The elements of Euclid explain'd. In a new, but most easie method: together with the use of every proposition through all parts of the mathematicks / Written in French, by that excellent mathematician, F. Claud. Francis Milliet de Chales ... And now made English. And a multitude of errors corrected, which had escap'd in the original.

Contributors

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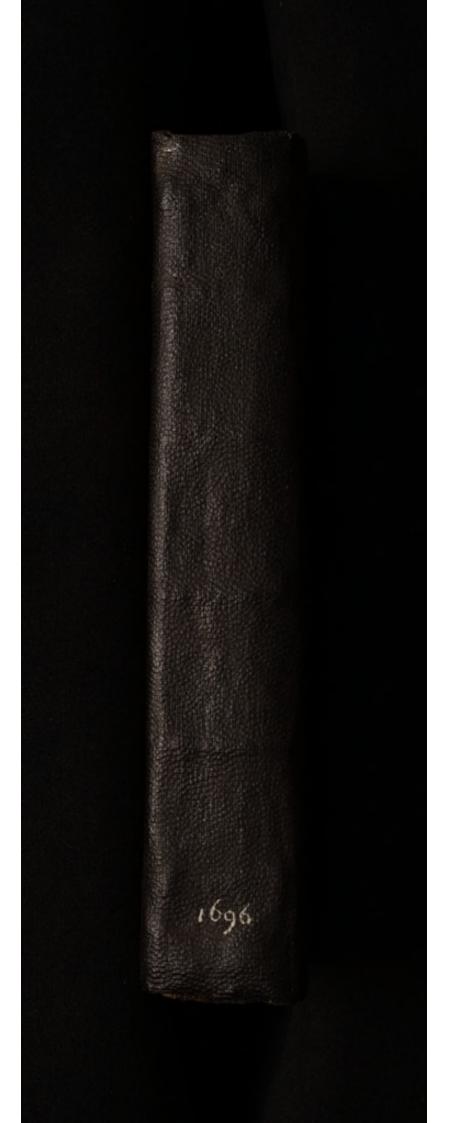
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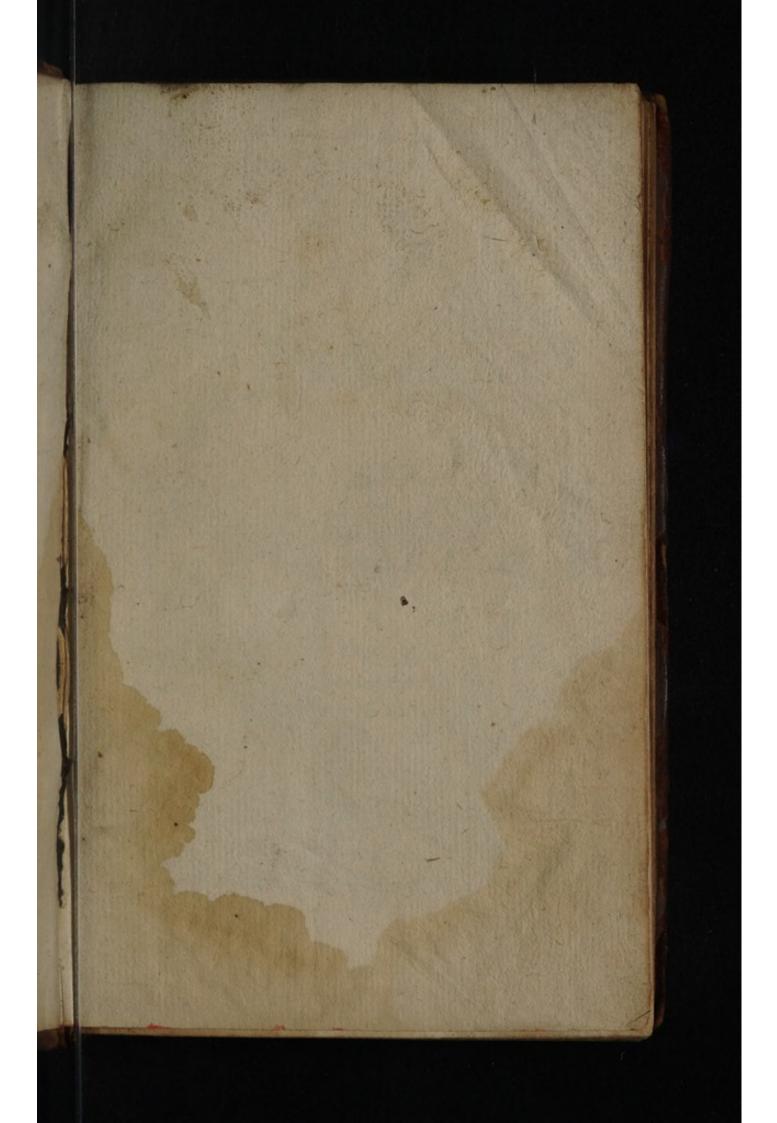
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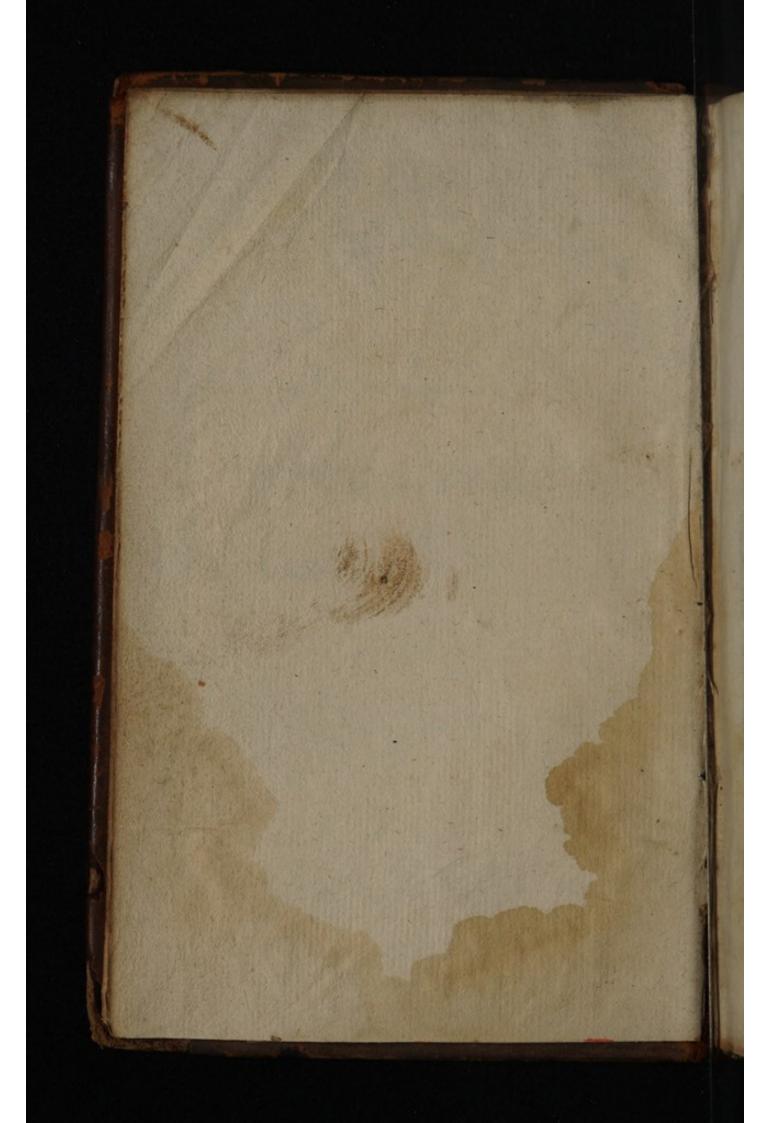
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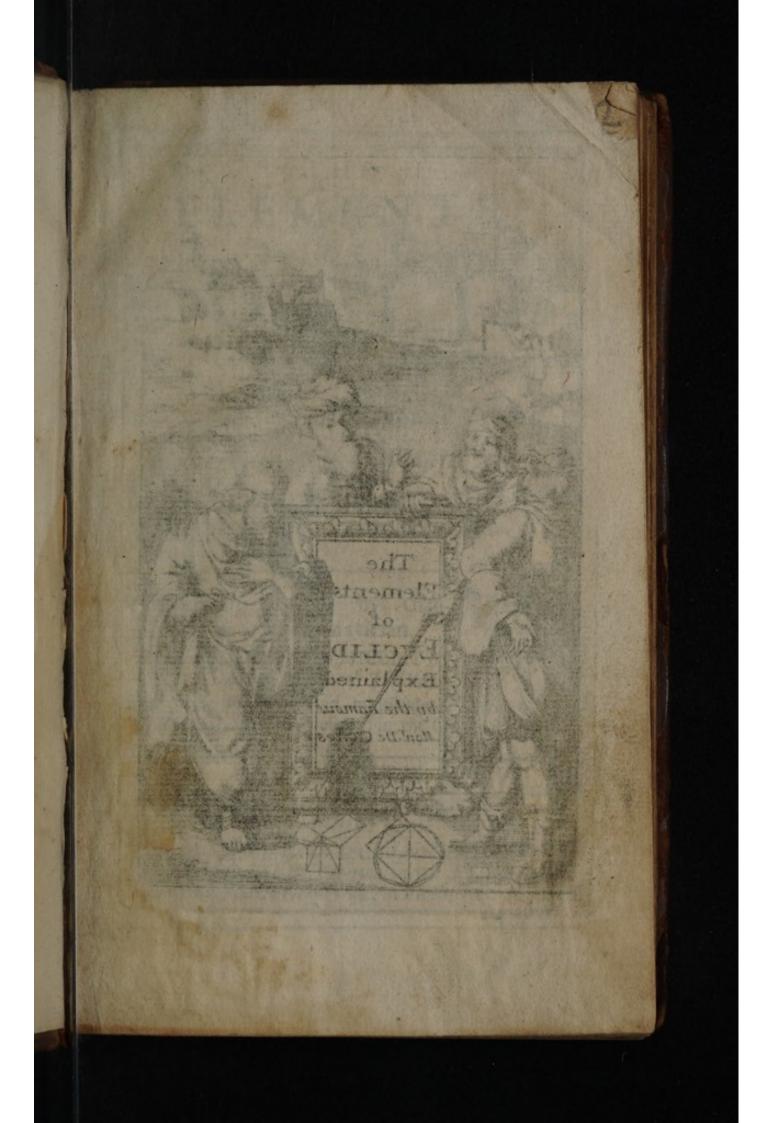
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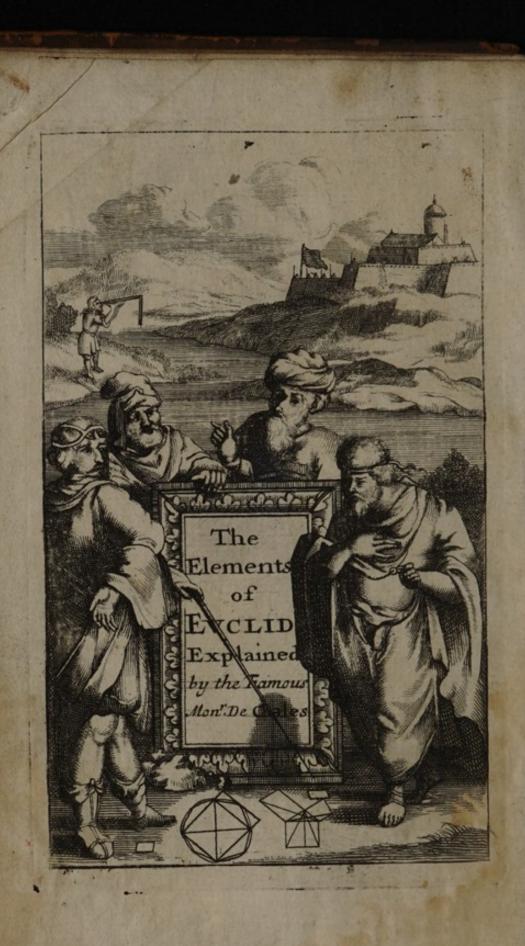
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THE

ELEMENTS

OF

EUCLID

Explain'd,

in a New, but most Easie method:

Together with

The Use of every Proposition through all parts of the Mathematicks.

Written in French, by that Excellent Mathematician,

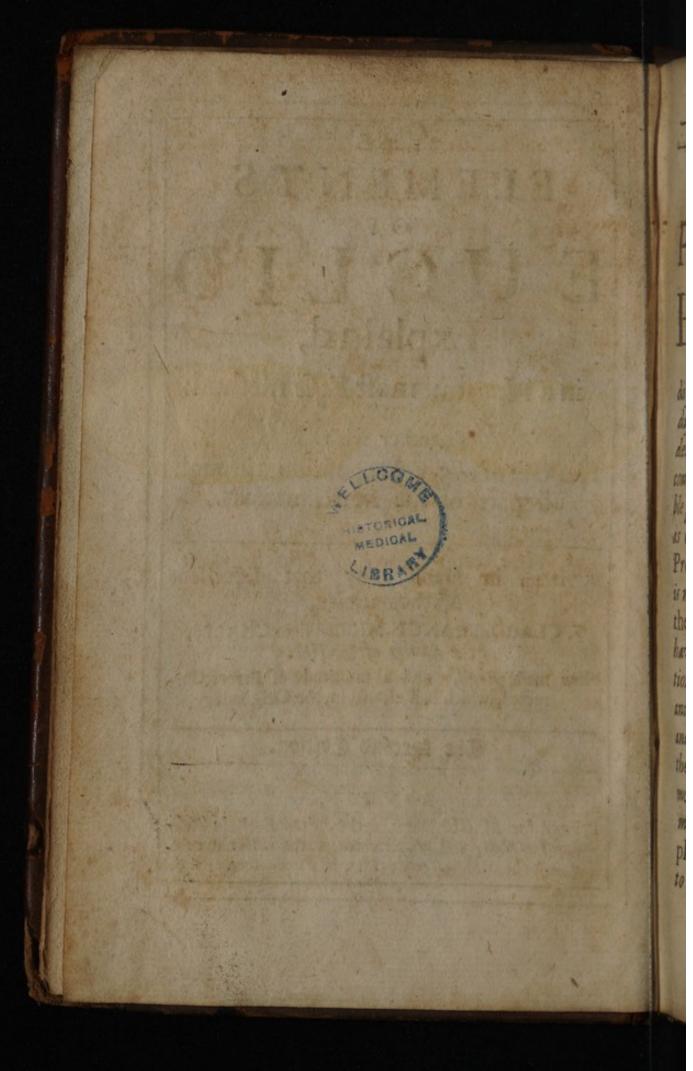
F. CLAUD. FRANCIS MILLIET de CHALES, of the Society of JESUS.

Now made English, and a multitude of Errors Corrected, which had escap'd in the Original.

The Second Edition.

LONDON,

Printed for M. Gillyslower at the Spread Eagle in Westminster-Hall, and W. Freeman at the Bible over against the Middle Temple Gate, in Fleet-street. 1696.



PREFACE,

Having for a long time observ'd, that most of those that take in hand the Elements of Euclid, are apt to dislike them, because they cannot presently discern, to what end those seemingly inconsiderable, and yet difficult Propositions, can conduce: I thought I should do an acceptable piece of service, in not only rendring them as easie as possible, but also adding to each Proposition a brief account of some Use, that is made of them in the other parts of the Mathematicks. In prosecuting which design, I have been oblig'd to change some Demonstrations, that seem'd too intricate and perplex'd, and above the ordinary capacity of Beginners, and to substitute others more intelligible in their stead. For the same reason, I have demonstrated the fifth Book after a method, much more clear, than that by Equimultiples, formerly used. I would not be thought to have set down all the Uses, that may be made

The Preface.

made of these Propositions: to have done that, would have oblig'd me to have compris'd the whole Mathematicks in this one Book; which would have render'd it both too large, and too difficult. But I have contented my self with the choice of such, as may serve to point out some of the Advantages they afford as, and are also in themselves most clear, and most easie to be apprehended. I have distinguish'd 'em by * Inverted Commas, that the Reader may know 'em; not desiring he should dwell too long upon 'em, or labour to understand 'em perfectly at sirst, since they depend on the Principles of the other Parts.

This therefore being the design of this small Treatise, I voluntarily offer it to the publick, in an Age, whose Genius seems more addi-Eted to the Mathematicks, than any that

has preceded it.

^{*} Instead of the Authors Italick Character.

eight Books of the Elements
of EUCLID, together
with the Use of the Propofitions.

THE FIRST BOOK.

HE design of EUCLID in this Book is to lay down the First Principles of Geometry; and to do it methodically, he begins with Definitions, and the explication of the most ordinary Terms. To these he adds some Postulata; and then Proposing those known Maxims, in which natural reason does instruct us, he pretends, not to advance a step farther without a Demonstration, but to convince every man, even the most obstinate, that will grant nothing, but what is extorted from him. In the first Propositions he treats of Lines, and the different Angles, which A 2

'are form'd by their concourse; and having occasion to compare divers Triangles together, 'in order to demonstrate the Properties of Angles, he makes that the business of the Eight first Propositions. Then tollow some Practical Instructions, how to divide an Angle and a Line into two equal parts, and to draw a Perpendicular. Next he shews the properties of a Triangle, together with those of Parallel "Lines; and having thus finish'd the Explication of this first figure, he passes on to Paral-'lelograms, teaching the manner of reducing any Polygone, or multangular figure into one "more regular. Lastly, he finishes the first Book with that famous Proposition of Pythagoras, That in every rectangular Triangle the Square of the * Base is equal to the Squares of both the other sides.

* He calls that the Base, which is commonly call'd the Hypotenuse, i. e. the Line that is opposite to the right

Angle.

DEFINITIONS.

This Definition must be understood in this sense: That quantity, which we conceive without distinguishing its parts, or so much as considering whether or no it has any, is a Mathematical point; which is therefore very dist-

different from those of Zeno, which were suppos'd to be absolutely indivisible, and therefore such, that we may reasonably doubt whether they are possible; but the former we cannot doubt of, if we conceive them aright.

2. A Line is length without breadth.

'The sense of this Definition is the same with the former That quantity, which we conceive as length, without reflecting on its breadth or thickness, is that, which we understand by a Line; though it be impossible to draw a real Line, which will not be of a certain breadth. Tis commonly said, that a Line is produc'd by the motion of a Point; which ought to be carefully observ'd; for motion may on that manner produce any quantity whatsoever: But here, we must imagine a Point to be only so mov'd, as to leave one trace in the space, through which it passes, and then, that trace will be a line.

3. The two Extreams of a Line are Points. 4. Aright Line is that, whose points are equal-

ly plac'd between the two Extreams.

Or thus. A right Line is the shortest that can be drawn from one point to another. Or yet. The Extreams of a right Line may cast a shadow upon the whole Line.

5. A Superficies or Surface is a quantity, to which is attributed length, and breadth, without

the consideration of any thickness.

A 4 6. The

7. A plane or right Superficies is that, whose lines are equally placed between its two Extreams; Or that, to which a right line may be every way apply'd.

A D

I have before observ'd, that motion may produce any quantity whatsoever: accordingly we say, when one line moves over another, it produces a Superficient or a Planta and all the perficient or a Planta and all the perfections are all the perfections are all the perfections are all the perfections are all the perfections and the perfection and the perfection and the perfection and the perfection are all the perfections are all the p

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perficies, or a Plane; and that that motion has a kind of affinity with Arithmetical Multiplication. Suppose then the 'line AB to pass along the line BC, retaining 'still the same situation, without any inclination to one side or other: the point A will describe s the line AD, the point B the line BC, and "the intermediate points the lines parallel to those, which will make up the Superficies A BCD. I add further, that this motion answers 6 to Arithmetical Multiplication; because did I know the number of points, that are contain'd 'in both those lines, AB, and DC; by multi-"plying them together, I should find a product, "which would give me the number of points, s which constitute the whole superficies ABCD. 'As for example, if AB contain'd four points, and BC lix, by faying four times fix make twenty four, I find, that the whole superficies ABCD confilts of twenty four points. Now

by a Mathematical point, may be understood any quantity whatsoever; e. g. a Foot, provided it be not subdivided into parts.

8. A plain Angle is the * distance or opening of two lines touching each other, so as not to compose.

only one line.

*Overture, Gall. wegs anninas unious. Eucl.

As the distance D betwixt the lines AB, and BC; which are not parts of the same line.

9. A Rectilineal Angle is the distance betwixt two right lines.

A C 'Tis chiefly of this fort of Angles that I would be understood at present; which I define by distance or opening, because Experience teaches, that the greatest part of Beginners deceive themselves in measuring the greatness of an Angle by that of the lines, within which it is contain'd.

The Angle that is more open, is the greater; that is, when the lines of one angle lie more apart from each other than those of another, taking them at the same di-

flance from the points of concourse, the former is greater than the latter. Accordingly, the angle A is greater than the angle E; because

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cause taking the points D and B as remote 'from the point A, as the points G and L are 'from the point E; the points D and B lie farther apart from each other, than the points G and L: from whence I infer, that if the lines "EG and EL were produc'd farther, the angle E would be always of the same largeness, and always less than the angle A.

We use three letters when we speak of an " Angle, of which the middlemost denotes the "point of concourse: as the angle BAD is the angle which by the lines BA and AD is form'd 'at the point A: the angle BAC is that made by the lines BA and AC: the angle CAD is

compris'd by the lines CA and AD.

A Circle is the measure of an Angle. Therefore to know the magnitude of the Angle BAD, I place the foot of the Compass upon the point A, and describe the circle BCD: the angle is so much the greater, by how many more parts of a circle the arch, that meafures it, contains: and because a circle is usueally divided into 360 parts, or degrees, therefore an angle is faid to have twenty, thirty, forty degrees, according as the arch, com-" pris'd betwixt the lines that form it, contains to many. So the angle is the greater, which contains more degrees, as the angle BAD is greater than the angle GEL. 'The line CA divides the angle BAD in the middle, because the

the arches BC and BD are equal; and the angle BAC is part of the angle BAD, because

the arch BC is part of the arch BD.

10. When one line falling upon another makes two equal angles, they are both right angles; and

the line perpendicular.

As for example: if the line 'AB, plac'd upon the line C D, make the angles ABC and ABD equal; that is, if, having describ'd a semicircle CAD from the center B, the arches AC and AD are equal: the angles ABC, and ABD are call'd right angles, and the line AB per-Therefore because the arch CAD pendicular. is a semicircle, the arches CA and AD are each of them a quarter of a circle, that is, the fourth part of three hundred and fixty degrees, that is, ninety.

11. An Obtuse angle is that which is greater

than a right one.

' As the angle EBD is an obtuse or blunt angle, because its arch EAD contains more than a quarter of a circle.

12. An Acute angle is that which is less than

a right one.

As the angle EBC is an acute, because the arch EC, which measures it, has less than ninety degrees.

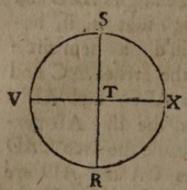
13. A Term is the extremity or end of any 14 A quantity

14. A Figure is a quantity comprehended by one or more Terms.

'That which is call'd a Figure ought to be

'limited and inclos'd on every side.

15. A Circle is a plain figure, terminated by the encompassing of one line, which is call d the Circumference; and is every where equally remote from the middle point.



'The Figure RVSX is a Circle, because all the lines TR, TV, TS, TX, drawn from the point T, to the line RVSX, are equal.

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16. The middle point is

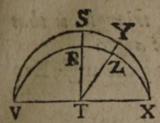
call'd the Center.

17. The Diameter of a Circle is any line pafsing through the Center, and terminated at the Circumference, dividing the Circle into two equal parts.

'As the lines VTX, and RTS.

But if any should dou, whether the line VTX does indeed divide the circle into two equal parts, so that the part VSX be equal to the part VRX; it may on this manner be prov'd.

Suppose the part VRX to be plac'd upon the other VSX: I say, they will not exceed one



the other. For if one, suppose VSX exceed the other VRX, the line TR will be less than TS; and in like manner TZ than TY, which is contrary to

the definition of a Circle, which affirms all the lines drawn from the center to the circumference to be equal.

18. A Semicircle is a figure terminated by the Diameter, and half the Circumference.

19. Rectilineal figures are such as are terminated by right lines, having three, or four, or five, or as many sides as you please.

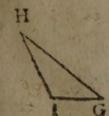
Euclid divides Triangles with respect either

to their angles, or fides.

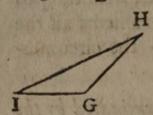
BAC

20. An Equilateral Triangle is that which has its three sides equal: ABC.

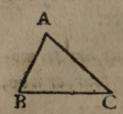
A 21: An Isosceles, or equicrural Triangle, is that which has two sides equal: As if the two sides AB, and AC be equal, the triangle ABC is an Isosceles.



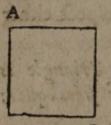
22. A Scalenum is a triangle having all the three sides unequal, as GHI. 23. A Rectangle triangle is that which has one right angle. ' As DEF, supposing the angle E to be a right one.



24. An Ambligone, or Obtusangle triangle is that which has one angle obtuse. As IGH.



25. Oxygone, or an Acutangle triangle is that whose angles are all acute. As ABC.

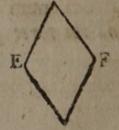


26. A Rectangle (properly so call'd) is a figure confifting of four sides, and baving all its angles right.

27. A Square has all its sides equal, and its angles right, as AB.

28. An Oblong Rectangle has its sides unequal, but its angles right: as CD.

29. A Rhombus, or Losange, has equal sides, but unequal angles: as E.F.



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30. A Rhomboides, or oblong Losange, bath both its sides and angles unequal: as GH. 31. Other irregular figures of four sides are call'd Trapelia. 33. Parallel lines are such, as being in the same D plane will never concur, keeping still an equal distance one from the other: as AB, CD. 33. A Parallelogram is a figure, whose two opposite sides are Parallels : 'as the Figure ABCD, whose fides AB, CD; 'and AC, BD, are parallels. 34. The Diameter of a Parallelogram is a right line drawn from one angle to another: as 35. The Complements are the two small Parallelograms, through which the Diameter does

DEMANDS, or SUPPOSITIONS.

not pass: as AFEH, and GDIE.

I. I'll suppos'd that a right line may be drawn from any point whatsoever to another.

2. That a right line may be continu'd to what length you please.

3. That

3. That from a Center given a Circle may be describ'd at any distance whatsoever.

MAXIMS, or AXIOMS.

1. Hose quantities that are equal to a third, are equal betwixt themselves.

2. If equal quantities be added to those that

are equal, the products will also be equal.

3. If equal quantities be taken away from those that are equal, the remainders will be equal.

4. If you add equal parts to quantities une-

qual, they will remain unequal.

5. If from equal quantities you take away unequal parts, the remainders will be unequal.

6. Quantities that are double, triple, quadruple, &c. in respect of the same, are equal among themselves.

7. Those quantities are said to be equal, which being apply'd one to the other, neither

exceeds.

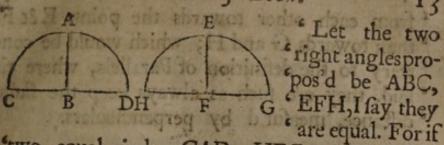
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8. Equal lines and angles being plac'd one upon another, do not surpass each other.

9. The whole is greater than its part.

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10. All right angles are equal to one anocher.



two equal circles CAD, HEG, be describ'd from the centers B and F; the fourth parts of those circles CA, HE, which are the measures of the angles, ABC, EFH, will be equal: therefore the angles ABC, EFH, having equal measures, will be equal.

The eleventh Maxim of

Euclid is to this effect. If two
lines AB, CD, being cut by a
third EF, make the internal
angles, BEF, DFE, less than
two right angles; the lines AB, CD being no

two right angles; the lines AB, CD being produc'd, will at length concur towards the points B and D.

Which, though it be true, is not clear enough to be received for a Maxim: therefore I have substituted another in its place.

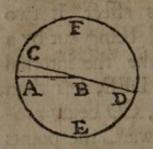
diculars contain'd betwixt them will be equal.

AE GB 'As for example, if the lines AB, 'CD, are parallels, the perpendicular lines FE, HG, are equal. For if EF was greater than GH, the lines AB and CD, would be more remote B

from each other towards the points E&F; than towards G and H; which would be contrary to the definition of Parallels, where tis faid, they are such as always keep the same distance, measur'd by perpendiculars.

12. Two right lines cannot enclose any space; that is to say, they cannot encompass it on all sides.

13: Two right lines cannot have one common segment.



By which I mean, that two right lines, suppose AB, and CB, meeting at the point B, cannot together make one sole line BD; but cutting one another separate again immedi-

ately after their rencounter. For, if you deficible a circle from the point B as a center, AFD will be a semicircle, because the right line ABD, passing through the center B, will divide the circle into two equal parts. The segment CFD will be also a semicircle, because CBD will be also a semicircle, because CBD will be also a right line, and will pass through the center B: therefore the segment CFD will be equal to the segment AFD. the part to the whole; which is repugnant to the ninth Maxim.

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There are two forts of Propositions: In some we have nothing but the bare Speculation of a Truth, without descending to Practice, which we call Theorems; in the other something is propos'd to be done, and those are call'd Problems.

The first number of the quotations denotes the Propositions, the second the Book. As by the 2 of the 3. that is, by the second Proposition of the third Book: but if only one number occur, it signifies such a Proposition of the book you are then upon.

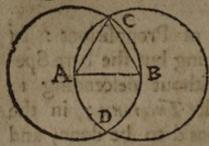
PROPOSITION I.

A PROBLEM.

To draw an Equilateral Triangle upon any line given.

Let T the line AB be propos'd for the base of an Equilateral Triangle; from the center A at the distance AB describe the circle BCD; and likewise from the center B at the distance BA describe the circle DAC cut-

ting the former at the point C. Then draw the lines AC and BC; and all the fides of the triangle ABC will be equal.



Demonstration The Lines AB, and AC being drawn from the same center A to the circumference of the circle BCD are e-

qual, by the Definition of a Circle; the lines BA, and BC are likewise equal being drawn from the center B to the circumference of the circle CAD. Lastly the lines AC and BC being equal to the same line AB, are also equal be-All the three fides therefore tween themselves. of the triangle ABC are equal.

The USE.

'The defign of Euclid in placing this Problem here was only to demonstrate the two following Propositions. But it may be also further ferviceable for the measuring

an inaccessible line, as for example, the line AB, which by reason of a River or Precipice cannot be approach'd. In fuch a case make

'a small Equilateral Triangle BDE, either of 'Wood, or Copper, or the like; and having placed it Horizontally upon B, observe the

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opint A, by the side BD, and any other point C, by the side BE. Then transfer your Triangle along the line BC, and place it upon divers parts of the same line; till at length you find a point C, upon which placing the Triangle you shall see the point B, by the side CG, and the point A by the side CF. I say the lines CB and CA are equal; so that by measuring the line BC, you may know the line AB. I might further demonstrate that the lines AB, and BC are equal; but let it suffice that in this Proposition you are taught the way of making an Instrument proper to take the dimensions of an inaccessible line.

PROPOSITION II.

Distribut A PROBLEM.

From a point given to draw a line equal to another line given.

ET the point proposed be B, from which a line is to be drawn equal to the line A. Take with the Compass the length of the line A, and at that interval, making B the Center, describe the circle CD. Drawing then from B 3

the point B to which side you please, a line BI, or BD, 'tis evident it will be equal to the

line A.

Euclid proposes a more mysterious and intricate method of demonstrating this Proposition; but in practice we always make use of this; in as much as, having taken with the compass the line A, tis as easie describing a circle from the center B, as from the center A.

PROPOSITION III.

A PROBLEM.

From a greater line to take a part equal to a less.

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Suppose you were to take from the line BC, a part BI, equal to the line A. Take betwixt the points of the compass the length of the line A, and at that distance, from the center B describe a circle, which shall cut the line BC at the point I. Tis certain the lines BI, and A, are equal.

The Use of these two preceding Propositions is sufficiently evident; for as much as we are frequently oblig'd in practical Geometry to draw one line equal to another; and to take a part of a greater line equal to a line that is less.

PROP-

PROPOSITION IV.

A THEOREM.

ſe

If two Triangles have two sides equal, each to the other respectively, and the angles also, form'd by those two sides, equal; their bases and other angles will be equal.

I E T the triangles A
BC, DEF, have two
fides equal each to the
other respectively; that
is to say, let AB be
equal to DE, and AC to

DF; and let the angles BAC, EDF, form'd by those sides be also equal; I say, the bases BC, EF, are equal, and the angles ABC, DEF; ACB, DFE, are equal; and lastly, the whole triangles equal in all respects,

Demonstration.

Suppose the triangle DEF to be plac'd upon the triangle ABC: the side DE being upon AB, they will not exceed each other, because they are suppos'd to be equal; so that the point E will be upon B, and the point D upon the point A. For the same reason the line DF will fall upon AC. For if it should fall on the out-

fide of it, the angle EDF would be greater than the angle BAC; and if it should fall within AC, the angle EDF would be less: and yet they are suppos'd to be equal. Therefore since the point D is upon the point A, and the line DF falls upon the line AC, to which it is equal, they will not exceed each other, but the point F will fall upon C. Lastly, fince the points E and F of the line EF, fall upon B and C; the line EF will fall upon BC; because it can neither fall higher as in BHC, nor lower as in BGC; for then two right lines would enclose space; which is contrary to the twelfth Maxim. Therefore the two triangles do not at all exceed each other; but not only the bases BC, EF, but also the angles ABC, DEF; and ACB, DFE, are equal.

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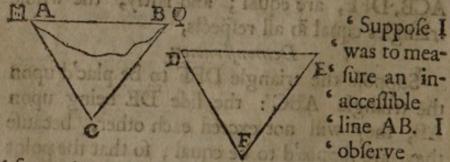
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Coroll. An Equilateral triangle hath all its

angles equal.

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from the point C the points A, and B; and then measure the angle C. This done, placing a Board horizontally, and observing successive-

lines according to the rule, which make the lines according to the rule, which make the angle C; and measure with a yard the lines AC, and BC, which are supposed accessible. Then going into an open field, and placing my Board again horizontally upon the point F, and observing the lines that I drew upon it, I make an angle DFE equal to the angle C; I make likewise FD, FE, equal to CA, CB, Then according to this Proposition the lines AB, and DE, are equal. So that measuring by the yard the accessible line DE, I shall know AB, which is inaccessible.

Another USE.

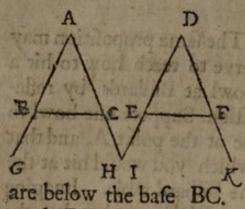
'ferve to teach how to hit a bowl at Billiards by reflection. Suppose one bowl to be at the point A, and that which you would hit at the point B, and CD the Billiard table. Imagine then a perpendicular BDE, and take the line DE equal to BD. I say, if you direct the bowl from the point A to E, the reflexion will carry it to B. For in the triangles BFD, EFD, the side FD being common, and the sides BD and DE equal; the angles BFD, EFD are equal, by this proposition. The angles

gles AFC, DFE, being opposite, are also equal, as I shall demonstrate hereafter. Therefore the angle of Incidence AFC, is equal to the angle of Reflection BFD; and by consequence the Reflection will be by AFB.

PROPOSITION V.

THEOREM.

In Isosceles, or Equicrural triangles, the angles that are above the Base are equal: as also those that are below it.



Et the Isosceles be ABC, that is to fay, let the fides AB, AC be equal. I say the angles ABC, ACB are equal; as also the angles GBC, HCB, that Suppose another triangle DEF, having the angle D equal to the angle A; and the sides DE, DF, equal to AB, AC. Since the fides AB, AC are equal, all the

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four lines AB, AC, DE, DF will be equal. Demonstration. Since the sides AB, DE; AC, DF, are equal; as also the angles A, and D: if the triangle DEF be plac'd upon ABC, they will not exceed each other, but the line DE will fall upon AB; DF upon AC; and EF upon BC (by the 4th.) therefore the angle DEF, will be equal to ABC. And because one part of the line DE falls upon AB, the whole line DI will be upon AG; otherwise two right lines would have a common fegment; therefore the angle IEF will be equal to GBC. Suppose then the triangle DEF turn'd, and apply'd another way to the triangle ABC, that is to fay, fo as DF may fall upon AB, and DE upon AC. Since the four lines AB, DF; AC, DE, are equal; as also the angles A and D: the triangles will likewise agree this way, and the angles ACB, DEF; HCB, IEF, will be equal. Now by the first comparing them it appear'd, that the angle ABC was equal to the angle DEF; GBC to IEF: therefor the angles ABC, ACB being equal to the fame DEF; and GBC, HCB, also equal to the fame IEF, they are equal among themfelves.

'I was unwilling to make use of Euclid's demonstration, because being very difficult,

it might discourage beginners,

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PROPOSITION. In live

apon BC (by the M.) the Lore the angle

If two angles of a triangle be equal, the triangle will be an Isosceles.

ET the angles ABC, ACB of the triangle ABC be equal: (see Fig. preced.) I say it is an Isosceles; that is to say, the two sides AB, AC, which are opposite to the equal angles, are equal. Suppose the triangle DEF to have a base EF equal to BC, and the angle DEF equal to ABC, as also DFE equal to ACB: fince the angles ABC, ACB are suppos'd to be equal, all the four angles ABC, ACB, DEF, DFE, will be equal. Suppose again therefore the base EF to be plac'd upon the base BC, so that the point E lie upon the point B, the bases being suppos'd equal it is evident they will not exceed each other. Further, the angle E being equal to the angle B, and the angle F to the angle C; the line ED will fall upon the line BA, and FD upon CA: fo that the lines ED and FD will meet at the point A. From whence it follows, that the line EC is equal to BA.

Let then the triangle DEF be turn'd to the other side, and be applied another way to the

tri-

triangle ABC: that is to fay, so that the point E lie upon C, and F upon B: the bases BC, FE will perfectly agree, being suppos'd to be equal: and because the angles F, and B; E, and C, are also suppos'd to be equal, the side FD will fall upon BA, and ED upon CA; and the point D upon A. Therefore the lines AC, DE will be equal. Whence it follows, that the sides AC, AB are equal between themselves, being equal to the same side DE.

The USE.

'This Proposition may serve for taking the dimensions of any fort of inaccessible lines.
'Tis said that Thales was the first that measur'd the heighth of Obelisks by their sha-

dows: it may be done by this Proposition.

For if you were to measure the height of the Obelisk AB; do but expect till the Sun be elevated 45 degrees above the Horizon; that is to say, till the angle ACB be 45 degrees; and, by the sixth Proposition, the shadow BC will be equal to the Obelisk AB. For since the angle ABC is a right angle, and the angle ACB half a right one, or of 45 degrees; the angle CAB will be half a right one, as I shall prove hereafter. Therefore the angles BCA,

BAC, are equal: and (by the 6.) the sides AB,

BC, are also equal. I can also measure the same height without making use of the shadow, by taking a stand so far from the point B, as that

the angle ACB may be half a right angle, which may be known by a Quadrant.

'These Propositions are of frequent use in

Trignometry, and in all other tracts.

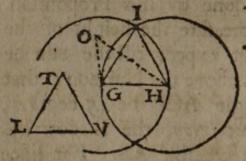
'The seventh Proposition may be omitted, because tis of no other use but to demonstrate

the eighth, which may be done without it.

PROPOSITION VIIL

THEOREM.

If two Triangles have all their sides equal, their opposite angles will also be equal.



ET the fide GI be equal to LT; HI, to VT; GH, to LV; I fay, that the angle GIH, will be equal to the angle LTV; IGH, man

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to the angle L; and IHG, to the angle V. From the center H, at the distance HI, describe the circle IG; and from the center G, at the distance GI, the circle HI.

Demon-

Demonstration.

Suppose the line LV brought upon HG: they would not exceed each other, because they are suppos'd to be equal. I add, that the point T will fall precisely upon the point I: For it ought to reach precifely to the circumference of the circle IG, because by the supposition the lines HI and VT are equal. It ought in like manner to reach to the circumference of the circle IH, because the lines GI and LT are equal. So then it will light upon the point I, being the point where those two circles cut each other. Indeed if it fell any where else, as upon O, the line HO, that is to fay VT, would be greater than HI; and the line GO, that is LT, would be less than GI; which is against the supposition. Whence I conclude, that the triangles will exactly correspond, and the angle GIH be equal to the Angle LTV.

П

The USE.

This Proposition is necessary for the proof of those that follow. And further, when we connot take the measure of an angle, because, the lines meeting in a solid body, we cannot apply our Instruments to it; we must take the three sides of the triangle, and make another upon a paper, whose angles we may measure. This is a very ordinary practice in

Gnomonicks, or Dialling; and in the Treatifes concerning cutting precious stones, so as to fit the pannels, and to retain the waters.

PROPOSITION

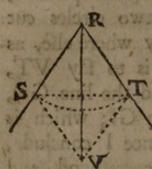
of the circle IG because by the supposition the

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To divide an Angle into two equal parts. quak, do then it will light upon the point I

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R ET the angle SRT be propos'd to be divided into two equal parts. Take the Compass, and from the center R, at any distance, draw the arch ST, cutting off two equal lines RS, RT. Then into hoth the fem age to a fee de Com

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draw the right line ST, and (by the 1.) describe an equilateral triangle STV. I say, the line VR divides the angle into two equal parts: that is to say, the angles VRT, and VRS, are equal.

Demonstration.

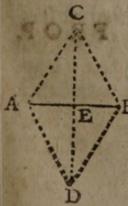
The triangles VRS, and VRT, have the fide VR common; and the fide RT was taken equal to the fide RS: the base also SV, is equal to VT, because the triangle SVT is equilateral. Wherefore (by the 8) the angles SRV, The VRT, are e ual.

Ine AB is cis on The USE is at SA and This Proposition is very useful to divide the fourth part of a circle into degrees: for tis the same thing to divide an arch, as an angle 'into two equal parts; and the line RV does both, that is, it divides both the arch ST, and the angle SRT. Having therefore apply'd the femidiameter to the fourth part of a circle, you cut off an arch of 60 degrees, which divided equally gives an arch of 30; and that 'again divided, makes one of 15 degrees. Tis true, to finish this division, we must divide an arch into three equal parts, but that is not to be done Geometrically. Pilots also divide the *Compass into 32 winds by the help of this Proposition only.

PROPOSITION X

A PROBLEM.

To divide a right line into two equal parts.



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CUppose the line AB was to be divided into two equal parts; upon the line AB describe an equilateral triangle AB C, (by the 1.) and divide the angle ACB into two equal parts by the line DC, (by the 9.) I say

30 The Elements of Euclid.

the line AB is divided equally at the point E; that is to fay, the lines AE and EB are equal.

Demonstration.

The triangles ACE, and BCE have the fide CE common, and the fides CA and CB are equal, because the triangle ACB is equilateral: and the angle ACB being divided equally, the angles ACE and BCE are also equal. Therefore (by the 4.) the bases AE and BE are equal.

the USE.

Great use is made of this Proposition, ordinary practices frequently requiring us to divide a line in the middle, which Geometricians require should be done exactly at the first dash, by a method that is infallible, and not by estays. This practice is likewise principally useful for dividing the parts.

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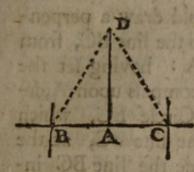
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PROPOSITION. XI.

A PROBLEM.

To draw a Perpendicular to a line given, upon a point of the same line.



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Suppose you were to raise a perpendicular upon the point A of the line BC. Take two equal lines AB and AC on both sides the point A, and make an equilateral triangle BDC up-

on the line BC, (by the 1.) I say the line AD is perpendicular, that is to say, the Angles BAD and CAD are equal.

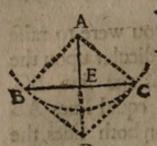
Demonstration:

The triangles BAD, and CAD have the fide AD common, the fides AC and AB are equal, and the bases BD and DC also equal: therefore (by the 8.) the angles BAD, and CAD, are equal; and (by the 10. def.) the line AD perpendicular to BC.

PROPOSITION. XIL

A PROBLEM.

To draw a perpendicular to a line given, from a point which is out of the line.



PROP

If you would draw a perpendicular to the line BC, from the point A: having set the foot of the compass upon A, describe the Circle BC, which shall cut the line BC, at the

points B and C. Then divide the line BC into two equal parts at the point E. I say the line AE is perpendicular to BC. Draw the lines AB, AC.

Demonstration. The triangles BEA, and CEA have the side AE common; and the sides EC and EB equal, the line BC having been equally divided at the point E; the bases AB and AC, being drawn from the center A to the circumference BC, are likewise equal: therefore the angles AEB, and AEC, are equal, (by the 8) and the line AE perpendicular, (by defin. 10.)

The method, in practice, of dividing the line BC in the middle, ir to describe two arches at D, at the same interval, from the centers B and C. 074

We have need of a Plummet or Squaringline almost in all our operations: no angles are of use in buildings but the right; and all chairs, benches, tables, buffets, and other moveables, are fram'd by the square. No survey of Land can be taken without making use of perpendicular lines; nor can Dialling be perform'd without them; The Carpenter's Level contains a right angle, and the same is preferr'd before any other, especially by the French, in Fortifications. Lastly, not only Mathematicians, but also the greatest part of practical Artisans, require that we should know how to draw a perpendicular.

PROPOSITION XIII.

A THEOREM.

One line falling upon another makes with it either two right angles, or two angles equal to two right ones.

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ine D, C.

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I Et the line AD fall upon BG; I say, 'twill make with it either two right angles; or two angles, one obtuse, and the other acute, G which joyn'd together shall be of equal value with two right ones, Demon-

Demonstration.

Suppose the line AD to fall perpendicularly upon BG, then tis evident (by defin. 10.) that the angles ADB, and ADG, are equal, and by confequence right angles.

Secondly, suppose the line ED not to fall perpendicularly upon BC, and draw a perpendicular AD (by the 11.) the angles ADB, and ADC are right angles, which are of e-

qual value with the three angles ADC, ADE, EDB. But the obtuse angle EDC, and the acute angle EDB, are of equal value with the three angles ADC, ADE, and EDB: therefore the angles EDC, and EDB, are of equal value

with two right ones.

This Proposition may be more easily demonstrated by describing a semicircle from the center D upon the line BC. For the angles EDB, and EDC, will require a semicircle for their measure, which is the measure of two right angles, as I have shewn before; in the 8 definition.

Coroll. 1. If the line AD falling upon BC, make one right angle ADC; it is evident the

other, ADB, will be also a right angle.

Coroll. 2. If the line ED, falling upon BC, make the angle EDB acute; the angle EDC will be obtuse.

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The USE.

By this means, when we know one of the angles which is made by one line falling upon another, we know also the other: as for example, if the angle EDB be one of 70 degrees, taking away 70 from 180, there will remain 110 for the angle EDC. This operation does frequently occur in Trigonometry; and also in Astronomy, for sinding the eccentricity of the circle through which the Sun anually passes.

PROPOSITION XIV.

A THEOREM.

If two lines meeting together at the same point of another line, make with it two angles equal to two right ones; they will make but one and the same line.

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Suppose the lines CA, and DA, to meet at the point A of the line AB; and that the angles adjoyning, CAB, and BAD, are equal to two right ones. I say, the lines CA and DA are but one and the same line; so that CA being continued, will fall precisely upon AD.

Imagine, if you please, that CA continu'd will pass on to E, and from the center A describe a circle.

Demonstration.

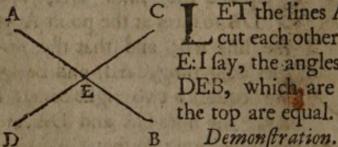
If you say that CAE is a right line, the arch CBE will be a semicircle. But 'tis suppos'd, that the angles CAB, and BAD are equal to two right ones, and that therefore their measure is a semicircle. Therefore the arches CBE, and CBD will be equal; which is impossible, one being a part of the other. Therefore the line CA being continu'd, will make but one and the fame line with AD.

PROPOSITION XV.

A THEOREM.

If two right lines cut each other, the opposite angles * at the top will be equal.

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ET the lines AB and CD cut each other at the point E: I say, the angles AEC, and DEB, which are opposite at the top are equal.

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The line CE falling upon the line AB, makes the the angles AEC and CEB equal to two right ones, (by the 13.) In like manner the line BE falling upon the line CD, makes the angle CE3 and BED equal to two right ones. Therefore the angles AEC, CEB, taken together, are equal to the angles CEB, BED; therefore taking away the angle CEB from both, the angle AEC will remain equal to DEB, (by the 3. Maxim.)

Coroll. 1. If two lines DE, and EC, concurring at the same point E of the line AB, form with it the opposite angles AEC, DEB equal,

DE and EC make but one right line.

Demonstration.

The line EC falling upon the line AB, makes the angles AEC, and CEB equal to two right ones, (by the 13.) 'Tis supposed likewise that the angle DEB is equal to the angle AEC. Therefore the angles DEB, BEC, are equal to two right ones. And (by the 14.) the lines CE and ED make but one right line.

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A The two preceding Propofitions are made use of to prove, that two lines make but one total. As for example, in Catopcricks or Perspectives, where that is required to prove, that all the lines thar

that can be drawn by reflexion from the point A to the point B, those are the shortest, which make the angle of Incidence equal to the angle of Reflexion. As for example; if the angles BED and AEF be equal, the lines AE, and EB, are shorter than AF, and FB. From the point B draw a perpendicular BD, and make the lines BD and CD equal; then draw EC, and FC. First in the triangles BED and " CED the side DE is common; and the sides BD, and DC being equal, as also the angles 'BDE, and CDE; the bases BE, and CE will be equal; as also the angles BED, and DEC, (by the 4.) In like manner I may prove, that BF, and CF are equal:

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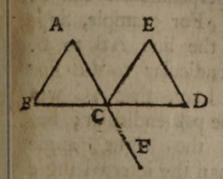
Demonstration.

"The angles BED and DEC are equal, and the angles BED and AEF are suppos'd likewise to be equal; therefore the opposite angles DEC and AEF will be equal; and (by the Coroll. of the 15.) AEC one right line; and by confequence AFC is a triangle, of which the fides AF and FC must be longer than AEC, that is to fay, than AE, and EB. But the lines AF and FG are equal to the lines AF and FB; therefore the lines AF and FB are longer than the lines AE and EB. And fince natural caue fes always act by the shortest lines, the Resles xion will always happen in such a manner, that * the angles of Reflexion and Incidence may be equal. FurFurther, because we can easily prove, that all the angles that can be made upon a plane about the same point, are equal to four right angles; for as much as in the first figure of this proposition, the angles AEC and AED are equal to two right ones, as also BEC and BED to two more: we make a general rule to determine what Polygones may be joyn'd in paving a Hall. Accordingly we say, that four squares, six triangles, and three hexagones, may be used for that purpose; and that therefore Bees are always observed to make their little cells of the last, that is, of sigures consisting of six sides.

PROPOSITION XVI.

A THEOREM.

The external angle of a triangle is greater than either of the internal opposite angles.



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Produce the fide BC of the triangle ABC I fay the external angle ACD, is greater than either of the internal opposite angles, ABC, or BAC. Suppose the tri-

angle ABC to be mov'd along the line BD,

and carry'd into the place of CED.

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'Tis impossible that the triangle ABC should be so mov'd, but the point A must come into the place of the point E; and then t'will appear, that the angle ECD, that is to say, ABC, is less than the angle ACD: therefore the internal angle ABC is less than the external ACD.

'Tis likewise easie to prove, that the angle A is less than the external angle ACD: for having prolong'd the side AC as far as F, the opposite angles BCF, and ACD, are equal (by the 15.) Therefore causing the triangle ABC to slide along the line ACF, I shall demonstrate, the angle BCF to be greater than the angle A.

The USE.

We may draw from this proposition many most useful conclusions. As first, that from a point given only one perpendicular can be drawn to the same line. For example, Sup-

A 'pose the line AB to be 'perpendicular to the line 'BC: I say, that AC will 'not be perpendicular; befor example, SupA 'pose the line AB to be 'perpendicular to the line 'BC: I say, that AC will 'not be perpendicular; because the right angle 'ABD must be greater than the internal angle 'ACB; therefore ACB cannot be a right angle,

nor AC a perpendicular. Se-

Secondly, that from the same point A cannot be drawn more than two equal lines; for example, AC, and AD; and if you draw a third as AE, it will not be equal to the former. For fince AC and AD are equal, the angles, ACD and ADC, are equal, (by the 5.) but in the triangle AEC, the external angle · ACB is greater than the internal AEC: and therefore likewise the angle ADE, is greater than the angle AED. Therefore the lines AE, and AD, are not equal; nor by consequence

. AC and AE.

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'Thirdly, that if the line AC makes the angle ACB acute, and ACF obtuse, the perpendicular drawn from the point A will fall on the side of the acute angle. For if you say that AE is a perpendicular, and that AEF is a right angle; the right angle AEF would be greater than the obtuse ACE. These conclufions are serviceable for measuring Parallelolograms, Triangles, and Trapelia, and to reduce them into rectangular figures.

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This Proposition is necessary to demon-

angles ABO, and ACE, to se less than two

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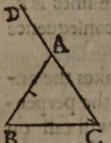
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PROPOSITION XVII.

A THEOREM.

Any two Angles of a triangle are less than two right ones.



L fay, that any two of its angles taken together, as BAC, and BCA, are less than two right ones. Produce the fide CA to the point D.

The internal angle C, is less than the external BAD, (by the 16.) Add therefore to both the angle BAC; the angles BAC, and BCA, will be less than the angles BAC, and BAD; yet those are but equal to two right ones, (by the 13.) therefore the angles BAC, and BCA,

After the same manner I can demonstrate the angles ABC, and ACB, to be less than two right ones, by producing the side BC.

Coroll. If one angle of a triangle be a right, or

obtuse angle, the others are acute.

are less than two right ones.

"This Proposition is necessary to demonstrate those that follow. \$ 100

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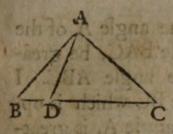
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PROPOSITION XVIII.

A THEOREM.

In every triangle whatsoever the greatest side is oppos'd to the greatest angle,



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Cuppose the side BC of the O triangle ABC, to be greater than the fide AC: I fay, the angle BAC, that is oppos'd to the side BC, is greater than the angle B, which is oppos'd to the side AC. Cut the line BC in D, fo that CD may be equal to AC; then draw the line AD.

Demonstration.

Since the sides AC, and CD, are equal, the triangle ACD will be an Isosceles, and (by the 5.) the angles CDA, and CAD, equal. Now the whole angle BAC is greater than the angle CA D: therefore the angle BAC is greater than the angle CDA; which yet, being an external angle in respect of the triangle ABD, is greater than the internal B, (by the 16.) Therefore the angle BAC is greater than the angle B.

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PROPOSITION XIX.

A THEOREM.

In every triangle the greatest angle is opposed to the greatest side.

and to A shill so TogET the angle A of the riangle BAC, be greater than the angle ABC. I c fay, the fide BC which is oppos'd to the angle A, is greater than the fide AC, that is oppos'd to the angle B. Demonstration. di OA of Con

If the fide BC be not greater than the fide AC, 'tis either equal; and then the angles A and B would be equal, (by the 5.) which is contrary to the supposition: or less; and if so, the side AC, being greater than BC, the angle B would be greater than the angle A, though the contrary be suppos'd. It remains therefore that the fide BC be greater than the fide AC.

The USE. The olders out

We may prove from these propositions 6 not only that no more than one perpendicular

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can be drawn from the same point to the same line; but also that it is the shortest of all. As

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nar can for example; if the line RV be perpendicular to ST, it will be less than RS: because the angle RVS being a right angle, the angle RSV will be an acute, (by the Coroll. of the 17.) and the line RV will be less than

RS, (by the preceding.) Therefore Geometricians do always make use of a perpendicular, when they take the dimensions of any thing, and reduce irregular figures to such as have one or more right angles. I add, that it being impossible that more than three perpendiculars should meet at the same point, it cannot be imagin'd that there should be more than three species or kinds of quantity, a line, a supersicies, and a solid body.

By these propositions we likewise prove, that a bowl exactly round cannot rest, but upon such a certain point. For example; let the

line AB represent a plane,
and C the center of the
earth, and that CA be
drawn perpendicular to
the line AB; I say, that a
bowl being plac'd upon

the point B, cannot rest there. For a heavy body cannot rest, when it may descend. Now

46 The Elements of Euclid.

the bowl B moving towards A continua lly defeends, and approaches nearer the center of the earth C; because in the triangle CAB, the

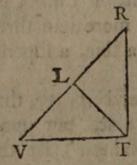
perpendicular CA is shorter than BC.
In like manner we prove, that a liquid body
must flow from B to A, and that its supersi-

cies must be round.

PROPOSITION. XX.

A THEOREM.

Any two sides of a triangle taken together are greater than the third.



I say that the two sides TL, LV, are greater than the side TV Some men prove this Proposition by the definition of a right line, which is the shortest that can be drawn from one point to another: therefore the line TV, is less than

the two lines TL and LV.

But it may also be demonstrated another way.

Continue the side VL to R, so that the lines

LR, and LT be equal; then draw the line

RT.

Demonstration.

The sides LT, and LR, of the triangle LTR, are equal; therefore the angles R, and LTR, are equal, (by the 5.) But the angle RTV is

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greater than the angle RTL: therefore the angle RTV is greater than the angle R: and (by the 19.) in the triangle RTV, the fide RV, that is to say, the sides LT and LV, are greater than the fide TV.

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PROPOSITION XXI.

A THEOREM.

If a small triangle be describ'd within a greater, upon the same base, the sides of the small one will be less than those of the greater; but they will form a grater angle.

ET the small triangle ADB be describ'd within the triangle ACB, upon the same base AB. I say B first, the tides AC and BC are greater than the fides AD and BD. Continue the line AD to E.

- Demonstration.

In the triangle ACE, the fides AC and CE, are greater than the fide AE alone, (by the 20) Therefore adding to them the fide EB; the sides AC, and CEB, are greater than the sides AE, and E3. In like manner in the triangle DBE, the two sides BE and ED are greater than the side BD alone, and adding the side AD, D 2

the fides ADE, and EB, will be greater than AD and BD.

I say further, that the angle ADB is greater than the angle ACB: for the angle ADB is an external angle in respect of the triangle DBE, and therefore greater than the internal DEB (by the 16.) In like manner the angle DEB, being an external angle in respect of the triangle ACE, is greater than the angle ACE, therefore the angle ADB is greater than the angle ACB.

The USE.

's By the help of this Proposition we demonfitrate in Opticks, that the Base AB view'd from the point C, will appear less, than when it is beheld from the point D; according to that principle, That quantities view'd under a greater angle will appear greater. Therefore

tis, that Vitruvius advises, not much to lessen the tops of very high Pillars, because they

being so remote from our sight, quickly appear slender enough without being dimi-

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PROPOSITION. XXII.

A THEOREM.

To describe a triangle, whose sides shall be equal to three sides given, provided that any two of them be greater than the third.

Let it be proposed to deficible a triangle, whose sides shall be equal to three lines given, AB, D, and E. Measure with the compass the line D, and setting one foot thereof upon the point B make an arch. Then take the line E, and placing the foot of your compass upon the point A, make another arch, cutting the former at the point C. Which done draw the lines AC, and BC. I say that the triangle ABC, is such a one as you desire.

Demonstration.

The fide AC is equal to the line E, because it reaches to the arch, which is drawn from the center A at the distance of the line E; and for the same reason the side BC is equal to the line D: therefore the three sides AC, BC, and AB, are equal to the lines ED, and AB.

I added a Proviso, that the two lines should be greater than the third: because otherwise fi

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50 The Elements of Euclid.

the lines D and E were less than the line AB, the Arches could not cut each other.

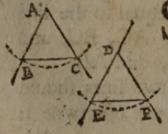
The USE.

'This Proposition may be useful for describing a figure equal or like to another: for having divided that, which is proposed to be equalled or imitated, into triangles; and made other triangles, having equal sides with the former; we shall have a figure exactly equal. But if we desire only one that is like, but less; as when we would describe a Plain, or Country upon paper; having divided it into triangles, and measur'd all their sides, we must make similar triangles; giving to each of their sides so many parts of a Scale, or line divided into equal parts, as the sides of the triangles proposed have of yards or feet.

PROPOSITION XXIII.

A PROBLEM.

To make an angle equal to another at a point of a line given.



Suppose you were to make an angle at the point A of the line AB, equal to the angle EDF. Describe from the points A and D as centers

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wo arches BC, and EF, at the same wideness of the compass; then take the distance EF, and having measur'd as much at BC, draw the line AC. I say the angles BAC, and EDF, are equal.

Demonstration.

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The triangles ABC, and DEF, have the fides AB, and AC, equal to the fides DE, and DF; fince the arches BC and EF v. re describ'd with the same wideness of the compass: the bases also BC and EF are equal, therefore the angles BAC and EDF are equal, (by the 8.)

The USE.

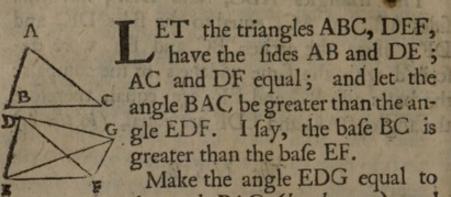
This Problem is so necessary in Geodesia, Fortifications, Perspective, Dialling, and all other parts of the Mathematicks, that the greatest part of their Operations would be impossible, if we did not know how to make one angle equal to another, or of fuch a number of degrees as we pleafe.

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PROPOSITION XXIV.

A THEOREM.

Of two triangles, having each two sides equal to two of the other, that which has the greatest angle, has also the greatest base.



the angle BAC, (by the 23) and the line DG equal to AC; then draw the line EG. First the triangles ABC and EDG, having the sides AB and DE, AC and DG, equal, and the angle EDG, equal to the angle BAC; their bases BC and EG will be equal (by the 4.) and the lines DG and DF being both equal to AC, will be equal betwixt themselves.

Demonstration.

In the triangle DGF, the sides DG and DF being equal, the angles DGF and DFG will be equal, (by the 5.) But the angle EGF is less than the angle DGF, and the angle EFG is greater than the angle DFG. Therefore in

the triangle EFG, the angle EFG will be greater than the angle EGF: and therefore (by the 18.) the line EG oppos'd to the greater angle EFG, will be greater than EF. Therefore BC, being equal to EG, is greater than EF.

PROPOSITION XXV.

A THEOREM.

Of two triangles, having each two sides equal to two of the other, that which has the greatest base, has likewise the greatest angle.

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ET the two triangles AB
C, DEF, have the fides
AB, DE; and AC, DF, equal;
and let the base BC be greatc er than the base EF. I say,
that the angle A will be great-

er than the angle D. .

Demonstration.

If the angle A be not greater than the angle D; it will be either equal, and then the bases BC, EF, will be equal, (by the 4) or it will be less, and the base EF greater than the base BC, (by the 24.) but both are contrary to the supposition,

"These Propositions are necessary to de-

" monstrate those that come after.

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PROPOSITION XXVI.

A THEOREM.

If one triangle has one side, and two angles, equal to those of another triangle; tis equal to it is all respects.

ACB, DFE, of the triangles

ABC, DEF, be equal; and the fides BC, and EF, which arebetween those angles, also equal.

I say, that the other sides are equal; for example, AC, and DF.

I magine, if you please, the side DF

to be greater than AC, and cutting GF equal to AC draw the line GE.

Demonstration.

The triangles ABC, GEF, have the sides EF, BC; AC, GE, equal; the angle C is also supposed to be equal to F. Therefore (by the 4) the triangles ABC, GEF, are equal in all respects; and the angles GEF, and ABC, are equal. But we supposed the angles ABC, DEF, to be equal: and so, the angles DEF, GEF, would be equal: that is, the whole to the part; which is impossible. Therefore the side DE will not be greater than the side AC, nor AC greater than DF,

DF, because the same demonstration may be

made in the triangle ABC.

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Again, suppose the angles A and D, C and F to be equal; and also the sides BC, and EF, oppos'd to the angles A and D, to be equal. I say, the other sides are equal. For if DF be greater than AC, cut GF equal to AC, and draw the

line GE. Demonstration.

The triangles ABC, GEF, having the sides EF, BC; FG, CA, equal, will (by the 4) be equal in all respects; and the angles EGF, BAC, will be equal. But we supposed, that the angles A and D were equal, therefore the angles D, and EGF, must be equal, which is impossible, since the angle EGF, being the external angle in respect of the triangle EGD, must be greater than the internal D, (by the 16.) therefore the side DF is not greater than AC.

The USE.

fure inaccessible distances. For example: the distance AD being propos'd, he would draw from the point A, the line AC perpendicular to AD;

then describing a semicircle at the point C, would measure the angle ACD, and take an-

the line AB, which was accessible, he could

know the other which was not. For the two triangles ADC, and ABC, have the right an-

gles CAD, and CAB equal, the angles ACD, and ACB are also taken equal; and the side

AC is common to both: therefore (by the 26.)

the fides AD and AB are equal. equal. but we supporte that

A LEMMA.

A line which is perpendicular to one of two parallels, is also perpendicular to the other.

E B f Let the parallel lines be AB, and CD, and let EF be perpendicu-'lar to CD. I fay, tis also perpendicular to AB. Cut the line CF e-' qual to FD, and upon the points

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C and D raise two perpendiculars to CD, which, by the definition of Parallels, will be equal to FE; then draw the lines EC and ED.

Demonstration.

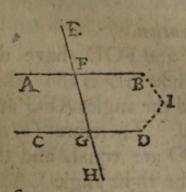
The triangles CEF, and FED, have the fide FE common, the fides CF, FD equal, and the angles CFE and EFD, right, and by consequence equal; therefore (by the 4.) the bases

bases EC, ED, and the angles FED, FEC, and FCE, FDE, will be equal; the two last of which being taken away from the right angles ACF and BDF, leave the two angles ACE, and BDE, equal; therefore the triangles CAE, DBE, will have (by the 4) the angles DEB, CEA, equal; which being added to the equal angles CEF, FED, make the angles FEB, and FEA, equal; therefore the line EF is perpendicular to AB.

PROPOSITION XXVII.

A THEOREM.

If a line, falling upon two others, makes with them the alternate angles equal, those two lines are parallel.



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ET the line EH, falling upon the lines AB and CD, make with them the alternate angles AFG, FGD equal, I say first, the lines AB, and CD, will never concur, though continu d as For suppose them to concur

far as you please. For suppose them to concur in I, and that FBI, and GDI, be two right lines.

Demonstration.

If FBI, and GDI, be two right lines, FIG

is a triangle; and (by the 16) the external angle AFG, is greater than the internal FGI. They cannot therefore be equal, if the lines AB and OD ever concur.

But because we have Examples of some crooked lines, which never concur, and yet are not parallels, approaching still nearer and

nearer to each other.

I say secondly, that if the line EH, falling upon the lines AB, and CD, makes

the alternate angles AFG, and FGD equal; the lines AB, CD, are parallel, or in all respects equally remote from each other, so that the perpendiculars between them will be equal. From the point G draw the perpendicular

GA to the line AB; and taking GD equal to AF, draw FD.

Demonstration.

The triangles AGF, and FGD, have the fide FG common; the fide GD is also taken equal to the fide AF, and the angles AFG and FGD suppos'd to be equal. Therefore (by the 4) the bases AG and FD are equal, and the angle CDF is equal to the right angle CAB; therefore FD is perpendicular. I add, that the line AB is parallel to CD: for the only parallel line that can be drawn from the point F to the line CD, ought to pass by the point A, according

lines

cording to the definition of Parallels; which requires, that the perpendicular lines AG and FD be equal.

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PROPOSITION XXVIII.

A THEOREM.

If a line, falling upon two others, makes the external angle, equal to the internal opposite angle on the same side; or the two internal angles on the same side equal to two right ones; those two lines will be parallel.

IN the precedent figure, suppose the line EH, falling upon AB, and CD, to make first the external angle EFB equal to the internal opposite angle on the same side FGD. I say, that the lines AB, and CD, are parallel.

Demonstration.

The angle EFB is equal to the angle AFG, being opposed to it at the top (by the 15.) and itis supposed that the angle FGD is also equal to the angle EFB; therefore the alternate angles AFG, FGD, will be equal; and (by the 27) the lines AB, and CD, will be parallel.

I say in the second place, that if the angles EFG, and FGD, which are the internal angles on the same side, be equal to two right ones, the lines AB and CD will be parallel.

Demonstr. The angles AFG and BFG are equal to two right angles, (by the 13.) and 'tis suppos'd that the angles BFG, and FGD, are also equal to two right angles; therefore the angles AFG, BFG, are equal to the angles BFG, and FGD; therefore taking away the angle BFG, which is common to both, the alternate angles AFG and FGD will be equal; and (by the 27.) the lines AB and CD will be parallel.

PROPOSITION XXIX.

A THEOREM.

If a line cut two parallels, the alternate angles will be equal; the external angle will be equal to the internal opposite angle; and the two internals on the same side will be equal to two right angles.

ET the line EH [see fig. preced.] cut the two parallels AB, and CD; I say first, the alternate angles AFG, and FGD, are equal. From the points F and G draw the perpendiculars GA, and FD, which by the definition of Parallels are equal.

Demonstration.
In the rectangle triangles AFG, and FGD,

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the fides FD and AG being equal, as also the right angles A and D, and the fide FG common to both. I say first, that the side GD is equal to AF. For if GD be greater; having cut the line DI equal to AF, and drawn the line FI; the triangles AFG, and FDI, would have their bases GF and FI equal, which is impossible. For fince the angle D is a right angle, the angle FID is an acute, and FIG an obtuse, (by the 13.) therefore (by the 18.) in the triangle FIG, the fide FG oppos'd to the obtuse angle, is greater than FI. Therefore DG is equal to AF; and the triangles AFG and FGD, having all their fides equal, will have the alternate angles AFG and FGD equal, as being opposed to the equal fides AG, and FD.

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I say again, that the external angle EFB is equal to the internal EGD, because (by the 15) it is equal to its opposite AFG, which is equal to its alternate FGD.

Lastly, since the angles AFG and GFB are equal to two right ones; taking away AFG, and substituting in its place its alternate FGD, the two internal angles GFB, and FGD, will be equal to two right angles.

The USE.

"a way of measuring the circuit or circumse-

rence of the Earth. In order to which he suppos'd two rays, proceeding from the center of
the Sun to two points of the earth, to be physically parallel; and also that at Syene, a town
in the higher parts of Egypt, the Sun comes
exactly to the Zenith upon the day of the Solstice, observing the Wells there to be then
illuminated to the very bottom: and likewise
computed the distance between Alexandria and
Syene by miles or furlongs.

E D

Let us therefore suppose Syene to be at the point A, and
Alexandria at B, where we
erect astyle BC perpendicular to the Horizon; and let
the two lines DF and EG re-

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present the two rays proceeding from the center of the Sun upon the day of the Solstice,
which are parallel to each other. DA,
which passes by Syene, is perpendicular, that
is, it passes through the center of the earth.
Having observed by the perpendicular style
BC the angle GCB, made by the ray of the
Sun EG; I say, the rays DA and EG being parallel, the alternate angles GCB and BFA are
equal; by which means we have got the angle
AtB, and its measure AB; which gives us in
degrees the distance between Alexandria and
Syene. And having supposed it to be known
in miles, the circumference of the earth may

be found by the simple Rule of Three, saying if so many degrees give so many miles, how many will 360 give?

PROPOSITION XIX.

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A THEOREM.

Lines parallel to a third, are also parallel among themselves.

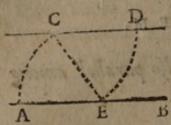
Suppose the lines AB, and FE to be parallel to the line CD: I say, they are parallel betwixt themselves. Let the line GL cut them all three.

For as much as the lines AB and CD are parallel, the alternate angles AHI, and HID, are equal (by the 29) and because the lines CD and FE are also parallel, the external angle HID will be equal to the internal ILE; [by the same] therefore the alternate angles AHI, and ILE, will be equal, and the lines AB and FE parallel [by the 27.]

PROPOSITION XXXI.

A PROBLEM.

To draw a line parallel to another by a point given.



a line by the point C, which shall be parallel to the line AB. Draw the line CE, and make the angle ECD e-CEA. I say the line CD is pa-

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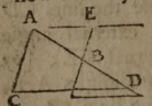
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qual to the angle CEA. I say the line CD is parallel to AB.

Demonstration.

The alternate angles DCE and CEA are equal: therefore the lines CD and AB are parallels.

The eleventh Maxim, i. e. If a line falling upon two others make the internal angles less than two right angles, those lines will concur, may also now be easily demonstrated.



Let the line AC, falling upon the lines AB and GD, make the internal angles ACD, and CAB, less than

the lines AB and CD will concur. Let the angles ACD and CAE be equal to two right angles: the lines AE and CD will be parallels (by

(by the 28.) Take the line AB as long as you please, and by the point B draw EF parallel to CA. Then take the line EB so oft as it is necessary, to make it reach lower than the line CD; as in the present figure I have taken it only twice; so that EB and BF are equal. By the point F draw a parallel FG equal to AE, and joyn the line GB. I say that the line ABG is only one line; and that therefore the line AB concurring in FG, if the line CD be continued, since it cannot cut its parallel FG, it will cut the line BG between B and G.

The triangles AEB and BFG have the fides AEF and FG, BE and BF, equal; as also the alternate angles AEB, and BFG, (by the 29.) therefore they are equal in all respects, (by the 4.) And the opposite angles ABE, and FBG, are equal; and by consequence [by the coroll. of the 15] AB and BG make but one right line.

The USE.

The use of parallel lines is very common; as in Perspectives, for as much as the appearances or images of lines parallel to the picture or table, are parallel among themselves. In Navigation, the lines of the same Rhomb of the wind are describ'd by Paralle's. Polar Dials have the hour lines Parallels. The Compass of Proportion is tounded also upon Parallels. E 3

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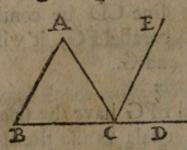
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PROPOSITION XXXII.

A THEOREM.

The external angle of a triangle is equal to both the internal opposite angles taken together; and all the three angles of a triangle are equal to two right angles.



ET the fide BC of the triangle ABC be produc'd to D. I say, that the external angle ACD is equal to both the internal angles A and B taken

together. By the point C draw the line CE parallel to the line AB.

Demonstration.

The lines AB and CE are parallels, therefore [by the 29.] the alternate angles ECA and CAB are equal; and by the same the external angle ECD is equal to the internal B. And by confequence the whole angle ACD, being equal to both the angles ACE, and ECD, of which it is compos'd, will be equal to both the angles A and B taken together.

The angles ACD and In the second place. ACB are equal to two right angles, (by the 13.) and I have demonstrated the angle ACD to be equal to both the angles A and B taken together;

the

ther; therefore the angles ACB, A, and B, that is to fay, all the angles of the triangle ABC, are equal to two right angles, or which is all one, to 180 degrees.

Corollary 1. All the three angles of one triangle are equal to all the three angles of another

triangle.

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Coroll. 2: If two angles of one triangle be equal to two angles of another triangle, their third angles are also equal.

Coroll. 3. If a triangle has one right angle, the other two will be acute; and taken toge-

ther will be equal to one right angle.

Coroll. 4. From a point given only one perpendicular can be drawn to the same line; because a triangle cannot have two right angles.

Coroll. 5. A perpendicular is the shortest of all the lines, that can be drawn from the same

point to the fame line.

Coroll. 6. In a rectangle triangle the right angle is the greatest angle, and the side oppos'd

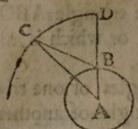
to it the greatest side.

Coroll. 7. Every angle of an equilateral triangle contains 60 degrees; that is to fay, the third part of 180.

The USE.

This Proposition is of use in Astronomy, to determine the Parallax. Suppose the point E 4

A to be the center of the earth; and that from



the point B upon the supersi-'cies be taken the angle DBC, 'that is to fay, the distance of a ' star from the Zenith D. If the earth was transparent the star

' would appear remote from the Ezenith D, according to the bigness of the an-'gle CAD, which is less than the angle CBD. For the angle CBD being an external angle 'in respect of the triangle ABC, it is (by the 6 32.) equal to both the opposite angles A and Therefore the angle C will be equal to the excess of the angle CBD above the angle A. Whence I infer, that if I can know by the Astronomical tables how far remote from the Zenith the star ought to appear to him that should be at the center of the earth, and observe it at the same time from the superficies, the difference of those two angles will be the Parallax BCA.

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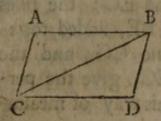
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PROPOSITION. XXXIII.

A THEOREM.

Two lines drawn towards the same parts, from the extremity of two other lines that are equal and parallel, are also themselves equal and parallel.



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ET the lines AB and CD be parallel and equal; and let the lines AC and BD be drawn from their extremities towards the fame parts;

I fay, that the lines AC and BD are equal and parallel. Draw the Diagonal line BC.

Demonstration.

Since the lines AB and CD are parallel, the alternate angles ABC and BCD will be equal; (by the 29) therefore in the triangles ABC and BCD, which have the fide BC common, and the fides AB and CD equal, together with the angles ABC and BCD equal also, the bases AC and BD will be equal, (by the 4) and also the angles DBC, and BCA; which being alternate angles, the lines AC and BD will be parallel, (by the 27)

The USE.

This Proposition is reduc'd to practice for the



the measuring the perpendicular hights, AG, of the vastest Mountains; and also their hori-Horital lines, CG, which are hid by their bulk. Take a large square ADB, and place it so at the point A, that the side DB may fall per-

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pendicularly; then measure the sides AD and DB. This done, do the same again at the point B, and measure BE and EC: the sides parallel to the horizon, AD, BE, added together give the horizontal line CG; and the perpendicular sides DB and EC, give the perpendicular hight AG. This way of measuring is called * Cultellation.

* Measuring by piece-meal.

PROPOSITION XXXIV.

A THEOREM.

The opposite sides and angles of a Parallelogram are equal; and the diameter divides it into two equal parts.

Suppose the figure ABDC [see the fig. of the preceding Prop.] to be a Parellelogram, that is to say, that the sides AB, CD; AC, and BD, are parallel. I say, the opposite sides AB, CD; AC, and BD, are equal; as also that the angles

A and D, ABD, and ACD; and that the diameter BC equally divides the whole figure.

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Demonstration.

The lines AB, and CD, are suppos'd to be parallels: therefore the alternate angles ABC and BCD will be equal, (by the 29.) In like manner the fides AC and BD being suppos'd to be parallels, the alternate angles ACB and CBD will be equal. And further, the triangles ABC, BCD, having the same side BC; and the angles ABC, BCD; ACB, and CBD equal, will be equal in all respects, (by the 26.) Therefore the fides AB, CD; AC, and BD, and the angles A and D, are equal: and the diameter divides the figure into two equal parts. And fince the angles ABC, BCD; ACB, and CBD, are equal, joyning together ABC and CBD; and likewise BCD and ACB, we infer that the opposite angles ABD, and ACD are equal.

The USE.

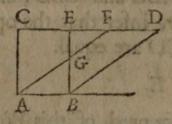
position for dividing grounds. If the field be a Parallelogram, they can divide it into two equal parts by the diameter AD. But if you be oblig'd to divide it by the point E: divide first the diameter into two equal parts by the point F, then draw the line EFG, which will divide the figure into two equal parts. For the

the triangles AEF, and GFD, having the alterternate angles EAF, FDG; and AEF, FGD; and the fides AF and FD equal, are equal, (by the 26) And fince the Trapezium BEFD with the triangle AEF; that is to fay, the triangle ADB, is half the parallelogram, (by the 34.) the fame Trapezium BEFD with the triangle DGF will be half the fame. Therefore the line EG divides it in the middle.

PROPOSITION XXXV.

A THEOREM.

Parallelograms, having the same base, and being between the same parallels, are equal.



ET the Parallelograms be ABEC, and ABDF, having the same base AB, and being between the same parallels AB and CD. Isay,

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they are equal. D. monstration.

The fides AB, CE are equal, (by the 34) as also AB, FD: therefore CE and FD are equal; and adding to them EF, the lines CF and ED will be equal. The triangles therefore CFA, and EDB, have the fides CA, EB, as also CF, and ED equal, together with the angles DEB, and FCA, (by the 29.) one being an external, and the

the other an internal angle on the same side. Therefore (by the 4) the triangle ACF and BED are equal; and taking from them both, that which is common, viz. the little triangle EGF, the Trapezium FGBD will be equal to the Trapezium CAGE: and adding to both the triangle AGB, the Parallelograms ABEC and ABDF will be equal.

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made use of this Proposition to prove, that Angels may extend themselves to what space they please. For supposing they can assume any ingure, provided they have not a greater extension: it is evident, that if an Angel should possess the space of the Parallelogram ABEC, it may likewise occupy the space of the Parallelogram ABDF; and because parallels may be continued in infinitum, (without end,) and Parallelograms may be still formed longer and longer, which will all be equal to ABEC; an Angel will be able to extend it self still farther and farther.

A Demonstration of the same Proposition by Indivisibles.

This method was lately invented by Cavalerius; which has found different acception in the the world, some approving, and others rejecting it. His method consists in this; that we imagine superficies's to be compos'd of lines, like so many threds. And tis certain, that two pieces of linnen will be equal, if they have both the same number of threds, of equal length, and equally compacted.

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'therefore ABEC, and ABDF, 'be propos'd, having the fame base AB, and being between the same parallels AB, CD. 'Divide the Parallelogram

ABEC into as many lines as you please, parallel to AB, which continue to the other Parallelogram ABDF. 'Tis evident there will
be no more in one, than in the other; and
that they will be of equal length, being all
equal to the base AB; and that they will not
be more closely compacted in one, than in the
other: therefore the Parallelograms will be
equal.

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PROPOSITION XXXVI

A THEOREM.

Parallelograms, upon equal bases, and between the same parallels are equal.

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ET the bases CB and OD of the parallelograms AC BF, ODEG, be equal; and let both be between the same parallels AE, CD. I say the

parallelograms are equal. Draw the lines CG, and BE. Demonstration.

The bases CB, and OD, are equal: OD, and GE, are also equal: therefore CB and GE are equal, and parallel; and by consequence (according to the 33.) CG and BE will be equal and parallel; and CBEG will be a parallelogram equal to CBFA, (by the 35.) having both the same base. In like manner, taking GE for the base, the parallelograms GODE and CBEG will be equal, (by the same.) Therefore the parallelograms ACBF, and ODEG, are equal.

The USE.

We oft reduce parellelograms, which have oblique angles, as CBEG, or ODEG, to referangles; as CBFA: so that measuring the latter,

latter, which is easie, being only to muliply

AC by CB, the product being equal to the

Parallelogram ACBF, we may by consequence

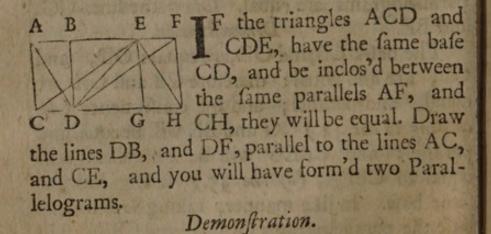
know the other Parallelograms CBEG, or

ODEG.

PROPOSITION XXXVII.

A THEOREM.

Triangles baving the same base, and being between the same parallels, are equal.



The Parallelograms ACDB, and ECDF, are equal (by the 35.) and the triangles ACD, CDE, are the halfs of those Parallelograms (by the 34.) Therefore the triangles ACD, CDE, are equal.

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PROPOSITION XXXVIII.

A THEOREM.

Triangles, that have equal bases, and are inclosed within the same parallels, are equal.

The triangles ACD, and EGH, [see fig. preced.] have equal Bases CD, and GH, and are inclos'd within the same parallels AF, and CH, they are equal. Draw the lines BD and HF parallel to the sides AC, and EG; and you will have form'd two parallelograms.

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Demonstration.

The Parallelograms ACDB, and EGHF, are equal; (by the 36.) and the triangles ACD and EGH are the halfs of those parallelograms, (by the 34.) therefore they are also equal.

The USE.

We have in these propositions directions for dividing a triangular field into two equal parts;

A for example the triangle ABC.

Divide the line which you will take for the base, as BC, into two equal parts in D: I say the triangles ABD, and ADC, are equal.

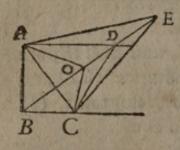
For if you suppose a line drawn by A, parallel to BC, those triangles will have equal bases, and

and be inclos'd within the same parallels, and by consequence will be equal. Other Divisions, grounded upon the same proposition, might be made; but I omit them, that I might not be tedious.

PROPOSITION XXXIX.

A THEOREM.

Equal trinagles, upon the same base, are within the same parallels.



DBC, having the same base BC, be equal; the line AD drawn by the tops will be parallel to the base. For if AD and BC be no paral-

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lel; if you draw a paralled by the point A, it will fall either below the line AD, as AO; or above it, as AE. Suppose it to fall above, and produce BD till it meet the line AE, at the point E; then draw the line CE.

Demonstration.

The triangles ABC and EBC are equal, (by the 38.) fince the lines AE and BC are parallel; 'tis likewise suppos'd that the tirangles ABC, and BDC, are equal: therefore the triangles

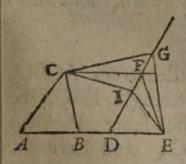
angles DBC and EBC would be equal; which is impossible, the first being part of the second. Whence I conclude, that a line parallel to BC

cannot be drawn above AD, as AE.

I add, that that parallel cannot be below AD, as AO: because the triangle BOC would be equal to the triangle ABC, and by consequence to the triangle DBC; that is to fay, the part would be equal to the whole. It must therefore be confess'd, that the line AD is parallel to the lineBC.

PROPOSITION. XL. A THEOREM.

Equal triangles, having equal bases, if they be taken upon the same line, are between the same parallels.



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F the equal triangles ABC and DEF, have equal bases AB and DE, taken upon the same line AE; the line CF drawn by their tops will be parallel to AE. For if it be not parallel, having drawn

by the point C a line parallel to AE, it will pass either above CF, as CG; or below it, as CI.

Demonstration.

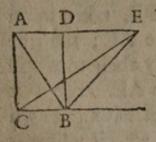
If it pass above CF, as CG, continue DF till

it meet with CG in G; and draw the line EG. The triangles ABC and DGE would be equal, (by the 38.) and ABC and DEF being suppos'd' to be equal, DEF, DGE, would be also equal; which, one being part of the other, cannot poffibly be: therefore the parallel cannot pass above CF. I add, that neither can it pass below it, as CI; because then the triangles ABG and DEI would be equal, and by confequence DEI, and DEF; the part and the whole. Therefore only CF can be parallel to AE.

PROPOSITION XLL

A THEOREM.

A Parallelogram will be double to a triangle, if they be between the same parallels, and have equal bales.



E F the Parallelogram ACBD, I and the triangle EBC, be between the same Parallels AE and BC; and have the fame base BC, or only equal bases; the Parallelogram will

be double the triangle. Draw the line AC. Demonstration.

The triangles ABC and BCE are equal, (by

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the 37.) But the Parallelogram ACBD is double the triangle ABC, (by the 34.) It is therefore double the triangle BCE. It would be also double a triangle, that, having a base equal to BC, should be between the same parallels.

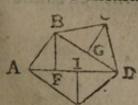
The USE.

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'The ordinary method of measuring the area or superficies of a triangle is built upon this Proposition. If the triangle ABC be proposed; from

the angle A we must draw AD perpendicular to the base BC; then multiplying the perpendicular AD by half the base BE, the product gives the area of the triangle: because multiplying AD, or what is the same, EF by BE, we have a rectangle BEFH, which is equal to the triangle ABC. For (by the 41.) the triangle ABC is half the rectangle HBC. G; and so likewise is the rectangle BEFH.

'We measure all sorts of rectilineal figures,



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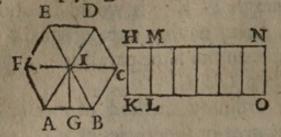
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'as ABCDE, by dividing them into triangles, as BCD, ABD, AED; drawing the lines AD,

'and BD; and the perpendi-'culars CG, BF, and El. For If of BD by CG, and half of

multiplying half of BD by CG, and half of AD by BF, and by EI, we have the area of all those triangles: adding which together the summ is equal to the rectilineal figure ABC DE.

We find the area of the regular Polygones, by multiplying half their circuit by a perpendi-



cular drawn

from their cen-

ter to one of

their sides. For

'multiplying A G by IG, we

fhall have a rectangle HKLM equal to the triangle AlB: and repeating the same for all the other triangles, taking always half of the bases, we shall have a rectangle HKON, which will have the side KO compounded of all the half bases, and by consequence equal to half the circumference; and the side HK equal to

the perpendicular IG.

Tis according to this principle, that Archimedes has demonstrated, that a circle is equal to a rectangle compris'd under the semidiameter, and a line equal to half the circumserence.

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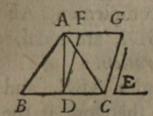
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PROPOSITION XLIL

A PROBLEM.

To make a Parallelogram equal to a triangle given, having one angle equal to an angle given.



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L ET a Parallelogram be defir'd, equal to the triangle ABC, and having an angle equal to the angle E, divide the base BC into two equal parts

at the point D; and draw the line AG parallel to BC, (by the 31.) then make the angle CDF equal to E, (by the 23.) and lastly draw the line CG parallel to DF: the figure FDCG is a Parallelogram, because the lines FG, DC; FD, and GC, are parallels, and its angle CDF is equal to the angle E; and farther, tis also equal to the triangle ABC.

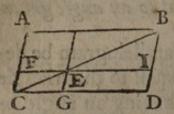
Demonstration.

The triangle ADC is half the parallelogram FDCG, (by the 41.) 'tis also half the triangle ABC; since the triangles ADC, and ADB, are equal, (by the 38.) Therefore the triangle ABC is equal to the Parallelogram FDCG.

PROPOSITION XLIL

A THEOREM.

The complements of a parallelogram are equal.



In the Parallelogram ABD C, the complements AF EH, and EGDI, are equal, Demonstration.

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The triangles ABC, and BCD, are equal, (by the 34.) therefore if the triangles HBE, and BIE; FEC, and CGE, which are also equal, (by the same,) be substracted, the complements AFEH, GDIE, which remain, will be equal.

PROPOSITION XLIV.

A PROBLEM.

To describe a parallelogram upon a line given, which shall be equal to a triangle, and have such a certain angle; i.e. equal to one given.



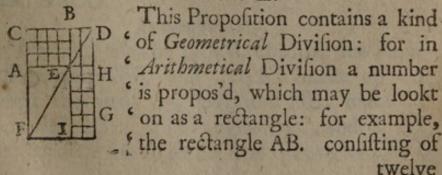
CUppose you be requir'd to make a parallelogram, which shall have one of its angles equal to the angle E, and one of its fides equal to the

the line D, and be equal to the triangle ABC. Make the Parallelogram BFGH, (by the 42.) which has the angle BFG equal to the angle E, and is equal to the triangle ABC. Produce the fides GF, and GH, so that HI may be equal to the line D; and draw the line IBN till it cuts GF produc'd to N; and from the point N draw the line NO parallel to GI, and IO parallel to BH; producing also the side FB to K, and HB to M. The parallelogram MK is that which you desire.

Demonstration.

GF and HM being parallels, the alternate angles GFB or the angle E, and FBM, are equal (by the 29.) In like manner the lines KB and MN being parallel, the alternate angles FBM, and BMO, are equal; therefore the angle BMO is equal to E, and the fide KB is equal to the line HI or D: and lastly, the Parallelogram MK is equal to the Parallelogram GFBH, (by the preceding,) and that was made equal to the triangle ABC. Therefore the Parallelogram MK is equal to the triangle ABC.

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twelve square feet, which is to be divided by another number, suppose, two; that is to say, another rectangle is defir'd to be made equal to AB, having one of its fides, BD equal to two; and the question is, how many feet the other fide ought to contain; which is, as it were, the quotient. This is done Geometrically by the Rule and Compass. Take BD consisting of two feet, and draw the Diagonal DEF: the line AF is that which you feek. For hav-'ing compleated the rectangle DCFG, the come plements EG, and EC, are equal, (by the 43.) and EG has for one of its sides EH, equal to BD, of two feet in length; and EI equal to AF. This kind of Division, is call'd Application, because the rectangle AB is apply'd to the line BD, or EH: and from hence tis, that 'all Division is frequently call'd Application; because the ancient Geometricians made smore "use of the Rule and Compass, than of Arithmetick.

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PROPOSITION XLV.

A PROBLEM.

To describe a parallelogram, which shall have a certain angle; and be equal to a rectilineal sigure given.

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L ET the rectilineal figure propos'd be AB CD, to which you are required to make an equal Parallelogram, which shall

have an angle equal to the angle E. Divide the rectilineal into triangles by the line BD: and (by the 42.) make a Parallelogram FGHI, which has the angle IHG equal to the angle E, and is equal to the triangle ABD; and (by the 44) make the Parallelogram IHKL equal to the triangle BCD, having one fide equal to IH, and the angle LIH equal to the angle E. The Parallelogram FGKL will be equal to the rectilineal ABCD.

Demonstration.

Nothing need be prov'd, but that the Parallelograms FGHI, and HKLI, make up but one; that is to fay, GH, and HK, make but one right line. The angles GHI, and LKH, are equal to the angle E. And the angles LKH, and

twelve square feet, which is to be divided by another number, suppose, two; that is to say, another rectangle is desir'd to be made equal to AB, having one of its fides, BD equal to two; and the question is, how many feet the other side ought to contain; which is, as it were, the quotient. This is done Geometrically by the Rule and Compass. Take BD confisting of two feet, and draw the Diagonal DEF: the line AF is that which you feek. For having compleated the rectangle DCFG, the come plements EG, and EC, are equal, (by the 43.) and EG has for one of its sides EH, equal to BD, of two feet in length; and EI equal to AF. This kind of Division, is call'd Application, because the rectangle AB is apply'd to the line BD, or EH: and from hence tis, that 'all Division is frequently call'd Application; because the ancient Geometricians made smore "use of the Rule and Compass, than of Arithmetick.

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PROPOSITION XLV.

A PROBLEM.

To describe a parallelogram, which shall have a certain angle; and be equal to a rectilineal sigure given.

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L ET the rectilineal figure propos'd be AB CD, to which you are required to make an equal Parallelogram, which shall

have an angle equal to the angle E. Divide the rectilineal into triangles by the line BD: and (by the 42.) make a Parallelogram FGHI, which has the angle IHG equal to the angle E, and is equal to the triangle ABD; and (by the 44) make the Parallelogram IHKL equal to the triangle BCD, having one fide equal to IH, and the angle LIH equal to the angle E. The Parallelogram FGKL will be equal to the rectilineal ABCD.

Demonstration.

Nothing need be prov'd, but that the Parallelograms FGHI, and HKLI, make up but one; that is to fay, GH, and HK, make but one right line. The angles GHI, and LKH, are equal to the angle E. And the angles LKH, and and KHI, are equal to two right angles, because KHIL is a Parallelogram. Therefore the angles GHI and KHI are equal to two right angles, and (by the 14.) GH and HK make one right line.

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The USE.

The use of this proposition is the same with the preceding; serving to measure the capa-

city of any figure whatfoever, by reducing it into triangles, and then making a rectangular

4 Parallelogram equal to them.

Tis easie likewise to make a rectangular Parallelogram upon a determinate side, which may be equal to many irregular figures. In

'like manner having many figures a rectangle may be described equal to their difference.

PROPOSITION. XLVI.

A PROBLEM.

To describe a square upon a line given.

D O describe a square upon the line AB, draw two perpendiculars AC and BD equal to AB, and draw the line CD.

Demonstration.

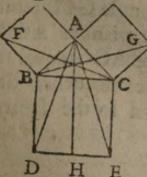
B The angles A and B being right angles,

angles, the lines AC and BD are parallels (by the 28) They are also equal; therefore the lines AB and CD are parallels and equals, [by the 33.] and the angles A and C equal to two right angles; as also B and D, [by the 29.] and since A and B are both right angles, the angles C and D will be so likewise. Therefore the sigure AD has all its sides equal, and all its angles right angles, and by consequence is a square.

PROPOSITION XLVII.

A THEOREM.

The square of the base of a rectangular triangle, is equal to the squares of both the other sides taken together.



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Suppose the angle BAC to be a right angle, and that squares were described upon all the sides BC, AB, and AC: that of the base BC, which is opposed to the right angle, will be equal to the squares or both the sides AB,

and AC. Draw the line AH parallel to BD, and CE; and joyn the lines AD, AE, FC, and BG. I will prove the square AF is equal to the rectangle BH, and the square AG to the rectan-

gel

Demonstration.

The triangles FBC, and ABD, have the fides AB, BF; BD, and BC, equal: and the angles FBC, and ABD, are equal, each containing the angle ABC more than their respective right angles. Therefore by the 4. the triangles ABD and FBC are equal. But the square AF is double the triangle FBC, [by the 41.] having the same base BF, and being between the same parallels BF, and AC. In like manner the re-Stangle BH is double the triangle ABD, having likewise the same base BD, and being between the same parallels BD and AH. Therefore the square AF is equal to the triangle BH. By the same method the triangles ACE, and GCB; may be prov'd to be equal, [by the 4] and the square AG to be double the triangle GCB; and the rectangle CH, double the triangle ACE, [by the 41.] therefore the square AG is equal to the rectangle CH; and by confequence the squares AF and AG are equal to the square BDEC.

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[&]quot;Tis said that Pythagoras, having sound out this Proposition, sacrifie'd a Hecatomb, i. e. a hundred Oxen, to the Muses, to return them thanks for their assistance; supposing it, it seems

feems, above the power of bare humane invention. Nor was his esteem thereof so irrational, as to some perhaps it may appear; this Proposition being the foundation of a very considerable part of the Mathematicks. For in the first place Trigonometry cannot possibly subsist without it, it being necessary to compose a table of all the lines that may be inferibed in a circle, as Chords, Sines, Tangents, Secants; as may appear by one example.

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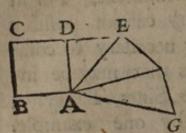
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Suppose the semidiameter AC to be divided into 100000 parts, and that the arch BC contains 30 degrees. Since the Chord, or line that subtends 60 degrees is equal

fine of 30 degrees, will be equal to half AC, and therefore contain 50000 parts. Now in the rectangular triangle ADB, the square of AB is equal to the squares of AD, and BD. Make therefore the square of AB, by multiplying 100000 by 100000, and from the product substract the square of 50000 or BD; the remainder will be the square of AD, or BF the sine of the complement: and extracting the square root of that number, you will have the line FB. This done, making as AD to BD, so AC to CE, you will have the tangent CE; then adding together the squares of AC and CE, the product [by the 47.] will give

the square of AE; extracting therefore from that number the square root, you will know the length of the line AE, which is the second.



'By this also we may aug'ment figures, as much as
'we please. For example;
'to double the square ABC
'D, continue the side CD,
'fo that AD and DE may
'be equal: the square of

AE will be double the square of ABCD; since (by the 47.) it is equal to both the squares of AD and DE. Making the right angle AEF, and taking EF equal to AB, the square of AF will be triple the square ABCD. Again, making the right angle AFG, and taking FG equal to AB, the square of AG will be quadruple, or sour times the square of ABCD. And that which I say of the square, may be understood of all similar sigures.

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PROPOSITION. XLVIII.

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If in a triangle the square of one side be equal to the squares of both the other sides, taken together; the angle opposite to that first side will be a right angle.

IF the square of the side NP be equal to both the squares of the sides NL, and LP, taken together; the angle NLP will be a right angle. Draw LR perpendicular to NL, and equal to LP; then draw the line NR.

Demonstration.

In the rectangular triangle NLR, the square of NR is equal to the squares NL, and RL, or LP, (by the 47.) Now the square of NP is also equal to the same squares of NL, and LP; therefore the square of NR is equal to the square of NP, and by consequence the lines NR and NP are equal. And because the triangles NLR, and NLP, have the side NL common; the sides LP and LR equal, and their bases NP and NR also equal; the angles NLP and NLR will be equal, (by the 8.) and the angle NLR being a right angle, the angle NLP must so so so.

THE SECOND BOOK

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EUCLID.

UCLID in this Book treats of the powers of right Lines; that is to fay, of their Squares; comparing the divers Rectangles, which are made upon a Line divided, as well with the Square, as the Rectangle, of the whole line. 'Tis a part exceeding useful, serving for the foundation of the principal Operations of Algebra. The three first Propositions demonstrate the third Rule, or Operation of Arithmetick, Multiplication. The fourth teaches to extract the ' square Root of any number whatsoever. Those that follow to the Eighth serve upon many oc-The rest instruct us in Ocasions in Algebra. e perations proper for Trigonometry. This Book feems at first view very difficult; because " men are apt to imagine there is something myfterious contain'd therein; nevertheless the greatgreatest part of its Demonstrations are grounded on this most evident Principle, That the whole is equal to all its parts taken together.

But it ought not to discourage any, if they should not at the first attempt fully compre-

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DEFINITIONS.

t. Arectangular parallelogram is compris dunder two lines, that form a right angle.

Bferve that henceforward by a rectangle we fhall intend fuch a parallelogram, whose angles are all right angles, distinguishing it by giving its longi-

tude and latitude, naming two of its sides, which contain one of its angles, as the lines AB and BC. For the rectangle ABCD is comprised under the lines AB and BC; BC denoting its longitude, and AB its latitude; and the other being equal to these it will not be necessary to name them. I have also formerly intimated that the line AB, remaining perpendicular to BC, and being moved from one extremity thereof to the other, produces the rectangle ABCD; and that that G 2

motion has some resemblance to Arithmetical multiplication: so that, as the line AB moving over the line BC, that is, taken so many times, as there are points in BC, composes the rectangle ABCD: so the multiplication also of AB by BC, will give the rectangle ABCD. As, suppose I knew the number of Mathematical points, that are in AB, for example 40, and that there were 60 in BC: it is evident that the rectangle ABCD will have so many lines equal to AB, as there are points in BC; and that multiplying 40 by 60, the product will be 2400, which is the

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onumber of Mathematical points in the rectangle ABCD.

'I may take what quantity I please for a Mathematical point; provided I do not afterwards · subdivide it; it must therefore be observ'd; that when I measure a line, for a Mathematical point I take that measure which best suits with my occasions; for example, when I say a line of five foot in length, my Mathemactical point is a line of a foot long; which I take without considering that it is compos'd of any parts. In measuring a superficies likewife I do the same, taking some known superficies, for example, a foot square; which I do not afterwards subdivide. I make use of a square rather than any other figure; because its length and bredth being equal there is no nee d

need of naming more than one of its dimensions to describe it. Accordingly when I would mark out the Area of the rectangle ABCD, I do not consider the sides as simple lines, but as rectangles of a determinate bredth: for example, when I say that the rectangle ABCD has the fide AB of four foot long, fince a foot ' is to me instead of a Mathematical point, I conceive the fide AB to have also a foot in bredth, and to be as the rectangle ABEF. Therefore knowing how many times the 'bredth BE is contain'd in the line BC, I 'shall know how many times the line AB is 'contain'd in the rectangle ABCD; that is to 'fay, multiplying AB which has four foot ' square, by 6, the product will be 24 foot square. In like manner knowing the magnitude of the rectangle ABCD to be 24 foot square, and one of its fides AB to be 4; dividing 24 by 4 the quotient will give me the other side BC, confifting of fix foot square. 2. Having drawn the diameter of a

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A E D 2. Having drawn the diameter of a rectangle, one of the lesser rectangles thro which it passes, together with the two complements, is call'd the B H C Gnomon. As the rectangle EG,

thro which the diameter BD paffes, together with the complements EF and GH, is call'd the Gnomon; their figure together: representing a Carpenters square.

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PROPOSITION I. A THEOREM.

If two lines be proposed, whereof one is divided in to divers parts, the rectangle contained under those two lines is equal to the rectangles contained under the line which is not divided, and the parts of the line divided.

E T the lines propos'd be
AB, and AC; and let AB
be divided into as many parts as
you please. The rectangle AD
contain'd under the lines AB and
AC, is equal to the rectangle AG contain'd
under AC and AE; to the rectangle EH contain'd under EG equal to AC, and EF; and to
the rectangle FD contain'd under FH equal
to AC and FB.

Demonstration.

The rectangle AD is equal to all its parts taken together; which are the rectangles AG, EH, and FD; and no other. Therefore the rectangle AD is equal to the rectangles AG, EH and FD taken togerher.

By Numbers.

This proposition holds true likewise in numbers. Suppose the line AC to be five foot long; AE two, EF four, FB three; and by consequence AB nine: the rectangle contained under

under AC five; and AB nine, that is to fay, five times nine, which makes forty five, is equal to twice five or ten, four times five or twenty, and three times five, or fifteen; for ten, twenty, and fifteen, make forty five.

The USE.

By this proposition is demonstrated the ordinary operation of multiplication. For example, if you were to multiply the number A, which is 53, by the number B, that is 8. Divide the number A into so many parts as there are characters: that is, two, 50, and 3; which multiply by 8, saying, eight times three is twenty sour; and so you make one rectangle. Then multiplying the number 50 by 8, the product will be 400. But its evident that the product of eight times 53, being 424, is equal to the product of 24, and the product 400 taken together.

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PROPOSITION II.

A THEOREM.

The square of any line is equal to the rectangles contain'd under the whole line, and all its parts.

CGHD T ET the line propos'd be AB, and its square ABCD. I say the square ABCD is equal to the rectangle contain'd under the whole line AEFB AB, and AE; another under AB and EF; and a third under AB and FB.

Demonstration.

The square ABCD is equal to all its parts taken together, which are the rectangles AG. EH, FD. The first AG is contain'd under AC equal to AB, and AE. The fecond EH s' contain'd under EG equal to AC or AB, and FE. The third FD is contain'd under FH equal to AB, and FB: and 'tis the fame thing to be contain'd under a line equal to AB, and to be contain'd under AB it self. Therefore the square of AB is equal to the rectangles contain'd under AB, and AE, EF, FB, the parts of AB;

By Numbers.

Let the line AB represent the number nine:

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its square will be 81: Let also the part AE be four; EF three, and EC two: nine times four make thirty fix; nine times three twenty feven, and nine times two eighteen, and 'tis plain that 36, 27, and 18 make 81.

The U.S.E.

'This Proposition serves likewise to prove "multiplication; as also for Equations in Algebra. und edi or laupe si DA bun ala rahun

PROPOSITION III.

A THEOREM.

If a line be divided into two parts, the rectangle contain'd under the whole line, and one of its parts, is equal to the square of the same part, and the rectangle contain'd under both the parts.

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T ET the line AB be divided into two parts at the point C; and let a rectangle be made B of the whole AB, and one of its parts AC, that is to fay, let AD be equal to AC;and then if the rectangle AF be compleated, it will be equal to the square of AC, and the rectangle contain'd under AC and CB. Draw the perpendicular GE.

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Demonstration.

The rectangle AF contain'd under AB and AD equal to AC, is equal to all its parts, which are the rectangles AE, and CF. The first AE is the square of AC, the lines AC and AD being equal; and the rectangle CF is contain'd under CB, and CE equal to AD or AC. Therefore the rectangle contain'd under AB and AC is equal to the square of AC, and the rectangle contain'd under AB and CB.

By Numbers.

Let AB be 2; and AC 3; and CB 5: the rectangle contain'd under AB and AC, will be three times eight, or 24: the square of AC 3, is nine; and the rectangle contain'd under AC 3, and GB 5, is three times 5 or 15. But it is evident that 15 and 9 make 24.

The USE.

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courag'd, if they do not presently apprehend these Propositions; which yet, in truth are not difficult, but as they are conceiv'd to contain some strange mystery.

PROPOSITION IV.

A THEOREM.

If a line be divided into two parts, the square of the whole line, will be equal to the squares of both the parts; and two rectangles contain d under the same parts.

The line AB be divided in C, and its square vided in C, and its square agonal also EB be drawn and K A CB a perpendicular cutting it CF: and by that point let the line GL be drawn parallel to AB. 'Tis evident that the square ABDE, is equal to the four rectangles GF, CL, CG, and LF. The two first of which are the squares of AC and CB: and the two Complements are contain'd under AC and CB.

Demonstration.

The sides AE and AB are equal: therefore the angles AEB, and ABE are half right angles

angles: and because the lines GL and AB are parallels; the angles of the triangles of the square GF (by the 29. 1.) will be equal; as also their sides (by the 6. 1.) Therefore GF is the square of AC. In like manner CL is the square of CB: the rectangle GC is contain'd under AC, and AG equal to BL or BC; and the rectangle LF is contain'd under LD equal to AC, and FD equal to BC.

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Coroll. If you draw the diagonal of a square

the rectangles which it cuts are squares.

The USE.

This Proposition teaches the method of extracting the square root of any number propos'd. Let the number be A, or 144, represented by the spuare AD, and its root by the line AB. I suppose it known from other principles that it requires two characters. I imagine therefore that the line AB is divided in C, fo that AC may represent the first character, and BC the second: Then searching the s root of the first character of the numbe 144, " which is 100, I find it to be 10: and making its square 100 represented by the square GF, 6 I substract it from 144; and there remains 44 for the rectangles GC, FL; and the square 6 CL: But because the figure of a Gnomon is not proper for this operation, I transport the rectangle FL unto KG, making one whole

rectangle KL, that is, 44. I know also already almost the whole side KB: for AC being 10, KC must be 20. I must therefore divide 44 by 20; that is to say, for my Divisor doubling the root found; I enquire then how many times 20 I can have in 44? and find twice; and therefore take 2 for the fide BL; and because 20 was not the intire side 'KB, but only KC; that two which came in the quotient I add to Divisor, making it 22; which number being found precisely twice in 44, adding 2 to the root before found, I conclude the whole square root of 144 to be 12. You see then that the square 144 is equal to the square of 10, which is 100, the 'square of 2, that is 4; and twice 20, which makes the two rectangles contain'd under two and ten.

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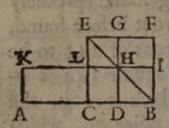
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A THEOREM.

If a line be divided into two equal parts and two parts that are unequal; the rectangle contain'd under the unequal parts, together with the square of the intermediate part; is equal to the square of half the line.



E G F TF the line AB be divided I into two equal parts in C; and two unequal parts in D; the rectangle AH contain'd under the unequal fegments

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AD, and DB, with the square CD, will be equal to the square of CB, that is, the square CF. Compleat the figure as you see; the rectangles LG and DI will be squares. (by the coroll. of the 4.) I will prove then that the rectangle AH, contained under AD, and DH equal to DB, with the square LG, is equal to the square CF.

Demonstration.

The rectangle AL is equal to the rectangle DF, both being contain'd under half the line AB, and DB, or DH, which is equal to it. Add to both the rectangle CH; the rectangle AH will be equal to the Gnomon CBG. Add theretherefore again to both the square LG; and the rectangle AH, with the square LG will be equal to the square CF.

By Numbers.

Let AB be 10; AC will be 5, and CB likewise; and let CD be 2, and DB 3. the rectangle contain'd under AD 7 and DB 3, that is to say 21, with the square of CD 2, that is 4, will be equal to the square CB 5, which is The USE.

This Proposition is very useful in the third Book: It is also us'd in Algebra, to demon-

ftrate the manner of finding the root of an af-

fected or impure square.

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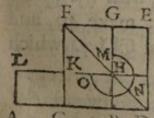
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PROPOSITION VI.

A THEOREM.

If a line be divided into two equal parts, and to it another line added; the rectangle contain'd under the line compounded of those two, and that which is added, with the square of half the divided line, is equal to the square of the line compounded of that half, and the line that is added.



F to the line AB, divided into two equal parts in C, be added the line BD; the rectangle AN, contain'd under the line AD, and DN equal to BD, with the square

of CB, is equal to the square of CD. Make the square of CD, and having drawn the diagonal FD, draw also BG parallel to EC, cutting FD at the point H, through which passes the line HN parallel to AD. KG will be the square of CB; and BN, that of BD.

Demonstration.

The rectangles AK, and CH, being upon equal bases AC and CB, are equal(by the 36.1.) The complements CH and HE are equal, (by the 43.1.) therefore the rectangles AK and HE are equal. Add to both the rectangle CN, and the square KG: the rectangles AK and CN that is, the rectangle AN, with the square KG, will be equal to the rectangles CN and HE, and the square KG, that is, the square CE.

By Numbers.

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Let AB consist of 8 parts,: AC of 4; and CB of 4; BD of 3. so that the whole AD be 11. Tis evident the rectangle AN is three times 11, that is 33; which with the square of KG, equal CB 4, that is 16; make 49, and therefore is equal to the square of CD 7. which is 49; for 7 times 7 make 49.

The USE.

Maurylocus, by the help of this Proposition measur'd the whole Earth at one single Observation.

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To effect which, he advises, that from the top of a mountain of known height A, you observe the angle BAC, made by the line AB, touching the superficies of the earth at B, and the line AC passing through the center: and

that in the triangle ADF, knowing the angle A, and the right angle ADF, you find by Trigonometry the fides AF and FD: and because tis easie to demonstrate that FB and FD are equal, you will then know the line AB, and also its square. Now we have demonstrated in the preceding Proposition, that the line ED being divided into equal parts in C, and the line AD added to it; the rectangle contain'd under EA, and AD, with the square of CD, or CB, is equal to the square of CA; and the angle ABC, being a right angle, (as is prov'd in the third book) the square of CA is equal to the squares of AB and BC; therefore the rectangle under AE and AD, with the square of BC, is equal to the squares of AB and BC. Take therefore from them both the square of BC, and the rectangle under AE, and AD, will be equal to the square of AB. Divide therefore the known square of AB, by the height of the mountain AD, and the Quotient will be the 'line AE; from which subtracting the hight

The Elements of Euclid.

of the mountain, the remainder will be the

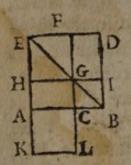
diameter of the earth DE.

We have made use likewise of the same proposition in our Algebra, to demonstrate the thirteenth proposition of the third Book, to find the root of a square equal to a more certain number of roots. The two that follow do also serve for the proof of the like operati-

PROPOSITION. VII.

A THEOREM.

If a line, be divided, the square of the whole line, with that of one of its parts, is equal to two rectangles contain'd under the whole line, and that first part together with the square of the other part.



ET the line AB be divided any where in C; the square AD of the line AB, with the square AL, will be equal to two right angles contain'd under AB and AC, with the square of CB. Make the square of AB, and hav-

ing drawn the diagonal EB, and the lines CF and HGI; prolong EA fo far, as that AK may be equal to AC: so AL will be the square of AC, and HK will be equal to AB; For HA is equal

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to GC, and GC is equal to CB, because CI is the square of CB, (by the Coroll. of the 4)

Demonstration.

Tis evident, that the squares of AD and AL are equal to the rectangles HL and HD, and the square CI. Now the rectangle HL is contain'd under HK equal to AB, and KL equal to AC. In like manner the rectangle HD is contain'd under HI equal to AB, and HE equal to AC. Therefore the squares of AB and AC are equal to two rectangles contain'd under AB and AC, and the square of CB.

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In Numbers.

Suppose the line AB to consist of 9 parts, AC of 4, and BC of 5. The square of AB 9 is 81, and that of AC 4 is 16; which 81 and 16 added together make 97. Now one rectangle under AB and AC, or 4 times 9, make 36, which taken twice is 72: and the square of CB 5 is 25; which 72 and 25 added together make also 97.

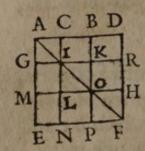
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PROPOSITION VIII.

A THEOREM.

If you divide a line, and add another to it equal to one of its parts, the square of the whole compounded line will be equal to four rectangles contain'd under the first line, and that part that is added together with the square of the other part.



any where in the point C, and BD equal to CB added to it: the square of AD will be equal to four rectangles contain'd under AB, and BC or BD, and

the square of AC. Make the square of AD, and having drawn the diagonal AE, draw likewise the perpendiculars BP, and CN, cutting the diagonal in I, and O: and also the lines MOH, and GIR, parallel to AB. The rectangles GC, LK, PH, MB, and NR, will be squares, (by the Coroll. of the 4.)

Demonstration.

The square ADEF is equal to all its parts; and the rectangles LB, OD, PM, are contain'd under lines equal to AB, and BC, and if you add the rectangle MI to the rectangle PH, they together will give you another rectangle contain'd under

under a line equal to AB, and another equal to CB or BD. Besides which there remains nothing but the square GC, which is the square of AC. Therefore the square AD is equal to four rectangles contain'd under AB and BD, and the square of AC.

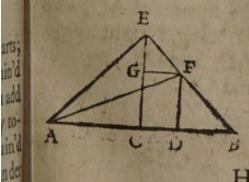
In Numbers.

Let the line AB confift of 7 parts, AC of 3, and CB of 4; as also BD: the square of AD 11 will be 121, And one rectangle under, AB 7 and BD 4, makes 28; which taken four times is 112; and those together with the square of 3, which is 9, make also 121.

PROPOSITION IX.

A PROBLEM.

If a line be divided into two equal parts, and two unequal, the squares of the unequal parts will be double the square of balf the line, and the square of the intermediate part.



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et the line be divided into two equal parts at the point C, and two unequal at the point D: the squares of the unequal parts AD, and DB will be double the squares of AC, which is half AB, and CD the intermediate part. Draw CE perpendicular to AB, and equal to AC; draw also the lines AE and BE, and the perpendicular DF, as likewise FG parallel to CD. Then joyn the line AF.

Demonstration.

The lines AC and CE are equal, and the angle C is a right angle: therefore (by the 5.1.) the angles CAE, and CEA, are equal; and confequently half right angles. In like manner, the angles CEB, CBF, GFE, and DFB, are half right angles; and the line GF and HE, DF and DB, equal, (by the 6.1.) and the whole angle AEF is a right angle. Now the square of AE(by the 47. 1.) is equal to the squares of AC and CE, which are equal: therefore it is double the square of AC. For the same reason, the square of EF is double the square of GF or CD. Now the square of AF is equal to the squares of AE, and EF, because the angle AEF is a right angle: therefore the square of AF is double the iquares of AC, and CD. The same square of AF is likewise equal to the squares of AD, and DF or DB, the angle D being a right angle. Therefore the squares of AD, and DB, are double the squares of AC, and CD.

Let AB be 10, AC 5, CD 3, and DB

2: the squares of AD 8, and DB 2, that is to say, 64 and 4, which make 68, are double the squares of AC 5, that is, 25, and of CD 3, which is 9: for 25 and 9 make 34, which is half of 68.

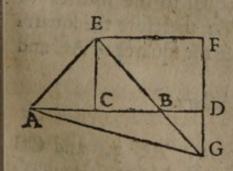
The USE.

"cept in Algebra; no more than that which follows.

PROPOSITION X.

A THEOREM.

If a line be added to another that is divided into two equal parts; the square of the line compounded of those two, with the square of that which is added, makes double the square of half the line, and the square of that which is compounded of half, and the line that is added.



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ET the line AB be divided in the middle at the point C, and the line BD added to it: the squares of G AD, and BD, will be double the squares of

AC, and CD. Draw the perpendiculars CE and DF equal to AC: and then draw the lines H 4 AE.

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AE, EF; and producing FD to G, so that DG may be equal to BD, joyn the lines AG, and EBG.

Demonstration.

The lines AC, CB, and CE being equal, and the angles at the point C being right angles: the angles CAE, AEC, CEB, and CBE, will be half right angles. In like manner the angle D being a right angle, and the lines BD and DG equal, the angles DBG, and DGB, will be half right angles; and so will likewise GEF, the angle F being a right angle; therefore the lines FG and FE are equal, (by the 6. 1.) and EF is equal to CD, (by the 33. 1.) Now the square of AE is double the square of AC, and the square of EG also double the square of EF, or CD, (by the 47. 1.) But the square of AG is equal to the squares of AE and EG, (by the same:) therefore the square of AG is double the squares of AC, and CD. The same square of AG is likewise (by the same) equal to the squares of AD, and DG equal to DB: therefore the squares of AD and DB are double the squares of AC and CD.

By Numbers.

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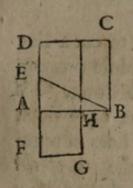
Let AB contain 6 parts, AC 3, and CB 3, BD 4; the square of AD 10 is 100; the square of BD 4 is 16, which make together 186. The square also of AC 3 is 9: the square

square of CD 7 is 49. Now 49 and 9 make 58, the half of 116.

PROPOSITION XI.

A PROBLEM.

To divide a line in such a manner, that the rectangle under the whole line, and one of its parts, shall be equal to the square of the other part.



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Suppose the line AB to be divided in such a manner, that the rectangle under the whole line AB, and BH, may be equal to the square of AH. Make the square of AB, (by the 46.1.) and dividing AD in the middle in E,

draw EB, and take EF equal to EB. Then make the square of AF, that is to say, let AF and AH be equal. I say, the square of AH will be equal to the rectangle HC, contain'd under HB, and BC equal to AB.

Demonstration.

The line AD is divided equally in the point E, and the line FA added to it, therefore (by the 6.) the rectangle DG contain'd under DF, and FG equal to AF, with the square of AE, is equal to the square of EF, equal to EB: now the square EB is equal to the squares of AE and AR

AB, (by the 47.1.) therefore the squares of AB and AE are equal to the rectangle DG, and the square of AE: and subtracting from both the square of AE; the square of AB, that is, AC, will be equal to the rectangle DG: taking away therefore the rectangle DH, which is common to both, the rectangle HC will be equal to the square of AH, that is, AG.

The USE.

This Proposition teaches how to cut a line according to the extreme and middle proportion, as will be shewn in the 6th Book, Tis also frequently made use of in the 14th Book of Euclid's Elements, to find the sides of regular Solids. It is useful also in the 11. of the 4. to inscribe a Pentagone in a circle, as also a Pentedecagone (or a figure with 15. angles.) You will see also other uses thereof in dividing lines on this manner, in the 30th Proposition of the 6.

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PROPOSITION XII.

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A THEOREM.

In an Obtuse triangle, the square of the side opposed to the obtuse angle, is equal to the squares of both the other sides, and two rectangles contained under the line upon which a perpendicular will fall, and the line which lies betwixt the triangle and the perpendicular.

Let the angle ACB, of the triangle ABC, be an obtuse, and let AD be drawn perpendicular to BC; the square of the side AB is equal to the squares of the sides AC and CB, and two rectangles contain'd under the side BC, and DC.

Demonstration.

The square of AB is equal to the squares of AD, and DB, (by the 47.1.) But the square of DB is equal to the squares DC, and CB, and two rectangles contain'd under DC and CB, (by the 4.) Therefore the square of AB is equal to the squares AD, DC, and CB. and two rectangles contain'd under DC and CB. In stead of the two first squares AD, and DC, put the square of AC, which is equal to them, (by the 47.) (1) The square AB will be equal to the squares AC.

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AC and CB, and two rectangles contain'd under DC and CB.

The USE.

"This Propolition is useful in Trignometry to measure the area of a triangle, whose sides 'are known. For Example, suppose the side AB to confift of twenty foot, AC of 13, BC of 11: the square of AB will be 400, that of 'AC 169, and that of BC 121. The fumm of the two last is 290, which subtracted from 400, there will remain 110 for the two rectangles under BC and CD. The half of which, 55, will make one half of those rectangles; dividing which number by BC, 11, we shall have 5 for the line CD; whose square 25 being subtracted from the square of AC, 169. leaves the square of AD, 144, whose root 12 will be the fide AD, which being multiplied by 52, the half of BC, will give the area of the triangle ABC, that is, 66 foot square.

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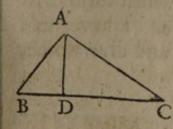
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PROPOSITION XIII.

A THEOREM.

In any triangle whatsoever, the square of the side opposed to the acute angle, with two rectangles contained under the side upon which the perpendicular falls, and the line which is between the perpendicular and that angle; is equal to the squares of both the other sides.



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Suppose the triangle to be ABC, and the acute angle C, and AD the perpendicular falling upon BC: the square of the side AB, oppos'd

to the acute angle C, with two rectangles contain'd under BC and CD, will be equal to the squares of AC, and BC.

Demonstration.

The line BC being divided in D, (by the 7) the squares of BC and DC are equal to two rectangles under BC and CD, and the square of BD. Add to both the square of AD: the squares of BC, DC, and AD, will be equal to two rectangles under BC, and CD, and the squares of BD, and AD. Instead of the squares of CD, and AD, put the square of AC, which is equal to them, (by the 47. 1) and instead of

the squares of BD, and AD, substitute the square of AB, which is equal to them, (by the same:) the squares of BC, and AC, will be equal to the square of AB, and two rectangles contain'd under BC, and CD

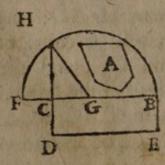
The USE.

These Propositions are very useful in Trigoonometry: I have made use of them in the eighth proposition of my third book, to prove, that in a triangle the Sine total has the same proportion to the fine of an angle, as the rectangle contain'd under the fides, which form that angle, to double the triangle. I have us'd them likewise in the seventh, and divers other propolitions.

PROPOSITION. XIV.

A PROBLEM.

To describe a square equal to a restilineal figure given.



O describe a square cqual to the rectilineal A, make (by the 45.1.) a rectangle BCDE equal to the rectilineal A. If the fides CD, and CB were equal, we should and

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should have what we distr'd: but being unequal; continue the line BC, so that CF may be equal to CD; and dividing the line BF, in the middle at the point G, describe the semicircle FH B; this done, prolong DC to H. The square of CH is equal to the rectilineal A. Draw the line GH.

Demonstration.

The line BF is divided into two equal parts in G, and two unequal in C: therefore (by the 5.) the rectangle contain'd under BC, CF, or CD, that is to fay, the rectangle BD, with the square of CG, is equal to the square of GB, or GH, which is equal to it. Now (by the 47. 1.) the square of GH is equal to the squares of CG, and CH: therefore the rectangle BD, and the square of CG, are equal to the squares of CG, and CH; and therefore taking away the square of CG, which is common to both, there will remain the square of CH equal to the rectangle BD, or, which is the same, the rectilineal A.

The USE.

This Proposition teaches us in the sirst place to reduce any rectilineal figures into squares; which being the chief measure of all superscies, because its dimensions are both equal, we can by this means take the magnitude of all sorts of rectilineal figures. Again it helps us to find a middle proportional betwixt two line

e lines given, as we shall see in the thirteenth

· Proposition of the Sixth Book.

flance of a Formal Definition: for, in his second book, de Anima, sect. 12. distinguishing betwixt a Formal and a Causal Definition, he explains them thus. If, when tis demanded What it is to square a Rectangle? answer be return'd, that it is to describe a square equal to a rectilineal; this answer contains the formal definition. But if it be said, that it is to find a middle proportional betwixt two lines; this gives the Causal definition. For to find a middle proportional is the cause of making a square equal to the rectilineal proposid.

This Proposition may also be farther useful for the squaring of crooked figures; and also, as far as is possible, even the Circle it self; for all forts of crooked figures may, at least as far as is discernible by sense, be reduced to rectificate. As for example, if we inscribe in a circle a Polygone consisting of a thousand fides, there will be no sensible difference betwixt it and the circle: therefore reducing this Polygone to a square, we do, as far as our sense served are capable of judging, square the circle.

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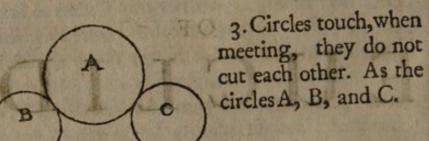
perties of a Circle, and compares divers lines which may be drawn within, or without it's circumference. It considers likewise the circumstances of circles, that cut each other, or touch a right line; and the differences of angles that are made either at the centers or circumferences. In fine it lays down the first principles for the establishing the practical part of Geometry; for which the circle is most commodiously made use of in almost all Treatises of the Mathematicks.

DEFINITIONS.

Those Circles are equal, whose diameters or semidiameters are equal.

2. A line is said to touch a cirle, when, meeting with its circumference, it does not cut it. As

the line AB.

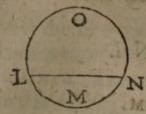


4. Those lines are equally remote from the center; when the perpendiculars, drawn from the center to the lines, are equal. As for example, if EF, and EG, perpendiculars to the lines AB, and CD,

be equal, AB and CD will be equally remote

from the center; because the distance ought

always to be measur'd by perpendicular lines.



5. A segment of a circle is a figure terminated on one side by a right line, and on the other by the circumference of a circle As LON, LMN.

6. The angle of the segment is the angle which the circumference makes with the right line. "As the angles LNO, NLM.

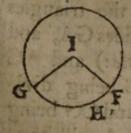


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7. An angle is in that segment in which are the lines that form it. 'As the angle FGH, is in the segment FGH.

8. An angle is upon that arch, to which it is opposed, or which is as its base. "As the "angle FGH, is upon the arch FIH.



9. The Sector is a figure contain'd under two semidiameters, and the arch which serves them for a base. 'As the figure FIGH.

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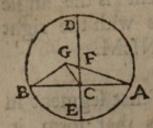
PROP.

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PROPOSITION I.

A PROBLEM.

To find the center of a Circle.



TO find the center of the circle AEBD, draw the line AB, and divide it in the middle at the point C; through which draw the perpendicular ED,

which also divide into two equal parts at the point F, and that point F will be the center of the circle. If it be not, suppose the point G to be the center; and draw the lines GA, GB, and GC,

Demonstration.

If the point G be the center, the triangles GAC, and GBC, will have the sides GA, and GB equal, (by the definition of a circle:) and AC and CB will be equal, the line AB being divided in the middle at the point C, and CG being common, the angles GCB, and GCA will be equal, (by the 8.1) and CG a perpendicular, not CD, which is contrary to the supposition. Therefore the center must of necessity be in the line GD. I add, that it must be at the point F, where it is divided into two equal parts: otherwise

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wife the lines drawn from the center to the cir-

cumference would not be equal.

Coroll. The center of a circle is in that line, which falling perpendicularly upon another, divides it into two equal parts.

The USE.

'This Proposition is necessary to demonstrate those that follow.

PROPOSITION II.

A THEOREM.

A right line drawn from one point of the circumference to another, falls wholly within the circle



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L ET a line be drawn from the point B to the point C. I say, it will be wholly contain'd within the circle. To prove that it cannot fall without the circle, as BVC; having found the cen-

ter of the circle A, draw the lines AB, AC, and AV.

Demonstration.

The fides AB, and AC, of the triangles ABCare equal: therefore (by the 5. 1.) the angle, ABC and ACB are equal, and fince the angle AVC is an external angle in respect of the tri-1 3 angle angle AVB, it is greater than the angle ABC, (by the 16. 1.) and then also it will be greater than the angle ACB. Therefore (by the 19.1.) in the triangle ACV, the side AC, oppos'd to the greater angle AVC, will be greater than AV: and by consequence AV ought not to reach to the circumference of the circle, if the line BVC was a right line.

The USE.

ftrate, that a circle can touch a right line but in one place. For if the line touch'd two points of the circumference, it would be drawn from one of its points to another: and by consequence, according to this Proposition, would enter the circle; though by its definition, the line that touches ought not to cut the circumference. Theodosius makes use of the same Demonstration to prove, that a Globe can touch a plane only in one point; for otherwise the plane would enter within the Globe.

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PROPOSITION III.

are equal, because the lines are equal, then the fourth a Rio and T A countries

If the Diameter divide a line, which does not pass through the center, into two equal parts, it will cut it at right angles; and if it cut it at right angles, it will divide it into two equal parts.



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IF the diameter AC, cut the line BD, which does not pass through the center F, into two equal parts at the point E, it will cut it at right angles. Draw the lines FB, and

FD. Demonstration.

In the triangles FEB; and FED, the side EF is common; the sides BE and ED are equal, because the line BD is equally divided in E, and their bases FB and FD are equal: therefore (by the 8. 1.) the angles BEF and DEF are equal, and by consequence right angles. I add, that if the angles BEF and DEF be right angles, the line BD will be divided into two equal parts at E, that is to say, the lines BE and ED will be equal.

Demonstration.

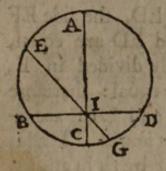
The triangles BEF and DEF are rectangular: therefore (by the 47. 1.) the square of the

fide DF will be equal to the squares of the sides ED, and EF. Now the squares of BF and FD are equal, because the lines are equal, therefore the squares of BE and EF are equal to the squares of DE and EF; and taking away the square of EF, the squares of BE, and ED will be equal, and by consquence the lines.

PROPOSITION IV.

ATHEOREM.

Two lines drawn within a circle cannot cut each other into two equal parts, unless they both pass through the center.



The lines AC and BD cut each other at the point I, which is not the center of the circle, they will not equally divide each other. First, if one of those lines, as AC, pass through the center, 'tis evi-

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dent it cannot equally be divided but at the center. But if neither pass through the center, as BD and EG, draw the line AIC through the center.

Demonstration.

If the line AC divide the line BD into two equal parts in I, the angles AID and AIB will be

be right angles, (by the 3) In like manner if the line EG was equally divided in I, the angle AIE would be a right angle; and consequently the angle AIB and AIE would be equal, which is impossible, one being part of the other. In a word the line AIC, which passes through the center, would be perpendicular to the lines BD and EG, if they were both equally divided at the point I.

The USE.

These two Propositions are us'd in Trigonometry, to demonstrate, that the half of a chord
of an arch is perpendicular to the semidiameter; and consequently, that it is the sine of
half the arch. By these also they demonstrate
that the sides of a triangle have the same proportion, as the sines of the opposite angles. We
also make use of it to find the Eccentricity of
the Circle, which the Sun describes in his annual motion.

PROPOSITION. V.

A THEOREM.

Circles that cut each other, have not the same center.



THE circles ABC, and ADC, which cut each other in A and C, have not the same center. If they had the same center, Sup.

The Elements of Euclid.

suppose E, the lines EA and ED would be equal, (by the definition of a circle;) as also the lines EA, and EB: therefore the lines ED and EB would be equal, which is impossible, one being part of the other.

PROPOSITION VI.

A THEOREM.

Two circles that touch each other on the inner side have not the same center.



THE circles BD and BC, which touch each other on the inner side at the point B, have not the same center. For should the point A be supposed

AB and AC, AB and AD, would be equal, (by the definition of a circle,) and consequently the lines AD and AC would be equal, which is impossible, one being part of the other.

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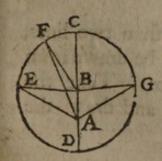
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PROPOSITION VIL

A THEOREM.

If many lines be drawn from any one point within the circle, which is not its center, to the circumference: I. that which passes through the center is the greatest: 2. the remainder of it, continued to the opposite part of the circumference, is the least: 3. that which is nearest to the greatest, exceeds those that are more remote: 4. There can be no more than two of them equal to each other.



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Suppose many lines to be drawn from the point A, being not the center of the circle, to the circumference; and the line AC to pass through the center B: I will demon-

strate, that it is greater than any of the other; for example, that it is greater than AF. Draw FB.

Demonstration.

The sides AB and BF of the triangle ABF, are greater than AF alone, (by the 20. 1) But BF and BC are equal, (by the definition of a circle:) therefore AB and BC, that is to say, A C, is greater than AF.

I add in the second place, that AD is the least;

least; for example, that it is less than AE,

Draw BE. Demnostration.

The sides EA and AB are greater than BE alone, but BE is equal to BD, therefore EA and AB are greater than BD: taking therefore from both that which is common AB, AE will remain greater than AD.

Further, AF, which is nearer AC than AE,

is also greater than it.

Demonstration.

The triangles FBA, and EBA, have the fides BF and BE equal, and BA is common to both: but the angle ABF is greater than the angle ABE: therefore (by the 24. 1.) AF is greater than AE,

Lastly, I say, that no more than two lines, that are equal to each other, can be drawn from the point A to the cicumference. Take the angles ABE and ABG equal; and draw the

lines AE and AG.

Demonstration.

The triangles ABG, and ABE, having the sides BE and BG equal; the side AB common to both, and the angles ABE and ABG equal; therefore their bases AE, and AG will be equal, (by the 4. 1.) But all the lines that can be drawn either on one side or the other, will be either nearer AC, than AE, and AG; or more remote from it; and accordingly will be either greater or less than AG. Therefore there can no more

more than two lines equal betwixt themselves be drawn from the point A to the circumserence.

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tion to prove, that if from any point of the superficies of a sphere, which is not the pole of any certain circle, divers arches of greater circles be drawn to the circumference of that circle, that which passes through its pole will be the greatest. For example: If from the pole of the world, which is distinct from the pole of the Horizon, (for the Zenith is its pole,) divers arches of greater circles be drawn to the circumference; the arch of the Meridian, which passes through the Zenith, will be the greatest arch. This Proposition is also brought to prove, that the Sun, when in his Apogaeum, is most remote from the Earth.

PROP.

PROPOSITION

A THEOREM.

If from a point taken without the circle, many lines be drawn to its circumference, 1. of all those that extend to the concave circumference, that which passes through the center is the greatest: 2. those that lye nearest to it, are greater than those that are more remote: 3. among those that fall upon the convex circumference, that which being continu'd passes through the center, is the least: 4. the nearer to that are less than those farther off: 5. there can be but two equal lines drawn from the Jame point either to the concave or convex circumference.



CUppose many lines drawn from the point A to the circumference of the circle GCDE.

First, the line AC, which paffes through the center B, is the greatest of all those that reach to the concave circumference; for

example, it is greater than AD. Draw the line Demonstration.

BD. In the triangle ABD, the fides AB and BD are greater than AD alone; but the fides AB and

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and BC are equal to AB and BD: therefore AB and BC, or AC, is greater than AD.

2. AD is greater than AE.

Demonstration.

The triangle ABD and ABE, have the side AB common to both, and the sides BD and BE equal, and the angle ABD is greater than the angle ABE: therefore (by the 24. 1.) the base AD is greater than the base AE.

3. AF, which being continu'd passes through the center, is the least of all those that are drawn to the convex circumference LFIK; for

example, it is less than AI. Draw IB.

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vere A to rcle Demonstration.

In the triangle AIB the sides AI and IB are greater than AB alone, (by the 20. 1.) therefore taking from both the equal lines BI and BF, AF will remain less than AI.

4. Al is less than AK. Draw the line BK.

Demonstration.

In the triangles AIB and AKB, the sides AK and KB are greater than the sides AI, and IB, (by the 21. 1.) therefore taking from both the equal sides BK, and BI, AI will remain less than AK.

5. There can be but two lines equal betwixt themselves drawn. Take the angles ABL, and ABK; as also ABE, and ABG equal.

Demonstration.

The triangles ABL, and ABK, will have their bases

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bases AL and AK equal, (by the 4. 1.) and by the same also AE and AG will be equal; but no other line can be drawn, that will not be either nearer to, or more remote from AF, or AC; and consequently, that will not be either greater or less than AK and AL, AE and AG.

PROPOSITION IX.

A THEOREM.

That point from whence three equal lines can be drawn to the circumference of a circle, is its center.

If the point were not the center of a circle, there could be but two equal lines drawn from it to the circumference, (by the 7, and 8.)

PROPOSITION X. A THEOREM.

Two circles cut each other only in two points.



IF two circles AEBD, and ABFD, should cut each other in three points A, B, and D; find (by the 1) the center C of the circle AEBD; and draw the lines CA, CB, and CD.

Demonstration.

The lines CA, CB, and CD, drawn from the center C to the circumference of the circle AEBD, are equal: but the same lines are also drawn to the circumference of the circle ABFD. therefore (by the 9) the point C will be the center of the circle ABFD. So that two circles, which cut each other, will have the same center; which is contrary to the fifth Proposition

PROPOSITION, XI.

A THEOREM.

If two circles touch each other on the inside, a line drawn through both the centers, will also pass through the point where they touch.



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F the two circles EAB and . EFG touch each other on B the inside, at the point E; a line drawn through both their centers will pass through the point E. For if the point D

was the center of the leffer circle, and C that of the greater, so that the line CD passing through both should not pass through the point E: draw the lines CE and DE.

Demonstration.

The lines DE, and DG, drawn from the

center of the lesser circle D to its circumference, would be equal: and adding the line CD, the lines ED, and DC, would be equal to CG. Now ED and DC are greater than EC alone, (by the 20. 1.) and so CG will be greater than CE: yet C being the center of the greater circle, CE and CB are equal: therefore CG will be greater than CB, which is impossible.

PROPOSITION XII.

A THEOREM.

If two circles touch each other on the outside, a line drawn through both their centers, will pass through the point where they touch.

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IF the line AB, which does not pass through the point C where the circles touch, be said to be drawn from the center A to the center B; draw the lines AC and BC.

Demonstration.

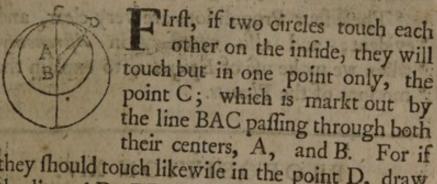
In the triangle ACB, the sides AC and BC would not be greater than the side AB alone, (which is contrary to the 20. 1.) because AD and AC, as also BE and BC, are equal.

PROP.

pair through the point C, where the cit-PROPOSITION XIII.

A THEORFM.

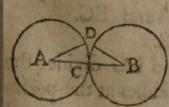
Two Circles can touch each other only in one point.



they should touch likewise in the point D, draw the lines AD, BD.

Demonstration.

The lines AC and AD, drawn from the center of the lesser circle to its circumference, are equal: and adding AB, the lines BA, AC, and BA and AD, would be equal, Now BC and BD, drawn from the center of the greater circle to its circumference, will be equal: therefore the fides BA and AD will be equal to the side BD alone, which is contrary to the 20. I.



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Secondly, if two circles touch each other on the outfide; drawing the line AB from one center to the other,

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it will pass through the point C, where the circles touch, (by the 12.) But if you say that they touch also at the point D: having drawn the lines AD and BD; the line BC and BD, AC and AD, being equal, the two sides of a triangle taken together, would be equal to the third, which is contrary to the 20. I.

The USE.

These four Propositions are very clear, and evident; and also necessary in Astronomy, when we make use of Epicycles, to explain the motions of the Planets.

PROPOSITION XIV.

A THEOREM.

Equal lines drawn within a circle, are equally remote from the center; and those that are equally remote from the center, are equal.



Supposing the lines AB and CD to be equal: I prove, that the perpendiculars EF and EG, drawn from the center, are also equal. Draw the lines EA and EC.

Demonstration.

The perpendiculars EF and EG divide the lines AB and CD in the middles at the points

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F and G, [by the 3.] therefore AE and CG are equal. The angles F and G are right angles: therefore [by the 47. 1.] the square of EA is equal to the squares of EF and FA; as also the square of EC is equal to the squares of EG and GC: but the squares of EA and EC are equal, because the lines EA and EC are equal: therefore the squares of EF and FA are equal to the squares of EG and GC: and taking away the equal squares AF and CG, there will remain the squares of EF and EG equal; and consequently the lines EF and EG, which are the distances of the lines AB and CD from the center, are equal.

But supposing the distances or perpendiculars EF and EG to be equal; I will prove after the same manner that the squares of EF and FA are equal to the squares of EG and GC; and taking away the equal squares of EF and EG, there will remain the squares of AF and CG equal. And therefore the lines AF and CG, and their

double AB and CD, are equal.

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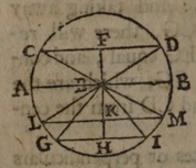
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PROPOSITION XV.

A THEOREM.

The Diameter is the greatest of all lines inscribed in a Circle; and of the rest that is the greatest which is nearest the Center.



THE diameter AB is the greatest of all lines that can be drawn in the circle GIDC. As for example, it is greater than CD; for draw the lines EC and ED.

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Demonstration.

In the triangle CED, the sides EC and ED are greater than CD alone, [by the 20.1.] but AE and EB, or AB, is equal to EC and ED; therefore the diameter AB is greater than CD.

Secondly, let the line GI be more remote from the center than the line CD; that is to fay, let the perpendicular EH be greater than the perpendicular EF. I say that CD is greater than GI. Draw the lines EC, and EG.

Demonstration.

The squares of CF and FE [by the 47. 1.] are equal to the square of EC: but the square of EC is equal to the square of EG, and the square of EG

EG equal to the squares of GH and HE: therefore the squares of CF and FE are equal to the squares of GH and HE; and taking from one side the square of HE, and from the other the square of EF, which is less than the square of HE, the square of GF will remain greater than the square of GH. Therefore the line CF will be greater than the line GH; and the whole line CD, the double of CF, will be greater than GI, the double of GH.

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Theodosius makes use of these two Propositions to demonstrate, that in a sphere the leffer circles are more remote from the center. I have also made use of them in Astrolabes. To these Propositions may likewise be referred that Mechanical proposition of Aristotle, by which he shews, that the Rowers at the middle of a Gally have greater force, than those that are at, either the fore, or hinder part thereof; because the sides of the Gally being crooked, the Oars of the middle part are longer, i.e. reach farther, than the rest. The Demonstrations relating to the Iris, or Rain-bow, do also suppose the truth of these propositions.

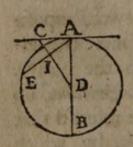
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PROPOSITION

A THEOREM.

A line drawn perpendicularly upon the extremity of the diameter, falls wholly on the out side of the eircle, and touches it. But any other line drawn betwixt that and the circumference of the circle, enters within the circle, and cuts it.



ET the perpendicular AC be drawn upon the point A, which is the extremity of the diameter AB: I say first, that all the other parts of the same line, for example the point C, fall on the outside of the circle. Draw the line DC.

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Demonstration.

Since the angle DAC of the triangle DAC is a right angle, DCA will be an acute: and (by the 19. 1.) the fide DC will be greater than the side DA; therefore the line DC reaches beyond the circumference of the circle.

I add, that the line CA touches the circle, because that meeting with it at the point A it does not cut it, but all its points are on the out-

fide of the circle.

I fay also that no other line can be drawn trom

from the point A below CA, which does not cut the circle. If there could, suppose EA to be such an one; and from the point D draw a perpendicular to it, DI.

Demonstration.

Since the angle DIA is a right angle, and the angle IAD an acute, AD will be greater than DI: therefore the line DI does not reach to the circumference, but the point I is within the circle.

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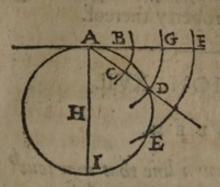
Some Philosophers use this Proposition, but altogether in vain, to prove, that quantity is not divisible in infinitum, or that there really are in the world such things as Zenonical, i.e. absolutely and in their own nature indivisible points. For the Proposition does not, as they would have it, prove, that a circle touches a 'right line in a Zenonical, but in a Mathematical point, which is nothing else but a quantity considered without distinction of parts, that is to fay, without conceiving them distinct and feparate one from the other, whether in reality it has fuch parts or not making no matter. We can therefore take any quantity whatfoever for a Mathematical point; which being once establish'd, our circle will consist of such points, and will be mathematically perfect, provided it touch not a right line, but in a part equal to that quantity which we have taken for a point. But if we afterwards take a e less part for our Mathematical point, the circle which was exactly perfect according to the first supposition, will be imperfect in the second, and degenerate into a Polygone. I believe, tis as impossible to describe a circle, that according to any supposition whatsoever fhall be most exactly perfect, as it is to con-

ceive the least possible quantity.

'Secondly, those consequences, which some men draw from this Proposition relating to the angle of contact, which they take to be less than any rectilineal angle, are grounded upon this miltake, that they imagine an angle to be a true quantity; the contrary of which may sappear from hence, That the lines, that constain an angle, being produc'd to any longitude, the angle becomes not at all the greater. Further, it ought to be duly consider'd, what we mean, when we fay, that one angle is greater than another; for this is all we underfland, that a circle being described from the point of concourse at any distance whatsoever the lines of that we call the greater angle will contain betwixt them a greater arch of that circle, than those of that which we call the e less; which is the sole meaning of the Excess of one angle above another. From whence I infer, that the angle of contact can no more be compar'd with a rectilineal angle, than a fuperficies

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superficies with a line, being at the same time both equal, and greater and less than a recti-



ineal angle. As for example; from the point A draw the line AD, making with AE a rectilineal angle; I say it is both greater and less than, and equal to, the angle of contact.

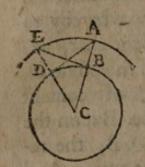
For if we suppose divers circles describ'd from the point A, as the center, whereby to measure those angles; it is evident that, according to the arch drawn beyond the point D, that is the arch EF, the angle of contact is greater than the rectilineal angle. But on the contrary according to the arch CB, the re-Etilineal angle is the greater of the two. And lastly, according to the arch DG, passing through the point in which AD cuts the circumference, they are both equal. From whence it follows, that the angle of contact is at the same time both less and greater than, and equal to, the rectilineal angle: and consequently, they ought not at all to be compar'd together. In a word, Angles are no quantities; nor are they call'd less or greater one than another, but with respect to the arches which they contain: so that all the disputes about the angle of contact, and all the Paradoxes

doxes, conclude nothing either for or against the divisibility of quantity; an Angle being no species, but only a property thereof.

PROPOSITION XVII.

A PROBLEM.

From a point given to draw a line that may touch a Circle.



TO draw a line from the point A touching the circle BD, draw the line AC to its center; and at the point B draw a perpendicular BE, which may cut an arch of a circle, describ'd from the center C through the

point A, at the point E. Draw also the lines EC, and AD. I say the line AD touches the circle in D.

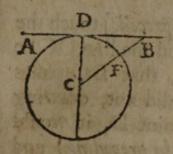
Demonstration.

The triangles EBC and ADC have the same angle C; and the sides CD and CB, CE and CA equal, (by the definit. of a Circle:) and therefore they are equal in all respects, (by the 4.1.) and the angles CBE and CDA are equal But the angle CBE is a right angle, therefore the angle CDA will be so too, and (by the 16.) the line AD will touch the circle.

PROPOSITION XVIII.

A THEOREM.

A line drawn from the center of a circle to the point where a right line touches it, is perpendicular to that line.



Haroline district

If the line CD be drawn from the center C to the point of contact D, CD will be perpendicular to AB. For if it be not, draw the line CB perpendicular to AB Demonstration.

Since the line CB is suppos'd to be perpendicular, the angle B will be a right angle, and consequently CDB an acute, (by the 32. 1.) Therefore the line CB, oppos'd to the sesser angle, will be less than CD, which is impossible; because CF, which is but part of CB, is equal to CD.

PROPOSITION XIX.

A THEOREM.

If a line, perpendicular to the tangent, be drawn from the point of contact, it will pass through the center of the circle.

ET the line AB [see Fig. preced.] touch the circle at the point D, and the line DC be perpendicular to AB. I say, that DC passes through the center. For if it did not, drawing a line from the center to the point D, it would be perpendicular to AB, (by the preceding) and so there would be two perpendiculars drawn to the same point D of the same line, which cannot be.

The USE.

The use of lines Tangents is very common in Trigonometry; upon which account it is that I have made a table, whereby to measure all sorts of triangles, as well spherical as rectilineal. In my Opticks likewise are divers propositions founded upon Tangents; as when is determined what part of a Globe is enlightened. The phases or Apparitions of the Moon are established also upon the same doctrine; and that samous Problem of Hipparchus's, by which he found the distance of the Sun, by the difference of

the true and apparent Quadratures. In Dial-Ing the Italian and Babylonian hours are fre-'quently describ'd by lines Tangents. Lastly we take the dimensions of the Earth by a line that touches its superficies; and in the art of Navigation, take a Tangent line for our Hori-

PROPOSITION. XX,

A THEOREM.

The angle at the center is double the angle at the circumference, which has the same arch for its base.

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F the angle ABC, which is at the center,, and the angle A DC, at the circumference, have the same arch AC for their base, A the first will be double the second. This Proposition has three

different cases: the first of which is, when the line ABD passes through the center B, the line AB in one triangle concurring with the line BD of the other.

oils siduels a Demonstration.

The angle ABC is the external angle in respect of the triangle BDC: therefore (by the 32. 1.) it is equal to both the angles D and C; which being equal, (by the 5. 1.) because their sides BC and BD are equal, the angle ABC is the double of either.

The second case is, when one angle incloses

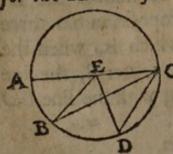


the other, but none of the lines that form them concur in one; as you see in the next figure. The angle BID is at the center, and the angle BAD at the circumference. Draw the line AIC through the center.

Demonstration.

The angle BIC is double the angle BAC, and CID is double the angle CAD, (by the preceding case:) therefore the angle BID is double the angle BAD,

The third case is, when it happens, that neither one angle incloses the other, nor does any of the lines that form them, concur in one. Which case is wholly omitted by my Author, but for the Readers satisfaction is here supplied



Let the angle at the center be BED, and the angle at the circumference BCD, having the same arch for their base BD. I say, the angle BED is double the angle BCD. Draw the line

EC, and continue it to the point A.

ter

Demonstration.

The angle AED is double the angle ACD, (by the 1. case;) and (by the same) the angle AEB is double the angle ACB: therefore the remainder of the one BED is double the remainder of the other BCD.

The USE.

That Problem, which is ordinarily propos'd, shewing how to describe an Horizontal Dial by one sole opening of the Compass, is built in part on this Proposition. And again, when we would determine the Apog aum of the Sun, or the excentricity of his Circle, by three observations, we suppose the angle at the center to be double that at the circumference. Ptelomey makes frequent use of this Proposition to determine both the excentrick circle of the Sun, and the Epicycle of the Moon. The first Proposition of the third book of Trigonometry

is grounded also upon this here.

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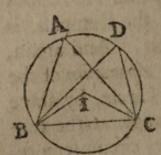
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PROPOSITION XXI

A THEOREM.

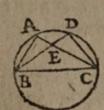
The angles, that are in the same segment of a circle, or that have the same arch for their base, are equal.



F the angles BAC and BDC are in the same segment of a circle, which is greater than a semicircle, they will be ec qual. Draw the lines BI and

Demonstration.

The angles A and D are each of them the half of the angle BIC, (by the preceding,) therefore they are equal. They have likewise the fame arch BC for their base.



Secondly, let the angles A and D be in the same segment BAD, which is less than a semicircle; they will nevertheless be equal.

Demonstration.

All the angles of the triangle ABE are equal to all the angles of the triangle DEC, (by 1. Coroll. of the 32. 1.) but the angles AEB and DEC are equal, (by the 15. 1.) Also the angles ECD and

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and ABE are equal, (by the preceding case,) being in the same segment ABCD, greater than a semicircle; therefore the angles BAE and EDC are equal; which, the angles at E being equal, and consequently (by the Coroll. of the 15.1.) the lines AE and EC, making but one right line, as likewise DE and EB another, are the angles A and D, in the same segment ABCD, and having the arch BC for their base.

The USE.

This Proposition is produc'd in Opticks to prove, that the line BC will appear of the same greatness, when tis view'd from A, and D, because it is seen in both cases under equal

angles.

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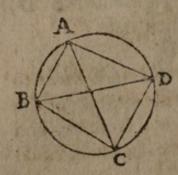
Co CC D nd

The same Proposition is us'd to describe large circles without having their centers: for example, if we would make large Copper basons of a spherical sigure, such as we might work upon in polishing Spectcales, and glasses to see at a great distance. For having made in Iron an angle BAC equal to that, which is contained in the segment ABC, and at the points B and C strongly fastn'd two small iron pins; if the triangle BAC be mov'd so, that the side AB may always touch the pin B, and the side AC the pin C, the point A will describe an arch of the circle ABCD. This manner of describing a circle may also be us'd in making great Astrolabes.

PROPOSITION. XXII.

A THEOREM.

Quadrilateral figures, inscrib'd in a circle, have their opposite angles equal to two right angles.



L ET a quadrilateral figure, or a figure of four fides, be inscribed in a circle, in such fort that all its angles may terminate at the circumference of the circle ABCD: I say the opposite

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angles BAD and BCD are equal to two right angles. Draw the diagonals AC, and BD.

Demonstration.

All the angles of the triangle BAD are equal to two right angles. In stead then of the angle A BD put the angle ACD, which is equal to it (by the 21.) being in the same segment ABCD: and instead of the angle ADB, put the angle ACB, which is in the same segment of a circle BCD A. Therefore the angles BAD, and the angles ACD and ACB, that is to say, the whole angle BCD, are equal to two right angles.

The USE.

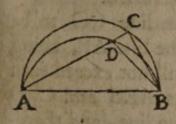
'Ptolomey makes use of this Proposition to frame the table of Chords, or lines subtending

ing arches. I have also us'd the same in my third book of Trigonometry, to prove, that the sides of an obtusangle triangle would have the same proportion among themselves as the sines of the opposite angles.

PROPOSITION XXIII.

A THEOREM.

Two similar segments of a circle, describ'd upon the same line, are equal.



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I call those similar segments of a circle, which contain equal angles; and I say, that if such be describ'd upon the same line AB, they will fall

one upon the other, and not exceed each other in any part. For if either did exceed each other, as do the segments ADB, and ACB, they would not be similar: to demonstrate which, draw the lines ADC, BD, and BC.

Demonstration.

The angle ADB is an external angle in respect of the triangle DBC: therefore (by the 16, 1.) it is greater than the angle ACB, and by consequence the segments ALB and ACB contain unequal angles, which I say is to be diffimilar.

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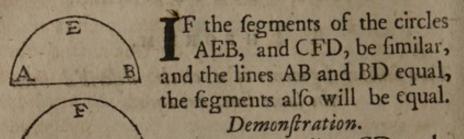
PROP.

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PROPOSITION XXIV:

A THEOREM.

Two similar segments of a circle describ'd upon equal lines, are equal.



Suppose the line CD to be plac'd upon the line AB, being suppos'd to be equal, they will not exceed each other; and then the segments AEB and CFD will be describ'd upon the same line, and therefore will be equal, (by the preceding.)

The USE.

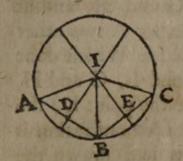
Crooked figures are frequently reduc'd to rectilineals by this Proposition. As for example: if two similar segments of a circle AEC, and ADB, be describ'd upon AB and AC, the equal sides of the triangle ABC: tis evident, that, transposing the segment AEC unto ADB, the

triangle ABC is equal to the figure ADBCEA.
PROP.

PROPOSITION XXV.

A PROBLEM.

To compleat a circle, of which we have but a part.



Having the arch ABC given, to compleat the circle we must find its center; to which end draw the lines AB and BC, which having divided in the middle

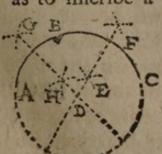
at the points E and D, draw their two perpendiculars EI and DI; which will meet at the point I, the center of the circle.

Demonstration.

The center is in the line DI, (by the coroll. of the t.) it is also in EI, (by the same;) therefore it must be at the point I.

The USE.

'This proposition occurs very frequently:
but sometimes it is express'd in other terms;
as to inscribe a triangle in a circle; or to de-



fcribe a circle through three points given, provided they be not plac'd in a right line. Let the points propos'd be A, B, and C; and placing the foot of the compass at the point C, describe two L 4

arches F and E, at any distance whatsoever. Then remove the foot of the compass to the 'point B, and at the same distance describe two other arches cutting the former in E and F; also from the point B, as the center, describe at any distance the arches G and H, and at the same distance from the center A two other arches cutting them in G and H. Which done draw the lines through F and E, G and H, ' which shall cut each other at the point D, the center of the circle. The Demonstration is obvious enough: for if you had drawn the ' lines AB, and BC, you had, by this operation, divided them equally and perpendicu-'larly. This Proposition is exceeding necessary to describe Astrolabes, and compleat circles, of which we have but three points. That Proposition in Astronomy, which teaches how to find the Apoganm, and excentricity of the circle of the Sun, virtually contains this. And 'I also have made frequent use of it in my Treatife concerning the Cutting of Stones.

PROPOSITION XXVI.

A THEOREM.

Equal angles, whether at the centers, or the circumferences of equal circles, have equal arches for their bases.





IF the angles D and I, at the centers of equal circles ABC, and EFG, be equal; the arches BC and FG will be equal. For if the arch BC was greater or less than the arch FG, since the angles are measur'd by arches, the angle D would be greater or less than the angle I.

But if the equal angles be suppos'd to be at the circumferences of equal circles, as A and E; the

angles which they enclose at the centers, as D and I, being their doubles, will be likewise equal, and consequently require equal arches for their bases, BC and FG; which arches are likewise the measures of the angles A and E.

PROPOSITION XXVII:

ATHEOREM

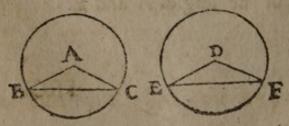
Angles, whether at the centers or circumferences of equal circles, having equal arches for their bases, are also equal.

The angles D and I (fig. preced.) at the centers of equal circles have equal arches BC and FG for their bases, they will be equal because their measures BC and FG are equal. And if the angles A and E, at the circumferences of equal circles have equal arches BC and FG for their bases, since the angles they enclose at the centers will be equal, they also that are the halves of those angles (by the 20.) will be equal.

PROPOSITION XXVIII.

A THEOREM.

Equal lines, within equal circles, answer to equal arches.



If the equal lines BC and EF be applied to equal circles, ABC, and

and DEF, they will be the chords of equal arches, BC, and EF. Draw the lines AB, AC,

ED, EF. Demonstration.

In the triangles ABC and DEF, the sides AB and AC, DE and DF are equal, being the semi-diameters of equal circles; and their bases BC and EF are supposed equal, therefore (by the 8.1.) the angles A and D will be equal; and (by the 26.) the arches BC and EF will be also equal.

PROPOSITION XXIX.

A THEOREM.

The lines that subtendequal arches of equal circles are equal.

IF the lines BC and EF (see fig. preced. Prop.)
fubtend (or are the chords of) equal arches
BC and EF in equal circles, they will be equal.

Demonstration.

The arches BC and EF are equal, and parts of equal circles; therefore (by the 27) the angles A and D will be equal. Therefore in the triangles ABC, DEF, the fides AB, AC, DE, and DF being equal, as also the angles A and D; the bases BC, EF will be equal, (by the 4. 1.)

The USE.

Theodosius by the 28 and 29 demonstrates, that

that the arches of the circles of the Italian and

' Babylonian hours, contain'd between two pa-'rallels, are equal. We have also demonstrat-

ed after the same manner, that the arches of

the circles of the Astronomical hours, contain'd

between two lines parallel to the Equator, are likewise equal. These Propositions are almost

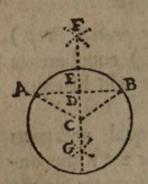
of continual use in spherical Trigonometry, and

also in Dialling.

PROPOSITION XXX.

A PROBLEM.

To divide an arch of a circle into two equal parts.



Suppose the arch AEB was to be divided into two equal parts, Place the foot of the compass at the point A, and describe two arches F and G; then removing it to the point B at the same distance describe other two arches,

cutting the former in F and G; the line GF will cut the arch AB equally at the point E. Draw the line AB.

Demonstration.

By this operation you have divided the line AB into two equal parts. For suppose there were drawn

drawn the lines AF, BF; AG, and EG; (which I have not done, least the figure should appear confus'd,) the triangles FGA and FGB would have all their sides equal, therefore (by the 8.1.) the angles AFD, & BFD would be equal. Again. the triangles DFA and DFB have the fide DF, common, the fides AF and BF equal, and the angles DFA and DFB equal: therefore (by the 4. I.) the bases AD and BD are equal, and also the angles ADF and BDF. We have therefore divided the line AB equally and perpendicularly at the point D. Therefore (by the 1.) the center of the circle is in the line FG: Suppose it then to be the point C, and draw the lines CA and CB; all the fides of the triangles ACD and BCD are equal: therefore [by the 8, 1.] the angles ACD and BCD are equal, and [by the 26.] the arches AE and EB.

The USE.

Having frequent occasion to divide an arch into two equal parts, the exercise of this Prosition is very common. Tis thus that we divide the Mariners compass into 32 winds: for having drawn two diameters cutting each other at right angles, we divide the circle into sour, and subdividing each quarter in the middle, we have eight parts; and again subdividing those twice, we make 32. We have also

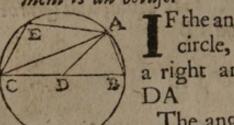
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also occasion for the same operation in the dividing a semicircle into 180 degrees; and because to compleat that division we are oblig'd to divide an arch into three equal parts, all Geometricians have sought after a method of doing that Geometrically, but have not yet been so happy as to find one.

PROPOSITION XXXI.

A THEOREM.

The angle in a semicircle is a right angle, that which is in a segment greater than a semicircle is an acute, and that which is in a lesser segment is an obtuse.



I F the angle BAC be in a semicircle, I will prove that it is a right angle. Draw the line DA Demonstration. 211

The angle ADB being an external angle in regard of the tri-

angle DAC, is equal to both the internals DAC, and DCA (by the 32.1.) and those being equal(by the 5.1.) because the sides DA and DC are equal it will be double the angle DAC. In like manner the angle ADC is double the angle DAB: therefore the two angles ADB, and ADC, which are equal to two right angles, are double the

the whole angle BAC, and confequently the

angle BAC is a right angle.

Secondly, the angle AEC, which is in the fegment AEC less than a semicircle, is an obruse angle. For in the quadrilateral figure AB CE, the two opposite angles E and B are equal to two right angles, (by the 22.) but the angle B is an acute; therefore the angle E will be an obtuse.

Thirdly, the angle B, which is in the fegment ABC greater than a semicircle, is an acute; because in the triangle ABC, the angle BAC is a right angle.

The USE.

Meebanicks make use of this Propositon

to try if their Squares be just; for having describ'd a semi-'circle BAD, they lay down ' the point A of their square B AD upon the circumterence, and one of its sides AB upon the point of the diameter B:

and then the other branch AD ought to pass precifely to the point D, which is the other

extreme of the diameter.

Ptolomey uses this Proposition to compose his rable of Chords or Subtendants, of which he has occasion in his Trigonometry.

· There is also a method of raising a perpendicular

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dicular at the end of a line, grounded upon

this Propolition: For example to raise a perpen-

dicular at the point A of the line AB. I place the foot of the compass upon the point C ta-

ken any where, and describe a circle through

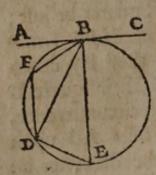
'the point A, cutting the line AB at the point Then I draw the line BCD; and so'tis

evident, that the line AD is in a semicircle.

PROPOSITION XXXII.

ATHEOREM

A line cutting a circle at the point of contact, makes, with the tangent, angles, equal to those in the alternate segments.



ET the line BD cut the circle at the point B, which is that where the line A B touches it. I fay the angle CBD, made by the line BD and the tangent ABC, is equal to the angle F in the alternate feg-

ment BFD; and that the angle ABD is equal

to the angle E in the segment BED.

First, if the line pass through the center, as the line BE, it will make with the tangent two right angles, (by the 18.) and the angles of the semicircles would be also right angles, (by the

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preceding,) therefore in this case the proposition would be true. But if the line do not pass through the center, as BD; draw the line EE through the center, and joyn the line DE.

Demonstration.

The line BE makes, with the tangent, two right angles; and all the angles of the triangle BDE are equal to two right angles, (by the 32. 1.) therefore taking away the right angles CDE, and D which is in the semicircle, and likewise the angle EBD which is common to both, there will remain the angle ABD equal to the angle E.

Again, the angle CBD is equal to the angle F; because in the quadrilateral figure BFDE, which is inscrib'd in a circle, the opposite angles E and F are equal to two right angles, [by the 22.] but the angles ABD and CBD are also equal to two right angles, [by the 13: 1.] and the angles ABD and E are equal, as I have now demonstrated: therefore the angles CBD and F are equal.

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The USE.

'This Proposition is necessary to prove that which follows,

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PROPOSITION XXXIII.

A PROBLEM.

Upon a line given to describe a segment of a circle capable of an angle given.



Let Tit be proposed to decle upon the line AB capable of the angle C. Make the angle BAD equal to the angle C, and draw AE perpendicular to AD; make also the angle ABF equal

to the angle BAE: and in fine, from the point F, where BF and AE concur; at the distance BF or FA, describe a circle. The segment BEA is capable of an angle equal to the angle C.

Demonstration.

The angles BAF and ABF being equal, the lines FA and FB are equal, [by the 6. 1.] and the circle, which is describ'd from the center F, by A, passes by B: Now the angle DAE being a right angle, the line DA touches the circle in A, [by the 16.] therefore the angle contain'd in the segment BEA, as the angle E, is equal to the angle DAB, that is the angle C, [by the preceding.] But if the angle given be an obtuse, we must take an acute, its complement to 180 degrees.

PROPOSITION XXXIV.

A PROBLEM.

A circle being given, to cut a segment in it capable of a certain angle.

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TO cut a segment of the circle BCE capable of the angle A, draw [by the 17.] the tangent BD, and make the angle DBC equal to the angle A. Tis evident [by the 32.]

that the segment BEC is capable of an angle equal to DBC, and consequently to the angle A.

The USE.

I have made use of this Proposition to find Geometrically the excentricity of the Annual circle of the Sun, and his Apogeum, having three observations given. Tis used likewise in Opticks, to find a point where two unequal lines propos'd may appear equal, or under equal angles, by making upon each line segments which will contain equal angles.

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PROP.

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PROPOSITION XXXV.

A THEOREM.

If two lines cut each other within a circle, the re-Etangle contain'd under the parts of one is equal to the rectangle contain'd under the parts of the other.

Irst, if the two lines cut each other in the center, they will be both equal, and both equally divided; so that in that case it is evident, the rectangle contain'd under the parts of one, will be equal to the rectangle contain'd under the parts of the other.



Secondly, if one of the lines pass through the center F, as AC, and divide the line BD, into two equal parts at the point E: I say the rectangle contain'd under AE and EC is equal to the rectangle con-

tain'd under BE and ED, that is to say, to the square of BE. The line AC is perpendicular to BD, [by the 3.]

Demonstration.

Since the line AC is divided equally in F, and unequally in E, the rectangle contain'd under AE, and EC, with the square of FE, is equal

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to the square of FC or FB [by the 5. 2] Now the angle E being a right angle, the square of FB is equal to the squares of BE and FE; therefore the rectangle under AE, EC, with the square of FE, is equal to the squares of BE and EF: and taking away the square of EF, there remains the rectangle under AE, EC, equal to the square of BE.



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Thirdly, let the line pass through the center F, and divide the line CD into unequal parts at the point E: draw FG perpendicular to CD, and [by the 3.] the lines CG and GD will be equal.

Demonstration.

Since the line AB is divided equally in F and unequally in E, the rectangle contain'd under AE, EB, with the square of EF, is equal to the square of FB, or FC, [by the 5. 2.] Instead of the square of EF put the squares of FG and GE, which are equal to it, [by the 47. 1.]

In like manner the line CD being divided equally in G, and unequally in E; the rectangle under CE, ED, with the square of GF, is equal to the square of GC. Add the square of GF; the rectangle under CE, ED, with the squares of GE and GF, will be equal to the squares of CG and GF, that is to say, [by the 47. 1.] to the square of FC. Therefore the rectangle un-

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der AE, EB, with the squares of EG and GF, is equal to the rectangle under CE, ED, with the same squares: and consequently taking away the same squares from both, the rectangle AE, EB, will be equal to the rectangle CE, ED.

Fourthly, if the lines CD and HI, cut each other in E, neither of the two passing through the center: I say, the rectangle CE, ED is equal to the rectangle HE, EI. For drawing the line AFB, it is plain the rectangles CE, ED, and HE, EI, are both equal to the rectangle AE, EB, [by the preceding case;] therefore they are equal betwixt themselves.

The USE.

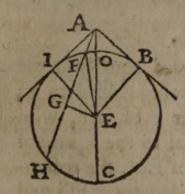
We are taught by this Proposition a method of finding a fourth proportional to three lines given, or a third proportional to two.

PROP.

PROPOSITION XXXVI.

A THEOREM.

If from a point taken without the circle a line be drawn to touch, and another to cut the circle; the square of the Tangent will be equal to the rectangle contain dunder the whole secant, and the external line.



Suppose the line AB to be drawn from the point A, taken without the circle, to touch the circle in B; and the line AC, or AH cutting it. The square of AB will be equal to the rectangle contain'd under AC, and AO

AF. If the secant pass through the center, as AC, draw the line EB.

Demonstration.

Since the line OC is divided in the middle at the point E, and the line AO added to it; the rectangle contain'd under AO and AC, with the square of OE or EB, will be equal to the square of AE, [by the 6.2.] Now the line AB is suppos'd to touch the circle at the point B; therefore [by the 18.] the angle B is a right an
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gle, and (by the 47. 1.) the square of AE is equal to the squares of EB and AB; therefore the rectangle under AC and AO, with the square of EB, is equal to the squares of EB and AB: and taking away the square of EB from both, the rectangle under AC, AO will be equal to the square of AB.

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Secondly, suppose the secant AH not to pass through the center; and draw the line EG perpendicular to FH, which will divide in the middle the line FH at the point G; draw also the

line EF.

Demonstration.

The line FH being divided equally at the point G, and the line AF being added to it; the rectangle contain d under AH, AF, with the square of I.G, will be equal to the square of AG. Add to both the square of EG: the rectangle under AH, AF, with the squares of FG and GE, that is (by the 47. 1.) the square of FE, or EB, will be equal to the squares of AG and GE, that is, (by the 47. 1.) the square of AE. Further, the square of AE (by the same) is equal to the squares of EB and AB: therefore the re-Stangle contain'd under AH, AF, with the square of BE, is equal to the squares BE and AB: and taking away the square of BE from both, the re-Etangle contain'd under AH, AF will be equal to the square of AB.

Coroll. 1. If you draw divers secants from the

fame point, as AC and AH, the rectangles under AC and AO, AH and AF, will be equal betwixt themselves, since they are both equal to

the square of AB.

Coroll. 2. If you draw two tangents from the fame point, as AB, AI, they will be equal: because the squares will be equal to the same rectangle under AC, and AO, and consequently betwixt themselves; as also the lines.

PROPOSITION XXXVII. A THEOREM.

If the rectangle contain'd under the secant and the external line be equal to the square of a line that falls upon the circle, that line will touch the circle.

Suppose the secant to be AC or AH, and the rectangle AC, AO; or AH, AF, (see fig. preced.) to be equal to the square of the line AB; the line AB will touch the circle. Draw the tangent AI, (by the 17.) and the line IE.

Demonstration.

Since the line AI touches the circle, the rectangle AC, AO; or AH, AF, will be equal to the square of AI. But the square of AB is supposed to be equal to either of those rectangles; therefore the squares of AI and AB are equal, and consequently the lines AI and AB. Therefore

the triangles ABE and AIE, having all sides equal, will be equiangular, (by the 8.1.) and because the angle AIE is a right angle (by the 18.) the line AI being a tangent, the angle ABE will be a right angle, and the line AB a tangent, (by the 16.)

The USE.

* Maurylocus makes use of this Proposition to find the diameter of the Earth. For observing from the top of a mountain OA, the superficies of the Earth by the line BA, he takes notice of the angle OAB, made by the line A. B, and a perpendicular AC: and by Trigonometry calculates the length of the line AB. Then multiplying AB by AB to have its square, he divides the product by AO the height of the mountain, which gives the quotient AC, the diameter of the earth, with the height of the mountain; from which having subducted AO, there will remain OC the diameter of the earth. This Proposition serves also to prove the fifth of the third book of Trigonometry.

THE FOURTH BOOK

OF THE

ELEMENTS

EUCLID.

This fourth Book is exceeding useful in Trigonometry. For by inscribing Polygons in a Circle, we learn the methods of composing the Table of Subtendants, Tangents, and Secants; a practice most necessary for taking all forts of Dimensions.

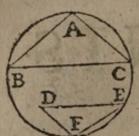
Again, by inscribing Polygons in a circle, we find the divers Aspects of the Stars, which also take their names from those Polygons.

Thirdly, the same Operations give us the Quadrature of the Circle, as exact as is needful. And by them we also demonstrate, that Circles are in the duplicate proportion of that of their Diameters.

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Fourthly, Military Architecture does frequently

DEFINITIONS.



A Rectilineal figure is infcrib'd in a circle, or a circle is describ'd about it, when all its angles are in the circumference of the same circle.

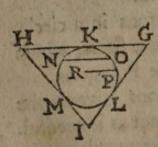
As the triangle ABC is infcrib'd in a circle, and the circle is describ'd

about the triangle; because its angles A, B,

and C, do all terminate at the circumference. The triangle DEF is not inscrib'd in the cir-

cle, because the angle D does not terminate at the circumference of the circle.

2. A rectilineal figure is describ'd about a circle, and the circle inscrib'd within that fi-



gure, when all the sides of the figure touch the circumference of the circle. "As the triangle GHI is described about the circle KLM because its fides touch the circumference of the circle in K, L, and M.

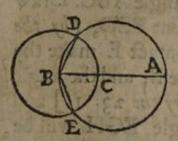
3. A line is apply'd to, or inscrib'd in a cir-

cle, when its two extreams touch the circumference of the circle. "As the line NO. But "the line RP is not inscrib'd in the circle.

PROPOSITION I:

A PROBLEM.

To inscribe in a circle a line, that does not exceed its Diameter.



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Let a line be propos'd to be inscrib'd in the circle AEBD, not exceeding its diameter. Take the length of the Line propos'd upon the diameter; for example, let it

be BC. Place the foot of the compass upon the point B, and describe a circle at the distance of BC, which may cut the circle AEBD in D and E. Then draw the line BD or BE. Tis evident they are equal to BC, (by the desinition of a circle.)

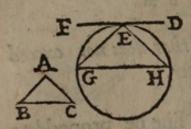
The USE.

"This Proposition is necessary for the performance of what is requir'd in the following.

PROPOSITION

A PROBLEM.

To inscribe in a circle a triangle equiangular to another triangle.



ET the circle be EGH, in which a triangle is to be inscrib'd, equiangular to the triangle ABC. Draw the tangent FED, (by the

17. 3.) and at the point of contact E make the angle DEH equal to the angle B, and the angle FEG equal to the angle C, [by the 23. 1.] and draw the line GH; the triangle EGH will be equiangular to the triangle ABC.

Demonstration.

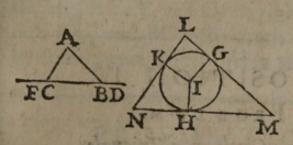
The angle DEH is equal to the angle EGH of the alternate segment, [by the 32. 3.] But the angle DEH is equal to the angle B, and consequently the angles B and G are equal. By the same reason the angles C and H are also equal, and [by Coroll. 2. of the 32. 1.] the angles A and GEH will be equal. Therefore the triangles E GH and ABG are equiangular.

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PROPOSITION III

A PROBLEM.

To describe a triangle about a circle equiangular to another triangle.



F you would describe a triangle equiangular to the trian-M gle ABC about the circle GKH

Continue one of the sides of the triangle given BC to Dand F, and make the angle GIH equal to the angle ABD, and HIK equal to the angle ACF: then draw the tangents LGM, LKN, and NHM, through the points G, K, and H. These tangents will concur; because the angles IKL and IGL being right angles, if you should draw a line KG, the angles KGL and GKL would be less than two right angles: therefore the lines GL and KL mult concur, [by the 11. Axiom

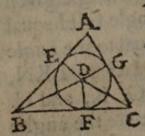
Demonstration. All the angles of the quadrilateral GIHM are equal to four right angles, because it may be divided into two triangles. The angles IGM and IHM, which are made by the rangents,

are right angles; therefore the angles M and I are equal to two right angles, as are also the angles ABC and ABD. But the angle GIH is equal to the angle ABD, therefore the angle M will be equal to the angle ABC. By the same reason the angles N and ACB are equal, and therefore the triangles LMN and ABC are equiangular.

PROPOSITION IV.

A PROBLEM.

To inscribe a Circle in a Triangle.



If you would inscribe a circle in the triangle ABC, divide the angles ABC and ACB into two equal parts, [by the 9. 1.] drawing the lines BD and CD, which will concur

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at the point D. This done, from the point D draw the perpendiculars DE, DF, and DG, which will be equal; so that a circle describ'd from the center D, at the distance DE, will pass through F and G.

Demonstration.

The triangles DEB and DFB have the angles DEB and DFB equal, being both right angles: the angles DBE and DBF are also equal, the angle

angle ABC having been divided into two equal parts; and the fide DB is common: therefore (by the 26. 1.) the triangles will be equal in all respects, and the fides DE and DF will be equal. After the same manner might I demonstrate the fides DF and DG to be equal. 'Tis possible therefore to describe a circle, which shall pass through the points E, F, and G; and because the angles E, F, and G are right angles, the sides AB, AC, and BC will touch the circle, which by consequence is inscrib'd in the triangle.

PROPOSITION V.

A PROBLEM.

To describe a Circle about a Triangle.

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IF you would describe a circle about the triangle ABC, divide the sides AB and BC into two equal parts, at the points D and E, drawing the perpendicu-

lars DF and EF, which will concur at the point F. Which done, if you describe a circle from the center F, at the distance FB, it will pass through A and C; that is to say, the lines FA, FB, and FC, are equal.

Demonstration.

The triangles ADF and BDF have the fide

The Elements of Euclid.

DF common, and the sides AD and DB equal, the side AB having been divided equally in D; and the angles at D are equal, being right angles. Therefore (by the 4. 1.) the bases AF and BF are equal; as also the bases BF and CF.

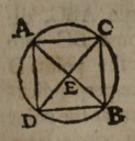
The USE.

I have frequent occasion to inscribe a triangle in a circle; as, for instance, in the first proposition of my 3. book of Trigonometry. This performance also is necessary for the measuring the area of a triangle; and upon many other occasions.

PROPOSITION VI.

A PROBLEM.

To inscribe a square in a circle.



To inscribe a square in the circle ACBD, draw the diameter AB, and perpendicular to it the line DC passing through the center E; then draw the lines AC, CB, BD, DA, and you will

being AC become gles right ECC the fam

have inscrib'd in the circle the square ACBD.

Demonstration.

The triangles AEC and CEB have their sides equal, and the angles AEC and CEB equal, being

being both right angles: therefore their bases AC and CB are equal, (by the 4. 1.) Further, because the sides AE and EC are equal, the angles EAC and ECA will be equal: and the angle E being a right angle, they will be half-right angles, (by the 32. 1.) therefore the angles ECB is half a right angle, and consequently the angle ACB will be a right angle. And the same reason holds for all the rest: therefore the sigure ACBD is a square.

PROPOSITION VIL

A PROBLEM.

To describe a square about a circle.

Having drawn the two diameters AB, and CD, which cut each other perpendicularly at the center E, draw the tangents FG, GH, HI, and IF, by the points A

D, B, C, and you will have describ'd the square FGHI, about the circle ADBD.

Demonstration.

The angles E and A are right angles, therefore (by the 27. 1.) the line, FG and CD are parallels. After the same manner I may prove that CD and HI, FI and AB, AB and DH, are parallels. Therefore the figure FCDG is a paralle.

PROPOSITION VIII.

A PROBLEM.

To inscribe a Circle in a Square.

F you would inscribe a Circle in the Square FGHI, [see the fig. preced.] divide the sides FG, GH, HI, IF, in the middles at the points A, D, B, C, and draw the lines AB and CD, which may cut each other at the point E. I demonstrate that the lines EA, ED, EB, and EC are equal, and the angles A, D, B, C, right angles: and that therefore you may describe a circle from the center E, which will pass through A, D, B, and C, and touch the sides of the square FGHI,

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Demonstration.

Since the lines AB and GH do conjoyn the lines AG and BH, which are parallel and equal, they also will be parallel and equal: therefore the figure AGDE is a parallelogram; and the lines AE and GD, AG and ED are equal: and AG and GD being equal, AE and ED will be equal also. Tis after the same manner that the lines AE, EC, EB, are prov'd equal. Further, AG and CD being parallel, and the angle G a right angle, the angle D will be so likewise. Therefore the circle ADBC may be describ'd from the center E, which will pass through the points A, D, B, C, and touch the sides of the square.

PROPOSITION IX.

A PROBLEM.

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To describe a Circle about a Square.

D, [see fig. in Prop. 6.] draw the diagonals AB, and CD, which will cut each other at the point E; the point E will be the center of the circle, which will pass through the points A, C, B, D. It ought therefore to be demonstrated, that the lines AE, EB, CE, and ED are equal.

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Demonstration.

The fides AC and CB are equal, and the angle C is a right angle; therefore the angles BAC and ABC are equal, (by the 5. 1.) and

half right angles, (by the 32. 1.)

After the same manner I demonstrate, that the angles ACD, ADC, BDC, and BCD, are half right angles. Therefore the triangle AEC having the angles EAC and ECA half right angles, and consequently equal, will have also (by the 6.1.) its sides AE and EC equal. The same may be prov'd of the lines EC and EB, EB and ED, that they likewise are equal.

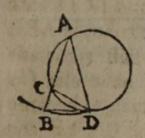
The USE.

We shew in the 12 Book, that Polygons inscrib'd in a circle, degenerate into circles; and as these Polygons are always in the duplicate proportion of that of their diameters, so likewise are circles. In practical Geometry we have frequent occasion to inscribe a square and other Polygons in a circle, or to describe them about it, to reduce a circle to a square.

PROPOSITION. X.

A PROBLEM.

To describe an Isosceles (or equicrural triangle) baving its angles at the base, each of them double to the third angle.



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To describe an Isosceles ABD, having each of its angles ABD and ADB, double the angle A, divide the line AB (by the 11.2.) so that the square of AC may be equal to the rectangle un-

der AB and BC; and from the center A at the distance AB describe the circle BD, in which inscribe BD equal to AC; and drawing the line DC describe a circle about the triangle ACD, (by the 5)

Demonstration.

Since the square of AC or BD is equal to the rectangle contain'd under AB and BC, the line BD will touch the circle ACD at the point D, (by the 37. 3.) therefore the angle BDC will be equal to the angle A, being in the alternate segment CAD, (by the 32. 3.) Now the angle BCD, being an external angle in respect of the triangle ACD, is equal to the angles A and N 4

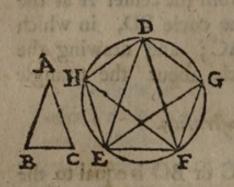
196 The Elements of Euclid.

CDA; therefore the angle BCD is equal to the angle BDA. Further, the angle ADB is equal to the angle ABD, (by the 5. 1.) therefore DCB and DBC are equal, and (by the 6. 1.) the fides BD and DC will be equal; and fince BD is equal to AC, the fides AC and CD will be equal; and fo likewise the angles A and CDA. Therefore the angle ADB is double the angle A.

PROPOSITION XL

A PROBLEM.

To inscribe a regular Pentagon in a circle.



TO inscribe a Regular Pentagon in a circle, describe (by the 10.) an Isosceles ABC, having each of its angles ABC, ACB, at the base, double the angle

A. Inscribe in the circle the triangle DEF equiangular to ABC: then divide the angles DEF and DFE into two equal parts, drawing the lines EG and FH. Lastly, joyn the lines DH, DG,GF, EH, and you will have describ'd a regular Pentagon; that is to say, a Pentagon having equal sides, and equal angles.

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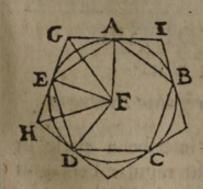
Demonstration.

The angles DEG, GEF, DFH, and HFE, being the halves of the angles DEF and DFE, each of which is double to the angle EDF, are equal to the angle EDF: and consequently the five arches, which are their bases, are equal, (by the 26. 3.) and the lines DH, HE, EF, FG, and GD, are equal, (by the 29. 3.) Secondly, the angles DGF, GFE, and so of the rest, having each three of those equal arches for its base, will be also equal, (by the 27. 3.) Therefore the sides and angles of the Pentagon DHEFG are equal.

PROPOSITION XII.

A PROBLEM.

To describe a Pentagon about a Circle



Micribe a regular Pentagon ABCDE in the gon ABCDE in the circle, (by the 11,) and having drawn tangents through the points A, B, C, D, E, (by the 17.3.) you will have describ'd a regular Pentagon

gon about the circle. Draw the lines FA, FG, FE, FH, FD.

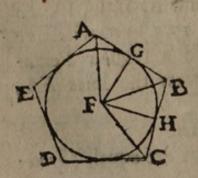
Demonstration.

The tangents GE and GA are equal, (by co-roll.

roll. 2. of the 36. 3.) as also EH and HD: the lines FA and FE are also equal (by the definit. of a circle;) therefore (by the 8. 1) the triangles FGA and FGE are equal in all respects; and the angles AFG and EFG are equal, as also the angles EFH and DFH. And because (by the 27. 3.) the angles EFA and EFD are equal, their halves EFH and EFG will be equal; and (by the 26. 1.) the triangles EFH and EFG will be equal in all respects, and the sides EG and After the same manner EH will be also equal. I can demonstrate all the sides to be divided into two equal parts; and confequently, fince the lines GE and GA are equal, GH and GI will be also equal. Further, the angles G and H being double the angles FGE, and FHE, are Therefore we have describ'd a realso equal. gular Pentagon about the circle.

PROPOSITION XIII. A PROBLEM.

To inscribe a circle in a regular Pentagon.



To inscribe a circle in the regular Pentagon ABCDE, divide the angles A and B into two equal parts by the lines AF and BF, which concur at the point

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point F. Then drawing the line FG perpendicular to AB, describe a circle from the center F at the distance FG. I say it will touch all the other sides; that is to say, having drawn FH perpendicular to BC, IH and FG will be equal.

Demonstration.

Since the equal angles A and B were divided into two equal parts, their halves GAF and GBF will be equal: And fince the angles at G are right angles, the triangles AFG and BFG will be equal in all respects, (by the 26.1.) therefore the lines AG and GB are equal.

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Further, I may prove the lines BG and BH, as also the lines FG and FH, to be equal; and the sides AB and BC of a regular pentagon being equal, the lines BH and HC will be equal, and consequently, the angles at the point H being also right angles, and equal, the triangles BFH and HFC will be equal in all respects, and the angles FBH and FCH will be equal. And since the angles B and C are equal, the angles FBH will be half the angle C. So passing from one to the other I will demonstrate, that all the perdendiculars FG and FH, and the rest, are equal.

PROPOSITION XIV.

A PROBLEM.

To describe a circle about a regular Pentagon.



TO describe a circle about the regular Pentagon ABCDE, divide equally two of its sides AB and BC at G and H, and draw the perpendiculars GF and HF. The circle drawn from the center F, at the distance

DG.

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FA, will pass through B, C, D, E.

Demonstration.

Suppose the circle describ'd, it is evident (by the 1.3.) that having divided the line AB in the middle in G, and drawn the perpendicular GF, the center of the circle must be in that perpendicular: It is also in HF; therefore it is at the point F.

The USE.

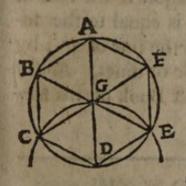
These Propositions are solely useful for the composing the table of Sines, and drawing the platforms of Gittadels, for their ordinary sigures are Pentagons. Observe also that these methods of describing Pentagons about a circle, may be apply'd likewise to other Polygons.

gons. But in my book of Military Architecture, I have shewn another way of inscribing a regular Pentagon in a circle.

PROPOSITION XV.

A PROBLEM.

To inscribe a regular Hexagon in a Circle.



TO inscribe a regular Hexagon in the circle ABC DEF, draw the diameter AD, and fixing the foot of the compass at the point D describe a circle at the distance of DG: then draw the diameter AD,

meters EGB, and CGF; and the lines AB, AF, FE, and the rest. Demonstration.

Tis evident, that the triangles CDG, and DGE, are equilateral; therefore the angles CGD, DGE, and those opposed to them at the top BGA, and AGF, are each of them the third part of two right angles; that is to say, contain 60 degrees. Now all the angles that can be made about the same point are equal to four right angles, that is to say, 360 degrees. Therefore taking away four times 60, that is 240, from 360, there will remain 120 for BGC and FGE; which therefore each contain

degrees. Therefore all the angles at the lenter being equal, all the arches and all the ides will be equal; and every angle as A, B, C, &c. will be compounded of two angles of 60 degrees each, that is, 120 degrees, and therefore will be equal.

Coroll. The fide of a Hexagon is equal to the

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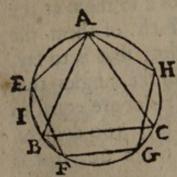
The USE.

Because the side of a Hexagon is the base of an arch of 60 degrees, and is equal to the semidiameter, its half will be the sine of 30; by which sine we begin the table of Sines, Euclider treats of Hexagons in the last Book of his Elements.

PROPOSITION XVI.

A PROBLEM.

To inscribe a regular Pentedecagon in a circle.



Inscribe in the circle an equilateral triangle ABC (by the 2) and a regular pentagon (by the 11.) so that the angles may meet at the point A. The lines BF, BI, and IE, will be the sides of

a Pentedecagon: and if you inscribe in the other

arches lines equal to BF and BI, you will com-

Demonstration.

Since the line AB is the fide of an equilateral triangle, the arch AEB will be the third part of the whole circle, that is, five fifteenths. But the arch AE being the fifth part, will contain three fifteenths; therefore the arch EB contains two: and if you divide it in the middle at the point I, each part will be a fifteenth.

The USE.

This Proposition serves only to open the way to other Polygons. We have in the Compass of Proportion some most easie methods of inscribing all ordinary Polygons, but they are grounded on this here. For it would be impossible to mark Polygons upon that instrument, if their sides were not first found by this, or other like Propositions.

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par'd with the less; whether it do really contain the less, or not.

A Part taken in general is ordinarily divided 'into (that which is call'd) an Aliquot part, and

an Aliquant part.

I. An Aliquot part (which alone Euclid defines in this Book) is a magnitude of a magnitude, a less of a greater, when it exactly meafures the greater. 'That is to fay, 'tis a leffer quantity compar'd with a greater, which precifely measures the greater. As a line two foot long taken three times, is equal to a line of fix feet in length.

2. A Multiple is a magnitude of a magnitude, a greater of a less, when it is exactly meafur'd by the less. "That is to say, a Multitiple is a greater quantity compar'd with a less, which it contains exactly so many times. For example, a line fix foot long is triple a

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the same

· line two foot long.

An Aliquant part, is a lesser quantity compar'd with a greater, which it does not exactly measure. As a line of four feet in length is an aliquant part of a line ten foot long. "In a word, An Aliquot part so many times repeated will equal the whole: but an Aliquant part, though it contains such a quantity of the whole, yet repeated as you please will never exactly equal, but either come short of, or exceed, the whole. Equi-

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Equimultiples are magnitudes which equally contain their aliquot parts. That is to fly, to many times. for example, if A contains

B as many times as C contains D, A and C will

be equimultiples of B and D.

3. * Proportion is a respect of one magnitude to another of the same kind.

* Grac. Noy G. Gall Raison.

4. Quanties are said to have a certain proportion to each other, when being mutiply'd they can exceed each other. "For which reafon they ought to be of the same kind. 'indeed a line has no proportion to a supersicies; because a line taken Mathematically is consider'd without any breadth at all; and therefore multiply'd as much as you please will never give any breadth, which yet a fue perficies contains.

For as much as Proportion is a respect or relation founded upon quantity, it ought to have two terms. That which some Philoso-' phers would call the Fundamentum, or Foundation, Mathematicians name the Antecedent, and the Term is call'd by them the Consequent. As if we were to compare the quantity A with the quantity B, that respect or proportion would have for the Antecedent the quantity A, and for the consequent the quantity B.

tecedent, and A for the consequent.

Proportion, or the respect of one quantity to another, is divided into Rational Proportion,

'and Irrational.

'Rational Proportion is the respect of one quantity to another, which is commensurable

to it, that is, when both the quantities have the fame common measure, by which both may be

exactly measur'd. As the proportion of a line four foot long to another that is six, is rational,

because a line two foot long may exactly meafure both. A when this happens, these quan-

tities have the same proportion as one number to another. For example, since the line two

foot long, which is their common measure, is

found twice in the four-foot line, and thrice in that which is fix foot long; the first has the

fame proportion to the second, as 2 to 3.

Irrational Proportion is betwixt two quantities of the same kind, which are incommen-

furable, i. e. have no common measure. As the proportion of the side of a square to its diago-

nal. For there is no measure so small, as will

precifely measure both.

Four quantities will be proportionals, when the proportion of the first to the second, is the fame with, or like to, that of the third to the fourth; so that, to speak properly, Proportionality onality is a similitude of proportions. But it is no easie matter to understand in what consists this similitude of proportions; that is to say, how two respects of relations may be alike. For Euclid has not given us a just definition thereof, or such an one as might explain the nature of the thing, but contented himself to set down some marks or signs, by which it may be known, whether or no quantities have the same proportion. And 'tis the obscurity of this definition, which has render'd the whole Book so difficult to be understood; which defect therefore I shall endeavour to supply.

5. Euclid makes four magnitudes to have the fame proportion, when taking the Equimultiples of the first and the third, and likewise the Equimultiples of the second and the fourth, according to any multiplication whatsoever: if the multiple of the first exceed that of the second, the multiple of the third will also exceed that of the fourth; and if it be equal to, or less than the second, the third will be equal to or less than the fourth. In such a case the first has the same proportion to the second, as the third to the fourth.

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A, B :: C, D 2, 4 :: 3, 6 E, F :: G, H 10, 8 :: 15, 12 K, L :: M, N 8, 8 :: 12, 12 O, P :: Q R 6, 16 :: 9, 24 As for example, if four magnitudes were proposed, A, B, C, D; having taken the Equimultiples of A and C, as their quintuples E and G; and F and H the doubles of B and D; In like manner taking K and M the quadruples of A and

G, and L and N the doubles of B and D; Again taking O and Q the triples of A and C,
and P and R the quadruples of B and D; Because E being greater than F, G is greater
than H; and K being equal to L, M is equal
to N; and lastly, O being less than P, Q is
less than R: therefore A will have the same
proportion to B, as C to D. But methinks Euclid ought to have demonstrated this Proposition, it being too perplex'd and obscure to pass
for a Principle.

To explain aright what Proportionality is, or how four magnitudes may be in the same proportion; though it may be sufficient to say in general, that the first ought to be a like part, or a like whole in respect of the second; as the third is, compar'd with the fourth: yet because this definition agrees not to the proportion of Equality, I shall give a more geneneral one; and to make it the more intelligible, explain first what is a similar or like Aliquot part.

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Similar Aliquot parts, then are such as are contain'd in their wholes as many times one as the other: as 3 in respect of 9, and 2 in respect of 6, are similar aliquot parts; because they are each contain'd three times in their respective wholes.

The first quantity will have the same proportion to the second, as the third to the fourth, if the first contains so many times such aliquot parts of the second, as the third contains the

A,B,C,D, 'like aliquot parts of the fourth. As 'if A contains the hundredth, thoufandth or hundred-thousandth

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part of B, as oft as C contains the hundredth, thousandth, or hundred thousandth part of D; (and the like may be said of all other aliquot parts imaginable:) there will be the same proportion of A to B, as of C to D.

To render this Definition still more clear, I will prove first that, if A has the same proportion to B as C to D, A will contain the aliquot parts of B, as oft as C does the like of D; and secondly, if A contain the aliquot parts of B as oft as C does the like of D, then there will be the same proportion of A to B, as of C to D.

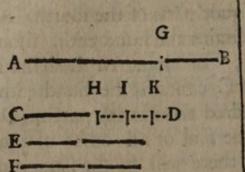
The first point seems sufficiently evident from the very notion of the terms: for if A contains the tenth part of B once more than an hundred times, and C contains the tenth part

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of D an hundred times only; the quantity A will be a greater Whole compar'd to B, than C compar'd to D; therefore it cannot be compar'd after the same manner, the respect or re-

lation being not the same.

"The second point seems more difficult, viz. if a quantity, suppose AB, contain the aliquot parts of another, CD, as oft as a third, E, contains the like of a fourth, F: there will be the same proportion of AB to CD, as of E to



'F. But if it be other wife, let us suppose AB to have a greater proportion to CD, ' than E has to F; that 'is to fay, that AB is too great to have the ' fame proportion to 001

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CD, that E has to F: Therefore a quantity less than AB, as AG, will have the same propor-'tion to CD, as E to F. Divide therefore CD in 'the middle in H, and HD in the middle in I, and ID in the middle in K; continuing the bike division till you arrive at an aliquot part of CD less than GB, which I will suppose to be KD.

Demonstration. Since there is the same proportion of AG to CD, as of E to F; AG will contain KD, an aliquot part of CD, as oft as E contains the like 'like aliquot part of F. Now AB will contain KD once more than E contains the like aliquot part of F; which is contrary to the supposition.

6. The first Quantity is said to have a greater proportion to the second, than the third to the sourth, when the first contains a certain aliquot part of the second oftner than the third contains the like aliquot part of the sourth: As 101 has a greater proportion to 10 than 200 to 20, because 101 contains the tenth part of, 10, that is, 1, once above a hundred times; and 200 contains the tenth part of 20, i.e. 2, a hundred times only.

7. Magnitudes or Quantities having the same

proportion, are call'd proportionals.

8. * Proportionality, or Analogy, is a fimilitude of Proportions or Respects.

* 'Avanoyia. Eucl.

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9. In each proportionality are required at the least three terms: "For that there may be a similitude of proportions, there must be two of them: and every proportion having two tems, an Antecedent and a Consequent, there seems to be a necessity of four terms; as when we say, that A has the same proportion to B as C to D; but because the consequent of the farst proportion may be the antecedent of the second, three terms may suffice; as when A is said to have the same proportion to B as B to C.

10. Magnitudes are said to be continually proportionals, when the intermediate terms are taken twice, i. e. both as antecedents and confequents. 66 As if there be the same proportion of A to B, as of B to C, and of C to D.

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11. In that ease A will have the duplicate proportion to C, and the triplicate to D, of what

it has to B.

But here it is to be observ'd, that there is a great deal of difference between double proportion, and duplicate. We say that the proportion of four to two is double, because four is the double of two; the number two giving the name to the proportion, or rather to the antecedent thereof. Accordingly double, triple, quadruple, quintuple, &c. are denominations drawn from the numbers two, three, four, five, &c. compar'd with unity; which I inflance in, because we more easily conceive the s proportion, the less are its terms. But, as I faid, these denominations do rather affect the antecedents, than the proportions themselves; for we call that double or triple proportion, whose antecedent is double or triple its confequent. But by duplicate proportion we understand such an one, as is compounded of two fimilar proportions. As, if there be the same proportion of two to four, as of four to eight: the proportion of two to eight being compounded of the proportion of two to four, and that

that of four to eight, which are similar an equal, will be the duplicate of each of them. So three to twenty seven is the duplicate proportion of that of three to nine. The proportion of two to four is call'd the sub-double, because two is the half of four; but that of two to eight is the duplicate of the sub-double: which is as much as to say, that two is the half of half of eight, as three is the third part of the third part of the third part of twenty seven; where you may observe, that the Denominators and are taken twice.

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In like manner the proportion of eight to two is a duplicate proportion of that of eight to four, because eight is the double of sour, but it is the double of the double of two. If there be four terms in continual proportion, the proportion of the first to the last is a triplicate of that of the first to the second; as in these four numbers, Two, Four, Eight, Sixteen; the proportion of two to sixteen is a triplicate of that of two to four, because two is the half of the half of the half of sixteen. So also the proportion of sixteen to two is a triplicate of that of sixteen to eight, because sixteen is the double of the double of the double of two.

quents to consequents, are call'd Homologous magnitudes. As if there be the same proportion

of A to B, as of C to D; A and C, B and D

are homologous.
The following Definitions explain the divers

manners of arguing from proportionals: for the demonstration of which this Book was princi-

pally compos'd.

pare the antecedent of the one with the antecedent of the other, and the consequent of the one with the consequent of the one with the consequent of the other. "As for example, if because there is the same proportion of A to B, as of C to D, I infer, that there is the same proportion of A to C as of B to D. This manner of argumentation holds only when all the four terms are of the same species or kind; i.e. either all lines, or all superficies's, or all solids. Tis demonstrated Prop. 16.

14.*Inverted proportion is the comparing of

the confequents with the antecedents.

Avanaly Eucl. Converse Gall.

As, if because there is the same proportion of A to B, as of C to D, I conclude that there is the same proportion of B to A, as of D to

. C. Coroll. of Propos. 16.

paring of the antecedent and the consequent taken together, with the consequent alone. As if, because there is the same proportion of A to B, as of C to D, I conclude that there is the same proportion of A and B to B, as of C and D to D. Prop. 18.

16. Divilion of Proportion is the comparing of the Excess of the antecedent above the consequent to the same consequent. " As, if there be the same proportion of AB to B, as of CD to D; from thence I infer, that there is the ' the same proportion of A to B, as of C to D. Proposition 17.

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17. Conversion of Proportion is the comparing of the antecedent with the Difference of the Terms: " As, if there be the same proportion of AB to B, as of CD to D. I thence conclude, that there is the same proportion of AB to A, as of CD to C. Coroll. of Prop. 18.

18. Proportion of Equality is the comparing of the extream magnitudes, and omitting those

in the middle. " As if, there being the same proportion of A to E.F.G.H. to B, as of E to F; and of B to 'C, as of F to G; and of C to D,

'as of G to H; I infer, that there is the same

proportion of A to D, as of E to H.

19. Proportionality or Equality orderly plac'd, is that in which the terms are compar'd together in the same order. "As in the fore-

going example. Prop. 22.

20. Proportionality of Equality disorderly plac'd, is that in which the terms are compar'd in a different order. ', As if, there being the fame proportion of A to B, as of G to H; and of B to C, as of F to G; and of C to D, as of

E to F; I conclude that there is the same pro-

portion of A to D, as of E to H. Prop. 23. See in short all the different manners of argumentation by Proportion.

" As A to B, fo C to D; therefore By Alternate proportion: as A to C, fo B

Inverted: As B to A, fo D to C

Composition. As AB to B, so CD to D.

Division. If as AB to B, so CD to D: then As A to B, fo C to D.

Conversion. As AB to A, so CD to C. Orderly Equality. If as A to B, so C to D; and as B to E, fo D to F: then as A to E, fo C to F.

Disorderly Equality. If as A to B, so D to F, and as B to E, fo C to D: then as A to E, fo C to F.

Euclid's fifth Book contains but 25 propositions, to which nine more have fince been added, and are commonly receiv'd. And the first fix in Euclid, serving only for the proof of those that follow by the method of Equimultiples, fince I intend not to make use of that method, I shall wholly omit; begining with the Seventh, without changing either the order or

number of the propolitions.

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Three quantities A, B, C, being propos'd, it is requir'd to be granted, that there is a fourth possible, to which the quantity C has the same proportion as A to B.

PROPOSITION VII.

ATHEOREM

Equal quantities have the same proportion to another quantity, and another quantity has the same proportion to quantities that are equal.

A, 8.

C. 4.

B, 8.

The quantities A and B be equal, they will have the same proportion to the third C.

Demonstration.

If one of the two, suppose A, had a greater proportion to C than B: A would contain any aliquot part of C, oftner than B could contain the same; and consequently A would be greater than B, which is contrary to what was supposed.

Again, I say, if the quantities A and B be equal, the quantity C will have the same proportion to A as to B.

Demonstration.

If the quantity C had a greater proportion to A, than to B, it would contain a certain aliquot part of A, oftner than the like part of B; which part therefore of A must be less than the like aliquot part of B. and consequently the quantity A would be less than B, which is contrary to the supposition.

PROPOSITION VIII.

A THEOREM.

The greater of two quantities has a greater proportion to the same, than the less; and the same quantity has a lesser proportion to the greater, than to the less.

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Suppose the quantities AB and C be compar'd with the same EF, and that

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AB exceeds C. I say, that AB will have a greater proportion to EF, than C will have to the same. Cut AD equal to C, and divide EF in the middle, and again one half in the middle, and so on, till you come to an aliquot part of EF less than DB, as GF.

Demon-

Demonstration.

AD and C being equal, AD has the same proportion to EF, as C to EF, (by the 7) and therefore AD will contain GF an aliquot part of EF, as oft as C will contain the same, (by defin. 5.) But AB contains the same aliquot part once more than AD, DB being greater than GF; therefore (by defin. 6.) the proportion of AB to EF is greater than that of C to the same EF.

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Secondly, I say, that EF has a less proportion to AB, than to C. Take a certain aliquot part, as the fourth, of C, as oft as you can in FF, which suppose to be five times; either there will remain something of the quantity EF, or nothing; if nothing remain, it is evident, that five times the fourth part of AB making a greater line than so many times the fourth part of C, the fourth part of AB could not be five times contain'd in EF. But if the fourth part of C taken five times reach no farther than G, the fourth part of AB taken so many times will proceed either as far as F, or to I, something short of F. If it reach as far as F, EF will have the same proportion to AB as EG to C. But (by the preceding part) EF has a greater proportion to C, than EG to the same C; therefore EF has greater proportion to C, than to AB. But if the fourth part of AB reach no farther than I, EI will have the same proportion to AB as EG to C.

C. But EI has a greater proportion to C, than EG to C; therefore EF, greater than EI, has a greater proportion to C, than the same EF to AB.

PROPOSITION IX.

A THEOREM.

Quantities that have the same proportion to another quantity, or to whom another quantity has the same proportion, are equal.

A, B, C,

IF the quantities A and B have the same proportion to a third quantity C, I say, A and B are equal.

Demonstration.

If one of the two, v. g. A, were greater than B, it would have a greater proportion to the quantity C, (by the 8.) which is contrary to the supposition.

Secondly, if the quantity C has the same proportion to the quantities A and B; I say A and B are equal. For if A were greater than B, C would have a greater proportion to the quantity B, than to A, (by the 8.) which is also contrary to the supposition.

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PROPOSITION X:

A THEOREM.

The quantity that has the greater proportion to an other; is the greater quantity; and that the lefser, to which that other quantity has the greater proportion.

F the quantity A has a greater pro-A, B, C, I portion to the quantity C, than B to the same C, I say, A is greater than B. For if A and B were equal, they would have both the same proportion to C; and if A were less than B, B would have a greater proportion to C, than A to the same C; both which are contrary to the supposition.

Secondly, if C has a greater proportion to B than to A, I say that A will be greater than B. For if A and B were equal. C would have the same proportion to both, [by the 7.) And if A were less than B, C would have a less proportion to B than to A, (by the 8.) both which are

contrary to what was suppos'd.

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224 The Elements of Euclid.

PROPOSITION XI.

A THEOREM.

Proportions that are equal one to unother, are also equal amongst themselves.

A, B; C, D; EF. 4, 2; 8, 4; 6,3. IF A has the same proportion to B, as C to D; and C the same proportion to D, as E to F;

I say, A will have the same proportion to B, as E to F.

Demonstration.

Since A has the same proportion to B, as C to D; A will contain any certain aliquot part of B, as oft as C contains the like aliquot part of D, (by defin. 5) And in like manner, as oft as C contains that aliquot part of D, so oft will E contain the like aliquot part of F. So that as oft as A contains any certain aliquot part of B, so many times also will E contain the like aliquot part of F. Therefore A has the same proportion to B, as E to F.

PROPOSITION. XII.

ATREOREM.

If many quantities be proportional, one Antecedent will have the same proportion to his consequent, as all the Antecedents taken together to all the Consequents.

A. B. as C to D; I say that A and C taken together will have the same proportion to B and D, as A to B.

Demonstration.

Since A has the same proportion to B, as C to D; the quantity A will contain any certain aliquot part of B, as oft as C contains the like aliquor part of D, (by defin. 5) suppose the fourth part. Now the fourth part of B and the fourth part of D, make the fourth part of BD; and accordingly AC will contain the fourth part of BD, as oft as A contains the fourth part of B; and the like may be said of any other aliquot parts. Therefore A has the same proportion to B, as AC to be BD.

PROPOSITION XIII,

A THEOREM.

If of two equal proportions one is greater than a third, the other will be so likewise.

A, B:C, D:E,F| D; but a greater proportion to B, than E to F: I say, that C also will have a greater proportion to D, than E to F.

Demonstration.

Since A has a greater proportion to B than E to F, A will contain a certain aliquot part of B, oftner than E contains the like aliquot part of F, (by defin. 6.) but C contains a like aliquot part of D, as oft as A contains that of B; because A has the same proportion to B, as C to D: and therefore C contains a certain aliquot part of D, oftner than E contains the like aliquot part of F; and consequently, C has, a greater proportion to D, than E to F, (by def. 6.)

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PROPOSITION XIV.

A THEOREM.

If the first quantity has the same proportion to the second, as the third to the fourth; according as the first is greater, or equal, or less, than the third, the second will be greater, or equal, or less, than the fourth.

A, B:C, D. I fay first, if A be greater than C, B will be also greater than D.

Demonstration.

Since A is greater than C, A will have a greater proportion to B, than C to the same B, (by the 8.) But there is the fame proportion of A to B, as of C to D: therefore C has a greater proportion to D, than C to B, and consequently (by the 10) B is greater than D.

Secondly, if A be equal to C, B will be also

equal to D. Demonstration.

Since A and C are equal, there will be the same proportion of A to B, as of C to the same B, (by the 7.) But as A to B, so C to D; therefore C has the same proportion to B, as the same C to D, and consequently B and D are equal, (by the 9.)

Thirdly, if A be less than C, B will also be

less than D. Demonstration.

Since A is less than C, A will have a less proportion to B, than C to the same B, (by the 8.) But as A is to B, so C to D: therefore C will have a less proportion to D, than the same C to B, and consequently B will be less than D, (by the 10)

PROPOSITION XV.

A THEOREM.

Equimultiples, and similiar aliquot parts, are in the same proportion.

A, B; C, D,

the Equimultiples of A and

E, 2; H, 3,

F, 2; l, 3,

G, 2, K, 3,

C into parts equal to A, v. g. E,

F, G, and the quantity D into parts equal to B. Because C and D are the equimultiples of A and B, there will be as many parts of one, as of other other. Let the parts of D therefore be H, I, K.

Demonstration.

E has the same proportion to H, and F to I, and G to K, as A to B, because they are all equal.

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qual. Therefore (by the 12.) as A to B, fo E,

F, G, to H, I, K, i. e. fo C to D,

Coroll. The same numbers of the aliquot parts of two quantities are in the same proportion that the quantities are For since E has the same proportion to H, as C to D; and F to I, as C to D; E and F will have the same proportion to H and I, as C to D.

PROPOSITION XVI.

A THEOREM.

Alternate Proportion.

If four magnitudes of the same kind be proportional, they will be also alternatively so.

IF A has the same proportion to B as C to D, and all the four quantities are of the same kind, that is, either all Lines, or all Supercifies's, or all Solids; A will have the same proportion to C, as B to D. For if not, suppose A to have a greater proportion to C, than B to D.

Demonstration.

Since tis supposed, that A has a greater proportion to C than B to D, the quantity A will contain a certain aliquot part of C, v.g. a third

part

part, oftner than B contains a third part of D. Let A therefore contain a third part of C four times, but B the third part of D only three times; having then divided A into four parts, each will contain one third part of C; but B being divided into four parts, they will not contain each of them a third part of D; therefore three fourths of A will contain three thirds of C, that is, the whole quantity of C; but three fourths of B will not contain three thirds, or the whole quantity of D. But on the contrary, fince there is the same proportion of A to B, as of C to D, there will be also the same proportion of three fourths of A to three fourths of B, as of C to D, (by the coroll. of the 15) and (by the 14) if three fourths of A be equal to C, three fourths of B will be equal to D; therefore A cannot have a greater proportion to C, than B to D.

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A LEMMA.

If the first have the same proportion to the second, as the third to the fourth; any aliquot part of the first will have the same proportion to the second, as the like aliquot part of the third to the (fourth.

16, A,	3; B;	32, 6, C, D
E,	AL Y	F.
4.		8.

If A has the fame proportion to B, as C to D; and E be an aliquot part of A, and F a like aliquot part of C: I say, that E will have the same proportion to B, as F to D.

Demonstration.

If E had a greater proportion to B than F to D, E would contain a certain aliquot part of B, oftner than F contains the like aliquot part of D; and consequently, E taken twice, thrice, or four times, would contain an aliquot part of B, oftner than F taken twice, thrice or four times contain'd the like aliquot part of D. But E taken four times is equal to A; and likewise F taken four times is equal to C; therefore A would contain an aliquot part of B oftner than C contain'd the like aliquot part of D, and by consequence A would have a greater proportion to B than C to D; which is contrary to the supposition.

A COROLLARY,

"which Euclid places after his 4th Proposition,

Inverted Proportion

If the first has the same proportion to the second, as the third to the fourth; the second will have the same proportion to the first, as the fourth to the third.

A, B; C, D, 4, 8; 12, 24, E, F, 1, 3.

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IF A has the same proportion to B as C to D, B will have the same proportion to A as D to C.

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Demonstration.

If B had a greater proportion to A than D to C, B would contain an aliquot part of A, suppole a fourth of E, oftner than D contain'd F the fourth part of C. Let us suppose then that B contains eight times the quantity E; D must contain but seven times the quantity F. Now fince A has the same proportion to B as C to D, E will have the same proportion to B as F to D, (by the preceding Lemma,) and (by the 15) E taken eight times will have the same proportion to B, as F taken eight times to D; but E taken eight times is contain'd in B, therefore F taken eight times must be contain'd in D, notwithstanding what was shewn to the contrary; therefore B cannot have a greater proportion to A, than D to C.

The USE.

The Followers of Averroes seem to have made use of a manner of argumentation not much unlike this, to prove that the world had existed from Eternity; urging, that there is the same proportion between an eternal act of the will of God, and the eternal production of the world, as between a temporal act, and a temporal effect; therefore by a kind of Alternation, there is the same proportion of a temporal act of the will, i. e. an act beginning in time, to an eternal effect; as of an eternal will to a temporal effect. Now tis evident, that the will

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will, or an act of the will that begins in time, cannot produce an eternal effect; therefore the eternal act of God could not produce an Effect in time. But this argument is faulty in two respects; first, in that it supposes it possible for an act of the divine will to begin in time; and secondly, in that it is drawn from Alternate proportion, though the terms be of a different kind or species.

PROPOSITION XVII.

A THEOREM.

The Division of Proportion.

If compounded quantities be proportional, they will be so likewise being divided.

AB,B; CD,D, I TF AB has the same propor-8, 3; 16, 6, A will have the same proportion to B as CD to D, A will have the same proportion to B, as C to D. Demonstration.

Since AB is supposed to have the same proportion to B as CD to D, AB will contain a certain aliquot part of B, as oft as CD contains the like aliquot part of D. Now that aliquot part must be found as oft in B, as the like aliquot part is found

found in D. Therefore taking away B from AB, and D from CD, A will contain as many aliquot parts of B, as C contains the like of D, and consequently A will have the same proportion to B, as C to D.

PROPOSITION XVIII.

A THEOREM.

The Composition of Proportion.

If quantities, being divided, be proportional, they will be so lik wife when compounded.

A, B; C, D, 5. 3; 10,6. AB,B; CD,D, 8, 3; 16, 6,

F A has the fame proportion to Bas C to D, AB also will have the same proportion to B as CD to D. Demonstration.

Since A is suppos'd to have the same proportion to B as C to D, A will contain any aliquot part of B, as oft as C contains the like aliquot part of D. Now the quantity B contains any of its own aliquot parts, as oft as D contains the like of his; therefore adding B to A, and D to C, AB will contain any aliquot part of B as oft as CD contains the like aliquot part of D, and consequently (by defin. 5.) AB will have the same proportion to B as CD to D.

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A COROLLARY.

The Conversion of Proportion:

If AB has the same proportion to B as CD to D, then AB will have the same proportion to A as CD to C. For (by the preceding) A has the same proportion to B, as C to D: and (by the Coroll. of the 16.) B will have the same proportion to A, as D to C; and therefore compounding them, AB will have the same proportion to A, as CD to C.

The USE.

"We have frequent use of this manner of argumentation in almost all parts of the Mathematicks.

PROPOSITION XIX.

A THEOREM.

If the Wholes be in the same proportion, as the parts that are taken away from them, the Remainders will be also in the same proportion.

AB, CD; B, D, IF the quantity AB has 16, 8; 4, 2, A, C; AB, CD, CD, as the part B to the part D: I fay, A will have

have the same proportion to C, as AB to CD.

Demonstration.

AB is supposed to have the same proportion to CD, as B to D: therefore alternatively (according to the 16.) AB has the same proportion to B, as CD to D; and by conversion of proportion, AB will have the same proportion to A, as CD to C; and again alternatively, there will be the same proportion of AB to CD, as of A to C.

The USE.

'This Propolition is commonly made use of in the rule of Fellowship. For instead of working by the rule of Three for every particular Associate or Partner, having done it for the rest, to the last they assign the Remainder of the Gain; supposing that if there be the same proportion of the whole sum of all the Principal of one Associate to his part of the Gain: there will be also the same proportion of the Principal pal that remains to the Remainder of the Gain.

The 20 and 21 Propositions are not neces-

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PROPOSITION XXII.

A THEOREM.

The proportion of Equality orderly plac'd.

If divers terms be propos'd, and an equal number of others compar'd with them, so that those which answer to each other in the same order be proportional; the firsts and the lasts will be also proportional.

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12, 6, 2; 6, 3, 1, IF the quantities A,B,C Land the quantities D,E F, be proportional; that is, if there be the same

proportion of A to B as of D to E, and of B to C as of E to F; A will also have the same proportion to C, as D to F.

Demonstration.

If A has a greater proportion to C than D to F, A will contain an aliquot part, v. g. the half of C, oftner than D can contain the half of F. Let us suppose then the half of C to be contain'd twelve times in A, and the half of F only eleven times in D. Now because B has the same proportion to C as E to F, the quantity B will contain the half of C, as oft as E contains the half of F: Suppose then those halves to be contain T

tain'd fix times in each, B and E. A, which contains the half of C twelve times, will have a greater proportion to B, which contains the same half of C six times, than D, which contains the half F eleven times only, to E, which contains the same half of F six times; and confequently A will have a greater proportion to B, than D to E, which is contrary to what was suppos'd.

PROPOSITION XXIII

A PROBLEM.

The proportion of Equality disorderly plac'd.

If two Orders of terms, be in the same proportion, disorderly plac'd: the first and last of both Orders will be proportional.

A, B, C. D, E, F. G, TF the quantities A, B, C, and the others D, E, 12, 6,3. 8, 4, 2. 1, F, equal in number, be in the same proportion, disorderly plac'd, that is, if A has the same proportion to B as E to F, and B to C as D to E: A will have the same proportion to C, as D to F. Suppose B to have the same proportion to C, as F to G.

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Demonstration.

Since there is the same proportion of A to B as of E to F, and of B to C as of F to G; A has the same proportion to C, as E to G, (by the 22.) Further, since B has the same proportion to C as D to E, and also as F to G; D will have the same proportion to E, as F to G, (by the 1 1.) and alternatively; (according to the 16.) D will have the same proportion to F, as E to G. Now we have before prov'd, that as E to G, so A to C; therefore, as A to C, so D to F.

PROPOSITION XXIV.

A THEOREM.

If the first quantity has the same proportion to the second, as the third to the fourth; and also the fifth to the second, as the sixth to the fourth: the first with the fifth will have the same proportion to the second, as the third with the sixth to the fourth.

E, F. 4, 6. 6, 2; 9, 3, A, B; C, D,

IF A has the same proportion to B as C to D, and E to B as F to D; AE will have the same proportion to B, as CF to D. Demonstration.

Since A has the same proportion to B as C to D.

D, A will contain any aliquot part of B, as oft as C contains the like aliquot part of D, (by defin. 5) In like manner, E will contain the same aliquot part of B, as oft as F contains the like of D; so that A and E will contain any aliquot part of B, as oft as C and F contain the like aliquot part of D: therefore AE will have the same proportion to B, as CF to D.

PROPOSITION XXV.

A THEOREM.

If four magnitudes be proportional, the greatest and the least will exceed the other two:

B, D, 8, 6, 4, 3; 4, 3, A,C; E,F, IF the four magnitudes AB, CD, E, F, be proportional; and AB the greatest and F the least; AB and F will exceed CD and E. Since AB has the

fame proportion to CD as E to F, and AB is fuppos'd greater than E; CD will be also greater than F, (by the 14.) Divide therefore AB so, that the magnitude A may be equal to E; and CD so, that the magnitude C may be equal to F.

Since AB has the same proportion to CD as A to C, B will also have the same proportion to D as AB to CD, (by the 19.) and AB being

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fuppos'd greater than CD, B will be greater than D. Now if to A and E, which are equal, be added F and C, which are also equal, A and F will be equal to C and E, and adding to the two first B, which is the greater, and to the two last D, which is the less, AB and F will be greater than CD and E.

The USE.

By this Proposition is demonstrated a propriety of Geometrical proportionality, whereby it is distinguish'd from that which is call'd Arithmetical. For in this latter the two middle terms are equal to the two extreams; but in the former, (as has been prov'd,) the greatest and the least exceed the two others,

^{&#}x27;Tho' the nine following Propositions are not Euclids; yet, because many make use of them, and quote them as if they were his, I thought I ought not to omit them.

PROPOSITION. XXVI.

A THEOREM.

If the first has a greater proportion to the second than the third to the fourth, the fourth will have a greater proportion to the third than the second to the first.

9, 4; 6, 3, A,B; C, D, E. 8. IF A has a greater proportion to B than C to D, D will have a greater proportion to C than B to A. Suppose E to have the same proportion to B as C to D

A will be greater than E, (by the 10.)

Demonstration.

There is the same proportion of E to B, as of C to D: therefore (by the Coroll. of the 16.) D has the same proportion to C, as B to E. But B has a greater proportion to E than to A, (by the 8.) therefore D has a greater proportion to C, than B to A.

PROPOSITION. XXVII.

A Theorem.

If the first has a greater proportion to the second than the third to the fourth, the first will also have a greater proportion to the third than the second to the fourth.

A, B; C, D,

A will have a greater proportion
to C than B to D. Let E have
the same proportion to B, as C to

D; in that case A must be greater than E.

Demenstration.

E has the same proportion to B, as C to D: therefore (by the 16.) E has the same proportion to C, as B to D. And because A is greater than E, the proportion of A to C will be greater than that of E to C Therefore the proportion of A to C, is greater than that of B to D.

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PROPOSITION XXVIII.

A THEOREM.

If the first has a greater proportion to the second than the third to the fourth, the first and the second will have a greater proportion to the second, than the third and the fourth to the fourth.

9, 4; 6, 3, A, B; C, D, E, 8.

F the Proportion of A to B be greater than that of C to D, the Proportion of AB to B will also be greater than that of CD to D. Suppose E to have the

some proportion to B, as C to D.

Demonstration.

E has the same proportion to B, as C to D: therefore (by the 18.) EB has the same proportion to B, as CD to D. And AB being greater than EB, AB will have a greater proportion to B, than EB to the same B, and consequently than CD to D.

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PROPOSITION XXIX.

A THEOREM.

If the first with the second has a greater proportion to the second, than the third with the fourth to the fourth; the first will have a greater proportion to the second, than the third to the fourth.

9,4;6,3. A,B; C,D, E. If the proportion of AB to B be greater than the proportion of CD to D, the proportion of A to B will be also greater than that of C to D. Suppose EB to have the

fame proportion to B, as CD to D: EB will be less than AB, and E less than A:

Demonstration.

Since EB has the same proportion to B, as CD to D; dividing them, E will have the same proportion to B, as C to D, (by the 17.) And A being greater than E, the proportion of A to B will be greater than that of E to the same B, and consequently than that of C to D.

PROPOSITION XXX.

A THEOREM.

If the proportion of the first with the second to the second, be greater than that of the third with the fourth to the fourth; the proportion of the first with the second to the first, will be less than that of the third with the fourth to the third.

A, B, C, D, be greater than that of CD to D, the proportion of AB to B to D, the proportion of AB to A will be less than that of CD to C.

Demonstration.

The proportion of AB to B is supposed to be greater than that of CD to D: therefore (by the 29.) the proportion of A to B will be greater than that of C to D; and (by the 26.) the proportion of D to C will be greater than that of B to A: therefore being compounded (by the 28.) the proportion of CD to C, will be greater than that of AB to A.

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PROPOSITION XXXI.

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If many quantities are in a greater proportion among themselves, that an equal number of other quantities, plac'd after the same manner; the first of the first order will be in a greater proportion to the last of that order, than the first of the second order to the last of that.

A, B, C, D, E, F.

16, 10,3, 9, 6, 2,

portion to B, than D to E; and a greater proportion to C, than E to F; A will have a greater proportion to C, than D to F.

Demonstration.

Since A has a greater proportion to B than D to E, A will also have a greater proportion to D than B to E; and because B has a greater proportion to C than E to F, B will also have a greater proportion to E than C to F. Therefore A will have a greater proportion to D than C to F: and Alternatively (by the 27.) A will have a greater proportion to C, than D to F.

PROPOSITION XXXII.

A THEOREM.

If many quantities are in greater proportion among themselves, than an equal number of other quantities, plac'd after a different manner; the first of the first order will have a greater proportion to the last of that Order, than the first of the second Order to the last of that.

13, 6 A, C, B,	, 2, E,	PR.	25	
В,	F	H	, I,	K,
12,	3	4.	2,	I

portion to C than I to K, and C a greater proportion to E than H to I; A will have a greater pro-

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portion to E, than H to K. Suppose B to have the same proportion to C as I to K, and C the same proportion to F as H to I; then A will be greater than B, and F than E.

Demonstration.

Since 'tis propos'd that B has the same proportion to C as I to K, and C to F as H to I; B will have the same proportion to F, as H to K, (by the 23.) But A has a greater proportion to F, than B to the same F, (by the 8.) and the proportion of A to E is greater than that of A to F, because F is greater than E: therefore the proportion of A to E, is greater than that of H to K.

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PROPOSITION XXXIII.

A THEOREM.

If the Whole has a greater proportion to the Whole than the part taken away to the part taken away, the Remainder will have a greater proportion to the Remainder than the Whole to the Whole.

TF AB has a greater proportion to CD than B to D, A will have a greater proportion to C than AB to CD.

Demonstration.

We suppose that the proportion of AB to CD is greater than that of B to D; therefore (by the 27.) the proportion of AB to B is greater than that of CD to D: and (by the 30.) the proportion of AB to A, is less than that of CD to C; therefore alternatively, the proportion of AB to CD is less than that of A to C:

PROPOSITION XXXIV.

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A THEOREM.

If two orders of magnitudes be proposed, and the proportion of the first of the first, to the first of the second, be greater than that of the second to the second; and that greater than that of the third to the third, &c. the whole first order will have a greater proportion to the whole second, than the whole first order except its first magnitude to the whole second order except its first magnitude. But a less proportion than that of the first magnitude of the second; and lastly, a greater proportion, than that of the last magnitude of the first order, to the first order, to the second.

The proportion of A to E be greater than that of B to F, and the proportion of B to F greater than that of C to G: I say, that A,B,C, will have a greater proportion to E, F, G, than the proportion of BC to FG.

Demonstration.

Tis suppos'd the proportion of A to E is greater than that of B to F; and therefore alternatively

ternatively, the proportion of A to B is greater than that of E to F; and by compounding them, the proportion of AB to B greater than that of EF to F; and again alternatively, the proportion of AB to EF greater than that of B to F. And because the proportion of the whole AB to EF is greater than that of the part B to the part F, the proportion of the Remainder A to the Remainder E will be greater than that of the whole AB to the whole EF: In like manner, I may prove the proportion of B to F greater than that of BC to FG, and consequenty that of A to E much greater than that of BC to FG. Therefore alternatively, the proportion of A to BC is greater than that of E to FG; and compounding them, the proportion of A, B, C, to BC, greater than that of E, F, G, to FG: therefore the proportion of A,B,C, to E, F G, will be greater than that of BC to FG.

Secondly, the proportion of A to E, is greater than that of A,B,C, to E, F, G.

Demonstration.

I have demonstrated, that the proportion of the whole A,B,C, to the whole E, F,G, is greater than that of the part BC to the part IG: therefore the proportion of the Remainder A to the Remainder E, will be greater than that of the whole A B, C, to the whole E, F, G, (by the 33.)

Thirdly, the proportion of A,B,C, to E,F,G,

is greater than that of C to G.

Demonstration.

The proportion of A to E is greater than that of B to F; and therefore alternatively, that of A to B is greater than that of E to F; and compounding them, the proportion of AB to B will be greater than that of EF to F; and again alternatively, that of AB to EF greater than that of B to F. But the proportion of B to F is greater than that of AB to EF is greater than that of C to G; and that of AB to C greater than that of C to G; and that of AB to C greater than that of EF to G; and therefore by compounding them, the proportion of A, B, C, to C, will be greater than that of E, F, G, to G; and that of A, B, C; to E, F, G, greater than that of C to G.

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THE SIXTH BOOK

The Elements of Lincold

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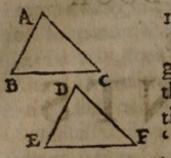
ELEMENTS

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EUCLID.

His Book begins to apply the Doctrine of Proportions, explain'd in general in the 'preceding, to particular matters; and taking its rise from the most simple figures, i.e. Tri-'angles, it gives Rules to determine not only the proportion of their sides, but also that of their capacity, Area or superficies. In the enext place we learn from it how to find out 'proportional lines, and to augment or diminish any figure, according to any proportion affign'd. "Here also is demonstrated the most useful Rule of Three; and the Forty-seventh of the First 'extended to any figure whatsoever. Lastly, it 'lays down the most facile and most certain Principles to conduct us in taking any manner of Dimensions. DE-

DEFINITIONS:



Retilineal figures are fimilar, when their angles are equal, and the fides, that form those angles, proportional. As the triangles ABC, DEF, will be similar, if the angles A and D, B and E, C

and F, be equal; and AB has the same proportion to AC as DE to DF, and AB to BC as DE to EF.

A C B E D E

2. Figures are reciprocal, when they may be so compar'd, that the antecedent of one proportion and the consequent of another are both found in the same Figure. That is, when

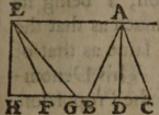
the Analogy begins and ends in the same figure. As if AB has the same proportion to CD, as DE to BF.

3. A line is divided according to the extreme and middle proportion, when the whole

line has the same proportion to the greater part, as the B greater part to the less. As if AB has the same proportion

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to AC, as AC to CB: the line AB is divided according to the extreme and middle proportion.



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4. The height of any figure is a perpendicular drawn from its fummity to its base. 'As in the triangles ABC, EFG, the perpendiculars EH and 'AD, whether they fall with-

in or without the triangles, are their heights. Hence it follows, that all triangles and parallelograms, that have equal heights, may be plac'd within the same parallels. For having set their bases upon the line HC, if the perpendiculars DA and HE be equal, the lines EA and HC will be parallels.

5. A Proportion is faid to be compounded

of many others, when the and or as

* quantities of those proporti- * Denominators.

ons being multiply'd, make wo which had be

another. To understand the true intent of this definition, it must be observed, that every

Proportion, at least, every * rational Proportion, takes its Denomination from a certain num* Expressible by true numbers.

ber, denoting that respect or relation that the antecedent of the proportion bears to the confequent. As if two magnitudes were proposed, one of twelve foot in length, and the other of fix, we should call that proportion of 12 to

6 the double proportion. In like manner if 4 and 12 were propos'd, we should give that the name of subtriple proportion, & being its Denominator; importing as much as that the proportion of 4 to 12, is the same as that of to unity, or as one to three. This Denomnator is call'd the quantity of the Proportion. Suppose therefore three terms were given, 12, 6, and 2; the first proportion of 12 to 6 being double, its Denominator is two; the second of 6 to 2 being triple, its Denominator is three; the proportion therefore of 12, to 2 is faid to be compounded of that of 12 to 6, and of 6 to 2, the double and the triple proportion. To find therefore the Denominator of the proportion of 12 to 2, multiply three by two, and the product fix will shew the proportion of 12 to 2 to be sextuple, i. e. as one to six. This is that which Mathematicians commonly un-'derstand by compounded Proportion, though methinks it might more properly have been eall'd Proportion multiply'd.

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PROPOSITION I.

A THEOREM.

Parallelograms, and Triangles, of the same hight, are in the same proportion as their bases.



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C Uppose the triangles AGC, and DEM, to have the same hight, and to be plac'd between the fame parallels, AD, and GM: I fay, they

will have the same proportion as their bases GC, and EM. Divide the base EM into as many equal parts as you please, and draw lines from the point D to each division, as DF, DH, DL. In like manner divide the line CG into parts equal to those of the line EM, and draw lines from its fummity A to those divisions, as AB, AI, &c. All those little triangles. being enclos'd within the same Parallels, and having equal bases, are equal, (by the 38 1.)

Demonstration. The base GC contains so many aliquot parts of EM, as there are parts found in it equal to EF; but as many parts equal to EF as are found in the base GC, so many little triangles are contain'd in the triangle AGC, equal to those

those contain'd in DEM; which being equal among themselves, are the aliquot parts of the triangle DEM. As oft therefore as the base GC contains those aliquot parts of EM, so oft does the triangle AGC contain the aliquot parts of the triangle DEM; which also will happen in every division whatsoever: therefore the triangle AGC has the same proportion to the triangle DEM, as the base GC to the base EM.

Now Parallelograms, describ'd upon the same bases, and enclos'd between the same parallels, are double the triangles, (by the 41.1.) therefore they are in the same proportion as the tri-

angles, i. e. as their bases.

The USE.

'This Proposition is not only serviceable in demonstrating those that follow, but also of great use in dividing large Fields, or Plains.
'As for example, suppose you were to take the

third part of the Trapesium
ABCD, having the sides AD
and BC parallel; produce the
line BC to E, so that CE
may be equal to AD; and
GC E taking BG the third part of

BE, draw AG: I say, the triangle ABG is the third part of the Trapesium

Demon

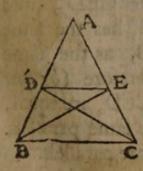
Demonstration.

'The triangles ADF, and FCE, are equiangular, because the lines AD and CE are parallels, and their sides AD and CE are equal: therefore (by the 26.1.) the triangles are equal, and consequently the triangle ABE is equal to the trapesium ABCD. But the triangle ABE, (by the preceding,) therefore the triangle ABE, (by the preceding,) therefore the triangle ABCD.

PROPOSITION II.

A THEOREM.

A line drawn in a triangle parallel to its base divides its sides proportionally; and the line that divides the sides of a triangle proportionally, will be parallel to its base.



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If in the triangle ABC the line DE be parallel to the base BC, the sides AB and AC will be divided proportionally, i. e. AD will have the same proportion to DB, as AE to EC. Draw the lines DC and BE.

The triangles DBE, and DEC, having the same base

base DE, and being enclos'd within the same Parallels DE and BC, are equal, (by the 37. 1.) Demonstration.

The triangles ADE and DBE have the same hight E, taking AD and DB for their bases, and therefore may be plac'd within the line AB and another parallel to it drawn through the point E, and consequently have the same proportion as their bases, (by the 1.) i. e. the triangle ADE has the same proportion to the triangle DBE, or its equal CED, as AD to DB. But the triangle ADE has likewise the same proportion to the triangle CED, as AE to EC; and therefore AD has the same proportion to DB, as AE to EC.

Secondly, suppose AE to have the same proportion to EC, as AD to DB, I say the lines DE and BC will be parallels.

Demonstration.

AD has the same proportion to DB, as the triangle ADE to the triangle DEB, (by the 1.) and AE has the same proportion to EC, as the triangle ADE to the triangle CED; and consequently the triangle ADE has the same proportion to the triangle DEB, as the same ADE to the triangle CED. Therefore (by the 9.5.) the triangles BDE, and CED, are equal, and (by the 39.1.) between the same parallels. Therefore the lines DE and BC are parallels.

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The USE.

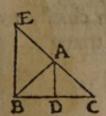
This Proposition is absolutely necessary for the demonstration of those that follow. It may also be serviceable in taking Dimensions. As, in the following figure, if you were to measure the hight of BE; having erected a staff or pole DA, there will be the same proportion of CD to DA, as of BC to BE.

PROPOSITION III.

A THEOREM.

A line that divides an angle of a triangle into two equal parts, divides its base into two parts, which have the same proportion as the sides.

And if it divide the base into two parts proportional to the sides, it will divide the angle into two equal parts.



IF the line AD divide the angle BAC into two equal parts, AB will have the same proportion to AC as BD to DC. Produce the side CA, and take AE equal to AB; then

draw the line EB.

Demonstration.

The external angle CAB is equal to the two internal angles AEB, and ABE: which being equal, (by the 5.1.) because the sides AE and AB

Secondly, if AB has the same proportion to AC as BD to DC, the angle BAC will be divided into two equal parts.

Demonstration.

AB or EA has the same proportion to AC, as BD to DC: therefore EB and AD are parallel; and (by the 29. 1.) the alternate angles EBA and BAD, the internal BEA, and the external DAC, will be equal: and the angles BEA and EBA being equal, the angles BAD and DAC will be also equal. Therefore the angle BAC will be divided into two equal parts.

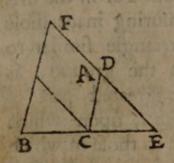
The USE.

We make use of this Proposition chiefly to find the proportion of the sides of a triangle.

PROPOSITION IV.

A THEOREM.

The sides of equiangular triangles are proportional.



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IF the triangles ABC, and DCE, be equiangular, i. e. if the angles ABC and DCE, BAC and CDE be equal, AB will have the same proportion to BC as DC to CE, and also AC will have the same pro-

portion to CB as DE to EC. Join the triangles after such a manner, that the bases BC and CE may be upon the same line, and produce the sides BA and ED till they meet in F; since the angles ACB and DEC are equal, the lines AC and FE are parallels, [by the 28. 1.] and by the same reason CD and BF are parallels, and therefore AFDC a Parallelogram.

Demonstration.

In the triangle BFE, AC is parallel to FE, therefore [by the 2.] AB has the same proportion to AF, or, which is equal to ir, CD, as BC to CE: and alternatively, AB has the same proportion to BC, as CD to CE. In like manner in the same triangle, CD being parallel to BF,

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BF, FD or AC has the same proportion to DE, as BC to CE, [by the 2.] and alternatively, AC has the same proportion to BC, as DE to CE.

The US E.

This Proposition is of so general use, that it may pass for a most universal principle in taking all manner of Dimensions. For in the first oplace, all the methods of measuring inaccessible Ines, by describing a small triangle similar to that which is form'd upon the ground, is founded upon it; as also the greatest part of those Mathematical Instruments, upon which are describ'd triangles, similar to those of which we defire to take the dimensions, as the Geo-" metrical Square, the Pantometer, the Arbalest or Cross-staff, &c. Nor could we know how to raise the Plane of any place, but by the help thereof. So that in fine, to unfold all the uses of this Proposition, it would be necessary to transcribe the whole first Book of Practical Geometry.

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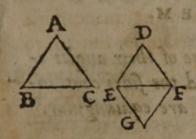
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PROPOSITION V.

A THEOREM.

Triangles, which have proportional sides, are equiangular.



The triangles ABC and DEF have proportional fides, i. e. if AB has the same proportion to BC as DE to EF, and also AB the same proportion to AC as DE to

DF; the angles ABC and DEF, A and D, C and F, will be equal. Make the angle FEG equal to the angle B, and EFG equal to the angle C.

Demonstration.

The triangles ABC and EFG have two angles equal, therefore (by Coroll. 2. of the 32. 1.) they are equiangular; and (by the 4.) AB has the same proportion to BC as GE to EF. Now 'tis suppos'd, that DE has the same proportion to EF as AB to BC, therefore DE has the same proportion to EF as EG to EF; and consequently (by the 9. 5.) DE and EG are equal. After the same manner DF may be provide equal to FG, and consequently (by the 8. 1) the triangles DEF and GEF are equiangular. But the angle

angle GEF was made equal to B, therefore the angle DEF is equal to the angle B, and the angle DFE to the angle C; and consequently, the triangles ABC and DEF are equiangular.

PROPOSITION VI.

A THEOREM.

Triangles, which have each one of their angles equal to one of the other, and the sides containing that angle proportional, are equiangular.

F the triangles ABC and DEF (see fig. preced.) I have the angles B and E equal, and the fide AB has the same proportion to BC as DE to EF, the triangles ABC and DEF will be equiangular. Make the angle FEG equal to the angle B, and the angle EFG equal to the angle C. Demonstration.

The triangles ABC and GEF are equiangular, (by Coroll. 2. of the 32. i.) therefore AB has the same proportion to BC as GE to EF, (by the 4) But as AB to BC, so is DE to EF; therefore DE has the same proportion to EF as GE to the same EF; and therefore (by the 9. 5.) DE and EG are equal; and the triangles DEF and GFE, having the angles DEF and GEF, each equal to the angle B, and the fides DE

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DE and EG equal, with the fide EF common to both, will be equal in all respects, (by the 4:

1.) therefore they will be equiangular; and the the triangle EGF being equiangular to ABC, the triangles ABC and DEF are equiangular.

The Seventh Proposition is of no use.

PROPOSITION VIII.

A THEOREM.

A perpendicular, drawn from the right angle of a rectangular triangle to the opposite side, divides the triangle into two others similar to it.

A D C will divide the

If the perpendicular BD be drawn from the right angle ABC to the opposite side AC, it will divide the rectangular triangle ABC into two triangles

ADB and BDC, which will be similar, or equiangular to the triangle ABC.

Demonstration.

The triangles ABC and ADB have the same angle A, and the angles ABC and ADB right angles, therefore they are equiangular, [by Coroll. 2. of the 32.1.] In like manner the triangles BDC and ABC have the angle C common to both, and the angles ABC and BDC right angles; therefore they also are equiangular. There-

Therefore the triangles ABC, BDC, and ADB, are similar triangles:

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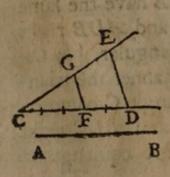
The USE.

'By the help of this Proposition inaccessible distances may be measured by a Carpenter's Rule. As for example, if I were to measure the distance DC; having drawn the perpendicular BD, and plac'd my square upon the point B in such a manner, that by one of its sides BC I could observe the point C, and by the other the point A: 'tis evident, there would be the same proportion of AD to DB, as of DB to DC. Therefore multiplying DB by its self, and dividing the product by AD, the Quotient would be DC.

PROPOSITION IX.

A PROBLEM.

Of a line given, to cut off what part you please.



Let the line propos'd be AB, from which you defire to take away three fifth parts. Make an Angle ECD, and upon one of its fides CD take five equal parts, three of which shall be contained in

CF: then taking CE equal to AB, draw the line DE, and another parallel to that FG: the line CG will contain three fifth parts of CE or

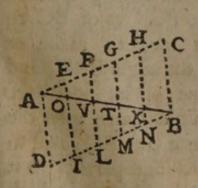
AB. Demonstration.

In the triangle ECD, FG being parallel to the base DE, CF will have the same proportion to FD as CG to GE, [by the 2.] and compounding them, [by the 18.5.] CE will have the same proportion to CG as CD to CF; and (by the Coroll. of the 16.5.) CG will have the same proportion to CE as CF to CD. But CF contains three fifths of CD, therefore CG will contain three fifths of CE or AB.

PROPOSITION. X.

A PROBLEM.

To divide a line after the same manner as another line given is divided.



IF you would divide the line AB after the same manner as AC is divided, make with the two lines the angle CAB of what magnitude you please; then draw the line BC, and parallel to

will be divided after the same manner that AC is.

Demonstration.

Since in the triangle BAC, HX is drawn parallel to the base BC, it will divide the sides AB and AC proportionally, (by the 2.) and the

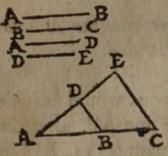
fame may be faid of all the rest.

To do this more easily, you may draw the line BD parallel to AC; and transferring the divisions of AC to BD, draw lines from one to the other.

PROPOSITION XI.

A PROBLEM.

Two lines being given, to find a third proportional.



IF you would find a third proportional to the lines AB and BC, i. e. that there may be the same proportion of AB to BC, as of BC to the line sought; make at pleasure the angle EAC, and upon one of its sides take

the lines AB and BC, one immediately after the other; and upon the other fide take AD equal to BC: then draw the line BD, and parallel to it the line CE; and the line DE will be that which you feek.

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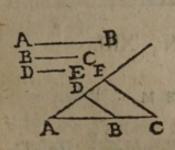
Demonstration.

In the triangle EAC the line DB is parallel to the base CE, therefore [by the 2.] there is the same proportion of AB to BC, as of AD or BC to DE.

PROPOSITION: XII.

A PROBLEM.

Three lines being given, to find a fourth propor-



pos'd, to which you are to find a fourth proportional, be AB, BC, and DE. Make at pleasure the angle FAC, and take upon AC the lines AB and BC, and upon AF

DB, and parallel to it FG: I say, that DF is the line sought, i. e. there is the same proportion of AB to BC, as of DE, or AD to DF, [by the 2.]

Demenstration.

In the triangle FAC the line DB is parallel to the base FC: there is therefore the same proportion of AB to BC, as of AD to DF, [by the 2.]

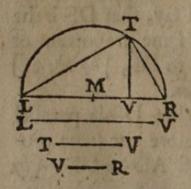
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The USE.
The use of the Compass of Proportion is grounded upon these four Propositions; for that Instrument teaching us to divide a line as we please; to make use of the Rule of Three, without the help of Arithmetick; to extract the Square, and Cubick roots; to double the Cube; to measure all sorts of Triangles; to find the capacity of Superficies's, and the solidity of bodies; and to augment or diminish any figure, according to what proportion we desire; all these operations are demonstrated by the preceding Propositions.

PROPOSITION XIII.

A PROBLEM.

Two lines being given, to assign a middle propor-



IF you desire a middle proportional between the lines LV, and VR; having plac'd them so, that they make but one right line LR, divide that line into two equal parts in M; and having

ing describ'd a semicircle LTR from the center M, draw the perpendicular VT, which will be a midle proportional between LV and VR. Draw the lines LT, and TR.

Demonstration.

The angle LTR, being describ'd in a semicircle, is a right angle, (by the 31. 3.) and (by the 8.) the triangles LVT and TVR are similar; therefore there is the same proportion of LV to VT in the triangle LVT, as of VT to VR in the triangle TVR, (by the 4.) therefore VT is a middle proportional between LV.

The USE.

By this Proposition any rectangular Paral-6 lelogram may be reduc'd to a Square. For example, in the Rectangle contain'd under LV and VR, the square of VT is equal to the Rectangle under LV and VR; as I all hereafter demonstrate. will have the lame proportion to use Par

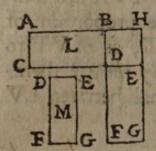
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A THEOREM.

Equal equiangular Parallelograms have their fides reciprocal; and equiangular Parallelograms, that have their fides reciprocal, are equal.



If the Parallelograms L and M be equiangular and equal, their fides will be reciprocal, i.e. CD will have the fame proportion to DE as FD to DB. Since they have e-

qual angles, they may be so joyn'd together, that the sides CD and DE, FD and DB, will concur in two right lines, (by the Coroll. of the 15. 1.) producing therefore the sides AB and GE, you compleat the Parallelogram BDEH.

Demonstration.

The Parallelograms L and M being equal, will have the same proportion to the Parallelogram BDEH. But the proportion of L to BD EH, is as the base CD to the base DE; and that of M, or DFGE, to BDEH, is as the base FD to the base DB, (by the 1.) Therefore CD has the same proportion to DE, as FD to DB.

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Secondly, if the Parallelograms L and M be equiangular, and have their fides reciprocal, they will be equal.

Demonstration.

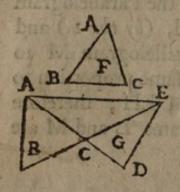
The sides of the Parallelograms are supposed to be reciprocal, i.e. that there is the same proportion of CD to DE, as of FD to DB: but as the base CD is to DE, so is the Parallelogram L to the Parallelogram BDEH, (by the 1.) and as FD to DB, so is the Parallelogram M to BDEH; therefore L has the same proportion to BDEH, as M to the same BDEH; therefore (by the 9.5.) the Parallelograms L and M are equal.

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PROPOSITION XV.

A THEOREM.

Equal triangles, that have one angle equal each to the other, have the sides that form that angle, reciprocal; and if those sides be reciprocal, they will be equal.



IF the triangles F and G, being equal, have the angles ACB and DCE equal, their sides that form those angles will be reciprocal, i.e. BC will have the same proportion to CE as CD to CA. Place the

triangles so, that the sides CD and CA may make one right line; and then because the angles ACB and DCE are supposed to be equal, BC and CE will also make one right line, (by Coroll. of the 15. 1.) draw the line AE.

Demonstration.

The triangle ABC has the same proportion to the triangle ACE, tas the triangle ECD, equal to the former, to the same ACE, (by the 7.5) But as ABC to ACE, so is the base BC to the base CE, [by the 1.] having both the same hight A; and as ECD to ACE, so is the base

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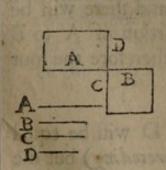
equal

CD to CA, (by the same:) therefore BC has the same proportion to CE, as CD to CA. But if the sides be supposed reciprocal, i. e. that BC has the same proportion to CE as CD to CA, the triangles ABC and CDE will be equal, because they will both have the same proportion to ACE.

PROPOSITION XVI.

A THEOREM.

If four lines be proportional, the rectangle contain'd under the first and the fourth, will be equal to the rectangle contain'd under the second and the third. And if the rectangle contain'd under the extremes be equal to that contain'd under the middle terms, the four lines will be proportional.



proportional, i. e. if as A to B, so C to D, the rectangle containd under the first A, and the fourth D, will be equal to the rectangle contain'd under B and C.

Demonstration.

The rectangles have one angle equal each to th' other, because 'tis a right angle in both; their sides also are reciprocal: therefore they are equal, [by the 14.]

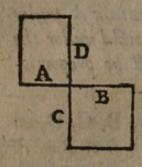
The Elements of Euclid. 278

In like manner, if they are equal, their fides will be reciprocal, i. e. A will have the same proportion to B as C to D.

PROPOSITION XVII.

A THEOREM.

If three lines be proportional, the rectangle contain'd under the first and the third, will be equal to the square of the middle term. And if the Square of the middle term be equal to the rectangle under the extremes, the three lines are proportional.



IF the three lines A, B, D, be proportional, the rectangle contain'd under A and D will be equal to the square of B. Take C equal to B, and there will be the same proportion of A to B as of C to D; therefore the four

lines are proportional.

Demonstration.

The rectangle under A and D will be equal to that under B and C, (by the preceding) but the last rectangle is a square, the lines B and C being equal: therefore the rectangle contain'd under A and D is equal to the square of B.

In like manner, if the rectangle under A and

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D be equal to the square of B, A will have the same proportion to B as C to D: and B and C being equal, A will have the same proportion to B, as B to D.

The USE.

By these four Propositions may be demonstrated that Rule in Arithmetick, which is commonly call'd the Rule of Three; and confequently, the Rules of Fellowship, of False, and all those others that depend upon Proportion. For example, Suppose Three numbers given, A 8, B 6, and C 4, and it be requir'd to find 'a fourth proportional number; which taking as found, I will call D. The rectangle then contain'd under A and D, is equal to that under B and C. But I may have this latter rectangle by multiplying B by C, i. e. fix by four, the product will be twenty four; therefore the rectangle contain'd under A and D is also twenty four; and therefore dividing that number by A, which is 8, the Quotient three will be the number fought.

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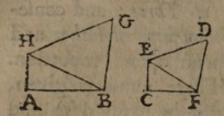
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PROPOSITION

A PROBLEM.

To describe a Polygon similar to another uton a line given.



Et AB be the line affign'd, upon which you are requir'd to describe a Polygon fimilar to the Polygon

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CFDE; and having divided the Polygon CFDE into triangles, upon the line AB make a triangle ABH fimilar to the triangle CFE, i. e. make the angle ABH equal to the angle CFE, and BAH equal to FCE, for then the triangles ABH and CFE will be equiangular, [by Corol.2. of the 32. Make also upon the line BH a triangle equiangular to FDE.

Demonstration.

Since the triangles, which are part of the Polygons, are equiangular, the two Polygons are equiangular. Further, fince the triangles ABH and CFE are equiangular, AB will have the same proportion to BH as CF to FE, [by the 4.] In like manner, the triangles HBG and EFD being equiangular, BH will have the same proportion to EG as FE to FD: and by equality, (accord[according to defin. 18.5.] AB will have the same proportion to BG, as CF to FD. And the same may be said of all the other sides. Therefore (by defin. 1.) the Polygons are similar.

The USE.

'Upon this proposition is grounded the greatest part of Practical Geometry, that relates to the raising the plane of any place, as of a building, field, forest, or a whole Country. For having divided a line into equal parts, to anfwer the feet or yards contain'd in the plane, ' you may describe a figure similar to, but less than the Original, in which you may fee the proportion of all its lines. And having by experience found it much more easie to travel upon paper, than to take a tedious journey either by land or water, this propolition will e likewife afford us affistance in this respect, informing us in almost all the parts of Geodesia, and Chorography; and giving Instructions how to compose Geographical Charts, and Maps; which are nothing else but methods of reducing great figures to small. Further, the use of this Proposition extends it self to almost all those Arts, that require the design and model of their works before-hand.

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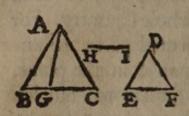
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PROPOSITION

A THEOREM.

Similar triangles are in the duplicate proportion of their homologous sides.



F the triangles ABC and DEF be fimilar or equiangular, they will be in the duplicate proportion of their homologous fides BC, EF,

ze the proportion of the triangle ABC to the triangle DEF will be the duplicate of the proportion of BC to EF; fo that finding a third proportional HI to the lines BC and EF, and making BC to have the same proportion to EF as EF to HI, the triangle ABC will have the fame proportion to DEF as the line BC to the line HI; which is to have to it a duplicate proportion, by defin. 11.5. Take BG equal to HI, and draw the line AG.

Demonstration.

The angles B and E of the triangles ABG and DEF are equal; and besides, since the triangles ABC and DEF are similar, AB will have the same proportion to DE as BC to EF, [by the fourth. But as BC to EF, so EF to HI or BG; therefore as AB to DE, so EF to BG; and

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and consequently, the sides of the triangles ABG and DEF being reciprocal, the triangles will be equal, [by the 15.] And [by the 1.] the triangle ABC has the same proportion to the triangle ABG, as BC to BG or HI: therefore the triangle ABC has the same proportion to the triangle DEF, as BC to HI.

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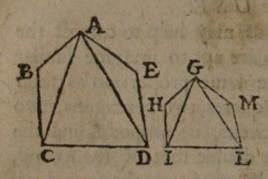
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These Propositions may help to correct the error of those, who are apt to imagine similar figures to have the same proportion as their sides. For if two squares, two pentagons, two hexagons, or two circles, be proposed, and the side of the first be double that of the second, the first figure will be quadruple the second: if the side of the first be triple that of the second, the first figure will be nine times greater than the second. Therefore to make a square triple to another, you must seek a middle proportional between one and three, and you'll find for the side of your triple figure almost 12.

PROPOSITION XX.

A THEOREM.

Similar Polygons may be divided into an equal number of triangles, and are in the duplicate proportion of their homologous sides.



IF the Polygons ABCDE and GHILM be similar, they may be divided into an equal number of similar triangles, which and

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will be the similar parts of their wholes. Draw the lines AC, AD, GI, GL.

Demonstration.

Since the Polygons are similar, their angles B and H will be equal; and AB will have the same proportion to BC as GH to HI, (by define 1.) therefore the triangles ABC and GHI are similar, (by the 6.) and (by the 4.) BC has the same proportion to CA as HI to GI. Further, because CD has the same proportion to BC as IL to IH, and BC the same to CA as HI to IG; by equality, CD will have the same proportion to CA as IL to GI. Now the angles

BCD and HIL being equal, if the angles ACB and GIH, which are equal, be taken from them, the angles ACD and GIL will remain equal. Therefore (by the 6.) the triangles ACD and GIL will be similar. In like manner, 'tis easie to run over all the triangles of the Polygons, and to prove them similar.

I add further, that the triangles are in the

same proportion as the Polygons.

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Since all the triangles are similar, their sides will be proportional, (by the 4) but each triangle to its similar is the duplicate proportion of the homologous sides, (by the 19) therefore every triangle of one Polygon to every triangle of the other is in the duplicate proportion of its sides; which being the same, the duplicate proportion must be the same; and there will be the same proportion of each triangle to its similar, as of all the triangles of one Polygon to all the triangles of the other Polygon, (by the 12.5.).

Coroll. 1. Similar Polygons are in the dupli-

cate proportion of their fides.

Coroll. 2. If three lines be in continual proportion, a Polygon describ'd upon the first will have the same proportion to a l'olygon describ'd upon the second, as the first line to the third, i. e. it will be in the duplicate proportion of that of the first line to the second

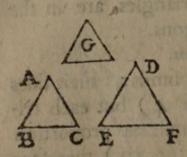
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PROPOSITION XXI.

A THEOREM.

Polygons, that are similar to another Polygon, are so also amongst themselves.



F two Polygons be similar to a third, they will be io also betwixt themselves. For they may each be divided into as many fimilar triangles, as are in the third.

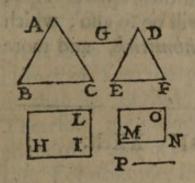
But triangles similiar to a third, are also similar amongst themselves; because angles equal to a third, are equal amongst themselves; and the angles of the triangles being equal, those of the Polygons being compounded of them must be so likewise.

I add, that the fides of the triangles being proportional, those of the Polygons must be so also, because they are the same.

PROPOSITION. XXII.

A Theorem.

Similar Polygons describ'd upon four proportional lines, are also proportional. And if the Polygons be proportional, the lines are so too.



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F BC has the fame proportion to EF as HI to MN, the Polygon ABC will also have the fame proportion to the similar Polygon DEF, as HL to its similar Polygon MO. Seek

a third proportional G to the lines EC and EF, and to the lines HI and MO another third proportional P, (by the II) Since BC has the same proportion to EF as HI to MN, and EF to G as MN to P; by equality, EC will have the same proportion to G, as HI to P: and this proportion will be the double of that of BC to EF, or HI to MN.

Demonstration.

The Polygon ABC to the Polygon DEF is in the duplicate proportion of that of BC to EF, (by the 20.) that is, as BC to G; and the Polygon HL has the same proportion to MO, as HI to P. Therefore ABC has the same proportion.

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portion to DEF, as HL to MO.

And if the similar Polygons be proportional, the lines being in the subduplicate proportion to them, will be also proportional.

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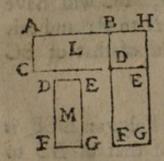
The USE.

A, B; C, D, This Proposition may be ea3, 2; 6, 4. 'sily apply'd to numbers, if the
9, 4; 36, 16, 'numbers A, B, C, D, be proE, F; G, H, 'portional, their squares E, F,
'G, H, will be so also; which
'is very serviceable in Arithmetick, and more
'in Algebra.

PROPOSITION XXIII.

A THEOREM.

Equiangular Parallelograms are in the proportion compounded of the proportions of their sides.



If the Parallelograms L and M be equiangular, the proportion of L to M will be compounded of that of AB to DE, and that of BD to DF. Joyn the Parallelograms, so that their

fides BD and DF may make but one right line, asalfo CD and DE another; which, the Paral-

rallelograms being equiangular, may be done, [by the Coroll. of the 15. 1.] and compleat the parallelogram BDEH.

Demonstration.

The parallelogram L has the same proportion to the parallelogram BDEH, as the base AB to the base BH or DE, [by the 1.] and the parallelogram BDEH has the same proportion to the parallelogram DFGE, i. e. M, as the base BD to DF. But the proportion of the parallelogram L to the parallelogram M is compounded of that of L to the parallelogram BDEH, and of that of BDEH to the parallelogram M. Therefore the proportion of L to M is compounded of that of AB to DE, and that of BD to DF. For example, let AB be 8, DE 5, BD 4, DF 7; and make as 4 to 7, so 5 to 83; by which means you will have three numbers, 8, 5, and 83, 8 to 5 being the proportion of the parallelogram I to BDEH, which is that of Ab to DE; and 5 to 84 that of the parallelogram BDEH to M. Taking away therefore the middle term five, there will remain 8 to 84 for the proportion compounded of the two.

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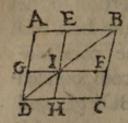
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PROPOSITION XXIV.

A THEOREM.

In all Parallelograms, those through which the diameter passes, are similar to the great one.



Suppose the diameter of the Parallelogram AC pass thro'the Parallelograms EF, GH: I say they are similar to the Paralleleogram AC.

Demonstration.

The Parallelograms AC and EF have the fame angle B: and because in the triangle BCD, IF is parallel to the base DC, the triangles BFI and BCD are equiangular. Therefore [by the fourth] BC has the same proportion to CD as BF to FI, and consequently the sides are in the same proportion. In like manner IH being parallel to BC; DH will have the same proportion to HI as DC to BC; the angles are also equal, all the sides being parallels: therefore [by defin. 1.] the parallelograms EF and GH are similar to the parallelogram AC.

The USE.

I have made use of this Proposition to demonstrate the 10th Proposition of my last Book of of Perspectives, where I have shewn a way to

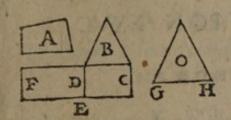
draw an Image similar to the Original, by a pa

' rallelogram compos'd of four Rules.

PROPOSITION XXV.

A PROBLEM.

To describe a Polygon similar to one Polygon given and equal to another.



If you desire to defcribe a Polygon equal to the rectilineal A, and similar to the Polygon B, make a paralle-

logram CE equal to the Polygon B, [by the 44.

1.] and upon the line DE make another parallelogram equal to the rectilineal A, [by the 45.

1.] Then find a middle proportional GH between CD and DF, [by the 13.] Lastly, make upon GH a Polygon O, similar to B, [by the 18.] which will be equal to the rectilineal A.

Demonstration.

Since CD, GH, and DF, are in a continual proportion, the rectilineal B describ'd upon the first, will have the same proportion to the rectilineal O describ'd upon the second, as CD to DF, [by coroll. 2. of the 20.] But as CD to DF, so is the parallelogram CE to FE, or B to A, which

which are equal to them. Therefore B has the same proportion to O as B to A, and consequently, [by the 9.5.] A and O are equal.

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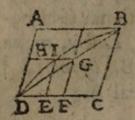
The US E.

This proposition teaches how to change one figure for another, retaining still its equality to a third; which is very useful in Practical Geometry, for the reducing all figures to squares.

PROPOSITION XXVI.

A THEOREM.

If in one angle of a parallelogram you describe a less, similar to the former, the diameter of the greater will fall upon the angle of the less.



IF in the angle D of the parallelogram AC you defcribe a lesser parallelogram DG, fimilar to the other, the diameter BG will pass by the point G.

For if it do not pass by that point, suppose it then to pass by the point I, and to make the line BID. Draw the line IE parallel to HD.

Demonstration.

The parallelogram DI would be fimilar to

the parallelogram AC, (by the 24.) But the parallelogram DG is also supposed similar to it, therefore the parallelograms DI and DG would be similar, which is impossible; for so HI would have the same proportion to IE or GF, as HG to the same GF; and (by the 9.5) the lines HI and HG would be equal.

PROPOSITION XXX.

A PROBLEM.

To divide a line according to the extreme and middle proportion.

A C B pos'd to be divided according to the extreme and middle proportion, i. e. so, that AB may have the same proportion to AC as AC to CB. Divide the line AB (by the 11.2) so, that the rectangle contain'd under AB and CB may be equal to the square of AC.

Demonstration.

Since the rectangle under AB and CB is equal to the square of AC, AB will have the same proportion to AC as AC to CB, (by the 17.)

The USE.

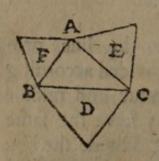
This proposition is necessary in the Thirteenth Book of Euclid, for the finding the side of

of five regular bodies. And Friar Lucas, of the Holy Sepulcher, has compos'd a whole Book concerning the properties of a Line divided according to the extreme and middle proportion.

PROPOSITION XXXI.

A THEOREM.

A Polygon describ'd upon the base of a rectangular triangle, is equal to the two similar Polygons describ'd upon the other sides of the same triangle.



If the angle BAC of the triangle ABC be a right angle, the polygon D, defcrib'd upon its base BC, will be equal to the two similar polygons F and E describ'd upon the sides AB and AC.

Demonstration.

The polygons D, E, and I, are amongst themselves in the duplicate proportion of their homologous sides BC, AC, and AB, (by the 20.) and if squares were describ'd upon the same sides, they also would amongst themselves be in the duplicate proportion of their sides; but [by the 47. 1.] the square of BC would be equal to the

the squares of AC and AB: therefore the polygon D describ'd upon the base BC, will be equal to the similar polygons E, and F, describ'd upon AB and AC.

The USE.

This proposition is made use of to augment or diminish all manner of figures, being more universal than the 47. 1. which yet is exceeding useful, in as much as almost all Geometry is grounded upon that principle.

The 32. Proposition is useless.

PROPOSITION XXXIII.

A THEOREM.

In equal circles, the angles as well at the center as circumference, as also the sector, are in the same proportion as the arches upon which they stand.



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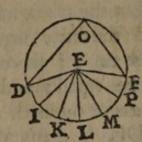
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IF the circles ANC and DOF are equal, the angle ABC will have the fame proportion to the angle

DEF as the arch AC to the arch DF. Suppose AG, GH, and HC, to be equal arches, and and consequently the aliquot parts of AC; and let DF be divided into as many parts, equal to AG, as it contains; and draw the lines EI, EK,

and the rest. Demonstration.

All the angles, ABG, GBH, HBC, DEI, IEK, and the rest, are equal, (by the 27.3.) so that AG, an aliquot part of the arch AC, will be contain'd in the arch DF, as oft as the angle ABG, an aliquot part of the angle ABC, is contain'd in the angle DEF; therefore the arch AC will have the same proportion to the arch DF, as the angle ABC to the angle DEF. And because N and O are the halves of the angles ABC and DEF, they will be in the same proportion as these: therefore the angle N has the same proportion to the angle O, as the arch AC to the arch DF.

The same holds likewise of the Sectors: for if you draw the lines AG, GH, HC, DI, IK, and the rest, they will be equal, (by the 29.3.) and each little sector will be divided into a triangle, and a segment. But the triangles will be equal, (by the 8.1.) and the little segments will also be equal, (by the 24.3.) therefore the whole little Sectors will be equal; and consequently, as many aliquot parts of the arch AC as are contain'd in the arch DF, so many aliquot parts of the sector ABC will be contain'd in the sector DEF. Therefore the arch has the same proportion to the arch, as the sector to the sector.

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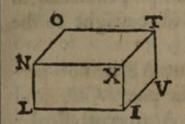
EUCLID.

His Book establishes the first principles relating to folid bodies; infomuch that it is impossible to know any thing certainly concerning the third species of quantity, without understanding what is herein taught. Upon which account the knowledge of it is absolute-'ly necessary to a through insight into the greatest part of Mathematical Treatises. In the first ' place, the Doctrin of the Sphere deliver'd by 'Theodofius does suppose a perfect knowledge of the whole. In like manner Spherical Trigo-' nometry, the third part of Practical Geometry, 'divers propositions of Staticks and Geography are built upon the principles of Solids The main difficulties in Gnomonicks, Conick Sections, ctions, and the Tracts concerning the cutting of pretious Stones, arifing chiefly from their eminencies and rais'd parts, not easily represented upon paper, and their being contain'd under many superficies, are render'd intelligible and easie by the previous knowledge of the doctrin of Solids.

'I have omitted the feventh, eighth, ninth, and tenth Books of the Elements of Euclid, being of little or no use in any part of the Mathematicks. And I have oft wondred how they obtain'd a place amongst the Elements, since 'tis evident Euclid compil'd them for no other end, but to settle the Doctrin of Incommensurables; which being little better than a vain curiofity, ought not to be receiv'd into the Books which treat of the First Principles of the Science, but to make a particular Treatise by its self. The fame may be faid of the thirteenth Book, and those that follow it. And therefore 'tis my oe pinion, that almost all parts of the Mathematicks may sufficiently be understood by the help of these eight Books of the Elements of Eu-" clid:

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DEFINITIONS.



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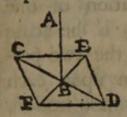
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A Solid body is a quantity, that hath length, breadth, and depth, or thickness. 'As the figure LT, whose length is

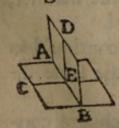
NX, breadth NO, and thickness LN.

2. The extremes or terms of a solid body are superficies's.



3. A Line is right, or perpendicular to a plane, when 'tis perpendicular to all the lines, which it meets in the plane. 'As the 'line AB will be right to the

plane CD, if it be perpendicular to the lines CD and FE, which being drawn upon the plane CD, pass by the point B, so that the angles ABC, ABD, ABE, and ABF, are right angles.

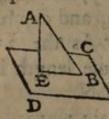


4. One plane is perpendicular to another, when a perpendicular line drawn upon one of them to the common section, is also perpendicular to the other.

We call the line that is com-

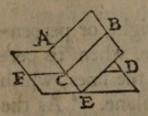
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'mon to both the planes the common section of the planes: As the line AB, which is as well in the plane AC, as in the other AD. If therefore the line DE, drawn on the plane AD, perpendicular to AB, be also perpendicular to plane AC, the plane AD will be right to the plane AC.



5. If the line AB be not perpendicular to the plane CD, and from the point A a perpendicular be drawn to it AE, and also the line BE; the angle ABE is the

Inclination of the line AB to the plane CD.



6. The Inclination of one plane to another, is the acute angle form'd by the two perpendiculars drawn upon each plane to their common fection.

As the Inclination of the plane AB to the plane

AD, is nothing else but the angle BCD, form'd by the lines BC and CD, drawn upon the two planes, perpendicular to their common

" fection AE.

7. Planes are inclin'd after the same manner,

if their angles of Inclination be equal.

8. Planes are parallel, if being continu'd as far as you please, they still retain the same distance one from the other.

9. Solid figures are similar, which are contain'd within; or terminated by, an equal number n of

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ber of similar planes; as two Cubes. "This "definition does not agree to those figures, "whose superficies's are crooked; as the Sphere the Cylinder, and the Cone.

tain'd within, or terminated by, an equal number of equal and similar planes. "Insomuch, that if they were supposed to penetrate each other, neither of them would exceed, having their sides and angles equal.

A folid angle is the concourse, or inclination, of divers lines, in different planes. "As the concourse of the lines AB, AC, and AD, which are in different planes.

by triangles, whose bases are in the same plane.
"As the figure ABCD.

13. A Parallelpipedon is a folid figure contain'd within fix quadilateral planes, of which the opposites are parallel.

gure, having two parallel planes fimilar and equal, and the others Parallelograms. "As the figure AB. Its opposite planes may be Polygons.

15. A Sphere is a solid figure, terminated by one only superficies. from which divers lines lines being drawn to a point in the middle of the figure, they will be all equal. 'Some define a Sphere by the motion of a semicircle, turn'd about upon its diameter, which remains immoveable.

16. The Axis of a Sphere is that immoveable line about which the semicircle is turn'd.

17. The Center of the Sphere is the same with that of the semicircle, by whose motion it is made.

18. The Diameter of a Sphere, is any line whatfoever passing through its center, and ter-

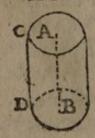
minated at the superficies.

19. If a line, immoveable at one of its points, taken above the plane of a circle, be mov'd about the circumference, it will describe a Cone. 'As if the line AB, being fix'd at the point A, be mov'd about the circumfe-

rence BED, it will describe the Cone ABED. The point A will be its summity or vertex,

and the circle BED its base.

20. The Axis of a Cone, is the line drawn from the vertex to the center of the base. " As « AC.



21. If a line be mov'd about the circumference of two parallel circles, so that it remains always parallel to a line drawn from the center of one of the circles to that of the o-

ther,

her, i. e. the Axis, it will describe a Cylinder 22. Cones are said to be right, when the Axis is perpendicular to the plane of the base. Also right cones are similar, when their axis's and the diameters of their bases are in the same proportion. But inclin'd cones are not similar, unless they have a third condition; that their axis's be equally inclin'd to the planes of their bases.

PROPOSITION I.

A THEOREM.

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Arightline cannot have one of its parts upon a plane, and the other above or below it.

F The line AB be upon the plane AD, it will not, being continu'd, either rise above or fall below it, but all its parts will lie upon the same. For if

it be possible that BC can be a part of AB continued, draw upon the same plane AD the line BD perpendicular to AB, and also BE perpendicular to D upon the same.

Demonstration.

The angles ABD and DBE are two right angles; therefore (by the 14 1) AB and BE make but one right line, and consequently BC

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is no part of the line AB continued: otherwise two right lines CB and EB would have the same part AB in common, which is repugnant to the 13. Axiom of the first Book.

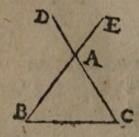
The USE.

'Upon this Proposition is built a principle in Gnomonicks, by which we prove, that the shadow of the Style cannot fall out of the plane of a great circle, in which is the Sun. For the extremity of the Style being taken for the centre of the heavens, and consequently of all the greater circles, and the shadow being always in a right line of a Ray drawn from the Sun to the opacous body, and this ray being in the plane of this great circle, the shadow must be so likewise.

PROPOSITION II.

A THEOREM.

Lines that cut each other, are in the same plane, as are also the parts of a triangle.



If the two lines BE and CD cut each other at the point A, and a triangle be form'd by drawing the base BC; I say, all the parts of the triangle ABC

are in the same plane, and also the lines BE and CD.

Demonstration.

ABC is in a plane, and another part of the fame triangle not in the same, but it must be also affirm'd, that one part of a right line is in a plane, and another part of the same line is not in the same plane; which is contrary to Prop. 1. And because the sides of the triangle must be in the same plane in which is the triangle, the lines BE and CD will be also in the same plane.

The USE.

'This Proposition sufficiently determines a plane, by the concourse of two right lines, or by a triangle. I have also made use of it in Opticks, to prove that objective parallel lines, which meet upon a Table, ought to be represented by lines that concur in a point.

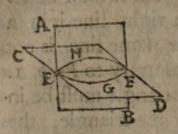
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PROPOSITION III.

A THEOREM.

The common section of two planes is one right line.



The planes AB and CD cut each other, their common fection EF will be one right line. For if not, take two points common to both planes, as E and F; and draw

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a right line from the point E to the point F upon the plane AB, which suppose to be EHF. Draw likewise upon the plane CD a right line from the same point E to F; and if it be not the same with the former, suppose it to be EGF.

Demonstration.

These lines drawn upon two planes are two different lines, and enclose space; which is contrary to the 12. Axiom of the 1. Therefore they will make but one right line, which being in both the planes will be their common section.

The USE.

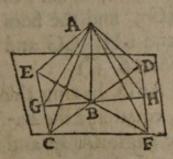
This is a fundamental proposition, supposed in divers parts of the Mathematicks, though it be not always quoted. Particularly, it is taken for granted in Gnomonicks, when the hour-lines

lines are represented upon Dials, by marking only the common section of their plane, and that of the wall.

PROPOSITION. IV.

A THEOREM.

If a line be perpendicular to two others that cut each other, it will be also perpendicular to the plane of the same lines.



If the line AB be perpendicular to the lines CD and EF, which cut each other at the point B, so that the angles AbC, ABD, ABE, and ABF, be right angles, (which

cannot conveniently be represented upon a plate,) it will be also perpendicular to the plane of the lines GD and EF, i. e. to all the lines that shall be drawn upon the same plane through the point B; as, for example, the line GBH. Let the lines BC, BD, BE, and BF, be equal, and draw the lines EC, DF, AC, AD, AE, AF, AG, and AH.

Demonstration.

The four triangles ABC, ABD, ABE, and ABF, have each a right angle at the point B; and the fides BC, BD, BE, and BF equal, with the

the fide AB common to all. Therefore their bases AC, AD, AE, and AF, are equal, (by

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the 4. 1.)

2. The triangles EBC and DBF will be in all respects equal, having their sides BC, BD, BE, and BF, equal; and the angles CBE and DBF, being oppos'd at the top, equal: therefore the angles BCE, BDF, BEC, and BFD, will be equal, (by the 4. 1.) and also the bases EC and DF.

3. The triangles GBC, and DBH, having the opposite angles CBG, and DBH, equal; as also the angles BDH, and BCG; and the sides BC and BD; the sides BG and BH, CG and DH, will be also equal, (by the 26. 1.)

4 The triangles ACE and AFD, having the fides AC, AD, AE, and AF, equal, and the bases EC and DF also equal; the angles ADF and

ACE will be equal, (by the 8. 1.)

5. The triangles ACG and ADH have the fides AC and AD, CG and DH equal, with the angles ADH and ACG; therefore their ba-

fes AG and AH are equal.

Lastly, the triangles ABH and ABG have all their sides equal; therefore (by the 8. 1.) the angles ABG and ABH will be equal, and the line AB perpendicular to GH. Accordingly the line AB will be perpendicular to any line drawn through the point B upon the plane of the lines CD and EF, which I call being perpendicular to their plane.

The

The USE.

Book of Theodosius: for example, to demonstrate that the Axis of the world is perpendicular to the plane of the Equinoctial. In like manner in Gnomonicks, its demonstrated by this proposition, that the Equinoctial line in Horizontal Dials is perpendicular to the Metridian. Nor is it less useful in other Mathematical Treatises; as those concerning Astrobabes, and the cutting of precious stones.

PROPOSITION V.

A THEOREM.

If a line be perpendicular to three others, which cut each other at the same point, they will be all three in the same plane.

A F B B If the line AB be perpendicular to three lines BC, BD, and BE, which cut each other at the fame point B, the lines BC, BD, and BE, are in the fame plane. Suppose the plane AE to be that

of the lines AB and BE, and CF that of the lines BC and BD. If BE be the common section of both the planes, it will be in the plane

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of the lines BC and BD, as was afferted: but if BE be not, let BG be their common section.

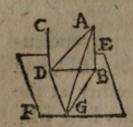
Demonstration.

AB is perpendicular to the lines BC and BD, therefore it is perpendicular to their plane CF, (by the 4.) and (by Defin. 3.) AB will be perpendicular to BG. But it is also supposed perpendicular to BE; therefore the Angles ABE and ABG are right angles, and consequently equal, though one be part of the other. Therefore the two planes can have no other common section but BE, BE is therefore in the plane CF.

PROPOSITION. VI.

A THEOREM.

Lines that are perpendicular to the same plane,
are parallel.



F the lines AB and CD be perpendicular to the same plane EF, they will be parallel. 'Tis evident, that the internal angles ABD and BDC are right angles; but that is not enough; it re-

mains to be prov'd, that AB and CD are in the same plane. Draw DG perpendicular to BD, and equal to AB; draw also the lines BG, AG, and AD.

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Demonstration.

The triangles ABD and BDG have the sides AB and DG equal, and BD common to both: and the angles ABD and BDG are right angles, therefore their bases AD and BG are equal, (by the 4. 1) Further, the triangles ABG and ADG have all their sides equal: therefore the angles ABG and ADG are equal; and ABG being a right angle, because AB is perpendicular to the plane, ADG is also a right angle. Therefore the line DG is perpendicular to three lines CD, DA, and DB, which consequently are in the same plane, (by the 5.) but the line AB is in the plane of the lines AD and DB, (by the 2.) therefore AB and CD are in the same plane.

Coroll. Two parallel lines are in the same

plane.

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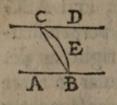
The USE.

By this proposition we demonstrate, that the hour-lines, in all planes that are parallel to the Axis of the World, as the Polars, Meridional, and others, are parallel among themselves.

PROPOSITION VII.

A THEOREM.

A line drawn from one Parallel to another, is in the same plane with them.



THE line CB, being drawn from the point B of the line AB to the point C of its parallel CD, is (I say) in the plane of the lines AB 2 119

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and CD.

Demonstration.

The parallels AB and CD are in the same plane: in which if you draw a right line from the point B to the point C, it will be the same with CB; otherwise two right lines would enclose space, contrary to 12. Axiom of the 1.

PROPOSITION VIII.

A THEOREM.

If one of two parallel lines be perpendicular to a plane, the other will be so also.

F of the two parallel lines AB and CD, [fee fig. prop. 6.] the one AB be perpendicular to the plane

plane EF, the other CD will be so also. Draw the line DB: since the angle ABD is a right angle, and AB and CD are supposed to be parallels, the angle CDB will be a right angle, (by the 29:

1.) therefore if I can prove, that the angle CDG is also a right angle, it will follow (by the 4.) that CD is perpendicular to the plane EF. Make a right angle BDG, and take DG equal to AB; then draw the lines BG and AG.

Demonstration.

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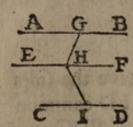
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The triangles ABD and BDG have the sides AB and DG equal, with the side BD common to both; and the angles ABD and BDG are right angles: therefore (by the 4.1.) their bases AD and BG are equal. The triangles ADG and ABG have all their sides equal, therefore (by the 8.1.) the angles ADG and ABG are equal: But the latter is a right angle, because AB is supposed to be perpendicular to the plane EF, therefore the angle ADG is a right angle; and the line DG being perpendicular to the lines DB and DA, will be perpendicular to the plane of the lines DB and DA, which is the same in which are the parallels AB and CD. Therefore the angle GDC is a right angle, (by defin.3.)

PROPOSITION IX.

A THEOREM.

Lines, that are parallel to a third, are also parallel among themselves, though not all in the same plane.



IF the lines AB and GD are parallel to the line EF, they will be parallel to each other, though all the three lines be not in the same plane. Upon the plane of the lines AB and EF the de die co

draw the line HG perpendicular to AB; which will be also perpendicular to EF, [by the Lemma after the 26. 1. In like manner upon the plane of the lines EF and CD draw the line HI perpendicular to EF and CD.

Demonstration.

The line EH being perpendicular to the lines GH and HI, is so also to the planes of the lines HG and HI, (by the 4.) therefore (by the 8.) the lines AG and CI are perpendicular to the plane of the lines HG and HI, and [by the 6.] parallel to each other.

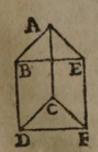
The USE.

'This propolition is frequently used in Perspectives, to determine the representation of · parallel parallel lines upon a table; as also in the cutting of precious Stones, to prove the sides of
the Pannels to be parallel among themselves,
because they are so to a line in a different
plane. In Gnomonicks likewise we are sometimes obliged to prove, that the Vertical circles ought to be describ'd on Walls by perpendicular lines; because the lines, that are the
common sections of them and the walls, are
parallel to a line drawn from the Zenith to the
Nadir.

PROPOSITION X:

A THEOREM.

If two lines, which concur, are parallel to two others concurring, of a different plane, they will make equal angles.



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If the lines AB and CD, AE and CF be parallel, though they be not all four upon the same plane, yet the angles BAE and DCF will be equal. Let the lines AB and CD, AE and CF be equal, and draw the

lines BE, DF, AC, BD, and EF.

Demonstration.

The lines AB and CD are suppos'd to be both parallel and equal, therefore [by the 33.1.]

the lines AC and BD are parallel and equal, as also AC and EF; and [by the preceding] BD and EF are parallel, and equal, and consequently [by the 33.1.] BE and DF will be also parallel and equal. Therefore the triangles BAE and DCF have all their sides equal: and [by the 8.1.] the angles BAE and DCF will be equal.

Coroll. Many the like Propositions might be made, which would not be altogether unuseful: as for example, if upon a parallel plane the line CD be drawn parallel to the line AB, and the angles BCE and DCF be equal, the lines AE

and CF will be parallel.

The USE.

By this Propolition we demonstrate, that the angles made by the planes of the hour-circles with a plane parallel to the Equator, are equal to the angles made by them with the plane of the Equator.

PRO-

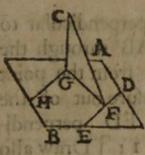
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PROPOSITION XI.

A PROBLEM.

To draw a perpendicular to a plane from a point given out of the plane.



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TF you defire to draw a per-L pendicular from the point C to the plane AB, draw the line EF at pleasure, and CF perpendicular to it, [by the 12. 1.] And again [by the II. I.] upon the plane AB draw FG perpen-

dicular to ED, and CG perpendicular to FG, I fay, CG will be perpendicular to the plane AB. Draw GH parallel to FE.

Demonstration.

The line EF being perpendicular to the lines CF and FG, will be perpendicular to the plane CFG, [by the 4.] and HG being parallel to EF, will be also perpendicular to the same plane, [by the 8.] And because CG is perpendicular to the lines GF and GH, it will be perpendicular to the plane AB, [by the 4.]

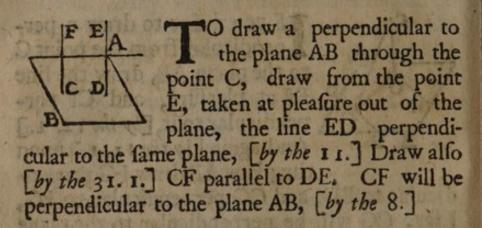
The Elements of Euclid.

PROPOSITION XII.

A Problem.

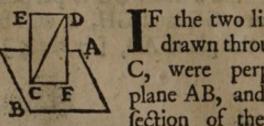
DC

To draw a perpendicular to a plane through a point of the same plane.



PROPOSITION XIII. A THEOREM.

Two lines perpendicular to a plane cannot be drawn through the same point.



F the two lines CE and CD, drawn through the same point C, were perpendicular to the plane AB, and CF the common section of the planes of those

lines, with the plane AB; the angles ECF and DCF DCF would be both right angles, which is im-

possible.

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I add, that two perpendiculars DC and DF to the plane AF cannot be drawn from the same point D: for having drawn the line CF, there would be two right angles, DCF and DFC, in the same triangle, contrary to the 32. 1.

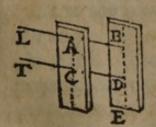
The USE.

'This Proposition is necessary to shew, that a perpendicular to a plane was sufficiently describ'd, in as much as but one such can be drawn through the same point.

PROPOSITION XIV.

A THEOREM.

Planes, to which the same line is perpendicular, are parallel.



If the line AB be perpendicular to the planes AC and BD they will be parallel, i. e. they will in all places be equally distant from each other. Draw

the line DC parallel to AB, [by the 31. 1.] and joyn the lines BD and AC.

Demonstration.

AB is supposed to be perpendicular to the X 2 planes

planes AC and BD, therefore the line CD, which is parallel to it, will be also perpendicular to them, [by the 8] and consequently the angles B and D, A and C, will be right angles; and [by the 28. 1.] the lines AC and BD will be parallels, and the figure ABDC a parallelogram. Therefore the lines AB and CD are equal, [by the 34. 1.] i. e. the planes in the points A and C, B and D. are equally distant. Accordingly the line CD may be drawn through any other point whatsoever; therefore the planes AC and BD are equally distant in all places the one from the other.

The USE.

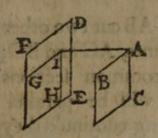
Theodosius demonstrates the circles, that have the same poles, as the Equator, and the two Tropicks, to be parallel, because the Axis of the World is perpendicular to their planes.

PRO-

PROPOSITION XIII.

A THEOREM.

If two lines, meeting at a point, be parallel to two lines of another plane, the planes of those lines will be parallel.



IF the lines AB and AC be parallel to the lines DE and DF, which are in another plane, the planes BC and FE are parallel. Draw AI perpendicular to the plane BC, [by

the 11.] and GI and IH parallel to FD and DE: they will be also parallel to the lines AB and AC [by the 9.)

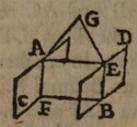
Demonstration.

The lines AB and GI are parallel, and the angle IAB is a right angle, AI being perpendicular to the plane BC: therefore [by the 29. 1.] the angle AIG is a right angle, as also the angle AIH. Therefore [by the 4.] the line AI is perpendicular to the plane CH; and being also perpendicular to the plane BC, the planes BC and GH, or FE, will be parallel, by the 14]

PROPOSITION

A THEOREM.

If a plane cut two others which are parallel, their common sections will together with them be parallel.



F the plane AB cut two other parallel planes, AC and BD; I say, their common sections AF and BE will be parallel. For if not, being continu'd they

would at length concur, e.g. at the point G.

Demonstration.

The lines AF and BE are upon the planes AC and BD; and therefore [by the 1.] can never be either above, or below it; therefore if they concur at the point G, the planes must do so likewise, and consequently they would not be parallel, which is contrary to what was suppos'd.

The US E.

By this Proposition we demonstrate, in the Treatise of Conick and Cylindrick Sections, that if the Cone or Cylinder be cut by a plane parallel to its base, the sections are circular. By the same we describe Astrolabes; and prove in Gnomonicks, that the angles, which 'the the hour-circles make with a plane parallel to a great circle, are equal to those which they make

in the circle it self; and again in Perspectives,

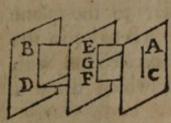
that the Images of the objective lines perpendi-

cular to the table, concur at the point of fight.

PROPOSITION XVII.

A THEOREM.

Two lines are divided proportionally by parallel planes.



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F the lines AB and CD be divided by parallel planes, I say, AE will have the same proportion to EB as CF to FD. Draw the line AD,

passing through the plane EF at the point G: Draw also AC, BD, FG, and GE.

Demonstration.

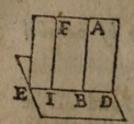
The plane of the triangle ABD cuts the three planes, therefore (by the 16.) the sections BD and EG are parallel; and (by the 2. 6.) AE has the same proportion to EB, as AG to GD. In like manner the plane of the triangle ADC cuts the planes EF and AC, therefore the sections AC and FG are parallel; and FC has the same proportion to FD as AG to GD, i.e. as AE to EB.

PRO-

PROPOSITION XVIII

A THEOREM.

If a line be perpendicular to a plane, all the planes, in which that line is found, are perpendicular to the same plane.



If the line AB be perpendicular to the plane ED, all the planes in which it is found will be perpendicular to the plane ED. Suppose AB to be in the

plane AE, having for a common section with the plane ED the line BE; to which draw a perpendicular FI.

Demonstration.

The angles ABI and BIF are right angles, therefore the lines AB and FI are parallel; and (by the 8.) FI will be perpendicular to the plane ED. Therefore the plane AE will be perpendicular to the plane ED, (by def. 4.)

The same may be proved of the plane AD.

The first Proposition in Gnomonicks, which may pass for a fundamental one, is built upon this proposition; which is also frequently made use of in Spherical Trigonometry, in Perfectives,

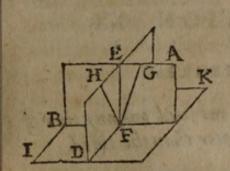
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spectives, and generally in all those Treatises which are oblig d to consider divers planes.

PROPOSITION XIX.

A THEOREM.

If two planes cutting each other be perpendicular to another, their common section will be perpendicular to the same.



IF the planes AB and ED, which cut each other, be perpendicular to the plane IK, their common fection EF is also perpendicular to the plane IK.

Demonstration.

If EF be not perpendicular to the plane IK, upon the plane AB draw the line GF perpendicular to the common fection BF: and the plane AB being perpendicular to the plane IK, the line GF will be perpendicular to the fame plane. Draw likewife FH perpendicular to the common fection DF; it will be also perpendicular to the plane IK. We shall have therefore two perpendiculars to the same plane, drawn through the same point F, (contrary to the

EF is perpendicular to the plane IK.

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The USE.

By this Proposition we demonstrate, that the circle which passes through the Poles of the World and the Zenith is the Meridian, and cuts all the diurnal arches into two equal parts; and that the Stars spend as much time in their motions from their risings to this circle, as from the circle to their settings.

PROPOSITION XX.

A THEOREM.

If three plain angles make one solid one, any two of them ought to be greater than the third.

IF the angles BAC, BAD, and CAD, make the folid angle A, and the angle BAC be the greatest angle; the two others, taken together, are greater than BAC.

Suppose the angle CAE to be equal to the angle CAD, and the lines AD and AE to be equal; and draw the lines CEB, CD, and BD.

Demonstration.

The triangle CAE and CAD have the fides AD and AE equal, and the fide AC common to both,

therefore (by the 4.1.) their bases CD and CE are equal. But the sides CD and DB are greater than the side CB alone, (by the 20.1.) therefore taking away the equal lines CD and CE, the line BD will be greater than BE. Further, the triangles BAE and BAD have the sides AE and AD equal, and the side BA common, and the base BD greater than the base BE: therefore (by the 18.1.) the angle BAD is greater than the angle BAE; adding therefore the equal angles CAD and CAE, the angles BAD and CAD will be greater than the angles CAE and BAE, i. e. the angle BAC.

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PROPOSITION XXI.

A THEOREM.

All the plain angles, that make one solid angle, are less than four right angles.

If the plain angles BAC, BAD, and CAD, make the folid angle A, they will be less than four right angles. Draw the lines BC, BD, and CD, and you will have

a pyramid, whose base is the triangle BCD.

Demonstration.

The folid angle at the point B, has the angles ABC and ABD greater than that of the base

base alone CBD. In like manner ACB and ACD are greater than BCD alone, and the angles ADC and ADB are greater than CDBalone. But all the angles of the base are equal to two right angles, therefore the angles ABC, ABD, ACB, ACD, ADC, and ADB, are greater than two right angles. And becanse all the angles of the three triangles BAC, BAD, and CAD, are equal to fix right angles; taking away more than two right angles, there will remain less than four for the angles made at the point A. But if the solid angle A consist of more than three plain angles, so that the base of the pyramid be a polygon, it may be divided into triangles; and the computation being made, you will find, that all the plain angles which make up the folid one, are always less than four right angles.

The USE.

These two propositions shew when many plain angles may make up one solid one, which is often necessary in the treatises of cutting of Stones, and in the following propositions.

The 22. and 23. Propositions are of no use.

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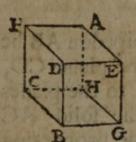
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PROPOSITION XXIV.

A THEOREM.

If a solid body be terminated by parallel planes, the opposite sides will be similar and equal parallelograms.



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IF the folid AB be terminated by parallel planes, the oppofite fuperficies's will be fimilar and equal parallelograms.

Demonstration.
The parallel planes AC and

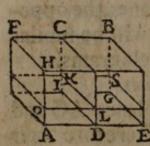
BE are cut by the plane FE: therefore their common fections are parallel, (by the 16.) and fo likewife DF and AE; therefore AD will be a parallelogram. After the same manner I may demonstrate, that AG, FB, CG, and the rest, are parallelograms. I add, that the opposite parallelograms, e. g. AG and FB, are similar and equal. The lines AE and EG are parallel to the lines FD and DB: therefore the angles AEG and FDB are equal, (by the 10.) Accordingly I may demonstrate all the sides and all the angles of the opposite parallelograms to be equal, therefore the parallelograms are similar and equal.

The Elements of Euclid.

PROPOSITION XXV.

A THEOREM.

If a Parallelepipedon be divided by a plane parallel to one of its planes, the two solid bodies which arise by that division, will have the same proportion as their bases.



If the parallelepipedon AB be divided by the plane CD, which is parallel to the planes AF and BE, the folid AC will have the fame proportion E to BD, as the base AI to the

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AH which shews the hight of the figure to be divided into as many equal parts as you please; for example, ten thousand; which we may take as indivisibles, i.e. without reflecting upon the possibility of their being further subdivided. Suppose also so many superficies's parallel to the base AI, as there are parts in the line AH; I have described only one OS: so that the solid AB be compounded of all those supposer is compounded of all its sheets and quires laid one upon another. Tis evident that so the solid AC will be compounded of ten thousand super-

superficies's equal to the base AI, (by the preceding,) and the solid DB will contain ten thousand superficies's equal to the base DG.

Demonstration.

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Every superficies of the solid AC has the same proportion to any of the superficies's of the solid DB, as the base AI to the base DG; because they are every one of them equal to their bases: therefore (by the 12.5.) all the superficies's of the solid AC, taken together, will have the same proportion to all those of the solid DB, as the base AI to the base DG. But all the superficies's of the solid AC make up the solid AC, which has no other parts but those superficies's; and all the superficies's of the solid DB are nothing else but the solid DB; therefore the solid AC has the same proportion to the solid DB, as the base AI to the base DG.

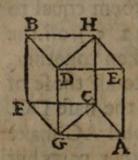
The USE.

'This is Cavalerius's demonstration; which is very clear, provided it be used as it ought; and that the line, by which is measur'd the thickness of the superficies's, be taken in the same respect in both the terms. I shall make use of it hereafter, to render some intricate and perplex'd demonstrations more facile and clear.

PROPOSITION XXVI:

A THEOREM.

A parallelepipedon is divided into two equal parts by the diagonal plane.



Suppose the parallelepipedon AB to be divided by the plane CD, drawn from one angle to another: I say it will be divided into two equal parts. Divide the line AE into as many parts as you please; and

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draw so many planes parallel to the base AF; each of those places is a parallelogram equal to the base AF, (by the 24.)

Demonstration.

All the parallelograms, that can be drawn parallel to the base AF, are divided into two equal parts by the plane CD; for the triangles which are form'd on both sides the plane CD, have their base common, in each equal to CD, and their sides equal, being those of a parallelogram. But 'tis evident, that the parallelepipedon is nothing else but those parallelograms, which are each divided into two equal triangles: therefore the parallelepipedon is divided into two equal vided into two equal parts by the plane CD.

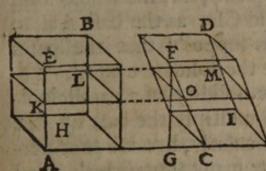
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The 27. and 28. Propositions are of no use according to this way of demonstrating.

PROPOS. XXIX, XXX, XXXI.

A THEOREM.

Parallelepipedons of the same hight, having the same or equal bases, are equal.



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F the Parallelepipedons AB
and CD be of the
fame hight according to the perpendiculars AE
and FG, and have

the same or equal bases AH and CI, they will be equal. Suppose the two bases to be set upon the same plane; since their perpendiculars are equal, the bases EB and FD will be in the same plane, which will be parallel to the plane of the bases AH and GI. Suppose then the line AE or FG to be divided into as many equal parts as you please, e. g. ten thousand, and according to them so many superficies's or planes drawn of the same thickness: I have describ'd only one for all, as K or M. Each superficies will form in these solids a parallel plane, similar and equal to the base, (b) the 24.) as KL, OM;

334 The Elements of Euclid.

and there will be as many in one solid as in the other; because their thickness, which I take perpendicularly according to their respective hights, is equal.

Demonstration.

The base AH has the same proportion to the base CI, as each plane KL to OM. But they being equal in number in both, all the antecedents (by the 15.5.) will have the same proportion to all the consequents, i.e. the whole solid AB to the whole solid CD, as the base AH to the base CI. But 'tis suppos'd that the bases are equal, therefore the solids are equal.

Coroll. To find the folidity of a parallelepipedon, 'tis usual to multiply the base by the hight taken perpendicularly, because that perpendicular shows how many superficies's equal to the base are contain'd in it. As for example, if I take a foot for my indivisible measure, i.e. which I will not afterwards subdivide; if the base contain twelve feet square, and the perpendicular hight ten, I shall have an hundred and twenty cubick feet for the folidity of the body AB. For the hight containing ten feet, I may make ten parallelograms equal to the bafe, having each a foot in thickness; but the base with one foot in thickness makes twelve cubick feet: the whole therefore will make an hundred and twenty, if the hight contain ten feet.

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PROPOSITION XXXII.

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A THEOREM.

Parallelepipedons of the same hight are in the same proportion as their bases.

Have prov'd this proposition in the preceding, demonstrating, that the parallelepipedon AB has the same proportion to the parallelepipedon CD, as the base AH to the base

CI. (See fig. preced.). Coroll. Parallelepipedons that have equal bases, are in the same proportion as their hights. As the parallelepipedons AB and AL, whose perpendicular hights are AK and AE. For if you divide the hight AK into as many aliquot parts as you please, and AE into as many as it contains equal to the former, and draw, according to each part, planes parallel to the base; as many as AE contains of the aliquot parts of AK, so many will the solid AB contain of the superficies's equal to the base, which are the aliquot parts of the folid AL; therefore (by defin. 5. 5.) the folid AB will have the same proportion to the folid AL, as the hight AE to the hight AK.

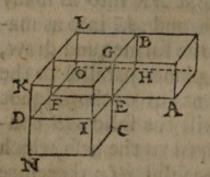
The USE.

'The three preceding propositions contain almost all the ways of measuring parallelepipedons, and may be esteem'd as first principles for that purpose. 'Tis after the same manner also that we take the dimensions of the solidity of Walls, by multiplying their bases by their hights.

PROPOSITION XXXIII.

A THEOREM.

Similar parallelepipedons are in the triplicate proportion of their homologous sides.



If the parallelepipedons AB and CD be timiliar, i. e. if all the planes of one be like those of the other; and all their angles equal, so that they may be plac'd POTT. AH

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in a right line, i. e. that AE and EF, HE and EI, GE and EC, may make right lines; and AE has the same proportion to EF as HE to EI, and as GE to EC: I say, that here are four solids in continual proportion according to the proportion of the side EA to that, which is homologous to it, EF or DI.

Demonstration.

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The parallelepipedon AB has the same proportion to EL of the same hight, as the base AH to the base EO, (by the 32.) But the base AH has the same proportion to the base EO, as AE to EF, (by the 1. 6.) In like manner, the proportion of the folid EL to the folid EK, is the same with that of the base EO to the base ED, i. e. that of HE to El. And lastly, the solid EK has the same proportion to the solid EN, as the hight GE to the hight EC, (by the coroll. of the 32.) or (taking the line EF for their common hight) as the base GI to the base CI, i. e. as GE to EC. But the proportion of AE to EF, of HE to EI, and of GE to EC, was suppos'd to be the same; and consequently, the folid AB has the same proportion to EL as EL to EK, and as EK to CD. Therefore (by defin. 11.5.) the proportion to AB to CD will be the triplicate proportion of that of AB to EL, or of AE to its homologous side EF.

Coroll. If four lines be in continual proportion, the parallelepipedon describ'd upon the first, has the same proportion to another similar parallelepipedon describ'd upon the second, as the first to the fourth; for the proportion of the first to the fourth, is the triplicate proportion of that of the first to the second.

The USE.

You may perceive by this proposition that Y 3 that

that famous problem of the duplication of the Cube, propos'd by the Oracle, confifts in finding two middle terms in continual proportion. For if you make the fide of the first cube the first term, and the double of that the fourth; having found twe middle proportionals, the cube describ'd upon the first line will have the fame proportion to that describ'd upon the fecond, as the first line to the fourth, i.e. as one to two. By this proposition also may be core rected their error, who fancy fimilar folids to have the same proportion as their sides; as if a cube of one foot in length was the half of a cube two foot long; when indeed it is but the eighth part thereof. This is likewise the foundation of the Rule concerning the fize of the bores of Canons; and is applicable not only to bullets, but to all forts of fimilar bodies. For example; should a man, about to build a Na-'vy, and resolving to retain the same proportion in all his Vessels, reason thus with him felf; If a ship of an hundred tun require fifty foot in Keel, another of two hundred tuns ought to have an hundred foot in Keel; he would be guilty of a great mistake: for inflead of making a Vessel twice as large as the former, he would make one eight times fo much. He ought to affign to the fecond Veffel fomewhat less than fixty three feet.

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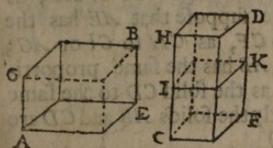
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PROPOSITION XXXIV.

A THEOREM.

Equal parallelepipedons have their bases's and hights reciprocal, and those that have their bases and bights reciprocal are equal.



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If the parallelepipedons

AB and CD be equal, their bafes and hights

will be recipro-

cal, i. e. the base AE will have the same proportion to the base CF, as the hight CH to the hight AG. Having made CI equal to AG, draw the plane IK parallel to the base CF.

Demonstration.

The parallelepipedon AB has the same proportion to CK, being of the same hight, as the base AE to the base CF, (by the 32.) But as AB to CK, so is CD to the same CK, because AB and CD are equal; and as CD to CK, which have both the same base, so is the hight CH to the hight CI, (by the Coroll. of the 32.) therefore as the base AE to the base CF, so is the hight CH to the hight CI or AG.

I add, that if the base AE has the same proportion to the base CF as the hight CH to the hight AG, the solids AB and CD will be equal.

Demonstration.

AB has the same proportion to CK, being of the same hight, as the base AE to the base CF, (by the 32.) Also the hight CH has the same proportion to the hight CI or AG, as CD to CK: But we suppose that AE has the same proportion to CF, as CH to CI or AG; therefore the solid AB has the same proportion to the solid CK as the solid CD to the same CK, and consequently the solids CK and CD are equal, (by the 9.5.)

The USE.

'This Reciprocation of the bases and hights makes the solid very easie to be measur'd. And the proposition seems to bear some analogy to the 14. Prop. of the 6. which asserts, That equiangular and equal parallelograms have their sides reciprocal; and the practice of the Rule of three may be demonstrated from both.

The 35. Prop. may be omitted.

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PROPOSITION XXXVI.

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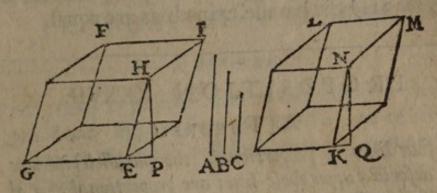
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A THEOREM.

If three lines be incontinual proportion, a parallelepipedon made of those three lines is equal to an equiangular parallelepipedon, which bas all its sides equal to the middle line.



The lines A, B, C, be in continual proportion, the parallelepipedon EF made of those three lines, the side FI being equal to the line A, HE equal to B, and HI equal to C, is equal to the equiangular parallelepipedon KL, whose sides LM, MN, and KN, are each of them equal to the line B. From the points H and N draw the lines HP and NQ perpendicular to the planes of the bases; which lines will be equal, because the solid angles E and K are supposed equal, (so that if they could penetrate, neither would exceed the other,) and the lines EH and

and KN are also supposed equal. Therefore the hights HP and NQ are equal.

Demonstration.

There is the same proportion of Ato B, or of FI to LM, as of B to C, or LM to HI: Therefore the parallelogram FH contain'd under FI and IH is equal to the parallelogram LN contain'd under LM and MN both equal to B, (by the 16.6.) therefore the bases are equal. But the hights HP and NQ are also equal; therefore (by the 31.) the parallelepipedons are equal.

PROPOSITION XXXVII.

A THEOREM.

If four lines be proportional, the parallelepipedons describ'd upon those lines are proportional: and if the similar parallelepipedons be proportional, their bomologous sides will be also proportional.

A B C D DF the line A has the fame proportion to B as C to D, the fimilar parallelepipedons, whose homologous fides are the lines A,B,C,D, will be in the fame proportion.

Demonstration.

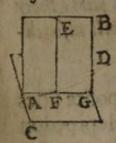
The parallelepipedon A is in the triplicate proportion to the parallelepipedon B, of that of

of the line A to the line B, or that of the line C to the line D. But the parallelepipedon C to the parallelepipedon D is also in the triplicate proportion of that of the line C to the line D, (by the 33.) Therefore the parallelepipedon A has the same proportion to the parallelepipedon B, as the parallelepipedon C to the parallelepipedon D.

PROPOSITION XXXVIII.

A THEOREM.

If two planes be perpendicular to each other, a perpendicular drawn from a point in one of the planes to the other will fall upon the common Section.



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B TF, the planes AB and CD being L perpendicular to each other, you draw from the point E in the plane AB a line perpendicular to the plane CD, it will fall upon the common section of the planes.

Draw EF perpendicular to the common fection Demonstration.

The line EF, perpendicular to AG, the common section of the planes, which are supposed to be perpendicular, will be perpendicular to the plane CD, (by defin. 3.) and because two

lines cannot be drawn from the point E perpendicular to the plane CD, (by the 13.) every perpendicular will fall upon the common fection AG.

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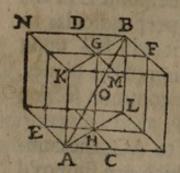
The USE.

This proposition ought to have follow dnext after the 17th. because it respects solids in general. Tis of use to us in the Treatise of Astrolabes, to prove that in the Analemma all the circles, perpendicular to the Meridian, ought to be markt by right lines.

PROPOSITION XXXIX.

A THEOREM.

If in a parallelepipedon be drawn two planes, which divide the opposite sides into two equal parts, their common section and the diameter will also divide each other into two equal parts.



Suppose the opposite sides of the parallelepipedon AB to be divided into two equal parts by the planes CD and EF, their common section GH and the diameter BA will equally divide

each other at the point O. Draw the lines BG, GK, AH and HL. I shall prove first, that the

two first of these, BG and GK, (and so like-wise AH and HL,) make but one right line. For the triangles DGB and KMG have their fides DB and KM equal, because they are the halves of equal fides; as also GD and GM. Further, DB and KM being parallel, the alternate angles BDG and GMK will be equal, (by the 29. 1.) and therefore (by the 4. 1.) the triangles DBG and KGM will be equal in all respects, and consequently the angles BGD and KGM: and [by the coroll. of the 15. 1.] BG and GK make but one right line, as also LH and HA: therefore ALBK is one plane, in which are found both the diamater AB, and the common fection of the planes GH. The plane ALBK cutting the parallel planes AN and CD, their common fections GH and AK will be parallel: And [by the 2.6.] BG will have the fame proportion to GK, as BO to OA; and therefore [by the 18. 5.] as BK to GK, fo BA to BO; and [by the 4. 6.] fo GH or AK to OG. But BK is double to BG, therefore BA is double to BO and AK, equal to GH, double to GO. Therefore the lines GH and AB divide each other equally at the point O.

Coroll. 1. All the diameters are divided at ..

the point O.

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Coroll. 2. Here we may add fome Corollaries, which depend upon divers propositions. As for example, that triangular prisms of the same 346 The Elements of Euclid.

fame hight are in the same proportion as their bases: For the parallelepipedons, of which they are the halves, are [by the 32.] in the same proportion as their bases: Therefore the halves of their bases, and the halves of the parallelepipedons, i. e. the prisms, will be in the same proportion.

Coroll. 3. Polygon prisms of the same hight are also in the same proportion as their bases, because they may be resolved into triangular ones, each of which will have the same pro-

portion as their bases.

Coroll. 4. The rest of the propositions concerning parallelepipedons are also applicable to prisms: as for example, that equal prisms have their hights and bases reciprocal; and that similar prisms are in the triplicate proportion of that of their homologous sides.

The USE.

'This proposition may help us to find out the center of Gravity in parallelepipedons; and to demonstrate some other propositions in the thirteenth and sourteenth books of Euclid.

PROP.

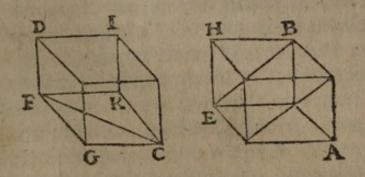
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PROPOSITION XL.

A THEOREM.

A Prism, that has a parallelegram for its base double to the triangular base of another prism, and of the same hight, is equal to it.

Let ABE and CDG be two triangular prisms, of the same hight; and the base of one the parallelogram AE, double to the triangle FGC, the base of the other prism: I say these prisms are equal. Suppose the parallelepipedons AH and GI were compleated.



Demonstration.

'Tis suppos'd, that the base AE is double to the triangle IGC, but the parallelogram GK is double to the same triangle, [by the 34. 1] therefore the parallelograms AE and GK are equal;

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equal; and confequently the parallelepipedons AH and GI, having the same bases and the same hights, are equal; and therefore the prisms that are the halves, [by the 26.] will be likewise equal.

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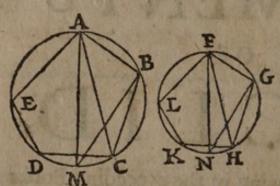
Uclid, after having in the preceding Books deliver'd the general principles of folid bodies, and explain'd the manner of measuring the most regular of them, that is, such as are terminated by plain superficies's; treats in this of fuch bodies as are contain'd in superfices's that are crooked, as the Cylinder, Cone, and 'Sphere: comparing one with the other, and 'giving rules, relating both to their folidity, and the manner of taking their dimensions. 'The Book is of exceeding great use, because in it we find the principles upon which the most learned Mathematicians have built so many famous demonstrations concerning the 'Cylinder, the Cone, and the Sphere. PRO-

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PROPOSITION I.

A THEOREM.

Similar polygons, inscrib'd in circles, are in the same proportion as the squares of the diameters of the same circles.



ABCDE, and ABCDE, and FGHKL, infcrib'd in circles, be similar, they will be in the same proportion as the squares

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of the diameters AM, FN. Draw the lines BM, GN, AC, and FH.

Demonstration.

'Tis suppos'd that the polygons are similar, that is to say, that the angles B and G are equal, and that AB has the same proportion to BC as FG to GH: from whence I infer, [by the 6.6.] that the triangles ABC and FGH are equiangular, and that the angles ACB and FHG are equal: so that likewise [by the 21.3] the angles AMB and FNG are equal. But the angles ABM and FGN, being in a semicircle, are right angles, [by the 31.3.] and consequently, the triangles ABM and FGN are equiangular.

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Therefore [by the 4.6.] AB has the same proportion to FG, as AM to FN: and [by the 22.6.] if two similar polygons be discrib'd upon AB and FG, as those that are propos'd; and two other similar polygons upon AM and FN, which shall be two squares; the polygon ABCDE will have the same proportion to the polygon FGHKL, as the square of AM to the square of FN.

"This proposition is necessary to demon-

ftrate that which follows.

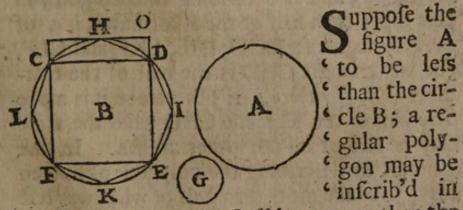
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LEMMA.

If a certain quantity be less than a circle, a regular polygon may be inscrib'd in the same circle greater than that quantity.



'the same circle, which shall be greater than the figure A. Let the figure G be the difference between the figure A and the circle, so that the figures

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The inscrib'd square is more than half of the circle, being half of the square describ'd about the circle; and in describing the octogon you take more than half of the Remainder, i. e. of the four segments CHD, DIE, EKF, and CLF, For the triangle CHD is the half of the rectangle CO, [by the 34. 1.] therefore it is more than half of the segment CHD; and the same may be said of all the other arches. In like manner, in describing the polygon of sixteen sides, you take more than half of what was left of the circle; and so in all the others. Therefore you will leave at last a less quantity than G. For tis evident, that two unequal quantities

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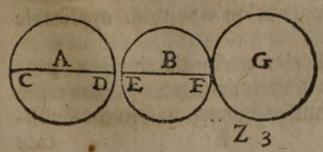
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ties being propos'd, if you take away more than half of the greater, and afterwards more than half of what remains, and again more than half of what is still left hehind; at length that which remains will be less than the second quantity. Suppose the second quantity to be 'contain'd in the first an hundred times: 'tis evident, that dividing the first into an hundred parts, in fuch fort, that the first part may have 'a greater proportion to the secondsthan two to one; the last will be less than the hundredth part: fo that at last you will obtain a polygon, which will be less exceeded by the circle, than the circle exceeds the figure A; that is to fay, that what will remain of the circle, when the polygon is taken away, will be less than G. Therefore the polygon will be greater than the 'figure A.

PROPOSITION

A THEOREM.

Circles are in the same proportion as the squares of their diameters.



Prove, that the circles A and B are in the same proportion, as the the squares of CD and EF. Suppose the figure G to have the same proportion to the circle B, as the square of CD to the square of EF: if the figure G be less than the circle A, [by the preceding Lemma,] a regular polygon may be inscrib'd in the circle A greater than G. Let a simular regular polygon be also inscrib'd in the circle B.

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Demonstration.

The polygon of the circle A will have the same proportion to the polygon of B, as the square of CD to the square of EF, i. e. the same as G to the circle B; but the quantity G is less than the polygon inscrib'd in A: accordingly therefore [by the 14.5.] the circle B must be less than the polygon inscrib'd in it, which is manifestly false. It must therefore be granted that the figure G, being less than the circle A, cannot have the same proportion to the circle B, as the square of CD to the square of EF; and confequently, that the circle A cannot have a greater proportion to the circle B, than the square of CD to the square of EF: nor can it be faid to have a less; for then the circle B would have a greater proportion to the circle A, and the same demonstration would be applicable to it.

Coroll. 1. Circles are in the duplicate proportion of that of their diameters; because the squares being similar figures, are in the duplicate proportion of that of their sides, [by the

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Coroll. 2. Circles are in the same proportion as the similar polygons, that are inscrib'd in them.

Coroll. 3. This ought to be well observ'd as a general rule: When similar figures, being inscrib'd in others, so that they may approach still nearer and nearer to them, and at last degenerate into the figures themselves, are in the same proportion; the figures that contain them are also in the same proportion. What I would fay is this; That simular regular polygons, infcrib'd in divers circles, are always in the same proportion as the squares of the diameters; and being made of more sides, so as to approach still nearer and nearer to the circles, they still retain the same proportion; and the circles themselves are in the same proportion as the squares of their diameters. This manner of measuring round bodies, by inscribing in them others, is of great use.

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This being a very general Proposition, enables us to argue about circles in the same manner as we do of squares. For example, we say in the 47. 1. I that in a rectangle triangle the square of the base alone is equal to the squares of both the sides taken together. We may say the same of circles, i. e. That the circle, described

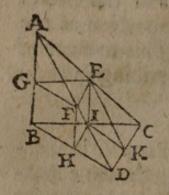
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'scrib'd upon the base of a rectangle triangle, is equal to the circles, whose diameters are the 'sides. And in the same manner may we augment or diminish circles, according to what proportion we please. We prove also by it in Opticks, that the light decreases in the duplicate proportion of that of the distances of the 'lucid bodies.

PROPOSITION III.

A THEOREM.

Every Pyramid, whose base is triangular, may be divided into two equal prisms, which make up more than half of the pyramid; and into two equal pyramids.



N the pyramid ABCD I may be found two equal prisms, EBFI, and EHKC, which will be greater than half the pyramid. Divide the fix fides of the pyramid equally at the points G,F,E,I, H,K, and draw the lines EG,

GF, FE, EI, HI, FH, IK, and EK.

Demonstration.

In the triangle ABD, AG has the same proportion to GB as AF to FD, because AB and

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AD are equally divided in G, and F; therefore [by the 2.6.] GF and BD are parallels; and GF will be the half of BD, i. e. equal to BH. In like manner, GE and BI, FE and HI, will be paarllels, and equal: and [by the 15.11.] the planes GFE and BHI will be parallel, and confequently EBFI will be a prism. The same may be faid of the figure HEKF, which will be also a prism equal to the other, [by the 40.11.] the parallelogram base HIKD being double the triangular BHI, [by the 41. 1.]

Secondly, I fay, the pyramids AEFG, and

ECKI, are fimilar and equal.

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Demonstration.

The triangles AFG and FDH are equal, (by the 8. 1.) as also FDH and EIK; and likewise AGE, and EIC, and so of all the other triangles of the pyramids: therefore the pyramids are equal, (by defin. 10.11.] They are also similar to the great pyramid ABDC: for the triangles AGE, and ECI are fimilar, (by the 2.6.) the lines GE and BC being parallels; and the like may be demonstrated of all the other triangles of the lesser pyramids.

Lastly, I say the prisms are more than half of the first pyramid. For if each was equal to one of the lesser pyramids, both would be equal to the half of the greater pyramid. But they are each of them greater than one of those pyramids; as the prism GHE contains the pyramid

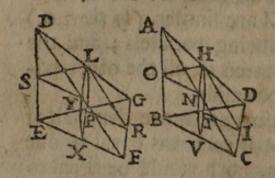
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GBHI, and somewhat more; and that pyramid is equal and similar to the others, having all their triangles equal and similar to those of the pyramid AGFE, as may be easily prov'd by the parallelism of their sides: from whence I infer, that the two prisms taken together are greater than the two pyramids, and consequently greater than half of the great pyramid.

PROPOSITION IV.

A THEOREM.

If two triangular pyramids of the same hight be divided into two prisms and two pyramids, and the latter pyramids subdivided after the same manner; all the prisms of one pyramid will have the same proportion to all those of the other, as the base of one pyramid to the base of the other.



If the two pyramids ABCD, mids ABCD, DEFG, of the fame hight, and having triangular bases, be divided into two prisms and two py-

ramids, according to the method laid down in the third proposition; and the two lesser pyramids be subdivided after the same manner, and

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fo in order, that, having made as many divisions of one as of the other, you have the same number of prisms in both; I say, that all the prisms of one will have the same proportion to all the prisms of the other, as their bases.

Demonstration.

The pyramids being of the same hight, the prisms, produc'd by the first division, will have also the same hight, because they have each the half of that of their pyramids. But prisms of the same hight are in the same proportion as their bases, (by the coroll. of the 39. 11.) The bases BTV and EPX are similar to the bases BDC and EGF; and having for their fides the half of those great bases, they can make but the fourth part of them, but they are in the same proportion as the great bases are; therefore the first prisms will have the same proportion as the great bases. After the same manner I may prove that the prisms produc'd by the second division, i.e. of the lesser pyramids, will be in the same proportion as the bases of those lesser pyramids, which are in the same proportion as the great bases. Therefore all the prisms of one have the same proportion to all the prisms of the other, as the base to the base.

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"These two propositions are necessary to compare pyramids together, and to take their dimensions.

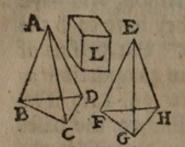
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PROPOSITION V.

A THEOREM.

Triangular pyramids of the same hight are in the same proportion as their bases.



THE pyramids ABCD and EFGH are in the fame proportion as their bafes. For if they were not, one of them, e. g. ABCD, would have a greater pro-

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portion to the pyramid EFGH, than the base BCD to the base FGH; so that a quantity less than ABCD would have the same proportion to the pyramid EFGH, as the base BCD to the base FGH. Divide the pyramid ABCD after the manner of the third proposition; divide also the pyramids, that result from that first division, into two prisms and two pyramids, and those again into two other prisms, continuing the division as long as there shall be occasion. Since the prisms of the first division are more than the half of the pyramid ABCD, (by the 3.) and the prisms of the second division more than half the remainder, i.e. of the two leffer pyramids, and those of the third division still more than the half of what is left; it is evident, that

fo many divisions may be made, that that which remains shall be less than the excess of the pyramid ABCD above the quantity L, that is, that all the prisms taken together shall be greater than the quantity L. Make as many divisions of the pyramid EFGH, so that you may have as many prisms as there are in ABCD.

Demonstration.

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The prisms of ABCD have the same proportion to the prisms of EFGH, as the base BCD to the base FGH: but the proportion of the base BGD to the base FGH is the same with that of the quantity L to the pyramid EF GH: therefore the prisms of ABCD have the same proportion to the prisms of EFCH, as the quantity L to the pyramid EFGH. But also the prisms of ABCD are greater than the quantity L: therefore (by the 14. 5.) the prisms contain'd in the pyramid EFGH would be greater than the same pyramid EFGH, which is evidently false, because the part cannot be greater than the whole. Therefore it must be granted, that no quantity less than one of the pyramids can have the fame proportion to the other as the base to the base, and consequently neither of the pyramids can have a greater proportion to the other than the base to the base.

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PROPOSITION VI.

A THEOREM.

All forts of pyramids, of the same bight, have the same proportion as their bases.





HE pyramids ABC and DEFG, of the fame hight, are in the fame proportion as the bases BC and EFG. Di-

vide the bases into triangles.

Demonstration.

The triangular pyramids AB and DE, being of the same hight, are in the same proportion as their bases, (by the 5.) So also the triangular pyramids AC and DF are in the same proportion as their bases. Therefore the pyramid ABC has the same proportion to the pyramid DEF as the base BC to the base EF, (by the 12.5.) Further, since the pyramid DEF has the same proportion to the pyramid ABC, as the base BC; and again, the pyramid DG has the same proportion to the pyramid ABC, as the base G to the base BC; the pyramid DEFG will also have the same proportion to the pyramid ABC, as the base G to the base EFG to the base BC.

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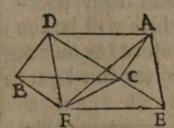
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PROPOSITION VII.

A THEOREM.

Every pyramid is the third part of a prism, being upon the same base, and of the same bight.



CUppose first the triangular prism AB be propos'd: I fay, a pyramid, having one of the triangles ACE or BDF for its base,

and being of the same hight, as the pyramid ACEF, will be the third part of the prism. Draw the three diagonals AF, DC, FC, of the three parallelograms.

Demonstration.

The prism is divided into three equal pyramids, ACFE, ACFD, and CFBD; therefore each will be the third part of the prism. The two first, having for their bases the triangles AEF and AFD, which (by the 34.1.) are equal, and for their hight, the prependicular drawn from the top C to the plane of their bases AF, will be equal, (by the preceding,) The pyramids ACFD, and CFBD, which for their bases have the equal triangles ADC and DCB, and the same top F, will be also equal, (by the preceding.) Therefore one of those pyramids, e. g. AFCE,

AFCE, having the same base BDF with the prism, and the same hight, which is the perpendicular drawn from the point F to the plane of the base ACE, is the third part of the same prism. If the prism be a polygon, it must be divided into divers triangular prisms; and the pyramid, which has the same base, and the same hight, will be also divided into as many triangular pyramids; each of which will be the third part of its prism. Therefore (by the 12.5.) the polygon pyramid will be the third part of the polygon prism.

PROPOSITION VIII.

A THEOREM.

Similar pyramids are in the triplicate proportion of that of their homologous sides.

If the pyramids be triangular, compleat the prisms, which will be also similar, because they will have certain planes the same with those of the pyramids. But the similar prisms are in the triplicate proportion of their homologous sides, by Coroll. 4. of the 39.11. Itherefore the pyramids, which (by the preceding are the third parts of the prisms, will be in the triplicate proportion of that their homologous sides.

If the pyramids be polygons, they must be re-

duc'd to triangular pyramids.

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PROPOSITION IX.

A THEOREM.

Equal pyramids have their hights and bases reciprocal, and those that have their heights and bases reciprocal are equal.

F two equal triangular pyramids be propos'd, . make prisms upon the same base, and of the same hight. Since every prism is triple his pyramid, (by the 7.) they will also be equal. But equal prisms have their bases and hights reciprocal, (by Coroll. 4. of the 39. 11.) therefore the bases and hights of the pyramids, which are the same with those of the prisins, will be also reciprocal.

Secondly, if the bases and hights of the pyramids be reciprocal, the prisms will be equal, as also the pyramids, which are the third parts

of the prisms. If the pyramids propos'd be polygons, they

must be reduc'd to triangular pyramids.

Coroll. Other propositions may be made concerning pyramids: as for example; That pyramids of the same hight, are in the same proportion as their bases; and those that have the same bases, are in the same proportion as their hights. The

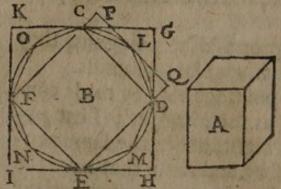
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rer of measuring pyramids, which is, by multiplying their bases by the third part of their hights. Other propositions may also be made, as, That if a prism be equal to a pyramid, the bases and the hight of the prism, with the third part of the hight of the pyramid, will be reciprocal; which is as much as to say, that if the base of the pyramid has the same proportion to the base of the prism, as the hight of the pyramid, to the third part of the hight of the pyramid, the prism and the pyramid will be equal.

A LEMMA.

If a quantity less than a Cylinder be proposed, a polygon prism may be inscribed in the Cylinder grater than that quantity.



A be less than the cylinder, whose base is the circle B, a polygon prism may be inscribed in the

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cylinder greater than the quantity A. The fquare

'square CDEF, inscrib'd in, and GHIK circum'scrib'd about the circle. CLDMENFO is an
'octogon inscrib'd. Draw the tangent PLQ: and
'suppose you had so many prisms as there are
'polygon bases, and all of the same hight with
'the cylinder. That which has the circumscrib'd
'square for its base, will encompass the cylin'der; and that whose base is the inscrib'd square,

'will be also inscrib'd in the cylinder.

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Demonstr. Prisms of the same hight are in the same proportion as their bases, (by coroll.3. of the 39. 11.) and the inscrib'd square being the half of that which is circumscrib'd, its prism will be the half of the other, and therefore more than the half of the cylinder. And making the prism with the octogon base, you 'take away more than half of what remain'd of 'the cylinder, after the prism of the inscrib'd 'square was taken from it, because the triangle CLD is the half of the rectangle CQ. And because prisms of the same hight are in the same propotion as their bases, the prism, whose base is the triangle CLD, will be the half of the prism, which for its base has the rectangle 'DCPQ: it will therefore be more than the 'half of that part of the cylinder, whose base is the fegment DLC. The same may be said of 'all the other segments. After the same manner I may demonstrate, that making a polygon prism of fixteen sides, I take away more than half A a 2

half of what remains of the cylinder, after the octogon prism is taken from it: so that there will remain at last a part of the cylinder, less than the excess of the cylinder above the quantity A. We shall have therefore a prism inscrib'd in the cylinder, which shall be less exceeded by the cylinder than the quantity A. i.e. which shall be greater than the quantity A. The same way of arguing will hold of the pyramids inscrib'd in a cone.

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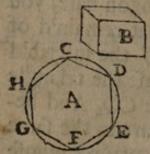
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PROPOSITION X.

A THEOREM.

A Cone is the third part of a cylinder, having the same base, and being of the same hight.



If a cone and a cylinder have the circle A for their base, and be of the same hight, the cylinder will be triple the cone. For if the proportion of the cylinder to the cone was greater than the triple proportion,

the quantity B less than the cylinder would have the same proportion to the cone as three to one: and (by the preceding Lemma) a polygon prism may be inscrib'd in the cylinder greater than the quantity B. Suppose that which has for its base the polygon CDEFGH to be such an one. Make also upon the same base a pyramid inscrib'd in the cone.

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Demonstr. The cylinder, the cone, the prism, and the pyramid, are of the same hight; therefore the prism is the triple of the pyramid, (by the 7.) But the quantity B is also the triple of the cone; therefore the prism has the same proportion to the pyramid, as the quantity B to the cone: and (by the 14.5.) the prism being greater than the quantity B, the pyramid would be greater than the cone, in which it is inscrib'd, which is impossible.

But if it be faid, that the cone has a greater proportion to the cylinder than one to three, the same method may be made use of to de-

monstrate the contrary.

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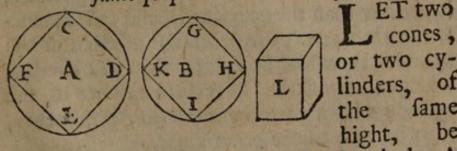
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PROPOSITION XI. A THEOREM.

Cylinders and Cones of the same bight are in the same proportion as their bases.



propos'd, having for their bases the circles A and B; I say, they are in the same proportion as their bases. For if not, one of them, e. g. the cylinder A would have a greater proportion to Aa3

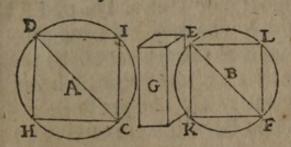
the cylinder B, than the base A has to the base B; suppose then that the quantity L, less than the cylinder A, has the same proportion to the cylinder B, as the base A to the base B. Therefore a polygon prism may be inscrib'd in the cylinder A, which shall be greater than the quantity L. Suppose it that therefore, whose base is the polygon CDEF; and inscribe a similar polygon GHIK in the base B, which is also the base of a cylinder of the same hight.

Demonstr. The prisms of A and B are in the same proportion as their polygon bases, (by coroll. 4. of the 39. 11) and the polygons are in the same proportion as the circles, (by coroll. 2. of the 2.) therefore the prism A has the same proportion to the prism B, as the circle A to the circle B . But as the circle A to the circle B, fo is the quantity L to the cylinder B: therefore as the prism A to the prism B, so is the quantity L to the cylinder B. But the prism A is greater than the quantity L, and confequently [by the 14. 5.] the prism B, inscribed in the cylinder B, would be greater than its cylinder, which is impossible. Therefore neither of the cylinders has a greater proportion to the other, than its base to the other's base.

Coroll. Cylinders are triple the cones, of the same hight, therefore cones of the same hight are in the same proportion as their bases.

A THEOREM.

Cylinders and Cones, that are similar, are in the triplicate proportion of that of the diameters of their bases.



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E T two cones or two cylinders, that are similar, be propos'd, having the circles

A and B for their bases; I say, that the proportion of the cylinder A to the cylinder B is the triplicate proportion of that of the diameter DC to the diameter EF. For if it be not the triplicate proportion, let the quantity G, less than the cylinder A, be, to the cylinder B, in the triplicate proportion of that of the diameter DC to the diameter EF; and inscribe a prism in the cylinder A greater than G, and another similar to it in the cylinder B: they will be of the same hight with the cylinder, because similar cylinders have their hights and the diameters of their bases proportional, as well as prisms, (by defin. 22. 11.)

Demonstr. The diameter DC has the same proportion to the diameter EF as the side DI to the side EL, or as DC to EF, as I have shewn

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in the first. But similar prisms are in the triplicate proportion of that of their homologous sides, (by coroll.4.0f 39.11.) therefore the prism A to the prism B is in the triplicate proportion of that of DC to EF. But we supposed that the quaintity G in respect of the cylinder B was in the triplicate proportion of that of DC to EF; therefore the prism A will have the same proportion to the prism B as the quantity G to the cylinder B; and (by the 14.5.) the prism A being greater than the quantity G, the prism B, inscrib'd in the cylinder B, will be greater than the cylinder B, which is impossible. Therefore similar cylinders are in the triplicate proportion of that of the diameters of their bases.

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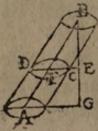
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Cones are the third parts of Cylinders, [by the 10.] therefore similar cones are in the triplicate proportion of that of the diameters of their ba-

fes.

PROPOSITION XIII. A THEOREM.

If a cylinder be cut by a plane, that is parellel to its base, the parts of its axis will be in the same proportion as the parts of the cylinder.



Let the cylinder AB be cut by the plane DC parallel to its base: I say, the cylinder AF will have the same proportion to the cylinder FB, as the line AF to the line

line FB. Draw the line BG perpendicular to the plane of the base A. Draw also upon the planes of the circles DC and A the lines FE

and AG. Demonstration.

The plane of the triangle BAG cuts the parallel planes A and DC; therefore the sections FE and AG are parallel, (by the 16.11.) So that AF has the same proportion to FB, as the hight GE to EB. Take any aliquot part of EB; and having divided GE and EB into parts equal to it, draw so many planes parallel to the base A; then will you have so many cylinders of the same hight; which, having their bases and hights equal, will be equal, (by the 11.)

Further, the lines AF and FB will be divided after the same manner as EG and EB, [by the 17.11] so that the line AF will as oft contain any aliquot part of the line FB, as the cylinder AF contains the like aliquot part of the cylinder FB; therefore the parts of the cylinder will be in the same proportion as the parts

of their axis.

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374 The Elements of Euclid. PROPOSITION XIV.

A THEOREM.

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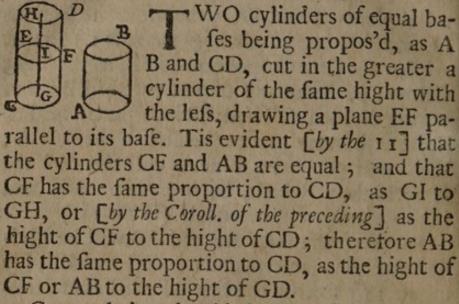
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Cylinders and cones, having the same bases, are in the same proportion as their hights.

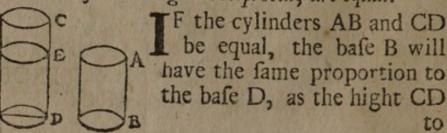


Cones, being the third parts of cylinders, if their bases be equal, will be also in the same proportion as their hights.

PROPOSITION XV.

A THEOREM.

Cylinders and cones that are equal, have their bafes and hights reciprocal: and those, that have their bases and hights reciprocal, are equal.



to the hight AB. Take the hight DE equal to

the hight AB.

Ta with partner in the AB of

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Fill

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CD

Demonstr. The cylinder AB has the same proportion to the cylinder DE, of the same hight, as the base B to the base D, [by the 11.] But as the cylinder AB is to the cylinder DE, so is the cylinder CD, equal to AB, to the cylinder DE; i. e. so is the hight CD to the hight AB or DE. Therefore as the base B to the base D,

fo is the hight CD to the hight AB.

Secondly, if the base B has the same proportion to the base D, as the hight CD to the hight AB, the cylinders AB and CD will be equal. For the cylinder AB is in the same proportion to the cylinder DE, as the base B to the base D: and the cylinder CD will have the same proportion to DE, as the hight CD to the hight DE: therefore AB has the same proportion to DE, as CD to DE; and [by the 9.5.] the cylinders AB and CD will be equal.

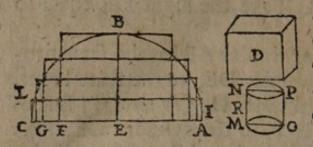
"The 16. and 17. Propositions are very difificult, and of no other use but to prove the
18. which may more easily be done by the

" following Lemma's.

LEMMA I.

If a quantity be proposed less than a sphere, cylinders of the same hight may be inscribed in the same sphere greater than that quantity.

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Suppose A BC to be a great semicircle of the sphere, where-

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and the quantity D to be the quantity lefs 'than that fphere: I fay, several cylinders of 'the fame hight may be inscrib'd in the sphere which taken together will be greater than the quantity D. For if the semi-sphere exceed the 'quantity D, it will exceed it by some magni-'tude; let it then be the cylinder MP, fo that the quantities D and MP taken together may be equal to the femi-sphere. Make a great cir-'cle of the fphere to have the same proportion to the base MO, as the hight MN to the hight 'R. Then divide the line EB into as many e-'qual parts as you please, each being less than 'R: and drawing parallels to the line AG, de-'scribe the inscrib'd and circumscrib'd paralleslograms. The number of the circumscrib'd will exceed that of the inscrib'd by one. But all 'the rectangles circumfcrib'd will furpass all the 'inscrib'd by the little rectangles through which the circumference of the circle passes: 'all which taken together are equal to the rect-Sangle AL. I imagine then the semicircle to be turn'd about upon the diameter EB; the feemicircle will by that motion describe a semi-Tphere,

'sphere, and the rectangles inscrib'd so many 'cylinders inscrib'd in the semi-sphere; and the 'circumscrib'd, other cylinders circumscrib'd.

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Demonstr. The circumscrib'd cylinders sur-'pass the inscrib'd more than the semi-sphere 'surpasses the same inscrib'd cylinders, it being 'contain'd within the circumscrib'd cylinders. But the circumscrib'd surpass the inscrib'd by the cylinder AL: therefore the semi-sphere 'will furpass those inscrib'd cylinders by less 'than the cylinder describ'd by the rectangle 'AL. But the cylinder AL is less than the cy-'linder MP: for there is the same proportion of a great circle of the sphere, which is the base of the cylinder AL, to MO, as of MN to 'R; therefore (by the preceding) a cylinder, which should have a great circle of the sphere for its base, and the hight R, would be equal to the cylinder MP: but the cylinder AL, tho it have the same base, yet its hight CL is less than R; therefore the cylinder AL is less than the cylinder MP. Consequently the semi-'sphere, that exceeds the quantity D by the cy-'linder MP, and the inscrib'd cylinders by a 'quantity less than AL, exceeds the inscrib'd cylinders by less, than it exceeds the quantity D; therefore the quantity D is less than the 'cylinders inscrib'd in the semi-sphere.

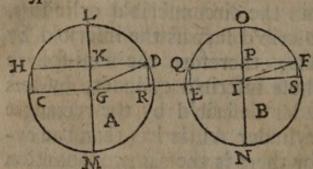
'That which I have faid of the femi-sphere, is

'applicable to an entire sphere.

LEM-

LEMMA II.

Similar cylinders, inscribd in two spheres, are in the triplicate proportion of the diameters of the Spheres.



F two fimilar cylinders CD and EF be 'inscrib'd in the fpheres and B,

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they will be in the triplicate proportion of the diameters LM and NO. Draw the lines

'GD and IF. Demonstration.

'The right cylinders CD and EF are fimilar; therefore HD has the same proportion to DR as QF to FS, as also KD has the same pro-'portion to DG as PF to FI. Consequently the triangles GDK and IFP are fimilar, (by the 6. 6.) therefore KD has the same proportion to PF as GD to IF, or LM to ON. But the si-' milar cylinders CD and EF are in the triplicate proportion of KD and PF, the semidiameters of their hases, (by the 12.) therefore the 'fimilar cylinders CD and EF, inscrib'd in the 'fpheres A and B, are in the triplicate proporstion of the diameters of the spheres.

PROPOSITION XVIII.

A THEOREM.

Spheres are in the triplicate proportion of their diameters.

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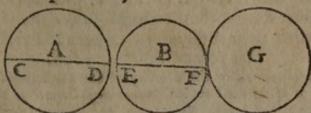
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proportion of that of their diameters CD and EF. For if not, one of the spheres, suppose A, will be in a greater proportion to B, than the triplicate of that of the diameters CD and EF; therefore the quantity G, less than the sphere A, will be in the triplicate proportion of that of CD to EF, to the sphere B; and then some cylinders may (according to the Lem. 1) be inscribed in the sphere A, greater than the quantity G. Inscribe an equal number of cylinders in the sphere B, similar to those in the sphere A.



Demonstr. The cylinders of the sphere A tothose of the sphere B are in the triplicate proportion of that of CD to EF, but the quantity G to the sphere B is also in the triplicate proportion of that of CD to EF: therefore the cylinders of the sphere A have the same proportion to the similar cylinders of the sphere B, as the the quantity G to the sphere B. Consequently the cylinders of A being greater than the quantity G, the cylinders of B, i.e. inscrib'd in the sphere B, will be greater than the sphere B, which is impossible. Therefore the spheres A and B are in the triplicate proportion of that of their diameters.

Coroll. Spheres are in the same proportion as the cubes of their diameters; because cubes, being similar solids, are in the triplicate proportion of their sides, [by the 33. 11.]

FINIS.

Printed for M. Gilliflower, and W. Freeman, these two Books following.

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