**Clavis geometrica catholica. The geometrical key: or the gate of equations unlock'd: a new discovery of the construction of all equations ... not exceeding the fourth degree; viz. of, linears, quadratics, cubics, biquadratics; and the finding of all their roots ... without the use of mesolable, trisection of angles; without reduction, depression, or any other previous preparation of equations, by a circle, and any ... parabole. And this, by one only general rule ... Fortified with demonstrations / [Thomas Baker].**

**Contributors**

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GEOMETRICAL
KEY

BAKER

1684

# THE GEOMETRY OF ALL EQUATIONS, CUBIC, and BIQUADRATIC, by a *Circle*, and any one only *Parabole*.

A

# *Clavis Geometrica Catholica:*
# SIVE
# JANUA ÆQUATIONUM
# *RESERATA:*

Methodus omnes Æquationes quomodolibet affectas, quartum gradum non Excedentes; nempe,

*LINEARES,*
*QUADRATICAS,*
*CUBICAS,*
*BIQUADRATICAS*
Conſtruendi;

Ipſarumque omnes radices, tam falſas quàm veras eliciendi; abſque ope *Meſolabii*, *Anguli Triſectionis*; abſque Æquationum *Reductione*, *Depreſſione*, vel quavis aliâ præparatione, per Circulum, & (quamlibet) unicam Parabolam.

*Idque*,

Ex unâ unicâ quidem Regulâ Generali; quâ ſimplicior, perfectior, generalior, intellectu facilior, praxi accomodatior, non eſt excogitanda, vel exoptanda.

Demonſtrationibus munita, Figuris ad quamlibet Æquationem inſignita, & Æquationibus numeralibus, pro varietate caſuum, ad quamlibet Figuram adaptatis Exemplificata.

*In uſum Tyronum; opus adhuc deſideratum.*

Ἔαν ἔσῃ φιλόμαθης, ἤσῃ πολύμαθης.

A THO. BAKER.

*LONDINI*, Typis *J. Playford*, & proſtant venales
apud *R. Clavel*, àd inſigne *Pavonis* in Cœmeterio
D. *Pauli*: M. DC. LXXX. IV.

# THE
# Geometrical Key:
## OR THE
## GATE of EQUATIONS
## *UNLOCK'D:*

A New diſcovery of the Conſtruction of all Equations, howſoever affected, not exceeding the fourth Degree; *viz.* of,

*LINEARS,*
*QUADRATICS,*
*CUBICS,*
*BIQUADRATICS;*

And the finding of all their Roots, as well falſe, as true; without the uſe of *Meſolabe, Triſection of Angles*; without *Reduction, Depreſſion*, or any other previous preparation of Equations, by a Circle, and any (and that but one only) Parabole.

*And this*,

By one only General Rule; than which a more ſimple, more perfect, more general, more eaſie to be underſtood, or more fit for practice, cannot be deviſed or wiſhed for.

Fortified with Demonſtrations, Illuſtrated with Figures, to each Equation; and Exemplified with numeral Equations, (according to all the varieties of caſes,) adapted to each Figure.

*For the uſe of Young Mathematicians, a Work hitherto deſired.*

Ἐαν ἐσῃ φιλόμαθης, ἔσῃ πολύμαθης.

By *THO. BAKER.*

*LONDON*, Printed by *J. Playford*, for *R. Clavel*, at
the *Peacock* in S. *Paul's* Church-yard: M. DC. LXXX. IV.

*Inclytis ac præſtantiſſimis viris, nobiliſſimis*

# MATHEMATICIS,

*Reverendiſſimo in Chriſto P. ac D.*

**D. SETHO**

*De* Sarum *Epiſcopo, primitias;*

Nobiliſſimo ac Honorando

*FRANCISCO ROBARTES,* Armig.

Tam Generis Nobilitate, quam Virtutum
ſplendore ornatiſſimo, Filio

*Honorandiſſimi D. D.*

**JOHANNIS**

Comitis de *Radnor*, &c.

*Spectatiſſimo & admodùm Colendo*

D.D. *JOSEPHO WILLIAMSONO,*

Societatis Regalis Præſidi vigilantiſſimo,

In Teſtimonium Gratitudinis & Obſervantiæ,
Lucubrationes haſce quales quales ſunt
humiliter devovet

*T. B.*

TO THE MOST
*Renowned and Excellent Men,*
*MOST FAMOUS*
**MATHEMATICIANS:**

*The Right Reverend Father in God,*
**SETH,**
Lord Biſhop of *Sarum*, theſe his firſt Fruits:

*The moſt Noble and Honourable,*
**FRANCIS ROBARTES,** *Eſq;*
A Zealous Fautor of Learning;

*Son of the Right Honorable,*
**JOHN,**
EARL of *Radnor*, &c.

The Right Worſhipful,
Sr *JOSEPH WILLIAMSON*, Kt
And Preſident of the Royal Society, in Teſtimonial of his Gratitude and Obſervance,

*Theſe his Mathematical Lucubrations (ſuch as they are) moſt humbly devotes,*
T. B.

### *TO THE YOUNG*

# MATHEMATICAL READER

## PARANETICΩS.

THERE *is none who is not fervently deſirous of knowledge; whom the love of truth doth not vehemently inflame, and ſet on fire: Now this enflamed deſire no Science gives ſuch ſatisfaction to, as* Matheſis *(the Princeſs of all Sciences) doth: Whoſe misfortune, yet whether it be more to be wondred at, or pitied, I cannot determine: For tho ſhe deſervedly challenges this privilege as peculiar to her ſelf, to careſs and affect her Votaries (upon the moſt clear and evident intuition of Truths) with more incredible and ſatisfactory pleaſure, than other Arts or Sciences can or dare pretend to, and with ſuch certainty, as leaves in mens minds, as little room as power to doubt; yet, of late years, to gain but very few Devoto's to her Shrine. The moſt certain skill of the moſt expert Phyſician is founded, but upon uncertain ſpeculations, and his ſecond conceptions of the Symptoms of a diſeaſe juſtle by his firſt Sentiments: Now* Galen's *Authority, anon* Paracelſus's *prevails; both by turns commended, condemned: And he will ſuppoſe himſelf to be no mean Artiſt, if from ſome hundreds of Concluſions, he can but fancy ſome of them more likely to be truer than their oppoſites; and can but hope that he is able eaſily to manage others of them againſt oppoſers; and oft-times hath none, on which he may rely, as exempted from all manner of Scruple. The* Old Ariſtotelian

*Lectori benevolo tyroni*

# MATHEMATICO
## PARANETICΩS.

NEMO prorsùs eſt, qui ſcire non unicè expetat; quem veri amor non vehementèr inflammet; huic verò tam inflammato deſiderio una propemodum *Matheſis* (omnium artium facilè princeps,) ſatisfacit; Cujus tamen infortunium an demirandum vel miſerandum magis ſit, planè ignoro: Quamvis enim quaſi propriam hanc ſibi palmam vendicet, ut Cultores ſuos mirum in modum afficiat & demulceat, (idque incredibili illâ voluptate, quæ ex claro & evidente veritatis intuitu exurgit, ut nullus omninò dubitandi locus ſupereſſe poſſit,) paucos tamen hujuſce deliciis captos, perpauciores nunc dierum unicè devotos conciliaverit. Certiſſima optimi medici peritia ſpeculationibus incertis nititur, primique ferè morbi ſypmtomatum & cauſarum conceptus ſecundis, & ſecundi tertiis cedunt. Apud quem nunc *Galeni*, nunc *Paracelſi* valet Authoritas; & uterque per vices excellit, vileſcit; cui hæreat, omninò ſubdubitat. Præclarè ſecum actum arbitrabitur, ſi ex aliquot Concluſionum Centuriis, alias putet vero-ſimiliores oppoſitis, alias ſperet tueri ſe haud difficultèr adverſus oppugnantes poſſe; & ſæpè habeat nullas, quibus ſecluſo omni dubitationis ſcrupulo acquieſcat. Exploſis ferè veteribus

*Ariſtote-*

*licorum* principiis, nova *Carteſiana* (ne dicam rationi magis congrua) ſubindè ſubeunt; ipſorumque Carteſianorum nonulla à *Boylianis* recentioribus (experientiâ comprobatis) peſſundantur. Hujuſce autem Triumviratûs quilibet, in rerum naturalium cauſis (quibus non omniò poſſit ſubeſſe falſum) aſſignandis, non poſſe toto cælo ſæpiſſimè errare, nullus dubito; quippe qui nullo Topico, (unde vel maximè probabilia eliciantur Argumenta,) ſuffigere poſſint, τὸ quod erat demonſtrandum. At horum aſſeclis ſcholæ ubique adeò arctantur, ut vix inibi reſpirantibus aer ſuppetit: Unica verò Mathematica (ubi non una aliqua expanditur veritas, ſed planè innumerabiles; eæque non vulgares & obviæ, ſed nominis ſublimioris eximiæ plerunque atque reconditæ, maximéque admirabiles perſpicuè demonſtrantur) vacua, araneoſa & contempta jacet. Ab hâc quidem, tanquam à loco peſtiſero (nunc-dierum) refugitum; ad illas verò, tanquam ad oraculum Delphicum (vel, ut ad Candida Tecta Columbæ,) turmatìm confugitum eſt.

Reges olim Principeſque hujus amænitate & ſimplicitate adeò ſunt allecti, ut (cæteris regnorum ſuorum deliciis ſuas ſibi res habere juſſis,) in Clientelam ſe ſupplices contulerunt, & nomina in Militiam ſolam Mathematicam dederunt, tanto pretio hanc ſcientiam redimentes. Si *Anacharſim* Schytam, & *Heraclitum* Epheſium commemorem, qui regna hæreditaria Contemplationi hujus poſtpoſuerunt; iiſdemque relictis, ad Philoſophorum ſedere pedes, quàm regiis inſidere ſoliis maluerunt: Si *Atlantem Mauritaniæ* Regem, quem (ob artem, quam inſigniter calluit, Aſtronomicam) humeris cælos ſuffulcientem, fabuloſa nobis exhibet Antiquitas: Si *Agathoclem Siciliæ* Regem, *Ptolomeum Philadelphum, Alphonſum* lian *Philoſophers Principles are exploded by the freſh ſtart of the new (I ſay not, more rational)* Carteſian; *and ſome of the* Carteſian *again baffled by the newer and truer experimental* Boylian: *And each of them, as far to ſeek of the true unerring Cauſes of natural things, as they are of the reaſon of the Magnetic vertue of the Load-ſtone; in aſmuch to any Topic (from which they draw their moſt concluding Arguments) they cannot ſubfix a* quod erat demonſtrandum. *And yet their Schools are ſo ſtuffed with Proſelytes, that they have ſcarce room to breath in: But the Mathematic (School) only, (in which, not ſome one Truth only is expanded, but even innumerable; and thoſe, not mean and obvious, but moſt high, admirable and myſterious are cleerly demonſtrated) lies orbate and neglected. From this they fly, as from ſome Peſt-houſe; but to thoſe, they troop, as to a Delphic Oracle, or as Doves to white Dove-houſes. Kings and Princes heretofore, have been ſo enamored with her ſimplicity and pleaſantneſs, that (forſaking all the delights of their Kingdoms) have made their addreſſes to her Shrines, paid Homage to her Altars; thus redeeming ſcience, at ſo great a price. Should I mention* Anacharſis *the Scythian, and* Heraclitus *the Epheſian, who prefer'd the Contemplation of Philoſophy before their hereditary Kingdoms, and choſe rather (leaving thoſe) to ſit at the feet of Philoſophers, than on their Kingly Thrones: Should I recount* Atlas *King of* Mauritania, *whom (for his Aſtronomic skill, wherein he excelled) Antiquity hath fabled to bear up the Heavens on his ſhoulders; or* Agathocles *King of* Sicily, Ptolomy *of* Philadelphia, Alphonſus, *of* Caſtile, *phonſum Caſtellæ, Fredericum Daniæ, Gulielmum Lantgravium Haſſiæ* , &c. Imo verò, ſi Imperatores, *Cæſarem* ſc. *Adrianum*, *Theodoſium*, &c. qui (ſtudiis hiſce, Imperatoribus quidem dignis, incumbentes) ſcriptis ſuis, quam bello geſtis (quamplurimis licet & inclytis) inſigniores evaſêre; non tantum opprobrium, ſed & pudorem ævo huic degeneri tacitè injicerem. Si quoſdam alios ordinis inferioris reputem, hujuſce gratiſſimis (pæne dixeram divinis) faſcinationibus mirum in modum extra ſe raptos: * Si *Archimedem*, ſpretâ Cuticulæ curâ, apud balneam cinere focario Figuras Geometricas exarantem, digito Lineas ducentem; thermiſque etiam nudum exilientem (cùm Aurifabri furtum deprehenderat) ſuumque ἕυρηκα ingeminantem: Si *Pythagoram*, (inveſtigatâ Trianguli Rectanguli proprietate) Hecatomben Muſis immolantem: Vix illos æquè debito honore proſequerer, ac nobiſmet ipſis acerrimè ſc. perſtringendo) ruborem non immeritò incuterem. Denique quamvis intrinſeca ejus forma & pulchritudo nonnullos etiam infimi ſubſelii homunciones, qui à limine ſolum ſalutarunt Arcana ſua neutiquam luſtrantes) eò adegerit, ut demirarentur; perpaucos certè videre eſt, aut illam penitiùs introſpexiſſe & calluiſſe, nedum Cultores ſuos remuneraſſe. Quâ de cauſâ gratiam, exiſtimationem & authoritatem apud Uulgus indies amitteret, formoſa hæc Dea, hariolari nequeam. Utrum eò quod cùm ſcientia liberalis (ſive ingenua) fuerit, ideoque tenaciſſimis illis Lucronibus, (in hominum numero vix cenſcendis) incongrua, qui (ad rem plus ſatis attenti) marſupia quàm animos locupletare malint; aut ideò, quòd ſpem omnem ad ἀκμὴν perveniendi, (quibus ſummis in votis eſt, aut Cæſares, aut nullos eſſe) abjiciunt;

* *Plutarch in vitâ Marcelli.* P. 397.

Frederic *of* Denmark, William Lantgrave *of* Haſſia, *&c. Yea, but ſhould I mention Emperors,* viz. Cæſar, Adrian, Theodoſius, *&c. who (devoting themſelves wholly to theſe Studies, worthy indeed of Emperors) rendred themſelves more Illuſtrious, by their Writings, than by their Warlike (tho many and great) Atchievements; I ſhould but ſilently ſhame and reproach this our degenerate Age. Nay, ſhould I but mention, how ſtrangely the minds of ſome others of a lower Sphere have been Captivated with its (I had almoſt ſaid, divine) charming delights: How it forced* * Archimedes, *ſometimes to forget his repaſt, and the care of his body, and at the bath on the heath to exarate Geometrical Figures, and to draw Lines with his Finger; nay, (upon the detection of the Gold-ſmiths theft) to leap naked out of his Bath, and to ingeminate his* ἕυρηκα: *How* Pythagoras *(upon the diſcovery of the propriety of a Rectangle Triangle) to immolate an Hecatomb to the Muſes,* &c. *I ſhould not more loudly honour them, then tacitly check and upbraid our ſelves. Laſtly, tho her intrinſic worth and beauty hath compelled others of the loweſt Orbe, (who (ſaluting her only at the threſhold) never entred, or had the leaſt glimps of her Arcana's or inner Rooms) to admire her; yet certain it is, very few are skilled in her myſteries; by which means it comes to paſs, that ſhe is as little regarded, as her Clients rewarded. For what cauſe this beautiful Goddeſs, ſhould thus ſuffer an Eclipſe in her glory and eſteem with the Vulgar, now a days, I cannot divine; Whether it be, ſhe being a liberal Science, and therefore (on that account) unſuitable to the humours of thoſe cloſe-fiſted Miſers, (who are ſcarce to be reckoned among the number of men) who love to have their purſes enriched rather than their minds: Or, whether*

*Plutarch in the life of Marcellus.p.307.

abjiciunt; Aut eò quòd ardua nimis & ſpinoſa artem hanc præ ſe ferre (quod faciliùs autumem) exiſtimant: Aut quæcunque tandem illa fuerint *μορμολύκεια*, quæ recentes poſtquam vel primori linguá Cupedias haſce libaverint, abſterrerint, (cum demum eorundem ſedulitatem & ſolertiam egregio Evidentiæ & Certitudinis præmio abundè compenſaſſet) indicare, nedum dijudicare non eſt meum. At verò hoc (Amice Lector) obſervanti expertoque mihi ſæpiuſculè occurrit; quòd cùm Tyrones nonnulli (non contemnendo tam operæ quam olei diſpendio) problema etiam ad Carceres uſque, juxta Artis Analyticæ Regulas inſequuti ſunt, videlicet ad Æquationem ad gradum elatiorem aſcendentem, quàm præſentiſcebant, ſc. ad tertiam, quartam, altioremve poteſtatem, (cujus reſolutio quidem Arithmetica ſubdifficilem, conſtructio verò Geometrica (quantum ſcivere) impoſſibilem eſſe, utpote toti orbi Mathematico peregrinum adhuc & ignotum) actum eſt derepentè de Matheſi, & quâ Problemati quà Arti valedixeruut. At hoc quidem (Lector) utrùm demirandum magis, an abſurdum meritò dubitandum; nempè, quod illud ipſum quod tibi animos adderet, animos adimeret, quod potiùs ut ulteriùs concitatiore impetu involando incitaret, te percellerit: Vere ardua (multo minus illa, de quibus præjudicare pro more noſtro plus ſatis ſolemus) non tantum ingenii aciem non retuderint, verum etiam virtutis animique tibi coti fuerint, conatuſque animaverint: Imò verò eo ipſo nomine, quod admiranda ſunt & difficilia (modò poſſibilia) ſpem animumque adderent, aciemque tuam exacuerint. Quò ſæpiùs humi abs *Hercule* proſtratus *Antæus*, eo vivacior exurgit, & fortior contendit *. Tam amiſſam gloriam reparandi

ſpes,

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* *Hom.* Il. pag. 704.

ther their deſpondency of ever arriving to any conſiderable eminency of height, (it being as good, to be nothing, as not a None-ſuch, or, but a Spy, to an Art:) or whether it be the fancied difficulty and knottineſs of the Study it ſelf, (which I have moſt cauſe to ſuſpect.) Or, what that ſuppoſed Mormo may be, that foreſtals and prejudiceth ſome newly entred, & ſcares others, who have taſted ſome of her ſweets from farther Eſſayes (which in fine, would have crowned their ſedulity and diligence, with evidence and certainty, (both which this Art carries, and no other doth, and which is reward enough to compenſate their pains) I ſhall not undertake to determine. Only this one thing (Reader) in my little experience, hath occurred to my obſervation oft, that when ſome Tyro's (with as great expence of pains, as of time) have according to the Rules of the Analytic Art, purſued a Problem to its end, viz. to an Equation aſcending to an higer degree, (viz. to the 3, 4, or higher Power) than they expected, whoſe Reſolution Arithmetical they conceived very difficult; but Geometrical Conſtruction impoſſible, (as being yet unknown to the Mathematic world; they have bid farewel, as to the Problem, ſo to the Art it ſelf too. But this (Reader) is as much abſurd, as ſtrange: viz. That what ſhould recommend this ſtudy to thy reaſon ſhould diſcourage thee; that what ſhould animate thy diligence, and quicken thee to a further Eſſay, ſhould decreſt and diſpirit thee. Real difficulties (much leſs conceived prejudices) ſhould be ſo far from blunting thy edge, that they ſhould rather be the whetſtone of vertue and ſharpen thy endeavours: Why may not the ſame things, which (for the excellency of them) are the objects of thy admiration, be (for their poſſibility) as well the object of thy hope, and the Encouragement of thy induſtry? Antæus recovered more ſtrength by each fall Hercules gave him. * The poſſible hopes of Redeeming or retrieving their

*Hom. Il, pag. 704.

loſt

ſpes, *Græcos*; quam compluſcularum victoriarum recordatio *Trojanos* ad prælia accendit. Non adeò ardua hujus artis pericula, & inexpugnabilia, quin (uti bello) aut Solertia aut Caſu, aut utroque ſunt ſuperabilia. Præclara *Alexandri* geſta *Cæſar* recolens, propriamque exinde exprobrans ignaviam, ad auſa inclyta auſpicanda novos ſumpſit animos, proſperéque pugnatum eſt. *Appelles* ſummâ incuriâ penecillum ſuum in Tabellam iratus projiciens, canis rabidi ſalivam (quam malè depinxerat, fortuitò ſed graphice correxit, & felici hoc infortunio, detur verbo venia) quàm arte clarior evaſit. Ludit in omnibus (etiam in ſtudiis humanis) fortuna Dea (verius dixerim divina providentia;) & qui non Artis, Aleæ ſæpiſſimè fit magiſter: Et quemadmodum plurimum laboribus & vigiliis, ita Caſui aliquando nonnihil tribuendum; qui tibi ſtudiis hiſce invigilanti nova aliquando & præter opinionem ſuggerat. Sagaces ſedulique Lapidis Philoſophici indagatores quamvis aurem propoſitum nequaquam attingerint, attamen non omnem plerumque ludunt operam; dum fortuitò in aliud quod tam laboribus quam inpenſis non indignum incidunt. Haud aliter mihi evenit in Regulæ hujus univerſalis inventione (de quâ hoc Tractatu agimus) nempè de Conſtruendis Æquationibus omnibus quartum gradum non excedentibus, & de determinandis illarum locis: quibus plurimi non vulgaris eruditionis Geometrici diu inſudarunt, multumque lucubrationibus ſuis oleum, exitu non ex æquo fælici, inſumpſerunt.

Non eò quod (fateor) mihi cor limante Minervâ,
Acriùs, & tenues finxerunt pectus *Athenæ*;
Sed quòd (dexterior ſolito) mihi dexter *Apollo*
Adſuit ——

Dum

*loſt honour enflamed the* Græcians, *as well as their being fleſht with frequent Victories did the* Trojans, *to fight. The difficulties of this Art are not ſo inſuperable, (but as in War) they may be overcome, either by induſtry or fortune, or both.* Cæſar *upbraided his own ſlackneſs with the memory of* Alexander's *conqueſts, even upon the bare ſight of his Picture only, and enſpirited himſelf to high bold and daring attempts, and proved ſucceſsful.* Appelles *in anger careleſly throwing his Pencil, accidentally (tho inartificially) well ſhaped the ill drawn Vomit of his Painted Dog; and this proſperous miſchance made him more famous, than his Art could doe. Fortune (or Providence rather) ſports it ſelf (pardon the word) in all things, (even in humane ſtudies) and he that is not Maſter of Art, may yet be ſo of a Chance. As (Reader) thou maiſt attribute much to thy ſedulity and induſtry; ſo ſomewhat ſometimes to fortune too, which may (if induſtrious in thy inqueſt) favour thee perhaps with ſome new invention beyond thy expectation. Thoſe buſie enquirers and ſearchers after the Philoſophers Stone tho they loſe their aim, yet not uſually all their labour; but ſtumble oft on ſomething worthy their diligence and expence. It chanced to me thus in the invention of this univerſal Rule, of which we now treat; namely, of the Conſtruction of all Equations not exceeding the fourth degree, and of determining their places; about which many Learned Geometricians have ſweated and ſpent much Oyl at their Lucubratory Tables, but perhaps not with equal ſucceſs. Not, that I had quicker brains, but better luck. For whiles buſying my ſelf (who pretend not to Learning, nor to the Profeſſion of the Mathematic Art, but one, who at ſome ſubciſive hours, for diverſions ſake, its ſtudy much delights) in an Analytic inqueſt (by way of Poriſma) of what* des Cartes *had*

Dum enim Analyticam diſquiſitionem illorum quæ Cl. *Carteſius* de eodem Argumento * Poriſmaticos me- *Geom. l. 3 pag. 85. ditarer, (qui me Mathematicum non profiteor, ſed quem (ſi quando vacat) horis ſucciſivis delectant Mathematices ſtudia) fortuna (ars nequaquam) conatibus favens noſtris, in methodum quæ Regulam ejus nimis ſtrictam, ampliorem, imo (quod & ipſe demiratus ſum) facillime eam univerſalem redderet, præter inſtitutum incidi. Quod dubiò procul Cl. illo & ſagaciſſimo viro perſpectum fuiſſet, ſi fors fauſta illum (prout me) eò duxerat, ut Circulum à quovis Puncto in poſitione dato, per verticem cujuſvis diametri in Parabolâ ductum (prout à vertice Axis fecit) deſcripſiſſet; & proprietatem quandam Parabolæ (huic inſtituto apprimè aptam natam) inſuper animadvertiſſet. At *Bernardus* non videt omnia.

Ad hoc tamen tam *Cl. Carteſius*, quàm alii eruditiſſimi Mathematici quà Antiqui quà Neoterici collimarunt: Horum verò ſpeculationes eò tantùm vergebant, ut Meſolabum & Angulorum Triſectionem, ſi ſors dederit, tandem demùm inveſtigarint.

*Vieta* (primus Analyſis ſpecioſæ Inventor) hanc rem non perperam videtur ventilaſſe; at poſt accuratiſſimam diſquiſitionem non potuiſſe expedire & effectam dare cuilibet innoteſcat Tractatus ejus de Pſeudo-meſolabo & ſupplemento Geometriæ conſulenti.

Cl. *Outhedus* (noſtras) poſtquam Æquationes nonnullas Cubicas prælibaverat * (quâ etiam ſolertiâ *Cl. Math. Chap. 18. (uti inquit) alias innumeras Analytices ſtudioſus poterit comminiſci) ſperat fore, ut illarum ope Meſolabum hactenus tenebris obvolutum in lucem tandem proferatur.

*Written on that Subject; Fortune (not Art) was pleaſed favourably to aſpect my weak Endeavours; and when not deſigning it, I happened to hit on ſuch an improvement of his Rule, as (to my great admiration) would render it univerſal: which (no doubt) that quick Lyncean-eyed man might have ſeen, and would have made a præ-diſcovery of, had it been his hap (as it was mine) to have deſcribed a Circle, from any Point in Poſition given, paſſing through the Vertex of any Diameter in a Parabole, as he did from the Vertex of its Axe; and withall had taken into conſideration a certain propriety (than which none could have ſo well ſuited his deſign) belonging to the Diameter of any Parabole. But* Bernardus non videt omnia.

*And as this (no doubt) was famous* Des Cartes'*s chief aim; ſo hath the Enquiry after this one thing eſpecially, been the Subject of the chiefeſt Study of the choiceſt and learnedeſt Mathematicians of the former and latter Ages; but taking different Meaſures, it proved not equally ſucceſsful. For their Speculations bended moſtly to the invention (if poſſible) of the Meſolabe and Triſection of Angles.*

Vieta *(the fiſt Inventor of Specious Analyſis) ſeems, not perfunctorily to have examined this matter; and after his moſt exquiſite ſearch, tacitly inſinuates his own ignorance, as may be ſeen in his two Tracts; the one de Pſeudo-meſalabo; the other, de ſupplemento Geometriæ.*

*Our Famous Mr.* Oughtred* *having prelibated ſome few Cubic Equations, hopes by their help, the Meſolabe hitherto involved in darkneſs, may at length be brought to light.*

* *Cl. Math.* Chap. 18.

Quod etiam *Carteſius* ipſe (unà cum Celeberrimis ejus Commentatoribus, *Franciſco* à *Schooten*, *Huddenio*, *Florimondo de Beaun*, *Johanne de Wit*, (in Elementis Curvarum) (qui omnes Carteſii veſtigiis preſſiſſimè inſiſterunt;) & alii nonnulli (ut *Cl. Fermatius*, *M. de la Hire*, *Sluſius* (qui cæteris palmam hâc in re præripere videtur) Conicarum Sectionum ope & Circuli aggreſſi ſunt: Quorum quidem omnium hâc de re Lucubrationes neutiquam ſunt contemnendæ; & uti quidem multis nominibus, ita eo præcipuè, quòd ad plana omnia & ſolida loca reperienda (quorum beneficio Æquationes ad elatiores gradus aſcendentes poſſint conſtrui) viam præſtravere.

Notandum verò Clariſſimum illum (*Carteſium* nempè) non ulteriores (quantum video) progreſſus feciſſe, quàm iſtiuſmodi Æquationes conſtruendo, quibus ſupponendum eſt (ut plurimùm ſaltem) ſecundum terminum deeſſe. Quem quidem (eâ de causâ opinor) nimis acrius perſtrinxiſſe videtur *Bartholinus*, inſimulando Cl. illum varia in hâc materiâ imperfecta reliquiſſe,*) quæ (ut ipſe innuit) ad perfectionem deducere conatus eſt vir eximius *Florimundus de Beaune*, ſed in medio curſu ſubſiſtens (morte præventus immaturâ) ultimam Authoris manum non paſſa ſunt: At effecta dedit, & numeris omnibus abſoluta nobis exhibuit idem *Bartholinus* (uti inquit) in duobus Tractatibus, de limitibus Æquationum, & dioriſticâ methodo: perperam (ni fallor) aſſerens, neceſſitatem priùs reſolvendæ, immò & reſolutæ priùs determinandæ & definiendæ cujuſvis Æquationis à Dioriſtice primò inventæ, quam ejuſdem conſtructionem quilibet auſus ſit aggredi.

* Barth. *Selecta Geom.* Ep. pag. 7.

Des Cartes *himſelf, with thoſe famous Commentators on him* Franciſcus â Schooten, Hudden, Florimond de Beaun, John de Wit *(in his Elements of Curves) (all moſt preſſingly inſiſting in his ſteps) and divers others (as* Fermat M. de la Hire, *and* Sluſius *(who ſeems to outdoe them all) have attempted it, by the help of Conical Sections and a Circle; all whoſe pains therein, being of ſingular uſe, are not to be deſpiſed; as in many other reſpects, ſo in this eſpecially; in that they find out all Plane and Solid places for the Conſtruction of all Equations of higher Degrees.*

*But Note, it reaches no further (for ought I can perceive) than to ſuch Equations, where the ſecond term moſtly muſt be ſuppoſed to be wanting. Upon which account, I ſuppoſe* Bartholinus * *is pleaſed to cenſure* Des Cartes, *that he had left many things, about this matter imperfect; which (as he inſinuates)* Florimond de Beaun *afterwards endeavoured to bring to perfection, but being prevented by an immature death, deſiſted in the midway: Which again after that* Bartholinus *himſelf (as he ſays) hath perfected in his two Tracts, the one, of the limits of Equations; the other, in his Dioriſtics: Strangely (if I miſtake not) concluding a neceſſity of every Equation to be reſolved; yea, and being reſolved of limiting and determining every of them, by a Dioriſtic firſt found, before ever any man may dare to attempt its Conſtruction.*

* Selecta Geometr. Ep. pag. 7.

For

Ob defectum cujus, magnum (inquit) animadverto in Arte hiatum, cui replendo, non mediocre ſufficit ingenium, quem ipſe (acumine quo pollet non vulgari) accuratiſſimè poſtmodùm implevit. Quæ quidem an tam eximio uſui fuerint in Æquationum radicibus Arithmeticè eliciendis non ediſſeram: At verò an tanti momenti, an tam prorſùs neceſſaria ad Æquationes Geometricè conſtruendas (ut arbitratur ille, & contendit) doctorum arbitrio, poſtquam Tractatum hunc percurrerint, relinquendum eſſe cenſeo. In quo nullus dubito, quin reperiant, omnes Æquationes quomodolibet affectas quartum gradum non excedentes, Dioriſtice nequaquam obſtetricante, ad amuſſim Geometricè conſtrui poſſe; immo quidem vel abſque ope Meſolabi, Sectionis Angulorum, ſive aberit ſive aderit ſecundus (ſive quivis alius) terminus, nulluſque ſupererit ubilibet hiatus; idque ſine præviâ quâlibet Reductione, depreſſione, limitatione, *&c.* omninò prout *Frontiſpicium* fuſiùs indicat.

*Adrian Rom. Probl. Reſp. pag. 315.*

Jam verò, ſi *Vieta*, ſub intuitu inventionis duorum Theorematum (quæ ſunt fundamenta omnis doctrinæ Angulorum Sectionum, ad rem verò noſtram obliquè tantum ſpectantia) eâ extaticâ lætitiâ afficeretur, ut exclamaverit, Tibi, ô Diva Meluſinis, oves centum pro unâ *Pythagoreâ* immolavi; liceat mihi (præ inventi hujus gaudio) qui non ovibus, quidem, ovationibus tamen cum Vietâ contendere. Abſit autem, ut ipſe mihi quicquam arrogem; quippe, ſi in hâc re Fortuna magis mihi (eruditionem neutiquam obtendenti) quàm aliis verè Lynceis & doctioribus arriſerit, & aſpiraverit; imo, ſi quivis alius in

*For defect of which Dioriſtic method, I find ſaith he *) a great gap in Art, which to fill up, a mean Art is not ſufficient; which yet afterwards his moſt acute wit (I confeſs) moſt accurately filled up: Which whether it may be of ſo great uſe in reference to the Arithmetical Reſolution of Equations, I ſhall not here determine; but whether it be of ſo great moment and ſo abſolutely neceſſary (as he poſſitively affirms) to the Geometrical Conſtruction of Equations, I ſhall leave to the judgment of the more Learned, when they ſhall have peruſed this Treatiſe: Wherein I doubt not, but they will find the Geometrical Conſtruction of all Equations, howſoever affected, not exceeding the fourth degree, without the Midwifery of a Dioriſtic to be exactly performed; nay, I may add, without the help of Meſolabe, Section of Angles, whether the ſecond or any other term be abſent or not, without leaving any hiatus any where, without any previous Reduction, depreſſion, limitation,* &c. *altogether as the Frontiſpiece at large declares.*

*And now, ſhall* Vieta, *upon the review and proſpect of having found two Theorems (which indeed are the Fundamentals of the whole Doctrine of Angular Sections, but obliquely only reſpects our buſineſs in hand) be traſported into ſuch an extaſy of joy, as to cry out * ô Diva, Meluſinis, tibi oves centum pro unâ* Pythagoreâ *immolavi? And ſhall the Author for the joy of this invention, vye with him in his joy, tho he cannot in Hecatombs? But far be it from me, to arrogate any thing to my ſelf; for if in this (or any other) inſtance, fortune ſhould ſmile on me (who pretend not to Learning) more than on others of greateſt eminency in Learning; nay, if even they too ſhould*

* Loco citato.

* Adriani Rom. Prob. Reſp. pag. 315.

in inventum quodvis inclytum (nomen ſuum immortalitati mandaturum) fortuitò inciderit; non eſt quòd exinde vel gloriolam ſibi arroget, aut magis inſoleſcat, quàm qui fortè fortunâ uno & unico globi miſſilis projectu totum ſimul Conorum Luſoriorum Enneada dejecerit; aut quàm qui *Hercule* colludens caſu haud prorſus abſimili illum proſtraret.

Si verò mihi vel tantillum (cui ne hilum quidem) gloriæ, hujus inventi cauſâ, tribuendum; plurimum equidem celeberrimo Carteſio primò impetiendum lubens agnoſco; cujus humeris (velut nanus quidam) inſidenti, longius diſſita illò paulo acutiùs perſpexiſſe conitgit; à cujus face lucernula hæc (qualis qualis ſit) lucem ſuam (velut à Sole Luna) mutuata eſt.

Non nihil etiam (quod gratitudinis ergô, amicitiæque devinctiſſimæ ſpecimen refero) D. *Thomæ Strode* de *Maperton* in agro *Somerſetenſi*, viro verè generoſo, præſtantiſſimoque Mathematico meritò reddendum; non tantum eò quod reperti hujus anſam præbuit; ſed quod ejuſdem Ideas quaſdam ſubminiſtravit Lectiſſimus iſte (qui penes eum eſt, & quem mecum humaniter communicavit) liber *M. S.* In quo proprietatem ad Diametrum Parabolæ ſpectantem ſuprà memoratam reperi; quâ ſine malè forſitan ſucceſſiſſent omnia, parumque abfuiſſet, quin invento excideram: Libellus ſanè utiliſſimus, magnique æſtimandus, utpotè, qui non ſolum propriis novis & abditis, ſed præclaris omnibus ex intimis Authorum ferè omnium Sectiones Conicas tractantium viſceribus erutis, refertiſſimus; cujus ſubtiliſſimæ de Hyperbolicis ceteriſque Coniſectionibus Propoſitiones ſi pro merito prædicarentur cunctorum Rhetorum Hyperbolas ſuperarent.

*ſhould chance to light on ſome famous invention, which might in a more eminent manner immortalize their names; yet have they no more ſhadow of reaſon to be proud of it, than he, who accidentally at one tip, ſhould ſtrike down the whole pack of Nine-pins; or, than he, who in ſporting or dallying with* Hercules *ſhould by a like chance foyl him.*

*But if any praiſe were (which is none) due to me, for the invention; a very great ſhare muſt redound to the honour, firſt of the famous* Des Cartes, *on whoſe Gigantic ſhoulders ſtanding, I chanced to ſee further than he; but I confeſs, this Candle (ſuch as it is) was lighted at his taper.*

*Another part (which I mention as a ſpecimen (ſuch as it is) of my gratitude and reſpects) is deſervedly due to that moſt worthy Gentleman and moſt Excellent Mathematician Mr.* Thomas Strode *of* Maperton *in the County of* Somerſet; *not only for the occaſion given of this invention (beſt known to him only) but for the light I received from his incomparable* M. S. *touching Conical Sections; wherein I found the propriety belonging to the Diameter of a Parabole above mentioned; without which the invention it ſelf might perhaps have proved abortive. A Treatiſe of ſuch worth and uſe, that beſides his diſcovery of many new and hidden things never extant, it ſeems to have engroſſed all that is excellent in every Author (that I know) that hath treated or is extant on that ſubject; and which needs the Rhetoricians Hyperboles, to recommend to the World the excellency of his* Geometrical *ones, and other Conical Sections.*

*But*

Ad rem verò: Diſquiſitionis hujus exitus non magis fœlix fauſtuſque fuit, quàm media (quibus hoc inveniendo uſus ſum) apprimè congrua; quæ ad calcem hujus Tractatûs appoſui; eoque potiſſimùm conſilio, ut non tantum te (Tyro) manuducerem, tibique animos adderem; ſed ut iiſdem (aut non abſimilibus) premens veſtigiis, propriis ſtudiis & laboribus altiora & non priùs audita moliaris.

Si enim applicetur methodus noſtra (quâ duce hæc invenimus) tam ad Hyperbolas & Ellipſes, quam ad Parabolas; imò ad paraboloeides, Hyperboloeides, Elliptoeides, ſive ad quaſvis alias elatioris gradus Curvas, quà ſimilis, quà diſſimilis Conſtitutionis (quas comminiſci poterint ſtudioſi) efformandas; haud dubiè particulares ſaltem (imò & univerſales) Regulæ, ad quamplurimas (etiam ad omnes) Æquationes, ad quodvis ulterius par graduum conſtruendas, emergent. Quod ſtudioſis relinquo; viriſque ingenio perſpicaci adnotaſſe ſufficiat, operæque forſan erit pretium.

Cuipiam idcirco muſſitanti vel ſuggerenti hoc ipſum Inventi hujus gloriam imminuiſſe, quòd caſu repertum, regero. Quo ad finem, non abnuo; attamen quoad media (quibus uſus ſum) dico; excogitatâ ratione me ea compoſuiſſe, & ad finem propoſitum aſſequendum adeò apta nata, ut aptiora nequaquam poſſint excogitari; tantique (hujuſce farinæ rebus indagandis) momenti, ut cuipiam κατὰ πόδας eadem inſequenti, res nominis ſublimioris hiſce à nobis inventis adeò facillimè aſſequi contingat, ut locus obtrectationi non relinquatur.

*But to return: The event or end however was not more lucky, than the means uſed, ſuitable; which I have on ſet purpoſe diſcovered at the heel of this Treatiſe, as well (Tyro) for thy Encouragement in this Study, as for the further improvement (if I miſtake not) of this Invention, by the induſtry of the more Learned, inſiſting in the ſame (or the like) method; by which means higher things may be diſcovered.*

*For aſſuredly, the application of this Method (which I have uſed in this diſcovery) to Hyperboles, Ellipſes, as well (as to Paraboles; nay, to Paraboloeids, Hyperboloeids, Elliptoeids, or to any other Curves of an higher degree, either of a like or different Conſtitution (which the Studious may find out) will (undoubtedly) diſcover particular (nay I may add, univerſal) Rules, for the Conſtruction of divers (nay, all) Æquations of the fifth and ſixth; or of any other pair of higher degrees, which I ſhall leave to thoſe that are Studious; to whom to have animadverted this, may ſuffice, and perhaps worth the while.*

*To any one therefore whiſpering or ſuggeſting, that it is a diminution of the Glory of this Invention, that it was found by chance, I reply; As to the end or event, indeed, I deny it not; but as to the Medium's I uſed, I ſay, that they are ſo well ſuited for the attaining of ſuch an end propoſed, as none could be more ſuitable; and of ſo high a moment in the ſearch of things of this nature, that if it be exactly purſued, things of an higher nature may ſo eaſily be attained, that it may retrieve the diſhonour, and procure more glory, than the chance of the invention can diminiſh.*

Porrò, non ignoro, quòd non deerunt Catones Cenſorii, qui Tractatum hunc nimiæ prolixitatis inſimulaverint, immo vellicaverint, ſuggerintque forſan, quòd aptiùs & peritiùs; ſi ſtrictiùs & ſuccinctiùs egerim; nempè, ſi duas vel tres ſolas Æquationes, totidem Demonſtrationibus muniveram, & Schemata vel Figuras ad quatuor vel ſex, totumque opuſculum ad 4 (plus minus) Schedas perſtrinxeram; quod ſatis abundè totum clarè & perſpicuè reddidiſſet, immo & Lectoribus ſagacioribus (emunctæ licet naris,) magis arriſiſſet. Non equidem inficias eo, palamquè profiteor, me in privatum uſum totum intra unius Schedulæ cancellos coercuiſſe.

At hoc tibi velim (Lector) innoteſcat; hæc nequaquam Veteranorum, qui ex pede *Herculem*, vel ex ungue Leonem probè nôrunt, exarata; at Tyronum & ſciolorum gratiâ, qui neque hunc vel illum, ex hoc, vel ex illo, niſi digito monſtrati, dignoſcere ſapiunt. Eo igitur potiſſimùm inſtituto iſta compoſuimus, ut illis commodo, uſui & adjumento eſſent, qui aut nolunt, aut (cùm craſſæ ſint Minervæ) nequeunt conciſam & Laconicam brevitatem capere.

Horologium tam mirè fabrefactum (quale quondam Reginæ dono datum) ut præanguſto annuli ſignatorii ambitu latitaret, utut Artificis ſolertiam & ingenium mirificè deprædicet, ſuæ tamen magis famæ, quam Reginæ uſui conſuluiſſe, nullus dubito.

Annon

*Moreover, I am not ignorant, that ſome cenſorious* Cato's *may implead and impeach this Treatiſe of too much prolixity; and will perhaps ſuggeſt, I had done more like an Artiſt, if I had been more ſuccinct and conciſe;* viz. *If I had fortified two or three (at the the moſt) Æquations with as many Demcnſtrations; and had confined the Schemes or Figures to* 4 *(more or leſs) and the whole, to as many ſheets, which would have rendred the whole ſufficiently intelligible, yea, and have better gratified the moſt curious Reader. True: I deny it not, and ingeniouſly confeſs, that for my private uſe, I have confined and contracted the whole to one ſheet.*

*But know this (Reader) withal; this was never deſigned by me, for ſuch as are Veteranes, perpolite Artiſts, who know Lions by their paws only, and* Hercules *by his foot; but for young, ſlow Mathematic Scioli's who know neither a Lyon nor* Hercules, *unleſs they are told ſo. I deſignedly ſpun it into a long thred, for the eaſe, uſe and encouragement of ſuch, who either will not take the pains, or have not the brains to apprehend ſuch Laconic conciſeneſs and brevity.*

*A Watch contrived within the narrow ſphere of the ſignet of a Ring (which was once preſented unto a Queen) may commend the skill and ingenuity of the Artificer; yet its uſefulneſs never as yet recommended it to the Worlds uſage.*

Annon *Homeri* Ilias literis majuſculis, folioque edita lectu (ideoque intellectu) faciliora; Annon *Johannis Tredeſcanti* cochlearia argentea vulgaria juſtæ magnitudinis utiliora; quam cùm hæc nuci, illa Ceraſi oſſiculo impacta aut damnata?

Annon *Euri* lenitèr ſpirantis mollis aura genus humanum magis recreat & refocillat; quàm quæ Globo *Ithaco*, vel Philoſophico concluſa?

*Carteſio* ipſi, quamvis etiamnum plurimos illum meritò demirantes detinet; plures tamen Lectores, paucioreſque Commentatores procul dubiò habuiſſe contigerit; ſi non ex profeſſo * (ut fatetur) conciſus ſuccinctuſque fuiſſet.

* *Geom.* l. 3. p. 105.

Sæpiſſimè animadverti, Brachygraphica concionum Adverſaria ſubitò repetendarum utiliſſima fuiſſe; attamen temporis progreſſu, Authores ipſos non eſſe relegendo, nedum intelligendo; viliſſimiſque (quibus ſolis tandem idonea comperta ſunt, (horreſco referens) uſubus addicta.

Alia inſuper gravis cauſa me inpulit paulò fuſiùs dilatandi. Obſervanti enim mihi occurrit, opuſcula quævis plus ſatis conciſa (elegantiſſima licet & concinna) eo ipſo nomine rariùs vænire; quia ſc. pauciſſimorum captui (quod maximè nunc dierum præcavendum) ſunt attemperata. Non enim de libris Mathematicis, quod de *Româ* quondam aſſerendum; ſc. Omnia *Romæ* eſſe vænalia.

*Are not* Homer's *Iliads Written in Capital Letters and enlarged unto a Folio, better legible (and therefore the more intelligible) and* John Tredeſcant's *common Silver Houſe-ſpoons more uſeful, than when the one are crammed into a Nut-ſhel; and the other, into a Cherry-ſtone?*

*Do not* Eurus's *gentle ſoft blaſts refreſh and cool more, then when impriſon'd within the Concave of the* Ithacan *(or Philoſophers) ball?*

Des Cartes *himſelf, had he not been ſo deſignedly conciſe and curt (as himſelf ſays he was*) tho he hath ſtill many Admirers, yet might he have had more Readers, and fewer Commentators.* * Geom. l.3.p.105.

*And I have often obſerv'd, tho Brachygraphical Sermon-notes have proved very uſeful for a ſudden Repetition; yet after ſome years have been as illegible, and unintelligible to the very Pen-men of them themſelves, as uſeful for ſome other employ, (which I tremble to relate) to which themſelves have condemned them.*

*Beſides, another weighty reaſon induced me, to enlarge on this Subject; for I have obſerved, too much conciſeneſs in any Treatiſes of this kind (tho embelliſht with never ſo much Elegancy, Art, and Concinnity) even in that reſpect, renders them the leſs vendible;* viz. *becauſe, not ſuited (which had need now-a-days, be aforehand conſidered of) to the capacity of the vulgar: What of old was attributed to* Rome, *may not now be attributed to Mathematic Books,* viz. *Omnia Romæ ſunt vænalia.*

Hujus equidem ævi moris non eſt, inopes, & ætatis provectioris (etiamſi pulchras) viduas, (qualis eſt *Matheſis*) (quæ tamen eò pulchrior, quò natu grandior) abſque ſummâ dote ſibi deſponſare; quâ quidem (& non contemnendâ) Tractatulum hunc (perſpicuitate licet, quoad potui, adornatum, quò magis vænalis redderetur) cumulari neceſſe priùs erat, quàm ſponſorem adipiſci. Tὸ Quantum dabitis, (ſi cum Typographis res habenda ſit Mathematica) primâ facie auribus injicitur, objiciunturque protinùs oculis quamplurimi exoleti, exeſi, & tineoſi hujus farinæ libri, invenundati, & (utì inquiunt) invendibiles: Si indotata Matheſis, faceſſe, hinc te ocùis proripe.

Aut ſi typis quævis id genus mandentur, ſi minus perſpicua, Luculenta, & intellectu facillia, mercis reſectanæ ad inſtar, in Bibliopolarum officinarum abacis obſcuris, vel angulis ſentis ſitu reponuntur. Quâ de cauſâ, (commonefactus etiam ab illis, quibus morem non gerere grande eſſet piaculum (opuſculum iſtud prodire bilingue neceſſum erat. Quibus rationibus adductus (Tyro Philomathematice) tuo etiam commodo omninò conſulens, tuoque modulo omnia attemperans, Deam hanc cultu prorſus vulgari (ſyrmate licet paulò productiore) veſtitam amplectere; cujus reverà non eſt (Rhetorum inſtar) bombycinis *Tyriis* adornari: Non enim convenit eis, qui in perpetuâ veritate verſantur, ampullas projicere, & ſeſquipedalia verba.

*For it is not the guiſe and humour of this Age to eſpouſe poor, Aged (tho fair) widowed Ladies (as (* Matheſis *is) (who yet, is by ſo much the fairer, by how much the more ancient) without a conſiderable Dowry: Which even this Treatiſe (tho it hath, the acceſſion (as much as I could) of perſpicuity and plainneſs to advantage its ſale) hath had (and all others, of the like Complexion) muſt have) ere an undertaker would, or can be had. To whom if you apply your ſelf, and the concern be Mathematic, you muſt expect your Ears to be ſtormed with a* Qantum dabitis: *And preſently produced to your view an infinite multitude of Exolete, half-moth-eaten Books, of this kind, unſold, and (as they will perſwade you) never will.*

*If no portion farewel; or if one, and it be Printed, it muſt be clad in ſuch a plain Garb or Dreſs, as may render it eaſie to be underſtood; or elſe, like braided wares in Shops, they will be placed in ſome naſty Corners, and lye upon the Book-ſellers hands. For which reaſon I was enforced alſo, and adviſed too by ſome (whoſe deſires are commands) to make it double-tongued. Thus (kind Reader) upon theſe inducements, conſulting they eaſe and profit, and having ſuited all things to thy capacity, embrace this Goddeſs, tho clad in a long robe, yet in a plain dreſs. Her nature inclines not to be arrayed (like Rhetoricians) in* Tyrian *Silks: For it is very unſuitable to thoſe, who are converſant in perpetual Truths, to project for bombaſt Language.*

Age ergò (Lector) ad menſam Lucubratoriam ſedeas; in Tabulâ Centrali ſive Synopſi Æquationem congruam, cujus Conſtructionem velis, quæras, quæ ad Regulam, Demonſtrationem, Figuram illi accommodam diriget: Circinum ſume ſcalamque digitorum (ea enim eſt, quâ κατὰ πάντα uſi ſumus) deſcriptâque ſecundum artem (prout capite hujus Tractatûs te edocuimus) Parabolâ, cujus latus Rectum ſit unitas, vel digitus unus, inſtituantur & applicentur omnia, juxta ibidem præſcripta; omniaque votis tuis ad amuſſim reſponſura, è veſtigio reperies.

Si ſerenâ fronte & ambabus ulnis hæc (qualia qualia) te excepiſſe noverim, ad altiora meditanda ſtimulos adjicies; quæ quidem jamdudum calamus ad umbilicum perduxit; ſin minùs æternùm de Tabulâ.

Spero (Lector) te Errata, quà Authoris, quà Typographi (ſi quæ irrepſerint) neutiquam ægrè Laturum: (quæ tamen (ſi quæ ſint) vel in Tractatu vel in Figuris) facillimè inter ſe conferendo, poſſint corrigi) cùm rem ſuam longo nimis intervallo ab invicem diſſiti, uterque egerit.

*Vale.*

*Sit down therefore at thy Study-Table (Reader) ſeek the Æquation whoſe Conſtruction thou deſigneſt, in the Central Table, or Synopſis, which will guide thee, to its Rule for its Conſtruction, its Demonſtration, Figure, or (at leaſt) to one ſuitable to it. Take thy Compaſs and the Scale of Inches (for that Scale only have I uſed through the whole) and having deſcribed according to Art (which in Chap.* 1. *is taught) a Parabole, let all things be applied accordingly, as we have preſcribed; and thou ſhalt find all things forthwith exactly to anſwer thy expectation.*

*If this (ſuch as it is) be kindly accepted at thy hands; as it will encourage me to meditate on things of a ſublimer nature, ſo will it to publiſh thoſe, which I have already finiſhed; if not, this is too much, which I have already done.*

*I hope (Reader) thou wilt be ſo Candid and Juſt, kindly to pardon, as the Lapſes (if any) of the Author, ſo of the Typographiſt, (which yet, both in the Treatiſe and the Schemes, may eaſily be Corrected, by comparing one with the other) who did his buſineſs at too remote a diſtance from the Author.*

Farewell.

## *Notæ ſeu Symbola, quibus in ſequentibus utor.*

Additionis +

Subductionis —

Cum non proponitur utra Magnitudo ſit Major, vel Minor, & tamen ſubductio facienda eſt nota Differentiæ eſt ∽, id eſt, Minus incertò; ut propoſita AO ∽ AD; Differentia erit AO — AD; vel AD — AO.

Multiplicationis, ×

Æquale, =

Majus ⊐

Minus ⊏

Parallela, SS.

Perpendicularis, ⊥

Quadratum, Q.

Quadratum Radii, Q. Rad.

Quadratum x, $x^2$

Quadratum NO, $NO^2$; &c.

Quadraticè involutum, Ⓠ

Cubus x, $x^3$

Quadrato-quadratum x, $x^4$

Ratio, ſive proportio ∷

Vel ∷. Ergo, g.

# *The Explication of the Notes or Symbols.*

| | |
|---|---|
| Addition | + |
| Subduction | — |
| The Difference between two quantities, when it is not propounded which of them is the Greater or Leſſer, and nevertheleſs the Subduction is to be made, | ∽ |
| Multiplication | × |
| Equality | = |
| The Greater | ⊐ |
| The Leſſer | ⊏ |
| Parallels | ∽∽ |
| A Perpendicular | ⊥ |
| The Square, | Q. |
| The Square of the Radius, | Q. Rad. |
| The Square of x, | $x^2$ |
| The Square of N O, | $NO^2$, *&c.* |
| The Involution of the Square | @ |
| The Cube of x, | $x^3$ |
| The Quadrato-quadrat of x, | $x^4$ |
| Ratio, or Proportion | :: |

# *A Catalogue of the Mathematical Works of the Learned Mr.* Thomas Baker, *Rector of* Biſhop Nympton *in* Devonſhire, *with a Propoſal about printing the ſame; and firſt one, intituled, The* GEOMETRICAL KEY, *or the Gate of Æquations Unlockt.*

A New Diſcovery of the Conſtructions of all Æquations howſoever affected, not exceeding the *4th* degree; *viz.* Of *Linears*, *Quadratics*, *Cubics*, and *Biquadratics*, and the finding of all their Roots as well true as negative, without the uſe of *Meſolabe*, and *Triſection of Angles*; without Reduction, Depreſſion, or any other prævious preparation of Æquations, by a Circle and any (and that but one only) *Parabole*: and this by one only general Rule. than which a more ſimple, more perfect, more general, more eaſy to be underſtood, or more fit for practiſe, cannot be deviſed or wiſhed for.

Fortified with Demonſtrations, Illuſtrated with Figures to each Æquation, which are Exemplified with numeral Æquations, (according to all the varieties of Caſes) adapted to each Figure, for the uſe of young Mathematicians: a work hitherto deſired.

THe Treatiſe conſiſts of about a Quire of Paper, the Diſcourſe whereof (but not the *Algebraick Calculus*) is both in *Latin* and *Engliſh*, the better to promote its forreign vend; and this doth not render it above three Sheets the larger than it would have been in one of theſe Languages. Beſides which, there is belonging to it diverſe Draughts of Schemes to be engraven, and one *Folio* Draught, whereto the literal *Calculus* for ſetling the *Center*, and finding the

*Radius* of the Circle that is to interſect the *Parabole* is expreſſed in readineſs for all Caſes.

How *Des Cartes* and all other famous *Analyſts* came to miſs this general Rule, and himſelf to fall upon it, he acquaints the Reader in the middle of his Diſcourſe; namely, that they conſidered the Axe of a *Parabole* and not its Diameter: and affirms, that if it had been his or their hap to have deſcribed a Circle from any Point in *Poſition* given, paſſing through the *Vertex* of any Diameter in the *Parabole*, and had taken into conſideration a certain propriety (than which none could ſo have ſuited the deſign) belonging to the Diameter of any *Parabole*, they could not but with greateſt eaſe, have made a full diſcovery of the Univerſal Rule.

The excellency of which Invention appears, in that it diſcovers not only the Geometrical Conſtruction of all Æquations as aboveſaid, by one only ſtanding meaſure and Scheme, and that by one only general rule, with the exact number of Roots as well true as negative, but alſo by giving a fair proſpect towards their *Arithmetical Calculus*, or numerous Reſolution, by making a Diſcovery of their two firſt figures or numbers; namely, by applying the Compaſſes to the ſeveral Roots Geometrically found in the Scheme, and comparing them with that very Scale from which the ſaid Scheme (ſuited to the propoſed Æquation) was drawn, the reſidue of which roots, (though not preciſely, yet ſufficient nearly approximating to the true) may diverſe ways in Decimals be found out, which the Author (as he intimated in a Letter of *April* 1682, to Mr. *Collins*) is willing to impart; but as to the Invention of theſe reſiduals (to be entail'd to the two firſt figures or Numbers of this Author thus findable.) The Learned Mr. *Iſaac Newton* Profeſſor of Mathematicks in *Cambridge* (in a Letter long ſince communicated to the aforeſaid Mr. *Collins*) hath as to this purpoſe performed the ſame (as is conceived) by a different method, namely, that when a root of any Æquation is by any Method (which by the Authors aforeſaid it may be) ſo near found, that it doth not differ above a tenth part of its ſelf from the true root ſought, the reſidue of the root inquired will be eaſily calculated by aid of ſome terms or Fractional parts of an infinite Series or rank of continual Proportionals, derived from the difference between the Reſolvend of the known part of the Root, and that whoſe Root is ſought. By which means by raiſing Reſolvends out of any aſſumed Roots with an eaſy approach, without raiſing the

the reſpective powers of the ſaid Roots, we are delivered from the moſt toilſom Drudgery of Mathematical Calculations, by finding the Roots of Æquations in numbers, by *Vietas* general method; a thing utterly unknown to the Ancients. However this is not ſaid to diſparage that Method which *Vieta* ſo greatly eſteemed, that when he had obtained it, he gave *Algebra* this high *Encomium*, that it did *Nullum non Problema ſolvere*, in his numerical Method Mr. *Oughtred* and *Harriot* have taken commendable pains. But now laſt of all, to perform it in Species as Mr. *Iſaac Newton* hath done, ſeems a new Invention never to be ſufficiently praiſed; for out of a literal Æquation of five Dimenſions, ſuppoſing all the terms extant and affirmed, he hath given a Series for the Root in Species, and ſuch a one as ſhall ſerve for finding the Roots of all Equations of 3, 4 or 5 Dimenſions, by only altering the ſigns according as as the Æquation is affected, and expunging ſuch parts as relate to Deficient terms in an incompleat Æquation propoſed.

Now that this admirable Doctrine may come to light, and the Learned Author (who hath many other Treatiſes worthy publick view) may be incited to impart the ſame, encouragements for the promoting thereof (ſeeing Undertakers are not to be had without) muſt be propounded.

It is therefore humbly offered, that the Royal Society by their Treaſurer &c. enter into Bond to ſuch Bookſeller as ſhall be the Undertaker, to take off 60 of theſe Books in Quires at 1½ *d.* each Sheet, and as much each Plate, as ſoon as printed.

*The Treatiſe it ſelf and the Propoſal, is approved and agreed to by the Council of the Royal Society.*

And in regard ſuch a Subſcription is not ſufficient to incite an Undertaker, that the reſpective Members endeavour by virtue of this Narrative, to obtain as many more Subſcribers as they can procure amongſt others that are not of the Society, each of them to advance half a Crown in hand, in part of the former price: upon which encouragements, *Robert Clavel* Bookſeller at the *Peacock* in St. *Pauls-Church-Yard*, is ready to give reciprocal ſecurity for the performance according to this Propoſal, hoping the like encouragement will be given towards printing the reſt of the Treatiſe of this moſt Learned Author, whereof take the enſuing Catalogue.

1. The *Hyperbolic Key*, or the Geometrical Conſtruction of *Cubic* and *Biquadratic* Æquations; by a Circle and an equilateral *Hyperbole*; to wit the one moiety (exactly) *viz.* of eight *Cubicks* and four and twenty *Biquadratics*; (as is expreſſed in the former treatiſe) namely,

1. Of all thoſe Cubicks; wherein

1. The quantity ($q$) is wanting; and $p$ and $r$ affected with divers Signes.

2. The quantity ($p$) is wanting, and in the Æquation be had $-q$.

3. None of the terms are wanting, and in the Æquation be had $+q$,

2. Of all thoſe *Biquadratic* Æquations in which are had $+s$. by the demiſſion of Perpendiculars, from the points of Interſections of the Circle and Hyperbole to the Aſſymptote; part of the other moiety, by the demiſſion of perpendiculars from the aforeſaid interſections to the axe; *&c.* with Schemes adapted to each Æquation, *&c.* with a Synopſis of the whole, wherein the Literal Rule for fixing the Center, and finding the Radius of the Circle, that is to interſect the Æquilateral Hyperbole (the eaſieſt way of the Conſtruction of which is likewiſe therein diſcovered) is expreſſed in readineſs for all Caſes.

This method of Conſtruction (were it not, that for every Caſe a new Hyperbole muſt be deſcribed) would not be inferior to that by a Parabole, but rather exceed it; in that the Circle doth not arcuate the ſame way which the figure doth, but croſſes it the other way; by which means a clearer diſcovery (as to the one moiety) of the points of interſection of the Circle with the Hyperbole is obtained, than what can poſſibly be had in any other Coni-ſection.

3. The Geometrical Conſtruction of ſome Æquations which aſcend to the *5th* and *6th* power, with the finding of their Roots, by a Curve of the third degree; namely by the firſt kind (for there are two kinds) of a Paraboleid and a Circle, illuſtrated with Schemes to each Æquation, and numeral Æquations adapted to them; with a Synopſis to the ſame for placing the Center and finding the Radius, and a general little Table, for the deſcribing of both kinds of Paraboleids.

8. The

4. The Conſtruction of all Cubick Æquations howſoever affected by a Circle only, Geometrically upon Suppoſition, that one *Poſtulatum* be granted to be Geometrical (which indeed is but a Supplement to Geometrical defects;) *viz.* That from any point aſſigned in the circumference of a Circle (that is normaly quadriſected) may be drawn a right Line, ſo that the parts intercepted both ways by the Circumference and Diameter, may be equal to the Radius of the Circle: this way (though not ſo purely Geometrical as the reſt) is not to be deſpiſed, ſith that theſe Lines may ſufficient preciſely be ſo drawn.

5. The Geometrical Conſtruction of all Cubic Æquations according to the Rule found out by ***Franciſcus a Shooten***, mentioned in his Commentaries on *Des Cartes*, *Lib.* 3. *Pag.* 328, 329, 330, illuſtrated with Figures and Numeral Æquations adapted to each Figure, *&c.*

6. The Reſolution of all Cubick Æquations in numbers, not only by a general Rule by the aſſiſtance of any Figure reſolving them Geometrically *&c.* but by a more particular method far exceeding any extant in Numbers or by help of Tables; illuſtrated with Figures and Examples in numbers, ſuited to each figure and Æquation.

7. Mixt or Compound *Trigonometry*; in many inſtances far exceeding the ſimple, as finding two *Quæſitas* (as it were) by one operation, or by two at moſt; with a Synopſis of the admirable harmony between Plain and Sphærical Triangles: for inſtance,

In plain Rectangular Triangles, the ▭*e* under half the ſum of the *Hypotenuſe* and one ſide: and half their difference, is equal to the Square of $\frac{1}{2}$ the other ſide, ſo in Sphærical Rectangular Triangles. The ▭ under the Tangents of half the ſum and half the difference of the *Hypotenuſe* and one ſide, is equal to the ſquare of the Tangent of half the other ſide.

Again in Obliquangular Plain Triangles.

$$\begin{matrix}cs,\\ s,\end{matrix}\tfrac{1}{2}Z\angle\angle.\begin{matrix}cs,\\ s,\end{matrix}\tfrac{1}{2}X\angle\angle:\tfrac{1}{2}\,\textit{Baſis}\left\{\begin{matrix}\tfrac{1}{2},\ Z\ \textit{crurum.}\\ \tfrac{1}{2},\ X\ \textit{crurum.}\end{matrix}\right.$$

Thus likewiſe in Sphærical Obliquangular Triangles.

$$\begin{matrix}cs,\\ s,\end{matrix}\tfrac{1}{2}Z\angle\angle\ \textit{ad Baſem.}\ \begin{matrix}cs,\\ s,\end{matrix}\tfrac{1}{2}X\angle\angle\ \textit{ad Baſem}::\tfrac{1}{2}\textit{Baſis.}\ \begin{matrix}t,\tfrac{1}{2}Z\ \textit{Crurum.}\\ t,\tfrac{1}{2}X\ \textit{Crurum.}\end{matrix}$$

Again in plain Triangles.

$\frac{1}{2}$ *Baſis*. $\frac{1}{2}$ Z *crurum* : $\frac{1}{2}$ X *crurum*. $\frac{1}{2}$ X of the Segments of the Baſe.

In

In Sphærical alſo.

$t$, $\frac{1}{2}$ *Baſis*. $t$, $\frac{1}{2}$ Z *Crurum*, $t$, $\frac{1}{2}$ X *Crurum*, $t$, $\frac{1}{2}$ X *Segmentorum baſis*, with infinite other alike harmonious.

To which is added the Geometrical Conſtruction of all Sphærical Triangles, by a moſt plain and eaſy uniform way, which is indeed of ſingular uſe.

Alſo a diſcovery of the Method by which *Vieta* (*Lib.* 8 *p.* 431 *&c.*) found out his Canonical Analogy of Sphærical Triangles, which he hath left undemonſtrated, but in this Treatiſe is diſcovered.

8. *Cardanus Promotus*, or *Cardans* Rules, or *Vieta*'s *duplicata Hypoſtaſis*, *in infinitum*, carried on with a Table for the compoſition *in infinitum* of ſuch Æquations. By which means ſuch Canons are generally compoſed for Æquations of two Nomes (and in many Caſes for more) equal to a Reſolvend given.

9. A Continuation of *Vietas Apollonius Gallus*, *Appendicula*; 1. And his Problemes otherwiſe demonſtrated, wherein the Baſe and Angle oppoſite to the Baſe are always two of the *Data's*, and the other, either the perpendicular or the difference of the Segments of the Baſe, or the difference of the ſquares of the ſides, or the ſum of the Squares of the ſides, or the Sum of the ſides, or the difference of the ſides, or their ▭, whoſe Geometrical effection was altogether unknown to the antient Analyſts, *Vieta ibid.*

10 *Vera & Genuina Symmetrica Climactiſmus*, by which means all *Aſymmetries* in *Algebraics* may be wiped off, and an Æquation found in any one of the unknown magnitudes propoſed, which ſhall never aſcend higher, than the double of the higheſt power firſt propoſed, by which alſo that moſt perplexing entangling inextricable way of *Vieta* may be laid aſide as uſeleſs, and inefficacious, though hitherto it hath been the only remedy. *Adverſus vitium Aſymmetriæ*, this treatiſe was many years ſince compoſed and laid aſide; but the Author lately meeting with the *opera Poſthuma* of *Monſieur de Fermat*, treating on the ſame Subject in his (*Varia opera Mathematica pag.* 58, 59, &c.) and finding that though he rightly hits the mark; yet that he goes not in a ſtreight Line to it, hath reviſed his old Copy, and compared it with *Fermats*; and which of the two, hath gone the Simpler way, the Author leaves to the judgment of others, being loath in the leaſt to take up the Gantlets againſt ſuch a famous man whom the world admires.

11. *Apollonius magnus Gregorianus*, or a Treatiſe of four Geometrical proportionals, wherein divers ways are found to ſolve that Grand problem, which hath ſo amuſed the world, (*viz.*)

Having the ſum of all the Squares, and the ſum of all the Cubes, of four Geometrical Proportionals to find the Proportionals themſelves; with queſtions of the like nature, by low Æquations, without aid of Analytick Store.

11. Of Triangular Sections by a different method than what *Anderſon* has performed it by, in *Vieta*, with a diſcovery of the falſhood (as to angular Sections) of Mr. *Oughtreds* 1*ſt.* Rule, in his *Clavis Mathem. c.* 16. *p.* 14.

12. The finding out of Æquations which may infinitely aſcend, whoſe Roots are either in Arithmetical or Geometrical proportion which may be found out in numbers by extracting the Square and Cube Root, with ſurd Canons adapted to that purpoſe, and to many other Æquations.

13. A Miſcellany of the ſolution of many knotty Problemes, namely, ſuch as have been found difficult to be brought to any Equation, or elſe would mount very high *in Ordine Scalæ*, with a new method of Depreſſing them, by aid of one or two Æquations, raiſed by altering the Data, and putting two unknown quantities, by which means the adjutant Æquations as having the ſame common root, depreſs the Æquation that otherwiſe ſhould be reſolved

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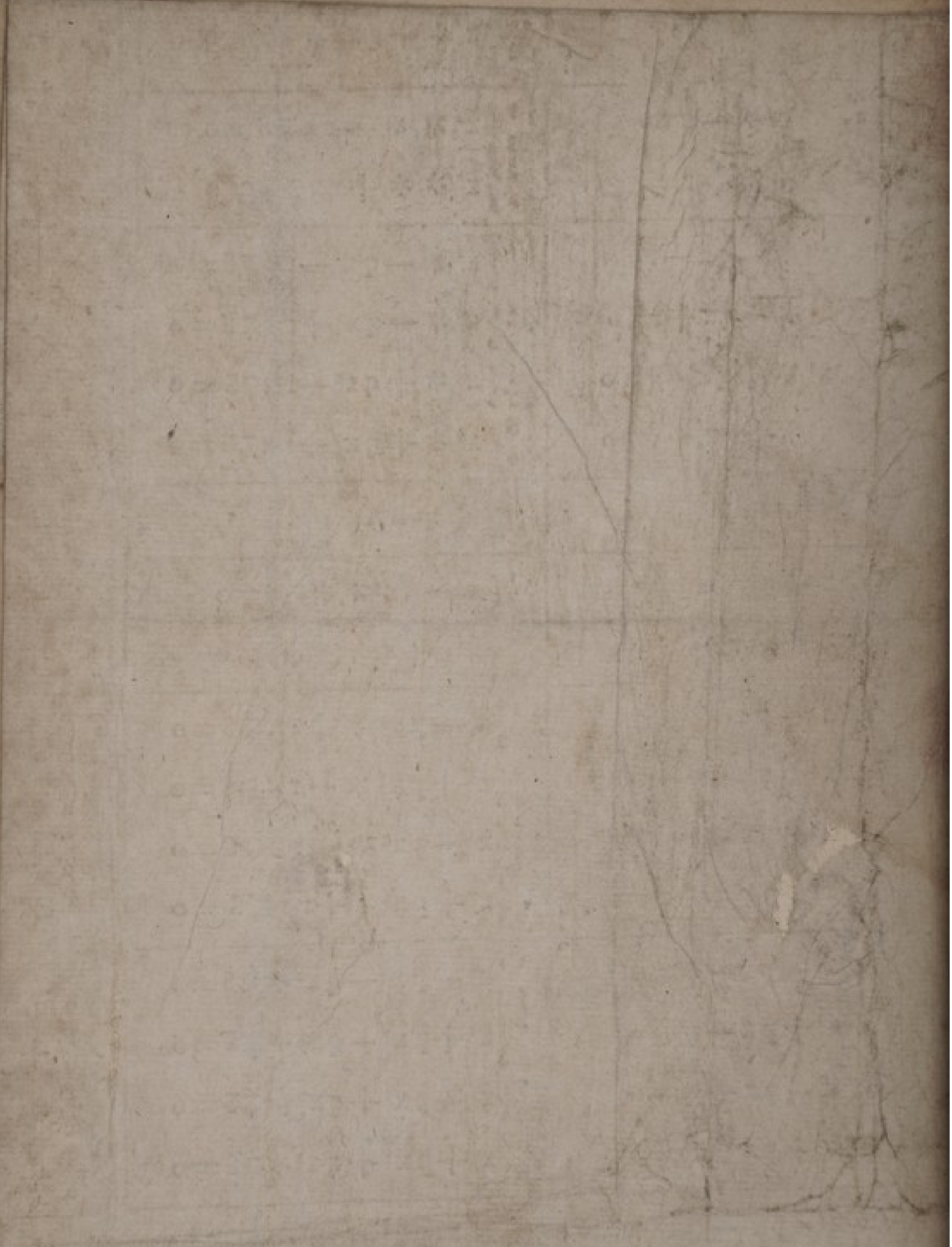
## *ADVERTISEMENT.*

THe Author herein ſuppoſeth the Reader to underſtand the uſe of common Symbols deſcribed in his firſt Book; *viz.* cs, for *Coſine*, s, for *Sine*, Z for *Sum*, X for difference, ∠ for *Angle*, ∠∠ for *Angles*. And the Reader muſt be informed, that as the whole ſeems novel, ſo a brief Demonſtration of thoſe Proportions in *Sec.* 7. to hold in Sphæricals is moſt deſirable; and if others be not wanting in their encouragements, it's not to be feared the Royal Society will be ſlow in theirs.

# SYNOPSIS.

| Claſſ. | *Æquationum.* | *Æquationum.* | *Regulæ Centralis.* |
|---|---|---|---|
| 1 | | 1. $x^4 \ast \ast \ast - S = 0$ | $\frac{L}{2} = b = AD.\quad 0 = d = DH$ |
| 2 | 1. $x^2 \ast - q = 0$<br>2. $x^2 \ast + q = 0$ impoſſibilis. | $\begin{Bmatrix}1\\3\end{Bmatrix} x^4 \ast - qx^2 \ast \mp S = 0$<br>$\begin{Bmatrix}2\\4\end{Bmatrix} x^4 \ast + qx^2 \ast \mp S = 0$ | $\left.\begin{array}{l}\frac{L}{2} + \frac{q}{2L}\\ \frac{L}{2} + \frac{q}{2L}\end{array}\right\} = b = AD.\ 0 = d = DH$ |
| 3 | 1. $x^3 \ast \ast - r = 0$<br>2. $x^3 \ast \ast + r = 0$ | $\begin{Bmatrix}1\\3\end{Bmatrix} x^4 \ast \ast - rx \mp S = 0$<br>$\begin{Bmatrix}2\\4\end{Bmatrix} x^4 \ast \ast + rx \mp S = 0$ | $\frac{L}{2} = b = AD$<br>$\frac{r}{2L^2} = d = DH$ |
| | 1. $x^3 \ast - qx - r = 0$<br>2. $x^3 \ast - qx + r = 0$<br>3. $x^3 \ast + qx - r = 0$<br>4. $x^3 \ast + qx + r = 0$ | $\begin{Bmatrix}1\\3\end{Bmatrix} x^4 \ast - qx^2 - rx \mp S = 0$<br>$\begin{Bmatrix}2\\4\end{Bmatrix} x^4 \ast - qx^2 + rx \mp S = 0$<br>$\begin{Bmatrix}5\\7\end{Bmatrix} x^4 \ast + qx^2 - rx \mp S = 0$<br>$\begin{Bmatrix}6\\8\end{Bmatrix} x^4 \ast + qx^2 + rx \mp S = 0$ | $\frac{L}{2} + \frac{q}{2L} = b = AD$<br>$\frac{r}{2L^2} = d = DH$<br>$\frac{L}{2} \backsim \frac{q}{2L} = b = AD$<br>$\frac{r}{2L^2} = d = DH$ |
| | $\frac{p}{2} = BA$ | $\frac{p}{2} = BA$ | $\frac{p}{4} = aE$, in Claſſ. 5, 6, 7, 8. |
| 5 | 1. $x - p = 0$<br>2. $x + p = 0$ | $\begin{Bmatrix}1\\3\end{Bmatrix} x^4 - px^3 \ast \ast \mp S = 0$<br>$\begin{Bmatrix}2\\4\end{Bmatrix} x^4 + px^3 \ast \ast \mp S = 0$ | $\frac{L}{2} + \frac{p^2}{8L} = b = AD$<br>$\frac{P}{4} + \frac{p^3}{16L^2} = d = DH$ |
| 6 | 1. $x^3 - px^2 \ast + r = 0$<br>2. $x^3 + px^2 \ast - r = 0$<br>3. $x^3 - px^2 \ast - r = 0$<br>4. $x^3 + px^2 \ast + r = 0$ | $\begin{Bmatrix}1\\3\end{Bmatrix} x^4 - px^3 \ast + rx \mp S = 0$<br>$\begin{Bmatrix}2\\4\end{Bmatrix} x^4 + px^3 \ast - rx \mp S = 0$<br>$\begin{Bmatrix}5\\7\end{Bmatrix} x^4 - px^3 \ast - rx \mp S = 0$<br>$\begin{Bmatrix}6\\8\end{Bmatrix} x^4 + px^3 \ast + rx \mp S = 0$ | $\frac{L}{2} + \frac{p^2}{8L} = b = AD$<br>$\frac{P}{4} + \frac{p^3}{16L^2} \backsim \frac{r}{2L^2} = d = DH$<br>$\frac{L}{2} + \frac{p^2}{8L} = b = AD$<br>$\frac{P}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = d = DH$ |
| 7 | 1. $x^2 - px - q = 0$<br>2. $x^2 + px - q = 0$<br>3. $x^2 - px + q = 0$<br>4. $x^2 + px + q = 0$ | $\begin{Bmatrix}1\\3\end{Bmatrix} x^4 - px^3 - qx^2 \ast \mp S = 0$<br>$\begin{Bmatrix}2\\4\end{Bmatrix} x^4 + px^3 - qx^2 \ast \mp S = 0$<br>$\begin{Bmatrix}5\\7\end{Bmatrix} x^4 - px^3 + qx^2 \ast \mp S = 0$<br>$\begin{Bmatrix}6\\8\end{Bmatrix} x^4 + px^3 + qx^2 \ast \mp S = 0$ | $\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD$<br>$\frac{P}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} = d = DH$<br>$\frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD$<br>$\frac{P}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} = d = DH$ |
| 8 | 1. $x^3 - px^2 - qx - r = 0$<br>2. $x^3 + px^2 - qx + r = 0$<br>3. $x^3 - px^2 - qx + r = 0$<br>4. $x^3 + px^2 - qx - r = 0$<br>5. $x^3 - px^2 + qx + r = 0$<br>6. $x^3 + px^2 + qx - r = 0$<br>7. $x^3 - px^2 + qx - r = 0$<br>8. $x^3 + px^2 + qx + r = 0$ | $\begin{Bmatrix}1\\3\end{Bmatrix} x^4 - px^3 - qx^2 - rx \mp S = 0$<br>$\begin{Bmatrix}2\\4\end{Bmatrix} x^4 + px^3 - qx^2 + rx \mp S = 0$<br>$\begin{Bmatrix}5\\7\end{Bmatrix} x^4 - px^3 - qx^2 + rx \mp S = 0$<br>$\begin{Bmatrix}6\\8\end{Bmatrix} x^4 + px^3 - qx^2 - rx \mp S = 0$<br>$\begin{Bmatrix}9\\11\end{Bmatrix} x^4 - px^3 + qx^2 + rx \mp S = 0$<br>$\begin{Bmatrix}10\\12\end{Bmatrix} x^4 + px^3 + qx^2 - rx \mp S = 0$<br>$\begin{Bmatrix}13\\15\end{Bmatrix} x^4 - px^3 + qx^2 - rx \mp S = 0$<br>$\begin{Bmatrix}14\\16\end{Bmatrix} x^4 + px^3 + qx^2 + rx \mp S = 0$ | $\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD$<br>$\frac{P}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} + \frac{r}{2L^2} = d = DH$<br>$\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD$<br>$\frac{P}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} - \frac{r}{2L^2} = d = DH$<br>$\frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD$<br>$\frac{P}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} \backsim \frac{r}{2L^2} = d = DH$<br>$\frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD$<br>$\frac{P}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} + \frac{r}{2L^2} = d = DH$ |



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# *Clavis Geometrica Catholica:*

# THE

# GEOMETRICAL KEY,

# OR THE

# Gate of Equations unlock'd.

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# *Clavis Geometrica Catholica.*

Prænoſcenda ſunt quædam cognitu quidem admodùm neceſſaria; nempè

Natura
Proprietates } Parabolæ.
Conſtructio

## CAP. I.

*Methodus Synthetica poſtulat, ut à Parabolâ, tanquam univerſali hujus Tractatûs Subjecto, ad partes, hoc eſt, ad Rectas quæ in Parabolæ partibus expenduntur, tanquàm principia & cauſæ; tum demùm ad Parabolæ affectiones ſive proprietates; (ut tandem ad ejus Conſtructionem) fiat proceſſus.*

*Fig.* 1. Cuilibet, vel in limine Matheſin ſalutanti notum eſt, Baſem ($bNc$) Coni ($abc$) eſſe circularem, punctumque ($a$) vocari ejus Verticem; & Rectam ($aZ$), à Vertice ($a$) ad centrum Baſeos Circularis ($Z$) perductam), appellari ejus Axem. Quibus agnitis,

*Fig.* 1. 1. Si Conus ($bac$) plano ſecetur per Axem ($aZ$,) reſultabit Triangulum ($abc$;) in cujus Plano ducatur recta $AO$ (cuilibet laterum, puta) lateri ($ac$) parallela. In plano Baſeos Circularis ($bNc$,) erigatur ad Diame-

# THE
# GEOMETRICAL KEY.

Some Things truly very neceſſary to be known, are to be premiſed; *viz.*

The Nature
The Properties of a Parabole.
The Conſtruction

## CHAP. I.

*Synthetical Method requires, that we proceed from a Parabole, as the univerſal Subject of this Treatiſe, unto the Parts, that is, unto thoſe Right Lines, which are conſidered in the parts of a Parabole, as the principles and cauſes; then at length, unto the affections and properties of a Parabole; that ſo way may be made for its conſtruction.*

IT's well known to every mean Mathematician, that the Baſe ( $bNc$ ) of the Cone ( $abc$ ) is circular; and the Point ( $a$ ) is called its Vertex, and the Right Line ( $aZ$ ) (which is drawn from the Vertex ( $a$ ), to the center of the Circular Baſe ( $Z$, ) is termed its Axe. Which being known, *Fig.* 1.

1. If the Cone ( $bac$ ) be cut with a Plane through its Axe ( $aZ$, ) there will reſult the Triangle ( $abc$; ) in whoſe Plane, draw $AO$ parallel ( to either of the ſides, ſuppoſe) to the Side ( $ac$. ) In the plane of the Circular *Fig.* 1.

Diametrum (*bc*,) perpendicularis *ON*. Sectus autem ſit idem Conus altero plano ſecundo, ſecante Baſim Circularem (*bNc*) ſecundum duas Rectas *ON*, *OA*. Sectio curva (*ANOR*) reſultans vocatur *Parabola.*

1\. 2. Recta *AO*, (quæ quidem omnes Lineas quæ in Parabolâ ducuntur ſibi invicem Parallelas (ut *NR*, *nR*, *NR*,) bifariam dividit (in *O*, *o*, *o*,) dicitur *Parabolæ Diameter*. Fig. 1.

2\. Et ſi Recta *AO* (omnes prædictas Parallelas bifariam dividens) ad Angulos Rectos ſecet, vocatur *Axis* (ſive *Diameter originaria:*) Sin ad Obliquos, vulgò (abſque ullo alio additamento) dicitur *Diameter*.

3\. Unaquæque Rectarum ſibi invicem Parallelarum, abs Axe vel Diametro bifariam diviſarum (nempe, *NR*, *nR*, *NR*,) vocatur uſitatiùs *Ordinata*; hoc eſt, Recta ad Axem vel Diametrum ordinatim applicata.

4\. Portio verò Axis vel Diametri (ut *AO*) inter ordinatam (*noR*), & Verticem Parabolæ (*A*) intercepta, vocatur *Abſciſſa* Axis vel Diametri.

3\. Parabolæ affectiones ſive proprietates (quæ ad rem noſtram ſpectant) ad hunc modum poſſint expiſcari. Fig. 1.

Supponamus eundem Conum (*abc*) ſectum eſſe Plano tertio (*DnH*,) Baſi Circulari (*bNc*) parallelo; liquebit, fore

1 $oH = Oc$; (ob $ac \parallel AO$; & $bc \parallel DH$.)

*Ob Circ.* 2 Et $bO \times Oc = ON^2$; & $Do \times oH = no^2$.

$$\begin{array}{ll} \triangle\ Sim. & 3 \\ 1. & 4 \end{array} \quad \text{Et} \left\{ \begin{array}{l} AO \,.\, bO :: Ao \,.\, Do \\ \qquad\ Oc \ = \ oH \end{array} \right\} \text{multipl.}$$

$$\begin{array}{ll} 3 \times 4 & 5 \\ 5,\ 2 & 6 \end{array} \quad \left. \begin{array}{l} AO \,.\, bO \times Oc :: Ao \,.\, Do \times oH \\ \text{h.e. } AO \,.\, ON^2 :: Ao \,.\, no^2. \end{array} \right\}$$

*Inverſè.* 7 $AO \,.\, Ao :: ON^2 \,.\, no^2$; quæ eſt propr. generalis.

7\. 8 $\left\{ \frac{NO^2}{AO} = \frac{no^2}{Ao} \right.$; (quarum quævis ſit) $= L$, quæ dicitor Latus Rectum.

8\. 9 g°, $L \times AO = NO^2$; & $L \times Ao = no^2$.

9\. 10 h. e. $\left\{ \begin{array}{l} L \,.\, NO :: NO \,.\, AO \\ L \,.\, no :: no \,.\, Ao \end{array} \right\}$ Atque hæc eſt proprietas prima ſpecialis.

Fig. 1.

Baſe *(bNc,)* erect *ON* perpendicular to the Diameter *(bc;)* and let the ſame Cone be cut with another ſecond Plane, cutting the Circular Baſe *(bNc)* according to the two Right Lines *ON*, *OA*. The crooked Section *(ANOR)* reſulting, is called a *Parabole*.

1. 2. The Right Line *AO*, (that which biſects all parallel Fig. 1.
Lines in a Parabole, (as *NR*, *nR*, *NR*,) in the points (*O*,*o*,*o*,)
2. is called, *The Diameter of the Parabole*. And if the Right
Line *AO* (biſecting all the aforenamed Parallels) cuts them, at Right Angles, then is it called, *The Axe*, (or, *The originary Diameter:*) But if at Oblique, then (without any other Additament) it is uſually called, *The Diameter*.

3. Each of thoſe abovenamed Parallels, biſected (as aboveſaid) by the Axe or Diameter, is called uſually, *An Ordinate*; *i. e.* a Right Line ordinately applied to the Axe or Diameter.

4. But that portion of the Axe or Diameter (as *Ao*), intercepted between the Ordinate *(nok)*, and the Vertex of the Parabole *(A,)* is called, *The Abſciſt*, of the Axe or Diameter.

3. Thoſe affections or properties of a Parabole, which concern our matter in hand, may thus be found out. Fig. 1.

Suppoſe we the ſame Cone *(abc)* to be cut with a third Plane *(DnH,)* parallel to the Circular Baſe *(bNc:)* It will be evident,

1 $oH = Oc$ (for $ac \approx AO$; and $bc \approx DH$.)

Circle. 2 And $bO \times Oc = NO^2$; and $Do \times oH = no^2$.

3, 4 And $\left\{\begin{array}{l} AO \,.\, bO :: Ao \,.\, Do \\ \qquad Oc \;=\; oH \end{array}\right\}$ multiply.

3×4 5 $AO \,.\, bO \times Oc :: Ao \,.\, Do \times oH$.

5, 2 6 h. e. $AO \,.\, NO^2 :: Ao \,.\, no^2$.

Inversè. 7 $AO \,.\, Ao :: NO^2 \,.\, no^2$; which is the general property. Fig. 1.

7. 8 $\left\{ \frac{NO^2}{AO} = \frac{no^2}{Ao} \right.$; (let each of them be) $= L$, which let be called, *The Right Side*.

8. 9 g°, $L \times AO = NO^2$; and $L \times Ao = no^2$.

9. 10 i. e. $\left\{\begin{array}{l} L \,.\, NO :: NO \,.\, AO \\ L \,.\, no :: no \,.\, Ao \end{array}\right\}$ And this is its firſt ſpecial property.

9 11 $\frac{Ny^2}{L} = ay$

9 12 $\frac{Ae^2}{L} = ae$ Fig.2.

11 ∽ 12 13 $$\frac{Ny^2 \backsim Ae^2}{L} = \begin{Bmatrix} \frac{\overline{Ny + Ae} \times \overline{Ny - Ae}}{L} \\ \frac{NO \times OR}{L} \end{Bmatrix} = \begin{Bmatrix} ay \backsim ae \\ NO \end{Bmatrix}$$

14 14 g°, u; $\begin{Bmatrix} L \,.\, NO :: OR \,.\, AO \\ L \,.\, no :: oR \,.\, Ao. \end{Bmatrix}$ Quæ eſt proprietas ſecunda ſpecialis.

Proprietas Parabolæ generalis (§ 7.) verbis enunciatur ſic; nempè, $AO \,.\, Ao :: No^2 \,.\, no^2$.

Quadrata Rectarum (*NO*, *no*,) ad Axem (vel Diametrum) ordinatim applicatarum, diſtantiis ſuis à vertice (*A*;) vel, (quod perinde eſt) Quadrata Ordinatarum (*NO*, *no*,) Abſciſſis ſuis (*AO*, *Ao*,) ſunt directè proportionalia.

1. Parabolæ proprietas prima ſpecialis verbis enunciatur ſic. (§ 10.)

ſc. $\begin{Bmatrix} L \,.\, NO :: NO \,.\, AO \\ L \,.\, no :: no \,.\, Ao. \end{Bmatrix}$ Fig. 1.

Recta (*NO*) ad Axem ordinatim applicata, eſt media proportionalis, inter Abſciſſam ejus (*AO*) & Latus Rectum (*L*.)

Vel, Ut Abſciſſa (*AO*,) eſt ad ejus Ordinatam, cujus eſt Abſciſſa (*NO*;) ita ipſa Ordinata (*NO*,) eſt ad Latus Rectum (*L*.)

2. Secunda verò proprietas ſpecialis (§ 14.) ſic:

Si ad Axem Parabolæ (*ay*,) ordinatim ſint applicatæ duæ Rectæ (*NR*, *BA*;) Dico,

Ut Latus Rectum (*L*,) eſt ad aggregatum ipſarum Rectarum; ita earundem differentia, ad differentiam ipſarum Abſciſſarum; nempe, $L \,.\, \begin{Bmatrix} Ny + EA \\ NO \end{Bmatrix} :: \begin{Bmatrix} Ny - EA \\ OR \end{Bmatrix} \,.\, \begin{Bmatrix} ay - aE \\ AO. \end{Bmatrix}$ Fig.2.

Vel ſic: Si ad Axem Parabolæ (*ay*) ordinatim applicata recta (*NR*,) ſecet aliam Diametrum (ut *Ao*, productam, ſi opus fuerit) in duo Segmenta (*NO*, *OR*;) Dico,

Rectan-

9 11 $\frac{Ny^2}{L} = ay$

9 12 $\frac{Ae^2}{L} = ae$

11 ∽ 12 13 $\left\{\begin{matrix} ay \backsim ae \\ u, AO \end{matrix}\right\} = \frac{Ny^2 \backsim Ae^2}{L} = \left\{\begin{matrix} \frac{\overline{Ny+Ae} \times \overline{Ny-Ae}}{L} \\ u, \frac{NO \times OR}{L}. \end{matrix}\right\}$

Fig. 2.

14 14 $\left\{\begin{matrix} L \,.\, NO :: OR \,.\, AO \\ L \,.\, no :: oR \,.\, Ao. \end{matrix}\right\}$ Which is its ſecond ſpecial property.

The general property of a Parabole (§ 7) is expreſſed in words, thus:

The Squares of the Right Lines (*No*, *no*,) ordinately applied to the Axe (or Diameter), to their diſtances from the Vertex (*A*): Or (which is all one), The Squares of the Ordinates (*No*, *no*,) to their Abſciſts (*AO*, *Ao*,) are directly proportional.

1. The firſt ſpecial property of a Parabole; is expreſſed in words, thus: (§ 10.)

A Right Line (as *NO*) ordinately applied to the Axe, is a Mean Proportional, between its Abſciſt (*AO*,) and the *Latus Rectum* (*L*.) Fig. 1.

Or, *As* the Abſciſt (*AO*,) *Is* to the Ordinate, of which it is the Abſciſt (*NO*:) *So* is the ſame Ordinate (*NO*,) *To* the *Latus Rectum* (*L*.) *viz.* $\left\{\begin{matrix} L \,.\, NO :: NO \,.\, AO \\ L \,.\, no :: no \,.\, Ao \end{matrix}\right\}$

2. But the ſecond ſpecial property (§ 14.) thus:

If to the Axe of a Parabole (*ay*,) be ordinately applied two Right Lines (*Nq*, *AE*;) I ſay,

That the *Latus Rectum*, is to the ſum of thoſe two Right Lines; as their difference, is to the difference of their Abſciſts; *viz.* $L \,.\, \left\{\begin{matrix} Ny+EA \\ NO \end{matrix}\right\} :: \left\{\begin{matrix} Ny-EA \\ OR \end{matrix}\right\} . \left\{\begin{matrix} ay-aE \\ Ao \end{matrix}\right\}$ Fig. 2.

Or thus: If any Ordinate (as *NR*) applied to the Axe of a Parabole, cut any other Diameter (as *Ao*, produced if need be) into two Segments, (as *NO*, *OR*;) I ſay,

The

Rectangulum ſub latere Recto, & Diametro interceptâ, eſſe æquale Rectangulo ſub Segmentis. Vel,

Ut Latus Rectum, ad unum Segmentorum; ſic alterum, ad Diametrum interceptam.

4. Atque hinc (per modum Conſectarii) eluceſcet modus Parabolam in plano conſtruendi, ex datis ejus Axe A O, & latere ejus Recto L.

1. Ad Rectam A O, applicentur ad Angulos Rectos infinitæ Parallelæ (ut hN N h, &c.)

*Fig.* 3.

Menſurata A I = L; & in Axe A O, dato quocunque puncto (verbi gratiá), O ſumpto, reperiatur inter duas Rectas (A I, A O,) media Proportionalis A L, applicanda Axi in Puncto O, (h. e. O N, vel B N =) A L; & ſic de aliis infinitis hujuſmodi Rectis modo prædicto reperiendis & conſtituendis, ex diverſis punctis Axis A O: Linea curva incedens per extrema dictarum parallelarum (verbi gratià inſtar omnium) per extremum N Rectæ O N (vel B N,) delineabit Parabolam.

*Fig.* 3.

2. Qui quidem Parabolam deſcribendi modus ſatis admodùm facilis ſit licet, expeditior tamen mihi videtur ille, qui a Triangulo Rectangulo Iſoſcele, (ſicuti Hyperbole abs obtuſangulo, & Ellipſis ab acutangulo Triangulo Iſoſcele derivata,) originem ſuam trahit.

*Fig.* 3.

Exponatur itaque Triangulum Iſoſceles (b a c), rectangulam ad (a,) cujus Latera (a b, a c) ſint æqualia: In
1 perpendiculari (a o) demiſſà, ſumatur a f = $\frac{L}{2}$, & biſecetur (a f) in A, (quod erit Vertex Parabolæ:) Ductis infinitis Rectis (h o h, &c.) Baſi (b c) parallelis; abs f, tanquam à Centro, intervallo verò infinitarum ipſarum Parallelarum (exempli gratià, inſtar omnium) intervallo (O h, vel) B h, deſcribatur Arcus ſecans ipſam Parallelam (B h,) in puncto N; (hoc eſt, ſtatuenda eſt f N = B h:) Et ſic abs f, infinitis verò hujuſmodi intervallis, deſcribantur infiniti alii Arcus, ipſas proprias Parallelas ſecantes in (N). Dico, Lineam curvam incedentem per omnes illas Interſectiones ad N, delineare Parabolam.

*Demonſtr.*

The Rectangle made of the *Latus Rectum*, and intercepted Diameter, is equal to the Rectangle made of both the Segments.

Or, *As* the *Latus Rectum*, *Is* to one of the Segments: *So* is the other, *To* the intercepted Diameter.

4. And hence (by way of Conſectary) may be found out a way, how to deſcribe a Parabole in Plano; having the Axe A o, and *Latus Rectum* (L) given.

1. To the Axe A O, let be applied to Right Angles infinite Parallels (as h N N h, *&c.*)

Make A I = L; and any Point (as o) being taken, in the given Axe A O, find out between the two Right Lines (A I, A O,) a mean Proportional (A L,) to be applied to the Axe, in the Point O, (*i. e.* O N, or B N) = A L; and ſo of infinite other Right Lines of this ſort, to be found after the ſame manner, and to be placed from divers points of the Axe A O. A crooked Line paſſing through the Extreams of the ſaid Parallels (for Example, one for all) through the Extream (N), of the Right Line O N (or B N), will deſcribe a Parabole. *Fig.* 3.

2. Although this way of deſcribing a Parabole, is eaſie enough; yet that way ſeems to me to be more expedite, which hath its origin from a rectangular Iſoſceles Triangle, (as an Hyperbole from an obtuſangular, and an Ellipſe from an acutangular Iſoſceles Triangle.) *Fig.* 3.

Let therefore an Iſoſceles Triangle (a b c) rectangular at (a) be made, whoſe Sides (ab, ac) are equal: In the Perpendicular (a o) let fal'n on the Baſe, let be taken a f $= \frac{1}{2}$; which being biſected in (A), will be the Vertex of the Parabole. Infinite Right Lines (as h o h, *&c.*) being drawn parallel to the Baſe (b c,) from f, (as from a Center) but at the diſtance of thoſe infinite Parallels, (for Example, one for all) at the diſtance of (O h, or) B h, let an Arch be deſcribed, cutting the ſaid Parallel (B h) in the Point N, (*i. e.* making f N = B h, or a B:) And ſo from f, at infinite other Diſtances of this ſort, infinite other Arches muſt be deſcribed, cutting their proper Parallels in (N:) I ſay, A crooked Line (as N A N) paſſing through all thoſe Interſections, will deſcribe a Parabole. *Fig.* 3.

*Demonſtr.*

1 | 2 | $af = \frac{L}{2}$. $aA = Af = \frac{L}{4}$; g°, $4\,Af = L$.

*Conſtr.* | 3 | $(aA + AB =)\ Af + AB = (aB = Bh =)\ fN$.

⊚ | 4 | $Af^2 + 2\,Af \times AB + AB^2 = fN^2$.

5 | $AB \backsim Af = fB$.

⊚ | 6 | $Af^2 - 2\,Af \times AB + AB^2 = fB^2$.

47, è 1. | 7 | $(BN^2, u)\ NO^2 = fN^2 - fB^2 = (4-6)\ 4\,Af \times AB = (\S\ 2,\ L \times AB,\ u)\ L \times AO$: hoc eſt;

7 | 8 | $L . NO :: NO . AO$; quæ eſt prima propr. ſpecial.

## CAP. II.

### *De Equationibus omnibus quartum gradum non excedentibus, quomodolibet affectis, conſtruendis, & ipſarum radicibus tam falſis quam veris reperiendis.*

QUO quidem artificio, Parabolâ ſuppoſitâ deſcriptâ, punctum reperiatur, à quo (tanquam à centro) intervallo quodam determinando, circulus poſſit deſcribi, qui ita ſecet vel tangat Parabolam, ut à punctis concurſûs rectæ eductæ omnes omnium Æquationum quartum gradum non excedentium, quomodolibet affectarum, radices tam falſas quàm veras determinent; ut hujus negotii præcipuus eſt cardo, & unicum illud maximè inquirendum; ita quod ad amuſſim præſtat Regula hæc ſubnexa; quam, diſtinctionis ergò, liceat appellare *Centralem*, vel *Locum*.

*Regula Centralis.*

$$\text{Pars}\left\{\begin{array}{l} 1\ \left\{\frac{L}{2} + \frac{p^2}{8L} \overset{+}{\backsim} \frac{q}{2L} = b = AD.\right. \\ 2\ \left\{\frac{p}{4} + \frac{p^3}{16L^2} \overset{+}{\backsim} \frac{pq}{4L^2} \overset{+}{\backsim} \frac{r}{2L^2} = d = DH.\right. \end{array}\right.$$

In

*Demonſtr.*

1 | 2 | $af = \frac{L}{2}$. $aA = Af = \frac{L}{4}$; g°, $4Af = L$.

Conſtr. | 3 | $(aA + AB =)\ Af + AB = (aB = Bh =)\ fN$.

Ǥ | 4 | $Af^2 + 2Af \times AB + AB^2 = fN^2$.

| 5 | $AB \backsim Af = fB$.

Ǥ | 6 | $Af^2 - 2Af \times AB + AB^2 = fB^2$.

47, è 1. | 7 | $(BN^2, u)\ NO^2 = fN^2 - fB^2 = (4-6)\ 4Af \times AB = (\S 2, L \times AB, u)\ L \times AO$: that is,

7 | 8 | $L . NO :: NO . AO$; which is the firſt ſpecial property of a Parabole.

## CHAP. II.

### *Of the Conſtraction of all Equations, not exceeding the fourth Degree, howſoever affected; as alſo the finding all their Roots, as well falſe as true.*

By what artifice, a Parabole being ſuppoſed to be deſcribed, a certain Point may be found, from which, (as from a Center) at a certain diſtance to be determined) a Circle may be deſcribed, which may ſo cut or touch a Parabole, that from the Points of their Meeting, Right Lines drawn may determine all the Roots, as well falſe as true, of all Equations, not exceeding the fourth Degree, howſoever affected: As it is the Hinge on which all this buſineſs hangs, and the only thing chiefly to be enquired after; ſo is it that, which this following Rule exactly performs, which for diſtinction-ſake we may call the *Central Rule*, or *Place*.

*The Central Rule.*

Part 1: $\frac{L}{2} + \frac{p^2}{8L} \overset{+}{\backsim} \frac{q}{2L} = b = AD.$

Part 2: $\frac{p}{4} + \frac{p^3}{16L^2} \overset{+}{\backsim} \frac{pq}{4L^2} \overset{+}{\backsim} \frac{r}{2L^2} = d = DH.$

In quâ eſt obſervanda occurrit, $\left\{\begin{array}{l}\text{Quantitatum}\\ \text{Signorum}\end{array}\right\}$ determinatio.

1. Si qua Quantitatum (p, q, r,) in Æquatione propoſitâ deficeret, quæ deficit, à Regulâ Centrali (ut neceſſe eſt) eſt abdicanda, & à reliquis determinanda eſt Regula.

2. Quod ad Signa determinanda ſpectat, notandum;

1. In Regulâ continuò habebitur $+\frac{r}{2L^2}$, niſi quum in Æquatione propoſitâ, p & r diverſis ſignis affici contigerit; quo in caſu, Signo negativo (nempè $-\frac{r}{2L^2}$) mulctari oportet.

2. Quocunque ſigno, in Æquatione propoſitâ, denotari acciderit Quantitas (q;) contrario quidem in Regulâ (aliâ implicata licet) deſignanda eſt.

Non quidem eſt hujus loci, nec tanti momenti, ut moneam, oportere aliquando fieri, inter demonſtrandum, (licet nunquam inter conſtruendum) Signorum Enallagen; nempè, quum exceſſus in Regulâ fuerit penes quantitates Signo Negativo adfectas; quum cuique figuram inſpicienti facilè videre eſt, an punctum D, citra vel ultra verticem Axis vel Diametri; vel punctum (H) ad dextram, vel ſiniſtram ejuſdem Axis vel Diametri cadere contigerit, necne.

Miſſâ autem hâc (forſan ſubobſcurâ) verborum ambage, Regulam centralem cuilibet Æquationum Claſſi propriam, brevi Synopſi oculis ſubjiciam. Vide Synopſin.

Hiſce benè perſpectis, & probè intellectis, ad altiora & penitiora hujuſmodi myſteria facilior erit aditus.

In which is to be obſerved, The determination of the $\left\{\begin{array}{l}\text{Quantities.}\\ \text{Signs.}\end{array}\right.$

1. If any one of the Quantities (p, q, r,) be wanting in the Æquation propoſed, that which wants, muſt (of neceſſity) be excluded the Central Rule; and the true Rule is to be determined by the remaining Quantities only.

2. For the determination of the Signs;

1. In the Rule muſt always be had $+\frac{r}{2L^2}$, unleſs in the Equation propoſed, it happens, that p and r are affected with divers Signs; in which caſe, it muſt be $-\frac{r}{2L^2}$.

2. With what Sign ſoever the Quantity (q) is noted in the Equation propoſed, it muſt be marked with its contrary Sign (although involved with another Quantity) in the Rule.

It pertains not to this place, neither is it of ſuch moment to notifie, That ſometimes, whiles demonſtrating (although never whiles working) a change of the Signs muſt be made; *viz.* When in the Rule the Negative Quantities happen to exceed the Affirmatives, ſeeing any one that inſpects the Figure may eaſily diſcover, whether or no the Point D happens to fall below or above the Vertex of the Axe or Diameter; or the Point (H) on the right or left ſide of the ſaid Axe or Diameter.

But paſſing by this (perhaps obſcure) way of Diſcourſe, we will preſent, in a brief Synopſis, the proſpect of each Central Rule proper to each Claſs of Equations. See the Synopſis.

Theſe things clearly perceived, and rightly underſtood, a more eaſie entrance will be had, to the more hidden and higher Myſteries of this kind.

## *Regulæ Centralis ad Parabolam applicatio; ſive* Regula Generalis.

DEſcribatur Parabola (N A M,) cujus Latus Rectum
ſit L (ſive 1), Axiſque (a y, vel) A y;
ad quem, ſi in Æquatione habeatur p, ordinatim applicetur
1 $BA = \frac{p}{2}$, occurrens Parabolæ in B & A;
& (ab alterutro, puta) ex A, ducatur recta Parallela
Axi A O: Si verò in Æquatione deficeret p, nulla
erit neceſſitas, vel applicandi B A ad Axem; vel ducendi
Diametrum A y.

Tum in $\left\{\begin{matrix}\text{Axe, vel}\\ \text{Diametro}\end{matrix}\right\}$ A y, ſi in Æquatione, p
2 $\left\{\begin{matrix}\text{abſuerit}\\ \text{deficerit}\end{matrix}\right\}$ ſumatur $AD = b$ (ſuprà inventæ, Æquationi
propoſitæ congruæ); cujus quælibet quantitas
Signo $+$ adfecta, vel denotata), aggregatìm vel ſingulatìm
in $\left\{\begin{matrix}\text{Axe, vel}\\ \text{Diametro}\end{matrix}\right\}$ deorſum, verſus y eſt diſponenda,
& exinde, Quantitas negativa (ſi qua fuerit,)
in eodem $\left\{\begin{matrix}\text{Axe, vel}\\ \text{Diametro}\end{matrix}\right\}$ (continuato, ſi opus fuerit,)
ſurſum verſus A eſt collocanda; inventumque
erit punctum D.

A quo Puncto (nempè D,) erigatur perpendicularis
3 ad A y, Recta $DH = d$ (ſuprà inventæ, Æquationi
propoſitæ congruæ); cujus etiam quælibet quantitas
4 Signo $+$ denotata, aggregatìm vel ſingulatìm,
verſus ſiniſtram eſt in ipſâ perpendiculari diſponenda,
& exinde, quælibet reliqua quantitas (ſi qua fuerit)
Signo (—) deſignata, eſt in ipſâ (continuatâ, ſi
5 opus fuerit,) verſus dextram collocanda; inventumque
erit (punctum, vel) Circuli centrum (H.)

Quo invento, & connexâ H A, oportet ex Centro
6 (H) circulum (H A M) deſcribere, cujus Semidiameter

## *The application of the Central Rule to a Parabole;* *or*, The General Rule.

LET a Parabole NAM be deſcribed, whoſe *Latus*
*Rectum* is L (or 1), and Axe (a y, or) A y;
to which, if in the Equation be found p, let there be
1 ordinately applied BA $= \frac{p}{2}$, meeting the Parabole in
B and A; and from (either of which, ſuppoſe) A,
let there be drawn parallel to the Axe, the Right Line
AO: But if p be wanting in the Equation, there will
be no need either of applying BA to the Axe, or of
drawing the Diameter AO.

Then in the $\left\{\begin{matrix}\text{Axe, or}\\ \text{Diameter}\end{matrix}\right\}$ AO, if in the Equation p
2 be $\left\{\begin{matrix}\text{had, or}\\ \text{wanting}\end{matrix}\right\}$, make AD = b (before found, proper
to the Equation propoſed); all whoſe Quantities noted
with the Sign +, aggregately or ſeverally are to be
diſpoſed on the $\left\{\begin{matrix}\text{Axe, or}\\ \text{Diameter}\end{matrix}\right\}$ downwards towards y;
and from thence, the Quantity (if any) noted with
a Negative Sign (—), is to be placed upwards towards
wards A, on the ſame $\left\{\begin{matrix}\text{Axe, or}\\ \text{Diameter}\end{matrix}\right\}$; and the Point D
will be found.

From which Point (*viz.* D) let be erected perpendicular
3 to A y, the Right Line DH = d (above
found, proper to the Equation propoſed); all whoſe
4 Quantities alſo, marked with the Sign +, are aggregately
or ſeverally in the ſaid Perpendicular, to be
diſpoſed towards the left hand; and from thence, thoſe
remaining Quantities (if any,) marked with the Sign
5 (—), are to be placed on it (continued, if need be)
towards the right hand; and the (Point, or) center
of the Circle (H) will be found.

Which being found, and HA connected, from the
6 Center (H) muſt be deſcribed a Circle (HAM,)
whoſe

diameter ſit HA, ſi Equatio, non ſit Biquadratica, hoc eſt, ſi non habeatur Quantitas S.

7 Quod ſi habeatur S, & Signo quidem negativo adfecta (nempe $- S$), oportet ulteriùs in hâc Lineâ
8 AH, utrinque productâ, ex unâ parte ſumere $AI = L$
9 (ſive 1), & ex alterâ parte $AK = \frac{S}{L_3}$; deſcripto-
10 que Semicirculo, cujus Diameter (IK,) erigere AL perpendicularem ad AH, quæ occurrat huic Semicirculo (ILK,) in Puncto L; quod illud ipſum eſt, per quod alter Circulus (NLM) tranſire debet.

11 Quod ſi verò habeatur $+ S$, oportet inſuper in alio Semicirculo, cujus Semidiameter eſt AH, inſcribere $AZ = AL$ inventæ; inventumque erit Punctum Z, per quod primus Circulus quæſitus tranſire debet.

Circulus igitur deſcriptus tranſiens per A, ſi defecerit S (ut ſuprà § 6.); vel ſi habeatur S; tranſiens
12 per L, ſi ſit $- S$ (ut ſuprà § 10.); per Z verò, ſi ſit $+ S$, (ut ſuprà § 11.) ſecare vel tangere poſſit Parabolam in 1, 2, 3, aut 4 punctis; à quibus, ſi ad Axem vel Diametrum demittantur perpendiculares, obtinebuntur omnes Æquationis radices, tam falſæ, quàm veræ; nimirum,

13 1. Si quidem in Æquatione defecerit p, & ſit $- r$; veræ radices erunt illæ harum Perpendicularium, quæ ad ſiniſtram partem Axis reperientur (ut NO); & reliquæ (ut MO) erunt falſæ.

14 2. Si verò in Æquatione habeatur p, & ſit $- p$; veræ radices erunt illæ, quæ ad ſiniſtram partem Axis (ut NO); falſæ verò (ut MO) quæ ad dextram reperientur.

15 Sed contrà; ſi ſit $+ p$, veræ quidem cadent ad dextram partem Axis vel Diametri (ut MO); falſæ verò (ut NO), ad ſiniſtram.

Notandum: Si hic Circulus neque ſecat, neque tangit Parabolam in aliquo puncto; indicio eſt, impoſſibilem eſſe Æquationem, nullamque admittere radicem, ſive veram ſive falſam, ſed tantum imaginarias.

whoſe Semidiameter H A, if it be not a Biquadratic
Æquation, (*i. e.*) if the Quantity S be wanting.

7 But if S be had, and it be — S, then further in
this Line A H, both ways produced, muſt be taken
8 on the one ſide A I = L (or 1), and on the other
9 ſide A K = $\frac{S}{L\,3}$; and a Semicircle being deſcribed,
10 whoſe Diameter I K muſt be erected A L perpendi-
cular to A H, which may meet this Semicircle (I L K,)
in the Point L; which is that very Point, through
which the other Circle (N L M) muſt paſs.

11 But and if be had + S, there muſt moreover in
another Semicircle, whoſe Diameter is A H, be in-
ſcribed A Z = A L found; and the Point Z will be
found, by which the firſt Circle ſought ought to
paſs.

A Circle therefore deſcribed paſſing through A, if
S be wanting (as before, § 6.); or if S be had, paſſing
12 through L, if it be — S (as above, § 10.); but through
Z, if it be + S, (as above, § 11.) may cut or touch
the Parabole in 1, 2, 3, or 4 Points; from which, if
Perpendiculars be demitted to the Axe or Diameter,
all the Roots, as well falſe as true, will be had; *viz.*

13 1. If in the Equation p be wanting, and it be — r;
thoſe of theſe Perpendiculars will be the true Roots,
which ſhall be found on the left ſide of the Axe (as
N O); and the reſt (as M O) falſe.

14 2. But if in the Equation p be had, and it be — p;
thoſe will be the true Roots, which ſhall be found on
the left ſide of the Axe (as N O); and thoſe falſe
(as M O), which on the right.

15 But contrariwiſe; if it be + p, thoſe will be the
true Roots, which ſhall fall on the right ſide of the
Axe or Diameter (as M O); and thoſe falſe (as N O),
which on the left.

*Note:* If this Circle neither cuts nor touches the
Parabole in any Point, it is a token of an impoſ-
ſible Equation, and that it admits of no Roots, whe-
ther true or falſe, but only imaginary ones.

Si ſumatur *Latus Rectum* L = 1 (ſive Unitati) dico L, cum omnibus ſuis gradibus omitti poſſe; quod ſemel annotâſſe ſufficiat.

Quorum omnium demonſtratio, ſingulas formulas Æquationum percurrendo, in ſequentibus innoteſcet. Priuſquàm verò rem aggreſſus fuero, liceat mihi pauliſper ἀπολογίζειν;

Non eſt, quòd Parabolam (ut veteres olim *Junonem Lucinam*), ad Claſſis primi, ſecundi, & partis prioris ſeptimi, nedum prioris partis quinti partum, in anxilium invocemus; cùm faciliore forſan nixu, famulante ſolo circulo, unico, partu ſatis admodum maturè levetur: Quàm dexterè autem & auſpicatò obſtetricis (ancillante ſolo Circulo) partes hâc in re poſſit agere (myſteriis quidem gravida) Parabola; non abs re, imo operæ forſan erit pretium, (ut Regulæ generalis Amplitudinem oſtendamus,) brevitèr perſtringere.

## CLAS. I.

*De Æquationibus quartæ Dimenſionis conſtruendis, ubi omnes termini (p, q, r,) deficiunt; vel, ubi affectis ſub nullo gradu Parodico.*

OMnes hujus cenſûs Æquationes ad unicam quidem ſolam formulam ſunt reducibiles.

1. $x^4 * * * - S = 0$

*Synopſ. Cl. 1.* *Regula Centralis.* $\frac{L}{2} = b = AD.\quad 0 = d = DH.$

*Reg. Gen.* Deſcribatur Parabola (NAM), cujus Latus Rectum ſit L (ſive 1), Axiſque A y; in quo ſumatur

2 1 $AD = AH = b = \frac{L}{2}$. Ex unâ parte AH (utrinque

8 2 productæ) ſumatur $AI = L$, & ex alterâ parte,

AK

If be taken the *Latus Rectum* L = 1 (or an Unity), I ſay, L with all its degrees may be omitted; which once to have noted may ſuffice.

The Demonſtration of all which, running through each particular form of Equations, will appear in our following Diſcourſe. Which before I ſhall attempt, I ſhall take leave to make this Apology.

Though no neceſſity of invoking a Parabole (as of old they did *Juno Lucina*), to midwife forth the two firſt Claſſes of Equations, as alſo the former part of the ſeventh, much leſs the former part of the fifth: Seeing without her aſſiſtance a Circle only may with more eaſe perhaps, and timely enough bring it to birth; yet how dextrouſly and luckily a Parabole (big with Myſteries) can in this buſineſs act the Midwife's part, by help of a Circle only, as it will not be altogether beſides our purpoſe, ſo perhaps worth our while to ſhew, ſith that it blazons the Amplitude of our General Rule.

## CLAS. I.

### *Of the Conſtruction of Equations of the fourth Dimenſion, where all the terms (p, q, r,) are wanting, or where affected under no Parodic Degree.*

ALL Equations of this kind are reducible to this one only form.

1. $x^4 * * * - S = 0$.

*Central Rule.* $\frac{L}{2} = b = AD. \quad 0 = d = DH.$

*Gen. Rule*

Let a Parabole (NAM) be deſcribed, whoſe *Latus Rectum* L (or 1), and Axe Ay; in which take

2 1 $AD = AH = b = \frac{L}{2}$. On the one part of AH (both *Fig. 4.*

8 2 ways produced) let be made AI = L; and on the

| | | |
|---|---|---|
| 9 | 3 | $AK=\frac{S}{L^3}$; deſcriptoque Semicirculo, cujus Diameter |
| 10 | 4 | ſit IK, erigenda eſt AL ad Axem perpendicularis, quæ occurrat huic Semicirculo (ILK), in puncto L. Centro quidem H, intervallo verò HL, deſcribatur Circulus (NLM), qui ſecabit Parabolam in punctis N & M: A quibus demiſſæ rectæ (NO, MO), erunt radices quæſitæ; quarum altera vera, altera falſa. |
| | | *Formula.* $x^4 * * * - S = 0$. |
| | | *Demonſtrat.* |
| 1 | 5 | AD, vel $AH = b = \frac{L}{2}$. *Fig.* 4. |
| 3×4 | 6 | $\begin{cases} AI \times AK = AL^2; \text{ ob circulum.} \\ (L \times \frac{S}{L^3} =)\ AL^2 = \frac{S}{L^2}. \end{cases}$ |
| 47, è 1.<br>Q. 5. + Q. 6. | 7 | $\begin{cases} AH^2 + AL^2 = (HL^2 =)\ \text{Q. Rad.} \\ b^2 + \frac{S}{L^2} = \text{Q. Rad.} \end{cases}$ |
| *Supp.* | 8 | $NO = x$. |
| *Supp.* | 8 | $MO = -x$. |
| *Ob para.* | 9 | $\begin{cases} L\ .\ NO :: NO\ .\ AO. \\ L\ .\ x :: x\ .\ \frac{x^2}{L} = AO. \end{cases}$ |
| *Ob para.* | 9 | $\begin{cases} L\ .\ MO :: MO\ .\ AO. \\ L\ .\ -x :: -x\ .\ \frac{x^2}{L} = AO. \end{cases}$ |
| 9 ∽ 5 | 10 | $\begin{cases} AO \backsim AD = DO. \\ \frac{x^2}{L}\ (-b, u) - \frac{L}{2} = DO. \end{cases}$ |
| ⊖ | 11 | $b^2 + \frac{x^4}{L^2} - x^2 = DO^2$. |
| Q. 8. | 12 | $x^2 = NO^2$. |
| Q. 8. | 12 | $x^2 = MO^2$. |

DO²

9 3 other ſide $AK = \frac{S}{L^3}$: And a Semicircle being deſcri-
10 4 bed, whoſe Diameter IK, erect AL perpendicular to the Axe, which may meet this Semicircle (ILK) in the Point L. Center H, and diſtance HL, let a Circle (NLM) be deſcribed, which will cut the Parabole in the Points N and M: From which Perpendiculars demitted to the Axe (as NO, MO,) will be the Roots deſired, whereof the one true, the other falſe.

*Form.* $x^4 \ast \ast \ast - S = o.$

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47, è 1. / 11 + 12 — 13 $\left\{\begin{array}{l} DO^2 + NO^2 = (DN^2 =)\ Q.\,Rad. \\ b^2 + \frac{x^4}{L^2} = Q.\,Rad. \end{array}\right.$

47, è 1. / 11 + 12 — 13 $\left\{\begin{array}{l} DO^2 + MO^2 = (DM^2 =)\ Q.\,Rad. \\ b^2 + \frac{x^4}{L^2} = Q.\,Rad. \end{array}\right.$

13 = 7 — 14 $\frac{x^4}{L^2} = \frac{S}{L^2}$; in $L^2$.

$\times L^2$ — 15 $x^4 = S.$

*Tranſp.* — 16 $x^4 \ast \ast \ast - S = o.$ *Q. e. d.*

*Illuſtrat.* *Central.*

$$\left\{\begin{array}{l} x^4 \ast \ast \ast - 6561 = 0 \\ x^4 \ast \ast \ast - 0.6561 = 0 \end{array}\right\} \frac{L}{2} = 0.5 = b.$$

Fig. 4.

$$\left\{\begin{array}{l} NO = x = 9 \\ MO = -x = -9 \end{array}\right\}$$

---

**CLAS.**

## CLAS. II.

### *De Æquationibus, 1. Secundæ Dimenſionis conſtruendis, ubi deficit ſecundus Terminus, nempe p: 2. Quartæ Dimenſionis, ubi deficit ſecundus & quartus Terminus, (nempe p & r;) vel affectis tantum ſub Quadrato, vel ſecundo Gradu Parodico.*

OMnes Æquationes $\left\{\begin{matrix}\text{ſecundæ}\\ \text{quartæ}\end{matrix}\right\}$ Dimenſionis, ad $\left\{\begin{matrix}1\\ 3\end{matrix}\right\}$ formulas poſſint reduci.

$$\left\{\begin{matrix}1.\ x^2 * -q=0 \left\{\begin{matrix}1\\3\end{matrix}\right\}\begin{matrix}x^4 * -qx^2 * -S=0\\ x^4 * -qx^2 * +S=0\end{matrix}\\ 2.\ x^2 * +q=0\ \text{impoſſibilis.}\ \}\ 2.\ x^4 * +qx^2 * -S=0.\end{matrix}\right.$$

*Regula Centralis.*

Synopſ. Cl. 2. Si $\left\{\begin{matrix}-q\\ +q\end{matrix}\right\} \frac{L}{2} \underset{\backsim}{+} \frac{q}{2L}=b=AD. \quad 0=d=DH.$

Reg.Gen. Suppoſitâ igitur deſcriptâ Parabolâ (NAM), cujus Latus Rectum ſit L (ceu 1), Axiſque Ay; opor-
1 1 tet facere $AB=\frac{L}{2}$ continuò; tum ſumptâ bD, vel
2 2 $bH=\frac{q}{2L}$; collocetur in Axe ulteriùs verſus y, ſi in
Æquatione habeatur $-q$; ſed ſurſum, in Axe (continuato, ſi opus fuerit) ſi habeatur $+q$.

Tum ex Centro H, oportet Circulum deſcribere Fig. 5.
6 3 (NAM) cujus Semidiameter ſit HA, ſi in Æquatione non habeatur Quantitas S.

7 4 Aſt ſi habeatur S; & ſit $-S$; oportet ulterius in hâc lineâ AH (productâ utrinque) ex unâ parte ſu-
8 5 mere $AI=L$, & ex alterâ parte $AK=\frac{S}{L^3}$; deſcrip- Fi. 6, 8, 9
9 6 toque

## CLAS. II.

### *Of the Conſtruction of Equations, 1. Of the ſecond Dimenſion, where the ſecond Term (p) is wanting. 2. Of the fourth Dimenſion, where the ſecond and fourth Term (viz. p and r) are wanting; or affected only under a Square, or the ſecond Parodic Degree.*

ALL Equations of the $\left\{\begin{array}{l}\text{ſecond}\\\text{fourth}\end{array}\right\}$ Dimenſion, are reducible to theſe $\left\{\begin{array}{c}1\\3\end{array}\right\}$ forms.

$$\left\{\begin{array}{l}1.\ x^2 * -q=0\ \left\{\begin{array}{l}1\\3\end{array}\right\}\begin{array}{l}x^4 * -qx^2 * -S=0\\x^4 * -qx^2 * +S=0\end{array}\Big\}\\2.\ x^2 * +q=0\Big\}\ 2.\ x^4 * +qx^2 * -S=0.\\\quad\ \text{impoſſible.}\end{array}\right.$$

*Central Rule.*

*Synopſ. Cl. 1.* $$\text{If}\left\{\begin{array}{c}-q\\+q\end{array}\right\}\frac{L}{2}+\backsim\frac{q}{2L}=b=AD.\quad 0=d=DH.$$

*Gen. Rule* A Parabole being therefore ſuppoſed to be deſcribed
(as NAM), whoſe *Latus Rectum* L (or 1), and
1 1 Axe Ay; make always $Ab=\frac{L}{2}$; then taking BD,
2 2 or $bH=\frac{q}{2L}$, let it be placed (from b,) farther downwards towards y, if in the Equation be had $-q$; but upwards on the Axe (continued, if need be) if in the Equation be had $+q$.
6 3 Then from the Center H, let a Circle (NAM) be deſcribed, whoſe Semidiameter HA, if in the Equation be not had the Quantity S. *Fig. 5.*
7 4 But if S be had, and it be $-S$, then muſt be taken farther in this Line AH (both ways produced), on
8 9 5 6 the one ſide $AI=L$, and on the other $AK=\frac{S}{L^3}$; and

a Semi-

toque Semicirculo (I L K), cujus Diameter I K, eri-
10 7 gere A L perpendicularem A H, quæ occurrat huic Semicirculo (I L K), in puncto L.

11 8 Quòd ſi verò habeatur + S; oportet inſuper in alio Semicirculo, cujus Diameter ſit A H, inſcribere *Fig.* 7.
9 A Z = A L inventæ.

Circulus igitur deſcriptus, tranſiens per A, ſi defecerit S (ut ſuprà § 3.) vel ſi habeatur S, tranſiens per L, ſi ſit — S (ut ſuprà § 7.); per Z verò ſi ſit + S, (ut ſuprà § 9.) ſecabit vel tanget Parabolam in 2 aut 4 punctis; à quibus ſi ad Axem demittantur perpendiculares, omnes radices tam falſæ, quàm veræ obtinebuntur; quarum veræ ex unâ, falſæ verò ex alterâ parte Axis contingerint.

*Impoſſ.*

$$\left\{\begin{array}{l} 1.\ x^2 * - q = 0 \left\{\begin{array}{l} 1 \\ 3 \end{array}\right\} \begin{array}{l} x^4 * - qx^2 * - S = 0 \\ x^4 * - qx^2 * + S = 0 \end{array} \\ 2.\ x^2 * + q = 0 \left\{\begin{array}{l} 2 \end{array}\right\} \begin{array}{l} x^4 * + qx^2 * - S = 0 \\ * * \quad * * \quad * \end{array} \end{array}\right.$$

—

*Caſ.* 1. Ubi — q. *Fi.* 5, 6, 7

*Demonſtrat.*

1 + 2 10 $\left\{\begin{array}{l} A b + (b D, u)\ b H = (A D, u)\ A H = b. \\ \frac{L}{2} + \frac{q}{2L} = (A D, u)\ A H = b. \end{array}\right.$

⊙ 11 $b^2 = (A D^2, u)\ A H^2 = $ Q. Rad. in quadratic. *Fig.* 5.

5 × 6 12 $\left\{\begin{array}{l} A I \times A K = (\text{ob circl.})\ A L^2 = (\text{p. conſtr.})\ A Z^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = A L^2 = A Z^2. \end{array}\right.$

47, è 1 11 + 12 13 $\left\{\begin{array}{l} A H^2 + A L^2 = (H L^2 =) \text{ Q. Rad.} \\ b^2 + \frac{S}{L^2} = \text{Q. Rad.} \end{array}\right.$ *Fig.* 6.

47, è 1 11 — 12 14 $\left\{\begin{array}{l} A H^2 - A Z^2 = (H Z^2 =) \text{ Q. Rad.} \\ b^2 - \frac{S}{L^2} = \text{Q. Rad.} \end{array}\right.$ *Fig.* 7.

NO

a Semicircle (ILK), whoſe Diameter IK being
10 | 7 | deſcribed, muſt be erected AL perpendicular to AH,
which may meet this Semicircle (ILK), in the
Point L.

11 | 8 | But if be had + S; then moreover in another *Fig.* 7.
Semicircle, whoſe Diameter AH, muſt be inſcribed
9 | AZ = AL found.

A Circle therefore deſcribed paſſing through A, if
S be wanting, (as § 3.) or if S be had, paſſing
through L, if it be — S (as § 7.); but through Z,
if it be + S, (as § 9.) will cut or touch the Parabole
in 2 or 4 Points; from which, if Perpendiculars
be demitted to the Axe, all the Roots, as well
falſe as true, will be had; of which, thoſe true on
the one part, on the other part falſe.

*Impoſſ.*

$$\left\{\begin{array}{l} 1.\ x^2 * - q = 0 \left\{\begin{array}{l} 1 \\ 3 \end{array}\right\} \begin{array}{l} x^4 * - qx^2 * - S = 0 \\ x^4 * - qx^2 * + S = 0 \end{array} \\ 2.\ x^2 * + q = 0 \left\{\begin{array}{l} 2 \\ \ \end{array}\right\} \begin{array}{l} x^4 * + qx^2 * - S = 0 \\ * * \quad * * \quad * \end{array} \end{array}\right.$$

---

*Supp.* | 15 | $NO = x.$

*Supp.* | 15 | $MO = -x.$

*Ob para.* | 16 | $\left\{\begin{array}{l} L \,.\, NO :: NO \,.\, AO. \\ L \,.\, x :: x \,.\, \frac{x^2}{L} = AO. \end{array}\right.$

*Ob para.* | 16 | $\left\{\begin{array}{l} L \,.\, MO :: MO \,.\, AO. \\ L \,.\, -x :: -x \,.\, \frac{x^2}{L} = AO. \end{array}\right.$

16 ∽ 10 | 17 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\ HO. \\ \frac{x^2}{L}(-b, u) - \frac{L}{2} - \frac{q}{2L} = (DO, u)\ HO. \end{array}\right.$

⊙ | 18 | $b^2 + \frac{x^4}{L^2} - x^2 - \frac{qx^2}{L^2} = (DO^2, u)\ HO^2.$

*Q.* 15. | 19 | $x^2 = NO^2.$

*Q.* 15. | 19 | $x^2 = MO^2.$

| | | | |
|---|---|---|---|
| 47, *e* 1.<br>18 + 19 | 20 | $\left\{\begin{array}{l} DO^2 + NO^2 = HN^2. \\ b^2 + \frac{x^4}{L^2} - \frac{qx^2}{L^2} = HN^2 = Q.\,Rad. \end{array}\right.$ | |
| 47, *e* 1.<br>18 + 19 | 20 | $\left\{\begin{array}{l} DO^2 + MO^2 = HM^2. \\ b^2 + \frac{x^4}{L^2} - \frac{qx^2}{L^2} = HM^2 = Q.\,Rad. \end{array}\right.$ | |
| 20 = 11 | 21 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} = 0$; in $\frac{L^2}{x^2}$. | |
| $\times \frac{L^2}{x^2}$ | 22 | $x^2 \ast - q = 0$. *Q. e. d.* in Quadratic. | *Fig.5.* |
| 20 = 13 | 23 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| × *L*² | 24 | $x^4 - qx^2 = S$. | |
| *Tranſp.* | 25 | $x^4 - qx^2 - S = 0$. *Q. e. d.* in Biquadr. | *Fig.6.* |
| 20 = 14 | 26 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| × *L*² | 27 | $x^4 - qx^2 = -S$. | |
| *Tranſp.* | 28 | $x^4 - qx^2 + S = 0$. *Q. e. d.* in Biquadr. | *Fig.7.* |

*Illuſtrat.*

*q.* *Central.*

$$\left\{\begin{array}{l} x^2 \ast - 144 = 0 \\ x^2 \ast - 1.44 = 0 \end{array}\right\} \qquad \frac{L}{2} = 0.5$$

$$\left\{\begin{array}{l} NO = x = 12 \\ MO = -x = -12 \end{array}\right\} \quad + \frac{q}{2L} = 0.72$$

$$\underline{b = 1.22} = AD.$$

*Fig.5.*

*q.* *s.* *Central.*

$$\left\{\begin{array}{l} x^4 \ast - 108x^2 \ast - 5184 = 0 \\ x^4 \ast - 1.08x^2 \ast - 0.5184 = 0 \end{array}\right\} \quad \frac{L}{2} = 0.5$$

$$\left\{\begin{array}{l} NO = x = 12 \\ MO = -x = -12 \end{array}\right\} \qquad + \frac{q}{L2} = 0.54$$

$$\underline{b = 1.04} = AD.$$

*Fig.6.*

$x^4$

$$\left\{\begin{array}{l} x^4 \ast - \overset{q.}{292}\,x^2 \ast + \overset{s.}{9216} = 0 \\ x^4 \ast - 2.92\,x^2 \ast + 0.9216 = 0 \end{array}\right\}$$

$$\left\{\begin{array}{l} NO = x = 16. \quad no = x = 6 \\ MO = -x = -16. \quad mo = -x = -6 \end{array}\right\}$$

*Central.* Fig.7.

$$\frac{L}{2} = 0.5$$

$$+\frac{q}{2L} = 1.46$$

$$b = 1.96 = AD.$$

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*Caſ.* 2. Ubi $+q$.

Ubi $\left\{\begin{array}{l} 1.\ \frac{L}{2} \sqsupset \frac{q}{2L} \\ 2.\ \frac{q}{2L} \sqsupset \frac{L}{2} \end{array}\right\}$ Fig.8, 9.

*Demonſtrat.*

1—2 | 10 | $\left\{\begin{array}{l} Ab - (bD, u)\ bH = (AD, u)\ AH. \\ \frac{L}{2} - \frac{q}{2L} = (AD, u)\ AH = b. \end{array}\right.$ Fig.8.

2—1 | 10 | $\left\{\begin{array}{l} (bD, u)\ bH - Ab = (AD, u)\ AH. \\ \frac{q}{2L} - \frac{L}{2} = (AD, u)\ AH = b. \end{array}\right.$ Fig.9.

⊚ | 11 | $b^2 = (AD^2, u)\ AH^2.$

5×6 | 12 | $\left\{\begin{array}{l} AI \times IK = (\text{ob Circl.})\ AL^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2. \end{array}\right.$

11+12 | 13 | $\left\{\begin{array}{l} AD^2 + AL^2 = (DL^2, u)\ HL^2 = Q.\ Rad. \\ b^2 + \frac{S}{L^2} = Q.\ Rad. \end{array}\right.$

| | | | |
|---|---|---|---|
| *Supp.* | 14 | $NO = x.$ | |
| *Supp.* | 14 | $MO = -x.$ | |
| *Ob para.* | 15 | $\left\{\begin{array}{l} L \,.\, NO :: NO \,.\, AO. \\ L \,.\, x :: x \,.\, \frac{x^2}{L} = AO. \end{array}\right.$ | |
| *Ob para.* | 15 | $\left\{\begin{array}{l} L \,.\, MO :: MO \,.\, AO. \\ L \,.\, -x :: -x \,.\, \frac{x^2}{L} = AO. \end{array}\right.$ | |
| 15 − 10 | 16 | $\left\{\begin{array}{l} AO - AD = (DO, u)\, HO. \\ \frac{x^2}{L}\, (-b, u) - \frac{L}{2} + \frac{q}{2L} = (DO, u)\, HO. \end{array}\right.$ | *Fig.*8. |
| 15 + 10 | 16 | $\left\{\begin{array}{l} AO + AD = (DO, u)\, HO. \\ \frac{x^2}{L}\, (+b, u) + \frac{q}{2L} - \frac{L}{2} = (DO, u)\, HO. \end{array}\right.$ | *Fig.*9. |
| ⊙ | 17 | $b^2 + \frac{x^4}{L^2} - x^2 + \frac{qx^2}{L^2} = (DO^2, u)\, HO^2.$ | |
| *Q.* 14. | 18 | $x^2 = NO^2.$ | |
| *Q.* 14. | 18 | $x^2 = MO^2.$ | |
| 47, *e* 1<br>17 + 18 | 19 | $\left\{\begin{array}{l} DO^2 + NO^2 = HN^2. \\ b^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} = HN^2 = Q.\, Rad. \end{array}\right.$ | |
| 47, *e* 1<br>17 + 18 | 19 | $\left\{\begin{array}{l} DO^2 + MO^2 = HM^2. \\ b^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} = HM^2 = Q.\, Rad. \end{array}\right.$ | |
| 19 = 13 | 20 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| × $L^2$ | 21 | $x^4 + qx^2 = S.$ | |
| *Transp.* | 22 | $x^4 + qx^2 - S = 0.$ *Q. e. d.* in Biquadratic. | *Fig.*8, 9. |
| | | *Illuſtrat.* | |
| | | $\left\{\begin{array}{l} \quad\quad\quad\;\; q. \quad\quad\quad\quad s. \\ x^4 * + 60x^2 * - 11421 = 0 \\ x^4 * + 0.60x^2 * - 1.1421 = 0 \end{array}\right\}$ | |
| | | $\left\{\begin{array}{l} NO = x = 9 \\ MO = -x = -9 \end{array}\right\}$ | *Central.* |

*Central.*

Fig.8.

$$\frac{L}{2}=0.5$$

$$-\frac{q}{2L}=0.30$$

$$b=0.20=AD.$$

$$\left\{\begin{array}{l} x^4 * + \overset{q.}{192} x^2 * - \overset{s.}{16384}=0 \\ x^4 * + 1.92 x^2 * - 1.6384=0 \end{array}\right\}$$

$$\left\{\begin{array}{l} NO = x = 8 \\ MO = -x = -8 \end{array}\right\}$$

*Central.*

Fig.9.

$$\frac{q}{2L}=0.96$$

$$-\frac{L}{2}=0.5$$

$$b=0.46=AD.$$

**C L A S.**

# CLAS. III.

## *Of the Conſtruction of Equations of three or four Dimenſion, where the ſecond and third Term* (viz. *p and q*) *are wanting ; or of Cubic Equations, affected under no Parodic Degree ; or of Quadrato-quadratic, affected under the firſt Parodic Degree.*

ALL Equations of theſe kinds may be reduced to theſe following forms.

*Fig.* 10. *Fig.* $\begin{cases} 11 \\ 12 \end{cases}$

$$\left\{\begin{array}{l} 1.\ x^3 * * - r = 0 \left\{\begin{array}{l} 1 \} x^4 * * - rx - S = 0 \\ 3 \} x^4 * * - rx + S = 0 \end{array}\right. \\ 2.\ x^3 * * + r = 0 \left\{\begin{array}{l} 2 \} x^4 * * + rx - S = 0 \\ 4 \} x^4 * * + rx + S = 0 \end{array}\right. \end{array}\right\}$$

*Central Rule.*

*Synopſ. Cl.* 3.

$$\frac{L}{2} = b = AD. \quad \frac{r}{2L^2} = d = DH.$$

*Gen. Rule* Suppoſe the Parabole (NAM) to be already deſcribed, whoſe *Latus Rectum* L (or 1), and Axe Ay;

2 1 in which let be taken $AD = \frac{L}{2}$; and erecting a Perpendicular

4 2 to the Axe (*viz.* DH) $= \frac{r}{2L^2}$, from the

Center H, muſt be deſcribed a Circle (NA), whoſe

6 3 Semidiameter HA, if it be only a Cubic Equation, *Fig.* 10.

(*i. e.*) if the Quantity S be not: But and if S be

7 4 had, and it be — S, then muſt be taken farther in this

8 5 Line AH, both ways produced, on the one ſide $AI = L$, *Fig.* 11.

9 6 and on the other ſide $AK = \frac{S}{L^3}$; and a Semicircle

(ILK) being deſcribed, whoſe Diameter IK muſt

10 7 be erected AL perpendicular to AH, which may

meet this Semicircle (ILK) in the Point L.

But

## CLAS. III.

### *De Æquationibus trium vel quatuor Dimenſionum conſtruendis, ubi deficit ſecundus & tertius Terminus (nempè, p & q); vel, de Æquationibus Cubicis ſub nullo; vel, de Quadrato-quadraticis, ſub primo tantùm Gradu Parodico affectis.*

OMnes hujus cenſûs Æquationes ad ſequentes formulas poſſint reduci.

Fig.10.

$$\left\{\begin{array}{l} 1.\ x^3 * * - r = 0 \\ 2.\ x^3 * * + r = 0 \end{array}\right. \left\{\begin{array}{l} 1 \\ 3 \\ 2 \\ 4 \end{array}\right. \begin{array}{l} x^4 * * - rx - S = \\ x^4 * * - rx + S = \\ x^4 * * + rx - S = \\ x^4 * * + rx + S = \end{array}$$

Fig. $\left\{\begin{array}{l} 11 \\ 12 \end{array}\right.$

*Regula Centralis.*

Synopſ. Cl. 3.

$$\frac{L}{2} = b = AD. \quad \frac{r}{2L^2} = d = DH.$$

*Reg.Gen.* Supponatur Parabolam (NAM) jam deſcriptam
eſſe, cujus Latus Rectum L (ceu 1), Axiſque A y;
2 1 in quo ſumatur $AD = \frac{L}{2}$; & erigendo ad Axem per-
4 2 pendicularem $DH = \frac{r}{2L^2}$, oportet ex Centro H, Cir-
6 3 culum deſcribere (NA), cujus Semidiameter ſit HA, *Fig.10.*
ſi Æquatio tantùm Cubica fuerit, hoc eſt, ſi non habea-
7 4 tur Quantitas S: Aſt ſi habeatur S, & ſit — S, opor-
tet ulteriùs in hâc lineâ AH, productâ utrinque, ex
8 9 5 6 unâ parte ſumere $AI = L$, & ex alterâ parte $AK = \frac{S}{L^3}$, *Fig.11.*
deſcriptoque Semicirculo (ILK), cujus Diameter IK,
10 7 erigere AL, perpendicularem ad AH, quæ occurrat
huic Semicirculo (ILK) in puncto L.

Quòd

11 8 Quòd ſi verò habeatur $+ S$; oportet inſuper in
alio Semicirculo, cujus Diameter eſt A H, inſcribere Fig. 12.
9 $AZ = AL$ inventæ. Circulus igitur deſcriptus tran-
ſiens per L, ſi ſit $- S$ (ut ſuprà § 7.); per Z vero ſi
ſit $+ S$ (ut § 9.); ſecabit vel tanget Parabolam in
1 vel 2 punctis; à quibus ſi ad Axem demittantur Per-
pendiculares, obtinebuntur omnes Æquationis radices,
tam falſæ, quàm veræ; nimirum, ſi in Æquatione ha-
13 beatur $- r$; veræ, (ut N O,) ad ſiniſtram partem Axis
cadent, & falſæ (ut M O) ad dextram: Sed contra, ſi
habeatur ibi $+ r$, veræ cadent ad dextram Axis par-
tem (ut M O), & falſæ (ut N O) ad ſiniſtram.

Fig. 10.
$$\left\{\begin{array}{l} 1.\ x^3 * * - r = 0 \\ 2.\ x^3 * * + r = 0 \end{array}\right. \quad \begin{array}{l} \left\{\begin{array}{l} 1 \\ 3 \end{array}\right\} \left\{\begin{array}{l} x^4 * * - r x - S = 0 \\ x^4 * * - r x + S = 0 \end{array}\right\} \\ \left\{\begin{array}{l} 2 \\ 4 \end{array}\right\} \left\{\begin{array}{l} x^4 * * + r x - S = 0 \\ x^4 * * + r x + S = 0 \end{array}\right\} \end{array}$$
Fig. $\left\{\begin{array}{l} 11 \\ 12 \end{array}\right.$

---

*Demonſtrat.*

1 10 $AD = \frac{L}{2} = b.$

2 11 $DH = \frac{r}{2L^2} = d.$

47, e 1 Q. 10. + Q. 11. 12 $\left\{\begin{array}{l} AD^2 + DH^2 = (HA^2 =)\ \text{Q. Rad.} \\ b^2 + d^2 = \text{Q. Rad. in Cubic.} \end{array}\right.$ Fig. 10.

5 × 6 13 $\left\{\begin{array}{l} AI \times AK = (\text{ob Circle})\ AL^2 = (\text{per conſtr.})\ AZ^2. \\ L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2. \end{array}\right.$

47, e 1 12 + 13 14 $\left\{\begin{array}{l} AH^2 + AL^2 = (HL^2 =)\ \text{Q. Rad.} \\ b^2 + d^2 + \frac{S}{L^2} = \text{Q. Rad. in Biquadr.} \end{array}\right.$ Fig. 11.

47, e 1 12 — 13 15 $\left\{\begin{array}{l} AH^2 - AZ^2 = (HZ^2 =)\ \text{Q. Rad.} \\ b^2 + d^2 - \frac{S}{L^2} = \text{Q. Rad. in Biquadr.} \end{array}\right.$ Fig. 12.

11 8 But if be had $+ S$; then moreover in another Semicircle, whose Diameter is A H, must be inſcribed *Fig.*12.
9 A Z = A L found. A Circle therefore deſcribed, paſſing through L, if it be — S (as above, § 7.); but through Z, if it be + S (as § 9.) will cut or touch the Parabole in 1 or 2 Points; from which, if Perpendiculars be demitted to the Axe, all the Roots of the Equation, as well falſe as true, will be had; *viz.* The true (as N O) will fall to the left ſide of the Axe, and the falſe (as M O) to the right, if in the Equation be had $- r$: But contrarily, if in it be had $+ r$, the true will fall to the right ſide of the Axe (as M O), and the falſe (as N O) to the left.

*Fig.*10.
$$\begin{cases} 1.\ x^3 \ast \ast - r = o \begin{cases} 1 \ \{ x^4 \ast \ast - rx - S = o \} \\ 3 \ \{ x^4 \ast \ast - rx + S = o \} \end{cases} \\ 2.\ x^3 \ast \ast + r = o \begin{cases} 2 \ \{ x^4 \ast \ast + rx - S = o \} \\ 4 \ \{ x^4 \ast \ast + rx + S = o \} \end{cases} \end{cases}$$
*Fig.* $\begin{cases} 11 \\ 12 \end{cases}$

*Supp.* 16 $NO = x$.

*Supp.* 16 $MO = - x$. *Fig.*11.

*Ob para.* 17 $\begin{cases} L \,.\, NO :: NO \,.\, AO. \\ L \,.\, x :: x \,.\, \frac{x^2}{L} = AO. \end{cases}$

*Ob para.* 17 $\begin{cases} L \,.\, MO :: MO \,.\, AO. \\ L \,.\, -x :: -x \,.\, \frac{x^2}{L} = AO. \end{cases}$

17 ∽ 10 18 $\begin{cases} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} (\backsim b, u) \backsim \frac{L}{2} = HP. \end{cases}$

Ⓖ 19 $b^2 + \frac{x^4}{L^2} - x^2 = HP^2$.

16 ∽ 11 20 $\begin{cases} NO \backsim (OP, u)\ DH = PN. \\ x \backsim (d, u) \backsim \frac{r}{2L^2} = PN. \end{cases}$

| | | | |
|---|---|---|---|
| 16 + 11 | 20 | $\left\{\begin{array}{l} MO + (OP, u)\ DH = PM. \\ -x\ (+d, u) + \frac{r}{2L^2} = PM. \end{array}\right.$ | |
| @ | 21 | $d^2 + x^2 - \frac{rx}{L^2} = PN^2.$ | |
| @ | 21 | $d^2 + x^2 - \frac{rx}{L^2} = PM^2.$ | |
| 47, e 1.<br>19 + 21 | 22 | $\left\{\begin{array}{l} HP^2 + PN^2 = (HN^2 =)\ Q.\ Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{rx}{L^2} = Q.\ Rad. \end{array}\right.$ | |
| 47, e 1.<br>19 + 21 | 22 | $\left\{\begin{array}{l} HP^2 + PM^2 = (HM^2 =)\ Q.\ Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{rx}{L^2} = Q.\ Rad. \end{array}\right.$ | *Fig.*11. |
| 22 = 12 | 23 | $\frac{x^4}{L^2} - \frac{rx^2}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 24 | $x^3 * * - r = 0$. *Q. e. d.* in Cubic. | *Fig.*10. |
| 22 = 14 | 25 | $\frac{x^4}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| × *L*² | 26 | $x^4 - rx = S.$ | |
| *Tranſp.* | 27 | $x^4 * * - rx - S = 0$. *Q. e. d.* in Biquadr. | *Fig.*11. |
| 22 = 15 | 28 | $\frac{x^4}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| × *L*² | 29 | $x^4 - rx = - S.$ | |
| *Tranſp.* | 30 | $x^4 * * - rx + S = 0$. *Q. e. d.* in Biquadr. | *Fig:*12. |
| *Supp.* | 16 | $MO = x.$ | *Fig.*11. |
| *Supp.* | 16 | $NO = - x.$ | |
| *Ob para.* | 17 | $\left\{\begin{array}{l} L\ .\ MO :: MO\ .\ AO. \\ L\ .\ x :: x\ .\ \frac{x^2}{L} = AO. \end{array}\right.$ | |
| *Ob para.* | 17 | $\left\{\begin{array}{l} L\ .\ NO :: NO\ .\ AO. \\ L\ .\ -x :: -x\ .\ \frac{x^2}{L} = AO. \end{array}\right.$ | |

| | | | |
|---|---|---|---|
| $10 \backsim 17$ | 18 | $\{AD \backsim AO = (DO, u)\ HP.$<br>$(b, u)\ \frac{L}{2} \backsim \frac{x^2}{L} = HP.$ | |
| ⊖ | 19 | $b^2 + \frac{x^4}{L^2} - x^2 = HP^2.$ | |
| $16 + 11$ | 20 | $\{MO + (OP, u)\ DH = PM.$<br>$x\ (+d, u) + \frac{r}{2L^2} = PM.$ | |
| $16 \backsim 11$ | 20 | $\{NO \backsim (OP, u)\ DH = PN.$<br>$-x\ (-d, u) - \frac{r}{2L^2} = PN.$ | |
| ⊖ | 21 | $d^2 + x^2 + \frac{rx}{L^2} = PM^2 = PN^2.$ | |
| 47, e 1<br>$19 + 21$ | 22 | $\{HP^2 + PM^2 = (HM^2 =)$ Q. Rad.<br>$b^2 + d^2 + \frac{x^4}{L^2} + \frac{rx}{L^2} =$ Q. Rad. | |
| $19 + 21$ | 22 | $\{HP^2 + PN^2 = (HN^2 =)$ Q. Rad.<br>$b^2 + d^2 + \frac{x^4}{L^2} + \frac{rx}{L^2} =$ Q. Rad. | |
| $22 = 12$ | 23 | $\frac{x^4}{L^2} + \frac{rx}{L^2} = 0;$ in $\frac{L^2}{x}.$ | |
| $\times \frac{L^2}{x}$ | 24 | $x^3 + rx = 0 = -NO.$ *Q.e.d.* in Cubic. | *Fig.* 16. |
| $22 = 14$ | 25 | $\frac{x^4}{L^2} + \frac{rx^2}{L^2} = + \frac{S}{L^2};$ in $L^2.$ | |
| $\times L^2$ | 26 | $x^4 + rx = S.$ | |
| *Transp.* | 27 | $x^4 ** + rx - S = 0.$ *Q.e.d.* in Biquadratic. | *Fig.* 11. |
| $22 = 15$ | 28 | $\frac{x^4}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2};$ in $L^2.$ | |
| $\times L^2$ | 29 | $x^4 + rx = -S.$ | |
| *Transp.* | 30 | $x^4 ** + rx + S = 0; -N = x.$ *Q.e.d.* in Biquadr. | *Fig.* 12. |

*Illuſtrat.*

$$\left\{\begin{array}{l} x^3 \ast \ast - \overset{r.}{1728} = 0 \\ x^3 \ast \ast - 1.728 = 0 \end{array}\right\} \ NO = x = 12.$$

*Central.*

$$\frac{L}{2} = 0.5 = b. \quad \frac{r}{2L^2} = 0.864 = d.$$

Fig. 10.

$$\left\{\begin{array}{l} x^3 \ast \ast + \overset{r.}{1728} = 0 \\ x^3 \ast \ast + 1.728 = 0 \end{array}\right\} \ NO = -x = 12.$$

*Central.*

$$\frac{L}{2} = 0.5 = b. \quad \frac{r}{2L^2} = 0.864 = d.$$

$$\left\{\begin{array}{l} x^4 \ast \ast - \overset{r.}{1600}x - \overset{s.}{7761} = 0 \\ x^4 \ast \ast - 1.600x - 0.7761 = 0 \end{array}\right\} \begin{array}{l} NO = x = 13. \\ MO = -x = -4.5 \end{array}$$

*Central,*

$$\frac{L}{2} = 0.5 = b. \quad \frac{r}{2L^2} = 0.800 = d = DH.$$

Fig. 11.

$$\left\{\begin{array}{l} x^4 \ast \ast + \overset{r.}{1600}x - \overset{s.}{7761} = 0 \\ x^4 \ast \ast + 1.600x - 0.7761 = 0 \end{array}\right\} \begin{array}{l} MO = x = 4.5 \\ NO = -x = -13. \end{array}$$

*Central.*

$$\frac{L}{2} = 0.5 = b. \quad \frac{r}{2L^2} = 0.800 = d = DH.$$

$$\begin{array}{l} \left\{\begin{array}{l} x^4 ** - \overset{r.}{2560}x + \overset{s.}{9984} = 0 \\ x^4 ** - 2.560x + 0.9984 = 0 \end{array}\right\} \begin{array}{l} NO = x = 12. \\ no = x = \phantom{1}4. \end{array} \end{array}$$

*Central.*

$$\frac{L}{2} = 0.5 = b. \quad \frac{r}{2L^2} = 1.280 = d = DH.$$

Fi

$$\begin{array}{l} \left\{\begin{array}{l} x^4 ** + \overset{r.}{2560}x + \overset{s.}{9984} = 0 \\ x^4 ** + 2.560x + 0.9984 = 0 \end{array}\right\} \begin{array}{l} NO = -x = -12. \\ no = -x = -\phantom{1}4. \end{array} \end{array}$$

*Central.*

$$\frac{L}{2} = 0.5 = b = AD. \quad \frac{r}{2L^2} = 1.280 = d = DH.$$

CLAS.

## CLAS. IV.

### *De Æquationibus trium vel quatuor Dimenſionum conſtruendis, ubi deficit ſecundus Terminus (p); vel, de Æquationibus Cubicis, tantum ſub primo; vel, de Quadrato-quadraticis, ſub primo & ſecundo Gradu Parodico affectis.*

OMnes hujus Claſſis Æquationes ad haſce formulas ſequentes reducuntur.

$$\left\{\begin{array}{l} 1.\ x^3 * - qx - r = 0 \left\{\begin{array}{l} 1 \\ 3 \end{array}\right\} \begin{array}{l} x^4 * - qx^2 - rx - S = 0 \\ x^4 * - qx^2 - rx + S = 0 \end{array} \\ 2.\ x^3 * - qx + r = 0 \left\{\begin{array}{l} 2 \\ 4 \end{array}\right\} \begin{array}{l} x^4 * - qx^2 + rx - S = 0 \\ x^4 * - qx^2 + rx + S = 0 \end{array} \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{q}{2L} = b = AD. \quad \frac{r}{2L^2} = d = DH.$$

$$\left\{\begin{array}{l} 3.\ x^3 * + qx - r = 0 \left\{\begin{array}{l} 5 \\ 7 \end{array}\right\} \begin{array}{l} x^4 * + qx^2 - rx - S = 0 \\ x^4 * + qx^2 - rx + S = 0 \end{array} \\ 4.\ x^3 * + qx + r = 0 \left\{\begin{array}{l} 6 \\ 8 \end{array}\right\} \begin{array}{l} x^4 * + qx^2 + rx - S = 0 \\ x^4 * + qx^2 + rx + S = 0 \end{array} \end{array}\right.$$

*Regula Centralis.*

*Synopſ. Cl. 4.* $$\frac{L}{2} \backsim \frac{q}{2L} = b = AD. \quad \frac{r}{2L^2} = d = DH.$$

*Reg.Gen.* Supponendo itaque Parabolam (NAM) jam de-
ſcriptam eſſe, cujus Latus Rectum eſſe L (ceu 1),
2 1 Axemque A y; in quo ſumendo A b $= \frac{L}{2}$, oportet
3 2 facere b D $= \frac{q}{2L}$; eamque ſumere in Axe continuato
3 deorſum verſus y, ſi in Æquatione habeatur — q; ſed
verſus alteram partem ſurſum, in eodem Axe conti-
4 nuato, ſi habeatur + q.

Porrò

## CLAS. IV.

### *Of the Conſtruction of Equations of three or four Dimenſions, where the ſecond Term* (viz. *p*) *is wanting; or of Cubic Equations, affected only under the firſt Degree; or of Quadrato-quadratic, affected under the firſt and ſecond Parodic Degree.*

ALL Equations of this Claſs are reduced to theſe following forms.

$$\left\{\begin{array}{l} 1.\ x^3 * -qx - r = 0 \\ 2.\ x^3 * -qx + r = 0 \end{array}\right. \left\{\begin{array}{l} 1 \\ 3 \\ 2 \\ 4 \end{array}\right. \left\{\begin{array}{l} x^4 * -qx^3 - rx - S = 0 \\ x^4 * -qx^2 - rx + S = 0 \\ x^4 * -qx^2 + rx - S = 0 \\ x^4 * -qx^2 + rx + S = 0 \end{array}\right\}$$

*Central Rule.*

$$\frac{L}{2} + \frac{q}{2L} = b = AD. \quad \frac{r}{2L^2} = d = DH.$$

$$\left\{\begin{array}{l} 3.\ x^3 * +qx - r = 0 \\ 4.\ x^3 * +qx + r = 0 \end{array}\right. \left\{\begin{array}{l} 5 \\ 7 \\ 6 \\ 8 \end{array}\right. \left\{\begin{array}{l} x^4 * +qx^2 - rx - S = 0 \\ x^4 * +qx^2 - rx + S = 0 \\ x^4 * +qx^2 + rx - S = 0 \\ x^4 * +qx^2 + rx + S = 0 \end{array}\right\}$$

*Central Rule.*

*Synopſ. Cl.* 4. $$\frac{L}{2} \backsim \frac{q}{2L} = b = AD. \quad \frac{r}{2L^2} = d = DH.$$

*Gen. Rule*

Suppoſing therefore a Parabole (NAM) already
deſcribed, whoſe *Latus Rectum* is L (or 1), and Axe Ay;
2 3 1 2 in which taking $Ab = \frac{L}{2}$, muſt be made $bD = \frac{q}{2L}$, and
3 to place it on the Axe continued (downwards) towards y, if in the Equation be had $-q$; but (upwards) towards the other part, on the ſame Axe con-
4 tinued, if be had $+q$.

More-

Porrò è puncto D, erigendo ad Axem perpendicularem
3 $DH = \frac{r}{2L^2}$, oportet ex Centro H, Circulum
5 deſcribere, cujus Semidiameter ſit HA, ſi Æquatio
tantùm Cubica fuerit, hoc eſt, ſi non habeatur Quantitas S.

7 Aſt ſi habeatur S, & ſit quidem — S, oportet
ulteriùs in hâc lineâ HA, utrinque productâ, ex
8 4 unâ parte ſumere $AI = L$, & ex alterâ parte $AK = \frac{S}{L^3}$,
9 5 deſcriptoque Semicirculo, cujus Diameter IK, erigere
10 6 AL, perpendicularem ad HA, quæ occurrat huic
Semicirculo (ILK) in puncto L.

11 Quòd ſi verò habeatur + S, oportet inſuper in
alio Semicirculo, cujus Diameter ſit AH, inſcribere
7 $AZ = AL$ inventæ. Circulus igitur deſcriptus tranſiens
12 per L, ſi ſit — S (ut ſuprà § 6.); per Z verò ſi
ſit + S (ut § 7.); ſecabit vel tanget Parabolam in tot
punctis quot Æquatio admittet radices; à quibus ſi ad
Axem demittantur perpendiculares, habebuntur omnes
Æquationes radices, tàm falſæ, quàm veræ; quarum
quidem veræ (ut NO) ad ſiniſtram cadent, & falſæ
13 (ut MO) ad dextram partem Axis, ſi habeatur — r:
Sed contra, ſi habeatur ibi + r; veræ cadent ad dextram
(ut MO), & falſæ (ut NO) ad ſiniſtram.

---

*Caſ.* 1. Ubi — q.

$$\begin{array}{l} 1. \\ 2. \end{array} x^3 * - qx \mp r = 0 \left\{ \begin{array}{l} 1 \\ 3 \end{array} \right\} \begin{array}{l} x^4 * - qx^2 \mp rx - S = 0 \\ x^4 * - qx^2 \mp rx + S = 0 \end{array} \left\} \begin{array}{l} 2 \\ 4 \end{array} \right.$$

*Demonſtrat.*

2 8 $\left\{ \begin{array}{l} Ab + bD = b = AD. \\ \frac{L}{2} + \frac{q}{2L} = b = AD. \end{array} \right.$

3 9 $\frac{r}{2L^2} = d = DH.$

$AD^2$

Moreover from the Point D, erecting a Perpendi-
3 cular to the Axe $DH = \frac{r}{2L^2}$, muſt a Circle be de-
6 ſcribed from the Center H, whoſe Semidiameter HA,
if it be only a Cubic Equation, *viz.* if the Quantity S
be wanting.

7 But if S be had, and it be $-S$, there muſt farther in
this Line AH, both ways produced, be taken on the one
8 4 ſide $AI = L$, and on the other ſide $AK = \frac{S}{L^3}$; and a
9 5
10 6 Semicircle being deſcribed, whoſe Diameter IK,
muſt be erected AL perpendicular to HA, which may
meet this Semicircle (ILK), in the Point L.

11 But if $+S$ be had; then moreover in another
Semicircle, whoſe Diameter is AH, muſt be inſcribed
7 $AZ = AL$ found. A Circle therefore deſcribed paſ-
12 ſing through L, if it be $-S$ (as above, § 6.); but
through Z, if it be $+S$, (as § 7.) will cut or touch
the Parabole, in as many Points, as the Equation will
admit Roots; from which, if Perpendiculars be de-
mitted to the Axe, all the Roots, as well falſe as
true, will be had; of which, the true (as NO) will
13 fall on the left ſide of the Axe, and the falſe (as MO)
on the right, if be had $-r$: But contrarily, if be had
$+r$; the true (as MO) will fall on the right, and
the falſe (as NO) on the left.

---

47, e 1 Q. 8. + Q. 9. 10 $\left\{ \begin{array}{l} AD^2 + DH^2 = HA^2 = Q.\,Rad. \\ b^2 + d^2 = Q.\,Rad.\ \text{in Cubic.} \end{array} \right.$ *Fig.* 13.

4 × 5 11 $\left\{ \begin{array}{l} AI \times AK = (\text{ob Circl.})\ AL^2 = (\text{per conſtr.})\ AZ^2. \\ L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2. \end{array} \right.$

47, e 1 10 + 11 12 $\left\{ \begin{array}{l} AH^2 + AL^2 = (HL^2 =)\ Q.\,Rad. \\ b^2 + d^2 + \frac{S}{L^2} = Q.\,Rad.\ \text{ſi} - S. \end{array} \right.$ *Fig.* 14.

| | | | |
|---|---|---|---|
| 47, *e* 1<br>10 — 11 | 13 | $\left\{\begin{array}{l} AH^2 - AZ^2 = (HZ^2 =)\ \text{Q.Rad.} \\ b^2 + d^2 - \frac{S}{L^2} = \text{Q. Rad. ſi} + S. \end{array}\right.$ | *Fig.* 15. |
| *Supp.* | 14 | $NO = x.$ | |
| *Supp.* | 14 | $MO = -x.$ | |
| *Ob para.* | 15 | $\left\{\begin{array}{l} L\ .\ NO :: NO\ .\ AO. \\ L\ .\ x :: x\ .\ \frac{x^2}{L} = A\odot. \end{array}\right.$ | |
| *Ob para.* | 15 | $\left\{\begin{array}{l} L\ .\ MO :: MO\ .\ AO. \\ L\ .\ -x :: -x\ .\ \frac{x^2}{L} = AO. \end{array}\right.$ | |
| 15 — 8 | 16 | $\left\{\begin{array}{l} AO \backsim AD = (DO,\ u)\ HP. \\ \frac{x^2}{L}\ (\backsim b,\ u) - \frac{L}{2} - \frac{q}{2L} = HP. \end{array}\right.$ | |
| ⊙ | 17 | $b^2 + \frac{x^4}{L^2} - x^2 - \frac{qx^2}{L^2} = HP^2.$ | |
| 14 ∽ 9 | 18 | $\left\{\begin{array}{l} NO \backsim (OP,\ u)\ DH = PN. \\ x \backsim (d,\ u) \backsim \frac{r}{2L^2} = PN. \end{array}\right.$ | |
| 14 + 9 | 18 | $\left\{\begin{array}{l} MO + (OP,\ u)\ DH = PM. \\ -x\ (+d,\ u) + \frac{r}{2L^2} = PM. \end{array}\right.$ | |
| ⊙ | 19 | $d^2 + x^2 - \frac{rx^2}{L^2} = PN^2.$ | |
| ⊙ | 19 | $d^2 + x^2 - \frac{rx}{L^2} = PM^2.$ | |
| 47, *e* 1.<br>17 + 19 | 20 | $\left\{\begin{array}{l} HP^2 + PN^2 = (HN^2 =)\ \text{Q.Rad.} \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = \text{Q.Rad.} \end{array}\right.$ | |
| 47, *e* 1.<br>17 + 19 | 20 | $\left\{\begin{array}{l} HP^2 + PM^2 = (HM^2 =)\ \text{Q.Rad.} \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = \text{Q. Rad.} \end{array}\right.$ | |
| | | | $\frac{x^4}{L^2}$ |

| | | | |
|---|---|---|---|
| 20=10 | 21 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} - \frac{rx^2}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 22 | $x^3 \ast - qx - r = 0$. *Q.e.d.* in Cubic. | *Fig.*13. |
| 20=12 | 23 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 24 | $x^4 - qx^2 - rx = S$. | |
| *Tranſp.* | 25 | $x^4 \ast - qx^2 - rx - S = 0$. *Q.e.d.* ſi—S. | *Fig.*14. |
| 20=13 | 26 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 27 | $x^4 - qx^2 - rx = - S$. | |
| *Tranſp.* | 28 | $x^4 \ast - qx^2 - rx + S = 0$. *Q.e.d.* ſi+S. | *Fig:*15. |

$$2.\ x^3 \ast - qx + r = 0 \left\{ \begin{matrix} 2 \\ 4 \end{matrix} \right\} \left\{ \begin{matrix} x^4 \ast - qx^2 + rx - S = 0 \\ x^4 \ast - qx^2 + rx + S = 0 \end{matrix} \right\}$$

*Demonſtrat.*

| | | |
|---|---|---|
| *Supp.* | 14 | $MO = x$. |
| *Supp.* | 14 | $NO = - x$. |
| *Ob para.* | 15 | $\left\{ \begin{matrix} L \,.\, MO :: MO \,.\, AO. \\ L \,.\, x :: x \,.\, \frac{x^2}{L} = AO. \end{matrix} \right.$ |
| *Ob para.* | 15 | $\left\{ \begin{matrix} L \,.\, NO :: NO \,.\, AO. \\ L : - x :: - x \,.\, \frac{x^2}{L} = AO. \end{matrix} \right.$ |
| $15 \backsim 8$ | 16 | $\left\{ \begin{matrix} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} (\backsim b, u) - \frac{L}{2} - \frac{q}{2L} = HP. \end{matrix} \right.$ |
| ⊙ | 17 | $b^2 + \frac{x^4}{L^2} - x^2 - \frac{qx^2}{L^2} = HP^2$. |
| 14+9 | 18 | $\left\{ \begin{matrix} MO + (OP, u)\ DH = PM. \\ x\ (+d, u) + \frac{r}{2L^2} = PM. \end{matrix} \right.$ |

| | | | |
|---|---|---|---|
| 14 $\backsim$ 9 | 18 | $\left\{\begin{array}{l} NO\ (\backsim OP, u)\ DH = PN. \\ -x\ (-d; u) - \frac{r}{2L^2} = PN. \end{array}\right.$ | |
| ⊙ | 19 | $d^2 + x^2 + \frac{rx}{L^2} = PM^2.$ | |
| ⊙ | 19 | $d^2 + x^2 + \frac{rx}{L^2} = PN^2.$ | |
| 47, e 1<br>17 + 19 | 20 | $\left\{\begin{array}{l} HP^2 + PM^2 = (HM^2 =)\ Q.\ Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = Q.\ Rad. \end{array}\right.$ | |
| 47, e 1<br>17 + 19 | 20 | $\left\{\begin{array}{l} HP^2 + PN^2 = (HN^2 =)\ Q.\ Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = Q.\ Rad. \end{array}\right.$ | |
| 20 = 10 | 21 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| × $\frac{L^2}{x}$ | 22 | $x^3 \ast - qx + rx = 0$. *Q.e.d.* in Cubic. | *Fig.* 13. |
| 20 = 12 | 23 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| × *L*² | 24 | $x^4 - qx^2 + rx = S.$ | |
| *Tranſp.* | 25 | $x^4 \ast - qx^2 + rx - S = 0$. *Q.e.d.* ſi $-S$. | *Fig.* 14. |
| 20 = 13 | 26 | $\frac{x^4}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| × L² | 27 | $x^4 - qx^2 + rx = - S.$ | |
| *Tranſp.* | 28 | $x^4 \ast - qx^3 + rx + S = 0$; *Q.e.d.* ſi $+S$. | *Fig.* 15. |

*Illuſtrat.*

$$\begin{array}{l} \qquad\quad q. \qquad\quad r. \\ \left\{\begin{array}{l} x^3 \ast - 300x - 1703 = 0 \\ x^3 \ast - 3.00x - 1.703 = 0 \end{array}\right\} \\ NO = x = 19.6 + \left\{\begin{array}{l} MO = -x = -13. \\ mo = -x = -6.6 + \end{array}\right. \end{array}$$

*Fig.* 13.

$$\begin{array}{l} \qquad\quad q. \qquad\quad r. \\ \left\{\begin{array}{l} x^3 \ast - 300x + 1703 = 0 \\ x^3 \ast - 3.00x + 1.703 = 0 \end{array}\right\} \\ \left\{\begin{array}{l} MO = x = 13. \\ mo = x = 6.6 \end{array}\right\} NO = -x = -19.6 + \end{array}$$

*Central.*

$$\left.\begin{array}{l}\frac{L}{2}=0.5\\ +\frac{q}{2L}=1.5\\ \hline b=2.5=AD\end{array}\right\}\quad \frac{r}{2L^2}=0.8515=d=DH.$$

$$\left\{\begin{array}{l} x^4 * - \overset{q.}{144}x^2 - \overset{r.}{1200}x - \overset{s.}{7600}=0\\ x^4 * - 1.44x^2 - 1.200x - 0.7600=0\end{array}\right\}$$

$NO = x = 15.8+$

$MO = -x = -10.$

Fig. 14.

$$\left\{\begin{array}{l} x^4 * - \overset{q.}{144}x^2 + \overset{r.}{1200}x - \overset{s.}{7600}=0\\ x^4 * - 1.44x^2 + 1.200x - 0.7600=0\end{array}\right\}$$

$MO = x = 10.$

$NO = -x = -15.8$

*Central.*

$$\left.\begin{array}{l}\frac{L}{2}=0.5\\ \frac{q}{2L}=0.72\\ \hline b=1.22=AD\end{array}\right\}\quad \frac{r}{2L^2}=0.600=d=DH.$$

$$\left\{\begin{array}{l} x^4 * - \overset{q.}{376}x^2 - \overset{r.}{960}x + \overset{s.}{18000}=0\\ x^4 * - 3.76x^2 - 0.960x + 1.8000=0\end{array}\right\}$$

$$\left\{\begin{array}{l} NO = x = 19.435+\\ no = x = 6.\end{array}\right\}\left\{\begin{array}{l} MO = -x = -15.435\\ mo = -x = -10.\end{array}\right\}$$

Fig. 15.

$$\left\{\begin{array}{l} x^4 * - \overset{q.}{376}x^2 + \overset{r.}{960}x + \overset{s.}{18000}=0\\ x^4 * - 3.76x^2 + 0.960x + 1.8000=0\end{array}\right\}$$

$$\left\{\begin{array}{l} MO = x = 15.435-\\ mo = x = 10.\end{array}\right\}\left\{\begin{array}{l} NO = -x = -19.435\\ no = -x = -6.\end{array}\right\}$$

Cen-

*Central.*

$$\frac{L}{2}+\frac{q}{2L}=0.5+1.88=2.38=b. \qquad \frac{r}{2L^2}=0.480=d.$$

---

*Caſ.* 2. Ubi $+q$.

$$\left\{\begin{array}{l} 1.\ \frac{L}{2} \sqsupset \frac{q}{2L} \\ 2.\ \frac{q}{2L} \sqsupset \frac{L}{2} \end{array}\right\}$$

$$3.\ x^3 * + qx - r = 0 \left\{\begin{array}{c} 5 \\ 7 \end{array}\right\} \left\{\begin{array}{l} x^4 * + qx^2 - rx - S = 0 \\ x^4 * + qx^2 - rx + S = 0 \end{array}\right\}$$

*Demonſtrat.*

| | | | |
|---|---|---|---|
| 1 — 2 | 8 | $Ab - bD = AD = b.$<br>$\frac{L}{2} - \frac{q}{2L} = b = AD.$    1. Ubi $\frac{L}{2} \sqsupset \frac{q}{2L}$. | |
| 3 | 9 | $\frac{r}{2L^2} = d = DH.$ | |
| 47, *e* 1<br>Q. 8. +<br>Q. 9. | 10 | $AD^2 + DH^2 = (HA^2 =)$ Q. Rad.<br>$b^2 + d^2 =$ Q. Rad. in Cubic. | *Fig.*16. |
| 4 × 5 | 11 | $AI \times AK =$ (ob Circl.) $AL^2 =$ (per conſtr.) $AZ^2$.<br>$(L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2.$ | |
| 47, *e* 1<br>10 + 11 | 12 | $AH^2 + AL^2 = (HL^2 =)$ Q. Rad.<br>$b^2 + d^2 + \frac{S}{L^2} =$ Q. Rad.    In Biquadr. ſi — S. | *Fig.*17. |
| 47, *e* 1<br>10 — 11 | 13 | $AH^2 - AZ^2 = (HZ^2 =)$ Q. Rad.<br>$b^2 + d^2 - \frac{S}{L^2} =$ Q. Rad.    In Biquadr. ſi + S. | *Fig.*18. |
| *Supp.* | 14 | $NO = x.$ | |
| *Supp.* | 14 | $MO = -x.$ | |

L . NO

| | | | |
|---|---|---|---|
| *Ob para.* | 15 | $\left\{\begin{array}{l} L\ .\ NO :: NO\ .\ AO. \\ L\ .\ x :: x\ .\ \frac{x^2}{L} = AO. \end{array}\right.$ | |
| *Ob para.* | 15 | $\left\{\begin{array}{l} L\ .\ MO :: MO\ .\ AO. \\ L\ .\ -x :: -x\ .\ \frac{x^2}{L} = AO. \end{array}\right.$ | |
| $15 \backsim 8$ | 16 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L}\ (\backsim b, u) - \frac{L}{2} + \frac{q}{2L} = HP. \end{array}\right.$ | |
| © | 17 | $b^2 + \frac{x^4}{L^2} - x^2 + \frac{qx^2}{L^2} = HP^2.$ | |
| $14 \backsim 9$ | 18 | $\left\{\begin{array}{l} NO \backsim (OP, u)\ DH = PN. \\ x\ (\backsim d, u) - \frac{r}{2L^2} = PN. \end{array}\right.$ | |
| $14+9$ | 18 | $\left\{\begin{array}{l} MO + (PO, u)\ DH = PM. \\ -x\ (+d, u) + \frac{r}{2L^2} = PM. \end{array}\right.$ | |
| © | 19 | $d^2 + x^2 - \frac{rx}{L^2} = PN^2.$ | |
| © | 19 | $d^2 + x^2 - \frac{rx}{L^2} = PM^2.$ | |
| 47, *e* 1<br>$17+19$ | 20 | $\left\{\begin{array}{l} HP^2 + PN^2 = (HN^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = Q.Rad. \end{array}\right.$ | |
| 47, *e* 1<br>$17+19$ | 20 | $\left\{\begin{array}{l} HP^2 + PM^2 = (HM^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = Q.Rad. \end{array}\right.$ | |
| $20 = 10$ | 21 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0;$ in $\frac{L^2}{x}.$ | |
| $\times \frac{L^2}{x}$ | 22 | $x^3 * + qx - r = 0.$ *Q.e.d.* in Cubic. | *Fig.*16. |
| $20 = 12$ | 23 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2};$ in $L^2$. | |
| $\times L^2$ | 24 | $x^4 + qx^2 - rx = S.$ | |
| *Tranſp.* | 25 | $x^4 * + qx^2 - rx - S = 0.$ *Q.e.d.* in Biquadr. ſi $-S$. | *Fig.*17. |

$\frac{x^4}{L^2}$

| | | | |
|---|---|---|---|
| 20 = 13 | 26 | $\frac{x^4}{L^2}+\frac{qx^2}{L^2}-\frac{rx}{L^2}=-\frac{S}{L^2}$; in $L^2$. | |
| × *L²* | 27 | $x^4+qx^2-rx=-S$. | |
| *Tranſp.* | 28 | $x^4 * + qx^2 - rx + S = 0$. *Q.e.d.* in Biquad. ſi $+S$. | *Fig.*18. |
| | | 4. $x^3 * + qx + r = 0 \left\{\begin{matrix}6\\8\end{matrix}\right\} \left\{\begin{matrix}x^4 * + qx^2 + rx - S = 0\\x^4 * + qx^2 + rx + S = 0\end{matrix}\right\}$ | |
| *Supp.* | 14 | $MO = x$. | |
| *Supp.* | 14 | $NO = -x$. | |
| *Ob para.* | 15 | $\left\{\begin{array}{l}L \,.\, MO :: MO \,.\, AO.\\ L \,.\, x :: x \,.\, \frac{x^2}{L} = AO.\end{array}\right.$ | |
| *Ob para.* | 15 | $\left\{\begin{array}{l}L \,.\, NO :: NO \,.\, AO.\\ L \,.\, -x :: -x \,.\, \frac{x^2}{L} = AO.\end{array}\right.$ | |
| 15 ∽ 8 | 16 | $\left\{\begin{array}{l}AO \backsim AD = (DO,\ u)\ HP.\\ \frac{x^2}{L}\ (\backsim b,\ u) - \frac{L}{2} + \frac{q}{2L} = HP.\end{array}\right.$ | |
| ◎ | 17 | $b^2 + \frac{x^4}{L^2} - x^2 + \frac{qx^2}{L^2} = HP^2$. | |
| 14 + 9 | 18 | $\left\{\begin{array}{l}MO + (OP,\ u)\ DH = PM.\\ x\ (+d,\ u) + \frac{r}{2L^2} = PM.\end{array}\right.$ | |
| 14 ∽ 9 | 18 | $\left\{\begin{array}{l}NO \backsim (OP,\ u)\ DH = PN.\\ -x\ (-d,\ u) - \frac{r}{2L^2} = PN.\end{array}\right.$ | |
| ◎ | 19 | $d^2 + x^2 + \frac{rx}{L^2} = PM^2$. | |
| ◎ | 19 | $d^2 + x^2 + \frac{rx}{L^2} = PN^2$. | |
| 47, e 1<br>17 + 19 | 20 | $\left\{\begin{array}{l}HP^2 + PM^2 = (HM^2 =)\ Q.Rad.\\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = Q.Rad.\end{array}\right.$ | |

| | | | |
|---|---|---|---|
| 47, e 1<br>17+19 | 20 | $\left\{\begin{array}{l} HP^2+PN^2=(HN^2=)\ Q.\ Rad. \\ b^2+d^2+\frac{x^4}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=Q.\ Rad. \end{array}\right.$ | |
| 20=10 | 21 | $\frac{x^4}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=0$; in $\frac{L^2}{x}$. | |
| $\times\frac{L^2}{x}$ | 22 | $x^3 * + qx + rx = 0$. *Q.e.d.* in Cubic. | *Fig.* 16. |
| 20=12 | 23 | $\frac{x^4}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=+\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 24 | $x^4+qx^2+rx=S$. | |
| *Tranſp.* | 25 | $x^4 * + qx^2 + rx - S = 0$. *Q.e.d.* in Biquadr. ſi $-S$. | *Fig.* 17. |
| 20=13 | 26 | $\frac{x^4}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=-\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 27 | $x^4+qx^2+rx=-S$. | |
| *Tranſp.* | 28 | $x^4 * + qx^2 + rx + S = 0$. *Q.e.d.* in Biquadr. ſi $+S$. | *Fig.* 18. |

*Illuſtrat.*

$$\left\{\begin{array}{l} x^3 * + \overset{q.}{50x} - \overset{r.}{912} = 0 \\ x^3 * + 0.50x - 0.912 = 0 \end{array}\right\} \quad NO = x = 8.$$

$$\left\{\begin{array}{l} x^3 * + \overset{q.}{50x} + \overset{r.}{912} = 0 \\ x^3 * + 0.50x + 0.912 = 0 \end{array}\right\} \quad NO = -x = -8.$$

*Fig.* 16.

*Central.*

$$\left.\begin{array}{r} \frac{L}{2} = 0.5 \\ -\frac{q}{2L} = 0.25 \\ \hline b = 0.25 = AD \end{array}\right\} \quad \frac{r}{2L^2} = 0.456 = d = DH.$$

$$\left\{\begin{array}{l} x^4 * + \overset{q.}{50x^2} - \overset{r.}{1200x} - \overset{s.}{13536} = 0 \\ x^4 * + 0.50x^2 - 1.200x - 1.3536 = 0 \end{array}\right\}$$

*Fig.* 17.

$NO = x = 12.$

$MO = -x = -7.1-$

*Fig.*17.

$$\begin{array}{l} \qquad\qquad\quad q. \qquad\qquad\quad r. \qquad\qquad\quad s. \\ \left\{\begin{array}{l} x^4 \ast + 050\,x^2 + 1200\,x - 13536 = 0 \\ x^4 \ast + 0.50\,x^2 + 1.200\,x - 1.3536 = 0 \end{array}\right\} \end{array}$$

$MO = x = 7.1$ *ferè.*

$NO = -x = -12.$

*Central.*

$$\left.\begin{array}{r} \frac{L}{2} = 0.5 \\ -\frac{q}{2L} = 0.25 \\ \hline b = 0.25 = AD \\ \hline \end{array}\right\} \frac{r}{2L^2} = 0.600 = d = DH.$$

$$\begin{array}{l} \qquad\qquad\quad q. \qquad\qquad\quad r. \qquad\qquad\quad s. \\ \left\{\begin{array}{l} x^4 \ast + 50\,x^2 - 2828\,x + 6000 = 0 \\ x^4 \ast + 0.50\,x^2 - 2.828\,x + 0.6000 = 0 \end{array}\right\} \end{array}$$

$NO = x = 12.$

$no = x = 2.2 +$

*Fig.*18.

$$\begin{array}{l} \qquad\qquad\quad q. \qquad\qquad\quad r. \qquad\qquad\quad s. \\ \left\{\begin{array}{l} x^4 \ast + 50\,x^2 + 2828\,x + 6000 = 0 \\ x^4 \ast + 0.50\,x^2 + 2.828\,x + 0.6000 = 0 \end{array}\right\} \end{array}$$

$NO = -x = -12.$

$no = -x = -2.2$

*Central.*

$$\left.\begin{array}{r} \frac{L}{2} = 0.5 \\ -\frac{q}{2L} = 0.25 \\ \hline b = 0.25 = b = AD. \\ \hline \end{array}\right\} \frac{r}{2L^2} = 1.414 = d = DH.$$

2. Ubi $\frac{q}{2L} \sqsupset \frac{L}{2}.$

3. $x^3 \ast + qx - r = 0 \left\{\begin{array}{l} 5 \\ 7 \end{array}\right\} \left\{\begin{array}{l} x^4 \ast + qx^2 - rx - S = 0 \\ x^4 \ast + qx^2 - rx + S = 0 \end{array}\right\}$

*Demonſtr.*

### *Demonſtrat.*

| | | | | |
|---|---|---|---|---|
| $2-1$ | 8 | $bD - Ab = b = AD.$<br>$\frac{q}{2L} - \frac{L}{2} = b = AD.$ | | |
| 3 | 9 | $\frac{r}{2L^2} = d = DH.$ | | |
| 47, *e* 1<br>*Q*. 8. +<br>*Q*. 9. | 10 | $AD^2 + DH^2 = (HA^2)$<br>$b^2 + d^2 = (HA^2 =)$ Q. Rad. in Cubic. | | *Fig*. 19. |
| $4 \times 5$ | 11 | $AI \times AK =$ (ob Circl.) $AL^2 =$ (per conſtr.) $AZ^2.$<br>$(L \times \frac{S}{L^3} =) \frac{S}{L^2} = AL^2 = AZ^2.$ | | |
| 47, *e* 1<br>$10+11$ | 12 | $AH^2 + AL^2 = (HL^2 =)$ Q. Rad.<br>$b^2 + d^2 + \frac{S}{L^2} =$ Q. Rad. | In Biquadr.<br>ſi $-$ S. | *Fig*. 20. |
| 47, *e* 1<br>$10-11$ | 13 | $AH^2 - AZ^2 = (HZ^2 =)$ Q. Rad.<br>$b^2 + d^2 - \frac{S}{L^2} =$ Q. Rad. | In Biquadr.<br>ſi $+$ S. | *Fig*. 21. |
| *Supp.* | 14 | $NO = x.$ | | |
| *Supp.* | 14 | $MO = -x.$ | | |
| *Ob para.* | 15 | $L . NO :: NO . AO.$<br>$L . x :: x . \frac{x^2}{L} = AO.$ | | |
| *Ob para.* | 15 | $L . MO :: MO . AO.$<br>$L . -x :: -x . \frac{x^2}{L} = AO.$ | | |
| $15+8$ | 16 | $AO + AD = (DO, u)\ HP.$<br>$\frac{x^2}{L}\ (+b, u) + \frac{q}{2L} - \frac{L}{2} = HP.$ | | |
| Ⓖ | 17 | $b^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} - x^2 = HP^2.$ | | |
| $9 \backsim 14$ | 18 | $(OP, u)\ DH \backsim NO = PN.$<br>$(d, u)\ \frac{r}{2L^2} - x = PN.$ | | |

| | | | |
|---|---|---|---|
| 14 + 9 | 18 | $\begin{cases} MO + (PO, u)\ DH = PM. \\ -x\ (+d, u) + \frac{r}{2L^2} = PM. \end{cases}$ | |
| ⊙ | 19 | $d^2 + x^2 - \frac{rx}{L^2} = PN^2.$ | |
| ⊙ | 19 | $d^2 + x^2 - \frac{rx}{L^2} = PM^2.$ | |
| 47, e 1<br>17 + 19 | 20 | $\begin{cases} HP^2 + PN^2 = (HN^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = Q.Rad. \end{cases}$ | |
| 47, e 1<br>17 + 19 | 20 | $\begin{cases} HP^2 + PM^2 = (HM^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = Q.Rad. \end{cases}$ | |
| 20 = 10 | 21 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 22 | $x^3 * + qx - r = 0.$ *Q.e.d.* in Cubic. | *Fig.*19. |
| 20 = 12 | 23 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| × $L^2$ | 24 | $x^4 + qx^2 - rx = S.$ | |
| *Tranſp.* | 25 | $x^4 * + qx^2 - rx - S = 0.$ *Q.e.d.* in Biquadr. ſi $-S$. | *Fig.*20. |
| 20 = 13 | 26 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| × $L^2$ | 27 | $x^1 + qx^2 - rx = -S.$ | |
| *Tranſp.* | 28 | $x^4 * + qx^2 - rx + S = 0.$ *Q.e.d.* in Biquad. ſi $+S$. | *Fig.*21. |
| | | 4. $x^3 * + qx + r = 0 \left\{ \begin{matrix} 6 \\ 8 \end{matrix} \right\} \left. \begin{matrix} x^4 * + qx^2 + rx - S = 0 \\ x^4 * + qx^2 + rx + S = 0 \end{matrix} \right\}$ | |
| *Supp.* | 14 | $MO = x.$ | |
| *Supp.* | 14 | $NO = -x.$ | |
| *Ob para.* | 15 | $\begin{cases} L\ .\ MO :: MO\ .\ AO. \\ L\ .\ x :: x\ .\ \frac{x^2}{L} = AO. \end{cases}$ | |

L . NO

| | | | |
|---|---|---|---|
| *Ob para.* | 15 | $\left\{\begin{array}{l} L \;.\; NO :: NO \;.\; AO. \\ L : -x :: -x \;.\; \frac{x^2}{L} = AO. \end{array}\right.$ | |
| $15 \backsim 8$ | 16 | $\left\{\begin{array}{l} AO \pm AD = (DO, u)\; HP. \\ \frac{x^2}{L}\;(+b, u) + \frac{q}{2L} - \frac{L}{2} = HP. \end{array}\right.$ | |
| ⊙ | 17 | $b^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} - x^2 = HP^2.$ | |
| $14 + 9$ | 18 | $\left\{\begin{array}{l} MO + (OP, u)\; DH = PM. \\ x\;(+d, u) + \frac{r}{2L^2} = PM. \end{array}\right.$ | |
| $9 \backsim 14$ | 18 | $\left\{\begin{array}{l} (OP, u)\; DH \backsim NO = PN. \\ (d, u)\; \frac{r}{2L^2} + x = PN. \end{array}\right.$ | |
| ⊙ | 19 | $d^2 + x^2 + \frac{rx}{L^2} = PM^2.$ | |
| ⊙ | 19 | $d^2 + x^2 + \frac{rx}{L^2} = PN^2.$ | |
| 47, e 1<br>$17 + 19$ | 20 | $\left\{\begin{array}{l} HP^2 + PM^2 = (HM^2 =)\; Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = Q.Rad. \end{array}\right.$ | |
| 47, e 1<br>$17 + 19$ | 20 | $\left\{\begin{array}{l} HP^2 + PN^2 = (HN^2 =)\; Q.Rad. \\ b^2 + d + \frac{x^4}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = Q.Rad. \end{array}\right.$ | |
| $20 = 10$ | 21 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0;$ in $\frac{L^2}{x}.$ | |
| $\times \frac{L^2}{x}$ | 22 | $x^3 \ast + qx + r = 0.$ *Q. e. d.* in Cubic. | *Fig.* 19. |
| $20 = 12$ | 23 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2};$ in $L^2$. | |
| $\times L^2$ | 24 | $x^4 + qx^2 + rx = S.$ | |
| *Tranſp.* | 25 | $x^4 \ast + qx^2 + rx - S = 0.$ *Q.e.d.* in Biquadr. ſi $-S$. | *Fig.* 20. |
| $20 = 13$ | 26 | $\frac{x^4}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2};$ in $L^2$. | |
| $\times L^2$ | 27 | $x^4 + qx^2 + rx = -S.$ | |
| *Tranſp.* | 28 | $x^4 \ast + qx^2 + rx + S = 0.$ *Q. e. d.* in Biquadr. ſi $+S$. | *Fig.* 21. |

*Illuſtrat.*

*Illuſtrat.*

$$\left\{\begin{array}{l} x^3 * + \overset{q.}{384}\,x - \overset{r.}{3584} = 0 \\ x^3 * + 3.84\,x - 3.584 = 0 \end{array}\right\} \quad NO = x = 8.$$

$$\left\{\begin{array}{l} x^3 * + \overset{q.}{384}\,x + \overset{r.}{3584} = 0 \\ x^3 * + 3.84\,x + 3.584 = 0 \end{array}\right\} \quad NO = -x = -8.$$

*Fig.*19.

*Central.*

$$\left.\begin{array}{r} \frac{q}{2L} = 1.92 \\ -\frac{L}{2} = 0.5 \\ \hline b = 1.42 = AD. \\ \hline \end{array}\right\} \quad \frac{r}{2L^2} = 1.792 = d = DH.$$

$$\left\{\begin{array}{l} x^4 * + \overset{q.}{396}\,x^2 - \overset{r.}{3256}\,x - \overset{s.}{17040} = 0 \\ x^4 * + 3.96\,x^2 - 3.256\,x - 1.7040 = 0 \end{array}\right\}$$

$NO = x = 10.$

$MO = -x = -$ 3.6 &c.

$$\left\{\begin{array}{l} x^4 * + \overset{q.}{396}\,x^2 + \overset{r.}{3256}\,x - \overset{s.}{17040} = 0 \\ x^4 * + 3.96\,x^2 + 3.256\,x - 1.7040 = 0 \end{array}\right\}$$

$MO = x =$ 3.6 &c.

$NO = -x = -10.$

*Fig.*20.

Hactenus de Æquationibus quartum gradum non excedentibus, ſub nullo extremo gradu Parodico affectis; vel, ubi deficit quantitas (p); in quibus omnes Rectæ ad Axem applicantur: Jam de reliquis, ubi omnes rectæ ad Diametrum ſunt applicandæ.

CLAS.

*Central.*

$$\left.\begin{array}{l} \frac{q}{2L} = 1.98 \\ -\frac{L}{2} = 0.5 \\ \hline b = 1.48 = AD. \\ \hline \end{array}\right\} \frac{r}{2L^2} = 1.628 = d = DH.$$

$$\left\{\begin{array}{l} x^4 * + \overset{q.}{200\,x^2} - \overset{r.}{3600\,x} + \overset{s.}{6000} = 0 \\ x^4 * + 2.00\,x^2 - 3.600\,x + 0.6000 = 0 \end{array}\right\}$$

$NO = x = 10.$

$no = x = 1.8 +$

Fig.20.

$$\left\{\begin{array}{l} x^4 * + \overset{q.}{200\,x^2} + \overset{r.}{3600\,x} + \overset{s.}{6000} = 0 \\ x^4 * + 2.00\,x^2 + 3.600\,x + 0.6000 = 0 \end{array}\right\}$$

$NO = -x = -10.$

$no = -x = -1.8 +$

*Central.*

$$\left.\begin{array}{l} \frac{q}{2L} = 1.00 \\ \frac{L}{2} = 0.5 \\ \hline b = 0.5 = AD. \\ \hline \end{array}\right\} \frac{r}{2L^2} = 1.800 = d = DH.$$

Hitherto have we treated of Æquations, not exceeding the fourth degree, affected under no extream Parodic; or, where the Quantity (p) is wanting; in which all Right Lines are referred to the Axe. Now, of the reſt, where p is had, where all Right Lines are to be applied to the Diameter.

CLAS.

# CLAS. V.

*De Æquationibus linearibus; & Quadrato-quadraticis conſtruendis, & Radicibus Geometricè inveſtigandis, affectis tantum ſub tertio Gradu Parodico; vel, de Æquationibus unius, vel quatuor Dimenſionum, ubi deficiunt tertius & quartus Terminus (q & r.)*

AD haſce ſequentes formulas omnes hujus generis Æquationes reduci poſſint.

$$\left\{\begin{array}{l} 1.\ x-p=0 \left\{\begin{array}{l} 1 \\ 3 \end{array}\right. \left\{\begin{array}{l} x^4-px^3 * * -S=0 \\ x^4-px^3 * * +S=0 \end{array}\right\} \\ 2.\ x+p=0 \left\{\begin{array}{l} 2 \\ 4 \end{array}\right. \left\{\begin{array}{l} x^4+px^3 * * -S=0 \\ x^4+px^3 * * +S=0 \end{array}\right\} \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2}+\frac{p^2}{8L}=b=AD. \quad \frac{p}{4}+\frac{p^3}{16L^2}=d=DH.$$

*Reg.Gen.* Supponatur itaque Parabolam (NAM) jam de-
ſcriptam eſſe, cujus Latus Rectum ſit L (ceu 1),
Axiſque (Ay); ad quem ordinatim applicetur Recta
1 1 $BA=\frac{p}{2}$, occurrens Parabolæ in B & A: Et ex A
(puta) ducatur Diameter, vel Axi parallela Ay; in
2 2 quâ ſumendo $Ab=\frac{L}{2}$, oportet facere $bD=\frac{p^2}{8L}$,
2 3 eamque ſumere in Diametro continuatâ deorſum ver-
ſus y; (vel ſic, ſumatur $AD=\frac{L}{2}+\frac{p^2}{8L}$, eamque
deorſum in Diametro, collocando.

Porrò

# CLAS. V.

*Of the conſtruction of Linear Equations, and of Quadrato-quadratics, and of the invention of their Roots geometrically, affected only under the third Parodic Degree; or of Equations, of one only, or of four Dimenſions, where the third and fourth Terms are wanting,* (viz. *q and r.*)

ALL Equations of theſe ſorts may be reduced to theſe following forms.

$$\left\{\begin{array}{l} 1.\ x-p=0\left\{\begin{array}{l}1\\3\end{array}\right.\left\{\begin{array}{l}x^4-px^3\ast\ast-S=0\\x^4-px^3\ast\ast+S=0\end{array}\right\} \\ 2.\ x+p=0\left\{\begin{array}{l}2\\4\end{array}\right.\left\{\begin{array}{l}x^4+px^3\ast\ast-S=0\\x^4+px^3\ast\ast+S=0\end{array}\right\} \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2}+\frac{p^2}{8L}=b=AD.\quad \frac{p}{4}+\frac{p^3}{16L^2}=d=DH.$$

*Gen. Rule* Let the Parabole (NAM) therefore be ſuppoſed to be already deſcribed, whoſe *Latus Rectum* is L (or 1), and Axe (Ay); to which, let the Ordinate

1 1 $BA=\frac{p}{2}$ be applied, meeting the Parabole in B and A: And from A (ſuppoſe) draw a Diameter, or a Parallel to the Axe (*viz.* Ay); in which, making

2 2 $Ab=\frac{L}{2}$, muſt be made $bD=\frac{p^2}{8L}$, ſetting it down-

2 3 wards on the Diameter continued towards y; (or you may make $AD=\frac{L}{2}+\frac{p^2}{8L}$, and place it downwards on the Diameter.

Porrò ex D, erigenda eſt ad Diametrum perpendi-
4 4 cularis $DH = \frac{p}{4} + \frac{p^3}{16L^2}$; (vel ſumatur $De = \frac{p}{4}$, &
5 ulterius ad ſiniſtram, $eH = \frac{p^3}{16L^2}$:) Tum ex Centro
H, oportet Circulum deſcribere, cujus Semidiame-
6 6 ter ſit H A, ſi in Æquatione non habeatur Quanti- *Fig.*22.
tas S.

7 Aſt ſi habeatur S, & ſit quidem $-S$, oportet ulte-
riùs in hàc lineâ H A, utrinque productâ, ex unâ
8 7 parte ſumere $AI = L$; & ex alterâ parte, $AK = \frac{S}{L^3}$;
9 8 deſcriptoque Semicirculo, cujus Diameter I K, eri-
10 9 gere A L perpendicularem ad A H, quæ occurrat huic
Semicirculo (I L K) in puncto L.

11 Quòd ſi verò habeatur $+S$; oportet inſuper in
alio Semicirculo, cujus Diameter ſit H A, inſcribere
10 $AZ = AL$, inventæ. Circulus igitur deſcriptus, tran- *Fig.*23.
12 ſiens per L, ſi ſit $-S$; per Z verò, ſi ſit $+S$, ſecabit *Fig.*24.
vel tanget Parabolam, in 1 ceu 2 punctis, à quibus ſi ad
Diametrum demittantur Perpendiculares, obtinebun-
tur omnes Æquationes radices, tam falſæ, quam veræ.
Quarum quidem veræ (ut N O) ad ſiniſtram cadent,
14 & falſæ (ut M O) ad dextram, ſi in Æquatione ha-
15 beatur $-p$: Sed contra, ſi habeatur ibi $+p$, veræ qui-
dem cadent (ut M O) ad dextram, falſæ verò (ut N O)
ad ſiniſtram.

$$1.\ x - p = 0 \left\{ \begin{matrix} 1 \\ 3 \end{matrix} \right\} \left\{ \begin{matrix} x^4 - px^3 * * - S = 0 \\ x^4 - px^3 * * + S = 0 \end{matrix} \right\}.$$

*Demonſtrat.*

2 + 3 11 $\left\{ \begin{array}{l} Ab + bD = b = AD. \\ \frac{L}{2} + \frac{p^2}{8L} = b = AD. \end{array} \right.$

4 + 5 12 $\left\{ \begin{array}{l} De + eH = d = DH. \\ \frac{p}{4} + \frac{p^3}{16L^2} = d = DH. \end{array} \right.$

Then from D, erect a Perpendicular to the Diameter
4 4 $DH = \frac{p}{4} + \frac{p^3}{16L^2}$; (or take $De = \frac{p}{4}$, and farther
5 towards the left hand make $eH = \frac{p^3}{16L^2}$ :) Then from
the Center H muſt a Circle be deſcribed, whoſe Semidiameter
6 6 is HA, if in the Equation the Quantity S be not had. Fig.22.

7 But if S be had, and it be −S, then muſt there farther
in this Line AH, both ways produced, be taken on
8 9 7 8 the one ſide $AI = L$, and on the other ſide $AK = \frac{S}{L^3}$;
and a Semicircle being deſcribed, whoſe Diameter IK,
10 9 muſt be erected AL perpendicular to AH, which may
meet this Semicircle (ILK) in the Point L.

11 But if + S be had; then moreover in another
Semicircle, whoſe Diameter is HA, muſt be inſcribed
10 $AZ = AL$ found. A Circle therefore deſcribed paſſing Fig.23.
12 through L, if it be − S; but through Z, if it be Fig.24.
+S, will cut or touch the Parabole in 1 or 2 Points;
from which, if Perpendiculars be demitted to the Diameter,
all the Roots of the Equation, as well falſe as
14 true, will be had; of which, the true (as NO) will
fall to the left hand, and the falſe (as MO) on the
right, if in the Equation be had −p: But if in it be
had +p, then the true Roots (as MO) will fall on
the right hand, but the falſe (as NO) to the left.

47, e 1 Q.11.+ Q.12. 13
$$\left\{\begin{aligned} &AD^2 + DH^2 = HA^2. \\ &b^2 + d^2 = HA^2. \end{aligned}\right.$$

7 × 8 14 Fig.22.
$$\left\{\begin{aligned} &AI \times AK = (\text{ob Circl.})\ AL^2 = (\text{per conſtr.})\ AZ^2. \\ &\left(L \times \frac{S}{L^3} =\right) \frac{S}{L^2} = AL^2 = AZ^2. \end{aligned}\right.$$

47, e 1 13 + 14 15 Fig.23.
$$\left\{\begin{aligned} &HA^2 + AL^2 = (HL^2 =)\ Q.\ Rad. \\ &b^2 + d^2 + \frac{S}{L^2} = Q.\ Rad. \end{aligned}\right\} \left\{\begin{aligned} &\text{In Biquadr.} \\ &\text{ſi} - S. \end{aligned}\right\}$$

| | | | | |
|---|---|---|---|---|
| 47, e 1<br>13 — 14 | 16 | $\left\{ \begin{array}{l} HA^2 - AZ^2 = (HZ^2 =)\ Q.Rad. \\ b^2 + d^2 - \frac{S}{L^2} = Q.Rad. \end{array} \right\}$ | In Biquadr.<br>ſi + S. | *Fig.24.* |
| *Supp.* | 17 | $NO = x.$ | | |
| *Supp.* | 17 | $MO = -x.$ | | |
| 17 — 1 | 18 | $\left\{ \begin{array}{l} NO - (OF, u)\ BA = (FN, u)\ OR. \\ x - \frac{p}{2} = OR. \end{array} \right.$ | | |
| 17 + 1 | 18 | $\left\{ \begin{array}{l} MO + (OF, u)\ BA = (FM, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{array} \right.$ | | |
| *Ob para.* | 19 | $\left\{ \begin{array}{l} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ x :: x - \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array} \right.$ | | |
| *Ob para.* | 19 | $\left\{ \begin{array}{l} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ -x :: -x + \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array} \right.$ | | |
| 19 ∽ 11 | 20 | $\left\{ \begin{array}{l} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} - \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p2}{8L} = HP. \end{array} \right.$ | | |
| ⊙ | 21 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{p^2x^2}{L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} = HP^2.$ | | |
| 17 ∽ 12 | 22 | $\left\{ \begin{array}{l} NO \backsim (OP, u)\ DH = PN. \\ x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} = PN. \end{array} \right.$ | | |
| 17 + 12 | 22 | $\left\{ \begin{array}{l} MO + (OP, u)\ DH = PM. \\ -x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} = PM. \end{array} \right.$ | | |
| ⊙ | 23 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} = PN'.$ | | |
| ⊙ | 23 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} = PN^2.$ | | |
| 21 + 23 | 24 | $\left\{ \begin{array}{l} HP^2 + PN^2 = (HN^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} = Q.Rad. \end{array} \right.$ | $\frac{x^4}{L^2}$ | |

| | | | |
|---|---|---|---|
| $24=13$ | 25 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}=0$; in $\frac{L^2}{x^3}$. | |
| $\times\frac{L^2}{x^3}$ | 26 | $x-p=0$. *Q. e. d.* in Linear. | *Fig.*22. |
| $24=15$ | 27 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}=+\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 28 | $x^4-px^3=S$. | |
| *Tranſp.* | 29 | $x^4-px^3 \ast \ast -S=0$. *Q. e. d.* in Biquadr. ſi $-S$. | *Fig.*23. |
| $24=16$ | 30 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}=-\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4-px^3=-S$. | |
| *Tranſp.* | 32 | $x^4-px^3 \ast \ast +S=0$. *Q. e. d.* in Biquad. ſi $+S$. | *Fig.*24. |
| | | $2.\ x+p=0\left\{\begin{array}{l}2 \\ 4\end{array}\right\}\left\{\begin{array}{l}x^4+px^3 \ast \ast -S=0 \\ x^4+px^3 \ast \ast +S=0\end{array}\right\}$ | |
| *Supp.* | 17 | $MO=x$. | |
| *Supp.* | 17 | $NO=-x$. | |
| $17+1$ | 18 | $\left\{\begin{array}{l}MO+(OF,u)\,BA=(FM,u)\,OR. \\ x+\frac{p}{2}=OR.\end{array}\right.$ | |
| $17-1$ | 18 | $\left\{\begin{array}{l}NO-(OF,u)\,BA=(FN,u)\,OR. \\ -x-\frac{p}{2}=OR.\end{array}\right.$ | |
| *Ob para.* | 19 | $\left\{\begin{array}{l}L\ .\ MO :: OR\ .\ AO. \\ L\ .\ x :: x+\frac{p}{2}\ .\ \frac{x^2}{L}+\frac{px}{2L}=AO.\end{array}\right.$ | |
| *Ob para.* | 19 | $\left\{\begin{array}{l}L\ .\ NO :: OR\ .\ AO. \\ L\ .\ -x :: -x-\frac{p}{2}\ .\ \frac{x^2}{L}+\frac{px}{2L}=AO.\end{array}\right.$ | |
| $19 \backsim 11$ | 20 | $\left\{\begin{array}{l}AO \backsim AD=(DO,u)\,HP. \\ \frac{x^2}{L}+\frac{px}{2L}\,(-b,u)-\frac{L}{2}-\frac{p^2}{8L}=HP.\end{array}\right.$ | |
| ⊙ | 21 | $\left\{\begin{array}{l}b^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{p^2x^2}{4L^2}-x^2-\frac{p^2x^2}{4L^2}-\frac{px}{2}-\frac{p^3x}{8L^2}=HP^2. \\ h\,e,\ b^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}-x^2-\frac{px}{2}-\frac{p^3x}{8L^2}=HP.\end{array}\right.$ | |

| | | | |
|---|---|---|---|
| 17 + 12 | 22 | $\begin{cases} MO + (OP, u)\ DH = PM. \\ x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16 L^2} = PM. \end{cases}$ | |
| 17 − 12 | 22 | $\begin{cases} NO - (OP, u)\ DH = PN. \\ -x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16 L^2} = PN. \end{cases}$ | |
| ☉ | 23 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3 x}{8 L^2} = PM^2.$ | |
| ☉ | 23 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3 x}{8 L^2} = PN^2.$ | |
| 47, e 1<br>21 + 23 | 24 | $\begin{cases} HP^2 + PM^2 = (HM^2 =)\ \text{Q. Rad.} \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} = \text{Q. Rad.}\ (= HN^2.) \end{cases}$ | |
| 24 = 13 | 25 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} = 0;$ in $\frac{L^2}{x^3}$. | |
| × $\frac{L^2}{x^3}$ | 26 | $x + p = 0.$ *Q. e. d.* in Linear. | *Fig.*22. |
| 24 = 15 | 27 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} = +\frac{S}{L^2};$ in $L^2$. | |
| × *L*² | 28 | $x^4 + px^3 = S.$ | |
| *Tranſp.* | 29 | $x^4 + px^3 * * - S = 0.$ *Q. e. d.* in Biquadr. ſi − S. | *Fig.*23. |
| 24 = 16 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} = -\frac{S}{L^2};$ in $L^2$. | |
| × *L*² | 31 | $x^4 + px^3 = -S.$ | |
| *Tranſp.* | 32 | $x^4 + px^3 * * + S = 0.$ *Q. e. d.* in Biquadr. ſi + S. | *Fig.*24. |

*Illuſtrat.*

$$\begin{cases} 1.\ x - \overset{p.}{1.6} = 0 \\ 2.\ x + 1.6 = 0 \end{cases} \quad \begin{matrix} NO = x = 16. \\ NO = -x = -16. \end{matrix}$$

$$\begin{cases} \frac{p}{2} = 0.8. \quad \frac{p^2}{4} = 0.64. \quad \frac{p^3}{8} = 0.512. \\ \frac{p}{4} = 0.4. \quad \frac{p^2}{8} = 0.32. \quad \frac{p^3}{16} = 0.256. \end{cases}$$ *Fig.*22.

*Central.*

Fig.22.

$$\left.\begin{array}{l}\dfrac{L}{2} = 0.5\\ \dfrac{p^2}{8L} = 0.32\\ \hline b = 0.82 = AD.\end{array}\right\}\qquad \begin{array}{l}\dfrac{p}{4} = 0.4\\ \dfrac{p^3}{16L^2} = 0.256\\ \hline d = 0.656 = DH.\end{array}$$

$$\left\{\begin{array}{l}x^4 - \overset{p.}{10}x^3 * * - \overset{s.}{3456} = 0\\ x^4 - 1.0x^3 * * - 0.3456 = 0\end{array}\right\}\ \begin{array}{l}NO = x = 12.\\ MO = -x = -6.\end{array}$$

$$\left\{\begin{array}{l}x^4 + \overset{p.}{10}x^3 * * - \overset{s.}{3456} = 0\\ x^4 + 1.0x^3 * * - 0.3456 = 0\end{array}\right\}\ \begin{array}{l}MO = x = 6.\\ NO = -x = -12.\end{array}$$

Fig.23.

$$\left\{\begin{array}{lll}\dfrac{p}{2} = 0.5. & \dfrac{p^2}{4} = 0.25. & \dfrac{p^3}{8} = 0.125\\ \dfrac{p}{4} = 0.25. & \dfrac{p^2}{8} = 0.125. & \dfrac{p^3}{16} = 0.0625.\end{array}\right.$$

*Central.*

$$\left.\begin{array}{l}\dfrac{L}{2} = 0.5\\ \dfrac{p^2}{8L} = 0.125\\ \hline b = 0.625 = AD\end{array}\right\}\qquad \begin{array}{l}\dfrac{p}{4} = 0.25\\ \dfrac{p^3}{16L^2} = 0.0625\\ \hline d = 0.3125 = DH.\end{array}$$

$$\left\{\begin{array}{l}x^4 - \overset{p.}{15}x^3 * * + \overset{s.}{2744} = 0\\ x^4 - 1.5x^3 * * + 0.2744 = 0\end{array}\right\}\ \begin{array}{l}NO = x = 14.\\ no = x = 7.\end{array}$$

Fig.24.

$$\left\{\begin{array}{l}x^4 + \overset{p.}{15}x^3 * * + \overset{s.}{2744} = 0\\ x^4 + 1.5x^3 * * + 0.2744 = 0\end{array}\right\}\ \begin{array}{l}NO = -x = -14.\\ no = -x = -7.\end{array}$$

*Central.*

$$\frac{L}{2} + \frac{p^2}{8L} = 0.78125 = b = AD.\ \frac{p}{4} + \frac{p^3}{16L^2} = 0.5859275 = d = DH$$

# CLAS. VI.

## *De Æquationibus Cubicis, affectis tantùm ſub Quadrato, vel ſecundo gradu Parodico; vel de Quadrato-quadraticis, affectis tantùm ſub Latere & Cubo; vel ſub primo & tertio gradu Parodico; ſeu de Æquationibus tertiæ vel quartæ Dimenſionis, ubi deficit tertius Terminus, vel Quantitas (q).*

HHjus cenſûs Æquationes ad ſequentes formulas reducuntur; ſc.

$$\left\{\begin{array}{l} 1.\ x^3 - px^2 \ast + r = 0 \left\{\begin{array}{l} 1 \left.\begin{array}{l} x^4 - px^3 \ast + rx - S = 0 \\ x^4 - px^3 \ast + rx + S = 0 \end{array}\right\} \\ 3 \end{array}\right. \\ 2.\ x^3 + px^2 \ast - r = 0 \left\{\begin{array}{l} 2 \left.\begin{array}{l} x^4 + px^3 \ast - rx - S = 0 \\ x^4 + px^3 \ast - rx + S = 0 \end{array}\right\} \\ 4 \end{array}\right. \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{p^2}{8L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{r}{2L^2} = d = DH.$$

$$\left\{\begin{array}{l} 3.\ x^3 - px^2 \ast - r = 0 \left\{\begin{array}{l} 5 \left.\begin{array}{l} x^4 - px^3 \ast - rx - S = 0 \\ x^4 - px^3 \ast - rx + S = 0 \end{array}\right\} \\ 7 \end{array}\right. \\ 4.\ x^3 + px^2 \ast + r = 0 \left\{\begin{array}{l} 6 \left.\begin{array}{l} x^4 + px^3 \ast + rx - S = 0 \\ x^4 + px^3 \ast + rx + S = 0 \end{array}\right\} \\ 8 \end{array}\right. \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{p^2}{8L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = d = DH.$$

*Reg. Gen.* Deſcribatur itaque Parabola (NAM), cujus Latus Rectum ſit L (ceu 1), & Axis (ay); ad quem or-
1 dinatìm applicetur Recta BA $= \frac{p}{2}$, occurrens Parabolæ in B & A: Ex A (puta) ducatur Diameter, vel
2 2 Recta Axi parallela (Ay); in quâ ſumendo Ab $= \frac{L}{2}$,

oportet

## CLAS. VI.

### *Of Cubic Equations, affected only under a Square, or the ſecond Parodic Degree; or of Quadrato-quadratics, affected only under a Side and a Cube, or under the firſt and third Parodic Degree; or of Equations of the third or fourth Dimenſion, where the third Term or Quantity (q) is wanting.*

ALL Equations of this kind are reduced to theſe following forms, *viz.*

$$\left\{\begin{array}{l} 1.\ x^3 - px^2 * + r = 0 \left\{\begin{array}{l} 1\ \} x^4 - px^3 * + rx - S = 0 \\ 3\ \} x^4 - px^3 * + rx + S = 0 \end{array}\right. \\ 2.\ x^3 + px^2 * - r = 0 \left\{\begin{array}{l} 2\ \} x^4 + px^3 * - rx - S = 0 \\ 4\ \} x^4 + px^3 * - rx + S = 0 \end{array}\right. \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2} + \frac{p^2}{8L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{r}{2L^2} = d = DH.$$

$$\left\{\begin{array}{l} 3.\ x^3 - px^2 * - r = 0 \left\{\begin{array}{l} 5\ \} x^4 - px^3 * - rx - S = 0 \\ 7\ \} x^4 - px^3 * - rx + S = 0 \end{array}\right. \\ 4.\ x^3 + px^2 * + r = 0 \left\{\begin{array}{l} 6\ \} x^4 + px^3 * + rx - S = 0 \\ 8\ \} x^4 + px^3 * + rx + S = 0 \end{array}\right. \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2} + \frac{p^2}{8L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = d = DH.$$

*Gen. Rule*

Let a Parabole (NAM) therefore be deſcribed, whoſe *Latus Rectum* L (or 1), and Axe (a y); to
1 1 which, let be ordinately applied $BA = \frac{p}{2}$, meeting the Parabole in B and A: From A (ſuppoſe) let there be drawn a Diameter, or a Right Line, parallel to
2 2 the Axe (*viz.* A y); in which, taking $Ab = \frac{L}{2}$,

2 3 oportet facere bD $= \frac{p^2}{8L}$, (vel AD $= \frac{L}{2} + \frac{p^2}{8L}$) eam-
que collocare in Diametro continuatâ verſus y.
Tum è Puncto D, erigatur ad Diametrum perpendi-
cularis DH $= \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2}$, ſi p & r iiſdem ſig-
nis ſint affectæ, quæ ad ſiniſtram eſt collocanda; ſin p
& r diverſis Signis fuerint affectæ, & $\frac{p}{4} + \frac{p^3}{16L^2} \sqsupset \frac{r}{2L^2}$,
tum fiat DH $= \frac{p}{4} + \frac{p^2}{16L^2} - \frac{r}{2L^2}$, quæ etiam ad ſini-
ſtram eſt collocanda: Quòd ſi $\frac{r}{2L^2} \sqsupset \frac{p}{4} + \frac{p^3}{16L^2}$; tum
fiat DH $= \frac{r}{2L^2} - \frac{p}{4} - \frac{p^3}{16L^2}$, quæ ad dextram eſt
collocanda.
Vel ſic: Erigendo ad Diametrum perpendicularem
4 4 (DH), oportet in eâ ſumere De $= \frac{p}{4}$, & ulteriùs
5 in eâ verſus ſiniſtram, oportet facere ef $= \frac{p^3}{16L^2}$; imo,
ſi in Æquatione p & r, iiſdem ſignis ſint adfectæ, ulte-
6 6 riùs ad ſiniſtram oportet facere fH $= \frac{r}{2L^2}$; ad dex-
tram verò (ex Puncto f), ſi p & r, diverſis Signis
ſint denotatæ.
Tum Centro H, intervallo verò HA, deſcribatur *Fig.*25.
Circulus (NAM), ſi Æquatio fuerit tantùm Cubica,
hoc eſt, ſi non habeatur Quantitas S.
Aſt ſi habeatur S, & ſit quidem $-$S, oportet ulte-
riùs in hàc lineâ HA, utrinque productâ, ex unâ
8 9 7 8 parte ſumere AI $=$ L; & ex alterâ parte, AK $= \frac{S}{L^3}$;
deſcriptoque Semicirculo, cujus Diameter IK, eri-
10 9 gere AL perpendicularem ad AH, quæ occurrat huic
Semicirculo (ILK) in puncto L.
11 Quòd ſi verò habeatur $+$S; oportet inſuper in
alio Semicirculo, cujus Diameter ſit HA, inſcribere
11 10 AZ $=$ AL, inventæ. Circulus igitur deſcriptus, tran-
12 ſiens per L, ſi ſit $-$S; per Z verò, ſi ſit $+$S, ſecabit
vel

2 3 muſt be made $bD = \frac{p^2}{8L}$, (or $AD = \frac{L}{2} + \frac{p^2}{8L}$) placing it in the Diameter continued towards y.

Then from the Point D, erect perpendicularly to the Diameter $DH = \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2}$, if p and r are affected with the ſame Signs, which is to be placed to the left hand; but if p and r are affected with divers Signs, and $\frac{p}{4} + \frac{p^3}{16L^2} \sqsupset \frac{r}{2L^2}$, then make $DH = \frac{p}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2}$, and place it alſo to the left: But if $\frac{r}{2L^2} \sqsupset \frac{p}{4} + \frac{p^3}{16L^2}$, then make $DH = \frac{r}{2L^2} - \frac{p}{4} - \frac{p^3}{16L^2}$, and place it to the right.

Or thus: Erecting a Perpendicular to the Diameter
4 4 (DH), take in it $De = \frac{p}{4}$, placing it to the left hand;
5 and farther thence to the left hand place $ef = \frac{p^3}{16L^2}$;
yea, and farther yet to the left hand muſt be made
6 6 $fH = \frac{r}{2L^2}$, if in the Equation p and r be affected with
the ſame Signs; but to the right (from the Point f),
if p and r are noted with divers Signs.

Then center indeed H, but diſtance HA, let the Circle (NAM) be deſcribed, if it be only a Cubic Equation, that is, if the Quantity S be not had. *Fig.* 25.

But if S be had, and it be $-S$, then muſt there farther in this Line HA, both ways produced, be taken on
8 7 the one ſide $AI = L$, and on the other ſide $AK = \frac{S}{L^3}$;
9 8
and a Semicircle being deſcribed, whoſe Diameter IK,
10 9 muſt be erected AL perpendicular to AH, which may
meet this Semicircle (ILK) in the Point L.

11 But if $+S$ be had; then moreover in another
Semicircle, whoſe Diameter is HA, muſt be inſcribed
11 10 $AZ = AL$ found. A Circle therefore deſcribed paſ-
12 ſing through L, if it be $-S$, but through Z, if $+S$,

vel tanget Parabolam, in 1, 2, 3, vel 4 Punctis, à quibus ſi ad Diametrum demittantur Perpendiculares, obtinebuntur omnes Æquationes radices, tam falſæ, quam veræ. Quarum quidem veræ (ut NO) ad ſiniſtram cadent, ſi in Æquatione habeatur $-p$: Sed contra, ſi habeatur ibi $+p$, veræ (ut MO) cadent ad dextram, falſæ verò (ut NO) ad ſiniſtram. 14

$$\left\{\begin{matrix} 1.\ x^3 - px^2 * + r = 0 \left\{\begin{matrix} 1 \\ 3 \end{matrix}\right\} \begin{matrix} x^4 - px^3 * rx - S = 0 \\ x^4 - px^3 * rx + S = 0 \end{matrix} \\ 2.\ x^3 + px^2 * - r = 0 \left\{\begin{matrix} 2 \\ 4 \end{matrix}\right\} \begin{matrix} x^4 + px^3 * rx - S = 0 \\ x^4 + px^3 * rx + S = 0 \end{matrix} \end{matrix}\right\}$$

1. Caſ. Ubi $\left\{\begin{matrix} -p+r \\ +p-r \end{matrix}\right\}$; & ſic, $\left\{\begin{matrix} 1 \\ 2 \end{matrix}\right\} \frac{p}{4} + \frac{p^3}{16L^2} \begin{matrix} \sqsupset \\ \sqsubset \end{matrix} \frac{r}{2L^2}$.

---

1. Ubi $\frac{p}{4} + \frac{p^3}{16L^2} \sqsupset \frac{r}{2L^2}$.

*Demonſtrat.*

$$Ab + bD = b = AD.$$

2 + 3 | 11 | $\frac{L}{2} + \frac{p^2}{8L} = b = AD.$

$$De + ef - fH = d = DH.$$

4 + 5 − 6 | 12 | $\frac{p}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2} = d = DH.$

47, e 1 | $AD^2 + DH^2 = HA^2.$

Q. 11. + Q. 12. | 13 | $b^2 + d^2 = (HA^2 =)$ Q. Rad. in Cubic. Fig. 25.

$AI \times AK =$ (ob Circl.) $AL^2 =$ (per conſtr.) $AZ^2.$

7 × 8 | 14 | $(L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2.$

47, e 1 | $AH^2 + AL^2 = (HL^2 =)$ Q. Rad.

13 + 14 | 15 | $b^2 + d^2 + \frac{S}{L^2} =$ Q. Rad. $\left.\right\}$ In Biquadr. ſi $-S$. Fig. 26.

47, e 1 | $AH^2 - AZ^2 = (HZ^2 =)$ Q. Rad.

13 − 14 | 16 | $b^2 + d^2 - \frac{S}{L^2} =$ Q. Rad. $\left.\right\}$ In Biquadr. ſi $+S$. Fig. 27.

will cut or touch the Parabole in 1, 2, 3, or 4 Points;
from which, if Perpendiculars be demitted to the Diameter, all the Roots of the Equation will be had, as
well falſe as true: Of which, the true Roots (as NO) will
14. fall to the left, if in the Equation be had $-p$: But
contrarily, if in it be had $+p$, the true (as MO)
will fall to the right, but the falſe (as NO) to the left.

$$\left\{\begin{array}{l} 1.\ x^3 - px^2 * + r = 0 \\ 2.\ x^3 + px^2 * - r = 0 \end{array}\right. \left\{\begin{array}{l} \left.\begin{array}{l}1\\3\end{array}\right\} \begin{array}{l} x^4 - px^3 * + rx - S = 0 \\ x^4 - px^3 * + rx + S = 0 \end{array} \\ \left.\begin{array}{l}2\\4\end{array}\right\} \begin{array}{l} x^4 + px^3 * - rx - S = 0 \\ x^4 + px^3 * - rx + S = 0 \end{array} \end{array}\right\}$$

*Caſ.* 1. Where $\left\{\begin{array}{l} -p+r \\ +p-r \end{array}\right\}$; and ſo $\left\{\begin{array}{l} 1 \\ 2 \end{array}\right\} \frac{p}{4} + \frac{p^3}{16L^2} \genfrac{}{}{0pt}{}{\sqsupset}{\sqsubset} \frac{r}{2L^2}.$

| | | |
|---|---|---|
| *Supp.* | 17 | $NO = x.$ |
| *Supp.* | 17 | $MO = -x.$ |
| 17 − 1 | 18 | $\left\{\begin{array}{l} NO - (OF, u)\ BA = (NF, u)\ OR. \\ x - \frac{p}{2} = OR. \end{array}\right.$ |
| 17 + 1 | 18 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{array}\right.$ |
| *Ob para.* | 19 | $\left\{\begin{array}{l} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ x :: x - \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ |
| *Ob para.* | 19 | $\left\{\begin{array}{l} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ -x :: -x + \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ |
| 19 ∽ 11 | 20 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} - \frac{px}{2L}\ (\backsim b, u) - \frac{L}{2} - \frac{p2}{8L} = HP. \end{array}\right.$ |
| ⊛ | 21 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} = HP^2.$ |

| | | | |
|---|---|---|---|
| 21 | 22 | h e, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} = HP^2$. | |
| 17 − 12 | 23 | $\begin{cases} NO - (PO, u)\ DH = PN. \\ x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} + \frac{r}{2L^2} = PN. \end{cases}$ | |
| 17 + 12 | 23 | $\begin{cases} MO + (OP, u)\ DH = PM. \\ -x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2} = PM. \end{cases}$ | |
| ⊙ | 24 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{rx}{L^2} = PN^2$. | |
| ⊙ | 24 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{rx}{L^2} = PN^2$. | |
| 47, e 1<br>22 + 24 | 25 | $\begin{cases} HP^2 + PN^2 = (HN^2 =)\ Q.\ Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = Q.\ Rad. \end{cases}$ | |
| 25 = 13 | 26 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x^3}$ | 27 | $x^3 - px^2 * + rx = 0$. *Q. e. d.* in Cubic. | *Fig.* 25. |
| 25 = 15 | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 29 | $x^4 - px^3 + rx = S$. | |
| *Tranſp.* | 30 | $x^4 - px^3 * + rx - S = 0$. *Q.e.d.* in Biquadr. ſi $-S$. | *Fig.* 26. |
| 25 = 16 | 31 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 32 | $x^4 - px^3 + rx = -S$. | |
| *Tranſp.* | 33 | $x^4 - px^3 * + rx + S = 0$. *Q.e.d.* in Biquad. ſi $+S$. | *Fig.* 27. |
| *Supp.* | 17 | $MO = x$. | |
| *Supp.* | 17 | $NO = -x$. | |
| 17 + 1 | 18 | $\begin{cases} MO + (OF, u)\ BA = (MF, u)\ OR. \\ x + \frac{p}{2} = OR. \end{cases}$ | |

| | | | |
|---|---|---|---|
| 17 − 1 | 18 | $\left\{\begin{array}{l} NO - (OF,\ u)\ BA = (NF,\ u)\ OR. \\ -x - \frac{p}{2} = OR. \end{array}\right.$ | |
| *Ob para.* | 19 | $\left\{\begin{array}{l} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ | |
| *Ob para.* | 19 | $\left\{\begin{array}{l} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ -x :: -x - \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ | |
| 11 ∽ 19 | 20 | $\left\{\begin{array}{l} AD \backsim AO = (DO,\ u)\ HP. \\ (b,\ u)\ \frac{L}{2} + \frac{p^2}{8L} - \frac{x^2}{L} - \frac{px}{2L} = HP. \end{array}\right.$ | |
| ⊙ | 21 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} = HP^2.$ | |
| 21 | 22 | $he,\ b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} = HP^2.$ | |
| 17 + 12 | 23 | $\left\{\begin{array}{l} MO + (OP,\ u)\ DH = PM. \\ x\ (+d,\ u) + \frac{p}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2} = PM. \end{array}\right.$ | |
| 17 − 12 | 23 | $\left\{\begin{array}{l} NO - (OP,\ u)\ DH = PN. \\ -x\ (-d,\ u) - \frac{p}{4} - \frac{p^3}{16L^2} + \frac{r}{2L^2} = PN. \end{array}\right.$ | |
| ⊙ | 24 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{rx}{L^2} = PM^2.$ | |
| ⊙ | 24 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{rx}{L^2} = PN^2.$ | |
| 47, e 1<br>22 + 24 | 25 | $\left\{\begin{array}{l} HP^2 + PM^2 = (HM^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = Q.Rad. \end{array}\right.$ | |
| 25 = 13 | 26 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = 0;\ in\ \frac{L^2}{x}.$ | |
| $\times\ \frac{L^2}{x^3}$ | 27 | $x^3 + px^2 * - rx = 0.$ *Q.e.d.* in Cubic. | *Fig.25.* |
| 25 = 15 | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2};\ in\ L^2.$ | |
| $\times\ L^2$ | 29 | $x^4 + px^3 - rx = S.$ | |
| *Tranſp.* | 30 | $x^4 + px^3 * - rx - S = 0.$ *Q.e.d.* in Biquadr. ſi − S. | *Fig.26.* |

25 = 16 | 31 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = -\frac{S}{L^2}$; in $L^2$.

$\times L^2$ | 32 | $x^4 + px^3 * - rx = - S.$

*Tranſp.* | 33 | $x^4 + px^3 * - rx + S = 0.$ *Q. e. d.* in Biquadr. ſi + S. *Fig.* 27.

*Illuſtrat.*

$$\begin{array}{l} \qquad\ \ p. \qquad\qquad\quad r. \\ \left\{\begin{array}{l} x^3 - 16x^2 * + 576 = 0 \\ x^3 - 1.6x^2 * + 0.576 = 0 \end{array}\right\} \end{array}$$

$$\left\{\begin{array}{l} NO = x = 12. \\ no = x = 9.2 \end{array}\right\} \quad MO = -x = -5.21.$$

$$\begin{array}{l} \qquad\ \ p. \qquad\qquad\quad r. \\ \left\{\begin{array}{l} x^3 + 16x^2 * - 576 = 0 \\ x^3 + 1.6x^2 * - 0.576 = 0 \end{array}\right\} \end{array}$$

$$MO = x = 5.21. \quad \left\{\begin{array}{l} NO = -x = -12. \\ NO = -x = -9.2. \end{array}\right.$$

*Fig.* 25.

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.8. & \frac{p^2}{4} = 0.64. & \frac{p^3}{8L^2} = 0.512. \\ \frac{p}{4} = 0.4. & \frac{p^2}{8} = 0.32. & \frac{p^3}{16L^2} = 0.256. \end{array}\right.$$

*Central.*

$$\begin{array}{l} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.32 \\ \hline b = 0.82 = AD. \end{array} \left\} \begin{array}{l} \frac{p}{4} = 0.4 \\ \frac{p^3}{16L^2} = 0.256 \\ \hline \qquad 0.656 \\ -\frac{r}{2L^2} = 0.288 \\ \hline d = 0.368 = DH. \end{array} \right.$$

$$\begin{array}{l} \qquad\ \ p. \qquad\qquad\quad r. \qquad\qquad\quad s. \\ \left\{\begin{array}{l} x^4 - 16x^3 * + 900x - 8912 = 0 \\ x^4 - 1.6x^3 * + 0.900x - 0.8912 = 0 \end{array}\right\} \end{array}$$

$NO = x = 14.6 +$ *Fig.* 26.

$MO = -x = -8.8$ *ferè.*

$x^4 +$

$$\left\{\begin{array}{l} \phantom{x^4+}\ \ \ \textit{p.} \qquad\qquad\quad \textit{r.} \qquad\qquad\quad \textit{s.} \\ x^4 + 16x^3 * - 900x - 8912 = 0 \\ x^4 + 1.6x^3 * - 0.900x - 0.8912 = 0 \end{array}\right\}$$

$MO = x = 8.8$ *ferè.*

$NO = -x = -14.6$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.8. & \frac{p^2}{4} = 0.64. & \frac{p^3}{8} = 0.512. \\ \frac{p}{4} = 0.4. & \frac{p^2}{8} = 0.32. & \frac{p^3}{16} = 0.256. \end{array}\right.$$

*Central.*

Fig.26.

$$\left.\begin{array}{l} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.32 \\ \hline b = 0.82 = AD. \\ \hline \end{array}\right\} \quad \begin{array}{l} \frac{p}{4} = 0.4 \\ \frac{p^2}{16L^3} = 0.256 \\ \hline \phantom{\frac{p^2}{16L^3} =}\ 0.656 \\ -\frac{r}{2L^2} = 0.45 \\ \hline d = 0.206 = DH. \\ \hline \end{array}$$

$$\left\{\begin{array}{l} \phantom{x^4-}\ \ \ \textit{p.} \qquad\qquad\quad \textit{r.} \qquad\qquad\quad \textit{s.} \\ x^4 - 24x^3 * + 1900x + 840 = 0 \\ x^4 - 2.4x^3 * + 1.900x + 0.0840 = 0 \end{array}\right\}$$

$$\left\{\begin{array}{l} NO = x = 18.\ \textit{ferè.} \\ no = x = 14. \end{array}\right\} \quad MO = -x = -7.6+$$

$$\left\{\begin{array}{l} \phantom{x^4+}\ \ \ \textit{p.} \qquad\qquad\quad \textit{r.} \qquad\qquad\quad \textit{s.} \\ x^4 + 24x^3 * - 1900x + 840 = 0 \\ x^4 + 2.4x^3 * - 1.900x + 0.0840 = 0 \end{array}\right\}$$

Fig.27.

$$MO = x = 7.6+ \quad \left\{\begin{array}{l} NO = -x = -18.\ \textit{ferè.} \\ no = -x = -14. \end{array}\right.$$

*Central.*

$$\frac{L}{2} + \frac{p^2}{8} = b = 1.22 = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2} = d = 0.514 = DH$$

2. Ubi $\frac{r}{2L^2} \sqsupset \frac{p}{4} + \frac{p^3}{16L^2}$.

*Demonſtrat.*

| | | | | |
|---|---|---|---|---|
| 2 + 3 | 11 | $\left\{\begin{array}{l} Ab + bD = b = AD. \\ \frac{L}{2} + \frac{p^2}{8L} = b = AD. \end{array}\right.$ | | |
| 6 — 5 — 4 | 12 | $\left\{\begin{array}{l} Hf - fe - eD = d = DH. \\ \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = d = DH. \end{array}\right.$ | | |
| Q.11. + Q.12. | 13 | $\left\{\begin{array}{l} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = (HA^2 =) \text{ Q. Rad. in Cubic.} \end{array}\right.$ | | *Fig.* 28. |
| 7 × 8 | 14 | $\left\{\begin{array}{l} AI \times AK = (\text{ob Circl.}) \; AL^2 = (\text{per conſtr.}) \; AZ^2. \\ (L \times \frac{S}{L^3} =) \; \frac{S}{L^2} = AL^2 = AZ^2. \end{array}\right.$ | | |
| 47, *e* 1<br>13 + 14 | 15 | $\left\{\begin{array}{l} AH^2 + AL^2 = (HL^2 =) \text{ Q. Rad.} \\ b^2 + d^2 + \frac{S}{L^2} = \text{Q. Rad.} \end{array}\right\}$ | In Biquadr.<br>ſi — S. | *Fig.* 29. |
| 47, *e* 1<br>13 — 14 | 16 | $\left\{\begin{array}{l} AH^2 - AZ^2 = (HZ^2 =) \text{ Q. Rad.} \\ b^2 + d^2 - \frac{S}{L^2} = \text{Q. Rad.} \end{array}\right\}$ | In Biquadr.<br>ſi + S. | *Fig.* 30. |
| *Supp.* | 17 | $NO = x.$ | | |
| *Supp.* | 17 | $MO = -x.$ | | |
| 17 — 1 | 18 | $\left\{\begin{array}{l} NO - (OF, u) \; BA = (NF, u) \; OR. \\ x - \frac{p}{2} = OR. \end{array}\right.$ | | |
| 17 + 1 | 18 | $\left\{\begin{array}{l} MO + (OF, u) \; BA = (MF, u) \; OR. \\ -x + \frac{p}{2} = OR. \end{array}\right.$ | | |
| *Ob para.* | 19 | $\left\{\begin{array}{l} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, x :: x - \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ | | |
| *Ob para.* | 19 | $\left\{\begin{array}{l} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, -x :: -x + \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ | | |

AO

| | | | |
|---|---|---|---|
| $19 \backsim 11$ | 20 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} - \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p2}{8L} = HP. \end{array}\right.$ | |
| @ | 21 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} = HP^2.$ | |
| 21 | 22 | h e, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} = HP^2.$ | |
| $17 + 12$ | 23 | $\left\{\begin{array}{l} NO + (OP, u)\ DH = PN. \\ x\ (+d, u) + \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN. \end{array}\right.$ | |
| $17 - 12$ | 23 | $\left\{\begin{array}{l} MO - (OP, u)\ DH = PM. \\ -x\ (-d, u) - \frac{r}{2L^2} + \frac{p^3}{16L^2} + \frac{p}{4} = PM. \end{array}\right.$ | |
| @ | 24 | $d^2 + x^2 + \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PN^2.$ | |
| @ | 24 | $d^2 + x^2 + \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PM^2.$ | |
| 47, e 1<br>$22 + 24$ | 25 | $\left\{\begin{array}{l} HP^2 + PN^2 = (HN^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = Q.Rad. \end{array}\right.$ | |
| $25 = 13$ | 26 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 27 | $x^3 - px^2 \ast + r = 0.$ *Q.e.d.* in Cubic. | *Fig.*28. |
| $25 = 15$ | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 29 | $x^4 - px^3 + rx = S.$ | |
| *Tranſp.* | 30 | $x^4 - px^3 \ast + rx - S = 0.$ *Q.e.d.* in Biquadr. ſi $-S$. | *Fig.*29. |
| $25 = 16$ | 31 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 32 | $x^4 - px^3 + rx = -S.$ | |
| *Tranſp.* | 33 | $x^4 - px^3 \ast + rx + S = 0.$ *Q.e.d.* in Biquad. ſi $+S$. | *Fig.*30. |
| *Supp.* | 17 | $MO = x.$ | |
| *Supp.* | 17 | $NO = -x.$ | |

| | | |
|---|---|---|
| 17 + 1 | 18 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ x + \frac{p}{2} = OR. \end{array}\right.$ |
| 17 − 1 | 18 | $\left\{\begin{array}{l} NO - (OF, u)\ BA = (NE, u)\ OR. \\ -x - \frac{p}{2} = OR. \end{array}\right.$ |
| *Ob para.* | 19 | $\left\{\begin{array}{l} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ |
| *Ob para.* | 19 | $\left\{\begin{array}{l} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ -x :: -x - \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ |
| 19 $\backsim$ 11 | 20 | $\left\{\begin{array}{l} AO - AD = (DO, u)\ HP. \\ \frac{x^2}{L} + \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} = HP. \end{array}\right.$ |
| ⊙ | 21 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} = HP^2.$ |
| 21 | 22 | h e, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} = HP^2.$ |
| 17 − 12 | 23 | $\left\{\begin{array}{l} MO - (OP, u)\ DH = PM. \\ x\ (-d, u) - \frac{r}{2L^2} + \frac{p^3}{16L^2} + \frac{p}{4} = PM. \end{array}\right.$ |
| 17 + 12 | 23 | $\left\{\begin{array}{l} NO + (OP, u)\ DH = PN. \\ -x\ (+d, u) + \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN. \end{array}\right.$ |
| ⊙ | 24 | $d^2 + x^2 - \frac{rx}{L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = PM^2.$ |
| ⊙ | 24 | $d^2 + x^2 - \frac{rx}{L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = PN^2.$ |
| 47, e 1<br>22 + 24 | 25 | $\left\{\begin{array}{l} HP^2 + PM^2 = (HM^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = Q.Rad. \end{array}\right.$ |
| 25 = 13 | 26 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = 0;$ in $\frac{L^2}{x}.$ |
| × $\frac{L^2}{x}$ | 27 | $x^3 + px^2 - r = 0.$ *Q. e. d.* in Cubic. |

Fig. 28.

$\frac{x^4}{L^2}$

25=15 | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$.

× $L^2$ | 29 | $x^4 + px^3 - rx = S$.

*Tranſp.* | 30 | $x^4 + px^3 * - rx - S = 0$. *Q.e.d.* in Biquadr. ſi $-S$. *Fig.*29.

25=16 | 31 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$.

× $L^2$ | 32 | $x^4 + px^3 - rx = - S$.

*Tranſp.* | 33 | $x^4 + px^3 * - rx + S = 0$. *Q.e.d.* in Biquadr. ſi $+S$. *Fig.*30.

*Illuſtrat.*

$$\left\{\begin{array}{l} x^3 - \overset{p.}{16} x^2 * + \overset{r.}{1536} = 0 \\ x^3 - 1.6\, x^2 * + 1.536 = 0 \end{array}\right\} \quad MO = - x = - 8.$$

$$\left\{\begin{array}{l} x^3 + \overset{p.}{16} x^2 * - \overset{r.}{1536} = 0 \\ x^3 + 1.6\, x^2 * - 1.536 = 0 \end{array}\right\} \quad MO = x = 8.$$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.8. & \frac{p^2}{4} = 0.64. & \frac{p^3}{8} = 0.512 \\ \frac{p}{4} = 0.4. & \frac{p^2}{8} = 0.32. & \frac{p^3}{16} = 0.256. \end{array}\right.$$

*Central.* *Fig.*28.

$$\left.\begin{array}{r} \frac{L}{2} = 0.5, \\ \frac{p^2}{8} = 0.32 \\ \hline b = 0.82 = AD. \end{array}\right\} \quad \begin{array}{r} \frac{r}{2L^2} = 0.768 \\ \hline - \frac{p^3}{16} = 0.256 \\ - \frac{p}{4} = 0.4 \\ \hline 0.656 \\ d = 0.112 = DH. \end{array}$$

$$\left\{\begin{array}{l} x^4 - \overset{p.}{16} x^3 * + \overset{r.}{1800} x - \overset{s.}{19712} = 0 \\ x^4 - 1.6\, x^3 * + 1.800\, x - 1.9712 = 0 \end{array}\right\} \quad \textit{Fig.}29.$$

$NO = x = 14.$

$MO = - x = - 11.4 -$

$$\left\{\begin{array}{l} x^4 + \overset{p.}{16\,x^3} \ast - \overset{r.}{1800\,x} - \overset{s.}{19712} = 0 \\ x^4 + 1.6\,x^3 \ast - 1.800\,x - 1.9712 = 0 \end{array}\right\}$$

$MO = x = 11.4-$

$NO = -x = -14.$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.8. & \frac{p^2}{4} = 0.64. & \frac{p^3}{8} = 0.512. \\ \frac{p}{4} = 0.4. & \frac{p^2}{8} = 0.32. & \frac{p^3}{16} = 0,256. \end{array}\right.$$

*Central.*

Fig.29.

$$\begin{array}{r} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.32 \\ \hline b = 0.82 = AD \\ \hline \end{array} \left.\vphantom{\begin{array}{c}a\\a\\a\\a\end{array}}\right\} \begin{array}{r} \frac{r}{2L^2} = 0.900 \\ \hline -\frac{p^3}{16} = 0.256 \\ -\frac{p}{4} = 0.4 \\ \hline 0.656 \\ d = 0.244 = DH. \\ \hline \end{array}$$

$$\left\{\begin{array}{l} x^4 - \overset{p.}{12\,x^3} \ast + \overset{r.}{2184\,x} + \overset{s.}{7232} = 0 \\ x^4 - 1.2\,x^3 \ast + 2.184\,x + 0.7232 = 0 \end{array}\right\}$$

$MO = -x = -8.$

$mo = -x = -3.6+$

$$\left\{\begin{array}{l} x^4 + \overset{p.}{12\,x^3} \ast - \overset{r.}{2184\,x} + \overset{s.}{7232} = 0 \\ x^4 + 1.2\,x^3 \ast - 2.184\,x + 0.7232 = 0 \end{array}\right\}$$

Fig.30.

$MO = x = 8.$

$mo = x = 3.6+$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.6. & \frac{p^2}{4} = 0.36. & \frac{p^3}{8} = 0.216. \\ \frac{p}{4} = 0.3. & \frac{p^2}{8} = 0.18. & \frac{p^3}{16} = 0.108. \end{array}\right.$$

*Central.*

$$\left.\begin{array}{r} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.18 \\ \hline b = 0.68 = AD \\ \hline \end{array}\right\} \quad \begin{array}{r} \frac{r}{2L^2} = 1.092 \\ \hline -\frac{p^3}{16} = 0.108 \\ -\frac{p}{4} = 0.3 \\ \hline 0.408 \\ d = 0.684 = DH. \\ \hline \end{array}$$

$$\left\{\begin{array}{l} 3.\ x^3 - px^2 * - r = 0 \left\{\begin{array}{l} 5 \left\}\ x^4 - px^3 * - rx - S = 0\right. \\ 7 \left\{\ x^4 - px^3 * - rx + S = 0\right. \end{array}\right. \\ 4.\ x^3 + px^2 * + r = 0 \left\{\begin{array}{l} 6 \left\}\ x^4 + px^3 * + rx - S = 0\right. \\ 8 \left\{\ x^4 + px^2 * + rx + S = 0\right. \end{array}\right. \end{array}\right.$$

*Central.*

$$\frac{L}{2} + \frac{p^2}{8L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = d = DH.$$

2. Caſ. Ubi $\left\{\begin{array}{l} -p-r \\ +p+r \end{array}\right\}$

*Demonſtrat.*

2+3 | 11 | $\left\{\begin{array}{l} Ab + bD = b = AD. \\ \frac{L}{2} + \frac{p^2}{8L} = b = AD. \end{array}\right.$

4+5+6 | 12 | $\left\{\begin{array}{l} De + ef + fH = d = DH. \\ \frac{p}{4} + \frac{p^3}{16 L^2} + \frac{r}{2L^2} = d = DH. \end{array}\right.$

47, e 1 Q.11.+ Q.12. | 13 | $\left\{\begin{array}{l} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = (HA^2 =) \end{array}\right.$ Q. Rad. in Cubic.

*Fig.* 31.

| | | | |
|---|---|---|---|
| $7 \times 8$ | 14 | $AI \times AK =$ (ob Circl.) $AL^2 =$ (per conſtr.) $AZ^2$.<br>$L \times \frac{S}{L^3} =) \frac{S}{L^2} = AL^2 = AZ^2$. | |
| 47, e 1<br>$13 + 14$ | 15 | $AH^2 + AL^2 =$ ($HL^2 =$) Q. Rad.<br>$b^2 + d^2 + \frac{S}{L^2} =$ Q. Rad. in Biquadr. ſi $- S$. | *Fig*. 32. |
| 47, e 1<br>$13 - 14$ | 16 | $AH^2 - AZ^2 =$ ($HZ^2 =$) Q. Rad.<br>$b^2 + d^2 - \frac{S}{L^2} =$ Q. Rad. in Biquadr. ſi $+ S$. | *Fig*. 33. |
| *Supp*. | 17 | $NO = x$. | |
| *Supp*. | 17 | $MO = - x$. | |
| $17 - 1$ | 18 | $NO -$ (OF, u) $BA =$ (NF, u) $OR$.<br>$x - \frac{p}{2} = OR$. | |
| $17 + 1$ | 18 | $MO +$ (OF, u) $BA =$ (MF, u) $OR$.<br>$- x + \frac{p}{2} = OR$. | |
| *Ob para*. | 19 | $L . NO :: OR . AO$.<br>$L . x :: x - \frac{p}{2} . \frac{x^2}{L} - \frac{px}{2L} = AO$. | |
| *Ob para*. | 19 | $L . MO :: OR . AO$.<br>$L . - x :: - x + \frac{p}{2} . \frac{x^2}{L} - \frac{px}{2L} = AO$. | |
| $19 \backsim 11$ | 20 | $AO \backsim AD =$ (DO, u) $HP$.<br>$\frac{x^2}{L} - \frac{px}{2L}$ (— b, u) $- \frac{L}{2} - \frac{p^2}{8L} = HP$. | |
| $\odot$ | 21 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{p^2x^2}{L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} = HP^2$. | |
| 21 | 22 | h e, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} = HP^2$. | |
| $17 - 12$ | 23 | $NO -$ (OP, u) $DH = PN$.<br>$x$ (— d, u) $- \frac{p}{4} - \frac{p^3}{16L^2} - \frac{r}{2L^2} = PN$. | |
| $17 + 12$ | 23 | $MO +$ (OP, u) $DH = PM$.<br>$- x$ (+ d, u) $+ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = PM$. | |

| | | | |
|---|---|---|---|
| ⊖ | 24 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{rx}{L^2} = PN^2.$ | |
| ⊖ | 24 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{rx}{L^2} = PM^2.$ | |
| 47, e 1<br>22 + 24 | 25 | $\left\{\begin{array}{l} HP^2 + PN^2 = (HN^2 =) \text{ Q. Rad.} \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{rx}{L^2} = \text{Q. Rad.} \end{array}\right.$ | |
| 25 = 13 | 26 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{rx}{L^2} = 0;$ in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 27 | $x^3 - px^2 \ast - r = 0.$ *Q. e. d.* in Cubic. | *Fig.* 31. |
| 25 = 15 | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2};$ in $L^2$. | |
| $\times L^2$ | 29 | $x^4 - px^3 - rx = S.$ | |
| *Tranſp.* | 30 | $x^4 - px^3 \ast - rx - S = 0.$ *Q. e. d.* in Biquadr. ſi $-S$. | *Fig.* 32. |
| 25 = 16 | 31 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2};$ in $L^2$. | |
| $\times L^2$ | 32 | $x^4 - px^3 - rx = -S.$ | |
| *Tranſp.* | 33 | $x^4 - px^3 \ast - rx + S = 0.$ *Q. e. d.* in Biquadr. ſi $+S$. | *Fig.* 33. |
| *Supp.* | 17 | $MO = x.$ | |
| *Supp.* | 17 | $NO = -x.$ | |
| 17 + 1 | 18 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ x + \frac{p}{2} = OR. \end{array}\right.$ | |
| 17 − 1 | 18 | $\left\{\begin{array}{l} NO - (OF, u)\ BA = (NF, u)\ OR. \\ -x - \frac{p}{2} = OR. \end{array}\right.$ | |
| *Ob para.* | 19 | $\left\{\begin{array}{l} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ | |
| *Ob para.* | 19 | $\left\{\begin{array}{l} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ -x :: -x - \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ | |

| | | | |
|---|---|---|---|
| 11 ∽ 19 | 20 | $\left\{\begin{array}{l} AD \backsim AO = (DO, u)\ HP. \\ (b, u)\ \frac{L}{2} + \frac{p^2}{8L} - \frac{x^2}{L} - \frac{px}{2L} = HP. \end{array}\right.$ | |
| ⊙ | 21 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{px}{2} - \frac{p^2x^2}{4L^2} - \frac{p^3x}{8L^2} = HP^2.$ | |
| 21 | 22 | h e, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} = HP^2.$ | |
| 17 + 12 | 23 | $\left\{\begin{array}{l} MO + (OP, u)\ DH = PM. \\ x\ (+ d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = PM. \end{array}\right.$ | |
| 17 — 12 | 23 | $\left\{\begin{array}{l} NO - (OP, u)\ DH = PN. \\ -x\ (- d, u) - \frac{p}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2} = PN. \end{array}\right.$ | |
| ⊙ | 24 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{rx}{L^2} = PM^2.$ | |
| ⊙ | 24 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{rx}{L^2} = PM^2.$ | |
| 47, e 1<br>22 + 24 | 25 | $\left\{\begin{array}{l} HP^2 + PM^2 = (HM^2 =)\ Q.Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{rx}{L^2} = Q.Rad. \end{array}\right.$ | |
| 25 = 13 | 26 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 27 | $x^3 + px^2 * + rx = 0.$ *Q. e. d.* in Cubic. | *Fig.* 31. |
| 25 = 15 | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times$ *L*² | 29 | $x^4 + px^3 + rx = S.$ | |
| *Tranſp.* | 30 | $x^4 + px^3 * + rx - S = 0.$ *Q. e. d.* in Biquadr. ſi $-S$. | *Fig.* 32. |
| 25 = 16 | 31 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times$ *L*² | 32 | $x^4 + px^3 + rx = - S.$ | |
| *Tranſp.* | 33 | $x^4 + px^3 * + rx + S = 0.$ *Q. e. d.* in Biquad. ſi $+S$. | *Fig.* 33. |

*Illuſtrat.*

$$\left\{\begin{array}{l} x^3 - \overset{p.}{16}\,x^2 * - \overset{r.}{648} = 0 \\ x^3 - 1.6\,x^2 * - 0.648 = 0 \end{array}\right\} \quad NO = x = 18.$$

$$\left\{\begin{array}{l} x^3 + \overset{p.}{16}\,x^2 * + \overset{r.}{648} = 0 \\ x^3 + 1.6\,x^2 * + 0,648 = 0 \end{array}\right\} \quad NQ = - x = - 18.$$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.8. & \frac{p^2}{4} = 0.64. & \frac{p^3}{8} = 0.512 \\ \frac{p}{4} = 0.4. & \frac{p^2}{8} = 0.32. & \frac{p^3}{16} = 0.256. \end{array}\right.$$

Fig. 31.

*Central.*

$$\left.\begin{array}{l} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.32 \\ \hline b = 0.82 = AD. \end{array}\right\} \quad \begin{array}{l} \frac{p}{4} = 0.4 \\ \frac{p^3}{16} = 0.256 \\ \frac{r}{2} = 0.324 \\ \hline d = 0.980 = DH. \end{array}$$

$$\left\{\begin{array}{l} x^4 - \overset{p.}{12}\,x^3 * - \overset{r.}{2400}\,x - \overset{s.}{16000} = 0 \\ x^4 - 1.2\,x^3 * - 2.400\,x - 1.6000 = 0 \end{array}\right\}$$

$NO = x = 20.$
$MO = - x = - 5.5 -$

Fig. 32.

$$\left\{\begin{array}{l} x^4 + \overset{p.}{12}\,x^3 * + \overset{r.}{2400}\,x - \overset{s.}{16000} = 0 \\ x^4 + 1.2\,x^3 * + 2.400\,x - 1.6000 = 0 \end{array}\right\}$$

$MO = x = 5.5 -$
$NO = - x = - 20.$

$$\begin{cases} \frac{p}{2} = 0.6. \quad \frac{p^2}{4} = 0.36. \quad \frac{p^3}{8\,L^2} = 0{,}216. \\ \frac{p}{4} = 0.3. \quad \frac{p^2}{8} = 0.18. \quad \frac{p^3}{16\,L^2} = 0.108. \end{cases}$$

*Central.*

Fig. 32.

$$\left.\begin{array}{l} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.18 \\ \hline b = 0.68 = AD \\ \hline \end{array}\right\} \qquad \begin{array}{l} \frac{p}{4} = 0.3 \\ \frac{p^3}{16} = 0.108 \\ \frac{r}{2} = 1.200 \\ \hline d = 1.608 = DH. \\ \hline \end{array}$$

$$\begin{cases} x^4 - \overset{p.}{12}x^3 \ast - \overset{r.}{500}x + \overset{s.}{6000} = 0 \\ x^4 - 1.2x^3 \ast - 0.500x + 0.6000 = 0 \end{cases}$$

$NO = x = 12.$
$no = x = 8.$ *ferè.*

Fig. 33.

$$\begin{cases} x^4 + \overset{p.}{12}x^3 \ast + \overset{r.}{500}x + \overset{s.}{6000} = 0 \\ x^4 + 1.2x^3 \ast + 0.500x + 0.6000 = 0 \end{cases}$$

$NO = -x = -12.$
$no = -x = -8.$ *ferè.*

$$\begin{cases} \frac{p}{2} = 0.6. \quad \frac{p^2}{4} = 0.36. \quad \frac{p^3}{8} = 0.216. \\ \frac{p}{4} = 0.3. \quad \frac{p^2}{8} = 0.18. \quad \frac{p^3}{16} = 0.108. \end{cases}$$

*Central.*

*Central.*

Fig. 33.

$$\left.\begin{array}{l} \frac{L}{2} = 0.5 \\ \frac{p^2}{8L} = 0.18 \\ \hline b = 0.68 = AD \\ \hline \end{array}\right\} \quad \begin{array}{l} \frac{p}{4} = 0.3 \\ \frac{p^3}{16} = 0.108 \\ \frac{r}{2} = 0.250 \\ \hline d = 0.658 = DH. \\ \hline \end{array}$$

CLAS.

## CLAS. VII.

### *De Æquationibus Quadraticis, vel duarum Dimenſionum, in quibus nullus deficit Terminorum; & de Quadrato-quadraticis, affectis ſub ſecundo & tertio gradu Parodico; vel de Æquationibus quatuor Dimenſionum, ubi deficit quartus Terminus.*

HOrum generum Æquationes ad ſequentes formulas reducuntur.

$$\left\{\begin{array}{ll} 1.\ x^2-px-q=0 & \left\{\begin{array}{ll} 1 & x^4-px^3-qx^2 \ast -S=0 \\ 3 & x^4-px^3-qx^2 \ast +S=0 \end{array}\right\} \\ 2.\ x^2+px-q=0 & \left\{\begin{array}{ll} 2 & x^4+px^3-qx^2 \ast -S=0 \\ 4 & x^4+px^3-qx^2 \ast +S=0 \end{array}\right\} \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}=b=AD. \quad \frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}=d=DH.$$

$$\left\{\begin{array}{ll} 3.\ x^2-px+q=0 & \left\{\begin{array}{ll} 5 & x^4-px^3+qx^2 \ast -S=0 \\ 7 & x^4-px^3+qx^2 \ast +S=0 \end{array}\right\} \\ 4.\ x^2+px+q=0 & \left\{\begin{array}{ll} 6 & x^4+px^3+qx^2 \ast -S=0 \\ 8 & x^4+px^3+qx^2 \ast +S=0 \end{array}\right\} \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2}+\frac{p^2}{8L}\backsim\frac{q}{2L}=b=AD. \quad \frac{p}{4}+\frac{p^3}{16L^2}\backsim\frac{pq}{4L^2}=d=DH.$$

*Reg.Gen.* Deſcribatur itaque Parabola (NAM), cujus Latus Rectum ſit L (ceu 1), & Axis (ay); ad quem
I 1 ordinatim applicetur $BA=\frac{p}{2}$, occurrens Parabolæ in B & A: Ex puncto A (puta) ducatur Diameter,
2 vel Axi parallela (Ay); in quâ ſumendo $Ab=\frac{L}{2}$, & bc=

## CLAS. VII.

### *Of Quadratics, or of Equations of two Dimenſions, in which neither of the Terms is wanting; and of Quadrato-quadratics, affected under the ſecond and third Parodic Degree; or of Equations of four Dimenſions, where the fourth Term is deficient.*

ALL Equations of both theſe kinds are reduced to theſe following forms.

$$\left\{\begin{array}{l} 1.\ x^2 - px - q = 0 \left\{\begin{array}{l} 1\ x^4 - px^3 - qx^2 * - S = 0 \\ 3\ x^4 - px^3 - qx^2 * + S = 0 \end{array}\right. \\ 2.\ x^2 + px - q = 0 \left\{\begin{array}{l} 2\ x^4 + px^3 - qx^2 * - S = 0 \\ 4\ x^4 + px^3 - qx^2 * + S = 0 \end{array}\right. \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} = d = DH.$$

$$\left\{\begin{array}{l} 3.\ x^2 - px + q = 0 \left\{\begin{array}{l} 5\ x^4 - px^3 + qx^2 * - S = 0 \\ 7\ x^4 - px^3 + qx^2 * + S = 0 \end{array}\right. \\ 4.\ x^2 + px + q = 0 \left\{\begin{array}{l} 6\ x^4 + px^3 + qx^2 * - S = 0 \\ 8\ x^4 + px^3 + qx^2 * + S = 0 \end{array}\right. \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} = d = DH.$$

*Gen. Rule* | Let a Parabole (NAM) therefore be deſcribed, whoſe *Latus Rectum* L (or 1), and Axe (ay); to

1 | 1 which, ordinately apply $BA = \frac{p}{2}$, meeting the Parabole in B and A: From the Point (ſuppoſe) A, draw the Diameter, or a Parallel to the Axe

2 (*viz.* Ay); in which, taking $Ab = \frac{L}{2}$, and

bc =

2 3 4 $bc = \frac{p^2}{8L}$, oportet facere $cD = \frac{q}{2L}$; eamque ſumere
in Diametro continuatâ verſus y, ſi in Æquatione ha-
beatur $-q$; ſed verſus alteram partem (ſurſum,) ſi
habeatur ibi $+q$.

3 Porrò, è Puncto D, (erigendo ad Diametrum per-
5 pendicularem DH,) oportet in eâ ſumere $De = \frac{p}{4}$,
4 6 7 & $ef = \frac{p^3}{16L^2}$; imo, & $fH = \frac{pq}{4L^2}$, ſi in Æquatione ha-
beatur $-q$ (collocanda ad ſiniſtram, &c.) Quod ſi ha-
5 7 beatur ibi $+q$; tum $fH = \frac{pq}{4L^2}$, ad dextram eſt collo-
canda, à Puncto f. Tum Centro quidem H, inter-
6 8 vallo verò HA, deſcribatur Circulus (NAM), ſi
Æquatio tantùm Quadratica fuerit, hoc eſt, ſi non
habeatur Quantitas S.

7 9 Aſt ſi habeatur S, & ſigno quidem $-$ adfecta (nempe
$-S$), oportet ulteriùs in hâc lineâ AH, utrinque
8 10 productâ, ſumere $AI = L$, ex unâ parte, & ex alterâ
9 11 $AK = \frac{S}{L^3}$; deſcriptoque Semicirculo, cujus Diameter
10 12 IK, erigere AL perpendicularem ad AH, quæ occur-
rat huic Semicirculo (ILK) in puncto L.
11 13 Quòd ſi verò habeatur $+S$; oportet inſuper in
alio Semicirculo, cujus Diameter ſit AH, inſcribere
11 14 $AZ = AL$ inventæ.

12 Circulus igitur deſcriptus, tranſiens per L, ſi ſit $-S$;
per Z verò, ſi ſit $+S$, ſecabit vel tanget Parabolam,
in tot Punctis, quot Æquatio diverſas admittet Radi-
ces; è quibus ſi ad Diametrum demittantur Perpendi-
culares, obtinebuntur omnes Æquationis radices, tàm
falſæ, quàm veræ. Quarum quidem veræ (ut NO)
ad ſiniſtram cadent, & falſæ (ut MO) ad dextram
14 Diametri, ſi in Æquatione habeatur $-p$: Sed contra,
15 ſi habeatur ibi $+p$, veræ (ut MO) cadent ad dex-
tram, falſæ verò (ut NO) ad ſiniſtram.

3 $bc=\frac{p^2}{8L}$, muſt be made alſo $cD=\frac{q}{2L}$, and be placed
2 4 in the ſame Diameter continued towards y, if in the Equation be had $-q$; but towards the other part (upward,) if be had there $+q$.

3 Moreover, from the Point D, (DH being erected perpendicular to the Diameter) muſt be made
5 5 6 $De=\frac{p}{4}$, and $ef=\frac{p^3}{16L^2}$; nay, and $fH=\frac{pq}{4L^2}$ (far-
7 ther to the left hand), if in the Equation be had $-q$;
7 but if $+q$, then from the Point f, $fH=\frac{pq}{4L^2}$ is to be placed to the right hand. Then center truly H, but
6 8 diſtance HA, let a Circle (NAM) be deſcribed, if it be only a Quadratic Equation, that is, if the Quantity S be not had.

7 9 But if S be had, and it be $-S$, then farther in this Line AH, both ways produced, muſt be taken on the
8 10 9 11 one ſide $AI=L$, and on the other ſide $AK=\frac{S}{L^3}$;
and a Semicircle being deſcribed, whoſe Diameter IK,
10 12 muſt be erected AL perpendicular to AH, which may meet this Semicircle (ILK) in the Point L.
11 13 But if $+S$ be had, there muſt moreover in another Semicircle, whoſe Diameter is AH, be inſcribed
11 14 $AZ=AL$ found.

12 A Circle therefore deſcribed, paſſing through L, if it be $-S$, but through Z, if it be $+S$, will cut or touch the Parabole in as many Points, as the Equation will admit Roots; from which, if Perpendiculars be demitted to the Diameter, all the Roots of the Equation, as well true as falſe, will be had: Of which, the true (as NO) will fall to the left hand, and the falſe
14 (as MO) to the right ſide of the Diameter, if in the
15 Equation be had $-p$: But contrarily, if be had there $+p$, the true (as MO) will fall to the right, but the falſe (as NO) to the left.

$$\begin{cases} 1.\ x^2 - px - q = 0 \begin{cases} 1 \ x^4 - px^3 - qx^2 \ast - S = 0 \\ 3 \ x^4 - px^3 - qx^2 \ast + S = 0 \end{cases} \\ 2.\ x^2 + px - q = 0 \begin{cases} 2 \ x^4 + px^3 - qx^2 \ast - S = 0 \\ 4 \ x^4 + px^3 - qx^2 \ast + S = 0 \end{cases} \end{cases}$$

*Demonſtrat.*

| | | | | |
|---|---|---|---|---|
| 2+3+4 | 15 | $\begin{cases} Ab + bc + cD = b = AD. \\ \frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD. \end{cases}$ | 1°. Ubi − q. | |
| 5+6+7 | 16 | $\begin{cases} De + ef + fH = d = DH. \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} = d = DH. \end{cases}$ | | |
| 47, e 1<br>Q.15. +<br>Q.16. | 17 | $\begin{cases} AD^2 + DH = HA^2. \\ b^2 + d^2 = (HA^2 =)\ \text{Q. Rad. in Quadrat.} \end{cases}$ | | *Fig.*34. |
| 10 × 11 | 18 | $\begin{cases} AI \times AK = (\text{ob Circl.})\ AL^2 = (\text{per conſtr.})\ AZ^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2. \end{cases}$ | | |
| 47, e 1<br>17 + 18 | 19 | $\begin{cases} AH^2 + AL^2 = HL^2. \\ b^2 + d^2 + \frac{S}{L^2} = (HL^2 =)\ \text{Q.Rad.} \end{cases}$ | In Biquadr.<br>ſi − S. | *Fig.*35. |
| 47, e 1<br>17 − 18 | 20 | $\begin{cases} AH^2 - AZ^2 = HZ^2. \\ b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =)\ \text{Q.Rad.} \end{cases}$ | In Biquadr.<br>ſi + S. | *Fig.*36. |
| *Supp.* | 21 | $NO = x.$ | | |
| *Supp.* | 21 | $MO = -x.$ | | |
| 21 − 1 | 22 | $\begin{cases} NO - (OF, u)\ BA = (NF, u)\ OR. \\ x - \frac{p}{2} = OR. \end{cases}$ | | |
| 21 + 1 | 22 | $\begin{cases} MO + (OF, u)\ BA = (MF, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{cases}$ | | |
| *Ob para.* | 23 | $\begin{cases} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ x :: x - \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{cases}$ | | |

L . MO

| | | | |
|---|---|---|---|
| *Ob para.* | 23 | $\left\{\begin{array}{l} L \,.\, MO :: \; OR \;.\; AO. \\ L \,.\, -x :: -x+\frac{p}{2} \,.\, \frac{x^2}{L}-\frac{px}{2L}=AO. \end{array}\right.$ | |
| 23 ∽ 15 | 24 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\, HP. \\ \frac{x^2}{L}-\frac{px}{2L}\,(-b, u)-\frac{L}{2}-\frac{p2}{8L}-\frac{q}{2L}=HP. \end{array}\right.$ | |
| @ | 25 | $b^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{p^2x^2}{L^2}-x^2-\frac{p^2x^2}{4L^2}-\frac{qx^2}{L^2}+\frac{px}{2}+\frac{p^3x}{8L^2}+\frac{pqx}{2L^2}=HP^2.$ | |
| 25 | 26 | $he,\, b^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}-x^2+\frac{px}{2}+\frac{p^3x}{8L^2}+\frac{pqx}{2L^2}=HP^2.$ | |
| 21 − 16 | 27 | $\left\{\begin{array}{l} NO-(OP, u)\, DH=PN. \\ x\,(-d, u)-\frac{p}{4}-\frac{p^3}{16L^2}-\frac{pq}{4L^2}=PN. \end{array}\right.$ | |
| 21 + 16 | 27 | $\left\{\begin{array}{l} MO+(OP, u)\, DH=PM. \\ -x\,(+d, u)+\frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}=PM. \end{array}\right.$ | |
| @ | 28 | $d^2+x^2-\frac{px}{2}-\frac{p^3x}{8L^2}-\frac{pqx}{2L^2}=PN^2.$ | |
| @ | 28 | $d^2+x^2-\frac{px}{2}-\frac{p^3x}{8L^2}-\frac{pqx}{2L^2}=PM^2.$ | |
| 47, e 1<br>26 + 28 | 29 | $\left\{\begin{array}{l} HP^2+PN^2=HN^2. \\ b^2+d^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}=(HN^2=)\ Q.Rad. \end{array}\right.$ | |
| 29 = 17 | 30 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}=0;\ in\ \frac{L^2}{x}.$ | |
| $\times\frac{L^2}{x^2}$ | 31 | $x^2-px-q=0.$ *Q.e.d.* in Quadratic. | *Fig.34.* |
| 29 = 19 | 32 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}=+\frac{S}{L^2};\ in\ L^2.$ | |
| $\times L^2$ | 33 | $x^4-px^3-qx^2=S.$ | |
| *Tranſp.* | 34 | $x^4-px^3-qx^2 * -S=0.$ In Biquadr. ſi $-S$. | *Fig.35.* |
| 29 = 20 | 35 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}=-\frac{S}{L^2};\ in\ L^2.$ | |
| $\times L^2$ | 36 | $x^4-px^3-qx^2=-S.$ | |
| *Tranſp.* | 37 | $x^4-px^3-qx^2 * +S=0.$ *Q.e.d.* in Biquadr. ſi $+S$. | *Fig.36.* |

| | | |
|---|---|---|
| *Supp.* | 21 | $MO = x.$ |
| *Supp.* | 21 | $NO = -x.$ |
| 21 + 1 | 22 | $\left\{\begin{array}{l} MO + (OF, u)\, BA = (MF, u)\, OR. \\ x + \frac{p}{2} = OR. \end{array}\right.$ |
| 21 — 1 | 22 | $\left\{\begin{array}{l} NO - (OF, u)\, BA = (NF, u)\, OR. \\ -x - \frac{p}{2} = OR. \end{array}\right.$ |
| *Ob para.* | 23 | $\left\{\begin{array}{l} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, x :: x + \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ |
| *Ob para.* | 23 | $\left\{\begin{array}{l} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, -x :: -x - \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ |
| 23 — 15 | 24 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\, HP. \\ \frac{x^2}{L} + \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} - \frac{q}{2L} = HP. \end{array}\right.$ |
| ⊚ | 25 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{qx^2}{L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2$ |
| 25 | 26 | $h\,c,\ b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$ |
| 21 + 16 | 27 | $\left\{\begin{array}{l} MO + (OP, u)\, DH = PM. \\ x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} = PM. \end{array}\right.$ |
| 21 — 16 | 27 | $\left\{\begin{array}{l} NO - (OP, u)\, DH = PN. \\ -x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} - \frac{pq}{4L^2} = PN. \end{array}\right.$ |
| ⊚ | 28 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = PM^2.$ |
| ⊚ | 28 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^3} + \frac{pqx}{2L^2} = PN^2.$ |
| 47, e 1<br>26 + 28 | 29 | $\left\{\begin{array}{l} HP^2 + PM^2 = HM^2 = Q.\,Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} = Q.\,Rad. \end{array}\right.$ |

$\frac{x^4}{L^2}$

|  |  |  |  |
|---|---|---|---|
| $29 = 17$ | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} = 0$; in $\frac{x^2}{L^2}$. |  |
| $\times \frac{L^2}{x^2}$ | 31 | $x^2 + px - q = 0$. *Q.e.d.* in Quadratic. | *Fig.* 34. |
| $29 = 19$ | 32 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} = + \frac{S}{L^2}$; in $L^2$. |  |
| $\times L^2$ | 33 | $x^4 + px^3 - qx^2 = S$. |  |
| *Tranſp.* | 34 | $x^4 + px^3 - qx^2 * - S = 0$. *Q.e.d.* in Biquad. ſi $-S$. | *Fig.* 35. |
| $29 = 20$ | 35 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} = - \frac{S}{L^2}$; in $L^2$. |  |
| $\times L^2$ | 36 | $x^4 + px^3 - qx^2 = - S$. |  |
| *Tranſp.* | 37 | $x^4 + px^3 - qx^2 * + S = 0$. *Q.e.d.* in Biquad. ſi $+S$. | *Fig.* 36. |

*Illuſtrat.*

$$\left\{\begin{array}{l} x^2 - \overset{p.}{12}x - \overset{q.}{64} = 0 \\ x^2 - 1.2x - 0.64 = 0 \end{array}\right\} \quad \begin{array}{l} NO = x = 16. \\ MO = -x = -4. \end{array}$$

$$\left\{\begin{array}{l} x^2 + \overset{p.}{12}x + \overset{q.}{64} = 0 \\ x^2 + 1.2x + 0.64 = 0 \end{array}\right\} \quad \begin{array}{l} MO = x = 4. \\ NO = -x = -16. \end{array}$$

*Fig.* 34.

*Central.*

$$\frac{p}{4} = 0.3. \qquad \frac{p^2}{8} = 0.18. \qquad \frac{p^3}{16} = 0.108.$$

$$\frac{L}{2} + \frac{p^2}{8} + \frac{q^2}{2L} = 1 = b. \qquad \frac{p}{4} + \frac{p^3}{16} + \frac{pq}{4} = 0.600 = d.$$

$$\left\{\begin{array}{l} x^4 - \overset{p.}{8}x^3 - \overset{q.}{40}x^2 * - \overset{s.}{8624} = 0 \\ x^4 - 0.8x^3 - 0.40x^2 * - 0.8624 = 0 \end{array}\right\}$$

*Fig.* 35.

$NO = x = 14.$

$MO = -x = - 8.8 +$

$x^4 +$

$$\left\{\begin{array}{l} \quad\;\; p. \qquad\quad q. \qquad\qquad\quad s. \\ x^4 + 8x^3 - 40x^2 * - 8624 = 0 \\ x^4 + 0.8x^3 - 0.40x^2 * - 0.8624 = 0 \end{array}\right\}$$

Fig.35.

$MO = x = 8.8+$

$NO = -x = -14.$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.4. & \frac{p^2}{4} = 0.16. & \frac{p^3}{8} = 0.064. \\ \frac{p}{4} = 0.2. & \frac{p^2}{8} = 0.08. & \frac{p^3}{16} = 0.032. \end{array}\right.$$

*Central.*

$$\left.\begin{array}{l} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.08 \\ \frac{q}{2} = 0.20 \\ \hline b = 0.78 = AD \end{array}\right\} \qquad \left.\begin{array}{l} \frac{p}{4} = 0.2 \\ \frac{p^3}{16} = 0.032 \\ \frac{pq}{4} = 0.080 \\ \hline d = 0.312 = DH. \end{array}\right.$$

$$\left\{\begin{array}{l} \quad\;\; p. \qquad\quad q. \qquad\qquad\quad s. \\ x^4 - 8x^3 - 78x^2 * + 4320 = 0 \\ x^4 - 0.8x^3 - 0.78x^2 * + 0.4320 = 0 \end{array}\right\}$$

$NO = x = 12.$

$no = x = 7.1+$

$$\left\{\begin{array}{l} \quad\;\; p. \qquad\quad q. \qquad\qquad\quad s. \\ x^4 + 8x^3 - 78x^2 * + 4320 = 0 \\ x^4 + 0.8x^3 - 0.78x^2 * + 0.4320 = 0 \end{array}\right\}$$

Fig.36.

$NO = -x = -12.$

$no = -x = - 7.1+$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.4. & \frac{p^2}{4} = 0.16. & \frac{p^3}{8} = 0.064. \\ \frac{p}{4} = 0.2. & \frac{p^2}{8} = 0.08. & \frac{p^3}{16} = 0.032. \end{array}\right.$$

*Central.*

*Central.*

$$\left.\begin{array}{l}\frac{L}{2}=0.5\\ \frac{p^2}{8}=0.08\\ \frac{q}{2}=0.39\\ \hline b=0.97=AD\end{array}\right\}\qquad\begin{array}{l}\frac{p}{4}=0.2\\ \frac{p^3}{16}=0.032\\ \frac{pq}{4}=0.156\\ \hline d=0.388=DH.\end{array}$$

Fig. 36.

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$$\left\{\begin{array}{l}3.\ x^2-px+q=0\left\{\begin{array}{l}5\ \left\{x^4-px^3+qx^2*-S=0\right\}\\ 7\ \left\{x^4-px^3+qx^2*+S=0\right\}\end{array}\right.\\ 4.\ x^2+px+q=0\left\{\begin{array}{l}6\ \left\{x^4+px^3+qx^2*-S=0\right\}\\ 8\ \left\{x^4+px^3+qx^2*+S=0\right\}\end{array}\right.\end{array}\right.$$

*Caſ.* 1. Ubi $\frac{L}{2}+\frac{p^2}{8L}\sqsupset\frac{q}{2L}$.

*Demonſtrat.*

2+3−4 | 15 | $\left\{\begin{array}{l}Ab+bc-cD=b=AD.\\ \frac{L}{2}+\frac{p^2}{8L}-\frac{q}{2L}=b=AD.\end{array}\right.$

5+6−7 | 16 | $\left\{\begin{array}{l}De+ef-fH=d=DH.\\ \frac{p}{4}+\frac{p^3}{16L^2}-\frac{pq}{4L^2}=d=DH.\end{array}\right.$

47, e 1, *Q*.15.+*Q*.16. | 17 | $\left\{\begin{array}{l}AD^2+DH^2=HA^2.\\ b^2+d^2=(HA^2=)\text{Q. Rad. in Quadr.}\end{array}\right.$ Fig. 37.

10 × 11 | 18 | $\left\{\begin{array}{l}AI\times AK=\text{(ob Circl.) }AL^2=\text{(per conſtr.) }AZ^2.\\ L\times\frac{S}{L^3}=)\ \frac{S}{L^2}=AL^2=AZ^2.\end{array}\right.$

47, e 1, 17+18 | 19 | $\left\{\begin{array}{l}AH^2+AL^2=HL^2.\\ b^2+d^2+\frac{S}{L^2}=(HL^2=)\text{Q.Rad.}\end{array}\right\}$ $\left\{\begin{array}{l}\text{In Biquadr.}\\ \text{ſi }-S.\end{array}\right\}$ Fig. 38.

AH²

| | | | |
|---|---|---|---|
| 47, e 1<br>17 — 18 | 20 | $\left\{\begin{array}{l} AH^2 - AZ^2 = HZ^2. \\ b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =)\ \text{Q.Rad.} \end{array}\right\}$ In Biquadr. si + S. | *Fig.* 39. |
| *Supp.* | 21 | $NO = x.$ | |
| *Supp.* | 21 | $MO = -x.$ | |
| 21 — 1 | 22 | $\begin{cases} NO - (OF, u)\ BA = (NF, u)\ OR. \\ x - \frac{p}{2} = OR. \end{cases}$ | |
| 21 + 1 | 22 | $\begin{cases} MO + (OF, u)\ BA = (MF, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{cases}$ | |
| *Ob para.* | 23 | $\begin{cases} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ x :: x - \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{cases}$ | |
| *Ob para.* | 23 | $\begin{cases} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ -x :: -x + \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{cases}$ | |
| 23 ∽ 15 | 24 | $\begin{cases} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} - \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} + \frac{q}{2L} = HP. \end{cases}$ | |
| ◎ | 25 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{p^2x^2}{L^2} - x^2 - \frac{p^2x^2}{L^2} + \frac{qx^2}{L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$ | |
| 25 | 26 | he, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$ | |
| 21 — 16 | 27 | $\begin{cases} NO - (OP, u)\ DH = PN. \\ x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} + \frac{pq}{4L^2} = PN. \end{cases}$ | |
| 21 + 16 | 27 | $\begin{cases} MO + (OP, u)\ DH = PM. \\ -x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} = PM. \end{cases}$ | |
| ◎ | 28 | $d^2 + x^2 - \frac{px}{2} - \frac{p^2x}{8L^2} + \frac{pqx}{2L^2} = PN^2.$ | |
| ◎ | 28 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{L^2} = PM^2.$ | |

Hp²

| | | | |
|---|---|---|---|
| 47, e 1<br>26 + 28 | 29 | $\left\{\begin{array}{l} HP^2 + PN^2 = HN^2. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} = (HN^2 =) \text{ Q. Rad.} \end{array}\right.$ | |
| 29 = 17 | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} = 0$; in $\frac{L^2}{x^2}$. | |
| $\times \frac{L^2}{x^2}$ | 31 | $x^2 - px + q = 0$. *Q. e. d.* in Quadratic. | *Fig.* 37. |
| 29 = 19 | 32 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 33 | $x^4 - px^3 + qx^2 = S$. | |
| *Tranſp.* | 34 | $x^4 - px^3 + qx^2 * - S = 0$. In Biquadr. ſi $-S$. | *Fig.* 38. |
| 29 = 20 | 35 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 36 | $x^4 - px^3 + qx^2 = - S$. | |
| *Tranſp.* | 37 | $x^4 - px^3 + qx^2 * + S = 0$. *Q. e. d.* in Biquadr. ſi $+S$. | *Fig.* 39. |
| *Supp.* | 21 | $MO = x$. | |
| *Supp.* | 21 | $NO = - x$. | |
| 21 + 1 | 22 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ x + \frac{p}{2} = OR. \end{array}\right.$ | |
| 21 − 1 | 22 | $\left\{\begin{array}{l} NO - (OF, u)\ BA = (NF, u)\ OR. \\ - x - \frac{p}{2} = OR. \end{array}\right.$ | |
| *Ob para.* | 23 | $\left\{\begin{array}{l} L . MO :: OR . AO. \\ L . x :: x + \frac{p}{2} . \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ | |
| *Ob para.* | 23 | $\left\{\begin{array}{l} L . NO :: OR . AO. \\ L . - x :: - x - \frac{p}{2} . \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ | |
| 23 ∽ 15 | 24 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} + \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} + \frac{q}{2L} = HP. \end{array}\right.$ | |
| ☉ | 25 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p'x^2}{4L^2} + \frac{qx^2}{L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2$ | |

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he,

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|---|---|---|---|
| 25 | 26 | he, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2$. | |
| 21 + 16 | 27 | $\left\{\begin{array}{l} MO + (OP, u)\,DH = PM. \\ x\,(+ d, u) + \frac{p}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} = PM. \end{array}\right.$ | |
| 21 − 16 | 27 | $\left\{\begin{array}{l} NO - (OP, u)\,DH = PN. \\ -x\,(- d, u) - \frac{p}{4} - \frac{p^3}{16L^2} + \frac{pq}{4L^2} = PN. \end{array}\right.$ | |
| $\odot$ | 28 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = PM^2$. | |
| $\odot$ | 28 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = PN^2$. | |
| 47, e 1<br>26 + 28 | 29 | $\left\{\begin{array}{l} HP^2 + PM^2 = (HM^2 =)\ Q.\,Rad. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = Q.\,Rad. \end{array}\right.$ | |
| 29 = 17 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = 0$; in $\frac{L^2}{x^2}$. | |
| $\times \frac{L^2}{x^2}$ | 31 | $x^2 + px + q = 0$. *Q.e.d.* in Quadratic. | *Fig.*37. |
| 29 = 19 | 32 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 33 | $x^4 + px^3 + qx^2 = S$. | |
| *Tranſp.* | 34 | $x^4 + px^3 + qx^2 * - S = 0$. *Q.e.d.* in Biquad. ſi $- S$. | *Fig.*38. |
| 29 = 20 | 35 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 36 | $x^4 + px^3 + qx^2 = - S$. | |
| *Tranſp.* | 37 | $x^4 + px^3 + qx^2 * + S = 0$. *Q.e.d.* in Biquad. ſi $+ S$. | *Fig.*39. |

*Illuſtrat.*

$$3\left\{\begin{array}{lll} & p. & q. \\ x^2 - & 28x + & 180 = 0 \\ x^2 - & 2.8x + & 1.80 = 0 \end{array}\right\}\quad \begin{array}{l} NO = x = 18. \\ no = x = 10. \end{array}$$

*Fig.*37.

$$4\left\{\begin{array}{lll} & p. & q. \\ x^2 + & 28x + & 180 = 0 \\ x^2 + & 2.8x + & 1.80 = 0 \end{array}\right\}\quad \begin{array}{l} NO = -x = -18. \\ no = -x = -10. \end{array}$$

$$\left\{\begin{array}{lll} \frac{p}{2} = 1.4. & \frac{p^2}{4} = 1.96. & \frac{p^3}{8} = 2.744. \\ \frac{p}{4} = 0.7. & \frac{p^2}{8} = 0.98. & \frac{p^3}{16} = 1.372. \end{array}\right.$$

*Central.*

Fig. 57.

$$\left.\begin{array}{r} \frac{L}{2} = 0.5 \\ \frac{p^2}{8} = 0.98 \\ \hline 1.48 \\ -\frac{q}{2} = 0.90 \\ \hline b = 0.58 = AD. \end{array}\right\} \quad \begin{array}{r} \frac{p}{4} = 0.7 \\ \frac{p^3}{16} = 1.372 \\ \hline 2.072 \\ -\frac{pq}{4} = 1.260 \\ \hline d = 0.812 = DH. \end{array}$$

$$5 \left\{\begin{array}{l} x^4 \overset{p.}{-\ 28}\,x^3 \overset{q.}{+\ 240}\,x^2 * \overset{s.}{-\ 6000} = 0 \\ x^4 - 2.8\,x^3 + 2.40\,x^2 * - 0.6000 = 0 \end{array}\right\}$$

$NO = x = 10.$
$MO = -x = -\ 4.0+$

Fig. 38.

$$6 \left\{\begin{array}{l} x^4 \overset{p.}{+\ 28}\,x^3 \overset{q.}{+\ 240}\,x^2 * \overset{s.}{-\ 6000} = 0 \\ x^4 + 2.8\,x^3 + 2.40\,x^2 * - 0.6000 = 0 \end{array}\right\}$$

$NO = -x = -\ 10.$
$MO = +x = 4.0+$

$$\left\{\begin{array}{lll} \frac{p}{2} = 1.4. & \frac{p^2}{4} = 1.96. & \frac{p^3}{8} = 2.744. \\ \frac{p}{4} = 0.7. & \frac{p^2}{8} = 0.98. & \frac{p^3}{16} = 1.372. \end{array}\right.$$

*Central.*

Fig.38.

$$\left.\begin{array}{r} \frac{L}{2}=0.5 \\ \frac{p^2}{8}=0.98 \\ \hline 1.48 \\ -\frac{q}{2}=1.20 \\ \hline b=0.28=AD \\ \hline \end{array}\right\} \quad \begin{array}{r} \frac{p}{4}=0.7 \\ \frac{p^3}{16}=1.372 \\ \hline 2.072 \\ -\frac{pq}{4}=1.680 \\ \hline d=0.392=DH. \\ \hline \end{array}$$

$$7\left\{\begin{array}{l} x^4 - \overset{p.}{28}x^3 + \overset{q.}{90}x^2 * + \overset{s.}{26112}=0 \\ x^4 - 2.8x^3 + 0.90x^2 * + 2.6112=0 \end{array}\right\}$$

$NO = x = 20. +$

$no = x = 16.$

$$8\left\{\begin{array}{l} x^4 + \overset{p.}{28}x^3 + \overset{q.}{90}x^2 * + \overset{s.}{26112}=0 \\ x^4 + 2.8x^3 + 0.90x^2 * + 2.6112=0 \end{array}\right\}.$$

$NO = -x = -20. +$

$no = -x = -16.$

Fig.39.

$$\left\{\begin{array}{lll} \frac{p}{2}=1.4. & \frac{p^2}{4}=1.96. & \frac{p^3}{8}=2.744. \\ \frac{p}{4}=0.7. & \frac{p^2}{8}=0.98. & \frac{p^3}{16}=1.372. \end{array}\right.$$

*Central.*

$$\left.\begin{array}{r} \frac{L}{2}=0.5 \\ \frac{p^2}{8}=0.98 \\ \hline 1.48 \\ -\frac{q}{2}=0.45 \\ \hline b=1.03=AD \\ \hline \end{array}\right\} \quad \begin{array}{r} \frac{p}{4}=0.7 \\ \frac{p^3}{16}=1.372 \\ \hline 2.072 \\ -\frac{pq}{4}=0.630 \\ \hline d=1.442=DH. \\ \hline \end{array}$$

*Caſ.* 2.

### Caſ. 2. Ubi $\frac{q}{2L} \sqsupset \frac{L}{2} + \frac{p^2}{8L}$.

*Demonſtrat.*

| | | | | |
|---|---|---|---|---|
| 4—3—2 | 15 | $cD - bc - Ab = b = AD.$<br>$\frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} = b = AD.$ | | |
| 7—6—5 | 16 | $fH - ef - De = d = DH.$<br>$\frac{pq}{4L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = d = DH.$ | | |
| 47, e 1<br>Q.15.+<br>Q.16. | 17 | $AD^2 + DH^2 = HA^2.$<br>$b^2 + d^2 = (HA^2 =)$ Q. Rad. in Quadr. | | *Impoſſib.* |
| 10 × 11 | 18 | $AI \times AK =$ (ob Circl.) $AL^2 =$ (per conſtr.) $AZ^2.$<br>$(L \times \frac{S}{L^3} =) \frac{S}{L^2} = AL^2 = AZ^2.$ | | |
| 47, e 1<br>17+18 | 19 | $AH^2 + AL^2 = HL^2.$<br>$b^2 + d^2 + \frac{S}{L^2} = (HL^2 =)$ Q.Rad. | In Biquadr.<br>ſi − S. | *Fig.* 40. |
| 47, e 1<br>17 − 18 | 20 | $AH^2 - AZ^2 = HZ^2.$<br>$b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =)$ Q.Rad. | In Biquadr.<br>ſi + S. | *Impoſſib.* |
| *Supp.* | 21 | $NO = x.$ | | |
| *Supp.* | 21 | $MO = -x.$ | | |
| 21 — 1 | 22 | $NO - (OF, u)\ BA = (NF, u)\ OR.$<br>$x - \frac{p}{2} = OR.$ | | |
| 21 + 1 | 22 | $MO + (OF, u)\ BA = (MF, u)\ OR.$<br>$-x + \frac{p}{2} = OR.$ | | |
| *Ob para.* | 23 | $L \,.\, NO :: OR \,.\, AO.$<br>$L \,.\, x :: x - \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO.$ | | |

| | | |
|---|---|---|
| *Ob para.* | 23 | $\left\{\begin{array}{l} L \,.\, MO :: \;OR \;.\; AO. \\ L \,.\, -x :: -x+\frac{p}{2} \,.\, \frac{x^2}{L}-\frac{px}{2L}=AO. \end{array}\right.$ |
| 23 + 15 | 24 | $\left\{\begin{array}{l} AO+AD=(DO,\ u)\ HP. \\ \frac{x^2}{L}-\frac{px}{2L}\ (+b,\ u)+\frac{q}{2L^2}-\frac{p^2}{8L}-\frac{L}{2}=HP. \end{array}\right.$ |
| © | 25 | $b^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{p^2x^2}{4L^2}+\frac{qx^2}{L^2}-\frac{p^2x^2}{4L^2}-x^2-\frac{pqx}{2L^2}+\frac{p^3x}{8L^2}+\frac{px}{2}=HP^2.$ |
| 25 | 26 | he, $b^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}-x^2-\frac{pqx}{2L^2}+\frac{p^3x}{8L^2}+\frac{px}{2}=HP^2.$ |
| 21 + 16 | 27 | $\left\{\begin{array}{l} NO+(OP,\ u)\ DH=PN. \\ x\ (+d,\ u)+\frac{pq}{4L^2}-\frac{p^3}{16L^2}-\frac{p}{4}=PN. \end{array}\right.$ |
| 21 — 16 | 27 | $\left\{\begin{array}{l} MO-(OP,\ u)\ DH=PM. \\ -x\ (-d,u)-\frac{pq}{4L^2}+\frac{p^3}{16L^2}+\frac{p}{4}=PM. \end{array}\right.$ |
| © | 28 | $d^2+x^2+\frac{pqx}{2L^2}-\frac{p^3x}{8L^2}-\frac{px}{2}=PN^2.$ |
| © | 28 | $d^2+x^2+\frac{pqx}{2L^2}-\frac{p^3x}{8L^2}-\frac{px}{2}=PM^2.$ |
| 47, e 1<br>26 + 28 | 29 | $\left\{\begin{array}{l} HP^2+PN^2=HN^2. \\ b^2+d^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}=(HN^2=)\ Q.\ Rad. \end{array}\right.$ |
| 29 = 17 | 30 | $\frac{x^4}{L^2}-\frac{px^3}{L2}+\frac{qx^2}{L^2}=0$; in $\frac{L^2}{x^2}$. |
| $\times\frac{L^2}{x^2}$ | 31 | $x^3-px+q=0$. *Q.e.d.* in Quadr. |
| 29 = 19 | 32 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}=+\frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 33 | $x^4-px^3+qx^2=S.$ |
| *Transp.* | 34 | $x^4-px^3+qx^2*-S=0$. *Q.e.d.* in Biquad. ſi —S. *Fig.*40. |
| 29 = 20 | 35 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}=-\frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 36 | $x^4-px^3+qx^2=-S.$ |
| *Transp.* | 37 | $x^4-px^3+qx^2*+S=0$. *Q.e.d.* in Biquadr. ſi +S. |

MO

| | | |
|---|---|---|
| *Supp.* | 21 | $MO = x.$ |
| *Supp.* | 21 | $NO = -x.$ |
| 21 + 1 | 22 | $\begin{cases} MO + (OF, u)\ BA = (MF, u)\ OR. \\ x + \frac{p}{2} = OR. \end{cases}$ |
| 21 — 1 | 22 | $\begin{cases} NO - (OF, u)\ BA = (NF, u)\ OR. \\ -x - \frac{p}{2} = OR. \end{cases}$ |
| *Ob para.* | 23 | $\begin{cases} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ |
| *Ob para.* | 23 | $\begin{cases} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ -x :: -x - \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{cases}$ |
| 23 + 15 | 24 | $\begin{cases} AO + AD = (DO, u)\ HP. \\ \frac{x^2}{L} + \frac{px}{2L}\ (+b, u) + \frac{q}{2L} - \frac{p2}{8L} - \frac{L}{2} = HP. \end{cases}$ |
| G | 25 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{p^2x^2}{4L^2} + \frac{qx^2}{L^2} - \frac{p^2x^2}{4L^2} - x^2 + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = HP^2.$ |
| 25 | 26 | he, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = HP^2.$ |
| 21 — 16 | 27 | $\begin{cases} MO - (OP, u)\ DH = PM. \\ x\ (-d, u) - \frac{pq}{4L^2} + \frac{p^3}{16L^2} + \frac{p}{4} = PM. \end{cases}$ |
| 21 + 16 | 27 | $\begin{cases} NO + (OP, u)\ DH = PN. \\ -x\ (+d, u) + \frac{pq}{4L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN. \end{cases}$ |
| G | 28 | $d^2 + x^2 - \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = PM^2.$ |
| G | 28 | $d^2 + x^2 - \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = PN^2.$ |
| 47, e 1<br>26 + 28 | 29 | $\begin{cases} HP^2 + PM^2 = HM^2. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = (HM^2 =)\ Q.\ Rad. \end{cases}$ |

| | | |
|---|---|---|
| 29 = 17 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = 0$; in $\frac{L^2}{x^2}$. |
| $\times \frac{L^2}{x^2}$ | 31 | $x^2 + px + q = 0$. *Q.e.d.* in Quadratic. |
| 29 = 19 | 32 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = + \frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 33 | $x^4 + px^3 + qx^2 = S$. |
| *Tranſp.* | 34 | $x^4 + px^3 + qx^2 * - S = 0$. *Q.e.d.* in Biquadr. ſi — S. |
| 29 = 20 | 35 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} = - \frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 36 | $x^4 + px^3 + qx^2 = - S$. |
| *Tranſp.* | 37 | $x^4 + px^3 + qx^2 * + S = 0$. *Q.e.d.* in Biquadr. ſi + S. |

*Fig.*40.

*Illuſtrat.*

$$5 \begin{Bmatrix} x^4 - \overset{p.}{16}x^3 + \overset{q.}{212}x^2 * - \overset{s.}{23616} = 0 \\ x - 1.6x^3 + 2.12x^2 * - 2.3616 = 0 \end{Bmatrix}$$

NO = x = 12.
MO = − x = − 7. +

$$6 \begin{Bmatrix} x^4 + \overset{p.}{16}x^3 + \overset{q.}{212}x^2 * - \overset{s.}{23616} = 0 \\ x^4 + 1.6x^3 + 2.12x^2 * - 2.3616 = 0 \end{Bmatrix}$$

*Fig.*40.

MO = x = 7. +
NO = − x = − 12.

$$\begin{cases} \frac{p}{2} = 0.8. \quad \frac{p^2}{4} = 0.64. \quad \frac{p^3}{8} = 0.512. \\ \frac{p}{4} = 0.4. \quad \frac{p^2}{8} = 0.32. \quad \frac{p^3}{16} = 0.256. \end{cases}$$

*Central.*

*Central.*

$+\frac{q}{2} = 1.06$

$-\frac{p^2}{8} = 0.32$

$-\frac{L}{2} = 0.5$

$0.82$

$b = 0.24 = AD.$

$+\frac{pq}{4} = 0.848$

$-\frac{p^3}{16} = 0.256$

$-\frac{p}{4} = 0.4$

$0.656$

$d = 0.192 = DH.$

CLAS.

## CLAS. VIII.

*De Æquationibus Cubicis, & Quadrato-quadraticis, ſub omnibus gradibus Parodicis aſſectis; vel, de Æquationibus trium & quatuor Dimenſionum, in quibus nullus deficit Terminorum.*

HUjus quidem cenſûs Æquationes ad octo, illius verò ad ſexdecem formulas reduci poſſint.

$$\left\{\begin{array}{lll} 1.\ x^3 - px^2 - qx - r = 0 & \left\{\begin{array}{l} 1 \\ 3 \end{array}\right. & \left\{\begin{array}{l} x^4 - px^3 - qx^2 - rx - S = 0 \\ x^4 - px^3 - qx^2 - rx + S = 0 \end{array}\right\} \\ 2.\ x^3 + px^2 - qx + r = 0 & \left\{\begin{array}{l} 2 \\ 4 \end{array}\right. & \left\{\begin{array}{l} x^4 + px^3 - qx^2 + rx - S = 0 \\ x^4 + px^3 - qx^2 + rx + S = 0 \end{array}\right\} \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} + \frac{r}{2L^2} = d = DH.$$

$$\left\{\begin{array}{lll} 3.\ x^3 - px^2 - qx + r = 0 & \left\{\begin{array}{l} 5 \\ 7 \end{array}\right. & \left\{\begin{array}{l} x^4 - px^3 - qx^2 + rx - S = 0 \\ x^4 - px^3 - qx^2 + rx + S = 0 \end{array}\right\} \\ 4.\ x^3 + px^2 - qx - r = 0 & \left\{\begin{array}{l} 6 \\ 8 \end{array}\right. & \left\{\begin{array}{l} x^4 + px^3 - qx^2 - rx - S = 0 \\ x^4 + px^3 - qx^2 - rx + S = 0 \end{array}\right\} \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} \backsim \frac{r}{2L^2} = d = DH.$$

$$\left\{\begin{array}{lll} 5.\ x^3 - px^2 + qx + r = 0 & \left\{\begin{array}{l} 9 \\ 11 \end{array}\right. & \left\{\begin{array}{l} x^4 - px^3 + qx^2 + rx - S = 0 \\ x^4 - px^3 + qx^2 + rx + S = 0 \end{array}\right\} \\ 6.\ x^3 + px^2 + qx - r = 0 & \left\{\begin{array}{l} 10 \\ 12 \end{array}\right. & \left\{\begin{array}{l} x^4 + px^3 + qx^2 - rx - S = 0 \\ x^4 + px^3 + qx^2 - rx + S = 0 \end{array}\right\} \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} \backsim \frac{r}{2L^2} = d = DH.$$

## CLAS. VIII.

*Of Cubic and Biquadratic Equations, affected under all their Parodic Degrees; or, of Equations of three and four Dimenſions, in which neither of their Terms is wanting.*

ALL Cubic Equations of this kind may be reduced to eight, but Quadrato-quadratics to ſixteen forms.

$$\left\{\begin{array}{l} 1.\ x^3-px^2-qx-r=0 \left\{\begin{array}{r}1\\3\end{array}\right\} \left.\begin{array}{l}x^4-px^3-qx^2-rx-S=0\\ x^4-px^3-qx^7-rx+S=0\end{array}\right\} \\ 2.\ x^3+px^2-qx+r=0 \left\{\begin{array}{r}2\\4\end{array}\right\} \left.\begin{array}{l}x^4+px^3-qx^2+rx-S=0\\ x^4+px^3-qx^2+rx+S=0\end{array}\right\} \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}=b=AD.\quad \frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}+\frac{r}{2L^2}=d=DH.$$

$$\left\{\begin{array}{l} 3.\ x^3-px^2-qx+r=0 \left\{\begin{array}{r}5\\7\end{array}\right\} \left.\begin{array}{l}x^4-px^3-qx^2+rx-S=0\\ x^4-px^3-qx^2+rx+S=0\end{array}\right\} \\ 4.\ x^3+px^2-qx-r=0 \left\{\begin{array}{r}6\\8\end{array}\right\} \left.\begin{array}{l}x^4+px^3-qx^2-rx-S=0\\ x^4+px^3-qx^2-rx+S=0\end{array}\right\} \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}=b=AD.\quad \frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}\backsim\frac{r}{2L^4}=d=DH.$$

$$\left\{\begin{array}{l} 5.\ x^3-px^2+qx+r=0 \left\{\begin{array}{r}9\\11\end{array}\right\} \left.\begin{array}{l}x^4-px^3+qx^2+rx-S=0\\ x^4-px^3+qx^2+rx+S=0\end{array}\right\} \\ 6.\ x^3+px^2+qx-r=0 \left\{\begin{array}{r}10\\12\end{array}\right\} \left.\begin{array}{l}x^4+px^3+qx^2-rx-S=0\\ x^4+px^3+qx^2-rx+S=0\end{array}\right\} \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2}+\frac{p^2}{8L}\backsim\frac{q}{2L}=b=AD.\quad \frac{p}{4}+\frac{p^3}{16L^2}\backsim\frac{pq}{4L^2}\backsim\frac{r}{2L^2}=d=DH.$$

$$\left\{\begin{array}{l} 7.\ x^3 - px^2 + qx - r = 0 \left\{\begin{array}{l} 13\ \{x^4 - px^3 + qx^2 - rx - S = 0 \\ 15\ \{x^4 - px^3 + qx^2 - rx + S = 0 \end{array}\right. \\ 8.\ x^3 + px^2 + qx + r = 0 \left\{\begin{array}{l} 14\ \{x^4 + px^3 + qx^2 + rx - S = 0 \\ 16\ \{x^4 + px^3 + qx^2 + rx + S = 0 \end{array}\right. \end{array}\right.$$

*Regula Centralis.*

$$\frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD.\quad \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} + \frac{r}{2L^2} = d = DH.$$

*Reg.Gen.* Deſcribatur itaque Parabola (N A M), cujus Latus
Rectum ſit L (ceu 1), Axiſque (a y); ad quem
1 1 ordinatìm applicetur recta $BA = \frac{p}{2}$, occurrens Parabolæ
in B & A: Ex puncto A (puta) ducatur Diameter,
vel Axi parallela (A y); in quâ ſumptâ (AD=b,
2 2 3 hoc eſt, $Ab = \frac{L}{2}$, & $bc = \frac{p^2}{8L}$, deorſùm continùo
verſus y ſunt collocandæ. Tum exinde (à Puncto c,)
2 4 oportet facere $cD = \frac{q}{2L}$, eamque quidem ulteriùs
deorſum verſus y collocare, ſi in Æquatione habeatur
— q; ſurſum verò, verſus alteram partem, ſi habeatur
2 ibi + q, inventumque erit Punctum D. A quo,
erigatur perpendicularis ad A y, recta (DH = d, h. e.)
3 5 6 $De = \frac{p}{4}$, & $ef = \frac{p^3}{16L^2}$, ad ſiniſtram continuò collocandæ.
4 7 Tum à Puncto f, oportet facere $fg = \frac{pq}{4L^2}$,
eamque exinde ulteriùs ad ſiniſtram collocare, ſi in
5 Æquatione habeatur — q; ad dextram vero ſi + q.
8 Denique ex Puncto g, oportet facere $gH = \frac{r}{2L^2}$, eamque
ulteriùs exinde ad ſiniſtram collocare, ſi in Æquatione
p & r, iiſdem ſignis ſint adfectæ; ad dextram
verò exinde, ſi diverſis; inventumque erit Punctum H,
6 ſive Circuli centrum. Quo invento, & connexâ H A,
oportet ex Centro H, Circulum (N A M), deſcribere,
cujus Semidiameter ſit H A, ſi Æquatio tantùm Cubica
fuerit, hoc eſt, ſi non habeatur Quantitas S.

Aſt

$$\left\{\begin{array}{l} 7.\ x^3 - px^2 + qx - r = 0 \left\{\begin{array}{l} 13\ \{x^4 - px^3 + qx^2 - rx - S = 0\} \\ 15\ \{x^4 - px^3 + qx^2 - rx + S = 0\} \end{array}\right. \\ 8.\ x^3 + px^2 + qx + r = 0 \left\{\begin{array}{l} 14\ \{x^4 + px^3 + qx^2 + rx - S = 0\} \\ 16\ \{x^4 + px^3 + qx^2 + rx + S = 0\} \end{array}\right. \end{array}\right.$$

*Central Rule.*

$$\frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD. \quad \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} + \frac{r}{2L^2} = d = DH.$$

*Gen. Rule* Let a Parabole (N A M) therefore be deſcribed,
whoſe *Latus Rectum* L (or 1), and Axe (a y); to
1 1 which, ordinately apply $BA = \frac{p}{2}$, meeting the Parabole in B and A: From the Point A (ſuppoſe),
draw the Diameter, or a Parallel to the Axe
(*viz.* A y); in which, let be taken ($AD = b$, *i. e.*)
2 2 $Ab = \frac{L}{2}$, and $bc = \frac{p^2}{8L}$, placing them both always
3 downwards towards y. Then from thence (from the
2 4 Point c,) make $cD = \frac{q}{2L}$, placing it indeed yet farther downwards towards y, if in the Equation be had
—q; but upward, towards the other ſide, if be had
+q, and the Point D will be found. From which
2 Point, erect perpendicular to A y ($DH = d$, *i. e.*)
3 5 $De = \frac{p}{4}$, and $ef = \frac{p^3}{16L^2}$, both which place always
6 to the left hand. Then from the Point f, make
4 7 $fg = \frac{pq}{4L^2}$, placing it from thence farther to the left,
5 if in the Equation be had —q; but on the right, if +q.
8 Laſtly, from the Point g, make $gH = \frac{r}{2L^2}$, placing it
thence farther to the left hand, if in the Equation p
and r are affected with the ſame Signs; but to the right
hand from thence, if with divers; and the Point H, or
the center of the Circle, will be found: Which found,
6 and H A connected, center H, Semidiameter H A, let
the Circle (N A M) be deſcribed, if it be only a Cubic
Equation, *i. e.* if the Quantity S be not had. But

7 Aſt ſi habeatur S, & ſit — S, oportet ulteriùs in
hàc lineâ A H, productâ utrinque, ex unâ parte ſu-
8 9 mere A I = L, & ex alterâ parte $AK = \frac{S}{L^3}$; deſcri-
9 10
10 11 ptoque Semicirculo, cujus Diameter I K, erigere A L
ad A H perpendicularem, quæ occurrat huic Semicir-
culo (I L K) in puncto L.

11 Quòd ſi verò habeatur + S; oportet inſuper in
alio Semicirculo, cujus Diameter ſit A H, inſcribere
11 12 A Z = A L inventæ.

12 Circulus igitur deſcriptus, tranſiens per L, ſi ſit — S;
per Z verò, ſi ſit + S, ſecare vel tangere poſſit Para-
bolam, in tot Punctis, quot Æquatio diverſas admittet
12 Radices; è quibus ſi ad Diametrum (A y) demittantur
Perpendiculares, habebuntur omnes Æquationis radi-
ces, tàm falſæ, quàm veræ. Quarum quidem veræ
14 (ut N O) ad ſiniſtram partem Diametri cadent, &
falſæ (ut M O) ad ejus dextram, ſi in Æquatione
15 habeatur — p; Sed contra, ſi habeatur ibi + p, veræ
quidem cadent ad dextram (ut M O), falſæ verò (ut
N O) ad ſiniſtram.

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$$\left\{\begin{array}{l} 1.\ x^3 - px^2 - qx - r = 0 \left\{\begin{array}{l} 1 \left\{ x^4 - px^3 - qx^2 - rx - S = 0 \right\} \\ 3 \left\{ x^4 - px^3 - qx^2 - rx + S = 0 \right\} \end{array}\right. \\ 2.\ x^3 + px^2 - qx + r = 0 \left\{\begin{array}{l} 2 \left\{ x^4 + px^3 - qx^2 + rx - S = 0 \right\} \\ 4 \left\{ x^4 + px^3 - qx^2 + rx + S = 0 \right\} \end{array}\right. \end{array}\right.$$

*Demonſtrat.*

2+3+4 13 $\left\{\begin{array}{l} Ab + bc + cD = b = AD. \\ \frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD. \end{array}\right.$

5+6+7+8 14 $\left\{\begin{array}{l} De + ef + fg + gH = d = DH. \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} + \frac{r}{2L^2} = d = DH. \end{array}\right.$

47, e 1
Q.13.+ 15 $\left\{\begin{array}{l} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = (HA^2 =)\ \text{Q. Rad. in Cubic.} \end{array}\right.$ Fig. 41.
Q. 14.

AI×

7 But if S be had, and it be —S, then muſt there be
farther in this Line A H, both ways produced, taken on
8 9 the one ſide $AI = L$, and on the other $AK = \frac{S}{L^3}$;
9 10
and a Semicircle being deſcribed, whoſe Diameter I K,
10 11 muſt be erected A L perpendicular to A H, which may
meet this Semicircle (I L K) in the Point L.
11 But if + S be had, there muſt moreover in another
Semicircle, whoſe Diameter is A H, be inſcribed
11 12 $AZ = AL$ found.
12 A Circle therefore deſcribed, whoſe Center H paſſing
through L if it be — S, but through Z if + S,
will cut or touch the Parabole in ſo many Points, as
the Equation will admit diverſity of Roots; from
which, if Perpendiculars be demitted to the Diameter
12 (A y), all the Roots of the Equation, as well falſe
as true, will be had: Of which, thoſe truly which are
14 true (as N O) will fall on the left ſide of the Diameter,
and the falſe (as M O) on the right, if in the
15 Equation be had — p: But on the contrary, if it be
+ p, the true indeed will fall on the right hand (as
M O), but the falſe (as N O) on the left.

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9 × 10 16 $$\left\{\begin{array}{l} AI \times AK = \text{(ob Circl.)}\ AL^2 = \text{(per conſtr.)}\ AZ^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2. \end{array}\right.$$

47, e 1 / 15 + 16 17 $$\left\{\begin{array}{l} AH^2 + AL^2 = HL^2. \\ \quad b^2 + d^2 + \frac{S}{L^2} = (HL^2 =)\ \text{Q.Rad.} \end{array}\right\} \begin{array}{c} \text{In Biquadr.} \\ \text{ſi} - S. \end{array} \Big\}$$ Fig. 42.

47, e 1 / 15 — 16 18 $$\left\{\begin{array}{l} AH^2 - AZ^2 = HZ^2. \\ \quad b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =)\ \text{Q.Rad.} \end{array}\right\} \begin{array}{c} \text{In Biquadr.} \\ \text{ſi} + S. \end{array} \Big\}$$ Fig. 43.

Supp. 19 $NO = x$.

Supp. 19 $MO = -x$.

| | | |
|---|---|---|
| $19 - 1$ | 20 | $\left\{\begin{array}{l} NO - (OF, u)\ BA = (NF, u)\ OR. \\ x - \frac{p}{2} = OR. \end{array}\right.$ |
| $19 + 1$ | 20 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{array}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ x :: x - \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ -x :: -x + \frac{p}{2}\ .\ \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ |
| $21 \backsim 13$ | 22 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} - \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} - \frac{q}{2L^2} = HP. \end{array}\right.$ |
| ◎ | 23 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2.$ |
| 23 | 24 | hc, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2.$ |
| $19 - 14$ | 25 | $\left\{\begin{array}{l} NO - (OP, u)\ DH = PN. \\ x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} - \frac{pq}{4L^2} - \frac{r}{2L^2} = PN. \end{array}\right.$ |
| $19 + 14$ | 25 | $\left\{\begin{array}{l} MO + (OP, u)\ DH = PM. \\ -x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} + \frac{r}{2L^2} = PM. \end{array}\right.$ |
| ◎ | 26 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} - \frac{rx}{L^2} = PN^2.$ |
| ◎ | 26 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} - \frac{rx}{L^2} = PM^2.$ |
| 47, e 1<br>$24 + 26$ | 27 | $\left\{\begin{array}{l} HP^2 + PN^2 = HN^2. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HN^2 =)\ \text{Q.Rad.} \end{array}\right.$ |
| $27 = 15$ | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. |
| $\times \frac{L^2}{x}$ | 29 | $x^3 - px^2 - qx - r = 0.$ *Q.e.d.* in Cubic. |

Fig. 41.

$\frac{x^4}{L^2}$

| | | | |
|---|---|---|---|
| 27 = 17 | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in L². | |
| × L² | 31 | $x^4 - px^3 - qx^2 - rx = S.$ | |
| *Tranſp.* | 32 | $x^4 - px^3 - qx^2 - rx - S = 0.$ in Biquadr. ſi — S. | *Fig.*42. |
| 27 = 18 | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in L². | |
| × L² | 34 | $x^4 - px^3 - qx^2 - rx = - S.$ | |
| *Tranſp.* | 35 | $x^4 - px^3 - qx^2 - rx + S = 0.$ *Q.e.d.* in Biquad. ſi + S. | *Fig.*43. |
| *Supp.* | 19 | MO = x. | |
| *Supp.* | 19 | NO = — x. | |
| 19 + 1 | 20 | MO + (OF, u) BA = (MF, u) OR.<br>$x + \frac{p}{2} = OR.$ | |
| 19 — 1 | 20 | NO — (OF, u) BA = (NF, u) OR.<br>$- x - \frac{p}{2} = OR.$ | |
| *Ob para.* | 21 | L . MO :: OR . AO.<br>$L \,.\, x :: x + \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO.$ | |
| *Ob para.* | 21 | L . NO :: OR . AO.<br>$L \,.\, - x :: - x - \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO.$ | |
| 21 ∽ 13 | 22 | AO ∽ AD = (DO, u) HP.<br>$\frac{x^2}{L} + \frac{px}{2L} (\backsim b, u) - \frac{L}{2} - \frac{p^2}{8L} - \frac{q}{2L} = HP.$ | |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$ | |
| 23 | 24 | he, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$ | |
| 19 + 14 | 25 | MO + (OP, u) DH = PM.<br>$x (+ d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} + \frac{r}{2L^2} = PM.$ | |

| | | | |
|---|---|---|---|
| 19 — 14 | 25 | $\begin{cases} NO - (OP, u)\ DH = PN. \\ -x\ (-d, u) - \frac{p}{4} - \frac{p^2}{16L^2} - \frac{pq}{4L^2} - \frac{r}{2L^2} = PN. \end{cases}$ | |
| ⊕ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} + \frac{rx}{L^2} = PM^2.$ | |
| ⊕ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} + \frac{rx}{L^2} = PN^2.$ | |
| 47, 51<br>24 + 26 | 27 | $\begin{cases} HP^2 + PM^2 = HM^2. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = (HM^2 =)\ Q.Rad. \end{cases}$ | |
| 27 = 15 | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 + px^2 - qx + r = 0$. *Q.e.d.* in Cubic. | *Fig.*41. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$. | 31 | $x^4 + px^3 - qx^2 + rx = S.$ | |
| *Tranſp.* | 32 | $x^4 + px^3 - qx^2 + rx - S = 0$. *Q.e.d.* in Biquad. ſi — S. | *Fig.*42. |
| 27 = 18 | 33 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| *Tranſp.* | 34 | $x^4 + px^3 - qx^2 + rx + S = 0$. *Q.e.d.* in Biquad. ſi + S. | *Fig.*43. |

*Illuſtrat.*

$$1\ \begin{Bmatrix} x^3 - \overset{p.}{8}\, x^2 - \overset{q.}{324}\, x - \overset{r.}{1440} = 0 \\ x^3 - 0.8\, x^2 - 3.24\, x - 1.440 = 0 \end{Bmatrix}$$

$$NO = x = 24. \quad \begin{cases} MO = -x = -10. \\ mo = -x = -6. \end{cases}$$

*Fig.*41.

$$2\ \begin{Bmatrix} x^3 + \overset{p.}{8}\, x^2 - \overset{q.}{324}\, x + \overset{r.}{1440} = 0 \\ x^3 + 0.8\, x^2 - 3.24\, x + 1.440 = 0 \end{Bmatrix}$$

$$\begin{Bmatrix} MO = x = 10. \\ mo = x = 6. \end{Bmatrix} \quad NO = -x = -24.$$

$$\begin{cases} \frac{p}{2} = 0.4. \quad \frac{p^2}{4} = 0.16. \quad \frac{p^3}{8} = 0.064. \\ \frac{p}{4} = 0.2. \quad \frac{p^2}{8} = 0.08. \quad \frac{p^3}{16} = 0.032. \end{cases}$$

*Central.*

Fig.41.

$$\left.\begin{array}{l} +\frac{L}{2} = 0.5 \\ +\frac{p^2}{8} = 0.08 \\ +\frac{q}{2} = 1.62 \\ \hline \quad b = 2.20 = AD. \end{array}\right\} \quad \begin{array}{l} +\frac{p}{4} = 0.2 \\ +\frac{p^3}{16} = 0.032 \\ +\frac{pq}{4} = 0.648 \\ +\frac{r}{2} = 0.720 \\ \hline \quad d = 1.600 = DH. \end{array}$$

$$1 \begin{cases} x^4 - \overset{p.}{6} x^3 - \overset{q.}{328} x^2 - \overset{r.}{2304} x - \overset{s.}{4608} = 0 \\ x^4 - 0.6 x^3 - 3.28 x^2 - 2.304 x - 0.4608 = 0 \end{cases}$$

$$NO = x = 24. \begin{cases} MO = -x = -8. \\ mo = -x = -6. \\ mo = -x = -4. \end{cases}$$

$$2 \begin{cases} x^4 + \overset{p.}{6} x^3 - \overset{q.}{328} x^2 + \overset{r.}{2304} x - \overset{s.}{4608} = 0 \\ x^4 + 0.6 x^3 - 3.28 x^2 + 2.304 x - 0.4608 = 0 \end{cases}$$

$$\left.\begin{array}{l} MO = x = 8 \\ mo = x = 6 \\ mo = x = 4 \end{array}\right\} NO = -x = -24.$$

Fig.42.

$$\begin{cases} \frac{p}{2} = 0.3. \quad \frac{p^2}{4} = 0.09. \quad \frac{p^3}{8} = 0.027. \\ \frac{p}{4} = 0.15. \quad \frac{p^2}{8} = 0.045. \quad \frac{p^3}{16} = 0.0135. \end{cases}$$

*Central.*

$$\left.\begin{array}{l} +\frac{L}{2}=0.5\\ +\frac{p^2}{8}=0.045\\ +\frac{q}{2}=1.64\\ \hline b=2.185=AD \end{array}\right\} \begin{array}{l} +\frac{p}{4}=0.15\\ +\frac{p^3}{16}=0.0135\\ +\frac{pq}{4}=0.4920\\ +\frac{r}{2}=1.152\\ \hline d=1.8075=DH. \end{array}$$

*Fig.42.*

$$3\left\{\begin{array}{lllll} & p. & q. & r. & s.\\ x^4 - & 6x^3 - & 381x^2 - & 326x + & 2208=0\\ x^4 - & 0.6x^3 - & 3.81x^2 - & 0.326x + & 0.2208=0 \end{array}\right\}$$

$$\left.\begin{array}{l} NO=x=23.\\ no=x=2. \end{array}\right\}\left\{\begin{array}{l} MO=-x=-16.\\ mo=-x=-3. \end{array}\right.$$

$$4\left\{\begin{array}{lllll} & p. & q. & r. & s.\\ x^4 + & 6x^3 - & 381x^2 + & 326x + & 2208=0\\ x^4 + & 0.6x^3 - & 3.81x^2 + & 0.326x + & 0.2208=0 \end{array}\right\}$$

$$\left.\begin{array}{l} MO=x=16.\\ mo=x=3. \end{array}\right\}\left\{\begin{array}{l} NO=-x=-23.\\ no=-x=-2. \end{array}\right.$$

$$\left\{\begin{array}{lll} \frac{p}{2}=0.3. & \frac{p^2}{4}=0.09. & \frac{p^3}{8}=0.027.\\ \frac{p}{4}=0.15. & \frac{p^2}{8}=0.045. & \frac{p^3}{16}=0.0135. \end{array}\right.$$

*Fig.43.*

*Central.*

$$\left.\begin{array}{l} +\frac{L}{2}=0.5\\ +\frac{p^2}{8}=0.045\\ +\frac{q}{2}=1.905\\ \hline b=2.450=AD \end{array}\right\} \begin{array}{l} +\frac{p}{4}=0.15\\ +\frac{p^3}{16}=00.135\\ +\frac{pq}{4}=0.5715\\ +\frac{r}{2}=0.163\\ \hline d=0.8980=DH. \end{array}$$

$$\begin{cases} 3.\ x^3-px^2-qx+r=0 & \begin{cases} 5 & x^4-px^3-qx^2+rx-S=0 \\ 7 & x^4-px^3-qx^2+rx+S=0 \end{cases} \\ 4.\ x^3+px^2-qx-r=0 & \begin{cases} 6 & x^4+px^3-qx^2-rx-S=0 \\ 8 & x^4+px^3-qx^2-rx+S=0 \end{cases} \end{cases}$$

*Caſ.* 1. Ubi $\frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}\sqsupset\frac{r}{2L^2}$.

*Demonſtrat.*

| | | | | |
|---|---|---|---|---|
| 2+3+4 | 13 | $Ab+bc+cD=b=AD.$<br>$\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}=b=AD.$ | | |
| 5+6+7−8 | 14 | $Dc+cf+fg-gH=d=DH.$<br>$\frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}-\frac{r}{2L^2}=d=DH.$ | | |
| 47, e 1<br>Q.13.+<br>Q.14. | 15 | $AD^2+DH^2=HA^2.$<br>$b^2+d^2=(HA^2=)$ Q. Rad. in Cubic. | | *Fig.*44. |
| 9 × 10 | 16 | $AI\times AK=$ (ob Circl.) $AL^2=$ (per conſtr.) $AZ^2.$<br>$(L\times\frac{S}{L^3}=)\ \frac{S}{L^2}=AL^2=AZ^2.$ | | |
| 47, e 1<br>15+16 | 17 | $HA^2+AL^2=HL^2.$<br>$b^2+d^2+\frac{S}{L^2}=(HL^2=)$ Q.Rad. | In Biquadr.<br>ſi − S. | *Fig.*45. |
| 47, e 1<br>15−16 | 18 | $HA^2-AZ^2=HZ^2.$<br>$b^2+d^2-\frac{S}{L^2}=(HZ^2=)$ Q.Rad. | In Biquadr.<br>ſi + S. | *Fig.*46. |
| *Supp.* | 19 | $NO=x.$ | | |
| *Supp.* | 19 | $MO=-x.$ | | |
| 19−1 | 20 | $NO-(OF,u)\ BA=(NF,u)\ OR.$<br>$x-\frac{p}{2}=OR.$ | | |
| 19+1 | 20 | $MO+(OF,u)\ BA=(MF,u)\ OR.$<br>$-x+\frac{p}{2}=OR.$ | | |

| | | | |
|---|---|---|---|
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, x :: x - \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, -x :: -x + \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ | |
| $21 \backsim 13$ | 22 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} - \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} - \frac{q}{2L} = HP. \end{array}\right.$ | |
| @ | 23 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2.$ | |
| 23 | 24 | $he,\ b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2.$ | |
| $19 - 14$ | 25 | $\left\{\begin{array}{l} NO - (OP, u)\ DH = PN. \\ x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} - \frac{pq}{4L^2} + \frac{r}{2L^2} = PN. \end{array}\right.$ | |
| $19 + 14$ | 25 | $\left\{\begin{array}{l} MO + (OP, u)\ DH = PM. \\ -x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} - \frac{r}{2L^2} = PM. \end{array}\right.$ | |
| @ | 26 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} + \frac{rx}{L^2} = PN^2.$ | |
| @ | 26 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} + \frac{rx}{L^2} = PM^2.$ | |
| 47, e 1<br>$24 + 26$ | 27 | $\left\{\begin{array}{l} HP^2 + PN^2 = HN^2. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = (HN^2 =)\ Q.Rad. \end{array}\right.$ | |
| $27 = 15$ | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 - px^2 - qx + r = 0.$ *Q.e.d.* in Cubic. | *Fig.* 44. |
| $27 = 17$ | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times$ *L*² | 31 | $x^4 - px^3 - qx^2 + rx = S.$ | |
| *Tranſp.* | 32 | $x^4 - px^3 - qx^2 + rx - S = 0.$ *Q.e.d.* in Biquad. ſi—S. | *Fig.* 45. |

$\begin{array}{c} x^4 \\ L^2 \end{array}$

| | | |
|---|---|---|
| 27 = 18 | 33 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}+\frac{rx}{L^2}=-\frac{S}{L^2}$; in $L^2$. |
| × $L^2$ | 34 | $x^4-px^3-qx^2+rx=-S.$ |
| *Tranſp.* | 35 | $x^4-px^3-qx^2+rx+S=0.$ *Q.e.d.* in Biquad. ſi + S. *Fig.*46. |
| *Supp.* | 19 | $MO=x.$ |
| *Supp.* | 19 | $NO=-x.$ |
| 19 + 1 | 20 | $\left\{\begin{array}{l} MO+(OF,u)\,BA=(MF,u)\,OR. \\ x+\frac{p}{2}=OR. \end{array}\right.$ |
| 19 − 1 | 20 | $\left\{\begin{array}{l} NO-(OF,u)\,BA=(NF,u)\,OR. \\ -x-\frac{p}{2}=OR. \end{array}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\,.\,MO :: OR\,.\,AO. \\ L\,.\,x :: x+\frac{p}{2}\,.\,\frac{x^2}{L}+\frac{px}{2L}=AO. \end{array}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\,.\,NO :: OR\,.\,AO. \\ L\,.\,-x :: -x-\frac{p}{2}\,.\,\frac{x^2}{L}+\frac{px}{2L}=AO. \end{array}\right.$ |
| 21 ∽ 13 | 22 | $\left\{\begin{array}{l} AO \backsim AD=(DO,u)\,HP. \\ \frac{x^2}{L}+\frac{px}{2L}\;(\backsim b,u)-\frac{L}{2}-\frac{p^2}{8L}-\frac{q}{2L}=HP. \end{array}\right.$ |
| ⊙ | 23 | $b^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}-\frac{qx^2}{L^2}+\frac{p^2x^2}{4L^2}-x^2-\frac{p^2x^2}{4L^2}-\frac{px}{2}-\frac{p^3x}{8L^2}-\frac{pqx}{2L^2}=HP^2$ |
| 23 | 24 | h e, $b^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}-\frac{qx^2}{L^2}-x^2-\frac{px}{2}-\frac{p^3x}{8L^2}-\frac{pqx}{2L^2}=HP^2.$ |
| 19 + 14 | 25 | $\left\{\begin{array}{l} MO+(OP,u)\,DH=PM. \\ x\,(+d,u)+\frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}-\frac{r}{2L^2}=PM. \end{array}\right.$ |
| 19 − 14 | 25 | $\left\{\begin{array}{l} NO-(OP,u)\,DH=PN. \\ -x-(d,u)\,\frac{p}{4}-\frac{p^3}{16L^2}-\frac{pq}{4L^2}+\frac{r}{2L^2}=PN. \end{array}\right.$ |
| ⊙ | 26 | $d^2+x^2+\frac{px}{2}+\frac{p^3x}{8L^2}+\frac{pqx}{2L^2}-\frac{rx}{L^2}=PM^2.$ |
| ⊙ | 26 | $d^2+x^2+\frac{px}{2}+\frac{p^3x}{8L^2}+\frac{pqx}{2L^2}-\frac{rx}{L^2}=PN^2.$ |

$HP^2$

| | | | |
|---|---|---|---|
| 47, *e* 1 | 27 | $HP^2 + PM^2 = HM^2.$ | |
| 24 + 26 | | $b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HM^2 =)$ Q.Rad. | |
| 27 = 15 | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 + px^2 - qx - r = 0$. *Q. e. d.* in Cubic. | *Fig.*44. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4 + px^3 - qx^2 - rx = S.$ | |
| *Tranſp.* | 32 | $x^4 + px^3 - qx^2 - rx - S = 0$. *Q.e.d.* in Biquad. ſi $-$S. | *Fig.*45. |
| 25 = 18 | 33 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 34 | $x^4 + px^3 - qx^2 - rx = -S.$ | |
| *Tranſp.* | 35 | $x^4 + px^3 - qx^2 - rx + S = 0$. *Q.e.d.* in Biquad. ſi $+$S. | *Fig.*46. |

*Illuſtrat.*

$$3 \left\{ \begin{array}{l} x^3 - \overset{p.}{21} x^2 - \overset{q.}{110} x + \overset{r.}{2662} = 0 \\ x^3 - 2.1 x^2 - 1.10 x + 2.662 = 0 \end{array} \right\}$$

$$\left. \begin{array}{l} NO = x = 20. \ \textit{proximè.} \\ no = x = 12.24 \ \textit{proximè.} \end{array} \right\} \quad MO = -x = -11.$$

$$4 \left\{ \begin{array}{l} x^3 + \overset{p.}{21} x^2 - \overset{q.}{110} x - \overset{r.}{2662} = 0 \\ x^3 + 2.1 x^2 - 1.10 x - 2.662 = 0 \end{array} \right\}$$

$$MO = x = 11. \quad \left\{ \begin{array}{l} NO = -x = -20. \ \textit{proximè.} \\ no = -x = -12.24 \ \textit{proximè.} \end{array} \right.$$ *Fig.*44.

$$\left\{ \begin{array}{lll} \frac{p}{2} = 1.05. & \frac{p^2}{4} = 1.1025. & \frac{p^3}{8} = 1.157625. \\ \frac{p}{4} = c.525. & \frac{p^2}{8} = 0.55125. & \frac{p^3}{16} = 0.5788125. \end{array} \right.$$

*Central.*

*Central.*

$$\left.\begin{array}{l} +\frac{L}{2}=0.5 \\ +\frac{p^2}{8}=0.55125 \\ +\frac{q}{2}=0.55 \\ \hline b=1.60125=AD \end{array}\right\} \quad \begin{array}{r} +\frac{p}{4}=0.525 \\ +\frac{p^3}{16}=0.5788125 \\ +\frac{pq}{4}=0.57550 \\ \hline 1.6793125 \\ \hline -\frac{r}{2}=1.3310 \\ \hline d=0.3483125=DH. \end{array}$$

Fig.44.

$$5\ \left\{\begin{array}{l} \overset{}{x^4}-\overset{p.}{18}x^3-\overset{q.}{205\tfrac{1}{4}}x^2+\overset{r.}{3033}x-\overset{s.}{5117}=0 \\ x^4-1.8x^3-2.05\tfrac{1}{4}x^2+3.033x-0.5117=0 \end{array}\right\}$$

$$\left.\begin{array}{l} NO=x=21.5 \\ no=x=8.5 \\ no=x=2. \end{array}\right\} \quad MO=-x=-14.$$

$$6\ \left\{\begin{array}{l} x^4+\overset{p.}{18}x^3-\overset{q.}{205\tfrac{1}{4}}x^2-\overset{r.}{3033}x-\overset{s.}{5117}=0 \\ x^4+1.8x^3-2.05\tfrac{1}{4}x^2-3.033x-0.5117=0 \end{array}\right\}$$

$$MO=x=14.\left\{\begin{array}{l} NO=-x=-21.5 \\ no=-x=-8.5 \\ no=-x=-2. \end{array}\right.$$

Fig.45.

$$\left\{\begin{array}{lll} \frac{p}{2}=0.9. & \frac{p^2}{4}=0.81. & \frac{p^3}{8}=0.729. \\ \frac{p}{4}=0.45. & \frac{p^2}{8}=0.405. & \frac{p^3}{16}=0.3645. \end{array}\right.$$

*Central.*

Fig.45.

$$\left.\begin{array}{l}+\frac{L}{2}=0.5\\ +\frac{p^2}{8}=0.405\\ +\frac{q}{2}=1.02625\\ \hline b=1.93125=AD\end{array}\right\}\quad\begin{array}{l}+\frac{p}{4}=0.45\\ +\frac{p^3}{16}=0.3645\\ +\frac{pq}{4}=0.923625\\ \hline \qquad 1.738125\\ \hline -\frac{r}{2}=1.5165\\ \hline d=0.221625=DH.\end{array}$$

$$7\;\left\{\begin{array}{l}x^4-\overset{p.}{12x^3}-\overset{q.}{195x^2}+\overset{r.}{550x}+\overset{s.}{3000}=0\\ x^4-1.2x^3-1.95x^2+0.550x+0.3000=0\end{array}\right\}$$

$$\left.\begin{array}{l}NO=x=20.\\ no=x=5.\end{array}\right\}\left\{\begin{array}{l}MO=-x=-10.\\ mo=-x=-3.\end{array}\right.$$

$$8\;\left\{\begin{array}{l}x^4+\overset{p.}{12x^3}-\overset{q.}{195x^2}-\overset{r.}{550x}+\overset{s.}{3000}=0\\ x^4+1.2x^3-1.95x^2-0.550x+0.3000=0\end{array}\right\}$$

$$\left.\begin{array}{l}MO=x=10.\\ mo=x=3.\end{array}\right\}\left\{\begin{array}{l}NO=-x=-20.\\ no=-x=-5.\end{array}\right.$$

Fig.46.

$$\left\{\begin{array}{lll}\frac{p}{2}=0.6. & \frac{p^2}{4}=0.36. & \frac{p^3}{8}=0.216.\\ \frac{p}{4}=0.3. & \frac{p^2}{8}=0.18. & \frac{p^3}{16}=0.108.\end{array}\right.$$

*Central.*

$$\left.\begin{array}{l}+\frac{L}{2}=0.5\\ +\frac{p^2}{8}=0.18\\ +\frac{q}{2}=0.975\\ \hline b=1.655=AD\end{array}\right\}\quad\begin{array}{l}+\frac{p}{4}=0.3\\ +\frac{p^3}{16}=0.108\\ +\frac{pq}{4}=0.585\\ \hline \qquad 0.993\\ \hline -\frac{r}{2}=0.275\\ \hline d=0.718=DH.\end{array}$$

*Caſ.* 2. Ubi $\frac{r}{2L^2} \sqsupset \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2}$.

*Demonſtrat.*

2+3+4 | 13 | $\left\{\begin{array}{l} Ab + bc + cD = b = AD. \\ \frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD. \end{array}\right.$

8-7-6-5 | 14 | $\left\{\begin{array}{l} Hg - gf - fe - eD = d = DH. \\ \frac{r}{2L^2} - \frac{pq}{4L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = d = DH. \end{array}\right.$

47, e 1 Q.13.+ Q.14. | 15 | $\left\{\begin{array}{l} AD^2 + DH^2 = HA^2. \\ \quad b^2 + d^2 = (HA^2 =) \text{ Q. Rad. in Cubic.} \end{array}\right.$ | *Fig.* 47.

9 × 10 | 16 | $\left\{\begin{array}{l} AI \times AK = \text{(ob Circl.) } AL^2 = \text{(per conſtr.) } AZ^2. \\ (L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2. \end{array}\right.$

47, e 1 15+16 | 17 | $\left\{\begin{array}{l} AH^2 + AL^2 = HL^2. \\ \quad b^2 + d^2 + \frac{S}{L^2} = (HL^2 =) \text{Q.Rad.} \end{array}\right\}$ In Biquadr. ſi — S. | *Fig.* 48.

47, e 1 15 — 16 | 18 | $\left\{\begin{array}{l} HA^2 - AZ^2 = HZ^2. \\ \quad b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =) \text{Q.Rad.} \end{array}\right\}$ In Biquadr. ſi + S. | *Fig.* 49.

*Supp.* | 19 | $NO = x.$

*Supp.* | 19 | $MO = -x.$

19 — 1 | 20 | $\left\{\begin{array}{l} NO - (OF, u)\ BA = (NF, u)\ OR. \\ \quad x - \frac{p}{2} = OR. \end{array}\right.$

19 + 1 | 20 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{array}\right.$

| | | | |
|---|---|---|---|
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, x :: x-\frac{p}{2} \,.\, \frac{x^2}{L}-\frac{px}{2L}=AO. \end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, -x :: -x+\frac{p}{2} \,.\, \frac{x^2}{L}-\frac{px}{2L}=AO. \end{array}\right.$ | |
| $21 \backsim 13$ | 22 | $\left\{\begin{array}{l} AO \backsim AD=(DO,u)\,HP. \\ \frac{x^2}{L}-\frac{px}{2L}\,(-b,u)-\frac{L}{2}-\frac{p^2}{8L}-\frac{q}{2L}=HP. \end{array}\right.$ | |
| @ | 23 | $b^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}+\frac{p^2x^2}{4L^2}-x^2-\frac{p^2x^2}{4L^2}+\frac{px}{2}+\frac{p^3x}{8L^2}+\frac{pqx}{2L^2}=HP^2.$ | |
| 23 | 24 | he, $b^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}-x^2+\frac{px}{2}+\frac{p^3x}{8L^2}+\frac{pqx}{2L^2}=HP^2.$ | |
| $19+14$ | 25 | $\left\{\begin{array}{l} NO+(OP,u)\,DH=PN. \\ x\,(+d,u)+\frac{r}{2L^2}-\frac{pq}{4L^2}-\frac{p^3}{16L^2}-\frac{p}{4}=PN. \end{array}\right.$ | |
| $19-14$ | 25 | $\left\{\begin{array}{l} MO-(OP,u)\,DH=PM. \\ -x\,(-d,u)-\frac{r}{2L^2}+\frac{pq}{4L^2}+\frac{p^3}{16L^2}+\frac{p}{4}=PM. \end{array}\right.$ | |
| @ | 26 | $d^2+x^2+\frac{rx}{L^2}-\frac{pqx}{2L^2}-\frac{p^3x}{8L^2}-\frac{px}{2}=PN^2.$ | |
| @ | 26 | $d^2+x^2+\frac{rx}{L^2}-\frac{pqx}{2L^2}-\frac{p^3x}{8L^2}-\frac{px}{2}=PM^2.$ | |
| 47, *e* 1<br>$24+26$ | 27 | $\left\{\begin{array}{l} HP^2+PN^2=HN^2. \\ b^2+d^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}+\frac{rx}{L^2}=(HN^2=)\,Q.Rad. \end{array}\right.$ | |
| $27=15$ | 28 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}+\frac{rx}{L^2}=0$; in $\frac{L^2}{x}$. | |
| $\times\frac{L^2}{x}$ | 29 | $x^3-px^2-qx+r=0$. *Q.e.d.* in Cubic. | *Fig.* 47. |
| $27=17$ | 30 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}-\frac{qx^2}{L^2}+\frac{rx}{L^2}=+\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4-px^3-qx^2+rx=S.$ | |
| *Tranſp.* | 32 | $x^4-px^3-qx^2+rx-S=0$. *Q.e.d.* in Biquad. ſi $-S$. | *Fig.* 48. |

| | | |
|---|---|---|
| $27 = 18$ | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{rx}{L^2} = -\frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 34 | $x^4 - px^3 - qx^2 + rx = -S.$ |
| *Tranſp.* | 35 | $x^4 - px^3 - qx^2 + rx + S = 0.$ *Q.e.d.* in Biquad: ſi $+S$. *Fig.* 49. |
| *Supp.* | 19 | $MO = x.$ |
| *Supp.* | 19 | $NO = -x.$ |
| $19 + 1$ | 20 | $\left\{\begin{array}{l} MO + (OF, u)\, BA = (MF, u)\, OR. \\ x + \frac{p}{2} = OR. \end{array}\right.$ |
| $19 - 1$ | 20 | $\left\{\begin{array}{l} NO - (OF, u)\, BA = (NF, u)\, OR. \\ -x - \frac{p}{2} = OR. \end{array}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, x :: x + \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, -x :: -x - \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ |
| $21 \backsim 13$ | 22 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\, HP. \\ \frac{x^2}{L} + \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} - \frac{q}{2L} = HP. \end{array}\right.$ |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} + \frac{p^2x^2}{L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2$ |
| 23 | 24 | h e, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$ |
| $19 - 14$ | 25 | $\left\{\begin{array}{l} MO - (OP, u)\, DH = PM. \\ x\ (-d, u) - \frac{r}{2L^2} + \frac{pq}{4L^2} + \frac{p^3}{16L^2} + \frac{p}{4} = PM. \end{array}\right.$ |
| $19 + 14$ | 25 | $\left\{\begin{array}{l} NO + (OP, u)\, DH = PN. \\ -x\ (+d, u) + \frac{r}{2L^2} - \frac{pq}{4L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN. \end{array}\right.$ |
| ⊙ | 26 | $d^2 + x^2 - \frac{rx}{L^2} + \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = PM^2.$ |
| ⊙ | 26 | $d^2 + x^2 - \frac{rx}{L^2} + \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} + \frac{px}{2} = PN^2.$ |

$HP^2$

| | | | |
|---|---|---|---|
| 47, e 1<br>24 + 26 | 27 | $HP^2 + PM^2 = HM^2.$<br>$b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HM^2 =)$ Q.Rad. | |
| 27 = 15 | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 + px^2 - qx - r = 0.$ *Q. e. d.* in Cubic. | *Fig.* 47. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4 + px^3 - qx^2 - rx = S.$ | |
| *Tranſp.* | 32 | $x^4 + px^3 - qx^2 - rx - S = 0.$ *Q. e. d.* in Biquad. ſi — S. | *Fig.* 48. |
| 27 = 18 | 33 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} - \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 34 | $x^4 + px^3 - qx^2 - rx = - S.$ | |
| *Tranſp.* | 35 | $x^4 + px^3 - qx^2 - rx + S = 0.$ *Q. e. d.* in Biquad. ſi + S. | *Fig.* 49. |

*Illuſtrat.*

$$3\ \begin{Bmatrix} x^3 - \overset{p.}{8}x^2 - \overset{q.}{240}x + \overset{r.}{2304} = 0 \\ x^3 - 0.8x^2 - 2.40x + 2.304 = 0 \end{Bmatrix}$$

$$\begin{rcases} NO = x = 12. \\ no = x = 12. \end{rcases} \quad MO = - x = - 16.$$

$$4\ \begin{Bmatrix} x^3 + \overset{p.}{8}x^2 - \overset{q.}{240}x - \overset{r.}{2304} = 0 \\ x^3 + 0.8x^2 - 2.40x - 2.304 = 0 \end{Bmatrix}$$

$$MO = x = 16. \quad \begin{cases} NO = - x = - 12. \\ no = - x = - 12. \end{cases}$$ *Fig.* 47.

$$\begin{cases} \frac{p}{2} = 0.4. \quad \frac{p^2}{4} = 1.16. \quad \frac{p^3}{8} = 0.064. \\ \frac{p}{4} = 0.2. \quad \frac{p^2}{8} = 0.08. \quad \frac{p^3}{16} = 0.032. \end{cases}$$

*Central:*

*Central.*

$$+\frac{L}{2} = 0.5$$
$$+\frac{p^2}{8} = 0.08$$
$$+\frac{q}{2} = 1.20$$
$$b = 1.78 = AD.$$

$$+\frac{r}{2} = 1.152$$
$$-\frac{p}{4} = 0.2$$
$$-\frac{p^3}{16} = 0.032$$
$$-\frac{pq}{4} = 0.480$$
$$0.712$$
$$d = 0.440 = DH.$$

*Fig.47.*

$$5\ \left\{\begin{array}{l} \overset{p.}{x^4 - 8x^3} - \overset{q.}{208x^2} + \overset{r.}{2432x} - \overset{s.}{6144} = 0 \\ x^4 - 0.8x^3 - 2.08x^2 + 2.432x - 0.6144 = 0 \end{array}\right\}$$

$$\left\{\begin{array}{l} NO = x = 12. \\ no = x = 8. \\ no = x = 4. \end{array}\right\} \quad MO = -x = -16.$$

$$6\ \left\{\begin{array}{l} \overset{p.}{x^4 + 8x^3} - \overset{q.}{208x^2} - \overset{r.}{2432x} - \overset{s.}{6144} = 0 \\ x^4 + 0.8x^3 - 2.08x^2 - 2.432x - 0.6144 = 0 \end{array}\right\}$$

$$MO = x = 16. \quad \left\{\begin{array}{l} NO = -x = -12. \\ no = -x = -8. \\ no = -x = -4. \end{array}\right.$$

*Fig.48.*

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.4. & \frac{p^2}{4} = 0.16. & \frac{p^3}{8} = 0.064. \\ \frac{p}{4} = 0.2. & \frac{p^2}{8} = 0.08. & \frac{p^3}{16} = 0.032. \end{array}\right.$$

*Cen-*

*Central.*

Fig.48.

$$\left.\begin{array}{l} +\frac{L}{2}=0.5 \\ +\frac{p^2}{8}=0.08 \\ +\frac{q}{2}=1.04 \\ \hline b=1.62=AD. \end{array}\right\} \quad \begin{array}{l} +\frac{r}{2}=1.216 \\ \hline -\frac{p}{4}=0.2 \\ -\frac{p^3}{16}=0.032 \\ -\frac{pq}{4}=0.416 \\ \hline \quad 0.648 \\ \hline d=0.568=DH. \end{array}$$

$$\begin{array}{l} \qquad\quad p. \qquad\quad q. \qquad\qquad r. \qquad\qquad s. \\ 7\left\{\begin{array}{l} x^4-6x^3-333x^2+2322x+9720=0 \\ x^4-0.6x^3-3.33x^2+2.322x+0.9720=0 \end{array}\right\} \\ \quad \left.\begin{array}{l} NO=x=15. \\ no=x=12. \end{array}\right\}\left\{\begin{array}{l} MO=-x=-18. \\ mo=-x=-3. \end{array}\right. \end{array}$$

$$\begin{array}{l} \qquad\quad p. \qquad\quad q. \qquad\qquad r. \qquad\qquad s. \\ 8\left\{\begin{array}{l} x^4+6x^3-333x^2-2322x+9720=0 \\ x^4+0.6x^3-3.33x^2-2.322x+0.9720=0 \end{array}\right\} \\ \quad \left.\begin{array}{l} MO=x=18. \\ mo=x=3. \end{array}\right\}\left\{\begin{array}{l} NO=-x=-15. \\ no=-x=-12. \end{array}\right. \end{array}$$

Fig.49.

$$\left\{\begin{array}{lll} \frac{p}{2}=0.3. & \frac{p^2}{4}=0.09. & \frac{p^3}{8}=0.027. \\ \frac{p}{4}=0.15. & \frac{p^2}{8}=0.045. & \frac{p^3}{16}=0.0135. \end{array}\right.$$

*Central.*

$$\left.\begin{array}{l} +\frac{L}{2}=0.5 \\ +\frac{p^2}{8}=0.045 \\ +\frac{q}{2}=1.665 \\ \hline b=2.210=AD. \end{array}\right\} \quad \begin{array}{l} +\frac{r}{2}=1.161 \\ \hline -\frac{p}{4}=0.15. \\ -\frac{p^3}{16}=0.0135 \\ -\frac{pq}{4}=0.4995 \\ \hline \quad 0.6630 \\ \hline d=0.4980=DH. \end{array}$$

$$\begin{cases} 5.\ x^3 - px^2 + qx + r = 0 \begin{cases} 9 \begin{cases} x^4 - px^3 + qx^2 + rx - S = 0 \end{cases} \\ 11 \begin{cases} x^4 - px^3 + qx^2 + rx + S = 0 \end{cases} \end{cases} \\ 6.\ x^2 + px^2 + qx - r = 0 \begin{cases} 10 \begin{cases} x^4 + px^3 + qx^2 - rx - S = 0 \end{cases} \\ 12 \begin{cases} x^4 + px^3 + qx^2 - rx + S = 0 \end{cases} \end{cases} \end{cases}$$

*Caſ.* 1. Ubi $\frac{L}{2} + \frac{p^2}{8L} \sqsupset \frac{q}{2L}$; & $\frac{p}{4} + \frac{p^3}{16L^2} \sqsupset \frac{pq}{4L^2} + \frac{r}{2L^2}$.

*Demonſtrat.*

| | | | | |
|---|---|---|---|---|
| 2+3−4 | 13 | $\begin{cases} Ab + bc - cD = b = AD. \\ \frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = b = AD. \end{cases}$ | | |
| 5+6−7−8 | 14 | $\begin{cases} De + ef - fg - gH = d = DH. \\ \frac{p}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} - \frac{r}{2L^2} = d = DH. \end{cases}$ | | |
| 47, e 1<br>*Q.*13.+<br>*Q.*14. | 15 | $\begin{cases} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = (HA^2 =) \text{ Q. Rad. in Cubic.} \end{cases}$ | | *Fig.*50. |
| 9×10 | 16 | $\begin{cases} AI \times AK = \text{(ob Circl.) } AL^2 = \text{(per conſtr.) } AZ^2. \\ (L \times \frac{S}{L^3} =) \frac{S}{L^2} = AL^2 = AZ^2. \end{cases}$ | | |
| 47, e 1<br>15+16 | 17 | $\begin{cases} HA^2 + AL^2 = HL^2. \\ b^2 + d^2 + \frac{S}{L^2} = (HL^2 =) \text{ Q.Rad.} \end{cases}$ | In Biquadr.<br>ſi − S. | *Fig.*51. |
| 47, e 1<br>15−16 | 18 | $\begin{cases} HA^2 - AZ^2 = HZ^2. \\ b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =) \text{ Q.Rad.} \end{cases}$ | In Biquadr.<br>ſi + S. | *Fig.*52. |
| *Supp.* | 19 | $NO = x.$ | | |
| *Supp.* | 19 | $MO = -x.$ | | |
| 19−1 | 20 | $\begin{cases} NO - (OF, u)\, BA = (NF, u)\, OR. \\ x - \frac{p}{2} = OR. \end{cases}$ | | |
| 19+1 | 20 | $\begin{cases} MO + (OF, u)\, BA = (MF, u)\, OR. \\ -x + \frac{p}{2} = OR. \end{cases}$ | | |

| | | | |
|---|---|---|---|
| *Ob para.* | 21 | $\begin{cases} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, x :: x - \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{cases}$ | |
| *Ob para.* | 21 | $\begin{cases} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, -x :: -x + \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{cases}$ | |
| 21 ∽ 13 | 22 | $\begin{cases} AO \backsim AD = (DO, u)\, HP. \\ \frac{x^2}{L} - \frac{px}{2L}\, (-b, u) - \frac{L}{2} - \frac{p^2}{8L} + \frac{q}{2L} = HP. \end{cases}$ | |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$ | |
| 23 | 24 | he, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} = HP^2.$ | |
| 19 − 14 | 25 | $\begin{cases} NO - (OP, u)\, DH = PN. \\ x\, (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} + \frac{pq}{4L^2} + \frac{r}{2L^2} = PN. \end{cases}$ | |
| 19 + 14 | 25 | $\begin{cases} MO + (OP, u)\, DH = PM. \\ -x\, (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} - \frac{r}{2L^2} = PM. \end{cases}$ | |
| ⊙ | 26 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} + \frac{rx}{L^2} = PN^2.$ | |
| ⊙ | 26 | $d^2 + x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} + \frac{rx}{L^2} = PM^2.$ | |
| 47, e 1<br>24 + 26 | 27 | $\begin{cases} HP^2 + PN^2 = HN^2. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = (HN^2 =)\ Q.Rad. \end{cases}$ | |
| 27 = 15 | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0;$ in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 - px^2 + qx + r = 0.$ *Q.e.d.* in Cubic. | *Fig.* 50. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2};$ in $L^2$. | |
| $\times L^2$ | 31 | $x^4 - px^3 + qx^2 + rx = S.$ | |
| *Tranſp.* | 32 | $x^4 - px^3 + qx^2 + rx - S = 0.$ *Q.e.d.* in Biquad. ſi − S. | *Fig.* 51. |

$\frac{x^4}{L^2}$

| | | |
|---|---|---|
| 27=18 | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. |
| ×$L^2$ | 34 | $x^4 - px^3 + qx^2 + rx = - S.$ |
| *Tranſp.* | 35 | $x^4 - px^3 + qx^2 + rx + S = 0.$ *Q.e.d.* in Biquad. ſi + S. *Fig.* 52. |
| *Supp.* | 19 | $MO = x.$ |
| *Supp.* | 19 | $NO = - x.$ |
| 19+1 | 20 | $\left\{\begin{array}{l} MO + (OF, u)\, BA = (MF, u)\, OR. \\ x + \frac{p}{2} = OR. \end{array}\right.$ |
| 19−1 | 20 | $\left\{\begin{array}{l} NO - (OF, u)\, BA = (NF, u)\, OR. \\ - x - \frac{p}{2} = OR. \end{array}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \;.\; MO :: OR \;.\; AO. \\ L \;.\; x :: x + \frac{p}{2} \;.\; \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \;.\; NO :: OR \;.\; AO. \\ L \;.\; - x :: - x - \frac{p}{2} \;.\; \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ |
| 21 ∽ 13 | 22 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\, HP. \\ \frac{x^2}{L} + \frac{px}{2L}\; (\backsim b, u) \backsim \frac{L}{2} \backsim \frac{p^2}{8L} + \frac{q}{2L} = HP. \end{array}\right.$ |
| ⊚ | 23 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2$ |
| 23 | 24 | h e, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2.$ |
| 19+14 | 25 | $\left\{\begin{array}{l} MO + (OP, u)\, DH = PM. \\ x\, (+ d, u) + \frac{p}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} - \frac{r}{2L^2} = PM. \end{array}\right.$ |
| 19−14 | 25 | $\left\{\begin{array}{l} NO - (OP, u)\, DH = PN. \\ - x\, (- d, u) - \frac{p}{4} - \frac{p^3}{16L^2} + \frac{pq}{4L^2} + \frac{r}{2L^2} = PN. \end{array}\right.$ |
| ⊚ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} - \frac{rx}{L^2} = PM^2.$ |
| ⊚ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} - \frac{rx}{L^2} = PN^2.$ |

| | | | |
|---|---|---|---|
| 47, e 1<br>24 + 26 | 27 | $\begin{cases} HP^2 + PM^2 = HM^2. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HM^2 =)\ \text{Q.Rad.} \end{cases}$ | |
| $27 = 15$ | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 + px^2 + qx - r = 0$. *Q.e.d.* in Cubic. | *Fig.*50. |
| $27 = 17$ | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times$ *L*² | 31 | $x^4 + px^3 + qx^2 - rx = S$. | |
| *Tranſp.* | 32 | $x^4 + px^3 + qx^2 - rx - S = 0$. *Q.e.d.* in Biquad. ſi $-S$. | *Fig.*51. |
| $27 = 18$ | 33 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| $\times$ *L*² | 34 | $x^4 + px^3 + qx^2 - rx = -S$. | |
| *Tranſp.* | 35 | $x^4 + px^3 + qx^2 - rx + S = 0$. *Q.e.d.* in Biquad. ſi $+S$. | *Fig.*52. |

*Illuſtrat.*

$$5\ \begin{cases} x^3 - \overset{p.}{24x^2} + \overset{q.}{20x} + \overset{r.}{1200} = 0 \\ x^3 - 2.4x^2 + 0.20x + 1.200 = 0 \end{cases}$$

$$\left.\begin{matrix} NO = x = 20. \\ no = x = 10. \end{matrix}\right\} \quad MO = -x = -6.$$

$$6\ \begin{cases} x^3 + \overset{p.}{24x^2} + \overset{q.}{20x} - \overset{r.}{1200} = 0 \\ x^3 + 2.4x^2 + 0.20x - 1.200 = 0 \end{cases}$$

$$MO = x = 6. \quad \begin{cases} NO = -x = -20. \\ no = -x = -10. \end{cases}$$

*Fig.*50.

$$\begin{cases} \frac{p}{2} = 1.2. & \frac{p^2}{4} = 1.44. & \frac{p^3}{8} = 1.728. \\ \frac{p}{4} = c.6. & \frac{p^2}{8} = 0.72. & \frac{p^3}{16} = 0.864. \end{cases}$$

*Central.*

*Central.*

Fig.50.

$$\left.\begin{array}{l} +\frac{L}{2} = 0.5 \\ +\frac{p^2}{8} = 0.72 \\ \hline + \qquad 1.22 \\ -\frac{q}{2} = 0.10 \\ \hline b = 1.12 = AD. \\ \hline \end{array}\right\} \qquad \begin{array}{l} +\frac{p}{4} = 0.6 \\ +\frac{p^3}{16} = 0.864 \\ \hline \qquad 1.464 \\ \hline -\frac{pq}{4} = 0.120 \\ -\frac{r}{2} = 0.600 \\ \hline \qquad 0.720 \\ \hline d = 0.744 = DH. \\ \hline \end{array}$$

$$9\left\{\begin{array}{l} x^4 - \overset{p.}{30\,x^3} + \overset{q.}{128\,x^2} + \overset{r.}{2016\,x} - \overset{s.}{11520} = 0 \\ x^4 - 3.0\,x^3 + 1.28\,x^2 + 2.016\,x - 1.1520 = 0 \end{array}\right\}$$

$$\left\{\begin{array}{l} NO = x = 20. \\ no = x = 12. \\ no = x = \ \ 6. \end{array}\right\} \quad MO = -x = -8.$$

Fig.51.

$$10\left\{\begin{array}{l} x^4 + \overset{p.}{30\,x^3} + \overset{q.}{128\,x^2} - \overset{r.}{2016\,x} - \overset{s.}{11520} = 0 \\ x^4 + 3.0\,x^3 + 1.28\,x^2 - 2.016\,x - 1.1520 = 0 \end{array}\right\}$$

$$MO = x = 8. \qquad \left\{\begin{array}{l} NO = -x = -20. \\ no = -x = -12. \\ no = -x = -\ 6. \end{array}\right.$$

$$\left\{\begin{array}{lll} \frac{p}{2} = 1.5. & \frac{p^2}{4} = 2.25. & \frac{p^3}{8} = 3.375. \\ \frac{p}{4} = 0.75. & \frac{p^2}{8} = 1.125. & \frac{p^3}{16} = 1.6875. \end{array}\right.$$

Cen-

*Central.*

Fig. 51.

$$\left.\begin{array}{l}
+\frac{L}{2}=0.5 \\
+\frac{p^2}{8}=1.125 \\
\hline
+\quad 1.625 \\
\hline
\quad\frac{q}{2}=0.64 \\
\hline
\quad b=0.985=AD.
\end{array}\right\}
\begin{array}{l}
+\frac{p}{4}=0.75 \\
+\frac{p^3}{16}=1.6875 \\
\hline
\quad 2.4375 \\
\hline
-\frac{pq}{4}=0.9600 \\
-\frac{r}{2}=1.008 \\
\hline
\quad 1.9680 \\
\hline
\quad d=0.4695 \pm DH.
\end{array}$$

$$\begin{array}{l}
\quad\quad\quad p. \quad\quad\quad\quad q. \quad\quad\quad\quad r. \quad\quad\quad\quad s. \\
11\left\{\begin{array}{l} x^4-26x^3+20x^2+1832x+3360=0 \\ x^4-2.6x^3+0.20x^2-1.832x+0.3360=0 \end{array}\right\} \\
\left.\begin{array}{l} NO=x=20. \\ no=x=14. \end{array}\right\}\left\{\begin{array}{l} MO=-x=-6. \\ mo=-x=-2. \end{array}\right.
\end{array}$$

$$\begin{array}{l}
\quad\quad\quad p. \quad\quad\quad\quad q. \quad\quad\quad\quad r. \quad\quad\quad\quad s. \\
\left\{\begin{array}{l} x^4+26x^3+20x^2-1832x+3360=0 \\ x^4+2.6x^3+0.20x^2-1.832x+0.3360=0 \end{array}\right\} \\
\left.\begin{array}{l} MO=x=6. \\ mo=x=2. \end{array}\right\}\left\{\begin{array}{l} NO=-x=-20. \\ no=-x=-14. \end{array}\right.
\end{array}$$

Fig. 52.

$$\left\{\begin{array}{lll}
\frac{p}{2}=1.3. & \frac{p^2}{4}=1.69. & \frac{p^3}{8}=2.197. \\
\frac{p}{4}=0.65. & \frac{p^2}{8}=0.845. & \frac{p^3}{16}=1.0985.
\end{array}\right.$$

*Central.*

$$\left.\begin{array}{l}
+\frac{L}{2}=0.5 \\
+\frac{p^2}{8}=0.845 \\
\hline
\quad 1.345 \\
\hline
-\frac{q}{2}=0.10 \\
\hline
\quad b=1.245=AD.
\end{array}\right\}
\begin{array}{l}
+\frac{p}{4}=0.65. \\
+\frac{p^3}{16}=1.0985 \\
\hline
\quad 1.7485 \\
\hline
-\frac{pq}{4}=0.1300 \\
-\frac{r}{2}=0.916 \\
\hline
\quad 1.0460 \\
\hline
\quad d=0.7025=DH.
\end{array}$$

*Caſ.* 2. Ubi $\frac{q}{2L} \sqsupset \frac{L}{2} + \frac{p^2}{8L}$ ; & $\frac{pq}{4L^2} + \frac{r}{2L^2} \sqsupset \frac{p}{4} + \frac{p^3}{16L^2}$.

*Demonſtrat.*

| | | | | |
|---|---|---|---|---|
| 4—3—2 | 13 | $\left\{ \begin{array}{l} Dc - cb - bA = b = AD. \\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} = b = AD. \end{array} \right.$ | | |
| 7+8-6-5 | 14 | $\left\{ \begin{array}{l} Hg + gf - fc - eD = d = DH. \\ \frac{pq}{4L^2} + \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = d = DH. \end{array} \right.$ | | |
| 47, e 1<br>Q.13.+<br>Q.14. | 15 | $\left\{ \begin{array}{l} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = (HA^2 =) \text{ Q. Rad. in Cubic.} \end{array} \right.$ | | *Fig.* 53. |
| 9 × 10 | 16 | $\left\{ \begin{array}{l} AI \times AK = \text{(ob Circl.) } AL^2 = \text{(per conſtr.) } AZ^2. \\ (L \times \frac{S}{L^3} =) \frac{S}{L^2} = AL^2 = AZ^2. \end{array} \right.$ | | |
| 47, e 1<br>15 + 16 | 17 | $\left\{ \begin{array}{l} HA^2 + AL^2 = HL^2. \\ b^2 + d^2 + \frac{S}{L^2} = (HL^2 =) \text{Q.Rad.} \end{array} \right\}$ | $\left\} \begin{array}{c} \text{In Biquadr.} \\ \text{ſi} - S. \end{array} \right\}$ | *Fig.* 54. |
| 47, e 1<br>15 — 16 | 18 | $\left\{ \begin{array}{l} HA^2 - AZ^2 = HZ^2. \\ b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =) \text{Q.Rad.} \end{array} \right\}$ | $\left\} \begin{array}{c} \text{In Biquadr.} \\ \text{ſi} + S. \end{array} \right\}$ | *Fig.* 55. |
| *Supp.* | 19 | $NO = x.$ | | |
| *Supp.* | 19 | $MO = -x.$ | | |
| 1 — 19. | 20 | $\left\{ \begin{array}{l} (OF, u)\ BA - NO = (NF, u)\ OR. \\ \frac{p}{2} - x = OR. \end{array} \right.$ | | |
| 19 + 1 | 20 | $\left\{ \begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ -x + \frac{p}{2} = OR. \end{array} \right.$ | | |

| | | | |
|---|---|---|---|
| *Ob para.* | 21 | $L \,.\, NO :: OR \,.\, AO.$<br>$L \,.\, x :: \frac{p}{2} - x \,.\, \frac{px}{2L} + \frac{x^2}{L} = AO.$ | |
| *Ob para.* | 21 | $L \,.\, MO :: OR \,.\, AO.$<br>$L \,.\, -x :: -x + \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO.$ | |
| 13 — 21 | 22 | $AD \backsim AO = (DO, u)\ HP.$<br>$(b, u)\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} - \frac{px}{2L} + \frac{x^2}{L} = HP.$ | |
| 13 + 21 | 22 | $AD + AO = (DO, u)\ HP.$<br>$(b, u)\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} + \frac{x^2}{L} - \frac{px}{2L} = HP.$ | |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - \frac{pqx}{2L^2} - \frac{p^2x^2}{4L^2} + \frac{p^3x}{8L^2} - x^2 + \frac{px}{2} = HP^2.$ | |
| 23 | 24 | he, $b^4 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} - x^2 + \frac{px}{2} = HP^2.$ | |
| 19 + 14 | 25 | $NO + (OP, u)\ DH = PN.$<br>$x\ (+ d, u) + \frac{pq}{4L^2} + \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN.$ | |
| 14 — 19 | 25 | $(OP, u)\ DH - MO = PM.$<br>$(d, u)\ \frac{pq}{4L^2} + \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} + x = PM.$ | |
| ⊙ | 26 | $d^2 + x^2 + \frac{pqx}{2L^2} + \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PN^2.$ | |
| ⊙ | 26 | $d^2 + x^2 + \frac{pqx}{2L^2} + \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PM^2.$ | |
| 47, e 1<br>24 + 26 | 27 | $HP^2 + PN^2 = HN^2.$<br>$b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = (HN^2 =)$ Q.Rad. | |
| 27 = 15 | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 - px^2 + qx + r = 0.$ *Q.e.d.* in Cubic. | *Fig.*53. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times$ *L*² | 31 | $x^4 - px^3 + qx^2 + rx = S.$ | |
| *Tranſp.* | 32 | $x^4 - px^3 + qx^2 + rx - S = 0.$ *Q.e.d.* in Biquad. ſi — S. | *Fig.*54. |

33

| | | |
|---|---|---|
| 27 = 18 | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = -\frac{S}{L^2}$; in $L^2$. |
| $\times L^2$ | 34 | $x^4 - px^3 + qx^2 + rx = -S.$ |
| *Tranſp.* | 35 | $x^4 - px^3 + qx^2 + rx + S = 0.$ *Q.e.d.* in Biquad. ſi + S. *Fig.* 55. |
| *Supp.* | 19 | $MO = x.$ |
| *Supp.* | 19 | $NO = -x.$ |
| 19 + 1 | 20 | $\left\{\begin{array}{l} MO + (OF, u)\, BA = (MF, u)\, OR. \\ x + \frac{p}{2} = OR. \end{array}\right.$ |
| 1 − 19 | 20 | $\left\{\begin{array}{l} (OF, u)\, BA - NO = (NF, u)\, OR. \\ \frac{p}{2} + x = OR. \end{array}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, x :: x + \frac{p}{2} \,.\, \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, -x :: \frac{p}{2} + x \,.\, -\frac{px}{2L} - \frac{x^2}{L} = AO. \end{array}\right.$ |
| 13 + 21 | 22 | $\left\{\begin{array}{l} AD + AO = (DO, u)\, HP. \\ (b, u)\, \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} + \frac{x^2}{L} + \frac{px}{2L} = HP. \end{array}\right.$ |
| 13 − 21 | 22 | $\left\{\begin{array}{l} AD - AO = (DO, u)\, HP. \\ (b, u)\, \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} + \frac{px}{2L} + \frac{x^2}{L} = HP. \end{array}\right.$ |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} + \frac{pqx}{2L^2} - \frac{p^2x^2}{4L^2} - \frac{p^3x}{8L^2} - x^2 - \frac{px}{2} = HP^2$ |
| 23 | 24 | h e, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - x^2 - \frac{px}{2} = HP^2.$ |
| 14 − 19 | 25 | $\left\{\begin{array}{l} (OP, u)\, DH - MO = PM. \\ (d, u)\, \frac{pq}{4L^2} + \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} - x = PM. \end{array}\right.$ |
| 19 + 14 | 25 | $\left\{\begin{array}{l} NO + (OP, u)\, DH = PN. \\ -x\,(+d, u) + \frac{pq}{4L^2} + \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN. \end{array}\right.$ |

⊙ 26 $d^2 + x^2 - \frac{pqx}{2L^2} - \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PM^2.$

⊙ 26 $d^2 + x^2 - \frac{pqx}{2L^2} - \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PN^2.$

47, e 1 / 24 + 26 27 $\begin{cases} HP^2 + PM^2 = HM^2. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HM^2 =)\ Q.Rad. \end{cases}$

27 = 15 28 $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$.

$\times \frac{L^2}{x}$. 29 $x^3 + px^2 + qx - r = 0$. *Q.e.d.* in Cubic. *Fig.* 53.

27 = 17 30 $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$.

$\times L^2$. 31 $x^4 + px^3 + qx^2 - rx = S.$

*Tranſp.* 32 $x^4 + px^3 + qx^2 - rx - S = 0.$ *Q.e.d.* in Biquad. ſi $-S$. *Fig.* 54.

27 = 18 33 $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$.

$\times L^2$ 34 $x^4 + px^3 + qx^2 - rx = -S.$

*Tranſp.* 35 $x^4 + px^3 + qx^2 - rx + S = 0.$ *Q.e.d.* in Biquad. ſi $+S$. *Fig.* 55.

### *Illuſtrat.*

5 $\begin{cases} x^3 - \overset{p.}{12}x^2 + \overset{q.}{200}x + \overset{r.}{4200} = 0 \\ x^3 - 1.2x^2 + 2.00x + 4.200 = 0 \end{cases}$

MO $= -x = -10.$

6 $\begin{cases} x^3 + \overset{p.}{12}x^2 + \overset{q.}{200}x - \overset{r.}{4200} = 0 \\ x^3 + 1.2x^2 + 2.00x - 4.200 = 0 \end{cases}$ *Fig.* 53.

MO $= x = 10.$

$$\begin{cases} \frac{p}{2} = 0.6. \quad \frac{p^2}{4} = 0.36. \quad \frac{p^3}{8} = 0.216. \\ \frac{p}{4} = 0.3. \quad \frac{p^2}{8} = 0.18. \quad \frac{p^3}{16} = 0.108. \end{cases}$$

## *Central.*

$+\frac{q}{2L} = 1.00$

$-\frac{p^2}{8} = 0.18$

$-\frac{L}{2} = 0.5$

$0.68$

$b = 0.32 = AD.$

$+\frac{pq}{4} = 0.600$

$+\frac{r}{2} = 2.100$

$2.700$

$-\frac{p^3}{16} = 0.108$

$-\frac{p}{4} = 0.3$

$0.408$

$d = 2,292 = DH.$

*Fig.* 53.

$$9 \begin{cases} x^4 - \overset{p.}{12} x^3 + \overset{q.}{216} x^2 + \overset{r.}{916} x - \overset{s.}{6168} = 0 \\ x^4 - 1.2 x^3 + 2.16 x^2 + 0.916 x - 0.6168 = 0 \end{cases}$$

$$\begin{cases} NO = x = 3.8 + \\ MO = -x = -6. \end{cases}$$

$$10 \begin{cases} x^4 + \overset{p.}{12} x^3 + \overset{q.}{216} x^2 - \overset{r.}{916} x - \overset{s.}{6168} = 0 \\ x^4 + 1.2 x^3 + 2.16 x^2 - 0.916 x - 0.6168 = 0 \end{cases}$$

$$\begin{cases} MO = 6 = x. \\ NO = -x = -3.8 + \end{cases}$$

*Fig.* 54.

$$\begin{cases} \frac{p}{2} = 0.6. \quad \frac{p^2}{4} = 0.36. \quad \frac{p^3}{8} = 0.216. \\ \frac{p}{4} = 0.3. \quad \frac{p^2}{8} = 0.18. \quad \frac{p^3}{16} = 0.108. \end{cases}$$

*Fig.* 54.

*Central.*

| | |
|---|---|
| $+\frac{q}{2L} = 1.08$ | $+\frac{pq}{4} = 0.648$ |
| $-\frac{p^2}{8} = 0.18$ | $+\frac{r}{2} = 0.458$ |
| $-\frac{L}{2} = 0.5$ | $1.106$ |
| $0.68$ | $-\frac{p^3}{16} = 0.108$ |
| $b = 0.40 = AD.$ | $-\frac{p}{4} = 0.3$ |
| | $0.408$ |
| | $d = 0.698 = DH.$ |

$$\begin{array}{l} \quad\quad\; p. \quad\quad\quad q. \quad\quad\quad\quad r. \quad\quad\quad\quad s. \\ 11\left\{\begin{array}{l} x^4 - 12\,x^3 + 200\,x^2 + 2344\,x + 2976 = 0 \\ x^4 - 1.2\,x^3 + 2.00\,x^2 + 2.344\,x + 0.2976 = 0 \end{array}\right\} \end{array}$$

$$\left\{\begin{array}{l} MO = -x = -6. \\ mo = -x = -1.47+ \end{array}\right.$$

$$\begin{array}{l} \quad\quad\; p. \quad\quad\quad q. \quad\quad\quad\quad r. \quad\quad\quad\quad s. \\ 12\left\{\begin{array}{l} x^4 + 12\,x^3 + 200\,x^2 - 2344\,x + 2976 = 0 \\ x^4 + 1.2\,x^3 + 2.00\,x^2 - 2.344\,x + 0.2976 = 0 \end{array}\right\} \end{array}$$

$$\left\{\begin{array}{l} MO = x = 6. \\ mo = x = 1.47+ \end{array}\right.$$

*Fig.* 55.

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.6. & \frac{p^2}{4} = 0.36. & \frac{p^3}{8} = 0.216. \\ \frac{p}{4} = 0.3. & \frac{p^2}{8} = 0.18. & \frac{p^3}{16} = 0.108. \end{array}\right.$$

*Central.*

| | |
|---|---|
| $+\frac{q}{2} = 1.00$ | $+\frac{pq}{4} = 0.600$ |
| $-\frac{p^2}{8} = 0.18$ | $+\frac{r}{2} = 1.172$ |
| $-\frac{L}{2} = 0.5$ | $1.772$ |
| $0.68$ | $-\frac{p^3}{16} = 0.108$ |
| $b = 0.32 = AD.$ | $-\frac{p}{4} = 0.3.$ |
| | $0.408$ |
| | $d = 1.364 = DH.$ |

*Caſ.* 3. Ubi $\frac{L}{2}+\frac{p^2}{8L} \sqsupset \frac{q}{2L}$; & $\frac{pq}{4L^2}+\frac{r}{2L^2} \sqsupset \frac{p}{4}+\frac{p^3}{16L^2}$.

*Demonſtrat.*

| | | | | |
|---|---|---|---|---|
| $2+3-4$ | 13 | $\begin{cases} Ab+bc-cD=b=AD. \\ \frac{L}{2}+\frac{p^2}{8L}-\frac{q}{2L}=b=AD. \end{cases}$ | | |
| $8+7-6-5$ | 14 | $\begin{cases} Hg+gf-fe-eD=d=DH. \\ \frac{r}{2L^2}+\frac{pq}{4L^2}-\frac{p^3}{16L^2}-\frac{p}{4}=d=DH. \end{cases}$ | | |
| 47, e 1<br>Q.13.+<br>Q.14. | 15 | $\begin{cases} AD^2+DH^2=HA^2. \\ b^2+d^2=(HA^2=)\ \text{Q. Rad. in Cubic.} \end{cases}$ | | *Fig*.56. |
| $9 \times 10$ | 16 | $\begin{cases} AI \times AK=(\text{ob Circl.})\ AL^2=(\text{per conſtr.})\ AZ^2. \\ (L \times \frac{S}{L^3}=)\ \frac{S}{L^2}=AL^2=AZ^2. \end{cases}$ | | |
| 47, e 1<br>$15+16$ | 17 | $\begin{cases} HA^2+AL^2=HL^2. \\ b^2+d^2+\frac{S}{L^2}=(HL^2=)\text{Q.Rad.} \end{cases}$ | In Biquadr.<br>ſi $-$S. | *Fig*.57. |
| $15-16$ | 18 | $\begin{cases} HA^2-AZ^2=HZ^2. \\ b^2+d^2-\frac{S}{L^2}=(HZ^2=)\text{Q.Rad.} \end{cases}$ | In Biquadr.<br>ſi $+$S. | *Fig*.58. |
| *Supp.* | 19 | $NO=x.$ | | |
| *Supp.* | 19 | $MO=-x.$ | | |
| $1-19$ | 20 | $\begin{cases} (OF,u)\ BA-NO=(NF,u)\ OR. \\ \frac{p}{2}-x=OR. \end{cases}$ | | |
| $19+1$ | 20 | $\begin{cases} MO+(OF,u)\ BA=(MF,u)\ OR. \\ -x+\frac{p}{2}=OR. \end{cases}$ | | |

L'. NO

| | | | |
|---|---|---|---|
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, x :: \frac{p}{2} - x \,.\, \frac{px}{2} - \frac{x^2}{L} = AO. \end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, -x :: -x + \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO. \end{array}\right.$ | |
| $21 + 13$ | 22 | $\left\{\begin{array}{l} AO + AD = (DO, u)\ HP. \\ \frac{px}{2L} - \frac{x^2}{L}\ (+b, u) + \frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = HP. \end{array}\right.$ | *Fig.* 57. |
| $21 - 13$ | 22 | $\left\{\begin{array}{l} AO - AD = (DO, u)\ HP. \\ \frac{x^2}{L} - \frac{px}{2L}\ (-b, u) - \frac{L}{2} - \frac{p^2}{8L} + \frac{q}{2L} = HP. \end{array}\right.$ | *Fig.* $\left\{\begin{array}{l} 56 \\ 57 \\ 58 \end{array}\right.$ |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} - x^2 - \frac{p^2x^2}{4L^2} = HP^2.$ | |
| 23 | 24 | he, $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{px}{2} + \frac{p^3x}{8L^2} - \frac{pqx}{2L^2} - x^2 = HP^2.$ | |
| $19 + 14$ | 25 | $\left\{\begin{array}{l} NO + (OP, u)\ DH = PN. \\ x\ (+d, u)\ \frac{r}{2L^2} + \frac{pq}{4L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN. \end{array}\right.$ | |
| $19 - 14$ | 25 | $\left\{\begin{array}{l} MO - (OP, u)\ DH = PM. \\ -x\ (-d, u) - \frac{r}{2L^2} - \frac{pq}{4L^2} + \frac{p^3}{16L^2} + \frac{p}{4} = PM. \end{array}\right.$ | |
| ⊙ | 26 | $d^2 + x^2 + \frac{rx}{L^2} + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PN^2.$ | |
| ⊙ | 26 | $d^2 + x^2 + \frac{rx}{L^2} + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PM^2.$ | |
| 47, e 1<br>$24 + 26$ | 27 | $\left\{\begin{array}{l} HP^2 + PN^2 = HN^2. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = (HN^2 =)\ Q.Rad. \end{array}\right.$ | |
| $27 = 15$ | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 - px^2 + qx + r = 0.$ *Q.e.d.* in Cubic. | *Fig.* 56. |
| $27 = 17$ | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4 - px^3 + qx^2 + rx = S.$ | |
| *Tranſp.* | 32 | $x^4 - px^3 + qx^2 + rx - S = 0.$ *Q.e.d.* in Biquad. ſi $-S$. | *Fig.* 57. |

33

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|---|---|---|---|
| 27=18 | 33 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=-\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 34 | $x^4-px^3+qx^2+rx=-S$. | |
| *Tranſp.* | 35 | $x^4-px^3+qx^2+rx+S=0$. *Q.e.d.* in Biquad. ſi $+S$. | *Fig.* 58. |
| *Supp.* | 19 | $MO=x$. | |
| *Supp.* | 19 | $NO=-x$. | |
| 19+1 | 20 | $\left\{\begin{array}{l} MO+(OF,u)\,BA=(MF,u)\,OR. \\ x+\frac{p}{2}=OR. \end{array}\right.$ | |
| 1−19 | 20 | $\left\{\begin{array}{l} (OF,u)\,BA-NO=(NF,u)\,OR. \\ \frac{p}{2}+x=OR. \end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\,.\,MO :: OR\,.\,AO. \\ L\,.\,x :: x+\frac{p}{2}\,.\,\frac{x^2}{L}+\frac{px}{2L}=AO. \end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\,.\,NO :: OR\,.\,AO. \\ L\,.\,-x :: \frac{p}{2}+x\,.\,-\frac{px}{2L}-\frac{x^2}{L}=AO. \end{array}\right.$ | |
| 21−13 | 22 | $\left\{\begin{array}{l} AO-AD=(DO,u)\,HP. \\ \frac{x^2}{L}+\frac{px}{2L}\,(-b,u)-\frac{L}{2}-\frac{p^2}{8L}+\frac{q}{2L}=HP. \end{array}\right.$ | *Fig.* 56<br>57<br>58 |
| 21+13 | 22 | $\left\{\begin{array}{l} AO+AD=(DO,u)\,HP. \\ -\frac{px}{2L}-\frac{x^2}{L}\,(+b,u)+\frac{L}{2}+\frac{p^2}{8L}-\frac{q}{2L}=HP. \end{array}\right.$ | *Fig.* 57. |
| ⊙ | 23 | $b^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}-x^2+\frac{p^2x^2}{4L^2}-\frac{px}{2}-\frac{p^3x}{8L^2}+\frac{pqx}{2L^2}-\frac{p^2x^2}{4L^2}=HP^2$ | |
| 23 | 24 | h e, $b^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}-x^2-\frac{px}{2}-\frac{p^3x}{8L^2}+\frac{pqx}{2L^2}=HP^2$. | |
| 19−14 | 25 | $\left\{\begin{array}{l} MO-(OP,u)\,DH=PM. \\ x\,(-d,u)-\frac{r}{2L^2}-\frac{pq}{4L^2}+\frac{p^3}{16L^2}+\frac{p}{4}=PM. \end{array}\right.$ | |
| 19+14 | 25 | $\left\{\begin{array}{l} NO+(OP,u)\,DH=PN. \\ -x\,(+d,u)+\frac{r}{2L^2}+\frac{pq}{4L^2}-\frac{p^3}{16L^2}-\frac{p}{4}=PN. \end{array}\right.$ | |

$d^2+$

| | | | |
|---|---|---|---|
| ⊙ | 26 | $d^2+x^2-\frac{rx}{L^2}-\frac{pqx}{2L^2}+\frac{p^3x}{8L^2}+\frac{px}{2}=PM^2$. | |
| ⊙ | 26 | $d^2+x^2-\frac{rx}{L^2}-\frac{pqx}{2L^2}+\frac{p^3x}{8L^2}+\frac{px}{2}=PN^2$. | |
| 47, e 1<br>$24+26$ | 27 | $\begin{cases} HP^2+PM^2=HM^2. \\ b^2+d^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}-\frac{rx}{L^2}=(HM^2=)\text{ Q.Rad.} \end{cases}$ | |
| $27=15$ | 28 | $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}-\frac{rx}{L^2}=0$; in $\frac{L^2}{x}$. | |
| $\times\frac{L^2}{x}$ | 29 | $x^3+px^2+qx-r=0$. *Q. e. d.* in Cubic. | *Fig*.56. |
| $27=17$ | 30 | $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}-\frac{rx}{L^2}=+\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4+px^3+qx^2-rx=+S$. | |
| *Tranſp.* | 32 | $x^4+px^3+qx^2-rx-S=0$. *Q.e.d.* in Biquad. ſi $-S$. | *Fig*.57. |
| $27=18$ | 33 | $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}-\frac{rx}{L^2}=-\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 34 | $x^4+px^3+qx^2-rx=-S$. | |
| *Tranſp.* | 35 | $x^4+px^3+qx^2-rx+S=0$. *Q.e.d.* in Biquad. ſi $+S$. | *Fig*.58. |

*Illuſtrat.*

$$5\quad \left\{\begin{array}{l} x^3-\overset{p.}{16}x^2+\overset{q.}{100}x+\overset{r.}{1072}=0 \\ x^3-1.6x^2+1.00x+1.072=0 \end{array}\right\}$$

$MO=-x=-6$.

$$6\quad \left\{\begin{array}{l} x^3+\overset{p.}{16}x^2+\overset{q.}{100}x-\overset{r.}{1072}=0 \\ x^3+1.6x^2+1.00x-1.072=0 \end{array}\right\}$$

*Fig*.56.

$MO=x=6$.

$$\begin{cases} \frac{p}{2}=0.8. & \frac{p^2}{4}=0.64. & \frac{p^3}{8}=0.512. \\ \frac{p}{4}=0.4. & \frac{p^2}{8}=0.32. & \frac{p^3}{16}=0.256. \end{cases}$$

*Central.*

$$\left.\begin{array}{rl} +\frac{L}{2} & = 0.5 \\ +\frac{p^2}{8} & = 0.32 \\ \hline & 0.82 \\ -\frac{q}{2} & = 0.50 \\ \hline b & = 0.32 = AD. \end{array}\right\} \quad \begin{array}{rl} +\frac{r}{2L^2} & = 0.536 \\ +\frac{pq}{4L^2} & = 0.400 \\ \hline & 0.936 \\ -\frac{p^3}{16} & = 0.256 \\ -\frac{p}{4} & = 0.4 \\ \hline & 0.656 \\ d & = 0.280 = DH. \end{array}$$

Fig. 56.

$$9\left\{\begin{array}{l} x^4 - \overset{p.}{12}x^3 + \overset{q.}{80}x^2 + \overset{r.}{500}x - \overset{s.}{3768} = 0 \\ x^4 - 1.2x^3 + 0.80x^2 + 0.500x - 0.3768 = 0 \end{array}\right\}$$

$$\left\{\begin{array}{l} NO = x = 5.16 - \\ MO = -x = -6. \end{array}\right.$$

$$10\left\{\begin{array}{l} x^4 + \overset{p.}{12}x^3 + \overset{q.}{80}x^2 - \overset{r.}{500}x - \overset{s.}{3768} = 0 \\ x^4 + 1.2x^3 + 0.80x^2 - 0.500x - 0.3768 = 0 \end{array}\right\}$$

Fig. 57.

$$\left\{\begin{array}{l} MO = x = 6. \\ NO = -x = -5.16 - \end{array}\right.$$

$$\left\{\begin{array}{lll} \frac{p}{2} = 0.6. & \frac{p^2}{4} = 0.36. & \frac{p^3}{8} = 0.216. \\ \frac{p}{4} = 0.3. & \frac{p^2}{8} = 0.18. & \frac{p^3}{16} = 0.108. \end{array}\right.$$

*Central.*

*Fig.* 57.

$$\left.\begin{array}{l} +\frac{L}{2}=0.5 \\ +\frac{p^2}{8}=0.18 \\ \hline \qquad 0.68 \\ -\frac{q}{2}=0.40 \\ \hline b=0.28=AD. \\ \hline \end{array}\right\} \quad \begin{array}{l} +\frac{r}{2}=0.250 \\ +\frac{pq}{4}=0.240 \\ \hline \qquad 0.490 \\ \hline -\frac{p^3}{16}=0.108 \\ -\frac{p}{4}=0.3 \\ \hline \qquad 0.408 \\ d=0.082=DH. \\ \hline \end{array}$$

$$11\left\{\begin{array}{l} x^4-\overset{p.}{16}x^3+\overset{q.}{40}x^2+\overset{r.}{1365}x+\overset{s.}{1998}=0 \\ x^4-1.6x^3+0.40x^2+1.365x+0.1998=0 \end{array}\right\}$$

$$\left\{\begin{array}{l} MO=-x=-6. \\ mo=-x=-1.6- \end{array}\right.$$

$$12\left\{\begin{array}{l} x^4+\overset{p.}{16}x^3+\overset{q.}{40}x^2-\overset{r.}{1365}x+\overset{s.}{1998}=0 \\ x^4+1.6x^3+0.40x^2-1.365x+0.1998=0 \end{array}\right\}$$

$$\left\{\begin{array}{l} MO=x=6. \\ mo=x=1.6- \end{array}\right.$$

*Fig.* 58.

$$\left\{\begin{array}{lll} \frac{p}{2}=0.8. & \frac{p^2}{4}=c.64. & \frac{p^3}{8}=0.512. \\ \frac{p}{4}=0.4. & \frac{p^2}{8}=0.32. & \frac{p^3}{16}=0.256. \end{array}\right.$$

*Central.*

$$\left.\begin{array}{l} +\frac{L}{2}=0.5 \\ +\frac{p^2}{8}=0.32 \\ \hline \qquad 0.82 \\ -\frac{q}{2}=0.20 \\ \hline b=0.62=AD. \\ \hline \end{array}\right\} \quad \begin{array}{l} +\frac{r}{2}=0.6825 \\ +\frac{pq}{4}=0.160 \\ \hline \qquad 0.8425 \\ \hline -\frac{p^3}{16}=0.256 \\ -\frac{p}{4}=0.4. \\ \hline \qquad 0.656 \\ d=0.1865=DH. \\ \hline \end{array}$$

$$\left\{\begin{array}{l} 7.\ x^3 - px^2 + qx - r = 0 \left\{\begin{array}{l} 13 \left\{ x^4 - px^3 + qx^2 - rx - S = 0 \right. \\ 15 \left\{ x^4 - px^3 + qx^2 - rx + S = 0 \right. \end{array}\right. \\ 8.\ x^3 + px^2 + qx + r = 0 \left\{\begin{array}{l} 14 \left\{ x^4 + px^3 + qx^2 + rx - S = 0 \right. \\ 16 \left\{ x^4 + px^3 + qx^2 + rx + S = 0 \right. \end{array}\right. \end{array}\right.$$

*Caſ.* 1. Ubi $\frac{L}{2} + \frac{p^2}{8L} \sqsupset \frac{q}{2L}$; & $\frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} \sqsupset \frac{pq}{4L^2}$.

### *Demonſtrat.*

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|---|---|---|---|---|
| $2+3-4$ | 13 | $Ab + bc - cD = b = AD.$<br>$\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = b = AD.$ | | |
| $5+6+8-7$ | 14 | $De + ef + fg - gH = d = DH.$<br>$\frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} - \frac{pq}{4L^2} = d = DH.$ | | |
| 47, e 1<br>Q.13.+<br>Q.14. | 15 | $AD^2 + DH^2 = HA^2.$<br>$b^2 + d^2 = (HA^2 =)$ Q. Rad. in Cubic. | | *Fig.*59. |
| $9 \times 10$ | 16 | $AI \times AK =$ (ob Circl.) $AL^2 =$ (per conſtr.) $AZ^2.$<br>$(L \times \frac{S}{L^3} =) \frac{S}{L^2} = AL^2 = AZ^2.$ | | |
| 47, e 1<br>$15+16$ | 17 | $HA^2 + AL^2 = HL^2.$<br>$b^2 + d^2 + \frac{S}{L^2} = (HL^2 =)$ Q.Rad. | In Biquadr.<br>ſi − S. | *Fig.*60. |
| 47, e 1<br>$15-16$ | 18 | $HA^2 - AZ^2 = HZ^2.$<br>$b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =)$ Q.Rad. | In Biquadr.<br>ſi + S. | *Fig.*61. |
| *Supp.* | 19 | $NO = x.$ | | |
| *Supp.* | 19 | $MO = -x.$ | | |
| $19-1$ | 20 | $NO - (OF, u)\ BA = (NF, u)\ OR.$<br>$x - \frac{p}{2} = OR.$ | | |
| $19+1$ | 20 | $MO + (OF, u)\ BA = (MF, u)\ OR.$<br>$-x + \frac{p}{2} = OR.$ | | |

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|---|---|---|---|
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, x :: x-\frac{p}{2} \,.\, \frac{x^2}{L}-\frac{px}{2L}=AO. \end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L \,.\, MO :: OR \,.\, AO. \\ L \,.\, -x :: -x+\frac{p}{2} \,.\, \frac{x^2}{L}-\frac{px}{2L}=AO. \end{array}\right.$ | |
| $21 \mp 13$ | 22 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\, HP. \\ \frac{x^2}{L}-\frac{px}{2L}\,(-b, u)-\frac{L}{2}-\frac{p^2}{8L}+\frac{q}{2L}=HP. \end{array}\right.$ | |
| Ⓖ | 23 | $b^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{p^2x^2}{4L^2}-x^2-\frac{p^2x^2}{4L^2}+\frac{px}{2}+\frac{p^3x}{8L^2}-\frac{pqx}{2L^2}=HP^2.$ | |
| 23 | 24 | he, $b^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}-x^2+\frac{px}{2}+\frac{p^3x}{8L^2}-\frac{pqx}{2L^2}=HP^2.$ | |
| $19-14$ | 25 | $\left\{\begin{array}{l} NO-(OP, u)\, DH=PN. \\ x\,(-d, u)-\frac{p}{4}-\frac{p^3}{16L^2}-\frac{r}{2L^2}+\frac{pq}{4L^2}=PN. \end{array}\right.$ | |
| $19+14$ | 25 | $\left\{\begin{array}{l} MO+(OP, u)\, DH=PM. \\ -x\,(+d, u)+\frac{p}{4}+\frac{p^3}{16L^2}+\frac{r}{2L^2}-\frac{pq}{4L^2}=PM. \end{array}\right.$ | |
| Ⓖ | 26 | $d^2+x^2-\frac{px}{2}-\frac{p^3x}{8L^2}-\frac{rx}{L^2}+\frac{pqx}{2L^2}=PN^2.$ | |
| Ⓖ | 26 | $d^2+x^2-\frac{px}{2}-\frac{p^3x}{8L^2}-\frac{rx}{L^2}+\frac{pqx}{2L^2}=PM^2.$ | |
| 47, e 1<br>$24+26$ | 27 | $\left\{\begin{array}{l} HP^2+PN^2=HN^2. \\ b^2+d^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}-\frac{rx}{L^2}=(HN^2=)\ Q.Rad. \end{array}\right.$ | |
| $27=15$ | 28 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}-\frac{rx}{L^2}=0;$ in $\frac{L^2}{x}.$ | |
| $\times\frac{L^2}{x}$ | 29 | $x^3-px^2+qx-r=0.$ *Q.e.d.* in Cubic. | *Fig.* 59. |
| $27=17$ | 30 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}-\frac{rx}{L^2}=+\frac{S}{L^2};$ in $L^2.$ | |
| $\times L^2$ | 31 | $x^4-px^3+qx^2-rx=S.$ | |
| *Tranſp.* | 32 | $x^4-px^3+qx^2-rx-S=0.$ *Q.e.d.* in Biquad. ſi $-S.$ | *Fig.* 60. |

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|---|---|---|
| 27=18 | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = -\frac{S}{L^2}$; in L². |
| $\times L^2$ | 34 | $x^4 - px^3 + qx^2 - rx = -S$. |
| *Tranſp.* | 35 | $x^4 - px^3 + qx^2 - rx + S = 0$. *Q.e.d.* in Biquad. ſi+S. *Fig.61.* |
| *Supp.* | 19 | $no = x$. |
| 1−19 | 20 | $\left\{\begin{array}{l} OF, u)\ BA - no = (nF, u)\ OR. \\ \frac{p}{2} - x = OR. \end{array}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\ .\ no :: OR\ .\ AO. \\ L\ .\ x :: \frac{p}{2} - x\ .\ \frac{px}{2L} - \frac{x^2}{L} = AO. \end{array}\right.$ |
| 21+13 | 22 | $\left\{\begin{array}{l} AO + AD = (DO, u)\ HP. \\ \frac{px}{2L} - \frac{x^2}{L} (+b, u) + \frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = HP. \end{array}\right.$ |
| G | 23 | &c. Vide pag. 124, &c. |
| *Supp.* | 19 | $MO = x$. |
| *Supp.* | 19 | $NO = -x$. |
| 19+1 | 20 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ x + \frac{p}{2} = OR. \end{array}\right.$ |
| 19−1 | 20 | $\left\{\begin{array}{l} NO - (OF, u)\ BA = (NF, u)\ OR. \\ -x - \frac{p}{2} = OR. \end{array}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\ .\ NO :: NO\ .\ AO. \\ L\ .\ -x :: -x - \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ |
| 21 ∽ 13 | 22 | $\left\{\begin{array}{l} AO \backsim AD = (DO, u)\ HP. \\ \frac{x^2}{L} + \frac{px}{2L} (\backsim b, u) - \frac{L}{2} - \frac{p^2}{8L} + \frac{q}{2L} = HP. \end{array}\right.$ |

b²+

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|---|---|---|---|
| ⊚ | 23 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - x^2 - \frac{p^2x^2}{4L^2} - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2$ | |
| 23 | 24 | h e, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 - \frac{px}{2} - \frac{p^3x}{8L^2} + \frac{pqx}{2L^2} = HP^2$ | |
| 19 + 14 | 25 | $\begin{cases} MO + (OP, u)\ DH = PM. \\ x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} - \frac{pq}{4L^2} = PM. \end{cases}$ | |
| 19 — 14 | 25 | $\begin{cases} NO - (OP, u)\ DH = PN. \\ -x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} - \frac{r}{2L^2} + \frac{pq}{4L^2} = PN. \end{cases}$ | |
| ⊚ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{rx}{L^2} - \frac{pqx}{2L^2} = PM^2.$ | |
| ⊚ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{rx}{L^2} - \frac{pqx}{2L^2} = PN^2.$ | |
| 47, *e* 1<br>24 + 26 | 27 | $\begin{cases} HP^2 + PM^2 = HM^2. \\ b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = (HM^2 =)\ \text{Q.Rad.} \end{cases}$ | |
| 27 = 15 | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 + px^2 + qx + r = 0.$ *Q. e. d.* in Cubic. | *Fig*.59. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| × $L^2$ | 31 | $x^4 + px^3 + qx^2 + rx = S.$ | |
| *Tranſp.* | 32 | $x^4 + px^3 + qx^2 + rx - S = 0.$ *Q.e.d.* in Biquad. ſi — S. | *Fig*.60. |
| 27 = 18 | 33 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| × $L^2$ | 34 | $x^4 + px^3 + qx^2 + rx = -S.$ | |
| *Tranſp.* | 35 | $x^4 + px^3 + qx^2 + rx + S = 0.$ *Q.e.d.* in Biquad. ſi + S. | *Fig*.61. |
| *Supp.* | 19 | $no = -x.$ | |
| ‡ — 19 | 20 | $\begin{cases} (OF, u)\ BA - no = (nF, u)\ OR. \\ \frac{p}{2} + x = OR. \end{cases}$ | |

L . no

*Ob para.* 21 $\begin{cases} L \,.\, no :: OR \,.\, AO. \\ L \,.\, -x :: \frac{p}{2} + x \,.\, -\frac{px}{2L} - \frac{x^2}{L} = AO. \end{cases}$

21 + 13 22 $\begin{cases} AO + AD = (DO, u)\ HP. \\ -\frac{px}{2L} - \frac{x^2}{L}\ (+b, u) + \frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = HP. \end{cases}$

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*Illuſtrat.*

7 $$\begin{cases} x^3 - \overset{p.}{40}x^2 + \overset{q.}{432}x - \overset{r.}{1152} = 0 \\ x^3 - 4.0x^2 + 4.32x - 1.152 = 0 \end{cases}$$

$$\begin{cases} NO = x = 24. \\ no = x = 12. \\ no = x = 4. \end{cases}$$

8 $$\begin{cases} x^4 + \overset{p.}{40}x^2 + \overset{q.}{432}x + \overset{r.}{1152} = 0 \\ x^4 + 4.0x^2 + 4.32x + 1.152 = 0 \end{cases}$$

*Fig.* 59.

$$\begin{cases} NO = -x = -24. \\ no = -x = -12. \\ no = -x = -4. \end{cases}$$

$$\begin{cases} \frac{p}{2} = 2.0. \quad \frac{p^2}{4} = 4.0. \quad \frac{p^3}{8} = 8.0. \\ \frac{p}{4} = 1.0. \quad \frac{p^2}{8} = 2.0. \quad \frac{p^3}{16} = 4.0. \end{cases}$$

*Central.*

*Central.*

$$\left.\begin{array}{r} +\frac{L}{2}=0.5\\ +\frac{p^2}{8}=2.0\\ \hline 2.5\\ -\frac{q}{2}=2.16\\ \hline b=0.34=AD.\\ \hline \end{array}\right\} \quad \begin{array}{r} +\frac{p}{4}=1.0\\ +\frac{p^3}{16}=4.0\\ +\frac{r}{2}=0.576\\ \hline 5.576\\ -\frac{pq}{4}=4.32\\ \hline d=1.256=DH.\\ \hline \end{array}$$

Fig. 59.

13 $$\left\{\begin{array}{l} x^4-\overset{p.}{34}x^3+\overset{q.}{344}x^2-\overset{r.}{704}x-\overset{s.}{3072}=0\\ x^4-3.4x^3+3.44x^2-0.704x-0.3072=0 \end{array}\right\}$$

$$\left.\begin{array}{l} NO=x=16.\\ no=x=12.\\ no=x=8. \end{array}\right\} \quad MO=-x=-2.$$

14 $$\left\{\begin{array}{l} x^4+\overset{p.}{34}x^3+\overset{q.}{344}x^2+\overset{r.}{704}x-\overset{s.}{3072}=0\\ x^4+3.4x^3+3.44x^2+0.704x-0.3072=0 \end{array}\right\}$$

Fig. 60.

$$MO=x=2. \quad \left\{\begin{array}{l} NO=-x=-16.\\ no=-x=-12.\\ no=-x=-8. \end{array}\right.$$

$$\left\{\begin{array}{lll} \frac{p}{2}=1.7. & \frac{p^2}{4}=2.89. & \frac{p^3}{8}=4.913.\\ \frac{p}{4}=0.85. & \frac{p^2}{8}=1.445. & \frac{p^3}{16}=2.4565. \end{array}\right.$$

Cen-

*Central.*

Fig.60.

$$\left.\begin{array}{l}
+\frac{L}{2}=0.5\\
+\frac{p^2}{8}=1.445\\
\hline
\qquad 1.945\\
-\frac{q}{2}=1.72\\
\hline
b=0.225=AD.
\end{array}\right\}
\begin{array}{l}
+\frac{p}{4}=0.85\\
+\frac{p^3}{16}=2.4565\\
+\frac{r}{2}=0.352\\
\hline
\qquad 3.6585\\
-\frac{pq}{4}=2.9240\\
\hline
d=0.7345=DH.
\end{array}$$

$$15\left\{\begin{array}{l}
\overset{p.}{} \qquad\qquad \overset{q.}{} \qquad\qquad \overset{r.}{} \qquad\qquad \overset{s.}{}\\
x^4-40x^3+400x^2-514x+4128=0\\
x^4-4.0x^3+4.00x^2-0.514x+0.4128=0
\end{array}\right\}$$

$$\left\{\begin{array}{l}
NO=23.8+\\
no=16.
\end{array}\right.$$

$$16\left\{\begin{array}{l}
\overset{p.}{} \qquad\qquad \overset{q.}{} \qquad\qquad \overset{r.}{} \qquad\qquad \overset{s.}{}\\
x^4+40x^3+400x^2+514x+4128=0\\
x^4+4.0x^3+4.00x^2+0.514x+0.4128=0
\end{array}\right\}$$

$$\left\{\begin{array}{l}
NO=-x=-23.8+\\
no=-x=-16.
\end{array}\right.$$

Fig.61.

$$\left\{\begin{array}{lll}
\frac{p}{2}=2.0. & \frac{p^2}{4}=4.0. & \frac{p^3}{8}=8.0.\\
\frac{p}{4}=1.0. & \frac{p^2}{8}=2.0. & \frac{p^3}{16}=4.0.
\end{array}\right.$$

*Central.*

$$\left.\begin{array}{l}
+\frac{L}{2}=0.5\\
+\frac{p^2}{8}=2.0\\
\hline
\qquad 2.5\\
-\frac{q}{2}=2.00\\
\hline
b=0.50=AD.
\end{array}\right\}
\begin{array}{l}
+\frac{p}{4}=1.0\\
+\frac{p^3}{16}=4.0\\
+\frac{r}{2}=0.257\\
\hline
\qquad 5.257\\
-\frac{pq}{4}=4.000\\
\hline
d=1.257=DH.
\end{array}$$

X

*Caſ.*

*Caſ.* 2. Ubi $\frac{q}{2L} \sqsupset \frac{L}{2} + \frac{p^2}{8L}$; & $\frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} \sqsupset \frac{pq}{4L^2}$.

### *Demonſtrat.*

| | | | |
|---|---|---|---|
| 4—3—2 | 13 | $Dc - cb - bA = b = AD.$<br>$\frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} = b = AD.$ | |
| 5+6+8-7 | 14 | $De + ef + fg - gH = d = DH.$<br>$\frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} - \frac{pq}{4L^2} = d = DH.$ | |
| 47, *e* 1<br>*Q*.13.+<br>*Q*.14. | 15 | $AD^2 + DH^2 = HA^2.$<br>$b^2 + d^2 = (HA^2 =)$ Q. Rad. in Cubic. | *Fig*.62. |
| 9 × 10 | 16 | AI × AK = (ob Circl.) $AL^2$ = (per conſtr.) $AZ^2$.<br>$(L \times \frac{S}{L^3} =)\ \frac{S}{L^2} = AL^2 = AZ^2.$ | |
| 47, *e* 1<br>15 + 16 | 17 | $HA^2 + AL^2 = HL^2.$<br>$b^2 + d^2 + \frac{S}{L^2} = (HL^2 =)$ Q.Rad. } In Biquadr. ſi — S. | *Fig*.63. |
| 47, *e* 1<br>15 — 16 | 18 | $HA^2 - AZ^2 = HZ^2.$<br>$b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =)$ Q.Rad. } In Biquadr. ſi + S. | *Fig*.64. |
| *Supp*. | 19 | $NO = x.$ | |
| *Supp*. | 19 | $MO = -x.$ | |
| 19 — 1 | 20 | NO — (OF, u) BA = (NF, u) OR.<br>$x - \frac{p}{2} = OR.$ | |
| 19 + 1 | 20 | MO + (OF, u) BA = (MF, u) OR.<br>$-x + \frac{p}{2} = OR.$ | |

L . NO

| | | | |
|---|---|---|---|
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\ .\ NO\ ::\ OR\ .\ AO. \\ L\ .\ x\ ::\ x-\frac{p}{2}\ .\ \frac{x^2}{L}-\frac{px}{2L}=AO. \end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\ .\ MO\ ::\ OR\ .\ AO. \\ L\ .\ -x\ ::\ -x+\frac{p}{2}\ .\ \frac{x^2}{L}-\frac{px}{2L}=AO. \end{array}\right.$ | |
| 21 + 13 | 22 | $\left\{\begin{array}{l} AO+AD=(DO,u)\ HP. \\ \frac{x^2}{L}-\frac{px}{2L}\ (+b,u)+\frac{q}{2L}-\frac{p^2}{8L}-\frac{L}{2}=HP. \end{array}\right.$ | |
| @ | 23 | $b^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{p^2x^2}{4L^2}-\frac{p^2x^2}{4L^2}-x^2-\frac{pqx}{2L^2}+\frac{p^3x}{8L^2}+\frac{px}{2}=HP^2.$ | |
| 23 | 24 | he, $b^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}-x^2-\frac{pqx}{2L^2}+\frac{p^3x}{8L^2}+\frac{px}{2}=HP^2.$ | |
| 19 — 14 | 25 | $\left\{\begin{array}{l} NO-(OP,u)\ DH=PN. \\ x\ (-d,u)-\frac{p}{4}-\frac{p^3}{16L^2}-\frac{r}{2L^2}+\frac{pq}{4L^2}=PN. \end{array}\right.$ | |
| 19 + 14 | 25 | $\left\{\begin{array}{l} MO+(OP,u)\ DH=PM. \\ -x+(d,u)\ \frac{p}{4}+\frac{p^3}{16L^2}+\frac{r}{2L^2}-\frac{pq}{4L^2}=PM. \end{array}\right.$ | |
| @ | 26 | $d^2+x^2-\frac{px}{2}-\frac{p^3x}{8L^2}-\frac{rx}{L^2}+\frac{pqx}{2L^2}=PN^2.$ | |
| @ | 26 | $d^2+x^2-\frac{px}{2}-\frac{p^3x}{8L^2}-\frac{rx}{L^2}+\frac{pqx}{2L^2}=PM^2.$ | |
| 47, e 1<br>24 + 26 | 27 | $\left\{\begin{array}{l} HP^2+PN^2=HN^2. \\ b^2+d^2+\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}-\frac{rx}{L^2}=(HN^2=)\ Q.Rad. \end{array}\right.$ | |
| 27 = 15 | 28 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}-\frac{rx}{L^2}=0$; in $\frac{L^2}{x}$. | |
| $\times\frac{L^2}{x}$ | 29 | $x^3-px^2+qx-r=0.$ *Q.e.d.* in Cubic. | *Fig.*62. |
| 27 = 17 | 30 | $\frac{x^4}{L^2}-\frac{px^3}{L^2}+\frac{qx^2}{L^2}-\frac{rx}{L^2}=+\frac{S}{L^2}$; in $L^2$. | |
| $\times L^2$ | 31 | $x^4-px^3+qx^2-rx=S.$ | |
| *Tranſp.* | 32 | $x^4-px^3+qx^2-rx-S=0.$ *Q.e.d.* in Biquad. ſi—S. | *Fig.*63. |

| | | | |
|---|---|---|---|
| $27 = 18$ | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = -\frac{S}{L^2}$; in $L^2$: | |
| $\times L^2$ | 34 | $x^4 - px^3 + qx^2 - rx = -S.$ | |
| *Tranſp.* | 35 | $x^4 - px^3 + qx^2 - rx + S = 0$. *Q.e.d.* in Biquad. ſi $+S$. | *Fig*.64. |
| *Supp.* | 19 | $no = x.$ | |
| $1 - 19$ | 20 | $\left\{\begin{array}{l} (OF, u)\ BA - no = (nF, u)\ OR. \\ \frac{p}{2} - x = OR. \end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\ .\ no :: OR\ .\ AO. \\ L\ .\ x :: \frac{p}{2} - x\ .\ \frac{px}{2L} - \frac{x^2}{L} = AO. \end{array}\right.$ | |
| $21 - 13$ | 22 | $\left\{\begin{array}{l} AO - AD = (DO, u)\ HP. \\ \frac{px}{2L} - \frac{x^2}{L}\ (-b, u) - \frac{q}{2L} + \frac{p^2}{8L} + \frac{L}{2} = HP. \end{array}\right.$ | |
| ⊚ | 23 | &c. Ut in pag. 131, &c. | |
| *Supp.* | 19 | $MO = x.$ | |
| *Supp.* | 19 | $NO = -x.$ | |
| $19 + 1$ | 20 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ x + \frac{p}{2} = OR. \end{array}\right.$ | |
| $19 - 1$ | 20 | $\left\{\begin{array}{l} NO - (OF, u)\ BA = (NF, u)\ OR. \\ -x - \frac{p}{2} = OR. \end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ -x :: -x - \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ | |
| $21 + 13$ | 22 | $\left\{\begin{array}{l} AO + AD = (DO, u)\ HP. \\ \frac{x^2}{L} + \frac{px}{2L}\ (+b, u) + \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} = HP. \end{array}\right.$ | |

$b^2 +$

| | | | |
|---|---|---|---|
| @ | 23 | $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - \frac{p^2x^2}{4L^2} - x^2 + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = HP^2$ | |
| 23 | 24 | h e, $b^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} - x^2 + \frac{pqx}{2L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = HP^2$. | |
| 19 + 14 | 25 | $MO + (OP, u)\ DH = PM$.<br>$x\ (+d, u) + \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} - \frac{pq}{4L^2} = PM$. | |
| 19 − 14 | 25 | $NO - (OP, u)\ DH = PN$.<br>$- x\ (-d, u) - \frac{p}{4} - \frac{p^3}{16L^2} - \frac{r}{2L^2} + \frac{pq}{4L^2} = PN$. | |
| @ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{rx}{L^2} - \frac{pqx}{2L^2} = PM^2$. | |
| @ | 26 | $d^2 + x^2 + \frac{px}{2} + \frac{p^3x}{8L^2} + \frac{rx}{L^2} - \frac{pqx}{2L^2} = PN^2$. | |
| 47, e 1<br>24 + 26 | 27 | $HP^2 + PM^2 = HM^2$.<br>$b^2 + d^2 + \frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = (HM^2 =)$ Q.Rad. | |
| 27 = 15 | 28 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = 0$; in $\frac{L^2}{x}$. | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 + px^2 + qx + r = 0$. *Q. e. d.* in Cubic. | *Fig*.62. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = + \frac{S}{L^2}$; in $L^2$. | |
| × $L^2$ | 31 | $x^4 + px^3 + qx^2 + rx = S$. | |
| *Tranſp.* | 32 | $x^4 + px^3 + qx^2 + rx - S = 0$. *Q.e.d.* in Biquad. ſi − S. | *Fig*.63. |
| 27 = 18 | 33 | $\frac{x^4}{L^2} + \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{rx}{L^2} = - \frac{S}{L^2}$; in $L^2$. | |
| × $L^2$ | 34 | $x^4 + px^3 + qx^2 + rx = - S$. | |
| *Tranſp.* | 35 | $x^4 + px^3 + qx^2 + rx + S = 0$. *Q.e.d.* in Biquad. ſi + S. | *Fig*.64. |

*Illuſtrat.*

*Illuſtrat.*

$$7\left\{\begin{array}{l} x^3 - \overset{p.}{20}\,x^2 + \overset{q.}{300}\,x - \overset{r.}{3776} = 0 \\ x^3 - 2.0\,x^2 + 3.00\,x - 3.776 = 0 \end{array}\right\}$$

$NO = x = 16.$

$$8\left\{\begin{array}{l} x^3 + \overset{p.}{20}\,x^2 + \overset{q.}{300}\,x + \overset{r.}{3776} = 0 \\ x^3 + 2.0\,x^2 + 3.00\,x + 3.776 = 0 \end{array}\right\}$$

$NO = - x = - 16.$

$$\left\{\begin{array}{lll} \frac{p}{2} = 1. & \frac{p^2}{4} = 1. & \frac{p^3}{8} = 1. \\ \frac{p}{4} = 0.5. & \frac{p^2}{8} = 0.5. & \frac{p^3}{16} = 0.5. \end{array}\right.$$

Fig.62.

*Central.*

$$\left.\begin{array}{l} + \frac{q}{2} = 1.50 \\ \hline - \frac{p^2}{8} = 0.5 \\ - \frac{L}{2} = 0.5 \\ \hline \qquad 1.0 \\ b = 0.5 = AD. \\ \hline \end{array}\right\} \quad \begin{array}{l} + \frac{p}{4} = 0.5 \\ + \frac{p^3}{16} = 0.5 \\ + \frac{r}{2} = 1.888 \\ \hline \qquad 2.888 \\ - \frac{pq}{4} = 1.5 \\ \hline d = 1.388 = DH. \\ \hline \end{array}$$

$$13\left\{\begin{array}{l} x^4 - \overset{p.}{50}\,x^3 + \overset{q.}{875}\,x^2 - \overset{r.}{6450}\,x - \overset{s.}{22456} = 0 \\ x^4 - 5.0\,x^3 + 8.75\,x^2 - 6.450\,x - 2.2456 = 0 \end{array}\right\}$$

$$\left\{\begin{array}{l} NO = x = 28. \\ MO = - x = - 2.5 - \end{array}\right.$$

Fig.63.

$$14\left\{\begin{array}{l} x^4 + \overset{p.}{50}\,x^3 + \overset{q.}{875}\,x^2 + \overset{r.}{6450}\,x - \overset{s.}{22456} = 0 \\ x^4 + 5.0\,x^3 + 8.75\,x^2 + 6.450\,x - 2.2456 = 0 \end{array}\right\}$$

$$\left\{\begin{array}{l} MO = x = 2.5 - \\ NO = - x = - 28. \end{array}\right.$$

$$\left\{\begin{array}{lll} \frac{p}{2}=2.5. & \frac{p^2}{4}=6.25. & \frac{p^3}{8}=15.625. \\ \frac{p}{4}=1.25. & \frac{p^2}{8}=3.125. & \frac{p^3}{16}=7.8125. \end{array}\right.$$

*Central.*

Fig.63.

$$\left.\begin{array}{l} +\frac{q}{2}=4.375 \\ \hline -\frac{p^2}{8}=3.125 \\ -\frac{L}{2}=0.5 \\ \hline \qquad 3.625 \\ b=0.750=AD. \\ \hline \end{array}\right\} \begin{array}{l} +\frac{p}{4}=1.25 \\ +\frac{p^3}{16}=7.8125 \\ +\frac{r}{2}=3.225 \\ \hline \qquad 12.2875 \\ -\frac{pq}{4}=10.9375 \\ \hline d=1.3500=DH. \\ \hline \end{array}$$

$$15\left\{\begin{array}{l} x^4 - \overset{p.}{50}x^3 + \overset{q.}{875}x^2 - \overset{r.}{8000}x + \overset{s.}{29044}=0 \\ x^4 - 5.0x^3 + 8.75x^2 - 8.000x + 2.9044=0 \end{array}\right\}$$

$$\left\{\begin{array}{l} NO=x=27.1 \textit{ ferè.} \\ no=x=7.+ \end{array}\right.$$

$$16\left\{\begin{array}{l} x^4 + \overset{p.}{50}x^3 + \overset{q.}{875}x^2 + \overset{r.}{8000}x + \overset{s.}{29044}=0 \\ x^4 + 5.0x^3 + 8.75x^2 + 8.000x + 2.9044=0 \end{array}\right\}$$

Fig.64.

$$\left\{\begin{array}{l} NO=-x=-27.1 \textit{ ferè.} \\ no=-x=-7.+ \end{array}\right.$$

$$\left\{\begin{array}{lll} \frac{p}{2}=2.5. & \frac{p^2}{4}=6.25. & \frac{p^3}{8}=15.625. \\ \frac{p}{4}=1.25. & \frac{p^2}{8}=3.125. & \frac{p^3}{16}=7.8125. \end{array}\right.$$

*Central.*

$$\left.\begin{array}{r} +\frac{q}{2} = 4.375 \\ \hline -\frac{p^2}{8} = 3.125 \\ -\frac{L}{2} = 0.5 \\ \hline 3.625 \\ b = 0.750 = AD \\ \hline \end{array}\right\} \quad \begin{array}{r} +\frac{p}{4} = 1.25 \\ +\frac{p^3}{16} = 7.8125 \\ +\frac{r}{2} = 4.0000 \\ \hline 13.0625 \\ -\frac{pq}{4} = 10.9375 \\ \hline d = 2.1250 = DH. \\ \hline \end{array}$$

Fig.64.

*Caſ.* 3. Ubi $\frac{q}{2L^2} \sqsupset \frac{L}{2} + \frac{p^2}{8L}$; & $\frac{pq}{4L^2} \sqsupset \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2}$.

*Demonſtrat.*

4—3—2 13 $\left\{\begin{array}{l} Dc - cb - bA = b = AD. \\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} = b = AD. \end{array}\right.$

7-8-6-5 14 $\left\{\begin{array}{l} Hg - gf - fe - eD = d = DH. \\ \frac{pq}{4L^2} - \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = d = DH. \end{array}\right.$

47, e 1 Q. 13.+ Q. 14. 15 $\left\{\begin{array}{l} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = (HA^2 =) \text{ Q. Rad. in Cubic.} \end{array}\right.$ Fig.65.

9×10 16 $\left\{\begin{array}{l} AI \times AK = (\text{ob Circl.}) \; AL^2 = (\text{per conſtr.}) \; AZ^2. \\ (L \times \frac{S}{L^3} =) \; \frac{S}{L^2} = AL^2 = AZ^2. \end{array}\right.$

47, e 1 15+16 17 $\left\{\begin{array}{l} HA^2 + AL^2 = HL^2. \\ b^2 + d^2 + \frac{S}{L^2} = (HL^2 =) \text{ Q.Rad.} \end{array}\right\}$ In Biquadr. ſi — S. Fig.66.

47, e 1 15—16 18 $\left\{\begin{array}{l} HA^2 - AZ^2 = HZ^2. \\ b^2 + d^2 - \frac{S}{L^2} = (HZ^2 =) \text{ Q.Rad.} \end{array}\right\}$ In Biquadr. ſi + S. Fig.67.

NO

| | | | |
|---|---|---|---|
| *Supp.* | 19 | $NO = x.$ | |
| *Supp.* | 19 | $MO = -x.$ | |
| 1—19 | 20 | $\left\{\begin{array}{l}(OF, u)\ BA - NO = (NF, u)\ OR.\\ \frac{p}{2} - x = OR.\end{array}\right.$ | |
| 19+1 | 20 | $\left\{\begin{array}{l}MO + (OF, u)\ BA = (MF, u)\ OR.\\ -x + \frac{p}{2} = OR.\end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l}L \,.\, NO :: OR \,.\, AO.\\ L \,.\, x :: \frac{p}{2} - x \,.\, \frac{px}{2L} - \frac{x^2}{L} = AO.\end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l}L \,.\, MO :: -OR - \,.\, AO.\\ L \,.\, -x :: -x + \frac{p}{2} \,.\, \frac{x^2}{L} - \frac{px}{2L} = AO.\end{array}\right.$ | |
| 13—21 | 22 | $\left\{\begin{array}{l}AD - AO = (DO, u)\ HP.\\ (b, u)\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} + \frac{x^2}{L} - \frac{px}{2L} = HP.\end{array}\right.$ | *Fig.* $\left\{\begin{array}{l}65\\66\\67\end{array}\right.$ |
| 13+21 | 22 | $\left\{\begin{array}{l}AD + AO = (DO, u)\ HP.\\ (b, u)\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} + \frac{x^2}{L} - \frac{px}{2L} = HP.\end{array}\right.$ | *Fig.* 67. |
| ⊙ | 23 | $b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} + \frac{p^2x^2}{4L^2} - \frac{pqx}{2L^2} - \frac{p^2x^2}{4L^2} + \frac{p^3x}{8L^2} - x^2 + \frac{px}{2} = HP^2.$ | |
| 23 | 24 | $he,\ b^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^3} - \frac{pqx}{2L^2} + \frac{p^3x}{8L^2} - x^2 + \frac{px}{2} = HP^2.$ | |
| 19+14 | 25 | $\left\{\begin{array}{l}NO + (PO, u)\ DH = PN.\\ x\ (+d, u) + \frac{pq}{4L^2} - \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} = PN.\end{array}\right.$ | |
| 14—19 | 25 | $\left\{\begin{array}{l}(OP, u)\ DH - MO = PM.\\ (d,)\ \frac{pq}{4L^2} - \frac{r}{2L^2} - \frac{p^3}{16L^2} - \frac{p}{4} + x = PM.\end{array}\right.$ | |
| ⊙ | 26 | $d^2 + x^2 + \frac{pqx}{2L^2} - \frac{rx}{L^2} + \frac{p^3x}{8L^2} - \frac{px}{2} = PN^2.$ | |
| ⊙ | 26 | $d^2 + x^2 + \frac{pqx}{2L^2} - \frac{rx}{L^2} - \frac{p^3x}{8L^2} - \frac{px}{2} = PM^2.$ | |

| | | | |
|---|---|---|---|
| 47, e 1<br>24 + 26 | 27 | $\left\{\begin{array}{l} HP^2 + PN^2 = HN^2. \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = (HN^2 =)\ \text{Q.Rad.} \end{array}\right.$ | |
| 27 = 15 | 28 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = 0;\ \text{in}\ \frac{L^2}{x}.$ | |
| $\times \frac{L^2}{x}$ | 29 | $x^3 - px^2 + qx - r = 0.$ *Q.e.d.* in Cubic. | *Fig.* 65. |
| 27 = 17 | 30 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = + \frac{S}{L^2};\ \text{in}\ L^2.$ | |
| $\times L^2$ | 31 | $x^4 - px^3 + qx^2 - rx = S.$ | |
| *Tranſp.* | 32 | $x^4 - px^3 + qx^2 - rx - S = 0.$ *Q.e.d.* in Biquad. ſi—S. | *Fig.* 66. |
| 27 = 18 | 33 | $\frac{x^4}{L^2} - \frac{px^3}{L^2} + \frac{qx^2}{L^2} - \frac{rx}{L^2} = - \frac{S}{L^2};\ \text{in}\ L^2.$ | |
| $\times L^2$ | 34 | $x^4 - px^3 + qx^2 - rx = - S.$ | |
| *Tranſp.* | 35 | $x^4 - px^3 + qx^2 - rx + S = 0.$ *Q.e.d.* in Biquad. ſi+S. | *Fig.* 67. |
| *Supp.* | 19 | $MO = x.$ | |
| *Supp.* | 19 | $NO = - x.$ | |
| 19 + 1 | 20 | $\left\{\begin{array}{l} MO + (OF, u)\ BA = (MF, u)\ OR. \\ x + \frac{p}{2} = OR. \end{array}\right.$ | |
| 1 — 19 | 20 | $\left\{\begin{array}{l} (OF, u)\ BA - NO = (NF, u)\ OR. \\ \frac{p}{2} + x = OR. \end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\ .\ MO :: OR\ .\ AO. \\ L\ .\ x :: x + \frac{p}{2}\ .\ \frac{x^2}{L} + \frac{px}{2L} = AO. \end{array}\right.$ | |
| *Ob para.* | 21 | $\left\{\begin{array}{l} L\ .\ NO :: OR\ .\ AO. \\ L\ .\ - x :: \frac{p}{2} + x\ .\ - \frac{px}{2L} - \frac{x^2}{L} = AO. \end{array}\right.$ | |
| 13 + 21 | 22 | $\left\{\begin{array}{l} AD + AO = (DO, u)\ HP. \\ (b, u)\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} + \frac{x^2}{L} + \frac{px}{2L} = HP. \end{array}\right.$ | *Fig.* 67. |
| 13 — 21 | 22 | $\left\{\begin{array}{l} AD - AO = (DO, u)\ HP. \\ (b, u)\ \frac{q}{2L} - \frac{p^2}{8L} - \frac{L}{2} + \frac{px}{2L} + \frac{x^2}{L} = HP. \end{array}\right.$ | *Fig.* $\left\{\begin{array}{l} 65 \\ 66 \\ 67 \end{array}\right.$ |

$b^2 +$

⊙ 23 $b^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{p^2x^2}{4L^2}+\frac{pqx}{2L^2}-\frac{p^3x}{8L^2}-\frac{p^2x^2}{4L^2}-\frac{px}{2}-x^2=HP^2$

23 24 h e, $b^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{pqx}{2L^2}-\frac{p^3x}{8L^2}-\frac{px}{2}-x^2=HP^2$.

14—19 25 $\begin{cases}(OP,u)\ DH-MO=PM.\\(d,u)\ \frac{pq}{4L^2}-\frac{r}{2L^2}-\frac{p^3}{16L^2}-\frac{p}{4}-x=PM.\end{cases}$

19+14 25 $\begin{cases}NO+(OP,u)\ DH=PN.\\-x(+d,u)+\frac{pq}{4L^2}-\frac{r}{2L^2}-\frac{p^3}{16L^2}-\frac{p}{4}=PN.\end{cases}$

⊙ 26 $d^2+x^2-\frac{pqx}{2L^2}+\frac{rx}{L^2}+\frac{p^3x}{8L^2}+\frac{px}{2}=PM^2$.

⊙ 26 $d^2+x^2-\frac{pqx}{2L^2}+\frac{rx}{L^2}+\frac{p^3x}{8L^2}+\frac{px}{2}=PN^2$.

47, e 1
24+26 27 $\begin{cases}HP^2+PM^2=HM^2.\\b^2+d^2+\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=(HM^2=)\text{ Q.Rad.}\end{cases}$

27=15 28 $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=0$; in $\frac{L^2}{x}$.

$\times\frac{L^2}{x}$ 29 $x^3+px^2+qx+r=0$. *Q.e.d.* in Cubic. *Fig.*65.

27=17 30 $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=+\frac{S}{L^2}$; in $L^2$.

*× L²* 31 $x^4+px^3+qx^2+rx=S$.

*Tranſp.* 32 $x^4+px^3+qx^2+rx-S=0$. *Q.e.d.* in Biquad. ſi—S. *Fig.*66.

27=18 33 $\frac{x^4}{L^2}+\frac{px^3}{L^2}+\frac{qx^2}{L^2}+\frac{rx}{L^2}=-\frac{S}{L^2}$; in $L^2$.

*× L²* 34 $x^4+px^3+qx^2+rx=-S$.

*Tranſp.* 35 $x^4+px^3+qx^2+rx+S=0$. *Q.e.d.* in Biquad. ſi+S. *Fig.*67.

*Illuſtrat.*

7 $\begin{cases}x^3-\overset{p.}{24}x^2+\overset{q.}{450}x-\overset{r.}{1775}=0\\x^3-2.4x^2+4.50x-1.775=0\end{cases}$

NO $=x=5$.

$$8\left\{\begin{array}{l} x^3 + \overset{p.}{24}\,x^2 + \overset{q.}{450}\,x + \overset{r.}{1775} = 0 \\ x^3 + 2.4\,x^2 + 4.50\,x + 1.775 = 0 \end{array}\right\}$$

$$NO = -x = -5.$$

$$\left\{\begin{array}{lll} \frac{p}{2} = 1.2 & \frac{p^2}{4} = 1.44 & \frac{p^3}{8} = 1.728 \\ \frac{p}{4} = 0.6. & \frac{p^2}{8} = 0.72. & \frac{p^3}{16} = 0.864. \end{array}\right.$$

*Central.*

Fig.65.

$$\left.\begin{array}{l} +\frac{q}{2} = 2.25 \\ \hline -\frac{p^2}{8} = 0.72 \\ -\frac{L}{2} = 0.5 \\ \hline \quad\; 1.22 \\ b = 1.03 = AD. \\ \hline \end{array}\right\} \begin{array}{l} +\frac{pq}{4} = 2.700 \\ \hline -\frac{r}{2} = 0.8875 \\ -\frac{p^3}{16} = 0.864 \\ -\frac{p}{4} = 0.6 \\ \hline \quad\;\; 2.3515 \\ d = 0.3485 = DH. \\ \hline \end{array}$$

$$13\left\{\begin{array}{l} x^4 - \overset{p.}{20}\,x^3 + \overset{q.}{500}\,x^2 - \overset{r.}{496}\,x - \overset{s.}{11520} = 0 \\ x^4 - 2.0\,x^3 + 5.00\,x^2 - 0.496\,x - 1.1520 = 0 \end{array}\right\}$$

$$\left\{\begin{array}{l} NO = x = 5.9 + \\ MO = -x = -4. \end{array}\right.$$

$$14\left\{\begin{array}{l} x^4 + \overset{p.}{20}\,x^3 + \overset{q.}{500}\,x^2 + \overset{r.}{496}\,x - \overset{s.}{11520} = 0 \\ x^4 + 2.0\,x^3 + 5.00\,x^2 + 0.496\,x - 1.1520 = 0 \end{array}\right\}$$

Fig.66.

$$\left\{\begin{array}{l} MO = x = 4. \\ NO = -x = -5.9 + \end{array}\right.$$

$$\left\{\begin{array}{lll} \frac{p}{2} = 1.0. & \frac{p^2}{4} = 1.00. & \frac{p^3}{8} = 1.000. \\ \frac{p}{4} = 0.5. & \frac{p^2}{8} = 0.50. & \frac{p^3}{16} = 0.500. \end{array}\right.$$

Cen-

*Central.*

Fig.66.

$$\left.\begin{array}{l} +\frac{q}{2}=2.50 \\ \hline -\frac{p^2}{8}=0.50 \\ -\frac{L}{2}=0.5 \\ \hline \qquad 1.00 \\ b=1.50=AD. \\ \hline \end{array}\right\} \begin{array}{l} +\frac{pq}{4}=2.50 \\ \hline -\frac{r}{2}=0.248 \\ -\frac{p^3}{16}=0.500 \\ -\frac{p}{4}=0.5 \\ \hline \qquad 1.248 \\ d=1.252=DH. \\ \hline \end{array}$$

$$15\left\{\begin{array}{l} x^4-\overset{p.}{24}x^3+\overset{q.}{450}x^2-\overset{r.}{2000}x+\overset{s.}{1125}=0 \\ x^4-2.4x^3+4.50x^2-2.000x+0.1125=0 \end{array}\right\}$$

$$\left\{\begin{array}{l} NO=x=5. \\ no=x=0.66 \textit{ ferè}. \end{array}\right.$$

$$16\left\{\begin{array}{l} x^4+\overset{p.}{24}x^3+\overset{q.}{450}x^2+\overset{r.}{2000}x+\overset{s.}{1125}=0 \\ x^4+2.4x^3+4.50x^2+2.000x+0.1125=0 \end{array}\right\}$$

$$\left\{\begin{array}{l} NO=-x=-5. \\ no=-x=-0.66 \textit{ ferè}. \end{array}\right.$$

$$\left\{\begin{array}{lll} \frac{p}{2}=1.2. & \frac{p^2}{4}=1.44. & \frac{p^3}{8}=1.728. \\ \frac{p}{4}=0.6. & \frac{p^2}{8}=0.72. & \frac{p^3}{16}=0.864. \end{array}\right.$$

Fig.67.

*Central.*

$$\left.\begin{array}{l} +\frac{q}{2}=2.25 \\ \hline -\frac{p^2}{8}=0.72 \\ -\frac{L}{2}=0.5 \\ \hline \qquad 1.22 \\ b=1.03=AD. \\ \hline \end{array}\right\} \begin{array}{l} +\frac{pq}{4}=2.700 \\ \hline -\frac{r}{2}=1.000 \\ -\frac{p^3}{16}=0.864. \\ -\frac{p}{4}=0.6 \\ \hline \qquad 2.464 \\ d=0.236=DH. \\ \hline \end{array}$$

And

At manum de Tabulâ; quandoquidem methodum quâ ad Regulam Catholicam exquirendam uſus ſum, (quò impenſiùs demiraretur Mathematicorum vulgus, & nonnullos altioris ſubſellii ſuſpenſos teneam) quaſi occultum quoddam myſterium preſſiſſe, in animo erat.

Apud Diophantum enim, alioſque quam plurimos, quà Veteranos quà Neotericos, ſepiuſcule artem celare (quod maximam artem autumant) conſuetudinem invaluiſſe non ignoro. Et hoc quidem de induſtriâ feciſſe ſtatueram, ut Tyronibus (in quorum uſum ſolum hæc exarata ſunt, & quibus ſolis hàc in re conſultum eſt) voluptatem illius, proprio Marte, inveſtigandæ, non præripiam. Diù quidem multùmque animo revolvi, quid agerem; anxius hærebam, & quò me verterem, planè neſciebam. Tandèm verò (diebus haud paucis elapſis) ſuſcepto conſilio non ſtare, ſed à propoſito reſilire decrevi: Et à ſententiâ priore ideò decedere viſum eſt, nempè quòd voluptatem quam à fontibus Geometricis haurire expectarent Tyrones, cruciatus, inter inquirendum (rebus etiam non ſemper auſpicatò ſuccedentibus), minime compenſaturos ſuſpicabar. Ab illis igitur mihi gratias habitum iri perſuaſum habui, ſi eos tanto onere levarem, tantiſque ex ambagibus & Mæandris manuducerem, methodum ipſam, quà Regulam Generalem ipſe excogitavi & comperi, brevitèr perſtringendo.

Sæpé numerò admirari ſoleo, quibus mediis, ſive veſtigiis in Æquationibus Cubicis & Biquadraticis (quibus ſecundus Terminus deeſt) conſtruendis, inſiſtebat Carteſius. At tantum non obſtupui, ſaltem non ſatis mirari potui, quomodo egregio illo viro (quo nihil majus habet orbis Geometricus, tanto ingenii acumine & perſpicacitate imbuto, ut omnes in ſui admirationem meritò rapiat), tam portentoſa & rara (ad hanc rem ſpectantia) perſpicienti, etiam alia ejuſdem farinæ, & perſpectu quidem æque facilia non deprehendiſſe contigiſſet; nempè, quòd non æque Parabolæ cujuſvis Diametrum ac Axem animadvertiſſet. Si enim à Puncto quovis intra vel extra Parabolam poſitione dato,

Cir-

And here I had determined to put a period to this Tract, intending to ſuppreſs the Method, by which I came in proſpect of the *General Rule*, and to reſerve it as ſome choice Secret; to the intent, that the meaner ſort of *Mathematicians* might be rapp'd into admiration, and that I might detain ſome others of an higher form in ſuſpence:

It being an uſual thing with *Diophantus*, and very many other *Veterans* as well as *Neoterics*, to conceal their Art, which they look upon as the greateſt point of Art. And this truly deſignedly I had determined to do, that I might not anticipate that pleaſure, which *Tyro*'s (for whoſe uſe and ſake only all is done what I have done) might, by their own induſtry, find in its ſearch very much; and a long while concerned I was, and at a ſtand what to do, either to conceal, or diſcover it: At length (tho long firſt) I reſolved not to ſtand to my former determinations, but to make a diſcovery; and that which ſwayed and prevailed with me moſt, was, That the pleaſure which *Tyro*'s might expect whiles buſied in its Inqueſt, I ſuſpected might not make them a ſuitable compenſation (things not always ſucceeding according to their expectation) for the pains they might ſuſtain in its ſearch. I conceived therefore I might do them an acceptable office, if I ſhould eaſe them of ſo great a burden, and lead them out of thoſe Mazes and Labyrinths, in which they might be toyled, by diſcovering that Method by which I my ſelf found out the General Rule.

I have oftentimes wondered what Mediums *Des Cartes* uſed in the finding out of the conſtruction of ſuch Cubic and Biquadratic Equations, wherein the ſecond Term is wanting: But I wondred much more, that ſo excellent a Man (than whom the Geometrical World hath none greater, one endued with ſo great ſharpneſs of wit and perſpicacity, that he deſervedly becomes the wonder of all), ſeeing and finding ſuch wonderful rare things as (touching this buſineſs) he did, ſhould not have been ſo happy as to ſee and find other things alſo of the ſame nature, and altogether as obvious; *viz.* That he had not as well conſidered the Diameter of a Parabole, as its Axe; for if it had been his hap, from any Point within or without any Parabole given in Poſition, to have

Circulum tam per verticem Diametri cujuſpiam poſitione ſimiliter datæ, quàm per verticem Axis tranſeuntem, deſcripſiſſe; & Operationem quam ad Axem ad Diametrum applicuiſſe, ſors dediſſet; rem (de quâ quærimus) acu tetigiſſe, nempè Regulam Generalem ad Æquationes omnes Cubicas & Biquadraticas, quomodolibet affectas conſtruendas perſentiſcere contigiſſet. Quid in cauſâ fuiſſet, quòd Diametrum non reſpexiſſe contigerit, me fugit; at verò, pace tanti viri, ſuſpicari liceat; ſcilicet, V. CL. proprietatem quandam Parabolæ ad hanc rem apprimè ſpectantem, (ut non dicam, neſciviſſe) paulò minùs penſiculatè trutinâſſe; quod ſi feciſſet, dubio procul (ſi modò iiſdem, quibus ipſe uſus ſum, mediis innixus fuiſſet, cùm tam nova, & priùs orbi inaudita protulit) Univerſalem Regulam nullo negotio indagâſſet.

have deſcribed a Circle, paſſing as well through the Vertex of any Diameter, (given likewiſe in Poſition) as through the Vertex of its Axe, and had applied that Operation to the Diameter, which he did to the Axe, he could not have miſſed that mark which we aim at, *viz.* have found a Method for the conſtruction of all Cubic and Biquadratic Equations, howſoever affected, as we have found. What the reaſon might be, he had no reſpect unto the Diameter, I know not; but (begging his pardon) I humbly conceive, This moſt excellent acute Man (I cannot ſay, was ignorant of, but) took not into his conſideration a certain propriety of a Parabole, (than which none could ſo aptly have ſuited his deſign) which if it had been his hap ſo to have done, no doubt (he uſing the ſame Mediums which I am about to diſcover, when he amuſed the World with ſuch rare Inventions, and things never before heard of) he could not but with greateſt eaſe have made a full diſcovery of the *Univerſal Rule.*

## De Regulâ Generali inveſtigandâ.

SI à puncto quolibet (puta H) poſitione dato, intra vel extra quamlibet Parabolam datam, & ſub eodem Plano, deſcribatur Circulus, tranſiens per Verticem $\left\{\begin{matrix}\text{Axis}\\ \text{Diametri}\end{matrix}\right\}$ poſitione datæ, ſecans Parabolam, in 1, 2, 3, ſeu 4 Punctis; & abs ipſis demittantur ad $\left\{\begin{matrix}\text{Axem}\\ \text{Diametrum}\end{matrix}\right\}$ Perpendiculares; non ſolum omnes Æquationum formulæ, quartum gradum non excedentium eluceſcent; ſed ad ipſas Conſtruendas Regulæ poſſint elici. Enimverò, Fig. $\left\{\begin{matrix}13\\ 50\end{matrix}\right.$

Si à Puncto H (poſitione dato,) demittatur ad $\left\{\begin{matrix}\text{Axem}\\ \text{Diametrum}\end{matrix}\right\}$ Perpendicularis H D, erunt D H, A D, poſitione ſimiliter datæ. Fig. $\left\{\begin{matrix}13\\ 50\end{matrix}\right.$

Et ſi à vertice Diametri A, (*Fig*. 50.) ducatur A B ∽ D H; erit B A poſitione data. *Fig*. 50.

Pro variâ poſitione Puncti H liquido conſtat, Punctum D, citra vel ultra $\left\{\begin{matrix}\text{Axis}\\ \text{Diametri}\end{matrix}\right\}$ verticem cadere poſſe.

Omnes varias poſitiones Puncti H, ſupervacaneum eſſet exponere; unicam tantùm, inſtar omnium proferam, & in exemplum dabo; nempe.

Fingatur Punctum H, ad lævam $\left\{\begin{matrix}\text{Axis}\\ \text{Diametri}\end{matrix}\right\}$ poſitione dari; & Punctum D citra eorundem vertices cadere. Fig. $\left\{\begin{matrix}13\\ 50\end{matrix}\right.$

## *The manner of finding out the General Rule.*

IF from any Point (as H) given in poſition, either within or without any given Parabole, and on the ſame Plane be deſcribed a Circle, paſſing through the Vertex of its $\left\{\begin{matrix}\text{Axe,}\\ \text{Diameter,}\end{matrix}\right\}$ given in poſition, *Fig.* $\left\{\begin{matrix}13\\ 50\end{matrix}\right.$ Cutting the Parabole in, 1, 2, 3, or 4 Points; and from them be demitted Perpendiculars to the $\left\{\begin{matrix}\text{Axe,}\\ \text{Diameter,}\end{matrix}\right\}$ as all forms of Equations, not exceeding the fourth degree, ſo Rules for their Conſtruction may eaſily be had. For,

If from the Point H, (given in poſition) be demitted H D, Perpendicular to the $\left\{\begin{matrix}\text{Axe,}\\ \text{Diameter,}\end{matrix}\right\}$ then will D H, *Fig.* $\left\{\begin{matrix}13\\ 50\end{matrix}\right.$ A D, be given in poſition.

And if from the Vertex of the Diameter (A) (*Fig.* 50.) be drawn A B ∥ D H, then will B A likewiſe be given in poſition. *Fig.* 50.

According to the various poſition of the Point H, it is evident, that the Point D, may happen below or above the Vertex of the $\left\{\begin{matrix}\text{Axe.}\\ \text{Diameter.}\end{matrix}\right\}$

It were needleſs to ſet down all the diverſe poſitions of the Point H; I ſhall inſtance in one only, for all, *viz.*

Let the Point H, be ſuppoſed to be poſited towards the left ſide of the $\left\{\begin{matrix}\text{Axe,}\\ \text{Diameter,}\end{matrix}\right\}$ and the Point D to *Fig.* $\left\{\begin{matrix}13\\ 50\end{matrix}\right.$ fall below either of their Vertex's.

## *SERIES* I.

| | | | |
|---|---|---|---|
| | 1 | Lactus Rect. $= L$, dat. | |
| | 2 | × × × | |
| | 3<br>4 | $\left.\begin{matrix} AD = b \\ DH = d \end{matrix}\right\}$ poſitione data. | |
| 47, *e* 1<br>$\underline{Q}.3 +$<br>$\underline{Q}.4.$ | 5 | $\left\{\begin{matrix} AD^2 + DH^2 = HA^2 \\ b^2 + d^2 = HA^2 \end{matrix}\right.$ | |
| *Supp.* | 6 | $NO = x.$ | *Fig.* 13. |
| | 7 | × × × | |
| *Ob para.* | 8 | $\left\{\begin{matrix} L \,.\, NO :: NO \,.\, AO. \\ L \,.\, x :: x \,.\, \frac{x^2}{L} = AO. \end{matrix}\right.$ | |
| $8 \backsim 3$ | 9 | $\left\{\begin{matrix} AO \backsim AD = (DO, u)\, HP. \\ \frac{x^2}{L} \backsim b = HP. \end{matrix}\right.$ | |
| @ | 10 | $b^2 + \frac{x^4}{L^2} - \frac{2bx^2}{L} = HP^2$ | |
| $6 - 4$ | 11 | $\left\{\begin{matrix} NO - (OP, u)\, DH = PN. \\ x - d = PN. \end{matrix}\right.$ | |
| @ | 12 | $d^2 + x^2 - 2dx = PN^2$ | |
| 47, *e* 1<br>$10 + 12$ | 13 | $\left\{\begin{matrix} HP^2 + PN^2 = HN^2 \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{2bx^2}{L} + x^2 - 2dx = HN^2\text{, reduc.} \end{matrix}\right.$ | |
| $13 = 5$ | 14 | $b^2 + d^2 + \frac{x^4}{L^2} - 2Lbx^2 - \frac{2L^2dx}{L^2} = HN^2 = (\S\, 5)\, b^2 + d^2$<br>$+ \frac{L^2x^2}{L^2}$ | *Fig.* 13. |

*S E-*

## SERIES 1.

| | | | |
|---|---|---|---|
| | 1 | Lactus Rect. $= L$, given. | |
| | 2 | × × × | |
| | 3 | $AD = b$ } given in poſition. | |
| | 4 | $DH = d$ } | |
| 47, e 1<br>Q.3 +<br>Q. 4. | 5 | $\left\{\begin{array}{l} AD^2 + DH^2 = HA^2 \\ b^2 + d^2 = HA^2 \end{array}\right.$ | |
| Supp. | 6 | $MO = x.$ | *Fig*.13. |
| | 7 | × × × | |
| Ob para. | 8 | $\left\{\begin{array}{l} L \,.\, MO :: M \,.\, AO. \\ L \,.\, x :: x \,.\, \frac{x^2}{L} = AO. \end{array}\right.$ | |
| 3 ∽ 8 | 9 | $\left\{\begin{array}{l} AD \backsim AO = (DO, u)\, HP. \\ b \backsim \frac{x^2}{L} = HP \end{array}\right.$ | |
| ⊖ | 10 | $b^2 + \frac{x^4}{L^2} - \frac{2bx^2}{L} = HP^2$ | |
| 6+4 | 11 | $\left\{\begin{array}{l} MO + (OP, u)\, DH = PM. \\ x + d = PM. \end{array}\right.$ | |
| ⊖ | 12 | $d^2 + x^2 + 2dx = PM^2$ | |
| 47, e 1<br>10+12 | 13 | $\left\{\begin{array}{l} HP^2 + PM^2 = HM^2 \\ b^2 + d^2 + \frac{x^4}{L^2} - \frac{2bx^2}{L} + x^2 + 2dx = HM^2 \end{array}\right.$; reduced, | |
| 13=5 | 14 | $b^2 + d^2 + \frac{x^4}{L^2} - 2Lbx^2 + \frac{2L^2dx}{L^2} = HM^2 = (\S\, 5)\, b^2 + d^2$ $+ \frac{L^2x^2}{L^2}$ | *Fig*.13. |

## *SERIES* 2.

| | | | |
|---|---|---|---|
| | 1 | Latus Rect. $= L$, dat. | |
| | 2 | $BA = a$ | |
| | 3 | $AD = b$ poſitione data. | *Fig*. 50. |
| | 4 | $DH = d$ | |
| 47, e 1<br>*Q*.3.+<br>*Q*.4. | 5 | $\left\{\begin{array}{l} AD^2 + DH^2 = HA^2. \\ b^2 + d^2 = HA^2. \end{array}\right.$ | |
| *Supp.* | 6 | $NO = x.$ | *Fig*. 50. |
| 6 — 2 | 7 | $\left\{\begin{array}{l} NO - (OF, u)\, BA = (NF, u)\, OR. \\ x - a = OR. \end{array}\right.$ | |
| *Ob para.* | 8 | $\left\{\begin{array}{l} L \,.\, NO :: OR \,.\, AO. \\ L \,.\, x :: x - a \,.\, \frac{x^2}{L} - \frac{ax}{L} = AO. \end{array}\right.$ | |
| 3 ∽ 8 | 9 | $\left\{\begin{array}{l} AD \backsim AO = (DO, u)\, HP. \\ b - \frac{x^2}{L} + \frac{ax}{L} = HP. \end{array}\right.$ | |
| ⊚ | 10 | $b^2 + \frac{x^4}{L^2} - \frac{2ax^3}{L^2} + \frac{a^2x^2}{L^2} - \frac{2bx^2}{L} + \frac{2abx}{L} = HP^2.$ | |
| 6 — 4 | 11 | $\left\{\begin{array}{l} NO - (OP, u)\, DH = PN. \\ x - d = PN. \end{array}\right.$ | |
| ⊚ | 12 | $d^2 + x^2 - 2dx = PN^2.$ | |
| 47, e 1.<br>10 + 12 | 13 | $\left\{\begin{array}{l} HP^2 + PN^2 = HN^2. \\ \left.\left\{\begin{array}{l} b^2 + d^2 + \frac{x^4}{L^2} - \frac{2ax^3}{L^2} + \frac{a^2x^2}{L^2} - \frac{2bx^2}{L} + x^2 \\ + \frac{2abx}{L} + 2dx. \end{array}\right.\right\} = HN^2, \text{ reduc.} \end{array}\right.$ | |
| 13 = 5 | 14 | $\left.b^2 + d^2 + \frac{x^4}{L^2} - \frac{2ax^3}{L^2} \begin{array}{ll} +a^2x^2 & +2Labx \\ -2Lb & -\frac{2L^2d}{L^2} \\ +\frac{L^2}{L^2} & \end{array}\right\} \begin{array}{l} = HN^2 = \\ = (\S\, 5)\, b^2 + d^2. \end{array}$ | *Fig*. 50. |

Hinc

## *SERIES* 2.

| | | | |
|---|---|---|---|
| | 1 | Latus Rect. $=$ L, given. | |
| | 2 | $BA=a$ | |
| | 3 | $AD=b$ given in poſition. | *Fig.* 50. |
| | 4 | $DH=d$ | |
| 47, e 1<br>*Q.*3. +<br>*Q.*4. | 5 | $AD^2+DH^2=HA^2.$<br>$b^2+d^2=HA^2.$ | |
| *Supp.* | 6 | $MO=x.$ | *Fig.* 50. |
| 6 + 2 | 7 | $MO+(OF,u)\,BA=(MF,u)\,OR.$<br>$x+a=OR.$ | |
| *Ob para.* | 8 | $L\,.\,MO :: OR\,.\,AO.$<br>$L\,.\,x :: x+a\,.\,\frac{x^2}{L}+\frac{ax}{L}=AO.$ | |
| 3 ∽ 8 | 9 | $AD \backsim AO=(DO,u)\,HP.$<br>$b-\frac{x^2}{L}-\frac{ax}{L}=HP.$ | |
| Ⓖ | 10 | $b^2+\frac{x^4}{L^2}+\frac{2ax^3}{L^2}+\frac{a^2x^2}{L^2}-\frac{2bx^2}{L}-\frac{2abx}{L}=HP^2.$ | |
| 6 + 4 | 11 | $MO+(OP,u)\,DH=PM.$<br>$x+d=PM.$ | |
| Ⓖ | 12 | $d^2+x^2+2dx=PM^2.$ | |
| 47, e 1<br>10 + 12 | 13 | $HP^2+PM^2=HM^2.$<br>$\left.\begin{array}{l} b^2+d^2+\frac{x^4}{L^2}+\frac{2ax^3}{L^2}+\frac{a^2x^2}{L^2}-\frac{2bx^2}{L}+x^2 \\ -\frac{2abx}{L}-2dx. \end{array}\right\\}=HM^2,$ reduce. | |
| 13 = 5 | 14 | $\left.b^2+d^2+\frac{x^4}{L^2}+\frac{2ax^3}{L^2}\begin{array}{l}+a^2x^2 \\ -2Lb \\ +L^2 \\ \hline L^2\end{array}\begin{array}{l}-2Labx \\ +2L^2d \\ \hline L^2\end{array}\right\\}\begin{array}{l}=HM^2=(\S\ 5 \\ =b^2+d^2.\end{array}$ | *Fig.* 50. |

Hence

Hinc (per § 14, utriuſque Seriei,) liquidò conſtat; ſi pro Homogeneo comparationis, fingatur.

*f* 1. $b^2 + d^2 = HA^2 =$ Q. Rad. tum $b^2 + d^2$ evaneſcere, Fig. $\left\{\begin{matrix}13\\50\end{matrix}\right.$ reſiduamque Æquationem (Multiplicatam in $\frac{L^2}{x}$,) ad Cubicam, (quâ quidem terminus ſecundus deficiet in primâ Serie; nullus verò in ſecundâ) deprimi.

*g* 2. $b^2 + d^2 \pm \frac{S}{L^2} =$ Q. Rad. tum $b^2 + d^2$ etiam evaneſcere, reſiduamque Æquationem in quarto gradu ſubſiſtere, hoc eſt fore Biquadraticam; quâ, in primâ ſerie terminum ſecundum; nullum verò terminorum, in ſecundâ, deficere continget.

Quomodo autem in figurâ conſtrui poſſit $b^2 + d^2 \pm \frac{S}{L^2} =$ Q. Rad. paucis expedire operæ erit pretium.

Deſcribantur duo diverſa paria Parabolarum, in quarum altero (pari,) (nempe, ut in *Fig.* 14, 15.) ſint AD, DH ſimiles & ſimiliter poſitæ, ac in *Fig.* 13; in altero verò (nempe ut in *Fig.* 51. 52.) ſint BA, AD, DH ſimiles & ſimiliter poſitæ, ac in *Fig.* 50. Jam,

1. Si à vertice $\left\{\begin{matrix}\text{Axis}\\\text{Diametri}\end{matrix}\right\}$ erigatur ad AH Perpendicularis $AL = \surd \frac{S}{L^2}$; vel (quod perinde eſt,) ſi Fig. $\left\{\begin{matrix}14\\51\end{matrix}\right.$
*h* in linea AH, productâ utrinque, ex unâ parte ſumatur
*i* $AI = L$, & ex alterâ parte $AK = \frac{S}{L^3}$, orietur $AL^2$
*k* $= \frac{S}{L^2}$.

Et

Hence it is manifeſt, (by § 14 of both Series's;) if for the Homogene of the compariſon, be ſuppoſed,

*f* 1. $b^2 + d^2 = HA^2 = Q$. Rad. then $b^2 + d^2$ will vaniſh, and the remaining Equation be depreſſed (being Multiplied into $\frac{L^2}{x}$,) to a Cubick; in which the ſecond term will be wanting in the firſt Series; but none of the terms, in the ſecond. *Fig.* $\left\{\begin{matrix}13\\50\end{matrix}\right.$

*g* 2. $b^2 + d^2 \pm \frac{S}{L^2} = Q$. Rad. then $b^2 + d^2$ will vaniſh, (as before,) and the remaining Equation to ſubſiſt in the fourth degree, that is, will be a Biquadratic; where, in the firſt Series, the ſecond term, but in the ſecond Series, neither of the terms will happen to be wanting.

How $b^2 + d^2 \pm \frac{S}{L^2}$ may be made $= Q$. Rad. I will briefly ſhew.

Let there be deſcribed two divers pairs of Paraboles; in the one of which (as in *Fig.* 14, 15.) let AD, DH be like, and a like poſited, as in *Fig.* 13; but in the other pair (as in *Fig.* 51. 52.) let BA, AD, DH be like, and a like poſited, as in *Fig.* 50. Now,

1. If from the Vertex of the $\left\{\begin{matrix}\text{Axe}\\\text{Diameter}\end{matrix}\right\}$ be erected *Fig.* $\left\{\begin{matrix}14\\51\end{matrix}\right.$
*h* to AH a Perpendicular $AL = \sqrt{\frac{S}{L^2}}$; or, (which is all
one,) if in the line AH, both ways produced, be taken
*i* on the one ſide $AI = L$, and on the other ſide $AK =$
*h x i* *k* $\frac{S}{L^3}$, then will $AL^2 = \frac{S}{L^2}$.

*m* Et $\left\{\begin{array}{l} HA^2 + AL^2 = HL^2 \\ b^2 + d^2 + \frac{S}{L^2} = \text{Q. Rad.} \end{array}\right\}$ q. e. f. 1. *Fig.* $\left\{\begin{array}{l}14\\51\end{array}\right.$

2. Si Diametro A H, deſcribatur Semicirculus, in quo, vel inſcribatur $AZ = \sqrt{\frac{S}{L^2}}$; vel (quod idem eſt) ſummatur $AZ = AL$ (ſupra (k) inventæ,) orietur

*n* $AZ^2 = \frac{S}{L^2}$.

*o* Et $\left\{\begin{array}{l} AH^2 - AZ^2 = HZ^2 \\ b^2 + d^2 - \frac{S}{L^2} = \text{Q. Rad.} \end{array}\right\}$ q. e. f. 2. *Fig.* $\left\{\begin{array}{l}15\\52\end{array}\right.$

Series prima continuata.

$14 = \left\{\begin{array}{l} m \\ o \end{array}\right.$ 15 $\frac{x^4}{L^2} * \begin{array}{l} -2Lbx^2 \\ +\frac{L^2x^2}{L^2} \end{array} - \frac{2L^2dx}{L^2} = \pm\frac{S}{L^2}$; in $L^2$, &

*Tranſp.* 16 $x^4 * \begin{array}{l} -2Lbx^2 \\ +L^2. \end{array} - 2L^2dx\ (\mp S) = 0.$

$14 = 5$ 17 $\frac{x^4}{L^2} * \begin{array}{l} -2Lbx^2 \\ +\frac{L^2x^2}{L^2} \end{array} - \frac{2L^2dx}{L^2} = 0$; in $\frac{L^2}{x}$. $14 = f$

$\times\frac{L^2}{x}$ 18 $x^3 * \begin{array}{l} -2Lbx \\ +L^2x. \end{array} - 2L^2d = 0.$

Series ſecunda continuata.

$14 = \left\{\begin{array}{l} m \\ o \end{array}\right.$ 15 $\left.\frac{x^4}{L^2} = \frac{2ax^3}{L^2} \begin{array}{l} +a^2x^2 \\ -2Lb \\ +\frac{L^2}{L^2} \end{array} \begin{array}{l} +2Labx \\ -\frac{2L^2d}{L^2} \end{array}\right\} = \pm\frac{S}{L^2}$; in $L^2$, &

*Tranſp.* 16 $\left. x^4 - 2ax^3 \begin{array}{l} +a^2x^2 \\ -2Lb \\ +L^2 \end{array} \begin{array}{l} +2Labx\ (\mp S) \\ -2L^2d \end{array}\right\} = 0.$

17

47, e 1 5+k *m* And $\left\{\begin{array}{l} HA^2+AL^2=HL^2 \\ b^2+d^2+\frac{S}{L^2}=Q.Rad. \end{array}\right\}$ w. w. d. *Fig.* $\left\{\begin{array}{l}14\\51\end{array}\right.$

2. If on the Diameter A H be a Semicircle deſcrib'd, and in it, either inſcribed $AZ=\sqrt{\frac{S}{L^2}}$, or (which is the ſame thing,) made $AZ=AL$ above (§ k) found, then will $AZ^2=\frac{S}{L^2}$. *n*

*o* And $\left\{\begin{array}{l} AH^2-AZ^2=HZ^2 \\ b^2+d^2-\frac{S}{L^2}=Q\,Rad. \end{array}\right\}$ w. w. d. *Fig.* $\left\{\begin{array}{l}15\\52\end{array}\right.$

The firſt Series continued.

$14=\left\{\begin{array}{l}m\\o\end{array}\right.$ 15 $\frac{x^4}{L^2} * \begin{array}{l}-2Lbx^2\\ \frac{+L^2x^2}{L^2}\end{array} + \frac{2L^2dx}{L^2} = (\pm\frac{S}{L^2};)$ in $L^2$, and

*Tranſp.* 16 $x^4 * \begin{array}{l}-2Lbx^2\\+L^2x^2.\end{array} + 2L^2dx\ (\mp S) = 0.$

14 = 5 17 $\frac{x^4}{L^2} * \begin{array}{l}-2Lbx^2\\ \frac{+L^2x^2}{L^2}\end{array} + \frac{2L^2dx}{L^2} = 0$; in $\frac{L^2}{x}$. 14 = *f*

$\times\frac{L^2}{x}$ 18 $x^3 * \begin{array}{l}-2Lbx\\+L^2x.\end{array} + 2L^2d = 0.$

The ſecond Series continued.

$14=\left\{\begin{array}{l}m\\o\end{array}\right.$ 15 $\left.\frac{x^4}{L^2} + \frac{2ax^3}{L^3} \begin{array}{l}+a^2x^2\\-2Lb\\ \frac{+L^2}{L^2}\end{array} \begin{array}{l}-2Labx\\ \frac{+2L^2d}{L^2}\end{array}\right\} = \pm\frac{S}{L^2}$; in $L^2$, and

*Tranſp.* 16 $x^4 + 2ax^3 \begin{array}{l}+a^2x^2\\-2Lb\\+L^2\end{array} \begin{array}{l}-2Labx\\+2L^2d\end{array} (\mp S) = 0.$

$14 = 5$ 17 $$\left.\begin{array}{l} \frac{x^4}{L^2} - \frac{2ax^3}{L^2} + a^2x^2 + 2Labx \\ \qquad\qquad -2Lb - \frac{2L^2d}{L^2} \\ \qquad\qquad +\frac{L^2}{L^2} \end{array}\right\} = 0;\ \text{in } \frac{L^2}{x}.$$ $14 = f$

$x.\frac{L^2}{x.}$ 18 $$\left.\begin{array}{l} x^3 - 2ax^2 + a^2x + 2Lab \\ \qquad\qquad -2Lb - 2L^2d \\ \qquad\qquad +L^2. \end{array}\right\} = 0.$$

## *In Æquationibus Conſtruendis.*

*Obſerv.* 1 1. Cubicis, ubi terminus ſecundus $\left\{\begin{array}{l}\text{deerit}\\\text{aderit}\end{array}\right\}$ Circulum quidem per Verticem $\left\{\begin{array}{l}\text{Axis.}\\\text{Diametri.}\end{array}\right\}$ *Fig.* $\left\{\begin{array}{l}13\\50\end{array}\right.$ *Fig.* $\left\{\begin{array}{l}13\\50\end{array}\right.$

2. Biquadraticis verò per extremum quidem L, rectæ $AL = \sqrt{\frac{S}{L^2}}$ (à vertice $\left\{\begin{array}{l}\text{Axis}\\\text{Diametri}\end{array}\right\}$ ad H A Perpendiculariter erectæ, ſi in Æquatione habeatur — S. *Fig.* $\left\{\begin{array}{l}14\\51\end{array}\right.$

Per extremum verò Z, rectæ $AZ = \frac{S}{L^2}$, in Semicirculo (cujus diameter ſit H A,) à vertice $\left\{\begin{array}{l}\text{Axis}\\\text{Diametri}\end{array}\right\}$ inſcriptæ, ſi ibi habeatur + S oportere tranſire, Quæ omnia ab Æquationibus ( 16, 18) patent. *Fig.* $\left\{\begin{array}{l}15\\52\end{array}\right.$

*Obſerv.* 2 Si Æquationis cujuſlibet propoſitæ ſigna, quæ quidem in ſecundo & quarto termino reperiantur, mutentur, (quæ in tertio verò invariatis,) diverſa quidem à propoſitâ evadet Æquatio, eaſdem cum ipſa habens radices; quarum quæ in unâ Æquatione ſunt veræ, in alterâ evadent falſæ, & contra; & conſequentèr utrique conſtruendæ ipſa eadem Regula (quæcunque fuerit) inſervire neceſſe eſt.

Patet

14=5 | 17 | $$\frac{x^4}{L^2}+\frac{2ax^3}{L^2}\begin{array}{l}+a^2x^2-2Labx\\ -2Lb+\frac{2L^2d}{L^2}\\ +\frac{L^2}{L^2}\end{array}\left.\right\}=o;\ \text{in}\ \frac{L^2}{x}.$$ 14 =

$\times\frac{L^2}{x}$ | 18 | $$x^3+2ax^2\begin{array}{l}+a^2x-2Lab\\ -2Lb+2L^2d\\ +L^2.\end{array}\left.\right\}=o.$$

*In the Conſtructions of Equations.*

*Obſerv.*1 1. Cubic, where the ſecond term is $\left\{\begin{array}{l}\text{wanting}\\ \text{not wanting}\end{array}\right\}$ the Circle, muſt paſs through the Vertex of the $\left.\begin{array}{l}\text{Axe.}\\ \text{Diameter.}\end{array}\right\}$ *Fig.* $\left\{\begin{array}{l}13\\ 50\end{array}\right.$ *Fig.* $\left\{\begin{array}{l}12\\ ,\end{array}\right.$

2. But in Biquadraticks, through the Point L, of the Right-line $AL=\sqrt{\frac{S}{L^2}}$ (from the Vertex of the $\left\{\begin{array}{l}\text{Axe}\\ \text{Dia-}\\ \text{meter}\end{array}\right\}$ to H A Perpendicularly erected,) if in the Equation be had — S. *Fig.* $\left\{\begin{array}{l}14\\ 15\end{array}\right.$

But through the Point Z, of the Right-line $AZ=\sqrt{\frac{S}{L^2}}$, inſcribed in a Semicircle (whoſe Diameter is H A,) from the Vertex of the $\left\{\begin{array}{l}\text{Axe}\\ \text{Diameter}\end{array}\right\}$ if be had + S. *Fig.* $\left\{\begin{array}{l}15\\ 52\end{array}\right.$ All which is evident, from Equations 16, 18, *&c.*

*Obſerv.*2 If of any Equation propoſed the ſigns be changed, which are found in the ſecond and fourth terms, (thoſe in the third remaining unchanged,) another Equation diſtinct from that propoſed will appear, having the ſame Roots with it; of which, thoſe which are true in the one Equation, will be falſe in the other; and conſequently the ſame Rule (whatſoever it be,) will ſerve for both their Conſtructions.

Patet ex comparatione Æquationum 16, 18, in primâ Serie, cum 16, 18, in ſecundâ: A quantitate enim S, fieri non poteſt, ut immutetur (quam non ingreditur) Regula; quemadmodum fuſiùs infrà patebit.

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*De inveſtigandâ Regulâ unicuique Æquationum formulæ Conſtruendæ inſerviente.*

Si utriuſque Seriei Æquationem 16, 18, (ſuprâ inventam,) cum aliâ ſimplici aſſumptâ ſimili comparemus, & unumquemque terminum illius, correſpondenti termino hujus adæquari fingamus; Regula Centralis ſub involucris Co-efficientium latitans, è latebris eruere cogetur.

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*Primo*, Æquationem 16 vel 18 (ubi ſecundus terminus deficere contingit,) ſuprà inventam, cum aliâ ſimili & æquali aſſumptâ (quâ ſimilitèr ſecundus terminus deficit) conferamus; nempe.

$$\begin{array}{r|} 16 \\ \\ 18 \\ \\ \end{array}\;\left\{\begin{array}{l} x^4 * -2Lbx^2-2L^2dx\,(\mp S)=0 \\ \qquad +L^2 \\ x^3 * -2Lbx-2L^2d=0. \\ \qquad +L^2 \end{array}\right\}\text{inventam.}$$

Comparandam cum,

$$\begin{array}{r|} 19 \\ 20 \end{array}\;\left\{\begin{array}{l} x^4 * \mp qx^2-qx\,(\mp S)=0 \\ x^3 * \mp qx-r=0. \end{array}\right\}\text{aſſumptâ.}$$

Primò fingamus,

$$\begin{array}{r} 20 \\ 19 \end{array}\quad\begin{array}{r|} 21 \\ \\ \end{array}\;\left\{\begin{array}{l} x^3 * -qx-r=0. \\ x^4 * -qx^2-rx\,(\mp S)=0 \end{array}\right\}\text{aſſumptam.}$$

Ex

This appears, by comparing the 16 and 18 Equations in the firſt Series, with the 16 and 18 in the ſecond: For it is impoſſible that the quantity S, ſhould make any change in that Rule, of which it is no Part; as hereafter ſhall more largely appear.

### *Of finding out a Rule, ſerving for the Conſtruction of each ſeveral form of Equations.*

If we compare the 16 or 18 Equations of both Series's, with another aſſumed ſimple and alike; and ſuppoſe each term of that, to be equal to its Correſpondent term of this; the Central Rule, which lies hid under the Covert of the Co-efficients, will eaſily be detected.

*Firſt*, Let us compare the 16 or 18 Equation before found, (where the ſecond term happens to be wanting) with another like and equal aſſumed, in which likewiſe the ſecond term is wanting: *viz.*

$$\begin{array}{l|l} \begin{array}{l}16\\ \\18\\ \\ \end{array} & \left.\left\{\begin{array}{l} x^4 * \begin{array}{l}-2Lb\\+L^2\end{array} x^2 + 2L^2dx\,(\mp S) = 0 \\ x^3 * \begin{array}{l}-2Lb\\+L^2.\end{array} x + 2L^2d = 0 \end{array}\right.\right\} \text{ fonnd.} \end{array}$$

To be compared with,

$$\begin{array}{l|l} \begin{array}{l}19.\\20\end{array} & \left.\left\{\begin{array}{l} x^4 * \mp qx^2 + rx\,(\mp S) = 0 \\ x^3 * \mp qx + r = 0. \end{array}\right.\right\} \text{ aſſumed.} \end{array}$$

Firſt, ſuppoſe we,

$$\begin{array}{l|l} 21 & \left.\left\{\begin{array}{l} x^3 * - qx + r = 0. \\ x^4 * - qx^2 + rx\,(\mp S) = 0 \end{array}\right.\right\} \text{ aſſumed.} \end{array}$$

It

Ex hypotheſi liquet.

$18 = 21$ 22 $-2Lb + L^2 = -q$, ejus correſpondenti.

*Tranſp.* 23 $L^2 + q = 2Lb$.

$\frac{23}{2L}$ 24 $\frac{L}{2} + \frac{q}{2L} = b = AD$, in Axem.

$18 = 21$ 25 Pari jure, $-2L^2d = -r$ ejus correſpond.

$\frac{25}{2L^2}$ 26 g° $\frac{r}{2L^2} = d = DH$, ad Axem $\perp$.

*Conſectar.* I.

Si proponeretur Æquatio conſtruenda,

21 27 $$\left\{\begin{array}{l} x^3 * - qx \mp r = 0. \\ x^4 * - qx^2 \mp rx \ (\mp S) = 0. \end{array}\right\}$$ Fig. $\left\{\begin{array}{l} 13 \\ 14 \\ 15 \end{array}\right.$

24, 26 $$\text{Orietur} \left\{\begin{array}{l} \frac{L}{2} + \frac{q}{2L} = b = AD \\ \frac{r}{2L^2} = d = DH \end{array}\right\} \text{Central.}$$

Secundo, fingamus,

20, 19 28 $$\left\{\begin{array}{l} x^3 * + qx - r = 0. \\ x^4 * + qx^2 - rx \ (\mp S) = 0 \end{array}\right\} \text{aſſumptam.}$$

Ex hypotheſi pate,

$18 = 28$ 29 $-2Lb + L^2 = +q$, ejus correſpondenti.

*Tranſp.* 30 $L^2 - q = 2Lb$.

$\frac{30}{2L}$ 31 $\frac{L}{2} - \frac{q}{2L} = b = AD$, in Axem.

$18 = 28$ 32 Æquo jure, $-2L^2d = -r$, ejus correſp.

$\frac{32}{2L^2}$ 33 g° $\frac{r}{2L^2} = d = DH$, $\perp$ ad Axem.

*Con-*

It is evident by our ſuppoſition.

18 = 20 | 22 | $-2Lb+L^2=-q$, its correſpondent.

*Tranſp.* | 23 | $L^2+q=2Lb$.

24 | $\frac{L}{2}+\frac{q}{2L}=b=AD$, upon the Axe.

18 = 20 | 25 | By parity of reaſon, $+2L^2d=+r$.

$\frac{25}{2L^2}$ | 26 | g°, $\frac{r}{2L^2}=d=DH$, ⊥ to the Axe.

*Conſectar.* 1.

If the Equation propoſed ſhould be,

21 | 27 | $\left\{\begin{array}{l} x^3 * -qx\pm r=0. \\ x^4 * -qx^2\pm rx\ (\mp S)=0. \end{array}\right\}$ Fig. $\left\{\begin{array}{l}13\\14\\15\end{array}\right.$

24, 26 | Then will $\left\{\begin{array}{l} \frac{L}{2}+\frac{q}{2L}=b=AD \\ \frac{r}{2L^2}=d=DH \end{array}\right\}$ Central.

Secondly, ſuppoſe we,

20, 19 | 28 | $\left\{\begin{array}{l} x^3 * +qx+r=0. \\ x^4 * +qx^2+rx\ (\mp S)=0 \end{array}\right\}$ aſſumed.

It is manifeſt by our hypotheſis,

18 = 28 | 29 | $-2Lb+L^2=+q$, its correſpondent.

*Tranſp.* | 30 | $L^2-q=2Lb$.

$\frac{30}{2L}$ | 31 | $\frac{L}{2}-\frac{q}{2L}=b=AD$, upon the Axe.

18 = 28 | 32 | By the ſame reaſon, $+2L^2d=+r=$ its correſp.

$\frac{32}{2L^2}$ | 33 | g°, $\frac{r}{2L^2}=d=DH$, ⊥ to the Axe.

*Conſectàr.* 2.

Si proponeretur Æquatio conſtruenda,

28 34 $$\left\{\begin{array}{l} x^3 * + qx \mp r = 0. \\ x^4 * + qx^2 \mp rx\,(\mp S) = 0 \end{array}\right\}$$ *Fig.* 16. *Fig.* $\left\{\begin{array}{l}17\\18\end{array}\right.$

31 33 $$\text{Orietur}\left\{\begin{array}{l} \frac{L}{2} \backsim \frac{q}{2L} = b = AD \\ \frac{r}{2L^2} = d = DH \end{array}\right\}\text{Central.}$$

Quæ duo Conſectaria quartum Claſſem complectuntur.

---

*Secundò*, Fingamus in Æquatione 16 vel 18 inventâ, & in 19, 20 aſſumptâ, tertium terminum etiam deficere; nempè,

16 35, 18 36 $$\left\{\begin{array}{l} x^4 * \left(\begin{array}{l}-2Lbx^2\\+L^2\end{array}\right) - 2L^2dx\,(\mp S) = 0 \\ x^3 * (=qx) - 2L^2d = 0. \end{array}\right\}\text{inventam.}$$

Comparandum cum,

19 37, 20 38 $$\left\{\begin{array}{l} x^4 * * \quad - rx\,(\mp S) = 0 \\ x^3 * * \quad - r = 0. \end{array}\right\}\text{aſſumptâ.}$$

Quandoquidem ex hypotheſi ſupponimus $q = 0$, g°,

35 = 37 39 $-2Lb + L^2 = 0$, ejus correſp. ($L^2 = 2Lb$, vel)

*Tranſp.* 40 $L = 2b$.

$\frac{40}{2}$ 41 $\frac{L}{2} = b = AD$, in Axem.

35 = 37 42 Eſt autem $\frac{r}{2L^2} = d = DH$, ⊥ ad Axem; (vide § 25, 26; vel § 32, 33.)

*Con-*

### *Conſectar.* 2.

If an Equation propoſed, were,

28 34 $\left\{\begin{array}{l} x^3 * + qx \pm r = 0. \\ x^4 * + qx^2 \pm rx\,(\mp S) = 0 \end{array}\right\}$ *Fig.* 16. *Fig.* $\left\{\begin{array}{l} 17 \\ 18 \end{array}\right.$

31 Then will $\left\{\begin{array}{l} \frac{L}{2} \backsim \frac{q}{2L} = b = AD \\ \frac{r}{2L^2} = d = DH \end{array}\right\}$ Central.

33

Which two Conſectaries comprehend the fourth Claſs.

*Secondly*, Suppoſe in the 19 and 20 Equations aſſumed, the third term (or q) to be alſo wanting, *viz.*

16 35 $\left\{\begin{array}{l} x^4 * \left(\begin{array}{l} -2Lbx^2 \\ +L^2 \end{array}\right) + 2L^2dx\,(\pm S) = 0 \\ x^3 * \left(\begin{array}{l} -2Lbx \\ +L^2 \end{array}\right) + 2L^2d = 0. \end{array}\right\}$ found,

18 36

To be compared with,

19 37 $\left\{\begin{array}{l} x^4 * * \quad + rx\,(\mp S) = 0. \\ x^3 * * \quad + r = 0. \end{array}\right\}$ aſſumed.

20 38

Foraſmuch as is ſuppoſed $q = 0$; g°,

35=37 39 $-2Lb + L^2 = 0$, its correſpondent,

*Tranſp.* 40 $(L^2 = 2Lb$, or$)\,L = 2b$.

$\frac{40}{2}$ 41 $\frac{L}{2} = b = AD$, on the Axe.

And $\frac{r}{2L^2} = d = DH$ (as before § 25, 26; and § 32, 33.)

### *Conſectar.* 3.

Si proponeretur Æquatio conſtruenda,

$$\begin{array}{rr} 38 & \\ & 43 \\ 37 & \end{array}\ \left\{\begin{array}{l} x^3 * * \mp r = 0. \\ x^4 * * \mp r x\ (\mp S) = 0. \end{array}\right\}$$

*Fig.* 10.
*Fig.* $\left\{\begin{array}{l} 11 \\ 12 \end{array}\right.$

$$\begin{array}{r} 41 \\ \\ 42 \end{array}\quad \text{Orietur}\left\{\begin{array}{l} \frac{L}{2} = b = AD \\ \frac{r}{2L^2} = d = DH \end{array}\right\}\text{Central.}$$

Quod tertiam Æquationum Claſſem complectitur.

*Tertiò*, Fingamus, in Æquatione 16, vel 18 inventâ, & in 19, 20 aſſumptâ, quartum terminum (præter ſecundum) deficere; nempè,

$$\begin{array}{rr} 16 & 44 \\ & \\ 18 & 45 \end{array}\ \left\{\begin{array}{l} x^4 * \begin{array}{l}-2Lb\\+L^2\end{array} x^2 (-2L^2 dx)\ (\mp S) = 0 \\ x^3 * \begin{array}{l}-2Lb\\+L^2\end{array} x (-2L^2 dx) = 0,\ \text{hoc eſt} \\ x^2 * \begin{array}{l}-2Lb\\+L^2\end{array} x \begin{array}{l}-2Lb\\+L^2\end{array} = 0. \end{array}\right\}\text{inventum,}$$

Comparandum cum,

$$\begin{array}{rr} 19 & 46 \\ 20 & 47 \end{array}\ \left\{\begin{array}{l} x^4 * \mp q x^2 * (\mp S) = 0. \\ x^2 * \mp q = 0. \end{array}\right\}\text{aſſumptâ.}$$

Primò fingamus,

$$\begin{array}{rr} 47 & \\ & 48 \\ 46 & \end{array}\ \left\{\begin{array}{l} x^2 * - q = 0. \\ x^4 * - q x^2 * (\mp S) = 0. \end{array}\right\}\text{aſſumptam.}$$

Ex

### *Conſectar.* 3.

If the Equation to be made, were,

Fig. 10.

Fig. $\left\{\begin{array}{l}11\\12\end{array}\right.$

$$\begin{array}{lll} 38 & & \left\{\begin{array}{l} x^3 * * \pm r = 0. \\ x^4 * * \pm rx\ (\mp S) = 0 \end{array}\right\} \\ 37 & 43 & \end{array}$$

$$\begin{array}{ll} 41 & \\ 42 & \end{array} \text{Then will} \left\{\begin{array}{l} \frac{L}{2} = b = AD \\ \frac{r}{2L^2} = d = DH \end{array}\right\} \text{Central.}$$

Which Comprehends all the Equations of the third Claſs.

---

*Thirdly*, Suppoſe (in the 16 or 18 Equation found, and in the 19, 20 aſſumed) the fourth term (beſide the ſecond) to be wanting; *viz.*

$$\begin{array}{lll} 16 & 44 & \left\{\begin{array}{l} x^4 * \begin{array}{l} -2Lb \\ +L^2 \end{array} x^2 (+2L^2 dx)\ (\mp S) = 0 \\ x^2 * \left.\begin{array}{l} -2Lb \\ +L^2 \end{array}\right\} = 0. \end{array}\right\} \text{found,} \\ 18 & 45 & \end{array}$$

To be compared with,

$$\begin{array}{lll} 19 & 46 & \left\{\begin{array}{l} x^4 * \mp qx^2 * (\mp S) = 0 \\ x^2 * \pm q = 0. \end{array}\right\} \text{aſſumed.} \\ 20 & 47 & \end{array}$$

First ſuppoſe,

$$\begin{array}{lll} 47 & & \left\{\begin{array}{l} x^2 * - q = 0. \\ x^4 * - qx^2 * (\mp S) = 0. \end{array}\right\} \text{aſſumed.} \\ 46 & 48 & \end{array}$$

It

Ex hypotheſi patet,

$44 = 48$ 49 $-2Lb + L^2 = -q$, ejus correſpond.

*Tranſp.* 50 $L^2 + q = 2Lb$.

$\frac{50}{2L}$ 51 $g^{\circ}$, $\frac{L}{2} + \frac{q}{2L} = b = AD$, in Axem.

Ex hypotheſi etiam liquet,

$44 = 48$ 52 $0 = q = 2L^2 d$, ejus correſpond.

52 53 $g^{\circ}$, $d = DH = 0$; adeòque, Punctum D & H in Axem co-incidere.

*Conſectar.* 4.

Si proponeretur Æquatio conſtruenda,

47, 46 54 $$\left\{\begin{array}{l} x^2 * - q = 0. \\ x^4 * - q x^2 * (\mp S) = 0. \end{array}\right\}$$ *Fig.* 5. *Fig.* {6. 7.

51, 53 $$\text{Orietur}\left\{\begin{array}{r} \frac{L}{2} + \frac{q}{2L} = b = AD \\ 0 = d = DH \end{array}\right\}\text{Central.}$$

Secondò finge,

47, 46 55 $$\left\{\begin{array}{l} x^2 * + q = 0. \\ x^4 * + q x^2 * (-S) = 0. \end{array}\right\}\text{aſſumptam.}$$

Ex hypotheſi patet,

$44 = 55$ 56 $-2Lb + L^2 = +q$.

*Tranſp.* 57 $L^2 - q = 2Lb$.

$\frac{57}{2L}$ 58 $\frac{L}{2} - \frac{q}{2L} = b = AD$, in Axem.

59 Eſt autem $d = DH = 0$, *&c.* (ut ſuprà § 53.)

*Con-*

It is evident by our ſuppoſition,

44=48 49 $-2Lb+L^2=-q$.
*Tranſp.* 50 $L^2+q=2Lb$.
$\frac{50}{2L}$ 51 g°, $\frac{L}{2}+\frac{q}{2L}=b=AD$, on the Axe.

It appears likewiſe by the ſuppoſition,

44=48 52 $o=q=2L^2d$, its correſpondent,
52 53 g°, $d=DH=o$, and g°, the Point D and H, to be co-incident on the Axe.

*Conſectar.* 4.

If the Equation to be made, were,

47, 46 54 $$\left\{\begin{array}{l} x^2 * -q=o \\ x^4 * -qx^2 * (\mp S)=o \end{array}\right\}$$ *Fig.* 5. *Fig.* {6. 7.

51, 53 Then will $$\left\{\begin{array}{r} \frac{L}{2}+\frac{q}{2L}=b=AD \\ o=d=DH \end{array}\right\}$$ Central.

Secondly, ſuppoſe,

47, 46 55 $$\left\{\begin{array}{l} x^2 * +q=o. \\ x^4 * +qx^2 * (-S)=o \end{array}\right\}$$ aſſumed.

By ſuppoſition it appears,

44=55 56 $-2Lb+L^2=+q$.
*Tranſp.* 57 $L^2-q=2Lb$.
$\frac{57}{2L}$ 58 g°, $\frac{L}{2}-\frac{q}{2L}=b=AD$, on the Axe.
59 Now $d=DH=o$, *&c.* (as above § 53.)

*Conſectar.* 5.

Si proponeretur Æquatio conſtruenda,

47 46 | 60 $$*\left\{\begin{array}{l} x^2 * + q = 0. \\ x^4 * + q x^2 * (-S) = 0. \end{array}\right\}$$ *Fig.* 8.

58 59 $$\text{Orietur}\left\{\begin{array}{r} \frac{L}{2} \backsim \frac{q}{2L} = b = AD \\ 0 = d = DH \end{array}\right\}\text{Central.}$$

Quæ quidem duo ultima Conſectaria omnes Æquationum ſecundi Claſſis formulas complectuntur.

---

*Quartò*, Fingamus in Æquatione aſſumptâ § 19, (cui Æquatio § 16 adæquari ſupponitur) quantitates, tum q & r (præter p) deficere, nempè

61 $x^4 * * * - S = 0$ aſſumpta.

16 = 61 | 62 Ex hypotheſi $q = 0$; g°, $-2Lb + L^2 = 0$, ejus correſpondenti; &

*Tranſp.* | 63 ($L^2 = 2Lb$, vel) $L = 2b$.

$\frac{63}{2}$ | 64 g°, $\frac{L}{2} = b = AD$, in Axem.

65 Eſt autem $d = DH = 0$, &c. (ut ſuprà, § 53.)

*Conſectar.* 6.

61 | 66 Si proponeretur Æquatio conſtruenda, *Fig.* 4.
$x^4 * * * - S = 0$.

64 65 $$\text{Orietur}\left\{\begin{array}{r} \frac{L}{2} = b = AD \\ 0 = d = DH \end{array}\right\}\text{Central.}$$

Conſectaria 5 & 6, primam Æquationum Claſſem concludunt.

Hactenùs de Regulâ Centrali inveſtigandâ, ubi ſecundus terminus continuò deficit.

*Secundò*,

### *Conſectar.* 5.

If the Equation propoſed to be made,

47 60 be $\left\{ \begin{array}{l} x^{2} * + q = 0. \\ x^{4} * + q x^{2} * (-S) = 0. \end{array} \right\}$ Fig. 8.
46

58 Then will $\left\{ \begin{array}{l} \frac{L}{2} \backsim \frac{q}{2L} = b = AD \\ \quad 0 = d = DH \end{array} \right\}$ Central.
59

Which two laſt Conſectaries comprehend all the forms of Equations in the ſecond Claſs.

---

*Fourthly*, Suppoſe in the aſſumed Equation § 19, (to which the 16th. is ſuppoſed to be equal,) the quantities both q and r (beſides p) to be wanting, *viz.*
61 $x^{4} * * * - S = 0$ aſſumed.

16=61 62 By ſuppoſition $q = 0$; g°, $-2Lb + L^{2} = 0$, its correſpondent; and
Transp. 63 $(L^{2} = 2Lb$, or) $L = 2b$.
$\frac{63}{2}$ 64 g°, $\frac{L}{2} = b = AD$, on the Axe.
65 Now $d = DH = 0$, &c. (as above § 53.)

### *Conſectar.* 6.

61 66 If the Equation propoſed to be made be, $x^{4} * * * - S = 0$. Fig. 4.

64 Then will $\left\{ \begin{array}{l} \frac{L}{2} = b = AD \\ 0 = d = DH \end{array} \right\}$ Central.
65

The 5 and 6 Conſectaries conclude the firſt Claſſis.

Hitherto of finding out the Central Rule, for the Conſtruction of all Equations, where the ſecond term is wanting.

*Secundò*, Æquationem ſuprà inventam § 16, 18, in ſecundâ Serie, (ubi ſecundus terminus non deficere contingit) cum aliâ ſimplici & Æquali aſſumptâ, (quâ ſimiliter ſecundus terminus aderit) inter ſe conſeramus: nempè,

16 | 67
18 | 68

$$\left.\begin{array}{ll} 67 & \left\{\begin{array}{l} x^4 - 2ax^3 + a^2x^2 + 2Labx\,(\mp S) = 0. \\ \qquad\qquad\quad -2Lb - 2L^2d \\ \qquad\qquad\quad +L^2 \end{array}\right. \\ 68 & \left\{\begin{array}{l} x^3 - 2ax^2 + a^2x + \left.\begin{array}{l} 2Lab \\ -2Lb - 2L^2d \end{array}\right\} = 0. \\ \qquad\qquad\quad +L^2 \end{array}\right. \end{array}\right\}\text{invent.}$$

Comparandam cum;

$$\left.\begin{array}{ll} 69 & x^4 - px^3 \pm qx^2 \pm rx\ (\mp S) = 0. \\ 70 & x^3 - px^2 \pm qx \pm r = 0. \end{array}\right\}\text{aſſumptâ.}$$

67 = 69 | 71 Manifeſtò apparet, $2a = p$, ejus correſp. ideòque

71 | 72 $\frac{p}{2} = a = BA$: ($\& \frac{p^2}{4} = a^2$.)

Hinc, ſi in Æquatione Conſtruendâ propoſitâ, quartum gradum non excedente, reperiatur quantitas p; *Obſerv.* 3 oportere ordinatim ad Axem continuò applicari rectam $BA = \frac{p}{2}$, occurrentem Parabolæ in B & A; à quorum alterutro Puncto concurſûs (ab A puta) agi rectam A y Axi Parallelam: vel (quod perinde eſt) à vertice Axis, erigi debere à dextrâ Parabolæ ad Axem Perpendicularem $E = \frac{p}{4}$; & ex E, (actâ E A ipſi Axi Parallelâ, donec occurrat Parabolæ in A) duci A B ipſi a E Parallelam: quo fiet, diſtantiam Diametri ab Axe reperiri continuò: quod quidem animadvertiſſe operæ forſan erit pretium.

Jam

*Secondly*, Compare we the Equation above found; § 16, or 18, in the ſecond Series, (where the ſecond term happens not to be wanting) with another like and equal aſſumed, in which likewiſe the ſecond term is had: *viz.*

16 67 / 18 68

$$\left.\begin{array}{l} \left\{\begin{array}{l} x^4+2ax^3+a^2x^2-2Labx\,(\mp S)=0. \\ \qquad\qquad\;\; -2Lb+2L^2d \\ \qquad\qquad\;\; +L^2 \end{array}\right. \\ \left\{\begin{array}{l} x^3+2ax^2+\left.\begin{array}{l} a^2x-2Lab \\ -2Lb+2L^2d \end{array}\right\}\mp 0. \\ \qquad\qquad\;\; +L^2 \end{array}\right. \end{array}\right\}\text{found}$$

To be Compared with,

69 / 70

$$\left.\left\{\begin{array}{l} x^4+px^3\pm qx^2\mp rx\,(\mp S)=0. \\ x^3+px^2\pm qx\mp rq=0. \end{array}\right.\right\}\text{aſſumed.}$$

Obſerv. 3 Hence, if in any Equation, whoſe Conſtruction is required, not exceeding the fourth degree, be found the quantity p, it follows, that there ought always be ordinately applied to the Axe, a Right-line (as) $BA=\frac{p}{2}$, meeting the Parabole in B and A; from either of which Points of meeting (as ſuppoſe from A) muſt be drawn A y Parallel to the Axe; or (which is the ſame) from the Vertex of the Axe, muſt (towards the Right-ſide of the Parabole) be erected the Perpendicular $aE=\frac{p}{4}$; and from E, (E A being drawn Parallel to the Axe, 'till it meets the Parabole in A) muſt be drawn A B, Parallel to the ſaid a E; by which means, the diſtance of the Diameter from the Axe will always be had; which to have taken notice of, may perhaps be worth the while.

Jam ſi in locum (a) (§ 67, 68.) ſubſtituatur ejus valor, nempè $\frac{p}{2}$; & in locum $a^2$, ejus valor, nempè $\frac{p^2}{4}$, orietur Æquatio nova Æquationi § 67, vel 68, inventæ ad æquata, nempè,

73
$$\left\{\begin{array}{l} x^4 - px^3 + \frac{p^2}{4}x^2 + Lpbx\,(\mp S) = 0. \\ \qquad\qquad\quad -2Lb - 2L^2d \\ \qquad\qquad\quad +L^2 \\ x^3 - px^2 + \frac{p^2}{4}x + Lpb = 0. \\ \qquad\qquad\quad -2Lb - 2L^2d \\ \qquad\qquad\quad +L^2 \end{array}\right\}\text{(inventa)}$$

Comparanda cum,

69, 70 | 74
$$\left\{\begin{array}{l} x^4 - px^3 \pm qx^2 \pm rx\,(\mp S) = 0. \\ x^3 - px^2 \pm qx \pm r = 0. \end{array}\right\}\text{aſſumptâ.}$$

Primò finge,

75
$$\left\{\begin{array}{l} x^4 - px^3 - qx^2 - rx\,(\mp S) = 0. \\ x^3 - px^2 - qx - r = 0. \end{array}\right\}\text{aſſumptam.}$$

73 = 75 | 76 Conſtat, $\frac{p^2}{4} - 2Lb + L^2 = -q$ ejus correſp.

*Tranſp.* | 77 $L^2 + \frac{p^2}{4} + q = 2Lb.$

$\frac{77}{2L}$ | 78 $\frac{L}{2} + \frac{p^2}{8L} + \frac{q^2}{2L} = b = AD$, in Diametrum.

73 = 75 | 79 Item, $+Lpb - 2L^2d = -r$, ejus correſpond. & ſi in locum b ſubſtituatur ejus valor (ſuprà inventus) ſc.

Now, if (in § 67, 68) in the place of (a) be ſubſtituted its valor, *viz.* $\frac{p}{2}$; and in the place $a^2$, its valor, *viz.* $\frac{p^2}{4}$, will ariſe a new Equation, equal to that found, § 67, or 68, *viz.*

73 $$\left\{\begin{array}{l} x^4 + px^3 + \frac{p^2}{4}x^2 - Lpbx\,(\mp S) = 0. \\ \qquad\qquad\;\; -2Lb + 2L^2d \\ \qquad\qquad\;\; +L^2 \\ x^3 + px^2 + \frac{p^2}{4}x - Lpb = 0. \\ \qquad\qquad\;\; -2Lb + 2L^2d \\ \qquad\qquad\;\; +L^2 \end{array}\right\} \text{found}$$

To be compared with,

69
70
74 $$\left\{\begin{array}{l} x^4 + px^3 \pm qx^2 \mp rx\,(\mp S) = 0. \\ x^3 + px^2 \pm qx \mp r = 0. \end{array}\right\} \text{aſſumed.}$$

First ſuppoſe,

75 $$\left\{\begin{array}{l} x^4 + px^3 - qx^2 + rx\,(\mp S) = 0. \\ x^3 + px^2 - qx + r = 0. \end{array}\right\}$$

73=75. 76 It is manifeſt, $+\frac{p^2}{4}\,2Lb + L^2 = -q$ its correſp.

*Tranſp.* 77 $L^2 + \frac{p^2}{4} + q = 2Lb.$

$\frac{77}{2L}$ 78 $\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD$, on the Diameter.

73=75 79 Again, $-Lpb + 2L^2d = +r$ its correſp.

*Tranſp.* 80 $Lpb + r = 2L^2d$; and if in the place of b, be ſubſtituted its valor, (above found) *viz.*

78

| | | |
|---|---|---|
| 78 | | $\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}$, fiet |
| 78, 79 | 80 | $\frac{L^2p}{2}+\frac{p^3}{8}+\frac{pq}{2}-2L^2d=-r.$ |
| *Tranſp.* | 81 | $\frac{L^2p}{2}+\frac{p^3}{8}+\frac{pq}{2}+r=2L^2d.$ |
| $\frac{81}{2L^2}$ | 82 | $\frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}+\frac{r}{2L^2}=d=DH$; $\perp$ ad Diam. |
| | | *Conſectar.* 1. |
| | | Si proponeretur Æquatio conſtruenda, |
| 75 | 83 | $\left\{\begin{array}{l} x^4 \mp px^3 - qx^2 \mp rx\,(\mp S)=0. \\ x^3 \mp px^2 - qx \mp r = 0. \end{array}\right\}$ *Fig.* $\left\{\begin{array}{l}42\\43\end{array}\right.$ *Fig.* 41. |
| 78<br>82 | | Orietur $\left\{\begin{array}{l} \frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}=b=AD \\ \frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}+\frac{r}{2L^2}=d=DH \end{array}\right\}$ Centr. |
| | | Secundò, Finge inventam § 73, comparandam |
| 74 | 84 | cum $\left\{\begin{array}{l} x^3 - px^2 - qx + r = 0 \\ x^4 - px^3 - qx^2 + rx\,(\mp S)=0. \end{array}\right\}$ aſſumptà. |
| 73 = 84 | 85 | Patet, $+\frac{p^2}{4}-2Lb+L^2=-q$ ejus correſpond. |
| *Tranſp.* | 86 | $L^2+\frac{p^2}{4}+q=2Lb.$ |
| $\frac{86}{2L}$ | 87 | $\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}=b=AD$; in Diametrum. |
| 73 = 84 | 88 | Æquo jure, $+Lpb-2L^2d=+r$, ejus correſp. |
| *Tranſp.* | 89 | $Lpb-r=2L^2d.$ |
| | | Si in locum b, ſubſtituatur ejus vālor (ſuprà inventus) ſc. |
| 87<br>89 | 90 | $\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}$; fiet, $\frac{L^2p}{2}+\frac{p^3}{8}+\frac{pq}{2}-r=2L^2d.$ |
| $\frac{90}{2L^2}$ | 91 | $\frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}-\frac{r}{2L^2}=d=DH$; $\perp$ ad Diametr. |

*Con-*

78 | | $\frac{L}{2}+\frac{p^2}{8L}+\frac{q^2}{2L}$: Then will be

78, 79 | 81 | $\frac{L^2p}{2}+\frac{p^3}{8}+\frac{pq}{2}+r=2L^2d$.

$\frac{81}{2L^2}$ | 82 | $\frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}+\frac{r}{2L^2}=d=DH$; ⊥ to the Diam.

*Conſectar.* 1.

If the Equation propoſed to be made,

75 | 83 | be $\left\{\begin{matrix} x^4 \pm px^3 - qx^2 \pm rx \left\{\begin{matrix}(\mp S)\\(\pm S)\end{matrix}\right\} = 0. \\ x^3 \mp px^2 - qx \pm r = 0. \end{matrix}\right\}$ Fig. $\left\{\begin{matrix}42\\43\end{matrix}\right.$ Fig. $\left\{\begin{matrix}42\\43\end{matrix}\right.$ Fig. 41.

78<br>82 | | Then will $\left\{\begin{matrix} \frac{L}{2}+\frac{p}{8L}+\frac{q}{2L}=b=AD \\ \frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}+\frac{r}{2L^2}=d=DH \end{matrix}\right\}$ Cent.

Secondly, ſuppoſe § 73, found to be compared

74 | 84 | with $\left\{\begin{matrix} x^3+px^2-qx-r=0. \\ x^4+px^3-qx^2-rx\ (\mp S)=0. \end{matrix}\right\}$ aſſumed.

73=84 | 85 | It is plain; $+\frac{p^2}{4}-2Lb+L^2=-q$, its correſp.

*Tranſp.* | 86 | $L^2+\frac{p^2}{4}+q=2Lb$.

$\frac{86}{2L}$ | 87 | $\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}=b=AD$; on the Diameter.

73=84 | 88 | Alſo, $-Lpb+2L^2d=-r$, its correſpond.

*Tranſp.* | 89 | $Lpb-r=2L^2d$.

If in the place of b, be ſubſtituted its valor (above found)

87<br>89 | 90 | *viz.* $\frac{L}{2}+\frac{p^2}{8L}+\frac{q}{2L}$; then will be, $\frac{L^2p}{2}+\frac{p^3}{8}+\frac{pq}{2}-r=2L^2d$.

$\frac{90}{2L^4}$ | 91 | $\frac{p}{4}+\frac{p^3}{16L^2}+\frac{pq}{4L^2}-\frac{r}{2L^2}=d=DH$; ⊥ to the Diam.

Con-

### *Conſectar.* 2.

Si proponeretur Æquatio conſtruenda, *Fig.* 44.

84 92 $\begin{cases} x^3 \mp px^2 - qx \pm r = 0. \\ x^4 \mp px^3 - qx^2 \pm rx \begin{Bmatrix} (\mp S) \\ (\mp S) \end{Bmatrix} = 0, \text{ fiet} \end{cases}$ *Fig.* $\begin{cases} 45 \\ 46 \end{cases}$ *Fig.* $\begin{cases} 45 \\ 46 \end{cases}$

87 / 91 $\left.\begin{cases} \frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} - \frac{r}{2L^2} = d = DH \end{cases}\right\}$ Central.

Tertiò, Finge inventam § 73, comparandam

74 93 cum $\left.\begin{cases} x^3 - px^2 + qx + r = 0. \\ x^4 - px^3 + qx^2 + rx\ (\mp S) = 0. \end{cases}\right\}$ aſſumptâ.

73 = 93 94 Liquet, $\frac{p^2}{4} - 2Lb + L^2 = + q$, ejus correſpond.

*Tranſp.* 95 $L^2 + \frac{p^2}{4} - q = 2Lb.$

$\frac{95}{2L}$ 96 $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = b = AD$; in Diametrum.

73 = 93 97 Item, $+ Lpb - 2L^2d = + r$ ejus correſpond.

*Tranſp.* 98 $Lpb - r = 2L^2d$; ſi in locum b, ſubſtituatur ejus valor (ſuprà inventus) nempè,

96 $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L}$; fiet,

98 99 $\frac{L^2p}{2} + \frac{p^3}{8} - \frac{pq}{2} - r = 2L^2d.$

$\frac{99}{2L^2}$ 100 $\frac{p^2}{4} + \frac{p^3}{15L^2} - \frac{pq}{4L^2} - \frac{r}{2L^2} = d = DH$; $\perp$ ad Diametr.

*Con-*

### *Conſectar.* 2.

If the Equation, whoſe Conſtruction is deſired, be

*Fig.* 44.

84 92 $\left\{\begin{array}{l} x^3 \pm px^2 - qx \mp r = 0. \\ x^4 \pm px^3 - qx^2 \mp rx (\mp S) = 0. \end{array}\right\}$ then will

*Fig.* $\left\{\begin{array}{l}45\\46\end{array}\right.$ *Fig.* $\left\{\begin{array}{l}45\\46\end{array}\right.$

87 91 $\left\{\begin{array}{l} \frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} \backsim \frac{r}{2L^2} = d = DH \end{array}\right\}$ Central.

Thirdly, Suppoſe § 73 found, to be compared

74 93 with $\left\{\begin{array}{l} x^3 + px^2 + qx - r = 0. \\ x^4 + px^3 + qx^2 - rx (\mp S) = 0. \end{array}\right\}$ aſſumed.

73=93 94 It is evident, $\frac{p^2}{4} - 2Lb + L^2 = +q$, its correſp.

*Tranſp.* 95 $L^2 + \frac{p^2}{4} - q = 2Lb.$

$\frac{95}{2L}$ 96 $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = b = AD$; on the Diameter.

73=93 97 Alſo, $-Lpb + 2L^2d = -r$ its correſpond.
*Tranſp.* 98 $Lpb - r = 2L^2d$; if in the place of b, be ſubſtituted its valor (before found) *viz.*
96 $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L}$; then will be,
98 99 $\frac{L^2p}{2} + \frac{p^3}{8} - \frac{pq}{2} - r = 2L^2d.$
$\frac{99}{2L^2}$ 100 $\frac{p^2}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} - \frac{r}{2L^2} = d = DH$; ⊥ to the Diam.

### *Conſectar.* 3.

Eſto ad Conſtruendum Æquatio propoſita,

93 101 $\begin{cases} x^3 \mp px^2 + qx \pm r = 0. \\ x^4 \mp px^3 + qx^2 \pm rx \begin{Bmatrix} (\mp S) \\ (\pm S) \end{Bmatrix} = 0; \text{ fiet} \end{cases}$ *Fig.* 50. *Fig.* $\begin{Bmatrix} 51 \\ 52 \end{Bmatrix}$

96 $\frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD$ *Fig.* $\begin{Bmatrix} 51 \\ 52 \end{Bmatrix}$

100 $\frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} \backsim \frac{r}{2L^2} = d = DH$ Central.

Quartò; Finge inventam § 73, comparari

74 102 cum $\begin{cases} x^3 - px^2 + qx - r = 0 \\ x^4 - px^3 + qx^2 - rx(\mp S) = 0. \end{cases}$ aſſumptâ.

73=102 103 Patet, $\frac{p^2}{4} - 2Lb + L^2 = +q$ ejus correſp.

*Tranſp.* 104 $L^2 + \frac{p^2}{4} - q = 2Lb.$

$\frac{104}{2L}$ 105 $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = b = AD$; in Diametrum.

73=102 106 Rurſùs, $+Lpb - 2L^2d = -r$, ejus correſp.

*Tranſp.* 107 $Lpb + r = 2L^2d$. Et ſi in locum b, ſubſtituatur ejus valor (ſuprà inventus) nempè

165 $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L}$; fiet,

107 108 $\frac{L^2p}{2} + \frac{p^3}{8} - \frac{pq}{2} + r = 2L^2d.$

$\frac{108}{2L^2}$ 109 $\frac{p}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} + \frac{r}{2L^2} = d = DH$; ⊥ ad Diam.

*Con-*

### *Conſectar.* 3.

Let the Equation propoſed to be made,

93 | 101 | be $\left\{\begin{matrix} x^3 \pm px^2 + qx \mp r = 0. \\ x^4 \pm px^3 + qx^2 \mp rx \left\{\begin{matrix}(\mp S)\\(\mp S)\end{matrix}\right\} = 0;\end{matrix}\right.$ then | *Fig* 50; *Fig.* $\left\{\begin{matrix}51\\52\end{matrix}\right.$ *Fig.* $\left\{\begin{matrix}51\\52\end{matrix}\right.$

96, 100 | will $\left\{\begin{matrix} \frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD \\ \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} \backsim \frac{r}{2L^2} = d = DH \end{matrix}\right\}$ Central.

Fourthly, ſuppoſe § 73 found, to be compared

74 | 102 | with $\left\{\begin{matrix} x^3 + px^2 + qx + r = 0. \\ x^4 + px^3 + qx^2 + rx\ (\mp S) = 0. \end{matrix}\right\}$ aſſumed.

73=102 | 103 | It is manifeſt; $\frac{p^2}{4} - 2Lb + L^2 = +q$, its correſp.

*Tranſp.* | 104 | $L^2 + \frac{p^2}{4} - q = 2Lb.$

$\frac{104}{2L}$ | 105 | $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = b = AD$; on the Diameter.

73=102 | 106 | Again, $-Lpb + 2L^2d = +r$, its correſpond.

*Tranſp.* | 107 | $Lpb + r = 2L^2d$. If in the place of b, be ſubſtituted its valor, (before found) *viz.*

105 | $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L}$; then will be,

107 | 108 | $\frac{L^2p}{2} + \frac{p^3}{8} - \frac{pq}{2} + r = 2L^2d.$

$\frac{p}{4} + \frac{p^3}{16L^2} - \frac{pq}{4L^2} + \frac{r}{2L^2} = d = DH$; ⊥ to the Diam.

### *Conſectar.* 4.

Si conſtruenda proponeretur Æquatio, Fig. 59.

102 | 110 $$\left\{\begin{array}{l} x^3 \mp p x^2 + q x \mp r = 0. \\ x^4 \mp p x^3 + q x^2 \mp r x \left\{\begin{array}{l} (\mp S) \\ (\mp S) \end{array}\right\} \begin{array}{l} = 0 \\ = 0 \end{array} \end{array}\right\} \text{fiet,}$$ Fig. $\left\{\begin{array}{l} 60 \\ 61 \end{array}\right.$ Fig. $\left\{\begin{array}{l} 60 \\ 61 \end{array}\right.$

105 / 109 $$\left\{\begin{array}{l} \frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD. \\ \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} + \frac{r}{2L^2} = d = DH. \end{array}\right\} \text{Central.}$$

Quæ quidem quatuor Conſectaria octavum Claſſem complectuntur.

*Secundò*, Finge, in Æquatione 73 inventâ, & in 74 aſſumptâ, quartum terminum deficere; & 73 comparari,

$$x^3 \mp p x^2 \mp q x = 0, \text{ hoc eſt}$$

111 $$\text{cum} \left\{\begin{array}{l} x^2 \mp p x \mp q = 0. \\ x^4 \mp p x^3 \mp q x^2 * (\mp S) = 0. \end{array}\right\} \text{aſſumptâ.}$$

Primò finge,

111 | 112 $$\left\{\begin{array}{l} x^2 - p x - q = 0. \\ x^4 - p x^3 - q x^2 * (\mp S) = 0. \end{array}\right\} \text{aſſumptâ.}$$

73=112 | 113 Liquet, $\frac{p^2}{4} - 2Lb + L^2 = -q$, ejus correſpond.

*Tranſp.* | 114 $L^2 + \frac{p^2}{4} + q = 2Lb.$

$\frac{114}{2L}$ | 115 $\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD$, in Diametrum.

Æquo

### *Conſectar.* 4.

If the Equation propoſed to be made

102 110 be $\left\{\begin{array}{l} x^3 \pm px^2 + qx \pm r = 0. \\ x^4 \pm px^3 + qx^2 \pm rx \left\{\begin{array}{l}(\mp S)\\(\mp S)\end{array}\right\} = 0 \end{array}\right.$; then Fig. 59. Fig. $\left\{\begin{array}{l}60\\61\end{array}\right.$ Fig. $\left\{\begin{array}{l}60\\61\end{array}\right.$

105, 109 will $\left\{\begin{array}{l} \frac{L}{2} + \frac{p^2}{8L} \backsim \frac{q}{2L} = b = AD. \\ \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{pq}{4L^2} + \frac{r}{2L^2} = d = DH \end{array}\right\}$ Cent.

Which four Conſectaries comprehend the eighth Claſſe of Æquations.

---

*Secondly*, Suppoſe in the 73d. Æquation found, and in the 74th. aſſumed, the fourth term to be wanting; and the 73d. to be compared.

$$x^3 \pm px^2 \mp qx = 0; \text{ that is}$$

111 with $\left\{\begin{array}{l} x^2 \pm px \mp q = 0. \\ x^4 \pm px^3 \mp qx^2 * (\mp S) = 0. \end{array}\right\}$ aſſumed.

First, ſuppoſe,

111 112 $\left\{\begin{array}{l} x^2 + px - q = 0. \\ x^4 + px^3 - qx^2 * (\mp S) = 0. \end{array}\right\}$ aſſumed.

73=112 113 It is evident, $\frac{p^2}{4} - 2Lb + L^2 = -q$; its correſp.

*Tranſp.* 114 $L^2 + \frac{p^2}{4} + q = 2Lb.$

$\frac{114}{2L}$ 115 $\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD$, on the Diameter.

Æquo jure; Quoniam ſupponimus quartum terminum, nempè r, deficere, hoc eſt $r = 0$;

73=112 116 $g, + Lpb - 2L^2d = 0$, ejus correſpond. &

*Tranſp.* 117 ($Lpb = 2L^2d$, vel) $pb = 2Ld$. Si igitur in locum b, ſubſtituatur ejus valor (ſuprà inventus) nempè,

115 $\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L}$, orietur,

117 118 $\frac{Lp}{2} + \frac{p^3}{8L} + \frac{pq}{2L} = 2Ld$; &

$\frac{118}{2L}$ 119 $\frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} = d = DH$; $\perp$ ad Diametr.

*Conſectar.* 5.

Si proponeretur Æquatio conſtruenda,

*Fig.* 34.

112 120 $\left\{\begin{array}{l} x^2 \mp px - q = 0. \\ x^4 \mp px^3 - qx^2 * \left\{\begin{array}{l} (\mp S) \\ (\mp S) \end{array}\right\} = 0. \end{array}\right\}$ Orietur, *Fig.* $\left\{\begin{array}{l} 35 \\ 36 \end{array}\right.$

*Fig.* $\left\{\begin{array}{l} 35 \\ 36 \end{array}\right.$

115, 119 $\left\{\begin{array}{l} \frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD. \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} = d = DH \end{array}\right\}$ Central.

Secundò, finge,

111 121 $\left\{\begin{array}{l} x^2 - px + q = 0. \\ x^4 - px^3 + qx^2 * (\mp S) = 0. \end{array}\right\}$ aſſumptá.

73=121 122 Liquet, $\frac{p^2}{4} - 2Lb + L^2 = + q$, ejus correſp.

*Tranſp.* 123 $L^2 + \frac{p^2}{4} - q = 2Lb$.

$\frac{123}{2L}$ 124 $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = b = AD$; in Diametrum.

Item,

By the ſame reaſon, foraſmuch as the fourth term (*viz.* r) is ſuppoſed to be wanting; that is $r = 0$;
73=112 116 g°, $-Lpb + 2L^2d = 0$, its correſpondent; *i. e.*
*Tranſp.* 117 ($Lpb = 2L^2d$ or) $pb = 2Ld$: If therefore in the place of b, be ſubſtituted its valor (above found,) *viz.*
$\frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L}$: Then will be,
117 118 $\frac{Lp}{2} + \frac{p^3}{8L} + \frac{pq}{2L} = 2Ld$; and
$\frac{118}{2L}$ 119 $\frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} = d = DH$; ⊥ to the Diameter.

### *Conſectar.* 5.

If the Equation to be effected, were

$$\begin{array}{r} 112 \quad 120 \end{array} \left\{\begin{array}{l} x^2 \pm px - q = 0. \\ x^4 \pm px^3 - qx^2 * \left\{\begin{array}{c} (\mp S) \\ (\pm S) \end{array}\right\} = 0; \end{array}\right. \text{then will be}$$

*Fig.* 34.
*Fig.* $\left\}\begin{array}{l}35\\36\end{array}\right.$
*Fig.* $\left\}\begin{array}{l}35\\36\end{array}\right.$

$$\begin{array}{r} 115 \\ 119 \end{array} \quad \left\{\begin{array}{l} \frac{L}{2} + \frac{p^2}{8L} + \frac{q}{2L} = b = AD \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{pq}{4L^2} = d = DH \end{array}\right\} \text{Central.}$$

Secondly, ſuppoſe,

$$\begin{array}{r} 111 \quad 121 \end{array} \left\{\begin{array}{l} x^2 + px + q = 0. \\ x^1 + px^3 + qx^2 * (\mp S) = 0. \end{array}\right\} \text{aſſumed.}$$

73=121 122 It is evident $\frac{p^2}{4} - 2Lb + L^2 = +q$, its correſp.
*Tranſp.* 123 and $L^2 + \frac{p^2}{4} - q = 2Lb$.
$\frac{123}{2L}$ 124 g°, $\frac{L}{2} + \frac{p^2}{8L} - \frac{q}{2L} = b = AD$; on the Diameter.

Again,

Item, Quoniam ex hypotheſi $r = 0$;

73 = 121 | 125 $g^{o}$, $+ L p b - 2 L^{2} d = 0$, ejus correſpond. &

126 $(L p b = 2 L^{2} d$ vel$)$ $p b = 2 L d$. Si igitur in locum $b$, ſubſtituatur ejus valor, nempè

124 | $\frac{L}{2} + \frac{p^{2}}{8 L} - \frac{q}{2 L}$; fiet

126 | 127 $\frac{L p}{2} + \frac{p^{3}}{8L} - \frac{p q}{2 L} = 2 L d$; &

$\frac{127}{2 L}$ | 128 $\frac{p}{4} + \frac{p^{3}}{16 L^{2}} - \frac{p q}{4 L^{2}} = d = DH$; ⊥ ad Diametrum.

*Conſectar*. 6.

Sit Æquatio conſtruenda propoſita,

Fig. 37

121 | 129 $\left\{\begin{array}{l} x^{2} \mp p x + q = 0. \\ x^{4} \mp p x^{3} + q x^{2} * \left\{\begin{array}{l} (\mp S) \\ (\mp S) \end{array}\right\} = 0; \end{array}\right.$ fiet — Fig. $\left\{\begin{array}{l} 38 \\ 39 \end{array}\right.$

Fig. $\left\{\begin{array}{l} 38 \\ 39 \end{array}\right.$

124, 128 | $\left\{\begin{array}{l} \frac{L}{2} + \frac{p^{2}}{8 L} \backsim \frac{q}{2 L} = b = A D \\ \frac{p}{4} + \frac{p^{3}}{16 L^{2}} \backsim \frac{p q}{4 L^{2}} = d = D H \end{array}\right\}$ Central.

Conſectaria 5 & 6, ſeptimum Claſſem Æquationum complectuntur.

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*Tertiò*, Finge in Æquatione 73 inventâ, & in 74 aſſumptâ, tertium terminum deficere; & 73 comparari.

130 cum $\left\{\begin{array}{l} x^{3} \mp p x^{2} * \pm r = 0. \\ x^{4} \mp p x^{3} * \pm r x (\mp S) = 0 \end{array}\right\}$ aſſumptâ.

Primò

Again, by the hypotheſis $r = o$,

73=121 125 $g^{o}$, $- L p b + 2 L^2 d = o$, its correſpond. And
126 ($L p b = 2 L^2 d$ or) $p b = 2 L d$. If then in the place of b, be ſubſtituted its valor, *viz.*

124 $\frac{L}{2} + \frac{p^2}{8 L} + \frac{q}{2 L}$; then will

126 127 $\frac{L p}{2} + \frac{p^3}{8 L} + \frac{p q}{2 L} = 2 L d$; and

$\frac{127}{2L}$ 128 $\frac{p}{4} + \frac{p^3}{16 L^2} - \frac{p q}{4 L^2} = d = DH$; $\perp$ to the Diameter.

*Conſectar.* 6.

If the Equation to be made, were

121 129 $\left\{\begin{array}{l} x^2 \pm p x + q = o. \\ x^4 \pm p x^3 + q x^2 * \left\{\begin{array}{l} (\mp S) \\ (\pm S) \end{array}\right\} = o \end{array}\right.$; then will Fig. 37 Fig. $\left\{\begin{array}{l} 38 \\ 39 \end{array}\right.$

Fig. $\left\{\begin{array}{l} 38 \\ 39 \end{array}\right.$

124
128
$\left.\left\{\begin{array}{l} \frac{L}{2} + \frac{p^2}{8 L} \backsim \frac{q}{2 L} = b = A D \\ \frac{p}{4} + \frac{p^3}{16 L^2} \backsim \frac{p q}{4 L^2} = d = D H \end{array}\right.\right\}$ Central.

The 5th. and 6th. Confectaries comprehend the ſeventh Claſs.

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*Thirdly*, Suppoſe in the 73d. Equation, found, and in the 74th. aſſumed, the third term to be wanting; and the 73d. to be compared

130 with $\left.\left\{\begin{array}{l} x^3 \pm p x^2 * \mp r = o. \\ x^4 \pm p x^3 * \mp r x (\mp S) = o \end{array}\right.\right\}$ aſſumed.

Primò finge,

130 | 131 $\left\{\begin{array}{l} x^3 - px^2 * + r = 0 \\ x^4 - px^3 * + rx^2 \ (\mp S) = 0 \end{array}\right\}$ aſſumptâ.

Conſtat, Quoniam ſupponitur $q = 0$;

73=131 | 132 $g^\circ, \frac{p^2}{4} - 2Lb + L^2 = 0$, ejus correſpond.

*Tranſp.* | 133 $\& L^2 + \frac{p^2}{4} = 2Lb$.

$\frac{133}{2L}$ | 134 $\frac{L}{2} + \frac{p^2}{8L} = b = AD$, in Diametrum.

73=131 | 135 Liquet etiam $+ Lpb - 2L^2d = + r$, ejus correſp.

*Tranſp.* | 136 $g^\circ$, $Lpb - r = 2L^2d$. Si itaque in locum b, ſubſtituatur ejus valor, nempè,

134 | $\frac{L}{2} + \frac{p^2}{8L}$; fiet,

136 | 137 $\frac{L^2p}{2} + \frac{p^3}{8} - r = 2L^2d$.

$\frac{137}{2L^2}$ | 138 $\frac{p}{4} + \frac{p^3}{16L^2} - \frac{r}{2L^2} = d = DH$; in Diametrum.

*Conſectar.* 7.

Sit Æquatio conſtruenda, *&c.*

*Fig.* 25.

131 | 139 $\left\{\begin{array}{l} x^3 \mp px^2 * \mp r = 0. \\ x^4 \mp px^3 * \mp rx \left\{\begin{array}{l} (\pm S) \\ (\mp S) \end{array}\right\} = 0; \end{array}\right.$ Orietur

*Fig.* $\left\{\begin{array}{l} 26 \\ 27 \end{array}\right.$ *Fig.* $\left\{\begin{array}{l} 26 \\ 27 \end{array}\right.$

134, 138 $\left\{\begin{array}{l} \frac{L}{2} + \frac{p^2}{8L} = b = AD \\ \frac{p}{4} + \frac{p^3}{16L^2} \backsim \frac{r}{2L^2} = d = DH \end{array}\right\}$ Central.

Secundò,

First, ſuppoſe,

130 131 $\left\{\begin{array}{l} x^3 + px^2 * - r = 0. \\ x^4 + px^3 * rx(\mp S) = 0. \end{array}\right\}$ aſſumed.

It is evident, in as much as is ſuppoſed $q = 0$;

73=131 132 $g^\circ, +\frac{p^2}{4} - 2Lb + L^2 = 0$; its correſpondent,

*Tranſp.* 133 and $L^2 + \frac{p^2}{4} = 2Lb$.

$\frac{133}{2L}$ 134 $\frac{L}{2} + \frac{p^2}{8L} = b = AD$, on the Diameter.

73=131 135 It is evident alſo, $-Lpb + 2L^2d = -r$, its correſp.
*Tranſp.* 136 $g^\circ, Lpb - r = 2L^2d$. If therefore in the place of b,
be ſubſtituted its valor, *viz.*
134 $\frac{L}{2} + \frac{p^2}{8L}$; then will,
136 137 $\frac{L^2p}{2} + \frac{p^3}{8} - r = 2L^2d$.
$\frac{137}{2L^2}$ 138 $\frac{p^2}{4} + \frac{p^3}{16L^2} - \frac{r}{\frac{1}{2}L^2} = d = DH$; on the Diameter.

*Conſectar.* 7.

Let this Equation be to be made, *viz.*

*Fig.* 25.
*Fig.* $\left\{\begin{array}{l} 26 \\ 27 \end{array}\right.$
*Fig.* $\left\{\begin{array}{l} 26 \\ 27 \end{array}\right.$

131 139 $\left\{\begin{array}{l} x^3 \pm px^2 * \mp r = 0. \\ x^4 \pm px^3 * \mp rx \left\{\begin{array}{l} (\mp S) \\ (\pm S) \end{array}\right\} = 0; \end{array}\right.$ then will

134, 138 $\left\{\begin{array}{l} \frac{L}{2} + \frac{p^2}{8L} = b = AD. \\ \frac{P}{4} + \frac{p^3}{16L^2} \backsim \frac{r}{2L^2} = d = DH \end{array}\right\}$ Central.

Secundò, finge,

130 140 $\left\{\begin{array}{l} x^3 - px^2 \ast - r = 0 \\ x^4 - px^3 \ast - rx(\mp S) = 0 \end{array}\right\}$ aſſumptam.

73=104 Liquet, (per § 132, 133, 134,)

134 141 $\frac{L}{2} + \frac{p^2}{8L} = b = AD$; in Diametrum.

73=140 142 Item, $+ Lpb - 2L^2d = - r$, ejus correſpond.
*Tranſp.* 143 g°, $Lpb + r = 2L^2d$; ſi igitur in locum b, ſubſtituatur ejus valor, nempè,
141 $\frac{L}{2} + \frac{p^2}{8L}$; fiet,
143 144 $\frac{L^2p}{2} + \frac{p^3}{8} + r = 2L^2d$.
$\frac{144}{2L^2}$ 145 $\frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = d = DH$; ⊥ ad Diametr.

### *Conſectar.* 8.

Eſto hæc Æquatio conſtruenda, ſc.

140 146 $\left\{\begin{array}{l} x^3 \mp px^2 \ast \mp r = 0. \\ x^4 \mp px^3 \ast \mp rx \left\{\begin{array}{l} (\mp S) \\ (\mp S) \end{array}\right\} = 0 \end{array}\right.$; fiet *Fig.* 31. *Fig.* $\left\{\begin{array}{l} 32 \\ 33 \end{array}\right.$

141 145 $\left\{\begin{array}{l} \frac{L}{2} + \frac{p^2}{8L} = b = AD \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = d = DH \end{array}\right\}$ Central. *Fig.* $\left\{\begin{array}{l} 32 \\ 33 \end{array}\right.$

Conſectaria 7 & 8, Claſſem ſextum Complectuntur.

Secondly, ſuppoſe,

130 140 $\left\{\begin{array}{l} x^3 + px^2 * + r = 0. \\ x^4 + px^3 * + rx\ (\mp S) = 0. \end{array}\right\}$ aſſumed.

73=140 It is plain (by § 132, 133, 134, that)

134 141 $\frac{L}{2} + \frac{p^2}{8L} = b = AD$; on the Diameter.

73=140 142 Alſo, $- Lpb + 2L^2d = + r$, its correſpond.

Tranſp. 143 g°, $Lpb + r = 2L^2d$; if then, in the place of b, be ſubſtituted its valor, *viz.*

141 $\frac{L}{2} + \frac{p^2}{8L}$; then will

143 144 $\frac{L^2p}{2} + \frac{p^3}{8} + r = 2L^2d$.

$\frac{144}{2L^2}$ 145 $\frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = d = DH$; ⊥ on the Diameter.

*Conſectar.* 8.

Let this Equation be to be made, *viz.*

140 146 $\left\{\begin{array}{l} x^3 \pm px^2 * \pm r = 0. \\ x^4 \pm px^3 * \pm rx \left\{\begin{array}{l} (\mp S) \\ (\pm S) \end{array}\right\} = 0; \end{array}\right.$ then will — *Fig.* 31. *Fig.* $\left\{\begin{array}{l} 32 \\ 33 \end{array}\right.$

141, 145 $\left\{\begin{array}{l} \frac{L}{2} + \frac{p^2}{8L} = b = AD \\ \frac{p}{4} + \frac{p^3}{16L^2} + \frac{r}{2L^2} = d = DH \end{array}\right\}$ Central. — *Fig.* $\left\{\begin{array}{l} 32 \\ 33 \end{array}\right.$

The 7th. and 8th. Conſectaries comprehend the ſixth Claſs of Equations.

*Fourthly*,

*Quartò*, Finge in Æquatione 73, inventâ, & in 74 aſſumptâ, tertium & quartum terminos deficere, & 73 comparari, cum

$$x^3 \mp p x^2 * * = 0;$$ h e,

147 $\left\{\begin{array}{l} x \mp p = 0 \\ x^4 \mp p x^3 * * (\mp S) = 0 \end{array}\right\}$ aſſumptâ.

Finge,

147 148 $\left\{\begin{array}{l} x - p = 0 \\ x^4 - p x^3 * * (\mp S) = 0 \end{array}\right\}$ aſſumptam.

Ex hypotheſi, eſt $q = 0$,

73=148 149 $g^{\circ}, \frac{p^2}{4} - 2Lb + L^2 = 0$, ejus correſp.

*Tranſp.* 150 $\& L^2 + \frac{p^2}{4} = 2Lb$.

$\frac{150}{2L}$ 151 ideóque $\frac{L}{2} + \frac{p^2}{8L} = b = AD$; in Diametrum.

Item, ex hypotheſi eſt $r = 0$,

73=148 152 $g^{\circ}, + Lpb - 2L^2 d = 0$, ejus correſpond. &

*Tranſp.* 153 $(Lpb = 2L^2 d$, vel$)$ $pb = 2Ld$; in locum b, ſubſtituatur igitur ejus valor; nempè,

151 $\frac{L^2}{2} + \frac{p^2}{8L}$, & orietur

153 154 $\frac{Lp}{2} + \frac{p^3}{8L} = 2Ld$, &

$\frac{154}{2L}$ 155 $\frac{p^2}{4} + \frac{p^3}{16L^2} = d = DH$; ⊥ ad Diametrum.

*Conſectar.*

*Fourthly*, In the 73d. Æquation found, and in the 74th. aſſumed, ſuppoſe the 3d. and 4th. terms to be wanting, and the 73d. to be compared with,

147 $\left\{\begin{array}{l} x \pm p = 0 \\ x^4 \pm px^3 * * (\mp S) = 0 \end{array}\right\}$ aſſumed.

Suppoſe,

147 148 $\left\{\begin{array}{l} x + p = 0 \\ x^4 + px^3 * * (\mp S) = 0 \end{array}\right\}$ aſſumed.

By ſuppoſition, $q = 0$;

73=148 149 $g^{\circ}, \frac{p^2}{4} - 2Lb + L^2 = 0$, its correſpondent,

*Tranſp.* 150 and $L^2 + \frac{p^2}{4} = 2Lb$.

$\frac{150}{2L}$ 151 and $\frac{L}{2} + \frac{p^2}{8L} = b = AD$; on the Diameter.

Alſo, by ſuppoſition is $r = 0$;

73=148 152 $g^{\circ}, -Lpb + 2L^2 d = 0$, its correſpondent, and
*Tranſp.* 153 ($Lpb = 2L^2 d$, or) $pb = 2Ld$; in the place of b, let be ſubſtituted its valor, *viz.*

151 $\frac{L}{2} + \frac{p^2}{8L}$ and will be,

153 154 $\frac{Lp}{2} + \frac{p^3}{8L} = 2Ld$; and

$\frac{154}{2L}$ 155 $\frac{p}{4} + \frac{p^3}{16L^2} = d = DH$; ⊥ to the Diameter.

*Conſectar.*

## *Conſectar.* 9.

Finge hanc Æquationem conſtruendam, nempe, *Fig.*22.

147 $$156 \left| \begin{cases} x \mp p = 0. \\ x^4 \mp p x^3 * * \begin{Bmatrix} (\mp S) \\ (\mp S) \end{Bmatrix} \end{cases} \right\} = 0;$$ fiet, *Fig.* $\begin{cases} 23 \\ 24 \end{cases}$ *Fig.* $\begin{cases} 23 \\ 24 \end{cases}$

151, 155 $$\left. \begin{cases} \frac{L}{2} + \frac{p^2}{8 L} = b = A D \\ \frac{p}{4} + \frac{p^3}{16 L^2} = d = D H \end{cases} \right\} \text{Central.}$$

Quod ultimum Conſectarium quintum Claſſem complectitur.

E quibus omnibus Conſectariis generalitèr obſervandum eſt:

*Obſerv.*4 Regulæ Centralis quantitates & ſigna ita omninò determinari oportere, quemadmodum in iis, quæ ad ipſam (ſuprà, *pag.* 8.) annotavimus, ſunt expoſita.

# *F I N I S.*

### *Conſectar.* 9.

Suppoſe this Æquation propoſed to be made, *viz.* *Fig.*22.

147 156 $$\left| \begin{cases} x \pm p = 0. \\ x^4 \pm p x^3 * * \begin{Bmatrix} (\mp S) \\ (\mp S) \end{Bmatrix} = 0; \text{ then will} \end{cases} \right.$$ *Fig.* $\begin{cases} 23 \\ 24 \end{cases}$ *Fig.* $\begin{cases} 23 \\ 24 \end{cases}$

151 155 $$\left. \begin{cases} \frac{L}{2} + \frac{p^2}{8L} = b = AD \\ \frac{p}{4} + \frac{p^3}{16L^2} = d = DH \end{cases} \right\} \text{Central.}$$

Which laſt Conſectary completes the 5th. Claſs.

From all which Conſectaries, it may be generally obſerved:

*Obſerv.*4 That the quantities and ſigns of the Central Rule, muſt be altogether ſo determined, as they are expoſed in thoſe things, which we have noted before, in the 8th. *pag.*

*THE END.*

## *Some Books Printed for, and Sold by* Robert Clavel, *at the Sign of the* Peacock *in St. Paul's* Church-yard.

THE Annals of King *James*, and King *Charles* the Firſt, Containing a faithful and impartial Account of the Great Affairs of State and Tranſactions of Parliaments in *England*, in *Folio.* Wherein ſeveral material Paſſages Relating to the late Civil Wars (not mentioned in former Hiſtories) are made known; in particular, ſome of Mr. *Ruſhorth*'s Miſtakes and Omiſſions. And firſt the Caſe of the Devorce of the Earl of *Eſſex* from his Counteſs, which had ſo great Influence on the enſuing Troubles, Related from the Original Proceedings in that Court.

2. The True Cauſe of the Troubles in our Church, *viz.* The Connivance of ſome Church-men at the Diſſenters from the Government of the Church, as Eſtabliſhed by Law, and the Favour found at Court from great Perſons there.

3. King *James* not in ſo much Influenced by *Gondamore*, as is Related by Mr. *Ruſhworth.*

4. The Three Eſtates in Parliament, who they were, in King *James*'s Speech in Parliament, 1620.

5. An Authentick and Impartial Account of the beginning of the Troubles in *Scotland*, and the Wars which enſued.

6. The True State of our late Civil Wars, their Beginnings, Cauſes, who the Aggreſſors, *&c.* The reſt are too large to take notice here, but may be ſeen in the Preface.

*Vare-*

*Varenius*'s *Geography* in *Folio*, *Engliſh*, Illuſtrated with many Copper Cuts.

Dr. *Willis*'s Works in *Folio*, *Engliſh*.

The Hiſtory of the *Iriſh Rebellion*, traced from many preceding Acts to the Grand Eruption, the 23d. of *October*, 1641. and thence purſued to the *Act of Settlement*, 1662.

Tracts Written by *John Selden* of the Inner-Temple, Eſq; and Tranſlated by the Eminent Dr. *A. L.* The 1ſt. *Jani Anglorum facies altera*, with large Notes thereupon. 2ly, *Englands Epinomis.* 3ly, Of the Original of Eccleſiaſtical Juriſdictions of Teſtaments. The 4th, Of the Diſpoſition or Adminiſtration of inteſtate Goods.

Mr. *Scrivener*'s Body of Divinity.

Dr. *Cumber* on the *Liturgy* in *Folio.*

Mr. *Sam*'s *Britannia.* *Ogleby*'s Hiſtory of *Affrica*, *Aſia*, and *America.*

Biſhop of St. *Davids*'s Vindication of the Biſhops Rights to Vote in Capital Caſes —— his ſeaſonable Corrective

The Compleat Catalogue to the end of *Eaſter Term*, 1684.

The Biſhop of *Lincoln*'s Obſervations, and Animadverſions on *Pope Pius* the V. his *Bull* againſt Queen *Elizabeth*: Whereunto is Annexed the *Bull* of *Pope Paul* the III. againſt King *Henry* the VIII.

Dr. *Cumber*'s Vindication of the Divine Right of Tyths.

Biſhop of *Cork*'s Perſwaſive to all Proteſtants.

Religion and Loyalty ſupporting each other, in Vindication of the Loyal Addreſſors.

Biſhop of St. *Davids*'s *Billa Vera*, or Argument of *Ignoramus* —— his ſhort way to a laſting Settlement, and Anſwer to *Sidney*'s *Speech* —— his Advice to a ſound Proteſtant and Proſelyte of *Rome* call'd back.

Three Sermons of Dr. *Standiſhes*.

Two of Mr. *Richard Werge* of *New-Caſtle*.

One Sermon of Dr. *Morice* before the King.

Two of Dr. *Dixons*'s, Prebend of *Rocheſter*.

Dr. *Ward*'s Sermon of *Blandford*.

*Ogleby*'s *Eſop* in *Engliſh*, adorn'd with 160 Sculptures.

A Diſcourſe of Natural and Moral Impotency.

Biſhop of St. *David*'s Anſwer to *Melius Inquirendum* —— his Anſwer to the *Proteſtant Reconciler*.

*Brown*'s Treatiſe of Preternatural Tumours.

*Mocket* s Tractatus de politia Eccleſ. Anglicanæ.

The Reduction of *Ireland* to the Crown of *England*.

*Smith*'s Rhetorick, the Fifth Edition.

*Humphrey*'s Reſolution of Conſcience.

Dr. *Byan*'s Eight Sermons, Preached before His Majeſty in his Exile.

Friendly Conference between a Miniſter and a Quaker, Two parts.

Dr. *Duport*'s *Poems*. *Seneca* with *Farnaby*. *Skicard*'s Hebrew Grammar.

*Eſop*'s Fables, Greek and Latin.

*Compend. Politicum*. An Account of the Troubles in the Regin of King *Henry* the 3d.

*Martindale*'s Book of Surveying.

Books of Riddles.

*FINIS.*

Fig. 1.

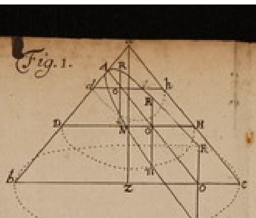


Fig. 2.

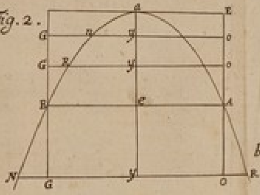


Fig. 3.

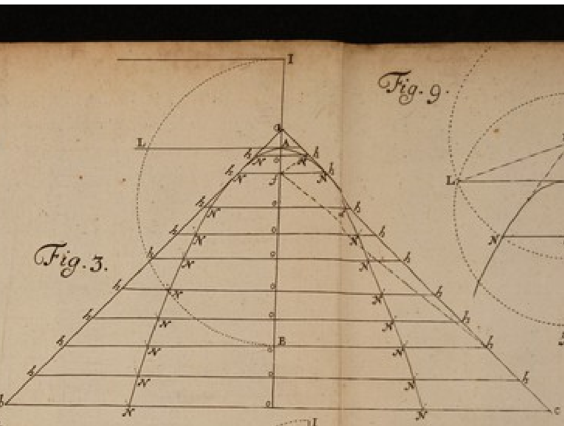


Fig. 9.

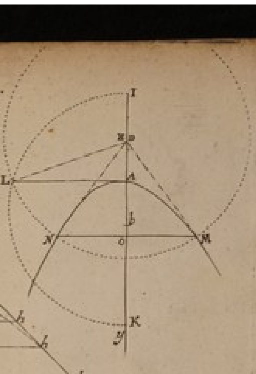


Fig. 4

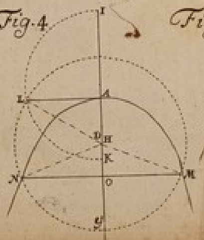


Fig. 5

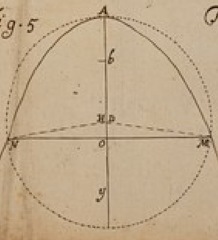


Fig. 6.

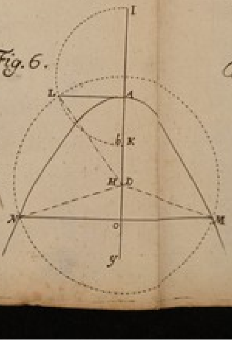


Fig. 8.

Fig:10

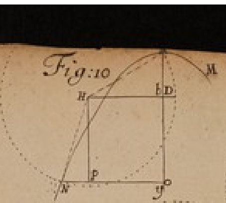


Fig:11.

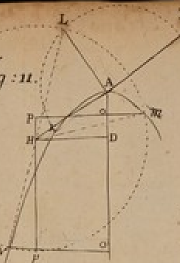
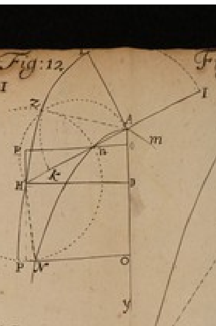


Fig:12



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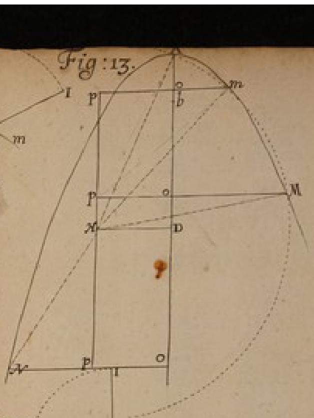

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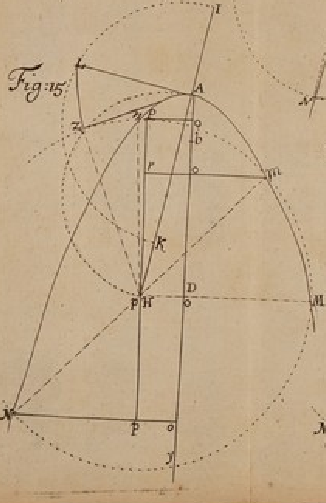

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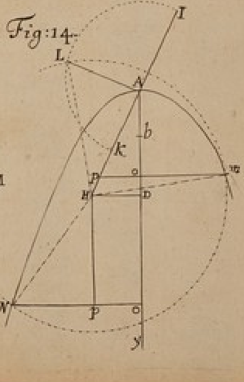

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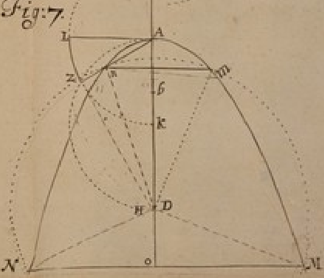

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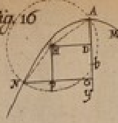
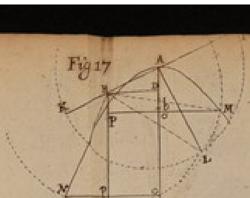


Fig 17



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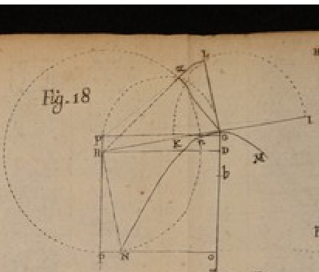

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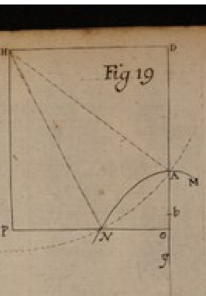

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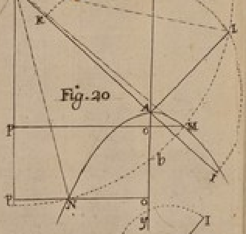

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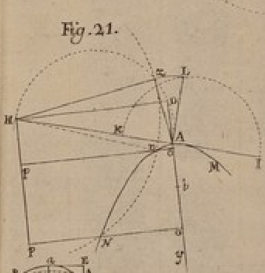

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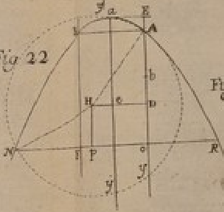

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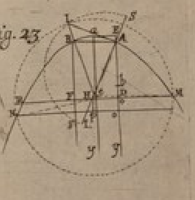

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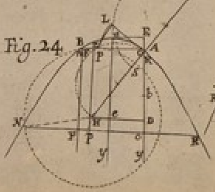


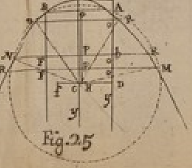
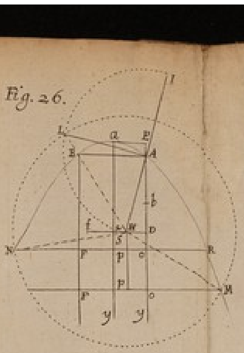


Fig. 25

Fig. 26.



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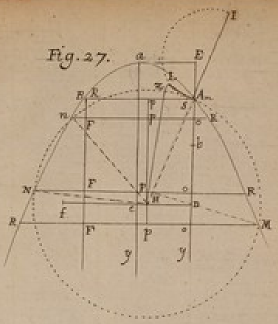

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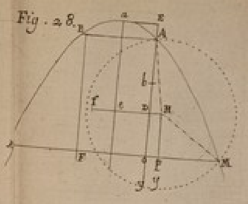

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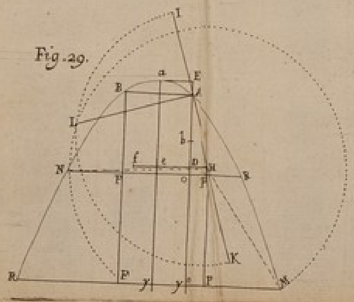

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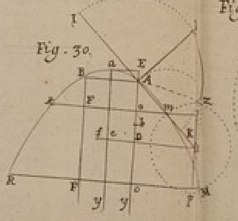

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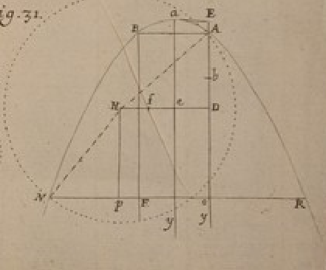

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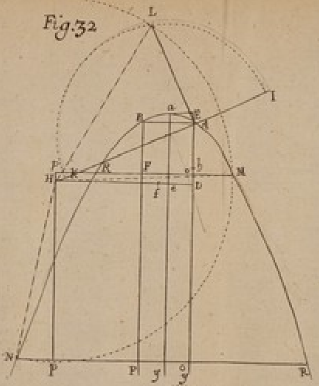

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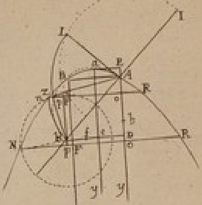

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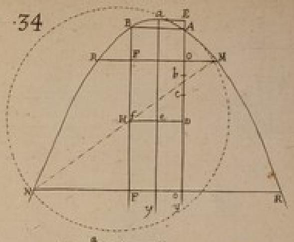

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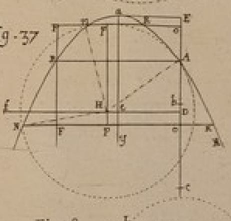

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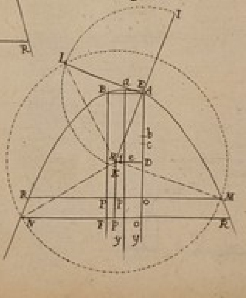

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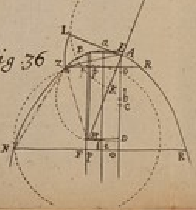

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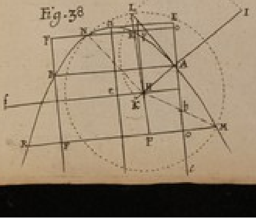

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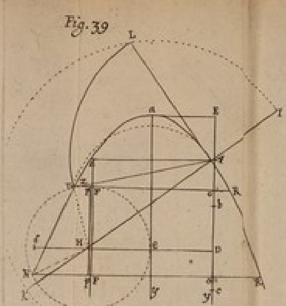

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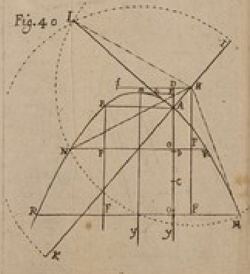

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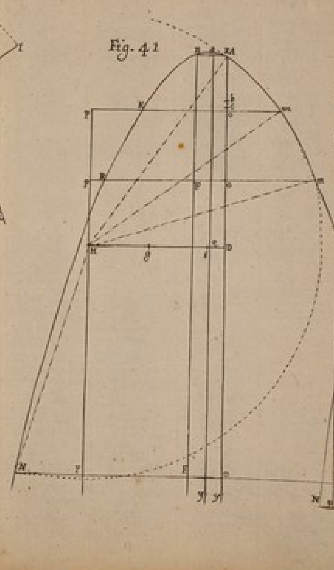

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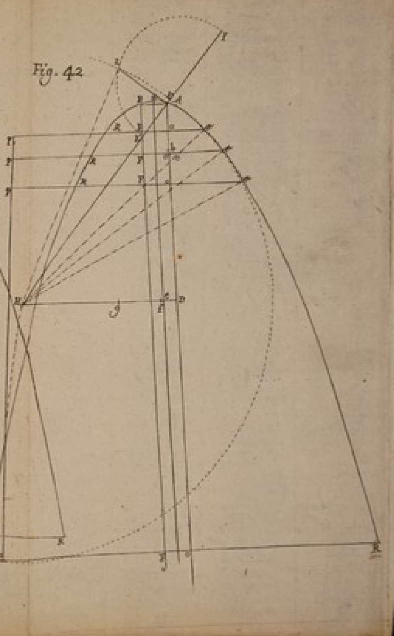

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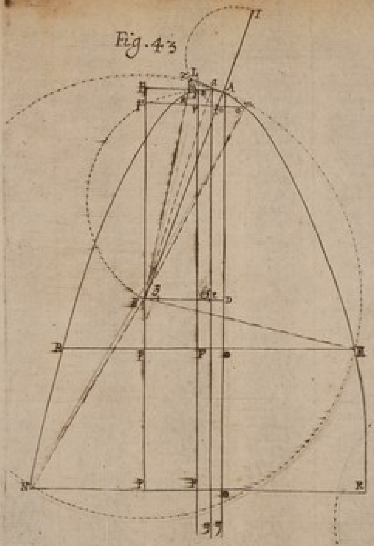

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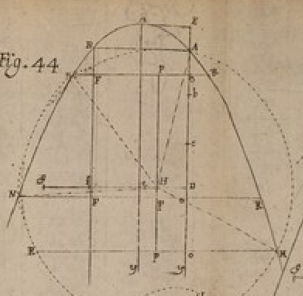

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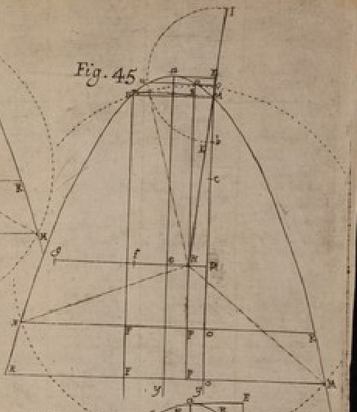

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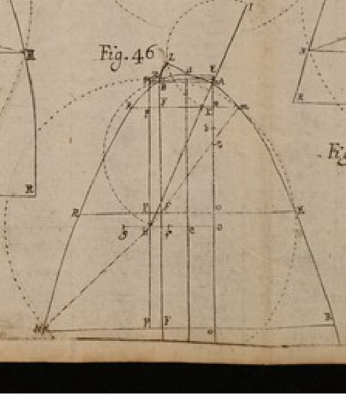

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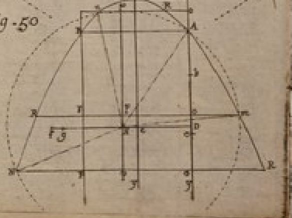

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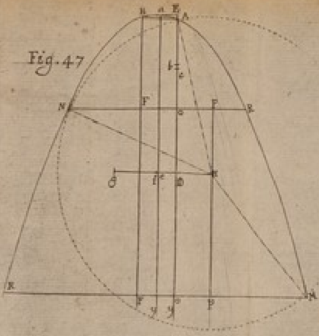

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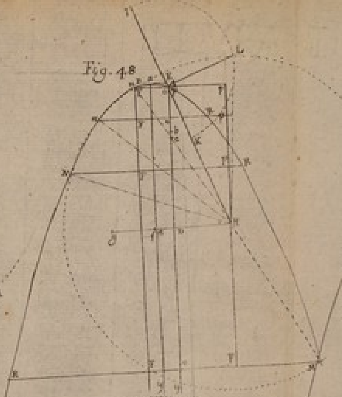

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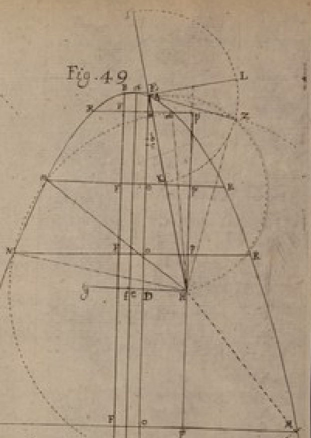

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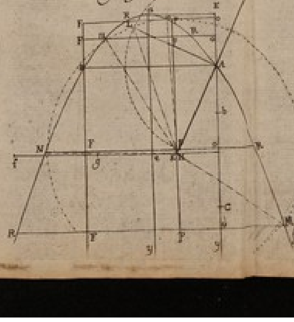

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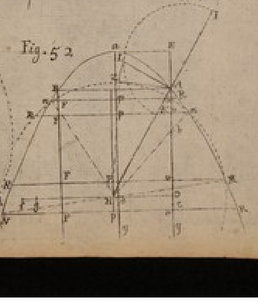

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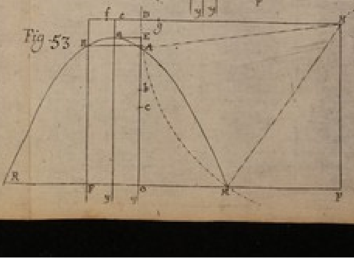

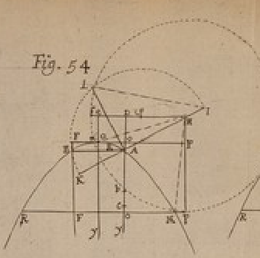
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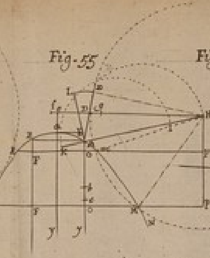
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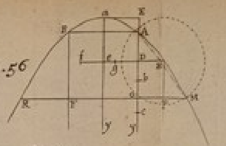
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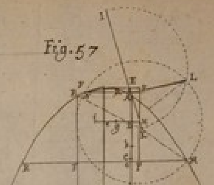
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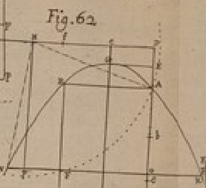
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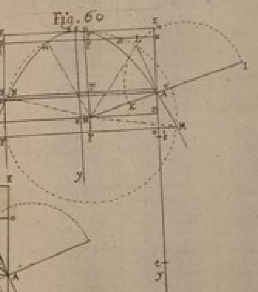
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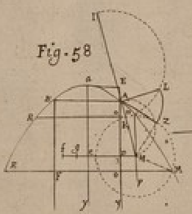
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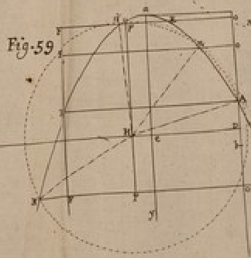
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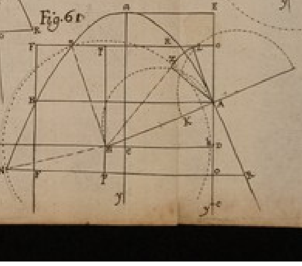
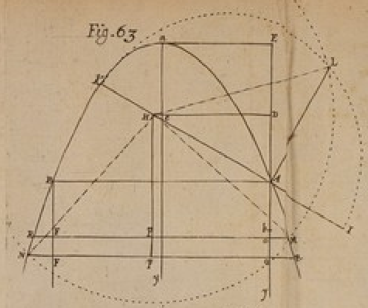


Fig. 61

Fig. 63



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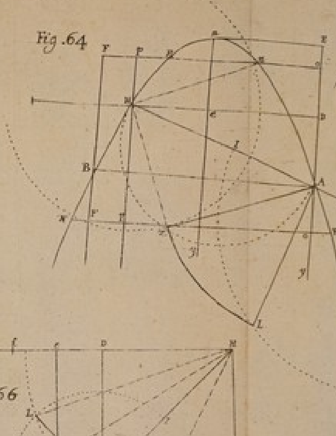

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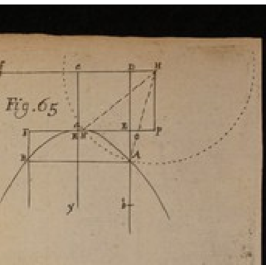

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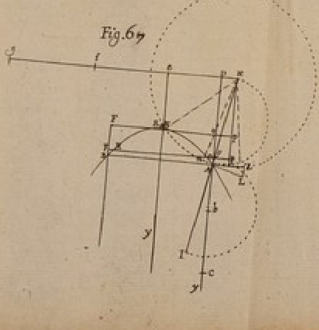

Fig. 66