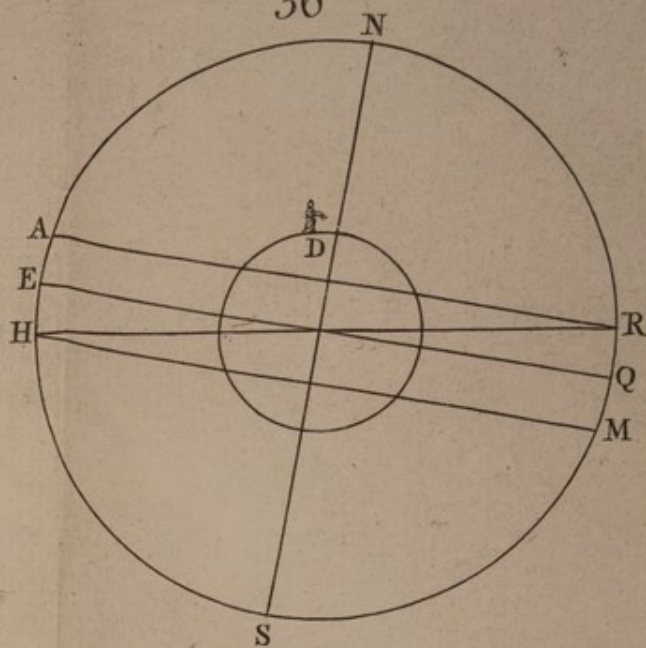
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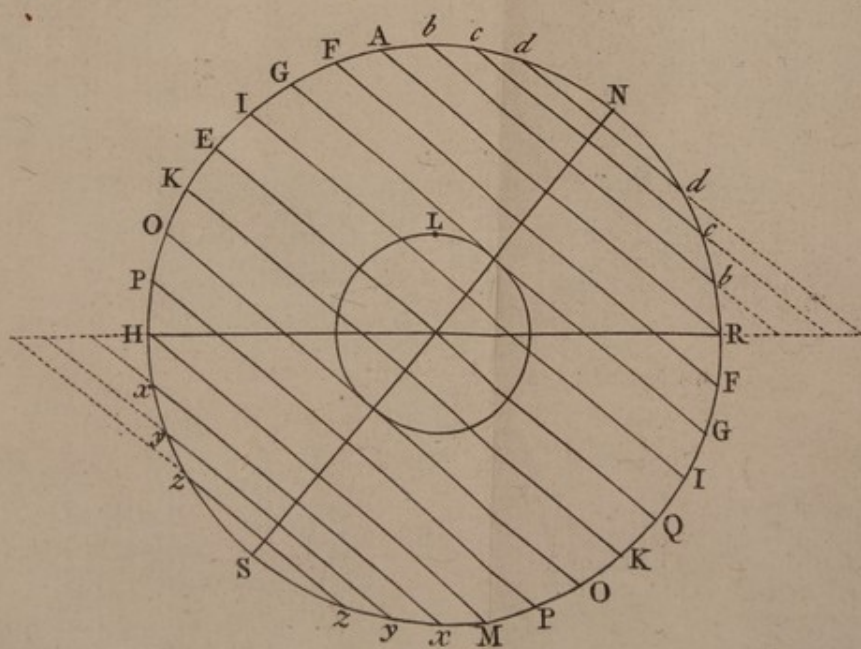
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36



37



ally changes its situation in the heaven, so as to go round, in a celestial parallel corresponding to the parallel of the place: thus, fig. 27, A the zenith of London goes round in the celestial parallel $AB C D E F G H$, according to the order of those letters: the cause of this change of place in the zenith of London is this; by the rotation of the earth, London is carried round in its parallel $abcd \&c$, according to the order of those letters, and by that means comes successively to be under different parts of the corresponding parallel $ABCD \&c$; that is, London has for its zenith, successively different points of that parallel: so that, when the place of London is at a , the zenith, or point over London will be A ; when London is at b , the zenith will be B ; when London is come in its parallel to c , the zenith will be c , $\&c$. By this motion of the zenith of London round the parallel $ABCD \&c$, from A to B , from C to D , from D to E , and so on, every point in that parallel will appear to come to the zenith of London, the contrary way; that is, the several points $A, B, C, D, E, \&c$, will appear to come successively, one after another, according to the order of those letters, to the zenith of London: first the point A , then B , after that c , then D , $\&c$.

343 By the rotation of the earth from west to east, every point in the heaven, except the celestial poles, which are immoveable, appears to go round from east to west, either in the celestial equator, or in a circle parallel to the celestial equator: thus, fig. 28, a point A , which is in the celestial equator, appears to go round in it, according to the order of the letters $ABCD \&c$; a point L in the heaven which is not in the celestial equator, will appear to be carried round in a parallel drawn through that point, such as is the circle $LMNOPQRS$, according to the order of those letters. The further any point in the heaven is from the celestial equator, the less is the parallel described by it; thus, the point p describes the parallel $pqrst$, less than the parallel $LMNOPQRS$.

CHAP. 7. THE PARALLEL, RIGHT, AND OBLIQUE SPHERE: THE ARCTIC, AND ANTARCTIC CIRCLE.

344 In each of the six following figures, the sphere of the heaven is represented, with the globe of the earth in the middle of it; N and s are the celestial poles, N the arctic, and s the antarctic pole; in fig. 29, 31, and 33, the celestial equator is represented by EQ , the north parallels by aa, bb, cc ; and the south parallels by xx, yy, zz : all strait lines, as they would appear, if they were to be viewed with the eye at an infinite distance, and in the plane of the celestial equator extended ^a, in such a view, only one half of the equator and of each parallel, viz. the convex semicircle ^b, can be seen; in fig. 30, 32, and 34,

^a § 254 and 258. ^b § 260.

FIG. the celestial equator and parallels, that they may be seen entire, are represented by oblong ellipses, such as they would appear, if they were to be viewed obliquely, with the eye a little elevated above the plane of the equator, and at an infinite distance^a; in these, and most of my other figures of this kind, in which the pictures of entire circles are given, such as they would appear drawn upon a glass sphere, and viewed obliquely, I have represented the concave semicircles^b, which are furthest from the eye, by pointed semiellipses; and the convex semicircles^c, which are nearest to the eye, by black semiellipses; that they may the more easily be distinguished from one another.

345 An inhabitant of the earth who lives at either of the poles, has one of the celestial poles always in his zenith, the other in his nadir; he has also the equator coincident with his horizon: and therefore, all the celestial parallels, being parallel to the equator, are parallel to his horizon: such a person is said to live in a *parallel sphere*, or to have a *parallel horizon*: fig. 29, 30.

346 An inhabitant of the earth who lives under the equator, has both the celestial poles in his horizon, and is said to live in a *right sphere*, or to have a *right horizon*; because the celestial equator EQ , and all the parallels aa , bb , cc , and xx , yy , zz , cut his horizon HR at right angles, fig. 31, 32.

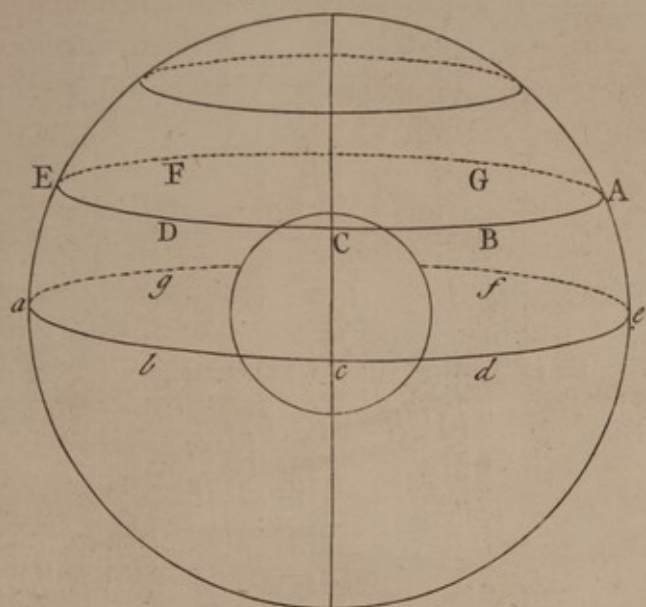
347 An inhabitant of the earth who lives between either of the poles and the equator, is said to live in an *oblique sphere*, or to have an *oblique horizon*; because the celestial equator cuts his horizon obliquely; and all the parallels in the sphere of heaven, have their planes oblique to the plane of his horizon, fig. 33, 34. In an oblique sphere, some of the parallels intersect the horizon at oblique angles; some are entire above, some entire below the horizon; and these also are all so situated, that their planes extended would intersect the plane of the horizon extended, at oblique angles: thus, let fig. 37 represent the earth surrounded with the sphere of the heaven; let an inhabitant of the earth be placed in north latitude at L ; his horizon is HR , which is intersected obliquely by the equator EQ , and by all the parallels between AR and HM , such as FF , GG , II , &c: the parallel AR is entire above the horizon, as are also all the parallels between AR and the north pole N ; such as bb , cc , dd : the parallel HM is entire below the horizon, as are also all the parallels between HM and the south pole s ; such as xx , yy , zz : if the planes of these were extended, as they are represented in the figure before us, by the pointed lines; they would intersect the plane of the horizon extended, at oblique angles.

348 The largest parallel which appears entire above the horizon of any place in north latitude, is by the ancient astronomers called the *arctic circle* of that place: within this circle, between it and the arctic pole, are compre-

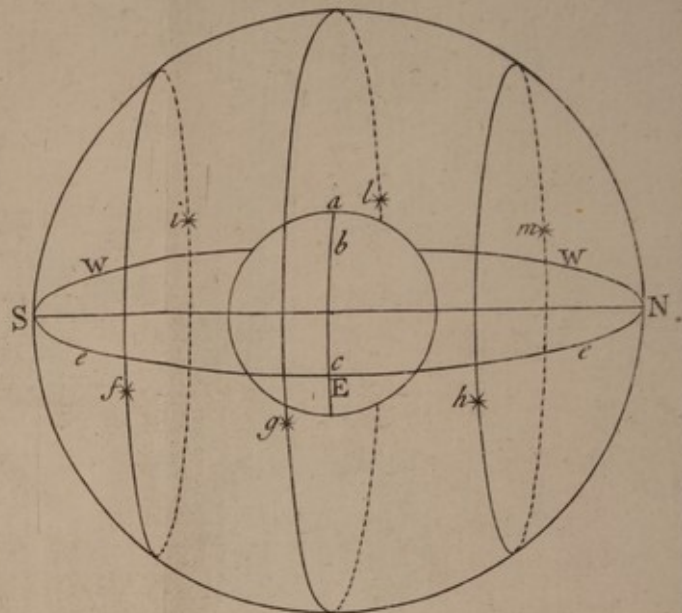
^a § 257 and 258. ^b § 260. ^c § 260.

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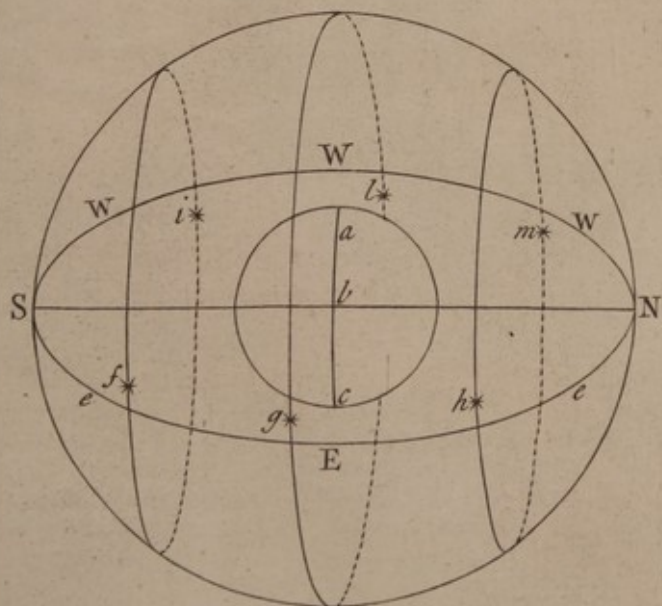
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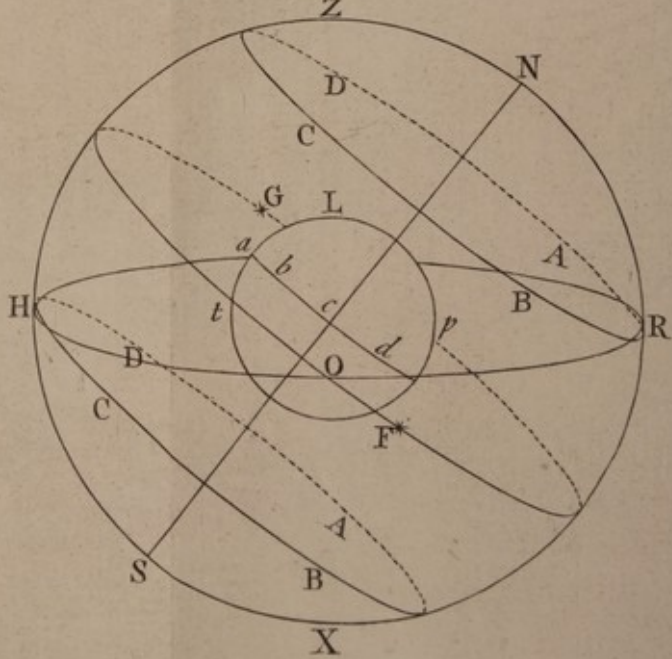
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41



hended all the stars which never set in that place, but are carried perpetually round above the horizon, in circles parallel to the equator. The largest parallel which is hid entire below the horizon of any place in north latitude, is the *antarctic circle* of that place, according to the ancients: this circle comprehends all the stars which never rise in that place, but are carried perpetually round below the horizon, in circles parallel to the equator.^a

349 In a parallel sphere, the equator may be considered as both *arctic* and *antarctic circle*^b; for, the equator being coincident with the horizon, all the parallels on one side of the equator are entire above the horizon, and all the parallels on the other side of the equator are entire below the horizon: thus, since an inhabitant under the north pole has the equator EQ coincident with his horizon, fig. 29, all the north parallels aa, bb, cc, are entire above his horizon, and visible; and all the south parallels xx, yy, zz, are entire below his horizon, and invisible to him: on the other hand, an inhabitant under the south pole has all the south parallels entire above his horizon, and all the north parallels entire below it; as will appear from a view of the 29th figure inverted.

350 In an oblique sphere, the nearer any place is to one of the poles, the larger are the *arctic* and *antarctic circles* in that place, as being nearer to the celestial equator, which is a great circle: thus, if a place c is in 30° of north latitude, fig. 35, the horizon of it is HR, the *arctic circle* is AR, the *antarctic* HM: if a place D is in 80° of north latitude, fig. 36, the horizon is HR, the *arctic circle* AR, the *antarctic* HM; both much greater than in the 35th figure.

351 In a right sphere, the *arctic* and *antarctic circles* have no place; because no parallel appears entire above the horizon, or is hid entire below it; but every parallel is divided into equal parts by the plane of the horizon: this appears by fig. 31, where the equator EQ and every parallel is divided into two equal parts by the plane of the horizon. Some modern geographers, in their maps and globes, by the *arctic* and *antarctic circles* understand two fixt circles, drawn parallel to the equator, at the distance of 23° 30' from each of the poles; but these are, properly speaking, the *polar circles*; and will hereafter be described under that name: § 371.

^a The *arctic circle* is called also *maximus semper apparentium*, and *circulus perpetus apparitionis*; and the *antarctic circle*, *maximus semper occultorum*, and *circulus perpetus occultationis*; vide scriptores graecos de sphaera, Geminum, Proclum, &c: vide etiam Scalig. in Manil. lib. 1. qui tamen ibi male eos reprehendit qui circulum arcticum eundem semper cum aequatore coelesti & tropicis polum habere statuunt; mutatur quidem aequatoris coelestis locus, per punctorum equinoctialium regressum, ut infra patebit; sed mutantur itidem tam tropici quam circulus arcticus & antarcticus, ita ut aequatori sint omnes semper paralleli.

^b Gemini. introd. cap. de circulis sphaera,

FIG.

CHAP. 8. THE DIFFERENCE IN THE APPARENT DIURNAL MOTION OF
THE SUN AND OTHER HEAVENLY BODIES, IN THE SAME HORIZON;
OR IN DIFFERENT POSITIONS OF THE HORIZON: THE TROPICS:
THE EQUINOXES: THE SOLSTICES.

352 By the definition of the horizon § 278, it appears, that the plane of the horizon of every place upon the earth is invariable, so that it always cuts the surface of the earth in the same points; and this section of the globe of the earth is the circumference of a great circle, from every point of which, the place whose horizon it is, is 90° distant: so that by the rotation of the earth, the plane of every horizon is carried round along with the earth, and therefore there will arise, from the different position of the horizon in respect of the equator and its parallels, the following differences in the apparent motion of the sphere of the heaven, and of the heavenly bodies.

38 353 To an inhabitant in a parallel sphere, fig. 38, by the plane of his horizon *abcdefgh* going round according to the order of those letters, the visible hemisphere of the heaven and all the heavenly bodies which are therein will appear to go round the contrary way, according to the order of the letters *ABCDEFGH*, above the horizon, in circles parallel to it: § 343 and 345.

354 In a right, and oblique sphere, the horizon of every place is carried round along with the earth in such a manner, that the east side of it is always successively sinking below those stars and other heavenly bodies which are in the eastern part of the heaven; by means whereof the stars, &c. appear to rise above it, on the east side: and the west side of the horizon is constantly rising above the stars, &c. and covering them; by means whereof, the stars, &c. in the western part of the heaven appear to sink down below the west side of the horizon, or to set.

355 And this rising and setting of the stars and other heavenly bodies, in a right sphere, is in circles which cut the horizon at right angles: thus, fig. 39 and 40 are pictures of a right sphere, *a* is the place of the inhabitant, *eee* *ww* the plane of his horizon extended to the heaven, and viewed obliquely, *eee* is the east side, and *ww* the west side of the horizon; *f*, *g*, *b*, are stars in the eastern hemisphere of the heaven, and *i*, *l*, *m*, stars in the western hemisphere: it is plain, that when the earth is in the situation represented in fig. 39, the stars *f*, *g*, *b*, are below the horizon, and invisible; and the stars *i*, *l*, *m*, are above the horizon, and visible: but when the earth, by the rotation round its axis *ns*, according to the order of the letters *abc*, is come into the situation represented in fig. 40, then, by the east side of the horizon *eee* sinking below the

1875
The first of the year
was a very dry one
and the crops were
much injured.

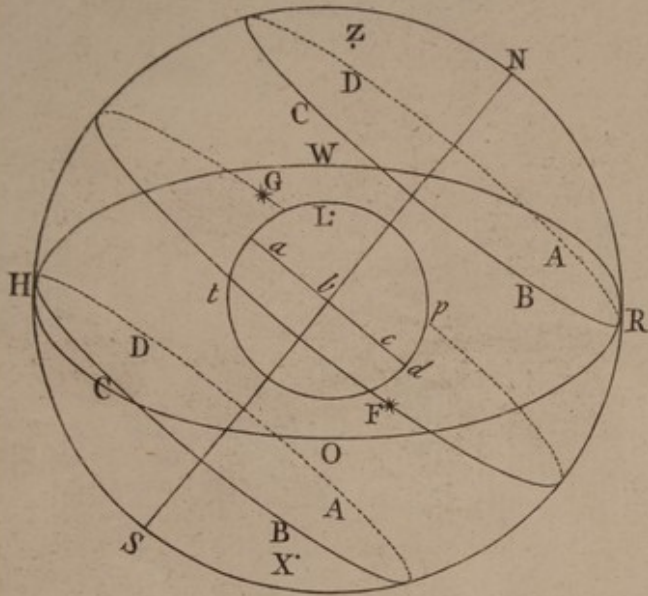
The second of the year
was a very wet one
and the crops were
much injured.
The third of the year
was a very dry one
and the crops were
much injured.
The fourth of the year
was a very wet one
and the crops were
much injured.
The fifth of the year
was a very dry one
and the crops were
much injured.
The sixth of the year
was a very wet one
and the crops were
much injured.
The seventh of the year
was a very dry one
and the crops were
much injured.
The eighth of the year
was a very wet one
and the crops were
much injured.
The ninth of the year
was a very dry one
and the crops were
much injured.
The tenth of the year
was a very wet one
and the crops were
much injured.



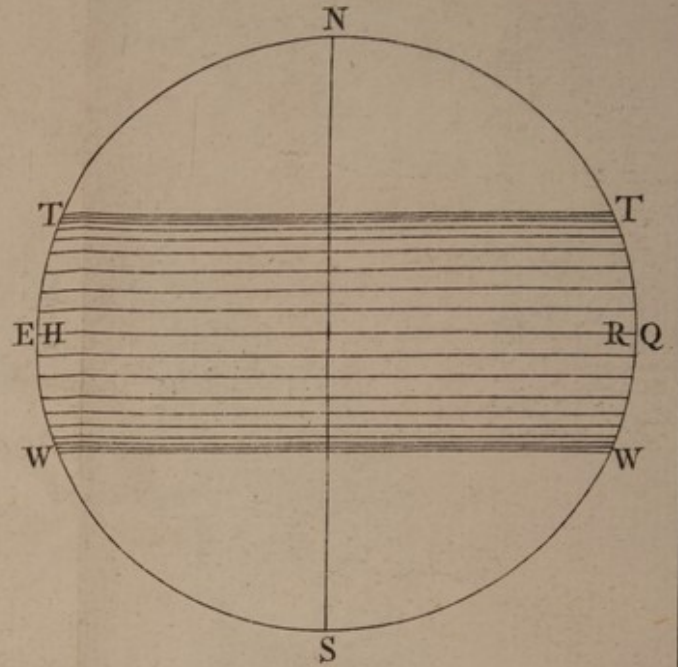
1876
The first of the year
was a very dry one
and the crops were
much injured.
The second of the year
was a very wet one
and the crops were
much injured.
The third of the year
was a very dry one
and the crops were
much injured.
The fourth of the year
was a very wet one
and the crops were
much injured.
The fifth of the year
was a very dry one
and the crops were
much injured.
The sixth of the year
was a very wet one
and the crops were
much injured.
The seventh of the year
was a very dry one
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much injured.
The eighth of the year
was a very wet one
and the crops were
much injured.
The ninth of the year
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much injured.
The tenth of the year
was a very wet one
and the crops were
much injured.

Book I.

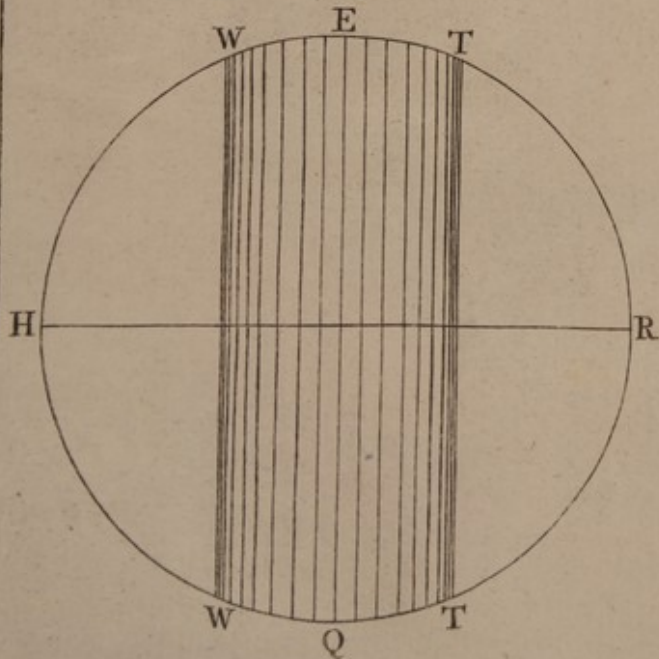
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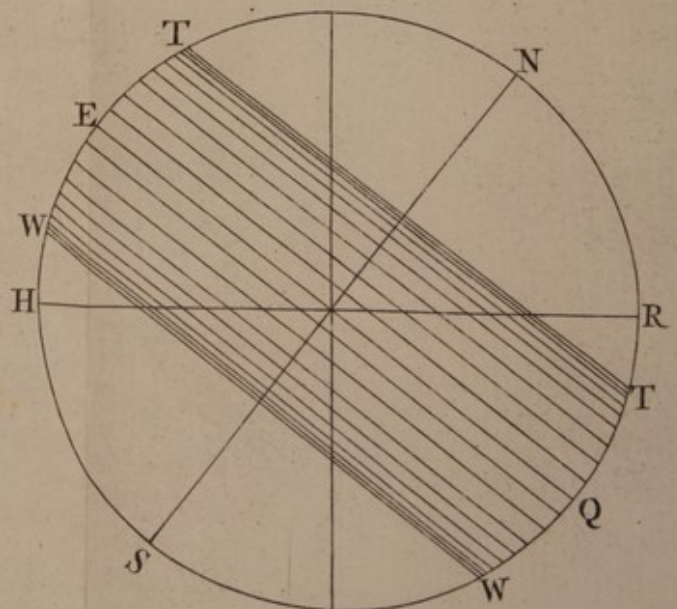
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44



45



the stars *f, g, h*, those stars will appear to have risen above the east side of the horizon; and by the west side of the horizon *wWw* rising above the stars *i, l, m*, and covering them, those stars will appear to be set below the west side of the horizon. FIG. 40

356 In an oblique sphere, the rising and setting of the heavenly bodies is in circles which cut the horizon at oblique angles: the 41 and 42 figures are pictures of an oblique sphere, in which, let *HZRX* represent the sphere of the heaven, *Lpct* the globe of the earth, let the point *L* be London, whereof *z* is the zenith, and *x* the nadir; let *HOR* be the plane of the horizon of London; if the earth be imagined to turn round upon its two poles *N* and *s*, according to the order of the letters *abcd*, so that the point *a* upon the earth moves into the place of the point *b*, and the point *b* moves into the place of *c*, the appearances produced by this rotation of the earth will be the same as they would be, if the sphere *HZRX* turned round upon the poles thereof *N* and *s*, the contrary way, according to the order of the letters *ABCD*: to be more particular, if the earth's rotation be such, that from the situation represented fig. 41, it comes into the situation in fig. 42; it is evident, that by the sinking of the east side of the horizon *HOR* obliquely below the star *F*, the star *F* will appear to rise obliquely above the east side of the horizon *HOR*; as also that, by the rising of the west side of the horizon *HWR* obliquely above the star *G*, and covering it, the star *G* will appear to set, or sink obliquely below the west side of the horizon: what is here said, to explain the rising and setting of the stars, is applicable to the sun, moon, or any of the heavenly bodies, and explains their rising and setting also. 41 42

357 The sun, seen from our earth, appears in a different point of the heaven, every day of the year: if the sun be observed only for a few days together at a time, this change of place is hardly discoverable, except by nice astronomical observations; and therefore, when we are considering the sun's place, only in order to explain his diurnal motion round the earth, his rising and setting, and causing the vicissitude of night and day, we may look upon him as continuing in the same point of the heaven, during the time of one or two natural days: the change of the sun's place in the heaven is however, in any considerable number of days, very obvious to every body; it is from hence, that he appears to rise and set in different parts of the horizon, at different times of the year; from hence it is, that his height at noon is so much greater in summer than in winter; in a word, it is to the change of the sun's place in the heaven, that we owe the difference we find in the length of the days and nights, and in the seasons of the year. The year is the time in which the sun goes through the several changes of his place in the sphere of the heaven, and returns to the same place again: the cause of the sun's appearing thus to change his

FIG. his place is really the motion of the earth round the sun, whereby we change our place, and view him from different stations; and consequently, at different times of the year, see him in different parts of the heaven: this annual motion of the earth round the sun will be more fully explained in the next book; it is sufficient for our present purpose to observe, that the sun, by this motion of the earth, appears sometimes in the equator, sometimes north, and sometimes south of the equator, and to consider the difference of the suns apparent diurnal motion arising from hence, either in the same horizon, or in different positions of the horizon.

358 When the suns place is in the celestial equator, his apparent diurnal motion is in the celestial equator it self; when the suns place is in any other point of the heaven, his diurnal motion is in a parallel drawn through that
28 point: § 343: fig. 28.

359 Twice a year the sun appears in the equator; then his apparent diurnal motion is in the celestial equator; all the rest of the year he is either in north declination, and then his apparent diurnal motion is in a north parallel; or in south declination, and then his apparent diurnal motion is in a south parallel. At the beginning of the spring, about the 10th of March, the sun is in the equator; from that time, he appears every day to decline more and more towards the north, and describes a parallel every day more northward than the parallel of the preceding day, till summer begins^a, about the 11th of June; then he is in his utmost northern declination, and his diurnal motion is in the most northern parallel in the sphere of the heaven which he ever appears in: from this time, his declination grows less and less, and his diurnal motion is in a parallel nearer every day to the equator than the parallel of the preceding day, till he comes to be in the equator again, about the 12th of September: from that time, he declines more and more south, till he comes to his utmost southern declination; there he describes the most southern parallel he ever appears in, about the 11th of December: from that time, his declination grows less and less, and his diurnal motion is in a parallel every day nearer and nearer to the equator, till he arrives at the equator again, the next spring.

360 The most northern and southern parallels the sun ever describes are called *the tropics*, from a greek word *tropè* which signifies turning; because in those two parallels the sun is in his utmost declination, and from thence begins to turn back towards the equator again. The northern tropic is called *the tropic of Cancer*, and *the summer tropic*; because it passes through that part of the heaven which is called the sign Cancer, or the Crab; and when the sun appears therein, the summer begins, to those who live in north latitude. The

^a Astronomers begin the four seasons in March, June, September, and December; as will appear when we come to treat of the seasons.

southern tropic is called *the tropic of Capricorn*, and *the winter tropic*; because FIG. it passes through a part of the heaven which is called the sign Capricorn; and when the sun appears therein, winter begins, to those in north latitude: in 43 fig. 43, 44, 45, EQ is the equator, TT, the summer, ww the winter tropic. 44

361 The time the sun is above the horizon of any place is called *an artificial day*; the time the sun is below the horizon of any place it is *night* in that place. By the day is sometimes meant the natural, and sometimes the artificial day; but this will not occasion any ambiguity in the expression, if the author be tolerably careful, and the reader any thing attentive. 45

362 In a right and oblique sphere, by the sinking of the east side of the horizon below the sun, the sun appears to rise above the east side of the horizon; and by the rising of the west side of the horizon above the sun, the sun appears to set below the west side of the horizon: in a right sphere, this rising and setting of the sun is in circles that make right angles with the horizon; in an oblique sphere, the sun rises and sets obliquely: all this is easily understood from what has been said of the rising and setting of the stars, § 355, 356.

363 In a parallel sphere, an inhabitant under the north pole has the sun above his horizon, and it is consequently perpetual day with him, all the time the sun is in north declination; that is, for six months together, from the 10th of March to the 12th of September, § 359; but the sun is below his horizon, and it is night with him, all the time the sun is in south declination; that is, for six months together, from the 12th of September to the 10th of March, § 359. On the other hand, it is day with an inhabitant under the south pole, for six months together, all the time the sun is in south declination; and night, six months together, all the time the sun is in north declination: fig. 43. 43

364 When one part of the circle in which the sun appears to go round the earth, in his diurnal motion, is above, and the other part below the horizon of any place, that part of the circle which is above the horizon is called *the suns diurnal arc*; because all the time the sun appears therein, it is day in that place; that part of the circle which is below the horizon, is *the suns nocturnal arc*; 44 and whilst the sun is moving therein, it is night in that place: fig. 44, 45. 45

365 In a right sphere, since the equator and every parallel is divided into two equal parts by the horizon, § 351, the diurnal arc is equal to the nocturnal, every day in the year: under the line therefore, there is perpetual *equinox*, that is, the days and nights are equal, throughout the year; every day being 12 hours long, and every night of the same length. See fig. 44. 44

366 Every place in an oblique sphere has the day equal to the night, when the sun is in the equator^a, about the 10th of March, and the 12th of September: for this reason, those two seasons are called *the equinoxes*; and the equator, *the*

^a Because the equator is divided into two equal parts by the horizon. § 177. *equinoctial*

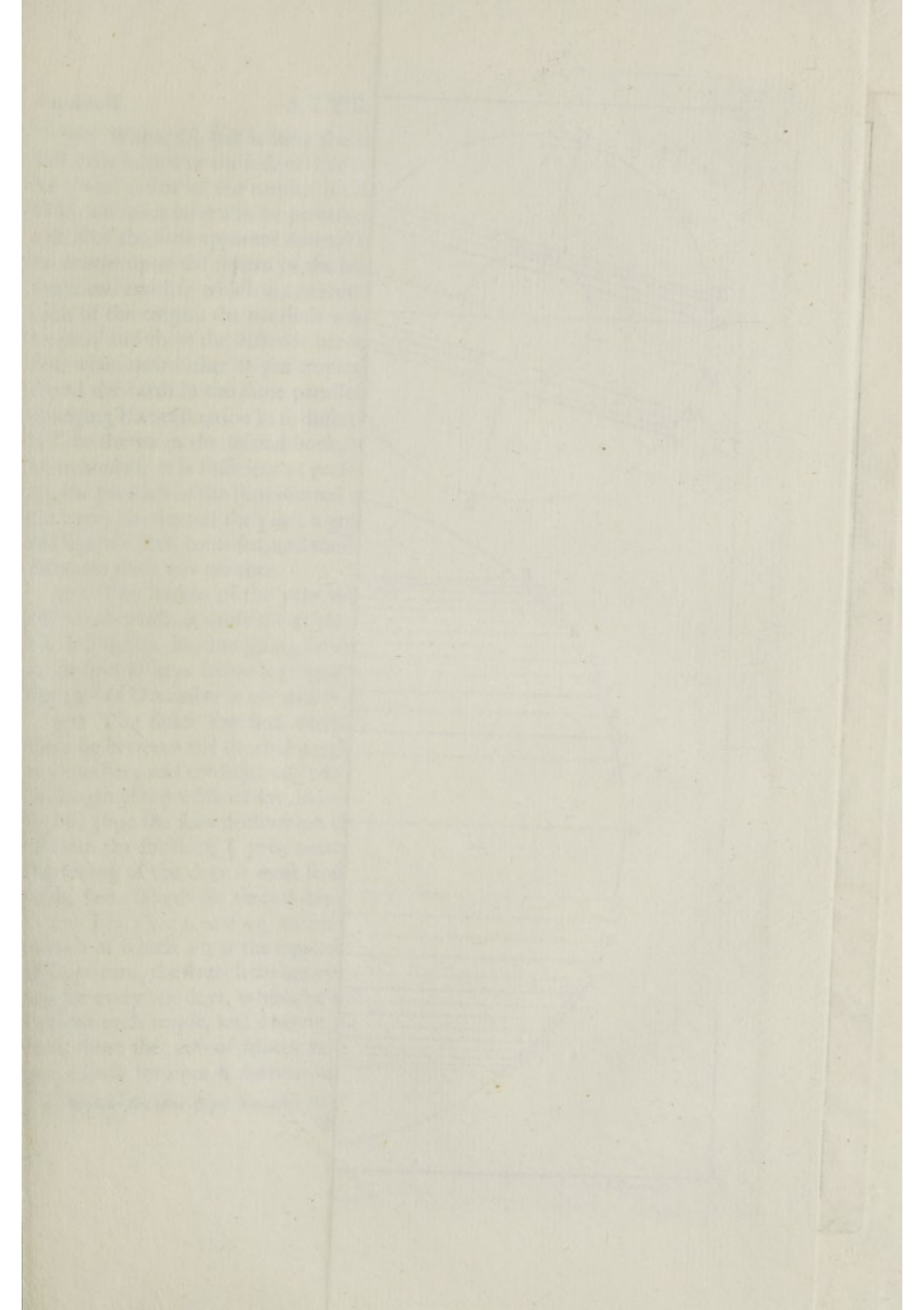
FIG. *equinoctial circle, or the equinoctial*: the 10th of March is called *the vernal*, the 12th of September *the autumnal equinox*; because one of these is the beginning of the spring, the other of the autumn: this time of spring and autumn is with regard to those who live in north latitude; as for those who live in south latitude, their spring begins in September, and their autumn in March: but it may be observed here, once for all, that all the astronomical books we have were written by authors who lived on the north side of the equator, and had the north pole elevated above their horizon; and therefore many terms and expressions used in astronomy, are such as are proper for persons in that situation: thus we call it the vernal equinox when our spring begins; we look upon the north as the upper part of the heaven; and call any motion in the heaven, from south to north, ascending; and from north to south, descending.

367 At all other times of the year when the sun is not in the equator, the days are not of the same length with the nights, in an oblique sphere; because every parallel which is intersected by the plane of the horizon is divided by it
45 into two unequal parts. See fig. 45.

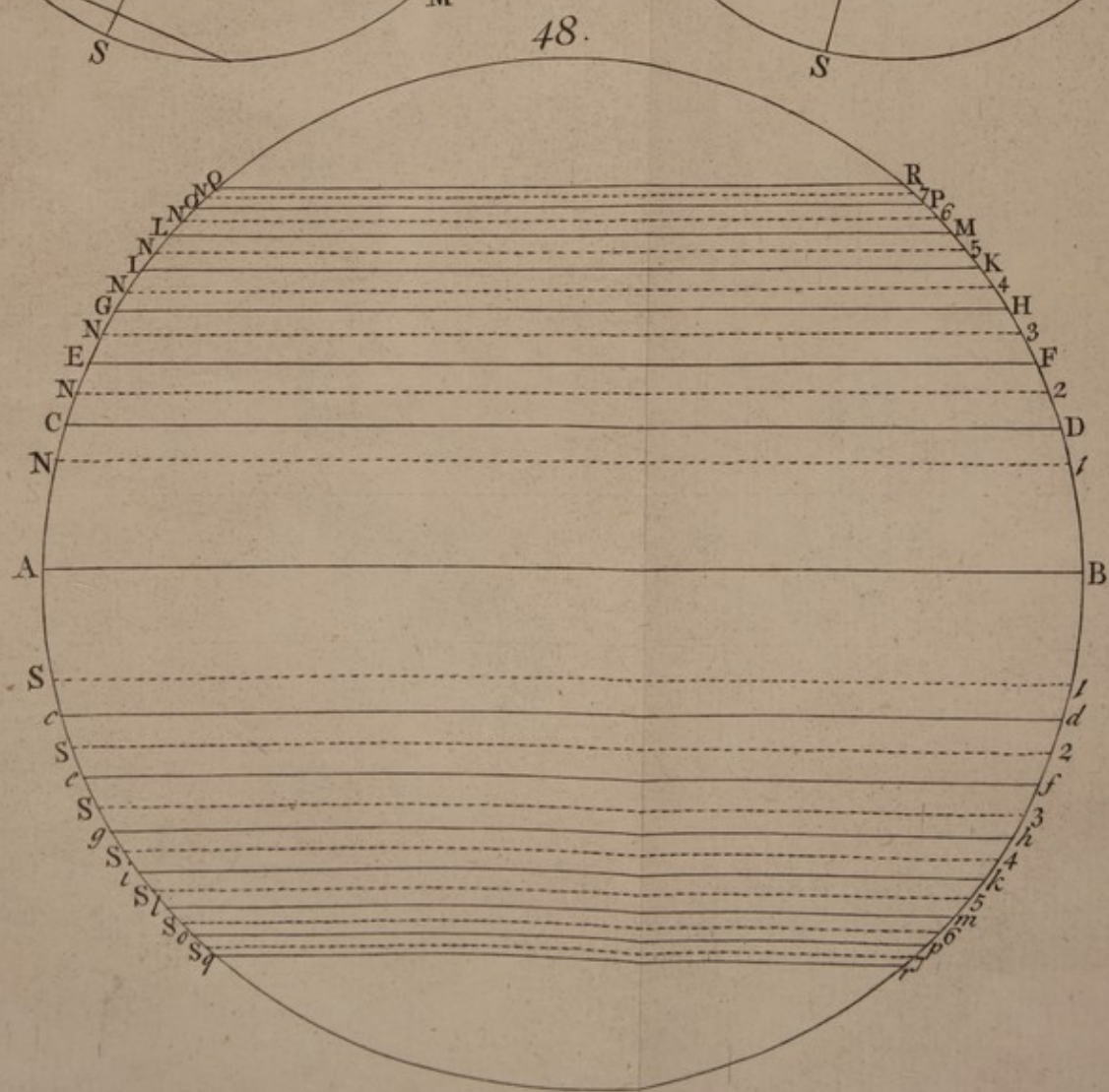
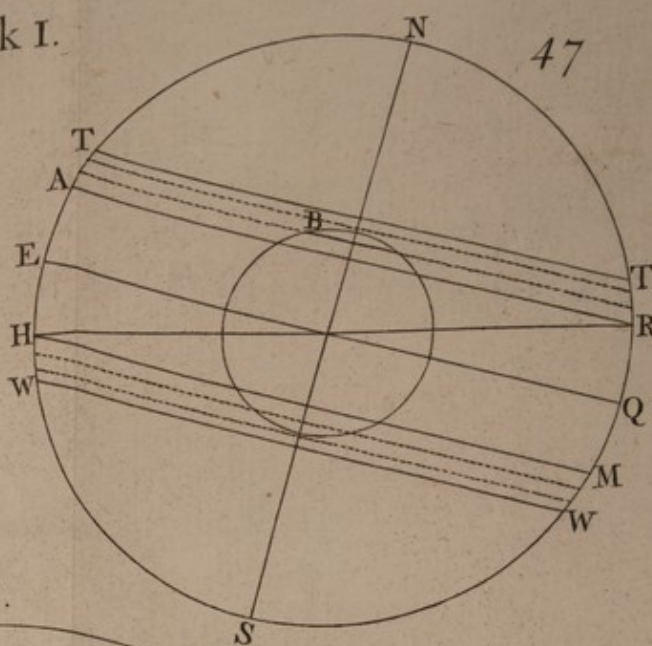
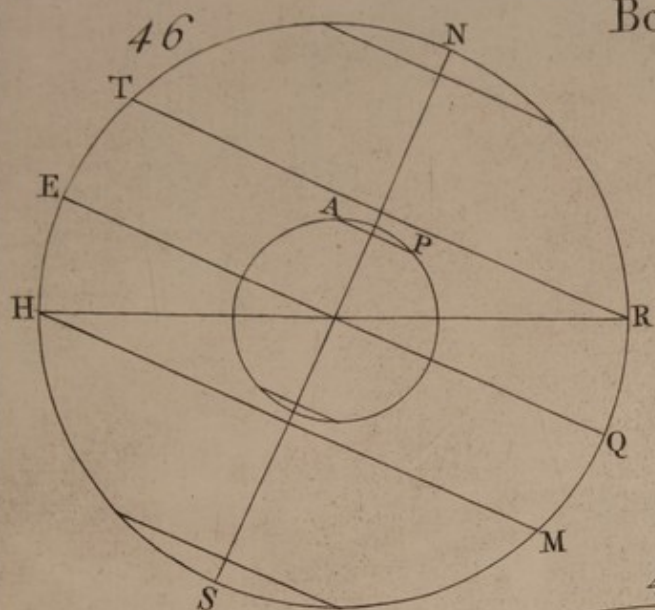
368 Every place in north latitude has the day longer than the night, all the time the sun is in north declination; and the day is longer, the greater that declination is: and the longest day is when the sun is in the tropic of Cancer:
45 the reason is evidently this; fig. 45, the north parallels are all divided unequally by the horizon, in such a manner, that the diurnal arc in every parallel exceeds the nocturnal: and the further north any parallel is, the more unequally is it divided; and the more does the diurnal exceed the nocturnal arc; and therefore, the tropic of Cancer being the furthest north of any of the parallels which the sun ever appears to describe, his diurnal arc does then most exceed his nocturnal arc when he is in that tropic; and consequently then is the longest day and the shortest night.

369 Every place in north latitude has the night longer than the day, all the time the sun is in south declination; and the day is the shorter and the night the longer, the greater the south declination of the sun is; and the shortest day and longest night is when the sun is in the tropic of Capricorn: the reason of
45 this is also easily seen by fig. 45, by which it appears, that all the south parallels are unequally divided by the horizon, in such a manner, that the diurnal arc in every one of them is less than the nocturnal; and the further south any parallel is, the more unequally is it divided, and the more does the nocturnal arc exceed the diurnal: the tropic of Capricorn therefore being the furthest south of any of the parallels the sun ever describes, the nocturnal arc does then most exceed the diurnal, and then will be the shortest day and the longest night when the sun is in that tropic,

370 When



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370 When the sun is near the celestial equator, his declination alters so fast, that in two or three days time it is easy to observe a change therein; when he is near either of the tropics, his declination changes so slowly, that no sensible difference in it can be perceived for several days together: if all the parallels of the suns apparent diurnal motion for every day of the year were to be drawn upon the sphere of the heaven, those parallels would be most distant from one another which are nearest to the equator; from the equator towards each of the tropics the parallels would be closer and closer, till very near the tropics; and there the distance between them would be insensible; so that the sun, when near either of the tropics, seems in his diurnal motion to be carried round the earth in the same parallel for several days together: the cause of his changing his declination in so different a manner, at different times of the year, will be shewn in the second book, when the position of the ecliptic comes to be treated of, it is sufficient at present to lay down the fact: in fig. 43, 44, and 45, the parallels of the suns diurnal motion are drawn at their proper distances, for every ten days of the year: a greater number of parallels would have made the figure a little confused, and these are enough to shew the inequality of their distances from one another.

371 The seasons of the year when the sun is in the tropics are called *the solstices*; because at those times the sun seems to be at a stand, with regard to his declination; neither going further from the equator, nor coming nearer to it, for several days following: near the 11th of June is *the summer*, and near the 12th of December is *the winter solstice*.^a

372 The faster the suns declination changes, the greater difference will there be between the diurnal parallels of any two days immediately following one another; and consequently the greater increase or decrease will there be in the length of the artificial day, in an oblique sphere, in the same compass of time, § 368, 369: the suns declination changes fastest near the equinoxes, and slowest near the solstices, § 370; near the equinoxes therefore the lengthening or shortening of the days is most sensible, near the solstices they seem to continue of the same length for several days together.

373 Fig. 43, 44, and 45, are pictures of a parallel, right, and oblique sphere, in each of which EQ is the equator, TT the tropic of Cancer, ww the tropic of Capricorn, the strait lines between the tropics are the suns diurnal parallels, one for every ten days, which he appears to describe going from the equator towards each tropic, and coming from the tropic towards the equator again: thus, about the 10th of March he describes the equator EQ, the next day he goes a little into north declination, and describes a parallel about one degree

^a *Solstitialis dies idem est qui brumalis, apud Gellium, lib. 9. cap. 7.*

FIG. north from the equator, the day following he goes a little further into north declination, and his apparent diurnal motion is in a parallel further north; and thus he continues going every day further into north declination, and describes in his diurnal motion a parallel every day further north than that of the preceding day, but though the sun continues all the time increasing in north declination, yet is this increase every day less and less, as he goes further from the equator, till he comes to his utmost north declination, then he describes the tropic of Cancer TR , about the 12th of December; from that time, his declination decreases every day, in such a manner, that he describes the same diurnal parallels over again in an inverted order, as he goes backwards towards the equator, at which he arrives about the 12th of September; then he goes into south declination, more and more every day, and describes every day a parallel more south than the parallel of the preceding day, till he comes to his utmost south declination, about the 12th of December; then he describes the tropic of Capricorn WM ; and from that time his declination decreases, and he describes every day a parallel nearer to the equator than the parallel of the preceding day, till he comes to describe the equator again, about the 11th of March following.

374 It has already been observed, § 350, that the nearer a place is to either of the poles, the nearer to the celestial equator will the arctic and antarctic circles be: if a place A in north latitude, fig. 46, be so near the north pole, that the northern tropic TR is the arctic circle; when the sun is in that tropic, he will be carried round therein above the horizon, and make it continual day in that place for 24 hours, § 348: and when the sun is in the southern tropic WM , which is also the antarctic circle in that place, he will be carried round below the horizon, and make it continual night, for 24 hours: § 348.

46 375 A parallel AP drawn through a place A , whose longest day is 24 hours, is called *the polar circle*: a parallel drawn in the heaven corresponding to this circle ^a is the polar circle in the sphere of the heaven: the polar circle is either north or south: the *north polar circle* is drawn upon the earth through those places in north latitude which have their longest day 24 hours, and the northern tropic for their arctic circle, and the southern for their antarctic: *the south polar circle* passes through those parts of the earth in south latitude which have their longest day 24 hours: the polar circles are at the same distances from the poles as the tropics are from the equator; namely, $23^{\circ} 30'$.

47 376 If a place B , fig. 47, is still further north than A was supposed to be in the foregoing section, so as to be within the polar circle, the arctic circle AR and the antarctic WM will be nearer to the equator than the tropics are; and will each of them comprehend more of the parallels of the sun's diurnal motion, besides the tropics: in this situation, all the time the sun is so far in north declination

^a § 341.

declination as to be within the arctic circle, he will be carried round above the horizon of that place, § 348, and it will be continual day there; and all the time the sun is so far in south declination as to be within the antarctic circle, he will be carried round below the horizon, § 348, and it will be continual night.

377 From what has been said of the sun's apparent motion in places situated in north latitude, in the three sections immediately preceding, it is easy to understand, that a place in south latitude, whose antarctic circle is the southern tropic, will have 24 hours continual day, when the sun is in that tropic; a place within the south polar circle will have continual day all the time the sun is within the antarctic circle, and continual night all the time he is within the arctic circle of that place: &c.

378 I have hitherto, in considering the appearances of the heavenly bodies, had no regard to refraction, by which all the rays that come from them, as they enter our air and pass through it to our eyes, are refracted or bent, §. 213: the nature of refraction will be shewn more at large, when the atmosphere which surrounds our earth comes to be treated of; it is sufficient for the present to observe, that thereby all the heavenly bodies are made to appear higher than they really are; and that the nearer they are to the horizon, the greater is the refraction: the sun therefore is, by means of refraction, made to appear longer above the horizon than he really is, and, by that means, the artificial day is longer in every part of the earth than it would be without refraction.

CHAP. 9. CLIMATES.

379 The ancients, considering the diversity there is in the rising and setting of the heavenly bodies, especially of the sun, and, in consequence thereof, the difference in the length of the days and nights in different places, according as they are more or less distant from the equator, divided the earth, so much of it as was then known to them, into climates: and, instead of the method now in use of setting down the latitude of places in degrees and minutes, they contented themselves with saying in what climate the place under consideration was situated; and when they had a mind to be more exact, they would tell us that it was in the beginning, the middle, or end of such a climate.

380 A *climate* is a space upon the surface of the earth contained between two parallels, so far distant from each other that the longest day in one differs half an hour from the longest day in the other parallel: the word climate

is derived from a greek word that signifies to bend or incline, the difference of climates arises from the different inclination or obliquity of the sphere^a: the ancients took the parallel wherein the length of the longest day is twelve hours and three quarters for the beginning of the first climate: as for those parts which are nearer to the equator than that parallel, they were not accounted to be in any climate; either because they may in a loose and general sense be considered as being in a right sphere, though, strictly speaking, only the parts directly under the equator are so; or because they were thought to be uninhabitable, by reason of the heat, and were besides unknown.

381 Climates are either on the north or south side of the equator: *the number of the northern climates* most generally received is seven; to which the like number of southern climates correspond: the names of the northern climates were not generally taken from the order in which they follow one another; but each climate was denominated from some remarkable place which was supposed to be situated in the middle of it: the names of the northern climates, according to the ancients were these:

the climate the middle
of which passes through

- 1 Meroe.
- 2 Syene in Egypt.
- 3 Alexandria in Egypt.
- 4 Rhodes.
- 5 Rome: according to others, through the Hellespont.
- 6 The Borysthenes; that is, through the mouth of that river.
- 7 The Riphean mountains.

The southern parts of the earth being then very little known; the southern climates received their names from the northern ones to which they did in such a manner correspond, that they were as far distant from the equator southward as the others were northward: their names were these^b;

the south climate which
corresponds to the north
climate through

- 1 Meroe.
- 2 Syene.
- 3 Alexandria.
- 4 Rhodes.
- 5 Rome, or the Hellespont.
- 6 The Borysthenes.
- 7 The Riphean mountains.

^a Qua græci κλίματα dicunt, inclinationes coeli vocat Vitruvius lib. 1. c. 1: & Gellius lib. 14. c. 1. coeli divergentias, Gellius ibid. ^b Gr. ἀντιδιαμετρὸς &c. ubi ἀντὶ in compositione similitudinem vel aequalitatem significat.

382 A parallel is said to pass through *the middle of a climate* when the longest day in that parallel differs a quarter of an hour from the longest day in either of the extrem parallel that bound the climate: this parallel does not divide the climate into 2 equal parts, but the part nearest to the equator is larger than the other; because the farther we go from the equator the less increase of latitude will be sufficient to increase the length of the longest day a quarter of an hour: in the middle parallel of the first climate the longest day is 13 hours: in the middle of the second climate 13 hours and an half: in the middle of the third 14 hours, &c. In fig. 48, let *AB* be the equator, *NI* 48 a north parallel where the longest day is 12 hours and $\frac{1}{4}$, *CD* a parallel where the longest day is 13 hours, *N2* a parallel where the longest day is 13 hours and $\frac{1}{4}$; the parallel *NI* is the beginning, *CD* is the middle of the first northern climate: *N2* is the end of the first and the beginning of the second climate: and all the space between the parallels *NI* and *N2* is said to be in the first northern climate: the parallels *N2* and *N3* contain the second northern climate; and the parallel *EF* is in the middle thereof, &c: if we take parallels at the like distances, on the south of the equator, we shall have the southern climates; thus, the parallels *s1* and *s2* contain the first southern climate; the parallels *s2* and *s3* contain the second, &c. in like manner, *cd* is said to pass through the middle of the first southern climate; *ef* through the middle of the second, &c.

383 We may observe that every climate has three parallels which mark the beginning, the middle, and end of it: and that the parallel which marks the end of every preceding climate is the beginning of that which is immediately subsequent: some of the ancients, divide the earth by these parallels, and sometimes by a parallel do not mean a meer linear circle but a space of some breadth; in which sense a *parallel* is the same as half a climate: and shews the difference of a quarter of an hour in the length of the longest day.

384 Some of the moderns reckon the different climates by the increase of half an hour, in the length of the longest day; as has been described, § 380, beginning at the equator, and going on till they come to the polar circle, where the longest day is 24 hours: from the polar circle towards the pole, they count the climates by the increase of a whole natural day in the length of the longest day; till they come to a parallel under which the day is of the length of 15 natural days, or half a month: from this parallel, they proceed to reckon the climates by the increase of half months or whole months in the artificial day; till they come to the pole it self, under which the length of the day, is six months. Those between the equator and polar circles are called *hour-climates*; those between the polar circles and the poles, *month-climates*.

385 Both ancient and modern writers differ very much from one another in the

the number of climates, because of the different views in which they consider them; the ancient astronomers^a considered the climates from the equator to each of the poles, in order to shew the difference in the apparent motion of the heavenly bodies, especially the sun, in all the different situations of places upon the earth which can be imagined; the geographers^b chiefly considered the parts of the earth then known; later writers, to take in new discoveries, increase the number of climates: this subject is treated of at large by *Ricciolus*, *geograph. reformat.* l. 7. c. 9, 10, &c.^c The use of a table of climates is, the climate in which any place is situated, or the length of the longest day in that place, being given, to find the latitude; or the latitude of a place being given, to find the climate, or the length of the longest day in that place.

386 In common speech, we use the word climate in a more loose and general sense; so that when two countries are said to be in different climates, it is vulgarly understood that the quality and temperature of the air, and the seasons of the year, are remarkably different in one from what they are in the other: this is not true of different climates, in the astronomical sense of the word, as defined § 380; for the difference of half an hour in the length of the longest days in two places is so far from being the occasion of any considerable difference either in the air, or inhabitants, that we may sometimes find a great likeness between the countries and inhabitants of different climates, and a great difference between those which are in the same parallel: the difference in the temperature of the air, and the constitutions and tempers of the inhabitants, may often be better accounted for, by the different situation of countries, as to their being high and mountainous, or low and level; near to the sea, or distant from it; their being woody, fenny, &c; as also by the nature of the soil and the water. *Hippocrates de aëre, aquis, & locis; fusè.*

387 The following table is taken out of *Ricciolus*, who in calculating it has made an allowance for the effects of refraction, which other writers have neglected to do, in their tables of climates: refraction, by making the sun appear higher than he is, keeps him longer above the horizon of every place upon the earth; and consequently makes the artificial day longer than it would be without refraction^d. In this table, the climates are distinguished by the increase of half an hour in the longest day, till it comes to be 16 hours; from thence, by the increase of one hour in the longest day, till it comes to be 20 hours; and from thence by the increase of 2 hours, till it comes to be 24 hours; there the month-climates begin, which are distinguished by the increase of 15 days in the length of the longest artificial day.

^a Hipparchus *apud* Strabon. l. 2. Ptolem. *magn. construat.* l. 2. c. 6.

^c See also Varenus *geograph.* l. 2. cap. 25. prop. 13.

^d § 378.

^b Strabo *loco citato.*

Climate	Parallel	Height of the pole.		Longest Day.		Middle of north Climate passes through	Middle of S. Clim. passes through
		D.	M.	H.	M.		
I	1		2	59	12	15	
	2	M	7	18	12	30	Mindanao Isl.
	3		11	29	12	45	Isl. Ascension.
II	4	M	15	36	13	0	Goa.
	5		19	33	13	15	Isl. St. Helena.
III	6	M	23	8	13	30	S. Lucar in California.
	7		26	50	13	45	Assumption in S. Amer.
IV	8	M	29	49	14	0	Cairo.
	9		32	48	14	15	Coquimbo in Chili.
V	10	M	35	35	14	30	C. di Chille in Mouth of R. de
	11		38	9	14	45	la Plata.
VI	12	M	40	32	15	0	Baldivia.
	13		42	41	15	15	Alcala d' Henares.
VII	14	M	44	42	15	30	Asti in Piedm.
	15		46	33	15	45	Coronatorum Lacus.
VIII	16	M	48	15	16	0	Brifac in Alface.
	17		51	14	16	30	Port Desire.
IX	18	M	53	46	17	0	Hamburg.
	19		55	55	17	30	Mid. of Str. of Magellan.
X	20	M	57	44	18	0	C. Horn.
	21		59	20	18	30	Jeroslaw.
XI	22	M	60	39	19	0	Ergimulum.
	23		61	47	19	30	In Tart.
XII	24	M	62	4	20	0	N. of Freeze-
	25		64	12	21	0	land. *
XIII	26	M	65	10	22	0	S. of Iceland.
	27		65	43	23	0	
XIV	28	M	65	54	24	0	Middle of Icel.
				Continual Day in North Lat.	Continual Night in North Latitude.	Continual day in South Latitude.	
XV	29		66	2	Days 15	Days 12	Days 14
	30	M	66	53	31	27	30
XVI	31		67	43	45	41	44
	32	M	69	30	62	58	60
XVII	33		71	8	77	71	74
	34	M	73	0	93	87	89
XVIII	35		75	56	108	101	104
	36	M	78	6	124	117	120
XIX	37		81	10	139	132	135
	38	M	84	0	156	148	150
XX	39		87	40	172	162	164
	40	M	90	0	188	180	178

* This Freezeland is no where to be found, *Baudrand* Vol. 2. pag. 442.

FIG.

CHAP. 10. ASTRONOMICAL TERMS ARISING FROM THE DIFFERENCE OF THE NOON SHADOWS, IN DIFFERENT PARTS OF THE EARTH; ASCII, AMPHISCII, HETEROSCII, PERISCII. ASTRONOMICAL TERMS ARISING FROM THE INHABITANTS OF THE EARTH LIVING IN THE SAME, OR IN DIFFERENT PARALLELS; SYNOECI, PERIOECI, ANTOECI, ANTICHTHONES, ANTIPODES. THE ZONES.

388 When the sun's apparent diurnal motion is in a circle which passes through the zenith of any place upon the earth, *the sun* is that day said to be *vertical* to that place; because the sun will that day at noon be in the zenith, directly over the head of an inhabitant of that place.

389 At the equinoxes, about the 10th of March and the 12th of September, the sun, being in the celestial equator, is vertical to all those who live under the line: fig. 31, 32.

390 When the sun is in any of the celestial parallels, he is that day vertical to those who live in a corresponding parallel upon the earth^a: thus, when the sun is in 10° north declination, he is that day vertical to those who live in 10° of north latitude: thus again, when the sun is in either of the tropics, he is that day vertical to all who live in a corresponding parallel upon the earth^a, which is one of the terrestrial tropics. The utmost declination of the sun, either north or south, is 23° 30', the celestial tropics are therefore parallels drawn 23° 30' from the celestial equator; and the terrestrial tropics, which are usually drawn upon the terrestrial globe, are the same number of degrees and minutes from the terrestrial equator, being corresponding parallels.^a

391 Since the sun, in his apparent diurnal motion, is never carried round in a parallel further from the celestial equator than the tropics, § 360, he is never vertical to those who live without the terrestrial tropics; that is, above 23° 30' from the terrestrial equator.

392 All who live between the tropics have the sun vertical twice a year: thus, the sun is vertical to those who live under the line, twice a year; in March, and September; § 389, thus again, if a place is in 10° north latitude, the sun will be vertical to it, when he is in 10° north declination, which he is twice a year; once as he goes, increasing in north declination, from the equator towards the northern tropic; the other time as, his declination decreasing, he comes back again from the northern tropic towards the equator.

393 All who live between the tropics have the sun at noon sometimes north, and sometimes south; thus, they who live in a place, which is in 20° north la-

itude, have the sun at noon to the north of their zenith, when the sun is in more than 20° north declination: at all other times, when the sun is either in south declination, or not so much north as 20° , he will be south from their zenith at noon.

394 Those places that lye without the tropics never have the sun vertical: but such of them as are in north latitude, have the sun at noon always to the south; and such of them as are in south latitude, have the sun at noon always to the north of their zenith.

395 When the sun at noon is in the zenith of any place, the inhabitants of that place were by the ancients called *asccii*, that is, without shadow; because the shadow of a man standing upright, when the sun is directly over his head, is not extended beyond that part of the earth which is directly under his body: in such a situation, the shadow of an upright pillar, whilst the sun is in the zenith, will fall upon that part of the earth upon which the pillar stands; and therefore will not be visible.

396 Since the shadow of every opake body is extended from the sun; when the sun at noon is southward from the zenith of any place, the shadow of an inhabitant of that place, and indeed of every other opake body, is extended towards the north: when the sun is northward from the zenith of any place, the shadow falls towards the south.

397 All who live between the tropics have the sun at noon, sometimes north, and sometimes south from their zenith, § 393; and consequently their shadows at noon fall at one time of the year towards the south, at another time of the year towards the north; these were called *amphiscii*, that is, having both kinds of meridian shadows.

398 They who live without the tropics have their noon shadows always the same way; and are therefore called *beteroscii*, that is, having but one kind of shadow: if they are in north latitude, their meridian sun being always south, § 394, their noon shadows will be always towards the north: this is the case of Europe in general, and of England in particular: they who live in south latitude, without the southern tropic, have the sun at noon always north, and consequently their noon shadow always falls towards the south.

399 When a place is so far distant from the equator that, when the days are longest in that place, the sun is carried round 24 hours or longer above the horizon of it^a; the inhabitants thereof, during that time, are called *periscii*, that is, having their shadows go round them: the nearer a place is to one of the poles, the longer are the inhabitants thereof *periscii*: if there be any inhabitants, under the poles, they are *periscii* for six months together: and indeed several days more, upon the account of refraction § 387.

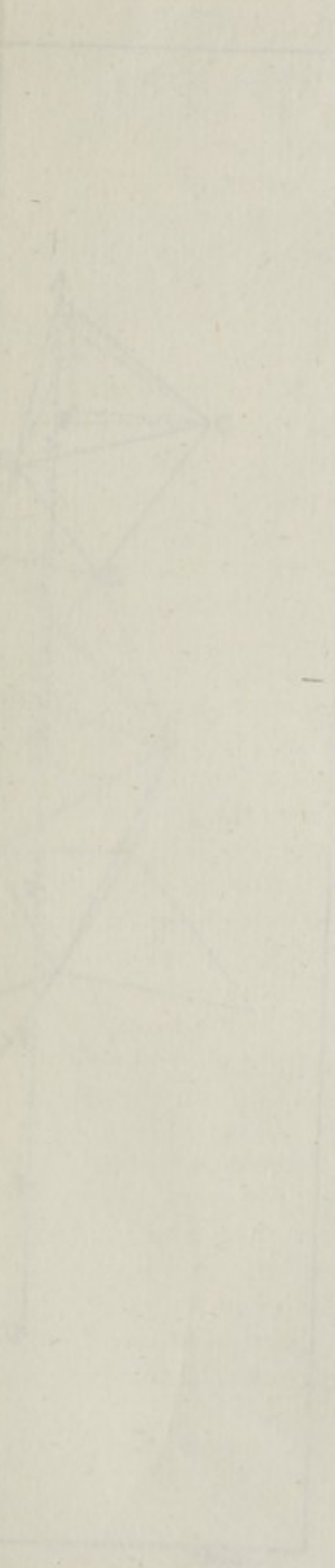
400 The inhabitants of different parts of the earth may be considered with regard to the parallels in which they live; in order to know whether the length of the days, or the times of the several seasons of the year, are the same, or different therein: if two places were so near together that there was no observable difference between their several horizons, the inhabitants of those places were by the ancients called *synæci*^a; that is, near neighbours: these, having the same horizon, have the nights and days, and the seasons of the year alike, and at the same time. The inhabitants of distant places in the same parallel were called *periæci*; that is, living in the same circle: these, being on the same side of the equator, have the seasons of the year alike, and at the same time; having also a like elevation of the pole, and a like horizon, the heavenly bodies appear in both to rise, to come to their meridians, and to set, in the same manner, *suprà* chap. 8, but at different times of the day, according as their meridians are more or less distant, § 311; they have therefore the same kind of days and nights, but at different hours. Some writers restrain the appellation of *periæci* to those, who live in opposite points of the same parallel; when it is noon in one of such places it is midnight in the other, § 302. When two places are in parallels which are equally distant from the equator, but one of them in north, the other in south latitude; the inhabitants were called *antæci*: these have a like elevation of the poles, but in one place the north, in the other the south pole is elevated; they have therefore a like increase of days and nights, and like seasons of the year, but at different times of the year; it being the longest day and the beginning of summer in one place, when it is the shortest day and the beginning of winter in the other: some will have the *antæci* to be under the same geographical meridian, so as to have the day or night at the same time, see § 302. If two places were in parallels equally distant from the equator, one in north, the other in south latitude, and in opposite meridians^b; the inhabitants of one place were, in respect of the inhabitants of the other place, called *antichthonæ*, that is, living upon opposite parts of the earth; or *antipodes*, that is, having their feet opposite: when two persons are *antipodes*, the zenith of one is the nadir of the other; § 277, and 281. They who are *antipodes* have a like elevation of the pole, but of different poles; they have also like days and nights, and like times of the year; but every thing at opposite hours of the day, and opposite seasons of the year: thus, when it is noon with us, it is midnight with our *antipodes*; when the days are longest, and summer begins with us, the days are shortest, and winter begins with them.

401 The zones are parts of the spherical surface of the earth, into which it is divided by the tropics, and polar circles: all that part of the earth which lyes

^a Cleomed. cap. περί οικήσεων.

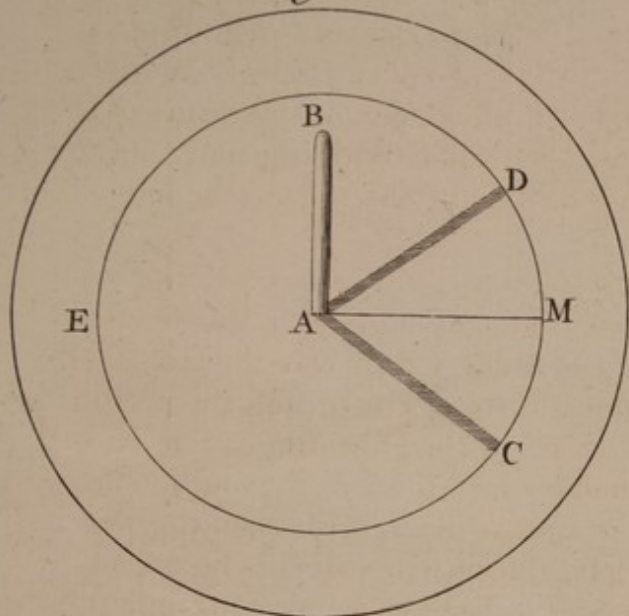
^b § 300.

1. The first part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom. It is shown that the structure of the atom is determined by the laws of quantum mechanics, and that the structure of the atom is determined by the laws of quantum mechanics.

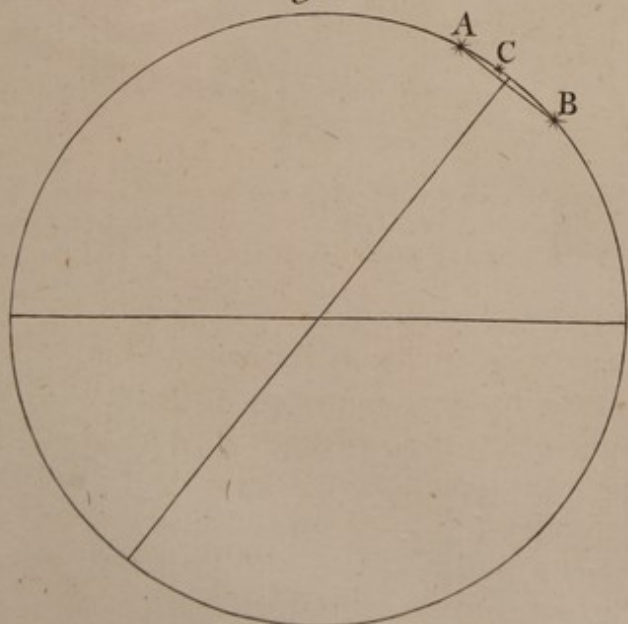


Book I.

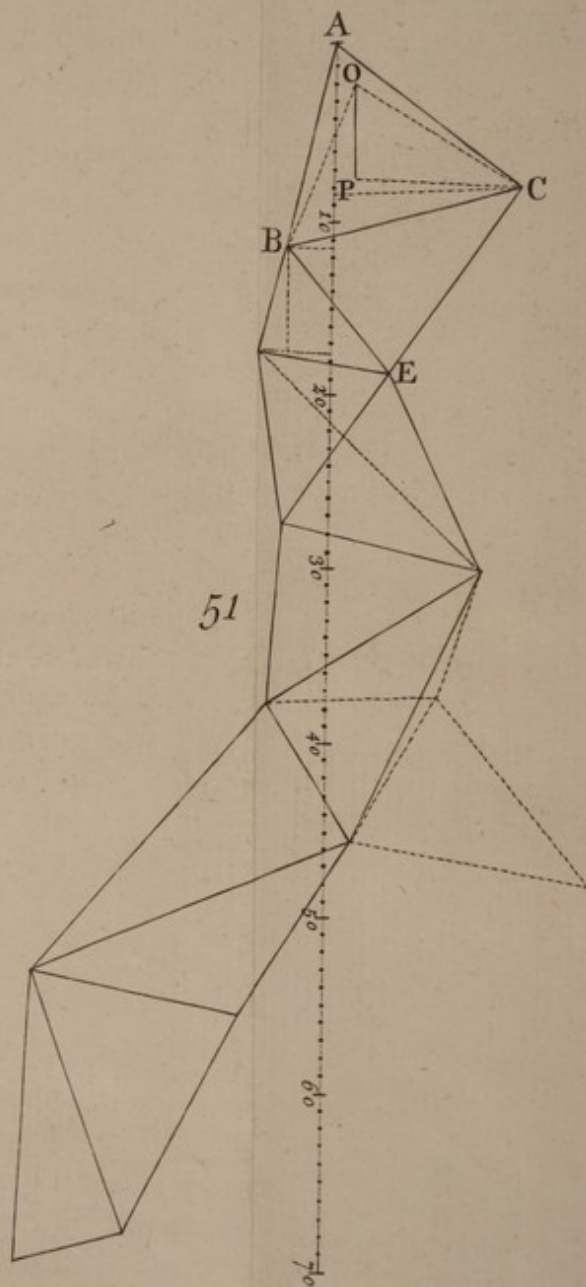
49



50



51



between the two tropics is called *the torrid zone*; this some of the ancients thought uninhabitable, by reason of the great heat, which they imagined must be there, when the sun is vertical: but it is now known, that many parts of it are inhabited, and that when the sun is vertical to them, his heat is usually qualified with cool breezes, and refreshing showers. Between each tropic and the polar circle next to it, lyes a *temperate zone*; the polar circles comprehend the *frigid zones*; these were supposed to be uninhabitable, by reason of the cold: the zones are generally said to be five; one torrid, two temperate, and two frigid: some make the number of them six, dividing the torrid zone into two by the equator.^a

402 When any *parts of the earth*, or of the *heaven* are said to be *on the right hand*, or *on the left*, these expressions have different meanings in different writers; to understand which it is necessary to know which way the face is supposed to be turned: geographers look towards the north, because the northern part of the earth is most known; with them therefore the east is on the right hand, the west on the left: astronomers look towards the south, observing the heavenly bodies as they come to the meridian; with them the west is on the right hand, and the east on the left: the ancient augurs, who pretended to divine by observing from what part of the heaven thunder and lightning came, which way birds flew, &c, set their faces towards the east; so that the south was on their right hand: the poets looked towards the fortunate islands in the west; and had therefore the north on their right hand.^b

The following verses may serve to help the memory,

To the north pole turns the geographer;

At the meridian looks th' astronomer;

The augur, where aurora smiles;

The poet tow'rs the happy isles.

CHAP. II. HOW TO DRAW A MERIDIAN LINE.

403 In a right sphere, every point in the heaven appears to rise at right angles above the east side of the horizon, and to go gradually higher and higher till it is at the meridian, there it is at the highest, and from thence it falls lower and lower, till it sets at right angles below the west side of the horizon.

In any place in an oblique sphere, every point in the heaven not com-

^a The word *zone* signifies a girdle or belt.

^b Ricciol. *geogr. reform.* p. 4. *ubi hoc distichon Brietii profert,*

Ad boream, terra; sed caeli mensor ad austrum;

Augur ad exortum movet; occasumque poetæ.

FIG. prehended within the arctic circle of that place appears to come obliquely above the east side of the horizon, and to rise gradually higher and higher, till it comes to the meridian, there it is at its greatest height, and from thence it descends obliquely lower and lower, till it sets below the west side of the horizon. In any place in an oblique sphere, every point in the heaven which is comprehended within the arctic circle of that place rises gradually from the lowest point of the parallel of its apparent motion, and goes higher and higher till it comes to the meridian, and from thence descends gradually, till it returns to the lowest point again: the lowest point of every parallel, being opposite to the highest, is in the opposite meridian.^a

404 The rotation of the earth, which is the cause of the apparent diurnal motion of the heavenly bodies, being supposed equable and uniform, as there is no reason to imagin it to be otherwise, the apparent altitude of every point in the heaven is the same at equal distances from the meridian, whether ascending towards the meridian, or descending from it; provided the point does not in the mean time change its declination, but continues all the while to move in the same parallel: from hence and from the preceding section is derived the following method of finding a meridian line.

405 *To find a meridian line by the sun*; upon an horizontal plane, at the point A, fig. 49, erect a perpendicular wire AB; an hour or two before noon, for example, about 10 in the morning, make a mark at D, where the shadow of the wire terminates; take away the wire, and from the center A through the point D draw a circle C M D E; replace the wire as before; as the sun rises higher, the shadow will grow shorter, till noon, when it is at the shortest; then as the sun grows lower, the shadow will grow longer; observe in the afternoon when the extremity of the shadow just touches the circle again, and make a mark there, as C; find the middle point M of the arc CD, and from the center A draw through it the line AM, this is the meridian line; or a line made by the common section of the planes of the meridian and horizon^b. The reason of the operation is this; at equal distances from noon, before and after it, the sun is of the same height, and in verticals equally distant from the meridian: therefore conversely, a circle drawn through the zenith equally distant from any two verticals, in both of which the sun is at the same height, is the meridian. The planes of two such verticals, or rather their common sections with the plane of the horizon, are found, by marking the shadows of the wire before and after noon, when they are of the same length, as AC and AD are, by § 25: for other methods of finding the meridian line, see *Ricciol. Almagest*, l. 10. § 4. *Tacquet astron.* l. 5. c. 4. *Suite des memoires de l'Acad. Royale pour l'ann. 1718.* pag. 22. edit. Par.

a § 300.

b § 160.

406 I have

406 I have here supposed the suns declination to continue the same, forenoon and afternoon; this operation therefore is to be performed at one of the solstices, when the suns declination does not sensibly alter for some time, § 371; and the summer solstice is much the best, because the sun is then highest, and changes his altitude fastest; whereas at the winter solstice he is so low, that the observations of him are liable to errors arising from refraction, and his motion above the horizon is so oblique, that he changes his vertical faster than he does his altitude; which is inconvenient in an operation that is to determine his vertical by his altitude.

There are moreover some other cautions to be observed in finding the meridian line by this method; 1. A clear day should be chosen, and the horizontal plane should be white, or of some light colour, that the shadow upon it may appear the more distinct; 2. The wire must not be too long, that the extremity of the shadow may be well defined; *Ricciolus* proposes to have it about 6 inches long; 3. The end of the wire must not be a sharp point, but about the thickness I have drawn it in the figure, that the end of the shadow may be distinct; 4. When accuracy is required, from the center A draw several concentric circles, that you may mark in each of them the points where the shadow of the wire terminates, at several times before and after noon; as at 9 in the morning, and 3 in the afternoon; at 11 in the morning, and 1 in the afternoon, &c: if the meridian line passes through the middle of all the arcs thus marked out, the agreement of so many several observations will be a good proof, that the meridian line, found the same by them all, is sufficiently exact.

407 *One meridian line being given, another may be drawn upon another horizontal plane, by the following method; hang a thread with a plummet exactly over the south end of the meridian line given, and another thread with a plummet over the south end of the plane upon which the meridian line is to be drawn; let one person observe at noon, the moment when the shadow of the first thread falls exactly upon the meridian given, and let another observer, at the same time, mark two distant points in the shadow of the second thread: a line drawn through those points is the meridian line required. By the same method may a meridian line be found upon a south wall, by marking two points in the shadow of a thread hung at a little distance from it: if the meridians are near, he that observes the shadow of the first thread, may let the other know the moment it falls upon the meridian line, by saying now: if they are far distant, it should be done by the motion of the hand, because sound takes some time to pass from one place to another.*

408 The meridian line is the very basis of astronomical observations: a meridian line being found, there may be placed over it a quadrant or sextant in such

FIG. such a manner, that though it be moved up or down to give it different elevations, in order to view through the sights of it the celestial bodies at different altitudes; yet the plane of that side of the instrument upon which the degrees are marked shall continue all the while in the plane of the meridian. *The mural arc in the Royal Observatory at Greenwich* is a wall built of black marble, one side of which, standing exactly in the plane of the meridian, has a large and accurately divided brass quadrant fixt to it, moveable round its center, and with telescopic sights. See the description of it in *Smith's Optics*.

CHAP. 12. TO OBSERVE THE TRANSITS OF THE HEAVENLY BODIES:
TO FIND THE HEIGHT OF THE POLE; THE MERIDIAN OF THE R. OBSERVATORY AT PARIS: GNOMONS, AND THEIR USE.

409 *To observe the transits, or passages of the heavenly bodies cross the plane of the meridian;* a meridian line being found by § 405, hang two threads with plummets exactly over it, at a little distance from one another, and they will be in the plane of the meridian; if you place your eye close to one of the threads, in such a manner that you make it cover the other, and both appear as one thread; when a star is behind the threads, it is in the meridian: by the same method may the sun be viewed, through a smoked glass; when the threads pass through his center, he is in the meridian: but the best way of observing either the sun, moon, stars or planets, is through a telescope placed in the meridian, with two cross hairs, one of which is in a vertical, the other in an horizontal position; when the vertical hair passes through the center of the sun, he is in the meridian.

410 *To find the elevation of the pole,* in any place; with an instrument fixt in the plane of the meridian by § 408, take, by one of the methods shewn § 284, 285, 286, the greatest and least height of some star which never sets in that place, but is comprehended within the arctic circle thereof, the middle height between these two extreams, is the elevation of the pole: thus, fig. 50, observe the height of the star A, when in the meridian, at its greatest height at A; suppose it found $60^{\circ} 30'$: twelve hours after, by the rotation of the earth, the same star will be in the opposite part of the meridian, at its least height at B; suppose this found $40^{\circ} 30'$: the difference between the greatest height and the least is 20° ; half this difference, viz. 10° , added to the least height, or taken from the greatest, gives the elevation of the pole, $52^{\circ} 30'$.

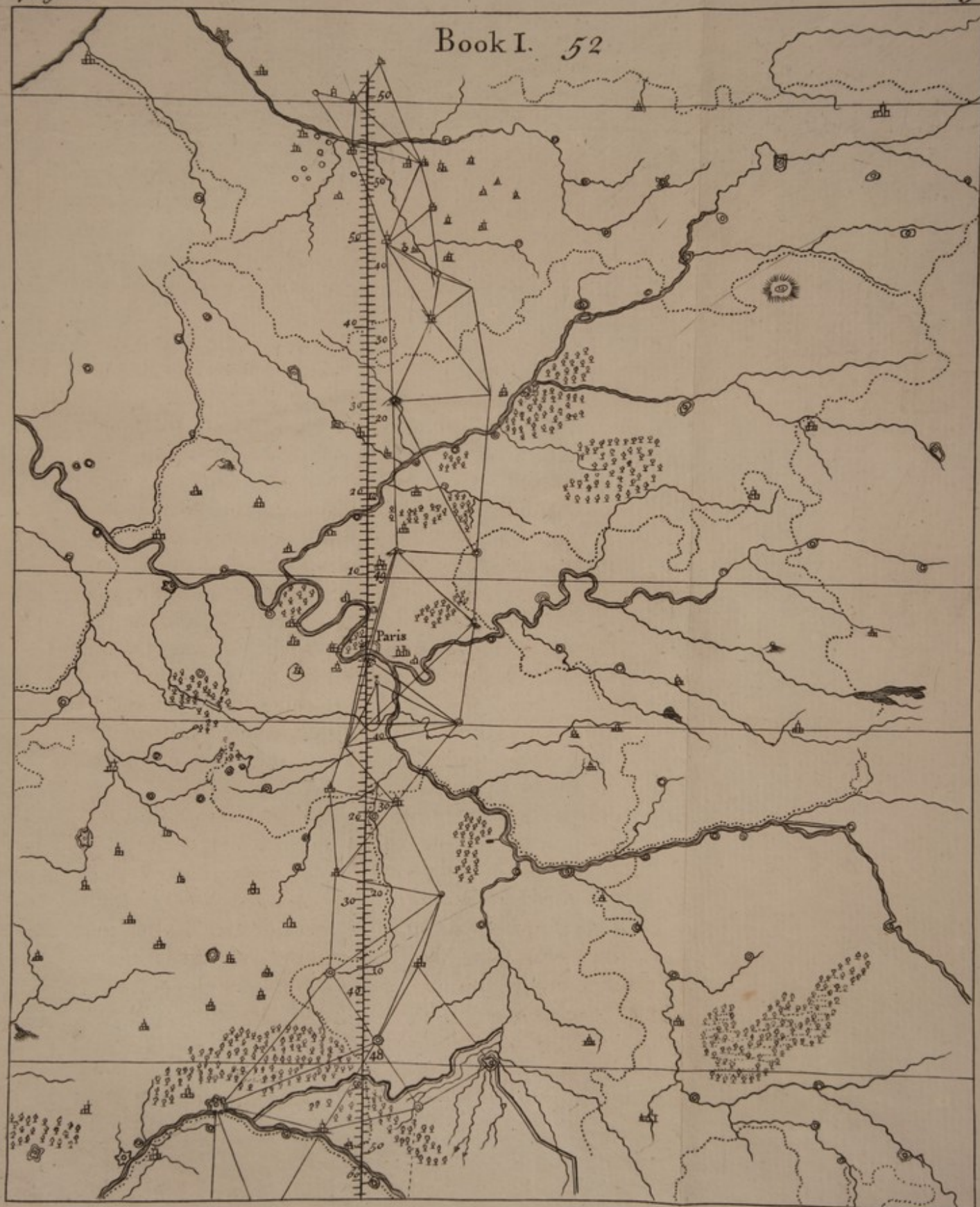
411 *The elevation of the pole may be found by one observation* of the height of a star in the meridian, if the declination of the star be known: thus, in the table of the declination of stars for the year 1730^a, I find the declination of

^a *Connaissance des temps, ann. 1730.*

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the pole star to be $87^{\circ} 52' 18''$: its distance from the pole, being the comple- FIG.
ment of its distance from the equator, is then $2^{\circ} 7' 42''$: suppose I find the
greatest height of this star, in the meridian at c, fig. 50, to be $54^{\circ} 38' 12''$: 50
take from thence the stars distance from the pole, $2^{\circ} 7' 42''$, and the remainder
is the height of the pole, $52^{\circ} 30' 30''$. The elevation of the pole may also be
found, by taking the least height of a star, and adding thereto the distance of
the star from the pole; but care must be taken, not to choose a star which
comes near the horizon when it is at the lowest: once for all, let it be remem-
bered, that those observations of the stars, &c are best which are made nearest
to the zenith; as being least liable to the errors arising from the uncertainty,
and variableness of refraction.

412 *To find the height of the equator*; take the meridian height of a star
which has no declination, but is in the equator; or take the meridian height
of the sun when he is in the celestial equator; the height of the equator is here
found at once: the height of the equator may also be found by the meridian
height of a star which has declination, or by the meridian height of the sun
when he is not in the equator, provided it be first known what and how much
the declination of the star or the sun is at the time of making the observation;
thus, suppose, in the year 1730, I am in a place where I want to know the
height of the equator; I find the star called *Aldebaran* or the *Bulls eye* is that
year in $15^{\circ} 56' 48''$ north declination^a; I observe the meridian height of it, and
find it $45^{\circ} 2'$; from $45^{\circ} 2'$, I take the stars declination, $15^{\circ} 56' 48''$; and the re-
mainder $29^{\circ} 5' 12''$ is the height of the equator. Thus again, suppose, Octob.
10th 1730, I observe the meridian height of the sun to be $31^{\circ} 4'$; the suns de-
clination that day is $6^{\circ} 38' 46''$ ^b, and, this declination being south, the sun is
to an inhabitant in north latitude so much lower than the equator; I must there-
fore add the suns declination to his height; the sum of these two numbers,
viz $37^{\circ} 42' 46''$ is the height of the equator.

413 *Scholium*, the height of the equator being found, the elevation of the
pole is known; or the elevation of the pole being found, the height of the e-
quator is known: for one of these is the complement to the other § 328, & 330.

414 Astronomers give us *tables of declination* for some of the stars, which
tables must be calculated for some particular time, because the declination
of the stars alters a little every year, as we shall see hereafter; we have in some
of the common almanacks, tables of the declination of some of the principal
stars, for the current year; these are very useful for sailors, that they may find
at any time of the night in what latitude they are, by the stars, in the man-
ner just now described, § 412: in some almanacks the suns declination is set
down for every day of the year; that, by his meridian height, the latitude may
be

^a ^b *Connoissance des temps*, ann. 1730.

FIG. be found in the day time. At sea, where they cannot have a fixt meridian line, they take the greatest height of the sun or a star for the meridian height.

415 A meridian line may be *verified*, *prolonged* to what distance we please, and the length of it *measured*. The meridian of the Royal Observatory at Paris being found, and an instrument with telescopic sights placed vertically^a therein, the north and south points of the visible horizon were observed through the sights, and upon the north point a pillar was erected; then, by another instrument placed horizontally, several distant objects, as steeples, &c were viewed, and the angles the visual lines made with the meridian line were observed: then, from the places of those objects, as from so many new stations, other objects were observed; and where proper ones did not occur, large poles or trees were set up to be viewed: by this means several triangles were formed along the meridian: in order to measure these triangles, a paved way near Paris, from *Villejuive* to *Jurvisy*, being strait and lying from north to south, was made choice of for the *fundamental base*. For actual measuring the length of this way, two poles were made use of, each of which was four *toises*^b long, and made of two pike-staves joyned together at the great ends by a screw: the method observed in measuring was as follows; one of the measuring poles was first laid upon the ground, and then the other was joyned to it end to end, along by a rope stretched from north to south; then the first pole was taken up and laid down at the end of the second, and so on successively: and for the greater ease in keeping the account, the measurer who laid down the second pole had ten little stakes given him, one of which he stuck into the ground at the end of his pole every time he laid it down; so that every stake marked 8 *toises*, and when all the 10 stakes were stuck into the ground they marked eighty *toises*: in this manner, the length of the high way between the middle of the mill of *Villejuive* and the pavilion of *Jurvisy* was twice measured; and was found to be 5663 *toises* and 4 feet, in going; and 5663 *toises* and one foot, in returning: but, as a nearer approach to exactness could not be hoped for, the round number of 5663 *toises* was pitched upon as the length of this *fundamental base*. This base is in fig. 51, represented by the line *OP*, and the calculations of the triangles were built upon it, in the following manner; from one end of the base, was observed the angle *COF*; from the other end *P*, the angle *COF* was observed; and from the station *C*, the angle *OCF*: and thus all the angles of the triangle *CPO* and the length of one side *OP* being known, the lengths of the remaining sides *OC* and *PC* were found by calculation. The next step was to observe all the angles of the triangle *OBC*, and from thence, and from the known length of the side *OC* to calculate the other side *OB* and

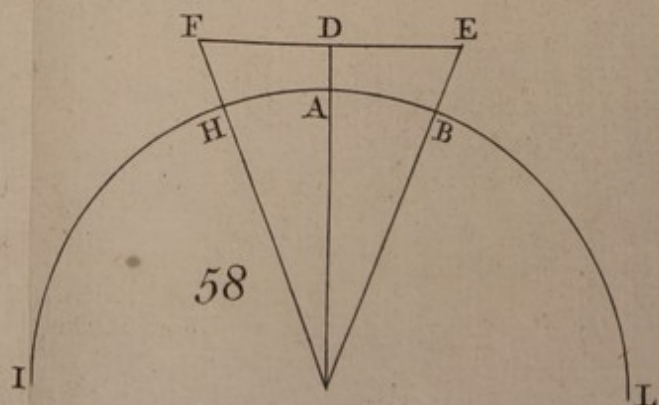
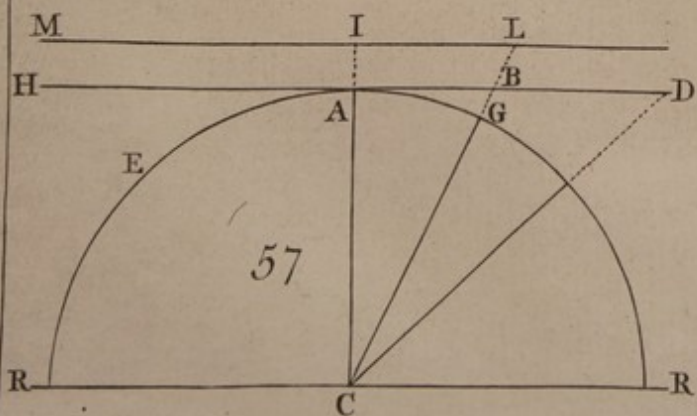
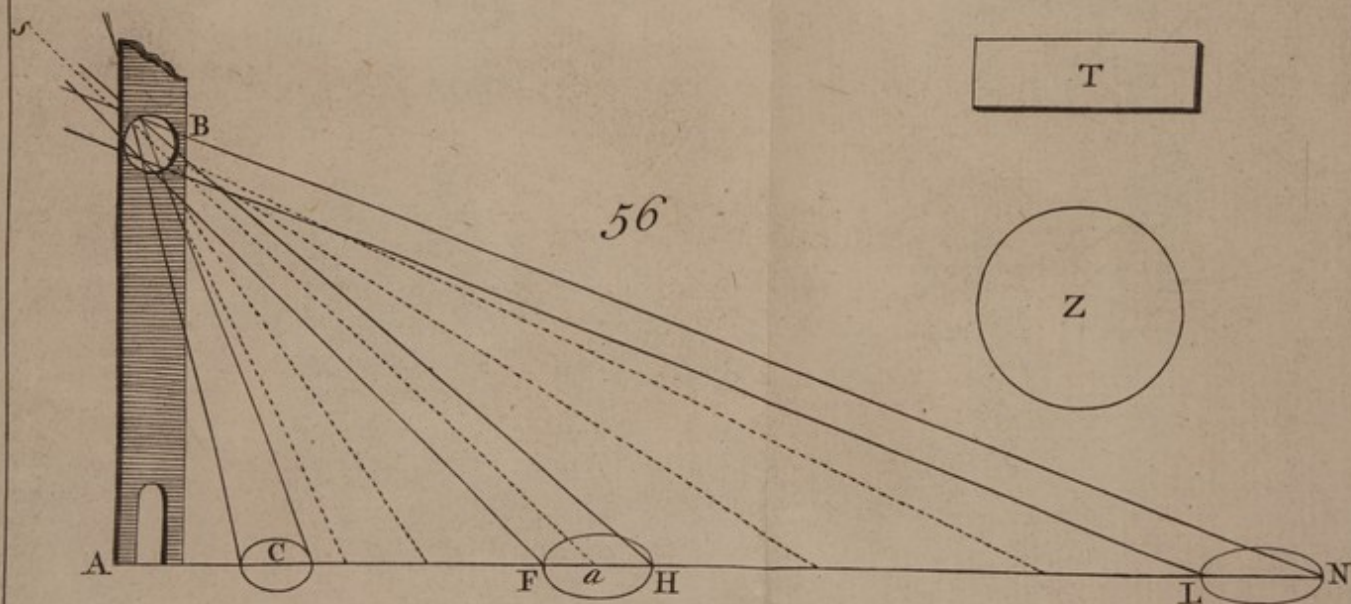
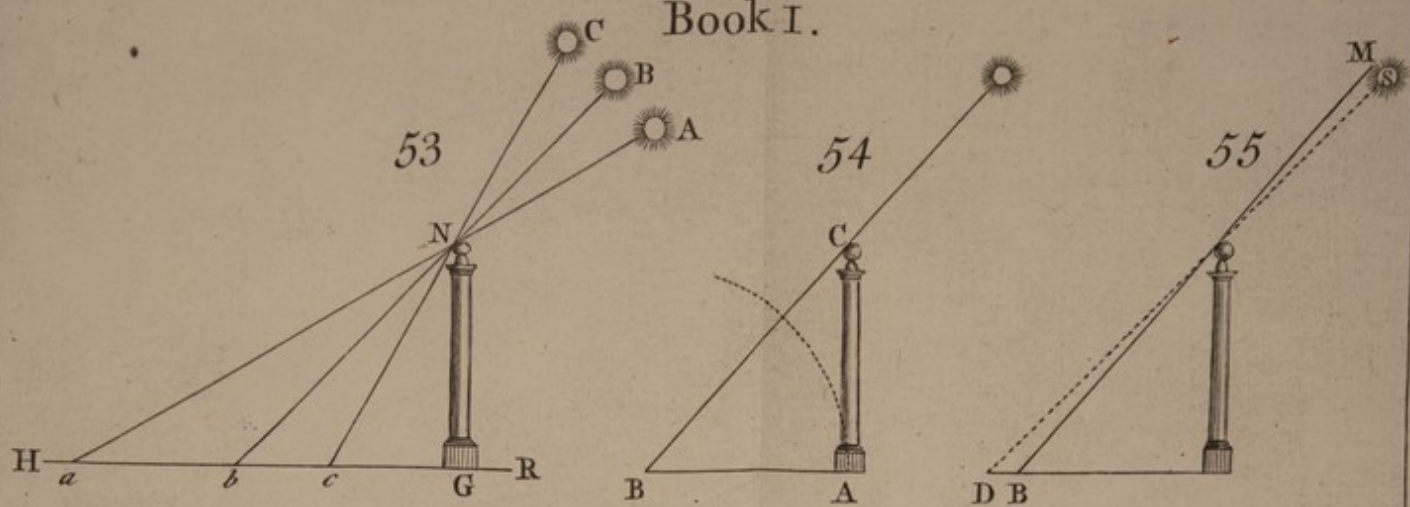
^a § 408. ^b A *toise* is six Paris feet.

B C. Then

1875
The first of the year was a
very dry one, and the
crops were much injured.
The second of the year was
a very wet one, and the
crops were much injured.
The third of the year was
a very dry one, and the
crops were much injured.
The fourth of the year was
a very wet one, and the
crops were much injured.
The fifth of the year was
a very dry one, and the
crops were much injured.
The sixth of the year was
a very wet one, and the
crops were much injured.
The seventh of the year was
a very dry one, and the
crops were much injured.
The eighth of the year was
a very wet one, and the
crops were much injured.
The ninth of the year was
a very dry one, and the
crops were much injured.
The tenth of the year was
a very wet one, and the
crops were much injured.



Book I.



B C. Then, all the angles being observed, and the side B C being known of the triangle ABC, which may be called the first or principal triangle of the meridian of the Observatory, the other sides A B and A C were found. Then, from one of the sides now known, and the angles observed, all the sides of the next adjoining triangle C B E were found. Thus they proceeded from one triangle to another, to the place where the meridian ended, in the south part of France; and there the last triangle was terminated by a base of the length of 7246 *toises*, which was actually measured, in order to verify the preceding operations.

416 The meridian line of Paris being prolonged, in the manner just now described, the situation of several other places in France was determined by trigonometry; and an accurate map of the country drawn; especially of those parts which lye near the meridian of Paris: we have this map, with the triangles formed along the meridian, in the *Suite des memoires de l'Acad. Royale des sciences, année 1718*: I have, in fig. 52, given an exact copy of so much of it as I thought necessary for my purpose, taken from the Paris edition: in the book just now cited we are told, that the longitudes of Montpellier and some other places in France were found to be the same, by astronomical observations, as the trigonometrical operations had determined them to be: this shewed at once the truth of those trigonometrical operations, and what exactness may be expected in determining the longitude of places by those astronomical observations: those observations were of emersions of the satellits of Jupiter.

417 There was one method of making observations very much used by the ancients, which is proper to be now mentioned; and that was by the length of the meridian shadows cast by an upright pillar of a known height upon a level pavement: a pillar erected for this purpose was called a *gnomon*^a, because thereby the height of the pole, and the seasons of the year, may be known: the center of the base of the pillar may be called *the center of the gnomon*: the greater the height of the sun is, the shorter is the shadow of a gnomon; and conversely, the shorter the shadow is, the greater is the suns height: thus, fig. 53, let the gnomon be GN, the pavement HR; when the sun is at A, the shadow reaches to *a*; when the sun is at B, the shadow reaches to *b*; when he is still higher at C, the shadow reaches only to *c*; in observations of this kind, it was usual to set down what proportion the length of the shadow bore to the height of the pillar; that is, the height of the pillar being divided into a certain number of equal parts, to say how many of those parts the length of the shadow, beginning to measure it from the center of the gnomon, amounted to, at the equinoxes or solstices: the most ancient observation of this kind, that we meet with, is that made by *Pytheas*, in the time of *Alexander the Great*, at

^a *Solis umbilicus quem gnomonem vocant.* Plin. N. H. l. 2. c. 72.

FIG. Marfeilles, where he found the height of the gnomon was to the meridian shadow, at the summer solstice, as $213\frac{1}{8}$ to 600; just the same as *Gassendus* found it to be, by an observation made in the same place, almost two thousand years after, in the year 1636.^a

418 To find the elevation of the pole by the shadow of a gnomon: it is sufficient for this purpose, to find the meridian height of the sun; for that being given, the height of the equator is found, by § 412; and consequently the elevation of the pole, by § 413: now the meridian height of the sun is found by the
54 gnomon in the following manner; in the fifty fourth figure, let the gnomon be AC, let the shadow be AB; and let CB be part of a ray drawn from the center of the sun, passing by the top of the gnomon and terminating the shadow at B; these three lines form a right-angled triangle BAC; whereof the two legs AC and AB are given; the number of feet, inches &c contained in each of them being found by actually measuring them: from hence either of the acute angles may be found in this manner; let one leg be radius, and the other will be tangent of the opposite angle, § 140: thus, if we make AB radius, AC will be tangent of the opposite angle ABC; this tangent is thus found by the golden rule; as the number of feet, inches, &c in AB is to the number of feet, inches, &c in AC; so is the radius or the whole sine 10000000, to a fourth number, which is the tangent required: this fourth number found in the table of tangents makes known the angle ABC, § 120: and this angle is the measure of the suns meridian height at the time of the observation: which was the thing required to be found.

419 The following example will serve to illustrate this proposition; *Pliny*^b tells us, that at Rome, at the equinoxes, the shadow is to the gnomon as 8 to 9: suppose then, making use of the figure last mentioned, the gnomon AC to be
54 90 feet, and the shadow AB 80 feet; if we make AB radius, AC will be tangent of the angle ABC: we may say then as AB of 80 feet, is to the whole sine 10000000; so is AC of 90 feet, to a fourth number: viz. 11250000, tangent of the angle ABC: the angle answering to this tangent in the table, § 120, is $48^{\circ} 22'$; this is therefore the height of the equator at Rome, and the elevation of the pole there is $41^{\circ} 38'$: § 112, 113.

420 *Ricciolus*^c takes notice of these defects in the observations of the suns height, made with the gnomon, by the ancients, and some of the moderns; 1, they did not reckon the suns parallax, which makes his apparent altitude $10''$ less than it would be, if the gnomon were placed at the center of the earth, as will be better understood when the suns parallax comes to be treated of; 2, they

^a *Gassendus de vita Peireskii ad ann. 1636. v. Ricciol. alm. l. 3. c. 14. & Geogr. reform. l. 7. c. 4. & 5.*
^b *Nat. hist. lib. 2. cap. 72.* ^c *Geogr. reformat. l. 7. c. 4.*

did not consider refraction, by which the apparent height of the sun is a little FIG. increased; 3, they made their calculations from the length of the shadow, as if it were terminated by a ray coming from *s*, the center of the suns disk, fig. 55, 55 such as is express'd by the pointed line *sD*; whereas the shadow is in reality terminated by the ray *MB*, coming from *M*, the upper edge of the suns disk: so that, instead of the height of the center of the sun, their calculations gave the height of the upper edge of his disk: the errors arising from hence are small, and of no consequence in some cases, but in order to find the height of the pole, the moment of the solstices or equinoxes, or the obliquity of the elliptic, with accuracy, they must be corrected in the following manner: to the altitude of the sun found by the gnomon, add the suns parallax $10''$, and take from it the semidiameter of the sun, about $16'$, and the refraction, which is different at different altitudes of the sun, as will be seen in the table of refractions, and you have the suns true altitude: thus, in order to correct the observation mentioned in § 419; to the meridian height of the sun at Rome, found by the gnomon at the equinox to be $48^{\circ} 22'$, add the suns parallax $10''$, which makes $48^{\circ} 22' 10''$; from this sum take the suns apparent semidiameter $16'$, and the refraction $1' 57''$, and the remainder $48^{\circ} 4' 13''$ is the true height of the suns center, and consequently the true height of the equator at Rome, the complement whereof $41^{\circ} 55' 47''$ is the elevation of the pole.

421 There are also other kinds of gnomons, which shew the height of the sun by letting his rays shine through a hole in a plate of metal, so as to form his picture upon a level pavement: in an instrument of this sort, *the center of the gnomon* is the point in the pavement which is exactly under the center of the hole: the way of finding the meridian height of the sun by one of these gnomons is the same as by the other first described; only, instead of the length of the shadow cast by the pillar, we here measure the distance of the suns picture from the center of the gnomon.

422 Of this kind was the gnomon made in the church of *St. Petronia* at Bologna, in the year 1576, by *Egnatio Dante*; the nature of which is easily understood, by fig. 56, taken out of *Ricciolus*^a: *AB* is the south wall of the church, 56 near the top of which is placed a brass plate of the thickness represented by the letter *T*; the bigness of the hole in the plate is shewn by the circle *z*; the plate was set inclined in the wall in an angle of about $45^{\circ} \frac{1}{2}$, which is the height of the equator in that place; the height of the hole in the plate from the ground is near 66 Bologna feet; the line *AN* is 169 such feet, drawn upon a marble pavement, not exactly in the meridian, but as near it as the pillars would admit of; the ellipsis *c* shews the place of the suns picture upon the

^a *Almagest. nov. l. 2. cap. 14.*

FIG. pavement, at the summer, the ellipsis LN, at the winter solstice; and the ellipsis
 56 FH at the equinoxes: when the distance of the suns picture from the center of the gnomon A is to be measured, we must take the center of the picture, which, for example, at the equinoxes is the point *a*, marked by the ray *sa*, drawn from the center of the sun.

423 Of this kind also was the gnomon made in the same church of *St. Petronia* at Bologna, in the year 1645, by *Dominico Cassini*; who placed the brass plate, through the hole of which the rays of the sun were to pass, in the roof of the church, and drew a meridian line 120 feet in length upon the pavement: the thing is so well represented in the reverse of a medal struck upon that occasion, that I thought it would be agreeable to the reader to see an exact copy thereof: I have therefore taken one out of the *Giornale de letterati*, and prefixed it as an head-piece before the first book of this work: a larger picture of it may be seen in *Wright's travels*.

424 *Bianchini* and *Maraldi* did in the same manner, draw a meridian line upon the pavement of the great hall of the baths of *Dioclesian*, now the church of the *Carthusians*, in Rome: an account of this is given by *Bianchini* in a treatise entituled *Blanchinus de nummo & gnomone Clementino*.

425 A gnomon may be made, and a meridian line drawn, after the manner of *Cassini*, by the following method; place the brass plate with a hole in it in the south end of the roof of the building; by a thread with a plummet at the end of it let down through the center of the hole, find the point in the pavement which is exactly under it, this point is *the center of the gnomon*; from this center, draw several concentric circles; an hour or two before and after noon, mark the points where the northern, as also where the southern edge of the suns picture touches these circles, and you will have several arcs; a line drawn from the center of the gnomon through the middle of these arcs is a meridian line: the reason of this operation is the same as of that described § 405.

426 These meridian lines now mentioned, are drawn upon long plates of brass, with which the marble pavement is inlaid; there are also drawn thereon lines crossing the meridians at right angles, to shew how far the center of the suns picture at noon reaches, at different times of the year: when the suns picture at noon is furthest from the center of the gnomon, the sun is then lowest, § 417, and consequently he is that day in his greatest south declination, and it is then our winter solstice; when the suns picture at noon is nearest to the center of the gnomon, the sun is highest, and consequently he is then in his greatest north declination, and it is our summer solstice.

427 It is obvious that the less the suns altitude is, the further from the center will the place of his picture be; for the same reason as the shadow of an upright pillar is longer, the higher the sun is: as was shewn § 417. 428 To

428 *To find the time of the solstice*; observe the distance of the suns picture at noon from the center of the gnomon, the day before, and the day after the solstitial day; if these distances are exactly equal, the meridian heights of the sun are for those two days exactly equal, and then the time of the suns being in the solstitial point is exactly at noon: if the distance of the suns picture from the center of the gnomon be greater the day before the solstice than it is the day after, the time of the solstice is before noon; if on the contrary, the time of the solstice is after noon.

It is exceedingly difficult by this method, to find the moment of the solstices within some hours; for at those times the suns declination, and consequently his meridian height, alters not above $15''$ in a natural day; and therefore an error of more than $15''$ in the observation of the suns meridian height, will occasion an error of a whole day in fixing the time of the solstice; an error of more than half $15''$, will cause an error of half a day; and so on in this proportion. *V. Ricciol. alm. l. 3. c. 14.* in *Gregory's astronomy l. 3. prop. 11.* we have a more accurate method of finding the solstices communicated by Dr. *Halley* in the *Philos. transactions* of the year 1695.

429 *To find the time of the equinox*; on the day of the equinox, find by a gnomon the meridian height of the sun, and the height of the equator, by § 419; if these be equal, the equinox is exactly at noon; if the height of the sun be different from that of the equator, then, as many minutes as the sun is higher than the equator, so many hours is the moment of the equinox before noon; as many minutes as the sun is lower than the equator, so many hours is the equinox after noon: the reason of this computation is this; at the equinoxes the declination of the sun alters at the rate of $24'$ in a natural day, which is $1'$ in an hour. From these two last sections it appears, that the equinoxes are much more easily observed than the solstices.

430 It is not improbable that some of those many obelisks^a erected in Egypt, and dedicated to the sun, were made use of as gnomons; as we are informed by *Pliny*^b one of them was, at Rome, which *Augustus* brought thither and set up in the *Campus Martius*: the height of this obelisk was 116 feet, it had a round ball placed on the top of it, that the end of the shadow might be seen more distinctly, and the stone pavement upon which the shadow fell was inlaid with brass rulers, which, by marking the different lengths of the meridian shadow, shewed the lengthning and shortning of the days. It is hardly worth mentioning in this place, that the Spaniards, at their conquest of Peru, found pillars of curious and costly workmanship set up in several places,

^a *Vid. Bargaei commentar. de obelisco, ap. Graev. Thes. R. ant. Tom. 4.*

^b *Nat. hist. l. 36. c. 10.*

by the meridian shadows whereof their *Amautas* or philosophers had, by long experience and repeated observations, learned to determine the time of the equinoxes: those seasons of the year were celebrated by them with great festivity and rejoicing, in honour of the sun, whom they fancied at those times to sit, in all his glory, upon his throne which they erected for him: and therefore on those days they presented him with rich offerings of gold, silver, jewels and other valuable things, adorning his throne, as they did also the pillars, with fragrant herbs and flowers. ^a

CHAP. 13. OF THE MEASURES OF LENGTH OF DIVERS NATIONS.

431 Before the dimension of the earth be attempted, it will be proper to ascertain the length of those measures, by which the magnitude thereof is to be exprest: and first we will consider the lesser measures of length, by which the greater are usually estimated; the lesser measures of length have been sometimes taken from the dimensions of the parts of an human body, as the foot, the cubit, the palm, &c; here, if we take a man well proportioned and of a middle stature, a *foot* is the distance between the heel and the tip of the great toe; a *cubit* is the distance between the elbow and the end of the middle finger; a *palm* is the breadth of the hand, or four fingers; and an *inch* is the breadth of the thumb; twelve of which are equal to the length of the foot: sometimes these measures are taken from other things, as the thickness of a middle sized corn of barley or wheat, and for a measure still smaller the thickness of a hair taken out of an horses or a camels tail: ^b but in all these measures there is an uncertainty, arising from the difference in the proportions and statures of men, and in the bigness of the grains of corn, &c; and therefore, in most civilized nations, it is usual to keep in some publick place authorized standards of those measures which are most in use, that recourse may be had thereto.

432 *Ricciolus* ^c proposes the following method to find the length of the ancient Roman foot; there was kept in the Capitol, for a standard, a measure of a cubic form, called from its shape *quadrantal*, or *amphora*; the dimension of the inside of this vessel was a foot every way; and it contained 80 pounds of wine or water ^d: the ancient pound contained 12 ounces, and the ounce 8 drachms ^e: the weight of the ancient Roman silver coin called *denarius* was a drachm ^f: several of these pieces still entire and not sensibly diminished by

^a *Garcilasso de la Vega* Royal history of Peru, l. 2. c. 22.

^b *Sic Arabes.* v. *Ricciol.* ^c *Geogr. reform.* l. 2. c. 1. & 2. ^d *Festus in voce quadrantal*, *Rhemnius Fannius de mens. & pond. ap. Graev. Tom. 2.* ^e *Uncia fit drachmis bis quatuor* ---- *Rhemnius Fannius, unciaque in libra pars est quæ mensis in anno.* *Rhemnius Fannius.* ^f *Drachma Attica denarii argentei habet pondus, inquit Plinius, nat. hist. l. 21. c. 34.*

time, were very carefully weighed, and found to weigh at the rate of 8 to a modern Roman ounce^a; from whence it may be concluded that the Roman ounce and pound are the same now as they were anciently: and consequently if a cubic vessel be made containing 80 modern Roman pounds of clean rain water, a side thereof will give us the length of the ancient Roman foot.

433 The *Congius* was the eighth part of the *amphora*, and contained 10 pounds of wine or water^b; a side of a vessel of this capacity of a cubic form would shew the length of half the Roman foot. *Villalpandus*^c has given us a print of a brass *congius* in the *Farneſe* collection, whose shape was such, that the dimensions of the several parts of it shewed the length of the Roman inch, half foot, and foot; it being so contrived, as he thinks, on purpose that it might serve for a standard of those several measures in length, as well as for a measure of capacity: particularly the thickness of the upper rim or lip of this vessel was an inch, and depth of it exactly a foot: the inscription upon it shews it to have been made in the time of *Vespasian*, according to the standard in the Capitol, and that it contained 10 pounds weight: the Roman foot taken from this is therefore called *the foot of Vespasian*, to distinguish it from other Roman feet which are next to be described.

434 There is at Rome an ancient sepulchral monument of *Cossutius*, upon which a foot-rule, pair of compasses, square, and mallet are cut *in relievo*: of this stone, which was formerly in the garden of *Angelo Coloti* a learned antiquary, from whence some writers call the foot taken from thence *the Colotian foot*, we have a print in *Lucas Pætus*^d, it is the opinion of this author, that the rule, as well as the rest of the tools, was placed there to shew of what trade *Cossutius* was, but that the length of it may be drawn at random, there being no reason for the workman to be exact about it; and that the Roman foot might better be taken from ancient brass rules, of which he had three exactly of the same length, marked with such divisions as agreed with the number of equal parts into which the Roman foot is by ancient writers said to be divided: though he measured some other ancient brass rules which were of a different length. There is in the Vatican gardens another like monument of *Statilius*, having a foot with its divisions cut upon it, which are not upon the foot of *Cossutius*:

^a Bp Hooper in his enquiry into the ancient measures observes, that under the Consuls the weight of the *denarius* was at the rate of 7 to an ounce; that under the first Emperours it was diminished so as to weigh but the eighth part of an ounce; and that afterwards under the *Gordians* and the following Emperours it rose to its first weight.

^b *Uti quadrantal vini octoginta pondo fiet. Congius vini decem pondo is fiet.* Fragment. veteris plebisciti apud Fellum, voc. pondera publica.

^c Tom. 3. apparat. urbis & templi Hierosolym. l. 3. c. 25.

^d *De mensur. & ponder. Rom. & Græc. ap. Græv. Theſ. Rom. ant. Tom. 2.*

a *quadrantal* made by this foot of *Statilius* was found to hold 80 Roman pounds of water; *Greaves*^a denies this experiment to have been made with accuracy, this author who seems to have surpassed all that went before him in exactness, shews there is a difference between the foot of *Cossutius* and that of *Statilius*, whereas other writers had carelessly affirmed them to be both exactly of a length: and he gives the following reasons why he takes that of *Cossutius* to be the ancient Roman foot; 1, the length of it is the same as that of several ancient brass rules, which he carefully measured; 2, he found almost all the stones of white marble in the pavement of the Pantheon to be exactly three *Cossutian* feet square, and the lesser ones of Porphyry one and a half square of the same foot: 3, the dimensions of many of the stones in the foundation of the Capitol, and the arcs of *Titus* and *Severus* were such, as made it probable they were cut out by this measure; 4, from the neatness of the sculpture, he thinks it not likely the carver would cut the rule too short or too long, when he might as easily give it the true length of a foot: 5, having made a *quadrantal* by this foot he measured the capacity of it, and found it contained exactly 7 and a half of the *Farnese congius*: and whereas it ought to have held just 8 *congi*, according to what is said of the *amphora* by *Festus* and *Rhemnius Farnius*, his opinion is, that when those authors affirm that the length of every one of the sides of the *amphora* was equal to the Roman foot, they write what was vulgarly believed upon tradition; not that they were precisely equal, but that they came the nearest to it of any known measure: in like manner he observes, both Greek and Roman writers speak of the Roman *denarius* and Grecian *drachma* as of the same value; whereas they were not exactly so, but only by common estimation, so that one passed for the other, not being much different in weight and value.

235 The learned Bp *Hooper* in his enquiry into the state of the ancient measures, p. 153, and 166, shews that the cubical relation of the *amphora* to a foot, and of the *congius* to the half foot, was very probably accidental only, and not primarily designed; and that it could not be designed by the *decemviri*, since the doubling the cube was afterward in *Plato's* time a problem not well understood by the Greeks themselves: and indeed by the *plebiscitum* it appears, that the capacity of the *amphora* was to be regulated by the weight of the water it was to contain, rather than by the dimension of its sides.

436 The length of the Roman foot being once found, their other measures are easily known, by variety of passages in ancient writers, which tell us what proportion there was between those measures and the foot: as when one

^a In his discourse of the Roman foot and *denarius*.

tells us that the palm was the fourth part of a foot; another, that the pace was five feet; a third, that the *stadium* was 625 feet; &c.

437 The *mile* was a Roman measure containing a thousand paces, from whence it had its name^a: the pace being five feet, the Roman mile was 5000 Roman feet. There was a gilded pillar erected in the *forum* by *Augustus*, which from its use was called *milliarium aureum*^b; it had inscriptions upon it shewing the distances from thence of the principal cities and towns in Italy; and in the great roads there was a stone set up at the end of every mile, upon which was marked the number of miles it was distant from the *milliarium*: from hence come those expressions, so common amongst Latin writers, *ad primum*, *secundum*, &c. *lapidem*, to signify the distance of so many miles; as when *Livy*, l. 5, says the Romans met the Gauls *ad undecimum lapidem*, eleven miles from Rome. The mile, though originally a Roman measure, is often made use of by such Greek writers as were acquainted with the Roman affairs: it is still in use amongst several nations of Europe, but in different places is of a different length.

438 *Cassini*^c attempted to find the length of the ancient Roman mile, and of the foot which measured it, by the following method; the distance between Narbonne and Nîmes is said by *Strabo*, l. 10, to be 88 miles; this distance, when the meridian of Paris was prolonged, in the manner described § 415, was measured trigonometrically, and found to be 67500 *toises*; if this number be divided by 88, the quotient will be 767, the number of *toises* in a mile; and if this last number be multiplied by 6, the product will be 4602, we may cast away the odd 2 feet as of little consequence in so large a number, and take the round number 4600 for the number of Paris feet in a Roman mile, of 5000 Roman feet: the Paris foot then is to the Roman, as 46 to 50, or as 23 to 25; this is the same proportion as there is between the Paris and the modern Roman foot, and consequently the Roman foot is the same now as it was anciently.

439 That this is the true proportion between these ancient and modern measures, is thus confirmed by the same author, by another instance: the distance between Bologna and Modena is set down in the itinerary of *Antoninus*^d, and in the tables published by *Peutinger*, 25 miles; this distance also has been measured trigonometrically, and found to be 19143 *toises*; this number divided

^a It is called by *Cicero* *milliare* and *milliarium*, but more frequently *mille passus*.

^b Dio. l. 54. *Plutarch. in Galba*, v. *Panciroli*. & *Donat. descript. urb. Romæ*, ap. *Graev. Thef. Rom. ant.* Tom. 3. Setting up of mile-stones in the Roman roads was long before begun by *C. Gracchus*. *Plutarch. in Gracchis*.

^c *Suite des mémoires de l'Académie Royale des sciences ann. 1718*, pag. 150 ed. *Par.*

^d *De itinerariis antiquis & recentibus* v. *Ricciol. Geogr. reform.* l. 3. c. 3.

by 25 makes the mile of 766 *toises*, but one *toise* less than it was found to be in the preceding section, thus the Roman mile and the foot which measured it seem to be pretty well ascertained.

440 The *stadium* is a measure of greek original, being the length of the race-ground at Pisa in Elis, where the Olympic games were celebrated; it was marked out by *Hercules*, 600 feet long, measuring it by the length of his own foot; and his stature being above the common size, his foot was proportionally longer than that in common use: *Columella*^a and *Pliny*^b expressly say, that a *stadium* was 625 Roman feet; this number is the eighth part of 5000; and accordingly 8 *stadia* are reckoned to a mile: the Olympic *stadium* of 600 Herculean feet is commonly to be understood when nothing is said to the contrary: in some parts of Greece the *stadium* was shorter, consisting of 600 common feet, of a less dimension than those of *Hercules*^c: this may occasion some uncertainty in some passages of ancient authors, where they measure distances by the *stadium*, because it does not always appear which *stadium* is meant.

441 There is great diversity in the modern mile, in different places: the present Roman mile contains, according to *Ricciolus*^d, 984 paces; which is but 16 paces less than the ancient Roman mile, of 1000 paces: in other parts of Italy, the miles are of different lengths: the mile of Ancona, according to the same author, contains 1375 Roman paces; the mile of Bologna, 1266 $\frac{2}{3}$; the mile of Florence, 1270; that of Ferrara but 898; &c.

442 The English mile is very uncertain, if we measure the distances of places by vulgar estimation of the number of miles; for then the mile is usually longer, the further from London: the English measured or statute mile is 5280 English feet, or 1056 English paces, each pace being five feet.^e

443 The French reckon distances by leagues, which are very different in different parts of France; *Cassini*^f reduces them to three sorts; 1, the league near Paris, of 2000 *toises*; 2, the common league, of 2282 *toises*; 3, the league near the coast of 2853 *toises*.

444 The ancient Grecian foot is generally supposed to be in proportion to the ancient Roman as 25 to 24, that is to exceed it half a Roman inch: thus we are told by *Greaves* in his dissertation of the Roman foot p.40, and *Arbutnot* in his tables of ancient coins, weights and measures; but this I think is true of the Olympic or Herculean foot: for from what *Gellius* says l. 2. c. 1. it appears that the common Grecian foot was less than this. *Philander* in his comment on *Vitruvius* mentions a porphyry pillar in Rome, upon which there

a Lib. 5. c. 1. b Lib. 2. c. 28. c *Gellius* lib. 2. cap. 1. *Censorinus de die natali* c. 11 tit. ult. *Ricciolus*, observes upon this occasion, that if the foot of *Hercules* was exactly a sixth part of his height, he was six Roman feet and 3 inches high. *Geogr. reform.* l. 2. c. 4. d *Geogr. reformat.* l. 2. c. 8.
e *Coke* 4 *inst.* chap. 61. f *Suite des memoires de l'Acad. Royale* anné 1718. p. 247.

was an inscription in greek letters, signifying the length of it to be 9 feet, he says that he measured it, and that the foot there made use of exceeded the Roman but the 9th part of an inch: this pillar is also mentioned by other authors, but was not to be found when *Greaves* was at Rome, nor several years before. *Ricciolus* takes the common greek foot to be the same with the Roman one deduced by *Villalpandus* from the *congius* of *Vespasian*.

445 *Cassini* gives us the following proportions between the modern foot of different parts of Europe, in the *Suite des memoires de l'Acad. Royale, année 1718*, p. 250. If the Paris foot be divided into 12 inches, every inch into 12 lines, and every line into 10 equal parts, the whole foot will be divided into 1440 equal parts, of which parts the Bologna foot contains 1682; the Rhinland or Leyden foot, 1390; the English foot, 1350; the Roman foot, 1306.

The *toise* is a measure which contains six Paris feet: as our fathom is six English feet, and the Grecian $\delta\rho\gamma\upsilon\alpha$ was six Grecian feet.

446 *Greaves* compared the English foot, taken from the iron standard at Guild-Hall in London, with the standards of divers nations: he gives us a table of them in his dissertation of the Roman foot, out of which I selected the following particulars.

Of such parts as the English foot contains

The Roman foot, or that on the monument of *Cossutius* in Rome, which *Greaves*^a found to be of the same length with the standard of the present Roman foot cut upon a stone in the Capitol, under the direction of *Lucas Pætus*, contains

The foot on the monument of <i>Statilius</i>	1000
The foot of <i>Villalpandus</i> deduced from the <i>Congius</i> of <i>Vespasian</i>	967
The Greek foot	972
The Paris foot	986
The Rhinland or Leyden foot used by <i>Snellius</i>	1007
Add to these out of <i>Arbutnot's</i> tables the Bologna foot	1068
	1033
	1250

Larger tables of the ancient and modern measures of divers nations may be seen in *Ricciolus Geogr. reformat. lib. 3. c. 4*: *Greaves & Cassini, loc. citat*: *Arbutnot's* tables: and in the authors quoted by those writers.

447 The proportion between these several feet being thus known, if any distance be expressed in one of these measures, it is easy to know what it is in any other of these measures, by the golden rule: thus, English feet may be reduced to French, or French to English, &c.

448 It is a common experiment, known to every body, that if a *pendulum* be put into motion, it will swing faster or slower according as the thread or

^a Dissert. of the Roman foot. v. etiam Fabrettum de aquis & aqueduct. dissert. 2. ap. Græv. T. 4.

wire of it is shorter or longer: a pendulum which measures three Paris feet 8 lines and a half from *the center of oscillation*, or upper end of the wire where its swing begins, to the center of the ball, will perform one swing in a second of time: this then is *an universal measure*, whose length we shall have, if we make a pendulum that will perform 60 swings in a minute, or 3600 in an hour: it has indeed been discovered, by the same clock going slower the nearer it was to the equator, that a pendulum kept of the same length will swing slower, the nearer it is to the equator, and faster the nearer it is to either of the poles: and consequently, that if we would have a pendulum swing seconds, it must be made shorter, the nearer we are to the equator; this difference of length, arising from the difference of latitude, is however so inconsiderable, that a pendulum which swings seconds may serve very well for an universal measure, for the greatest part of Europe.^a

449 Such an universal measure as has now been described, would serve to shew what proportion there is between the different measures of different nations, as well as if the original standards were compared with one another: it would likewise so ascertain those standards, that it would easily be discovered if in time to come any change should be made in them.

450 It would indeed be very well if all measures of length were taken from the universal measure: thus the length of a pendulum which swings seconds might be called the universal yard: a third part of this might be the universal foot: five such feet the universal pace: 6 such feet the universal *toise* or fathom: 1000 universal paces might be the universal mile.

CHAP. 14. THE DIMENSIONS OF THE EARTH.

451 It has already been proved that the earth is round^b; we are now to shew how the magnitude of this globe of the earth, its circumference, diameter, superficial, and solid content, may be found in miles, yards, feet, or any other known measure.

452 *To find the measure of a great circle upon the earth:* draw a meridian line at any place which you make choice of for your first station^c; and take the height of the pole there^d; prolong the meridian line^e, either northwards, till you come to a place where the pole is elevated one degree more, or southwards, till the height of the pole is one degree less than at the first station; find the distance between these two places in miles, yards, feet, or any other known measure^f; and you have then the number of miles, yards, &c, contain-

^a Picard *mesure de la terre*. ^b § 273. ^c By § 405. ^d By § 410, 411, 412. ^e & ^f By § 415.

ed in one degree, or the 360th part of a great circle upon the earth; this number multiplyed by 360 gives the whole circumference of a great circle upon the earth, in the same measure, and shews the number of miles, yards, feet, &c, contained therein.

Scholium, If the meridian be further prolonged, so that there is a difference of several degrees between the heights of the pole in the two places, which are made choice of for the first and last stations, the errors to which the observations of those heights of the pole may be liable, will then be divided amongst all those degrees; and will therefore be of less consequence, than when they are placed to the account of one degree only: with this view, the meridian of Paris was so far prolonged, that there was the difference of about $8^{\circ}\frac{1}{2}$ between the elevations of the pole, at the two extremities thereof; for at the north end of it, in the parallel of Dunkirk, and near that place, the height of the pole was $50^{\circ} 51'$; and at the south end, at Collioure, near Rouffillon, it was but $42^{\circ} 20'$: the difference measured upon the meridian is $8^{\circ} 31' 11''\frac{5}{8}$; the distance between the parallels of Collioure and Dunkirk measured by trigonometry, in the manner above described^a, was found, all proper allowances being made, to be 486156 *toises*; this number divided by $8^{\circ} 31' 11''\frac{5}{8}$ will give 57061 *toises* to a degree; instead of this, *Cassini* thought proper to take *Picard's* measure of 57060 to a degree, being a round number, and very little different from his own: the earth is here supposed to be exactly spherical, so that all the degrees measured in a great circle thereof are equal.^b

453 Since there are 57060 *toises* in a degree, if this number be multiplyed by 360, the product 20541600 will be the number of *toises* contained in a great circle upon the earth.

454 The circumference of a circle being given, its diameter may be found, by § 68; thus, from the circumference of a great circle upon the earth in the preceding section, the diameter of the earth is found to be 6538594 *toises*; the semidiameter then is 3269297 *toises*: these measures are given us by *Cassini*, in the *Suite des memoires de l'Acad. R. ann. 1718. pag. 247 edit. Par.* and are the same as were before given by *Picard*, in his *mesure de la terre; art. 11.*

455 There being six Paris feet in a *toise*, if the foregoing numbers be multiplyed by 6, we shall have the following dimensions of the earth in Paris feet:

	Paris feet.
One degree of a great circle upon the earth is	342360
The circumference of a great circle upon the earth is	123249600
The diameter of the earth is	39231564
The semidiameter of the earth is	19615782

^a By § 415.

^b *Suite des mem. de l'Acad. R. ann. 1718. p. 247-*

456 Since, according to *Greaves's* table,^a the Paris foot is to the English foot, as 1068 to 1000; we may by the golden rule find the following dimensions of the earth in English feet:

	English feet
A degree of a great circle upon the earth is	365640
A minute of a degree is	6094
A second is	$101\frac{1}{2}$
The circumference of a great circle upon the earth is	131630573
The diameter of the earth is	41899310
The semidiameter of the earth is	20949655

457 If these English feet be reduced to English miles, each mile containing 5280 feet, we shall have the following dimensions of the earth in English statute miles:

	English miles
A degree of a great circle upon the earth is	$69\frac{1}{5}$
A minute of a degree is	$1\frac{2}{6}$
The circumference of the earth is	24930
The diameter of the earth is	$7935\frac{2}{5}$
The semidiameter of the earth is	$3967\frac{1}{2}$

458 The circumference of the earth being given, by § 457, the surface of the earth will by § 204 be found to be 200,000,000 square English miles.

459 The surface of the earth being given by § 458, the solid content of the earth will by § 205 be found to be 266,000,000,000 cubic, English miles.

460 To find the measure of the earth is a problem which, in all likelihood, has been studied by astronomers, ever since the earth was believed to be spherical: *Laërtius*^b mentions *Anaximander*, the disciple of *Thales*, as the first amongst the Greeks who wrote upon this subject: *Archytas*^c of Tarentum an eminent Pythagorean famous for his skill in mathematics and mechanics, seems to have made some attempts this way: perhaps these were the authors of the most ancient opinion we have of the magnitude of the earth, recorded by *Aristotle*, in his book *de cælo*^d, viz. that the circumference of the earth is four hundred thousand *stadia*: this is almost twice as much as it has since been determined to be; but such a mistake, made when astronomy was in its infancy, is very pardonable: it is probable *Aristarchus* of Samos considered the magnitude of the earth as well as of the sun, and moon: *Archimedes*^e mentions the ancients who held the circumference of the earth to be 300000 *stadia*: what methods were used by these Geometers does not at present appear, but that

a § 446. b γῆς καὶ θαλάσσης μέγεθος ἀπὸ τοῦ Ἰωνοῦ. *Laertius in Anaximandro.*

c Hor. Od. 28. lib. 1. *Te maris & terra numeroque carentis arena*

Mensuræm sobibent Archyta.

d Lib. 2. cap. ult. e *In arenario.*

they were observations of stars in the zenith, or in the horizon, and actual measuring some part of the circumference of the earth, is very probable, from what the writer *de cælo* remarks in the place just now quoted; that we have different stars pass through our zenith, according as our situation is more north or south; and that in the southern parts of the earth we have stars come above our horizon, which if we go northward will no longer be visible to us.

These different appearances of the stars point out two ways of measuring the earth; one of which is by observing stars which pass through the zenith of one place, and do not pass through the zenith of another; the other by observing some stars which come above the horizon in one place, and are observed at the same time to be in the horizon of another place.

461 *Eratosthenes*, at Alexandria in Egypt, made use of the first of these methods; he knew, that at the summer solstice the sun was vertical to the inhabitants of Syene, a town situated in the confines of Ethiopia, under the tropic of cancer, where they had a well built for that purpose, on the bottom of which the rays of the sun fell perpendicularly on the day of the summer solstice; he observed, by the shadow of a wire set perpendicular in an hemispherical basin, how much the sun was on the same day at noon distant from the zenith of Alexandria, and found that distance to be a 50th part of the circumference of a great circle in the heaven; supposing then Alexandria and Syene to be under the same meridian, he concluded the distance between them to be a 50th part of a great circle upon the earth; and this distance being by measure 5000 *stadia*, that the circumference of the earth is 250000 *stadia*: but because this number divided by 360 would give $694\frac{4}{9}$ *stadia* to a degree, either *Eratosthenes* himself, or some of his followers, assigned the round number 700 *stadia* to a degree; which multiplied by 360 makes the circumference of the earth 252000 *stadia*: and this is the reason why different authors give us these two different numbers for the measure of *Eratosthenes*.

462 *Pofidonius*, who lived in the time of *Pompey* the great, attempted to measure the circumference of the earth, by the other method of horizontal observations: he knew that the star called Canopus was but just visible in the horizon of Rhodes, and that at Alexandria its meridian height was a 48th part of a great circle in the heaven, or $7^{\circ}\frac{1}{2}$; this shews what part of a great circle upon the earth the distance between those places amounts to; supposing them both to be under the same meridian, and the distance between them to be 5000 *stadia*, the circumference of the earth will be 240000 *stadia*: this is the first measure of *Pofidonius* related by *Cleomedes*^a, a writer of the same age, who remarks that a less measure is to be taken, if the distance between Rhodes and Alexandria

^a Lib. 1. cap. ult.

is found to be less than it was just now said to be. *Strabo*^a makes the circumference of the earth, according to *Posidonius*, to be 180000 *stadia*, at the rate of 500 *stadia* to a degree; the occasion of this diversity is thought to be this; the same author in another place^b relates, that *Eratosthenes* measured the distance between Rhodes and Alexandria, and found it to be but 3750 *stadia*; taking this for a 48th part of the earth's circumference, according to *Posidonius*, the whole circumference will be 180000 *stadia*: this measure, in which the number of degrees assigned by *Posidonius* and the number of *stadia* measured by *Eratosthenes* are made use of, was received by *Marinus*^c of Tyre and others, and is generally ascribed to *Ptolemy*, because he makes use of it in his geography, where he often follows *Marinus*.^d

463 The method of *Posidonius* is justly rejected, because of the uncertainty of refraction, in stars that are near the horizon: the method of *Eratosthenes* is explained at large, and corrected by *Ricciolus*, *geograph. reformat.* lib. 5. c. 3. see also *Varenius's geography*, l. 1. c. 4.

464 *Cassini* remarks, that, taking exactly the mean between the last dimensions of *Eratosthenes* and *Posidonius*, a degree of a great circle upon the earth will be 600 *stadia*, and a minute of a degree 10 *stadia*, that is a mile and a quarter of the ancient Roman measure; which is just a mile of the modern Italian measure; for the modern Italian mile is equal to a mile and a quarter of the ancient measure, as appears by the distance between Bologna and Modena, which is 20 modern Italian miles, and was 25 ancient Roman miles: so that, according to this way of reckoning, a degree of the circumference of the earth is 60 modern Italian miles, and 75 ancient Roman miles: the circumference of the earth is 21600 modern, and 27000 ancient miles: and allowing 3 ancient miles to a mean league, a degree will be 25 leagues; and the whole circumference 900 leagues^e.

465 After *Eratosthenes* and *Posidonius*, several others have made use of the different heights of the pole in distant places under the same meridian, to find the dimensions of the earth: that great encourager of learning among the Arabians *Almamoon* Calif of Babylon, about the year of Christ 800, had the distance measured between two places situated two degrees asunder, and under the same meridian, in the plains of Sinjar near the red sea: the result was, that the mathematicians employed in this affair, whether they measured each degree

a Lib. 2. pag. 95. ed. Casaub.

b Lib. 2. pag. 126. ed. Casaub.

c Ptolem. *geogr.* l. 1. cap. 7.

d V. Ricciol. *geogr. ref.* l. 5. c. 8. & *Cassini Suite des memoires de l'Acad. R.* annee 1718, p. 16.

e *Suite des memoires de l'Acad. R.* annee 1713. pag. 16. ed. Par,

separately,

separately, or went over the whole way more than once, found at one measuring 56 miles; at another 56 miles $\frac{1}{3}$; or as some have it 56 $\frac{2}{3}$ in a degree ^a.

466 *Fernelius*^b a learned French Physician, in the year 1525, made the next attempt to find out the circumference of the earth: for this purpose, he first took the height of the pole at Paris, then, going from thence directly northwards, till he came to the place where the height of the pole was one degree more than at Paris, he measured the length of the way; by the number of the revolutions of the wheel of his coach in which he travelled; and, after proper allowances for the risings, declivities, and turnings, which he met with in the road, he concluded 68 Italian miles to be contained in a degree.

467 After this *Snellius*^c, an eminent Dutch mathematician, took the heights of the pole at Alcmaer and Bergen-op-zoom, and found the difference between them $1^{\circ} 11' 30''$. he measured the distances between the parallels of those two places, by taking several stations, and forming several triangles; by means whereof he found one degree to contain 341676 Leyden feet: he measured also the distance between the parallel of Leyden and Alcmaer, which differ in the elevation of the pole half a degree, and by this measuring he found 342120 Leyden feet were contained in a degree: he took the mean number between these two, and assigned 342000 Leyden feet to a degree; which, according to *Picard*, reduced to French measure, make 55021 *toises*.

468 Our countryman *Norwood*, in the year 1635, having taken the height of the pole at London and York, and measured the distance between the parallels of those two places, assigns 367200 English feet, which is 69 miles $\frac{1}{2}$ and 14 poles, to a degree; each pole being 16 feet $\frac{1}{2}$: the method used by *Norwood* is described by him, in a small treatise called the sea-mans practice.

469 *Ricciolus*^d, after the year 1654, having made use of several methods to determine the circumference of the earth, assigns to one degree 64363 Bologna paces; which make 61650 French *toises*.

470 About the year 1669, the Royal Academy of Sciences at Paris, thought it worth the while to make a new trial for the solution of this famous problem; and the rather, because in the former attempts no body had made use of telescopic sights, which are much the best for this purpose; for by them the view may be directed to an object at a greater distance, and to a point of an object with more certainty; and consequently the triangles for measuring distances may be formed with greater accuracy, than otherwise can be done.

^a Alfragani *element. astron.* c. 10. & *ibi* Christmannus. Ricciol. *geogr. reformat.* lib. 5. c. 11. *Picard. mesure de la terre. Suite des memoires ann.* 1718. chap. 1.

^b Fernelius *Cosmographie* c. 1. Ricciol. *geogr. reform.* l. 5. c. 10. *Suite des memoires ann.* 1718. ch. 1.

^c Snellius in *Eratoſthene Batavo* lib. 2. Ricciol. *Geogr. reformat.* lib. 5. cap. 11. *Suite des memoires de l'Acad. R. ann* 1718. p. 17. *éd. Par.*

^d Ricciol. *Geogr. reform.* lib. 3.

The fundamental base of the trigonometrical operations was much longer than either *Snellius* or *Ricciolus* had made use of, it has already been described § 415; the distance measured was between the parallels of Soudon and Malvoisine; the difference between the heights of the pole in these two places was a little more than one degree; the result of the whole was, that one degree was concluded to contain 57060 *toises*. *Picard* in a treatise intituled *mesure de la terre*, gives an account of the whole process in this essay to find the dimensions of the earth, together with a description of the instruments made use of therein, and the manner of adjusting them: this piece was afterwards translated into English by *R. Waller* F. R. S. there is also a large extract out of it published by *Cassini*, *Suite des memoires de l'Acad. R. ann. 1718, part. 2. chap. 6.*

471 The members of the Royal Academy at Paris, knowing that the longer portion of the meridian was measured, the more accurately would the circumference of the earth be found^a, prolonged the meridian of the R. Observatory quite cross the kingdom of France, and measured it trigonometrically, in the manner already described, § 415, and in the beginning of this chapter: this work was begun in the year 1683, but no progress was made therein till the year 1700, and it was not finished till the year 1718: they made use of the fundamental base of *Picard*, as being measured with sufficient accuracy; an account of the whole process was published by *Cassini* F. in the year 1720; under this title, *Suite des memoires de l'Academie Royale des sciences, année 1718*; in this work it is shewn, that there are mistakes in *Snellius*, in the calculations of some of his triangles, so that his dimensions of the earth are not to be depended upon^b: it is likewise shewn what defects there are in the methods used by *Ricciolus*^c, the chief occasions of which were his taking too short a fundamental base, and not sufficiently considering refraction.

472 Though *Snellius* had made some mistakes in his calculations, there is no room to question the accuracy of his observations; Holland is, by reason of its flatness, the fittest country in Europe to measure an arc of the meridian therein, and *Snellius* had this uncommon opportunity of finding the exact length of his fundamental base, *viz.* the distance between one tower at Leyden and another at Souterwode; a frost happened, just after the country round Leyden had been overflowed, this enabled him to take two stations upon the ice, the distance between which he carefully measured three times over, and then from those stations observed the angles the visual rays pointing at those towers made with the strait line thus taken upon the ice: these considerations induced *Peter Van Muschenbroek* professor of philosophy and mathematics at Utrecht to make new observations, and form triangles upon the fundamental

^a See the *Scholium* to § 452. ^b *Suite des memoires partie 2. c. 8.* ^c *Suite des memoires partie 2. c. 9. base*

base of *Snellius*, which he did in the year 1700^a: the result of his operations is FIG. this, he assigns 5703 *toises* to a degree, which is but 27 *toises* less than the French astronomers had done. I have here given an account of all the attempts to measure the earth which are worth relating: that of the French astronomers was made with such care and exactness, that I think we may very well acquiesce therein.*

CHAP. 15. THE DIFFERENCE BETWEEN THE APPARENT LEVEL AND THE TRUE: THE EXTENT OF THE VISIBLE HORIZON.

*473 *A truly level surface* is a segment of any spherical surface which is concentric to the globe of the earth: the surface of the sea or of any large piece of water, when still, forms it self into a truly level surface. *A true line of level* is an arc of a great circle which we imagine to be described upon a truly level surface: *The apparent level* is a strait line drawn tangent to an arc or line of true level. Every point of the apparent level, except the point of contact, is higher than the true level: thus, fig. 57, let *EAG* be an arc of a great circle drawn upon the earth, to a person who stands upon the earth at *A* the line *HD* is the apparent level, parallel to his rational horizon *RR*; but this line, the further it is extended from his station *A*, the further it recedes from the center; for *BC* is longer than *AC*, and *DC* is longer than *BC*, &c. 57

*474 Since the semidiameter of the earth is known^b, it is easy to compute how much the apparent level is above the true, if it be known what the distance of the object looked at horizontally is from the station of the observer: thus, fig. 57, let the observer standing at *A* look through a telescope placed horizontally, at the object *B*; here *BAC* is a right-angled triangle^c, in which if *AC* be made radius, *AB* will be tangent, and *CB* secant of the angle *ACB*^d; now to find this tangent, say, as the number of feet in *AC* the semidiameter of the earth is to the number of feet in *AB* the distance of the object; so is *AC* as radius to *AB* as tangent; that is, so is 10000000 to a fourth number; which is the tangent required: the tangent *AB* being found in the table, we have the secant *CB*, from which if the radius *CG* be taken, the remainder *GB* is the excess of the secant above the radius; or the height of the apparent level above the true. See the following table. 58

^a *Muschenbroek Dissertat. de magnitudine terra.*

^b § 456.

^c § 123.

^d §. 140.

* Other geometrical methods of finding out the diameter of the earth, without any celestial observations, may be seen in *Kepler*, *Varenus*, *Ricciolus*, and others: but, from the variableness of refraction, they are less certain than the method described §. 452, for which reason I have omitted them. See *Cassini observ. faites en div. endr.* p. 16. & seq.

A TABLE SHEWING THE HEIGHT OF THE APPARENT LEVEL ABOVE THE TRUE.

Seconds	Feet	Inch.	Inch.	Sec.	Feet	Inch.	Inch.
<p>If the distance of the object from the place of the spectator be which measured in a great circle upon the earth amounts to</p>	1	101	6. 8	<p>If the distance of the object from the place of the spectator be which measured in a great circle upon the earth amounts to</p>	31	3148	6. 8
	2	203	1. 6		32	3250	1. 6
	3	304	8. 4		33	3351	8. 4
	4	406	3. 2		34	3453	3. 2
	5	507	10. 0		35	3554	10. 0
	6	609	4. 8		36	3656	4. 8
	7	710	11. 6		37	3757	11. 6
	8	812	6. 4		38	3859	6. 4
	9	914	1. 2		39	3961	1. 2
	10	1015	8. 0		40	4062	8. 0
	11	1117	2. 8		41	4164	2. 8
	12	1218	9. 6		42	4265	9. 6
	13	1320	4. 4		43	4367	4. 4
	14	1421	11. 2		44	4468	11. 2
	15	1523	6. 0		45	4570	6. 0
	16	1625	0. 8		46	4672	0. 8
	17	1726	7. 6		47	4773	7. 6
	18	1828	2. 4		48	4875	2. 4
	19	1929	9. 2		49	4976	9. 2
	20	2031	4. 0		50	5078	4. 0
	21	2132	10. 8		51	5179	10. 8
	22	2234	5. 6		52	5281	5. 6
	23	2336	0. 4		53	5383	0. 4
	24	2437	7. 2		54	5484	7. 2
	25	2539	2. 0		55	5586	2. 0
	26	2640	8. 8		56	5687	8. 8
	27	2742	3. 6		57	5789	3. 6
	28	2843	10. 4		58	5890	10. 4
	29	2945	5. 2		59	5992	5. 2
	30	3047	0. 0		60	6094	0. 0
the height of the apparent level above the true will be				the height of the apparent level above the true will be			
0. 074				4. 746			
0. 296				7. 409			
1. 186				10. 680			
2. 670							

THE CONTINUATION OF THE FOREGOING TABLE.

Min.	Feet.	Feet. Inch.	Min.	Feet.	Feet. Inch.
1	6094	0 10. 680	31	188914	851 9. 828
2	12188	3 6. 580	32	195008	907 8. 532
3	18282	7 11. 853	33	201102	965 3. 528
4	24376	14 1. 812	34	207196	1024 7. 884
5	30470	22 1. 932	35	213290	1085 9. 600
6	36564	31 11. 412	36	219384	1148 8. 676
7	42658	42 5. 436	37	225478	1213 5. 112
8	48752	56 9. 384	38	231572	1277 10. 908
9	54846	71 9. 876	39	237666	1348 2. 064
10	60940	88 7. 728	40	243760	1417 1. 764
11	67034	107 2. 940	41	249854	1496 11. 388
12	73128	127 7. 512	42	255948	1569 10. 452
13	79222	149 9. 444	43	262042	1638 9. 084
14	85316	173 8. 736	44	268136	1716 0. 108
15	91410	199 4. 320	45	274230	1794 11. 424
16	97504	226 9. 264	46	280324	1875 7. 032
17	103598	255 11. 568	47	286418	1958 0. 000
18	109692	286 11. 232	48	292512	2042 2. 328
19	115786	319 7. 188	49	298606	2128 2. 016
20	121880	354 0. 504	50	304700	2215 6. 792
21	127974	390 4. 248	51	310794	2305 5. 472
22	134068	428 5. 352	52	316888	2396 9. 240
23	140162	468 10. 224	53	322982	2489 10. 368
24	146256	510 6. 084	54	329076	2584 8. 856
25	152350	553 11. 232	55	335170	2681 4. 704
26	158444	599 1. 776	56	341264	2779 9. 912
27	164538	646 1. 680	57	347358	2880 0. 480
28	170632	694 10. 944	58	353452	2982 0. 408
29	176726	745 5. 568	59	359546	3085 8. 628
30	182820	797 8. 484	60	365640	3191 2. 208

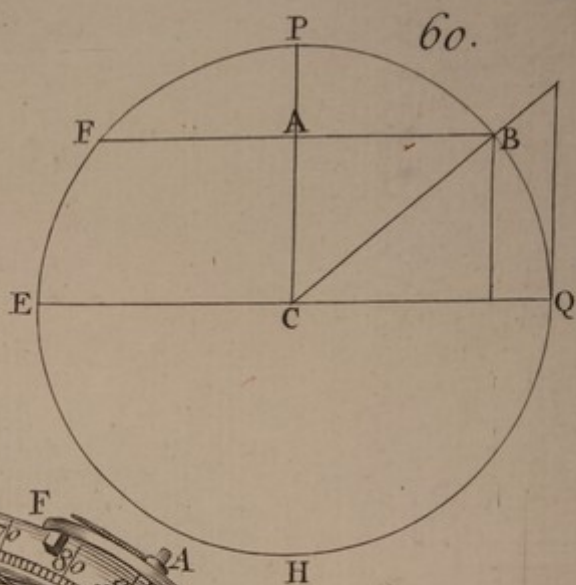
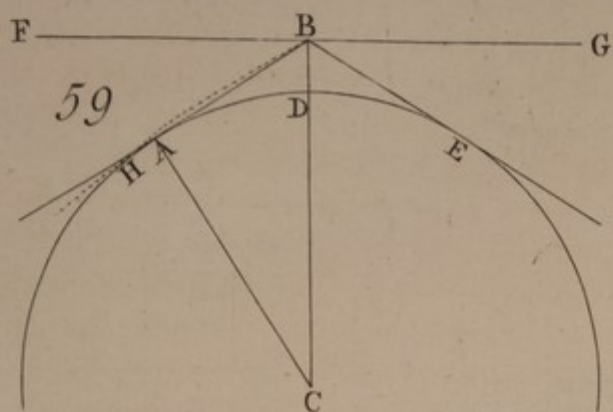
475 The

475 The foregoing table is taken from *Cassini*^a, only his French toises, feet, inches, and lines are here turned into English feet, inches, and decimal parts of an inch^b: one *use of the table* is, an arc of a great circle upon the earth being given in seconds or minutes, to find the length thereof in English measure; thus, an arc of 8" is 312 feet 6 inches and 4 tenths of an inch; thus again, an arc of 20' is 121880 English feet. *Another use of the table* is, an arc of a great circle upon the earth being given, in seconds or minutes, or in feet and inches, to find what is the excess of the secant above the radius, or the height of the apparent level above the true: this is so little in very small arcs, that it is here given only for these numbers of seconds, 5", 10", and from thence for every 10": in the table of minutes, it is given for every minute, from 1' to 60', and then for every 10', from 1°, to 2°: thus, if an arc be 20", or 2031 feet 4 inches, the height of the apparent level above the true is 1 inch 186: thus again, if the distance of the place of an object from the place of the observer be 30', or 182820 feet, the height of the apparent level above the true is 797 feet 8 inches: *Another use of the table* is, that the distance of any object which is viewed through sights placed horizontally being given, the height of it may be found; or, conversely, the height of the object being given, its distance may be found. Thus, if the distance of a mountain whose top is in the sensible horizon be 15', or 91410 feet, the height of the mountain is 199 feet 4 inches; thus, conversely, if the height of a tower whose top appears in the sensible horizon be 199 feet 4 inches, the distance of the tower from the place of the spectator is 15', or 91410 feet.

476 If the distance of an object given be a number of feet which is not in the table, take the number of feet in the table which is nearest to the number given, and say, as the square of the number thus taken, is to the square of the number given; so is the height of the apparent level above the true, corresponding to the number taken, to the height of the apparent level which corresponds to the number given: thus, if it be enquired what is the height of the apparent level above the true, when the distance of the object is 200000 feet, the nearest number to this in the table is 201102, the height of the level corresponding thereto is 965 feet, say then, as the square of 201102 is to the square of 200000, so is 965 to a fourth number of feet, by which the apparent level exceeds the true at the distance of 200000 feet. I have hitherto supposed the line of level to be a tangent to an arc of a great circle drawn upon the surface of the earth, whereas, in levelling, the eye is usually some height, suppose

^a *Suite des mèm.* part. 2. chap. 5.

^b The decimal parts arise from turning French measure into English, but may be neglected in any measure which exceeds 2 or 3 feet.



four feet, above the surface of the earth; but this makes no difference in levelling: FIG. 57
 for as the height of the eye must be added to the radius AC, it must be added
 also to the secant CB, because ML is supposed in levelling to be parallel to HD:
 there is indeed a difference between the length of AI and BL, but it is quite
 insensible. *Ricciol.* l. 6. c. 24.

477 *Another use of the table* is for levelling, in order to convey water from
 one place to another. *Levelling* is the art of finding out whether two given
 points above the surface of the earth are of equal height, that is equally di-
 stant from the center of the earth or not; and, if they are of different heights,
 what that difference is: the common methods of levelling are sufficient for
 laying pavements or walks, for conveying water to small distances, for placing
 horizontal dials, or astronomical instruments: but in levelling the bottoms of
 canals which are to convey water to the distance of many miles, the differ-
 ence between the apparent and true level must be taken into the account:
 thus, fig. 58 let IAL be an arc of a great circle upon the earth, let it be re- 58
 quired to cut a canal whose bottom shall be a true level from A to B, of the
 length of 5078 feet or 50", the common method is to place the levelling in-
 strument in the bottom of the canal at A, and looking through the sights plac-
 ed horizontally at a stick set up perpendicular at B, to make a mark where
 the visual ray or line of the apparent level points, as E; and then to sink the
 bottom of the canal at B as much below E as A is below D: but this will not
 give us a true level, for, by the table, at the distance of 50" or 5078 feet, the
 apparent level is 7 inches above the true; and therefore, to make a true level,
 B must be sunk 7 inches lower than the apparent level directs: so that if A
 be 4 feet below D, B must be 4 feet 7 inches below the mark E. I have here
 mentioned the error which will arise from placing the level at one end of the
 line to be levelled, and shewn how to correct it; but in most cases, it is bet-
 ter to take a station in the middle of the line to be levelled; thus, if the points
 H and B are to be levelled, place the instrument in the middle at A, and set- 58
 ting up sticks perpendicular at H and B, make marks upon each stick where
 the apparent level points, as E and F; those points are level: and if you sink H
 as much below F as B is below E, HAB will be a true level. When the bottom
 of a canal is thus truly level, if water be let in at one end, it will rise to the
 same height at the other. If water be required to run with any velocity, that
 is of another consideration: a river will run, though very slowly, which hath
 not above six inches descent below the true level for a mile in length: if a ri-
 ver whose water is foul be required to run, with such a velocity as to carry
 its foulness into the sea, sixteen inches, or at the least one foot fall below the
 true level, in a mile running, has been thought sufficient, by persons skilful
 in

FIG. in that affair. *Vide Ricciol. geogr. reformat. l. 6. c. 24.* The visible horizon hath been already described, and hath been shewn to have a different extent at a different height of the eye^a; I now proceed to a further consideration of it.

478 The semidiameter of the earth and the height of the eye being given,
 59 the extent of the visible horizon is thus found; let ADE be a piece of a great circle upon the earth, C the center of the earth, B the eye of the observer, BD the height of the eye, BA and BE lines drawn from the eye touching the surface of the earth at A and E, and terminating the visible horizon, the length of BA is required; in order to find it, proceed in this manner; add DB the height of the eye, which I will here suppose to be 5 feet, to DC the semidiameter of the earth, which is 20949655 feet^b, and you have the length of CB, 20949660 feet; draw CA, and you have a triangle BAC, whose angle at A is a right one, by § 28, make the hypotenuse CB radius, and CA will be the sine of the opposite angle ABC, § 140 case 1; say then, as CB is to CA, so is the whole sine or radius to the sine of the angle ABC: this angle being found, its complement ACB is known^c, and consequently the arc AD is also known, and may be found in feet or miles by the table^d, or by § 456; thus, in the example before us, as 20949660 is to 2094655, so is the radius 10000000 to a 4th number, viz. 9999993; which number, is the sine of an angle of 89° 56': the angle ABC then is 89° 56', and therefore its complement ACD is 4', and the arc DA is 4' ^e, that is, by the table^f 24376 feet.

479 The depression of the horizon of the sea at a given height of the eye may
 59 be thus found by calculation: in fig. 59 if the eye be at B, the sensible horizon is FG, the depression of the horizon of the sea is the angle FBA, which, being the complement of ABC, is equal to ACD, that is 4', by §. 478.

480 Since the depression of the horizon of the sea, or the angle FBA, fig.
 59 59, may be found by observation, §. 294; the extent of the visible horizon, at any height of the eye, may also be found by observation: for its semidiameter
 59 DA differs not sensibly from an arc of a great circle upon the earth of the same number of minutes and seconds as the angle of depression is observed to be^g; and the number of feet contained in that arc may be found in the table^h, thus, if the depression of the horizon is 30', the semidiameter of the visible horizon is also 30', that is, by the table, 182820 feet. Different authors give us different extents of the visible horizon: either because they differ in the measure of the earth's semidiameter from whence it is computed, or in their account of the measures which they make use of: thus, when they differ in the number of *stadia*, it may be because they differ in the measure of the *stadium*: *Vide Ricciol. geogr. reformat. l. 6. c. 22. Varen. l. 3. c. 34.*

a § 293 & 294 b §. 456. c §. 55. cor. 5. d §. 474. e §. 30. f §. 474. g §. 479. h §. 474.

1871
The following is a list of the
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Education, for the year 1871-72.

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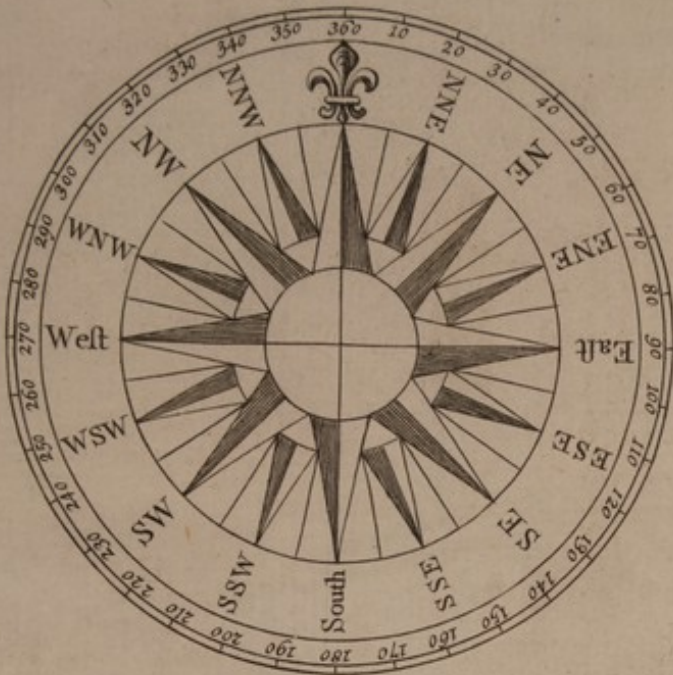
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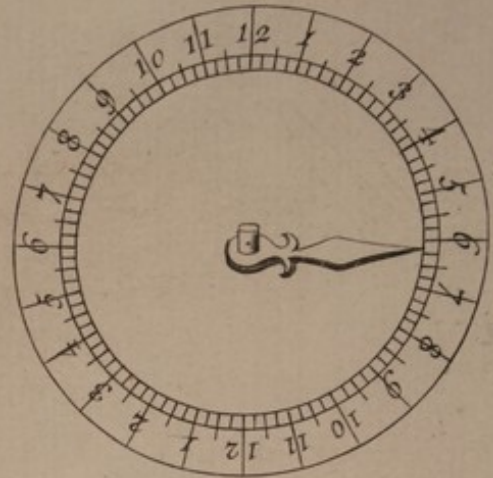
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Book I.

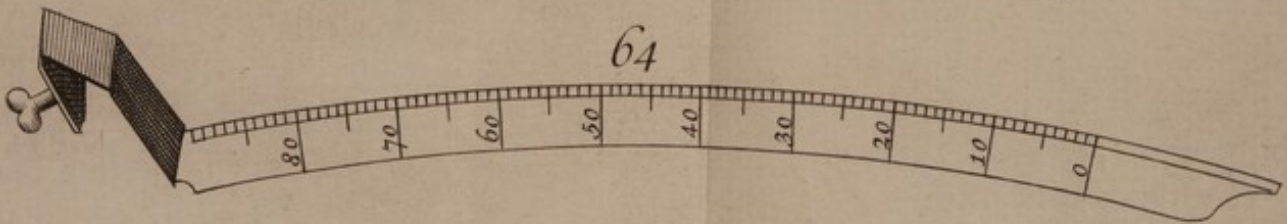
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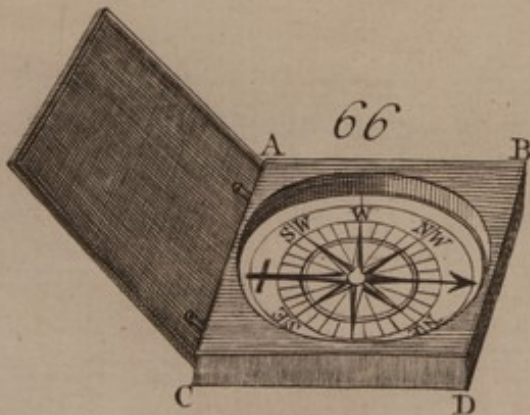
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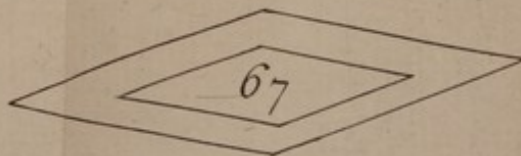
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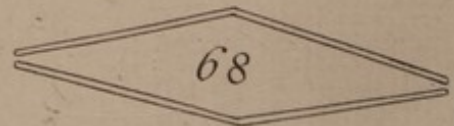
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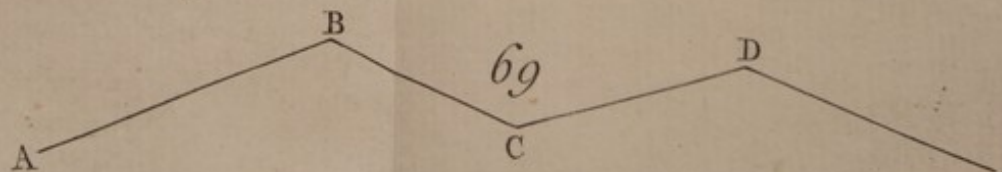
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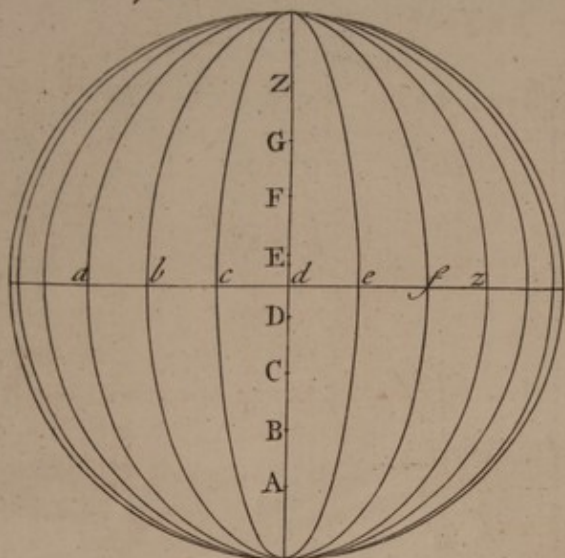
481 The following table shews the different depression of the horizon of the sea at different heights of the eye, as it comes out, calculated from the dimension of the earth, by the method taught § 294; and as it was found by observation, with the difference between these two, which is owing to refraction: it is taken from *Cassini*^a, only I have turned his French into English feet.

The height of the eye above the surface of the sea.		The depression of the horizon of the sea.	
Feet	Inches	" "	
1157	6, 9	$\left\{ \begin{array}{l} 32 \ 30 \\ 36 \ 18 \end{array} \right.$	$\left\{ \begin{array}{l} \text{by observation} \\ \text{by calculation} \end{array} \right.$
Difference by refraction		$\begin{array}{r} 3 \ 48 \end{array}$	
775	2, 3	$\left\{ \begin{array}{l} 27 \ 0 \\ 29 \ 36 \end{array} \right.$	$\left\{ \begin{array}{l} \text{by observation} \\ \text{by calculation} \end{array} \right.$
Difference by refraction		$\begin{array}{r} 2 \ 36 \end{array}$	
571	11, 0	$\left\{ \begin{array}{l} 24 \ 0 \\ 25 \ 25 \end{array} \right.$	$\left\{ \begin{array}{l} \text{by observation} \\ \text{by calculation} \end{array} \right.$
Difference by refraction		$\begin{array}{r} 1 \ 25 \end{array}$	
387	3, 4	$\left\{ \begin{array}{l} 19 \ 45 \\ 20 \ 54 \end{array} \right.$	$\left\{ \begin{array}{l} \text{by observation} \\ \text{by calculation} \end{array} \right.$
Difference by refraction		$\begin{array}{r} 1 \ 9 \end{array}$	
288	4, 3	$\left\{ \begin{array}{l} 15 \ 0 \\ 17 \ 1 \end{array} \right.$	$\left\{ \begin{array}{l} \text{by observation} \\ \text{by calculation} \end{array} \right.$
Difference by refraction		$\begin{array}{r} 2 \ 1 \end{array}$	
187	0, 9	$\left\{ \begin{array}{l} 13 \ 0 \\ 14 \ 41 \end{array} \right.$	$\left\{ \begin{array}{l} \text{by observation} \\ \text{by calculation} \end{array} \right.$
Difference by refraction		$\begin{array}{r} 1 \ 41 \end{array}$	
9	7, 3	$\left\{ \begin{array}{l} 3 \ 20 \\ 3 \ 18 \end{array} \right.$	$\left\{ \begin{array}{l} \text{by observation} \\ \text{by calculation} \end{array} \right.$

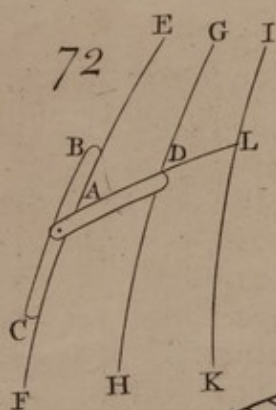
^a *Observations astronomiques faites en divers endr.* p. 18. Ed. Par.

Book I.

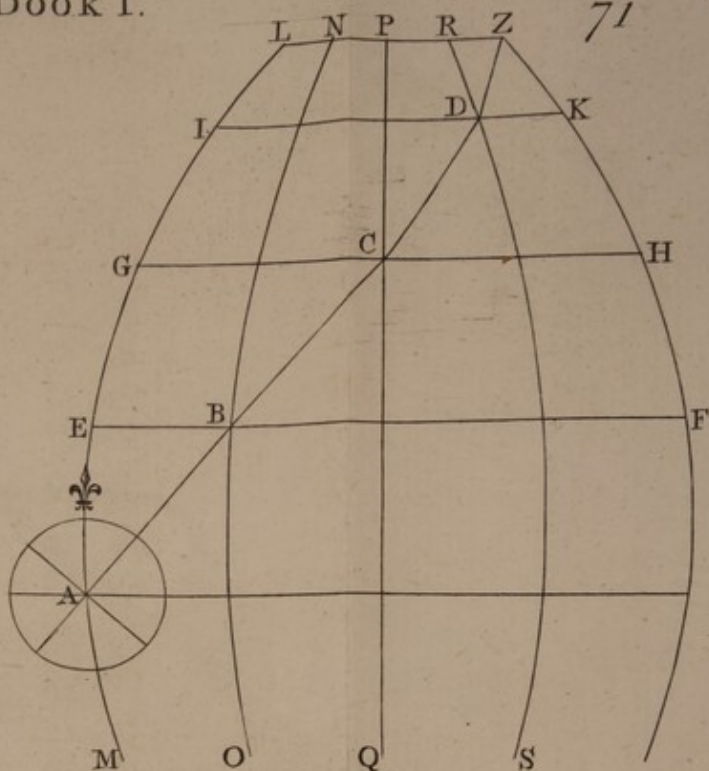
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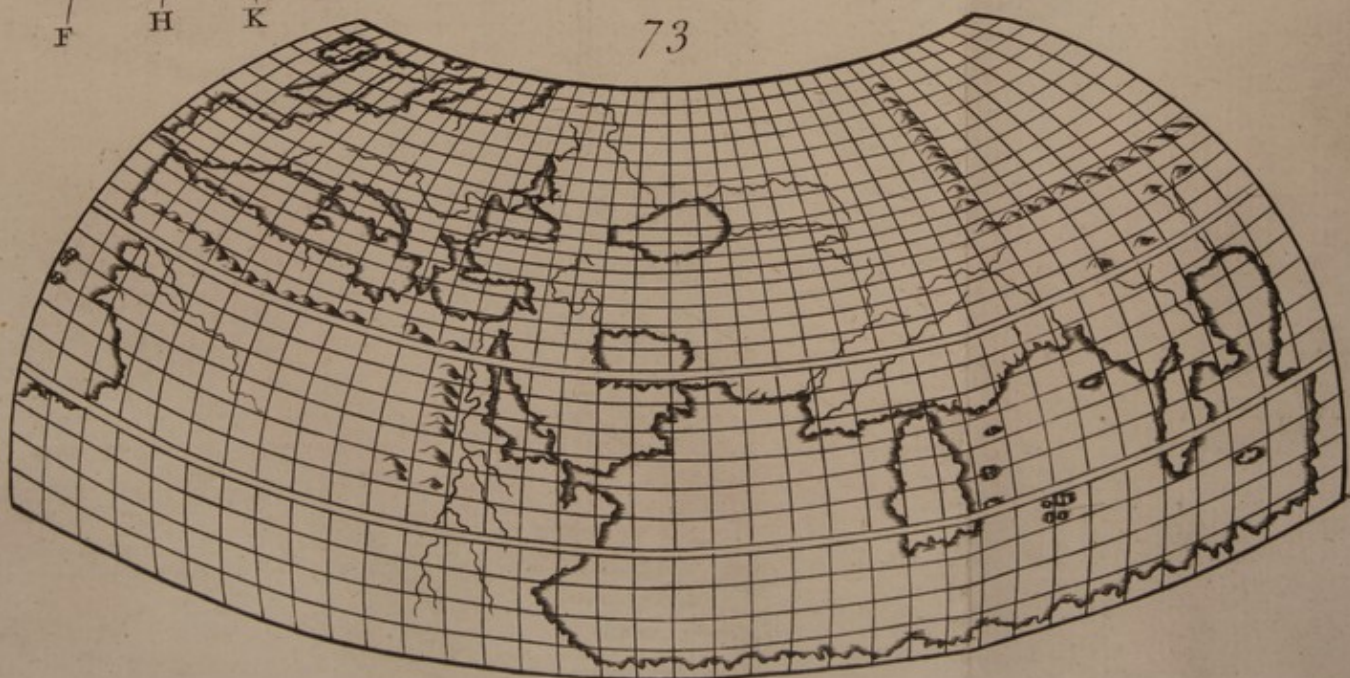
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A TABLE SHEWING THE MEASURE OF A DEGREE OF LONGITUDE
IN DIFFERENT LATITUDES.

Deg. of lat.	Engl. feet.	Geogr. miles.	Deg. of lat.	Engl. feet.	Geogr. miles.
0	365640	60.	36	295809	48. 54
1	365584	59. 99	37	292013	47. 92
2	365417	59. 96	38	288128	47. 28
3	365139	59. 92	39	284156	46. 63
4	364749	59. 85	40	280096	45. 96
5	364249	59. 77	41	275952	45. 28
6	363637	59. 67	42	271723	44. 59
7	362914	59. 55	43	267412	43. 88
8	362082	59. 42	44	263019	43. 16
9	361138	59. 26	45	258547	42. 43
10	360085	59. 09	46	253995	41. 68
11	358922	58. 90	47	249366	40. 92
12	357650	58. 69	48	244661	40. 15
13	356269	58. 46	49	239881	39. 36
14	354779	58. 22	50	235029	38. 57
15	353181	57. 96	51	230105	37. 76
16	351476	57. 68	52	225110	36. 94
17	349663	57. 38	53	220048	36. 11
18	347744	57. 06	54	214918	35. 27
19	345719	56. 73	55	209722	34. 41
20	343589	56. 38	56	204463	33. 55
21	341354	56. 01	57	199142	32. 68
22	339016	55. 63	58	193760	31. 80
23	336573	55. 23	59	188319	30. 90
24	334029	54. 81	60	182820	30. 00
25	331382	54. 38	61	177266	29. 09
26	328635	53. 93	62	171658	28. 17
27	325788	53. 46	63	165997	27. 24
28	322841	52. 98	64	160286	26. 30
29	319796	52. 48	65	154526	25. 36
30	316654	51. 96	66	148719	24. 40
31	313415	51. 43	67	142867	23. 44
32	310080	50. 88	68	136971	22. 48
33	306652	50. 32	69	131034	21. 50
34	303129	49. 74	70	125056	20. 52
35	299515	49. 15			

FIG. The measure of a degree in any parallel is found by the following proportion: as the radius is to the cosine of the latitude of any parallel, so is the number of feet, yards, miles &c. in a degree of a great circle, to the number of
 60 feet, yards, miles &c. in a degree of that parallel. *Demonstration*, let $EPQH$ be a meridian drawn upon the earth, EQ the equator, FB a parallel, suppose of 40° : I say the circumference of the equator is to the circumference of the parallel FB , and consequently a degree in the equator or in any great circle is to a degree in that parallel, as CB the semidiameter of the equator is to AB the semidiameter of the parallel: but AB is cosine of the arc BQ which is the latitude of the parallel FB ; therefore a degree in the equator is to a degree in the parallel, as the radius is to the cosine of the latitude. Q. E. D. By this proposition the foregoing table is calculated, which shews the measure of a degree of longitude in the several parallels drawn through every degree of latitude, from the equator to the latitude of 70° . In the first column of the table is the degree of latitude, the second column shews the measure of a degree in english feet, the third column shews it in geographical miles and decimal parts: a *geographical mile* is the sixtieth part of a degree of a great circle upon the earth.

CHAP. 17. THE USE OF THE TERRESTRIAL GLOBE.

485 *The terrestrial globe* is an image of the earth in little, which exhibits to our view the surface of the earth and the parts thereof, as seas, continents, islands, lakes, rivers, countries, kingdoms and states, &c. in their proper di-
 61 mention, shape and situation. See fig. 61.

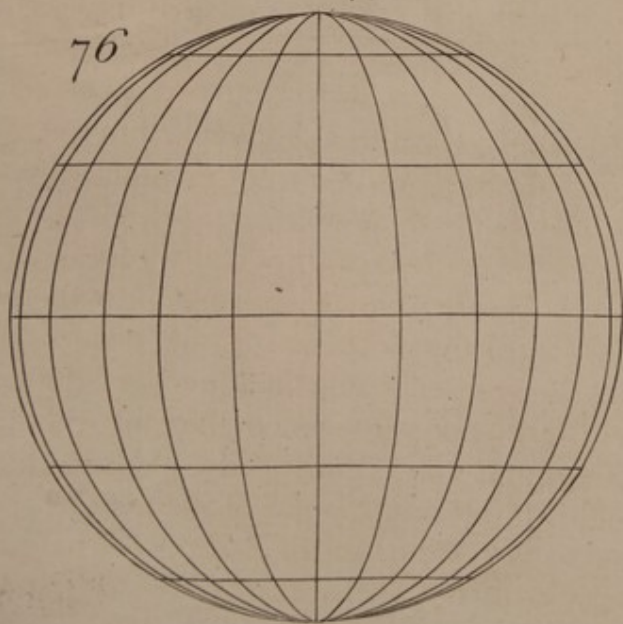
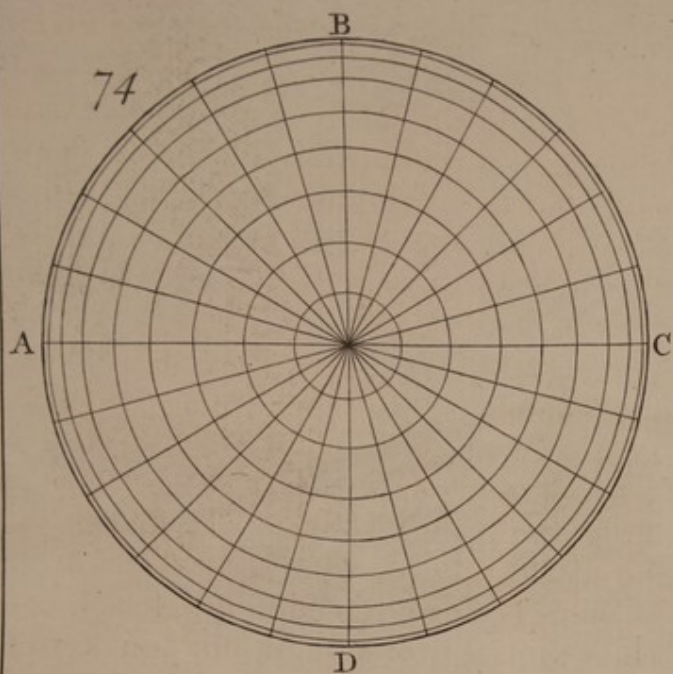
486 It hath already been observed, that we may imagine what circles we please to be drawn upon the surface of the earth: some of these imaginary circles, as the equator with some of its parallels and secondaries, are actually drawn upon the globe; others, as circles of distance, or position, the horizons and meridians of different places, may be represented by the apparatus or furniture of the globe.

487 *The equator*, described § 298, is drawn with double and sometimes with treble lines, and is divided into 360 degrees of longitude, beginning at the first meridian: in large globes, each degree is subdivided into halves and quarters of a degree: the equator shews at once whether any given place upon the globe is in north or south latitude. § 326.

488 *Secondaries of the equator* or *meridians* described § 299, are, upon small globes, usually twelve in number, so that there are twenty four geographical



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graphical meridians, described § 300, drawn at equal distances, one through every fifteenth degree of the equator; *the first meridian*, described § 309, is distinguished from the rest by double lines. These meridians give a general view of the longitude of places, but shew it accurately in those places only through which they are drawn.

489 *Terrestrial parallels*, described § 335, are usually drawn on each side of the equator, through every tenth degree of the first meridian: they give a general view of the latitude of places, but shew it accurately in those places only through which they are drawn. In large globes, both meridians and parallels may be more numerous. The circle which cuts the equator obliquely will be considered hereafter.

490 *The tropics*, described § 390, and the *polar circles*, described § 375, are drawn with double lines, to distinguish them from the other parallels. *The zones*, described § 401, are easily seen upon the globe. *The furniture of the globe* is now to be considered.

491 *The axis of the globe* is a strait wire passing through the globe at the poles, and represents the axis of the earth, described §. 298.

492 *The meridian of the globe* is the brass circle in which the globe is suspended by the axis: It hath one side graduated, or divided into 360 degrees, the innermost edge of this graduated side is properly the meridian, described § 299, and divides the globe into the east and west hemispheres.

493 The meridian consists of two semicircles; one of them ABC, which is commonly made use of to express *the geographical meridian*, described § 300, is divided into two quadrants, the degrees whereof are numbered 10, 20, 30, &c. to 90; beginning at the equator, and proceeding towards each of the poles, in order to shew the latitude of places: the other semicircle ADC which represents *the opposite meridian*, described § 300, is also divided into two quadrants, the numbers thereon 10, 20, 30, &c. proceed from each pole towards the equator, in order to shew the elevation of the pole.

494 The geographical meridian is generally meant; when we speak of the meridian in the use of the globes. *The zenith point of the meridian* is the point directly over the center of the horizon of the globe, as L is, fig. 61.

495 *The horizon of the globe* is the wooden circle on the top of the frame which supports the meridian, with the globe suspended therein: the upper side of this circle is properly the horizon, and represents the plane of the horizon of any place which is brought to the zenith point of the meridian; and when this is done, the globe is said to be rectified to the horizon of that place: thus, fig. 61, London stands exactly over the center of the horizon, and the globe

FIG. globe is rectified to the horizon of London. How to rectify the globe to the horizon of any place will be shewn §. 509.

The horizon hath two notches for the meridian to slide in, when different elevations of the pole are to be exhibited: there is also, on the bottom of
61 the frame, a little piece of wood marked H, with a notch in it cut in such a manner, that the meridian, sliding therein, may always intersect the horizon at right angles, and be supported at a proper height, so that exactly half the globe and half the meridian may be always above the horizon. In using the globe, the graduated side of the meridian is to be turned towards the east side of the horizon, as in fig. 61.

496 The horizon of the globe hath several circles upon it, among which I shall at present only take notice of the circle of the winds, leaving the rest to a more proper place. *The circle of the winds* hath the 4 cardinal points, described §. 299 and 304, as also the intermediate points, in all 32, with their names, or the initial letters of their names, these are called *the points of the compass*: it is also divided into 360 degrees, in order to shew lesser deviations from the cardinal points. The circle of the winds may be drawn upon paper, a card,
62 or any horizontal plane, and is then usually called *the compass*: fig. 62.

497 *To set or rectify the compass*: if *the meridian line of a compass*, that is the line SN which is drawn through the north and south points, be placed exactly over, or parallel to a meridian line of the place where we are, its points will then shew us the respective points in the horizon of that place; its north point will stand towards the north, its east point towards the east, and so of the rest.

61 498 *The hour circle* is a small circle of brass, FG, so placed upon the meridian as to have one of the poles in its center, and that usually the north pole, because the northern parts of the earth fall most frequently under our consideration. It is divided into twice 12 or 24 equal parts, to answer the several hours of the natural day, the two figures of 12 are to be placed exactly
61 over the meridian, the 12 on the south side at F expresses 12 at noon, the other 12 at G stands for 12 at night. The numbers on the west side shew the hours from noon to midnight, the numbers on the east side shew the hours
63 from midnight to noon. See fig. 63.

499 *The hour index* is a small brass pointer, to be put upon that end of the axis which passes through the center of the hour circle: if the globe be turned round in the meridian, the axis will carry the hour index round, so as to make it point successively to all the divisions of the hour circle: however, the index is moveable upon the axis, so that we may hold the globe still and turn the index till it points at what hour we please.

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500 *The quadrant of altitude* is a thin narrow slip of brass bent into a quarter of a circle, fig. 64, with one edge divided into 90 degrees, it hath at one end, which may be called *the center of the quadrant*, a screw, to fasten it to the zenith point of the meridian, the other end is at liberty, and may be carried round the horizon.

501 *To find the distance between two places* upon the globe: lay the quadrant of altitude upon the globe, so that the graduated edge may pass through both the places, and observe how many degrees and minutes are between them: or thus, with a pair of compasses, set one foot upon one place, and open the compasses till the other foot stands upon the other place, then, carry this distance to the equator, and, setting one foot of the compasses upon the first meridian, observe how many degrees and minutes it is, which you may turn into english miles, by §. 457.

502 *To find all places that are at a given distance from a place given:* for example, to find all places that are ten degrees from London: open a pair of compasses till they measure ten degrees upon the equator, then, setting one foot upon London, turn the other round, and it will pass through all the places that are ten degrees from London. By the same method, we may find all places that are at an equal distance from any given place: thus, setting one foot of a pair of compasses upon London, and turning the other round through Rome, we may see all places that are at the same distance from London that Rome is; as also what places are nearer to, or further from London than Rome is.

503 *To find the latitude of any place* upon the globe: bring the place to the meridian, and the degree and minute under which it lies shews its latitude: thus, bring London to the meridian, as in fig. 61, and it appears to be in $51^{\circ} 31'$ north latitude. If a place lying under the equator be brought to the meridian, the mark over it will be 00, because it has no latitude, § 326.

504 *To find all places which have the same latitude with a given place:* suppose the given place be London, turn the globe round, and all places that pass under the same point of the meridian that London doth, have the same latitude with London. *To find the difference of latitude* of two given places upon the globe, as London and Paris: find the latitude of each place, by § 503, and the difference is easily known.

505 *To find the longitude of any place* upon the globe: bring the place to the meridian, and the degree and minute of the equator the meridian then passes through is the longitude: thus, if Rome be brought to the meridian, its longitude will appear to be $12^{\circ} 45'$, upon our english globes, which have the first meridian drawn through London.

FIG. 506 *To find all places which have the same longitude with a given place: as Naples, it is sufficient to bring Naples to the meridian; for all places then under the meridian have the same longitude as Naples: § 307. To find the difference of longitude of two places: find the longitude of each place by § 505 and the difference is easily known.*

507 *To find a place upon the globe, its longitude and latitude being given: let the place be Aracta, the longitude of which I find, in the table §. 331, to be $44^{\circ} 55'$ east from London, and latitude $36^{\circ} 6'$ north: bring $44^{\circ} 55'$ of the equator to the meridian, and under the 36° of N. latitude is Aracta, or the place where it ought to be: for this method will serve to insert a place upon the globe, its longitude and latitude being given.*

508 *To rectify the globe to the latitude of any place: let the place be London, which, by the table §. 331, is in $51^{\circ} 31'$ N. latitude: move the brass meridian in the notches till the north pole of the globe is elevated $51^{\circ} 31'$ above the north side of the horizon, that is, till the elevation of the pole is equal*

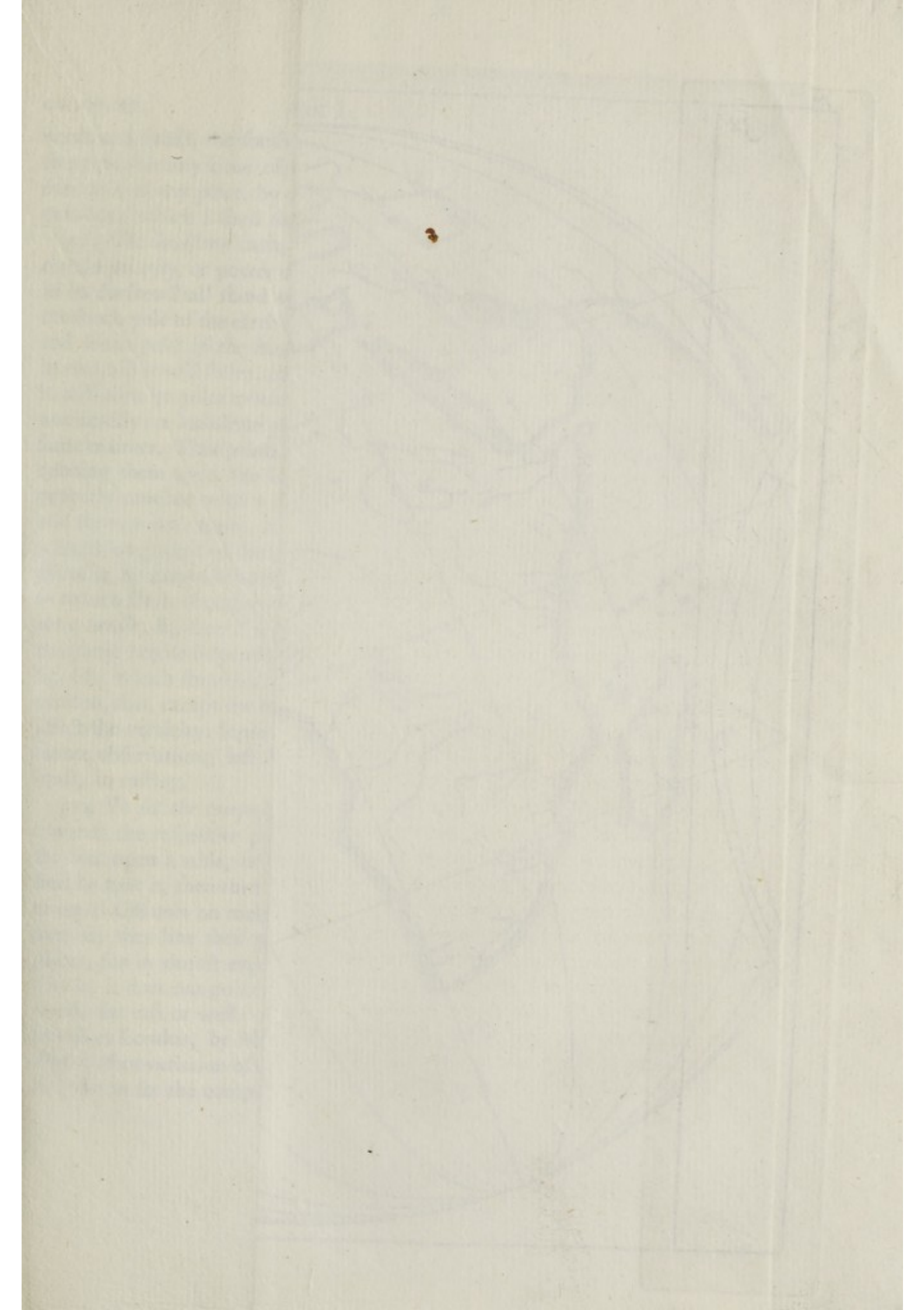
61 to the latitude: as in fig. 61.

509 *To rectify the globe to the horizon of any place, as London: rectify the globe to the latitude of London, by §. 508, and bring London to the meridian; the horizon of the globe will then represent the horizon of London.*

510 *To find towards what point of the compass any place lyes from a given place: if it be enquired towards what point of the compass Lisbon lyes from London, rectify the globe to the horizon of London, by §. 509, screw the center of the quadrant of altitude to the zenith point of the meridian, and turn the other end round till the graduated edge passes through Lisbon; then observe what point of the horizon in the circle of the winds the graduated edge passes through, which in the present case will be S. W. by S, we say then Lisbon lyes South West by South from London.*

511 *To find the angle of position between any place and a given place: if it be enquired what is the angle of position of Lisbon and London, that is, what angle a great circle drawn through these two places makes with the meridian of London, do every thing directed by § 510, and observe how many degrees of the horizon are contained between the graduated edge of the quadrant of altitude and the meridian; which in the present case will be $33^{\circ} 45'$: the angle of position of Lisbon with regard to London is then $33^{\circ} 45'$.*

512 *To rectify the globe to the meridian of the place where we are, which is the same thing as to rectify the globe to the situation the earth is in every day when it is noon in that place: suppose the place is London, rectify the globe to the horizon of London, by § 509, then turn the frame of the globe about till the north and south points of the horizon of the globe are towards the*
north



79



north and south: the south may be known by the sun at noon, the north, by FIG. the pole star any time of the night: the globe may also be rectified to the meridian of any place, by a meridian line, or by a magnetic needle in a compass-box, which I shall therefore now describe.

513 *The loadstone* hath, besides its power of attracting iron and steel, a certain *polarity*, or power of turning it self, in such a manner, that one point in its surface shall stand towards the north, and the opposite point towards the south pole of the earth; for which reason, these points are called the north and south *poles of the loadstone*: this may be seen, by inclosing a loadstone in cork till it will swim, and putting it into water, where, if rightly balanced, it will turn its poles towards the respective poles of the earth, nearly though not exactly; a loadstone properly suspended by a thread will also turn in the same manner. This polarity or *verticity* is communicated to iron or steel, by rubbing them upon the loadstone, so that if a needle or strait piece of wire properly touched with a loadstone be thrust through a round piece of cork, and thrown into water, it will, as it swims, turn its ends north and south: a small long piece of steel thus touched, with a little brass socket in the middle of it, by means whereof it may be suspended in an horizontal position, so as to turn freely about, upon a sharp pointed pin set perpendicular, is *the magnetic needle*, fig. 65: if a compass be drawn at the bottom of a box, and the 65 magnetic needle suspended over its center, it makes *the common compass-box*, fig. 66; which should be made of brass, wood, glass, marble, &c. with this 66 caution, that, except the needle, there be no iron or steel about it, lest it should affect the verticity: some have thought that a brass box is not proper for accurate observations, lest filings of iron should chance to be mixed with the brass, in casting.

514 *To set the compass-box*, so that the points of the compass shall stand towards the respective points of the horizon of the place where we are: set the box upon a table, or any level plane, with this caution, that no iron or steel be near it, then turn the box gently about, till the needle, after vibrating to equal distances on each side the meridian line of the compass, rests exactly over it; that line then points north and south. This is true only in a few places, for in almost every part of the earth, there is a *variation of the needle*; that is, it does not point exactly north, but declines from the meridian, towards the east or west: thus, this present year 1736, the variation was observed at London, by Mr. *Graham*, as he informs me, to be $15^{\circ}.15'$, west. The present variation of the needle in the place where we are must be known, in order to set the compass-box exactly.

FIG. 515 The variation of the needle is not only different in different parts of the earth, so as to be east in some, and west in others, at the same time, and more east or west in one place than another; but there is also a *variation of the variation*, that is, the variation in the same place changes in a tract of time: thus, in the year 1576, when the variation was first discovered by *Norman* at London, it was $11^{\circ}.15'$, eastward; in the year 1665, it was $1^{\circ}.22'.30''$, westward; in the year 1666, $1^{\circ}.35'.36''$, west: in the year 1722, $14^{\circ}.17'$, west: in the year 1736, $15^{\circ}.15'$, west. See *Whiston's Historical Preface* to his treatise of the dipping needle, p. 7. See also *Phil. Transact.* N. 40.

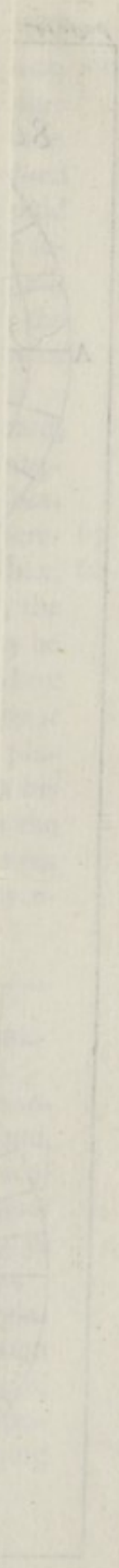
516 There is also a *very small variation of the variation* amounting only to a few minutes of a degree, in the same place, at different hours of the same day, but this is discoverable only by very accurate observations, no notice can be taken of it at sea, and the setting the globe doth not require such exactness; I only mention it here, because I am speaking of the variation. By *Mr. Graham's* observations made in London from Feb. the 6th, 1722, to May the 10, 1723, the greatest variation was $14^{\circ}.45'$, west; the least $13^{\circ}.50'$, west: See *Phil. Transact.* N. 383. It has also been observed, that different needles, especially if touched with different loadstones, will differ a few minutes in their variation. *V. Poleni epist. Phil. Transact.* N. 421.

517 To find the variation: set the meridian line of the compass-box over a meridian line, or, if the compass-box be square, lay one of the sides to which
66 the meridian line of the compass is parallel, as AB or CD, upon a meridian line, and the needle will shew the variation.

518 Some large globes have a compass-box upon the pedestal, as 1 K, fig.
61 61, if the compass-box be set by § 514, the points of the horizon of the globe will be directed towards their respective points in the horizon of the place where the globe is, and the globe is then rectified to the meridian of the place.

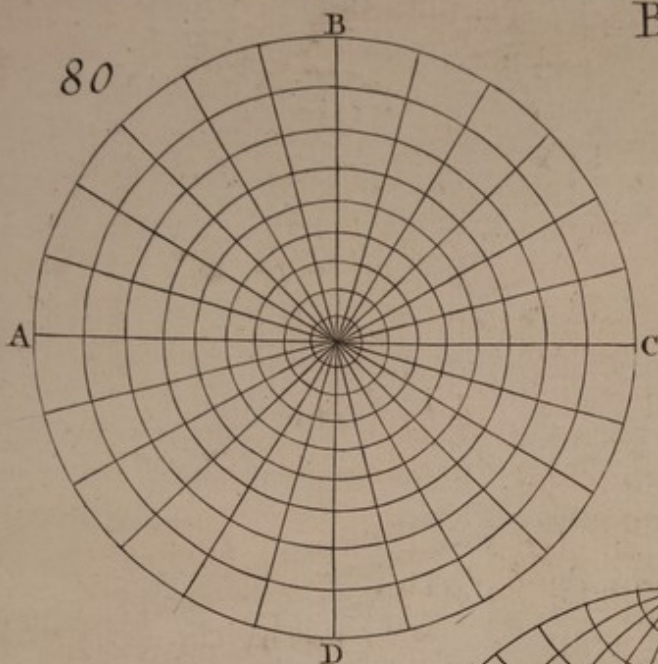
519 To rectify the globe to the present situation of the earth, at any hour of the day: if you be in London, and would bring the globe into the situation the earth is in at four in the afternoon: rectify the globe to the meridian of London, by §. 512: since the rotation of the earth carries the meridian and hour-circles of London upon the globe coinciding with the like circles upon the earth, the situation of the globe will then correspond to that of the earth, and if it stands in the sun, it will be illuminated as the earth is.

520 The hour of the day being given at one place, as London, to find what hour it is at any other place upon the globe, as Naples: rectify the globe to the horizon of London, by § 509, set the hour index at 12 at noon, turn the globe round till Naples is at the meridian, the index will then shew what hour it is at London, when it is noon at Naples: thus, if it points at 11 in
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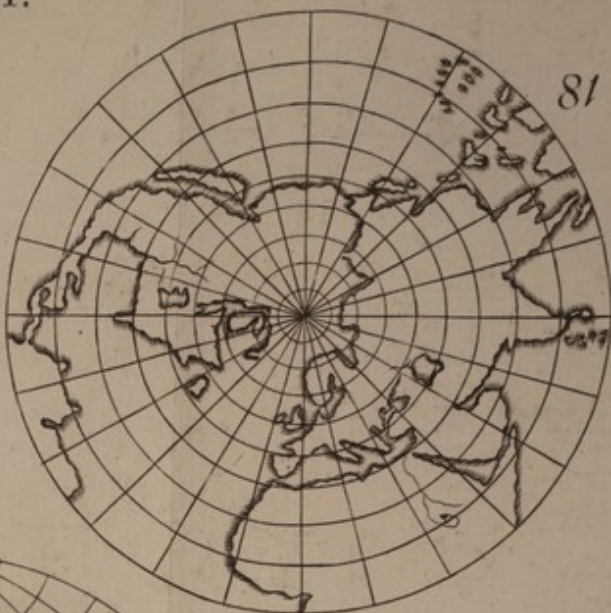


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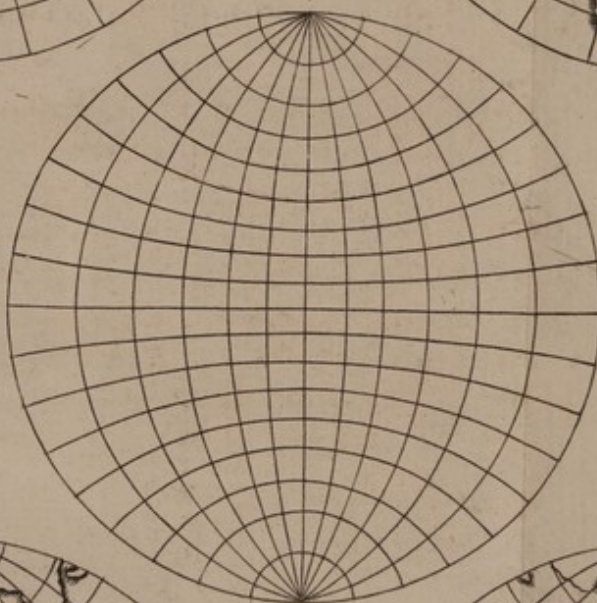
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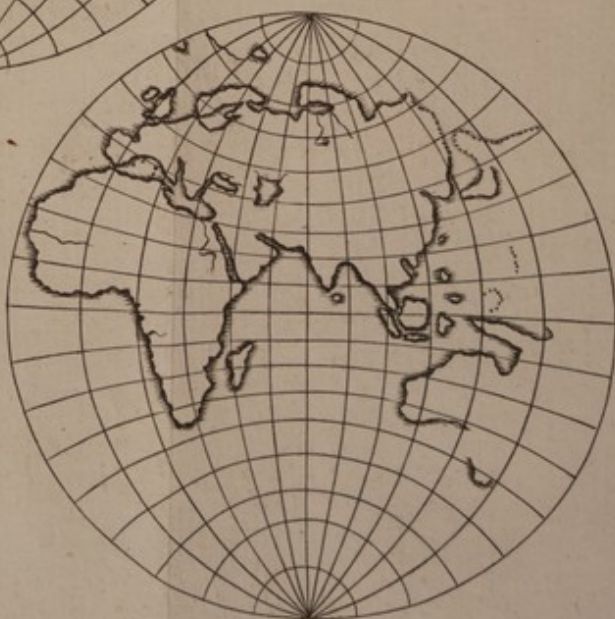
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the morning, it shews that it is 11 in the morning at London, when it is noon at Naples; and consequently, whatever is the time given at London, the time of the day at Naples is an hour forwarder in the day; as if it be 2 in the afternoon at London, it is 3 in the afternoon at Naples &c: this problem may be resolved more accurately without the globe, by § 333, and the table § 331; these would shew us that the time of the day at Naples is fifty nine minutes and one second forwarder than at London: it may be proper to observe once for all, that though globes serve very well to help the imagination, in conceiving the nature of problems to be solved, yet they cannot come up to the accuracy of calculation, in the solutions they help us to.

521 In the *seaman's compass*, the circle of the winds is drawn upon a card, on the under side of which, under the meridian line of the compass, the magnetic needle is fixt, which is then a thin plate of steel of the shape represented fig. 67, or only two wires, fig. 68, with a small brass socket, by means where- of the card is suspended horizontally upon a sharp pin set upright in a box, and directed by the verticity of the needle, so that, by looking thereon, the points of the compass in the horizon of the place where the ship is may be known, allowance being made for the variation: the seaman's compass-box hath a weight at the bottom of it, and is hung in two brass circles, to keep it in an horizontal position, notwithstanding the motion of the ship; it is placed in the forepart of the ship, where the pilot sits, and by looking upon his compass is able to know, every moment, the points of the compass in the horizon of the place where the ship is; and consequently can steer her towards what point he pleases. How to find the variation at sea shall be shewn hereafter.

CHAP. 18. NAVIGATION EXPLAINED BY THE GLOBE: THE RHUMBS.

522 *Navigation* is the art of guiding a ship at sea from one place to another, in the safest and most convenient manner: in order to attain this end, four things are especially necessary; 1 to know the situation and distance of the places: 2 to know, at all times, the points of the compass: 3 to know the line in which the ship is to be directed from one place to the other: 4 to know, in any part of the voyage, what point of the globe the ship is upon.

523 For the first, the knowledge of the distance and situation of the places between which a voyage is to be made implies, besides a general notion of geography, our being acquainted in particular with the islands, rocks, sands, straits, rivers, &c. near which we are to sail; the bending in, or run-

FIG. ning out of the shores, the signs of being near land, which are, in some places, the appearance of some sorts of birds, the floating of weeds upon the surface of the sea, and very near land, the depth and colour of the water: moreover, the knowledge of the time of the winds setting in, particularly of the trade-winds or monsoons: the seasons when storms and hurricanes are to be expected, and the signs of their approach: the motion of currents, but especially of the tides: all which is to be learned, by the use of good sea-charts and journals of voyages, but chiefly by observation and experience.

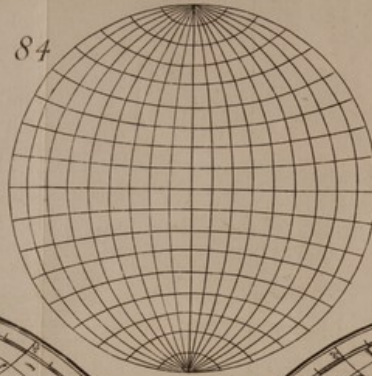
524 The second thing required in navigation is, to know at all times the points of the compass in the horizon of the place where the ship is: the ancients, to whom the polarity of the loadstone was unknown, as well as its power to communicate that property to iron and steel, used in the night to find the north, by the pole star; in the day time they found the east or west, by the rising or setting of the sun; the method of doing this shall be shewn hereafter. Another way they had of knowing the points of the compass was, by a view of the land: for their maps shewed the bearings of one head-land or cape from another: for this reason, the ancients could not venture far from shore, where they could have nothing to direct them in bad weather, when neither sun nor stars were to be seen. The moderns, besides the methods used by the ancients, have the advantage of the seaman's compass, described in the foregoing chapter; by means whereof, they can, at any time, in the wide ocean, and in the darkest night, know where the north is, and consequently, the rest of the points of the compass.

525 The third thing required to be known in navigation is, the line which a ship describes upon the globe of the earth, in going from one place to another: this will be treated of in the remaining part of this chapter.

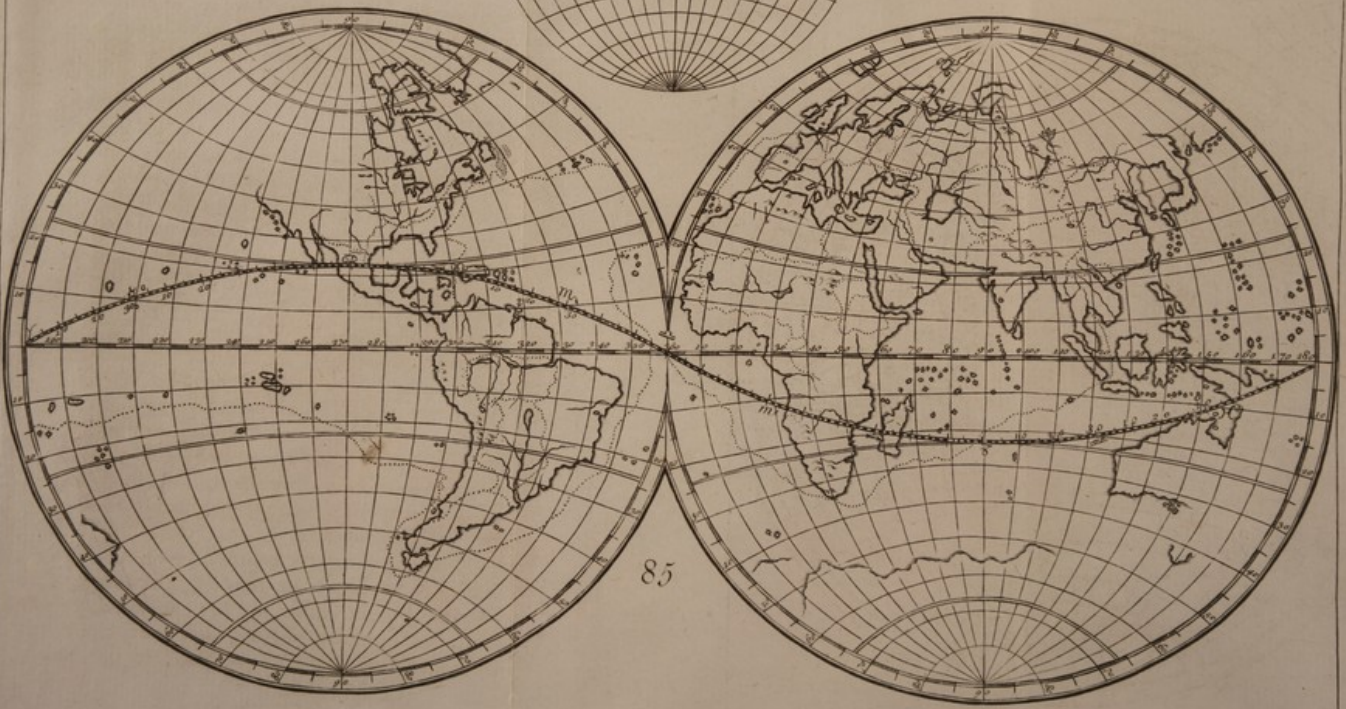
526 *The shortest way* from one place upon the earth to another is an arc of a great circle drawn through the two places: *The most convenient way for a ship* is that by which we may sail from one place to another, directing the ship all the while towards the same point of the compass: a ship is guided by *steering her*, or directing her progress towards some point of the compass: the line wherein a ship is directed is called *the ships course*, which is named from the point towards which she sails: thus, if she sails towards the N. E. point, her course is said to be north east: in long voyages, a ships way may consist of several different courses; as from A to B, from B to C, from C to D, fig. 69: when we speak of a ships course, we consider one of these at a time: navigation would be exceedingly perplexing, if the course were every moment to be changed; the seldomer it is changed, the more easily is a ship guided: and therefore that course is the most convenient for sailing, wherein the



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the ship is directed, all the while, towards the same point of the compass. FIG.

527 *If two places A and z lye under the same meridian*, the course from one to the other is due north or south: thus, fig. 70, let Az be part of a meridian, if A lyes south from z, the course from A to z is north; and the course from z to A is south: the truth of this is evident from the nature of the meridian, that it marks upon the horizon the north and south points, and that every point of any meridian is north or south from every other point of it: thus, fig. 70, if A be south from z, it is south from all the intermediate points, as B, C, D, E, &c. as may be seen upon the globe, by bringing any of those points to the zenith point of the meridian; and consequently, if we are to go from z to A, when we are come to any of those points, we must continue the course south, to arrive at A: again, if z be north from A, it is north from all the intermediate points, and therefore, if we are to go from A to z, when we are come to any of those points, we must continue our course north, in order to arrive at z. From this proposition, we may, by way of converse, deduce the following corollary; *if a ship sails due north or south*, she will continue in the same meridian.

528 *If two places lye under the equator*, the course from one to the other is an arc of the equator, and is due east or west: thus, fig. 70, let az be part of the equator, if a be west from z, the course from a to z is east; and the course from z to a is west: for, since the equator marks the east and west points upon the horizon, every point of the equator lyes east or west from every other point of it: thus, fig. 70, if a be west from z, it is also west from every intermediate point, b, c, d, e, &c; as may be seen upon the globe, by placing it as for a right sphere, by § 346, and bringing a or z or any of the intermediate points to the zenith of the meridian, and consequently, if we are to go from z to a, when we are arrived at any of those points, we must continue our course due west, in order to arrive at a: again, if z be east from a, it is east likewise from every intermediate point; and therefore, if we are to go from a to z, when we are come to any of those points, we must continue our course due east, in order to arrive at z. From hence, by way of converse, we have the following corollary; *if a ship under the equator sails east or west*, she will continue under the equator. In this case, and the foregoing, § 527, the course, being an arc of a great circle, viz. the meridian or equator, is the shortest way, as well as the most convenient.

529 *If two places lye under the same parallel*, the course from one to the other is due east or west: this may be seen upon the globe, by the following method, bring any point of a parallel to the zenith, and stretch a thread over it perpendicular to the meridian; the thread will then be a tangent to the parallel, and stand east and west from the point of contact: therefore, the converse

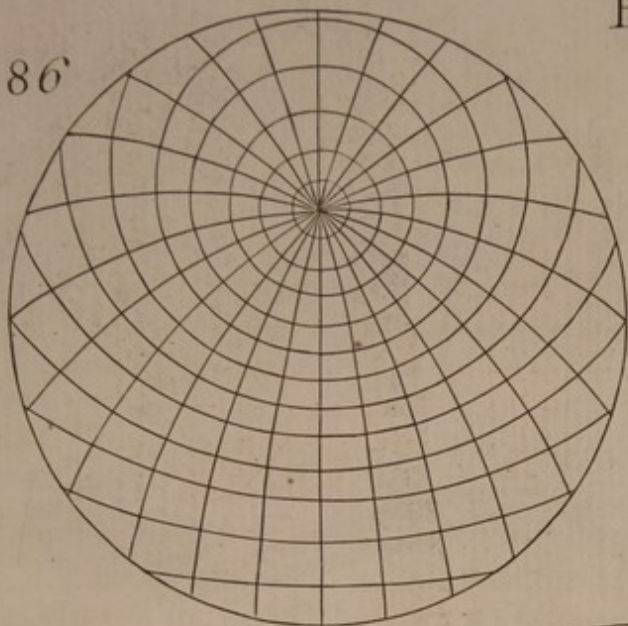
FIG. verfe is true, that if a ſhip in any point of a parallel be directed eaſt or weſt, ſhe will go in a tangent to that parallel; and, ſince ſhe muſt go upon the ſurface of the globe of the earth, ſhe will be always in that point of the tangent which touches the parallel: that is ſhe will always be in the parallel. Hence this corollary, *If a ſhip in any parallel ſails due eaſt or weſt, ſhe will continue in the ſame parallel.* In this caſe, the moſt convenient courſe, though not the ſhorteſt, from one to the other is to ſail due eaſt or weſt.

530 *If two places A and z lye neither under the equator, nor in the ſame meridian, nor in the ſame parallel,* the moſt convenient, though not the ſhorteſt, 71 courſe from one to the other is in a rhumb, fig. 71: If we ſhould in this caſe attempt to go the ſhorteſt way, in a great circle drawn through the two places, we muſt be perpetually changing our courſe: for, whatever the bearing of z is from A, the bearings from all the intermediate points, as B, C, D, E, &c. will be different from it, as well as different from each other; as may eaſily be ſeen upon the globe, by bringing firſt the point A, and then the points B, C, D, E, &c. ſucceſſively to the zenith of the meridian, and obſerving the bearing of z from each of them: thus, ſuppoſe, when the globe is rectified to the horizon of A, the bearing of z from A be north eaſt, and the angle of poſition of z with regard to A 45° ; if we bring B to the zenith of the meridian, we ſhall have a different horizon, and the bearing and angle of poſition of z from B will be different from the former: and if we go on, to bring the other points, C, D, E, &c. to the zenith of the meridian, we ſhall for each of them have a different horizon, and z will have a different bearing and angle of poſition: from hence we may draw this *corollary*, when two places lye one from the other towards a point not cardinal, if we ſail 71 from one place towards the point of the others bearing, we ſhall never arrive at the other place: thus, if z lyes north eaſt from A, if we ſail from A towards the north eaſt, we ſhall never arrive at z.

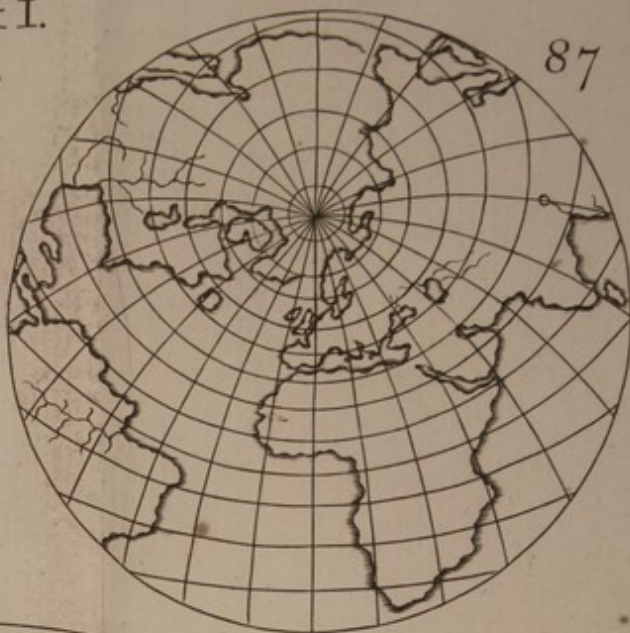
71 531 *A rhumb* upon the globe is a line drawn from a given place A, fig. 71 ſo as to cut all the meridians it paſſes through at equal angles: the rhumbs are denominated from the points of the compaſs, in a different manner from the winds: thus, at ſea, the north eaſt wind is that which blows from the north eaſt point of the horizon, towards the ſhip in which we are; but we are ſaid to ſail upon the N. E. rhumb when we go towards the north eaſt: ſometimes the lines drawn towards the 4 cardinal points are called whole rhumbs, and the middle points between them half rhumbs, and theſe are again divided into quarter rhumbs: ſometimes they divide the 32 points into 4 quarters, and call the rhumb next the eaſt the firſt rhumb, the next to that the ſecond rhumb, &c. Every rhumb bears a certain relation to the meridians through

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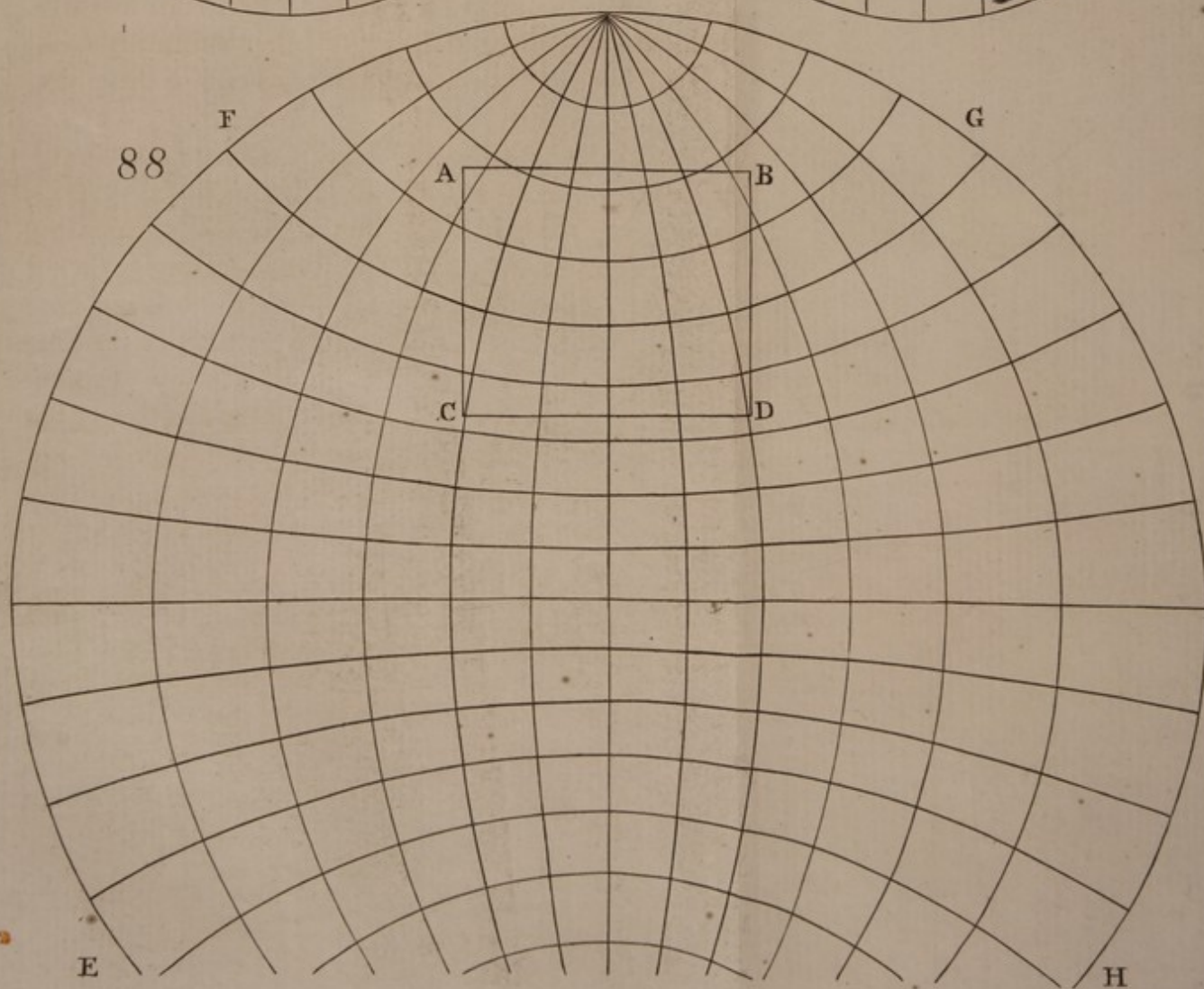
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through which it is drawn, thus, the north rhumb is said to make an angle infinitely small with the meridian, that is, it is coincident with it. the east rhumb makes a right angle, the half rhumbs make half right angles with all the meridians they pass through &c. it is from this relation that the definition of a rhumb is taken, and from hence it is said to be one essential property of a rhumb that it makes equal angles with all the meridians it passes through, thus, fig. 71, the rhumb $ABCDZ$ passing through the meridians LM , NO , PQ , makes the angles LAB , NBC , PCD , equal, from whence it follows, that the direction of a rhumb is, in every part of it, towards the same point of the compass: thus, from every point of a north east rhumb upon the globe the direction is towards the north east, and that rhumb makes an angle of 45° with every meridian it is drawn through: we have, on large globes, the compass, with rhumb lines proceeding from the center of it, drawn here and there upon the sea: if the center of a compass be in the equator, the four whole rhumbs are arcs of great circles, namely the meridian and equator: if the center of a compass be out of the equator, the north and south rhumbs only, are arcs of a great circle, namely the meridian: every other rhumb is a sort of *belix* or spiral line, which continually approaches nearer to one of the poles, but never can come to it.

532 Another property of the rhumbs is, that equal parts of the same rhumb are contained between parallels of equal difference of latitude; so that a ship, continuing in the same rhumb, will run the same number of miles, in sailing from the parallel of 10° to the parallel of 30° , as she does in sailing from the parallel of 30° to that of 50° , or from the parallel of 50° to that of 70° , &c: thus, fig. 71, let LM , NO , PQ , RS be meridians, EF the parallel of the latitude of 10° , GH the parallel of 30° , IK the parallel of 50° , if we draw through them the rhumb $ABCD$, the lines AB , BC , and CD will be equal, upon the globe: this rule is true, as far as the rhumbs need be considered in navigation, but not of any use near the poles, because the rhumbs never reach the poles^a.

533 A rhumb may be drawn upon a globe by two rulers bent according to the curvature of the globe, if one of them AD be joyned at one end A to the middle of the other fixt ruler CB , so as to be moveable to any angle; the following example shews the method of doing it: if it be required to draw from the point A the north east rhumb, move the ruler AD till the angle BAD is 45° , the angle that rhumb makes with the meridian, then apply the edge of the ruler BC to the meridian EF , with the vertex of the angle at A , which is to be the center of the compass, the line AD drawn by the edge of the move-

^a Metius in *doctrina spherica* l. 5. c. 4. Vide etiam *de rhumbis*, Varen. *geogr.* l. 3. c. 39.
Ricciol. *geogr. reformat.* lib. 10. c. 21.

FIG. able ruler is the rhumb required from A to D; which, by applying the edge of the first ruler to the next meridian GH, with the vertex of the same angle A at D, may be continued from D to L in the next meridian IK, &c: a curve drawn through these points A, D, L, &c. will be the rhumb required. The more numerous the meridians are, the more exact will the rhumb-lines be: if a compass with rhumbs be thus drawn upon a globe, with the center of the compass upon a given point A, every rhumb will shew towards what point of the compass we must sail, to go from A to any other place, as B, C, D, &c. through which that rhumb passes: thus, fig. 71, if the rhumb ABCD be north east, we must sail north east, to go from A to B, or to C, or D.

The fourth thing mentioned to be required in navigation was, to know at any time, what point of the globe a ship is upon: but before we proceed to this we shall consider the nature and use of maps, because maps are used in the practice of navigation, though the theory be learned by the globe.

CHAP. 19. OF MAPS AND THEIR USE.

534 A map is a representation of some part of the surface of the earth, delineated upon a plane: the earth being round, no part of the spherical surface of it can be accurately exhibited upon a plane, and therefore some have proposed the making of *globular maps*^a; for this purpose, a plate of brass might be hammered, or, at a less expence, a piece of pasteboard might be formed into a segment of a sphere, and covered on its convex side with a map projected in the same way as the papers of the common globes are: a map made in this method would shew every thing in the same manner, as it would be seen upon a globe of the same diameter with the sphere upon the segment of which it was delineated: and indeed maps of this sort would be in effect segments of such a globe, but they are not in common use.

535 Maps are either general or particular; the ancients sometimes described all the parts of the then known earth in *one general map*^b, as fig. 73: in this view, one of them compares the shape of the earth to the leather of a sling^c, whose length exceeds its breadth: the length of the then known parts of the earth from east to west was considerably greater than from north to south; for which reason, the former of these was called the longitude, and the other the latitude^d.

536 The modern *general maps* are such as give us a view of an entire hemisphere, or half of the globe; and are projected upon the plane of some great

^a Ricciol. *geogr. reform.* l. 10. c. 28.
perieges. v. 7.

^d Ptolem. *geogr.* l. 1. c. 6.

^b Magini *geogr. vet. & nov. part.* 2.

^c Dionys.

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circle, which terminates the projected hemisphere, and divides it from the other half of the globe, as the equator, or the meridian, or horizon of some place: from this circle the *projection* is denominated, and said to be *equatoreal*, *meridional*, or *horizontal*.

537 *Particular maps* are such as exhibit to us less than an hemisphere; of this sort are maps of the great parts into which the earth is divided, as Europe, Asia, Africa, North America, South America; or maps of particular kingdoms, provinces, countries, or of lesser districts.

538 *In maps three things are chiefly required: the first* is, to shew the longitude and latitude of places, or the situation they are in with regard to the circles of the globe: this all maps in general, which are laid down from tables of longitude and latitude, perform, by means of the meridians and parallels delineated upon them, in what manner soever they be projected: and they do this the more accurately, the greater number they have of meridians and parallels.

539 *The second* requisite is, to exhibit the shape of countries the same, and the extent of them in the same proportion, as upon the globe: this cannot be done accurately in any general maps, except globular ones; but in particular maps, which take in but a small part of the earth, as a province, or county, the defect will not be sensible.

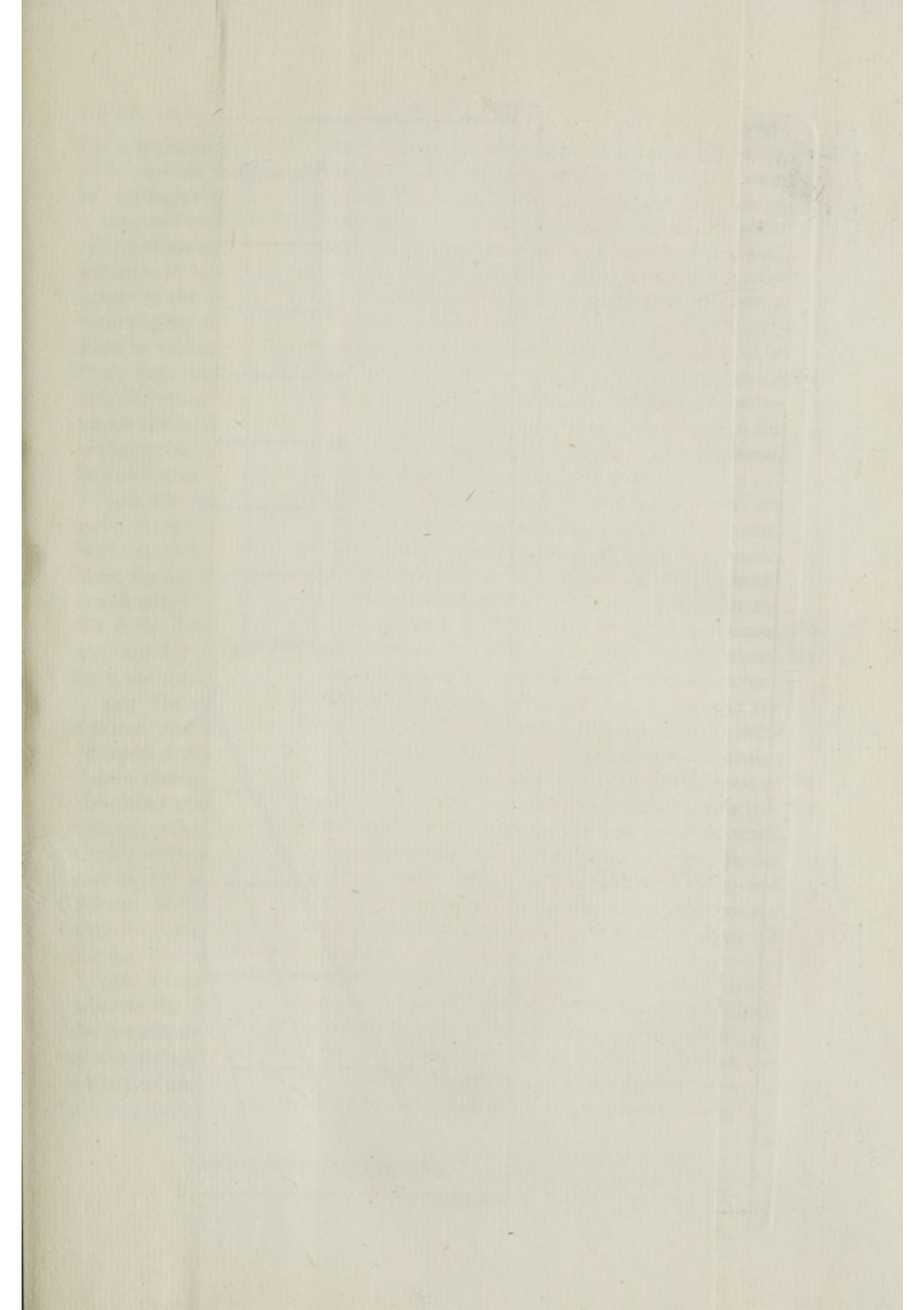
540 *The third* is, that the bearings and distances of places from one another be truly shewn: this in globular maps would be done in the same manner as upon the globe: it is also done in the true chart described § 553, so as to serve the purposes of navigation: but that chart is very deficient in the second requisite, which was the shewing countries in their true shape and proportion.

541 *In general maps*, the circles of the globe are projected, according to the rules of perspective, the most useful of which to our present purpose have been already laid down in the introduction, from § 250 to the end. The *projections* of the circles in general maps are of two sorts, one *convex*, the other *concave*: to understand which, we may imagine the globe upon which the circles are delineated to be of thin glass, and that half of it is viewed at a time; now we may be conceived to view this hemisphere, either on the convex or concave side, and we may suppose it to be placed at different distances from the eye; from which diversity of our view, there will arise different projections, or pictures of it in a map: if the eye be supposed to view the hemisphere at an infinite distance, the *projection* of its circles, whether it be viewed on the convex or concave side will be the same, namely the *orthographic*, described § 254: in this projection, the parts about the middle are very well exhibited, but the extream parts are very much contracted: the reason

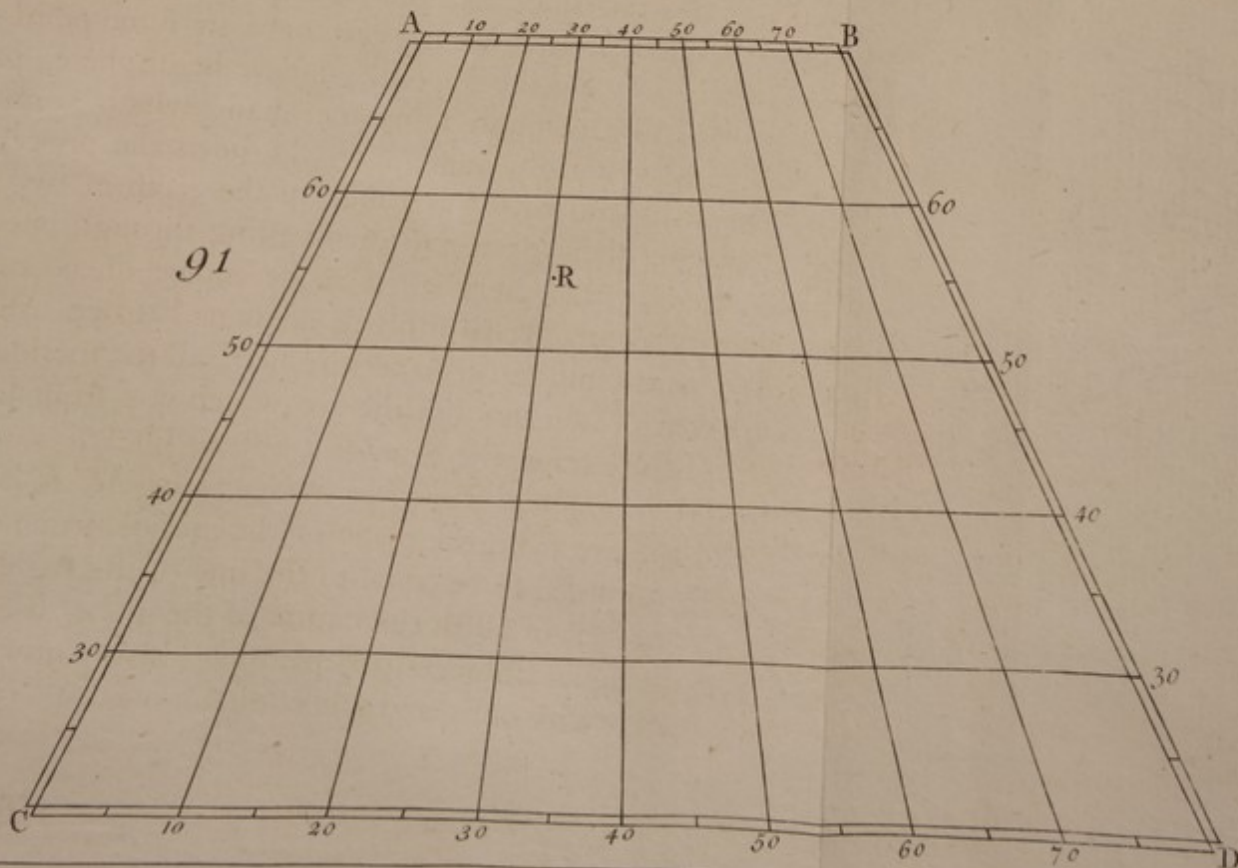
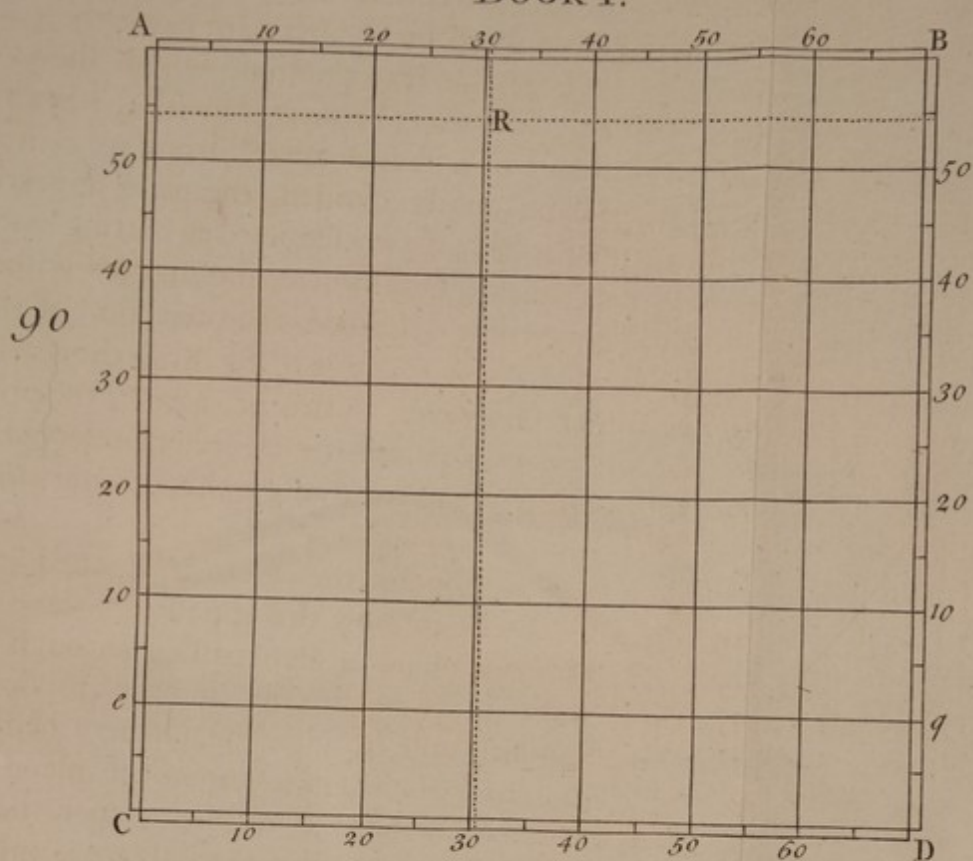
FIG. of which appears, if we consider fig. 147 and 148 of the introduction, which shew us, that the orthographic projections of convex and concave semicircles divided into equal parts are similar, and if the planes of them continued pass through the eye, they are straight lines, unequally divided, so as to have their divisions much less near their extremities than about the middle. If we imagine two semicircles cutting each other at right angles to be drawn upon a glass hemisphere, with divisions marked at every tenth degree, and viewed at an infinite distance, with the eye in such a situation that the planes of both semicircles continued pass through it, the orthographic projection of them, whether
 74 concave or convex, will be the lines AC and BD, fig. 74, with the unequal divisions, through which I have drawn circles. To shew the *orthographic equatorial projection*, fig. 74 is the orthographic projection of the circles upon the plane
 75 of the equator; and fig. 75 is a general map of the northern hemisphere, formed upon that projection: here, the meridians are all straight lines, and the parallels complete circles. For an example of the *orthographic meridional projection*,
 76 fig. 76 is an orthographic projection of the circles upon the plane of a meri-
 77 dian; and fig. 77 is a general map of an hemisphere, formed upon that projection: here, all the meridians are projected into semiellipses, by § 258, except that the plane of which produced passes through the eye, and that is a straight line, by § 258: this projection has one advantage not to be found in any other general map, namely, that the decrease of the parallels from the equator to the poles appears therein in true proportion.

542 The *orthographic horizontal projection* has great variety, according to
 78 the different latitude of places, and is pretty difficult to execute; fig. 78, though not perfectly exact, will serve to give any person an idea of it; the center of this map is in Chaldea, about the place where I suppose Ur of the Chaldees to have been situated, it is therefore projected upon the horizon of that place, or near the matter: it was delineated by me in the *camera obscura*,
 79 from Senex's great globe, as was also the map fig. 79: in a map drawn this way, the extremest parts are a little more contracted than in the orthographic projection, from which, notwithstanding it differs very little in the main, though in the orthographic projection the globe is supposed to be at an infinite distance, but in this is placed at a moderate distance from the eye.

543 The great contraction of the extremest parts in the orthographic projection makes it so deficient in the second quality required in maps, that it is seldom used, except when the disk of the earth, in a solar eclipse, is to be exhibited; of which in its proper place: the method commonly made use of in the construction of maps is the *stereographic projection*, wherein the eye is supposed, from some point in the surface of a sphere, to view the concave of
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the opposite hemisphere: in this projection, the parts about the middle are a little contracted, so as to be too small for the extream parts, the reason whereof, will appear from fig. 156 of the introduction, which shews us, that if a concave semicircle with equal divisions be viewed, with the eye in the plane of it continued, at the distance of a semidiameter from its center, the projection will be a strait line unequally divided, the parts appearing less the nearer to the center; consequently, if two semicircles cutting one another at right angles, drawn upon a concave hemisphere, and marked with equal divisions be viewed in the manner now described, the projection of them will be strait lines unequally divided, as AC and BD, fig. 80; the parts appearing less, the nearer to the center: however, in this projection, the proportion between the middle and the extream parts is much better preserved than in the orthographic. *The stereographic projection* is also either *equatoreal*, *meridional*, or *horizontal*.

544 *The equatoreal projection* supposes the eye to be situated in one of the poles of the earth, and from thence to view the opposite concave hemisphere, with its circles, projected upon a plane of glass passing through the equator: thus, the eye is supposed to be in the north pole, in order to view the southern hemisphere; and in the south pole, to view the northern hemisphere: fig. 80 is the stereographic projection of the circles upon the plane of the equator, and fig. 81 is a map of the northern hemisphere, upon that projection: here, the meridians are all strait lines, and the parallels are compleat circles.

545 *The meridional projection* supposes the eye to be in some point of the equator, and from thence to view the opposite concave hemisphere, projected upon a glass plane passing through some meridian, which a meridian drawn through the eye cuts at right angles: fig. 82 shews the projection of the circles upon a meridian, and fig. 83 is a map of the earth in two hemispheres, projected upon the plane of a meridian passing through one of the Canary islands called Ferro: this meridian is made choice of, because, by making the projection thereon, one hemisphere contains Europe, Asia, and Africa; and the other, north and south America: here, all the meridians are arcs of circles, except that drawn through the eye which is a strait line; all the parallels also are arcs of circles, the equator is a strait line.

546 There is another *meridional projection* invented by *M. de la Hire*, wherein the distance of the eye from the plane of the meridian upon which the projection is made is supposed to be equal to the sine of the angle of 45° : this projection comes the nearest of all to the nature of the globe, because the meridians are set therein at equal distances, the parallels also are nearly equidistant; I see no good reason why our geographers do not make use of it: fig.

FIG. fig. 84 is the projection of an hemisphere with its circles, according to *M. de la*
 84 *Hire's* method; and fig. 85 exhibits a map of the two hemispheres of the earth,
 85 according to that projection. *Bion, usage des globes, l. 3. c. 1. § 2.*

547 In the *horizontal projection*, the eye is supposed to be in that point of the surface of the globe which is diametrically opposite to the place upon the horizon of which the projection is made, and from thence to view the opposite concave hemisphere with its circles, projected upon a glass plane passing through that horizon: thus if we project a map of this sort upon the horizon of London, the eye is supposed to be at the place of the Antipodes to London, and from thence to view the concave hemisphere: there is great variety in this sort of projection, for if the place the horizon of which it is upon be under the pole, the horizontal projection is the same as the equatorial, § 544: if the place be under the equator, it is the same as the meridional, § 545: if the place be in an oblique sphere, the projection will be different according to the different latitude of the place: for it will approach nearer to the equatorial or meridional projection; according as the place is nearer to one of the poles, or to the equator: fig. 86 is an horizontal projection for the
 86 latitude of $51^{\circ} 31'$, and fig. 87 is a map projected upon the horizon of London,
 87 whose latitude is $51^{\circ} 31'$: the chief advantage of this kind of maps is this, that the place upon the horizon of which the map is projected is always in the center of it, and we may see the bearings of places from it.

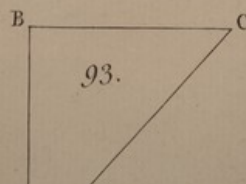
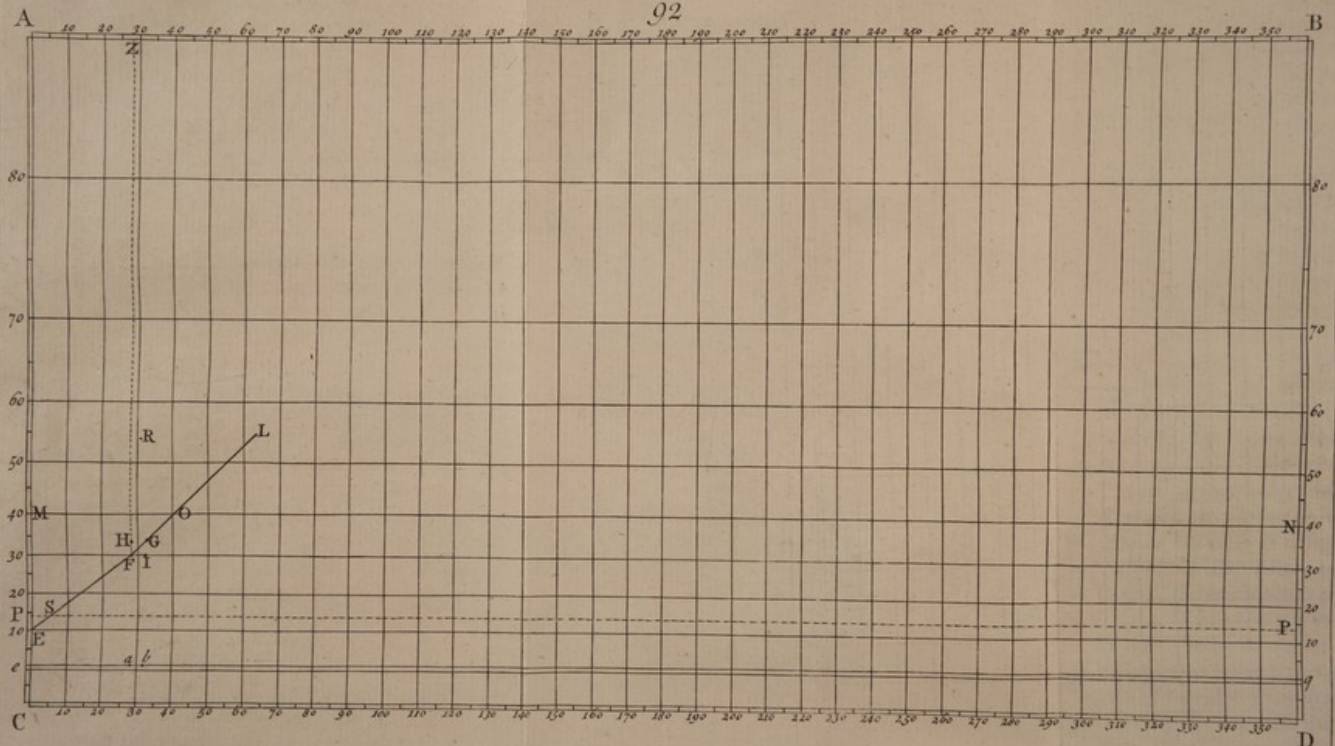
548 A *particular map* is a part of a general one, and may be made upon the same principles, as, by projecting a large hemisphere, and taking so much
 88 of it as the map is designed to contain: thus, fig. 88, from the hemisphere EFGH we may take ABCD; when a large part of the earth is to be represented, the meridional projection described § 545 is commonly made use of.

549 When we are to delineate a map of a smaller part of the earth, if it be near the equator, the meridians and parallels may be represented by equidistant straight lines: as fig. 90, if at some distance from the equator, the parallels may be equidistant straight lines, and the meridians straight lines a little
 90 converging towards the nearest pole; as fig. 91: or the meridians may be straight lines converging towards the nearest pole, and the parallels circular: of
 91 this kind is the map of Europe, fig. 89, taken from *Senex*.

550 When we are to make a map of a very small district, as of a county, or hundred, whatever part of the earth it be in, the meridians and parallels may be equidistant straight lines, drawn through every minute, or every $4'$, $10'$, or $15'$, &c. of longitude, according as the largeness of the map will allow: for as a small part of a great circle upon the earth does not differ sensibly
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from a strait line, so a small part of the spherical surface of the earth differs FIG. not sensibly from a plane.

551 *Sea charts* are particular maps, for the use of navigation, wherein is exhibited some part of the sea, with the shores that bound it: the inland parts are often omitted as of no use to the sailor, except where there are large rivers for him to go up, or sea-marks for him to see at a distance; but the parts of the sea near the shore are carefully laid down, with marks signifying rocks, sands, or flats, and figures expressing the *soundings*, or depth of the water: sea-charts are of two sorts, as in the two following sections.

552 *The plain chart* is, where meridians and parallels are expressed by equidistant strait lines, as fig. 90: this shews the longitude and latitude of places 90 very well, but is erroneous in shewing the rhumb which must be sailed upon, to go from one place to another, except when the places are very near the equator; because no allowance is made therein for the decrease of the parallels abovementioned, chap. 17.

553 In *Wright's chart*, commonly called *Mercator's*, the meridians and parallels are both expressed by strait lines, but from the equator towards the poles the length of a degree upon the meridians is increased, in the same proportion as the length of a degree decreases in the parallels, upon the globe. To understand the nature of this chart, we may suppose the globe, with its usual delineations upon it, to be put into an hollow cylinder of the same diameter with the equator, with its axis coincident with the axis of the cylinder, and then to swell, as if it were blown like a bladder, all the parts stretching uniformly, so that the meridians lengthen in the same proportion as the parallels do, till every part of its surface touches the concave surface of the cylinder: then would each parallel have the same diameter with the equator, the meridians would be strait lines, parallel to each other, and consequently in all latitudes would have the same distance, as on the equator, they would also be lengthened in the proportion before mentioned, from the equator towards each pole: and the rhumbs would unbend, and be projected into strait lines upon the surface of the cylinder: if we imagine the surface of this cylinder to be cut open, along one of the meridians; and unbent into a parallelogram, the projection of the meridians and parallels would appear as in fig. 92: if places are laid down therein, from a table of longitude and latitude, the map will indeed be very deficient in the third particular required in maps, but is nevertheless vastly more convenient for navigation than any other sort of maps whatsoever, because the rhumbs are here reduced to strait lines, so that we need only draw a strait line between any two places, in order to find what rhumb we must sail upon, to go from one place to another.

FIG. 554 Maps have usually a compass drawn upon them, by which the north side may be known: when we use a map upon a table, we generally lay it with the north side farthest from us; and when a map is hung upon a wall, the north is placed uppermost: in both these cases, the east side of the map is on the right hand, the west on the left, I need not say the south is opposite to the north: the lines drawn from east to west are parallels, the two extremest parallels as AB and CD, fig. 89, 90, 91, 92, are divided into degrees, and each degree into halves and quarters of a degree, or into 5', 10', or single minutes, according to the largeness of the map: the lines from north to south are meridians, the two extremest meridians AC and BD are divided into degrees of latitude, and each degree into halves, quarters, &c.

555 *To find the latitude of a place in a map:* stretch a thread over the place, so that it may cut the same degree and minute of latitude, in the two outside meridians AC and BD; the degree and minute marked by the thread in those meridians is the latitude of the place: thus, fig. 90, 91, 92, the latitude of the place R is $54^{\circ}. 30'$.

556 *To find the longitude of a place in a map:* stretch a thread over the place, so that it may cut the same degree and minute in the two extremest parallels AB and CD, fig. 90, 91, 92; the degree and minute cut by the thread is the longitude; thus, the longitude of R is $30^{\circ}. 15'$: we may use the edge of a ruler instead of a thread.

557 *To find a place in a map, its longitude and latitude being given:* stretch one thread over the degree and minute of latitude in the two extremest meridians AC and BD, fig. 90, 91, 92, and another thread over the degree and minute of longitude in the two extremest parallels AB and CD; the point where the threads cross one another will shew the place. This method will also serve to insert a place in a map, its longitude and latitude being given.

558 What has been said in the three sections immediately preceding, is applicable only to such maps as have either the meridians, or parallels, or both, represented by straight lines; as fig. 89, 90, 91, and 92: in a map where the parallels are expressed by curves, as fig. 89, *to find the latitude of any place;* draw with a pencil, or imagine to be drawn, a parallel through it, and the degree which is cut in the extremest meridians AC and BD is the latitude of that place: in a map where the meridians are expressed by curves, as fig. 88, *to find the longitude of any place;* draw with a pencil, or imagine to be drawn, a meridian through it, and the degree where it cuts the equator, or one of the extremest parallels AB or CD, shews the longitude of that place.

559 The fourth thing necessary to be known in navigation was, *to find at any time what part of the globe a ship is upon:* the chief problem in navigation

tion is, to find in what course a ship must sail, in order to arrive at the place to which she is bound: for this purpose it is previously necessary to know, in what place the ship is. The port from whence a ship begins her voyage is supposed to be known, as to longitude and latitude, and to be set down in the chart: when the ship has sailed some time, as 24 hours, it is usual to find the point upon the chart where the ship is then supposed to be: this point is marked by some with the prick of a pin: this point being found, a line drawn from it to the place we are bound to shews the course to be sailed upon; and we may find what rhumb it is, if we observe what angle that line makes with the meridians in the chart: thus, fig. 92 if a ship sets out from E, and is found to be arrived at F, if we are to go to G, the line FG is the course to be sailed, and the angle which this line makes with the meridian is easily found by the protractor, introd. § 33; or if a compass drawn upon a thin transparent piece of horn be laid with its center upon F, and its meridian upon the meridian of F, the edge of a ruler layed from F to G will shew the rhumb to be taken: the angle HFG in the example before us is 45° , therefore the rhumb is north-east. See the table § 570.

560 *The finding the place where the ship is*, which is usually done every day at 12 a clock, depends upon four things; viz. 1 the longitude, and 2 the latitude of that place, 3 the course the ship has run, 4 the distance, that is the way she has made, or the number of leagues or miles she has run in that course, from the place of the last observation: now any two of these being known, the rest may easily be found.

561 *If the longitude and latitude be known*, nothing more is requisite; for the place may be marked upon a globe, by § 507, or upon the chart, by § 557: but how to find the longitude at sea, with certainty, is not yet discovered; what attempts have been made towards it, shall be shewn in its proper place: in the mean time, we will leave it out of the present consideration, except so far as it may be found by the course or distance, as in § 562, case 1 and 2: we proceed now with the course, distance, and latitude.

562 *From the course and distance to find the ship's place*: here are four cases, 1, if the course has been under the same meridian, as § 527, then the distance in geographical leagues or miles turned into degrees and minutes will give us the latitude, and consequently the ship's place; the longitude being the same as at the last observation: thus, fig. 92, if a ship in sailing from F towards Z, has run 40 leagues, or 120 geographical miles, which are those in use at sea, whatever the latitude of F was, the latitude of the place she is now come to will be 2° different from it; that is, it will be 2° less if she has sailed towards the equator, and 2° more if from the equator: thus, if the latitude

FIG. 92. titude of F was 30° north, and the ship has sailed 40 leagues north, she will be in 32° north latitude; and since she continues in the same meridian FZ , her place is that point of this meridian which is cut by the parallel of 32° north latitude: to find this point, set one foot of a pair of compasses upon the 30° in one of the extream meridians in the chart, and the other upon the 32° of the same, then, with this opening, set one foot upon F , and turn the other north, and where it falls upon the same meridian, as at H , there is the ship's place.

Case 2, *If the course has been in the equator*, as § 528, the distance in leagues or miles turned into degrees and minutes gives us the difference of longitude: thus, fig. 92, if a ship sailing from a towards q has run 40 leagues or 120 miles east, her place is then 2° east from a , that is, at b .

Case 3, *If the course has been in a parallel*, as § 529, turn the distance in leagues or miles into degrees and minutes, and take the measure of those degrees and minutes upon one of the graduated meridians, with a pair of compasses, setting one foot as much above the parallel as the other below it; then carry this distance to the parallel, and set it off from the place sailed from, towards the east or west, according as the course has been: thus, fig. 92, if a ship sailing from F in the latitude of 30° has run 180 miles, that is, 2° , east, take the distance of 2° with a pair of compasses, setting one foot upon the meridian AC in the 29° , and opening the other to 31° ; carry this distance to the course, set one foot upon F , and turn the other east, and where it falls, as at the point I , there is the ship's place.

Case 4, *If the course has been oblique*, that is, towards a point not cardinal, turn the distance in leagues or miles into degrees and minutes, then set one foot of a pair of compasses upon one of the graduated meridians, in the latitude sailed from, and extend the other north or south, according as the course has been, till it takes in the number of degrees and minutes the ship has run; carry this distance and set it off upon the line of the course, from the place of the last observation, and you will have the ship's place, without any considerable error: thus, if F the place sailed from, be in the latitude of 30° , and the ship has run 60 leagues or 180 miles, which is equal to 3° , in a north-east course FL , set one foot of a pair of compasses on the graduated meridian AC in 30° , and extend the other northward to 33° , then, with this opening set one foot of the compasses upon F , and at G , where the other foot falls upon the line FL , there is the ship's place.

563 *From the course and latitude given*, to find the ship's place: draw the line of the course, and lay the edge of a ruler over the degree of latitude in the two graduated meridians; the point where the edge of the ruler cuts the line

line of the course is the ship's place; thus, fig. 92, suppose a ship sailing from *R* north-east is come into 40° of north latitude, for the line of the course draw *FL*, making an angle of 45° with the meridian, where this line is cut by the line *MN*, the parallel of 40° , namely at *O*, is the place of the ship.

564 *From the distance and latitude given, to find the ship's place:* lay the edge of a ruler over the degree of latitude in the two graduated meridians, take the distance turned into degrees and minutes, with a pair of compasses, by § 562, case 4, set one foot upon the place the ship sailed from, and where, with this opening of the compasses, the other foot touches the edge of the ruler, there is the place of the ship: thus, fig. 92, if a ship sailing from *E* is come into the latitude of 14° north, and the distance in leagues amounts to $5^{\circ}\frac{1}{4}$, measure $5^{\circ}\frac{1}{4}$ from *E* northwards upon the meridian *AC*, with the compasses, then set one foot upon *E*, where the other touches the line *PP*, namely at *S*, is the place of the ship: in finding the place of the ship, by the foregoing propositions, we have supposed two of these three things to be exactly known, 1 the course, 2 the distance, 3 the latitude of the place the ship is arrived at; if there is uncertainty in any of these *data*, it will occasion a like uncertainty in the place of the ship: the method of finding each of them, and how far it may be depended upon, will be shewn in the following sections.

565 *To find the course:* this is known by the sea-compass, § 521 and 524, but with these uncertainties, 1 the variation cannot be always exactly known, for want of observations, 2 a current, or 3 the ebbing and flowing, or 4 the rolling of the sea, may carry the ship out of the line in which she is directed, 5 the wind, when the ship does not sail exactly before it, which is seldom the case, drives the ship out of the course in which she is directed; this alteration of her course is called her *lee-way*, and allowance for it must be made, in order to find the true course: but this lee-way is different, according to the different angles the wind makes with the intended course, the different strength of the wind, and the number of sails made use of.

566 *To find the distance, or number of miles the ship has run:* this is sometimes guessed at, by knowing how far the ship will sail in such a time, with such a wind, which is found by experience, by observing how far she has sailed in such a time, under the same meridian, or near it, when the distance may be pretty exactly measured, by the difference of latitude: but the most certain way of reckoning the distance is by *the log-line*: *the log* is a triangular piece of board, poized with lead at one corner, to make it swim upright: it is fastened to the end of a line which is divided into equal parts, the divisions are called *knots*, because every one of them is marked with a knot, that they may be counted in the dark, by feeling them with the fingers: a knot

should be an aliquot part, it is usually the 120th part, of a geographical mile, that it may bear the same proportion to a mile, as half a minute does to an hour: the line is wound upon a reel which turns round easily, and is placed in the hinder part of the ship: when they *heave the log*, as they commonly do once every hour, they let it down into the water, and when ten fathoms are gone from off the reel, the log is supposed to be out of the eddy of the *ship's wake*, then the half minute glass is turned up, and when it is run out, they stop the reel, and observe how much line is wound off, to find how far the log is run from the ship, or rather the ship from the log, in that time; and conclude, that as many knots as run off from the reel in half a minute, so many miles the ship sails in an hour: thus, if 6 knots run off in half a minute, the ship sails at the rate of 6 miles an hour: but here likewise must be some uncertainty, from the rolling of the sea when it runs high, from the drift of currents, and from the difference of the wind, which seldom continues exactly in the same tenor for an hour together.

567 *Vitruvius l. 10. c. 14.* mentions a method used by the ancients of measuring a ship's way by wheels on the outside, which being turned round by the water, turned other wheels with their indexes within the ship, in the same manner as the wheel of a coach is made to turn the wheels of a *way-wiser*, and shew the distance travelled by land^a: *De Chales, Navigat. l. 5. prop. 6.* describes a machine by which the strength of the wind may be estimated, this also would be of use to measure a ship's way^b.

568 *To find the latitude*: this is done by taking with instruments the meridian height of the sun, or stars; but cannot be done so accurately at sea as at land, by reason of the motion of the ship: however, the latitude is found with much greater certainty than either the course or distance, and should therefore be often taken when opportunity serves, to correct them thereby:

569 Since *sailing in a parallel* requires only the keeping the course due east or west, § 529, and the latitude is found with greater certainty than either the course or distance, § 568; it is usual, in sailing between the tropics to get into the latitude of the place to which the ship is bound, and steer directly east or west, according as the place lyes from the ship: for this purpose, the *trade-winds* or *monsoons* are to be observed. See monsoon in the index.

570 Since, in the construction of the compass, the 360° of the circle are divided into 32 equidistant points, the distance of any two adjoining points must be $11^\circ 15'$; for 32 times $11^\circ 15'$ is 360 : from hence, the angle which the line of any course makes with the meridian being given, the rhumb sailed upon may be known; or conversely, the rhumb being given, the angle of the course may be found, by the following table.

^a v. & *Phil. transact.* N. 391 & 408. ^b v. & *Ricciol. geograph. reformat. l. 1. c. 3. & l. 10. c. 23.*

A TABLE OF THE ANGLES WHICH EVERY RHUMB MAKES WITH THE MERIDIAN.

North.	South.	Rhumbs	D.	M.	North.	South.
		$\frac{1}{4}$	02	49		
		$\frac{1}{2}$	05	37		
		$\frac{3}{4}$	08	26		
North by East	South by East	1	11	15	N. by W.	S. by W.
		$\frac{1}{4}$	14	04		
		$\frac{1}{2}$	16	52		
		$\frac{3}{4}$	19	41		
N. N. E.	S. S. E.	2	22	30	N. N. W.	S. S. W.
		$\frac{1}{4}$	25	19		
		$\frac{1}{2}$	28	07		
		$\frac{3}{4}$	30	56		
N. E. by N.	S. E. by S.	3	33	45	N. W. by N.	S. W. by S.
		$\frac{1}{4}$	36	34		
		$\frac{1}{2}$	39	22		
		$\frac{3}{4}$	42	11		
North East.	South East.	4	45	00	North West.	South West.
		$\frac{1}{4}$	47	49		
		$\frac{1}{2}$	50	37		
		$\frac{3}{4}$	53	26		
N. E. by E.	S. E. by E.	5	56	15	N. W. by W.	S. W. by W.
		$\frac{1}{4}$	59	04		
		$\frac{1}{2}$	61	52		
		$\frac{3}{4}$	64	42		
E. N. E.	E. S. E.	6	67	30	W. N. W.	W. S. W.
		$\frac{1}{4}$	70	19		
		$\frac{1}{2}$	73	07		
		$\frac{3}{4}$	75	56		
East by North	East by South	7	78	45	W. by N.	W. by S.
		$\frac{1}{4}$	81	34		
		$\frac{1}{2}$	84	22		
		$\frac{3}{4}$	87	11		
East	East.	8	90	00	West.	West.

571 Thus I have entered upon the use of the terrestrial globe, and shewn the nature and use of maps and charts: the mechanical methods of delineating them I omit, as foreign to my purpose; they may be seen in *Varenius's* Geography, and in a treatise of maps and globes printed A. D. 1717. I have likewise explained the nature of the rhumbs, and shewn how to find the place of a ship upon the chart, mechanically, which is sufficient to give the reader a general notion of navigation: in the writers upon that subject, we have rules to find the place of a ship, by trigonometrical calculation: the foundation of which calculation is this; in oblique sailing, the line of the ships way together with the meridian of the place from whence she sailed and the parallel of the place where she is arrived form a *rhumb-triangle*; which, though curvilinear upon the globe, may, when the distance sailed is small, be considered as rectilinear, without sensible error, and is accordingly drawn so upon both the plain chart and *Mercators*: thus, fig. 93, suppose a ship sails from A to C, draw AB to represent the meridian of A, for the parallel of C draw CB perpendicular to AB, and there will be formed the rhumb-triangle ABC, whose angle at B is a right one by construction, the angle at A is the angle of the course, the hypotenuse AC is the distance, AB is the difference of latitude, and BC is in plain sailing called the departure, and in *Mercators* sailing the longitude: now this triangle being right angled, if one acute angle and one side thereof be known, the other two sides and the remaining acute angle may be found, by § 143: or if two sides be given, the remaining side and the two acute angles may be found by § 98, here it is to be remarked, that though the calculation of any problem is more exact than construction, or drawing it in lines geometrically; yet navigation by trigonometry must be attended with all the uncertainty which arises from the errors to which the *data*, upon which it is founded, viz. the course, distance, and latitude, are liable: since there are no methods of finding these besides those laid down in this chapter.

CHAP. 20. THE HEIGHT OF MOUNTAINS, THE DEPTH OF THE SEA.

572 Mountains are so considerable parts of the earth as to demand some notice should here be taken of them: many of them furnish the spring-heads of large rivers, some are famous for the mines of rich metals contained in their bowels, others are remarkable for the dreadful eruptions of fire, smoke, stones and ashes to which they are subject; these and many other particulars relating to mountains I leave to those writers who describe the countries wherein

wherein they are found, or write the natural history of them, see *Varen. geog. l. 1. c. 10.* my present design leads me only to enquire into the height of mountains, in order to find what proportion it bears to the diameter of the earth, and how far it may be objected to what has been said of its roundness.

573 The height of mountains has often engaged the attention of the inquisitive, though we have little of certainty upon that subject delivered to us by the ancients: sometimes authors content themselves with only telling us how many days it takes to go from the bottom of a mountain to the top: thus, *Strabo*^a, speaking of some of the highest mountains in Greece, writes that an able traveller might walk from the bottom of any of them to the top in a days time, whereas it would take him above five days to go up one of the Alps: sometimes they say how long the sun may be seen to enlighten the top of a mountain, before his rising in the morning, and after his setting in the evening; to this purpose, we have accounts of the mountains Caucasus and Casius, in *Aristotle*^b and other writers: sometimes they relate how far off the top of a mountain may be seen, or to what distance the prospect from thence reaches, or in general, that the top of a mountain is above the middle region of winds and rain: in proof whereof, it is said of some mountains which had anciently sacrifices offered upon the tops of them, that the ashes have been observed to lye there undisturbed from one years end to another, so that characters drawn therein were not effaced^c: mount Athos is particularly taken notice of for casting a shadow to the distance of above 80 miles, as far as Myrine a town in the Island of Lemnos^d, in the market-place of which, there was erected the statue of an heifer that used to be covered by the shadow, according to a celebrated verse upon that occasion,

Ἀθὼς καλύπτει πλεῖστα Λημνίας βοός.

The Lemnian heifer is by Athos cover'd.

Atlas is said to hide his head in the clouds, and by the poets is therefore feigned to support the heaven: but I find only three mountains recorded to have had their height taken with accuracy, probably by trigonometry, by any of the ancients: Olympus and Pelion in Thessaly, and Cyllene in Arcadia have not only served to embellish the writings of the poets, but have also employed the skill of geometers. Olympus was found by *Xenocrates* to rise above ten *stadia* higher than the temple of *Apollo* built at the foot of it, he seems to have used a good deal of care in this affair, because, in an epigram of his extant in *Plutarch*^e, he says the height of it is ten *stadia* and a sixth part of a *stadium* wanting two feet, and pleads the merit of the performance to recommend him-

^a *Strabo geogr. l. 4.*

^b *Arist. Meteor. l. 1. c. 63. Solin. c. 37.*

^c *de Atho refert Solinus*

^d *c. 16. de Cyllene Gemin. c. de signif. astrorum. Plin. nat. hist. l. 4. c. 10.*

^e *Plut. in vita P. Æmilii.*

self to the favour of *Apollo*. The other two were, by order of some princes, measured by *Dicearchus*^a, the height of Pelion was found by him to be 10 *stadia*, of Cyllene 15 *stadia*. For the measure of the *stadium* see § 440.

574 Amongst modern travellers, the Pic of Teneriff^b is famous for its height: we are told it may be seen at sea to the distance of 4°, which is about 270 English miles; it is by many thought to be the highest mountain upon the whole earth: others think the mountains which run through south America exceed all others in height; for whereas the air on the tops of several very high mountains has been said, by reason of its thinness, to occasion reachings, difficulty of breathing, and sometimes vomiting^c, the air here is so unfit for respiration, that upon a party of Spaniards crossing over them at the only pass which is practicable, many of their men and horses died suddenly. *Varen. Geogr. l. 1. c. 10.*

575 The height of a mountain may be considered either absolutely, that is how much its elevation is above the surface of the sea, or relatively, i. e. how much higher it is than the ground upon which the stations are taken to measure it. *Ricciolus* in his *geographia reformat* l. 6. has a large account of mountains, and the several attempts which have been made to measure the height of them, both by the ancients and moderns; he shews several trigonometrical methods of doing it, as does also *Varenius* l. 1. c. 9. one of the principal of these I have explained § 475, but they are all liable to uncertainty, by reason of refraction: it is probable the ancients did not take refraction into the account, for that reason the heights of mountains which they give us are greater than they ought to be; the moderns have considered refraction, but it is so variable, that it is hardly possible to ascertain at any time what allowance ought to be made for it.

576 The mercurial barometer seems to promise better success in this affair than can be expected by any other means: it is now universally agreed on, that the quicksilver is sustained in the tube of the barometer by the pressure of the air upon that quicksilver wherein the lower end of the tube is immersed, that the farther from the center of the earth the barometer is placed, the shorter is the column of air, and the less its weight by the pressure whereof the quicksilver is supported in the tube, and consequently, the lower will it subside: thus, whatever is the height of the quicksilver in the tube at the bottom of a well, it will stand lower when the barometer is brought up to the surface of the earth, and will subside still lower if it be carried up to the top of an high tower; and this subsiding of the quicksilver is in a certain propor-

^a *Plin. nat. hist. l. c.*

^b *Ricciol. geogr. reformat. lib. 6. c. 13.*

^c Mr Edens found nothing of this on the top of Teneriff, v. *Phil. transf. N. 345.*

tion to the height of the place where the experiment is made. When the gentlemen of the academy of sciences at Paris drew the meridian line mentioned § 415, they made use of barometrical observations to determine the height of some of the highest mountains in France, and of some of the Pyrenees, which separate that kingdom from Spain: by the same method, *Dr. Scheuchzer* took the heights of several of the Alps; from the detail of his observations, which may be seen in the *philosophical transactions* for the year 1728, n. 405 and 406, it appears that the highest of the Alps, the tops whereof are covered with everlasting snow, do not rise above 10000 Paris feet, which is 10680 English feet, or 2 English miles 120 feet, above the level of the sea, and these mountains, which are undoubtedly the highest in Europe, that gentleman is of opinion may equal the height of any upon the globe.

577 *The depth of the sea* has been thought by some to answer to the height of mountains, as if the earth had been once a perfect globe, and the mountains consisted of what was taken out of the cavities made for the reception of the waters, but this is mere conjecture without any solid foundation: the depth of the sea is in many places unknown, being unfathomable by the line, though great lengths have been made use of for that purpose. *Mr. Boyle in his treatise of subterranean and submarine regions* § 2, mentions a curious traveller who, upon enquiring whether he had taken notice of any extraordinary deep soundings in the vaster seas, answered that on the other side of the line about the latitude of 35° they let down 400 fathoms of line, and that the lead which weighed about 30 or 35 pounds by going so deep appeared when drawn up again intensely cold like ice: there are in the same treatise several proofs that the sea-water is colder the deeper we go. *Varenius^a* tells us the depth of the sea has in some places been found a German mile which is equal to four English miles. *Ricciolus^b* writes of some englishmen having sounded with lines to the depth of 3000 ells, but he speaks of it by hearsay. *Olaus magnus^c* speaks of some parts of the northern seas being so deep that a ship load of line would not suffice to sound them to the bottom, but this I suppose is said at random. *Cesar^d* is said to have found the depths of some parts of the Mediterranean and Adriatic not to exceed 15 *stadia*, which is almost two English miles: it is probable that it was from this relation that *Pliny^e* says the depth of the sea is in some places 15 *stadia*. In the *philosophical transactions* for the year 1728 N. 405 we have a description of a machine to measure the greatest depths of the sea, but if any experiments have been made therewith they are not yet communicated to the public.

^a *Geogr. l. 1. c. 12.*

^b *Geograph. reformat. l. 6. c. 21. & l. 10. c. 1.*

^c *l. 2. c. 10.*

^d *Priscianus apud Majol. dier canicular. colloq. 10.*

^e *N. Hist. l. 2. c. 102.*

578 Mr. Boyle, in the place before cited, says he was very inquisitive about *the inequality of the bottom of the sea*, by the information he received of profest divers, and other curious Persons who had opportunities of seeing divers at their work and conversing with them, or who had sounded the sea in several places, it appears that the floor of the sea is in many places exceedingly unequal, being here flat and there craggy, rocks rising to a considerable height in some parts, and valleys sinking as deep in others: that generally the depth of the sea is found to increase or decrease gradually, but in some places is so very unequal that in casting the lead on one side of the ship they find it 20, 30, or perhaps 40 fathoms deeper than they had found it just before when they sounded from the other side: particularly he was told by an ancient and experienced pilot that not far from the mouth of our channel he has sometimes found the bottom of the sea so abrupt, that in sailing twice the length of his ship the water has deepened from thirty to an hundred fathoms, if not much more; and this sudden alteration of depth is chiefly observable near the shore, though there are instances of the like far out at sea: but as in sailing far from land there is not so much occasion for sounding, it is only known in general that the depths in the wide ocean are very great, the particular depths are known but in very few parts of it.

579 In sailing near the shore, the sounding line with the lead is often cast out, to find the depth of the water: the lead has often tallow or grease upon the lower end of it, that it may bring up with it sand, gravel, shells, &c. in order to shew what kind of soil is at the bottom, the knowledge of the depth of the water and the kind of soil is very useful to find exactly where the ship is, to prevent her running aground, and carry her safe into port

580 *The proportion of the surface of the earth to that of the sea*, so far as discoveries have been made, may be found, by taking the paper of a terrestrial globe and separating the land from the sea with a pair of scissars, and comparing the respective weights of each: this experiment supposes the delineation of the globe to be correct, the paper whereon it is printed should also be of an equable thickness as near as possible; weighing thus the papers of Mr. Senex's globe of 16 inches diameter, the weight of the paper whereon the sea was represented was 349 grains, that of the land 124 grains: so the surface of the sea is almost three times as great as that of the land hitherto discovered. I omitted weighing the parts contained within the polar circles, because it is not known to any degree of exactness how much of them is land and how much is sea.

ASTRONOMY. BOOK II.

CHAP. I. THE SYSTEM OF THE WORLD, THE FIXT STARS, THE PLANETS, THE TWINKLING OF THE STARS, THEIR DIFFERENT MAGNITUDES,

581 *The system of the world* is the whole collection of those great bodies which compose the universe of material beings, and comprehends the stars, the sun, the planets, and comets, together with the indefinitely extended space wherein they are contained^a: from the author whose opinion is followed, in the description of the number, magnitude, situation, motion, and other affections of those bodies, it is called the *Ptolemaic system*, the *Tycho-nic*, the *Copernican* &c. I shall, before I have done, endeavour to give the reader a general view of the several systems which have been advanced by the most celebrated philosophers and astronomers, in order to account for the appearances of the heavenly bodies; but I shall in the first place treat of that system which was anciently taught by the disciples of *Pythagoras*^b, after long disuse revived about 200 years ago by *Copernicus*, cultivated afterwards by *Galileo*, *Kepler*, and others, and greatly improved by the discoveries of *Sir Isaac Newton*. The method I propose to make use of is this, taking it at present for granted that this system is the true one, I shall lay down the several parts of it, and shew under each head how the appearances of the heavenly bodies hitherto observed naturally result from such an order and disposition of them as is here supposed: this exact correspondence of the *phenomena* with the *hypothesis* is alone so strong an argument in favour of the truth of it, that we might very well acquiesce therein; but we shall meet with other proofs in the progress of this work.

582 The stars, as well those seen by the naked eye as those which are discoverable by the telescope, keep always the same situation in respect of one another, and are therefore called *fixt stars*: thus, if we consider any number of stars which appear near to one another and to our view form a triangle, square, or any other figure, regular or irregular, we have reason to believe,

^a Κόσμος ἐστὶ σύνθεμα τῶν ἄστρον καὶ γῆς καὶ τῶν μεταξὺ φύσεων. Diodorus ap. Achill. Tat. p. 129. edit. Petavii: iisdem fere verbis, Cleomedes l. 1. κόσμος, οὗ τῆς διακοσμήσεως, primus appellavit Pythagoras, Achill. Tatius ibid. τὸ ὅλον λίσσιν [Stoici] τὸν κόσμον, πᾶν δὲ μετα τῆ καὶ. id. ibid. universum, universitatem, & rerum universitatem vocat Cicero.

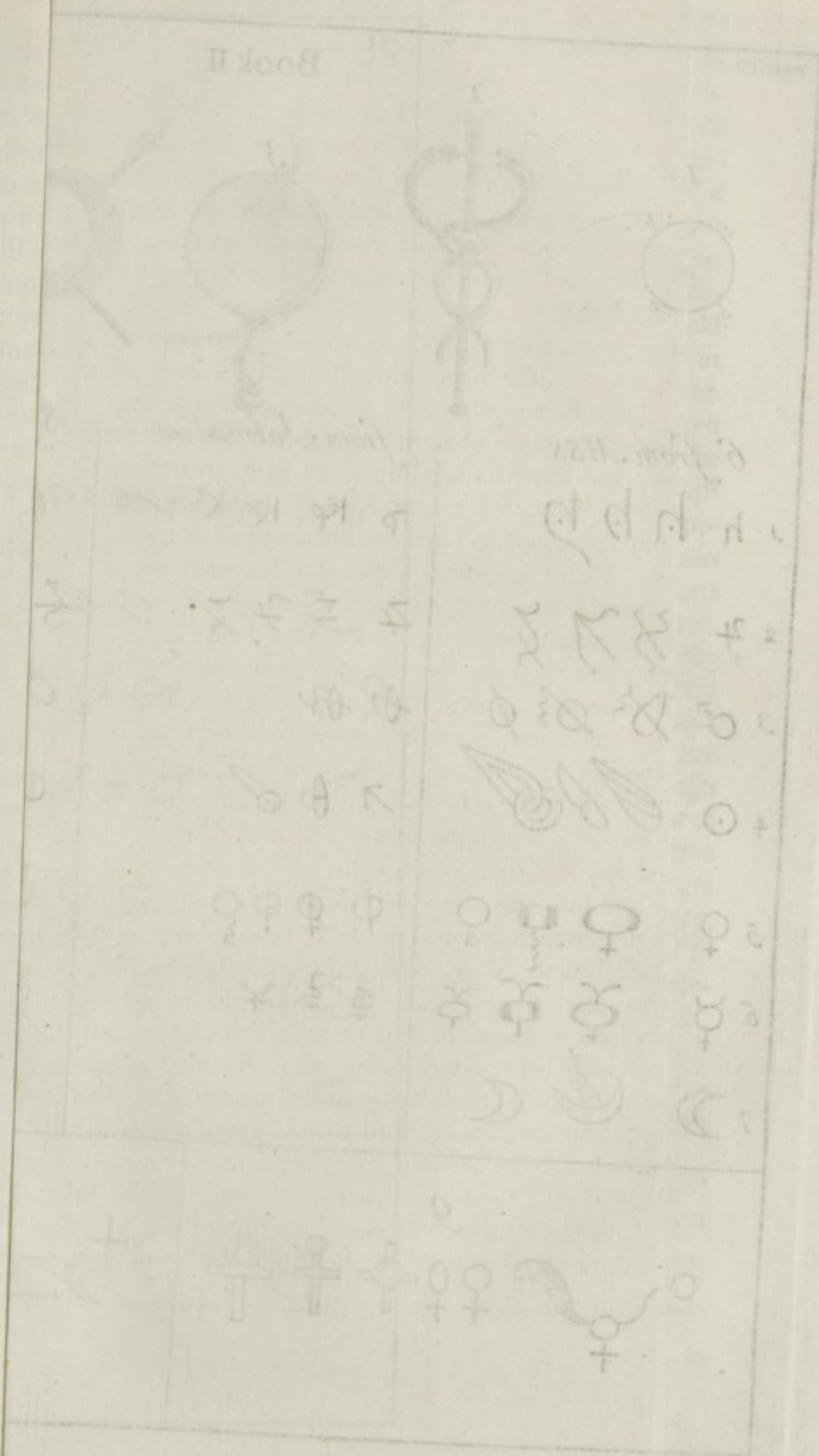
^b See the history of astronomy at the end of this work.

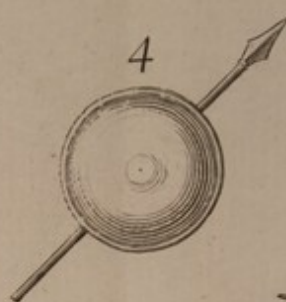
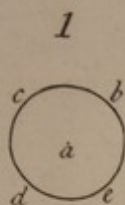
that ever since the creation of our earth, they have always appeared to its inhabitants to form the same figure, as to shape and dimensions, as they do at present, and will probably continue to do so as long as our earth shall last: this is true of the fixt stars in general, as far as we are able to judge, by comparing the latest observations of the stars with the most ancient ones whereof we have any account remaining; as for particular changes which have been observed in some stars, and may be looked upon as exceptions to the general rule, they shall be treated of in a chapter upon that subject.

583 Besides the fixt stars, there are to be seen in the heaven five *planets*, which by common spectators are not distinguish'd from the stars, though they be of a very different nature; I shall treat of them more particularly by and by, it is sufficient at present to take notice that they do not keep their places, as the fixt stars are observed to do, but are seen sometimes in one part of the heaven and sometimes in another: from this change of place they are called planets or wanderers^a; from hence also they came first to be distinguished from the fixt stars, and to have the natures and motions of them enquired into.

584 There is another difference between the fixt stars and the planets, which may very properly be taken notice of in this place, *the fixt stars twinkle*^b, whereas the planets shine with a steady light: the cause of this difference seems to be this, the fixt stars are at such an immense distance from us, that their apparent diameters are exceedingly small, so that every little particle of dust that floats in the air and comes between our eye and any fixt star is big enough to cover it, and hide it for a moment from our sight: now our air near the earth is so full of various kinds of particles which are in continual motion, that some one or other of them is perpetually passing between us and any star that we look at, which makes us every moment alternately see it and lose the view of it; and this twinkling of the stars is greatest in those stars which are nearest the horizon, because they are viewed through a good deal of thick air, where the intercepting particles are most numerous, whereas stars that are near the zenith do not twinkle so much, because we do not look at them through so much thick air, and therefore the intercepting particles, being fewer, come less frequently before them: as for the planets, they, by reason of their being much nearer to us than the stars, have a sensible apparent magnitude, and that so great as not to be covered by the small particles before mentioned, and therefore they do not twinkle, but shine with a steady light. That this is a true account of this matter, any one may be easily satisfied by the following experiment; lay a small flat piece of glass of about the bigness of a silver penny upon the ground between your self and the

a By Milton *wandering fires*. *Errantes vocat* Nigidius, ap. Gellium. b Ricciol. l. 6 c. 2. Tacquet *astron.* *fun,*





6 from MSS.

h h h h

2 4 3 7 2

3 ♂ ♂ ♂ ♂

4 ○ ○ ○

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7 ☾ ☾ ☾

7 from Salmasius

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8 from Du Fresne

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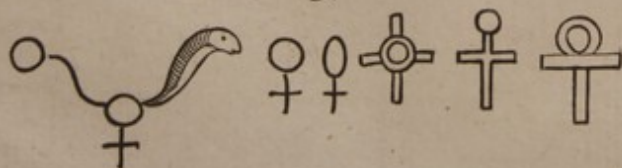
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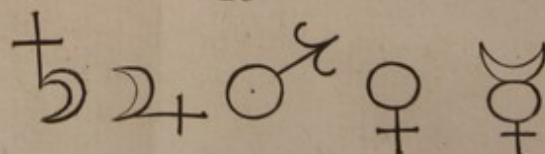
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9



10



sun, with the plane of it a little declining from the sun, so that retiring from it three or four hundred yards you may see the light of the sun shining upon it to that distance or farther: if you come so near the glass that the image of the sun reflected from it appears bigger than one of the largest stars in the heaven, you will see it shine with a steady light, and look like a planet; but if you keep your eye upon it and go from it to the distance of three or four hundred yards, till its apparent diameter is no bigger than that of a small star, you will perceive it twinkle in the same manner as the stars do: in like manner, if you turn your face towards the sun, and look upon a plowed field where there are many small stones, you may observe some of them that are at a distance to twinkle, whilst others that are nearer shine with a steady light.

585 It must be confessed, that some of the stars appear so large to the naked eye, that one would be apt to think they were too big to be covered by the small dusty particles just now mentioned; but it is to be considered, that all luminous bodies appear larger than other bodies of the same dimensions, because the force wherewith the particles of light emitted from them fall upon the retina, is such as to cause a trembling or vibration, not only in those parts upon which the picture is painted, but also in the parts round about: thus, fig. 1, suppose the light emitted from a star should really fall upon only the point *a* of the retina ^a, yet if it comes with such force as to cause a trembling round about that point as far as the circle *b c d e* reaches, the star will appear of the same magnitude as if its picture on the retina took up the area of that circle: if the force of the emitted rays be abated, as they are by passing through glasses, whereby many of them are reflected, the luminous object loses some of that adventitious apparent magnitude, and paints its picture nearer to its true dimensions^b. And thus it is that the fixt stars looked at through a telescope are stripped of that glare of rays which to the naked eye seem to stream every way, and then though the glasses magnify them perhaps an hundred times, yet so immensely great is their distance from us as to make their apparent diameters even thus magnified still insensible, so that they appear like points: the case is very different if we view any of the planets through a telescope, their distances are moderate in comparison of that of the stars, and therefore a telescope that magnifies an hundred times will very sensibly increase their apparent magnitudes.

586 *The fixt stars appear to us of different magnitudes:* this may arise from one of these two causes, either they differ in their bigness, and those stars are

^a I mean here a physical point, v. §. 223:

^b Luminous bodies appear larger even through a telescope than might be expected from their dimensions and distances: in an experiment made by the R. Academy of Sciences at Paris, a fire of three feet diameter, at the distance of 31897 toises, appeared under an angle of 8 seconds; whereas by calculation it should appear under an angle of 3 seconds.

in reality the largest which appear so to us and shine with the strongest lustre, or their distances from us may be unequal, and those stars may be farthest off which seem smallest to the eye and give the faintest light; or in some instances these two causes may both combine to make that difference which we perceive among the stars, and some of them may excel in brightness as well as appear larger than others, because they are both bigger in themselves and nearer to us: however this be, astronomers have from their apparent magnitudes distinguished the stars into six orders or classes: the first contains those of the largest size, the second those which appear next in bigness, and so on, till we come to stars of the sixth magnitude, among which are counted the smallest that can be seen with the bare eye unassisted with glasses: as for those which cannot be discerned without the telescope, and are therefore called *telescopic stars* they are not reckoned into the six orders. Now although this distribution of the stars be generally received, we are not to imagine that all the stars of any one of these classes are exactly of the same bigness, we may rather say there are almost as many orders of stars as there are stars, there is such a difference in the bigness, colour, and brightness of them: thus, some of those which are reckoned of the first magnitude are much larger than others which are placed in the same class, so that there is for that reason a disagreement among the astronomers in this respect, and the same stars are counted by some of the first magnitude, which others have assigned to the second: there is also the like uncertainty with respect to the other classes.

CHAP. 2. THE CONSTELLATIONS, UNFORMED STARS, CATALOGUES OF THE STARS, THE NUMBER OF THE STARS, THE MILKY WAY.

587 The sphere of the fixt stars was by the ancients divided into several *constellations* or *asterisms*: a constellation is a collection of stars which in the heaven appear near to one another, and may be imagined to represent the figure of some real or imaginary animal, or of some other known visible thing: as a ram, a lion, a centaur, an harp, a crown &c. the number of the ancient constellations is 48, the number upon our present globes is about 70: I call those the ancient constellations which we have received from the Greeks, and particularly from *Ptolemy*^a: we find some of these occasionally mentioned by *Homer* and *Hesiod*, but *Aratus*^b professedly treats of them all, except two or three which were invented after his time, and that in the following method: 1 he shews how each constellation is situated in respect of those that are near it, 2 what position it is in with regard to the principal cir-

a In his *Almagest* or *μεγάλη σύνταξις* in fine lib. 7, & initio lib. 8.

b v. *Arati phenomena*

cles of the sphere, 3. what other constellations rise or set with it: this falls far short of the accuracy of *Hipparchus* the Rhodian and of *Ptolemy*, as to setting down the places of the stars; but was sufficient for the use of sailors, and the purposes of husbandry, which were the ends chiefly proposed by this author, who lived about 270 Years before the birth of our Saviour, and is the poet cited by *St. Paul*, Acts 17. 28. *Aratus* has been in such esteem, as to be translated by *Cicero*, *Cesar Germanicus*, and *Ovid*, paraphrased by *Avienus*, and illustrated by about 50 commentators^a. *Hipparchus* the Bythynian^b has shewn, by several passages quoted from the writings of them both, that he followed the descriptions of *Eudoxus*, who flourished about 100 years before him, and it is very probable that the Greek astronomers who succeeded him continued to use the same figures of the constellations, all along down to *Ptolemy*, though with some variations and additions.

588 *Ptolemy's* work has been always in such esteem among the astronomers, that almost all who have written since his time have agreed in drawing the figures of the constellations, or supposing them to be drawn, so as to answer his description of them, as far as possible: and indeed this is necessary, to avoid confusion in comparing ancient and modern observations together: thus for example, the great Bear should be so delineated that every star in that constellation which is taken notice of by *Ptolemy* may be in the same part of the figure, viz. the snout, the eye, the forehead, &c. as he describes it. It does not appear that *Ptolemy* published any pictures of the constellations, if he did they are not come to our hands; the most ancient pictures we have of them are those published out of an ancient manuscript by *Grotius*^c; but these he justly observes, though venerable for their antiquity, are by no means agreeable to the originals in the heaven: as for the figures printed in some editions of *Hyginus*^d, the designers of them seem to have followed their own fancies in delineating them, without any regard to the descriptions of the author.

589 One use of the constellations was to distinguish the stars from one another: thus, if we mention the star in the bull's northern eye, an astronomer knows what star is meant, as well as if we pointed it out to him in the heaven. In the infancy of astronomy, only the principal stars were taken notice of: thus, at first only seven stars were reckoned in the little bear, and the same number in the great bear, and they were then called the two wains or waggons, each of these having four stars in the form of a parallelogram, which represent the wheels, as the three stars lying almost in a strait line do the beasts of draught: agreeably to this the 7 principal stars in the great bear are at this day commonly called Charles's wain.

^a Scaliger Not. in Manilium p. 25. & Grotii Syntagma Arateorum in epist. dedicat. ^b a Petavio editus in Uranologia. ^c in Syntagm. Arateorum. ^d Poeticum Astronomicum.

590 The names of the constellations and the number of stars observed in each of them by different astronomers are as follows.

The ancient constellations.		Ptolemy	Tycho	Hevel.	Flamst.
Urfa Minor	the little Bear	8	7	12	24
Urfa Major	the great Bear	35	29	73	87
Draco	the Dragon	31	32	40	80
Cepheus	Cepheus	13	4	51	35
Bootes	Bootes	23	18	52	54
Corona Bor.	the N. Crown	8	8	8	21
Hercules, <i>Engonasin</i>	Hercules kneeling	29	28	45	113
Lyra	the Harp	10	11	17	21
Cygnus, <i>Gallina</i>	the Swan	19	18	47	81
Cassiopea	Cassiopea	13	26	37	55
Perseus	Perseus	29	29	46	59
Auriga	the Waggoner	14	9	40	66
Serpentarius	Serpentarius	29	15	40	74
Serpens	the Serpent	18	13	22	64
Sagitta	the Arrow	5	5	5	18
Aquila, Vultur	the Eagle	15	12	23	71
<i>Antinous</i>	<i>Antinous</i>		3	19	
Delphin	the Dolphin	10	10	14	18
Equulus, <i>Equi sectio</i>	the Horses head	4	4	6	10
Pegasus, <i>Equus</i>	the Horse	20	19	38	89
Andromeda	Andromeda	23	23	47	66
Triangulum	the Triangle	4	4	12	16
Aries	the Ram	18	21	27	66
Taurus	the Bull	44	43	51	141
Gemini	the Twins	25	25	38	85
Cancer	the Crab	23	15	29	83
Leo	the Lion	35	30	49	95
<i>Coma Berenices</i>	<i>Berenices Hair</i>		14	21	43
Virgo	the Virgin	32	33	50	110
Libra, <i>Chelæ</i>	the Scales	17	10	20	51
Scorpius	the Scorpion	24	10	20	44
Sagittarius	the Archer	31	14	22	69
Capricornus	Capricorn	28	28	29	51
Aquarius	the Water Bearer	45	41	47	108
Pisces	the Fishes	38	36	39	113
Cetus	the Whale	22	21	45	97
Orion	Orion	38	42	62	78
					Eridanus

The ancient constellations.		Ptolemy	Tycho	Hevel.	Flamst.
Eridanus, <i>Fluvius</i>	Eridanus	34	10	27	84
Lepus	the Hare	12	13	16	19
Canis major	the great Dog	29	13	21	31
Canis minor	the little Dog	2	2	13	14
Argo, Navis	the Ship	45	3	4	64
Hydra	the Hydra	27	19	31	60
Crater	the Cup	7	3	10	31
Corvus	the Crow	7	4		9
Centaurus	the Centaur	37			35
Lupus, <i>fera</i>	the Wolf	19			24
Ara	the Altar	7			9
Corona Australis	the S. Crown	13			12
Piscis Australis	the S. Fish	18			24

The new southern constellations.

Columba Noachi	Noah's Dove	10
Robur Carolinum	the Royal Oak	12
Grus	the Crane	13
Phoenix	the Phenix	13
Indus	the Indian	12
Pavo	the Peacock	14
Apus, Avis Indica	the Bird of Paradise	11
Apis, Musca	the Bee, or Fly	4
Chamæleon	the Chameleon	10
Triangulum Aust.	the S. Triangle	5
Piscis volans, Passer	the flying Fish	8
Dorado, Xiphias	the Sword Fish	6
Toucan	the American Goose	9
Hydrus	the Water Serpent	10

Hevelius's constellations made out of unformed stars.

		Hevel.	Flamst.
Lynx	the Lynx	19	44
Leo minor	the little Lyon		53
Asterion & Chara	the Greyhounds	23	25
Cerberus	Cerberus	4	
Vulpecula & Anser	the Fox and Goose	27	35
Scutum Sobieski	Sobieski's Shield	7	
Lacerta	the Lizard	10	16
Camelopardalus	the Camelopard	32	58
Monoceros	the Unicorn	19	31
Sextans	the Sextant	11	41

591 The constellations are so delineated as to take in the stars, especially the most remarkable ones, into such parts of the figures as are most easily distinguished: thus, *the bull* is drawn so as to have one star in his southern, another in his northern eye; one in the tip of his northern, another in the tip of his southern horn &c: besides the stars which form the constellations, there are others which could not be comprehended in the figures, but are contained in the interstices between them; these, because not reduced into any certain figure, are called *unformed stars*^a: some of these which were reckoned amongst the unformed by the ancients have been from time to time made into new constellations by later astronomers: thus, to console the *Queen of Ptolemy Euergetes*, for the loss of a lock of her hair which was stolen out of the temple of *Venus*, where she had dedicated it on account of a victory obtained by the King her husband, *Conon* an astronomer out of some unformed stars near the tail of the lion made the constellation of *Berenice's hair*^b: thus *Antinous*^c the favourite of *Adrian*, besides being deified by that Emperor, was placed amongst the stars, and a constellation made out of some unformed stars near the eagle received its name from him: in later times, the celebrated *Hevelius* made several new constellations out of the unformed stars, which are now drawn upon our globes, but in fainter lines, in order to distinguish them from the ancient ones: I shall mention but one instance more, that Sir *Charles Scarborough* physician to *Charles 2*, gave the name of *Cor Caroli* to a single star of the 2d magnitude near the great bear, in memory of that excellent but unfortunate Prince *Charles 1*.

592 Since the discovery of South-America, the Portuguese and Dutch navigators, crossing the line, and having a view of the stars near the south pole, have furnished our globes with new southern constellations which were unknown to the ancients: as *el Dorado* the golden fish, *el Cruzero* the cross, which we call the Crofiers &c.

593 In the numbers set down in the foregoing table, the unformed stars are reckoned to the several constellations to which they belong: *Coma Berenices* and *Antinous* are printed in a different character, because, though both of them be mentioned by *Ptolemy*, the first of these is counted by him among the unformed of the lion, the latter among the unformed of the eagle. The numbers of *Tycho* are taken from his *progymnasmata*, those of *Hevelius*, not having his *prodromus* at hand, I transcribed from *Flamsteed's Historia Cælestis* vol. 3, as I did also the numbers of the new southern constellations, *Flamsteed's* numbers of some of the old southern constellations I have encreased by the addition of those stars of them which cannot be seen in *England*,

^a *Stella informes, ἀμύφωτοι* Ptolomeo.

^b Ricciol. *almagest.* l. 6 c. 4.

^c Xiphilinus in Adriano.

but have been observed at Sta. *Helena* by Dr. *Halley*. The names in the first column are those in common use, seven of *Ptolemy's* names are different from them, these are added in italic: thus *Hercules* is by him called *Engonasin*, i. e. a man kneeling: instead of *the swan* he has a word which signifies a fowl in general, or particularly one of the hen-kind: *Pegasus* in *Ptolemy* is *the horse*: in the room of *Libra*, he has *Chelæ*, *the claws*; for among the ancients the *Scorpion* was drawn larger than at present, so that the claws of it took up that part of the heaven which is now assigned to *Libra*: *Eridanus*, is by him called *the river*: and in lieu of *the wolf* he has *the wild beast*. Several of the constellations have more names than one: thus, *Bootes* is also called *Arctophylax*: and *Ophiuchus* is another name for *Serpentarius*: not to mention other names which sometimes occur, especially among the poets.

594 Besides the additions mentioned, § 591 and 592, there have also at different times been some alterations made in the forms of the constellations: thus, *Ptolemy*, who chiefly followed *Hipparchus*, says, l. 7. c. 5. of his *Almagest*, that he does not always make use of the same figures with those before him, but had, for the sake of giving his figures a truer proportion and adapting them better to the situation of the stars, made many alterations therein, as the astronomers before his time had done, in the constellations of those that were more ancient: and he instances particularly in some stars, which, by reason of their great distance from her head, he placed in the ribs of *Virgo*, whereas *Hipparchus* had placed them in her shoulders. This observation will explain to us how it comes to pass that we find a difference amongst ancient authors, in their descriptions of the constellations: thus, the bull is drawn according to *Hipparchus* and *Ptolemy* with only head and neck and fore-part, as we have him upon our present globes; whereas *Vitruvius*, *Pliny*, and others describe him entire, and place the *Pleiades* in his tail.

595 Besides the names of the constellations, the ancient Greeks, who were imitated herein by the Romans and Arabians, gave names to some small collections of stars, as also to some of the most remarkable single stars, on account of their being supposed to have some particular significancy and influence: thus, the cluster of small stars in the neck of the bull was called the *pleiades*: five stars in the bull's face the *hyades*: a cloudy star in the breast of the crab was called *præsepe*, *the manger*; and two stars near it *aselli*, *the asses*: four stars at the extremity of the right hand of *Aquarius* were called *the urn*: fourteen stars between the tails of the fishes were called *the lines*: whereof five stars were reckoned in the *northern*, and nine in the *southern line*, and the bright star between them was called *the knot*: the *southern crown* was called by some *uraniscus*, by *Hipparchus*, *the herald's rod*, such as *Mercury* is usually drawn with;

with: the preceding star at the feet of *Gemini* was called *propus*: a bright star in the breast of *Leo*, the *lion's heart*, and *basiliscus*, or the royal star: a large star between the knees of *Bootes* was called *Arcturus*: one in the ear of corn which *Virgo* holds in her left hand, *spica Virginis*: one in her right wing, *ωεστυρίτης*, *vindemiator*: the *Pleiades* were by the Romans called *Vergiliæ*, and the *Hyades*, *Suculæ*: the biggest star in the *harp* often takes the name of the constellation, being called *Lyra* by the Greeks, and by the Romans *Fidicula*.

596 As the Europeans first learned astronomy from the Arabians, many *arabick names of some of the principal stars* are still retained by them, though some of them are so wretchedly mangled by ignorant transcribers that it has cost learned men a good deal of pains to set them right, others they have not been able to make any thing of at all, but this very thing, their being uncouth or unintelligible, may perhaps have recommended those names to our astrologers; otherwise, it seems to be as agreeable to good sense for an english writer to call a star *the fishes mouth*, as to call it *fomahaut*, an arabick word of the same signification: the like may be said of the rest, however, since custom has so prevailed that we cannot help meeting with those names in books, and upon our globes, I shall give those which occur most frequently, as correctly as I can, in the index.

597 It is very probable the stars, especially some of the most remarkable collections of them, such as *Charles's wain*, the *Pleiades*, *Orion*, &c. were formed into constellations and had names given them, in the early ages of the world: some of them, by their different appearances, serve to mark out the seasons of the year, and upon that account were not only taken notice of as directors when to begin plowing, sowing, and other works of husbandry^a, but were also thought to have a great influence over the temperature of the air, and the fruitfulness of the earth; so that, from their being signs, pointing out the times of the year when heat or cold, dryness or moisture are usually predominant, they began to be looked upon as the causes of those qualities in natural bodies, and were imagined to have dominion over minerals, vegetables, and animals, over the complections, and constitutions, and even the tempers and dispositions of mankind: this opinion found credit the more easily, because the sun, moon, stars, and planets were believed to be of a divine nature^b, some held they were inhabited by an inferior kind of deities^c,

^a Hesiod. *Oper. & dies*. l. 2. v. 1 & 2.

^b Vossius *de origine & progressu Idololatriæ*. l. 2. c. 30.

^c Εὐκλείδης Οὐρανὸν vocat Alcinous, de doctrina Platonis. Singulis astris singulas animas tribuit Plato, in *Timæo* p. 41. ed. Serrani tam cælum quam stellas animalia esse, ex eorum motu, credabant philosophi vet. Achilles Tatius in *Arat.* p. 128. ed. Petav. Pythagoræi existimabant planetas esse corpora divina, *Gemin* αὐτῶν γ. c. 1.

which

which governed their motions and directed their influences: others that they were animals, and had each of them a living soul, which was guided by an intelligence: others, that they were animated by a part of the substance of the supream being: each of these notions led mankind to pay them religious worship. The most ancient idolaters are with great probability thought by some learned men to have received the name of *Zabii*, from worshipping the host of heaven^a: this they could hardly do, without dividing the stars into parcels, as we find their aptest scholars in idolatry the Egyptians did, who divided the heaven into several regions, which they called the stations or mansions of their gods^b.

598 *The Egyptians worshipped the heavenly Bodies*, but especially the sun and moon, which they called their great gods: they thought the sun, whom they called *Osiris*, a proper representative of the spirit of nature, the soul of the world, or the supream being, who is every where present, exercising his power over every part of the universe^c. The moon as she receives her light from the sun was looked upon as female, and called *Isis*^d, which goddess is sometimes made to signify universal material nature, considered as passive, and susceptible of various impressions, forms and qualities; but the moon considered in another view, as active, distributing light and warmth over the face of the earth, was said to be male: thus *Isis* and the moon were counted of both sexes^e. They found, or imagined themselves to find, in various

^a צבא השמים *zeba' hasbamaim* Hyde *de relig. vet. Persar.* c. 3. Pocock. *specim. Hist. Arab.* alias adjert etymologias Spencerus, *de leg. Hebr.* l. 2. c. 1. ipse, Scaligerum secutus, ex צבא, voce Arabica, quæ orientales denotat, nomen derivari contendit. When we are told, what Spencer has sufficiently made out, that many of the observances of the Mosaic law which seem to be trifling, or to serve only to exercise the obedience of the Jews, were instituted by God, in order to preserve that people from the idolatry and superstition of the *Zabii*; I think we may understand by that name, not only the *Chaldeans*, but all other nations also, who were guilty of the like practices, by whom the Israelites were in danger of being infected: especially the *Egyptians*, among whom they lived so long, and the inhabitants of those countries whose land they were to possess, or in whose neighbourhood they were to be settled. Statuerunt (*Zabii*) nullum esse Deum præter stellas — stellas esse divinas (vel Deos minorum gentium) & Solem esse Deum magnum — reliquos quinque planetas esse Deos, sed duo luminaria esse majores: — Solem regere mundum s. perierem & inferiorem, philosophi apud eos existimarunt — Deum esse spiritum sphaera, vel orbis caelestis, orbis caelestes & planetas esse corpora, & Deum Opt. Max. esse spiritum illorum — crediderunt antiquitatem (i. e. aternitatem) mundi, quia caeli juxta eos sunt Deus. Maimonid. *More Nevoch.* l. 3 c. 29 De *Zabiiis*, D' Herbelot, *Bibliothèque Orientale*, in voce *Sabi*.

^b Kircherus *Oedip. Aegyptiac.* T. 2. part. 2. pagg. 153, 195, 241. ^c Εἰσὶν δύο θεοὶ αἰθέρος καὶ πρώτους τῆς ἀλλοῦ καὶ τῆς σελήνης, Diodor. Sic. l. 1. Vossius *de Idololatr.* l. 1. c. 37. hinc ο θεος pro sole, η θεα pro luna, apud Hor. *Apoll.* l. 1. hierogl. 5. & 10, & alios, hinc Sol vocatur αὐρανός τις & διαποτὴς κόσμου, Onomacrito (vulgo Orpheo) in hymn. & ἡνὼν παρρηγοῦσθαι θεῶν Proclo hymn. in Solem. Deos omnes ad solem, ad lunam Deas omnes referri, ostendit Kircher. *Obelisci.* Pamph. l. 3. c. 13. Solem *Osiridis*, lunam *Isidis* nomine intelligi, idem ibid. & l. 3. c. 7. & *Oed. Aegypt.* T. 1. p. 172. & T. 3. p. 139. Vossius *de Idololatr.* ^d *Isis* ab ἰση derivatur a Vossio *Idol.* l. 1. c. 38. de *Iside* Apuleius *Metamorph.* l. 11. ^e Hinc θελὺς καὶ ἀρσὴν Onomacrito hymn. in lunam, hinc Deus *Lunus*. v. & Voss. *de Idololatr.* l. 2. c. 24.

animals, some properties or qualities corresponding to the motions, appearances or influences of the sun, moon, and some of the stars: this induced them not only to use those animals in their hieroglyphic representations of their deities, but also to pay them divine honours: thus, they called *the beetle*^a a living image of the sun, they pretended that those animals roll up their eggs in little round pellets of dirt, which they turn towards the west, while they themselves continue creeping on towards the east; by the first of these motions exhibiting the diurnal, by the second, the annual motion of the sun and planets. They made *the hawk*^b, upon account of his piercing eye, strength, swiftness, and some other qualities, an hieroglyphic of the sun, and sometimes of the supream being. By *the ram*^c, a prolific animal, they represented the genial fertilizing warmth of the sun in the spring; as they did his violent scorching heat in the summer, by the hot and furious beast *the lion*^d. *The bull*^e was looked upon by them as another emblem of the various powers of the sun, in forwarding the business of agriculture, wherein this creature is very serviceable, the worship of a living bull with some peculiar marks, to which they gave the name *Apis*, in one part of Egypt, and *Mnevis* in another, is mentioned by a great number of authors^f; the Jews golden calf is thought to be an imitation of this piece of Egyptian idolatry. *The goat*^g had divine honours paid him in some parts of Egypt, as a representative of the tendency of universal nature to propagate the several species of beings; from hence the Greeks took their image of *Pan*^h. They deified *the Nile*ⁱ whose overflowing is so beneficial to the land of Egypt, and as that river was observed to begin to swell at the rising of *the dog-star*; this made them have a particular veneration for that star, as if its benign influence contributed to that fruitfulness which the Nile occasioned: they called the star *Isis*^k, the name of their great goddess, and *Thoth*^l, the name of their god *Mercury*, whom for his sagacity in the invention of letters, arts and sciences, they sometimes painted in the form of a dog, or of a man with a dogs head, and worshipped him under the name of *Anubis*^m. They called the same star *Sothis*, or *Siothi*, which in the Egyptian language, according to *Kircher*, signifies holyⁿ. *Sibor*^o and *Siris* are other names for the Nile, from whence *Vof-*

a Hor. Apoll. l. 1. hierogl. 10. Kirch. Obel. Pamph. p. 122, 308, 335. b Hor. Apoll. l. 1. hieroglyph. 6. Kircher. Obelisc. Pamph. p. 309. c Kircher. ibid. p. 269. d Id. ibid. p. 282. e Id. ibid. p. 256. f Kircher. Oedip. Ægypt. T. 1. p. 194. Voss. de Idololatr. l. 1. c. 29. Selden. de Diis Syris Syntagm. 1. c. 4. g Kircher. Obelisc. Pamph. p. 274. h Kircher. ibid. Vossius de Idol. l. 3. c. 74. i Kircher. Oed. Ægypt. T. 1. p. 48. Salmas. Plinian. exercit. p. 427. de Nilo etiam v. Selden. de Diis Syr. Syntagm. 1. c. 4. & Voss. de Idol. k Hor. Apoll. l. 1. hier. 3. l Isis Ægyptiis matrem denotat, Salmas. de annis climacter. p. 567; Thoth patrem, Chamberlain, oration. Dominic. p. 30, hinc forte Θις, nam frigida sunt etyma vulgo allata. m v. Kircher. & Vossium de Idol. n Prodrum. Copt. p. 147. a Σιθ Αδαμī filio trahit Selden. de Diis Syr. Prolegom. c. 3. o Sibor Hebr. niger, a limo sc. hinc olim αὐρον, postea Melo vocatus.

fius^a derives *Sirius*, the greek name of this star, which was imagined to have such dominion over that river; as also *Osiris*, the name of one of their principal deities. *The wolf*^b, a rapacious devouring animal, was looked upon by them as an emblem of the consuming power of the sun. *Virgo* was a representation of their goddess *Isis*. *The Scorpion*^c was an emblem of the destructive power of the malignant genius *Typhon*. *The triangle* was a picture of the land of Egypt, which is of a triangular shape; and therefore *Kircher*^d derives *deltoton*, another name for that constellation, from *deltoti*, which in the Egyptian language signifies a good country, rejecting the common etymology taken from its resembling the greek letter Δ. *The balance* was the hieroglyphic of justice, and therefore *libra* was thought a proper constellation for that part of the heaven wherein the sun appears at one of the equinoxes, when he distributes equal day and night to the whole earth: or, if it be considered with regard to the land of Egypt, it may intimate the equal distribution of the Nile to the several parts of the country, by the genius of the waters; for the figure of this constellation was among the Egyptians a man holding a pair of scales^e. To mention no more, the Egyptians worshipped *serpents* and *fishes*^f. *The altar*^g is also said to have been an hieroglyphic of the land of Egypt. I have here confined myself chiefly to such animals and other figures as we meet with among our present constellations, which I think were most of them originally Egyptian.

599 That the Egyptians worshipped all the animals, and used the figures abovementioned as hieroglyphics of their deities, *Kircher*^h proves at large by the testimony of several ancient authors; to which we may add a remarkable passage in *Herodotus*ⁱ, who says, that in Egypt all sorts of beasts, as well wild as tame, were accounted sacred: and indeed I am inclined to think they worshipped the host of heaven and all sorts of animals, while the children of *Israel* lived among them, from the caution *Moses* gives that people, *Deut.* 4. 15. 'Take heed lest you corrupt *your selves*, and make you a graven image, the similitude of any figure, the likeness of male or female, the likeness of any beast that is on the earth, the likeness of any fowl that flieth in the air, the like-

a *De Idol.* l. 2. c. 74. v. Selden. *de Diis Syr. Proleg.* c. 3. & *Syntag.* 1. c. 3. *Siri Ægyptiis filius*, Salmas. *de annis Climaeter.* p. 566. b *Kirch. Obel. Pamph.* p. 296. *Id. Oed. Æg. T.* v. p. 242. c *Scorpionem pingebant tauri testiculos rodentem, quod Sol in hoc signo, terrorem adurens, fertilitati, quam in tauro promovebat, inimicas vires exerat: ejus rei imagines vide ap. Hyde de rel. vet. Persar.* c. 4. d *Kirch. Ob. Pamph.* p. 325. *prodr. Copt.* p. 233. e *Kircher. Oed. Æg. T.* 2. part 2. p. 156. *Manil. l. 2. Humana est facies libra* — in quem locum ita Scaliger: *libripens enim in astrothesia figurabatur, alii a virgine gestari volunt, ut Kalendaria ambo rustica Romana: adulatores poeta Augusto in manus tradunt.* v. *locum.* f *De Serpentibus*, *Kircher. Obel. Pamph.* p. 347. *Vossius de Idololat.* l. 4. c. 63. *de piscibus*, *Vossius ib.* l. 4. c. 51. *ubi tamen pisces a sapientioribus pro deorum symbolis potius quam pro diis cultos fuisse innuit, quod etiam de ceteris animalibus existimandum est.* g *Hor. Apoll. l. 1. hierogl.* 22. h *locis supra citatis.* i *In Euterpe.*

'ness of any thing that creepeth upon the ground, the likeness of any fish that
'is in the waters beneath the earth, and lest thou lift up thine eyes unto hea-
'ven, and when thou seest the sun and the moon and the stars, even all the host
'of heaven, shouldest be driven to worship them, and serve them, which the
'Lord thy God hath divided unto all nations under the whole heaven. But the
'Lord thy God hath taken you and brought you forth out of the iron furnace,
'even out of Egypt. The mention here made of their being brought out of
the land of Egypt seems to be intended, to put the Jews in mind of their
deliverance from the idolatrous practises of that country, as well as from the
slavery they had endured therein.

600 The Greeks, who learned astronomy of the Egyptians^a, retained several of their figures, as *the ram, the bull, the lion, the dog, the triangle, &c.* but accommodated almost all of them to the fabulous history of their gods and heroes, whom in this manner they placed among the stars^b; but there is a great disagreement in their fabulous relations: thus, the Twins are by some supposed to represent *Castor and Pollux*, by others *Apollo and Hercules*, or *Triptolemus* and *Jasion*: I do not think it worth the while to enter into the detail of the history, whether true or fabulous, to which the constellations are said to relate; enough of it may be seen in *Hyginus*^c, if any one is not satisfied with what he finds there, let him read *Ricciolus Almagest. l. 6. c. 3, 4, 5*, and the authors there quoted, as also *Sherburne's* notes upon *Manilius*. Sir *Isaac Newton*^d observes that *Musæus*, who is said to have made the first globe among the Greeks, was father to *Orpheus*, one of the *Argonauts*; and that the greatest part of the figures upon the celestial globe are made to relate to things or persons concerned in the Argonautic expedition, and none of them are supposed by any authors to bear relation to any transaction of later date: this is very true, but the great disagreement there is among the mythologists in their accounts of those figures shews them to be of greater antiquity, and that the constellations were received some time among the Greeks, before their poets, according to their several fancies, applied them to different fables^d.

601 It must be owned the stars are capable of being reduced to figures very different from those now in use among us, those stars which are formed into the lion might have made an horse or any other animal, in fact the constel-

^a Lucian. *de astrologia*. ^b Ut Græci omnia suis heroibus tribuunt, ita & suis Ægyptii: inquit Vossius, *de Idol. l. 1. c. 28*. Græci hieroglyphicas Ægyptiorum imagines Diis eorum fabulosis accommodabant, Kircher. *Oed. Æg. T. 2. part 2. p. 196*. ^c *Poeticon Astronomic.* ^d *Chronology. p. 84*. ^d Nigidius duo volumina scripsit, 1 de sphaera Græcica, 2 de sphaera Barbarica; in hoc historiam Syderum ex mente Ægyptiorum contexuit, in illo juxta sententiam Græcorum. w. Scalig. in *Manil. p. 368*. & Salmaf. *de annis Climacter. p. 592*.

lations of the Chinese and Japonese are very different from ours^a: some superstitious Arabians, though they received their astronomy from the Greeks, have given some of their constellations different figures; this they are said to have done because they thought it unlawful to draw any human figure, and therefore they changed all such on the celestial globe into some other form, and very often absurdly enough: thus, instead of *Aquarius* they give us the figure of a mule saddled carrying two barrels, for *Gemini* two peacocks, *Virgo* with them is a wheat-sheaf, the *centaur* is an horse and a bear fighting, *Auriga* is a mule saddled, with a bridle drawn so as to have some stars upon it, whereas there are none upon the bridle of *Auriga* as drawn by the Greeks: for *Ophiuchus* they draw a stork or crane, for *Hercules* they make a camel with his furniture kneeling, for *Sagittarius* they draw only a quiver, for *Cassiopea* they place a bitch in a chair, for *Andromeda* a sea-calf, and for *Cepheus* a dog, instead of *Bootes* they put a monstrous figure that *Scaliger* says he could not tell what to make of. *not. in sphaeram Barbaric. Manil. p. 484.*

602 Some christian astronomers^b, displeased to see the heaven of the fixt stars possess'd by the fabulous heathen deities and heroes, have been for making a reformation in that point, and have propos'd to retain the ancient figures, referring them to some scripture history: in this view they would have the ram to be a memorial of that which was offered instead of *Isaac*, *Virgo* to represent the *B. Virgin*, &c. Others would new model all the constellations, and put them into different forms: thus, for the twelve constellations of the *Zodiac*, they give us the twelve Apostles, &c. but this last project would bring intolerable confusion into astronomy, in comparing the ancient and modern observations; to prevent which, the best way is to draw the figures as near as possible to the descriptions of *Ptolemy*.

603 There are two ways of delineating the figures of the constellations, either such as they appear in the sphere of the heaven, or, which are the reverses of the former, such as are drawn upon the common celestial globe: if a large room were built with its inside of a spherical form, the constellations might be drawn thereon as they appear in the sphere of the heaven it self: in this manner they may be seen on the concave surface of my glass sphere, if one looks into it: in the same manner they may also be drawn upon two concave hemispheres, which would contain them all, or, which would be better, segments of spheres might be made of brass or pastboard^c, of such di-

^a *Martinii Atlas Sinic. prae fat. p. 18. Kempfer hist. of Japan.* The Persian and Indian constellations are also different from ours: *Albumasar, introduct. l. 6. c. 1. Souciet, observat. mathemat. astronomiques & geograph. faites en Chine p. 247.* ^b *Schillerus in caelo Christiano, Skilskardus in astroscopio, Bartchius in globo caelesti. v. Ricciol. Almagest. l. 6. c. 6. & Sherburne's Appendix to Manilius p. 140.* ^c *v. § 534.*

mentions that each segment should contain a single constellation upon its concave surface. The convex surface of my glass sphere shews the constellations the same as the common celestial globe does, that is, such as they would appear if we imagine the stars all to be fixt in the surface of a sphere, and to be viewed by a spectator on the outside of it looking upon its convex surface.

604 Sometimes the *constellations* are *drawn* upon two *planispheres* projected upon a great circle, this way gives us the pictures of the two concave hemispheres *in plano*: the nature of these projections may be understood from § 541, but as we have observed those maps of the earth to represent it in the truest proportion which shew us a small part of it, § 539, so those maps of the heaven will shew us the stars in the truest situation which take in only one constellation entire, with some parts of the adjoining ones.

605 The first *use* of all these concave *spheres*, *planispheres*, *globes* and pictures of the constellations is, by comparing them with the originals, to know the stars in the heaven: it has been already observed book 1. chap. 2. that, by the rotation of the earth round her axis one way, the heaven with all the stars, &c. appears to revolve the contrary way; so that, though the fixt stars keep their situation with respect to one another, the sphere of the fixt stars is changing its position with respect to us, every moment of the natural day: therefore, to compare the stars upon a concave hemisphere, globe, planisphere, or picture, with the stars in the heaven; we must place the globe, hemisphere, &c. in such a position as to correspond with the present situation of the heaven: I shall hereafter shew how to *rectify the celestial globe to any time of the night*, that is, to place it so that every star upon the globe may point at its corresponding star in the heaven; and then the constellations may be known, by comparing the heaven and the globe with one another: thus, if I would find a star called *Arcturus* in the heaven; the globe being rectified to the time of night, I find *Arcturus* thereon, then if I imagine a line to be drawn from the center of the globe through that star, that line continued will point at *Arcturus* in the heaven. Suppose, on the other hand, I see some bright star in the heaven which I want to know upon the globe, I first rectify the globe to the time of night, and then if I imagine a line to be drawn from the star to the center of the globe, it will point at the corresponding star on the surface of the globe. Maps of the concave surface of the heaven, especially such as take in only one constellation with some parts of those which surround it, are very useful for the purpose of being acquainted with the stars: of this sort are the figures given us by *Bayer* in his *Uranometria*, and those of *Flamsteed* published after his death by *Hodgson*; the manner of using them will be seen by the following example: if I would know the stars in the

the great bear, I turn the figure about, till the principal stars thereof are in the same situation with regard to upper or under, right or left, as they appear in the heaven, at the time of making my observation; when this is done, it is easy, by looking first on the heaven and then upon the map, to see to what parts of the figure, whether to the eye, snout, &c. the rest of the stars are to be referred.

606. *A catalogue of the stars* contains both the constellations and the unformed stars, with the number of stars in both, and the place of each star: the first of this kind amongst the Greeks was made by *Hipparchus* of Rhodes, who flourished about 120 years before Christ: he, as *Pliny* informs us, *Nat. Hist. l. 2. c. 26*, upon the appearance of a new star, began to doubt whether there might not be changes amongst the fixt stars, and therefore made a catalogue of them, setting down the place and magnitude of each star, that, if in time to come any new stars should come into view, or any of those observed by him should increase or be diminished in magnitude, or should totally disappear, such changes might be known to after ages: here we have one use of a catalogue of the stars pointed out to us; another use is to determine thereby the true places and motions of the heavenly bodies.

607 The most ancient catalogue now remaining is that of *Ptolemy*^a, copied chiefly from *Hipparchus*, but with some few alterations, as was said § 594, he settles the places of the stars as he himself observed them, in the beginning of the reign of *Antoninus Pius*, about A. D. 140; the number of stars in this catalogue is 1026. In imitation of *Ptolemy*, several astronomers have since his time made catalogues of the stars, computing their places for some year in or near their own time: this perhaps may want some explanation, for which purpose it is to be observed, that the place of a star is denoted by setting down its longitude and latitude^b, and that, though the fixt stars keep the same situation in respect of one another, the sphere of the heaven appears to have a slow motion, by which the longitude of all the stars is altered, at the rate of about 50" in a year, their latitude continuing the same; this makes it necessary for any one who makes a catalogue of the stars and sets down their longitudes and latitudes, to tell us for what point of time the catalogue is calculated.

608 The Arabians are the first who after *Ptolemy* observed the stars, and noted down their places: the learned *Hyde*^c mentions several of their catalogues, and published the most considerable one of them in arabic with a latin translation, it was made by *Ulug Beigh* grandson to *Tamerlane*, from his

^a *Almagest. l. 7 § 8.*

^b What the longitude and latitude of stars is, will be shewn hereafter.

^c *Præfat. ad tabb. Ulug Beighi.*

own observations, at *Samarcand*: the number of the stars contained therein is 1022, the places of the stars are reduced to A. D. 1437.

609 The noble *Tycho Brahe*^a tells us, that he had, by his own observations, rectified the places of 1000 stars, and that he put that number upon his globe of ten feet diameter; notwithstanding this, his catalogue published in his *progymnasmata*, A. D. 1602, a year after his death, and afterwards by *Longomontanus* in his *Astronomia Danica*, contains but 777 stars, with their places for the year 1600: we have therein some stars omitted by *Ptolemy*, though the whole number falls short of his, because several southern stars, which could be seen by *Ptolemy* at Alexandria in Egypt, are not visible in *Tycho*'s more northern situation at Uraniburg.

610 About the same time with *Tycho*, *William Landgrave* of Hesse, assisted by *Rothmannus* and *Byrgius*, observed the stars, and inserted 400 of them in a catalogue, rectifying their places to the year 1593.

611 *Bayer* in his *Uranometria* published a catalogue of 1160 stars, compiled chiefly from *Ptolemy* and *Tycho*, together with very beautiful figures of the constellations^b: one thing which makes this work valuable is this, that, besides describing the stars in the manner used by *Ptolemy*, every star is marked with some letter, in such a method that the biggest star of every constellation is denoted by the first letter of the greek alphabet, the next biggest star by the second &c: and when any constellation contains a greater number of stars than that alphabet will reach, those that remain are marked in like manner by letters of the Roman alphabet: by this means every star is as easily distinguished, as if it had a proper name given to it: thus, if mention is made of the star in the constellation of the ram marked by *Bayer* with the letter γ , every astronomer knows as well what star is meant, as if it were pointed out to him in the heaven: this invention is so useful, that *Flamsteed* has in his own catalogue taken in *Bayer*'s letters, as far as they go; as *Senex* also has done upon his globes of the largest size, as also upon his planispheres.

612 *Kepler*, who had the use of *Tycho*'s observations, published a catalogue of 1000 stars observed by *Tycho*, at the end of his *Tabulæ Rudolphinæ*; he adds thereto those stars of *Ptolemy*'s catalogue which *Tycho* had omitted, together with those of the new southern constellations, from other authors: so that his whole catalogue amounts to above 1160: their places are computed for the year 1600.

^a In *astron. instaurat. mechanica*.

^b *Flamsteed*, after others, justly finds fault with *Bayer*'s figures, for being inverted; it seems the author himself made the same complaint, laying the fault upon the designer or engraver. *Ricciol. l. 6. pag. 411.*

613 The celebrated *Hevelius* is next to be mentioned; he composed a catalogue of 1888 stars, whereof 1553 were observed by himself: the places of them were computed by him for the year 1660.

614 The largest and most complete of all is the British catalogue of the stars, owing to the labours of Mr. *Flamsteed*, Royal professor of astronomy at Greenwich; it contains no less than 3000 stars, many of which are so small that they cannot be seen without the help of a telescope, the places of them are rectified to the year 1689: we have it in the 3d volume of his *Historia Cælestis*, wherein, besides that of *Ptolemy* which is to be met with in a great many other books of astronomy, we have also the above mentioned catalogues of *Ulug Beigh*, *Tycho*, the *Prince of Hesse*, and *Hevelius*, reprinted; together with an account of each of them in the *Prolegomena*.

615 The greatest number of stars is to be seen in a winter's night, when the air is clear, and no moon appears: but astronomers^a tell us that a good eye can hardly reckon above 1000 stars at a time in the visible hemisphere: for though in a clear night without moon-shine they seem to be innumerable, this is a deception of the sight, arising from their strong sparkling, and our looking at them in a general and confused manner without reducing them into any order: whereas, if we come to view them distinctly, and consider a small part of the sphere of the heaven at a time, we shall not easily discover a single star but the same has been observed by some astronomer, and is inserted in his catalogue, and has found a place upon the celestial globes of the largest size. Some persons are able at the same time to see a greater number of stars than others, by reason of the difference there is in mens eyes: I have my self more than once seen seven, and a learned astronomical friend assures me he has seen eight stars among the *Pleiades*, where common eyes can discover but six, according to that often quoted verse of *Ovid*,

Quæ septem dici, sex tamen esse solent.

Which sev'n are call'd, though only six appear:

but this is nothing to what *Kepler*^b says of his tutor *Mæstlinus*, that he could reckon 14 stars in the *Pleiades* without any glasses, and of an ecclesiastic, who could in a clear night count to the number of 40 in the buckler of *Orion*. We must therefore, notwithstanding what was said in the beginning of this section, be forced to acknowledge the number of the stars to be vastly great, though there be no reason to think it infinite, as *Jordano Bruno*^c would have us believe.

^a Keil's *Astron. lect.* 6. v. Ricciol. *Almagest.* l. 6. p. 411.

^b ap. Ricciol. l. 6. p. 411

^c ap. Ricciol. *Alm.* l. 6. pag. 412.

616 An ordinary telescope will, in several parts of the heavens, discover ten times as many stars as are visible to the naked eye: *Hook* in his *Micrographia* page 241, says, that with a telescope of 12 feet he counted 78 stars among the *Pleiades*, and with a more perfect telescope was able to see a great many more: *Galileo* found 21 in a cloudy star in *Orion*, and reckoned at least 80 in the space between his belt and his sword, and above 500 in another part of him within the compass of one or two degrees square; so that he was discouraged from proceeding in his intention of observing all the stars of that constellation: this makes the account of *Antonius Maria de Rheita*^a not improbable, who affirms that he counted therein about 2000.

617 That part of the heaven which is called *the milky-way* is found by the telescope to owe its whiteness to a great number of stars scattered therein, too small to appear distinct to the naked eye: the like is true of several bright spots in the heaven, which appear like small white clouds: the greater perfection telescopes are brought to, the greater number of stars will they discover; and yet there is no question to be made, but that, whatever perfection they can be brought to, there will still remain out of their reach a great number of stars scattered through the vast abyss of immeasurable space, so that we have reason to believe, that only that infinitely wise and powerful Being who created them is able to tell the number of the stars, and to call them all by their names.

CHAP. 3. THE SOLAR SYSTEM: A PLURALITY OF WORLDS.

618 *The solar system* consists of the sun with the planets and comets which perform their revolutions round him: and may be thus described, in the midst of a vast cavity surrounded every way with stars, which are all at an immense distance from the sun and from one another, is placed *the sun*, a vast globe of fire, fixt as to his situation, whilst our earth with the five planets and several comets perform their revolutions round him, at different distances from him, and in different periods of time. The planets may be considered as so many earths, which are in different degrees enlightened and warmed by the sun; perhaps they may be inhabited by creatures both rational and irrational, though very different from those upon our earth. To an inhabitant of any of the planets our earth would appear like one of the rest of the planets: in this view, we may reckon the earth among the planets. *The names of the planets* are these, the nearest to the sun mercury, the

^a *In radio Sydereomyft.* p. 197 Ricciol. l. 6. p. 413. See Sherburne's *Notes upon Manilius* p. 157.

next to him in order venus, then the earth, mars, jupiter, saturn. For a further account of the names of the planets see the remarks at the end of this chapter.

619 *The distances of the planets from the sun* are, in round numbers, in the following ratio: if we imagine the distance of the earth from the sun to be divided into 10 equal parts, the distance of mercury from the sun is 4, the distance of venus 7 of those parts, the distance of mars 15, of jupiter 52, of saturn 95. How these proportional or comparative distances are found out, will be the subject of enquiry in another place; as will also what the real absolute distances of the planets from the sun are, in some known measure, and by what means we discover them. When we compare the distances of the several planets from the sun, in a looser and more general way, we call those superior which are further from the sun, those inferior which are nearer to him: thus, in respect of our earth, venus and mercury are *the inferior*, mars, jupiter and saturn *the superior planets*: in order to explain the appearances of the inferior planets, it will be sometimes sufficient to consider only one of them, the case of the other being so much the same, as not to want a particular explanation: in like manner, there is so great a similitude between the several cases of the superior planets, that an illustration of one will make it easy to understand the rest.

620 *The periodical times of the planets*, in which they go round the sun, are in round numbers, as follows: mercury goes round the sun in 3 months, venus in 7 months, the earth in 12 months, mars in 2 years, jupiter in 12 years, saturn in 30 years.

621 The abovementioned are called *primary planets*, to distinguish them from others of an inferior order; for some of the primary planets, in their revolutions round the sun, are attended with other globular bodies smaller than themselves, which are called *secondary planets*, *moons*, or *satellites*: thus, our earth is attended by the moon, jupiter has four moons or satellites, which are not visible without the telescope, and saturn has five, which cannot all be seen without the help of very long telescopes in a very clear air.

622 Though *the moon* be less than any of the primary planets, yet she is so much nearer to us, that she appears much larger than any of them do, and gives a great deal more light than all the stars and planets together: on account of this, and of her appearing nearly of the same dimensions with the sun, the sun and moon are called *two great lights*, and the moon is called *the lesser light*^a, as being vastly inferior to the sun in brightness as well as magnitude. The moon, by enlightning the darkness of the night, which she is therefore said to rule, as the sun is to rule the day, and by her various

^a Gen. 1. 16.

appearances dividing the year into parts, which from the moon are called *months*, is of great consideration to the inhabitants of the earth: not to mention at present the certain influence she has upon the tides, and that less certain upon the weather.

623 *The characters of the sun, moon, and planets*, which are often put to signify them, are these that follow: the sun ☉, the moon ☾, mercury ☿, venus ♀, the earth ⊕, or ⊖, mars ♂, jupiter ♃, saturn ♄. For an account of these characters see the remarks.

624 *The number of comets* which go round the sun is not yet ascertained: so many different ones have been observed, as to assure us there are at the least 20 of them, but of these there are only 3 or 4 whose periodical times and distances from the sun are certainly known: it is probable the number of comets belonging to the solar system is greater than was just now mentioned, for some of them may be too small to be seen without glasses, and the discovery of such must be owing to their accidentally falling within the telescope of some astronomer, who is looking at the heaven upon some other account; others, that in some part of their course approach near enough to the earth to be visible to us, may at the same time happen to be so near the sun, as for that reason to escape observation; for the splendor of his rays is such, that the brightest star or planet disappears when it is immersed therein, and requires to be at some distance from him, in order to be discernable to us.

625 *Every fixt star* may be a *sun* surrounded with its proper planets and comets: the fixt stars seem to be of the same nature with the sun, shining with their own light as he does, many of them may equal and even surpass him in magnitude; the reason of their appearing so small is their exceeding great distance from us. When we speak of a *plurality of worlds*, the word world does not signify the universe, as in the definition § 581, but is taken in a more restrained sense: for we may either mean that the several planets are habitable in like manner as our earth is, which in common speech we sometimes call the world; or we may by the world understand the whole solar system, and consider the *starry systems*, that is, the several fixt stars with the planets and comets belonging to each of them, as so many different worlds: in one or other of these senses some of the ancient philosophers, as well as the moderns, have held a plurality of worlds: but this speculation will be better resumed hereafter, when we have considered the planets of the solar system more distinctly, and seen how well they appear fitted for the habitation of animals.

Remarks

Remarks upon the names of the planets in the foregoing chapter.

626 The abovementioned names of the planets are the roman names of heathen gods: as idolatry was propagated at first from the east, we must search for the origin of both the greek and the roman names of their deities among the eastern languages, as learned men have done with good success^a. I make no question, but the planets had originally other names given them, which were expressive of the nature of their motions, or appearances, or of the influences they were thought to have: some ancient greek names, which we meet with in several authors^b, seem to be of this sort: there we find saturn, whose common greek name *χρονος cronos*, time, was given him upon account of the slowness of his motion^c, was also called *φαινωρ phaenon*, i. e. appearing or shining^d: he is thought to have received this appellation, because of all the planets he is the seldomest hid by the rays of the sun, or else, his light being the dimmest of them all, he was called so out of a superstitious custom the ancients had of giving good names to bad things for the sake of good luck: his ancient hebrew name is thought to be *khiun*^e. He was counted unfortunate, and of a malign influence, for no better reason, I believe, than for his being thought cold and dry by reason of his great distance from the heat of the sun and the vapours of the earth, and his appearing of a pale leaden hue: for the astrologers judged of the nature of the planets by their colour^f.

Φαινωρ phaetbon is an ancient greek name of the planet jupiter, it is taken from his brightness, *Homer* applies the same word to the sun as an epithet^g. His ancient hebrew name is thought to be *gad*, he has other names *khokbab zedec*, the star of justice, and *mazal tob*^h, good fortune: but these are of later date. The astrologers pretend he is of a benevolent nature, as being of a middle temperament, between the heat of mars and the coldness of saturn.

Mars was by the ancient greeks called *πυροεις pyrois*, fiery, his persian and arabic names, *axur* and *azer*, signify fireⁱ, all from his red fiery appearance; which I believe is the best reason the astrologers have for imagining this planet unfortunate, and particularly to be a promoter of war and bloodshed^k.

^a Vossius, Selden, Hyde. ^b *Autor de mundo* inter *Aristot. opera*, *Cicero de nat. deor.* l. 2. *Hyginus, Martianus Capella & alii.* ^c *Κρονος* dicitur qui est idem *χρονος*, id est temporis spatium, *Cicer. de nat. deor.* l. 2. ^d *Φαινωρ λαμπρος.* *Hefych.* ^e *כִּיּוֹן* Amos 5. 25. ^f *Stellarum naturas Chaldaei* *ficcas, frigidas, humidas, juxta colorum diversitatem deprehendisse sibi videntur, inquit Albumasar, introduc.* l. 4. c. 4. ubi plura hic spectantia. ^g *Ηελιος* *Φαινωρ* *Il.* λ. v. 734. ^h Selden. *de Diis Syr.* *Syntag.* 1. c. 1. ⁱ Hyde *de relig. vet. Persar.* pag. 63. ab *azer* fortasse *azni*, per metathesin, vel ab *אֶרֶז* *ariz, fortis,* v. Selden. *de Diis Syr.* *Syntag.* 1. c. 6. ^k *Astrologis stella jovis aut veneris conjuncta cum luna ad ortus puerorum salutaris, saturni martisque contraria.* *Cicero divinat.* l. 1. *prosperus & salutaris fulgur jovis, rutilus horribilisque martis.* *Cic. somn. Scipion.*

Venus was called *εωσφορος* *eosphoros*, the morning star, and *φωσφορος* *phosphoros*, the harbinger of light, when she appears in the morning, rising before the sun; and *εσπερος* *hesperos*, or the evening star, when she sets after him: under the first and last of these names she is mentioned by *Homer*, who justly styles her, as *Milton* does also, Fairest of stars^a — It may be observed upon this occasion, that both greek and roman writers frequently comprehend under the name of stars all the heavenly bodies in general, which appear as such to common observers: we do the same in english, complying herein with the vulgar, who are unacquainted with the difference between the stars and planets. Venus is thought, upon the account of her beautiful light, to have been the first of the planets to which religious worship was paid; she was adored in the east under various names^b. The astrologers count her fortunate, the ancients thought her to be moderately hot, by reason of her nearness to the sun, and moist, because they were such philosophers as to fancy she was near enough to the earth to be within reach of the vapours ascending from thence; upon account of these two qualities, they imagined this to be a prolific planet.

Mercury is generally so near the sun that he is seldom seen, this occasioned him to be unknown to the Greeks, and consequently unnamed by them, for some time; they called him *ελλαν* *stilon*, from his bright shining, his light being exceedingly vivid and sparkling beyond the rest of the planets: the astrologers think him to be of an indifferent nature, of good or bad influence according to the quality of the stars and planets with which he is in conjunction.

The sun was called by the Phenicians *beel famen*^c, the lord of heaven, being the most glorious of the heavenly bodies; his common greek name *ηλιος* *elios* is derived from *אל* *el*, one of the names of God; his most ancient Hebrew name is *shemeš*, a minister or servant, from his being constantly employed in the service of the world, say the lexicographers, but I rather think from his being the minister or servant of the creator, by whom he was made and appointed to rule the day, to divide the light from the darkness, and to distribute heat and light over the earth: he had other names from his heat, as *חמה* *khamma*, *pire*^d, or from his light, as *orus*, *phæbus*, *phanes*^e; the name *mithra*, by which he was known among the Persians, is by *Vossius*^f derived from *mither*, which in their language signifies great, but *Hyde*^g, whose

^a *Iliad*. X. v. 318. *Εσπερος ος καλλιστος εν ποσειδονι ισταται αστηρ.* ^b *Vossius Idololatr.* *Selden. de Diis Syris.* *Hyde de rel. vet. Persar. c. 3.* ^c v. *Selden. de Diis Syr. Syntag. 2. c. 1. Sol dux, princeps & moderator reliquorum luminum.* *Cic. somn. Scipion.* ^d *Dicunt Aegyptii solem πην, ab ignea virtute.* *Kircher. Prodrom. Copt. p. 146.* ^e *Orus ab אל lux.* *Φανης Sol, Voss de Idol. l. 2. c. 7.* *φως, καθαρς, λαμπρς.* *Hesych.* ^f *De Idololatr. l. 2. c. 9.* ^g *De rel. v. Pers. c. 4.*

consummate

consummate skill in the persian learning makes his opinion of great weight, derives it from *mibr*, love or mercy, because, says he, the whole world is cherished by the sun, and feels the effects, as it were, of his love: that learned writer, from their own books, has shewn at large that the Persians did not worship the sun, as they are by a great number of authors said to have done, but only considered him as a glorious and beautiful light, and the fittest symbol of the divinity which is to be met with among the visible parts of the creation; and therefore they paid him, not divine adoration, but only a civil respect, as they did also to the fire and other elements: and that their turning themselves towards the rising sun, or their sacred fire, in their devotions was the occasion of their being thought to worship them. And yet I am apt to believe that, whatever the wiser sort among them might practice themselves or teach others, many of the common people even among the Persians worshipped both the sun and the fire: the experience of all times and places shews how apt men are, especially the vulgar, to pass from the use of symbolical representations of the divinity to the worship of the symbols themselves. But whatever the Persians did, it is certain that other nations worshipped the sun by the name of Mithra: the Greeks are said to have had a temple at Alexandria where they offered human sacrifices to him, and there are several roman inscriptions remaining wherein Mithra is stiled the invincible, almighty and most holy god. *v. Hyde de relig. vet. Persar. p. 112.*^a

The names of the moon *σεληνη selene* in greek, and *luna* in latin, are both derived from words which signify light; one of her names in hebrew *לבנה lebanah* is from her whiteness, another *יָרֵאֲכָב yareakh* is, as some tell us, from a word which signifies to refresh; because she was thought to refresh the earth with her influences.

The greek names of the planets abovementioned are egyptian names expressed in greek words of like import; it is common when names have a known signification in their original language, for an author who writes in another language to express the meaning of them in words of a like signification, though of a very different sound: there are many examples of this in the translation of the old testament by the seventy. The words *φωνων, Φαεθων &c.* are manifestly greek, and yet these are expressly said to be egyptian names by the authors quoted in the margin^b.

627 We have in *Hyde* the chinese names from a learned native of China^c: some of them are taken from such things as they believe to be under the par-

^a *vide etiam inscriptiones a Grutero et Reinesio editas.*

^b Achilles Tatius *p. 136. ed. Petav. Manetho apud authorem chronici Alexandrini, p. 108. ed. Raderi, τοι γαρ λεγομενοι κρονου απερα εμελουν τον λαμπρυνται τον δε Διος τον Φαιθοντα τον δε Αρειος τον πυρωδη τον δε Αφροδιτης τον χαλλισεν τον δε Ερμου τον σελειοντα. c de relig. vet. Persar. p. 221.*

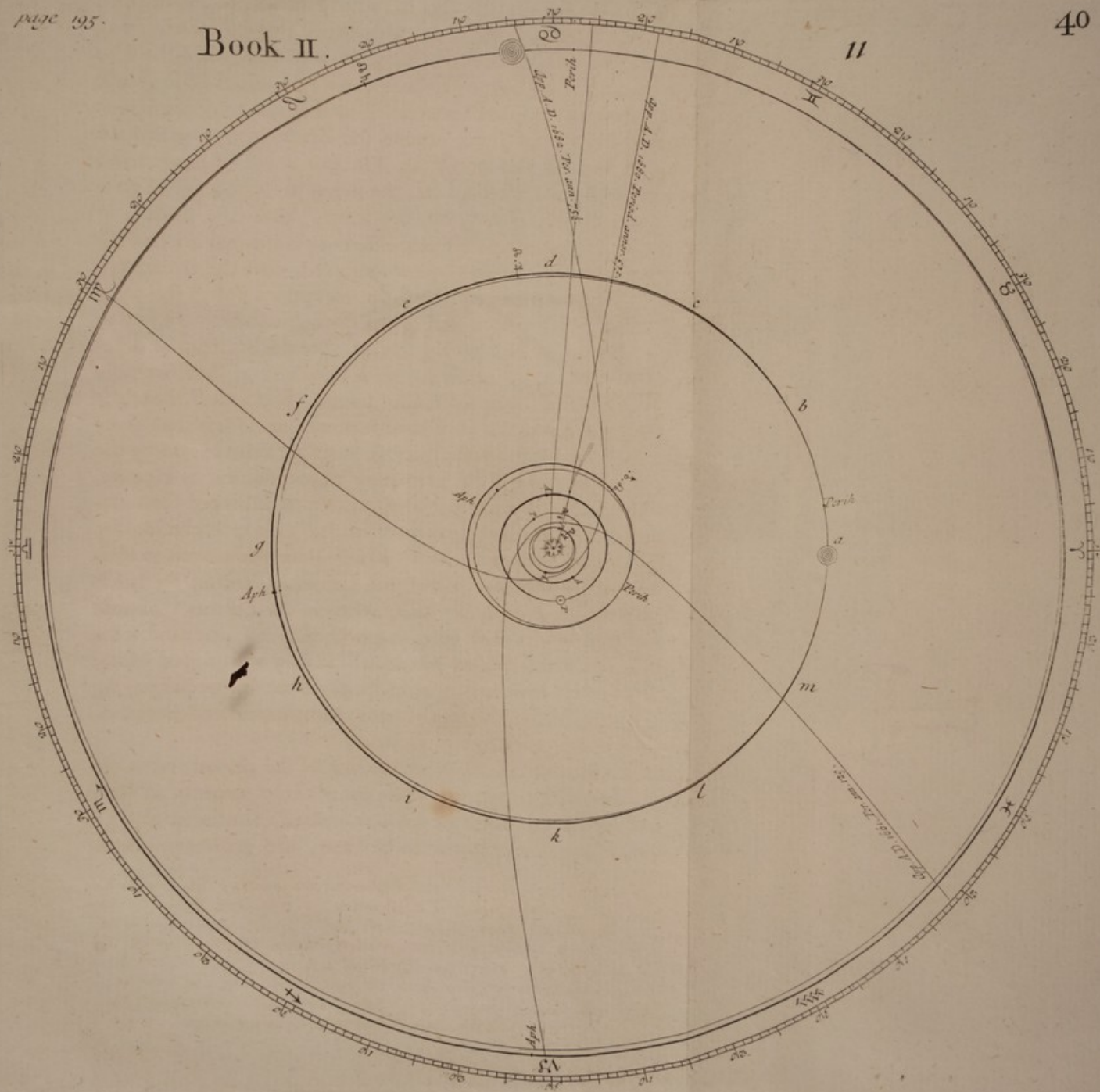
ticular dominion of each planet; thus, they call mercury *shiu* water: why that planet is thought to preside over that element we are not told, it may be perhaps from his being visible only when near the horizon, in the morning or evening, when the air is commonly moist: he is also called *tchin* the morning star: one name for venus is *kin* gold, another *tai pe* very white: they call mars *bo* fire, and *yungb* flame: jupiter is called *mo* wood, and *çui* a year; for what reason it is not very easy to guess: saturns name is *tu* or *tien* i. e. earth, perhaps from the dullness of his aspect.

628 The dominion of the heavenly bodies was fancied by superstitious antiquity to be so extensive, that the sun, moon and planets had each of them a particular constellation or portion of the heaven assigned to it, wherein when they were posited they were supposed to be most powerful in their influence; and were therefore said to be in their exaltation: as when in the opposite point of heaven they were said to be in their fall or detriment, and thought to be weak. Besides this, every one of them had its peculiar colour, metal, stone, tree, plant, fruit, flower, animal, number, day, hour, &c. assigned to it; from whence are derived the supposed virtues of amulets and talismans: for the dealers in these fooleries pretend that there is a sympathy between the heavenly bodies and those things which are under their dominions, so that, for instance, that plant, metal, colour &c. over which any planet presides has reciprocally a power of attracting the influences of that planet. Hyde *de relig. vet. Persar.* p. 63 & 129.

629 I have before observed, § 597, that zabiism or the worship of the host of heaven was the most ancient kind of idolatry^a, the custom of deifying dead men was later, though that also is very ancient^b: when this practice began, it was usual to make the people believe the departed hero was gone to take up his residence in the heaven, or in one of the heavenly bodies, which upon this occasion was consecrated to him^c: from hence came the *personal names of the planets*, as we may call those in common use, because they were taken from persons deified; though after the planets had received those personal names from deified heroes, the reverse of this became customary, and princes and great men frequently took their names from the planets^d. The most ancient author^e now remaining who mentions any names of the

a v. § Maimonid. *de Idololatria*, cap. 1. b Some think Misraim was the first who deified his father Ham, whose name **מִצְרַיִם** has the same signification with *Ζεύς* the greek name of Jupiter. *Cumberland's Sanchoniatho* p. 57, 156, 199. c We may see something like this practised in later times, in the deifying *Julius Caesar* and *Antinous*, v. Sueton. in *Julio*. c. 88. & Xiphilin. in *Adriano*. d v. *chronicon Alexandrin.* p. 84. 86. 89. 90. & Hyde *de relig. vet. Persar.* p. 65. e *Timæus Locrus de anima mundi, inter opera Platonis*, p. 99. ed. Serrani, & ex eo *Plato in Timæo*, p. 38. *autor de mundo, apud quem hæc occurrunt prope initium*, ο του Φαιδοντ^{ου}, αμα ημυ κρητου καλουμενος κυκλος, φειξος δε ο Φαιδοντς, Διος λεγομενος. μεθ' ο πυρις, ο Ηρακλειους π και Αριος ατταρρενομενος. εξος δε ο σιλωτ, οι ιερει Ερμιο υ καλουσι ειοι, πνις δε Απελλανος. μετ' οι ο του
planets,





planets, calls that nearest the sun the star sacred to Hermes, others called it the star of Apollo. Mars was by the Egyptians called the star of Hercules, by the Babylonians *Thurras*^f, *Homer* uses a word very like this last for an epithet of Mars as a deity^g, his name in the ancient Etruscan language was *Turan*^h, but it is not certain the planet was called by that name. Venus, which was by some named the star of Juno, among the Egyptians was called the star of Isis, as jupiter was the star of Osiris. The sun as one of the heathen deities had the name of Apollo, Phœbus &c. the moon as a goddess had great variety of names, *Isis*, *Diana*, *Hecate*, &c. See p. 179, note c.

630 This diversity of names arises from different persons having had the same star consecrated to them: it was before observed, that the Greeks applied those stories to their own heroes which the Egyptians did to theirs; it may be observed further, that a great deal of the confusion which we find in the histories of the heathen gods is owing to this, that the ancients, at different times, deified various persons under the same name: thus, there were several Mercuries, several Marses, several Venuses, several Apolloses &c.ⁱ, and such a number of Jupiters, that *Varro* is said to have reckoned up no less than three hundred^k. Again, when any of the eastern or northern nations are said by the ancients to have worshipped any of the grecian or roman deities, as Hermes, Jupiter, Apollo, Saturnus; we must not imagine they worshipped them under the greek or roman names, but the writer in that case calls the idol by the name of some deity of his own country, which came the nearest to it in the fabulous account of its actions, power and influence, and in the kinds of worship which used to be paid to it: thus, whereas the northern nations anciently adored the god whom they imagined to preside in battle, and to be the disposer of victory, a roman writer would tell us they worshipped Mars; in like manner, if a grecian historian were to speak of the Egyptians paying divine honours to Thoth, as the inventor of letters, arithmetic and several other arts, he would say they worshipped Hermes.

I shall conclude these remarks about the names of the planets, with observing that the consecrating *Cronus* into the planet saturn appears to have been mentioned in very ancient history^l, see *Cumberland's Sancboniatho* p. 37. that learned prelate, p. 113, takes *Cronus* to be Ham, and derives this name

Φερσεφονεύς, or Αφροδίτης, or δι' Ηραος ὠνομαζομένης.

^f *Chronicon Alexandrin.* p. 88. v. etiam Sal-

mas. *Plinian. exercit.* p. 1235. ^g Οὐρανός *iliad.* E. & *passim.*

^h v. *Gorii Musæum Etruscum* vol. 2. p. 113. *taran.* *Cambro-Britannis tonare significat.* *Voss. Idol.* l. 2. c. 33. *præter Jovem alios etiam Deos & in his Martem tonare, & sibi propria habere fulmina, credebant Etrusci.* *Plinius N. Hist.* l. 2. c. 20 & 52. *Voss. Idol.* l. 3. c. 8. *Gorius Musæi Etrusci* vol. 2. p. 79.

ⁱ *Cicero de nat. Deor.* l. 3.

^k *Jovis nomine fabulosa antiquitas dignata est reges & principes, qui sui ordinis ceteros potentia antistarent: itaque videas nullo prope seculo defuisse suum Jovem, usque ad tempora belli Trojani.*

Voss. de Idol. l. 1. c. 14. ^l *Philo. ap. Euseb. Præp. Evang.* p. 40 & 156. *ed. Vigeri.*

FIG. of him from רן a word which imports such an illustrious person as a king. *Rephas* is the name of saturn in the coptic language, *Kircher. Prodr. Copt.* p. 147, the seventy render *Chiun* in *Amos* 5. 25, by *Rephas*; some copies seem to have had *Rempban*, *Acts* 7. 43; the Bp. p. 221, derives it from רפא a word which imports that healthful constitution that produces long life, growth to great stature, and strength: this is more probable than what some learned men think, that the seventy read רן instead of רנן , the bottom of the letter נ being effaced in their copy. Ham was especially deified and adored of all the giants before and after the flood, the mightiest men of his race gloried in their likeness to him, and therefore affected to be called *Rephaim*. The Bp. p. 199, thinks *Sem* the best son of *Noah* to be the same as *Melchizedec*, that he was called *the just* (*Melchizedec* signifies a righteous king) and that some of his idolatrous posterity consecrated him into the planet jupiter, which was called מלכ *zedec* by the eastern people in ancient times, as is shewn by *Bochart* in his *Canaan* l. 2. c. 2. if the reader desires to see more about the names of the planets, he may consult the authors referred to in the margin^a.

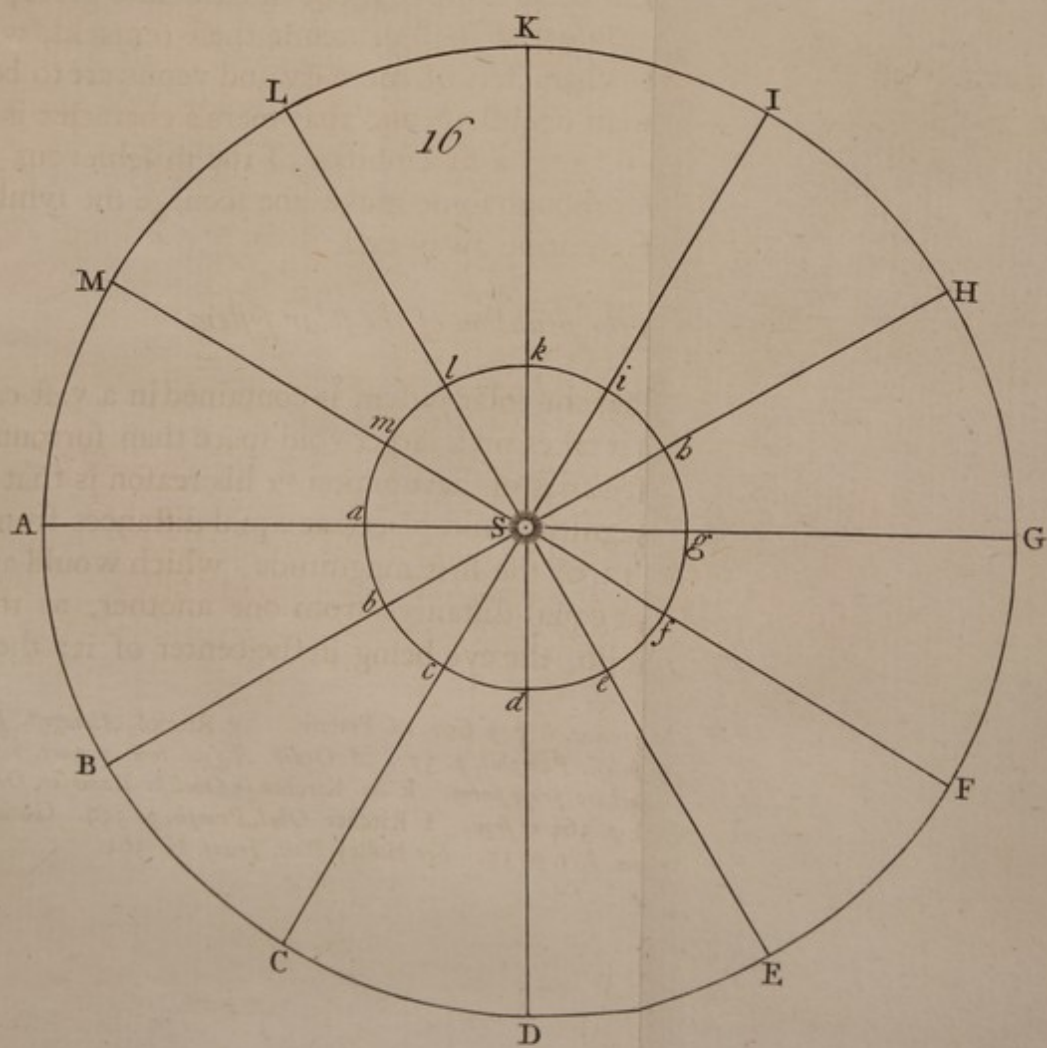
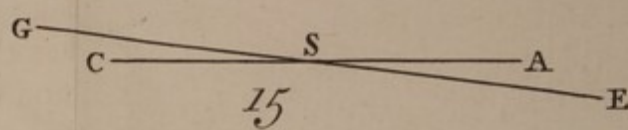
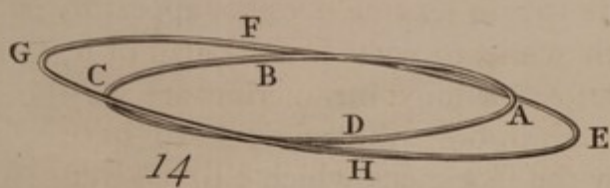
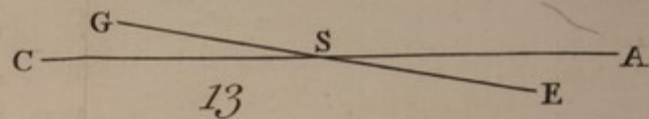
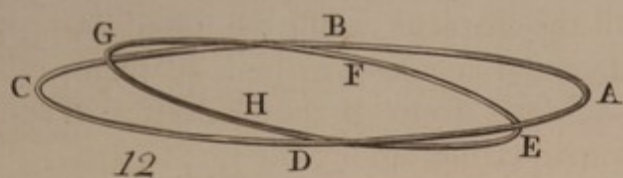
Remarks upon the characters of the sun, moon and planets.

631 The use of characters in astronomy is probably of great antiquity, especially among the prognosticators, who, besides the conveniency of its being compendious, would like it for giving a mysterious air to their writings: the character of the sun \odot is thought to be the picture of a buckler, whereof the middle point represents the *umbo* or boss, the bucklers of the ancients used to be bright, in order to dazzle the eyes of their enemies^b; *Hyde* tells us the Persians call the sun by a word which signifies a buckler^c: the character in manuscripts is often a buckler seen in a side view, often a cone; see 6 fig. 6, 7, 8, n. 4: a cone was sacred to the sun, *Porphyrius ap. Euseb. præparat. evangel.* p. 98, a circle is mentioned as an egyptian character of the sun, 8 by *Clemens* of Alexandria^d, as is also the crescent or common character of the moon \smile , which is the picture of the moon when she is about 4 days old.

632 As to the characters of the planets, the common opinion^e is that they were taken from the symbols of those deities whose names they bear, in this

^a Ricciol. *Almagest*. l. 7. c. 1. Kircher. *Prodrom. Copt.* p. 147. *id. Oedip. Ægypt.* tom. 2. part. 2. p. 168, 179. Selden. *de Diis Syris*. Vossius *de Idololatr.* Hyde *de relig. vet. Persar.* ^b *Hinc avaris* *Quoniam ab Homero passim vocatur, huc spectant illa militis gloriosi ap. Plautum act. 1. sc. 1.*

Curate ut splendor meo sit Clypeo clarior
Quam solis radii esse olim, quom fudum'it, solent:
Ut, ubi usus veniat, contra conferta manu
Præstringat oculorum aciem in acie hostibus.



view, the character of mercury ☿ is his *caduceus* or rod with serpents twist-
 ed about it, of venus ♀ a looking-glass with a handle, for such we are told
 the looking-glasses of the ancients were, the character of mars ♂ is a dart
 and buckler, of saturn ♄ a sickle, that of jupiter ♃ is generally agreed to be a
 Z the first letter of his name in greek, with a stroke through it as a mark
 of abbreviation. *Salmasius*^f will have all the characters to be the initial let-
 ters of the greek names of the planets, I cannot acquiesce in some of them,
 especially in his deriving the character of mars from ♂ the first letter of
 ♂υγεης, because that never was his common name among the greek astro-
 nomers. *Kircher*^g compounds the characters of the planets out of ☉ and ☿, a
 cross+ the mark used for the four elements, and ♈, the character of *aries*, which
 he says denotes fruitfulness; the manner of combining these marks into the
 characters of the planets may be seen figure 10: his scheme does not appear to
 me satisfactory enough to make it worth while to enter into a detail of it.
 One objection against it is that the characters in some manuscripts are very
 different from those in common use, though in other manuscripts they have
 a good deal of likeness, as may appear by the specimens which I have given
 of them partly out of a greek manuscript in the library of Trinity College
 in Cambridge of an astrological piece translated out of latin into greek, part-
 ly out of *Salmasius*^h and *Du Fresne*ⁱ. I shall conclude these remarks, when I
 have first taken notice that the characters of mercury and venus are to be met
 with upon some of the Egyptian obelisks^k, and that mars's character is often
 drawn with a scourge, as fig. 6. n. 3, a fit emblem of the mischievous influ-
 ence he was supposed to have, though some make the scourge the symbol of
 the *dii averrunci*^l, deities which drive away evil.

Remarks upon the situation of the solar system.

633 When I said, § 618, that the solar system is contained in a vast cavity,
 I did not intend to assert that it takes up a larger void space than surrounds any
 of the fixt stars, though *Kepler* is of that opinion^m; his reason is that if all
 the fixt stars were equal in magnitude and placed at equal distances from one
 another, we should see just 12 of the first magnitude, which would appear
 to us in a regular form and at equal distances from one another, as the angles
 of an icosaedron would do, the eye being in the center of it: the next

c *De rel. vet. Persar.* p. 106.

d *Stromat.* l. 5. p. 657. ed. Potteri.

e *Ricciol. Almagest.* l. 7. c. 1.

f *Plinian. exercitat.* p. 1237.

g *Obelisc. Pamph.* p. 37, & *Oedip. Egypt.* tom. 2. part. 2. p. 402.

h *Ubi supra.* i *In glossario græco-barbaro prope finem.* k ap. *Kircher. in Obelisco Rameseo, Oed. Eg.*

tom. 3. p. 161. v. etiam *Obelisc. Pamph.* p. 364 et sequ. l *Kircher. Obel. Pamph.* p. 359. *Gorius Musæi*

Etrusc. V. 2. p. 103. m *Epit. Astronom.* l. 1. p. 35. See *Halley Phil. Transf.* N. 364.

order

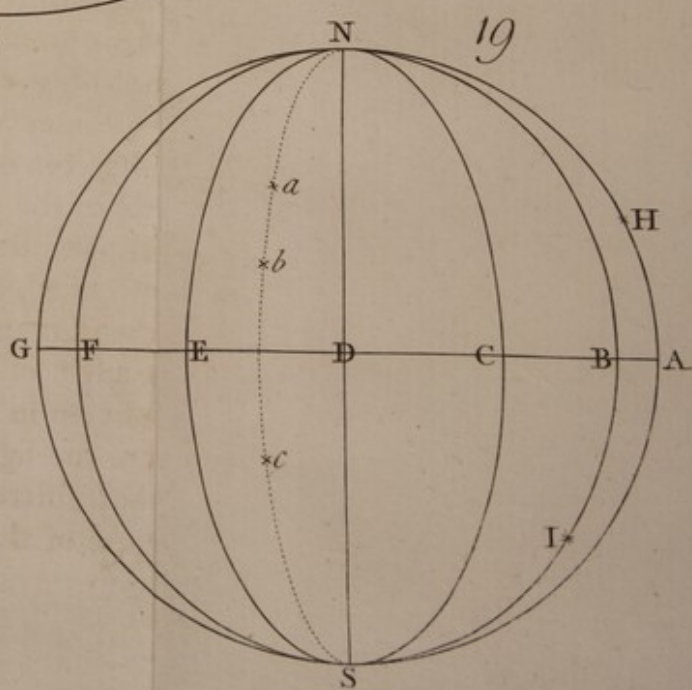
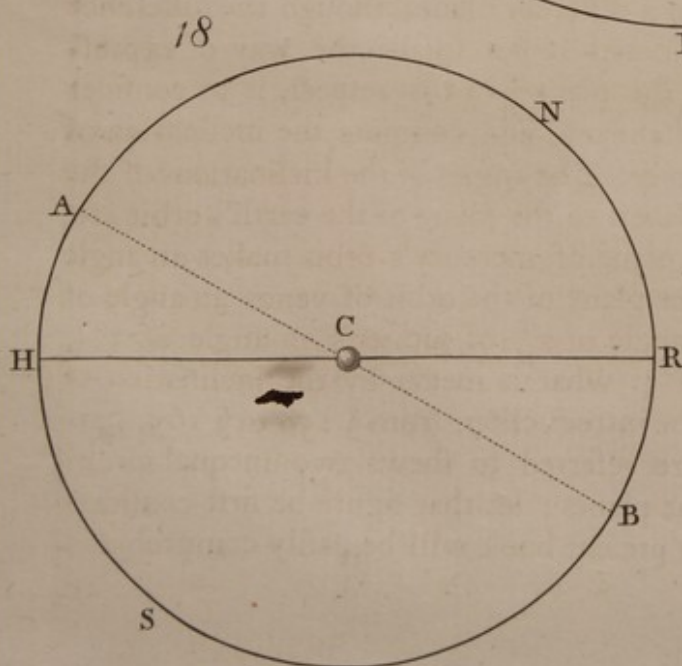
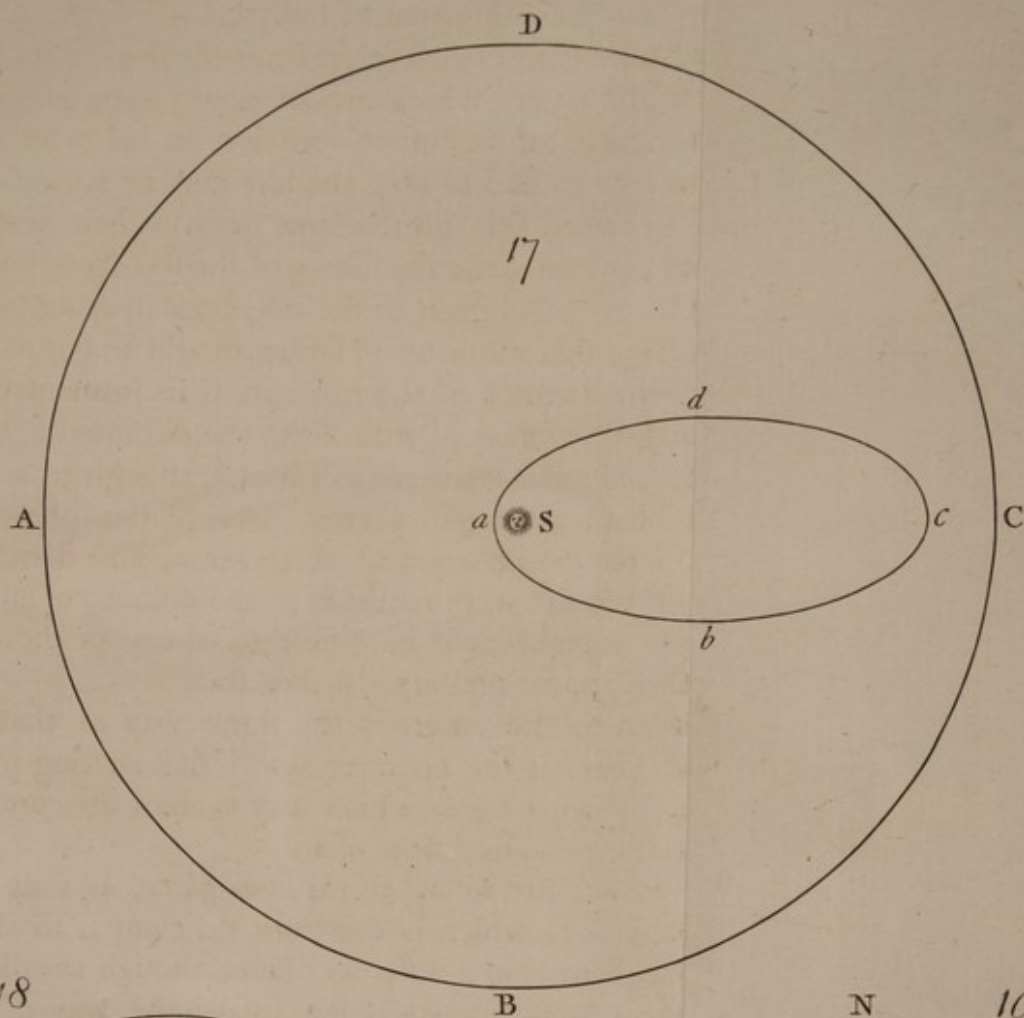
FIG. order of stars would be about 52, which would also appear regularly dispersed, at a distance double to the former: the distance of the third order of stars would be treble that of the first, and their apparent magnitudes would be proportionably diminished, and these also would be uniformly dispersed, and so of the rest: in short all stars of the same magnitude would appear uniformly dispersed upon the surface of the heaven; whereas this is so far from being the case, that in some parts of the heaven we see 3 or 4 large stars near to one another, but in other parts of equal dimensions, we find none but little ones. If we may offer our conjectures about things so far out of our reach, it is probable the several starry systems, as well as the stars in the centers of them, are very different from one another in dimensions, though they may be similar in their disposition, and in the uses they are designed for: we may observe a diversity as well as an uniformity in those parts of the creation which lye more open to our view: such a mixture of uniformity and diversity gives a beauty to the creation, displays to us the wisdom and power of the creator, and is a proof that this universal frame was neither the product of a fortuitous concourse of atoms, nor the work of different deities taking their several parts: there is uniformity enough to shew that all was brought into being and contrived by one mind, and there is variety enough to manifest the wisdom and power of that mind to be infinite.

CHAP. 4. THE ORBITS OF THE PLANETS AND COMETS.

634 The way that a planet or comet describes round the sun is called its orbit: *the orbits of the planets* may at present be considered as concentric circles, having the sun in their common center: in this view they are well enough represented in their several proportions by the 11th figure, where the sun is in the center, the least circle round him is the orbit of mercury, the next in bigness that of venus, the next to that the orbit of the earth, the orbits of mars, jupiter and saturn follow in order, and are easily distinguished, as well by their several dimensions as by being marked with the characters of their respective planets.

635 *The orbits of comets* are oblong ellipses, in which they go round the sun so as to pass quite through the planetary system: the three pieces of ellipses in the 11 figure represent a part of each of the orbits of those three comets whose periods and distances from the sun are known, the remaining parts of the ellipses are to be conceived extended a good way beyond the limits of the paper. Comets in their revolutions round the sun pass quite through





through the planetary system: and as in some parts of their orbits they come FIG.
very near the sun, much nearer than mercury, so in other parts they go to a
great distance from him, far exceeding that of saturn.

636 *The motion of all the planets in their orbits is the same way:* namely according to the order of the letters which are set down upon the orbit of jupiter, in the eleventh figure: thus jupiter's motion in his orbit is from II
a to *b*, from *b* to *c*, from *c* to *d*, and so on; the like may be said of the rest of the planets, they all go round the sun the same way in their orbits: the outermost circle of the figure represents the sphere of the fixt stars, but it was not possible to have this in any proportion to the rest, since in order to do that it ought to be drawn so large that the orbit of saturn should be but as a point in comparison of it; for the distance of the fixt stars is so immensely great, that if the orbit of saturn were removed from us to the distance of the nearest star, it would become insensible, that is its diameter, though prodigiously large in itself, would subtend an arc in a great circle of the sphere of the heaven too small to be liable to astronomical observation, and therefore we may easily conceive that if the sun were removed to the distance of any of the stars, no part of the solar system would be visible to us except the body of the sun, which would then appear no bigger than a star.

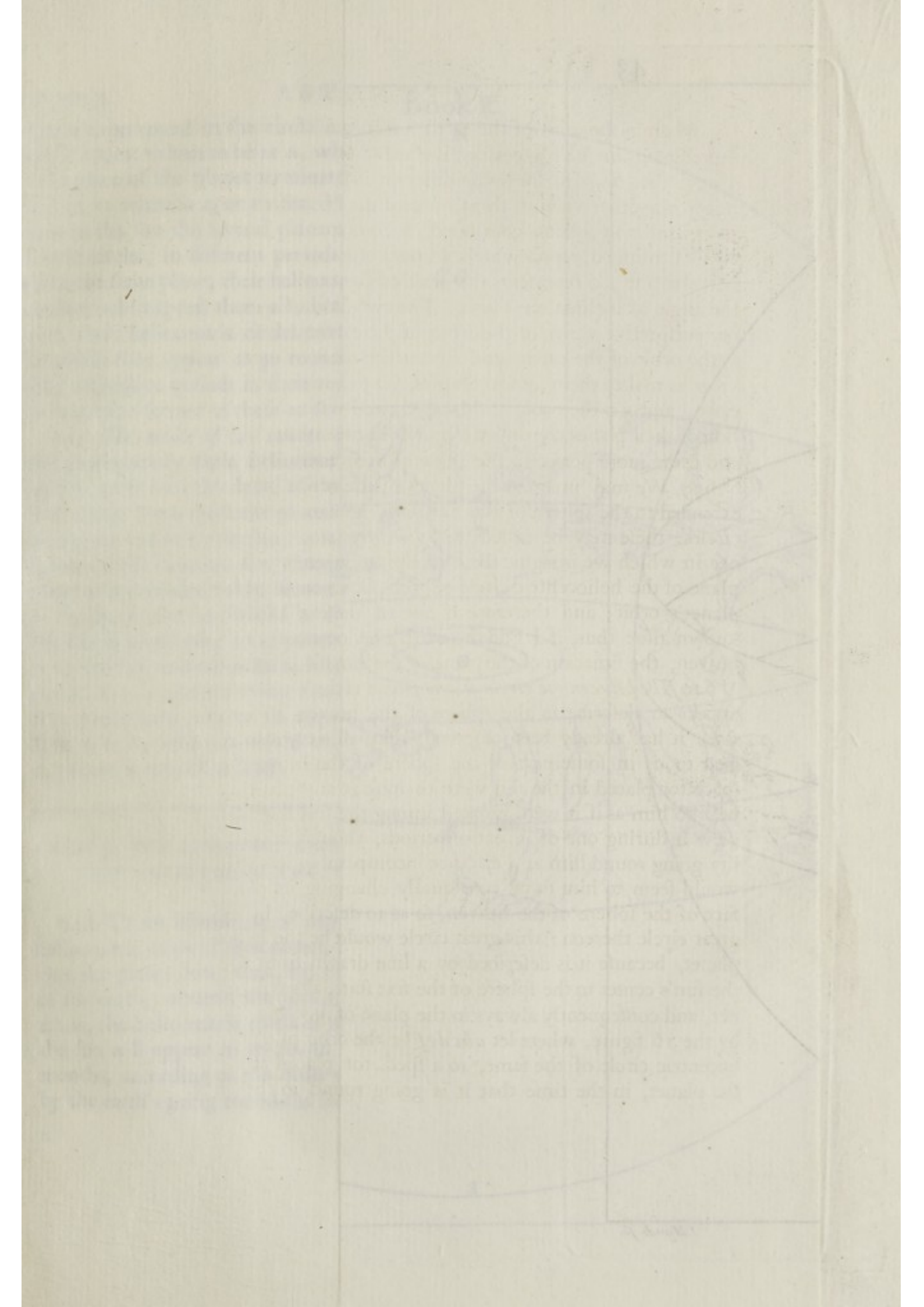
637 *The motion of some of the comets is the same way as that of the planets; the motion of others is the contrary way:* the writing upon the three orbits of comets in the figure shews which way each of the comets goes round the sun, and tells the periodical time of it.

638 *The orbits of the planets are not all in the same plane,* as they are represented in the eleventh figure, which is therefore deficient as to that particular, but every one of them is in a different plane, though the difference of the inclinations of their several orbits is but small: the way of expressing the situation of the orbits of the planets in this respect, is to consider the plane of the earth's orbit as a standard, and compute the inclination of the several planetary orbits from thence. The angles of the inclinations of the planes of the several orbits of the planets to the plane of the earth's orbit are, in round numbers, as follows: the plane of mercury's orbit makes an angle with the orbit of the earth of 7° , the plane of the orbit of venus an angle of $3^{\circ} \frac{1}{2}$, the plane of mars's orbit an angle of 2° , of jupiter's an angle of $1^{\circ} \frac{1}{2}$, the plane of saturn's an angle of $2^{\circ} \frac{1}{2}$: what is meant by the inclination of planes to one another is shewn in the introduction, from § 158 to § 165, particularly in § 164, where the figure referred to shews two unequal circles having the same center but different planes: let that figure be first considered, and then the 12 figure of this present book will be easily comprehend- 12
ed,

FIG. ed, wherein the orbit of the earth $ABCD$ and the orbit of venus $EFGH$ are represented in such a perspective view as makes them appear ellipses, by § 257, but so as to shew the different inclination of their planes. The 13 figure is another view of them, where the eye being supposed in the line of their common section continued, is consequently in the planes of both the orbits continued, upon which account they appear as straight lines, by § 258: here the line AC represents the orbit of the earth, EG the orbit of venus, and the angle of inclination is ASE . The two following figures are in like manner perspective views of the orbits of the earth and of mars; in fig. 14 $ABCD$ is the orbit of the earth, and $EFGH$ the orbit of mars, in such a perspective view as makes them appear ellipses, § 257: in fig. 15, AC is the orbit of the earth, and EG the orbit of Mars, viewed with the eye in the common section of their planes continued; by which means they appear straight lines § 258, and shew more perfectly the angle of their inclination, which is ASE .

639 We may imagine the planes of the orbits of the several planets to be extended to the sphere of the fixt stars, and to mark thereon so many great circles: these may be called *the heliocentric circles of the planets*. The manner in which we imagine these circles to be generated demonstrates that the plane of the heliocentric circle of each planet coincides with the plane of the planet's orbit; and therefore if one of these planes be found, the other is known also: thus, if I find the earth's heliocentric circle in the sphere of the heaven, the situation of the plane of the earth's orbit is thereby discovered.

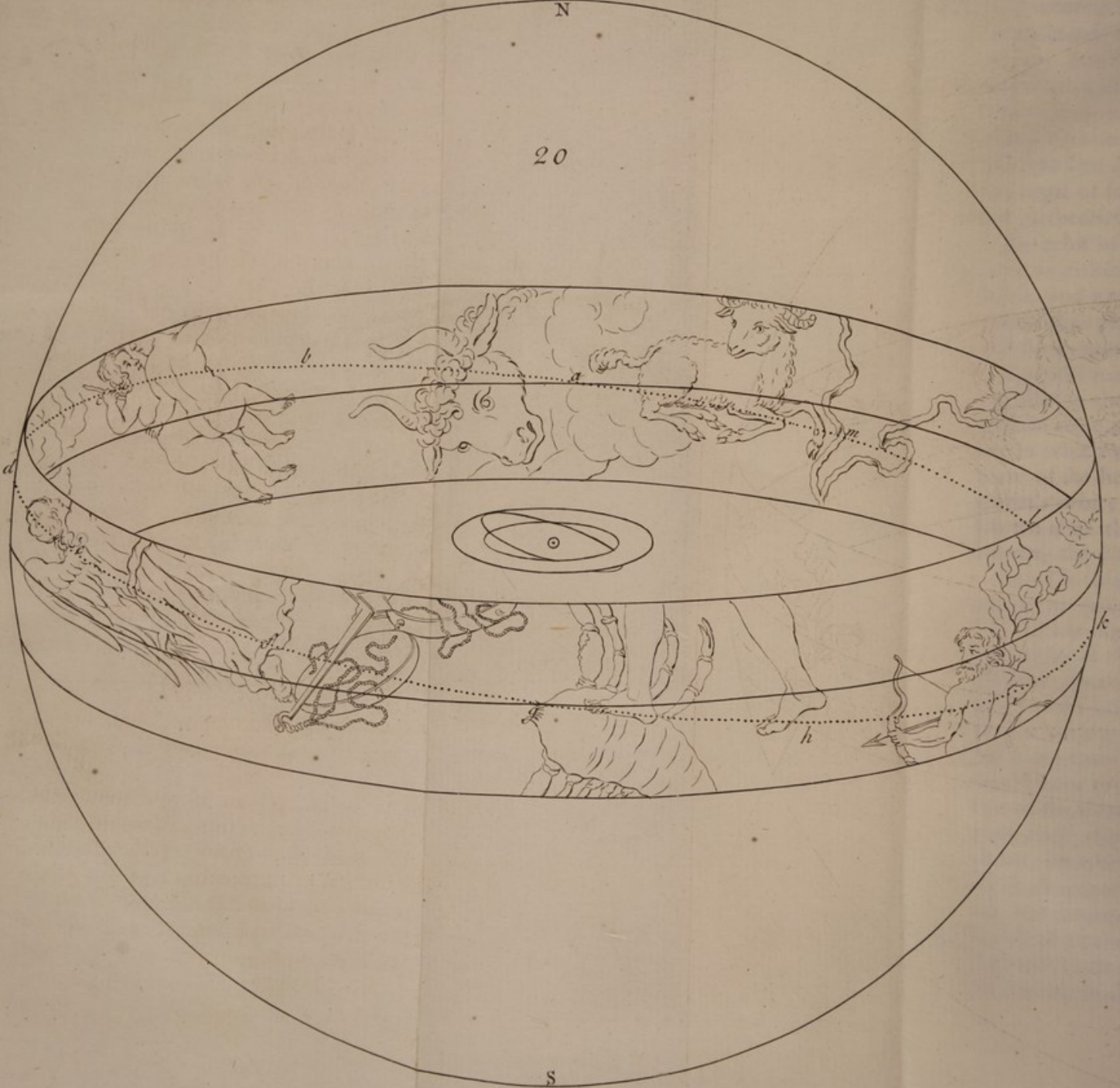
640 *The heliocentric circle of any planet* is the circle which the planet would appear to describe in the sphere of the heaven, to a spectator placed in the sun: it has already been observed, that all very distant objects always appear to us in some part of the sphere of the heaven, § 248; if therefore a spectator placed in the sun were to look at any planet, it would always appear to him as if it were situated among the fixt stars; and if he continued to view it during one of its entire periods, though it were all the while in reality going round him at a distance incomparably less than that of the stars, it would seem to him to be continually changing its place in the concave surface of the sphere of the heaven, so as to describe, by its apparent motion, a great circle thereon: this great circle would be the heliocentric circle of that planet, because it is described by a line drawn from the eye in the place of the sun's center to the sphere of the fixt stars, always passing through the planet, and consequently always in the plane of its orbit. This may be illustrated by the 16 figure, where let $abcdef$ be the orbit of a planet, $ABCDEF$ the heliocentric circle of the same, to a spectator placed at s the center of the sun, the planet, in the time that it is going round in the circle $abcdef$, would appear



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20



appear to go round in the circle $ABCDEF$: for when the planet is at a , it FIG.
 would appear to him to be at A , where a line drawn from his eye at s through 16
 a the place of the planet terminates: when the planet is at b , it would ap-
 pear at B : when at c , at C : &c. We may say then in general, that to a spec-
 tator in the sun the several planets would appear to describe different helio-
 centric circles, in different periods of time. If the orbits of the planets were
 all in the same plane, their heliocentric circles would all coincide, that is, one
 circle would express them all; but as the planes of their orbits are all diffe-
 rent, their heliocentric circles must be different also: and since their periods
 in which they appear to go round their heliocentric circles are the same as
 their respective periods in their real orbits, there must be the same difference
 between the former of these as there is between the latter.

641 *The orbits of the comets are in planes different from one another,*
 and consequently their heliocentric circles are different also: I shall not at
 present enter into the detail of their several inclinations, but leave it to ano-
 ther place. To a spectator placed at the sun every comet, if it could be seen
 during an entire revolution, would appear to describe a great circle on the
 sphere of the heaven: for, though its orbit be in reality a long ellipsis, the
 plane of it extended to the heaven would mark a great circle thereon where-
 of the eye at the sun would be the center; only, as the comet moves round
 the sun in an ellipsis, its apparent magnitude would increase as it came near-
 er to the sun, and diminish as it went farther off: thus, fig. 17, to a spectator 17
 at the sun s , a comet moving round in its orbit $abcd$ would appear to go round
 in a great circle $ABCD$: its apparent magnitude would increase as it went
 from c to A , where it would be greatest; and decrease as it went from A to
 c , where it would be least.

CHAP. 5. THE ECLIPTIC: ZODIAC: CIRCLES OF LATITUDE: HELIOCEN-
 TRIC LATITUDE OF THE PLANETS: THEIR NODES AND LIMITS.

642 To an inhabitant of any planet the sun appears to go round in the
 heliocentric circle of that planet, in the periodical time thereof, and the same
 way the planet does: thus, fig. 16, let s be the sun, $abcdefghijklm$ the orbit 16
 of the earth, wherein she goes round the sun in 12 months, $ABCDEFGHI$
 KLM , the heliocentric circle of the earth; I say to an inhabitant of the earth
 the sun will appear to go round in the circle $ABCDEFGHIKLM$, in twelve
 months, according to the order of the letters, from A to B , from B to C , &c.
 by the earth's going round the sun in the circle $abcdefghijklm$, in that time:

C c

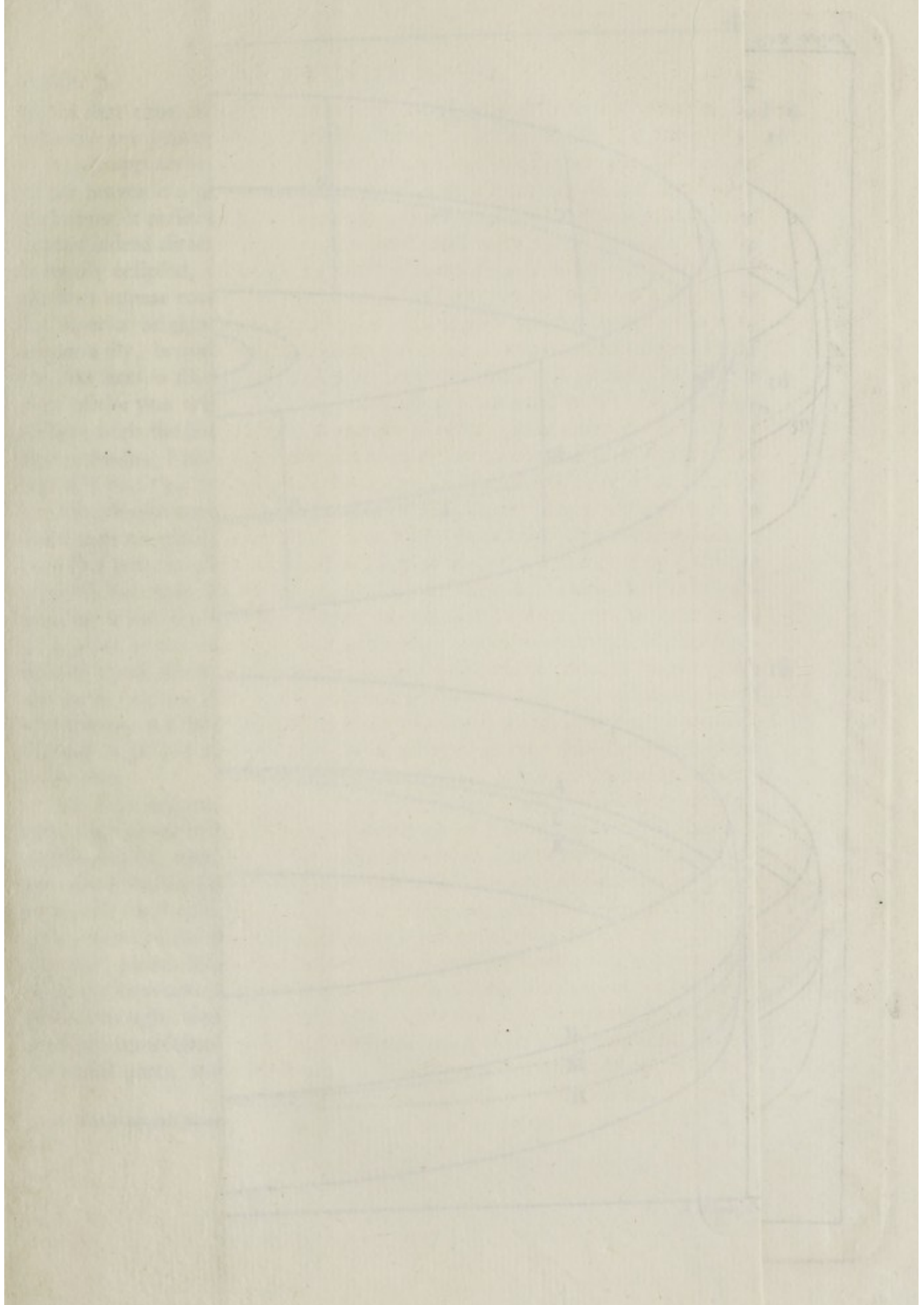
thus,

FIG. thus, when the earth is at *a*, if we draw a line from *a* through the sun at *s*,
 16 the point *G* in the sphere of the heaven where the line terminates is the place
 where the sun then appears to an inhabitant of the earth: in a months time
 the earth will be got from *a* to *b*, draw a line then from *b* through the sun,
 and where it terminates, at *h*, is the apparent place of the sun at that time:
 in like manner, if we draw lines from the earth in the 12 several situations
 in which she is here represented for the 12 several months, we shall find
 that when the earth is at *c*, the sun will appear at *i*: when the earth is at *d*,
 the sun's apparent place will be at *k* &c: in like manner if there be any in-
 habitants in venus, the sun appears to them to go round in the heliocentric
 circle of venus in seven months: the same is to be understood of the rest of
 the planets, if they have any inhabitants.

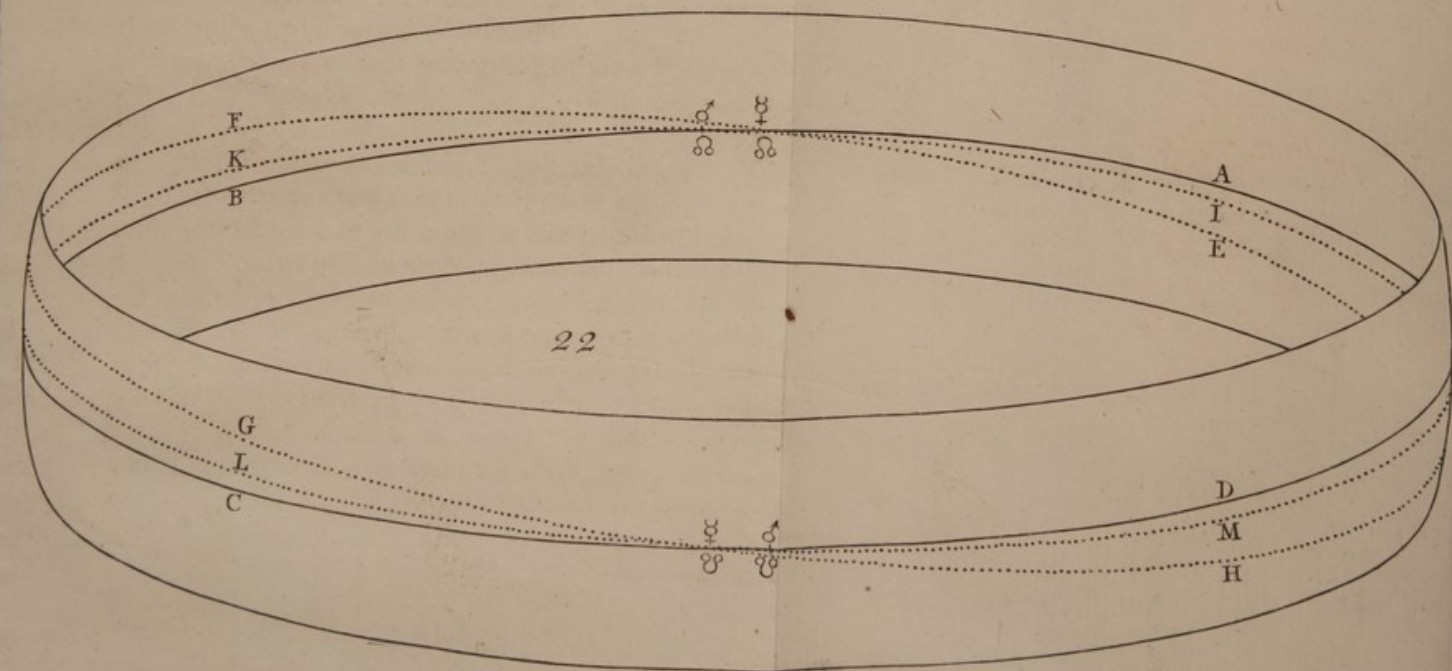
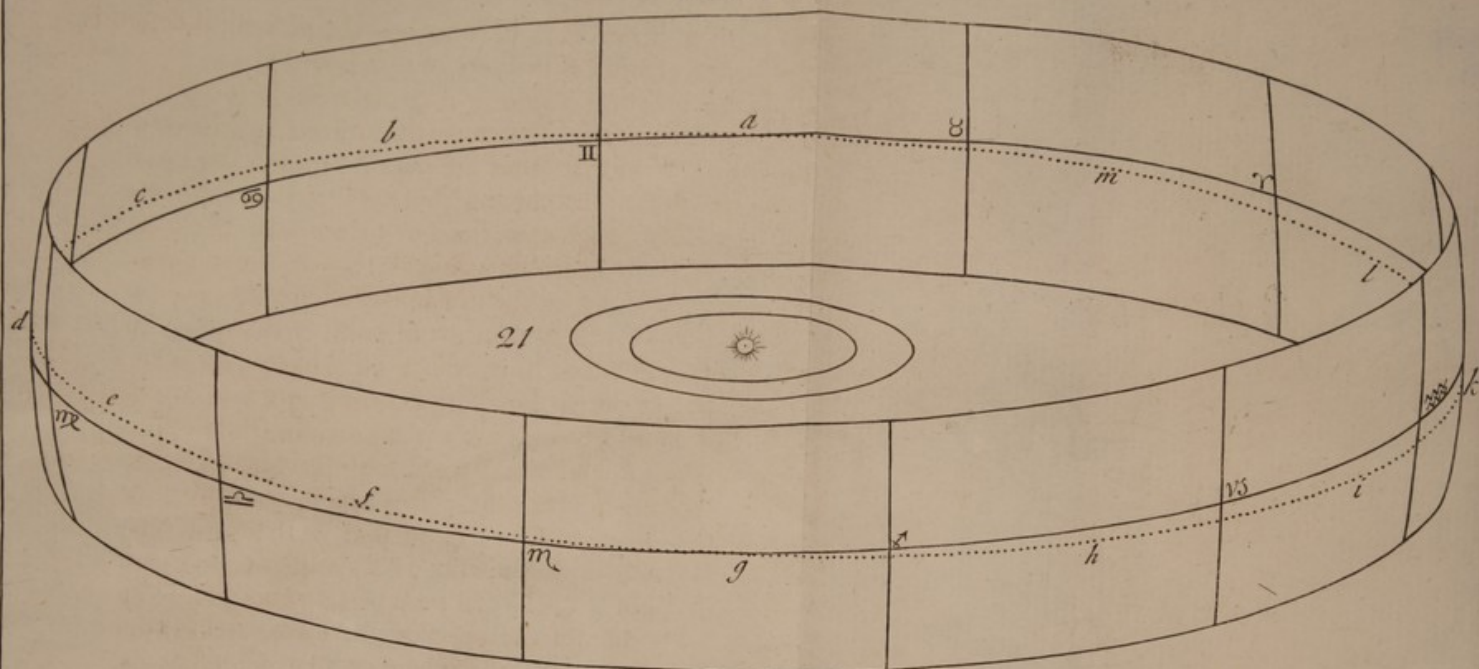
643 The heliocentric circle of the earth is called *the ecliptic*, because e-
 clipses of the luminaries cannot happen but when the moon is in or near that
 circle, as will appear hereafter, when eclipses come to be treated of: it is
 called by some greek writers κυκλος ηλιακος the circle of the sun, because it is
 described by his apparent annual motion: it is called κυκλος λοξος^a the oblique
 circle, because it is cut by the celestial equator at oblique angles: in *Ptole-*
my it has the name of ο διαμεισων των ζωδιων κυκλος the circle which passes through
 the midst of the animals, because the twelve constellations through which
 the ecliptic is drawn were anciently all of them animals or parts of animals;
 and are all so now, except the balance, which takes up the place that for-
 merly belonged to the claws of the scorpion. For this reason, a belt or hoop
 taken in the sphere of the heaven of the breadth of about ten degrees on each
 side of the ecliptic, twenty degrees in all, is called *the Zodiac*; from ζωδιον
zodion an animal: and the constellations through which the ecliptic is drawn
 are *the constellations of the zodiac*.

16 644 The ecliptic which is represented by the circle ABCD &c. fig. 16, is of
 great consideration to us: *the year* is measured by the time wherein the sun
 goes quite round the ecliptic, or rather appears so to do, by our earth going
 round him, whilst he all the while keeps his situation: the distance of the
 earth from the sun is so exceedingly great, that if the earth were viewed from
 the sun it would appear no bigger than a point, its diameter would be insensi-
 ble: from this point it is that we must always view the sun, and therefore
 all the inhabitants of the earth who see the sun at the same time, having their
 eyes as to sense in the same point, must see him in the same place among the
 fixt stars, or, to be the more express, in the same point of the ecliptic; and, if
 they observe him for the space of a year, will see him go round in the eclip-

^a Hinc Apollo vocatur Ασζηας; Aristoph. Plat. 1. 1. v. 2. & Suid.



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tic in that time, from A to B, from B to C, &c. — the periodical time wherein any planet goes round the sun is the year of that planet^a. FIG. 16

645 It appears by § 642, 643, that *the ecliptic* or path of the sun in the sphere of the heaven is a *fixt circle*, whose place therein is determined by the constellations it passes through, and the stars that are near it on each side: we cannot indeed directly see the sun's place among the stars, except when he is totally eclipsed, and his light is so screened from us by the moon as to let the stars appear round him, which otherwise would be rendered invisible by his superior brightness; but we may know the sun's place indirectly, or consequentially, because we can know at any time what point of the sphere of the fixt stars is diametrically opposite to him: thus, fig. 16, suppose it the time of the year when the earth is in that point of her orbit which is marked here with the letter *g*, if I know the point *G* is then diametrically opposite to the sun, I know *A* which is opposite to *G* is the sun's place: and therefore if I find that in the course of the year the points *GHIKLMABCDEFGHI* are successively opposite to the sun, I may be assured that in that time, the sun's apparent place was successively in the opposite points *ABCDEFGHIKLM*: now the point in the heaven opposite to the sun may be known at 12 every night in the year, because he is then in the opposite meridian, and his depression below the horizon is easily found, and whatever that depression is, that point in the heaven is opposite to the sun which is then in the meridian as much above the horizon as the sun is below it: thus fig. 18, let *c* be the earth, *SHAN* the meridian of the observer continued to the sphere of the heaven, *HR* his horizon, *RB* the depression of the sun, at 12 at night, suppose it 30° ; I say the point *A* which is 30° high above *H* is opposite to the sun. 18

646 The ecliptic being thus found, we may imagine as many great circles as we please to be drawn upon the sphere of the heaven, perpendicular to the ecliptic, and intersecting each other at the poles of the ecliptic: these are called *secondaries of the ecliptic*, or *circles of latitude*. The *latitude of a star or planet* or any of the heavenly bodies is its distance from the ecliptic in degrees, minutes and seconds, measured upon a circle of latitude drawn through that star, planet, &c. and may be *northern* or *southern*. The ecliptic is drawn upon the common celestial globe, by which we may see what constellations it passes through, there are also usually 6 circles of latitude, which by their mutual intersections shew the poles of the ecliptic, as well as divide it into 12 equal parts, answerable to the number of months in a year: fig. 19 is 19

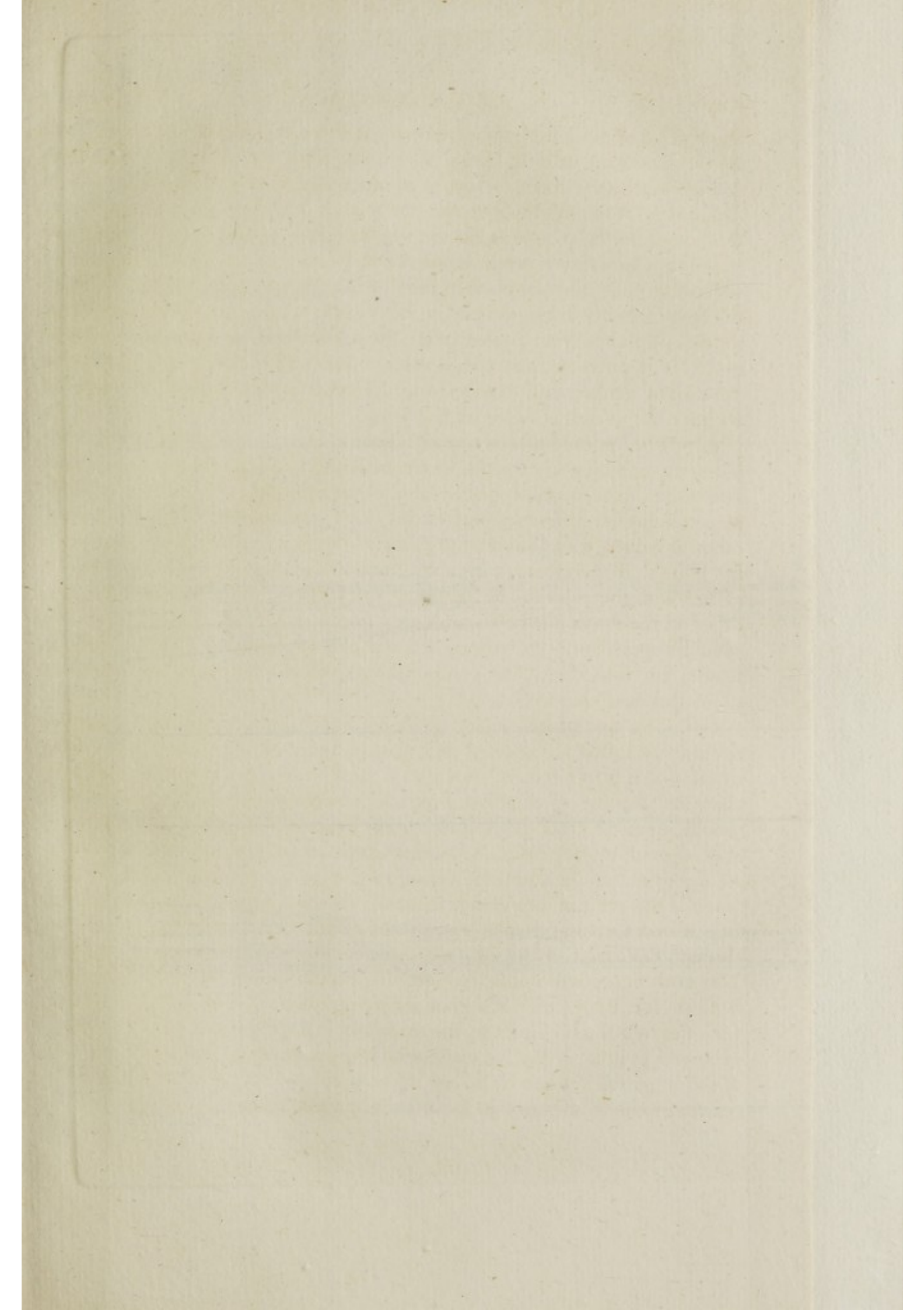
^a *Periodus cujusvis planeta ipsius planeta annus vocatur.* Macrobi. in somn. Scipion. l. 2. c. 11.

FIG. the picture of a celestial globe, where AG is the ecliptic, N the north, s the south pole of the ecliptic, NAS, NBS, NCS, NDS, &c. are circles of latitude, or rather halves of them, which is as much as can be seen at one view upon the convex of the solid globe: the star H is in so many degrees, minutes and seconds of north latitude as the arc HA amounts to, the star I is in south latitude, the quantity whereof is measured by the arc IB. The different notions of latitude when applied to stars in the heaven or places upon the earth must be carefully distinguished: in the heaven, or upon the celestial globe, latitude is distance from the ecliptic; upon the earth, or upon the terrestrial globe, it is distance from the equator: sometimes indeed we consider the distance of the heavenly bodies from the celestial equator, but this is called declination, as has been already said § 339.

647 The latitude of a planet is either heliocentric or geocentric; the *heliocentric latitude of a planet* at any time is the latitude it would appear in at that time, to a spectator placed in the sun: the heliocentric circles of the planets being all different great circles, any two of them must divide each other in halves, § 177; and consequently intersect one another in two opposite points: if therefore we consider the heliocentric circle of any planet together with the ecliptic, we shall find one half of it is on the north side, the other on the south side of the ecliptic: so that any planet seen from the sun would appear in north latitude, for one half of its period; and in south latitude, for the other. The geocentric latitude of the planets will be treated of in the next chapter.

648 The two points where the heliocentric circle of any planet cuts the ecliptic are called *the nodes* of that planet: and that which the planet passes through as it goes into north latitude is *the ascending node*, whereof this is the character Ω : the opposite point to this, where the planet seen from the sun would appear to cross the ecliptic, going into south latitude, is its *descending node*, expressed by this mark \oslash : a line drawn from one node to the other is *the line of nodes*: the common section of the plane of the ecliptic and the plane of the orbit of any planet extended to the sphere of the heaven is the line of nodes of that planet: the planes of the orbits of all the planets pass through the center of the sun, and therefore all their lines of nodes do so too.

20 The 20th figure will make the foregoing particulars very easy to be comprehended, it is the picture of a glass sphere representing the sphere of the heaven, the two small ellipses in the middle of it are the orbits of the earth and of mercury, in a perspective view, which makes them appear ellipses, § 257; of the larger ellipses the black one is the ecliptic, the pointed one the heliocentric circle of mercury in a perspective view also, N is the north, s the south

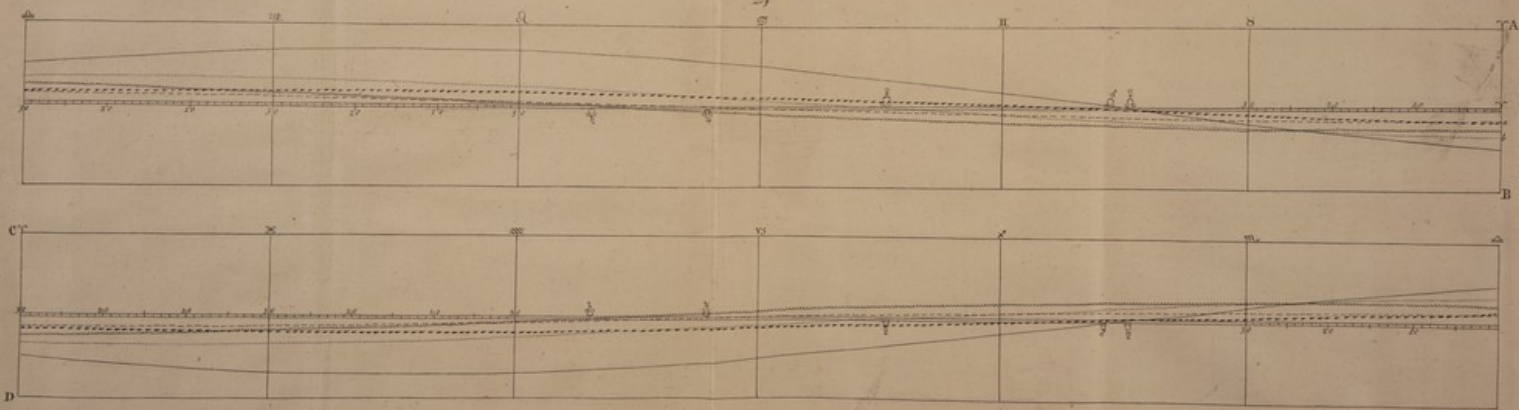


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- a black line $\frac{1}{2}$
- - a dotted line $\frac{1}{2}$
- - - a pointed line $\frac{1}{2}$
- - - - a dashed line $\frac{1}{2}$
- - - - - an indented line $\frac{1}{2}$

24



south pole of the ecliptic, the way of mercury's heliocentric motion is shewn FIG. by the order of the letters *abcdefghiklm*: now suppose the eye at *s* the center of the sun, as mercury goes from *a* his ascending node, he would appear to deviate more and more northwards from the ecliptic, till he comes to *d*, where he is in his greatest north latitude, or *northern limit*: from thence his latitude gradually diminishes, till he comes to his descending node *g*: from thence he gradually deviates more and more southward from the ecliptic, till he comes to his greatest southern latitude or *southern limit k*: from thence he draws nearer to the ecliptic, till he comes again into it, at his ascending node. the heliocentric latitude and motion of a superior planet may be shewn by a figure like the preceding: fig. 21 is the picture of the zodiac taken out of a 21 glass sphere, the two small ellipses in the middle of it are the orbits of the earth and of mars, in a perspective view; of the larger ellipses the black one is the ecliptic, the pointed one the heliocentric circle of mars, whose motion is according to the order of the letters *abcd*, &c. his ascending node is at *a*, his descending at *g*, his northern limit at *d*, his southern at *k*.

649 From the two preceding figures, one of which shews the heliocentric motion of a superior the other that of an inferior planet, the heliocentric motions of all the planets are easily understood: to a spectator at the center of the sun they would appear in the ecliptic only when in their nodes; in all other parts of their orbits their north or south latitude would be greater, the nearer they are to their north or south limits, where their latitude would be greatest of all: the greatest heliocentric latitude of each planet is the same as the inclination of its orbit to the orbit of the earth: thus, the greatest heliocentric latitude of mercury is 7° , of venus $3^{\circ}\frac{1}{2}$, of mars 2° &c. see § 638.

650 The heliocentric circles of the several planets cut the ecliptic in different places: thus, the ascending nodes of mercury, venus, &c. are all in different points of the ecliptic; this difference arises from hence, that the common sections of the several planetary orbits and of the earth's orbit are all different, and consequently their several lines of nodes are different also. To make this plain, let fig. 22 be the zodiac taken out of a glass sphere, upon 22 which is drawn the ecliptic *ABCD*, the heliocentric circle of mercury *EFGH*, the heliocentric circle of mars *IKLM*, all appearing ellipses, as being drawn in a perspective view, § 257: the figure shews their ascending nodes to be in different points of the ecliptic; the same figure shews their descending nodes and limits to be in different points also.

651 I shall now shew the heliocentric circles of all the planets upon the zodiac: they should have been delineated in different colours, if they could have been printed in that manner from copper plates; but that being impracticable,

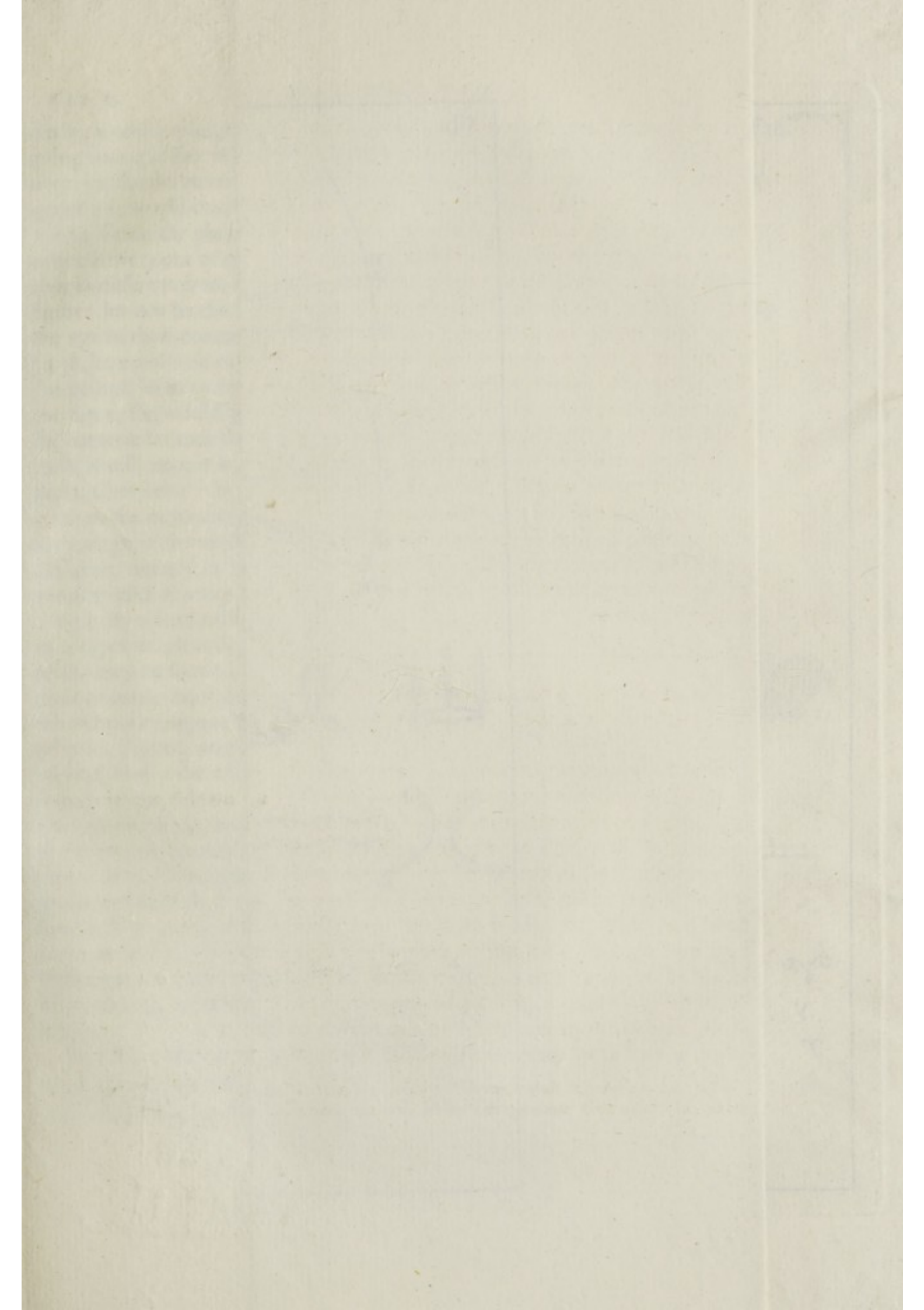
- FIG. practicable, I have, to avoid confusion, drawn them in lines of different kinds, to which I have therefore given the different names of black, dotted, 23 pointed, dashed, indented lines; as may be seen fig. 23. This being premised, imagine the ecliptic and the heliocentric circles of all the planets to be drawn upon a glass sphere in these different sorts of lines; the zodiac, or an hoop taken in that sphere of the breadth of ten degrees on each side of the ecliptic, would be more than broad enough to contain them all: imagine this hoop to be cut open in a circle of latitude, and to be unbent into a plane; it would then be an oblong parallelogram, whereof one side represents the zodiac as it appears upon the concave, the other as upon the convex surface of the sphere: 24 the 24 figure shews the zodiac drawn upon the concave surface of the sphere, and thus unbent into a parallelogram $ABCD^a$: the double line in the middle is the ecliptic, divided into 12 equal parts: every twelfth part has a piece of a circle of latitude drawn through it, which in this view is a strait line perpendicular to the ecliptic: the black curve shews the heliocentric circle of mercury: the dotted curve the heliocentric circle of venus: the pointed curve that of mars: the dashed curve the heliocentric circle of jupiter: the indented curve that of saturn: the places of the ascending and descending nodes, and of the north and south limits of the several planets, are easily seen upon their respective curves.

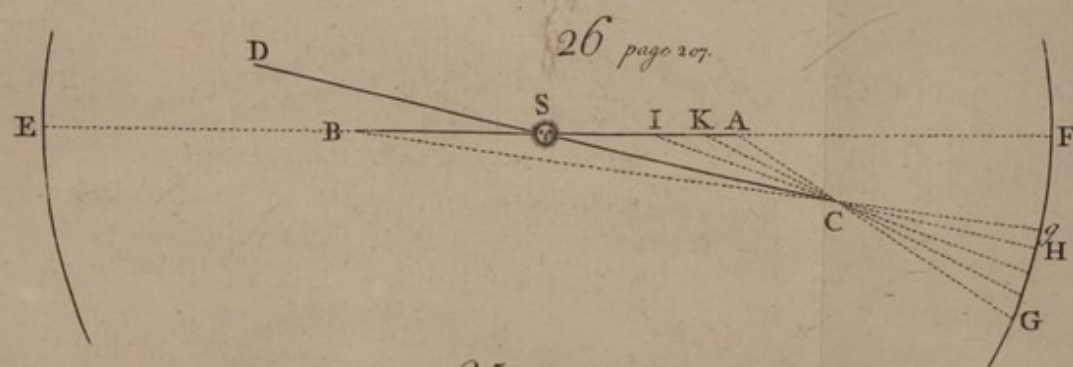
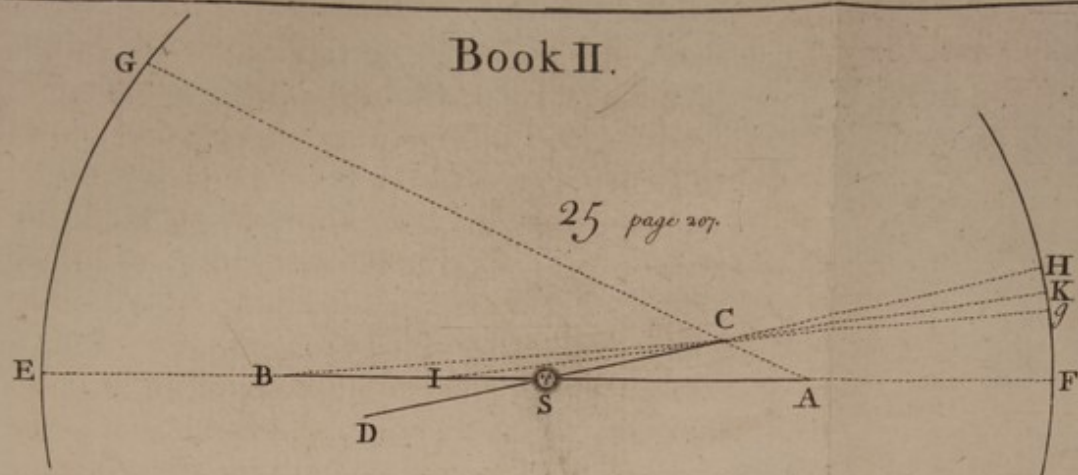
CHAP. 6. THE GEOCENTRIC LATITUDE OF THE PLANETS: SIGNS
OF THE ZODIAC: LONGITUDE OF THE HEAVENLY BODIES.

652 *The geocentric latitude of a star or planet* is the latitude it appears in to an inhabitant of the earth: the sphere of the fixt stars is so immensely great, that compared therewith the orbit of the earth is but as a point: that is, the diameter of the earth's orbit at the distance of the fixt stars would be insensible, too small to be measured by astronomical observation: for this reason, whatever part of her orbit the earth is in, the stars always appear to us in the same position as they would do if we were to view them from the sun; and consequently there is no difference between the geocentric and heliocentric latitude of the stars. The case is otherwise with regard to the planets: the orbit of the earth bears a considerable proportion to their distances from us; and therefore their apparent places in the heaven are altered by our change of place, as well as by their own motions: so that if a planet were to stand still

^a No part of a spherical surface can form a plane, and therefore an hoop taken out of a sphere would not unbend into an exact parallelogram; but the difference is inconsiderable in the case before us.

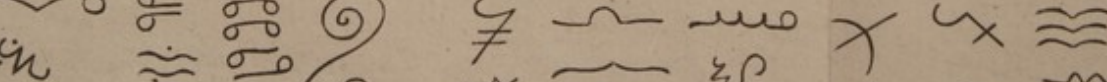
during





25 page 212.





during a whole year, our continual change of place, occasioned by the earth's going round in her orbit, would cause an apparent change of place in the planet: we should be continually viewing it from different stations; and consequently it would continually be seen in different points of the heaven. FIG.

653 Since the plane of the earth's orbit is different from the planes of the orbits of every one of the planets, the geocentric latitude of any planet is almost always different from its heliocentric latitude, as will appear by the following figure: let AB be the orbit of the earth, CD the orbit of venus, viewed with the eye in their common section continued, whereby they appear strait lines, § 258, let E and F be two opposite points of the ecliptic: suppose venus to be in the point c in her utmost north limit; if she were at that time viewed from the sun s , she would appear in the sphere of the heaven at the point h ; her heliocentric latitude then is fh ; but let venus in c be viewed from the earth, and she will appear in different places, according as the earth is in different parts of her orbit: for if the earth be at B , a line drawn from thence through c shews the apparent place of venus to be at g ; and her geocentric latitude is then fg : if the earth be at A , the apparent place of venus will then be very different, namely in G ; and her geocentric latitude EG , if the earth be at I venus would appear at k , &c. 25

654 By a figure similar to the preceding, the different geocentric latitude of a superior planet, when viewed from the earth in different parts of her orbit, may be shewn: thus fig. 26 let AB be the orbit of the earth, CD the orbit of mars, both viewed with the eye in their common section continued, whereby they appear strait lines by § 258, let E and F be opposite points of the ecliptic: suppose mars to be in his south limit at c if he were at that time viewed from s the center of the sun, he would appear in the sphere of the heaven at the point h ; his heliocentric latitude then is fh : but let mars in c be viewed from the earth, and he will appear in different places according as the earth is in different parts of her orbit: for if the earth be at B , a line drawn from B through c shews the apparent place of mars to be at g ; and his geocentric latitude is then fg , if the earth be at A , the apparent place of mars will be in G ; and his geocentric latitude EG , if the earth be in any other part of her orbit, as at the point I or k , it is easy to see, by a line drawn from either of those points through mars at c , that he will appear in different places in the sphere of the heaven, and would be in different geocentric latitudes. 26

655 *The zodiac is astral or local^a*: the astral zodiac is divided into twelve unequal parts, because it consists of the twelve constellations, some of which

^a Gemini *elementa Astronomiæ* p. 1. ed. Petav. Cassini *astronomie Indienne* p. 37. ed. Par.

are larger and take up more room in the heaven than others; as will appear upon a view of the common celestial globe: the astral zodiac always continues invariably the same, because the same stars go now to the making up the figure of the ram, the bull and the rest, as did anciently; except some little alterations already mentioned, § 594. The local zodiac is divided into twelve equal parts, each containing 30° , which are called *signs*, or *dodecatemoria* twelfth parts, beginning at the point where the ecliptic is cut by the celestial equator near the constellation of the ram, of which point this is the mark γ , and proceeding in order according to the apparent annual motion of the sun in the ecliptic. The first sign, containing 30° measured upon the ecliptic, is called aries: the second, containing also 30° , taurus: &c. *signum* a sign is often put to signify any constellation in ancient writers; but those of the zodiac being thought of principal consideration, custom has prevailed to restrain the name sign to the parts of the zodiac in the manner just now mentioned. *The order, names, and characters of the twelve signs* are as followeth: 1 aries γ , 2 taurus δ , 3 gemini Π , 4 cancer $\var�$, 5 leo Ω , 6 virgo \varnothing , 7 libra ζ , 8 scorpio \mathfrak{M} , 9 sagittarius \dagger , 10 capricornus ♄ , 11 aquarius ♁ , 12 pisces ♈ . Aries is made the first sign of the zodiac, because when the sun enters the first point of aries, which is about the 10th of march old style, it is the vernal equinox to all who live in north latitude, see § 366: this is reckoned the beginning of the astronomical year, because it is the entrance into the spring, when our earth begins to appear with a new aspect; the plants growing thereon rise again from a sort of death which they suffered by the coldness of the winter; and the animals, by the returning warmth of the sun, are invigorated with new life and strength.

656 Any *motion* in the heaven from any part of γ towards δ , from δ towards Π , &c. is said to be *according to the order of the signs*, in latin *motus in consequentia*, sc. *signa*, motion towards the following signs: such is the apparent annual motion of the sun in the ecliptic; such also would the motions of all the planets appear to be in their several heliocentric circles, if we were to view them from the sun. Any motion the contrary way to this, as from any part of γ towards ♈ , from ♈ towards ♁ , from ♁ towards ♄ is said to be contrary to the order of the signs, *motus in antecedentia*, i.e. *signa*, motion towards the preceding signs: such is sometimes the apparent motion of the planets seen from our earth; as will be more fully shewn hereafter.

657 The local zodiac is not always invariably the same as to the places of the several signs, though the whole zodiac always takes up the same place in

* These latin names are so much in use with english writers that they seem adopted into our language: the english of them may be seen page 174.

the sphere of the heaven, without any alteration; namely ten degrees on each side of the ecliptic, which circle was before said to be fixt and invariable: the points where the celestial equator cuts the ecliptic have a retrograde motion of about $50''$ in a year: that is, the celestial equator every year cuts the ecliptic in two points which are distant $50''$ contrary to the order of the signs from the intersecting points of the year immediately preceding: this change of place of the first point of the ecliptic, from whence the signs begin to be reckoned, occasions a like change of place in the signs themselves; which, though not sensible in a few years, in a long tract of time becomes very considerable: thus, since the first cultivating of astronomy by the Greeks above 2000 years ago, the first point of the ecliptic is removed backward above a whole sign; and, whereas it was then about the middle of the constellation of the ram, is now about the middle of the constellation of the fishes: notwithstanding this, it retains its ancient mark γ ; and the signs keep their ancient names, though they be every one of them got into different constellations from what they were in anciently, when they received these denominations: thus, the first sign is still called aries, though it be now in the constellation of the fishes: the second sign is called taurus, though in the constellation of the ram: the third sign gemini, though in the constellation of the bull: and so of the rest.

658 The local zodiac is generally meant, when we speak of the zodiac: the ancients made the breadth of the zodiac but twelve degrees^a, sufficient to take in the moon in her greatest deviations on either side of the ecliptic, which amount to near six degrees of latitude; as will be shewn when the moon's motion comes to be treated of: the moderns make the zodiac twenty degrees broad^b, in order to take in mars and venus in their greatest geocentric latitude, in which they appear to us near ten degrees north or south from the ecliptic. See fig. 22 and 24.

659 *The longitude of a phenomenon in the heaven* is the number of degrees, minutes and seconds contained upon the ecliptic, between the first point thereof and the point where it is cut by a circle of latitude drawn through the phenomenon: the longitude is reckoned beginning at the first point of the ecliptic, and proceeding according to the order of the signs.

660 Some writers, as *Copernicus* and *Streete*, make the first point of the ecliptic invariable: in order to this, they imagine a circle of latitude to be drawn through a star in the head of the ram, which is marked by *Bayer* with the letter γ ; where that circle cuts the ecliptic is the first point from whence they reckon the longitude; and the star, from its use, is called the first star of the

^a Plin. *nat. hist.* l. 2. c. 15. Manilius l. 1. v. 680. Proclus *de Sphaera*. Martiaus Capella, l. 8. c. de *signifero*.
^b Kepler. *epit. astronom.* p. 159. Gregor. *astron.* l. 2. prop. 1.

FIG. ram: when this method is made use of, the longitude of a phenomenon is said to be so many degrees, or so many signs, degrees, minutes and seconds from the first star of the ram: thus, in *Streete's* Caroline tables, the longitude of jupiter's ascending node is two signs and eight degrees from the first star of aries: which is thus exprest, *Long.* ♈ ♀ 1 * γ 2^s. 8°.

661 The common way of reckoning the longitude of a phenomenon in the heaven is, to take γ for the first point of the ecliptic, and not to number the degrees quite round that circle in a continued series, but to make a new beginning at the first point of every sign, and to reckon from thence, according to the order of the signs, only as far as 30, the number of degrees contained in a sign: the degrees of the ecliptic upon the common celestial globe are numbered in this manner; as are also the degrees of the ecliptic in 24 the 24th figure. When this method is made use of, the longitude of a phenomenon is exprest by saying it is in such a degree and minute of such a sign: thus, in the picture of the zodiac in the 24th figure, the longitude of the ascending node of mercury is 8 14° $\frac{2}{3}$.

662 Any phenomenon in the heaven, in whatsoever part it be, whether near 19 the ecliptic or not, is referred to some sign: thus, fig. 19, all the space contained upon the celestial globe between the semicircles of latitude NGS and NFS is within the sign aries; all between the semicircles NFS and NES belongs to the sign taurus, &c: in what degree of the sign any phenomenon is, a circle of latitude drawn through it determines, as was said before: thus, fig. 19, the stars *a*, *b*, *c*, are in 15° of ♈.

663 The place of a phenomenon in the heaven is exprest by setting down its longitude and latitude; saying it is in such a degree and minute of such a sign, and in such a degree and minute of north or south latitude: in this manner the places of the stars are set down in the catalogues; as are also the places of the heavenly bodies in the common almanacks: — by the place of any of the heavenly bodies is meant the place of its center.

664 Since the ecliptic is a fixt circle, § 645, the latitudes of the fixt stars, and of all phenomena in the heaven whose places are fixed, continue always invariably the same: since the point γ hath a retrograde motion of 50" a year, the longitudes of the fixt stars and of all phenomena whose places are fixed are altered 50" a year, in the common way of beginning to reckon the longitude from γ: and this is the reason why, in the several catalogues of stars calculated for different times, we find the longitudes of the same stars to be different; but their latitudes to be the same: as was mentioned § 607.

Remarks

Remarks upon the signs of the zodiac in the foregoing chapter.

665 The ancient Egyptians had several divisions of the heavens, and particularly of the zodiac, to which the rest of the heaven was supposed to have relation; and accordingly assigned a greater or less number of deities to preside therein, all under the direction of the supreme being: one division of the heaven was into two hemispheres, one on the north, the other on the south side of the ecliptic: one division of the zodiac was into the four parts which the sun occupies during the four seasons: another was into the twelve signs, wherein they imagined some of the most powerful genii to reside; I suppose because their principal deities the sun moon and planets are always to be found among those signs: observing that the earth with the plants and inhabitants thereof are differently affected at different times of the year, they imagined this to be owing to the different natures and influences of the signs through which the sun passes; and therefore they represented those signs by hieroglyphical figures expressing their supposed natures and influences: they did not think this enough, but subdivided every sign into three parts, so as to make thirty six parts in all, every one of these parts also was supposed to have its particular genius of a lower order, upon which account the influences of the heavenly bodies were thought to be different in different parts of the same sign^a: they had also another way of dividing so much of the heaven as was referred to one sign into four parts, so as to make 48 in all; which was therefore the number of the constellations; to this number the Greeks long after superstitiously adhered, though the figures of the constellations are of such different dimensions, as by no means to answer to the original regular division of the heaven into 48 parts: I should not have mentioned these particulars, but only to shew upon what kind of notions judicial astrology seems to have been originally founded; namely upon a belief that the stars were ministerially concerned in the government of this sublunary world, by virtue of their influences, which were under the direction of inferior deities; to whom therefore the ancients used to address themselves, in prayers, sacrifices, and other superstitious rites, in order to draw down their benevolent influences, or avert those that were hurtful and unfortunate.

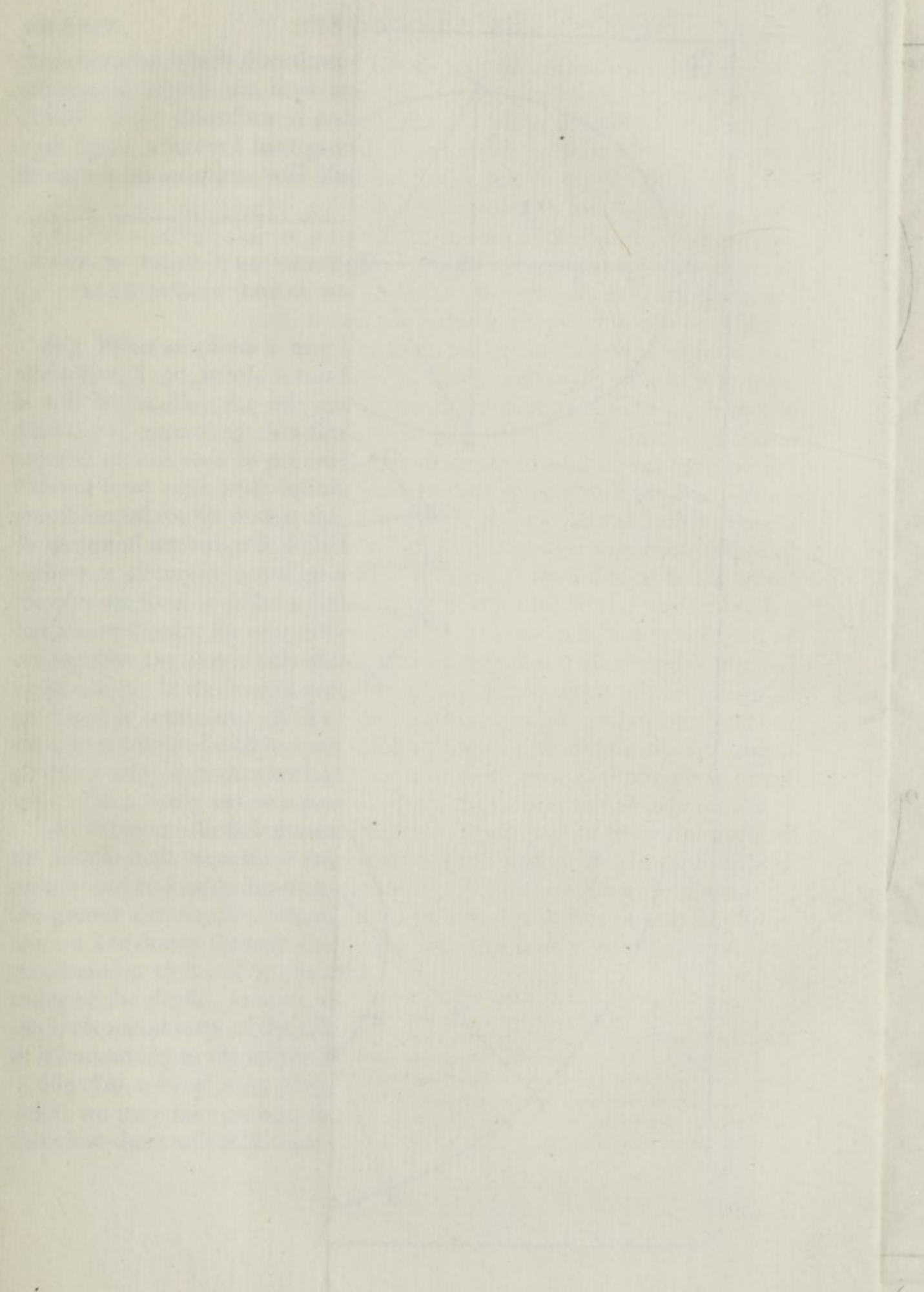
Remarks upon the characters of the signs.

666 These characters are thought to be very ancient^b; they are most of them drawn so as in some measure to resemble the figures they are intended

^a Kircher. *Oed. Æg. T. 2. part. 2. c. 2, 3, & 5.* Scalig. *in Manil. p. 371.* ^b Scaliger. *ap. Kirch. loco citat.*

FIG. to signify^a, as may be seen fig. 25, where I have given, chiefly from *Kircher*^b,
 25 such pictures of the constellations of the zodiac, or of parts of them, as serve
 best to shew the likeness of the characters to the originals from whence they
 were taken: γ represents the horns of the ram: δ the head and horns of
 the bull: Π the figure of the twins, as drawn according to some authors
 joyning hands and feet, or rather as thus described by *Plutarch*^c; the Spartans,
 says he, call the ancient statues of the *Dioscuri* $\delta\epsilon\kappa\alpha\upsilon\alpha$ beams, they are two
 pieces of wood joyned together by two cross pieces—these words are plainly
 a description of the character Π , as others have also observed^d. The charac-
 ter Θ is a little different from what we find it in manuscripts, transcribers
 have brought it pretty near to the numeral figures 6 and 9; it is not unusual
 for people to alter characters as well as words which they do not understand
 to something that is familiar to them; *Kircher* thought the Egyptians in-
 tended by this mark to represent the head of the ibis and the tail of the crab;
 I have given his pictures of them, though I am not of his opinion: I rather
 think it was the intention of the inventor of this character to represent the
 change of the sun's declination from north to south, by two lines drawn so
 as to point contrary ways: Ω is the tail of the lion, a symbol of his courage,
 for he is said to lash himself with his tail when he is angry^e: \mathfrak{M} was origi-
 nally the three ears of corn which virgo used antiently to be drawn holding
 in her hand: Ξ is the beam of the balance: \mathfrak{M} was at first a picture of the
 scorpion which hasty transcribers have gradually changed into the figure now
 in use: \mathfrak{A} is the arrow of the archer; in manuscripts we often meet with
 the bow and arrow: \mathfrak{B} , which is the character for capricorn used by the I-
 talian, French, and most European writers, represents in some measure the
 figure of the constellation; for which reason I have chosen it here, rejecting
 \mathfrak{W} in common use with us, as a barbarous character, which owes its original
 to a bad imitation of the other: \approx is a natural representation of the wavy
 surface of water: \times is the picture of two fishes tyed together back to back:
 in manuscripts we have sometimes instead, of this, two fishes heads. I shall
 only add that Ξ and \approx are to be met with upon the ancient Egyptian obe-
 lisks now at Rome, whereof *Kircher* has published the draughts^f: that ano-

^a Salmaf. *Plin. exercit.* p. 1236. ^b *Oed. Æg. T. 2. part. 2. c. 3.* ^c *De fraterno amore, initio, δεκαυα παρα-
 γογίς α δεικνύει. διος παύσατο δεικνύει, λονισος, η δεχομένη δελαδη τα σπινυμνα* Eustath. in *Il. ε. v.* 744. ubi &
 hunc locum Plutarchi laudat. ^d *Xylander in locum. Palmerius exercitat. in Plut.* Shuckford *sacr. and
 propb. hist.* vol. 1. p. 348. It may be observed here that before sculpture was in use the ancient idolaters
 made use of rough hewn logs of wood or stone for images of their gods; by degrees they gave them hu-
 man shape, but still with their legs joyned together: *Dedalus* was the first who attempted to form them
 with their legs asunder, and was therefore said to make walking statues. *Gorius Musæi Etrusci* vol. 2.
 p. 32. ^e *Homer. iliad. γ. v.* 170. *Plin. nat. hist. l. 8. c. 16.* ^f *Obelise. Pamphil. & Oed. Æg. T. 3.*



ther character which sometimes occurs in manuscripts for ϑ seems to be an FIG. attempt to express the head and fore-legs of the bull: and that sometimes instead of the characters ϑ and m we meet with abbreviated words. The 25th figure, wherein I have given the characters of the signs as they are found 25 in ancient manuscripts, will illustrate these remarks.

CHAP. 7. THAT THE ORBITS OF THE PLANETS AND COMETS ARE ELLIPTICAL: THEIR MOTIONS IN THEIR ORBITS NOT EQUABLE.

667 What an *ellipse* is may in some measure be understood from the introduction, § 190, where it was shewn to arise from the section of a cone; but it will be necessary for our present purpose to consider another method of drawing or generating that figure, which is as followeth: let a thread tied together at both ends be put over two pins fixed upright upon a plane, at any distance from each other less than the string thus tied will reach; a pen or pencil carried round within the string, so as to keep it always stretched out to its utmost tension, will describe upon the plane a curve, which is the periphery or circumference of an ellipse, fig. 26: each of the points s F wherein 26 the pins are fixed is called a *focus* or *umbilicus* of the ellipse: the farther the focuses are asunder the more oblong will the ellipse be, the nearer the focuses are together the nearer will the ellipse be to a circle, if the same thread be made use of: if the focuses coincide or meet in a point, the string will then go round it continuing all the while of the same length, and consequently the curve described will be a compleat circle whereof that point is the center: all this is easily apprehended by a view of the preceding figure, and may be tried with a string and two pins as above.

668 When an ellipse is drawn, there arise these terms; a line drawn through the focuses both ways till it reaches the circumference is called *the axis*, the *greater axis* or longest diameter of the ellipse, as PA , fig. 27: a point taken in 27 the greater axis equally distant from each focus is *the center* of the ellipse, as c : a line drawn through the center perpendicular to the greater axis and terminated at each end by the circumference is the *lesser axis* or shortest diameter of the ellipse, as mm : the distance between the center and either focus is *the excentricity* of the ellipse, as cs or CF : the excentricity is greater or less according as the ellipse is more or less oblong.

669 *The orbit of every planet is an ellipse* having the sun in one of its focuses: we may now consider the ellipse fig. 27 as the orbit of a planet, and in 27 this view there will arise some astronomical terms which must be explained:

The

FIG. The axis of any planets elliptic orbit, as P A fig. 27 is called *the line of the abfi-*
 27 *des*: the point A, where the planet is at its greatest distance from the sun, is
 called its *aphelion*, *aux*^a, or *higher abfis*^b: the point P where the planet is at its
 least distance from the sun is its *perihelion*, *oppositum augis*, or *lower abfis*: the
 extrean points of the lesser axis of the elliptic orbit of a planet M and m are the
 places of its *mean* or middle *distance*: and a line drawn from either of those
 points to the sun as M s or m s is the *line of the planets mean distance*. When we
 would exprefs what the excentricity of the orbit of any planet is, we suppose
 the line of its mean distance to be divided into 1000 equal parts, and say
 to what number of such parts the excentricity is equal.

If we imagin the mean distance of	Saturn	} to be divided into	1000	} equal parts,	the excentricity is	55	} of those parts.
	Jupiter		1000			48	
	Mars		1000			93	
	The Earth		1000			17	
	Venus		1000			7	
	Mercury		1000			210	

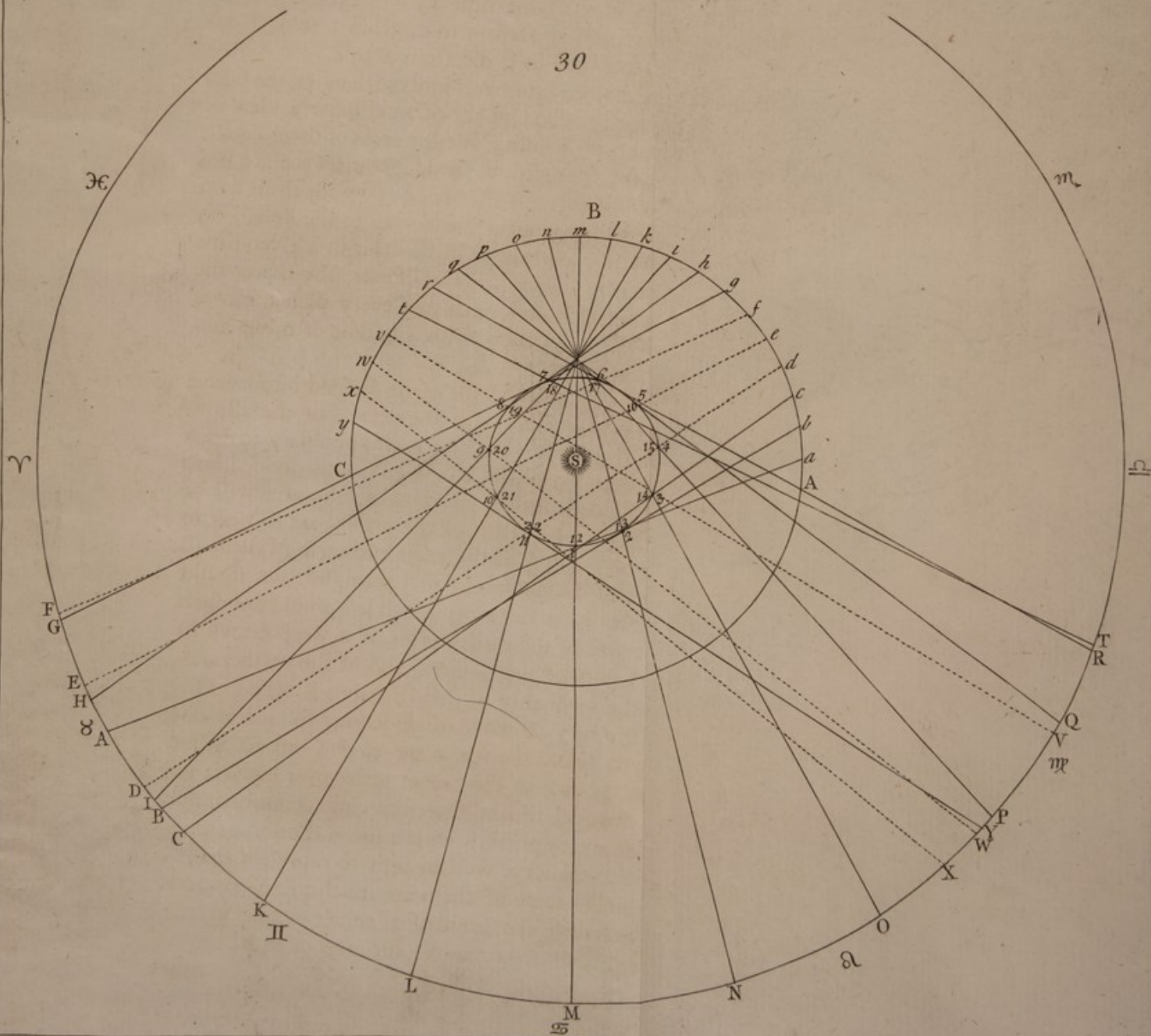
670 The orbits of the planets differ very little from circles, as may be seen
 11 fig. 11, where they are drawn in their true shapes: the orbits of mercury,
 mars, jupiter and saturn, are there exprest by double lines; the outermost of
 which are the lines of the planets motions; the innermost are compleat circles
 whereof the sun is the center, and the radius of each circle is the least distance
 of its respective planet from the sun; and consequently the distance between the
 two lines shews how much any planet is farther from the sun in any other part
 of its orbit than it is at the perihelion. The excentricities of venus and the
 earth are so small, that instead of double ones, their orbits are exprest by sin-
 gle lines, drawn gradually thicker and thicker from the perihelion to the
 aphelion; and thinner and thinner from the aphelion to the perihelion: by
 this means the different distances of these planets from the sun in different
 parts of their orbits is exhibited: thus, the earth is as much farther from the
 sun in any point of her orbit than she is at the perihelion, as the thickness
 of the line in that point of her orbit amounts to.

671 *The motion of the planets in their orbits is not equable*; but every one
 of them observes this rule, that a line drawn from the sun to the planet moves
 over equal areas upon the plane of its orbit in equal times: thus, let the el-
 28 lipsis, fig 28, be the orbit of a planet, if the planet moves in a certain time,
 for example in a month, from the point A to B; we may say that in a months
 time the line s A drawn from the sun to the planet has swept or moved over
 the triangular area A s B: to this make the triangular area B s c equal, and a
 line drawn from the sun to the planet will move over this area in a time equal

^a *Aux origine Perficum, altitudo. Golii lexie.*

^b *Plin. N. H l. 2. c. 15. al. apsis.*

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to the former, that is, in another month the planet will move from B to C: FIG. and so for the rest of the orbit, if the plane of it be divided into the mixt tri- 28 angles CSD, DSE, ESF, &c. in such a manner that the area of every one of them be equal to the area of ASB, a line drawn from the sun to the planet will sweep each of these triangles in the same time as it does the triangle ASB; and consequently the planet will go from B to C, from C to D, from D to E, from E to F, &c. in the same time as it did from A to B.

672 The figure before us being divided into twelve mixt triangles, the sides 28 of each triangle consist of two strait lines and an arc of an ellipsis: a view of the figure is sufficient to shew, that if we would make the areas of them equal, the shorter the two strait lines of any triangle are, the longer must the arc be; the longer the strait lines are, the shorter must the arc be: now the strait lines are shorter the nearer to the perihelion, longer the nearer to the aphelion; and consequently the arcs described by any planet in its orbit in a given time are longer, the nearer the planet is to its perihelion; shorter, the nearer the planet is to its aphelion. From hence it followeth that every planet moves swiftest in its perihelion, slowest in its aphelion, with a middle or mean motion at its mean distances.

673 The 28th figure represents the orbit of the earth divided into twelve 28 parts or arcs, answering to the twelve months in the year; but the ellipsis is much more excentric than its true figure requires: it was necessary to draw it thus, the better to shew the inequality of the arcs of an ellipsis divided into equal triangles: if a circle be divided into equal triangles, the arcs will be exactly equal: as appears by fig. 16: the orbits of the planets are so near to 16 circles, that if the orbit of mercury which is the most excentric of them all were drawn in its true figure, and divided into equal triangles, we should scarcely perceive any difference between the arcs; much less would any such difference be perceptible in the arcs of the earth's orbit, if it were drawn in its true proportion. Since the orbits of the planets, and of the earth particularly, are so near to circles, the inequality of their motions is very small.

674 *The orbit of every comet is a very excentric ellipsis* having the sun in one of its focuses: *the motion of comets round the sun is not equable*, but according to this rule, a line drawn from the sun to the comet passes over equal areas upon the plane of its ellipsis in equal times: therefore comets move much swifter at their perihelia than at their aphelia; with a mean motion at their mean distances. The 26th figure will serve well enough to represent the or- 26 bit of a comet, though the orbits of some of them are much more excentric than that ellipsis: the more excentric the orbit of a comet is, the greater difference will there be between the arcs it moves through in equal times; that

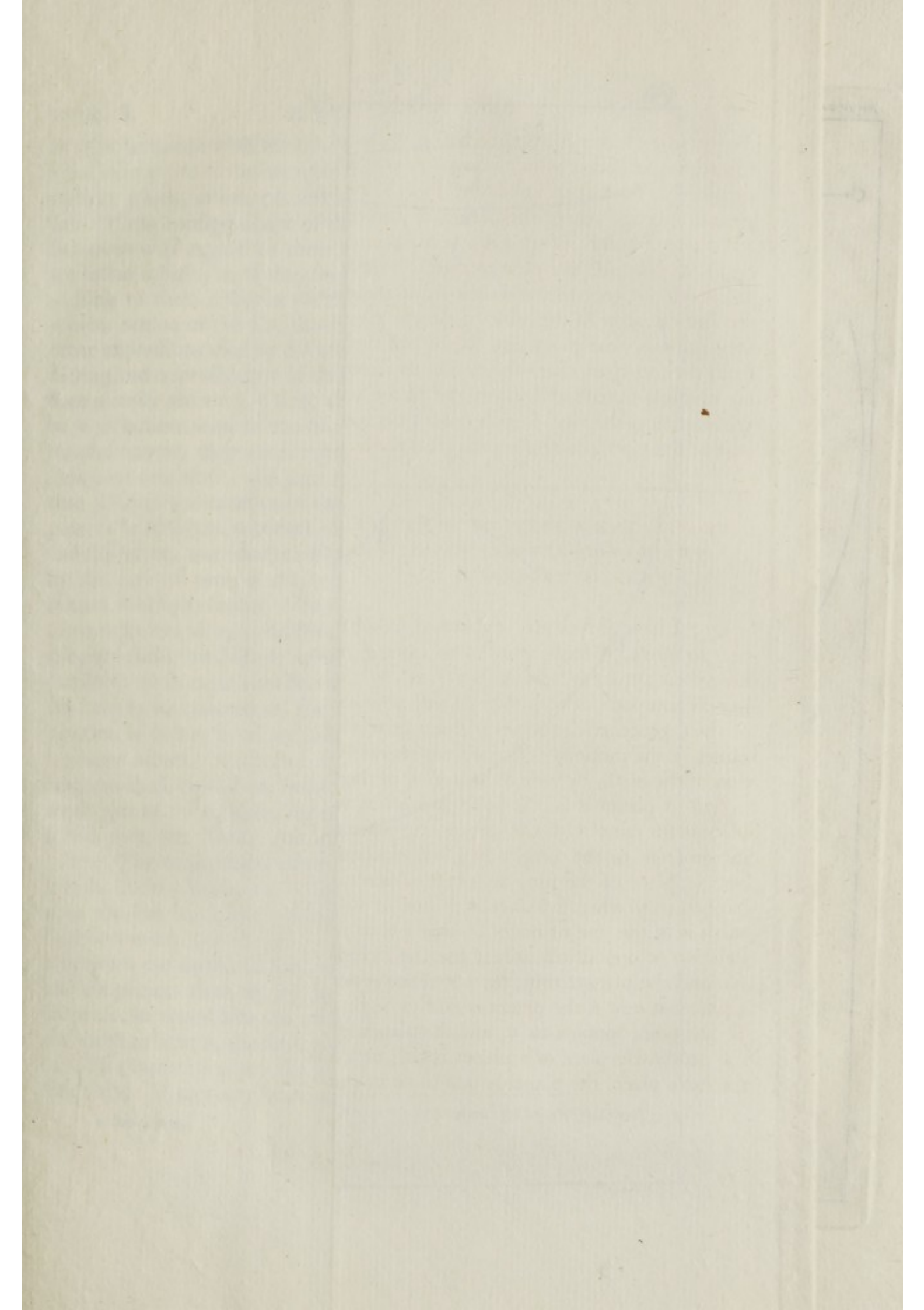
FIG. that is, the greater inequality will there be in the motion of the comet: for this reason, a comet which moves pretty fast near its perihelion may move exceedingly slow at its aphelion; so as to make its entire period to take up a great number of years: the period of the comet which appeared in the year 11 1680 is 575 years; its orbit is a very excentric ellipsis, as appears by the 11th figure, as does also the near approach it made to the sun in its perihelion; in its aphelion it goes to a distance from the sun about eight times as great as the semidiameter of the orbit of saturn. Comets are visible to us only when they come as near to the sun as the orbit of jupiter; when they are farther from the sun than that, their distance from the earth is so great that they are invisible to us^a: and this is the reason we so rarely see any of them, notwithstanding they are so numerous; because they are for much the greatest part of their periods moving very slowly at immense distances from us.

CHAP. 8. THE ASPECTS OF THE INFERIOR PLANETS WITH THE SUN: THEIR GEOCENTRIC MOTIONS: THEIR APPEARING DIRECT, STATIONARY, RETROGRADE.

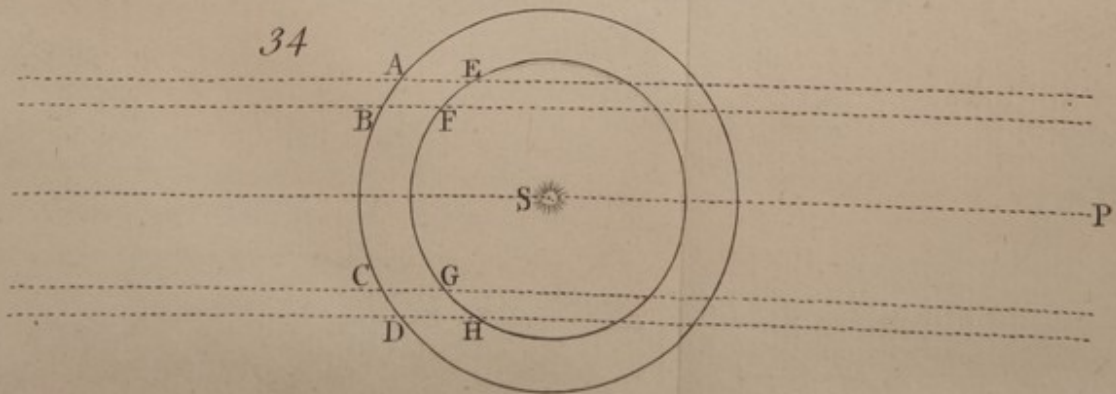
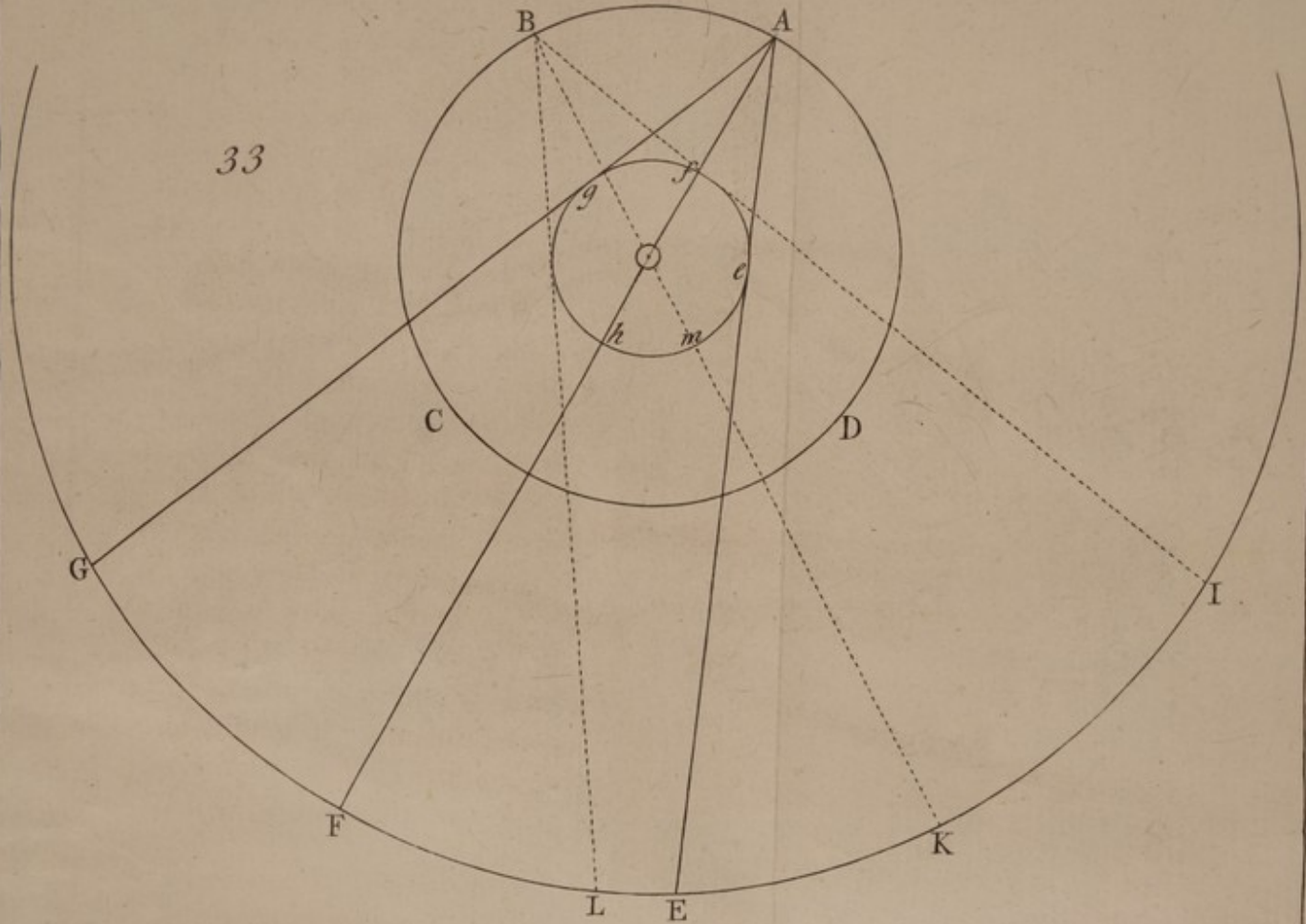
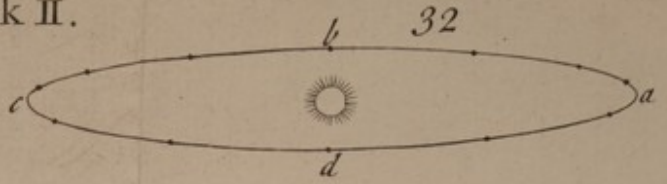
675 I have sufficiently explained in what manner the planets would appear to move, if they were to be viewed from the sun; I have also shewn how their apparent latitude is varied, by their being viewed from the earth in different parts of her orbit: I shall now enter into a more particular detail of their geocentric motions, which arise from a combination of these two causes, 1 the motions of the planets themselves in their orbits, and 2 the motion of the earth, by which the station of the observer is continually changing.

676 A planet is said to be *in conjunction* with the sun, when its geocentric place is the same with the geocentric place of the sun: a planet is said to be *in opposition* to the sun, when its geocentric place is opposite to the geocentric place of the sun: an exact or central conjunction or opposition can happen only when a planet is in one of its nodes at the same time that the earth is in the line of nodes of that planet; but it is generally called a conjunction or opposition, when the same circle of latitude passes through the 24 sun and the planet: thus, fig. 24, if the geocentric place of the sun be at γ , of a planet at a or b , the planet is said to be in conjunction: if the sun appears at ε , the point opposite to γ , and the planet at a or b , it is in opposition: when the geocentric place of a planet is 90° , or a quarter of a circle, distant from the sun's place, the planet is said to be in *quadrature*, or in a quartile aspect,

^a *v.* Newtoni Princip. l. 3. lemma 4.



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or to be in square with the sun: these terms are used in a like sense when applied to any two of the heavenly bodies: thus, the sun and moon, or the moon and any planet, or two planets may be in conjunction, opposition, or quadrature. These configurations of the heavenly bodies, arising from their mutual situations with regard to their different longitudes measured on the ecliptic, are called *aspects*; as if they looked at one another in a different manner according to their different positions: the word seems to be taken from the ancient notion of the sun, moon and planets being animated, as do also some other expressions used by the astrologers; as when they talk of the planets afflicting and oppressing, or of their aiding and strengthening one another. It has been already observed, § 626, that the favourers of judicial astrology would have us believe some of the heavenly bodies to be of a benign, others of an hurtful nature; they teach moreover that planets of the same nature in some situations cooperate, and joyn their influences; and those of a contrary nature obstruct one another in their several effects; and that they exert these powers in different manners and degrees, according to their several aspects: and therefore, according to *Kepler's* definition, an aspect is an angle formed by the rays of two planets meeting upon the earth, having a power to influence sublunary beings. The ancients reckoned only five aspects, which are often represented by characters, as may be seen in the common almanacks: they are these that follow, conjunction σ , opposition $\var�$, quartile \square , trine \triangle , sextile \ast : with regard to their supposed influences, the opposition and square are held to be malevolent, the trine and sextile benevolent aspects: the conjunction is said to be of an indifferent nature: some of the moderns take in a greater number of aspects; as the decile, when the difference in longitude between those two of the heavenly bodies which are under consideration is a tenth part of the ecliptic; the octile, when an eighth part; the quintile, when a fifth part, &c. *Kepler. epit. astronom. p. 842.*

677 The conjunction of an inferior planet with the sun is of two sorts; 1 in its inferior semicircle, or that part of its orbit which is nearer to our earth than the sun is; 2 in its superior semicircle^a, or that part which is farther from our earth than the sun is: in the former of these conjunctions the planet is between our earth and the sun; in the latter the sun is between our earth and the planet: thus, fig. 29, let $OPQR$ be part of the ecliptic, s the sun, the three circles round him the orbits of mercury, venus and the earth, suppose the earth to be at A , the sun's geocentric place will be at Q , if mercury be then at I , his geocentric place also is at Q , and he is in conjunction in his inferior semicircle: if mercury be at M , his geocentric place is Q , and he is in con-

^a See § 270.

FIG. junction in his superior semicircle: in like manner venus at *E* is in conjunction in her inferior semicircle; at *C* in her superior.

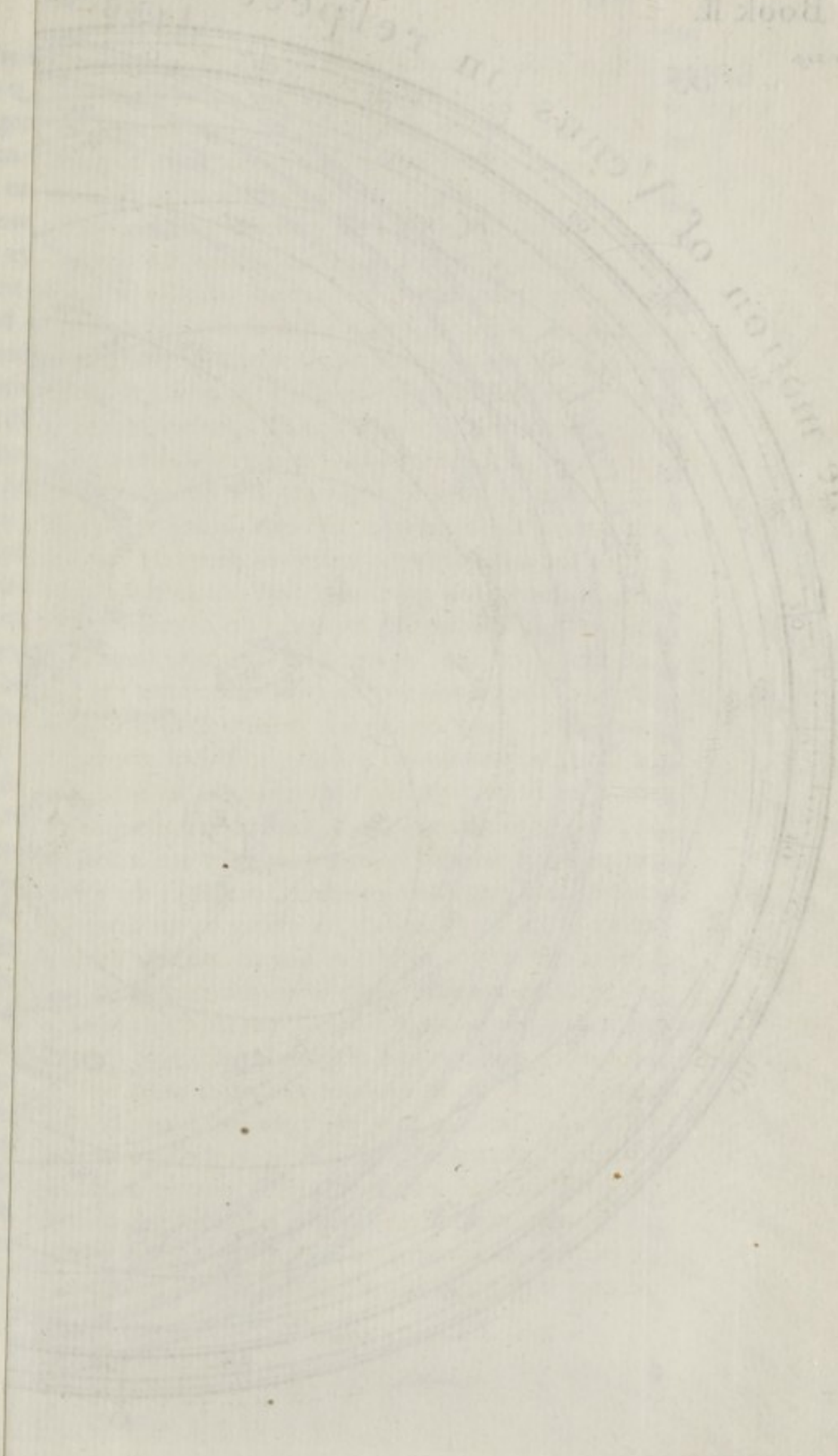
678 The inferior planets can never be in opposition to the sun, nor indeed can they ever appear at a great distance from him: this distance is called their
29 *elongation*: thus, fig. 29, if the earth be at *A*, venus at *H*, her geocentric place is *T*, her elongation *QT*: the greatest elongation of an inferior planet is when the visual line, that is a line drawn from the earth to the planet, is a tangent to its orbit: thus mercury at *N* and venus at *H* are in their greatest elongations: this is speaking in a general and inaccurate way; for the elongation of a planet is then greatest of all when the planet is in its aphelion at the time when the visual line is a tangent to its orbit: thus the greatest possible elongation of mercury is *QP*, when he is in aphelion, as at *L*: it is found by observation that the greatest elongation of mercury is about 28° , of venus 48° ^a. The inferior planets in their elongations are sometimes eastward, sometimes west-
29 ward from the sun: thus, the earth being at *A*, when venus appears in any part of the arc *QO* she is westward from the sun, and therefore rises before him, is seen in the morning, and is called *the morning star*: when she appears any where in the arc *QT*, she is eastward from the sun, and therefore sets after him, is seen in the evening, and is then *the evening star*.

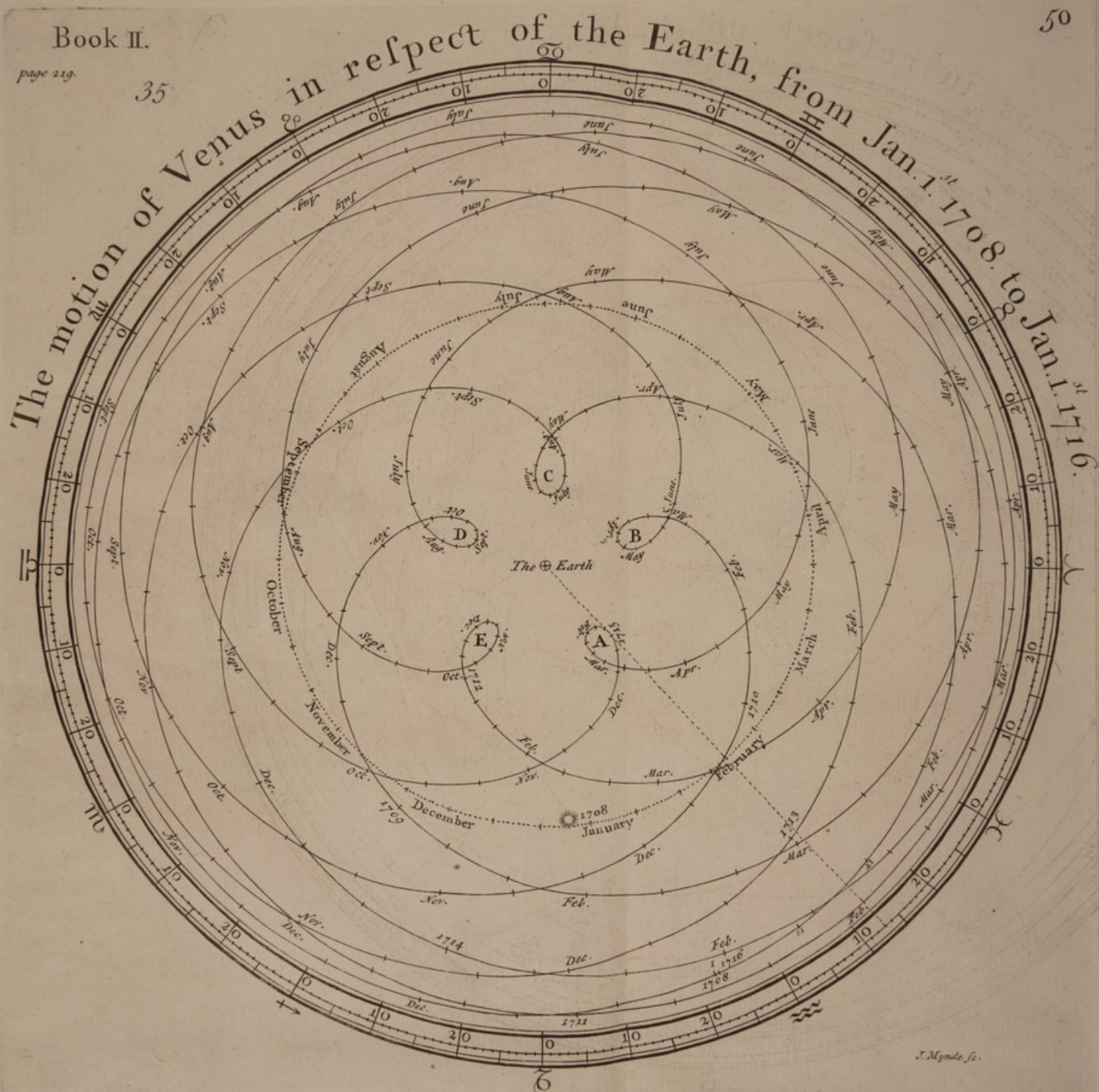
679 If the planets were to be viewed from the sun they would always appear direct, going on in their heliocentric circles, according to the order of the signs, as was shewn § 640; and their motions, though not exactly so, would be nearly equable: whereas, to us who view them from the earth, they appear sometimes *direct*, sometimes *retrograde*; and both these with great inequality of velocity: sometimes they are *stationary*, for a little while not sensibly changing their geocentric places in the sphere of the heaven: this diversity of their appearances is owing to the several combinations of the motions of the earth and of the planets in their orbits; and therefore, in order to a full explanation, both these motions must be taken into the account: but it will not be amiss first to consider them separately; and, since the earth is longer in going round the sun in her orbit than either of the inferior planets in theirs, let us consider the earth as at rest in some part of her orbit, whilst an inferior planet, for example, mercury, goes round in his; and see what his appearances would be upon this supposition.

680 Whilst an inferior planet is moving in its superior semicircle its geocentric motion is direct; whilst it is moving in its inferior semicircle its geocentric motion is retrograde: thus, fig. 29, suppose the earth at rest at *A*, whilst
29 mercury is going in his orbit from *N* to *I*, from *I* to *L*, his motion appears to

^a Mem. d' Acad. Royale ann. 1709. 7 Aoust.

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an inhabitant of the earth to be retrograde, contrary to the order of the signs; *FIG.* namely from R to Q, from Q to P: when mercury is moving from L to M, 29 from M to N his geocentric motion is direct, according to the order of the signs; namely from P to Q, from Q to R.

681 When our earth is in the line of nodes of an inferior planet its orbit in a perspective view appears a strait line, as *fig. 31*; because the plane of it passes through the eye of an observer upon the earth, § 258; and therefore the planets apparent motion is then in a strait line: thus, let the earth be in the line of nodes of mercury, the projection of his orbit is, as in *fig. 31*; a strait line A c, extended on each side of the sun B, to the distance of the planets greatest elongation: if mercury be then in his superior semicircle, his apparent motion is direct, from A to c, in a strait line, but unequal, going the unequal divisions of the line in equal times, faster the nearer to the sun by § 268^a, if a conjunction happens, he passes behind the sun, from *a* to *b*. If when the earth is in the line of his nodes, mercury be in his inferior semicircle, his apparent motion is retrograde, in the same strait line as before, from c to A, but more unequal than in the preceding case, § 268^b; going faster the nearer to the sun, and in conjunction he passes between the sun and our eye, and appears to go over the sun's disk from *b* to *a*: in this manner mercury has often been seen to pass over the sun's disk, venus was never observed to do so but once, Novemb. 24. 1639. When the earth is out of the line of nodes of an inferior planet its orbit appears an ellipsis more or less excentric according as our eye is differently elevated above its plane, § 268^c: the apparent motion therefore of the planet is then in an elliptic curve: thus, let the earth be as far as possible, that is 90°, out of the line of nodes of mercury, the projection of his orbit will be such an elliptic curve, as *fig. 32*, wherein he will appear to move according to the order of the letters, direct when in his superior semicircle, from *a* to *b*, from *b* to *c*, being above the sun at *b*, in conjunction: but in his inferior semicircle his apparent motion will be retrograde, from *c* to *d*, from *d* to *a*; and in conjunction he will be below the sun at *d*: in these cases also his apparent motion will be unequal, faster the nearer to conjunction, but most unequal in the inferior semicircle, going through the unequal arcs into which the elliptic curve is divided, in equal times. See § 268, 269.

682 The geocentric motion of the inferior planets both direct and retrograde is sometimes swifter and sometimes slower; swifter the nearer to the sun, slower the nearer to their greatest elongations: the inequality now spoken of is not owing to the small inequality of their motions in their orbits,

a *Introduc.* *fig.* 153.

b *Introduc.* *fig.* 154.

c *Introduc.* *fig.* 152.

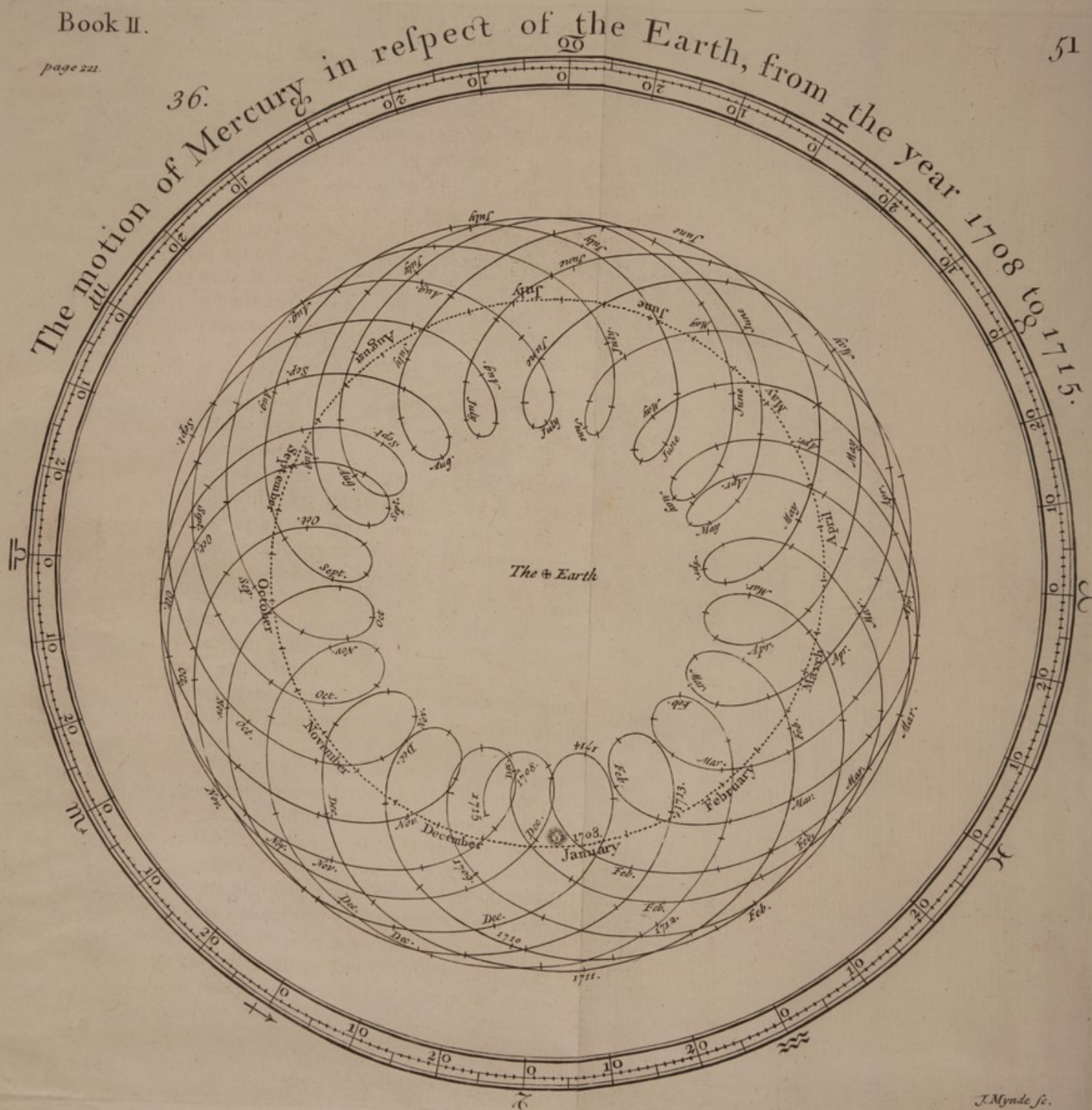
FIG. which was mentioned § 672, but to their orbits being viewed obliquely, and by that means projected into oblong ellipses or strait lines with the sun in the middle of them, as fig. 31 and 32; in which projections a body moving with an uniform velocity round a circle will appear to move irregularly, swifter the nearer to the middle; slower the nearer to the extremities *ac* or *Ac*: see this more fully explained in the introduction, § 268, 269.

683 An inferior planet is stationary while its motion is changing from direct to retrograde, or from retrograde to direct; that is it will appear not to change its place sensibly for some time: now if the earth stood still the places of being stationary would be at the planets greatest elongation; for though it be the nature of a tangent to touch its circle only in a point, yet when the circle is large the receding of the circle from the tangent is not perceptible in the parts very near the point of contact: thus, fig. 29, suppose the earth to be at rest at *A* venus would appear stationary, her geocentric place continuing at *T*, all the while she is going in her orbit from *a* to *b*: because her deviation from the visual line *AT* would scarce be perceptible so near the point of contact *H*. Thus to an inhabitant of the earth the inferior planets appear always near the sun, alternately going from him and returning to him, in strait lines or elliptic curves, first on one side and then on the other: sometimes so near the sun as to be rendred invisible by his stronger light: sometimes, when in or near their nodes, they pass behind the sun, in their superior semicircles; or come between the sun and our eyes, in their inferior semicircles: and in this last case appear like spots moving over the disk of the sun.

684 Hitherto, for the more easy comprehension of them, I have considered the geocentric motions of the inferior planets, supposing the earth to stand still in some part of her orbit, while they go round the sun in theirs; the motion of the earth makes some alterations which shall now be shewn: if the earth were to stand still in any point of her orbit as at *A*, the places of conjunction both in the inferior and superior semicircle, as also of greatest elongation, and consequently the places of direct and retrograde motion and of the stations of an inferior planet would be always in the same parts of the heaven: thus, fig. 29, upon this supposition, the places of mercury's stations would be always the points *P* and *R*, the arc of his direct motion *PR*, and the arc of his retrograde motion *RP*: whereas now, the places wherein these appearances happen are continually advancing forward, in the ecliptic, according to the order of the signs, by the motion of the earth in her orbit: thus, fig. 33, let *ABCD* be the orbit of the earth, *efgb* the orbit of mercury, ☉ the sun, *GKI* an arc of the ecliptic; when the earth is at *A* the sun's geocentric place is at *F*, and mercury in order to a conjunction must be in the

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the line *AF*, that is; in his orbit he must be at *f* or *b*; suppose him to be at *f* in his inferior semicircle, if the earth stood still at *A*, mercury's next conjunction would be when he is in his superior semicircle at *b*, the places also of mercury's greatest elongation from the sun would in his orbit be *e* and *g*, and in the ecliptic *E* and *G*, but let the earth go on in her orbit, suppose from *A* to *B*, the sun's geocentric place is now at *K*, and mercury in order to be in conjunction should be in the line *BK* at *m*. As by the motion of the earth the places of mercury's conjunctions with the sun are thus continually carried round the ecliptic according to the order of the signs, so the places also of his utmost elongation must go forward the same way: thus, when the earth is at *A* the places of mercury's greatest elongation from the sun are in the ecliptic *E* and *G*; the motion of the earth from *A* to *B* advances them forward, from *G* to *I*, from *E* to *I*.

685 But the geocentric motion of mercury will be better seen in the following figure, which is much easier to be comprehended than upon a transient view it may appear to be. In figure 30 we have part of the ecliptic marked γ δ Π &c, in the center of which is the sun *s*, and round him the orbits of mercury and the earth: the orbit of mercury is divided into 11 equal parts, each part containing such an arc as mercury goes through in 8 days, the points of division are marked with numeral figures 1 2 3 4 &c, part of the orbit of the earth *ABC* is likewise divided into 22 equal arcs, each arc being so much as the earth goes through in 8 days time, the points of division are marked with the letters *abcdef* &c, and shew so many several stations from whence mercury may be observed by an inhabitant of the earth: suppose then mercury to be at that point of his orbit which is marked with the figure 1, and at the same time the earth to be at *a*; draw a line from *a* through 1 and it shews mercury's geocentric place at *A*: in 8 days mercury will be got to 2, the earth to *b*; draw a line from *b* through 2 and it points out mercury's geocentric place at *B*: in another 8 days mercury will be at 3, the earth at *c*; a line drawn from *c* through mercury at 3 shews his place at *C*: in this manner going through the figure, and drawing lines from the earth at *d*, *e*, *f*, *g*, &c, through the figures in mercury's orbit 4, 5, 6, 7, &c, we shall find his geocentric places successively at *D*, *E*, *F*, *G*, &c; where we may observe that from *A* to *B*, from *B* to *C* mercury is direct, from *C* to *D* from *D* to *E* retrograde. I have in the figure marked 22 points in the earth's orbit, for so many stations from whence mercury may be viewed; to answer to these stations, there must be 22 places taken in the orbit of mercury: for this purpose I have marked the place of that planet at the end of every 8 days, for two of his periodical revolutions; and this is the reason of two numeral

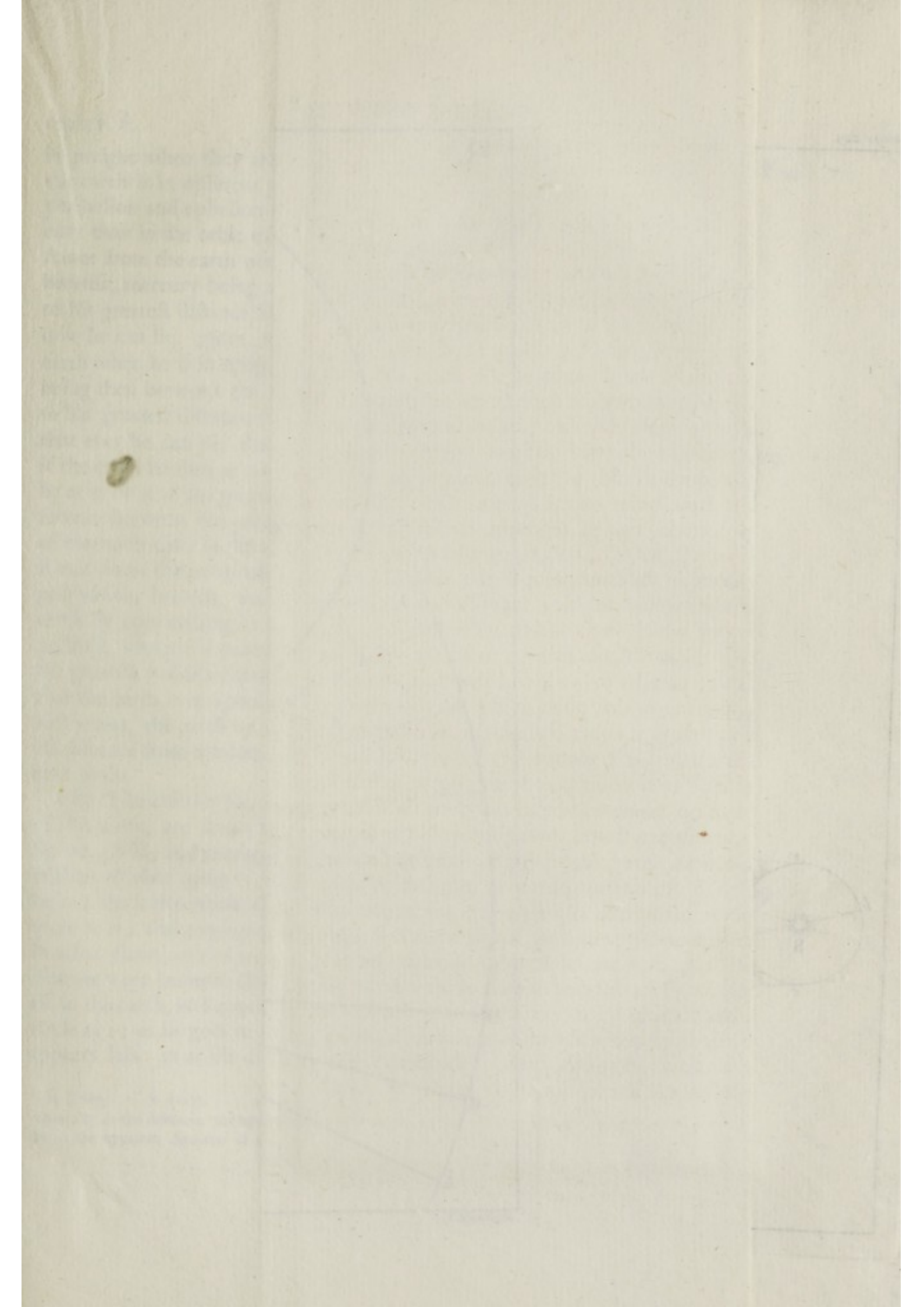
FIG. numeral figures being placed at each division, to shew that we are to go over
 30 his orbit twice in considering the scheme before us.

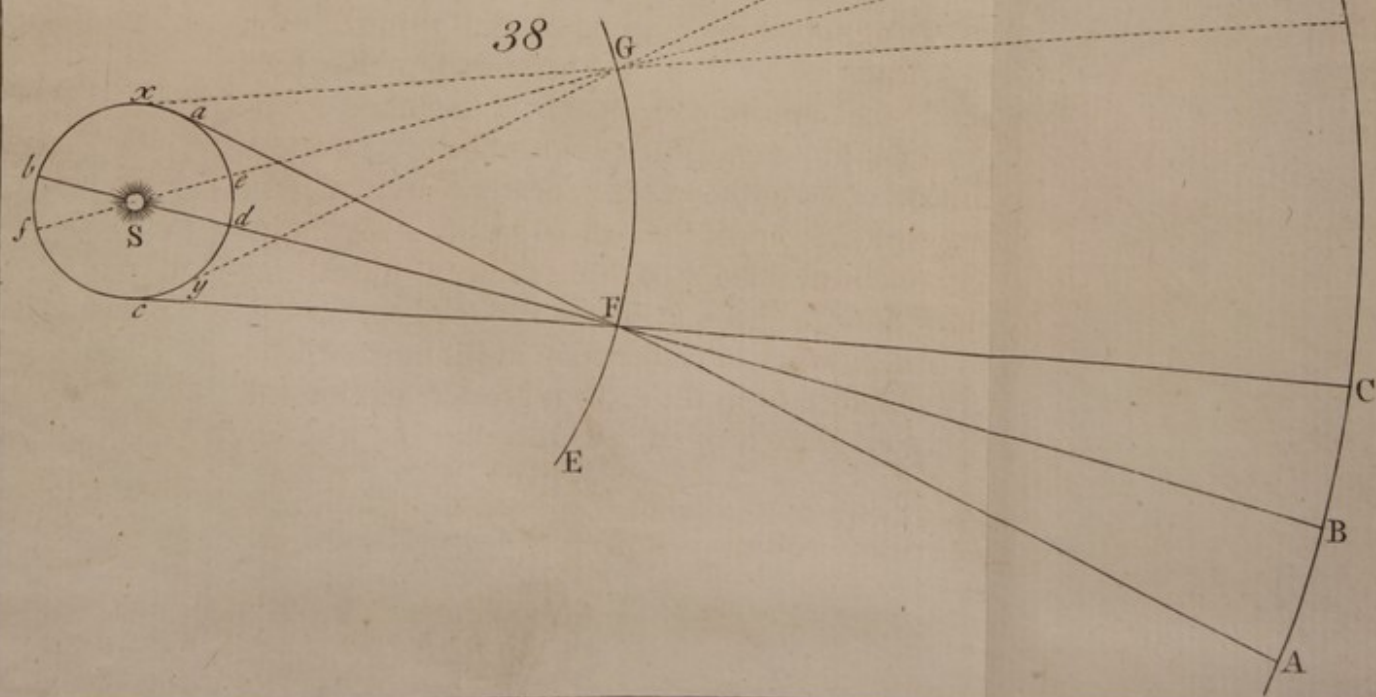
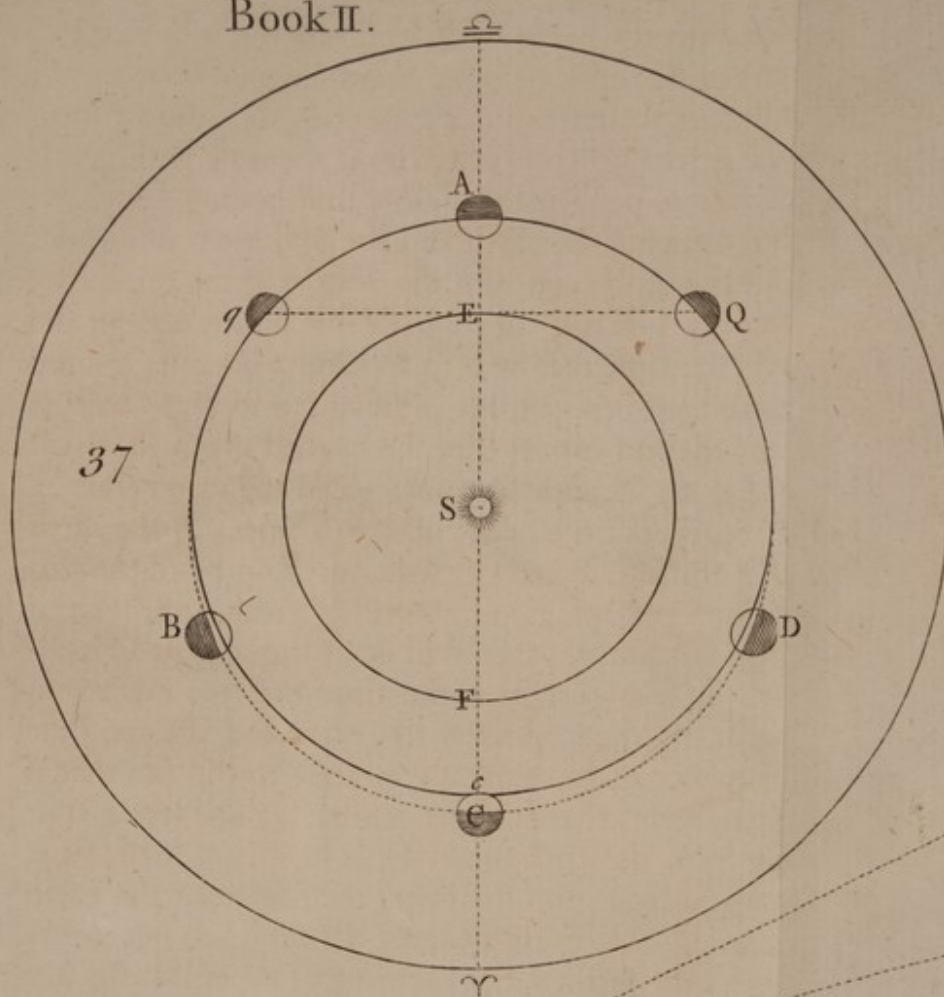
686 The geocentric motion of venus may be explained by a figure similar to that made use of to shew the motion of mercury; only, as mercury goes through his orbit quicker than venus, conjunctions, oppositions, elongations, stations, and the changes of motion from direct to retrograde or from retrograde to direct are all more frequent in mercury than in venus.

687 In order to an explanation of the stations of the planets, it may be remembred that I have before laid down this position, § 636, that, in comparison of the distance of the stars, the orbit of saturn is but as a point; much more may the same be said of the orbit of the earth; and consequently, if we consider any two parallel lines which are not at a greater distance from one another than the diameter of the orbit of the earth amounts to, such lines continued would, as far as could be perceived by observation, terminate in the same point of the sphere of the heaven: this being premised, let the
 34 two circles fig. 34 represent the orbits of venus and of the earth, let the lines AE, BF, CG, DH, be parallel to SP, I say they will all, if continued, terminate in the same point in the heaven, as to sense, as the line SP extended thither would do: now suppose venus at E when the earth is at A, the visual ray by which venus is seen is the line AE: suppose while venus goes from E to F the earth goes from A to B, the visual ray in which venus is now seen is BF, parallel to AE; and therefore venus will be all that time stationary, appearing in that point of the heaven where SP extended would terminate: this station is at her changing from direct to retrograde: again, suppose when the earth is at C venus is at G, the visual line is CG; if while the earth goes from C to D venus goes from G to H, so that she is seen in the line DH parallel to CG, venus will be all that time stationary, appearing in that point where a line drawn from S through P would terminate: this station is at her changing from retrograde to direct: both stations are in her inferior semicircle.

688 An inferior planet in conjunction with the sun in its inferior semicircle is said to be in *perigee*, in conjunction in its superior semicircle it is said to be in *apogee*^a: thus fig. 29, if when the earth is at A venus be at E, mercury at I, they are in perigee; if venus be at G, mercury at M, they are in apogee. The distance of each of the inferior planets from the earth when the planet is in perigee is variable; as is also its apogee distance: the variation of these distances of the inferior planets is owing partly to the excentricities of their orbits and of the orbit of the earth; and partly to the motions of those planets and the motion of the earth; by which it comes to pass that they are

^a The word *perigee* signifies near the earth; *apogee* distant from the earth.





in perigee when they are in different parts of their orbits, as well as when the earth is in different parts of her orbit—Since the difference between the perihelion and aphelion distances from the sun is greater in the orbit of mercury than in the orbit of the earth^a, mercury will be at the least possible distance from the earth when he is in perigee at the time that he is in aphelion: because, mercury being then between the earth and the sun, his running out to his greatest distance from the sun brings him the nearest to the earth that ever he can be: again, mercury is at the greatest possible distance from the earth when he is in apogee at the time that he is in aphelion; because, the sun being then between the earth and mercury, his running out in that situation to his greatest distance from the sun carries him the farthest from the earth that ever he can be: thus, fig. 29, suppose mercury to be at his aphelion L, 29 if the earth be then at B mercury is at the least possible distance; if the earth be at D he is at the greatest possible distance from the earth.—Since the difference between the perihelion and aphelion distances is greater in the orbit of the earth than in the orbit of venus^b, venus will be at the least possible distance from the earth when she is in perigee at the time that the earth is in perihelion; because, venus being then between the earth and the sun, the earth by approaching in that part of her orbit the nearest to the sun that is possible, comes the nearest to venus that ever she can be: again, venus is at the greatest possible distance from the earth when she is in apogee at the time that the earth is in aphelion; because, the sun being then between the earth and venus, the earth by running out, in that part of her orbit, to her greatest distance from the sun, goes the farthest from venus that is possible for her ever to do.

689 The inferior planets, going round the sun in orbits less than the orbit of the earth, are sometimes nearer to us than at other times, as has been shewn, § 688; and consequently their apparent diameters are sometimes greater than at other times^c: thus, fig. 29, suppose the earth to be at A, if venus 29 be at E she is the whole diameter of her orbit, *viz.* EG, nearer to us than if she were at G; and consequently her disk appears much larger at E than at G: in other places, as F or H, her distances are intermediate; and consequently her diameters are between the least and the greatest. Thus also mercury is nearest to the earth, and appears largest, when in conjunction in his inferior semicircle at I; as he goes to L his distance from the earth is greater, and his disk appears less: at M his distance is greatest of all, and therefore the apparent

a § 669. b § 669. c Since the apparent diameters of an object at different distances are reciprocally as the distances, the apparent diameter of venus in perigee is to her diameter in apogee, as 17 to 3; the apparent diameter of mercury in perigee to his diameter in apogee as 14 to 6.

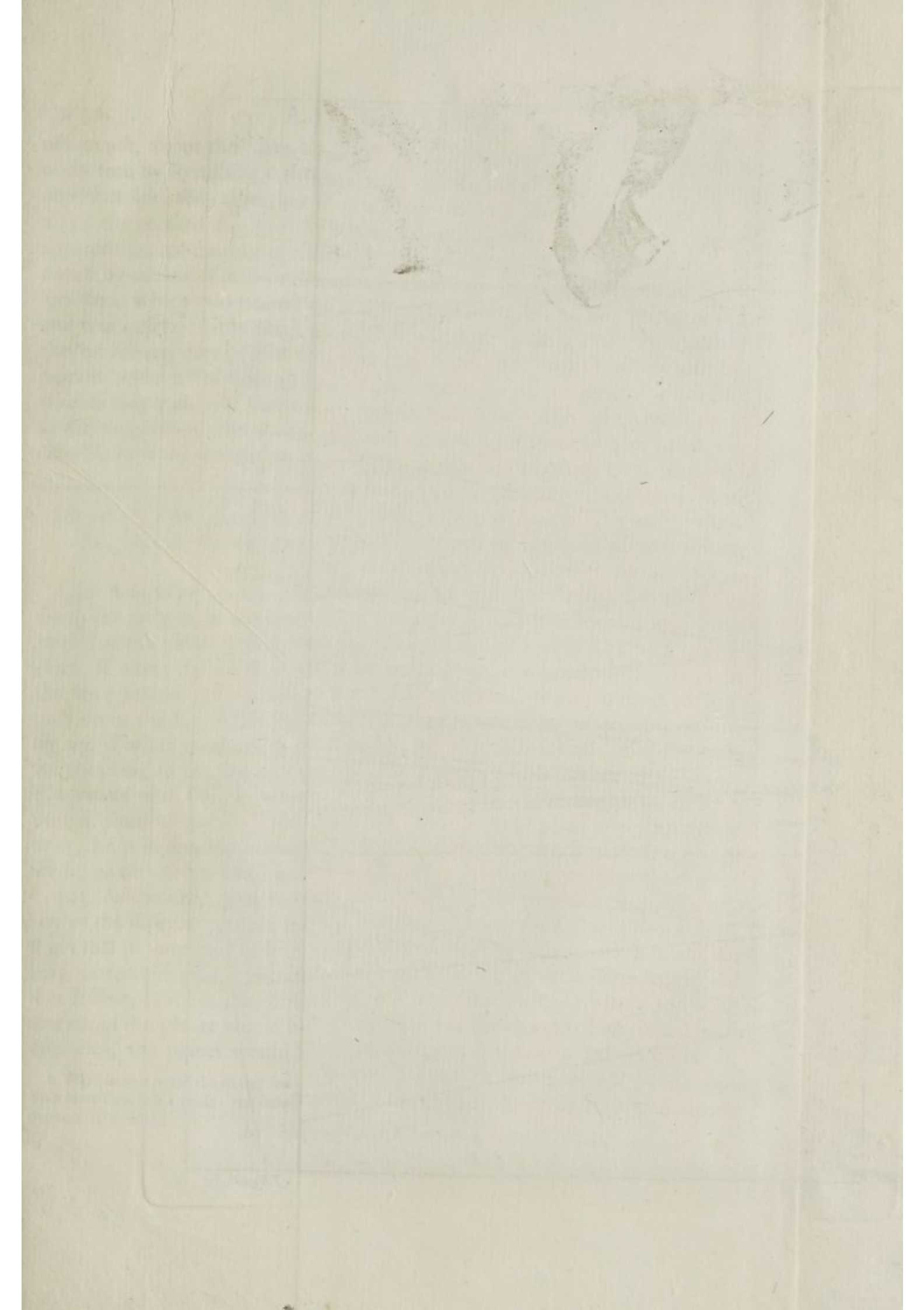
FIG. diameter of his disk is then least. As by the different distances of the inferior planets from us, at different times, their apparent diameters are different; so the difference of their apparent diameters shews their distances to be different^a: how the apparent diameters of the planets are measured will be shewn hereafter.

690 A common spectator may observe an inferior planet alternately to approach nearer and nearer to the sun, till it is in conjunction with him; and recede farther and farther from the sun, till it is at its greatest elongation; and this will be first on one side of the sun and then on the other: but if we observe the change of the apparent place of an inferior planet in the sphere of the heaven, its direct motions, stations, and retrogradations, and moreover frequently measure its disk with the micrometer, we shall find, by the difference of its apparent diameter at different times, that the planet at some times comes nearer and nearer to us, and at other times recedes farther and farther from us; in such a manner that, taking the whole of its apparent motion into the account, its course round the earth appears to be in a complicated curve, which for its resemblance thereto may be called a *spiral*: and in such a curve must the real motion of the planet be, to account for all its appearances, if the earth remained perpetually fixed in the same place.

35 691 The 35th and 36th figures, taken from *Cassini*^b, shew in what kind
36 of spirals the motions of venus and mercury must be, upon this supposition that the earth does not change her place: an explanation of the 35th figure will be sufficient to make the other easily understood. The outermost circle is the ecliptic, the earth \oplus is in the center of it, as it always appears to be, § 248, 249: the spiral line marked with the names of the months abbreviated shews the motion of venus, such as it appears to an inhabitant of the earth, for eight years successively; the time for which *Cassini* drew these figures was from the first of january 1608 to jan. 1, 1716: the short strokes cross the spiral shew the places of venus for the first day, as also for the 10th and 20th day of every month, during that time: the months and years are set down in their proper places: thus the spiral exhibits the direct motions, stations and retrogradations of venus, with the times of them: it shews also her nearest approaches to the earth, and her receding to the greatest distances from the earth, together with her several intermediate distances: a thread stretched from \oplus through any of the places of venus in the spiral to the ecliptic will shew her geocentric longitude at that time; thus, the pointed line $\oplus A$ shews the longitude of venus, january 1. 1715, to be in 13° of π : that line appears to touch the spiral for some length on each side the point

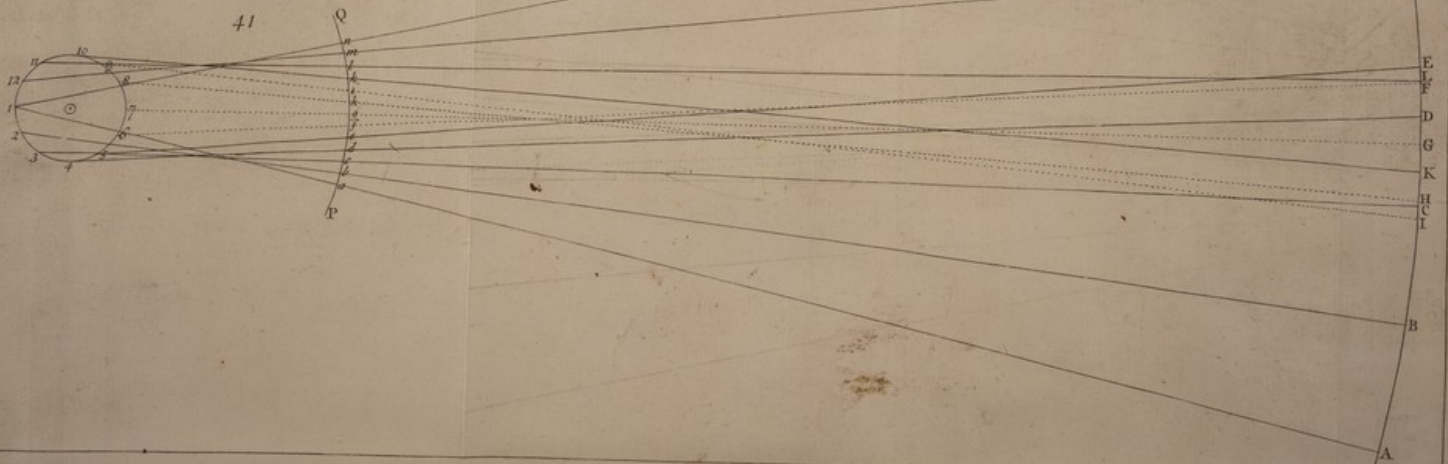
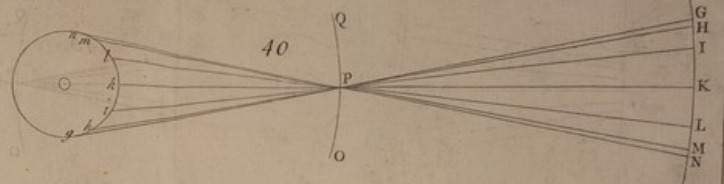
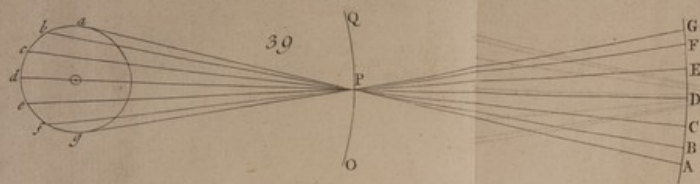
^a § 237, 239, 242.

^b *Memoires d' Acad. Royale, ann. 1709.*



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33



of contact, venus therefore then appeared stationary: her other stations are FIG. easily seen by stretching a thread from \oplus in the same manner, so as to touch, 35 on either side, the curve that encloses those little oval spaces marked A, B, C, D, E: the pointed circle with the names of the months at length shews the apparent annual motion of the sun, which is not sensibly different in 8 years time: by means of this circle it may be seen, for any part of the time here specified, which was nearest to the earth venus or the sun; and how much one was nearer to the earth than the other: the elongations of venus from the sun for any part of that time may also be seen by the same figure; if two threads be stretched from \oplus to the ecliptic, in such a manner that one of the threads may pass over the place of venus, the other over the place of the sun at the time given, the arc of the ecliptic between the two threads so stretched will shew the elongation of venus from the sun.

CHAP. 9. THE ASPECTS OF THE SUPERIOR PLANETS WITH THE SUN, THEIR APPEARING DIRECT, STATIONARY, RETROGRADE.

692 A superior planet, going round the sun in a larger orbit than the earth does, can only be in conjunction with the sun when the sun is between the earth and the planet: thus, let fig. 37 represent the orbits of the earth and of 37 mars, if when the earth is at E mars be at C, mars is in conjunction with the sun; for the geocentric place of both is at γ . A superior planet is in opposition to the sun when the earth is between the sun and the planet: thus, fig. 37, if when the earth is at E mars be at A, he is in opposition to the sun; 37 for the place of the sun is γ , the place of mars α . A superior planet is in quadrature with the sun when the geocentric place of it is 90° from the geocentric place of the sun: thus, fig. 37, if when the earth is at E mars be at q 37 or Q, he is in quadrature with the sun; for the lines QE and SE form a right angle, as do also qE and sE.

693 As the earth goes round the sun in less time and in a less orbit than any of the superior planets, it will not be amiss to suppose a superior planet to stand still in some part of its orbit, while the earth goes once round the sun in hers; and consider the appearances the planet would then have: which are these that follow, 1. while the earth is in her most distant semicircle, the apparent motion of the planet would be direct: 2. while the earth is in her nearest semicircle^a, the planet would be retrograde: 3. while the earth is near the

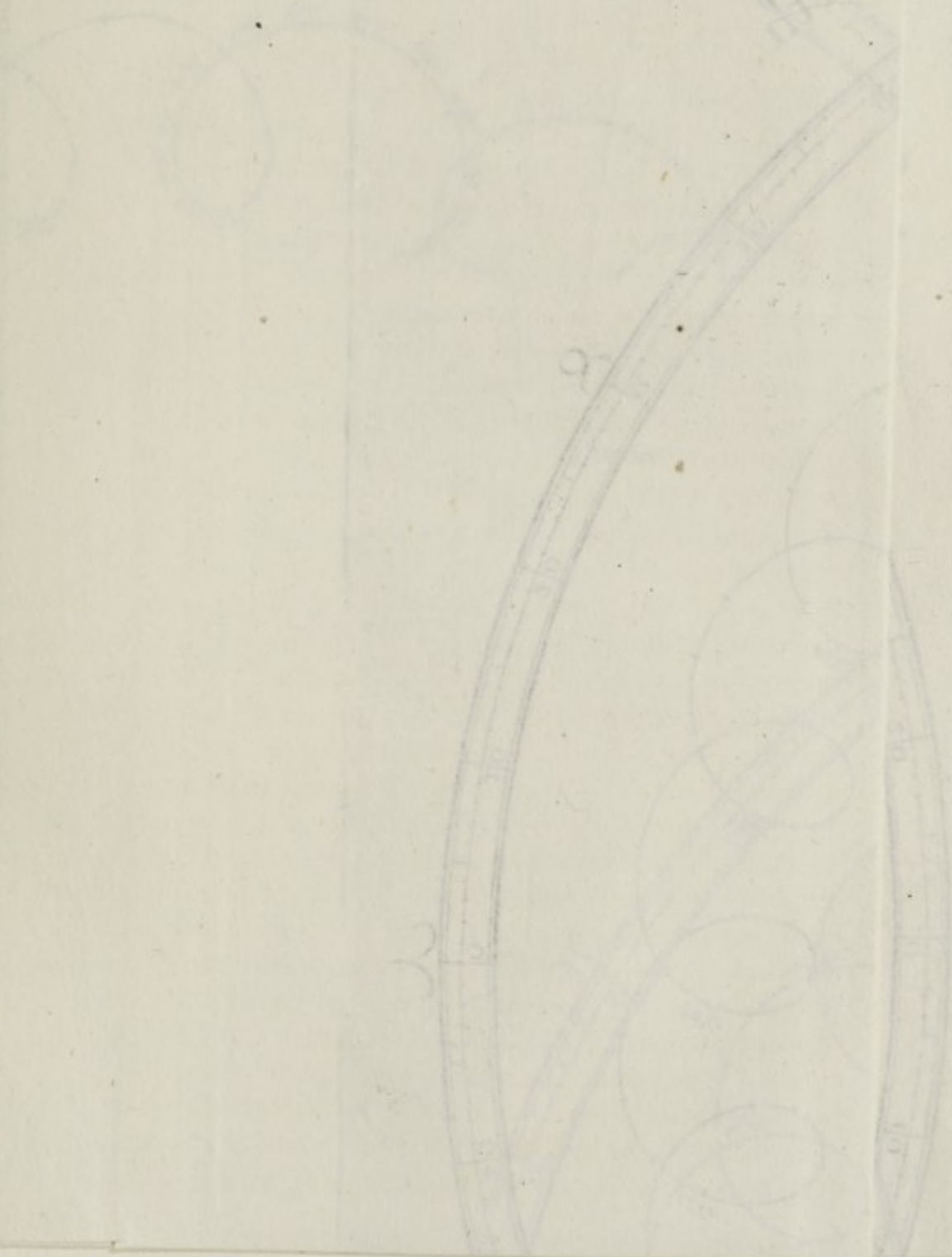
^a What is here called the nearest semicircle is a little less, what is called the most distant semicircle is a little more than half a circle: the deficiency and excess are greater in mars than in jupiter, greater in jupiter than in saturn.

FIG. points of contact of a line drawn from the planet so as to be a tangent to the
 38 orbit of the earth, the planet would be stationary: thus, fig. 38, let the circle *abcd* be the orbit of the earth, *s* the sun, *EFG* an arc of the orbit of jupiter, *ABC* an arc of the ecliptic, suppose jupiter to continue at *F* while the earth goes round in her orbit according to the order of the letters *abcd*; I say while the earth is in the semicircle most distant from jupiter, going from *a* to *b*, from *b* to *c*, his motion in the heaven would appear direct; from *A* to *B* from *B* to *C*: but while the earth is in her nearest semicircle *cda*, the motion of jupiter would appear retrograde, from *C* to *B* from *B* to *A*: for *a, b, c, d* may be considered as so many several stations from whence an inhabitant of the earth would view jupiter, at different times of the year; and a strait line drawn from each of these stations through *F* the place of jupiter, and continued to the ecliptic, would shew his apparent places there to be successively at *A, B, C, B, A*. While the earth is near the points of contact *a* and *c*, jupiter would appear stationary, because the visual ray drawn from the earth through jupiter does not sensibly differ from the tangent *Fa* or *Fc*. When the earth is at *b*, a line drawn from *b* through *s* and *F* to the ecliptic shews jupiter to be in conjunction with the sun at *B*: when the earth is at *d*, a line drawn from *d* through *s* continued to the ecliptic would terminate in a point opposite to *B*; which shews jupiter to be then in opposition to the sun: thus it appears that jupiter's motion is direct when he is in conjunction; retrograde when in opposition.

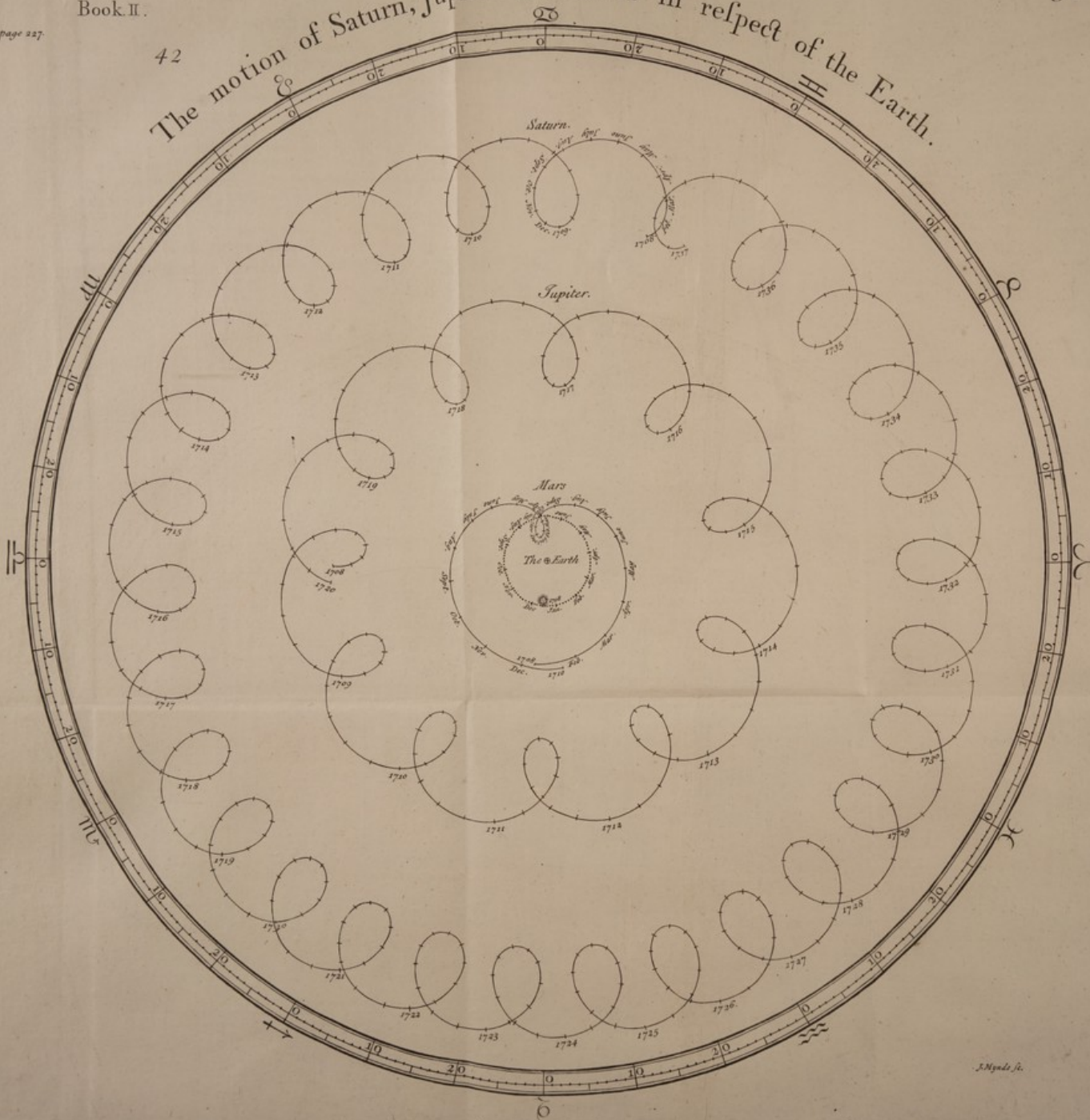
694 The direct motion of a superior planet is swifter the nearer it is to
 39 conjunction, slower the nearer to quadrature with the sun: thus, fig. 39, let \odot be the sun, the little circle round it the orbit of the earth, whereof *abcdefg* is the most distant semicircle, *OPQ* an arc of the orbit of jupiter, *ABCDEFG* an arc of the ecliptic; if we suppose jupiter to stand still at *P*, by the earth's motion from *a* to *g*, jupiter would appear to go direct from *A* to *G*, describing the unequal arcs *AB, BC, CD, DE, EF, FG*, in equal times: when the earth is at *d* jupiter is in conjunction with the sun at *D*, and there his direct motion is swiftest; when the earth is in the point of contact of a line drawn from jupiter tangent to her orbit, as in the points *a* or *g*, jupiter is nearly in quadrature with the sun, and the nearer the earth is to either of those points the slower is the geocentric motion of jupiter: for the arcs *CD* and *DE* are greater than *BC* or *EF*, the arcs *BC* and *EF* are greater than *AB* or *FG*.

695 The retrograde motion of a superior planet is swifter the nearer it is to
 40 opposition, slower the nearer it is to quadrature with the sun: thus, fig. 40, let \odot be the sun, the little circle round it the orbit of the earth, whereof *gbiklmn* is the nearest semicircle, *OPQ* an arc of the orbit of jupiter, *NKG*
 an

der, and Mr. W. respect of the



The motion of Saturn, Jupiter and Mars in respect of the Earth.



an arc of the ecliptic: if we suppose jupiter to stand still at p , by the earth's motion from g to n , jupiter would appear to go retrograde from G to N , describing the unequal arcs GH , HI , IK , KL , LM , MN in equal times: when the earth is at k jupiter appears at K , in opposition to the sun, and there his retrograde motion is swiftest; when the earth is at either g or n , the points of contact of the tangents pg and pn , jupiter is nearly in quadrature with the sun; and the nearer he is to either of those points the slower is his retrogradation: for the arcs IK and KL are greater than HI or LM , and the arcs HI and LM are greater than GH or MN . Since the direct motion is swiftest when the earth is at d , and continues diminishing till it changes to retrograde; near the time of change it must be insensible: in like manner, the retrograde motion, being swiftest when the earth is at k , and diminishing gradually till it changes to direct, must also at the time of that change be insensible: for any motion gradually decreasing till it changes into a contrary motion gradually increasing must at the time of the change be insensible.

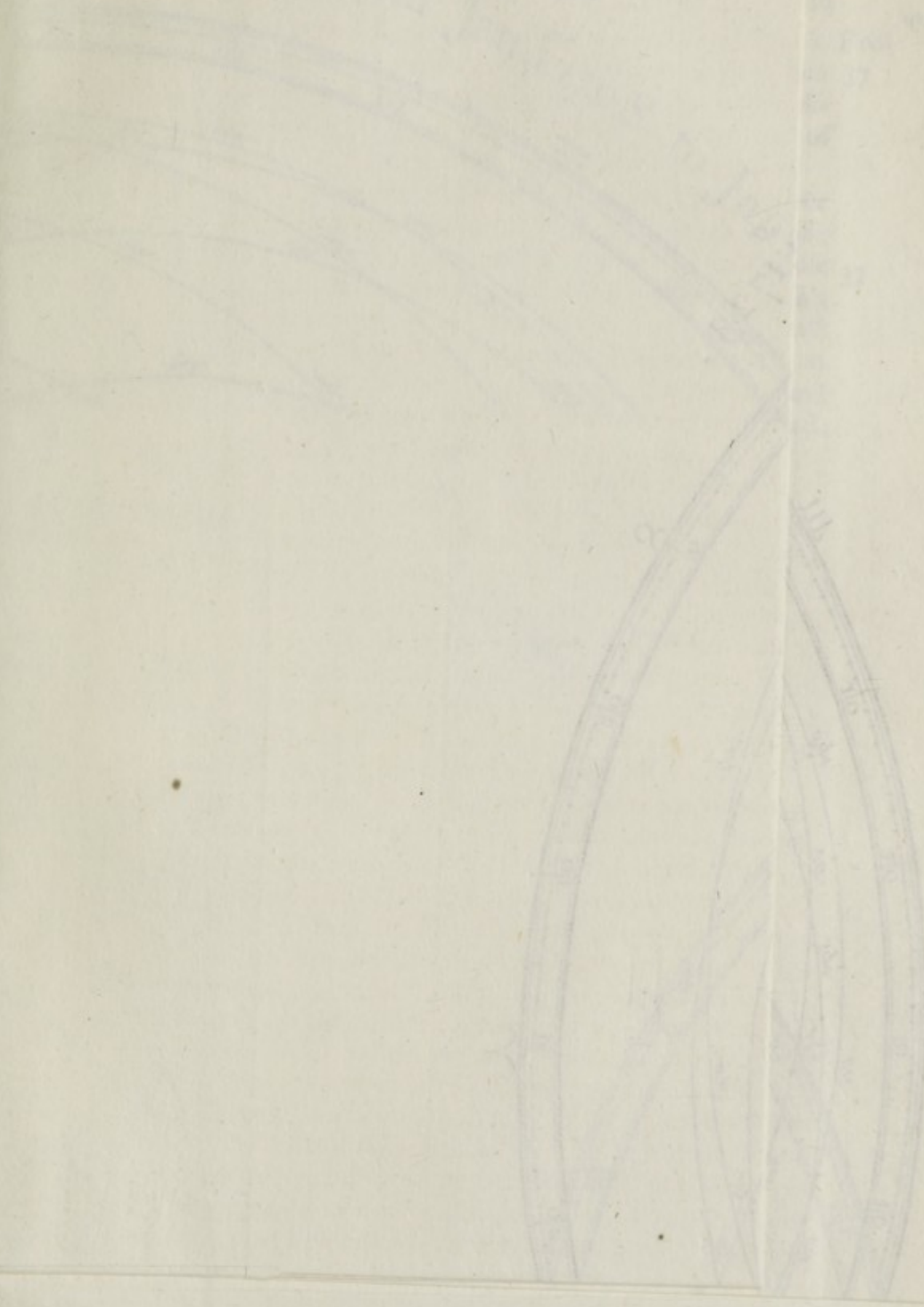
696 All these changes in jupiter's apparent motion, caused by the earth's going round in her orbit, will happen in the same manner if we now suppose jupiter to go on slowly in his orbit; only they will every year fall out when the earth is in different parts of her orbit, and consequently at different times of the year: thus, fig. 38, suppose while the earth goes round her orbit jupiter goes from F to G , the points of the earth's orbit from whence jupiter must be viewed to appear stationary will now be x and y ; and consequently his stations must be at a time of the year different from the former: moreover, the conjunction of jupiter with the sun will now be when the earth is at f ; and his opposition when she is at e : and therefore these also will fall out at different times of the year from those of the conjunction and opposition immediately preceding. As to the other two superior planets, saturn's motion is so very slow, that it alters but little the times when, and consequently the places where he is in conjunction, in opposition, or stationary: the motion of mars is so much swifter than that of jupiter, that the times when, and places where he is in conjunction, in opposition, or stationary, are much more changed thereby than the times and places of the same appearances are in jupiter.

697 But the geocentric motion of jupiter, arising from the combination of his motion in his orbit with the motion of the earth in hers, is best exemplified by the 41st figure; where \odot is the sun, the circle 1234 &c. is the orbit of the earth, divided into twelve equal arcs for the twelve months of the year, pQ an arc of the orbit of jupiter, containing so much as he goes through

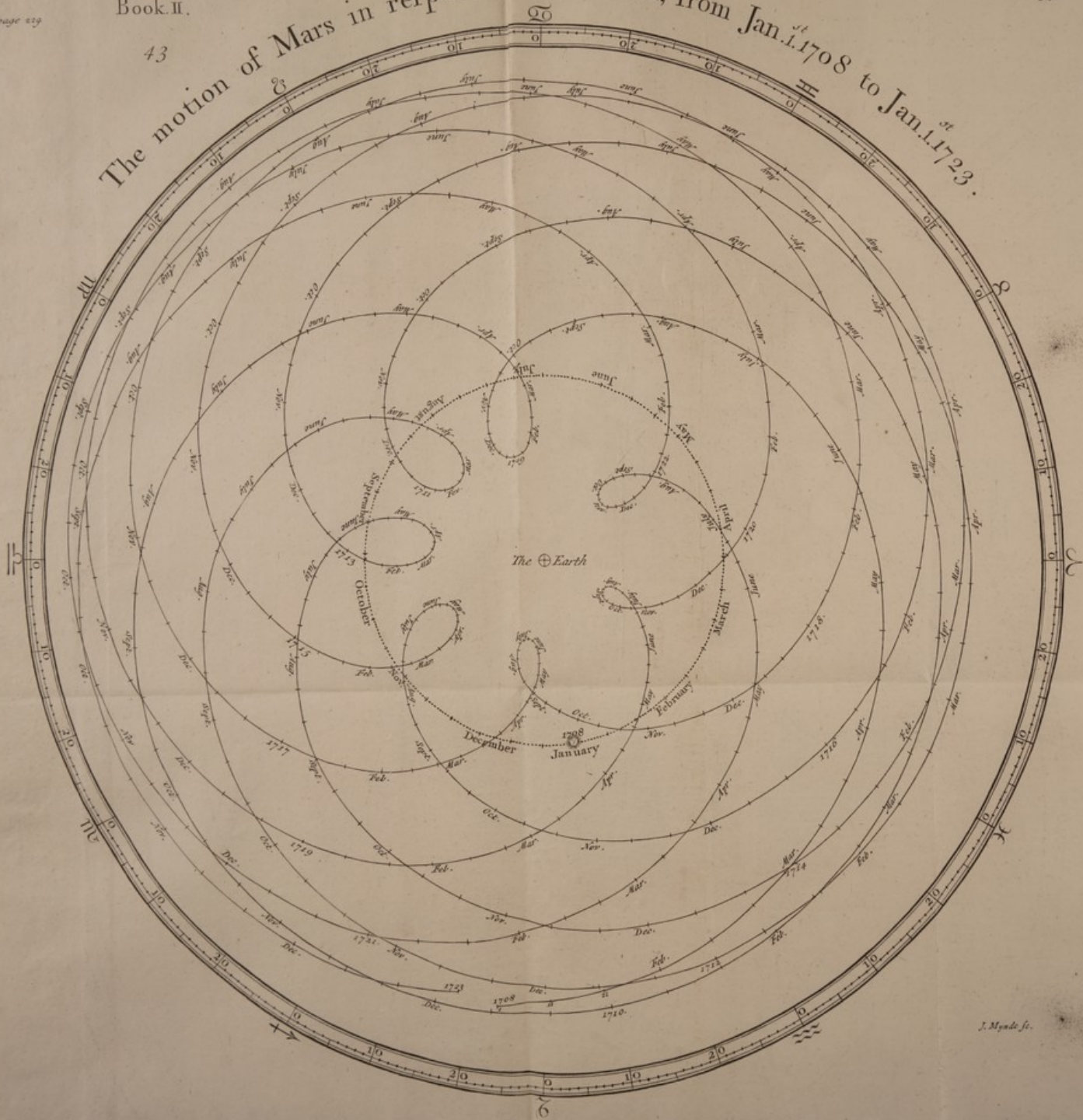
FIG. through in a year, divided into 12 equal arcs, each of them so much as jupiter goes through in a month's time: now suppose the earth to be at the numeral figure 1 when jupiter is at a , a line drawn from 1 through a shews jupiter's place in the ecliptic to be at A: in a month's time the earth will be got from 1 to 2, jupiter from a to b , a line drawn from 2 through b to the ecliptic shews his geocentric place to be at B: in another month the earth will be at 3, jupiter at c , a line drawn from 3 through c points out his geocentric place at C: it is easy to go through the figure in the same manner, and find jupiter's geocentric places, for the rest of the months, at D, E, F, &c: it is easy also to observe jupiter's geocentric motion to be direct in the arcs AB, BC, CD, DE; retrograde in the arcs EF, FG, GH, HI; direct again in IK, KL, LM, MN: the figure shews likewise the inequality of the arcs of the geocentric motion of jupiter.

698 A superior planet in conjunction with the sun is said to be in *apogee*; in opposition to the sun it is said to be in *perigee*: thus, fig. 37, if when the earth is at E mars be at c , he is in apogee; if when the earth is at E mars be at A, he is in perigee: the distance of each of the superior planets from the earth in apogee is variable, as is also its perigee distance: the variation of these distances of the superior planets is owing partly to the excentricities of their orbits, and of the orbit of the earth, and partly to the motions of those planets and of the earth; by which it comes to pass that they are in perigee or in apogee when they are in different parts of their orbits, as well as when the earth is in different parts of her orbit.

699 Since the excentricity of every one of the superior planets is greater than the excentricity of the earth, § 669, every superior planet will be at the least possible distance from the earth when he is in perigee at the same time that he is in his perihelion; because, the earth being between the sun and a superior planet in perigee, the planet coming then nearest to the sun approaches the nearest to the earth that ever he can be: thus, fig. 37, if when mars is at A in perihelion the earth be at E, mars is in the nearest perigee that is possible: it is true, if when mars is at c the earth be at F, mars is in perigee; but the perigee distance CF exceeds the perigee distance AE so much as the difference between the perihelion and aphelion distances of mars from the sun amounts to: which in the figure before us is the short line cc . For the same reason, of their excentricities being greater than that of the earth, every superior planet is at its greatest distance from the earth when he is in apogee at the same time that he is in aphelion: because, the sun being between the earth and a superior planet in apogee, the planet then running out to the greatest distance from the sun will be at the greatest distance also from



The motion of Mars in respect of the Earth, from Jan. 1st 1708 to Jan. 1st 1723.



from the earth that ever he can be: thus, fig. 37, if when the earth is at E FIG. mars be at c, he is in the most distant apogee that is possible: indeed if when 37 the earth is at F mars be at A, he is then also in apogee, but the apogee distance FA is less than the apogee distance EC by so much as the length of the line CC amounts to.

700 The superior planets, going round the sun in larger orbits than our earth does, are sometimes nearer us than at other times, as was shewn § 698; and consequently their apparent diameters are variable: thus, fig. 37, suppose 37 the earth to be at E, if mars be at A, he is the whole diameter of the earth's orbit EF nearer to us than if he were at c; and consequently his disk must appear much larger at A than it would if he were at c: in other places, as at q, B, P, Q, the distances of mars from the earth at E are intermediate; and consequently his apparent diameters are between the greatest and the least. The diameter of the earth's orbit bears a greater ratio to the diameter of the orbit of mars than it does to the diameter of the orbit of jupiter; and a greater ratio to the diameter of the orbit of jupiter than it does to the diameter of the orbit of saturn, § 619: and consequently the difference between the greatest and least apparent diameters is greater in mars than in jupiter, greater in jupiter than in saturn.

701 Since the apparent diameters of an object at different distances are reciprocally as the distances, the apparent diameter of mars in perigee is to his apparent diameter in apogee as 25 to 5: the apparent diameter of jupiter in perigee to his apparent diameter in apogee as 62 to 42: of saturn as 105 to 85. See § 619.

702 Since the areas of circles are as the squares of their diameters, the apparent magnitude of the disks of the planets in perigee to their apparent magnitudes in apogee are as followeth,

$$\begin{array}{l} \text{Mercury} \\ \text{Venus} \\ \text{Mars} \\ \text{Jupiter} \\ \text{Saturn} \end{array} \left. \vphantom{\begin{array}{l} \text{Mercury} \\ \text{Venus} \\ \text{Mars} \\ \text{Jupiter} \\ \text{Saturn} \end{array}} \right\} \text{ of } \left\{ \begin{array}{l} 196 \\ 289 \\ 625 \\ 3844 \\ 11025 \end{array} \right\} \text{ as } \left\{ \begin{array}{l} 36 \\ 9 \\ 25 \\ 1764 \\ 7225 \end{array} \right\} \text{ to } \left\{ \begin{array}{l} 5\frac{1}{2} \\ 32 \\ 25 \\ 2\frac{1}{4} \\ 1\frac{1}{2} \end{array} \right\} \text{ or as } \left\{ \begin{array}{l} 5\frac{1}{2} \\ 32 \\ 25 \\ 2\frac{1}{4} \\ 1\frac{1}{2} \end{array} \right\} \text{ to 1 nearly.}$$

703 What was said of the inferior planets, § 690, is applicable also to the superior; if we observe the changes of their apparent places in the sphere of the heaven, their direct motions, stations, and retrogradations, and measure their disks with the micrometer, we shall find, by the difference of their apparent diameters at different times, that, taking the whole of their apparent

FIG. rent motion into the account, their courses round the earth appear to be in complicated curves, which for their resemblance thereto may be called *spirals*: and in such curves must the real motions of the superior planets be, to account for all their appearances, if the earth remained perpetually fixed
 42 in the same place: the 42d and 43d figures shew in what kind of spirals
 43 the motion of the superior planets must be, upon this supposition that the earth does not change her place: they want no explanation, if what was said § 691 be attended to.

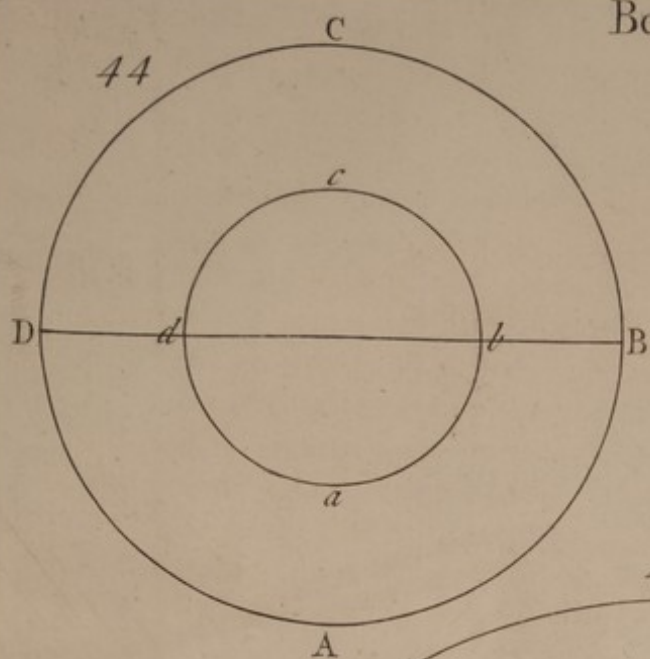
CHAP. 10. THE GEOCENTRIC MOTIONS OF THE INFERIOR AND
 SUPERIOR PLANETS FARTHER EXPLAINED.

704 The geocentric motions of the planets, both inferior and superior, being some of the most difficult appearances in astronomy to represent to the imagination, I shall endeavour to illustrate them farther, by the following familiar examples, of ships in motion upon the sea; premising to each instance a proper *lemma*, or general position taken from optics; lemma 1, A body moving along the same way with the eye, with a velocity equal to the velocity of the eye, will appear to be at rest: thus, if two ships are sailing eastward in parallel lines, at the same rate, to a spectator in one of the ships the other ship will continually appear in the same situation, and he will all along have the same view of it, as if both the ships stood still. 2 A body moving the same way with the eye, with a greater velocity than that of the eye, will appear to go forward the way that it is really carried, but with a less velocity than if the eye were at rest: thus, if two ships sail in parallel lines, eastward, with unequal velocities, the swifter ship will to a spectator in the slower ship appear to go eastward, but with a slower motion than if he stood upon land. 3 A body moving along the same way with the eye, with a less velocity than that of the eye, will appear to go the contrary way to what its real motion is: thus, if two ships sail with unequal velocities, eastward, to a spectator in the swifter ship the slower ship will appear to go westward. 4 If a moving body be carried one way and the eye the contrary way, that body will appear to go the same way that its real motion is, with a greater velocity than if the eye were at rest: thus, if two ships sail in parallel lines, one eastward, the other westward, to a spectator in the ship sailing eastward the other ship will appear to go westward, with a greater velocity than it would do if he stood upon land. 5 If a body at rest be viewed by the eye carried along, the body will appear to move with a velocity equal

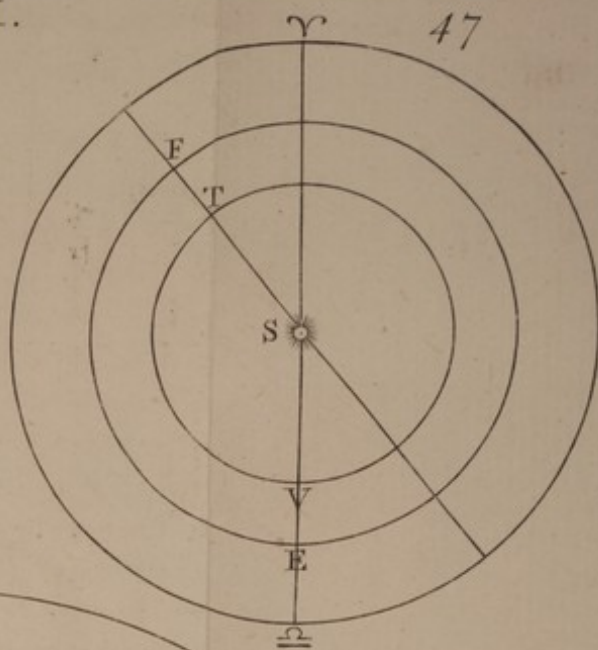


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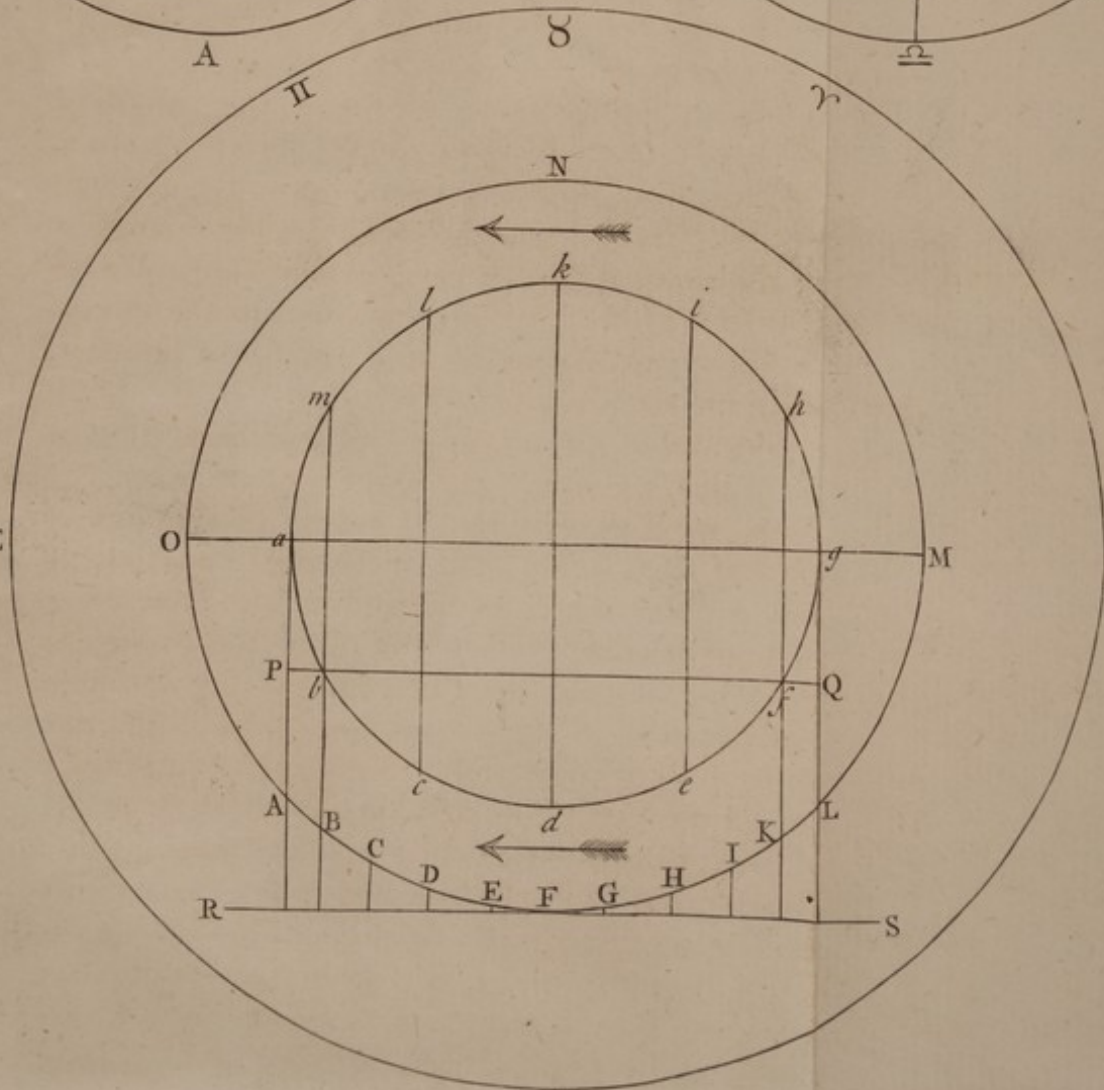
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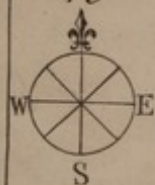
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to that of the eye, but in a contrary direction: thus, to a spectator in a ship FIG. which sails eastward the shore seems to go westward, with a velocity equal to that of the ship.

705 Let there be two concentric circles $abcd$ and $ABCD$, fig. 44, if there 44 be drawn any where through the biggest circle a diameter as BD , the semicircles bcd and BCD considered together may be called semicircles on the same side; as may also the semicircles dab and DAB . The semicircles dab and BCD may be called opposite; as may also the semicircles dcb and DAB .

706 If a body be moved round in one circle and the eye be carried round the same way in another concentric circle, when the eye and the moving body are in the semicircles on the same side, they may be considered as both going in parallel lines, in the same direction; but when the eye and the moving body are in opposite semicircles, they may be considered as going in parallel lines, in contrary directions: thus, suppose fig. 45 to shew the points 45 of the compass, the flower de luce being the north point, if two ships sail round in the concentric circles $abcd$ &c. and $ABCD$ &c, fig. 46, according 46 to the order of the letters, while the ships are in the semicircles, $abcdefg$ and OFM , they may be considered as both going in parallel lines PQ and RS , eastward; while they are in the semicircles $ghiklmn$ and MNO , as going both in parallel lines QP and SR , westward: when the two ships are in opposite semicircles, they may be considered as going in parallel lines, one eastward the other westward: it must be here observed, that equable circular motion thus reduced to rectilinear will appear unequable, as in the figure before us, by § 262. See fig. 148 of the introduction.

707 If we suppose the velocities of the two ships sailing in these concentric circles to be unequal, and the ship which sails in the least circle abc &c. 46 to be the swifter of the two, we shall have all the variety of appearances which are mentioned § 704, of ships sailing in parallel lines: thus, suppose one ship at a the other at A , and that whilst one goes from a to b the other goes from A to B , to a spectator in either of them the other ship would appear to stand still, by lemma 1; because, as to appearance, they are both carried eastward with equal velocities, in the parallel lines PQ and RS : the like appearance will again happen, when one ship goes from f to g in the same time that the other goes from K to L ; because there also they appear to advance with equal velocities eastward, in the lines PQ and RS .

708 The same suppositions of the motions of the ships being made, since whilst the ship in the less circle goes from b to c from c to d the ship in the 46 greater circle goes from B to C from C to D , &c, it is plain the two ships are both carried eastward with unequal velocities; and therefore to a spectator carried

FIG. carried in the slower ship from *B* to *c* from *c* to *D* the other ship will appear to be carried eastward, but more slowly than if he stood upon land, by lemma 2: to a spectator in the swifter ship who is carried from *b* to *c* from *c* to *d* the other ship being carried really eastward from *c* to *D* from *D* to *E* will appear to go westward, by lemma 3: if the ships are in opposite semicircles, as if whilst one is going from *B* to *c*, from *c* to *D*, the other is going from *b* to *i*, from *i* to *k*, to a spectator in the ship going eastward the other ship will appear to go westward, with greater velocity than if he stood upon land, by lemma 4.

709 What has been said of two ships sailing in concentric circles will help the imagination, in conceiving the geocentric motion of the planets; to instance first in one of the inferior planets, let fig. 46 represent the orbits of the earth and of venus, wherein they are carried perpetually round, according to the order of the letters, with different velocities, venus going round the circle *abcd* &c. in much less time than the earth does in the circle *ABCD* &c, § 670: whilst the earth is in any part of the semicircle *OFM* we may consider her motion as in the line *RS*, in the direction from *R* to *S*; if venus be then any where in her inferior semicircle, or that on the same side *abcdefg*, her motion may be considered as in the line *PQ*, in the direction from *P* to *Q*; here, since the circular motions of venus and the earth reduced to rectilinear are in the same directions, contrary to the way that the arrows in the figure point, which direction referred to the ecliptic and viewed from the earth situated in or near *F* is from γ towards ν , contrary to the order of the signs, venus will appear retrograde, but with a slower motion than if the earth stood still, by § 704, lemma 2.

46 710 If while the earth is going along any of the arcs *AB*, *BC*, *CD*, *DE*, *EF*, &c, venus be in her superior or opposite semicircle *ghiklma*, her motion reduced to rectilinear is now from *Q* to *P*, contrary to the direction of the earth's motion, and therefore, by lemma 3, venus will appear to go in the direction *QP*, according to the pointing of the arrows, and swifter than if the earth from whence she is viewed were at rest: the apparent motion of venus in this case referred to the ecliptic is from ν towards γ , according to the order of the signs.

711 If venus be near the point of contact of a tangent to her orbit drawn from the earth, her motion does not carry her forward in the same direction with the earth nor in a contrary direction; but only brings her nearer to, or carries her farther from the earth: venus may at those times be considered as at rest, and therefore if she be then viewed from the earth she will appear to go in a direction contrary to that of the earth, by lemma 5: thus,
if

if while venus is near the point a the earth be at A , the motion of the earth Fig. from A to B will make venus appear to go in a contrary direction, which in 46 the ecliptic will be from ϑ towards Π ; the like appearance of venus will happen, if when venus is at g the earth be at L .

712 If when venus is at a the earth be at A , and venus goes from a to b 46 in the same time the earth goes from A to B , venus and the earth appear to be carried in the same direction, with equal velocities; venus will therefore appear *stationary*, by lemma 1: by the same lemma, venus will have the like appearance, if while she is going from f to g the earth be going from K to L .

713 Since when venus is at her utmost eastern elongation going into her inferior semicircle her apparent motion in the same direction as the earth's is as nothing, § 706, and from thence increases till she comes into conjunction with the sun, where she is carried with her greatest velocity in the same direction as the earth is; there must be an intermediate place wherein the motion of venus thus increasing comes to equal the motion of the earth, and consequently where she appears *stationary*; as when venus goes from a to b 46 while the earth goes from A to B : again, since the motion of venus is swiftest at her inferior conjunction, in the same direction with the earth's motion, and from thence decreases till venus is at her utmost western elongation, where it is again as nothing; there must be some intermediate place as f , where the motion of venus thus decreasing comes to be equal to the earth's motion: as when venus is at f the earth at K , and venus goes through the arc fg whilst the earth goes through the arc KL . Thus we see one station of venus is a little after her utmost eastern elongation, when she begins to be retrograde; the other a little before her utmost western elongation, when she begins to be direct: as was said § 687.

714 Since the times of the conjunctions, utmost elongations, direct or retrograde motions as also of the stations of the inferior planets depend upon the combinations of their motions in their orbits with the motion of the earth in her orbit; any of these appearances will be more frequent in mercury than in venus: because mercury, going with a swifter motion through his orbit than venus, will in any given time pass more frequently than venus through the places where he is in conjunction, utmost elongation, direct, retrograde, stationary. How to find the time when an inferior planet will come again into a given situation with respect to the sun and the earth, may be seen by the following examples; let fig. 47 represent the orbits of venus and the earth, 47 let the earth be at E venus at v , venus is then in her inferior conjunction with the sun in γ : from the sun s venus and the earth would appear in conjunction

G g

FIG. junction in ϵ : let venus go round her orbit and return to v , the earth, taking longer time to go round the sun than venus, will in the mean time go from ϵ through only a part of her orbit, and venus must overtake the earth before she can have another inferior conjunction, that is she must, besides an entire revolution in her orbit, which is equal to four right angles, § 42, go through as much more angular motion round the sun as the earth has done in the mean time, so as to be in a right line drawn between the sun and the earth; suppose this to happen when the earth is got to F and venus to τ : the angular motions of the earth and venus performed in the same time are reciprocally as their periodical times: and therefore as the periodical time of the earth is to the periodical time of venus, so is the angular motion of venus, which is equal to four right angles added to the angular motion of the earth in the time between two like conjunctions of venus, to the angular motion of the earth in the same time; and therefore, by division of proportion^a, as the difference between the periodical times of venus and the earth is to the periodical time of venus, so are four right angles or 360 degrees to a fourth quantity, namely to the angular motion or number of degrees which the earth goes in her orbit from the time of one conjunction of venus to the next conjunction of the same kind: now the periodical time of the earth is 365 days 6 hours, or 8766 hours: the period of venus 224 days 16 hours, or 5392 hours: the difference is 3374 hours: say then, as 3374 is to 5392, so are four right angles or 360 degrees to a fourth number, which will come out 575° : this number of degrees the earth goes through in a year and 218 days: if therefore venus were this day in an inferior conjunction, it would be a year and 218 days before she came again into another inferior conjunction: and this alteration of the time occasions a proportionable change in the place, so that if one conjunction be in τ the next like conjunction will be in m .—The time between any situation of mercury with respect to the sun and the earth and another like situation may be found by the same method: the periodical time of the earth is 8766 hours: the period of mercury 87 days 23 hours, or 2111 hours: the difference 6655 hours: say then, as 6655 hours is to 2111, so are four right angles or 360° to a fourth number, which comes out 114° : this number of degrees the earth goes through in 116 days: if therefore mercury were to be this day in his inferior conjunction, it would be 116 days before he would be again in a like conjunction^b.

715 This problem is commonly resolved by astronomers in another manner; for they compute the diurnal heliocentric motions of venus and of the

^a Euclid. *elem.* I. 5 *prop.* 17.

^b See Keil's *astronomy*, lecture 15.

earth; the difference of these motions is the diurnal motion of venus from FIG. the earth, or the quantity by which venus would be seen to recede from the earth every day by a spectator placed in the sun: thus the middle motion of the earth is every day about 59 minutes and 8 seconds; the middle motion of venus in a day is 1 degree 36 minutes and 8 seconds, the difference is 37 minutes: say therefore as 37 minutes is to 360 degrees, or to 21600 minutes, so is one day to the space of time wherein venus having left the earth recedes from her 360 degrees; that is, to the time in which she returns to the earth again, or the time between two conjunctions of the same kind: this will be found to be about 583 days, as before § 714^a.

716 These times are here computed according to the middle motions of the planets, supposing them to move always equably, this is therefore called a *mean conjunction*: but because venus and the earth are really carried in elliptic orbits, in which their motions are constantly variable, sometimes faster, and sometimes slower; it may be, that the true conjunction shall happen some few days sooner or later than by this computation. *The time of the true conjunction* is to be computed from the time of the mean conjunction in this manner: find by astronomical tables the true places of the earth and venus in the ecliptic; from whence we shall have the distance of the earth and venus seen from the sun: compute also for the same time the angular motions of these two planets for any given time; suppose 6 hours; the difference of these two motions will give the access of venus to the earth, or her recess from it in 6 hours: then say, as this difference is to the arc between the places of venus and the earth at the time of the mean conjunction, so is 6 hours to the time between the mean conjunction and the true; this time added to, or subtracted from the time of the mean conjunction, according as venus is in antecedence or consequence from the earth, shews the time of their true conjunction^b.

717 If we would apply what was said in the beginning of this chapter to the *geocentric motions of the superior planets*, we may make use of the 46th figure in the following manner: suppose *abcde* &c. to be the orbit of the earth and *ABCDE* &c. to be the orbit of mars; when the earth and mars are in semicircles on the same side, as when the earth is going in the semicircle *abcde* while mars is going in the semicircle *oFM*, the earth and mars may be considered as going in the same direction, the earth in the direction *PQ*, mars in the direction *RS*; but the earth being carried with greater velocity than mars, to a spectator upon the earth mars will, by lemma 2, appear to go in the direction *SR*, contrary to his true direction, which refer-

^a Keil's *astronomy*, lecture 15.

^b Keil. *ibid.*

FIG. red to the ecliptic is retrograde: when the earth and mars are in opposite semicircles, as when the earth is in the semicircle *gbiklma* while mars is in the semicircle *oFM*, the earth and mars are carried in contrary directions; mars will therefore, by lemma 3, appear then to be carried the way he really goes, which referred to the ecliptic is direct: and he will appear to go faster than if the earth were at rest.

718 *The stations of the superior planets* may also be explained by the 34th figure, which was made use of, § 687, for the inferior planets, premising as was there done, that parallel lines which are not at a greater distance from one another than the diameter of the earth's orbit terminate, as to appearance, in the same point of the sphere of the heaven: let the two circles fig. 34, now represent the orbits of mars and of the earth, let the lines *AE*, *BF*, *CG*, *DH*, be parallel to *SP*; I say, they will all, if continued, terminate in the same point of the heaven, as to sense, as the line *SP* would do: now suppose mars at *A* when the earth is at *E*, the visual ray by which mars is seen is *EA*; suppose while mars goes from *A* to *B* the earth goes from *E* to *F*, the visual ray in which mars is now seen is *FB*; parallel to *EA*; and therefore mars will be all that time stationary, appearing in that point of the heaven where *SP* extended would terminate: this station of mars is at his changing from retrograde to direct: again, suppose when mars is at *C* the earth is at *G*, the visual line is *GC*; if while the earth goes from *G* to *H*, mars goes from *C* to *D*, he will be stationary, appearing in that point where a line drawn from *P* through *S* extended to the heaven would terminate: this station of mars is at his changing from direct to retrograde.

719 Since *the times of the conjunctions, oppositions, direct and retrograde motions*, as also of the *stations of the superior planets* depend upon the combinations of their motions in their orbits with the motion of the earth in her orbit; any of these appearances will be more frequent in saturn than in jupiter, in jupiter than in mars: because, the slower the motion of the planet is, the sooner will the earth overtake it, so as to have it again in any given situation: thus, suppose saturn to be in conjunction with the sun in γ , if saturn were to stand still, in one revolution of the earth, that is in one year, saturn would be again in conjunction in γ ; but as saturn goes on slowly according to the order of the signs, at the rate of about twelve degrees in a year, § 620, the earth must go through almost thirteen degrees more than an entire revolution, so that there will be a year and about thirteen days, between any conjunction of saturn with the sun and the conjunction immediately following: as jupiter goes faster in his orbit than saturn, the earth in going from one conjunction of jupiter to another must have a proportionably

nably longer time added to the year: the motion of mars being still swifter than that of jupiter, the time must be still longer between any two conjunctions immediately following one another.

720 The time when a superior planet will come again into a given situation with respect to the sun and the earth, may be found by the methods before used for the inferior planets, § 715: thus, the mean diurnal motion of the earth is about 59 minutes and 8 seconds; the mean motion of *saturn* in a day is 2 minutes, the difference is 57 minutes 8 seconds: say therefore as 57 minutes 8 seconds is to 360 degrees, or to 21600 minutes, so is one day to the space of time wherein the earth having left saturn recedes from him 360 degrees, that is, to the time wherein she returns to saturn again, or the time between two conjunctions, oppositions, or other like aspects &c: this time will be found to be 378 days, or 1 year and 13 days; so that if saturn were to be in opposition to the sun this day, it would be a year and 13 days before he would be again in opposition. — The middle motion of *jupiter* in a day is 4 minutes 59 seconds; the difference between this and the earth's diurnal motion is 54 minutes and 9 seconds: say then as 54 minutes 9 seconds is to 360 degrees, or to 21600 minutes, so is one day to the space of time wherein the earth having left jupiter recedes from him 360 degrees, that is, to the time wherein she returns to jupiter again, this time will be found to be 398 days, or 1 year and 33 days; so that if jupiter were to be in opposition to the sun this day, it would be a year and 33 days before he would be again in opposition. — The middle motion of *mars* in a day is 31 minutes 27 seconds; the difference between this and the earth's diurnal motion is 27 minutes 41 seconds: say then as 27 minutes 41 seconds is to 360 degrees or to 21600 minutes, so is one day to the space of time wherein the earth having left mars recedes from him 360 degrees, that is to the time wherein she returns to him again, this time will be found 780 days, or 2 years and 50 days; so that if mars were to be in opposition to the sun this day, it would be 2 years and 50 days before he would be again in opposition to him. The true conjunctions &c. may be found from the mean, as § 716.

721 Since the planets are all nearer to us than the fixt stars, they may sometimes come between us and some of those stars which lie near their heliocentric orbits: several such *occultations of fixt stars by the planets* are recorded by astronomers both ancient and modern: an inferior planet may also sometimes pass between us and a superior planet, and cover it for a little while: but these appearances are very rare. *Aristot. meteor. l. 1. c. 16. Ptolem. almagest. l. 10. c. 4 & 9. l. 11. c. 3. Kepler. astron. part. optic. p. 304. Streete astronomia Carolina p. 107. Ricciol. l. 7. c. 10.*

CHAP. II. THAT THE PLANETS ARE OPAKE BODIES AND BORROW
ALL THEIR LIGHT FROM THE SUN.

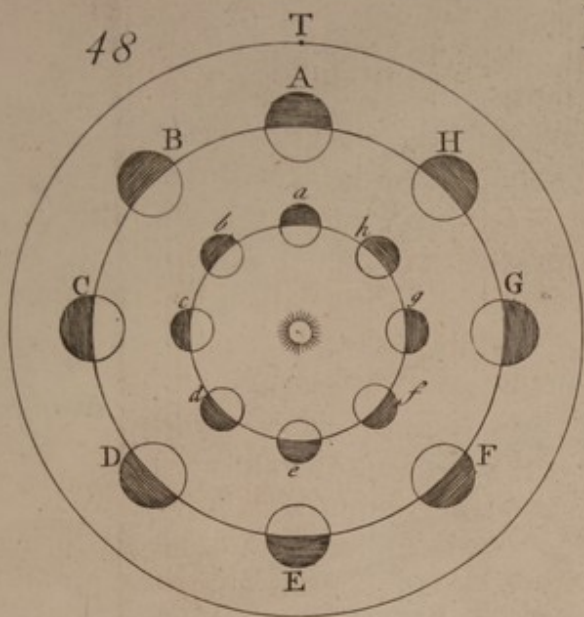
722 The planets are globular opake rough bodies which receive all their light from the sun: it is owing to the roughness of the surfaces of the planets that they reflect light to us from every part in the manner they do; optical writers shew that the image of any object reflected from a globular *speculum* or looking-glass is diminished more and more, the greater the distance of the eye is from the speculum: if therefore the surfaces of the planets were smooth and polished they would be invisible to us, the image of the sun reflected by any of them would be too small to strike our eye sensibly, or if it did it would appear only a lucid point: whereas now their diameters are of a sensible magnitude, because, their surfaces being rough, every point of that hemisphere which is enlightened by the sun reflects light every way, § 223; and consequently makes the body of the planet visible in its proper dimensions. What I have now said may be illustrated by the following experiment, place a silver globe perfectly polished of about 2 inches diameter in the sun, the rays which fall thereon being reflected variously according to their several incidences upon the convex surface, we shall have them come to our eye only from one point of the globe, which will therefore appear a small bright spot, but the rest of its surface will appear dark; let this globe be boyled in the liquor made use of to whiten silver, and placed in the sun, and it will then appear of its full dimensions all over white or luminous: for the effect of that liquor is to take off the smoothness of the polish, and make the surface rough, so that every point of it shall reflect the rays of light of every kind every way.

723 Every planet has one half of its surface illuminated; this illuminated hemisphere is always that which is turned towards the sun, the other hemisphere of the planet is dark: if we would speak accurately the sun, being larger than any of the planets, illuminates a little more than half of every planet: thus, in fig. 136 of the introduction, let $ABCD$ be the sun, $EFGI$ a planet, the illuminated part EIG will be a little bigger than EFG ; but this difference between the enlightened and the obscure part is insensible, because the distance of the sun from any of the planets is so great that his light may be considered as coming to them in lines physically parallel, having the same effects as if it came in lines exactly parallel.

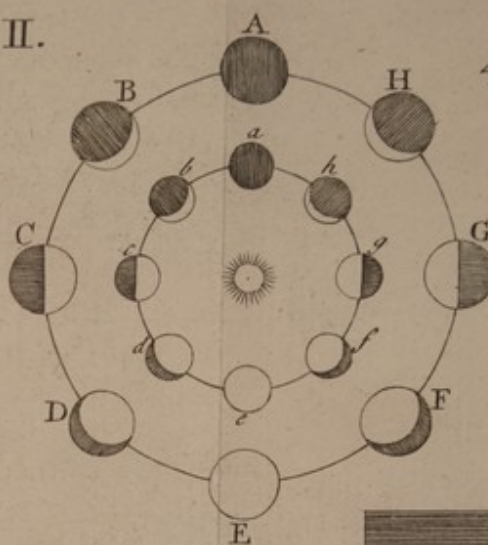
724 *The inferior planets*, going round the sun in less orbits than our earth does, will sometimes have more sometimes less of their illuminated hemispheres

Book II.

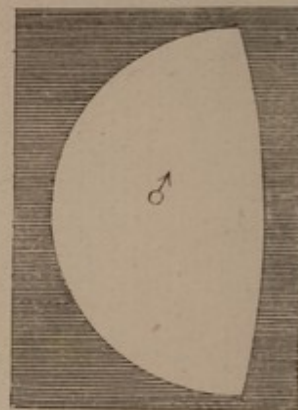
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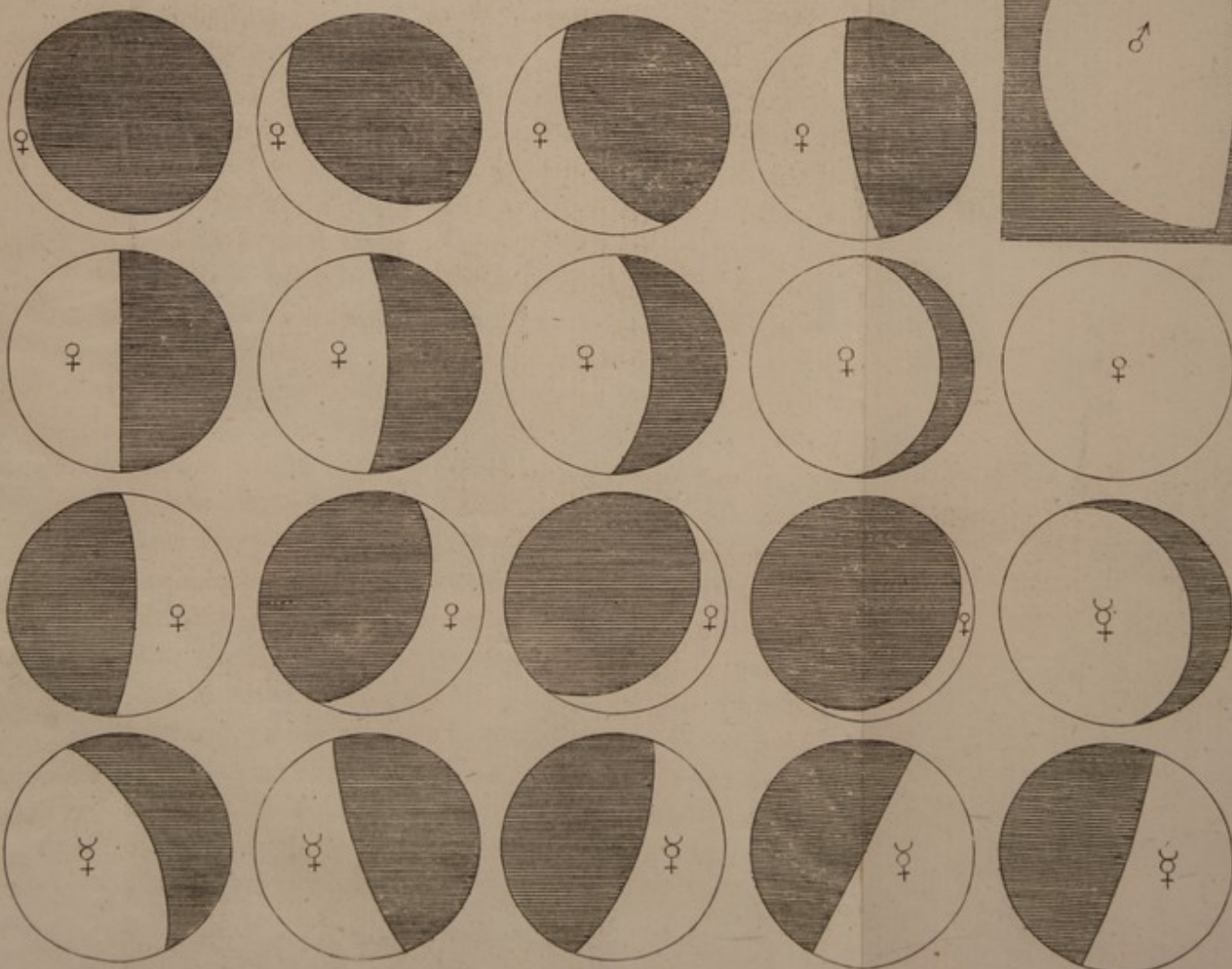
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50



spheres turned towards us, and consequently, since the illuminated part only **FIG.** is visible to us, through a good telescope they will appear to go through all the changes which we see in the moon, being sometimes horned like the new moon, like a half-moon, like the moon nearer the full &c: the 48th figure represents the orbits of mercury, venus and the earth, with the sun in the center of them; the planets mercury and venus are drawn in eight different situations, with their illuminated hemispheres towards the sun; if we suppose the earth to be all the while at τ , when venus is at A her dark hemisphere is towards the earth, and she is therefore invisible, except this conjunction happens in her node, for then she appears like a dark spot upon the disk of the sun, as has been already said § 681: when venus is at B a little of her enlightened hemisphere is turned towards the earth, and therefore she appears sharp horned: when she is at C about half her enlightened hemisphere is towards our earth, and she appears like an half moon: at D more than half her enlightened hemisphere is towards the earth, and she appears like the moon about 3 days before she is at the full: at E the whole enlightened hemisphere is towards the earth, venus is then either behind the sun or so very near him that she can hardly be seen, but if she could she would appear round, like the full moon: at F venus will appear like the moon about 3 days after the full: at G like a half moon again: at H sharp horned again, but the points of the horns are turned the contrary way to what they were in B: all this is applicable to mercury. The 49th figure exhibits the appearances of venus and mercury corresponding to their several situations in the 48th figure; thus when venus is at A in fig. 48th, she is quite dark, as at A in fig. 49: when she is at B in fig. 48, she appears as at B fig. 49, &c: so when mercury is at a fig. 48, he appears as at a fig. 49, &c: fig. 50 exhibits several views of mercury and venus actually taken with the telescope, and delineated by the accurate *Hévelius: selenograph. p. 70.*

725 The inferior planets appearing with the several phases of the moon is a proof of their going round the sun in less orbits than our earth does: when *Copernicus* first published his account of the solar system, it was objected to him that it could not be true, because if it were the inferior planets must have different phases, according to their different situations with respect to the sun and the earth; whereas they always appear round to us: the answer made by him was, that they appear round to the eye by reason of the great distance, but that, if we could have a nearer and more distinct view, we should see in them the same variety of shapes as we do in the moon: the telescope invented since the death of *Copernicus* has verified this solution of the objection, to the satisfaction of every body. The distance of objects, especially

FIG. specially such as are luminous, hinders us from seeing their true shapes; the flame of a torch or a candle, though really of a conic figure, has at a distance the round appearance of a star or planet.

726 *The superior planets*, going round the sun in larger orbits than our earth does, always turn much the greatest part of their enlightened hemispheres towards the earth; and therefore *appear round like the full moon*, except *mars* who sometimes appears like the moon at a little distance from the full, and in quadrature almost bisected; according to *Hevelius*^a, from
 51 whom the picture of mars fig. 51 is taken. For when mars is in quadrature
 37 as at *q* or *Q*, fig. 37, a good deal of his dark hemisphere is towards the earth at *E*: when mars is in conjunction at *C*, or in opposition at *A*, his whole enlightened hemisphere is towards the earth; and therefore he must in both those cases appear exactly round: in other situations, which are at any distance from being in quadrature, as at *B* and *D*, there is so little of the dark hemisphere of mars towards the earth that he wants very little of appearing round. The orbit of the earth is so small in comparison of the orbits of jupiter and saturn, that they turn very nearly the same hemispheres towards us as they do towards the sun; for which reason those planets always appear round through the telescope.

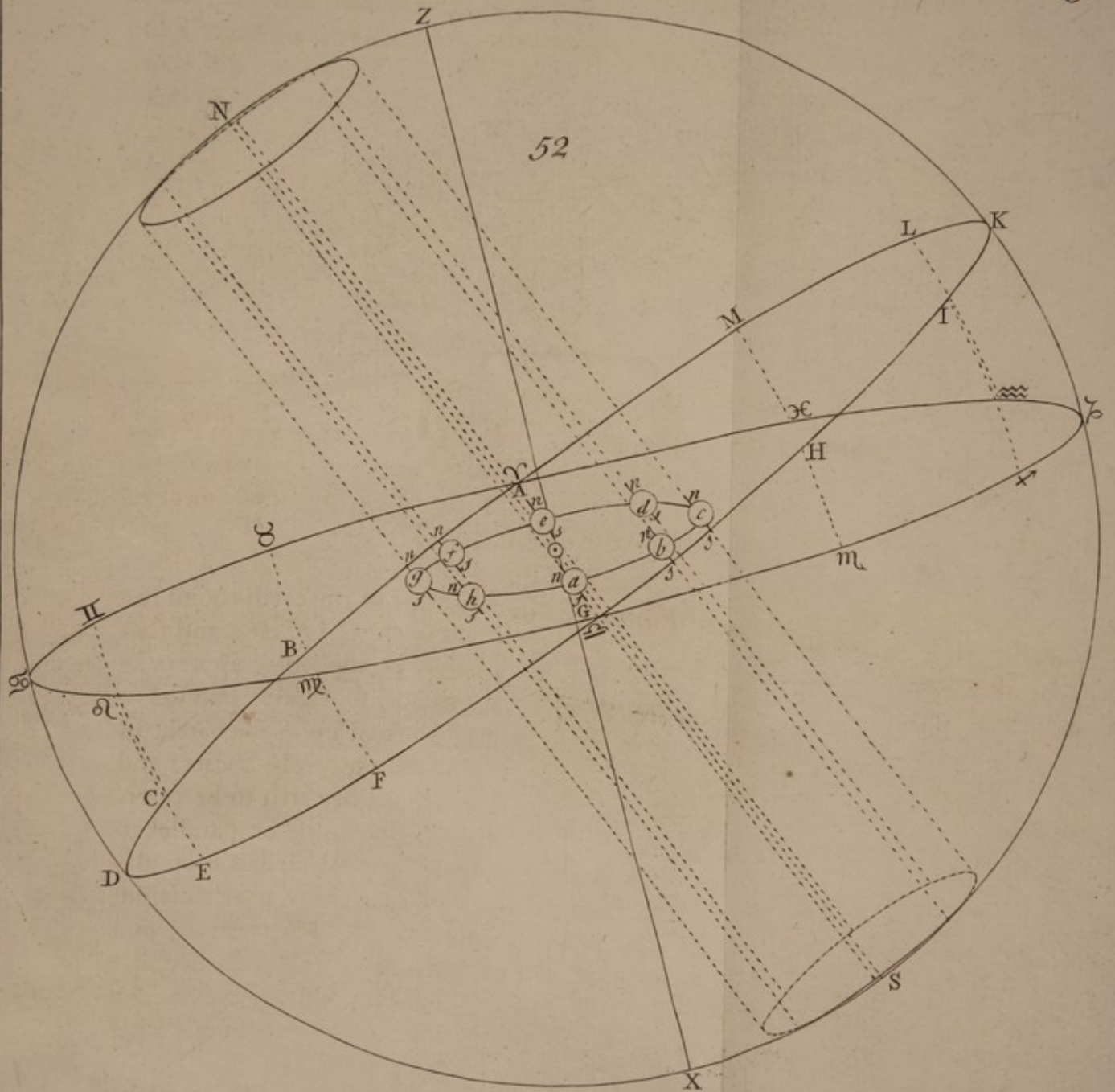
727 The inferior planets do not shine brightest when they are full or round viewed through the telescope: thus, *venus* does not appear in her
 47 *greatest brightness* in her superior conjunction at *A*, fig. 47, though we then see her illuminated hemisphere entire: the reason is this, the light which comes from venus to us in that situation is more diminished by her being then at her greatest distance from us^b, than it is increased by her turning her whole enlightened disk towards us: Dr. *Halley* has shewn^c, that venus is brightest when her elongation from the sun is about 40 degrees; she does indeed in that situation shew but about a fourth part of her enlightened disk to the inhabitants of the earth, so that she would through the telescope look like the moon about 3 days old; but she is then so near us that this fourth part contains a larger area and sends us more light than her whole enlightened disk does when she is at her greatest distance from us: in this situation with respect to the sun venus is often seen in the day time, in sunshine, and is by the vulgar sometimes taken for a new star; and in the night her light is so strong that she casts a shadow, which none of the rest of the heavenly bodies ever do, except the sun and moon. By the method made use of by *Halley* for venus, it will be found that *mercury* is in his *greatest brightness* when very near his utmost elongation.

^a *Selenographia* pag. 42 & 67.

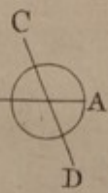
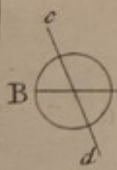
^b §. 88.

^c Jones *Phil. Trans. abr.* vol. 4. p. 300.

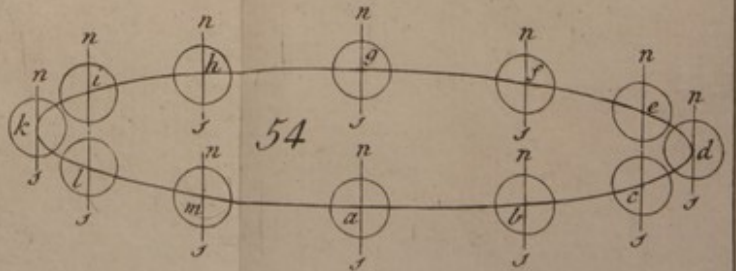
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728 All the planets appear white or luminous, because their surfaces reflect the rays of light of all kinds, red, yellow, green, &c together; for it is the mixture of all these which produces whiteness^a: there is however some little difference in *the colours of the planets*, as their surfaces are differently modified, so as to reflect the rays of one colour in greater quantity than others: thus mars appears of a reddish hue, the light of venus is a little inclined towards yellow, that of jupiter very white, saturn a little livid, and more dim than the rest, one reason of which may be his great distance from us. We may also observe some difference in *the colours of the fixt stars*: astrologers tell us such stars are of the nature of mars, such of venus &c, the ground of these fancies seems to be the different colours of the stars, wherein some resemble one planet and some another^b. *Kepl. astr. opt. p. 261.* FIG.

CHAP. 12. THE ROTATION OF THE EARTH AND HER REVOLUTION
ROUND THE SUN CONSIDERED TOGETHER: THE INCLINATION
OF THE EARTH'S AXIS TO THE PLANE OF HER ORBIT
THE CAUSE OF THE DIVERSITY OF SEASONS.

729 It has been already shewn, that by the rotation of the earth round her axis all the heavenly bodies appear to turn round us in 24 hours^c: and that from the same cause every point in the heaven except the poles appears in the natural day to describe either the celestial equator, or a circle parallel to it^d: and that according as any place is differently situated upon the earth, in a parallel, right or oblique sphere, every one of the heavenly bodies, and amongst the rest the sun, appears to the inhabitants of the earth to be carried round them either in a circle coincident with the horizon, or parallel to it, or that cuts the horizon at right, or at oblique angles^e. It has been also observed, that the sun appears in different points of the heaven at different times of the year, sometimes in the equator, sometimes very near it, and sometimes farther off from it, either north or south; and that this declination from the equator is within certain limits, which the sun never exceeds^f: the cause of all this is the inclination of the earth's axis to the plane of her orbit, which is now to be more particularly considered.

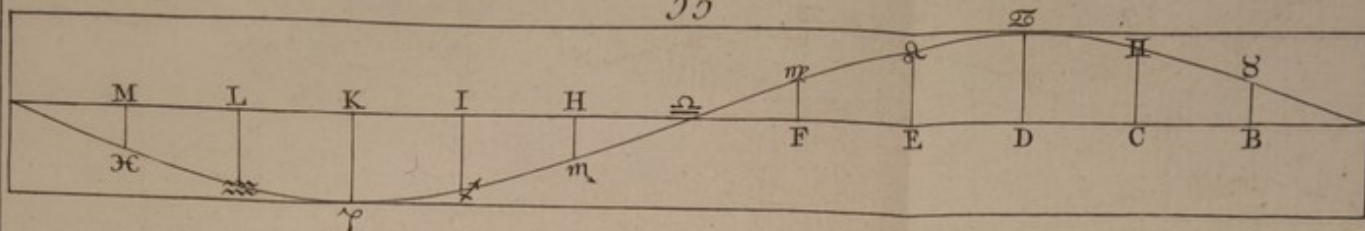
730 *The axis of the earth's diurnal rotation* is in every part of the earth's orbit *parallel to it self* in every other part of her orbit: thus, fig. 52, let 52
N D S K be a glass sphere, or the sphere of the heaven, viewed at a great di-

a § 222. b Chaldei, ab elementis eorumque commixtionibus ad stellas progredientes, naturas earum per colores judicant: Albumasar, *introduc.* l. 1. c. 2. c § 296. d § 343. e book 1 chap. 8. f book 1 chap. 8.

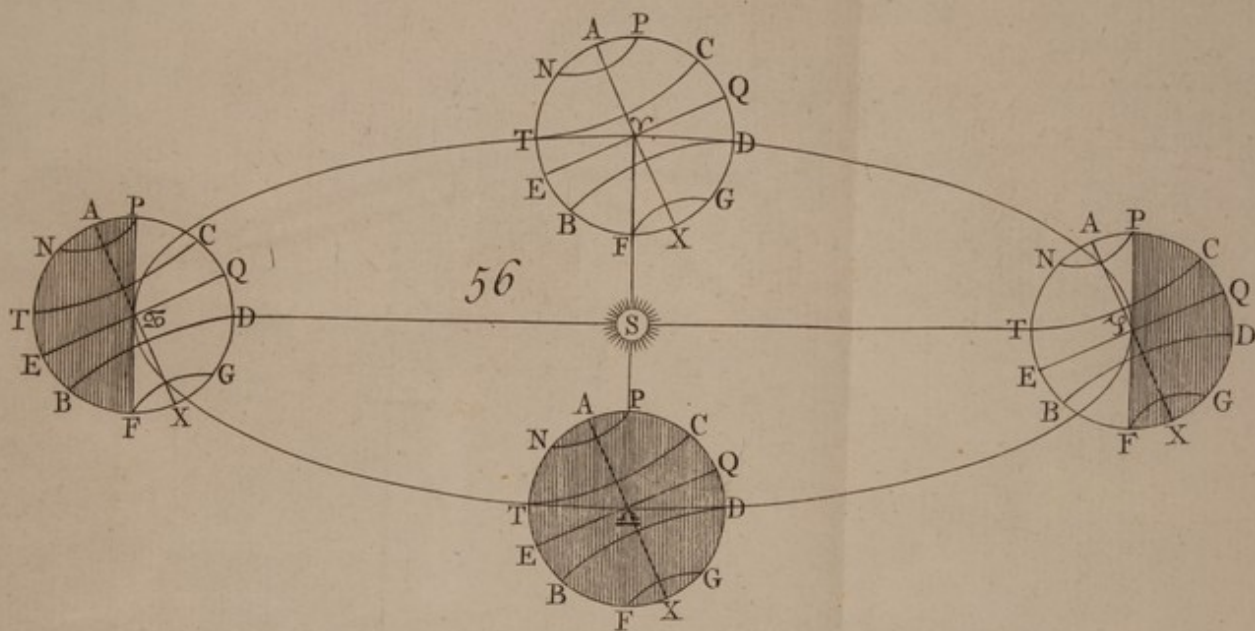
FIG. stance, so as to have the eye a little elevated above the plane of the ecliptic,
 52 which circle, marked with the characters of the signs γ ϑ Π &c, is here
 seen in a perspective view, as is also the plane of the equator, $ABCDEFGHIH$
 $IKLM$, and the orbit of the earth, $abcdefghg$, in the middle whereof is the
 sun \odot ; ns is the axis of the earth, if the position of it be observed when the
 earth is at a , and a line drawn parallel to it as ns , I say that when the
 earth is at b or c or d &c, her axis is in every one of these places parallel
 to ns , as it is represented in the figure. Some writers, in explaining the Co-
 pernician system, mention this parallelism of the earth's axis as caused by a
 third motion of the earth, which they call *the motion of inclination*, as if, be-
 sides the diurnal rotation and annual motion, another motion were impress-
 ed upon the earth, whereby the inclination of her axis to the plane of her
 orbit were continued in the same angle; whereas this is only the consequence
 of the diurnal and annual motions continuing independent upon one ano-
 ther, the one no way disturbing the other: suppose a globe to be carri-
 53 ed along so that the center of it goes in the line AB , when the globe is at
 A , among the infinite number of diameters of the globe imagine some one
 inclined to the line AB in any angle to be marked out, so as to be distin-
 guished from the rest, as CD ; if the globe has no other motion impressed
 upon it but a progressive one in the line AB , when it is come from A to B
 the diameter CD will be in the situation cd , parallel to its former situation:
 if at the same time with the progressive motion a rotation round the axis CD
 be impressed upon the globe, these motions are independent upon one ano-
 ther and one will have no effect upon the other, but the axis of rotation will
 at B , as also at all the intermediate places between A and B , be parallel to CD .
 If instead of a rectilinear motion we suppose the globe to go in a curve, as in a
 circle or ellipsis, as the earth does round the sun, the axis of rotation will then
 also continue parallel to it self, undisturbed by the circular motion: these
 motions of a globe are here supposed to be *in vacuo*, and not in a medium
 where they meet with any friction or resistance, for then the axis of rotation
 might be changed, according to the different impulse of the medium upon its
 surface: now this is the case of the earth and planets in their revolutions
 round the sun, that they move in spaces void of all resisting matter; or if it
 has any resistance it is so small as not to have any sensible effect in many ages.

731 *The axis of the earth extended both ways terminates as to sense, in
 the same points of the heaven, throughout the year: in reality the axis of
 the earth extended describes in the heaven round each of the points N and s
 a circle equal to the circumference of the earth's orbit; but the sphere of the
 heaven is so immensely large, that in comparison of it the orbit of the earth*
 is

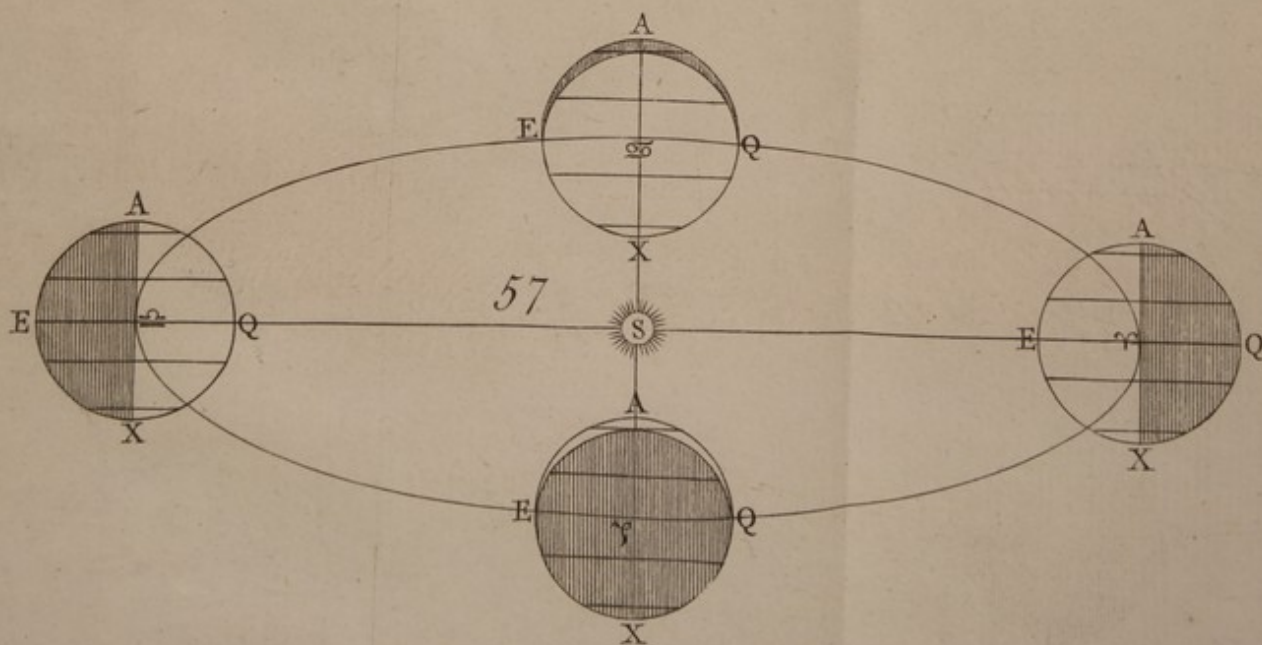
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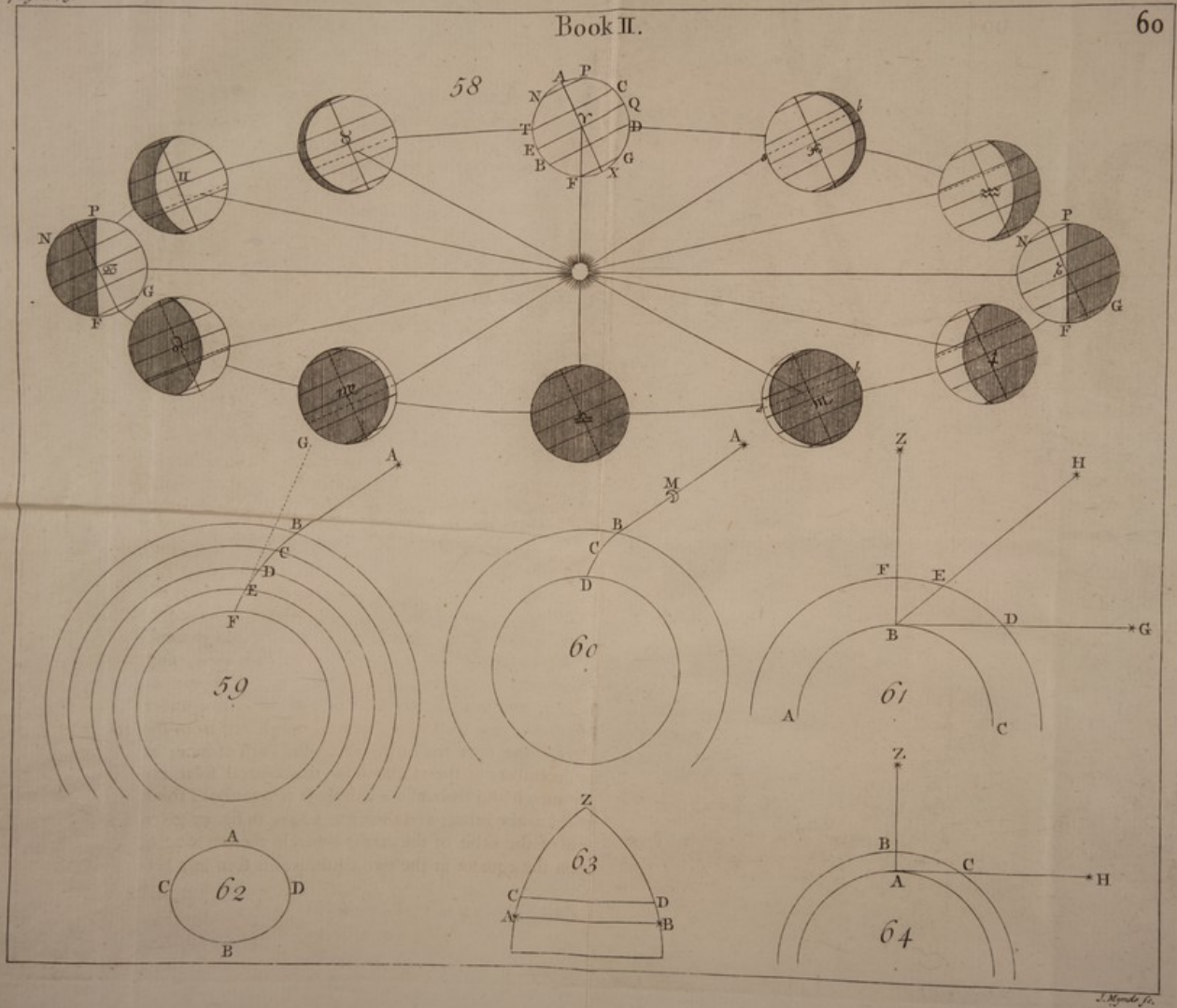


is but as a point: and consequently the axis of the earth extended both ways FIG.
 terminates, as to sense, throughout the year in the points *N* and *s*; and those 52
 points are the poles round which the heaven appears every day to turn.
 From hence it comes to pass, that every point in the heaven appears to de-
 scribe either the equator or a parallel to the equator, in the space of a natu-
 ral day. It has already been observed, § 359 and 366, that the sun is but
 twice a year in the equator, at the two equinoxes; and that every other day
 of the year he is more or less in north or south declination: and accordingly
 appears to describe a parallel more or less north or south of the equator:
 what diversity the declination of the sun thus varying occasions in the length
 of the days and nights, at different times of the year in the same horizon, or
 at the same time of the year in different horizons, has been before explained,
 book 1. chap. 8. and 10. I shall now shew from whence this difference in
 the sun's declination arises.

732 *If the axis of the earth were perpendicular to her orbit*, the planes of
 the earth's equator and of her orbit would be coincident; and then, since the
 sun is in the center of the earth's orbit, the plane of the earth's equator ex-
 tended would pass through the sun, and the sun would appear to be carried
 round in the celestial equator every natural day in the year: thus, fig. 54 ex- 54
 hibits the orbit of the earth in a perspective view, the eye being at a great
 distance and a little elevated above the plane of it: if the axis of the earth
ns were perpendicular to it, as it is here represented, the plane of the earth's
 equator in every one of the situations *a, b, c, d, &c* would pass through the
 sun; and then the sun's apparent place in the heaven must be in the celestial
 equator; and consequently we should have perpetual equinox: the sun indeed
 would, by the progress of the earth through her orbit, appear every day in a
 different point of the equator, so as to go through that circle in the space of a
 year; but this would make no difference in the seasons of the year: for every
 place of the earth would then enjoy the light and heat of the sun every day
 alike, and that in the same manner as it does now at the equinoxes.

733 *The axis of the earth is inclined to the plane of the earth's orbit*, so as
 to make an angle with it of $66^{\circ} 31'$: hence the plane of the earth's equa-
 tor makes with the plane of her orbit an angle of $23^{\circ} 29'$, the complement
 to $66^{\circ} 31'$: and consequently the celestial equator and ecliptic intersect each
 other in an angle of $23^{\circ} 29'$. The ecliptic is cut by the celestial equator in
 the points γ and ϵ , and therefore when the sun appears in either of those
 points he appears in the celestial equator also; in every other point of the e-
 cliptic he is in north or south declination: let fig. 52 be the glass sphere, or 52
 the sphere of the heaven represented as seen at a great distance in a perspec-

FIG. tive view, $ABCD$ &c is the celestial equator, $\gamma \delta \Pi \varpi$ &c the ecliptic, $abcde$
 52 fg the orbit of the earth, N the arctic, s the antarctic pole, z the north, x the
 south pole of the ecliptic: about the 10th of march, the earth being at a , the
 sun's place to an inhabitant of the earth is at γ ; and the rotation of the earth
 for that natural day makes him appear to be carried round in the celestial e-
 quator: the next day, by the earth going about one degree in her orbit from
 a towards b , the sun's place will appear to be advanced about one degree from
 γ towards δ ; this carries him a little into north declination, and he that day
 appears to go round in a parallel a little north of the equator. As the earth
 goes on from a to b , from b to c , from c to d , &c, the sun appears to pro-
 ceed in the ecliptic; changing his declination according as that circle devi-
 ates from or approaches to the celestial equator: thus, from γ towards δ
 and so on according to the order of the signs, his northern declination every
 day increaseth; for about the 10th of april when the sun is at δ his declinati-
 on is δB ; about the 10th of may when he is at Π it is ΠC ; about the 10th of
 june when he is at ϖ it is ϖD ; and this is his greatest north declination,
 amounting to $23^{\circ} 29'$: that day the sun appears to be carried round in the
 tropic of cancer, and is vertical at noon to those who live in a correspond-
 ing parallel upon the earth, namely in the northern tropic, in $23^{\circ} 29'$ north
 latitude: from thence the sun's declination continually decreases; so that a-
 55 bout the 10th of july his place is at Ω , his declination is ΩE : about the
 10 of august he is at φ , his declination φF : about the 10th of september
 he is again in the celestial equator at ϵ : for the remaining months he is in
 south declination; in october at μ : in november at τ : in december at ζ ,
 in his greatest south declination, when he describes the tropic of capricorn,
 and is vertical at noon to those who live under that circle, or in $23^{\circ} 29'$ south
 latitude: in january the sun is at κ : in february at λ : and about the re-
 55 turning 10th of march in γ again. In the 55th figure we have the celestial
 equator and ecliptic in another view; it represents an hoop containing all
 that part of the sphere of the heaven which is between the tropics; suppo-
 sing it to be cut open and unbent into a plane, in the same manner as we
 considered the zodiac, § 65 1: it shews the change of the sun's declination in
 his progress through the twelve signs: in april it is δB ; in may ΠC ; &c.
 52 734 The 52 and 55 figures shew how it comes to pass that the sun's de-
 55 clination changes faster the nearer he is to one of the points γ and ϵ , where
 the equator and ecliptic intersect one another; for there those circles recede
 fastest from one another: the same figures shew also why the sun's declinati-
 on changes more slowly the nearer he is to one of the solstitial points ϖ and
 ζ ; because there the ecliptic touching the two tropics does not recede sensibly



bly from them for some space: and therefore all the while the sun is near ei- FIG.
ther of the solstitial points the parallels in which he every day appears to
be carried round by the earth's rotation are not sensibly different from one
another, but are in a manner coincident with the nearest tropic: and so long
the change of the sun's declination is scarce perceptible.

735 *The change of the seasons*, arising from the inclination of the earth's
axis to the plane of her orbit, may be very well shewn in another manner:
the 56 figure, which is commonly made use of for this purpose is taken o- 56
riginally from *Galileo*^a; the ellipsis $\gamma \approx \mathcal{L}$ represents the orbit of the earth
in a perspective view, the earth with such circles as are requisite for the pre-
sent occasion is pictured in the four several situations which she is in at the
beginning of the four seasons of the year, AX is the axis of the earth, pa-
rallel to it self in all the situations, EQ is the equator, TC the northern tropic,
 NP the north polar circle, BD the southern tropic, FG the south polar: now
if we imagine a line to be drawn from s the center of the sun to the center
of the earth in each situation, this *central solar ray*, as it may be called, will
about the 10th of june, when the earth is at \mathcal{L} , terminate upon the surface
of the earth at the northern tropic TC ; and consequently, by the earth's ro-
tation round her axis, the inhabitants of every part of the circle TC will
successively have the sun in their zenith, or in other words he will be vertical^b
to them that day at noon, and the sun appears that day to be carried round
in the tropic of cancer: about the 10th of september the earth is at γ , the
central solar ray $s\gamma$ terminates upon the surface of the earth in the equator;
and therefore the sun appears to be carried round in the celestial equator, and
is successively vertical to those who live under that circle: about the 10th
of december, when the earth is at \approx , the central solar ray $s\approx$ terminates on
the surface of the earth in the southern tropic BD ; and for that day the sun
appears to be carried round in the tropic of capricorn, and is vertical to those
who live under that circle: about the 10th of march the earth is at \simeq , and
the ray $s\simeq$ points at the equator again; and consequently the sun appears a-
gain in the celestial equator, and is again vertical to those who live under
the line.—The 56 figure shews very well how a central ray drawn from the 56
sun points at the northern tropic upon the earth, about the 10th of june; at
the southern tropic, in december: it shews also how this central solar ray
points at the equator, in march and september: but these two seasons I think
are not so well represented as the other; and therefore I have in fig. 57 given 57
another perspective view of the orbit of the earth, wherein the pointing of
the central solar ray upon the equator at the two equinoxes is seen in a bet-
ter manner. 736

a v. *Galilei Systema cosmicum dialog.* 3.

b See § 388 to § 395.

FIG. 736 Fig. 58 exhibits the orbit of the earth in another perspective view; the earth is drawn in the twelve several places which she is in about the 10th day of each of the twelve months, keeping the parallelism of her axis in those several situations: now if a line be drawn from the center of the sun to the center of the earth in each situation, in march when the earth is at ϵ , it points at the equator: in april it points at the parallel here exprest by the dotted line ab , which is a little in north latitude: in may at a parallel farther north, which is also exprest in the figure by a dotted line: and in june at the northern tropic: in july and august the line points at parallels less and less north, marked here by dotted lines: in september it points at the equator again: for the rest of the months the line points at parallels in south latitude, marked in the figure by dotted lines; going more and more south from september till december, when it points at the southern tropic: from thence it points every day at a parallel nearer and nearer to the equator; till about the 10th of the returning march it points at the equator again: it is easy to see that according as the central solar ray points any day of the year, at the equator, or at some parallel upon the earth more or less north or south, the sun appears to be carried round that day either in the equator, or in some celestial parallel more or less north or south, corresponding to the terrestrial parallel.

737 The sun is not only twice a year vertical to the inhabitants under the line, namely at the two equinoxes; but also to the inhabitants of any parallel situated between the tropics: thus the sun is vertical to the inhabitants of the parallel ab in april, as the earth is going from the equinoctial point ϵ towards the solstitial point ζ ; and in august, as she is going from the solstitial point ζ towards the equinoctial point γ : in like manner, the figure shews the central solar ray to point at the same parallel in july as in may: in october as in february: in november as in january: from hence appears the reason why the days are of the same length with us, as also in any place in an oblique sphere, in august as in april: in july as in may: in november as in january: in october as in february: the only difference is that in april, may, january, and february, the days are increasing; but in july, august, october, and november, they are decreasing.

738 If we imagine a plane perpendicular to the line drawn from the center of the sun to pass through the center of the earth, the section of the globe of the earth by that plane will be a great circle, which may be called *the circle of illumination*; because it marks out that half of the earth which is towards the sun and is enlightened by him, wherein it is day, and divides it from the other half which is in darkness, wherein it is night: the circle of illumination always divides the equator into two equal parts, § 177; and therefore

therefore all who live under the equator have the days and nights equal FIG.
throughout the year.

739 About the 10th of march, when the earth is at the point ϵ , and the 56
10th of september, when she is at ν , the plane of the circle of illumination 57
passes through the axis of the earth, and divides not only the equator but 58
every parallel also both north and south into two equal parts; and consequently every point of the earth's surface is by her rotation carried just as long in light as in darkness: those times therefore are the *two equinoxes*, when the nights and days are equal over the face of the whole earth^a, and the circle in which the sun then appears to be carried round is called *the equinoctial*.

740 In every other situation of the earth, every parallel through which the plane of the circle of illumination passes is divided by it unequally, and more unequally the farther the earth is from the equinoctial points ν and ϵ ; 58
and consequently, at all other times of the year except the equinoxes, the *days and nights are unequal*, in every place situated in an oblique sphere: and the greater the distance of time is from the equinox, the greater is this inequality of the day and night: the greatest inequality therefore is at the *two solstices*; when the earth is at ω or ζ .

741 In every situation of the earth except at the two equinoxes at ν and ϵ , 58
there may be taken parallels near each pole which are not cut by the circle of illumination, but are some of them entirely in the light, others entirely in the dark: so long as any parallel is entirely in the light it is *continual day* without night in every place of the earth in that parallel; as it is also in every place situated between that parallel and the nearest pole: so long as any parallel continues entirely in the dark it is *continual night* without day to all the inhabitants therein; as it is also in every place situated between that parallel and the nearest pole.

742 About the 10th of june, in the summer solstice, when the earth is 56
at ζ , the largest parallel NP which can be taken entirely in the light is the 58
north polar circle; the largest parallel FG which can be taken entirely in the dark is the *south polar circle*: at that time therefore it is continual day every where within the north polar, and continual night in all places within the south polar circle: about the 10th of december, at the winter solstice, when the earth is at ω , the reverse of this happens; the north polar circle NP is then entirely in the dark, the south polar FG entirely in the light, as appears by

^a In a parallel sphere the sun is in reality then carried round in the horizon, half his body being above and half below the plane of that circle: notwithstanding, to a spectator under either of the poles his whole body would be seen all the while above the horizon; because refraction always makes him appear higher than his true place.

FIG. the 56 and 58 figures: at that time therefore it is continual day, within the 56 south polar, and continual night within the north polar circle.

58 743 From the summer solstice, when the earth being at \mathcal{B} fig. 58, the north polar NP is entirely in the light, the south polar FG entirely in the dark, the greatest north parallels which can be taken entirely in the light and the greatest south parallels which can be taken entirely in the dark daily grow less and less, till the earth comes to γ , at the autumnal equinox; when, the circle of illumination cuts all the parallels equally, and every point of the earth's surface is by her rotation continued as long in the light as in the dark. From the autumnal equinox the parts about the north pole begin to go entirely into the dark, and the parts about the south pole entirely into the light; and the circle of illumination cuts the surface of the earth in such a manner as to encrease every day the darkness about the north pole, and the light about the south, till the winter solstice; when, the earth being at \mathcal{C} , the whole north polar is in the dark, and the whole south polar in the light. From the winter solstice the greatest entirely dark northern parallels and the greatest entirely light southern parallels grow every day less and less, till the vernal equinox; when, the earth being at \mathcal{D} , all the parallels are again divided into equal parts by the circle of illumination. From the vernal equinox the parts about the north pole come every day more and more entirely into the light, and the parts about the south pole more and more entirely into the dark, till at the summer solstice the earth is again at \mathcal{B} ; and the circle of illumination cuts the surface of the earth in such manner as to throw the whole north polar into the light, and the whole south polar 58 into the dark. The 58 figure makes what I have now said easy to be comprehended; shewing the gradual successive gaining and losing of light and darkness, in different months, upon the parts about each of the poles.

744 From the 10th of march to the 10 of september, whilst the earth is going in her orbit through $\mathcal{C} \ \mathcal{M} \ \mathcal{I} \ \mathcal{B} \ \mathcal{K}$, and by that motion makes the sun appear to go through the northern signs $\gamma \ \delta \ \Pi \ \mathcal{C} \ \Omega \ \mathcal{A}$, the circle of illumination divides the parallels through which it passes, unequally, in such a manner that the greater part of every north parallel is in the light, and the greater part of every south parallel in the dark: during that time therefore the days are longer than the nights to all who live in north latitude; and the nights are longer than the days to those who live in south latitude. In the other half year, from the 10th of september to the 10th of march, when the earth is going in the other part of her orbit through $\gamma \ \delta \ \Pi \ \mathcal{C} \ \Omega \ \mathcal{A}$, and the sun is in the southern signs $\mathcal{C} \ \mathcal{M} \ \mathcal{I} \ \mathcal{B} \ \mathcal{K}$, the parallels through which the plane of the circle of illumination passes are so divided by it that the greater part

part of every north parallel is in the dark, and the greater part of every south parallel in the light: during all that time therefore all who live in north latitude have the night longer than the day; but all who live in south latitude have the day longer than the night.

CHAP. 13. OF THE ATMOSPHERE, REFRACTION, THE TWILIGHT.

745 Our earth is surrounded every way with a sphere of air wherein various kinds of particles are continually floating; the whole taken together is called *the atmosphere*, from its spherical shape, and from the greek word *atmos* which signifies a mist, vapor or smoke: The *air* is a fluid *sui generis*, having some properties not to be found in other fluids; it is compressible, and elastic, capable of having its parts squeezed closer together, or of expanding it self so as to take up a greater space: a cubic foot of air may be compressed into a 60th part of that dimension; on the other hand, a cubic inch of air, if all pressure were taken off, would expand it self so as to fill a space many millions of times as large^a. Air has this in common with all other material substances that it is weighty, though in a less degree than most bodies with which we are acquainted; a glass bottle filled with air weighs something heavier than it will do if the air is exhausted out of it by the air-pump; we may also make a bottle filled with air in its natural state still heavier, by forcing more air into it.

746 The gravity of the air makes the lowest parts of the atmosphere to be the heaviest as well as the most dense, because they are pressed upon by the weight of all above them; and for the same reason the higher the air is the thinner it grows, till at the extream or highest parts of the atmosphere it is exceedingly thin and light: and this is the reason why clouds and vapours can rise but to a certain height, the higher air being specifically lighter; and consequently unable to support them. What the texture is of the particles of which air consists we must be contented to be ignorant of; some have thought they might resemble the spring of a watch, which unbends its coils when left to it self, but may be forced to wind them up into a narrow compass. Sir *Isaac Newton*^b supposes that some bodies may have their smallest parts endued with a repulsive power, which makes them recede from one another, and that air may be of this sort. How the upper and lower air come to be of a different density may be illustrated by a familiar instance of an heap of wool or sponge, or the like compressible matter, where the pressure of what is above will

^a Newt. *princip.* l. 3 *prop.* 41. opt. book 3 qu. 28. Gregor. *astron.* l. 5 *prop.* 3. ^b Opt. book 3 qu. 31.

make the lower parts lye more compact and close together, in proportion to the weight which lies upon them: thus the atmosphere may be considered as consisting of an indefinite number of spheres of different densities, enclosing one another as the several coats of an onion do.

747 The atmosphere is not only admirably fitted for the respiration and nourishment of animals, the growth of vegetables, the production and propagation of sounds &c; but helps also to make our habitable earth that beautiful scene of variety which it now is. If it were not for the atmosphere the sun and stars would give us no light but just when we turn our eyes upon them, the brightness of the sun would indeed be greater than it is, but if we turned our eyes from him towards any other part of the heaven, it would appear as dark and full of stars as in a bright star-light night in winter. As for the various bodies which are upon the earth, they would all, without the atmosphere, appear to us as dark as at midnight; except only those parts of them which happened to be in such a position that the rays of the sun falling upon them were reflected to our eyes. It is easy to imagine how much of the beauty of the visible creation would be lost in these circumstances, besides the insupportable prejudice to the eyes of all creatures, by passing so suddenly from pitchy darkness to excessive light. The numberless small particles of various kinds which float in the air receive the light from the sun, and like so many small *specula* or looking-glasses reflect and scatter it through the air^a, and this occasions that light which we see in the day-time, by which our eyes are affected so strongly as to render the fainter light of the stars insensible. By this means the sky is illuminated all round us by the sun, not only whilst he is above our horizon, but also for some time before his rising, and after his setting; so long as any of his rays can either directly or by refraction reach any part of the atmosphere within our visible horizon: for the air, as well as all other mediums which transmit light, refracts or bends the rays of it, if they come into it from a different medium: this property of the air is exceedingly beneficial to the inhabitants of the earth, as it lengthens the days by the addition of the twilight, but it gives astronomers some trouble, as it changes a little the places of all the heavenly bodies, and makes them appear higher than they really are, and must therefore be taken into the account, if we would be exact: the ancients were not well acquainted with the refraction of the air, which renders many of their observations of the heavenly bodies, especially near the horizon, liable to uncertainty.

^a Thus if the sun shines into a dark room through an hole in a door or shutter of a window, we perceive it in a long stream of light: which appears brighter the fuller the room is of dust or smoke.

748 *The rays of light* coming out of a thinner medium into a thicker are **FIG.** *refracted* towards the perpendicular, whether the surface of the thicker medium be plane or curve; as has been already observed, § 213 and 215: a ray of light coming from any of the heavenly bodies to the eye of an inhabitant of the earth passes first through a space either perfectly void of all matter, or very nearly such, in a strait line, till it reaches the top of our atmosphere; upon the spherical surface of which if it falls obliquely, it will be a very little bent or refracted, at its entrance into the exceedingly thin air, and will be continually more and more bent as it passes through thicker and thicker air, till it arrives at the earth.

749 If we imagine the atmosphere to be divided into any number of concentric spherical spaces of different densities, for example into four, as is done in fig. 59 by a like number of spherical surfaces, a ray of light coming **59** from a star at A would proceed in a strait line till it falls upon the outward surface at B^a, there it would be a little bent, and go on from thence in a strait line to C, where, falling upon a surface of thicker air, it would receive a greater refraction, and go on in the strait line CD, at D still thicker air would give it a new and greater refraction, into the line DE, at E the air being more dense would give it the greatest refraction, into the line EF: as all vision is made in a right line, a spectator upon the earth at F would see the star in the line FE continued; so that its refracted or apparent place is G, higher or nearer to the zenith than its true place A: refraction therefore makes all the heavenly bodies appear higher than their true places.

750 If instead of four we imagine the atmosphere to be divided into an indefinite number of spherical spaces containing air of different densities, it is easy to see that, instead of going in four several strait lines, a ray of light passing through the atmosphere will go in a curve line, whose curvature continually increases till it falls upon the earth, as is represented by the curve **60** ABCD, fig. 60.

751 If a star be in the zenith of a spectator upon the earth, a ray coming from it to his eye falls perpendicularly upon the spherical surface of the atmosphere, and will therefore proceed in a strait line, without being refracted, by § 212: if a star be in the horizon of an observer, a ray coming from it to his eye suffers the greatest refraction of all, for two reasons, 1 the ray falls upon the surface of the atmosphere with the greatest obliquity, 2 it passes through the largest space of the lower and denser part of the air: thus, fig.

a I call that the outward surface of the atmosphere where the air begins to be dense enough to cause a sensible refraction; the height of this is about 40 or 50 miles above the earth: how much higher the thinner air may reach is difficult to determin.

FIG. 61, let ABC be part of the surface of the earth, B the place of an observer,
 61 if a star be in his zenith at Z , the line ZB drawn from the star to his eye falls perpendicular upon the spherical surface of the atmosphere at F , and consequently will be perpendicular to all the concentric spherical surfaces which we can imagine in the atmosphere: if a star be in his horizon at G , the ray GD , by which the star is seen, falls more obliquely upon the atmosphere at D , than the ray HE from a star at H does at E ; or than a ray coming from any star between G and Z to the eye at B : moreover the line DB is longer than FB , or EB , or than any other line which can be drawn from the outside of the atmosphere to the eye at B , between DB and FB .

752 The *refraction* of the heavenly bodies is not only *greatest in the horizon* and *none at all in the zenith*, but decreases from the horizon as their altitudes increase, till near the zenith it becomes scarcely sensible: astronomers give us tables of refraction, which serve chiefly for the country where they were made: the air is condensed, and consequently refraction increased by cold; and therefore refraction is greater in cold countries than in hot: it is also greater in cold weather than in hot, in the same country, and the morning refraction is greater than the evening; because the air is rarified by the heat of the sun in the day, and condensed by the coolness of the night: refraction is also subject to some small variation, at the same time of the day in the finest weather; see § 483: but these niceties are, I suppose, not considered in the tables, or rather they give us the mean refractions between the greatest and the least.

753 The *refraction of all the heavenly bodies is the same* at the same apparent height above the horizon: thus the refraction of the sun, moon, or any star is the same, at the same apparent altitude; because the rays come from them all in strait lines till they enter our atmosphere: if the moon which is the nearest to our earth be of the same apparent altitude with a star, a ray coming from the moon goes through the same space of air as a ray from the
 60 star does: thus, fig. 60, if a star be at A , the moon at M , a ray from either comes alike in the line AB and is alike refracted into the line $ABCD$.

754 The horizontal refraction being the greatest is the cause of *the sun appearing of an oval form, at his rising and setting*: for the lower edge of the
 62 sun B is more refracted than the upper edge A , by which means they are brought nearer together, that is the perpendicular diameter AB is shortened: whereas refraction does not shorten the transverse diameter CD : moreover, the lower edge suffering the greatest refraction, the horizontal sun does not appear a compleat ellipsis, but the lower half CBD is part of a more oblong oval than the upper half CAD . What I have now said of the sun is applicable

cable also to the rising and setting moon. For the same reason, if we take with FIG. an instrument the distance of two stars when they are in the same vertical and near the horizon, we shall find it considerably less than if we measure it when they are both at such an height as to suffer little or no refraction; because the lower star is more lifted up than the higher. There is also another alteration made by refraction in the apparent distance of stars: if two stars are in the same almicanarah, or circle parallel to the horizon, their apparent distance is less than the true; for since refraction makes each of them higher in the vertical or azimuth in which they appear, it must bring them into parts of the vertical where they come nearer to one another: thus, let the star A be in the vertical A Z, the star B in the vertical B Z, their true distance 63 is A B, an arc of a great circle: suppose A to be lifted up by refraction to C, and B to be lifted up to D, their apparent distance is now C D, less than A B; for since all verticals converge so as to meet in the zenith Z, it is manifest the distance between any two verticals is diminished as they grow nearer the zenith: this contraction of distance, according to Dr. Halley^a, is at the rate of at least one second in a degree; so that, for example, if the distance between two stars in a position parallel to the horizon measures 30° , it is at most to be reckoned but $29^{\circ} 59' 30''$.

755 I have here set down four *tables of refraction*, that of Tycho Brahe^b, which is the oldest we have, was made from his own observations; it goes no higher than to 45° , because at greater altitudes the author tells us he did not find any sensible refraction: Tycho's table of the refraction of the stars is not much different from those of Flamsteed and Newton, except near the horizon; he makes the refractions of the sun and moon about $4'$ greater than that of the stars, probably from some false hypothesis^c, for by comparing his own observations in his *historia cœlestis*, they come out the same, making allowance for the small unavoidable errors to which observations are liable. The table of Cassini is taken from *les elemens d'astronomie verifiez*, p. 21: Flamsteed's from his *historia cœlestis*, vol. 1 p. 396, and is the result of his own observations: Sir Isaac Newton's table was first published in the transactions, n. 368, may 1721.

756 The use of the following tables: the apparent altitude of any of the heavenly bodies being given, take from thence the refraction, and what remains is the true altitude: thus, if the altitude of the sun, moon, or a star be observed 15° , take from thence, by Flamsteed's table, $3'$, the true altitude is $14^{\circ} 57'$.

a Phil. transact. n. 368, may &c. 1721.

b v. Tychonis *prozymasmata*. p. 79, v24, & 280.

c Perhaps from his making the parallaxes of the sun and moon about 3 minutes too great.

A TABLE OF REFRACTIONS.

Appar. Altit.	Refraction according to					
	Newt.	Flamst.	Cassini	Tycho		
	° ' "	° ' "	° ' "	⊙ ° ' "	☾ ° ' "	* ° ' "
0 0	33 45	33 0	32 20	34 0	33 0	30 0
0 15	30 24					
0 30	27 35	26 38				
0 45	25 11					
1 0	23 7	23 22	27 56	26 0	25 0	21 30
1 15	21 20					
1 30	19 46	20 17				
1 45	18 22					
2 0	17 8	17 26	21 4	20 0	20 0	15 30
2 30	15 2	15 15				
3 0	13 20	13 23	16 6	17 0	17 0	12 30
3 30	11 57	11 53				
4 0	10 48	10 39	12 48	15 30	15 20	11 0
4 30	9 50	9 38				
5 0	9 2	8 48	10 32	14 30	14 20	10 0
5 30	8 21					
6 0	7 45	7 26	8 55	13 30	13 50	9 0
6 30	7 14					
7 0	6 47	6 25	7 44	12 45	12 45	8 15
7 30	6 22					
8 0	6 0	5 37	6 47	11 15	12 0	6 45
8 30	5 40					
9 0	5 22	5 2	6 4	10 30	11 20	6 0
9 30	5 6					
10 0	4 52	4 33	5 28	10 0	10 45	5 30
11 0	4 27	4 6	4 58	9 30	10 10	5 0
12 0	4 5	3 45	4 32	9 0	9 35	4 30
13 0	3 47	3 29	4 12	8 30	9 0	4 0
14 0	3 31	3 13	3 54	8 0	8 30	3 30
15 0	3 17	3 0	3 36	7 30	8 0	3 0
16 0	3 4	2 48	3 24	7 0	7 30	2 30
17 0	2 53	2 38	3 11	6 30	7 0	2 0
18 0	2 43	2 29	3 0	5 45	6 30	1 15
19 0	2 34	2 21	2 49	5 0	6 0	0 30
20 0	2 26	2 14	2 39	4 30	5 30	0 0

A TABLE OF REFRACTIONS.

ap. alt.	Refraction according to						ap. alt.	Refraction									
	New.		Fla.		Caff.			Tycho		New.		Fla.		Caff.			
	°	' "	°	' "	°	' "		°	' "	°	' "	°	' "	°	' "		
21	2	18	2	7	2	31	4	0	5	0	56	0	36		0	40	
22	2	11	2	1	2	25	3	30	4	35	57	0	35		0	38	
23	2	5	1	55	2	18	3	10	4	10	58	0	34		0	37	
24	1	59	1	50	2	12	2	50	3	45	59	0	32		0	35	
25	1	54	1	45	2	6	2	30	3	20	60	0	31	0	29	0	34
26	1	49	1	40	2	0	2	15	3	0	61	0	30		0	33	
27	1	44	1	36	1	55	2	0	2	40	62	0	28		0	31	
28	1	40	1	31	1	51	1	45	2	20	63	0	27		0	30	
29	1	36	1	27	1	46	1	35	2	0	64	0	26		0	28	
30	1	32	1	23	1	42	1	25	1	45	65	0	25		0	27	
31	1	28	1	20	1	38	1	15	1	30	66	0	24		0	26	
32	1	25	1	17	1	34	1	5	1	20	67	0	23		0	25	
33	1	22	1	14	1	30	0	55	1	10	68	0	22		0	24	
34	1	19	1	11	1	27	0	45	1	0	69	0	21		0	22	
35	1	16	1	9	1	23	0	35	0	50	70	0	20	0	19	0	21
36	1	13	1	7	1	20	0	30	0	45	71	0	19		0	20	
37	1	11	1	5	1	18	0	25	0	40	72	0	18		0	19	
38	1	8	1	2	1	15	0	20	0	35	73	0	17		0	18	
39	1	6	1	0	1	12	0	15	0	30	74	0	16		0	17	
40	1	4	0	58	1	10	0	10	0	25	75	0	15		0	16	
41	1	2	0	56	1	7	0	9	0	20	76				0	14	
42	1	0	0	54	1	5	0	8	0	15	77				0	13	
43	0	58	0	52	1	3	0	7	0	10	78				0	12	
44	0	56	0	50	1	1	0	6	0	5	79				0	11	
45	0	54	0	48	0	59	0	5	0	0	80		0	9	0	10	
46	0	52	0	46	0	58					81				0	9	
47	0	50	0	45	0	56					82				0	8	
48	0	48	0	44	0	54					83				0	7	
49	0	47	0	42	0	52					84				0	6	
50	0	45	0	40	0	50					85				0	5	
51	0	44			0	49					86				0	4	
52	0	42			0	47					87				0	3	
53	0	40			0	45					88				0	2	
54	0	39			0	43					89				0	1	
55	0	38	0	34	0	41					90		0	0	0	0	

Ricciolus, besides treating largely of refraction, in his *almagest*, gives us there and in his *astronomia reformata*, another table; wherein, from his own observations and those of *Tycho* and others corrected, he sets down the different refractions of the heavenly bodies, at three different times of the year, summer, winter, and the equinoxes.

757 Refraction lengthens the day and shortens the night; by making the sun appear above the horizon a little before his rising, and a little after his setting: refraction also makes the moon and stars appear to rise sooner and set later than they really do. The apparent diameter of the sun or moon is about half degree; the horizontal refraction is a little more than that, as appears by the tables; this makes both sun and moon appear entire above the horizon when they are entirely below it.

758 The rays of the sun being scattered over some of those parts of the air which are within our visible horizon cause *the twilight*^a, both morning and evening: the morning twilight begins when the sun is not more than about 18° below our rational horizon, for then his rays first reach the eastern parts of the air within our visible horizon: as he grows nearer rising, his light spreads farther round, and enlightens a larger portion of our air, and it grows lighter and lighter, till sun-rise: in like manner, after sun-set the light gradually decreases, till the sun is got so low that none of his rays can reach the western parts of the air within our visible horizon, or not enough to cause any sensible light there; and then the evening twilight ends: this happens when the sun's depression below the rational horizon is about 18° .

759 There is some little difference in *the duration of the twilight*, arising from the different density and height of the atmosphere; by which the vapours and other particles that reflect the light of the sun are carried up to a greater height at some times than at others; for the higher the reflecting particles are raised, the lower may the sun be depressed and yet have his rays reach them. For this reason the evening twilight ends at a lower depression of the sun than the morning twilight begins at; because the heat of the day, by rarifying the air, raises the reflecting particles to a greater height, whereas the coolness of the night, by condensing the air, brings them down nearer to the earth: this causes the evening twilight to be longer than the morning, at the same time of the year in the same place. For the same reason the twilight is longer in hot weather than in cold, as also in hot countries than in cold, o-

^a It is called the twilight as being between or partaking of two lights, the light of the sun and that of the stars; *v. Skinneri etymolog.* The latin word *crepusculum* is from its being doubtful whether it be day or night: *res dubia crepera vocantur*, Censorinus: *v. Vossii etymolog.* The beginning of the morning twilight is commonly called the day-break, day-spring, or dawning of the day.

ther circumstances being the same: *Ricciolus* at Bologna observed, at the equinoxes, the duration of the twilight in the morning to be 1 hour and 47 minutes; in the evening 2 hours: and that the evening twilight did not end till the sun was above 20° below the horizon^a: however the differences in the same country arising from this cause are so small that they are generally not taken notice of.

760 Besides these *physical differences*, as they may be called, in the duration of the twilight, arising from the different constitution of the air, there are other *astronomical differences*; which are owing to the different situations of places upon the earth, or to the difference of the sun's place in the heaven: if a place is situated in a parallel sphere, or near to it, the apparent motion of the sun being parallel to the horizon or nearly parallel, he will be carried round for some months at a less depression than 18° ; and therefore so long will the twilight last: in a right sphere, the twilight is shortest; because, the sun rising and setting at right angles to the horizon, the changes of his depression are made in the shortest times: in any place in an oblique sphere, the nearer the place is to one of the poles the longer are the twilights; because the case comes the nearer to that of a parallel sphere. As to the different places of the sun, the twilights are longest in all places which are in north latitude, when the sun is in the tropic of cancer; in south latitude, when he is in the tropic of capricorn: the time of the shortest twilight is different in different latitudes: with us in England, the shortest twilights are about the latter end of september, and the latter end of february^b.

761 *Ricciolus*^c gives us a table of the duration of the twilight, at different times of the year, in different latitudes: but, as the differences of latitude are therein set down in no orderly progression, I have here inserted a new and more methodical table, calculated upon a supposition that the twilight begins and ends when the sun is 18° below the horizon: the letters c. d. signify that it is then continual day without night: c. n. continual night without either day or twilight: w. n. that the twilight then lasts the whole night: thus for instance, when the sun is in \times or m , that is, about the 10th of february, and about the 10th of october; the twilight, in the latitude of $52\frac{1}{2}^{\circ}$ is 2 hours: in the latitude of 70° it is 3 hours 41'.

The table shews how the duration of the twilight increases as the latitude is greater; which is a very providential relief to the inhabitants of the countries near the pole, in their very long winter nights.

^a *almagest*. l. 1 c. 31.
book 2 prop. 41: and *Kzil*, lect. 20.

^b How to find the shortest twilight, may be seen in *Gregory's astronomy*
^c *almagest*. l. 1 c. 31.

A table of the duration of the twilight in different latitudes.

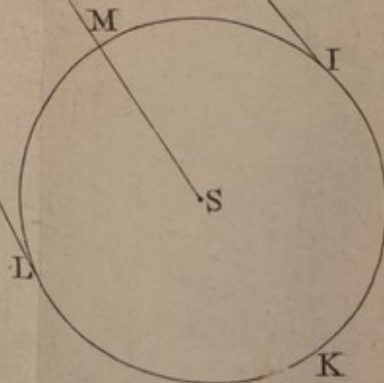
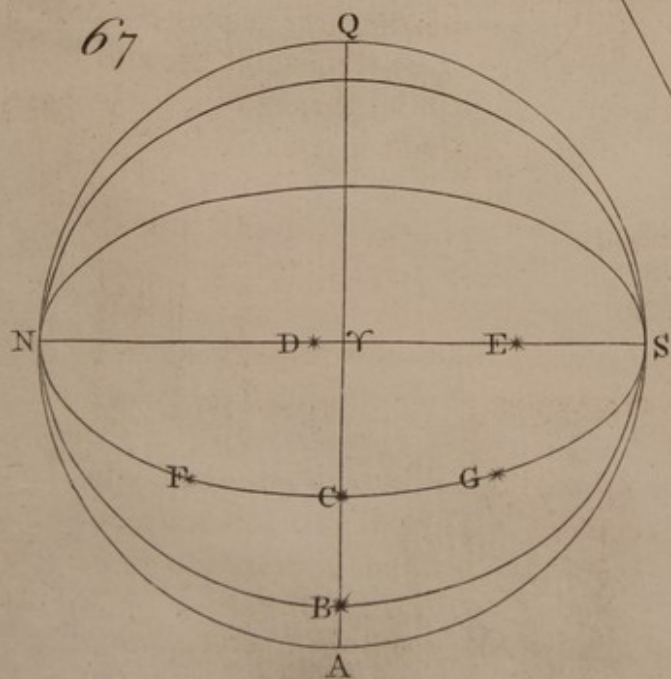
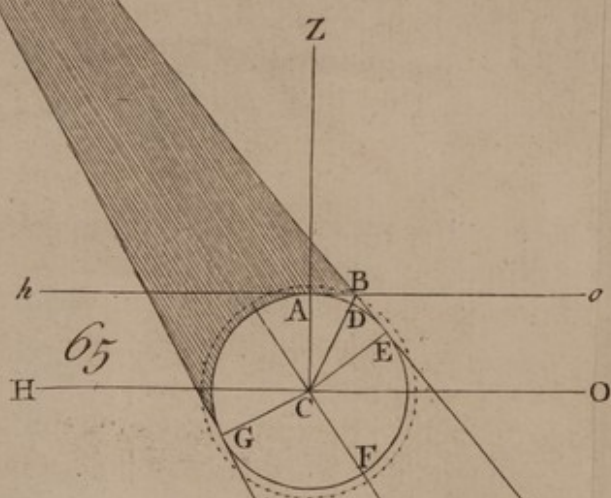
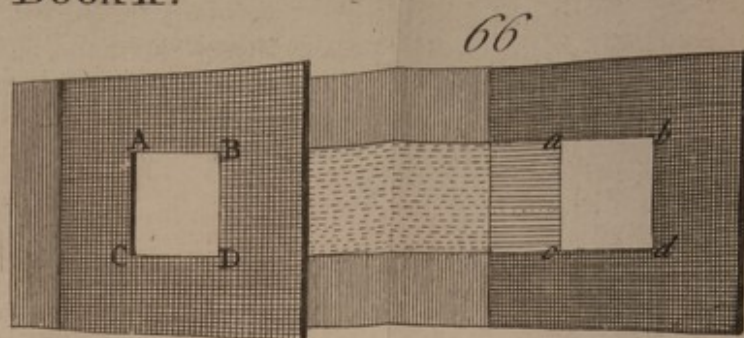
Alt. Pole	0	10	20	30	40	45	50	52 $\frac{1}{2}$
☉ enters	H. '	H. '	H. '	H. '	H. '	H. '	H. '	H. '
☉	1 18	1 21	1 28	1 41	2 8	2 39	w. n.	w. n.
♈ ♏	1 16	1 19	1 25	1 36	1 58	2 19	3 3	w. n.
♉ ♏	1 13	1 15	1 20	1 28	1 43	1 55	2 12	2 25
♊ ♎	1 12	1 13	1 17	1 24	1 35	1 44	1 55	2 2
♋ ♎	1 13	1 14	1 18	1 24	1 35	1 43	1 54	2 0
♌ ♎	1 16	1 17	1 21	1 28	1 40	1 49	2 1	2 8
♍	1 18	1 19	1 23	1 30	1 43	1 53	2 6	2 15

The table continued.

Alt. Pole	55	60	65	70	75	80	85	90
☉ enters	H. '	H. '	H. '	H. '	H. '	H. '	H. '	H. '
☉	w. n.	w. n.	w. n.	c. d.	c. d.	c. d.	c. d.	c. d.
♈ ♏	w. n.	w. n.	w. n.	c. d.	c. d.	c. d.	c. d.	c. d.
♉ ♏	2 41	3 55	w. n.	w. n.	w. n.	c. d.	c. d.	c. d.
♊ ♎	2 10	2 33	3 8	4 18	w. n.	w. n.	w. n.	w. n.
♋ ♎	2 8	2 27	2 56	3 41	5 2	17 32	w. n.	w. n.
♌ ♎	2 18	2 43	3 26	11 38	11 14	10 32	8 38	c. n.
♍	2 26	2 57	4 4	10 24	9 30	7 46	c. n.	c. n.

762 The twilight has hitherto been considered only as it is caused by the light of the sun himself reflected to us; but, besides this, the body of the sun is encompassed with a sphere of light, which is either the *ether* that immediately surrounds him heated to such a degree as to become luminous, or *the sun's atmosphere*, consisting of fiery particles emitted from his body, but retained near him by attraction: this, being of larger dimensions than the sun, must rise before him, and set after him; and consequently lengthens the twilight, by illuminating our air, when the sun is at too low a depression to reach it with his own light: this is also the cause that the rising sun is preceded by a luminous segment of a circle, in the east, different from the light which the atmosphere reflects from the body of the sun: the like to this may be observed in the west, just after sun-set. *Greg. astr. book 2 prop. 8.*

763 *The height of the atmosphere*, that is of such parts of it as are dense enough to reflect the sun's light, may be determined, by knowing at how low a depression of the sun the twilight begins or ends: *v. Ricciol. almagest. l. 1 c. 31.* supposing that depression of the sun to be 18°, the height of the atmosphere



sphere is about 40 miles^a. *Cassini*^b is of opinion that, in settling the duration of the twilight, astronomers have made it too long; by mistaking for part of it a light which accompanies the sun in his progress through the ecliptic^c: he asserts the depression of the sun at the beginning of the morning and end of the evening twilight to be but 17° : the height of the atmosphere would from this supposition be but 37 miles: and it should seem that this is still less, because of the sun's atmosphere, § 762. FIG.

Additions to the foregoing chapter.

764 There are three reasons why the horizontal refraction is the greatest: 1 a ray from any of the heavenly bodies when they are in the horizon falls with the greatest obliquity upon the spherical surface of the atmosphere: 2 an horizontal ray passes through the thickest vapours: 3 it passes through the largest space of thick vapours: thus, fig. 64, a ray coming from a star at *z* in the zenith passes only through the line *BA* of the atmosphere, whereas a ray coming from a star at *H* in the horizon passes through the line *CA* of the atmosphere, which is almost 14 times as long as the line *BA*: this excess of *CA* above *BA* is not represented in any tolerable proportion in the figure, nor indeed can be; because in so small a compass the earth cannot be exhibited large enough for the atmosphere. 64

765 The great length which an horizontal ray goes through thick vapours is the reason that the small stars are sometimes invisible near the horizon; and that the sun may be looked upon without hurting our eyes, at rising and setting; because much of his light is reflected back: the sun near the horizon looks red, because the red-making rays which are least refrangible are also least reflexible; and therefore pass in the greatest quantity through those vapours by the particles of which the rest of the sun's light is reflected back again into the *ether*.

766 In § 757 I said refraction lengthens the days: this it does a little in every place, but near either of the poles this effect of refraction is very considerable; causing the sun to appear some days above the horizon, when he is really below it: almost every body who writes upon this subject takes notice of that extraordinary refraction, amounting to several degrees, which gave some Hollanders who wintered A. D. 1596 in Nova Zembla, in the latitude of 76° , a sight of the sun 17 days sooner than by the theory he ought to have appeared to them: some have thought, that the Hollanders misreckoned either their latitude or their time; but their account appears too accu-

^a See § 767. ^b *Decouverte de la lumiere celeste* &c pag. 52 ed. Par. ^c Of which I shall treat hereafter.

FIG. rate to admit of such a suspicion: the cause of so great a refraction was probably the very great condensation of the air, in that extremely cold region, during a night of above three months continuance. *Kepl. astr. opt. p. 138.*

Remarks upon § 763.

65 767 In the 65th figure let $A E F G$ be the earth, the pointed circle surrounding it the outward surface of the atmosphere, the height of which $D B$ is to be found: let A be the place of an observer, $b o$ his sensible, $H O$ his rational horizon: let $I K L M$ be the sun, $E N G$ the shadow of the earth, $I B$ a ray from the upper edge of the sun, touching the earth in E , and falling upon the outward surface of the atmosphere at B , from whence it is reflected to the eye of the observer at A , in the line of his sensible horizon $B A$; the sun, being larger than the earth, illuminates a little more than an hemisphere thereof, represented by the arc $E F G$, which is $180^{\circ} 32'$ *, whose half $E F$ is $90^{\circ} 16'$, as is also the angle $E C S$: $O C S$ is the sun's depression, which is 18° by observation, this being taken from $E C S$ leaves $E C O 72^{\circ} 16'$: now since $\angle C O$ or $A C O$, by the definition of the horizon, is 90° , $A C E$ complement to $E C O$ is $17^{\circ} 44'$: and the half of $A C E$ viz. $D C A$ † is $8^{\circ} 52'$: here then is a right-angled triangle $B A C$, with one acute angle $B C A$ and one side $A C$ known; and therefore, by § 143, the ratio of any one side to any other may be found. If $A C$ be made radius $B C$ will be the secant of the angle $B C A$ of $8^{\circ} 52'$, say then as 10000000 to 10120948, so is $3967\frac{1}{2}$ the number of miles contained in the earth's semidiameter to a fourth number, which is $4015\frac{1}{2}$, the number of miles from the center of the earth to the outward surface of the atmosphere; from which take the earth's semidiameter $3967\frac{1}{2}$ miles, the remainder 48 miles is the height of the atmosphere above the surface of the earth. In this calculation no allowance is made for refraction, which in the horizon is about $34'$, so much therefore must the angle $B C A$ be diminished,

65 * The angles $E C F$ and $E C N$ together are equal to two right angles or 180° , *Eucl. I. 1 prop. 13.* $E C F$ must therefore exceed 90° as much as $E C N$ wants of 90° : now in the right angled triangle $C E N$ the angle $E C N$ must want as much of a right angle as $C N E$ amounts to, § 55, that is as much as the apparent semidiameter of the sun $I N S$ viewed from N the vertex of the cone of the earth's shadow would be: the shadow of the earth reaches but a little farther from the earth than the moon, which distance from the earth is in a manner insensible, compared with the vast distance of the sun from the earth; so that we may assume the sun's apparent semidiameter viewed from N the same as we find it viewed from the earth at F , this by observation is at the sun's middle distance $16'$; therefore $E C F$ is $90^{\circ} 16'$.

65 † The triangles $B A C$ and $B E C$ are right angled by § 28, the sides $A C$ and $E C$ are equal by § 25, the side $B C$ is common, and hypotenuse in both, whose square is equal to the squares of $B E$ and $E C$ and to the squares of $B A$ and $A C$ by *Eucl. book 1 prop. 47*, but $A C$ and $E C$ being equal their squares are equal, wherefore the squares of $B E$ and $B A$ must be equal; and consequently the sides $B E$ and $B A$ themselves are equal: so that the triangles $B E C$ and $B A C$ are mutually equilateral, and therefore mutually equiangular, by *Eucl. book 1 prop. 8*; that is, the angle $B C E$ is equal to the angle $B C A$.

this

this brings it to $8^{\circ} 18'$, the secant of which is 10105851; from whence the height of the atmosphere comes out 42 miles.

CHAP. 14. THE SUN'S DISTANCE FROM THE EARTH VARIABLE;
THE SUN'S APOGEE AND PERIGEE: THE CAUSES OF THE HEAT
IN SUMMER: THE SUN'S APPARENT DIAMETER VARIABLE.

768 How the earth goes round the sun in an ellipsis, in one of the focuses of which the sun is placed, has been explained, book 2 chap. 7; one consequence of this is, that the sun's apparent annual motion is the same as if he were carried round the earth in an ellipsis, in one of the focuses of which the earth were placed. As we call the point of the earth's elliptic orbit where she is farthest from the sun her aphelion, so we call the point in the ecliptic wherein the sun then appears *the sun's apogee*; because he is then farthest from the earth: in like manner, when the earth is in her perihelion, or nearest to the sun, we call the point of the ecliptic wherein the sun then appears *the sun's perigee*; because he is then at his least distance from the earth: it is easy to see that the sun's apogee is opposite to the earth's aphelion; and the sun's perigee opposite to the earth's perihelion. As the longitude of the sun's apogee is reckoned from the point γ , § 661, it must increase $50''$ in a year; because the point γ moves so much backwards contrary to the order of the signs, as was said § 657^a. The place of the sun's apogee is this present year 1740 in $8^{\circ} 25'$ of \mathfrak{z} ; the place of his perigee in $8^{\circ} 25'$ of \mathfrak{h} : the sun is therefore in apogee in june; in perigee in december: thus it appears that the sun is nearer the earth in our winter than in our summer. The sun being nearer to the earth in winter than in summer, it may seem strange to some persons that the winter should be colder than the summer; the causes of this are now to be enquired into.

769 One cause of the heat of the sun being greater with us in summer than in winter is, his having then a greater altitude above our horizon; so that his rays fall nearer to a perpendicular upon those parts of the earth which are round about us: we may consider the heating particles that come from the sun as an innumerable multitude of inconceivably little balls, continually shot from his body, every way: if they cause heat in the bodies upon which they fall, by striking upon their surfaces, the more directly they

^a Whether the place of the earth's aphelion, and consequently of the sun's apogee, be invariable in respect of the fixt stars or not, is controverted by astronomers. v. Ricciol. *alm. l. 3. c. 25.*

FIG. strike upon them, the greater is the force of the percussion; and the more intense will the heat be: on the other hand, the more obliquely they fall upon the surfaces of bodies, the weaker is the stroke; and the less the heat caused thereby. See the remarks § 777.

770 Another reason why any body is less heated the more obliquely the heating particles fall upon its surface is, that they are the more thinly spread
66 thereon, by such obliquity: thus, let $ABCD$ be an hole of a foot square in a board placed so as to receive the sun's rays perpendicular upon it; if a plane of larger dimensions be placed behind the board, near it and parallel to it, so as to receive the rays coming through the hole perpendicular also, they will cover an area a foot square, as $abcd$: if the plane be held obliquely to the rays, they will cover an area of the same breadth as before, but of a greater length; suppose the plane to be held so as to receive the rays with such obliquity that they shall cover an area one foot in breadth, and two feet in length, in this case it is manifest the heating particles falling upon an area twice as large must be spread proportionably thinner thereon; and consequently they will cause but half so much heat.

771 Another reason why the weather is hotter in summer than in winter is this, that heat is not produced in bodies by the sun instantaneously, or all at once, nor do the effects of the heat of the sun cease immediately, upon his rays being intercepted; as is the case of light, which is spread or stopped in a moment: the heating particles operate gradually, and their force decreases also gradually; and therefore the longer any body is exposed to the rays of the sun the hotter it grows; and the hotter any body is the longer it is in cooling: now in summer there are more hours in the day for the sun to heat the earth in, than the night has to cool it again; in the winter, on the contrary, the nights are long, and the days short: and this will account for a good deal of the difference as to heat and cold between those two seasons.

772 Since the heating power of the sun is greatest when his rays fall most directly upon us, § 769, 770, and when the days are longest, § 771, it may be imagined that the hottest weather ought to be about the 10th of June, when the sun is in the tropic of cancer; for then the sun comes nearest to our zenith, and continues the longest above our horizon: but we find by experience that our greatest heats are generally above a month after this, about the end of July, when the sun's altitude is a little diminished, and the days sensibly shortened: this time is called *the dog-days*, because the dog-star, the largest and brightest star in the firmament, rising with the sun at that time, was by most of the ancients believed to be the cause of the extraordinary heats of that season; though some of them, with better judgement, thought his thus
rising

rising only pointed out the time of the year when the weather is usually hottest. I shall treat farther of the dog-days hereafter.

773 To find the true cause of this, we must remember what was said § 771, that the earth is not heated by the sun instantaneously, nor is it cooled all at once: but the heating particles streaming from the sun fall upon the parts of any solid body, or penetrate into its pores, in a continual succession; and therefore every moment an addition is made to the heat before excited: this addition is made, by the heating particles coming on faster than they fly off, or than their heating power is destroyed; or by their continuing to communicate farther degrees of motion to the already agitated particles of the heated body: perhaps both these causes combine to increase heat. In the absence of the sun, the heat of the earth gradually decreases; because some of the heating particles fly off, and are reflected back: others are absorbed into the pores of the bodies heated by them, and attracted by and closely united to their solid parts; and so gradually lose their motion, which, by being continually communicated to more and more particles of matter, becomes at last insensible. Now in whatever degree the parts of the earth round the place where we inhabit are heated, at the summer solstice, by the days continually lengthening from the preceding winter to that time, their heat will continue to be still farther increased for so long time as there is every day an addition of a greater quantity of heat than is lost in the night immediately preceding; though the additions are every day less and less: this is usually for above a month after the longest day. From the foregoing reasoning, we may see also how it comes to pass that the hottest time of the day is not at noon, though the sun be then highest; but about two hours after.

774 When I speak of the heating particles streaming from the sun, I know that some imagin fire to be a subtle matter, diffused through the universe; and that the action of the sun upon it causes the heat which we feel in sun-shine: I have no occasion to enter into this controversy, the force of the sun will be greater the higher he is, and the longer he is above our horizon; whether he pours upon us heating particles thrown off from his own body, or only impells the particles of fire diffused through our air.

775 The sun's apparent diameter is greater the nearer he is to the earth; and consequently greatest in december, least in june, by § 768.

The following table shews the sun's apparent diameter for every 10th day throughout the year: it is taken from *connaissance du temps*, only I have set down the days of the month in old style. v. & *Cassini observat. astronomiques*, p. 22, & *Flamsteed. histor. cælest.*

A table

A table of the apparent diameters of the sun.

d.	'	"	d.	'	"	d.	'	"	d.	'	"
Jan. 9	32	40	April 9	31	56	July 9	31	40	Octo. 9	32	24
19	32	38	19	31	50	19	31	44	19	32	30
30	32	36	29	31	46	30	31	46	30	32	34
Feb. 9	32	30	May 9	31	44	Aug. 9	31	50	Nov. 9	32	36
19	32	24	19	31	42	19	31	56	19	32	38
27	32	18	30	31	40	30	32	0	29	32	42
Mar. 9	32	13	June 9	31	38	Sept. 9	32	6	Dec. 9	32	44
19	32	8	19	31	38	19	32	12	19	32	44
30	32	2	29	31	38	29	32	18	30	32	42

Remarks upon § 768.

776 In computing the sun's place in the ecliptic, astronomers often reckon his distance from the point of his apogee; which distance is called *the sun's anomaly*^a: and is either the mean or the true. *The sun's mean anomaly* is his distance from his apogee, computed upon a supposition that his motion has been all the while equable: *the sun's true anomaly* is his true distance from his apogee, computed by making allowance for the unequability of his motion: the sun's true motion being sometimes swifter sometimes slower than his mean motion; in order to find his true from his mean anomaly, we must sometimes add to his mean motion, and sometimes subtract from it: this operation is called *prosthaphæresis*, a greek compound word which signifies addition and subtraction.

Remarks upon § 769.

777 If two equal bodies strike upon a plane with equal forces, but in different angles, the strength of their strokes will be as the sines of those angles: now the radius or whole sine is the greatest of all the sines, and therefore a perpendicular stroke is the strongest; and the strength of any oblique stroke, *cæteris paribus*, decreases in the same ratio as the sine of the angle does: the meridian height of the sun at the summer solstice is with us at *Cambridge* $61^{\circ} 19'$, his height at the winter solstice is $14^{\circ} 21'$ ^b, the sine of $61^{\circ} 19'$ is 8772858,

^a The word signifies irregularity, or unequability, in greek *ανωμαλια*, and is used in this case because the sun's apparent motion is more or less equable according to his distance from his apogee; for it is slowest in his apogee, quickest in perigee, and mean at his mean distances from the earth.

^b I here suppose the latitude of *Cambridge* $52^{\circ} 10'$, as found by late observations; that in the table § 331 was from *Streete's astronomia Carolina*, but he does not tell us his author.

the sine of $14^{\circ} 21'$ is 2478445; the sun's heat then at noon at the summer solstice is to his heat at noon at the winter solstice as 8772858 to 2478445; that is nearly as 100 to 28.

CHAP. 15. THE COLURES: RIGHT ASCENSION: OBLIQUE ASCENSION: ASCENSIONAL DIFFERENCE: THEIR USES.

778 We have already seen, that many of the motions of the heavenly bodies are not equable; we may however reasonably suppose the rotation of the earth round her axis to be equable: no discovery having been hitherto made to the contrary. *Kepler*^a imagined the earth's rotation was quicker in her perihelion than in her aphelion; this opinion was owing partly to his believing the sun's rotation round his axis to be the cause of the rotation of the earth, and partly to some irregularities in the motions of the moon, which he could not otherwise account for: but these irregularities will appear to belong to the moon herself, when we come to treat of her motions. Since *Kepler*'s time, clocks have been brought to such a degree of perfection, by the inventions of *Huygens*, as to prove that the rotations of the earth are all performed throughout the year in equal times^b. It may perhaps admit of some doubt, whether, since the parts of the earth towards her surface are not every where of the same density, the attractions of the sun and moon do not cause some inequality in the rotation of the earth; so as to accelerate it when some parts of the earth are coming towards the sun or moon, and retard it when they are turning from them. *Hook*^c seems to have thought he had discovered some inequality in the diurnal motion of the earth; but refers the matter to farther experiments, to be made by observing with an accurate clock, whether stars, when very near the zenith, and consequently out of the reach of sensible refraction, go through equal arcs of the parallel of their apparent diurnal motion in equal times, or not: if there be any such inequality it would, how small soever, hinder the settling the places of the stars exactly by their transits.

779 From the equability of the rotation of the earth it follows, that the apparent diurnal motion of the heaven is equable also: this motion therefore, which is sometimes called the motion of the *primum mobile*, and is made in the plane of the equator and circles parallel to it, is very proper for

^a *Kepl. epit. astron. p. 108, 286, 550, 574, 722. theoria.*

^c See *Hook's* posthumous works, p. 545.

^b *Halleius in tractatu de emendatione lunaris*

a standard, by which other motions in the heaven may be measured: for this reason, and others that will appear in the sequel, the celestial equator is of great account, in our enquiries into the places and motions of the heavenly bodies: that circle is supposed to be divided into 360 degrees, beginning at the point γ , or the point of the vernal equinox, proceeding round according to the order of the signs; and each degree to be subdivided into minutes and seconds, &c.

780 We may imagine a secondary of the equator, or circle of declination, to be drawn through any point of the equator: the principal secondaries of the equator are the two *colures*, one of which is drawn through the equinoctial points γ and ϵ , and called *the equinoctial colure*; the other is drawn through the solstitial points ϖ and \varnothing , and called *the solstitial colure*: the use of the colures is to mark out the four cardinal points of the ecliptic; at which when the sun arrives the four seasons of the year begin. Since the point γ changes its place $50''$ every year, the other cardinal points must also change their places as much; and consequently the colures also change their situations in the sphere of the heaven; and this change, though not sensible in a few years, becomes in a great number of years very considerable: thus the situation of the colures, is found to be very different now from what it was 2000 years ago^a. The colures have that name from the greek word *καλος* maimed, and *ουρα* a tail; because, in every place in north latitude, their lower parts that are within the antarctic circle, described § 348, are cut off by the horizon; so as never to be seen. *Achilles Tatius cap. 27 de coluris*.

781 The motion of the equinoctial points, described § 657, is called *the precession of the equinoxes*^b; because it is in *antecedentia*, that is from a following sign towards a preceding one, as from aries towards pisces, &c: by some it is called the *regression* or going back of the equinoxes, because it is contrary to the order of the signs: this retrograde motion, by carrying the equinoctial points to meet the sun as he comes along the ecliptic in his apparent annual motion, makes him arrive at them and cause the general equinoxes, a little sooner every year than he would do if those points continued immoveable: thus, the arc of regression being $50''$ a year makes the equinoxes happen $20'$ in time sooner than they otherwise would do.

782 Another consequence of the precession of the equinoxes, and what is chiefly to be considered in this place is, that the celestial equator and its poles are continually changing their situations among the fixt stars; and therefore,

^a *Newton's chronology* p. 88, 91.

^b Hipparcho apud Ptolemæum, μεταπτώσις των σημειων ισημερινου. Copernico, *anticipatio vel precessio æquinoctiorum*.

whenever

whenever we speak of the distance of any star from the equator or poles, we must be understood to mean the equator and poles as they are situated at the time of the observation: but, the change now mentioned is so very slow, that, in most cases of common use, we may consider the equator and poles as if for a small number of years they continued immoveably the same.

783 The use of secondaries of the equator is to shew the places of stars, or of any points in the heaven, in respect of the equator; in the same manner as the secondaries of the ecliptic shew their places in respect of the ecliptic: the arc of a secondary of the ecliptic, or circle of latitude, contained between a star and the ecliptic is called the latitude of that star; the arc of a secondary of the equator, or circle of declination, contained between a star and the equator is called the declination of that star: an arc of the ecliptic reaching, according to the order of the signs, from the point γ to the point where the ecliptic is cut by a circle of latitude drawn through a star is called the longitude of that star; an arc of the equator reaching, according to the order of the signs, from the point γ to the point where the equator is cut by a circle of declination drawn through a star is called the *right ascension* of that star: thus we see that declination and right ascension are the same in respect of the equator, as latitude and longitude are in respect of the ecliptic^a. Latitude of stars has been treated of, § 646: longitude, § 659: and declination, § 340: right ascension only remains to be farther explained.

784 When astronomers consider what point of the equator comes to the eastern part of the horizon, and consequently rises at the same time with a star, they say, the *ascension* of a star is an arc of the equator which, proceeding from the point γ , or the point of the vernal equinox, according to the order of the signs, is contained between the point γ and the point of the equator that rises with that star: ascension is, according to the different position of the sphere, right, or oblique.

Scholium. This is a compendious way of speaking, as *Kepler epit. astron.* p. 240, observes: the greek words *συναναβολη* and *συγχηλιδυσις*, which may be translated coascension and codecension, are more properly applicable to a point of the equator, than to an arc of it which is terminated at one end by that point: however we must be content to take commonly received terms of art as we find them.

785 The *right ascension* of a star is an arc of the equator which, proceeding according to the order of the signs, reaches from the point γ , the point

^a Declination is called latitude from the equator by *Ptolemy*: right ascension might have been called longitude upon the equator. *Veteres longitudinem et latitudinem stellarum penes aequatorem considerasse, ostendit Petavius, variar. differt. l. 2. c. 2.*

FIG. of the vernal equinox, to the point of the equator that rises with the star in
 67 a right sphere: let fig. 67 be a projection or picture of the eastern half of the
 concave sphere of the heaven, let $E Q$ be the equator, $s N$ the horizon, which,
 in a right sphere as is here drawn, is cut by the equator at right angles, by
 § 346; let N be the arctic, s the antarctic pole, which in a right sphere are
 in the plane of the horizon; the curves $N A s$, $N B s$, &c, and the strait line
 $N D s$ are projections of semicircles of declination: now if we imagin the
 sphere to move upon the poles $N s$ according to the order of of the letters A
 $B C D$, a star at D or at E or any point in the semicircle of declination $N D s$
 comes to the horizon and rises above it at the same time with the point γ ;
 and therefore we say its right ascension is 0: a star at F or G or any point in
 the semicircle of declination $N C s$ rises at the same time with the point c ,
 and its right ascension is the arc γc , or so many degrees minutes and seconds
 as that arc contains: a star in the semicircle $N B s$ rises with the point B , and
 its right ascension is the arc γB , &c.

786 The term right ascension is taken from a right sphere^a, as is also the
 common definition of it given § 785: but we are not therefore necessitated
 to go under the line, in order to find the right ascensions of the heavenly bo-
 dies; they may be observed in any place, by an instrument fixed in the plane
 of the meridian, as described § 408, and a good clock, to observe the times
 of their *transits*: see § 409. For, since the plane of every meridian extended
 passes through the celestial poles, § 299, &c; and since, in a right sphere, the
 plane of the horizon extended passes also through the poles, it follows, that
 every meridian is coincident with the horizon of some place or other in a right
 67 sphere: thus, fig. 67, the plane of the meridian, $N A s$ is coincident with the
 horizon of the place under the line which, in the situation of the heaven
 here represented, has E for its zenith: for this reason, the point of the equa-
 tor which comes to the plane of any meridian^b, at the same time with a star
 gives us the right ascension of that star^c.

787 It is true, the point γ , as well as every other point of the equator,
 is invisible; and therefore the right ascensions of stars, which must be reck-
 oned from γ , cannot be found directly and immediately; but they may by
 the help of the sun; because the theory of his motion is so well ascertained,
 that his place in the ecliptic at any time is easily known, from whence his
 right ascension may be calculated; and this being given, *the right ascension of*

^a Ptolemy calls it ascension in a right sphere. ^b § 303. ^c Since all secondaries of the equator and all meri-
 dians are drawn through the poles; when any one point of a secondary of the equator is in the plane of
 the meridian, every other point of that secondary is also in the plane of the meridian: and therefore the
 definition of right ascension in § 783, expresses the thing very well.

any star may be thus found by observation: let the motion of a clock be so adjusted that the hand may go round the 24 hours exactly in the time of a rotation of the earth, that is, in the time which passes between any fixt star leaving the plane of the meridian and returning to it again: set the hand of the clock thus regulated at 12 when the sun is in the meridian; observe the hour minute and second when the fixt star whose right ascension is required comes to the meridian; and the intermediate time, turned, by the table § 316, into degrees minutes and seconds of the equator, is the difference between the right ascensions of that star and of the sun when before observed in the meridian: add to this difference the right ascension of the sun at that time, and you have the right ascension of the star: thus, this present year 1740, june the first at noon, the right ascension of the sun is $80^{\circ} 54' 53''$, Arcturus comes to the meridian at $8^h 40' 16''$ * in the evening, this turned into degrees minutes and seconds is $130^{\circ} 4' 0''$, which added to the right ascension of the sun gives the right ascension of Arcturus $210^{\circ} 58' 53''$.

If the hour circle of a clock be divided into 360° , and, when the sun is in the meridian, the hands be set at the degree minute and second which mark the right ascension of the sun; they will, when a star comes to the meridian, point out its right ascension directly, without the trouble of reducing time to degrees minutes and seconds of the equator.

788 The foregoing section shews how to find the right ascension of a star, by the time of its transit; the converse of this is to find the time of the night, by observing the transit of a star whose right ascension is known: this is done by turning the arc of difference between the right ascensions of the sun and the star into time: thus, I find by an ephemeris, that june the first 1740, the right ascension of the sun is $80^{\circ} 54' 53''$, the right ascension of Arcturus $210^{\circ} 58' 53''$, the difference $130^{\circ} 4' 0''$, the time answering to $130^{\circ} 4' 0''$ is $8^h 40' 16''$: so much then does Arcturus come to the meridian after the sun*. The time of the night may also be found by the height of a star whose right ascension and declination is known, if we have first the latitude of the place of observation: but this requires trigonometrical calculation. By either of these methods a clock may be corrected. The latitude of the place where we are may be found by the meridian altitude of a star whose declination is known; or of the sun, if his declination be known for the time of observation: see § 411 to 415. This is of use at sea, see § 568.—We have in some almanacks a table of the declination of the sun, for every day of the year: and a table of the right ascensions and declinations of the principal stars; which, will serve, for the use of sailors, for several years, without sensible error.

* The time used § 787, 788, & 789, is sydercal time, which I shall shew how to reduce to common time, hereafter.

FIG. 789 The *difference of right ascensions of two stars is found*, by turning the time between their coming to the meridian into degrees minutes and seconds of the equator, by the table § 316: thus, fig. 67, if the star B comes two hours later to the meridian than the star C, the difference of their right ascensions is 30° ; and so much does the right ascension of B exceed that of C.—It is easy to see, that all stars in the same semicircle of declination come to the meridian at the same time; and consequently have the same right ascension. The *right descension* of a star is the same with its right ascension; for, in a right sphere, the same point of the equator that rises with a star sets with it also.

790 *Oblique ascension* of a star is an arc of the equator reaching, according to the order of the signs, from the point γ to the point of the equator which rises with the star, in an oblique sphere: the difference between the right and oblique ascension of a star is called its *ascensional difference*. Oblique ascension and ascensional difference are relative to some given latitude: for the farther any place is removed from a right sphere, the greater is the ascensional difference of any star in that place. *Oblique descension* of a star is an arc of the equator reaching, according to the order of the signs, from γ to the point that sets with the star, in an oblique sphere: oblique descension is also relative to some latitude, in the same manner as oblique ascension is. The oblique ascension or descension of a star may be found by calculation, for any place, if the latitude of the place and the right ascension and declination of the star be first known.

791 We have in some books of astronomy *tables of oblique ascensions* for the several degrees of the ecliptic, for several latitudes^a: by means of these, since the sun's place in the ecliptic is known for every day in the year, we may find what his oblique ascension is, any day of the year, in any of the latitudes for which the tables are made: some authors give us also *tables of ascensional differences*^b: The arc of ascensional difference of the sun for any day of the year in a given latitude, turned into time, shews, for that latitude, how many hours and minutes before or after fix the sun rises that day: the same arc, turned into time, shews also, for the same day and latitude, how many hours and minutes before or after fix the sun sets: it is easy from hence to find the length of the artificial day, that is, how many hours and minutes the sun is above the horizon; and if this time be taken from the natural day of 24 hours, the remainder is the length of the night: but this is speaking astronomically, without regard to refraction; for that lengthens the day and shortens the night, as was before observed, § 757.

^a Ricciol. l. 10. pag. 562.

^b Ricciol. ib. pag. 561. Gregory's astronomy book 2. prop. 60.

792 The right ascensions and declinations of the stars being known, their longitudes and latitudes may be found by calculation; and conversely, their longitudes and latitudes being known, their right ascensions and declinations may be found by calculation. In order *to make a catalogue of the stars*, their right ascensions and declinations are found by observations made with proper instruments; and from thence their longitudes and latitudes are calculated, and set down in the catalogue. The places of stars are set down in the catalogues according to their longitudes and latitudes, rather than according to their right ascensions and declinations; because their latitudes continue invariably the same, and their longitudes change regularly, at the rate of 50" in a year, see § 607, and 657, so that the longitude of any star is easily found for any year past present or to come, by any catalogue: whereas both the right ascensions and declinations of stars are, by the precession of the equinoctial points altering the situation of the celestial equator, changed every year a little, and that irregularly; the change not being the same in all the stars, but greater in some than in others, according as they are differently situated in respect of the ecliptic and the equator.

793 When I say *the latitudes of the stars are invariable*, it is upon a supposal that two positions are true, which are both controverted amongst the astronomers: one of these is, that the stars are so fixed as to keep their places immoveably; the other, that the orbit of the earth continues unalterably in the same plane, and consequently that the ecliptic is invariable: as to the first of these positions, *Ptolemy*^a has a chapter on purpose to prove that the stars are fixed, and for that end instances in a considerable number of stars which he found to lye in right lines with one another, or to form triangles and other figures, exactly in the same manner as *Hipparchus* had described them, above 260 years before him. *Tycho*^b not only found those stars to keep their places in respect of one another, but several others also to answer so exactly to that description, that he declares it to be his opinion, that all the differences, which he found between the distances of some stars from one another, as they were observed by himself and as they are set down by the ancients, are owing, either to the inaccuracy of the observations of the ancients, and the imperfections of their instruments; or to the mistakes of transcribers, who, in some instances, have manifestly given us wrong numbers, and very probably in many more. On the other hand, Dr. *Halley*^c has observed, that the Bull's eye, Sirius, and Arcturus are now found to be above half a degree

^a *almagest. l. 7. c. 1.* ^b *epist. astronom. p. 43. progymnasmat. p. 234.* ^c *Philos. transact. n. 355. Jones's abridg. p. 225.*

more southerly than the ancients reckoned them: that this difference cannot arise from the errors of transcribers; because the declinations of these stars, set down by *Ptolemy*, as observed by *Timocharis*, *Hipparchus* and himself, shew their latitudes given by him are such as those authors intended: that it is scarce to be believed, that those three observers could be deceived in so plain a matter. To this he adds, that the bright star in the shoulder of Orion has in *Ptolemy* almost a whole degree more southerly latitude than at present: that an ancient observation, made A. D. 509, at Athens, as *Bullialdus* supposes, of an appulse of the moon to the Bull's eye, shews that star to have had less latitude at that time than it now has: that as to Sirius, it appears by *Tycho's* observations that he found him 4 minutes and an half more northerly than he is at this time. All these observations, compared together, seem to favour an opinion, that *some of the stars have a motion of their own*, which changes their places in the sphere of the heaven: this change of place in the stars, if there be any such, may, as Dr. *Halley* observes, shew it self in so long a time as 1800 years, though it be entirely imperceptible in the space of one single century: and it is likely to be soonest discovered in such stars as those just now mentioned; because they are all of the first magnitude, and may therefore probably be some of the nearest to our solar system.

794 As to the second position, that *the ecliptic is invariable*, *Tycho*^a brings several instances of stars whose latitudes he found to be different from what they were in *Ptolemy's* catalogue; and says that, according to his observations, the latitudes of the northern stars has increased, and the latitudes of the southern stars decreased; and that this increase and decrease of latitude is greater or less in different stars, according as they are nearer to or farther from the solstitial colure. This would imply such a change in the plane of the earth's orbit as causes the northern parts of the ecliptic to recede from the north pole, and the southern parts to recede from the south pole: *Tycho*^b indeed confesses, that his own observations of some of the biggest stars, and particularly of the Bull's eye and Sirius, compared with the numbers of *Ptolemy*, were against this opinion of the change of the ecliptic; but, to silence this objection, he has recourse again to the inaccuracy of the ancient observations, and the mistakes of transcribers. *Ricciolus* takes notice that *Tycho* seems to allow the observations of the ancients to be good, or reject them as faulty, according as they favour, or stand against his opinion about the change of the ecliptic^c. If we consider the mutual attraction of the heavenly bodies,

^a *epist. astronom.* p. 30, 65, 66, 69, 70, 86. *progymnasim.* p. 233, ad 240.

^b *progymnasim.* p. 246.

^c *almag.* l. 6. c. 15.

of which more in another place, we cannot think it impossible that both the plane of the earth's orbit, and the planes of the rest of the planets should be subject to some alterations; but that there has been an actual change in any of these, we cannot reasonably conclude, without proof from observations: in this enquiry, there are four things to be considered in the orbit of every planet; 1 its excentricity, 2 the place of its aphelion, 3 the place of its ascending node, 4 the inclination of its plane to the plane of the orbit of any of the other planets: it has been the opinion of many great astronomers^a, that all these continue invariable in respect of the fixt stars: some assign to the nodes and aphelia a very slow motion, but this opinion seems to have been built upon some preconceived hypothesis, rather than any actual discovery of such motion^b: perhaps the aphelion of saturn may be excepted from this remark, which, according to *Maraldi*, has a motion of about a minute in a year, according to the order of the signs^c; and the nodes of jupiter, which the same author makes to go backwards at the rate of about 37" in a year^d. As to the planes of the planetary orbits, we have no observations to shew that any of them was ever inclined to the rest in a different angle from what it is at present: the change of latitude of some stars supposed by *Tycho* does not appear to be made out clearly enough to conclude from thence any change of the plane of the ecliptic: on the other hand, two occultations of the same star by the planet mars, with a space of 1934 years between, the earth being at the time of both observations very nearly in the same place of her orbit, make it extremely probable that the planes of both the orbit of mars and of the earth have continued in the same situation during that long interval^e.

795 For the excentricities of the orbits of the planets see § 670 and 669: their aphelia and perihelia are marked in the 11th figure, by the letters A and P. The inclinations of the planetary orbits to the plane of the ecliptic have been given in round numbers, § 638; and, together with their ascending nodes, are shewed in the 24th figure, explained § 651: I shall now set them down to a greater exactness, with the place of the aphelion and ascending node of each planet, for the year 1700; they are all taken from the tables of *De la Hire*, except those of mercury, in which I follow the accurate determination of Dr. *Halley*^f.

^a *Streete, astronomia Carolina*, p. 45. *Du Hamel, Reg. scientiar. Acad. histor. l. 4. c. 4.* *Newton. princip. lib. 3. prop. 14.* *Jones's Phil. trans. abridg. vol. 4. p. 305 & 320.* *v. et Ricciol l. 3. c. 26.*

^b *Kepler. tabb. Rudolphin.* ^c *Memoires d'Acad. Royale 26 novemb. ann. 1704,* ^d *Mem. d'Acad. 20 fevrier 1706.* ^e The first of these occultations is mentioned by *Ptolemy almag. l. 10. c. 9.* the second is in the *Phil. trans. Jones's abr. vol. 4. p. 305.* ^f *Phil. trans. n. 386.*

	Aphelion.	Ω	Inclination.
γ —————	\ddagger 29° 15' —————	\mathfrak{S} 21° 56' —————	2° 33' 30"
ν —————	\ddagger 10 17 —————	\mathfrak{S} 7 12 —————	1 19 20
δ —————	\mathfrak{M} 0 35 —————	\mathfrak{S} 17 25 —————	1 51 0
ϱ —————	\equiv 6 56 —————	Π 13 54 —————	3 23 5
ζ —————	\ddagger 12 44 —————	\mathfrak{S} 14 42 —————	6 59 20

CHAP. 16. WHETHER ANY OF THE STARS CHANGE THEIR PLACES OR NOT: THE OBLIQUITY OF THE ECLIPTIC VARIABLE.

796 The subject of § 793 in the foregoing chapter is of so curious a nature, that I thought it would deserve a farther discussion, if we could find materials for it: if the distances of several stars from one another were at any time to be taken exactly, and, after a considerable number of years, the same distances were to be carefully measured again, it seems to be a likely way to discover whether the stars so observed change their places or not: the ancients have given us nothing of this kind; *Tycho*^a and the *Prince of Hesse*^b were the first who left us any distances taken with good instruments, and their accounts agree so well together, as to give us reason to believe they are as exact as could be expected from plain sights. I have compared a great number of their distances, particularly those of the brightest stars, with observations of the same distances taken by later astronomers, especially *Hevelius*^c and *Flamsteed*^d, and can find no such change in any of them as will shew that either Capella, Lyra, the Bull's eye, the bright star in the shoulder of Orion, the Lyon's heart, Spica \mathfrak{M} , or Sirius have, in the interval of time between the observations of *Tycho* and *Flamsteed*, which is about 100 years, gone out of their places: but in Arcturus the case at first appeared to be different; *Hevelius* sets down the distance of that star from Lyra 4' greater than *Tycho* had observed it, 72 years before him; and *Flamsteed*, 22 years after, measured the distance between the same two stars still 3' greater than *Hevelius* found it; so that, if Lyra stood still all the while, there was an appearance of Arcturus having gone 7' out of his place in 100 years: it seemed some confirmation of this motion of Arcturus, that *Flamsteed* found his distance from the head of Hercules 3' greater than it is set down by the *Prince of Hesse*;

^a *epist. astron.* p. 50 & 51, *progymnasn.* p. 229 & 230, & in *Tychon. histor. cælest. passim.*

^b *v. lib. 12 histor. cælest.* Tychon. a Lucio Barretto (i. e. Alberto Curtio) edit. nam hoc anagrammate cum nomen suum texisset ait Ricciol. *astron. reformat.* l. 4. c. 8. p. 219. ^c *machin. cælest. part. 4.*

^d *histor. cælest. vol. 1.*

and that his distance from the Lyon's tail was a little decreased: to make these observations consistent with one another, Arcturus must have gone almost directly towards the ecliptic, so as to decrease his latitude about $7'$; and indeed his latitude in *Flamsteed's* catalogue is $5\frac{1}{2}'$ less than in that of *Tycho*: thus far every thing favoured the opinion of Arcturus having a motion almost directly towards the ecliptic; upon this, I considered several other stars to the north of Arcturus, from which he must have grown more distant, if he had really had any such motion; and several stars to the south of him, to which such a motion would have made him approach nearer: the distances of Arcturus from these stars being calculated from their longitudes and latitudes in the different catalogues of *Tycho*, the *Prince of Hesse*, *Hevelius* and *Flamsteed*, I found that very few of them agreed with the supposed motion of Arcturus, but that the greatest part of them came out such as shewed he had kept his place; for I seldom found the difference of the distances computed from the several catalogues to exceed $2'$, and these differences fell out so, that they could not all be true; since from some of them Arcturus appeared to have gone towards the south, and from others towards the north: so that, upon the whole, after a pretty long and careful enquiry, I am inclined to think the stars keep their places immoveably, and this will appear the more probable, if we consider the fixt stars as so many suns: see § 625.

797 It has been a subject of controversy amongst astronomers, whether the obliquity of the ecliptic be variable or not; that is, whether the angle between the planes of the ecliptic and equator, mentioned § 733, continues always unchangeably the same, or is subject to alteration: it is also disputed among those writers who hold it liable to change, whether that change be regular or irregular. It is certain that the obliquity of the ecliptic is at present found to be above a third part of a degree less than *Ptolemy* has given it us: if we enter into a detail of all the observations of it which were made from *Ptolemy's* time down to *Tycho*, we shall find that later observers have generally found it less than those who went before them; but this has not always been the case: for there are some instances where, on the contrary, later observations have made it a few minutes greater than some preceding ones. They who will have the obliquity of the ecliptic to have been always the same that it is at present get rid of all these difficulties, by throwing upon the inaccuracy of the observations of the ancients, and the imperfection of their instruments, all the differences that are found between them and the moderns in this affair: others think this is treating the ancients with too little respect; for, though it appears by *Ptolemy* that they did not pretend to observe more nicely than to a 6th part of a degree, and their instruments, as far as we

have any account of them, fall far short of those made use of by the moderns; yet it can hardly be imagined they could be so greatly mistaken in a matter so easy to be known, and so well deserving their attention. As for those who suppose that the obliquity of the ecliptic is subject to a sort of libration, increasing in some ages and decreasing in others, they certainly attribute a greater degree of exactness to the observations upon which they ground this fancy, than there is any good reason to do. Some have thought that the obliquity of the ecliptic has been always gradually decreasing, the planes of the ecliptic and equator growing continually nearer to coincidence: this opinion, besides its being grounded upon a comparison of ancient and modern observations, receives a good deal of support from the consideration of the shape of the earth, as we shall see by and by, § 802.

798 The obliquity of the ecliptic is equal to the distance of either tropic from the equator, which is the same with half the distance between the two tropics: the best way of finding it, because least liable to error from refraction, is to take the height of the sun when he is in the summer tropic; the height of the pole, and consequently the complement thereof, the height of the equator, being first known in the place of observation: this gives us the distance of the tropic from the equator directly. There is another way of finding this, the only one made use of by the ancients, and that is, to observe the meridian height of the sun in each tropic: but then refraction must be taken into the account; for refraction increases the meridian altitude of the sun, when he is low in the winter solstice, more than it does when he is high and more out of the reach of vapours in the summer solstice; thus the tropics are in appearance brought nearer to one another by refraction, in the same manner as the upper and lower edge of the sun's disk were shewn to be, § 754; for want of being aware of this, the ancients measured the distance of the tropics less than it really was: they fell also into another mistake of supposing the parallax^a of the sun too great, which operated the same way, and diminished also the distance between the tropics, and that about twice as much as they lessened it by not considering refraction^b; these errors of the ancients, of supposing a false parallax, and neglecting refraction, are no hindrance to our making use of their observations, in this disquisition; because we know what consequences they draw after them, and can therefore tell how to make proper allowances for them: now upon the whole, after proper allowances, the obliquity of the ecliptic comes out from the observations of the ancients above a third part of a degree greater than the modern astronomers have found it. *Louville*, after having compared all the ancient observa-

^a Parallax will be explained in the next chapter.

^b See the remarks.

tions he could meet with, was of opinion that the obliquity of the ecliptic decreased at the rate of 1' in an hundred years, and from a great number of observations made by himself about the year 1711, set it down $23^{\circ} 28' \frac{1}{2}^a$, which is half a minute less than *Flamsteed* and *De la Hire* made it: he goes still farther and thinks he discovered it to be 5 seconds less in the year 1720 than he had observed it in the year 1714: but, notwithstanding the great care with which his observations appear to have been made, it seems hardly credible that any one can, in an affair of this nature, be certain he has arrived to such a degree of exactness as is here pretended.

Louville, for the farther support of his opinion, fetches arguments from the times of fabulous antiquity, upon a presumption however that fable has generally some truth for its foundation. He cites *Herodotus* as saying that the ancient Egyptians had a tradition among them, that the ecliptic had once been perpendicular to the equator; this is so odd a fancy, that he thinks it hard to imagin how it should ever have come into their heads, except they had actually found by observation that those two circles were grown nearer to one another than they had formerly been: if they had made such a discovery, it was natural for them to suppose that there had been a time when the ecliptic and equator were at the greatest distance from coincidence that was possible; they might then set down what they had only thus supposed, as a fact which had been observed by them: but this is all mistake; *Herodotus* only says that the Egyptians affirmed the sun, in the space of 11340 years, during the reigns of their ancient kings, to have altered his course four times, twice rising where he then set and setting where he rose^b: which cannot be explained by the change of the obliquity; and seems to be only an idle amusing story, invented to support their vain pretensions to antiquity, but fit to pass only among persons ignorant of astronomy^c.

He goes on to tell us from *Diodorus Siculus*, that the Chaldeans pretended to have astronomical observations of 403000 years before the time of *Alexander* the great: this extravagant account will have some foundation, if we allow the Chaldeans to have found out that the obliquity of the ecliptic diminished at the rate just now mentioned, of one minute in 100 years; for, this being supposed, if we run back 402942 Chaldean years from the time of *Alexander's* entry into Babylon, we shall come to the time when the ecliptic and equator must have intersected each other at right angles: and this is but 58 years short of the time from which the Chaldeans pretended to date their first ob-

^a *Histoire d'Acad. Royale pour l'ann. 1714 sur l'équinoxe du printemps de 1714. Histoire d'Acad. ann. 1716. v. & memoires pour l'ann. 1721.* ^b in Euterpe. ^c See the remarks.

servations. It is remarked upon this occasion, that there cannot be given a more probable account of the fabulous antiquity of the Egyptians and Chaldeans than this, that, from observing some very slow motions in the heaven they calculated the periods of such motions, and fixed upon some time when they supposed them to begin, and dated from thence the origin of the world, and of their own nation, which they were willing to have believed to be of equal antiquity with the world it self: we have some thing like this fanciful way of reasoning among some Christian astronomers, who have been of opinion that the sun's apogee was in the point γ at the time of the creation, and pretend from thence to fix the age of the world, which by that method comes out not many years different from the numbers in the translation of the bible called *the septuagint*.

799 We are told^a that, notwithstanding these reasons of *Louville*, the rest of the astronomers of the French academy were of the contrary opinion: it was alledged by *De la Hire*, that *Ptolemy* thought the obliquity of the ecliptic unchangeable; and that therefore in determining it he had less regard to his own observations than to those astronomers who preceded him, as *Hipparchus* and *Eratoſthenes*: that he was more attentive to the theory than to the practical part of astronomy; and more a mathematician than an observer; and that therefore he settled the obliquity, manifestly upon the credit of the ancients^b. In answer to this, it must be owned that *Ptolemy* paid such a deference to *Hipparchus*, whom he commends greatly for his fidelity and industry, as to take his obliquity, and calculate his tables according to it; but the question here is not what was *Ptolemy's* opinion in this matter, but what is the result of his observations: which, as to this particular, appear to have been made with a good deal of care and exactness. He describes two instruments proper for that purpose, he recommends one of them as more commodious than the other; he tells us, he took meridian altitudes of the sun, and that, from such altitudes, and especially those observed by him at many returns of the solstices, he found the distance between the tropics to be 47 degrees and more than half but less than three fourths of a degree; which he says is nearly the same as *Eratoſthenes* had determined it, and *Hipparchus* had made use of^c. Now, according to them, if the solstitial colure be divided into 83 equal parts, the distance between the tropics contains 11 of those parts: thus the obliquity of the ecliptic comes out from the observations of *Eratoſthenes* $23^{\circ} 51' 20''$, from those of *Ptolemy* $23^{\circ} 48' 45''$.

^a *Histoire d' Acad. pour l'annee 1716.*

^b *memoires pour l'annee 1716.*

^c *almageſt l. 1. c. 10.*

800 I here follow the greek text of *Ptolemy*, and his commentator *Theon*^a, who lived also at Alexandria: there are, in this and many other places, faults and omissions in the translation of *Trapezuntius*, which it is evident *De la Hire* made use of, without consulting the original: and this may be one reason of his having so mean an opinion of the above mentioned observations of *Ptolemy*; for who ever looks only into the latin translation will imagin his quadrant with which he took the meridian altitudes of the sun was divided only into 90 degrees; whereas it is manifest, by the greek, and by *Theon*'s comment, that each degree was subdivided, though we are not told into how many parts: *Theon* says, into as many parts as possible; from *Ptolemy*'s determining the distance between the tropics, to have exceeded the number of whole degrees, more than half and less than three fourths, as above, I am apt to think each degree was divided at least into 4 equal parts: and surely with such an instrument a careful observer could not easily be mistaken a quarter of a degree. But to give us an instance of *Ptolemy*'s want of accuracy, we are told that he settled the latitude of Alexandria, his own observatory, 13' less than *Chazelles*, who went thither on purpose in the year 1700 with proper instruments, found it to be; and lest it should be thought this difference might arise from the situation of Alexandria being different now from what it was formerly, it is said that the contrary to this appears from the ruins of the ancient buildings still remaining^b: now in answer to this, not to mention that *Greaves* took the latitude of that place 7' less than *Chazelles*, and therefore differed but 6' from *Ptolemy*, it may be remarked, that though it appears from *Ptolemy*'s *almagest*, l. 1 c. 10, that he knew a much better way of observing the latitude than *De la Hire* fancies he made use of, yet he might more easily mistake 13' therein, than be out two thirds of a degree in the distance between the tropics.

801 It is objected, that *Pappus* of Alexandria^c, 270 years after *Ptolemy*, supposes such a ratio between the diameters of the ecliptic and of the tropic as brings out the obliquity of the ecliptic but $23^{\circ} 30'$, and it is concluded from hence that this was the well known measure of it in the time of *Pappus*, and that consequently *Ptolemy*'s measure must have been then found to be erroneous by some later astronomers; since it is not likely it should have decreased from $23^{\circ} 51'$ to $23^{\circ} 30'$ in so short a time: but this is an hasty conclusion, for *Pappus* says nothing of any observations of the obliquity, nor does his subject lead him to it, he had only occasion to prove that the diameter of the ecliptic was less than double the diameter of the tropic; and, in

a edit. Basil. anno 1538. b *memoires pour l'ann.* 1716. c In *collectionib. mathemat.* l. 6. prop. 35.

order to this, he assumes, that the distance between the centers of the ecliptic and of the tropic is to the semidiameter of the tropic as 10 to 23: now these numbers are probably the same as he would have made use of if he had acquiesced in *Ptolemy's* measure of the obliquity, being sufficient for his purpose, which did not require larger or more exact numbers; so that no consequence can be drawn from hence of *Pappus* being of a different opinion from *Ptolemy*: but, to put the matter out of all doubt, we have extant great part of the comment of *Pappus* upon the 5th book of *Ptolemy's almagest*^a, wherein he mentions the same obliquity of the ecliptic that *Ptolemy* made use of, $23^{\circ} 51'$, without any hint of his dissenting from it: this deserves the more to be taken notice of, because not only *De la Hire*, but other writers also, lay a great stress upon the passage quoted out of *Pappus* in the beginning of this section: add to this, that *Theon*^b, who wrote after *Pappus*, and *Proclus* called *Diadochus*^c used the same measure; which the Alexandrian astronomers seem so long to have taken from *Eratoſthenes*, without any suspicion of its being subject to alteration.

802 *De la Hire* objects farther against *Ptolemy's* observations, that he has in many instances, given us the latitudes of stars above a quarter of a degree different from what they are now found to be: but these errors are easily accounted for, if we consider that his observations of the stars were made with a zodiacal *armilla*, an instrument that by its construction must be liable to great errors; and accordingly we find that *Tycho*, having made such an instrument at a great expence, was induced, after a little experience of it, to lay it aside as useless: whereas the meridian altitudes of the sun were taken by *Ptolemy* with a quadrant drawn upon a smooth stone, (marble I suppose) placed in the plane of the meridian, divided into degrees with those degrees farther subdivided as has been said: they who are for the immutability of the obliquity will alledge, that, though *Tycho* settles it $23^{\circ} 31' 30''$, greater by $2\frac{1}{2}'$ than it is at present, *Flamsteed*, from the observations of *Tycho*, after proper allowances, brings out the same obliquity as he does from his own observations made 100 years after: in answer to this it may be remarked, that, according to the theory just now mentioned, the obliquity ought to decrease but $1'$ in the time between *Tycho* and *Flamsteed*, and it may very well be supposed that *Tycho*, with his plain sights, might mistake as much as that; nay it is certain that, in his meridian altitudes of the sun taken the same day with different instruments there is often a difference of above one, and sometimes of more than two minutes. *Tychon. hist. cælest. passim.*

^a inter Theonis commentar. edit. Basil. supra citat. pag. 258. ^b edit. Basil. p. 60. ^c about A. D. 500.

A TABLE OF THE OBLIQUITY OF THE ECLIPTIC.

	Authors Names.	Years before Christ	Obliquity.
L	<i>Pytheas</i>	320	23° 49' 23"
	<i>Eratoſthenes</i>	230	23 51 20
		after Christ	
	<i>Ptolemy</i>	140	23 48 45
F	<i>Almamon</i>	832	23 35
	<i>Albatennius</i>	890	23 35
F	<i>Thebet</i>	911	23 33 30
F	<i>Abul Waſi and Abu Hamed</i>	999	23 35
F	Perſian tables in <i>Chryſococca</i>	1004	23 35
F	<i>Albatrunius</i>	1007	23 35
F	<i>Arzakel</i>	1104	23 33 30
F	<i>Almeon</i>	1140	23 33 30
F	<i>Cbojab Naſſir Oddin</i>	1290	23 30
F	<i>Prophatius the Jew</i>	1300	23 32
F	<i>Ebn Shatir</i>	1363	23 31
	<i>Ulug Beigh</i>	1437	23 30 17
R	<i>Purbachius and Regiomontanus</i>	1460	23 28
R	<i>Waltherus</i>	1476	23 30
R	<i>Wernerus</i>	1510	23 28 30
	<i>Copernicus</i>	1525	23 28 24
R	<i>Egnatio Danti</i>	1570	23 29
	<i>Prince of Heſſe</i>	1570	23 30
	<i>Tycho Brahe</i>	1584	23 31 30
	<i>Wright</i>	1594	23 30
	<i>Ricciolus</i>	1646	23 30 20
F	<i>Hevelius</i>	1653	23 30 20
L	<i>Caffini</i>	1655	23 29 15
	<i>De la Hire</i>	1686	23 29
	<i>Flamſteed</i>	1690	23 29
L	<i>Bianchini</i>	1703	23 28 25
*	<i>Roemer</i>	1706	23 28 41
	<i>Louville</i>	1715	23 28 24
	<i>Godin</i>	1730	23 28 20

* Godin. *mem. d' Acad. ann.* 1734.

The foregoing table, taken chiefly from *Ricciolus*^a, *Flamsteed*^b and *Louville*, shews what the obliquity of the ecliptic was observed to be, by different astronomers, in the several ages of the world, from the most ancient times wherein we have any records of its having been measured, down to the latest observations thereof: a view of it may induce us to think, that the angle between the planes of the equator and ecliptic is less now than formerly, and that it is continually decreasing; *Louville*, having corrected the observations of the ancients, by making proper allowances for refraction not considered, and for parallax supposed to great, finds them very favourable to his opinion mentioned § 798, that the decrease of the obliquity is about 1' in 100 years; his dissertation, which is very curious, is published in latin, in the *acta eruditor. Lipsiæ*, A. D. 1719: the history of the academy of sciences for the year 1716 gives a short view of it; but at the same time tells us that the other astronomers of the academy were still of opinion the obliquity of the ecliptic was invariable: perhaps this was the reason why this dissertation did not find a place in their *memoires*.

803 M. *Godin* another member of the French academy of sciences resumed this subject, in the *memoires* of the year 1734: he remarks that *Louville*, notwithstanding his great exactness, made use of the table of refractions for Paris, to correct the observations of the ancient astronomers who lived in places where refraction might be different; and that the height of the pole at Marseilles, which is one of the chief grounds he goes upon, is not even now accurately enough determined; since *Gassendus*, *Cassini*, and *de la Hire* differ about 4' in settling it: this induced him to make use of such modern observations only in this enquiry as may be depended upon for their exactness, and which were made in places where the refraction is well known: he proceeds in this manner; in the year 1655 the obliquity of the ecliptic came out $23^{\circ} 29' 15''$, from the observations of *Cassini* by the gnomon at Bologna: now it is certain, by all the observations made at Paris and other places, that since 1730 the obliquity is no more than $23^{\circ} 28' 20''$; and that there cannot be an error of 5" in this determination of it: it has therefore grown 55" less in 80 years; which is very near 1' in 90 years: and this agrees, within 5", with the observations of *Richer*, *De la Hire*, *Roemer*, and *Bianchini*. Whatever may be thought of *Godin*'s objection to correcting the observations made at Alexandria and other places by the table of refractions for Paris, I see no reason for his questioning *Louville*'s determination of the height of the

^a *almagest. l. 3. c. 27, & astronom. reformat. l. 1. c. 6.*
See also *Philos. transact. n. 163.*

^b *prolegom. in histor. celest. vol. 3.*

pole at Marseilles; since it appears by his dissertation to have been very carefully taken, with a quadrant of 3 feet radius furnished with telescopic sights.

804 *Godin* will have it that the ecliptic approaches nearer to the equator, and that the equator is immoveable: he supposes the axis of this motion to pass through the equinoctial points, he does not say whether he means the places of them as now situated, or as they are continually changing by the precession; but seems to mean the latter: he supposes also that the orbits of all the rest of the planets are invariable, whilst the orbit of the earth only is perpetually changing: these are suppositions not very easy of digestion: if they be granted, it is certain the latitudes of stars must change in proportion to the motion of the ecliptic; but their declinations no otherwise than from the precession of the equinox: he thinks he has, from the declinations of stars set down by *Ptolemy* compared with what they are at present, found that the greatest part of them have changed their latitudes, since the time of that astronomer, as much as his supposed motion of the ecliptic would cause them to do; but owns at the same time that several stars favoured the contrary opinion^a: he goes on to shew that this motion of the ecliptic would cause that circle to cut the orbits of the planets successively in different points, but different orbits in a different manner; by reason of their different inclinations: the consequence of this is that their inclinations to the ecliptic must be changed, and their nodes must have a retrograde motion, a little different from one another, as well as different from what arises from the motion of the equinoctial points.

805 If there be any change in the obliquity of the ecliptic, which I think to be very probable, it may be caused either 1 by a change in the inclination of the earth's axis, the plane of the earth's orbit continuing immoveable: or 2 by a change in the plane of the earth's orbit, the axis of the earth continuing parallel to it self: or 3 by a change both in the inclination of the axis and in the situation of the orbit of the earth. The first of these seems most likely to be the case; for if we consider the shape of *the earth*, that it is *an oblate spheroid*, or a globe a little flattened like a turnep, having its diameter at the poles a little shorter than at the equator; and that, by reason of this figure of the earth, the attraction of the heavenly bodies will act upon it in the same manner as if it had a ring round the equator; and that the sun, being in the plane of the ecliptic, is perpetually pulling this ring, or the protuberant parts about the equator of the earth, towards a coincidence with the plane of the ecliptic; which action of the sun, together with the like attraction of the moon, is the cause of the precession of the equinoxes, as I shall

^a See § 794.

shew more at large when I come to treat of the causes of the planetary motions; if I say we consider all this, it will not appear improbable, that these actions of the sun and moon, which have a continual tendency to bring the plane of the earth's equator nearer to the plane of the ecliptic, should gradually diminish the angle between those planes. The moon indeed, being nearer to the earth, contributes more than the sun towards this effect, by her attraction; for though she is but twice a month in the ecliptic, yet since she is always near it, never deviating much above 5° from that circle, and these deviations are as much towards the north in one half of her period round the earth as they are towards the south in the other, they will so balance one another, that the effect of the whole attraction of the moon will be pretty much the same as if she were carried round the earth exactly in the plane of the ecliptic. I do not here absolutely exclude all change in the position of the ecliptic^a; for, as the planets move round the sun in different planes, their mutual attractions must have a tendency to change the situation of all their planes, and bring them nearer to a coincidence with one another: but then the distances of the planets from each other is so great, that the change here mentioned may be so exceedingly small as not to become sensible in many ages.

Remarks upon § 798 note ^b pag. 276.

806 Most of the ancient observations, which are come down to us, were made at Alexandria in Egypt, where, to speak in round numbers, which are sufficient in this case, the height of the pole is 31° ; the height of the equator 59° ; the height of the sun in the winter solstice 35° , at which height refraction is about $1' \frac{1}{2}$; the height of the sun at the summer solstice 82° , at which height refraction is about $7''$; so that refraction, in that latitude, by increasing the sun's apparent meridian altitude above $1'$ more at the winter than at the summer solstice, diminishes the distance between the tropics as much; and consequently lessens the obliquity about half that quantity: we know that *Ptolemy* determines the sun's horizontal parallax to be very near $3'$; whereas it does not amount to above $10''$, as we shall see in the following chapter: if the ancients considered *Ptolemy's* parallax in this affair, as I believe they did not^b, this at the altitude of 35° is almost $2' \frac{1}{2}$, and at the altitude of 82° not more than $21''$; and then these mistakes about the parallax and refraction, would occasion their settling the obliquity of the ecliptic about $1' \frac{1}{2}$ less than they would otherwise have done.

^a See § 794.

^b It is certain *Ptolemy* did not; see his *almagest* L. 1 c. 10.

Remarks upon § 798 note ^c pag. 277.

807 It is impossible to explain this tradition of the Egyptians of the diurnal rising and setting of the sun; but if they meant it of his annual rising and setting, and by that expression understood his entrance into the points of the vernal and autumnal equinox, some sense may be made of it^a: they supposed the equator immovable, and, if they had found by observation that the obliquity of the ecliptic gradually diminished, they might naturally conclude the ecliptic, or circle of the sun's apparent annual motion, would in time be coincident with the equator; at which time there would be a perpetual equinox over the face of the whole earth: that thence forward the ecliptic would decline from the equator the contrary way; so that the semicircle of the ecliptic which is now on the north, would be then on the south side of the equator: this would change the seasons of the year, in such a manner that the summer would happen, in Egypt, when the sun was in north, the winter when he was in south declination; the spring would begin when the sun entered into ϵ , and the autumn when he entered into γ : it is easy to see that, supposing this motion of the plane of the ecliptic to be constant, it would cause the vicissitudes of the spring beginning alternately, though with a very long interval of time between, when the sun was in γ and ϵ : the Egyptians, resolving to raise their pretensions to antiquity high enough, gave it out that four such changes had been actually observed by their nation.

Remarks upon the table.

808 The most ancient observation here mentioned is that of *Pytheas* of Marseilles, not that he was the first who is recorded to have observed the obliquity of the ecliptic, for we are told *Thales*^b and *Anaximander*^c did it near 300 years before him; but we are not informed what they found it to be: the only ancient account of the observation of *Pytheas* is in *Strabo* l. 2, where he is said to have found that, at Marseilles, at the summer solstice, the length of the gnomon was to the length of its shadow, as 120 to 42 wanting a 5th: by the 5th I suppose, with *Gassendus* and *Louville*, is meant a 5th of one of the 120 parts into which the gnomon was divided; the gnomon then was to the shadow, in whole numbers, as 600 to 209: say then, as 600 to 209, so is radius to the cotangent of the height of the sun's upper edge; which comes out $70^{\circ} 47' 41''$: this corrected by taking away for refraction $20''$, the semidiameter of the sun $15' 49''$, and adding parallax $3''$, gives the true height of

^a Fracastorius in *homocentricis*, § 3 c 8.

^b Laertius in *Thalete*.

^c Laertius in *Anaximandro*:

the sun's center $70^{\circ} 31' 35''$: from whence take away the height of the equator at Marfeilles, observed by *Louville* $46^{\circ} 42' 12''$, there remains the obliquity of the ecliptic in the time of *Pytheas* $23^{\circ} 49' 23''$.

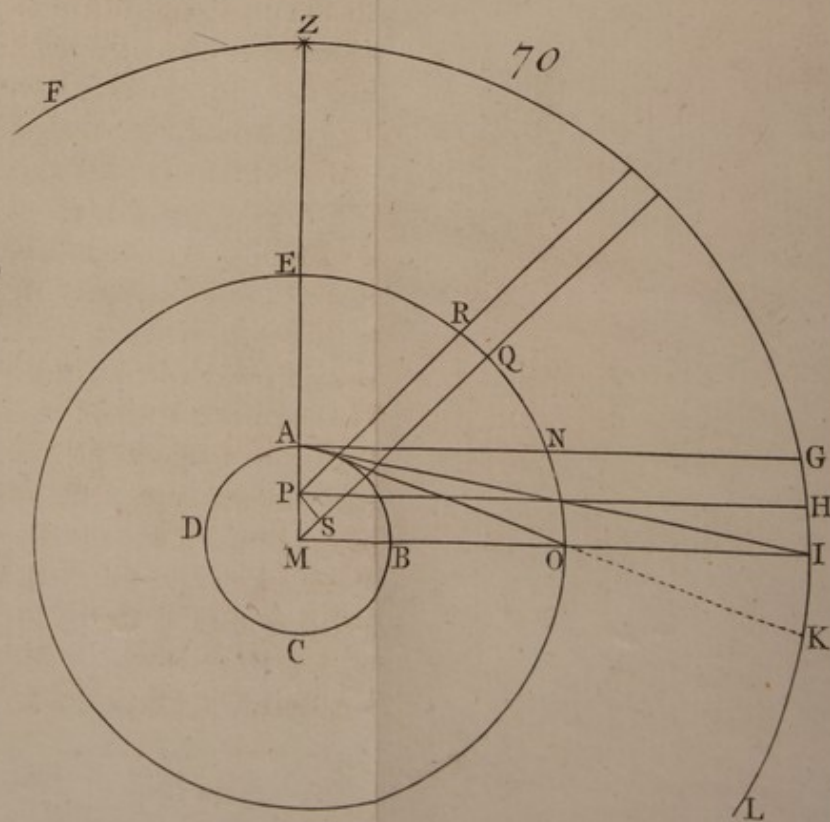
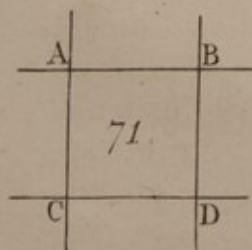
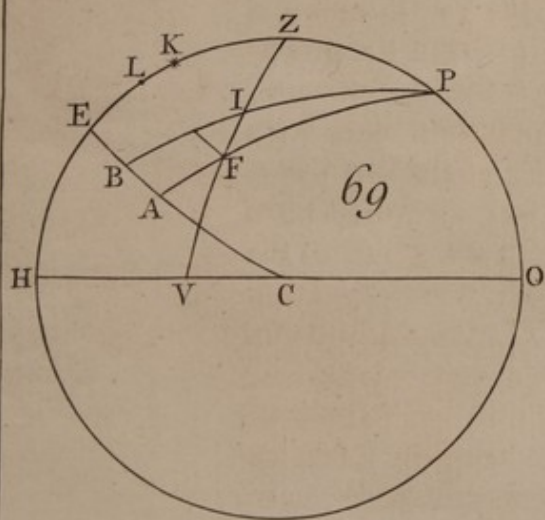
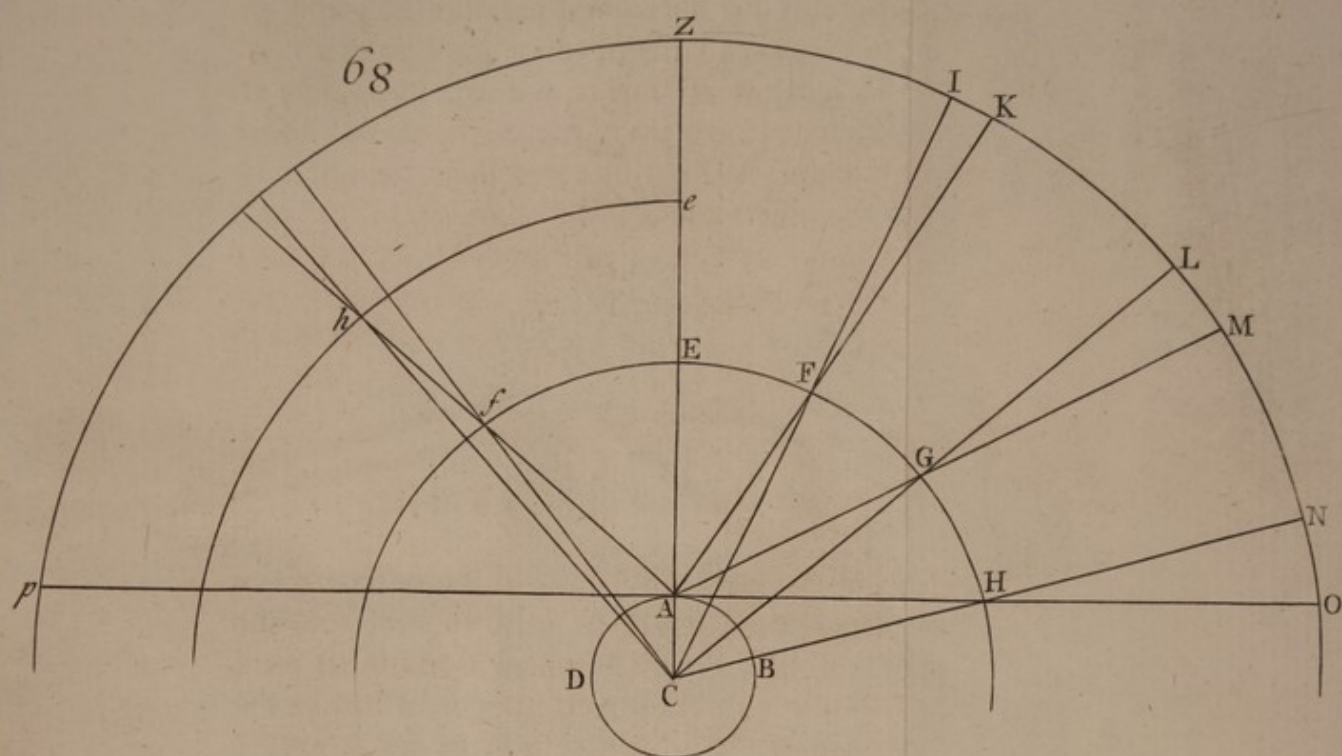
809 Since *Ptolemy* all astronomers seem, after his example, to have retained the obliquity settled by *Eratosthenes*, till the time of *Almamon*: that Prince, assisted by many learned men, found the greatest meridian height of the sun at Bagdat to be $80^{\circ} 15'$; the least $33^{\circ} 5'$: the least being subtracted from the greatest, the remainder $47^{\circ} 10'$ is the distance between the two tropics; half of which, $23^{\circ} 35'$, is the obliquity. *Albatennius*, about the same time, brought out the same obliquity, from his own observations: I have set down his own words^a because his book is not common; it appears from them, that he did not consider the parallax of the sun at all, though some writers tell us the contrary. *Arzakel*, who is much commended for his abilities by *Aben-Ex-ra*^b, found the obliquity $1'$ less than *Albatennius*, for his numbers are $23^{\circ} 34'$, according to most writers, though *Flamsteed* sets them down half a minute less: from thence forward we find the obliquity lessening, till the time of *Tycho*, when refraction came first to be considered in this affair; though *Walther* and even *Regiomontanus*^c had before observed refraction to alter the altitudes of the heavenly bodies.

The letter F on the side of the table intimates the observation to be taken from *Flamsteed*, R from *Ricciolus's almagest* or *astronomia reformatata*, L from *Louville*, those which have no letter prefixed are from the authors themselves: the measure of *Eratosthenes* as well as that of *Ptolemy* is from *Ptolemy's almagest* l. 1 c. 10. As to the years set down in the table, I have followed the authors from whence I took the observations: there are in the table a few instances where the angle of the obliquity of the ecliptic seems to increase, being greater by a subsequent observation than more ancient ones had determined it; but this increase never rises much above $2'$, and so small a difference may very well be attributed to the inaccuracy of the observers: whereas the difference of above $20'$ between the most ancient and the latest observations is too great to be accounted for any other way, than by an actual diminution of the angle between the equator and ecliptic: if we will allow

^a Nos autem in hoc nostro tempore cum Albidada longissima, & latere, quorum opus & doctrina in *Almagesti* libro docetur, post partium diminutionem (i. e. divisionem) & positionis instrumenti verificationem, tam optimam quam esse possit, frequenter observavimus, solisque propiorem ascensum puncto zenith capitis in medij diei circulo in *Araffa* civitate 12 graduum, & 26 minutorum, remotiorem autem ejus elongationem 59 graduum & 36 minutorum esse deprehendimus. Per hoc ergo probatum est quantitatem arcus inter duo solstitia 47 graduum, & 10 minutorum existere, declinationemque circuli signorum ab æquinoctiali circulo, non nisi harum partium medietatem quod est 23 graduum & 35 minutorum obtinere. De scientia stellarum c. 4.

^b apud Scaliger. de emendat. tempor.

^c Snellius in appendice ad observationes Hassiacas.



the ancients to have had eyes and instruments, and to have used any tolerable care in their observations. I shall here give some of the numbers corrected by the parallax of the sun, supposing his horizontal parallax $10''$, and allowing for refraction according to *Newton's* table of refraction § 765; where it must be owned we cannot be so accurate as is to be wished, by reason of the difference of refraction in different places upon the earth, and at different times of the year in the same place. With this correction, the obliquity of the ecliptic comes out from the observations of *Eratosthenes* $23^{\circ} 51' 51''$, of *Ptolemy* $23^{\circ} 49' 16''$, of *Albatennius* $23^{\circ} 35' 35''$, of *Arzakel* $23^{\circ} 34' 42''$, of *Prophatius* $23^{\circ} 32' 42''$, of *Ulug Beigh* $23^{\circ} 30' 58''$, of *Copernicus* $23^{\circ} 30' 7''$, of *Walther* $23^{\circ} 29' 15''$, of the *Prince of Hesse* $23^{\circ} 30' 20''$, of *Tycho* $23^{\circ} 29' 0''$, of *Hevelius* $23^{\circ} 29' 0''$.

CHAP. 17. OF THE PARALLAX OF THE HEAVENLY BODIES.

810 In speaking of the places of the heavenly bodies in the sphere of the heaven, we have hitherto considered them as if they were viewed from the center of the earth; or, which would be the same thing, as if the earth were but a point; that is, as if the distance between the center and surface of the earth were insensible, when compared with the distances of the heavenly bodies from us: this is true with regard to the fixt stars, see § 274, and in a great measure also in respect of the sun, and all the planets except the moon: in fig. 68, let ABD be the earth, pzo part of the sphere of the heaven, now, though we are sure, by geometrical reasoning, that if two strait lines were to be drawn from any two different points of the surface of the earth as A and B to the center of the sun or any of the planets at H , they would form an angle AHB ; and if they were prolonged from thence to the sphere of the heaven they would cross one another and terminate in two points thereof N and O , which would be at some distance from one another; yet this distance is, for the most part, too small to be taken by any instrument: this distance between the points N and O is called the *parallax*, which is said to be *sensible* when great enough to be discoverable by observation; *insensible* when too small to be measured by any instrument: the word *parallax* in greek signifies any kind of change in general; but, in astronomy, the *parallax of a phenomenon* is the change of its apparent place in the sphere of the heaven, caused by its being seen from different points of view.

811 The *diurnal parallax* of a phenomenon, which is generally understood when the word is used without any epithete, is the arc of a great circle

FIG. cle in the sphere of the heaven, contained between the point wherein it appears to a spectator upon the surface of the earth and the point wherein it would appear if it were viewed by a spectator at the center of the earth: the former of these points is called *the apparent place*, the latter *the true place* of the phenomenon: it is called diurnal parallax to distinguish it from the annual, and because it arises from the semidiameter of the earth whose revolution round her axis causes the vicissitude of day and night, or because it is alterable by the diurnal rotation of the earth which changes the apparent altitudes of the heavenly bodies and consequently their parallax.

812 A phenomenon which at different altitudes continues at the same distance from the center of the earth has the greater parallax the farther it appears from the zenith of the observer: therefore the parallax is greatest in the horizon. A phenomenon that appears in the zenith, whatever its distance
68 from the earth is, has no parallax: thus, if a phenomenon be at H, in the visible horizon of a spectator at A, its parallax is the arc NO; its apparent place is O; its true place N: if it be at some height above the horizon, as at G, its parallax is LM; which is less than NO: if it be higher, as at F, its parallax is IK; less than LM: if it be at E or e, it appears in the zenith, at Z; and has then no parallax: for, the lines AZ and CZ coinciding, its apparent place is the same with its true place.

813 *Parallax always diminishes the altitude* of a phenomenon, or makes it appear lower than it would do from the center of the earth: thus, O the apparent place of H is farther from the zenith Z than its true place N is; in like manner, M the apparent place of G is lower than its true place L; and K the apparent place of F is lower than its true place I. This change of the altitude by parallax may, according to the different situation of the ecliptic and equator in respect of the horizon of the spectator, cause an apparent *parallax* or change of the *latitude, longitude, declination, and right ascension*, of
69 the phenomenon; or it may change only some of them: thus, fig. 69, let HO be the horizon, EC part of the ecliptic, P the pole of the ecliptic, Z the zenith, ZV an arc of a vertical drawn through I the true place, and F the apparent place of a planet, let DA be a circle of latitude drawn through the planet's true place, and DB a circle of latitude drawn through its apparent place; it is manifest that the parallax of altitude FI, by changing the place of the planet from I to F, alters its latitude from BI to AF; it alters its longitude also as much as the arc AB amounts to § 659. If the place of a planet be in a vertical circle which cuts the ecliptic in the ninetieth degree from the horizon, that is which cuts the ecliptic at right angles, the parallax will change the latitude of the planet, but not its longitude: thus, if a planet be
in

in the point κ of the vertical $z e h$, this vertical being perpendicular to the ecliptic, and consequently a circle of latitude, by the definition § 646, the parallax of latitude κl will be the same with the parallax of altitude; but, since the same circle of latitude $p e h$ passes through the true place κ and the apparent place l , the parallax of altitude κl does not change the longitude. Let $e c$, which before represented the ecliptic, be now the equator, p the pole of the equator, $p e h$ the meridian, $p i b$ a circle of declination passing through the true place, and $p f a$ a circle of declination passing through the apparent place of the planet; the point b shews the true, and the point a the apparent right ascension of the planet, and $b a$ is the parallax of right ascension: $i b$ is its true, $f a$ its apparent declination, and their difference is the parallax of declination: if the star be in the meridian as at κ , the parallax of declination will be κl the same as the parallax of altitude; but there will be no parallax of right ascension: because the same circle of declination $p e h$ passes through the apparent place l and the true place κ .

814 The parallaxes of any two phenomena as f and b whose distances from the center of the earth are different, and apparent distances from the zenith equal, are reciprocally as their distances from the center of the earth: thus, if the distance of b from c be double the distance of f from c , the parallax of f will be double the parallax of b . The converse of this is true; if the parallax of f be double the parallax of b the distance of b is double the distance of f from the earth.

815 One use of the knowledge of *parallaxes* is to find the true place of a phenomenon: in all observations of the heavenly bodies, it is necessary to know how much they are depressed by parallax, that by adding so much to their apparent we may have their true altitudes: indeed in all of them except the moon the parallax is so small, that, in most cases, we need not take any account of it, but then to know it to be thus small is of good use, to prevent our falling into the mistakes, and help us to correct the observations of those who have supposed it too great; as all astronomers till of late have done: thus, *Ptolemy* settled the horizontal parallax of the sun when in perigee about $3'$; as *Tycho* did also: *Kepler* diminished it to $1'$: but, by the latest and best observations, it is found not to exceed $10''$.

Another use of *parallaxes* is to find the distance of the heavenly bodies from the earth: thus, the horizontal parallax of any one of them being known, its distance from the earth may be found; by § 143: see the moon's distance found by her horizontal parallax, § 147. The parallax of a phenomenon at any given altitude being known its distance from the earth may be found: see the remarks, § 825.

FIG. 816 The moon being the nearest to the earth of all the heavenly bodies; her parallax is the greatest, and consequently the most easy to be determined; it amounts to about a whole degree, as we shall see when the moon comes to be treated of. The sun and all the primary planets, except mars and venus when in perigee, are at so great distances from the earth, that their parallax is too small to be observed. When mars is in opposition to the sun, his distance from the earth is but half so much as the sun's distance from the earth; and consequently his parallax is then double to that of the sun: *Tycho* in the year 1582 endeavoured with incredible diligence to discover the parallax of mars in opposition; but *Kepler*, having examined *Tycho's* observations, concluded from them that mars's parallax was scarcely sensible^a.

817 The horizontal *parallax* of a phenomenon would be found directly, if it were to be seen by two observers, distant from each other 90° of a great circle upon the earth, and so situated that one of them saw the phenomenon in his zenith, the other in his horizon: the distance between the two points in the sphere of the heaven in which the phenomenon appeared to them would be its parallax: thus, fig. 68, let a phenomenon be at H, to an observer at B it appears in his zenith at N; to one at A it appears in his horizon at O; its horizontal parallax is NO. This method is mentioned for the easiness of conceiving it, but is not likely to be put in practice, and, if it were, horizontal observations are very uncertain, by reason of the variableness of refraction in or near the horizon: but the parallax of a phenomenon at any known altitude being found, its horizontal parallax may be computed from thence, in the manner set down in the remarks^b. Also if two observers not so far distant as 90° observe, at the same time, meridian altitudes of a phenomenon, the difference of declination which comes out from those altitudes is the parallax owing to the difference of latitude of the places of observation, this being known the horizontal parallax which would arise from a distance of 90° may be computed from it: thus the meridian altitudes of mars in perigee were observed at the same time at Paris by *Cassini*, and at Cayenne^c, by *Richer*; and the horizontal parallax of mars computed from thence: the best way to have found the parallax of mars, by observations made at

^a Ricciol. *almagest* l. 7 c. 3.

^b § 825, corollary 2.

^c Cayenne is an island on the coast of N. America, in the latitude of $4^\circ 56' 18''$, distant from Paris $3^h 39'$ in longitude, westward, thither *Richer* went by order of the R. Academy of sciences to make observations, amongst other things, of the meridian altitudes of mars and venus in perigee, and of some stars near them: the Messieurs of the Academy resolving to do the same at Paris and other places in France, in order to discover the parallaxes of those two planets. I have here given the result of their observations, which may be seen more at large, in *Observations Astronomiques & Physiques faites en l'isle de Cayenne par Richer* p. 14. & *elémens d'Astronomie vérifiés, par Cassini*, p. 5. & seqq.

those two places, would have been to have observed with the telescope an occultation of some fixt star by mars^a; if this had appeared exactly the same in both places, it would have shewn that mars had no sensible parallax; if he had, the moment that the upper edge of mars's disk appeared at Cayenne to touch the star, it would at Paris have appeared a little lower than the star; and when the lower edge of the planet appeared at Paris to touch the star, it would at Cayenne have appeared a little higher than the star: and these distances of mars from the star seen in one place and not in the other would have been owing to the parallax.

818 As they had not an opportunity of making such an observation in both places at the same time^b, they took, the same day at Paris and Cayenne, meridian altitudes of mars and of a fixt star in aquarius nearly of the same declination as that planet then had; in order to find whether the difference between the altitudes of mars and that star, were the same at Cayenne, as at Paris: if it were, since the star has no parallax, the declination of mars would come out the same from the observations in both places; and his parallax would be insensible: if, on the contrary, the difference between the altitudes of mars and of the star were observed to be greater at one place than at the other, this would shew a difference between the declination of mars as calculated from the observations of him in one place and his declination as it came out from the observations in the other place; which difference would be the parallax of mars, arising from his being viewed by two observers distant from each other as much as the arc of a great circle contained between the parallels of Paris and Cayenne amounts to.

Scholium. It is easy to see that, in order to find whether the declinations of mars would come out the same from his meridian altitudes in Paris and Cayenne, the difference between the latitudes of those places must be known: but, besides this, their difference in longitude must also be considered: now this is such, that any of the heavenly bodies comes to the meridian of Cayenne 3^h 40' later than to the meridian of Paris: in this interval of time, mars, by his motion in his orbit, changes his declination; and, though this change be very small, it is necessary to know exactly what it is, in an affair of so great nicety as that before us; this is known by comparing the meridian height of mars taken the night immediately before with his meridian height taken the night after the observation of him from whence the parallax is sought: for this shews what the change of his declination is in the interval of 24 hours; from whence, by the rule of proportion, it is easily found what it is in 3^h 40'.

^a v. les elemens d'astronomie verifiez p. 34.

^b ibid. p. 35.

819 The three observations of the meridian altitudes of mars, made the same nights at Paris and Cayenne, by which his parallax was found, were in the month of september 1672: by the first observation, sept. 5, mars appeared $12''$; by the second observation, sept. 9, $13''$; by the third, sept. 24, $17''$ more southerly, at Paris; than he would have done, at the same moment, under the same meridian, in the latitude of Cayenne: *Cassini* says that the third number should have been less than the second, because mars was a little farther from the earth at the time of the third observation, than at the second; and therefore his parallax should have then been less: but an error of a few seconds is unavoidable in such observations: he thought the best way was to split the difference between the second and third observations, and make the parallax of mars, owing to the difference between the latitudes of Paris and Cayenne, $15''$; from whence the horizontal parallax of mars at that time is by calculation $25\frac{1}{3}''$. *v. les elemens d'astronomie verifiez p. 39.*

820 Mars is nearest to the earth and his parallax the greatest of all, when he is in opposition at the same time that he is in perihelion; such was his situation in the year 1672, when *Cassini*, by the methods here mentioned, § 817 and 822, from his own observations at Paris and those of *Richer* at Cayenne, determined his horizontal parallax to be $25\frac{1}{3}''$: those observations of *Cassini* were confirmed by those of *Maraldi*, made in the year 1704^a, and again in the year 1719^b; from which he concluded the sun's parallax to be $10''$, as *Cassini* had settled it in several places of his writings^c: at the time of *Cassini*'s observations the distance of mars from the earth was to the mean distance of the sun from the earth, as $25\frac{1}{3}$ to $9\frac{1}{2}$; the parallaxes of any two phenomena are reciprocally as their distances^d, and therefore, if the parallax of mars be $25\frac{1}{3}''$, the sun's parallax at his mean distance comes out $9\frac{1}{2}''$: instead of which the round number $10''$ is usually assigned for the sun's parallax. There are other methods of finding the sun's parallax made use of by the ancients and some of the moderns, but they are not so accurate as those just now mentioned; they require also some knowledge of the moons theory, in order to the more easily understanding them: I shall therefore reserve the explanation of them to a more proper place.

821 Venus in perigee is so near the sun, that her parallax cannot easily be discovered by comparing her with any fixt stars^e. Venus in perigee may sometimes be so near one of her nodes as to appear upon the disk of the sun; thus she was seen nov. 24. A. D. 1639, only at Hoole in Lancashire by *Hor-*

^a *Histoire d'acad. ann. 1706.*

^b *Memoires d'acad. ann. 1722.*

^c *Decouverte de la lumiere*

celeste p. 22. Elemens d'astronomie p. 21.

^d § 814.

^e *Hamel. hist. acad. p. 109. 199. Mar-*

aldi memoires d'acad. 1722.

160x, and at Manchester by his friend *Crabtree*, to whom he had beforehand given notice of that uncommon appearance^a: *venus* will not be again visible upon the sun's disk, till may 26. 1761, and then, if the time of her transit be observed by different persons, in different parts of the earth, in the manner described by *Halley* in the philosophical transactions n. 348, her parallax from the sun, *i. e.* the difference between her parallax and the sun's, may be determined, to a degree of accuracy not to be hoped for from any other method. Mercury appears oftner upon the disk of the sun than *venus*, but his distance from the earth at that time wants so little of being equal to the sun's, that his parallax from the sun is too little to be easily discovered.

822 The parallax of a phenomenon, if sensible, may be found by one observer only, by marking the apparent difference of its situation, in respect of fixt stars that are near it, at different hours of the same night: if I make use of this method, a fixt star and a clock that beats seconds will supply the place of another observer, distant from me any number of degrees upon the earth less than a quarter of a circle; for all that he could tell me would be, that at the same moment when I viewed the phenomenon at a certain distance from the star, it appeared to him at a different distance from it; which is the parallax owing to the distance of our two places of observation: suppose that distance to be 45 degrees, the rotation of the earth, in a certain time which may easily be computed in any latitude, suppose the time 5 hours, will carry me into a situation 45 degrees distant from that wherein I am at present, and consequently, supposing the phenomenon and the star to have no motion in the mean time besides the apparent diurnal motion, I shall, at these two different times of observation, one now the other five hours hence, find the same difference between the distances of the phenomenon from the star as would be found by two observers distant 45° from each other, viewing it at the same moment of time. By this method, which, from the authors here quoted^a, is more at large explained in the remarks, § 826, *Cassini* found the parallax of mars nearly the same as by that mentioned § 817: by the same, *Flamsteed* at one time determined the parallax of mars to be less than 30" ^b; and at another time not to exceed 25" ^c: by the same method, *Bianchini* makes the parallax of mars to amount to near 40": so difficult is it in observations of this nature to be certain we are within a few seconds of the truth.

^a *Cassini de la comete ann. 1680*: *Bianchini op. cæta err. ditor. Lipsie ann. 1685*: *Whiston's 7th astronomical lecture*. *Digges* an English gentleman, invented several methods of finding out the parallax by one observer, published in his *ale seu scale mathematicæ* 1573; from whom subsequent authors seem to have borrowed.

^b *Philosoph. transact.* n. 89 and 96.

^c *ibid.*

FIG. 823 *The annual parallax* of a phenomenon is the change of its apparent place in the sphere of the heaven, which is caused by its being viewed from the earth in different parts of her orbit: the annual parallax of all the heavenly bodies except the fixt stars is considerable: it is the motion of the earth through the different parts of her orbit that causes the sun to appear every day of the year in a different point of the ecliptic; as was shewn § 642: and it is from the same cause, namely our continually changing our station, that the geocentric places of the planets are almost always different from what they would be if they were viewed from any fixt point in the orbit of the earth: as has been shewn in the 8th, 9th and 10th chapters of this book. The reason of this is plain, the diameter of the earth's orbit, bears a considerable proportion to the semidiameters of the orbits of the most distant planets, being more than a 5th part of the semidiameter of the orbit of saturn; and above a 3d of the semidiameter of jupiter's orbit: but it bears so small a proportion to the semidiameter of the sphere of the heaven, or the distances of the fixt stars from us, that if they have any sensible parallax it is exceedingly small: what attempts have been made to discover it, and with what success, shall be the subject of the next chapter.

Remarks upon § 812.

824 In order to see how it comes to pass that the parallax of a phenomenon is greater the less its altitude is, we may take it in the following view;
 68 a line AC drawn from the center of the earth to the place of the spectator is a semidiameter of the earth, which, if we suppose it to be viewed from the phenomenon whose parallax is considered, will appear under an angle that may be called the angle of the semidiameter: thus, if the phenomenon be at H , the angle of the semidiameter is CHA ; if at G , it is CGA , &c.—The angle of the semidiameter is equal to the angle of parallax, because vertical to it: thus, CHA is equal to NHO ; and therefore either of these is called the parallactic angle: in like manner, CGA is equal to LGM ; and CFA to IFK . The angle of the semidiameter is less, the more obliquely the semidiameter CA is viewed: thus, from H the line CA would be seen in the most direct view; and then it appears under the greatest angle CHA : from G the line CA is viewed more obliquely, and therefore appears under a less angle CGA : from F it is viewed still more obliquely, and appears under a still less angle CFA : from E or e the line AC is viewed end-wise, and appears a point.

Remarks upon § 815.

825 The distance of a phenomenon from the center of the earth is in the same

same ratio to the semidiameter of the earth, as the sine of its apparent distance from the zenith is to the sine of its parallax: it is a known property of every plane triangle, that the sines of the angles are as the sides which subtend them: or, conversely, the sides are as the sines of the angles subtended by them: thus, fig. 68, CG the distance of the phenomenon G from the center of the earth is to AC the semidiameter of the earth, as the sine of the angle CAG , or ZAM † which measures the apparent distance of G from the zenith, is to the sine of CGA the angle of the parallax of G . FIG. 68

Corollary 1. Hence the parallax of a phenomenon at any altitude being given, its distance from the center of the earth may be known: and conversely.

Corollary 2. The sines of the apparent distances of a phenomenon from the zenith are as the sines of its parallax: therefore as the sine of any given distance of a phenomenon from the zenith is to the sine of its parallax at that altitude, so is the sine of 90° to the sine of its horizontal parallax.

Remarks upon § 822.

826 In fig. 70, let $ABCD$ be the equator of the earth, A the place of an observer, imagin the plane of the equator to pass through mars at E , and to be extended every way till it reaches the sphere of the fixt stars, and marks thereon the celestial equator, part whereof is represented by FHL : suppose the celestial equator divided into 24 equal parts, beginning at Z the zenith of the observer, and a plane to be drawn from M the center of the earth to each division: these planes will mark 24 hour-circles, whereof MAZ is the meridian of the observer, whom we suppose to see mars, when at E , in conjunction with a fixt star Z , in his zenith: if neither mars nor the star had any other motion besides the apparent diurnal, they would both appear to go round in 24 hours; and, if this diurnal motion be equable, mars and the star would in six hours after being in the meridian MAZ be in MOI the plane of the sixth hour-circle, which is also the rational horizon of the place A : now a spectator at M the center of the earth would see mars and the star go round from Z to I continuing all the while in exact conjunction, but it will happen otherwise to the observer at A ; for he will see mars in exact conjunction with the star, only while they are both in the plane of his meridian MAZ , but six hours after, when mars is at O , and the star at I , so that both of them are in the plane MOI , the star will to him appear to be in the plane AI , and mars in the plane AK : it is to be farther observed, that mars, being 70

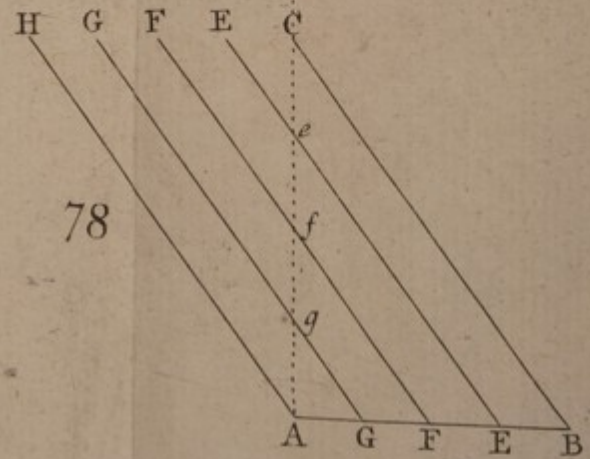
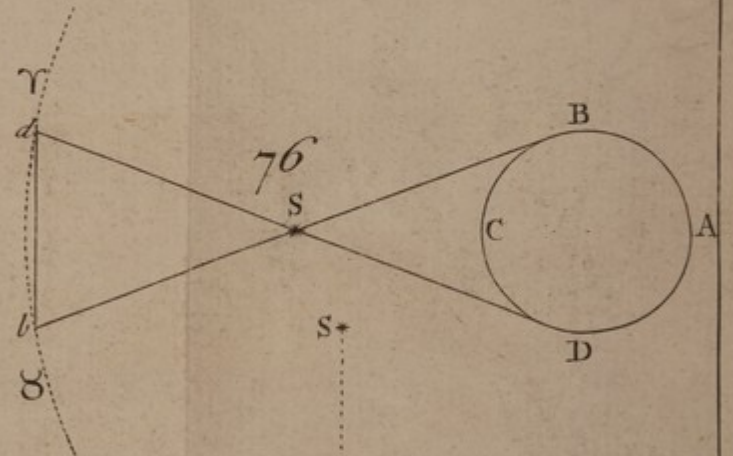
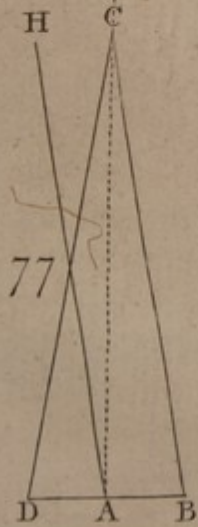
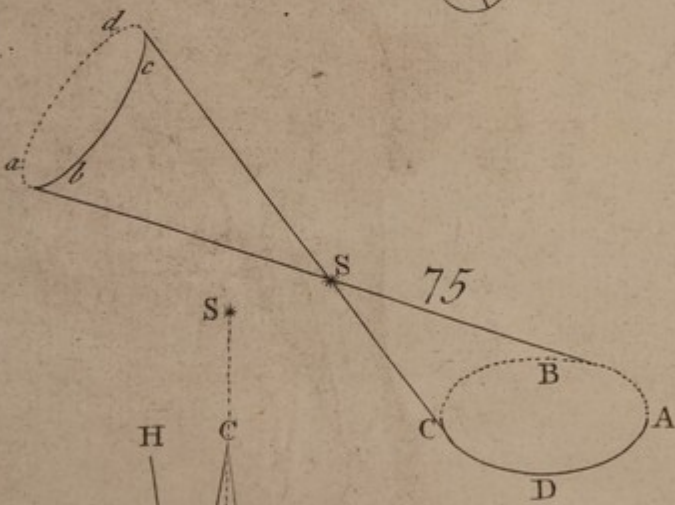
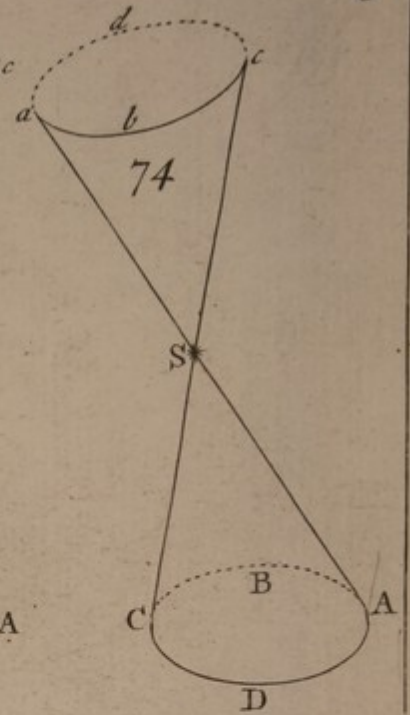
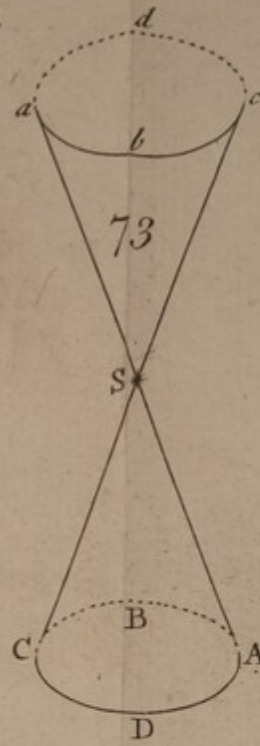
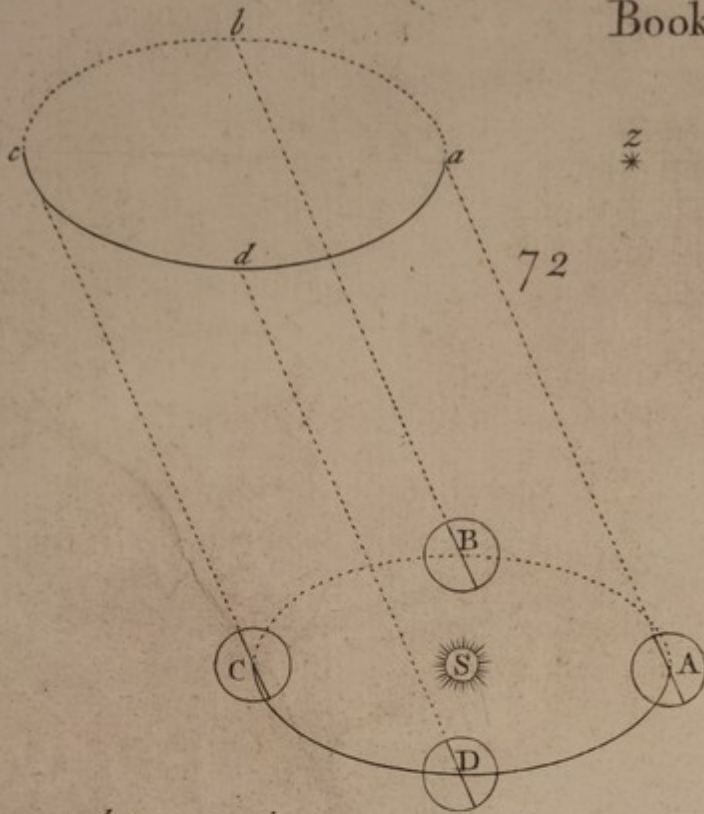
† The obtuse angle CAG and the acute angle ZAM have the same sine; because either of these angles is supplement to the other: § 105.

thus

FIG. thus depressed by a sensible parallax, will appear to come to ANG the sensible horizon of A , a few seconds before he really comes to the rational horizon MOI ; and consequently before the expiration of six hours after his being in the meridian; whereas, the star will appear to go from the meridian to the horizon exactly in six hours; the sensible and rational horizon being coincident in respect of the stars § 275: now the time that mars appears to come to the horizon before the star is his parallax in time; and is equal to the time mars takes in going the arc NO , from the sensible horizon to the rational: and may be easily turned into measure, in order to have his parallax in an arc of a great circle. We shall also have directly the parallax of mars in an arc of a great circle, if, when he is in the sensible horizon, we take his apparent distance from the star, with the micrometer. The parallax of mars being thus found to be the arc GK , or the angle NAO , to which the alternate MOA is equal; we have a right angled triangle AMO , whereof the acute angle at O is known; and consequently, by § 143, the ratio of AM the semidiameter of the earth to MO the distance of mars from the earth may be known.

827 In order to find the difference of time between the coming of mars and that of the star to any hour-circle, there must be four fine hairs or very
71 small wires crossing at right angles, as in fig. 71, placed in the focus of the telescope; and the telescope must be turned round till some star near mars appears by the diurnal motion to pass along one of the hairs, as AB , which hair will then be parallel to the equator, and the hairs AC and BD perpendicular to it will then be meridians, by the coming to which the right ascension of any star may be known, see § 786: mark the time then when mars and the star are both in your meridian, and six hours after, the star will come to the sixth hour circle, but mars will come to it a little sooner; against that time therefore, having the telescope ready with the two cross hairs AB and CD parallel to the equator, observe the moment that mars comes to one of the hairs AC or BD , and count how many seconds later the star comes to the same hair.

828 It must be owned several circumstances have been now mentioned to attend the observer, which he may more easily suppose in theory, than be favoured with in practice; but trigonometry will here come in aid: for 1 suppose the observer to be at P in some parallel at a good distance from the equator, the parallax of right ascension will be found less than it would under the equator, but in a ratio easy to be known; for it will be in the ratio of PM to AM or as the sine of the height of the equator to the whole sine; 2 horizontal observations being difficult, instead of the sixth let the different
appulses



appulses of mars and the star to the third hour-line be observed; here also FIG. will be two third hour-lines, the true one MQ drawn through the center of 70 the earth, and the apparent one PR drawn parallel to it though P the place of the observer: and as PS is to PM , or the sine of the half right angle ZMQ contained between the meridian and the third hour-circle of the place P is to the whole sine, so is the parallax found at the third hour-circle to the parallax at the sixth: 3 suppose mars not to be in the equator, but in some parallel; his parallax then found, in order to know what it would be in the equator, must be increased just so much as the semidiameter of the equator exceeds the semidiameter of the parallel wherein mars then is, or as the whole sine is to the sine complement of mars's declination: 4 as mars has a motion in his orbit, this must be taken into the account; and since this is either equable, or equably accelerated or retarded, which of these is the case may be found by preceding and subsequent observations of mars, so as to enable us to know, with sufficient exactness, at any moment of time, the true place of mars wherein he would appear to a spectator at the center of the earth; and, by comparing it with his apparent place, discover his parallax as well as if he had no other motion besides the diurnal.

CHAP. 18. OF THE ANNUAL PARALLAX OF THE FIXT STARS: THEIR
DISTANCE FROM US: THE VELOCITY OF LIGHT.

829 Since the revival of the ancient opinion of the earth's annual motion round the sun by *Copernicus*, several astronomers have endeavoured to find out whether the fixt stars are subject to any annual parallax or not: this enquiry very well deserves our attention, upon two accounts; 1 if such parallax could be discovered, it would demonstrate the truth of the *Copernican* system: 2 if the quantity of the annual parallax of any of the fixt stars could be found, the distances of those stars from our earth might be known^a.

The axis of the earth extended, being carried parallel to it self during the earth's annual revolution round the sun, in reality describes a circle in the sphere of the fixt stars equal to the orbit of the earth: thus, fig. 72, 72 let $ABCD$ be the orbit of the earth, s the sun, the dotted lines the axis of the earth extended, this axis when the earth is at A points at a in the sphere of the heaven; when the earth is at B points at b ; when at C points at c ; when at D points at d : so as in a years time to describe $abcd$, equal to $ABCD$: but though the orbit of the earth, and consequently the circle $abcd$ be

^a See the remarks.

FIG. immensely large, many millions of miles in diameter, it is but a point in comparison of the sphere of the heaven; that is, the angle under which its diameter appears to an inhabitant of the earth is insensible, less than can be measured by any observation; and therefore the celestial poles appear to be in the same points of the heaven throughout the year: thus, in the figure before us, though when the earth is at A the place of the celestial north pole is a , and when the earth is at c the place of the north pole is c , and a star at z is nearer the point a than it is to the point c by the whole length of the line ac yet, if this line ac viewed from the earth should take up no sensible space in the sphere of the heaven, as if it should measure no more than 2 or 3 seconds, which is no more than the errors of the nicest observations may amount to, the star at z will appear at the same distance from the pole throughout the year; and consequently it will have no annual parallax.

830 We have seen the parallax of the sun to be no more than $10''$, this smallness of the sun's diurnal parallax is owing to this, that the semidiameter of the earth bears an exceedingly small proportion to the semidiameter of the earth's orbit, being not more than a 20000th part of it, for so many semidiameters of the earth is the sun distant from us: things are great or little only in comparison to other things of the like sort; the distance of the sun may be inconsiderable when compared with the vastly greater distance of the stars: if we suppose the distance of the nearest star to be 20000 times as great as the sun's distance from us, the annual parallax of that star would be no more than $10''$; too little to be discovered by any of the methods commonly made use of to take the height of the pole by the stars: the distance of the nearest stars may be much greater than this, and their parallax consequently much less than $10''$; it is no wonder therefore that astronomers have found the height of the pole in every place to be the same throughout the year: but some have been too hasty in concluding from thence that the earth has no such annual motion round the sun as the followers of *Copernicus* suppose.

831 If the annual parallax of any fixt star were sensible, that star would appear to change its place so as to describe a very small ellipsis in the sphere of the heaven, in a years time: thus, fig. 73 and 74, let $ABCD$ be the orbit of the earth, s the star to be observed, if we imagin a strait line to be drawn from the earth in any part of her orbit as A through the star to a point in the heaven as a , that visual line Aa , being carried along with the earth in her annual motion, will describe two conic surfaces Asc and asc , having their common vertex at the star s ; and the motion of the earth round $ABCD$ will make the star appear to go round $abcd$. If the star s were in the pole of the ecliptic, as fig. 73, the ellipsis described by it would have the same excentricity

ty with the orbit of the earth; and consequently would differ very little from FIG. a circle: if the star be at any distance from the pole of the ecliptic, the greater that distance is the more oblong will the ellipsis be, see fig. 74 and 75: if 74 the star be in the plane of the ecliptic, the ellipsis may be said to be infinite- 75 ly small in breadth; for it will become a strait line^a, wherein the star will appear to go, according to the order of the signs, one half of the year, and contrary to the order of the signs, during the other half: see fig. 76. 76

832 The greater the distance of the star *s* is from the earth the less will 73 *asc* and *asc* the angles of the cones be; and consequently the less will the ellipsis *abcd* appear: if the distance of the star be so great that the angles of the cones are too small to be measured by astronomical observation, the ellipsis *abcd* becomes as a point, and the annual parallax is said to be insensible; that is no change of place is discoverable in the star, throughout the year. It is the opinion of many astronomers that the stars appearing to us of different magnitudes is in a great measure owing to their being placed at different distances from us: if this be true, the parallax will be greatest, and consequently most easily to be found, in those stars which are of the greatest apparent magnitude.

833 I shall not enter into a detail of all the variety of changes which their aberrations in an elliptic curve, mentioned § 831, must cause in different stars, according as they are differently situated in the sphere of the heaven in respect of the ecliptic and equator; enough may be seen upon this subject in the author quoted in the margin^b: it is sufficient for our present purpose to remark, that such aberration of any star must change its longitude and latitude; now both these are found not directly and immediately, but by its declination and right ascension, as was said § 792: the declination of a star is found by taking its meridian altitude, the height of the equator being first known: the right ascension of a star is found by the time of its coming to the meridian, § 787^c. Thus we have pointed out to us two methods of enquiring into the annual aberration or parallax of the stars: one by observing whether or no any change can be discovered in the meridian altitudes of the same star at different times of the year; the other by examining whether the intervals of time between any two stars coming to the meridian are equal throughout the year: if there be any sensible change of declination in any of the stars, it must be greatest, and consequently most easily discovered, in those

^a § 258.

^b Eustachius Manfredius *de annuis inerrantium stellar. aberrationib.* Bononia 1729.

^c Both declination and right ascension of stars are perpetually changing, by the annual precession of the equinox, § 792: this change, though very small, must be taken into the account in our enquiry into the annual parallax of the fixt stars.

that are near the pole of the ecliptic: but the change of right ascension here enquired after must be greatest in stars in the solstitial colure, and nearest the pole of the equator: I shall treat of both these methods in their order.

834 *Galileo* seems to be the first who thought of trying whether any annual parallax of the stars were discoverable or not: to this purpose, supposing the stars to be at different distances from us, and those stars to be nearest to us that appear the biggest, he suggests, that by observing with a telescope two stars very near to each other, one of the greatest the other of the least magnitude, their apparent distance from one another might perhaps be found to vary, as they were viewed, at different times of the year, from the earth in different parts of her orbit^a. The same author proposes also a method whereby we may not only discover whether a star has any sensible annual parallax or not, but determine the quantity of it also, and consequently know the distance of the star from us^b: the method is this, upon the top of some very high building let a small strait bar be fixed parallel to the horizon and perpendicular to the meridian; recede from it towards the north till you can see the bar just cover or pass exactly through the center of some star when it is in the meridian; then upon a wall or some strong post well fixed place a mark exactly at the height of your eye; thus you will have in effect a very large instrument with sights placed in the meridian, sufficient to discover the least alteration in the meridian height of the star, by observing it from month to month, but chiefly when it is at its utmost north and south limits: but in taking meridian altitudes by this or any other instrument in order to discover the annual parallax, the stars observed must be such as pass our meridian not far from the zenith; otherwise the variable-ness of refraction will render the observations too uncertain for that purpose: and therefore *Halley* justly remarks that, whereas a difference of about 6 seconds in the meridian height of Sirius had been observed at Paris, no conclusion can from thence be drawn about the annual parallax; because at Paris the meridian height of Sirius is under 25° , at which height refraction will vary 7 or 8 seconds, which is more than the whole parallax supposed to be observed amounts to^c.

835 Another way to attempt the discovery of the annual parallax by the change of declination of the fixt stars is to view, through a telescope placed perpendicular to the horizon, some star in or near the solstitial colure that passes through the zenith or very near to it; if the parallax of the star be sensible, there will at different times of the year appear a difference in its me-

^a *Galilei Systema cosmic. dialog.* 3.

^b *idem ibid.*

^c *Phil. transf. n.* 346. abridg. vol. 6. p. 165.
ridian

meridian altitudes; and its altitudes at the two solstices will differ most from each other: thus a star that in june passes through the zenith of any place in north latitude will in december pass southward from the zenith, and a star that in december passes through the zenith will in june pass northward from it, as we shall see by and by^a: *Hook* was the inventor of this method, and in the year 1669 put it in practice at Gresham College with a telescope 36 feet long, and gave an account of the process in one of his Cutlerian lectures^b: his first observation was july 6, when he tells us the bright star in the head of draco marked by *Bayer* with the letter γ passed about $2' 12''$ northward from the zenith: the second was july 9, when that star passed at the same distance from the zenith as before: the 3d observation was august 6, when the star passed northward from the zenith about $2' 6''$: the fourth and last was october 21, when the transit of the star was $1'$ and 48 or 50 seconds north from the zenith.

836 In 1689 *Flamsteed* began the observations from whence he deduced the annual parallax, a detail whereof we have in a letter of his to Dr. *Wallis*^c: he there remarks that neither *Tycho* nor *Ricciolus* could discover any annual parallax^d, because their instruments with plain sights were liable to errors of a minute or more, as appears by many places in *Tycho's historia cælestis*: he describes his own instrument which had fixed to it a telescope of 7 feet, with cross hairs: he says that, by seven years continual observations, he found the pole-star nearer to the pole in december than in the months april, may, july, august, or september: and that its apparent distance from the pole was greater in april than in september, and greater in july and may than in april: he could not observe that star in june, because it then comes to the meridian near the same time the sun does; but concludes, from his observations in other months, that its apparent distance from the pole in june must be different from that in december about $40''$ or $45''$ ^e: we ought here to take notice that, though some writers have laid a great stress upon these observations, as if they were sufficient to demonstrate a sensible annual parallax, *Flamsteed* himself speaks of them with a good deal of diffidence; he says indeed that he could be sure of taking the height of the star so near as $5''$, provided he could be certain the divisions of his instrument were accurate enough;

^a See the remarks. ^b *v.* An attempt to prove the motion of the earth from observation, by *R. Hook*.

^c *inter Wallisii opera vol. 3.* ^d Neither *Tycho* nor *Ricciolus* thought of endeavouring to find the annual parallax of the stars, both of them believing the earth kept her situation in the center of the solar system.

^e The medium of this is about $42''$; from hence the annual parallax of a star at the pole of the ecliptic would come out about $47''$, supposing its distance from us to be the same with that of the pole-star. See the remarks.

but

but this he justly makes a doubt of^a, and recommends the use of much larger instruments, as necessary for the purpose: he owns that the wall upon which his mural arc was fixed sunk a little out of its true position; and that he could not be certain he was sufficiently exact in the allowance he made for the errors caused thereby: he adds that he thought he discovered an annual parallax in some of the southern stars also; but rightly remarks that the variableness of refraction might occasion all the differences he found in their meridian altitudes: he mentions also some observations of Sirius, from which the annual parallax of that star would come out near $30''$; but declares he will not answer for the exactness of those observations which were many of them made by his assistant.

837 Mr. *Whiston*^b, from the observations of *Hook* and *Flamsteed*, computes that the greatest annual parallax, or that which a star in the pole of the ecliptic would have, is $47''$; from whence he calculates the distance of the stars to be about 9000 semidiameters of the orbit of the earth, or 50000000000 miles: a prodigious distance this, but Mr. *Bradley* in a letter to Dr *Halley*^c tells him, he believes he may venture to say that in two of the stars observed by him the parallax does not amount to 2 seconds; and that he is of opinion if it had been so much as one second he should have discovered it, by the great number of observations made by him, especially of the star γ *draconis*: this, instead of 9000 times, makes the distance of the stars from us 400000 times as great as that of the sun; which is above 40 times as great as the distance calculated from *Flamsteed*'s observations.

838 In the year 1725 Mr. *Molyneux*, assisted by Mr. *Bradley*, with a telescope of above 24 feet placed perpendicularly at his house at Kew, began to observe the same star in Draco that *Hook* had done, in hopes of verifying what he had communicated to the public: but their observations came out so contrary to what they expected, as to be matter of great surprise to them, and give them reason to call in question the accuracy of *Hook*'s observations; since they must have been liable to an error of at least 30 seconds: but indeed, besides this, had they been more exact, they were too few to conclude any thing from them, in so nice an affair as the annual parallax.

839 The observations at Kew were as followeth: from december 3, 1725, the star did not sensibly change its distance from the zenith, at the

^a He says the subtense of an angle of $5''$ was in his instrument no more than the 600th part of an inch, and no instrument can be divided so nicely that we can be certain none of the divisions are the 600th part of an inch too great or too small.

^b *Whiston*'s astronomical lectures, lect. 4.

^c This letter, which I am now going to make use of, is published in the phil. trans. n. 406. and in the abridgment, vol. 6. p. 167.

time of its passing over the meridian for several days: december 17, it passed a little more southerly, and so continued gradually to pass more and more southerly at every *transit* over the meridian, till about the beginning of march, when it was found to pass $20''$ more southerly than at the time of the first observation, and seemed to have arrived at its utmost southern limit; because in several observations made about this time no sensible difference was found in its situation: by the middle of april it appeared to be returning back towards the north; and about the beginning of june it passed over the meridian at the same distance from the zenith as it had done in december when it was first observed. From that time the star appeared gradually more and more northerly at every transit, till september following, when it again became stationary, being then near $20''$ more northerly than in june, and no less than $30''$ more northerly than it was in march: from september the star returned towards the south, till it arrived in december to the same situation it was in at that time twelve months, allowing for the difference of declination on account of the precession of the equinox.

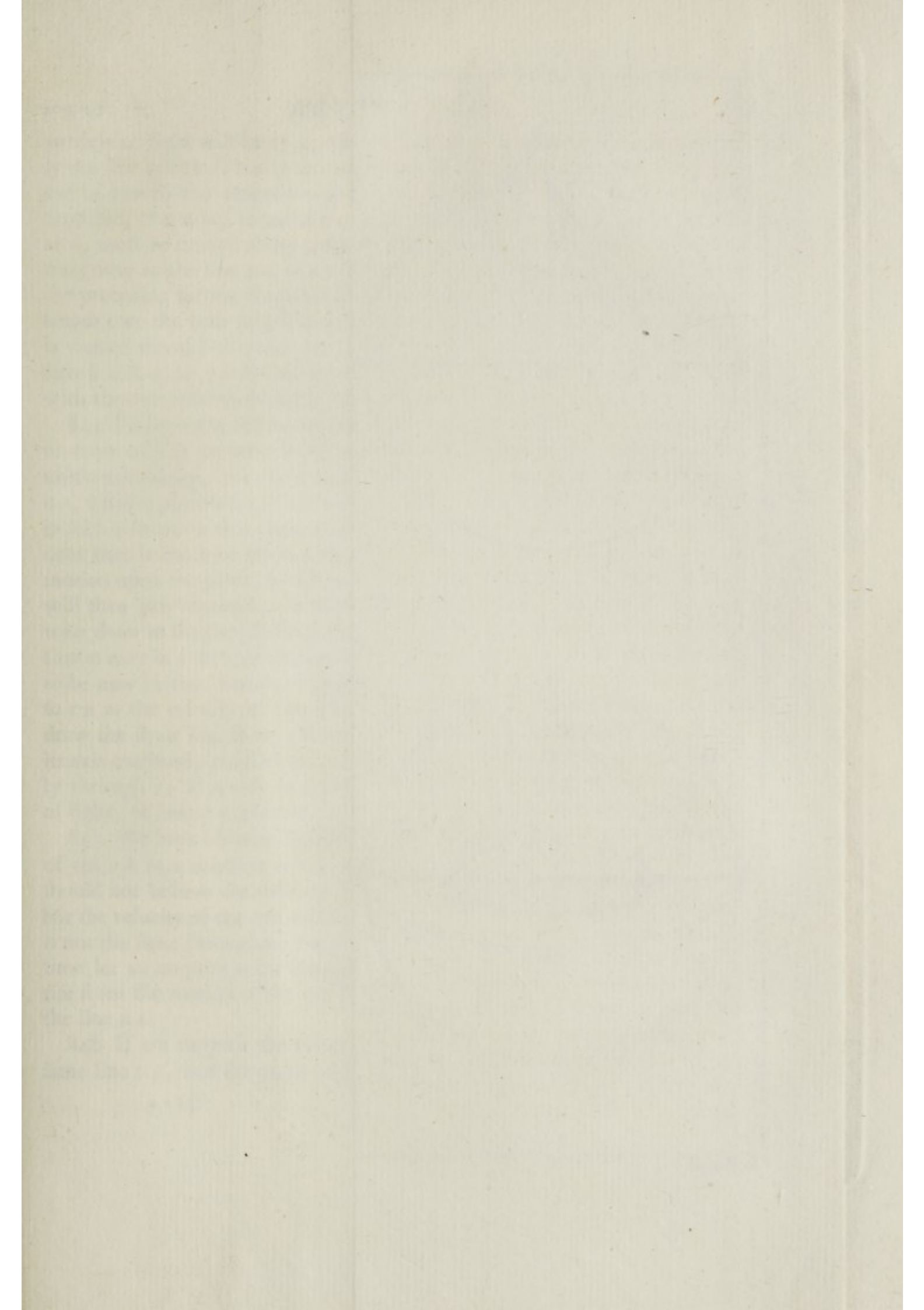
840 The like observations were made upon a small star almost opposite in right ascension to the star γ in the dragon and about the same distance from the north pole of the equator; this star did not change its declination above half so much as the star γ in the dragon did, in the same time; which was a sufficient proof that the apparent change of place in these stars was not owing to a *nutation of the earth's axis*; since, if that had been the cause, the alteration in both the stars would have been nearly equal, they both being situated near the solstitial colure.

841 In the year 1727 Mr. *Bradley*, with a telescope of $12\frac{1}{2}$ feet, began the like observations at Wanstead in Essex; he tells us, that, not having a place convenient for the use of so long a telescope as that of Mr. *Molyneux*, he contented himself with one of but little more than half that length, which by frequent trials he found sufficient for his purpose; since its situation may be securely depended upon to half a second, when it is carefully adjusted: that the instrument was suspended so as to enable him to observe six degrees and $\frac{1}{4}$ on each side his zenith, which gave him an opportunity of taking in several stars very different in magnitude and situation, and among others *Capella* a star of the first magnitude:

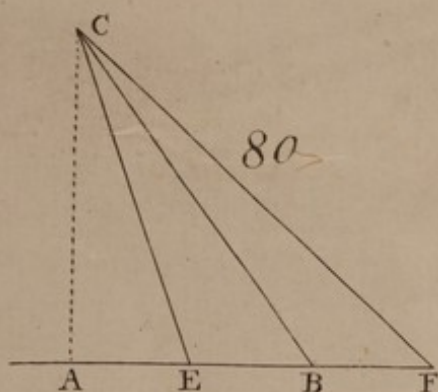
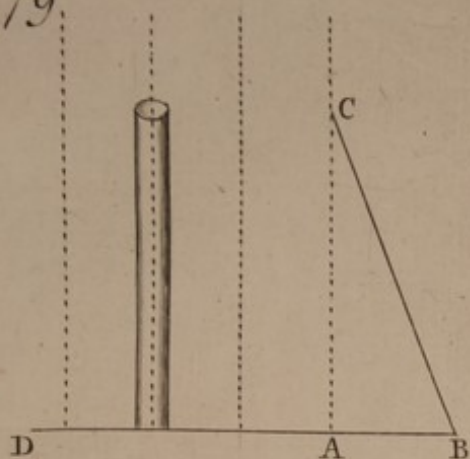
842 After a years observations *Bradley* conjectured this change of place in stars to arise from these two causes, 1 the progressive motion of light being in time, 2 the annual motion of the earth in her orbit: he considered this affair in the following manner: if the eye be at rest, or if it be in motion along a strait line directly towards an object, or directly from it, the appa-
rent

FIG. 77. rent place of the object will continue the same, whether light be propagated in time or instantaneously; but if the eye be in motion in any other direction than in a strait line from the object or towards it, and if light takes up any time in its progress from one place to another, the place of an object will be changed, and that change will be different according as those causes are differently combined: thus, fig. 77 let CA be a ray of light falling perpendicularly from a star at C upon the line BD ; if the eye be at rest at A , or moves along the line AC , in the direction AC or CA , the apparent place of the star will continue the same, for it will be seen by the visual ray AC , whether the progress of light be in time or instantaneous: but if the eye be carried in any other direction as BD , and light be propagated in time, with a certain velocity which we will, for example, suppose to be in such proportion to the velocity of the eye that a particle of light runs from C to A , in the same time that the eye is carried from B to A ; then that particle of light by which the star is seen by the eye when arrived at A must have been at C when the eye was at B : for joyn the points C and B , and imagin the line BC to be a tube of such a diameter as to let but one particle of light pass through it at a time (or if instead of so small a tube we take the axis of a larger the case will be the same) it is easy to conceive that the particle of light which is at C when the eye is at B and arrives at A at the same time with the eye will pass through the tube carried along with the velocity here supposed all the way parallel to it self, or inclined to BD in an angle equal to DBC ; and that it cannot pass through the tube thus carried if it be inclined to BD in any other angle: from hence it follows that, although the star C be perpendicular to the line BD in which the eye is moving, and would be seen in the direction AC if the progress of light were instantaneous, yet by reason of the motion of light being in time, its apparent place will be in the direction of the tube through which it is seen, so that when the eye is arrived at A the star will appear to be at H .

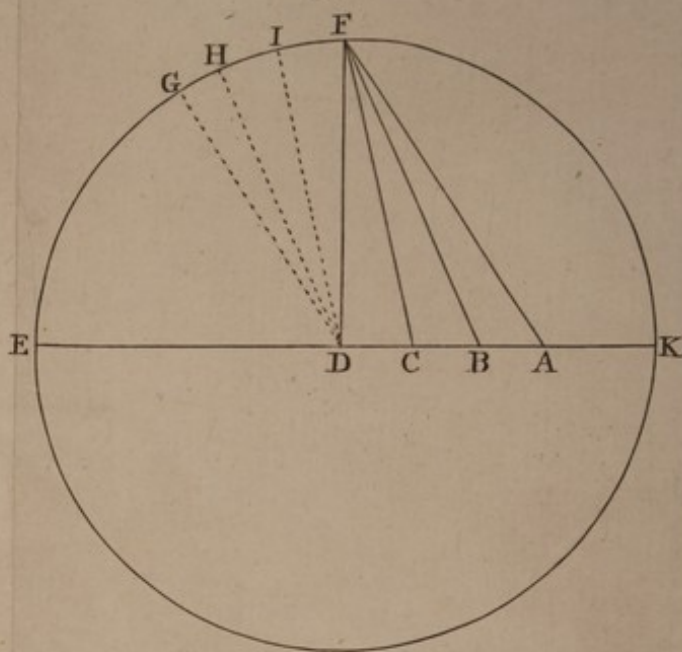
843 This may be farther illustrated, if we imagine the time to be divided into any number of equal parts, as 4, and the lines CA and BA to be each divided into the like number of equal parts, as in fig. 78: let us, for example, suppose light to move from C to A in 4 minutes, and the eye with the tube to be carried from B to A in the same time; it is manifest that at the end of the first minute the tube being carried into the situation EE , the particle of light, having run through one of the equal parts, will be in the tube at e ; at the end of the second minute the tube will be in the situation FF , and the particle of light at f ; at the end of the third minute the tube will be in the situation GG , the particle of light at g ; at the end of the fourth minute the particle



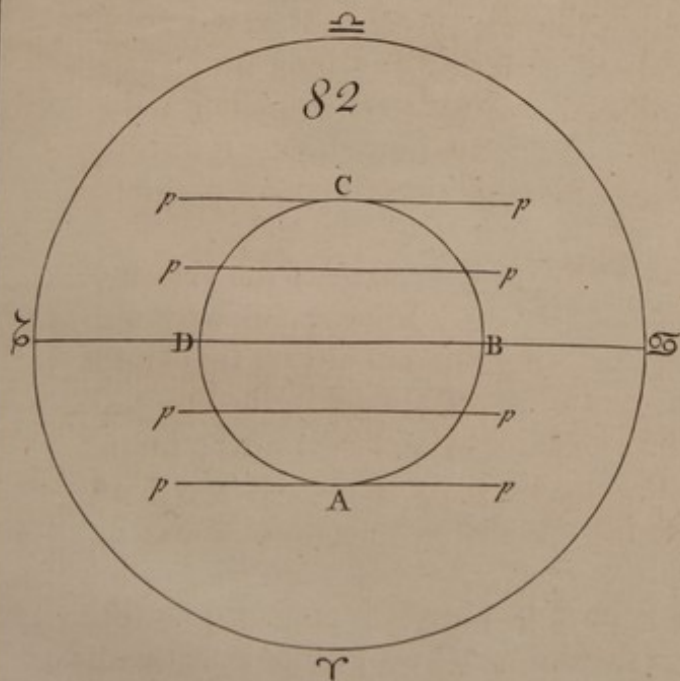
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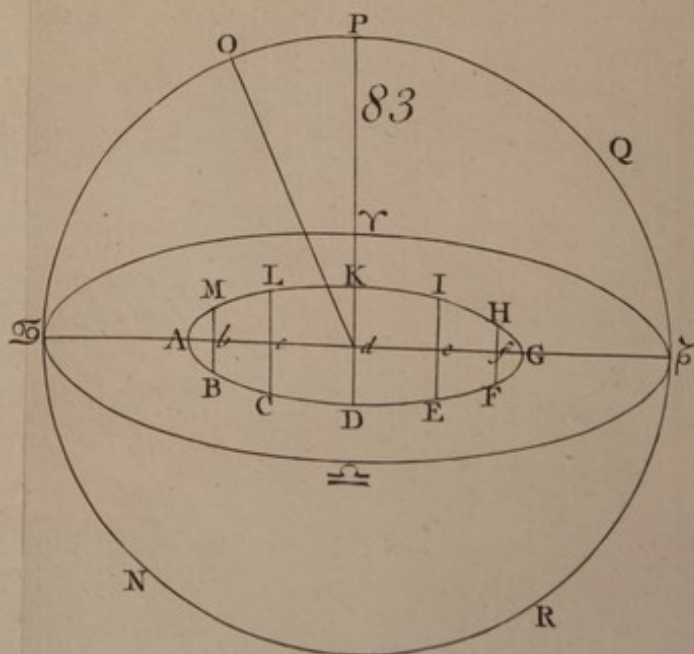
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82



83



particle of light will be at A, the tube in the situation AH; and consequent-
ly the star which is really at c will appear at H. It is easy to see that if the
eye be moved the contrary way from D towards A with the velocity before
supposed, the tube, to make the star c visible through it to the eye arrived
at A, must be carried along parallel to DC, or all the while inclined the con-
trary way to the line DA in an angle equal to BDC. — We have in this and
the preceding section supposed the star to be at c and the tube to be of the
length CB; the case will be the same if we suppose the star to be at s which
is viewed through the tube CB: for the particle of light by which the star is
seen is still to be considered as running through the line CA while the tube
with the eye looking through it is carried along the line BA.

844 Perhaps the following familiar instance may make the matter before
us more easy to be conceived: let us suppose drops of rain to fall, with an
uniform velocity, in a continual succession, and all in a direction parallel to
CA, upon a plane BD; if we would have a tube at rest upon the plane to be
in such a situation that some of the rain may pass strait through it, it is evi-
dent that it must be set in a direction parallel to CA: but if the tube be in
motion upon the plane, as if it be carried along the line BD, none of the drops
will then pass through it as before, because the sides of the tube will meet
with them as they are falling; except we give the tube such a different direc-
tion as may in a manner compensate for its motion: what that direction ought
to be may be thus found; suppose a drop at c is falling towards A, take AB
to CA as the velocity of the tube is to the velocity of the falling drop, and
draw the strait line from c to B; I say the tube must be carried along with
its axis constantly parallel to CB in order to have the falling drop pass direct-
ly through it; it is easy to apply this to the case of the temporary progress
of light: as before explained.

845 We have hitherto supposed the velocity of light to be to the velocity
of the eye in a constant ratio, or as CA to BA: there is no reason why we
should not believe the velocity of light to be uniform and always the same,
but the velocity of the eye which we here consider as moving in the line BA
is not the same throughout the year, as we shall see by and by^a: in the mean
time let us enquire what change of the apparent place of the star c will a-
rise from the motion of the eye being sometimes swifter sometimes slower in
the line BA.

846 If we suppose the velocity of light to be always represented by the
same line CA, and the mean velocity of the eye by BA, a greater velocity of

a § 852

FIG. the eye than the mean must be represented by a longer line than BA ; a less
77 velocity of the eye by a shorter line than BA .

847 If the line by which the velocity of the eye is express'd be longer than
80 BA , the angle of the tube's inclination through which the star C must be view-
ed by the eye arriv'd at A will be less than ABC : thus, fig. 80, if to express
the velocity of the eye instead of BA we take a longer line FA , the tube then
will be represented by the line FC , and the angle of its inclination will be
 AFC , less than ABC^a . On the other hand, if the velocity of the eye be ex-
press'd by a shorter line than BA , the angle of the inclination of the tube must
be greater than ABC : thus if instead of BA the velocity of the eye be express'd
by a shorter line as EA , the tube will be represented by EC , and the angle
of its inclination will be AEC , which is greater than ABC^b . It is all along
suppos'd here that the velocity of light is represented by CA .

81 848 If we imagin a great circle EFK , fig. 81, to be drawn in the sphere
of the heaven with its plane passing through the star F to be observed and
the line AD in which the eye is carried according to the order of the letters
 $ABCD$, we may call E , that point of the circle towards which the motion of
the eye is directed, *the point of the eye's direction*.

The less the angle of the tube's inclination is at any time, the more will
a star seen through it be depressed from its true situation towards the point of
81 the eye's direction: thus, fig. 81, let E be the point of the eye's direction
towards which the eye is carried in the line $ABCD$ according to the order of
the letters, F the star to be observed; if the tube be carried from A to D in-
clined to AD in the angle DAF , draw DG parallel to AF , and G will be
the place where the star appears through the tube, when the eye is arriv'd
at D : if the tube be inclined in a greater angle as DBF , draw DH parallel to
 BF , and H will be the star's place seen by the eye at D ; and the distance HE
is greater than GE : if the angle of the tube's inclination be still greater as
 DCF , the place of the star will be I , which is still farther distant from E .
This change of place is called *the aberration of the stars*^c.

849 From hence it follows, 1, that the aberration of a star always makes
it appear nearer to the point of the eye's direction than its true place: 2, that
the aberration of a star is greater or less towards any point in the heaven ac-
cording as the eye is carried towards that point with a greater or less veloci-
81 ty: 3, that if the motion of the eye towards any point as E be gradually ac-
celerated and retarded by turns, the aberration towards that point will be
gradually increased and diminished by turns: 4, that if the direction of the
eye's motion be contrary to the former, as from D to B the aberration will

^a Eucl. I. 1. prop. 16.

^b Eucl. *ibid.*

^c voyez mem. d'Acad. R. ann. 1737.

be also the contrary way towards the point κ which is diametrically opposite to ϵ , fig. 81. FIG. 81

850 I shall now apply what has been said in the foregoing sections to the motion of the eye of a spectator carried along with the earth in her annual revolution round the sun, having first laid down this *postulatum*, that the diameter of the earth's orbit measured in the sphere of the heaven is insensible, and therefore all parallel lines the distances of which from each other do not exceed that diameter may be look'd upon as physically coincident: thus, fig. 82, let $ABCD$ be the orbit of the earth, $\gamma \approx \approx \delta$ the ecliptic, all the lines pp parallel to BD are physically coincident with BD , and if continued both ways to the sphere of the heaven would terminate in the same points \approx and δ , as to sense, as the line BD does; this is true as far as can be found by observation, since no annual parallax is yet discovered. 82

851 First let us suppose that the star whose aberration in declination is to be observed is situated in the pole of the ecliptic; that pole is in the solstitial colure, and it is easy to see, by inspecting the celestial globe, that when any point of the solstitial colure is in any meridian the planes of that meridian and of the solstitial colure are coincident; the time of observing the declinations of stars is when they are in the meridian, the method is by their altitudes, § 833; for all change of meridian altitude found in any star is just so much change in the declination of that star: now the declination of a star in the solstitial colure will be changed in the same manner by the motion of the earth in her orbit as it would be if she were carried backward and forward with different velocities in the common section of the solstitial colure and the ecliptic, that is in the line $\approx \delta$ fig. 83: for let $NOPQR$ be the solstitial 83 colure, P the north pole of the ecliptic, O the pole of the equator, let $\gamma \approx \approx \delta$ be the ecliptic viewed obliquely, the line $\approx \delta$ is the common section of the solstitial colure and the ecliptic, let $ABCD$ &c be the orbit of the earth wherein she is carried round according to the order of the letters, her place being at A in december, at B in january, at C in february, &c: this motion, since it sometimes makes the eye approach nearer to the point δ , and sometimes carries it the contrary way towards the point \approx , will change the declination of any star in the solstitial colure, in the same manner as it would do if the earth were carried in the line $\approx \delta$, from A to G in the half year from december to june; and from G to A in the other half year from june to december.

852 The motion of the earth thus considered is unequable, some times 83 faster than other, as is shewn by the unequal parts into which the line AG is divided, each part answering to the same space of time, or one month:

FIG. thus, the motion of the earth from A to B in the first month, considered with
 83 regard to her approach towards the point \mathcal{B} , is reduced to the line Ab ; her motion from B to C in the second month is reduced to the line bc ; her motion the third month from C to D is reduced to the line cd , &c: now since the lines Ab , bc , cd , &c, which measure the earth's approach to the point \mathcal{B} , in each month, increase in length while the earth goes from A to D, or from december to march; and decrease in the same ratio as the earth goes from D to G, that is from march to june: it is manifest that the earth is carried towards the point \mathcal{B} , from december to march, with a velocity continually increasing; from march to june, with a velocity continually decreasing: in like manner, it is easily seen by the figure, that the earth is carried towards the opposite point \mathcal{A} , from G to f , from f to e , &c, with a velocity which continually increases from june to september; and decreases in the same ratio from september to december: as also that in june and december the earth is stationary in respect of the points \mathcal{A} and \mathcal{B} ; because when her place is at A or G the direction of her motion is not towards either of those points, but is at right angles to the line $\mathcal{A}\mathcal{B}$.

853 Though the annual motion of the earth be circular, we may at any moment of time consider it as being in the direction of a strait line which I shall here call *the earth's tangent*; that is a line drawn through the center of the earth, at that moment, tangent to her orbit; as the earth is continually going round in her orbit according to the order of the signs, the direction of this tangent is continually changing, so as to point successively round the ecliptic; and therefore the earth goes with her whole velocity towards all the points of the ecliptic successively: if we imagin a line to be drawn at any time through the center of the ecliptic parallel to and in the same direction with the earth's tangent, it will shew the point of the ecliptic wherein the earth's tangent terminates, and towards which the earth is carried with her whole
 84 velocity at that time: thus, fig. 84, let ABCD be the orbit of the earth wherein she is carried round according to the order of those letters, at the vernal equinox when the earth is at A she is carried with her whole velocity towards the point \mathcal{B} in the direction of the tangent AE parallel to and therefore, by § 850, coincident with $s\mathcal{B}$: at the autumnal eqinox when the earth is at c she goes with her whole velocity towards the point \mathcal{A} in the direction cF coincident with $s\mathcal{A}$.

854 Thus we see the earth is carried with the greatest velocity towards the
 84 point \mathcal{B} at the vernal, and towards \mathcal{A} at the autumnal equinox: 2, that in december when the earth is at D and in june when she is at B her motion is in a direction perpendicular to the line $\mathcal{A}\mathcal{B}$, and consequently she does not approach

approach to either of the points α or β , but, in respect of them, may be con- FIG.
sidered as stationary: 3, at the summer solstice when the earth is at B her 84
whole velocity is towards γ ; at the winter when at D it is towards ϵ .

855 From what has been premised it follows, 1, that from december to
march the star ρ appears to recede from the point α , and consequently 83
grows farther and farther from the intermediate point o , the pole of the e-
quator, and is farthest from that pole, or has least northern declination in
march: 2, from march to june the star approaches gradually to the point α
and consequently to the pole of the equator o , so that, its declination con-
tinually increasing during these three months in the same proportion as it de-
creased in the three months immediately preceding, the star comes to have
the same declination in june as in the foregoing december; allowance being
made for the precession of the equinox in that half year: 3, from june to
september the star, going now a contrary direction, recedes gradually from
the point β ; and in september is farthest from that point, and consequently
nearest to the pole o , or has greatest north declination: 4, from september
to december the star approaches to the point β , and recedes farther from the
pole o , so that, its declination continually decreasing in the same ratio as it
had before been increasing, the star appears to have the same declination a-
gain in december, as it had that time twelve months; allowing always for
the change of declination caused by the precession of the equinox.

856 Having seen what change of declination the motion of the earth and
the temporary progress of light would cause in a star situated in the pole of
the ecliptic, let us next consider how these causes would affect the declinati-
on of a star situated in any other point of the solstitial colure. If a star were
in the point α or β , which are in the plane of the ecliptic, the motion of 83
the earth, being considered as in the line $\alpha\beta$, is in a strait line either direct-
ly towards the star, or directly from it; and therefore will not cause any
change in the stars declination, as was said § 842.

857 Since the greatest change of declination would happen to a star in
the pole of the ecliptic, and no change to a star in the point α or β in
the plane of the ecliptic; it is easy to see, that the change of declination will
be greater or less in any intermediate star in the solstitial colure, according
as its true place is nearer to the pole of the ecliptic, or to the plane of the
ecliptic; the aberrations in declination of any two stars in the solstitial co-
lure are as the sines of their latitudes: therefore the aberration of any star in
the solstitial colure whose latitude is given being known, the aberration which
a star in the pole of the ecliptic would have may be found; and conversely,
the aberration in declination of a star in the pole of the ecliptic being known,
the

FIG. the aberration incident to any other star in the solstitial colure may be calculated, if its latitude be given: thus the greatest change of declination of γ *draconis* which is near the pole of the ecliptic was found by *Bradley* to be $39''$, whereas the greatest change of declination of a small star almost opposite to it in right ascension and situated about 60 degrees distant from the pole of the ecliptic, was no more than about $19''$. From observations made of the change in these and several other stars, *Bradley* settles the greatest change in declination which would happen to a star in the pole of the ecliptic at $40''.4$; for its aberration from its true place would be $20''.2$ towards γ at the vernal equinox, and $20''.2$ towards ω at the autumnal.

858 Hitherto we have taken notice of the aberration of those stars only that are in the solstitial colure; in order to see what is the aberration of other stars, we may consider the whole affair in another and more general view. The point of the ecliptic towards which the earth's tangent is directed is always 90 degrees behind the sun's place: thus, fig. 84, let $ABCD$ be the orbit of the earth, $\gamma \omega \approx \gamma$ the ecliptic; when the earth is at A the sun's geocentric place is γ , the earth's tangent is AE , which being parallel to and therefore coincident with $s\gamma$, is directed towards the point γ ; in like manner, when the earth is at B the sun's place is ω , the direction of the tangent BT parallel to $s\gamma$ is towards the point γ : when the earth is at C the sun's place is \approx , the direction of the tangent CF is towards the point ω , &c.

859 The aberration of stars caused by the temporary progress of light and annual motion of the earth is such, that a star in the pole of the ecliptic would at all times appear, nearer to that point of the ecliptic towards which the earth's tangent is directed than its true place is, and just so much nearer as the aberration amounts to; because the eye is always carried towards that point with the whole velocity of the earth's motion in her orbit, § 853: the earth's tangent is directed towards every point of the ecliptic successively according to the order of the signs, § 853: we may in this case look upon the orbit of the earth as a perfect circle, the excentricity thereof mentioned § 669 being so small that it need not be taken into this account; we may also suppose the velocity of the earth in her orbit to be equable, for the small inequability mentioned § 673 may here be disregarded. From these premises it follows, that a star in the pole of the ecliptic would annually describe a circle round it, appearing always at the distance of $20''.2$ from its true place: thus the longitude of a star in that situation would be continually changing, so as to go quite round the ecliptic according to the order of the signs in a year's time, but its latitude would be always the same; for it would be always $89^\circ 59' 39''.8$; wanting $20''.2$ of 90° , its true latitude.

860 From what has been said it is manifest, 1, that the aberration of a *Fig.* star in the pole of the ecliptic would be $20''2$ towards the point γ at the vernal, and $20''2$ towards the point ϖ at the autumnal equinox; so that its place at one equinox would be $40''4$ distant from its place at the other: and consequently its declination would be $40''4$ less in september than in march. 2, for the same reason, the aberration of a star in the pole of the ecliptic, would at the summer solstice, carry it $20''2$ nearer to the point γ than its true place; and as much nearer to the point ϖ , at the winter solstice: thus *fig. 85*, let $ABCD$ be the orbit of the earth, $\gamma \varpi \varpi \gamma$ the ecliptic, $F \varpi E \gamma$ the 85 solstitial colure; at the vernal equinox when the earth is at C , the aberration of a star at E the pole of the ecliptic would carry it $20''2$ from E nearer to the point γ , towards which the earth's tangent is then directed; but at the autumnal equinox, when the earth is at A , the aberration would carry the star $20''2$ nearer to the point ϖ , towards which the earth's tangent is then directed: in june when the earth is at D the aberration is towards γ ; in december when the earth is at B the aberration is towards ϖ .

861 The aberration of any *star not in the pole of the ecliptic* is in an ellipsis whose longest axis is equal to the diameter of the circle which a star in the pole of the ecliptic would appear to describe, and is at right angles to the circle of latitude or secondary of the ecliptic drawn through the star; the shortest axis is to the longest as the sine of the stars latitude is to radius: thus, the longest axis being parallel to the ecliptic and always the same, the shortest axis is less the less the latitude of the star is; so that the ellipsis of aberration of a star in the ecliptic becomes a small arc of the ecliptic of $40''4$, which seen from the earth would appear a strait line, and may be considered as such.

862 For since the aberration always carries a star nearer to that point in the heaven towards which the earth's tangent is directed, and the earth's tangent is always in the plane of the ecliptic, the aberration of a star in the ecliptic can only be in longitude. The greatest aberration of *a star in the plane of the ecliptic* is $20''2$ forward, when the earth's tangent points 90 degrees before the true place of the star; and $20''2$ backward, when the earth's tangent is directed towards the point in the ecliptic 90 degrees behind the star: thus, *fig. 84*, suppose a star's true place were in the point γ , when 84 the earth is at C and the earth's tangent points at ϖ , the aberration of the star will be $20''2$ forward, from γ towards ϖ ; when the earth is at A and the tangent points at γ , the aberration will be $20''2$ backward, from γ toward γ . A star in the plane of the ecliptic has no aberration when the point
towards

FIG. towards which the earth's tangent is directed is in quadrature with the star:
 84 thus when the earth is at A or C the star at γ would appear in its true place.

863 If we enquire how the aberration of a star stands with regard to its place compared with the place of the sun, we shall from § 858, 861, and 862 have the following corollaries: 1, a star in the plane of the ecliptic
 84 would appear in its true place when in quadrature with the sun: thus, fig. 84, if a star were in the point γ , when the earth is at B, the sun's place at \oplus ; the earth's tangent BT coincident with S γ is directed towards the star: when the earth is at D, the sun's place at \ominus , the tangent is directed towards the point \oplus , opposite to the star: in both these cases, by § 842, there is no aberration. 2, A star in the plane of the ecliptic, would have its greatest aberration forward, or according to the order of the signs, when in opposition to the sun; and its greatest aberration backward when in conjunction: thus,
 84 fig. 84, a star in the point γ , would have its greatest aberration from γ towards \oplus when the earth is at C, and the sun at \oplus ; but its greatest aberration from γ towards \ominus would be when the earth is at A, the sun's place at γ . 3, A star neither in the pole nor plane of the ecliptic, would appear in one of the extreme points of the shortest axis of its ellipsis of aberration, when in quadrature with the sun: and consequently it would be in its true longitude, but in its least or greatest latitude: in least latitude when the sun's place is 90 degrees forwarder than the point of the ecliptic which shews the longitude of the star; and in greatest latitude when the sun's place is 90 degrees
 85 behind the star's longitude: thus, fig. 85, if a star be at G and its longitude marked by the point \oplus , when the earth is at A and the sun's place is \oplus , 90° before \oplus , the star will have least latitude; for it will have its greatest aberration towards \oplus : but when the earth is at C and the sun's place is γ , 90° behind \oplus , the star will have the greatest latitude; for it will have its greatest aberration from \oplus towards the pole of the ecliptic. 4, A star neither in the pole nor plane of the ecliptic would appear in one of the extreme points of the longest axis of the ellipsis of aberration when in conjunction or opposition with the sun; and consequently would be in its true latitude, but in
 85 its greatest or least longitude: fig. 85, a star at G the longitude of which is in the point \oplus , will have its longitude most increased or carried forward, when the earth is at B and the sun's place in the point \ominus ; and most decreased, when the sun is in \oplus : the same star G would be in its true longitude when the sun's place is γ or \oplus .

864 If a star be in the solstitial colure, all change of latitude will be so much change of declination; that is the difference between its greatest and least declination will be equal to the shortest axis of the ellipsis of aberration:

on, see the remarks § 877: in any star not in the solstitial colure, the change FIG. of declination is equal to the distance between two circles drawn parallel to the equator, and tangents on each side to the ellipsis of aberration: thus, fig. 86, let EQ be the equator, $\varepsilon \zeta$ the ecliptic, N the pole of the equator, P the 86 pole of the ecliptic, AB the ellipsis of aberration of a star; draw the parallels CD and FG , the distance between those parallels, measured by the arc CF , is the difference between the greatest and least declination of the star.

865 We have seen what change in declination is caused by the aberration of stars; it is time to consider what *change in right ascension* may be expected from the same cause: if a star were in the pole of the equator, or so near it that the pole were surrounded by the ellipsis of aberration; it is easy to see that the star would appear to go round the pole, and consequently its right ascension would be varied through all the points of the equator, according to the order of the signs, in a years time: but we know of no star that is not more than $40''$ distant from the pole, and therefore too far off to surround it with its ellipsis of aberration; so that we may say *the aberration in right ascension* of any star is an arc of the equator which measures the angle made at the pole of the equator by two arcs of great circles drawn tangents on each side to the ellipsis of aberration of that star: thus, fig. 87, let PQ be the sol- 87 ftitial colure, $\gamma \varepsilon$ the ecliptic, $\gamma CD \varepsilon$ the equator, P the pole of the ecliptic, N the pole of the equator, AB the ellipsis of aberration of a star the true place whereof is in the center of that ellipsis, draw the arcs NC and ND touching the ellipsis in A and B , the arc CD is the whole aberration in right ascension of that star; for when the apparent place of the star is A the point of its right ascension is C : when the star appears at B its right ascension is D .

866 Of two stars in the solstitial colure that star will have the greatest aberration in right ascension which is nearest to the pole of the equator: for since the angle made by the tangent arcs is in both cases subtended by the longest axis of the ellipsis of aberration, which is always the same, $40''4$, the angle will be greater the nearer the subtending line is to the vertex of it, that is to the pole of the equator: thus, fig. 87, if the true place of a star be in the 87 center of the ellipsis AB , its aberration in right ascension is the arc CD , or the angle CND ; if a star be in the center of the ellipsis ab , its aberration in right ascension is the arc EF , less than CD ; or the angle ENF less than CND .

867 Of two stars equally distant from the pole of the equator that star will have the greatest aberration in right ascension which is nearest the solstitial colure: for, if a star be in the solstitial colure, the angle made at the pole of the equator by the tangent arcs is subtended by the longest axis of the

R r

ellipsis

FIG. ellipsis of aberration; but if a star be not in the solstitial colure, the angle made by the tangent arcs will be subtended by some other diameter of the ellipsis; and, the farther the star is from the solstitial colure, the shorter will be the diameter of the ellipsis which subtends the angle that measures the aberration in right ascension: thus, fig. 87, if a star were in the solstitial colure PQ in the center of the ellipsis AB , the angle ANB is subtended by AB the longest axis of the ellipsis: but if the star be in the center of the ellipsis rs , the angle of aberration in right ascension rNs will be subtended by some other diameter, which is shorter than the longest axis.

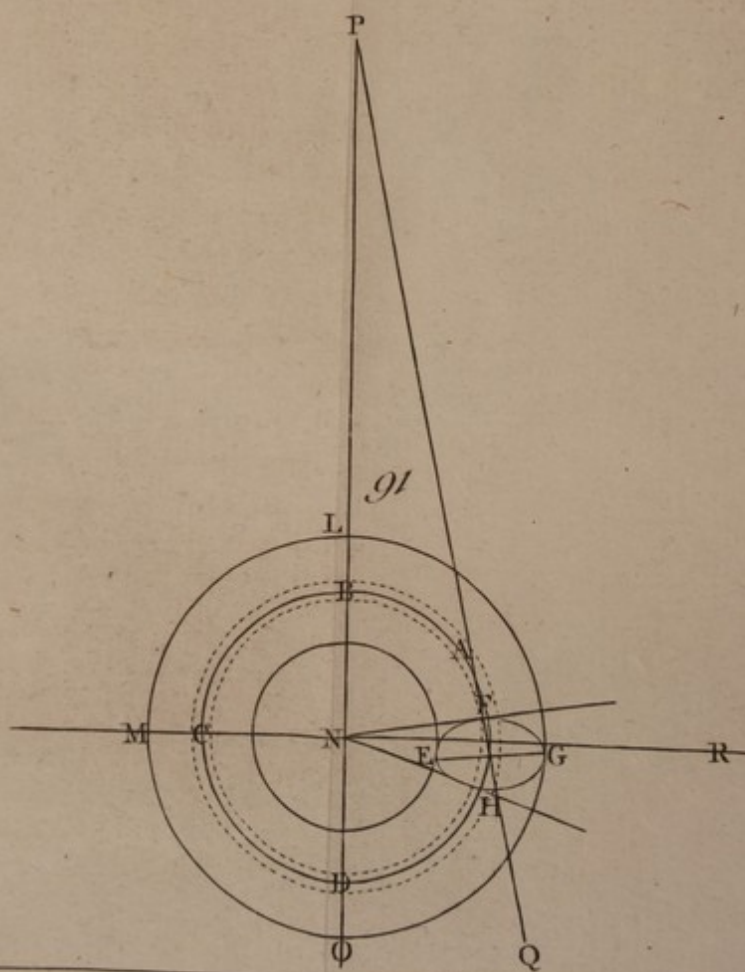
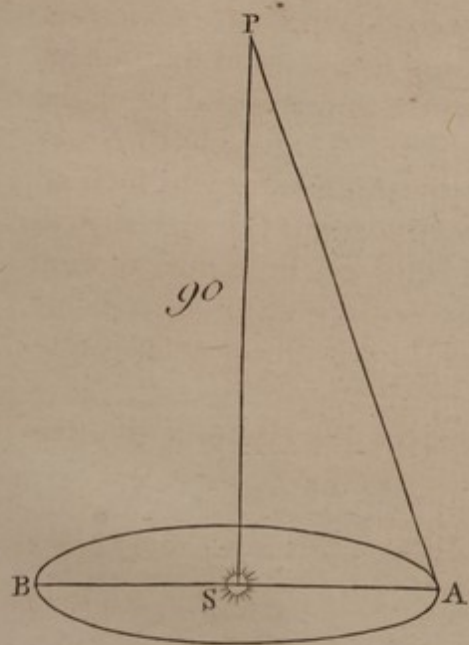
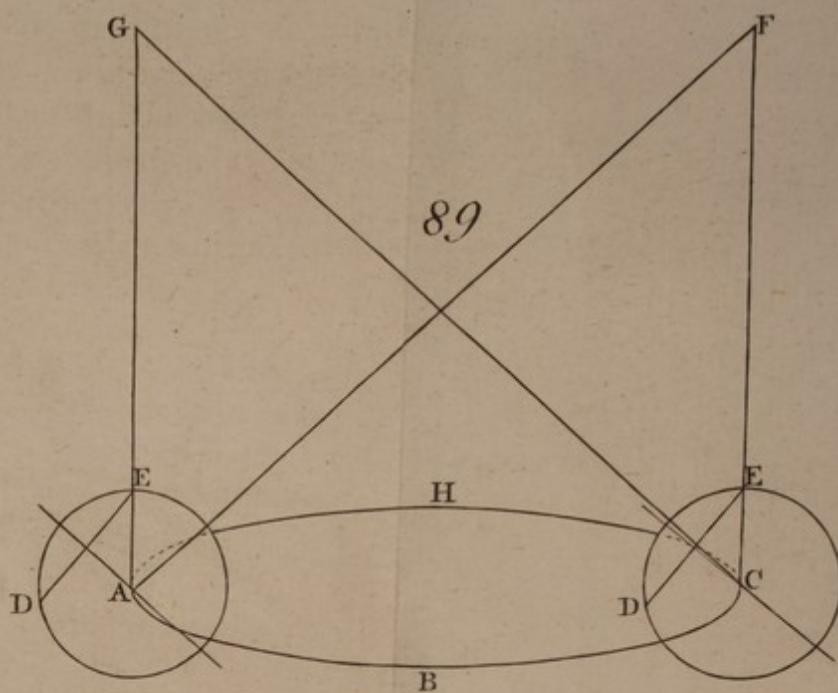
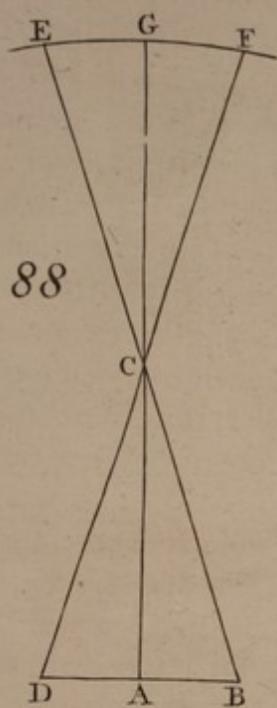
Scholium. Though, *ceteris paribus*, the aberration in right ascension is greatest in stars nearest the pole of the equator, yet may it more easily be observed in those that are at some distance from the pole: for stars that are very near it appear to go round in very small circles, and with a very slow motion, and therefore the time of their passing the meridian cannot be ascertained with exactness, without much longer telescopes than are required for stars near to the equator: the more the telescope magnifies, the more easily may the transit of a star be determined^a.

868 How to find the right ascension of a star, by comparing it with the sun, has been shewn, § 787; but the method there pointed out, though sufficient for the common uses of astronomy and navigation, will not serve in so nice an affair as the aberration of stars; for the right ascension of the sun, which is changing every day of the year, cannot be known for any particular time, within a few seconds; the theory of the sun's motion is not ascertained accurately enough for that, and if we were to attempt to deduce the sun's right ascension, by taking his meridian altitude every day, we could not do it with such exactness as the present case requires; as well by reason of the unavoidable small errors to which observations are liable, as because some of the elements of calculation are not determined to a sufficient degree of certainty. Nor can we, for this purpose, take some one star from which as from a fixed point we may begin to reckon the right ascension of other stars; because that star which we here suppose fixed will also have its aberrations, as well as those stars which we compare with it. The best method of finding the aberration of stars in right ascension is, to chuse two stars which differ considerably in right ascension, and observe whether the intervals of time between their transits cross the meridian are invariably the same throughout the year or not^b.

869 In order to make these observations, a clock must be had that goes as equably as possible; or rather two clocks, that they may correct one ano-

^a Manfredius *de annuis inerrantium stellarum observationib.* c. 9.

^b *idem ibid.*



ther: the intervals between the transits of the stars are to be turned into mean time^a: we should be well assured that the instrument with which we observe is always in the same plane at the times of observation; for this purpose, instead of one telescope moveable upon a mural arc, it would be better to have two, fixed to a strong old-built wall, in such a manner that one of them may point at the parallel of one of the stars to be observed, the other at the parallel of the other star; each telescope being furnished with cross hairs, one of which is to be perpendicular to the horizon. It is of no consequence whether the mural instrument or the telescopes be fixed precisely in the plane of the meridian or not, provided we be sure they continue all the time immoveable; since the intention here is not to find the true right ascension of either star, but to observe whether the difference of their right ascensions be constantly the same, throughout the year, or be different at different times of the year^b.

870 Observations of this sort have been made by several astronomers^c; by some with a view of finding out whether any annual parallax of the fixt stars could be discovered by a variation in their right ascensions, or not: thus they have observed with great care what number of seconds Lyra comes to the meridian after Capella; as also how many seconds Capella comes to the meridian after Lyra, throughout the year: the like observations have been made of the transits of Capella and Arcturus, of Arcturus and Aldebaran, of Arcturus and Sirius, and of several other pairs of stars: in all which there has been found an inequality in the intervals of time between their transits; and this inequality has been observed to have its regular periods of increase and decrease, which it went through in the space of a year; and that not in such order and degree as was to be expected from the annual parallax of the stars, but, in many instances, quite the contrary; and so agreeably to *Bradley's* hypothesis, in the greatest number of stars, that *Manfredi* owns such an agreement could by no means be looked upon as accidental^d: though that astronomer at the same time declares against the belief of the annual motion of the earth, and seems unwilling to admit of the progress of light being temporary, and is for suspending his judgment till some other causes can be thought of for the solution of these appearances.

We may now go on to shew in what manner, the aberration of the stars

a How to turn apparent into mean or sydereal time will be shewn hereafter.

b v. Manfredium loco citato.

c Romer, Cassini, Maraldi, Manfredi: v. Horrebovii *basin astronomia*, cap. 10, de parallaxi orbis annui: et Manfredii epistolam de novissimis circa fixorum siderum errores observationibus, inter Academicorum opuscula edita Bononia 1731.

d Manfredius in epistola novissime citata.

FIG. being known, the velocity of light may be found; having first premised the following lemma.

871 The velocity of light being to the velocity of the eye as CA to BA , § 77 842, requires the tube to be carried along always parallel to BC , or constantly inclined to BD in an angle equal to ABC , in order to have the star s seen through it by the eye arrived at A : the converse is true, that if the star s be seen by the eye at A , through the tube BC carried parallel to it self along with the eye from B to A constantly inclined in the angle ABC ; then the velocity of light is to the velocity of the eye as CA is to BA , fig. 77.

872 It has been shewn, § 857, that the greatest aberration in declination which a star that were situated in the pole of the ecliptic would have amounts 88 to $40''.4$; in fig. 88, let G be the true place of a star in the pole of the ecliptic, E its apparent place in march, and F in september; the arc EF or the angle ECF is the greatest difference of the star's declination, found to be $40''.4$, § 857: the angles BCD and ECF , being vertical, are equal; therefore ACB the half of BCD is $20''.2$: the angle BAC is a right one, here then is a right-angled triangle BAC with one acute angle known; and consequently, by § 141, the ratio of BA to CA , that is the ratio of the velocity of the eye to the velocity of light, may be known: now BA will be found to be to CA , as 1 to 10210, so that the velocity of light is to the velocity of the eye (which in this case is the same as the velocity of the earth in her orbit^a) as 10210 to 1: the earth runs through her orbit in $365^d 5^h 48' 57''$, or 31556937 seconds, which carries her through a length equal to the semidiameter of her orbit in 5022442 seconds; but light goes 10210 times as fast, and therefore passes through that semidiameter, or as far as from the sun to the earth, in 492 seconds, that is in 8 minutes and 12 seconds.

873 *Romer*, who first discovered the temporary progress of light, by apparent inequalities in the times of the returns of the eclipses of the satellits of jupiter, (which will be treated of hereafter) determined the velocity of light to be such that it passes from the sun to us in about 11 minutes; others, from the same eclipses, have concluded that it runs that length in about 7 minutes of time: the velocity of light here deduced from the aberrations of the stars is as it were a mean betwixt what has at different times been determined from the eclipses of jupiter's satellits.

874 From what has been said, *Bradley* draws these conclusions: 1, that these two different methods of finding the velocity of light thus agreeing in the result, is a confirmation of the truth of the hypothesis which accounts for the annual aberration of the stars, and inequalities in the times of the re-

^a § 853.

turns of the eclipses of the satellites of jupiter, by the temporary progress of light and the annual motion of the earth in her orbit: 2, that the light of all the stars arrives at the earth with the same velocity; for the annual aberration is the same in stars of the 5th or 6th as in stars of the 2d or 3d magnitude, which is a proof that the lights of them all enter the tube of the observer upon the earth with the same velocity: 3, that light is propagated from all the stars with an uniform velocity, so as to run through equal spaces in equal times, at all distances from the stars: this follows from the 2d conclusion, since the light from some stars when it arrives at the earth must have run through a vastly greater length than that from other stars, except we suppose all the stars to be at the same distance from us, which is highly improbable, on account of the difference in their apparent magnitude and brightness: 4, if the light of the sun be propagated with the same velocity as that of the stars (which we have no reason to doubt of) light moves with the same velocity after reflection as before it; since the light of the sun reflected from a satellite of jupiter traverses the orbit of the earth in the same space of time as the light does which is emitted directly from the stars.

875 *Bradley* having found, by comparing the observations of several stars, that they all conspired to prove that the *maximum*, or greatest aberration of a star in the pole of the ecliptic would be about 40 or 41 seconds; supposes it to be $40\frac{1}{2}$: this, from the agreement of his observations, he was induced to think could not err so much as a second from the truth, and that therefore the velocity of light is determined from these observations within 5 or 6 seconds; which is such a degree of exactness as we can never hope to attain from the eclipses of jupiter's satellites.

876 Having thus determined what the aberration of a star in the pole of the ecliptic would be, he calculated, upon that supposition, what change of declination the stars observed by him ought to have at several times of the year; and compared them with the changes which he observed them to have: and here he found such an agreement between the hypothesis and the observations, through all the parts of the year, as amounts to a kind of demonstration that the hypothesis gives at least the true law of the aberration of different stars; and, since it does this without allowing any thing at all for annual parallax of the fixt stars, he concludes that parallax must be much smaller than hath been hitherto supposed by those who have pretended to deduce it from their observations: and says he is of opinion he should have discovered it, if it amounted to so much as a single second.

Remarks

FIG.

Remarks upon § 835.

877 If the annual parallax of the fixt stars were discoverable by any change of their declination, that change would be greatest, and consequently most easy to be observed, in those stars that are nearest the pole of the ecliptic. If a star be in the solstitial colure, any change in declination is just so much change in latitude; and the converse is true: but the change of declination and latitude, though it be of the same quantity, is not always the same way, but in some stars it is the contrary way, as will easily be seen by the 86 figure; where ENQ is the solstitial colure, $\ominus \mathcal{L}$ the ecliptic, EQ the equator, P the north pole of the ecliptic, N the north pole of the equator: 1, if a star be in the arc PQ between the pole of the ecliptic and the nearest point of the equator, all increase of declination is so much increase of latitude, and all decrease of declination is so much decrease of latitude; and conversely: thus, suppose the true place of a star be D , if at any time its apparent distance from Q the nearest point of the equator were to be increased any number of seconds, as for example if the apparent place of the star were to be at H , its apparent distance from \mathcal{L} the nearest point of the ecliptic must at the same time be increased the same number of seconds, or the same arc DH : in like manner, if the star D were ever to appear nearer to Q , it must at the same time appear nearer to \mathcal{L} : 2, what I have now said is also true of any star that is situated in the arc $N\ominus$, between the pole of the equator and the nearest point of the ecliptic, that all increase or diminution of declination is so much increase or diminution of latitude; and conversely: 3, but if the place of a star be in the arc PN , between the pole of the ecliptic and the pole of the equator, all increase of declination is so much decrease of latitude; and all decrease of declination so much increase of latitude; and conversely: 4, this last proposition is also true of stars situated in the arcs $Q\mathcal{L}$ and $E\ominus$; but they are too far from the pole of the ecliptic to fall under our present consideration.

878 If any star had a sensible annual parallax, its parallactic ellipsis would have the same shape and position, whether it were of the same magnitude or not, as its ellipsis of aberration; but there would be the distance of three months between the time when the star appears in any given point of its ellipsis of aberration, and the time when it would appear in a similar point of its parallactic ellipsis: thus, whereas a star neither in the pole nor plane of the ecliptic would be in one of the extreme points of the shortest axis of its ellipsis of aberration, and consequently have the greatest or least latitude, when in quadrature with the sun, § 863; the same star would appear in that extreme of the shortest axis of its parallactic ellipsis which is nearest the ecliptic

cliptic, and have least latitude, when in conjunction with the sun: and in FIG. the opposite extreme, and have greatest latitude, when in opposition to the sun. Since the shortest axis of the parallaëtic ellipsis of a star in the solstitial colure coincides with that colure, the times of the greatest and least latitude of such star are the two solstices; and these are also the times of its greatest and least declination by, § 877.

879 In fig. 89, let ABCD be the orbit of the earth, A the place of the earth 89 in december, c in june; a star at F that in june is seen in the line cF, and passes through the zenith of any place situated in the parallel DE, will, in december when the earth is at A, be seen in the line AF, passing southwards from G which is then the zenith, to the distance of GF: in like manner, a star at G that passes through the zenith of a place in the parallel DE in december, when the earth is at A, will pass northwards from the zenith in june, being then seen in the line cG.

Remarks upon § 836, and 837.

880 If the parallax in latitude of a star whose distance from the pole of the ecliptic is given were found, the parallax that a star at the same distance from us in the pole of the ecliptic would have might be known: for the parallaxes in latitude of two such stars are as the sines of their latitudes.

881 The parallax of a star in the pole of the ecliptic being given, *the distance of such a star from the earth* may be thus found: in fig. 90, let AB be 90 the orbit of the earth, s the sun, P a star in the pole of the ecliptic, draw PS and PA, in the triangle PSA, the angle at s is a right one by the construction, and therefore, if the parallaëtic angle SPA be known, the proportion of As the semidiameter of the orbit of the earth to Ps the distance of the star P from the sun may be known by § 143.

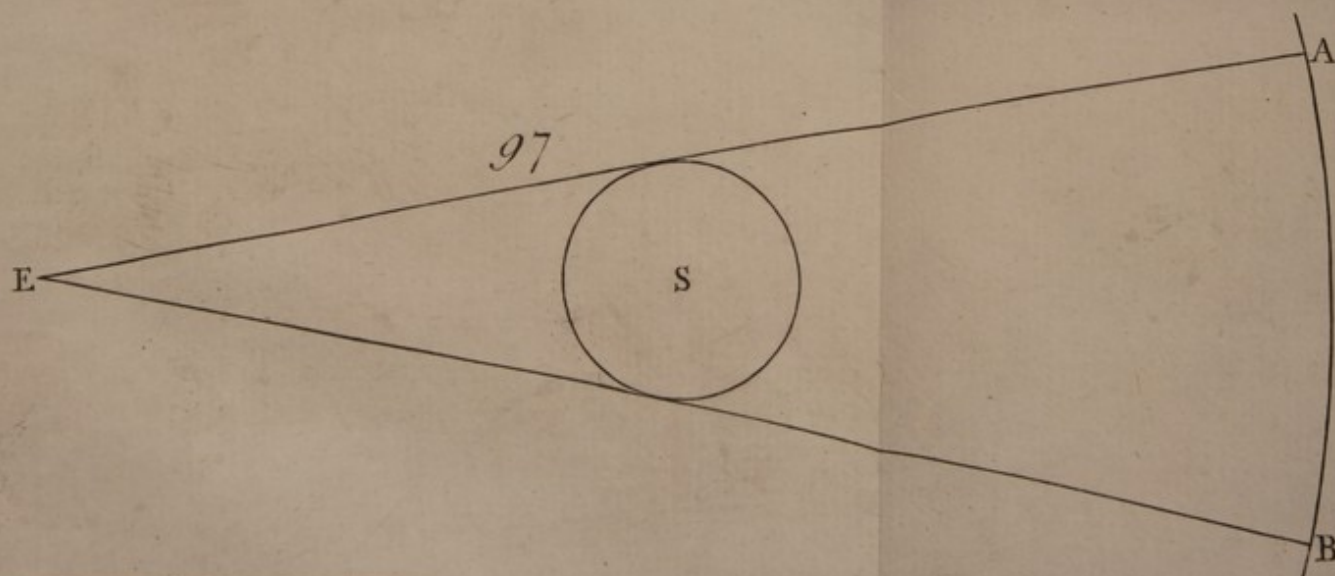
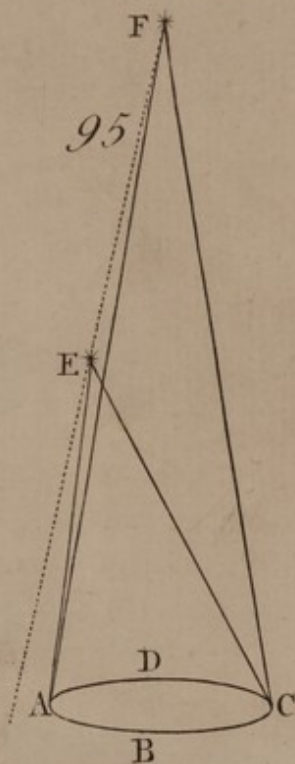
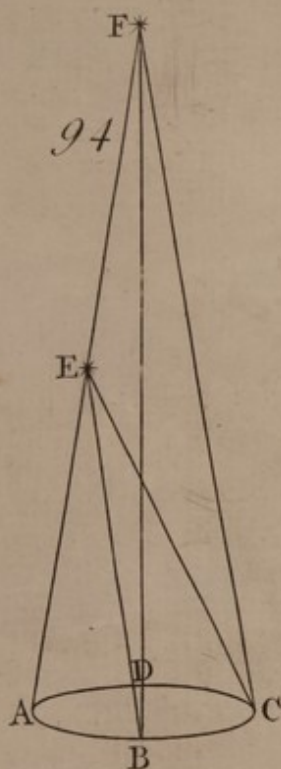
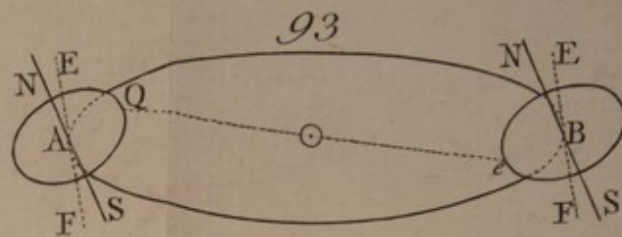
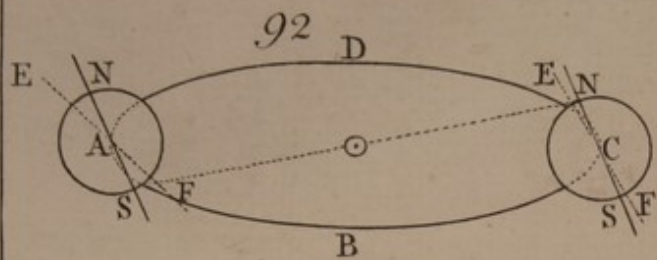
882 It is easy to conceive that any change of latitude or longitude in a star arising from its appearing to go round in a parallaëtic ellipsis would cause a change in declination, or right ascension, or in both; and that in a greater or less degree, according as the axes of that ellipsis are situated in respect of the equator: 1, if the shortest axis, the extrem points whereof are the places of greatest and least latitude, coincides with a secondary of the equator, which is the case only of stars in the solstitial colure; the times of greatest and least declination are the solstices, as has been shewn § 878: 2, if the longest axis is nearer than the shortest axis to coincidence with a secondary of the equator, which is the case of stars near the pole of the equator and nearer to the equinoctial colure than to the solstitial; the times of greatest and least declination will be nearer to the equinoxes than to the solstices: but the
times

FIG. times of greatest and least right ascension would be nearer to the solstices than to the equinoxes. The pole star is very near the equinoctial colure; it was no more than $21'$ distant from that colure, and about $2^\circ 26'$ from the solstitial, when *Flamsteed* observed it: he found its declination in december about $40''$ greater than its declination in june; such a change agrees as well as could be expected with the observations and theory of *Bradley*, but hath by mistake^a been thought to proceed from an annual parallax: had that been the cause, the greatest difference of declination would have been found at the two equinoxes, as may be seen by fig. 91; where let MR be the equinoctial colure, PO the solstitial, P the pole of the ecliptic, N the pole of the equator, EFGH the parallactic ellipsis of the pole star, FH the shortest axis, coincident with PQ a secondary of the ecliptic passing through the star; when the sun is in conjunction with the star, which is a little before the summer solstice, the stars apparent place is at H, in least latitude; when the sun is in opposition to the star, which is a little before the winter solstice, the stars place is at F, in greatest latitude: but the position of the line FH is such, that it is nearly a tangent to the circle ABCD the parallel of the stars diurnal revolution; so that, whatever be the change of the stars latitude measured by the line FH, its change of declination measured by the distance between the two pointed parallels will be much less. If any sensible change of declination were caused in the pole star by annual parallax, it would be most easily discovered about the equinoxes; for near those times that star is in one of the extreme points of the line EG the longest axis of its parallactic ellipsis, which axis is nearly coincident with the equinoctial colure: thus the star is near the point G in least declination a little before the vernal equinox; and near the point E, in greatest declination, a little before the autumnal: if we imagin a parallel GLMO to be drawn tangent to the parallactic ellipsis near the point G, and another parallel to be drawn tangent to the ellipsis near the point E; the distance between these two parallels is the difference between the stars declination at one equinox and its declination at the other: but this difference is too small to be sensible. The greatest parallax of the pole star, at the solstices is in right ascension; this in fig. 91 is measured by the angle FNH: the parallax in right ascension of the pole star is to its parallax in latitude, very nearly as the arc of the equator which measures the angle FNH is to the arc FH: or, as radius to the sine of the star's distance from the pole: but this parallax has not yet been observed to be sensible by any astronomer.

^a This mistake was taken notice of by *Romer*, in a letter to *Flamsteed*: *ap. Horrebovium in basi astron.* c. 10: and by *Cassini*, *mem. d'acad. ann.* 1699.

Book II.

67



883 The learned *Gregory* remarks^a, that *Flamsteed's* method of determi- FIG.
ning the annual parallax of the pole star, by its distance from the pole being
greater near the summer than it was near the winter solstice, supposes the
axis of the earth to be parallel to it self at both those times; which it would
not be, if the southern hemisphere of the earth should happen to be more
dense than the northern: for in that case, the south pole, being nearer to the
sun in december than in june, would be more attracted by him; and this at-
traction of the sun would cause such an alteration in the position of the earth's
axis, at the two solstices, as would make stars in north declination and north
latitude appear at those times to change their declination the same way as
they ought to do from annual parallax: thus, fig. 92, let *ABCD* be the orbit 92
of the earth, *A* and *C* the earth; if the south pole *s* be heavier than the north
pole *N*, it is nearer to the sun, and consequently more attracted towards him
in the direction *s* ☉, in december, when the earth is at *A*; than in june,
when the earth is at *C*: and consequently, at the winter solstice, the axis of
the earth will incline from the situation *NS* towards the situation *EF*, more
than it does at the summer solstice. Now to this, besides what has been said
by *Whiston*^b, it may be answered, that such a nutation or dipping of the north
pole as is here supposed would cause a like change in all the stars that are in
or near the solstitial colure; which is contrary to *Bradley's* observations.

884 There is indeed another nutation of the earth's axis, demonstrated *a*
priori, from the earth being an oblate spheroid, as was mentioned § 805;
but this nutation is the same way at both solstices: thus fig. 93, in december 93
when the earth is at *A*, the protuberant parts about the equator, which may
be considered as a ring round the earth, are most attracted where they are
nearest to the sun, as at *Q*, and that in the direction *Q* ☉; in june, when the
earth is at *B*, they are nearest the sun at *e*, and most attracted in the directi-
on *e* ☉; but both these attractions tend to change the situation of the axis
the same way, from *NS* to *EF*: this nutation *Whiston* has calculated to be
not above 42''', and consequently too little to fall under the nicest observa-
tion^c.

885 There are several stars, as the first in the head of the ram, the head
of Castor, the middle one in the sword of Orion, the star in the breast of
Virgo marked by *Bayer* with the letter γ , &c. which, appear single to the
bare eye, but are by the telescope discovered to be double; and some authors^d

a astronom. book 3, prop. 55. b astronom. lect. 4. p. 12. mathemat. lect. 21. c astron. lect. 4.
d *Gregory* in his astronomy book 3 prop. 54, says *Cassini* discovered that the first of aries and some other
stars appear sometimes divided into two stars; this expression implies that at other times they appear single,
but *Cassini* says no such thing: the truth is, these stars appear single and perhaps a little oblong, through a
short telescope, but a long one shews them double.

FIG. tell us that the same star appears single at one time and double at another: if this last assertion be true of any star seen through the same telescope, it must be owing to one of these three causes, either 1, one or both the stars so viewed change their situations, which is not very probable^a: or 2, one of them has one side much darker than the other, so as to be visible or invisible to us at different times, according as the rotation of that star round its axis turns its bright or dark side towards us^b: or 3, the diameter of the earth's orbit is not quite insensible, in comparison of the distance of those stars from us; but causes an annual parallax in them: if this last were the case, there would be stated times of such stars appearing single or double, according as the earth is in such or such part of her orbit; but I do not find this asserted by any astronomer; had such an observation ever been made, it would decide in favour of the annual parallax; upon this consideration, I was myself induced to observe all the double stars here mentioned, at different times of the year, through two very good telescopes, one of 14 and another of 17 feet: I have not yet repeated my observations often enough to determine positively in the affair, but from those I have made I am persuaded they will be found to appear always the same, without any sensible change of distance or situation: this will not seem strange, after what has been said about the smallness of the annual parallax, § 876.

886 How the annual parallax would be discovered by a star appearing
 94 single at one time of the year and double at another, may be seen by fig. 94; where let $ABCD$ be the orbit of the earth, E and F two stars at considerably different distances from the solar system; if a line EF drawn through them and continued touch the orbit of the earth in any point as A , when the earth is in that point they will appear as one star: when the earth is in any other point as B , they must be seen by two visual lines BE and BF ; and their apparent distance is measured by the angle EBF : when the earth is in the point C opposite to A , their apparent distance measured by the angle ECF is the greatest possible; and then, if ever, they will appear as two stars: if the distance of the nearest of these stars E from the solar system be so great in comparison of the diameter of the orbit of the earth that the angle ECF becomes insensible, there will be the appearance but of one single star throughout the
 95 year. By the same way of reasoning, fig. 95, when a line continued through

^a See § 796.

^b The stars seem to be of the same nature with the sun: the sun, as we shall see hereafter, has sometimes a considerable number of dark spots upon his surface; the stars may have some of them: and some stars may be so crusted over with spots as to be rendered wholly or in part dark and invisible to us: the solar spots discover the sun to have a rotation round his axis; the stars may, some of them at least, have a like rotation.

two such stars E and F as are at a great distance from each other as well as FIG. from the solar system does not touch the orbit of the earth, but comes near 95 some point thereof A, if the distance of the nearest star E from the solar system be so great in comparison of the diameter of the earth's orbit that the difference between the angles FAE and FCE, even magnified by the telescope, is insensible, though they appear through it as two stars very near together, there will be no sensible variation in the apparent distance between them, throughout the year; and to the eye unassisted they may always have the appearance of a single star.

887 The middle star in the sword of Orion is found to be a cluster of three stars, which form a triangle having two of its sides nearly equal, and the angle between those sides a little obtuse; the star at the obtuse angle is much larger than either of the other two, which are nearly equal; these three stars, together with four others adjacent, are surrounded by a faint hazy light like that of a bright whitish cloud; this light is brightest near the stars, and grows gradually more faint and diluted at a distance from them, till it becomes insensible. *Hugens* who first discovered this wonderful appearance, as he justly calls it, has given us a draught of it, but the stars are all drawn nearly of equal magnitude, and the luminous space is more defined than it ought to be; which faults were in all likelihood owing to the mistakes of the engraver: I have therefore, in fig. 96, given another scheme of it, such as I have 96 often seen it through a telescope of 17 feet, and have expressed therein the apparent magnitudes of the several stars.

I shall conclude this chapter, when I have told the reader that a general account of Mr. *Bradley's* instrument with a figure of it may be seen in *Smith's* optics vol. 2 p. 454; and that a more particular description of a like instrument, made portable for the use of the members of the R. Academy at Paris who went in the year 1736 to measure a degree upon the meridian within the N. polar circle, may be found in a book intitled *degre du meridiën entre Paris et Amiens determine &c.* 8^{vo} Paris 1740.

CHAP. 19. OF THE DISTANCE AND MAGNITUDE OF THE FIXT STARS.

888 The distances of the stars from us and their magnitudes are very proper to be considered together, because the knowledge of either of these will help us to determine the other: for, supposing the apparent magnitude of an object, or the angle under which it appears to be known, geometry teaches us, if the distance of the object be also known, how to find its real magnitude;

S f 2

or,

FIG. or, if its real magnitude be known, how to find its distance from us. How the distance of a star would be found, if its annual parallax could be ascertained, has been shewn in the chapter immediately preceding^a: we have there seen that the distance of the stars comes out from *Bradley's* observations to be at least 400000 times as great as the distance of the sun from us^b; an amazing distance this, and beyond the power of the imagination to reach: perhaps we may have a better idea of its vastness, if we measure it by the velocity of some moving body, as *Hesiod* does the height of heaven, when he tells us that an iron anvil would be nine days in falling from heaven to earth, and would not reach the earth till the tenth day^c. A cannon ball has been found to go about 100 french toises in a second, which is a little more than 7 english miles and $\frac{1}{4}$ in a minute^d: found, according to many accurate experiments of Mr. *Derham*, goes at the rate of an english mile in 9 half seconds and $\frac{1}{4}$, or 13 miles in a minute^e: the distance of the sun from the earth, supposing his parallax $10''$ as before settled, is 10300 diameters of the earth, or 82 millions of english miles; so that a cannon ball going with the velocity above mentioned would be above 21 years in passing from our earth to the sun: found would go this length in 12 years: light is found to run from the sun to our earth in a little more than 8 minures. If the distance of a star from the earth be 400000 times as great as the distance of the sun, its light must take up above six years in coming to us: found would be 4800000 years, and a cannon ball 8600000 in going through that space.

889 This distance of the stars is concluded from a supposition that the parallax is such, that the whole annual parallax of a star in the pole of the ecliptic and at the same distance from us with those observed by *Bradley*
90 would be one second; in which case the angle SPA , fig. 90, would be half a second: that accurate astronomer does not say that he found the parallax of any of those stars so much as one second, but that he thinks it cannot be more than that: from whence we may reasonably conclude, that the distance of the stars from us is at least as great as was just now mentioned: we are not sure that it is not much greater, since, for aught appears to the contrary, the parallax of the stars may be much less than a second. The stars observed by *Bradley* were some of them of the 2d and 3d magnitude; how

a § 880 and 881. b § 837. c in *Theogonia* v. 722. *Hesiod* here gives us a much more grand idea of the height of heaven than *Homer*, who makes *Vulcan* fall from thence to earth in a day; *Iliad* A. v. 592: but perhaps those poets had different views; *Hesiod* had a mind to represent the great height of heaven; *Homer* to shew the swiftness of *Vulcan's* fall, when thrown down from thence by *Jupiter*. *Milton* copies after *Hesiod*, in describing the fall of the rebellious angels;

Nine days they fell——— *Paradise lost* book 6. v. 871.

d *Hugenii cosmothecor.* l. 2.

e *Jones Phil. trans. abr.* vol. 4. pag. 396.

much greater then, in all probability, is the distance of those of the 5th and 6th magnitude? and how much greater still is that of those telescopic stars which cannot be seen without very long telescopes, and of numberless others which we may very reasonably believe to be so far off as to be out of the reach of any telescopes that can ever be made? We cannot indeed conclude that the stars are all of equal dimensions, and that all the difference in their apparent magnitudes is owing to their being at different distances from us; nor have we any reason to believe they are all at the same distance from us, and that those which appear largest are all proportionably of greater real magnitude: it seems most probable, that, though they are of various sizes, and scattered through the vast expanse of space at various distances from us and from one another; yet many of them are nearly of the same size, so that, in general, and as to a great number of them, the reason why some appear less than others is because they are placed at a greater distance from us. From the brightness of the stars, the exceeding smallness of their diameters when divested of spurious rays by the telescope, and their vastly great distance from the sun and from the earth, we may certainly conclude, that their light is not received from the sun and reflected to us, as that of the planets is; but they are self-shining bodies like our sun: they are placed so far from one another, that if every star be, like the sun, surrounded with a number of planets and comets, there is no reason to believe that one of these systems does disturb or otherwise affect any of the rest; and particularly their immense distance from the solar system makes it utterly improbable that they should have any kind of influence upon our earth, or any of the planets.

890. Some astronomers, despairing to discover the annual parallax, have thought of other methods of finding out the distance of the stars; which, though they may be looked upon as conjectural, deserve to be mentioned, for the ingeniousness of their contrivance. The apparent diameters of the sun and planets are easily measured with sufficient exactness, the distance of the sun and of any of the planets from our earth may at any time be computed, near enough to the truth for the purpose now before us; from these *data* the distance of a star may be found, supposing it equal to the sun in its real dimensions, and that its apparent diameter is equal, or in a known ratio to one of the planets: thus *James Gregory*, by supposing *Sirius* to be of the same bigness with the sun, and to have the same apparent diameter with *Jupiter* in opposition, investigated the distance of that star; as may be seen at large in *Gregory's astronomy*, book 3, prop. 57 &c. The learned *Hugens* tells us that he found it so difficult to measure the apparent diameters of the fixt stars, by reason of their smallness, in as much as they appear even through
the

the telescope but as lucid points without any sensible magnitude, when the glare of the spurious rays that surround them is taken away; that he was induced to try some way of diminishing the sun's apparent diameter, till it was reduced to the smallness of a star: in order to this, he covered one end of a twelve foot empty tube with a very thin plate of brass, wherein he had pierced a very small hole not above the 140th part of an inch in diameter, and, with his eye at the other end of the tube, looked at the sun, which by being thus viewed was diminished to an 182d part of his whole diameter, but still appeared much brighter than *Sirius* does by night: in order therefore to diminish still farther the sun's apparent diameter, he took from a microscope a small globule of glass of about the same bigness with the hole, and placed it therein; this, according to calculation, reduced the sun's diameter to an 152d part of that 182d part which was seen before through the bare hole, or to a 27664th part of his whole apparent diameter; and yet this particle of the sun was equal in brightness to *Sirius* : from whence he concludes that, if the sun were at 27664 times his present distance from us, he would appear equal to *Sirius* ; and consequently, if *Sirius* be of the same magnitude with the sun, the distance of that star must be 27664 times as great as the sun's distance from us^a: the force of this reasoning will be better understood, if we consider the following *lemma*.

891 If two globes of equal dimensions be at different distances from us, their apparent diameters will be reciprocally as their distances, see § 242 and 243: thus, if the distance of a star which is of the same bigness as the sun be 30000 times the distance of the sun from us, the apparent diameter of that star will be but a 30000th part of the apparent diameter of the sun: from whence the converse follows, that if the apparent diameter of a star which is of the same bigness as the sun be a 30000th part of the sun's apparent diameter, the distance of that star from us is 30000 times as great as the distance of the sun.

892 *Cassini* endeavoured to find out the distance of *Sirius* by the following method^b: to abate the sparkling of that star, he viewed it through a telescope of thirty four feet, covering the object glass with a thin pastboard which had in the middle a round hole of an inch and an half diameter; through the telescope thus prepared, the disk of *Sirius* was seen very well defined, and free from those glaring rays which appear to the bare eye: *Jupiter* was above the horizon at the same time, and, being viewed through the same telescope, and by several methods compared with *Sirius* , the diameter of that planet was judged to be ten times as great as the diameter of *Sirius* ;

^a *Hugenii cosmtheor. l. 2.*

^b *memoires d'acad. ann. 1717.*

the apparent diameter of jupiter was at that time 50 seconds, from whence it follows that the diameter of firius was about 5 seconds: if we suppose then, says that author, firius to be equal to the sun in bigness, the distance of the sun from us being about 10000 diameters of the earth, and the apparent diameter of firius being to that of the sun as 1 to 384, the distance of firius from the earth comes out 3840000 diameters of the earth. Thus *Cassini* makes the distance of firius but 384 times as great as the sun's distance, which is much less than *Hugens* brings it out, who yet makes it but a fifteenth part of what the distance of the stars observed by *Bradley* is concluded to be, from their parallax: it is very probable firius is much nearer to us than those stars, since he appears so much bigger. The methods made use of by *Gregory*, *Hugens* and *Cassini* may well be called conjectural, because they depend upon two uncertain suppositions; one, that the sun and firius are equal in magnitude; the other, that the apparent diameter of firius is determined with sufficient exactness: I need not say any thing of the uncertainty of the first of these suppositions, and as to the second, *Cassini* himself has given us an observation of his in another place^a, which makes it evident that we may easily be deceived in measuring the apparent diameter of a fixt star, so as to determine it to be much greater than it is; as I am apt to think that learned astronomer has done in the present case.

893 The account he gives of his observation is as follows; there is a star in the breast of Virgo called γ by *Bayer*, which, though single and round to the bare eye, through a telescope of 11 feet, seems oblong; and through one of 16 feet, appears divided into two stars, so near together that the distance between them is no more than equal to the apparent diameter of one of them: it was known that the 21st day of april, 1720, new style, about midnight, the moon was to pass over these stars, and eclipse them; so curious a sight deserved to be attended to; the moon was then near the full, and shone so bright as to cause many of the stars then above the horizon to disappear; this must also diminish the apparent magnitude of these stars, which through a telescope of 16 feet were seen stript of their spurious rays, with their diameters very well defined: as the dark edge of the moon drew near to them, it caused no change in their colour or situation; at 0^h 25' 14" after midnight, the occultation of the most westerly of the two stars, by the appulse of the dark edge of the moon, was observed through the 16 feet telescope; and at 0^h 25' 44", the moon covered also the most easterly star; each of them disappeared in an instant, or half a second, without any preceding diminution of magnitude or brightness: at 0^h 51' 16", they both came together into

^a *memoires d' acad. ann. 1720.*

view again, from behind the bright edge of the moon, to which they were then parallel: the reason of there being such an interval between the times of their occultations was, their being then in an oblique position to the dark edge of the moon's disk.

894 There were two circumstances attending this observation that deserve to be taken notice of: the first is, that each star disappeared in an instant, or in about half a second; whereas there was an interval of 30 seconds in time between the two occultations: from the difference between the time the moon took up in hiding each star and the time she was going through the space between the two stars, it is evident, that the distance between the centers of these stars is to the diameter of one of them as 30 to $\frac{1}{2}$, or as 60 to 1; from whence it follows, that the apparent diameter of one of the stars (being equal to half the distance between their centers) is to its true diameter as 30 to 1: thus we see that, notwithstanding the diminution of the glare of those stars by the telescope, their apparent diameters were still 30 times greater than they ought to be; and consequently their apparent disks were magnified to 900 times their true dimensions: we may judge from hence, how extremely small the fixt stars would appear, if we could see them perfectly freed from that false light which so greatly increases their apparent magnitude: and from this smallness of their apparent diameters, if they be nearly equal in magnitude to the sun, it will follow that their distance is exceedingly great, see § 891.

895 The other remarkable was, that this observation makes it probable that there is not any atmosphere round the moon; since, if there had been any, one of these stars, falling obliquely into it before the other, ought, by refraction, to have suffered some change in its colour, or in its distance from the other star which was not yet entered into the atmosphere: but no such alteration could be perceived, though the observation was performed with the utmost attention to that particular, and was very proper to have made such a discovery; because, those two stars being of the same magnitude and brightness, if any change had happened to either of them, it would have been easy to have taken notice of it; but this we shall have occasion to consider again, when we come to treat of the moon.

896 *Cassini*, in order to determine the real magnitude of Sirius, having first mentioned the great difficulty of finding the distance of the stars from annual parallax, supposes Sirius to be at no greater distance from the earth than saturn, which says he is about 100000 diameters of the earth, and is the least that can be assigned by whatever hypothesis we explain the motions of the heavenly bodies; since all astronomers agree that the stars are farther
off

off than the planets: from this distance of Sirius, and its apparent magnitude 5", he calculates the true diameter of that star to be more than two diameters of the earth^a. But this distance of Sirius is certainly much too small, and consequently his real magnitude deduced from thence must be too small likewise, if we allow his apparent diameter to be so much as 5". The comet that appeared in 1680 is computed, when in aphelion, to be at a distance from the sun 14 times as great as the distance of saturn^b: we cannot but suppose the nearest stars to be at a greater distance from the sun than any comet of the solar system ever goes to: if we suppose sirius to be but 20 times the distance of saturn from the sun, and his apparent diameter to be 5", his real diameter will be 48 diameters of the earth.

897 The apparent diameter of 5" here assigned to sirius, though less than it has been estimated by many astronomers^c, is certainly too great; perhaps the error is caused by *Cassini* using, as he tells us he did, a telescope with a small aperture at the object glass; for this will make any of the larger stars seem to have a sensible diameter^d; whereas, if, instead of contracting the aperture, the eye-glass be thinly smoked, they appear like lucid points, without any sensible magnitude^e: to this purpose Dr. *Halley* says as follows, 'The diameters of *spica virginis* and *aldebaran* are so small, that when they happen to immerge behind the dark edge of the moon, they are so far from losing their light gradually, as they must do if they were of any sensible magnitude, that they vanish at once with all their lustre; and emerge likewise in a moment, not small at first, but at once appear with their full light, even though the emerfion happen very near the cusp, where, if they were four seconds in diameter, they would be many seconds of time in getting entirely separated from the limb: but the contrary appears to all those who have observed the occultations of these bright stars. Now though sirius be bigger than either of them, yet he is by far less than two of them, and consequently his diameter to one of theirs is less than the square root of 2 to 1, or than 14 to 10; so that, in *Cassini's* telescope which shewed the diameter of sirius 5", one of these stars ought to appear about 4 seconds in diameter: whereas we are otherwise certain that they are less than one single second in diameter; the great strength of their native light forming the resemblance of a body, when it is nothing else but the spissitude of their rays^f,

^a *Elements d'astronomie par Mr. Cassini in 4to Paris 1740. liv. 1. chap. 5.*

Halley, the aphelion distance of this comet from the sun is a little more than 138 times the mean distance of the earth from the sun, *v. Newton. princip. l. 3. prop. 41.*

^c *Tycho* makes the apparent diameter of a star of the first magnitude two minutes, *v. Ricciol. almag. l. 6. c. 9.*

^d *Flamsteed*, committing the same fault of lessening the aperture, makes the diameter of sirius 15", *Hist. celest vol. 1. p. 17.*

^e *Hugens* makes this remark, *system. saturn.*

^f *Phil. transact. abridg. vol. 6. p. 166.*

FIG. 898 Since the apparent diameters of the stars, even those of the first magnitude, are thus small, not amounting to a single second; we cannot hope to determine the measure of them with any certainty: we may assign them a measure which we certainly know they do not exceed; but we cannot be sure that measure is not too great for them: all luminous objects appear larger than opaque ones of the same dimensions; thus, mercury appears larger when in all his brightness than when he is seen like a dark spot upon the sun^a; though he be nearest us in the situation last mentioned. The apparent diameters of the fixt stars are much smaller than they have been generally determined by those who have considered them, and yet small as they are, the real magnitudes of them are probably very great; otherwise they would not be visible to us at all, by reason of their immense distance from us: if we suppose the apparent diameter of any of the stars observed by *Bradley* to be equal to a 400000th part of the sun's apparent diameter, or a 200th part of a second, and this does not seem too much to assign for the apparent diameter of one of them, which is a star of the second magnitude^b; it will follow from that supposition and from what has been said of the distance of that star, that it equals the sun in bigness. In short, we have no proofs that any of the stars are less than the sun; it is very probable many of them equal and many surpass that luminary in their real dimensions: and this may be true, not only of those which are of the greatest lustre, but even of those which are scarcely visible to us. How great soever be the real magnitude of any body, we may assign it a distance from us large enough to make it appear under an angle of any given smallness: thus, if the sun were 10000 times as big as he is, we might suppose him to be removed to such a distance from us, as to appear no bigger than the smallest telescopic star; or even to be out of the reach of any glasses that can ever be made.

CHAP. 20. THE APPARENT DIAMETERS OF THE SUN AND PRIMARY PLANETS.

899 The apparent diameter of the sun, or of any planet is the angle under which the sun or the planet appears to an inhabitant of the earth: thus, fig. 97 97, let *s* be the sun, *E* the eye of an observer upon the earth, *EA* and *EB* lines drawn from the eye of the observer tangents to the globe of the sun and terminated in the sphere of the heaven at *A* and *B*; the angle *AEB* is the apparent diameter of the sun, the measure of that angle is *AB*, an arc of a

^a *Flamsteed* measured his diameter in the former situation 16", *Hist. celest. vol. 1. p. 17.* *Bradley* in the latter but 10" 45", *Phil. trans. abr. vol. 6. p. 238.*

^b The star α in the great bear. great

great circle in the sphere of the heaven, and the apparent diameter of the sun is said to be so many minutes and seconds as that arc contains: see § 235 to 244, see also § 292.

900 The apparent diameter of the sun was pretty well known to the ancients; we shall see by the table that some of them were not out much above a minute, in determining it: when they set down what it was at the sun's greatest distance, and what at his least distance from us; I do not think they had any method of finding out the difference by immediate observation; but they first supposed the sun to move equably in a circle, and, finding the space of time from the vernal equinox to the autumnal was several days longer than from the autumnal to the vernal, they considered how much the earth must be placed out of the center of the sun's circle^a to cause such an apparent inequality in his motion; and how much nearer he must be to us in winter than in summer: and from thence inferred how much greater his apparent diameter must be in winter than in summer. The moderns have been able to observe directly what difference there is in the sun's apparent diameter, at different times of the year; and from thence have concluded what change there is in his distance from us: and this is done, either by looking at the sun, where the telescope and micrometer are of great use; or by projecting the sun's picture upon a white plane in a dark room.

901 The sun's vertical or perpendicular diameter is found by taking the height of the upper and lower edge of his disk, as mentioned § 292: this is most conveniently done when the sun is in or near the meridian; because there is then no sensible change in his altitude, during the space of two or three minutes: the height of each edge must be corrected by proper allowance for parallax and refraction, and we shall have the true diameter, which is the difference between the true height of the upper and lower edge. This method is very simple, and gives us the sun's apparent diameter with as much exactness as can be expected from the divisions of the instrument, which should be a large quadrant, furnished with a telescope with two cross-hairs^b, one perpendicular, the other horizontal.

902 The second method of determining the sun's apparent diameter is to observe by a good clock the time the sun's disk takes in passing through the plane of the meridian, or some other hour circle. At or very near either of the equinoxes, when the sun's apparent diurnal motion is in the equator, or

^a I speak here of those among the ancients who supposed the earth to be placed in the center of the universe, and the sun and planets to move round the earth in excentric circles, in a manner that will be explained hereafter.

^b They are now generally very fine silver wires; though called hairs, perhaps because hairs were first in use.

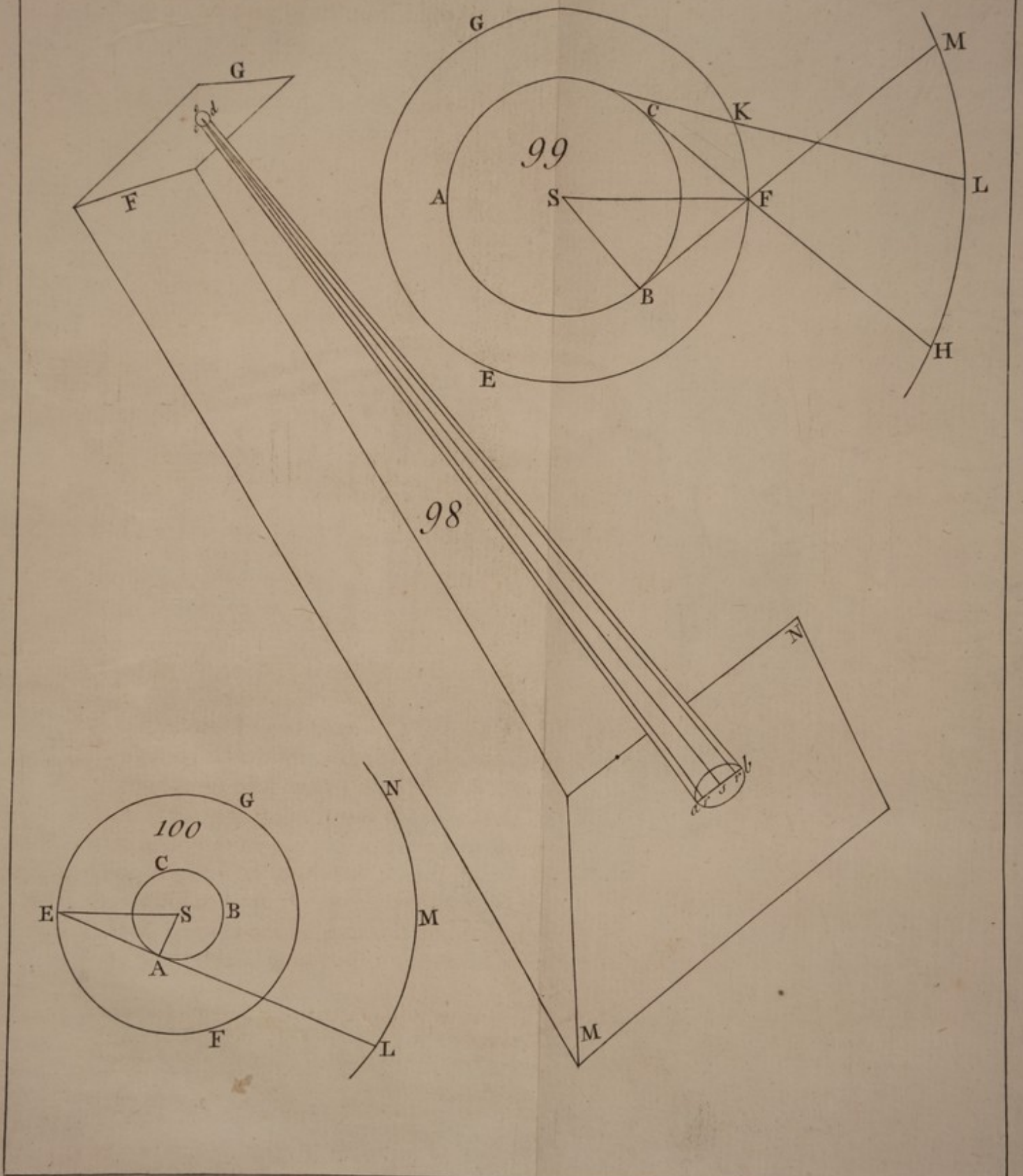
in a parallel very near that circle, say, as the time between the sun's leaving the meridian or any other hour circle and returning to it again, is to 360 degrees; so is the time of the sun's transit, to the number of minutes and seconds of a degree contained in his apparent diameter. The time of the sun's transit may also be turned into parts of a great circle, by the table page 79. At any other time of the year, when the sun is in a parallel at some distance from the equator, his diameter measures a greater number of minutes and seconds in that parallel than it would do in a great circle, and takes up proportionally more time in passing over the meridian; we may then use this analogy; as the whole sine, is to the cosine of the sun's declination; so is the time the sun is passing through the meridian reduced to minutes and seconds of a degree, to the arc of a great circle which measures the sun's apparent horizontal diameter. This method is easily put in practice by two persons, one of which is to look through a telescope with cross hairs, and give a signal by pronouncing quick the word *now*, or some other monosyllable, at the moment the sun's western edge, as also when his eastern edge appears to touch the perpendicular hair; the other is to measure the interval of time between those two moments, by the swings of a pendulum, or the hand of a clock that shews seconds: the whole may also be performed by one observer, if he makes use of a stop-watch; or has a clock placed near enough for him to hear the beats, whilst he is viewing the transit.

903 In the year 1732 on the 23, 24 and 30 of december, new style, which is the time of year when the sun is nearest us, *Cassini*^a measured, with a micrometer carefully adjusted, the vertical diameter of the sun; and after proper allowances for parallax and refraction, found his greatest apparent diameter to be $32' 37'' 24'''$, or $32' 37\frac{1}{2}''$. In the year 1735 on the 30 of june, new style, when the sun is at the greatest distance from us, the same *Cassini* found his true apparent diameter $31' 32'' 24'''$, or $31' 32\frac{1}{2}''$: *Cassini* here takes notice that his observations agree so exactly with those made some years before by *Louville*, that there is not the difference of a single second between them. *Louville* appears to have taken the apparent diameter of the sun with the utmost care; he measured the horizontal diameter by the time of the sun's passage by the vertical hair of his telescope, he made use of a clock which beat seconds, and a stop-watch which made five beats in a second: he took the vertical diameter of the sun with a micrometer; and found it just so much less than the horizontal, as it ought to be by the different refraction of the upper and lower edge: upon the whole, he makes the sun's greatest apparent diameter $32' 37'' 7'''$, and from thence computes his least to be $31' 32'' 50'''$ ^b.

^a *Elements d'astronomie liv. 2. chap. 5.*

^b *memoires d'Acad. R. ann. 1724.*

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The greatest and least apparent diameters of the sun, according to *Cassini* FIG. the father and *Flamsteed*, are something different from those here set down: see § 775, and the following table.

A table of the sun's apparent diameter at his greatest, mean and least distance; according to several authors^a.

Authors names.	greatest distance of ☉			mean distance of ☉			least distance of ☉		
	'	"	'''	'	"	'''	'	"	'''
<i>Aristarchus, Archimedes</i>	30	0		30	0		30	0	
<i>Ptolemy</i>	31	20		32	18		33	20	
<i>Albatennius</i>	31	20		32	28		33	40	
<i>Regiomontanus</i>	31	0		32	27		34	0	
<i>Copernicus</i>	31	40		32	44		33	54	
<i>Tycho</i>	30	0		31	0		32	0	
<i>Ricciolus</i>	31	0		31	40		32	8	
<i>Job. Domin. Cassini</i> ^b	31	40		32	13		32	46	
<i>Gascoigne</i> ^c	31	40					32	50	
<i>Flamsteed</i> ^d	31	30					32	48	
<i>De la Hire</i> ^e	31	38		32	11		32	44	
<i>Louville</i> ^f	31	32	50	32	4	36	32	37	7
<i>M. Cassini F.</i> ^g	31	32	30	32	5	0	32	37	30

904 In order to take the sun's apparent diameter by the projection of his picture, let the sun shine into a dark room through a hole in a thin plate of metal *FG*, fig. 98, the diameter of the hole *cd*, which should not exceed 98^o two inches, must be known in equal parts of as small a scale as is possible; for instance let it be found how many hundredth parts of an inch it contains: let the plate *FG*, be so placed at the time of the observation, that a ray from the center of the sun may fall perpendicularly upon it: receive the circular image of the sun upon a plate *MN*, which should be white, and placed parallel to the plate *FG*; the distance between the two plates *es*, must be known, in equal parts of the same scale, by which the diameter of the hole *cd*, was measured: let the diameter of the sun's image *ab*, be taken with a pair of

^a v. Ricciol. *almagest. nov. l. 3. c. 10.* ^b *astronom. reformat. l. 1. c. 12.*

astronomiques, p. 22.

^c *Flamsteed. hist. celest. vol. 1. p. 4.*

^d *Hist. celest. vol. 1. p. 23.*

^e *Tabb. astronom. edit. 2. p. 17.*

^f *memoires d'Acad. R. ann. 1724.*

^g *Elements d'astrono-*

mie, liv. 2. chap. 5.

compasses,

FIG. compasses, and measured upon the same scale; if the diameter of the hole cd 98 be subtracted from it, there will remain the true apparent diameter of the sun, viz. rr , in hundredth parts of an inch; and consequently the semidiameter rs , is known: imagin the line es drawn, and you have the right angled triangle esr , two sides of which viz. es , and sr , and the right angle rse , are known, and consequently the remaining angles may be known by trigonometry; and therefore it may be known what the angle ser is, by which the sun's semidiameter sr , is measured.

905 One method of finding *the apparent diameter of a planet* is, to compare it with, and estimate what proportion it bears to, the diameter of the moon, or of a spot in the moon, or the interval between two stars near to one another, and not far distant from the planet. 2, another way is to place between the planet and the eye a thin plate of metal with a small hole in it, at such a distance from the eye, that the entire disk of the planet may appear exactly to fill the hole; and then, by measuring the diameter of the hole and the distance of the plate from the eye, to determine by trigonometry the visual angle subtended by the diameter of the hole, which in this observation is the same as the apparent diameter of the planet. 3, a third method similar to the second is, to suspend a thread or wire between the eye and the planet, at such a distance from the eye that it may just cover the disk of the planet, and to determine trigonometrically the visual angle then subtended by the thickness of the thread, which is done by measuring its distance from the eye, and finding its real thickness, by winding it close round a rule and observing how many threads cover an inch in length: by this method *Galileo* attempted to measure the apparent diameter of *Lyra*, and computed it to be 5 seconds^a. 4, a fourth way is to measure the picture of the planet cast through a telescope upon white paper in a dark room, and compare it with the diameter of the sun or moon cast in the same manner. 5, to mention no more such uncertain methods^b, the diameters of the planets are best taken by the micrometer: *Venus* and *Mercury* have been seen in the sun, so that their diameters have been compared with the diameter of the sun: the apparent diameter of *Jupiter* may be found by reducing into seconds of a degree the time a satellit, or, which is better, the time the shadow of a satellit takes in passing over his disk. 6, the disk of a planet may also be measured, when there is an occultation of it by the moon, by counting the number of seconds that pass between the appulse of the edge of the moon's disk to the edge of the disk of the planet and the time of the planet being totally hid by the moon: but then the horary motion of the moon and of the planet at the time

^a *System. cosmic. dial.* 3.

^b *v. Ricciol. almag. nov. l. 7. § 6. c. 9.*

of observation must be known, as also whether the edge of the moon's disk comes upon the planet directly or obliquely; and, if obliquely, with what degree of obliquity.

906 The smallest object, as the finest hair or wire, placed in *the focus of a telescope*^a appears greatly magnified and very distinct: if a thin plate of metal be placed therein having a round hole of a diameter a little less than the diameter of the eye-glass, it will distinctly circumscribe so much space in the heaven as is visible through the telescope at one view: this space may be measured, either by finding two stars whose distance from each other is exactly equal to its apparent diameter, and is otherwise known; or by observing the time a star in or very near the equator takes in passing over it, see § 902. Some astronomers have determined the apparent diameters of the planets by estimating how many times they were contained in the apparent diameter of the space thus circumscribed by the telescope^b. *Hugens* advises to prepare two or three long narrow brass plates, with sides very strait, but a little converging, and, having two slits so made in the opposite sides of the telescope, that the plane of a long plate sliding therein may touch the plane of the plate with the round hole, to thrust it in till the planet be just hid; and, observing in what part of the long plate the breadth of it is just sufficient to cover the whole planet, to take that breadth with a fine pair of compasses, and compare it with the diameter of the hole: from whence the apparent diameter of the planet is easily found^c. The diameters of planets measured by *Hugens's* method are something bigger than they ought to be, because the rays of light are inflected as they pass by the edges of solid bodies, as was mentioned § 220.

907 *The micrometer* is an instrument now very well known, it is so contrived that two parallel fine wires being placed in the focus of a telescope, one fixed the other moveable, or both of them moveable, they may be made to approach or recede one from the other till they appear exactly to touch two opposite points in the disk of the planet; and then the index shews the apparent diameter of the planet in minutes and seconds: see the description of this instrument in *Smith's optics* book 3 chap. 8, see also *Phil. transact.* abr. vol. 1 p. 218. *Flamsteed* made use of a micrometer with two plates, one of which was made to approach to or recede from the other, till the planet appeared to be clasped between the edges of the two plates: this method gave the apparent diameters of the planets as much too small as *Hugens'* gave them too great; and that by reason of the rays being inflected the contrary way, see

^a The focus of the telescope is a place within the tube where the focus of the object-glass and the focus of the eye-glass coincide.

^b *v. Ricciol. alm. l. 7. § 6. c. 9.*

^c *Hugenii system. saturn.*

§ 220: upon this account, *Whiston* very advisedly chose the mean between the measures of *Hugens* and those of *Flamsteed*.

908 I shall now give a table of the apparent diameters of the primary planets, as they are stated by different authors^a: nothing exact can here be expected from the ancients; among the moderns, those marked with an asterisk are such as may best be depended upon.

Authors names.	Diam.	♂			♀			♂			♀		
		'	"	'''	'	"	'''	'	"	'''	'	"	'''
<i>Albatennius</i> ^b	mean	1	44	28	2	36	40	1	34	0	3	8	0
<i>Alfraganus</i>	mean	1	44	28	2	36	40	1	34	0	1	34	0
<i>Tycho</i>	mean	1	50		2	45		1	40		3	15	
<i>Hortensius</i>	least	0	31	0	0	38	30	0	9	0	0	15	20
	mean	0	37	0	0	50	0	0	36	0	0	53	0
	greatest	0	42	40	1	1	40	1	4	0	1	40	0
<i>Kepler</i>	greatest	0	30		0	50		6	30		7	6	
<i>Ricciolus</i>	least	0	22	0	0	38	18	0	10	6	0	33	30
	mean	0	26	40	0	49	46	0	22	0	1	4	12
	greatest	0	36	0	1	8	46	1	32	0	4	8	0
* <i>Hugens</i> ^c	greatest	0	30		1	4		0	30		1	25	
* <i>Flamsteed</i> ^d	greatest	0	25		0	54		0	33		1	12	
* <i>Horrox</i> ^e	greatest										1	18	30
* <i>Crabtree</i> ^f	greatest										1	9	
* <i>Pound</i> ^g	mean	0	18		0	37	15						
* <i>Bradley</i> ^h	greatest												0 10 45

Remarks upon the sun's apparent diameter.

909 The continual streaming of the particles of light from the body of the sun make it reasonable to believe he must waste in bulk: Sir *Isaac Newton* remarks, that comets which in their revolution round the sun come so near him as to go through his atmosphere, may by the resistance thereof, have their motions so retarded, that, approaching nearer and nearer to him every revolution, they may at last fall into the sun: such a supply, though very

^a v. Ricciol. *almag. nov.* l. 7. § 6. c. 9. & *astronom. reformat.* l. 10. c. 7. ^b *Albatennius* and *Alfraganus* determine the apparent diameter of each planet by estimating to how great a part of the sun's apparent diameter it is equal: thus they say the apparent diameter of jupiter is equal to a 12th part of the sun's apparent diameter, &c; *Ricciolus* collects their measures as here set down from the sun's apparent diameter as settled by them. ^c *System. saturn.* ^d *Hist. celest.* vol. 1. p. 8, & seq. ^e *Venus in sole visa.* ^f *ibid.* ^g *Newton's princip.* book 3 phenom. 1, 2. ^h *Phil. transact.* abr. vol. 6. p. 238.

small and happening but rarely, if we consider the inconceivable minuteness of the particles of light, may be thought sufficient to keep up the bulk of the sun for many ages, so that no diminution thereof should be perceivable by the inhabitants of the system: but however that be, the most ancient and modern observations of the sun's apparent diameter are quite insufficient for determining whether it be diminished or not. See *Smith's optics* p. 415. FIG.

CHAP. 21. THE DISTANCES OF THE PLANETS FROM THE SUN: THE
REAL MAGNITUDES OF THE SUN AND PLANETS.

910 The parallaxes of the sun and planets and their apparent diameters are so well determined, that the distances of the planets from the sun and the real magnitude of the sun and planets are known to a tolerable degree of certainty: the horizontal parallax of the sun is now generally agreed to be about $10''$, see § 820; from hence the distance of the sun from the earth may be found by § 143.

911 In the *Copernican* system, the proportion between the femidiameters of the planetary orbits, or between the distances of the several planets from the sun is found by trigonometry; so that if the distance of any one of them, as for instance if the distance of the earth from the sun be given, the distances of all the rest of the planets from the sun may be found: in any of the superior planets, the ratio its distance from the sun bears to the distance of the earth from the sun is known, by measuring the angle of the planets retrogradation: thus, fig. 99, let s be the sun, ABC the orbit of the earth, EFG 99 the orbit of mars, HLM part of a great circle in the sphere of the heaven wherein mars appears to go retrograde, let the place of mars be at F ; by the earth passing in her orbit from B to C , mars will appear to move retrograde from M to H : the angle then of the retrogradation of mars namely MFH is known by observation, to this the vertical angle BFC is equal: draw sf and sB , and you have the triangle sBF , wherein the angle sBF is a right one, by § 28; the angle sFB is equal to half the angle of retrogradation, and consequently known; so that in this triangle two angles are known, and therefore the remaining angle FsB may be found, § 55 cor. 3: and, all the angles being known, the ratio of any one side to any other, as the ratio of sB the distance of the earth from the sun to sF the distance of mars from the sun may be found, § 141.

Scholium. The angle of the retrogradation of mars is here expressed as if mars stood still at F , whilst the earth went from B to C ; whereas mars in the

FIG. mean time moves forward in his orbit, from F to K, so that his real retrogradation will be only ML less than MH: but since the quantity of mars's motion in his orbit may be found, during the time of his retrogradation, it may be known how much the angle of his retrogradation is thereby diminished, and consequently what his retrogradation would have been, if he had stood still all the while at F.

912 In either of the inferior planets, the ratio of the semidiameter of its orbit to the semidiameter of the orbit of the earth is easily found, by the angle of the planet's greatest elongation: thus, fig. 100, let s be the sun, ABC the orbit of mercury, EFG the orbit of the earth, LMN an arc of a great circle in the sphere of the fixt stars; A the place of mercury when seen at L in his greatest elongation from the sun; draw SE, and SA, and you have a triangle, whereof the angle SAE is a right one, SEA is the angle of mercury's elongation, known by observation, and therefore the remaining angle ASE may be found: and consequently the ratio of ES the distance of the earth from the sun to AS the distance of mercury from the sun may be known.

913 The two sections immediately preceding shew how to find the comparative distances of the planets from the sun, but only in a general and less accurate way; because neither the excentricities of the planetary orbits nor the inclinations of their planes are there taken into the account: more exact methods may be seen in *Gregory's astronomy* book 3 prop. 19, 21 and 22; and *Cassini elements d'astronomie* liv. 4. chap. 6.

914 The distances of the planets from the sun are found to be in a certain ratio to the periodical times of their revolutions round the sun; the squares of their periodical times are as the cubes of their distances: this analogy, discovered by *Kepler*^a, furnishes us with a method of finding the ratio between the distances of the planets from the sun, to greater exactness than can be done by any other method; because the periodical times of the planets are known to great exactness.

915 The periodical times of the planets were given before in round numbers; in the following table they are set down more accurately^b.

		days	hours	min.
mercury	} revolves round the sun in {	87.	23.	16
venus		224.	16.	49
the earth		365.	6.	9
mars		686.	23.	27
jupiter		4332.	12.	20
saturn		10759.	6.	36

^a *epit. astron.* l. 4. *Gregory astron.* book 1 prop. 40. & 67. *Streete astron. Carolin.* ^b *Streete ibid.* 916

916 *The comparative mean distances of the planets from the sun* were before set down only in round numbers, § 619; the following table has them with greater exactness^a.

	♄	♀	♂	♁	♀	♃
According to <i>Kepler</i>	951000.	519650.	152350.	100000.	72400.	38806.
According to <i>Bullialdus</i>	954198.	522520.	152350.	100000.	72398.	38585.
From the periodical times	953806.	520116.	152369.	100000.	72333.	38710.

917 From hence, *the absolute distances of the planets from the sun*, supposing the sun's parallax 10", are as follows;

The distance of	mercury	from the sun is	8000	semidiameters of the earth, or	32,000000	english miles.
	venus		14900		59,000000	
	the earth		20600		82,000000	
	mars		31400		125,000000	
	jupiter		107300		426,000000	
	saturn		196700		780,000000	

918 From their apparent diameters and these real distances, we have *the real diameters of the sun and primary planets*, as follows^b;

The diameter of	the sun	is equal to	100	diameters of the earth, or	793,000	english miles.
	mercury		$\frac{1}{100}$		2,600	
	venus		1		7,900	
	the earth		$\frac{1}{100}$		7,935	
	mars		$\frac{1}{9}$		4,800	
	jupiter		$\frac{1}{8}$		77,000	
	saturn		$\frac{1}{8}$		67,000	

919 *The superficial contents of the sun and primary planets* are as follows, § 86;

The superficial content of	the sun	is equal to	10000	surfaces of the earth.
	mercury		$\frac{1}{100}$	
	venus		1	
	mars		$\frac{1}{9}$	
	jupiter		93	
	saturn		72	

^a *Newton's princip.* book 3. phenom. 4. The near agreement of the comparative distances computed from the periodical times with those determined from observations by *Kepler* and *Bullialdus* is a good proof of the analogy mentioned § 914.

^b In the apparent diameters, I follow *Bradley* for mercury; the mean between *Horrox* and *Crabtree* for venus; *Flamsteed* for mars; and *Pound* for jupiter and saturn.

920 *The solidities of the sun and primary planets are as follows, § 87;*

The solidity of	the sun	is equal to	1000000	times the solidi- ty of the earth.
	mercury		$\frac{1}{27}$	
	venus		1	
	mars		$\frac{1}{5}$	
	jupiter		899	
	saturn		612	

921 The mean distances of the planets from the sun, or the femidiameters of their orbits being given; it is easy to compute the circumferences of those orbits; by § 68. The circumference of the orbit of a planet, and the periodical time in which it goes round therein being given; we may easily compute the mean velocity with which the planet goes round the sun in its orbit. *The mean horary velocities of the primary planets are as follows;*

mercury	goes round the sun at the rate of	94000	miles in an hour.
venus		69000	
the earth		59000	
mars		48000	
jupiter		26000	
saturn		19000	

Scholium. The horary motion of every planet when in perihelion is swifter than is here set down, and slower when in aphelion: as was said § 672.

922 The distances of the planets from the sun above set down suppose the parallax of the sun to be precisely 10", as it was determined by the parallax of mars, § 820 and 822: but since it is impossible to be certain there is not an error of 2 or 3 seconds in the parallax of mars, arising from the imperceptible errors in the observations upon which the calculation of the parallax of that planet is founded; and since the difference of 3" in the parallax of mars would occasion a difference of 1000 femidiameters of the earth in his distance; it is impossible to determine the distance of mars from the earth with certainty, so near as to 1000 femidiameters of the earth: and consequently the distance of the sun from the earth cannot be determined nearer than within about 2000 or 3000 femidiameters of the earth. If the parallax of mars were greater, it would be more easy to determine his distance with certainty; the difficulty of determining the distances of phenomena increases in a duplicate proportion of their distances, so that a distance 20 times as great as another is 400 times as difficult to be determined with equal certainty. When astronomers give us the distances of the planets in leagues and miles, as they may do

do, since the semidiameters of the earth may be reduced to leagues and miles with accuracy enough, as when they tell us the distance of the earth from the sun is about 30 millions of leagues; we can have no greater certainty the number is exact within 2 or 3 millions of leagues, than we have that in a distance by common estimation of 30 leagues upon the earth there is not a mistake of 2 or 3 leagues. *Cassini observations astronomiques*, § 37.

923 Those among the ancients who placed the earth in the center of the universe determined the moon's distance by her parallax, which they could discover; as for the distances of mars, jupiter and saturn, they conjectured them to be proportional to their periods: and since the moon, which they found to be nearest to the earth, performed her circuit in the shortest time; they concluded saturn, whose period was longest, to be proportionally the most distant planet from the earth: this rule could not be applied to venus or mercury; because they are always near the sun. Some of them placed mercury farther from the earth, others nearer to the earth than venus: others, observing that mercury and venus were never very far distant from the sun, imagined the orbits of those two planets to have the sun for their center, and that they accompanied the sun in his annual revolution round the earth.

924 The ancients determined the distance of the sun from the earth by eclipses of the moon; or by observations of the moon when she appears half illuminated by the sun: and, from the distance of the sun thus found, computed his parallax; which they made much greater than the observations of the moderns have shewn it to be, as was said § 815.

Pythagoras conjectured the distances of the several planets from the earth to be in certain harmonical proportions^a, and was herein followed by some of the Platonic philosophers^b and others.

CHAP. 22. CHANGES IN THE FIXT STARS: CLOUDY STARS.

925 There are several stars now to be seen in the heaven, which are not mentioned in the ancient catalogues; and others are now not to be found, though the places in which they formerly appeared have been set down by astronomers who observed them: there are also some stars whose apparent

^a *Pythagoras prodidit hunc totum mundum musica factum ratione; septemque stellas inter cœlum & terram vagas, quæ mortalium geneses moderantur, motum habere suppositor, & intervalla musicis diastematis habere congrua, sonitusque varios reddere pro sua quaque altitudine ita concordēs, ut dulcissimam quidem concinant melodiam, sed nobis inaudibilem propter vocis magnitudinem, quam non capiant aurium nostrarum angustia.* Censorin. *de die natalis* c. 11. Vide *Ciceronis somnium Scipionis*.

^b *Macrob. in somn. Scip. l. 2. c. 3. V. Ricciol. alm. l. 9. § 5. c. 7.*

magnitude is periodically increased and diminished: as to those stars which are omitted in the ancient catalogues but inserted in those of later date, it is probable many were overlooked at first by reason of their smallness: some of the ancients mention the pleiades as consisting of six stars only, others as of seven: this has induced some authors to count the seventh of the pleiades among the new stars; but this difference in reckoning may perhaps be owing to the difference in the eyes of the observers; or to the different constitution of the air: it may not be improper upon this occasion to mention, that several astronomers have given us figures of the pleiades, as they appear through the telescope: thus *Galileo* gave us one containing 36 stars^a; *Hook* has one of 78^b; *De la Hire* has a scheme of 64^c: and *Maraldi* one of 56 stars^d: as to some little difference between their several draughts, it may easily be accounted for, by the great difficulty there is in placing those stars in their true situation. v. *Ricciol. l. 8. § 2. c. 1.* Phil. transf. abr. vol. 1. p. 246.

926 The most ancient observation of a new star is that by *Hipparchus*, about 120 years before Christ; which occasioned his setting about a catalogue of the stars, as was said, § 606: it is pity we have not some account in what part of the heaven this star was seen: another new star is said to have appeared about A. D. 130: a third about A. D. 389: a fourth in the ninth century in the 15° of scorpio, which is said to have cast as much light as a quarter of the moon does: a fifth A. D. 945 between the constellations of Cassiopea and Cepheus: a sixth in the year 1264 near the constellation of Cassiopea: but as to all these stars, the accounts we have of them are so imperfect, that nothing can be built upon them^e.

927 The first new star of which we have any good account is that which in the beginning of november 1572 appeared in Cassiopea, making nearly a rhombus with the three stars $\alpha \beta \gamma$ of that constellation, where it was seen 16 months successively, without any change of place among the fixt stars: this star had no hair round it or tail as comets have; but shone with the same lustre as the other fixt stars do; surpassing *firius* or *lyra* in brightness and magnitude: it appeared even bigger than *jupiter*, which at that time was near his perigee; and by some was thought equal to *venus* when she is in her greatest lustre: it shone forth at once in all its brightness; and continued to do so all november, so as to be seen, by those who had very good eyes, even at noon-day; which none of the stars or primary planets can be, except *venus*: and in the night it might be perceived through thin clouds by which other stars were hid: it did not continue long of the same

^a in nuncio sydereo.

^b micrographia pag. 241.

^c memoires d'acad. ann. 1693.

^d memoires d'acad. ann. 1708.

^e v. Tychon. progymnasmat. cap. 3. & Ricciol. l. 8. § 2. c. 1.

apparent

apparent magnitude, for in december it seemed to be but equal to jupiter: in january 1573 it was less than that planet, but a little bigger than stars of the first magnitude: to which it was equal in february and march: thus gradually diminishing, in april and may it was but of the second magnitude; in june july and august it was only equal to the biggest stars in Cassiopea, which are counted of the third: in september october and november, it was no bigger than a star of the 4th order; and in december it equalled the star called α by *Bayer* to which it was near: towards the end of the year 1573, and in the month january of the year following, it was but little superior to stars of the fifth; in february it was no bigger than a star of the sixth magnitude: and in march became quite invisible. It was also subject to divers changes in brightness and colour, as it diminished in bigness: for when it appeared biggest, its light was white and brilliant: from that time it grew a little yellowish: and in the beginning of the spring 1573 was something of the colour of mars, being reddish like the bull's-eye, and a little less bright than the right shoulder of Orion: in may it was of a pale lividish white, like saturn: which colour continued to the last, as did its sparkling also; only growing more dim and faint, as it was nearer to its disappearing. This star was observed by several astronomers, but especially by *Tycho Brahe*, who wrote a treatise upon it, wherein he determines its longitude and latitude, and concludes it to be in the region of the fixed stars, at a greater distance from the earth than the moon, or even any of the planets; by reason of its having no sensible parallax: he gives us also the observations of it and opinions about it of the *Prince of Hesse*, *Hainzelius*, *Mæstlinus*, and others, with his own remarks upon them. Some have thought this star to be the same which is said, § 926, to have appeared in A. D. 945, and 1264; its place in the heaven seems to favour this opinion.

928 August 13 1596, *David Fabricius* observed a star in the neck of the whale, in $25^{\circ} 45'$ of aries, with a south latitude of $15^{\circ} 54'$: it disappeared after the month october of the same year: *Bayer*, in his maps of the heaven printed in 1603, marks it with the letter σ , as a star of the 4th magnitude: *Phocylides Holwarda* observing it in the year 1637, and, not knowing what had been done before, thought that the first time of its being taken notice of; he saw it come into view again, after having disappeared nine months: it has been since found to be every year pretty regular in its period; except that from october 1672 to december 1676 *Hevelius* could not see any thing of it. *Bullialdus*, in a treatise printed at Paris in 1667, having compared together the observations which had been made of it from the year 1638 to 1666, determined the periodical time between this stars appearing in its greatest brightness

ness and returning to it again to be 333 days: he found also that about 120 days pass between the time when it is first seen of the 6th magnitude and its disappearing: that it continues in its greatest lustre for about 15 days: that after its first appearance of the 6th magnitude it increases in bigness much faster till it comes to be of the 4th, than it does from that time to its being of the third: and that from its being of the third its increase to the second magnitude is by still slower degrees. He accounts for these appearances, by supposing this star to be a globe having the greatest part of its surface dark, the rest being luminous; and that, having a rotation round its axis, it turns sometimes its dark sometimes its luminous part towards the earth ^a.

929 A simple rotation will not explain the variations that appear in this star: for 1, it does not every year arrive at the same apparent magnitude; but some years it surpasses stars of the second order and other years does not equal those of the third: 2, the time of its appearance is not always the same; but some years it is visible four months, and in other years no longer than three months: 3, the time between its first appearance and its coming to its greatest brightness is not always equal to the time between its arriving at its greatest brightness and its disappearing; but sometimes its apparent magnitude increases faster than it decreases, and sometimes it decreases faster than it increases: 4, according to *Hevelius*, it was not to be seen at all for four whole years, from october 1672 to december 1676. *Cassini* makes the mean period of this star 334 days, allowing it to be sometimes 3 or 4 days more or less than that: he accounts for the variations observed in it, by supposing a change in the position of the axis of the stars rotation ^b: I think it is more probable, that the surface of the star is subject to change, being sometimes covered over with more spots than at other times.

930 Three changeable stars have been discovered in the constellation of the swan; the first was seen by *Gulielmus Janssonius* in 1600, at the setting on of the neck, near that in the breast called γ by *Bayer*, who mentions it as a new star: it is not in the catalogue of *Tycho*, though he sets down several other stars near it which are less: in 1606 *Kepler* published a small treatise of this star annexed to a larger piece about the new star that appeared in *Serpentarius*: he therein evidently proves it to be a new star, against some who at that time were of another opinion: he tells us that it had for six years at least continued constantly immoveable as to its place among the fixed stars, and unchangeable as to its magnitude; being reckoned by all who observed it of the third order, something less than the star in the breast, and a little bigger than that in the bill of the swan; he determined its place to be in $16^{\circ} 18'$ \approx

^a *Cassini elements d'astronomie* l. 1. c. 6.

^b *idem ibidem*.

with

with $55^{\circ} 30'$ or $32'$ north latitude: from the year 1606 I find no account of it for 10 years; but in 1616, 1621, 1624 and 1629, *Ricciolus* says it was seen by himself and others: that author, who seems to think it appeared all the intermediate time from the year 1606 to 1629, could not find any writer who had taken notice of the time of its disappearing: but is very positive that it was invisible in the last years between 1640 and 1650: in 1655 *Domenicho Cassini* saw it again; when it grew bigger, so as in 1657, 1658 and 1659 to be of the 3d magnitude, according to *Hevelius*; who from the beginning of the year 1660, with surprize, found it gradually less, and at the end of 1661 to be quite invisible: in november 1665 it began to appear again, and in september 1666 *Hevelius* saw it exactly in the same place as before, with his naked eye, but exceedingly small: in 1670, 1671, 1672, 1675 and 1677 it appeared of about the 6th magnitude: in 1681 it could not be seen without a telescope: in 1715 it appeared of about the 6th magnitude, as it does this present year 1741.

931 The second changeable star in the swan was discovered june 20 1670, near the head on the side towards the arrow; it was of the 3d magnitude, and kept its bigness to the 3d of july; but its light was then sensibly fainter: the 11 of that month it was scarce of the 4th magnitude: on the 10 of august it did not exceed the 5th; in september it diminished very fast; and oct. 14 *Hevelius* could not see it, though he looked for it very diligently: this star, after having disappeared near six months, was seen again march the 17th 1671, being then of the 4th magnitude: the 3d of april it was judged to be bigger than either of the stars which are at the bottom of the harp, and are of the third magnitude; but was a little less than that in the bill of the swan: on the 4th of april it was looked upon to be almost equal in bigness to, and much brighter than the star in the bill: on the 9th it seemed a little less, and nearly equal to the biggest of the two stars on the bottom of the harp: on the 12th it was equal to the least of those two stars; but on the 15th it was grown bigger, and was again equal to the biggest of those two stars: between the 16 and 27 of april it appeared of various magnitudes; sometimes equal to the biggest of those two stars, sometimes to the least, and sometimes of a middle size between them both: the 27 and 28 of the same month it was become as big as the star in the bill of the swan, and grew bigger than that, and nearly equal to the star in the breast, but its light was a little dimmer and redder, from the 30 of april to the 6 of may: on the 15 it was less than the swan's bill; and the 16 it was of a middle size between the two stars in the bottom of the harp: from that time it continued to grow less and less, so that on the 2d of august it was judged by *Hevelius* to

be scarcely of the 6th magnitude, and on the 11th of september it could not be perceived by him at all: this star appeared twice in its greatest splendor in the space of one month; the first time on the 4th of april, the second in the beginning of may 1671, which is not known to happen to any other star. By comparing the observations of these two years, it seemed at first to return periodically to the same phase in about 10 months, so that it ought to have appeared again in february 1672, whereas it was hardly visible to the bare eye on the 6th of march in that year, and on the 29 it was scarce of the 6th magnitude, and it has never appeared since. This star is not to be found in any ancient catalogue, though several much smaller stars near it are set down in them. Its longitude was $1^{\circ} 52' 26''$ of π , and its latitude $47^{\circ} 25' 22''$ north. *The days in this § are all new style.*

932 The third changeable star in the swan is the star χ of that constellation, which in the year 1686 was discovered by *Kirchius* to change its apparent magnitude: by 20 years observations of its periods, this star was found to return to the same phase in about 13 months and $\frac{1}{2}$, or 405 days; though subject to great physical alterations, for in some years it arrived at a greater magnitude than in others; but rarely exceeded stars of the 5th: and in the years 1699, 1700 and 1701 it could not be seen at all, even at those seasons when, according to the observations of other years, it might be expected to appear in its greatest lustre. Its longitude from the first star in γ is about $9^{\circ} 6' 30''$, with $52^{\circ} 40'$ north latitude.

933 In the year 1604, in the latter end of september old style, a new star appeared near the heel of the right foot of Serpentarius: there were in that part of the heaven at that time the three superior planets, very near one another; a sight which so engaged the attention of astronomers, that no appearance thereabouts could long have escaped them: on sept. 17, *Kepler*, who wrote a treatise about this star, from which this account is taken, carefully observed the three superior planets: on the 23 he again viewed mars and jupiter, then drawing near to conjunction; as a scholar of his did also on the 27: on the 28 those planets were observed by several persons: on the 29, which was the day when mars and jupiter were in conjunction, they were observed by *Mæstlinus*; but none of these persons as yet saw any thing of the new star: on the 30 the sudden breaking of the clouds gave one of *Kepler's* friends an opportunity of having a very short view of it; for, in looking for mars and jupiter, besides those two planets he saw a bright star near them, but soon lost sight of it again, by the clouds intervening: on the 2, 3, 4, and 6, of october, it was seen by several persons in different places; but, by reason of the cloudy weather at Prague, *Kepler* could not have a view of it till the 7
of

of that month. He tells us that all who observed it agreed in this, that it was exactly round, without any beard or tail, with one of which comets are always attended; and that it was exactly like one of the fixt stars, except that in the vividness of its lustre and quickness of its sparkling it exceeded any thing they had ever seen before: as to its colour, both *Kepler* and *Mæstlinus* took notice that it was every moment changing into some of the colours of the rainbow, as yellow, orange, purple, and red; but was generally white, when it was at a little height above the vapours near the horizon. As to the magnitude of this star at its first appearance, every body who saw it was of opinion that it was not only bigger than any of the fixt stars, but surpassed saturn and mars, and even jupiter; which last planet was near it all the month of october; and was, by his steady light, easily distinguished from this vehemently sparkling star: *Tycho* pronounced the star in Cassiopea to be something bigger than jupiter, who was at that time in perigee; the star in Serpentarius was much bigger than jupiter, but jupiter was then in apogee, and consequently in his least apparent magnitude. It continued for the whole month of october nearly of the same bigness; except that, as the sun began to grow nearer to the star than to jupiter, it did not seem to exceed that planet in magnitude so much as before: and yet oct. 30 it was so much brighter than jupiter, that *Kepler* could see the star, when he could not see jupiter; though that planet was at the same time farther off from the sun's beams than the star was: novemb. 6 *Kepler* saw it for the last time before it was got too far into the evening twilight; for cloudy weather followed: another person saw it on the 8: on the 12, though its place might be marked by the moon, which was then near it, yet no new star was to be seen: it was not however covered by the moon, because her latitude was then above 3 degrees, whereas the star had not so much as 2 degrees latitude: at Prague therefore the star was invisible, by the sun being too near it, some time between the 6 and the 12 of november; but at Turin, where the height of the pole is less than at Prague, it was seen the 13 of november, by those who had very good eyes: decemb. 24, the sky being clear after a good deal of cloudy weather, *Kepler* had the first certain view of it again, after its being hid by the twilight; it sparkled exceedingly, but was much diminished in magnitude; however it still seemed bigger than scorpion's heart, though deeper in the twilight than scorpion's heart: but it appeared less than arcturus; perhaps because arcturus was out of the sun's beams: for on the 3d of january, being quite out of the twilight, it seemed bigger than either arcturus, or saturn, which planet had then been three days out of the twilight. From this time the star grew less and less; for on the 10 of march, being carefully viewed, it appeared much less than sa-

turn; and but little bigger than the stars of the 3d magnitude in the knees of Serpentarius. April 11 it seemed equal to the bright star in the right knee of Serpentarius; and continued nearly of that magnitude, may and june: august the 2 and 4, being carefully compared with some adjacent stars, it was judged to be equal to the nearest star in the leg of Serpentarius, which is counted of the 3d magnitude, but is less than that in the knee: the new star continued still to sparkle more strongly than any other fixt star: aug. 19, by reason of the lightfomeness of the night at that time of the year, it could hardly be determined whether the new star appeared bigger than that in the leg or not; but it was more easily discerned than that, by reason of its greater sparkling: sept. 3 it was less than that in the leg: sept. 28, which was a year after its first coming into view, it was more easy to be seen than the star in the leg, and yet it was not very easily seen, because it was now got a little within the evening twilight again, and was this year hid by the nearness of the sun's light 40 days sooner than in the preceeding year; from which circumstance, it may be conjectured how much greater its magnitude was at its first appearance than now. In january 1606 the sky was seldom clear in the morning, however, on the 25 in the morning *Kepler* could see the place of it, but could not see the star. Jan. 27 he was uncertain whether he did not see some glimmering of the new star or not; so that, if it was not quite extinct, it was grown too small to be perceived in the morning twilight. In march its place was so far out of the sun's beams, that it rose high enough above the horizon before the appearance of the morning twilight, but there was not the least glympse of the star to be seen. Upon the whole, *Kepler* concludes that it disappeared sometime between october 1605 and the february following; but on what day is uncertain. He says it had no parallax nor motion; and remarks, that all the planets were successively in conjunction with this star, during the time of its appearance.

934 Besides these, several other changes have been observed in the stars: *Montanari* found two stars of the 2d magnitude in the constellation of the ship, marked β and γ by *Bayer*, to be wanting: he says, they were seen by himself and others in 1664, on occasion of the comet which appeared that year; when they first disappeared he does not know, only he is sure that on the 10th of april 1668 there was not the least glympse of them to be seen; and yet the rest about them, even of the 4th and 5th magnitudes, remained unchanged: he says, he has observed above an hundred other changes among the fixt stars, though none of them so great as those. *Domenicko Cassini* discovered a new star of the 4th and two of the 5th magnitude in Cassiopea: five new stars in the same constellation, whereof 3 have since disappeared

peared: two in the beginning of the constellation Eridanus; one of the 4th, the other of the 5th magnitude: four of the 5th or 6th order, near the north pole: that great astronomer has also remarked that the star placed by *Bayer* near the star ϵ of the little bear is no longer visible: that the star A in Andromeda which had disappeared came again into view in 1695: that in the same constellation instead of one in the knee marked ν there are two others come more northerly; and that the star ξ is grown much less: that the star placed by *Tycho* at the end of the chain of Andromeda as of the fourth magnitude was grown so small that it could scarcely be seen: that the star which in *Tycho's* catalogue is the 20th of the fishes is no longer visible, except it can be imagined to have changed its place and gone above 4 degrees lower, to the place marked \circ by *Bayer*: thus *Cassini*, in his *elements d'astronomie*, pag. 73, but the truth is, there is a false print in *Tycho's* catalogue, where this last mentioned star is set down in north latitude instead of south: *Flamsteed* has set it right in his edition: it is very probable, some other supposed changes in the stars are, like this, only mistakes of the transcriber or printer.

935 In 1672, *Dom. Cassini* saw a star in the neck of the bull which he thought was not visible in *Tycho's* time, nor when *Bayer* made his figures. *Halley*, in his catalogue of the southern stars, says, that two stars one in the pastern the other in the knee of the left fore-leg of Sagittary, reckoned of the 2d magnitude by *Ptolemy* and *Bayer*, were by him found scarcely of the 4th: that a star set down by *Ptolemy* in the left thigh, and another in the bending of the right hind-leg are now invisible: and that four of the unformed stars near the southern fish, set down by *Ptolemy* of the 3d magnitude, appeared to him of the 6th, or at most not above the 5th. In the year 1679 *Flamsteed* could not with the naked eye see the star (f) in aquarius, set down by *Ptolemy* of the 5th magnitude, and by *Tycho* and *Bayer* of the 6th: and in the year 1680 the star (n) in virgo was in vain sought for by him.

936 Many other changes in the stars have been taken notice of by *Maraldi*; as that the star α in the left leg of Sagittarius, marked by *Bayer* as of the third magnitude, appeared but of the 6th in 1671: in 1676 it was found again to be of the 3d by *Halley*; in 1692 it could hardly be perceived: but in 1693 and 1694 it was of the 4th magnitude. In the same constellation the star in the right arm reckoned by *Halley* of the 3d magnitude has been observed to be grown much less. That in the thigh called θ had disappeared; *Maraldi* saw it, from 1699 to 1709, of the 6th magnitude; he found it to be composed of two stars, distant from each other $35'$ in latitude^a. The star θ in the tail of the serpent, counted by *Tycho* of the 3d, was found by *Montanari*

^a Thus the *memoires*, and *Cassini* from them, but it must be a mistake; I suppose it should be 35 seconds.

of the 5th; but grew bigger in the following years. The star ρ in Serpentarius did not appear from the time of its being observed by *Montanari* to 1695. The star ψ in the lion, after disappearing, was seen by *Montanari* in 1667; *Maraldi* saw it in 1691, but very small. The star ξ of the same constellation, marked by *Tycho* and *Bayer* of the 3d magnitude, was hardly visible in 1693. The star (i) in the breast of the lion of the 6th order was not visible in 1709; but there might be seen thereabouts 8 stars not set down in the catalogues. The star β in Medusa's head was found to be of different magnitudes in different years by *Montanari*; *Maraldi* could perceive no change in it in 1693, but in 1694 it varied considerably, so as to be sometimes of the 2d magnitude, at other times of the 3d or 4th. The star γ in the ear of the great dog is set down by *Tycho* and *Bayer* of the 3d magnitude: it was invisible in the year 1670; but in 1692 and 1693 appeared of the 4th magnitude. *Maraldi* discovered 4 new stars in the great dog which are not in *Bayer's* catalogue.

937 In 1704, *Maraldi* discovered a star in the Hydra to be periodical; its position is in a right line with those two in the tail marked π and γ , at the same distance from π easterly as π is from ψ : this star had been described by *Montanari* in 1670, but was not visible in april 1702: *Maraldi* saw it for the first time in the beginning of march 1704, in the same place where *Montanari* had found it 34 years before: it appeared of the 4th magnitude, and brighter than the star ψ , and continued nearly in the same state till the beginning of april: it then gradually diminished till the end of may, when it could no longer be seen by the naked eye, but was visible through the telescope for a month longer: it could not be seen again till towards the end of november 1705, when that part of the heaven began to get out of the sun's rays; it was very faint, and grew less and less till the end of february 1706, and could then be scarce perceived even with a telescope: it did not appear again till the 18 of april N.S. 1708; when it was bigger than stars of the 6th order: it was then increasing, so as on the 11 of may to be equal to the last star but one of the Hydra: on the 16 and 20 it was still bigger; but the 5 of june, after being hid several days by cloudy weather and moon-shine, it was found to be grown less, and continued decreasing the following days; by reason of the evening twilight, it could not be seen without a telescope, which shewed it equal in magnitude to the brightest of the two stars which compose the last but one of the Hydra; this makes it probable, that it might have been seen some time longer, if it had not been among the vapours which are near the horizon: november the 23 1709 it appeared again of the same magnitude with the last star but one of the Hydra: in december it was equal to a star which is near to that last mentioned and cannot be seen without

a telescope: february the 7 1710 it could hardly be seen with the telescope: may the 24 1712 this star appeared again for the 5th time; it was a little smaller than the last but one of the Hydra: june the 9 it was equal to an unformed star near it, and could be seen in moon-shine: june 16 it was less, and afterwards disappeared. This star was seen by *Hevelius* in 1662 of the 5th magnitude: it seems to be a *periodical star*, like those in the whale and the swan: *Maraldi*, after comparing the several observations which had been made of it, judged the time between its appearing with its greatest lustre, or equal to a star of the 4th magnitude, and its return to the same phase again to be about 2 years; though with considerable variations: as is likewise the case of the periodical stars of the swan and the whale, which are liable to variations in their periods.

938 Besides these, in the *memoires* for the year 1709, *Maraldi* says he has observed other changes in the stars; of which these four are the most considerable: 1, of two stars set down by *Bayer* under the southern hand of virgo one of the 6th magnitude the other of the 5th, the last is still to be seen of the same magnitude; but that of the 6th, which was a degree more south, could not be seen with the telescope. 2, *Ricciolus* sets down a star of the 6th magnitude in the northern thigh of virgo which *Bayer* has omitted, and is not now to be seen. 3, A star of the 6th magnitude set down by *Bayer* in the western scale, in $12^{\circ} 20'$ of η with 3° north latitude, is invisible. 4, In 1666, *Hevelius* says he could not find a star of the 4th magnitude in the eastern scale of libra, taken notice of by *Tycho* and *Bayer*, but *Maraldi* in 1709 says he had seen it for near 15 years last past, smaller than *Tycho* and *Bayer* had found it, but brighter than some other stars near it set down by *Hevelius*. A star of the 4th magnitude first discovered by *Dom. Cassini* near the constellation of the hare appeared in the same state in 1709.

939 *Gregory*, in his astronomy, book 2 prop. 30, says that 'Some stars of the 6th, 5th and 4th magnitude set down in the ancient catalogues and observed by *Tycho*, are now quite vanished; we have several instances of this in the catalogue of *Hevelius*: he mentions four in his *prodromus astronomicus*; as one in the left thigh of Aquarius, the first of the two adjoining stars in the tail of Capricorn, the second of the belly of the whale, and the first of the unformed stars behind the scales of libra. The French astronomers have made farther observations of these stars. There have been also remarkable changes in the brightness of some stars of the first 2d and 3d order, as appears by the difference amongst authors in assigning their magnitude.' To name no more, a notable change has happened to a star in the great bear marked δ by *Bayer*, within these few years; it is now scarce

of

of the 3d magnitude, whereas it is set down by *Tycho* and *Bayer* as of the second.

940 *Cloudy stars* are small luminous spots in the heaven, some of which appear to the naked eye like dim stars surrounded by an hazy light; others like little whitish clouds, pretty much resembling the milky way in brightness and colour. *Ptolemy* sets down five cloudy stars: 1, one at the extremity of the right hand of *Perseus*: 2, one in the middle of the crab, called *præsepe* or the manger: 3, one unformed, near the sting of the scorpion: 4, the eye of *Sagittary*, which he calls cloudy and double: 5, one in the head of *Orion*. The first of these, that in the hand of *Perseus*, appears through the telescope so thick set with stars, that I believe the like is hardly to be found in any other part of the heaven: *Galileo*^a counted 21 stars in that in the head of *Orion*; and above 40 in the *præsepe*: in the eye of *Sagittary* two stars may be seen in a clear sky by the naked eye, the telescope shews several more. *Flamsteed* takes notice of a cloudy star before the bow of *Sagittary*, which consists of a great number of small stars: and that the star (d) above the right shoulder of *Sagittary* is encompassed with small stars after the manner of a cloudy star. *Dom. Cassini* discovered a little patch between the great and little dog, which he says is very full of stars; the same was observed also by *Flamsteed*. The two whitish spots near the south pole called *the Magellanic clouds*, which seen by the bare eye exactly resemble the milky-way, viewed by *Halley* through a telescope appeared to be a mixture of small clouds and small stars.

941 There are also in the heaven little whitish spots, which through our best glasses appear magnified and more luminous, but without any stars: of this sort is a spot near the most northern star in the girdle of *Andromeda*, first taken notice of in 1612 by *Simon Marius*; it appeared to his naked eye like a little cloud, but through the telescope he saw white rays which were brightest near the center of the spot: its diameter was about a quarter of a degree. *Bullialdus* observed it in 1664, and judged it liable to such changes as to be sometimes invisible: it is remarked in confirmation of this opinion, that it was represented in some figures of the constellations drawn about the year 1500, but is not set down either by *Tycho* or *Bayer*; that it was seen in 1612, but from that time till 1664 it was not taken notice of by any astronomer; and that in the year 1667 it was less bright than the year before: ever since the year last mentioned it has been constantly visible, and is so this present year 1741 to the naked eye; but is now not near so much as a quarter of a degree in diameter. — Another cloudy spot is near the ecliptic, between the head and bow of *Sagittary*, not far from the point of the

^a in nuncio sydereæ.

winter solstice: it is small but very luminous.—A third is upon the back of the centaur, marked ω by *Bayer*: but cannot be seen in England.—A fourth is a small obscure spot, but there is a star shines through it, which makes it more bright: it precedes the right foot of Antinous, it was discovered by *Kirchius*, who could not see it with the bare eye, but saw it with a telescope of 4 feet.—A fifth is in the constellation of Hercules, nearly in a right line between the stars ζ and η , somewhat nearer ζ than η : it is but a little spot, but visible to the bare eye when the sky is clear and the moon absent.

942 But of all the cloudy stars hitherto taken notice of none is so remarkable as that described by *Hugens* in words to this effect^a; ‘Looking accidentally in the year 1656 through a telescope at the middle star in Orion’s sword, I saw twelve stars instead of one, which was no new thing; their position was such as is shewn in the figure annexed: seven of these stars, of which three are very close together, seemed to shine through a cloud, so that a space round them of the shape represented in the figure appeared much brighter than any other part of the heaven; which, being very serene and very black, looked here as if there were an opening through which one had a prospect into a brighter region: I have since often viewed this wonderful appearance, which continues the same without any change of place or shape to this time, (1659) This phenomenon, which was observed by *Hugens* with an excellent telescope of 23 feet, I have often viewed with one of 17 feet, through which I could see very well all the 12 stars mentioned by him: see what is said § 887; to which I shall only add, that the luminous space has sometimes appeared to me nearly of the same shape with the figure which is formed by the seven stars within it.

There are undoubtedly more of these spots, which have not yet been taken notice of: *Cassini* is of opinion, that the brightness of them proceeds from stars so minute as not to be distinguished by the best glasses^b; others more probably think they are great spaces in the *ether* through which a lucid medium is diffused: they appear indeed small to us, most of them but of few minutes in diameter; yet since they are among the fixt stars, that is, since they have no annual parallax, they cannot fail to occupy spaces immensely great, and perhaps not less than our whole solar system: in all these so vast spaces it should seem, that there is a perpetual uninterrupted day, which may furnish matter of speculation, as well to the curious naturalist, as to the astronomer^c.

943 I have here given as full an account as I could of changeable and cloudy stars, from several writers; by comparing of all which together, I

^a *System. saturn.* p. 8.

^b *elements d’astronomie* l. 1. c. 7.

^c *Phil. trans. abr.* vol. 4 p. 225.
have

have corrected the mistakes that some of them had fallen into: I have sometimes quoted my author, but have not done it in every instance, to avoid being tedious: for the rest, these that follow may be consulted; Tychonis *progymnasmata*: Keplerus *de nova stella in pede Serpentarii*: Ricciol. *almag.* l. 8. § 2. c. 1. Hevelii *historia miræ stellæ: ejusdem cometographia*, pag. 374 *et sequ.* & 400: Cassini *elements d'astronomie* l. 1. c. 6 & 7: Phil. transf. abr. vol. 1: *Acta eruditor. Lipsiæ ann.* 1688: Flamsteedii *historia cælestis* vol 1: Halleii *catalogus stellarum australium: Memoires de l'Acad. R. des sciences ann.* 1706 & 1709.

944 When the new stars were first observed, the *Aristotelians*, who held that the heavens and heavenly bodies were not subject to any change, would have it that they were only meteors generated in our atmosphere, and raised up to a great height therein^a: but their having no sensible parallax was an evident demonstration to the contrary, and shewed that they were far above the moon: and their agreeing with the stars in some other particulars as well as in their great distance from the earth, has induced the generality of philosophers to think they are of a similar nature.

945 How the changes in those stars that are periodical may be effected, has been in some measure shewn, § 928 and 929. *Maupertuis*^b has another solution of these appearances, which is to this effect: 1, he supposes many of the stars, like the sun, to have a rotation round their own axes: 2, that this rotation causes them to deviate from an exact globular shape into that of an oblate spheroid, or globe flattened: 3, that there may be infinite variety in the degrees of this flatness, so that some may be quite thin and flat like a round trencher: 4, that a star of this shape may have a large planet going round it, in a plane different from that of the stars equator: in this case, the planet would be twice in every revolution round the star in the plane of the star's equator continued, at which times it would cause no change in the situation of that plane; but at all other times it would, by its attraction, change more or less the situation of the plane of the star's equator, and most of all at the two seasons when it is farthest out of the plane continued: by this change, he supposes that such a flat star as at some times cannot be seen by us, when only its thin edge is turned towards our earth, may at other times, by turning a good deal of its flat side towards us, become visible. *He-*

^a *Bullialdus* gives us a pleasant instance of the philosophical bigotry of a Florentine Physician, who would not be prevailed upon to look at any of the heavenly bodies through a telescope himself, for fear of seeing something which should oblige him to desert some of *Aristotle's* opinions: and as for any discoveries made by glasses, which others told him of, he would have no regard at all to them. *apud Hevelium in cometograph. p.* 329.

^b *Discours sur les différentes figures des astres. v. histoire de l'Acad. R. des sciences ann.* 1732. & *acta eruditor. mensis julii* 1733.

velius is of opinion, 1, that the sun and stars are surrounded with atmospheres: 2, that, whirling round their axes with great rapidity, they throw off great quantities of matter into those atmospheres, and thereby cause great changes therein: 3, and that thus it may come to pass, that a star, which when its atmosphere is clear shines out with great lustre, may at another time, when it is full of clouds and thick vapours, appear greatly diminished in brightness and magnitude, or even become quite invisible^a.

946 As for such stars as, after having been for many years invisible, break out suddenly into shining all at once, like those in Cassiopea and Serpentarius, it has been thought that, as the sun may have now and then a comet fall into it, in the manner described § 909; the same may also happen to any of the stars which like the sun have planets and comets revolving round them: and then a star which had for a long time been so covered over with spots as not to be visible to us may, when a comet falls into it, be made to blaze out suddenly, by the access of that new fuel; and may for a while continue to shine with great lustre; till, by the spots returning and covering its surface, it appears gradually to diminish in brightness and magnitude, and at last becomes again quite invisible. *Fontinelle*, upon mentioning changeable stars, has this reflection^b; 'The fixt stars being so many suns, it is very possible that among the infinite number of them there may be some *half-suns*^c; our sun has often spots upon his surface, which, if they were permanent and more extensive, would reduce him to an half-sun: if all the suns are surrounded with planets inhabited, it is easy to imagine what a fright the inhabitants must be in, when they loose sight of their sun for a considerable time; or rather with what calmness they behold a sight which is familiar to them but would be dreadful to us. The stars which appear and disappear without any regular periods may be suns that are sometimes covered with very large spots, from which at other times they are perfectly free: the changes in the spots upon the sun give us room to think there may be great variety of causes in this affair:' but whatever be the case of periodical stars, which may have planets inhabited by creatures fitted for such vicissitudes of light and darkness as are allotted to them, I do not think it probable that a star which for a whole century does not cast light enough to be seen by us should be appointed to do the office of a sun, or have any planetary worlds to be enlightened and cherished by it: what other ends it may be made for, it may be in vain for us to conjecture: in the works of creation, as well as in the ways of providence, the designs of infinite wisdom, and the methods by which infinite power brings about those designs are often to man unsearchable.

^a *Hevelii cometograph. p. 380.*

^b *histoire de l'acad. ann. 1706.*

^c *demi-soleils.*
A D D E N D A.

A D D E N D A.

Since the sixteenth chapter of the second book was printed off, wherein is shewn, that there is great probability of the obliquity of the ecliptic continually decreasing, two books have come to hand which have confirmed me in that opinion; one of them is *de Bononiensi scientiarum et artium instituto atque Academia commentarii*, Bononiæ 1731: together with *Academicorum quorundam opuscula varia*, in 4to: in page 258, there is an account of a review of the meridian line at Bologna mentioned § 423, by *Eustachio Manfredi*: that line, which is drawn upon brass plates inlaid into the pavement, was found by him to have continued exactly in the plane of the meridian, but, as to its horizontal situation, it was risen up too high in some places, and worn down too low in others, by peoples walking over it: in order to make allowances for this change, *Manfredi* made a table, by which he corrected his observations: after these corrections, the height of the pole there came out $44^{\circ} 29'$ and $20''$ or $25''$, the same as *Ricciolus* and *Cassini* had formerly settled it; but he found the angle between the equator and ecliptic about $30''$ less than *Cassini* had observed it, at the time of his first drawing that line, which was between 60 and 70 years before: he recommends drawing a meridian line upon an upright wall, in order to avoid the alterations one made upon a pavement is subject to, by its being trodden upon. The treatise of *Manfredi* upon this subject, read at the Academy in the year 1722, is among the *opuscula*, page 596. The other book is *Elements d'astronomie*, par Mr. *Cassini*, Paris 1740, wherein l. 2. chap. 3. that author says, that, comparing together a great number of observations made in the R. Observatory for 66 years past, the obliquity of the ecliptic is found to have decreased about $30''$ in that space of time. He mentions the observations of the ancients, and gives a table of the obliquity according to several authors, both as they observed it, and as it would have come out according to a regular annual decrease: I have occasion to add nothing from him to my table given pag. 281, except that the obliquity was observed by *Richer* in the year 1672 to be $23^{\circ} 28' 54''$, and at the R. Observatory 1738, $23^{\circ} 28' 20''$. He does not say by whom this last observation was made; the numbers are the same as those of *Godin* in 1730.

The End of the first Volume.

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