Daniel
Henry
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EUCLIDIS
ELEMENTORUM
Libri xv. breviter demonſtrati,

Opera

Is. BARROW, *Cantabrigienſis*,
Coll. TRIN. *Soc.*

Καθαρμοὶ ψυχῆς λογικῆς εἰσὶν αἱ μαθηματικαὶ ἐπιστῆμαι. HIEROCL.

LONDINI,
Excudebat R. DANIEL, Impenſis
GUIL. NEALAND Bibliopolæ
Cantabrig. clↄ Iↄc LIX.

Nobilissimis & Generosissimis Adolescentibus,

D^no^ EDOUARDO CECILIO,

Illustriss. Comitis Sarisburiensis *Filio*;

D^no^ *IOHANNI KNATCHBVL*,

Et

D. *FRANCIS. WILLOVGHBY*,

ARMIGERIS.

UNicuique vestrum (Optimi Adolescentes) tantum me debere reputo, quantum homo homini debere potest. Mea enim sententia, ultra sincerum amorem non est quod

quiſpiam de alio bene mereri poſſit. Hunc autem jamdiu eſt quo ex ſingulari veſtra bonitate mihi indultum experior ; ejuſque ſenſus, intimis animi medullis inhærens, ipſi ardens ſtudium impreſſit quovis honeſto modo reciprocos affectus prodendi. Quandoquidem vero ea fortunarum mearum tenuitas, ea veſtrarum amplitudo, exiſtit, ut nec ego alia quam gratæ alicujus agnitionis ſignificatione uti queam, nec vos aliam admittere velitis ; ea propter haud illibenter hanc occaſionem arripio, honoris & benevolentiæ, quibus vos proſequor, publicum hoc & dura-
bile

bile μνημόσυνον edendi. Etſi cum oblati anathematis exilitatem, & libellum veſtris nominibus conſecratum, quam is longe infra veſtrorum meritorum dignitatem ſubſidat, attentius conſidero, timor ſubinde aliquis & dubitatio animum inceſſant, ne hoc ſtudium erga vos meum vobis dehoneſtamento ſit potius quam ornamento; ſcilicet memor cum ſim, ut malæ cauſæ, ſic & mali libri patrocinium in patroni contumeliam magis quam in gloriam cedere. Sed quum veſtrarum virtutum id robur, eam fore ſoliditatem, recognoſcerem, quæ veſtrum decus, meo quantumvis labefactato, inconcussum sustinere possint; idcirco non dubitavi vos in aliquatenus commune mecum periculum induere. Virtutes illas intelligo, quibus nemo unquam in vestra ætate aut in vestro ordine, saltem me judice, majores deprehendit; quæ vos insigniter gratos omnibus & amabiles reddunt; eximiam modestiam, sobrietatem, benignitatem animi, morum comitatem, prudentiam, magnanimitatem, fidem, præclaram insuper ingenii indolem, quæ vos ad omnem ingenuam scientiam non tantum excellenti captu, sed & appetitu forti ac sincero, instruxit. Quas vestras præclarissimas dotes prout nemo est fortassis qui me melius novit, aut pro consuetudine, quam jamdudum vobiscum dulcissimam coluisse ex vestro favore mihi contigit, penitus introspexit, ita nemo est qui impensius miratur & suspicit; aut qui ipsas libentius prædicare ac celebrare vellet, si non cum eloquii mei vires supergrederentur, tum etiam quæ in singulis vobis elucent, prolixi alicujus commentarii aut panegyricæ orationis libertatem, potius quam præstitutas hujusmodi salutationibus angustias, exposcerent. Quin potius divinam clementiam imploro, ut vos earundem virtutum sancto tramiti insiſtere, atque hos egregios fructus vernæ veſtræ ætatis felicibus incrementis matureſcere concedat; vitamque vobis in hoc ſeculo ingenuam, innocentem, jucundam, & in futuro beatam ac ſempiternam tranſigere largiatur. Minime autem dubito, ne pro conſueto veſtro in me candore hoc ultimum fortaſſis quod vobis præſtare potero, benevolentiæ erga vos & obſervantiæ teſtimonium, alacriter accepturi ſitis; quod vobis propenſiſſimo affectu offert

Veſtri in æternum amantiſsimus,

& obſervantiſsimus,

I. B.

Benevolo LECTORI.

SI quid in hac elementorum editione præstitum sit, scire desideras, amice Lector, accipe, pro genio operis, breviter. Ad duos præcipue fines conatus meos direxi. Primum, ut cum requisita perspicuitate summam demonstrationum brevitatem conjungerem, quo eam libello molem compararem, quæ commode absque molestia circumferri posset. Id quod assecutus videor, si absentem Typographi cura non frustretur. Concinnius enim quispiam meliori ingenio aut majori peritia excellens, at nemo forsan brevius plerasque propositiones demonstraverit; præsertim cum in numero & ordine propositionum ipse nihil immutarim, nec licentiam mihi assumpserim quamcunque propositionem Euclideam procul ablegandi tanquam minus necessariam, aut quasdam faciliores in axiomatum censum referendi; quod nonnulli fecerunt: inter quos peritissimus Geometra Andr. Tacquetus, quem ideo etiam nomino, quod quædam ex eo desumpta agnoscere honestum duco; post cujus elegantissimam editionem, ipse nihil atten-

tare

tare voluissem, si non visum fuisset doctissimo viro non nisi octo Euclidis *libros suâ curâ adornatos publico communicare, reliquis septem, tanquam ad elementa Geometriæ minus spectanctibus, omnino quasi spretis atque posthabitis. Mihi autem jam ab initio alia provincia demandata fuit, non elementa Geometriæ utcunque pro arbitrio conscribendi, verum* Euclidem *ipsum, eumque totum, quam possem brevissime, demonstrandi. Quod enim quatuor libros spectat, septimum, octavum, nonum, decimum, quamvis illi ad Geometriæ planæ & solidæ elementa, ut sex præcedentes & duo subsequentes, non tam prope pertineant; quod tamen ad res Geometricas admodum utiles sint, tam propter Arithmeticæ & Geometriæ valde propinquam cognationem, quam ob notitiam commensurabilium & incommensurabilium magnitudinum ad figurarum tam planarum quam solidarum intellectum apprime necessariam, nemo est è peritioribus Geometris qui ignorat. Quæ vero in tribus ultimis libris continetur, 5 corporum regularium nobilis contemplatio, illa non nisi injuria prætermitti potuit; quando nempe illius gratia noster* στοιχειωτὴς, *Platonicæ familiæ philosophus, hoc elementorum systema universum condidisse perhibetur;*

uti

uti testis est * Proclus, *iis verbis*, Ὅθεν δὴ καὶ τῆς συμπάσης στοιχειώσεως τέλος προεστήσατο τὴν τῶν καλουμένων πλατωνικῶν σχημάτων σύστασιν. *Præterea facile in animum induxi ut opinarer, nemini harum scientiarum amanti non futurum esse cordi penes se habere integrum* Euclidæum *opus, quale passim ab omnibus citatur & celebratur. Quare nullum librum nullamque propositionem negligere volui earum quæ apud* P. Herigonium *habentur; cujus vestigiis presse insistere necesse habui, quoniam ejusce libri schematismis maxima ex parte uti statutum erat, quod præviderem mihi ad novas describendas tempus non suppetere; etsi nonnunquam id facere praoptassem. Eadem de causa nec alias plerasque quam* Euclidæas *demonstrationes adhibere volui, succinctiori forma expressas, nisi forte in 2, & 13, & parce in 7, 8, 9 libris; ubi ab eo nonnihil deflectere operæ pretium videbatur. Bona igitur spes est saltem in hac parte cum nostris consiliis, tum studiosorum votis, aliquo modo satisfactum iri. Nam quæ adjecta sunt in Scholiis problemata quædam & theoremata, sive ob suum frequentem usum ad naturam elementarem accedentia, sive ad eorum quæ sequuntur expeditam demonstrationem conducentia, seu quæ regula-*

* lib. 2.

rum practicæ Geometriæ quarundam præcipuarum rationes innuunt ad suos fontes relatas, per ea, ut spero, libellus ultra destinatam molem magnopere non intumescet.

Alter scopus ad quem collineatum est, eorum desideriis consuluit qui demonstrationibus symbolicis potius quam verbalibus delectantur. In quo genere cum plerique apud nos Guilielmi Oughtredi *symbolis assueti sint, ea plerumque usurpare consultius duximus. Nam qui* Euclidem *hâc viâ tradere & interpretari aggressus sit, hactenus, quod ego sciam, præter unum* P. Herigonium, *repertus est nemo. Cujus viri longe doctissimi methodus, sane in multis egregia, ac ejus peculiari proposito admodum accomodata, duplici tamen defectu laborare mihi visa est. Primo, quod cum Propositionum ad unius alicujus theorematis aut problematis probationem adductarum posterior à priori non semper dependeat; quando tamen illæ inter se cohærent, quando non, nec ex ordine singularum, nec ullo alio modo, satis prompte innotescere potest: unde ob defectum conjunctionum & adjectivorum* (ergo, rursus, *&c.) non raro difficultas & dubitandi occasio, præsertim minus exercitatis, inter legendum oboriri solent. Deinde sæpenumero evenit, ut prædicta methodus supervacaneas repetitiones effugere nequeat, à quibus demonstrationes est quando prolixæ, aliquando*

&

& magis intricata, evadunt. Quibus vitiis noster modus facile per verborum signorumq; arbitrariam mixturam medetur. Atque hæc de opellæ hujus intentione & methodo dicta sufficiant. Cæterum quæ in laudem Matheseos in genere, aut Geometriæ ipsius; & quæ de historia harum scientiarum, ideoque de Euclide *horum elementorum digestore, dici possent, & reliqua hujusmodi ἐξωτερικά, cui hæc placent, apud alios interpretes consulere potest. Neque nos angustias temporis quod huic operi impendi potuit, nec interpellationes negotiorum, nec adjumentorum ad hæc studia apud nos egestatem, & quædam alia, ut liceret non immerito, in excusationem obtendemus; metu scilicet inducti, ne hæc nostra omnibus minus satisfaciant. Verum quæ ingenui Lectoris usibus elaboravimus, eadem in solidum ipsius censuræ ac judicio submittimus; probanda si utilia sibi compererit; sin omnino secus, reiicienda.*

I. B.

Ad amicissimum Virum, *I. B.* de *EVCLIDE* contracto

Εὐφημισμός.

FActum bene! didicit Laconice loqui
Senex profundus, & aphorismos induit.
Immensa dudum margo commentarii
Diagramma circuit minutum; utque Insula
Problema breve natabat in vasto mari.
Sed unda jam detumuit; & glossa arctior
Stringit Theoremata: minoris anguli
Lateribus ecce totus Euclides *jacet,*
Inclusus olim velut Homerus in nuce;
Pluteoque sarcina modo qui incubuit, levis
En fit manipulus. Pelle in exigua latet
Ingens Mathesis, matris ut in utero Hercules,
In glande quercus, vel Ithaca Eurus in pila.
Nec mole dum decrescit, usu fit minor;
Quin auctior jam evadit, & cumulatius
Contracta prodest erudita pagina.
Sic ubere magis liquor è presso effluit;
Sic pleniori vasa inundat sanguinis
Torrente cordis Systole; sic fusius
Procurrit æquor ex Abylæ angustiis.
Tantilli operis ars tanta referenda unice est
BAROVIANO nomini, ac solertiæ.
Sublimis euge mentis ingenium potens!
Cui invium nil, arduum esse nil solet.
Sic usque pergas prospero conamine,
Radiusque multum debeat ac abacus tibi;
Sic crescat indies feracior seges,
Simili colonum germine assiduo beans.
Specimen futuræ messis hic stet liber,
Magnæque famæ illustria hæc præludia.
Iuvenis dedit qui tanta, quid dabit senex?

Car. Robotham, *CANTAB.*
Coll. Trin. Sen. *Soc.*

In novam *Elementorum*

EVCLIDIS

Editionem à D. *IS. BARROW*, *Collegii SS.* TRIN. *Socio*, viro opt. & eruditissimo, adornatam.

BEnigne Lector! si uspiam auditum est tibi,
Quantus tenella Nix Geometres siet;
Quæ mille radiis, mille ludit angulis,
Totumque puro ducit Euclidem sinu:
Amabis ultro candidissimum Virum,
Cui plena nivium est indoles, sed quas tamen
Præclarus ardor mentis urget Enthea;
Et usque blandis temperat caloribus:
Quo suavius nil vivit, & melius nihil.
Is, dum liquentes pectore excutit nives,
Et inde & inde spargit, en aliam tibi,
Lector benigne, è nivibus Geomertiam!

G.C. *A.M. C.E.S.*

Notarum explicatio.

=	æqualitatem.	significat.
⊏	majoritatem.	
⊐	minoritatem.	
+	plus, vel addendum esse.	
—	minus, vel subtrahendum esse.	
∹	differentiam vel excessum; item quantitates omnes, quæ sequuntur, subtrahendas esse, signis non mutatis.	
×	multiplicationem, vel ductum lateris rectanguli in aliud latus.	
	Idem denotat conjunctio literarum, ut AB=A×B.	
√	Latus, vel radicem quadrati, vel cubi, &c.	
	Q. & q quadratum. C. & c cubum.	
	Q. Q. rationem quadrati numeri ad quadratum numerum.	

Reliquas, quæ ubicunque occurrunt, vocabulorum abbreviationes ipse Lector per se facile intelliget; exceptis iis, quas tanquam minus generalis usus, suis locis explicandas relinquimus.

LIB.

LIB. I.

Definitiones.

I. PUnctum est cujus pars nulla est.

II. Linea vero longitudo latitudinis expers.

III. Lineæ autem termini sunt puncta.

IV. Recta linea est, quæ ex æquo sua interjacet puncta.

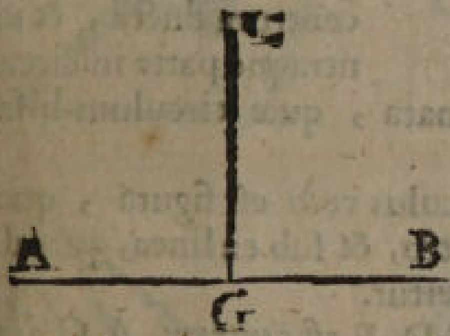
V. Superficies est, quæ longitudinem, latitudinemque tantum habet.

VI. Superficiei autem extrema sunt lineæ.

VII. Plana superficies est, quæ ex æquo suas interjacet lineas.

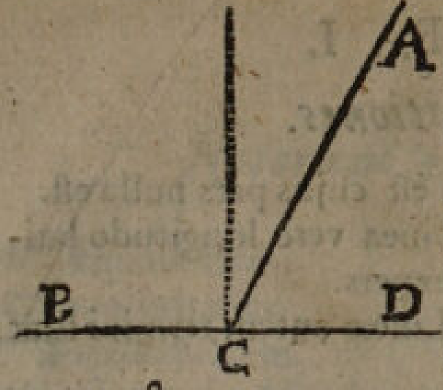
VIII. Planus vero angulus est, duarum linearum in plano se mutuo tangentium, & non in directum jacentium alterius ad alteram inclinatio.

IX. Cum autem quæ angulum continent, lineæ, rectæ fuerint, rectilineus ille angulus appellatur.



X. Cum vero recta linea C G super rectam lineam A B consistens, eos qui sunt deinceps angulos C G A, C G B æquales inter se fecerit, rectus est uterque æqualium angulorum, & quæ insistit recta linea C G, perpendicularis vocatur ejus (A B) cui insistit.

Not. *Cum plures anguli ad unum punctum: (ut ad* G) *exsistunt, designatur quilibet angulus tribus literis, quarum media ad verticem est: illius de quo agitur: ut angulus quem rectæ* C G, A G *efficiunt ad partes* A *vocatur* C G A, *vel* A G C.



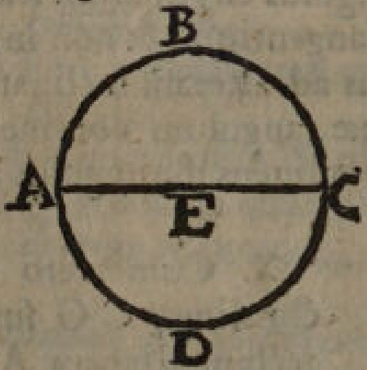
XI. Obtusus angulus est, qui recto major est, ut A C B.

XII. Acutus vero, qui minor est recto, ut A C D.

XIII. Terminus est, quod alicujus extremum est.

XIV. Figura est, quæ sub aliquo, vel aliquibus terminis comprehenditur.

XV. Circulus est figura plana, sub una linea comprehensa, quæ peripheria appellatur, ad quam ab uno puncto eorum, quæ intra figuram sunt posita, cadentes omnes rectæ lineæ inter se sunt æquales.



XVI. Hoc vero punctum centrum circuli appellatur.

XVII. Diameter autem circuli est recta quædam linea per centrum ducta, & ex utraque parte in circuli peripheriam terminata, quæ circulum bifariam secat.

XVIII. Semicirculus vero est figura, quæ continetur sub diametro, & sub ea linea, quæ de circuli peripheria aufertur.

In circulo E A B C D. E *est centrum,* A C *diameter,* A B C *semicirculus.*

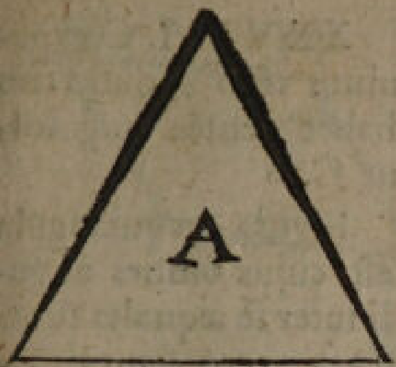
XIX. Rectilineæ figuræ sunt, quæ sub rectis lineis continentur.

XX. Trilateræ quidem, quæ sub tribus.

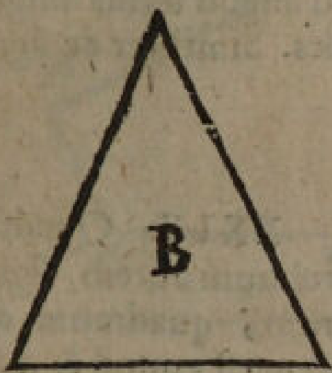
XXI. Quadrilateræ vero, quæ sub quatuor.

XXII. Multilateræ autem, quæ sub pluribus, quam quatuor rectis lineis comprehenduntur.

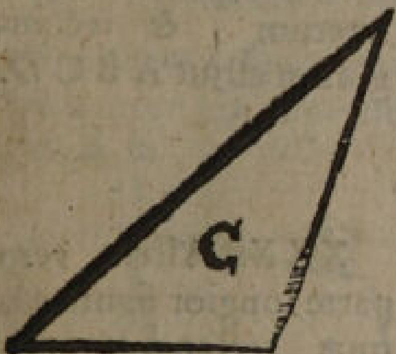
XXIII.



XXIII. Trilaterarum autem figurarum, æquilaterum eſt triangulum, quod tria latera habet æqualia, ut triangulum A.



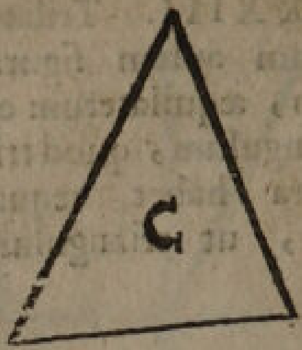
XXIV. Iſoſceles autem, quod duo tantum æqualia habet latera, ut triangulum B.



XXV. Scalenum vero, quod tria inæqualia habet latera, ut C.

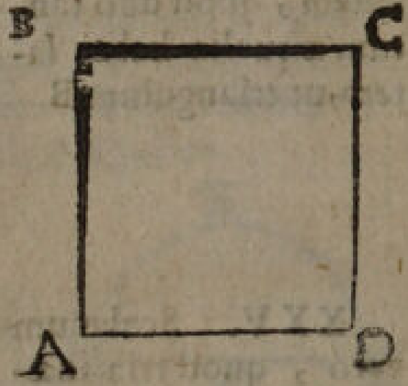
XXVI. Adhæc etjam trilaterarum figurarum, rectangulum quidem triangulum eſt, quod rectum angulum habet, ut triangulum A.

XXVII. Amblygonium autem, quod obtuſum angulum habet, ut B.

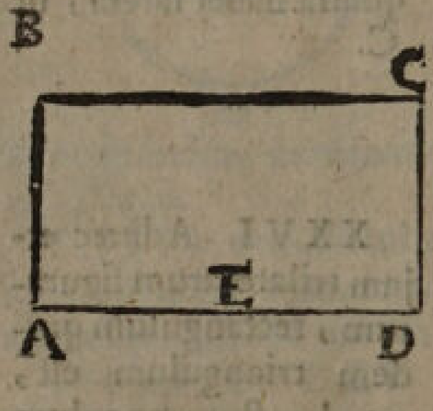


XXVIII. Oxygonium vero, quod tres habet acutos angulos, ut C.

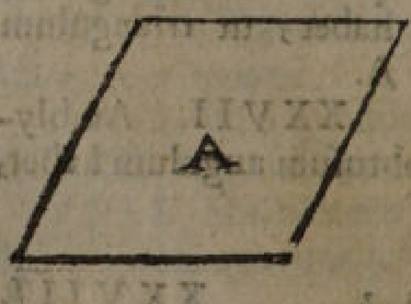
Figura æquiangula eſt, cujus omnes anguli inter ſe æquales ſunt. Duæ vero figuræ æquiangulæ ſunt; ſi ſinguli anguli unius ſingulis angulis alterius ſint æquales. Similiter de figuris æquilateris concipe.



XXIX. Quadrilaterarum autem figurarum, quadratum quidem eſt, quod & æquilaterum, & rectangulum eſt; ut A B C D.

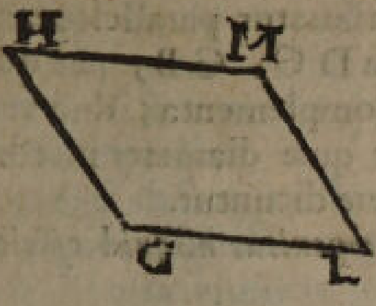


XXX. Altera vero parte longior figura eſt, quæ rectangula quidem, at æquilatera non eſt, ut A B C D.



XXXI. Rhombus autem, quæ æquilatera, ſed rectangula non eſt, ut A.

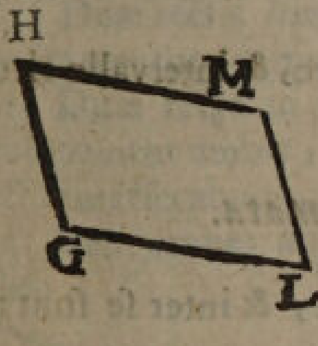
XXXII.



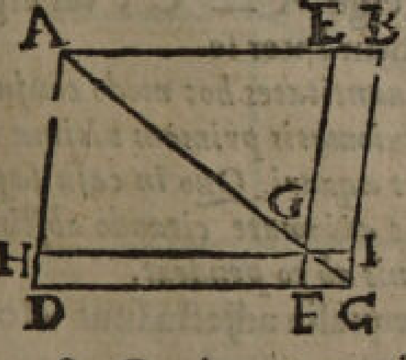
XXXII. Rhomboydes vero, quæ adversa & latera, & angulos habens inter se æquales, neque æquilatera est, neque rectangula, ut G L M H.

XXXIII. Præter has autem reliquæ quadrilateræ figuræ trapezia appellentur; ut G N D H.

XXXIV. Parallelæ rectæ lineæ sunt, quæ cum in eodem sint plano, & ex utraque parte in infinitum producantur, in neutram sibi mutuo incidunt, ut A, & B.



XXXV. Parallelogrammum est figura quadrilatera, cujus bina opposita latera sunt parallela, seu æquidistantia, ut G L H M.



XXXVI. Cum vero in parallelogrammo A B C D diameter A C ducta fuerit, duæque lineæ E F, H I, lateribus parallelæ secantes diametrum in uno eodemque puncto G, ita ut parallelogrammum ab hisce parallelis in quatuor distribuatur parallelogramma; appellantur duo illa D G, G B, per quæ diameter non transit, Complementa; duo vero reliqua H E, F I, per quæ diameter incedit, circa diametrum consistere dicuntur.

Problema est, *cum proponitur aliquid efficiendum.*

Theorema est, *cum proponitur aliquid demonstrandum.*

Corollarium est *consectarium, quod è facta demonstratione tanquam lucrum aliquod colligitur.*

Lemma est *demonstratio præmissæ alicujus, ut demonstratio quæsiti evadat brevior.*

Postulata.

1. POstuletur, ut à quovis puncto ad quodvis punctum rectam lineam ducere concedatur.

2. Et rectam lineam terminatam in continuum recta producere.

3. Item, quovis centro, & intervallo circulum describere.

Axiomata.

1. Quæ eidem æqualia, & inter se sunt æqualia.

ut A = B = C. ergo A = C, vel ergo omnes A, B, C, æquantur inter se.

Nota, *Cum plures quantitates hoc modo conjunctas invenias, vi hujus axiomatis primam ultimæ & quamlibet earum cuilibet æquari. Quo in casu sæpe, brevitatis causa, ab hoc axiomate citando abstinemus; etsi vis consecutionis ab eo pendeat.*

2. Et si æqualibus æqualia adjecta sunt, tota sunt æqualia.

3. Et

3. Et si ab æqualibus æqualia ablata sunt, quæ relinquuntur sunt æqualia.

4. Et si inæqualibus æqualia adjecta sint, tota sunt inæqualia.

5. Et si ab inæqualibus æqualia ablata sint, reliqua sunt inæqualia.

6. Et quæ ejusdem vel æqualium sunt duplicia, inter se sunt æqualia. Idem puta de triplicibus, quadruplicibus, &c.

7. Et quæ ejusdem, vel æqualium sunt dimidia, inter se sunt æqualia. Idem concipe de subtriplis, subquadruplis, &c.

8. Et quæ sibi mutuo congruunt, ea inter se sunt æqualia.

Hoc axioma in rectis lineis, & angulis valet conversum, sed non in figuris, nisi illæ similes fuerint.

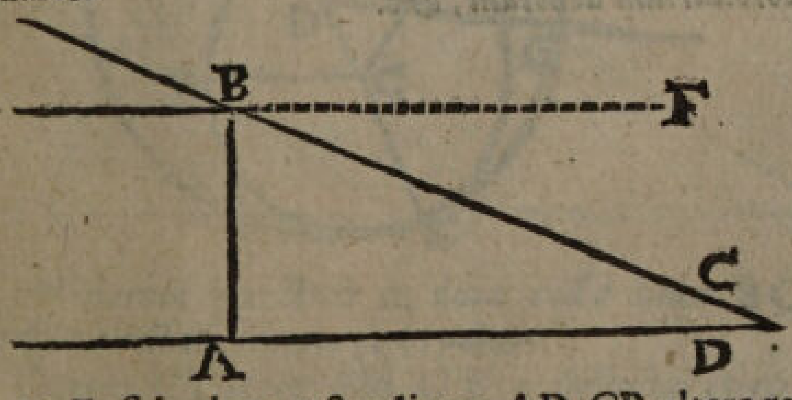
Cæterum, magnitudines congruere dicuntur, quarum partes applicatæ partibus, æqualem vel eundem locum occupant.

9. Et totum sua parte majus est.

10. Duæ rectæ lineæ non habent unum & idem segmentum commune.

11. Duæ rectæ in uno puncto concurrentes, si producantur ambæ, necessario se mutuo in eo puncto intersecabunt.

12. Item omnes anguli recti sunt inter se æquales.



13. Et si in duas rectas lineas AD, CB, altera recta BA incidens, internos ad easdemque partes

angulos B A D, A B C duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ ſibi mutuo incident ad eas partes, ubi ſunt anguli duobus rectis minores.

14. Duæ rectæ lineæ ſpatium non comprehendunt.

15. Si æqualibus inæqualia adjiciantur, erit totorum exceſſus adjunctorum exceſſui æqualis.

16. Si inæqualibus æqualia adjungantur, erit totorum exceſſus exceſſui eorum, quæ à principio, æqualis.

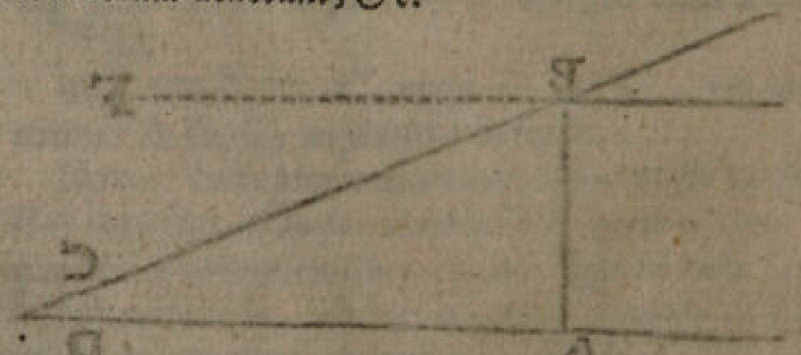
17. Si ab æqualibus inæqualia demantur, erit reſiduorum exceſſus, exceſſui ablatorum æqualis.

18. Si ab inæqualibus æqualia demantur, erit reſiduorum exceſſus exceſſui totorum æqualis.

19. Omne totum æquale eſt omnibus ſuis partibus ſimul ſumptis.

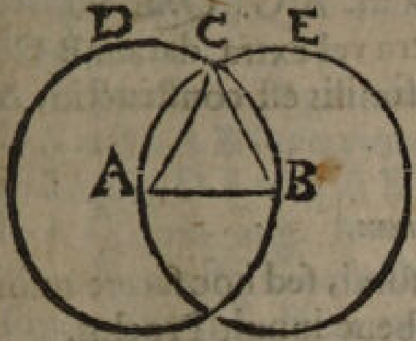
20. Si totum totius eſt duplum, & ablatum ablati, erit & reliquum reliqui duplum. Idem de reliquis multiplicibus intellige.

Citationes intellige ſic. Cum duo numeri occurrunt, prior deſignat propoſitionem, poſterior librum. Ut per 4. 1. intelligitur quarta propoſitio primi libri, atque ita de reliquis. *Cæterum, ax. axioma, poſt. poſtulatum, def. definitionem, ſch. ſcholium, cor. corollarium denotant, &c.*



LIB. I.

PROP. I.



S*Vper data recta linea terminata* A B, *triangulum æquilaterum* A B C *constituere.*

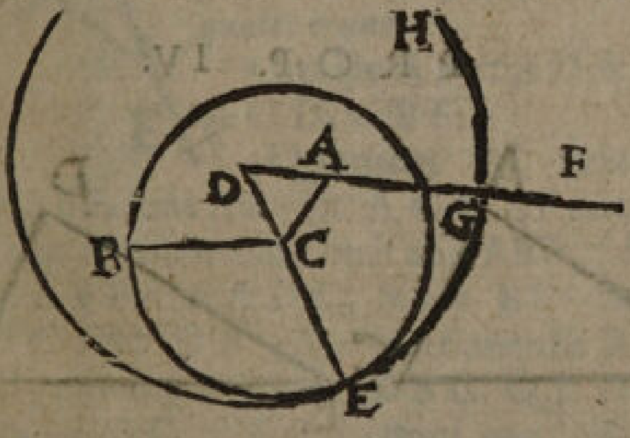
Centris A & B, eodem intervallo A B, vel B A *a* describe duos circulos se intersecantes in puncto C, ex quo *b* duc rectas C A, C B. Erit A C *c* = A B *c* = B C *d* = A C. *e* Quare triangulum A C B est æquilaterum. Quod Erat Faciendum.

a 3.*post.*
b 1.*post.*
c 15.*def.*
d 1.*ax.*
e 23.*def.*

Scholium.

Eodem modo super A B describetur triangulum Isosceles, si intervalla æqualium circulorum majora sumantur, vel minora, quam A B.

PROP. II.



Ad datum punctum A *datæ rectæ lineæ* B C *æqualem rectam lineam* A G *ponere.*

Centro C, intervallo C B *a* describe circulum C B E. *b* Iunge A C, super qua *c* fac triangulum æquilaterum A D C. *d* produc D C ad E. cen-

a 3.*post.*
b 1.*post.*
c 1.1.
d 2.*post.*

centro D, spatio D E, describe circulum D E H: cujus circumferentiæ occurrat D A *e* protracta ad G. Erit A G = C B.

e 2. *post.*
f 15. *def.*
g *constr.*
h 3. *ax.*
k 15. *def.*
l 1. *ax.*

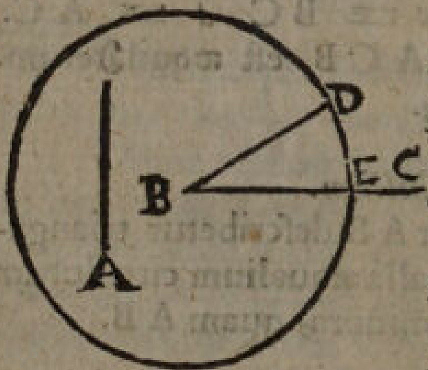
Nam D G *f* = D E, & D A *g* = D C, quare A G *h* = C E *k* = B C *l* = A G. Q. E. F.

Positio puncti A, intra vel extra datam B C, casus variat, sed ubique similis est constructio, & demonstratio.

Scholium.

Poterat A G circino sumi, sed hoc facere nulli postulato respondet, ut bene innuit Proclus.

PROP. III.



Duabus datis rectis lineis A, & B C, de majore BC minori A æqualem rectam lineam BE detrahere.

a 2. 1.

Ad punctum B *a* pone rectam B D = A. Circulus centro B, spatio B D descriptus auferet B E *b* = B D *c* = A *d* = B E. Q. E. F.

b 15. *def.*
c *constr.*
d 1. *ax.*

PROP. IV.

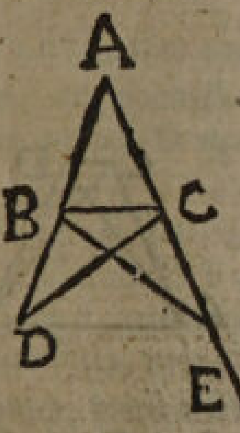
Si duo triangula B A C, E D F *duo latera* B A, A C *duobus lateribus* E D, D F *æqualia habeant, utrumque utrique* (hoc est B A = E D, & A C = D F) *habeant vero angulum* A, *angulo* D *æqualem,*

lem, sub æqualibus rectis lineis contentum, & basim BC basi EF æqualem habebunt; eritque triangulum BAC triangulo EDF æquale, ac reliqui anguli B, C reliquis angulis E, F æquales erunt, uterque utrique, sub quibus æqualia latera subtenduntur.

Si punctum D puncto A applicetur, & recta DE rectæ AB superponatur, cadet punctum E in B, quia DF *a* = AB. Item recta DF cadet in AC, quia ang. A *a* = D. Quinetiam punctum F puncto C coincidet, quia AC *a* = DF. Ergo rectæ EF, BC, cum eosdem habeant terminos, *b* congruent, & proinde æquales sunt. Quare triangula BAC, EDF; & anguli B, E; itemque anguli C, F etiam congruunt, & æquantur. Quod erat Demonstrandum.

a hyp.
b 14. ax.

PROP. V.



Isoscelium triangulorum ABC qui ad basim sunt anguli ABC, ACB inter se sunt æquales. Et productis æqualibus rectis lineis AB, AC qui sub base sunt anguli CBD, BCE inter se æquales erunt.

a Accipe AF = AD, & *b* junge CD, ac BF.

Quoniam in triangulis ACD, ABF, sunt AB *c* = AC, & AF *d* = AD, angulusq; A communis, *e* erit ang. ABF = ACD; & ang. AFB *e* = ADC, & bas. BF *e* = DC; item FC *f* = DB. ergo in triangulis BFC, BDC *g* erit ang. FCB, = DBC. Q.E.D. Item ideo ang. FBC = DCB. atqui ang. ABF *h* = ACD. ergo ang. ABC *k* = ACB. Q.E.D.

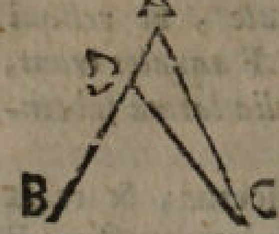
a 3. 1.
b 1. post.
c hyp.
d constr.
e 4. 1.
f 3. ax.
g 4. 1.
h pr.
k 3. ax.

Corollarium.

Hinc, Omne triangulum æquilaterum est quoque æquiangulum.

PROP.

PROP. VI.



Si trianguli A B C *duo anguli* A B C, A C B *æquales inter se fuerint, & sub æqualibus angulis subtensa latera* A B, A C *æqualia inter se erunt.*

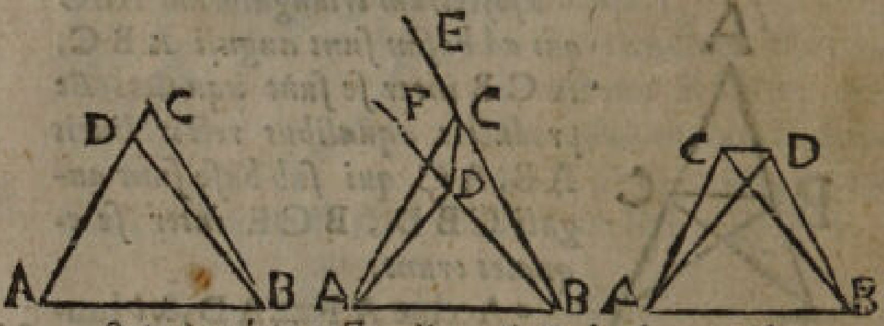
Si fieri potest, sit utravis
a 3. 1. B A $\sqsubset$ C A, *a* Fac igitur B D $=$ C A, & *b* duc
b 1. *post.* C D.

In triangulis DBC, ACB, quia BD *c* $=$ CA,
c *suppos.* & latus BC commune est, atque ang. DBC *d* $=$
d *hyp.* A C B. *e* erunt triangula D B C, A C B æqualia
e 4. 1. inter se, pars & totum, *f* Quod Fieri Nequit.
f 9. *ax.*

Coroll.

Hinc, Omne triangulum æquiangulum est quoque æquilaterum.

PROP. VII.



Super eadem recta linea A B *duabus eisdem rectis lineis* A C, B C, *aliæ duæ rectæ lineæ æquales* A D, B D, *utraque utrique* (*hoc est,* A D $=$ AC, & B D $=$ B C) *non constituentur ad aliud punctum* C, *atque aliud* D, *ad easdem partes* C, *eosdemque terminos* A, B *cum duabus initio ductis rectis lineis habentes.*

a 9. *ax.* 1. *Cas.* Si punctum D statuatur in A C. *a* liquet non esse A D $=$ A C.

2. *Cas.* Si punctum D dicatur intra triangulum A C B, duc C D, & produc B D F, ac B C E.
b 5. 1. Iam vis A D $=$ A C. ergo ang. ADC *b* $=$ ACD;
c *suppos.* item quia BD *c* $=$ BC, erit ang. FDC *b* $=$ ECD.

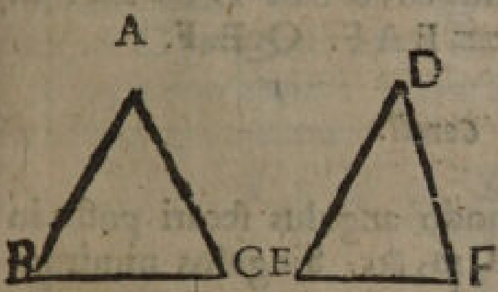
ergo

ergo ang. F D C *d* ⊏ A C D, id est ang. F D C d 9.*ax.*
⊏ A D C *d* Q. F. N.

3. *Caſ.* Sin D cadat extra triangulum A C B, jungatur C D.

Rurſus, ang. B C D *e* = BDC, & BCD *e* = e 5. 1.
B D C *f* ergo ang. A C D ⊐ B D C. & proinde f 9. *ax.*
multo magis ang. B C D ⊐ B D C. Sed erat ang. B C D = B D C. Quæ repugnant. Ergo, &c.

Prop. VIII.



Si duo triangula A B C, D E F *habuerint duo latera* A B, A C *duobus lateribus* D E, D F, *utrumque utrique æqualia; habuerint vero & baſim* B C, *baſi* E F, *æqualem: angulum* A *ſub æqualibus rectis lineis contentum angulo* D *æqualem habebunt.*

Quia B C *a* = E F, ſi baſis B C ſuperponatur a *hyp.*
baſi EF, illæ *b* congruent. ergo, cum AB *c* = DE, b 8. *ax.*
& AC *c* = D F, cadet punctum A in D. (nam c *hyp.*
in aliud punctum cadere nequit, per præcedentem) ergo angulorum A, & D latera coincidunt. *d* quare anguli illi pares ſunt. Q. E. D. d 8. *ax.*

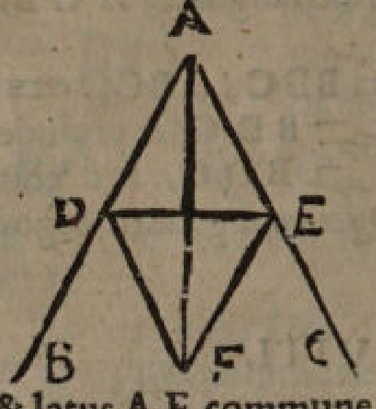
Coroll.

1. Hinc triangula ſibi mutuo æquilatera, etiam mutuo *x* æquiangula ſunt. x 4. 1.

2. Triangula ſibi mutuo æquilatera *y* æquentur inter ſe. y 4. 1.

PROP.

P R O P. IX.



Datum angulum rectilineum B A C *bifariam secare.*

a 3. 1. b 1. 1. a Sume A D = A E; duc D E, super qua b fac triang. æquilat. D F E.

Ducta A F angulum B A C bisecabit.

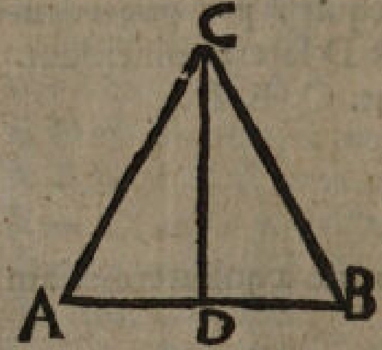
c constr. d 8. 1. Nam A D c = A E, & latus A F commune est, & bas. D F c = F E. d ergo ang. D A F = E A F. Q. E. F.

Coroll.

Hinc patet quomodo angulus secari possit in æquales partes 4, 8, 16, &c. Singulos nimirum partes iterum bisecando.

Methodus vero regula & circino angulos secandi in æquales quotcunque hactenus Geometras latuit.

P R O P. X.

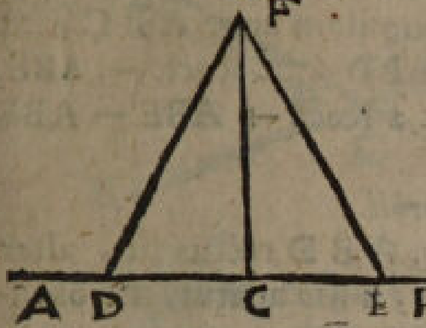


Datam rectam lineam A B *bifariam secare.*

a 1. 1. b 9. 1. Super data A B a fac triang. æquilat. A B C. ejus angulum C b biseca recta C D. Eadem datam A B bisecabit.

c constr. d 4. 1. Nam A C c = B C, & latus C D est commune; & ang. A C D c = B C D, d ergo A D = B D. Q. E. F. Praxin hujus & præcedentis, constructio primæ hujus libri satis indicat.

P R O P. XI.



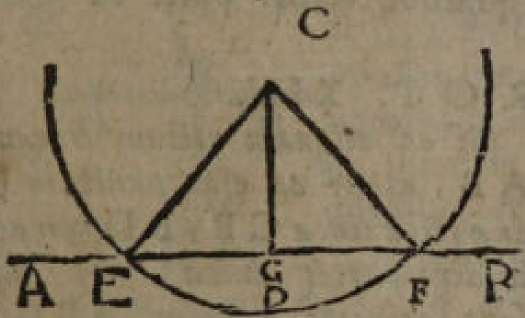
Data recta linea A B, & puncto in ea dato C, rectam lineam C F ad angulos rectos excitare.

a Accipe hinc inde C D = C E. Super D E *b* fac triang. æquilat. D F E. Ducta F C perpendicularis est. a 3. 1. b 1. 1.

Nam triangula D F C, E F C sibi mutuo *c* æquilatera sunt. *d* ergo ang. D C F = E C F. *e* ergo F C perpendicularis est. Q. E. F. c *constr.* d 8. 1. e 10. *def.*

Praxis tam hujus, quam sequentis expeditur facillime ope normæ.

P R O P. XII.

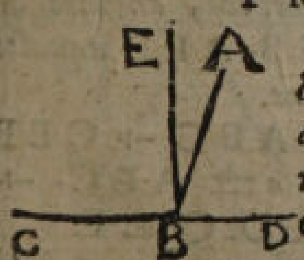


Super datam rectam lineam infinitam A B, à dato puncto C quod in ea non est, perpendicularem rectam C G deducere.

Centro C *a* describe circulum, qui secet datam A B in punctis E & F *b* biseca E F in G. ducta C G perpendicularis est. a 3. *post.* b 10. 1.

Ducantur enim C E, C F. Triangula E G C, F G C, sibi mutuo *c* æquilatera sunt. *d* ergo anguli E G C, F G C, æquales, & *e* proinde recti sunt. Q. E. F. c *constr.* d 8. 1. e 10. *def.*

P R O P. XIII.



Cum recta linea A B, super rectam lineam C D consistens, facit angulos A B C, A B D; aut duos rectos, aut duobus rectis æquales efficiet.

a 10 def. Si anguli ABC, ABD pares sint *a* liquet illos
b 11. 1. rectos esse; sin inæquales sint, ex B *b* excitetur
c 19. ax. perpendicularis B E. Quoniam ang. A B C *c* =
d 3. ax. Rect. + ABE; & ang. ABD *d* = Rect. − ABE;
e 2. ax. erit ABC + ABD *e* = 2 Rect. + ABE − ABE
= 2 Rect. Q. E. D.

Coroll.

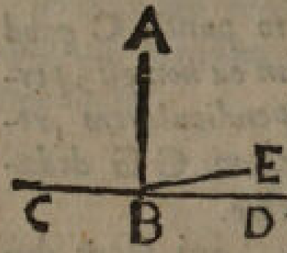
1. Hinc, si unus ang. A B D rectus sit, alter A B C etiam rectus erit; si hic acutus, ille obtusus erit, & contra.

2. Si plures rectæ quam una ad idem punctum eidem rectæ insistant, anguli fient duobus rectis æquales.

3. Duæ rectæ invicem secantes efficiunt angulos quatuor rectis æquales.

4. Omnes anguli circa unum punctum constituti conficiunt quatuor rectos. patet ex Coroll. 2.

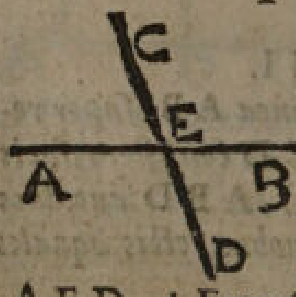
PROP. XIV.



Si ad aliquam rectam lineam A B, *atque ad ejus punctum* B *duæ rectæ lineæ* C B, B D *non ad easdem partes ductæ, eos qui sunt deinceps angulos* A B C, A B D *duobus rectis æquales fecerint, in directum erunt inter se ipsæ rectæ lineæ* C B, B D.

a 13. 1. Si negas, faciant C B, B E unam rectam. ergo
b hyp. ang. A B C + ABE *a* = 2 Rect. *b* = ABC +
c 9. ax. A B D. *c* Quod Est absurdum.

PROP. XV.

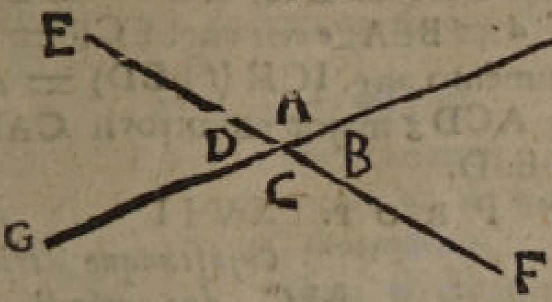


Si duæ rectæ lineæ A B, C D *se mutuo secuerint, angulos ad verticem* C E B, A E D *æquales inter se efficient.*

Nam ang. A E C + C E B
a 13. 1. *a* = 2 Rect. *a* = A E C +
b 3. ax. A E D. *b* Ergo C E B = A E D. Q. E. F.

Schol.

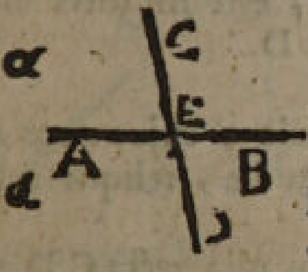
Schol.



Si ad aliquam rectam lineam G H, atque ad ejus punctum, A duæ rectæ lineæ E A, A F non ad easdem partes sumptæ, angulos ad verticem D, & B æquales fecerint, ipsæ rectæ lineæ E A, A F in directum sibi invicem erunt.

Nam 2 Rect. = *a* D + A *a* = B + A. *b* ergo EA, AF sunt in directum sibi invicem. Q.E.D. a 13. 1. b 14. 1.

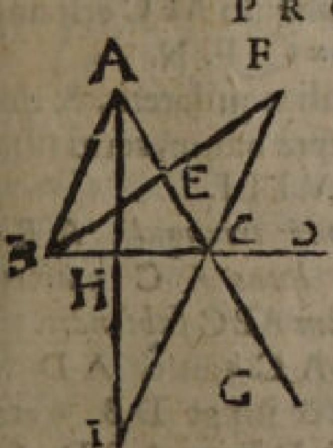
Schol. 2.



Si quatuor rectæ lineæ E A, E B, E C, E D ab uno puncto E exeuntes, angulos oppositos ad verticem æquales inter se fecerint, erunt quælibet duæ lineæ A E, E B, & C E, E D in directum positæ.

Nam quia ang. AEC + AED + CEB + DEB *a* = 4 Rect. erit A E C + A E D *b* = CEB + DEB = 2 Rect. *c* ergo CED, & AEB sunt rectæ lineæ. Q. E. D. a 4 Cor. 13. 1 b 13. 1. & 2. ax. c 14. 1.

PROP. XVI.



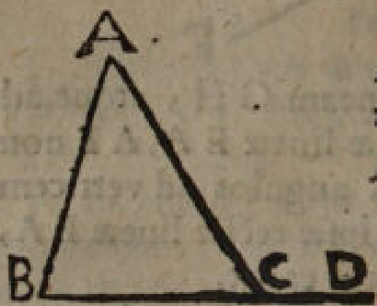
Cujuscunque Trianguli A B C *uno latere* B C *producto, externus angulus* A C D *utrolibet interno & opposito* C A B, C B A, *major est.*

Latera A C, B C *a* bisecent rectæ A H, B E, è quibus productis *b* cape EF = B E, *b* & H I = A H, a 10. 1. & 1. post. b 3. 1.

Conjuganturque F C, I. B Quo-

Quoniam C E *c* = E A, & E F *c* = E B, &
c constr. *d 15.1.* ang. F E C *d* = BEA; *e* erit ang. ECF = EAB.
e 4. 1. Simili argumento ang. ICH (*f* FCD) = ABH.
f 15. 1. *g 9. ax.* ergo totus ACD *g* major est utrovis CAB, &
ABC. Q. E. D.

PROP. XVII

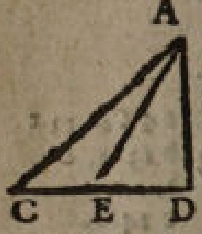


Cujuscunque trianguli ABC duo anguli duobus rectis sunt minores, omnifariam sumpti.

Producatur latus B C. Quoniam ang. ACD + ACB *a* = 2 Rect. & ang,
a 13. 1. *b 16. 1.* ACD *b* ⊏ A, *c* erit A + ACB ⊐ 2 Rect. Eo-
c 4. ax. dem modo erit ang. B + ACB ⊐ 2 Rect. Denique producto latere A B, erit similiter ang. A + B ⊐ 2 Rect. Quæ E. D.

Coroll.

1. Hinc, in omni triangulo, cujus unus angulus fuerit rectus, vel obtusus, reliqui acuti sunt.

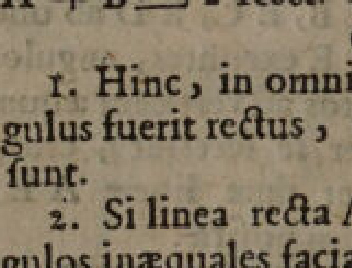


2. Si linea recta A E cum alia recta C D angulos inæquales faciat, unum AED acutum, & alterum A E C obtusum, linea perpendicularis A D ex quovis ejus puncto A ad aliam illam CD demissa, cadet ad partes anguli acuti AED.

Nam si AC ad partes anguli obtusi ducta, dicatur perpendicularis, in triangulo AFC erit ang.
x 17. 1. AEC + ACE ⊏ 2 Rect. *x* Q. F. N.

3. Omnes anguli trianguli æquilateri, & duo anguli trianguli Isoscelis, supra basim, acuti sunt.

PROP. XVIII.



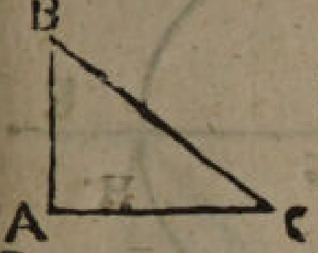
Omnis trianguli A B C *majus latus* A C *majorem angulum* ABC *subtendit.*

Ex A C *a* aufer A D =
a 3. 1. AB, & junge DB. *b* ergo
b 5. 1. ang. A D B = A B D. Sed

c ADB

e ADB ⊏ C. ergo ABD ⊏ C. d ergo totus ang. ABC ⊏ C. Eodem modo erit ABC ⊏ A. Q. E. D. c 16. 1. d 9 ax.

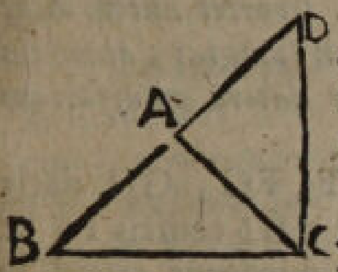
P R O P. XIX.



Omnis trianguli A B C *major angulus* A *majori lateri* BC *subtenditur.*

Nam si dicatur A B = BC, a erit ang. A = C. contra Hypoth. & si A B ⊏ BC, b erit ang. C ⊏ A, contra hyp. quare potius B C ⊏ A B. & eodem modo B C ⊏ A C. Q. E. D. a 5. 1. b 18. 1.

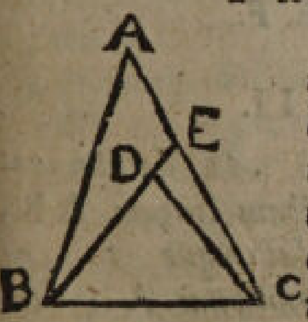
P R O P. XX.



Omnis trianguli A B C *duo latera* B A, A C *reliquo* B C *sunt majora quomodocunque sumpta.*

Ex B A producta a cape A D = A C, & duc D C. b ergo ang. D = A C D. c ergo totus B C D ⊏ D d ergo B D (e B A + A C) ⊏ B C. Q. E. D. a 3. 1. b 5. 1. c 9. ax. d 19. 1, e constr. & 2. ax.

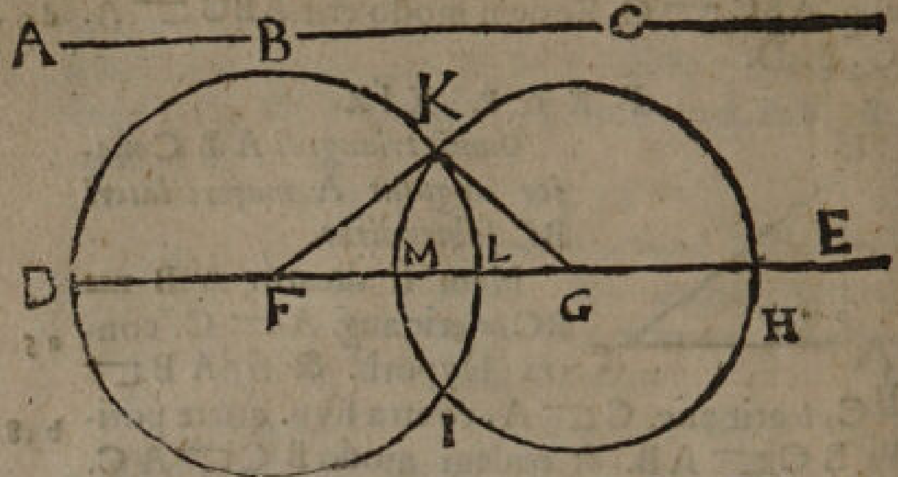
P R O P. XXI.



Si super trianguli A B C *uno latere* BC, *ab extremitatibus duæ rectæ lineæ* B D, C D, *interius constitutæ fuerint, hæ constitutæ reliquis trianguli duobus lateribus* B A, C A *minores quidem erunt, majorem vero angulum* BDC *continebunt.*

Producatur BD in E. estque CE + ED a ⊏ CD adde commune B D, b erit BE + EC ⊏ BD + DC. Rursus BA + AE a ⊏ BE; b ergo BA + AC ⊏ BE + EC. quare B A + AC ⊏ B D + D C. Q. E. D. 2. Ang. B D C c ⊏ DEC c ⊏ A. ergo ang. BDC ⊏ A. Q. E. D. a 20 1. b 4. ax. c 16. 1.

Prop. XXII.

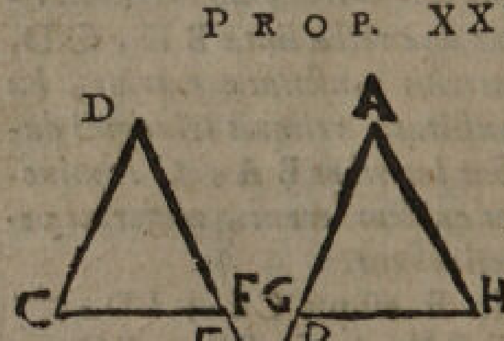


Ex tribus rectis lineis F K, F G, G K, *quæ sint tribus datis rectis lineis* A, B, C, *æquales, triangulum* F K G *constituere. Oportet autem duas reliqua esse majores omnifariam sumptas; quoniam uniuscujusque trianguli duo latera omnifariam sumpta reliquo sunt majora.*

a 3. 1. b 3. post. Ex infinita DE *a* sume DF, FG, GH datis A, B, C ordine æquales. Tum si *b* centris F, & G, intervallis F D, & G H ducantur circuli se intersecantes in K; junctis rectis K F, K G constituetur triangulum F K G, *c* cujus latera F K, F G, G K tribus DF, F G, G H, *d* id est tribus datis A, B, C æquantur. Q. E. F.

c 15. def. d 1. ax.

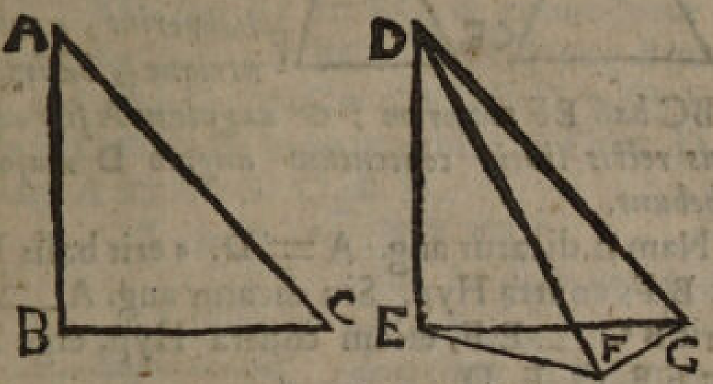
Prop. XXIII.



Ad datam rectam lineam A B, *datumque in ea punctum* A, *dato angulo rectilineo* D *æquale angulum rectilineum* A *constituere.*

a 1. post. b 3. 1. c 22. 1. *a* Duc rectam C F secantem dati anguli latera utcunque. *b* Fac AG = CD. Super AG *c* constitue triangulum alteri C D F æquilaterum, ita ut ut AH = DF, & GH = CF; & habebis ang. A *d* = D. Q. E. F. d 8. 1.

PROP. XXIV.



Si duo triangula A B C, D E F *duo latera* A B, A C *duobus lateribus* D E, D F *æqualia habuerint, utrumque utrique; angulum vero* A *angulo* EDF *majorem ſub æqualibus rectis lineis contentum, & baſim* BC, *baſi* EF, *majorem habebunt.*

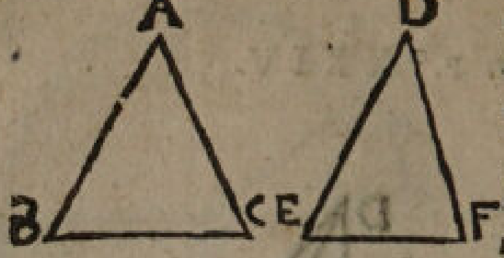
a Fiat ang. EDG = A, & DG *b* = D F *c* = A C, connectanturque E G, F G. a 23. 1. b 3. 1. c *hyp.*

1. *Caſ.* Si E G cadit ſupra E F. Quia A B *d* = D E, & A C = *e* DG, & ang. A *e* = EDG, *f* erit B C = E G. Quia vero D F *e* = D G, *g* erit ang. DFG = DGF. *h* ergo ang. DFG ⊏ EGF; *h* & proinde ang. EFG ⊏ EGF. *k* quare EG (B C) ⊏ E F. Q. E. D. d *hyp.* e *conſtr.* f 4. 1. g 5. 1. h 9. *ax.* k 19. 1.

2. *Caſ.* Si baſis E F baſi E G coincidat, *l* liquet E G (B C) ⊏ E F. l 9 *ax.*

3. Sin E G Cadat infra E F. Quoniam D G + G E *m* ⊏ D F + F E, ſi hinc inde auferantur D G, D F, æquales, manet E G (B C) *n* ⊏ E F. Q. E. D. m 21. 1. n 5. *ax.*

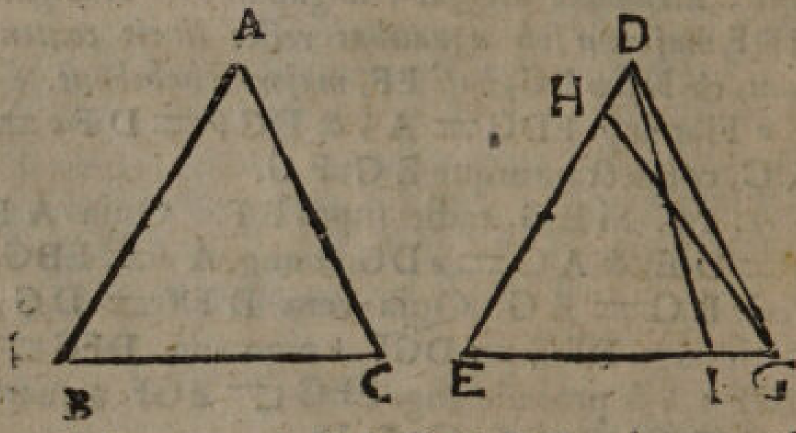
PROP. XXV.



Si duo triangula ABC, DEF *duo latera* AB, AC *duobus lateribus* DE, DF *æqualia habuerint, utrumq; utrique, basim vero* BC *basi* EF *majorem; & angulum* A *sub æqualibus rectis lineis contentum angulo* D *majorem habebunt.*

a 4. 1. Nam si dicatur ang. A $=$ D. *a* erit basis BC $=$ EF, contra Hyp. Sin dicatur ang. A $\sqsupset$ D,
b 24. 1. *b* erit BC $\sqsupset$ EF, etiam contra Hyp. ergo BC $\sqsubset$ EF. Q. E. D.

PROP. XXVI.



Si duo triangula BAC EDG, *duos angulos* B, C, *duobus angulis* E, DGE, *æquales habuerint, utrumque utrique, unumque latus uni lateri æquale, sive quod æqualibus adjacet angulis, seu quod uni æqualium angulorum subtenditur: reliqua latera reliquis lateribus æqualia, utrumque utrique, & reliquum angulum reliquo angulo æqualem habebunt.*

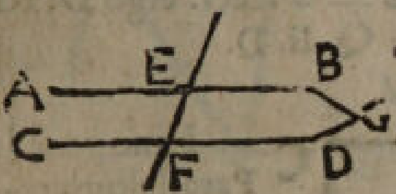
1. *Hyp.* Sit BC $=$ EG. Dico BA $=$ ED, & AC $=$ DG, & ang. A $=$ EDG. Nam si dicatur
a 3. 1. ED $\sqsubset$ BA, *a* fiat EH $=$ BA, ducaturque GH.

Quoniam

Quoniam AB *b* = HE, & BC *c* = EG, & ang. B *c* = E, erit ang. EGH *d* = C *e* = DGE. *f* Q. E. A. ergo AB = ED. Eodem modo AC = DG. *d* quare etiam ang. A = EDG.
b suppos. c hyp. d 4 1. e hyp. f 9. ax.

2. *Hyp.* Sit AB = ED. Dico BC = EG; & AC = DG & ang. A = EDG. Nam si dicatur EG ⊏ BC, fiat EI = BC, & connectatur DI. Quia AB *g* = ED, & BC *h* = EI, & ang B *g* = E, erit ang. EID *k* = C *m* = EGD. *n* Q. E. A. ergo BC = EG. ergo ut prius, AC = DG, & ang. A = EDG. Q. E. D.
g hyp. h suppos. k 4. 1. m hyp. n 16. 1.

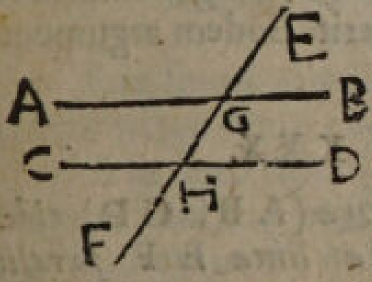
Prop. XXVII.



Si in duas rectas lineas AB, CD recta incidens linea EF alternatim angulos AEF, DFE, æquales inter se fecerit, parallelæ erunt inter se illæ rectæ lineæ AB, CD.

Si AB, CD dricantur non esse parallelæ; conveniant productæ, nempe in G. quo posito angulus externus AEF interno DFE *a* major erit, cui tamen ponitur æqualis. Quæ repugnant. a 16. 1.

Prop. XXVIII.

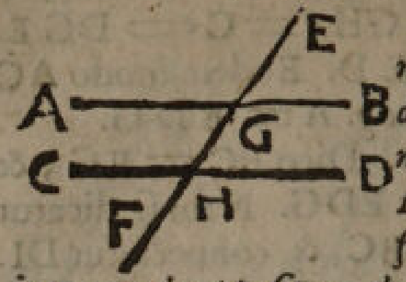


Si in duas rectas lineas AB, CD recta incidens linea EF externum angulum AGE interno & opposito, & ad easdem partes CHG æqualem fecerit, aut internos & ad easdem partes AGH, CHG duobus rectis æquales; parallelæ erunt inter se ipsæ rectæ lineæ AB, CD.

1. *Hyp.* Quia per hyp. ang. AGE = CHG, *a* erit altern. BGH = CHG. *b* parallelæ igitur sunt AB, CD. Q. E. D.
a 15. 1. b 27. 1.

2. *Hyp.* Quia ex hyp. Ang. AGH + CHG = 2 Rect. *a* = AGH + BGH, *b* erit CHG = BGH. Ergo *c* AB, CD parallelæ sunt. Q. E. D.
a 13. 1. b 3. ax. c 27. 1.

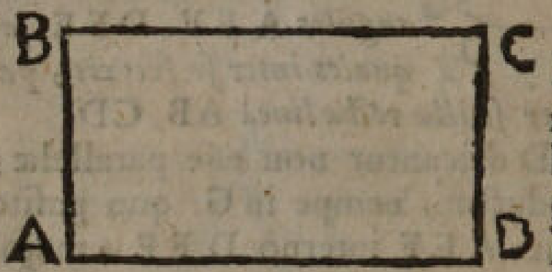
PROP. XXIX.



In parallelas rectas lineas A B, C D, *recta incidens linea* E F, *& alternatim angulos* D H G, A G H *æquales inter se efficit; & externum* B G E *interno, & opposito, & ad easdem partes* DHE *æqualem; & internos & ad easdem partes* A G H, C H G *duobus rectis æquales facit.*

a 13. ax. Liquet A G H, + C H G = 2 Rect. *a* alias A B, C D non essent parallelæ, contra hyp. Sed
b 13. 1. & ang. DHG + CHG *b* = 2 Rect. ergo DHG
c 13. ax. *c* = A G H *d* = B G E. Q. E. D.
d 15. 1.

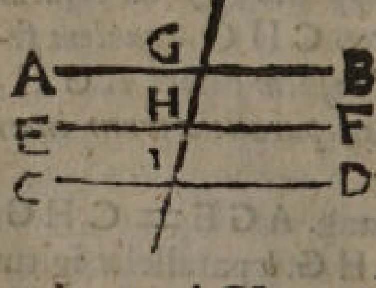
Coroll.



Hinc omne Parallelogrammum A C habens unum angulum rectum A, est rectangulum.

29. 1. Nam A + B *a* = 2 Rect. ergo cum A re-
a 3. ax. ctus sit, *b* etiam B rectus erit. Eodem argumento D, & C recti sunt.

PROP. XXX.

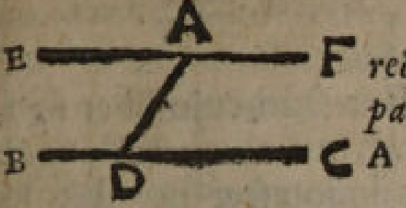


Quæ (A B, C D) *eidem rectæ lineæ* E F *parallelæ, & inter se sunt parallelæ.*

Tres rectas secet utcunque recta G I. Quoniam A B, E F parallelæ sunt,
a 29. 1. *a* erit ang. AGI = EHI, Item propter CD, EF
b 1. ax. parallelas, *a* erit ang. EHI = DIG. *b* ergo ang.
c 27. 1. AGI = D I G. *c* quare AB, CD parallelæ sunt. Q. E. D.

PROP.

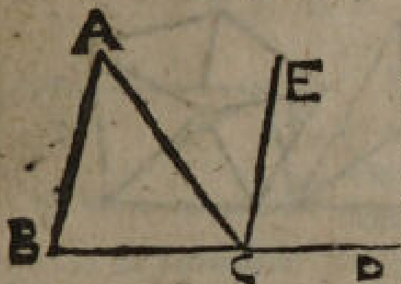
PROP. XXXI.



A dato puncto A datæ rectæ lineæ B C ducere parallelam rectam lineam A E.

Ex A ad datam BC duc rectam utcunque AD. ad quam, ejusque punctum a 23. 1.
A *a* fac ang. D A E = A D C. *b* erunt A E, BC b 27. 1.
parallelæ. Q. E. F.

PROP. XXXII.



Cujuscunque trianguli A B C *uno latere* B C *producto, externus angulus* A C D *duobus internis, & oppositis,* A B *est æqualis. Et trianguli tres interni anguli,* A, B, A C B *duobus sunt rectis æquales.*

Per C *a* duc C E parall. B A. Ang. A *b* = a 31. 1.
ACE. & ang. B *b* = ECD. ergo A + B *c* = b 29. 1.
A C E + E C D *d* = A C D. Q. E. D. Pono c 2. ax. d 19. ax.
ACD + ACB *e* = 2 Rect. *f* ergo A + B + e 13. 1.
A C B = 2 Rect. Q. E. D. f 1. ax.

Corollaria.

1. Tres simul anguli cujusvis trianguli æquales sunt tribus simul cujuscunque alterius. Unde

2. Si in uno triangulo duo anguli (aut singuli, aut simul) æquales sint duobus angulis (aut singulis, aut simul) in altero triangulo, etiam reliquus reliquo æqualis est. Item, si duo triangula unum angulum uni æqualem habeant, reliquorum summæ æquantur.

3. In triangulo si unus angulus rectus sit, reliqui unum rectum conficiunt. Item, angulus, qui duobus reliquis æquatur, rectus est.

4. Cum in Isoscele angulus æquis cruribus contentus rectus est, reliqui ad basim sunt semirecti.

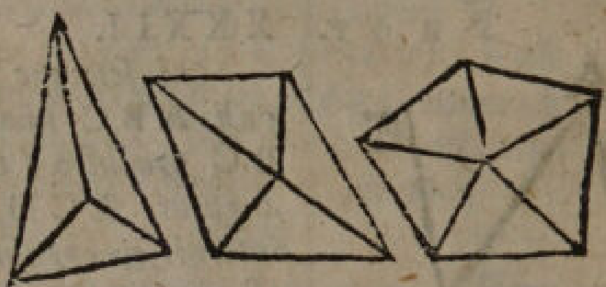
5. Tri-

5. Trianguli æquilateri angulus facit duas tertias unius recti, nam $\frac{1}{3}$ 2 Rect. $=\frac{2}{3}$ Rect.

Schol.

Hujus propositionis beneficio, cujuslibet figuræ rectilineæ tam interni quam externi anguli quot rectos conficiant, innotescet per duo sequentia theoremata.

THEOREMA I.



Omnes simul anguli cujuscunque figuræ rectilineæ conficiunt bis tot rectos demptis quatuor, quot sunt latera figuræ.

Ex quovis puncto intra figuram ducantur ad omnes figuræ angulos rectæ, quæ figuram resolvent in tot triangula quot habet latera. Quare cum singula triangula conficiant duos rectos, omnia simul conficient bis tot rectos, quot sunt latera. Sed anguli circa dictum punctum conficiunt quatuor rectos. Ergo, si ab omnium triangulorum angulis demas angulos circa id punctum, anguli reliqui qui componunt angulos figuræ conficient bis tot rectos demptis quatuor, quot sunt latera figuræ. Q. E. D.

Hinc *Coroll.* Omnes ejusdem speciei rectilineæ figuræ æquales habent angulorum summas.

THEOREMA 2.

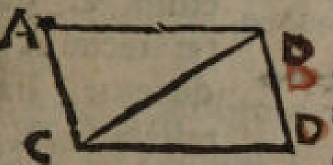
Omnes simul externi anguli cujuscunque figuræ rectilineæ conficiunt quatuor rectos.

Nam singuli figuræ interni anguli cum singulis externis conficiunt duos rectos. Ergo interni

terni simul omnes, cum omnibussimul externis conficiunt bis tot rectos, quot sunt latera figuræ. Sed(ut modo ostensum est,)interni simul omnes etiam cum quatuor rectis efficiunt bis tot rectos, quot sunt latera figuræ. Ergo externi anguli quatuor rectis æquantur. Q. E. D.

Coroll. Omnes cujuscunque speciei rectilineæ figuræ æquales habent externorum angulorum summas.

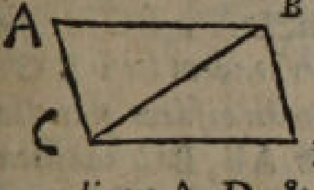
PROP. XXXIII.



Rectæ lineæ AC, BD, *quæ æquales & parallelas lineas* AB, CD, *ad partes easdem conjungunt, & ipsæ æquales ac parallelæ sunt.*

Connectatur C B. Quoniam ob AB, CD parallelas. ang. ABC a = BCD, & per hyp. AB = CD, & latus CB commune est, b erit AC = BD, b & ang. ACB = DBC. c ergo AC, BD etiam parallelæ sunt. Q. E. D. a 29. 1. b 4. 1. c 27. 1.

PROP. XXXIV.



Parallegrammorum spatiorum ABDC *æqualia sunt inter se quæ ex adverso latera* AB, CD; ac AC, BD; *angulique* A, D, & ABD, ACD, *& illa bifariam secat diameter* C B.

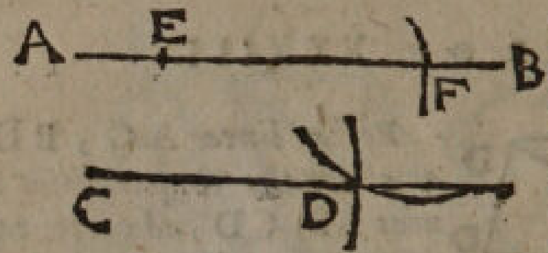
Quoniam AB, CD a parallelæ sunt, b erit ang. ABC = BCD. Item ob AC, DB a parallelas, b erit ang. ACB = CBD. c ergo toti anguli A CD, ABD æquantur. Similiter ang. A = D. Porto, cum communi lateri CB adjaceant anguli ABC, ACB, ipsis BCD, CBD pares d, erunt AC = BD, d & AB = CD. adeoque etiam triang. ABC = CBD. Quæ E. D. a hyp. b 29. 1. c 2. ax. d 26. 1.

SCHOL.

SCHOL.

Omne quadrilaterium A B D C *habens latera opposita æqualia, est parallelogrammum.*

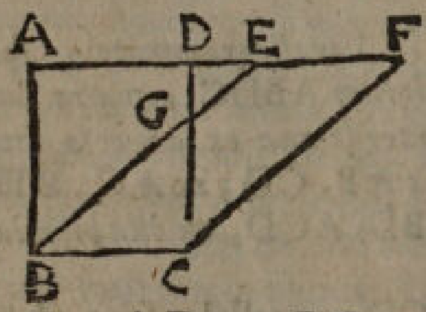
Nam per 8. 1. ang. A B C = B C D. *a* ergo
a 17. 1. A B, C D parallelæ sunt. Eadem ratione ang. B C A = C B D; *a* quare A C, B D etiam paralle-
b 35. def. 1. læ sunt. *b* Ergo A B D C est parallelogrammum. Q. E. D.



Hinc expeditius per datum punctum C datæ rectæ A B ducetur parallela C D.

Sume in A B quodvis punctum E. centris E. & C ad quodvis intervallum duc æquales circulos E F, C D. centro vero F, spatio E C duc circulum F D, qui priorem C D secet in D. Erit ducta C D parall. A B. Nam ut modo demonstratum est, C E F D est parallelogrammum.

PROP. XXXV.



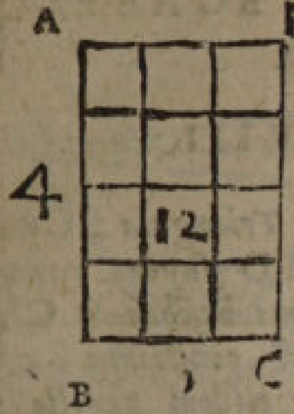
Parallelogramma BCDA, BCFE *super eadem basi* BC, *& in eisdem parallelis* AF, BC *constituta, inter se sunt æqualia.*

a 34. 1.
b 2. ax.
c 29. 1.
d 4. 1.
e 3. ax.
f 2. ax.

Nam A D *a* = B C *a* = E F. adde communem DE, *b* erit AE = DF. Sed & AB *a* = DC; & ang. A *c* = C D F. *d* ergo triang. A B E = D C F. aufer commune D G E, *e* erit Trapez. ABGD = EGCF. adde commune BGC, *f* erit Pgr. ABCD = EBCF, Q. E. D. Reliquorum casuum non dissimilis, sed simplicior & facilior est demonstratio.

Scho-

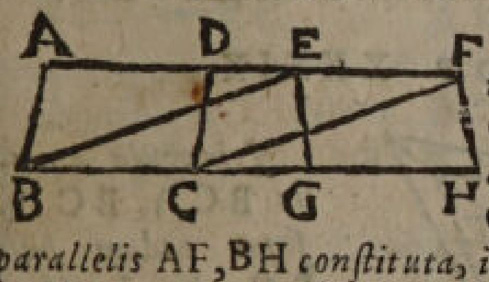
Scholium.



Si latus A B parallelogrammi rectanguli A B C D ferri intelligatur perpendiculariter per totam B C, aut B C per totam A B, producetur eo motu area rectanguli A B C D. Hinc rectangulum fieri dicitur ex ductu seu multiplicatione duorum laterum contiguorum. Sit exempl. gr. B C pedum 3, A B 4. Duc 3 in 4; proveniunt 12 pedes quadrati pro area rectanguli.

Hoc supposito, ex hoc theoremate cujuscunq; parallelogrammi (* EBCF) habetur dimensio. Illius enim area producitur ex altitudine B A ducta in basim B C. Nam area rectanguli A C parallelogrammo E B C F æqualis, fit ex B A in B C. ergo, &c. * v. fig. propos. 35.

PROP. XXXVI.

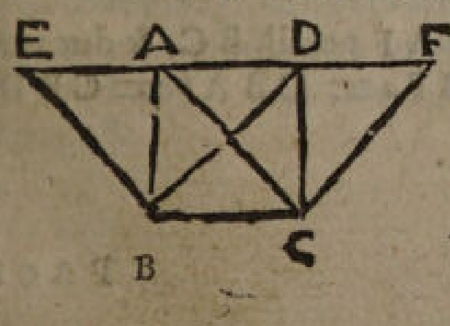


Parallelogramma B C D A, G H F E *super æqualibus basibus* BC, G H, *& in eisdem parallelis* AF, BH *constituta, inter se sunt æqualia.*

Ducantur BE, CF. Quia BC *a* = GH *b* = EF, *c* erit BCFE parallelogrammum. ergo Pgr. BCDA *d* = BCFE *d* GHFE. Q. E. D.

a hyp.
b 34. 1.
c 33. 1.
d 35. 1.

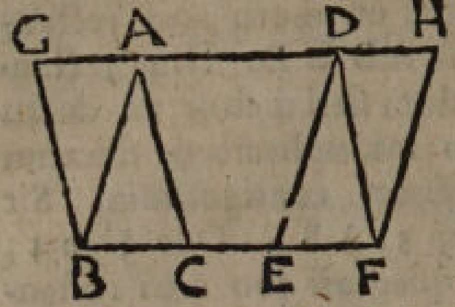
PROP. XXXVII.



Triangula BCA, B G D *super eadem basi* BC *constituta, & in eisdem parallelis* B C, E F *inter se sunt æqualia.*

a 31. 1. b 34. 1 c 35. 1. & 7. ax. a Duc BE parall. CA, a & CF parall. BD. Erit triang. BCA $b = \frac{1}{2}$ Pgr. BCAE $= c\ \frac{1}{2}$ BDFC $b =$ BCD. Q.E.D.

PROP. XXXVIII.



Triangula BCA, EFD *super æqualibus basibus* BC, EF *constituta, & in eisdem parallelis* GH, BF, *inter se sunt æqualia.*

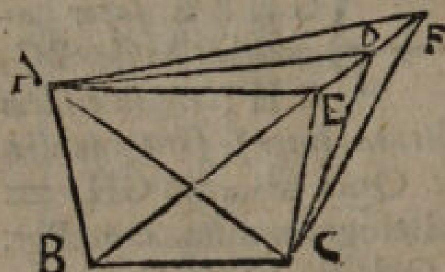
Duc BG parall. CA. & FH parall. ED. erit triang. BCA $a = \frac{1}{2}$ Pgr. BCAG $b = \frac{1}{2}$ EDHF $c =$ EFD. Q.E.D.

a 34. 1. b 36. 1 & 7. ax. c 34. 1.

Schol.

Si basis BC ⊏ EF, liquet triang. BAC ⊏ EDF. & si BC ⊐ EF, erit BAC ⊐ EDF.

PROP. XXXIX.

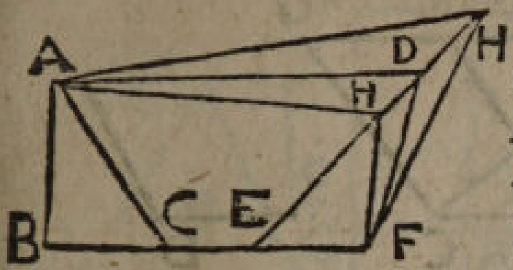


Triangula æqualia BCA, BCD, *super eadem basi* BC, *& ad easdem partes constituta, etiam in eisdem sunt parallelis* AD, BC.

Si negas, sit altera AF parall. BC; & ducatur CF. ergo triang. CBF $a =$ CBA $b =$ CBD. c Q.E.A.

a 37. 1. b hyp. c 9. ax.

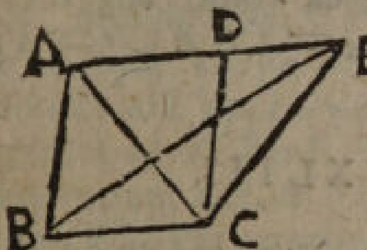
PROP. XL.



Triangula æqualia B C A, E F D *super æqualibus basibus* B C, E F, *& ad easdem partes constituta, & in eisdem sunt parallelis* A D, B F.

Si negas, sit altera A H parall. BF. & ducatur F H. ergo triang. E F H *a* = B C A *b* = EFD. *c* Q. E. A. a 38. 1. b hyp. c 9. ax.

PROP. XLI.



Si parallelogrammum A B C D *cum triangulo* B C E *eandem basim* B C *habuerit, in eisdemque fuerit parallelis* A E, B C, *duplum erit parallelogrammum* ABCD *ipsius trianguli* BCE.

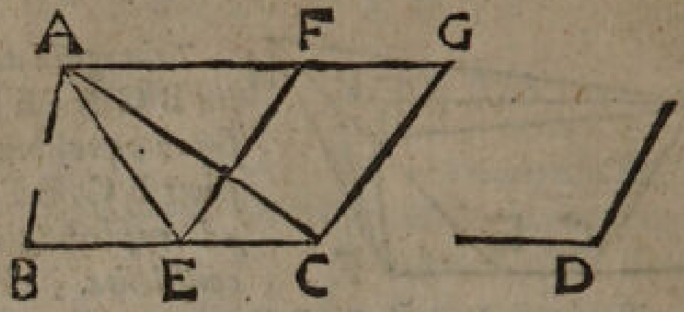
Ducatur AC. Triang. BCA *a* = BCE. ergo Pgr. ABCD *b* = 2 BCA *c* = 2 BCE. Q. E. D. a 37. 1. b 34. 1. c 6 ax.

Scholium.

Hinc habetur area cujuscunq; trianguli BCE. Nam cum area parallelogrammi ABCD producatur ex altitudine in basim ducta; producetur area trianguli ex dimidia altitudine in basim ducta, vel ex dimidia basi in altitudinem. ut si basis B C sit 8, & altitudo 7; erit trianguli B C E area, 28.

PROP.

Prop. XLII.

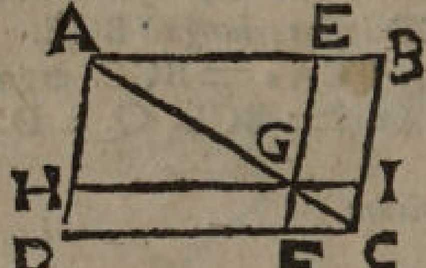


Dato triangulo A B C *æquale parallelogrammum* E C G F *constituere in dato angulo rectilineo* D.

a 31. 1. Per A *a* duc A G parall. B C. *b* fac ang. B C G
b 23. 1. = D. basim B C *c* biseca in E. *a* duc E F parall.
c 10. 1. C G. Dico factum.

Nam ducta A E. erit ex constr. ang. E C G
d 38. 1. = D, & triang. B A C *d* = 2 A E C *e* = Pgr.
e 41. 1. E C G F. Q. E. F.

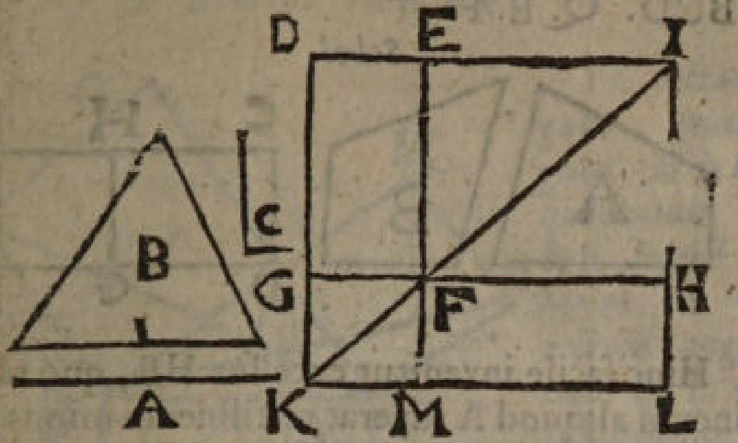
Prop. XLIII.



In omni parallelogrammo ABCD *complementa* D G , G B *eorum quæ circa diametrum* AC *sunt parallelogrammorum* HE, F I *inter se sunt æqualia.*

a 34. 1. Nam Triang. A C D, = *a* A C B. & triang.
A G H *a* = A G E. & triang. G C F *a* = G C I.
b 3. ax. *b* ergo Pgr. D G = G B. Q. E. D.

PROP. XLIV.

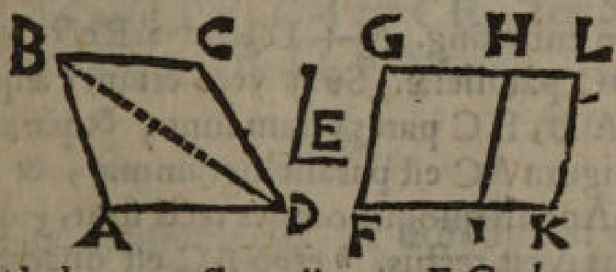


Ad datam rectam lineam A, *dato triangulo* B, *æquale parallelogrammum* FL *applicare in dato angulo rectilineo* C.

a Fac Pgr. FD = triang. B, ita ut ang. GFE = C. & pone lateri GF in directum FH = A. Per H b duc IL parall. EF; cui occurrat DE producta ad I. per IF ductæ rectæ occurrat DG protracta ad K. Per K b duc KL parall. GH; cui occurrant EF, & IH prolongatæ ad M, & L. Erit FL. Pgr. quæsitum. a 42. 1. b 31. 1.

Nam Pgr. FL c = FD = B d & ang. MFH = GFE = C. Q.E.F. c 43. 1. d 15. 1.

PROP. XLV.

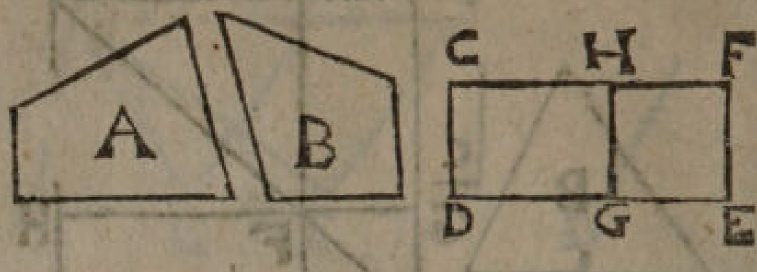


Ad datam rectam lineam FG *dato rectilineo* ABCD *æquale parallelogrammum* FL *constituere, in dato angulo rectilineo* E.

Datum rectilineum resolve in triangula BAD, BCD. a = Fac Pgr. FH = BAD ita ut ang. F = E. producta FI a fac (ad HI) Pgr. a 44. 1.

b 19. ax. c constr. IL = BCD. erit Pgr. FL = b FH + IL c = ABCD. Q. E. F.

Schol.



Hinc facile invenitur excessus HE, quo rectilineum aliquod A superat rectilineum minus B; nimirum si ad quamvis rectam CD applicentur Pgr. DF = A. & DH = B.

PROP. XLVI.

A data recta linea AD quadratum AC describere.

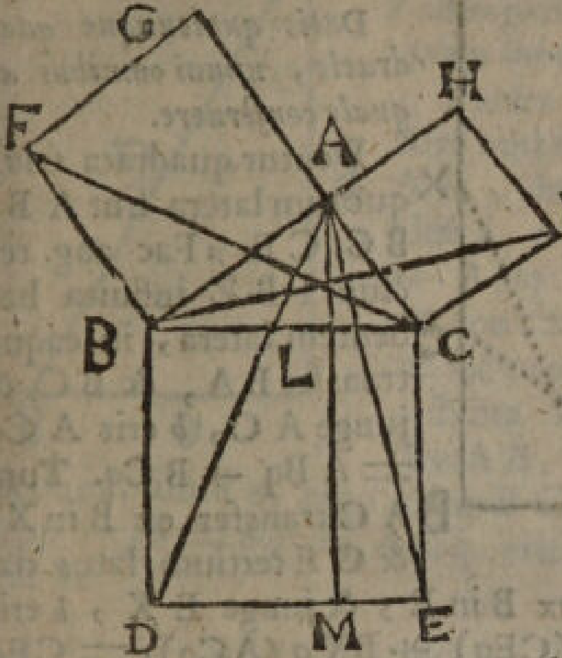
a 11. 1. b 3. 1. a Erige duas perpendiculares, AB, DC b æquales datæ AD; & junge BC. dico factum.

c constr. d 28. 1. e constr. f 34. 1. Cum enim ang. A + D c = 2 Rect. d erunt AB, DC parallelæ. Sunt vero etiam e æquales, f ergo AD, BC pares etiam sunt, & parallelæ. ergo Figura AC est parallelogramma, & æquilatera. g Sch. 29. 1. h 29. def. Anguli quoque omnes recti sunt, g quoniam unus A est rectus. h ergo AC est quadratum. Q. E. F.

Eodem modo facile describes rectangulum, quod sub datis duabus rectis contineatur.

PROP.

PROP. XLVII.



In rectangulis triangulis BAC *quadratum* BE, *quod à latere* BC *rectum angulum* BAC *subtendente describitur, æquale est eis*, BG, CH, *quæ à lateribus* AB, AC *rectum angulum continentibus describuntur.*

Iunge AE, AD; & duc AM. parall. CE.

Quoniam ang. DBC a = FBA; adde communem ABC, erit ang. ABD = FBC. Sed & AB b = FB, & BD b = BC. c ergo triang. ABD = FBC. atqui Pgr. BM. d = 2 ABD; & Pgr. BG d = 2 FBC (nam GAC est una recta per hyp. & 14. 1.) e ergo Pgr. BM = BG. Simili discursu Pgr. CM = CH. Totum igitur BE = BG + CH. Q. E. D.

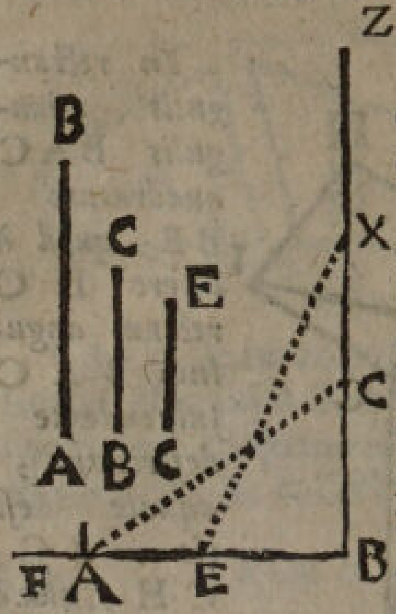
a 12. ax.
b 29. def.
c 4. 1.
d 41. 1.
e 6. ax.

Schol.

Hoc nobilissimum, & utilissimum theorema ab inventore Pythagora, Pythagoricum dici meruit. Ejus beneficio quadratorum additio, & substractio perficitur; quo spectant duo sequentia problemata.

PROBL. 1.

Andr. Tacq.

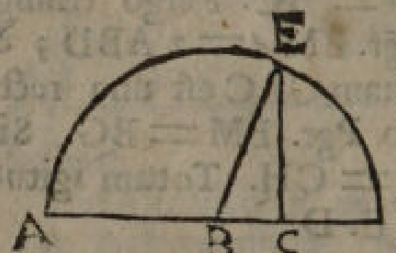


Datis quotcunque quadratis, unum omnibus æquale construere.

Dentur quadrata tria, quorum latera sint A B, B C, C E. a Fac ang. rectum F B Z infinita habentem latera, in eaque transfer B A, & B C, & junge A C, b erit A Cq = A Bq + B Cq. Tum A C transfer ex B in X; & C E tertium latus datum transfer ex B in E, & junge E X, b erit EXq = EBq (CEq) + BXq (ACq) c = CEq + ABq + BCq. Q. E. F.

a 11. 1.
b 47. 1.
c 2 ax.

PROBL. 2.



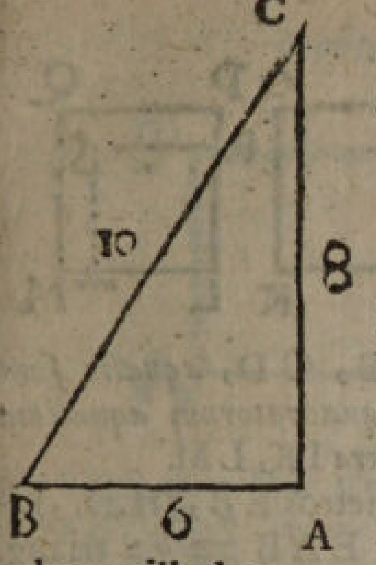
Datis duabus rectis inæqualibus A B, B C, exhibere quadratum, quo quadratum majoris A B excedit quadratum minoris B C.

Centro B intervallo BA describe circulum. ex C erige perpendicularem C E occurrentem peripheriæ in E. & ducatur B E. a Erit B Eq (BAq) = BCq + CEq. b ergo BAq − BCq = CEq. Q. E. F.

a 47. 1.
b 3. ax.

PROBL.

PROBL. 3.

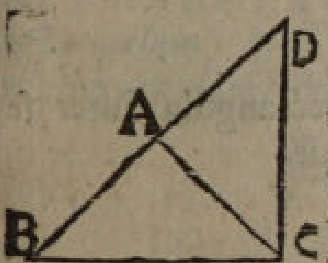


Notis duobus quibuscunque lateribus trigoni rectanguli ABC, reliquum invenire,

Latera rectum angulum ambientia sint A C, A B, hoc 6. pedum, illud 8. ergo cum A C q + A Bq = 64 + 36 = 100 = B Cq. erit BC = √ 100 = 10. 47. 1.

Nota sint deinde latera A B, B C, hoc 10. pedum, illud 6. ergo cum B Cq − A Bq = 100 − 36 = 64 = A Cq. erit Acq = √ 64 = 8. 47. 1.

PROP. XLVIII.



Si quadratum quod ab uno latere B G trianguli describitur, æquale sit eis quæ à reliquis trianguli lateribus A B; A C describuntur quadratis, angulus B A C comprehensus sub A B, A C reliquis duobus trianguli lateribus, rectus est.

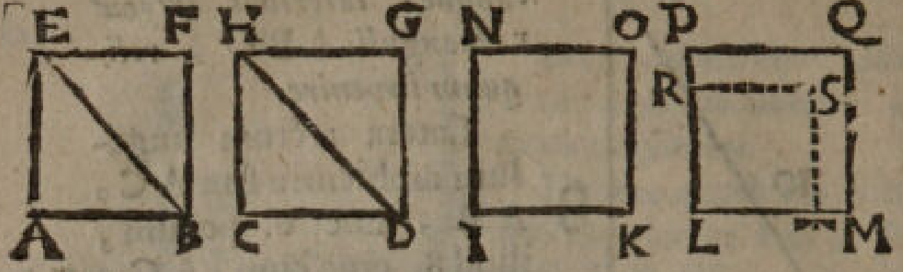
Duc ad AC perpendicularem D A = A B, & junge C D.

Iam C Dq a = A Dq + A Cq = A Bq + A Cq = B Cq. * ergo C D = B C. ergo triangula C AB, CAD, sibi mutuo æquilatera sunt; quare ang. CAB b = CAD c = Rect. Q.E.D. a 47. 1. * Vide seq. Theor. b 8. 1. c hyp.

Schol.

Assumpsimus exinde quod CDq. = BCq, sequi CD = BC. Hoc vero manifestum fiet ex sequenti theoremate.

THEOREMA.



Linearum æqualium A B, C D, *æqualia sunt quadrata* A F, C G; *& quadratorum æqualium* N K, P M *æqualia sunt latera* I K, L M.

Pro 1 Hyp. Duc diametros E B, H D. Liquet A F = *a* 2 triang. E A B = *b* 2 triang. H C D = *a* C G. Q. E. D.

a 34. 1.
b 4. 1. & 6. ax.
a 46. 1.
b 1. part.
c hyp.
d 9. ax.

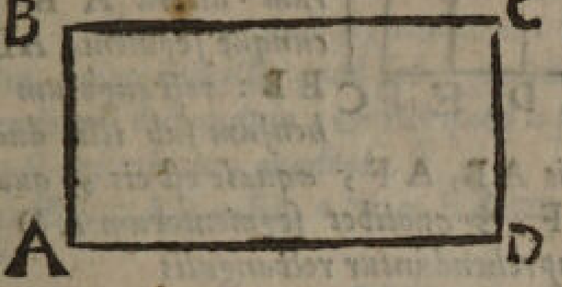
2. Hyp. Si fieri potest, sit L M ⊏ I K. fac L T = I K; *a* sitque L S = L Tq. ergo L S *b* = NK *c* = LQ. *d* Q. E. A. ergo LM = IK.

Coroll.

Eodem modo quælibet rectangula inter se æquilatera æqualia ostendentur.

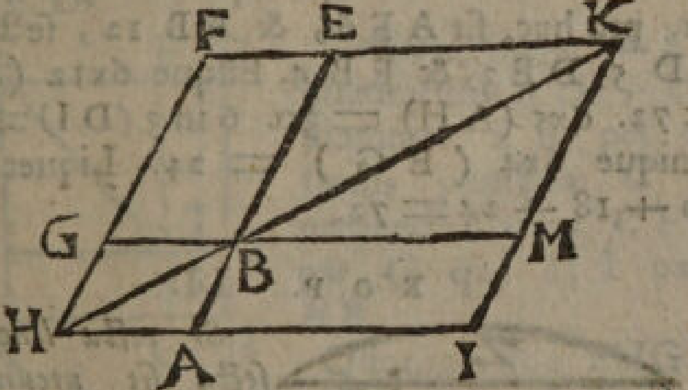
LIB. II.

Definitiones.



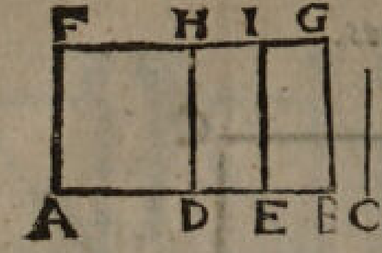
I. OMne parallelogrammum rectangulum A B C D contineri dicitur sub rectis duabus A B, A D, quæ rectum comprehendunt angulum.

Quando igitur dicitur rectangulum sub B A, A D*; vel brevitatis causa, rectangulum* B A D, *vel* B A x AD, (*vel* Z A *pro* Z x A) *designatur rectangulum, quod continetur sub* B A, & A D *ad rectum angulum constitutis.*



II. In omni parallelogrammo spatio FHIK unumquodq; eorum, quæ circa diametrum illius sunt, parallelogrammorum, cum duobus complementis Gnomon vocetur. *ut Pgr* FB + BI + GA (EHM) *est Gnomon. item* Pgr. FB + BI + EM (GKA) *est Gnomon.*

PROP. I.



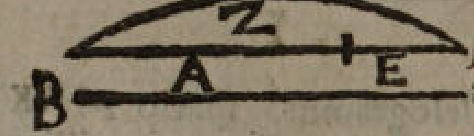
Si fuerint duæ rectæ lineæ A B, A F, seceturque ipsarum altera A B in quotcunque segmenta AD, DE, EB: rectangulum comprehensum sub illis duabus rectis lineis AB, AF, æquale est eis, quæ sub insecta AF, & quolibet segmentorum AD, DE, EB comprehenduntur rectangulis.

a 11.1. *a* Statue AF, perpendicularem ad AB. *a* per F duc infinitam FG perpendicularem ad AF. *a* Ex D, E, B erige perpendiculares DH, EI, BG. erit AG rectangulum sub AF, AB, &
b 19 ax.1. *b* est æquale rectangulis AH, DI, EG, hoc est
c 34.1. (quia DH, EI, AF *c* pares sunt) rectangulis sub AF, AD; sub AF, DE; sub AF, EB. Q. E. D.

Schol.

Propositiones decem primæ hujus libri valent etiam in numeris. Reliquas quilibet tyro examinet. pro hac, sit AF 6, & AB 12, sectus in AD 5, DE 3, & EB 4. Estque 6x12 (AG) $= 72$. 6x5 (AH) $= 30$. 6 in 3 (DI) $= 18$. denique 6x4 (EG) $= 24$. Liquet vero $30 + 18 + 24 = 72$.

PROP. II.

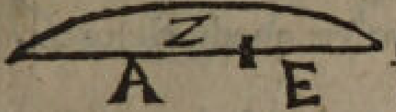


Si recta linea Z secta sit utcunque; rectangula, quæ sub tota Z, & quolibet segmentorum A, E comprehenduntur, æqualia sunt ei, quod à tota Z fit, quadrato.

Dico ZA + ZE = Zq. Nam sume B = Z.
a 1.2. *a* Estque BA + BE = BZ; hoc est (ob B = Z) ZA + ZE = Zq. Q. E. D.

PROP.

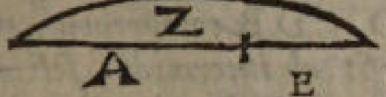
PROP. III.



Si recta linea Z secta sit utcunque; rectangulum sub tota Z, & uno segmentorum E comprehensum, æquale est illi, quod sub segmentis A, E comprehenditur, rectangulo, & illi quod à prædicto segmento E describitur, quadrato.

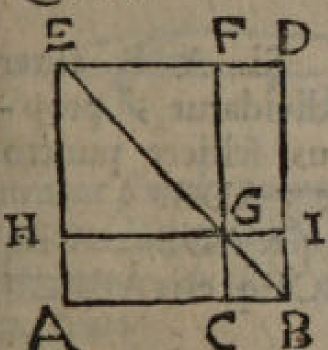
Dico. ZE=AE+Eq. [a] Nam EZ = EA + EE. a 1. 2.

PROP. IV.



Si recta linea Z secta sit utcunque; Quadratum, quod à tota Z describitur, æquale est, & illis quæ à segmentis A, E describuntur, quadratis, & ei, quod bis sub segmentis A, E comprehenditur, rectangulo.

Dico Zq=Aq.+Eq.2 AE.[a] Nam ZA=Aq+AE.[a] & ZE =Eq + AE. quum igitur Z A + ZE[b] = Zq, [c] erit Zq = Aq + Eq + 2 A E. Q. E. D. a 3. 2. b 2. 2. c 1. ax.



Aliter. Super AB fac quadratum AD, cujus diameter EB. per divisionis punctum C duc perpendicularem CF; & per G duc HI parall. AB.

Quoniam ang. EHG=A rectus est, & AEB [d] semirectus, [e] erit reliquus HGE etiam semirectus. Ergo HE [f]= HG [g]= EF [g] = AC. [h] proinde HF quadratum est rectæ AC. eodem modo CI est CB [k]. ergo AG, GD rectangula sunt sub AC, CB. Quare totum quadratum AD [k]= ACq + CBq + 2 ACB. Q. E. D. d 4. Cor. 32. 1. e 32. 1. f 6. 1. g 34. 1. h 29. def. 1. k 19. ax. 1.

Coroll.

Coroll.

1. Hinc liquet parallelogramma circa diametrum quadrati esse quadrata.

2. Item diametrum cujusvis quadrati ejus angulos bisecare.

3. Si $A = \frac{1}{2} Z$; erit $Zq = 4\,Aq$, & $Aq = \frac{1}{4} Zq$. item è contra, si $Zq = 4\,Aq$. erit $A = \frac{1}{2} Z$.

PROP. V.

Si recta linea AB *secetur in æqualia* AC *&* CB, *& non æqualia* AD, DB, *rectangulum sub inæqualibus segmentis* AD, DB *comprehensum una cum quadrato, quod fit ab intermedia sectionum* CD, *æquale est ei, quod à dimidia* CB *describitur, quadrato.*

Dico CBq = ADB + CDq.

a 4. 2.
b 3. 2.
c hyp.
d 1. 2.

Æquantur enim ista { CBq. / aCDq + CDB + DBq + CDB / CDq + bCBD(cAC × BD) + CDB / CDq + dADB.

Scholium.

Si AB aliter dividatur, propius scilicet puncto bisectionis, in E; dico AEB ⊏ ADB.

a 5. 2 & 3. ax.

Nam AEBa = CBq − CEq. & ADBa = CBq − CDq. ergo quum CDq ⊏ CEq, erit AEB ⊏ ADB. Q. E. D.

Coroll.

b 4. 2.

Hinc ADq + DBq ⊏ AEq + EBq. Nam ADq + DBq + 2 ADBb = ABq b = AEq + EBq + 2 AEB. ergo quum 2 AEB ⊏ 2 ADB, erit ADq + DBq ⊏ AEq + EBq. Q. E. D.

c 3. ax.

Unde 2. ADq + DBq − AEq c + EBq = 2 AEB − 2 ADB.

PROP. VI.

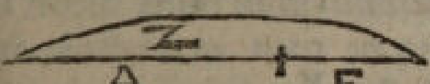
Si recta linea A *bifariam secetur, & illi recta quæpiam linea* E *in directum adjiciatur; rectangulum comprehensum sub tota cum adjecta* (sub. A + E), *& adjecta* E, *una cum quadrato, quod à dimidia* $\frac{1}{2}$ A, *æquale est quadrato à linea, quæ tum ex dimidia, tum ex adjecta componitur, tanquam ab una* $\frac{1}{2}$ A + E *descripto.*

Dico $\frac{1}{4}$Aq (a Q. $\frac{1}{2}$A) + AE + Eq = Q. $\frac{1}{2}$ A + E. a Nam Q. $\frac{1}{2}$ A + E = $\frac{1}{4}$ Aq + Eq + AE. a 4. & 3. Cor. 4 2.

Coroll.

Hinc si tres rectæ E, E + $\frac{1}{2}$ A, E + A sint in proportione Arithmetica, rectangulum sub extremis E, E + A contentum, una cum quadrato excessus $\frac{1}{2}$ A, æquale erit quadrato mediæ E + $\frac{1}{2}$ A.

PROP. VII.



Si recta linea Z *secetur utcunque; Quod à tota* Z, *quodque ab uno segmentorum* E, *utraque simul quadrata, æqualia sunt illi, quod bis sub tota* Z, *& dicto segmento* E *comprehenditur, rectangulo, & illi, quod à reliquo segmento* A *fit, quadrato.*

Dico Zq + Eq = 2 ZE + Aq. Nam Zq a = Aq + Eq + 2 AE. & 2 ZE b = 2 Eq. + 2 AE. a 4. 2. b 3, 2.

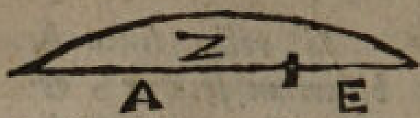
Coroll.

Hinc, quadratum differentiæ duarum quarumcumque linearum Z, E, æquale est quadratis utriusque minus duplo rectangulo sub ipsis.

c Nam Zq + Eq − 2 ZE = Aq = Q. Z − E. c 7. 2. & 3. ax.

PROP.

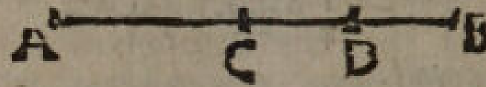
PROP. VIII.



Si recta linea Z secetur utcunque; rectangulum quater comprehensum sub tota Z, & uno segmentorum E, cum eo, quod à reliquo segmento A fit, quadrato, æquale est ei, quod à tota Z, & dicto segmento E, tanquam ab una linea Z+E describitur, quadrato.

a 7. 2. & 3. ax. Dico 4 ZE+Aq=Q. Z+E. Nam 2 ZE a= Zq+Eq− Aq. ergo 4 ZE+Aq=Zq+Eq+ 2
b 4. 2. ZE b=Q. Z+E. Q. E. D.

PROP. IX.



Si recta linea AB *secetur in æqualia* AC, CB, *& non æqualia* AD, DB. *quadrata, quæ ab inæqualibus totius segmentis* AD, DB *fiunt, simul duplicia sunt, & ejus, quod à dimidia* AC, *& ejus, quod ab intermedia sectionum* CD *fit, quadrati.*

Dico ADq+DBq = 2 ACq+2 CDq. Nam
a 4. 2. ADq+DBq a=ACq+CDq+ 2ACD+DBq:
b hyp. atqui 2 ACD (b 2 BCD) + DBq
c 7. 2. c= CBq
d 2. ax. (ACq) + CDq. d ergo ADq+DBq = 2 ACq + 2 CDq. Q. E. D.

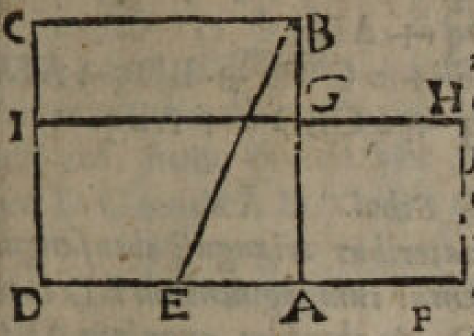
PROP. X

Si recta linea A *secetur bifariam, adjiciatur autem ei in rectum quæpiam linea; Quod à tota* A *cum adjuncta* E, *& quod ab adjuncta* E, *utraque simul quadrata, duplicia sunt & ejus, quod à dimidia* $\frac{1}{2}$ A; *& ejus, quod à composita ex dimidia, & adjuncta, tanquam ab una* $\frac{1}{2}$ A + E, *descriptum est, quadrati.*

a 4. 2. Dico Eq+Q. A+E, hoc est a Aq+2 Eq+2
b Cor. 4. 2. AE=2 Q. $\frac{1}{2}$ A+2 Q. $\frac{1}{2}$ A+E. Nam 2 Q. $\frac{1}{2}$ A b
c 4. 2. = $\frac{1}{2}$ Aq. & 2 Q. $\frac{1}{2}$ A+E c= $\frac{1}{2}$ Aq+2Eq+2 AE.

PROP.

PROP. XI.



Datam rectam lineam AB secare in G, ut comprehensum sub tota AB, & altero segmentorum BG rectangulum, æquale sit ei, quod à reliquo segmento AG sit, quadrato.

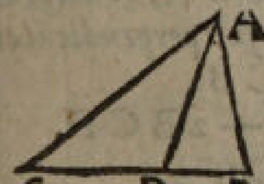
Super AB *a* describe quadratum AC. latus AD *b* biseca in E. duc EB. ex EA producta cape EF = EB. ad AF *a* statue quadratum AH. Erit AH = ABxBG. a 46. 1. b 10. 1.

Nam protracta HG ad I; Rectang. DH + EAq *c* = EFq *d* = EBq *e* = BAq + EAq. ergo DH *f* = BAq *d* = quad. AC. subtrahe commune AI; *f* remanet quad. AH = GC; *d* id est AGq = ABx BG. Q. E. F. c 6. 2. d constr. e 47. 1. f 3. ax.

Scholium.

Hæc Propositio numeris explicari nequit; * neque enim ullus numerus ita secari potest, ut productum ex toto in partem unam æquale sit quadrato partis reliquæ. * vid. 6. 13.

PROP. XII.



In amblygoniis triangulis ABC quadratum, quod fit à latere AC angulum obtusum ABC subtendente, majus est quadratis, quæ fiunt à lateribus AB, BC obtusum angulum ABC comprehendentibus, rectangulo bis comprehenso, & ab uno laterum BC, quæ sunt circa obtusum angulum ABC, in quod, cum protractum fuerit, cadit perpendicularis AD, & ab assumpta exterius linea BD sub perpendiculari AD prope angulum obtusum ABC.

Dico

Dico $ACq = CBq + ABq + 2\,CB \times BD$.

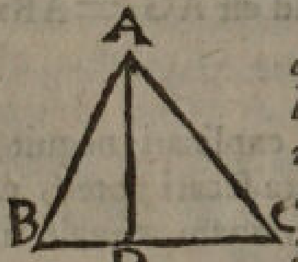
Nam ista æqualia sunt inter se:
- ACq.
- a 47. 1. — a CDq + ADq.
- b 4. 2. — b CBq + 2 CBD + BDq + ADq
- c 47. 1. — CBq + 2 CBD c + ABq.

Schol.

Hinc, *cognitis lateribus trianguli obtusanguli* ABC, *facile invenientur tum segmentum* BD *inter perpendicularem* A D, *& obtusum angulum* ABC *interceptum, tum ipsa perpendicularis* A D.

Sic; Sit AC 10, AB 7, CB 5; unde ACq 100, ABq 49, CBq 25. Proinde ABq + CBq = 74. hunc deme ex 100, manet 26 pro 2 CBD. unde C B D erit 13. hunc divide per C B 5, provenit $2\frac{3}{5}$ pro BD. quare AD invenitur per 47. 1.

Prop. XIII.



In oxygoniis triangulis ABC *quadratum à latere* A B *angulum acutum* A C B *subtendente, minus est quadratis, quæ fiunt à lateribus* A C, C B *acutum angulum* A C B *comprehendentibus, rectangulo bis comprehenso, & ab uno laterum* B C, *quæ sunt circa acutum angulum* ACB, *in quod perpendicularis* AD *cadit, & ab assumpta interius linea* D C *sub perpendiculari* A D, *prope angulum acutum* A C B.

Dico $ACq + BCq = ABq + 2\,BCD$.

Nam æquantur ista:
- A Cq + B Cq.
- a 47. 1. — a A Dq + D Cq + B Cq.
- b 7. 2. — b A Dq + B Dq + 2 B C D.
- c 47. 1. — c A Bq + 2 B C D.

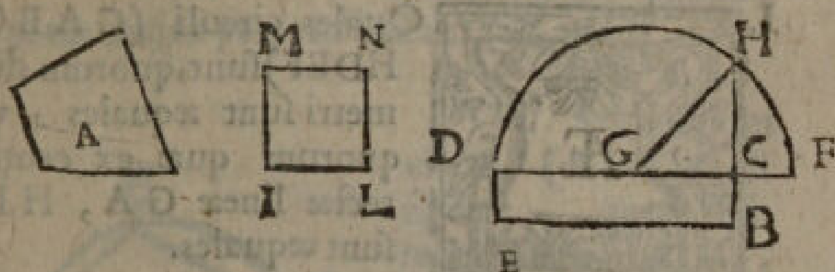
Coroll.

Hinc etiam *cognitis lateribus trianguli* A B C, *invenire est tam segmentum* DC *inter perpendicula-*

rem A D, & acutum angulum A B C interceptum, quam ipsam perpendicularem A B.

Sit AB 13, A C 15, B C 14. Detrahe A Bq (169) ex A Cq + B Cq hoc est ex 225 + 196 = 421; remanet 252 pro 2 BCD; unde BCD erit 126. hunc divide per B C 14, provenit 9 pro D C. unde A D = $\sqrt{}$: 225 − 81 = 12.

PROP. XIV.



Dato rectilineo A æquale quadratum M L invenire.

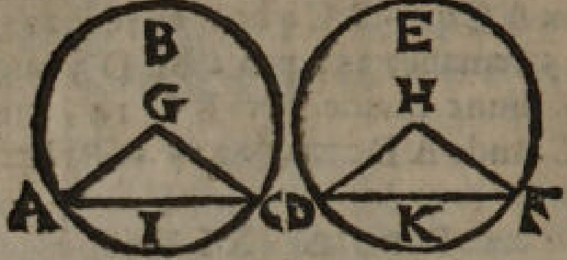
a Fac rectangulum D B = A, cujus majus latus D C produc ad F, ita ut C F = C B. b Biseca D F in G, quo centro ad intervallum G F describe circulum F H D, producatur C B, donec occurrat circumferentiæ in H. Erit C Hq = * M L = A a 45. 1. b 10. 2. * 46. 1.

Ducatur enim G H. Estque A c = D B c = D C F d = G Fq − G Cq e = H Cq c = M L Q. E. F. c constr. d 5. 2. & 3. ax. e 47. 1. & 3. ax.

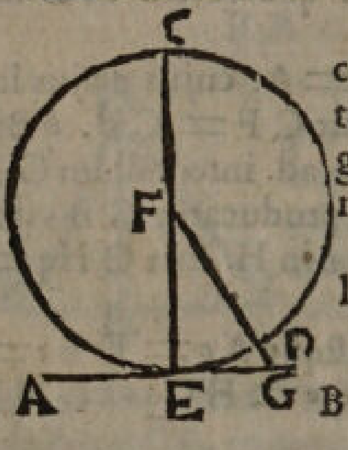
LIB.

LIB. III.

Definitiones.

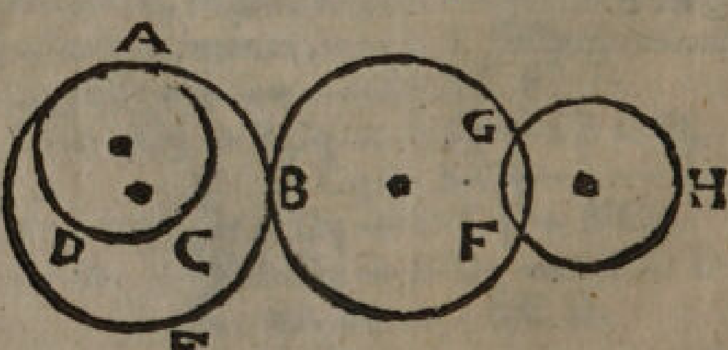


I. ÆQuales circuli (G A B C, HDEF) sunt, quorum diametri sunt æquales, vel quorum quæ ex centris rectæ lineæ G A, H D, sunt æquales.



II. Recta linea A B circulum F E D tangere dicitur, quæ cum circulum tangat, si producatur circulum non secat.

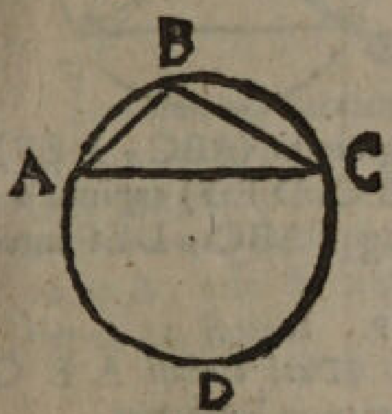
Recta F G secat circulum F E D.



III. Circuli D A C, A B E (item F B G, A B E) se mutuo tangere dicuntur, qui se mutuo tangentes sese mutuo non secant.

Circulus BFG secat circulum FGH. In

IV. In circulo GABD æqualiter distare à centro dicuntur rectæ lineæ FE KL, cum perpendiculares GH, GN quæ à centro G in ipsas ducuntur, sunt æquales. Longius autem abesse illa BC dicitur, in quam major perpendicularis GI cadit.

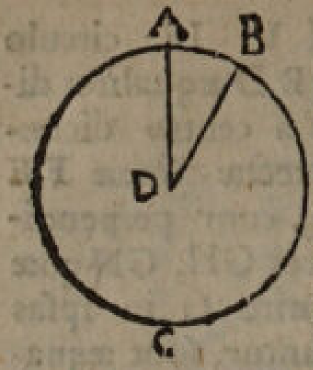


V. Segmentum circuli (ABC) est figura, quæ sub recta linea AC, & circuli peripheria ABC comprehenditur.

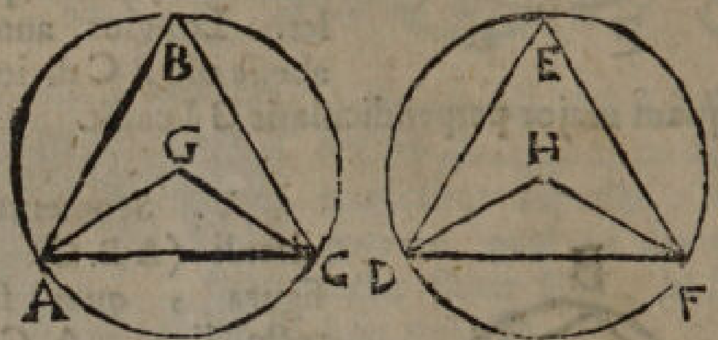
VI. Segmenti autem angulus (CAB) est, qui sub recta linea CA, & circuli peripheria AB comprehenditur.

VII. In segmento autem (ABC) angulus (ABC) est, cum in segmenti peripheria sumptum fuerit quodpiam punctum B, & ab illo in terminos rectæ ejus lineæ AC, quæ segmenti basis est, adjunctæ fuerint rectæ lineæ AB, CB, is inquam angulus ABC ab adjunctis illis lineis AB, CB comprehensus.

VIII. Cum vero comprehendentes angulum ABC, rectæ lineæ AB, BC aliquam assumunt peripheriam ADC, illi angulus ABC insistere dicitur.

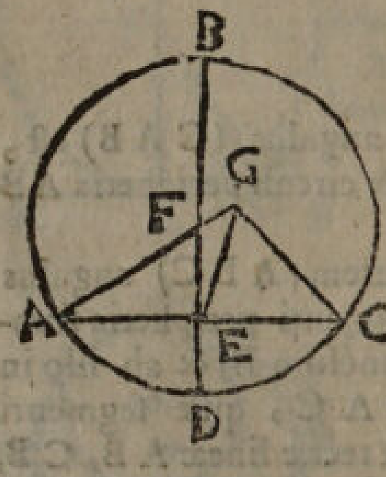


IX. Sector autem circuli (ADB) est, cum ad ipsius circuli centrum D constitutus fuerit angulus ADB; comprehensa nimirum figura ADB. & à rectis lineis AD, BD angulum continentibus, & à peripheria AB ab illis assumpta.



X. Similia circuli segmenta (ABC, DEF) sunt, quæ angulos (ABC, DEF) capiunt æquales; aut in quibus anguli ABC, DEF inter se sunt æquales.

PROP. I.



Dati circuli ABC *centrum* F *reperire*.

Duc in circulo rectam AC utcunq;,quam bisseca in a E. per E duc perpendicularē DB. hanc bisseca in F. erit F centrū.

a 10.1.

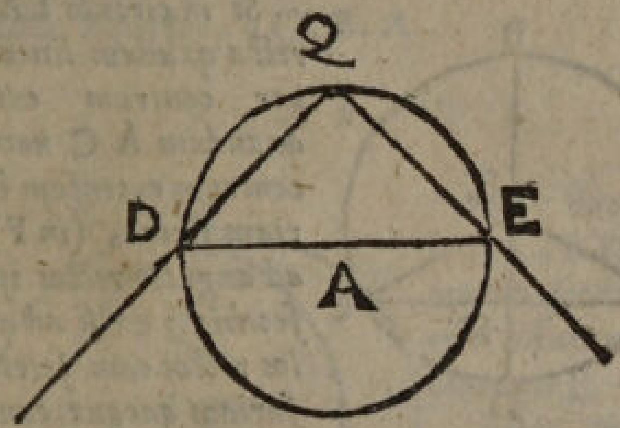
Si negas, centrum esto G, extra rectam DB (nam in ea esse non potest, cum ubique extra F dividatur inæqualiter) ducanturque GA, GC, GE. Vis G centrum esse; a ergo GA = GC; & per constr. AE = EC, latus vero GE commune est; b ergo anguli GEA, GEC pares, & c proinde recti sunt. d ergo ang. GEC = FEC rect. e Q. E. A.

a 15. def. 1.
b 8. 1.
c 10. def. 1.
d 12. ax.
e 9. ax.

Coroll.

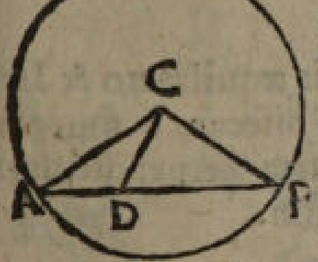
Coroll.

Hinc, ſi in circulo recta aliqua linea B D aliquam rectam lineam A C biſariam & ad angulos rectos ſecet, in ſecante B D erit centrum.



Facillime per normam invenitur centrum vertice Q ad circumferentiam applicato. Si enim recta D E jungens puncta D, & E, in quibus normæ latera Q D, Q E peripheriam ſecant, biſecetur in A, erit A centrum. Demonſtratio pendet ex 31. hujus. *Andr. Tacq.*

P R O P. II.



Si in circuli C A B *peripheria duo quælibet puncta,* A, B *accepta fuerint; recta linea* A B, *quæ ad ipſa puncta adjungitur, intra circulum cadet.*

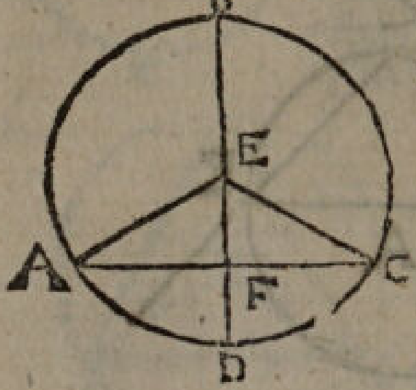
Accipe in recta A B quodvis punctum D, & ex centro C duc C A, C D, C B. & quoniam C A *a* = C B, *b* erit ang. A = B. Sed ang. C D B *c* ⊏ A; ergo ang. C D B ⊏ B. *d* ergo C B ⊏ C D. atqui C B tantum pertingit ex centro ad circumferentiam; ergo C D eouſque non pertingit. ergo punctum D eſt intra circulum. Idemque oſtendetur de quovis alio puncto rectæ A B. Tota igitur A B cadit intra circulum. Q. E. D.

a 15. *def.* 1.
b 5. 1.
c 16. 1.
d 19. 1.

Coroll.

Hinc, Recta circulum tangens, ita ut eum non secet, in unico puncto tangit.

Prop. III.



Si in circulo EABC *recta quædam linea* BD *per centrum extensa quandam* A C *non per centrum extensam bifariam secet, (in* F*) & ad angulos rectos ipsam secabit; & si ad angulos rectos eam secet, bifariam quoque eam secabit.*

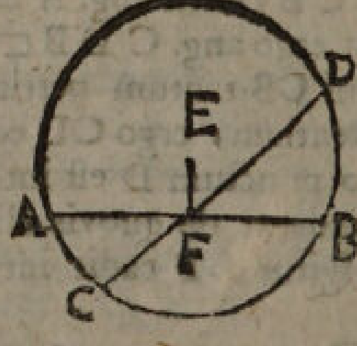
Ex centro E ducantur E A, E C.

a hyp. b 15.def.1. 1.*Hyp.* Quoniam AF *a* = FC, & EA*b* = EC,
c 8.1. latusque EF commune est, *c* erunt anguli EFA,
d 10 def.1. EFC pares, & *d* consequenter recti. Q. E. D.
e hyp. & 11.ax. 2.*Hyp.* Quoniam ang. EFA *e* = EFC, & ang.
f 5.1. EAF *f* = ECF, latusque EF commune, *g* erit
g 26.1. AF = FC. Bisecta est igitur A C. Q. E. D.

Coroll.

Hinc, in triangulo quovis æquilatero & Isoscele linea ab angulo verticis bisecans basim, perpendicularis est basi. & contra perpendicularis ab angulo verticis bisecat basim.

Prop. IV.



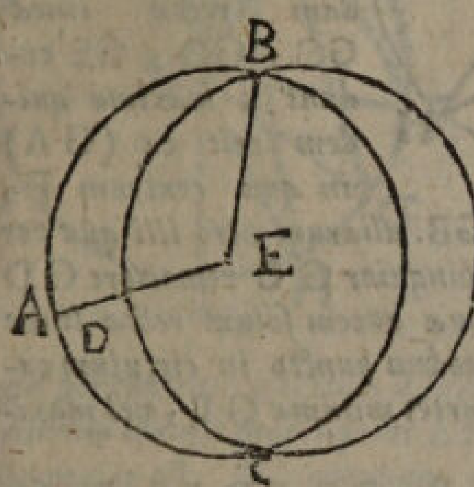
Si in circulo A C D *duæ rectæ lineæ* A B, C D *sese mutuo secent non per centrum* E *extensæ, sese mutuo bifariam non secabunt.*

Nam si una per centrum

trum transeat, patet hanc non bisecari ab altera, quæ ex hyp. per centrum non transit.

Si neutra per centrum transit, ex E centro duc E F. Si jam ambæ A B, C D forent bisectæ in F, anguli EFB, EFD *a* ambo essent recti, & a 3. 3.
proinde æquales. *b* Q. E. A. b 9. ax.

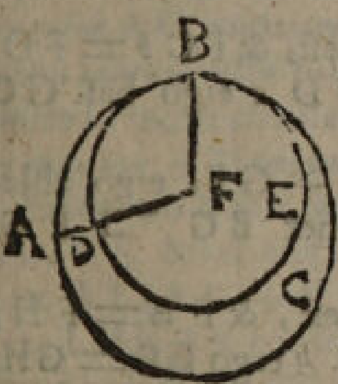
Prop. V.



Si duo circuli BAC, BDC *sese mutuo secent, non erit illorum idem centrum* E.

Alias enim ductis ex communi centro E rectis E B, E D A, essent E D *a* = E B *a* = a 15. def. 1.
E A. *b* Q. E. A. b 9. ax.

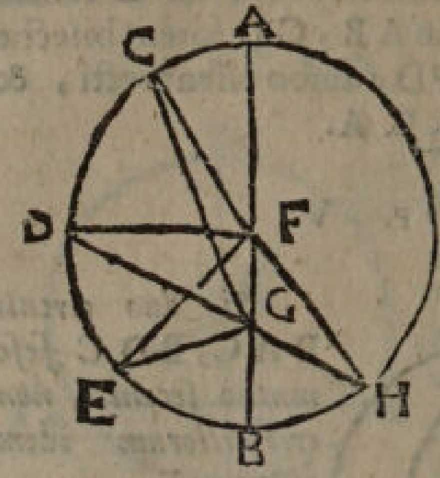
Prop. VI.



Si duo circuli BAC, BDE, *sese mutuo interius tangant* (in B) *eorum non erit idem centrum* F.

Alias ductis ex centro F rectis F B, F D A, essent F D *a* = F B *a* = F A. a 15. defin.
b Q. F. N. b 9 ax.

PROP. VII.



Si in AB *diametro circuli quodpiam sumatur punctum* G, *quod circuli centrum non sit, ab eoque puncto in circulum quædam rectæ lineæ* GC, GD, GE *cadunt; maxima quidem erit ea* (GA) *in qua centrum* F, *minima vero reliqua* GB. *aliarum vero illi, quæ per centrum ducitur, propinquior* GC *remotiore* GD *semper major est. Duæ autem solum rectæ lineæ* GE GH *æquales ab eodem puncto in circulum cadunt, ad utrasque partes minimæ* GB, *vel maximæ* GA.

a 23. 1. Ex centro F duc rectas FC, FD, FE; & *a* fac ang. BFH = BFE.

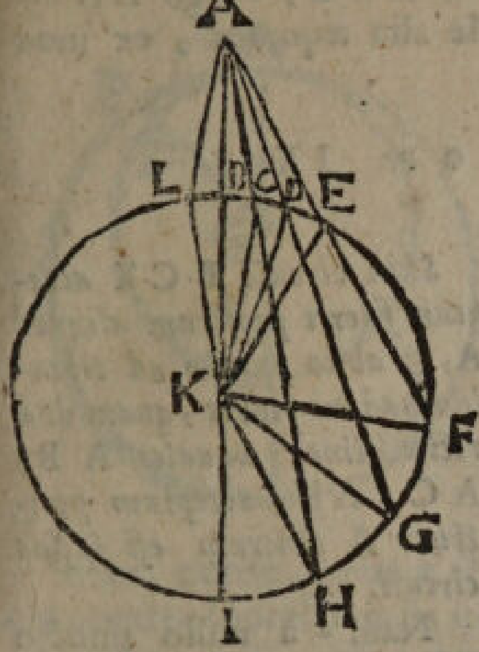
a 20. 1. 1. GF + FC (hoc est GA) *a* ⊏ GC. Q. E. D.

b 15. def. 1. c 9. ax. d 24. 1. 2. Latus FG commune est, & FC *b* = FD, atque ang. GFC *c* ⊏ GFD *d* ergo baf. GC ⊏ GD. Q. E. D.

e 20. 1. f 5. ax. 3. FB (FE) *e* ⊐ GE + GF. ergo ablato communi FG *f* remanet BG ⊐ EG. Q. E. D.

g constr. h 4. 1. 4. Latus FG commune est, & FE = FH; atque ang. BFH *g* = BFE. *h* ergo GE = GH. Quod vero nulla alia GD ex puncto G æquetur ipsi GE, vel GH, jamjam ostensum est. Q. E. D.

PROP. VIII.



Si extra circulum sumatur punctum quodpiam A, ab eoque puncto ad circulum deducantur quædam lineæ AI, AH, AG, AF, quarum una quidem AI per centrum K protendatur, reliquæ vero ut libet; in cavam peripheriam cadentium rectarum linearum maxima quidem est illa AI, quæ per centrum ducetur, aliarum autem ei quæ per centrum transit propinquior AH remotiore AG semper major est. In convexam vero peripheriam cadentium rectarum linearum minima quidem est illa AB, quæ inter punctum A, & diametrum BI interponitur; aliarum autem ea, quæ est minimæ propinquior AC remotiore AD semper minor est. Duæ autem tantum rectæ lineæ AC, AL æquales ab eo puncto in ipsum circulum cadunt, ad utrasque partes minimæ AB, vel maximæ AI.

Ex centro K duc rectas KH, KG, KF; KC, KD, KE. & fac ang. AKL = AKC.

1. AI (AK + KH) *a* ⊏ AH. Q. E. D. a 20. 1.

2. Latus AK commune est; & KH = KG; atque ang. AKH ⊏ AKG. *b* ergo baf. AH ⊏ AG. Q. E. D. b 24. 1.

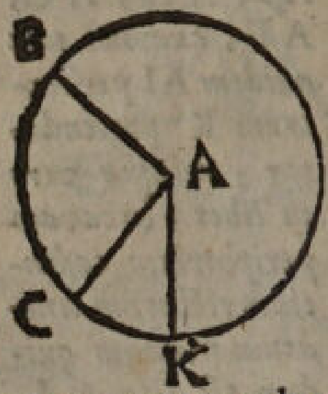
3. KA *c* ⊐ KC + CA. aufer hinc inde æquales KC, KB, *d* erit AB ⊐ AC. c 20. 1. d 5. ax.

4. AC + CK *e* ⊐ AD + DK. aufer hinc inde æquales CK, DK, *f* erit AC ⊐ AD. Q. E. D. e 21. 1. f 5. ax.

5. Latus K A est commune & K L = K C atque ang. A K L g = A K C, b ergo L A = C A. hisce vero nulla alia æquatur, ex mox ostensis. ergo, &c.

g constr.
h 4. 1.

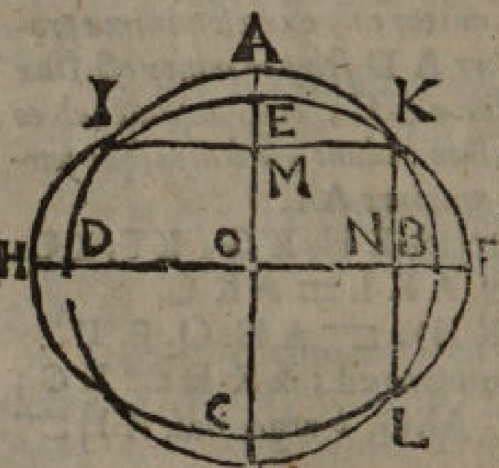
Prop. IX.



Si in circulo B C K acceptum fuerit punctum aliquod A, & ab eo puncto ad circulum cadant plures, quam duæ rectæ lineæ æquales A B, A C, A K, acceptum punctum A centrum est ipsius circuli.

a 7. 3. Nam a à nullo puncto extra centrum plures quam duæ rectæ lineæ æquales duci possunt ad circumferentiam. Ergo A est centrum. Q. E. D.

Prop. X.

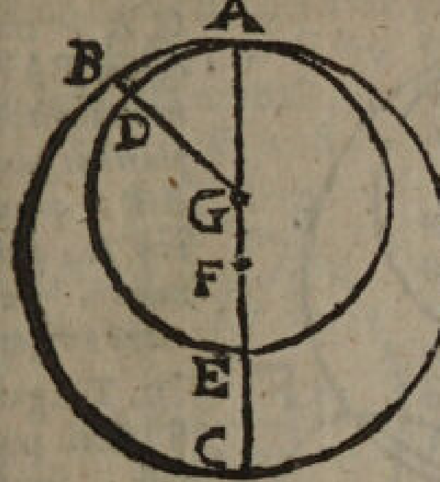


Circulus I A K B L circulum I E K F L in pluribus quam duobus punctis non secat.

Secet, si fieri potest, in tribus punctis I K L. Iunctæ I K K L. bisecentur in M & N. a Ambo circuli centrum habent in singulis perpendicularibus M C, N H, & proinde in earum intersectione O. ergo secantes circuli idem centrum habent. b Q. F. N.

a cor. 1. 3.
b 5. 3.

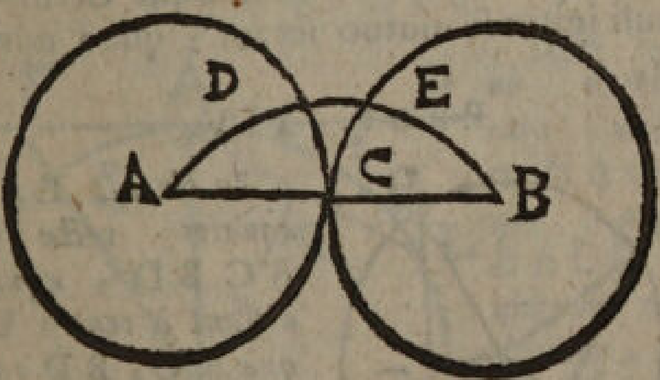
PROP. XI.



Si duo circuli GADE, FABC *sese intus contingant, atque accepta fuerint eorum centra* G, F; *ad eorum centra adjuncta recta linea* F G, *& producta, in* A *contactum circulorum cadet.*

Si fieri potest, recta F G protracta secet circulos extra contactum A, sic ut non F G A, sed FGDB sit recta linea. ducatur G A. Et quia GD [a] = GA, & GB [b] ⊐ GA, (cum recta FGB transeat per F centrum majoris circuli) erit G B ⊐ GD. [c] Q. E. A. a 15.def.1. b 7.3. c 9.ax.

PROP. XII.

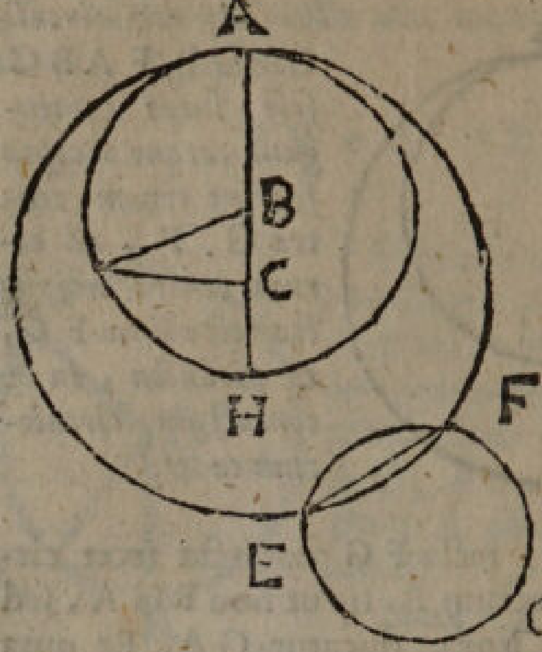


Si duo circuli ACD, BCE *sese exterius contingant, linea recta* A B *quæ ad eorum centra* A, B *adjungitur, per contactum* C *transibit.*

Si fieri potest, sit recta ADEB secans circulos extra contactum C in punctis D, E. Duc A C, CB. erit AD + EB (AC + CB) [a] ⊏ AD+EB. [b] Q. E. A. a 20.1. b 9.ax.

PROP.

PROP. XIII.



Circulus C A F *circulum* BAH *non tangit in pluribus punctis, quam uno* A, *sive intus, sive extra tangat.*

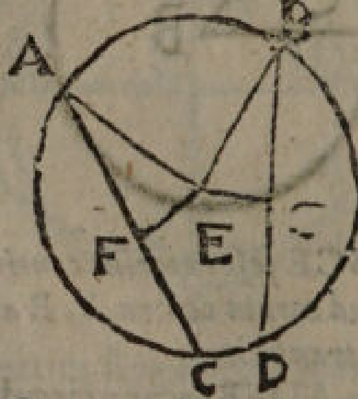
1. Tangat, si fieri potest, intus in punctis A, H. a ergo recta C B centra connectens, si producatur cadet tam in A, quam in H. Quoniam igitur CH b = CA, & BH ⊏ CH. erit BA (c BH) ⊏ CA. Q. E. A.

a 11. 3.
b 15. def. 1.
c 15. def. 1.
d 9. ax.

2. Sin dicatur exterius contingere in punctis E & F, e ducta recta E F in utroque circulo erit. Circuli igitur se mutuo secant, quod non ponitur.

e 2. 3.

PROP. XIV.



In circulo E A B C *æquales rectæ lineæ* A C B D, *æqualiter distant à centro* E, *&* *quæ* A C, B D *æqualiter distant à centro, æquales sunt inter se.*

Ex centro E duc perpendiculares E F, E G : a quæ bisecabunt A C, D B. connecte E A E B.

a 3. 3.

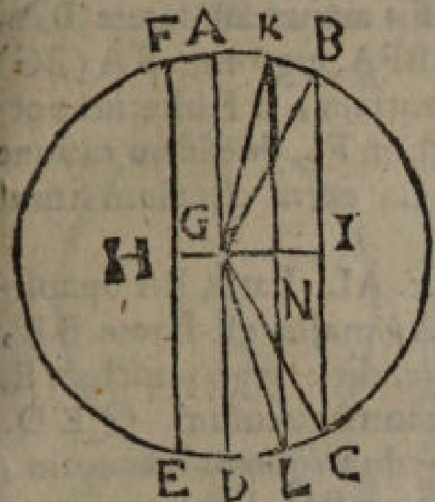
1. *Hyp.* AC = BD. ergo A F b = B G. sed & E A

b 7. ax.

EA = EB. ergo FEq *c*= EAq − AFq = EBq − BGq*c*= EGq. *d* ergo FE = EG. Q.E.D. c 47.1. & 3.ax. d Schol. 48.1.

2. *Hyp.* EF = EG. ergo AFq *c*= EAq − EFq = EBq − EGq *c*= GBq. ergo AF *d*= GB. *e* proinde AD = BC. Q.E.D. e 6 ax.

PROP. XV.

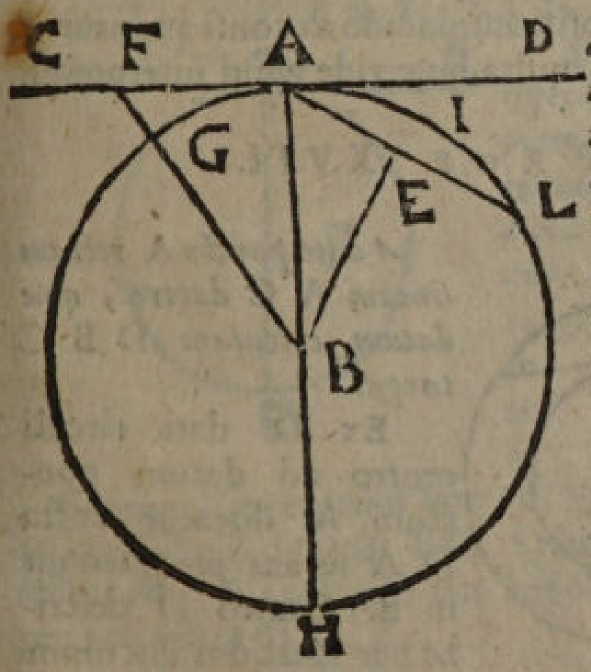


In circulo GABC *maxima quidem linea est diameter* AD; *aliarum autem centro* G *propinquior* FE *remotiore* BC *semper major est.*

1. Duc GB, GC. Diameter AD (*a* GB + GC) *b* $\sqsubset$ BC Q.E.D. a 15. def. 1. b 20. 1.

2. Sit distantia GI $\sqsubset$ GH. accipe GN = GH. per N duc KL perpend. GI. junge GK, GL. & quia GK = GB, & GL = GC; estque ang. KGL $\sqsubset$ BGC, *c* erit KL (FE) $\sqsubset$ BC. Q.E.D. c 24. 1.

PROP. XVI.



Quæ CD *ab extremitate diametri* HA *cujusque circuli* BALH *ad angulos rectos ducitur, extra ipsum circulum cadet, & in locum inter ipsam rectam lineam, & peripheriam comprehen-*

prehensum altera recta linea A L *non cadet*, & *semicirculi quidem angulus* BAI *quovis angulo acuto rectilineo* B A L *major est*; *reliquus autem* D A I *minor*.

1. Ex centro B ad quodvis punctum F in recta A C duc rectam B F. Latus B F subtendens angulum rectum B A F *a* majus est latere B A, quod opponitur acuto BFA. ergo cum B A (BG) pertingat ad circumferentiam, B F ulterius porrigetur, adeoque punctum F, & eadem ratione quodvis aliud rectæ A C, extra circulum situm erit. Q. E. D.

a 19. 1.

2. Duc BE perpendic. AL. Latus BA oppositum recto angulo B E A *b* majus est latere B E, quod acutum BAE subtendit: ergo punctum E, adeoque tota E A cadit intra circulum. Q. E D.

b 19. 1.

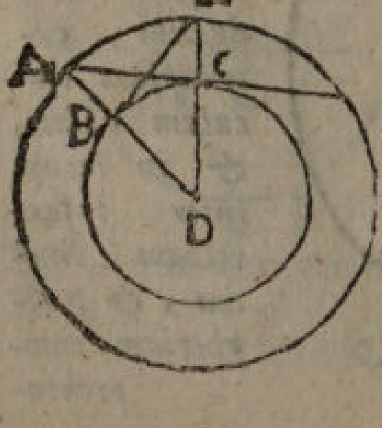
3. Hinc sequitur angulum quemvis acutum, nempe EAD angulo contactus DAI majorem esse. Item angulum quemvis acutum B A L angulo semicirculi BAI minorem esse. Q.E.D.

Coroll.

Hinc, recta à diametri circuli extremitate ad angulos rectos ducta ipsum circulum tangit.

Ex hac propositione paradoxa consequuntur, & mirabilia bene multa, quæ vide apud interpretes.

PROP. XVII.



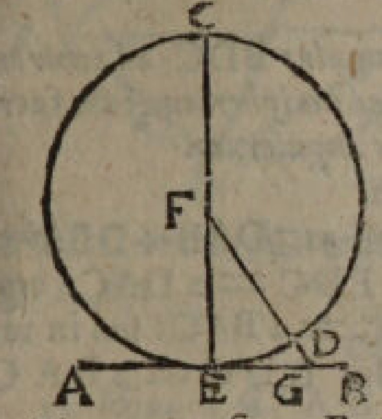
A dato puncto A *rectam lineam* A C *ducere, quæ datum circulum* D B C *tangat*.

Ex D dati circuli centro ad datum punctum A ducatur recta D A secans peripheriam in B. Centro D describe per A alium circulum A E;

A E; & ex B duc perpendicularem ad A D, quæ occurrat circulo A E in E. duc E D occurrentem circulo B C in C. ex A ad C ducta recta tanget circulum D B C.

Nam D B *a* = D C, & D E *a* = D A, & ang. a 15. def. 1.
D communis est: *b* ergo ang. ACD = EBD, b 4. 1.
rect. *c* ergo A C tangit circulum C. Q. E. F. c cor. 16. 3.

P R O P. XVIII.

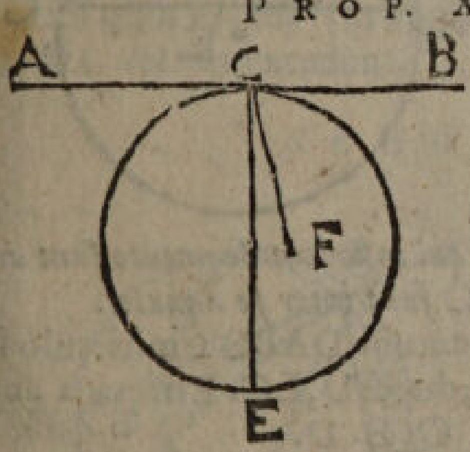


Si circulum FEDC tangat recta quæpiam linea A B, à centro autem ad contactum E adjungatur recta quædam linea F E; quæ adjuncta fuerit F E ad ipsam contingentem A B perpendicularis erit.

Si negas, sit ex F centro alia quædam F G perpendicularis ad contingentem, *a* secabit ea circulum in D. Quum igitur ang. F G E rectus dicatur *b* erit ang. F E G acutus. *c* ergo F E (F D) ⊏ F G. *d* Q. E. A.

a 2. def. 3.
b cor. 17. 1.
c 19. 1.
d 9 ax.

P R O P. XIX.



Si circulum tetigerit recta quæpiam linea A B, à contactu autem C recta linea C E ad angulos rectos ipsi tangenti excitetur, in excitata C E erit centrum circuli.

Si negas, sit centrum extra C E in F, & ab F ad contactum ducatur F C. Igitur ang. F C B rectus est; & *a* proinde par angulo E C B recto per hypoth. *b* Q. E. A.

a 11. ax.
b 9. ax.

PROP.

PROP. XX.

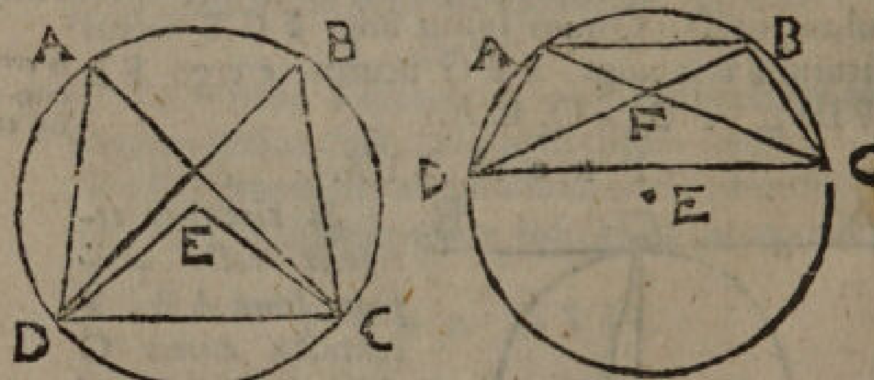
In circulo DABC, *angulus* BDC *ad centrum duplex est anguli* BAC *ad peripheriam, cum fuerit eadem peripheria* BC *basis angulorum.*

Duc diametrum ADE.

a 32. 1. b 5. 1. Externus angulus BDE a= DAB + DBA b= 2 DAB. Similiter ang. EDC = 2 DAC. ergo in primo casu totus BDC = 2 BAC; sed in tertio

c 20. ex. casu c reliquus angulus BDC = 2BAC. Q. E. D.

PROP. XXI.



In circulo EDAC *qui in eodem segmento sunt anguli,* DAC *&* DBC *sunt inter se æquales.*

1. *Cas.* Si segmentum DABC semicirculo sit majus, ex centro E, duc ED, EC. Eritque 2 ang.

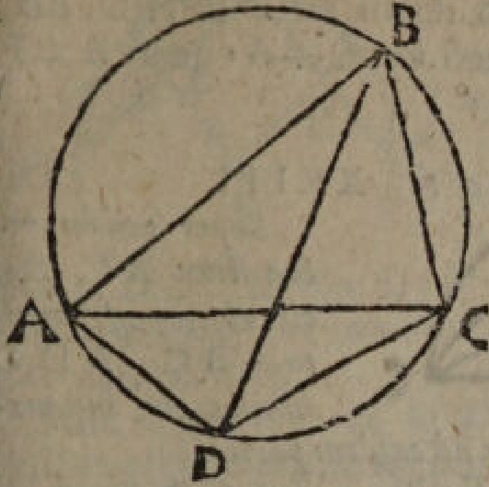
a 20. 3. A a= E a= 2 B. Q. E. D.

2. *Cas.* Sin segmentum semicirculo majus non fuerit, summa angulorum trianguli ADF æquatur summæ angulorum in triangulo BCF. Demantur hinc inde

b 15. 1. c per 1. ax. AFD b= BFC, & ADB c= ACB, remanent DAC = DBC. Q. E. D.

PROP.

PROP. XXII.



Quadrilaterorum ABCD *in circulo descriptorum anguli* ADC, ABC, *qui ex adverso, duobus rectis sunt æquales.*

Duc AC, BD. Ang. ABC + BCA + BAC *a* = 2 Rect. Sed BDA *b* = BCA, & BDC *b* = BAC. ergo ABC + ADC = 2 Rect. Q. E. D.

a 32. 1.
b 21. 3.
c 1. ax

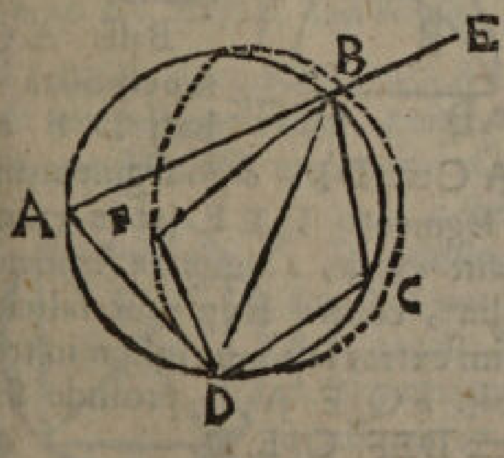
Coroll.

1. Hinc, si * AB unum latus quadrilateri in circulo descripti producatur, erit angulus externus EBC æqualis angulo interno ADC, qui opponitur ei ABC, qui est deinceps externo EBC. ut patet ex 13. 1. & 3. ax.

* *vide seq. diagram.*

2. Item circa Rhombum circulus describi nequit; quia adversi ejus anguli vel cedunt duobus rectis, vel eos excedunt.

SCHOL.

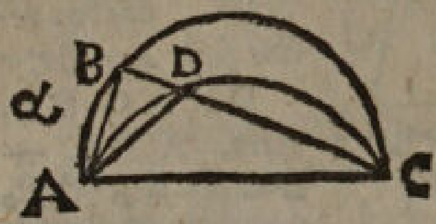


Si in quadrilatero ABCD *anguli* A, & C *qui ex adverso duobus rectis æquantur, circa quadrilaterum circulus describi potest.*

Nam circulus per quoslibet

bet tres angulos B, C, D transibit (ut patebit ex 5.4.) dico eundem per A transire. Nam si neges, transeat per F. ergo ductis rectis BF, FD, BD;
a 22. 3. ang. C+F *a*=2 Rect. *b*=C+A *c* quare A=F.
b *hyp.*
c 3 *ax.*
d 21. 1. *d* Q. E. A.

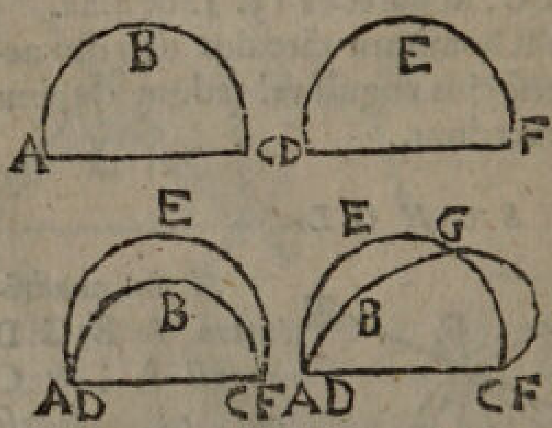
PROP. XXIII.



Super eadem recta linea A C duo circulorum segmenta A B C, A D C similia & inæqualia non constituentur ad easdem partes.

Nam si dicantur similia, duc C B secantem circumferentias in D, & B, & junge A D, ac
a 10 *def.* 3. AB. Quia segmenta ponuntur similia, *a* erit ang.
b 16. 1. ADC = ABC *b* Q. E. A.

PROP. XXIV.

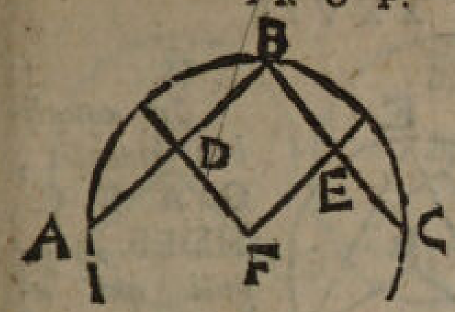


Super æqualibus rectis lineis A C, D F similia circulorum segmenta ABC, D E F sunt inter se æqualia.

Basis A C superposita basi D F ei congruet, quia A C = DF. ergo segmentum ABC congruet segmento D E F (alias enim aut intra cadet, aut extra, *a* atque ita segmen-
a 23 3. ta non erunt similia, contra Hyp. aut saltem partim intra, partim extra, adeoque ipsum in tribus punctis secabit. *b* Q. E. A.) *c* proinde se-
b 10 3.
c 8. *ax.* gmentum. ABC = DEF. Q. E. D.

PROP.

PROP. XXV.

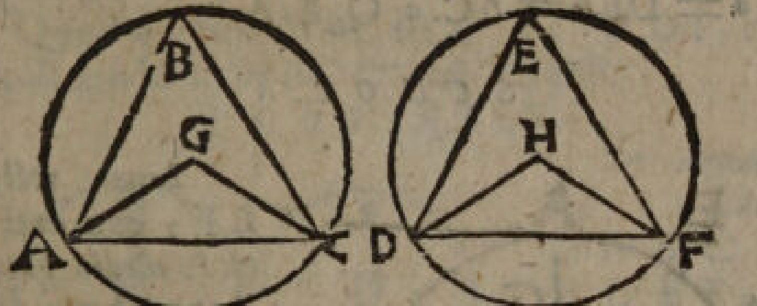


Circuli segmento ABC dato, describere circulum, cujus est segmentum.

Subtendantur utcunque duæ rectæ AB, BC, quas biseca in D, & E. Ex D, & E duc perpendiculares DF, EF occurrentes in puncto F. Hoc erit centrum circuli.

Nam centrum [a] tam in DF, quam in EF exsistit. ergo in communi puncto F. Q. E. F. a Cor. 1. 3.

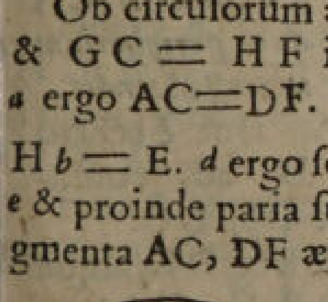
PROP. XXVI.



In æqualibus circulis GABC, HDEF æquales anguli æqualibus peripheriis AC, DF insistunt, sive ad centra G, H, sive ad peripher. B, E constituti insistant.

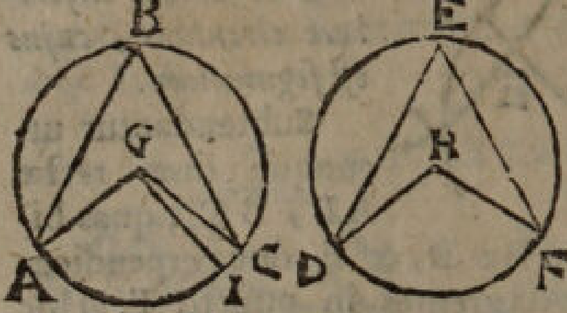
Ob circulorum æqualitatem, est GA = HD, & GC = HF item per hyp. ang. G = H. [a] ergo AC = DF. Sed & ang. B [b] = $\frac{1}{2}$ G = [c] $\frac{1}{2}$ H [b] = E. [d] ergo segmenta ABC, DEF similia, [e] & proinde paria sunt. [f] ergo etiam reliqua segmenta AC, DF æquantur. Q. E. D. a 4. 1. b 20. 3. c hyp. d 10. def. 3. e 24. 3. f 3. ax.

Scholium.



In circulo ABCD, sit arcus AB par arcui DC; erit AD parall. BC. Nam ducta AC, [a] erit ang. ACB = CAD. quare per 27. 1. a 26. 3.

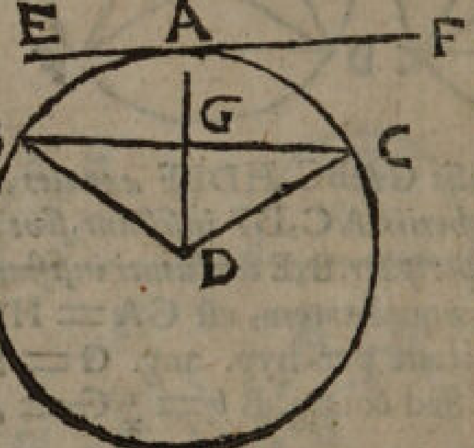
PROP. XXVII.



In æqualibus circulis, GABC, HDEF, *anguli qui æqualibus peripheriis* AC, DF *insistunt, sunt inter se æquales, sive ad centra* G, H, *sive ad peripherias* B, E *constituti insistant.*

Nam si fieri potest, sit alter eorum AGC ⊏ DHF. fiatque AGI = DHF. ergo arcus
a 26. 3. b hyp. c 9 ax. AI *a* = DF *b* = AC. *c* Q. E. A.

SCHOL.



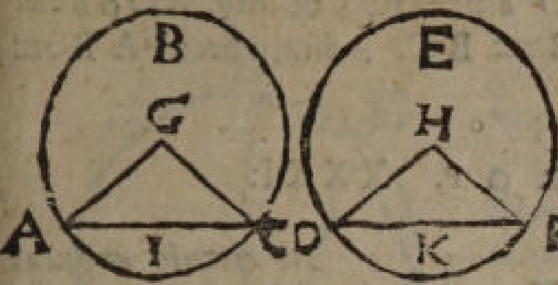
Linea recta EF, *quæ ducta ex* A *medio puncto peripheriæ alicujus* BC, *circulum tangit, parallela est rectæ lineæ* BC, *quæ peripheriam illam subtendit.*

Duc è centro D ad contactum A rectam DA, & connecte DB, DC.

Latus DG commune est; & DB = DC, atque
a 17. 3. ang. BDA *a* = CDA (ob arcus BA, CA *b* æ-
b hyp. quales) *c* ergo anguli ad basim DGB, DGC
c 4. 1. d 10 def 1. æquales, & *d* proinde recti sunt. Sed interni an-
e hyp. guli GAE, GAF *e* etiam recti sunt. *f* ergo BC,
f 28. 1. EF sunt parallelæ. Q. E. D.

PROP.

Prop. XXVIII.

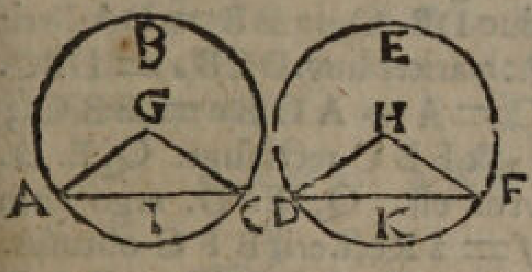


In æqualibus circulis GABC, HDEF, *æquales rectæ lineæ* AC, DF *æquales peripherias auferunt; majorem quidem* ABC *majori* DEF, *minorem autem* AIC *minori* DKF.

E centris G, H, duc GA, GC; & HD, HF. Quoniam GA = HD, & GC = HF, atque AC *a* = DF; *b* erit ang. G = H. *c* ergo arcus AIC = DKF. *d* proinde reliquus ABC = DEF. Q. E. D. a hyp. b 8. 1. c 16. 3. d 3. ax.

Quod si subtensa AC sit ⊏ vel ⊐ DF, erit simili modo arcus AC ⊏ vel ⊐ DF.

Prop. XXIX.

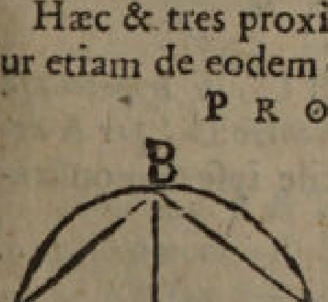


In æqualibus circulis GABC, HDEF, *æquales peripherias* ABC, DEF *æquales rectæ lineæ* AC, DF *subtendunt*.

Duc GA, GC; & HD, HF. Quia GA = HD; & GC = HF; & (ob arcus AC, DF *a* pares) etiam ang. G *b* = H; *c* erit bas. AC = DF. Q. E. D. a hyp. b 27. 3. c 4. 1.

Hæc & tres proxime præcedentes intelligantur etiam de eodem circulo.

Prop. XXX.



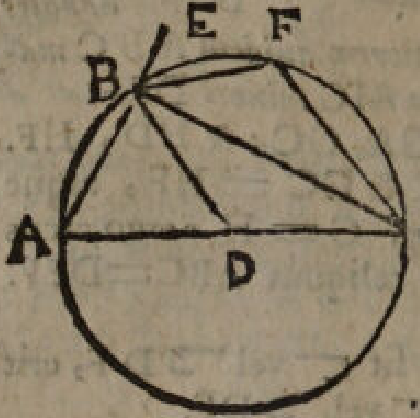
Datam peripheriam ABC *bifariam secare.*

Duc AC; quam biseca in D. ex D duc perpendicularem DB occurrentem arcui in B. Dico factum.

Iungantur enim A B, C B. Latus D B commune est; & A D *a* = DC; & ang. ADB *b* = CDB. *c* ergo A B = B C. *d* quare arcus A B = B C. Q. E. F.

a *const.*
b 12. *ax.*
c 4. 1.
d 28. 3.

PROP. XXXI.



In circulo angulus A B C, *qui in semicirculo, rectus est; qui autem in majore segmento* BAC, *minor recto; qui vero in minore segmento* B F C, *major est recto. Et insuper angulus majoris segmenti recto quidem major est, minoris autem segmenti angulus, minor est recto.*

Ex centro D duc DB. Quia DB = DA, erit ang. A *a* = DBA. pariter ang. DCB *a* = DBC. *b* ergo ang. A B C = A + A C B *c* = E B C, *d* proinde A B C, & E B C recti sunt. Q. E. D. *e* ergo B A C acutus est. Q. E. D. ergo cum B A C + B F C *f* = 2 Rect. erit B F C obtusus. denique angulus sub recta C B, & arcu B A C major est recto A B C. factus vero sub C B, & B F C peripheria minoris segmenti, recto E B C *g* minor est. Q. E. D.

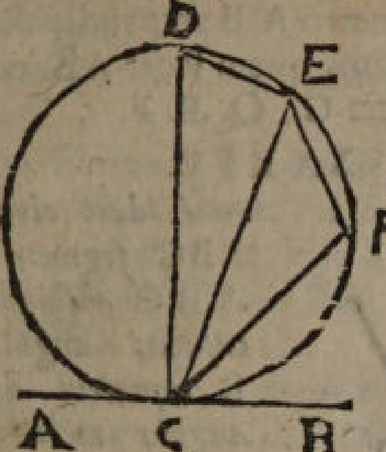
a 5. 1.
b 2. *ax.*
c 32. 1.
d 10. *def.* 1.
e *cor.* 17. 1.
f 22. 3.
g 9. *ax.*

SCHOLIUM.

In triangulo rectangulo A B C, *si hypotenusa* A C *bisecetur in* D, *circulus centro* D, *per* A *descriptus transibit per* B. ut facile ipse demonstrabis ex hac, & 21. 1.

PROP.

PROP. XXXII.



Si circulum tetigerit aliqua recta linea AB, *à contactu autem producatur quædam recta linea* CE *circulum secans: anguli* ECB, ECA, *quos ad contingentem facit, æquales sunt iis, qui in alternis circuli segmentis consistunt, angulis* EDC, EFC.

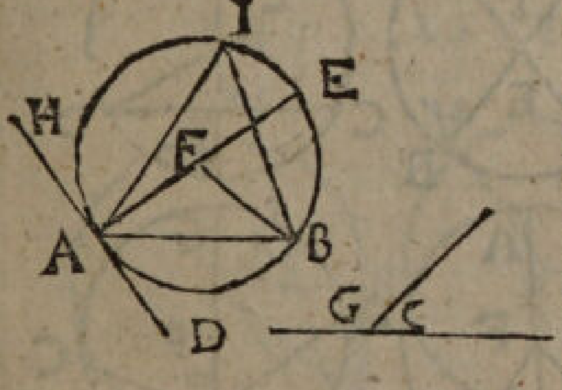
Sit CD latus anguli EDC perpendiculare ad AB (*a* perinde enim est) *b* ergo CD est diameter. *c* ergo ang. CED in semicirculo rectus est. *d* ergo ang. D + DCE = Rect. *e* = ECB + DCE. *f* ergo ang. D = ECB. Q. E. D.

a 26. 3.
b 19. 3.
c 31. 3.
d 32. 1.
e constr.
f 3. ax.

Cum igitur ang. ECB + ECA *g* = 2 Rect. *h* = D + F; aufer hinc inde æquales ECB, & D, *k* remanent ECA = F. Q. E. D.

g 13. 1.
h 22. 3.
k 3. ax.

PROP. XXXIII.



Super data recta linea AB *describere circuli segmentum* AIEB, *quod capiat angulum* AIB *æqualem dato angulo rectilineo* C.

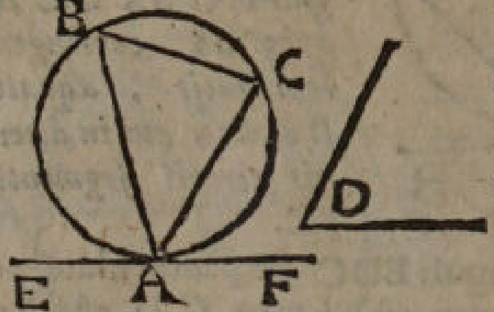
a Fac ang. BAD = C. per A duc AE perpendicularem ad HD. ad alterum terminum datæ AB fac ang. ABF = BAF. cujus alterum latus secet AE in F. centro F per A describe circulum, quod transibit per B (quia ang. FBA *b* =

a 23. 1.

b constr. b = F A B, c ideoque F B = F A); segmentum
c 6. 1. AIB est id quod quæritur.

Nam quia H D diametro A E perpendicularis
d cor.16.3. est, d tangit HD circulum, quem secat AB. ergo
e 32. 3. f constr. ang. AIB e = BAD f = C. Q. E. F.

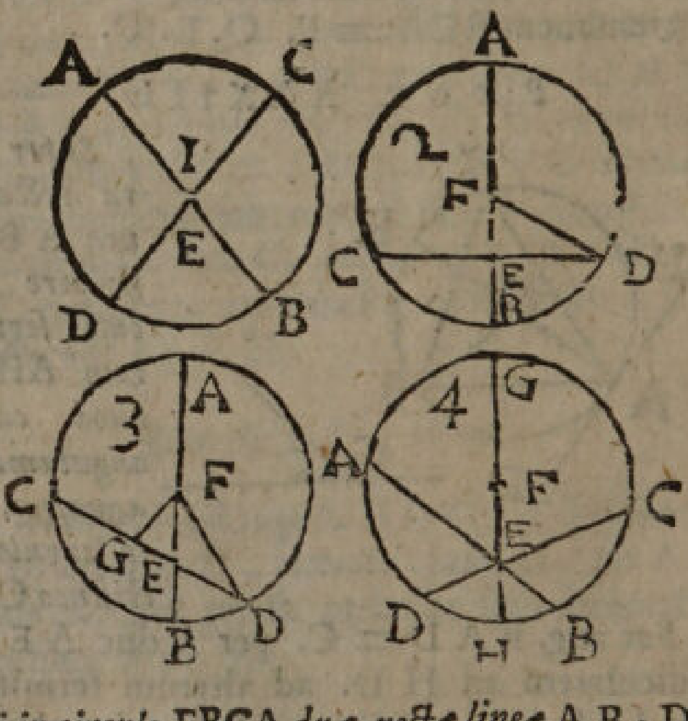
PROP. XXXIV.



A dato circulo ABC segmentum ABC abscindere capiens angulum B æqualem dato angulo rectilineo D.

a 17. 3. a Duc rectam EF, quæ tangat
b 23. 1. datum circulum in A. b ducatur item AC faciens ang. FAC = D. Hæc auferet segmentum ABC
c 32. 3. d constr. capiens angulum B c = CAF d = D. Q. E. F.

PROP. XXXV.



Si in circulo FBCA duæ rectæ lineæ A B, D C sese mutuo secuerint, rectangulum comprehensum sub

sub segmentis A E, E B unius, æquale est ei quod sub segmentis C E, E D alterius comprehenditur, rectangulo.

Cas. 1. Si rectæ sese in centro secent, res clara est.

2. Si una A B transeat per centrum F, & reliquam C D bisecet, duc F D. Estque Rectang. AEB + FEq [a] = FBq [b] = FDq [c] = EDq + FEq [d] = CED + FEq. [e] ergo Rectang. AEB = CED. Q. E. D. a 5. 2. b sch. 48. 1. c 47. 1. d hyp. e 3. ax.

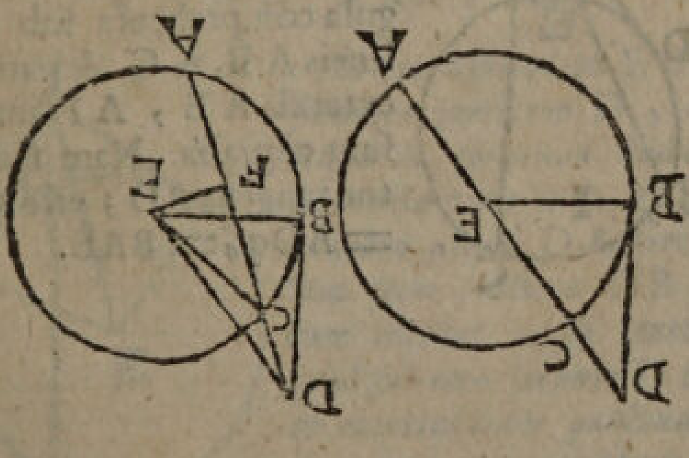
3. Si una A B diameter sit, alteramque C D secet inæqualiter, biseca C D per F G perpendicularem ex centro.

Æquantur ista { Rectang. AEB + FEq.
[f] FBq (FDq) f 5. 2.
[g] FGq + GDq. g 47. 1.
FGq + [h] GEq + Rectang. CED. h 5. 2.
[k] FEq + CED. k 47. 1. }

[l] Ergo Rectang. AEB = CED. l 3. ax.

4. Si neutra rectarum A B, C D per centrum transeat, per intersectionis punctum E duc diametrum G H. Per modo demonstrata Rectang. AEB = GEH = CED. Q. E. D.

PROP. XXXVI.



[a] *Si extra circulum E B C sumatur punctum aliquod D, ab eoque puncto in circulum cadant duæ rectæ lineæ DA, DB; quarum altera DA circulum*

ſecet, altera vero D B tanget; Quod ſub tota ſecante D A, & exterius inter punctum D, & convexam peripheriam aſſumpta D C comprehenditur rectangulum, æquale erit ei, quod à tangente D B deſcribitur, quadrato.

1. *Caſ.* Si ſecans A D tranſeat per centrum E, junge E B; a faciet hæc cum D B rectum angulum; quare D Bq + E B Q (E Cq) b = E Dq c = A D x D C + E Cq d ergo A D x D C = D Bq. Q. E. D.

a 18. 3. b 47. 1. c 6. 2. d 3. ax.

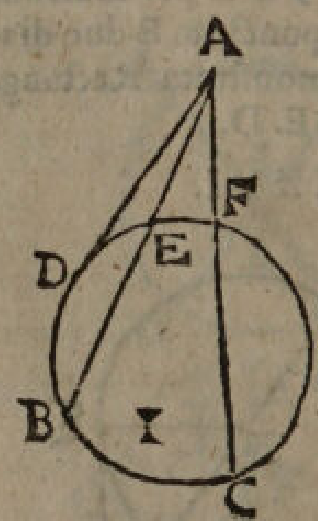
2. *Caſ.* Sin A D per centrum non tranſeat, duc EC, EB, ED; atque EF perpend. A D, quare a biſecta eſt A C in F.

a 3. 3.

Quoniam igitur BDQ + EBq b = DEq b = EFq + FDq c = EFq + ADC + FCQ d = ADC + C Eq (E Bq); e erit B Dq = A D C. Q. E. D.

b 47. 1. c 6. 2. d 47. 1. e 3. ax.

Coroll.



1. Hinc, ſi à puncto quovis A extra circulum aſſumpto, plurimæ lineæ rectæ A B, A C circulum ſecantes ducantur, rectangula comprehenſa ſub totis lineis A B, A C, & partibus externis A E, A F inter ſe ſunt æqualia. Nam ſi ducatur tangens AD; erit CAF = ADq a = BAE.

a 36. 3.

2. Con-

2. Constat etiam duas rectas A B, A C ab eodem puncto A ductas, quæ circulum tangant, inter se æquales esse.

Nam si ducatur A E secans circulum; erit A Bq a = EAF b = ACq. a 36. 3. b 36. 3.

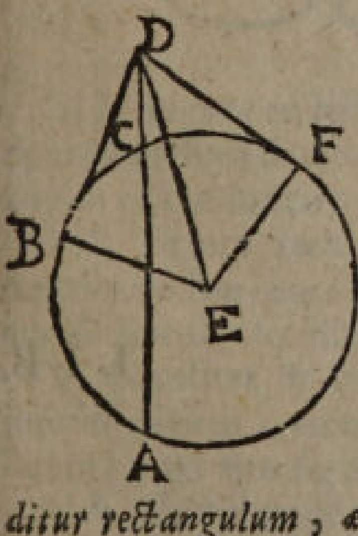
3. Perspicuum quoque est ab eodem puncto A extra circulum assumpto, duci tantum posse duas lineas, A B, A C quæ circulum tangant.

Nam si tertia A D tangere dicatur, erit A D c = A B c = A C. d Q. F. N. c 2. cor. d 8 3.

4. E contra constat, si duæ rectæ æquales A B, A C ex puncto quopiam A in convexam peripheriam incidant, & earum una A B circulum tangat, alteram quoque circulum tangere.

Nam si fieri potest, non A C, sed altera A D circulum tangat. ergo A D e = AC f = A B. g Q. E. A. e 2. cor. f hyp. g 8 3.

PROP. XXXVII.



Si extra circulum E B F *sumatur punctum* D, *ab eoque in circulum cadant duæ rectæ lineæ* D A, D B; *quarum altera* D A *circulum secet, altera* D B *in eum incidat; sit autem quod sub tota secante* D A, *& exterius inter punctum, & convexam peripheriam assumpta* D C, *comprehenditur rectangulum, æquale ei, quod ab incidente*

DB

DB describitur quadrato, incidens ipsa DB circulum tanget.

a 17. 3. b hyp. c 36. 3. d 1. ax & sch. 48, 1. e 8. 1. f 12. ax. g cor. 16. 3.

Ex D a ducatur tangens DF; atque ex E centro duc ED, EB, EF. Quia DBq b = ADC c = DFq, d erit DB = DF. Sed EB = EF, & latus ED commune est; e ergo ang. EBD = EFD. Sed EFD rectus est, f ergo EBD etiam rectus est. g ergo DB tangit circulum. Q. E. D.

Coroll.

h 8 1.

Hinc, h ang. EDB = EDF.

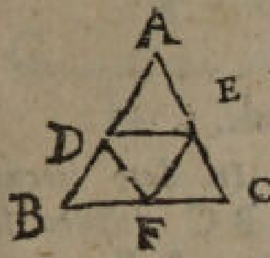
LIB.

LIB. IV.

Definitiones.

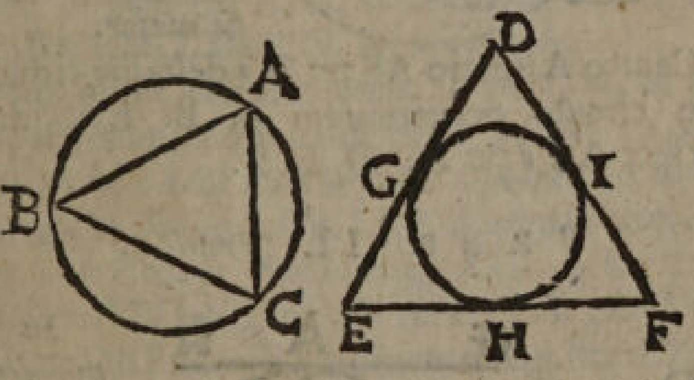
I F·Igura rectilinea in figura rectilinea inscribi dicitur, cum singuli ejus figuræ, quæ inscribitur, anguli singula latera ejus in qua inscribitur, tangunt.

Sic triangulum DEF *est inscriptum in triangulo* ABC.



II. Similiter & figura circa figuram describi dicitur, cum singula ejus, quæ circumscribitur, latera singulos ejus figuræ angulos tetigerint, circa quam illa describitur.

Ita triangulum A B C *est descriptum circa triangulum* DEF.



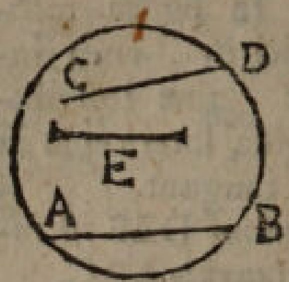
III. Figura rectilinea in circulo inscribi dicitur, cum singuli ejus figuræ, quæ inscribitur, anguli tetigerint circuli peripheriam.

IV. Figura vero rectilinea circa circulum describi dicitur, cum singula latera ejus, quæ circumscribitur, circuli peripheriam tangunt.

V. Similiter & circulus in figura rectilinea inscribi dicitur, cum circuli peripheria singula latera tangit ejus figuræ, cui inscribitur.

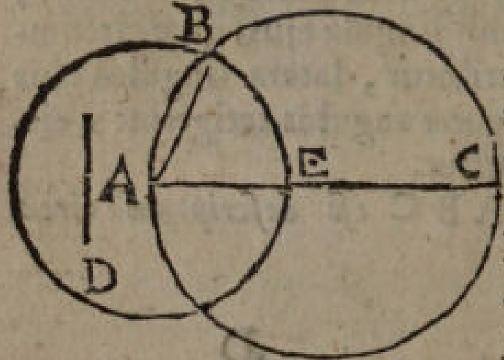
VI. Circulus autem circa figuram describi di-

dicitur, cum circuli peripheria singulos tangit ejus figuræ, quam circumscribit, angulos.



VII. Recta linea in circulo accommodari, seu coaptari dicitur, cum ejus extrema in circuli peripheria fuerint; ut recta linea A B.

PROP. I. *Probl.* 1.

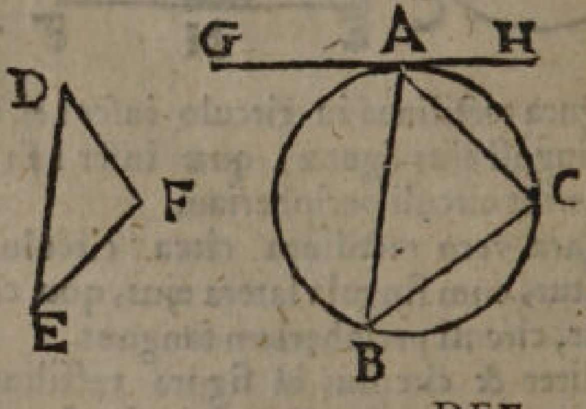


In dato circulo ABC *rectam lineam* AB *accommodare æqualem datæ rectæ lineæ* D, *quæ circuli diametro* AC *non sit major.*

a 3. *post.* & 3. 1. b 15. *def* 1. c *constr.*

Centro A, spatio AE = D *a* describe circulum dato circulo occurrentem in B. Erit ducta AB *b* = AE *c* = D. Q. E. F.

PROP. II. *Probl.* 2.



In dato circulo ABC *triangulum* ABC *describere dato triangulo* DEF *æquiangulum.*

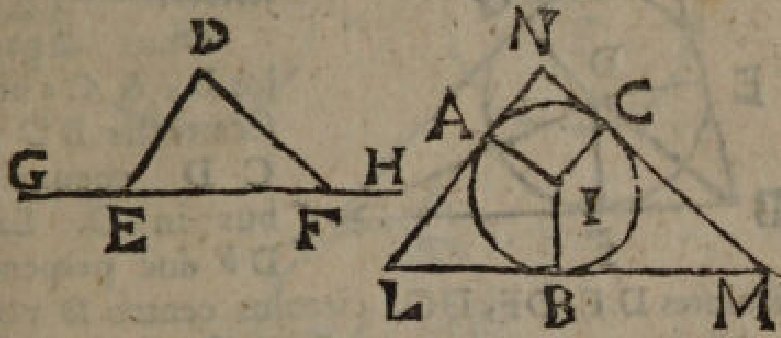
a 17. 3. b 23. 1.

Recta GH circulum datum *a* tangat in A. *b* Fac ang. HAC = E; *b* & ang. GAB = F, & junge BC. Dico factum.

Nam

Nam ang. B*c* = HAC*d* = E ; & ang. c 32. 3.
C*c* = GAB*d* = F ; *e* quare etiam ang. BAC = D. d *constr.*
ergo triang. BAC circulo inſcriptum triangulo e 32. 1.
DEF æquiangulum eſt. Q. E. F.

PROP. III *Probl.* 3.

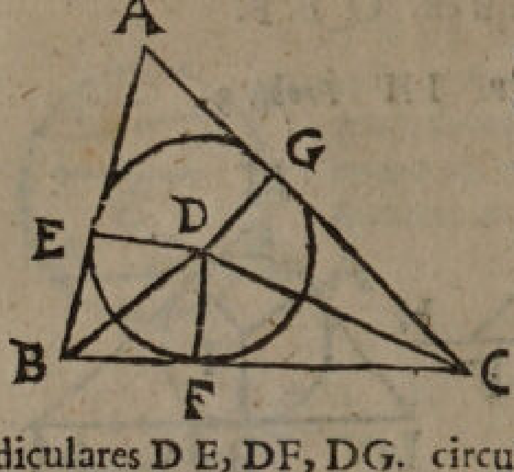


Circa datum circulum IABC *triangulum* LNM *deſcribere, dato triangulo* DEF *æquiangulum.*

Produc latus EF utrinque. *a* Fac ad centrum I ang. AIB = DEG. & ang. BIC = DFH. deinde in punctis A, B, C circulum *b* tangant tres rectæ LN, LM, MN. Dico factum. a 23. 1. b 17. 3.

Nam quod coibunt rectæ LN, LM, MN, atque ita triangulum conſtituent, patet ; *c* quia anguli LAI, LBI *d* recti ſunt, adeoque ducta AB angulos faciet LAB, LBA duobus rectis minores. Quoniam igitur ang. AIB + L *e* = 2 Rect. *f* = DEG + DEF ; & AIB *g* = DEG *h* erit ang. L = DEF. Simili argumento ang. M = DFE. *k* ergo etiam ang. N = D. ergo triang. LNM circulo circumſcriptum dato EDF eſt æquiangulum. Q. E. F. c 13. *ax.* d 18. 3. e *Schol.* 32. 1. f 13. 1. g *conſtr.* h 3. *ax.* k 32. 1.

PROP. IV. *Probl.* 4.



In dato triangulo ABC, *circulum* EFG *inscribere.*

a 9. 1. Duos angulos B, & C *a* biseca rectis BD, CD coeuntibus in D. Ex

b 12. 1. D *b* duc perpendiculares DE, DF, DG. circulus centro D per E descriptus transibit per G, & F, tangetque tria latera trianguli.

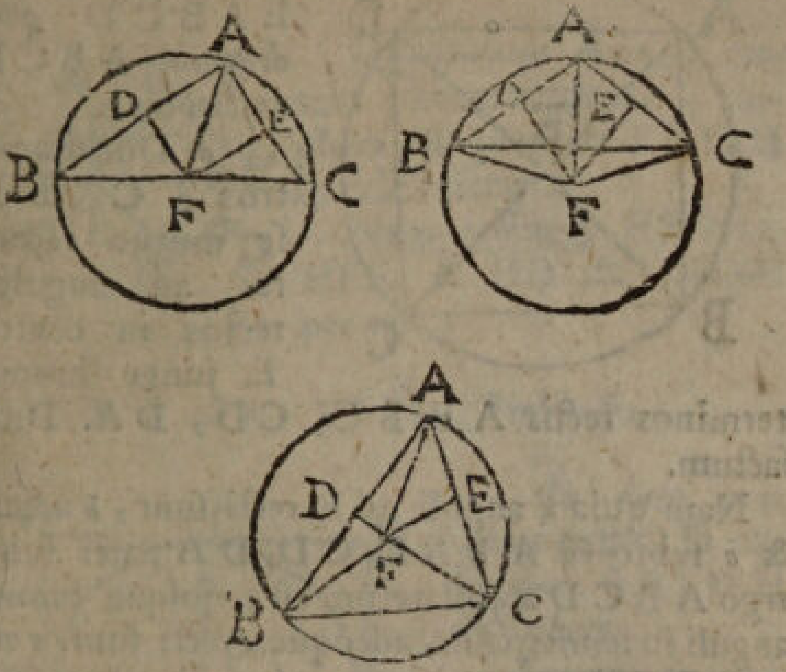
c constr. d 12. ax. e 26. 1. Nam ang. DBE *c* = DBF; & ang. DEB *d* = DFB; & latus DB commune est: *e* ergo DE = DF. Simili argumento DG = DF. Circulus igitur centro D descriptus transit per E, F, G; & cum anguli ad E, F, G sint recti, tangit omnia trianguli latera. Q. E. F.

Scholium.

Petr. Herig. Hinc, *cognitis lateribus trianguli, invenientur eorum segmenta, quæ fiunt à contactibus circuli inscripti.* Sic,

Sit AB 12, AC 18, BC 16. Erit AB + BC = 28. ex quo subduc 18 = AC = AE + FC, remanet 10 = BE + BF. ergo BE, vel BF = 5. proinde FC, vel CG = 11. quare GA, vel AE = 7.

PROP. V. *Probl. 5.*



Circa datum triangulum ABC *circulum* FABC *describere.*

Latera quævis duo B A , A C *a* biseca perpendicularibus D F , E F concurrentibus in F. Hoc erit centrum circuli, a 10,& 11.1.

Nam ducantur rectæ FA, FB, FC. Quoniam AD*b*=DB ; & latus D F commune est ; & ang. FDA *c* = FDB, *d* erit FB=FA. eodem modo F C = F A. ergo circulus centro F per dati trianguli angulos B, A, C transibit. Q. E. F. b *constr.* c *constr.* & 11. *ax.* d 4. 1.

Coroll.

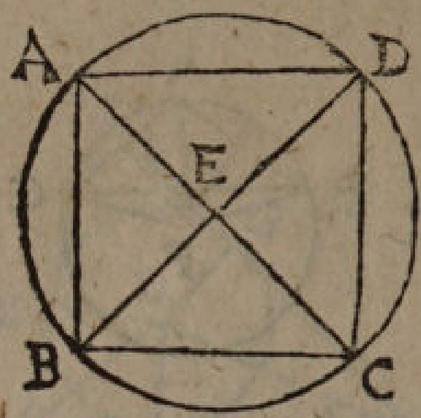
* Hinc , si triangulum fuerit acutangulum , centrum cadet intra triangulum; si rectangulum, in latus recto angulo oppositum ; si denique obtusangulum, extra triangulum. * 31. 3.

Schol.

Eadem methodo describetur circulus , qui transeat per data tria puncta , non in una recta linea existentia.

PROP.

PROP. VI. *Probl. 6.*



In dato circulo EABCD quadratum ABCD inscribere.

a 11, 1. *a* Duc diametros A C, B D se mutuo secantes ad angulos rectos in centro E. junge harum terminos rectis A B, B C, C D, D A. Dico factum.

b 26. 3. c 29. 3. Nam quia 4 anguli ad E recti sunt, *b* arcus, & *c* subtensæ A B, B C, C D, D A pares sunt. ergo A B C D æquilaterum est; ejusque omnes
d 31. 3. e 29. def. 1. anguli in semicirculis, adeoque *d* recti sunt. *e* ergo A B C D est quadratum, dato circulo inscriptum. Q. E. F.

PROP. VII. *Probl. 7.*

Circa datum circulum EABCD quadratum FHIG describere.

a] 17. 3. Duc diametros A C, B D se mutuo secantes perpendiculariter. per harũ extrema *a* duc tangentes concurrentes in F, H, I, G. Dico factum. Nam ob
b 18 3. c 28. 1. angulos ad A, & C *b* rectos, *c* erit F G parall. HI. eodem modo FH parall. GI. ergo FHIG est parallelogrammum; & quidem rectangulum. sed & æquilaterum, quia FG *d*= HI *d*= BD *e*=
d 34. 1. e 15. def. 1. f 29. def. 1. CA *d*= FH *d*= GI. quare FHIG est *f* quadratum, dato circulo circumscriptum. Q. E. F.

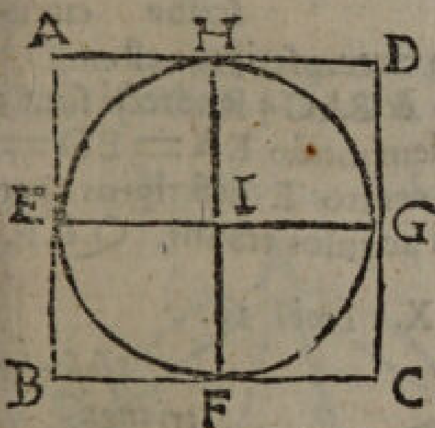
SCHOL.

SCHOL.

Quadratum ABCD circulo circumscriptum, duplum est quadrati EFGH circulo inscripti.

Nam rectang. HB = 2 HEF. & HD = 2 HGF. per 41. 1.

PROP. VIII. *Probl.* 8.

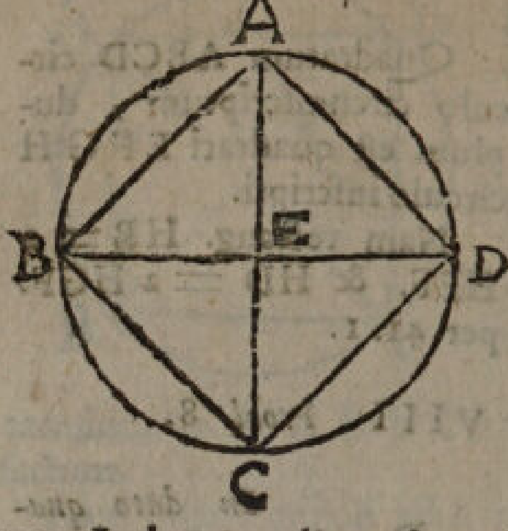


In dato quadrato ABCD *circulum* IEFGH *inscribere.*

Latera quadrati biseca in punctis H, E, F, G; junge HF, EG sese secantes in I. circulus centro I per H descriptus quadrato inscribetur.

Nam quia AH, BF *a* pares ac *b* parallelæ sunt, *c* erit AB parall. HF parall. DC. eodem modo AD parall. EG parall. BC. ergo IA, ID, IB, IC sunt parallelogramma. Ergo AH*d*=AE*e*=HI=EI=IF=IG. Circulus igitur centro I per H descriptus transibit per H, E, F, G, tangetque quadrati latera, cum anguli ad H, E, F, G sint recti. Q. E. F. a 7. ax.
b 34. 1.
c 33. 1.
d 7. ax
e 34. 1.

PROP. IX. *Probl.* 9.

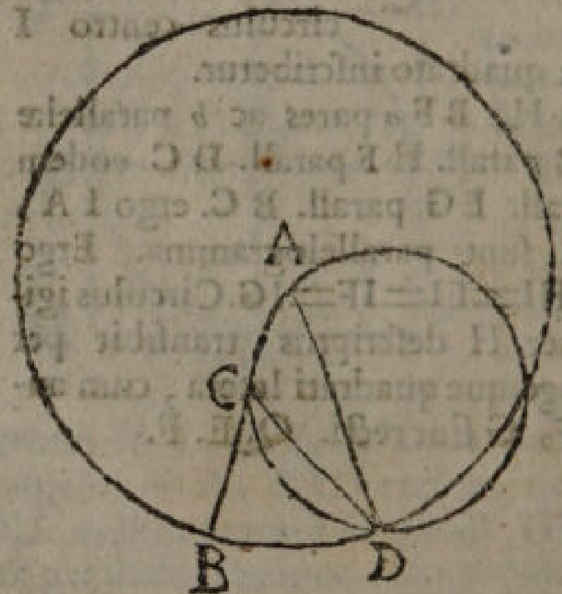


Circa datum quadratum A B C D *circulum* E A-B C D *describere.*

Duc diametros A C, B D secantes in E. centro E per A describe circulum. Is dato quadrato circumscriptus est.

a 4 cor. 32. 1. b 6. 1. Nam anguli ABD, & BAC a semirecti sunt; b ergo EA = EB. eodem modo EA = ED = EC. Circulus igitur centro E descriptus per A,B,C,D dati quadrati angulos transit. Q.E.F.

PROP. X. *Probl.* 10.



Isosceles triangulum A B D *constituere, quod habeat utrumque eorum quæ ad basim sunt angulorum* B & A D B *duplum reliqui* A.

Accipe quamvis rectam AB, quam a seca in C, ita ut AB x BC = ACq. Centro A per B describe circulum A B D;

a 11. 2. in

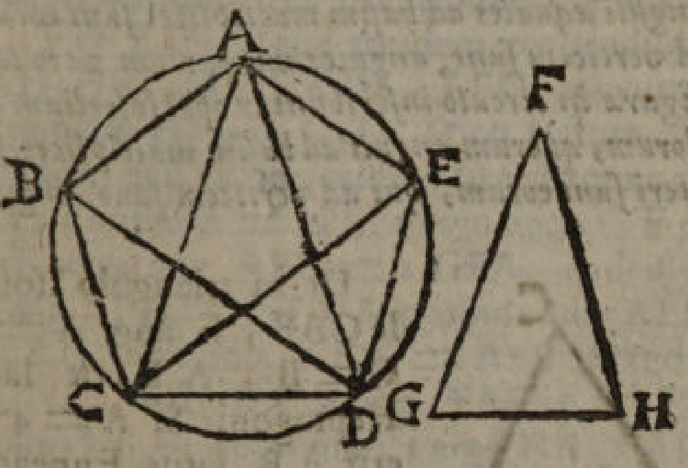
in hoc *b* accommoda B D = A C, & junge AD. erit triang. ABD quod quæritur. b 1. 4.

Nam duc D C; & per C D A *c* deſcribe circulum. Quoniam AB x BC = ACq. *d* liquet BD tangere circulum A C D, quem ſecat C D. *e* ergo ang. BDC = A. ergo ang. BDC + CDA *f* = A + CDA = BCD. ſed BDC + CDA = BDA *h* = CBD. *k* ergo ang. BCD = CBD. ergo DC *l* = DB *m* = AC. *n* quare ang. CDA = A = BDC. ergo ADB = 2 A = ABD. Q. E. F. c 5. 4. d 37. 3. e 32. 3. f 2. ax. g 32. 1. h 5. 1. k 1. ax. l 6. 1. m conſtr. n 5. 1.

Coroll.

Cum omnes anguli A, B, D *o* conficiant $\frac{5}{5}$ 2 Rect. (2 Rect.) liquet A eſſe $\frac{1}{5}$ 2 Rect. o 32. 1.

P R O P. XI. *Probl.* II.



In dato circulo ABCDE *pentagonum æquilaterum & æquiangulum* ABCDE *inſcribere.*

a Deſcribe triangulum Iſoſceles FGH, habens utrumque angulorum ad baſim duplum anguli ad verticem. *b* Huic æquiangulum C A D inſcribe circulo. Angulos ad baſim ACD, & ADC *c* biſeca rectis D B, C E occurrentibus circumferentiæ in B, & E. connecte rectas CB, BA, AE, ED. Dico factum. a 10. 4. b 2. 4. c 9. 1.

Nam ex conſtr. liquet quinque angulos CAD,
d 26 3. CDB, BDA, DCE, ECA pares eſſe; quare d
e 29 3. arcus e & ſubtenſæ DC, CB, BA, AE, DE æquantur. Pentagonum igitur æquilaterum eſt.
f 27. 3. Eſt vero etiam æquiangulum, f quia ejus anguli
g 2. ax. BAE, AED, &c. inſiſtunt arcubus g æqualibus BCDE, ABCD, &c.

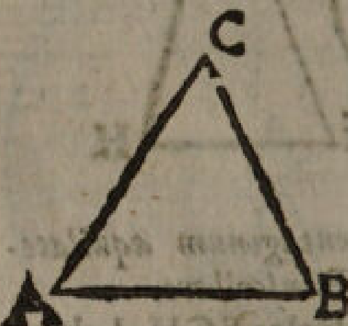
Hujus problematis praxis facilior tradetur ad 10. 13.

Coroll.

Hinc, angulus pentagoni æquilateri & æquianguli æquatur $\frac{3}{5}$ 2 Rect. vel $\frac{6}{5}$ Rect.

Schol.

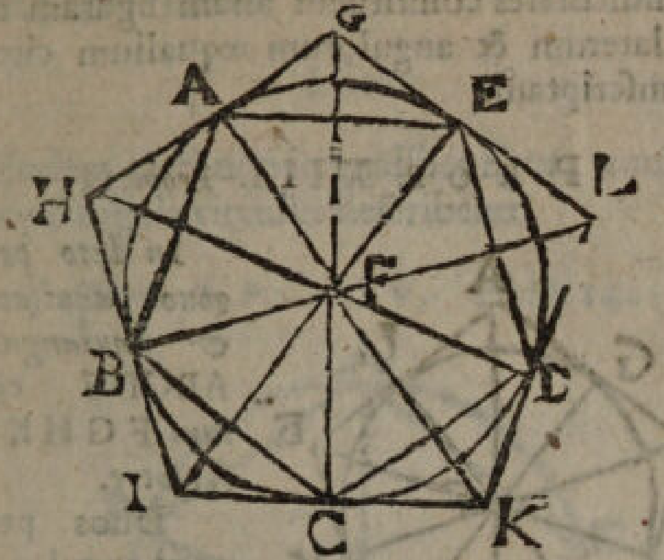
Petr. Herig. *Vniverſaliter figuræ imparium laterum inſcribuntur circulo beneficio triangulorum Iſoſcelium, quorum anguli æquales ad baſim multiplices ſunt eorum, qui ad verticem ſunt, angulorum; parium vero laterum figuræ in circulo inſcribuntur ope Iſoſcelium triangulorum, quorum anguli ad baſim multiplices ſeſquialteri ſunt eorum, qui ad verticem ſunt, angulorum.*



Ut in triangulo Iſoſcele CAB, ſi ang. A = 3 C = B; A B erit latus Heptagoni. Si A = 4 C; erit A B latus Enneagoni, &c. Sin vero A = $1\frac{1}{2}$ C, erit A B latus quadrati. Et ſi A = $2\frac{1}{2}$ C, ſubtendet AB ſextam partem circumferentiæ: pariterque ſi A = $3\frac{1}{2}$ C; erit AB latus octagoni, &c.

PROP.

PROP. XII. *Probl.* 12.



Circa datum circulum FABCDE *pentagonum æquilaterum & æquiangulum* HIKLG *describere.*

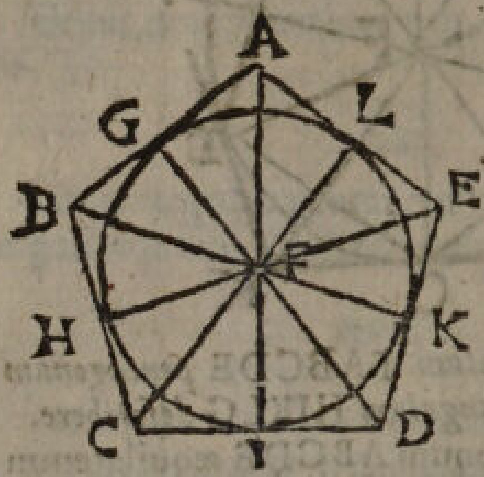
a Inscribe pentagonum ABCDE æquilaterum & æquiangulum; duc è centro rectas FA, FB, FC, FD, FE, iisque totidem perpendiculares GAH, HBI, ICK, KDL, LEG concurrentes in punctis H, I, K, L, G. Dico factum. Nam quia GA, GE ex uno puncto G b tangunt circulum, c erit GA = GE. d ergo ang. GFA = GFE. ergo ang. AFE = 2 GFA. eodem modo ang. AFH = HFB; & proinde ang. AFB = 2 AFH. Sed ang. AFE e = AFB. f ergo ang. GFA = AFH. sed & ang. FAH g = FAG; & latus FA est commune, h ergo HA = AG = GE = EL, &c. k ergo HG, GL, LK, KI, IH latera pentagoni æquantur: sed & anguli etiam, utpote l æqualium AGF, AHF, &c. dupli; ergo, &c.

a 11. 4
b cor. 16. 3.
c 1 cor. 36. 3.
d 8. 1.
e 27. 3.
f 7. ax.
g 12. ax.
h 26. 1.
k 2. ax.
l 32. 1.

Coroll.

Eodem pacto, si in circulo quæcunque figura æquilatera & æquiangula describatur, & ad extrema semidiametrorum ex centro ad angulos ductarum, excitentur lineæ perpendiculares, hæ perpendiculares constituent aliam figuram totidem laterum & angulorum æqualium circulo circumscriptam.

PROP. XIII. *Probl.* 13.



In dato pentagono æquilatero & æquiangulo ABCDE *circulum* FGHK *inscribere.*

Duos pentagoni angulos A, (a 9. 1.) & B *a* biseca rectis AF, BF concurrentibus in F. Ex F duc perpendiculares FG, FH, FI, FK, FL. Circulus centro F per G descriptus tanget omnia pentagoni latera.

(b hyp.) Duc FC, FD, FE. Quoniam BA *b* = BC; (c constr.) & latus BF commune est; & ang. FBA *c* = (d 4. 1.) FBC, *d* erit AF = FC; & ang. FAB = FCB. (e hyp.) Sed ang. FAB *e* = $\frac{1}{2}$ BAE *e* = $\frac{1}{2}$ BCD. ergo ang. FCB = $\frac{1}{2}$ BCD. eodem modo anguli totales C, D, E omnes bisecti sunt. Quum igitur (f 12. ax. 1) ang. FGB *f* = FHB, & ang. FBH = FBG, & (g 26. 1.) latus FB sit commune, *g* erit FG = FH. similiter omnes FH, FI, FK, FL, FG æquantur. Ergo circulus centro F per G descriptus transit per (h cor. 16. 3.) H, I, K, L; *h* tangitque pentagoni latera, cum anguli ad ea puncta sint recti. Q. E. F.

Coroll.

Hinc, si duo anguli proximi figuræ æquilateræ & æquiangulæ bisecentur, & à puncto, in quo coeunt lineæ angulos bisecantes, ducantur rectæ

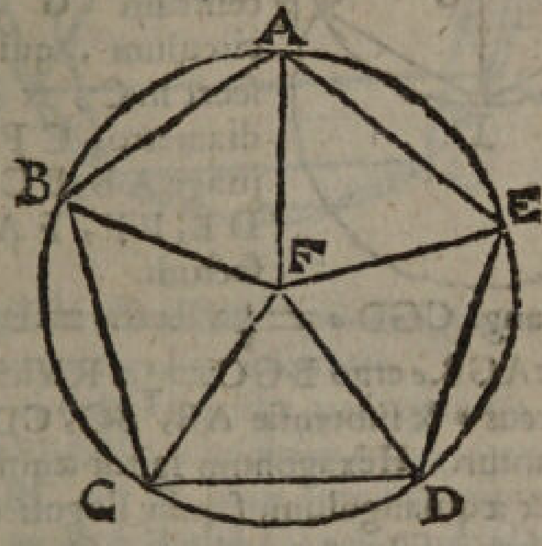
lineæ

lineæ ad reliquos figuræ angulos, omnes anguli figuræ erunt bisecti.

Schol.

Eadem methodo in qualibet figura æquilatera & æquiangula circulus describetur.

PROP. XIV. *Probl.* 14.



Circa datum Pentagonum æquilaterum & æquiangulum ABCDE *circulum* FABCD *describere.*

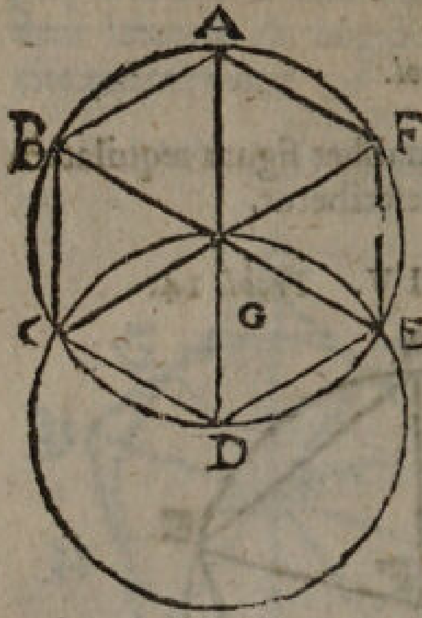
Duos pentagoni angulos biseca rectis AF, BF concurrentibus in F. Circulus centro F per A descriptus pentagono circumscribitur.

Ducantur enim FC, FD, FE. *a* Bisecti itaque sunt anguli C, D, E. *b* ergo FA, FB, FC, FD FE æquantur. ergo circulus centro F descriptus, per A, B, C, D, E, pentagoni angulos transibit. Q. E. F. a cor. 13. 4. b 6. 1.

Schol.

Eadem arte circa quamlibet figuram æquilateram & æquiangulam circulus describetur.

PROP. XV. *Probl.* 15.



In dato circulo G-ABCDEF *hexagonum & æquilaterum & æquiangulum* AB-CDEF *inſcribere.*

Duc diametrum AD; centro D per centrum G deſcribe circulum, qui datum ſecet in C, & E. duc diametros CF, EB. junge AB, BC, CD, DE, EF, FA. Dico factum.

a 32. 1.
b 15. 1.
c cor. 13. 1.
d 26. 3.
e 29. 3.
f 27. 3.

Nam ang. CGD *a* $= \frac{1}{3}$ 2 Rect. *a* = DGE *b* = AGF *b* = AGB. *c* ergo BGC = $\frac{1}{3}$ 2 Rect. = FGE. *d* ergo arcus *e* & ſubtenſæ AB, BC, CD, DE, EF æquantur. Hexagonum igitur æquilaterum eſt: ſed & æquiangulum, *f* quia ſinguli ejus anguli arcubus inſiſtunt æqualibus. Q. E. F.

Coroll.

1. Hinc latus Hexagoni circulo inſcripti ſemidiametro æquale eſt.

2. Hinc facile triangulum æquilaterum ACE in circulo deſcribetur.

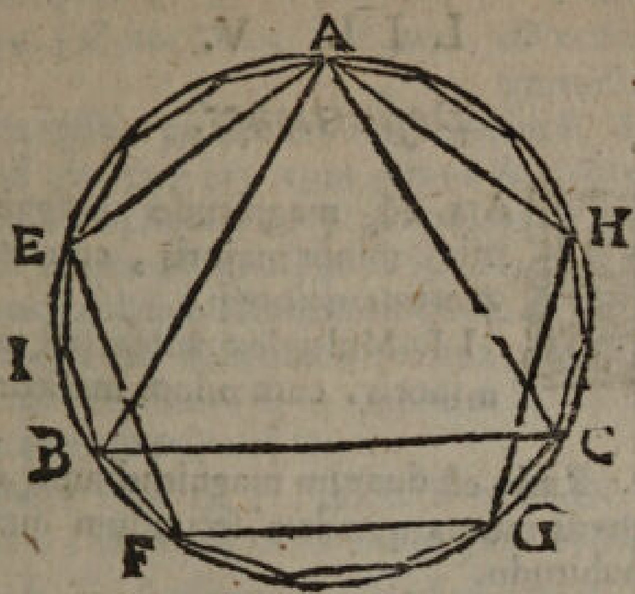
Schol. Probl.

Andr. Tacq.
a 1. 1.

Hexagonum ordinatum ſuper data recta CD *ita conſtrues.* *a* Fac triangulum CGD æquilaterum ſuper data CD. centro G per C, & D deſcribe circulum. Is capiet Hexagonum ſuper data CD.

PROP.

PROP. XVI. *Probl. 16.*



In dato circulo AEBC *quindecagonum æquilaterum & æquiangulum inſcribere.*

Dato circulo ᵃ inſcribe pentagonum æquilaterum AEFGH ; ᵇ itemque triangulum æquilaterum ABC. erit BF latus quindecagoni quæſiti. a 11. 4. b 3 4.

Nam arcus A B ᶜ eſt $\frac{1}{3}$, vel $\frac{5}{15}$ peripheriæ, cujus AF eſt $\frac{2}{5}$ vel $\frac{6}{15}$. ergo reliquus B F = $\frac{1}{15}$ periph. ergo quindecagonum, cujus latus B F, æquilaterum eſt ; ſed & æquiangulum ,ᵈ cum ſinguli ejus anguli arcubus inſiſtant æqualibus, quorum unuſquiſque eſt $\frac{13}{15}$ totius circumferentiæ. ergo, &c. c conſtr. d 27. 3.

Schol.

Circulus dividitur Geometrice in partes
- 4, 8, 16, &c. per 6, 4, & 9, 1.
- 3, 6, 12, &c. per 15, 4, & 9, 1.
- 5, 10, 20, &c. per 11, 4, & 9, 1.
- 15, 30, 60, &c. per 16, 4 & 9, 1.

Cæterum diviſio circumferentiæ in partes datas etiamnum deſideratur; quare pro figurarum quarumcunq; ordinatarum conſtructionibus ſæpe ad mechanica artificia recurrendum eſt, propter quæ Geometræ practici conſulendi ſunt.

LIB.

LIB. V.

Definitiones.

I. PArs est magnitudo magnitudinis, minor majoris, cum minor metitur majorem.

II. Multiplex autem est major minoris, cum minor metitur majorem.

III. Ratio est duarum magnitudinum ejusdem generis mutua quædam secundum quantitatem habitudo.

In omni ratione ea quantitas, quæ ad aliam refertur, dicitur antecedens rationis; ea vero, ad quam alia refertur, consequens rationis dici solet. ut in ratione 6 ad 4; antecedens est 6, & consequens 4.

Nota.

Cujusque rationis quantitas innotescit dividendo antecedentem per consequentem. ut ratio 12 ad 5 effertur per $\frac{12}{5}$. *item quantitas rationis* A *ad* B *est* $\frac{A}{B}$. *Quare non raro brevitatis causa, quantitates rationum sic designamus,* $\frac{A}{B} \sqsubset$, *vel* $=$, *vel* $\sqsupset \frac{C}{D}$; *hoc est, ratio* A *ad* B *major est ratione* C *ad* D, *vel ei æqualis, vel minor. Quod probe animadvertat, quisquis hæc legere volet.*

Rationis, sive proportionis species, ac divisiones vide apud interpretes.

IV. Proportio vero est rationum similitudo.

Rectius quæ hic vertitur proportio, proportionalitas, sive analogia dicitur; nam proportio idem denotat quod ratio, ut plerisque placet.

V. Rationem habere inter se magnitudines dicuntur, quæ possunt multiplicatæ se mutuo superare.

VI. In

E, 12.	A, 4. B, 6.	G, 24.	VI. In eadẽ ratione magnitudines di-
F, 30.	C, 10. D, 15.	H, 60.	

cuntur esse, prima A ad secundam B, & tertia C ad quartam D, cum primæ A, & tertiæ C æquemultiplicia E, & F à secundæ B, & quartæ D æquemultiplicibus G, & H, qualiscunque sit hæc multiplicatio, utrumque E, F ab utroque G, H, vel una deficiunt, vel una æqualia sunt, vel una excedunt, si ea sumantur E, G; & F, H quæ inter se respondent.

Hujus nota est ::. ut A. B :: C. D. *hoc est* A *ad* B, *&* C *ad* D *in eadem sunt ratione. aliquando sic scribimus* $\frac{A}{B} = \frac{C}{D}$ *id est*, A. B :: C. D.

VII. Eandem autem habentes rationem (A.B :: C.D) proportionales vocentur.

E, 30.	A, 6. B, 4.	G, 28.	VIII. Cum vero æquemultiplicium, E mul-
F, 60.	C, 12. D, 9.	H, 63.	

tiplex primæ magnitudinis A excesserit G multiplicem secundæ B; at F multiplex tertiæ C non excesserit H multiplicem quartæ D; tunc prima A ad secundam B majorem rationem habere dicetur, quam tertia C ad quartam D.

Si $\frac{A}{B} \sqsubset \frac{C}{D}$, *necessarium non est ex hac definitione, ut* E *semper excedat* G; *quum* F *minor est quam* H; *sed conceditur hoc fieri posse.*

IX. Proportio autem in tribus terminis paucissimis consistit. *Quorum secunda est instar duorum.*

X. Cum autem tres magnitudines A, B, C proportionales fuerint, prima A ad tertiam C duplicatam rationem habere dicetur ejus, quam habet ad secundam B: at quum quatuor magnitudines A, B, C, D, proportionales fuerint, prima A ad quartam D triplicatam rationem habere

dicetur

dicetur ejus, quam habet ad secundam B; & semper deinceps uno amplius, quamdiu proportio extiterit.

Duplicata ratio exprimitur sic $\frac{A}{C} = \frac{A}{B}$ *bis. Hoc est, ratio* A *ad* C *duplicata est rationis* A *ad* B. *Triplicata autem sic* $\frac{A}{D} = \frac{A}{B}$ *ter.* id est, *ratio* A *ad* D *triplicata est rationis* A *ad* B.

∺ *denotat continue proportionales.* ut A, B, C, D; *item* 2, 6, 18, 64 sunt ∺

XI. Homologæ, seu similes ratione, magnitudines dicuntur, antecedentes quidem antecedentibus, consequentes vero consequentibus.

Ut si A. B :: C. D; *tam* A *&* C, *quam* B *&* D *homologæ magnitudines dicuntur.*

XII. Alterna ratio, est sumptio antecedentis ad antecedentem, & consequentis ad consequentem.

Vt sit A. B :: C. D. *ergo alterne, vel permutando, vel vicißim,* A.C :: B. D. *per* 16. 5.

In hac definitione, & 5. *sequentibus imponuntur nomina sex modis argumentandi, quibus mathematici frequenter utuntur; quarum illationum vis innititur propositionibus hujus libri, quæ in explicationibus citantur.*

XIII. Inversa ratio, est sumptio consequentis ceu antecedentis, ad antecedentem velut ad consequentem.

Vt A. B :: C. D. *ergo inverse*, B.A :: D. C. *per* cor. 4. 5.

XIV. Compositio rationis, est sumptio antecedentis cum consequente, ceu unius, ad ipsam consequentem.

Vt A. B :: C. D. *ergo componendo*, A+B.B :: C+D.D. *per* 18. 5.

XV. Divisio rationis, est sumptio excessus, quo consequentem superat antecedens, ad ipsam consequentem.

Vt

Vt A. B :: C. D. *ergo dividendo*, A-B. B :: C-D. D. *per* 17.5.

XVI. Converſio rationis, eſt ſumptio antecedentis ad exceſſum, quo ſuperat antecedens ipſam conſequentem.

Vt A. B :: C. D. *ergo per converſam rationem*, A. A-B :: C. C-D. *per* cor. 19.5.

XVII. Ex æqualitate ratio eſt, ſi plures duabus ſint magnitudines, & his aliæ multitudine pares, quæ binæ ſumantur, & in eadem ratione; cum ut in primis magnitudinibus prima ad ultimam, ſic & in ſecundis magnitudinibus prima ad ultimam ſeſe habuerit. Vel aliter; ſumptio extremorum, per ſubductionem mediorum.

XVIII. Ordinata proportio eſt, cum fuerit quemadmodum antecedens ad conſequentem, ita antecedens ad conſequentem: fuerit etiam ut conſequens ad aliud quidpiam, ita conſequens ad aliud quidpiam.

Vt ſi A. B :: D. E. *item* B. C :: E. F. *erit ex æquo* A. C :: D. F. *per* 22. 5.

XIX. Perturbata autem proportio eſt; cum tribus poſitis magnitudinibus, & aliis, quæ ſint his multitudine pares, ut in primis quidem magnitudinibus ſe habet antecedens ad conſequentem, ita in ſecundis magnitudinibus antecedens ad conſequentem: ut autem in primis magnitudinibus conſequens ad aliud quidpiam, ſic in ſecundis magnitudinibus aliud quidpiam ad antecedentem.

Vt ſi A. B :: F. G. *item* B. C :: E. F. *erit ex æquo perturbate* A. C :: E. G. *per* 23. 5.

XX. Quotlibet magnitudinibus ordine poſitis, proportio primæ ad ultimam componitur ex proportionibus primæ ad ſecundam, & ſecundæ ad tertiam, & tertiæ ad quartam, & ita deinceps, donec extiterit proportio.

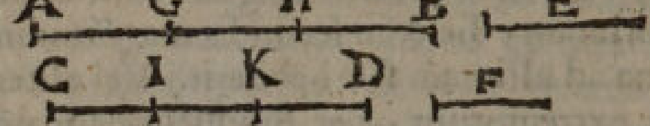
Sint

Sint quotcunque A, B, C, D; ex hac def. $\frac{A}{D} = \frac{A}{B} + \frac{B}{C} + \frac{C}{D}$.

Axioma.

Æquemultiplices eidem multiplici, sunt quoq; inter se æquemultiplices.

Prop. I.

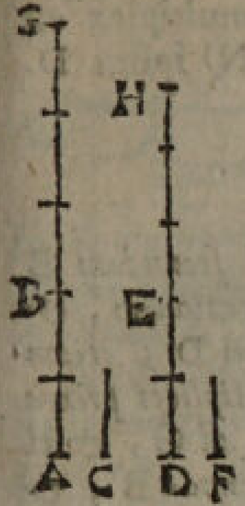


Si sint quotcunque magnitudines AB, CD, *quotcunque magnitudinum* E, F *æqualium numero, singulæ singularum, æquemultiplices; quam multiplex est unius* E *una magnitudo* AB, *tam multiplices erunt & omnes* AB+CD *omnium* E+F.

Sint AG, GH, HB partes quantitatis AB ipsi E æquales. item CI, IK, KD partes quantitatis CD ipsi F pares. Harum numerus illarum numero æqualis ponitur. Quum igitur
a 3. ax. AG+CI a = E+F; a & GH+IK = E+F; a & HB+KD = E+F, liquet AB+CD æque multoties continere E+F, ac una AB unam E continet. Q. E. D.

Prop.

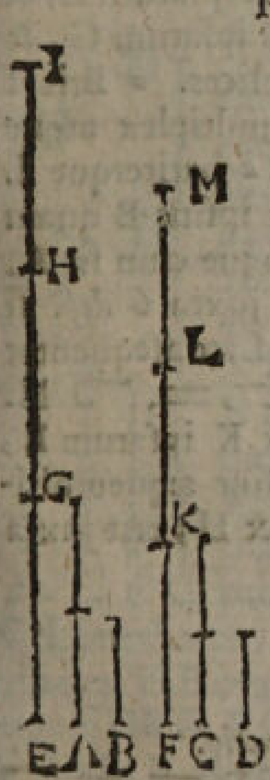
PROP. II.



Si prima A B *secundæ* C *æque fuerit multiplex, atque tertia* D E *quartæ* F ; *fuerit autem & quinta* B G *secundæ* C *æque multiplex, atque sexta* E H *quartæ* F, *erit & composita prima cum quinta* (A G) *secundæ* C *æque multiplex, atque tertia cum sexta* (DH) *quartæ* F.

Numerus partium in A B ipsi C æqualium æqualis ponitur numero partium in D E ipsi F æqualium. Item numerus partium in B G ponitur æqualis numero partium in E H. *a* ergo numerus partium in A B+B G æquatur numero partium in DE+E H. hoc est tota AG æquemultiplex est ipsius C, atque tota G H ipsius F. Q. E. D. a 2.ax.

PROP. III.



Sit prima A *secundæ* B *æquemultiplex, atque tertia* C *quartæ* D ; *sumantur autem* E I, F M *æquemultiplices primæ & tertiæ ; erit & ex æquo, sumptarum utraque utriusque æquemultiplex : altera quidem* E I *secundæ* B, *altera autem* F M *quartæ* D.

Sint E G, G H, H I partes multiplicis E I ipsi A pares ; item F K, K L, L M partes multiplicis F M ipsi C æquales. *a* Harum numerus illarum numero æquatur. porro A, id est E G, vel G H, vel G I ipsius B ponitur æquemultiplex atque C, vel FK, &c. ipsius D. a 1.p.

b ergo

b 2. 5. *b* ergo E G + G H æquemultiplex eſt ſecundæ
c 2. 5. B, atque F K + K L quartæ D. *c* Simili argumento E I (E H + H I) tam multiplex eſt ipſius B, quam F M (F L + L N) ipſius D. Q. E. D.

PROP. IV.

Si prima A ad ſecundam B eandem habuerit rationem, & tertia C ad quartam D; etiam E & F æquemultiplices primæ A, & tertiæ C ad G, & H æquemultiplices ſecundæ B, & quartæ D, juxta quamvis multiplicationem, eandem habebunt rationem, ſi prout inter ſe reſpondent, ita ſumptæ fuerint. (E. G :: F. H.)

Sume I, & K ipſarum E, & F; item L & M ipſarum G, & H æquemultiplices. *a* Erit I ipſius A æquemultiplex atque
a 3. 5. K ipſius C; *a* pariterque L tam multiplex ipſius B quam M ipſius D. Itaque cum ſit A. B *b* :: C. D; juxta 6 *def.* ſi
b hyp. I ⊏, =, ⊐ L; conſequenter pari modo K ⊏, =, ⊐ M. ergo cum I, & K ipſarum E, & F ſumptæ ſint æquemultiplices, atque L, & M ipſarum G & H; erit juxta 7. *def.* E. G :: F. H. Q. E. D.

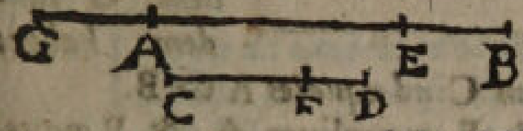
Coroll.

Hinc demonſtrari ſolet inverſa ratio.

Nam quoniam A. B :: C. D, ſi E ⊏, =, ⊐
c 6 def. 5. G; *c* erit ſimiliter F ⊏, =, ⊐ H. ergo liquet, quod

quod si G $\sqsubset$, $=$, $\sqsupset$ E, esse H $\sqsubset$, $=$, $\sqsupset$ F.
[d] ergo B. A :: D.C. Q. E. D. d 6.def.5.

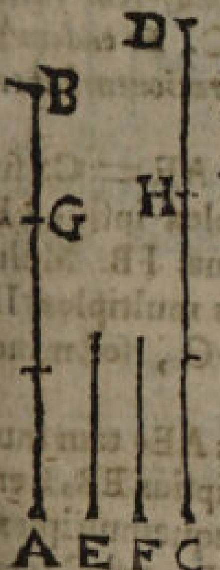
PROP. V.



Si magnitudo A B *magnitudinis* C D *æque fuerit multiplex, atque ablata* AE *ablata* CF; *etiam reliqua* EB *reliquæ* FD *ita multiplex erit, ut tota* AB *totius* CD.

Accipe aliam quandam GA, quæ reliquæ FD ita sit multiplex, atque tota AB totius CD, vel ablata AE ablatæ CF. [a] ergo tota GA+AE totius CF + FD æquemultiplex est, ac una AE unius CF, hoc est, ac AB ipsius CD. [b] ergo GE = AB. [c] proinde, ablata communi AE, manet GA = EB. ergo, &c. a 1.5. b 6.ax. c 3.ax.

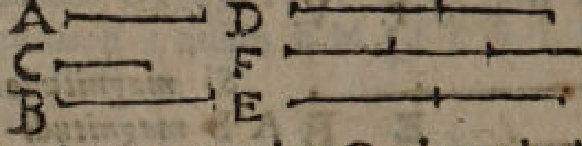
PROP. VI.



Si duæ magnitudines A B, C D *duarum magnitudinum* E, F *sint æquemultiplices; & detractæ quædam sint,* AG, *&* CH, *earundem* E, *&* F *æquemultiplices; & reliquæ* G B, HD *eisdem* E, F *aut æquales sunt, aut æque ipsarum multiplices.*

Nam quia numerus partium in A B ipsi E æqualium ponitur æqualis numero partium in CD ipsi E æqualium; item numerus partium in A G æqualis numero partium in C H. si hinc A G, inde CH detrahatur, [a] remanet numerus partium in reliqua G B æqualis numero partium in H D. ergo si GB sit E semel, erit HD etiam C semel. si GB sit E aliquoties, erit H D etiam C toties accepta. Q. E. D. a 3.ax.

PROP. VII.



Æquales A & B ad eandem C eandem habent rationem; & eadem C ad æquales A & B.

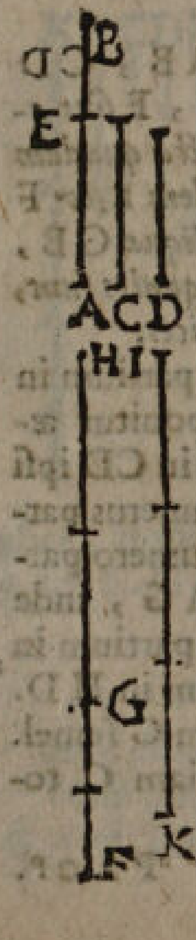
Sumantur D & E æqualium A & B æquemultiplices, & F utcunque multiplex ipsius C; a erit D = E. quare si D $\sqsubset$, =, $\sqsupset$ F, erit similiter E $\sqsubset$, =, $\sqsupset$ F. b ergo A. C :: B. C. inverse igitur C. A c :: C. B. Q.E.D.

a 6 ax.
b 6 def. 5.
c cor. 4 5.

Schol.

Si loco multiplicis F sumantur duæ æquemultiplices, eodem modo ostendetur æquales magnitudines ad alias inter se æquales eandem habere rationem.

PROP. VIII.



Inæqualium magnitudinum AB, C, major AB ad eandem D majorem rationem habet, quam minor C. Et eadem D ad minorem C majorem rationem habet, quam ad majorem AB.

Ex majori AB aufer AE = C. sumatur HG tam multiplex ipsius AE, vel C, quam GF reliquæ FB. Multiplicetur D, donec ejus multiplex IK major evadat quam HG, sed minor quam HF.

Quoniam HG ipsius AE a tam multiplex est, quam GF ipsius EB, b erit tota HF totius AB æquemultiplex, atque una HG unius AE, vel C. ergo cum HF $\sqsubset$ IK (quæ multiplex est ipsius D) sed HG $\sqsupset$ IK, c erit $\frac{AB}{D} \sqsubset \frac{C}{D}$ Q.E.D.

a const.
b 1. 5.
c 8. def. 5.

Rursus

Rursus quia IK $\sqsubset$ HG, at IK $\sqsupset$ HF (ut prius dictum) *d* erit $\frac{D}{C} \sqsubset \frac{D}{AB}$ Q. E. D.

PROP. IX.

Quæ ad eandem eandem habent rationem, æquales sunt inter se. Et ad quas eadem eandem habet rationem, eæ quoque sunt inter se æquales.

1. *Hyp.* Sit A. C :: B. C. dico A = B. Nam sit A $\sqsubset$, vel $\sqsupset$ B, *a* erit ideo $\frac{A}{C} \sqsubset$, vel $\sqsupset \frac{B}{C}$. contra Hyp. a 8. 5.

2. *Hyp.* Sit C. B :: C. A. dico A = B. nam sit A $\sqsubset$ B. *b* ergo $\frac{C}{B} \sqsubset \frac{C}{A}$. contra Hyp. b 8. 5.

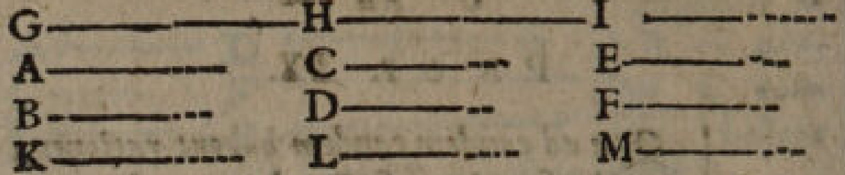
PROP. X.

Ad eandem magnitudinem rationem habentium, quæ majorem rationem habet, illa major est: ad quam vero eadem majorem rationem habet, illa minor est.

1. *Hyp.* Sit $\frac{A}{C} \sqsubset \frac{B}{C}$. Dico A $\sqsubset$ B. Nam si dicatur A = B, *a* erit A. C :: B. C. contra Hyp. Sin A $\sqsupset$ B, *b* erit $\frac{A}{C} \sqsupset \frac{B}{C}$ etiam contra Hyp. a 7. 5. b 8. 5.

2. *Hyp.* Sit $\frac{C}{B} \sqsubset \frac{C}{A}$. Dico B $\sqsupset$ A. Nam dic B = A. *c* ergo C. B :: C. A. contra Hyp. vel dic B $\sqsubset$ A. *d* ergo $\frac{C}{A} \sqsubset \frac{C}{B}$ etiam contra Hyp. c 7. 5. d 8. 5.

Prop. XI.



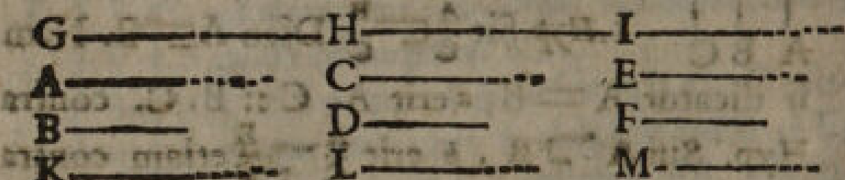
Quæ eidem sunt eædem rationes, & inter se sunt eædem.

Sit A.B :: E. F. item C. D :: E.F. dico A. B :: C.D. sume ipsarum A, C, E æquemultiplices G, H, I; atque ipsarum B, D, F æquemultiplices K, L, M. Et quoniam *a* A.B :: E.F. si G ⊏, =,
a hyp. b 6. def. 5. ⊐ K, *b* erit pari modo I ⊏, =, ⊐ M. pariterque quia *a* E.F :: C. D. si I ⊏, =, ⊐ M, *b* erit H similiter ⊏, =, ⊐ L. ergo si G ⊏, =, ⊐ K, erit similiter H ⊏, =, ⊐ L. *c* quare
c 6. def. 5. A. B :: C. D. Q. E. D.

Schol.

Quæ eisdem rationibus sunt eædem rationes, sunt quoque inter se eædem.

Prop. XII.



Si sint magnitudines quotcunque A, & B; C & D; E, & F *proportionales; quemadmodum se habuerit una antecedentium* A *ad unam consequentium* B, *ita se habebunt omnes antecedentes,* A, C, E *ad omnes consequentes,* B, D, F.

Sume antecedentium æquemultiplices G, H, I; & consequentium K, L, M. Quoniam quam
a 1. 5. multiplex est una G unius A, *a* tam multiplices sunt omnes G, H, I omnium A, C, E; pariterque quam multiplex est una K unius B, *a* tam multiplices sunt omnes K, L, M omnium B, D, F; si G ⊏, =, ⊐ K, erit similiter

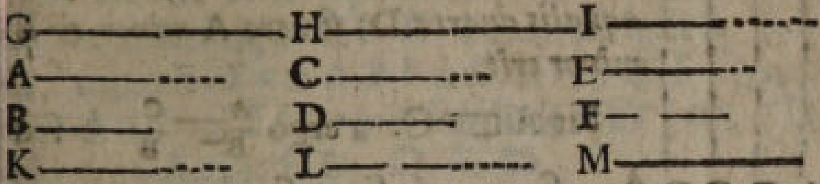
G+

$G+H+I \sqsubset, =, \sqsupset K+L+M$. *b* quare $A.B : A+C+E.B+D+F$. Q. E. D. b 6. def. 5.

Coroll.

Hinc, si similia proportionalia similibus proportionalibus addantur, tota erunt proportionalia.

PROP. XIII.



Si prima A ad secundam B eandem habuerit rationem, quam tertia C ad quartam D; tertia vero C ad quartam D majorem habuerit rationem, quam quinta E ad sextam F; prima quoque A ad secundam B majorem rationem habebit, quam quinta E ad sextam F.

Sume ipsarum A, C, E æquemultiplices G, H, I: ipsarumque B, D, F æquemultiplices K, L, M. Quia $A.B :: C.D$; si $H \sqsubset L$, *a* erit $G \sqsubset K$. Sed quia $\frac{C}{D} \sqsubset \frac{E}{F}$, *b* fieri potest ut sit $H \sqsubset L$, & I non $\sqsubset M$. ergo fieri potest ut $G \sqsubset K$, & I non $\sqsubset M$. *c* ergo $\frac{A}{B} \sqsubset \frac{E}{F}$. Q. E. D. a 6. def. 5. b 8. def. 5. c 8 def. 5.

SCHOL.

Quod si $\frac{C}{D} \sqsupset \frac{E}{F}$, erit quoque $\frac{A}{B} \sqsupset \frac{E}{F}$. Item si $\frac{A}{B} \sqsubset \frac{C}{D} \sqsubset \frac{E}{F}$, erit $\frac{A}{B} \sqsubset \frac{E}{F}$. & si $\frac{A}{B} \sqsupset \frac{C}{D} \sqsupset \frac{E}{F}$, erit $\frac{A}{B} \sqsupset \frac{E}{F}$.

PROP. XIV.

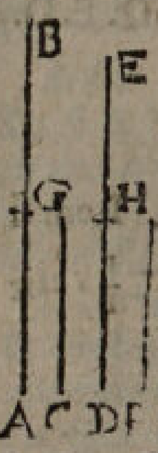
Si prima A ad secundam B eandem habuerit rationem, quam tertia C ad quartam D; prima vero A, quam tertia C major fuerit; erit & secunda B major quam quarta D. Quod si prima A fuerit æqualis tertiæ C, erit & secunda B æqualis quartæ D; si vero A minor, & B minor erit.

a 8.5. Sit A ⊏ C. *a* ergo $\frac{A}{B}$ ⊏ $\frac{C}{B}$. *b* sed
b hyp.
c 13.5. A B C D $\frac{A}{B} = \frac{C}{D}$. *c* ergo $\frac{C}{D}$ ⊏ $\frac{C}{B}$. *d* ergo B ⊏ D.
d 10.5.
e 7.5. Simili argumento si A ⊐ C, *d* erit B ⊐ D. Sin
f hyp.
g 11. & 9.5. ponatur A = C; ergo C. B *e* :: A. B *f* :: C. D. *g* ergo B = D. Quæ E. D.

SCHOL.

A fortiori, si $\frac{A}{B}$ ⊐ $\frac{C}{D}$, atque A ⊏ C, erit B ⊏ D. Item si A = B, erit C = D. Et si A ⊏, vel ⊐ B, erit pariter C ⊏, vel ⊐ D.

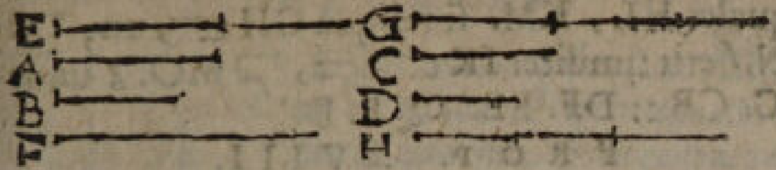
PROP. XV.



Partes C & F cum pariter multiplicibus AB, & DE in eadem sunt ratione, si prout sibi mutuo respondent, ita sumantur. (AB. DE :: C. F.)

Sint AG, GB partes multiplicis AB ipsi C æquales: item DH, HE partes multiplicis DE ipsi F æquales. *a* Harum numerus illarum nume-
a hyp. ro æquatur. ergo quum *b* AG. C ::
b 7.5. DH. F; *b* atque GB. C :: HE. F. *c* erit
c 12.5. AG + GB (AB.) DH + HE (DE) :: C. F. Q. E. D.

PROP. XVI.



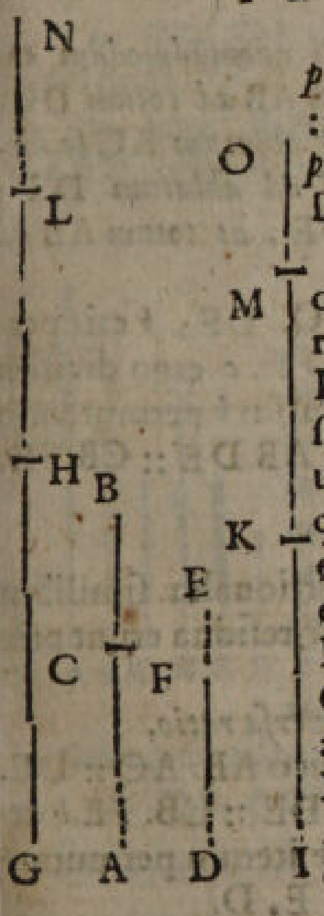
Si quatuor magnitudines A, B, C, D *proportionales fuerint ; & vicissim proportionales erunt.* (A.C :: B. D.)

Accipe E & F æquemultiplices ipsarum A & B. ipsarumque C & D æquemultiplices G & H. Itaque E. F [a] :: A.B.[b] :: C.D [a] :: G.H Quare si E ⊏, =, ⊐ G, [c] erit similiter F ⊏, =, ⊐ H.[d] ergo A.C :: B.D. Q. E. D. a 15.5. b hyp. c 11.5 & 14.5. d 6.def.5.

SCHOL.

Alterna ratio locum tantum habet, quando quantitates ejusdem sunt generis. Nam Heterogeneæ quantitates non comparantur.

PROP. XVII.

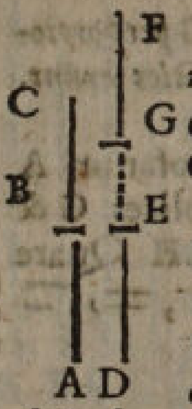


Si compositæ magnitudines proportionales fuerint (AB.CB :: DE. FE;) *hæ quoque divisæ proportionales erunt.* (AC.CB :: DF. FE.)

Accipe GH, HL, IK, KM ordine æquemultiplices ipsarum AC, CB, DF, FE. item LN, MO æquemultiplices ipsarum CB, FE. Tota GL totius AB [a] tam multiplex est, quam una GH unius AC, [b] id est quam IK ipsius DF ; [c] hoc est quam tota IM totius DE; Item HN (HL + LN) ipsius CB [d] æquemultiplex est, ac KO (KM + MO) ipsius FE. Quum igitur per hyp. AB, BC :: DE. EF. si GL ⊏, =, ⊐ HN, etiam similiter. a 1. 5. b constr. c 1. 5. d 2. 5.

e 6.def.5. militer e erit IM ⊏, =, ⊐ KO. aufer hinc inde
f 5.ax. æquales HL, KM. ſi reliqua GH ⊏, =, ⊐
g 6.def.5. LN, f erit ſimiliter IK ⊏, =, ⊐ MO. g unde
AC. CB :: DF. FE. Q. E. D.

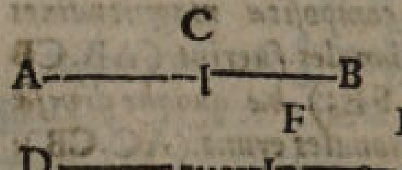
PROP. XVIII.



Si diviſæ magnitudines ſint proportionales (AB.BC :: DE. EF,) hæ quoque compoſitæ proportionales erunt (AC. CB :: DF. FE.)

Nam ſi fieri poteſt, ſit AB. CB ::
a 17. 5. DF. FG ⊐ FE. a ergo erit diviſim
b hyp. & 11. 5. AB. BC :: DG. GF. b hoc eſt DG.
GF :: DE. EF. ergo cum DG ⊏ DE,
c 14. 5. c erit GF ⊏ EF. Q. E. A. Simile
d 9.ax. abſurdum ſequetur, ſi dicatur AB. CB :: DE. GF ⊏ FE.

PROP. XIX.



Si quemadmodum totum AB ad totum DE, ita oblatum AC ſe habuerit ad ablatum DF; & reliquum CB ad reliquum FE, ut totum AB ad totum DE, ſe habebit.

a hyp. Quoniam a AB. DE :: AC. DF, b erit per-
b 16. 5. mutando AB. AC :: DE. DF. c ergo diviſim
c 17. 5. AC. CB :: DF. FE. quare rurſus b permutando
d hyp. & 11. 5. AC. DF :: CB. FE; d hoc eſt AB. DE :: CB. FE.
Q. E. D.

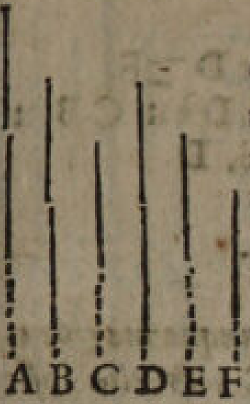
Coroll.

1. Hinc, ſi ſimilia proportionalia ſimilibus proportionalibus ſubducantur, reſidua erunt proportionalia.

2. *Hinc demonſtrabitur converſa ratio.*

Sit AB. CB :: DE. FE. Dico AB. AC :: DE.
a 16. 5. DF. Nam a permutando AB. DE :: CB. FE. b er-
b 19. 5. go AB. DE :: AC. DF. quare iterum permutando, AB. AC :: DE. DF. Q. E. D.

Prop. XX.



Si sint tres magnitudines A, B, C; *& aliæ* D, E, F *ipsis æquales numero, quæ binæ & in eadem ratione sumantur* (A. B :: D. E, *atque* B. C :: E. F;) *ex æquo autem prima* A *major fuerit, quam tertia* C; *erit & quarta* D *major quam sexta* F. *Quod si prima* A *tertiæ* C *fuerit æqualis; erit & quarta* D *æqualis sextæ* F. *Sin illa minor, hæc quoque minor erit,*

Hyp. Si A $\sqsubset$ C. quoniam a E. F :: B.C. (a hyp.)

b erit inverse F. E :: C.B. c Sed $\frac{C}{B} \sqsupset \frac{A}{B}$ d ergo $\frac{F}{E} \sqsupset \frac{A}{B}$ vel $\frac{D}{E}$. e ergo D $\sqsubset$ F. Q.E.D. (b cor. 4 5. c hyp. & 8. 5. d schol. 13. 5. e 10. 5.)

2. *Hyp.* Simili argumento, si A $\sqsupset$ C, ostendetur D $\sqsupset$ F.

3. *Hyp.* Si A = C. quoniam F. E :: C. B :: f A. B :: D. E. g erit D = F. Q.E.D. (f 7. 5. g 11. 5. & 9. 5.)

Prop. XXI.

Si sint tres magnitudines A, B, C; *& aliæ* D, E, F *ipsis æquales numero, quæ binæ & in eadem ratione sumantur, fueritque perturbata eorum proportio,* (A. B :: E. F. *atque* B. C :: D. E;) *ex æquo autem prima* A *quam tertia* C *major fuerit; erit & quarta* D *quam sexta* F *major. Quod si prima fuerit tertiæ æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.*

1. *Hyp.* A $\sqsubset$ C. Quoniam a D. E :: B. C, (a hyp.)

invertendo erit E.D :: C.B. atqui $\frac{C}{B}$ b $\sqsupset$ $\frac{A}{B}$. (b 8. 5.)

c ergo

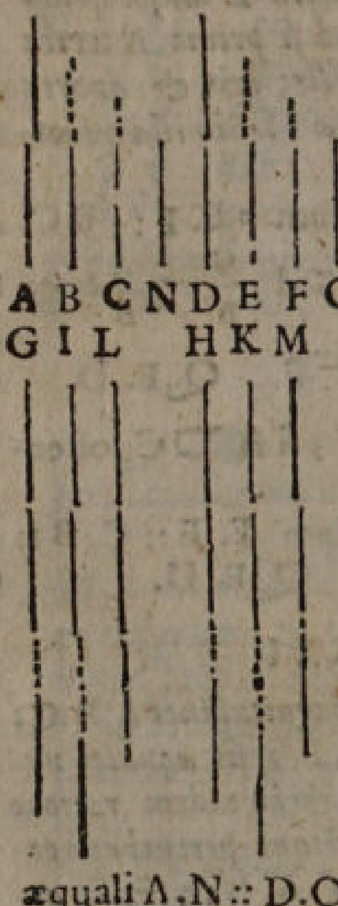
e schol. 13. 5. d 10 5. [e] ergo $\frac{E}{D} \sqsupset \frac{A}{B}$, hoc est $\frac{E}{F}$. [d] ergo D $\sqsubset$ F.

Q. E. D.

2. *Hyp.* Similiter, si A $\sqsupset$ C, erit D $\sqsupset$ F.

e 7. 5. f hyp. g 9 5. 3. *Hyp.* Si A = C. quoniam E.D [e] :: C.B :: [e] A.B :: [f] E. F. [g] erit D = F. Q. E. D.

PROP. XXII.



Si sint quotcunque magnitudines A, B, C; & aliæ ipsis æquales numero D, E, F, quæ binæ & in eadem ratione sumantur (A.B :: D.E. & B. C :: E. F;) & ex æqualitate in eadem ratione erunt (A.C :: D.F.)

Accipe G, H ipsarum A, D; & I, K ipsarum B, E; item L, M ipsarum E, F æquemultiplices.

a hyp. b 4 5. Quoniam [a] A. B :: D.E. [b] erit G. I :: H. K. eodem modo, erit I. L :: K.M. ergo si G $\sqsubset$, =, $\sqsupset$ L, [c] erit

c 20. 5. d 6. def 5. H. $\sqsubset$, =, $\sqsupset$ M; [d] ergo A.C :: D.F. Eodem pacto si ulterius C. N :: F.O, erit ex æquali A.N :: D.O. Q. E.D.

PROP.

PROP. XXIII.

Si sint tres magnitudines A, B, C, *aliæque* D, E, F *ipsis æquales numero; quæ binæ in eadem ratione sumantur; fuerit autem perturbata earum proportio.* (A. B :: E. F. & B. C :: D. E.) *etiam ex æqualitate in eadem ratione erunt.*

Sume G, H, I, ipsarum A, B, D; item K, L, M ipsarum C, E, F æquemultiplices. erit G. H *a* :: A. B *b* :: E. F *a* :: L. M. porro quia *b* B. C :: D. E. erit *c* H. K :: I. L. ergo G, H, K; & I, L, M habent se juxta 21. 5. quare si G ⊏, =, ⊐ K, erit similiter I ⊏, =, ⊐ M. *d* proinde A. C :: D. F. Q. E. D. a 15. 5. b hyp. c 4. 5. d 6. def. 5.

Eodem modo si plures fuerint magnitudinibus tribus, &c.

Coroll.

Ex *his sequitur, rationes ex iisdem rationibus compositas esse inter se easdem. item, earundem rationum easdem partes inter se easdem esse. * 22 & 23. 5. & 10. def. 6.

PROP. XXIV.

A————·—|·——·
C————· B G
D————·—|·——
F————· E H

Si prima AB *ad secundam* C *eandem habuerit rationem quam tertia* DE *ad quartam* F; *habuerit autem & quinta* BG *ad secundam* C *eandem rationem, quam sexta* EH *ad quartam* F; *etiam composita prima cum quinta* (AG) *ad secundam* C *eandem habebit rationem, quam tertia cum sexta* (DH) *ad quartam* F.

Nam quia *a* AB. C :: DE. F. atque ex hyp. & inverse C. BG :: F. EH, erit *b* ex æquali AB. BG :: DE. EH. ergo componendo AG. BG :: DH. EH. *c* item BG. C :: EH. F. *b* ergo rursus ex æquo, AG. C :: DH. F. Q. E. D. a hyp. b 22. 5. c hyp.

PROP.

PROP. XXV.

Si quatuor magnitudines proportionales fuerint, (AB. CD :: E. F.) maxima AB & minima F reliquis CD & E majores erunt.

Fiant AG = E ; & CH = F. Quoniam AB. CD *a* :: E. F *b* :: AG. CH; *c* erit AB. CD :: GB. HD. *d* sed AB $\sqsubset$ CD. *e* ergo GB $\sqsubset$ HD. atqui AG + F = E + CH. ergo AG + F + GB $\sqsubset$ E + CH + HD, hoc est AB + F $\sqsubset$ E + CD. Q. E. D.

a hyp.
b 7. 5.
c 19. 5.
d hyp.
e schol. 14. 5.

Quæ sequuntur propositiones non sunt Euclidis ; sed ex aliis desumptæ, ob frequentem earum usum Euclidæis subjungi solent.

PROP. XXVI.

A———————C——————
B——————— D——————
E———————————

Si prima ad secundam habuerit majorem proportionem, quam tertia ad quartam; habebit convertendo, secunda ad primam minorem proportionem, quam quarta ad tertiam.

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{B}{A} \sqsupset \frac{D}{C}$. Nam concipe $\frac{C}{D} = \frac{E}{B}$. *a* ergo $\frac{A}{B} \sqsubset \frac{E}{B}$. *b* quare A $\sqsubset$ E. *c* ergo $\frac{B}{A} \sqsupset \frac{B}{E}$, *d* vel $\frac{D}{C}$. Q. E. D.

a 13. 5.
b 10. 5.
c 8. 5
d cor 4. 5.

PROP. XXVII.

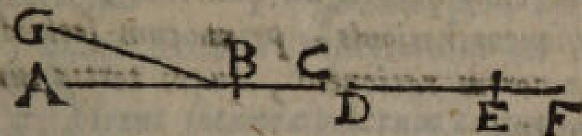
A———————C——————
B——————— D——————
E———————————

Si prima ad secundam habuerit majorem proportionem, quam tertia ad quartam; habebit quoque vicissim prima ad tertiam majorem proportionem, quam secunda ad quartam.

Sit

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{A}{C} \sqsubset \frac{B}{D}$. Nam puta $\frac{E}{B} = \frac{C}{D}$.
a ergo A ⊏ E. *b* ergo $\frac{A}{C} \sqsubset \frac{E}{C}$, *c* vel $\frac{B}{D}$. Q. E. D. a 10. 5. b 8. 5. c 16. 5.

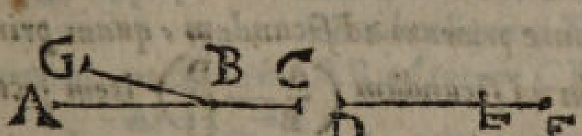
PROP. XXVIII.



Si prima ad secundam habuerit majorem proportionem, quam tertia ad quartam; habebit quoque composita prima cum secunda ad secundam majorem proportionem, quam composita tertia cum quarta ad quartam.

Sit $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Dico $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Nam cogita $\frac{GB}{BC} = \frac{DE}{EF}$. *a* ergo AB ⊏ GB. adde utrinque BC, a 10. 5.
b erit AC ⊏ GC. *c* ergo $\frac{AC}{BC} \sqsubset \frac{GC}{BC}$. *d* hoc est $\frac{DF}{FE}$. b 4. ax. c 8. 5. d 18. 5.
Q. E. D.

PROP. XXIX.

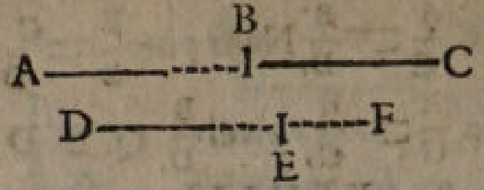


Si composita prima cum secunda ad secundam majorem habuerit proportionem, quam composita tertia cum quarta ad quartam; habebit quoque dividendo prima ad secundam majorem proportionem quam tertia ad quartam.

Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Intellige $\frac{GC}{BC} = \frac{DE}{EF}$. *a* ergo AC ⊏ GC. aufer commune a 10. 5.
BC, *b* erit AB ⊏ GB. *c* ergo $\frac{AB}{BC} \sqsubset \frac{CB}{BC}$ *d* vel $\frac{DE}{EF}$. b 5. ax. c 8. 5. d 17. 5.
Q.E.D.

PROP.

PROP. XXX.



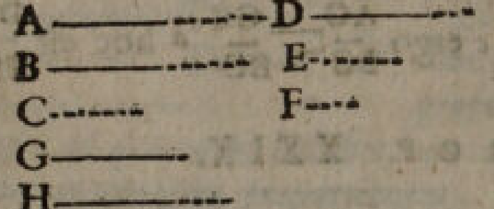
Si composita prima cum secunda ad secundam habuerit majorem proportionem, quam composita tertia cum quarta ad quartam; habebit, per conversionem rationis, prima cum secunda ad primam minorem rationem, quam tertia cum quarta ad tertiam.

Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Nam quia

a hyp. b 19. 5. c 16. 5. d 28. 5.

$\frac{AC}{BC}$ [a] $= \frac{DF}{FE}$, [b] erit dividendo $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. [c] convertendo igitur $\frac{BC}{AB} \sqsupset \frac{EF}{DE}$. [d] ergo componendo $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Q. E. D.

PROP. XXXI.



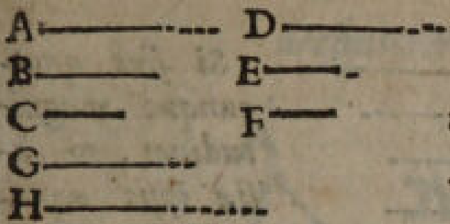
Si sint tres magnitudines A, B, C, & aliæ ipsis æquales numero D, E, F; sitque major proportio primæ priorum ad secundam, quam primæ posteriorum ad secundam $\left(\frac{A}{B} \sqsubset \frac{D}{F};\right)$ *item secundæ priorum ad tertiam major, quam secundæ posteriorum ad tertiam* $\left(\frac{B}{C} \sqsubset \frac{E}{F};\right)$ *erit quoque ex æqualitate major proportio primæ priorum ad tertiam, quam primæ posteriorum ad tertiam* $\left(\frac{A}{C} \sqsubset \frac{D}{F}.\right)$

a 10. 5. b 8. 5. c 13. 5. d 10. 5. e 8. 5. f 22. 5.

Concipe $\frac{G}{C} = \frac{E}{F}$. [a] ergo $E \sqsubset G$. [b] ergo $\frac{A}{C} \sqsubset \frac{A}{B}$.
Rursus puta $\frac{H}{G} = \frac{D}{E}$. [c] ergo $\frac{H}{G} \sqsupset \frac{A}{B}$; [d] ergo fortius $\frac{H}{G} \sqsupset \frac{A}{G}$. [d] quare $A \sqsubset H$. [e] proinde $\frac{A}{C} \sqsubset \frac{H}{C}$, vel $\frac{D}{F}$.
Q. E. D.

PROP.

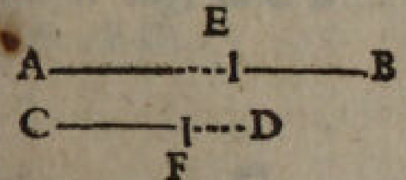
PROP. XXXII.



Si sint tres magnitudines A, B, C; *& aliæ ipsis æquales* D, E, F; *sitque major proportio primæ priorum ad secundam, quam secundæ posteriorum ad tertiam* $\left(\frac{A}{B} \sqsubset \frac{E}{F};\right)$ *item secundæ priorum ad tertiam major quam primæ posteriorum ad secundam* $\left(\frac{B}{C} \sqsubset \frac{D}{E};\right)$ *erit quoque ex æqualitate major proportio primæ priorum ad tertiã, quam primæ posteriorum ad tertiam* $\left(\frac{A}{C} \sqsubset \frac{D}{F}.\right)$

Hujuſce demonſtratio plane ſimilis eſt demonſtrationi præcedentis.

PROP. XXXIII.



Si fuerit major proportio totius AB *ad totum* CD, *quam ablati* AE *ad ablatum* CF; *erit & reliqui* EB *ad reliquum* FD *major proportio, quam totius* AB *ad totum* CD.

Quoniam $\frac{AB}{CD}$ [a] $\sqsubset \frac{AE}{CF}$, *b* erit permutando a *hyp.* b 17. 5.

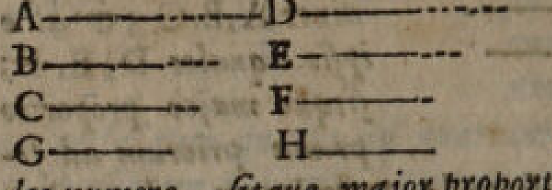
$\frac{AB}{AE} \sqsubset \frac{CD}{CF}$. *c* ergo per converſionem rationis c 30. 5.

$\frac{AB}{EB} \sqsupset \frac{CD}{FD}$. permutando igitur $\frac{AB}{CD} \sqsupset \frac{EB}{FD}$.

Q. E. D.

PROP.

PROP. XXXIV.



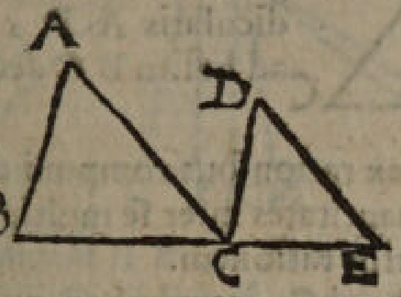
Si sint quotcunque magnitudines, & aliæ ipsis æquales numero, sitque major proportio primæ priorum ad primam posteriorum, quam secundæ ad secundam; & hæc major quam tertiæ ad tertiam, & sic deinceps: habebunt omnes priores simul ad omnes posteriores simul, majorem proportionem, quam omnes priores, relicta prima, ad omnes posteriores, relicta quoque prima; minorem autem, quam prima priorum ad primam posteriorum; majorem denique etiam, quam ultima priorum ad ultimam posteriorum.

Horum demonstratio est penes interpretes; quos adeat, qui eam desiderat. nos omisimus, brevitatis studio; & quia illorum nullus usus in his elementis.

LIB.

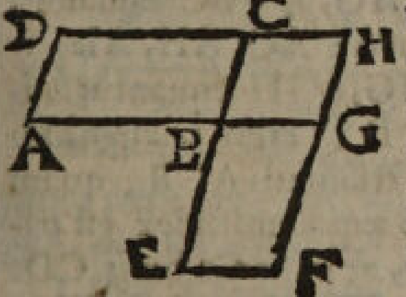
LIB. VI.

Definitiones.



I. **S**Imiles figuræ rectilineæ sunt (ABC, DCE,) quæ & angulos singulos singulis æquales habent; atque etiam latera, quæ circum angulos æquales, proportionalia.

Ang. B = DCE; *&* AB. BC :: DC. CE. *item ang.* A = D; *atque* BA. AC :: CD. DE. *denique ang.* ACB = E. *atque* BC. CA :: CE. ED.

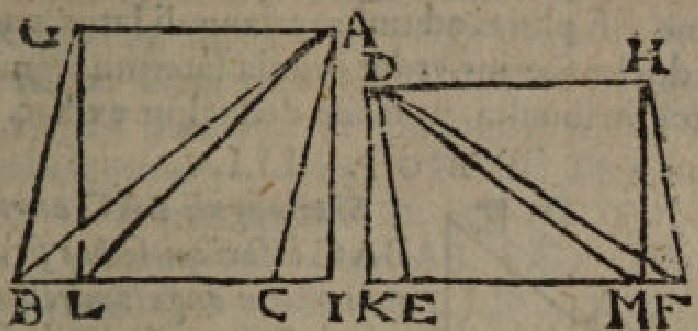


II. Reciprocæ autem sunt (BD, BF,) cum in utraque figura antecedentes, & consequentes rationum termini fuerint. (hoc est, AB. BG :: EB. BC.)

III. Secundum extremam & mediam rationem recta linea AB secta esse dicitur, cum ut tota AB ad majus segmentum AC, ita majus segmentum AC ad minus CB se habuerit. (AB. AC :: AC. CB.)

 IV. Alti-

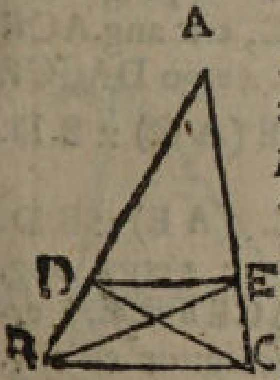
Schol.



Hinc, *triangula* A B C, D E F, *& parallelogramma* A G B C, D E F H, *quorum æquales sunt bases* BC, EF, *ita se habent ut altitudines* AI, DK.

a Sume IL = CB; & KM = EF; ac junge LA, LG, MD, MH. liquet esse triang. ABC. DEF :: *b* ALI. DKM :: *c* AI. DK :: *d* pgr. AGBC. DEFH. Q. E. D. a 3. 1. b 7. 5. c 1. 6. d 41. 1. & 15. 5.

PROP. II.



Si ad unum trianguli ABC *latus* BC, *parallela ducta fuerit recta quædam linea* DE, *hæc proportionaliter secabit ipsius trianguli latera* (AD. BD :: AE. EC.) *Et si trianguli latera proportionaliter secta fuerint* (AD. BD :: AE. EC) *quæ ad sectiones* D, E *adjuncta fuerit recta linea* DE, *erit ad reliquum ipsius trianguli latus* BC *parallela.* Ducantur CD, BE.

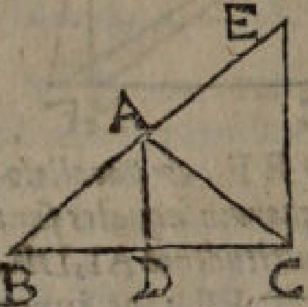
1. *Hyp.* Quia triang. DEB *a* = DEC; *b* erit triang. ADE. DBE :: ADE. ECD. atqui triang. ADE. DBE *c* :: AD. DB. & triang. ADE. DEC *c* :: AE. EC. *d* ergo AD. DB :: AE. EC. a 37. 1. b 7. 5. c 1. 6. d 11. 5.

2. *Hyp.* Quia AD. DB :: AE. EC. *e* hoc est triang. ADE. DBE :: ADE. ECD; *f* erit triang. DBE = ECD. *g* ergo DE, BC sunt parallelæ. Q. E. D. e 1. 6. f 9. 5. g 39. 1.

Schol.

Imo, si plures ad unum trianguli latus parallelæ ductæ fuerint, erunt omnia laterum segmenta proportionalia, ut facile deducitur ex hac.

PROP. III.



Si trianguli BAC *angulus* BAC *bifariam sectus sit, secans autem angulum recta linea* AD *secuerit & basim, basis segmenta eandem habebunt rationem quam reliqua ipsius trianguli latera* (BD. DC :: AB. AC.) *Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera* (BD. DC :: AB. AC.) *recta linea* AD *quæ à vertice* A *ad sectionem* D *ducitur, bifariam secat trianguli ipsius angulum* BAC.

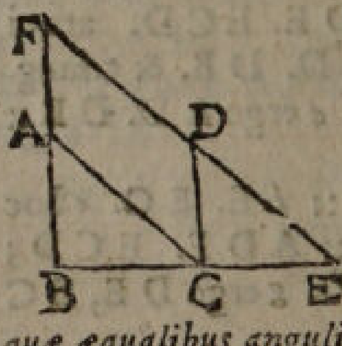
Produc BA; & fac AE = AC. & junge CE.

1. *Hyp*. Quoniam AE = AC, erit ang. ACE *a* = E *b* = $\frac{1}{2}$ BAC *c* = DAC. *d* ergo DA, CE parallelæ sunt. *e* quare BA. AE (AC) :: BD. DC. Q. E. D.

2. *Hyp*. Quoniam BA. AC. (AE) :: BD. DC. *f* erunt DA, CE parallelæ: *g* ergo ang. BAD = E; & ang. DAC *g* = ACE *h* = E. *k* ergo ang. BAD = DAC. bisectus igitur est ang. BAC. Q. E. D.

a 5. 1.
b 32. 1.
c hyp.
d 27. 1.
e 2. 6.
f 2. 6.
g 29. 1.
h 5. 1.
k 1. ax.

PROP. IV.



Æquiangulorum triangulorum ABC, DCE *proportionalia sunt latera, quæ circum æquales angulos* B, DCE (AB. BC :: DC. CE, &c.) *& homologa sunt latera* AB, DC, &c. *quæ æqualibus angulis* ACB, E, &c. *subtenduntur*.

Statue

Statue latus B C in directum lateri C E, & produc B A, ac E D donec *a* occurrant. a 32. 1. & 13. ax.

Quoniam ang. B *b* = ECD, *c* sunt B F, C D parallelæ. Item quia ang BCA *b* = CED, *c* sunt C A, E F parallelæ. Figura igitur C A F D est parallelogramma. *d* ergo A F = C D; *d* & A C = F D. Liquet igitur A B. A F (C D) *e* :: B C. CE. *f* permutando igitur AB. BC :: CD. CE. *e* item B C. C E :: F D. (A C) D E. *f* ergo permutando BC. AC :: CE. DE. quare etiam *g* ex æquo AB. AC :: CD. DE. ergo, &c.

b hyp. c 28. 1. d 34. 1. e 2. 6. f 16. 5. g 22. 5.

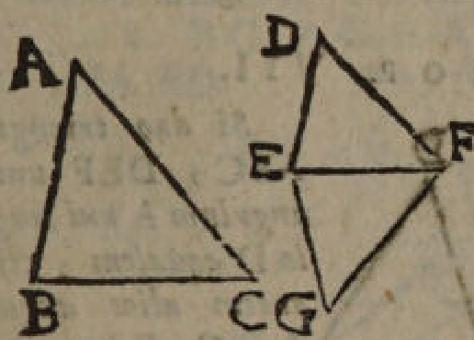
Coroll.

Hinc AB. DC :: BC. CE :: AC. DE.

Schol.

Hinc si in triangulo FBE ducatur uni lateri FE parallela AC; erit triangulum ABC simile toti FBE.

PROP. V.

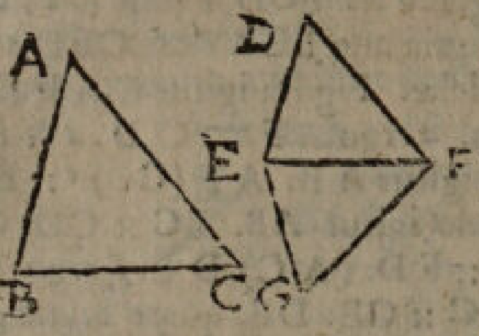


Si duo triangula A B C, D E F *latera proportionalia habeant* (A B. BC :: D E. E F. & A C. B C :: D F. E F. item A B. A C :: D E D F) *æquiangula erunt triangula, & æquales habebunt eos angulos, sub quibus homologa latera subtenduntur.*

Ad latus E F *a* fac ang. F E G = B; *a* & ang. EFG = C, *b* quare etiam ang. G = A. ergo G E. EF *c* :: A B. B C :: *d* D E. E F. *e* ergo GE = D E. Item GF F E *c* :: A C. C B *d* :: DF. FE. *e* ergo GF = DF. Triangula igitur DEF, GEF sibi mutuo æquilatera sunt. *f* ergo ang. D = G = A. *f* & ang. FED = FEG = B. *g* proinde & ang. DFE = C. ergo, &c.

a 23. 1. b 32. 1. c 4. 6. d hyp. e 11. 5. & 9. 5. f 8. 1. g 32. 1.

Prop. VI.

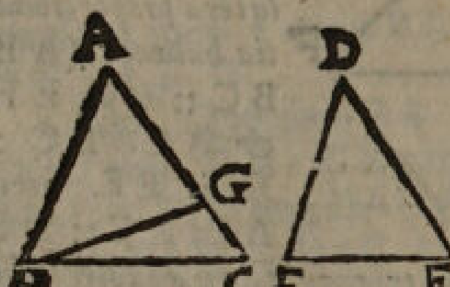


Si duo triangula A B C, D E F *unum angulum* B *uni angulo* D E F *æqualem, & circum æquales angulos* B, D E F *latera proportionalia habuerint* (A B. B C :: D E. E F,) *æquiangula erunt triangula* A B C, D E F; *æqualesque habebunt angulos, sub quibus homologa latera subtenduntur.*

a 32. 1. b 4. 6. c hyp. d 9. 5. e hyp. f constr. g 4. 1. h 32. 1.

Ad latus E F fac ang. FEG = B, & ang. EFG = C. *a* unde & ang. G = A. ergo GE. EF *b* :: AB. BC *c* :: DE. EF. *d* ergo DE = GE. atqui ang. D E F *e* = B *f* = G E F. *g* ergo ang. D = G = A. *h* proinde etiam ang. E F D = C. Q. E. D.

Prop. VII.



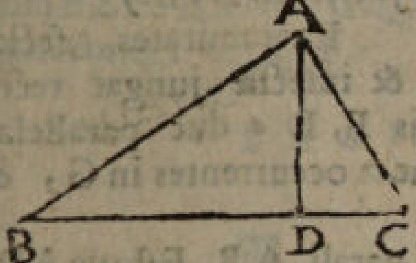
Si duo triangula ABC, DEF *unum angulum* A *uni angulo* D *æqualem, circa autem alios angulos* A B C, E *latera proportionalia habeant* (A B. B C :: D E. E F;) *reliquorum autem simul utrunque* C, F *aut minorem aut non minorem recto, æquiangula erunt triangula* ABC, DEF, *& æquales habebunt eos angulos circum quos proportionalia sunt latera.*

a hyp. b 32. 1. c 4. 6. d hyp. e 9. 5. f 5. 1. g cor. 17. 1.

Nam si fieri potest, sit ang. ABC ⊏ E. fac igitur ang. ABG = E; ergo cum ang. A *a* = D, *b* erit etiam ang. AGB = F. ergo AB. BG *c* :: DE. EF :: AB. BC. *e* ergo BG = BC. *f* ergo ang. BGC = BCG. *g* ergo ang. BGC. vel C minor

minor est recto; g proinde ang. AGB, vel F recto major est. ergo anguli C & F non sunt ejusdem speciei, contra Hyp. g cor. 13. 1.

Prop. VIII.



Si in triangulo rectangulo ABC, *ab angulo recto* B A C *in basin* B C *perpendicularis* A D *ducta est; quæ ad perpendicularem triangula* A D B, ADC, *tum toti triangulo* ABC, *tum ipsa inter se, similia sunt.*

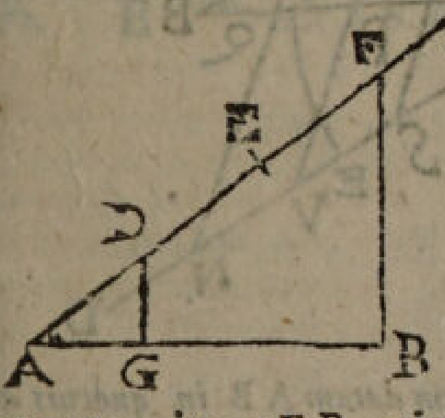
Nam ang. B A C a = B D A a = C D A. & ang. B A D b = C. & C A D b = B. ergo per 4. 6. & 1 def. 6. a 12. ax. b 32. 1.

Coroll.

Hinc 1. BD. DA c :: DA. DC. c 1. def. 6.

2. BC. AC :: AC. DC. & C B. BA :: BA. BD.

Prop. IX.

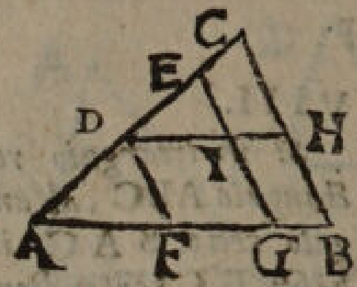


A data recta linea A B *imperatam partem* $\frac{1}{3}$ (A G) *auferre.*

Ex A duc infinitam A C utcunq; in qua a sume tres AD, DE, EF æquales utcunque, junge F B, cui ex D b duc parallelam DG. Dico factum. a 3. 1. b 31. 1.

Nam GB. AG c :: FD. AD. ergo d componendo AB. AG :: AF. AD. ergo cum AD = $\frac{1}{3}$ AF, erit AG = $\frac{1}{3}$ AB. Q. E. F. c 2. 6. d 18. 5.

PROP. X.



Datam rectam lineam A B *insectam similiter secare* (in F, G,) *ut data altera* A C, *secta fuerit* (*in* D, E.)

Extremitates sectæ & insectæ jungat recta

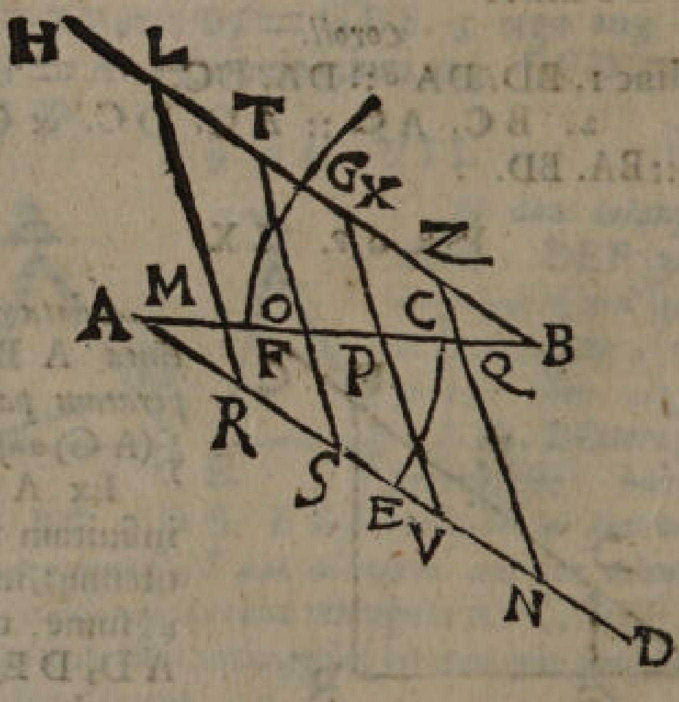
a 31. 1. B C. Huic ex punctis E, D a duc parallelas E G, D F rectæ secandæ occurrentes in G, & F. Dico factum.

a Ducatur enim DH parall. A B. Estque AD.

b 2. 6. DE b :: A F. F G, & D E. E C b :: D I. I H c ::

c 34. 1. & 7. 5. FG. GB. Q. E. F.

Scholium.



Hinc *discimus rectam datam* A B *in quotvis æquales partes* (puta 5.) *secare.* id quod facilius præstabitur sic;

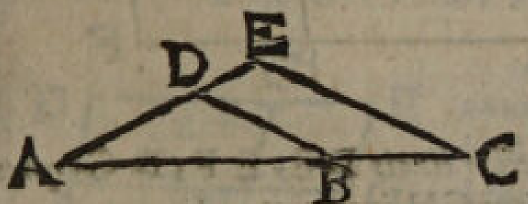
Duc infinitam AD, eique parallelam BH etiam infinitam. Ex his cape partes æquales AR, RS, SV, VN; & BZ, ZX, XT, TL; in singulis una

pau-

pauciores, quam desiderentur in AB; tum rectæ ducantur LR, TS, XV, ZN. hæ quinquisecabunt datam A B.

Nam RL, ST, VX, NZ [a] parallelæ sunt. ergo quum AR, RS, SV, VN [b] æquales sint, [c] erunt AM, MO, OP, PQ æquales. Similiter quia BZ = ZX, erit BQ = QP. ergo A B quinquisecta est. Q. E. F. a 33. 1. b constr. c 2. 6.

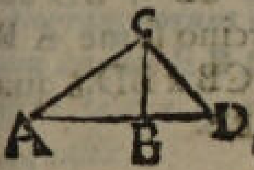
PROP. XI.



Datis duabus rectis lineis A B, A D, *tertiam proportionalem* D E *invenire*.

Junge B D, & ex A B protracta sume B C = A D. per C duc C E parall. B D. cui occurrat A D producta in E. Erit D E expetita.

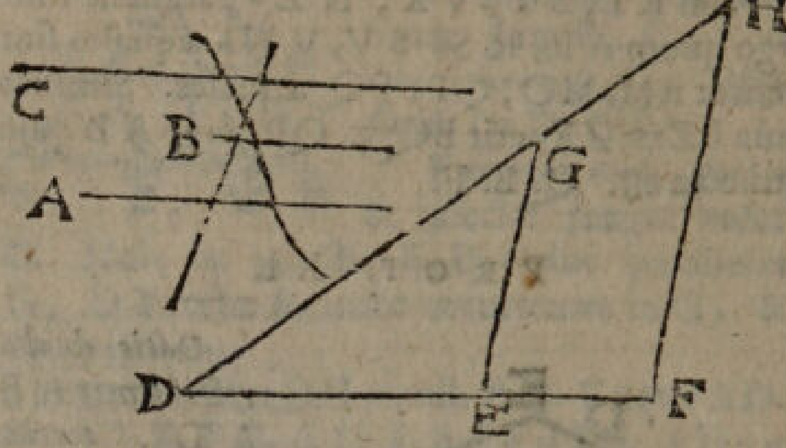
Nam AB. [a] BC. (AD) :: AD. DE. Q. E. F. a 2. 6.



Vel sic, fac ang. A B C rectum, & ang. A C D etiam rectum. [b] erit A B. B C :: B C. B D. b cor. 9.

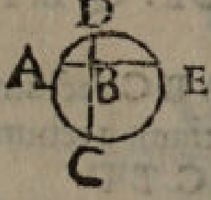
PROP.

PROP. XII.



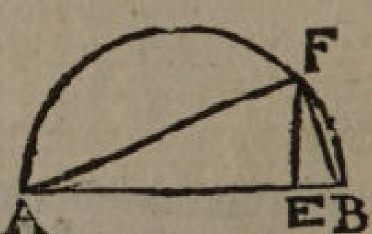
Tribus datis rectis lineis D E, E F, D G, *quartam proportionalem* GH *invenire*.

Connectatur E G. per F duc F H parall. E G, cui occurrat D G producta ad H. liquet esse
a 2. 6 DE. EF a :: DG. GH. Q. E. F.



Vel ita. CD = CB + BD adapta circulo. Circino sume A B.
a 35. 3. Erit AB x BE = CB x BD. b quare AB. CB :: BD. BE.
b 16. 6.

PROP. XIII.



Duabus datis rectis lineis A E, E B, *mediam proportionalem* E F *adinvenire*.

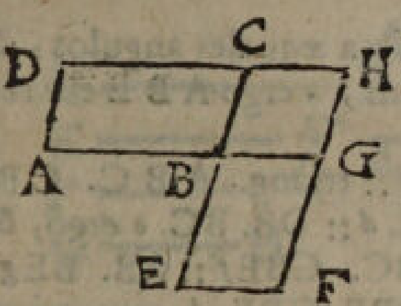
Super tota A B diametro describe semicirculum A F B. Ex E erige perpendicularem E F occurrentem peripheriæ in F. Dico A E. E F :: E F. E B. Du-
a 31. 3. cantur enim A F, & F B. Ex trianguli a rectanguli

guli A F B recto angulo deducta est F E basi perpendicularis ; *b* ergo A E. F E :: F E. E B. Q. E. F. b *cor.* 8. 6.

Coroll.

Hinc, linea recta, quæ in circulo à quovis puncto diametri, ipsi diametro perpendicularis ducitur ad circumferentiam usque, media est proportionalis inter duo diametri segmenta.

PROP. XIV.



Æqualium, & unum A B C *uni* E B G *æqualem habentium angulum, parallelogrammorum* B D, B F, *reciproca sunt latera quæ circum æquales angulos.* (A B. B G :: E B. B C:) *Et quorum parallelogrammorum* B D, B F, *unum angulum* A B C *uni angulo* E B G *æqualem habentium, reciproca sunt latera quæ circum æquales angulos, illa sunt æqualia.*

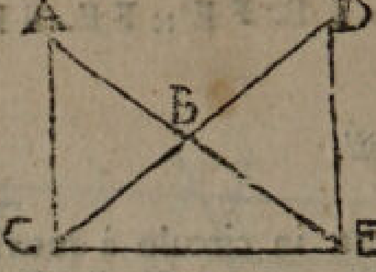
Nam latera A B, B G circa æquales angulos faciant unam rectam: *a* quare E B, B C etiam in directum jacebunt. Producantur FG, DC; donec occurrant. a *sch.* 15. 1.

1. *Hyp.* AB. BG *b* :: BD. BH *c* :: BF. BH *d* :: BE. BC. *e* ergo, &c. b 1. 6. c 7. 5. d 1. 6.

2. *Hyp.* BD. BH *f* :: AB. BG *g* :: BE. BC *h* :: BF. BH, *k* ergo Pgr. BD = BF. Q. E. D. e 11. 5. f 1. 6. g *hyp.* h 1. 6. k 11. & 9. 5.

PROP.

PROP. XV.



Æqualium, & unum ABC, *uni* DBE *æqualem habentium angulum triangulorum* ABC, DBE, *reciproca sunt latera, quæ circum æquales angulos* (AB. BE :: DB. BC) : *Et quorum triangulorum* ABC, DBE, *unum angulum* ABC *uni* DBE *æqualem habentium reciproca sunt latera, quæ circum æquales angulos* (AB. BE :: DB. BC.) *illa sunt æqualia.*

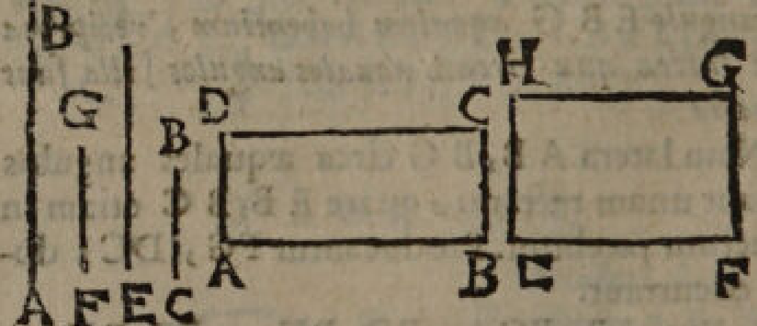
Latera CB, BD circa æquales angulos, statuantur sibi in directum; *a* ergo ABE est recta linea. ducatur CE.

a *sch.* 15. 1.
b 1. 6.
c 7. 5.
d 1. 6.
e 11. 5.
f 1. 6.
g *hyp.*
h 1. 6
k 11, & 9. 5.

1. *Hyp.* AB. BE *b* :: triang. ABC. CBE *c* :: triang. DBE. CBE. *d* :: DB. BC. *e* ergo, &c.

2. *Hyp.* Triang. ABC. CBE *f* :: AB. BE *g* :: DB. BC *h* :: triang. DBE. CBE. *k* ergo triang. ABC = DBE. Q. E. D.

PROP. XVI.



Si quatuor rectæ lineæ proportionales fuerint (AB. FG :: EF. CB,) *quod sub extremis* AB, CB *comprehenditur rectangulum* AC, *æquale est ei, quod sub mediis* EF, FG *comprehenditur, rectangulo* EG. *Et si sub extremis comprehensum rectangulum* AC *æquale fuerit ei, quod sub mediis comprehenditur, rectangulo* EG, *illæ quatuor rectæ lineæ proportionales erunt* (AB. FG :: EF. CB.)

1. *Hyp.*

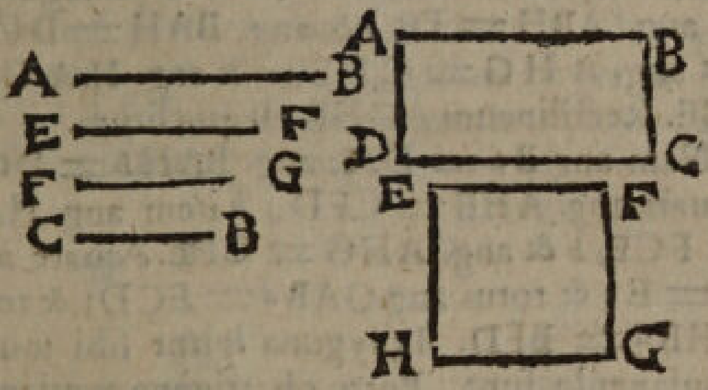
1. *Hyp.* Anguli B & F recti, ac *a* proinde pares sunt; atque ex hyp. AB. FG :: EF. CB. *b* ergo rectang. AC = EG. Q. E. D. a 12. *ax.*

2. *Hyp.* *c* Rectang. AC = EG; atque ang. B = F; *d* ergo AB. FG :: EF. CB. Q. E. D. b 14 6. c *hyp.* d 14 6.

Coroll.

Hinc ad datam rectam lineam AB facile est datum rectangulum EG applicare, *e* faciendo AB. EF :: FG. BC. e 12. 6.

PROP. XVII.



Si tres rectæ lineæ sint proportionales (AB. EF :: EF. CB,) *quod sub extremis* AB, CB *comprehenditur rectangulum* AC, *æquale est ei, quod à media* EF *describitur, quadrato* EG. *Et si sub extremis* AB, CB *comprehensum rectangulum* AC, *æquale sit ei, quod à media* EF *describitur, quadrato* EG, *illæ tres rectæ lineæ proportionales erunt* (AB. EF :: EF. CB.)

Accipe FG = EF.

1. *Hyp.* AB. EF *a* :: EF (FG.) CB. ergo Rectang. AC *b* = EG *c* = EFq. Q. E. D. a *hyp.* b 16 6 c 29. *def.* 1.

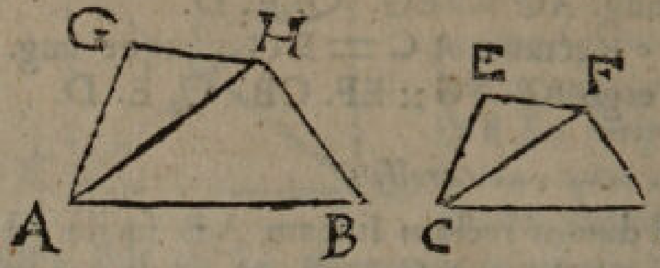
2. *Hyp.* Rectang. AC *d* = quadr. EG = EFq. *e* ergo AB. EF :: FG (EF.) BC. d *hyp.* e 16 6.

Coroll.

Sit A in B = Cq. ergo A. C :: C. B.

PROP.

PROP. XVIII.

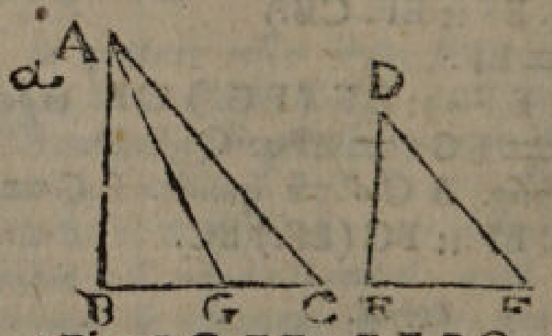


A data recta linea AB *dato rectilineo* CEFD *simile similiterque positum rectilineum* AGHB *describere.*

Datum rectilineum resolve in triangula. *a* fac ang. ABH = D; *a* & ang. BAH = DCF;
a 23. 1. *a* & ang. AHG = CFE; *a* & ang. HAG = FCE. Rectilineum AGHB est quæsitum.

b constr. Nam ang. B *b* = D. & ang. BAH *b* = DCF.
c 32. 1. *c* quare ang. AHB = CFD; *b* item ang. HAG = FCE, *b* & ang. AHG = CFE. *c* quare ang.
d 2. ax. G = E; & totus ang. GAB *d* = ECD; & totus GHB *d* = EFD. Polygona igitur sibi mutuo æquiangula sunt. Porro ob trigona æquiangula,
e 4. 6. AB. BH *e* :: CD. DF. & AG. GH. *e* :: CE. EF. item AG. AH. *e* :: CE. CF. & AH. AB *e* ::
f 22. 5. CF. CD. *f* unde ex æquo AG. AB :: CE. CD.
g 6. def. 6. eodem modo GH HB :: EF. FD. *g* ergo polygona ABHG, CDFE similia similiterque posita existunt. Q. E. F.

PROP. XIX.



Similia triangula A B C, D E F *sunt in duplicata ratione laterum homologorum* B C E F.

a 11. 6. *a* Fiat BC. EF :: EF. BG. & ducatur AG.

Quia

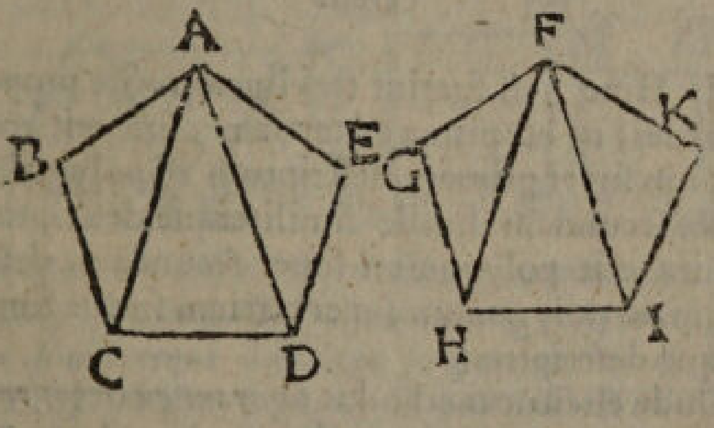
Quia AB. DE *b* :: BC. EF *c* :: EF. BG. & ang. B $=$ E; *d* erit triang. ABG $=$ DEF. verum triang. ABC. ABG *e* :: BC. BG; & *f* $\frac{BC}{BG} = \frac{BC}{EF}$ bis; ergo triang. $\frac{ABC}{ABG}$ hoc est $\frac{ABC}{DEF}$ *g* $= \frac{BC}{EF}$ bis. Q. E. D.

b cor. 4. 6.
c constr.
d 15. 6.
e 1. 6
f 10. def. 5.
g 11. 5.

Coroll.

Hinc, si tres lineæ BC, EF, BG proportionales fuerint; ut est prima ad tertiam, ita est triangulum super primam BC descriptum ad triangulum super secundam EF simile similiterque descriptum. vel ita est triangulum super secundam EF descriptum ad triangulum super tertiam simile similiterque descriptum.

PROP. XX.



Similia polygona ABCDE, FGHIK *in similia triangula* ABC, FGH; *&* ACD, FHI, *&* ADE, FIK *dividuntur, & numero æqualia, & homologa totis.* (ABC. FGH :: ABCDE. FGHIK :: ACD. FHI :: ADE. FIK.) *Et polygona* ABCDE, FGHIK *duplicatam habent eam inter se rationem, quam latus homologum* BC *ad homologum latus* GH.

1. Nam

a hyp. 1. Nam ang. B ^{a} = G; & A B. BC ^{a} :: F G.
b 6. 6. G H. ^{b} ergo triangula ABC, FGH æquiangula sunt. eodem modo, triangula A E D, F K I assimilantur. cum igitur ang. BCA ^{b} = GHF; & ang. ADE ^{b} = FIK; totique anguli BCD,
c hyp. GHI; atque toti CDE, HIK ^{c} pares sint, ^{d} re-
d 3. ax. manent ang. ACD = FHI; & ang. ADC =
e 32. 1. FIH; ^{e} unde etiam ang. CAD = HFI. ergo triangula ACD, FHI similia sunt. ergo, &c.

2. Quoniam igitur triangula B C A, G H F
f 19. 6. similia sunt, ^{f} erit $\frac{BCA}{GHF} = \frac{BC}{GH}$ bis. ob eandem causam $\frac{CAD}{HFI} = \frac{CD}{HI}$ bis. denique triang. $\frac{DEA}{IKF} =$
g hyp & 16. 5. $\frac{DE}{IK}$ bis. quare cum B C. G H ^{g} :: C D. H I ^{g} ::
h sch. 23. 5. DE. IK, ^{h} erit triang. B C A. G H F :: C A D.
k 12. 5. H F I :: D E A. I K F :: ^{k} polyg. A B C D E. FGHIK :: $\frac{BC}{GH}$ bis.

Coroll.

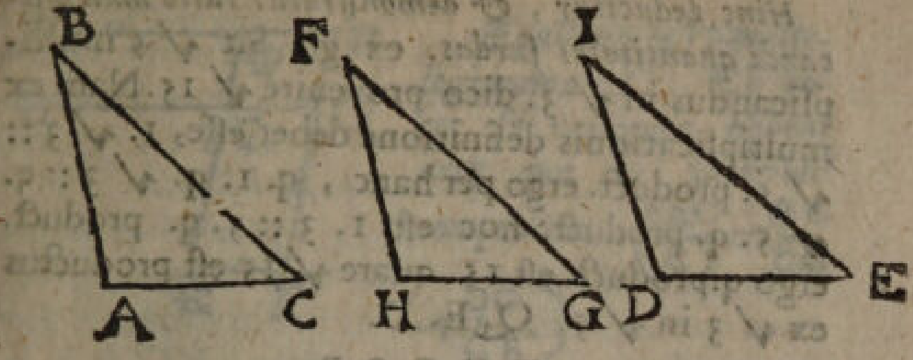
I. Hinc, si fuerint tres lineæ rectæ proportionales; ut est prima ad tertiam, ita erit polygonum super primam descriptum ad polygonum super secundam simile similiterque descriptum. vel ita erit polygonum super secundam descriptum ad polygonum super tertiam simile similiterque descriptum.

Unde elicitur methodus *figuram quamvis rectilineam augendi vel minuendi in ratione data. Vt si velis pentagoni, cujus latus* C D, *aliud facere quintuplum. inter* AB, *&* 5 AB *inveni mediam propor-*
* 18. 6. *tionalem. Super hac* * *construe pentagonum simile dato. hoc erit quintuplum dati.*

II. Hinc etiam, si figurarum similium homologa latera nota fuerint, etiam proportio figurarum innotescet; nempe inveniendo tertiam proportionalem.

PROP.

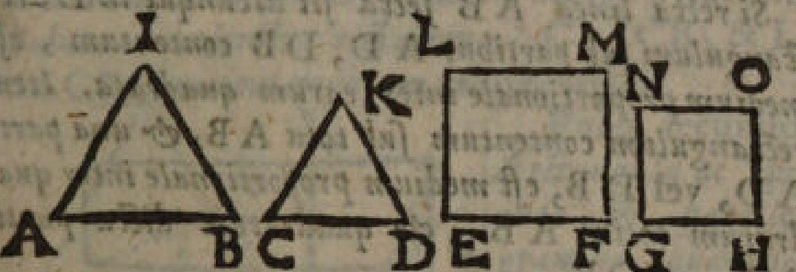
PROP. XXI.



Quæ (ABC, DIE) *eidem rectilineo* HFG *sunt similia, & inter se sunt similia.*

Nam ang. A a = H a = D. & ang. C a = G a = E; & ang. B a = F a = I. a item AB.AC :: HF. HG :: DI. DE. a & AC. CB :: HG. GF :: DE. EI. & AB. BC :: HF. FG :: DI. IE. a ergo ABC, DIE similia sunt. Q.E.D. a 1. def. 6.

PROP. XXII.



Si quatuor rectæ lineæ proportionales fuerint (AB. CD :: EF. GH.) *& ab eis rectilinea similia similiterque descripta proportionalia erunt.* (ABI. CDK :: EM. GO.) *Et si à rectis lineis similia similiterque descripta rectilinea proportionalia fuerint* (ABI.CDK :: EM.GO) *ipsæ etiam rectæ lineæ proportionales erunt.* (AB.CD :: EF.GH.)

1. *Hyp.* $\frac{ABI}{CDK}$ a = $\frac{AB}{CD}$ bis = a $\frac{EF}{GH}$ bis a $\doteq$ $\frac{FM}{GO}$. a 19. 6.

d ergo ABI. CDK :: EM. GO. Q. E. D.

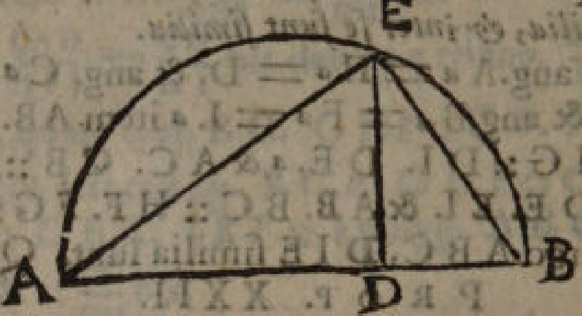
2. *Hyp.* $\frac{AB}{CD}$ bis = a $\frac{ABI}{CDK}$ b = $\frac{FM}{GO}$ c = $\frac{EF}{GH}$. b hyp. c 20. 6.

bis. ergo AB. CD :: EF. GH. Q. E. D. d cor. 23 5.

Schol.

Hinc deducitur, & demonstratur ratio multiplicandi quantitates surdas. ex gr. Sit $\surd 5$ multiplicandus in $\surd 3$. dico provenire $\surd 15$. Nam ex multiplicationis definitione debet esse, 1. $\surd 3$ :: $\surd 5$. product. ergo per hanc, q. 1. q. $\surd 3$ :: q. $\surd 5$. q. product. hoc est. 1. 3 :: 5. q. product. ergo q. product. est 15. quare $\surd 15$ est productus ex $\surd 3$ in $\surd 5$. Q. E. D.

THEOR.



Petr. Herig. *Si recta linea* A B *secta sit utcunque in* D, *rectangulum sub partibus* A D, D B *contentum, est medium proportionale inter earum quadrata. Item rectangulum contentum sub tota* A B, *& una parte* A D, vel D B, *est medium proportionale inter quadratum totius* A B, *& quadratum dictæ partis* A D, vel D B.

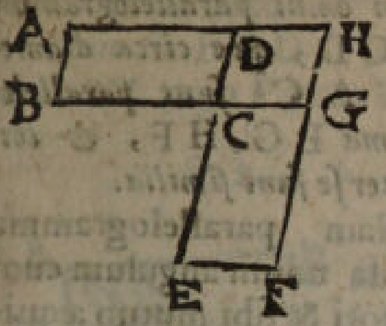
Super diametrum A B describe semicirculum. ex D erige normalem DE occurrentem peripheriæ in E. junge AE, BE.

a cor. 8 6. b 22. 6 c 17 6. Liquet esse A D. D E [a] :: D E. D B. [b] ergo A Dq. D Eq :: D Eq. D Bq. [c] hoc est, A Dq. ADB :: ADB. DBq. Q. E. D.

d cor. 8 6. e 22. 6. f 17. 6. Porto, B A. A E [d] :: A E. A D. [e] ergo BAq. AEq :: AEq. ADq. [f] hoc est B Aq. B A D :: B A D. ADq. Eodem modo ABq. ABD :: ABD. BDq. Q. E. D.

a 1. 6. Vel sic; sit Z = A + E. liquet esse Aq. AE :: [a] A. E :: [a] AE. Eq. item Zq. ZA :: [a] Z. A. :: [a] ZA. Aq. & Zq. ZE :: [a] Z.E :: ZE. Eq.

PROP. XXIII.



Æquiangula parallelogramma AC, CF *inter se rationem habent eam quæ ex lateribus componitur.* $\left(\frac{AC}{CF} = \frac{BC}{CG} + \frac{DC}{CE}.\right)$

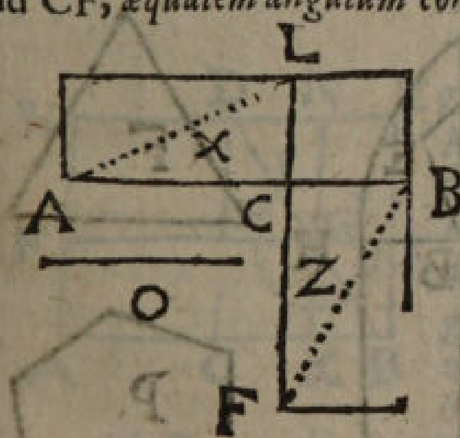
Latera circa æquales angulos C *a* sibi in directum statuantur; & compleatur parallelogrammum CH. *a sch. 15.*

Ratio. $\frac{AC}{CF}{}^{b} = \frac{AC}{CH} + \frac{CH}{CF}{}^{c} = \frac{BC}{CG} + \frac{DC}{CE}.$ b 20 *def.* 5. c 1. 6.

Q. E. D.

Coroll.

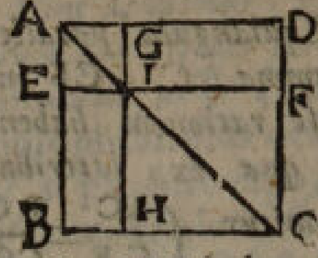
Hinc & ex 34. 1. patet primo, *Triangula, quæ unum angulum* (ad C) *æqualem habent, rationem habere ex rationibus rectarum*, AC ad CB, & LC ad CF, *æqualem angulum continentium.* *Andr. Tacq. 15. 5.*



Patet secundo, *Rectangula ac * proinde & parallelogramma quæcunque rationem inter se habere compositam ex rationibus basis ad basim, & altitudinis ad altitudinem.* Neque aliter de triangulis ratiocinaberis. ** 35. 1.*

Patet tertio, *Quomodo triangulorum ac parallelogrammorum proportio exhiberi possit.* Sunto parallelogramma X & Z; quorum bases AC, CB; altitudines vero CL, CF. Fiat CL. CF :: CB. O. * erit X. Z :: AC. O. ** 14. 6. & 1. 6.*

PROP. XXIV.

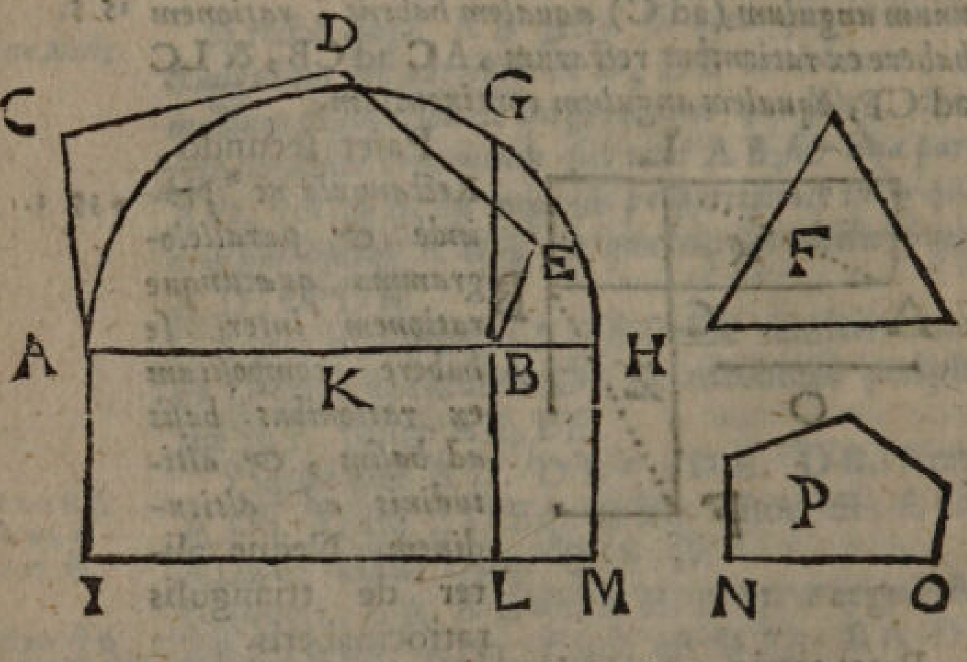


In omni parallelogrammo ABCD, quæ circa diametrum AC sunt parallelogramma EG, HF, & toti & inter se sunt similia.

Nam parallelogramma EG, HF habent singula unum angulum cum toto communem. *a* ergo toti & sibi mutuo æquiangula sunt. *a* Item tam triangula ABC, AEI, IHC, quam triangula ADC, AGI, IFC sunt inter se æquiangula. *b* ergo AE. EI :: AB. BC, *b* atque AE. AI :: AB. AC; *b* & AI. AG :: AC. AD. *c* ex æquali igitur, AE. AG :: AB. AD. *d* ergo Pgra. EG, BD similia sunt. eodem modo HF, BD similia sunt. ergo, &c.

a 29. 1.
b 4 6.
c 22. 5.
d 1 def. 6

PROP. XXV.



Dato rectilineo ABEDC simile similiterque positum P, idemque alteri dato F æquale, constituere.

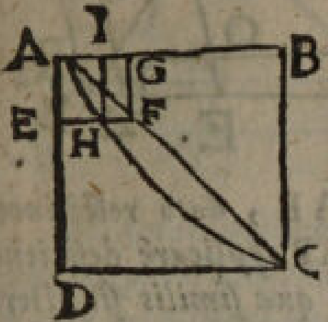
a Fac rectang. AL = ABEDC. *b* item super BL fac triang. BM = F. Inter AB, BH *c* inveni mediam proportionalem NO. super NO *d* fac

a 45. 1.
b 44. 1.
c 13. 6.

d fac polygonum P simile dato ABEDC. Erit hoc æquale dato F, d 18 6. e cor 20.6. f 1.6. g 14. 5. h constr.

Nam ABEDC (AL.) P :: e AB. BH f :: AL. BM. ergo P g = BM h = F. Q. E. F.

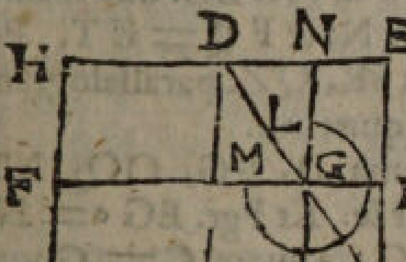
Prop. XXVI.



Si à parallelogrammo ABCD parallelogrammum AGFE ablatum sit, & simile toti, & similiter positum, communem cum eo habens angulum EAG, hoc circa eandem cum toto diametrum AC consistet.

Si negas AC esse communem diametrum, esto diameter AHC secans EF in H. & ducatur HI parall. AE. Parallelogramma EI, DB a similia sunt. b ergo AE. EH :: AD. DC c :: AE. EF. d proinde EH = EF. f Q. E. A. a 24 6. b 1. def. 6. c hyp. d 9. 5. f 9. ax.

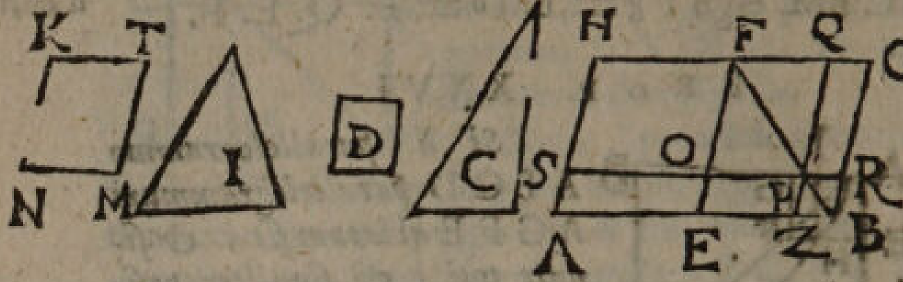
Prop. XXVII.



Omnium parallelogrammorum AD, AG secundum eandem rectam lineam AB applicatorum, deficientiumque figuris parallelogrammis CE, KI similibus, similiterque positis, ei AD, quod à dimidia describitur, maximum est AD, quod ad dimidium est applicatum, simile exsistens defectui KI.

Nam quia GE a = GC, addito communi KI, b erit KE = CI c = AM. adde commune CG, d erit AG = Gnom. MBL. sed Gnom. MBL e ⊐ CE (AD.) ergo AG ⊐ AD. Q. E. D. a 43. 1. b 2. ax. c 36. 1. d 2. ax. e 9. ax.

PROP. XXVIII.



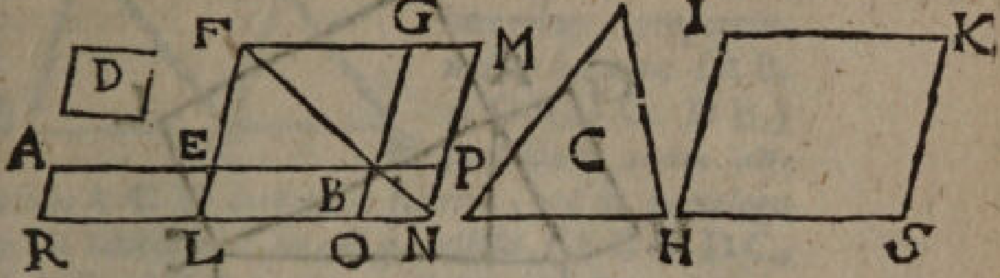
*Ad datam rectam lineam AB, dato rectilineo C æquale parallelogrammum AP applicare deficiens figura parallelogramma ZR, quæ similis sit alteri parallelogrammo dato D. * Oportet autem datum*
27.6. *rectilineum C, cui æquale AP applicandum est, non majus esse eo AF, quod ad dimidiam applicatur, similibus exsistentibus defectibus, & ejus AF quod ad dimidiam applicatur, & ejus D, cui simile deesse debet.*

a 28.6. Biseca AB in E. Super EB a fac Pgr. EG
b sch.45.1. simile dato D. b sitque EG = C + I. c fac pgr.
c 25.6. NT = I, & simile dato D, vel EG. duc diametrum FB. fac FO = KN; & FQ = KT. Per O, & Q duc parallelas SR, QZ. parallelogrammum AP est id quod quæritur.

Nam parallelogramma D, EG, OQ, NT,
d constr. & 24.6 ZR d sunt similia inter se. Et Pgr. EG e = NT
e constr. + C e = OQ + C; f quare C = Gnom.
f 3.ax. OBQ g = AO + PG h = AO + EP = AP.
g 2.ax. h 43.1. Q.E.F.

PROP.

PROP. XXIX.

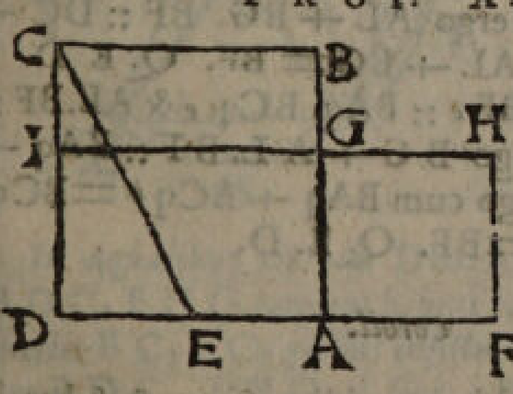


Ad datam rectam lineam AB, *dato rectilineo* C *æquale parallelogrammum* AN *applicare, excedens figura parallelogramma* OP, *quæ similis sit parallelogrammo alteri dato* D.

Biseca AB in E. super EB *a* fac Pgr. EG simile dato D. *b* sitque pgr. HK = EG + C, & simile dato D vel EG. fac FEL *c* = IH; *c* & FGM = IK. per LM duc parallelas RN, MN. & AR parall. NM. Produc ABP, GBO. Duc diametrum FBN. Pgr. AN est quæsitum. a 18. 6. b 25. 6. c 3. 1.

Nam parallelogramma D, HK, LM, EG *d* similia sunt. *e* ergo pgr. OP simile est pgro LM, vel D. item LM *f* = HK *f* = EG + C. *g* ergo C = Gnom. ENG. atqui AL *h* = IB *k* = BM. *l* ergo C = AN. Q. E. F. d constr. e 24. 6. f constr. g 3 ax. h 36. 1 k 43. 1. l 2. & 1. ax

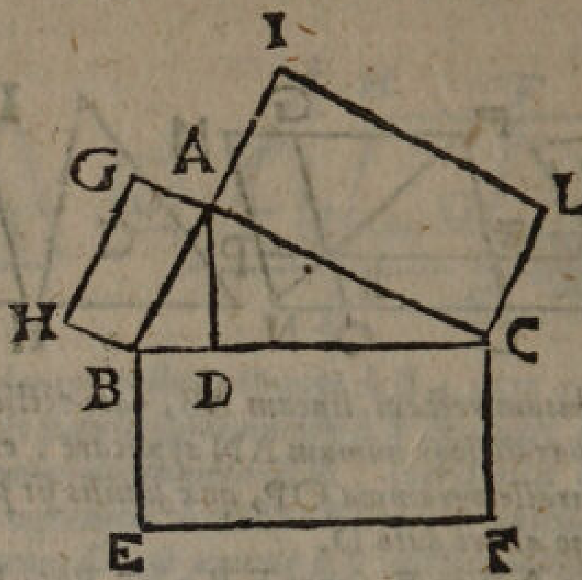
PROP. XXX.



Propositam rectam lineam terminatam AB, *extrema ac media ratione secare.* (AB. AG :: AG. GB.)

a Seca AB in G, ita ut AB x BG = AGq. *b* ergo BA. AG :: AG. GB. Q. E. F. a 11. 2. b 17. 6.

PROP. XXXI.



In rectangulis triangulis B A C, *figura quævis* B F *à latere* B C *rectam angulum* B A C *subtendente, descripta, æqualis est figuris* BG, AL, *quæ priori illi* B F *similes, & similiter positæ à lateribus* B A, A C *rectum angulum continentibus describuntur.*

Ab angulo recto B A C demitte perpendicu-
a cor. 8. 6 larem A D. Quoniam C B. C A *a* :: C A. D C.
b cor. 20. 6. *b* erit B F. A L :: C B. D C; inverseque A L.
BF :: DC. CB. Item quia BC, BA *a* :: BA. DB.
b erit BF. BG :: BC, DB; ac invertendo, BG.
c 24. 5. d schol. 24. 5. BF :: DB. BC. *c* ergo AL + BG. BF :: DC +
e 22. 6. DB. BC. *d* ergo AL + BG = BF. Q. E. D.

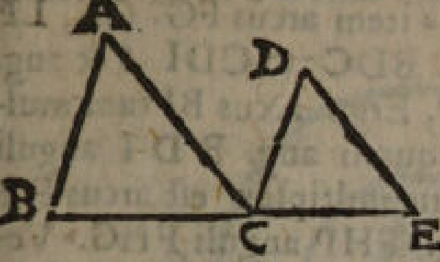
Vel sic. BG. BF *e* :: BAq. BCq. *e* & AL. BF ::
f 24. 5. ACq. BCq. *f* ergo BG + AL. BF :: BAq +
g sch. 24. 5. ACq. BCq. *g* ergo cum BAq + ACq *h* = BCq,
h 47. 1. *b* erit BG + AL = BF. Q. E. D.

Coroll.

Ex hac propositione, addi possunt, & subtrahi figuræ quævis similes, eadem methodo, qua quadrata adduntur & subtrahuntur, in schol. 47. 1.

PROP.

PROP. XXXII.

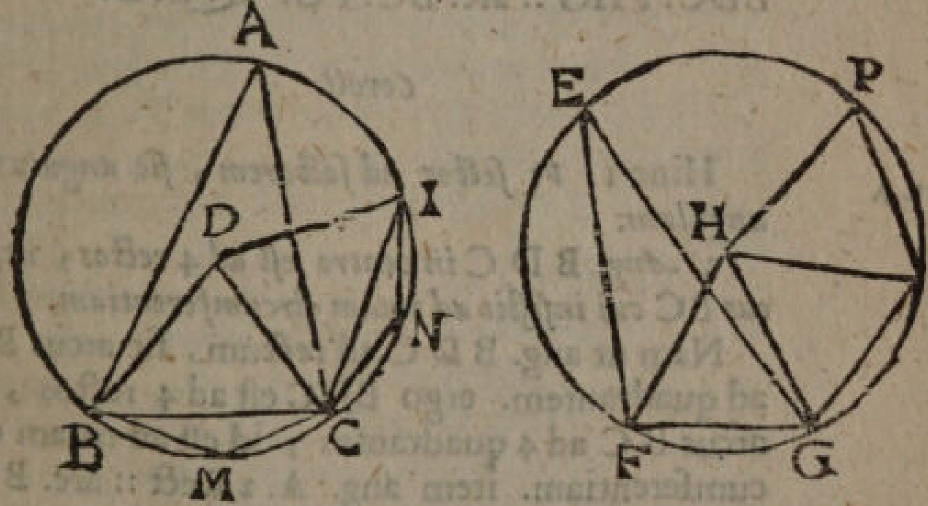


Si duo triangula ABC, DCE, *quæ duo latera duobus lateribus proportionalia habeant* (AB. AC :: DC. DE,) *secundum unum angulum* ACD *composita fuerint, ita ut homologa eorum latera sint etiam parallela* (AB ad DC, & AC ad DE) *tum reliqua illorum triangulorum latera* BC, CE *in rectam lineam collocata reperientur.*

Nam ang. A *a* = ACD *a* = D; & AB. AC *b* :: DC. DE. *c* ergo ang. B = DCE. ergo ang. B + A *d* = ACE. sed ang. B + A + ACB *e* = 2 Rect. *f* ergo ang. ACE + ACB = 2 Rect. *g* ergo BCE est recta linea. Q. E. D.

a 29. 1.
b hyp.
c 6. 6.
d 2. ax.
e 32. 1.
f 1. ax.
g 14. 1.

PROP. XXXIII.



In æqualibus circulis DBCA, HFGP, *anguli* BDC, FHG *eandem habent rationem cum peripheriis* BC, FG, *quibus insistunt; sive ad centra* (ut BDC, FHG,) *sive ad peripherias* A, E *constituti insistant: insuper vero & sectores* BDC, FHG, *quippe qui ad centra consistant.*

Duc

Duc rectas BC, FG. Accommoda CI=CB; & GL=FG=LP; & junge DI, HL, HP.

a 28. 3. b 27. 3. Arcus BC a=C I, a item arcus FG, GL, LP æquantur. b ergo ang. BDC=CDI b & ang. FGH=GHL=LHP. Ergo arcus BI tam multiplex est arcûs BC, quam ang. BDI anguli BDC. pariterque æquemultiplex est arcus FP arcûs FG, atque ang. FHP anguli FHG. Verum si arcus BI ⊏, =, ⊐ F P, c erit similiter ang. BDI ⊏, =, ⊐ FHP. ergo arc. BC. FG d:: ang. BDC. FHG e :: BDC/2 . FHG/2 f :: A.E.

c 27. 3. d 6. def 5. e 15. 5. f 20. 3.

Q. E. D.

g 27. 3. h 24. 3. k 4. 1. l 2 ax. Rursus ang. BMC g = CNI; h atque idcirco segm. BCM = CIN. k item triang. BDC = CDI. l ergo sector BDCM = CDIN. Simili ratione sectores FHG, GHL, LHP æquantur. Quum igitur prout arcus BI ⊏, =, ⊐ FGP, ita similiter sector BDI ⊏, =, ⊐ FHP. m erit sect. BDC. FHG :: arc. BC. FG. Q. E. D.

m 6. def. 5.

Coroll.

11. 5. Hinc 1. *Vt sector ad sectorem, sic angulus ad angulum.*

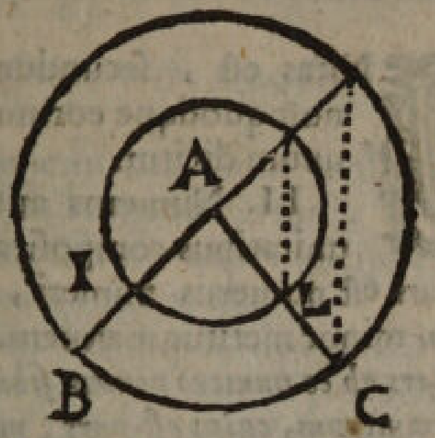
2. *Ang.* B D C *in centro est ad 4 rectos, ut arcus* BC *cui insistit ad totam circumferentiam.*

Nam ut ang. B D C ad rectum, sic arcus B C ad quadrantem. ergo BDC est ad 4 rectos, ut arcus B C ad 4 quadrantes, id est ad totam circumferentiam. item ang. A. 2 Rect :: arc. B C. periph.

Hinc 3. *Inæqualium circulorum arcus* IL, BC, *qui æquales subtendunt angulos, sive ad centra, ut* IAL & BAC, *sive ad peripheriam, sunt similes.*

Nam IL. periph. :: ang. IAL, (BAC.) 4 Rect. item arc. B C. periph :: ang. B A C. 4. Rect.

4. Rect. ergo IL. periph :: BC. periph. proinde arcus IL, & BC sunt similes. Unde



4. *Duæ semidiametri* AB, AC *à concentricis peripheriis arcus auferunt similes* IL, BC.

LIB.

LIB. VII.

Definitiones.

I. UNitas est, secundum quam unumquodque eorum quæ sunt, unum dicitur.

II. Numerus autem est, ex unitatibus composita multitudo.

III. Pars est numerus numeri, minor majoris, quum minor metitur majorem.

Omnis pars ab eo numero nomen sibi sumit, per quem ipsa numerum, cujus est pars, metitur; ut 4 dicitur tertia pars numeri 12, quia metitur 12 per 3.

IV. Partes autem, cum non metitur.

Partes quæcunque nomen accipiunt à duobus illis numeris, per quos maxima communis duorum numerorum mensura utrumque eorum metitur. ut 10 dicitur $\frac{2}{3}$ numeri 15, eo quod maxima communis mensura, nempe 5, metitur 10 per 2, & 15 per 3.

V. Multiplex vero major minoris, cum majorem metitur minor.

VI. Par numerus est, qui bifariam dividitur.

VII. Impar vero numerus, qui bifariam non dividitur; vel, qui unitate differt à pari.

VIII. Pariter par numerus est, quem par numerus metitur per numerum parem.

IX. Pariter autem impar est, quem par numerus metitur per numerum imparem.

X. Impariter vero impar numerus est, quem impar numerus metitur per numerum imparem.

XI. Primus numerus est, quem sola unitas metitur.

XII. Primi inter se numeri sunt, quos sola unitas, communis mensura, metitur.

XIII.

XIII. Compoſitus numerus eſt, quem numerus quiſpiam metitur.

XIV. Compoſiti autem inter ſe numeri ſunt, quos numerus aliquis communis menſura metitur.

In hac definitione & præcedenti unitas non eſt numerus.

XV. Numerus numerum multiplicare dicitur, cum toties compoſitus fuerit is qui multiplicatur, quot ſunt in ipſo multiplicante unitates, & procreatus fuerit aliquis.

Hinc, in omni multiplicatione unitas eſt ad multiplicatorem ut multiplicatus ad productum.

Nota, quod ſæpe cum multiplicandi ſunt quivis numeri, puta A *in* B, *literarum conjunctio productum denotat. Sic* AB = A *in* B. *item* CDE = C *in* D *in* E.

XVI. Cum autem duo numeri ſeſe multiplicantes aliquem fecerint, qui factus erit, planus appellabitur; Qui vero numeri ſeſe mutuo multiplicarint, latera illius dicentur. *Sic* 2 (C) in 3 (D) = 6 = C D *eſt numerus planus.*

XVII. Cum vero tres numeri mutuo ſeſe multiplicantes fecerint aliquem, qui procreatus erit, ſolidus appellabitur; Qui autem numeri mutuo ſeſe multiplicarint, latera illius dicentur. *Sic*, 2 (C) *in* 3 (D) *in* 5 (E) = 30 = CDE *eſt numerus ſolidus.*

XVIII. Quadratus numerus eſt, qui æqualiter æqualis, vel qui ſub duobus æqualibus numeris continetur. *Sit* A *latus quadrati; quadratus ſic notatur*, AA, *vel* Aq.

XIX. Cubus vero, qui æqualiter æqualis æqualiter, vel qui ſub tribus æqualibus numeris continetur. *Sit* A *latus cubi; cubus notatur ſic*, AAA, *vel* Ac.

In hac definitione, & tribus præcedentibus, unitas eſt numerus.

XX. Nu-

XX. Numeri proportionales sunt, cum primus secundi, & tertius quarti æquemultiplex est, vel eadem pars; vel deniq; cum pars primi secundum, & eadem pars tertii æque metitur quartum, vel vice versa. A. B :: C. D. hoc est, 3. 9 :: 5. 15.

XXI. Similes plani, & solidi numeri sunt, qui proportionalia habent latera.

Latera nempe non quælibet, sed quædam.

XXII. Perfectus numerus est, qui suis ipsius partibus est æqualis.

Ut 6. & 28. *Numerus vero qui suis ipsius partibus minor est, abundans appellatur: qui vero major, diminutus. ut* 12 *est abundans,* 15 *est diminutus.*

XXIII. Numerus numerum metiri dicitur per illum numerum, quem multiplicans, vel à quo multiplicatus, illum producit.

In divisione, unitas est ad quotientem, ut dividens ad divisum. Nota, quod numerus alteri lineola interjecta subscriptus divisionem denotat. Sic $\frac{A}{B}$ = A divis. per B. *item* $\frac{CA}{B}$ = C *in* A divis. per B.

Termini sive radices proportionis dicuntur duo numeri, quibus in eadem proportione minores sumi nequeunt.

Postulata.

1. Postuletur, cuilibet numero quotlibet sumi posse æquales, vel multiplices.

2. Quolibet numero sumi posse majorem.

3 Additio, subtractio, multiplicatio, divisio, extractionesque radicum, seu laterum, numerorum quadratorum, & cuborum concedantur etiam, tanquam possibilia.

Axio-

PROP. I.

A.....E..G.B 8 5 3
C...F..D $\frac{5}{3}$ $\frac{3}{2}$ $\frac{2}{1}$
H---

Si duobus numeris inæqualibus propositis (AB, CD) *detrahatur semper minor* CD *de majore* AB (& reliquus EB de CD &c.) *alterna quadam detractione, neque reliquus unquam præcedentem metiatur, quoad assumpta sit unitas* GB; *qui principio propositi sunt numeri* AB, CD *primi inter se erunt.*

Si negas, habeant AB, CD communem mensuram, numerum H. Ergo H metiens CD, *a* etiam AE metitur; proinde & reliquum FB; *a* ergo & CF, atque *b* idcirco reliquum FD; *a* quare & ipsum EG. sed totum EB metiebatur; *b* ergo & reliquum GB metitur, numerus unitatem. *c* Q. E. A.

a 11 ax. 7.
b 12. ax. 7.
c 9 ax. 1.

PROP. II.

9 6
A.........E.......B 15 9 6
6 3
C.......F...D $\frac{9}{6}$ $\frac{6}{3}$ $\frac{3}{0}$
G---

Duobus numeris datis AB, CD *non primis inter se, maximam eorum communem mensuram* FD *reperire.*

Detrahe minorem numerum CD ex majori AB, quoties potes. Si nihil relinquitur, *a* patet ipsum CD esse maximam communem mensuram. Si relinquitur aliquid EB, deme hunc ex CD; & reliquum FD ex EB, & sic deinceps, donec aliquis FD præcedentem EB metiatur. (nam *b* hoc fiet antequam ad unitatem perveniatur.) Erit FD maxima communis mensura.

a 6. ax 7.
b 1. 7.

Nam FD *c* metitur EB, *d* ideoque & CF; *e* proinde & totum CD; *d* ergo ipsum AE; atque idcirco totum AB metitur. Liquet igitur FD communem esse mensuram. Si maximam esse negas,

c constr.
d 11. ax 7.
e 12. ax. 7.

gas,

gas, sit major quæpiam G. ergo G metiens CD, *d* metitur A E, *e* & reliquum E B, *d* ipsumque C F. *e* proinde & reliquum F D, *g* major minorem. *h* Q. E. A. g *suppos.* h 9 *ax.* 1.

Coroll.

Hinc, numerus metiens duos numeros, metitur quoque maximam eorum communem mensuram.

PROP. III.

A 12	
B 8	
D 4	
C 6	
E .. 2	
F - - -	

Tribus numeris datis A, B, C non primis inter se, maximam eorum communem mensuram E reperire.

Inveni D maximam communem mensuram duorũ A, B. Si D metitur tertium C, liquet D maximam esse trium communem mensuram. Si D non metitur C, erunt saltem D, & C compositi inter se, ex coroll. præcedentis. Sit igitur ipsorum D, & C maxima communis mensura E. erit E is quem quæris.

Nam E *a* metitur C, & D; *a* ac D ipsos A, & B metitur; *b* ergo E metitur singulos A, B, C; nec major aliquis (F) eos metietur; nam si hoc affirmas, *c* ergo F metiens A, & B, eorum maximam communem mensuram D metitur. Eodem modo, F metiens D, & C, *c* eorum maximam communem mensuram E, *d* major minorem, metitur. *e* Q. E. A. a *constr.* b 11. *ax.* 7. c *cor.* 1. 7. d *suppos.* e 9 *ax* 1.

Coroll.

Hinc, numerus metiens tres numeros, maximam quoque eorum communem mensuram metitur.

PROP. IV.

A...... 6
B....... 7
B................. 18
B......... 9.

Omnis numerus A, omnis numeri B, minor majoris, aut pars est, aut partes.

a 4. def. 7. b 3. def. 7. c 4. def. 7.

SI A & B primi sint inter se, *a* erit A tot partes numeri B, quot sunt in A unitates. (ut $6 = \frac{6}{7} 7$.) Sin A metiatur B, *b* liquet A esse partem ipsius B. (ut $6 = \frac{1}{3} 18$.) denique si A & B aliter compositi inter se fuerint, *c* maxima communis mensura determinabit, quot partes A conficiat ipsius B; ut $6 = \frac{2}{3} 9$.

PROP. V.

A...... 6   D.... 4
6   6   4   4
B...... G...... C 12.   E.... H.... F 8

Si numerus A numeri B C pars fuerit, & alter D alterius E F eadem pars; & simul uterque (A + D) utriusque simul (B C + E F) eadem pars erit, quæ unus A unius B C.

a hyp. b constr. & 2. ax. 1. c 2. ax. 1.

Nam si B C in suas partes B G, G C ipsi A æquales; atque E F in suas partes F H, H F ipsi D æquales resolvantur; *a* erit numerus partium in B C æqualis numero partium in E F. Quum igitur A + D *b* = BG + EH = GC + HF, erit A + D toties in B C + E F, quoties A in B C. Q. E. D.

Vel sic brevius. Sit $a = \frac{x}{2}$ & $b = \frac{y}{2}$. *c* ergo

$$a + b = \frac{x}{2} + \frac{y}{2} = \frac{x + y}{2}.$$ Q. E. D.

PROP. VI.

3 3 4 4	
A ... G ... B 6. D .... H .... E 8	*Si numerus* AB
C .... 9 F 12	*numeri* C

partes fuerit; & alter DE *alterius* F *eædem partes; & simul uterq;* (AB + DE) *utriusq; simul* (C + F) *eædem partes erit, quæ unus* AB *unius* C.

Divide A B in suas partes A G, G B; & D E in suas D H, H E. Partium in utroque A B, D E æqualis est multitudo, ex hypoth. Quum igitur A G *a* sit eadem pars numeri C, quæ D H numeri F, *b* erit A G + D H eadem pars compositi C + F, quæ unus A G unius C. *b* Eodem modo G B + H E eadem pars est ejusdem C + F, quæ unus G B unius C; *c* ergo AB + DE eædem partes est ipsius C + F, quæ AB ipsius C. Q. E. D.

a hyp. b 5. 7. c 2. ax. 7.

Vel sic. Sit $a = \frac{2}{3}x$. & $b = \frac{2}{3}y$. *d* ergo $a + b = \frac{2}{3}x + \frac{2}{3}y = \frac{2}{3}y + {}^{3}x$. Q. E. ^{3}D.

a 1. ax. 1.

PROP. VII.

5 3	*Si numerus*
A ..... E ... B 8	A B *numeri*
6 10 6	C D *pars fuerit*,
G...... C .... F ...... D 16	*qualis ablatus* A E *ab-*

lati C F; *& reliquus* E B *reliqui* F D *eadem pars erit, qualis totus* AB *totius* CD.

a Sit E B eadem pars numeri G C, quæ A B ipsius CD, vel AE ipsius CF. *b* ergo AE + EB eadem est pars ipsius CF + GC, quæ AE ipsius CF, vel AB ipsius CD. *c* ergo GF = CD. aufer communem CF, *d* manet GC = FD. *e* ergo EB eadem est pars reliqui FD (GC) quæ totus AB totius CB. Q. E. D.

a 1. post. 7. b 5. 7. c 6 ax 1. d 3. ax. 1. e 2. ax 7.

Vel sic. Sit $a + b = x$, & $c + d = y$; atque tam $x = 3\,y$, quam $a = 3\,c$; dico $b = 3\,d$. Nam $3\,c + 3\,d$ *f* $= 3\,y = x$ *g* $= a + b$. aufer utrinq; $3\,c$ *g* $= a$, & *b* remanet $3\,d = b$. Q. E. D.

f 1. 2. g hyp.

PROP. VIII.

```
  6    2   4    2   2
A......H.. G.... E.. L.. B 16
       18          6
C.................. F...... D 24
```

Si numerus AB *numeri* CD *partes fuerit, quales ablatus* AE *ablati* CF ; *& reliquus* EB *reliqui* ED *eædem partes erit, quales totus* AB *totius* CD.

Seca AB in AG, GB partes numeri CD; item AE in AH, HE partes numeri CF; & sume GL = AH = HE; *a* quare HG = EL. & quia *b* AG = GB, *c* etiam HG = LB. Cum igitur totus AG eadem sit pars totius CD, quæ ablatus AH ablati CF; *d* erit reliquus HG, vel EL, eadem etiam pars reliqui FD, quæ AG ipsius CD. Eodem pacto, quia GB eadem pars est totius CD, quæ HE, vel GL, ipsius CF, *d* erit reliquus LB eadem pars reliqui FD, quæ GB totius CD; ergo EL + LB (EB) eædem est partes reliqui FD, quæ totus AB totius CD. Q. E. D.

a 3. ax. 1.
b constr.
c 3. ax. 1.
d 7. 7.

Vel sic facilius. Sit $a + b = x$. & $c + d = y$. Item tam $y = \frac{2}{3} x$, quam $c = \frac{2}{3} a$; vel *e* quod idem est, $3y = 2x$; & $3c = 2a$. Dico $d = \frac{2}{3} b$. Nam $3c + 3d$ *f* $= 3y = 2x$ *f* $= 2a + 2b$. *g* ergo $3c + 3d = 2a + 2b$. aufer utrinque $3c$ *h* $= 2a$; & *k* manet $3d = 2b$. *l* ergo $d = \frac{2}{3} b$. Q. E. D.

e 9. ax. 7.
f 1. 2.
g 1. ax. 1.
h hyp.
k 3 ax. 1.
l 8. ax. 7.

PROP. IX.

```
      A .... 4
   4      4
B .... G .... C 8
    5 D ..... 5
E ..... H ..... F 10
```

Si numerus A *numeri* BC *pars fuerit, & alter* D *alterius* EF *eadem pars; & vicissim quæ pars est, aut partes primus* A *tertii* D, *eadem pars erit, vel eædem partes, & secundus* BC *quarti* EF.

Poni-

Ponitur A ⊐ D. Sint igitur B G, G C, & E H, H F partes numerorum B C, E F, hæ ipsi A, illæ ipsi D pares. Utrinque multitudo partium æqualis ponitur. Liquet vero B G a eandem esse partem, aut easdem partes ipsius E H, quæ G C ipsius H F; b quare B C (B G + G C) ipsius E F (E H + H F) eadem pars est aut partes, quæ unus B G (A) unius E H (D.) Q. E. D. a 1, ax. 7. & 4 7. b 5, vel 6. 7.

Vel sic; Sit $a = \frac{b}{3}$ & $c = \frac{d}{3}$. dico

$\frac{c}{a} = \frac{d}{b}$. Nam $\frac{c}{a}$ a $= \frac{3d}{3b} = \frac{d}{b}$. a 1, ax. 7.

P R O P. X.

A .. G .. B 4
C...... 6
5 5
D .... H ..... E 10
F.............. 15

Si numerus A B numeri C partes fuerit, & alter D E alterius F eædem partes; & vicissim quæ partes est primus A B tertii D E, aut pars, eædem partes erit & secundus C quarti F, aut pars.

Ponitur A B ⊐ D E, & C ⊐ F. Sint A G, G B, & D H, H E partes numerorum C, & F, tot nempe in A B, quot in D E. Constat A G ipsius C eandem esse partem, quæ D H ipsius F. a quare vicissim A G ipsius D H, pariterque G B ipsius H E, & b proinde conjunctim A B ipsius D E eadem pars erit, aut partes, quæ C ipsius F. Q. E. D. a 9. 7. b 5, & 9. 7.

Applicare potes secundam præcedentis demonstrationem etiam huic.

P R O P. XI.

4 3
A .... E ... B 7
8 6
C........ F...... D 14

Si fuerit, ut totus A B ad totum C D, ita ablatus A E ad ablatum C F; & reliquus E B ad reliquum

F D *erit*, *ut totus* A B *ad totum* C D.

a 4. 7. Sit primo A B ⊐ C D; *a* ergo A B vel pars
b 10, *def.* est, vel partes numeri C D; *b* eademque pars est,
c 7. vel 8. 7. vel partes ipse A E ipsius CF; *c* ergo reliquus EB reliqui F D eadem pars est, aut partes, quæ totus AB totius C D. *b* ergo A B. C D :: E B. F D. Sin fuerit A B ⊏ C D; eodem modo erit juxta modo ostensa, C D. A B :: F D. E B. ergo invertendo, AB. CD :: EB. FD.

PROP. XII.

A, 4. C, 2. E, 3.
B, 8. D, 4. F, 6.

Si sint quotcunque numeri proportionales (A. B :: C. D :: E. F) *erit quemadmodum unus antecedentium* A *ad unum consequentium* B, *ita omnes antecedentes* (A + C + E) *ad omnes consequentes* (B + D + F.)

Sint primo, A, C, E minores quam B, D, F.
a 20. *def.* 7. ergo (propter easdem rationes) *a* erit A eadem
b 5, & 6. 7. pars aut partes ipsius B, quæ C ipsius D. *b* ergo conjunctim A + C eadem erit pars aut partes ipsius B + D, quæ unus A unius B. Similiter A + C + E eadem pars est, aut partes *ipsius
c 20. *def.* 7. B + D + F, quæ A ipsius B. *c* ergo A + C + E. B + D + F :: A. B. Q. E. D. Sin A, C, E, ipsis B, D, F majores ponantur, idem ostendetur invertendo.

PROP. XIII.

A, 3. C, 4.
B, 5. D, 12.

Si quatuor numeri proportionales sint (A. B :: C. D. *& vicissim proportionales erunt* (A. C :: B. D.)

Sint primo A & C ipsis B & D minores,
a 10. *def.* 7. atque A ⊐ C. Ob eandem proportionem, *a* erit A eadem pars, aut partes ipsius B, quæ C ipsius
b 9. & 10. 7. D. *b* ergo vicissim A ipsius C eadem pars est, aut partes, quæ B ipsius D. ergo A. C :: B. D. Sin

A ⊏

A ⊏ C; atque A & C majores statuantur, quam B & D, eadem res erit, proportiones invertendo.

PROP. XIV.

A, 9. D, 6.
B, 6. E, 4.
C, 3. F, 2.

Si sint quotcunque numeri A, B, C, *& alii totidem* D, E, F *illis æquales multitudine, qui bini sumantur, & in eadem ratione* (A. B :: D. E. & B. C :: E. F) *etiam ex æqualitate in eadem ratione erunt.* (A. C :: D F.)

Nam quia A. B :: D. E, *a* erit vicissim, A. D :: B. E :: *a* C. F. *a* ergo iterum permutando, A. C :: D. F. Q. E. D. a 13. 7.

PROP. XV.

1. D.
B ... 3. E ...... 6.

Si unitas numerum quempiam B *metiatur; æque autem alter numerus* D *alterum quendam numerum* E *metiatur; & vicissim æque unitas tertium numerum* D *metietur, & secundus* B *quartum* E.

Nam quia 1 est eadem pars ipsius B, quæ D ipsius E, *a* erit vicissim 1 eadem pars ipsius D, quæ B ipsius E. Q. E. D. a 9. 7.

PROP. XVI.

B, 4. A, 3.
A, 3. B, 4.
AB, 12. BA, 12.

Si duo numeri A, B *sese mutuo multiplicantes fecerint aliquos* AB, BA, *geniti ex ipsis* AB, BA *æquales inter se erunt.*

Nam quia AB = A in B, *a* erit 1 in A toties, quoties B in AB. *b* ergo vicissim 1 in B toties erit, quoties A in AB. atqui quoniam BA = B in A, *a* erit 1 in B toties, quoties A in BA. ergo quoties 1 in AB, toties 1 in BA; & *c* proinde AB = BA. Q. E. D. a 15. def. 7. b 15. 7. c 4. ax. 7.

PROP. XVII.

A, 3.
B, 2. C, 4.
AB, 6. AC, 12.

Si numerus A *duos numeros* B, C *multiplicans fecerit aliquos* AB, AC; *geniti ex ipsis eandem rationem habebunt, quam multiplicati.* (A B. A C :: B. C.)

a 15.def.7. Nam quia AB = A in B, *a* erit 1 toties in A, quoties B in AB. *a* item quia AC = A in C, erit 1 toties in A, quoties C in A C. ergo quoties B in A B, toties C in A C. quare B. A B ::
b 20.def.7.
c 13.7. C. A C. *c* ergo viciſſim, B. C :: A B. A C. Q. E. D.

PROP. XVIII.

C, 5. C, 5.
A, 3. B, 9.
$\overline{AC}$, 15. $\overline{BC}$, 45.

Si duo numeri A, B, *numerum quempiam* C *multiplicantes fecerint aliquos* A C, B C; *geniti ex ipsis eandem rationem habebunt, quam multiplicantes.* (A. B :: AC. BC.)

a 16.7. Nam AC *a* = CA; & BC *a* = CB; ſic idem C multiplicans A & B producit A C, & B C.
b 17.7. *b* ergo A. B :: AC. BC. Q. E. D.

Schol.

Ex his pendet modus vulgaris reducendi fractiones ($\frac{3}{5}$, $\frac{7}{9}$) ad eandem denominationem. Nam duc 9 tam in 3, quam in 5, proveniunt $\frac{27}{45} = \frac{3}{5}$. quoniam ex his, 3. 5 :: 27. 45. item duc 5 in 7, & 9, prodeunt $\frac{35}{45} = \frac{7}{9}$. quia 7. 9 :: 35. 45.

PROP. XIX.

A, 4. B, 6. C, 8. D, 12.
A D, 48. BC, 48.

Si quatuor numeri proportionales fuerint, (A B :: C. D;) *qui ex primo & quarto fit numerus* A D, *æqualis eſt ei, qui ex ſecundo & tertio fit, numero* BC.

BC. *Et si qui ex primo & quarto fit numerus* AD, *æqualis sit ei, qui ex secundo & tertio fit, numero* B C, *ipsi quatuor numeri proportionales erunt.* (A. B :: C. D.) a 17. 7.

1. *Hyp.* Nam A C. A D[a] :: C. D[b] :: A. B[c] :: AC. BC. [d] ergo AD = BC. Q. E. D. b hyp. c 18. 7. d 9. 5.

2. *Hyp.* Quoniam [e] A D = B C, erit A C. AD[f] :: A C. B C. Sed A C. A D[g] :: C. D. & AC. BC[h] :: A.B. [k] ergo C.D. :: A.B. Q. E. D. e hyp. f 7. 5. g 17. 7. h 18. 7. k 11. 5.

Prop. XX.

A.	B.	C.
4.	6.	9.
AC, 36.	BB, 36.	
	D, 6.	

Si tres numeri proportionales fuerint (A. B :: B. C.) *qui sub extremis continetur* (A C) *æqualis est ei, qui à medio efficitur* (BB.) *Et si qui sub extremis continetur* (AC) *æqualis fuerit ei* (Bq) *qui sub medio, ipsi tres numeri proportionales erunt* $(\frac{A}{B} :: \frac{B}{C}.)$

1. *Hyp.* Nam sume D = B. [a] ergo A. B :: D (B.) C. [b] quare A C = B D, [a] vel B B. Q. E. D. a 1. ax. 7. b 19. 7.

2. *Hyp.* Quia AC[c] = BD, [d] erit A. B :: D (B.) C. Q. E. D. c hyp. d 19. 7.

Prop. XXI.

A ... G .. B 5. E .... 10.
C .. H. D 3. F .... 6.

Numeri A B, C D *minimi omnium eandem cum eis rationem habentium* (E, F) *metiuntur æque numeros* E, F *eandem cum eis rationem habentes, major quidem* A B *majorem* E, *minor vero* C D *minorem* F.

Nam A B. C D[a] :: E. F. [b] ergo vicissim A B. E :: C D. F. [c] ergo A B eadem pars est, vel partes ipsius E, quæ C D ipsius F. Non partes; nam si ita, sint A G, G B partes numeri E; & CH, H D partes numeri F. [e] ergo A G. E :: a hyp. b 13. 7. c 10. def. 7.

CH.

d 13. 7. e hyp. C H. F; & permutando, A G. C H d :: E. F e :: AB. CD. ergo AB, CD non sunt minimi in sua ratione, contra hypoth. ergo, &c.

PROP. XXII.

A, 4. D, 12.
B, 3. E, 8.
C, 2. F, 6.

Si fuerint tres numeri A, B, C, & alii ipsis multitudine æquales D, E, F, qui bini sumantur, & in eadem ratione; fuerit autem perturbata eorum proportio (A.B :: E.F & B.C :: D.E;) *etiam ex æqualitate in eadem ratione erunt* (A.C :: D.F.)

a hyp. b 19. 7. c 1. ax. 1. d 19 7. Nam quia A. B a :: E. F, erit A F = B E; & quia B. C :: a D. E, b erit B E = CD. c ergo AF = CD. d quare A. C :: D. F. Q. E. D.

PROP. XXIII.

A, 9. B, 4.
C - - - - D - - -
E - -

Primi inter se numeri A, B, minimi sunt omnium eandem cum eis rationem habentium.

Si fieri potest, sint C & D minores quam A & B, atque in eadem ratione. a 21. 7. a ergo C metitur A æque, ac D metitur B, puta per eundem numerum E: quoties igitur 1 in E, b 13 def 7. b toties erit C in A. c 15. 7. c quare vicissim quoties 1 in C, toties E in A. simili discursu quoties 1 in D, toties E in B. ergo E utrumque A & B metitur; qui proinde inter se primi non sunt, contra Hypoth.

PROP. XXIV.

A, 9. B, 4.
C - - -
D - - - E - -

Numeri A, B, minimi omnium eandem cum eis rationem habentium, primi inter se sunt.

Si fieri potest, habeant A & B communem mensuram C; is metiatur A a 9. ax 7. b 17. 7. per D, & B per E; a ergo CD = A, b & CE = B.

b quare

b quare A. B :: D. E. Sed D & E minores sunt quam A & B, utpote eorum partes. Ergo A & B non sunt minimi in sua ratione, contra hypoth. b 17. 7.

PROP. XXV.

A, 9. B, 4.
C, 3. D--

Si duo numeri A, B primi inter se fuerint, qui unum eorum A metitur numerus C, ad reliquum B primus erit.

Nam si affirmes aliquem D numeros B & C metiri, a ergo D metiens C, metitur A. ergo A & B non sunt primi inter se, contra Hypoth. a 11. ax. 7.

PROP. XXVI.

A, 5. C, 8.
B, 3.
AB, 15. E----
F----

Si duo numeri A, B ad quempiam C primi fuerint, etiam ex illis genitus AB ad eundem C primus erit.

Si fieri potest, sit ipsorum AB, & C communis mensura numerus E. sitque $\frac{AB}{E} = F$; a ergo $AB = EF$; b quare E. A :: B. F. Quia vero A primus est ad C quem E metitur, c erunt E & A primi inter se; d adeoque in sua proportione minimi, & e proinde æque metiuntur B, & F; nempe E ipsum B, & A ipsum F. Quum igitur E utrumque B, C metiatur, non erunt illi primi inter se, contra Hypoth. a 9. ax. 7. b 19. 7. c 25. 7. d 23. 7. e 21. 7.

PROP. XXVII.

A, 4. B, 5.
Aq, 16.
D, 4.

Si duo numeri, A, B, primi inter se fuerint, etiam ex uno eorum genitus (Aq) ad reliquum B primus erit.

Sume $D = A$; ergo a singuli D, & A primi sunt ad B. b quare AD, vel Aq, ad B primus est. Q. E. D. a 1. ax. 7. b 26. 7.

PROP. XXVIII.

A, 5. C, 4.
B, 3. D, 2.
$\overline{AB}$, 15. $\overline{CD}$, 8.

Si duo numeri A, B ad duos numeros C, D, uterque ad utrumque, primi fuerint, & qui ex eis gignentur AB, CD, primi inter se erunt.

a 16. 7. Nam quia A & B ad C primi sunt, *a* erit AB ad C primus. Eadem ratione erit AB ad D
b 26. 7. primus. *b* ergo AB ad CD primus est. Q.E.D.

PROP. XXIX.

A, 3. B, 2.
Aq, 9. Bq, 4.
Ac, 27. Bc, 8.

Si duo numeri A, B primi inter se fuerint, & multiplicans uterque seipsum fecerit aliquem (Aq, & Bq;) & geniti ex ipsis (Aq, Bq) primi inter se erunt; & si qui in principio A, B genitos ipsos Aq, Bq multiplicantes fecerint aliquos (Ac, Bc;) & hi primi inter se erunt: & semper circa extremos hoc eveniet.

a 27. 7. Nam quia A primus est ad B, *a* erit Aq ad B primus. & quia Aq primus ad B, *a* erit Aq ad Bq primus. Rursus quia tam A ad B, & Bq;
b 28. 7. quam Aq ad eosdem B, & Bq primi sunt, *b* erit A x Aq, id est Ac, ad B x Bq, id est Bc, primus. Et sic porro de reliquis.

PROP. XXX.

8 5
A .... B ..... C 13. D - - - -

Si duo numeri AB, BC primi inter se fuerint, etiam uterque simul (AC) ad quemlibet illorum AB, BC primus erit. Et si uterque simul AC ad unum aliquem illorum AB primus fuerit, etiam qui in principio numeri AB, BC primi inter se erunt.

1. *Hyp.* Nam si AC, AB compositos velis,
a 12. ax 7. sit D communis mensura. *a* Is metietur reliquum BC. ergo AB, BC non sunt primi inter se, contra Hypoth.

2. *Hyp.*

2. *Hyp.* Positis AC, AB inter se primis, vis D ipsorum AB, BC communem esse mensuram. b Is igitur totum AC metitur. quare AC, AB non sunt primi inter se, contra Hypoth. b 10.ax.7.

Coroll.

Hinc numerus, qui ex duobus compositus, ad unum illorum primus est, ad reliquum quoque primus est.

PROP. XXXI.

A 5, B,8. *Omnis primus numerus A ad omnem numerum B, quem non metitur, primus est.*

Nam si communis aliqua mensura metiatur utrumque A, B, a non erit A primus numerus, contra Hypoth. a 11.def.7.

PROP. XXXII.

A, 4. D, 3.
B, 6. E, 8.
$\overline{AB}$, 24.

Si duo numeri A, B, se mutuo multiplicantes fecerint aliquem AB; genitum autem ex ipsis AB metiatur aliquis primus numerus D; is etiam unum eorum, qui à principio, A, vel B metietur.

Pone numerum D non metiri A; sit vero $\frac{AB}{D} = E$. a ergo AB = DE. b quare D. A :: B. E. c est vero D ad A primus. d ergo D, & A minimi sunt in sua ratione; e proinde D metitur B, æque ac A metitur E. liquet igitur propositum. a 9.ax.7. b 19.7. c hyp. & 31.7 d 23.7. e 21.7.

PROP. XXXIII.

A, 12. B, 2. *Omnem compositum numerum A, aliquis primus numerus B metitur.*

Unus vel plures numeri a metiantur A, quorum minimus sit B. is primus erit. a 13.def.7.

nam

a 13. def. 7. b 11. ax. 7. nam si dicetur compositus, [a] eum minor aliquis metietur, [b] qui proinde ipsum A metietur; quare B non est mininus eorum, qui A metiuntur; contra Hypoth.

PROP. XXXIV.

Omnis numerus A, *aut primus est, aut*
A, 9. *eum aliquis primus metitur.*

Nam A necessario vel primus est, vel compositus. Si primus, hoc est quod asserimus. a 33. 7. Si compositus, [a] ergo eum aliquis primus metitur. Q. E. D.

PROP. XXXV.

A, 6. B, 4. C, 8.
D, 2.
E, 3. F, 2. G, 4.

H - - I - - K - - - -
L - - -

Numeris datis quotcunque A, B, C *reperire minimos omnium* E, F, G *eandem rationem cum eis habentium.*

Si A, B, C primi sint inter se, ipsi in sua ratione a 23. 7. minimi [a] erunt. Si compositi sint, [b] esto b 3. 7. eorum maxima communis mensura D, qui ipsos metiatur per E, F, G. Hi minimi erunt in ratione A, B, C.

c 9. ax. 7. Nam D ductus in E, F, G [c] producit A B C. d 17. 7. [d] ergo hi & illi in eadem sunt ratione. Iam puta, e 21. 7. alios H, I, K minimos esse in eadem; [e] qui propterea æque metientur A, B, C nempe per numerum I. [f] ergo L in H, I, K ipsos A, B, C f 9 ax. 7. procreabit. [g] ergo ED = A = HL. [h] unde E. g 1. ax. 1. H :: L. D. Sed E [k] ⊏ H; [l] ergo L ⊏ D. ergo h 19. 7. k suppos. l 20. def. 7. D non est maxima communis mensura ipsorum A, B, C; contra Hypoth.

Coroll.

Hinc, maxima communis mensura quotlibet nume-

numerorum metitur ipsos per numeros, qui minimi sunt omnium eandem rationem cum ipsis habentium. Ex quo patet methodus vulgaris reducendi fractiones ad minimos terminos.

PROP. XXXVI.

Duobus numeris datis A, B, reperire, quem illi minimum metiuntur, numerum.

A, 5. B, 4.
AB, 20.
D - - - - - -
E - - - F - - -

1. *Cas.* Si A, & B primi sint inter se, est AB quæsitus. Nam liquet A & B metiri AB. Si fieri potest, metiantur A & B aliquem $D \sqsupset AB$; a 9 ax 7. & 1. ax. 1.
puta per E, & F. *a* ergo $AE = D = BF$. *b* quare b 19.7.
A. B :: F. E. Quia vero A, & B *c* primi sunt c hyp.
inter se, *d* adeoque in sua ratione minimi, *e* æque d 23.7.
metientur A ipsum F, ac B ipsum E. Atqui e 21.7. f 17.7.
B. E *f* :: AB. AE (D.) *g* ergo AB etiam me- g 20. def 7.
tietur D, seipso minorem. Q. E. A.

A, 6. B, 4. F - - - -
C, 3. D, 2. G - - - H - - -
AD, 12.

2. *Cas.* Sin A, & B inter se compositi fuerint, *h* reperiantur C, & D minimi in eadem ratione. *k* ergo h 35.7. k 19.7.
$AD = BC$. Erit AD, vel BC quæsitus. l 7. ax. 7.

Nam *l* liquet B, & A ipsum AD, vel BC metiri. Puta A, & B metiri $F \sqsupset AD$, nempe
A per G, & B per H. *m* ergo $AG = F = BH$. m 9. ax 7.
n unde A. B :: H. G *o* :: C. D. *p* proinde æque n 19.7. o constr.
metitur C ipsum H, ac D ipsum G. atqui D. G p 21.7. q 17.7.
q :: AD. AG (F.) ergo AD *r* metitur F, major r 20 def 7.
minorem. Q. E. A.

Coroll.

Hinc, si duo numeri multiplicent minimos eandem rationem habentes, major minorem, & minor majorem, producetur numerus minimus, quem illi metiuntur.

PROP. XXXVII.

A, 2. B, 3.
E, ... 6.
C - - - - F - - - D

Si duo numeri A, B numerum quempiam CD metiantur; etiam minimus E, quem illi metiuntur, eundem CD metietur.

Si negas, aufer E ex CD, quoties fieri potest, & relinquatur FD ⊐ E. quum igitur A & B a metiantur E, b & E ipsum CF, c etiam A, & B metiuntur CF; a metiuntur autem totum CD; d ergo etiam reliquum FD metiuntur. ergo E non est minimus, quem A, & B metiuntur, contra hyp.

a hyp.
b constr.
c 11. ax. 7.
d 12. ax. 7.

PROP. XXXVIII.

A, 3, B, 4, C, 6.
D, 12.

Tribus numeris datis A, B, C, reperire minimum, quem illi metiuntur.

a Reperi D minimum, quem duo A, & B metiuntur; quem si tertius C metiatur, patet D esse quæsitum. Quod si C non metiatur D, sit E minimus, quem C, & D metiuntur. Erit E requisitus.

a 36. 7.

A, 2. B, 3. C, 4.
D, 6. E, 12.
F. - -

Nam singulos A, B, C metiri E constat ex 11. ax. 7. Quod vero nullum alium F minorem metiantur, facile ostenditur. Nam si affirmas, b ergo D metitur F; b proinde E eundem F metitur, major minorem. Quod est absurdum.

b 37. 7.

Coroll.

Hinc, si tres numeri numerum quempiam metiantur; etiam minimus, quem illi metiuntur, eundem metietur.

PROP.

PROP. XXXIX.

A, 12. B, 4, C, 3.

Si numerum A quispiam numerus B metiatur, ille A quem B metitur, partem habebit C, à metiente B denominatam.

Nam quia $\frac{A}{B}$ [a] = C, [b] erit A = BC. [c] ergo $\frac{A}{C}$ = B. Q.E.D. a hyp. b 9.ax.7. c 7.ax.7.

PROP. XL.

A, 15. B, 3. C, 5.

Si numerus A partem habuerit quamlibet B, metietur illum numerus C, à quo ipsa pars B denominatur.

Nam quia BC [a] = A, [b] erit $\frac{A}{C}$ = B. Q.E.D. a hyp. & 9.ax.7. b 7.ax.7.

PROP. XLI.

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ G, 12. H ---

Numerum reperire G; qui minimus cum sit, habeat datas partes, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.

[a] Inveniatur G minimus, quem denominatores 2, 3, 4 metiuntur. [b] Liquet G habere partes, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. Si fieri potest, H $\sqsubset$ G habeat easdem partes; [c] ergo 2, 3, 4 metiuntur H, & proinde G non est minimus, quem 2, 3, 4 metiuntur. contra constr. a 38.7. b 39.7. c 40.7.

Prop. I.

A, 8. B, 12. C, 18. D, 27.
E - F - - G - - - H - - - -

Si fuerint quotcunque numeri deinceps proportionales A, B, C, D; extremi vero ipsorum A, D primi inter se fuerint; ipsi A, B, C, D minimi sunt omnium eandem cum eis rationem habentium.

Nam, si fieri potest, sint alii totidem E, F, G, H
a 14. 7. minores in illa ratione. *a* ergo ex æquali A. D ::
b 23. 7. E. H. ergo A, & D primi numeri, *b* adeoque in
c 21. 7. sua ratione minimi, *c* æque metiuntur E, & H, seipsis minores. Q. E. A.

Prop. II.

1.
A, 2. B, 3.
Aq, 4. AB, 6. Bq, 9.
Ac, 8. AqB, 12. ABq, 18. Bc, 27.

Numeros reperire deinceps proportionales minimos, quotcunque jusserit quispiam, in data ratione A ad B.

Sint A, & B minimi in data ratione. Erunt Aq, AB, Bq tres minimi deinceps in ratione A ad B.

a 17. 7. Nam AA. AB *a* :: A. B *a* :: AB. BB. item
b 24. 7. quia A, & B *b* primi sunt inter se, *c* erunt
c 29. 7. Aq, Bq inter se primi; *d* proinde Aq, AB, Bq
d 1. 8. sunt ∺ minimi in ratione A ad B.

Dico porro, Ac, AqB, ABq, Bc in ratione A ad B quatuor esse minimos. Nam AqA:
e 17. 7. AqB *e* :: A. B *e* :: ABA (AqB.) ABB. *e* atque A. B :: ABq. BBq. (Bc) Quum igitur Ac, & Bc

c f inter se primi sint, g erunt Ac, AqB, f 19.7.
ABq, Bc quatuor ∺ minimi in ratione A ad B. g 1.8.
odem modo quotvis proportionales investiga-
is. Q. E. F.

Coroll.

1. Hinc, si tres numeri minimi sunt propor-
onales, extremi quadrati erunt; si quatuor,
ubi.

2. Extremi quotcunque proportionales per
anc propos. inventi in data ratione minimi, in-
er se primi sunt.

3. Duo numeri, minimi in data ratione, me-
untur omnes medios quotcunque minimorum
n eadem ratione; quia scilicet producuntur ex
lorum multiplicatione in alios quosdam nu-
neros.

4. Hinc etiam liquet ex constructione, series
umerorum 1, A, Aq, Ac; 1, B, Bq, Bc; Ac,
AqB, ABq, Bc, constare æquali multitudine
umerorum; ac proinde extremos numeros
uotcunque minimorum continue proportiona-
ium, esse ultimos totidem continue proportio-
nalium ab unitate. ut extremi Ac, Bc continue
roportionalium Ac, AqB, ABq, Bc, sunt ultimi
otidem proportionalium ab unitate 1, A, Aq,
Ac; & 1, B, Bq, Bc.

5. 1, A, Aq, Ac; & B, BA, BAq; ac Bq, ABq
unt ∺ in ratione 1 ad A. item, B, Bq, Bc; &
A, AB, ABq; ac Aq, AqB sunt ∺ in ratione
ad B.

Prop. III.

A, 8. B, 12. C, 18. D, 28. *Si sint quot-cunque numeri A, B, C, D deinceps proportionales, minimi omni-um eandem cum eis rationem habentium; illorum ex-tremi A, D sunt inter se primi.*

 Nam

a 2. 8. Nam si *a* inveniantur totidem numeri minimi in ratione A ad B, illi non alii erunt, quam A, B, C, D; ergo juxta 2. coroll. præcedentis extremi A & D primi sunt inter se. Q. E. D.

PROP. IV.

A, 6. B, 5. C, 4. D, 3.
H, 4. F, 24. E, 20. G, 15.
I - - K - - L - - -

Rationibus datis quotcunque in minimis terminis, (A ad B, & C ad D) *reperire numeros deinceps minimos in datis rationibus.*

a 36. 7. *a* Reperi E minimum, quem B, & C metiuntur;
b 3. post 7. & B ipsum E *b* æque metiatur, ac A alterum F, puta per eundem H. *b* item C ipsum E, ac D alterum G æque metiantur: erunt F, E, G minimi in datis rationibus. Nam A H *c* = F; &
c 9 ax 7. d 18. 7. B H *c* = E. *d* ergo A. B :: A H. B H *e* :: F. E.
e 7. 5 Similiter C. D :: E. G. sunt igitur F, E, G deinceps proportionales in datis rationibus. Imo minimi sunt in iisdem: nam puta alios I, K, L minimos esse. *f* ergo A & B ipsos I & K,
f 21. 7. *f* pariterque C & D ipsos K & L æque metiuntur.
g 37. 7. ergo B, & C eundem K metiuntur. *g* Quare etiam E eundem K metitur, seipso minorem. Q. E. A.

A, 6. B, 5. C, 4. D, 3. E, 5. F, 7.
H, 24. G, 20. I, 15. K, 21.

Datis vero tribus rationibus A ad B, & C ad D, ac E ad F. reperi, ut prius, tres H, G, I minimos deinceps in rationibus A ad B, & C ad D. tunc si E numerum I metiatur,

h 3. post 7. *h* Sume alterum K, quem F æque metiatur; erunt quatuor H, G, I, K, deinceps minimi, in datis rationibus; quod non aliter probabimus, quam in priori parte.

A, 6.

A, 6. B, 5. C, 4. D, 3. E, 2. F, 7.
H, 24. G, 29. I, 15.
M, 48. L, 40. K, 30. N, 105.

Sin E non metiatur I, sit K minimus, quem E, & I metiuntur; & quoties I ipsum K, toties G ipsum L, & H ipsum M metiatur. quoties vero E ipsum K, toties F ipsum N metiatur. Erunt M, L. K, N minimi deinceps in datis rationibus; quod demonstrabimus, ut prius.

PROP. V.

C, 4. E, 3.
D, 6 F, 16 ED, 18.
CD, 24 EF, 48.

Plani numeri CD, EF rationem habent ex lateribus compositam.

$(\frac{CD}{EF} = \frac{C}{E} + \frac{D}{F})$

Nam quia CD. ED *a* :: C. E; *a* & ED. EF :: D. F. atque $\frac{CD}{EF}$ *b* $= \frac{CD}{ED} + \frac{ED}{EF}$, *c* erit ratio $\frac{CD}{EF} = \frac{C}{E} + \frac{D}{F}$. Q. E. D.

a 17. 7.
b 20. def. 5.
c 11. 5.

PROP. VI.

A, 16. B, 24. C, 36. D, 54. E, 81.
F, 4. G, 6. H, 9.

Si sint quotcunque numeri deinceps proportionales A, B, C, D, E; *primus autem* A *secundum* B *non metiatur, neque alius quispiam ullum metietur.*

Quoniam A non metitur B, *a* neque quilibet proxime sequentem metietur; quia A. B :: B. C :: C. D, &c. *b* Accipe tres F, G, H minimos in ratione A ad B. quoniam igitur A non metitur B, *a* neque F metietur G. *c* ergo F non est unitas. sed F, & H inter se primi sunt; ergo quum *e* sit ex æquo A. C :: F. H, & F non metiatur H, *d* neque A ipsum C metietur; proinde nec B ipsum D, nec C ipsum E, &c. quia A. C *e* :: B. D *e* :: C. E, &c. Eodem modo

a 20. def. 8.
b 35. 7.
c 5 ax. 7.
d 3. 3.
e 14. 7.

 sumptis

sumptis quatuor vel quinque minimis in ratione A ad *B*, ostendetur A ipsos D, & E; ac *B* ipsos E, & F non metiri, &c. Quare nullus alium metietur. Q. E. D.

PROP. VII.

A, 3. *B*, 6. C, 12. D, 24. E, 48.

Si sint quotcunnque numeri deinceps proportionales A, B, C, D, E; *primus autem* A *extremum* E *metiatur; is etiam metitur secundum B.*

a 6.7. Si negas A metiri B, *a* ergo nec ipsum E metietur, contra Hypoth.

PROP. VIII.

A, 24. C, 36. D, 54. *B*, 81.
G, 8. H, 12. I, 18. K, 27.
E, 32. L, 48. M, 72. F, 108.

Si inter duos numeros A, B *medii continua proportione ceciderint numeri* C, D; *quot inter eos medii continua proportione cadunt numeri, tot & inter alios* E, F *eandem cum illis habentes rationem, medii continua proportione cadent.* (L, M.)

a 35.7. b 14.7. c hyp. d 3.8. e 21.7. *a* Sume G, H, I, K minimos ∺ in ratione A ad C; *b* erit ex æquali, G. K :: A. B *c* :: E. F. Atqui G, & K *d* primi sunt inter se; *e* quare G æque metitur E, ac K ipsum F. per eundem numerum metiatur H ipsum L, & I ipsum M.

f constr. *f* itaque E, L, M, F ita se habent ut G, H, I, K; hoc est ut A, B, C, D. Q. E. D.

PROP. IX.

1.
E, 2. F, 3.
G, 4. H, 6. I, 9.
A, 8. C, 12. D, 18. B, 27.

Si duo numeri A, *B*, *sint inter se primi, & inter eos medii continua proportione ceciderint numeri,* C, D; *quot inter eos medii continua*

tinua proportione ceciderint numeri, totidem (E,G, & F, I) & inter utrumque eorum ac unitatem medii continua proportione cadent.

Constat 1, E, G, A; & 1, F, I, B esse ∺; & totidem quot A, C, D, B, nimirum ex 4 coroll. 2. 8. Q. E. D.

PROP. X.

A, 8. I, 12. K, 18. B, 27.
E, 4. DF, 6. G, 9.
D, 2. F, 3.
1.

Si inter duos numeros A, B, & unitatem continue proportionales ceciderint numeri (E, D, & F, G,) *quot inter utrumque ipsorum, & unitatem deinceps medii continua proportione cadunt numeri, totidem & inter ipsos medii continua proportione cadent,* I, K.

Nam E, D F, G; & A, D q F (I,) D G (K,) B sunt ∺, per 2. 8. ergo, &c.

PROP. XI.

A, 2. B, 3.
Aq, 4. AB, 6. Bq, 9.

Duorum quadratorum numerorum Aq, Bq *unus medius proportionalis est numerus* A B. *& quadratum* Aq *ad quadratum* Bq, *duplicatam habet lateris* A *ad latus* B *rationem.*

a Liquet Aq, AB, Bq, esse ∺. b proinde etiam $\frac{Aq}{Bq} = \frac{A}{B}$ bis. Q. E. D. a 17 7. b 10. def. 5.

PROP. XII.

Ac, 27. AqB, 36. ABq, 48. Bc, 64. *Duorum*
A, 3. B, 4 *cuborum nu-*
Aq, 9. AB, 12. Bq, 16. *merorum* Ac,
Bc *duo me-*
dii proportionales sunt numeri AqB, ABq. *Et cubus* Ac *ad cubum* Bc *triplicatam habet lateris* A *ad latus* B *rationem.*

a 2. 1. b 10 def. 5. *a* Nam Ac. AqB, ABq, Bc sunt ∺ in ratione A ad B. *b* proinde $\frac{Ac}{Bc} = \frac{A}{B}$ ter. Q. E. D.

PROP. XIII.

A, 2. B, 4. C, 8.
Aq, 4. AB, 8. Bq, 16. BC, 32. Cq, 64.
Ac, 8, AqB, 16, ABq 32. Bc, 64, BqC, 128, BCq, 256. Cc, 512.

Si sint quotlibet numeri deinceps proportionales, A, B, C; *& multiplicans quisque seipsum faciat aliquos; qui ab illis producti fuerint* Aq, Bq, Cq *proportionales erunt; & si numeri primum positi* A, B, C *multiplicantes jam factos* Aq, Bq, Cq, *fecerint aliquos* Ac, Bc, Cc; *ipsi quoque proportionales erunt. & semper circa extremos hoc eveniet.*

a 2. 8. b 14. 7. Nam Aq, AB, Bq, BC, Cq *a* sunt ∺ *b* ergo ex æquo Aq. Bq :: Bq Cq. Q. E. D.

a Item Ac, AqB, ABq, Bc, BqC, BCq, Cc sunt ∺, *b* ergo iterum ex æquo, Ac. Bc :: Bc. Cc. Q. E. D.

PROP. XIV.

Aq, 4. AB, 12. Bq, 36. *Si quadratus nu-*
A, 2. B, 6. *merus* Aq *quadra-*
tum numerum Bq *metiatur, & latus unius* (A) *metietur latus alterius* (B:) *& si unius quadrati latus* A *metietur latus alterius* B, *& quadratus* Aq *quadratum* Bq *metietur.*

a 2. & 11. 8. 1. *Hyp.* Nam Aq. AB *a* :: AB. Bq; cum igitur ex hyp. Aq metiatur Bq; idem Aq secundum

cundum AB *b* metietur. atqui Aq AB :: A. B. *c* ergo etiam A metitur B. Q. E. D. b 7. 8. c 10. def. 7.

2. *Hyp.* A metitur B. *c* ergo tam Aq ipsum AB, *c* quam AB ipsum Bq metitur; *d* & proinde Aq metitur Bq. Q. E. D. d 11. ax. 7.

PROP. XV.

A, 2. B, 6.
Ac, 8. AqB, 24. ABq, 72. Bc, 216.

Si cubus numerus Ac *cubum numerum* Bc *metiatur, & latus unius* (A) *metietur latus alterius* (B:) *Et si latus* A *unius cubi* Ac *latus* B *alterius* Bc *metiatur, & cubus* Ac *cubum* Bc *metietur.*

1. *Hyp.* Nam Ac, AqB, ABq, Bc *a* sunt ∺. ergo Ac, *b* metiens extremum Bc, *c* etiam secundum AqB metietur. atqui Ac. AqB :: A. B. *d* ergo etiam A metietur B. Q. E. D. a 2. & 12. 8. b hyp. c 7. 8.

2. *Hyp.* A metitur B; *d* ergo Ac metitur AqB, isque ABq, & hic Bc; *e* ergo Ac metietur Bc. Q. E. D. d 10. def. 7. e 11. ax. 7.

PROP. XVI.

A, 4. B, 9.
Aq, 16. Bq, 81.

Si quadratus numerus Aq *quadratum numerum* Bq *non metiatur, neque* A *latus unius alterius latus* B *metietur: & si* A *latus unius quadrati* Aq *non metiatur* B *latus alterius* Bq, *neque quadratus* Aq *quadratum* Bq *metietur.*

1. *Hyp.* Nam si affirmes A metiri B, *a* etiam Aq ipsum Bq metietur, contra hyp. a 14. 8.

2. *Hyp.* Vis Aq metiri Bq; *a* ergo A ipsum B metietur, contra hyp.

PROP.

PROP. XVII.

A, 2. B, 3. Ac, 8. Bc, 27.

Si cubus numerus Ac *cubum numerum* Bc *non metiatur, neque* A *latus unius latus* B *alterius metietur. Et si latus* A *unius cubi* Ac *latus* B *alterius* Bc *non metiatur, neque cubus* Ac *cubum* Bc *metietur.*

a 15.8. 1. *Hyp.* Dic A metiri B; *a* ergo Ac metietur Bc. contra Hypoth.

2. *Hyp.* Dic Ac metiri *Bc*; *a* ergo A ipsum B metietur. contra Hyp.

PROP. XVIII.

C, 6. D, 2. CD, 12. E, 9. F, 3. DE, 18. EF, 27.

Duorum similium planorum numerorum CD, EF, *unus medius proportionalis est numerus* DE: *& planus* CD *ad planum* EF *duplicatam habet lateris* C *ad latus homologum* E *rationem.*

* 21.def.7. Quoniam * ex hyp. C. D :: E. F; permu-
a 17.7. tando erit C. E :: D. F. atqui C. E *a* :: CD.
b 11.5. DE; *a* & D. F :: DE. EF. *b* ergo CD. DE :: DE. EF. Q. E. D.

c 10.def.5. *c* Ergo ratio CD ad EF duplicata est rationis CD ad DE; hoc est rationis C ad E, vel D ad F.

Coroll.

Hinc perspicuum est, inter duos similes planos cadere unum medium proportionalem, in ratione laterum homologorum.

PROP.

PROP. XIX.

CDE, 30. DEF, 60. FGE. 120. FGH, 240.
CD, 6. DF, 12. FG, 24.
C, 2. D, 3. E, 5. F, 4. G, 6. H, 10.

Duorum similium solidorum CDE, FGH, duo medii proportionales sunt numeri DFE, FGE. Et solidus CDE ad solidum FGH triplicatam rationem habet lateris homologi C ad latus homologum F.

Quoniam ex* hyp. C.D :: F.G ; & D.E :: G.H, erit *a* permutando C.F :: D.G *a* :: E.H. atqui CD. DF *b* :: C.F; & DF.FG *b* :: D.G. *c* quare CD. DF :: DF. FG :: E. H. *d* ergo CDE. DFE :: DFE. FGE :: E.H :: FGE. FGH. ergo inter CDE, FGH cadunt duo medii proportionales, DFE, FGE. Q.E.D. *e* Liquet igitur rationem CDE ad FGH triplicatam esse rationis CDE ad DFE, vel C ad F. Q.E.D.

* 21. def. 7. a 13. 7. b 17. 7. c 11. 5. d 17. 7. e 10. def. 5.

Coroll.

Hinc, inter duos similes solidos cadunt duo medii proportionales, in ratione laterum homologorum.

PROP. XX.

A, 12. C, 18. B, 27.
D, 2. E, 3. F, 6. G, 9.

Si inter duos numeros A, B, unus medius proportionali cadat numerus C, similes plani erunt illi numeri, A, B.

a Accipe D, & E minimos in ratione A ad C, vel C ad B. *b* ergo D æque metitur A, ac E ipsum C, puta per eundem F. *b* item D æque metitur C ac E ipsum B, puta per eundem G. *c* ergo DF = A, & EG = B. *d* quare A, & B plani sunt numeri. Quia vero EF *c* = C *e* = DG; *e* erit D.E :: F.G, & vicissim D.F :: E.G. *f* ergo plani numeri A, & B etiam similes sunt. Q.E.D.

a 35. 7. b 21. 7. c 9. ax. 7. d 16. def. 7. e 19. 7. f 21. def. 7.

PROP.

PROP. XXI.

A, 16. C, 24. D, 36. B, 54.
E, 4. F, 6. G, 9.
H, 2. P, 2. M, 4. K, 3. L, 3. N, 6.

Si inter duos numeros A, B *duo medii proportionales cadant numeri* C, D; *similes solidi erunt illi numeri*, A, B.

a 21. 8. *a* Sume E, F, G minimos ∵ in ratione A ad
b 20 8. C. *b* ergo E, & G sunt numeri plani similes.
c 22. def 7. hujus latera sint H & P; illius K & L: *c* ergo H.
d cor. 18 8. K :: P. L :: *d* E. F. Atqui E, F, G ipsos A, C,
e 21. 7. D *e* æque metiuntur, puta per eundem M; iidemque ipsos, C, D, B æque metiuntur, puta
f 9. ax 7. per eundem N. *f* ergo A = E M = H P M, *f* &
g 17 def. 7. B = G N = K L N; *g* quare A & B solidi sunt numeri. Quoniam vero C *f* = F M; & D *f* =
h 17. 7. F N, erit M. N *h* :: F M. F N *k* :: C. D *l* :: E.
k 7. 5. F :: H. K :: P. L. *m* ergo A, & B sunt numeri
l constr. m 21 def. 7. solidi similes. Q. E. D.

PROP. XXII.

A, 4. B, 6. C, 9.

Si tres numeri A, B, C *deinceps sint proportionales, primus autem* A *sit quadratus, & tertius* C *quadratus erit.*

Inter A, & C cadit medius proportionalis.
a 20. 8. *a* ergo A, & C sunt similes plani; quare *b* cum A
b hyp. quadratus sit, erit C etiam quadratus. Q. E. D.

PROP. XXIII.

A, 8. B, 12. C, 18. D, 27.

Si quatuor numeri A, B, C, D *deinceps sint proportionales; primus autem* A *sit cubus, & quartus* D *cubus erit.*

a 21 8. Nam A, & D *a* similes solidi sunt; ergo
b hyp. *b* cum A cubus sit, erit D cubus. Q. E. D.

Prop. XXIV.

A, 16. 24. B, 36.
C, 4. 6. D, 9.

Si duo numeri A, B rationem habeant inter se, quam quadratus numerus C ad quadratum numerum D, primus autem A sit quadratus; & secundus B quadratus erit.

Inter C, & D numeros quadratos, * adeoque inter A, & B eandem rationem habentes, a cadit unus medius proportionalis. Ergo b cum A quadratus sit, c etiam B quadratus erit. Q.E.D. * 8. 8. a 11. 8. b hyp. c 22. 8.

Coroll.

1. Hinc si fuerint duo numero similes AB, CD (A. B :: C. D) primus autem AB sit quadratus, etiam secundus CD quadratus erit. * 11. & 8. 8

* Nam AB. CD :: Aq. Cq.

2. Liquet ex his, proportionem cujusvis numeri quadrati ad quemlibet non quadratum, exhiberi nullo modo posse in duobus numeris quadratis. unde non erit, Q. Q :: 1.2. nec 1.5. :: Q. Q. &c.

Prop. XXV.

C, 64. 96. 144. D, 216.
A, 8. 12. 18. B, 27.

Si duo numeri A, B rationem inter se habeant, quam cubus numerus C ad cubum numerum D, primus autem A sit cubus, & secundus B cubus erit.

a Inter C, & D cubos, b adeoque inter A & B eandem rationem habentes, cadunt duo medii proportionales. ergo propter A c cubum, d etiam B cubus erit. Q. E. D. a 12. 8. b 8. 8. c hyp. d 23. 8.

Coroll.

1. Hinc etiam si fuerint duo numeri ABC, DEF (A. B :: D. E. & B. C :: E. F;) primus autem ABC cubus fuerit, etiam secundus DEF cubus erit. * 11. & 19. 8.

* Nam ABC. DEF :: Ac = Dc.

2. Patet etiam ex his, proportionem cujusvis un-

numeri cubi ad quemlibet numerum non cubum non posse reperiri in duobus numeris cubis.

PROP. XXVI.

A, 20. C, 30. B, 45.
D, 4. E, 6. F, 9.

Similes plani numeri A, B rationem inter se habent, quam quadratus numerus ad quadratum numerum.

a 18. 8. b 2. 8. Inter A, & B *a* cadit unus medius proportionalis C. *b* sume tres D, E, F minimos ∺ in ratione A ad C. Extremi D, F *b* quadrati erunt. atqui ex æquali A. B *c* :: D. F. ergo A. B ::
c 14. 7. Q. Q. Q. E. D.

PROP. XXVII.

A, 16. C, 24. D, 36. B, 54.
E, 8. F, 12. G, 18. H, 27.

Similes solidi numeri A, B, rationem habent inter se, quam cubus numerus ad cubum numerum.

a 19. 8. b 2. 8. *a* Inter A, & B cadunt duo medii proportionales, puta C & D: *b* sume quatuor E, F, G, H minimos ∺ in ratione A ad C. *b* Extremi E,
c 14. 7. H cubi sunt. At A. B *c* :: E. H :: C. C. Q. E. D.

Schol.

Vide Clavium. 1. Ex his infertur, nullos numeros habentes proportionem superparticularem, vel superbipartientem, vel duplam, aut aliam quamcunque multiplam non denominatam à numero quadrato, esse similes planos.

2. Nec duo quivis primi numeri, neque duo quicunque inter se primi, qui quadrati non sint, similes esse possunt.

LIB.

LIB. IX.

PROP. I.

A, 6. B, 54.
Aq, 36. 108. AB, 324.

SI duo similes plani numeri A, B *multiplicantes se mutuo faciant quendam* A B, *productus* AB *quadratus erit.*

Nam A. B *a* :: Aq. AB; cum igitur inter A, & B *b* cadat unus medius proportionalis, *c* etiam inter Aq, & AB cadet unus med. proport. ergo cum primus Aq sit quadratus, *d* etiam tertius AB quadratus erit. Q. E. D. a 17. 7. b 18. 8. c 8. 8. d 22. 8

Vel sic. Sint ab, cd similes plani, nempe a.b :: c.d. *x* ergo a d = bc. quare abcd, vel adbc *y* = adad = Q: ad. x 19. 7. y 1. ax. 7.

PROP. II.

Si duo numeri A, B *se mutuo multiplicantes faciant* AB *quadratum, similes plani erunt,* A, B.

A, 6. B, 54.
Aq, 36. AB, 324.

Nam A. B *a* :: Aq. AB ; quare cum inter Aq, AB *b* cadat unus medius proportionalis, *c* etiam unus inter A, & B medius cadet. *d* ergo A, & B sunt similes plani. Q. E. D. a 17. 7. b 11. 8. c 8. 8. d 20. 8.

PROP. III.

A, 2. Ac, 8 Acc, 64.

Si cubus numerus Ac *seipsum multiplicans procreet aliquem* Acc, *productus* Acc *cubus erit.*

Nam 1. A *a* :: A. Aq *b* :: Aq. Ac. ergo inter 1, & Ac cadunt duo medii proportionales. Sed 1. Ac *a* :: Ac. Acc. *c* ergo inter Ac, & Acc cadunt etiam duo a 15 *def.* 7. b 17. 7. c 8. 8.

medii

medii proportionales. Proinde cum Ac sit cubus,
d13.8. *d* erit Acc cubus. Q. E. D.

Vel sic ; aaa (Ac) in se ductus facit aaaaaa. (Acc;) hic cubus est, cujus latus aa.

Prop. IV.

Ac, 8. Bc, 27. Acc, 64. AcBc, 216. *Si cubus numerus Ac cubum numerum Bc multiplicans, faciat aliquem AcBc, factus AcBc cubus erit.*

a17.7. b11.8. c8.8. Nam Ac. Bc *a* :: Acc. AcBc. sed inter Ac & BC *b* cadunt duo medii proportionales; *c* ergo inter Acc, & AcBc totidem cadunt. itaque cum
d13.8. Acc sit cubus, *d* erit AcBc etiam cubus. Q.E.D.

Vel sic. AcBc = aaabbb (ababab) = C: ab.

Prop. V.

Ac, 8. B, 27. Acc, 64. AcB, 216. *Si cubus numerus Ac numerum quendam B multiplicans, faciat cubum AcB; & multiplicatus B cubus erit.*

a17.7. b12.8. c8.8. Nam Acc. AcB *a* :: Ac. B. Sed inter Acc, & AcB *b* cadunt duo medii proportionales. *c* ergo totidem cadent inter Ac, & B. quare cum Ac cu-
d13.8. bus sit, *d* etiam B cubus erit. Q. E. D.

Prop. VI.

A, 8. Aq, 64. Ac, 512. *Si numerus A seipsum multiplicans faciat Aq cubum; & ipse A cubus erit.*

a hyp. b19 def.7. c5.9. Nam quia Aq *a* cubus, & AqA (Ac) *b* cubus, *c* erit A cubus. Q. E. D.

Prop. VII.

A, 6. B, 11. AB, 66. D, 2. E, 3. *Si compositus numerus A numerum quempiam B multiplicans, quempiam faciat AB, factus AB solidus erit.*

Quoniam

Quoniam A compositus est, *a* metitur eum a- a 13. def.
liquis D, puta per E. *b* ergo A = D E; *c* quare b 9 ax. 7.
DEB = AB solidus est. Q. E. D. c 17. def. 7.

Prop. VIII.

1. a, 3. a^2, 9. a^3, 27. a^4, 81. a^5, 243. a^6, 729.

Si ab unitate quotcunque numeri deinceps proportionales fuerint (1, a, a^2, a^3, a^4, &c.) *tertius quidem ab unitate* a^2 *quadratus est; & unum intermittentes, omnes* (a^4, a^6, a^8, &c.): *quartus autem* a^3 *est cubus; & duos intermittentes omnes* (a^6, a^9, &c.) *septimus vero* a^6, *cubus simul & quadratus; & quinque intermittentes omnes* (a^{12}, a^{18}, &c.)

Nam 1. a^2 = Q. a. & a^4 = aaaa = Q. aa. & a^6 = aaaaaa = Q. aaa, &c.

2. a^3 = aaa = C. a. & a^6 = aaaaaa = C. aa. & aaaaaaaaa = C. aaa, &c.

3. a^6 = aaaaaa = C. aa = Q. aaa. ergo, &c.

Vel juxta Euclidem; quia 1. a *a* :: a. a^2, *b* erit a hyp.
a^2 = Q: a. ergo cum a^2, a^3, a^4 sint ∺ *c* erit b 20. 7.
tertius a^4 etiam quadratus. pariterq; a^6, a^8, &c. c 22. 8.
Item quia 1. a *a* :: a^2. a^3. erit a^3 *b* = a^2 in a = C: a. *d* ergo quartus ab a^3, nempe a^6, etiam cubus erit, &c. ergo a^6 cubus simul & quadratus existit, &c. d 23. 8.

Prop. IX.

1. a, 4. a^2, 16. a^3, 64. a^4, 256, &c.
1. a, 8. a^2, 64. a^3, 512. a^4, 4096.

Si ab unitate quotcunque numeri deinceps proportionales fuerint (1, a, a^2, a^3, &c.); *qui vero* (a) *post unitatem sit quadratus; & reliqui omnes,* a^2, a^3, a^4, &c. *quadrati erunt. At si* a, *qui post unitatem, sit cubus, & reliqui omnes* a^2, a^3, a^4, &c. *cubi erunt.*

1. *Hyp.* Nam a^2, a^4, a^6, &c. quadrati sunt ex præc. item quia a ponitur quadratus, *a* erit a 22. 8.
tertius a^3 quadratus, pariterque a^5, a^7, &c. ergo omnes.

 2. *Hyp.*

b 13. 8. 2. *Hyp*. a cubus ponitur, [b] ergò a^4, a^7, a^{10}
c 20. 7. cubi sunt: atqui ex præced. a^3, a^6, a^9, &c. cubi
d 3. 9. sunt. denique quia 1. a :: a. aa, [c] erit $a^2 = Q$:
e 23. 8. a. cubus autem in se [d] facit cubum; ergo a_2 cubus est, & [e] proinde ab eo quartus a^5, pariterq; a^8, a^{11}, &c. cubi sunt. ergo omnes. Q. E. D.

Clarius forsitan sic; Sit quadrati a latus b. ergo series a, a^2, a^3, a^4, &c. aliter exprimetur sic, bb, b_4, b6, b8, &c. liquet vero hos omnes quadratos esse; & sic etiam exprimi posse; Q: b, Q: bb, Q: bbb, Q: bbbb, &c.

Eodem modo, si b latus fuerit cubi a, series ita nominari potest; b_3, b^6, b_9, b^{12}, &c. vel C: b, C: b^2, C: b3, C: b4, &c.

Prop. X.

1, a, a^2, a^3, a^4, a^5, a^6.
1, 2, 4, 8, 16, 32, 64.

Si ab unitate quotcunque numeri deinceps proportionales fuerint (1, a, a^2, a^3, &c.); *qui vero post unitatem* (a) *non sit quadratus, neque alius ullus quadratus erit, præter* a_2 *tertium ab unitate, & unum intermittentes omnes* (a^4, a^6, a_8.) *At si a, qui post unitatem, non sit cubus, neque ullus alius cubus erit præter* a^3 *quartum ab unitate, & duos intermittentes omnes,* a^6, $a9$, a^{12}, *&c.*

a 1. *Hyp*. Nam si fieri potest, sit a^5 quadratus numerus. quoniam igitur a. a_2 a :: a4. a_5, atq;
a hyp. inverse a5. a_4 :: a2. a; sintque a5, & a_4 [b] qua-
b suppos. & 8. 9. drati, primusque a_2 quadratus, [c] erit a etiam
c 24. 8. quadratus, contra Hyp.

2. *Hyp*. Si fieri potest, sit a^4 cubus. quoniam igitur [d] ex æquo a4. a^6 :: a. a3, atque in-
d 14. 7. verse a 6. a_4 :: a_3. a; [b] sintque a6, & a_4 cubi, & primus a_3 cubus, [e] etiam a cubus erit, con-
e 25. 8. tra Hypoth.

PROP.

PROP. XI

1, a, a_2, a_3, a_4, a_5, a_6.
1, 3, 9, 27, 81, 243, 729.

Si ab unitate quotcunq; numeri deinceps proportionales fuerint (1, a, a_2, a_3, &c.) *minor majorem metitur per aliquem eorum qui in proportionalibus sunt numeris.*

Quoniam 1. a :: a. aa, a erit $\frac{aa}{a} = a = \frac{aaa}{aa}$ — a 5. ax. 7. & 20. def. 7.

Item quia 1. aa b :: a. aaa. a erit $\frac{aaa}{a} = aa = \frac{a^4}{aa} = \frac{a^5}{a^3}$ &c. denique quia 1. a^3 b :: a. a^4, a erit $\frac{a^4}{a} = a^3 = \frac{a^6}{a^3}$ &c. — b 14. 7.

Coroll.

Hinc, si numerus qui metitur aliquem ex proportionalibus, non sit unus proportionalium, neque numerus per quem metitur, erit aliquis ex proportionalibus.

PROP. XII.

1, a, a_2, a_3, a_4,
1, 6, 36, 216, 1296.
B, 3.

Si ab unitate quotcunq; numeri deinceps proportionales fuerint (1, a, a_2, a_3, a_4); *quicunque primorum numerorum* B *ultimum* a_4 *metiuntur, iidem* (B) *& eum* (a) *qui unitati proximus est, metientur.*

Dic B non metiri a, a ergo B ad a primus est; b ergo B ad a^2 primus est; & c proinde ad a^4 quem metiri ponitur Q. E. A. — a 31. 7. b 27. 7. c 26. 7.

Coroll.

1. Itaq; omnis numerus primus ultimum metiens, metitur quoq; omnes alios ultimum praecedentes.

 2. Si

2. Si aliquis numerus non metiens proximum unitati, metiatur ultimum, erit numerus compositus.

3. Si proximus unitati sit primus numerus, nullus alius primus numerus ultimum metietur.

PROP. XIII.

1, a, a^2, a^3, a^4,
1. 5, 25, 125, 625.
H - - G - - F - - E - -

Si ab unitate quotcunque numeri deinceps proportionales fuerint (a, a^2, a^3, &c.), *qui vero post unitatem* (a) *primus sit; maximum nullus alius metietur, præter eos qui sunt in numeris proportionalibus.*

Si fieri potest, alius quispiam E metiatur $a4$, a cor. 12. 9. nempe per F; ᵃ erit F alius extra a, a^2, $a3$. b 2 cor 12. 9. Quia vero E metiens $a4$ non metitur a, ᵇ erit c 33. 7. E numerus compositus; ᶜ ergo eum aliquis pri- d 11. ax. 7. mus metitur, ᵈ qui proinde ipsum $a4$ metitur; e 3 cor 12. 9. ᵉ ideoque alius non est, quam a. ergo a metitur E. Eodem modo ostendetur F compositus numerus, metiens $a4$, adeoque a ipsum F metiri. f 9 ax. 7. itaque quum EF ᶠ$= a4 = a$ in $a3$ ᵍ erit a.E :: F. g 19. 7. h 20. def. 7. $a3$. ergo cum a metiatur E, ʰ æque F metietur k cor. 11. 9. a^3, puta per eundem G. ᵏ Nec G erit a, vel a^2. ergo, ut prius, G est numerus compositus, & a eum metitur. quum igitur FG ᶠ$= a3 = a2$ in a, ᵍ erit a. F :: G. $a2$; & proinde, quia A metitur F, ʰ æque G metietur a^2, scilicet per eundem H; ᵏ qui non est a. ergo quum G H $= a2 = aa$. l 10. 7. ˡ erit H. a :: a. G. ergo quia a metitur G (ut m 10 def. 7. prius) ᵐ etiam H metietur a, numerum primum. Q. E. N.

PROP. XIV.

A, 30.
B, 2. C, 3. D, 5.
E -- F -- .

Si minimum numerum A primi numeri B, C, D metiantur; nullus alius numerus primus E illum metietur, præter eos, qui à principio metiebantur.

Si fieri poteſt, ſit $\frac{A}{E}$ = F. *a* Ergo A = E F. *b* Ergo ſinguli primi numeri B, C, D ipſorum E, F unum metiuntur; non E, qui primus ponitur; ergo F, minorem ſcilicet ipſo A; contra Hypoth.

a 9 ax. 7.
b 31. 7.

PROP. XV.

A, 9. B, 12. C, 16.
D, 3. E, 4.

Si tres numeri A, B, C deinceps proportionales, fuerint minimi omnium eandem cum ipſis rationem habentium; duo quilibet compoſiti, ad reliquum primi erunt.

a Sume D, & E minimos in ratione A ad B. *b* ergo A = Dq, *b* & C = Eq; *b* & B = DE. Quia vero D ad E *c* primus eſt, *d* erit D + E primus ad ſingulos D, & E. * ergo D in D + E *e* = D + DE (*f* A + B) ad E primus eſt, ideoque ad C vel Eq. Q. E. D. Pari pacto DE + Eq (B + C) ad D primus eſt, & proinde ad A = Dq. Q. E. D. Denique quia B ad D + E *h* primus eſt; is ad hujus quadratum *k* Dq + 2 DE + Eq (A + 2 B + C) primus erit. *l* quare idem B ad A + B + C, *i* adeoque ad A + C primus erit. Q. E. D.

a 35. 7.
b 2. 8.
c 24 7.
d 30 7.
* 26 7.
e 3. 2.
f prius.
g 27. 7.
h 26 7.
k 4 2.
l 30 7.

PROP. XVI.

A, 3. B, 5. C--- *Si duo numeri* A, B *primi inter se fuerint; non erit ut primus* A *ad secundum* B, *ita secundus* B *ad alium quempiam* C.

Dic A. B :: B. C. ergo quum A & B in sua
a 23. 7. ratione *a* minimi sint, A *b* metietur B æque ac B
b 21. 7. ipsum C; sed A *c* seipsum etiam metitur; ergo
c 6. ax. 7. A & B non sunt primi inter se, contra Hypoth.

PROP. XVII.

A, 8. B, 12. C, 18. D, 27. E---

Si fuerint quotcunque numeri deinceps proportionales A, B, C, D, *extremi autem ipsorum* A, D *primi inter se sint; non erit ut primus* A *ad secundum* B, *ita ultimus* D *ad alium quempiam* E.

Dic A. B :: D. E. ergo vicissim A. D :: B. E.
a 23. 7. ergo quum A & D in sua ratione *a* minimi sint,
b 21. 7. *b* metietur A ipsum B; *c* quare B ipsum C, & C
c 20. def. 7. sequentem D, *d* adeoque A eundem D metietur.
d 11. ax. 7. Ergo A & D non sunt primi inter se, contra Hypoth.

PROP. XVIII.

A, 4. B, 6. C, 9. Bq, 16. *Duobus numeris datis* A, B, *considerare an possit ipsis tertius proportionalis* C *inveniri*.

a 9. ax. 7. Si A metiatur Bq per aliquem C, *a* erit AC
b per 20. 7. =Bq. unde *b* liquet esse A.B :: B.C. Q. E. F.

A, 6. B, 4. Bq, 16. Sin A non metiatur Bq, non erit aliquis tertius proportionalis.

c 7. ax. 7. Nam dic A.B :: B.C. *a* ergo AC=Bq. *c* proinde $\frac{Bq}{A}$=C. Scilicet A metitur Bq, contra Hypoth.

PROP.

Prop. XIX.

A, 8. B, 12. C, 18. D, 27.
EC, 216.

Tribus numeris datis A, B, C, considerare an possit ipsis quartus proportionalis D inveniri.

Si A metiatur BC per aliquem D, a ergo AD = BC; b constat igitur esse A. B :: C. D. Q. E. F. a 9.ax.7. b ex 19.7.

Sin A non metiatur BC, non datur quartus proportionalis; quod ostendetur, prout in praecedenti.

Prop. XX.

A, 2. B, 3. C, 5.
D, 30. G----

Primi numeri plures sunt omni proposita multitudine primorum numerorum A, B, C.

a Sit D minimus, quem A, B, C metiuntur. si D+1 primus sit, res patet; si compositus, b ergo aliquis primus, puta G, metitur D+1, qui non est aliquis trium A, B, C; nam si ita, quum is c totum D+1, & d ablatum D metiatur, e idem reliquam unitatem metietur. Q. E. A. Ergo propositorum primorum numerorum multitudo aucta est per D+1; vel saltem per G. a 38.7. b 33.7. c suppos. d constr. e 12.ax 7.

Prop. XXI.

5 5 3 3 2 2
A ..... E ,.... B ... F ... C .. G .. D 20.

Si pares numeri quotcunque AB, BC, CD componantur, totus AD par erit.

a Sume EB = $\frac{1}{2}$ AB, & FC = $\frac{1}{2}$ BC, & GD = $\frac{1}{2}$ CD. b liquet EB + FC + GD = $\frac{1}{2}$ AD. c ergo AD est par numerus. Q. E. D. a 6.def.7. b 12.7. c 6.def.7.

PROP. XXII.

```
                 I        I        I
A........F.B......G.C....H.D..L. E 22.
     9        7        5       3
```

Si impares numeri quotcunque AB, BC, CD, DE *componantur, multitudo autem ipſorum ſit par, totus* AE *par erit.*

a 7. def. 7. Detracta unitate ex ſingulis imparibus, *a* manebunt AF, BG, CH, DL numeri pares, &
b 21. 9. *b* proinde compoſitus ex ipſis par erit; adde his
c hyp. *c* parem numerum conflatum ex reſiduis unitati-
d 21. 9. bus, *d* totus idcirco AE par erit. Q. E. D.

PROP. XXIII.

```
   7      5        I
A.......B.....C..E.D 15.
                   3
```

Si impares numeri quotcunque AB, BC, CD *componantur, multitudo autem ipſorum ſit impar; & totus* AD *impar erit.*

Nam dempto CD uno imparium, reliquorum
a 22. 9. aggregatus AC *a* eſt par numerus. huic adde
b 21. 3. CD − 1; *b* totus AE eſt etiam par; quare reſti-
c 7. def. 7. tuta unitate totus AD *c* impar erit. Q. E. D.

PROP. XXIV.

```
   4     5    I
A....B.....D. C 10.
         6
```

Si à pari numero AC *par* AB *detrahatur, & reliquus* BC *par erit.*

Nam ſi BD (BC − 1) impar fuerit, *a* erit BC (BD + 1) par. Q. E. D.
a 7. def. 7. Sin BD parem dicas, propter AB *b* parem, *c* erit
b hyp. AD par; *a* ideoque AC (AD + 1) impar, con-
c 21. 9. tra Hypoth. ergo BC eſt par. Q. E. D.

PROP.

Prop. XXV.

6 1 3
A ...... D . C ... B 10.
7

Si à pari numero AB impar AC detrahatur, & reliquus CB impar erit.

Nam AC – 1 (AD) *a* est par. *b* ergo DB est par. *c* ergo CB (DB – 1) est impar. Q.E.D. *a* 7.def.7. *b* 24 9. *c* 7.def 7.

Prop. XXVI.

4 6 1
A .... C ...... D . B 11.
7

Si ab impari numero AB impar CB detrahatur, reliquus AC par erit.

Nam AB – 1 (AD) & CB – 1 (CD) *a* sunt pares. *b* ergo AD – CD (AC) est par. Q. E. D. *a* 7.def 7. *b* 24 9.

Prop. XXVII.

1 4 6
A . D .... C ...... B 11.
5

Si ab impari numero AB par detrahatur CB, reliquus AC impar erit.

Nam AB – 1 (DB) *a* est par; & CB ponitur par. *b* ergo reliquus CD par est. *c* ergo CD + 1 (CA) est impar. Q. E. D. *a* 7.def.7. *b* 24.9. *c* 7.def.7.

Prop. XXVIII.

A, 3.
B, 4.
$\overline{AB}$, 12.

Si impar numerus A parem numerum B multiplicans fecerit aliquem AB, factus AB par erit.

Nam AB *a* componitur ex impari A toties accepto, quoties unitas continetur in B pari. *b* ergo AB est par numerus. *a* hyp. & 15. def.7. *b* 21.9.

Schol.

Eodem modo, si A sit numerus par, erit AB par.

Prop.

PROP. XXIX.

A, 3.
B, 5.
AB, 15.

Si impar numerus A, imparem numerum B multiplicans fecerit aliquem AB; factus AB impar erit.

a 15. def. 7.
b 23. 9.

Nam AB a componitur ex B impari numero toties accepto, quoties unitas includitur in A etiam impari. b ergo A B est impar. Q.E.D.

Scholium.

B, 12 (C, 4.
A, 3.

1. *Numerus A impar numerum B parem metiens, per numerum parem C eum metitur.*

a 9. ax 7.
b 29. 9.

Nam si C impar dicatur, quoniam a B=AC, erit B impar, contra Hypoth.

B, 15 (C, 5.
A, 3

2. *Numerus A impar numerum B imparem metiens, per numerum C imparem eum metitur.*

a 28. 9.

Nam si C dicatur par; a erit A C, vel B par, contra Hypoth.

B, 15 (C, 5.
A, 3

3. *Omnis numerus (A & C) metiens imparem numerum B, est impar.*

a 28. 9.

Nam si utervis A, vel C dicatur par, a erit B numerus par, contra Hypoth.

PROP. XXX.

B, 24 (C, 8. D, 12 (E, 4.
A, 3 A, 3

Si impar numerus A parem numerum B metiatur, & illius dimidium D metietur.

a hyp.
b 1. Schol. 29 9.
c 9. ax. 7.
d 1. 2.
e hyp.
f 7. ax. 1.
g 7. ax. 7.

a Sit $\frac{B}{A} = C$. b ergo C est numerus par. Sit igitur $E = \frac{1}{2}C$, erit B c$= CA$ d$= 2EA$ e$= 2D$. f ergo $EA = D$; & g proinde $\frac{D}{A} = E$. Q.E.D.

PROP.

Prop. XXXI.

A, 5. B, 8. C, 16. D - - -

Si impar numerus A ad aliquem numerum B primus ſit; & ad illius duplum C primus erit.

Si fieri poteſt, aliquis D metiatur A, & C. a ergo D metiens imparem A impar erit, b ideoque ipſum B paris C ſemiſſem metietur. ergo A, & B non ſunt primi inter ſe, contra Hypoth.

a 3. ſchol. 29.9.
b 30.9.

Coroll.

Sequitur hinc, numerum imparem, qui ad aliquem numerum progreſſionis duplæ primus eſt, primum quoque eſſe ad omnes numeros illius progreſſionis.

Prop. XXXII.

1. A, 2. B, 4. C, 8. D, 16.

Numerorum A, B, C, D, &c. à binario duplorum unuſquiſque pariter par eſt tantum.

Conſtat omnes 1, A, B, C, D a pares eſſe; atque b ∺ nimirum in ratione dupla, & c proinde quemque minorem metiri majorem per aliquem ex illis. d Omnes igitur ſunt pariter pares. Sed quoniam A primus eſt, e nullus extra eos eorum aliquem metietur. Ergo pariter pares ſunt tantum. Q. E. D.

a 6 def. 7.
b 20 def. 7.
c 11. 9.
d 8. def. 7.
e 13. 9.

Prop. XXXIII.

A, 30. B, 15.
D - - - E - -

Si numerus A dimidium B habeat imparem, A pariter impar eſt tantum.

Quoniam impar numerus B a metitur A per 2 parem, b eſt B pariter impar. Dic etiam pariter parem. c ergo eum par aliquis D per parem E metitur. unde 2 B d = A d = D E. e quare 2. E ::

a hyp.
b 9. def. 7.
c 8. def. 7.
d 9 ax. 7.
e 19. 7.

f 6. def. 7. g 20. def. 7. E :: D. B. ergo ut 2 *f* metitur parem E, *g* sic D par imparem B metitur. Q. E. N.

PROP. XXXIV.

A, 24. *Si par numerus A, neque à binario duplus sit, neque dimidium habeat imparem; pariter par est, & pariter impar.*

Liquet A esse pariter parem, quia dimidium imparem non habet. Quia vero si A bifarietur; & rursus ejus dimidium, & hoc semper fiat, tandem incidemus in aliquem *a* imparem (quia non in binarium, quoniam A à binario duplus non ponitur) is metietur A per parem numerum (nam *b* alias ipse A impar esset, contra Hypoth.) ergo A est etiam pariter impar. Q. E. D.

a 7. def. 7. b 1. sch. 19. 9.

PROP. XXXV.

A .... 8.

4 8

B .... F .... G 12.

C .... 18.

9 6 4 8

D .... H ...... L .... K .... N 27.

Si sint quotcunque numeri deinceps proportionales A, BG, C, DN, detrahantur autem FG à secundo, & KN ab ultimo, æquales ipsi primo A; erit ut secundi excessus BF ad primum A, ita ultimi excessus DK ad omnes A, BG, C ipsum antecedentes.

Ex DN deme NL = BG, & NH = C.

a hyp. b 17. 5. Quoniam DN. C. (HN) *a* :: HN. BG. (LN) *a* :: LN. (BG) A. (KN.) *b* erit dividendo ubique, DH. HN :: HL. LN :: LK. KN. *c* quare DK. C + BG + A :: LK (*d* BF.) KN. (A.) Q. E. D.

c 12. 5. d 3. ax. 1.

Coroll.

e 18. 5. Hinc *e* componendo, DN + BG + C. A + BG + C :: BG. A.

PROP.

PROP. XXXVI.

1. A, 2. B, 4. C, 8. D, 16.
E, 31. G, 62. H, 124. L, 248. F, 496.
M, 31. N, 465.
P - - - - Q - - -

Si ab unitate quotcunque numeri 1, A, B, C, D, deinceps exponantur in dupla proportione, quoad totus compositus E fiat primus, & totus hic E in ultimum D multiplicatus faciat aliquem F; factus F erit perfectus.

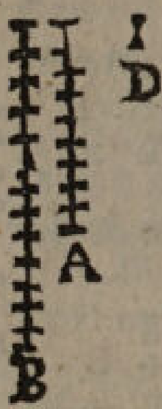
Sume totidem, E, G, H, L etiam in proportione dupla continue; ergo [a] ex æquo A. D :: E. L. [b] ergo AL = DE [c] = F. [d] ergo L = $\frac{F}{2}$ quare E, G, H, L, F sunt ∺ in ratione dupla. Sit G − E = M, & F − E = N. [e] ideo M. E :: N. E + G + H + L. [f] at M = E. [g] ergo N = E + G + H + L. ergo F = 1 + B + C + D + E + G + H + L = E + N. Quinetiam quia D [k] metitur DE (F,) [l] etiam singuli 1, A, B, C [m] metientes D, [m] nec non E, G, H, L metiuntur F. Porro nullus alius eundem F metitur. Nam si aliquis, sit P, qui metiatur F per Q. [n] ergo PQ = F = DE. [o] ergo E. Q :: P. D. ergo cum A primus numerus metiatur D, & [p] proinde nullus alius P eundem metiatur, [q] consequenter E non metitur Q. quare cum E primus ponatur, [r] idem ad Q primus erit. [s] ergo E & Q in sua ratione minimi sunt, & [t] propterea E ipsum P ac Q ipsum D æque metiuntur. [u] ergo Q est aliquis ipsorum A, B, C. Sit igitur B; ergo cum ex æquo sit B. D :: E. H; [x] ideoque BH = DE = F = PQ. [x] adeoque Q. B :: H. P. [y] erit H = P. ergo P est etiam aliquis ipsorum A, B, C, &c. contra Hypoth. ergo nullus alius præter numeros prædictos eundem F metietur: [z] proinde F est numerus perfectus. Q. E. D.

a 14. 7. b 19. 7. c hyp. d 7. ax. 7. e 35. 9. f 3. ax. 1. g 4. 5. h 2. ax. 1. k 7. ax. 7. l 11. ax. 7. m 11. 9. n 9. ax. 7. o 19. 7. p 13. 9. q 20 def. 7. r 31. 7. s 23. 7. t 21. 7. u 13. 7. x 19. 7. y 14. 5. z 22. def. 7.

LIB. X.

Definitiones.

I. Commensurabiles magnitudines dicuntur, quas eadem mensura metitur.

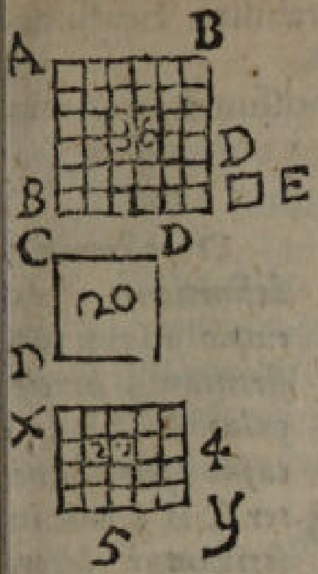


Commensurabilitatis nota est ⊔, *ut* A ⊔ B; *hoc est, linea* A 8 *pedum commensurabilis est lineæ* B 13 *pedum; quia* D *linea unius pedis singulas* A *&* B *metitur. Item* $\sqrt{18}$ ⊔ $\sqrt{50}$; *quia* $\sqrt{2}$ *singulas* $\sqrt{18}$, *&* $\sqrt{50}$ *metitur. Nam* $\sqrt{\frac{18}{2}} = \sqrt{9} = 3$. *&* $\sqrt{\frac{50}{2}} = \sqrt{25} = 5$. *quare* $\sqrt{18} . \sqrt{50} :: 3 . 5$.

II. Incommensurabiles autem sunt, quorum nullam communem mensuram contingit reperiri.

Incommensurabilitas significatur nota ⊔. *ut* $\sqrt{6}$ ⊔ $\sqrt{25}$ (5;) *hoc est* $\sqrt{6}$ *incommensurabilis est numero* 5, *vel magnitudini hoc numero designatæ; quia harum nulla est communis mensura, ut postea patebit.*

III. Rectæ lineæ potentia commensurabiles sunt, cum quadrata earum idem spatium metitur.



Hujusce commensurabilitatis nota est Ⴏ, *ut* AB Ⴏ CD; *h.e. linea AB sex pedum potentia commensurabilis est lineæ* CD, *quæ exprimitur per* $\sqrt{}$ 20. *quia spatium* E *unius pedis quadrati metitur tam* ABq (36) *quam rectangulum* XY (20,) *cui æquale est quadratum lineæ* CD($\sqrt{}$20.) *Eadem nota* Ⴏ *nonnunquam valet potentia tantum commensurabilis.*

IV. Incommensurabiles vero potentia, cum quadratis earum nullum spatium, quod sit communis eorum mensura, contingit reperiri.

Hujusmodi incommensurabilitas denotatur sic ; Ⴏ *v*$\sqrt{}$ 8; *hoc est, numeri vel lineæ* 5, *&* *v*$\sqrt{}$ 8 *sunt incommensurabiles potentia ; quia harum quadrata* 25, *&* $\sqrt{}$ 8 *sunt incommensurabilia.*

V. Quæ cum ita sint, manifestum est cuicunque rectæ propositæ, rectas lineas multitudine infinitas, & commensurabiles esse, & incommensurabiles; alias quidem longitudine & potentia, alias vero potentia solum. Vocetur autem proposita recta linea Rationalis.

Hujus nota est ῥ.

VI. Et huic commensurabiles, sive longitudine & potentia, sive potentia tantum, Rationales, ῥ.

VII. Huic vero incommensurabiles Irrationales vocentur.

Hæ sic denotantur ῤ.

VIII. Et quadratum, quod à proposita recta fit, dicatur Rationale ῥv.

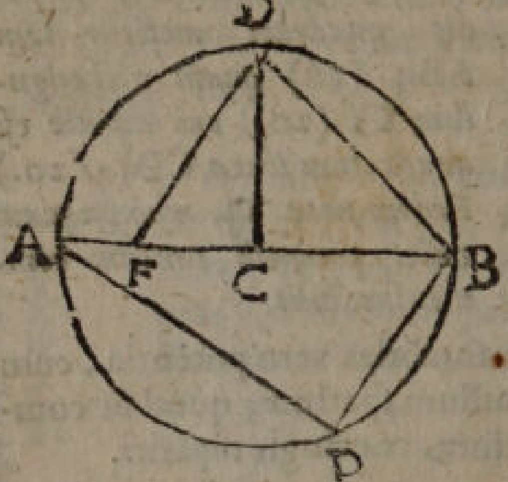
IX. Et huic commensurabilia quidem Rationalia ῥα.

X. Huic

X. Huic vero incommensurabilia, Irrationalia dicantur, ρα.

XI. Et rectæ, quæ ipsa possunt, Irrationales, ρ.

Schol.



Vt postremæ 7 definitiones exemplo aliquo illustrentur, sit circulus ADBP, *cujus semidiameter* CB; *huic inscribantur latera figurarum ordinatarum, Hexagoni quidem* BP, *Trianguli* AP, *quadrati* BD, *pentagoni* FD. *Itaque si juxta 5 defin. semidiameter* CB *sit Rationalis exposita, numero 2. expressa, cui reliquæ* BP, AP, BD, FD *comparandæ sunt,* *erit* BP [a] = BC = 2. *quare* BP *est* ρ ⊔ BC, *juxta 6. def. Item* AP [b] = $\surd$ 12 (*nam* ABq (16) − BPq (4) = 12) *quare* AP *est* ρ ⊔ BC, *etiam juxta 6. def. atque* APq (12) *est* ρν, *per def. 9. Porro* BD [b] = $\surd$ DCq + BCq = $\surd$ 8; *unde* BD *est* ρ ⊔ BC; & BDq ρν. *Denique*, FDq = 10 − $\surd$ 20 (*ut patebit ex praxi ad 10. 13. tradenda*) *erit* ρν, *juxta 10 def.* & *proinde* FD = $\surd$:10 − $\surd$ 20 *est* ρ, *juxta 11 defin.*

a cor. 15. 4.
b 67. 1.

Postulatum.

POstuletur, quamlibet magnitudinem toties posse multiplicari, donec quamlibet magnitudinem ejusdem generis excedat.

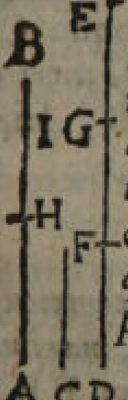
Axiomata.

1. MAgnitudo quotcunque magnitudines metiens, compoſitam quoque ex ipſis metitur.

2. Magnitudo quamcunque magnitudinem metiens, metitur quoque omnem magnitudinem quam illa metitur.

3. Magnitudo metiens totam magnitudinem & ablatam, metitur & reliquam.

Prop. I.

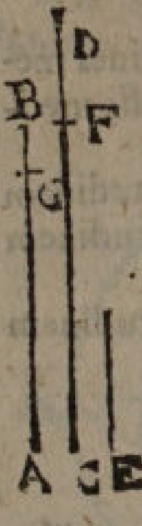


Duabus magnitudinibus inæqualibus A B, C *propoſitis, ſi à majore* A B *auferatur majus quam dimidium* (A H) *&* *ab eo* (HB) *quod reliquum eſt, rurſus detrahatur majus quam dimidium* (H I,) *& hoc ſemper fiat; relinquetur tandem quædam magnitudo* I B, *quæ minor erit propoſita minore magnitudine* C.

[a] Accipe C toties, donec ejus multiplex DE proxime excedat AB; ſintque DF = FG = GE = C. Deme ex A B pluſquam dimidium AH, & à reliquo HB pluſquam dimidium HI; & ſic deinceps, donec partes AH, HI, IB æque multæ ſint partibus DF, FG, GE. Iam liquet FE, quæ non minor eſt quam $\frac{1}{2}$ DE, majorem eſſe quam HB, quæ minor eſt [2] quam $\frac{1}{2}$ AB ⊐ DE. Pariterque G E quæ non minor eſt quam $\frac{1}{2}$ FE, major eſt quam IB ⊐ $\frac{1}{2}$ HB. ergo C, vel [2] GE ⊏ IB. Q. E. D. a post. 10.

Idem demonſtrabitur, ſi ex AB auferatur dimidium AH, & ex reliquo HB rurſus dimidium HI, & ita deinceps.

Prop. II.



Si duabus magnitudinibus inæqualibus propositis (AB, CD) *detrahatur semper minor* AB *de majore* CD, *alterna quadam detractione, & reliqua minime præcedentem metiatur; incommensurabiles erunt ipsæ magnitudines.*

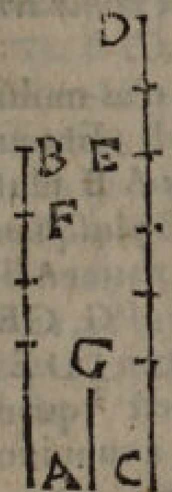
Si fieri potest, sit aliqua E communis mensura. Quoniam igitur A B detracta ex CD, quoties fieri potest, relinquit aliquam F D se minorem, & FD ex A B relinquit G B, & sic deinceps, *a* tandem relinquetur aliqua

a 1. 10.
b hyp.

GB ⊐ E. ergo E *b* metiens AB, *c* ideoque CF, *b* & totam C D; *d* etiam reliquam FD, metitur. *c* proinde & AG; *d* ergo & reliquam GB, seipsa minorem. Q. E. A.

c 2 ax.10
d 3.ax.10

Prop. III.



Duabus magnitudinibus commensurabilibus datis, A B, C D, *maximam earum communem mensuram* F B *reperire.*

Deme A B ex C D, & reliquum ED ex A B, & F B ex E D, donec FB metiatur ED; (quod tandem fiet, *a* quia per Hyp. AB ⊏ CD) erit FB quæsita.

a 2. 10.

Nam FB *b* metitur ED, *c* ideoque ipsam AF; sed & seipsam, *d* ergo etiam AB, & *c* propterea CE, *d* ideoque & totam C D. Proinde F B communis est mensura ipsarum AB, CD. Dic G communem quoq; esse mensuram, hac majorem; ergo G metiens AB, & CD, *e* metitur CE, & *f* reliquam E D, *e* ideoque A F, & *f* proinde reliquam F B major minorem. Q. E. A.

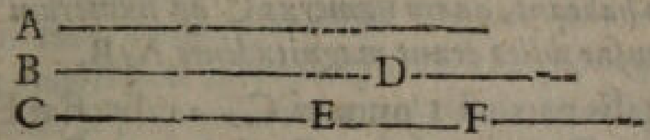
b constr.
c 1 ax.10.
d 1 ax.10.
e 2.ax.10.
f 3.ax.10.

Corol.

Coroll.

Hinc, magnitudo metiens duas magnitudines, metitur & maximam earum mensuram communem.

Prop. IV.



Tribus magnitudinibus commensurabilibus datis A, B, C; maximam earum mensuram communem invenire.

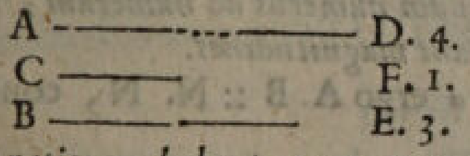
a Inveni D maximam communem mensuram duarum quarumcunque A, B; *a* item E ipsarum D & C maximam communem mensuram; erit E quæsita. a 3. 10.

a Nam perspicuum est E metiens D & C *b* metiri tres A, *B*, C. Puta aliam F hac majorem easdem metiri. *c* ergo F metitur D; *c* proinde & E, ipsorum D, C maximam communem mensuram, major minorem. Q. E. A. b *constr* & 2. *ax.* 10. c *cor.* 3. 10.

Coroll.

Hinc quoque, magnitudo metiens tres magnitudines, metitur quoque maximam earum communem mensuram.

Prop. V.



Commensurabiles magnitudines A, B inter se rationem habent, quam numerus ad numerum.

a Inventa C ipsarum A, B maxima communi mensura; quoties C in A & B, toties 1 continéatur in numeris D & E. *b* ergo C. A :: 1. D; quare inverse A. C :: D. 1. *b* atqui etiam C. a 3. 10. b 10. *def.* 7.

c 22. 5. B :: 1. E. *c* ergo ex æquali A. B :: D. E :: N. N. Q. E. D.

Prop. VI.

E ———— F. 1.

A ———————— C. 4.

B ———————— D. 3.

Si duæ magnitudines A, B inter se proportionem habeant, quam numerus C ad numerum D; commensurabiles erunt magnitudines A, B.

a sch. 10 6.
b constr.
c hyp.
d 22. 5.
e 5. ax. 7.
f 20. def. 7.
g constr.
h 1. def. 10.

Qualis pars est 1 numeri C, *a* talis fiat E ipsius A. Quoniam igitur E. A *b* :: 1. C. atque A. B *c* :: C. D; *d* ex æquo erit E. B :: 1. D. ergo quum 1 *e* metiatur numerum D, *f* etiam E metitur B; sed & ipsum A *g* metitur. *h* ergo A ⊐⊏ B. Q. E. D.

Prop. VII.

A ————————

B ——————

Incommensurabiles magnitudines A, B inter se proportionem non habent, quam numerus ad numerum.

a 6 10. Dic A. B :: N. N. *a* ergo A ⊐⊏ B, contra Hypoth.

Prop. VIII.

A ————————

B ——————

Si duæ magnitudines A, B inter se proportionem non habeant, quam numerus ad numerum, incommensurabiles erunt magnitudines.

a 5. 10. Puta A ⊐⊏ B *a* ergo A. B :: N. N, contr. Hypoth.

PROP.

PROP. IX.

A ————————
B ——————
E, 4.
F, 3.

Quæ à rectis lineis longitudine commensurabilibus fiunt quadrata, inter se proportionem habent, quam quadratus numerus ad quadratum numerum: & quadrata inter se proportionem habentia, quam quadratus numerus ad quadratum numerum, & latera habebunt longitudine commensurabilia. Quæ vero à rectis lineis longitudine incommensurabilibus fiunt quadrata, inter se proportionem non habent, quam quadratus numerus ad quadratum numerum: & quadrata inter se proportionem non habentia, quam quadratus numerus ad quadratum numerum, neq; latera habebunt longitudine commensurabilia.

1. *Hyp.* A. ⊔ B. Dico Aq. Bq :: Q. Q.

Nam [a] ſit A. B :: num. E. num. F. ergo $\frac{Aq}{Bq}$ ([b] $\frac{A}{B}$ bis)[c] = $\frac{E}{F}$ bis. [d] = $\frac{Eq}{Fq}$ [e] ergo Aq. Bq :: Eq. Fq :: Q. Q. Q. E. D.

a per 5. 10.
b 20. 6.
c ſch. 23. 5.
d 11, 8.
e 11. 5.

2. *Hyp.* Aq. Bq :: Eq. Fq :: Q. Q. Dico A ⊔ B. Nam $\frac{A}{B}$ bis ([f] $\frac{Aq}{Bq}$)[g] = $\frac{Eq}{Fq}$ [h] = $\frac{E}{F}$ bis. [i] ergo A. B :: E. F :: N. N. [k] quare A ⊔ B. Q. E. D.

f 10. 6.
g hyp.
h 11. 8.
i ſch. 23. 5.
k 6. 10.

3. *Hyp.* A ˈ⊔ B. Nego eſſe Aq. Bq :: Q. Q. Nam dic Aq. Bq :: Q. Q. Ergo A ⊔ B, ut modo oſtenſum eſt, contra Hypoth.

4. *Hyp.* Non Aq. Bq :: Q. Q. Dico A ˈ⊔ B. Nam puta A ⊔ B; ergo Aq. Bq :: Q. Q. ut modo diximus, contra Hypoth.

Coroll.

Lineæ ⊔ ſunt etiam ⊓; at non contra. Sed lineæ ˈ⊔ non ſunt idcirco ˈ⊓. Lineæ vero ˈ⊓ ſunt etiam ˈ⊔.

PROP. X.

Si quatuor magnitudines proportionales fuerint (C. A :: B. D;) prima vero C secundæ A fuerit commensurabilis; & tertia B quartæ D commensurabilis erit. Et si prima C secundæ A fuerit incommensurabilis, & tertia B quartæ D incommensurabilis erit.

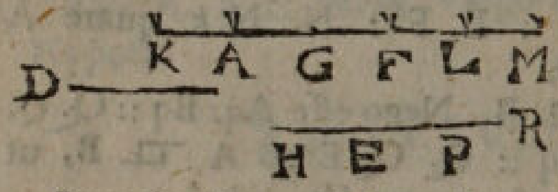
Si C ⊤⊥ A, *a* ideo erit C. A :: N.
a 5. 10. N*b* :: B. D. *b* ergo B ⊤⊥ D. Sin C
b 6. 10. '⊤⊥ A, ergo *c* non erit C. A :: N.N :: B. D.
c 7. 10. *d* quare B '⊤⊥ D. Q. E. D.
d 8. 10.

LEMMA 1.

Duos numeros planos invenire, qui proportionem non habeant, quam quadratus numerus ad quadratum numerum.

Huic Lemmati satisfacient duo quilibet numeri plani non similes, quales sunt numeri habentes proportionem superparticularem, vel superbipartientem, vel duplam; vel etiam duo quivis numeri primi. vid. Schol. 27. 8.

LEMMA 2.



B, 5, C, 3.

Invenire lineam HR, *ad quam data recta linea* KM *sit in ratione datorum numerorum* B, C.

Sch. 10 6. *a* Divide K M in partes æquales æque multas unitatibus numeri B. harum tot, quot unitates
3. 1. sunt in numero C, *b* componant rectam H R. liquet esse KM. HR :: B. C.

LEMMA 3.

Invenire lineam D, *ad cujus quadratum datæ rectæ* K M *quadratum sit in ratione datorum numerorum* B, C.

Fac

Fac *B*. C *a* :: KM. HR. ac inter KM, & a 2. lem. 10. 10.
HR *b* inveni mediam proportionalem D. Erit b 13. 6.
KMq. Dq *c* :: KM. HR *d* :: B. C. c 20. 6. d constr.

PROP. XI.

A ——————— B. 20.
E ———————— C. 16.
D ——————

Propositæ rectæ lineæ A invenire duas rectas lineas incommensurabiles; alteram quidem D longitudine tantum, alteram vero E etiam potentia.

1. Sume numeros B, C, *a* ita ut non sit B. C :: Q. Q. *b* fiatque B. C :: Aq. Dq. *c* liquet A ⊏⊐ D. Sed Aq *d* ⊏⊐ Dq. Q. E. F. a 2. lem. 10. 10. b 3. lem. 10. 10. c 9. 10. d 6. 10

2. *d* Fac A. E :: E. D. Dico Aq ⊏⊐ Eq. Nam A. D *e* :: Aq. Eq. ergo cum A ⊏⊐ D, ut prius, *f* erit Aq ⊏⊐ Eq. Q. E. F. d 13. 6. e 10. 6. f 10. 10.

PROP. XII.

Quæ (A, B) eidem magnitudini C sunt commensurabiles, & inter se sunt commensurabiles.

D, 18. E, 8.
F, 2. G, 3.
H, 5. I, 4. K, 6.

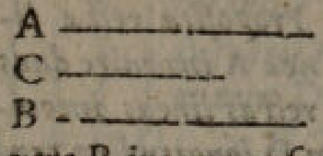
Quia A ⊏⊐ C, & C ⊏⊐ B, *a* sit A. C :: N. N :: D. E. atque C. *B* :: N. N :: F. G. *b* sumantur tres numeri H, I, K minimi ∺ in rationibus D ad E, & F *c* ad G. Iam quia A. C *c* :: D. E *c* :: H. I. ac C. B *c* :: F. G. *c* :: I. K. *d* erit ex æquali A. B :: H. K :: N. N. *e* ergo A ⊏⊐ B. Q. E. D. a 5. 10. b 4. 8. c constr. d 22. 5. e 6. 10.

Schol.

Hinc, omnis recta linea rationali lineæ commensurabilis, est quoque *f* rationalis. Et omnes rectæ rationales inter se commensurabiles sunt, saltem potentia. Item, omne spatium rationali spatio commensurabile, est quoque rationale; & omnia spatia rationalia inter se com- 12. 10 & def. 6. def. 9.

mensurabilia sunt. Magnitudines vero, quarum altera est rationalis, altera irrationalis, sunt inter se incommensurabiles. *def 7. & 10.*

PROP. XIII.



Si sint duæ magnitudines A, B; & altera quidam A eidem C sit commensurabilis, altera vero B incommensurabilis; incommensurabiles erunt magnitudines A, B.

a hyp. Dic B $\overline{\sqcup}$ A. ergo cum C [a] $\overline{\sqcup}$ A, [b] erit C
b 12. 10. $\overline{\sqcup}$ B, contra Hypoth.

PROP. XIV.

Si sint duæ magnitudines commensurabiles A, B; altera autem ipsarum A magnitudini cuipiam C incommensurabilis fuerit; & reliqua B eidem C incommensurabilis erit.

a hyp. Puta B $\overline{\sqcup}$ C. ergo cum A $\overline{\sqcup}$ [a] B,
b 12. 10. [b] erit A $\overline{\sqcup}$ C, contra Hyp.

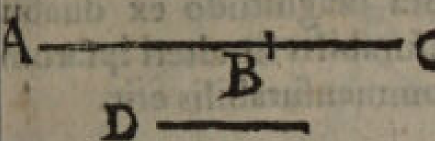
PROP. XV.

Si quatuor rectæ lineæ proportionales fuerint (A. B :: C. D;) prima vero A tanto plus posit quam secunda B, quantum est quadratum rectæ lineæ sibi commensurabilis longitudine; & tertia C tanto plus poterit, quam quarta D, quantum est quadratum rectæ lineæ sibi longitudine commensurabilis. Quod si prima A tanto plus posit quam secunda B, quantum est quadratum rectæ lineæ sibi incommensurabilis longitudine; & tertia C tanto plus poterit, quam quarta D, quantum est quadratum rectæ lineæ sibi longitudine incommensurabilis.

a hyp. Nam quia A. B [a] :: C. D. [b] erit Aq. Bq ::
b 21. 6.
c 17. 5. Cq. Dq. [c] ergo dividendo Aq − Bq. Bq :: Cq − Dq.

Dq.Dq. *d* quare √ : Aq − B¦. B :: √:Cq − Dq. d 22.6.
D. *c* invertendo igitur B.√ : Aq − Bq :: D. √ : e cor.4.5. f 22.5.
Cq − Dq. *f* ergo ex æquali A. √ : Aq − Bq ::
C. √ : Cq − Dq. proinde si A ⊤⊥, vel ⊤⊥ √
Aq − Bq, *g* erit similiter C ⊤⊥, vel ⊤⊥ √ : g 10.10.
Cq − Dq. Q. E. D.

PROP. XVI.



Si duæ magnitudines commensurabiles AB, BC *componantur, & tota magnitudo* AC *utrique ipsarum* AB, BC *commensurabilis erit: quod si tota magnitudo* AC *uni ipsarum* AB, *vel* BC *commensurabilis fuerit; & quæ à principio magnitudines* AB, BC *commensurabiles erunt.*

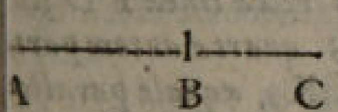
1. *Hyp.* *a* Sit D ipsarum AB, BC communis mensura. *b* ergo D metitur AC. *c* ergo AC ⊤⊥ AB, & BC. Q. E. D. a 3.10. b 1.*ax*.10. c 1.*def*.10.

2. *Hyp.* *a* Sit D communis mensura ipsarum AC, AB; *d* ergo D metitur AC − AB (BC;) proinde AB ⊤⊥ BC. Q. E. D. d 3.*ax*.10.

Coroll.

Hinc etiam, si tota magnitudo ex duabus composita, commensurabilis sit alteri ipsarum, eadem & reliquæ commensurabilis erit.

PROP. XVII.



Si duæ magnitudines incommensurabiles AB, BC *componantur, & tota magnitudo* AC *utrique ipsarum* AB, BC *incommensurabilis erit: Quod si tota magnitudo* AC *uni ipsarum* AB *incommensurabilis fuerit, & quæ à principio magnitudines* AB, BC *incommensurabiles erunt.*

1. *Hyp.*

1. *Hyp.* Si fieri poteſt, ſit D ipſarum A C

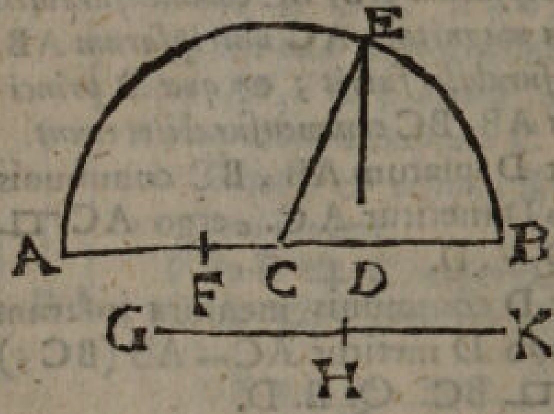
a 3 ax. 10. AB communis menſura. *a* ergo D metitur
b 1. def. 10. AC — AB (BC.) *b* ergo AB $\sqcap\!\!\sqcup$ BC, contra Hypoth.

c 16, 10. 2. *Hyp.* Dic A B $\sqcap\!\!\sqcup$ B C. *c* ergo A C $\sqcap\!\!\sqcup$ AB, contra Hypoth.

Coroll.

Hinc etiam, ſi tota magnitudo ex duabus compoſita, incommenſurabilis ſit alteri ipſarum eadem & reliquæ incommenſurabilis erit.

PROP. XVIII.

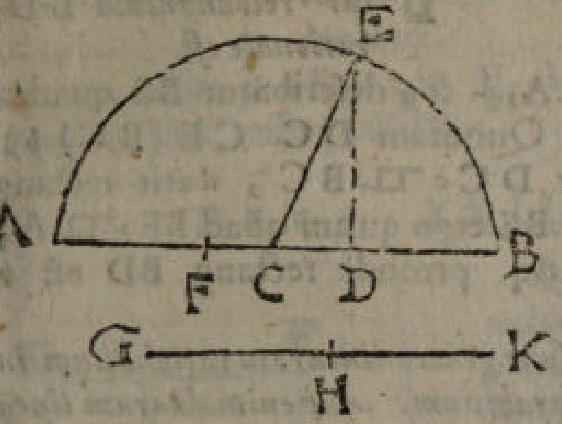


Si fuerint duæ rectæ lineæ inæquales AB, GK; *quartæ autem parti quadrati, quod fit à minori* GK *æquale parallelogrammum* ADB *ad majorem* AB *applicetur, deficiens figura quadrata, & in partes* AD, DB *longitudine commenſurabiles ipſam dividat; major* AB *tanto plus poterit quam minor* GK, *quantum eſt quadratum rectæ lineæ* FD *ſibi longitudine commenſurabilis.* *Quod ſi major* AB *tanto plus poſſit, quam minor* GK, *quantum eſt quadratum rectæ lineæ* FD *ſibi longitudine commenſurabilis; quartæ autem parti quadrati, quod fit à minori* GK, *æquale parallelogrammum* ADB *ad majorem* AB *applicetur; deficiens figura quadrata, in partes* AD, DB *longitudine commenſurabiles ipſum dividet.*

a 10. 1. *a* Biſeca GK in H; & *b* fac rectang. ADB =
b 28 6. GHq: abſcinde AF = DB. Eſtque ABq *c* =
c 8. 2.
d conſtr. & 4. 2. 4 ADB *d* (4 GHq, vel GKq) + FDq. Iam primo

rimo, Si AD ⊐⊏ DB, erit AB *e* ‾⊐⊏ DB *e* ⊐⊏ e 16. 10.
DB *f* (AF + DB, vel AB − FD) *g* ergo f constr.
B ⊐⊏ FD. Q. E. D. Sin secundo, AB ‾⊐⊏ g cor. 16 10
D, *h* erit ideo AB ⊐⊏ AB − FD (2 DB) h cor. 16. 10.
, ergo AB ⊐⊏ DB. *l* quare AD ⊐⊏ DB. k 12. 10. l 16 10.
Q. E. D.

Prop. XIX.



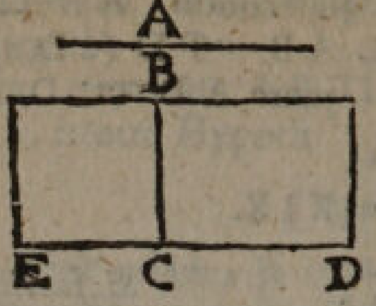
Si fuerint duæ rectæ lineæ inæquales, AB, GK; quartæ autem parti quadrati, quod fit à minore GK, æquale parallelogrammum ADB ad majorem AB applicetur, deficiens figura quadrata; & in partes incommensurabiles longitudine AD, DB, ipsam AB dividat; major AB tanto plus poterit, quam minor GK, quantum est quadratum rectæ lineæ FD, sibi longitudine incommensurabilis. Quod si major AB tanto plus possit, quam minor GK, quantum est quadratum rectæ lineæ FD sibi longitudine incommensurabilis; quartæ autem parti quadrati, quod fit à minore GK, æquale parallelogrammum ADB ad majorem AB applicetur, deficiens figura quadrata; in partes longitudine incommensurabiles AD, DB ipsam AB dividet.

Facta puta, & dicta eadem, quæ in præcedenti. Itaque primo, Si AD ⊐⊏ DB, *a* erit propterea AB ⊐⊏ DB; *b* quare AB ⊐⊏ 2 DB (AB − FD) *c* ergo AB ⊐⊏ FD. Q. E. D. a 17. 10. b 13. 10. c cor. 17 10.

Secundo, Si AB ⊐⊏ FD; *c* ergo AB ⊐⊏ AB − FD (2 DB;) *d* quare AB ⊐⊏ DB, & proinde AD ⊐⊏ DB. Q. E. D. d 13. 10. e 17. 10

Prop.

PROP. XX.



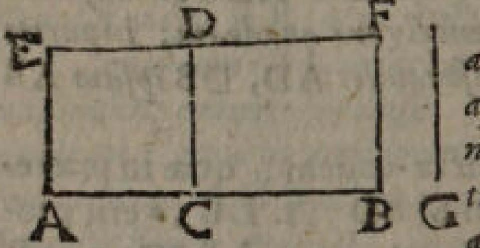
Quod sub rationalibus longitudine commensurabilibus rectis lineis B C, C D, *secundum aliquem prædictorum modorum, continetur rectangulum* B D; *rationale est.*

a 46. 1. Exponatur A, ῥ. & *a* describatur BE quadra-
b 1. 6. tum ex B C. Quoniam D C. C E (B C) *b* ::
c hyp. B D. B E. & D C *c* ⊓⊔ B C; *d* erit rectang.
d 10. 10. BD ⊓⊔ quad. BE. ergo quum quad. BE *e* ⊓⊔ Aq;
e hyp & 9. def. 10. *f* erit BD ⊓⊔ Aq. proinde rectang. BD est ῥν.
f 12. 10. Q. E. D.

Not. *Tria sunt genera linearum rationalium inter se commensurabilium. Aut enim duarum linearum rationalium longitudine inter se commensurabilium altera æqualis est expositæ rationali; aut neutra rationali expositæ æqualis est, longitudine tamen ei utraque est commensurabilis; aut denique utraque expositæ rationali commensurabilis est solum potentia. Hi sunt modi illi, quos innuit præsens theorema.*

In numeris, sit BC, $\sqrt{8}$ ($2\sqrt{2}$) & CD, $\sqrt{18}$ ($3\sqrt{2}$,) erit rectang. BD $= \sqrt{144} = 12$.

PROP. XXI.



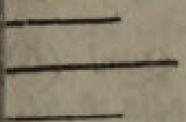
Si rationale D B *ad rationalem* D C *applicetur, latitudinem* C B *efficit rationalem, & ei* D C *ad quam applicatum est* DB, *longitudine commensurabilem.*

a 1. 6. Exponatur G, ῥ. & describatur D A quadra-
b hyp. tum ex B C. quoniam B D. DA *a* :: B C. C A;
c sch. 12. 10. atque, BD DA *b* sunt ῥα, *c* ideoque ⊓⊔; *d* erit
d 10. 10. BC

⊤⊓ CA. at CD (CA) *b* est ρ́. *e* ergo BC ρ́. Q. E. D. *e sch. 12. 10.*

In numeris, sit rectang. DB, 12; & DC, √ 8. et CB, √ 18. atqui √18 = 3 √ 2. & √ 8 = 2 √ 2.

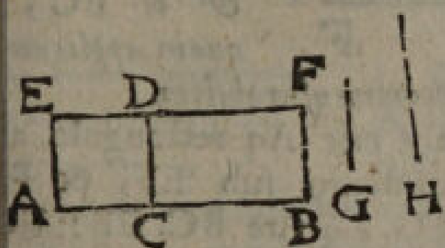
LEMMA.



Duas rectas rationales potentia solum commensurabiles invenire

Sit A exposita ρ́. *a* Sume B ⊤⊓ A, *b* & C ⊤⊓ B. liquet B, & C esse quæsitas. *a 11. 10.* *b sch. 12. 10.*

PROP. XXII.



Quod sub rationalibus DC, CB potentia solum commensurabilibus rectis lineis continetur rectangulum DB, irrationale est; & recta linea H ipsum potens, irrationalis; vocetur autem Media.

Sit G exposita ρ́. & describatur DA quadratum ex DC; sitque Hq = DB. Quoniam AC. CB *a* :: DA. DB. *b* atque AC ⊤⊓ CB, *c* erit DA ⊤⊓ DB (Hq). *d* atqui Gq ⊤⊓ DA. *e* ergo Hq ⊤⊓ Gq. *f* ergo H est ρ́. Q. E. D. vocetur autem Media. quia AC. H :: H. CB.

a 1. 6. *b hyp.* *c 10. 10.* *d hyp. & 9. def. 10.* *e 13. 10.* *f 11. 10.*

In numeris, sit DC, 3; & CB, √ 6. erit rectangulum DB (Hq) √ 54. quare H est υ√ 54.

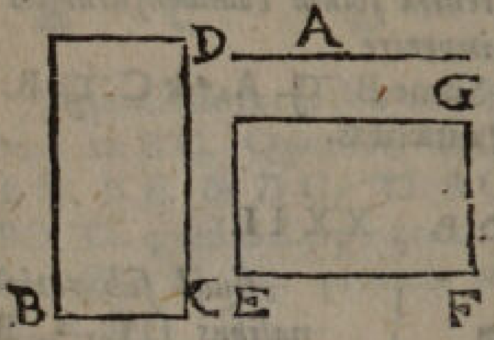
Mediæ nota est μ, Medii vero $\mu\nu$; pluraliter $\mu\alpha$.

SCHOL.

Omne rectangulum, quod potest contineri sub duabus rectis rationalibus potentia solum commensurabilibus, est Medium; quamvis contineatur sub duabus rectis irrationalibus: atque omne

omne Medium poteſt contineri ſub duabus recti rationalibus potentia tantum commenſurabilibus, ut exemp. gr. √ 24 eſt μν. quia continetur ſub √ 3, & √ 8, qui ſunt ρ̇ ⊔. etſi poſſet contineri ſub v√ 6, & v√ 96 irrationalibus; nam √ 94 = v√ 576 = v√ 6 in v√ 96.

PROP. XXIII.



Quod (BD) à media A fit, ad rationalem BC applicatum, latitudinem CD rationalem efficit & ei BC, ad quam applicatum eſt BD longitudine incommenſurabilem.

a ſch 12. 10. Quoniam A eſt μ, a erit Aq rectangulo al

b 1 ax. 1. cui (EG) æquale contento ſub EF, & FG

c 14. 6. ρ̇ ⊔. b ergo BD = EG. c quare BC. EF :: FG

d 22. 6. CD. d ergo BCq. EFq :: FGq. CDq. ſed BCq

e hyp. & EFq e ſunt ρ̇α; f ideoque ⊓. g ergo FGq ⊓

f ſch. 12. 10.

g 10. 10

h ſch. 12. 10. CDq. Ergo quum FG ſit ρ̇, h erit CD ρ̇. Por

k 1. 6. ro, quia EF. FG k :: EFq. EG (BD); o

l 10. 10. EF ⊓ FG, l erit EFq ⊓ BD. verum EF

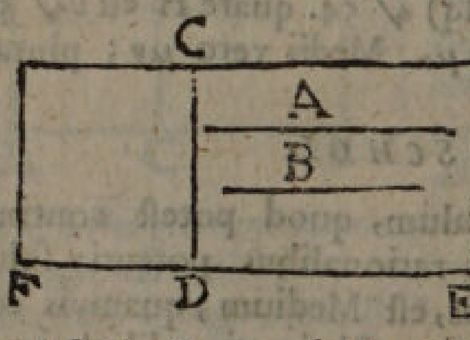
m ſch. 12. 10. m ⊓ CDq. n ergo rectang. BD ⊓ CDq

n 13. 10. quum igitur CDq. BD o :: CD. BC. p erit CD

o 1. 6.

p 10. 10. ⊓ BC. ergo, &c.

PROP. XXIV.



Mediæ A commenſurabilis B, media eſt.

a 11. 6. Ad CD a fac rectang CE = Aq; a & rectang. CF = Bq. Quoniam

b hyp. Aq (CE) eſt μν, b & CD ρ̇, c erit latitudo

c 23. 10 DE

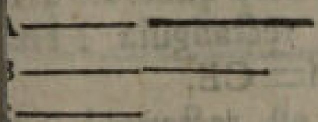
E ρ̇ ⊓ CD. Quoniam vero CE. CF *d* :: d 1. 6.
D. DF, & CE *e* ⊔ CF, *f* erit ED ⊔ DF. e *hyp.*
ergo DF eſt ρ̇ ⊔ CD. *h* ergo rectang. CF f 10. 10.
Bq) eſt μν. & proinde B eſt μ. Q. E. D. g 12. & 13 10.
h 22. 10.

Nota *quod ſignum* ⊓ *plerumque valet potentia tantum commenſurabile, ut in hac demonſtratione, & in præced. &c. quod intellige, ut ex uſu erit, & juxta citationes.*

Coroll.

Hinc liquet ſpatium medio ſpatio commenſurabile medium eſſe.

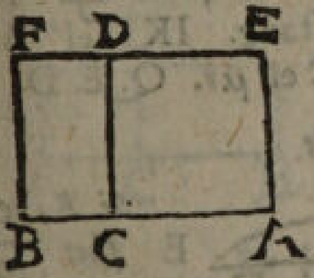
LEMMA.



Duas rectas medias A, B *longitudine commenſurabiles ; item duas* A, C *potentia tantum commenſurabiles invenire.*

a Sit A μ quævis; ſume B ⊔ A; *c*& C ⊓ A. a *lem* 22. 10. & 13. 6
Factum eſſe liquet. b 2. *lem.* 10. 10.
c 3 *lem.* 10. 10.
d *conſtr.* & 24. 10.

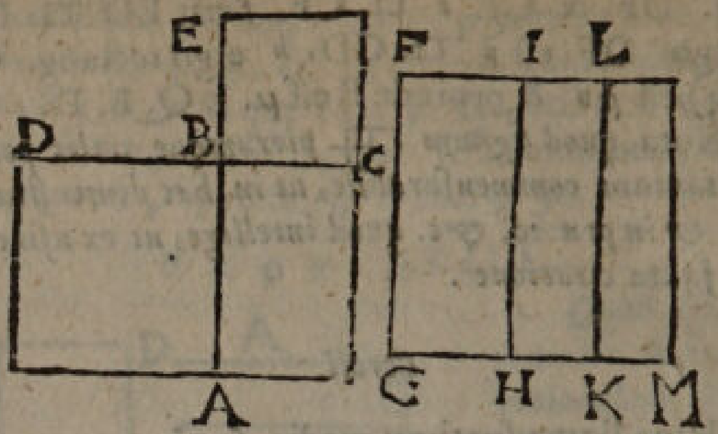
PROP. XXV.



Quod ſub DC, CB *mediis longitudine commenſurabilibus rectis lineis continetur rectangulum* DB, *medium eſt.*

Super DC conſtruatur quadratum DA. Quoniam a 1. 6.
AC. (DC) CB *a* :: DA. DB. & DC ⊔ CB; b 10. 10.
b erit DA ⊔ DB. *c* ergo DB eſt μν. Q. E. D. c 24. 10.

PROP. XXVI.



Quod sub mediis potentia tantum commensurabilibus rectis lineis A B, BC *continetur rectangulum* AC, *vel rationale est, vel medium.*

Super rectas AB, BC *a* describe quadrata AD
a 46 1. CE. atque ad FG ρ̇, *b* fac rectangula FH=
b cor. 16. 6. AD, *b* & IK=AC, *a b* & LM=CE.

Quadrata AD, CE, hoc est, rectangula FH
LM *c* sunt μα, & ⊓; ergo eandem habentes
c hyp. & 24. 20. rationem GH, KM sunt *d* ρ̇, & *e* ⊓. *f* ergo
d 23. 10. GH×KM est ῥν. atqui quia AD, AC, CE
e 10. 10. hoc est FH, IK, LM *g* sunt ∺; & *h* proinde
f 20. 10.
g sch. 22. 6. GH, HK, KM etiam ∺, *k* erit HKq=GH×
h 1. 6. KM; *l* ergo HK est ρ̇, vel ⊓, vel ⊓̸ IF
k 17. 6.
l 12. 10. (GF); si ⊓, *m* ergo rectang. IK vel AC
m 20. 10. est ῥν. Sin ⊓̸. *n* ergo AC est μν. Q. E. D.
n 22. 10.

LEMMA.

Si A, & E *sint* ⊓̸ *tantum;*

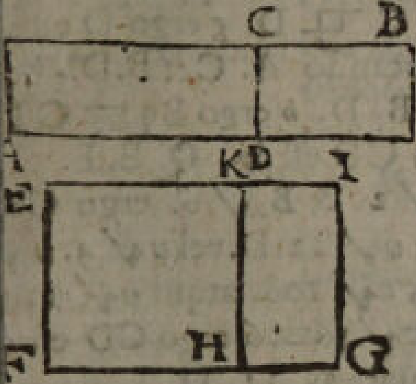
a hyp & 16. 10. *Erunt primo,* Aq, Eq, Aq+Eq, Aq−Eq *a* ⊓.
Erunt secundo, Aq, Eq, Aq+Eq, Aq−Eq ⊓
b 1, 6. AE, & 2 AE. Nam A. E *b* :: Aq. AE *b* :: AE
c hyp. Eq. ergo cum A *c* ⊓̸ E. *d* erit Aq ⊓̸ AE, *e* &
d 10. 10.
e 14. 10. 2 AE. item Eq *d* ⊓̸ A E, *e* & 2 AE. æquare cum
Aq+Eq ⊓ Aq, & Eq; & Aq−Eq ⊓ Aq, &

Eq

1, *f* erunt Aq+Eq, *f* & Aq−Eq ⊥ AE, & f 14 10.
AE.

Hinc erunt tertio, Aq, Eq, Aq+Eq, Aq−Eq, AE *g* ⊥ Aq+Eq+2AE; & Aq+Eq−2AE. g 14, 16, & 17 10.
& Aq+Eq+2AE ⊥ Aq+Eq−2AE. h cor. 7. 10.
(Q. A−E.)

PROP. XXVII.

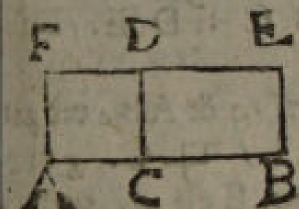


Medium AB *non superat medium* AC *rationali* DB.

Ad EF ρ', *a* fac EG = AB, *a* & EH = AC. Rectangula AB, AC, hoc est, EG, EH *b* sunt μα, *c* ergo FG, & FH sunt ρ' ⊥ EF. a cor. 16. 6. b hyp. c 23. 10.

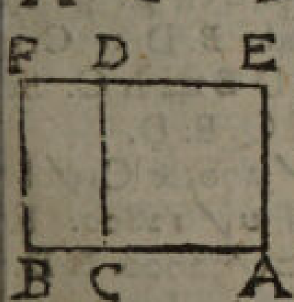
Atque si KG, *d* id est DB sit ρ'ν, *e* erit HG ⊥ HK; *f* quare HG ⊥ FH. *g* ergo FGq ⊥ FHq. *h* ergo FG est ρ'. verum prius erat FG ρ'. Quæ repugnant. d 3. ax. 1. e 21. 10. f 13. 10. g lem. 26 10. h sch. 12. 10.

SCHOL.



1. *Rationale* AE *superat rationale* AD *rationali* CE.

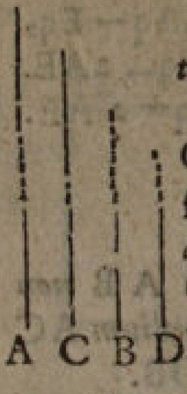
Nam AE *a* ⊥ AD; *b* ergo AE ⊥ CE. *c* quare CE est ρ'ν. Q. E. D. a hyp. b cor. 16. 10. c sch. 12. 10.



2. *Rationale* AD *cum rationali* CF *facit rationale* AF.

Nam AD *a* ⊥ CF; *b* quare AF ⊥ AD, & CF. *c* proinde AF est ρ'ν. Q. E. D. a sch. 12. 10. b 16 10. c sch. 12. 10.

PROP. XXVIII.



Medias invenire (C, & D) quæ rationale CD contineant.

a lem. 21. 10.
b 13. 6.
c 12. 6.
d 22. 10.
e conſt.
f 10. 10.
g 24. 10.
h 17. 6.
h ſch. 12. 10.

a Sume A, & B ρ̄ ⊓. b fac A. C :: C. B. c atque A. B :: C. D. Dico factum. Nam AB (Cq) d eſt μν; d unde C eſt μ. quum vero A. B e :: C. D, f erit C ⊓ D. g ergo D eſt μ. porro permutando A. C :: B. D. e hoc eſt C. B :: B. D. h ergo Bq = CD. atqui B q e eſt ρ̄ν. h ergo CD eſt ρ̄ν. Q. E. F.

In numeris, ſit A, √ 2; & B, √ 6. ergo C eſt u√ 12. fac √ 2. √ 6 :: u√ 12. D. vel u√ 4. u√ 36 :: u√ 12. D. erit D, u√ 108. atqui u√ 12 in u√ 108 = u√ 1296 = √ 36 = 6. ergo CD eſt 6. item C. D :: 1. √ 3. quare C ⊓ D.

PROP. XXIX.

Medias invenire potentia tantum commenſurabiles D, & E, quæ medium DE contineant.

a lem 21. 10.
b 13. 6.
c 12. 6.
d 17. 6.
e 22. 10.
f conſtr.
g 10. 10.
h 24. 10
k conſtr. & cor. 4. 5.
l 16. 6.
m 22. 6.

a Sume A, B, C ρ̄ ⊓. Fac A. D b :: D. B. c & B. C :: D. E. Dico factum.

Nam AB d = Dq & AB e eſt μν. ergo D eſt μ. & B f ⊓ C. g ergo D ⊓ E. h ergo E eſt μ. porro B. C f :: D. E, & permutando B. D :: C. E k hoc eſt D. A :: C. E. l ergo DE = AC. Sed AC m eſt μν. ergo DE eſt μν. Q. E. D.

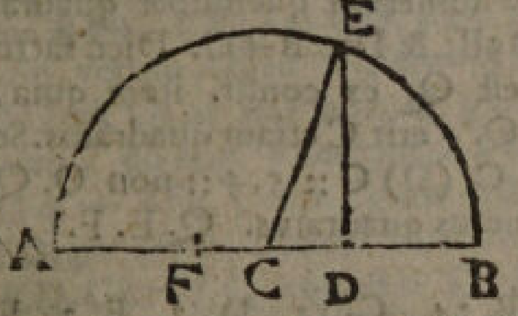
In numeris ſit A, 20; & B, √ 200; & C, √ 80. Ergo D eſt √√ 80000; & E u√ 12800. Ergo DE = √√ 1024000000 = √ 32000. & D. E :: √ 10. 2. quare D ⊓ E.

Schol

S C H O L.

, 6.	C, 12.	*Invenire duos numeros pla-*
, 4.	D, 8.	*nos similes vel dissimiles.*
B,24.	CD,96.	Sume quoscunque quatu-
		or numeros proportionales,
, 6.	C, 5.	A.B :: C. D. liquet AB, &
, 4.	D, 8.	CD esse similes planos. Pla-
B,24.	CD,40.	nos autem dissimiles quot-
		cunque reperies ope scholii
		27. 8.

L E M M A.



1. *Duos numeros quadratos* (DEq & CDq) *venire, ita ut compositus ex ipsis* (CEq) *quadra-us etiam sit.*

Sume AD, DB numeros planos similes (quo-um ambo pares sint, vel ambo impares) nimi-um AD, 24. & DB, 6. Horum summa, (AB) st 30; differentia (F D) 18, cujus semissis CD) est 9. *a* Habent vero plani similes A D, DB unum medium numerum proportionalem, empe D E. patet igitur singulos numeros C E, CD, DE rationales esse; proinde CEq (*b* CDq + DEq) est numerus quadratus requisitus. *a* 18. 8. *b* 47. 1.

Facile itaque invenientur duo numeri quadra-i, quorum excessus sit quadratus, vel non qua-ratus numerus. nempe ex eadem constructione, erit CEq — CDq = DEq. *c* 3 ax. 1.

Quod si A D, D B sint numeri plani dissimi-

 les,

les, non erit media proportionalis (DE) numerus rationalis; proinde quadratorum CEq CDq excessus (DEq) non erit numerus quadratus.

LEMMA 2.

2. *Duos numeros quadratos* B, C *invenire, ita ut compositus ex ipsis* D, *non sit quadratus. item quadratum numerum* A *dividere in duos numeros* B, C *non quadratos.*

A, 3. B, 9. C, 36. D, 45.

1. Sume numerum quemlibet quadratum B, sitque C=4B; & D=B+C. Dico factum.

Nam B est Q. ex constr. item quia B. C :: 1.4 :: Q. Q. *a* erit C etiam quadratus. Sed quoniam B+C. (D) C :: 5.4 :: non Q. Q. *b* non erit D numerus quadratus. Q. E. F.

a 24. 8.
b cor. 24. 8.

A, 36. B, 24. C, 12. D, 3. E, 2. F, 1.

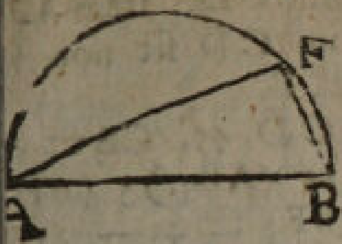
2. Sit A numerus quivis quadratus. Accipe D, E, F numeros planos dissimiles, sitque D=E+F. fac D. E :: A. B. & D. F :: A. C. Dico factum.

Nam quia D. E+F :: A. B+C. & D=E+F *a* erit A=B+C. Iam dic B quadratum esse *b* ergo A & B, & *c* proinde D & E, sunt numeri plani similes, contra Hypoth. idem absurdum sequetur, si C dicatur quadratus. ergo &c.

a 24. 5.
b 21. def. 7.
c 26. 8.

PROP

PROP. XXX.



Invenire duas rationales A B, A F *potentia tantum commensurabiles, ita ut major* A B *plus posſit, quam minor* A F, *quadrato rectæ lineæ* B F *longitudine ſibi commenſurabilis.*

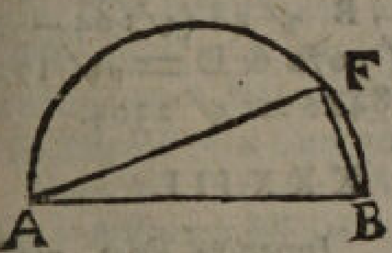
C.... E..... D

Exponatur AB, ρ'. *a* Sume CD, CE numeros a 1.lem.29.
uadratos, ita ut CD — CE (ED) ſit non Q. 10.
Fiatque CD. ED :: ABq. AFq. In circulo b 3.lem.10.
iper AB diametrum deſcripto *c* aptetur AF, 10.
ucaturque BF. Sunt AB, AF, quas petis. c 1.4.

Nam ABq. AFq *d* :: CD. ED. *e* ergo ABq d conſtr.
⊓ AFq. verum AB eſt ρ'. *f* ergo AF eſt ρ'. ſed e 6. 10.
uia CD eſt Q: at ED non Q: *g* erit AB ⊓ f ſch.12.10.
F. porro, ob ang. *h* rectum AFB, eſt ABq g 9. 10.
= AFq + BFq; cum igitur ABq. AFq :: h 31.3.
D. ED. per converſionem rationis erit ABq. k 47. 1.
BFq :: CD. CE :: Q. Q. *l* ergo AB ⊓ BF. Q.E.F. l 9. 10.

In numeris; ſit AB, 6; CD, 9, CE, 4; quare
ED, 5. Fac 9. 5 :: 36. (Q: 6) AFq. erit AFq
20. proinde AF √20. ergo BFq = 36 — 20 =
16. quare BF eſt 4.

PROP. XXXI.



Invenire duas rationales AB, AF *potentia tantum commenſurabiles, ita ut major* A B *plus poſsit, quam minor* AF, *quadrato rectæ lineæ* B F *ſibi longitudine incommenſurabilis.*

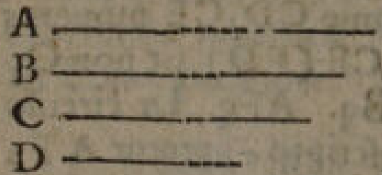
C......... E.... D

Exponatur AB, ρ'. *a* accipe numeros CE, ED a 2.lem.29.
quadratos, ita ut CD = CE + ED ſit non Q. 10.
& in reliquis imitare conſtructionem præcedentis. Dico factum.

Nam, ut ibi, AB, AF sunt $\dot{\rho}$ ⊐⊏. item AB: BFq :: CD. ED. ergo cum CD sit non Q
b 9. 10. b erunt AB, BF ⊐⊏. Q. E. F.

In numeris, sit AB, 5. CD, 45. CE = 36 ED = 9. Fac 45. 9 :: 25 (ABq.) 5 (AFq) ergo AF = √5. proinde BFq = 45 − 25 = 20. quare BF = √20.

PROP. XXXII.

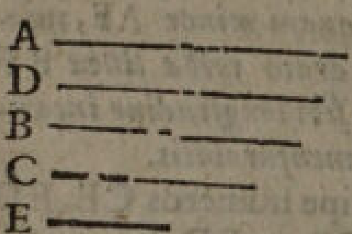


Invenire duas medias C, D potentia tantum commensurabiles, quæ rationale CD contineant, ita ut major C plus possit, quam minor D quadrato rectæ lineæ sibi longitudine commensurabilis.

a 30. 10. a Accipe A, & B $\dot{\rho}$ ⊐⊏; ita ut √Aq − Bq ⊐⊏
b 13. 6. A. b Fiatque A. C :: C. B, c atque A. B :: C
c 12. 6. D. Dico factum.
d constr.
e 21. 10. Nam quia A, & d B sunt $\dot{\rho}$ ⊐⊏, e erit C (f √
f 17. 6. AB) μ. item g ideo C ⊐⊏ D. h ergo D etiam
g 10. 10. μ. porro quia A. B d :: C. D; & permutatim A
h 24. 10. C :: B. D :: C. B; & Bq d est $\dot{\rho}v$, erit CD
k 17. 6. k (Bq) $\dot{\rho}v$. Denique quia √Aq − Bq d ⊐⊏
l 15. 10. A, l erit √Cq − Dq ⊐⊏ C. ergo, &c. Sin √ Aq − Bq ⊐⊏ Aq, erit √Cq − Dq ⊐⊏ C.

In numeris, sit A, 8; B, √48 (√: 64 − 16) ergo C = √AB = v√3072. & D = v√1728 quare CD = v√5308416 = √2304.

PROP. XXXIII.



Invenire duas medias D, E potentia solum commensurabiles, quæ medium DE contineant, ita ut major D plus possit, quam minor E, quadrato rectæ lineæ sibi longitudine commensurabilis.

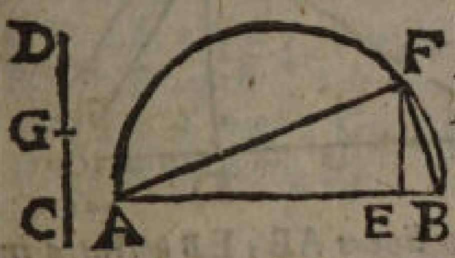
Sume

[a] Sume A, & C ρ^c, ⊤⊥; ita ut $\surd$ Aq − Cq ⊓⊔ A. [b] sume etiam B ⊤⊥ A, & C; & fac A. D [c] :: D. B [d] :: C. E. Erunt D, & E quæsitæ. a 30. 10. b lem. 21. 10. c 13. 6. d 12. 6.

Nam quoniam A, & C [e] sunt ρ^c, [e] & B ⊤⊥ A & C, [f] erit B ρ^c, & D ($\surd$ AB) [g] erit μ. Quia vero A. D :: C. E. erit permutando A. C :: D. E. ergo cum A ⊤⊥ C, [h] erit D ⊤⊥ E. ergo E est μ. porro, [l] quia D. B :: C. E; [l] & BC est $\mu\nu$, etiam DE ei [m] æquale est $\mu\nu$. deniq; propter A. C :: D. E. [e] quia $\surd$ Aq − Cq ⊓⊔ A, [n] erit $\surd$ Dq − Eq ⊓⊔ D. ergo, &c. Sin $\surd$ Aq − Cq ⊓⊔ A, erit $\surd$ Dq − Eq ⊓⊔ Eq. e constr. f sch. 12. 10. g 22. 10. h 10. 10. k 14. 10. l 22. 10. m 16 6 n 15. 10.

In numeris, sit A, 8; C, $\surd$ 48; B, $\surd$ 28. erit D $\surd$ 3072; & E $\surd$ 588. quare D. E :: 2. $\surd$ 3. & DE = $\surd$ 1344.

PROP. XXXIV.



Invenire duas rectas lineas AF, BF potentia incommensurabiles, quæ faciant compositum quidem ex ipsarum quadratis rationale, rectangulum vero sub ipsis contentum, medium.

[a] Reperiantur AB, CD ρ^c ⊤⊥; ita ut $\surd$ ABq − CDq ⊓⊔ AB. [b] biseca CD in G. [c] fac rectang. AEB = GCq. Super AB diametrum duc semicirculum AFB. erige perpendicularem EF. duc AF, BF. Hæ sunt quæ indagandæ erant. a 31. 10. b 10. 1. c 28. 6. d 12. 6. e cor. 8. 6 & 17. 6. f 7. 5.

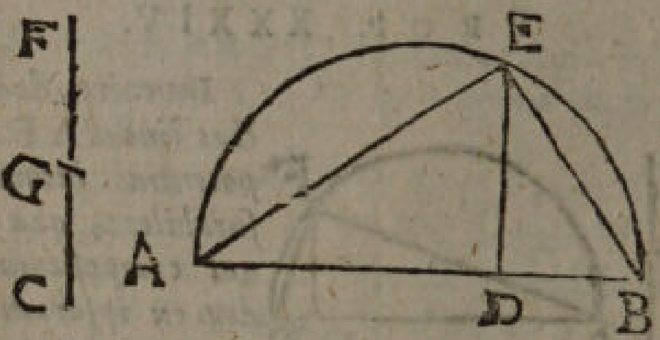
Nam AE. BE [d] :: BA x AE. AB x BE. Sed BA x AE [e] = AFq; [e] & AB x BE = FBq. [f] ergo AE. EB :: AFq. FBq. ergo cum AE [g] ⊓⊔ EB, [h] erit AFq ⊓⊔ FBq. Quinetiam ABq ([k] AFq + FBq) [l] est $\rho^c\nu$. denique EFq [l] = AEB [l] = CGq. [m] ergo EF = CG. ergo CD x AB = 2 EF x AB. atqui CD x AB [n] est $\mu\nu$. [o] ergo AB x EF, [p] vel AF x FB, est $\mu\nu$. Q. E. D. g 19. 10. h 10 10. k 31. 3. & 47. 1. l constr. m 1. ax. 1. n 22. 10. o 24. 10. p sch. 22. 6.

Explicatio per numeros.

Sit AB, 6. CD, $\sqrt{}$ 12. quare CG = $\sqrt{\frac{12}{4}}$ = $\sqrt{}$ 3. Est vero AE = 3 + $\sqrt{}$ 6. & EB = 3 - $\sqrt{}$ 6. & unde AF erit $\sqrt{}$: 18 + 216. Et FB, $\sqrt{}$ 18 - $\sqrt{}$ 216. item AFq + FBq est 36, & AF FB = $\sqrt{}$ 108.

Cæterum AE invenitur sic. Quia BA (6) AF :: AF. AE; erit 6 AE = AFq = AEq + 3 (EFq.) ergo 6 AE - AEq = 3. pone 3 + e = AE. ergo 18 + 6e - 9 - 6e - ee, hoc est 9 - ee = 3. vel ee = 6. quare e = $\sqrt{}$ 6 proinde AE = 3 + $\sqrt{}$ 6.

PROP. XXXV.



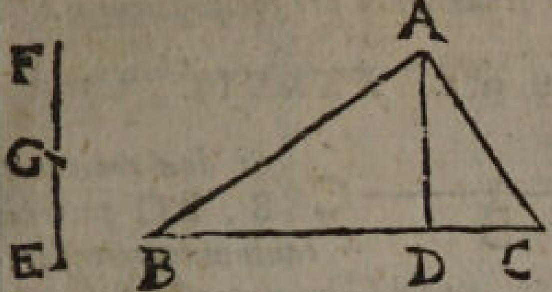
Invenire duas rectas lineas AE, EB *potentia incommensurabiles, quæ faciant compositum quidem ex ipsarum quadratis medium, rectangulum vero sub ipsis contentum, rationale.*

a 32. 10. *a* Sume AB, & CF μ ⊤, ita ut AB x CF sit ῥ, atque $\sqrt{}$ ABq - CFq ⊓ AB. & reliqua fiant, ut in præcedenti. erunt AE, EB, quas petis.

Nam, ut isthic ostensum est, AEq ⊓ EBq: item ABq (AEq + EBq) est μν. & denique AB x CF *b* est ῥ, idcirco & AB x DE, *d* hoc est, AE x EB, est ῥ. ergo, &c.

b *constr.* c *schol.* 12. 10 d *schol.* 22. 6.

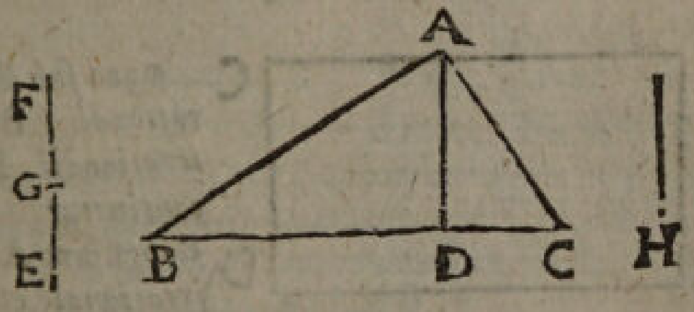
Prop. XXXVI.



Invenire duas rectas lineas BA, AC *potentia incommensurabiles, quæ faciant & compositum ex ipsarum quadratis medium, & rectangulum sub ipsis comprehensum medium, incommensurabileque composito ex ipsarum quadratis.*

a Accipe BC & EF μ ⊐⊏; ita ut BC x EF sit μν. & √ Bcq – EFq ⊐⊏ BC. & reliqua fiant, ut in præcedentibus. Erunt B A, A C exoptata. Nam, ut prius, B Aq ⊐⊏ ACq; item BAq + ACq est μν. & BA x AC est μν. Denique BC *b* ⊐⊏ EF, atque *c* ideo BC ⊐⊏ EG; estque BC. EG *d* :: BCq. B C x E G, (B C x A D, vel BA x A C.) *e* ergo B Cq (A Bq + A Cq) ⊐⊏ BA x A C. ergo, &c. a 33. 10. b constr. c 13. 10. d 1. 6. e 14. 10.

Schol.



Invenire duas medias longitudine & potentia incommensurabiles.

a Sume BC μ. sitque BA x AC μν, & ⊐⊏ BCq (BAq + ACq.) *b* Fac B A. H :: H. AC. Sunt B C, & H μ ⊐⊏. Nam B C est μ. *c* & BA x AC (*c* Hq) est μν. quare H est etiam μ. a 36. 10. b 13. 6. c 17. 6.

d 14.10. μ. d item BA x AC $\sqcup$ BCq; ergo Hq $\sqcup$ BCq. ergo, &c.

Principium senariorum per compositionem.

PROP. XXXVII.

Si duæ rationales AB, BC potentia tantum commensurabiles componantur, tota AC irrationalis est; vocetur autem ex binis nominibus.

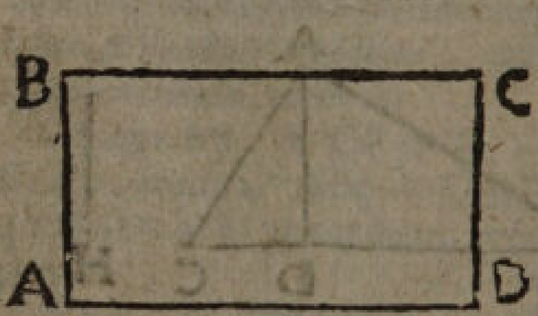
a hyp. b lem.16.10. c 11.def.10. Nam quia AB a $\sqcup$ BC, b erit ACq $\sqcup$ ABq. Sed AB a est ρ^{ϵ}. c ergo AC est ρ^{λ}. Q. E. D.

PROP. XXXVIII.

Si duæ mediæ AB, BC potentia tantum commensurabiles componantur, quæ rationale contineant, tota AC irrationalis est; vocetur autem ex binis mediis prima.

a hyp. b lem.16.10 c 11.def.10. Nam quoniam AB a $\sqcup$ BC, b erit ACq $\sqcup$ AB x BC, $\rho^{\epsilon}\iota$. c ergo AC est ρ^{λ}. Q. E. D.

LEMMA.

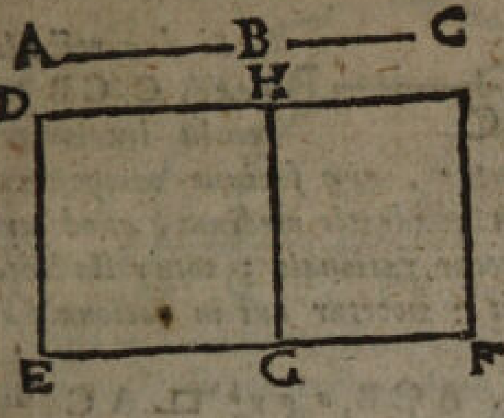


Quod sub linea rationali AB, & irrationali BC continetur rectangulum AC, irrationale est.

a hyp. b 21.10. Nam si rectang. AC dicatur $\rho^{\epsilon}\nu$; quum AB sit ρ^{ϵ}; b erit latitudo BC etiam ρ^{ϵ}. contra Hyp.

PROP.

PROP. XXXIX.



Si duæ mediæ AB, BC *potentia tantum commensurabiles componantur, quæ medium contineant, tota* AC *irrationalis erit; vocetur autem ex binis mediis secunda.*

Ad expositam DE ρ' *a* fac rectang. DF = ACq; *b* & DG = ABq + BCq. *a cor 16 6. b 47. 1. & 11. 6.*

Quoniam ABq *c* ⊔ BCq, *d* erit ABq + BCq, hoc est DG ⊔ ABq; sed ABq *e* est μν. *e* ergo DG est μν. verum rectang. ABC ponitur μν; *e* ideoque 2 ABC (*f* HF) est μν; *g* ergo EG, & GF sunt ρ'. quia vero DG *h* ⊔ HF; atque DG. HF :: *k* EG. GF *l* erit EG ⊔ GF. *m* ergo tota EF est ρ'. *n* quare rectang DF est ρ'ν. *o* ergo √DF, id est AC, est ρ'. Q.E.D. *c hyp. d 16 10. e 14. 10. f 4. 2. g 23. 10. h lem. 26. 10. k 1. 6. l 10 10. m 37. 10. n lem. 38, 10. o 11. def. 10.*

PROP. XL.

Si duæ rectæ lineæ AB, BC *potentia tantum commensurabiles componantur, quæ faciant compositum quidem ex ipsarum quadratis rationale, quod autem sub ipsis continetur, medium; tota recta linea* AC, *irrationalis erit: vocetur autem major.*

Nam quia ABq + BCq *a* est ρ'ν, & *b* ⊔ 2 ABC *c* μν, & proinde ACq (*d* ABq + BCq + 2 ABC) *e* ⊔ ABq + BCq ρ'ν, *f* erit AC ρ'. Q.E.D. *a hyp. b sch. 12. 10. c hyp. & 14. 10. d 4. 2. e 17. 10. f 11. def. 10.*

PROP.

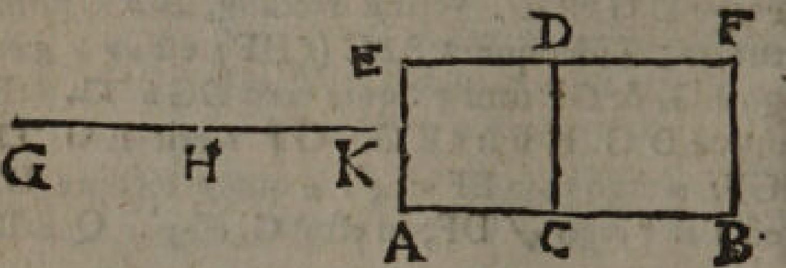
PROP. XLI.

Si duæ rectæ lineæ AC, CB *potentia incommensurabiles componantur, quæ faciant compositum quidem ex ipsarum quadratis medium, quod autem sub ipsis continetur, rationale; tota recta linea* AB *irrationalis erit: vocetur autem rationale ac medium potens.*

a hyp. & sch. 12. 10.
b sch. 12. 10.
c hyp.
d 17. 10.
e 11. def. 10.

Nam 2 rectang. ACB, a $\rho\nu$ b $\sqcup$ ACq + CBq c $\mu\nu$. d ergo 2 ACB d $\sqcup$ ABq. quare e AB est ρ'. Q. E. D.

PROP. XLII.



Si duæ rectæ lineæ GH, HK *potentia incommensurabiles componantur, quæ faciant & compositum ex ipsarum quadratis medium, & quod sub ipsis continetur medium, incommensurabileque composito ex quadratis ipsarum; tota recta linea* GK *irrationalis erit: vocetur autem bina media potens.*

a hyp.
b 23. 10.
c 4. 1.
d 1. 6.
e 10. 10.
f 37. 10.
g lem 38. 10.
h 11. def. 10.

Ad expositam FB ρ', fiant rectang. AF = GKq, & CF = GHq + HKq. Quoniam GHq + HKq (CF) a est $\mu\nu$; latitudo CB b erit ρ'. Item quia 2 rectang. GHK (c AD) a est $\mu\nu$, etiam AC b erit ρ'. Porro quia rectang. AD a $\sqcup$ CF, d atque AD. CF :: AC. CB, e erit AC $\sqcup$ CB. f Quare AB est g ρ'. ergo rectang. AF, id est, GKq est $\rho'\nu$. h proinde GK est ρ'. Q. E. D.

PROP.

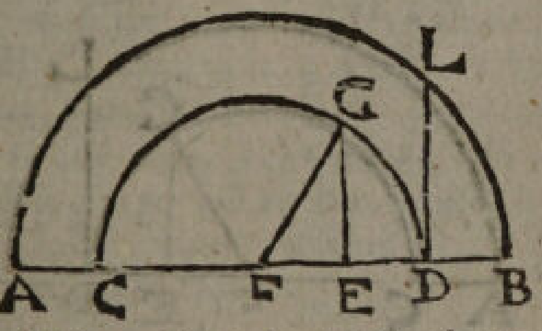
Prop. XLIII.

Quæ ex binis nominibus A B, *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Si fieri poteſt, binomium A B alibi in E ſecetur in alia nomina A E, E B. Liquet A B ſecari utrobique inæqualiter, quia AD $\sqsupset$ DB, & AE $\sqsupset$ EB.

Quoniam rectangula ADB, AEB *a* ſunt μα; *a* & ſingula ADq, DBq, AEq, EBq ſunt ῥα; *b* a- a 37. 10. b ſch. 27. 10
deoque ADq + DBq, *b* & AEq + EBq etiam ῥα, *b* idcirco ADq + DBq – : AEq + EBq. *c* hoc eſt, 2 AEB – 2 ADB eſt ῥν. *d* ergo AEB c ſch. 5. 2.
– ADB ῥν. ergo μν ſuperat μν per ῥν. *e* Q.E.A. d ſch. 12. 10 e 27. 10.

Prop. XLIV.

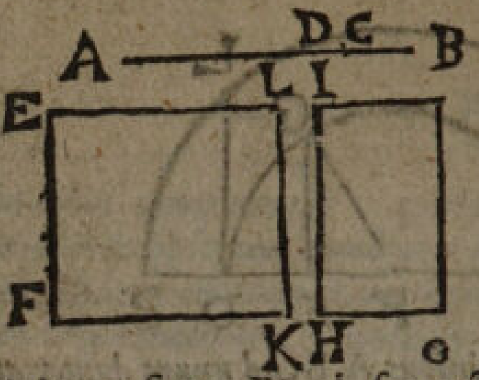


Quæ ex binis mediis prima A B, *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Puta AB dividi in alia nomina AE, EB. quo
poſito, ſingula ADq, DBq, EBq, *a* ſunt μα; *a* & a 38. 10.
rectangula ADB, AEB, eorumque dupla, ſunt b ſch. 27. 10.
ῥα. *b* ergo 2 A E B – 2 A D B, *c* hoc eſt ADq c ſch. 5. 2.
+ DBq – : AEq + EBq eſt ῥν. *d* Q.E.A. d 17. 10.

PROP.

PROP. XLV.



Quæ ex binis mediis ſecunda AB; *ad unum duntaxat punctum* C *dividitur in nomina* AC, CB.

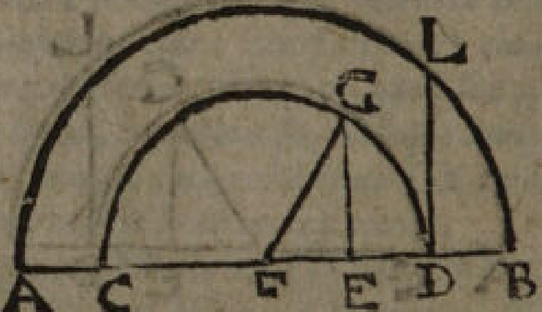
Dic alia eſſe nomina AD, DB.

Ad expoſitam EF ρ, fac rectang. EG = ABq. & EH = ACq + CBq; item EK = ADq + DBq.

a 39. 10.
b 16 & 24. 10.
c 23. 10.
d 24. 10.
e 4. 2.
f lem. 16. 10.
g 1. 6.
h 10. 10.
k 37. 10.

Quoniam ACq, CBq a ſunt μν ⊓; b erit ACq + CBq (EH) μν. c ergo latitudo FH eſt ρ. a quin & rectang. ACB, d ideoque 2 ACB e (IG) eſt μν: c ergo HG, eſt etiam ρ. Cum igitur EH f ⊓ IG, g atque EH. IG :: FH. HG, h erunt FH, HG ⊓. k ergo FG eſt binomium; cujus nomina FH, HG. Simili argumento FG eſt bin. cujus nomina FK, KG, contra 43. hujus.

PROP. XLVI.



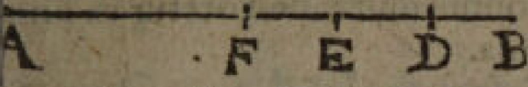
Major AB *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

a 40. 10.
b ſch. 27. 10.
c ſch. 5. 2.
d 17. 10.

Concipe alia nomina AE, EB. quo poſito rectangula ADB, AEB a μν; a & tam ADq + DBq, quam AEq + EBq ſunt ρ. b ergo ADq + DBq — : AEq + EBq, c hoc eſt 2 AEB — 2 ADB eſt ρ. d Q. F. N.

PROP.

PROP. XLVII.

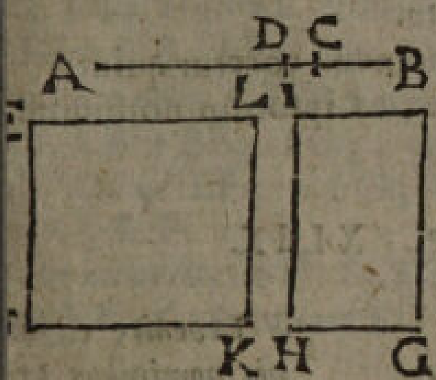


Rationale ac medium potens AB, *ad unum duntaxat punctum D dividitur in nomina* AD, DB.

Dic alia nomina AE, EB. *a* ergo tam AEq + EBq, quam ADq + DBq sunt μα. *a* & rectangula AEB, ADB, sunt ρ'α. *b* ergo 2 AEB − 2 ADB, *c* hoc est, ADq + DBq − : AEq + EBq est ρ'ν. Q.E.A.

a 41. 10. b sch. 27. 10. c sch. 5. 2. d 27. 10.

PROP. XLVIII.



Bina media potens AB, *ad unum duntaxat punctum* C *dividitur in nomina* AC, CB.

Vis AB dividi in alia nomina AD, DB. Ad expositam EF ρ', fiant rectang. EG = ABq, & EH = ACq + CBq, & EK = ADq + DBq. Quoniam ACq + CBq, nempe EH, *a* est μν, *b* erit latitudo FH ρ'. Item quia 2 ACB, *c* hoc est, HG, est *a* μν, *b* erit HG etiam ρ'. Ergo cum EH ⊓ IG, atque EH. IG *d* :: FH. HG, *e* erit FH ⊓ HG. *f* ergo FG est bin. cujus nomina FH. HG. Eodem modo ejusdem nomina erunt FK, KG; contra 43 hujus.

a 42. 10. b 23. 10. c 4. 2. d 1. 6. e 10. 10. f 37. 10.

Definitiones secundæ.

Expositâ rationali, & quæ ex binis nominibus, divisa in nomina; cujus majus nomen plus possit quam minus, quadrato rectæ lineæ sibi longitudine commensurabilis;

I. Siquidem majus nomen expositæ rationali com-

commensurabile sit longitudine, vocetur tota ex binis nominibus prima.

II. Si vero minus nomen expositæ rationali longitudine sit commensurabile, vocetur ex binis nominibus secunda.

III. Quod si neutrum ipsorum nominum sit longitudine commensurabile expositæ rationali, vocetur ex binis nominibus tertia.

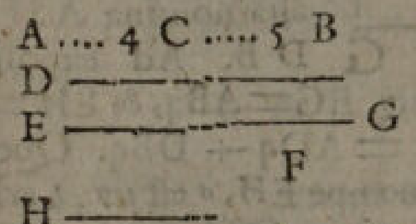
Rursus, si majus nomen plus possit quam minus, quadrato rectæ lineæ sibi longitudine incommensurabilis;

IV. Si quidem majus nomen expositæ rationali commensurabile sit longitudine, vocetur ex binis nominibus quarta.

V. Si vero minus nomen, vocetur quinta.

VI. Quod si neutrum ipsorum nominum, vocetur sexta.

PROP. XLIX.



Invenire ex binis nominibus primam, E G.

a Sume AB, AC numeros quadratos, quorum excessus CB non Q. exponatur D ρ́. b accipe quamvis E F ⊐⊏ D. c fac A B. C B :: EFq. FGq. erit EG bin. 1.

Nam E F d ⊐⊏ D. e ergo E F ρ́. f item E Fq ⊐⊏ F Gq. g ergo F G est etiam ρ́. item d quia EFq. FGq :: AB. CB :: Q. non Q. h erit E F ⌞⊐⊏ F G. denique quia per conversionem rationis EFq. EFq − FGq :: AB. AC :: Q. Q. k erit E F ⊐⊏ √ EFq − FGq. l ergo E G est bin. 1. Q. E. F.

a sch. 19. 10.
b 1. lem. 10. 10.
c 3 lem. 10. 10.
d constr.
e 6. def 10.
f 6. 10.
g sch. 12. 10
h 9. 10.
k 9. 10.
l 1. def. 48. 10.

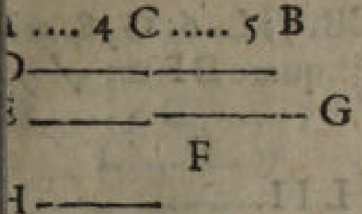
Explicatio per numeros.

Sit D, 8. E F, 6. A B, 9. C B, 5. quare cum

9. 5.

5 :: 36 : 20. erit FG, √20. proinde EG est 6 + √20.

PROP. L.



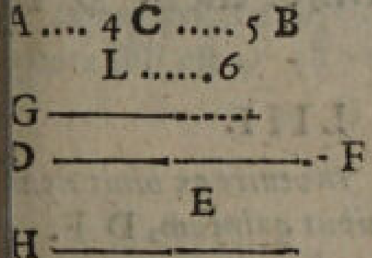
Invenire ex binis nominibus secundam, EG.

Accipe AB, & AC numeros quadratos, quorum excessus CB sit non Q. Sit D exposita ρ. sume FG ⊔ D. Fac CB. AB :: FGq. EFq. Erit EG quæsita. *Proba ut præcedentem*

Nam FG ⊔ D, quare FG est ρ. item EFq ⊔ FGq. ergo EF est etiam ρ. item quia FGq. EFq :: CB. AB :: non Q. Q. est FG ⊔ EF. denique quia CB. AB :: FGq. EFq, inverseque AB. CB :: EFq. FGq, erit ut in præcedenti, EF ⊔ √ EFq − FGq. [a] è quibus EG est bin. 2. Q. E. F. a 2. def. 48. 10.

In numeris, sit D, 8; FG 10; AB, 9; CB, 5. erit EF, √180. quare EG est 10 + 180.

PROP. LI.



Invenire ex binis nominibus tertiã, DF.

[a] Sume numeros AB, AC quadratos, quorum excessus CB non Q. Sitq; L numerus non Q, proxime major quam CB, nempe unitate, vel binario. sit G exposita ρ. [b] Fac L. AB :: Gq. DEq. [b] & AB. CB :: DEq. EFq. erit DF bin. 3. a sch. 29. 10. b 3. lem. 10. 10.

Nam quia DEq [c] ⊔ Gq, [d] est DE ρ. item Gq. DEq :: L. AB :: non Q. Q. [e] ergo G ⊔ DE. item quia DEq [e] ⊔ EFq, [d] etiam EF est ρ. quinetiam quia DEq. EFq :: AB. CB :: Q. non Q. [f] est DE ⊔ EF. porro, quia per c constr. 10. d sch. 12. 10. e 6. 10. f 9. 10.

constr. & ex æquali Gq. EFq :: L. CB :: non Q.
g sch.27.8. Q. (nam g L, & CB non sunt similes plani numeri) h erit G etiam ⊓⊔ EF. denique ut in
h 9.10. præced. $\sqrt{}$ DEq – EFq ⊓⊔ DE. k ergo DF est
k 3 def.48. 10. bin.3. Q. E. F.

In numeris, sit AB, 9; CB, 5; L, 6; G, 8. erit DE, $\sqrt{96}$ & EF, $\sqrt{\frac{480}{9}}$. quare DF = $\sqrt{96} + \sqrt{\frac{480}{9}}$.

PROP. LII.

A ... 3 C ...... 6 B
G ———-----
D ———-——— F
E
H ———

Invenire ex binis nominibus quartam, DF.

a sch.29.10. a Sume quemvis numerum quadratum AB, à quem divide in AC, CB non
b 2.lem 10. quadratos. sit G exposita ρ. b accipe DE ⊓⊔
c 3.lem.10. 10. G. c Fac AB. CB :: DEq. EFq. erit DF bin.4.

Nam ut in 49. hujus, DF ostendetur bin. item, quia per constr. & conversionem rationis DEq. DEq – EFq :: AB. AC :: Q. non Q.
d 9.10. d erit DE ⊓⊔ $\sqrt{}$ DEq – EFq. e ergo DF est
e 4 def. 48 10. bin. 4. Q. E. F.

In numeris, sit G, 8; DE, 6. erit EF $\sqrt{24}$ ergo DF est 6 + $\sqrt{24}$.

PROP. LIII.

A ... 3 C ...... 6 B
G ———--
D ———-——— F
E
HF

Invenire ex binis nominibus quintam, D F.

Accipe quemvis numerum quadratum AB cujus segmenta AC, CB sint non Q. sit G exposita ρ. sume EF ⊓⊔ G. fac CB. AB :: EFq. DEq. erit DF bin.5.

Nam ut in 50. hujus, erit DF bin. & quia per constr. & invertendo DEq. EFq :: AB.
a 9.10. CB, ideoque per conversionem rationis DEq.
b 5.def.48. 10. DEq – EFq :: AB. AC :: Q. non Q. a erit

D

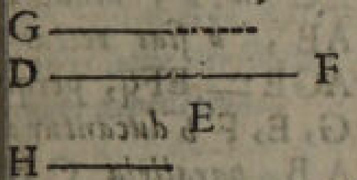
DE ⊓ $\sqrt{}$ DEq − EFq. [b] ergo DF est bin. 4. Q. E. F.

In numeris, sit G, 7; EF, 6. erit DE $\sqrt{54}$. quare DF est $6 + \sqrt{54}$.

Prop. LIV.

A ..... 5 C .... 7 B

L 9



Invenire ex binis nominibus sextam.

Accipe AC, CB primos numeros utcunque, sic ut AC + CB (AB) sit non Q. sume etiam quemvis L num. Q. sit G expos. ρ. [a] fiatque L. AB :: Gq. DEq. atque AB. CB :: DEq. EFq. erit DF. bin. 6. — a 3. lem. 10. 10.

Nam ut in 51. hujus, DF ostendetur bin. item quod DE, & EF ⊓ G. denique igitur quia per constr. & conversionem rationis DEq. DEq − EFq :: AB. AC :: non Q. Q. (Nam AB primus est ad AC, [b] ideoque ei dissimilis) [c] ergo DE ⊓ $\sqrt{}$ DEq − EFq. [d] ergo DF est bin. 6. Q. E. F. — b sch. 27. 8. c 9. 10. d 6. def. 48. 10.

In numeris, sit G, 6; DE $\sqrt{48}$. erit EF $\sqrt{28}$. quare DF est $\sqrt{48} + \sqrt{28}$.

LEMMA.

Sit AD *rectangulum, cujus latus* AC *secetur inæqualiter in* E; *bisectumque sit segmentum minus* EC *in* F; *atque ad* AE,
a 28. 6. *a fiat rectang.* AGE = EFq; *perque* G, E, F
b 31. 1. *b ducantur ad* AB *parallelæ* GH, EI, FK. *c Fiat autem*
c 14. 2. *quadratum* LM = *rectang.* AH, *atque ad* OMP *productam c fiat quadratum* MN = GI; *rectæque* LOS, LQT, NRS, NPT *producantur.*

Dico 1. MS, MT *sunt rectangula.* Nam ob quadratorum angulos OMQ, RMP rectos
a sch.15.1. *a* erit QMR recta linea. *b* ergo anguli RMO,
b 13. 1. QMP recti sunt. quare pgra MS, MT sunt rectangula.

c 2. ax. 2. 2. *Hinc patet* LS *c*= LT; *& proinde* LN *esse quadratum.*

3. *Rectangula* SM, MT, EK, FD *æquales*
d hyp *sunt.* Nam quia rectang. AGE *d* = EFq, *e* erit
e 17. 6. AE. EF :: EF. GE. *f* ideoque AH. EK :: EK.
f 1. 6. GI. hoc est per constr. LM. EK :: EK. MN
g sch. 22. 6. *g* verum LM. SM :: SM. MN. ergo EK *h* =
h 9. 5. SM *k* = FD *l* = MT.
k 36. 1.
l 43. 1.
m 2. ax. 1. 4. *Hinc* LN *m* = AD.
n 16. 10. 5. *Quia* EC *bisecta est in* F, *n patet* EF, FC
EC ⊏⊐ *esse.*

o 18. & 6. Si AE ⊏̸⊐ EC, & AE ⊏⊐ √ AEq -
16. 10. ECq, *o* erunt AG, GE, AE ⊏⊐. item, qu

AG

G. GE :: AH. GI *p* erunt AH, GI ; hoc est p 10. 10.
M, MN ⊏⊐. item iisdem positis,

7. OM ⊏⊐ MP. Nam per Hyp. AE, ⊏⊐ C, *q* ergo EC ⊏⊐ GE. *q* quare EF ⊏⊐ GE. q 14. 10.
ed EF. GE :: EK. GI. *r* ergo EK ⊏⊐ GI, r 10. 10.
oc est SM ⊏⊐ MN. atqui SM. MN :: OM. MP. *r* ergo OM ⊏⊐ MP.

8. Sin ponatur AE ⊏⊐ √ AEq − ECq, patet AG, GE, AE esse ⊏⊐. unde LM ⊏⊐ f 19. & 17.
MN. nam AG. GE :: AH. GI :: LM. MN. 10.

His bene perspectis, facile sex sequentes Propositiones expediemus.

PROP. LV.

Si spatium AD *contineatur sub rationali* AB, *& ex binis nominibus prima* AC, (AE + EC;) *recta linea* OP *spatium potens irrationalis est, quæ ex binis nominibus appellatur.*

Suppositis iis, quæ in lemmate proxime præcedenti descripta, & demonstrata sunt, liquet rectam OP posse spatium AD. *a* item AG, GE, AE sunt ⊏⊐. ergo cum AE *b* sit ρ ⊏⊐ AB, *c* erunt AG, & GE, ρ ⊏⊐ AB. *d* ergo rectangula AH, GI, hoc est quadrata LM, MN sunt a hyp. & lem. 54. 10.
ρ. ergo OM, MP sunt ρ *e* ⊏̶⊐. *f* proinde OP b hyp. c sch. 12. 10.
est bin. Q. E. D. d 20. 10.

In numeris, sit AB, 5; AC, 4 + √ 12. quare e lem. 54. 10. f 37. 10.
rectang. AD = 20 + √ 300 = quadr. LN. ergo OP est √ 15 + √ 5; nempe bin. 6.

PROP. LVI.

Si spatium AD *contineatur sub rationali* AB, *& ex binis nominibus secunda* AC (AE + EC;) *recta linea* OP *spatium* AD *potens, irrationalis est, quæ ex binis mediis prima appellatur.*

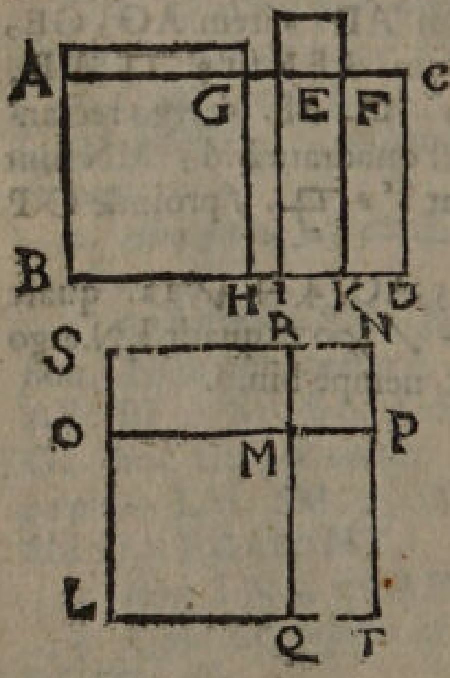
Rursus adhibito lemmate ad 54. hujus, erit OP = √ AD. *a* item AE, AG, GE sunt ⊓. ergo quum AE *b* sit $\dot{\rho}$, ⊓ AB, *c* erunt AG, GE etiam $\dot{\rho}$ ⊓ AB. ergo rectangula AH, GI; hoc est OMq, MPq *d* sunt μμ. *e* quinetiam OM ⊓̣ MP. denique EF ⊓ EC, & EC *f* ⊓ AB. *g* quare EF est $\dot{\rho}$ ⊓ AB. *g* ergo EK; hoc est SM, vel OMP est $\dot{\rho}\nu$. *h* Proinde OP est 2 μ prima. Q. E. D.

a hyp. & lem. 54. 10.
b hyp.
c sch. 12. 10.
d 22. 10.
e lem. 54 10.
f hyp. 12. 10.
g 20. 10.
h 38. 10.

In numeris, sit AB, 5; & AC, √ 48 : + 6. ergo rectang. AD = √ : 1200 + 30 = OPq. ergo OP est v√ 675 + v√ 75; nempe bimed. 1.

Vide Schem. 57.

PROP. LVII.



Si spatium AD *contineatur sub rationali* AB, *& ex binis nominibus tertia* AC (AE + EC;) *recta linea* OP *spatium* AD *potens, irrationalis est, quæ ex binis mediis secunda dicitur.*

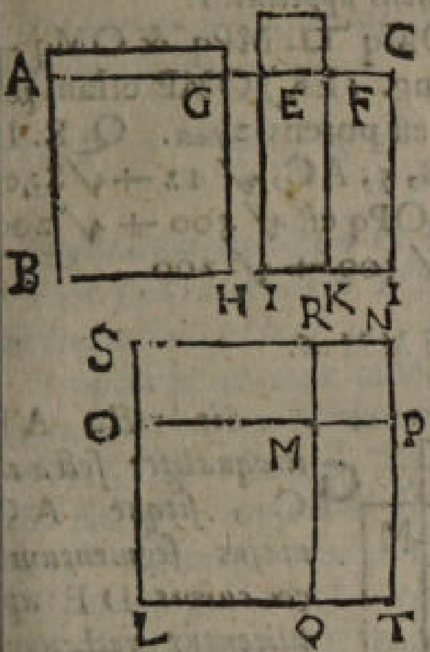
Ut prius, OPq = AD. item rectangula AH, GI, hoc est OMP, MPq sunt μμ. *a* item EK, vel OMP est μν. *b* ergo OP est bimed. 2.

a hyp. & 22. 10.
b 39. 10.

In

In numeris, ſit AB,5; AC,$\sqrt{32}+\sqrt{34}$. quare AD eſt $\sqrt{800}+\sqrt{600}$=OPq. proinde OP eſt $v\sqrt{450}+v\sqrt{50}$; hoc eſt bimed. 2.

PROP. LVIII.



Si ſpatium A D *contineatur ſub rationali* A B, *& ex binis nominibus quarta* AC (AE + EC;) *recta linea* OP *ſpatium potens, irrationalis eſt, quæ vocatur major.*

Nam iterum, OMq [a] ⊐ MPq. rectang. vero AI, hoc eſt OMq + MPq [b] eſt ῥν. [c] item EK, vel OMP eſt μν. [d] ergo OP ($\sqrt{AD}$) eſt major. Q. E. D.

a lem. 54. 10. b hyp. & 20. 10. c hyp. & 22. 10. d 40. 10.

In numeris, ſit AB 5; & AC, 4 + 8. ergo rectang. AD eſt 20 + $\sqrt{200}$. quare OP eſt $\sqrt{:}$ 20+$\sqrt{200}$.

PROP. LIX.

Si ſpatium AD *contineatur ſub rationali* AB, *& ex binis nominibus quinta* AC; *recta linea* OP *ſpatium* AD *potens, irrationalis eſt, quæ rationale & medium potens appellatur.*

Rurſus OMP ⊐ MPq. rectang. vero AI, vel OMq + MPq eſt μν. [a] item rectang. EK, vel OMP eſt ῥν. [b] ergo OP ($\sqrt{AD}$) eſt potens ῥν, & μν. Q. E. D.

a ut in præc. b 41. 10.

In numeris, ſit AB, 5; & AC 2 + $\sqrt{8}$. ergo rectang. AD = 10+$\sqrt{200}$=OPq. quare OP eſt $\sqrt{:}$ 10+$\sqrt{200}$

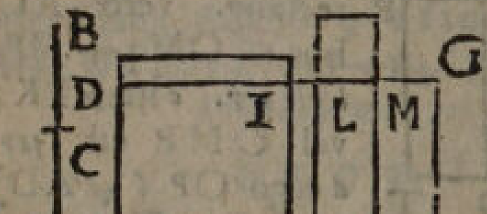
PROP. LX.

Si spatium A D *contineatur sub rationali* A B *& ex binis nominibus sexta* BC (AE + EC;) *recta linea* OP *spatium* AD *potens, irrationalis est, quæ bina media potens appellatur.*

Ut sæpe prius, OMq ⊥ MPq. & OMq + MPq est μν. & rectang. (EK) OMP etiam μ. *a* ergo OP = √ AD est potens 2 μα. Q. E. D.

42.10. *In numeris*, sit AB, 5; AC, √ 12 + √ 8; ergo rectang. AD, vel OPq est √ 300 + √ 200. proinde OP est √: √ 300 + √ 200.

LEMMA.



Sit recta AB *inæqualiter secta in* C, *sitque* AC *majus segmentum; & cuivis* DE *applicentur rectangula*, DF = ABq, & DH = ACq, & IK = CBq. *sitque* LG *bisecta in* M, *ducaturque* MN *parall.* GF.

Dico 1. *Rectang.* ACB = LN, vel MF.

a 4.2. & 3. *a* Nam 2 ACB = LF.

ax. 1. 2. DL ⊏ LG. nam DK (ACq + CBq)

b 7. 2. *b* ⊏ LF (2 ACB) ergo cum DK, LF sint æque

c 1. 6. alta, *c* erit DL ⊏ LG.

d 16. 10. 3. Si AC ⊤ CB, *d* erit rectang. DK ⊤ ACq, & CBq.

4. *Item*, DL ⊥ LG. nam ACq + CBq *e* ⊥ 2 ACB: hoc est DK ⊥ LF. sed DK.

e lem. 26. 10. LF *e* :: DL. LG. *f* ergo DL ⊥ LG.

f 10. 10. 5. *Ad hæc*, DL ⊥ √ DLq − LGq. Nam

g 1. 6. ACq. ACB *g* :: ACB. CBq. hoc est DH. LN ::

N :: LN. IK. c quare DI. LM :: LM. IL.
ergo DI x IL = LMq. ergo cum ACq k ⊔ h 17. 6.
Sq. hoc est DH ⊔ IK, & l proinde DI ⊔ k hyp
L, m erit DL ⊔ √ DLq − LGq. Q. E. D. l 10. 10 m 18. 10.
6. *Sin ponatur* ACq ʼ⊔ CBq, n *erit* DL ʼ⊔ n 19. 10.
√ DLq − LGq.

Hoc lemma præparationis vicem subeat pro 6. sequentibus propositionibus.

PROP. LXI.

Quadratum ejus quæ ex binis nominibus (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus primam.*

Suppositis iis, quæ in lemmate proxime antecedenti descripta & demonstrata sunt. Quoniam AC, CB a sunt ρ̇ ⊤⊔, b erit rectang. DK a hyp.
⊔ ACq; c ergo DK est ρ̇ν. d ergo DL ⊔ b lem 60. 10. c sch. 12. 10.
DE ρ̇. rectang. vero ACB, ideoque 2 ACB d 21. 10.
(LF) e est μν. f ergo latitudo LG est ρ̇ ʼ⊔ e 22. & 24. 10.
DE. g ergo etiam DL ʼ⊔ LG. h item DL ⊔ f 23. 10.
√ DLq − LGq. ex quibus k sequitur DG esse bin. 1. Q. E. D. g 13. 10. h lem 60. 10. k 1. def. 48. 10.

PROP. LXII.

Quadratum ejus, quæ ex binis mediis prima (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus secundam.*

Rursus adhibito lemmate proxime præcedenti; Rectang. DK ⊔ ACq. a ergo DK est a 24. 10.
μν. b ergo latitudo DK est ρ̇ ʼ⊔ DE. Quia ve- b 23. 10.
ro rectang. ACB, ideoque LF (2 ACB) c hyp. & sch. 12. 10.
c est ρ̇ν, d erit LG ρ̇ ⊔ DE. e ergo DL, d 21. 10.
LG sunt ʼ⊔. f item DL ⊔ √ DLq − e 13. 10.
LGq. g ex quibus patet DG esse bin. 2. Q. E. D. f lem 60. 10. g 2. def. 48. 10.

PROP.

PROP. LXIII.

Quadratum ejus, quæ ex binis mediis secunda (AC+CB) ad rationalem DE applicatum, facit latitudinem DG ex binis nominibus tertiam.

Ut in præced. DL est ῥ ⊤⊥ DE. porro quia
a hyp. & 24. 10. rectang. ACB, ideoque LF (2 ACB) *a* est
b 23. 10. μν, *b* erit LG ῥ ⊤⊥ DE. *c* quinetiam DL ⊤⊥
c lem. 60. 10. LG. *c* itemque DL ⊤⊥ √ DLq − LGq. *d* er-
d 3. def. 48. 10. go DG est bin. 3. Q. E. D.

PROP. LXIV.

Quadratum Majoris (AC+CB) ad rationalem DE applicatum, facit latitudinem DG ex binis nominibus quartam.

a hyp. & sch. 12. 10. Rursus ACq + CBq, hoc est DK *a* est ῥν.
b 21. 10. *b* ergo DL est ῥ ⊤⊥ DE. item ACB, ideoque
c hyp & 24. 10. LF (2 ACB) *c* est μν. *d* ergo LG est ῥ ⊤⊥
d 23. 10. DE. *e* proinde etiam DL ⊤⊥ LG. denique
e 13. 10. quia AC ⊤⊦ BC, *f* erit DL ⊤⊥ DLq −
f lem 60. 10. LGq. *g* unde DG. est bin. 4. Q. E. D.
g 4. def. 48. 10.

PROP. LXV.

Quadratum ejus, quæ rationale ac medium potest, (AC+CB) ad rationalem DE applicatum, facit latitudinem DG ex binis nominibus quintam.

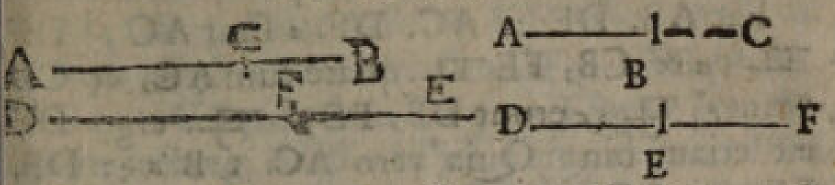
a 23. 10. Iterum, DK est μν. *a* ergo DL est ῥ ⊤⊥
b 21. 10. DE. item LF est ῥν. *b* ergo LG est ῥ ⊤⊥ DE.
c 13. 10. *c* ergo DL ⊤⊥ LG. *d* item DL ⊤⊥ √ DLq −
d lem 60. 10. LGq. *e* proinde DG est bin. 5.
e 5. def. 48. 10.

PROP. LXVI.

Quadratum ejus, quæ bina media potest (AC + CB) ad rationalem DE applicatum, facit latitudinem DG ex binis nominibus sextam.

Ut prius, DL & LG sunt ῥ ⊤⊥ DE. a hyp.
Quia vero ACq + CBq (DK) a ⊤⊥ ACB, b 14. 10.
ideoque DK ⊤⊥ LF (2 ACB) estque DK. c 1. 6.
LF c :: DL, LG. d erit DL ⊤⊥ LG. e denique d 10. 10. e lem. 60. 10.
DL ⊤⊥ √ DLq – LGq. f ex quibus liquet f 6. def. 48. 10.
DG esse bin. 6. Q. E. D.

LEMMA.



Sint AB, DE ⊤⊥; *fiatque* AB. DE :: AC DF.

Dico 1. AC ⊤⊥ DF. ut patet ex 10. 10. item CB ⊤⊥ FE. a quia AB. DE :: CB. FE. a 19. 5.

2. AC. CB :: DF. FE. Nam AC. DF :: AB. DE :: CB. FE. ergo permutando AC. CB :: DF. FE.

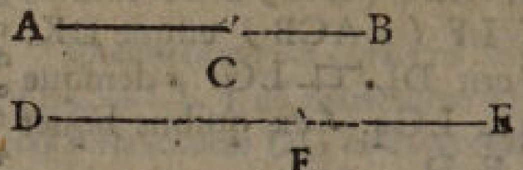
3. *Rectang.* ACB ⊤⊥ DFE. Nam ACq. ACB b :: AC. CB c :: DF. EF :: DFq. DFE. quare permutando ACq. DFq :: ACB. DFE. ergo cum ACq ⊤⊥ DFq, d erit ACB ⊤⊥ DFE. b 1. 6. c prius. d 10. 10.

4. ACq + CBq ⊤⊥ DFq + FEq. Nam quia ACq. CBq e :: DFq. FEq. erit componendo ACq + CBq. CBq :: DFq + FEq. FEq. ergo cum CBq ⊤⊥ FEq, f erit ACq + CBq ⊤⊥ DFq + FEq. e 22. 6. f 10. 10.

5. Hinc, si AC ⊤⊢, vel ⊤⊢ CB, g erit pariter DE ⊤⊢, vel ⊤⊢ EF. g 10. 10.

PROP.

PROP. LXVII.



Ei, quæ ex binis nominibus (AC + CB) *longitudine commensurabilis* DE, *& ipsa ex binis nominibus est, atq; ordine eadem.*

Fac AB. DE :: AC. DF. a sunt AC, DF ⊥; a & CB, FE ⊥. quare cum AC, & CB b sunt ρ' ⊤, c erunt DF, FE ρ' ⊤. ergo DE est etiam bin. Quia vero AC. CB a :: DF. FE. si AC ⊥, vel ⊥ √ ACq − BCq, d etiam similiter DF ⊥, vel ⊥ √ DFq − FEq. item si AC ⊥, vel ⊥ ρ' expos. e erit similiter DF ⊥, vel ⊥ ρ' expos. at si CB ⊥ vel ⊥ ρ', e erit pariter FE ⊥ vel ⊥ ρ'. Sin vero utraque AC, CB ⊥ ρ', erit utraq; etiam DF, FE ⊥ ρ'. g Hoc est, quodcunque binomium fuerit AB, erit DE ejusdem ordinis. Q. E. D.

a lem. 66. 10.
b hyp.
c lem. 66. 10. & sch. 12. 10.
d 15. 10.
e 12. 10. & 14. 10.
g Per def. 48. 10.

PROP. LXVIII.

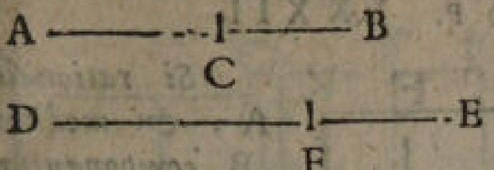
Ei, quæ ex binis mediis (AC + CB) *longitudine commensurabilis* DE, *& ipsa ex binis mediis est, atque ordine eadem.*

a Fiat AB. DE :: AC. DF. b ergo AC ⊥ DF, & CB ⊥ FE. ergo cum AC & CB c sint μ, d etiam DF, & FE erunt μ. & cum AC c ⊤ CB, e erit FD ⊤ FE. f ergo DE est 2μ. Si igitur rectang. ACB sit ρ'ν, quia DFE b ⊥ ACB, g etiam DFE est ρ'ν; et si illud μν, h hoc etiam erit μν. k Id est, sive AB sit bimed. 1. sive bimed. 2. erit DF ejusdem ordinis. Q. E. D.

a 12. 6.
b lem 66. 10.
c hyp.
d 24. 10.
e 10. 10.
f 38. 10.
g sch. 12. 10.
h 24. 10
k 38. vel 39. 10.

PROP.

PROP. LXIX.



Majori (AC + CB) *commensurabilis* DE, & *ipsa Major est.*

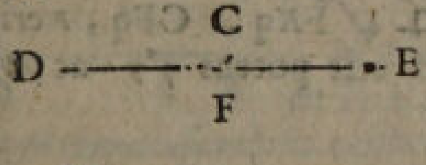
Fac AB. DE :: AC. DF. Quoniam AC [a] ⊓ CB, [b] erit DF ⊓ FE. item ACq + a *hyp.*
CBq [a] est ῥν; proinde cum DFq + FEq [b] ⊓ b *lem.* 66. 10.
ACq + CBq, [c] etiam DFq + FEq est ῥν. de- c *sch.* 12. 10.
nique rectang. ACB [a] est μν. [d] ergo rectang. DFE d 24. 10.
est μν (quia DFE [b] ⊓ ACB.) [e] Quare DE est e 40. 10.
major. Q. E. D.

PROP. LXX.

Rationale ac medium potenti (AC + CB) *commensurabilis* DE, *&* *ipsa rationale ac medium potens est.*

Iterum fac AB. DE :: AC. DF. Quia AC
[a] ⊓ CB, [b] etiam DF ⊓ FE. item quia a *hyp*
ACq + CBq [a] est μν, [c] erit DFq + FEq μν. b *lem* 66 10.
denique quia rectang. ACB [c] est ῥν, [d] etiam c 24 10.
DFE est ῥν. [e] ergo DE est potens ῥν, ac μν. d *sch.* 12. 10.
Q. E. D. e 41. 10.

PROP. LXXI.



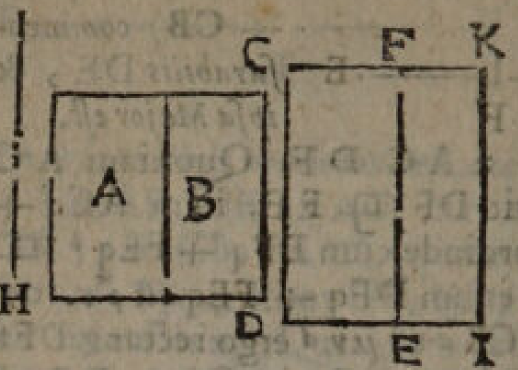
Bina media potenti (AC + CB) *commensurabilis* DE, *&* *ipsa bina media potens est.*

Divide DE, ut in praeced. Quia ACq [a] ⊓ a *hyp*
CBq, [b] erit DFq ⊓ FEq. item quia ACq b *lem* 65. 10.
+ CBq [a] est μν, [c] erit DFq + FEq etiam μν. c 24 10.
pariterque quia ACB [a] est μν, [d] etiam DFE est d 24. 10.
μν. denique quia ACq + CBq ⊓ ACB,

e erit

e 14. 10. *e* erit DFq + FEq ⊔⊓ DFE. *f* è quibus sequi-
f 42. 10. tur DE esse potentem 2 μσ. Q. E. D.

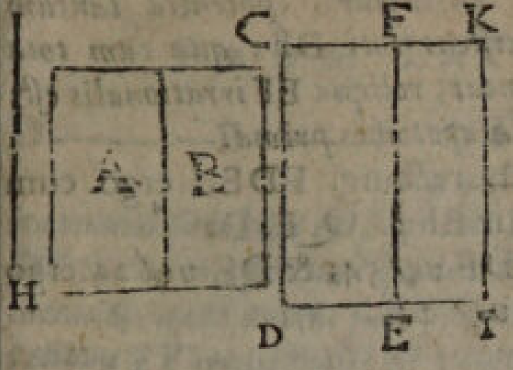
PROP. LXXII.



Si rationale A, & medium B componantur, quatuor irrationales fiunt; vel ea quæ ex binis nominibus, vel quæ ex binis mediis prima, vel major, vel rationale ac medium potens.

Nimirum si Hq = A + B, erit H una 4 linearum, quas theorema designat. Nam ad CD
a cor. 16. 6. expositum ρ', *a* fiat rectang. CE = A; item FI
b 2. ax. 1. = B; *b* ideoque CI = Hq. Quoniam igitur A
c 21. 10. est ρ'ν, etiam CE est ρ̑ν. *c* ergo latitudo CF est 'ρ ⊔⊓ CD. & quia B est μν, erit FI μν.
d 23. 10. *d* ergo FK est 'ρ ⊔⊓ CD. *e* ergo CF, FK sunt
e 13. 10. ρ ⊔⊓. Tota igitur CK *f* est bin. Si igitur A
f 37. 10 ⊏ B, hoc est CE ⊏ FI, *g* erit CF ⊏ FK. er-
g 1. 6. go si CF ⊔⊓ √ CFq − FKq, *h* erit CK bin.
h 1. def. 1. & proinde H = √ CI *k* est bin. Si ponatur
48. 10. CF ⊔⊓ √ CFq − FKq, *l* erit CK bin. 4.
k 55. 10. quare H (√ CI) *m* est major. Sin A ⊐ B;
l 4. def. *g* erit CF ⊐ FK; proinde si FK ⊔⊓ √ FKq −
48 10 CFq, *n* erit CK bin. 2. *o* quare H est 2 μ. pri-
m 58. 10. ma. denique si FK ⊔⊓ √ FKq − CFq, *p* erit
n 2. def. CK bin. 5. *q* unde H erit potens ρ'ν ac μν.
48. 10. Q. E. D.
o 56. 10.
p 5 def.
48. 10.
q 59. 10.

PROP. LXXIII.

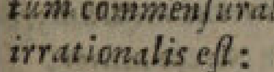


Si duo media A, B, inter se incommensurabilia componantur, duæ reliquæ irrationales fiunt; vel ex binis mediis secunda, vel bina media potens.

Nempe H potens A + B eſt una dictarum irrationalium. Nam ad CD expoſ. ρ', fac rectang. CE = A, & FI = B. unde Hq = CI. Quoniam igitur CE, & FI [a] ſunt μα, [b] erunt a *hyp.*
latitudines CF, FK ῥ [c] ⊔ CD. item quia CE b 13. 10.
[a] ⊔ FI; eſtque CE. FI [c] :: CF. FK, [d] erit c 1. 6.
CF ⊔ FK. [e] ergo CK eſt bin 3. nempe, ſi d 10. 10.
CF ⊔ √ CFq – FKq. unde H = √ CI e 3. *def.* 48. 10.
[f] erit 2 μ 2a. Sin vero CF ⊔ √ CFq – FKq, f 57. 10.
[g] erit CK bin. 6. & [h] proinde H eſt potens 2 μα. g 6. *def.* 48. 10
Q. E. D. h 60. 10.

Principium Senariorum per detractionem.

PROP. LXXIV.



Si à rationali DF rationalis DE auferatur, potentia tantum commenſurabilis exiſtens toti DF; reliqua EF irrationalis eſt: vocetur autem apotome.

Nam EFq [a] ⊔ DEq; ſed DEq [b] eſt ρ'ν. a *lem* 16. 10.
[c] ergo EF eſt ρ'. Q. E. D. b *hyp.*
c 10. & 11. *def.* 10.

In numeris, ſit DF, 2; DE, √ 3. EF erit 2 – √ 3.

PROP.

PROP. LXXV.

D E F *Si à media* DF *media* DE *auferatur, potentia tantum commensurabili existens toti* DF, *quæ cum tota* DF *rationale contineat; reliqua* EF *irrationalis est; vocetur autem mediæ apotome prima.*

a *sch.* 16 10. Nam EFq *a* ⊐ rectang. FDE. ergo cum
b *hyp.* FDE *b* sit ῥν, *c* erit EF ῥ. Q. E. D.
c 20. & 11. *In numeris*, sit DF $v\sqrt{54}$; & DE $v\sqrt{24}$. ergo
def. 10. EF est $v\sqrt{54} - v\sqrt{24}$.

PROP. LXXVI.

D E F *Si à media* DF *media* DE *auferatur, potentia tantum commensurabilis existens toti* DF, *quæ cum tota* DF *medium contineat; reliqua* EF *irrationalis est; vocetur autem mediæ apotome secunda.*

a *hyp.* Quia DFq, & DEq *a* sunt μα ⊐,
b 16. 10. *b* erit DFq + DEq ⊐ DEq. *c* quare DFq
c 24. 10. + DEq est μν. item rectang. FDE, *c* ideoque
d *cor.* 7. 2. 2 FDE *a* est μν. ergo EFq (*d* DFq + DEq − 2 FDE) *e* est ῥ'ν quare EF est ῥ'. Q. E. D.
e 27. 10.

In numeris, sit DF, $v\sqrt{18}$; & DE, $v\sqrt{8}$. erit EF $v\sqrt{18} - v\sqrt{8}$.

PROP. LXXVII.

A B C *Si à recta linea* AC *recta auferatur* AB, *potentia incommensurabilis existens toti* BC, *quæ cum tota* AC *faciat compositum quidem ex ipsarum quadratis rationale, quod autem sub ipsis continetur medium; reliqua* BC *irrationalis est: vocetur autem minor.*

a *hyp.* Nam ACq + ABq *a* est ῥ'ν. at rectang. ACB
b *sch.* 12. 10. *a* est μν. *b* ergo 2 CAB ⊐ ACq + ABq
c 7 2. (2 *c* CAB + BCq;) *d* ergo ACq + ABq ⊐
d 17 10. BCq. *e* ergo BC est 'ῥ. Q. E. D.
e 11. *def.* 10.

In

In numeris, sit AC, $\sqrt{:}\ 18. + \sqrt{\ }108$. AB $\sqrt{:}\ 18 - \sqrt{\ }108$. ergo BC est $\sqrt{:}\ 18 + \sqrt{:}\ 108. - \sqrt{:}\ 18 - \sqrt{\ }108$.

PROP. LXXVIII.

D———E———F *Si à recta linea* DF *recta auferatur* DE *potentia incommensurabilis existens toti* DF, *quæ cum tota* DF *faciat compositum quidem ex ipsarum quadratis medium, quod autem sub ipsis continetur, rationale; reliqua* EF *irrationalis est: vocetur autem cum rationali medium totum efficiens.*

Nam 2 FDE *a* est $\mathring{\rho}\nu$. *b* & DFq + DEq est $\mu\nu$. *c* ergo 2 FDE ⊏ DFq + DEq *d* (2 FDE + EFq) *e* ergo EF est ρ'. Q. E. D.

In numeris sit DF, $\sqrt{:}\sqrt{\ }216 + \sqrt{\ }72$. DE, $\sqrt{:}\sqrt{\ }216 - \sqrt{\ }72$. ergo EF est $\sqrt{:}\sqrt{\ }216 + \sqrt{\ }72 - \sqrt{:}\sqrt{\ }216 - \sqrt{\ }72$.

a *hyp.* & *sch.* 12. 10.
b *hyp.*
c *sch.* 12. 10.
d 7. 2.
e *sch.* 12. 10. & 11. *def.* 10.

PROP. LXXIX.

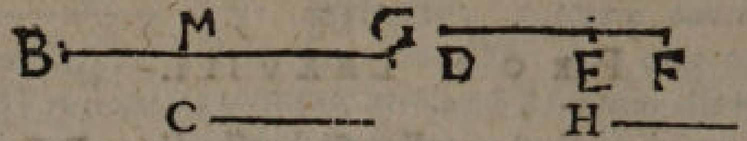
D E F *Si à recta* DF *recta auferatur* DE, *potentia incommensurabilis existens toti* DF, *quæ cum tota faciat & compositum ex ipsarum quadratis, medium; & quod sub ipsis continetur, medium, incommensurabileque composito ex quadratis ipsarum, reliqua irrationalis est: vocetur autem cum medio medium totum efficiens.*

Nam 2 FDE, & DFq + DEq *a* sunt $\mu\alpha$; *b* ergo EFq (*c* DFq + DEq − 2 FDE) est $\rho'\nu$. *d* proinde EF est ρ'. Q. E. D.

Exempl. gr. sit DF, $\sqrt{:}\sqrt{\ }180. + \sqrt{\ }60$. DE, $\sqrt{:}\sqrt{\ }180 - \sqrt{\ }60$. EF erit $\sqrt{:}\sqrt{\ }180. + \sqrt{\ }60 - \sqrt{:}\sqrt{\ }180. - \sqrt{\ }60$.

a *hyp.* & 24. 10.
b 27. 10.
c *cor.* 7. 2.
d 11. *def.* 10.

LEMMA.



Si idem sit excessus inter primam magnitudinem BG, *& secundam* C (MG) *qui inter tertiam magnitudinem* DF, *& quartam* H (EF;) *erit & vicissim idem excessus inter primam magnitudinem* BG, *& tertiam* DF, *qui inter secundam* C, *& quartam* H.

a hyp.
a 15. ax. 1.

Nam quia a æqualibus BM, DE adjectæ sunt æquales MG, EF, a hoc est C, H; erit excessus totorum BG, DF, b æqualis excessui adjectorum, C, H. Q. E. D.

Coroll.

Hinc, quatuor magnitudines Arithmetice proportionales, vicissim erunt Arithmetice proportionales.

PROP. LXXX.

Apotomæ AB *una tantum congruit recta linea rationalis* BC, *potentia tantum commensurabilis existens toti* AB.

a 22. 10.
b 24. 10.
c cor. 7. 2.
d lem. 79. 10.
e hyp. & 27. 10.
f sch. 12. 10.
g 27. 10.

Si fieri potest, alia BD congruat. a ergo rectangula ACB, ADB; b ideoq; eorum dupla sunt μα. cum igitur ACq + BCq − 2 ACB c = ABq c = ADq + DBq − 2 ADB. ergo vicissim ACq + BCq −: ADq + BDq d = 2 ACB −: 2 ADB. Sed ACq + BCq −: ADq + BDq e est ῥν. f ergo 2 ACB −: 2 ADB est ῥν. Q. E. A.

PROP.

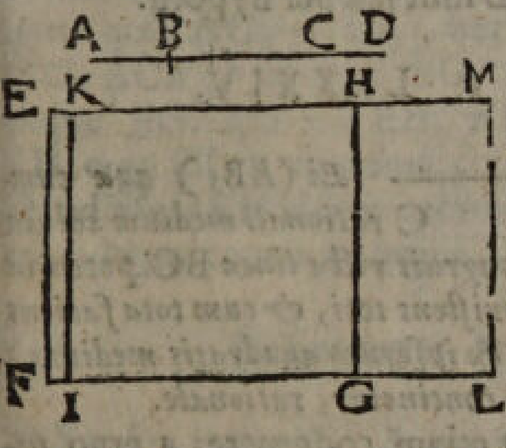
Prop. LXXXI.

A B D C *Mediæ Apotomæ primæ* AB *una tantum congruit recta linea media* BC, *potentia solum commensurabilis existens toti;& cum tota rationale continens.*

Dic etiam BD congruere. igitur quoniam tam ACq, & BCq; quam ADq, & BDq *a* sunt μα ⊓. *b* etiam ACq + BCq, & ADq + BDq erunt μα. *c* sed rectangula ACB, ADB; *d* adeoque 2 ACB, & 2 ADB sunt ρ'α. *e* ergo 2 ACB −: 2 ADB; *f* hoc est ACq + BCq −: ADq + BDq est ρ'ν. *g* Q.E.A.

a hyp. b 16.& 24. 10. c hyp. d sch.12. 10. e sch.27. 10. f 7. 2. & lem.79. 10. g 27. 10.

Prop. LXXXII.



Mediæ Apotomæ secundæ AB *una tantum congruit recta linea media* BC, *potentia solum commensurabilis existens toti, & cum tota medium continens.*

Si fieri potest, congruat alia BD. Ad EF ρ fiant rectang. EG = ACq + BCq; item rectang. EL = ADq + BDq. Item EI = ABq. Jam 2 ACB + ABq = ACq + BCq = EG, ergo cum EI = ABq, *a* erit KG = 2 ACB. porro ACq, & BCq *b* sunt μα ⊓. *c* Ergo EG (ACq + BCq) est μν. *d* ergo latitudo EH ρ ⊓ EF. *e* Quinetiam rectang. ACB; *f* ideoque 2 ACB (KG) est μν. *d* ergo KH est etiam ρ ⊓ EF. denique quia ACq + BCq, id est, EG, *g* ⊓ 2 ACB (KG) estque

a 4. 2. & 3. ax. 1. b hyp. c 24. 10. d 23. 10. e hyp. f 24. 10. g lem. 26 10.

h 1.6. k 80.10. l 74.10.

EG.KG :: [h] EH. KH [k] erit EH ⊓ KH. [l] ergo EK est aptome, cujus congruens KH. simili argumento erit KM ejusdem EK congruens; contra 80 hujus.

PROP. LXXXIII.

A B D C

Minori AB, una tantum congruit recta linea (BC) potentia incommensurabilis existens toti, & cum tota faciens compositum quidem ex ipsarum quadratis rationale; quod autem sub ipsis continetur medium.

a hyp. b lem. 97. 10. c sch. 27. 10. d 27. 10.

Puta alium BD congruere. Cum igitur ACq + BCq, & ADq + BDq [a] sint ρ'α, eorum excessus (2 [b] ACB —: 2 ADB) [c] est ρ'ν, [d] Q.E.A; quia ACB, & ADB sunt μα per hypoth.

PROP. LXXXIV.

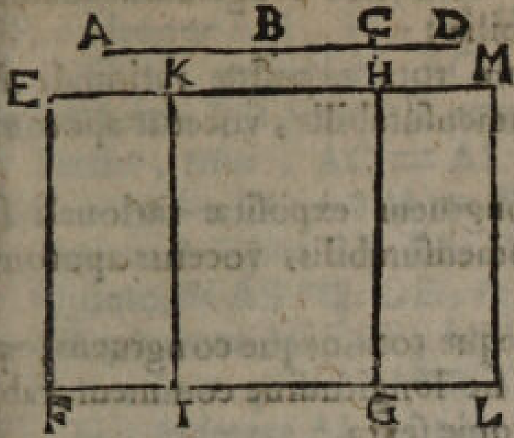
A B D C

Ei (AB,) quae cum rationali medium totum facit, una tantum congruit recta linea BC, potentia incommensurabilis existens toti, & cum tota faciens compositum quidem ex ipsarum quadratis medium; quod autem sub ipsis continetur, rationale.

a hyp. b sch. 12. 10. c lem. 79. 10. d sch. 27. 10.

Dic aliam BD etiam congruere. [a] ergo rectangula ACB, ADB. [b] ideoque 2 ACB, & 2 ADB sunt ρ'α. ergo 2 ACB —: 2 ADB; [c] hoc est, ACq + BCq —: ADq + BDq [d] est ρ'ν. Q.E.A: quum ACq + BCq, & ADq + BDq sint μα per hypoth.

PROP. LXXXV.



Si (AB,) quæ cum medio medium totum facit una tantum congruit recta linea BC potentia incommensurabilis existens toti, & cum tota faciens & compositum ex ipsarum quadratis medium, & quod sub ipsis continetur, medium, incommensurabileque composito ex ipsarum quadratis.

Suppositis iis quæ facta & ostensa sunt in 82 hujus; liquet EH, & KH esse ρ [c] ⊓⊔ EF. Porro igitur quia ACq + CBq, hoc est, rectang. EG [a] ⊓⊔ ACB, [b] ideoque EG ⊓⊔ 2 ACB (KG) estque EG. KG :: [c] EH. KH; erit EH ⊓⊔ KH. ergo EK est apotome, cujus congruens KH. Haud aliter KM eidem apotomæ EK. congruere ostendetur; contra 80 hujus.

a hyp.
b 14. 10.
c 1. 6.

Definitiones tertiæ.

EXposita rationali, & apotoma, si tota plus possit quam congruens quadrato rectæ lineæ sibi longitudine commensurabilis;

I. Si quidem tota expositæ rationali longitudine sit commensurabilis, vocetur apotome prima.

II. Si vero congruens expositæ rationali longitudine sit commensurabilis, vocetur apotome secunda.

III. Quod si neque tota, neque congruens expositæ rationali sit longitudine commensurabilis, vocetur apotome tertia.

Rursus, si tota plus possit quam congruens quadrato rectæ sibi longitudine incommensurabilis;

IV. Si quidem tota expositæ rationali sit longitudine commensurabilis, vocetur apotome quarta.

V. Si vero congruens expositæ rationali sit longitudine commensurabilis, vocetur apotome quinta.

VI. Quod si neque tota, neque congruens, expositæ rationali sit longitudine commensurabilis, vocetur apotome sexta.

PROP. LXXXVI, 87, 88, 89, 90, 91.

A.... 4 C..... 5 B
D———————
E————-----F
G
H————

Invenire apotomen primam, secundam, tertiam, quartam, quintam, sextam.

Apotomæ inveniuntur, subductis minoribus binomiorum nominibus ex majoribus. Exemp. gr. Sit $6 + \sqrt{20}$, bin. 1. erit $6 - \sqrt{20}$, apot. 1. &c. Quare de earum inventione plura repetere nihil est necesse.

LEMMA.

Sit rectangulum AC *sub rectis* AB, AD. *producatur* AD *ad* E, *&* *bisecetur* DE *in* F. *sitque rectang.* AGE = FEq. *&* *compleantur rectangula* AI, DK, FH. *Fiant vero quadratum* LM = AH, *&* *quadratum* NO = GI, *producanturque* NSR, OST.

Dico primo, rectangul. AI = LM + NO = TOq + SOq. ut patet ex constr.

Se-

Secundo, *Rectang.* DK = LO. Nam quia rectang. A G E *a* = FEq, *b* sunt A G, F E, G E ∺, *c* adeoque AH, FI, GI ∺; *a* hoc est, LM, FI, NO ∺. atqui LM, LO, NO *d* sunt ∺; ergo FI = *e* LO *f* = DK = *g* NM. a *constr.* b 17. 6. c 1. 6. d *sch.* 22. 6. e 9. 5. f 36. 1.

Tertio, *Hinc*, AC = AI − DK − FI = LM + NO − LO − NM = TR. g 43. 1

Quarto, *h Liquet* DF, FE, DE esse ⊔. h 16. 10.

Quinto, *Si* AE ⊔ DE, & AE ⊔ $\sqrt{}$ AEq − DEq, *k erunt* AG, GE, AE ⊔. k 18. 10. &

Sexto, *Item, quia* AE *l* ⊔ DE, *m erunt* AE, FE ⊔. *n ideoque* AI, FI; *hoc est*, LM + NO & LO *sunt* ⊔. 10. 10. l *hyp.* m 13. 10.

Septimo, *Item quia* AG * ⊔ GE, *n erunt* AH GI, *hoc est*, LM, NO ⊔. n 1. 6. & 10. 10. * prius.

Octavo, *Sed quia* AE *l* ⊔ DE, *o erunt* FE, GE ⊔, *n ideoque rectang.* FI ⊔ GI, *hoc est* LO ⊔ NO. *quare cum* LO. NO *p* :: TS, SO. *q erunt* TS. SO ⊔. o 14. 10. p 1. 6. q 10. 10.

Nono, *Sin ponatur* AE ⊔ $\sqrt{}$ AEq − DEq; *r erunt* AG, GE, AE ⊔. r 19. 10. & 17. 10.

Decimo, *s Quare rectang.* AH, GI, *hoc est* TOq, SOq *erunt* ⊔. s 1. 6. & 10. 10.

PROP. XCII.

Si spatium AC contineatur sub rationali AB, & Apotoma prima AD (AE − DE;) recta linea TS spatium AC potens, apotome est.

Adhibe lemma proxime antecedens pro præparatione ad demonstrationem hujus. Igitur TS = √AC. item AG, GE, AE sunt ⊓⊔; ergo cum AE ⊓⊔ *a* AB ρ'; *b* erunt AG, & GE ⊓⊔ AB. *c* ergo rectangula AH & GI, hoc est TOq & SOq sunt ρ'α. *d* item TO, SO sunt ρ' ⊓⊔, *e* proinde TS est apotome. Q. E. D.

a *hyp.*
b 12. 10.
c 20. 10.
d *lem.* 91. 10.
e 74. 10.

PROP. XCIII.

Vide Schem. præced.

Si spatium AC contineatur sub rationali AB, & apotoma secunda AD (AE − DE;) recta linea TS spatium AC potens; mediæ est apotome prima.

Rursus juxta lemma antecedens, AG, GE, AE sunt ⊓⊔. cum igitur AE *a* sit ρ' ⊓⊔ AB, *b* erunt AE, GE etiam ρ' ⊓⊔ AB. *c* ergo rectangula AH, GI, hoc est TOq, SOq, sunt μμ; *d* item TO ⊓⊔ SO. Denique quia DE *e* ⊓⊔ AB. ρ'. *f* erit rectang. DI, ejusque semissis DK, vel LO, hoc est TOS ρ'ν *g* è quibus sequitur TS (√AC) esse mediæ apot. 1. Q. E. D.

a *hyp.*
b 13. 10.
c 22. 10.
d *lem.* 74 10.
e *hyp.*
f 20. 10.
g 75. 10.

PROP.

PROP. XCIV.

Vide idem.

Si spatium AC contineatur sub rationali AB, & apotoma tertia AD (AE − DE;) recta linea TS spatium AC potens, mediæ est apotome secunda.

Ut in præcedenti TO, & SO sunt μ. Quoniam igitur DE *a* est ρ^c ⊓⊔ AB, *b* erit rectang. a hyp.
DI, *c* ideoque DK; vel TOS μν. *d* ergo TS b 22. 10. c 14. 10.
= √ AC est mediæ apot. 2. Q. E. D. d 76. 10.

PROP. XCV.

Vide idem.

Si spatium AC contineatur sub rationali AB, & apotoma quarta AD (AE − DE) recta linea TS spatium AC potens, minor est.

Rursus TO *a* ⊓⊔ SO. Quoniam igitur AE a lem 91. 10.
b est ρ^c ⊓⊔ AB, *c* erit AI, (TOq + SOq) ρ^c ν. b hyp. c 20 10.
atqui ut prius rectang. TOS est μν. *d* ergo TS d 77. 10.
= √ AC est minor. Q. E. D.

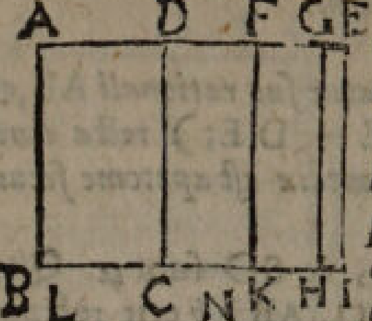
PROP. XCVI.

Vide idem.

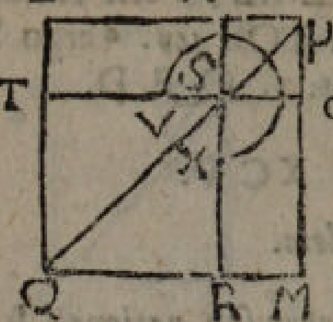
Si spatium AC contineatur sub rationali AB, & apotoma quinta AD (AE − DE;) recta linea TS spatium AC potens, est quæ cum rationali medium totum efficit.

Rursus enim TO ⊓⊔ SO. itaque cum AE
a sit ρ^c ⊓⊔ AB, *b* erit AI, hoc est TOq + SOq
μν. Sed prout in 93 rectang. TOS est ρ^c ν. *c* proinde a hyp. b 22. 10.
TS = √ AC est quæ cum ρ^c ν facit totum c 78. 10.
μν. Q. E. D.

PROP. XCVII.



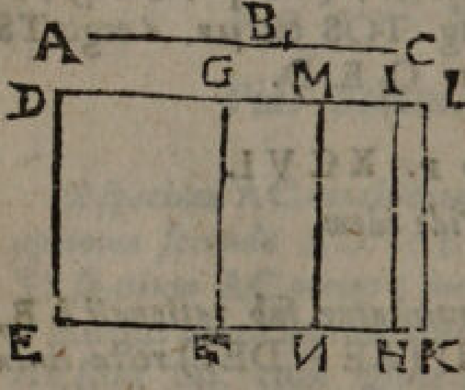
Si spatium A C *contineatur sub rationali* A P, *& apotoma sexta* A D (A E − D E;) *recta linea* T S *spatium* A C *potens, est quæ cum medio medium totum efficit.*



Itidem, ut sæpe prius, TO ⊔⊓ SO. item ut in 96, TOq + SOq est μν. rectang. vero TOS est ρ'ν, ut in 94. a denique TOq + SOq ⊔⊓ TOS. b ergo TS = √ AC est quæ cum μν facit totum μν. Q. E. D.

a lem 91. 10.
b 79. 10.

LEMMA A.



* cor. 166.

Ad rectam quamvis D E * *applicentur rectang.* DF = ABq, *&* DH = ACq, *&* IK = BCq; *& sit* GL *bisecta in* M; *ductaque sit* M N *parall.* GF.

Erit primo, *Rectang.* DK = ACq + BCq, *ut constructio indicat.*

Secundo, *Rectang.* ACB = GN, *vel* MK. Nam DK a = ACq + BCq b = 2 ACB + ABq. at ABq a = DF. ergo GK c = 2 ACB. & d proinde GN, vel MK = ACB.

a constr.
b 7. 2.
c 3 ax. 1.
d 7. ax. 1.

Tertio, *Rectang.* DIL = MLq. Nam quia ACq. ACB e :: ACB. BCq; hoc est DH. MK

e 1. 6

MK :: MK. IK, *e* erit DL. ML :: ML. IL. *f* ergo DIL = MLq. f 17. 6.

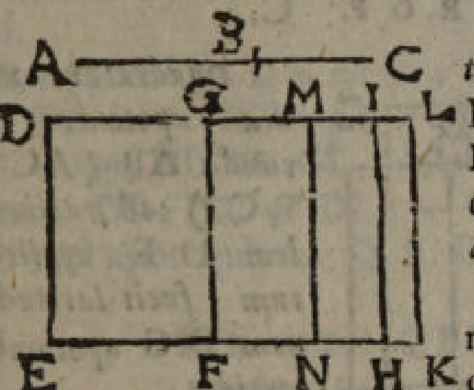
Quarto, *Si ponatur* AC ⊐⊏ BC, *erit* DK ⊓⊔ ACq. Nam ACq + BCq (DK) *g* ⊓⊔ ACq. g 16. 10.

Quinto, *Item*, DL ⊓⊔ √ DLq − GLq. Nam quia DH (ACq) ⊓⊔ IK (BCq) *h* erit DI ⊓⊔ IL. *k* ergo √ DLq − GLq ⊓⊔ DL. h 10. 10. k 18. 10.

Sexto, *Item* DL ⊐⊏ GL. Nam ACq + BCq ⊓⊔ *l* 2 ACB; hoc est, DK ⊓⊔ GK. *m* ergo DL ⊓⊔ GL. l lem. 16. 10. m 10. 10.

Septimo, *Sin ponatur* AC ⊐⊏ BC, *n erit* DL ⊓⊔ √ DLq − GLq. n 19. 10.

PROP. XCVIII.



Quadratum apotomæ AB (AC − BC) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen primam.*

Fac ut in lemmate proxime præcedenti.

Quoniam igitur AC, BC *a* sunt ρ^c ⊐⊏, *b* erit DK (ACq + BCq) ⊓⊔ ACq; *c* ergo DK est $\rho^c v$. *d* quare DL est ρ^c ⊓⊔ DE. *e* item rectang. GK (2 ACB) est μv. *f* ergo GL est ρ^c ⊓⊔ DE. *g* proinde DL ⊓⊔ GL; *h* sed DLq ⊓⊔ GLq. *k* ergo DG est apotome, & *l* quidem prima (quia *m* AC ⊐⊏ BC, & propterea DL ⊓⊔ √ DLq − GLq.) Q. E. D.

a hyp. b lem. 97. 10. c sch. 12. 10. d 21. 10. e 22, & 24. 10. f 23. 10. g 13. 10. h sch. 12. 10. k 74. 10. l 1 def. 85. 10. m lem. 97. 10.

PROP. XCIX.

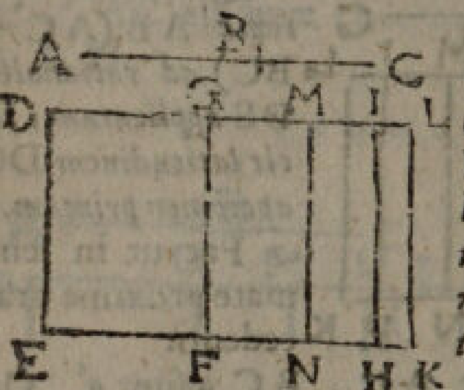
Vide Schema ſubſequens.

Quadratum mediæ apotomæ primæ AB (AC — BC) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen ſecundam.*

a hyp. b lem 97. 10. c 24. 10. d 23. 10. e hyp. & ſch. 12. 10. f 21. 10. g 13. 10. h ſch. 12. 10. k 74. 10. l lem 97. 10. m 2. def. 85. 10.

Rurſus (ſuppoſito lemmate præcedenti) quia AC, & BC [a] ſunt μ ⊓ [b], erit DK (ACq + BCq) ⊔ ACq; [c] quare DK eſt μν. [d] ergo DL eſt ῥ ⊔ DE. [c] item GK (2. ACB) eſt [e] μν [f] ergo GL eſt [e] ῥ ⊔ DE; [g] quare DL ⊔ GL. [h] Sed DLq ⊔ GLq. [k] ergo DG eſt apotome. quia vero DL [l] ⊔ $\surd$ DLq — GLq, [m] erit DG apotome ſecunda. Q. E. D.

PROP. C.



Quadratum mediæ [e] apotomæ ſecundæ AB (AC — BC) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen tertiam.*

a 23. 10. b lem. 16. 10. c 1. 6. & 10. 10. d ſch. 12. 10. e 74. 10. f 3. def. 85. 10. g lem. 97. 10.

Iterum DK eſt μν, [a] quare DL eſt [e] ῥ ⊔ DE. item GK eſt μν. [a] unde GL eſt ῥ [e] ⊔ DE; [b] item DK ⊔ GK, [c] quare DL ⊔ GL; [d] at DLq ⊔ GLq. [e] ergo DG eſt apot. & quidem [f] 3ª. [g] quia DL ⊔ $\surd$ DLq — GLq. Q. E. D.

PROP. CI.

Vide Schema præced.

Quadratum minoris AB (AC — BC) *ad rationalem* *[rati]onalem DE applicatum, facit latitudinem DG a[p]otomen quartam.*

Ut prius, ACq + BCq, hoc est DK est $\acute{\rho}\nu$; [a] ergo DL est $\acute{\rho}$ ⊓⊔ DE. at rectang ACB, ideque GK (2 ACB) * est $\mu\nu$, b quare GL est $\acute{\rho}$ ⊓⊔ DE. c ergo DL ⊓⊔ GL. d at DLq ⊓⊔ GLq. quia vero * ACq ⊓⊔ BCq, e erit DL ⊓⊔ $\surd$ DLq − GLq: f ergo DG conditiones habet [a]potomæ quartæ. Q. E. D. — a 21. 10. * hyp. b 23. 10. c 13. 10. d sch. 12. 10. e lem 97 10 f 4. def 85. 10.

PROP. CII.

Vide Schem. præced.

Quadratum ejus AB (AC − BC,) quæ cum [r]ationali medium totum efficit, ad rationalem DE [a]pplicatum, facit latitudinem DG apotomen quin[t]am.

Rursus enim, DK est $\mu\nu$, a quare DL est $\acute{\rho}$ ⊓⊔ DE. item GK est $\acute{\rho}\nu$, b unde GL est $\acute{\rho}$ ⊓⊔ DE. c ergo DL ⊓⊔ GL, d sed DLq ⊓⊔ GLq. [p]orro, DL e ⊓⊔ $\surd$ DLq − GLq. ex quibus, DG f est apot. quinta. Q. E. D. — a 23. 10. b 21. 10. c 13. 10. d sch 12 10. e lem. 97. 10. f 5 def 85. 10.

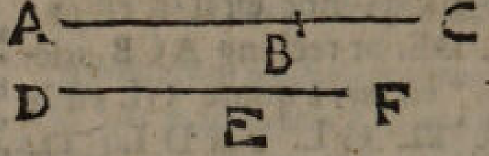
PROP. CIII.

Vide Schema idem.

Quadratum ejus AB (AC − BC,) quæ cum medio medium totum efficit, ad rationalem DE applicatum, facit latitudinem DG apotomen sextam.

Haud aliter, quam antea, DK, & GK sunt $\mu\alpha$; a quare DL & GL sunt $\acute{\rho}$ ⊓⊔. DE. item DK b ⊓⊔ GK, c quare DL ⊓⊔ GL. d ergo DG est apot. b cum igitur ACq ⊓⊔ BCq, ideoque DL ⊓⊔ $\surd$ DLq − GLq, e erit DG. apot. sexta. Q. E. D. — a 23. 10. b hyp & lem. 97. 10. c 10 10. d 74 10. e 6 def 85. 10.

PROP. CIV.



Recta linea DE *apotomæ* AB (AC — BC) *longitudine commensurabilis, & ipsa apotome est, atque ordine eadem.*

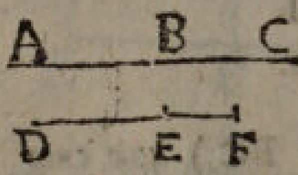
LEMMA.

Sit AB. DE :: AC. DF. & AB ⊓⊔ DE. Dico AC + BC ⊓⊔ DF + EF.

Nam AC. BC [a] :: DF. EF. ergo componendo AC + BC. BC :: DF + EF. EF. ergo per-
a lem 66 10. b 10.10. mutando AC + BC. DF + EF :: BC. EF. [a] at BC ⊓⊔ EF. [b] ergo AC + BC ⊓⊔ DF + EF. Q. E. D.

a 12 6 b lem. 103. 10. c hyp. d 67. 10. e Præcedentes ad 85. 10. [a] Fac AB. DE :: AC. DF. [b] igitur AC + BC ⊓⊔ DF + EF. ergo cum AC + BC [c] binomium sit, [d] erit DF + EF ejusdem ordinis binomium: [e] quare DF — EF ejusdem ordinis apotome est, cujus AC — BC. Q. E. D.

PROP. CV.

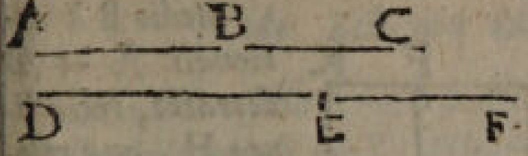


Recta linea DE *mediæ apotomæ* AB (AC — BC) *commensurabilis, & ipsa mediæ apotome est, atque ordine eadem.*

a 12. 6 b lem. 103. 10. c 68 10 d 75, & 76. 10. Iterum [a] fac AB. DE :: AC. DF. [b] quare AC + BC ⊓⊔ DF + EF. [c] ergo DF + EF est bimed. ejusdem ordinis, cujus AC + BC. [d] proinde & DF — EF mediæ apotome erit ejusdem classis, cujus AC — BC. Q. E. D.

PROP.

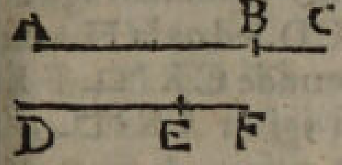
PROP. CVI.



Recta linea DE *Minori* AB (AC − BC) *commensurabilis, & ipsa minor est.*

Fiat AB. DE :: AC. DF. *a* estque AC + BC a lem. 103. 10.
TL DF + EF. atqui AC + BC *b* est Major, b hyp.
c ergo DF + EF quoque Major est. *d* & proinde c 69. 10. d 77. 10.
DF − EF est Minor. Q. E. D.

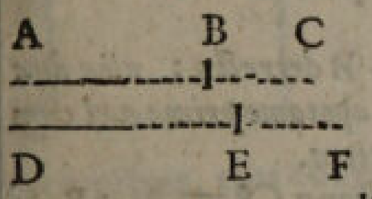
PROP. CVII.



Recta linea DE *commensurabilis ei* AB (AC − BC) *quæ cum rationali medium totum efficit, & ipsa cum rationali medium totum efficiens est.*

Nam ad modum præcedentium ostendemus DF + EF esse potentem ῤν, & μ̓ν. *a* ergo DF a 78. 10.
− EF est ut dicitur.

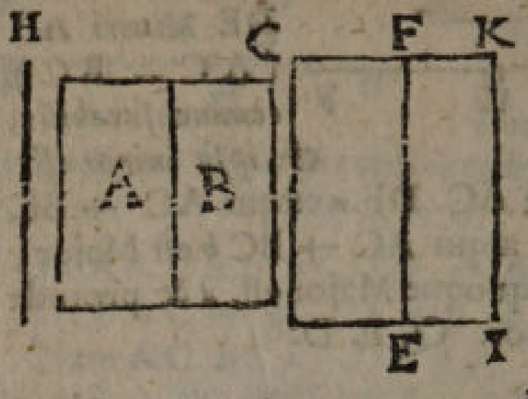
PROP. CVIII.



Recta linea DE *commensurabilis ei* AB (AC − BC) *quæ cum medio medium totum efficit, & ipsa cum medio medium totum efficiens est.*

Nam, ad normam præcedentium, erit DF + EF potens 2 μ̓α. *a* ergo DF − EF erit ut in prop. a 79. 10

PROP. CIX.



Medio B à rationali A + B detracto, recta linea H, quæ reliquum spatium A potest, una ex duabus irrationalibus fit, vel apotome, vel Minor.

Ad CD ρʹ, fac rectang. CI = A + B; & FI = B. quare CE *a* = A: (Hq) Quoniam igitur CI *b* est ῤν, *c* erit CK ρʹ ⊔ CD. sed quia FI *b* est μν, *d* erit FK ρʹ ⊔ CD. *e* unde CK ⊔ FK *f* ergo CF est apotome. Si igitur CK ⊔ √ CKq − FKq. *g* erit CF apot. prima; *h* quare √ CE (H) est apotome. sin CK ⊔ √ CKq − FKq, *k* erit CF apot. quinta. & proinde H (√ CE) *l* erit Minor. Q. E. D.

a 3.ax 1.
b hyp. & constr.
c 21.10
d 23.10.
e 13 10.
f 74 10.
g 1.def.85. 10.
h 92.10
k 4 def.85. 10.
l 95.10.

PROP. CX.

Vide Schem. præced.

Rationali B à medio A + B detracto; aliæ duæ irrationales fiunt, vel mediæ apotome prima, vel cum rationali medium totum efficiens.

Ad CD expos. ῤ fiant rectang. CI = A + B; & FI = B, *a* unde CE = A = Hq. Quoniam igitur CI *b* est μν: *c* erit CK ρʹ ⊔ CD. sed quia FI *b* est ῤν, *d* erit FK ρʹ ⊔ CD. *e* unde CK ⊔ FK. *f* ergo CF est apot. *g* nempe secunda; si CK ⊔ √ CKq − FKq, *h* quare H (√ CE) est mediæ apot. prima. Sin vero CK ⊔ √ CKq − FKq, *k* erit CF apot. quinta. & proinde H (√ CF) erit faciens μν cum ῤν. Q. E. D.

a 3 ax. 1.
b hyp. & constr.
c 23.10.
d 21.10.
e 13.10.
f 74 10.
g 2.def.85. 10.
h 93.10.
k 5.def.85. 10.
96 10.

PROP.

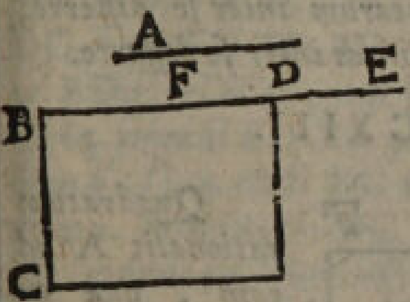
Prop. CXI.

Vide Schema idem.

Medio B à medio A + B detracto, quod sit incommensurabile toti A + B; reliquæ duæ irrationales sunt, vel mediæ apotome secunda, vel cum medio medium totum efficiens.

Ad CD ρ' fiant rectang. CI = A + B; & FI = B, *a* quare CE = A = Hq. Quoniam a 3 ax. 1.
igitur CI est μν. *b* erit CK ρ' ⊓ CD. eodem b 23. 10.
modo erit FK ρ' ⊓ CD. item quia CI *c* ⊓ c hyp.
FI, *d* erit CK ⊓ FK; *e* quare CF est apoto- d 10. 10.
ne, *f* tertia scilicet, si CK ⊓ √ CK − FKq, e 74. 10
g unde H (√ CE) erit mediæ apot. secunda. f 3 def. 85. 10.
verum si CK ⊓ √ CKq − FKq, *h* erit CF g 94. 10.
apot. sexta. *k* quare H erit faciens μν cum μ. h 6 def. 85. 10.
Q. E. D. k 97. 10.

Prop. CXII.



Apotome A non est eadem, quæ ex binis nominibus.

Ad expos. BC ρ', fiat rectang. CD = Aq. Ergo cum A sit apotome, *a* erit BD apot. prima. ejus congruens sit DE. *b* quare BE, DE sunt ρ' ⊔. *c* & BE ⊓ BC. Vis A esse bin. ergo BD est bin. 1. ejus nomina sint BF, FD; sitque BF ⊏ FD; *d* ergo BF, FD sunt ρ' ⊔; & BF *e* ⊓ BC. ergo cum BC ⊓ BE, *f* erit BE ⊓ FB. *g* ergo BE ⊓ FE. *h* ergo FE est ρ'. item quia BE ⊓ DE, *k* erit FE ⊓ DE. *l* quare FD est apotome, *l* adeoque FD est ῤ. sed ostensa est ρ'. quæ repugnant. ergo A male dicitur binomium. Q. E. D.

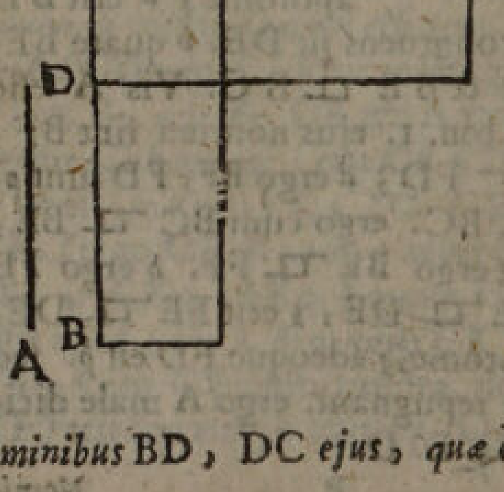
a 98. 10. b 74. 10. c 1. def 85. 10. d 37. 10. e 1. def. 48. 10. f 12. 10. g cor. 16. 10. h sch. 12. 10. k 14. 10. l 74. 10.

Nomina 13 linearum irrationalium inter se differentium.

1. Media.
2. Ex binis nominibus, cujus 6 ſpecies
3. Ex binis mediis prima.
4. Ex binis mediis ſecunda.
5. Major.
6. Rationale ac medium potens.
7. Bina media potens.
8. Apotome, cujus etiam 6 ſpecies.
9. Mediæ apotome prima.
10. Mediæ apotome ſecunda.
11. Minor.
12. Cum rationali medium totum efficiens.
13. Cum medio medium totum efficiens.

Cum latitudinum differentiæ arguant differentias rectarum, quarum quadrata ſunt applicata ad aliquam rationalem, ſitque demonſtratum in præcedentibus, latitudines quæ oriuntur ex applicationibus quadratorum harum 13 linearum inter ſe differre, perſpicue ſequitur has 13 lineas inter ſe differre.

PROP. CXIII.



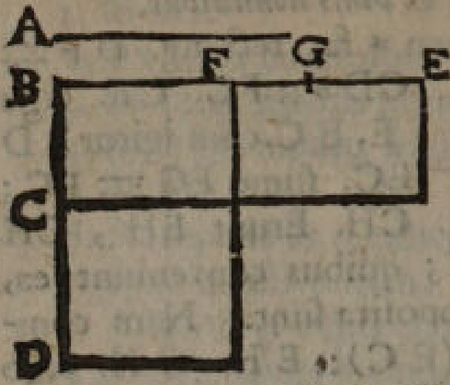
Quadratum rationalis A *ad eam, quæ ex binis nominibus* BC (BD + DC) *applicatum, latitudinem facit apotomen* EC, *cujus nomina* EH, CH *commenſurabilia ſunt nominibus* BD, DC *ejus, quæ ex binis nominibus &*

& in eadem proportione (EH. BD :: CH. DC;) *& adhæc, apotome* EC *quæ fit, eundem habet ordinem, quem ea* BC, *quæ ex binis nominibus.*

Ad DC minus nomen [a] fac rectang. DF = Aq = BE. quare BC. CD [b] :: FC. CE. ergo dividendo BD. DC :: FE. EC. cum igitur BD [c] ⊏ DC, [d] erit FE ⊏ EC. sume EG = EC; fiatque FG. GE :: EC. CH. Erunt EH, CH nomina apotomæ EC; quibus conveniunt ea, quæ in theoremate proposita sunt. Nam componendo FE. GE. (EC) :: EH. GH. ergo FH. EH [e] :: EH. CH [f] :: FE. EC [f] :: BD. DC. quare cum BD [g] ⊐ DC, [h] erit EH ⊐ CH; [h] & FHq ⊏ EHq. ergo, quia FHq. EHq [k] :: FH. CH. [k] erit FH ⊏ CH, [l] ideoque FC ⊏ CH. Porro CD [g] est ρ́, & DF (Aq) [g] est ρ́ν, [m] ergo FC est ρ́ ⊏ CD, quare etiam CH est ρ́ ⊏ CD. [n] igitur EH CH sunt ρ́, ac ⊐ ut prius. [o] ergo EC est apotome, cui congruit CH. porro EH. CH [f] :: BD. DC, ideo permutando EH. BD :: CH. DC. unde quia CH [f] ⊏ DC, [p] erit EH ⊏ BD. quinimo pone BD ⊏ √BDq − DCq; [q] erit ideo EH ⊏ √EHq − CHq. item si BD ⊏ ρ́ expos. erit EH ⊏ eidem ρ́; [s] hoc est si BC sit bin. 1. [r] erit EC apot. prima. Similiter si DC ⊏ ρ́ expos. [t] erit CH ⊏ eidem ρ́. [u] hoc est si BC sit bin. 2. [x] erit EC apot. 2. & si hæc bin. 3. illa erit apot. 3, &c. Sin BD ⊏ √BDq − DCq, [y] erit EH ⊏ √EHq − CHq; si igitur BC sit bin. 4, vel 5, vel 6. erit EC similiter apot. 4, vel 5, vel 6. Q. E. D.

a cor. 16. 6.
b 14. 6.
c hyp.
d 14. 5.
e 12. 5.
f Prius.
g hyp.
h 10. 10.
k cor. 20. 6.
l 16. 10.
m 21. 10.
n sch. 12. 10.
o 74. 10.
p 10. 10.
q 15. 10.
r 12. 10.
s 1. def.
48. 10.
t 1. def.
85. 10.
u 2. def.
48. 10.
x 2. def.
85. 10.
y 15. 10.

PROP. CXIV.

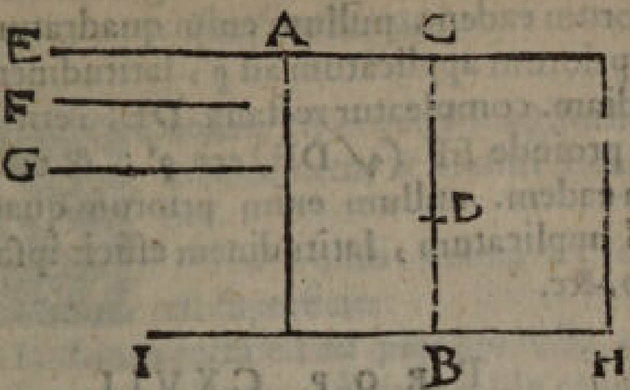


Quadratum rationalis A *ad apotomen* BC (BD—DC) *applicatum, facit latitudinem* BE *eam, quæ ex binis nominibus; cujus nomina* BE, GE *commensurabilia sint apotomæ* BC *nominibus* BDDC, *& in eadem proportione; & adhuc, quæ ex binis nominibus fit* (BE,) *eundem habet ordinem, quem ipsa apotome* BC.

a cor.16.6. *a* Fac rectang. DF = Aq; & BE. FE *b* ::
b 12. 6. EG. GF. Quoniam igitur DF = Aq = CE,
c 14.6. *c* erit BD. BC :: BE. BF. ergo per conversionem rationis BD. CD :: BE. FE :: EG. GF ::
d 19. 5. *d* BG. EG. sed BD *e* ⊐⊏ CD. *f* ergo BG ⊐⊏
e hyp. GE. ergo quia BGq. GEq *g* :: BG. GF. *h* erit
f 10 10. BG ⊓⊔ GF. *k* ideoque BG ⊓⊔ BF. porro
g cor.20. 6. BD *e* est ρ́, & rectang. DF (Aq) *e* est ρ́ν. *l* er-
h 10. 10. go BF est ρ́ ⊓⊔ BD. *m* ergo etiam BG est ρ́ ⊓⊔
k cor.16. 10. BD. *n* ergo BG, GE sunt ρ́ ⊐⊏. *o* quare BE
l 21.10. est bin. denique igitur quia BD. CD :: BG.
m 12. 10. GE; & permutando BD. BG :: CD. GE; sitque
n sch.12.10. BD ⊓⊔ BG; *p* erit CD ⊓⊔ GE. ergo si CB sit
o 37. 10. apot. prima; erit BE bin. 1. &c ut in anteceden-
p 10, 10. ti. ergo, &c.

PROP.

PROP. CXV.



Si spatium AB contineatur sub apotoma AC (CE − AE,) & ea, quæ ex binis nominibus CB; cujus nomina CD, DB commensurabilia sint apotomæ nominibus CE, AE, & in eadem proportione (CE. AE :: CD. DB.;) recta linea F spatium AB potens, est rationalis.

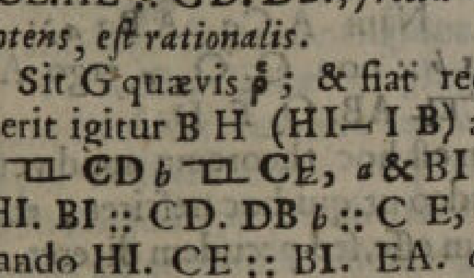
Sit G quævis ῥ; & fiat rectang. CH = Gq. a erit igitur BH (HI − IB) apotome; & HI a ⊓⊔ CD b ⊓⊔ CE, a & BI ⊓⊔ DB; a atque HI. BI :: CD. DB b :: CE, EA. ergo permutando HI. CE :: BI. EA. c ergo BH. AC :: HI. CE :: BI. EA. ergo cum HI d ⊓⊔ CE, e erit BH ⊓⊔ AC. f ergo rectang. HC ⊓⊔ BA. Sed HC (Gq) b est ῥν. g ergo BA (Fq) est ῥν. proinde F est ῥ. Q. E. D.

a 113. 10.
b hyp.
c 19. 5.
d 12. 10.
e 10. 10.
f 1. 6. & 10. 10
g sch. 12. 10.

Coroll.

Hinc, fieri potest, ut spatium rationale contineatur sub duabus rectis irrationalibus.

PROP. CXVI.

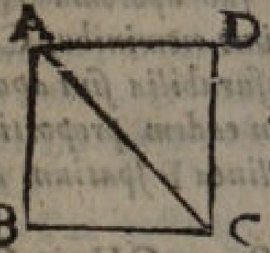


A media AB fiunt infinitæ irrationales BE, EF, &c. & nulla alicui antecedentium est eadem.

Sit AC expos.

a lem. 38. 10. b 11. 10. ρ́. ſitque AD ſpatium ſub AC, AB. *a* ergo AD eſt ρ'ν. Sume BE = √ AD. *b* ergo BE eſt ρ', nulli priorum eadem. nullum enim quadratum alicujus priorum applicatum ad ρ́, latitudinem efficit mediam. compleatur rectang. DE; *a* erit DE ρ'ν; & *b* proinde EF (√ DE) erit ρ'; & nulli priorum eadem. nullum enim priorum quadratum ad ρ́ applicatum, latitudinem efficit ipſam BE. ergo, &c.

Prop. CXVII.



Propoſitum ſit nobis oſtendere, in quadratis figuris BD, diametrum AC lateri AB incommenſurabilem eſſe.

a 47. 1. b cor. 24. 8. c 9. 10. Nam ACq. ABq *a* :: 2. 1 *b* :: non Q. Q. *c* ergo AC ⊥ AB. Q. E. D.

Celebratiſſimum eſt hoc theorema apud veteres philoſophos, adeo ut qui hoc neſciret, eum Plato non hominem eſſe, ſed pecudem diceret.

LIB.

LIB. XI.

Definitiones.

I. Solidum est, quod longitudinem, latitudinem, & crassitudinem habet.

II. Solidi autem extremum est superficies.

III. Linea recta est ad planum recta, cum ad rectas omnes lineas, à quibus illa tangitur, quæque in proposito sunt plano, rectos angulos efficit.

IV. Planum ad planum rectum est, cum rectæ lineæ, quæ communi planorum sectioni ad rectos angulos in uno plano ducuntur, alteri plano ad rectos sunt angulos.

V. Rectæ lineæ ad planum inclinatio est, cum à sublimi termino rectæ illius lineæ ad planum deducta fuerit perpendicularis; atque à puncto quod perpendicularis in ipso plano effecerit, ad propositæ illius lineæ extremum, quod in eodem est plano, altera recta linea fuerit adjuncta; est, inquam, angulus acutus insistente linea, & adjuncta comprehensus.

VI. Plani ad planum inclinatio, est angulus acutus rectis lineis contentus, quæ in utroque planorum ad idem communis sectionis punctum ductæ, rectos cum sectione angulos efficiunt.

VII. Planum ad planum similiter inclinatum esse dicitur, atque alterum ad alterum, cum dicti inclinationum anguli inter se fuerint æquales.

VIII. Parallela plana sunt, quæ inter se non conveniunt.

IX. Similes solidæ figuræ sunt, quæ similibus planis continentur, multitudine æqualibus.

X. Æquales & similes solidæ figuræ sunt, quæ ſimilibus planis multitudine & magnitudine æqualibus continentur.

X I. Solidus angulus eſt plurium quam duarum linearum, quæ ſe mutuo contingunt, nec in eadem ſunt ſuperficie, ad omnes lineas inclinatio.

Aliter.

Solidus angulus eſt, qui pluribus quam duobus planis angulis in eodem non conſiſtentibus plano, ſed ad unum punctum conſtitutis, continetur.

X I I. Pyramis eſt figura ſolida, planis comprehenſa, quæ ab uno plano ad unum punctum conſtituuntur.

X I I I. Priſma eſt figura ſolida, quæ planis continetur, quorum adverſa duo ſunt & æqualia, & ſimilia, & parallela; alia vero parallelogramma.

X I V. Sphæra eſt, quando ſemicirculi manente diametro, circumductus ſemicirculus in ſeipſum rurſus revolvitur unde moveri cœperat, circumaſſumpta figura.

Coroll.

Hinc radii omnes à centro ad ſuperficiem ſphæræ inter ſe ſunt æquales.

X V. Axis autem ſphæræ, eſt quieſcens illa recta linea, circum quam ſemicirculus convertitur.

X V I. Centrum ſphæræ eſt idem quod & ſemicirculi.

X V I I. Diameter autem ſphæræ, eſt recta quædam linea per centrum ducta, & utrinque à ſphæræ ſuperficie terminata.

X V I I I. Conus eſt, quando rectanguli trianguli manente uno latere eorum, quæ circa rectum angulum, circumductum triangulum in ſeipſum rurſus revolvitur unde moveri cœperat, circumaſſumpta figura. Atque ſi quieſcens recta linea

linea æqualis ſit reliquæ, quæ circa rectum angulum continetur, orthogonius erit conus; ſi vero minor, amblygonius; ſi vero major, oxygonius.

XIX. Axis autem coni, eſt quieſcens illa linea, circa quam triangulum vertitur.

XX. Baſis vero coni eſt circulus qui à circumducta recta linea deſcribitur.

XXI. Cylindrus eſt, quando rectanguli parallelogrammi manente uno latere eorum, quæ circa rectum angulum, circumductum parallelogrammum in ſeipſum rurſus revolvitur unde cœperat moveri, circumaſſumpta figura.

XXII. Axis autem cylindri, eſt quieſcens illa recta linea, circum quam parallelogrammum convertitur.

XXIII. Baſes vero cylindri ſunt circuli à duobus adverſis lateribus, quæ circumaguntur, deſcripti.

XXIV. Similes coni & cylindri ſunt, quorum & axes, & baſium diametri proportionales ſunt.

XXV. Cubus eſt figura ſolida ſub ſex quadratis æqualibus contenta.

XXVI. Tetraedrum eſt figura ſolida ſub quatuor triangulis æqualibus & æquilateris contenta.

XXVII. Octaedrum eſt figura ſolida ſub octo triangulis æqualibus & æquilateris contēta.

XXVIII. Dodecaedrum eſt figura ſolida ſub duodecim pentagonis æqualibus & æquilateris & æquiangulis contenta.

XXIX. Icoſaedrum eſt figura ſolida ſub viginti triangulis æqualibus & æquilateris contenta.

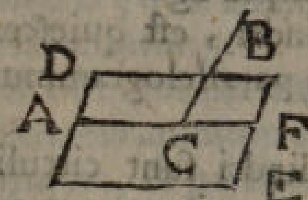
XXX. Parallelepipedum eſt figura ſolida ſex figuris quadrilateris, quarum quæ ex adverſo parallelæ ſunt, contenta.

XXXI. So-

XXXI. Solida figura in solida figura dicitur inscribi, quando omnes anguli figuræ inscriptæ constituuntur vel in angulis, vel in lateribus, vel denique in planis figuræ, cui inscribitur.

XXXII. Solida figura solidæ figuræ vicissim circumscribi dicitur, quando vel anguli, vel latera, vel denique plana figuræ circumscriptæ tangunt omnes angulos figuræ, circum quam describitur.

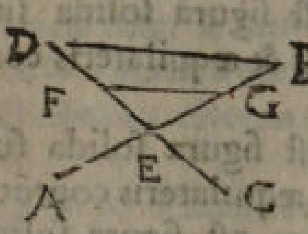
Prop. I.



Rectæ lineæ pars quædam A C non est in subjecto plano, quædam vero CB in sublimi.

Producatur A C in subjecto plano usque ad F. vis CB esse in directum ipsi AC; ergo duæ rectæ AB, AF habent commune segmentum A C.
a 10.ax.1. a Q. F. N.

Prop. II.



Si duæ rectæ lineæ A B, C D se mutuo secent, in uno sunt plano: atque triangulum omne DEB in uno est plano.

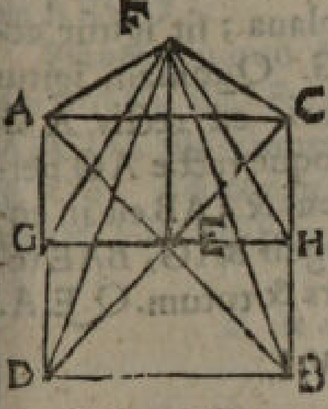
Puta enim trianguli DEB partem EFG esse in uno plano, partem vero FDGB in altero. ergo rectæ ED pars EF est in subjecto plano, pars vero FD in sublimi, a Q. E. A. ergo triangulum EDB in uno est plano; proinde & rectæ ED, EB;
a 1. 11. a quare & totæ A B, D C in uno plano existunt. Q. E. D.

Prop. III.

Si duo plana A B, C D *se mutuo secent, communis eorum sectio* E F *est recta linea.*

Si E F communis sectio non est recta linea, *a* ducatur in plano A B recta EGF, *a* & in plano CD recta EHF. duæ igitur rectæ EGF, EHF claudunt spatium. *b* Q. E. A. a 1. post. 1. b 14. ax. 1.

Prop. IV.



Si recta linea E F *rectis duabus lineis* AB, CD *se mutuo secantibus in communi sectione* E *ad rectos angulos insistat: illa ducto etiam per ipsas plano* A C B D *ad angulos rectos erit.*

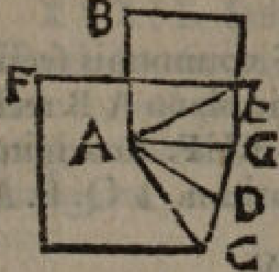
Accipe EA, EC, EB, ED æquales, & junge rectas AC, CB, BD, AD. per E ducatur quævis recta G H; junganturque FA, FC, FD, FB, FG, FH. Quoniam A E *a* = E B; & D E *a* = E C; & ang. A E D *b* = C E B, *c* erit A D. = C B. *c* pariterque AC = DB. *d* ergo A D parall. C B. *d* & A C parall. DB. *e* quare ang. GAE = EBH. *e* & ang. AGE = EHB. sed & AE *f* = EB *g* ergo GE = EH, & *g* AG = BH. quare ob angulos rectos, ex hyp. & proinde pares ad E, *h* bases FA, FC, FB, FD æquantur. Triangula igitur ADF, FBC sibi mutuo æquilatera sunt, *k* quare ang. DAF = CBF. ergo in triangulis AGF, FBH latera FG, FH *l* æquantur; & proinde etiam triangula FEG, FEH sibi mutuo æquilatera sunt. *m* ergo anguli FEG, FEH æquales ac *n* propterea recti sunt. Eodem modo F E cum omni-

a constr. b 15. 1. c 4. 1. d sch 34. 1. e 29. 1. f constr. g 26. 1. h 4. 1. k 8. 1. l 4. 1. m 8. 1. n 10. def. 1.

omnibus in plano A D B C per E ductis rectis lineis rectos angulos constituit, *o* ideoque eidem plano recta est. Q. E. D.

o 3. def. 11.

PROP. V.

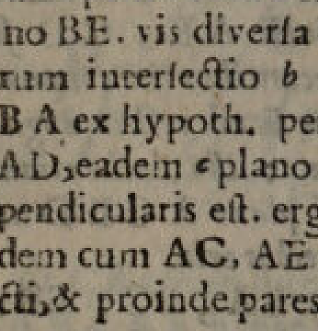


Si recta linea AB rectis tribus lineis AC, AD, AE se mutuo tangentibus in communi sectione ad rectos angulos insistat; illæ tres rectæ in uno sunt plano.

Nam A C, A D *a* sunt in uno plano FC. *a* item AD, AE sunt in uno plano BE. vis diversa esse hæc plana; sit igitur eorum intersectio *b* recta A G. Quoniam igitur B A ex hypoth. perpendicularis est rectis A C, AD, eadem *c* plano FC, *d* ideoque rectæ AG perpendicularis est. ergo (siquidem & *a* AB est in eodem cum AC, AE plano) anguli BAG, BAE recti, & proinde pares sunt, pars & totum. Q. E. A.

a 2. 11.
b 3. 11.
c 4. 11.
d 3. def. 11.

PROP. VI.



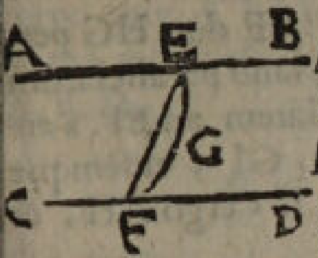
Si duæ rectæ lineæ A B, DC eidem plano EF ad rectos sint angulos; parallelæ erunt illæ rectæ lineæ A B, D C.

Ducatur AD, cui in plano EF perpendicularis sit DG = AB; junganturque BD, BG, AG. Quia in triangulis BAD, ADG anguli DAB, ADG *a* recti sunt; atque AB *b* = DG; & AD communis est; *c* erit BD = A G; quare in triangulis AGB, BGD sibi mutuo æquilateris ang. BAG *d* = BDG; quorum BAG rectus cum sit, erit BDG etiam rectus. atqui ang. GDC rectus ponitur; ergo recta GD tribus DA, DB, CD recta est; *e* quæ ideo in uno sunt plano, *f* in quo A B existit; cum

a hyp.
b constr.
c 4. 1.
d 8. 1.
e 5. 11.
f 2. 11.

um igitur AB, & CD ſint in uno plano, & an-
uli interni BAD, CDA recti ſint, g erunt AB, g 18. 1.
CD parallelæ. Q. E. D.

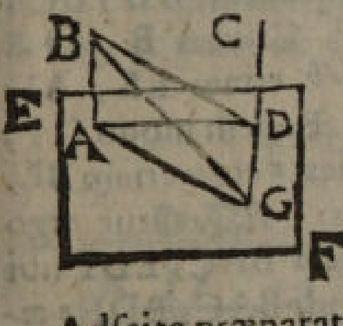
PROP. VII.



Si duæ ſint parallelæ rectæ lineæ AB, CD, *in quarum utraque ſumpta ſint quælibet puncta* E, F; *illa linea* E F, *quæ ad hæc puncta adjungitur, in eodem eſt cum parallelis plano* ABCD.

Planum in quo A B, C D, ſecet aliud planum per puncta E, F. ſi jam E F non eſt in plano ABCD, illa communis ſectio non erit. Sit ergo E G F. a hæc igitur recta eſt linea. duæ ergo rectæ EF, EGF ſpatium claudunt. b Q. E. A. a 3. 11. b 14 ax. 1.

PROP. VIII.

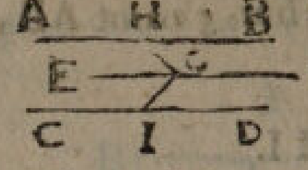


Si duæ ſint parallelæ rectæ lineæ A B, C D, *quarum altera* A B *ad rectos cuidam plano* EF *ſit angulos; & reliqua* C D *eidem plano* E F *ad rectos angulos erit.*

Adſcita præparatione & demonſtratione ſextæ hujus; anguli G D A & G D B recti ſunt. a ergo GD recta eſt plano per AD, DB (b in quo etiam A B, C D exiſtunt.) c ergo G D ipſi C D eſt perpendicularis; atqui ang. CDA etiam d rectus eſt. e ergo CD plano EF recta eſt. Q. E. D. a 4. 11. b 7. 11. c 3 def. 11. d 29. 1. e 4. 11.

PROP.

PROP. IX.

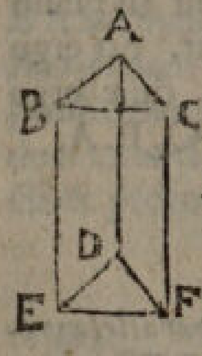


Quæ (AB, CD) *eidem rectæ lineæ* EF *sunt parallelæ, sed non in eodem cum illa plano, hæ quoque sunt inter se parallelæ.*

In plano parallelarum AB, EF duc HG perpendicularem ad EF. item in plano parallelarum EF, CD duc IG perpendicularem ad EF. *a* ergo EG recta est plano per HG, GI, eidemque plano *b* rectæ sunt AH, & CI. *c* ergo AH, & CI parallelæ sunt. Q. E. D.

a 4. 11.
b 8. 11.
c 6. 11

PROP. X.

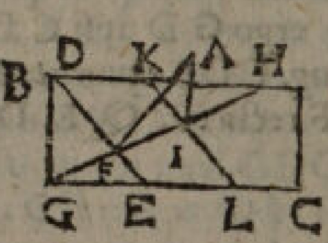


Si duæ rectæ lineæ AB, AC *se mutuo tangentes ad duas rectas* ED, DF *se mutuo tangentes sint parallelæ, non autem in eodem plano, illæ angulos æquales* (BAC, EDF) *comprehendent.*

Sint AB, AC, DE, DF æquales inter se, & ducantur AD, BC, EF, BE, CF. Cum AB, DE *a* sint parallelæ & æquales, *b* etiam BE, AD parallelæ sunt, & æquales. Eodem modo CF, AD parallelæ sunt, & æquales. *c* ergo etiam BE, FC sunt parallelæ & æquales. *d* Æquantur ergo BC, EF. Cum igitur triangula BAC, EDF sibi mutuo æquilatera sint, anguli BAC, EDF *e* æquales erunt. Q. E. D.

a hyp. & constr.
b 33. 1.
c 2. ax. 1. & 30. 1.
d 33. 1.
e 8. 1.

PROP. XI.



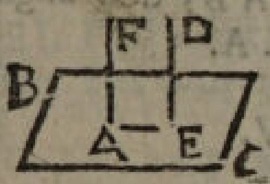
A dato puncto A *in sublimi ad subjectum planum* BC *perpendicularem rectam lineam* AI *ducere.*

In plano BC duc quamvis DE, ad quam ex A *a* duc perpendicularem AF. ad eandem per

in plano BC *b* duc normalem FH. tum ad FH a 12. 1.
demitte perpendicularem AI. erit AI recta pla- b 11. 1.
no BC.

Nam per I *c* duc KIL parall. DE. Quia DE c 31. 1.
recta est ad AF, & FH, *e* erit DE recta plano d *constr.*
FA; adeoque & KL eidem plano *f* recta est. e 4. 11.
ergo ang. KIA rectus est. atqui ang. AIF f 8. 11.
etiam *h* rectus est. *l* ergo AI plano BC recta est. g 3. *def.* 11.
Q. E. D. h *constr.* l 4. 11.

PROP. XII.

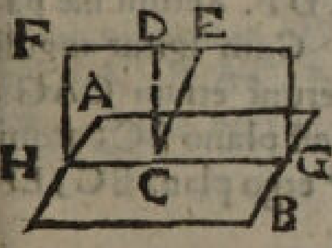


Dato plano BC, à puncto A, quod in illo datum est, ad rectos angulos rectam lineam AF excitare.

A quovis extra planum puncto D *a* duc DE rectam plano BC; & juncta EA *b* duc AF parall. DE. *c* perspicuum est AF plano BC rectam esse. Q. E. F. a 11. 11. b 31. 1. c 8. 11.

Practice perficiuntur hoc, & præcedens problema, si duæ normæ ad datum punctum applicentur, ut patet ex 4. 11.

PROP. XIII.

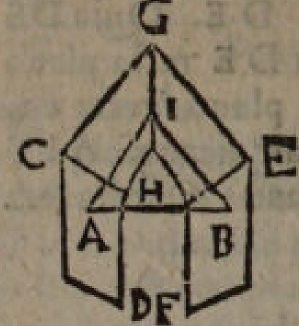


Dato plano AB, à puncto D, quod in illo datum est, duæ rectæ lineæ CD, CE ad rectos angulos non excitabuntur ab eadem parte.

Nam utraque CD, CE plano AB *a* recta esset, eædemque adeo parallelæ forent, quod parallelarum definitioni repugnat. a 6. 11.

PROP. XIV.

valet hæc conversa.

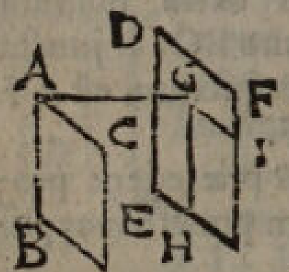


Ad quæ plana C D, F E, *eadem recta linea* A B *recta est; illa sunt parallela.*

Si negas, plana C D, F E concurrant, ita ut communis sectio sit recta G H; sume in hac quodvis punctum I, ad quod in propositis planis ducantur rectæ I A, I B. unde in triangulo I A B, duo anguli IAB, IBA *a* recti sunt. *b* Q. E. A.

a hyp. & 3. def. 11.
b 17. 1.

PROP. XV.



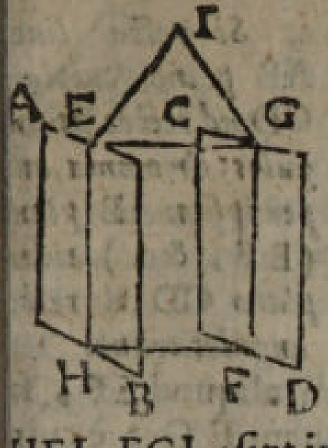
Si duæ rectæ lineæ A B, A C *se mutuo tangentes, ad duas rectas* D E, D F *se mutuo tangentes sint parallelæ, non in eodem consistentes plano; parallela sunt, quæ per illa dicuntur, plana* BAC, EDF.

Ex A *a* duc A G rectam plano E F. *b* Sintque G H, G I parallelæ ad D E, D F. *c* erunt hæ parallelæ etiam ad A B, A C. Cum igitur anguli IGA, HGA *d* sint recti, *e* erunt etiam CAG, BAG recti. *f* ergo GA recta est plano BC; atqui eadem recta est plano E F. *h* ergo plana BC, EF sunt parallela. Q. E. D.

a 11. 11.
b 31. 1.
c 30. 1.
d 3. def. 11.
e 29. 1.
f 4. 11.
g constr.
h 14. 11.

PROP.

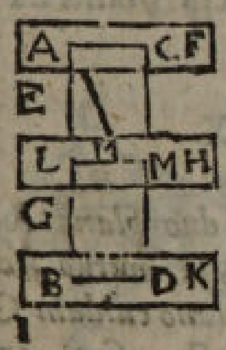
PROP. XVI.



Si duo plana parallela AB, CD, *plano quopiam* HEIGF *secentur, communes illorum sectiones* EH, GF *sunt parallelæ.*

Nam si dicantur non esse parallelæ, cum sint in eodem plano secanti, convenient alicubi, puta in *I.* quare cum totæ HEI, FGI *a* sint in planis AB, CD productis, a 1. 11.
etiam hæc convenient, contra hypoth.

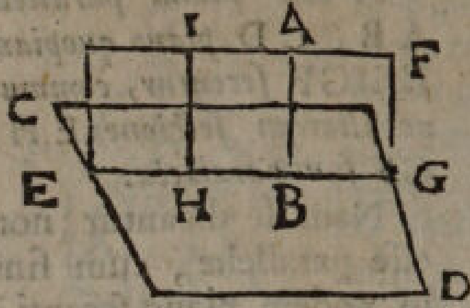
PROP. XVII.



Si duæ rectæ lineæ ALB, CMD *parallelis planis* EF, GH, IK *secentur, in easdem rationes secabuntur* (AL. LB :: CM. MD.)

Ducantur in planis EF, IK rectæ AC, BD. item AD occurrens plano GH in N; junganturque NL, NM. Plana triangulorum ADC, ADB faciunt sectiones BD, LN; & AC, NM *a* parallelas. ergo AL. a 16. 11.
LB *b* :: AN. ND *b* :: CM. MD. Q. E. D. b 2. 6.

PROP. XVIII.

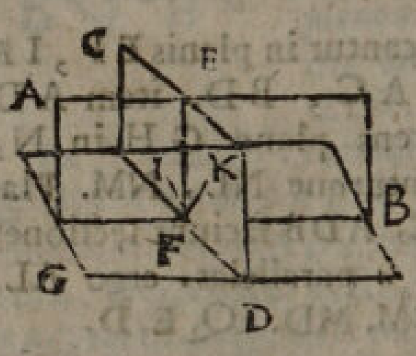


Si recta linea AB plano cuipiam CD ad rectos sit angulos; & omnia, quæ per ipsam AB plana (EF, &c.) eidem plano CD ad recto angulos erunt.

Ductum sit per AB planum aliquod EF, faciens cum plano C D sectionem E G; è cuju aliquo puncto H, in plano EF a ducatur HI parall. AB. b erit HI recta plano CD; paritérque aliæ quævis ad EG perpendiculares. c ergo planum EF plano CD rectum est; eademque ratione quævis alia plana per AB ducta plano EF recta erunt. Q. E. D.

a 31. 1.
b 8. 11.
c 4. def. 11.

PROP. XIX.



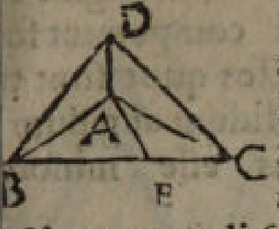
Si duo plana AB, CD, se mutuo secantia, plano cuidam GH ad rectos sint angulos, communis etiam illorum sectio EF ad rectos eidem plano (GH) angulos erit.

Quoniam plana AB, CD ponuntur recta plano GH, patet ex 4. def. 11. quod ex puncto F in utroque plano AB, CD duci possit perpendicularis plano GH; quæ a unica erit, & propterea eorundem planorum communis sectio Q. E. D.

a 13. 11.

PROP

PROP. XX.



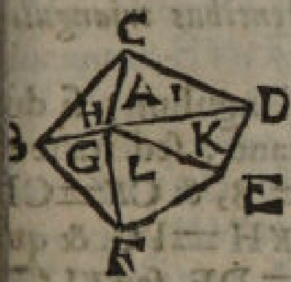
Si solidus angulus ABCD tribus angulis planis BAD, DAC, BAC contineatur; ex his duo quilibet, utut assumpti, tertio sunt majores.

Si tres anguli sunt æquales, patet assertio; si æquales, maximus esto BAC. ex quo *a* aufer AE = BAD; & fac AD = AE; ducanturque EC, BD, DC. a 23. 1.

Quoniam latus BA commune est, & AD *b* = E; & ang. BAE *b* = BAD; *c* erit BE = BD. d BD + DC *d* $\sqsubset$ BC. *e* ergo DC $\sqsubset$ EC. cum itur AD *b* = AE, & latus AC commune est, DC $\sqsubset$ EC *f*, erit ang. CAD $\sqsubset$ EAC. *g* ergo ng. BAD + CAD $\sqsubset$ BAC. Q. E. D.

b constr. c 4. 1. d 20. 1. e 5. ax. 1. f 25. 1. g 4. ax. 1.

PROP. XXI.



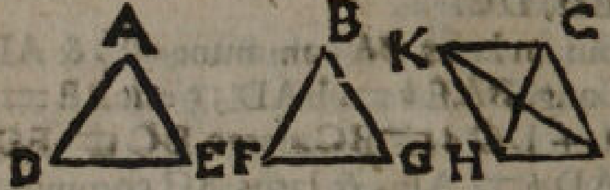
Omnis solidus angulus sub minoribus, quam quatuor rectis angulis planis, continetur.

Esto solidus angulus A; planis angulis illum componentibus subtendantur rectæ BC, CD, DE, EF, FB in uno plano existentes. Quo facto constituitur yramis, cujus basis est polygonum BCDEF, ertex A, totque cincta triangulis quot plani nguli componunt solidum A. Jam vero quia luo anguli ABF, ABC *a* majores sunt uno FBC, & duo ACB, ACD majores uno BCD, & ic deinceps, erunt triangulorum G, H, I, K, L irca basim anguli simul sumpti omnibus simul ngulis basis B, C, D, E, F majores. *b* sed anguli baseos una cum quatuor rectis faciunt bis tot ectos, quot sunt latera, sive quot triangula. *c* Ergo omnes triangulorum circa basim anguli una

a 20. 11. b sch. 32. 1. c 4. ax. 1.

cum 4 rectis conficiunt amplius quam bis tot rectos quot sunt triangula. sed iidem anguli circa basim una cum angulis qui componunt solidum,
d 32. 1. componunt *d* bis tot rectos quot sunt triangula. liquet ergo angulos solidum angulum A componentes quatuor rectis esse minores. Q. E. D.

PROP. XXII.

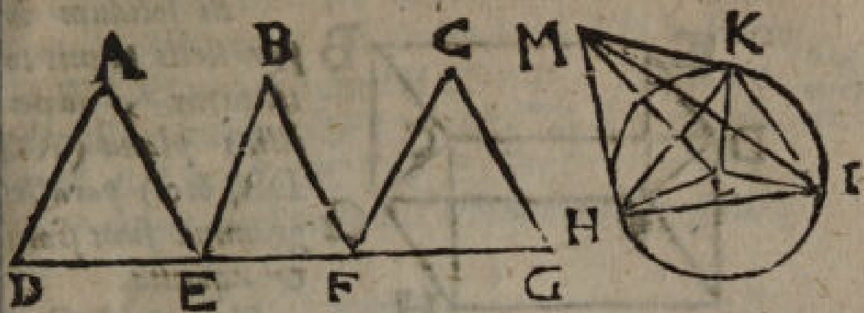


Si fuerint tres anguli plani A, B, HCI; *quorum duo utlibet assumpti reliquo sint majores; comprehendant autem ipsos rectæ lineæ æquales* AD, AE, FB, &c. *fieri potest, ut ex rectis lineis* DE, FG, HI, *æquales illas rectas connectentibus triangulum constituatur.*

a 22. 1. Ex iis *a* constitui potest triangulum, si duæ quælibet reliqua majores existant; sed ita se res habet. Nam *b* fac ang. HCK = B, & CK = CI,
b 23. 1. ducanturque HK, IK. *c* ergo KH = FG. & quia
c 4. 1. ang. KCI *d* ⊏ A; erit KI ⊏ DE. sed KI *f* ⊐
d hyp. HI + KH (FG;) ergo DE ⊐ HI + FG. Simili
e 24. 1. argumento quævis duæ reliqua majores
f 20. 1. ostendentur; & proinde ex iis triangulum *a* constitui potest. Q. E. D.

PROP.

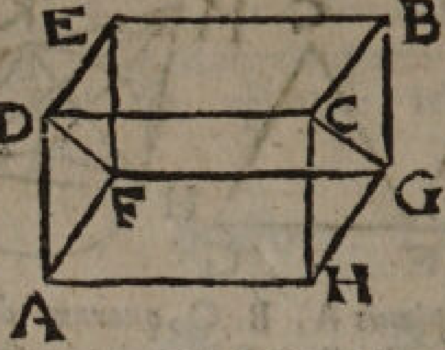
PROP. XXIII.



Ex tribus angulis planis A, B, C, *quorum duo* *uomodocunque assumpti reliquo sunt majores, soli-* *um angulum* MHIK *constituere.* * *Oportet autem* * 21.11. *llos tres angulos quatuor rectis minores esse.*

Fac AD, AE, BE, BF, CF, CG æquales nter se. Ex subtensis DE, EF, FG (hoc est, x æqualibus HI, IK, KH) *a* fac triang. HKI. a 22. 11. & 22. 1. *Quo-irca quod *b* describatur circulus LHKI. * Quo- b 5. 4. iiam vero AD ⊏ HL; *c* sit ADq = HLq + vid. Clavium. LMq. *d* sitque LM recta plano circuli HKI; & c seb 47. 1. lucantur HM, KM, IM. Quoniam igitur ang. d 12. 11. HLM *e* rectus est, *f* erit MHq = HLq + LMq e 3. def. 11. f 47. 1. = ADq. ergo MH = AD. simili argumento g constr. MK, MI, AD (id est, AE, EB, &c.) æquantur; ergo cum HM = AD, & MI = AE; & DE *h* = h constr. HI, *k* erit ang. A = HMI; *k* similiter ang. IMK k 8. 1. = B. *k* & ang. HMK = C. Factus est igitur ngulus solidus ad M ex tribus planis datis. Q. E. F. Brevitatis causa assumptum est, esse AD ⊏ HL, id quod in variis casibus demon-tratum vide apud Clavium.

PROP. XXIV.

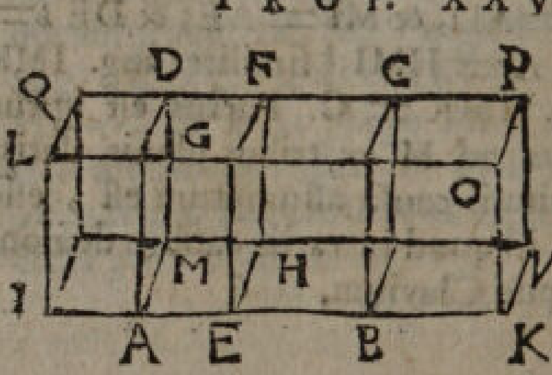


Si solidum A *parallelis planis contineatur, adversa illius plana* (AG, DB, &c.) *parallelogramma sunt similia & æqualia.*

Planum AC secans plana parallela AG, DB, *a* facit sectiones AH, DC parallelas. Eadem ratione AD, HC parallelæ sunt. Ergo ADCH est parallelogrammum. Simili argumento reliqua parallelepipedi plana sunt *b* parallelogramma. Quum igitur AF ad HG, & AD ad HC parallelæ sint, *c* erit ang. FAD = CHG ergo ob AF *d* = HG, & AD *d* = HC, ac propterea AF. AD :: HG. HC, triangula FAD, GAH *g* similia sunt & *h* æqualia; proinde & parallelogramma AE, HB similia sunt & *k* æqualia. idemque de reliquis oppositis planis ostendetur. ergo, &c.

a 16. 11.
b 35. def. 1.
c 10. 11.
d 34. 1.
e 7. 5.
g 6. 6.
h 4. 1.
k 6. ax. 1.

PROP. XXV.



Si solidum parallelepipedum ABCD *plano* EF *secetur adversis planis* AD, BC *parallelo, erit quemadmodum basis* AH *ad basim* BH, *ita solidum* AHD *ad solidum* BHC.

Concipe Ppp. ABCD produci utrinque. accipe AI=AE, & BK = EB; & pone plana IQ, KP planis AD, BC parallela. parallelogramma

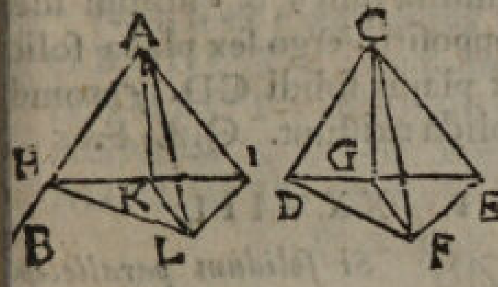
amma IM, AH, a & DL, DG, b & IQ, AD, a 36. 1. & 1. def. 6.
F, &c. a similia ac æqualia sunt; c quare Ppp. b 24. 11.
Q = AF; atque eadem ratione Ppp. BP = c 10. def. 11.
F. ergo solida IF, EP solidorum AF, EC æ-
uemultiplicia sunt, ac bases IH, KH basium
H, BH. Quod si basis IH ⊏, =, ⊐ KH, d e- d 14. 11. & 9. def. 11.
t similiter solidum IF ⊏, =, ⊐ EP. e proin- e 6. def. 5.
e AH. BH :: AF. EC. Q. E. D.

Hæc eadem omni prismati accommodari possunt; nde

Coroll.

Si prisma quodcunque secetur plano oppositis lanis parallelo, sectio erit figura æqualis, & similis planis oppositis.

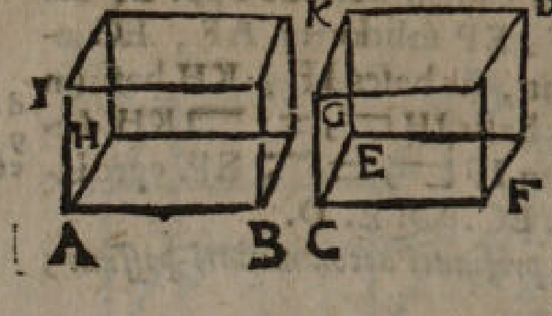
PROP. XXVI.



Ad datam rectam lineam AB, *ejusque punctum* A, *constituere angulum solidum* AHIL, *æqualem solido angulo dato* CDEF.

A puncto quovis F a demitte FG plano DCE a 11. 11.
rectam; ducanturque rectæ DF, FE, EG, GD, CG. Fac AH = CD, & ang. HAI = DCE. & AI = CE; atque in plano HAI, fac ang. HAK = DCG, & AK = CG. Tum erige KL rectam plano HAI, & sit KL = GF. ducaturque AL. erit angulus solidus AHIL par dato CDEF. Nam hujus constructio illius constitutionem penitus æmulatur, ut facile patebit examinanti. ergo factum.

PROP. XXVII.



A data recta linea AB, *dato solido parallelepipedo* CD *simile & similiter positum parallelepipedum* AK *describere.*

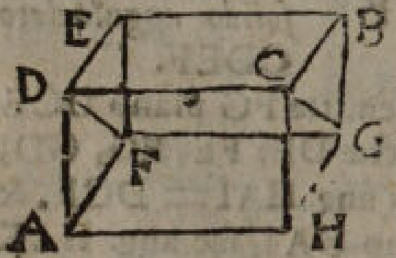
Ex angulis planis BAH, HAI, BAI, qui æquales sint ipsis FCE, ECG, FCG, *a* fac angulum solidum A solido C parem. item *b* fac FC. CE :: BA. AH. *b* ac CE. CG :: AH. AI (*c* unde erit ex æquali FC. CG :: BA. AI;) & perficiatur Ppp. AK. erit hoc simile dato.

a 26. 11.
b 12. 6.
c 22. 5.

Nam per constr. Pgra *d* BH, FE; *d* & HI, EG; & *d* BI, FG similia sunt, & *e* horum ideo opposita illorum oppositis. ergo sex plana solidi AK similia sunt sex planis solidi CD. *f* proinde AK, CD similia solida existunt. Q. E. F.

d 1. def. 6.
e 24. 11.
f 9. def. 11.

PROP. XXVIII.



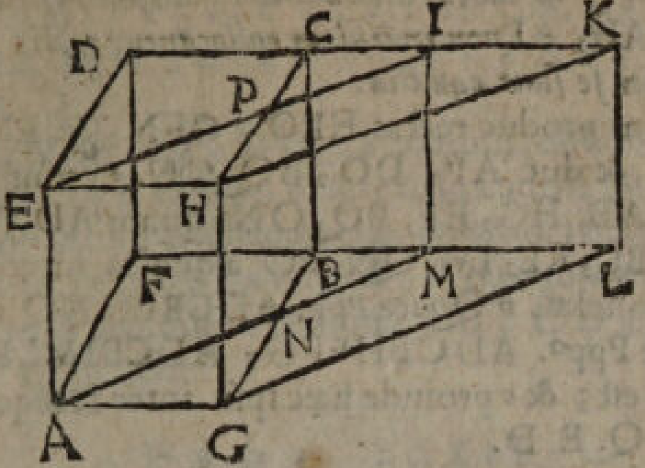
Si solidum parallelepipedum AB *plano* FGCD *secetur per diagonios* DF, CG *adversorum planorum* AE, HB, *bifariam secabitur solidum* AB *ab ipso plano* FGCD.

Nam quia DC, FG *a* æquales & parallelæ sunt, *b* planum FGCD est Pgr. & propter *a* Pgra AE, HB æqualia, & similia, *b* etiam triangula AFD, HGC, CGB, DFE æqualia & similia sunt. Atqui Pgra AC, AG ipsis FB, FD *a* etiam æqualia & similia sunt. ergo prismatis FGCDAH omnia plana æqualia sunt, & similia planis omnibus prismatis FGCDEB; & *c* proinde hoc prisma illi æquatur. Q. E. D.

a 24. 11.
b 34. 1.
c 9. def. 11.

PROP.

PROP. XXIX.



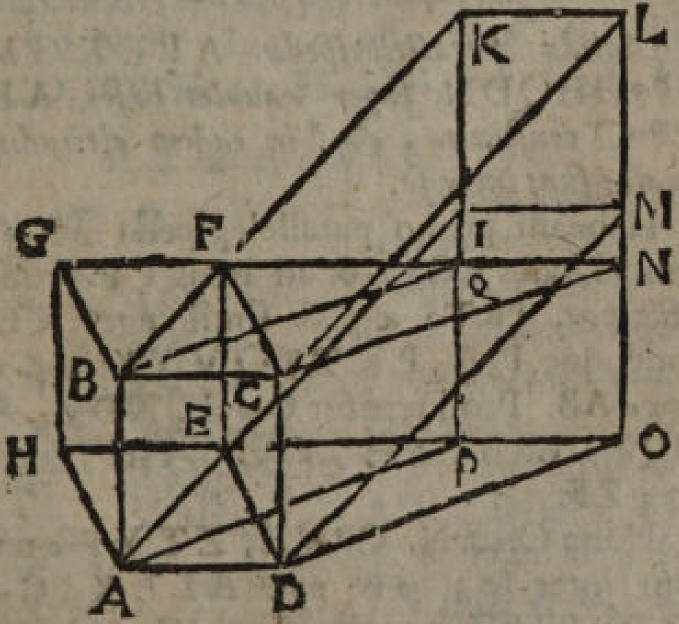
Solida parallelepipeda AGHEFBCD, AGHEMLKI *super eandem basim* AGHE *constituta, & * in eadem altitudine; quorum insistentes lineæ* AF, AM *in iisdem collocantur rectis lineis* AG, FL, *sunt inter se æqualia.*

* *Id est, inter parallela plana AGHF, FLKD. & sic intellige in sequent.*

Nam si ex a æqualibus prismatis AFMEDI, GBLHCK commune auferatur prisma NBMPCI, addaturque utrinque solidum AGNEHP, b erit Ppp. AGHEFBCD = AGHEMLKI. Q. E. D.

a 10. *def.* 11. & 35. 1.
b 3. & 2. *ax.* 1.

PROP. XXX.



Solida parallelepipeda ADBCHEFG, AD-

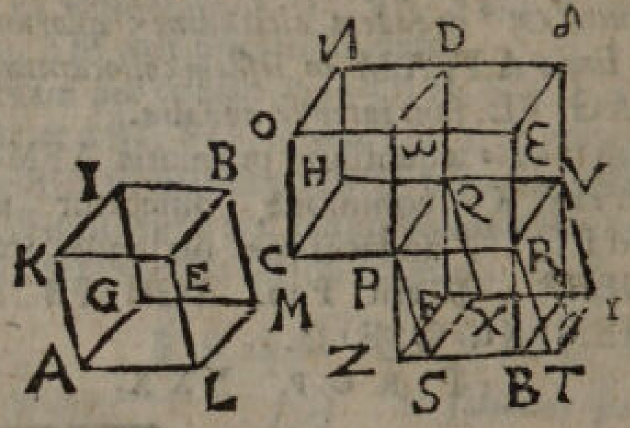
ADCBIMLK super eandem basim ADCB constituta, & in eadem altitudine, quorum insistentes lineæ AH, AI non in iisdem collocantur rectis lineis, inter se sunt æqualia.

a 34. 1. Nam produc rectas HEO, GFN, & LMO, KIP; & duc AP, DO, BQ, CN. *a* erunt tam DC, AB, HG, EF, PQ, ON; quam AD, HE, GF, BC, KL, IM, QN, PO æquales inter sese

b 29. 11. & parallelæ. *b* Quare Ppp. ADCBPONQ utrique Ppp°. ADCBHEFG, ADCBIMLK æ-

c 1. ax. 1. quale est; & *c* proinde hæc ipsa inter se æqualia sunt. Q. E. D.

PROP. XXXI.



*Solida parallelepipeda ALEKGMBI, CPωOHQDN super æquales bases ALEK, CPωO constituta, & * in eadem altitudine, æqualia sunt inter se.*

* *A titulo, est perpendicularis à plano basis ad planum oppositum.*

Habeant primo parallelepipeda AB, CD latera ad bases recta; & ad latus CP productum

a 18 6. *a* fiat pgr. PRTS æq. & simile pgr° KELA;

b 27 11. & 10. def. 11. *b* adeoque Ppp. PRTSQVYX æq. & sim. Pppo AB. Producantur OωE, NDδ, ωPZ, DQF, ERB, δVγ, TSZ, YXF; & duc Eδ, Bγ, ZF.

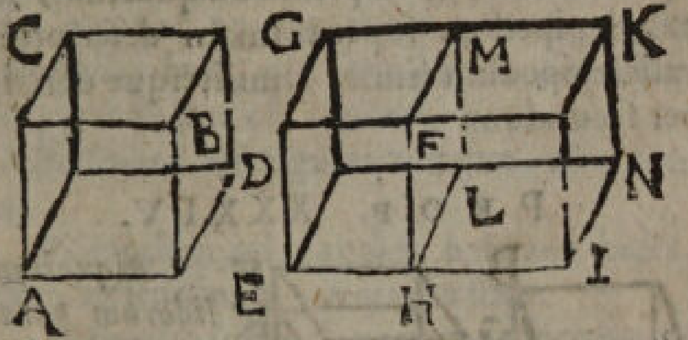
c 30. def. 11. Plana OεδN, CRVH, ZTYF *c* parallela

d hyp. & 35. 1. sunt inter se; *d* & pg^a ALEK, CPωO, PRTS, PRBZ æqualia sunt. Cum igitur Ppp. CD.

CD. PVδω e :: pgr. Cω (PRBZ.) Pte :: Ppp. e 25. 11.
PRBZQVγF. PVδω, f erit Ppp. CD f = f 9. 5.
PRBZQVγF g = PRVQSTYX h = AB. g 19. 11.
Q. E. D. h constr.

Sin Ppp^a AB, CD latera basibus obliqua habeant; super easdem bases, & in eadem altitudine, ponantur parallelepipeda, quorum latera basibus sint recta. k Ea inter se, & obliquis æqualia erunt; m proinde & obliqua AB, CD æquantur. Q. E. D. k 19. 11. m 1. ax. 1.

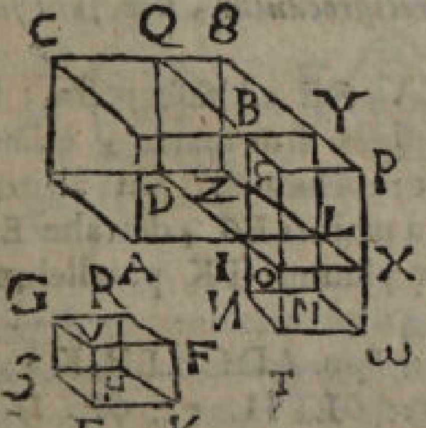
PROP. XXXII.



Solida parallelepipeda ABCD, EFGL *sub eadem altitudine, inter se sunt ut bases* AB, EF.

Producta EHI, a fac pgr. FI = AB, & b comple Ppp. FINM. Liquet esse Ppp. FINM. (c ABCD.) EFGL d :: FI. (AB) EF. Q. E. D. a 45. 1. b 31. 1. c 31. 11. d 25. 11.

PROP. XXXIII.



Similia solida parallelepipeda, ABCD, EFGH, *inter se sunt in triplicata ratione homologorum laterum* AI, EK.

Producantur rectæ AIL, DIO, BIN. & a fiant IL, IO, IN ipsis EK, KH, KF æquales, b adeoque a 3. 1. b 17. 11.

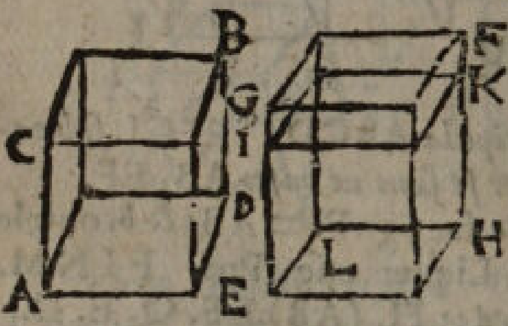
&

& Ppp. IXMT æq. & sim. Ppp° EFGH.
c31.1. c Perficiantur Ppp a IXPB, DLYQ. Itaque d e-
d hyp. rit AI. IL. (EK) :: DI. IO (HK) :: BI. IN.
e 1.6 (KF;) hoc est Pgr. AD. DL :: DL. IX ::
f 32. 11. BO. IT; f id est Ppp. ABCD. DLQY ::
g constr. DLQY. IXBP :: IXBP. IXMT. (g EFGH.)
h 10. def. 5. h ergo ratio ABCD ad EFGH triplicata est ra-
k 1.6. tionis ABCD ad DLQY, k vel AI ad EK. Q. E. D.

Coroll.

Hinc, si fuerint quatuor lineæ rectæ continue proportionales, ut est prima ad quartam, ita est parallelepipedum super primam descriptum ad parallelepipedum simile similiterque descriptum super secundam.

PROP. XXXIV.



Æqualium solidorum parallelepipedorũ ADCB, EHGF *bases & altitudines reciprocantur* (AD. EH :: EG. AC.) *Et quorum solidorum parallelepipedorum* ADCB, EHGF *bases & altitudines reciprocantur, illa sunt æqualia.*

Sint primo latera CA, GE ad bases recta; si jam solidorum altitudines sint pares, etiam
a 3. 1. bases æquales erunt. & res clara est. Sin altitu-
b 31. 1. dines inæquales sint, à majori EG a detrahe EI
c 32. 11. = AC. & per I b duc planum IK parallelum
d 17. 5. basi EH. itaque
e 1. 6. 1. *Hyp.* AD. EH c :: Ppp. ADCB. EHIK d ::
f constr. Ppp. EHGF. EHIK e :: GL. IL e :: GE. IE.
g 11. 5. (f AC;) g liquet igitur esse AD. EH :: GE. AC. Q. E. D.

2. *Hyp.*

2. *Hyp.* ADCB. EHIK *h* :: AD. EH *k* :: EG. EI *l* :: *GL. IL* *m* :: Ppp. EHGF. EHIK, *n* quare Ppp. ADCB = EHGF. Q. E. D. h 32. 11. k hyp. l 1. 6. m 32. 11. n 9. 5.

Sint deinde latera ad bases obliqua. Erigantur super iisdem basibus, in altitudine eadem, parallelepipeda recta. Erunt obliqua parallelepipeda his æqualia. Quare cum hæc per 1. partem reciprocent bases & altitudines, etiam illa reciprocabunt. Q. E. D.

Coroll.

Quæ de parallelepipedis demonstrata sunt Prop. 29, 30, 31, 32, 33, 34. *etiam conveniunt prismatis triangularibus, quæ sunt dimidia parallelepipeda, ut patet ex* Pr. 28. *Igitur*,

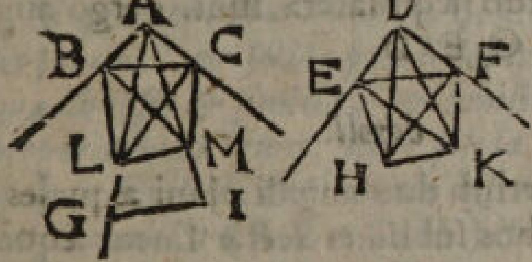
1. Prismata triangularia æque alta sunt ut bases.

2. Si eandem vel æquales habeant bases, & eandem altitudinem, æqualia sunt.

3. Si similia fuerint, eorum proportio triplicata est proportionis homologorum laterum.

4. Si æqualia sunt, reciprocant bases & altitudines. & si reciprocant bases & altitudines, æqualia erunt.

PROP. XXXV.



Si fuerint duo plani anguli BAC, EDF *æquales, quorum verticibus* A, D, *sublimes rectæ lineæ* AG, DH *insistant, quæ cum lineis primo positis angulos contineant æquales, utrumq; utriq;* (ang. GAB = HDE; & GAC = HDF.) *in sublimibus autem lineis* AG, DH *quælibet sumpta fuerint puncta* G, H; *&*

& ab his ad plana BAC, EDF, *in quibus consistunt anguli primum positi* BAC, EDF, *ductæ fuerint perpendiculares* GI, HK; *à punctis vero* I, K *quæ in planis à perpendicularibus fiunt, ad angulos primum positos adjunctæ fuerint rectæ lineæ* AI, DK; *hæ cum sublimibus* AG, DH *æquales angulos* GAM, HDK *comprehendent,*

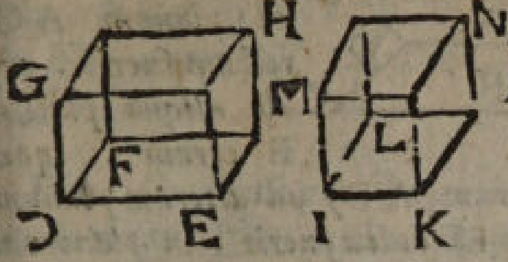
Fiant DH, AL æquales, & GI, LM parallelæ; & MC ad AC, MB ad AB, KF ad DF, KE ad DE perpendiculares, ducanturque rectæ
a 8. 11. BC, LB, LC, atque EF, HF, HE; *a* estque LM
b 3. def. 11. recta plano BAC; *b* quare anguli LMC, LMA, LMB; eademque ratione anguli HKF, HKD,
c 47. 1. *H*KE recti sunt. Ergo $ALq \overset{c}{=} LMq + AMq \overset{c}{=} LMq + CMq + ACq \overset{c}{=} LCq + ACq$;
d 48. 1. *d* ergo ang. ACL rectus est. Rursus $ALq \overset{e}{=}$
e 47. 1. $LMq + MAq \overset{e}{=} LMq + BMq + BAq \overset{e}{=} BLq + BAq$. *d* ergo ang. ABL etiam rectus est. Simili discursu anguli DF*H*, DE*H* recti
f 26. 1. sunt; *f* ergo $AB = DE$; *f* & $BL = EH$; *f* & $AC = DF$; & $CL = FH$. *g* quare etiam BC
g 4. 1. $= EF$, *g* & ang. $ABC = DEF$ *g* & ang. ACB
h 3. ax. 1. $= DFE$. unde reliqui è rectis anguli CBM, BCM reliquis FEK, EFK æquantur. *k* ergo
k 26. 1.
l 47. 1. $CM = FK$, *l* ideoque & $AM = DK$. ergo si
m constr. ex $LAq \overset{m}{=} HDq$. auferatur $AMq = DKq$,
n 47. 1. & 3. ax. *n* remanet $LMq = HKq$ quare trigona LAM,
o 8. 1. HDK sibi mutuo æquilatera sunt. *o* ergo ang. $LAM = HDK$. Q. E. D.

Coroll.

Itaque si fuerint duo anguli plani æquales, quorum verticibus sublimes rectæ lineæ æquales insistant, quæ cum lineis primo positis angulos contineant æquales, utrumque utrique; erunt à punctis extremis linearum sublimium ad plana angulorum primo positorum demissæ perpendiculares inter se æquales; nempe $LM = HK$

PROP.

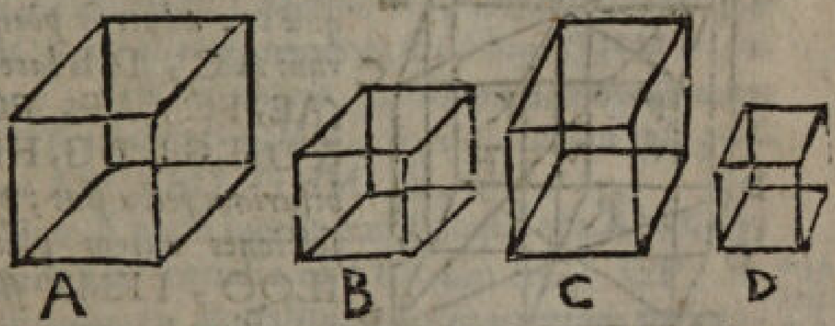
Prop. XXXVI.



Si tres rectæ lineæ DE, DG, DF *proportionales fuerint; quod ex his tribus fit solidum parallelepipedum* DH, *æquale est descripto à media linea* DG (IL) *solido parallelepipedo* IN, *quod æquilaterum quidem sit, æquiangulum vero prædicto* DH.

Quoniam DE. IK *a* :: IL. DF, *b* erit pgr. LK = FE. & propter angulorum planorum ad E & I, ac linearum GD, IM æqualitatem, etiam altitudines parallelepipedorum æquales sunt, ex coroll. præced. *c* ergo ipsa inter se æqualia sunt. Q. E. D. a hyp. b 14. 6. c 31. 11.

Prop. XXXVII.

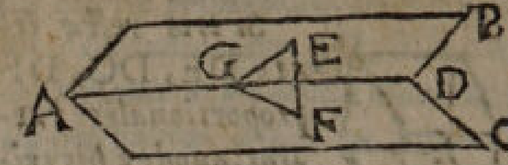


Si quatuor rectæ lineæ A, B, C, D *proportionales fuerint, & solida parallelepipeda* A, B, C, D *quæ ab ipsis & similia, & similiter describuntur, proportionalia erunt. Et si solida parallelepipeda, quæ & similia, & similiter describuntur, fuerint proportionalia* (A.B :: C. D.) *& ipsæ rectæ lineæ* A,B,C,D *proportionales erunt.*

Nam rationes parallelepipedorum *a* triplicatæ sunt rationum, quas habent lineæ. ergo si A.B :: C. D. *b* erit Ppp. A. Ppp. B :: Ppp. C. Ppp. D. & vice versa. a 33. 11. b sch 23. 5.

Prop.

PROP. XXXVIII.

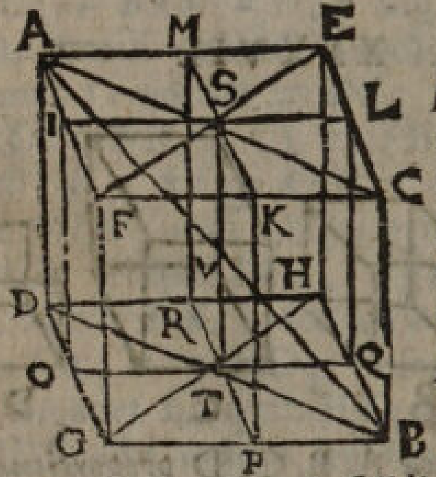


Si planum AB *ad planum* AC *rectum fuerit, & ab aliquo puncto* E *eorum, quæ sunt in uno planorum* (AB) *ad alterum planum* AC *perpendicularis* EF *ducta fuerit, in planorum communem sectionem* AD *cadet ducta perpendicularis* EF.

Si fieri potest, cadat F extra intersectionem AD. In plano AC *a* ducatur FG perpendicularis ad AD, jungaturque EG. Angulus FGE *b* rectus est; & EFG rectus ponitur. ergo in triangulo EFG *c* sunt duo anguli recti. Q. E. A.

a 11.1.
b 4,& 3. def.11.
c 17.1.

PROP. XXXIX.



Si solidi parallelepipedi AB, *eorum quæ ex adverso planorum* AC, DB *latera* (AE, FC, AF, EC, & DH, GB, DG, HB) *bifariam secta sint; per sectiones autem plana* ILQO, PKMR *sint extensa; planorum communis sectio* ST, *& solidi parallelepipedi diameter* AB, *bifariam se mutuo secabunt.*

Ducantur rectæ SA, SC, TD, TB. Propter *a* latera DO, OT lateribus BQ, QT, *b* angulosque alternos TOD, TQB æquales, *c* etiam bases DT, TB, & anguli DTO, BTQ æquantur. *d* ergo DTB est recta linea. eodem modo ASC recta est linea. Porro *e* tam AD ad FG, *e* quam FG ad CB; *f* ideoque AD ad CB, *g* ac proinde AC ad DB parallelæ & æquales sunt.

a 34.1.
b 29.1.
c 4.1.
d sch 15.1.
e 34.1.
f 9.11.& 1 ax.
g 33.1.

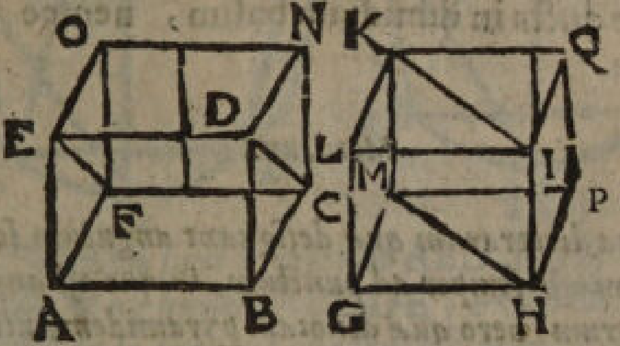
h quare

quare A B, & S T in eodem plano ABCD exsistunt. Itaque cum anguli AVS, BVT ad verticem, & alterni ASV, BTV æquentur; k & AS = BT; erit AV = BV, l & SV = VT. Q. E. D. b 7. 11. k 7. ax. 1. l 26. 1.

Coroll.

Hinc, in omni parallelepipedo diametri omnes se mutuo bisecant in uno puncto, V.

PROP. XL.



Si fuerint duo prismata ABCFED, GHMLIK *æqualis altitudinis, quorum hoc quidem habeat basim* ABCF *parallelogrammum, illud vero* GHM *triangulum; duplum autem fuerit parallelogrammum* ABCF *trianguli* GHM; *æqualia erunt ipsa prismata* ABCFED, GHMLIK.

Nam si perficiantur parallelepipeda AN, GQ, erunt hæc æqualia ob b basium AC, GP, & altitudinum æqualitatem. d ergo etiam prismata, e horum dimidia, æqualia erunt. Q. E. D. a 31. 11. b 34. 1. & 7. ax. c hyp. d 28. 11. e 7. ax. 1.

Schol.

Ex hactenus demonstratis habetur dimensio prismatum triangularium, & quadrangularium, seu parallelepipedorum, si nimirum altitudo ducatur in basim. Andr. Tacq.

Ut si altitudo sit 10 pedum, basis vero pedum quadratorum 100 (mensurabitur autem basis per ch. 35. 1. vel per 41. 1.) multiplica 100 per 10;

proveniunt 1000 pedes cubici pro ſoliditate priſmatis dati.

Vide ſchol. 35. 1. Nam quemadmodum rectangulum, ita & parallelepipedum rectum producitur ex altitudine ducta in baſim. Ergo quodvis parallelepipedum producitur ex altitudine in baſim ducta, ut patet ex 31. hujus.

Deinde cum totum parallelepipedum producatur ex altitudine in totam baſim, ſemiſſis ejus (hoc eſt priſma triangulare) producetur ex altitudine ducta in dimidiam baſim, nempe triangulum.

Monitum.

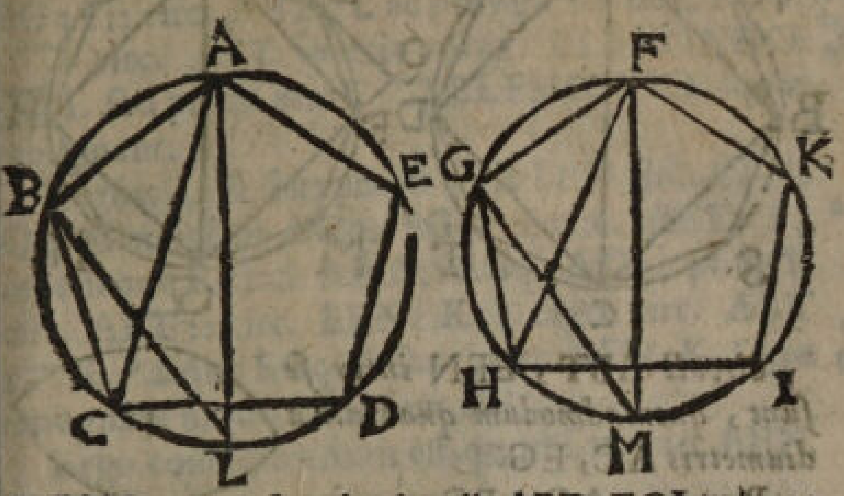
Nota, litterarum quæ deſignant angulum ſolidum primam eſſe ſemper ad punctum, in quo eſt angulus; litterarum vero quæ denotant pyramidem, ultimam eſſe ad verticem pyramidis.

Ex. gr. Angulus ſolidus ABCD eſt ad punctum A; pyramidis quoque BCDA vertex eſt ad punctum A, & baſis triangulum BCD.

LIB.

LIB. XII.

PROP. I.



QVæ sunt in circulis ABD, FGI *polygona similia* ABCDE, FGHIK, *inter se sunt, ut quadrata à diametris* AL, FM.

Ducantur AC, BL, FH, GM. Quoniam *a* ang. ABC = FGH, *a* atque AB. BC :: FG. GH, *b* erit ang. ACB (*c* ALB) = FHG (*c* FMG.) anguli autem ABL, FGM *d* recti, ac proinde æquales sunt. *e* ergo triangula ABL, FGM æquiangula sunt. *f* quare AB. FG :: AL. FM. *g* ergo ABCDE. FGHIK :: ALq. FMq.

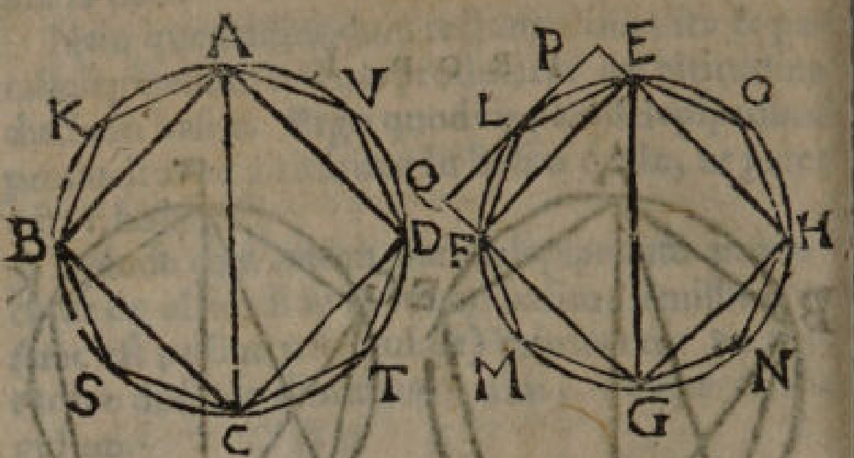
a 1. def. 6.
b 6. 6.
c 21. 3.
d 31. 3.
e 32. 3.
f cor. 4. 6.
g 22. 6.

Coroll.

Hinc (quia AB. FG :: AL. FM :: BC. GH, &c.) polygonorum similium circulo inscriptorum *h* ambitus sunt ut diametri.

h 1. 12. & 12. 5.

PROP. II.



Circuli ABT, EFN *inter se sunt, quemadmodum quadrata à diametris* AC, EG.

Ponatur ACq. EGq :: circ. ABT. I. Dico I = circ. EFN.

Nam primo, si fieri potest, sit I $\sqsupset$ circ. EFN, sitque excessus K. Circulo EFN inscribatur
a sch. 7. 4. quadratum EFGH, *a* quod dimidium est circumscripti quadrati, adeoque semicirculo majus.
b 30. 3. *b* Biseca arcus EF, FG, GH, HE, & ad puncta bisectionum junge rectas EL, LF, &c. per L
c sch. 27. 3. duc tangentem PQ (*c* quæ ad EF parallela est,) & produc HEP, GFQ; estque triangulum
d 41. 1 ELF *d* dimidium parallelogrammi EPQF, adeoque majus dimidio segmenti ELF; pariterque reliqua triangula ejusmodi reliquorum segmentorum dimidia superant. Et si iterum bisecentur arcus EL, LF, FM, &c. rectæque adjungantur, eodem modo triangula segmentorum semisses excedent. Quare si quadratum EFGH è circulo EFN, & è reliquis segmentis triangula
e 1. 10. detrahantur, & hoc fiat continuo, tandem *e* restabit magnitudo aliqua minor quam K. Eousque perventum sit, nempe ad segmenta EL, LF, FM, &c. minora quam K, simul sumpta.

ta. ergo I (f circ. EFN − K) ⊐ polyg. ELFMGNHO (circ. EFN − segm. EL + LF &c.) Circulo ABT inscriptum g puta simile polygonum AKBSCTDV. itaque quum AKBSCTDV. ELFMGNHO h :: ACq. EGq k :: circ. ABT. I. ac polyg. AKBSCTDV ⊐ circ. ABT. m erit polyg. ELFMGNHO ⊐ I. sed prius erat I ⊐ ELFMGNHO. quæ repugnant. f hyp. & 3. ax. g 10. 3. & 1 post. 1. h 1. 12. k hyp. 19 ax. 1. m 14. 5.

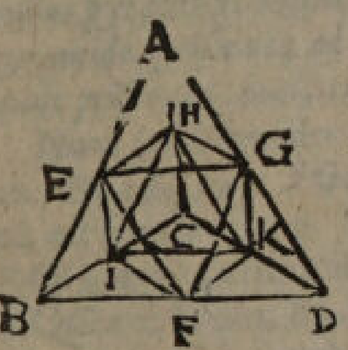
Rursus, si fieri potest, sit I ⊏ circ. EFN. Quoniam igitur ACq. EGq n :: circ. ABT. I; inverseque I. circ. ABT :: EGq. ACq. pone I. circ. ABT :: circ. EFN. K. o ergo circ. ABT ⊏ K. p atque EGq. ACq :: circ. EFN. K. Quæ repugnare modo ostensum est. n hyp. o 14. 5. p 11. 5.

Ergo concludendum est, quod I = circ. EFN. Q. E. D.

Coroll.

Hinc, ut circulus est ad circulum, ita polygonum in illo descriptum ad simile polygonum in hoc descriptum.

Prop. III.



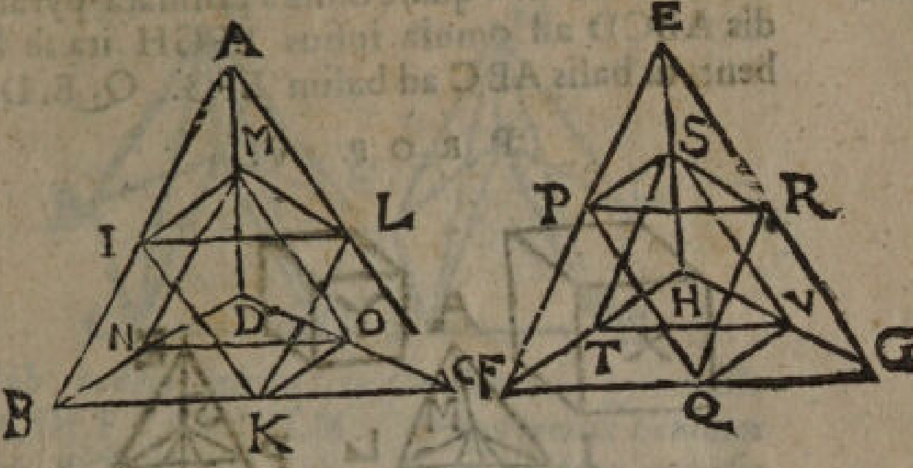
Omnis pyramis ABDC *triangularem habens basim, dividitur in duas pyramides* AEGH, HIKC *æquales & similes inter se, triangulares habentes bases, & similes toti* ABDC; *& in duo prismata æqualia* BFGEIH, EGDIHK; *quæ duo prismata majora sunt dimidio totius pyramidis* ABDC.

Latera pyramidis bisecentur in punctis E, F, G, H, I, K; junganturque rectæ EF, FG, GE, EI, IF, FK, KG, GH, HE. Quoniam latera

pyramidis proportionaliter secta sunt, *a* erunt
a 2. 6. HI, AB; & GF, AB; & IF, DC; atque HG,
DC, &c. parallelæ; proinde & HI, FG; & GH,
FI parallelæ sunt. liquet igitur triangula ABD,
b 29. 1. AEG, EBF, FDG, HIK *b* æquiangula esse; &
c 26. 1. quatuor ultima *c* æquari. eodem modo triangula
ACB, AHE, EIB, HIC, FGK æquiangula sunt;
& quatuor postrema inter se æqualia. similiter
triangula BFI, FDK, IKC, EGH; & denuo
triangula AHG, GDK, HKC, EFI, similia sunt
& æqualia. Quinetiam triang. HIK ad ADB, &
EGH ad BDC, & EFI ad ADC, & FGK ad
ABC *d* parallela sunt. Ex quibus perspicue sequi-
d 15. 11. tur primo, pyramides AEGH, HIKC æquales
e 10. def. 11. esse; totique ABDC, & inter se *e* similes. deinde
solida BFGEIH, FGDIHK prismata esse, &
quidem æque alta, nempe sita inter parallela pla-
na ABD, HIK. verum basis BFGE basis FDG
f 1. ax. 1. *f* duplex est. quare dicta prismata æqualia sunt.
g 40. 11. quorum alterum BFGEIH pyramide BEFI, hoc
est, AEGH majus est, totum sua parte; proinde
duo prismata majora sunt duabus pyramidibus,
totiusque adeo pyramidis ABDC dimidium ex-
cedunt. Q. E. D.

PROP.

Prop. IV.



Si fuerint duæ pyramides ABCD, EFGH *ejusdem altitudinis, triangulares habentes bases* ABC, EFG; *sit autem illarum utraque divisa & in duas pyramides* (AILM, MNOD; & EPRS, STVH) *æquales inter se, & similes toti; & in duo prismata æqualia* (IBKLMN, KLCNMO; & PFQRST, QRGTSV;) *ac eodem modo divisa sit utraque pyramidum, quæ ex superiore divisione natæ sunt, idque semper fiat; erit ut unius pyramidis basis ad alterius pyramidis basim, ita & omnia, quæ in una pyramide, prismata ad omnia, quæ in altera pyramide prismata, multitudine æqualia.*

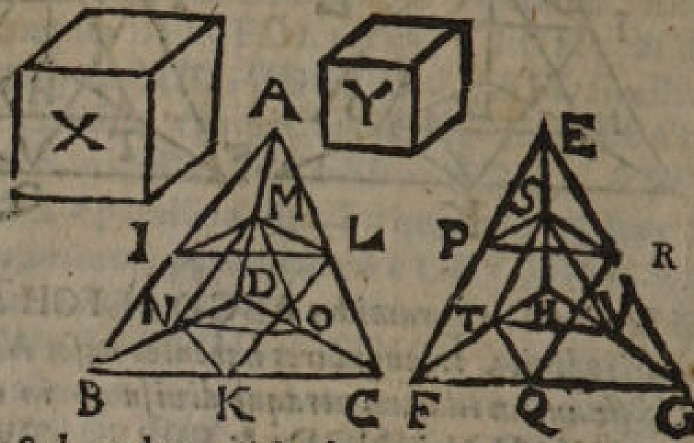
Nam (adhibendo constructionem præcedentis) BC. KC *a* :: FG. QG. *b* ergo triang. ABC est ad simile triang. LKC, ut EFG ad *c* simile RQG. ergo permutando ABC. EFG *d* :: LKC. RQG *e* :: Prism. KLCNMO. QRGTSV (nam hæc æque alta sunt) *f* :: IBKLMN. PFQRST. *g* quare triang. ABC. EFG :: Prism. KLCNMO + IBKLMN. Prism. QRGTSV + PFQRST. Q. E. D.

a 15. 5.
b 22. 6.
c 2. 6. &c.
d 16. 5.
e *sch.* 34. 11.
f 7. 5.
g 12. 5.

Sin ulterius simili pacto dividantur pyramides MNOD, AILM; & EPRS, STVH, erunt quatuor nova prismata hic effecta ad quatuor isthic producta, ut bases MNO & AIL ad base STV & EPR, hoc est ut LKC ad RQG, vel u

b 12. 5. ABC ad EFG. *b* quare omnia prismata pyramidis ABCD ad omnia ipsius EFGH ita se habent, ut basis ABC ad basim EFG. Q. E. D.

PROP. V.



Sub eadem altitudine existentes pyramides ABCD, EFGH, *triangulares habentes bases* ABC, EFG, *inter se sunt ut bases* ABC, EFG.

Sit triang. ABC. EFG :: ABCD. X. Dico X = pyr. EFGH. Nam, si possibile est, sit X ⊐ EFGH; sitque Y excessus. Dividatur pyramis EFGH in prismata & pyramides, & reliquæ pyramides similiter, *a* donec relictæ pyramides EPRS, STVH minores evadant solido Y.

a 1. 10. Quum igitur pyr. EFGH = X + Y; liquet reliqua prismata PFQRST, QRGTSV solido X majora esse. Pyramidem ABCD similiter divisam concipe; *b* eritque prism. IBKLMN + KLCNMO. PFQRST + QRGTSV :: ABC. EFG. *c* :: pyr. ABCD. X. *d* ergo X ⊏ prism. PFQRST + QRGTSV; quod repugnat prius affirmatis.

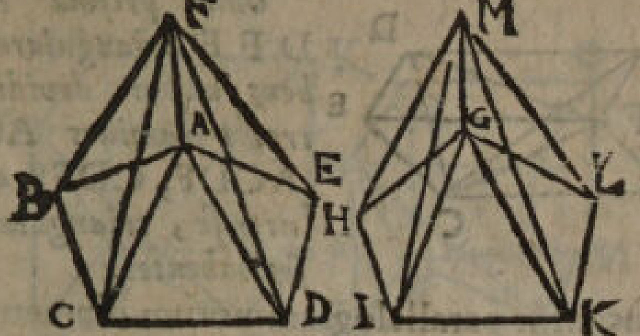
b 4. 12. c hyp. d 14. 5.

Rursus, dic X ⊏ pyr. EFGH. pone pyr. EFGH. Y :: X. pyr. ABCD *e* :: EFG. ABC. quia EFGH *f* ⊐ X, *g* erit Y ⊐ pyr. ABCD, quod fieri nequit, ex jam dictis. Concludo igitur, quod X = pyr. EFGH. Q.E.D.

e hyp. & cor. 4 5. f suppos. g 14. 5.

PROP.

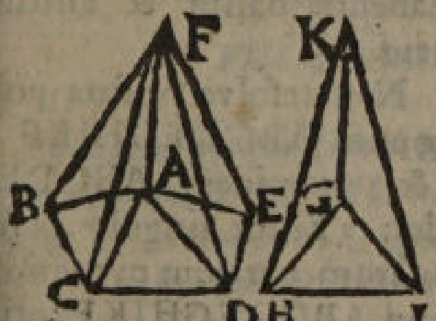
PROP. VI.



Sub eadem altitudine existentes pyramides ABCDEF, GHIKLM, *& polygonas habentes bases* ABCDE, GHIKL, *inter se sunt ut bases* ABCDE, GHIKL.

Duc rectas AC, AD, GI, GK. Est bas. ABC. ACD *a* :: pyr. ABCF. ACDF. *b* ergo composite ABCD. ACD :: pyr. ABCDF. ACDF. *a* atqui etiam ACD. ADE :: pyr. ACDF. ADEF. *c* ergo ex æquali ABCD. ADE :: ABCDF. ADEF. *b* ergo componendo ABCDE. ADE :: pyr. ABCDEF. ADEF. porro ADE. GKL *d* :: pyr. ADEF. GKLM; ac, ut prius, atque inverse GKL. GHIKL :: pyr. GKLM. GHIKLM. *c* ergo iterum ex æqualibus, ABCDE. GHIKL :: Pyr. ABCDEF. GHIKLM. Q. E. D.

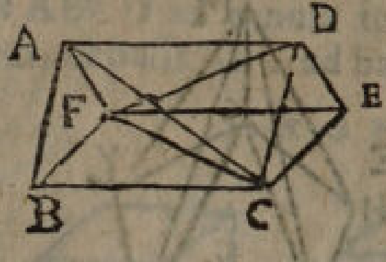
a 5. 12.
b 18 5.
c 22. 5.
d 5. 12.



Si bases non habent latera æque multa, demonstratio sic procedet. Bas. ABC. GHI *e* :: pyr. ABCF. GHIK. *e* atque ACD. GHI :: pyr. ACDF. GHIK. *f* ergo bas. ABCD. GHI :: pyr. ABCDF. GHIK. *f* Quinetiam bas. ADE. GHI :: pyr. ADEF. GHIK. *f* ergo bas. ABCDE. GHI :: pyr. ABCDEF. GHIK.

e 5. 12.
f 24. 5.

PROP. VII.

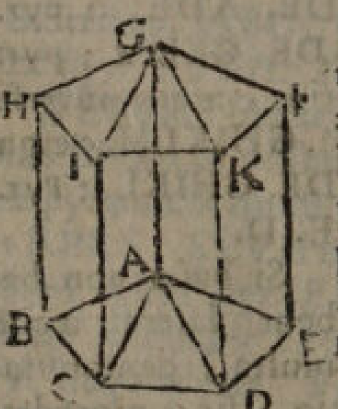


Omne prisma ABCDFE *triangularem habens basim, dividitur in tres pyramides* ACBF, ACDF, CDFE *æquales inter se, triangulares bases habentes.*

Ducantur parallelogrammorum diametri AC, CF, FD. Triang. ACB $a =$ ACD. b ergo æque altæ pyramides ACBF, ACDF æquantur. eodem modo pyr. DFAC = pyr. DFEC. atqui ACDF, & DFAC una eademque sunt pyramis. c ergo tres pyramides ACBF, ACDF, DFEC, in quos divisum est prisma, inter se æquales sunt. Q. E. D.

a 34. 1.
b 5. 12.
c 1. ax. 1.

Coroll.



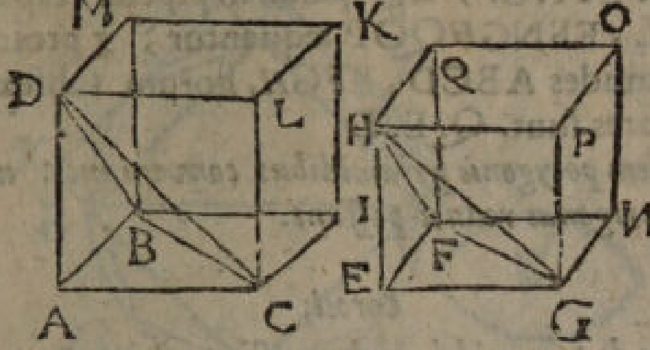
Hinc, quælibet pyramis tertia est pars prismatis eandem cum illa habentis & basim & altitudinem: sive, prisma quodlibet triplum est pyramidis eandem cum ipso habentis basim & altitudinem.

Nam resolve prisma polygonum ABCDEGHIKF in trigona prismata, & pyramidem ABCDEH in trigonas pyramides. a Erunt singulæ partes prismatis triplæ singularum partium pyramidis. b proinde totum prisma ABCDEGHIKF totius pyramidis ABCDEH triplum est Q. E. D.

a 7. 12.
b 1. 5.

PROP.

Prop. VIII.



Similes pyramides ABCD, EFGH, *quæ triangulares habent bases* ABC, EFG, *in triplicata sunt ratione homologorum laterum* AC, EG.

a Perficiantur parallelepipeda ABICDMKL, EFNGHQOP; quæ b similia sunt & pyramidum ABCD, EFGH c sextupla; d ideoque in eadem cum ipsis ratione ad se invicem, e hoc est in triplicata homologorum laterum. Q. E. D. a 17. 11. b 9. *def* 11. c 28. 11. & 7. 12. d 15. 5. e 33. 11.

Coroll.

Hinc, etiam similes polygonæ pyramides rationem habent laterum homologorum triplicatam; ut facile probabitur resolvendo has in trigonas pyramides.

Prop. IX.

Vide Schema præced.

Æqualium pyramidum ABCD, EFGH, *& triangulares bases* ABC, EFG *habentium, reciprocantur bases & altitudines. & quarum pyramidum triangulares bases habentium reciprocantur bases & altitudines, illæ sunt æquales.*

1. *Hyp.* Perfecta parallelepipeda ABICDMKL, EFNGHQOP æqualium pyramidum ABCD, EFGH (utrumque utriusque) a sextupla sunt, ac æqualia ideo inter se, ergo alt. (H.) a 28. 11. & 7. 12.

alt.

b 34. 11. c 15. 5. alt. (D) *b* :: ABIC. EFNG *c* :: ABC. EFG. Q. E. D.

d hyp. e 15. 5. f 34. 11. g 6. ax. 1. 2. *Hyp.* Alt. (H.) alt. (D) *d* :: ABC. EFG *e* :: ABIC. EFNG. *f* ergo parallelepipeda ABIC-DMKL, EFNGHQOP æquantur; *g* proinde & pyramides ABCD, EFGH, horum subsextuplæ, pares sunt. Q. E. D.

Eadem polygonis pyramidibus conveniunt: nam hæ ad trigonas reduci possunt.

Coroll.

Quæ de pyramidibus demonstrata sunt Prop. 6, 8, 9. etiam conveniunt quibuscunque prismatis, cum hæc tripla sint pyramidum eandem basim & altitudinem habentium. itaque 1. Prismatum æque altorum eadem est proportio, quæ basium.

2. Similium prismatum proportio triplicata est proportionis laterum homologorum.

3. Æqualia prismata reciprocant bases & altitudines; & quæ reciprocant, sunt æquales.

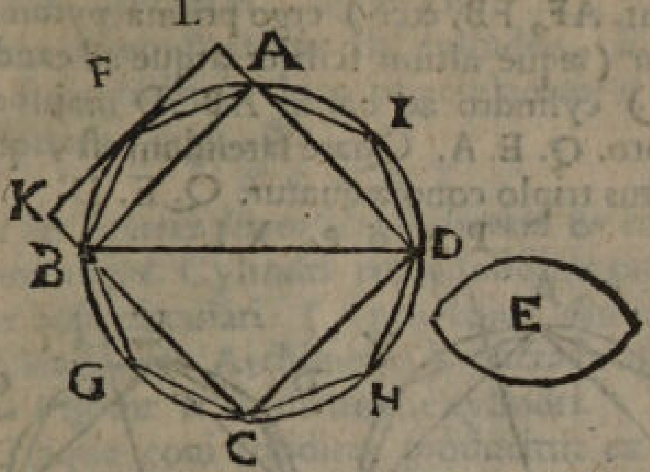
Schol.

Ex hactenus demonstratis elicitur dimensio quorumcunque prismatum & pyramidum.

a cor. 1. hujus; & sch. 40. 11. b 7. 12. *a* Prismatis soliditas producitur ex altitudine in basim ducta; *b* itaque & pyramidis ex tertia altitudinis parte ducta in basin.

PROP.

PROP. X.



Omnis conus tertia pars est cylindri habentis eandem cum ipso basin ABCD , *& altitudinem æqualem.*

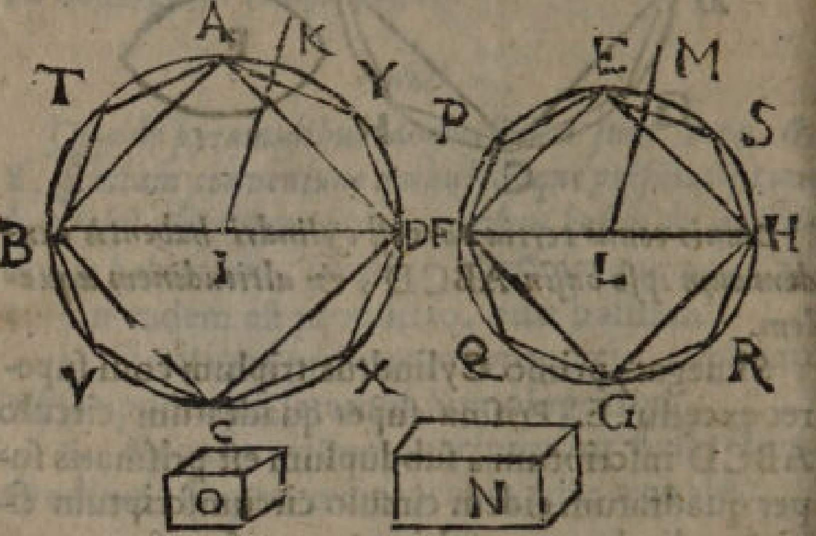
Si negas, primo Cylindrus triplum coni superet excessu E. Prisma super quadratum circulo ABCD inscriptum *a* subduplum est prismatis super quadratum eidem circulo circumscriptum sibi & cylindro æque alti. ergo prisma super quadratum ABCD superat cylindri semissem. eodem modo prisma super basim AFB cylindro æque altum segmenti cylindrici AFB *b* dimidio majus est. Continuetur bisectio arcuum , & detrahantur prismata , donec segmenta cylindri relicta, nempe ad AF , FB , &c. minora evadant solido E. Itaque cylind. — segment. AF,FB,&c. (prisma ad basim AFBGCHDI) *c* majus est quam cylind. — E (*d* triplum coni.) ergo pyramis dicti prismatis *e* pars tertia (ad eandem basim sita , ejusdemque altitudinis) cono æque alto ad basim ABCD circulum major est ; pars toto. Q. E. A.

Vide fig. 2. hujus.
a sch. 7. 4. & cor. 9. 12.
b sch. 27. 3. & cor. 9. 12.
c 5 ax. 1.
d hyp.
e cor. 7. 12.

Sin conus tertia parte cylindri major dicatur, sit itidem excessus E. Ex cono detrahe pyramides, ut in priori parte prismata ex cylindro , donec restent coni segmenta aliqua , puta ad AF ,

FB, BG, &c. minora solido E. ergo con. — E
f *hyp.* ($f\frac{1}{3}$ cylindr.) ⊐ pyr. AFBGCHDI (con. —
segment. AF, FB, &c.) ergo prisma pyramidis triplum (æque altum scilicet atque ad eandem basim) cylindro ad basim ABCD majus est, pars toto. Q. E. A. Quare fatendum est, quod cylindrus triplo cono æquatur. Q. E. D.

PROP. XI.



Sub eadem altitudine existentes cylindri, & coni ABCDK, EFGHM, *inter se sunt ut bases* ABCD, EFGH.

Sit circ. ABCD. circ. EFGH : : con. ABCDK. N. Dico N = con. EFGHM.

Nam si fieri potest, sit N ⊐ con. EFGHM, sitque excessus O. Suppoſita præparatione, & argumentatione præcedentis; erit O majus segmentis conicis EP, PF, FQ, &c. ideoque so-
a 30. 3. & 1. *post.* lidum N ⊐ pyr. EPFQGRHSM. *a* Fiat in cir-
b 6 12. culo ABCD simile polygonum ATBVCXDY.
c *cor* 2. 12. Quia pyr. ABVYK. pyr. EFQSM *b* : : polyg.
d *hyp.* e 14 5. ATBVY. polyg. EPFQS *c* : : circ. ABCD. circ. EFGH *d* :: con. ABCDK. N. *e* erit pyram. EPFQGRHSM ⊐ N. contra modo dicta.

Rursus dic N ⊏ con. EFGHM. pone con. EFGHM. O : : N. con. ABCDK *f* : : circ. EFGH. ABCD. *g* ergo O ⊐ con. ABCDK,

quod

uod absurdum est, ex ostensis in priori parte. Itaque potius dic, ABCD. EFGH :: con. BCDK. EFGHM. Q. E. D. (f hyp. & invertendo. g 14. 5.)

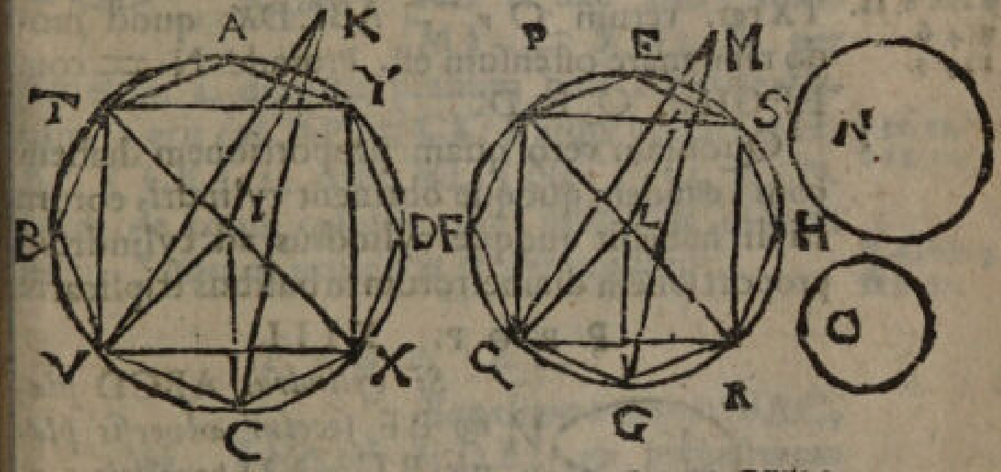
Idem demonstrabitur de cylindris, si conorum & pyramidum loco concipiantur cylindri et prismata. ergo, &c.

SCHOL.

Ex his habetur dimensio cylindrorum & conorum quorumcunque. Cylindri rectæ soliditas producitur ex base circulari (a pro cujus dimensione consulendus est Archimedes) ducta in altitudinem. b igitur & cujuscunque cylindri. (a 1. Prop. de dimens. circ. b 11. 12.)

c Itaque coni soliditas producitur ex tertia parte altitudinis ducta in basim. (c 10. 12.)

PROP. XII.



Similes coni & cylindri ABCDK, EFGHM, *in triplicata ratione sunt diametrorum* TX, PR, *quæ in basibus* ABCD, EFGH.

Habeat conus A ad aliquod N rationem triplicatam TX ad PR. dico N = con. EFGHM: Nam si fieri potest, sit N ⊐ EFGHM; sitque excessus O. ergo ut in Prioribus, N ⊐ pyr. EPFQGRHSM. Sint axes conorum IK LM, adducanturque rectæ VK, CK, VI, CI; & QM, GM, QL, GL. Quoniam coni similes sunt, a est VI. IK :: QL. LM. anguli vero VIK, QLM b recti sunt. c ergo trigona VIK, (a 24. def. 11. b 18. def. 11. c 6. 6.)

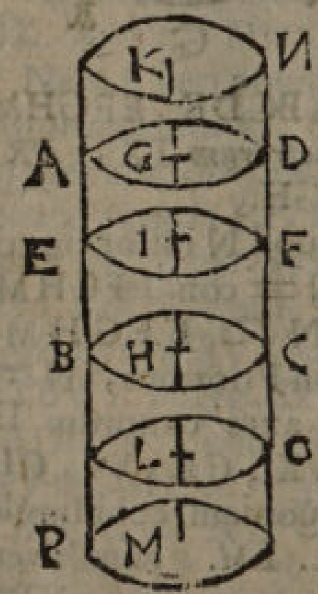
QLM,

d 4.6. QLM æquiangula sunt; d unde VC. VI :: QG. QL. item VI. VK :: QL. QM. ergo ex æ-
e 7.5. quali VC. VK :: QG.QM. e quinetiam VK. CK :: QM. MG. ergo rursus ex æquo VC.
f 5.6. CK :: QG. GM. f ergo triangula VKC, QMG similia sunt; similique argumento reliqua
g 9. def. 11. hujus pyramidis triangula reliquis illius. g quare
h cor. 8. 12. pyramides ipsæ similes sunt. h sunt vero hæ in
k 4.6. triplicata ratione VC ad QG, k hoc est VI ad
l 15.5. RL, l vel TX ad PR. m ergo Pyr. AIBVC-
m hyp. & 11.5 XDYK.pyr. EPFQGRHSM :: con. ABCDK.
n 14.5. N. n unde pyr. EPFQGRHSM ⊐ N; quod repugnat prius dictis.

Rursus, dic N. ⊏ con. EFGHM. sit con.
o Prius, & inversè. EFGHM. O :: N. con. ABCDK o :: pyr.
p cor. 8. 12. EPRM. ATCK p :: GQ. VC ter :: q PR.
q 4.6. TX ter. verum O r ⊐ ABCDK. quod mo-
r 14.5. do repugnare ostensum est. Proinde N = con. EFGHM. Q. E. D.

Quoniam vero quam proportionem habent coni, eandem quoque obtinent cylindri, eorum tripli, habebit quoque cylindrus ad cylindrum proportionem diametrorum in basibus triplicatã.

PROP. XIII.

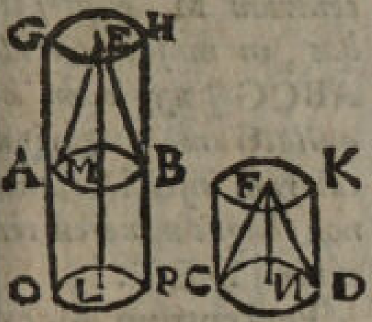


Si cylindrus ABCD *plano* EF *secetur adversis planis* BC, AD *parallelo; erit ut cylindrus* AEFD *ad cylindrum* EBCF, *ita axis* GI *ad axem* IH.

a 3. 1. Producto axe, a sume GK = GI, & HL = IH = LM. & concipe per puncta K, L, M, plana duci circulis AD, BC parallela. b ergo cylind. ED = cyl. AN. & cylin. EC b =
b 11. 12. BO b = OP. itaque cylindrus

drus EN cylindri ED æque multiplex eſt, ac axis IK axis IG. pariterque cylindrus FP æque multiplex eſt cylindri BF, ac axis IM axis IH. prout vero IK =, ⊏, ⊐ IM, *c* ſic cylindr. c 11. 12.
EN =, ⊏, ⊐ EP. *d* ergo cyl. AEFD. cyl. d 6. def. 5.
EBCF :: GI. IH. Q. E. D.

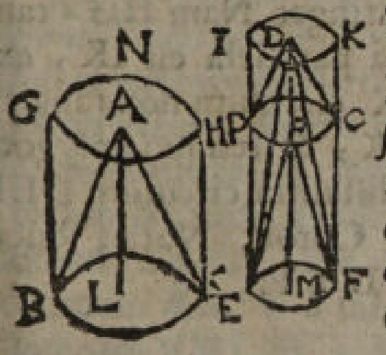
Prop. XIV.



Super æqualibus baſibus AB, CD *exiſtentes coni* AEB, CFD, *& cylindri* AH, CK, *inter ſe ſunt ut altitudines* ME, NF.

Productis cylindro HA & axe EM, ſume MI = FN; & per punctum L ducatur planum baſi AB parallelum. *a* erit cyl. AP = CK. *b* atqui cylind. AH. a 11. 12.
AP. (CK) :: ME. ML. (NF.) Q. E. D. b 13. 12.
Idem de conis cylindrorum ſubtriplis dictum puta. * imo de priſmatis & pyramidibus. * Adhibe 9. & 7. 12.

Prop. XV.



Æqualium conorum BAC, EDF, *& cylindrorum* BH, EK, *reciprocantur baſes & altitudines* (BC: EF :: MD. LA:) *& quorum conorum, & cylindrorum reciprocantur baſes & altitudines, illi ſunt æquales.*

Si altitudines pares ſint, etiam baſes pares erunt; & res clara eſt. Sin altitudines ſint impares, aufer MO = LA. a 14. 12.

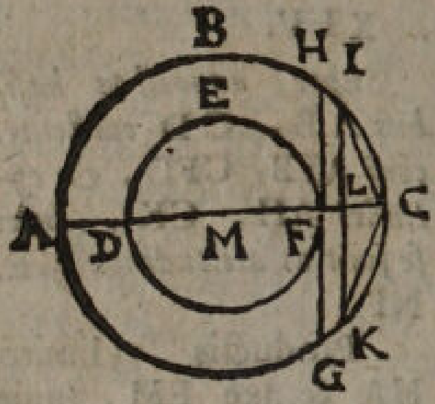
1. *Hyp.* Eſtque MD. MO (*a* LA) *b* :: cyl. b conſtr. c hyp.
EK (*c* BH.) EQ *d* :: circ. BC. EF. Q. E. D. d 11. 12.

a hyp. f 11. 12. g 11. 5. h 11. 12. k 9. 5.

2. Hyp. BC. EF e :: DM. OM (L A) f :: Cyl. EK. EQ g :: BC. EF h :: BH. EQ. k Ergo cylind. EK = BH. Q. E. D.

Simili argumento utere de conis.

PROP. XVI.



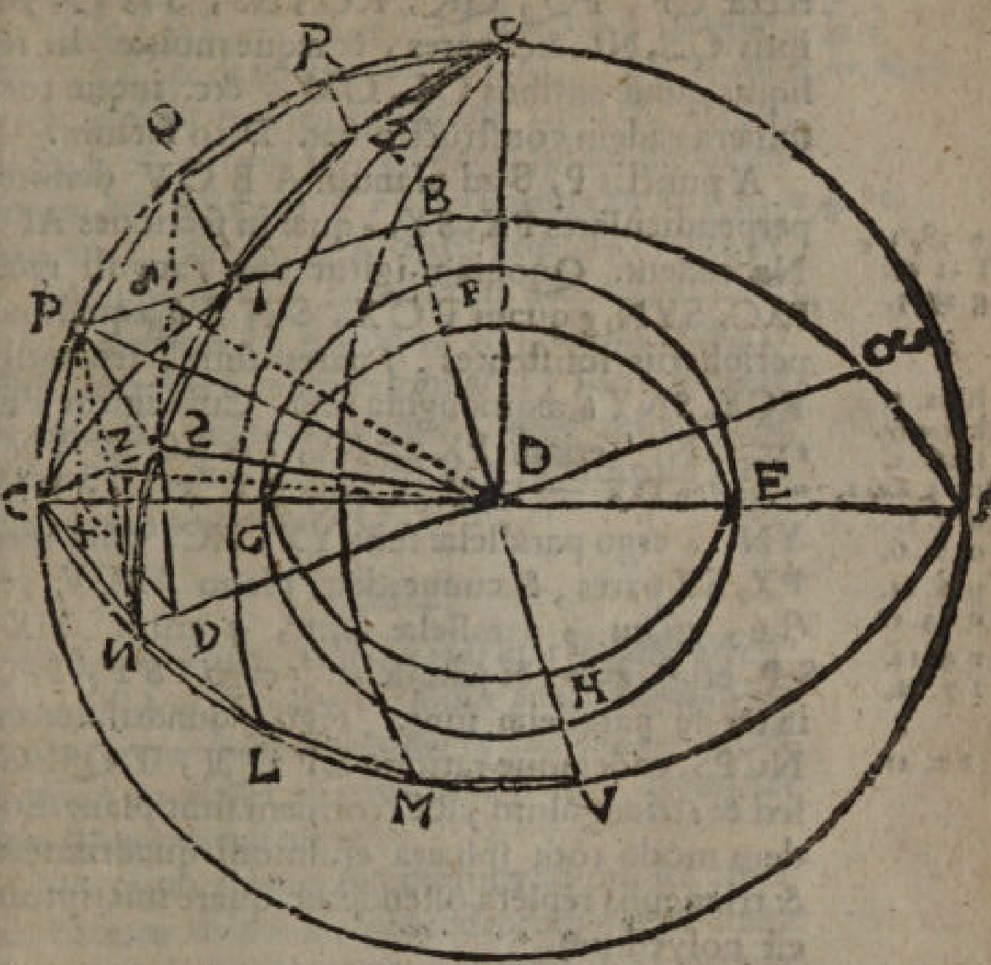
Duobus circulis ABCG, DEF circa idem centrum M existentibus, in majori circulo ABCG polygonum æquilaterum, & parium laterum inscribere, quod non tangat minorem circulum DEF.

Per centrum M extendatur recta A C secans circulum D E F in F. ex quo erige perpendicularem FH. a Biseca semicirculum ABC, ejusque semissem BC, atq; ita continuo, b donec arcus IC minor evadat arcu HC. ab I demitte perpendicularem IL. Liquet arcum IC totum circulum metiri, numerumque arcuum esse parem, adeoque subtensam IC latus esse c polygoni inscriptibilis, quod circulum DEF minime continget. Nam HG d tangit circulum DEF; e cui parallela est IK, extraque sita, f quare IK circulum non tangit, multoque magis GI, CK, & reliqua polygoni latera, longius à centro distantia, circulum DEF non tangunt. Q. E. F. Coroll. Nota; quod IK non tangit circulum D E F.

a 30. 3. b 1. 10. c sch. 16. 4. d cor. 16. 3. e 28. 1. f 34. def. 1.

PROP.

PROP. XVII.



Duabus sphæris ABCV, EFGH *circa idem centrum* D *existentibus, in majori sphæra* ABCV *solidum polyedrum inscribere, quod non tangat superficiem minoris sphæræ* EFGH.

Secentur ambæ sphæræ plano per centrum faciente circulos EFGH, ABCV. ducanturque diametri A C, BV secantes perpendiculariter. Circulo ABCV *a* inscribatur polygonum æquilaterum VMLNC, &c. circulum E F G H minime tangens. ducta diametro Nα, erectaque DO recta ad planum ABC. per DO, perq; diametros A C, Nα erigi concipiantur plana DOC, DON, quæ ad circulum ABCV *b* recta erunt, ideoque in superficie sphæræ *c* quadrantes

a 16. 12.
b 18. 11.
c cor. 33. 6.

 effici-

d 4.1. efficient [a] DOC, DON. in quibus d aptentur
rectæ CP, PQ, QR, RO, NS, ST, Tγ, γO
ipsis CN, NL, &c. pares, & æque multæ. In reliquis quadrantibus OL, OM, &c. inque tota sphæra eadem constructio fiat. Dico factum.

A punctis P, S ad planum A B C V demitte
e 38. 11. perpendiculares PX, SY, e quæ in sectiones AC,
f 12 ax. Nα cadent. Quoniam igitur tam f anguli recti
g 27. 3. PXC, SYN, g quam P C X, S N Y h æqualibus
peripheriis insistentes, f pares sunt, triangula
h 32. 1. PCX, SNY h æquiangula sunt. Cum igitur PC
k constr. k = SN, l etiam PX = SY, l & XC = YN;
l 26. 1.
m 3. ax. 1. m quare DX = DY. n ergo DX. XC :: DY.
n 7. 5. YN. o ergo parallelæ sunt YX, NC. quia vero
o 2. 6.
p 6. 11. PX, SY pares, & cum eidem plano ABCV rectæ, etiam p parallelæ sunt, q erunt Y X,
q 33. 1.
r 9. 11. S P etiam pares & parallelæ. r ergo S P, N C
s 7. 11. inter se parallelæ sunt. ergo s quadrilaterum
t 2. 11. NCPS, eademque ratione S P Q T, T Q R G,
sed & t triangulum γRO totidem sunt plana. Eodem modo tota sphæra ejusmodi quadrilateris & triangulis repleta ostendetur. quare inscriptum est polyedrum.

u 11. 11. A centro D u duc DZ rectum plano NCPS; & junge ZN, ZC, ZS, ZP. Quoniam D N.
x 4. 6. NC x :: DY.YX; est NC y ⊏ YX (SP;) pa-
y 14. 5. riterque SP ⊏ TQ, & TQ ⊏ γR. Et quia
z 3. def. 11. anguli DZC, DZN, DZS, DZP, z recti sunt,
a 15. def. 1. latera vero DC, DN, DS, DP a æqualia, &
b 47. 1. DZ commune, b erunt ZC, ZN, ZS, ZP æquales inter se; proinde circa quadrilaterum
c 15. def. 1. NCPS c describi potest circulus, in quo (ob NS, NC, CP d æquales, & NC ⊏ SP) NC
d constr. e plusquam quadrantem subtendit. f ergo ang.
e 28. 3. NZC ad centrum obtusus est. g ergo NCq ⊏
f 33. 6. 2 ZCq (ZCq + ZNq.) Sit NI ad AC nor-
g 12. 2.
h 32. 1. malis. ergo cum ang. A D N (h D N C +
k 9. ax. 1. DCN) sit k obtusus, l erit semissis ejus D C N
l 5. 1.

recti

ecti semisse major; proptereaque eo minor est
eliquus è recto ang. CNI. [n] unde IN $\sqsubset$ IC. n 19. 1.
rgo NCq (NIq + ICq) [o] $\sqsupset$ 2 INq. itaque o 47. 1.
N $\sqsubset$ ZC. & consequenter DZ [p] $\sqsubset$ DI. atqui p 47. 1.
unctum I est [q] extra sphæram EFGH. ergo q cor. 16. 12.
unctum Z potiori jure est extra ipsam. adeoque
lanum NCPS (cujus [r] proximum centro pun- r 47. 1.
ctum est Z) sphæram EFGH non contingit. Et
i ad planum SPQT demittatur perpendicularis
D δ, punctum δ, adeoque & planum SPQT
adhuc ulterius à centro elongatur; idemque est
de reliquis polyedri planis. ergo polyedrum
ORQPCN, &c. majori sphæræ inscriptum, mi-
norem non contingit. Q. E. F.

Coroll.

Hinc sequitur, *Si in quavis alia sphæra describatur solidum polyedrum, simile prædicto solido polyedro, proportionem polyedri in una sphæra ad polyedrum in altera esse triplicatam ejus quam habent sphærarum diametri.*

Nam si ex centris sphærarum ad omnes angulos basium dictorum polyedrorum rectæ lineæ ducantur, distribuentur polyedra in pyramides numero æquales & similes, quarum homologa latera sunt semidiametri sphærarum; ut constat, si intelligatur harum sphærarum minor intra majorem circa idem centrum descripta. congruent enim sibi mutuo lineæ rectæ ductæ à centro sphæræ ad basium angulos, ob similitudinem basium, ac propterea pyramides efficientur similes. Quare cum singulæ pyramides in una sphæra, ad singulas pyramides illis similes in altera sphæra [a] habeant proportionem triplicatam laterum homologorum, hoc est, semidiametrorum sphærarum; sint autem [b] ut una pyramis ad unam pyramydem, ita omnes pyramides, hoc est, solidum polyedrum ex his compositum, ad omnes pyra-

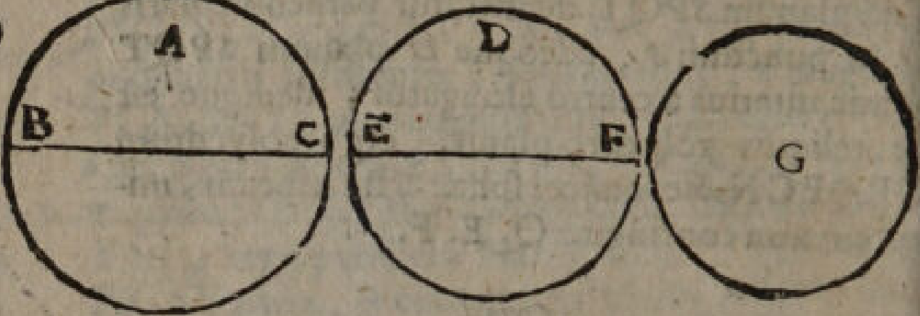
a cor. 8. 12.

b 12. 15.

mides, id est, ad solidum polyedrum ex illis constitutum; habebit quoque polyedrum unius sphæræ ad polyedrum alterius sphæræ proportio-
c 15. 5. nem triplicatam semidiametrorum, *c* atque adeo diametrorum.

PROP. XVIII.

HO



Sphæræ BAC, EDF *sunt in triplicata ratione suarum diametrorum* BCEF.

Sit sphæra BAC ad sphæram G in triplicata ratione diametri BC ad diametrum EF. Dico G = EDF. Nam si fieri potest, sit G ⊐ EDF. & cogita sphæram G concentricam esse ipsi EDF.
a 17. 12. Sphæræ EDF *a* polyedrum sphæram G non tan-
b cor. 17. 12. gens, sphæræque BAC simile polyedrum inscri-
c hyp. batur. *b* Hæc polyedra sunt in triplicata ratione
d 14. 5. diametrorum BC, EF, *c* id est, sphæræ BAC ad G. *d* Proinde sphæra G major est polyedro sphæræ EDF inscripto, pars toto

Rursus, si fieri potest, sit sphæra G ⊏ EDF. Sitque ut sphæra EDF ad aliam sphæram H, ita
e hyp. invers. G ad BAC, *e* hoc est in triplicata ratione dia-
f 14. 5. metri EF ad BC; cum igitur BAC *f* ⊏ H, incurrimus absurditatem prioris partis. Quin potius sphæra G = EDF. Q. E. D.

Coroll.

Hinc, ut sphæra ad sphæram, ita est polyedrum in illa descriptum ad polyedrum simile in hac descriptum.

LIB. XIII.

PROP. I.

Si recta linea z secundum extremam & mediam rationem secetur (z.a :: a.e;) majus segmentum a assumens dimidium totius z, quintuplum potest ejus, quod à dimidia totius z describitur, quadrati.

Dico Q. $a + \frac{1}{2}z$ = 5 Q: $\frac{1}{2}z$. ^a hoc est $aa + \frac{1}{4}zz + za = zz + \frac{1}{4}zz$. ^b vel $aa + za = zz$. Nam $ze + za$ ^c $= zz$. & ze ^d $= aa$. ^e ergo $aa + za = zz$. Q.E.D.

a 4. 2.
b 3. ax. 1.
c 2. 2.
d hyp. & 16. 6.
e 1. ax. & 1. ax.

PROP. II.

Si recta linea $\frac{1}{2}z + a$ *sui ipsius segmenti* $\frac{1}{2}z$ *quintuplum possit, duplæ prædicti segmenti (z) extrema ac media ratione sectæ majus segmentum est a, reliqua pars ejus quæ à principio rectæ* $\frac{1}{2}z + a$.

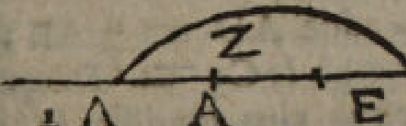
Dico z.a :: a.e. Nam quia per hyp. * $aa + \frac{1}{4}zz + za = zz + \frac{1}{4}zz$; vel $aa + za = zz$ ^a $= ze + za$. ^b erit $aa = ze$. ^c quare z. a :: a. e. Q.E.D.

* 4. 2.
a 2. 2.
b 3 ax. 1.
c 17. 6.

Vide fig. præced.

PROP. III.

Si recta linea z secundum extremam ac mediam rationem secetur (z. a :: a.e;) minus segmentum e assumens dimidium majoris segmenti a, quintuplum potest ejus, quod à dimidia majoris segmenti a describitur, quadrati.

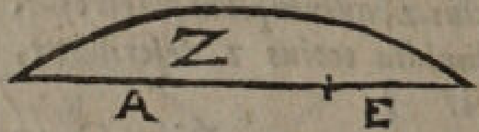


Dico Q: $e + \frac{1}{2}a$ = 5 Q: $\frac{1}{2}a$. ^a hoc est $ee + \frac{1}{4}aa + ea = aa + \frac{1}{4}aa$. ^b vel $ee + ea = aa$. Nam $ee + ea$ ^c $= ze$ ^d $= aa$. Q.E.D.

a 4 2.
b 3. ax.
c 3 2.
d hyp. & 17. 6.

PROP. IV.

Si recta linea z secundum extremam ac mediam rationem secetur (z. a :: a. e;) quod à tota z, quodque à minori segmento e, utraque simul quadrata, tripla sunt ejus, quod à majori segmento a describitur, quadrati.



Dico $zz + ee = 3\,aa$.

a 4. 2. [a] vel $aa + ee + 2\,ae + ee = 3\,aa$.

b 3. 2. Nam $ae + ee =$
c 17. 6.
d 2. ax. ze [c] $= aa$. [d] ergo $aa + 2\,ae + 2\,ee = 3\,aa$. Q. E. D.

PROP. V.

Si recta linea AB *secundum extremam & mediam rationem secetur in* C, *apponaturque ei* AD *æqualis majori segmento* AC; *tota recta linea* DB *secundum extremam ac mediam rationem secatur, & majus segmentum est quæ à principio recta linea* AB.

a hyp. Nam quia AB. AD [a] :: AC. CB, invertendoque AD. AB :: CB. AC; erit componendo DB. AB :: AB. AC. (AD.) Q. E. D.

Schol.

Quod si fuerit BD. BA :: BA. AD. erit BA. AD :: AD. BA − AD. Nam dividendo est BD − BA (AD) BA :: BA − AD. AD. ergo inverse, BA. AD :: AD. BA − AD. Q. E. D.

PROP. VI.

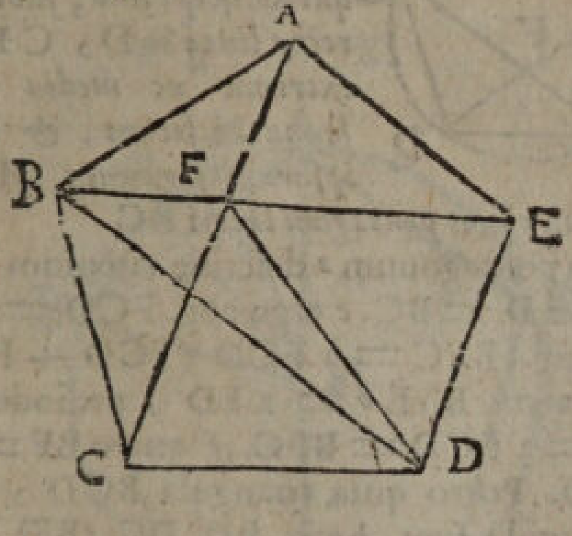
Si recta linea rationalis AB *extrema ac media ratione secetur in* C; *utrumque segmentorum* (AC, CB) *irrationalis est linea, quæ vocatur apotome.*

a 3. 1. Majori segmento AC [a] adde $AD = \frac{1}{2}\,AB$;
b 1. 13. [b] ergo DCq = 5 DAq. [c] ergo DCq ⊏⊐ ² DAq.
c 6. 10. proinde cum AB, [e] ideoque ejus semissis DA
d hyp.
e sch. 12. 10. sint ρ', etiam DC est ρ'. Quia vero 5. 1 :: non

Q.

Q.Q. *f* est DC ⊏⊐ DA. *g* ergo DC − AD, id est AC est apotome. Insuper quia ACq *h* = AB x BC, & AB est ρ', *k* etiam BC est apotome. Q.E.D. f 9. 10. g 74 10. h 17. 6. k 98. 10.

PROP. VII.



Si pentagoni æquilateri ABCDE *tres anguli, sive qui deinceps* EAB, ABC, BCD, *sive* EAB, BCD, CDE *qui non deinceps sint, æquales fuerint, æquiangulum erit ipsum pentagonum* ABCDE.

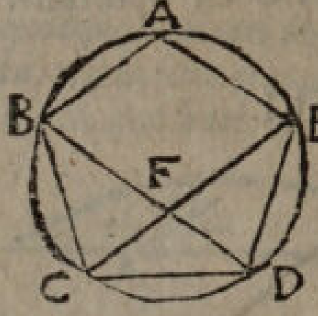
Paribus deinceps angulis subtendantur rectæ BE, AC, BD.

Quoniam latera EA, AB, BC, CD, angulique inclusi *a* æquantur, *b* erunt bases BE, AC, BD, *c* angulique AEB, ABE, BAC, BCA pares. *d* quare BF = FA, & *e* proinde FC = FE. ergo triangula FCD, FED sibi mutuo æquilatera sunt; *f* unde ang. FCD = FED, *g* proinde ang. AED = BCD. Eodem pacto ang. CDE reliquis æquatur. quare pentagonum æquiangulum est. Q.E.D. a hyp. b 4. 1. c 4. & 5. 1. d 6. 1. e 3. ax. 1. f 8. 1. g 2. ax. 1.

Sin anguli EAB, BCD, CDE, qui non deinceps, statuantur pares, *h* erit ang. AEB = BDC, & BE = BD, *k* ideoque ang. BED = BDE; *l* totus proinde ang. AED = CDE. ergo propter angulos A, E, D deinceps æquales, ut prius, pentagonum æquiangulum erit. Q.E.D. h 4. 1. k 5. 1. l 2. ax.

PROP.

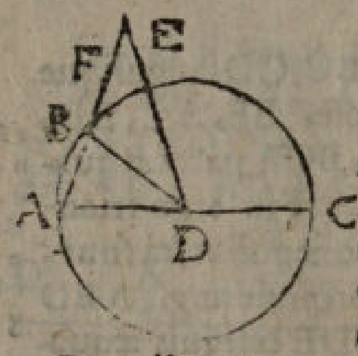
PROP. VIII.



Si pentagoni æquilateri & æquianguli ABCDE *duos angulos* BCD, CDE, *qui deinceps sint, subtendant rectæ lineæ* BD, CE; *hæ extrema ac media ratione se mutuo secant, & majora ipsarum segmenta* BF, *vel* EF *æqualia sunt pentagoni lateri* BC.

a 14. 4. Circa pentagonum [a] describe circulum ABD.
b 28. 3. [b] Arcus ED = BC. [c] ergo ang. FCD = FDC.
c 27. 3. [d] ergo ang. BFC = 2 FCD (FCD + FDC.)
d 32. 1. Atqui arcus BAE [b] = 2 ED, proinde ang.
e 33. 6. BCF [e] = 2 FCD = BFC. [f] quare BF = BC.
f 6. 1. Q. E. D. Porro quia triangula BCD, FCD
g 27. 3. [g] æquiangula sunt, [h] erit BD. DC (BF) :: CD
h 4. 6. (BF.) FD. pariterque EC. EF :: EF. FC. Q. E. D.

PROP. IX.



Si hexagoni latus BE, *& decagoni* AB, *in eodem circulo* ABC *descriptorum componantur, tota recta linea* AE *extrema ac media ratione secatur*. (AE. BE :: BE. AB.) *& majus ejus segmentum est hexagoni latus* BE.

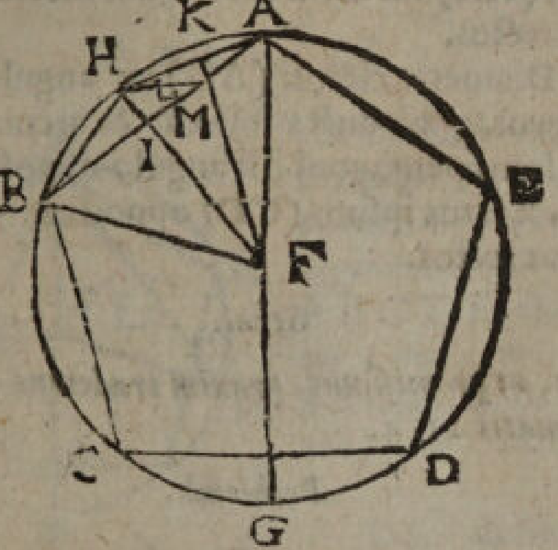
a hyp. & 37. 3. Duc diametrum ADC, & junge rectas DB, DE. Quoniam ang. BDC [a] = 4 BDA, estque
b 32. 1. ang. BDC [b] = 2 DBA (DAB + DBA,) erit
c 7 ax. 1. DBA ([b] BDE + BED) [c] = 2 BDA [d] = 2 BDE.
d 5. 1. proinde ang. DBA, vel DAB [e] = ADE. Itaque
e 1. ax. 1. trigona ADE, ADB æquiangula sunt, [f] quare
f 4. 6 AE. AD. ([g] BE) :: AD. (BE.) AB. Q. E. D.
g cor. 15. 4.

Coroll.

Coroll.

Hinc, si latus hexagoni alicujus circuli secetur extrema ac media ratione; majus illius segmentum erit latus decagoni ejusdem circuli. *sch* 5. 13.

P R O P. X.



Si in circulo ABCE *pentagonum æquilaterum* ABCDE *describatur; pentagoni latus* AB *potest & hexagoni latus* FB, *&* *decagoni latus* AH, *in eodem circulo descriptorum.*

Duc diametrum AG. Biseca arcum AH in K. Et duc FK, FH, FB, BH, HM.

Semicirc. AG – arc. AC *a* = AG – AD. a 28. 3. &
hoc est, arc. CG = GD *b* = AH = HB. ergo 3. *ax.*
arc. BCG = 2 BHK; *c* adeoque ang. BFG = 2 b *hyp.* & 7. *ax.*
BF*K*. *d* sed ang. BFG = 2 BAG. *e* ergo ang. c 33. 6.
BFK = BAG. Trigona igitur BFM, FAB *f* æ- d 20. 3. e 1. *ax.* 1.
quiangula sunt. *g* quare AB. BF :: BF. BM. f 32. 1.
h ergo AB x BM = BFq. Rursus ang. AF*K* *k* = g 4. 6 h 17. 6.
HF*K*; & FA = FH; *m* quare AL = LH, *m* & k 17. 3.
anguli FLA, FLH pares, ac proinde recti sunt. m 4. 1.
ergo ang. LHM *m* = LAM *n* = HBA. Trigo- n 17. 3. o 32. 1.
na igitur AHB, AMH *o* æquiangula sunt. *p* qua- p 4. 6.

q 17. 6. re AB. AH :: AH. AM.[q] ergo AB x AM = AHq. Quum igitur ABq[r] = AB x BM + AB x AM,[f] erit ABq=BFq + AHq. Q. E. D.

r 2. 2.
f 2. ax.

Coroll.

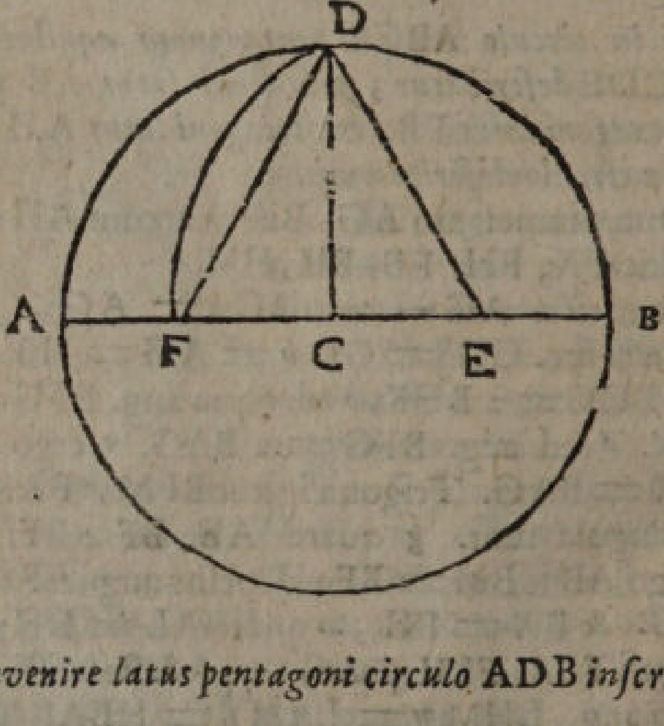
1. Hinc, linea recta (FK) quæ ex centro (F) arcum quempiam (H A) bisecat, etiam rectam (HA) illi arcui subtensam bisecat ad angulos rectos.

2. Diameter circuli (AG) ex angulo quovis (A) pentagoni ducta bisecat & arcum (CD,) quem latus pentagoni illi angulo oppositum subtendit, & latus ipsum (CD) oppositum, idque ad angulos rectos.

Schol.

Hic, ut promisimus, praxim trademus expeditam problematis 11. 4.

Problema.



Invenire latus pentagoni circulo ADB *inscribendi.*

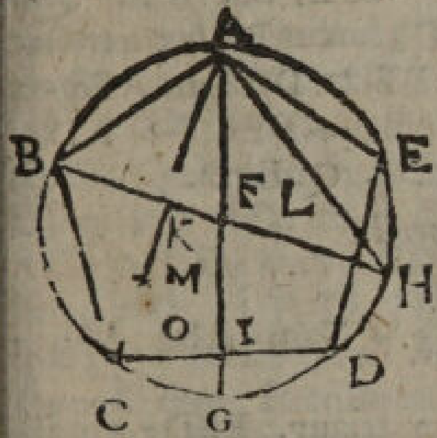
Duc diametrum A B. cui perpendicularem

C D

C D ex centro C erige. Biſeca CB in E. Fac
EF = ED. Erit DF pentagoni latus.

Nam BF x FC + ECq *a* = EFq *b* = EDq a 6. 2.
= DCq + ECq. *d* ergo BF x FC = DCq, vel b conſtr.
BCq. *e* quare BF. BC :: BC. FC. ergo quum BC c 47. 1.
ſit latus hexagoni, *f* erit FC latus decagoni, d 3. ax.
proinde DF *h* = √DCq + FCq *g* eſt latus pen- e 17. 6.
tagoni. Q. E. F. f 9. 13.
g 10. 13.
h 47. 1.

PROP. XI.



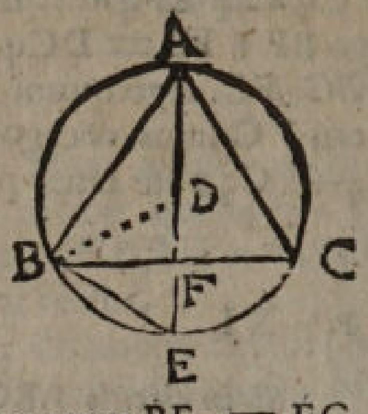
Si in circulo ABCD *rationalem habente diametrum* AG, *pentagonum æquilaterum* ABCDE *deſcribatur; pentagoni latus* AB *irrationalis eſt linea, quæ vocatur minor.*

Duc diametrum BFH, rectaſque AC, AH; & * fac FL = $\frac{1}{4}$ radii FH, & CM = $\frac{1}{4}$ CA. * 10. 6.

Ob angulos AKF, AIC *a* rectos, & commu- a cor. 10. 13.
nem CAI, trigona AKF, AIC *b* æquiangula b 32. 1.
ſunt; *c* ergo CI. FK *c* :: CA. FA (FB) *d* :: c 4. 6.
CM. FL. ergo permutando FK. FL :: CI. CM d 15. 5.
d :: CD. CK (2 CM.) *e* componendo igitur CD e 18. 5.
+ CK. CK :: KL. FL. *f* proinde Q: CD + CK f 22. 6.
(*g* 5 CKq.) CKq :: KLq. FLq. ergo KLq g 1. 13.
= 5 FLq. Itaque ſi BH (ρ̇) ponatur 8, erit FH
4; FL 1. & FLq. 1. BL 5. & BLq 25. KLq 5. è
quibus liquet BL, & KL eſſe ρ̇ *h* ⊔. *k* ideoque h 9. 10.
BK eſſe Apotomen; cujus congruens KL. cum ve- k 74. 10.
ro BLq − KLq = 20, *l* erit BL ⊓ √BLq − l 9. 10.
KLq. *m* unde BK erit apotome quarta. Quo- m cor. 8. 6 & 17. 6.
niam igitur ABq *m* = HB x BK, *n* erit AB minor. n 95. 10.
Q. E. D.

PROP.

PROP. XII.



Si in circulo ABEC triangulum æquilaterum ABC describatur, trianguli latus AB potentia triplum est ejus lineæ AD, quæ ex D centro circuli ducitur.

Protracta diametro ad E, duc BE. Quoniam arcus BE a = EC, arcus BE sexta est pars circumferentiæ. b ergo BE = DE. hinc AEq c = 4 DEq (4 BEq) d = ABq + BEq (+ ADq.) e proinde ABq = 3 ADq. Q. E. D.

a cor. 10. 13.
b cor. 15 4.
c 4 2.
d 47. 1.
e 3. ax 1.

Coroll.

1. AEq. ABq :: 4. 3.

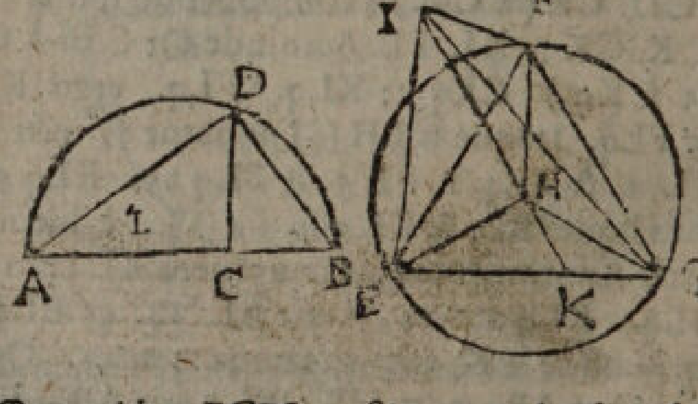
2. ABq. AFq :: 4. 3. f Nam ABq. AFq :: AEq. ABq.

3. DF = FE. Nam triang. EBD g æquilaterum est; h & BF ad ED perpendicularis. h ergo EF = FD.

4. Hinc AF = DE + DF = 3 DF.

f cor 8. 6. & 21 6.
g cor. 15. 4.
h cor. 3. 3.

PROP. XIII.



Pyramidem EGFI constituere, & data sphæra complecti; & demonstrare quod sphæræ diameter

AB

AB *potentia sit sesquilatera lateris* E F *ipsius pyramidis* EGFI.

Circa AB describe semicirculum ADB. a 10. 6.
a sitque AC = 2 CB. ex puncto C erige perpendicularem CD; & junge AD, DB. Tum radio HE = CD describe circulum HEFG; cui b inscribe triangulum æquilaterum EFG. b cor. 15. 4.
ex H c erige IH = CA rectum plano EFG, c 12. 11.
produc IH ad K; d ita ut IK = AB. rectasque d 3. 1.
adjunge IE, IF, IG. erit EFGI pyramis expetita.

Nam quia anguli ACD, IHE, IHF, IHG e recti sunt; & CD, HE, HF, HG e pares, e atque e constr.
IH = AC; f erunt AD, IE, IF, IG æquales inter se. Quia vero AC (2 CB.) CB g :: ACq. f 47. 1. g 20. 6.
CDq. erit ACq = 2 CDq. itaque ADq f = ACq + CDq h = 3 CDq = 3 HEq k = EFq. h 2. ax. k 12. 13.
l ergo AD, EF, IE, IF, IG pares sunt, adeoque pyramis EFGI est æquilatera. Quod si punctum C super H collocetur, & AC super HI, l 1. ax. 1.
rectæ AB, IH m congruent, utpote æquales. quare semicirculus ADB axi AB vel IK circumductus n transibit per puncta, E, F, G, * adeoque pyramis EFGI sphæræ inscripta erit. Q. E. F. m 8 ax. 1. n 15 def. 1. * 31. def. 11
liquet vero esse BAq. ADq o :: BA. AC p :: 3. 2. o cor. 8. 6.
Q. E. D. p constr.

Corollaria.

1. ABq. HEq :: 9. 2. Nam si ABq ponatur 9, erit ACq (EFq) 6. q proinde HEq erit 2. q 12. 13.

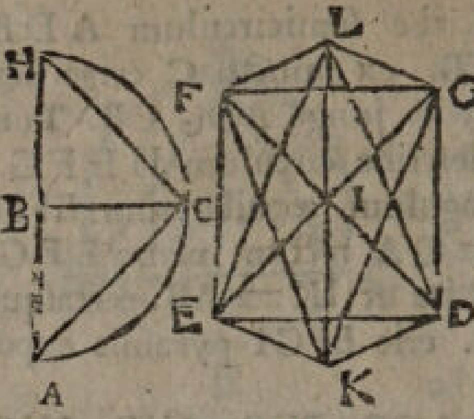
2. Si L centrum fuerit, erit AB. LC :: 6. 1. Nam si AB ponatur 6, erit AL, 3; r ideoque AC 4; quare LC erit 1. Hinc r constr.

3. AB. HI :: 6. 4 :: 3. 2. unde

4. ABq. HIq :: 9. 4.

PROP.

Prop. XIV.



Octaedrum KEFGDL *constituere; & data sphæra complecti, qua & pyramidem; & demonstrare, quod sphæra diameter* AH *potentia sit dupla lateris* AC *ipsius Octaedri.*

Circa AH describe semicirculum ACH. ex centro B erige perpendicularem BC. duc AC,
a 46. 1. HC. Super ED=AC *a* fac quadratum EFGD, cujus diametri DF, EG secantes in centro I. ex
b 12. 11. c 3. 1. I duc IL=AB *b* rectam plano EFGD. produc IL, *c* donec IK=IL. Connexis KE, KF, KG, KD, LE, LF, LG, LD ; erit KEFGDL octaedrum quæsitum.

Nam AB. BH, FI, IE, &c. æqualium quadratorum semidiametri æquales sunt inter se. *d* qua-
d 4. 1. re triangulorum rectangulorum LIE, LIF, FIE, &c. bases LF, LE, FE, &c. æquantur. proinde octo triangula LFE, LFG, LGD, LDE, KEF,
e 27. def. 11. KFG, KGD, KDE æquilatera sunt, *e* atque octaedrum constituunt, quod sphæræ cujus centrum I, radius IL, vel AB, inscribi potest. (quo-
f constr. niam AB, IL, IF, IK, &c. *f* æquales sunt.) Q. E. F. porro liquet AHq (LKq) *g* = 2 ACq
g 47. 1. (2.LDq.) Q. E. D.

Corollaria.

1. Hinc manifestum est, in Octaedro tres diametros EG, FD, LK se mutuo ad angulos rectos secare in centro sphæræ.

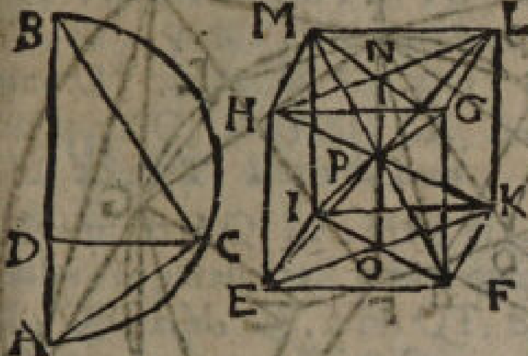
2. Item, tria plana EFGD, LEKG, LFKD esse quadrata, se mutuo ad angulos rectos secantia.

3. Octa-

3. Octaedrum dividitur in duas pyramides similes & æquales EFGDL, & EFGDK, quarum basis communis est quadratum EFGD.

4. Denique, bases octaedri oppositæ, inter se parallelæ sunt. 15. 11.

PROP. XV.



Cubum EFGHIKLM *constituere, & sphæra complecti, qua & priores figuras; & demonstrare, quod sphæræ diameter* AB *potentia sit tripla lateris* EF *ipsius cubi.*

Super AB describe semicirculum ACB; & [a] fac AB = 3 DA. ex D erige perpendicularem DC, & junge BC ac AC. Tum super EF = AC [b] construe quadratũ EFGH, cujus plano rectæ insistant EI, FK, HM, GL ipsi EF pares, quas connecte rectis IK, KL, LM, IM. Solidum EFGHIKLM cubus est, ut satis constat ex constructione.

a 10. 6.
b 46. 1.

In quadratis oppositis EFKI, HGLM duc diametros EK, FI, HL, MG, per quas ducta plana EKLH, FIMG se intersecent in recta NO. Hæc diametros cubi EL, FM, GI, HK [c] bisecabit in P, centro cubi. [d] ergo P centrum erit sphæræ per puncta cubi angularia transeuntis. Porro ELq [e] = EKq + KLq [e] = 3 KLq, [f] vel 3 ACq. atqui ABq. ACq [g] :: BA. DA [f] :: 3. 1. [h] ergo AB = EL. Quare cubum fecimus, &c. Q. E. F.

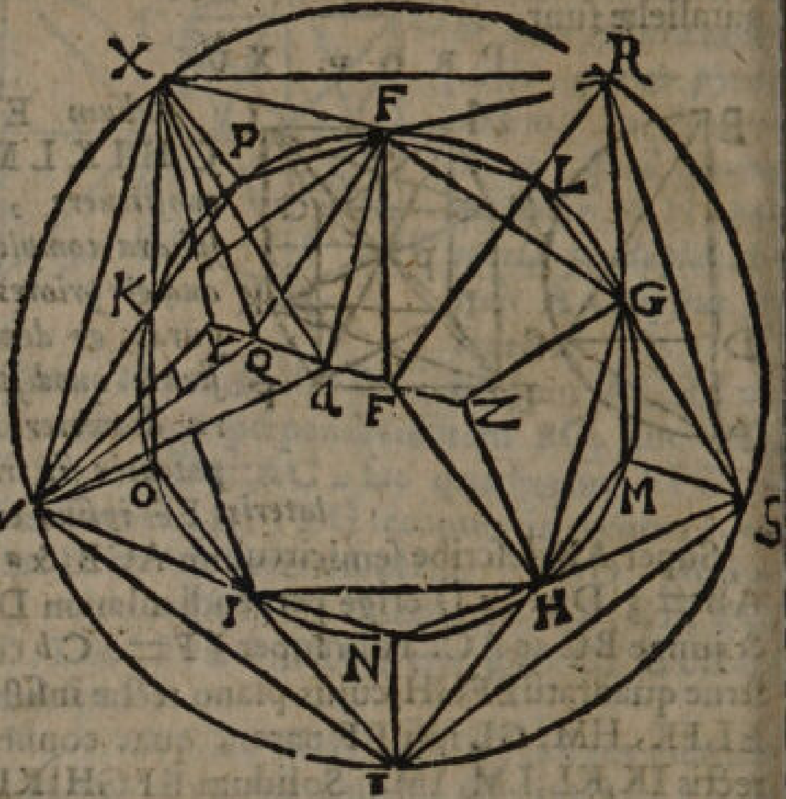
c cor. 39. 11.
d 15. def. 1. & 14. def. 11.
e 47. 1.
f constr.
g cor. 8 6.
h 14 5.

Coroll.

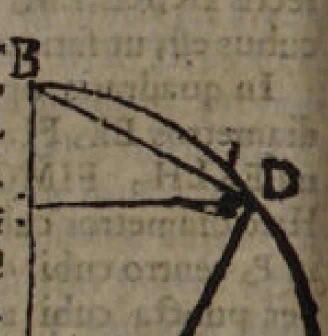
1. Hinc, omnes diametri cubi inter se æquales sunt, seseque mutuo in centro sphæræ bisecant. Eademque ratione rectæ quæ quadratorum oppositorum centra conjungunt, bisecantur in eodem centro.

k 47. 1. 2. Diameter ſphæræ poteſt latus tetraedri, & l 13. 13. m 15. 13. cubi. nempe ABq k = l BCq + m ACq.

PROP. XVI.



Icoſaedrum ZGHIKF-YVXRST *conſtituere, & ſphæra complecti, qua & antedictas figuras; & demonſtrare, quod icoſaedri latus* F G *irrationalis eſt linea, quæ vocatur minor.*



Super A B diametrum ſphæræ deſcribe ſemicirculum ADB; & a fac AB = 5 BC. ex C erige normalem C D, & duc A D ac B D. Ad intervallum E F = B D deſcribe circulum E F K N G;

a 10. 6.

cui inscribe pentagonum æquilaterum FKIHG. b 11. 4.
Biseca arcus FG, GH, &c. ac connecte rectas FL, LG, &c. latera nempe decagoni. Tunc c erige EQ, LR, MS, NT, OV, PX ipsi EF æquales, rectasque plano FKNG. & connecte RS, ST, TV, VX, XR; item FX, FR, GR, GS, HS, ST, HT, IT, IV, KV, KX. Denique producta EQ, sume QY = FL; & EZ = FL; rectasque duci concipe ZG, ZH, ZI, ZK, ZF; ac YV, YX, YR, YS, YT. Dico factum. c 12. 11.

Nam ob EQ, LR, MS, NT, OV, PX d æquales e & parallelas, etiam quæ illas jungunt, EL, QR, EM, QS, EN, QT, EO, QV, EP, QX f pares & parallelæ sunt. Item ideo LM (vel FG,) RS, MN, ST, &c. æquales sunt inter se. g ergo planum per EL, EM, &c. plano per QR, QS, &c. æquidistans, h & circulus QXRSTV è centro Q, circulo EPLMNO æqualis est; atque RSTVX est pentagonum æquilaterum. Duci vero intellectis EF, EG, EH, &c. ac QX, QR, QS, &c. quia FRQ k = FLq + LRq, l vel EFq m = FGq, n erunt FR, FG, ideoque omnes RS, FG, FR, RG, GS, GH, &c. æquales inter se. Proinde 10 triangula RFX, RFG, RGS, &c. æquilatera sunt & æqualia. Rursus ob ang. XQY o rectum, erit XYq p = QXq + QYq q = VXq vel FGq. quare XY, VX, hisque similiter YV, YT, YS, YR, ZG, ZH, &c. æquantur: Ergo alia decem trigona constituta sunt æquilatera, & æqualia, tam sibi mutuo, quam decem prioribus; ac proinde factum est Icosaedrum.

d constr. e 6. 11. f 33. 1. g 15. 11. h 1. def. 3. k 47. 1. l constr. m 10. 13. n sch. 48. 1. & 1. ax. o cor. 14. 11. p 47. 1. q 10. 13.

Porro, bisecta EQ in α, duc rectas αF, αX, αV; & propter QX r = QV, & commune latus αQ, angulosque EQX, EQV rectos; s erit αX = αV. similique argumento omnes, αX, αR, αS, αT, αV, αF, αG, αH, αI, αK æquantur. r 15. def. 1. s 4. 1.

t 9. 13. Quoniam autem ZQ. QE [t] :: QE. ZE, erit
u 3. 13. Zαq [u] = 5 Eαq [x] = EQq (EFq) + Eαq, [y] = αFq.
x 4. 2.
y 47. 1. ergo Zα = αF. [z] pari pacto αF = Yα. ergo ſphæra, cujus centrum α, radius αF, per 12 puncta icoſaedri angularia tranſibit.

Denique, quia Zα. αE :: ZY. QE; [a] ideoque
z 15. 5. Zαq. αEq :: ZYq. QEq. [b] erit ZYq = 5 QEq,
a 22. 6. vel 5 BDq: atqui ABq. BDq [c] :: AB. BC :: 5.
b 14. 5. 1. [d] ergo ZY = AB. Q. E. F.
c cor 8. 6.
d 1. ax. 1.
e ſch. 12. 10. Itaque ſi AB ponatur ρ [e], erit EF = √ ABq
f 11. 13 etiam ρ; proinde FG pentagoni, idemque Icoſaedri 5 latus, [f] eſt minor. Q. E. D.

Coroll.

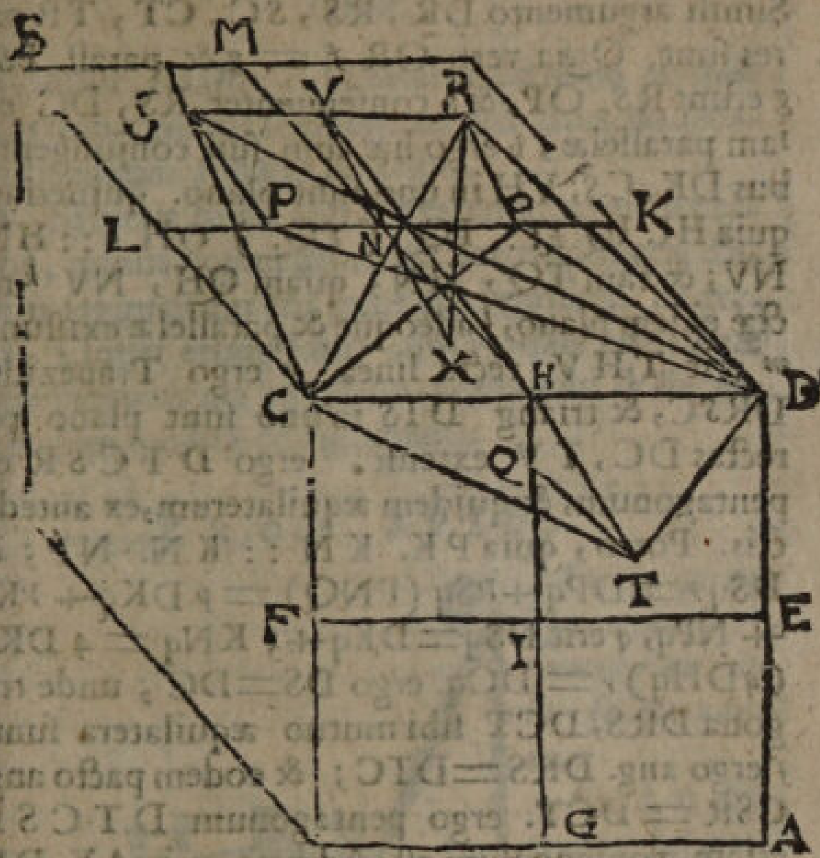
1. Ex dictis infertur, ſphæræ diametrum eſſe potentia quintuplum ſemidiametri circuli quinque latera icoſaedri ambientis.

2. Item manifeſtum eſt, ſphæræ diametrum eſſe compoſitam ex latere hexagoni, hoc eſt, ex ſemidiametro, & duobus lateribus decagoni circuli ambientis quinque latera icoſaedri.

3. Conſtat denique latera icoſaedri oppoſita,
a 33. 1. qualia ſunt RX, HI, eſſe parallela. Nam RX [a] pa-
b ſch. 16. 3. rall. LP. [b] parall. HI.

PROP.

Prop. XVII.



Dodecaedrum constituere, & sphæra complecti, ua & prædictas figuras; & demonstrare, quod do- ecaedri latus RS *irrationalis est linea, quæ vocatur potome.*

Sit AB cubus datæ sphæræ inscriptus, cujus tera omnia bisecentur in punctis E, H, F, G, , L, &c. rectæque adjungantur KL, MH, IG, EF. *a* Fac HI. IQ :: IQ. QH; & sume a 30. 6.
N O, N P pares ipsi I Q. Erige O R, P S rectas lano DB, & QT plano AC. sintque OR, PS, QT ipsis IQ, NO, NP æquales. Connexis DR, S, SC, CT, DT, erit DRSCT pentagonum Dodecaedri expetiti. Nam duc NV parall. OR, protracta NV ad occursum cum cubi centro X, connecte rectas DS, DO, DP, CR, CP, a 47. 1.
HV, HT, RX. Quia DOq *a* = DKQ (*b* KNq) b 7. ax. 1.
c 4. 13.
+ KOq *c* = 3 ONq (3 ORq) *d* erit DRq d 47. 1.

e 4. 2.1 = 4 ORq e = OPq, vel RSq. ergo DR = RS. Simili argumento DR, RS, SC, CT, TP pares sunt. Quia vero OR f = g & parall. PS
f constr. 9.6. 21.
g 33. 1. g erunt RS, OP, & h consequenter RS, DC etiam parallelæ; h ergo hæ cum suis conjungentibus DK, CS, VH in uno sunt plano. quinetiam quia HI. IQ k :: IQ (TQ) QH k :: HN. NV; & tam TQ, HN, quam QH, NV k rectæ eidem plano, l adeoque & parallelæ existunt, m erit THV recta linea. n ergo Trapezium DRSC, & triang DTS in uno sunt plano per rectas DC, TV extensc. ergo DTCSR est pentagonum, & quidem æquilaterum, ex antedictis. Porro, quia PK. KN :: KN. NP; & DSq p = DPq + PSq (PNQ) = p DKq + PKq + NPq, q erit DSq = DKq + 3 KNq = 4 DKq (4DHq) r = DCq. ergo DS = DC; unde trigona DRS, DCT sibi mutuo æquilatera sunt. s ergo ang. DRS = DTC; & eodem pacto ang. CSR = DCT. ergo pentagonum DTCSR etiam æquiangulum est. Ad hæc, quia AX, DX, CX, &c. sunt cubi semidiametri, t erit XN = IH, vel KN, u adeoque XV = KP. unde ob angulum x rectum RVX, z erit RXq = XVq + RVq (NPq) = KPq + NPq a = 3 KNq b = AXq, vel DXq, &c. ergo RX, AX, DX, & eadem ratione XS, XT, AX æquales sunt inter se. Et si eadem methodo, qua constructum est pentagonum DTCSR, fabricentur 12 similia pentagona tangentia duodecim cubi latera, ea Dodecaedrum constituent; ac per eorum puncta angularia transiens sphæra, cujus radius AX, vel RX, Dodecaedrum complectetur. Q. E. F.
h 9. 1.
k 7. 11.
k constr.
l 6. 11.
m 32. 6.
n 1. & 2. 11.
o 5. 13.
p 47. 1.
q 1. ax. 1.
& 4. 13.
r 4. 2.
s 8. 1.
t 15. 13.
u 1. ax. 1.
x 29. 1.
z 47. 1.
a 4. 13.
b 15. 13.

Denique, quia KN. NO c :: NO. OK, d erit KL. OP :: OP. OK + PL. Itaque si sphæræ diameter AB ponatur $\acute{p}$, erit KL e = $\sqrt{\frac{AB}{3}}$ f etiam $\acute{p}$. g unde OP, vel RS latus dodecaedri apotome erit. Q. E. D.
c constr.
d 15. 5.
e 15. 13.
f sch 12. 10.
g 6. 13.

Coroll.

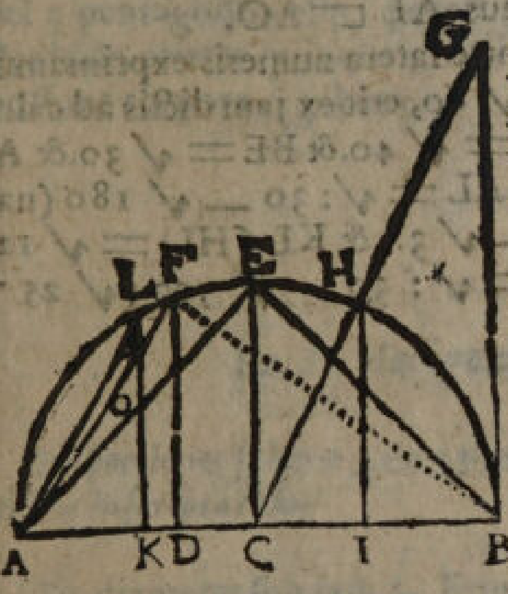
Coroll.

1. Hinc, si latus cubi secetur extrema ac me-
a ratione, majus segmentum erit latus dodeca-
ri in eadem sphæra descripti.

2. Si rectæ lineæ sectæ extrema ac media ra-
one, minus segmentum sit latus dodecaedri,
ajus segmentum erit latus cubi ejusdem sphęrę.

3. Liquet etiam latus cubi æquale esse lineæ
æ subtendenti angulum pentagoni dodecae-
i eadem sphæra comprehensi.

PROP. XVIII.



Latera quinq; figurarum exponere, & inter se comparare.

Sit AB diameter sphæræ, ac AEB semicirculus. sitq; AC a = $\frac{1}{2}$AB, a 10. 1.
& AD b = $\frac{1}{3}$ AB. Erige perpendiculares CE, DF, & b 10. 6.
GG = AB. junge AF, AE, BE, BF, CG. ex H
emitte perpendicularem HI, & sumpta CK =
CI, ex K erige perpendicularem KL, & conne-
cte AL. Denique c fac AF. AO :: AO. OF. c 30. 6.

Itaque 3. 2 d :: AB. BD e :: ABq. BFq, latus d constr.
Tetraedri. & 2. 1 :: a AB. AC :: ABq. BEq, f la- e cor. 8. 6.
tus Octaedri. f 14. 13.

Item 3. 1 d :: AB. AD e :: ABq. AFq, g la- g 15. 13.
tus Hexaedri.

Porro, quia AF. AO h :: AO. OF. k erit h constr.
k cor. 17. 13

l 4.6. AO latus Dodecaedri. denique BG (2 BC.)
m 24.5. BC [l] :: HI.IC. [m] ergo HI = 2CI [n] = KI. ergo
n constr. HIq [o] = 4CIq. proinde CHq = [p] 5 CIq. [q] ergo
o 4.2. ABq = 5 KIq. [r] itaque KI, vel HI, est radius cir-
p 47.1. culi circumscribentis pentagonum icosaedri; &
q 15.5. AK, vel IB, [s] est latus decagoni eidem circulo in-
r cor.16.13. scripti. unde AL erit latus pentagoni, [t] idemque
s 10.13. Icosaedri latus. Ex quibus liquet BF, BE, AF
t 16.13. esse ρ' ⊐. & AL, AO esse ρ' ⊐; atque BF $\sqsubset$ BE; & BE $\sqsubset$ AF; ac AF $\sqsubset$ AO. Quia vero 3 AFq = ABq [u] = 5 KLq. ac AF x AO

u 1.6. $\sqsubset$ AF x OF, [x] ideoque AF x AO + AF x OF
x 4.ax.1. $\sqsubset$ 2 AF x OF, [y] hoc est AFq $\sqsubset$ [z] 2 AOq. [a] e-
y 1.2. rit 3 AFq (5 KLq) $\sqsubset$ 6 AOq. proinde KL
z 17.6. $\sqsubset$ AO; & fortius, AL $\sqsubset$ AO.
a 47.1.

Jam vero ut haec latera numeris exprimamus, si AB ponatur $\sqrt{60}$, erit ex jam dictis ad calculum exactis, BF = $\sqrt{40}$. & BE = $\sqrt{30}$. & AF = $\sqrt{20}$. item AL = $\sqrt{:30 - \sqrt{180}}$ (nam AK = $\sqrt{15} - \sqrt{3}$. & KL (HI) = $\sqrt{12}$.) denique AO = $\sqrt{:30 - \sqrt{500}}$ ($\sqrt{25} - \sqrt{5}$.)

SCHOL.

SCHOL.

Præter jam dictas figuras nullam dari posse figuram solidam regularem (nempe quæ figuris planis ordinatis & æqualibus continëatur) admodum perspicuum est. Nam ad anguli solidi constitutionem requiruntur ad minimum tres anguli plani; *a* hique omnes simul 4 rectis minores esse debent. *b* Atqui 6 anguli trigoni æquilateri, 4 quadratici, & 3 hexagonici, sigillatim 4 rectos exæquant; quatuor vero pentagonici, 3 heptagonici, 3 octagonici, &c. 4 rectos excedunt. ergo solummodo ex 3, 4, vel 5 triangulis æquilateris, ex 3 quadratis, vel 3 pentagonis, effici potest angulus solidus. Proinde, præter quinque prædicta, nulla existere possunt corpora regularia.

a 21. 11.

b *Vid. schol.* 32. 1.

Ex P. Herigonio.

Proportiones sphæræ, & 5 figurarum regularium eidem inscriptarum.

Sit diameter sphæræ 2. Erunt

Peripheria circuli majoris, 6 |28318.

Superficies circuli majoris, 3 |14159.

Superficies sphæræ, 12 |56637.

Soliditas sphæræ, 4 |1879.

Latus tetraedri, 1 |62299.

Latus

Superficies tetraedri, 4 |6188.

Soliditas tetraedri, 0 |15132.

Latus hexaedri, 1 |1547.

Superficies hexaedri, 8.

Soliditas hexaedri, 1 |5396.

Latus octaedri, 1 |41421.

Superficies octaedri, 6 |9282.

Soliditas octaedri, 1 |33333.

Latus dodecaedri, 0 |71364.

Superficies dodecaedri, 10 |51462.

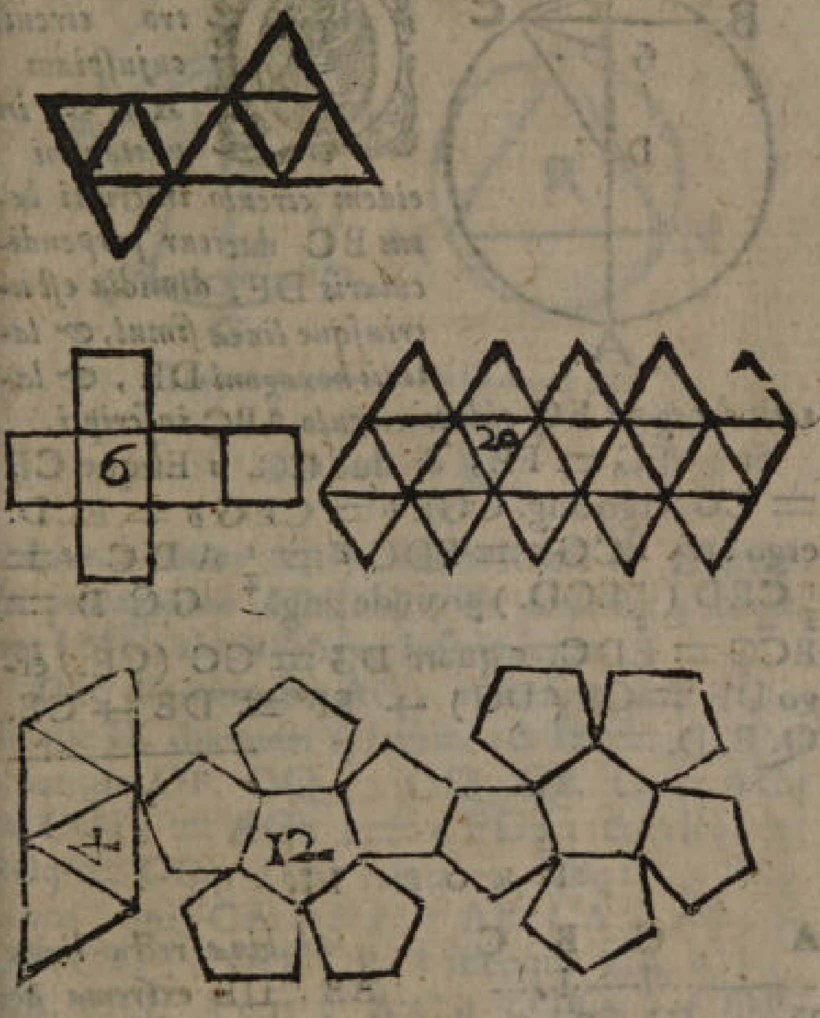
Soliditas dodecaedri, 2 |78516.

Latus Icosaedri, 1 |05146.

Superficies Icosaedri, 9 |57454.

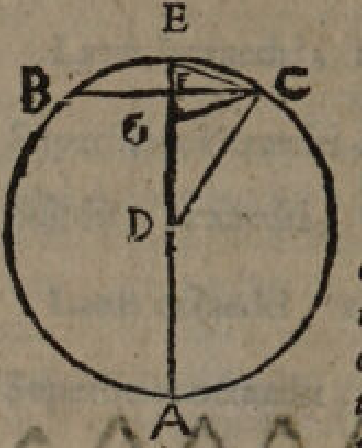
Soliditas Icosaedri, 2 |53615.

Quod si ex charta conficiantur quinque figuræ æquilateræ & æquiangulæ similes his quæ sunt in subjecta figura, componentur quinque figuræ solidæ, si rite complicentur.



LIB.

PROP. I.



QVæ ex D *centro circuli cujuſpiam* ABC *in pentagoni eidem circulo inſcripti latus* BC *ducitur perpendicularis* DF, *dimidia eſt utriuſque lineæ ſimul, & lateris hexagoni* DE, *& lateris decagoni* EC *eidem circulo* ABC *inſcripti.*

a 4. 1.
b 5. 1.
c 32. 1.
d hyp. & 33. 6.
a 20. 13.
f 7. ax.
g 6. 1.

Sume FG = FE, & duc CG. *a* Eſtque CE = CG. ergo ang. CGE *b* = CEG *b* = ECD. ergo ang. ECG *c* = EDC *d* = $\frac{1}{4}$ ADC *e* = $\frac{1}{2}$ CED ($\frac{1}{2}$ ECD.) proinde ang. *f* GCD = ECG = EDC. *g* quare DG = GC (CE.) ergo DF = CE (DG) + EF = $\frac{DE + CE}{2}$. Q. E. D.

PROP. II.

A G B C

D H E F

Si binæ rectæ lineæ AB, DE *extrema ac media ratione ſecentur* (AB. AG :: AG. GB. *&* DE. DH :: DH. HE;) *ipſæ ſimiliter ſecabuntur, in eaſdem ſcilicet proportiones.* (AG. GB :: DH. HE.)

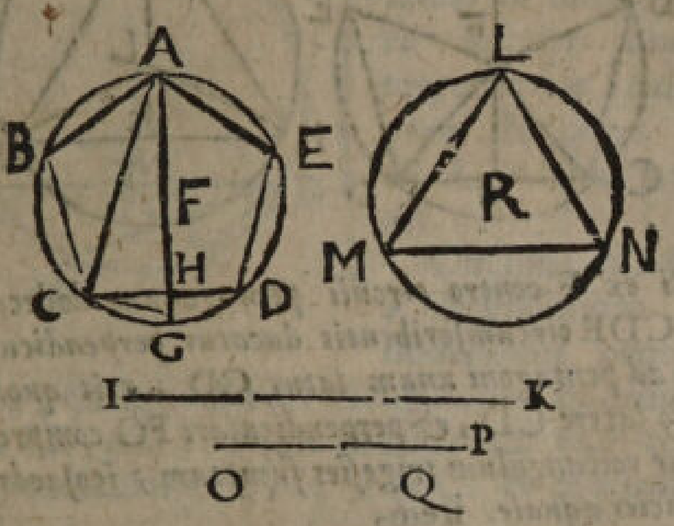
a 17. 6.
b 8. 2.
c 2. ax. 1.
d 22. 5 & 22. 6.

Accipe BC = BG & EF = EH. Eſtque AB x BG *a* = AGq. quare ACq *b* = 4 ABG + AGq *c* = 5 AGq. Similiter erit DFq = 5 DHq. *d* ergo AC. AG :: DF. DH. componendo igitur AC + AG. AG :: DF + DH. DH.

DH. hoc est 2 AB. AG :: 2 DE. DH. *e* proinde AB. AG :: DE. DH. unde *f* dividendo AG. GB :: DH. HE. Q. E. D.

e 22. 5.
f 17. 5.

PROP. III.



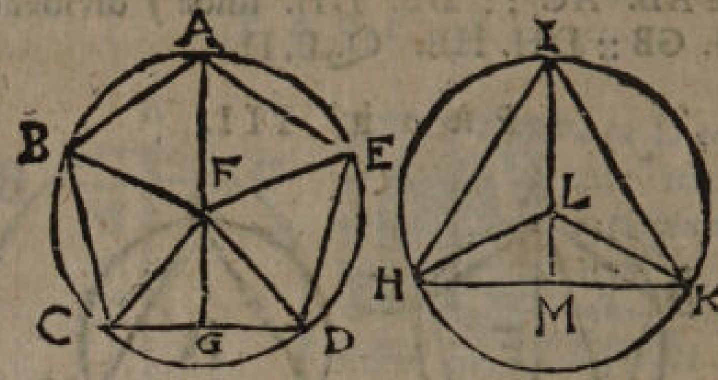
Idem circulus ABD *comprehendit & Dodecaedri pentagonum* ABCDE, *& Icosaedri triangulum* LMN, *eidem sphæræ inscriptorum.*

Duc diametrum AG, rectasque AC, CG. a sch. 47. 1.
Sitque IK diameter sphæræ, *a* & IKq = 5 OPq. b 30. 6.
b fiatque OP. OQ :: OQ. QP. Quia ACq c 47. 1.
+ CGq *c* = AGq *d* = 4 FGq; & ABq *e* = d 4. 2.
FGq = CGq. *f* erit ACq + ABq = 5 FGq. e 10. 13.
porro, quia CA. AB *g* :: AB. CA − AB; ac f 1, & 3. ax.
OP. OQ :: OQ. QP. *h* ideoque CA. OP :: g 8. 13.
AB. OQ. *k* erit 3 ACq (*l* IKq.) 5 OPq h 2. 13. & 16. 5.
(*m* IKq) :: 3 ABq. 5 OQq. ergo 3 ABq = 5 k 21. 6 & 4. 5
OQq. Verum ob ML *n* latus pentagoni circu- l 15. 13.
lo inscripti, cujus radius OP, erunt 15 RMq m constr.
o = 5 MLq *p* = 5 OPq + 5 OQq = * 3 n cor. 16. 13.
ACq + 3 ABq *q* = 15 FGq. *r* ergo RM o 12. 13.
= FG. *s* proinde circ. ABD = circ. LMN. p 10. 13.
Q. E. D.

q 15. 5.
* Prius.
r 1. ax. 1. & sch. 48 1.
s 1. def. 3.

PROP.

PROP. IV.



Si ex F centro circuli pentagonum dodecaedri ABCDE *circumscribentis ducatur perpendicularis* FG *ad pentagoni unum latus* CD ; *erit quod sub dicto latere* CD , *& perpendiculari* FG *comprehenditur rectangulum trigesies sumptum , icosaedri superficiei æquale.* item,

Si ex centro L *circuli triangulum icosaedri* HIK *circumscribentis , perpendicularis* L M *ducatur ad trianguli unum latus* H K ; *erit quod sub dicto latere* HK , *& perpendiculari* L M *comprehenditur rectangulum trigesies sumptum, icosaedri superficiei æquale.*

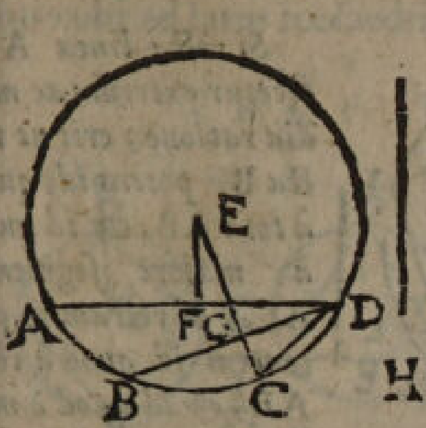
a 8. 1. Duc FA, FB, FC, FD, FE. *a* Erunt triangula CFD, DFE, EFA, AFB, BFC æqualia. at-
b 41. 1. qui CDxFG *b* = 2 triang. CFD. ergo 30 CD
c 15. 5. x GF *c* = 60 CFD *d* = 12 pentag. ABCDE *e* =
d 6. ax. superf. dodecaedri. Q. E. D.
e 17. 3. Duc LI, LH, LK. estque HKxLM *f* = 2
f 41. 1. triang. LHK. ergo 30 HKxLM *g* = 60 HLK
g 15. 5. *h* = 20 HIK *h* = superfic. icosaedri. Q. E. D.
h 16. 13.

Coroll.

k 15. 5. CDxFG. HKxLM *k* :: superfic. dodecaed. ad superf. icosaedri.

PROP.

PROP. V.



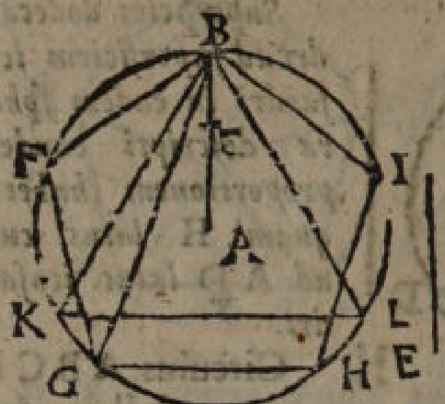
Superficies dodecaedri ad superficiem icosaedri in eadem sphæra descripti eandem proportionem habet, quam H latus cubi ad AD latus icosaedri.

Circulus ABCD [a] circumscribat tam dodecaedri pentagonum, quam icosaedri triangulum; quorum latera BD, AD; ad quæ demittantur ex E centro perpendiculares EF, EGC. & connectatur CD. a 3. 14.

Quoniam EC+CD. EC [b] :: EC. CD. erit b 9. 13.
EG ([c] $\frac{1}{2}$ EC + CD.) EF ([d] $\frac{1}{2}$ EC) [e] :: EF. c 1. 14. d cor. 12. 13
EG − EF ($\frac{1}{2}$ CD.) atqui H. BD [f] :: BD. H− e 15. 5.
BD. [g] ergo H. BD :: EG. EF. proinde H x EF f cor. 17. 13. g 2. 14.
= BD x EG. quum igitur H. AD [h] :: H x EF. h 1. 6.
AD x EF. erit H. AD :: BD x EG. AD x EF k 7. 5. l cor. 4. 14.
:: [l] superfic. dodecaedri ad superfic. icosaedri.
Q. E. D.

PROP.

PROP. VI.



Si recta linea AB secetur extrema ac media ratione; erit ut recta BF potens id, quod à tota AB, & id quod à majori segmento AC, ad rectam E, potentem id quod à tota AB, & id quod à minori segmento BC; ita latus cubi BG ad latus icosaedri BK eidem sphæræ cum cubo inscripti.

Circulo, cujus semidiameter AB, inscribantur dodecaedri pentagonum BFGHI, & icosaedri
a cor. 17. 13. triangulum BKL. *a* quare BG latus cubi erit ei-
b 12. 13. dem sphæræ inscripti. igitur BKq *b* = 3 ABq;
c 4. 13. & Eq *c* = 3 ACq. ergo BKq. Eq *d* :: ABq. ACq
d 15. 5. *e* :: BGq. BFq. permutando igitur BGq. BKq ::
e 2. 14. BFq. Eq. *f* unde BG. BK :: BF. E. Q. E. D.
f 22. 6

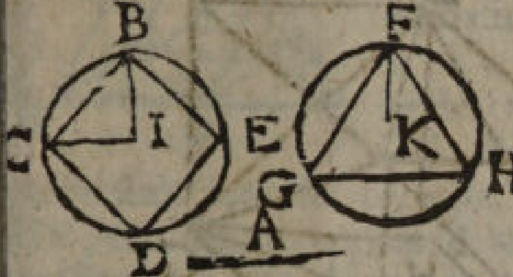
PROP. VII.

Dodecaedrum est ad Icosaedrum, ut cubi latus ad latus Icosaedri, in una eademque sphæra inscripti.

a 3. 14. Quoniam *a* idem circulus comprehendit & do-
b 47. 1. decaedri pentagonum & icosaedri triangulum, *b* erunt perpendiculares à centro sphæræ ad plana pentagoni & trianguli ductæ inter se æquales. itaque si dodecaedrum & icosaedrum intelligantur esse divisa in pyramides, ductis rectis à centro sphæræ ad omnes angulos, omnium pyramidum altitudines erunt inter se æquales.
c 5, & 6. 12. Cum igitur pyramides æque altæ *c* sint ut bases, & superficies dodecaedri sit æqualis 12 pentagonis, superficies vero icosaedri 20 triangulis; erit

rit dodecaedrum ad icosaedrum, ut superficies
dodecaedri ad superficiem icosaedri, *d* hoc est, ut   d 5. 14.
latus cubi ad latus icosaedri.

PROP. VIII.

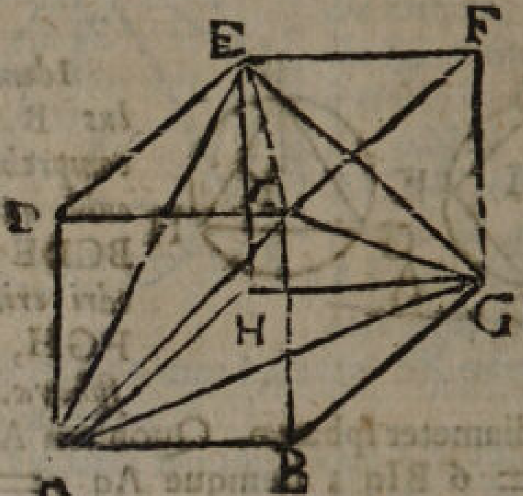


Idem circulus BCDE *comprehendit & cubi quadratum* BCDE *& octaedri triangulum* FGH, *ejusdem sphæræ.*

Sit A diameter sphæræ. Quoniam Aq *a* = 3   a 15. 13.
BCq *b* = 6 BIq; itemque Aq *c* = 2 GFq   b 47. 1.
= 6 KFq; erit BI = KF. *e* ergo circulus CBED   c 14. 13.
= GFH. Q. E. D.   d 12. 13.
e 2. def. 1.

LIB. XV.

PROP. I.



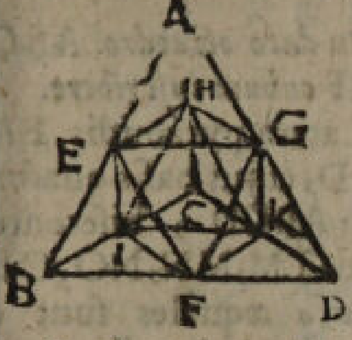
IN *dato cubo* ABGHDCFE *pyramidem* AGEC *describere.*

Ab angulo C duc diametros CA, CG, CE; Easque connecte diametris AG, GE, EA. Hæ omnes inter se *a* æquales sunt, utpote æqualium quadratorum diametri. ergo triangula CAG, CGE, CEA, EAG æquilatera sunt, ac æqualia: proinde AGEC est pyramis, quæ cubi angulis insistit, eique idcirco *b* inscribitur. Q. E. F.

a 47. 1.
b 31. def. 11.

PROP.

PROP. II.

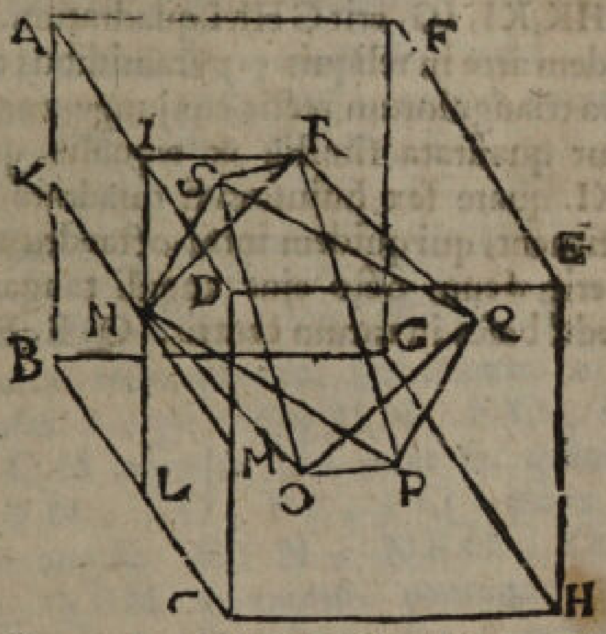


In data pyramide ABDC *octaedrum* EGKIFH *describere.*

a Biseca latera pyramidis in punctis E, I, F, K, G, H; quæ connecte 12 rectis EF, FG, GE, &c. Hæ omnes *b* æquales sunt inter se. proinde 8 triangula EHI, HK, &c. æquilatera sunt & æqualia, adeoque constituunt *c* octaedrum *d* in data pyramide descriptum. Q. E. F.

a 10. 1.
b 4. 1.
c 27. def. 11.
d 31. def. 11.

PROP. III.

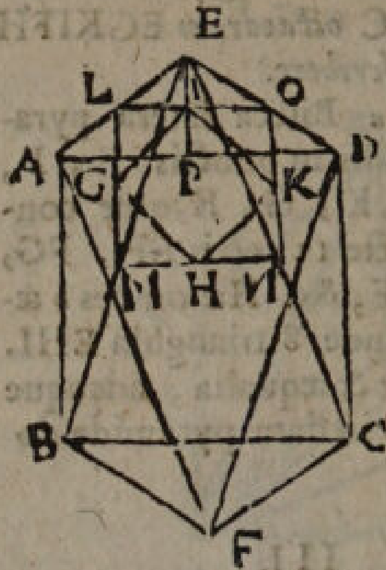


In dato cubo CHGBDEFA *octaedrum* NPQSOR *describere.*

Connecte quadratorum * centra N, P, Q, S, O, R, 12 rectis NP, PQ, QS, &c. quæ *a* æqualia sunt inter se, ideoque 8 triangula efficiunt æquilatera & æqualia. proinde *b* inscriptum est cubo Octaedrum NPQSOR. Q. E. F.

* 8. 4.
a 4. 1.
b 31. & 27. def. 11.

PROP. IV.



In dato octaedro ABCDEF cubum inscribere.

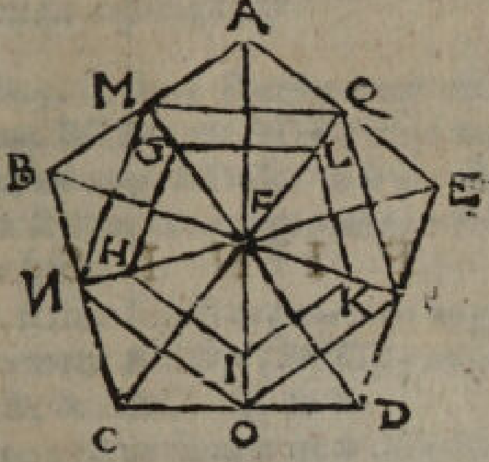
Latera pyramidis EABCD, cujus basis quadratum ABCD, bisecentur rectis LM, MN, NO, OL; quæ *a* æquales sunt & *b* parallelæ lateribus quadrati ABCD. *c* ergo quadrilaterum LMNO est quadratum.

a 4. 1.
b 2. 6.
c 29. def. 1.

Eodem modo, si latera quadrati LMNO bisecentur in punctis G, H, K, I, & connectantur GH, HK, KI, IG, erit GHKI quadratum. Quod si eadem arte in reliquis 5 pyramidibus octaedri centra triangulorum rectis conjungantur, describentur quadrata similia & æqualia quadrato GHKI. quare sex hujusmodi quadrata cubum constituent, qui quidem intra octaedrum descriptus erit, *d* cum octo ejus anguli tangant octo octaedri bases in earum centris. Q. E. F.

d 31. def. 11.

Prop. V.



In dato Icoſaedro Dodecaedrum inſcribere.

Sit A B C D E F pyramis Icoſaedri, cujus aſis pentagonum ABCDE; centra autem tri-ngulorum G, H, I, K, L; quæ con- ⁎5. 4.
ectantur rectis GH, HI, IK, KL, LG. rit GHIKL pentagonum dodecaedri inſcri-endi.

Nam rectæ FM, FN, FO, FP, FQ, er centra triangulorum tranſeuntes, *a* biſe- a cor. 3. 3.
ant baſes. *b* ergo rectæ MN, NO, OP, b 4. 1.
PQ, QM æquales ſunt inter ſe. quinetiam FM, FN, FO, FP, FQ *c* pares ſunt.
d ergo anguli MFN, NFO, OFP, c 4. 1. d 8. 1.
PFQ, QFM æquantur. pentagonum igi-ur GHIKL æquiangulum eſt; *e* proinde & e 4. 1.
æquilaterum, cum FG, FH, FI, FK, FL *f* pares f 12. 13.
int. Quod ſi eadem arte in reliquis undecim pyramidibus icoſaedri, centra triangulorum re-tis lineis connectantur, deſcribentur pentagona æqualia & ſimilia pentagono GHIKL. quam-obrem 12 hujuſmodi pentagona dodecaedrum

 con-

conſtituent; quod quidem in icoſaedro erit deſcriptum, cum viginti anguli dodecaedri in centris viginti baſium icoſaedri conſiſtant. Quapropter in dato icoſaedro dodecaedrum deſcripſimus. Q. E. F.

FINIS.

Innotationes in Elementa Euclidis *nuper edita, in quibus obscura illustrantur, errata emendantur, plurimaque quæ conducant ad Geometriæ rudimenta facilius percipienda adjiciuntur.*

*pag.*13.*lin.*5. scribe, Rursus ang. ACD e = e 5. 1.
DC; & ang. BCD e = BDC, ergo ang. ACD 19 ax.
= BDC, id est ang. ADC = BDC. Q. F. N.

*p.*17 *l.*ult. scribe, conjunganturque FC, IC, & roducatur ACG.

*p.*18. *l.*3. scribe, simili argumento ang. ICH = BH. ergo totus ACD, f (BCG) g major est utroque CAB, & ABC. Q. E. D.

*p.*21. apponantur figuræ quæ desunt.

*p.*40. *lin.*18. scribe, *Schol.*

Imo si fuerint duæ rectæ, secenturque ambæ in quotcunque partes, idem provenit ex ductu totius in totum, & partium in partes.

Nam sit Z = A + B + C, & Y = D + E; quia
DZ a = DA + DB + DC, & EZ a = EA + EB a 1. 2.
+ EC, & YZ a = DZ + EZ, b erit ZY = DA b 2. ax.
+ DB + DC + EA + EB + EC. Q. E. D.

Hinc patet ratio ducendi rectas compositas in compositas. Nam omnia partium rectangula accipere oportet, & habetur rectangulum ex totis.

Sin linearum in se ducendarum signis + admisceantur signa −, etiam signorum ratio habenda est. Quippe ex + in − provenit −; at ex − in − provenit +. Nam sit + A ducenda in B − C. & quoniam + A non affirmatur de toto B, sed de ejus parte tantum, qua superat C, debet AC manere negata. quare prodibit AB − AC. Vel sic; quia B constat partibus C, & B − C, * erit AB * 1. 2.
= AC + A in B − C; aufer utrinque AC, erit AB − AC = A in B − C. Similiter si − A ducenda sit in B − C, quoniam ex vi signi − non nega-

tur A de toto B; sed de ejus solummodo excessu supra C, debet AC manere affirmata. proveniet ergo − AB + AC. Vel sic; quia AB * = AC + A in B − C; tolle utrinque omnia, erit − AB = AC − A − in B + C; adde AC utrinque, eritq; − AB + AC = A in B − C.

Atque ex his rite perspectis, quæ subsequuntur 9. propositiones, aliæque ejusmodi innumeræ, ex linearum in se ductarum comparatione emergentes (quas apud Vietam, & alios Analystas in numerato habes) nullo negotio demonstrantur, rem plerumque quasi ad simplicem calculum exigendo.

° 19. ex. Porro, * liquet productum ex quapiam magnitudine in numeri cujuslibet partes æquari producto ex eadem in totum numerum. Ut 5 A + 7 A = 12 A. & 4 A in 5 A + 4 A in 7 A = 4 A in 12 A. quare quæ in hoc loco de rectarum in se ductu dicta sunt, eadem de numerorum in se multiplicatione intelligi possunt. proinde etiam quæ in 9 sequentibus theorematis de lineis affirmantur, eadem valent de numeris accepta; quippe cum istæ omnes ab hac prima immediate dependeant, & deducantur.

p. 42. inter demonstr. & Schol. propositionis quintæ, scribe.

Hoc theorema paulo aliter effertur, & facilius demonstratur, sic; Rectangulum ex summa & differentia duarum rectarum A, E, *æquatur differentiæ ex ipsis.*

° fch. 1. 2. Nam si A + E ducatur in A − E, * provenit Aq − AE + EA − Eq = Aq − Eq. Q. E. D.

p. 44. post demonstrationem prop. 9. scribe,

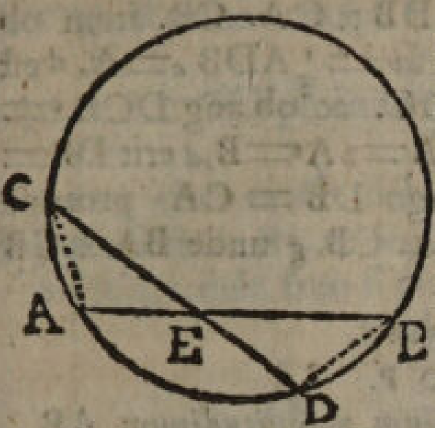
Aliter effertur & facilius demonstratur, sic;

Aggregatum quadratorum ex summa, & differentia duarum rectarum A, E, *æquatur duplo quadratorum ex ipsis.*

a 4. 2 Nam Q: A + E *a* = Aq + Eq + 2 AE. & Q: A

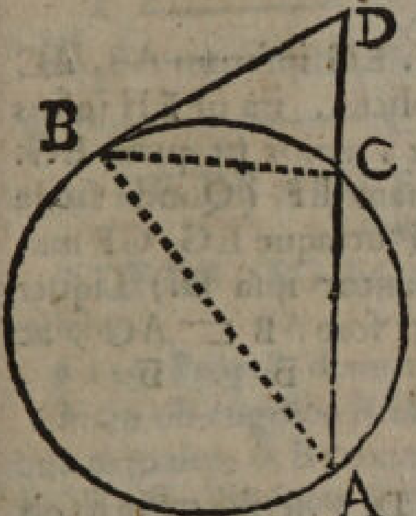
b sch. 7. 2. − E *b* = Aq + Eq − 2 AE. Hæc collecta faciunt 2 Aq + 2 Eq. Q. E. D.

p.67. post demonstrationem prop.28 scribe; Quod si subtensa AC ⊏ vel ⊐ DF, erit simili modo arcus AC ⊏, vel ⊐ DF.



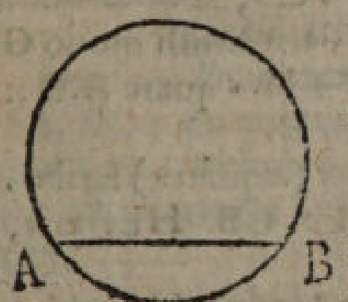
p.71. post demonstrationem prop. 35. scribe, Facilius sic, & universaliter; connecte AC & BD. atque ob angulos *a* CEA, DEB, *b* ipsosque C, B (super eodem arcu AD) pares; trigona CEA, BED, *c* æquiangula sunt. *d* ergo CE. EA :: EB. ED *e* proinde CE×ED=EA×EB. Q. E. D. a 15 1. b 21. 3. c cor. 32 1. d 4 6 e 16. 6.

Quæ ex 6. lib. citantur, tam hic quam in seq. ab hac minime pendent; quare iis uti licuit.



p. 71. Inter demonstr. & coroll. prop. 36. scribe, Facilius ac universalius sic;

Duc AB, & BC. ac ob angulos A, DBC *a* pares, & D communem, triangula BDC, ADB *b* æquiangula sunt. *c* ergo AD. DB :: DB. CD. *d* quare AD×DC=DBq. Q. E. D. a 32. 3. b 32. 1. c 4 6. d 17 6.



p.76. ad def. 7. 4. substitue figuram hanc.

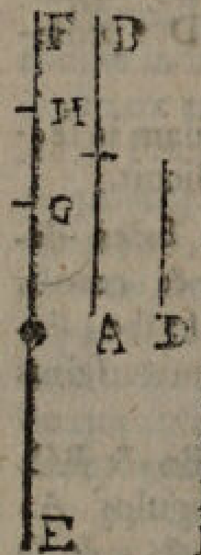
*pag.*82. post demonstrationem propos. 10. 4. scribe sic,

Hæc constructio Analytice indagatur sic; Factum sit; & angulum BDA bisecet recta DC. a ergo DA. DB :: CA. CB. item ob ang. CDA $b = \frac{1}{2}$ ADB $c =$ A, d est CA = DC. ac ob ang. DCB e = A + CDA = 2 A e = B, d erit DB = DC. f ergo DB = CA. proinde DA. (= BA.) CA :: CA. CB. g unde BA × CB = CAq.

a 3 6.
b constr.
c hyp.
d 6. 1.
e 32. 1.
f 1 ax.
g 17. 6.

p. 98. scribe Prop. 8. 5. sic.

PROP. 8.



Inæqualium magnitudinum AB, AC, *major* AB *ad eandem* D *majorem habet rationem, quam minor* AC: *& eadem* D *ad minorem* AC *majorem rationem habet, quam ad majorem* AB.

Sume EF, EG, ipsarum AB, AC æquemultiplices, ita ut EH ipsius D multiplex, major sit quam EG, at minor quam EF. (Quod facile continget, si utraque EG, GF majores accipiantur ipsa D.) Liquet juxta 8 def. 5. fore $\frac{AB}{D} \sqsubset \frac{AC}{D}$; ac $\frac{D}{AB} \sqsupset \frac{D}{AC}$. Quæ E. D.

b hyp.
c 6 def. 5

p. 100 *lin.* ult. post B, D, F. scribe, Porro ob A. B b :: C. D b :: E. F, c si G ⊏, =, ⊐ K, erit similiter H ⊏, =, ⊐ L; & I ⊏, =, ⊐ M. ac proinde si G ⊏, =, ⊐ K, erit simili modo G + H + I ⊏, =, ⊐ K + L + M. c quare A. B :: A + C + E. B + D + F. Q. E. D.

pag. 102. circa 23 *lin.* post (æquatur) scribe, Ergo, quum AG. DH :: C. F :: GB. HE. erit, &c. ut sequitur ibi.

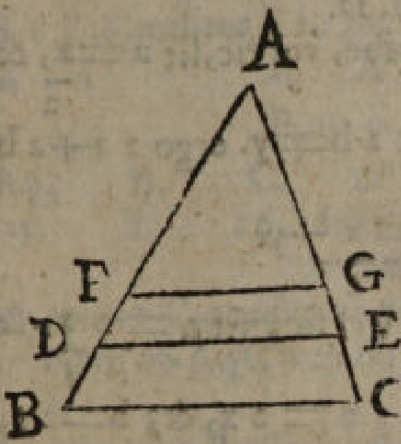
p. 104. *lin.* 1. post KO scribe, Itaque ablatis hinc inde communibus HL, KM, &c. ut ibi sequitur.

p.111. *l*.12. dele, Hujusce demonstratio, &c. & scribe, Intellige $\frac{G}{C} = DE$. [a] ergo $B \sqsubset G$. [b] ergo [a] 10. 5. [b] 8. 5. [c] *sch*.13 5.
$\frac{A}{G} \sqsubset \frac{A}{B}$. Rursus concipe $\frac{H}{G} = \frac{E}{F}$. [c] ergo $\frac{H}{G} \sqsupset \frac{A}{G}$.
[a] quare $A \sqsubset H$. [b] proinde $\frac{A}{C} \sqsubset \frac{H}{C}$ [d] vel $\frac{D}{F}$. Q.E.D. [d] 13 5.

p.114. circa 25.*lin*. dele, cum igitur, & scribe, Verum si HC, &c. ut sequitur.

p.116.*l*.2. dele Imo si plures, &c. & scribe sic.

Schol.



Imo si plures DE, FG, *ad unum latus* BC *parallelæ fuerint, erunt omnia laterum segmenta proportionalia.*

Nam DF.FA [a]:: EG. GA; & componendo, invertendoque FA.DA :: GA. EA; [a] ac DA. DB :: EA. EC. ergo ex æquo DF. DB :: EG. EC. Q. E. D. [a] 2. 6.

Coroll.

Si DF.DB :: EG.EC; [a] erunt BC, DE, FG parallelæ.

p.119. Prop.8. demonstretur sic.

Nam ob angulos BAC, ADB [a] rectos, [b] ideoque æquales, & B communem, trigona BAC, ADB [c] similia sunt. Simili discursu, similia sunt triangula BAC, ADC. [d] proinde ADB, ADC similia erunt. Q. E. D. [a] *hyp*. [b] 12.*ax*. [c] 32 & 4 6. [d] vid. 21.6.

Coroll. &c. ut sequitur.

pag.121. *lin.antepen*. scribe, Vel sic; Datæ sint AB, BC; ex quibus fac angulum rectum ABC. duc AC, & huic normalem CD, cui occurrat AB protracta in D. [a] estque AB. BC :: BC. BD. [a] *cor*.8. 6.

pag. 122. dele figuram istam furciferam.

ibid.

ibid. lin. 6. dele, vel ita; CD = CB. & quæ seq. cum sua figura.

pag. 123. post *lin.* 3. scribe, Vel (in eadem figura) sint AB, BF duæ datæ, *b* liquet esse A B. BF :: BF. BE.

p. 136. Propos. 31. [a] demonstretur sic.

a cor. 8. 6. b cor. 20. 6. c 24 5. d sch. 24 5. Ab angulo recto BAC demitte perpendicularem AD. Quoniam DC. CA :: [a] CA. CB, [b] erit AL. BF :: DC. CB. Item ob DB. BA :: [a] BA. BC, [b] erit BG. BF :: DB. BC. [c] ergo AL + BG. BF :: DC + DB (BC.) BC. ergo AL + BG = BF. Q. E. D.

pag. 146. *lin.* penult. scribe, vel sic, sit $a = \frac{x}{2}$, & $b = \frac{y}{2}$. quare $2a = x$, & $2b = y$. ergo $2a + 2b = x + y$. ergo $a + b = \frac{x+y}{2}$.

p. 147. *lin.* 17. scribe, Vel sic, sit $a = \frac{2x}{3}$, & $b = \frac{2y}{3}$, & $x + y = g$. ob $3a = 2x$, & $3b = 2y$, est $3a + 3b = 2x + 2y = 2g$. ergo $a + b = \frac{2}{3}g = \frac{2}{3} : x + y$.

p. 149. *l.* 9. scribe; Vel sic; sit $a = \frac{b}{3}$, & $c = \frac{d}{3}$, vel $3a = b$, & $3c = d$, estque $\frac{c}{a}$ * $= \frac{3c}{3a} = \frac{d}{b}$. * 5. 15.

ibid. lin. 27. dele, Applicare potes, &c. & scribe, Vel sic; sit $a = \frac{2b}{3}$, & $c = \frac{2d}{3}$, vel $3a = 2b$, & $3c = 2d$. Est $\frac{c}{a} = \frac{3c}{3a} = \frac{2d}{2b} = \frac{d}{b}$.

LEMMA.

AE,	BF,	CG,	DH,
A,	B,	C,	D,
E,	F,	G,	H.

Si proportionales numeri A, B, C, D *proportionales* * *numeros* AE, BF, CG, DH

DH metiantur per numeros E, F, G, H, *erunt ei* [E, F, G, H] *proportionales*.

Nam ob AEDH ^{a}= BFCG, ^{a} & AD = BC, ^{b} erit $\frac{AEDH}{AD} = \frac{BFCG}{BC}$, ^{c} hoc est EH = FG. a 19. 7. b 9. ax. 7. c 9 ax. 7.

^{a} ergo E . F :: G . H. Q. E. D.

Coroll.

Hinc $\frac{Bq}{Aq} = \frac{B}{A}$ in $\frac{B}{A}$. ^{d} Nam 1 . B :: B . Bq. ^{d} & d 15. def. 7.

1 . A :: A . Aq. ^{e} ergo 1 . $\frac{B}{A}$ :: $\frac{B}{A}$. $\frac{Bq}{Aq}$. ^{d} ergo $\frac{Bq}{Aq}$ = e lem. prae.

$\frac{B}{A} \times \frac{B}{A}$. Similiter $\frac{B}{Ac}$ in $\frac{Bq}{Ac} = \frac{BC}{Acc}$. & sic de reliquis.

PROP. 22.

Aq,	B,	C.	
4,	8,	16.	

Si tres numeri, Aq, B, C *deinceps sint proportionales, primus autem* Aq *sit quadratus; & tertius* C *quadratus erit*.

Nam ob AqC ^{a}= Bq, ^{b} erit C = $\frac{Bq}{Aq}$ ^{c}= Q. $\frac{B}{A}$. a 20. 7. b 7. ax. 7. c cor. lem. prae.

Liquet vero $\frac{B}{A}$ esse numerum, ^{d} ob $\frac{Bq}{Aq}$, vel C numerum. ergo si tres, &c. d hyp. & 14. 8.

PROP. 23.

Ac,	B,	C,	D.
8,	12,	18,	27.

Si quatuor numeri Ac, B, C, D *deinceps sint proportionales, primus autem* Ac *sit cubus; & quartus* D *cubus erit*.

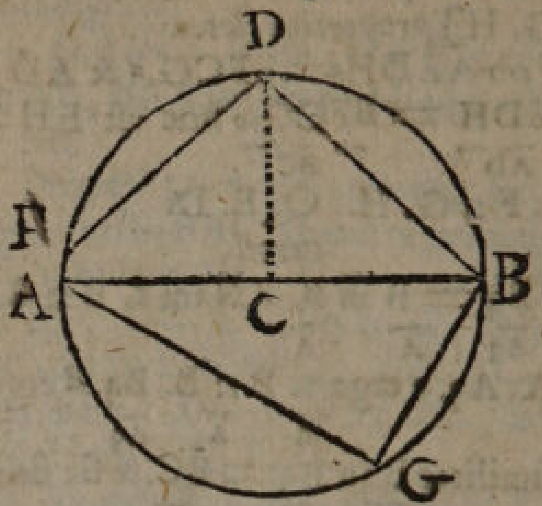
Nam quia AcD ^{a}= BC, ^{b} erit D = $\frac{BC}{Ac}$ a 19. 7. b 7. ax. 7.

^{c}= $\frac{B}{Ac}$ x C; hoc est (ob Ac C = ^{d} Bq, & ^{b} proinde C = $\frac{Bq}{Ac}$) D = $\frac{B}{Ac} \times \frac{Bq}{Ac}$ ^{e}= $\frac{BC}{Acc}$ ^{e}= C: $\frac{B}{Ac}$. c cor. lem. prae. d 20. 7.

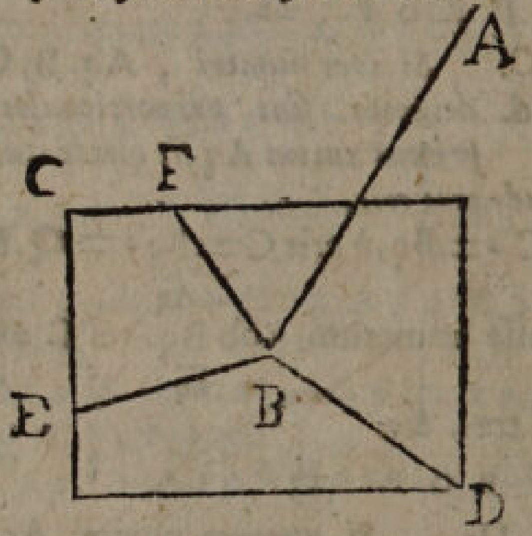
^{e} liquet vero ipsum $\frac{B}{Ac}$ esse numerum, quia $\frac{BC}{Acc}$, vel D numerus ponitur; ergo si quatuor numeri, &c. e 15. 8

p. 192.

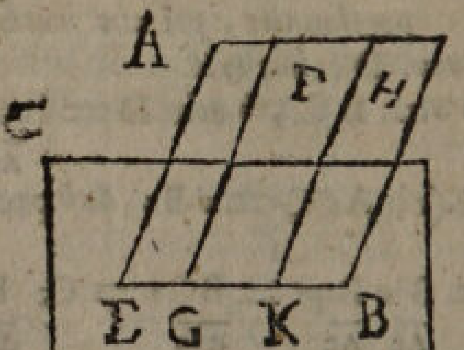
p.192. substitue hanc figuram.



p. 263. ad def. 3. scribe sic.

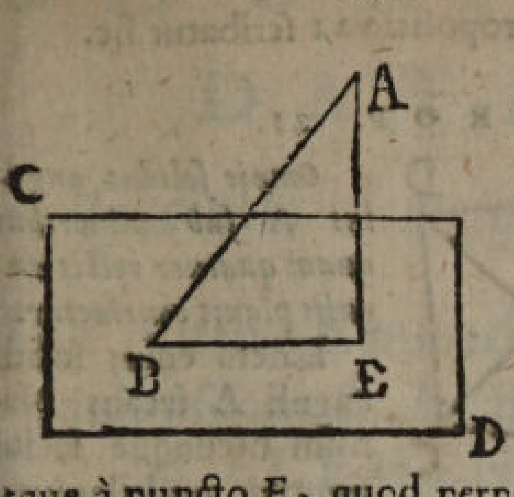


3. Linea recta AB est ad planum C D recta, cum ad rectas omnes lineas BD, BE, B F, à quibus illa tangitur, quæque in proposito sunt plano, rectos efficit angulos ABD, ABE, ABF.

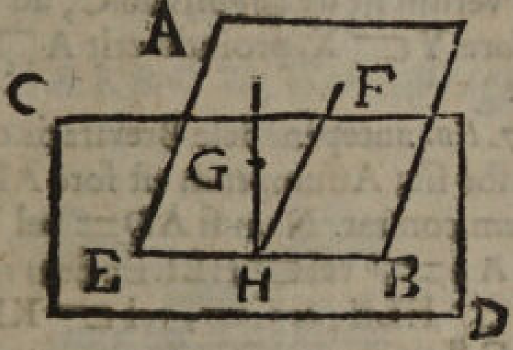


4. Planum AB ad planum C D rectum est, cum rectæ lineæ FG, HK, quæ communi planorum sectioni E B ad rectos angulos in uno plano A B ducuntur, alteri plano C D ad rectos sunt angulos.

5. Rectæ



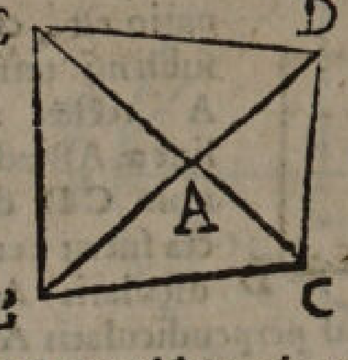
5. Rectæ lineæ AB ad planum CD inclinatio est, cum à sublimi termino A rectæ alius lineæ AB ad planum CD deducta fuerit perpendicularis AE; atque à puncto E, quod perpendicularis AE in ipso plano CD fecerit, ad propositæ illius lineæ extremum B, quod in eodem est plano, altera recta linea EB fuerit adjuncta: est, inquam, angulus acutus ABE insistente linea AB, & adjuncta EB comprehensus.



6. Plani AB ad planum CD inclinatio, est angulus acutus FHG rectis lineis FH, GH contentus, quæ in utroque planorum AB, CD ad idem communis sectionis BE punctum H ductæ, rectos cum sectione BE efficiunt angulos FHB, GHB.

pag. 275. Propositio 21 scribatur sic.

P R O P. 21.



Omnis solidus angulus A sub minoribus quam quatuor rectis angulis planis continetur.

Latera enim solidi anguli A secans planum utcunque faciat figuram multilateram BCDE, & totidem triangula ABC, ACD, ADE, AEB. Omnes angulos polygoni voco X; & summam angulorum ad trigonorum bases voco Y; quare X + 4 Rect. *a* = Y + A. Quia vero (ex angulis ad B) *b* est ang. ABE + ABC ⊏ CBE;
a 32. 1. & sch 32. 1. *b* 20. 11. *c* 5. ax. 1.
idemque verum sit de angulis ad C, ad D, ad E. *c* liquet fore Y ⊏ X. proinde erit A ⊐ 4 *Rect.* Q. E. D.

p. 277. *lin.* antepen. dele Brevitatis causa ass. &c. & scribe sic; Assumptum est fore AD ⊏ HL. Hoc autem constat. Nam si AD = vel ⊐ HL, erit ang. A *a* =, *b* vel ⊏ HLI. Eodem modo erit B =, vel ⊏ HLK, & C =, vel ⊏ KLI. quare A + B + C * quatuor rectos aut exæquabunt, aut excedent, contra hypoth. quin potius sit AD ⊏ HL. Q. E. D.
a constr. & 8. 1. *b* 21. 1. * 4 cor. 13. 1.

FINIS.

EUCLIDIS DATA

ſuccincte demonſtrata;

Una cum Emendationibus quibuſdam & Additionibus ad ELEMENTA.

EUCLIDIS nuper edita.

Opera

M^ri. Is. BARROW, *Cantabrigienſis*, Coll, *Trin.* Soc.

LONDINI

Excudebat *R. Daniel*, 1659.

Ornatissimo viro D. IACOBO STOCK, amico suo & patrono singulari.

NEc publica, nec tui nominis luce dignum censeo hunc paucorum dierum partum pusillum & præmaturum. Qui quidem quod se mundo, quodque Tibi, spectandum obtulerit, duplici nomine arrogantiæ speciem incurrit. Sed utrinque parata est excusatio qualiscunque. Nam amico obtemperatum oportuit jubenti mitterem hunc libellum Euclidæis (quæ cognatione proxima attingit) Elementis subjungendum. In eum quicquid est in publicum aut peccati aut meriti protinus rejicio, facti cujus author fuit, rationem redditurum. In Te autem delictum quod maxime aggravat, idem potenter extenuat, Tibi tantum debere. Nam cum iis, qui Diis ipsis sacrificia, ac modica magnis Regibus donaria offerre non dubitarunt, satius esse credo, etiam pro immensis beneficiis parum, quam nihil rependere. Sufficiat igitur regesisse, me Tibi multis magnisque nominibus obstrictum fore; vices, quas potuero maximas, referre debere; ultra vota & grates nihil posse; illa privatim, has publice persolutas præcellere; quibus agendis, quam jamdiu spe & studio aucuper, occasionem nondum comparere; præstare hanc

oblatam præbendere, quamvis exilem, quam elapsam
nequicquam pœnitentia prosequi. Esto igitur hæc
oblatio pignus quoddam & præludium futuræ am
plioris, in qua meritorum in me Tuorum historia u
berior ac distinctior commemoranda occurret. Qu
simpliciter agnoscere, non aut fuse describere, au
digne prædicare, præsentis est instituti. Ac rever
jam brevis sum ἑκὼν ἀέκοντί γε θυμῷ, necessitate po
tius coactus, quam inductus consilio. Nam me vel
ventis turgentia alio avocant; ac vereor ne hæc p
ne currenti calamo exequentem, quæ hæc ad te perf
ret, amica manus, importuna patientia præstoletu
Quid superest igitur, nisi ut te domi studiis ac rebu
honestis animum intendentem salutari præsentia tu
tetur, eum exorem venerandi ac ἀῤῥήτου nominis
quem tantæ beneficentiæ benignum remuneratores
jugibus votis exopto; idemque me extemplo sup
Tyrrhenos, Ionios, Ægeosque fluctus longinqua
profectionem suscepturum comitetur. Obtestor auten
ne tenuis opellæ patrocinium respuas, quod ultro im
pertire dignatus es

Tibi devinctissimo

& obsequentissimo,

I. B.

EVCLIDIS Data.

Definitiones.

DAta magnitudine dicuntur ſpatia, lineæ, anguli, quibus æqualia poſſumus invenire.

II. Ratio dari dicitur, cui poſſumus eandem invenire.

III. Rectilineæ figuræ ſpecie dari dicuntur, [e]arum & ſinguli anguli dati ſunt, & laterum ra[ti]ones ad invicem datæ ſunt.

Hinc, datæ ſunt ſpecie figuræ, quibus ſimiles [in]veniri poſſunt.

IV. Poſitione dari dicuntur puncta, lineæ, [an]gulique, quæ eundem ſitum ſemper obtinent.

V. Circulus magnitudine dari dicitur, cujus [linea] quæ ex centro datur magnitudine.

VI. Poſitione & magnitudine dari dicitur [ci]rculus, cujus datur centrum poſitione, & ea [qu]æ ex centro magnitudine.

VII. Circuli ſegmenta magnitudine dari [d]icuntur, in quibus dati ſunt magnitudine angu[li] & ſegmentorum baſes.

VIII. Poſitione & magnitudine dari dicun[tu]r circuli ſegmenta, in quibus anguli magnitu[d]ine dati ſunt, & ſegmentorum baſes poſitione [&] magnitudine.

IX. Magnitudo magnitudine major eſt data, [q]uando ablata data, reliqua eidem æqualis eſt.

X. Magnitudo magnitudine minor eſt data, [q]uando adjuncta data, tota eidem æqualis eſt.

Ut ſi A data ſit, erit A + B ⊏ B data. At B ⊐ A + B data.

XI. Magnitudo magnitudine major eſt data [ali]quam in ratione, quando ablata data, reliqua ad eandem habet rationem datam.

XII. Magnitudo magnitudine minor est data quam in ratione, quando adjuncta data tota ad eandem rationem habet datam.

Ut si A data sit, & $\frac{B}{C}$ detur, erit A+B ⊏ C, data q. in r. sin $\frac{A+B}{C}$ detur, erit B ⊐ C data q. in r.

PROP. 1.

A. B. *Datarum magnitudinum A, B,*
a. b. *ad invicem datur ratio.*

* hyp.
a 1.def.
b sch.7.5.
c 1.def.

Nam quia A * datur, [a] inveniri potest aliqua a = A. Eodem jure sume b = B. [b] estque a. b :: A. B. [c] quare ratio $\frac{A}{B}$ data est. Q. E. D.

PROP. 2.

A. B. *Si data magnitudo A ad aliam*
a. b. *aliquam B habeat rationem datam datur etiam hæc alia magnitudine.*

* hyp.
a 1.def.d.
b 2.def.d.
c 9.5.

Nam ob A * datam, [a] sume a = A; ac ob $\frac{A}{B}$ * datam, [b] sit $\frac{a}{b} = \frac{A}{B}$. [c] ergo b = B. [a] quare B datur. Q.E.D.

PROP. 3.

A. B. *Si quotlibet datæ magnitudine*
a. b. *A, B componantur, etiam ea A+B quæ ex his componitur, data erit.*

a 1.def.
b 2.ax.1.

Nam [a] cape a = A, & b = B; [b] estque a + b = A+B. [a] quare A+B datur. Q. E. D

PROP. 4.

A. B. *Si à data magnitudine A aufera-*
a. b. *tur data magnitudo B, etiam reliqua A − B dabitur.*

a 1.def.d.
b 3.ax.1.

[a] Sint enim a = A, & b = B. ergo A − B = a − b. [a] proinde A − B datur. Q. E. D.

PROP.

PROP. 5.

A. B. *Si magnitudo* A *ad sui-ipsius ali-*
C. D. *quam partem* B *habeat rationem datam, etiam ad reliquam* A−B *habebit rationem datam.*

Nam, quia $\frac{A}{B}$ a data est, b sit A. B :: C. D. — a hyp. b 2.def d.
ergo A. A − B :: C. C − D. b proinde $\frac{A}{A-B}$ datur. Q. E. D. — c cor.9.5.

PROP. 6.

A. B. *Si componantur duæ magnitudi-*
C. D. *nes* A, B, *habentes ad invicem rationem datam, etiam quæ ex his componitur magnitudo* A+B, *habebit ad utramque* A *& B rationem datam.*

Nam a sit A. B :: C. D. b ergo A + B. B :: C + D. D. c quare $\frac{A+B}{B}$ datur. Similiter $\frac{B+A}{A}$ datur. Q. E. D. — a 2.def.d. b 18 5. c 2.def.d.

PROP. 7.

A. B. *Si data magnitudo* A + B *data ratione secetur, utrumque segmentorum* A, *&* B *datum est.*

Nam ob $\frac{A}{B}$ * datam, a erit $\frac{A+B}{A}$ data. b ergo A datur. Eodem modo B datur. Q. E. D. — * hyp. a 6 dat. b 2.dat.

PROP. 8.

A. C. B. *Quæ* A, B *ad idem* C *rationem*
D. E. F. *habent datam, habebunt ad invicem rationem datam.*

Nam a sit A. C :: D. E. a & C. B :: E. F. quare ex æquali A. B :: D. F. a ergo $\frac{A}{B}$ datur. Q. E. D. — a 1.def.d.

Coroll.

Rationes ex datis rationibus compositæ, datæ sunt. Ut $\frac{A}{B}$ fit ex $\frac{A}{C}$, & $\frac{C}{B}$ datis.

PROP. 9.

A. B. C. *Si duæ, pluresve magnitudines*
D. E. F. *A, B, C ad invicem habeant rationem datam, habeant autem illæ magnitudines A, B, C ad alias quasdam D, E, F rationes datas, etsi non easdem; illæ aliæ magnitudines D, E, F etiam ad invicem habent rationes datas.*

a 20.def 5. b hyp. c cor.8.dat.

Nam ratio $\frac{D}{E}$ a fit ex b datis $\frac{D}{A}$, $\frac{A}{B}$, $\frac{B}{E}$; c ergo $\frac{D}{E}$ datur. Eadem de causa datur $\frac{E}{F}$. Q.E.D.

PROP. 10.

A. B. C. *Si magnitudo magnitudine major fuerit data, quam in ratione; & simul utraque illa eadem major erit data quam in ratione. Sin autem simul utraq; magnitudo eadem magnitudine major fuerit data, quam in ratione; & reliqua illa eadem major erit data quam in ratione; aut reliqua data est cum consequente, ad quam habet altera magnitudo rationem datam.*

a 6.dat. b 11.def.d.

1. Sint A, & $\frac{B}{C}$ datæ. a erit $\frac{B+C}{C}$ data. b ergo A + B + C ⊏ C data q. in r. Q.E.D.

c 17.5.

2. Sint A, & $\frac{B+C}{C}$ datæ: c ergo $\frac{B}{C}$ datur. proinde A + B ⊏ C data q. in r. Q.E.D.

d 5.d.

3. Sint A + B, & $\frac{C}{B+C}$ datæ. d Liquet $\frac{B}{B+C}$ dari. Q.E.D.

PROP. 11.

A. B. C. *Si magnitudo magnitudine major sit data quam in ratione, eadem simul utraque major erit data quam in ratione. Et si eadem simul utraque major sit data quam in ratione, eadem reliqua magnitudine major erit data quam in ratione.*

1. A, & B dantur. [a] ergo $\frac{B}{C}$ datur. proinde $\frac{B}{B+C}$ [b] A + B ⊏ B + C data q. in r. Q. E. D.

2. A, & B dantur. [c] ergo $\frac{B}{C}$ datur, proinde $\frac{B+C}{C}$ [b] A + B ⊏ C data q. in r. Q. E. D.

a 6. dat.
b 11. def. d.
c 5. dat.

PROP. 12.

A. B. C. *Si fuerint tres magnitudines A, B, C, & prima cum secunda (A + B) data sit, secunda quoque cum tertia (B + C) data sit; aut prima A tertiæ C æqualis est, aut altera altera major data.*

Nam si A + B, & B + C pares sint, [b] liquet A & C æquari; sin istæ impares fuerint, [b] liquet excessum A — C, vel C — A dari. Q. E. D.

a 4 ax 1.
b 4 dat.

PROP. 13.

D, A + B, C. E. *Si fuerint tres magnitudines D, A + B, C, & earum prima D ad secundam A + B habeat rationem datam; secunda autem A + B tertia C major sit data quam in ratione; prima quoque D major erit tertia C data quam in ratione.*

Sint A, & B, ac D datæ; [a] sitque $\frac{A}{C}$ $\frac{+ B}{A + B}$. D :: A. E [b] :: B. D — E. ergo [c] E, [d] & $\frac{B}{D - E}$ & (ob B datam) [e] $\frac{C}{C}$ dantur. [f] quare D (E + : $\frac{}{D - E}$ D — E) ⊏ C data q. in r. Q. E. D.

a 2. def. d.
b 19. 5.
c 2. dat.
d 2. def. d.
e 8. dat.
f 11. def. d.

PROP. 14.

A. C. B. D. E. *Si duæ magnitudines A & C ad invicem habeant rationem datam, utrique autem illarum adjiciatur data magnitudo B & D; totæ A + B, C + D, aut habent rationem datam, aut altera A + B altera C + D major erit data quam in ratione.*

Nam

a 12. 5. b hyp. c 2. def. d. Nam si A. C :: B. D[a] :: A + B. C + D ob $\frac{A}{C}$ [b] datam, [c] liquet $\frac{A+B}{C+D}$ dari.

d 2. def. d. e 2. dat. f 4. dat. g 11. def. d. Saltem [d] sit A. C :: E. D. [a] :: A + E. C + D. Ergo [c] $\frac{A+E}{C+D}$, ac [e] E, [f] ideoque B — E dantur. [g] proinde A + B (A + E : + B — E) ⊏ C + D data q. in r. Q. E. D.

PROP. 15.

A. C.
B. D.
E.

Si duæ magnitudines A & C habeant ad invicem rationem datam, & ab utraque harum aufetur data magnitudo B & D; reliquæ magnitudines A — B, C — D ad invicem habebunt aut rationem datam, aut altera A — B, alra C — D major erit data quam in ratione.

a 10. 5. b hyp. c 2 def. d. [b] Nam si A. C :: B. D[a] :: A — B. C — D. ob $\frac{A}{C}$ datam, [c] liquet $\frac{A-B}{C}$ dari.

d 2 def. 2. e 2. dat. f 4 dat. g 11. def. d. Saltem [d] sit A C :: E. D[a] :: A — E. C — D. Ergo [e] $\frac{A-E}{C-D}$, & [e] E, ac [f] ideo E — B dantur. [g] proinde A — B (A — E : + E — B) ⊏ C — D data q. in r. Q. E. D.

PROP. 16.

B. C.
A. D.
E.

Si duæ magnitudines B, C habeant rationem datam, & ab una quidem illarum C auferatur data magnitudo D, alteri autem B adjiciatur data magnitudo A; tota A + B residua C — D major erit data quam in ratione.

a 1. def. d. b 19. 5. c 2. def. d. d 2. dat. e 3. dat. f 11. def. d. Sit enim C. B[a] :: D. E[b] :: C — D. B — E. ergo [c] $\frac{C-D}{B-E}$, & [d] E, ac [e] ideo E + A dantur. [f] proinde B + A (E + A : + B — E) ⊏ C — D data q. in r. Q. E. D.

PROP.

PROP. 17.

A+B. D+E. C.

Si fuerint tres magnitudines A+B, C, D+E; & prima quidem A+B secunda C major sit data quam in ratione, tertia quoque D+E eadem secunda C major sit data quam in ratione; prima A+B ad tertiam D+E aut rationem habebit datam, aut altera altera major erit data quam in ratione.

Nam ob A, D, & B E [a] datas, [b] erit B data. [a] *hyp.* [b] 8. *dat.*
C, C E

ergo per 14. hujus.

PROP. 18.

A+C. E. G.
B+D. F. H.

Si fuerint tres magnitudines, atque ex his una utraque reliquarum major sit data quam in ratione; reliquæ duæ aut datam rationem habebunt ad invicem, aut altera altera major erit data quam in ratione.

Datæ sint A, B, C D ac sit A+C=B+D.
E, F;

Sitque C.E [a] :: A.G [b] :: C+A. E+G. itemque [a] 1. *def. d.* [b] 12. 5
D. F [a] :: B. H [b] :: D+B. F+H. [c] ergo [c] 1. *def. d.*
C+A [d] hoc est B+D, [c] & B+D, ac [e] idcirco [d] 7. 5. [e] 8. 5.
E+G, E+G, F+H [f] 2. *dat.*
E+G quin & G ac H [f] dantur. ergo per 15.
F+H; (hujus.

PROP. 19.

A+B. E.
C+D. F.

Si fuerint tres magnitudines, & prima quidem magnitudo secunda magnitudine major sit data quam in ratione, sit quoque secunda major tertia data quam in ratione; prima magnitudo tertia magnitudine major erit data quam in ratione.

Sint A, C, & C+D, D datæ; dico A+B
B E

E data q. in r.

Nam

a 2. def. d. Nam sit C+D.B a :: C.F b : : D. B − F. er-
b 19. 5. go c C & d F, ac e ideo F + A, & c D f ideoque
c 2. def. d. F B − F,
d 2. dat.
e 3. dat. E dantur. g proinde A+B (F+A :+B − F)
f 8. dat.
g 11. def. d. B − F

⊏ E data q. in r. Q. E. D.

P R O P. 20.

A. C. E. *Si datæ fuerint duæ magnitu-*
B. D. *dines A, C; & auferantur ab ipsis magnitudines B, D habentes ad invicem rationem datam; residuæ magnitudines A − B, C − D aut habebunt ad invicem rationem datam, aut altera A − B altera C − D major erit data quam in ratione.*

a 19. 5. Nam si A. C :: B. D a :: A − B. C − D; b li-
b 2. def. d. quet A − B dari.
C − D

Saltem sit D. B b :: C. E a :: C − D. E − B.
c 2. dat. ergo b C & c E, ac d propterea A − E, b itemque
d 4. dat. E
e 11. def. d. C − D datæ sunt. e ergo A − B (A − E : + E
E − B
− B) ⊏ C − D data q. in r. Q. E. D.

P R O P. 21.

A. C. E. *Si datæ fuerint duæ magnitu-*
B. D. *nes A, C; & adjiciantur ipsis aliæ magnitudines B, D habentes ad invicem rationem datam; totæ A + B, C + D aut habebunt ad invicem rationem datam, aut altera A + B altera C + D major erit data quam in ratione.*

a 12. 5. Nam si B. D :: A. C a : : A + B. C + D, b li-
b 2. def. d. quet A + B dari.
C − D

Saltem sit B. D b :: E. C a :: B + E. D + C.
c 2. dat. ergo c E, d ideoque A − E, & b B + E dantur.
d 4. dat. D + C

e ergo

e ergo A+B (B+E :: +A−E) ⊏ C+D data q. in r. Q.E.D. e 11. def.

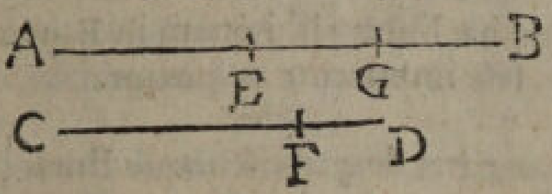
P R O P. 22.

A. B. C. *Si duæ magnitudines* A, B *ad aliam aliquam magnitudinem* C *habeant rationem datam, & simul utraque* A + B *ad eandem* C *habebit rationem datam.*

Nam ob $\frac{A}{C},\frac{B}{C}$ a datas, b erit $\frac{A}{B}$ data. c quare $\frac{A+B}{B}$, b ideoque $\frac{A+B}{C}$, data est. Q.E.D.

a hyp.
b 8 d.
c 6 d.

P R O P. 23.

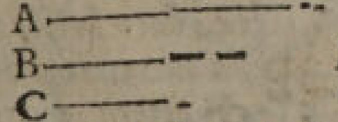


Si totum AB *ad totum* CD *habeat rationem datam, habeant autem & partes* AE, EB *ad partes* CF, FD *rationes datas (etsi non easdem;) habebunt omnia ad omnia rationes datas.*

Nam sit AE. CF a :: AG. CD b :: GE. FD. c ergo $\frac{GE}{FD}$ datur. quare (ob $\frac{EB}{FD}$ c datam) d erit $\frac{GE}{EB}$, ac e ideo $\frac{EB}{GB}$ data. ergo quum c $\frac{AB}{CD}$, & a $\frac{AG}{CD}$, d ideoque $\frac{AB}{AG}$, ac proinde e $\frac{AB}{GB}$, dentur, d erit $\frac{EB}{AB}$, data. Quare e $\frac{AB}{AE}$, & d $\frac{AE}{EB}$, & e $\frac{EB}{CF}$, dantur. Q.E.D.

a def. d.
b 19. 5.
c hyp.
d 8. dat.
e 5. dat.

P R O P. 24.

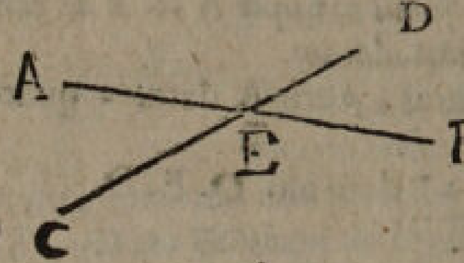


Si tres rectæ lineæ, A, B, C, *proportionales fuerint; prima autem* A *ad tertiam* C *habeat rationem datam; & ad secundam* B *habebit rationem datam.*

Nam

a cor.20.6. b 2.def.d. c 1.d. Nam A. C a :: Aq. Bq. b ergo $\frac{Aq}{Bq}$ data est. proinde $\frac{A}{B}$ c datur. Q. E. D.

PROP. 25.



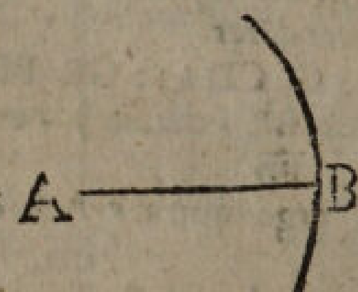
Si duæ rectæ lineæ, AB, CD positione datæ se mutuo secuerint, punctum E, in quo se invicem secant, positione datum est.

a 4.def.d. a Nam hæ lineæ alibi quam in E, neutrius situ mutuo, sese intersecare nequeunt.

Schol.

a Idem patet de quibuscunque lineis positione datis, seque in unico puncto intersecantibus: ut de circuli arcu, & recta, &c.

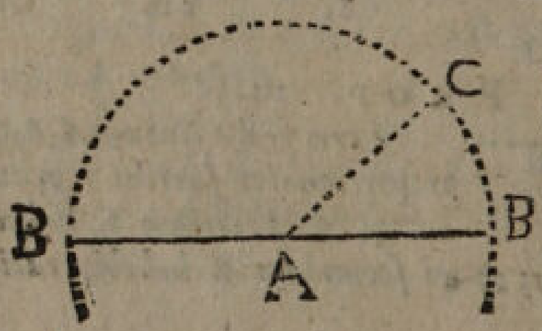
PROP. 26.



Si rectæ lineæ AB extremitates A, B, positione datæ sint, recta AB positione & magnitudine data est.

a 14 ax. Positione quidem, a quia inter eosdem terminos unica recta duci potest: & b 1.def.d. magnitudine, b quia si centro A per B ducatur circulus, hujus omnes radii ipsi AB æquantur.

PROP. 27.



Si rectæ lineæ AB positione & magnitudine datæ, data fuerit una extremitas A; & altera extremitas B data erit.

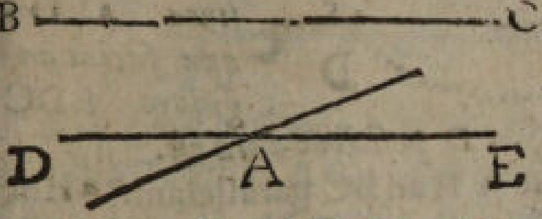
Nam

Nam si centro A, spatio AC a = AB b ducatur circulus, cui data recta c occurrat in B, d erit extremitas B data. a 1. def. d. b 3 post. c 2. post. d cor, 25.

Schol.

Vides partes puncti B determinandas esse.

PROP. 28.

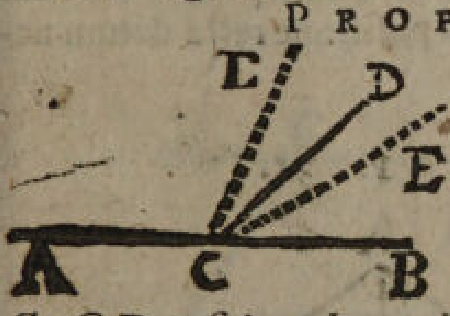


Si per datum punctum A contra datam positione rectam BC agatur recta linea DE, acta recta DE positione data est,

Nam a dic alteram per A ad BC fore parallelam. Hæc idcirco ad DE b parallela erit. c Quod repugnat. a 4. def. d. b 30. 1. c 34. def. 1.

Nota, Vocabulum *contra* in hoc libro parallelismum significare.

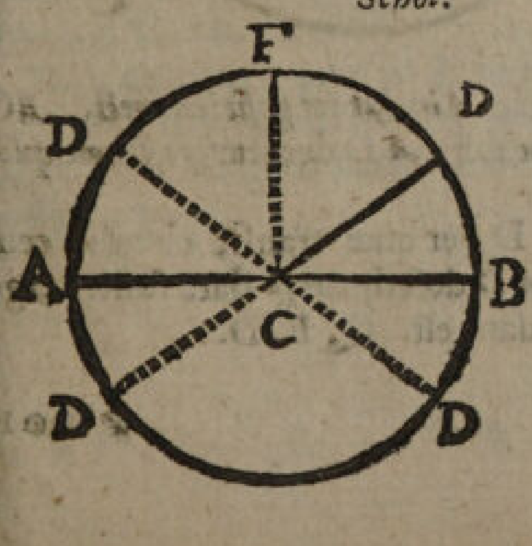
PROP. 29.



Si ad positione datam rectam AB, datumque in ea punctum C, agatur recta linea CD, quæ faciat angulum DCB datum; acta recta CD positione data erit.

a Nam quævis alia CE angulum b efficiet majorem, vel minorum dato BCD. a 4. def. d. b 9. ax. 1.

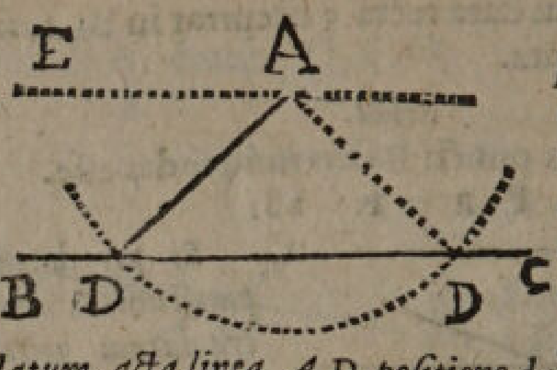
Schol.



Determinari debet situs anguli dati tam respectuperpendicularis CF, quam ipsiusAB, ut cernis in apposita figura.

PROP.

PROP. 30.



Si à dato puncto A in datam positione rectam BC agatur recta linea AD, quæ faciat angulum ADC datum, acta linea AD positione data est.

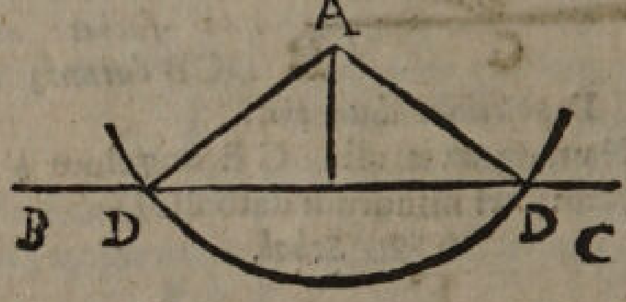
a 28. dat.
b 1. def. d.
c 29 dat.

Nam per A duc AE ad BC parallelam. a Hęc positione datur. Item ang. DAE par dato alterno ADC b datus est. c ergo recta AD positione data est. Q. E. D.

Schol.

Hinc praxim discimus à dato puncto ducendi rectam, quæ cum data positione recta datum angulum effici[t].

PROP. 31.



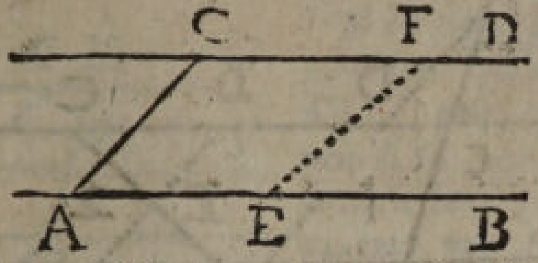
Si à dato puncto A in datam positione rectam BC data magnitudine recta AD ducatur, positione quoque data erit.

a 1. def. d.
b sch. 25. d.
c 26. d.

Nam puncta D, per quæ transit circulus centro A, a spatio AD descriptus, b data sunt. c ergo AD positione data est. Q. E. D.

PROP

PROP. 32.

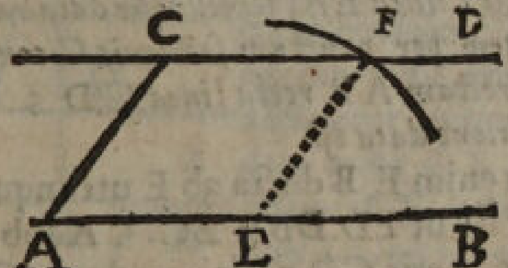


Si in datas positione parallelas rectas AB , CD *agatur recta linea* AC , *quæ faciat angulos datos* BAC , ACD , *acta recta* AC *magnitudine data est.*

Nam ad E (quodvis punctum in AB) fac ang. BEF = *a* BAC. liquet rectas EF, AC *b* parallelas , & *c* pares fore. *d* quare AC data est. Q. E. D.

a 1. def. d.
b 29. 1.
c 34. 1
d 2. def. d.

PROP. 33.

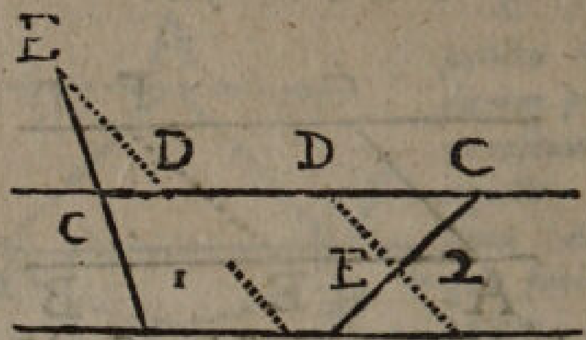


Si in datas positione parallelas rectas AB , CD *agatur magnitudine data recta* AC , *faciet angulos* BAC, ACD *datos.*

Nam ex quovis puncto E in AB , spatio EF *a* = AC describe circulum occurrentem rectæ CD in F. *b* Liquet EF, & AC parallelas esse posse. *c* ergo.

a 1. def. d.
b 34. 1.
c 29. 1.

PROP. 34.



Si in datas positione parallelas rectas AB, CD à dato puncto E agatur recta linea ECA, secabitur data ratione.

a 2. 6.
b 2. def. d.

Nam ab E duc rectam E B utcunque parallelis occurrentem in D, & B. *a* liquet esse EC. CA :: ED. DB. *b* quare $\frac{FC}{CA}$ datur. Q. E. D.

PROP. 35.

Si à dato puncto E in datam positione rectam AB agatur recta linea EA, seceturque data ratione; agatur autem per punctum sectionis C contra datam positione rectam AB recta linea CD; acta linea CD *positione data est.*

a 10. 6.
b 28. dat.

Recta enim E B ducta ab E utcunque in A B, *a* secetur sic ut ED.DB :: EC. CA. ob punctum D datum, *b* erit CD positione data. Q. E. D.

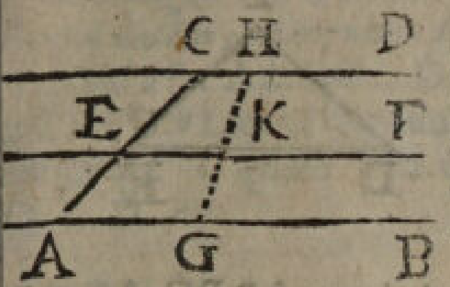
PROP. 36.

Si à dato puncto E in datam positione rectam lineam AB *agatur recta linea* EA; *adjiciatur autem ipsi aliqua recta* EC, *quæ ad illam* (EA) *habeat rationem datam; per extremitatem autem* C *adjectæ lineæ* EC *agatur contra datam positione rectam* AB *recta linea* CD; *acta linea* CD *positione data est.*

Demonstratio parum differt à præcedenti. *Vide fig.* 2.

PROP.

PROP. 37.

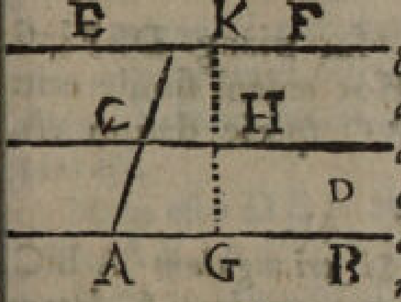


Si in datas positione parallelas rectas AB, CD, *agatur recta linea* AC, *& secetur ratione data; agatur autem per sectionis punctum* E *contra datas positione rectas* AB, CD *linea recta* EF; *acta recta* EF *positione data est.*

Nam duc rectam GH utcunque occurrentem parallelis. Hæc [a] secta sit in K ita ut GK. KH :: AE. EC. [b] Punctum K parallelæ (EF) situm determinat. Q. E. F.

a 2. def. d
b 28. dat., & sch. 2. 6.

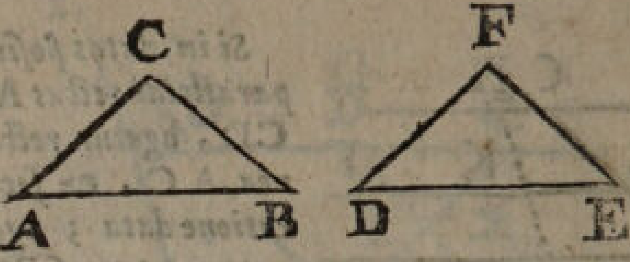
PROP. 38.



Si in datas positione rectas parallelas AB, CD *agatur recta linea* AC; *adjiciatur autem ipsi quædam recta* CE, *quæ ad actam* AE *habeat rationem datam; per extremitatem autem* E *adjectæ* CE *agatur contra datas positione parallelas* AB, CD *recta linea* EF; *acta recta linea* EF *est data positione.*

Demonstratio persimilis est præcedenti. Cerne & compara figuras.

PROP. 74.



Si trianguli ABC *singula latera* AB, BC, AC *magnitudine data sint, triangulum* ABC *specie datum est.*

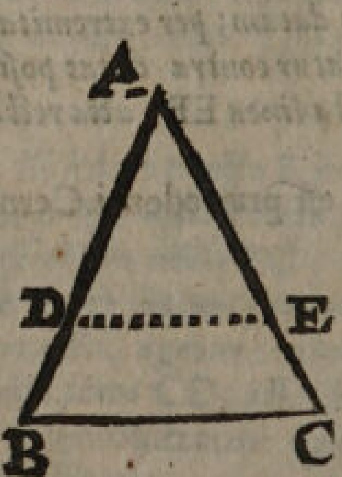
a 22. 1. b 5. 6. c 3. def. d. Nam *a* fac triang. DEF ipsi ABC æquilaterum. Hoc eidem *b* æquiangulum erit. *c* ergo ABC specie datum est. Q. E. D.

PROP. 40.

Si trianguli ABC *singuli anguli,* A, B, C *magnitudine dati sint, triangulum* ABC *specie datum est.*

a 23. 1. b 4. 6. c 3. def. d. Nam ad quamvis DE *a* fac triang. DEF ipsi ABC æquiangulum. *b* Hoc eidem simile erit. *c* proinde trigonum ABC specie datum est. Q. E. D.

PROP. 41.



Si triangulum ABC *unum angulum* A *datum habeat; circa datum autem angulum* A *duo latera* AB, AC *ad invicem habeant rationem datam; triangulum* ABC *specie datum est.*

a 1. def. d. b 6. 6. c 3. def. d. Nam in uno latere dati anguli sume quampiam AD; & *a* sit AB. AC :: AD. AE. & duc DE. *b* Liquet trigonum ADE ipsi ABC simile fore. *c* Quare ABC specie datum est. Q. E. D.

PROP.

PROP. 42.

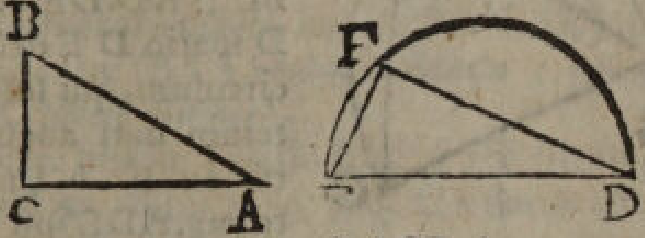
Si trianguli ABC *latera ad invicem habeant rationem datam, triangulum* ABC *specie datum est.*

Nam *a* fac AB. BC :: DE. EF. *a* & BC. CA :: EF. FD. *b* Liquet trigonum DEF trigono ABC assimilari. *c* quare ABC specie datum est. Q. E. D.

a 12. 6.
b 5. 6.
c 3. def. d.

Vide fig. 39.

PROP. 43.

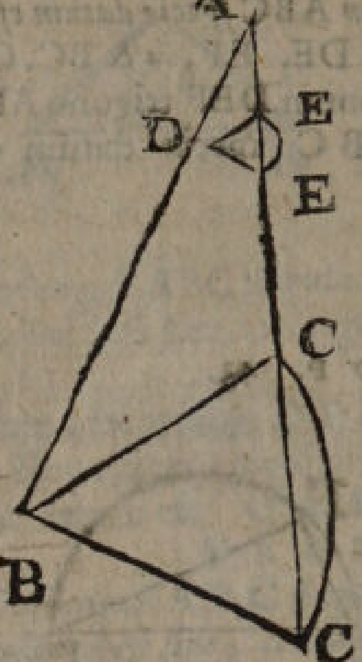


Si trianguli rectanguli ACB *circa unum acutorum angulorum* A *latera* AB, AC *ad invicem rationem habeant datam, triangulum* ACB *specie datum est.*

Nam esto DEF semicirculus utcunque; & *a* fac AB. AC :: DE. DF. inventamque DF *b* adapta in semicirculo; & duc EF. *c* Liquet triang. DFE ipsi ACB assimilari; & *d* proinde ipsum ACB specie dari. Q. E. D.

a 12. 5.
b 1. 4.
c 32. 1. & 4. 6.
d 3. def. d.

PROP. 44.



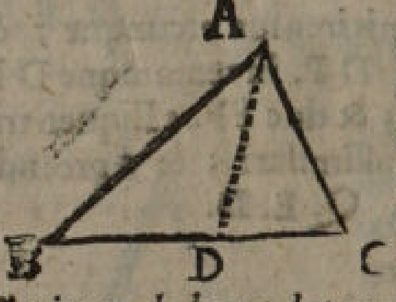
Si triangulum ABC *habeat unum angulum* A *datum ; circa alium autem angulum* ABC *latera* AB, BC *ad invicem habeant rationem datam ; triangulum* ABC *specie datum est.*

Nam in crure dati anguli sume quamlibet A D. & a fac A B. BC :: AD.DE. centro D spatio D E describe circulum, qui secet alterum dati anguli latus in E. b Eritque triang. ADE ipsi ABC simile. c quare datur specie triang. ABC. Q.E.D.

a 2. def. d.

b 7. 6.
c 3. def. d.

PROP. 45.



Si triangulum BAC *unum angulum* BAC *datum habeat ; circa datum vutem angulum* BAC *latera simul utraque tanquam unum* (BA+AC) *ad reliquum latus* (BC) *rationem habeant datam ; triangulum* BAC *specie datum est.*

Datum angulum B A C a bisecet recta A D. b ergo B A. A C :: B D. D C. & componendo BA + AC. AC :: BC. DC. permutando igitur BA + AC. BC :: AC. DC. ergo ob $\frac{BA + AC}{BC}$ c datam, d erit $\frac{AC}{DC}$ data. item ang. DAC sub-

a 9. 1.
b 3. 6.

c hyp.
d 2. def. d.

duplus

duplus dati B A C e datur. f ergo ang. C datur. e 2. dat.
g proinde trigonum ABC specie datum est. f 44. dat. g 40. dat.

Coroll.

Hinc in triangulo, datis uno latere AB, uno angulo BAC, & ratione aggregati laterum ad basim (R ad S;) datur triangulum. Nam datum angulum biseca, & fac R.S :: AB. BD. & centro B spatio B D duc circulum occurrentem rectæ bisecanti in D; & produc BDC. habes triangulum.

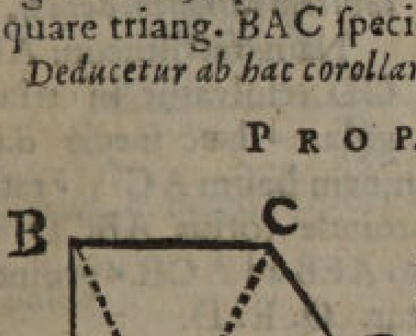
PROP. 46.

Si triangulum B A C *unum angulum C datum habeat; circa alium autem angulum* BAC *latera simul utraque tanquam unum* (BA+AC) *habeant ad reliquum* (B C) *rationem datam; triangulum* BAC *specie datum est.*

Nam bisecto angulo BAC, erit (ut in præcedenti) $\frac{AC}{DC}$ data. item ang. C a datus est. ergo a hyp.
ang. D A C, b proinde & duplus B A C datur. b 2. dat.
c quare triang. BAC specie datur. Q. E. D. c 40. dat.

Deducetur ab hac corollarium simile præcedenti.

PROP. 47.

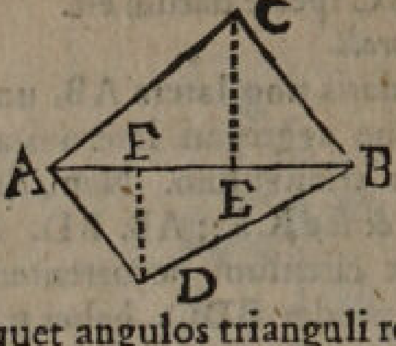


Data specie rectilinea ABCDE *in data specie triangula* BAE, CDE BCE *dividuntur.*

Nam ob ang. B, & $\frac{BA}{AE}$ a dat. b erit triang. BAE specie datum. Simili discursu triang. CDE specie datur. c quare ang. DCE datus est; Hunc deme ex dato BCD, d estque reliquus BCE datus. Similiter ang. CBE datur. e ergo triang. BCE etiam specie datum est. Q. E. D.

a hyp. & 3. def. d
b 41. dat.
c 3. def. d.
d 4 dat.
e 40. dat.

PROP. 48.

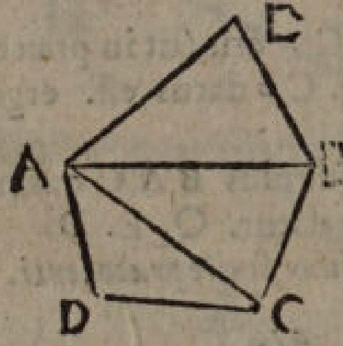


Si ab eadem recta AB describantur triangula ACB, ADB data specie, habebunt ad invicem rationem datam.

Duc enim perpendiculares CE, DF. Liquet angulos trianguli rectanguli CEB, ᵃ proinde & $\frac{CE}{CB}$ dari. ergo (quum $\frac{AB}{CB}$ ᵇ data sit) ᶜ erit $\frac{CE}{AB}$ data. Simili discursu datur $\frac{DF}{AB}$; ᶜ quare $\frac{CE}{DF}$, ᵈ hoc est triang. $\frac{ACB}{ADB}$ datur. Q. E. D.

a 40. dat.
b hyp.
c 8. dat.
d sch. 1. 6.

PROP. 49.

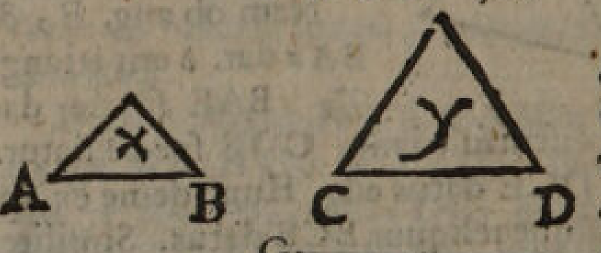


Si ab eadem recta linea AB duo rectilinea quælibet ABCD, AEB data specie describantur, habebunt ad invicem rationem datam.

Nam rectilineum ABCD resolvatur in triangula. ᵃ hæc specie data sunt. ergo ob communem basim AC, ᵇ ratio ADC ad ACB & ᶜ proinde totius ABCD ad ACB datur. ᵇ item ratio AEB ad ACB. ᵈ proinde & ABCD ad AEB datur. Q. E. D.

a 47. dat.
b 48. dat.
c [illegible] dat.
d 8. dat.

PROP. 50.

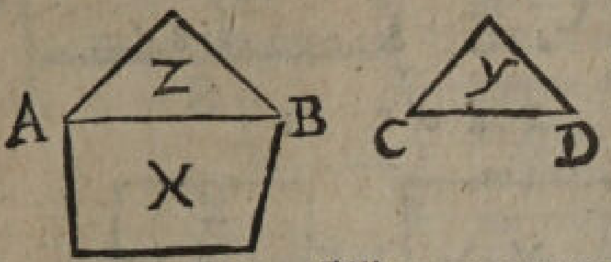


Si duæ rectæ lineæ AB CD ad invicem habeant rationem datam; & ab illis similia, similiterque descripta rectilinea X, Y habebunt ad invicem rationem datam.

Nam

Nam sit AB. CD :: [a] CD. G. [d] liquet AB ad G, [e] hoc est X ad Y dari. Q. E. D. a 11. 6. b 8. dat. c cor 20. 6.

PROP. 51.



Si duæ rectæ lineæ AB, CD habeant ad invicem rationem datam; & ab illis rectilinea quæcunque X, Y specie data describantur; habebunt ad invicem rationem datam.

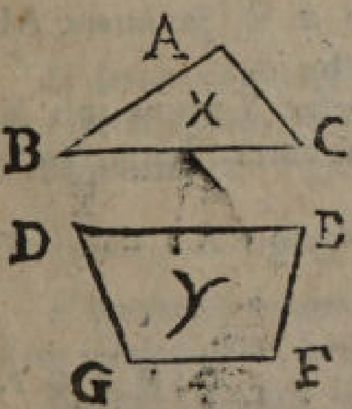
Nam [a] fac Z simile ipsi Y. Ac ob [b] $\frac{Z}{X}$, [c] & $\frac{Z}{Y}$ datas, [d] liquet $\frac{X}{Y}$ dari. Q. E. D. a 18. 6. b 49. dat. c 50. dat. d 8. dat.

PROP. 52.

Si à data magnitudine recta AB figura X specie data describatur, descripta figura X magnitudine data est.

Nam ABq [a] datur specie, & magnitudine; & [b] $\frac{ABq}{X}$ datur. [c] ergo X datur. a 3. & 1. def d. b 49. dat. c 2. dat.

PROP. 53.



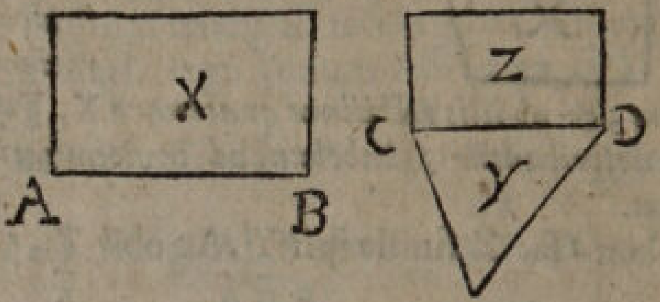
Si duæ figuræ X, Y specie datæ fuerint; & unum latus unius BC ad unum latus alterius DE habuerit rationem datam; reliqua quoque latera AB ad reliqua EG habebunt rationem datam.

Nam

a 3.def.d.
b hyp.

Nam { a AB / b $\overline{BC}$ / a $\overline{DE}$ / $\overline{EF}$ / a $\overline{FG}$ } dantur. &c. ergo per 8. dat.

PROP. 54.

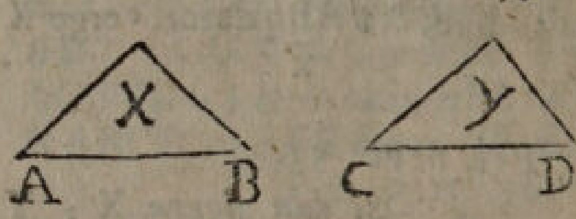


Si duæ figuræ X, Y *specie datæ ad invicem habuerint rationem datam, etiam latera* (AB, CD, &c.) *habebunt ad invicem rationem datam.*

a 18. 6.
b 3. def d.
c 49. dat.
d hyp.
e 8. dat.
f cor 20.6 & 24. dat.

Nam ad CD [a] fiat Z ipsi X similis.[b] Hæc specie datur.[c] ergo $\frac{Y}{Z}$ datur. Proinde ob $\frac{Y}{X}$ [d] datam, [e] datur $\frac{X}{Z}$. [f] ergo $\frac{AB}{CD}$ datur. ergo per præcedentem.

PROP. 55.



Si spatium X *magnitudine & specie datum fuerit, ejus latera* (AB &c.) *magnitudine data erunt.*

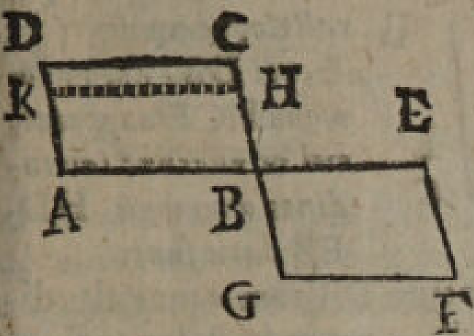
a 18. 6.
b 1. dat.
c 54. dat.
d 2. dat.

Nam ad quamvis C D [a] fac Y simile ipsi X. hoc specie & magnitudine datur. [b] ergo $\frac{Y}{X}$ datur. [c] quare $\frac{CD}{AB}$ datur. [d] ergo AB data est.

Q. E. D.

PROP.

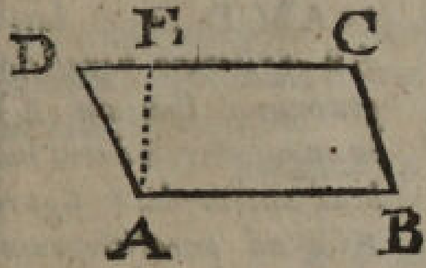
PROP. 56.



Si duo æquiangula parallelogramma AC, BF habueint ad invicem rationem datam, est ut primi latus AB ad secundi latus BE, ita reliquum secundi latus BG ad eam BH, ad quam alterum primi latus BC habet rationem datam, quam habet parallelogrammum AC ad parallelogrammum BF.

Nam duc HK parall. AB. Liquet esse BC. BH *a* :: AC. AH *b* :: AC. BF. Q. E. D. a 1. 6. b 14. 6. & 7. 5.

PROP. 57.



Si datum spatium AC ad datam rectam AB applicatum fuerit, in angulo BAD dato, datur applicationis altitudo AD.

a Erige perpendicularem AE. estque AB. AE *b* :: ABq. AB x AE *c* :: ABq. pgr. AC. *d* ergo AE datur. quare per E duc parallelam DC, *e* hæc abscindet quæsitam AD. Q. E. F. a 11. 1. b 1. 6. c 35. 1. d 1. & 2. dat. e 28. & 25. dat.

PROP. 58.

Si datum ad datam rectam applicetur, deficiens data specie figura, latitudines defectus datæ sunt.

Non differt à vigesima octava sextæ.

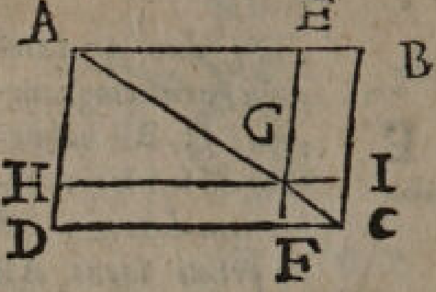
PROP. 59.

Si datum ad datam rectam applicetur, excedens data specie figura, latitudines excessus datæ sunt.

Eadem est cum vigesima nona sextæ.

PROP.

PROP. 60.



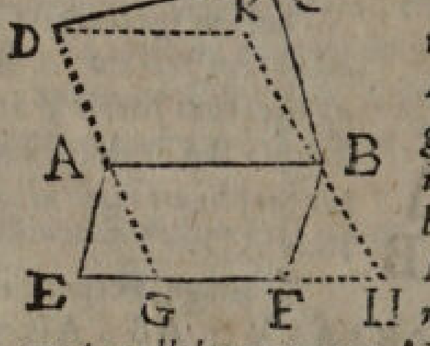
Si datum specie parallelogrammum (HE, vel DB) dato gnomone HCE augeatur, vel minuatur; latitudines gnomonis HD, EB datæ sunt.

a 3. dat
b 24.6.
c 55. dat.
d hyp.
e 4. dat.

1. *Hyp.* Liquet totum DB tam *a* magnitudine, quam *b* specie dari, *c* proinde & latitudines AB, AD; è quibus aufer *d* datas AE, AH, *e* manent EB, HD datæ. Q. E. D.

2. *Hyp.* Liquet HE *b* specie, & *a* magn. *c* dari, *c* quare & latera AE, AH; hæc deme ex *d* datis AB, AD: *e* remanent EB, HD datæ. Q. E. D.

PROP. 61.

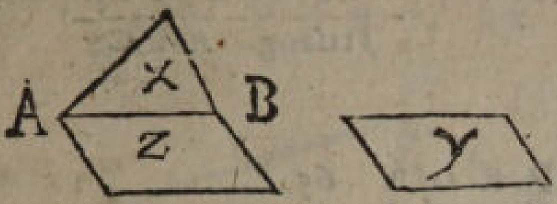


Si ad datæ specie figuræ ABCD unum latus AB applicetur parallelogrammum spatium AF in angulo BAE dato; habeat autem data figura AC ad parallelogrammum AF rationem datam; parallelogrammum AF specie datum est.

a 3. def. d.
b 49. dat.
c 8. dat.
d 35. 1.
e 1 6.
f hyp. & 4. dat.
g 40. dat.
h 3. def. d.

Ad DAG protractam duc (per B) parallelam, cui occurrant EFH, & DK parall. AB. Ac ob AD, & ang. BAD *a* dat. *a* liquet $\frac{\text{pgr.}}{AB}$ AK specie dari. *b* ergo $\frac{AK}{AC}$ & *c* proinde $\frac{AK}{AF}$, *d* vel $\frac{AK}{AH}$, *e* hoc est $\frac{AD}{AG}$ dantur. *e* ergo $\frac{AB}{AG}$ datur. Item ob angulos E, & GAE *f* notos, *g* datur $\frac{AE}{AG}$; *c* ergo $\frac{AB}{AE}$ datur. *h* unde pgr. AF specie datur. Q.E.D.

PROP. 62.



Si duæ recta AB, CD *ad invicem habeant rationem datam; & ab una quidem data specie figura* X *descripta sit, ab altera autem spatium parallelogrammum* Y *in angulo dato; habeat autem figura* X *ad parallelogrammum* Y *rationem datam; parallelogrammum* Y *specie datum est.*

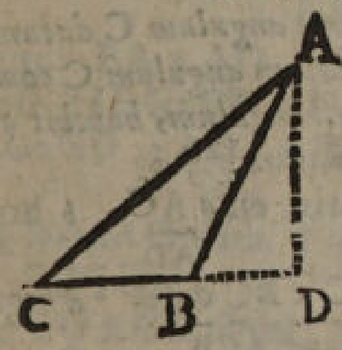
Nam ad AB sit pgr. Z simile ipsi Y. a Hujus ratio ad Y, & b proinde ad X datur. c ejusque anguli dantur. d ergo Z specie datur. e proinde & Y. Q. E. D. a 50. dat. b 8. dat. c hyp d 61. dat. e 3 def. d.

PROP. 63.

Si triangulum specie datum sit, quod ab unoquoq; laterum describitur quadratum, ad triangulum habebit rationem datam.

Sequitur ex 49. hujus.

PROP. 64.



Si triangulum ABC *angulum obtusum* ABC *datum habeat; illud spatium, quo latus* AC *obtusum angulum subtendens magis potest quam latera* AB, CB *obtusum angulum* ABC *ambientia, ad triangulum* ABC *habebit rationem datam.*

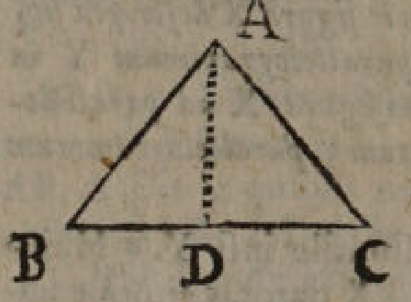
Nam demittatur AD perpendicularis productæ CBD. atque ob angulos a ABD, & D datos, b datur $\frac{BD}{AD}$, c hoc est $\frac{BD \times CB}{AD \times CB}$. d ergo a 4. dat. b 47 dat. c 1. 6. d 8. dat.

2 BD

e 12. 2.
f 41. 1.

$\frac{2\,BD \times CB}{\frac{1}{2}\,AD \times CB}$, hoc est, e $\frac{ACq - ABq - CBq}{f\ \text{triang. } ABC}$ datur. Q. E. D.

PROP. 65.



Si triangulum ACB *angulum acutum* C *datum habeat; illud spatium, quo latus* AB *angulum* C *subtendens minus potest, quam latera* AC, CB *angulum acutum* C *ambientia, habebit ad triangulum* ACB *rationem datam.*

a 40 dat.

Nam duc perpendicularem AD. Datur a $\frac{CD}{AD}$,

b 1. 6.
c 8. dat.

b hoc est $\frac{CD \times BC}{AD \times BC}$. c ergo $\frac{2\,CD \times BC}{\frac{1}{2}\,AD \times BC}$, hoc

d 13. 2.
e 41. 1.

est d $\frac{ACq + BCq - ABq}{e\ \text{triang. } ACB}$ datur. Q. E. D.

PROP. 66.

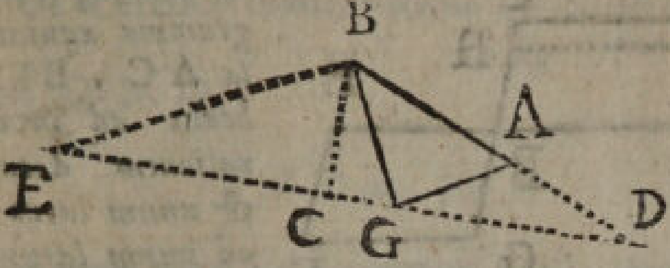
Si triangulum ACB *habuerit angulum* C *datum; quod sub rectis* AC, CB *datum angulum* C *comprehendentibus, continetur rectangulum, habebit ad triangulum* ACB *rationem datam.*

a 40 dat.
b 1. 6
c 41. 1.
d 8. dat.

Nam in figura præcedentis, est a $\frac{AC}{AD}$, b hoc est, c $\frac{AC \times BC}{AD \times BC}$, c hoc est $\frac{AC \times BC}{2\ \text{triang. } ACB}$ data. d ergo $\frac{AC \times BC}{\text{triang. } ACB}$ datur. Q. E. D.

PROP.

PROP. 67.



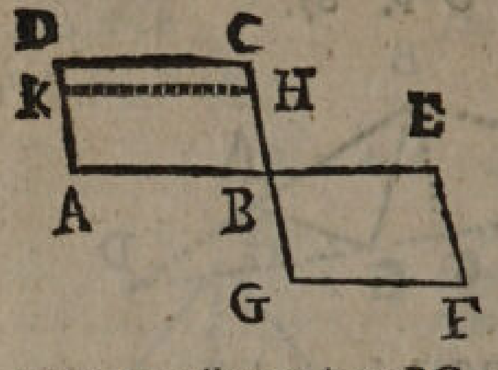
Si triangulum ABG *habuerit datum angulum* BAG; *illud ſpatium, quo duo datum angulum* BAG *comprehendentia latera tanquam una recta* BA + AG, *plus poſſunt, quam quadratum à reliquo latere* BG, *ad triangulum* ABG *habebit rationem datam.*

Produc BA ita ut AD = AG. per B duc BE parall. AG; cui occurrat DGE. denique duc normalem BC.

Liquet ang. D *a* = AGD *b* = E. *c* quare BE = BD, ideoque EC = CD. *e* ergo EG x GD + CGq = CDq. proinde BDq *f* (CDq + BCq) *g* = EG x GD + CGq + BCq = EG x GD * + BGq. Iam ob angulos AGD, & D *h* ſubduplos dati BAG, liquet *k* $\frac{AD}{DG}$, ideoq; $\frac{ADq}{DGq}$ dari. Cum igitur BA x AD. ADq *l* :: BA. AD *m* :: EG. GD :: *l* EG x GD. GDq, & permutando BA x AD. EG x GD :: ADq. GDq; *n* erit $\frac{BA x AD}{EG x GD}$; *o* hoc eſt $\frac{BA x AG}{EG x GD}$ data. *p* Atqui $\frac{BA x AG}{triang. AGB}$ datur; *q* ergo $\frac{EG x GD}{triang. AGB}$ datur. Q. E. D.

a 5. 1.
b 29. 1.
c 6. 1.
d cor. 3. 3.
e 5. 2.
f 47. 1.
g 2. ax. 1.
* 47. 1.
h 32. 1.
k 40. dat.
l 1. 6.
m 2. 6.
n 2. def. d.
o conſtr.
p 66 dat.
q 8. dat.

PROP. 68.

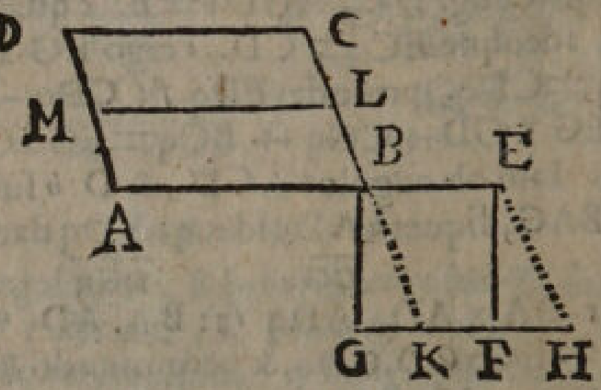


Si duo parallelogramma æquiangula AC, BF *habeant ad invicem rationem datam, & unum latus* AB *ad unum latus* BE *habeat rationem datam; & reliquum latus* BC *ad reliquum latus* BG *habebit rationem datam.*

a 2. def. d.
b 56. dat.
c 8. dat.

Nam ſit AB. BE :: BG. BH. *a* ergo $\frac{BG}{BH}$ datur. *b* item $\frac{BC}{BH}$ datur. *c* ergo $\frac{BC}{BG}$ datur.

PROP. 69.



Si duo parallelogramma AC, BF *datos angulos habeant, & ad invicem rationem datam; habeat autem & unum latus* AB *ad unum latus* BE *rationem datam; & reliquum latus* BC *ad reliquum latus* BG *habebit rationem datam.*

Latera AB, BE jaceant in directum. produc CBK, ac GFH ad occurſum cum EH parall. CK.

a Hyp.

Ob *a* ang. KBE (ABC) & pgr. *a* $\frac{AC}{BF}$, vel

AC

$\frac{AC}{BH}$ & [a] $\frac{AB}{BE}$ datas, [c] liquet $\frac{KB}{BC}$ dari. item ob ang. G, & GBK [d] datos, [e] datur $\frac{KB}{BG}$. [f] quare $\frac{BC}{BG}$ datur. Q. E. D.

b 35. 1.
c 68. dat.
d hyp. & 4 dat.
e 40. dat.
f 8. dat.

PROP. 70.

Si duorum parallelogrammorum (AC, BH, *vel* BF) *circa æquales angulos* (ABC, KBE) *aut circa inæquales quidem* (ABC, GBE) *datos tamen; latera* (AB, BE, & BC, BK, & BC, BG) *ad invicem habeant rationem datam; & ipsa parallelogramma* (AC, BH, & AC, BF) *habebunt ad invicem rationem datam.*

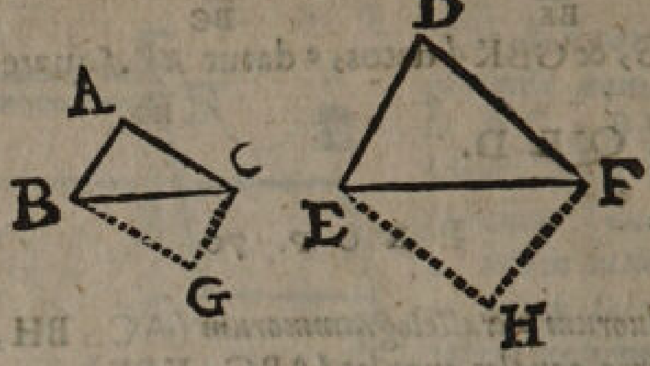
Nam (in fig. præced.) sit AB. BE :: KB. BL. & duc LM parall. BA.

Primo, Quia [a] $\frac{AB}{BE}$ [b] id est $\frac{KB}{BL}$, [a] ac $\frac{KB}{CB}$ datæ sunt, [c] erit $\frac{CB}{BL}$, [d] hoc est $\frac{AC}{AL}$, [e] vel pgr. $\frac{AC}{BH}$ data. Q. E. D.

Secundo, Ob angulos G, & GBK [f] datos, [g] datur $\frac{BK}{BG}$; item [h] $\frac{CB}{BG}$ data est. [c] ergo $\frac{CB}{BK}$ datur. proinde, ut prius, $\frac{AC}{BH}$, hoc est pgr. $\frac{AC}{BF}$ datur. Q. E. D.

a hyp.
b constr.
c 8. dat.
d 1. 6.
e 14. 6.
f hyp. & 4. dat.
g 40. d.
h 35. 1.

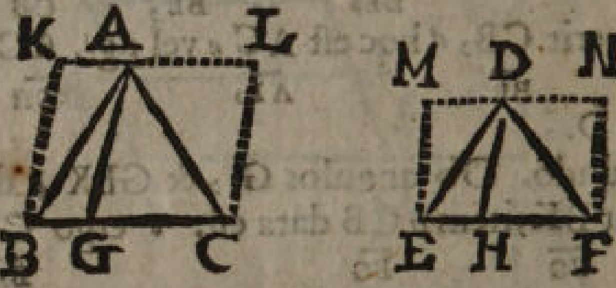
PROP. 71.



Si duorum triangulorum ABC, DEF, *circa æquales angulos, aut circa inæquales quidem, datos tamen* (A, & D) *latera* AB, DE, & AC, DF *ad invicem habeant rationem datam; & ipsa triangula* ABC, DEF *habebunt ad invicem rationem datam.*

a 70. dat. b 15. 5. c 34. 1. Nam compleantur pgra. AG, DH. *a* hæc datam habent rationem, *b* proinde & trigona ABC, DEF illorum *c* ſubdupla. Q. E. D.

PROP. 72.



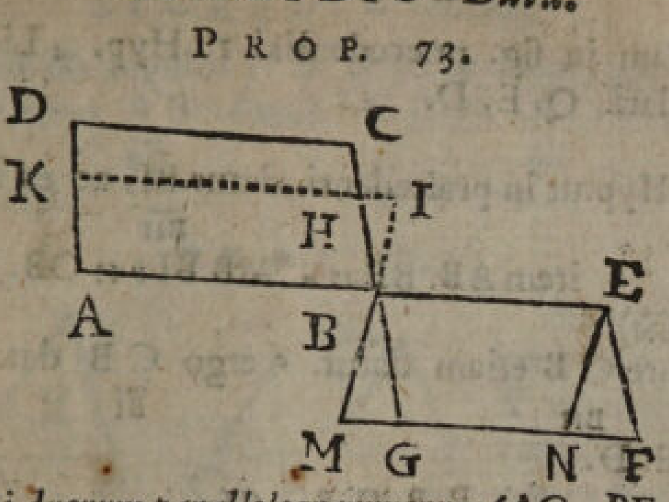
Si duorum triangulorum ABC, DEF *& baſes* BC, EF *fuerint in ratione data, & actæ ab angulis ad baſes* (AG, DH,) *quæ faciant ang.* AGC, DHF *æquales, aut inæquales quidem, ſed tamen datos, habeant ad invicem rationem datam; & ipſa triangula* ABC, DEF *habebunt ad invicem rationem datam.*

Nam duc BK ad AG, ac EM ad DH parallelas, & comple pgra. CK, FM. Hæc ſe habent juxta 70. hujus; quare triangula eorum

* 34. 1. * ſubdupla ABC, DEF rationem habent datam. Q. E. D.

PROP.

PROP. 73.



Si duorum parallelogrammorum (AC, BF, *vel* AC, BN) *circa æquales angulos, aut circa inæquales quidem, sed tamen datos, latera ad invicem ita se habeant, ut sit quemadmodum primi latus* AB *ad secundi latus* BE, *ita reliquum secundi latus* (BG, *vel* BM) *ad aliam aliquam rectam* (BH, *vel* BI;) *habeat autem & reliquum primi latus* BC *ad eandem rectam* (BH *vel* BI) *rationem datam; & ipsa parallelogramma* (AC, BF, *vel* AC, BN) *habebunt ad invicem rationem datam.*

Nam 1. Hyp. liquet [a] $\frac{CB}{BH}$, [b] id est $\frac{AC}{AH}$ [c] $\left(\frac{}{BF}\right)$ dari. Q. E. D. a hyp. b 1. 6. c 14. 6.

2. Hyp. Duc parallelam IHK. [a] Liquet angulos IBH (GBM) & BHI (ABH) dari. [b] ergo $\frac{BH}{BI}$ datur. item $\frac{CB}{BI}$ [a] data est. [c] proinde $\frac{CB}{BH}$, hoc est pgr. $\frac{AC}{BF}$ [d] vel $\frac{AC}{BN}$, datur. Q. E. D. a hyp. & 4 dat. b 40. dat. c 8 dat. d 35. 1.

PROP. 74.

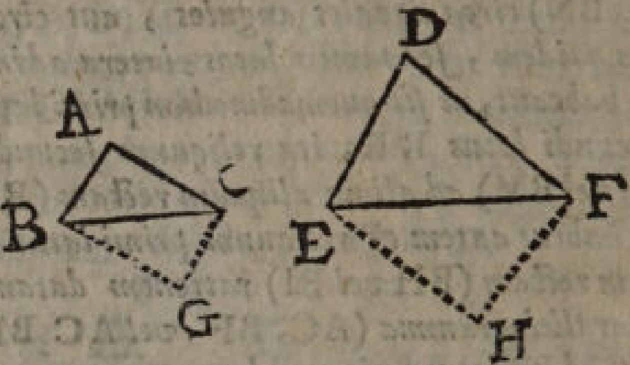
Si duo parallelogramma datam rationem habeant, aut in æqualibus angulis (*ut* AC, BF) *aut inæqualibus quidem, sed tamen datis* (*ut* AC, BN;) *erit ut primi latus* AB *ad secundi latus* BE, *ita alterum secundi latus* (BG, *vel* BM) *ad eam* (BH, *vel* BI) *ad quam reliquum primi latus* BC *rationem habet datam.*

a 56. dat. Nam in fig. præcedentis. 1. Hyp. a Liquet $\frac{CB}{BH}$ dari. Q. E. D.

2. Hyp. ut in præcedenti, datur $\frac{BI}{BH}$, ac ex hyp.

* hyp. b 4 6. c 8. dat. $\frac{AC}{BF(BN)}$ item AB. BE :: a *MB. BI b :: GB. BH. a quare $\frac{CB}{BH}$ etiam datur. c ergo $\frac{CB}{BI}$ data est. Q. E. D.

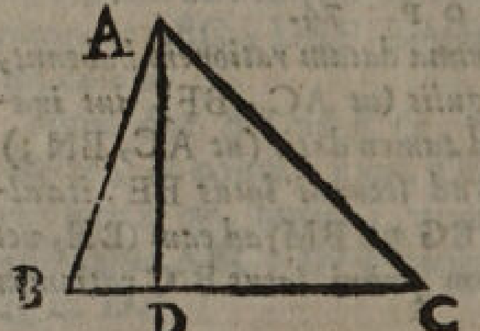
PROP. 75.



Si duo triangula ABC, DEF *ad invicem habeant rationem datam, aut in angulis* (A, D) *æqualibus, aut inæqualibus quidem sed tamen datis, erit ut primi latus* AB *ad secundi latus* DE, *ita alterum secundi latus* DF *ad eam rectam, ad quam reliquum primi latus* AC *habet rationem datam.*

Nam compleantur pgr. AG, DH. Ergo per præcedentem.

PROP. 76.

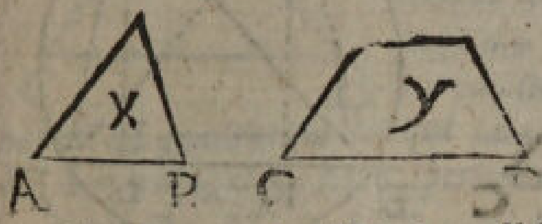


Si à trianguli ABC *specie dati vertice* A *linea perpendicularis* AD *agatur ad basim* BC, *acta linea* AD *ad basim* BC *habebit rationem datam.*

Nam

Nam ob angulos, * B, & ADB datos, a datur $\frac{AB}{AD}$; a item $\frac{AB}{BC}$ datur. b Ergo $\frac{AD}{BC}$ datur. Q. E. D. * hyp. & 3. def. d. a 40. dat. b 8 dat.

PROP. 77.



Si datæ figuræ specie X, Y ad invicem habeant rationem datam, & quodlibet latus unius AB *ad quodlibet alterius latus* CD *habebit rationem datam.*

Nam a $\frac{ABq}{X}$, & b $\frac{Y}{X}$, ac c proinde $\frac{ABq}{Y}$ datur; item $\frac{CDq}{Y}$ datur. c ergo $\frac{ABq}{CDq}$, ac ideo $\frac{AB}{CD}$ datur. Q. E. D. a 49. dat. b hyp. c 8. dat.

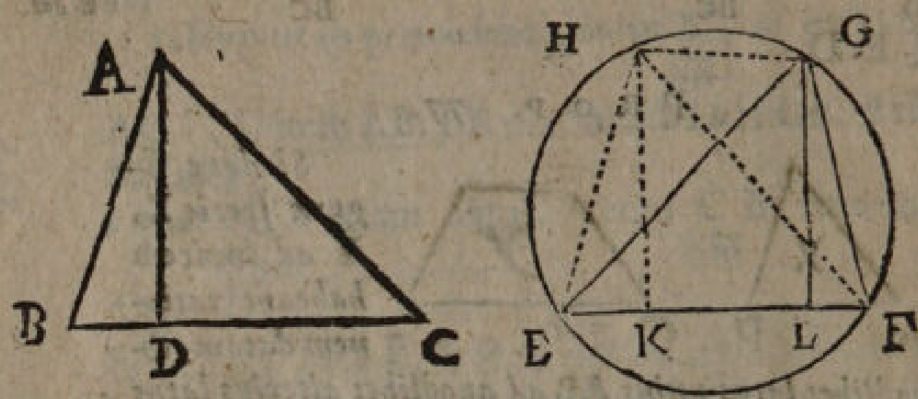
PROP. 78.

Si data figura specie X *ad aliquod rectangulum* DCE *habeat rationem datam; habeat autem & unum latus* AB *ad unum latus* DC *rationem datam; rectangulum* DCE *specie datum est.*

Sit DC. AB :: AB. CF. a ergo $\frac{DC}{CF}$ datur. a 8. dat.

Item ob b $\frac{X}{ABq}$, & c $\frac{X}{DCE}$ datas. d erit $\frac{ABq}{DCE}$, d hoc est f $\frac{DC \times CF}{D\ CE}$, vel e $\frac{CF}{CE}$ data. proinde e $\frac{DC}{CE}$ datur. quare rectang. DCE specie datur. Q. E. D. b 49. dat. c hyp. d 17. 6. e 1. 6. f 3. def. d.

PROP. 79.



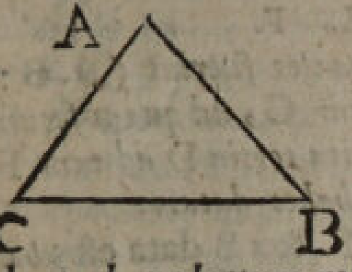
Si duo triangula ABC, GEF *unum angulum* BAC *uni angulo* EGF *æqualem habeant; ab æqualibus autem angulis* BAC, EGF *ad bases* BC, EF *perpendiculares agantur* AD, GL; *sitque ut primi trianguli basis ad perpendicularem, ita & alterius trianguli basis ad perpendicularem* (BC.AD :: EF.GL;) *illa triangula* ABC, EGF *æquiangula sunt.*

Circa triang. GEF describe circulum. Fac ang. FEH = B. Connecte HF, HG; & demitte perpendicularem HK.

Liquet triangula ABC, HEF, & ABD, HEK, ac ACD, HFK æquiangula fore. Proinde EK.
a 4. 6. KH :: BD.DA. *a* & FK.KH :: CD.DA.
b 24. 5. *b* quare EF.KH :: BC.DA :: *c* EF.LG.
c *hyp.* *d* quare KH = LG. *e* ergo HG parall. KL. *f* un-
d 9. 5.
e 33. 1. de ang. EGH = GEF. *g* ergo arcus EH, FG,
f 29. 1. *h* ideoque anguli EFH, GEF æquantur. *k* Item
g 26. 3.
h 27. 3. ang. EHF = EGF. *l* ergo trigona EHF, EGF;
k 21. 3. *m* proinde & trigona EGF, ABC sibi mutuo æ-
l 32. 2.
m 21. 6. quiangula sunt. Q. E. D.

PROP.

PROP. 80



Si triangulum ABC unum angulum A datum habuerit; quod autem sub lateribus AB, AC datum angulum comprehendentibus continetur rectangulū, habeat ad quadratum reliqui lateris BC rationem datam; triangulum ABC specie datum est.

Nam Q: AC + AB: − CBq vocetur X. a ergo $\frac{X}{\text{triang.ABC}}$; b & $\frac{ACxAB}{\text{triang.ABC}}$; & c propterea $\frac{X}{ACxAB}$ data est. d item $\frac{ACxAB}{CBq}$ datur. e ergo $\frac{X}{CBq}$, ideoq; $\frac{X+CBq}{CBq}$, f hoc est $\frac{Q: AC + AB,}{CBq}$ datur. g proinde triang. ABC specie datur. Q. E. D.

a 67. dat.
b 66. dat.
c 8. dat.
d hyp.
e 6. dat.
f hyp.
g 46. dat.

PROP. 81.

A. D.
B. E.
C. F.

Si tres rectæ proportionales A, B, C tribus rectis proportionalibus D, E, F extremas A, D, & C, F habuerint in ratione data; medias quoque B, E habebunt in ratione data. Et si extrema A ad extremam D, & media B ad mediam E habeat rationem datam; & reliqua C ad reliquam F habebit rationem datam.

Nam primo, ob $\frac{A}{D}$ & $\frac{C}{F}$ datas, a datur $\frac{AC}{DF}$, b hoc est, $\frac{Bq}{Eq}$. ergo $\frac{B}{E}$ datur. Q. E. D.

Secundo, ob c $\frac{Bq}{Eq}$, b hoc est $\frac{AC}{DF}$ datam, & c $\frac{A}{D}$ datam, d datur $\frac{C}{F}$. Q. E. D.

a 70. dat.
b 17. 6.
c hyp.
d 68. dat.

Prop. 82.

A. B :: D. E.
B. C :: E. F.

Si quatuor rectæ proportionales fuerint (A. B :: D. E) *erit ut prima* A *ad eam* C, *ad quam secunda* B *rationem habet datam, ita tertia* D *ad eam* F, *ad quam quarta* E *rationem habet datam.*

a hyp. b 2. def. d. — Nam quia B. C :: a E. F. & a $\frac{B}{C}$ data est; b erit $\frac{E}{F}$ data. atqui ex æquali A. C :: D. F. ergo, &c.

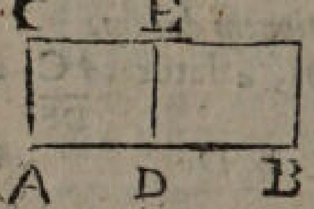
Prop. 83.

A. B. C. D.
F. E.

Si quatuor rectæ A, B, C, D *ita ad invicem se habeant, ut tribus ex iis, quibuscunque sumptis* A, B, C, *& quarta ipsis proportionali accepta* E, *ad quam reliqua* D *ex quatuor rectis proportionem habet datam; erit ut quarta* D *ad tertiam* C, *ita secunda* B *ad eam* F, *ad quam habet prima* A *rationem datam.*

a 16. 6. b hyp. c 1. 6. d 7. 5. — Nam AE a = BC b = DF. & datur b $\frac{D}{E}$, c hoc est $\frac{AD}{AE}$, d vel $\frac{AD}{DF}$, e vel $\frac{A}{F}$. ergo, &c.

Prop. 84.



Si duæ rectæ AB, AC *datum spatium comprehendant in angulo* A *dato; sit autem altera* AB *altera* AC *major data* DB; *etiam unaquæque ipsarum* AB, AC *data erit.*

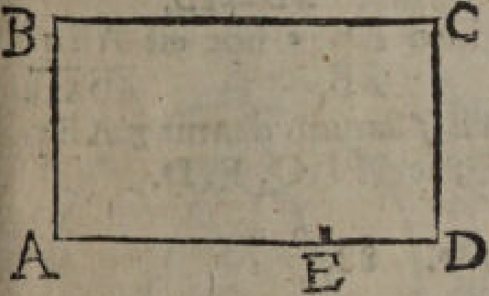
a 3. def. d. b hyp. c 59. d. d 3. dat. — Nam comple quadratum AE. a Hoc specie datum est. b item pgr. CB, & recta DB dantur. c ergo AC, vel AD, & tota d proinde AB datur. Q. E. D.

PROP. 85.

Si duæ rectæ BD, DE *datum spatium comprehendant in angulo* BDE *dato, sit autem simul utraq;* (BD+DE) *data; & earum quoque unaquæque* BD, *&* DE *data erit.*

Nam sume DA=DE, & comple quad. DC. Hoc specie datur; item pgr. BE, & recta BA *a* dantur. *b* ergo AD (DE) & *c* reliqua DB dantur. Q. E. D. a hyp. b 58. dat. c 4. dat.

PROP. 86.



Si duæ rectæ AB, AD *datum spatium* BD *comprehendant in angulo dato; quadratum autem unius* AD *quadrato alterius* AB *majus sit dato quam in ratione* (nempe ut sit ADxAE datum, & * reliqui ADxED ad ABq ratio data;) *& utraque ipsarum* AB, AD *data erit.* * 2. 2.

Nam ob BD, & DAxAE *a* data, *b* datur $\frac{BD.}{DAxAE}$ *c* ergo $\frac{AB}{AE}$ *d* ideoque $\frac{ABq}{AEq}$ datur. *e* item $\frac{ABq}{ADxED,}$ datur. *f* ergo $\frac{AEq}{ADxED,}$ ideoque $\frac{AEq}{4ADxED,}$ *g* & $\frac{AEq}{4ADxED+AEq}$ *h* hoc est $\frac{AEq}{Q:AD+ED}$ datur. *k* ergo $\frac{AE}{AD \quad ED;}$ & *l* componendo $\frac{AE}{2AD,}$ * ideoq; $\frac{AE}{AD,}$ *m* hoc est $\frac{AEq}{ADxAE}$ datur. denique igitur ob *e* datum ADxAE, *n* erit AEq data. *o* ergo AE, & *p* proinde AD, ac AB datæ sunt. Q.E.D.

a hyp. b 1. dat. c 69. dat. d 51. dat. e hyp. f 8. dat. g 6. dat. h 8. 2. k 54 dat. l 6 dat. * 8. dat. m 1. 6. n 2 dat. o 55. dat. p 57 dat.

PROP.

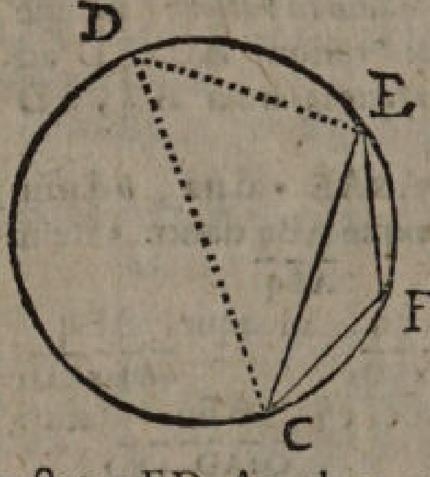
PROP. 87.

Si duæ rectæ AB, AD *datum spatium comprehendant in angulo dato, quadratum autem unius* AD *quadrato alterius* AB *majus sit dato* (AD x AE;) *earum utraque* AB, AD *data erit.*

a 2. dat.
b 69. dat.
c hyp. & 8. 2.
d 8. & 6. dat.
e 8. 2.
d 6. dat.
e 1 6.
f hyp.
g 2. dat.
h 55. dat.
k 57. dat.

Nam ob $\frac{BA \times AE}{BD}$ a datum, b erit $\frac{AE}{AB}$, ideoq; $\frac{AEq}{ABq}$, c hoc est $\frac{AEq.}{AD \times ED}$, d ac idcirco $\frac{AEq}{AEq + 4 AD \times ED}$, e hoc est $\frac{AEq}{Q:AD + ED}$, ac proinde $\frac{AE}{AD + ED}$; & d componendo $\frac{AE}{2AD}$, e ac ideo $\frac{AE}{AD}$, e hoc est $\frac{AEq}{AD \times AE}$ data. ergo ob AD x AE f datum, dantur g AEq, & h AE, ac k ideo AD, ac AB. Q. E. D.

PROP. 88.



Si in circulum CFED magnitudine datum acta sit recta linea CE, quæ segmentum auferat, quod datum angulum F comprehendat; acta recta linea CE magnitudine data est.

a hyp
b 4. dat.
c 40. dat.
d hyp & 5. def d.
e 2. dat.

Nam ducatur diameter CD; & connectatur ED. Ac ob ang. F a datum, b erit ang. D (reliquus è 2 rectis) datus. item rectus CED datur. c quare $\frac{CE}{CD}$ datur. ergo ob d datam CD, e erit CE data. Q. E. D.

PROP.

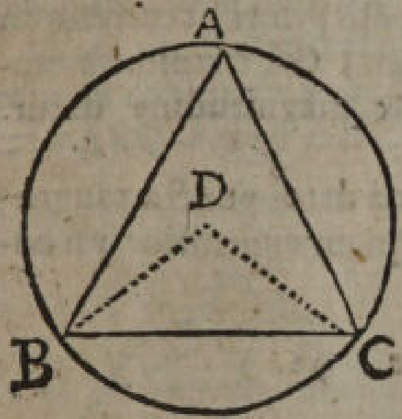
PROP. 89.

Si in datum magnitudine circulum CFED data magnitudine recta CE acta fuerit, auferet segmentum quod angulum (CFE) datum comprehendet.

Nam (in fig. præcedentis) quia $\frac{CE}{CD}$, & ang. CED dantur, *a* erit ang. D datus. *b* ergo ang. F *c* (1 Rect. — D) datus erit. Q. E. D.

a 43. dat.
b 4. dat.
c 22. 3.

PROP. 90.



Si in circuli positione dati circumferentia BAC datum fuerit punctum B, ab eo autem puncto B ad circumferentiam circuli inflexa fuerit recta BAC quæ datum angulum A efficiat; inflexæ rectæ altera extremitas C data erit.

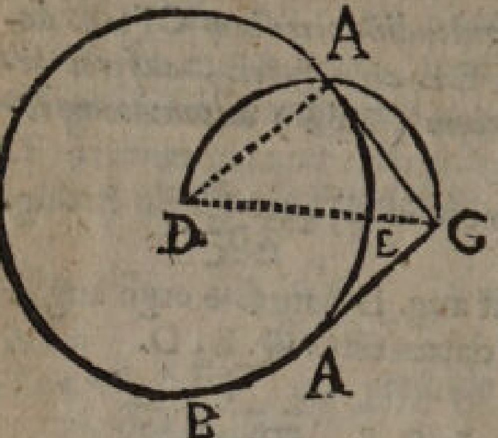
Ad *a* centrum D duc BD, & CD; *b* datusque est ang. D dati A *c* duplus. quare ob BD *d* datam, *e* erit DC data. *f* ergo punctum C datum est. Q. E. D.

a 1. 3.
b 2. dat.
c 20. 3.
d 26. dat.
e 19. dat.
f sch. 25. d.

Si ang. A obtusus fuerit; sume reliquum è 2 rectis acutum; ejus subsidio punctum C invenies, juxta dicta.

PROP.

PROP. 91.



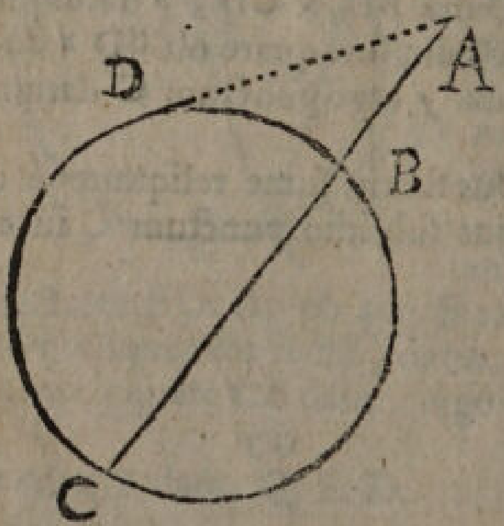
Si à dato puncto G acta fuerit recta GA, quæ datum positione circulum B E A contingat; acta linea GA positione & magnitudine data est.

Nam centrum D & punctum G connectat recta D G. super qua descriptus sit semicirculus D A G circulo priori occurrens in A. Ob ang. D A G *a* rectum, GA circulum *b* tangit. *c* ergo G A situ & magnitudine datur. Q. E. D.

a 31. 3.
b cor. 16. 3.
c 26. dat.

Hinc modus discitur à dato puncto tangentem ducendi, eo nonnunquam expeditior qui habetur ad 17. 3.

PROP. 92.



Si extra circulum positione datum BCD accipiatur aliquod punctum A, à dato autem puncto A in circulum producatur quædam recta AC; datum est id quod sub acta linea AC, & ea A B, quæ inter punctum A & convexam peripheriam B comprehenditur rectangulum CAB.

Nam

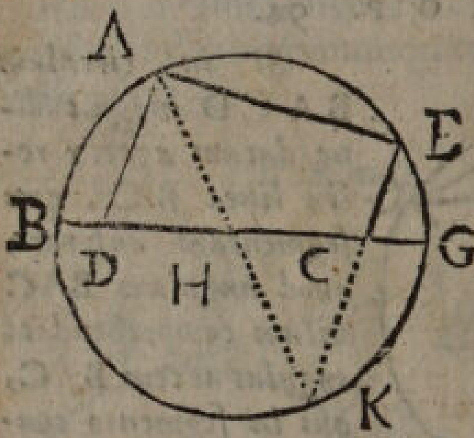
a 88. dat. * 1. dat. b 3. 6. c 12. 5. * 4. 6. d 2. def. d.

Duc CD; & primo ob angulos BAC, CAD datos, [a] dantur subtensæ BC, CD,* ideoque $\frac{CB}{DC}$ datur. Cum igitur CA. AB :: [b] CE. EB, & permutando CA. CE :: AB. EB :: (CA + AB. CB ::) * AD. DC. (Nam * ob ang. BAE = CAD; & D = BD; trigona ABE, ADC similia sunt) ac rursus permutando CA + AB. AD :: CB. DC. [d] erit $\frac{CA + AB}{AD}$ data. Q. E. D.

e 21. 3. b 4. 6. c prius. d 16. 6. e 52. dat. f 1. def. d.

Secundo, ob triangula AEB, DEC [e] similia; [b] erit CD, DE :: AB. BE [c] :: CA + AB. CB. [d] ergo CA + AB in DE = CD in CB. atqui CD x CB [e] datur [f] ergo CA + AB in DE datum est. Q. E. D.

PROP. 95.



Si in circuli BAG *positione dati diametro* BG *sumatur datum punctum* D; *à puncto autem* D *in circulum producatur quædam recta* DA, *& agatur à sectione* A *ad rectos angulos in productam rectam* DA *linea* AE; *per punctum autem* E, *in quo linea* AE, *quæ ad rectos angulos consistit, occurrit circumferentiæ circuli, agatur parallela* (ECK) *productæ rectæ* DA; *datum est illud punctum* C, *in quo parallela* EK *occurrit ipsi diametro* BG; *& quod sub parallelis lineis* AD, EC *comprehenditur rectangulum, datum est.*

a 31. 3.

Nam connectatur AK. [a] estque AB (ob angulum E, vel DAE rectum) diameter. ergo in-

interſectio H eſt centrum. [b] ergo DH datur. Atqui ob KH. HA [c] :: CH. HD, [d] eſt CH = HD. [e] ergo CH datur. [f] ergo punctum C datur. Q. E. D. [g] ergo KC x CE, hoc eſt [d] AD x CE datur. Q. E. D.

b 16. dat.
c 4. 6.
d 9. 5.
e 1. def. d.
f 27. dat.
g 93. dat.

F I N I S.