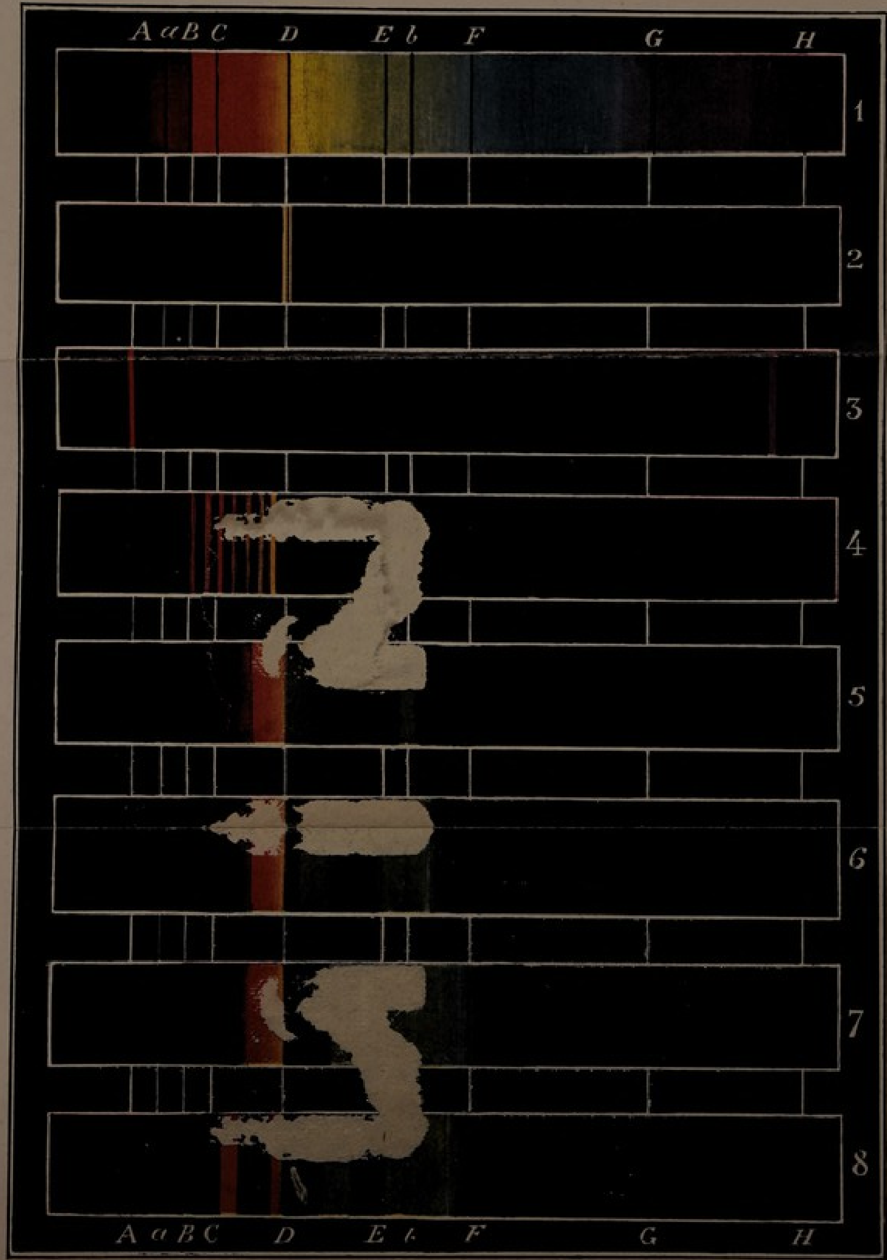
Mary Drabble.
April 1920.

A MANUAL OF PHYSICS FOR MEDICAL STUDENTS



VARIOUS TYPES OF SPECTRA.

1, Continuous Spectrum with Fraunhofer's lines; 2, Spectrum of Sodium; 3, Do. of Potassium; 4, Do. of Strontium; 5, Absorption Spectrum of Arterial blood, diluted 1 in 250; 6, Do. diluted 1 in 400; 7, Same as No. 6, but deprived of Oxygen; 8, Absorption of Spectrum of Chlorophyll in Alcohol.

A MANUAL OF PHYSICS

THEORETICAL AND PRACTICAL

FOR MEDICAL STUDENTS

BY

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SECOND EDITION, ENLARGED

WITH A COLOUR FRONTISPIECE AND
280 FIGURES IN THE TEXT

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PREFACE TO THE SECOND EDITION

This Manual is intended to be a companion volume to the "Manual of Chemistry," and it is believed that the medical student will find in the two books all the theoretical and practical information prescribed by any Syllabus for the First Examination in Physics and Chemistry.

For the present edition the text has been revised throughout and considerably extended. In the new matter frequent reference has been made to the therapeutic applications of radiant heat, light, and electricity. Simple geometrical proofs of the fundamental formulæ relating to prisms and lenses have been added to supplement, but not to replace, their experimental verification. Without practice in working numerical problems and in performing experiments, a knowledge of Physics can never have the necessary precision and reality. The written exercises now added to each chapter, and the numerous practical exercises of Part VI., have been carefully selected to provide this practice.

Further Tables have been added to the Appendix, and new Illustrations have been inserted in the text.

H. C. H. C.

January, 1918.

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A
MANUAL OF PHYSICS
FOR MEDICAL STUDENTS

PART I.—GENERAL PHYSICS

CHAPTER I

ELEMENTS OF KINEMATICS

Distinction between Physical and Chemical Properties of Matter—Units of Measurement—Fundamental Units—Units of Length, Mass, and Time—Derived Units—Units of Velocity and of Acceleration—Falling Bodies—Momentum—Various Kinds of Motion—Angular Measurement—Exercises.

Physical and chemical properties of matter.—The sister sciences of Chemistry and Physics are so closely related that many phenomena are common to both. In the introduction to a companion volume* to the present one, Chemistry is described as one branch of the larger Science of Matter. Physics is another important branch of the same science. Either science will be seen in more correct perspective if this fact is realized at the first.

The study of Chemistry begins with the recognition of certain chemical elements. We have not yet recognized a corresponding physical element,

* "A Manual of Chemistry." By Arthur P. Luff and Hugh C. H. Candy.

though it would seem more logical to begin our study with some conception of the kind. The chemical element is certainly not a physical element, but has often a very complex physical character, as the spectroscope shows. We are, however, concerned now with the state and condition of matter and with the properties that belong to all matter in a greater or less degree, rather than with those more special properties which serve to distinguish one chemical element from another.

Two properties, *weight* and *extension*, belong to every kind of matter; other properties are possessed only by certain kinds of matter and, therefore, help us to distinguish one kind of matter from another. Some of these properties can be recognized in the kind of matter which is under observation, without making any change in the composition of the matter. *Specific gravity* and *specific heat* are properties of this nature. The student will presently learn how they are demonstrated and measured, and will observe that the experiments involve no change in the substance examined. There are, however, other properties whose presence we can only discover or demonstrate by experiments in which the composition of the substance is changed. The metal magnesium possesses the property, when heated in air or in oxygen, of emitting a white light of dazzling brilliance, often employed for photographic purposes. In the act of exhibiting this property the magnesium is converted into a white powder, magnesium oxide; this substance differs entirely from the original metal. Many other substances, when heated in air, undergo a similar change in composition, though the change is not often attended with such brilliant effects.

The study of all changes in the *composition* of any kind of matter belongs to the science of Chemistry,

and all properties of matter which are not manifested without the assistance of such changes are, therefore, regarded as *chemical* properties. The property of combining with oxygen to form a new substance is a chemical property. Specific heat and specific gravity, on the other hand, are *physical* properties. They can be recognized and even measured without making any change in the composition of the substance. It is true that a substance may undergo some change in regard to these *properties* while remaining unchanged in *composition*. Water, for instance, is known to us in three different physical states; in the solid state as *ice*, in the liquid state as *water*, in the gaseous state as *steam*. Ice, water, and steam differ in specific heat and in specific gravity, but all consist of the same kind of matter, and possess exactly the same chemical composition. When water freezes or boils it undergoes a *physical* change only. The study of physical changes and the measurement of physical properties may be said to constitute the whole science of Physics.

Units of measurement.—The measurement of properties—or of magnitudes of any kind—necessitates the selection of units of measurement. The unit selected must be a magnitude of the same kind as those which it is intended to measure. The unit employed to measure other quantities of heat must itself be some definite quantity of heat. The unit employed to measure other quantities of matter must itself be some definite quantity of matter. The unit employed to measure other intervals of temperature must itself be some definite interval of temperature.

We shall, therefore, require as many different units as there are different kinds of magnitudes to be measured. These units, however, need not all be entirely independent of each other. We might, for

instance, choose as our *unit quantity of heat* that amount of heat which would raise the chosen unit quantity of matter through the chosen unit interval of temperature. The heat unit would thus depend upon the other two units, and would be fixed as soon as they had been selected. This is found to be convenient in practice, and units are accordingly divided into (1) fundamental units and (2) derived units.

Fundamental units.—There are three independent units, which have been arbitrarily chosen, and differ somewhat in different systems. These are :

1. The unit of length.
2. The unit of mass.
3. The unit of time.

1. **The unit of length.**—In the British system this is commonly called a *foot*. The yard is a definite length, and has been defined by statute to be "the distance between the centres of the transverse lines in the two gold plugs in the bronze bar deposited in the office of the Exchequer, when the temperature of the bar is 62° F." The foot is one-third of this length.

In the C.G.S. (centimetre-gramme-second) system the unit of length is a *centimetre*. The metre is the length, at 0° C., of a certain platinum bar which, with duplicates, is preserved in Paris. When made it was believed to be the ten-millionth part of a quadrant of the meridian of longitude which passes through Paris. It is now known to be slightly less than this. The centimetre is the hundredth part of the length of the standard metre bar. For purposes of conversion a metre may be regarded as equal to 39·37 inches, and a foot as equal to 30·48 centimetres. For road distances in France the *kilometre*, a thousand metres, is commonly used ; 8 kilometres are nearly equal to 5 miles.

A *micron* or *micrometre* (= $\frac{1}{10^6}$ metre) is a small unit employed in bacteriology and generally in microscopic measurements; it is denoted by μ.

$$1 \text{ in.} = 25{\cdot}4 \text{ mm.} = 25{,}400\ \mu.$$

The micron is a thousandth of a millimetre, and might therefore be called a milli-millimetre. In medical literature it is unfortunately often called a *micromillimetre,* and this error is the cause of some confusion. A micromillimetre is really the millionth of a millimetre, and is denoted by $\mu\mu$.

A *tenth-metre* (= $\frac{1}{10^{10}}$ metre) is a still smaller unit employed in the expression of such minute distances as the wave lengths of different lights—

$$1 \text{ micron } (\mu) = 10{,}000 \text{ tenth-metres.}$$

$$1\ \mu\mu = 10 \text{ tenth-metres.}$$

The *mil* (= 0·001 in.) is also used.

On the other hand, astronomical distances are often expressed in terms of the mean radius of the Earth's orbit; this "astronomical unit" is equal to 92,900,000 miles. A still larger unit—the "light year"—is sometimes employed; this is the distance travelled by light in a year.

1 *light year* = 63,000 *astronomical units.*

2. **The unit quantity of matter, or mass.**—The British unit of mass is a standard mass of platinum, also preserved in the Exchequer, and is called a *pound.* A standard mass of platinum is also preserved in Paris and is called a *kilogramme.* The C.G.S. unit of mass is one-thousandth part of this and is called a *gramme.* It was once believed to be the mass of a cubic centimetre of water at 4° C. More recent determinations have shown that one gramme of this standard water really fills 1·000028 c.c.

This volume is conveniently called a *millilitre*, and is denoted by the abbreviation *mil.* A *litre* will therefore be the volume occupied by a kilogramme of water at 4° C. and will be 1000·028 c.c. The cubic centimetre (1 c.c.) is a derived unit and is simply the volume of a cube whose edge is one centimetre long; we cannot therefore vary this volume without changing the unit of length. For purposes of conversion a kilogramme may be regarded as equal to 2·2 lb., and 1 lb. as equal to 453·6 grm.

Large masses are sometimes measured in terms of the *metric ton*. For instance, in the important Report on the Food Supply of the United Kingdom,* the quantities of cereals and other foodstuffs annually available are stated in metric tons. This large unit is equal to 1,000 kilogrammes and nearly equal to 2,205 lb.; the English *ton* is equal to 2,240 lb.

3. **The unit interval of time.**—We become conscious of the lapse of time by noting the succession of events, and the sequence of day and night was probably the earliest clock. The solar day, or interval from noon to noon, is still the basis of our time reckoning. Owing, however, to the position and motion of the earth relatively to the sun, this interval is not always of the same length throughout the year. We therefore use instead an average or *mean solar day* of invariable length. This interval is divided into 24 hours, the hour is divided into 60 minutes, the minute into 60 seconds. This second, the $\frac{1}{24 \times 60 \times 60}$th part of a mean solar day, is the unit of time adopted both in the British system and in the C.G.S. system.

* Cd. 8421. 1917.

Derived units.—From observation of different bodies we soon learn that some do not alter their position with regard to surrounding objects, and these we describe as being *at rest*. Others evidently undergo some displacement or change of position with regard to surrounding objects, and these we regard as being *in motion*. We use the terms in a relative and not in an absolute sense. We speak of the earth, houses, trees, etc., as at rest, although we know that the earth and the bodies on it are moving, just as we speak of a passenger as sitting *still* in an express train. For practical purposes we can generally disregard any motion that is common to all bodies belonging to the system considered; we shall only have to consider, as a rule, problems in which one of the bodies is in motion relatively to the others. The problem will be most simple when we are concerned not with the cause of the motion, or with the mass of the moving body, but only with the motion *per se*. When this is the case the problem belongs to **kinematics** (*κινήματα*, movements), a limited branch of kinetics (p. 18), and we only require to express—

(*a*) The *duration* of the motion. This is some period of time and, therefore, can be expressed in terms of the unit of time already selected.

(*b*) The *extent* of the motion; that is, the distance traversed by the body in the time. This is some length and, therefore, can be expressed in terms of the unit of length already selected.

(*c*) The *velocity* of the motion; this is a new kind of magnitude and we require a new unit in which to measure it.

We have defined motion as *change of position*; we shall now define velocity as the *time-rate of*

change of position. The *unit of velocity* will evidently be the velocity of a mass which moves the unit of length in the unit of time, and is thus derived from two units which have already been selected. In the British system the unit of velocity is a velocity of 1 foot per second. The C.G.S. unit is a velocity of 1 centimetre per second. If motion continues for t units of time and in this period the body changes its position, or moves, by l units of length, then it has moved at an average rate of $l \div t$ units of length per unit of time, and has an average velocity of $\frac{l}{t}$ units of velocity.

This average velocity need not be the actual velocity of the body, which may move more slowly at some moments and more rapidly at others. A train, for instance, which stops at two stations, 30 miles apart, and goes from one to the other in an hour, has evidently an *average* velocity of 30 miles per hour (44 ft. per second). At the commencement of the journey the actual velocity is much less than this, but rapidly increases till the value *exceeds* the average, and then diminishes again till the train finally comes to rest at the second station. We must, therefore, distinguish between—

(*a*) Uniform velocity;
(*b*) Variable velocity.

(*a*) **Uniform velocity.**—A velocity is called uniform when it undergoes no change in magnitude throughout the period of motion considered; in this case the average velocity and the actual velocity have evidently the same value. If that value be denoted by v, we know that the body moves through v units of length in *each* unit of time, and therefore through $v\,t$ units of length in t units of time; this distance,

the total space traversed by the moving body in the whole period under consideration, is usually denoted by s, and is evidently equal to $v\,t$. We see, therefore, that when a body is moving *uniformly* the three quantities specified on p. 6 are connected by the relation

$$s = v\,t \quad .. \quad .. \quad .. \quad .. \quad (1)$$

And from this equation, when any two of the quantities are known, we can find the value of the third.

(*b*) **Variable velocity.**—When a velocity varies, it may vary regularly or irregularly ; we shall only need to consider the case in which the variation is regular or uniform. In this case the velocity is uniformly increased or diminished by the same amount in the same time. The velocity of a falling body, for instance, regularly increases by about 32 ft. per second, every second. Conversely, the velocity of a body projected vertically upwards from the earth is uniformly *diminished* every second by the same amount. This change in the velocity, per second, whether increase or decrease, is called *acceleration ;* in this particular case it is due to gravity and is usually denoted by g. We may define acceleration in general as the *time-rate of change of velocity.* If, during an interval of time, t, the velocity of a body change *uniformly* from an initial value, v_1, to a final value, v_2, the total change in the interval is $v_2 - v_1$, and the rate of change per unit of time is therefore $\frac{v_2 - v_1}{t}$. This is the acceleration as defined above ; it is usually denoted by a. The definition, when stated in symbols, leads to the equation :

$$a = \frac{v_2 - v_1}{t} \quad .. \quad .. \quad .. \quad .. \quad (2)$$

The *unit of acceleration* is that acceleration which

changes the velocity by one unit of velocity in one unit of time. It is therefore derived from units already selected. The British unit changes the velocity by 1 foot per second, in one second; the C.G.S. unit by 1 cm. per second, in one second. When a varying velocity is subject only to *uniform* acceleration, it is equivalent, during the period considered, to a single uniform velocity whose value is intermediate between the minimum and maximum values of the varying velocity; these will evidently be the values at the beginning and end of the period: this uniform, average velocity, the mean of the initial and final velocities, will therefore be denoted by $\frac{v_1 + v_2}{2}$. The distance s actually described by the body in the period t will therefore be the same as if it moved all the time with this *uniform* velocity. We may therefore write (*see* p. 9 (1)):

$$s = \frac{v_1 + v_2}{2}\, t \qquad \ldots \quad \ldots \quad \ldots \quad (3)$$

By multiplication we may easily deduce from equations (2) and (3) that—

$$a\, s = \frac{{v_2}^2 - {v_1}^2}{2} \qquad \ldots \quad \ldots \quad \ldots \quad (4)$$

Problems which relate only to the motion of a moving body, without reference to its mass or to the cause of the motion, belong to the science of *kinematics*.

The equations now obtained will enable the student to solve any problems of this kind with which he is likely to meet. Their application is illustrated by the following examples:—

Ex. 1: A body falls to the ground from a height H; it is required to find (t) the time of descent, and (v_2) the velocity with which it strikes the ground.

In this case we have given—

$$v_1 = o$$
$$a = g$$
$$s = H$$

and we require to find v_2 and t. Substituting the given values in equation (4), we obtain—

$$g\ H = \frac{v_2{}^2 - o}{2}$$
$$\therefore v_2{}^2 = 2g\ H$$
$$\therefore\ v_2 = \sqrt{2g\ H}$$

Substituting now in equation (2), we obtain—

$$g = \frac{\sqrt{2g\ H} - o}{t}$$
$$\therefore t = \sqrt{\frac{2H}{g}}$$

It is convenient to have these results in this general form for reference; particular values can be readily derived from them if required. For instance, if H is 100 ft. and $g = 32$, we find—

$$v_2 = 80 \text{ ft. per second}$$
$$t = 2\tfrac{1}{2} \text{ seconds}$$

so that a body falling from a height of 100 ft. reaches the earth in $2\frac{1}{2}$ sec. and strikes it with a velocity of rather more than 54 miles per hour.

Ex. 2: A body projected vertically upwards just reaches a height H; it is required to find (v_1) the velocity of projection, and (t) the time of ascent.

In this case we have given—

$$v_2 = o$$
$$a = -g$$
$$s = H$$

and proceeding as in the previous example, we now find—

$$v_1 = \sqrt{2gH}$$
$$\text{and } t = \sqrt{\frac{2H}{g}}$$

We thus learn (1) that the velocity which must be given to a body to enable it to reach a certain height is the same as it would acquire in falling to the earth from that same height; and (2) that the times of ascent and descent are the same.

Ex. 3: What is the average velocity possessed by a falling body during the first n secs. of the fall?

We know
$$v_1 = o$$
$$a = g$$
$$t = n$$

$\therefore$ from equation (2) (p. 9) $v_2 = n\,g$

$\therefore$ average velocity $\dfrac{v_1 + v_2}{2} = \dfrac{n\,g}{2}$

Ex. 4: What is the average velocity possessed by a falling body during the n^{th} sec. of the fall?

At the *beginning* of the n^{th} sec. the body has been falling for $(n - 1)$ secs., and has thus acquired a velocity $v_1 = (n - 1)\,g$; at the *end* of the n^{th} sec. the velocity $v_2 = n\,g$, as in the previous example.

$\therefore$ the *average* velocity for this n^{th} second, $\dfrac{v_1 + v_2}{2}$,

$$\text{is } \frac{(n - 1)\,g + n\,g}{2} = \frac{(2n - 1)\,g}{2}$$

Momentum.—The quantity of motion, or, as it is now called, the *momentum*, of a moving body depends upon the mass of the body and also upon the velocity of motion. A small mass, such as a shot, though moving with a high velocity, will have less momentum than a goods train has when moving quite slowly. The unit of momentum is a derived unit. It is the momentum possessed by a unit mass moving with unit velocity; m units of mass moving with v units of velocity will, therefore, have $m\,v$ units of momentum.

Various kinds of motion.—We have hitherto supposed the motion of a moving body to be one

of *translation.* In this the body is moved as a whole from one part of space to another, but different parts or points of the body do not alter their positions relatively to each other; all points move with equal and parallel velocities. A rigid body sliding down a smooth inclined plane illustrates this type of motion. The motion of a body may, however, be restricted owing to the fact that one or more points of the body are fixed in space; a door, for instance, is only free to move about a fixed straight line which passes through its hinges; the movement possible is one of *rotation.* Translation and rotation are combined in the motion of a point in the tyre of a bicycle which is being ridden along a road. The motion may also be restricted to one of *oscillation* about a mean position. Imagine, for instance, that a rigid rod O A (Fig. 1) without weight is suspended, at its extremity O, from a fixed point, and that a heavy particle A fixed to the other extremity is slightly displaced from the vertical to a new position B. When released, the particle will move along the arc B A with gradually *increasing* velocity due to the influence of gravity. After passing A, however, gravity will begin to *diminish* the velocity and will bring the particle to rest at B′ (A B′ = A B). Continuing to act,

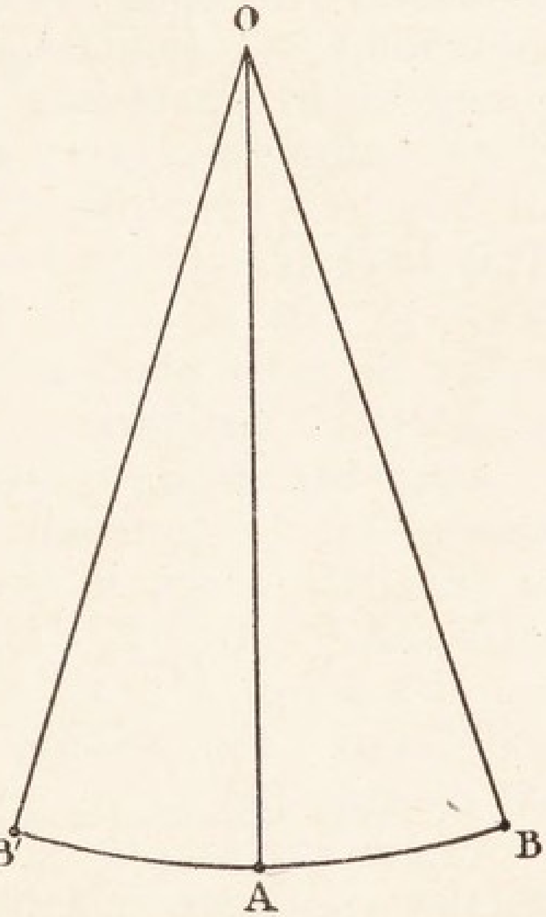


Fig. 1.—Oscillation about a mean position.

gravity will now cause the particle to return along B′ A to A, which it will reach with the same maximum velocity as before, and will be carried to rest at its original position B. The double journey, or "swing-swang," is called a *complete vibration*, and the time of executing it is called the *period*. The length of the arc A B, or A B′, measures the *amplitude* of the vibration. The motion of a pendulum is a familiar instance of this type. The prong of a tuning-fork also vibrates when struck. These vibrations are communicated to particles of air, and give rise to waves of sound (p. 194). This kind of motion owes its interest and importance to the fact that it is intimately related to many physical phenomena, as will be seen in later chapters of this book.

Angular measurement.—The unit of angular measurement commonly employed in this country is an angle which is one-ninetieth part of Euclid's right angle (Bk. I., Def. 10). This unit angle is called a *degree*, $1°$. It is subdivided into 60 minutes $(60')$, and each minute is further subdivided into 60 seconds $(60'')$.

Angles at the centre of a circle are proportional to the arcs upon which they stand. A method of measuring angles based upon this fact is sometimes used, and is known as the method of *circular measurement*. For this purpose a unit angle is chosen which, at the centre of a circle, stands upon an arc equal in length to the radius of the circle. This unit angle is called a *radian*. The length of the whole circumference of a circle is 2π times the length of the radius; therefore the angle subtended by the whole circumference must equal 2π radians. But the whole circumference subtends four right angles, or $360°$. Therefore, $360° = 2\pi$ radians, or $1° =$

$\frac{\pi}{180}$ radians. We can, therefore, convert the measure of an angle in degrees to its measure in radians by multiplying by the factor $\frac{\pi}{180}$.

$$\pi = \frac{22}{7} \text{ or, more nearly, } 3{\cdot}1416$$

It will be convenient here to define certain **ratios** of an angle to which reference is often made. These are really the ratios which exist between the lengths of the three sides of a right-angled triangle to which the particular angle also belongs. If, for instance, B A C (Fig. 2) be the angle, we obtain the required right-angled triangle by drawing from *any* point P on A B a straight line P M at right angles to A C.

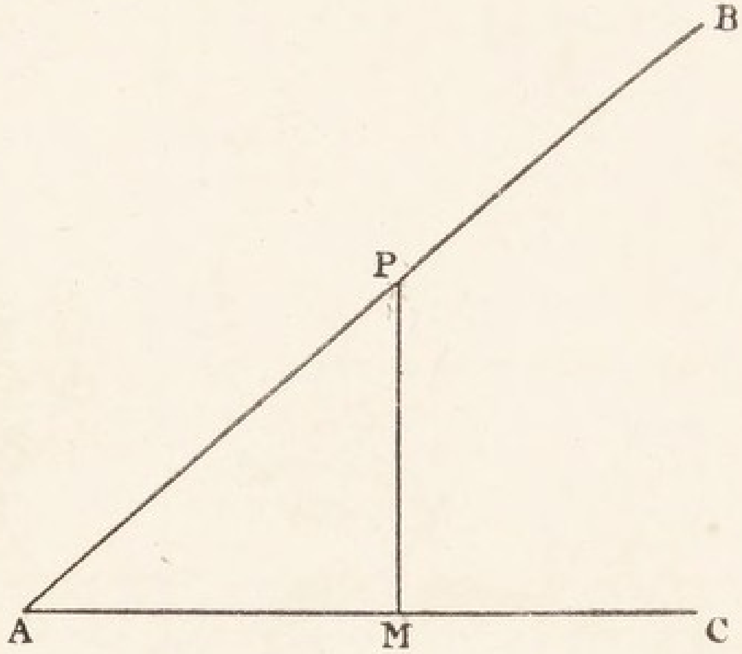


Fig. 2.—Ratios of an angle.

If P moves up or down A B the actual lengths of the sides P M, A M, A P alter, but always in the same proportion, so that the *ratios* between them remain unchanged so long as the angle B A C does not alter. They are, therefore, functions of this angle and serve to identify it.

The ratio $\frac{PM}{AP}$ or $\frac{\text{length of side opposite the angle}}{\text{length of side opposite the right angle.}}$ is called the *sine* of the angle;

The ratio $\frac{AM}{AP}$ or $\frac{\text{length of side adjacent to the angle.}}{\text{length of side opposite the right angle.}}$ is called the *co-sine* of the angle;

" " $\frac{PM}{AM}$ or $\frac{\text{length of side opposite the angle}}{\text{length of side adjacent to the angle}}$ is called the *tangent* of the angle.

That side of a right-angled triangle which is opposite the right angle is called the *hypotenuse*.

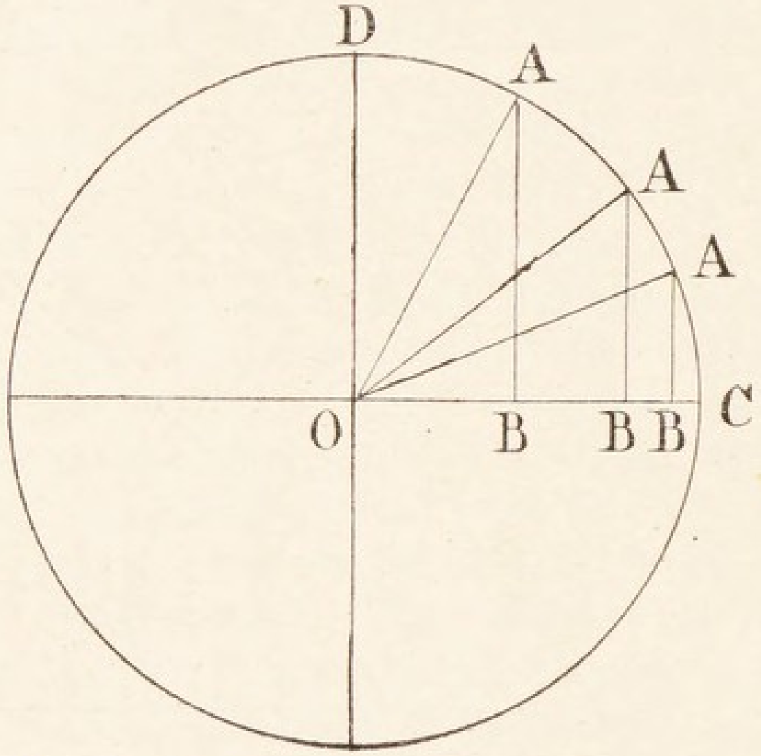


Fig. 3.—Comparison of θ and sin θ.

When we are concerned only with *small* angles we shall sometimes assume that the circular measure of the angle is numerically equal to its sine; the following argument shows to what extent the assumption is justified:—

Let θ radians be the circular measure of the angle AOC (Fig. 3);

$$\text{then } \theta = \frac{\text{arc } AC}{OA}$$

$$\text{and sin } = \frac{AB}{OA}$$

These fractions will be equal when

$$\text{arc } AC = AB$$

This is not quite the case until A coincides with C and the angle A O C is zero, but it is nearly true as long as θ is a very small angle, as the values in Table V (p. 432) show.

It will be convenient to notice here that when A moves round to D (Fig. 3), the angle A O B becomes 90° and at the same time A B becomes D O, so that then

$$\frac{AB}{OA} = \frac{DO}{OA} = 1$$

$$\therefore \sin 90° = 1$$

It is evident also that the sine of an angle can never be greater than unity.

Exercises

1. What is the average velocity possessed by a body falling from rest, (*a*) during the third second of the fall, (*b*) during the first three seconds of the fall? Through what distance will it fall in each period?

2. In a system of units in which the unit of length is a yard and the unit of time is a minute, state in words (*a*) the unit of velocity, (*b*) the unit of acceleration. Express the numerical values of these units in terms of the British foot-second units.

3. A bullet passes in succession through three screens at distances of 1,000 ft. apart. It is found by an electrical method that the time taken by the bullet to pass from the first screen to the second is 0·8 sec., and from the second to the third is 0·86 sec. Assuming that the motion of the bullet is uniformly retarded, find the value of the acceleration. [*First M.B.*]

(For Answers, *see* p. 388.)

CHAPTER II

KINETICS

Force—Newton's Laws of Motion—Gravitation—Atwood's Machine—Energy—Kinetic Energy—Potential Energy — Work — Power — Pressure — Tension — Graphic Representation of Forces and Velocities—Exercises.

Force.—When our observation extends beyond the motion itself, and we begin to inquire into the *cause* of the motion, the problem is no longer one of kinematics but belongs to *kinetics* (κῑνητικός, putting in motion), a larger division of the comprehensive science of Dynamics (δύναμις, force). This science of dynamics investigates the laws which regulate the action of Forces, whether this results in motion or not. Force always *tends* to produce motion, but it may not always succeed, because the action of one force may, as we shall see later, be exactly counteracted by the effect of another. When, as the result of this balance of forces, the body remains at *rest*, the problem belongs to the branch science of *Statics* (στατικός, causing to stand still). The relation of the parent science to its various subdivisions is therefore :—

Dynamics

Kinetics (Chap. II.)	Statics (Chap. III.)
Kinematics (Chap. I.)	

Sir Isaac Newton (b. 1642, d. 1727) clearly recognized that the cause of motion is force. His famous "Laws of Motion" were first published, in Latin, in 1687. The **first law of motion** may be literally translated thus :—

> Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by impressed force.

This statement recognizes that matter possesses a property, *inertia*, which tends to preserve the *status quo* by opposing, as it were, a passive resistance to any disturbance of this.

Newton regarded the statement as an axiom : (1) if a body be at rest it will remain at rest unless some force acts on it ; if in motion, the velocity of motion must continue uniform unless some force causes it to increase or diminish ; (2) the direction of motion must continue unchanged, and therefore rectilinear, unless some force cause it to be diverted. This law, therefore, supplies us with a definition of force which is often expressed as follows :—

> Force is that which produces, or tends to produce, motion or change of motion.

The **second law of motion** may be translated as follows :—

> The change of motion (produced) is proportional to the impressed force producing it, and pursues the direction in which that force is impressed.

This law leads to a method of measuring forces. When we change the velocity with which a mass is moving, we also change its momentum (p. 12). The change in the momentum will serve to measure the

force. It seems obvious that whatever change in the momentum is produced by a force, twice the force will produce twice the change, and three times the force will produce three times the change, and so on. In other words, the change is directly proportional to the force. For a given mass, m, change of momentum, $m\,v$, means change of velocity; the change of velocity per unit of time is acceleration, a; the change of momentum *per unit of time* is therefore $m\,a$. This will, therefore, be proportional to the acting force (F); so that—

$$F = k\,m\,a$$

where k is some constant. The most convenient value for this constant to have is unity. In order that it may have this value we must choose for our unit of force that force which will generate unit change of momentum in unit time, or, in other words, will give to unit mass unit acceleration. If this choice is made, then $F = 1$, $m = 1$, $a = 1$, and therefore $k = 1$.

On this condition the British (absolute) unit of force will be the force which will give to a mass of 1 lb. an acceleration of 1 ft. per second, in 1 sec. Now we know that its own weight, 1 lb. wt., gives to 1 lb. mass an acceleration of g ft. per second, in 1 sec., ∴ a force equal to 1 lb. wt. is evidently equal to g absolute units of force. In fact, the British absolute unit of force must be $\frac{1}{g}$ of 1 lb. wt., or equal to the weight of half an ounce (about). This unit is called a **poundal.**

Similarly, to make the constant k equal to unity in the C.G.S. system, we must choose for unit of force that force which will give to a mass of 1 grm. an acceleration of 1 cm. per second, in 1 sec. Since

the weight of 1 grm. gives to the mass of 1 grm. an acceleration of 981 cm. per second, in 1 sec., we see that the unit force required is $\frac{1}{981}$ of the 1 grm. weight; this force, however, is not called a *gramal*, it is called a **dyne.** The poundal and dyne are often called *absolute units of force*, while the lb. wt. and the grm. wt. are known as *gravitation units of force.* It is only when we employ absolute units that we have $k = 1$, and therefore

$$F = M a \quad .. \quad .. \quad .. \quad .. \quad (5)$$

If we employ gravitation units, we must put $k = \frac{1}{g}$, but the student will find it more convenient to work in absolute units and employ the simpler equation (5).

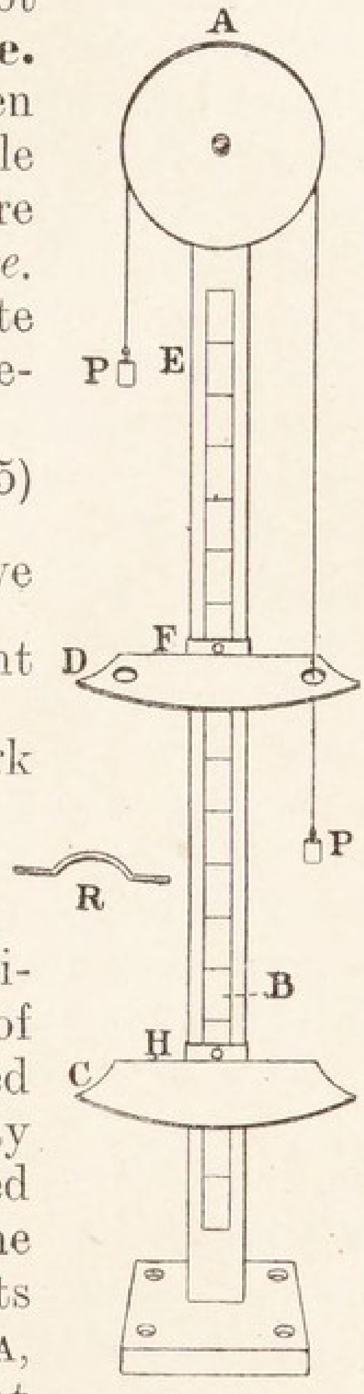


Fig. 4.—Atwood's machine.

This equation is of great importance. We have deduced it directly from Newton's second law of motion, of which it may be regarded as a symbolic expression. It may however, be experimentally verified by the use of Atwood's machine (Fig. 4). The apparatus consists essentially of a grooved wheel A, above a vertical pillar B. A light string, to the ends of which two equal masses, P, P, are attached, passes round A. A ring D and a platform C can be fixed at any desired point on B. A series of brass riders, R, of various weights is provided; they are so made that they will rest on P when in

motion, but will be removed by the ring when P in its descent passes through D.

Experiments may be performed with the apparatus as follows:—

One of the masses P is raised to E and is there placed on a drop platform, or otherwise detained, so that it can be instantly but smoothly released at a given signal. On this mass rests a rider R. This additional weight will cause P, when released, to descend with a velocity which will continually increase until R is removed. This happens when P passes through D. At this moment the time of motion from E to F must be noted by a stop-watch; let this be t_1 seconds. Throughout this time the weight of R has been employed in accelerating the velocity of P.

We must now measure the final velocity acquired by P at the end of this time. To do so, we must note carefully the time, t_2, which elapses between the removal of R and the impact of P upon the platform C fixed at a selected point H. There is usually a graduated scale on B which enables us to read the distance F H at once. Suppose that this distance is l feet. Then we know that P has acquired a velocity of l ft. per t_2 sec., and therefore of $\frac{l}{t_2}$ ft. per second. We know, too, that this velocity has been generated by the weight of R acting on a total mass of $2P + R$ for t_1 seconds; therefore the velocity generated per second, that is the acceleration, is $\frac{1}{t_1}$ of $\frac{l}{t_2}$

$$\therefore a = \frac{l}{t_1 \cdot t_2}$$

If the experiment is so arranged that each interval of time is exactly one second, then evidently $a = l$,

and the calculation is simplified ; this is often recommended, but the student will derive more benefit from less restricted experiments. In each experiment the impressed force, F, is the known weight of the added rider ; the mass moved, M, is the known sum of the two masses P, P and the mass of the rider, and the acceleration, a, is determined by the experiment as described. We have, therefore, all the data required for the verification of the important equation (5).

It is convenient in practice to perform one series of experiments in which the value of M remains constant while that of F is doubled, trebled, etc., and then to perform another series of experiments in which the value of F remains constant while that of M is doubled, trebled, etc. ; this is easily arranged by adjusting the values of P and R : thus, in the first series, the new values, P′ and R′, must be so chosen that we still have

$$\begin{aligned} 2\text{P}' + \text{R}' &= M = 2\text{P} + \text{R} \\ \text{but} \quad \text{R}' &= 2\text{R} \\ \therefore 2\text{P}' &= 2\text{P} - \text{R} \\ \therefore \text{P}' &= \text{P} - \frac{\text{R}}{2} \end{aligned}$$

In the second series the new values will be determined by the equations—

$$\begin{aligned} 2\text{P}' + \text{R}' &= 2M = 4\text{P} + 2\text{R} \\ \text{and} \quad \text{R}' &= \text{R} \\ \therefore 2\text{P}' &= 4\text{P} + \text{R} \\ \therefore \text{P}' &= 2\text{P} + \frac{\text{R}}{2} \end{aligned}$$

The first series of experiments will show that the acceleration varies directly as the force, or

$$a \propto F \text{ when } M \text{ is constant.}$$

The second series will show that the acceleration varies inversely as the mass, or

$$a \propto \frac{1}{M} \text{ when } F \text{ is constant.}$$

Therefore, when both F and M vary,

$$a \propto \frac{F}{M}$$

$$\text{or } F \propto M a$$

And this is equivalent to

$$F = k \times M a$$

where k is some constant factor which, by suitable choice of the absolute unit of force already defined —the poundal or the dyne—is made equal to unity, so that the relation becomes

$$F = M a$$

By combining this equation with some of those already obtained, we arrive at some interesting and useful results. Thus, from (2) and (5) we at once find

$$F = \frac{M v_2 - M v_1}{t} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

Or, in other words, force is measured by the change of momentum it produces per unit of time in the mass on which it acts.

Again, from (4) and (5) combined, we deduce—

$$F s = M a s$$

$$\therefore F s = \tfrac{1}{2} M v_2^2 - \tfrac{1}{2} M v_1^2 \quad \ldots \quad \ldots \quad (7)$$

The expression $\frac{1}{2} M v^2$ represents the *kinetic energy* of a mass, M, moving with velocity v. $\frac{1}{2} M v_1^2$ is evidently the kinetic energy at the *beginning* of the path s, and $\frac{1}{2} M v_2^2$ is the kinetic energy at the end of the path. The expression on the right-hand side

of (7) is therefore the change in kinetic energy which has taken place while the mass has travelled the distance s. Dividing both sides of the equation by s, we see that

$$F = \frac{\frac{1}{2} M v^2{}_2 - \frac{1}{2} M v_1{}^2}{s}$$

or, in words, force is measured by the change of the kinetic energy it produces in a mass, per unit of space traversed.

Energy and work.—The various forms of **energy** associated with matter are sometimes divided into two classes. One class includes all forms in which the energy is due to motion of matter. This energy is called **kinetic** energy and was referred to in the previous section. It may, however, reside in a mass apparently at rest, for every visible mass is really composed of a number of invisible masses, or molecules, and these are in motion even when the whole mass appears to us to be still. This internal molecular motion is also a form of kinetic energy. The most familiar manifestation of it to our senses is heat

All energy other than kinetic is frequently described as **potential.** Energy of this kind is due to the position or arrangement of the matter with which it is associated. Thus, every mass which occupies a position vertically above the surface of the earth has potential energy. It is attracted towards the earth by the force of gravity; if allowed to fall it loses this potential energy but acquires kinetic energy. The energy which resides in a loaded gun is potential; it is due to the arrangement of the materials of the charge, and their position relatively to the parts of the gun. When fired, much of this potential energy is converted into the kinetic energy of the projected bullet, but some is transferred as

heat to the gun and the products of explosion. The energy in a wound-up watch-spring is potential, and continues to be gradually exchanged for kinetic as long as the watch is working; owing, however, to the friction inherent in machinery in motion, some of the converted energy is again dissipated as heat. Zinc and dilute sulphuric acid, when not in contact, possess potential energy by virtue of their ability to react chemically with each other when contact takes place; this chemical energy may, by the assistance of suitable supplementary apparatus, be expended in producing a current of electricity; this electrical energy may be converted into the kinetic energy associated with heat, light, or mechanical motion; or it may be employed to decompose a chemical compound and thus restore the potential energy which the separate constituents possessed before combination took place. These illustrations sufficiently indicate, but by no means exhaust, the many transformations which this mysterious principle called Energy can undergo.

We can neither create nor destroy energy; the same is true of matter; but energy is *not* matter. It is always associated with matter, of which it is, as it were, the informing spirit. Matter devoid of energy would be the most complete conception of a corpse. The statement that energy can neither be created nor destroyed is known as the law of *conservation of energy.* Although in accordance with this law the transformations of energy to which we have referred lead to no absolute loss of energy, yet we rarely obtain the full exchange value to which the law entitles us. Owing to imperfections inherent in the mechanism by which the transformation is effected, the initial energy is not transformed entirely into the kind required. Some is often

unavoidably transformed into heat, as in some of the instances already quoted. In this way energy, although not really destroyed, may be so distributed as to cease to be effective for purposes useful to us. The medium by which the exchange, or transformation, of energy is effected is **work.** When a weight is raised by muscular effort, work is said to be done against gravity; the energy of the muscular tissue, previously derived from the consumption and assimilation of food, is transformed into the potential energy gained by the weight. If a weight, W, be thus raised to a vertical height, h, the work done against gravity is said to be $W h$, and the potential energy gained is valued in the same terms. More generally, if work is done by or against a force, F, while the point of application of the force moves through a distance, s, then the value of the work is $F s$. This expression forms one side of equation (7) on p. 24, which, therefore, states that the work done by the motive force is equivalent to the kinetic energy produced. The body falling from rest at a height, h, is really a particular illustration of this. The initial potential energy is $W h$ or $m g h$. The final kinetic energy at the earth's surface is $\frac{1}{2} m v^2$. The work done by gravity transforms one into the other, and we have

$$m g h = \tfrac{1}{2} m v^2$$

whence $v^2 = 2g h$, the same result as we obtained before (p. 11) by a different method. The unit of work is a derived unit; it is the work done by, or against, the unit force, while its point of application moves through the unit length. The British absolute unit is a **foot-poundal.** The British gravitation unit is a **foot-pound.** The C.G.S. absolute unit, however, is not called a centimetre-dyne but an

erg. The joule, $= 10^7$ ergs, is a practical unit to which reference will frequently be made.

It is sometimes convenient to express not merely the total quantity of work performed, but the *time rate of working*, or **power** of the agent. A practical unit often employed for this purpose is a "*horse-power*"; it is equal to 550 foot-pounds per second. A convenient practical unit in the C.G.S. system is the *watt*, which is equal to 1 joule per second. It may be useful to add here that 1 horse-power = 746 watts, and that 1 watt = 0·00134 horse-power.

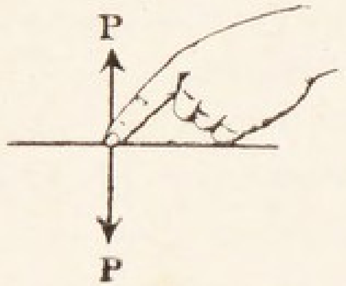


Fig. 5. — To illustrate pressure.

The third law of motion. — Newton's third law is as follows:—

> "To every action there is an equal and contrary reaction."

In its earlier and more limited applications this law recognized the dual aspect of force. When the finger presses the table, the finger is also pressed by the table (Fig. 5). At the point of contact a force, P, acts *from* the finger *towards* the table, and an equal counterforce reacts *from* the table *towards* the finger. To this dual stress we give the name **pressure.**

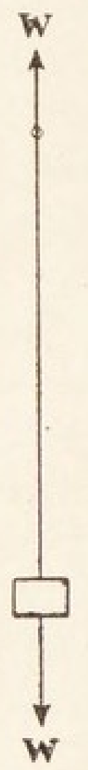


Fig. 6. — To illustrate tension.

Similarly, when a weight is suspended by a string from a nail, it is obvious that the reaction of the nail must exert an equal and opposite force. In this case (Fig. 6) the two forces act *from* each other. To this dual stress, which keeps the string taut, we give the name **tension.**

In its widest interpretation the third law of motion is a great generalization as far-reaching and important as that of the conservation of energy (p. 26); by the operation of Newton's law the physical and chemical equilibrium of every material system is maintained or recovered; many more recent laws, e.g. the law of mass action, Lenz's law (p. 373), etc., may really be regarded as particular examples of this great general law.*

Graphic representation of forces and velocities.—A straight line may be drawn (1) of any length, and (2) in any direction. We can, therefore, employ straight lines to represent graphically, *in magnitude and direction*, forces, velocities, etc. We shall often find it convenient to do so.

It will be convenient to tabulate here for reference the units hitherto discussed :—

Unit of	*British*	*C.G.S.*	*See p.*
Length .	1 ft.	1 cm.	4
Mass . .	1 lb.	1 grm.	5
Time . .	1 sec.	1 sec.	6
Velocity .	1 ft. per sec.	1 cm. per sec.	8
Acceleration	1 ft. per sec. per sec.	1 cm. per sec. per sec.	10
Force .	1 poundal	1 dyne	20
Work .	1 ft.-poundal	1 erg.	27
Energy .	—	—	27
Momentum	1 lb.-ft. sec.	1 grm.-cm. sec.	12
Power .	1 h.p. (= 550 ft. lb. per sec.)	1 watt (= 10^7 ergs per sec.)	28

EXERCISES

1. Two bodies are in motion. The mass of one is 10 lb. and its motion is accelerated at the rate of 12 ft. per sec.

* *See* "Manual of Chemistry" (Luff and Candy).

per sec.; for the other the corresponding values are 4,000 grm. and 400 cm. per sec. per sec. Find the force acting on each.

2. A body, whose mass is 12 lb., has an acceleration of 4 in. per sec. per sec. What is the magnitude of the force acting upon the body? [*First M.B.*]

3. The weights on an Atwood machine move 100 cm. in 5 sec., starting from rest. One weight weighs 1 grm. more than the other. Given $g = 981$, calculate the mass of each weight. [*Ibid.*]

4. A body has kinetic energy amounting to 10,000 ergs, and a momentum of 500 grm.-cm. per sec. Find its velocity and its mass. [*Ibid.*]

5. A bullet moving at the rate of 1,000 ft. per sec. is stopped after penetrating 3 ft. into a sandbank. Assuming the motion to be regularly retarded during the short time of its stoppage, find (1) how long it takes, (2) how great the retarding force is compared with the weight of the bullet. [*Ibid.*]

6. A stream of water from a fire hose is delivered at the rate of 5 lb. per sec., and strikes a wall perpendicularly at a speed of 60 ft. per sec. What is the momentum of the water arriving per sec., and what is its energy? Assuming the water not to rebound, what force does the water exert on the wall? In what units are the results stated?

(For Answers, *see* p. 388.)

CHAPTER III

STATICS

Centre of Gravity—Equilibrium—Composition and Resolution of Forces and Velocities—Friction—Mechanical Powers—Parallel Forces—Exercises.

Centre of gravity.—A force is fully determined when we know (1) its magnitude, (2) its direction, (3) its point of application. We have already represented the weight of a body as a force of definite magnitude W, acting in a vertical direction; we shall now show that this force may be said to be applied at a certain point called the *centre of gravity* of the body. Let the triangular board A B C (Fig. 7) be freely suspended at the angular point A by string attached to the nail P. The board can only remain at rest

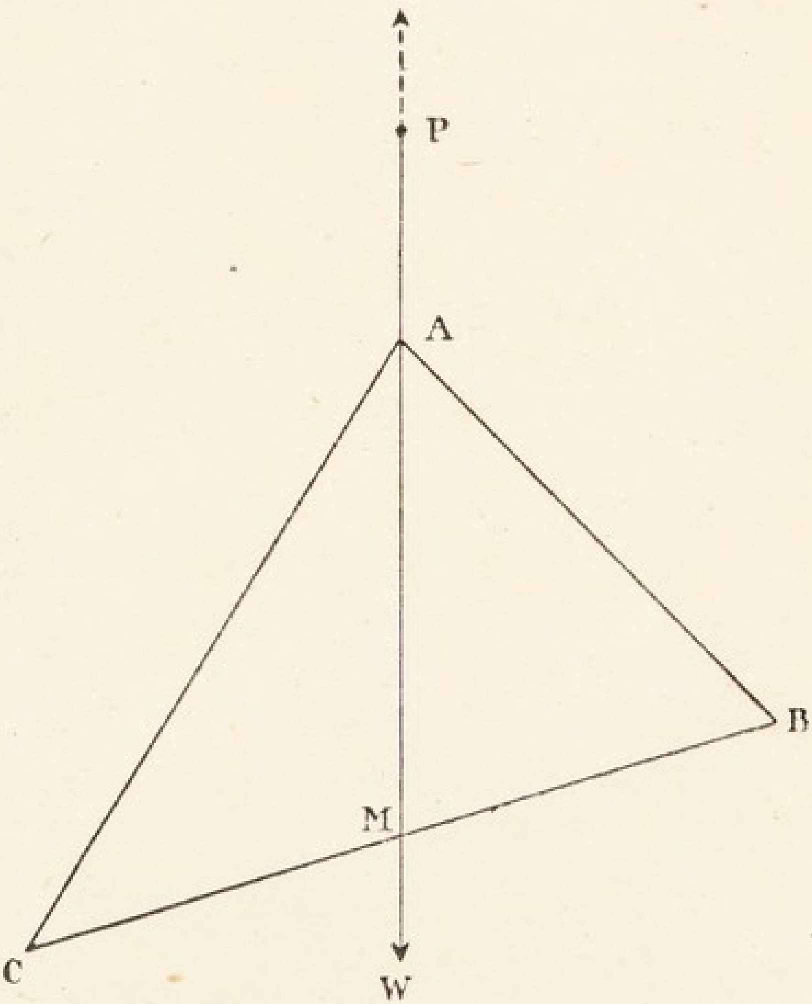


Fig. 7.—To illustrate the centre of gravity.

when W acts in the vertical direction P A M. Mark the line A M on the board. Let the board be now suspended at B (Fig. 8). It will come to rest when P B N is vertical. Mark the line B N; B N and A M intersect in the point G, which indicates the position of the centre of gravity. The line of action of W always passes through this point, which may therefore always be regarded as its point of application.

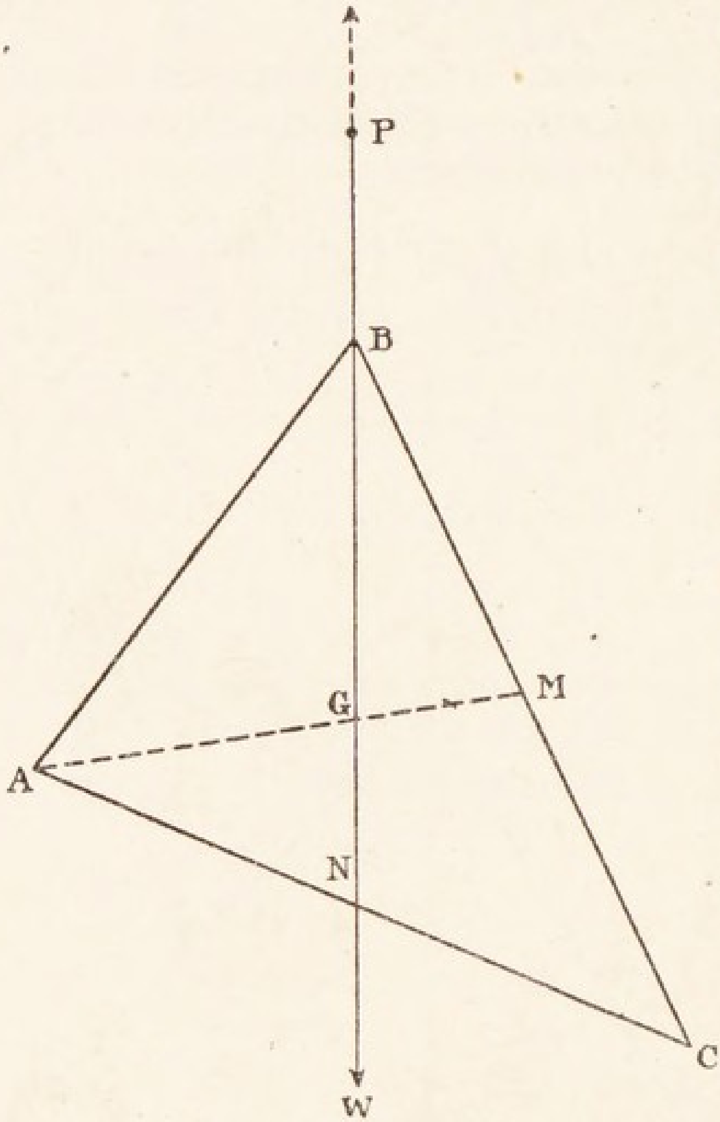


Fig. 8.—To illustrate the centre of gravity.

Equilibrium.—The positions of rest assumed by the board in Figs. 7 and 8 are positions of *stable* equilibrium, because if the board be displaced from either of these positions the forces tend to restore it. This is always the case when G is vertically below the point of suspension P. It would be possible for the board to be in equilibrium if resting on a nail passing loosely through A with G vertically *above* A. Such a position, however, would be one of *unstable* equilibrium, since if displaced from it the board would not return to this position, but would depart still farther from it, and assume after a time the stable position

in which G is vertically below A. When the point of suspension is neither above nor below but *coincides* with the centre of gravity, the position is one of *neutral* equilibrium. When displaced from this position the board remains equally in equilibrium in its new position.

Composition and resolution of forces and velocities.—When two forces act on a body, we can often find a single force which will produce the same effect on the body as the two forces combined. This force is called the *resultant*, and the two forces are called its *components*. A force which will completely counteract the two forces is sometimes called the *anti-resultant*. When the two components, P, Q, act in the same straight line and in the same direction the resultant R is evidently equal to their sum P + Q, and must act in the same direction. When the two components act in opposite directions, R is equal to their difference P − Q, and acts in the direction of the greater. Suppose, however, that the directions of P and Q are inclined to each other and meet at a point O (Fig. 9). The resultant may then be found by the following construction: On the direction of P measure a length O A, to represent the magnitude of P on some convenient scale; then O A represents the force P, both in magnitude and direction. Similarly, measure O B to represent Q in magnitude and direction. Complete the parallelogram O A C B. The diagonal O C of this parallelogram will

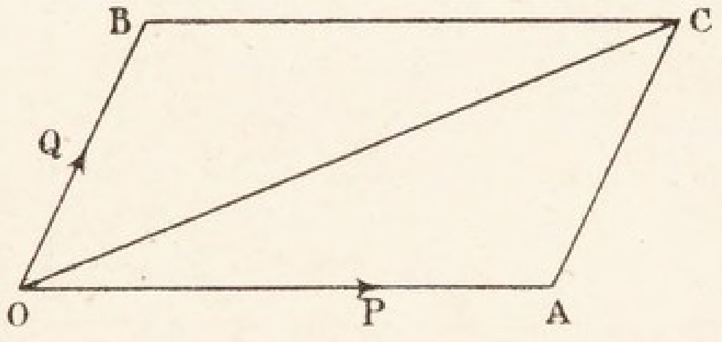


Fig. 9.—Parallelogram of forces.

represent the resultant R in magnitude and direction. This proposition is known as the Parallelogram of Forces. It may be verified experimentally with the apparatus shown in Fig. 10, and this experiment is described later (p. 405).

If O C (Fig. 9) represents the resultant of P and Q, then C O will represent their anti-resultant; also, A C is parallel and equal to O B, and therefore can also represent Q in magnitude and direction; hence three forces which are in equilibrium (P, Q, and their anti-resultant) are represented in magnitude and direction by the three sides of a triangle O A C, taken in order one way round, O A, A C, C O. This is true of any three concurrent forces in equilibrium. The statement forms part of the proposition known as the Triangle of Forces. The converse is also true and completes the proposition: if three concurrent forces can be so represented they are in equilibrium.

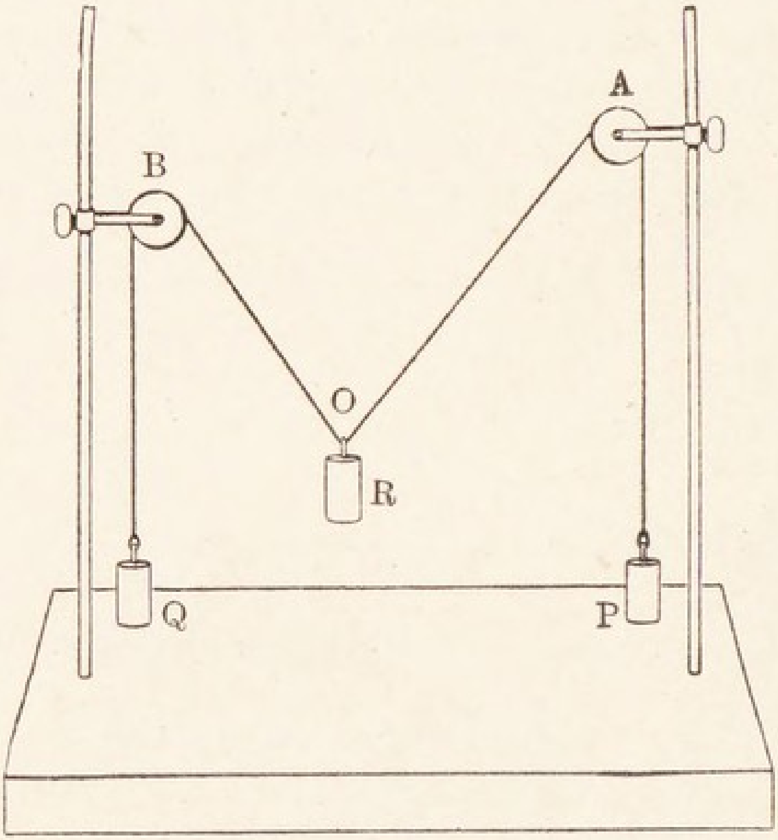


Fig. 10.—To illustrate parallelogram of forces.

Just as two forces may be *compounded* into one resultant force, so a single force may be *resolved* into two components. In this case we draw the line O C (Fig. 9) to represent the single force in

magnitude and direction; we then draw the two adjacent sides, O A, O B, in any desired directions, and finally complete the parallelogram by drawing through C one straight line parallel to A O, meeting O B in B, and another straight line parallel to B O, meeting O A in A. The points A and B are thus fixed, and the required components are represented by O A and O B.

The resolution and composition of *velocities* is effected in exactly the same way; the parallelogram and triangle propositions, in fact, apply to all directed quantities, or *vectors*, as they are usually called.

Since many different parallelograms may be constructed with the same diagonal O C, it is clear that the same force, or velocity, may be resolved into two components in a variety of ways. Of these the one most useful in practice is when the two components are at right angles to each other. This is illustrated in the following discussion of friction.

Friction.—When a mass remains at rest on the surface A C of an inclined plane (Fig. 11), the reaction, R, of the plane must be equal and opposite to W, the weight of the mass, which acts vertically downwards. Resolving as in the figure, we see that R is really the resultant of two components—N, normal, or perpendicular to the plane, and F, tangential, or along the plane. The component F is due to friction, and tends to prevent the mass from slipping. So long as the mass remains at rest, the three forces F, N, and W must be in equilibrium, and they are therefore proportional respectively to the sides O H, H L, L O of the triangle O L H (*see* Triangle of Forces, p. 34). But this triangle is equiangular, and therefore similar, to the plane triangle A B C; for the alternate angles L O H and C A B are equal, and also the right angles O H L and A B C; therefore,

the third angles O L H and A C B are equal; the sides, therefore, which subtend the equal angles must stand in the same ratio, that is—

$$OH : HL : LO :: AB : BC : CA$$

$$\text{Hence, } \frac{F}{N} = \frac{OH}{HL} = \frac{AB}{BC} = \frac{\text{height of plane}}{\text{base of plane}}.$$

This ratio (p. 16) is the tangent of the angle A C B.

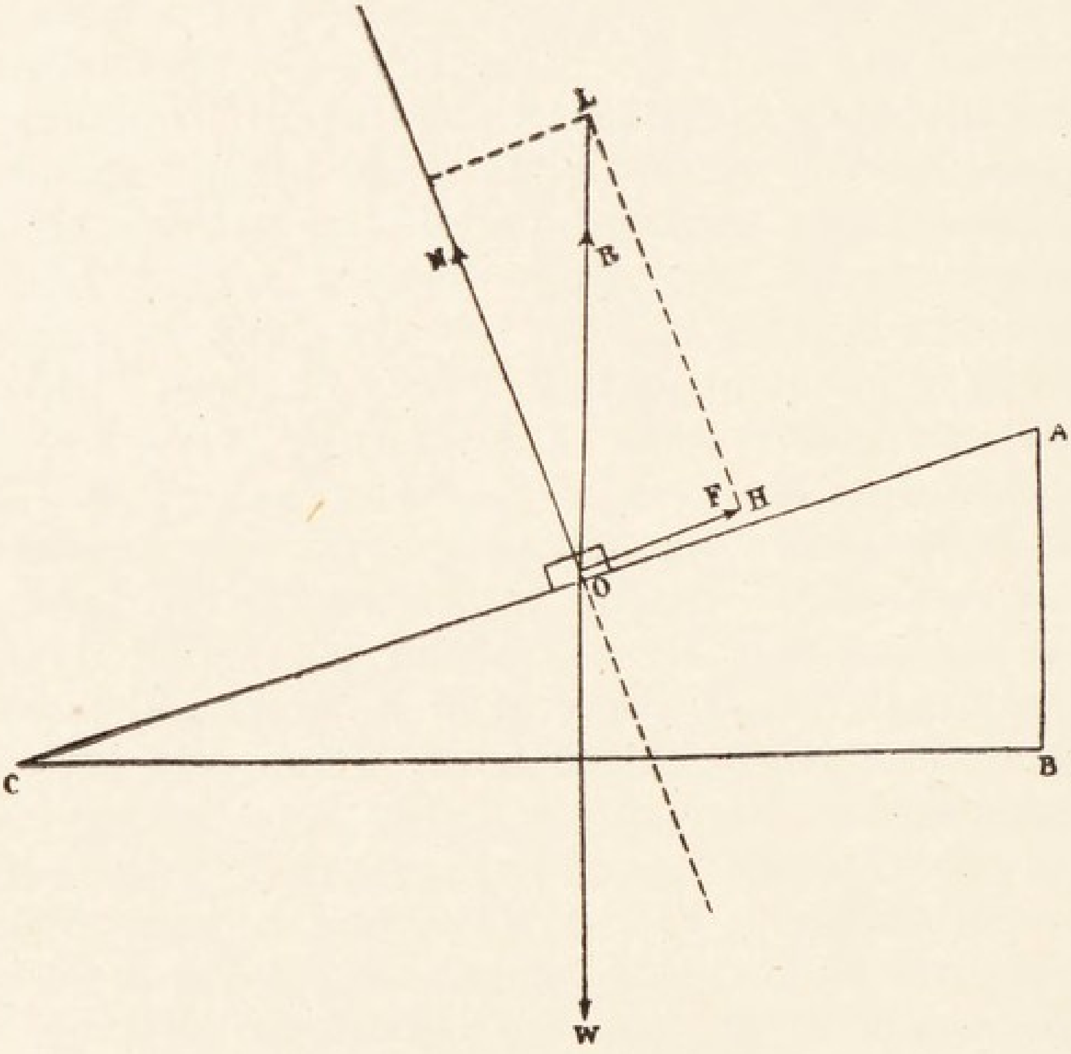


Fig. 11.—To illustrate friction.

If the inclination of the plane be such that the mass is just on the point of slipping, the ratio $\frac{F}{N}$ is called the **coefficient of friction** for the two surfaces in contact. We can, therefore, determine it experimentally, with suitable apparatus, by tilting

A C about C till the slipping point is reached, and then measuring height and base of the plane. If θ be the value of the angle A C B at this moment, we see that the coefficient of friction is tan θ. It is sometimes denoted by μ, and we then have $F = \mu N$. This is clearly the maximum value of F, and occurs when the body is on the point of sliding. It bears to the force N, acting normally to the contact surface, a ratio μ, which is constant for any particular pair of surfaces. The value of F is evidently not affected by the extent of the surface in contact. The total work done against friction by a body in sliding down the plane must be F $\times$ A C (p. 27), and is clearly independent of the *velocity* with which the body slides.

The conclusions we have here reached constitute the *laws of sliding friction*.

The resolution of the wind pressure on a sail surface into two components—the normal component at right angles to the surface, and the tangential component along the surface—enables a ship to sail and an aeroplane to rise and remain above the earth. For the sailing ship Nature supplies the wind; the aeroplane must to a great extent make its own wind. When driven through the air at a high velocity by its petrol engine in the direction O A (Fig. 12) the resistance of the air reacts in the direction A O on the sail surface. If the sail plane is set at a small angle to this direction, the reacting force of the wind is resolved into (1) a normal component O N and (2) a tangential component O T: O N may be further resolved into a vertical component O V and a horizontal component O H; O V must be opposite to W, the weight of the aeroplane and load, and at least equal to it in magnitude, or the machine will not be supported in the air, and

it evidently cannot rise until O V is greater than W. The magnitude of the original wind force, and therefore of each component derived from it, is directly proportional to the square of the velocity with which the plane travels. This velocity really creates the wind, and must therefore evidently reach a certain minimum value for any particular machine in order that O V may be greater than W.

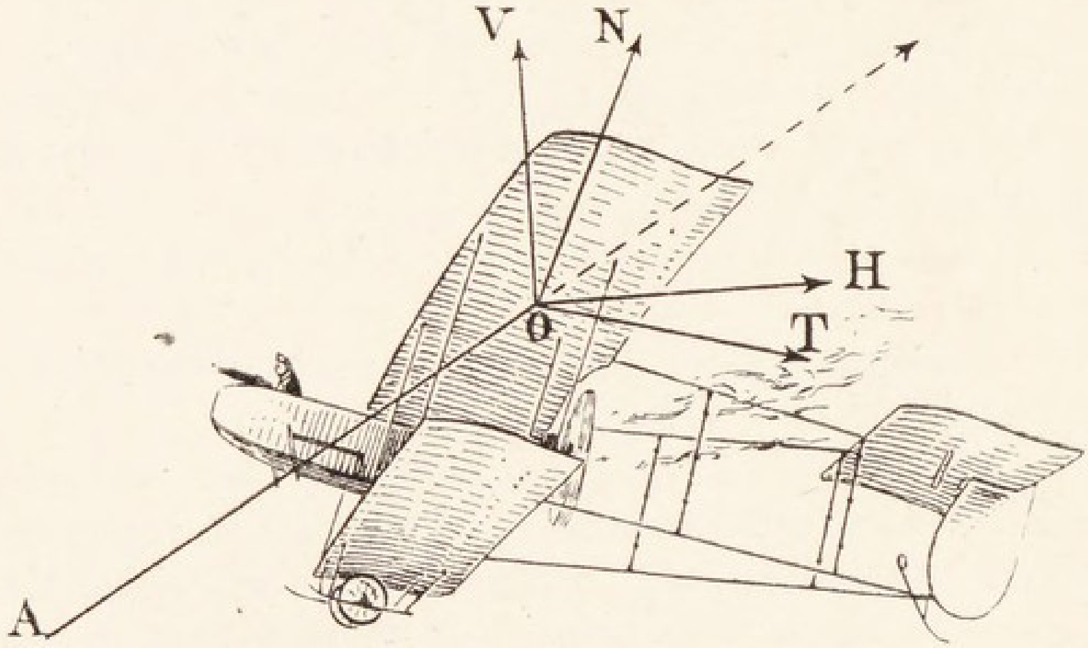


Fig. 12.—To illustrate the lifting component of the wind force acting on the sail surface of an aeroplane.

Mechanical powers.—Certain machines, or, as they are called, "mechanical powers," enable a weight W to be raised by a power P of less magnitude than W. They, therefore, confer "mechanical advantage," which is measured by the ratio $\frac{W}{P}$.

The following are often employed :—

1. **The lever.**—This is a rigid rod which, in use, rotates about a fixed point C (Fig. 13), called the *fulcrum.* The segments of the rod, C A, C B, between the fulcrum and the points of application of P and W are called the *arms* of the lever.

The power of a force to produce rotation depends upon (1) the magnitude of the force, and (2) the perpendicular distance between the line of action of the force and the point, or line, about which rotation takes place. This power is called the *moment* of the force about the turning-point or line. It is measured by the product of these

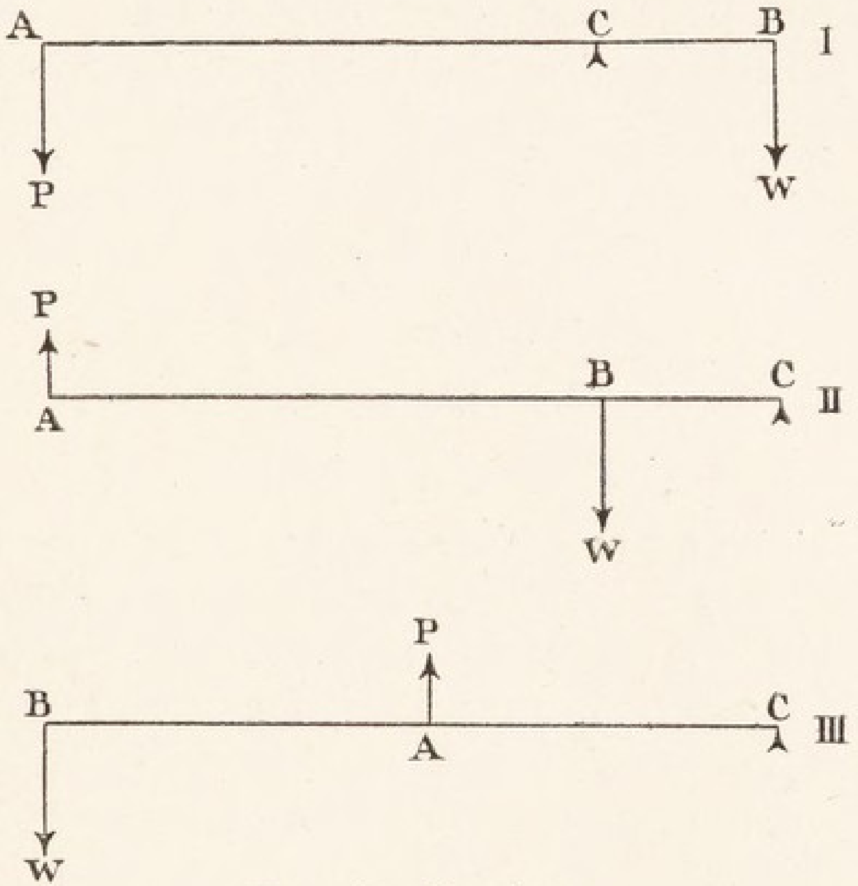


Fig. 13.—The lever.

two factors, (1) and (2). Thus (Fig. 13) the moment of P is P × C A, and the moment of W is W × C B. They tend to produce rotation about C in *opposite* directions, and, therefore, will be in equilibrium if

$$P \times CA = W \times CB$$

The mechanical advantage $\frac{W}{P}$ is therefore $= \frac{CA}{CB}$.

Levers are often classified in three orders, or classes, illustrated by the three diagrams in Fig. 13.

The common balance is an example of *Class* I. The arms are of equal length in the pair of scales, but in the "steelyard" type a constant force P is applied at different points on C A to balance different values of W.

In the beam and scales we can find the true weight of P, even if the arms are not equal, by placing it first in one scale and then in the other. Suppose that it is balanced in the first case by W_1, and in the second by W_2; we know then that

$$P \times CA = W_1 \times CB$$

and $$P \times CB = W_2 \times CA$$

Therefore, by multiplication,

$$P^2 = W_1 \times W_2$$

and $$P = \sqrt{W_1 \times W_2}$$

We can also find the true weight of P with a false balance of this type by the method of *counterpoise* or substitution. Place P in one pan and counterpoise it with sand, etc., in the other pan; then remove P and substitute weights sufficient to balance the undisturbed counterpoise. The sum of these weights is the true weight of P.

A pair of nut-crackers, or a lemon-squeezer, is an example of *Class* II. The hinge is at C, the nut or lemon at B, and the pressure of the hand is applied at A.

The human forearm, when raising a weight supported on the palm of the hand, is an example of *Class* III. The fulcrum C is at the elbow, where the ulna articulates with the humerus; the power is applied by the biceps muscle at its point of attachment between the elbow and hand.

2. **The wheel and axle** (Fig. 14).—When this machine is worked, A, the point of application of P, describes the circumference of a circle whose radius

is C A; B, the point of application of **W**, describes at the same time the circumference of a smaller circle whose radius is C B. Equating the work of each, we have (p. 27)—

$$P \times 2\pi \, . \, C\,A = W \times 2\pi \, . \, C\,B$$

$$\therefore \quad \frac{W}{P} = \frac{C\,A}{C\,B}$$

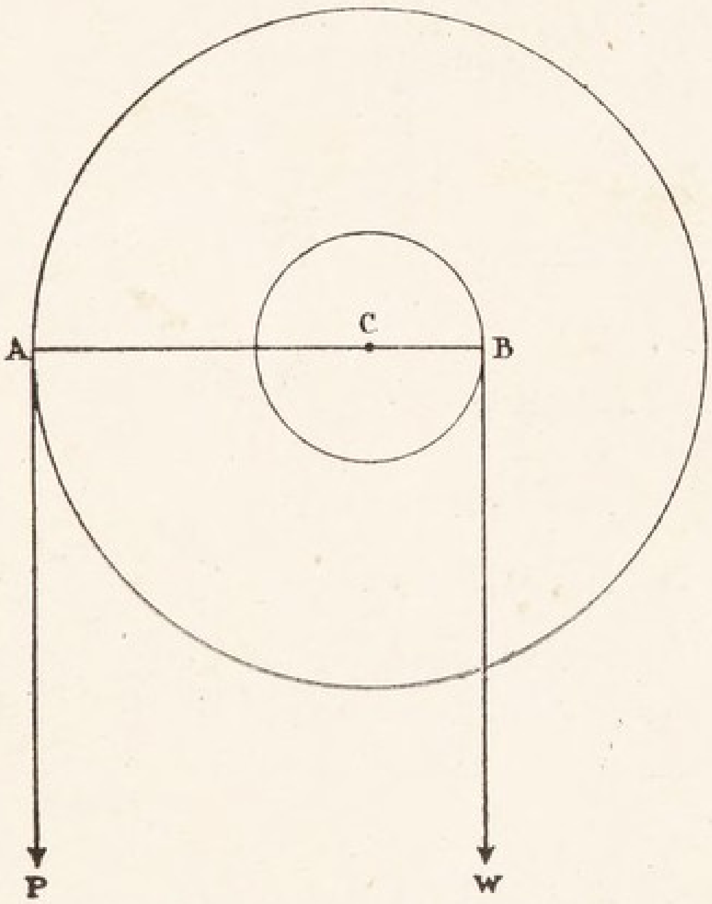


Fig. 14.—The wheel and axle.

3. **The inclined plane** (Fig. 11, p. 36).—If we suppose the plane to be *smooth* and a force P to be applied parallel to the plane, it is clear that while P pulls the body from C to A along C A, W is raised through the vertical height B A; equating the work of each, we have, therefore—

$$P \times C\,A = W \times B\,A$$

$$\therefore \quad \frac{W}{P} = \frac{C\,A}{B\,A} = \frac{\text{length of plane}}{\text{height of plane}}.$$

4. **The screw** (Fig. 15).—In structure, this machine is essentially an inclined plane wrapped round a cylinder whose circumference is equal in length to the base of the plane; the "pitch" of the screw—the distance between two consecutive threads—is equal to the height of the plane. A, the point of application of P, moves through a distance 2π C A, while the pressure W is made to move through a distance equal to the pitch of the screw. Equating the work, we have, therefore—

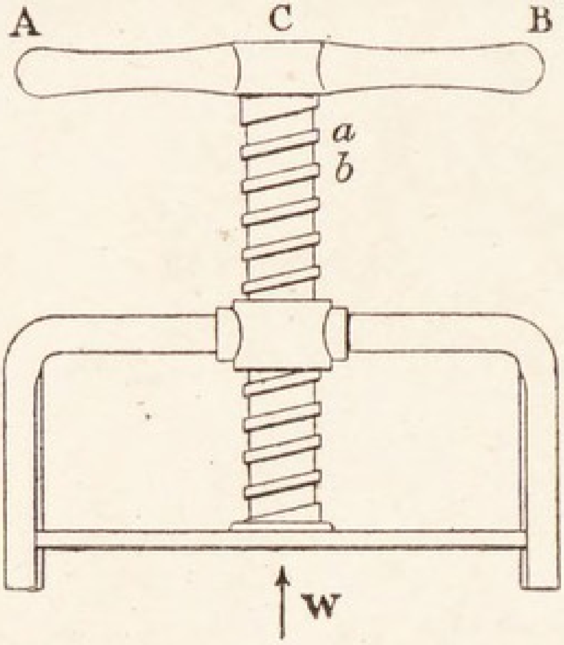


Fig. 15.—The screw.

$$P \times 2\pi \,.\, C\,A = W \times a\,b$$

If, as is often the case, P is applied at B also, then we must add $P \times 2\pi$ C B, and have, therefore—

$$P \times 2\pi \,.\, A\,B = W \times a\,b$$

$$\text{or } \frac{W}{P} = \frac{2\pi \,.\, A\,B}{a\,b}$$

5. **The pulley.**—In its simplest form, that of a grooved wheel round which a rope is passed (Fig. 16), this machine confers no mechanical advantage, but may conveniently alter the *direction* of P. Thus a downward force P may serve

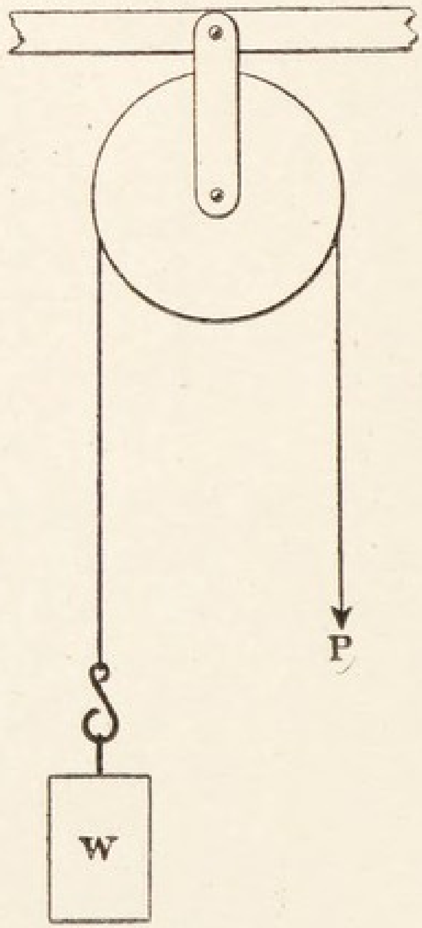


Fig. 16.—The pulley.

to raise W *upwards*. A combination of two or more pulleys may also confer mechanical advantage. In all cases the advantage may be calculated by the principle of equality of work already applied. If P descends and W ascends, we must have—

$$P \times \text{distance descended} = W \times \text{distance ascended}$$

$$\therefore \frac{W}{P} = \frac{\text{vertical descent of } P}{\text{vertical ascent of } W}$$

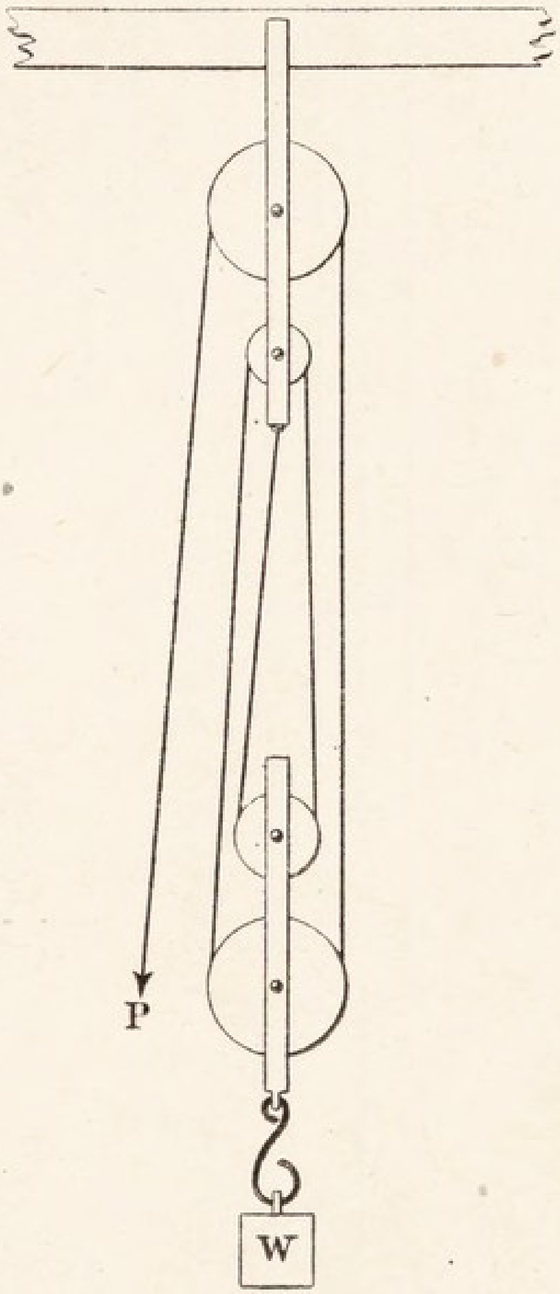


Fig. 17.—First system of pulleys.

Three **systems of pulleys** are generally described :—

(*a*) In the *first* system of pulleys (Fig. 17) the same string passes round every wheel. It is obvious that the descent of P is equally divided between the four parts of the string at the lower block, from which W hangs. W will, therefore, ascend by $\frac{1}{4}$ of the distance through which P descends, or $\frac{W}{P} = \frac{4}{1}$. Similarly, if there are n parts of string at the lower block, we shall have

$$\frac{W}{P} = \frac{n}{1}$$

(*b*) In the *second* system of pulleys (Fig, 18) all

the pulleys are movable, except one. A separate string passes round each movable pulley (A_1, etc.), and is attached at one end to the beam. It is clear that, if all the strings remain taut, when W ascends a distance h, A_1 must ascend h, and, therefore, the string on each side of A_1 must shorten by h, which means that a length of string $= 2h$ must be passed round, and therefore A_2 must ascend $2h$. Similarly,

A_3 must ascend $2 \times 2h = 2^2h$

A_4 must ascend $2 \times 2^2h = 2^3h$

and P must *descend* $2 \times 2^3h = 2^4h$. Hence by the equation of work—

$$W \times h = P \times 2^4h$$

$$\text{or } \frac{W}{P} = \frac{2^4}{1}$$

Similarly, if there are n movable pulleys, we shall have

$$\frac{W}{P} = \frac{2^n}{1}$$

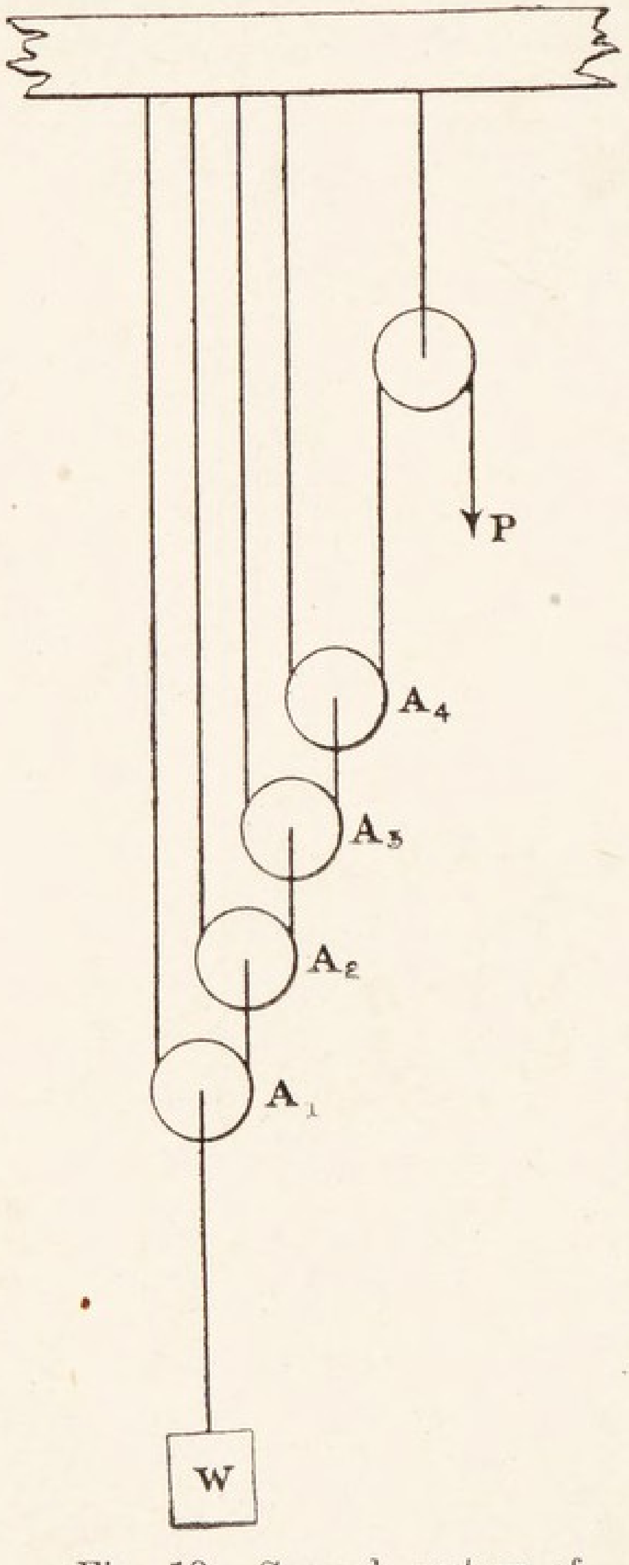


Fig. 18.—Second system of pulleys.

(c) In the *third* system of pulleys (Fig. 19), a separate string passes round each pulley, and is

attached at one end to the *weight*. If W ascends by a distance h, and all the strings remain taut, A_1 remains fixed, A_2 descends a distance h, and therefore $2h$ of A_2's rope passes on as well as the length h due to the rise of W; therefore A_2 descends by $2h + h$; similarly, A_4 descends by $2(2h + h) + h = 2^2h + 2h + h$, and P descends by $2(2^2h + 2h + h) + h = 2^3h + 2^2h + 2h + h = (2^4 - 1)\,h$.

$$\therefore\ W \times h = P \times (2^4 - 1)h$$

$$\text{or, } \frac{W}{P} = \frac{2^4 - 1}{1}$$

Similarly, if there are n pulleys, we shall have

$$\frac{W}{P} = \frac{2^n - 1}{1}$$

The weights of the pulleys themselves have been disregarded in all our examples, but in *this* system it should be noticed that they *assist* P.

Parallel forces.—When the two forces are parallel, as P and W (Fig. 13), the magnitude of the resultant is the algebraic sum of the forces; the direction of the resultant is parallel to that of the forces, and passes through a point C (Fig. 13) defined by the equation (p. 39) $P \times CA = W \times CB$. When the two parallel forces are equal and *unlike*, their algebraic sum is zero; they have therefore no resultant to produce translation and can only produce rotation. Two such forces constitute a

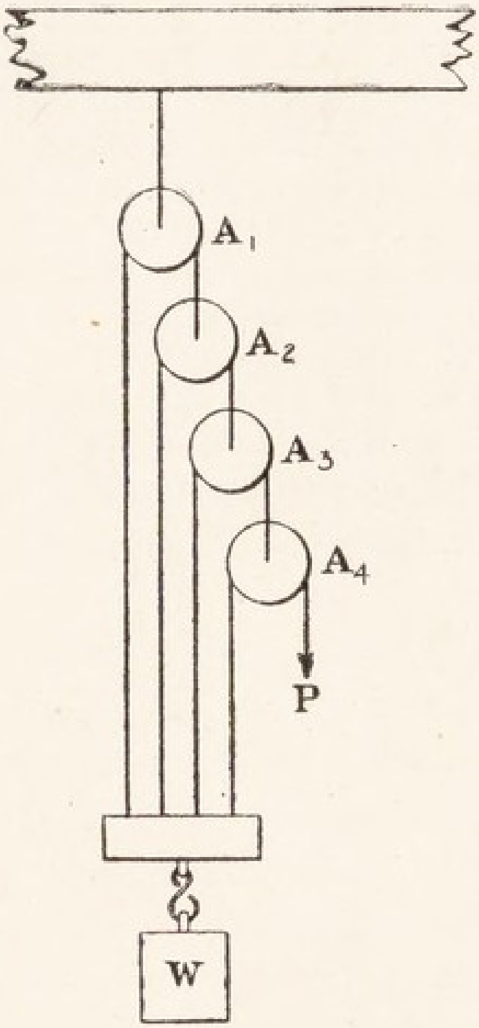


Fig. 19.—Third system of pulleys.

couple; the perpendicular intercepted between them is the *arm* of the couple; the product of one force and the arm is the *moment* of the couple. A screwdriver in use, or a screw press worked by both hands, illustrates the action of a couple (*see also* p. 313).

Exercises

1. The bob of a simple pendulum is deflected so that the string makes an angle of 60° with the vertical. Determine the direction and magnitude of the acceleration of the bob at the moment when it is released. [*First M.B.*]

2. A screw having a pitch of 1 cm. is worked by a power arm 50 cm. long. What is the theoretical mechanical advantage, and what force must be exerted on the power arm to lift 1,000 kilograms? [*Ibid.*]

3. If a man can exert at the handle of a windlass a force of 20 lb. and move the handle through 3 ft. a sec., calculate his power in proper units. [*First Professional.*]

(For Answers, *see* p. 388.)

CHAPTER IV

HYDROSTATICS

Properties of Matter—Mass and Weight—The Value of g—Extension—The Three States of Matter—Density and Specific Gravity—Methods of determining the Specific Gravity of Solids, Liquids, and Gases—Exercises.

IN considering the action of one or more forces upon a mass, we have hitherto supposed the mass to be solid. This limitation is unreal; forces act also on liquids, but the distinction is recognized by the prefix *hydro* (ὕδωρ, water) in the corresponding terms hydrodynamics, hydrokinetics, hydrostatics (cf. p. 18).

Every state of matter is subject to the action of gravity, since this property belongs to matter *per se.* Other properties—rigidity, viscosity, density, etc.—are possessed in very different degrees not only by different kinds of matter, but even by the same kind of matter when in different states; such properties considerably modify the action of gravity and other forces on the mass. We shall therefore begin our study of the statics of liquids with a brief account of these and other properties.

It has already been stated (p. 2) that the properties *weight* and *extension* belong to every kind of matter, in whatever physical state it may be. We shall now consider these two properties more closely, and shall also notice in this connection some points of difference between solids, liquids, and gases.

Weight.—The fact that an apple, when detached

from its tree, does not remain at rest in the air, but *begins to move*, tells us, in accordance with the first law of motion (p. 19), that the apple is acted upon by some force. The fact that it moves in a definite direction tells us, in accordance with the second law of motion, that this is the direction in which the force acts. We speak of this force in general as the force of gravity, but in a particular instance like the present we often describe it as the weight of the apple.

It is really due to a property inherent in matter. By virtue of this property every mass in the universe attracts every other mass with a certain force. The value of this force varies *directly* with the product of the two attracting masses, but *inversely* with the square of the distance between their respective centres of gravity. Between two masses of m_1 and m_2 grm., whose centres of gravity are separated by a distance of r cm., this mutual force of attraction is $\frac{m_1 \times m_2}{r^2} \times \frac{6 \cdot 658}{10^8}$ dynes; between m_1 lb. and m_2 lb. at a distance of r ft. the force is nearly $\frac{m_1 \times m_2}{r^2} \times \frac{1 \cdot 067}{10^9}$ poundals. The numerical factor in each case is the corresponding *constant of gravitation*. We are at present concerned only with those instances in which the earth is one attracting mass and the other is the apple or other mass in the earth's neighbourhood. Experiment has proved that at the same place the acceleration, g, produced by this attractive force *is the same in all masses:* the *magnitude* of the force must, therefore, always be proportional to the mass. In fact, if we denote the mass by M, and its weight, the attractive force, by W (in absolute units), we must have—

$$W = Mg$$

in accordance with equation (5), p. 21, of which this is simply a particular example.

The value of g may also be deduced from the time of oscillation of a simple pendulum. This ideal is represented in the actual experiment by a small, uniform metal sphere suspended by a light string. If O (Fig. 1, p. 13) be the point of suspension and A represents the centre of gravity of the sphere, then OA $(= l)$ is the length of the equivalent simple pendulum. The time, t, of a complete vibration is found by experiment, and g is then calculated from the known relation

$$t = 2\pi\sqrt{\frac{l}{g}}$$

As $\pi = 3{\cdot}1416$ and the value of g is constant at any particular place, it follows that $t \propto \sqrt{l}$, or $t^2 \propto l$.

Hence a long pendulum swings more slowly than a shorter one. Pendulum clocks, therefore, tend to lose in hot weather and to gain in cold weather. A "seconds pendulum" is one that makes a *complete vibration* in 2 secs., and the length of it is therefore given by

$$2 = 2\pi\sqrt{\frac{l}{g}}$$

$$\text{or } 1 = \pi^2 . \frac{l}{g}$$

$$\text{whence } l = \frac{g}{\pi^2} = \frac{g}{9{\cdot}87}$$

Accordingly, if $g = 32{\cdot}2$ ft. per sec. per sec., $l = 3$ ft. $3\frac{1}{4}$ in. nearly, which is, therefore, about the length of the seconds pendulum in London. If $g = 981$ C.G.S. units, then $l = 99{\cdot}39$ cm.; this may, therefore, be taken as the length of the seconds pendulum at Paris.

Careful experiments show that the value of g is *not* exactly the same at all places on the earth's surface. In C.G.S. units it varies from 978·10 at the Equator to 983·11 at the Pole. It is greatest at the Poles because the earth is somewhat flattened there and a mass on the surface is therefore nearer to the centre of the earth. The magnitude of the force varies inversely as the square of the distance between the centre of the earth and the centre of gravity of the mass attracted. This law of inverse squares appears to operate widely in nature. The force between two electrified bodies obeys the same law (p. 292). Although a mass really weighs more at the Poles than at the Equator we should not detect the difference if we used a pair of scales, because the "weights" themselves alter for the same reason and in the same proportion; but if we weigh out 1 lb. of shot at the Equator with a spring balance, we shall find that it weighs in London (with the spring balance) $\frac{32 \cdot 2}{32 \cdot 09}$ lb.

Since the weight can alter while the mass remains the same, it is clear that the two quantities are different, although they are so closely associated in common experience that the distinction between them is apt to become obscure. The student must not forget that mass is a quantity of matter, and that weight is a force; the two, therefore, differ in kind.

Extension is the property by virtue of which all matter takes up room. The occupation of a limited portion of space gives to a mass its shape and size, or, in more technical language, its *form* and *volume*. Matter is known to us only in the solid or in the fluid state; both form and volume of a *solid* are well defined, and force, often considerable, must be applied

to alter either. The perfect solid possesses a characteristic property of *rigidity* by which the connection between constituent parts of the mass is not broken without the exercise of force. A block of wood can be picked up by the hand without portions becoming detached, and is not divided into smaller portions without effort. A mass of mercury or of water cannot be similarly handled, and the difficulty is to *prevent* portions from becoming detached. In sawing wood, or any solid, considerable resistance is experienced, but the saw moves through water with no appreciable resistance if moved in the same direction as in sawing; if, however, the saw is moved in a direction at right angles to the surface of the blade, the resistance of the water is at once experienced. This resistance on the blade of an oar is utilized in the act of rowing. The pressure of a perfect fluid on any surface with which it is in contact is wholly normal to that surface, and the fact that there is no tangential component is the characteristic property of the perfect fluid; it is equivalent to saying that $F = 0$ (p. 36) and that a perfect fluid cannot exert friction.

We distinguish two varieties of fluid—the liquid and the gas.

Liquids offer almost as much resistance to compression as do solids, and therefore the volume of a *liquid* is not easily altered, but the form is readily changed owing to the lack of rigidity. A liquid, in fact, takes the form of the vessel in which it is contained, and can be given any form by pouring it into a vessel of the required contour. Some liquids, like treacle, only adapt themselves slowly to the new form; these are said to be *viscous*. A viscous liquid approximates to a soft solid. If the change of form takes place spontaneously, under the action

of gravity alone, however slowly, the substance is considered a liquid; sealing-wax is classed as a viscous liquid because when a straight stick is supported horizontally on two vertical uprights the portion between the uprights will after some days be found to have become curved, the central portion having descended. A tallow candle, which, in the same circumstances, remains straight, is a soft solid. A *gas* readily acquires not only the form but also the volume of any space in which it is enclosed. This characteristic tendency to expand, if allowed to do so, is illustrated in the following experiment:—

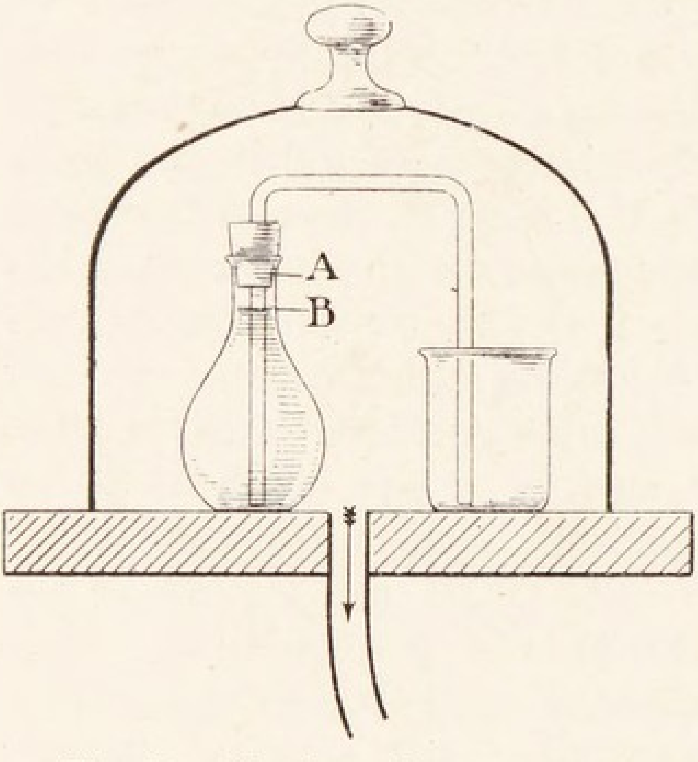


Fig. 20.—To show the expansion of a gas.

Air is contained in the small space between A and B in a flask (Fig. 20). The flask is furnished with a cork, through which passes a doubly-bent glass tube. One leg of this reaches to the bottom of the flask, which is nearly filled with water; the other leg reaches to the bottom of an empty beaker. The whole apparatus is placed under the bell-jar of an air-pump. If the pump be worked, the pressure on the confined air in the flask is removed and the volume of gas A B expands until it fills the whole of the flask, driving the water over into the beaker. If the air be readmitted into the bell-jar, the gas shrinks to its original volume.

The student has learned (p. 25) that every visible mass is an aggregate of invisible *molecules*. The molecules, in their turn, appear to be aggregates of still smaller material units. The aggregation of units is much closer in some kinds of matter than in others. To this relative closeness of aggregation we give the name **relative density** or **density.** The distinction is analogous to the relative density of population in different districts. It no doubt influences the relative mobility of the individual molecules, and is, therefore, associated with the general differences observed in the properties of solids, liquids, and gases. The density of the same kind of matter is different in the three states of aggregation. The solid is generally denser than the liquid, and the liquid much more dense than the gas.

Water is somewhat exceptional, and is of maximum density at 4° C., when still liquid. The density of ice at 0° C. is only about $\frac{11}{12}$ of this. Water at 4° C. is often chosen as the density standard, and this maximum density is therefore represented by unity. The mass of 1 c.c. of this water is practically 1 grm. (p. 5); if, therefore, the mass of 1 c.c. of any other substance be d grm., the density of the substance relative to this standard will be d. The mass M of V c.c. of this substance will evidently be $V \times d$ grm. We can compare masses by weighing them *at the same place* and, therefore, where g has the same value: for any mass M (p. 48)

$$W = M g$$

and for any other mass M'

$$W' = M' g$$

$$\therefore \frac{W}{W'} = \frac{M\ g}{M'\ g} = \frac{M}{M'}$$

If M and M' are the masses of the *same* volume V of each substance, then

$$\frac{M}{M'}=\frac{Vd}{Vd'}=\frac{d}{d'}$$

Definition.—The relative **density** of a substance is the ratio of the mass of any volume of the substance to the mass of the *same* volume of the standard substance.

The numerical value (d) of this ratio is independent of our units, and is therefore the same in all systems. The *absolute density* of a substance is the *mass of unit volume* of the substance, and will therefore depend upon the unit of mass and the unit of volume; if the relative density be d, the absolute density will be *d grm. per c.c.* if 1 c.c. of the standard water has a mass of 1 grm., but will be 1,000 *d oz. per cub. ft.* when 1 cub. ft. of the standard water has a mass of 1,000 oz.

Although the ratio of two masses is the same as the ratio of their weights, taken at the same place, we have seen (p. 50) that mass and weight are different things. Substances, of course, which differ in the closeness or density of their mass will show a similar difference in the weight which the mass possesses owing to the attraction of the earth, or gravity. We recognize this difference in the term specific gravity.

Definition.—The **specific gravity** of a substance is the ratio of the weight of any volume of the substance to the weight of the same volume of the standard substance.

Since the volume of every mass varies with temperature, this condition must be specified or understood; in simple laboratory experiments the comparison is usually made at room temperature.

Density is the ratio between two masses, and specific gravity is the ratio between two weights. They are referred to the same standard, and therefore they are represented by the same number. For liquids and solids the standard is generally water at 4° C., though water at 0° C. and water at room temperature 15·5° C. are standards frequently adopted. As the numerical value evidently depends upon the standard referred to, this must be clearly stated or understood. The density and specific gravity of a *gas* are usually referred to (1) dry air, or (2) hydrogen. In either case both the gas and the standard are valued at N. T. P., that is, at 0° C., and under a pressure of 760 mm. of mercury.

To find the specific gravity of any substance, we must therefore know (1) the weight W of some volume V of the substance; (2) the weight of the same volume of the standard. We shall now describe some of the methods by which these two quantities are determined.

Specific Gravity of a Solid

1. If the solid has a regular figure, as a cylinder, cone, cube, sphere, etc., V may be found from direct measurement of its dimensions, and W by weighing. Suppose that V c.c. of the substance weigh W grm., we know that V c.c. of standard water weigh V grm.

$\therefore$ Sp. gr. s of the substance $= \dfrac{W}{V}$ *referred to water at* 4° C.

2. If the solid has not a regular figure, V may be found by displacement.

Water stands at the level A in a graduated jar (Fig. 21). When the solid is placed in the jar, the water level rises to B. The difference between the two readings is the value of V.

3. Without finding the actual value of V we can find the weight W' of this volume of water; and then the specific gravity, since

$$s = \frac{W}{W'}$$

The method employed for this purpose is based upon a proposition which is known as the *principle of Archimedes*. It appears to have been first enunciated by this Greek philosopher, who was born at Syracuse about B.C. 287. When a body actually *floats*, wholly or partially immersed in water, it is obvious that the water must in some way exert upon it a resultant force equal in magnitude to the weight of the body, and acting vertically *upwards*. When the body *sinks*, it is obvious that this resultant upward force, if exerted, is at any rate not equal to the weight of the body. Indeed, it is in this case not quite so obvious that any upward force is exerted. It is, however, a fact that *every body immersed in water is acted on by an upward vertical force which is equal in magnitude to the weight of the water displaced by the body.*

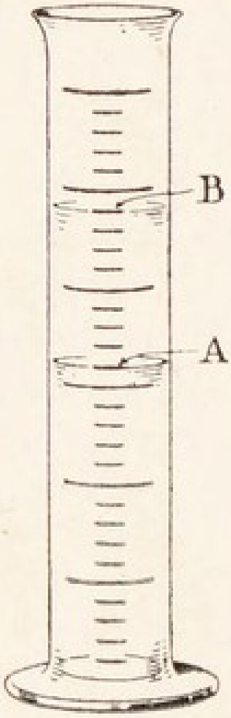


Fig. 21.—Volume of a solid found by displacement.

We might be led to this conclusion by the following argument. The beaker A (Fig. 22) is partly filled with water, in which a solid B is supposed to be entirely immersed. The space now occupied by B was formerly occupied by water; this water did not move, but remained *at rest*, though it has weight W'. This weight must, therefore, have been balanced by

an equal and opposite force *exerted by the surrounding water;* this will still be exerted on any solid, B, which displace the portion of water considered. B is therefore acted on by two forces—its own weight, W, acting vertically downwards, and a force, W', equal to the weight of the water it has displaced, acting vertically upwards.

The existence of this upward pressure is demonstrated by the following experiment: A small ground-glass plate is held by a string against the ground surface of a bell-jar (Fig. 23). The plate and jar are then sunk beneath the water and the string is released, but the glass plate is supported by the upward pressure of the fluid. Water may now be poured into the bell-jar, but the glass plate will not sink until the levels of the water inside and outside are nearly equal. The deeper the plate is immersed, the greater is the amount of water *inside* the jar; the upward pressure supporting it must, therefore, *increase with the depth.*

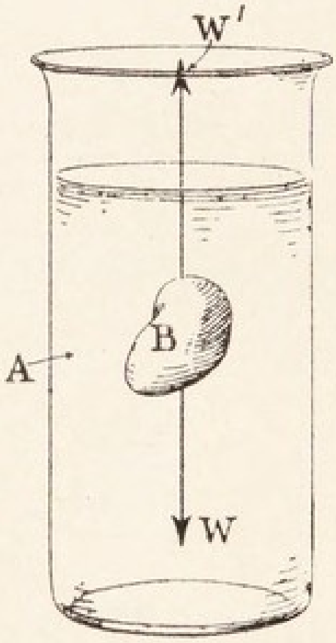


Fig. 22.—To illustrate the principle of Archimedes.

The truth of the principle of Archimedes can be demonstrated thus: A solid cylindrical piece of brass exactly fits a metal bucket, so that the bucket, when filled with water, contains a volume of water exactly equal to that of the brass. The brass is placed in the left pan of a balance, and the bucket filled with water in the right pan. Shot are then added till equilibrium is obtained, a fine silk thread is attached to the brass, which is suspended from the left hook of the balance and wholly immersed in distilled

water. The right-hand pan of the balance is now much too heavy, but if the water in the bucket be thrown away and the dry empty bucket be replaced, equilibrium will be restored, proving that the loss of weight of the brass when immersed in water is equal to the weight of its own volume of water.

The method by which the weighing in water is

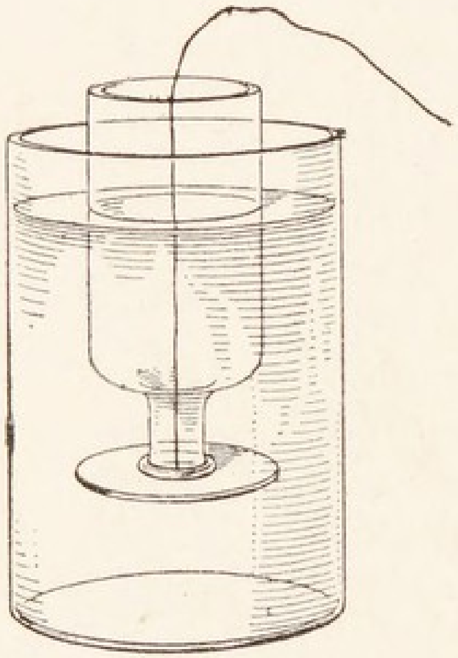


Fig. 23.—Glass plate supported by upward fluid pressure.

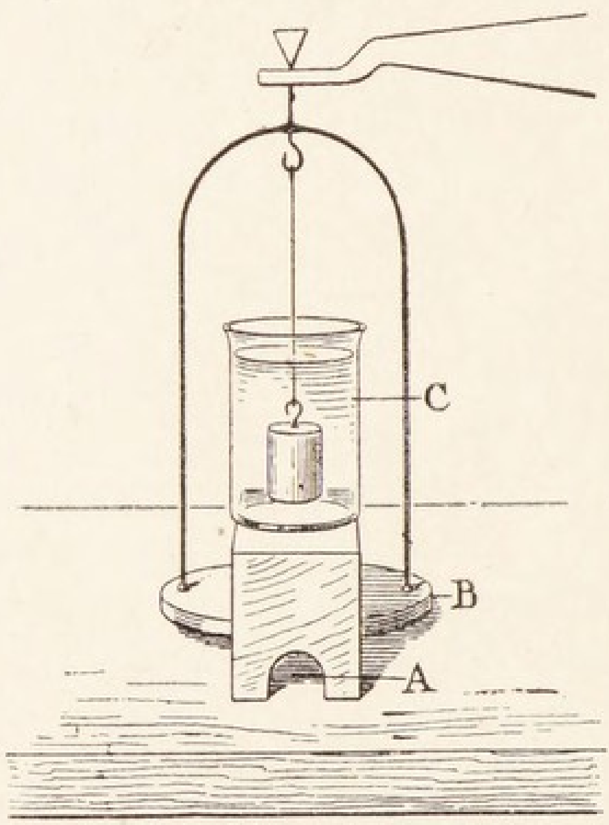


Fig. 24.—Method of weighing a substance in water.

effected is shown in Fig. 24. A little wooden platform A is arranged so that its legs straddle over the balance pan B without touching it; on this platform rests the beaker of water C, and in this, suspended by a fine silk thread from the hook on the balance, the piece of brass is entirely immersed.

Ex.: *Determination of the specific gravity of a solid insoluble in water:* (A) *When the solid is heavier than water.*—A piece of brass weighs in air 200 grm.; when wholly immersed in water, as shown in Fig. 24, it weighs 176·2 grm.; the

loss in weight is therefore $200 - 176{\cdot}2 = 23{\cdot}8$ grm. By the principle of Archimedes this is the weight of a quantity of water equal in *volume* to the brass, which weighs 200 grm. The sp. gr., s, of the brass, *referred to this water as standard*, is therefore (p. 54) $s = \dfrac{200}{23{\cdot}8} = 8{\cdot}4$.

(B) *When the solid is lighter than water.*—In this case a "sinker" of lead or some other heavy metal must be attached to the substance (e.g. wax or cork) to secure its complete immersion. The sp. gr. is then determined as indicated in the following example:—

1. Weight of wax + sinker in air . =	15 grm.
,, ,, + ,, water =	3·98 ,,
Loss A (= weight of water displaced by sinker *and* wax) . . .	11·02 ,,
2. Weight of sinker in air . . . =	5 grm.
,, ,, water . . =	4·4 ,,
Loss B (= weight of water displaced by sinker)	0·6 ,,
3. Weight of wax in air . . . =	10 grm.

Then sp. gr. of wax $= \dfrac{\text{weight of wax}}{\text{weight of water displaced by wax}}$

$$= \frac{\text{weight of wax}}{\text{Loss A} - \text{Loss B}} = \frac{10}{11{\cdot}02 - 0{\cdot}6} = 0{\cdot}96$$

If the solid is in fragments, as shot, sand, filings, etc., it cannot be suspended as in Fig. 24. To find the weight of the solid in water in this case we may use a specific gravity bottle (p. 63) and proceed as follows:—

1. Weigh the shot in air; let this be W_1.

2. Place the shot in a sp. gr. bottle, fill up with water, weigh, deduct the weight of the empty bottle, and so obtain W_2.

3. Weigh the sp. gr. bottle full of water, deduct the weight of the empty bottle, and so obtain W_3.

Then $W_1 + W_3 - W_2$ = weight of water displaced by the shot, and sp. gr. $= \dfrac{W_1}{W_1 + W_3 - W_2}$.

The displacement method (2, p. 55) may also be used. Thus, 50 grm. of lead shot caused the level of the water in a burette to rise 4·5 c.c.

$$\text{Sp. gr. of lead} = \frac{50}{4{\cdot}5} = 11{\cdot}1$$

When the solid is *soluble in water* it must be wholly immersed in some other liquid in which it is insoluble, and the experiment may then be conducted as before. As the result of the experiment, however, we shall now obtain the specific gravity, s_1, of the solid *referred to the new liquid as standard*. We only know, therefore, that the solid, bulk for bulk, is s_1 times as heavy as this liquid. We shall presently see, however, that the specific gravity, d, of this liquid referred to water is easily found. We shall then know that this liquid is d times as heavy as water. Hence it must follow that the solid is $s_1 \times d$ times as heavy as water. The sp. gr., s, of the solid referred to water is, therefore, $s_1 \times d$.

Alcohol, turpentine, etc., are sometimes employed for this purpose. A saturated aqueous solution of the substance may also be used, since the solid must, by hypothesis, be insoluble in this.

The specific gravity of a solid can also be found by means of **Nicholson's hydrometer**, which enables us to weigh the solid in air and in water without the aid of an ordinary balance. The instrument consists (Fig. 25) of a hollow brass cylinder A connected by a rigid wire stem to an upper pan B, and a lower pan C. The upper stem carries a mark (often a ring of platinum wire) at D. When employed for this experiment, the instrument is immersed in water, and weights are placed in the upper pan until the mark D is level with the surface of the water.

Ex.: 35·1 grm. are required. The weights are removed, and the solid is placed in B with any additional

weights required to sink the hydrometer to the same mark, D. Suppose 25·65 grm. are required; then the weight of the solid in air is 35·1 – 25·65 = 9·45 grm. The solid is now placed in the pan C under water, and weights are added till the mark is again level

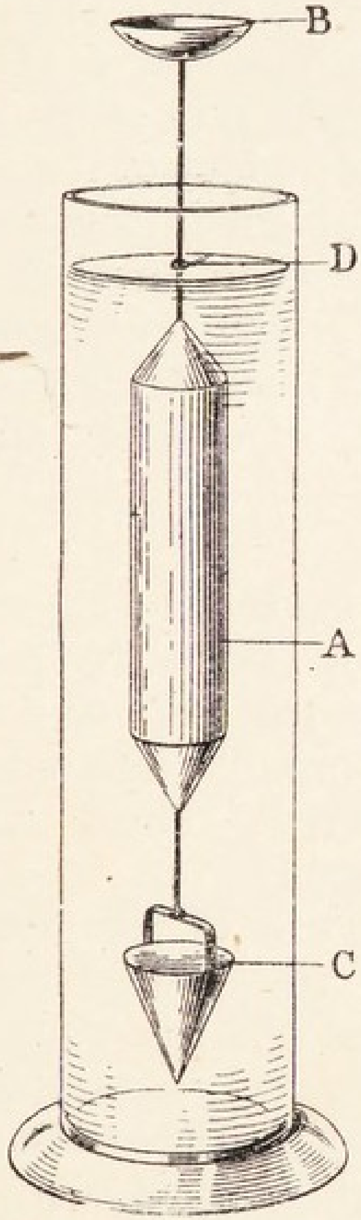


Fig. 25.—Nicholson's hydrometer.

Fig. 26.—Common hydrometer.

with the surface of the water. If this requires 26·70 grm., the weight of the solid in water must be 35·1 — 26·7 = 8·4 grm., and its loss when weighed in water is therefore 9·45 – 8·4 = 1·05 grm. Hence, the sp. gr. of the solid

$$= \frac{9 \cdot 45}{1 \cdot 05} = 9.$$

Specific Gravity of a Liquid

Nicholson's hydrometer may also be employed to determine the specific gravity of a liquid. For this purpose we require to know the weight of the hydrometer. Let this be C, and suppose that an additional weight, W, must be placed in the pan B (Fig. 25) to sink the instrument to the mark D in *water*. Then $W + C$ is evidently equal to the weight of the volume of *water* displaced by the instrument in this position. If, when the instrument is placed in another liquid, W' is required instead of W, then we know that $W' + C$ is the weight of the *same* volume of this liquid. Therefore the sp. gr., s, of this liquid $= \dfrac{W' + C}{W + C}$.

Hydrometers of this type, made by the addition of different weights to sink to the *same* mark in different liquids, are called hydrometers of constant immersion. Hydrometers of variable immersion are now more frequently employed. These sink to different depths in different liquids.

The **common hydrometer** is of this type and consists of a hollow glass or brass vessel weighted at the bottom and furnished at the top with a long graduated stem (Fig. 26). The weight in the lower bulb (mercury or shot) is so adjusted that the instrument will float in a vertical position with some point on the graduated stem in the surface of any liquid for which it is designed. The scale reading at this point indicates the specific gravity. It is usual to have several of these hydrometers; thus, one would read from 1·000 to 1·050, another from 0·950 to 1·000, and so on; by this means a stem of inconvenient length is avoided. Hydrometers are called urinometers, lactometers, alcoholometers, etc.,

according to the liquid for which they are specially suited. The specific gravity of the same liquid is different at different temperatures. Hydrometers are usually graduated for 60° F. or 15·5° C., and the liquid should, therefore, be at that temperature when the reading is taken. This must be remembered in the case of urine.

When important conclusions are to be drawn from the readings, the graduation of the hydrometer should be carefully checked by comparison with the

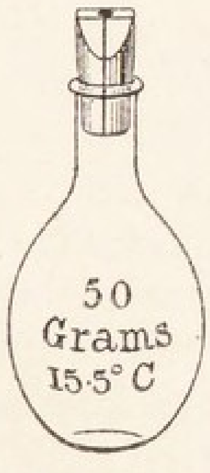


Fig. 27.—Specific gravity bottle.

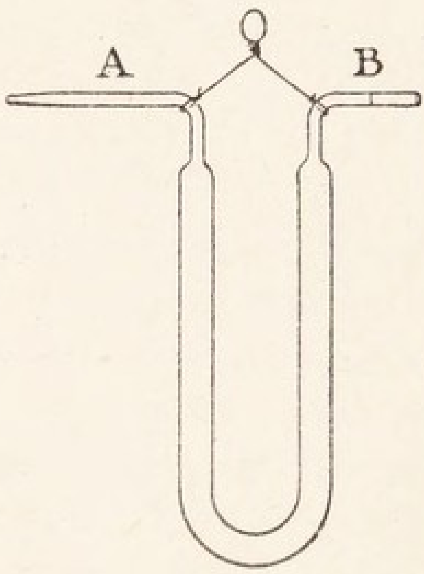


Fig. 28.—Sprengel tube.

results obtained by the following method, in which the specific gravity bottle (Fig. 27) is employed. This bottle has usually a capacity of from 20 to 50 c.c., and is fitted with a solid stopper drilled with a bore of fine calibre. The bottle is rinsed out with the liquid, then filled with it, and the stopper inserted, taking care to avoid air-bubbles. The excess of fluid is forced out through the perforated stopper. The bottle is then carefully wiped dry and weighed. Let this weight be W_1. Repeat the experiment with distilled water, and let the weight in this case be W_2. Let the weight of the bottle,

when dry and empty, be W. Then $W_1 - W$ is the weight of the *liquid* which filled the bottle, and $W_2 - W$ is the weight of the *same volume of water*; therefore the sp. gr. of the liquid $= \dfrac{W_1 - W}{W_2 - W}$.

We can also obtain an accurate volume of fluid by means of the **Sprengel tube** (Fig. 28). This is a light U-tube made of glass. The two ends are drawn out and bent as shown. A mark is made at B, and A is drawn out into a capillary tube, the end of which is broken off. The tube is filled by dipping the end A in the liquid, and sucking at B till the tube is filled and the liquid is sucked beyond the mark B. The volume is finally adjusted by holding a piece of filter paper at the end A, when the excess of liquid is soaked up by the filter paper and the level brought exactly to the mark B. The tube is then carefully wiped and weighed. The further course of the experiment is similar to that with the specific gravity bottle. The specific gravity bottle and the Sprengel tube are sometimes called **pyknometers.**

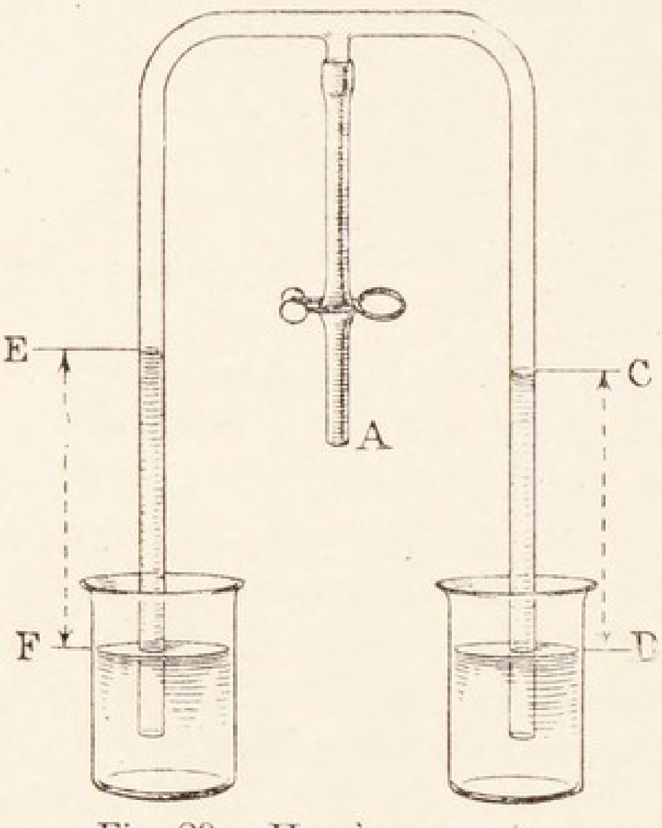


Fig. 29.—Hare's apparatus.

Hare's apparatus.—The specific gravity of liquids can also be conveniently compared by means of the apparatus shown in Fig. 29. This consists of two long tubes, the ends of which dip into the two

liquids. At the top these tubes are connected by a **T**-tube bearing an indiarubber tube and a spring clip. On sucking gently at A while the clip is open, the liquids are drawn up into the tubes. The clip is then closed, and the heights of the columns of liquids above the levels of the liquids in their respective cisterns measured; these heights are inversely as their respective specific gravities. Thus, a specimen of dilute alcohol was compared with distilled water: the alcohol column from E to F measured 100 mm., the water column from C to D 90 mm. The sp. gr. of alcohol to water was 0·9 to 1·0.

SPECIFIC GRAVITY OF A GAS

The specific gravity of a **gas** is determined on the same principle as that of a liquid, but in the first place the gas must be enclosed; and secondly, the volume of a gas varies so rapidly with alterations in temperature and pressure that special precautions must be taken; lastly, a gas is so light, and displaces so much air, that there is a sensible difference between its weight in air and its weight *in vacuo*—a difference which, in the case of ordinary solids and liquids, is so small that it can be neglected.

The gas is contained in a small glass balloon or spherical flask (Fig. 30), furnished with a well-made brass stopcock, to which a small hook can be attached. The air is removed from the flask by an air-pump, the stopcock closed, and the flask connected with a reservoir of the pure gas whose specific gravity is to be determined. On opening the stopcock the gas rushes in and fills the flask, and the process of exhaustion and filling is repeated so as to ensure the removal of all air. The flask, still in connection with the reservoir of gas, is immersed in a

beaker of water of known temperature for two or three minutes, the stopcock is then closed, and the barometer at once read. We have thus succeeded in enclosing a known volume of the gas at a known temperature and pressure. The flask is dried and suspended from one end of the balance. A similar flask of equal weight and displacement is suspended as a counterpoise from the other end. In this way the error due to the air displaced is obviated. The weight of the gas is then found as described under the

Fig. 30.—Flask for taking specific gravity of gases.

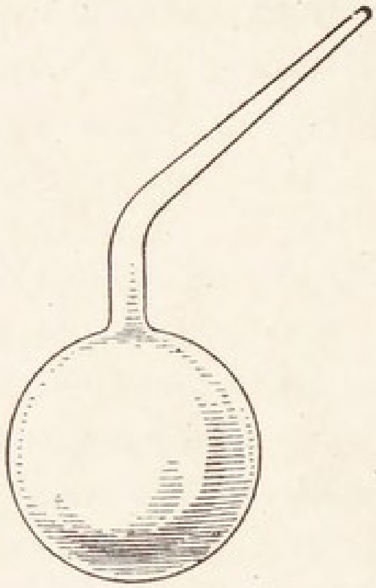


Fig. 31.—Dumas' flask for vapour density.

specific gravity of liquids, and the sp. gr. at 0° C. and 760 mm. calculated (p. 132).

The **specific gravity of a vapour** can be ascertained either (*a*) by finding the weight of a known volume of the vapour at a known temperature and pressure, or (*b*) by finding the volume occupied by a known weight of the substance when converted into gas or vapour. As an example of the former (*a*), we may cite **Dumas' method.** In this a glass bulb (containing about 200 c.c.), furnished with a long drawn-out neck (Fig. 31), is partially filled with the substance the specific gravity of whose

vapour is to be determined. The bulb is then plunged into a heated liquid, so that the substance boils violently, and its vapour drives out all the air from the bulb. When this has been effected and the bulb is full of the vapour, the end of the drawn-out neck is fused up by a blowpipe, the temperature of the heated liquid and the barometer being simultaneously noted. The bulb is then withdrawn, cleaned, and weighed. The weight of the empty bulb is deducted, and thus we get the weight of a known volume of the vapour at a known temperature and pressure: from this, after certain corrections, we can calculate the specific gravity of the vapour at 0° and 760 mm.

The method usually employed belongs to the second class (*b*). It is known as **Victor Meyer's method.** A tube about 2 ft. long is expanded at its lower end into a bulb A (Fig. 32); it is closed at its upper end with an indiarubber cork, and has a bent delivery tube B inserted a short distance below the cork; the delivery tube ends in a trough D filled with water. The bulbed tube A is surrounded by a second and larger tube which contains water, anilin, or other liquid of suitable boiling-point.

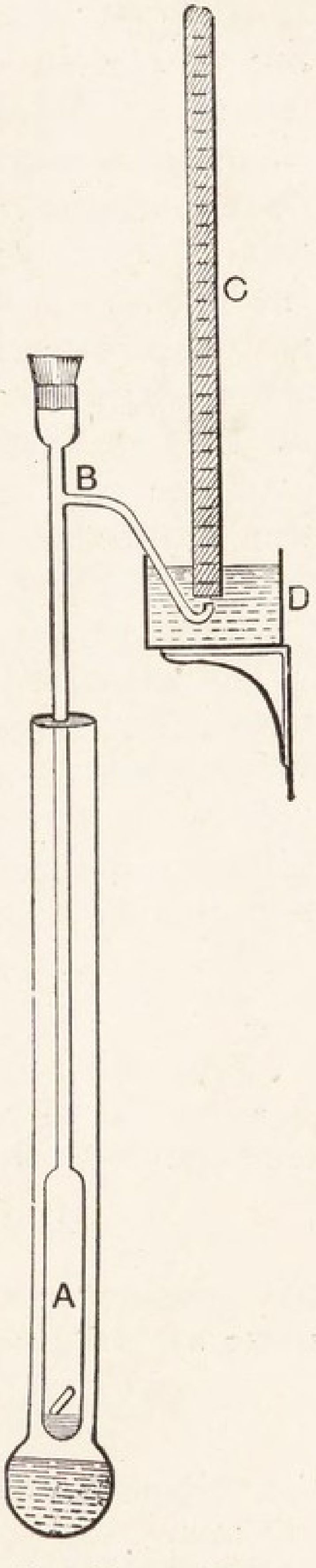


Fig. 32. — Victor Meyer vapour-density apparatus.

This liquid is caused to boil, and its vapour, heating the air in A, causes it to expand, and bubbles escape by the delivery tube into the air. As soon as bubbles cease to escape, the cork is removed and a weighed quantity W of the substance contained in a small bulb is dropped into A, and the cork immediately replaced. The substance is at once converted into vapour, which displaces some of the air, and this displaced air is collected in the graduated tube C, which has been placed over the end of the delivery tube. The liquid in the outer tube should have a boiling-point 20° to 30° higher than that of the substance; about 0·1 grm. of the substance should be taken.

The measured volume of the gas collected is corrected for temperature and pressure to the true volume at 0° and 760 mm. We know that this volume of the vapour weighs W. The weight W' of this same volume of hydrogen is then calculated, and finally the specific gravity of the vapour $= \frac{W}{W'}$.

Ex.: 0·073 grm. of ether displaced 25·3 c.c. of air, measured at 21·5° C. and 718·6 mm.; correcting for temperature (p. 132),

$$25{\cdot}3 \times \frac{273}{273 + 21{\cdot}5} = 23{\cdot}45 \text{ c.c.}$$

the pressure was 718·6 − 19·1 (the vapour tension of water at 21·5°) = 699·5 mm.

$$23{\cdot}45 \times \frac{699{\cdot}5}{760} = 21{\cdot}5 \text{ c.c.}$$

the volume of gas displaced when reduced to 0° C. and 760 mm.

Now, this volume of hydrogen weighs

$$\frac{21{\cdot}5 \times 0{\cdot}0896}{1{,}000} = 0{\cdot}00192 \text{ grm.}$$

and the sp. gr. of ether vapour, referred to hydrogen,

$$= \frac{0 \cdot 073}{0 \cdot 00192} = 37 \cdot 9$$

Exercises

1. A cork lifebuoy weighs 20 lb. What is the greatest weight that it could support in water, the specific gravity of cork being 0·3 ? [*First M.B. Lond.*]

2. A piece of wood of specific gravity 0·8, weighing 50 grm., floats in water. What proportion of its volume is submerged ? How much lead of specific gravity 11·4 must be attached to the wood in order that the two may just sink ? [*First Professional.*]

3. A solid weighs in air 14·86 grm., in water 8·67 grm., and in a second liquid 9·85 grm. Calculate the densities of the solid and of the second liquid. [*Ibid.*]

4. Apply the principle of Archimedes to calculate the force required to lift a mass of 30 tons of iron when submerged in sea water. A cubic foot of fresh water weighs 62·5 lb., and the specific gravities of sea water and iron are 1·03 and 7·6 respectively. [*Ibid.*]

5. What must be the volume of a balloon which when inflated with hydrogen, is able to lift a load of 800 kilogrammes, if the mass of a cubic metre of air is 1,290 grm. and that of a cubic metre of hydrogen is 90 grm. ? [*Ibid.*]

(For Answers, *see* p. 388.)

CHAPTER V

DIFFUSION

Diffusion of Solids and Liquids—Colloids—Dialysis—Osmotic Pressure—Diffusion of Gases—Graham's Law—Capillary Phenomena—Exercises.

THE relative mobility of particles in solids, liquids, and gases already illustrated in previous chapters is further exemplified by the phenomena of diffusion.

Diffusion of solids takes place slowly, at temperatures far below their melting-points. When a bright sheet of lead was laid on an ingot of gold and the two were maintained at the temperature of boiling water for some months, analysis proved that some lead had diffused into the gold and some gold into the lead.

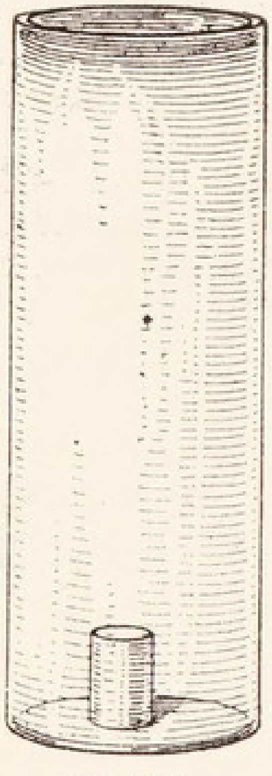


Fig. 33.
Diffusion of brine.

The **diffusion of solids in solution** is illustrated in the following experiment: A strong solution of common salt contained in a small jar is placed at the bottom of a deeper and larger jar (Fig. 33). The small jar is then covered with a glass plate while the large jar is gently filled with water by means of a funnel. The cover is then carefully removed from the small jar, leaving the heavy solution of brine in contact with the pure water above. The

heavy brine does not, however, remain at the bottom without mixing with the lighter water above. The molecules of the salt seem in solution to range farther and farther from each other, just as do the molecules of a gas, when the pressure is diminished, and to penetrate or diffuse into the water. This process of diffusion proceeds until, after the lapse of some time, the salt is evenly distributed throughout the water. The rate at which this diffusion of a substance takes place depends (1) on the nature of the substance; (2) on the strength of the solution, a 3 per cent. solution diffusing three times as quickly as a 1 per cent. solution; (3) on the temperature, the rate increasing as the temperature rises.

Relative *times* of diffusion of equal amounts of—

Albumin	49·00 units.
Magnesium sulphate . .	7·00 ,,
Sodium chloride . . .	2·33 ,,
Hydrochloric acid . .	0·1 unit.

So that hydrochloric acid diffuses 490 times as quickly as albumin.

Graham observed that substances which diffused quickly were crystalline, and that those which diffused slowly were not, so he divided substances into *crystalloids*, such as salt, and *colloids* (from κολλώδης = viscous or glue-like), such as gelatin, albumin, etc.

We now recognize that many substances which would not have been classed by Graham as colloids can be made to assume a colloidal state, e.g. ferric hydroxide, arsenious sulphide, gold, platinum; the word now denotes a condition of matter rather than a distinct class of bodies. Many phenomena, indeed, suggest that we have in this colloidal condition a sort of half-way house between liquid and solid—a state of molecular aggregation intermediate between that

of a substance in *solution* and that of a definite *precipitate;* solid particles, of ultra-microscopic dimensions, capable of sustained existence in *suspension*. The presence of these multiple molecules would be quite consistent with the low osmotic pressure of colloidal solutions. The classifications of our text-books are more rigid and precise than those of Nature. Natural divisions merge almost imperceptibly one into another, and the border line is often very difficult to define. Graham's experiments have, however, been confirmed and extended. The warm aqueous solution of some colloids, like gelatin, seems to "set," or solidify, but the solid formed will again dissolve on warming or diluting; others, however, seem to be coagulated, and the clot or *gel* is not subsequently redissolved.

If a water-tight membrane, such as a bladder or vegetable parchment, be tied tightly over the little jar in the previous experiment, the salt will still diffuse into the water, in spite of the bladder. If we mix a crystalloid, such as salt, with some white of egg (albumin), and place the *mixture* in a tube of vegetable parchment suspended in water, or in a drum of bladder stretched over a hoop floating in water (Fig. 34), the salt will diffuse rapidly, the albumin very slowly, through the membrane. Now, it is obvious that the process can be stopped at a time when nearly all the salt has passed through with but little albumin. If we evaporate the water we recover the salt almost pure; and if we repeat the process of diffusion with the residual albumin, we can remove practically all the salt, and on allowing the solution of albumin to evaporate in the sun we obtain the albumin free from salt. So, by taking advantage of the different rates at which substances diffuse through membranes, we can separate them from

each other. This process of separating dissolved substances by diffusion is called **dialysis.**

This movement of solids in solution, if resisted, tends to continue in spite of the resistance. So much pressure is, in fact, developed that few membranes will sustain the pressure without leaking.

The most successful of the earlier artificial membranes was prepared by a botanist, Pfeffer, during his investigations on the rise of sap in plants. A small vessel of porous earthenware, after thorough

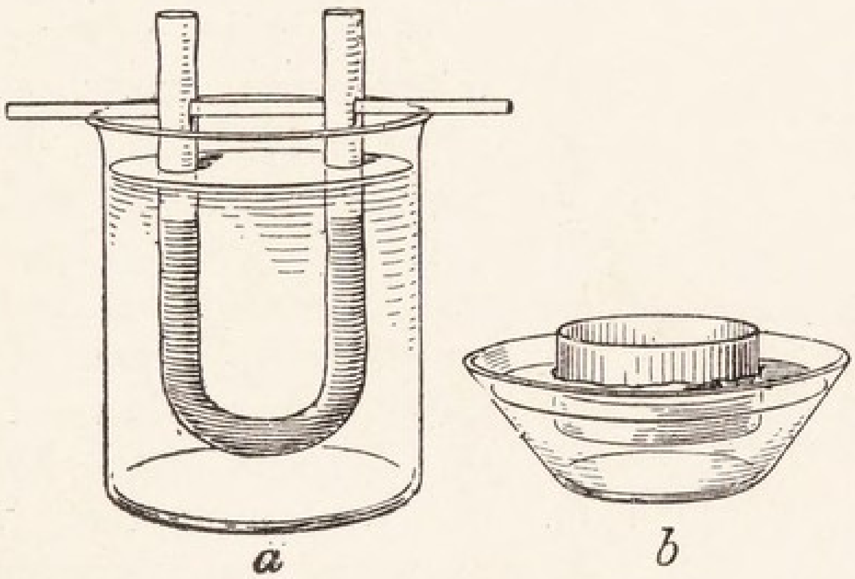


Fig. 34.—Diffusion of salt and albumin, (*a*) through parchment tube suspended from glass rod, (*b*) through layer of vegetable parchment stretched over hoop.

washing and drying, was soaked first in a solution of copper sulphate, and afterwards in a solution of potassium ferrocyanide. As the solutions came into contact in the pores of the pot, a gelatinous precipitate of copper ferrocyanide was thrown down, and this gelatinous precipitate, supported by the porous structure of the earthenware, formed an almost perfect membrane. The open mouth of the porous pot was closed by a glass tube A (Fig. 35), firmly cemented in; the glass tube had a T-tube B connecting it with a long bent U-tube C containing

mercury. By filling the porous pot with various solutions and immersing it in pure water, it was found that the water passed in much more rapidly than the solution came out. Pressure was thereby developed, and the mercury rose in the tube to considerable heights. Such pressure is termed **osmotic pressure** and the process **osmosis.**

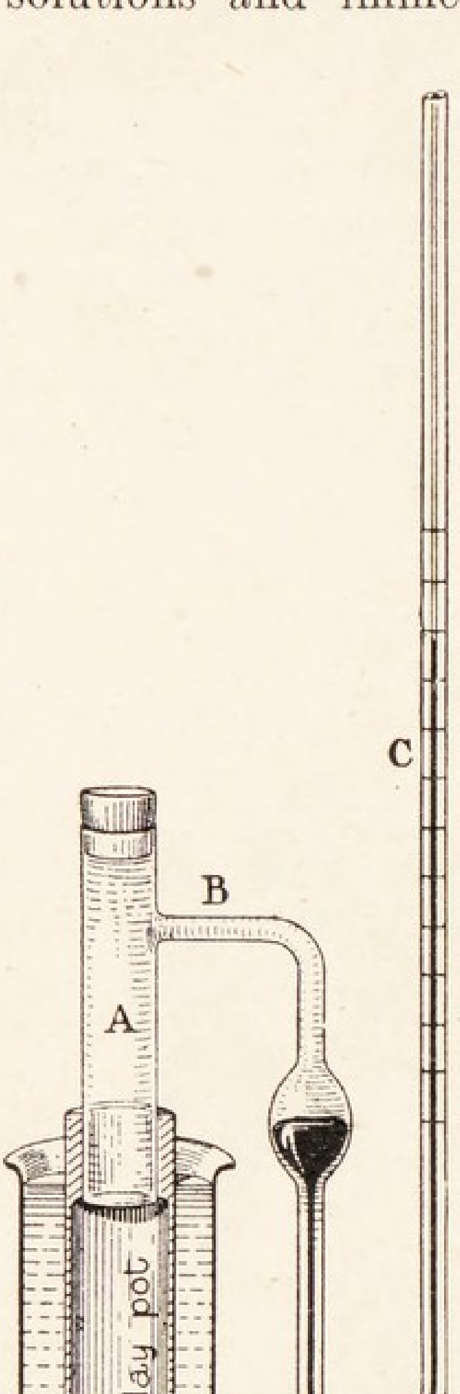


Fig. 35.—Pfeffer's apparatus for estimation of osmotic pressure.

Thus, with a 6 per cent. solution of sugar, a height of over 10 ft. (307·5 cm.) of mercury was attained.

It is generally believed at the present time that solids in dilute solutions obey the three great gas laws of Boyle, Gay-Lussac, and Avogadro, if we interpret volume, pressure, and temperature as meaning, in the case of dissolved substances, the volume of the solution in which the mass, *M*, of the substance is contained, the osmotic pressure of this solution, the temperature of this solution. Thus, by Pfeffer's results, it was shown—

(1) That the osmotic pressure increases with the strength of the solutions. If you double the strength of the solution, the osmotic pressure is also, roughly, doubled, thus—

1	per cent.	cane sugar	gave	o.p. =	53·5	cm.
2	,,	,,	,,	,,	101·6	,,
4	,,	,,	,,	,,	208·2	,,

This is really Boyle's law (p. 105); i.e. if you squeeze twice as many molecules into the same space you double the pressure.

(2) That the osmotic pressure varies with the absolute temperature. A solution of cane sugar gave—

At 32° C. or 305 absolute temp. an o.p. of 54·4 cm.
,, 14·1° C. or 287·1 ,, ,, 51·2 ,,

And $\dfrac{54{\cdot}4 \times 287{\cdot}1}{305} = 51{\cdot}2.$

This is the law of Charles and Gay-Lussac (p. 132).

(3) That molecular weights of various substances give the same osmotic pressure.

Cane sugar ($C_{12}H_{22}O_{11}$) has a molecular weight of 342; alcohol (C_2H_6O) has a molecular weight of 46: it was found that a solution containing 3·42 per cent. of cane sugar gave the same osmotic pressure as one containing 0·46 per cent. of alcohol. In other words, equal volumes of liquid which give equal osmotic pressures contain the same number of molecules. This is Avogadro's law.*

Diffusion of gases.—If a jar of hydrogen be placed mouth downwards over a jar of oxygen (Fig. 36), notwithstanding the fact that oxygen is sixteen times as heavy as hydrogen, the heavy oxygen does not remain in the lower jar, but mixes with the light hydrogen, and the latter passes downwards and mixes with the heavy oxygen, so that if, after an hour, a light be applied separately to each jar, it will be found that the gas explodes.

Graham discovered the law which governs the

* *See* Luff and Candy's "Manual of Chemistry."

rate at which diffusion of different gases takes place. The velocity of diffusion of a gas varies inversely as the square root of its density. Thus, hydrogen and oxygen will diffuse with relative velocities of

Fig. 36.—Diffusion of oxygen and hydrogen.

$$\frac{1}{\sqrt{1}} : \frac{1}{\sqrt{16}}$$

or, in other words, hydrogen diffuses four times as fast as oxygen.

This diffusion takes place when the gases are separated by a porous partition of clay or dry plaster-of-Paris, and thus pressure may be developed, as in the experiment shown in Fig. 37.

A Woulffe's bottle, containing some coloured liquid, is fitted with two corks. Through one passes a long glass tube, A, nearly to the bottom of the bottle. Through the other passes a short tube B which, at its upper end, is corked firmly into a porous pot C, filled, at first, with ordinary air. C is covered with an inverted beaker D, which can be filled with hydrogen by the tube H. As soon as the hydrogen enters the beaker D, diffusion takes place through the porous pot C; air passes into the hydrogen, and hydrogen passes into the air; but nearly four times as much hydrogen passes in as air passes out, so pressure is developed in the Woulffe's bottle, and the coloured liquid is forced up the tube A.

Capillarity.—We have already (p. 35) considered the force of friction between the surfaces of two solids in contact. When one of the two is a fluid, it is usual to assume that the component F (Fig. 11) has no value, but that the whole reaction between the surfaces is normal and represented by

N. This assumption is not strictly true of any known fluid, but it may be said to define the characteristic of an ideal perfect fluid. As *N* and *W* alone cannot be in equilibrium unless A C becomes horizontal, it follows that the surface of a liquid at rest must be a horizontal plane. We usually find this to be the case, and the surface of mercury in an open dish is often employed as an artificial horizon. When, however, the liquid is contained in narrow tubes, we find that the free surface is not horizontal. It is *concave* when the liquid is one that *wets* the tube, e.g. water (Fig. 38), but it is *convex* in the case of mercury, which does not wet the tube. Even in larger tubes, or vessels, a similar tendency to curvature is observed in immediate proximity to the sides of the vessel. The same phenomenon may also be seen where the free surface of a liquid is in contact with the fine stem of a hydrometer floating in it.

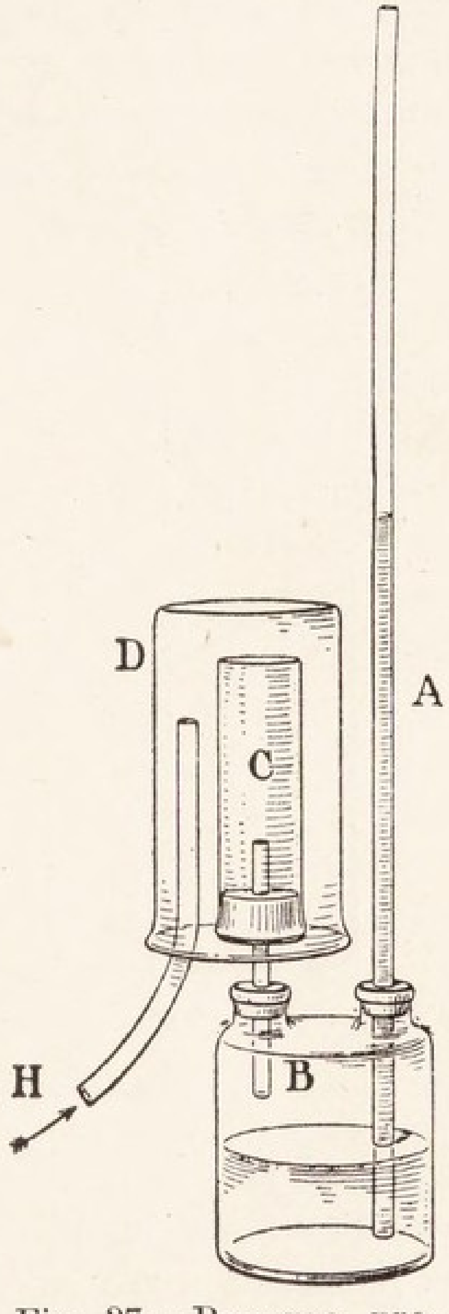


Fig. 37.—Pressure produced by diffusion of hydrogen into air.

These and many other effects, some of which will now be mentioned, have been ascribed to the existence of a stress to which the name *surface tension* has been given. A drop of liquid depending from a pipette does not hang flat as a chain would hang, but assumes a more or less spherical

form; nor does a drop lie perfectly flat on a horizontal surface as it would if the force of gravity were entirely unopposed. The curious movements executed by a fragment of camphor placed on the surface of warm water are due to local differences of surface tension produced by partial solution of the camphor. Similar local differences due to difference of temperature can produce movement in a single substance: when a horizontal metal plate, on which rests a

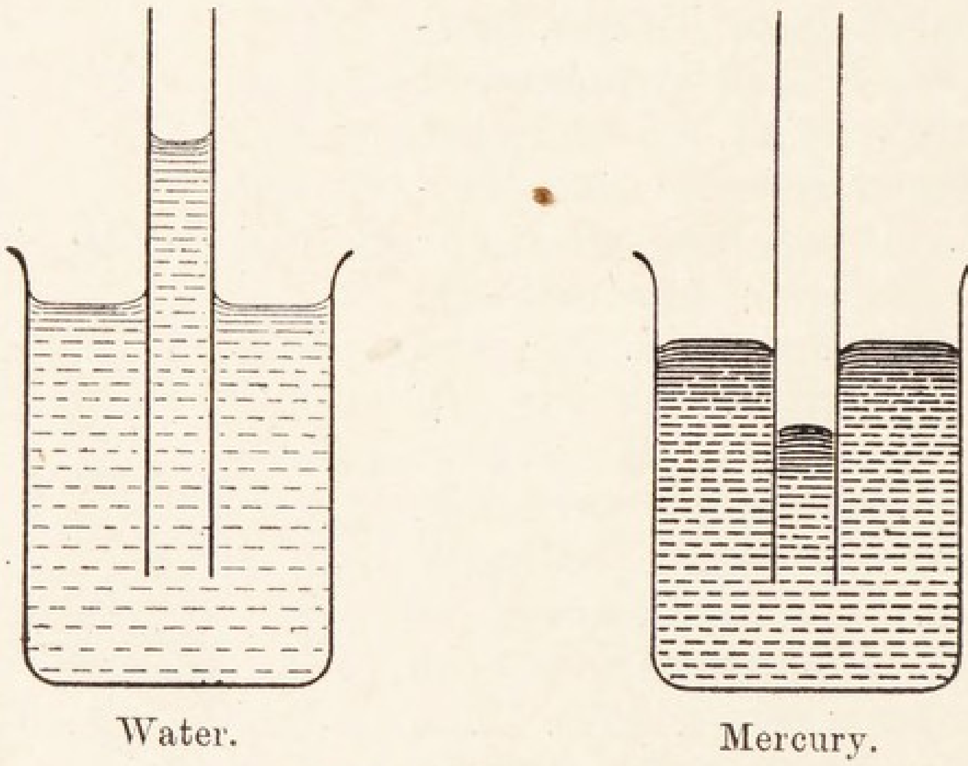


Fig. 38.—To illustrate capillarity.

layer of oil, is heated from beneath, not uniformly but at a selected spot, the oil over this spot will soon be dragged away in all directions, leaving the spot bare; the surface tension of a liquid diminishes as the temperature rises, and ultimately vanishes at the critical temperature. The "tears of strong wine" afford another familiar illustration; their formation is ultimately due to the fact that the surface tension of water is much higher than that of alcohol.

If a drop of bisulphide of carbon, coloured pink with a trace of iodine, be suspended in a mixture of sulphuric acid and water of the same specific gravity, the cohesion of the liquid particles will cause the drop to take the form of a sphere which floats in the liquid; the tension thus reduces the surface of the given volume to a minimum. If a clean metallic hoop or ring be dipped in a solution of soap and withdrawn, a film of the liquid remains stretched across the ring. If a thread moistened with soap solution be placed on this film it will form a loop of any desired shape as long as the film is intact, but if the film be broken it immediately springs into a circle. This is due to the surface tension acting on one side of the loop only, and thus dragging it out into a circle. If a soap bubble be blown on a tube and the mouth be withdrawn, the bubble will be seen to contract and become smaller owing to the action of the surface tension. If from a small pipette we deliver a bubble of air some distance below the surface of a liquid, the behaviour of the bubble on reaching the surface depends on the nature of the surface. In ordinary water the bubble, more or less flattened in its film of water, may travel for a short distance across the surface, but appears unable to rise from the surface and soon collapses. In soapy water, bubbles are more stationary and more permanent, and may separate from the surface unbroken; the surface is in this case more viscous, while the tension of the soapy film is less, so that the film is not ruptured. Liquids of this character readily froth when shaken, and advantage is taken of this circumstance in measuring the hardness of water. When the frothing is troublesome, as sometimes in pharmacy or in the examination of urine, the addition of a few drops of ether or alcohol will often so reduce the surface

viscosity that the trouble ceases without prejudice to the work in hand.

If a clean glass tube be drawn out into a *capillary tube* (Fig. 39), and one end of the capillary tube be immersed in some ink, the liquid will be seen to rush up the capillary, and will remain permanently above the level of the main bulk of the fluid.

The height to which the fluid will rise depends (1) on the nature of the tube, (2) on the nature of the fluid, (3) on the medium in which the experiment is made, (4) on the size of the tube, varying *inversely*

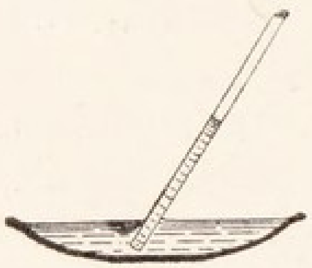


Fig. 39.—Capillary tube.

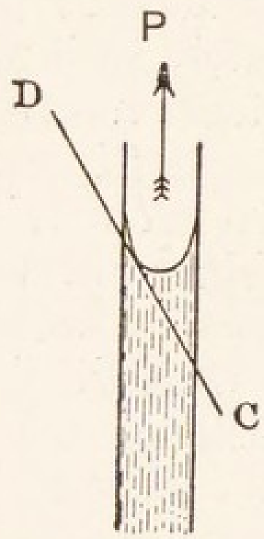


Fig. 40.—Surface tension.

as the radius (*see* below). If the capillary be dipped in mercury, the mercury will be depressed. Similarly, if a glass tube be smeared with vaseline and dipped into water, the level of the water will be depressed in the greasy tube.

The cause of these changes in level is surface tension (Fig. 40). The surface tension on the inside of a tube gives rise to a force directed along the tangent to the surface of the fluid at C D. If θ be the angle which this tangent makes with the vertical, and T be the tension per unit length of the circumference of contact, $T \cos \theta$ will be the vertical component *per unit length*,

and therefore $2\pi r . T \cos\theta$ will be the vertical component P for the whole circumference, when r is the internal radius of the capillary tube. Also, if the mean capillary elevation be h, and if w be the weight of a unit volume of the liquid, the weight supported by P is $\pi r^2.h.w.$

$$\therefore \quad 2\pi r . T \cos\theta = \pi r^2.h.w.$$

$$\therefore \quad \frac{2T \cos\theta}{r.w} = h$$

Careful experiments have confirmed this result.

When the liquid *wets* the tube, contact is so close that we may regard θ as zero, and in that case $\cos\theta = 1$, and the direction of T is vertical; the value thus becomes

$$T = \frac{h.r.w}{2} \text{ (gravitation units)}$$

$$= \frac{h.r.w.g}{2} \text{ (absolute units)}$$

Capillarity produces a slight error in the reading of the level of the mercury in the barometer, which becomes greater as the tube becomes narrower. Capillary tubes afford a most convenient method of collecting small samples of pathological fluids, vaccine, etc.

Exercises

1. If the surface tension of water at 20° C. be 74 dynes per cm., to what height will the liquid rise in a capillary tube of diameter 0·053 cm. ?

2. If water of unit density is found to rise to a height of 3·43 cm. in a tube of diameter 0·084 cm., what is the value of the surface tension of the liquid ?

(For Answers, *see* p. 388.)

CHAPTER VI

FLUID PRESSURE—PNEUMATICS

Transmission of Fluid Pressure—Pascal's Principle—Flow of Liquids in Closed Tubes—Atmospheric Pressure—Barometers—Water-Pumps—Siphon—Air-Pumps—Boyle's Law—Exercises.

Transmission of fluid pressure.—We have already referred (p. 56) to the upward vertical pressure exerted by a fluid on a solid which is immersed in the fluid. We were then only concerned with *vertical* forces, but it is not difficult to show that a fluid also exerts pressure in a lateral direction. If, for instance, a small area in the side of the beaker A (Fig. 22, p. 57) were cut round, the pressure of the liquid would force the piece outwards unless it were kept in place by an equal and opposite force. Experiment also shows that this force must be proportional (1) to the area, and (2) to the depth of the centre of gravity of this area below the surface of the liquid. In fact, *the pressure of a fluid on a surface immersed in that fluid* is equal to the weight of a column of fluid whose cross-section has the area of the surface immersed, and whose height is the vertical distance between the surface of the fluid and the centre of gravity of the immersed surface.

This property of transmitting pressure *equally in all directions* was recognized as a property of fluids by Blaise Pascal (1623–1662), the eminent French mathematician, and is therefore often called "Pascal's principle."

If we take a flat block of wood resting on a table and press it vertically downwards, it will have no tendency to move sideways; a solid only transmits pressure in the direction in which this is applied. If, however, we take a vessel filled with water, having tubes inclined in various directions, and apply vertical downward pressure by means of the piston A (Fig. 41), the liquid will rise to the same *vertical* height in *all* the tubes, irrespective of their

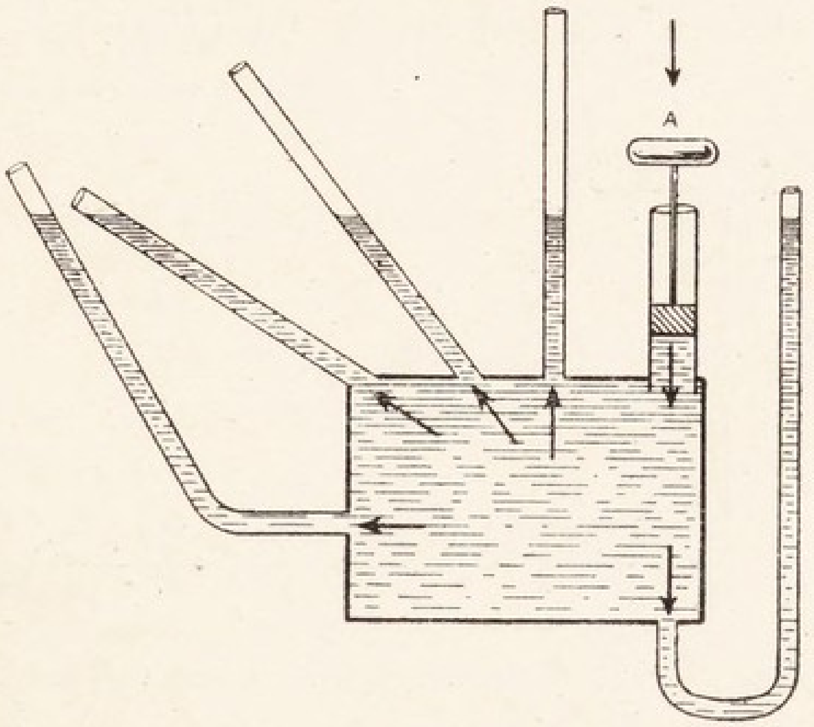


Fig. 41.—Transmission of pressure by fluids in all directions.

directions. Moreover, if the tube in which the piston works has a cross section of 4 sq. in., and each of the small tubes has a cross section of 1 sq. in., it will be found that if a force of 4 lb. wt. be applied by the piston, a force of 1 lb. wt. must be applied at *each* of the other apertures to prevent the fluid from rising in the tubes or the piston from descending. The pressure *per square inch* is therefore equal. Conversely, if a pressure of 1 lb. wt. be applied to an area of 1 sq. in. it will produce by transmission to an area of 10 sq. in. a pressure of 10 lb. wt. Theoretically,

there is no limit to this multiplication of pressure, but in practice it is limited by the fact that the mechanical apparatus by which the pressure is transmitted and applied must also support the strain. The improvements of Bramah and others have, however, enabled us to transmit in this way enormous pressures, and the *hydraulic press* is an instrument of the greatest value in engineering operations.

Flow of liquids in closed tubes.—In our consideration (p. 83) of the direction and magnitude of the pressure exerted by a liquid on a surface,

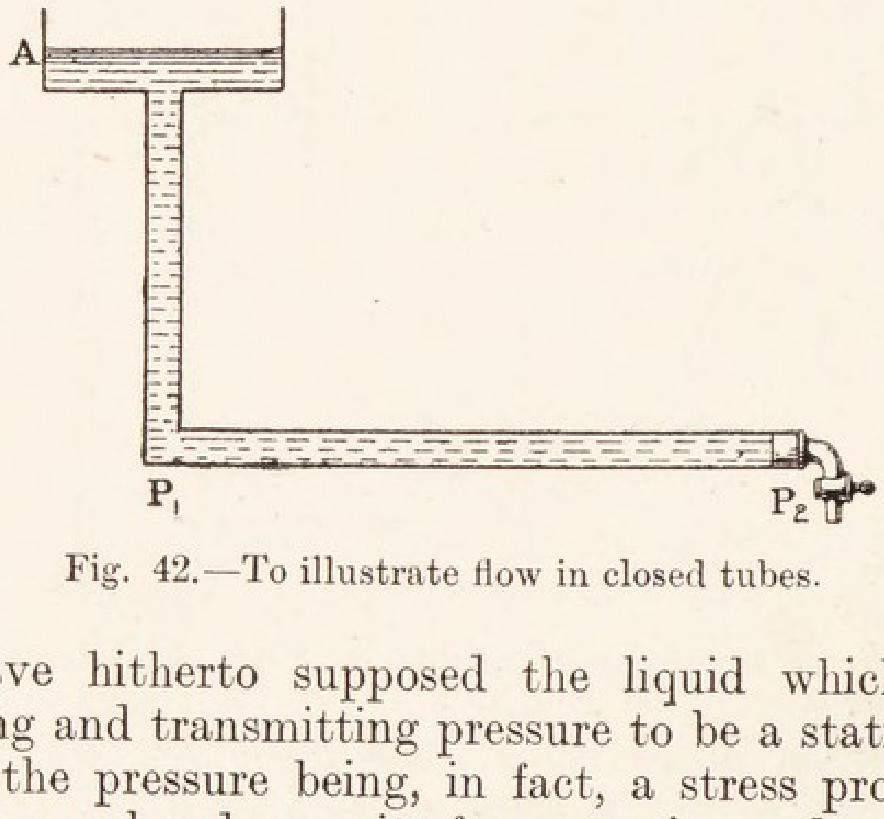


Fig. 42.—To illustrate flow in closed tubes.

we have hitherto supposed the liquid which was exerting and transmitting pressure to be a stationary mass, the pressure being, in fact, a stress produced by two equal and opposite forces, action and reaction (p. 28). If this condition is not fulfilled, but the pressure, p_1, at one point, P_1, is greater than the pressure, p_2, at another point, P_2, then this difference of pressure, $p_1 - p_2$, will result in a force tending to produce *motion* or *flow*.

In Fig. 42 the reservoir A is connected with a tube which at first descends vertically to P_1 and then proceeds at right angles to P_2. So long as the tap in

connection with the end P_2 is closed, the force tending to produce flow is balanced by an equal and opposite force reacting from the closed tap, and the pressures at P_1 and P_2 are equal. Directly the tap is opened this reacting force is removed, and the difference $p_1 - p_2$, due to the vertical height (H) of the water level in A above P_1, at once becomes effective and causes the water to flow from the tap. Similarly, if, without opening the tap, local difference of pressure can be created in a *closed* tube, or system of tubes, flow will take place and continue. The velocity of flow will depend upon the ratio of the constant difference $p_1 - p_2$ to the distance $P_1 P_2$, and is therefore proportional to $\frac{p_1 - p_2}{P_1 P_2}$, which is the difference of pressure *per unit of length*. This velocity is therefore greater from short tubes than from long ones, other things being equal.

Torricelli's law.—If H is maintained constant, the maximum velocity V of outflow is given by $V^2 = 2gH$, and this must occur when the tube $P_1 P_2$ is the shortest possible, as, for instance, when the aperture is in the side of a vessel. If the frictionless ideal (p. 77) were realized, so that no energy was spent in overcoming any resistance due to the viscosity of the liquid or to the surface of the tube, etc., then the velocity would be the same at P_2 whether the horizontal arm $P_1 P_2$ were long or short. In practice, however, these resistances do exist, and the velocity therefore diminishes with the length. The actual velocity v_2 at P_2 is therefore given by $v_2^2 = 2g\,h_v$ where h_v is some vertical height less than H. H is sometimes called the "head of water" and h_v the "velocity-head"; at P_2 the tube pressure is nil, but at the beginning of the tube it may be represented as due to a vertical column of the

liquid of height h_p, and is called the "pressure-head."

The three "heads" are connected by the relation $H = h_p + h_v$. We have supposed the tube of flow to be straight, uniform, and rigid. Departure from these conditions renders the problem more complex. As might be expected, *bends in the tube* act to a certain extent as obstacles and, by increasing the resistance, diminish the velocity. *If the tube is not uniform in diameter*, the changes of pressure and velocity will not be uniform. The velocity of flow will be greater in the narrow parts and less in the wide parts of the circuit. Conversely, since h_v is greater, h_p will be less; that is, the pressure will diminish in the narrow places and increase in the wide ones, into which their more rapid flow is directed.

In a system of branched tubes originating from a main tube, both bends and variation in calibre may be found. In such a system the sum total of the cross sections of the various branches will generally be (a) greater or (b) less than the cross section of the parent tube. In the former case, other things being equal, the resistance is diminished and the velocity may therefore increase. In the latter case the resistance is increased and the velocity diminished.

Other things, however, are not often equal, and what is gained by extended cross section in a system of many branches may be lost by the increased resistance offered by the narrow calibre of the individual tubes. On the other hand, a system with few branches may be compensated by the efficiency of each branch. Thus, in a blood system of either type we may find the same blood-pressure (h_p) in the aorta.

Lastly, if the tubes are not rigid but elastic, we must expect the features of the flow to be modified by the change.

In the **human vascular system** the three departures from simplicity now noticed are all present. Moreover, the initial pressure is not a continuous but a periodic one. The system as a whole is also so sensitive to nervous impulses that it presents many features not found in the simple hydraulic exemplifications to which reference has been made. A full discussion of its features would here be out of place, but one or two may now be mentioned which serve to illustrate what has been said above.

The muscular contraction of the left ventricle of the heart forces blood into the aorta, the trunk tube of the arterial system. The pressure so produced, transmitted laterally, distends the elastic wall of the artery, which, by reaction, recovers its original form and passes the impulse on. The pressure continues to be propagated in this way, weakening as it goes, till it reaches the capillaries, the network of fine tubes into which the trunk tube has by continued branching been converted. Their total capacity far exceeds that of the parent tube, and in the widened bed the stream flows more slowly; the velocity is, in fact, found to diminish from the aorta to the capillaries. The capillaries now gradually unite into veins which have a diminished total capacity, and the stream again flows in its narrower bed with increased velocity. The mean pressure diminishes gradually from the aorta to the capillaries. In the veins it is very slight, so that the effective difference of pressure is always directing flow from arteries to veins.

Atmospheric pressure.—This property of transmitting pressure equally in all directions is possessed

by all fluids, gases as well as liquids. The pressure exerted by the atmosphere, for instance, is transmitted to any surface with which it is in contact. This pressure is caused by the weight of the superincumbent atmosphere. Any instrument which enables us to measure this pressure is called a **barometer.**

The simplest form of barometer (Fig. 43) consists of a glass tube closed at one end, filled with mercury, and inverted in a cistern of mercury. The tube must be at least 32 in. long. It need not be absolutely uniform in diameter. In this tube the vertical height of the mercury column, above the surface of the mercury in the cistern, will be about 30 in., if measured at sea level. The space above the mercury is practically a vacuum, as it contains nothing but a small quantity of mercury vapour—it is termed the *Torricellian vacuum*. The reason why the mercury usually stands about 30 in. at the sea level is that a vertical column of mercury of this height balances the weight of the atmosphere pressing downwards on the surface of the mercury in the cistern. The vertical downward pressure of the atmosphere on the surface of mercury in the cistern is transmitted through the fluid, and presses the mercury in the barometer tube vertically up-

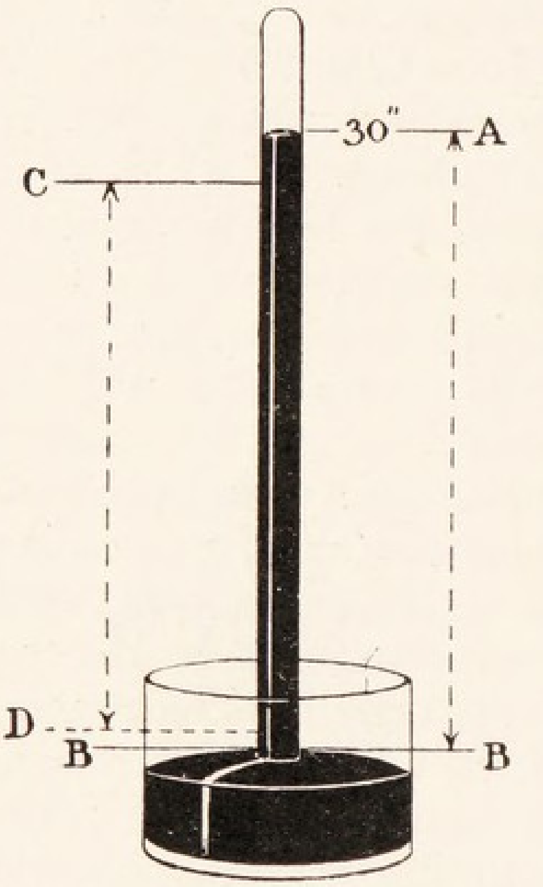


Fig. 43.—Barometer tube and cistern.

wards until the column of mercury balances the atmospheric pressure.

Numerical values of the atmospheric pressure.—We can therefore calculate the pressure of the atmosphere per sq. in. It supports a column of 30 cub. in. of mercury; mercury is 13·6 times as heavy as water, and 1 cub. in. of water weighs $\frac{1,000}{1,728}$ oz.; therefore, atmospheric pressure per sq. in.

$$= 30 \times 13{\cdot}6 \times \frac{1,000}{1,728} \text{ oz. wt.}$$
$$= 14{\cdot}75 \text{ lb. wt.}$$

In the C.G.S. system the normal height of the barometer is 76 cm., and the pressure per sq. cm. is therefore the weight of 76 c.c. of mercury—

$$= 76 \times 13{\cdot}6 \text{ grm. wt.}$$
$$= 76 \times 13{\cdot}6 \times 981 \text{ dynes}$$
$$= 1,013,961{\cdot}6$$

or, in round numbers, one million dynes (a *megadyne*) per sq. cm.

Height of the atmosphere if homogeneous.—The specific gravity of dry air at the surface of the earth, compared to water, is 0·001293, and compared to mercury will therefore be $\frac{0{\cdot}001293}{13{\cdot}6}$. To balance a column of 30 in. of merccury, a column of this air must have a height of $30 \times \frac{13{\cdot}6}{0{\cdot}001293}$ in., or nearly five miles.

This is therefore the height to which the atmosphere would extend if it were all composed of dry air of the same density as at the surface of the earth. In reality the air is not homogeneous, but becomes less dense as we ascend above the earth, and it therefore extends to a much greater height than five miles.

Variations in atmospheric pressure.—When the atmospheric pressure decreases, the

mercury sinks in the tube; when it increases, the mercury rises.

The three principal causes of these variations are—changes in temperature, changes in the amount of aqueous vapour, and mechanical movement of the atmosphere upwards or downwards. If the temperature rises the air expands, and the weight of a given column is less. If aqueous vapour displaces dry air the pressure is also diminished, since, referred to hydrogen, the specific gravity of aqueous vapour is 9, but that of air is 14·4. When the circulation of the air is cyclonic there is an *upward* suck in the centre of the cyclone which diminishes the pressure, whereas in an anticyclone there is a *downward* movement of the air in the centre and the pressure is increased. In this part of the globe, during the winter, the high barometer is over the cold plains of Russia; but in the summer the high barometer will be found over the Atlantic, which is cool as compared with the heated continent of Europe. Roughly speaking, in this country change of temperature may produce a variation of half an inch in the height of the barometer. Change in the amount of aqueous vapour may be the cause of a variation of about equal magnitude, while mechanical movement may alter the height to the extent of about $1\frac{1}{2}$ in.

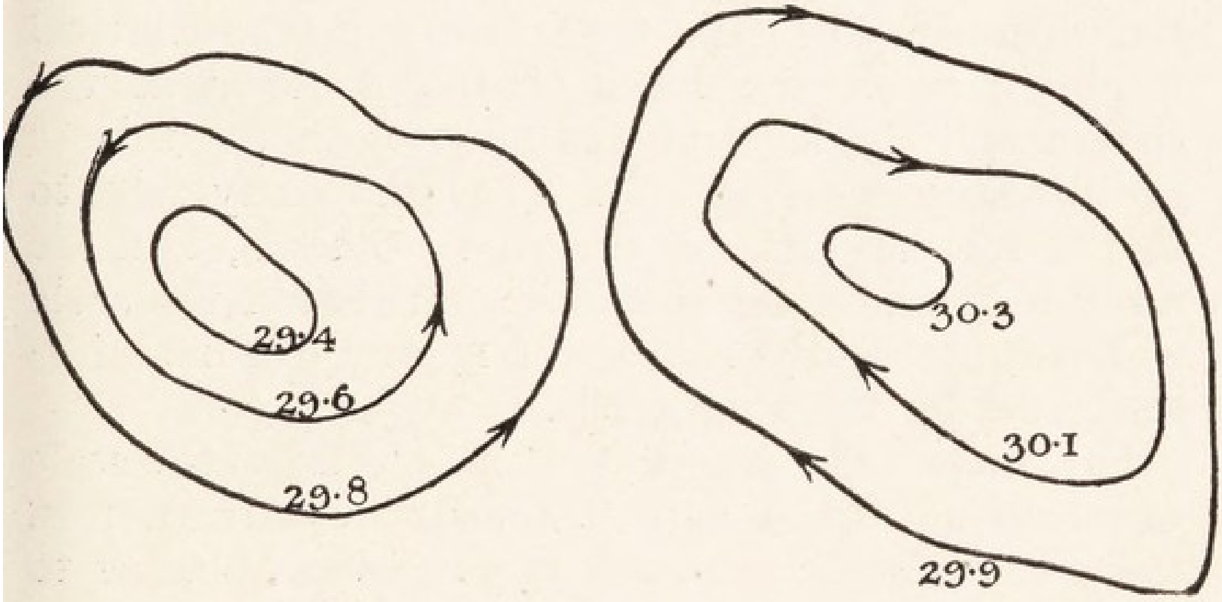
In weather maps, lines of equal barometric readings are marked: these are termed *isobaric* lines. In a cyclone (Fig. 44) the *lowest* barometer is in the centre, and the winds circulate round the centre in a direction opposite to that in which the hands of a clock revolve. In an anticyclone the *highest* barometer is in the centre, and the winds circulate in the same direction as that taken by the hands of a clock. In the southern hemisphere

the direction of the wind in a cyclone and in an anticyclone is reversed.

As the height of the barometric column depends on the weight of the atmosphere, it is obvious that, as we ascend a mountain and leave more and more of the air beneath us, the barometer will go down. If the atmosphere were homogeneous, we have seen (p. 89) that approximately

30 in. of mercury = 5 miles of atmosphere
or, 1 ,, ,, = $\frac{1}{6}$ mile ,,

therefore the mercury barometer would fall 1 in. for



Cyclone. Anticyclone.

Fig. 44.—Barometric pressure and direction of wind.

every ascent of $\frac{1}{6}$ mile, or 880 ft. As the atmosphere is not really homogeneous but gets lighter, a rather greater ascent is required; the mercury column falls 1 in. for every rise of 933 ft. approximately. So we can determine the height of a mountain by noting the difference between the barometer readings at its base and at its summit.

Corrections to be applied in reading an ordinary barometer.—Since the height of the barometer at any particular station will evidently

depend to some extent (*a*) upon the height of the station above sea level, and (*b*) upon the temperature of the mercury, it is usual, for purposes of comparison, to reduce the observed height to the value it would have at 0° C. and at sea level. The reading is also liable to be reduced by capillarity and, in some forms of barometer, by **cistern error.** This arises owing to the brass scale of inches or millimetres being fixed, while the level of the mercury in the cistern varies. If the atmospheric pressure diminishes, the mercury falls in the barometer tube, runs out into the cistern, and raises its level. Now, the distance we wish to measure is the vertical height between the level of the mercury in the cistern and the level of the mercury in the tube—the distance A B (Fig. 43). If the mercury falls to C, the mercury in the cistern rises to D, and we want to measure the distance C D; but if the scale is fixed, we really measure C B, and the result is too great by the length D B.

One way of avoiding this error is to make the scale of inches untrue, so as to allow for the alteration in the cistern level; but the more general plan is to have the bottom of the cistern made of leather so as to be movable, a device we owe to Fortin. In Fig. 45 will be found a diagram giving the construction of the cistern: A is the barometer tube, B the scale which ends in a white ivory point P, C C the upper part of the cistern, which is made of glass; into this fits a boxwood tube D D, to the lower end of which is wired a leather bag E E. The whole of the cistern and the bag is filled with mercury. Before reading the barometer the screw F is turned until the surface of the mercury touches the point P, which is the zero of the scale. The reading then gives the correct height of the barometer above the cistern.

The vernier used with this instrument is described later (p. 391).

Temperature error.—This is due to the fact that brass (of which the scale is usually made) and mercury expand at different rates when heated. The error is corrected by means of tables which have been calculated for the purpose (p. 430).

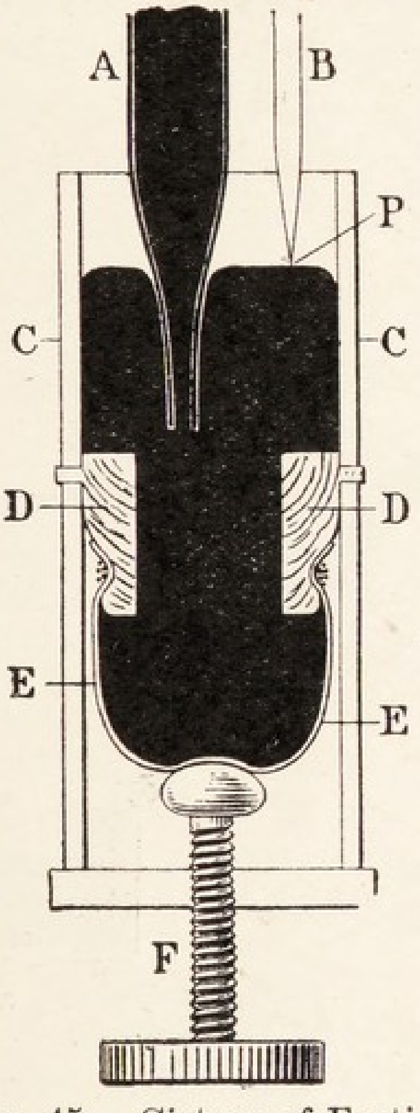


Fig. 45.—Cistern of Fortin's barometer.

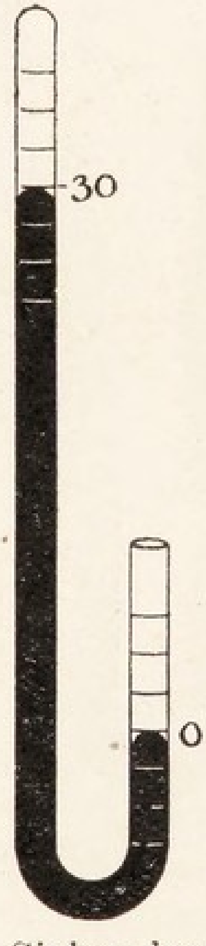


Fig. 46.—Siphon barometer.

Capillarity error.—Mercury is depressed in glass (p. 80), so that in a narrow tube it stands lower than in a wide one. This is also corrected by tables.

Error due to height above sea level.—As a height of 933 ft. causes a difference of 1 in. in the height of the barometer, it is obvious that the height of

the observing station must be known. This can be ascertained by consulting the ordnance maps of the district, or by comparing the height with that of some known station by means of some form of portable barometer.

Siphon barometer.—In this form the open end of the barometer tube is turned up to form a cistern (Fig. 46). The vertical distance between the two

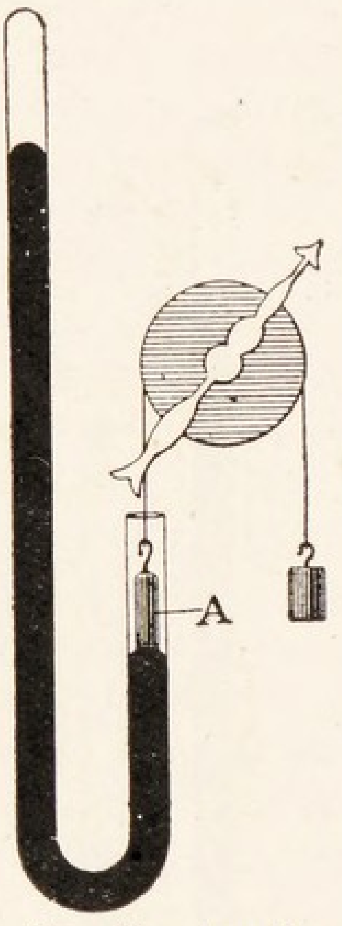


Fig. 47.—Construction of a wheel barometer.

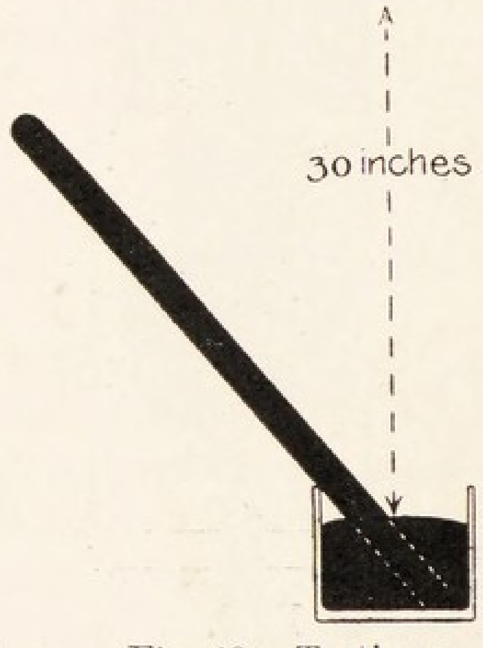


Fig. 48.—Testing a barometer.

ends of the barometric column indicates the atmospheric pressure. This distance is measured by means of a scale attached to the instrument.

The **wheel barometer** is a siphon barometer, and the movements of the mercury are, more or less, indicated by the oscillations of a counterpoised glass float A, which, as it rises and sinks with the mercury, turns the index on the face of the barometer (Fig. 47).

A barometer tube can be tested as to its freedom from air by gently inclining it. If there is no air the mercury runs up with a metallic click and fills the tube; if there is any air, the bubble at once becomes evident (*see* Fig. 48).

Glycerin barometer.—Other liquids instead of mercury have been used to fill barometer tubes. The height of a column of water equivalent to the 30-in. column of mercury must be $30 \times 13{\cdot}6$ in., which is equal to 34 ft. This would, therefore, be the ordinary height of a water barometer. Glycerin is about 1·27 times as heavy as water; the height of a glycerin barometer would therefore be $\frac{34}{1{\cdot}27}$ ft., or rather more than 26 ft. 9 in. The variations in the glycerin barometer are therefore about eleven times as great as when mercury is used, but no particular advantage seems to be gained by this increased movement, as small variations are easily and accurately read by means of the vernier (p. 391). There is a glycerin barometer at the Geological Museum in Jermyn Street.

The density of the atmosphere diminishes in geometrical, as the altitude increases in arithmetical, progression. Thus at 18,100 ft. the air has doubled its volume; at twice that height (36,200 ft.) the volume of the original mass is quadrupled.

The **aneroid barometer**, so called because it has no liquid (α, priv.; νηρός, a liquid), consists of a flat, metallic circular box (A, Fig. 49), with a corrugated lid thinner than the bottom. This box is partly exhausted of air. When the atmospheric pressure increases, the box lid is forced in; and when the pressure diminishes, the elasticity of the air inside forces the top out. The rest of the instrument consists of a mechanism for rendering this minute move-

ment of the top of the box visible. A stud of brass B is fixed on the lid and bolted to a strong lever C near its fulcrum D; the other end of the lever is pressed upwards by a spiral spring E, which tends to lift the top of the box. The movement of the top is thus magnified, and, by means of a series of bell crank levers, etc., turned into the rotary motion of the hand on the dial.

The graduations on the dial of an aneroid are made by comparing its indications with those of a mercury barometer. The mechanism is somewhat delicate and may become displaced, rendering the instrument inaccurate. When, therefore, the reading

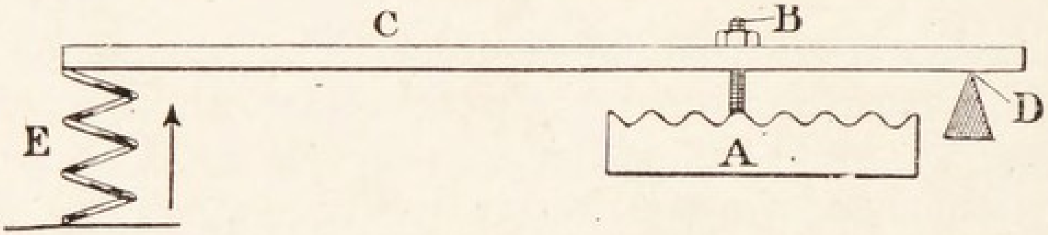


Fig. 49.—Construction of aneroid barometer.

of an aneroid barometer is of importance, as in fixing the height of a mountain, it should always be checked by comparison with a standard mercury barometer.

The terms "stormy," "set fair," etc., engraved on the dials of barometers are of but little significance, except to indicate that if the barometer falls the weather tends to become wet, warm, and windy; if it rises it is likely to be fine and dry. In barometers for use at about the sea level, "change" is usually placed at 29·5 in., and "fair" at 30·2 in.

Ordinary Pumps

The action of ordinary pumps depends on the atmospheric pressure.

Lift pump.—This consists of a barrel (Fig. 50) in which works a piston A having a valve open-

ing upwards. To the bottom of the barrel is connected the pipe passing down into the well. At the top of this pipe is a second valve B, also opening upwards. The pump has usually to be "primed" by pouring in water until the barrel is full. When the piston descends, the valve A opens, and the water passes through and is at the end of the stroke above the piston; at the up-stroke the valve A closes, the atmospheric pressure, acting on the surface of the water in the well, forces the water through the valve B, and fills the barrel, while the water which was above the piston is lifted up and runs out at the spout. It is obvious that the valve B must not be much more than 30 ft. above the level of the water in the well. If it were over 34 ft. the pump would be converted into a water barometer, and the atmospheric pressure would not be able to fill the upper part with water. The height to which water can be raised by the lift pump is limited, because, as the distance of the spout above the piston increases, the weight of water on the piston during the lift becomes so great that the pump is unworkable.

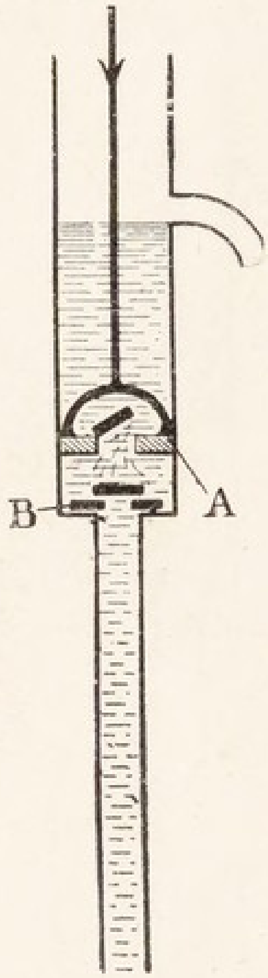


Fig. 50.—Lift pump.

Force pump.—In this pump (Fig. 51) the piston is solid, and the water is forced up a side pipe C, which has at its lower end a valve A opening outwards. The pipe to the well has at its upper end a valve B opening upwards. When the piston descends the water is forced through the valve A

and up the pipe C. During the up-stroke the barrel is filled with water by the atmospheric pressure, as in the lift pump. The valve B must not be much over 30 ft. above the well, but the pipe C can be of any reasonable height. In this pump the flow of water is discontinuous. When a continuous flow is required, as in a fire-engine, an air chamber is introduced (Fig. 52). Usually two force pumps deliver water through the tubes A and B, each furnished with a valve opening inwards. The strong chamber C is only partly filled with water, and each stroke of the pumps compresses the air in the upper part of C. This compressed air forces a fairly continuous stream of water up the pipe D.

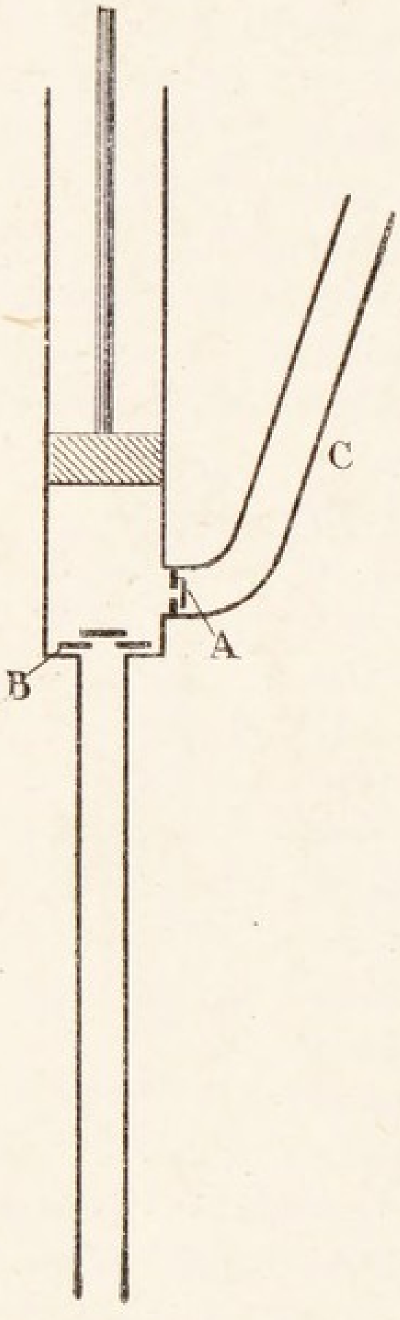


Fig. 51.—Force pump.

The **siphon** is another instrument in which the atmospheric pressure is utilized. This apparatus consists of a bent tube of glass, metal, etc. (Fig. 53), one leg of the bend being longer than the other. It is used for transferring fluid from one vessel to another at a lower level. The siphon is filled with the fluid and the short leg immersed in the vessel to be emptied. Now consider a particle at the top of the bend at A; it is acted on by two forces—(1) the pressure of the atmosphere transmitted up the column C A and therefore reduced by the vertical

height A Q of the liquid; (2) the pressure of the atmosphere transmitted up the column B A and therefore reduced by the vertical height A P of the liquid. If H is the height of a true barometer containing this liquid, (1) $= H - A\,Q$, and (2) $= H - A\,P$, (1) urges the particle from A to B, but (2) urges it from A to C, and, since (2) is the greater force, the particle moves towards C, and the atmospheric pressure forces the fluid up the

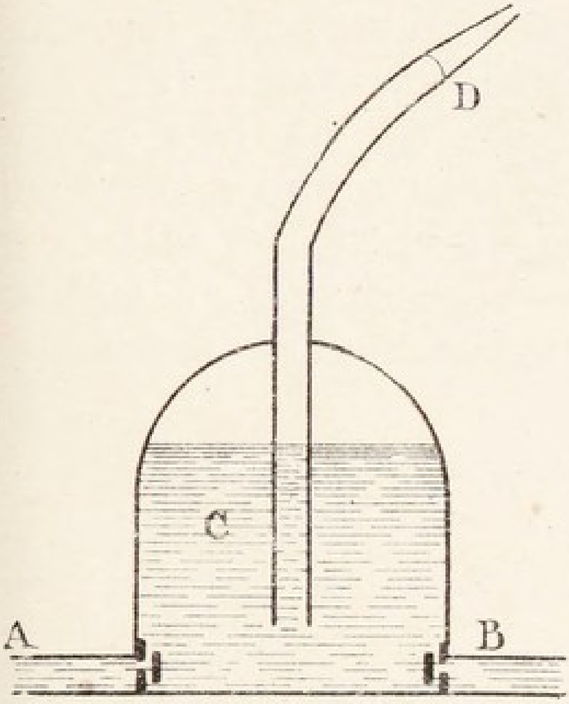


Fig. 52.—Air chamber for continuous flow.

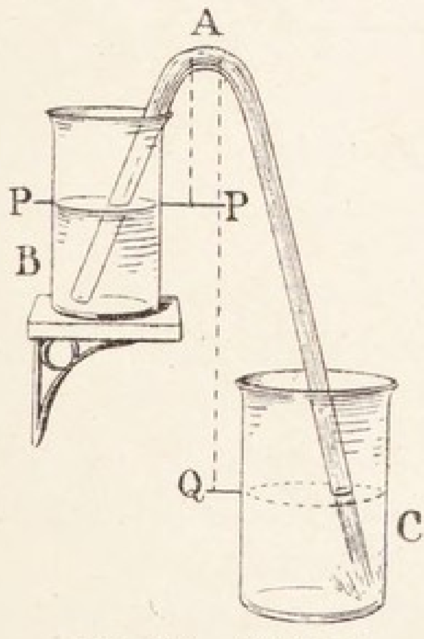


Fig. 53.—Siphon.

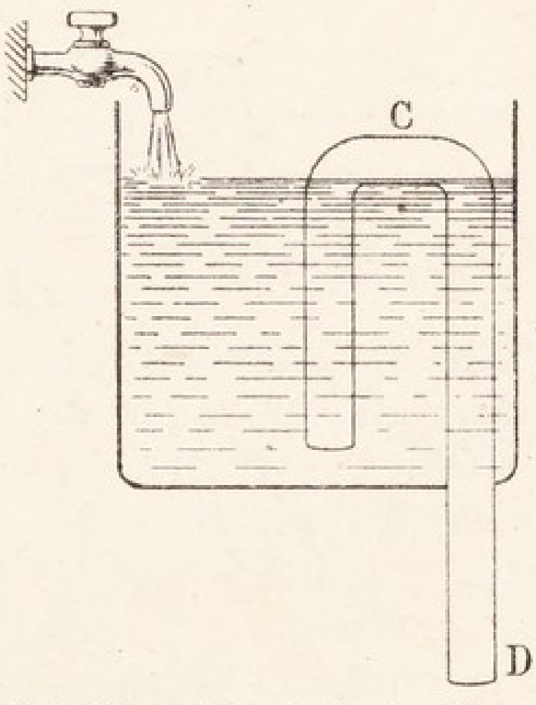


Fig. 54.—Automatic flushing cistern.

short limb until the vessel is emptied. It is clear that there will be no effective force (2) unless A P

is less than H; hence, if the liquid is water, A must be less than 34 ft. in vertical height above the level in B.

The siphon is very useful for automatic flushing. A small stream of water flows into a cistern (Fig. 54), and the water rises until it fills the top bend of the siphon at C. The siphon then acts, and a powerful rush of water issues from D until the cistern is emptied, when the process is repeated. A similar construction is employed in the Soxhlet extraction apparatus.

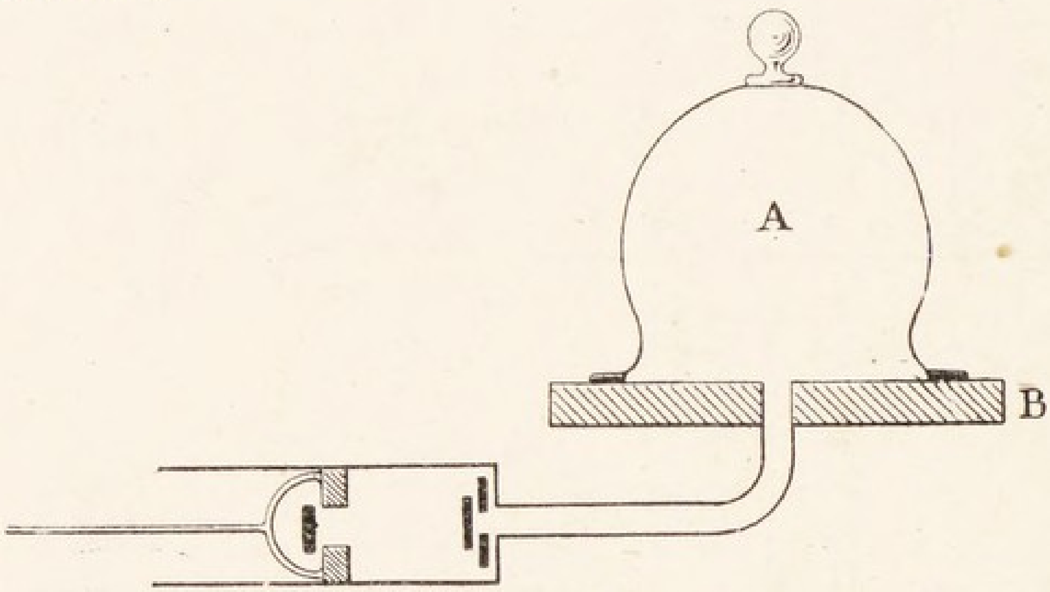


Fig. 55.—Common air pump.

Air Pumps

The ordinary air pump acts exactly like a lift pump, but the valves are made of oil silk. Instead of lifting water out of a well, it sucks air out of a glass bell-jar (Fig. 55), called a receiver A, which rests on a brass plate B ground flat and smeared with grease.

The common bicycle pump (Fig. 56) is still simpler in construction. Instead of a rigid piston with a movable valve, the piston itself is made of thin copper, or stouter indiarubber, sufficiently flexible to serve as a valve. The piston has somewhat the

shape of a bell or inverted cup. At the *up*-stroke a vacuum tends to form below P, but the air *above* forces its way past the piston, which becomes slightly more cylindrical (1) as the air passes between it and the wall of the pump. At the *down*-stroke the flexible bell spreads out (2) and drives the air before it through a valve leading *into* the tyre.

Fig. 56.—Common bicycle pump.

The **Tate air pump** (Fig. 57) is a more effective form of pump. The barrel is about twice as long as in the common pump, and there are two solid pistons A and B firmly connected together by the piston rod C. At each end of the barrel is a valve opening outwards, D and E. If the pistons be pulled outwards, the air between B and E is forced out through the valve E and escapes. As A moves with B the D valve shuts, and the space between D and A is a vacuum, until the

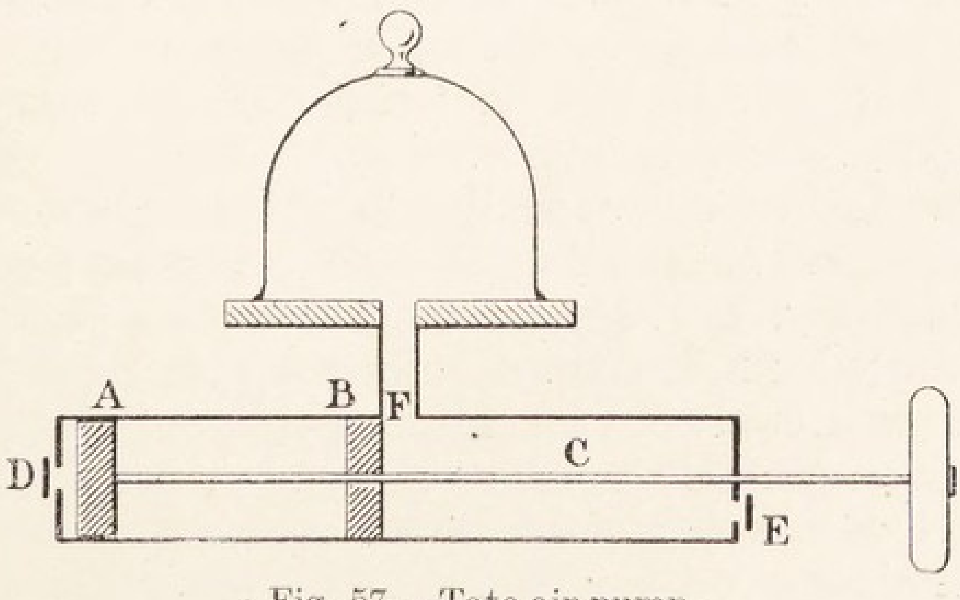


Fig. 57.—Tate air pump.

piston A passes the hole F leading to the receiver, when air rushes in from the bell-jar, to be forced out at the valve D when the pistons are pushed inwards. A similar action takes place with piston B and the space B E.

Ex.: If V be the volume of the bell jar and v the volume of air removed by the pump at each stroke, it is clear that a volume V of air expands to $V + v$, and therefore becomes rarefied. If the original density p_o is reduced to p_1, we have

$$V \times p_o = (V + v) \times p_1$$

$$\text{and, therefore, } p_1 = p_o \times \frac{V}{V + v}$$

Similarly, the density p_2, after a second stroke,

$$= p_1 \times \frac{V}{V + v} = p_o \times \left(\frac{V}{V + v}\right)^2$$

and finally, after n strokes,

$$p_n = p_o \times \left(\frac{V}{V + v}\right)^n$$

In the pumps described above the valves are opened by the compressed air, and when the air is much rarefied it has not elastic force enough to open the valve, and the pump ceases to act. As a consequence such pumps cannot reduce the air pressure below 1 mm. or 2 mm.—i.e. if they are connected with a vertical glass tube, the lower end of which dips into mercury, they cannot raise the mercury to more than 758 or 759 mm.

A much better vacuum can be obtained by using a valve worked mechanically, as in the **Fleuss pump.** In this (Fig. 58) the piston has a boss A, which lifts the valve B mechanically in the up-stroke. The valve is closed by the spiral spring C. Both the valve and the piston have a layer of oil O O. In the up-stroke, as soon as the piston passes the aperture D, the air above the piston is forced out at the valve

B, which is lifted by the boss A. At the down-stroke the valve B is closed by the spring C; a vacuum is formed, and, as soon as the piston is below the aperture D, air rushes in from the receiver, which is connected with E through the aperture D, to be ejected at the next up-stroke. The oil serves to perfect the fitting of the valve and the piston, and prevents any leakage of air.

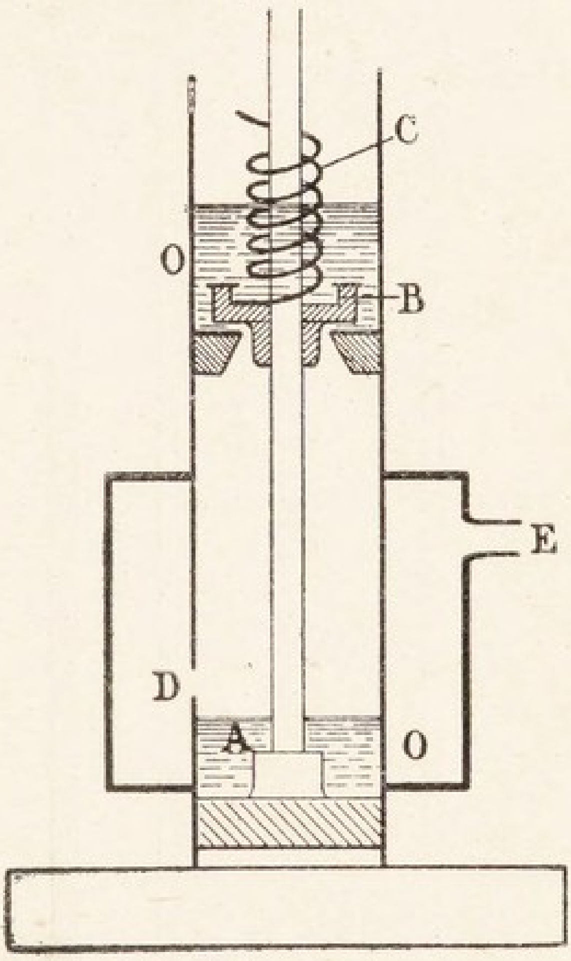


Fig. 58.—Fleuss air pump. (*After Lehfeldt.*)

Sprengel pump.—A much more perfect vacuum can be obtained by this pump (Fig. 59). Mercury falls from a funnel A through a piece of thick-walled india-rubber tubing B, which can be compressed with a screw pinchcock C, down a glass U-tube D, and falls over E into a glass tube F, which must be from 6 ft. to 7 ft. long, and has a T-tube G in the upper end. The bore of the glass tube must be small, a little over 1 mm. The screw pinchcock is so adjusted that the mercury as it falls into F breaks up into little threads, which act as pistons as they fall and suck in air or other gas from any vessel duly connected with the side tube G. If an upright glass tube, the lower end of which dips into mercury, be attached to G, the mercury will rise as the air is removed, until it stands at about 760 mm. When the vacuum is practically

perfect, the pellets of mercury fall with a metallic click owing to the absence of air.

It is to the Sprengel pump that we owe the

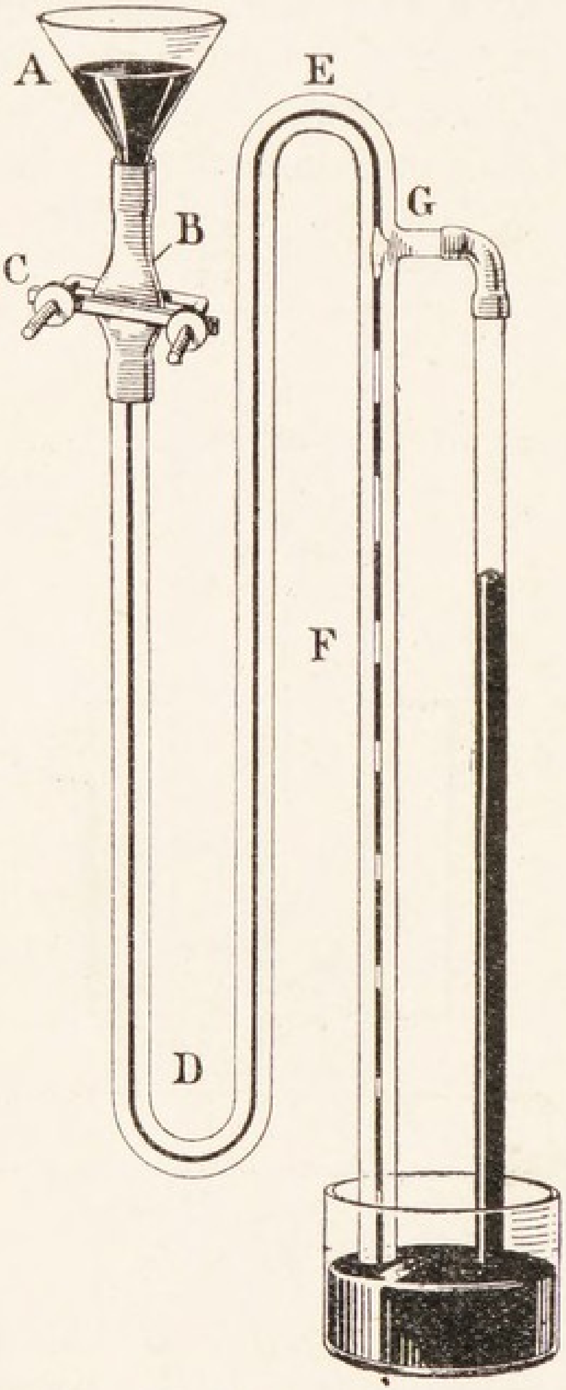


Fig. 59.—Sprengel pump.

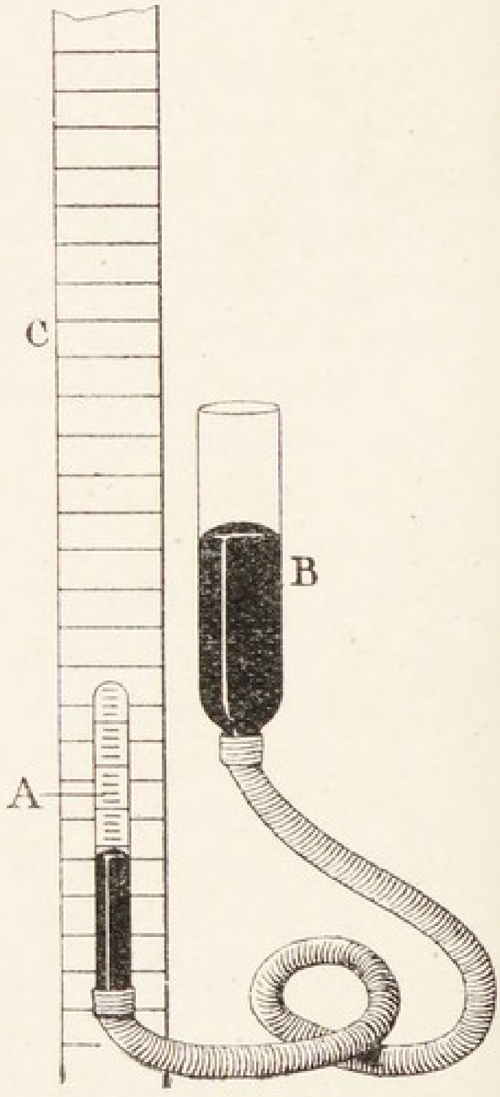


Fig. 60. — Apparatus for verifying Boyle and Mariotte's law.

discovery of the phenomena in high vacua investigated by Sir William Crookes, the Röntgen tube, etc., and it is largely used for exhausting the bulbs of incandescent electric lamps.

Boyle's law, sometimes called the law of Boyle and Mariotte, states the relation between the volume of a gas and the pressure to which it is exposed. *The volume of a gas varies inversely as the pressure, when the temperature remains constant.* Thus, if we double the pressure, the volume of a gas is halved, etc., so that the total product, *pressure* $\times$ *volume*, remains the same at any fixed temperature. This law enables us to reduce the volumes of gases to what they would be at 760 mm.

Ex.: A volume of hydrogen measures 240 c.c. at a pressure of 620 mm. of mercury; its volume, V, at 760 mm. will be found from the equation

$$V \times 760 = 240 \times 620$$

and will, therefore, be $\dfrac{240 \times 620}{760} = 195{\cdot}7$ c.c.

This law can be directly verified with a simple barometer tube, as shown in Fig. 43, but the apparatus shown in Fig. 60 is frequently employed for this purpose. A graduated glass tube A, closed at the upper end, contains the gas; the lower part of A is connected by a long length of thick-walled indiarubber tubing with the reservoir of mercury B. The latter can be moved up and down, and a scale C enables the height of the mercury level in B above that in A (which is fixed to the scale C) to be read off. The mercury is levelled (by moving B) in the tubes A and B, and the volume of gas in A read off (say it is 40 c.c.), the gas being then subject only to the atmospheric pressure (760 mm.). The reservoir B is then raised till its level is 200 mm. above the level in A. The gas is now exposed to a pressure of $760 + 200 = 960$ mm., and its volume will be found to be $\dfrac{40 \times 760}{960} = 31{\cdot}6$ c.c.

Exercises

1. Hydrogen gas is collected in a graduated tube over water. The volume of gas collected is 60 c.c., and the level of the water in the tube is 27·2 cm. above the water level in the tank. Calculate the mass of hydrogen in the tube, given the following data: Temperature, 14° C.; atmospheric pressure, 752 mm.; pressure of aqueous vapour at 14° C., 12 mm.; density of mercury, 13·6; mass of 1 litre of hydrogen at 0° C. and 760 mm., 0·09 grm. [*First M.B.*]

2. A cylinder 20 cm. long, provided with a well-fitting piston, is filled with air at a pressure of 2 atmospheres. How far must the piston be pushed in if the pressure is to be increased to 5 atmospheres? [*First Professional.*]

3. The density of hydrogen at 0° C. under a pressure of 76 cm. of mercury is 0·09 grm. per litre. What would be the density if the pressure were changed to 95 cm. of mercury? [*Ibid.*]

4. If the height of a barometer filled with mercury (sp. gr. = 13·6) be 30 in., what would be the height of a barometer filled with glycerin (sp. gr. = 1·26)? [*Ibid.*]

5. Assuming the specific gravity of blood to be 1·06 and a cubic foot of water to weigh 62·5 lb., find the difference in pressure in the blood due to difference in level between the head and the feet of a man 6 ft. high, standing upright. [*Ibid.*]

(For Answers, *see* p. 388.)

PART II.—HEAT

CHAPTER I

TEMPERATURES AND THERMOMETERS

Nature of Heat—Distinction between Temperature and Quantity of Heat—Measurement of Temperature—Construction of a Thermometer—Graduation—Different kinds of Thermometers—Exercises.

Nature of heat.—The student has already learned (p. 25) that Heat is a form of kinetic energy possessed by the invisible molecules of which every visible mass is composed. When we supply heat to a body, we therefore supply this form of energy, and the molecular vibrations become more and more rapid, and their effects more and more perceptible. These effects are very varied. Often the increased mobility of the particles results in a visible change from the solid to the liquid, or gaseous, state. Often the vibrations become so rapid as to affect other sense organs; the solid becomes luminous, emitting at first a dull *red* light, and finally a *white* light, when the solid is, as we say, "white-hot." These changes are accompanied by a rise in the *temperature* of the body.

Even though this kinetic energy be not sensible to us as heat, it is probable that no body is entirely deprived of it which is at a temperature above absolute zero (p. 132). The student must carefully distinguish between *temperature* and *quantity of heat*. The distinction is analogous to that between velocity

and momentum or quantity of motion (p. 12). One grm. of boiling water is hotter, but contains much less heat, than 10 grm. of tepid water.

Temperature is the degree or intensity of heat; it is independent of the mass of the body, and depends on the velocity of molecular agitation already referred to.

The **quantity of heat** possessed by a mass depends not only (1) on the temperature of the mass, but also (2) on the magnitude of the mass, and (3) on the nature of the mass.

Two bodies are said to be at the same temperature if, when free to exchange heat, the exchange which takes place is equal. Heat tends to pass from points at a higher temperature to points at a lower. The exchange of heat between two bodies which are at different temperatures results, if not prevented, in a gain of heat to the colder body, and a net loss of heat from the hotter body, until they finally arrive at the same temperature.

Thermometers

Temperatures are measured by a thermometer. In the construction of this instrument advantage is taken of the fact that most substances expand when heated, and, within certain limits, the expansion is sufficiently uniform to indicate the degree of heat.

One of the most common effects of heat on a mass is to make it increase in volume, but exceptions are met with in water at 0° C. and in indiarubber. Both of these substances contract when heated. A brass ball which slips easily through a ring when cold will, if heated for a few minutes, expand so much that it will rest on the ring (Fig. 61). The expansion of a liquid, when heated, is also easily seen.

In a thermometer, for instance, the mercury

evidently rises in the tube. The effect is still more obvious in the case of a gas, and is easily shown by Leslie's differential air thermometer (Fig. 62). If we warm the bulb A with the hand, the expansion of the air is at once indicated by the downward movement of the liquid in B, and the upward movement in C.

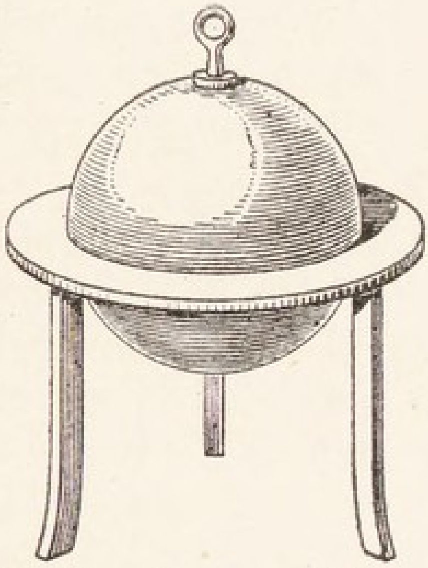


Fig. 61.—Gravesande's ball and ring.

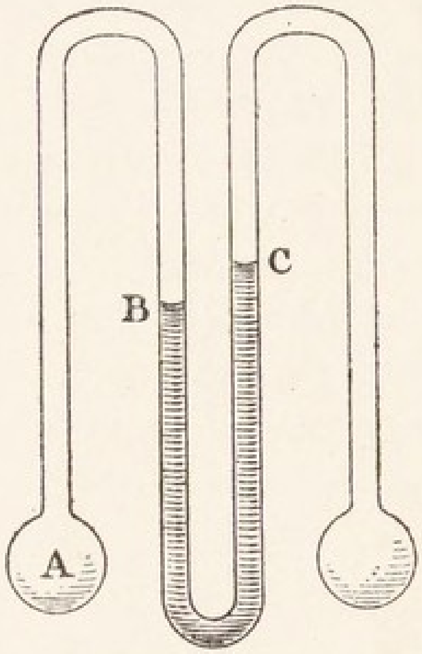


Fig. 62.—Leslie's differential air thermometer.

The **construction of a thermometer** is an important exercise and will be described at some length.

A glass tube of small bore is selected and is then calibrated to see if the bore is uniform. This is accomplished by introducing a short column of mercury (Fig. 63), and accurately measuring its length in various parts of the tube. If the tube be uniform in bore, the mercury column will be of the same length in all parts. A tube with uniform bore having been selected, a bulb is blown at one end

Fig. 63.—Calibration.

and a piece of wider tube attached at the other so as to form a funnel.

The liquids used for filling thermometers are usually mercury or coloured alcohol. Mercury is very suitable for this purpose, because (1) it is easily obtained in a pure state by distillation; (2) its expansion is almost uniform; (3) it remains liquid through a wide range of temperature, freezing at $-38{\cdot}8°$ C., and boiling at 350° C.; (4) it has a low specific heat; (5) it is a good conductor of heat. By virtue of these last two properties the mercury quickly acquires the temperature to be measured, while absorbing very little heat itself.

The liquid is poured into the funnel and the bulb is then gently heated. This causes the air to expand and escape through the mercury in the funnel. The bulb is then allowed to cool, and, as the air contracts, some of the mercury is sucked back into the bulb (Fig. 64). The bulb is again heated till the fluid boils, when its vapour expels all the air, and, on allowing the bulb to cool, it rapidly fills with mercury as the vapour condenses.

The tube having been filled, the thermometer is heated slightly above the highest temperature it is to indicate, and the end of the tube near the funnel drawn off and sealed by the blowpipe. The thermometer is then laid aside for some months to allow the bulb, which contracts slightly when freshly blown, to attain its final volume, when the instrument is ready for graduation.

Graduation of a thermometer.—As temperatures only indicate *degrees* of heat, they must have reference to some standard level, from which their difference is estimated; just as all heights are measured by their difference from sea level. The temperature often chosen for this purpose is that

of melting ice. To obtain, however, a standard *interval of temperature* we require *two* standard temperatures. The higher temperature now generally selected is the *boiling-point* of water, when *the barometer stands at* 760 *mm. or* 30 *in.* In the earlier thermometers of Newton and Fahrenheit the upper temperature selected was perhaps the more natural one, of the human body, or " blood heat." To determine the lower fixed point, the thermometer is placed carefully in melting ice (Fig. 65), and, when the mercury no longer falls, the level at which it

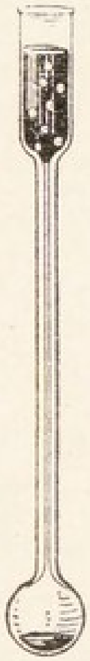


Fig. 64.—Filling thermometer bulb.

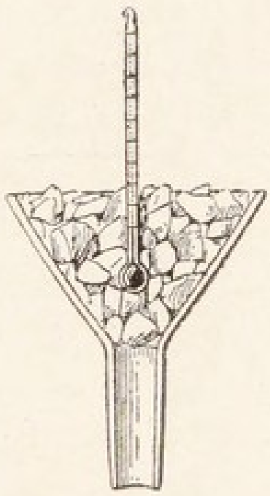


Fig. 65.—Verifying zero of thermometer.

remains stationary is marked as the freezing-point. To determine the upper fixed point, the thermometer is placed in a " hypsometer " (Fig. 66). The mercury, when stationary, just appears above the cork, the bulb of the thermometer being surrounded by the steam, which finally escapes at A. If the barometer does not stand at 760 mm. a correction must be made before the boiling-point is marked on the thermometer. A variation of 27 mm. practically alters the boiling-point by 1° C.

If, for instance, the barometer reading is 742 mm., or 18 mm. *less* than 760, then we *deduct* $\frac{18}{27} = \frac{2}{3}$ of a degree from 100° C. and say that the temperature of the steam is $99\frac{1}{3}$° C. If the barometer reading is 769 mm. we should estimate the steam temperature at $100\frac{1}{3}$° C.

Thermometrical scales.—In the earlier thermometers not only was a different interval of temperature selected, but this interval was differently subdivided. Newton, in 1701, adopted the interval between the

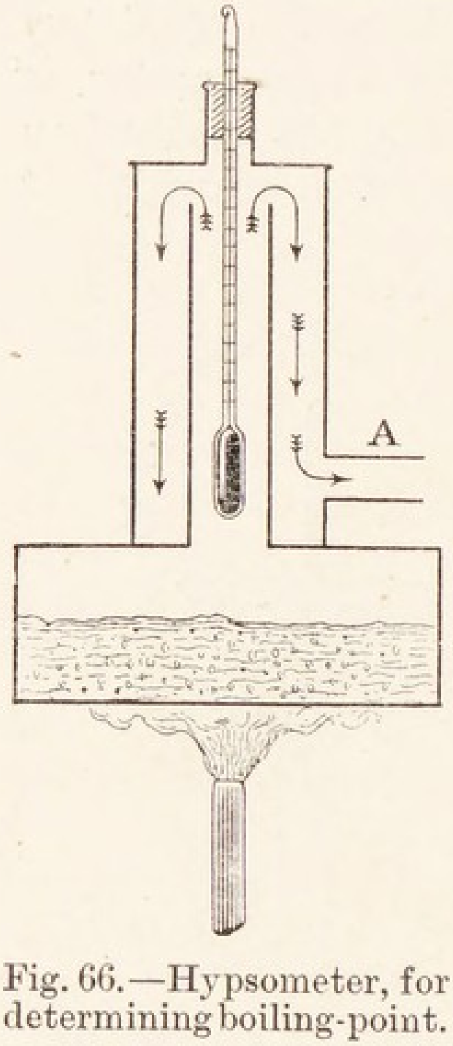


Fig. 66.—Hypsometer, for determining boiling-point.

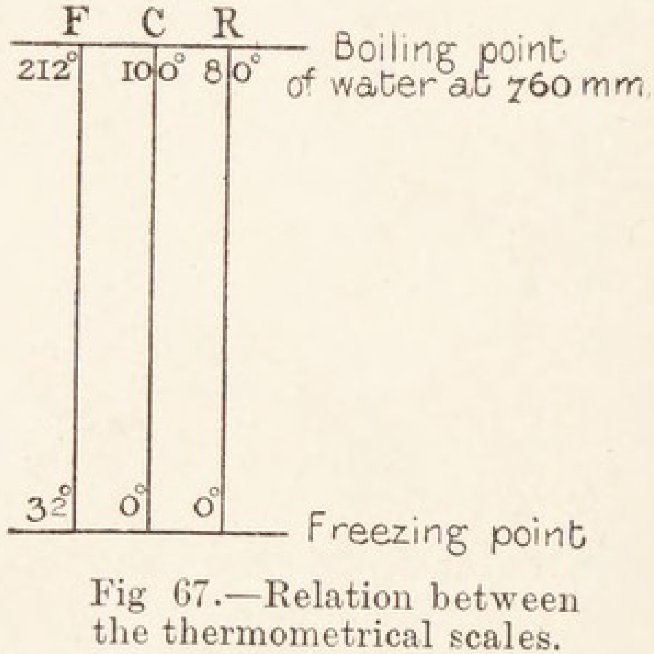


Fig 67.—Relation between the thermometrical scales.

freezing-point of water and "blood heat," and divided this into 12 equal parts. The freezing-point of salt water is lower than that of fresh water, and mixtures of ice and salt are often employed as freezing mixtures (p. 156). Fahrenheit employed the lowest temperature he thus obtained as his lower limit or zero, and "blood heat" as his upper limit. Like Newton, he at first divided his interval into 12 equal parts, but one such division

was inconveniently large for finer measurements, and to diminish it he increased the number of divisions to 96; blood heat was therefore indicated by 96 (instead of 12); it is now usually represented by 98·4 degrees on his scale, but has a range of about 1·2 degrees on either side of this point, according to the time of day or night and the situation of the recording thermometer when the temperature is taken. The freezing-point of pure water is represented by 32 on this scale, and the normal boiling-point by 212 when the graduation is extended to that temperature.

In 1731 the French scientist Réaumur constructed a thermometer by filling the bulb and a portion of the stem with alcohol at the freezing-point of water—his zero—and then graduating the stem above this point into divisions each of which had a volume equal to $\frac{1}{1000}$ of the volume of the alcohol at the zero. When this thermometer was brought to the temperature at which water normally boils, the alcohol expanded till it reached the 80th division. Réaumur therefore called the boiling-point of water 80, and it is still called 80 on this type of thermometer, but the instrument is now constructed by first determining the two fixed points (as usual), and then dividing the interval into 80 equal parts. The volume of each division is not now $\frac{1}{1000}$ of the volume below zero, but about $\frac{1}{6480} \times \frac{100}{80}$ if mercury replaces the original alcohol (*see* p. 129).

In 1742 Celsius, a Swedish astronomer, described the now familiar centigrade thermometer.

All three scales are used at the present time: *Fahrenheit's* in the British Isles, in Canada, the United States, etc.; the *Centigrade* in France, Germany, etc.; and *Réaumur's* in Italy, Russia, etc. The

relation between the scales is shown in the diagram (Fig. 67). The same standard interval is graduated into 180 divisions on the Fahrenheit scale, 100 divisions on the Centigrade scale, and 80 divisions on the Réaumur scale. Hence,

	180 F. degrees	=	100 C. degrees	=	80 R. degrees
or	9 ,,	=	5 ,,	=	4 ,,

The freezing-point of water is marked 32° in the Fahrenheit scale and 0° in the Centigrade and Réaumur. Water therefore boils at 212° F., 100° C., and 80° R., in presence of the standard atmosphere.

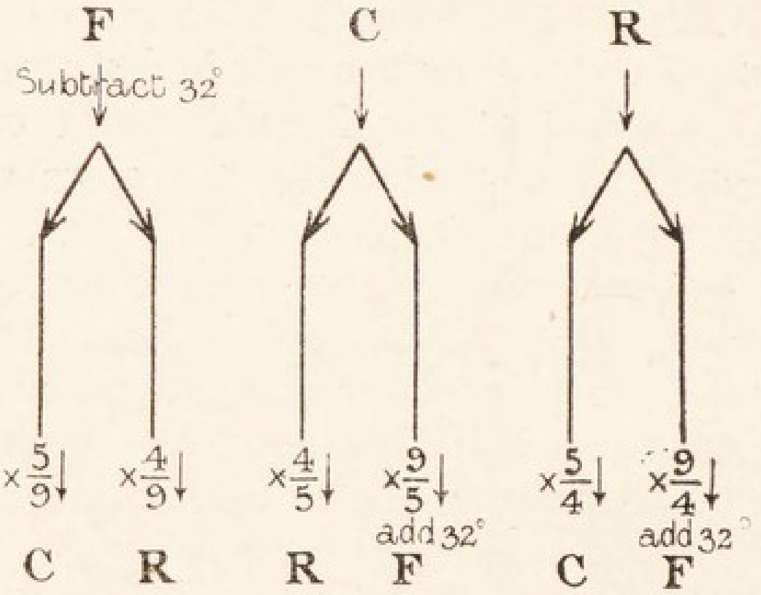


Fig. 68.—Conversion of thermometrical scales.

Conversion of scales.—To convert one scale into another, the scheme illustrated in Fig. 68 will be found useful. Thus, to convert 40° C. into Fahrenheit, $40 \times \frac{9}{5} = 72 + 32 = 104°$ F.; to convert 82° F. into Réaumur, subtract $32 = 50 \times \frac{4}{9} = 22{\cdot}2°$ R. In cheap, common thermometers the scales are made first and the thermometer tubes and bulbs are blown to fit the scale. The best clinical thermometers in this country have usually the "Kew certificate"—that is, they have been verified at the Physical Laboratory.

The *delicacy* of a thermometer is the smallest fraction of a degree it will show, and depends on the relative sizes of the bulb and the bore. The larger the bulb and the smaller the bore the more delicate is the thermometer.

The *sensitiveness* of a thermometer—that is, the rapidity with which it will acquire a temperature—depends on the size and shape of the bulb, and on the specific heat and the conductivity of the thermometric substance employed. A cylindrical bulb is

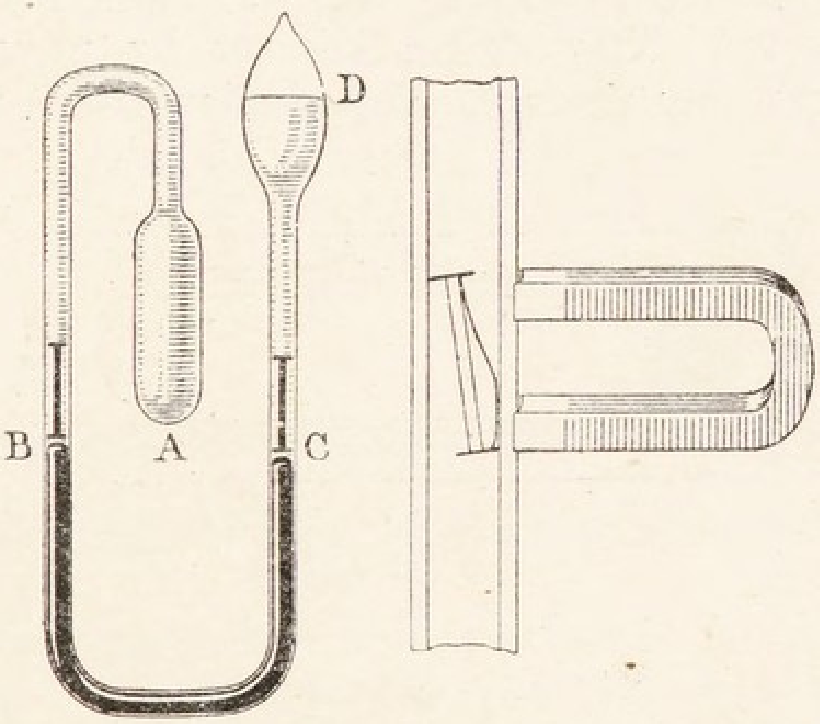


Fig. 69.—Six's thermometer, showing use of magnet for setting it.

more sensitive than a spherical one of the same capacity, but the smaller the bulb the more sensitive the thermometer, other things being equal.

Registering thermometers are thermometers which show the highest or lowest temperature to which they have been exposed.

Six's thermometer is an alcohol thermometer (Fig. 69). The bulb is at A, the alcohol column of the thermometer ends at B; the tube is filled with mercury up to C, and the rest with alcohol, which

only half fills the bulb D. The maximum and minimum temperatures are registered by two little indices of iron wire at B and C, which are kept in position by a very delicate spring. When the temperature goes up, the mercury rises at C and pushes up the index until the maximum temperature is attained; when the mercury falls, the spring keeps the index in its maximum position. As the temperature sinks the mercury forces up the iron index at B, so that the lower end of the index B gives the minimum, and the lower end of C the maximum, temperature. The indices are set by means of a small horseshoe magnet, whose hollowed-out ends are

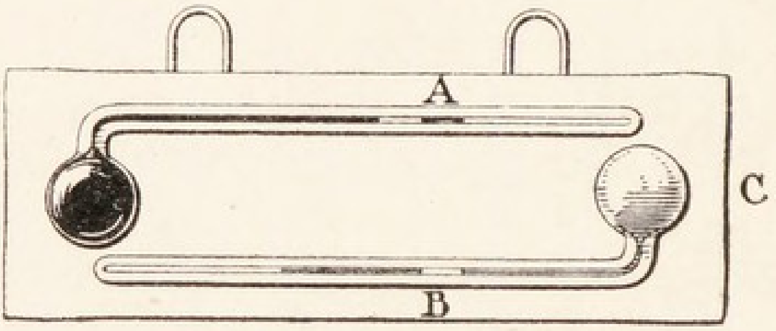


Fig. 70.—Rutherford's maximum and minimum thermometer.

moved along the glass tube, and bring the indices again into contact with the mercury.

Rutherford's maximum and minimum thermometer.—The maximum thermometer contains mercury, and the maximum temperature is registered by a little index of iron wire A (Fig. 70), which is pushed forward by the mercury as it expands, and so marks the maximum temperature.

The minimum thermometer contains alcohol, and the index is a piece of glass B, which allows the alcohol to pass it as the temperature rises; but when, in cooling, the surface of the liquid reaches the index, the surface tension pulls the index back with it, and so the minimum temperature is registered. In order

to set the instrument the end C is raised, when the indices fall to the surfaces of the liquids. When in use the instrument should be horizontal.

Negretti & Zambra's maximum thermometer.—In this instrument (Fig. 71) the bore is narrowed close to the bulb. The mercury will only pass

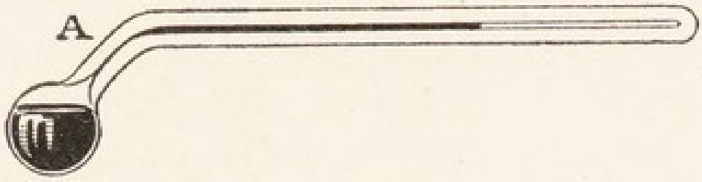


Fig. 71.—Negretti & Zambra's maximum thermometer.

through this constricted glass tube A when forced to do so. As the temperature rises, the expansion of the mercury in the bulb forces the liquid through the narrow tube; but when the temperature falls, the column breaks at A, one portion retreats into the

Fig. 71A.—Clinical thermometer, the mercury below body temperature.

bulb, and the other registers the maximum temperature. The thermometer is set by swinging it vertically downwards.

This form is much used for clinical thermometers (Fig. 71A).

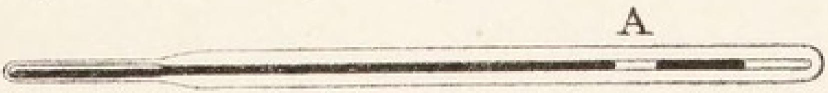


Fig. 72.—Philipps's maximum thermometer.

Philipps's maximum thermometer.—In this instrument (Fig. 72) the thread of mercury is broken by a bubble of air A. When the temperature rises, this bubble forces the index column of mercury along the tube, but when the temperature falls, the index

column is left indicating the maximum temperature. It is set in the same way as Negretti & Zambra's.

For temperatures below $-38°$ C., alcohol can be used down to $-130°$ C., when it freezes.

Temperatures beyond the range of the ordinary mercury thermometer can be determined by filling the space above the mercury with hydrogen under pressure. The boiling-point of mercury is thus raised (p. 172), and the thermometer, which must be made of hard glass, can be used up to 550° C.

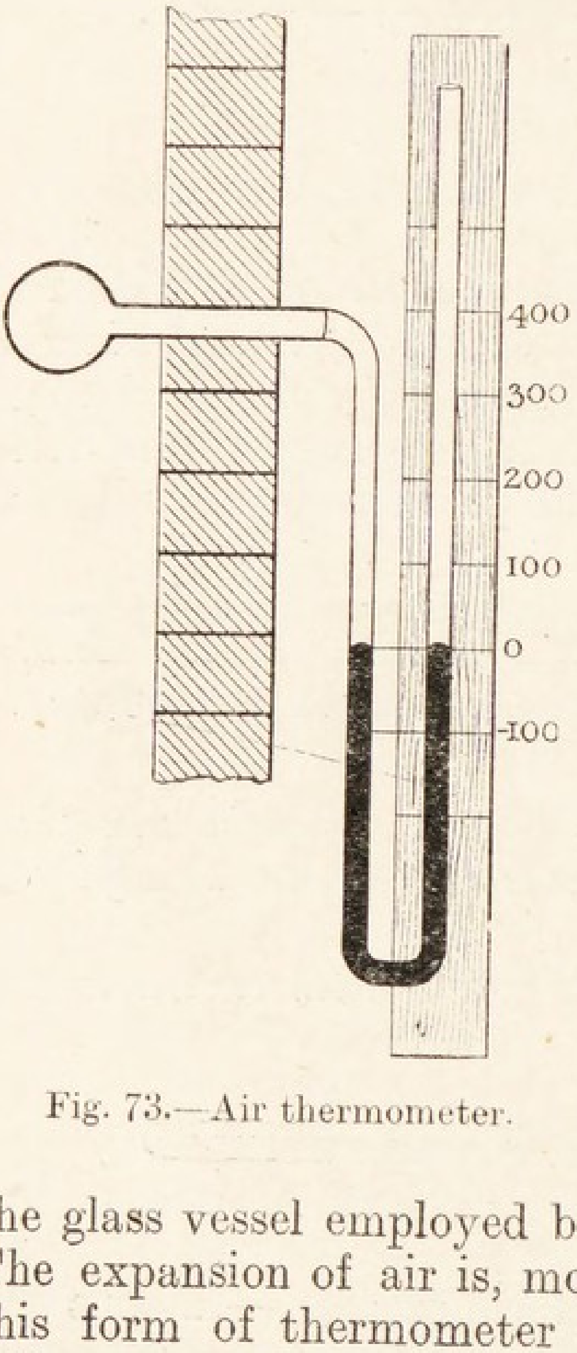


Fig. 73.—Air thermometer.

Air thermometer.—The coefficient of expansion of dry air is $\frac{1}{273}$, and is therefore about 20 times that of mercury, and about 140 times that of glass. The apparent expansion is therefore not much less than the real expansion, and small variations in the quality and calibre of the glass vessel employed become of less importance. The expansion of air is, moreover, so uniform that this form of thermometer can be used for a very wide range of temperature. For use in furnaces a bulb of porcelain or platinum is employed (Fig. 73), and is connected with a bent glass tube outside, which contains mercury. The thermometer is graduated by

plunging the bulb (1) into melting ice, and (2) into steam (at 760 mm.), noting the expansion, and then, if the tube is uniform, marking the rest of the scale. A correction is made for the increase of pressure produced by the rising column of mercury.

Temperatures, both high and low, can also be determined by **electrical means:** (1) by the variation of the resistance of a platinum wire, the resistance increasing at a known rate as the temperature rises; (2) by measuring, with the aid of a galvanometer (p. 349), the current developed on heating a junction of two dissimilar metals, such as platinum and a platinum-iridium alloy.

VARIOUS TEMPERATURES

		° C.	Degrees absolute.
Absolute zero		−273	0
Helium liquefies		−269	4
Hydrogen solidifies		−259	14
Hydrogen liquefies		−253	20
Oxygen liquefies		−183	90
Alcohol freezes		−130	143
Greatest natural cold (Nares)		− 57·7	215·3
Mercury freezes		− 38·8	234·2
Mercury boils		356·7	629·7
Red heat just visible	about	526	
Dull red heat	,,	700	
Yellow heat	,,	1,100	
White heat	,,	1,400	
Silver melts	,,	960	
Cast-iron melts	,,	1,550	
Highest temperature of a wind furnace	,,	1,805	
Platinum melts	,,	1,750	
Electric furnace	,,	3,500	
Sun's temperature	,,	5,550	

Very small amounts of impurities in a metal alter the melting-point considerably, and different samples

therefore give different results. This is probably the principal cause of the variation in the values obtained by different observers.

Exercises

1. Express in each of the three thermometric scales (p. 113) the following temperatures: Blood heat; the point of maximum density of water; the freezing-point of mercury; the boiling-point of mercury; normal *room-temperature.*

2. Mention three physical properties that change when the temperature of substances is altered, and explain in each case how the change may be employed for the measurement of temperature. [*First Professional.*]

3. A faulty thermometer reads $-1°$ C. when placed in melting ice, and 102° C. when in steam, on a day when the atmospheric pressure is 77 cm. What is the correct temperature when the thermometer indicates 75° C.? (2·7 cm pressure changes the boiling-point of water by 1° C.)

(For Answers, *see* p. 388.)

CHAPTER II

EXPANSION OF SOLIDS, LIQUIDS, AND GASES

Expansion of Solids—Square Expansion—Cubic Expansion—Expansion of Liquids—Expansion of Gases—Exercises.

Expansion of solids.—That different solids expand in different degrees can be shown by heating two rods, one of iron and one of brass, and magnifying their expansion by levers or some other suitable mechanical device. One simple method is illustrated in Fig. 74. The rod is placed in a bath, one end resting against one end of the bath and the other pressing against a lever L. A small mirror M is cemented on the axis at C; a beam of light A B is reflected from the mirror M and received on a scale. When the bar expands, the beam of light moves on the scale. The bath can be filled with water at various temperatures.

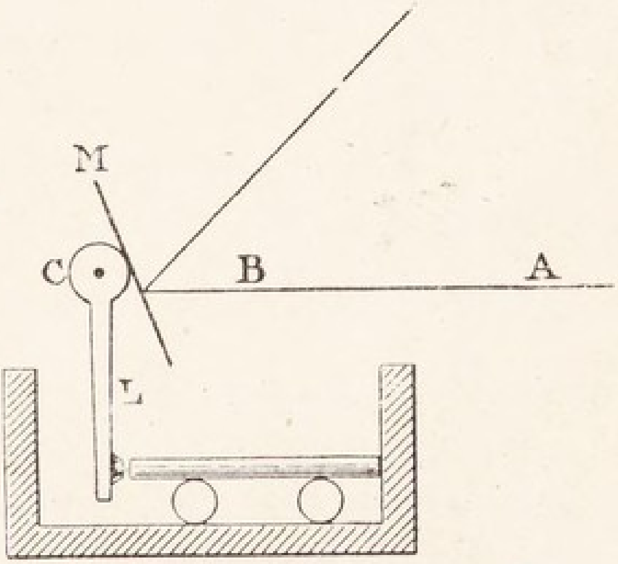


Fig. 74.—Expansion of a rod of metal.

The exact increase in length of a bar of metal for a given rise in temperature can be determined by the spherometer (p. 395), or by a travelling microscope. A bar of brass one metre in length E rests on a solid

foundation F (Fig. 75), and is surrounded by a glass tube A, through which water at any temperature can be passed by the tubes B C. On the top of the framework rests a piece of plate glass D D, through which a hole is bored large enough to receive the end of the rod of brass E. Water is run through, say, at 10° C., and the level of the end of the brass rod, relative to the plate glass, is determined by the spherometer. The temperature of the water is raised to 50°, and the increase in length found by a second reading of the spherometer.

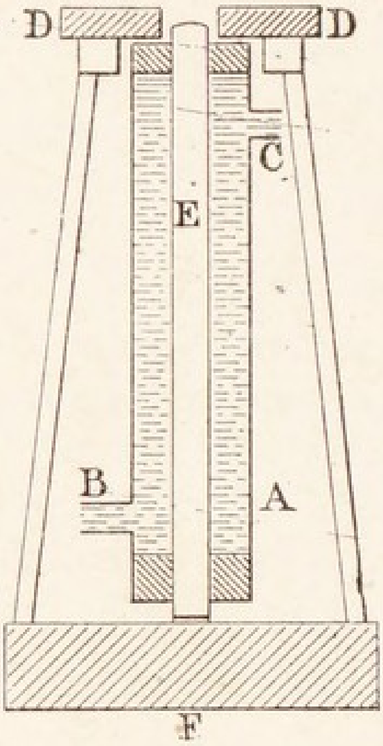


Fig. 75.—Determination of increase of length of brass rod.

What is technically called the *expansion* is not this increase in length, but the ratio $\frac{\text{increase in length}}{\text{original length}}$; if this expansion be due to a rise in temperature of 40° C., and if it may be regarded as uniform, then the expansion for a rise of 1° C. will be $\frac{1}{40}$ of $\frac{\text{increase in length}}{\text{original length}}$. This would be the *coefficient of linear expansion* of the brass between the temperatures 10° C. and 50° C. It is often more convenient to choose two temperatures which can more easily be maintained constant for a long time, such as 0° C. and 100° C. In that case, if a length, L, at 0° C. becomes $L + l$ at 100° C., the coefficient of linear expansion between these limits is $\frac{1}{100}$ of $\frac{l}{L}$.

This coefficient for	iron	is	0·000012
,, ,,	brass	,,	0·000019
,, ,,	zinc	,,	0·000030
,, ,,	slate	,,	0·000014
,, ,,	glass	,,	0·000009
,, ,,	platinum	,,	0·000009
,, ,,	pine wood	,,	0·0000005

It will be noticed that glass and platinum expand at the same rate, so that when a platinum wire is fused into a glass globe the joint remains sound when cold.

Ex.: Calculate l, the increase in length of 50 miles of *iron rails* for a rise in temperature of 20° C. We see from the foregoing explanation that

$$\frac{1}{20} \text{ of } \frac{l}{50} = 0{\cdot}000012$$

$$\therefore l = 0{\cdot}012 \text{ miles}$$

Similarly, if an *area*, A, at 0° C. becomes an area, $A + a$, at 100° C., then the mean coefficient of *superficial* expansion between these limits of temperature is $\frac{1}{100}$ of $\frac{a}{A}$.

Also, if a volume, V, at 0° C. become $V + v$ at 100° C., then the mean coefficient of *cubical* expansion between these limits is $\frac{1}{100}$ of $\frac{v}{V}$.

If we consider the increase in size of a *sheet* of metal, taking the coefficient of linear expansion to be a, and supposing the sheet to be 1 ft. square, each side will increase (for 1°) to $1 + a$, and therefore the new area will be $(1 + a)^2 = 1 + 2a + a^2$. Now, a^2 is so small that it may be neglected, and the new area then becomes $1 + 2a$.

The *square expansion* per unit area—that is, the *coefficient of superficial expansion*—is therefore reckoned as twice the linear.

Similarly, the volume of a cube of unit side becomes, for a rise of 1° C., $(1 + a)^3 = 1 + 3a + 3a^2 + a^3 = 1 + 3a$, since the other terms are so small that they may for practical purposes be neglected. The coefficient of *cubic expansion* is therefore taken as three times the linear.

The expansion of metals has to be taken into account in many directions. The wooden patterns for brass castings have to be made larger than the brass articles are intended to be. The iron tyres of wheels are made smaller than the wooden wheel,

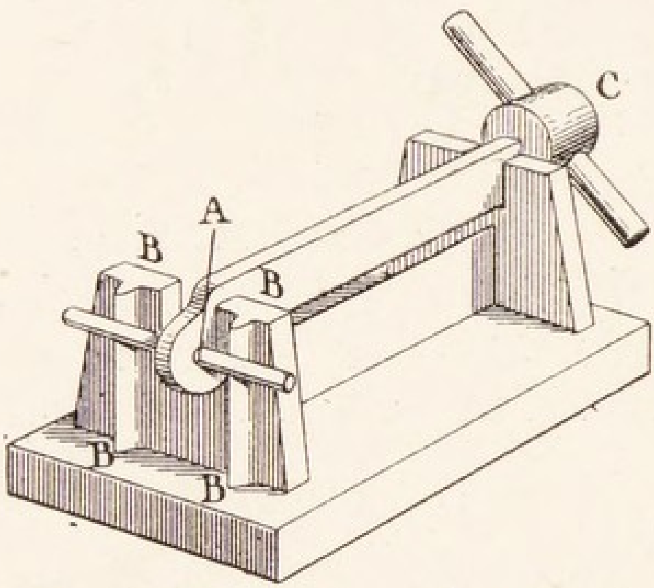


Fig. 76.—Apparatus to show the force of contraction when cooling.

so that when made red hot they can just slip over, and as they cool contract and bind the wheel firmly together. The rails on the line are laid so as to leave a small gap between the rails to allow room for expansion. In a long iron bridge like the Forth Bridge the difference in length in summer and in winter may amount to a foot, and the ends of the girders are mounted on rollers to allow this movement to take place.

The force with which this contraction or expansion takes place is enormous, as can be shown by the apparatus in Fig. 76. An iron bar is heated, and

while it is hot a rod of cast iron, about $\frac{1}{4}$ in. in diameter, is passed through the hole A, and screwed up tightly by the nut C against the jaws B B. As the bar cools it contracts and breaks the rod.

Two other interesting illustrations may be mentioned—the *compensating pendulum* and the *compensating balance wheel*.

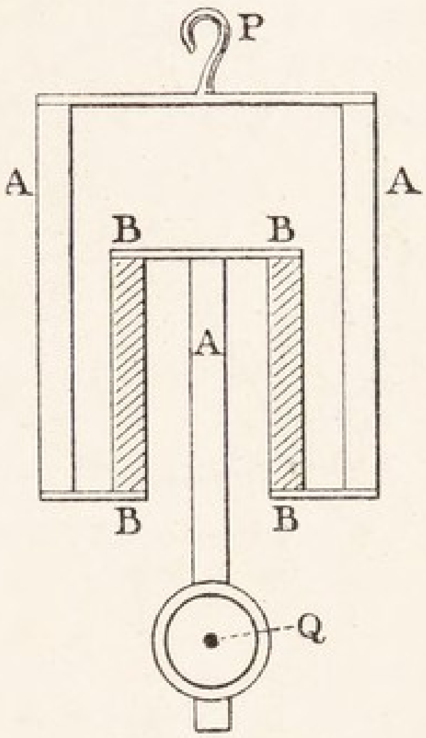


Fig. 77.—Gridiron pendulum, zinc and iron.

Compensating pendulum.—A simple pendulum in summer lengthens, and so the clock loses (p. 49); the reverse is the case in winter. The compensating pendulum is designed to keep the distance from the point of suspension to the centre of gravity of the heavy bob unchanged, however the temperature may vary. The bars A A A in Fig. 77 are of iron and the bars B B of zinc. As the temperature rises, the iron bars A A A expand and the bob tends to drop; but as the zinc bars B B expand more than the iron, they tend to raise the bob, and if the relative lengths be inversely as their coefficients of linear expansion (zinc : iron : : 12 : 30), the distance between the point of suspension P and Q will remain unaltered. This contrivance is sometimes called the *gridiron* pendulum, from its obvious resemblance to that domestic implement.

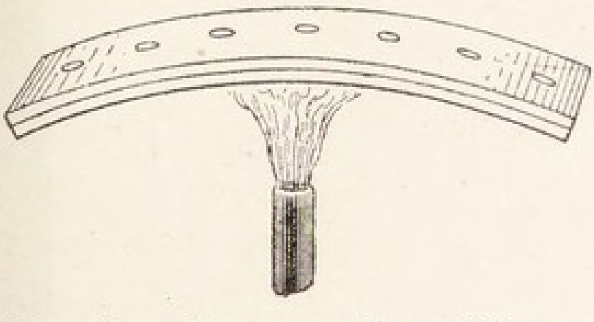


Fig. 78.—Compound bar of brass and iron bending when heated.

If two flat pieces of brass and iron respectively be riveted together at the ordinary temperature, the compound bar, when heated, will become curved. The brass expands more than the iron, and therefore forms the outer and longer curve (Fig. 78). This fact is utilized in the construction of *the compensating balance wheel*, which has a compound rim of brass and iron, the brass being on the outside (Fig. 79).

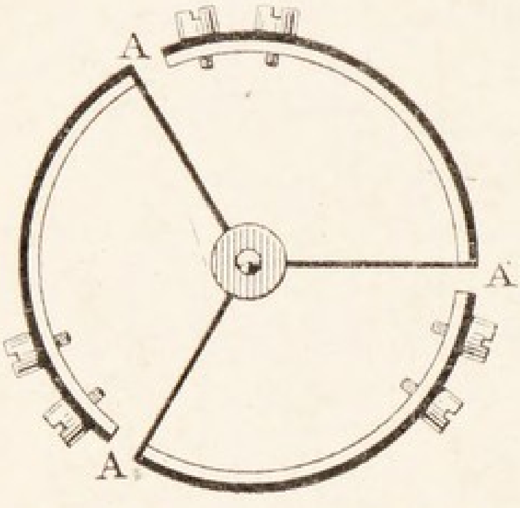


Fig. 79.—Compensating balance wheel.

The rim is divided in three places, A A A; at the free ends little weights are fixed. As the temperature rises, the spokes increase in length, and the rim is at a greater distance from the centre, tending to make the watch lose; but this tendency is compensated, if the weights are properly adjusted, by the rim curving inwards and bringing the weights at A A A nearer the centre.

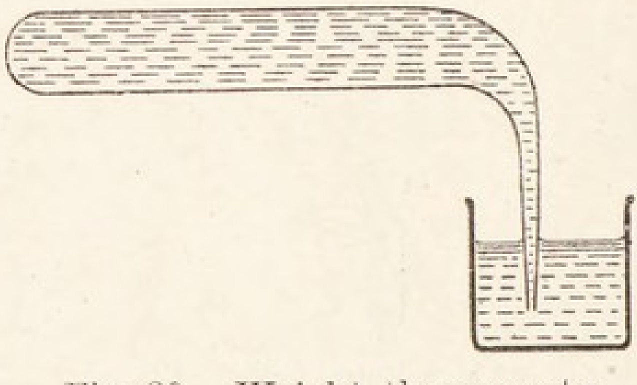


Fig. 80.—Weight thermometer.

Expansion of liquids.—We need only consider the *cubical* expansion of liquids; it is usually much greater than that of solids. Each liquid has its own coefficient of cubical expansion. That of mercury is 0·00018. The coefficient of expansion is usually determined by finding the specific gravity of the liquid at different temperatures by means of the specific gravity bottle (p. 63).

To find the coefficient of apparent expansion of a liquid :—

Ex. : Suppose that the sp. gr. is found to be S_1 when the liquid is at $t_1°$ C., and S_2 when it is at a higher temperature, $t_2°$ C. A weight, W, of this liquid which fills a volume, V_1, at the lower temperature will fill a larger volume, V_2, at the higher temperature, such that (p. 55)

$$S_1 V_1 = W = S_2 V_2$$

$$\therefore \quad \frac{S_1}{S_2} = \frac{V_2}{V_1}$$

$$\therefore \quad \frac{S_1}{S_2} - 1 = \frac{V_2}{V_1} - 1$$

$$\therefore \quad \frac{S_1 - S_2}{S_2} = \frac{V_2 - V_1}{V_1}$$

$$\therefore \quad \frac{1}{t_2 - t_1} \text{ of } \frac{S_1 - S_2}{S_2} = \frac{1}{t_2 - t_1} \text{ of } \frac{V_2 - V_1}{V_1}$$

= the mean coefficient of cubical expansion between t_1 and t_2 (p. 123).

In this case we can make a further simplification ; we only require the *ratio* of S_1 to S_2, and this is independent of the standard substance chosen ; we may therefore choose as our standard the liquid itself at the temperature t_1 ; then $S_1 = 1$ and $S_2 = \dfrac{W_2}{W_1}$, where W_2 and W_1 are the weights of the liquid which fill the same bottle at the respective temperatures t_2 and t_1.

$$\therefore \quad \frac{S_1}{S_2} = \frac{1}{\dfrac{W_2}{W_1}} = \frac{W_1}{W_2}$$

$$\therefore \quad \frac{S_1}{S_2} - 1 = \frac{W_1}{W_2} - 1$$

$$\therefore \quad \frac{S_1 - S_2}{S_2} = \frac{W_1 - W_2}{W_2}$$

and, on comparing this with the result previously

obtained, we see that the mean coefficient of cubical expansion between t_1 and t_2 is

$$\frac{1}{t_2 - t_1} \text{ of } \frac{W_1 - W_2}{W_2}$$

$W_1 - W_2$ is evidently the weight of liquid which overflows from the bottle during the rise of temperature from t_1 to t_2. In fact, the bottle is really employed in this experiment as a **weight thermometer.** This instrument often has the form of an elongated bulb (Fig. 80) with drawn-out neck which is bent down so as to dip into a small vessel of the liquid.

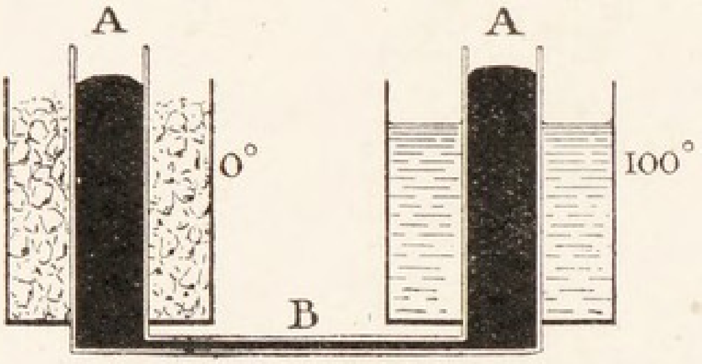


Fig. 81.—Apparatus for determining the absolute expansion of mercury.

Ex.: The empty bulb is first weighed = 20 grm.; it is then filled with mercury (or any other liquid) at 0°, re-weighing = 48 grm. It is next heated to 100°, and weighs 47·55 grm., so that at 0° the bulb contains 28 grm. and at 100° 27·55 grm.; the weight which has overflowed is therefore 0·45 grm.

Hence the mean coefficient of expansion is equal to $\frac{1}{100}$ of $\frac{0\cdot25}{27\cdot55}$ = 0·00016 nearly.

As the mercury is contained in a glass bulb, which also expands when heated, the *apparent expansion* as determined above only shows the *difference* between the real expansion of the mercury and the expansion of the glass. This is obviously less than the *absolute* expansion.

That the glass bulb does expand can be readily shown by plunging a thermometer, with a large bulb, suddenly into hot water, when the mercury will be

seen to drop for a moment and then rapidly rise in the stem. The absolute expansion of mercury can be determined (Fig. 81) by connecting two vertical glass tubes A A, filled with mercury, by a long narrow tube B. One of the vertical tubes is surrounded with ice, the other with hot water. The difference in their levels is read off. As in Hare's apparatus (p. 64), the specific gravities will be inversely as their heights, or $\dfrac{S_0}{S_{100}} = \dfrac{h_{100}}{h_0}$

$$\text{but} \quad \frac{S_0}{S_{100}} = \frac{V_{100}}{V_0} \text{ (p. 127)}$$

$$\therefore \quad \frac{V_{100}}{V_0} = \frac{h_{100}}{h_0}$$

and the absolute coefficient of expansion, $\dfrac{1}{100}$ of $\dfrac{V_{100} - V_0}{V_0}$, is therefore equal to $\dfrac{1}{100}$ of $\dfrac{h_{100} - h_0}{h_0}$ and can be determined by reading the heights of the two columns. If, for instance, the height in the tube at 0° is 1,000 mm., and that in the tube at 100° is 1,018 mm., the value of the coefficient will be $\dfrac{1}{100}$ of $\dfrac{18}{1000} = 0{\cdot}00018$.

The relation between the capacity of the bulb of a thermometer, together with any portion of the stem below zero, and the capacity of one division of the stem is evidently connected with the apparent coefficient of expansion (a). Let the capacity of bulb, etc., be equal to that of n divisions of the stem; these n divisions apparently expand to $n + 1$ divisions for a rise of 1°, therefore

$$a = \frac{1}{n}$$

In the case of mercury, a is about $\dfrac{1}{6480}$.

Therefore the bulb of a mercury thermometer should have a capacity of about 6,480 divisions of the stem.

To find the relation between a, the apparent, and m, the real, expansion of mercury we must remember that the volume of a stem division has altered owing to the real expansion g of the glass.

Let v be the capacity of a division at 0°
and v_1 „ „ „ „ „ „ 1°

$$\text{Then}\quad a = \frac{v_1 - v}{v}$$

$$= \frac{v_1}{v} - 1$$

$$\text{Therefore}\quad 1 + g = \frac{v_1}{v}$$

$$\text{Also,}\quad m = \frac{(n+1)\,v_1 - nv}{nv}$$

$$= \frac{(n+1)\,v_1}{nv} - 1$$

$$\text{Therefore}\quad 1 + m = \frac{(n+1)\,v_1}{nv}$$

$$= \frac{n+1}{n} \cdot \frac{v_1}{v}$$

$$= \left(1 + \frac{1}{n}\right) \cdot \frac{v_1}{v}$$

$$= (1 + a)(1 + g)$$

$$= 1 + a + g + ag$$

$$\text{Therefore}\quad m = a + g + ag$$

or, if we neglect the very small quantity ag, $m = a + g$. From this we might deduce

$$g = m - a$$

$$= \frac{1}{5550} - \frac{1}{6480}$$

$$= 0{\cdot}000180 - 0{\cdot}000154$$

$$= 0{\cdot}000026$$

which shows that the real expansion of glass is about $\frac{1}{7}$th of the real expansion of mercury.

The **expansion of water** is anomalous: 1,000 c.c. of water at 0°, when heated, *contract* to 999·88 c.c. at 4°. At 8° the volume is again 1,000, at 16° 1,000·85, and the expansion continues as the temperature rises. Water, therefore, is not, like most liquids, heaviest at its freezing-point, but at 4° C. above it.

This physical fact has a far-reaching effect on natural phenomena. If water continued to become heavier until it froze, ice would begin to form at the *bottom* of lakes, rivers, etc. They would be frozen solid during a severe winter, and would probably not completely melt during the summer. But as water when cooled below 4° expands, the colder and lighter water rises to the top and ice forms on the top, while the heavy layer of water at 4° rests on the bottom.

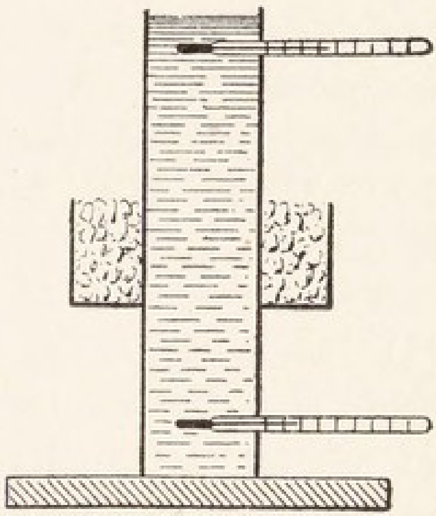


Fig. 82.—Hope's experiment.

This can be verified by Hope's experiment. A tall cylinder (Fig. 82) has two thermometers fitted in its side, one at the top and one at the bottom; the middle is cooled by a gallery containing ice and salt. The thermometer beneath will be found to sink to 4° C. and no lower, while the thermometer at the top falls till ice begins to form.

When water freezes it expands $\frac{9}{100}$ of its volume, so that ice is lighter than water, its sp. gr. being 0·9175. This expansion takes place with enormous force, bursting iron bottles, water-pipes, disintegrating rocks, etc.

Expansion of gases.—All gases have the same coëfficient of expansion as dry air (p. 118), and therefore expand by $\frac{1}{273}$ of their volume at 0° C. for

a rise of 1° C. in temperature. Similarly, a true gas will contract by $\frac{1}{273}$ of its volume at 0° for a fall of 1° C. If it continues to do so as the temperature falls, it will occupy no volume at all at −273° C., because it will have contracted $\frac{273}{273}$ of its volume. This does not really happen. Before this temperature is reached, every substance ceases to be gaseous and therefore no longer obeys this, or any, gaseous law. Though never actually reached, it is regarded as the lowest possible temperature, and is often called the *absolute zero*. Temperatures reckoned from this point are known as *absolute* temperatures. They can evidently be obtained by adding 273 to the Centigrade temperature. Thus, 100° C. is 373° absolute, 0° C. is 273° absolute, and t° C. is $(273 + t)$° absolute.

The expansion of a gas is regulated by two laws—(1) Boyle's law (p. 105), and (2) the law of *Charles and Gay-Lussac*, which states that "the volume of a gas varies directly as the absolute temperature when the pressure of the gas remains constant."

If V and V' be the volumes respectively occupied by the *same mass* of gas at two absolute temperatures, T and T', then

$$\frac{V}{V'} = \frac{T}{T'}$$

provided that the pressure, P, does not change. If, however, P changes to P' but T remains unchanged, then we know by Boyle's law that $\frac{V}{V'} = \frac{P'}{P}$.

Hence, *when both P and T change*, we must have

$$\frac{V}{V'} = \frac{T}{T'} \times \frac{P'}{P}$$

$$\text{or} \quad \frac{VP}{T} = \frac{V'P'}{T'}$$

This relation enables us to find what volume a quantity of gas collected at one temperature and pressure would occupy at any other temperature and pressure.

If, while the temperature rises, a gas be not allowed to expand, the pressure of the gas must be increased in proportion to the absolute temperature, for we have then

$$V = V'$$

$$\text{and} \quad \therefore \quad \frac{P}{T} = \frac{P'}{T'}$$

$$\text{or} \quad \frac{P'}{P} = \frac{T'}{T}$$

If the range of temperature chosen for the experiment be from 0° C. to t° C. we see that

$$\frac{P'}{P} = \frac{273 + t}{273}$$

$$\therefore \quad \frac{P'}{P} - 1 = \frac{273 + t}{273} - 1$$

$$\therefore \quad \frac{P' - P}{P} = \frac{t}{273}$$

$$\therefore \quad \frac{1}{t} \text{ of } \frac{P' - P}{P} = \frac{1}{273}$$

The expression on the left-hand side is the *coefficient of increase of pressure* of the gas when kept at constant volume; it is evidently equal to the coefficient of expansion (p. 118), and might be determined by experiments with the air thermometer (Fig. 73). For this purpose mercury would be constantly added to the open limb of the U to keep the mercury in the other limb always at the zero level in spite of the rise of temperature; the height (h) of the mercury in the open limb above the zero level will be proportional to the temperature, and the instrument might be graduated accordingly, for it can easily be shown that

$$\frac{t}{h} \doteq \frac{273}{H}$$

where H is the barometer reading when the mercury in both limbs stands at the zero level and the dry

air enclosed is at 0° C. H is therefore constant as long as the mass of air, or other gas, employed as the thermometric substance is not altered ; consequently

$$t = k\,h$$

where k is some constant. The instrument thus graduated becomes a *constant-volume air thermometer.*

On the expansion of air, when heated, depends the ventilation of rooms, collieries, etc. The hot air rises and escapes by any opening at the top of the room, the cold, fresh air flowing in along the floor.

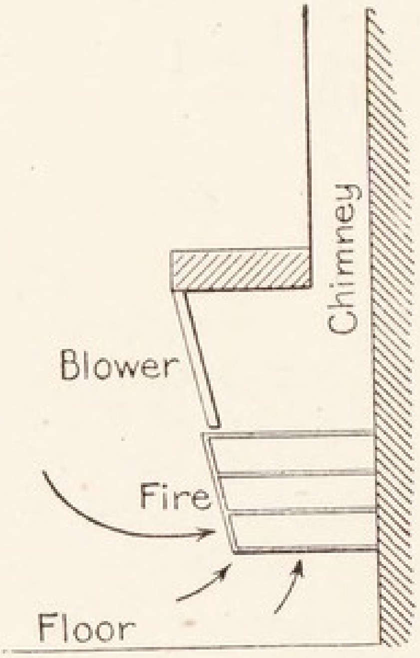


Fig. 83.—A "blower."

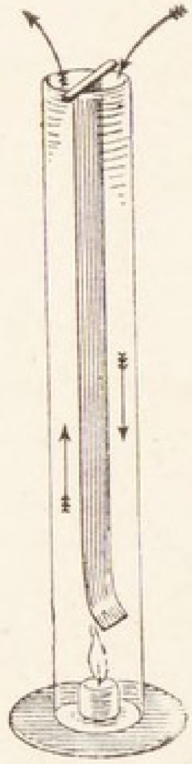


Fig. 84.--Effect of partition.

It is advantageous to remember this fact when a rescue is being attempted from a burning building: a cold, fresh current of air fairly free from smoke can usually be found by crawling along the floor.

To the expansion of air is due the draught in a chimney. The draught can be increased by using a thin sheet of metal to shut out all the cold air which is usually sucked in between the mantelpiece and the fire-bars. Such a contrivance is called a

"blower" (Fig. 83). It forces all the air to pass through the fire, thus quickening combustion; at the same time it ensures that all the air passing into the chimney is well heated. A great difference between the specific gravity of the gases in the chimney and the air outside is thus set up, and the draught thereby greatly increased.

Sometimes it is of great use to divide the incoming and the outgoing currents by a partition. Thus, a nightlight at the bottom of a tall, narrow glass jar soon flickers and goes out, but if a loose partition be inserted the flame burns brightly, and the two currents of air, instead of obstructing each other, pass on opposite sides of the partition (Fig. 84).

Exercises

1. Calculate the apparent coefficient of expansion of the alcohol used by Réaumur in his original thermometer from the information given on p. 113.

2. A flask full of air at 10° C. is heated to 100° C. and then corked up, and it is found that as a result of the heating 1 grm. of air has escaped. What weight of air was originally in the flask?

3. In an experiment with a constant-volume air thermometer the initial temperature is 15° C. and the initial pressure of the air is 75 cm. of mercury. The bulb is then immersed in the vapour of boiling alcohol, and the result is that the pressure of the air becomes 91·4 cm. of mercury. What value does this give for the boiling-point of alcohol? [*First M.B. Lond.*]

4. Two similar bars, one of steel, the other of brass, are laid side by side. What must be the length of each bar at 0° C., in order that the difference between their lengths shall be exactly 20 cm. at all temperatures?

Coefficient of linear expansion of steel $= 0{\cdot}0_4 12$

" " " " brass $= 0{\cdot}0_4 18$

5. A clock with an iron pendulum is adjusted to keep correct time at 20° C. What will be the rate of the clock if the temperature falls to 0° C.?

Coefficient of linear expansion of iron $= 0{\cdot}0_4 12$

(For Answers, *see* p. 389.)

CHAPTER III

TRANSFERENCE OF HEAT

Conduction—Convection—Radiation—Exercises.

HEAT has already been defined (p. 25) as a form of kinetic energy. It has also been stated (p. 108) that when two parts of a material system are not at the same temperature there will be, if not prevented, a net transference of heat from the hotter part to the colder till thermal equilibrium is produced. This transference of heat is effected by one or more of three processes which are distinguished by the terms *conduction*, *convection*, *radiation*. In the first two processes the parts or bodies between which the transference of heat occurs must be in actual *contact*; in radiation they are separated by an intervening medium which does not itself become warmed by the heat transmitted through it. Each process must now be considered in some detail.

Conduction is the process by which heat is transmitted in solids. Here the essential condition of *contact* is clearly fulfilled. Accordingly, when one part of a solid is made hotter than another, a portion of the heat supplied is passed on to other portions with which the heated portion is in actual contact, and so on. The molecules are set in vibration, and the vibration is transmitted from one particle to the next *without any perceptible movement of the particles*. If the *end* of a poker, for instance, be placed in a fire, the handle will presently be found to be hot, and every part of the poker between the end and the

handle will also be found to be hot. The extent to which this conduction of heat takes place differs considerably in different solids.

The metals are by far the best conductors of heat. Next come marble and various stones, whilst wood, wool, furs, cotton, etc., are such bad conductors that they are usually termed non-conductors.

Even among **metals**, considerable differences in conductivity are found. The best conductors are silver and copper, whilst platinum and German silver are among the worst. Representing the conductivity of silver as 100, that of copper has been estimated at 85, and of platinum at 8·4.

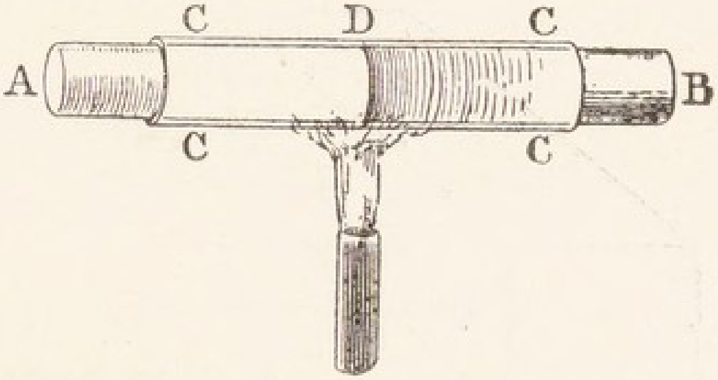


Fig. 85.—Conduction of brass and of wood.

Fig. 85 shows an experiment illustrating the fact that *brass* is a much better conductor than wood. A cylinder, half brass, half wood, has a sheet of thin paper tightly gummed round it. When the bar is heated by a Bunsen flame at the junction of the brass and wood, the paper over the wood chars, but the brass conducts away the heat so rapidly that the paper is not scorched.

The conducting power of *copper* is well shown by placing a piece of fine copper gauze about an inch above a Bunsen burner. When the gas is turned on and a light applied *above* the gauze, the gas will burn above, but remain unlighted below, the gauze. The copper conducts the heat away so rapidly that

the gas *below* the gauze is not raised to the temperature at which ignition takes place.

The miner's "*safety lamp*" depends on the same principle. The flame of the lamp is surrounded by fine copper gauze. If the lamp be placed in an atmosphere containing "fire damp," the marsh gas may burn inside the lamp, but the flame is at first so cooled down by the copper gauze that the gas outside is not raised to the ignition point. The miner has therefore time to retreat from the dangerous atmosphere.

Non-conductors of heat — wool, furs, flannel, feathers, etc.—have been naturally selected for cloth-

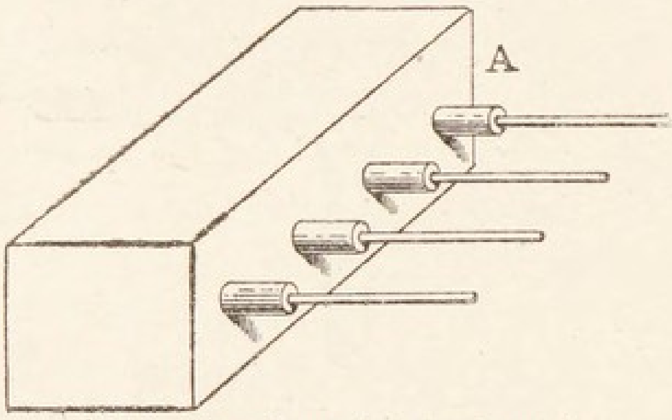


Fig. 86.—Ingenhousz trough.

ing. Ice is surrounded with non-conductors to prevent it from absorbing heat during the summer time.

The **relative conductivity** of substances for heat can be shown by means of the *Ingenhousz trough* (Fig. 86), in which rods of various substances coated with wax are fixed by corks in the side A, having their ends projecting into the trough. Hot water is poured into the trough, and the relative conductivity is shown by the lengths of wax which are melted. The best conductor is that on which the wax melts *farthest*, and is not necessarily that on which it melts *first*. The substance on which the wax melts first has the highest diffusivity—a property

which is inversely proportional to the specific heat of the substance.

The **absolute conductivity**, k, of a substance may be defined as "the quantity of heat transferred in unit time across unit area of a plate of unit thickness composed of that substance and having unit difference of temperature between the two faces of the plate." From this definition it follows that, when the difference of temperature between the two faces of the plate is $\theta°$,

Calories		Sec.		Sq. cm.		Cm.	
$k\theta$	pass in	1	across	1	of a plate	1	thick
$\therefore \dfrac{k\theta.t.\mathrm{A}}{l}$	,,	t	,,	A	,, ,,	l	,,

Liquids, except mercury, are *bad conductors* of heat. Water can be boiled in the upper part of a test tube without any marked rise being produced in a thermometer whose bulb is at the bottom of the liquid (Fig. 87).

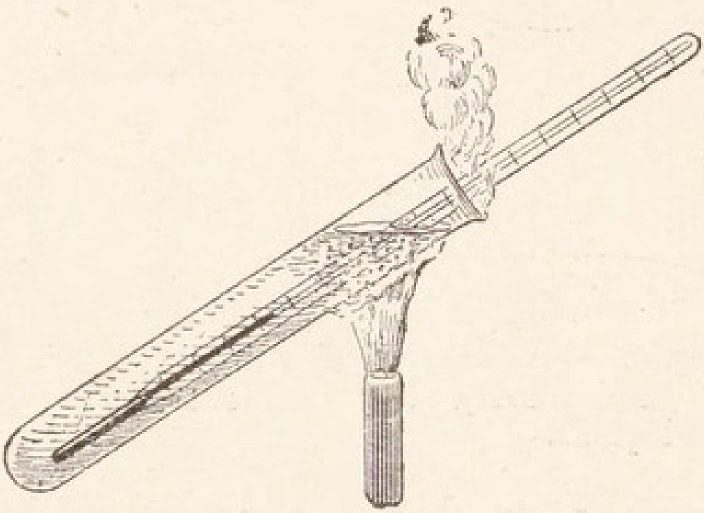


Fig. 87.—Non-conductivity of water.

The relative conductivity of liquids can be investigated by floating a tin of hot water on the surface and placing a thermometer at a definite distance below (Fig. 88).

Convection is the process by which liquids and gases are usually heated. In this process the final transference of heat from the hotter particle to the colder is probably effected by conduction, but the necessary contact is brought about by means of

currents, and these form the distinguishing feature of convection. Their existence can be demonstrated by placing a few crystals of magenta at the bottom of a large beaker of water and heating with a small flame (Fig. 89). As soon as the heat penetrates the glass the particles of water expand and rise to the surface. As they are coloured, their motion can

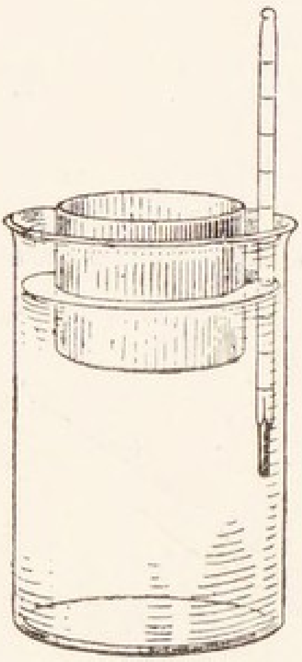


Fig. 88.—Relative conductivity of liquids.

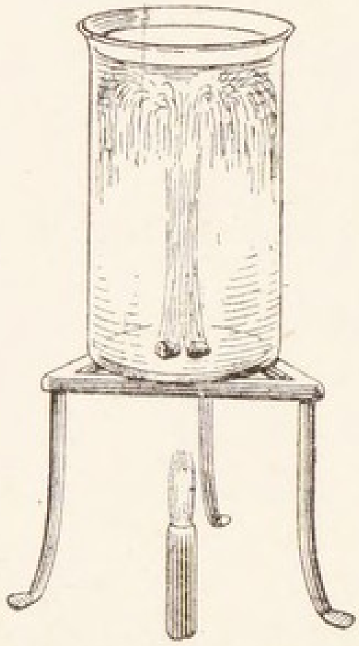


Fig. 89.—Convection currents.

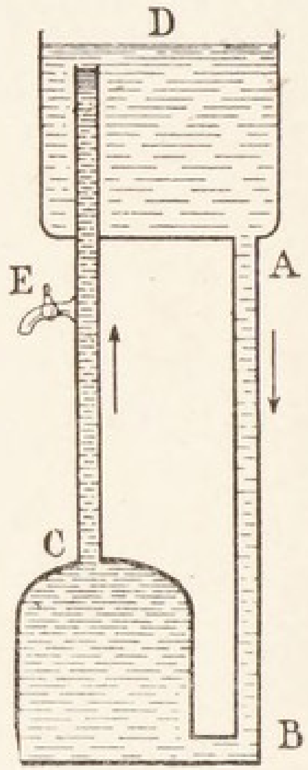


Fig. 90.—Hot-water system.

be seen, so that in convection there is *a visible movement of the particles.*

The **circulation of hot water** in the ordinary domestic supply is due to convection. There is a closed boiler in the lower part of the house (Fig. 90), and a cistern at the top. One pipe A passes from the bottom of the cistern and ends close to the bottom of the boiler at B, and a second tube passes from the top of the boiler C and ends near the surface of the water in the cistern at D. The whole system is filled with water and the boiler heated,

when a current of hot water passes up C D and a return current of cold water flows through A B. If a tap be inserted at E, hot water can be drawn off, and if a coil of pipes be inserted in C D it will be filled with hot water, and can be used for warming a room.

To the convection established by hot-air currents we owe the "land and sea" breezes and the trade winds.

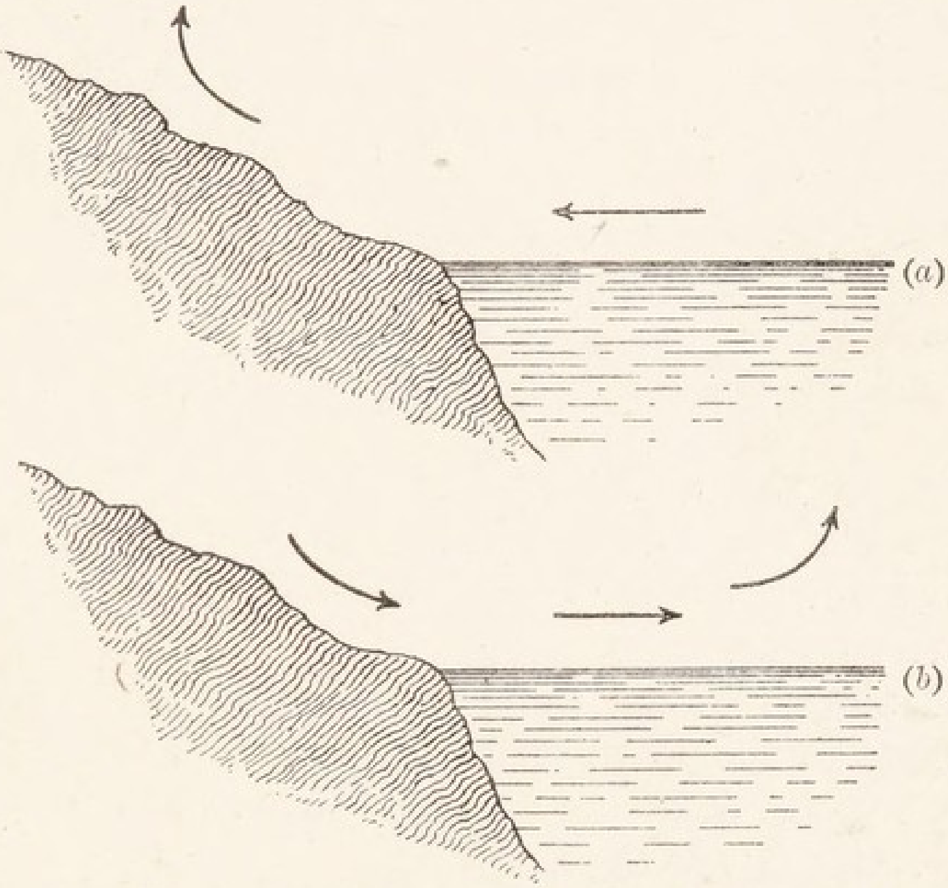


Fig. 91.—Land and sea breeze circulation (*a*) by day, (*b*) by night.

Land and sea breezes.—In the tropics, where the atmosphere is clear, there is a great difference between the day and the night temperature of the land; whereas the temperature of the ocean remains fairly constant. In the day when the sun is shining the land becomes intensely heated, and the hot air ascending from the heated land is replaced by cooler air from above the sea (Fig. 91, *a*). The movement landwards of this cool air is the *sea breeze*. After the

sun sets, the land cools rapidly by radiation into space, while the temperature of the ocean remains almost unaltered. The air above the land then becomes the colder, and the movement is reversed (Fig. 91, *b*). The warmer air ascends from the sea, and is replaced by cool air off the land, the seaward movement constituting a *land breeze.*

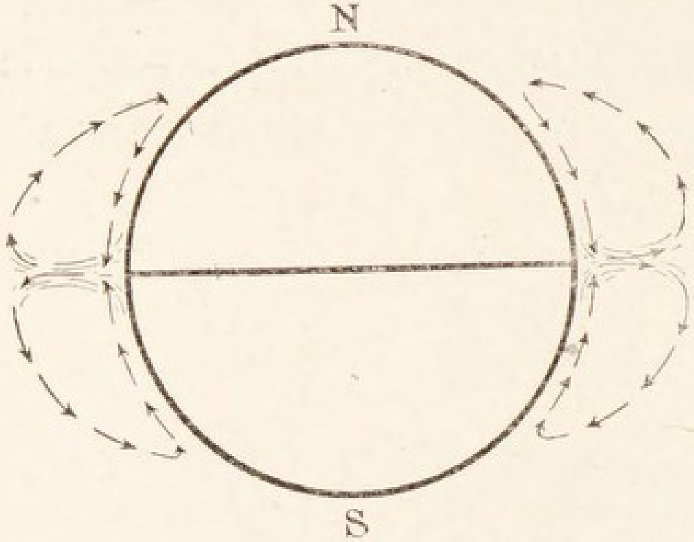


Fig. 92.—Origin of trade winds.

Trade winds. — A similar origin may be ascribed to the trade winds. The sun warms the earth most powerfully at the equator, and the hot air, rising up, flows off towards the poles, while a return current of air flows in the reverse direction along the surface of the earth.

If the earth did not revolve on its axis the upper current in the northern hemisphere would set towards the north, and the return current to the south (Fig. 92); but as the earth rotates from west to east, the particles of air (which move with the earth) travel in this direction most rapidly at the equator, and their velocity diminishes as we approach the poles. A particle of air starting from the equator towards the North Pole gains on the particles underneath, and so the upper current

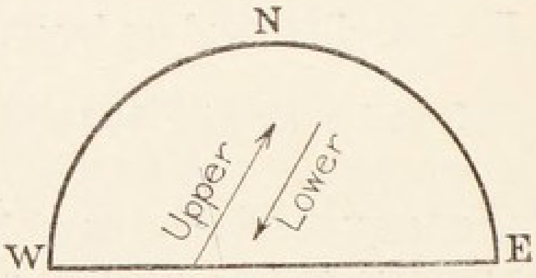


Fig. 93.—Deflection of trade winds from rotation of the earth.

flows to the north-east (Fig. 93), and for a similar reason the return current flows to the south-west. Occasionally a volcanic eruption at Teneriffe shoots up a cloud of ashes with such energy that it reaches the upper trade wind and demonstrates the existence of the latter, as it moves along in a direction opposed to the wind at the earth's surface.

The return "trades" in the northern hemisphere flow from north-east to south-west in a zone north of the equator—e.g. from North-west Africa to the northern coast of South America.

Radiation.—This process is, as already indicated, clearly distinguished from the two previous ones: (1) because the two bodies between which the transference of heat occurs are not in contact, but are separated by a medium; (2) because this intervening medium is not itself warmed by the passage of the heat. Heat so transmitted is often called *radiant heat*. The sun warms the earth by radiation. We are warmed by radiation when we stand in front of a fire. The radiant heat of the sun is, in fact, the main source of our heat supply. This heat does not warm the highly vacuous and intensely cold regions through which it travels on its way to this earth. The rays of the sun passing through a window and falling on the window-sill often make this quite hot, though the glass of the window remains cold. Radiant heat warms those objects through which it does *not* pass, objects which are *opaque* to heat rays; it does not warm those bodies through which it passes, bodies which are *transparent* to heat rays and are therefore called *diathermanous*.

There is a close analogy between radiant heat and other forms of radiant energy, such as *light* and *electricity*. Radiation of heat is, in fact, a sort of

wireless thermal telegraphy. In all these cognate phenomena the exciting cause is the more or less rapid vibration of the molecules of the radiating body. The propagation of this vibration through space by means of the ether which fills all space constitutes a *wave;* when the wave reaches a suitable receiver which is, so to speak, "*in tune*" with it, the energy is more or less absorbed by the receiver, which then emits the appropriate response; this response, when translated by our senses, is described by us as heat, light, electricity, etc.

Absorption of radiant heat.—Some substances absorb radiant heat much better than others. The sun's heat, as already mentioned, passes through the atmosphere without perceptibly warming it; but when it falls on the earth the heat is absorbed and the hot surface warms the air. The top of a mountain is much colder than the ground at its foot, because, although the mountain is a little nearer the sun, it is constantly dissipating its heat by radiation into space; whereas the plain is receiving the heat radiated from the higher ground which surrounds it on every side.

A dull, black surface is the best absorber of heat rays, and a polished metal surface the worst. On the other hand, the polished metal surface *reflects* heat better than a dull, black surface.

It is obvious that the heat which falls on a surface, not diathermanous, will be partly absorbed, raising the temperature of the body, and partly reflected. If the absorption is great the portion reflected will be small, and vice versa.

A surface which absorbs heat well will form a good radiator.

These facts can be demonstrated by a *Leslie's cube* (Fig. 94)—a tin box filled with hot water. One

side is polished, one painted, one varnished, and the fourth smoked a dead black. The sides are all maintained by the hot water at the same temperature, but it will be found that they radiate very different amounts of heat, the smoked surface being by far the best radiator. The radiation from each surface may be compared by presenting that surface to a thermopile (p. 370) connected with a mirror galvanometer. The superior absorption by a smoked surface can be shown (Fig. 95) by exposing two tin plates, one polished and one smoked, to the radiation from a red-hot iron ball. The smoked surface rapidly

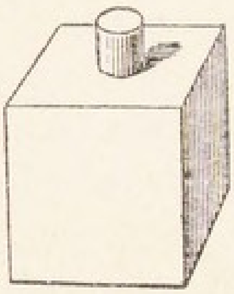


Fig. 94.—Leslie's cube.

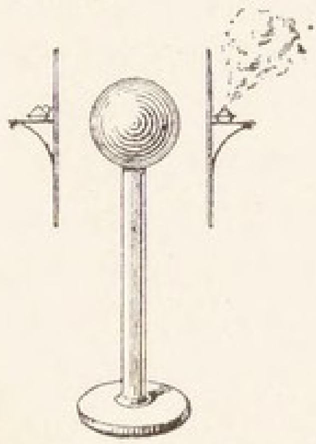


Fig. 95.—Absorption by black and by polished plate.

Fig. 96.—Dewar's vacuum vessel.

rises in temperature and lights a piece of phosphorus placed behind it.

To heat a saucepan placed *in front* of a fire, its surface should be blackened. On the other hand, to boil a kettle on a hot plate, the bottom of the kettle should be bright and free from soot, because this substance greatly hinders the *conduction* of heat from the plate to the kettle. Hot-water pipes should be dead black, teapots should be brightly polished, as in the former the object is to promote radiation, in the latter to prevent it.

A red-hot iron ball cooling on a metal stand illustrates the three methods by which heat is conveyed.

Thus, it loses heat by *conduction* along the metal stand, by *convection* from the currents of cold air which are constantly carrying off heat as they get warmed and rise, and by *radiation* as it emits radiant heat through the air in all directions.

We must, therefore, if we wish **to prevent a hot substance from losing heat,** (1) place it in contact, if possible, only with non-conducting substances, such as wood, wool, felt, paper; (2) protect it, if possible, from air currents (the vacuum vessel, Fig. 96, does this most successfully); (3) screen it, if possible, from radiation—by surrounding it, for instance, with a bright, polished, metal surface.

We must adopt precisely the same precautions if we wish **to prevent a cold body from gaining heat**—e.g. to keep ice from melting in a warm room.

Sir James Dewar found, when working with liquid air, that most of the loss by evaporation of this intensely cold liquid ($-180°$ C.) was due to the heating by convection currents of the air, which warmed the outside of the flask, while the particles of air, cooled by contact with the flask, continually descended, giving place to others. He therefore invented his *vacuum vessel* or cup (Fig. 96), now well known to all workers at low temperatures. In this apparatus, convection is completely stopped by placing the outside wall of the vessel containing the liquid air C in a vacuum, in which, as no particles of air are present, no convection can take place. The space A A is exhausted by a pump and sealed off at B.

The analogy between radiant heat and light has already been illustrated by the parallel between diathermanous and transparent substances. Colourless *rock salt* is the most diathermanous substance known. It allows heat from the sun, from a lamp, from boiling water, to pass equally well. Some

substances are diathermanous only to certain heat rays. The radiant heat emitted by a body which is not luminous, and would be invisible in a dark room, is called "*dark heat*," to distinguish it from the radiant heat emitted by a luminous body and called, for this reason, "*light heat*." The same substance is often not equally diathermanous to both these kinds of radiant heat.

Glass allows the light heat to pass, but is very opaque to dark heat. Thus, a glass screen in front of a blazing fire allows the luminous heat rays to pass, but absorbs the non-luminous heat rays given off by the bars, etc. These constitute by far the largest proportion of the heat, and the screen therefore becomes very hot.

The excessive heat in a conservatory, with a glass roof, when the sun is shining, is explained by the property that glass has of allowing the luminous heat rays of the sun to pass in, while it stops the non-luminous heat rays from the interior walls, shelves, etc., from passing out.

Substances may be quite opaque to light and yet be diathermanous—e.g. vulcanite, a strong solution of iodine in carbon disulphide, etc.

The following table shows the proportion of heat rays which pass through various substances from different sources of heat, the total number of heat rays emitted in each case being taken as 100 :—

PLATES 0·1 IN. THICK OF	SOURCE OF HEAT			
	Oil lamp	*White-hot platinum*	*Copper at* 400°	*Copper at* 100°
Rock salt .	92·3	92·3	92·3	92·3
Glass . .	39	24	6	0
Quartz . .	38	28	6	3
Alum . .	9	2	0	0
Ice . . .	6	5	0	0

Aqueous vapour absorbs a considerable amount of heat. In damp climates the radiation at night is largely stopped by the aqueous vapour. In dry climates, such as those of South Africa and Australia, the absence of aqueous vapour leads to a rapid cooling by radiation, so that even after the hottest day the temperature soon falls to the freezing-point.

Exercises

1. Supposing the body to be covered with a layer of woollen clothing on the average 3 mm. thick, the inner surface at the temperature of the body (37° C.) and the outer at the temperature of the air (15° C.), and the area of the body surface to be 1·5 sq. metres, calculate the number of calories lost from the skin in 24 hours, the conductivity of wool being 0·00003. [*First M.B. Lond.*]

2. An iron boiler is 6 mm. thick. Find the difference of temperature between the inside and outside surfaces if 10 kg. of water at 100° C. are evaporated per hour per square metre, the conductivity of the iron being 0·2 and the latent heat of steam 540.

3. A vacuum flask of effective area 300 sq. cm. and a total wall-thickness of 7 mm. is filled with 20 grm. of melting ice. Steam is continually passed round the vessel, and after 20 hours it is found that all the ice has melted. Find the heat transmitted, and express the insulating qualities of the flask as a thermal conductivity. (The latent heat of ice is 80.)

4. How much heat would escape in an hour through an unlagged steam-pipe of copper, the steam being at 140° C., the exterior surface of the pipe at 138° C., the thickness of the metal 2 mm., its conductivity 0·9, and the total external surface 1 sq. metre ? How much steam will have condensed in the pipe during this time, the latent heat of the steam being 510 calories per grm. ?

5. What different types of radiation may be emitted from a heated body ? In what respects do they differ, and how may each be obtained separate from the others ? [*First Professional.*]

6. Explain how the transference of heat between a calorimeter and surrounding objects can be reduced to a minimum. Why is this reduction desirable ? [*Ibid.*]

(For Answers, *see* p. 389.)

CHAPTER IV

CALORIMETRY

Latent Heat of Fusion—Calories—Calorimetry—Melting-point—Freezing Mixtures—Latent Heat of Steam—Specific Heat—Exercises.

It was stated (p. 107) that the increased mobility of the particles of a body which is produced by the application of heat often results in a visible change of state. A solid melts and becomes liquid; a liquid boils and assumes the gaseous state. If we

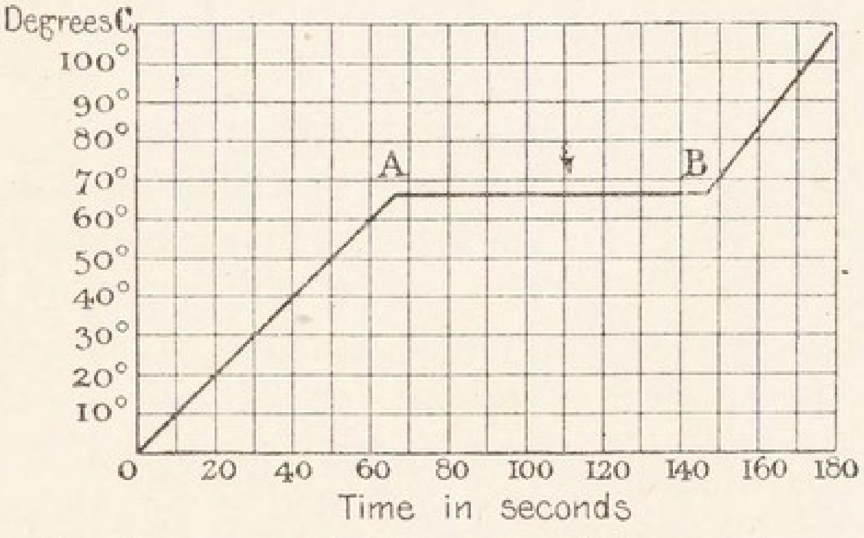


Fig. 97.—Graphic representation of rise of temperature on heating a solid, showing the stationary position of the thermometer during the melting, A—B.

keep a thermometer in contact with a lump of bees-wax in a beaker, or porcelain basin, to which heat is gradually applied, we shall find that the temperature rises until the solid begins to melt, and then remains unaltered until all the solid has melted (Fig. 97). During the time the thermometer remains stationary, the heat contributed to the substance

evidently does not make the body any hotter, but seems to become concealed or *latent*. It was therefore termed **latent heat** by Joseph Black, M.D. (1728–1799), who first demonstrated its existence, and in certain cases estimated its value, e.g. in the change of water into steam.

We now know that the latent heat is absorbed in producing that additional molecular kinetic energy which distinguishes the new state from the old. The heat absorbed in the passage from solid to liquid is frequently called the latent heat of *fusion*. Its value is often considerable. If two exactly similar vessels be taken, one containing 1 lb. of *ice* at 0° C., and the other 1 lb. of *water* at 0°, and if these vessels be heated with exactly similar burners starting at the same moment, the temperature of the water will begin to rise at once, but that of the ice will remain at 0° till all the ice has melted, if the mixture of ice and water be thoroughly stirred. By this time the thermometer in the water will be at 79° (Fig. 98).

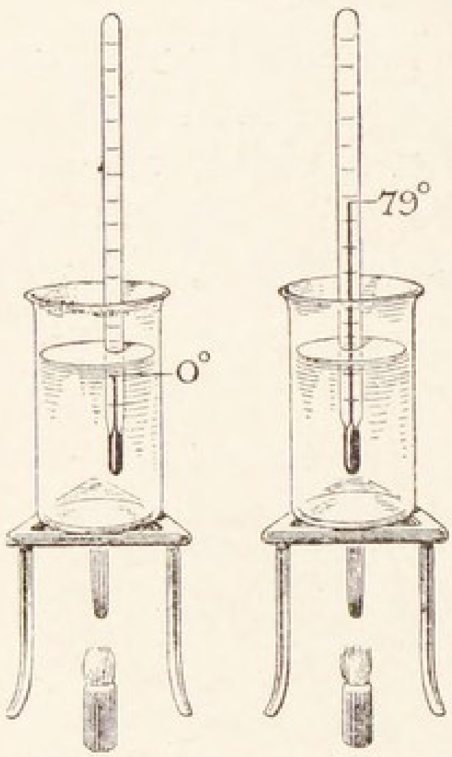


Fig. 98.—Latent heat of water.

In other words, as much heat is required to convert 1 lb. of ice at 0° into 1 lb. of water at 0° as would raise 1 lb. of water from 0° to 79° C., or from 32° F. to 174·2° F. The latent heat of water is therefore said to be 79 when the unit interval of temperature is 1° C., and 142·2 when this unit is 1° F. The first value, or 80, is generally used.

The **unit quantity of heat** has already been

stated (p. 4) as a derived unit. We shall now only specify two—

(1) The *gramme-degree-Centigrade*

unit of heat or *calorie*: this is the quantity of heat which will raise the temperature of a gramme of water by 1° C.; 79 calories are required to convert 1 grm. of ice at 0° C. to water at 0° C.

(2) The *pound-degree-Fahrenheit*

unit of heat, or British Thermal Unit (B.Th.U.): this will raise the temperature of 1 lb. of water by 1° F.; 142·2 such units are required to convert 1 lb. of ice at 0° C. to water at 0° C.

The unit of heat thus defined is not really quite invariable, and in strictness we distinguish three calories:

(1) The *mean calorie*, which is one-hundredth of the heat required to raise 1 grm. of water from 0° C. to 100° C.

(2) The *zero calorie*, which will raise 1 grm. of water from 0° to 1° C.

(3) The *common calorie*, which will raise 1 grm. of water through 1° C., between 15° C. and 17° C., which is practically at ordinary room-temperature. This is the smallest unit of the three.

1,000 common cals. = 992 zero cals. = 987 mean cals.

The *large calorie* or *major calorie* (Cal.) must be carefully distinguished from the smaller unit (cal.).

1 Cal. = 1,000 cals.

The Cal. therefore produces the same thermal change in 1 kg. of water that the cal. produces in 1 grm. of the same water.

Thermal processes may be regarded as reversible: a body gives out when its temperature falls by 1° C. exactly the same amount of heat as it must absorb to raise its temperature by 1° C. When water freezes, an amount of heat is evolved equal

to the latent heat which would be absorbed in melting the mass of ice formed. The latent heat of fusion of ice cannot be accurately measured by the experiment of Fig. 98. The conditions assumed are somewhat ideal and serve the purpose of illustration rather than of measurement. We shall, however, often require to measure this and other quantities of heat, and shall now indicate a method of doing so.

Calorimetry.—Since 1 calorie raises the temperature of 1 grm. of water by 1° C.,

$\therefore$ m calories raise the temperature of m grm. of water by 1° C.,

$\therefore$ mt calories raise the temperature of m grm. of water by t° C.

If, therefore, we have m grm. of water in a vessel, and if by heating it in any way we raise the temperature by t° C., we know that the water must have gained mt calories. We know also that more than this must have been contributed, because we cannot warm the water without warming the vessel, and some heat will be used in doing this. The same is true of the thermometer itself and of any stirrer which may be used in the experiments we shall make. If the heat employed in warming the vessel and these accessories could be utilized in warming water instead, we could raise by t° C. the temperature of *more* than m grm. of water; let the additional quantity be x grm. Then the vessel and accessories, in any thermal change, absorb (or evolve) as much heat as x grm. of water would in the *same* thermal change. For thermal purposes they are equivalent to x grm. of water; x is called their "water-value," or "water equivalent." It is determined by the following experiment:—

While some water is being heated over a burner, weigh

the dry vessel, place in it a convenient measure of cold water, and weigh again; say

Wt. of vessel + cold water	= 49	grm.
Wt. of ,,	= 24·14	,,
∴ Wt. of cold water	= 24·86	,,

Take the temperature of the cold water with a thermometer graduated to $\frac{1^\circ}{10}$, say 17·72° C. Remove the hot water from the burner, take the temperature of it, say 42·5° C., and *immediately* pour a convenient quantity into the cold water, stir rapidly, and note the thermometer; the temperature rises rapidly, remains stationary for an appreciable time, and then slowly falls. Note carefully the stationary temperature, say 27·65° C.; finally, weigh the vessel and contents—

Wt. of vessel + contents . . .	= 67·15	grm.
Wt. of same before addition of hot water	= 49	,,
∴ Wt. of hot water added . .	= 18·15	,,

We have now all the necessary data, and can deduce the value of x, the required water equivalent, as follows:—

$(24{\cdot}86+x)$ grm. of cold water at 17·72° have been raised to 27·65°, and must therefore have received $(24{\cdot}86 + x) \times 9{\cdot}93$ calories.

18·15 grm. of hot water have fallen from 42·5° to 27·65°, and must therefore have given out $18{\cdot}15 \times 14{\cdot}85$ calories.

Assuming that no heat leaves the vessel and its contents, the loss and gain must balance, or

$$(24{\cdot}86 + x) \times 9{\cdot}93 = 18{\cdot}15 + 14{\cdot}85$$

$$\therefore\ 24{\cdot}86 + x = \frac{18{\cdot}15 \times 14{\cdot}85}{9{\cdot}93}$$

$$= 27{\cdot}143$$

$$\therefore\ x = 2{\cdot}283$$

The water equivalent of the vessel and thermometer, *when used as in this experiment*, is therefore 2·283 grm.

Having now found the water equivalent of our vessel, etc., we can employ the same to determine the latent

heat of fusion of ice. A known mass of water at a known temperature is contained in the vessel. A lump of dry ice is added and the mixture carefully stirred till the ice is all melted, when the temperature is immediately noted. A final weighing gives the mass of the ice added. The latent heat can then be deduced as in the following record of an experiment made by the same student, with the same apparatus, under circumstances as similar as possible :—

Wt. of vessel and water . .	$=105 \cdot 23$ grm.
„ „	$= 24 \cdot 15$ „
∴ Wt. of water . .	$= 81 \cdot 08$ „
Water value of vessel, etc. .	$= 2 \cdot 283$ „
∴ Water + equivalent .	$= 83 \cdot 363$ „
Temperature of water before addition of ice	$20 \cdot 7°$ C.
Temperature of water after addition of ice	$18 \cdot 0°$ C.
Final weighing	$=107 \cdot 53$ grm.
∴ Wt. of ice added . .	$= 2 \cdot 3$ „
Heat lost by water and apparatus in falling from $20 \cdot 7°$ to $18°$ $= 83 \cdot 363 \times 2 \cdot 7$ . . .	$= 225 \cdot 0801$ cals.

This amount of heat (1) changes $2 \cdot 3$ grm. of ice to water at 0°, and (2) raises $2 \cdot 3$ grm. of water from 0° to 18°.

But we know that (2) requires $18 \times 2 \cdot 3 = 41 \cdot 4$ cals. Therefore (1) must require $225 \cdot 08 - 41 \cdot 4 = 183 \cdot 68$ cals. Therefore, to convert 1 grm. of ice to water at 0° must require

$$\frac{183 \cdot 68}{2 \cdot 3} = 79 \cdot 86 \text{ cals.}$$

This is, therefore, the value given by this experiment for the latent heat of fusion of ice.

A vessel employed in the measurement of quantities of heat is termed a *calorimeter*. The vessel used in the previous experiment was a copper calorimeter. To prevent as far as possible external loss, or gain,

of heat, subsidiary apparatus is often added. The nature of this has been indicated in the discussion of conduction, convection, and radiation (p. 146).

Melting-point. — The stationary temperature indicated by the thermometer during the change of a substance from the solid to the liquid state is called the melting-point of the substance. When the supply of the substance is adequate, this temperature is most easily and accurately determined as indicated on p. 149. When, however, only a small quantity of the substance is available, so that we cannot immerse the bulb of the thermometer in the melting mass, the following method is employed :—

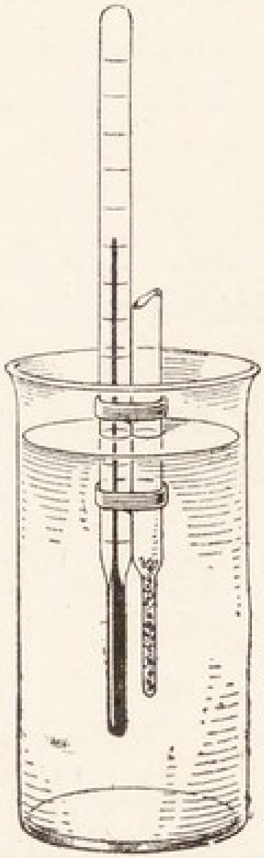


Fig. 99.—Determination of melting-point.

A few fragments of the substance are placed in a small tube, which is fixed to the bulb of a thermometer by a platinum wire or indiarubber ring (Fig. 99). The two are then immersed in a vessel of water, the temperature of which is *slowly* raised till the substance melts. In this method no stationary temperature is indicated by the thermometer. We must either use so little of the substance that the whole melts simultaneously, or we must take as the true melting-point the temperature when melting *begins*. Glycerol, strong sulphuric acid, etc., may replace the water if the solid does not melt below 100° C.

Some substances pass through an intermediate pasty state before they become liquid. This pasty state enables us to unite two pieces of wrought-iron or platinum by welding, also to work glass

into a great variety of useful and ornamental articles.

Alloys often melt at a lower temperature than their constituents. A notable example is *Wood's fusible metal.* This alloy melts at 60·5° C., though it is composed of 4 parts by weight of bismuth (melting at 269·2° C.), 2 parts of lead (melting-point, 327·7°), 1 of tin (melting-point, 231·9°), and 1 of cadmium (melting-point, 320·7°).

Freezing mixtures.—The action of many freezing mixtures depends on the absorption of latent heat during the sudden and enforced liquefaction of a solid. Thus, 2 parts of crushed ice or snow, mixed with 1 part of salt, form a liquid brine having a temperature of Fahrenheit's zero, or about − 18° C. Brine does not freeze at 0° C., and so, as soon as the ice and salt come into contact, the ice melts to form the solution of salt, and the sudden absorption of the latent heat required to melt the ice causes the fall in temperature. In some cases, instead of ice we use a solid salt containing a large quantity of so-called *water of crystallization,* as *Glauber's salt* ($Na_2SO_4 + 10H_2O$); the so-called water is solid, just as ice is solid. When this salt is mixed with hydrochloric acid the chemical action forms a solution of salt, and the ten molecules of solid water rapidly liquefy, causing a great fall in temperature. The sudden solution of a large quantity of a very soluble salt, as sal-ammoniac (NH_4Cl), in water also brings about a great fall in temperature.

A considerable quantity of heat becomes latent when a substance is converted from the liquid to the gaseous state at the same temperature. In ordinary circumstances, if water be heated the temperature begins to rise, and continues to do so until it reaches 100° C. or 212° F. At this temperature the

water begins to boil and the thermometer ceases to rise. In spite of the continued application of heat the temperature remains stationary until all the water has been converted into steam. The heat contributed is absorbed in converting the liquid water at 100° into gaseous water or steam at 100°; it is termed the latent heat of steam, and is 537·2°. That is to say, to convert 1 grm. of water at 100° into steam at the same temperature, 537·2 calories are required — an amount of heat which would raise 537·2 grm. of water 1° C.

Latent heat may be defined as *heat which produces a change in the physical state of a substance without altering its temperature.*

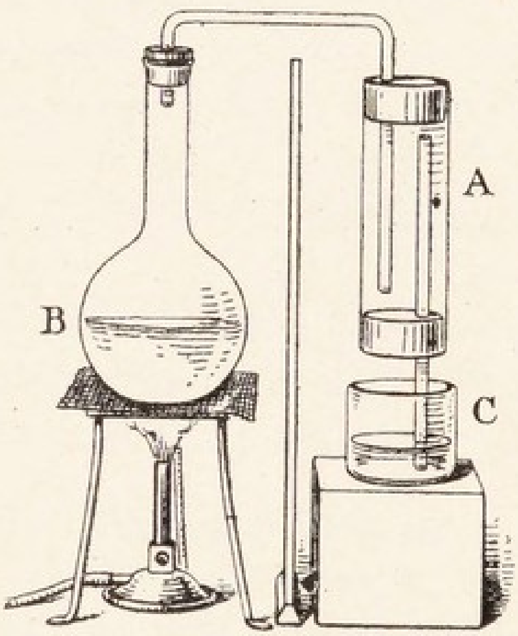


Fig. 100. —Experimental determination of latent heat of steam.

The latent heat of steam can be estimated by passing a known weight of steam into a calorimeter containing a known weight of water at a known temperature, and observing the rise in temperature produced. It is essential that only *steam* should enter the calorimeter C (Fig. 100). A short tube A is therefore interposed as a trap to intercept any water formed by premature condensation of steam, and when only *steam* appears to be issuing from the exit tube the calorimeter is placed in position. The weight gained by the calorimeter C is the weight of the steam used.

The following record of an actual experiment illustrates the calculation :—

Wt. of calorimeter and water	. =	90·58 grm.
Wt. of calorimeter . .	. =	24·15 ,,
∴ Wt. of water .	. =	66·43 ,,
Temperature of this water	. =	20·9° C.
Wt. of calorimeter, water, and steam	. =	92·52 grm.
Wt. of calorimeter and water	. =	90·58 ,,
∴ Wt. of steam .	. =	1·94 ,,
Temperature of mixture	. =	37·8° C.
Water equivalent of calorimeter as determined (p. 153)	. =	2·283 grm.

Let L be the latent heat of the steam:

$$\text{Heat lost by steam} = 1{\cdot}94 \times (L + 62{\cdot}2)$$

$$\text{Heat gained by water, etc.} = 68{\cdot}713 \times 16{\cdot}9 = 1161{\cdot}2497$$

$$\therefore 1{\cdot}94 \times (L + 62{\cdot}2) = 1161{\cdot}2497$$

$$\therefore L + 62{\cdot}2 = \frac{1161{\cdot}2497}{1{\cdot}94} = 598{\cdot}58$$

$$\therefore L = 536{\cdot}38$$

Specific heat.—The quantity of heat which becomes latent when a unit mass of a substance undergoes a change of state varies with the substance. Indeed the quantity of heat absorbed or evolved in *any* thermal change is influenced by the nature of the substance; mt calories (p. 152) are absorbed or evolved when the temperature of m grm. of *water* rises or falls by $t°$ C., but the same thermal change in the same mass of alcohol, copper, etc., will engage or release a different number of calories in every instance. The property of matter to which this difference is due is called *specific heat.* The difference is evidently one of degree, and measurements of specific heat are therefore referred to some standard substance. This is generally water, and the specific heat of any substance is simply the ratio between two quantities

of heat—(1) the quantity involved in a certain thermal change of a mass of the substance; (2) the quantity involved in the *same* thermal change of the *same* mass of water.

If, for instance, H calories are absorbed or evolved when the temperature of m grm. of a substance, A, rises or falls by $t°$ C., then the specific heat, s, of $A = \frac{H}{m\,t}$. H, the quantity of heat engaged in the change, is therefore $m\,s\,t$ calories; $m\,s$, the value of H when $t = 1°$, is called the *thermal capacity* of the mass, m; s, the specific heat of the substance, is therefore also the thermal capacity of unit mass of it.

The influence of specific heat is seen in the following experiments:—

Two balls, one of iron and one of bismuth, of equal weight (Fig. 101), are heated in boiling water for some time, and then laid on a cake of wax. Although these masses are of equal weight and at the same temperature, it will be seen that they evolve different quantities of heat; the iron obviously melts much more wax than the bismuth. The specific heat of iron is much greater than that of bismuth.

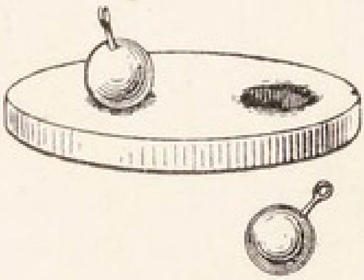


Fig. 101. — Greater heating power of iron compared with bismuth.

Again, if equal weights of water and iron, heated to 100°, be cooled in equal weights of cold water, the hot water evolves much more heat than the equal weight of iron. The difference is quite perceptible to the hand, and can be demonstrated (Fig. 102) by immersing the bulbs of a differential air thermometer in the two beakers. The specific heat of water is much greater than that of iron.

In fact, water has a high specific heat, but its value is represented by unity, since water is itself the standard adopted.

SPECIFIC HEATS OF VARIOUS SUBSTANCES

Hydrogen	3·409	Steam	0·48
Water, mixed with 20 per cent. alcohol	1·045	Glass	0·187
Water	1·000	Iron	0·113
Alcohol	0·615	Copper	0·095
Glycerin	0·612	Zinc	0·095
Ice	0·5	Bismuth	0·031
		Lead	0·031

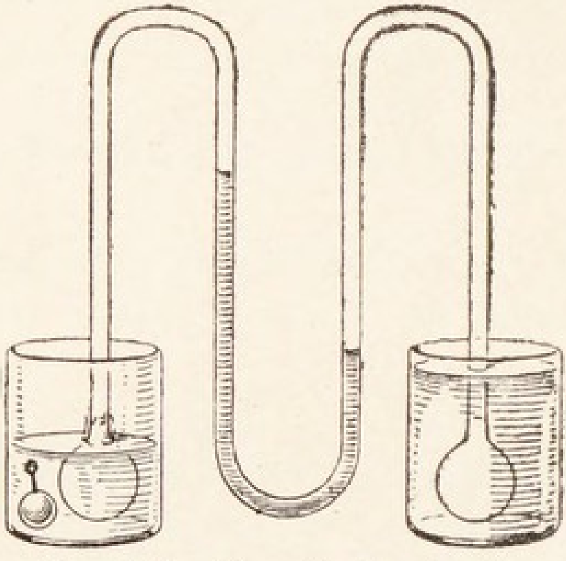


Fig. 102.—Specific heats of iron and water.

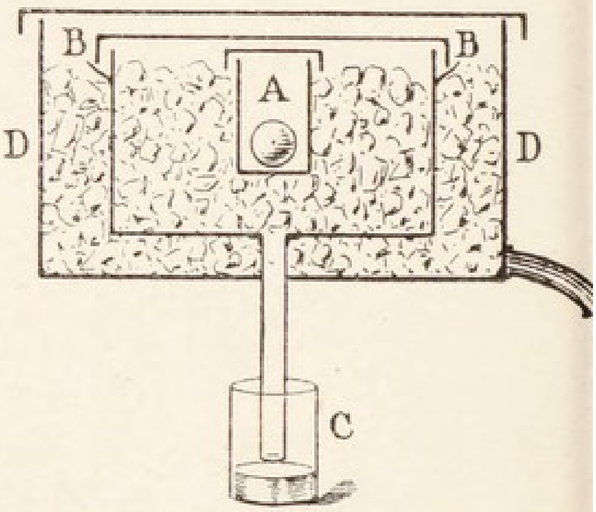


Fig. 103.—Lavoisier and Laplace calorimeter.

It has been shown* that the specific heat of a solid element $\times$ its atomic weight is equal to a constant, about 6·4.

Ex.: Iron, $0{\cdot}113 \times 56$ . . $= 6{\cdot}34$
Bismuth, $208{\cdot}5 \times 0{\cdot}031$. $= 6{\cdot}45$

So that the specific heat of any of the metals can be found by dividing 6·4 by the atomic weight, or the approximate atomic weight by dividing 6·4 by its specific heat.

Determination of specific heat.—The specific

* *See* "A Manual of Chemistry" (Luff and Candy).

heat of a substance may be determined by one of the following methods:—

1. **Method of Lavoisier and Laplace.**—A small, thin copper box A (Fig. 103) is surrounded by a second vessel B containing ice, which drains into a beaker C; outside B is a third vessel D also containing melting ice, so as to protect B from any heat from the outside. A weighed mass of the solid is raised to a known temperature in a steam oven, and then dropped into the box A, and the lids are put on; the beaker C has been previously dried and weighed. The heat rapidly melts the ice; the resulting water collects in C, and is weighed. If m grm. of solid at $t°$ C. are placed in A and fall to 0° C.; if w grm. of ice are melted and the latent heat of ice be 79; then we know $m\,s\,t = 79\,w$, and therefore s can be calculated.

Black previously used a rougher and less accurate method by boring a hole in a block of ice, inserting the heated substance, and mopping up, with a weighed dry rag, the water produced. The greater the specific heat, the more ice will be melted.

2. **Regnault's method.**—In this method the hot substance is *mixed* with the cold substance, and on this account it is frequently called the *method of mixtures*. The apparatus (Fig. 104) consists of two parts, the heating arrangement and the calorimeter. The heating arrangement consists of a tube of thin metal A, which is heated by steam. In this tube the substance B is suspended with a thermometer; the substance is preferably made in the form of a thick ring, inside which the thermometer is placed. The heating is continued until the thermometer remains constant for about twenty minutes. In the meantime the little carriage D on which the calorimeter is placed with its stirrer S, a delicate thermometer

T, and a weighed quantity of water, is wheeled away, so as not to be affected by the heat from the steam, and its temperature carefully read off. As soon as the temperature of the heated weight is steady, the calorimeter F is run under the tube A, a sliding door E, which closes the lower end of A, is opened and the heated substance lowered into the calorimeter, which is at once removed, constantly stirred, and the

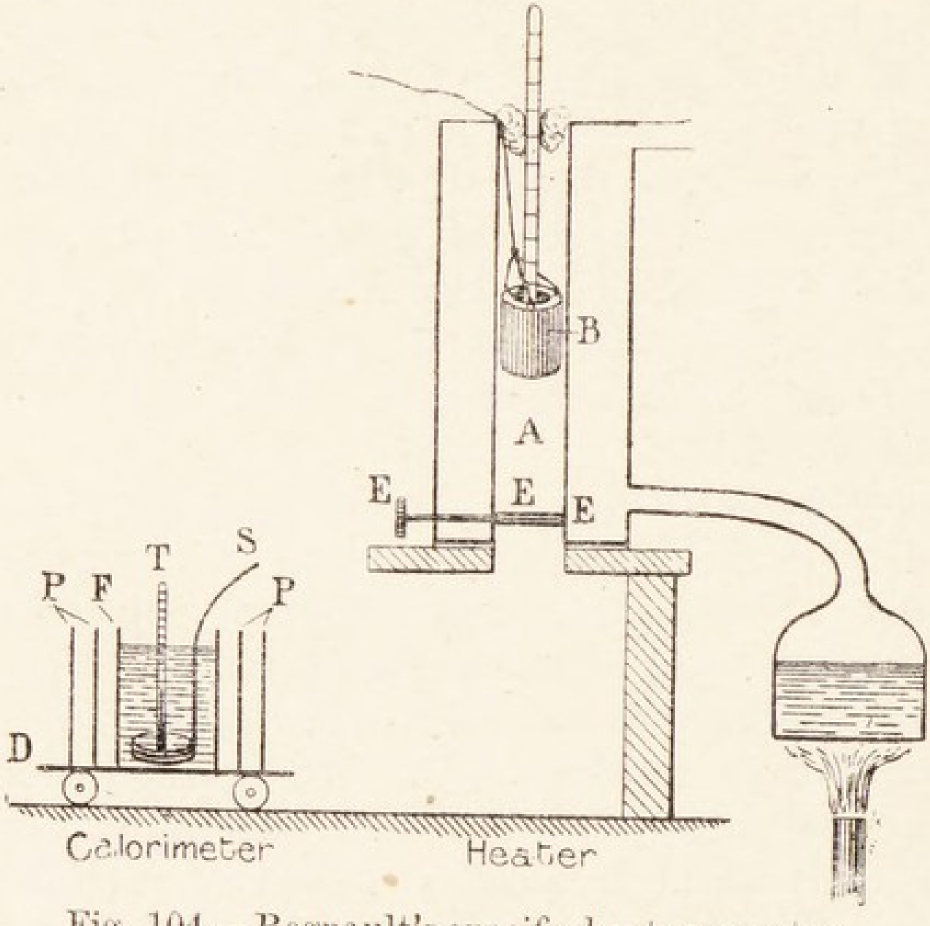


Fig. 104.—Regnault's specific-heat apparatus.

rise in temperature carefully noted. The calorimeter is surrounded with several polished screens P P, and with non-conducting material to prevent, as far as possible, its temperature from being affected by surrounding objects.

In this and all similar transfers of heat we have $m\,s\,t$ calories of heat transferred from the hot body to the cold, or the weight of the substance heated $\times$ its specific heat, $\times$ its fall in temperature $=$ (the

weight of water in the calorimeter plus its water value) × specific heat of water × its rise.

As the specific heat of water = 1, we have—

$$\text{Sp. ht.} = \frac{\text{(Wt. of water, plus water value of calorimeter)} \times \text{its rise}}{\text{Wt. of substance} \times \text{its fall}}.$$

Ex. 1: A mass of copper weighing 500 grm. is heated to 100° and plunged into a calorimeter containing 1,000 grm. of water at 12°; the resulting temperature is 15·7°. Water value of calorimeter = 4·5 grm.

$$\left.\begin{array}{r}\text{Sp. ht.}\\ \text{of copper}\end{array}\right\} = \frac{1004{\cdot}5 \times 3{\cdot}7}{500 \times 84{\cdot}3} = 0{\cdot}088$$

Ex. 2 (with the Lavoisier and Laplace calorimeter): 500 grm. of copper at 100° melted 57 grm. of ice, therefore (p. 161)—

$$s = \frac{w \times 79}{m\,t} = \frac{57 \times 79}{500 \times 100} = 0{\cdot}09$$

Several other small corrections have to be made to allow for the loss of heat by radiation, etc., during the experiment, in order to obtain rigidly accurate results.

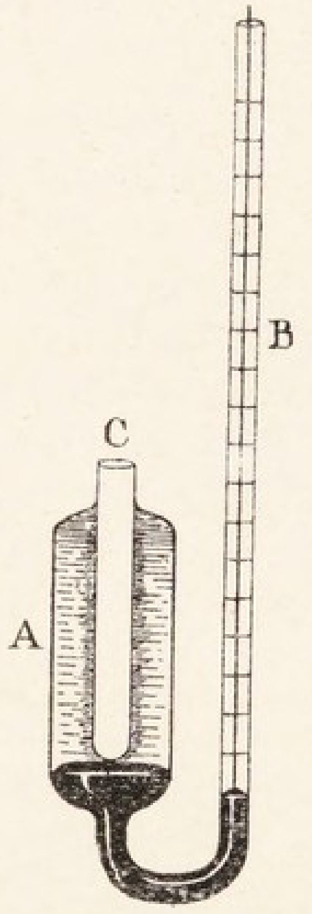


Fig. 105.—Bunsen's calorimeter.

3. **Bunsen's method.**—In this method the specific heat is determined by measuring the amount of ice melted by the heated body, from the *contraction* which takes place when ice is melted. The instrument (Fig. 105) consists of a long glass bulb A, attached to a graduated capillary tube B; a small test tube C is fused into the bulb A. Mercury fills the capillary tube, but the bulb, above the mercury, is filled with water from which all air has previously

been expelled by boiling. Some ether is poured into C, and, by passing a rapid current of air through the ether, a thin layer of ice is formed round C in the water in A. The ether is now emptied out, some water placed in C, and the whole allowed to stand till it has attained the temperature of 0° C., which is indicated by the column of mercury in the capillary remaining stationary. The heated body is then introduced into C, when some of the ice is melted, and from the contraction of the mercury in the capillary tube the amount can be calculated.

The **specific heat of a liquid** can be found by the foregoing methods, and is most often determined by the method of mixtures. It can, however, be also found by the *method of cooling*. If m grm. of water under certain circumstances cool through t°, and therefore lose $m\,t$ calories, we may assume that m grm. of another liquid *in the same circumstances* will also lose $m\,t$ calories. If we find, therefore, that this liquid cools through t_1° and call its specific heat s, we can say that $m\,s\,t_1 = m\,t$, and therefore that

$$s = \frac{t}{t_1}$$

which shows that for the same mass, cooling under the same circumstances, the specific heat varies *inversely* as the fall of temperature. We need not even use the same mass m of the liquid, though it is convenient to do so; the argument still holds if we take a known mass m_1 of the liquid, but we must then say

$$m_1\,s_1\,t_1 = m\,t$$

$$\text{and } \therefore\ s_1 = \frac{m\,t}{m_1\,t_1}$$

The **specific heat of a gas** has two values, which refer to two different conditions—

(1) When the gas remains at constant *volume*.

(2) When the gas expands and remains at constant *pressure*.

By allowing a known quantity of dry air to circulate through a spiral tube surrounded by a known mass of water, Regnault found the specific heat (2) of dry air to be 0·2375. The difference between this and the value of specific heat (1) is calculated in a later chapter (p. 186), and found to be nearly 0·069. The value of (1) is therefore

$$0{\cdot}2375 - 0{\cdot}069$$
$$= 0{\cdot}1687$$

Exercises

1. Dry steam at 100° C. is passed into a calorimeter containing 100 grm. of water at 20° C. What is the maximum weight of steam condensed ? The steam supply is then cut off. How much ice at 0° C. must be added in order to bring the calorimeter and its contents back to 20° C. ? Neglect the thermal capacity of the calorimeter. (Latent heat of steam = 540 ; of ice = 79.)

2. A calorimeter whose water equivalent is 15 contains 200 grm. of oil at 20° C. Find how much ice at 0° C. must be added to the oil in order to cool it to 12·5° C. (The specific heat of the oil is 0·6, and the latent heat of ice 79.)

3. 20 grm. of steam are passed into a calorimeter containing 100 grm. of water at 20° C. How much ice will have to be added simultaneously if the final temperature is still to be 20° C. ? (Latent heat of steam = 540 ; of ice = 80.)

4. In determining the latent heat of steam by the method of mixture, 453 grm. of water at 12° C. were placed in a calorimeter of water equivalent 12 grm. Steam was then passed in for a minute, and the temperature rose to 25° C. The increase of weight noted was 10 grm. A previous experiment showed that 0·2 grm. of water condensed in the supply tube per minute, and would have been carried on by the steam. Calculate the latent heat of steam from these data.

5. Why is scalding steam at 100° C. more destructive to animal tissue than boiling water at the same temperature ? [*First Professional.*]

6. Define the term latent heat. How many calories must be taken from 50 grm. of benzene to cool it from 20° C. to 0° C., given that benzene freezes at 5° C., that the latent heat of fusion is 30, and that the specific heat when liquid is 0·34, and when solid is 0·25? [*Ibid.*]

7. What is meant by the statement that the specific heat of iron is 0·1? If 100 grm. of ice-cold iron cools some water in a vessel from 15° C. to 14° C., how much lead at 20° will restore the original temperature of the water, the iron having been removed? (The specific heat of lead is 0·03.) [*Ibid.*]

(For Answers, *see* p. 389.)

CHAPTER V

VAPOUR PRESSURE

Evaporation and Ebullition distinguished—Vapour Pressure — Boiling - Point — Distillation—Hygrometry — Dew-Point—Hygrometers—Exercises.

THE gaseous state of a substance which, at ordinary temperatures, is liquid or solid is generally described by the term **vapour.** We speak, for instance, of alcohol vapour, ether vapour, water vapour, arsenic vapour, etc.

Evaporation distinguished from ebullition.—Steam is aqueous vapour at 100° C. Aqueous vapour, however, exists at all ordinary temperatures, and is always present in the atmosphere. If a little water be left in a shallow dish exposed to the air, it will, after a time, be found to have entirely disappeared, leaving the dish dry. It has passed into the atmosphere in the form of vapour; it has "evaporated." Evaporation takes place at all temperatures, and is therefore distinguished from ebullition, or boiling, which takes place only at a fixed temperature—the boiling-point. Vapours obey the usual gaseous laws, but less strictly than do the more permanent gases. They diffuse, tend to expand, and fill any space in which they are confined, and exert pressure on any surface in contact with them.

If we introduce a small drop of ether at the closed end of a U-tube containing mercury (Fig. 106) and plunge it into a vessel of hot water, the ether is

converted into vapour, which proves its elasticity by supporting a heavy column of mercury in the open leg A. Another striking experiment is to place a small quantity of water in a tin canister, the mouth of which can be closed by a cork. The water is boiled vigorously for some minutes to allow the steam to expel the air. The outside of the tin is subject to the atmospheric pressure, but it does not collapse, because this is balanced by the pressure of the steam inside. If we now cork the canister, at the same time removing the flame, and condense the steam by pouring on cold water, this internal pressure is instantly reduced to a value far below the atmospheric pressure of 15 lb. on the square inch, the canister is at once crushed in, and falls as a shapeless mass.

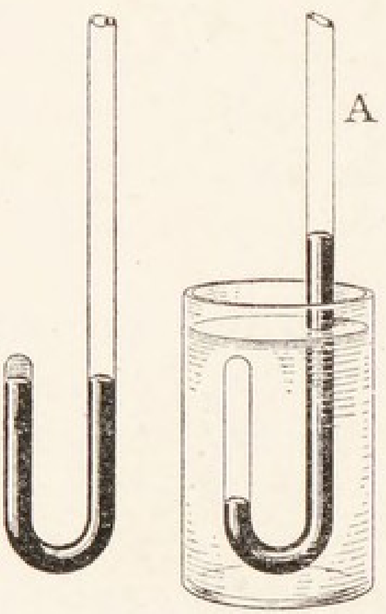


Fig. 106.—Vapour pressure of ether.

The **vapour pressure** of a liquid can be measured by the fall which it produces in the mercury column, when passed up into a barometric vacuum (Fig. 107). If, for instance, some alcohol, ether, or water be introduced into a barometer, the level of the mercury instantly falls, and the amount of the depression enables us to measure the vapour pressure in fractions of an inch, or of a millimetre of mercury. As the temperature rises, the vapour pressure increases until the vapour pressure, by itself, supports the atmospheric pressure, and the mercury stands at the same level in the barometer tube and in the cistern. The temperature at which this takes place is the *boiling-point* of the fluid.

In all measurements of vapour pressure in this way, some of the liquid must remain visible on the surface of the mercury; in other words, the space above the mercury must be *saturated* with the vapour. If a very small quantity of liquid be passed up into the barometer it will be completely converted into vapour, the space will be *unsaturated*, and the maximum vapour pressure will not be developed. The vapour pressure depends on the nature of the

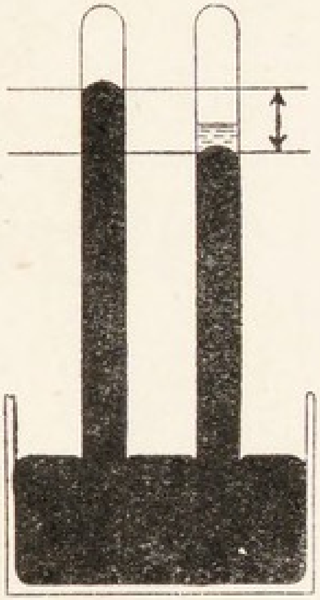


Fig. 107.—Depression of barometer by vapour.

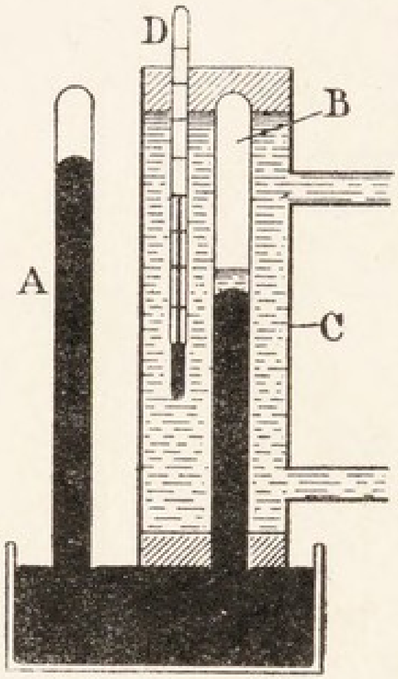


Fig. 108.—Apparatus for estimating vapour pressure.

liquid and the temperature, but is independent of the atmospheric pressure.

The vapour pressure of a liquid at various temperatures can be ascertained by the apparatus shown in Fig. 108. A is an ordinary barometer, B is the barometer tube containing the liquid. This is surrounded by a second tube, C, through which water at different temperatures can be passed; the thermometer, D, indicates the temperature. The pressure exerted by the vapour in B is equal to that which would be exerted by a column of mercury whose

height is the *difference* between the height of the column in A and the *corrected* height of the column in B. The correction required in the latter case is a small one, due to the column of the liquid above the mercury in B; the length of this liquid stratum must be reduced to its equivalent of mercury and the reduced length added to the actual height of the mercury in B. The following are some values for water and alcohol :—

VAPOUR PRESSURE

TEMP. IN ° C.	WATER	ALCOHOL
	Mm. of mercury	*Mm. of mercury*
0	4·6	12·2
10	9·17	23·8
20	17·4	44·0
40	54·9	133·7
60	148·9	350·0
80	354·6	812·0
100	760·0	1692·0
200	11689·0	22182·0

It has just been stated that the temperature at which the vapour pressure of a substance is equal to the atmospheric pressure is called the **boiling-point** of the substance. It follows that at the top of a mountain 15,000 ft. high, where the atmospheric pressure would be about 354 mm., water would boil a little below 80° C., instead of at 100° C. This becomes serious when an army is sent on a mountaineering expedition, as the water cannot be made hot enough, in open vessels, to cook properly, and the troops suffer from indigestion. The fall in the boiling-point of water in an open dish

has been used to determine the height of a mountain, 1° C. indicating an ascent of 1,080 ft., or 1° F. = 600 ft.; but, according to Whymper, this method does not give such accurate results as the barometer.

The fall in the boiling-point can easily be demonstrated by placing some hot water under the air pump; as the pump is worked and the pressure diminished the water boils vigorously.

The same fact can be shown without an air pump, as follows: Water is boiled in a round-bottomed

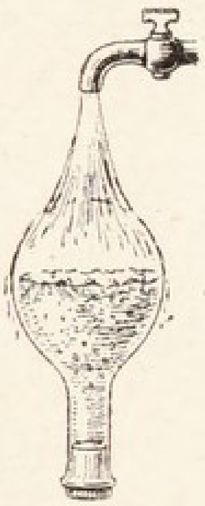


Fig. 109.—Water boiling under reduced pressure.

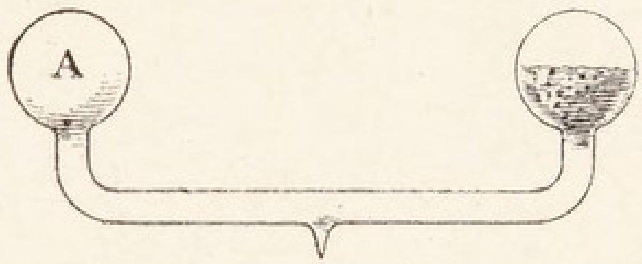


Fig. 110.—Ether boiling with the warmth of the hand.

flask till the steam has expelled the air. If the flask is then corked and inverted, the water can again be made to boil (Fig. 109) by pouring cold water over the flask. This condenses the atmosphere of steam in the flask and creates a partial vacuum, in which the water readily boils at temperatures considerably below 100° C.

Ether, if sealed up in a glass tube free from air, boils when warmed by the hand, owing to the diminished pressure (Fig. 110); the bulb A is held in the hand. A practical application is found in the vacuum pans used for the evaporation of sugar

syrup; by covering the pans and reducing the pressure inside, the boiling-point of the syrup is reduced from 110° to 65·5°.

On the other hand, by increasing the pressure we can raise the boiling-point, thus:—

PRESSURE IN ATMOSPHERES	BOILING-POINT OF WATER (° C.)
$1\frac{1}{2}$	112·2°
2	120·6
4	144
8	171
10	180·3
20	213
28	231

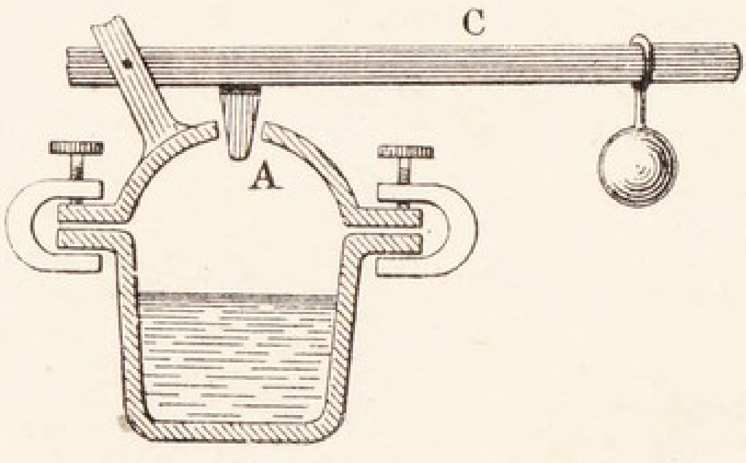


Fig. 111.—Papin's digester.

Increase in pressure is utilized for this purpose in **Papin's digester** (Fig. 111). An iron saucepan has a turned iron flange on which rests a lid with a similar flange, and the two are clamped so as to make an air-tight joint; the lid is perforated with a conical hole A in which fits a valve; this is kept in its place by the weighted lever C. Before the water can boil, the vapour has to lift this valve, in addition to the atmospheric pressure; the boiling-point is therefore raised and the solvent power of the water increased. In this way bones, calves' feet, etc., are quickly

converted into gelatin. The vulcanizer used by the dental mechanic is constructed on the same principle.

The boiling-point also depends, to a small extent, on the *nature* of the vessel containing the liquid. If its inner surface be smooth and clean the liquid may be heated slightly above its boiling-point. Then, if the vessel be shaken or the liquid stirred, a sudden rush of vapour ensues, and the temperature sinks to its normal boiling-point. This often causes the phenomenon of "bumping."

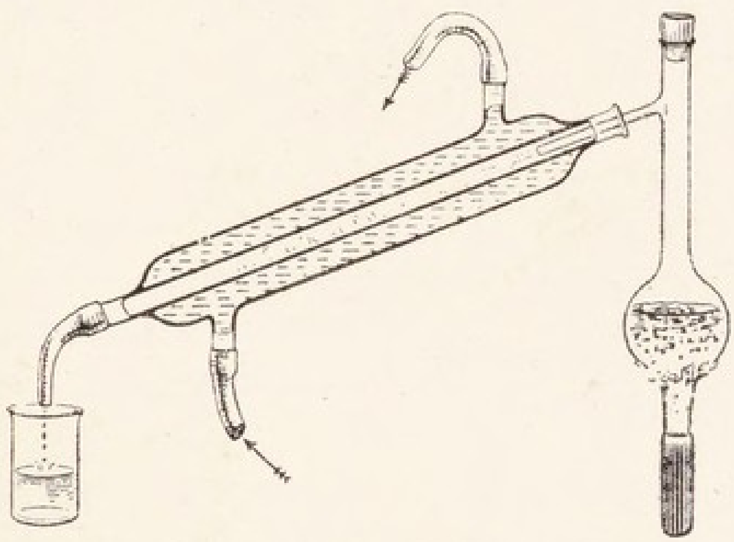


Fig. 112.—Distillation with Liebig's condenser.

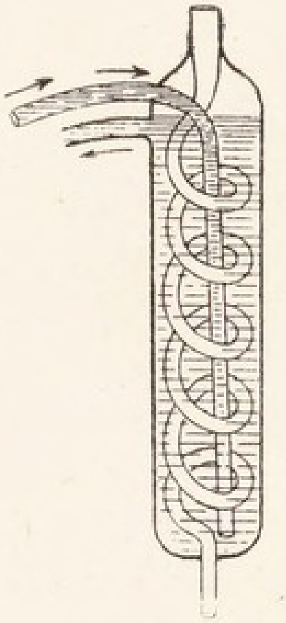


Fig. 113.—Spiral condenser.

The boiling-point of water is also raised by dissolving in it various substances, as common salt, calcium chloride, etc.

Distillation.—When two vessels A and B, connected together, contain liquid at different temperatures, the vapour from the hotter region A will exert a higher pressure than the vapour from B. Vapour will therefore tend to move from A to the colder region B, where it will condense. Vapour will rise continuously from A, and be condensed in B as

long as the difference in temperature is maintained. In practice the usual plan is to heat the distilling flask.

One form of " still " or " condenser " largely used is the **Liebig condenser** (Fig. 112) ; another form, the **spiral condenser,** has the condenser tube, whether metal or glass, twisted in a spiral called the " worm " (Fig. 113).

We can also distil by artificially cooling the receiving vessel or *receiver*. This is the principle of the **cryophorus** (Fig. 114). In this instrument we have

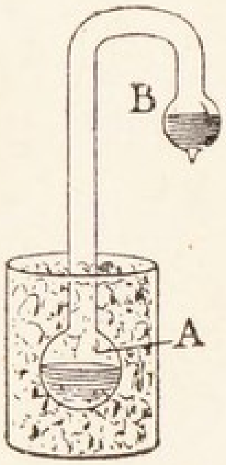


Fig. 114.—Cryophorus.

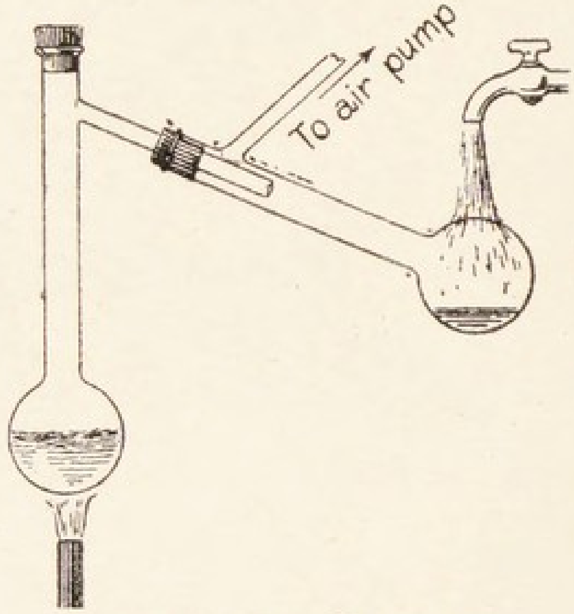


Fig. 115.—Distillation at reduced pressure.

two bulbs, A and B, connected by a tube containing water and water vapour. The air has been boiled out and the apparatus sealed by the blowpipe at B. The water is transferred to the bulb B and the lower bulb A immersed in a mixture of ice and salt, the water vapour in A is condensed rapidly, vapour rises from B, the water in B cools, and eventually freezes owing to the loss of heat from the enforced distillation.

Some organic liquids decompose when distilled under the atmospheric pressure, so that it is advisable

to distil them under diminished pressure, and thus lower the boiling-point. An apparatus for effecting this, made out of two distilling flasks, is shown in Fig. 115.

Cooling by rapid evaporation is often used on the large scale for producing cold; one form of apparatus is shown in Fig. 116. A and B are two iron vessels; in B is placed a solution of ammonia saturated at 0° C. This is gently heated, and the ammonia gas is driven off and condenses as a liquid in the interspace of the double-walled vessel A, which is kept in cold water. When sufficient ammonia has distilled over, the substance to be frozen is placed in C, A is taken out of the water and surrounded with some non-conducting substance, while B is plunged into cold water; the ammonia is rapidly reabsorbed by the cold water in B, and this enforces a rapid evaporation of the liquid in A, with a corresponding abstraction of heat from C.

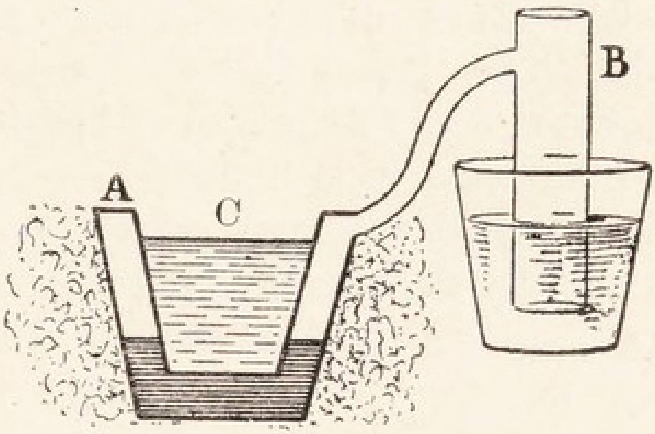


Fig. 116.—Freezing with ammonia.

Hygrometry

As already stated (p. 167), *some* aqueous vapour is always present in the atmosphere. The quantity varies from day to day within certain limits, but there is a maximum amount which cannot be exceeded at any given temperature. When this amount is present the atmosphere is said to be saturated. As the maximum increases with the temperature,

saturated air, if cooled to a lower temperature, will not be able to retain its full quota of aqueous vapour. Some of this vapour, hitherto invisible, will therefore be condensed, and may become visible as mist or fog, or may be deposited on cold surfaces in the form of dew. If the fall in temperature is sufficiently extended, this condensation may result in *rain* or *snow*. Rain-drops after formation may be suddenly frozen in passing through a very cold stratum of air, and become hail-stones. Dew does not fall; it is formed *in situ* on cold surfaces in contact with saturated air. The explanation of the formation of dew we owe to Dr. Wells. When the sun sets, the temperature of the surface of the earth rapidly falls, heat being radiated into space. At last the portion of the atmosphere close to the earth becomes so cold that it is unable to retain all its moisture in the form of vapour; some of it therefore condenses as visible drops on the cold surface. If the sky is clouded, free radiation into space is much hindered, and no dew is formed. If there is much wind, little dew is deposited, because as fast as one portion of air is cooled it is carried away by the wind, and its place taken by uncooled air. The largest formation of dew is always observed on calm, clear, cold nights. Obviously, the best radiators cool the most quickly; the minimum temperature is found on the grass. On a frosty morning the wooden sleepers on a railway line will be covered with hoar-frost, whilst the polished "metals" remain free. The temperature at which the deposition of dew begins is called the **dew-point.** At this temperature the air is saturated by the amount of aqueous vapour actually present. As all vapours exert pressure (p. 167), some of the atmospheric pressure recorded by the barometer must be due to the pressure of the

aqueous vapour in the air. The pressure of aqueous vapour at any temperature is, as already stated (p. 169), the pressure it exerts in a space *saturated* with it at that temperature. In fact the temperature must be the dew-point. To find, therefore, what pressure the aqueous vapour in the air is exerting at any moment, we must (1) find the dew-point at the moment; (2) find (from tables) the recorded pressure of aqueous vapour at this dew-point temperature. This is the pressure required.

If the actual temperature of the air, $t°$, is above the dew-point, then the air is not saturated. If it absorbed more and became saturated, still remaining at $t°$, then the aqueous vapour would exert a proportionately increased pressure. It would, in fact, exert the recorded pressure of aqueous vapour at $t°$. The ratio between these two pressures is therefore also the ratio between a, the amount of aqueous vapour actually present in the air at $t°$, and b, the maximum amount of aqueous vapour which could be present in the air at the same temperature. This ratio defines the *relative humidity* of the air, which is generally expressed as a percentage of the possible maximum, thus—

$$\text{Relative humidity} = \frac{\text{V.P. at dew-point}}{\text{V.P. at air temperature}} \times 100$$

Ex.: Air temperature = 55° F., dew-point 46·5° F.

Vapour pressure at 55° = 0·433 in., at 46·5° = 0·317 in.

$$\text{Relative humidity} = \frac{0{\cdot}317}{0{\cdot}433} \times 100 = 73 \text{ per cent.}$$

The measurement of the moisture of the atmosphere is called hygrometry, and instruments employed for this purpose are called **hygrometers.** As most of the measurements are based upon a knowledge of the dew-point, hygrometers are generally designed

to determine this temperature. The following are familiar types :—

Daniell's hygrometer (Fig. 117) consists of two glass bulbs connected by a large bent tube. The longer limb of this tube encloses a small thermometer A, and the other bulb B is covered with muslin. The bulbs contain ether, and this had been boiled for some time before the sealing of the apparatus, which now contains only ether and ether vapour. Before using the instrument the ether must be transferred to bulb A, so as to surround the thermometer; the bulb B is then cooled by pouring ether on the muslin outside; the vapour in B condenses, fresh vapour from A distils over, the thermometer falls, and at last dew is deposited on A. At this moment the temperature of A is observed and the artificial cooling of B is discontinued. The temperature is again observed when the dew has just disappeared. The mean of these temperatures is the dew-point. The bulb A is usually gilt to render the dew more visible.

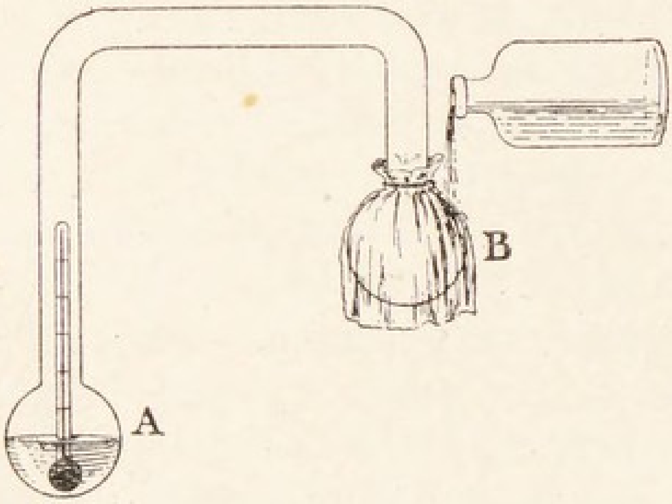


Fig. 117.—Daniell's hygrometer.

Dines' hygrometer.—In this apparatus (Fig. 118) iced water is run into a little metal box, in the top of which is inserted a thin plate of black glass A; underneath this is a delicate thermometer B. The cold water is turned on by the tap C, and the temperature at which the black glass gets dim noted, as with the previous instrument.

Regnault's hygrometer.—Some ether is placed in a thin glass tube (Fig. 119) furnished with a cork and two tubes—one long, one short—and a thermometer. Air is drawn through the ether by connecting the short tube with an aspirator; the temperature falls, and the readings are taken as before. The lower part of the glass tube is generally encased in a polished silver cap. A precisely similar tube containing no ether is often included in the apparatus. The two tubes are mounted, side by side, on a stand. Comparison makes it more easy to see when dew first appears on the ether-cooled cap.

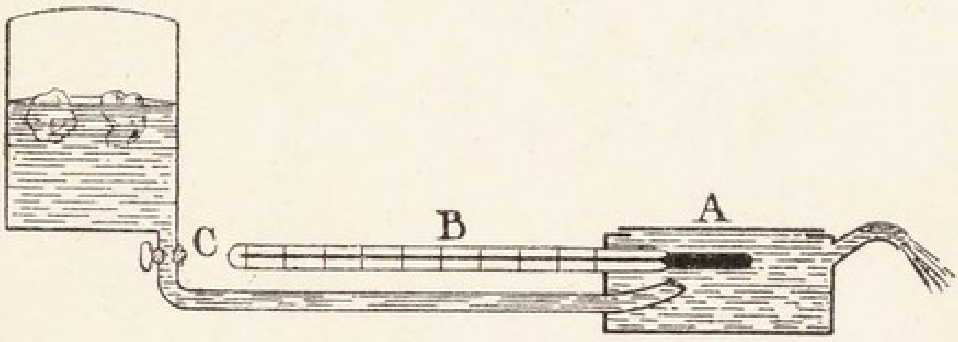


Fig. 118.—Dines' hygrometer.

The thermometer in the comparison tube indicates the temperature of the air. That in the ether tube indicates the dew-point.

Wet and dry bulb.—This hygrometer does not indicate the dew-point directly. It consists of two thermometers (Fig. 120) exactly alike, except that the bulb of one is surrounded with a piece of muslin, to which moisture is constantly conveyed by means of some lamp-wick from a small vessel containing distilled water. If the air is saturated the two thermometers read alike, but in dry air the water on the wet bulb evaporates rapidly, and the thermometer has a lower reading than the dry bulb,

so that the drier the air the greater is the difference between the thermometers.

Glaisher compared these differences with the readings of a Daniell's hygrometer taken under similar conditions, and has given a set of factors (p. 429) which, when multiplied by the difference between the readings, yields a number that, subtracted from the dry bulb reading, gives the dew-point.

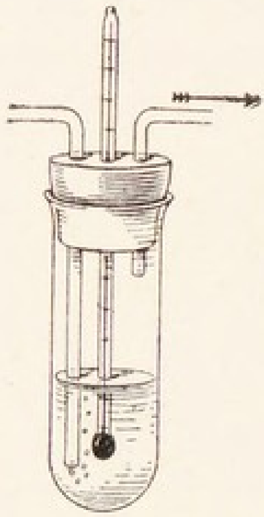


Fig. 119.—Regnault's hygrometer.

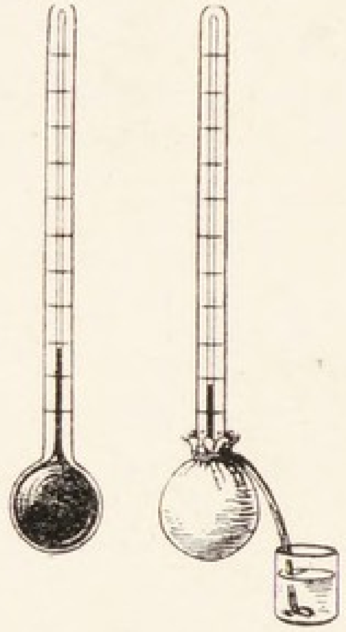


Fig. 120.—Wet and dry bulb.

Ex.: Dry bulb 50° F.
Wet bulb 45° F.

$$5^\circ \times 2{\cdot}06 \text{ (factor for } 50^\circ \text{ F.)} = 10{\cdot}3$$
$$50^\circ - 10{\cdot}3^\circ = 39{\cdot}7^\circ \text{ dew-point.}$$

The methods described lead to the determination of the dew-point and therefore (p. 177) of the *relative humidity*. The actual weight in grams of aqueous vapour in 1 litre of the air at t° C. when the temperature of the dew-point is d° C. may be found as follows :—

Let p be the pressure of aqueous vapour at d° C. The aqueous vapour whose weight is required fills 1 litre at

$t°$ C., and at a pressure p. If it could exist at 0° C. and 760 mm., it would then fill (p. 133) $\frac{1 \times p \times 273}{760 \times (273 + t)}$ litres, and would weigh $\frac{p \times 273 \times 0{\cdot}804}{760 \times (273 + t)}$ grm., since 1 litre of aqueous vapour at 0° C. and 760 mm. weighs 0·804 grm.

The weight of aqueous vapour in a given amount of air can also be determined experimentally by

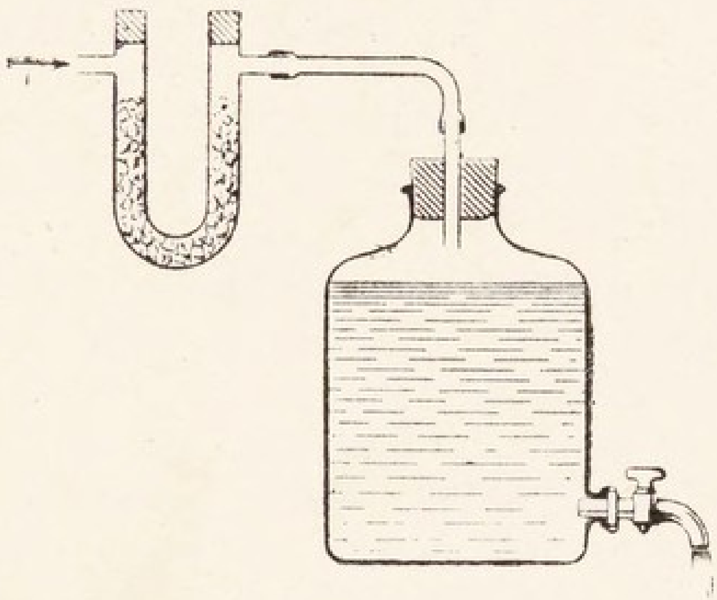


Fig. 121.—Direct determination of weight of aqueous vapour in air.

drawing the air slowly through a weighed **U**-tube containing strong sulphuric acid, when the increase in weight of the **U**-tube gives the weight required (Fig. 121).

Exercises

1. Explain the effect of increase of pressure on the boiling-point of a liquid. Give two examples in which this change of boiling-point is utilized. [*First Professional.*]

2. Distinguish between evaporation and ebullition. A sealed glass tube, in the form of an inverted **U**, contains a small quantity of water in each limb, and water vapour only in the remaining space. One limb is placed in a freezing mixture. Describe the changes that occur in each part of the tube. [*Ibid.*]

3. Define relative humidity. How can you determine the mass of water vapour present in a cubic metre of air? [*Ibid.*]

4. Air at 15° is 60 per cent. saturated with water vapour. What is the dew-point, given maximum pressure of water vapour at 15° = 1·270 cm., at 8° = 0·802 cm., and at 7° = 0·749 cm.?

5. When wet clothes are allowed to dry on the person, why is the "chill" felt by the wearer more when the clothes are getting dry than when they were getting wet?

(For Answers, *see* p. 389.)

CHAPTER VI

THERMODYNAMICS

Relation between Heat and Work — The Mechanical Equivalent of Heat—Heat of Combustion—Liquefaction of Gases—Exercises.

SINCE heat is a form of kinetic energy, the production of heat by the expenditure of kinetic energy should not be difficult. In point of fact, the destruction of kinetic energy by friction or other causes always results in a more or less complete transformation of it into heat. If the transformation of energy be complete, it is always found to accord with a fixed rate of exchange. This exchange value of the thermal unit has been measured in a variety of mechanical ways, and is usually expressed in corresponding gravitation or absolute units of work. The value, so expressed, is called **the mechanical equivalent of heat.**

In 1798 Count Rumford boiled 20 lb. of water, in about $2\frac{1}{2}$ hours, by the friction of a solid plunger pressing on the bottom of an iron cylinder, the plunger being turned by horses. It is also well known that savages obtain fire by the friction of a piece of hard wood on soft wood. In these cases kinetic energy is really converted into heat.

We have another example in the sparks formed when the brake is applied to a rapidly moving train ; so much energy of motion is converted into heat that the flakes of iron on the tyre become ignited.

When a gas is compressed, heat is evolved. If this

heat remains in the gas, its temperature is raised. If a gas, previously compressed but no longer retaining the heat produced by the compression, be suddenly allowed to expand, heat energy is absorbed in the expansion. If no external heat is supplied it must be drawn from the gas itself. The temperature of the gas is therefore *lowered.* This is

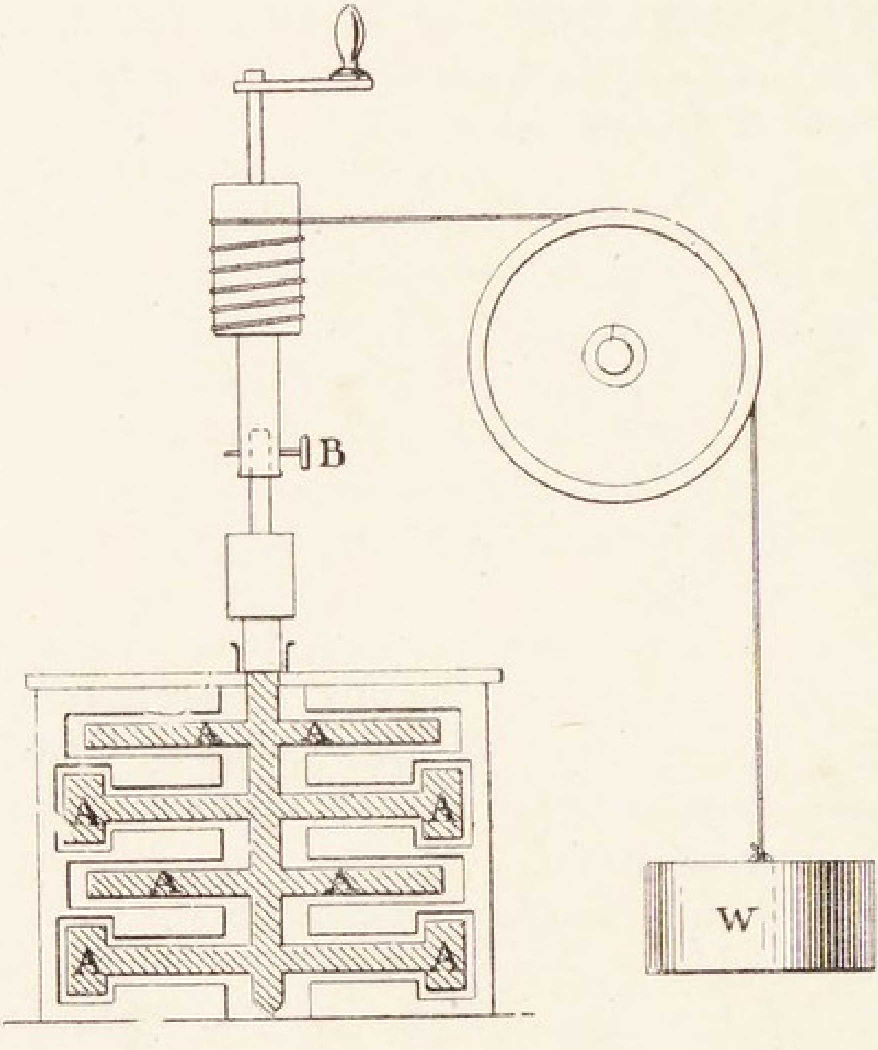


Fig. 122.—Joule's apparatus.

the basis of the most effective method of producing the cold necessary for the liquefaction of gases. When the conditions of a thermal phenomenon allow no heat to enter or leave the system concerned, they are described as *adiabatic;* when, on the other hand, the conditions secure external compensation, they are described as *isothermal.* When a soda-water

bottle is opened in the winter, the sudden absorption of heat owing to the expansion of the gas sometimes causes the water to freeze.

Determination of the mechanical equivalent of heat.—In 1845 Joule determined by the friction of a paddle-wheel in water the amount of energy required to produce a certain quantity of heat. Joule originally deduced from his experiments the value 772 foot-pounds at Manchester. More recently it has been concluded that a weight of 778 lb. falling 1 ft. would produce enough energy—if converted into heat—to raise 1 lb. of water 1° F., or 1,400 lb. falling 1 ft. would raise 1 lb. of water 1° C. The principle of Joule's experiments is illustrated diagrammatically in Fig. 122; the paddles A A are caused to rotate by the falling weight W, which is lifted by turning the handle at the top after taking out the pin B, and is allowed to fall many times. The rise in temperature of the water in the calorimeter is shown by a delicate thermometer.

This determination of the mechanical equivalent of heat, besides being of great theoretical interest, has been of immense practical value, as by means of it we can compare the actual amount of work performed by a steam engine with the energy produced in the boiler by the burning of the coals. The comparison is not satisfactory, even in the best steam engines, only about 20 per cent. of the energy evolved being converted into horse-power. The efficiency of the gas engine is greater, being about 35 per cent.

The mechanical equivalent of heat can also be calculated from the difference in the heat required to raise the temperature of a given volume of air (1) when its volume is kept constant, and (2) when it is allowed to expand. Let the specific heat (p. 165)

of dry air in case 1 be denoted by s_1, and in case 2 by s_2. By experiment, Regnault found $s_2 = 0{\cdot}2375$. From the value of the velocity of sound in dry air (p. 203) we can deduce that the ratio $\frac{s_2}{s_1} = 1{\cdot}408$.

We must therefore have

$$s_1 = \frac{0{\cdot}2375}{1{\cdot}408} = 0{\cdot}1687 \text{ nearly},$$

and therefore $s_2 - s_1 = 0{\cdot}069$ nearly.

This really means that an additional quantity of heat, equal to 0·069 calories, is required to raise the temperature of 1 grm. of dry air by 1° C. when it is allowed to expand and so remain at constant *pressure*, over and above the quantity required when the air is confined to a constant volume. This additional quantity of heat must therefore be the thermal equivalent of the work done by the gramme of air in expanding. We can calculate this work.

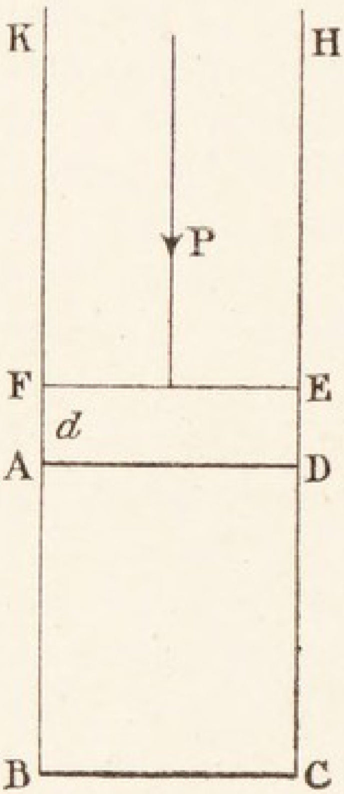


Fig. 123.—Graphical representation of work done by a gas in expanding under constant pressure.

Let the vessel K B C H (Fig. 123) be fitted with a movable piston, which presses on a volume of dry air inclosed in the vessel with a force represented by P units. Let the area of a cross section of the vessel be S square units. Suppose that, by expanding, the air, at first confined in the space A B C D, pushes the piston back from A D to a new position F E, through a vertical distance d. The added volume, v, is therefore $d \times S$.

If p be the pressure *per unit of area*, $p\,S = P$. Now the work done (p. 27)

$$\begin{aligned} &= P \times d \\ &= p\,S \times d \\ &= p \times v \end{aligned}$$

When we apply this general result to the particular case of 1 grm. of dry air at N.T.P. expanding for 1° C., we know (p. 89) that $p = 1{,}013{,}961{\cdot}6$ dynes, and also, since

1 litre of dry air at N.T.P. weighs 1·293 grm.

∴ 1 grm. ,, ,, ,, fills $\dfrac{1}{1{\cdot}293}$ litre,

and, since the coefficient of expansion is $\dfrac{1}{273}$, v, the added volume for 1° C., is $\dfrac{1}{273} \times \dfrac{1000}{1{\cdot}293}$ c.c.

∴ the work done, expressed in absolute *C.G.S.* units, is

$$\frac{1013961{\cdot}6 \times 1000}{273 \times 1{\cdot}293} \text{ ergs}$$

This must be equal to 0·069 calories.

$$\begin{aligned} \therefore 1 \text{ calorie} &= \frac{1013961{\cdot}6 \times 1000 \times 1000}{273 \times 1{\cdot}293 \times 69} \text{ ergs} \\ &= 41{\cdot}6 \times 10^6 \text{ ergs} \\ &= 42 \text{ million ergs, nearly.} \end{aligned}$$

This mechanical equivalent of heat, or, as it is sometimes called, *Joule's equivalent*, is frequently denoted by the symbol J. It must not be confounded with a *joule* (p. 28). The numbers derived from the experiments (p. 185) really lead to the same result if the necessary change of units is made, thus :

$$
\begin{aligned}
1 \text{ lb. } ^\circ\text{F. unit of heat} &= 778 \text{ ft.-lb. of work} \\
\therefore 1 \text{ lb. } ^\circ\text{C.} \quad ,, \quad ,, &= \tfrac{9}{5} \times 778 = 1{,}400 \text{ ft.-lb. of work} \\
\therefore 1 \text{ grm. } ^\circ\text{C.} \quad ,, \quad ,, &= 1{,}400 \text{ ft.-grm. of work} \\
&= 1{,}400 \times 30{\cdot}5 \text{ cm.-grm. of work (p. 4)} \\
& 1{,}400 \times 30{\cdot}5 \times 981 \text{ ergs} \\
&= 41{,}888{,}700 \text{ ergs}
\end{aligned}
$$

or, 1 calorie = nearly 42 million ergs, as before.

We shall generally say that Joule's equivalent of

$$
\begin{aligned}
1 \text{ calorie} &= 4{\cdot}18 \times 10^7 \text{ ergs} \\
&= 4{\cdot}18 \text{ joules}
\end{aligned}
$$

Heat of combustion.—The heat of combustion of a substance is the quantity of heat units expressed in calories which is given out during the combustion of 1 grm. of that substance. Combustion is only a particular instance of chemical combination. When the formation of a chemical compound is attended by the evolution of heat, the compound is described as *exothermic*, and the heat evolved when a molecular weight in grammes (1 grm. molecule) of the compound is formed from its elements is called the *heat of formation* of the substance. Thus:

$$C + O_2 = CO_2 + 94{,}300 \text{ calories}$$

means that when 12 grm. of carbon are burnt with 32 grm. of oxygen to form 44 grm. of carbon dioxide an amount of heat is evolved which would raise 94,300 grm. of water 1° C. If carbon is burnt to carbonic oxide the heat evolved is much less:

$$C + O = CO + 26{,}300 \text{ calories}$$

Again, $H_2 + O = H_2O + 68{,}300$ calories indicates that 2 grm. of hydrogen when burnt to water liberate enough heat to raise 68,300 grm. of water 1° C.

If we know the heat of formation of the products

formed by the combustion of a chemical compound and the heats of combustion of its constituent elements, we can calculate the *heat of formation* of the compound.

Ex.: Marsh gas, methane, CH_4, when burnt forms CO_2 and $2H_2O$:

Heat of formation of CO_2	= 94,300
,, ,, ,, $2H_2O$ . . .	= 136,600
	230,900

Heat of combustion of a molecular weight of CH_4 = 213,800 and 230,900 − 213,800 = 17,100. This must be the heat spent in separating the compound CH_4 into its elements, previous to their combustion.

In other words, 17,100 calories were evolved when 12 grm. of carbon combined with 4 grm. of hydrogen to form 16 grm. of marsh gas.

Heat of combustion of food.—The heat of formation of more complex bodies such as the fat, starch, and protein included in an ordinary mixed diet is not known. The heat of combustion has been measured, and is usually stated in large Calories [1 large Calorie (Cal.) = 1,000 small calories (cals.)]. By complete combustion—

1 grm. of protein evolves about	..	5·65 Calories
1 ,, ,, fat ,, ,,	..	9·4 ,,
1 ,, ,, carbohydrates evolves about		4·15 ,,

It must, however, be remembered that the full equivalent of this heat of combustion is not obtained by the consumer when these substances are consumed in human diet. The process of combustion in the body is not carried to the uttermost limit. The food is not burnt to an ash as in artificial combustion. The solids of the fæces and the urine still possess some unused heat of combustion. This must be valued and deducted from the above numbers if we

wish to obtain the heat equivalent metabolized in the body, or the *fuel value* of the food. The best results in this direction have been obtained by Atwater, and the calculations of "energy value" in the Report* on the Food Supply of the United Kingdom (p. 3) are based on his values,† "modified in accordance with the special characteristics of the British supply." With this modification it is reckoned that, when consumed in human diet, the fuel value, or energy value, of

1 grm. of protein	is 4·1	Calories
1 ,, ,, fat	,, 9·3	,,
1 ,, ,, carbohydrate ..	,, 4·1	,,

With these factors we can calculate the total fuel value of any given diet. About 80 per cent. of this total seems to be spent as heat, and about 20 per cent. in work. The fuel value required from a daily diet varies widely with the occupation and other personal factors of the consumer, but an average standard is probably somewhere between 3,000 and 4,000 Calories.

Heat of combustion of coal, etc.—The heat liberated in the burning of a sample of coal can be estimated as follows: The coal is finely powdered, and 2 grm. of the powder are mixed with about 24 grm. of oxygen mixture [$KClO_3$ (3 parts) + KNO_3 (1 part)]. The mixture is then pressed into a stout copper cylinder. In the mouth of this cylinder is inserted a dry piece of cotton wick previously soaked in a solution of potassium nitrate, to serve as a slow match. A calorimeter, containing a sufficient quantity of water, about 2 litres, is prepared. When all is

* Cd. 8421. 1917.

† "The Chemical Composition of American Food Materials," Bulletin No. 28, U.S. Department of Agriculture. Revised edition, Washington, 1906.

ready, the slow match is lighted and a little copper diving bell fixed over the copper cylinder containing the mixture of coal and potassium salts, and the whole is quickly sunk in the water of the calorimeter. As soon as the spark in the slow match reaches the mixture the latter deflagrates, the coal burning violently; when the combustion is finished, the water is mixed, and its rise in temperature noted with a delicate thermometer; the number of calories evolved can then be calculated after the necessary corrections have been made.

In a similar way the heat evolved during the combustion of starch, sugar, and other substances can be estimated. If the combustible substance is a gas, it is burnt at a jet (oxygen being supplied) in a copper calorimeter under water; the jet is lighted by a platinum wire made white hot by passing a current of electricity through it.

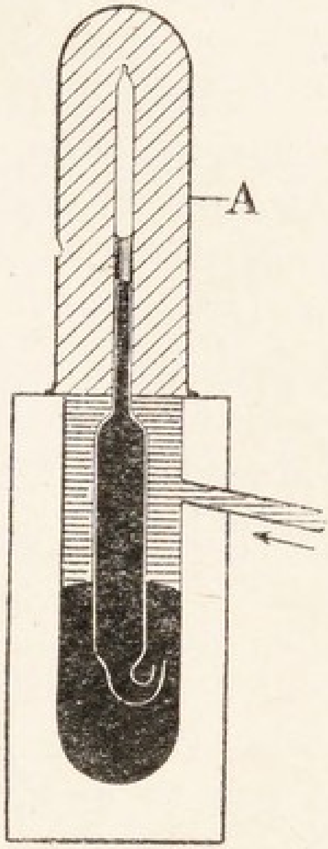


Fig. 124.—Andrews' experiment to show critical point of a gas.

Liquefaction of gases.—Andrews first proved that the temperature of a gas must be brought below a certain point, called its *critical point*, before pressure will liquefy it. Above this critical temperature no amount of pressure will transform the gas into a liquid. He enclosed some carbon dioxide in a thick-walled glass tube A (Fig. 124), closed at its upper end, but connected below with a reservoir of mercury to which great pressure could be applied. At a temperature of 13·1° C. a pressure of 48 atmospheres caused some

carbon dioxide to liquefy; when the temperature was 21° the pressure had to be raised to 60 atmospheres before any liquid was formed; but when the temperature rose to 31·9° the line of demarcation between the liquid and the gas became hazy, the liquid disappeared, and by no amount of pressure could it be made to reappear until the temperature fell below 31·9°, the critical point.

Those gases whose critical points are low—oxygen 119, nitrogen 146, hydrogen 223—for a long time resisted all attempts to liquefy them, since no known means of attaining such low temperatures existed. Finally, Pictet and Cailletet liquefied oxygen about the same time (1877). Pictet succeeded by a methodical lowering of temperatures. He first liquefied sulphur dioxide, and then utilized the cold produced by the rapid evaporation of the liquid to liquefy large quantities of carbon dioxide; rapidly evaporating this, in its turn, he reached the critical point, and with the aid of great pressure liquefied oxygen. Cailletet compressed oxygen to an enormous extent, and then, by opening a stopcock and allowing the gas to expand suddenly, a sufficient degree of cold was produced to form a mist of liquid oxygen.

Since that time liquid air has become a commercial article, and can be purchased by the litre. This result we owe chiefly to Dewar and Linde. The air is subjected to a pressure of 200 atmospheres, and allowed to escape through a fine orifice. As the air expands, heat is absorbed, but it is nearly all regenerated by the friction, and if air were a perfect gas no cooling would result, but air not being a perfect gas, the heat *lost* by the expansion is somewhat greater than that produced, and so the temperature falls, the gas is cooled and is used to cool the air coming to the fine aperture. This in its turn

is cooled to a lower temperature, so a sort of cumulative cooling is set up, and the liquid air begins to drop from the orifice at $-180°$ C.

Exercises

1. An average day's work for a man is 300 foot-tons. What is the minimum number of calories of heat that must be produced by the food which he consumes per diem, allowing that—apart from the work done—$\frac{99}{100}$ of the energy of the food is lost as heat from the surface of the body? (Assume J = 1,400 in foot-pounds and pound-degree-Centigrade calories.) [*First M.B.*]

2. A block of ice falls from the end of a glacier which is just melting, and 0·5 per cent. of the ice is thereby melted. From what height must the ice have fallen? (Latent heat of ice = 80; g = 980; J = $4{\cdot}2 \times 10^7$ ergs.) [*Ibid.*]

3. How much heat is produced when a mass of 500 kilos falls 5 metres on to the head of a pile? (g = 981; J = $4{\cdot}2 \times 10^7$.) [*Ibid.*]

4. Describe an experiment to show that air is heated by compression. Where does the energy represented by the heat that appears in the gas come from? Why does a current of air blowing up a mountain slope become cooled? [*First Professional.*]

(For Answers, *see* p. 389.)

PART III.—SOUND

Origin of Sound—Sound Waves—Amplitude—Pitch—Musical Intervals — Velocity of Sound — Vibration Rate of Tuning-Forks — Resonance — Timbre or Quality—Exercises.

Origin of sound.—When a pistol is fired, the heated gases liberated by the explosion compress the particles of air in their immediate neighbourhood. These in turn impinge upon others, squeezing them against particles still farther away, and so on. A local crowding or condensation is thus transmitted continually from one point to another. Owing to the elasticity of air the rear particles after impact rebound, while the front ones swing forward a little. A local rarefaction therefore replaces the previous local condensation, and is similarly transmitted from point to point. The transmission of this state of alternating compression and rarefaction constitutes a wave of sound, which, eventually reaching the ear of a bystander, shakes his tympanum. The vibration is transmitted by nerves to his brain, and he "hears" the explosion. The sounds that we usually hear are thus caused by waves in the air.

It is important to distinguish the *motion of the particles of the air* from the *motion of the wave* itself. Each individual particle moves but a short distance to and fro in the line of motion, but the wave may traverse a long distance. We can see a similar phenomenon if we watch the effect of a gust of wind

passing over a field of corn. Each ear of corn moves backwards and forwards perhaps but an inch or two, but the wave passes from one end of the field to the other. The wave is a transmission, not of matter, but of a certain state, or condition.

Sound and vibration.—Sound waves generally have their origin in the vibration of an elastic solid. When a tuning-fork is struck (Fig. 125), each arm begins to vibrate transversely, swinging first to *a*, then back to *a′*, and so on until the motion gradually ceases. As A swings to *a* it compresses the particles of air in its neighbourhood, and squeezes them together; they pass on the pressure, and themselves swing back and become more widely separated than they were in their normal state. This can be best seen from the next diagram (Fig. 126), representing the successive positions of particles of air as a sound wave passes. There are nine particles of air *a* to *i* represented at rest in the top line. In phase 1, a sound wave strikes particle *a* and pushes it towards *b*. In phase 2, *b* is pushed towards *c*, *a* continuing its forward movement. In phase 3, *a* is swinging back, but *b* swings on, and *c* has moved towards *d*; and so the compression passes, marked by the black particles, until in the eighth phase it has reached the last three particles *g*, *h*, and *i*, while *a* has just finished one complete vibration (p. 14), and returned to its normal position. Particles at the normal distance from each other are lightly shaded; those squeezed together are black; and those which are farther apart than usual are left unshaded.

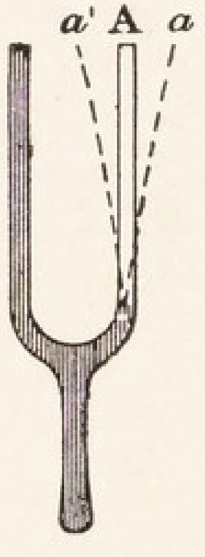


Fig. 125.—Tuning-fork vibrating.

Motion of sound waves.—The sound waves spread out in circles, just as a ripple spreads out in a pond when its surface is disturbed by a stone being thrown in, except that the vibrations of the individual particles of the sound wave are executed in the same direction as that in which the wave

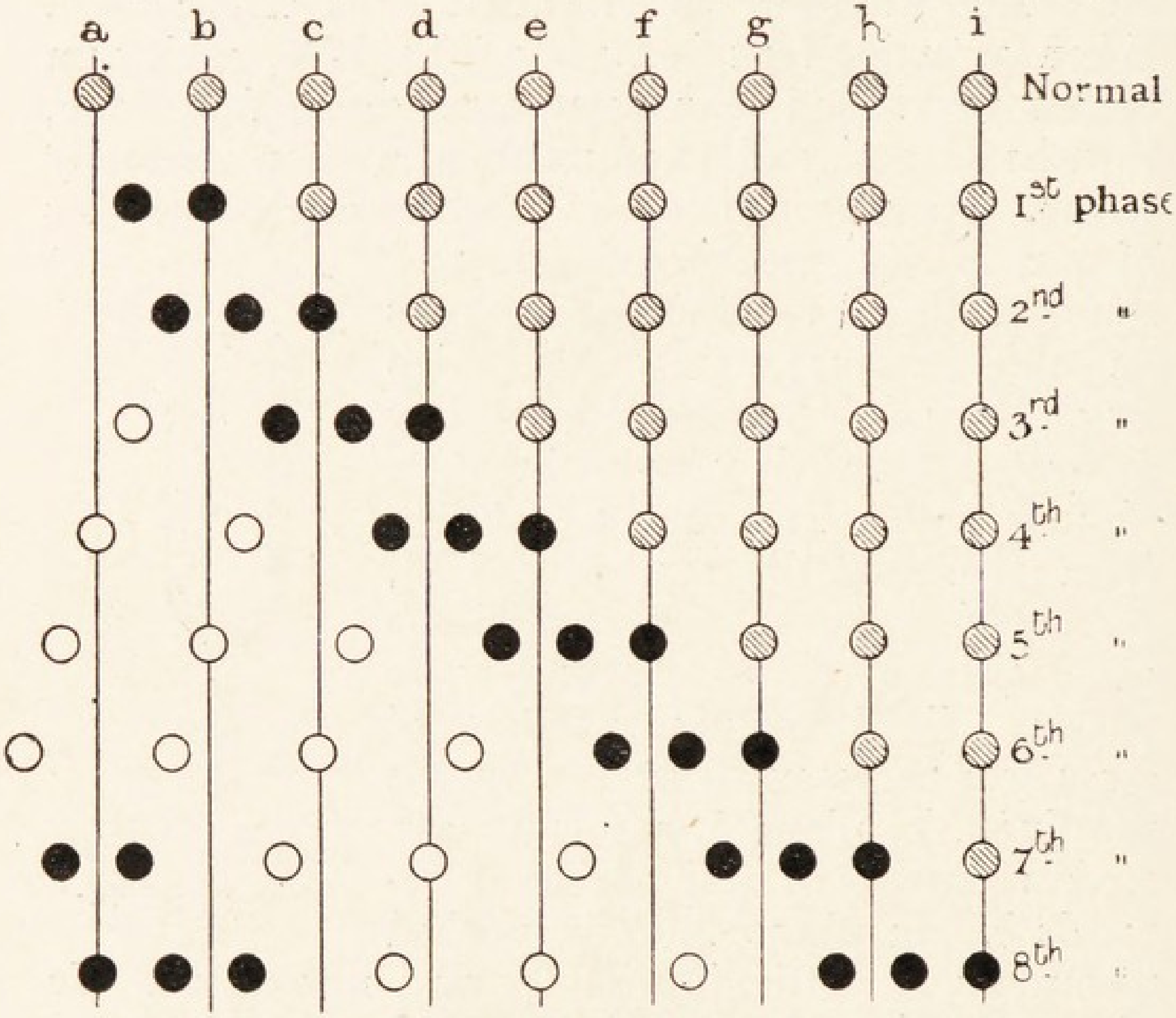


Fig. 126.—Successive phases of particles of air during passage of a sound wave.

progresses, whereas the particles of water move up and down at right angles to the direction of the wave. The former vibrations are described as *longitudinal*, the latter as *transverse.* The sound wave therefore produces alternate circles of compression and rarefaction, as seen in Fig. 127, instead of transverse crests and troughs.

If the sound waves be confined by a smooth tube, as in a speaking-tube, the distance at which the sound is audible is greatly increased. Sound, like light, can be reflected from plane, and focused by curved, surfaces.

Amplitude and pitch.—The number, n, of vibrations executed in a second is called the *frequency*,

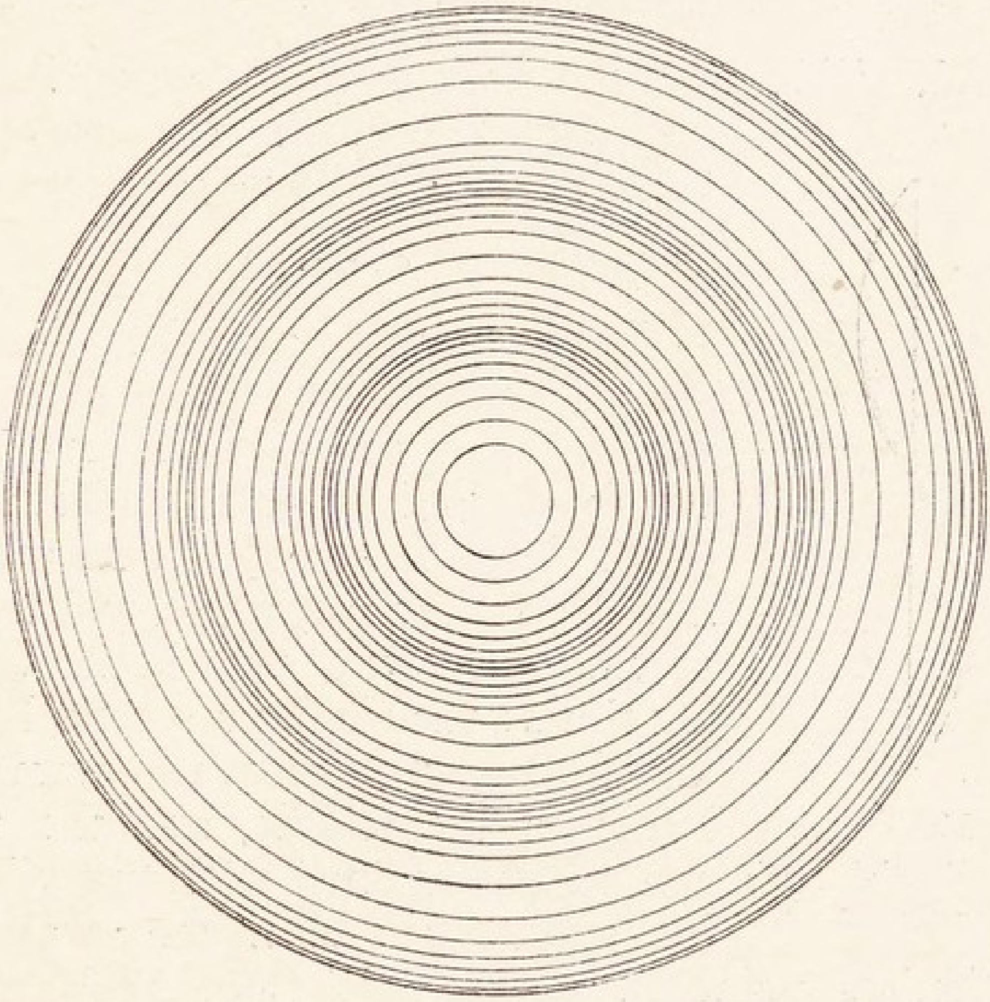


Fig. 127.—Passage of sound wave through air.

or *rate of vibration*. It will evidently vary inversely as the *period* (p. 14), which will therefore be $\frac{1}{n}$th of a second. The **pitch** of a note—i.e. whether it is high or low—varies directly with the frequency. The frequency of the middle C of a piano is 256. The C of an octave higher has a frequency of 512, while

the C of an octave lower has a frequency of 128. The intensity, or *loudness*, of a note is proportional to the square of the **amplitude** (p. 14) of the vibrations. The wave length (λ) is the distance which the wave travels in a period; it is also equal to the distance from any one particle to the next in the same phase as itself. Since the wave travels a distance λ in $\frac{1}{n}$th of a second, it will travel a distance $n\lambda$ in one second. This is the velocity, v, of sound in the medium in which the wave is travelling. We have therefore the relation

$$v = n\lambda$$

Fig. 128.—String vibrating with central node.

Wave motion can be demonstrated with a long piece of india-rubber tube or rope. If one end be fixed, and a sudden shake be given to the free end, a hump will be raised on the rope, which will travel to the fixed end and back again. By timing the shakes a second hump may be raised to meet the returning hump in the middle, and the rope can be maintained with two vibrating segments, and a comparatively motionless point in the middle (Fig. 128). This stationary point, or point of minimum displacement, is called a *node*. A string may vibrate as a whole, or it may divide into two or more vibrating segments. This can be very well demonstrated by varying the tension of a light string attached to the prong of an electro-magnetic tuning-fork. As the string is pulled tighter, the length of each vibrating segment increases, but the number of segments diminishes, till the whole length of string affected forms only one

segment. The extremities of every segment are nodes; the centre of each segment is an *antinode*, or region of maximum displacement. Owing to the persistence of impressions on the retina, the string appears to form loops, as if the hump at a place were formed on both sides at once instead of successively. If we halve the length of a string it vibrates twice as fast as the original string, and we get the higher octave of the fundamental note. This vibration of 2 : 1—i.e. when two notes are simultaneously vibrating, one twice as fast as the other—is very agreeable to the ear. The interval of a fifth consists

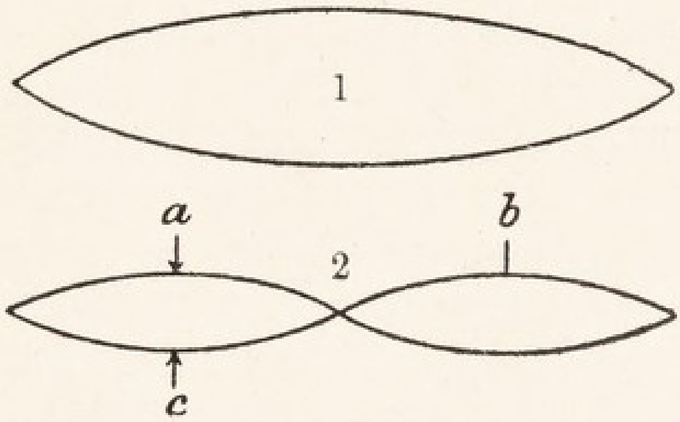


Fig. 129.—1. String vibrating as a whole. 2. When damped in the middle, giving the upper octave.

of two notes whose frequencies are as 3 : 2; of a fourth, 4 : 3; and of a third, 5 : 4. In fact, the simpler the ratio the more pleasant is the combined effect on the ear.

The ratios of the vibration rate of the notes in the scale are :—

1st	2nd	3rd	4th	5th	6th	7th	8th
doh	*ray*	*me*	*fah*	*sol*	*la*	*te*	*doh*
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

If the first vibrates 100 times a second, the fifth will vibrate $100 \times \frac{3}{2} = 150$ times a second.

The distance between two *consecutive* nodes—or antinodes—is half the wave length; the distance between node and antinode is one-quarter of the wave length. *Fixed* points must always be nodes. In Fig. 129 the string is represented (1) when sounding its fundamental, or lowest possible, note; it has then only two nodes, and these are at the two fixed ends. The length, l, of the string is therefore $= \frac{\lambda}{2}$, or $\lambda = 2l$, and therefore (p. 198)

$$v = n \times 2l$$

The string is also represented (2) when sound-

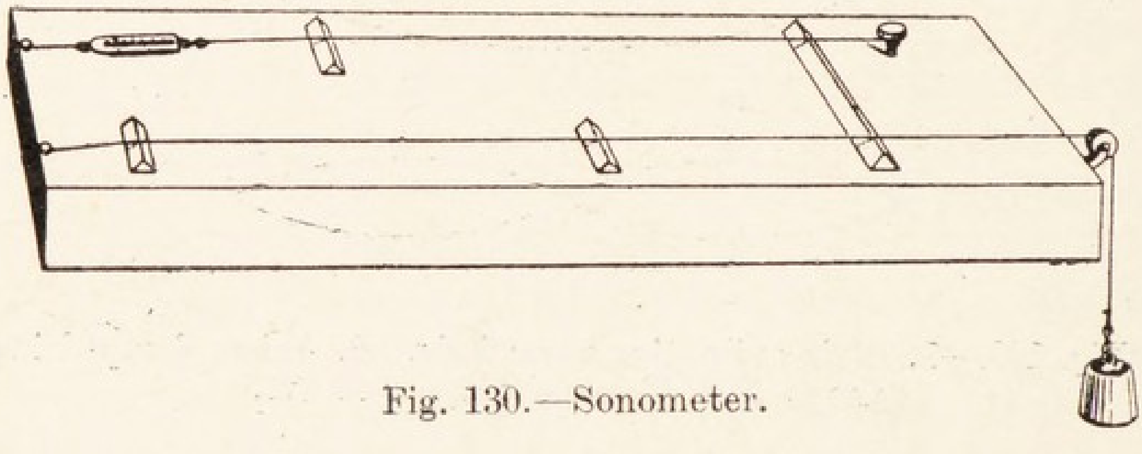


Fig. 130.—Sonometer.

ing the octave of its fundamental note; it has then an additional node in the middle, and l is now $= \lambda$. Consecutive antinodes are formed at a and b; the maximum displacement, $a\,c$, is twice the amplitude.

The velocity, v, with which the wave travels in the string, when plucked transversely, is given by the formula

$$v = \sqrt{\frac{t}{m}}$$

in which t represents the force keeping the string

taut, and m is the mass of unit length of the string.

We also know (p. 198) that $v = n\lambda$, and that $\lambda = 2l$, where l is the length of a loop; we have therefore

$$n \times 2l = \sqrt{\frac{t}{m}}$$

or,
$$n = \frac{1}{2l}\sqrt{\frac{t}{m}}$$

This formula can be verified by experiments with the sonometer (Fig. 130).

Air and sound.—Air is necessary for the transmission of ordinary sound. This can be shown by allowing a clock to strike in the receiver of an air-pump, the clock being placed on a soft cushion to prevent the sound being transmitted through the air-pump plate. If the pump be exhausted, the sound becomes very feeble; and if the receiver be filled with hydrogen and again exhausted, the sound is almost inaudible. Moreover, no sounds reach us from the sun, the source of so much light and heat. Sound is therefore not transmitted by the ether, which transmits the waves of light, electricity, and radiant heat.

Velocity of sound.—In ordinary *air* sound travels nearly 1,120 feet per second when the temperature is 15° C. We shall see presently that the velocity varies directly as the square root of the absolute temperature, and therefore increases by about 2 ft. per second for a rise in temperature of 1° C. The velocity in *dry* air at 0° C. may be taken as 1,084·7 ft., or 330·6 metres, per second. Wertheim and others determined values in different media; some of these are quoted here.

VELOCITY OF SOUND IN VARIOUS MEDIA IN FEET PER SECOND

Water at 15° C.	4,710
Lead at 20° C.	4,030
Iron	16,828
Steel	15,470
Wood along the fibre—	
Fir	15,218
Pine	10,900
Oak	12,622

The formula for the velocity of sound in terms of the density, D, and elasticity, E, of a medium is:

$$\text{velocity} = \sqrt{\frac{E}{D}}.$$

E represents the coefficient of elasticity. This is measured by the ratio between the stress applied and the strain produced by it, or $E = \frac{\text{stress}}{\text{strain}}$. In the case of the elasticity of volume of a gas, we may take pressure as the stress, and the compression produced by it as the strain. If a certain mass of the gas fills a volume, V, at a pressure, P, and a small additional pressure, p, reduces the volume to $V - v$, then, *if the temperature remains constant*, we know by Boyle's law that

$$\begin{aligned} VP &= (V - v) \times (P + p) \\ &= VP + Vp - pv - vP \\ \text{or, } 0 &= Vp - pv - vP \end{aligned}$$

Neglecting $p\,v$, which will be a very small quantity, we find

$$Vp = vP$$

$$\text{or } \frac{Vp}{v} = P$$

Now, the *compression* is $\frac{v}{V}$ (p. 122), and therefore

$$E = \frac{\text{stress}}{\text{strain}} = \frac{p}{\frac{v}{V}} = \frac{Vp}{v} = P$$

The formula for the velocity of sound in air then becomes, velocity $= \sqrt{\frac{P}{D}}$.

To find the value in cm. per second we must put $P = 1{,}013{,}961{\cdot}6$ dynes (p. 89) and $D =$ mass of 1 c.c. dry air $= 0{\cdot}001293$ grm.

This leads to the value for the velocity in air at N.T.P. of about 280 metres. By actual experiment, however, the value is found to be about 332 metres. Laplace showed that the discrepancy between the calculated and observed values is due to the fact that the calculation supposes that the temperature of the air remains constant during the passage of the sound wave. If the phenomenon is not isothermal but adiabatic (p. 184), then E will not equal P. This is really the case; the temperature of the air is raised by the compressions and lowered by the rarefactions, but the passage of the sound wave is too rapid for these changes to be adjusted by external influences. Both the heat produced by the compression in front of any particle and the cold produced behind it increase the force resisting compression, and therefore increase the velocity with which the wave travels. We recognize, therefore, two values of E, corresponding to the two values of the specific heat of air (p. 186). The ratio between them is the same, and in fact is found from the ratio between the observed value of the velocity of sound in air and the value as calculated from $\sqrt{\frac{P}{D}}$.

We therefore have

$$E \text{ (adiabatic)} = E \text{ (isothermal)} \times 1{\cdot}41$$
$$= P \times 1{\cdot}41$$
$$\therefore \text{ Velocity of sound } = \sqrt{1{\cdot}41 \frac{P}{D}}$$

This correction of Newton's formula by Laplace is completely confirmed by experiment.

We see from this formula that (1) if the temperature remains constant, a change in P will not alter the velocity, since it makes the same change in D; if the pressure on a certain mass of gas be doubled, its volume is halved, and therefore its density is also doubled. (2) If P remains constant, an increase in T (p. 132) will diminish D in the same proportion and therefore will increase the value of the velocity; in fact, at two absolute temperatures, T_1 and T_2, we have $\frac{T_1}{T_2} = \frac{D_2}{D_1}$, and therefore

$$\frac{(\text{Velocity at } T_1)^2}{(\text{Velocity at } T_2)^2} = \frac{1{\cdot}41 \frac{P}{D_1}}{1{\cdot}41 \frac{P}{D_2}} = \frac{D_2}{D_1} = \frac{T_1}{T_2}$$

(3) The presence of aqueous vapour in the air tends slightly to increase the velocity, since moist air is lighter than dry air at the same pressure. (4) In different gases, under the same circumstances, the velocity varies inversely as the square root of the density; sound would therefore travel four times as fast in hydrogen ($D = 1$) as in oxygen ($D = 16$).

The velocity of sound in air can be directly determined by observing the time which elapses between the flash and report of a gun, after carefully measuring the distance between the gun and the observer. Neither amplitude nor frequency of vibration makes

any difference in the velocity of sound. This is obvious to anyone listening to a band at a distance: the sound from the shrill piccolo arrives at the same instant as the deep notes of a bassoon.

If the ear be placed at one end of a long iron rail or pipe, which is struck at some distant point with a hammer, two sounds will reach the ear, the first travelling through the iron, the second through the air. The great conductivity of wood for sound is taken advantage of in the ordinary stethoscope.

Knowing the velocity of sound in air, we are able to calculate the distance of an explosion or signal gun, etc., of which the flash can be seen. Thus, if a clap of thunder is heard 4·8 seconds after the flash of lightning is seen, the distance of the flash may be taken to be $1{,}120 \times 4{\cdot}8 = 5{,}376$ ft., or a little over a mile. The velocity of light (p. 266) is so great that we need make no allowance for the time it takes to travel any terrestrial distance. Again, if you stand in front of a cliff, or when at sea in front of an iceberg, and shout, if you hear the reflected sound or echo in 6 seconds, as the sound has travelled to the cliff *and back*, the distance of the cliff is

$$\frac{1{,}120 \times 6}{2} = 3{,}360 \text{ ft.}$$

Vibration rate of tuning-forks.—The rate of vibration of a tuning-fork can be easily ascertained by attaching to one of its limbs a thin piece of metal or card and causing it to write on a smoked surface revolving at a uniform and known rate. Suppose we have a surface of smoked paper stretched round a cylinder and rotating so that the surface moves at the rate of 20 in. per second: after making a tracing (Fig. 131) we count the number of waves, measuring from crest to crest, say in 5 in., and we find there are 25 vibrations; in 20 in. there will

therefore be 100, and the rate of vibration is 100 per second.

Wave length.—Since $v = n\lambda$, we know that $\lambda = \frac{v}{n}$. The wave length can therefore be determined by dividing the velocity of sound per second by the number of vibrations per second. In the above example the sound wave, starting from the fork, will have traversed 1,120 ft. in the second, and during

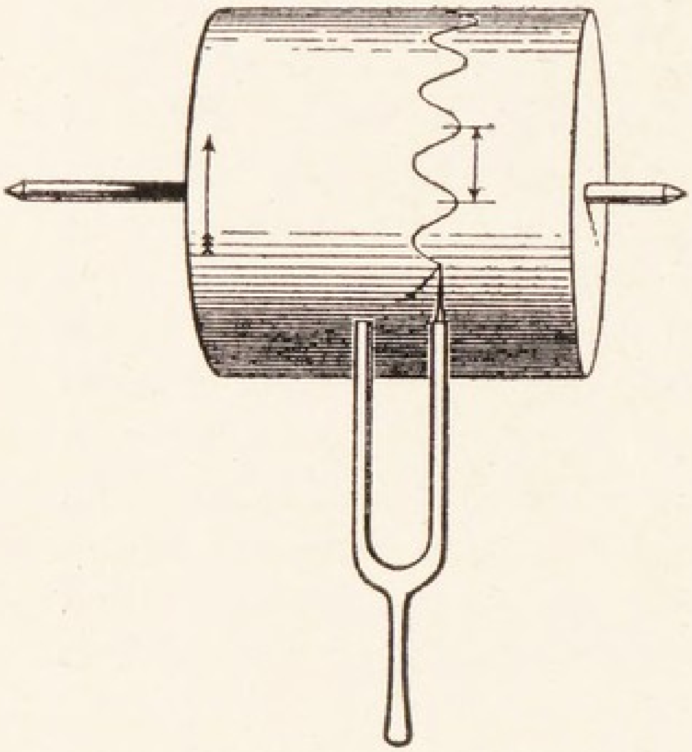


Fig. 131.—Tracing of tuning-fork.

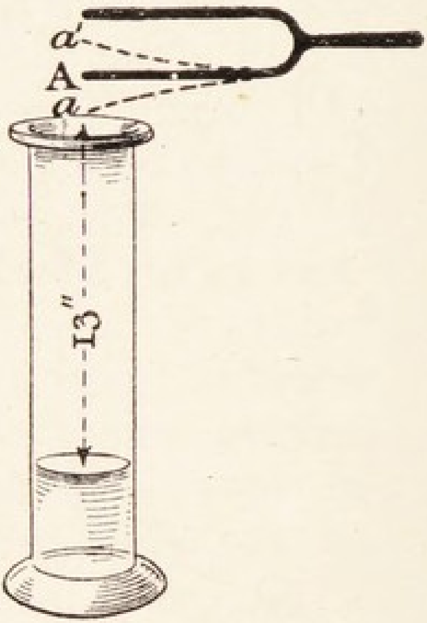


Fig. 132.—Resonating jar.

this time will have made 100 complete vibrations, so that the wave length of each will be $\frac{1,120}{100} = 11{\cdot}2$ ft.

Resonance.—If a tuning-fork, after being struck, be held in the fingers, the sound is feeble, but if the stem be held firmly on a table the sound is at once strengthened. The explanation is simple: the table is thrown into vibration, and, having a much larger surface than the limbs of the fork, a much larger volume of air is set in vibration.

If a tuning-fork be struck and held over a deep glass jar, and the jar be filled up slowly with water, a point will be reached when the sound of the fork will swell out and become very much louder. The air in the jar is, in fact, vibrating in unison with the fork, forming a kind of extempore organ pipe (Fig. 132).

To secure maximum resonance the wave must travel the distance, *d*, to the water level, be reflected, and return to the starting-point just in time to reach a particle there at the moment that it is about to swing upwards, that is when it has completed half a period; but in this time the wave must travel half a wave length; therefore $2d$ must $= \frac{\lambda}{2}$, or $d = \frac{\lambda}{4}$.

If we use a fork which vibrates 256 times a second, and whose wave length is therefore $\frac{1{,}120}{256} = 52$ in., and determine the conditions under which the above reinforcement of the note takes place, we shall find the distance from the surface of the water to the top of the jar to be nearly 13 in., or one-quarter of the wave length. The prong A swings to *a* (Fig. 132) and compresses the air in the jar, starting a sound wave which proceeds to the surface of the water, and back again (a distance of 26 in.) just as the prong is starting to swing back to a', so the two vibrations keep time. Maximum resonance will also be secured if a period and a half has elapsed, that is, if $2d = \frac{3\lambda}{2}$, or $d = \frac{3\lambda}{4}$, or, in fact, if *d* is any odd multiple of $\frac{\lambda}{4}$. If the tube were long enough, a series of levels

might be found corresponding to these values of d. They are all nodal planes and are separated by $\frac{\lambda}{2}$. The open end is an antinode. This furnishes a simple method of finding the wave length of a note by multiplying the shortest resonating length by 4; by dividing this into the velocity of sound in air we obtain the frequency. If the tube is long enough to give two nodes at distances d_1 and d_2, we can find λ from the fact that

$$d_2 - d_1 = \frac{\lambda}{2}$$

If we can find only the first node, a small correction is usually made for the open end. The corrected length may be taken to be $d + \frac{3r}{5}$, where r is the radius of the tube.

Range of audibility.—The extreme range of audibility appears to differ in individual cases. Tyndall quotes instances in proof of this. This perhaps accounts for the difference in the estimates by different observers of the limits of hearing. It seems probable from the experiments of Helmholtz that the lower limit corresponds pretty nearly to the value $n = 30$; while the higher limit seems to be in the neighbourhood of $n = 24{,}000$. This gives a range of nearly ten octaves. In practice the usual musical range is approximately from 32 to 4,000, or about seven octaves. The C′, the middle C of a soprano, is about 512. In the time of Handel it was 507. The C′ on the Albert Hall organ is 541, so that our B is about the same pitch as Handel's C.

Noise, strictly speaking, is any audible sound, whether musical or not; the musical sound results from regular and rhythmical vibrations, a definite number of impulses striking the ear at regular

intervals. How these vibrations are produced is immaterial. They may be a series of taps, as when a card is held against a revolving cog-wheel; or a series of puffs, as in the siren; or the vibrations of strings, the air in pipes, etc. Sounds which result from a medley of vibrations, whose frequencies bear no simple ratio to each other, have not, as a rule, that pleasing effect on the ear which we associate with music; but when the motive of the composer is to startle rather than to please, such sounds do find place in technical "music."

Quality of a note.—Besides differing in pitch and in intensity, a note may vary in *quality* or *timbre*. Thus a note of exactly the same pitch and loudness may be sounded on a flute, on a piano, and on a violin, but an educated ear has no difficulty in distinguishing the notes by their various qualities. The explanation of this difference in quality is to be found in the fact that hardly any note consists exclusively of one set of vibrations—i.e. it is not pure. This can be demonstrated by pressing down the *forte* pedal of a piano, which will raise the dampers from the strings, and then inducing someone with a powerful baritone voice to sing his top C into the piano; the C string, in unison, will be heard vibrating, but in addition there will be heard the C an octave above, and the fifth—i.e. the G—above that; so that the note sung was a mixture of the vibrations of all these, and several others more difficult to hear, with the fundamental note. Helmholtz has shown that any desired quality can be produced by mixing suitable overtones of suitable strengths with the fundamental note.

In an open organ pipe (Fig. 133) the wind is projected from a fine slit, S, against a sharp edge of wood, P, producing a flutter of immature sounds. In

reed pipes the effect is produced by wind driven against the edge of a metal *reed*, which more or less perfectly covers a longitudinal orifice in the wall of the pipe. In each case the pipe selects the sound to which its column of air can resonate and raises it to the dignity of a musical note. If the pipe be overblown it gives overtones. When sounding its fundamental note an open organ pipe has a node in the middle and an antinode at each open end. The

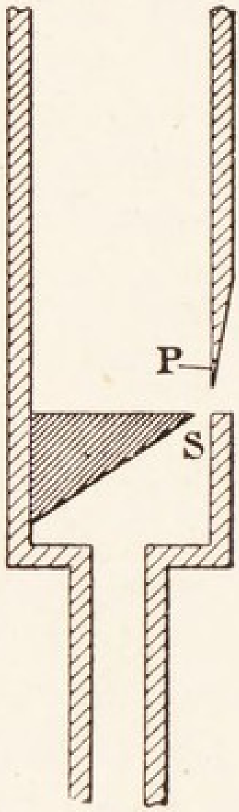


Fig. 133.—Organ pipe.

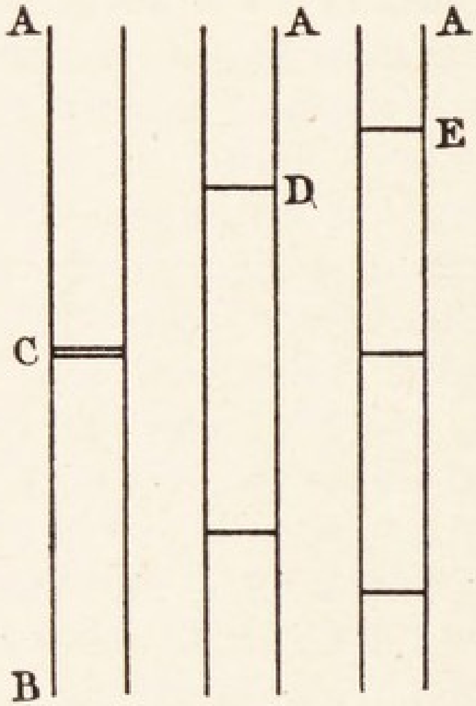


Fig. 134.—Overtones in organ pipe.

length, l, of the pipe is therefore equal to half a wave length, or $\lambda = 2l$. For the first harmonic or overtone the frequency is doubled and therefore the wave length must be halved; hence $\lambda = l$ and $\frac{\lambda}{4} = \frac{l}{4} = AD$ (Fig. 134).

Nodes and antinodes will therefore succeed each other at this distance, commencing with an antinode at the open end. Similarly, for the second overtone

n is trebled, and therefore $\lambda = \frac{2l}{3}$, and $\frac{\lambda}{4} = \frac{l}{6} = A\,E$ (Fig. 134). Nodes and antinodes will now alternate at this distance, the first antinode being at A and the first node at E.

If an open pipe were cut in half at C and the ends at C were closed, two pipes closed at one end would be produced, each of which would give a note of the same pitch as the open pipe A B. A closed end must always be a node, and an open end an antinode, so that the length, l, of a pipe, closed at one end, is $\frac{\lambda}{4}$ for the fundamental note. Hence $\lambda = 4l$. It will therefore be an octave lower than the fundamental note of an open pipe of the same length. In a piano string also the first overtone forms a node in the centre, and is therefore an octave above the fundamental note; the second has two nodes, and is a fifth above the last.

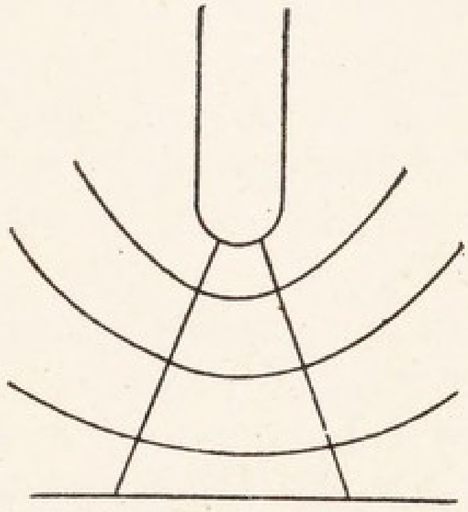


Fig. 135.—Nodes in bar and tuning-fork.

When any point on a string is struck or plucked, all the overtones which require that point for a node disappear. The hammers of a piano are so arranged that they strike the string at a point where the production of the most harmonious overtones is encouraged, while those which are discordant are not formed. The production of notes of pleasing quality in singing depends largely on so shaping the mouth, etc., that those overtones which combine harmoniously find suitable resonating cavities.

In a bar or rod the fundamental note is sounded

when it vibrates with two nodes. The first overtone has three nodes. In the tuning-fork, as the bar of steel is bent, the nodes approach until they are close together (Fig. 135).

Beats.—Reinforcement of sound as in resonance (p. 206) may also occur when the waves from two tuning-forks whose frequencies are near together simultaneously affect the same air particle. If, for instance, one fork makes n, and the other $n + 2$, vibrations per second, it is clear that in half a second the second fork makes exactly one vibration more than the first; each would therefore be in exactly the same phase once in half a second. A particle vibrating under the influence of the forks will be subject to the same conditions, and the amplitude of its vibration will therefore once in every half-second be a maximum one due to the combined waves. The loudness due to this maximum amplitude produces a "beat." In this case there will be two beats per second, and in general the number of beats will correspond to the numerical difference in the frequencies of the two forks. By counting the beats we can therefore determine n for one fork if we know the frequency of the other. The method is of practical use for this purpose.

Interference between two waves, however, produces destruction as well as reinforcement of sound. In the above example the influence of one wave only completely supplements the other at the beats; at all other moments they are more or less antagonistic, as the particle is required to be in different phases under the two impulses. The antagonism is most complete when the difference of phase corresponds to a difference in time of half a period or a difference in length of half a wave length. If the sound waves from the same fork can reach

the ear by two paths, the note is most completely destroyed if the paths differ in length by $\frac{\lambda}{2}$. This principle has been employed in practice to measure λ.

Exercises

1. Into a tube of about 1 cm. radius, water is poured. When the water level is at 33 cm. from the top, maximum resonance is obtained from a fork of frequency 256. On shortening the air column to 16·25 cm. maximum resonance occurs with a fork of frequency 512. Calculate from these data the velocity of sound in the air of the tube. [*First M.B.*]

2. Given that the velocity of sound in air at 0° C. is 1,090 ft. per sec., calculate what it would be in the air of a tube railway, temperature 20° C. and pressure 77 cm. of mercury. [*Ibid.*]

3. A tuning-fork of pitch 512 is set vibrating and held over a tube full of air, closed at the lower end by water. Calculate the lengths of the two shortest columns of air that give resonance with the fork, when the velocity of sound in air is 1,120 ft. per sec. [*Ibid.*]

4. Calculate the velocity of sound in air at 0° C. and 770 mm. pressure. The ratio of the specific heats of air is 1·40, and the density of air at 0° C. and 760 mm. is 0·00129 grm. per c.c. [*Ibid.*]

5. A wire, the mass of which per unit length is 0·006 grm. per cm., is fixed at two points 50 cm. apart, and is vibrating in two segments. If the wire is stretched by a weight of 5 kg., calculate the frequency of the note emitted. [*Ibid.*]

(For Answers, *see* p. 389.)

PART IV.—LIGHT

CHAPTER I

REFLECTION

Light Vibrations—Propagation of Light—Intensity of Light—Photometers—Laws of Reflection—Concave and Convex Mirrors—Formation of Images—Laryngoscope — Ophthalmoscope — Bronchoscope, Cystoscope, Gastroscope—Exercises.

LIGHT has already been mentioned among the phenomena of radiant energy referred to on p. 143. The vibrations from which it originates are transverse; the frequency is nearly a million million times that of a sound vibration. The range of the human eye is much more limited than that of the ear (p. 208). For lights of different colours the value of n varies approximately from 4×10^{14} (red light) to $7{\cdot}6 \times 10^{14}$ (violet light), and has therefore a range of about one octave. Although sound becomes inaudible in an exhausted receiver (p. 201), light does not become invisible. Light vibrations are transmitted by the ether which is assumed to fill all space—even so-called vacuous regions. In this imponderable medium the light waves travel with the remarkable velocity of about 186,000 miles per second, or nearly 3×10^{10} cm. per second. The wave length (λ_R) of red light will therefore be given in centimetres by the equation :

$$3 \times 10^{10} = 4 \times 10^{14}\, \lambda_R \text{ (p. 198)}$$

$$\text{or, } \lambda_R = \frac{3}{4} \times \frac{10^{10}}{10^{14}} \text{ cm.}$$

$$= 0{\cdot}75 \times \frac{0{\cdot}1}{10^4}\,\text{cm.}$$

$$= 0{\cdot}75 \times \frac{1}{10^6}\ \text{metres}$$

$$= 7{,}500\ \text{tenth-metres (p. 5)}$$

Similarly, the wave lengths of violet light will be about 3,947, or roughly 4,000 tenth-metres. The red light shown in the spectrum of potassium (Frontispiece, 3) has a wave length of 7,668 tenth-metres; the violet light in the same spectrum has a wave length of 4,047 tenth-metres. These wave lengths, although so minute, have been actually measured. We have seen (p. 212) that two sound waves may either reinforce each other or produce silence. Similarly, two light waves may either reinforce each other or produce darkness. If the two waves emanate from the same source but travel to the same spot by two different paths, reinforcement is produced when the two paths differ in length by an *even* number of half wave lengths, and darkness when they differ by an *odd* number of half wave lengths. In this interference we have therefore a clue to the wave length of the light examined.

The velocity of light has also been measured by various methods, and some of these are described later (p. 266).

Bodies like the sun and stars, which are themselves independent sources of light, are called *self-luminous*.

The earth, the other planets, as well as most terrestrial objects, are not self-luminous, and are only visible when they reflect light which comes to them from the sun or other self-luminous source.

Substances, as glass, water, etc., which transmit light, and through which objects can be clearly

distinguished, are termed *transparent;* substances which, like tracing-paper and ground glass, transmit light, but do not allow objects to be clearly distinguished, are called *translucent;* substances like steel, slate, marble, etc., which transmit no light, are termed *opaque.*

Law of inverse squares.—Light travels

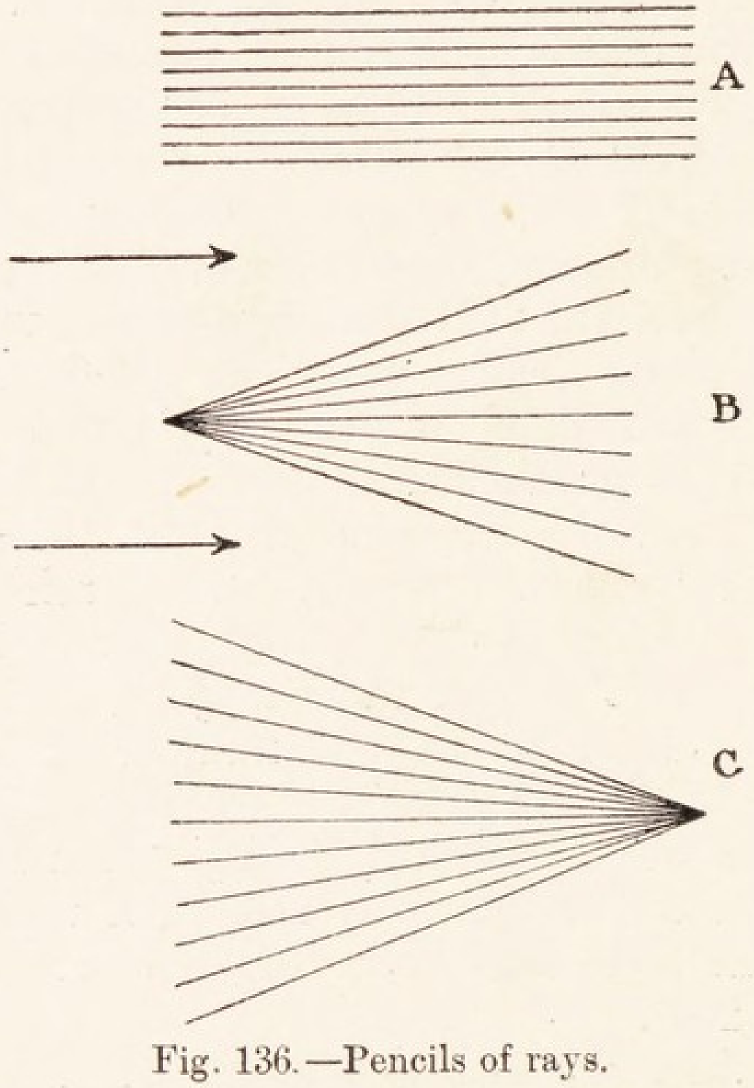


Fig. 136.—Pencils of rays.

through homogeneous media in straight lines, which radiate in all directions from the luminous point, and are called *rays* (*radius*, a ray).

A collection of rays is called a pencil (Fig. 136). The pencil may be *parallel*, A, *divergent*, B, or *convergent*, C. The light from a luminous point at the centre of a hollow sphere (Fig. 137) is therefore equally distributed over the whole surface. The

area of this surface is proportional to the square of the radius (r) and is equal to $4\pi r^2$ (p. 14). If the quantity of light on the whole surface be Q, the quantity *per unit of area* must be $\frac{Q}{4\pi r^2}$; this is the *intensity of illumination*, and evidently varies *inversely* as the square of the distance (r) from the luminous point to the illuminated area. The phenomenon is therefore regulated by the *law of inverse squares* (p. 50). Thus, for one and the *same* source of light at O (Fig. 137),

$$\frac{\text{intensity of illumination at A}}{\text{intensity of illumination at B}} = \frac{\overline{\text{O B}}^2}{\overline{\text{O A}}^2}$$

A second and *different* source of light placed at O will, however, produce at B intensity of illumination equal to that produced at A by the first source, if Q, Q', the quantities of light similarly emitted by the first and second sources respectively, fulfil the relation

$$\frac{Q}{4\pi\,\overline{\text{O A}}^2} = \frac{Q^1}{4\pi\,\overline{\text{O B}}^2}$$

$$\text{or,}\quad \frac{Q}{Q^1} = \frac{\overline{\text{O A}}^2}{\overline{\text{O B}}^2}$$

that is, if

$$\frac{\text{illuminating power of first source}}{\text{illuminating power of second source}} = \frac{\overline{\text{O A}}^2}{\overline{\text{O B}}^2}.$$

If, therefore, different sources of light equally illuminate a surface from which they are situated at different distances, we know that their illuminating powers must be *directly* proportional to the squares of these distances. The application of this fact to

measure illuminating power is the basis of *photometry* (φωτός, of light, μέτρον, measure).

Any apparatus by which the measurement is effected may be called a *photometer*.

Photometers.—Two types of instrument in common use are :

1. **Rumford's shadow photometer.**—An upright rod is placed in front of a white screen. Two shadows of the rod are cast on the screen by the two lights under comparison, say a candle flame

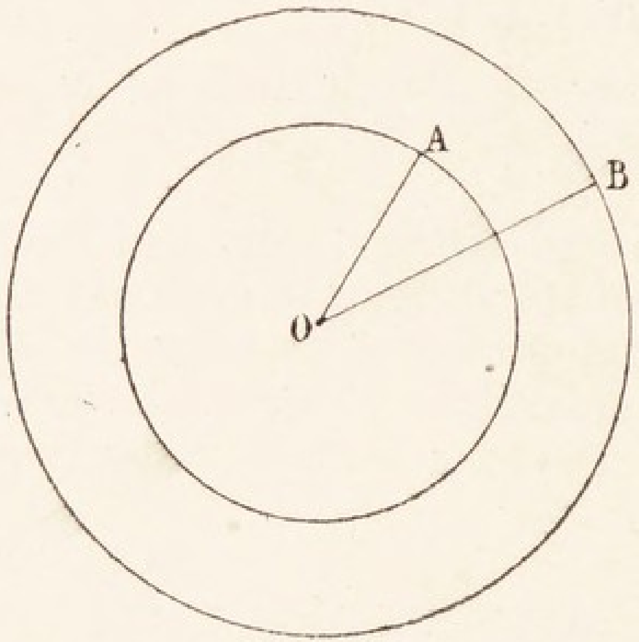


Fig. 137.—Distribution of light.

and a gas flame. The shadow cast by the candle flame is illuminated by the gas flame, and vice versa. The positions of the lights are adjusted until the two shadows are side by side and appear of equal strength. If the respective distances of the candle flame and the gas flame from the screen are then 1 ft. and 4 ft., we know that

$$\frac{\text{illuminating power of gas flame}}{\text{illuminating power of candle flame}} = \frac{4^2}{1^2} = \frac{16}{1}$$

or the *candle power* of this gas flame is 16.

The experiment may easily be modified to verify

the law of inverse squares; if, for instance, one candle be placed 1 ft. from the screen, it will be found that four similar candles placed at 2 ft. from the screen, or nine similar candles at 3 ft., etc., will produce equal illumination of the screen.

2. **Bunsen's grease-spot photometer.**—A small piece of wax is melted by a hot iron into the centre of a piece of ordinary white paper; the grease fills up the pores and renders the paper translucent, so that, when light reaches the paper, this portion transmits more and reflects less than the rest of the surface. Hence, if one side only is illuminated, the greased portion appears brighter than the rest if viewed from the *other* side, but darker than the rest if viewed from the illuminated side. Similarly, if the two sides are unequally illuminated, this portion will appear brighter than the rest when viewed from the side which is *least* illuminated. This distinction will only vanish when both sides are equally illuminated. The paper with the grease spot, called the *photometer disc*, is therefore moved backwards and forwards between the two sources of light, say a standard candle and a lamp, until the surface appears uniformly bright when viewed from either side. If the distances of the respective lights from the disc be then measured and squared, the numbers will give their relative illuminating powers—e.g. if from disc to candle the distance be 1 ft., and from disc to lamp be 3 ft., the

$$\frac{\text{illuminating power of lamp}}{\text{illuminating power of candle}} = \frac{3^2}{1^2} = \frac{9}{1}$$

or, the *candle power* of the lamp is 9.

Umbra and penumbra.—In consequence of the rectilinear propagation of light, a black shadow, the *umbra*, R (Fig. 138), is formed when the light

from a luminous *point* is intercepted by an opaque body. If, however, the source of light be a body of appreciable size, a partial shadow, or *penumbra*, surrounds the umbra (P, Fig. 138). Thus, the space C D is completely shaded, receiving no light from either the edge A or the edge B, while C E is in half shadow, receiving no light from B, and D F is similarly situated as regards A.

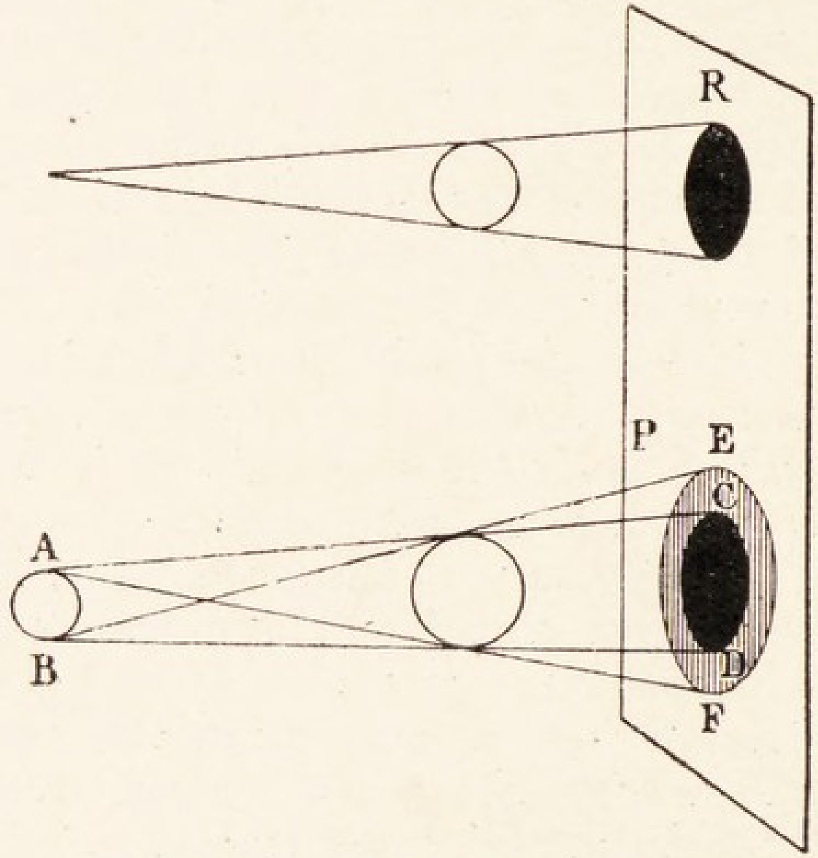


Fig. 138.—Umbra and penumbra.

Laws of reflection.—Some of the light which falls upon a polished surface is reflected from that surface. The direction of the reflected light is determined by the following laws :—

1. *The incident ray, the reflected ray, and the normal to the surface at the point of incidence, are in the same plane.*
2. *The angles which the incident and reflected rays make with the normal are equal.*

Thus, if A B (Fig. 139) be a ray incident at the point B on the reflecting surface C D, then (1) the reflected ray B E and the normal N B are in the same plane with A B, and (2) the angle of reflection, E B N, is equal to the angle of incidence, A B N.

It follows from this that a ray incident normally, as N B, is reflected back along the same line, B N.

It will be a useful exercise for the student to verify this law by the following experiment: On a drawing-board, furnished with paper, place an unframed looking-glass so that the reflecting surface

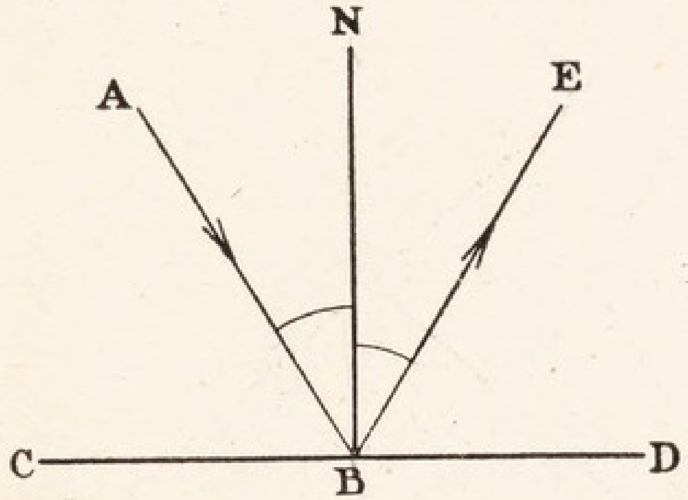


Fig. 139.—The angle of reflection is equal to the angle of incidence.

is perpendicular to the plane of the paper. At P (Fig. 140), a point in front of the glass, place a pin. On looking at the glass from a near point R, the image of P will be seen in a certain direction; mark this direction by placing pins at two points on it, R, S, so that these two pins and the image of P appear to be in one straight line. While looking along this line, place another pin at some point Q, so that its image also is seen in the same straight line with the other three, R S produced. The observer should now be able to see apparently *four* pins in a row, if he looks either along P Q, or along R S.

Remove the pins, and join P Q and R S. Similarly, by looking at the image of P from another position X, whence it is seen in the direction X O, the observer can, while looking along X O, place another pin at L, so that its image is also seen on X O produced. Four pins should now be seen by looking along P L or X O. Remove these pins, and join P L and X O.

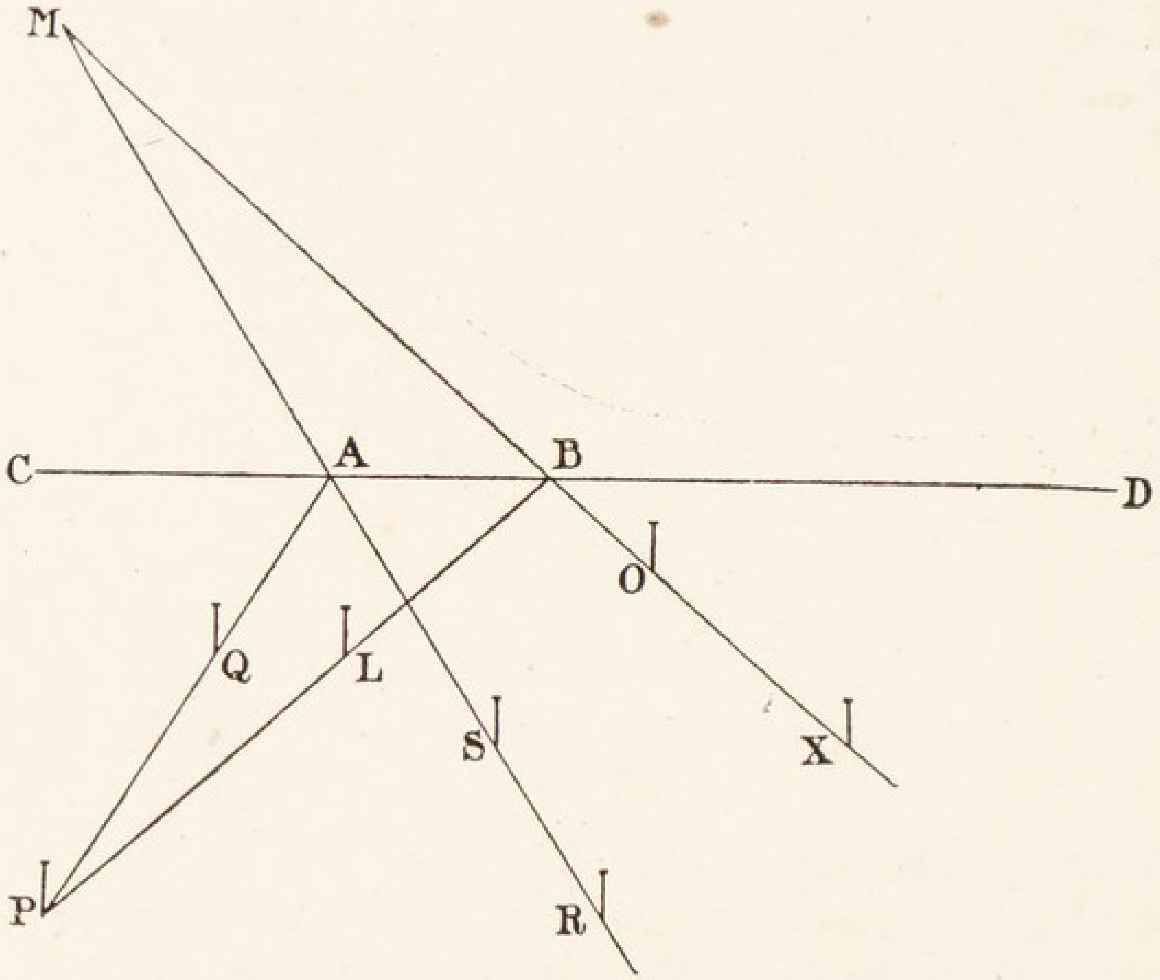


Fig. 140.—Verification of the laws of reflection.

Finally remove the mirror. Produce P Q and R S to meet at A. Produce P L and X O to meet at B. A and B are evidently points on the reflecting surface; join A B and produce it both ways, to C and D. At A draw A K (Fig. 141) perpendicular to C D. With centre A and any convenient radius, describe a circle cutting A P in H, A K in K, A R in T. Join H K and K T. These chords when measured should be found

to be of equal length. Then, angle of incidence H A K = angle of reflection K A T by Euclid I. 8.

We can also, from Fig. 140, locate the image of P. Since it is both in R S produced and in X O produced, it must be at M where these directions meet. Join P M. If the construction is perfectly true, P M will be perpendicular to C D and will be bisected by it.

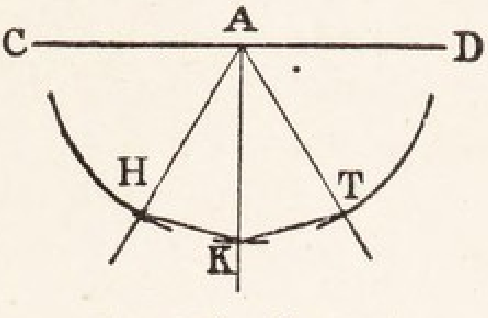


Fig. 141.—Law of reflection.

The position of M (Fig. 140) may also be found by the important **method of parallax.** If two points, A, B (Fig. 142), are viewed from a distant point E, in the line A B produced, they may appear to be equidistant from E, though they are not really so. If, however, the observer moves his eye to E′, a little distance on his *right*, they are no longer seen in the same straight line ; A is seen in the direction E′ A, and B is seen in the direction E′ B B′, so that, relatively to A, B appears to have moved to the *left*. Similarly, if the observer moves his eye to E″ on his *left*, B will evidently be seen by him in the direction E″ B B″, and appear on the *right* of A. The alternation of position thus produced is called *parallax* (Gk. παράλλαξις, alternation). If, for instance, A be the real position

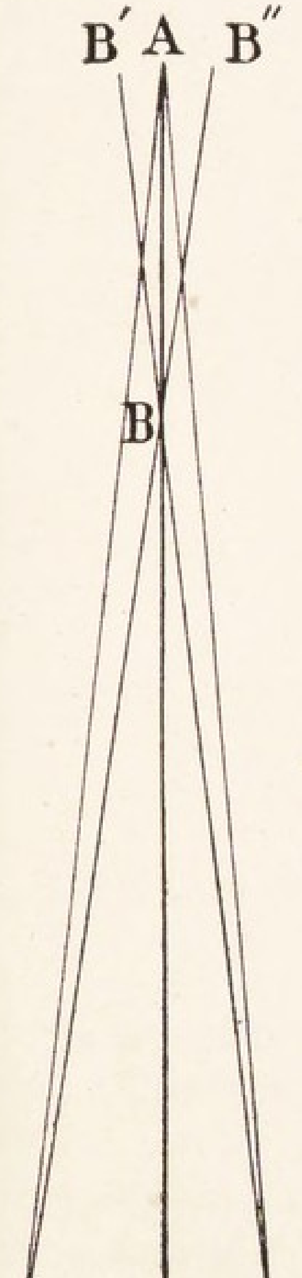


Fig. 142.—Method of parallax.

of an image formed in air, and B a trial position in which the observer places some object—e.g. a pin—he can, by slightly moving his head, tell whether B is nearer than A or not, and then adjust the trial position till no parallax occurs. B is then at A. By using a low mirror C D (Fig. 140), placing a taller pin at P, and moving a second tall pin *behind* the mirror till it appears by this test to be continuous and

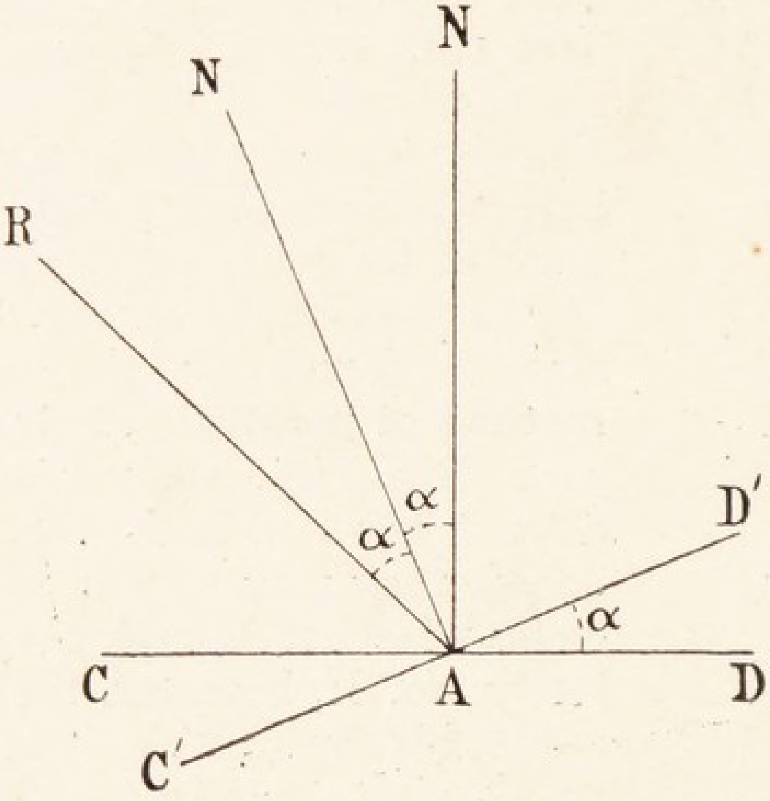


Fig. 143.—Rotation of mirror causes double rotation of the reflected ray.

coincide with the image of P, the position of M can be located. The student should certainly perform the experiment.

If the mirror be rotated through any angle, about an axis perpendicular to the plane of incidence and passing through the point of incidence, the reflected ray will be rotated through twice that angle, measured in the same direction. This important proposition may be verified experimentally by the pin method already employed,

and is easily established by the following geometrical proof:—

The ray N A (Fig. 143) incident normally on the mirror C D is reflected back along A N. When C D is turned through an angle a into the new position C′ D′, the normal must also be turned through the same angle into the new position A N′; the angle of incidence is now N A N′ $= a$, therefore the angle of reflection, N′ A R, is also $= a$, and the reflected ray

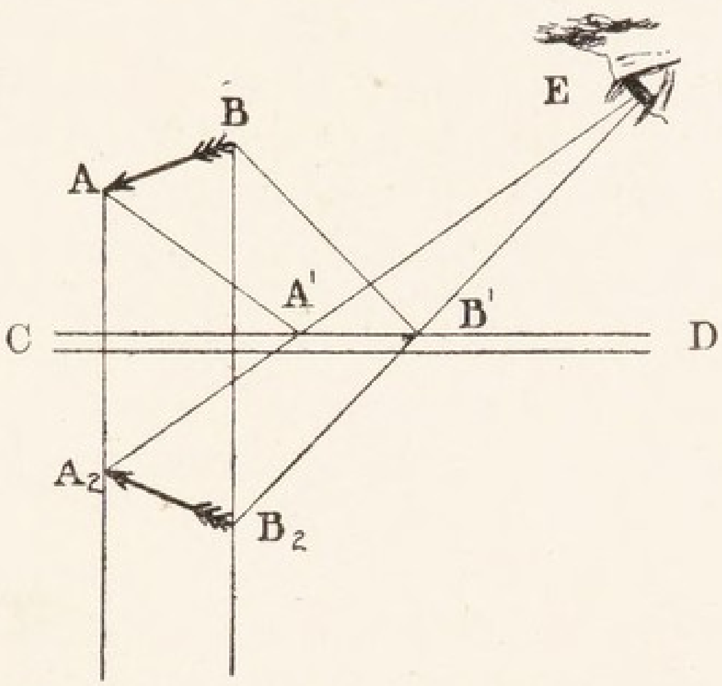


Fig. 144.—Image formed by reflection.

A N has therefore been turned through $2a$ into the new position A R.

The image of an object produced by reflection from a *plane* mirror is a *virtual* image. Rays do not really proceed to, or from, such an image. The reflected ray A T (Fig. 141) does not really come from the point where the image of H appears to be, though the straight line T A, *if produced*, would pass through this point. A virtual image, therefore, though visible to the eye, cannot be shown on a screen. Thus, rays from an arrow A B (Fig. 144) are reflected from the looking-glass C D, and enter

the eye in the direction A′ E, B′ E, and appear to an observer to come from points A_2, B_2, behind the mirror. Produce E A′, E B′, and draw normals to the mirror from A and B. Normal rays will be reflected normally (Law 2, p. 220), and the image of A will therefore appear to an observer to be at some point in A A_2, and also at some point in E A_2. The only point common to both directions is A_2, which there-

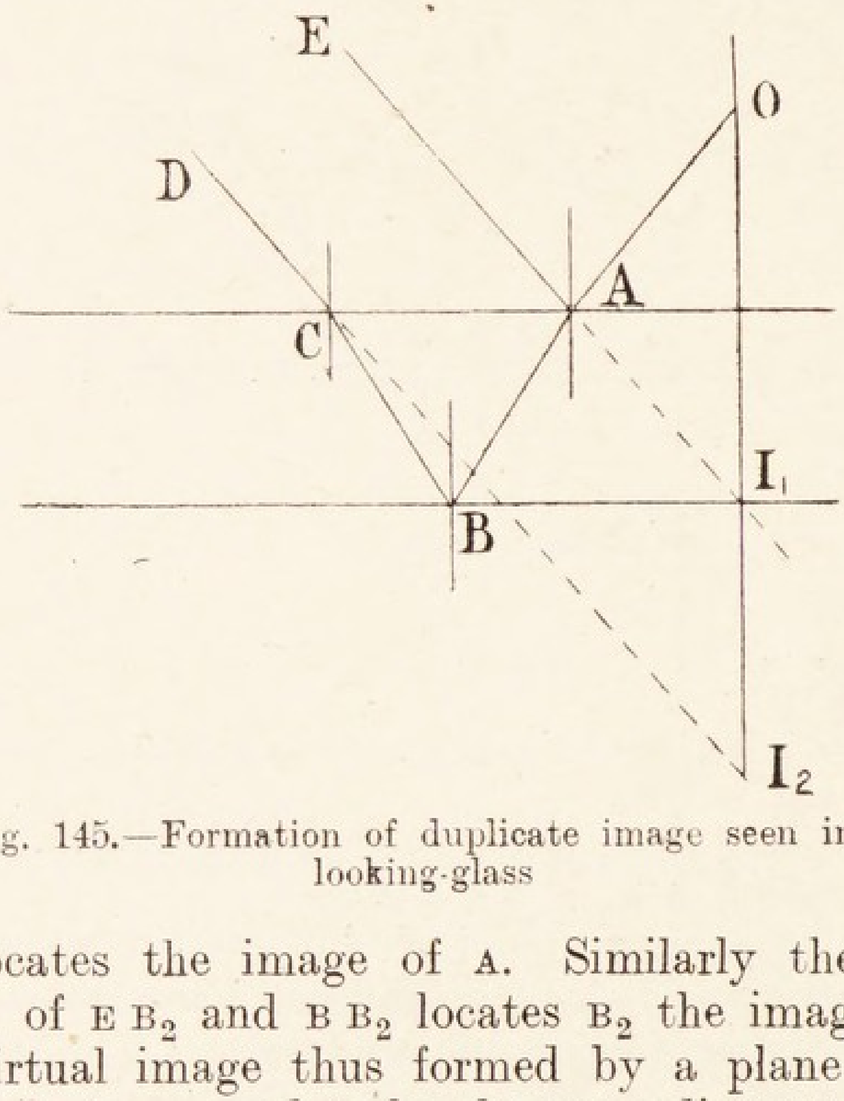


Fig. 145.—Formation of duplicate image seen in looking-glass

fore locates the image of A. Similarly the intersection of E B_2 and B B_2 locates B_2 the image of B. The virtual image thus formed by a plane mirror can easily be proved to be the same distance behind the mirror as the object is in front. This explains why when you approach, or recede from, a mirror your image appears to do the same.

When the plane mirror has appreciable thickness (Fig. 145), as in the case of an ordinary looking-glass, we often see more than one image of a single object

placed in front of the mirror. If all the light which falls on the front surface were reflected we should only see one image as before, but this is not the case. Some of the incident light from the object enters the glass, undergoing refraction (p. 240), and the refracted ray A B falling on the silvered surface at the back is again reflected to the front surface at C, where it emerges and undergoes a second refraction; it finally reaches the observer in a direction C D parallel to that pursued by the portion A E reflected from the front surface in the first instance. Although parallel in direction the rays do not coincide, but are separated by a distance which varies with the thickness of the glass, so that the two images, I_1, I_2, appear nearly side by side. As reflection may occur more than once, the journey may be repeated, and in that case we see a row of images.

When *two* mirrors are inclined to each other at any angle, and a source of light is situated between them, the rays after reflection from one mirror may fall on the other and be *again* reflected to the first, and so on. These successive reflections will give rise to a series of images. If the angle between the mirrors is $n°$, the number of images together with the object is equal to $\frac{360}{n}$. If n is 60, the number is 6. Let M C and M B (Fig. 146) be the two mirrors and A a luminous point. From the centre M describe a circle through A. The first image in the mirror M B will be at A_1, and an image of A_1 will be formed by reflection from M C at A_3, and this in its turn will form one at A_5. Now let us follow the course of a ray reflected first from M C. This will form an image at A_2; its second is A_4, and its third A_6, which coincides with A_5, the third image from the other mirror. So that any further reflections will

coincide with images already formed, and we shall see the object and five images arranged more or less symmetrically, and forming a pattern. This is the principle of the familiar *kaleidoscope.*

If the mirrors are parallel, the object and images are reflected backwards and forwards until they become too faint to be seen. This is known as the *endless gallery.*

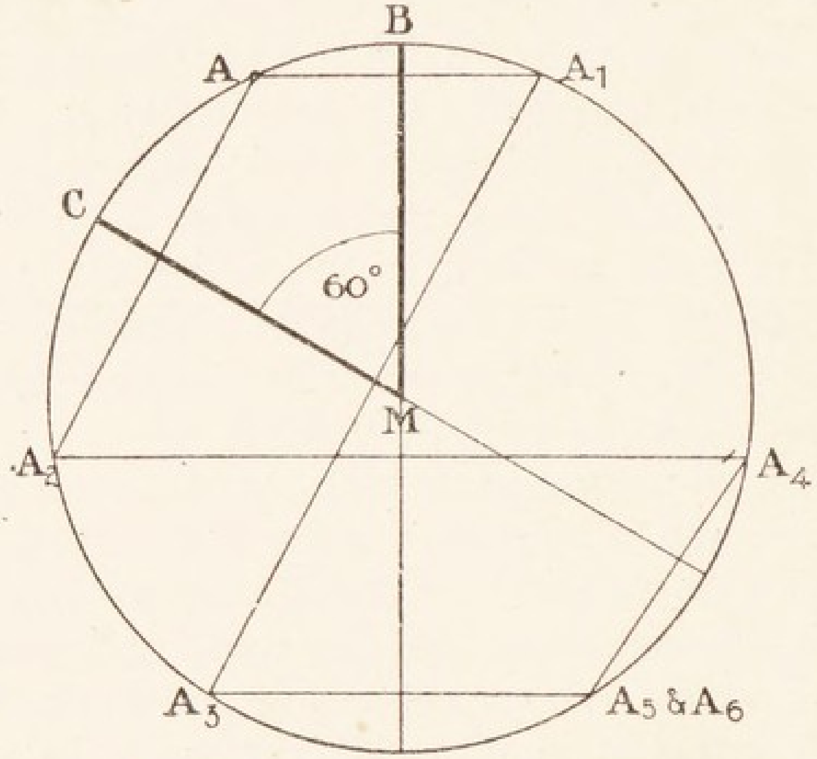


Fig. 146.—Principle of the kaleidoscope,

Formation of images by a small hole: pin-hole camera.—If, in a room with one window, the window be covered with an opaque screen in which a small pin-hole is made, a picture of the objects outside will appear on the wall opposite the pin-hole. This image is real and inverted (Fig. 147), and, in fact, photographs have been taken in this way without any lens. If a screen be held near the pin-hole, the picture will become smaller and brighter. The smaller the pin-hole the sharper the image. If two pin-holes are made, there will be two pictures;

if three, there will be three; and so on until the images overlap and obscure each other, and we lose all definition and have only a plain illuminated surface with no definite image. The same fact can be shown by replacing the front lens of a magic lantern by a brown paper cap with a pin-hole.

Mirrors, if not plane, may be either convex or concave; they may be made of polished metal or of silvered glass. Reflection in plane mirrors has already been studied. If the mirror presents a hollow surface to the incident rays it is termed concave; if the reverse, convex.

Concave mirrors.—From a point C describe a portion of a circle to represent a concave spherical mirror of which C is the *centre of curvature* (Fig. 148).

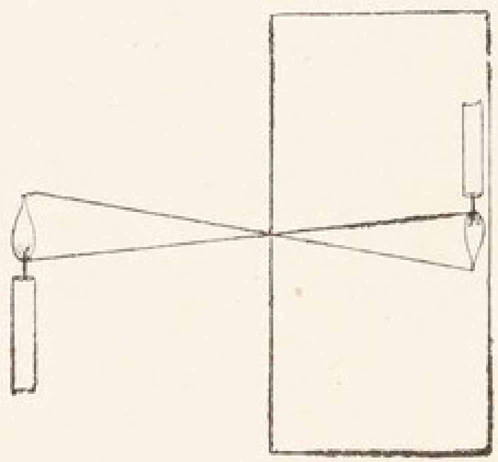


Fig. 147.—Image formed by pin-hole.

1. Parallel rays—i.e. rays which come from an infinite distance — after reflection come to a focus at a point half-way between C and the mirror. This is usually lettered F, and is called the *principal focus* of the mirror. If C F produced meet the mirror at A, C A is called the principal axis of the mirror.

2. Diverging rays from L come to a focus between F and C, as *l*. As the object L approaches the mirror, *l* recedes from it, so that they both move towards C. At C the incident ray coincides with the normal C O, and is therefore reflected on itself, and L and *l* coincide with C.

3. When L passes C, and is nearer to the mirror, *l* moves farther away till L reaches F, when the reflected rays are parallel and never meet. F is

therefore the best position for the light in a lighthouse reflector. The point *l* is then at

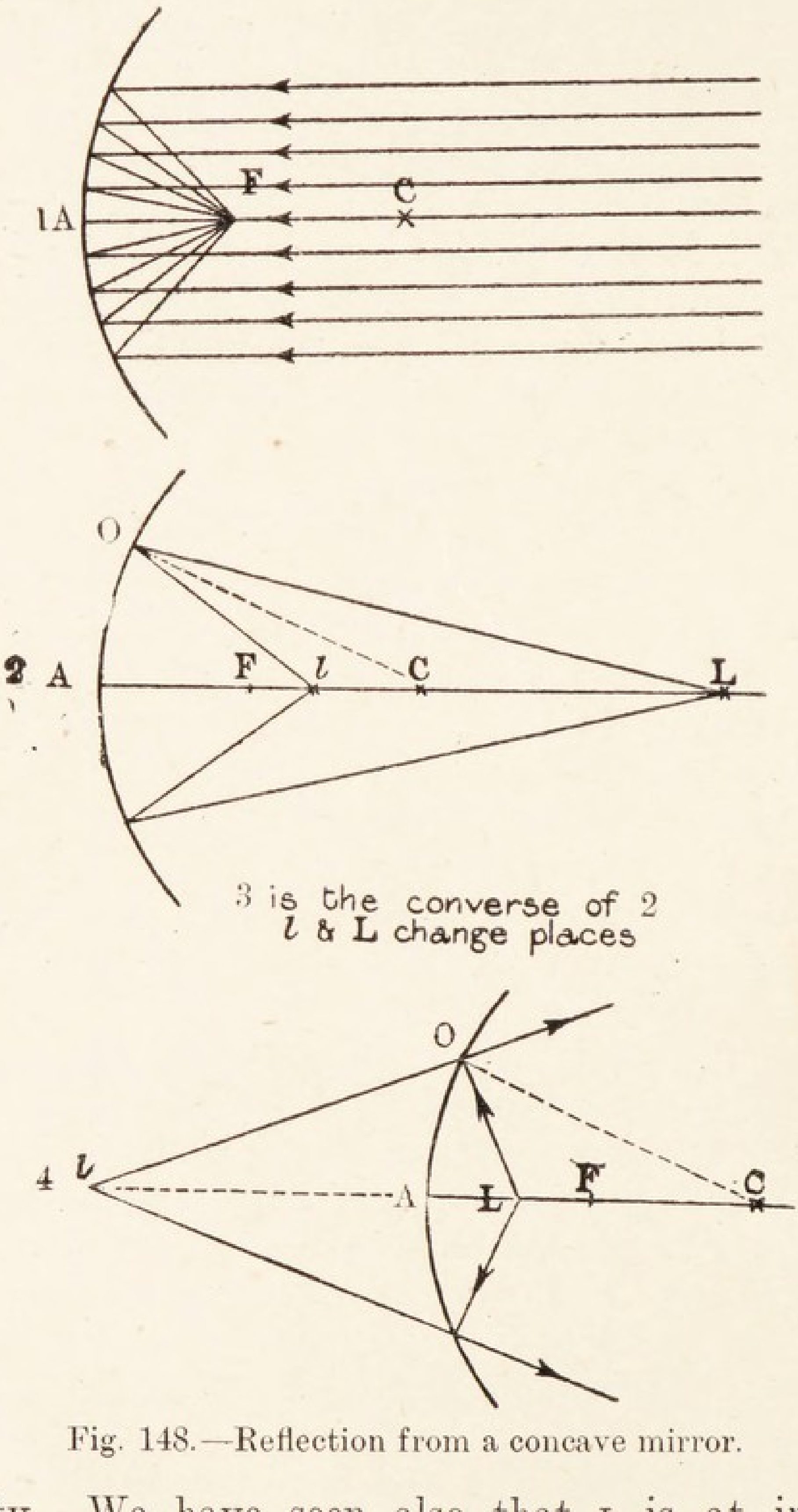


Fig. 148.—Reflection from a concave mirror.

infinity. We have seen also that L is at infinity when *l* is at F.

4. When L passes F the reflected rays diverge, and cannot therefore meet in a point; but if the directions of the rays, *after reflection,* be produced backwards, they will be found to meet behind the mirror at a point l, which is therefore the virtual focus corresponding to L. A pair of points such as L and l are termed *conjugate* foci. If the object is at either of these points the image is at the other. Their distances from the mirror are connected by the relation

$$\frac{1}{l\,A} + \frac{1}{L\,A} = \text{a constant.}$$

Convex mirrors.—These form only virtual images.

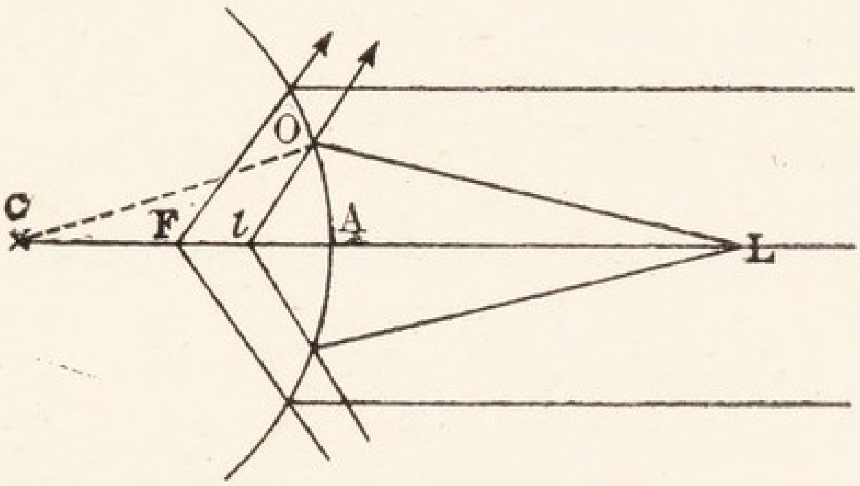


Fig. 149.—Reflection from a convex mirror.

Parallel rays are reflected from the convex surface; the reflected rays themselves cannot, of course, pass through the mirror, but their *directions,* if produced backwards, all meet in a *virtual* focus F (Fig. 149) *behind* the mirror. This is the principal focus, and is half-way between the centre of curvature and the mirror. If the object is at L, the virtual image appears to be at l between F and the mirror.

In this case the distances are connected by the relation

$$\frac{1}{l\,A} - \frac{1}{L\,A} = \text{a constant.}$$

In both cases the constant can be shown to be $\frac{2}{C\,A}$.

For a concave mirror the relation may be established by experiment (p. 414). Several positions are found giving paired values of $L A$ and $l A$, and the sum of their reciprocals is found to be practically constant. Among these pairs, particular importance attaches to two, namely

(i.) When $L A = \infty$ (infinity) and $l A = F A = f$; in this case, the relation becomes

$$\frac{1}{\infty} + \frac{1}{f} = \text{the constant},$$

$$\text{but } \frac{1}{\infty} = 0, \text{ therefore the constant is } \frac{1}{f}$$

$$\text{therefore always } \frac{1}{L A} + \frac{1}{l A} = \frac{1}{f}$$

(ii.) When $L A = l A = r$; in this case the relation becomes $\frac{1}{r} + \frac{1}{r} =$ the constant,

$$\text{therefore the constant is } \frac{2}{r}$$

This is the case when object and image coincide at C, the centre of curvature of the mirror; r is $C A$, the radius of curvature, and we see from (i.) and (ii.) that

$$\frac{1}{f} = \frac{2}{r}$$

$$\text{or } f = \frac{r}{2}$$

That is, F bisects $C A$.

The relation can also be established by geometry. Since by Law 2 (p. 220) the normal $C O$ bisects the angle $L O l$ (Fig. 148), or its supplement (Fig. 149), therefore, by Euclid VI., 3,

$$\frac{L C}{C l} = \frac{L O}{l O}$$

$$\text{or} \quad \frac{LA - CA}{CA - lA} = \frac{LA}{lA} \text{ very nearly.}$$

Put $LA = u, lA = v$, and $CA = r$, then the relation becomes

$$\frac{u - r}{r - v} = \frac{u}{v}$$

$$\therefore \; uv - rv = ur - uv$$

$$\therefore \; ur + rv = 2uv$$

divide by ruv,

$$\therefore \; \frac{1}{v} + \frac{1}{u} = \frac{2}{r}\left(= \frac{1}{f}\right).$$

Similarly the relation for a *convex* mirror may be proved to be

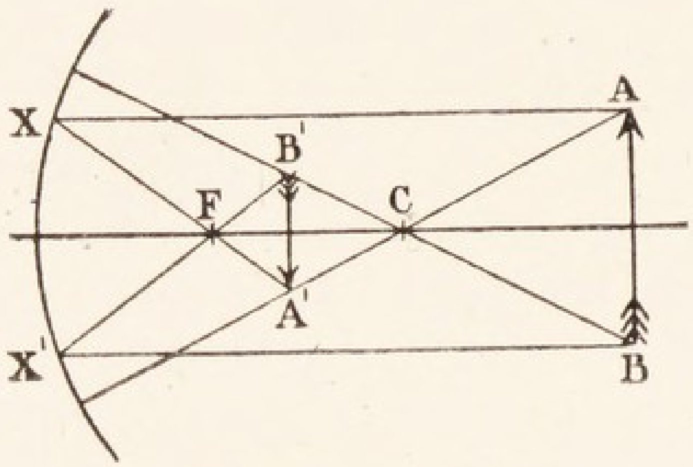
$$\frac{1}{v} - \frac{1}{u} = \frac{2}{r}\left(= \frac{1}{f}\right)$$

We have hitherto regarded the object as a luminous point, and have found its point-image. When the object is linear and finite we generally find the images of its two extremities, and by joining these obtain the image of the whole object. These constructions are of great assistance in the solution of problems, and we shall therefore consider them more fully.

Images in mirrors. *Concave mirrors.*—It will be convenient to consider four cases:

1. When the object is beyond C, as A B (Fig. 150): to find the position of the image (1) join A C and produce the line; the image will be somewhere in this line, which is normal to the mirror, and a ray in this direction is therefore reflected on itself. (2) Draw A X parallel to C F; a ray in this direction is reflected in the direction X F, and we know therefore that the image of A will be somewhere on X F, produced if necessary; it must therefore be at A′,

where the directions of A C and X F intersect. (3) Similarly, from B draw the two rays B C and B X′, whose directions, after reflection from the mirror, will intersect in B′, which is therefore the image of B. (4) Finally, join A′ B′ and obtain the image of A B. In this case we see that the image is *real*, *diminished*, and *inverted*.



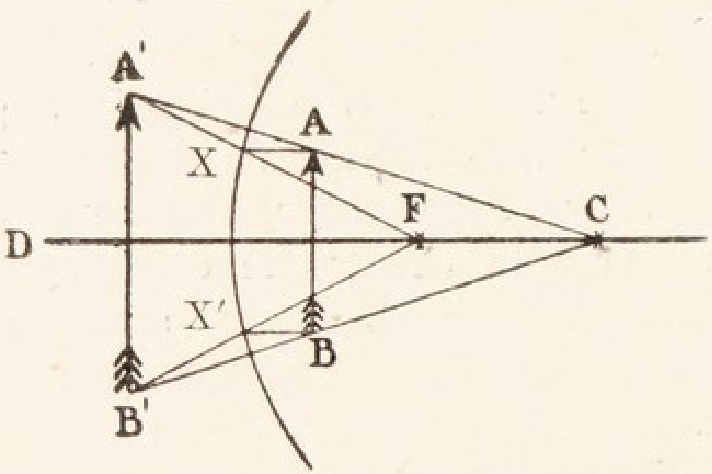


Fig. 150.—Images in concave mirror.

2. If the object is at C the object and image coincide.

3. If the object is between C and F it is the converse of 1; the object and image change places, and the image is *real*, *inverted*, and *magnified*.

4. If the object is between F and the mirror the image is *virtual* (behind the mirror), *erect*, and *magnified* (Fig. 150, D).

So if one looks into a concave mirror, at some distance, the face appears small and inverted. As the mirror is brought nearer, the image becomes larger, then suddenly vanishes, but reappears, upright and magnified, as the face approaches still nearer to the mirror.

Convex mirrors.—The image formed by a convex mirror is always virtual, diminished, and erect (Fig. 151).

In all these cases it is not difficult to prove

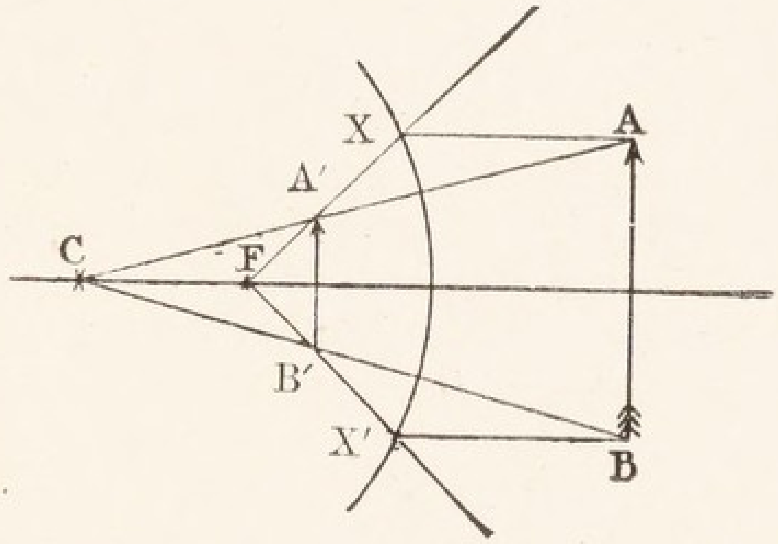


Fig. 151.—Image in convex mirror.

either graphically or by the consideration of similar triangles that

$$\frac{\text{Size of object}}{\text{Size of image}} = \frac{\text{distance of object from C}}{\text{distance of image from C}}$$

$$= \frac{u - r}{r - v}$$

$$= \frac{u}{v} \text{ (p. 233)}$$

Ex.: An object is 15 cm. in front of a concave mirror of 30 cm. focal length: find the position of the image and its size.

The object is between the mirror and F, as in Fig. 150, D. The image will therefore be virtual, erect, and magnified,

and will be formed behind the mirror. By drawing the figure to scale on squared paper the problem is readily solved; by application of the formula (p. 233), we have

$$\frac{1}{v} + \frac{1}{15} = \frac{1}{30} \therefore \frac{1}{v} = \frac{1}{30} - \frac{1}{15} = -\frac{1}{30}$$

the negative sign shows the image to be 30 cm. *behind* the mirror, and therefore *virtual;* also,

$$\frac{v}{u} = \frac{30}{15} = 2$$

The image is, therefore, twice as large as the object.

Ex.: An object 3 cm. long is 20 cm. in front of a convex mirror 12 cm. focal length: find the position of the image.

$$\frac{1}{v} - \frac{1}{20} = \frac{1}{12}$$

$$\therefore \frac{1}{v} = \frac{1}{12} + \frac{1}{20}$$

$$= \frac{8}{60}$$

$$\therefore v = \frac{60}{8} = 7{\cdot}5$$

The mirror is a convex one, and the image is therefore virtual, and is formed 7·5 cm. *behind* the mirror.

Both plane and convex mirrors are employed, either separately or in combination, in the construction of certain clinical instruments. In the **laryngoscope** a small plane mirror is attached, at an angle of about 45°, to a handle by which it can be held at the back of a patient's mouth. The plane of the mirror is so placed that, when illuminated, the observer sees in it an image of the patient's larynx. The requisite illumination is obtained from a concave mirror worn on the observer's forehead by means of a strap. This mirror reflects light from a lamp to the plane mirror, whence it is again reflected down the larynx.

The essential feature of the simplest form of **ophthalmoscope** is a concave mirror about 3 cm. in diameter, and having a central aperture about 4 mm. in diameter. The radius of curvature of this mirror is about 50 cm. The observer holds the back of this mirror in front of his own eye and looks through the aperture, into the eye of the patient, which is at the same time internally illuminated by light reflected from the mirror. A normal but magnified image of this illuminated surface is seen by the observer's eye, which is quite close to the eye examined in this *direct* method. In the *indirect* method the observer's eye and mirror are about 50 cm. from the patient's eye. A convex lens of 14 D is then held in front of the latter and at a distance of about 7 cm., the focal length of the lens. Rays from the illuminated interior of the eye will therefore on emergence pass through this lens and form a real inverted image of it between the lens and the mirror. This will be viewed by the observer through the aperture. To adapt the instrument to a variety of optical measurements, accessary apparatus is frequently added. The concave mirror is often reversible and the reverse side is a plane mirror. A second, and smaller, concave mirror is frequently added. This is 2 cm. in diameter, has a central aperture of 2 mm., and is tilted. A series of positive and negative lenses is also added to the refraction ophthalmoscope. These are so mounted that each in its turn can be rapidly brought to the central aperture, and the observer can read the number of the lens which enables him to get the direct image most clearly. From this he can diagnose and estimate the refractive error of the eye observed, after making due allowance for any known error of his own eye.

An eye may also be examined for errors of refraction

without the interposition of the convex lens, in the indirect method (p. 237). For this purpose the eye is illuminated by light reflected from the concave or plane mirror, and is viewed through the aperture by the observer placed at a distance of rather more than a metre (about arm's length).

The application of reflecting mirrors to obtain a view of an obscure interior has been widely extended in recent years, and has produced the **broncho-scope,** the **cystoscope,** and the **gastroscope.** These instruments are designed for the visual exploration of the trachea, bladder, and stomach, respectively.

Exercises

1. Light from an object at a distance of 20 cm. from a spherical mirror is reflected so as to form upon a screen an image four times the length of the object. Find the nature and focal length of the mirror, and illustrate by a diagram the formation of the image. [*First M.B.*]

2. A pin 3 cm. long is placed 48 cm. from a concave mirror and a real image is found to be produced at a distance of 16 cm. from the mirror. If the pin be moved 24 cm. towards the mirror, what changes in the position and size of the image will take place ? Give careful diagrams showing the path of the rays of light in the two cases. [*First Professional.*]

3. Two lamps of 32 and 16 candle-power are placed 100 cm. apart. Find the positions, on a line joining them, where a screen would be equally illuminated by the two lamps.

(For Answers, *see* p. 389.)

CHAPTER II

REFRACTION

Refraction of Light — Laws of Refraction — Critical Angle — Prisms — Convex and Concave Lenses — Formation of Images — Combinations of Lenses — Telescope—Microscope — Long and Short Sight — Velocity of Light—Exercises.

Refraction.—We have seen that light travels in a straight line in any medium. When, however, light passes from one medium to another the direction of this straight line is changed, except when that direction is normal to the common surface. A ray incident normally on this surface continues its rectilinear course without deviation. A ray incident obliquely on this surface instantly changes its course and assumes a new rectilinear direction. This new direction either approaches the normal more nearly than before or departs more widely from it, according as the second medium is denser or rarer than the first. When passing from a rare to a dense medium, as from air to water, the ray is always bent *towards* the normal; when passing from a dense to a rare medium, as from glass to air, the

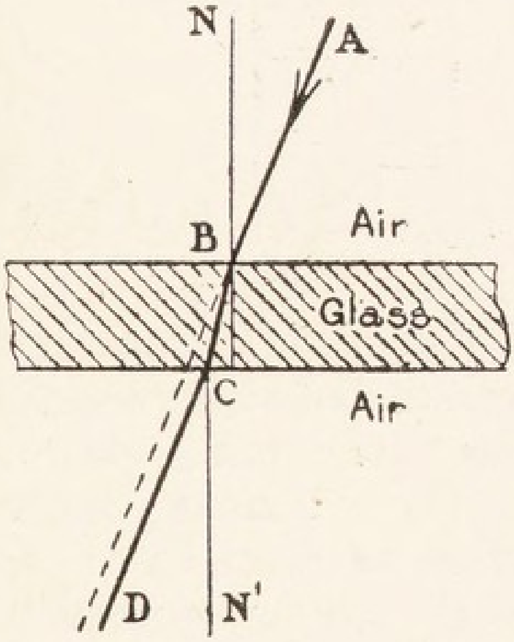


Fig. 152.—Course of refracted ray.

ray is always bent away from the normal. This change of course or bending of the ray is called refraction (*re*, back, *fraction*, broken). If the second medium is a stratum bounded by parallel surfaces, the direction of the ray after emergence is parallel to its direction before incidence.

Thus a ray A B (Fig. 152) strikes the parallel-sided piece of glass at B. Instead of continuing its course along the dotted line, the ray is refracted towards the normal N B and pursues the new direction B C. On emerging from the glass it is bent away from the normal C N in the direction C D parallel to A B.

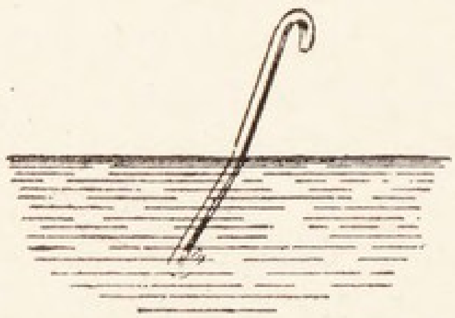


Fig. 153.—Bent appearance of stick under water.

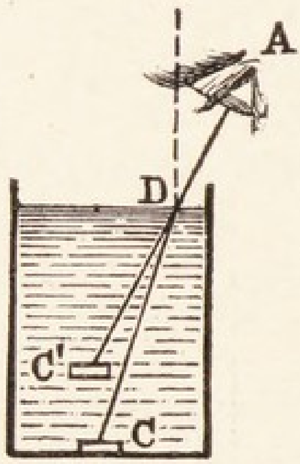


Fig. 154.—Coin under water.

It is owing to refraction that the blade of an oar or a portion of a stick when seen under water seems bent (Fig. 153). A coin or fish under water appears nearer to the surface than it really is; clear ponds appear to be only three-quarters as deep as they really are. If a coin be placed in an empty white jam-pot at C (Fig. 154) it cannot be seen by an eye at A because the view is blocked by the top of the pot. If water be poured in, the coin will become v sible, for the rays from C when they reach the surface at D are bent away from the perpendicular, so that they reach the eye in the direction D A, and the coin is seen at some point, C′, in A D produced.

Laws of refraction.—The direction of the refracted ray is determined by two laws: (1) The incident ray, the refracted ray, and the normal to the surface at the point of incidence are in the same plane; (2) the sine of the angle of incidence bears to the sine of the angle of refraction a ratio which is constant for any two media.

Let A B (Fig. 155) be the direction of a ray travelling in air and incident at B on the surface of a denser medium. The ray will be bent towards the normal, and pursue the course B C. From the centre B describe a circle cutting B A at D and B C at E; draw D N and E M perpendicular to the normal N B M. Then (1) A B, B C, and N M are in the same plane; and (2) the ratio $\frac{\text{N D}}{\text{E M}}$ is constant for any two media, whatever the angle of incidence A B N may be.

The second law of refraction is sometimes known as *Snell's law of sines*.

This ratio $\frac{\text{sine of angle of incidence}}{\text{sine of angle of refraction}}$ is called the relative *refractive index* from the first medium to the second, and is usually indicated by the Greek letter μ (*mu*). When the air is replaced by a vacuum the ratio is slightly increased, and becomes the *absolute index of refraction* for the second medium. If μ_1 and μ_2 are the absolute indices of two media, it can be shown that the relative index μ from (1) to (2) $= \frac{\mu_2}{\mu_1}$.

If the ray proceeds from a rare to a dense medium μ is greater than 1; from a dense to a rare medium μ is less than 1. Thus, air to water, $\mu = \frac{4}{3}$; air to glass, $\frac{3}{2}$.

This law can be verified thus: A rectangular block

of glass, such as is used for a letter-weight or a cutting shape, C G F D (Fig. 156), is laid on a sheet of white paper; a pin placed at Y is viewed through the glass and appears to be at Z. Insert another pin at P, so that P A Z seem to be in a line. Then draw the line G F and remove the glass block; through A draw the normal N N′, join A with P and with Y; from A

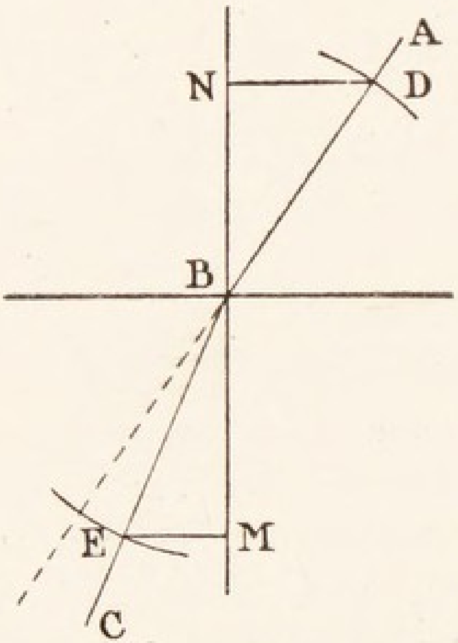


Fig. 155.—Snell's law.

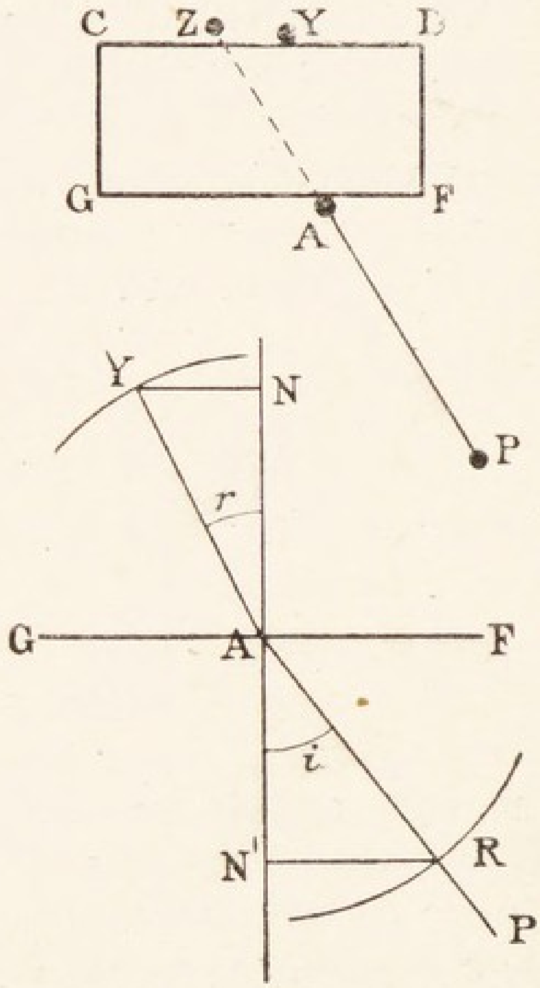


Fig. 156.—Proof of Snell's law.

describe a circle cutting A P at R and A Y at Y; drop the perpendiculars R N′ and N Y. P A Y would be the course of a ray travelling from air to glass, and for this ray N′ A R is the angle of incidence and N A Y the angle of refraction, and

$$\mu = \frac{\text{sine } i}{\text{sine } r} = \frac{N' R}{N\ Y} = \text{(in one case) } \frac{15}{10} \text{ cm.} = \frac{3}{2}$$

To draw the refracted ray.—The following

geometrical construction will enable us to determine the direction taken by a given incident ray, after refraction, when μ, the index of refraction, is known. Let the ray O A (Fig. 157) be incident at A on the surface C D. At the point A draw A N normal to this surface. On O A take *any* point O as centre,

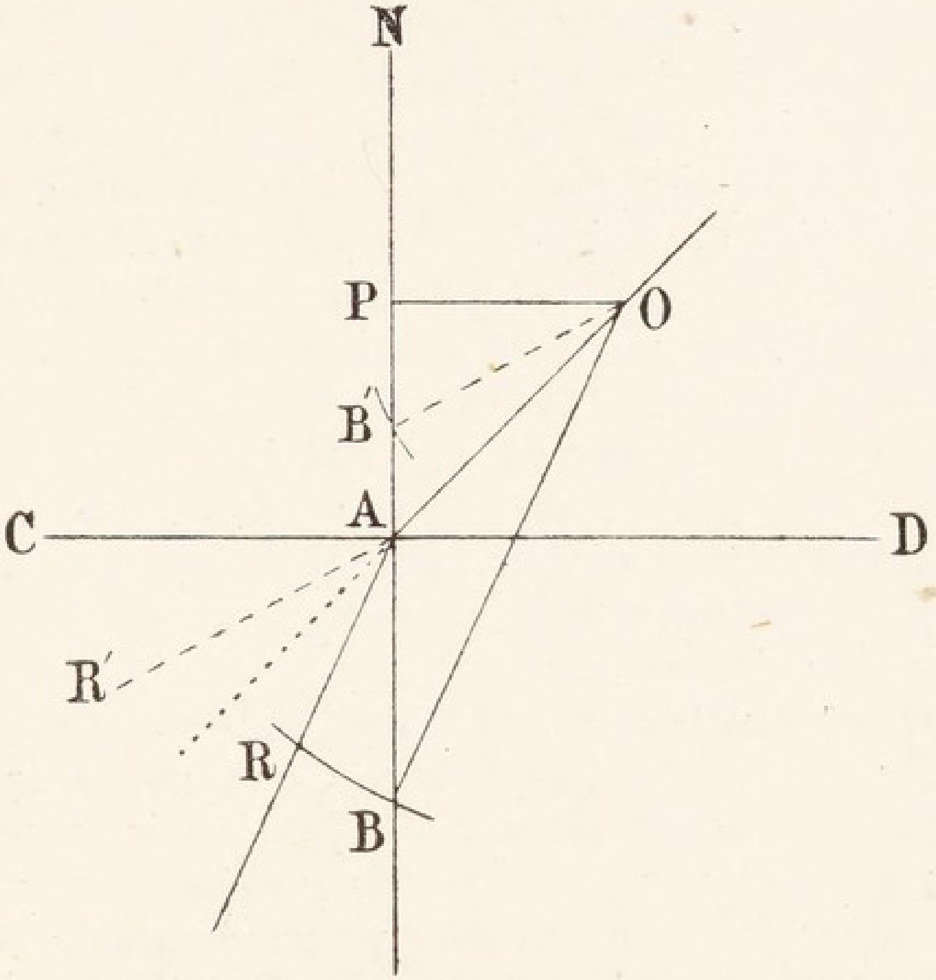


Fig. 157.—Construction of the refracted ray.

and with radius μ O A describe a circle cutting the normal in B. Join O B. Draw A R parallel to O B; this will be the direction of the ray O A after refraction. Draw O P perpendicular to A N.

Proof :—

$$\frac{\sin. P A O}{\sin. R A B} = \frac{\sin. P A O}{\sin. P B O} = \frac{\dfrac{O P}{O A}}{\dfrac{O P}{O B}}$$

$$= \frac{OB}{OA} = \frac{\mu\, OA}{OA} = \mu$$

and PAO is the angle of incidence
$\therefore$ RAB is the angle of refraction
$\therefore$ AR is the refracted ray.

We have assumed in the above case that the ray passes from the rarer medium to the denser, but the construction is similar if the incident ray is in the denser medium; in that case the radius O B is *less* than O A, and will therefore cut the normal *above* C D, as at B′. If we join O B′, and draw A R′ parallel to O B′, we obtain the refracted ray as before. There is a special case when the radius is *equal* to O P; in that case the circle *touches* A N at P, and A C is the direction of the refracted ray. If the radius is *less* than O P, the construction fails because in this case there is no refracted ray, but the incident ray is *internally reflected.* This phenomenon will now be further considered.

Critical angle.—We have seen that a ray incident normally on the surface between two media is not refracted. Thus the ray A O A′ (Fig. 158), in emerging at O in a direction normal to the surface D′ F, suffers no deviation. When we look perpendicularly down at a stone in a pond, we see it in its true direction. A ray B O (Fig. 158) incident in the denser medium (water) will, on emerging at O into the rarer medium (air), be refracted *away* from the normal and take a direction O B′ such that the angle A′ O B′ is *greater* than the angle A O B. Similarly, a ray C O will be refracted along O C′. Finally, a limiting position will be reached when the ray D O will be refracted along O D′ at right angles to the normal O A′. It clearly cannot be more refracted than this. Any ray E O. making an angle *greater* than A O D

with the normal O A, will not emerge at all, but is totally reflected from the under surface F D′ to E′ and would be seen by an eye under water as in a looking-glass. A bright metal spoon in a glass of water, held a little above the observer's eye, shows this image very well. An empty test-tube held under water and seen from above looks as if it were silvered, and small solid objects placed in the tube become invisible as if they were behind a looking-

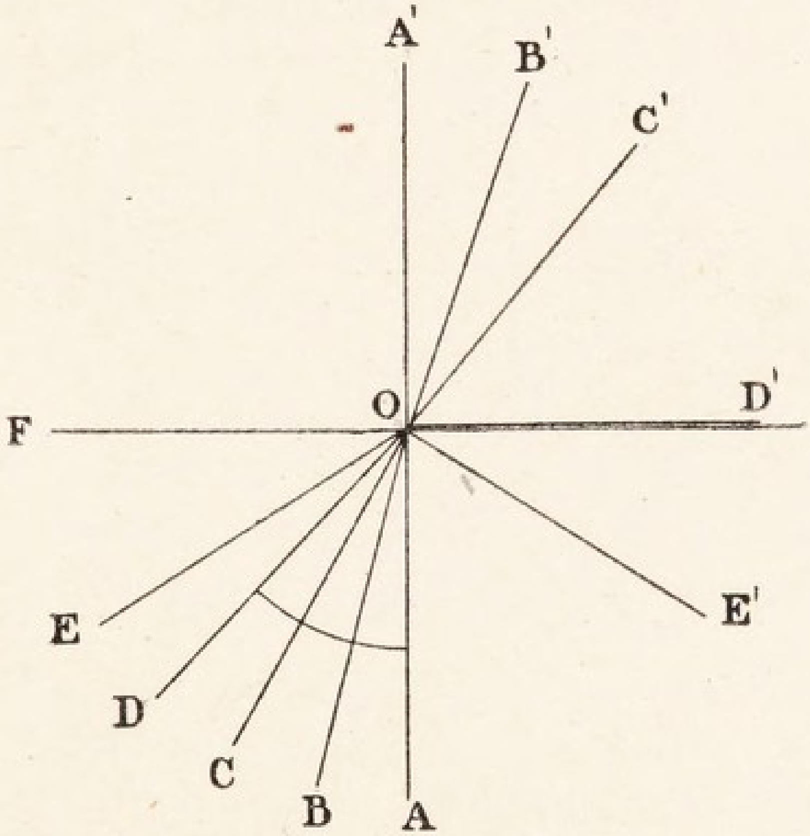


Fig. 158.—Critical angle.

glass. The light passing downwards through the water is totally reflected from the surface of the test-tube on which it falls. This total reflection is very perfect and brilliant, as practically no light is lost. The phenomenon can evidently only occur when light is passing from the denser medium to the rarer. The angle D O A at which total reflection just begins is called the *critical angle*, because if the angle of incidence is greater than the critical angle the ray

is totally reflected, and if less the ray is refracted in the ordinary way.

The critical angle, water to air, = 48° 30′; glass to air, 41° 75′; diamond to air, 23° 41′. The relation between the refractive index μ and the critical angle (δ) is $\mu = \frac{\sin 90°}{\sin \delta} = \frac{1}{\sin \delta}$ or $\sin \delta = \frac{1}{\mu}$. An object at A (Fig. 159) is seen at A′ by light which has been totally reflected in the glass prism, where the angle of incidence is 45° and therefore exceeds the critical angle.

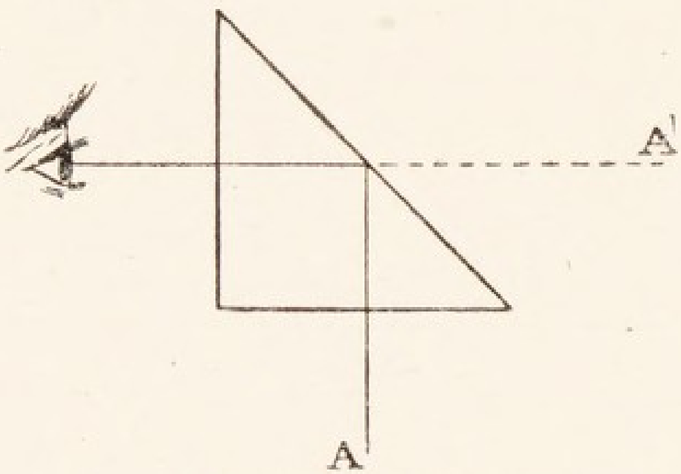


Fig. 159.—Total reflection.

The *mirage* can be explained as the result of the combined action of refraction and total internal reflection. It appears *in the air* when the lowest layers of the air are cooler than those above, and *below the ground level* if the air is hottest near the surface, as in Fig. 160, which gives a diagram of the formation of the image of a tree in a hot sandy desert. If we trace the course of a ray from a point O as it passes from the dense to the rarer air, it is bent more and more from the normal until it passes the critical angle, when it is internally reflected and reaches the eye in the direction O′ A, so that the tree appears inverted at O′. In all cases of mirage there must be comparatively still layers of air of different densities.

Prisms.—If we look through a glass prism at an object P (Fig. 161), it appears to be at P′. The rays from P striking the prism are bent towards the normal. On emerging into the air they are again

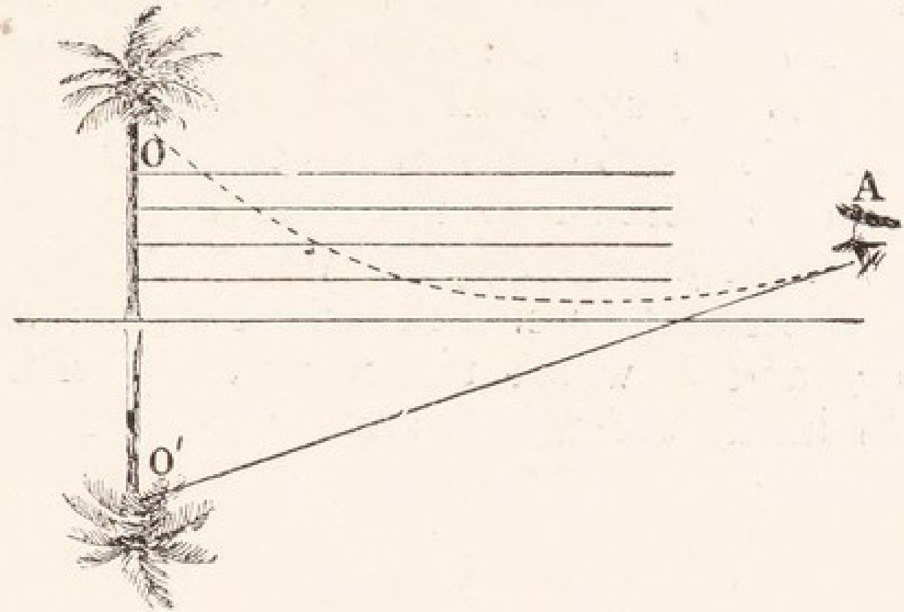


Fig. 160.—Mirage.

refracted, and enter the eye in the direction P′ E. By gently turning the prism the image P′ will be seen to move farther away, and then (still turning the

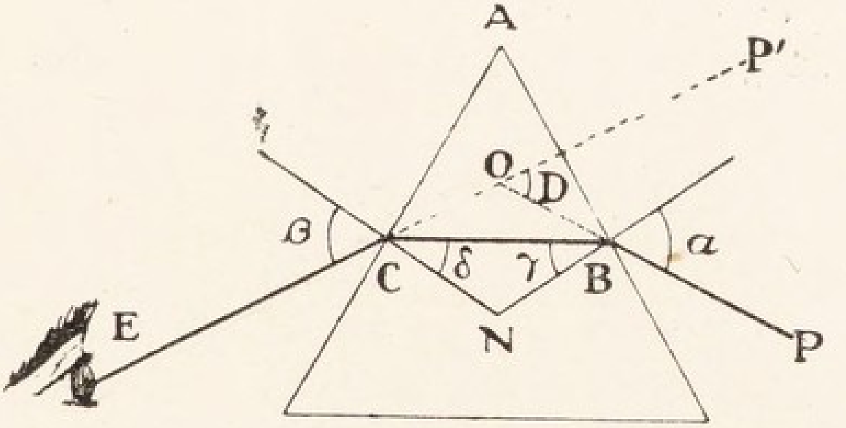


Fig. 161.—Refraction through a prism.

prism in the same direction) come back again. The position in which the image P′ and the object P are nearest to each other is termed the position of *minimum deviation.* The angle P O P′ is the angle

of deviation of the ray from its original direction P O. The deviation (D) depends upon the angle (A) of the prism and also upon the index of refraction (μ) of the substance of which the prism is made. It can be proved that in general, when the incident angle is small, $D = (\mu - 1) A$. The position of minimum deviation is the one most often employed in practice; in this position the ray B C, passing through the glass, is parallel to the base of the isosceles prism and the angles α and β, which the ray makes with the normals N B, N C, in air, are equal; also the angles γ and δ, which the ray makes with these normals inside the prism, are equal. When D has this minimum value, it can be proved that

$$\mu \sin \frac{A}{2} = \sin \frac{A + D}{2}$$

The following is a simple geometrical proof of these important relations:—

$$\begin{aligned} D &= \text{angle } OCB + \text{angle } OBC \text{ (Eucl. I., 32)} \\ &= (\beta - \delta) + (\alpha - \gamma) \\ &= (\beta + \alpha) - (\delta + \gamma) \end{aligned}$$

The angles $A C N$, $A B N$ are right angles

$\therefore$ the angles $C A B$, $C N B$ equal two right angles

$$\therefore \delta + \gamma = C A B = A \text{ (Eucl. I., 32)}$$

$$\therefore D = (\beta + \alpha) - A$$

also

$$\frac{\sin \alpha}{\sin \gamma} = \mu = \frac{\sin \beta}{\sin \delta}$$

$\therefore$ when the angles are *small* (p. 16)

$$\frac{\alpha}{\gamma} = \mu = \frac{\beta}{\delta}$$

$$\text{or } \alpha = \mu \gamma \text{ and } \beta = \mu \delta$$

$$\therefore \alpha + \beta = \mu(\gamma + \delta) = \mu A$$

$$\text{and } D = (\mu - 1) A$$

In the special case of minimum deviation we have $\alpha = \beta$ and $\gamma = \delta$;

$$\therefore \alpha = \frac{\alpha + \beta}{2} = \frac{A + D}{2}$$

$$\text{and} \quad \gamma = \frac{\gamma + \delta}{2} = \frac{A}{2}$$

$$\therefore \mu = \frac{\sin \alpha}{\sin \gamma} = \frac{\sin \dfrac{A + D}{2}}{\sin \dfrac{A}{2}}$$

By this formula the index of refraction of any substance which can be cut into a prism of known angle can be determined.

Lenses.—If two similar triangular prisms be placed base to base, the rays from a distant source

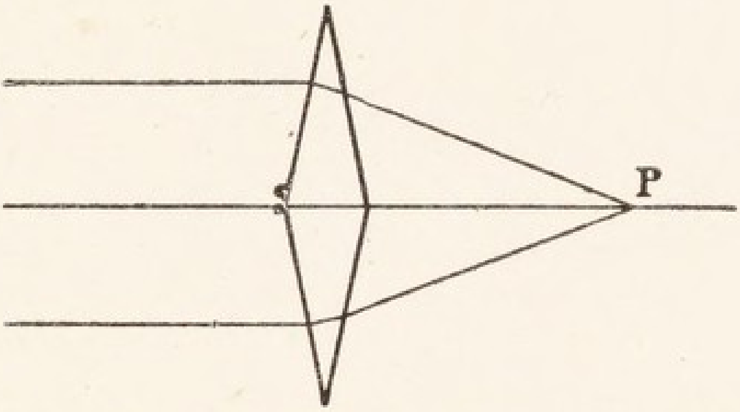


Fig. 162.—Parallel rays and double prism.

of light will be refracted, and converge to a point P (Fig. 162). If the surfaces be ground into curves we have a convex lens touched at four points by the two prisms. If the prisms be inverted so that their *thin* ends meet, the arrangement is equivalent to a diverging or *concave* lens.

Lenses are broadly distinguished as *convex* or *concave*, but several varieties of each class are recognized (Fig. 163). In the convex group the lens is thickest at the centre; in the concave group the greatest thickness is at the edges.

Optical centre of a lens.—If C_1 and C_2 (Fig. 167)

be the centres of curvature of the two faces of a lens, the straight line $C_1 C_2$, produced both ways, is the *principal axis* of the lens. A point, Q, can be found in $C_1 C_2$ such that $\frac{C_1 Q}{C_2 Q} = \frac{r_1}{r_2}$, where r_1 is the radius of curvature of the circle whose centre is C_1 and r_2 of that whose centre is C_2. This point, Q, is the optical centre of the lens. All rays which traverse the lens and pass through Q emerge in directions parallel to those which they had before entering the lens. Neglecting

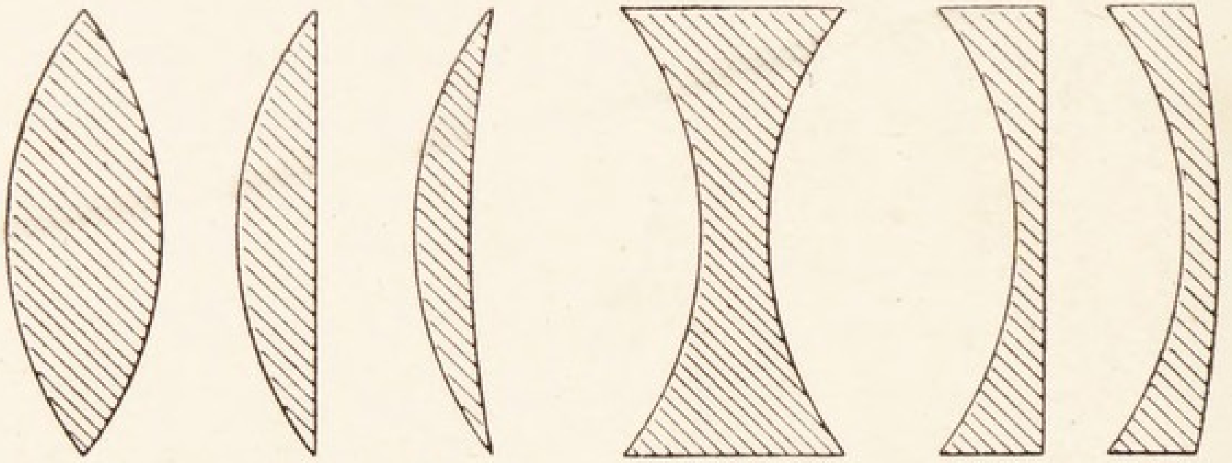


Fig. 163.—Lenses.

the thickness of the lens, we regard their course as straight.

Double convex lens.—1. Rays from a point at an infinite distance form a parallel pencil and converge after refraction to a point F on the principal axis (Fig. 164), termed the *principal focus*. The distance between the lens and the principal focus is called the *focal length* of the lens.

2. If the object O moves nearer, but is still beyond F, the rays come to a conjugate focus at F′ (Fig. 164, 2).

3. If the object is at F, the rays emerge parallel (the converse of 1)

4. If the object is between F and the lens, only a virtual focus is formed at F′ (Fig. 164, 4).

Images formed by convex lenses.—There is a close general resemblance between the formation of images by a convex lens and by a concave mirror (p. 233), though the former is due to *refraction*, the

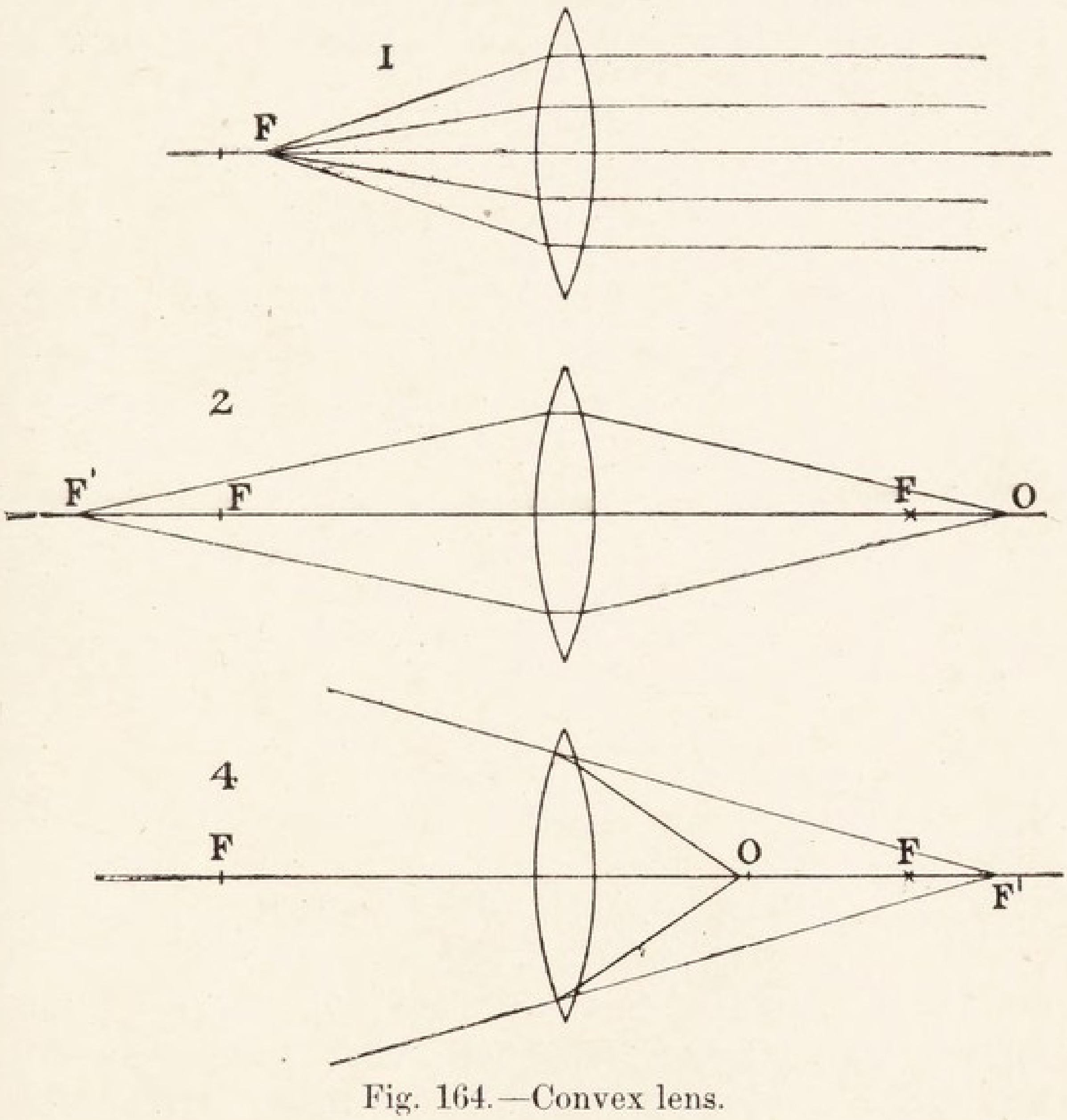


Fig. 164.—Convex lens.

latter to *reflection*. The position of the image can be found by a construction very similar to that employed before (p. 233).

1. When the object is beyond the principal focus, as the arrow A B (Fig. 165, 1), the position of the

image is found as follows: (*a*) Draw lines from A and B through C, the (optical) centre of the lens: the images of A and B will be somewhere on A C and B C produced, since rays through C undergo no deviation. (*b*) Through A and B draw lines parallel to the principal axis to cut the lens at x and x'. Since parallel rays, after refraction, pass through F, the images of A and B will be somewhere on x F

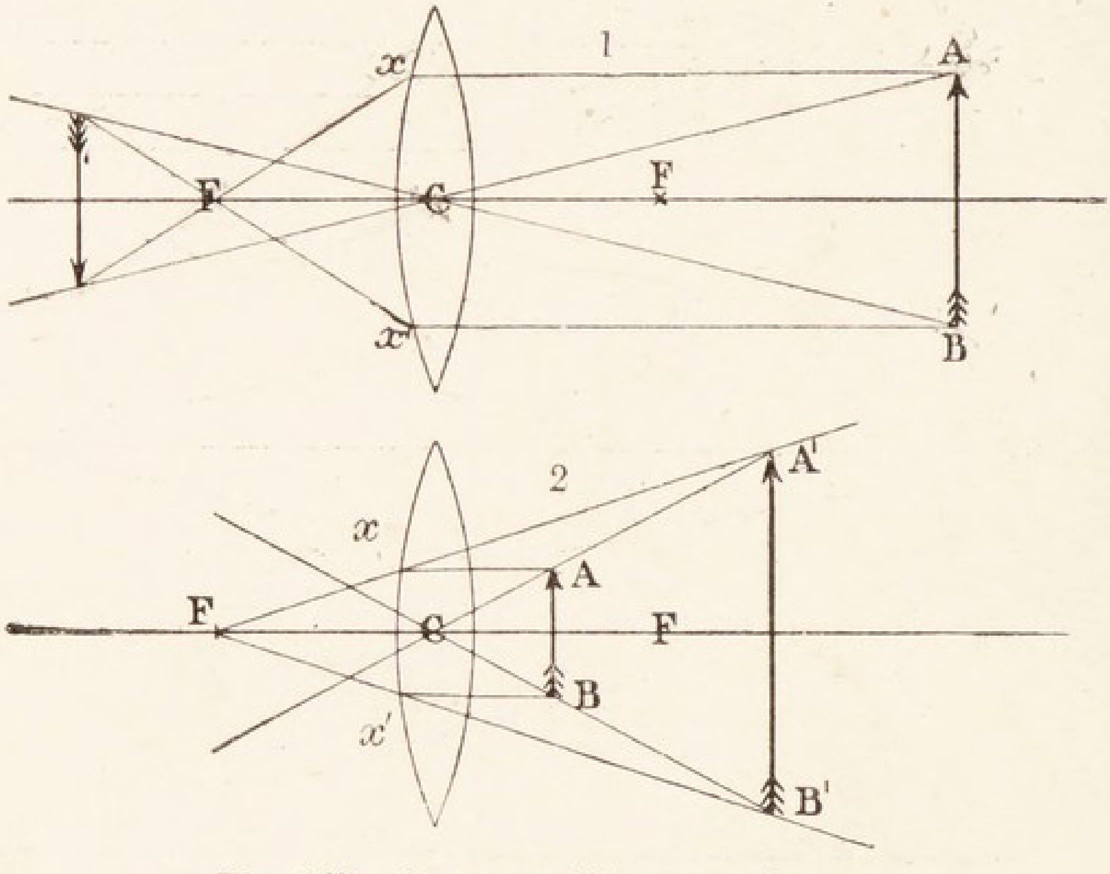


Fig. 165.—Images with convex lens.

and x' F, produced; the image will therefore be formed where these two lines, when produced, cut A C and B C, also produced. The image is real and inverted (camera lens).

2. When the object is between F and the lens, we make a similar construction (Fig. 165, 2). In this case, however, x F and A C would not meet if produced through F and C. If produced backwards they meet in A′, which would therefore be the virtual

image of A, seen by an observer so placed that his eye receives the rays x F, A C. Similarly, B′ will be the virtual image of B, to an eye which receives the rays x' F, B C. The image A′ B′ is erect, magnified, and virtual. Such an image is seen when the lens is used as a magnifying glass.

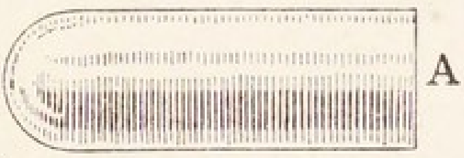


Fig. 166.—Stanhope lens.

The *Stanhope lens* consists of a piece of glass rod, one end of which is ground flat and polished, while the other has a convex surface forming a lens whose focal length is such that objects held on the flat face are in the focus of the lens. It is often used for magnifying small photographs cemented on the flat surface A (Fig. 166).

If the constructions of Fig. 165 are drawn to scale, graphical solutions of problems relating to convex lenses can be deduced from them. The distances of the object and image from the lens are, however, connected by the relation

$$\frac{1}{u} + \frac{1}{v} = \text{a constant.}$$

This relation can be verified by experiment for several positions of object and image with the apparatus of Fig. 175. But we know that when $u = \infty$, $v = f$; we must therefore have

$$\frac{1}{\infty} + \frac{1}{f} = \text{this constant;}$$

but $\frac{1}{\infty} = o$, $\therefore$ the constant $= \frac{1}{f}$, and we must have, *always*,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

This important formula may also be deduced from

the formula $D = (\mu - 1) A$ (p. 248). If P B C E (Fig. 167) be the course of a ray through the convex lens, and we draw tangent planes at B, C, meeting in A, we have then the prism of Fig. 161 ; the normals at B, C are now the radii of curvature r_1, r_2 ; P B $= u$, E C $= v$; from B, C draw perpendiculars x_1, x_2, to

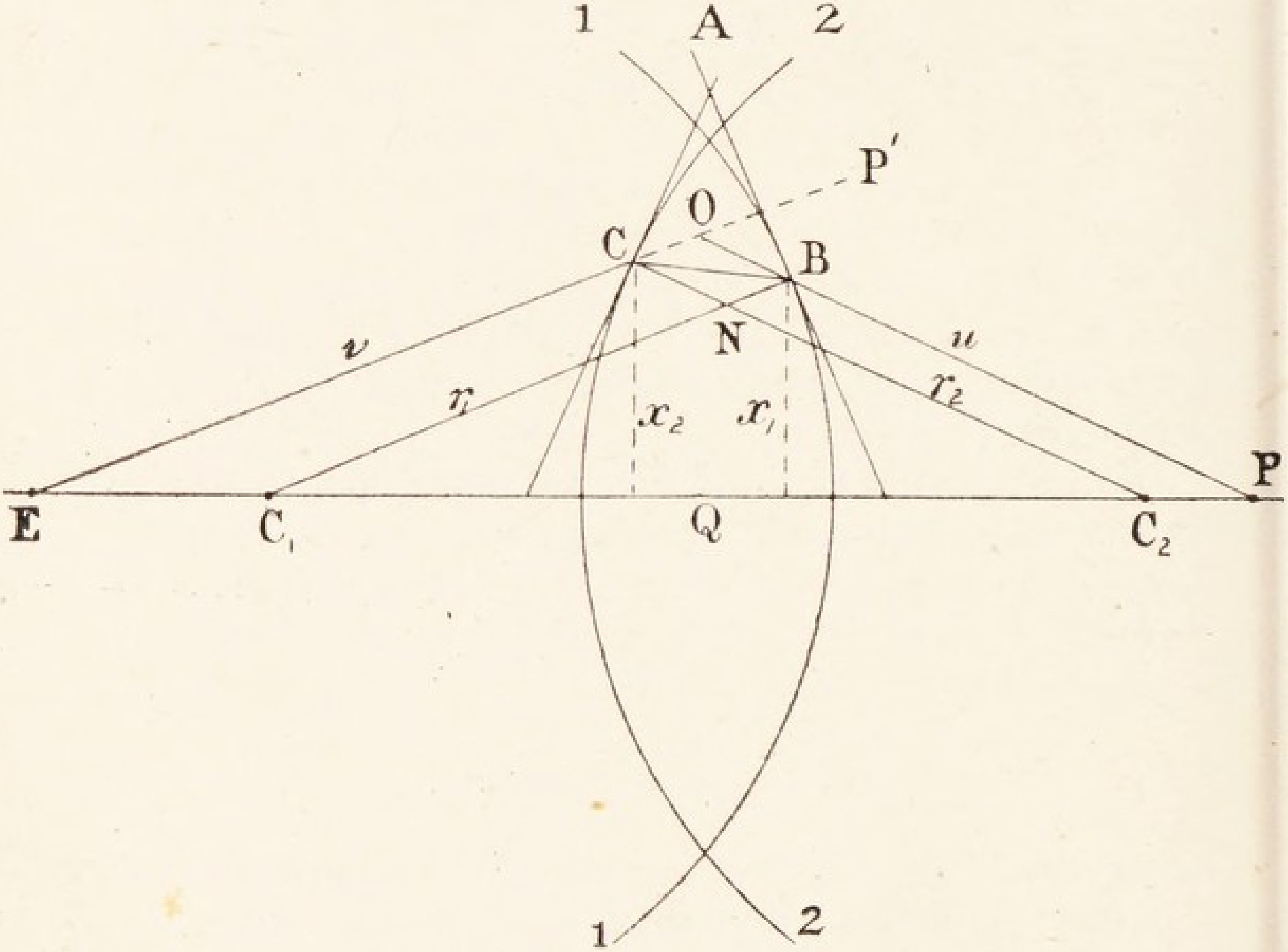


Fig. 167.—Geometry of refraction by convex lens.

the principal axis of the lens. We know (p. 247) that

$$
\begin{aligned}
D &= \text{angle } P\,O\,P' \\
&= O\,E\,P + O\,P\,E \text{ (Eucl. I., 32)} \\
&= \sin O\,E\,P + \sin O\,P\,E \text{—if the angles are small (p. 16)} \\
&= \frac{x_2}{v} + \frac{x_1}{u}
\end{aligned}
$$

We also proved that

$$\begin{aligned} A &= \text{angle } N\,C\,B + \text{angle } N\,B\,C \\ &= C_2\,N\,B \text{ (Eucl. I., 32)} \\ &= N\,C_1\,C_2 + N\,C_2\,C_1 \\ &= \frac{x_1}{r_1} + \frac{x_2}{r_2} \end{aligned}$$

$$\therefore \quad \frac{x_2}{v} + \frac{x_1}{u} = (\mu - 1)\left(\frac{x_1}{r_1} + \frac{x_2}{r_2}\right)$$

But if the lens is very thin, so that B C is practically parallel to the principal axis, x_1 will be equal to x_2, and we can divide through by this factor and so obtain

$$\frac{1}{v} + \frac{1}{u} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

= a constant which depends only on the curvature and index of refraction of the lens.

As before, if, when u is ∞, we denote the corresponding value of v by f, we now obtain

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Concave lens.—In this case the corresponding construction leads to the result

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{1}{f}$$

$$\text{or} \quad \frac{1}{u} - \frac{1}{v} = \frac{1}{-f}$$

if we give the negative sign to f. No real image is formed by a concave lens; the image is always virtual and erect (Fig. 168).

Combinations of lenses. — The **compound microscope** consists essentially of two convex lenses: (1) a very powerful, but small, lens, termed the *object glass*, or *objective*, O (Fig. 169), which forms an inverted magnified and real image at A, the object

a b being placed close to the objective; (2) a second convex lens E, termed the *eye-piece*, which acts as a magnifying glass and produces at B B a more enlarged and virtual image of the real image A.

The *magnifying power* is practically determined by

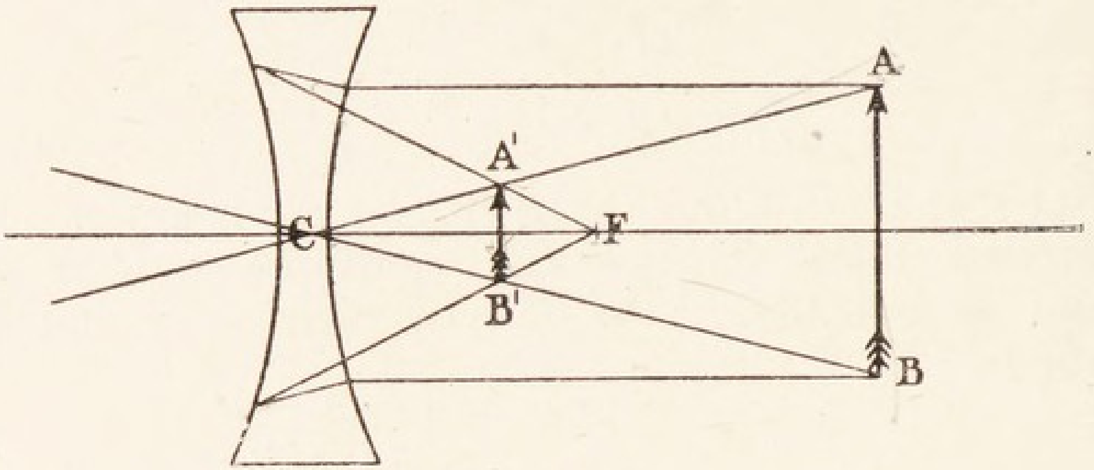


Fig. 168.—Image in concave lens.

viewing with one eye a very finely divided scale (*stage micrometer*) through the microscope and comparing this with an ordinary scale (held at the distance of distinct vision, about 10 in.) seen by the other eye.

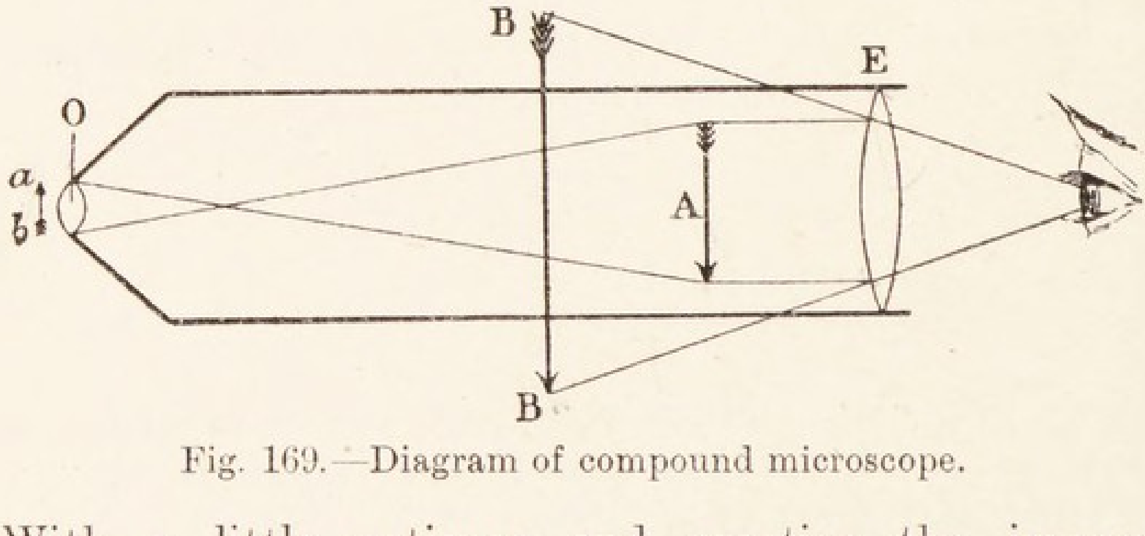


Fig. 169.—Diagram of compound microscope.

With a little patience and practice the images seen by the two eyes can be made to overlap and can thus be compared. If $\frac{1}{100}$ millimetre, when viewed through the microscope, appears to be of the same length as 9 millimetres seen by the un-

aided eye, the magnifying power is as $\frac{1}{100}$: 9, or as 1 : 900. This, in fact, gives the ratio of B B to *a b* (Fig. 169).

Astronomical telescope.—This consists, like the compound microscope, of two convex lenses. The object glass O (Fig. 170) forms an inverted real image, which the eye-piece magnifies, forming a virtual image. The difference between the two instruments is that, as the objects—sun, moon, etc.—viewed by the telescope cannot be brought near the object glass, the entire magnification is performed by the eye-piece. In theory the magnifying power of the telescope is represented by the ratio, focal

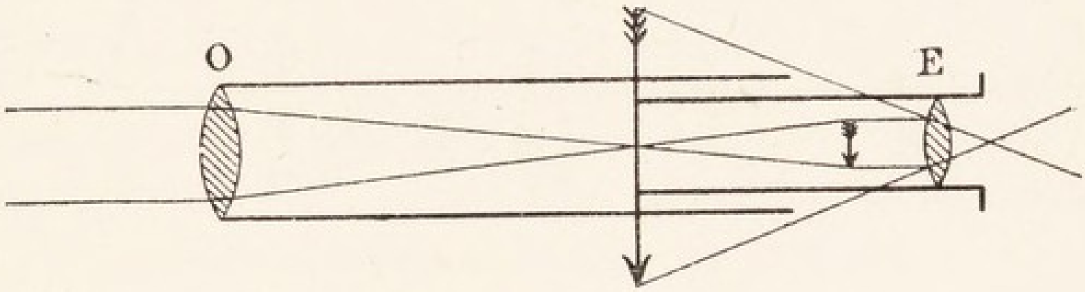


Fig. 170.—Astronomical telescope.

length of field lens : focal length of eye lens. It may also be experimentally determined by viewing divisions on a distant scale which can be seen both with the telescope and without it. Viewed in this way simultaneously with different eyes, direct comparison shows that one magnified division is equal to n unmagnified divisions ; the instrument therefore magnifies n times.

As the rays from the circumference of a lens do not come to a focus in quite the same spot as those from the centre, it is usual, both in the microscope and the telescope, to insert behind the eye-piece a thin circular plate of metal with a circular hole, called a *stop* or *diaphragm ;* this cuts off the outer

rays and gives a sharper image, although some light is lost. In the microscope there is also an adjustable stop to the objective.

When the telescope is employed to view terrestrial objects, and it is therefore desired to obtain an erect image, two additional convex lenses, R, R (Fig. 171), are introduced between the object glass and the eyepiece, so that the image is re-inverted and becomes erect. These extra lenses do not magnify, and the image loses somewhat in brightness owing to the slight loss of light in passing through two lenses. The object glass forms the image I, which is rendered erect at II, and magnified at III.

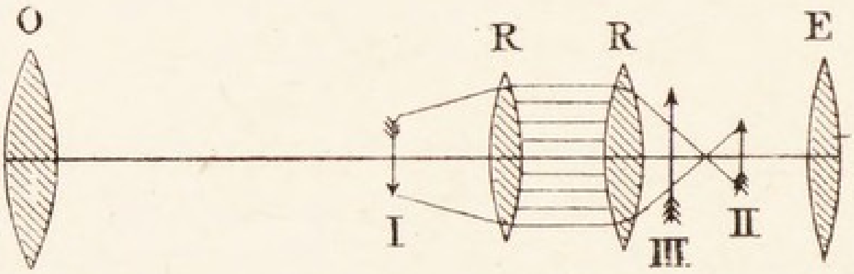


Fig. 171.—Diagram of terrestrial telescope.

Galileo's telescope (*opera glass*).—This consists of one convex and one concave lens, separated by a distance equal to the difference between their focal lengths. No real image is formed, but the combination is short, handy, and gives a well-lighted image.

The object glass O (Fig. 172) would form an inverted real image at C, but the ray A B is refracted by the concave lens E, and forms a virtual image at A′, which is erect and magnified.

The **magic lantern** contains two lenses, one a large convex or plano-convex lens C C, in the principal focus of which is placed the illuminant L, so that parallel rays are thrown on the object B. In front is a convex lens A, the *projecting lens*, which forms a

magnified inverted image on the screen, as seen in Fig. 173.

Long sight and short sight.—In a normal eye the extent of refraction which rays of light undergo in their passage through the cornea, aqueous humour,

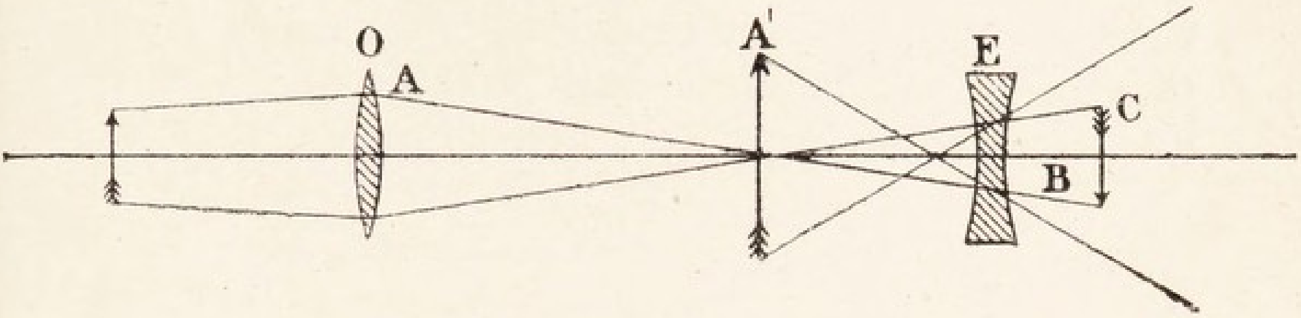


Fig. 172.—Galileo's telescope.

crystalline lens, and vitreous humour is just what is required to bring them to a focus at the distance of the retina and produce there a sharp inverted image of the object viewed. As the distance, v, of the image from the lens is practically constant,

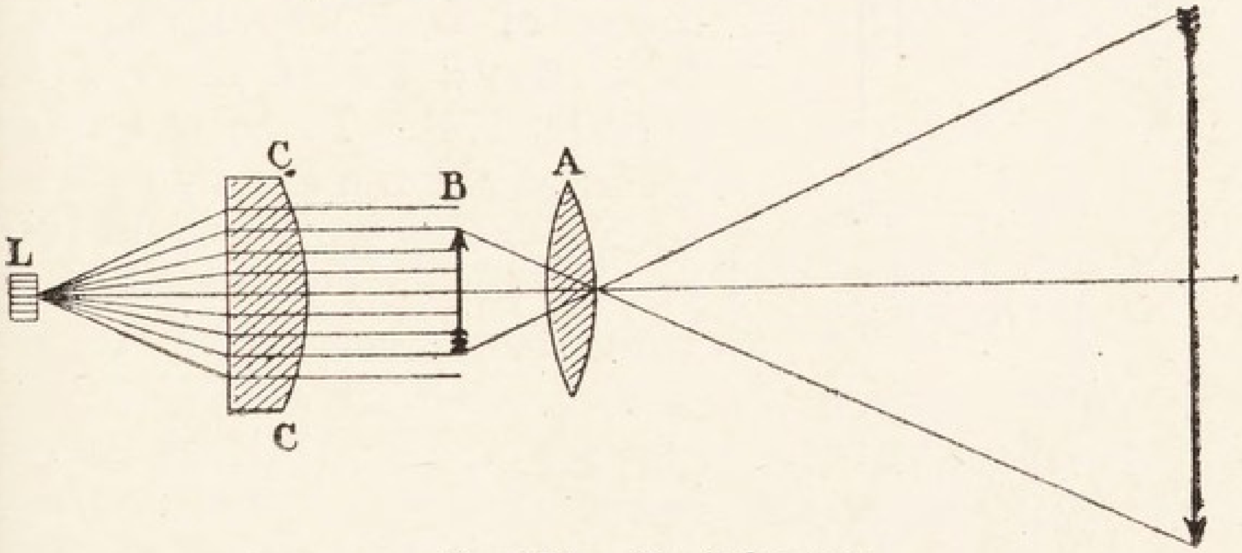


Fig. 173.—Magic lantern.

we should only be able to see objects clearly at one distance, u, if the lens were rigid and unyielding as the glass lens we have hitherto considered. This however, is not the case, and the ciliary muscles are able to alter the curvature of the anterior surface

of the lens, making it more convex to view near objects and less convex to view distant objects. So long as the muscular and other tissues retain sufficient of the elasticity which distinguishes them in youth, the eye will retain this power of *accommodation* and possess considerable *depth of focus*. It can then produce without conscious effort sharp pictures of objects at distances both greater and less than its geometrical focus. As age comes on, the tissues lose their elasticity, the lens becomes flatter, and the individual becomes *long-sighted* (presbyopia). He can form sharp images of distant objects, but the lens is not convex enough to focus near objects; he begins to hold his newspaper at arm's length, instead of the normal distance of 10 in. The image of near objects is, in fact, formed *behind* the retina (I., Fig. 174). To correct this we must aid the eye by means of a convex lens, which increases the convergence of the rays and brings them to a sharp focus *on* the retina R.

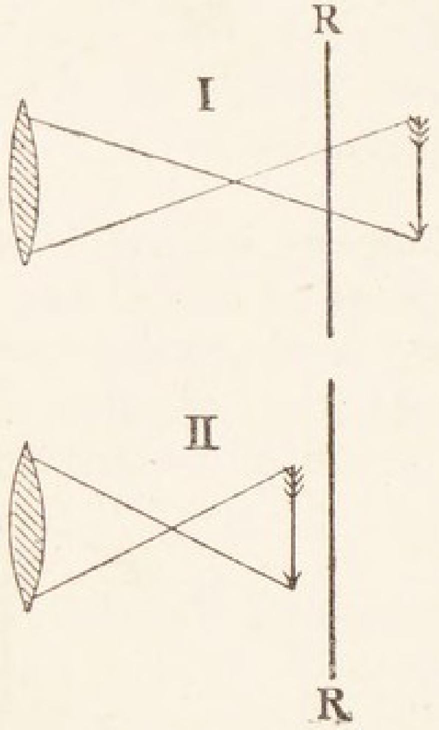


Fig. 174.—I., Long sight; II., short sight.

In *short sight*, or myopia (II., Fig. 174), we have the converse of the above. The lens is too convex and forms an image *in front* of the retina R. In some cases, as the person gets older, this is partially remedied by the flattening of the cornea; but it is usually necessary to correct this defect with a concave glass, which diminishes the convergence of the rays and so brings them to a focus on the retina.

Dioptres.—Lenses are numbered by opticians in

terms of a unit lens of 1 metre focal length, which is described as one dioptre, 1D. A lens of n dioptres, nD, has a focal length $= \frac{1}{n}$ m. $= \frac{100}{n}$ cm. There is some convenience in this method, because it gives to the *stronger* lens—with the smaller focal length—the *larger* number. As convex and concave lenses are also distinguished as positive and negative, a *concave* lens whose focal length is 20 cm. is numbered -5D.

To determine the focal length of a convex lens.—1. Focus the image of a distant object—the sun, a window, tree, etc.—on a piece of white paper:

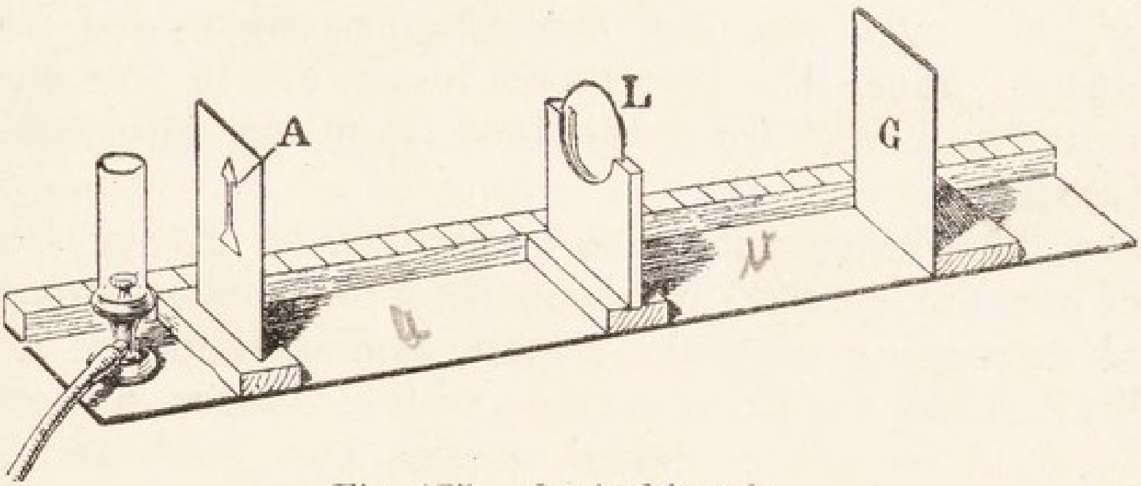


Fig. 175.—Optical bench.

when the image is sharp, the perpendicular distance between the image and the lens is the focal length.

$$u = \infty, \text{ and therefore } v = f$$

2. If v be the distance of the image from the lens, and u that of the object from the lens, and f the distance of the principal focus, we have stated (p. 255) that

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Arrange the lens L (Fig. 175) on a graduated scale; provide also a screen of ground glass G and well-illuminated arrow A, or cross, cut out of thin sheet zinc; place them as shown in the illustration, shift

the lens and screen so as to get sharp images, and calculate the value of f from various determinations of u and v.

3. Adjust the distances of object and image from the lens, till they are equal; we have then, since $u = v$,

$$\frac{1}{u} + \frac{1}{u} = \frac{1}{f}$$

$$\text{or } \frac{2}{u} = \frac{1}{f}$$

$$\therefore \; u = 2f$$

Hence in this position *the focal length is one-fourth of the total distance between the object and the image.* Since the object and image are in this case equally distant from the lens, they are also equal in size.

4. The image may also be located by the method of parallax (p. 223). This method has the advantage of dispensing with the screen, and is therefore not limited like the previous ones to real images, but can be used to locate virtual images also, such as the erect magnified image of Fig. 165 (2).

5. The focal length of a convex (or concave) lens may be determined by measuring the curvature of its faces when the index of refraction of the lens substance is known. If r_1 and r_2 be the radii of curvature of the two faces of the lens, and if μ be the refractive index of the glass of which the lens is made,

$$\frac{1}{f} = (\mu - 1)\left\{\frac{1}{r_1} + \frac{1}{r_2}\right\}$$

r_1 and r_2 are found by the spherometer (p. 395).

In the case of an equiconvex (or equiconcave) lens, $r_1 = r_2 = r$, and therefore, when $\mu = \frac{3}{2}$, we shall

have
$$\frac{1}{f} = \frac{1}{2} \times \frac{2}{r}$$
$$\text{or } f = r$$

To distinguish between a convex and a concave lens.—The image of a distant object viewed through a *convex* lens is *nearer* to the observer than is the lens; the image of the same object viewed through a *concave* lens is on the same side as the object, and therefore is *farther* from the observer than is the lens. It follows, therefore, from the method of parallax (p. 223), that if, while con-

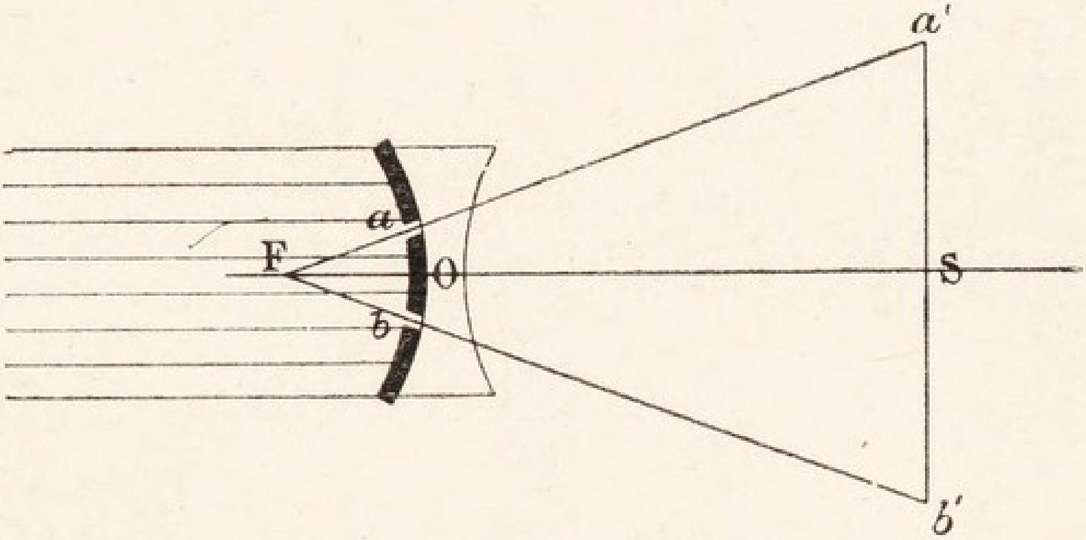


Fig. 176.—Determination of focus of concave lens.

tinuing to view the image, we slowly move the lens to the right, the image, if the lens is *convex*, will move to the left, but, if the lens is concave, will move to the right. This is a delicate and useful test.

To determine the focal length of a concave lens.—1. Combine it with a convex lens so that the combination behaves as a flat glass plate. If the focal length of the convex lens be known, that of the concave is obviously the same but negative.

2. Locate the image by the method of parallax; measure u and v and use the formula (p. 253), giving

a negative sign to v as the image is now on the *same* side as the object; f will be found to have a negative value also, indicating that the lens is concave.

3. Cover the concave lens with a piece of thin opaque paper (Fig. 176) and make two pin-holes, a and b, equidistant from the centre; throw a parallel beam so as to obtain two spots of light on a screen; then—

$$\frac{a\,b}{a'\,b'} = \frac{F\,O}{F\,S} = \frac{F\,O}{F\,O + O\,S} = \frac{f}{f + O\,S}$$

measure $a\,b$, $a'\,b'$, and $O\,S$.

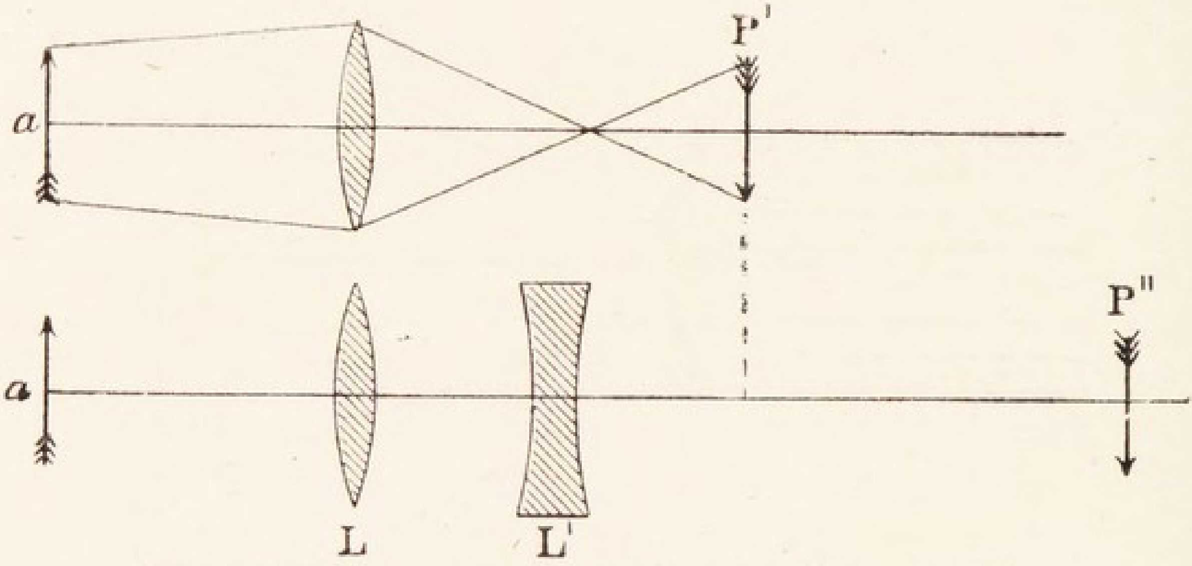


Fig. 177.—Determination of focus of concave lens.

4. Take a convex lens of shorter focus (Fig. 177), and in the optical bench determine (1) the position P′ of the image of an object a, formed by the convex lens alone, (2) the position P″ of the image of a, formed by the combination of the convex and concave lenses; then—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ and } L'\ P' = u \text{ and } L'\ P'' = v$$

5. If two convex lenses of focal length f_1, and f_2, be combined, and the focal length of the combination be F, then—

$$F = \frac{f_1 f_2}{f_1 + f_2 - a}$$

where a is the distance between the lenses. If $a = o$, the lenses are actually in contact and the equation may be written thus—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

We must therefore, as in the preceding method, (1) select a convex lens of focal length (f_2) shorter than the focal length ($-f_1$) of the given concave lens, and determine f_2; we must (2) determine F the focal length of a converging combination of the two lenses; we can then find f_1 from the foregoing formula.

6. By the spherometer method (5, p. 262).

To determine the magnifying power of a convex lens.—If we suppose that the image A′ B′ (Fig. 165, 2) is formed at 10 in. from C, we can prove by similar triangles that—

$$\frac{u}{10} = \frac{A\,B}{A'\,B'} = \frac{x\,x'}{A'\,B'} = \frac{f}{f + 10}$$

∴ the magnifying power of the lens

$$= \frac{A'\,B'}{A\,B} = \frac{f + 10}{f} = 1 + \frac{10}{f}$$

By looking at some squared paper, without the lens, from the *nearest* distance of distinct vision, and then interposing the lens at the *farthest* distance from the paper consistent with distinct vision of the magnified image thus seen, it is easy to determine the magnification by direct comparison of object and image: they are seen simultaneously, with different eyes; if one image-square is seen to be equal to n object-squares, the lens evidently magnifies n times.

Velocity of light.—The first recorded estimate

of the velocity of light seems to have been made by a Danish astronomer, Roemer, who communicated the discovery to the French Academy of Sciences in 1675. The planet Jupiter has four moons or satellites which revolve round it, and from time to time become eclipsed, to an observer on the earth, as they pass into the umbra (p. 219) cast by Jupiter. This happens about every $42\frac{1}{2}$ hours to the moon which is nearest Jupiter. The interval is not quite constant, and Roemer observed that it regularly increased as the earth and Jupiter moved farther apart, and regularly diminished as they approached nearer to each other. He therefore rightly concluded that the small variations observed in the length of the interval were due to the fact that light took longer to reach the earth when it was farther away from Jupiter. The total variation amounts to 16 min. 26·6 sec. This must therefore be the time which light takes to travel a distance equal to the diameter of the earth's orbit, or about 184,000,000 miles; its velocity is therefore very nearly 186,500 miles per second.

This value was practically confirmed by James Bradley, an English astronomer, who announced to the Royal Society, in 1729, his discovery of the aberration of light. Light travels in a straight line, and will therefore pass centrally down a straight tube whose axis lies in this straight line, *if the tube is stationary*; but if the tube is moving, the axis must be inclined to this straight line at such an angle that each successive point of the axis crosses the line just at the moment that the light has travelled to the same crossing. Similarly, we have to hold an umbrella at a certain angle to the vertical when *walking* in the rain, although the rain is falling vertically at the time; the faster we walk the greater the angle must be. An observer on the earth, pointing his tele-

scope at a star, is similarly affected by the motion of the earth, and points the telescope not at the true position of the star but at a position slightly displaced from that. This displacement measures the aberration, and evidently depends on the ratio between the velocity of the earth's motion in its orbit and the velocity of light. This ratio is found from the aberration to be about 1 to 10,000, and since the velocity of the earth is nearly 18·5 miles per second, the velocity of light is nearly 185,000 miles per second. From the relation $s = v\ t$ (p. 9) it is clear that when v is so large s must also be large unless t is some small fraction of a second. By selecting astronomical distances for s we have been able to give an appreciable value to t for the observation and measurement of this great velocity. The value has, however, been measured more recently by methods in which only small terrestrial distances, accurately measured, are employed. Two of these must be briefly described.

Fizeau's method.—In this method a toothed wheel revolves between a source of light and a plane mirror. The rate of revolution is known, and just permits the light which passes through the space between two teeth to be stopped, after reflection from the mirror, by one of these teeth. The observer, therefore, sees no returning beam of light. The light thus travels from the wheel to the mirror and back while the wheel turns through the width of a tooth. Both distance and time being known, the velocity of light is readily calculated.

Foucault's method.—In this method a beam of light incident on a plane mirror is reflected to a concave mirror. The plane mirror is rotating at a known rate about an axis in its own plane. The centre of curvature of the concave mirror is situated

in this axis so that the beam is reflected back from the concave to the plane mirror, and from this to the original source. If the plane mirror is rotating with sufficient rapidity, the image formed by the returning beam will be sensibly displaced. This displacement is a measure of the angle through which the plane mirror has turned while the light travelled twice the distance between the mirrors. From this angle and the known speed of rotation (400 revolutions per second) we can deduce the time occupied in the double journey. We have therefore values of s and t, and can find the velocity, v, from the usual equation $s = v\,t$ (p. 9).

The results obtained by these terrestrial methods show that the value of v is very close to 300×10^6 metres per second.

Exercises

1. A converging lens of 4 cm. focal length is used as a magnifying glass, the nearest distance of distinct vision being 25 cm. Where must the object be placed when the lens is quite close to the eye, and what is then the magnification ?

2. A convex lens of 6 in. focal length is employed to project a magnified image on a screen placed 3 ft. from the object. Where must the lens be placed ? Describe the image.

3. If the refractive index of glass is 1·6, show how to construct the smallest angle of incidence at which total reflection in the glass occurs.

4. A convex lens has a focal length of 25 cm. What are the positions of the image corresponding to the three following distances of the object from the lens : i. 50 cm., ii. 25 cm., iii. 12·5 cm. ? [*First Professional.*]

5. If a person is unable to see clearly objects that are nearer to him than 1 metre, what kind of lens would enable him to read a book held at a distance of 30 cm. ? What should be the focal length, and the *power*, of the lens ?

6. A man is so short-sighted that, without spectacles, he cannot see distinctly objects which are more than 6 in. from his eye. To what defect of his eye is this due ? What glasses would enable him to see a star distinctly ?

(For Answers, *see* p. 389.)

CHAPTER I

DISPERSION

Prismatic Dispersion — Spectra — Spectroscopes—Nature of Colour—Double Refraction—Polarization of Light—Saccharimeter—Exercises.

Dispersion.—We have already seen that a beam of white light, when passed through a prism, is refracted and bent away from the thin end of the prism. Moreover, the light is no longer *white*, but coloured. Every image we observe through a prism, even the image of the pin (p. 247), is more or less rainbow-tinted. The prism not only refracts, but *analyses* the composite white light and decomposes it into its constituent colours. In an optical sense the number of these colours is legion, for in this sense there are as many colours as there are light waves of different length in the visible spectrum ; even the two sodium lines (Frontispiece, 2) represent *two* yellows, in this sense, for their wave lengths differ by 6 tenth-metres. Our sense of vision is by no means so refined, and physiologically we appear to have only three fundamental colour sensations, which correspond roughly to red, green, and blue-violet. This number appears to be reduced to two in about 4 per cent. of people whose vision has been tested. Dalton, the famous chemist, accidentally discovered that both he and his brother were unable to distinguish between red and green. His form of *colour blindness* has been called *Daltonism*, and the subject has received considerable attention

since his paper appeared. The phenomenon, however, is still obscure. Dalton's account somewhat suggests that in his case the range of colour sensation was narrowed at the red end, and perhaps slightly extended at the blue end, compared to the normal range. The range of audibility certainly differs in individuals (p. 208), and the sense of tune —a sort of sound colour—is almost absent in some. To Dalton blood appeared to be "not unlike that colour called *bottle-green*." He also states that to

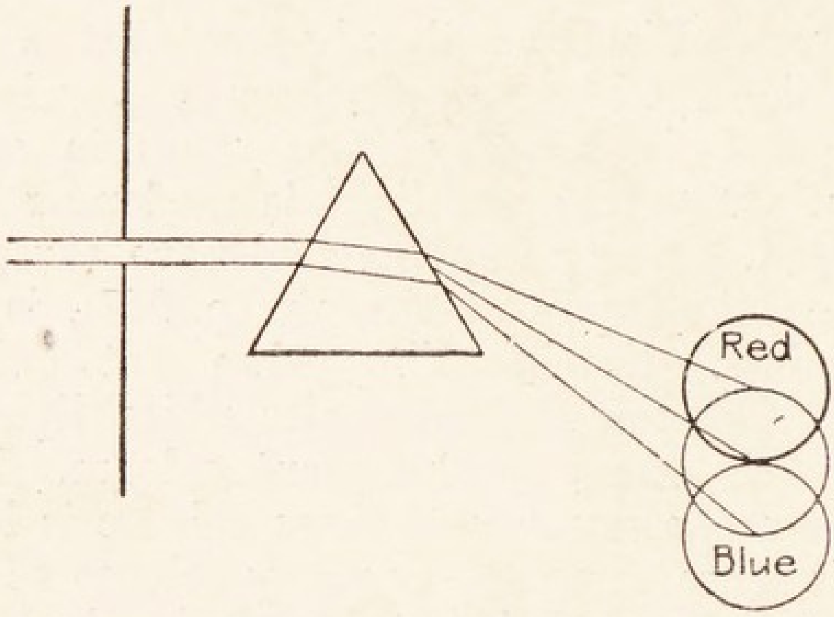


Fig 178.—Decomposition of a circle of white light.

him "the face of a laurel leaf (*Prunus laurocerasus*) is a good match to a stick of red sealing-wax; and the back of the leaf answers to the lighter red of wafers"; also, "a red soil just turned up by the plough" resembled the green woollen cloth such as is used to cover tables. In fact, the lower end of the normal spectrum to him was green. In a normal sense of vision, however, white light appears to stimulate three primary colour sensations—red, green, and blue-violet. Each of these colours is refracted to a different extent—the red least, the violet most.

If the source of light is a spot, we have three images of the spot—one red, one green, one blue-violet; but as these overlap, the colours are not pure (Fig. 178); where the three circles are superposed we have white light. If, however, we take as our source of light a narrow slit, the overlapping is slight, and we get a pure spectrum, a continuous image of the slit in all colours—red, yellow, green, blue, violet. This is called the **continuous spectrum**, and we obtain it whether the source of light is the sun, an electric lamp, a limelight, or a gas or candle flame. We can recompose these colours so as to reproduce a slit of white light, by passing the beam

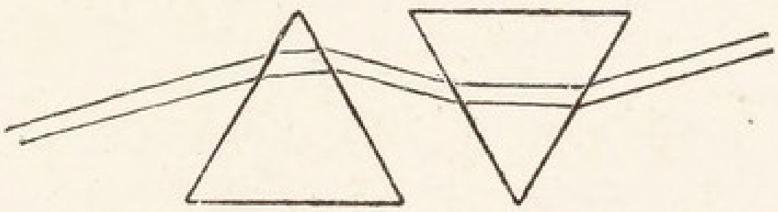


Fig. 179.—Recomposition of white light.

through a similar prism placed in the reverse position (Fig. 179).

It will be remembered that the red rays vibrate the most slowly—400 million million times in one second; the violet rays the most rapidly—760 million million times in one second—and these are the most refrangible. If D_v, D_r represent the respective deviations of the violet and red rays, then $D_v - D_r$ is the angle subtended by the whole spectrum of colour, and is called the *dispersion* of the prism. If μ_v, μ_r represent the corresponding refractive indices, we know (p. 248) that

$$D_v = (\mu_v - 1)\,A, \text{ and } D_r = (\mu_r - 1)\,A$$
$$\therefore \text{ dispersion } = (\mu_v - \mu_r)\,A$$

The *dispersive power* (Δ) is the ratio of the dispersion to the mean deviation D—say for the sodium

line—which is $(\mu - 1) A$, where μ is the corresponding mean refractive index,

$$\therefore \Delta = \frac{\text{dispersion}}{D}$$

$$\therefore \text{dispersion} = \Delta \times D$$

but $D = \frac{x}{f}$ (p. 255), where x is the distance of the incident ray in the lens from the principal axis,

$$\therefore \text{dispersion} = \Delta \times \frac{x}{f}$$

We shall presently make use of this formula.

The vibrations emitted from the sun, or other luminous source, are not all included in the visible spectrum. Below the red end are dark-heat rays —the *infra-red* rays—and beyond the violet end there are other rays of still shorter wave length—the *ultra-violet* rays—also invisible to the eye. The existence of the ultra-red rays can be demonstrated by a delicate thermometer, or the thermopile, and the ultra-violet rays can be rendered visible by receiving them on a screen moistened with an acid solution of quinine sulphate, which at once shines with a beautiful blue fluorescence when exposed beyond the violet.

The character of the spectrum varies with the source of light. The spectrum of the incandescent electric lamp (p. 104) is relatively rich in rays at the red (thermal) end, but poor in the ultra-violet (actinic) rays ; this lamp is therefore employed for therapeutic purposes in the *radiant-heat bath.* On the other hand, the light from an arc lamp is relatively rich in the ultra-violet rays, which have a special influence on the skin and neighbouring tissues, and for this reason the arc lamp has been largely employed in the Finsen treatment, etc., though this has been

recently to some extent replaced by X-ray treatment. The light of the mercury vapour (*Uviol*) lamp is particularly rich in violet and ultra-violet rays, and practically destitute of red. Workers with these rays suffer with conjunctivitis unless the eyes are protected by reddish-yellow glasses; these should also be used, instead of blue glasses, to protect the eyes from the effects of snow-glare, which are mainly due to the bluer elements in the sunlight reflected from the snow at high altitudes; ordinary sunlight is not rich in the ultra-violet rays, as these are to a large extent absorbed in passing through the earth's atmosphere. Quartz is more transparent to ultra-violet rays than is glass, and therefore replaces this substance in the lenses, etc., employed in operating these rays.

Line spectrum.—If the narrow slit be illuminated, not by ordinary white light but by the yellow light of a sodium flame, we see only one narrow yellow band instead of the continuous spectrum: the light from the sodium flame is not composite but *monochromatic*. If we illuminate the slit with the lavender flame of potassium we see one band in the red and one in blue-violet (*see* Frontispiece). These two hues mixed give us the lavender flame of potassium.

Absorption spectrum.—If we look at a continuous spectrum from, say, an electric light, and interpose between the electric light and the slit a mass of sodium vapour, we see the continuous spectrum, but with a black line in the place where the yellow sodium line should be. This is an *absorption band.* The sodium vapour absorbs the same light as it would emit if incandescent; just as the C string of a piano will take up the same sound which it would emit when struck. The black lines in the spectrum of the sun therefore indicate the

presence in the sun's atmosphere of substances which, if incandescent, will emit bright lines occupying the same positions.

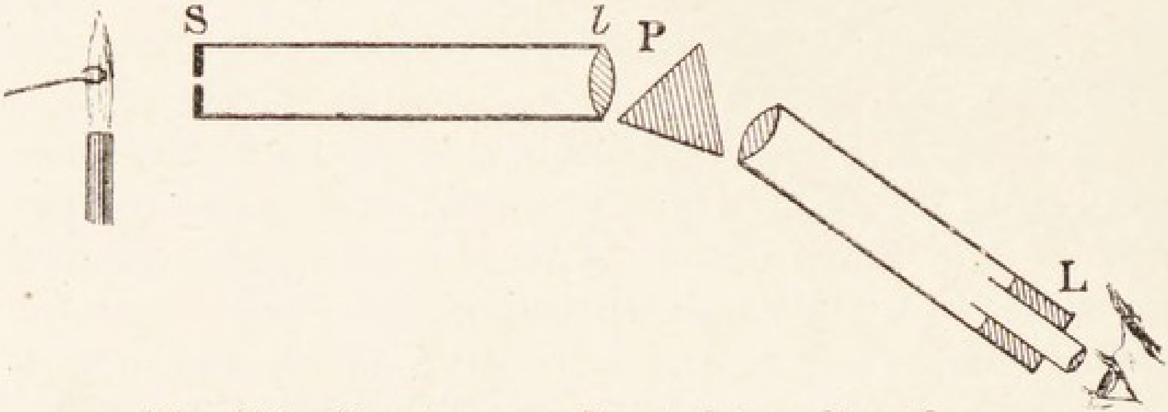


Fig. 180.—Spectroscope directed to sodium flame.

Spectroscopes.—An instrument designed for the observation of spectra is called a spectroscope and consists of three essential parts: (1) a narrow parallel-sided slit; (2) one or more triangular prisms of solid

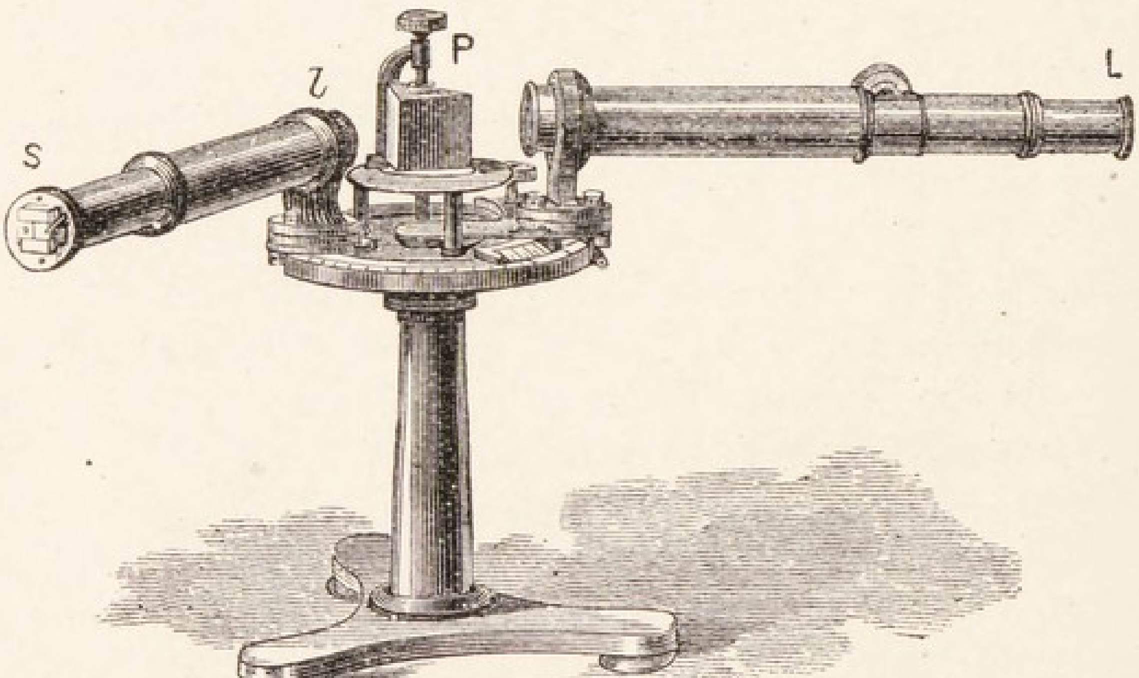


Fig. 181.—Spectroscope.

glass, or hollow glass prisms filled with carbon bisulphide, or other transparent, highly refracting, liquid; and (3) a lens or lenses to produce distinct vision.

The ordinary **table spectroscope** (Figs. 180 and 181)

consists of a brass tube bearing a parallel-sided slit at S, the light from which is focused by the lens *l*, falls on the prism P, and is viewed by a telescope of low magnifying power at L. The space between *l* and the telescope is often covered by a piece of black velvet when the instrument is used, to prevent light reaching the prism from any other source but the slit.

When the circumference of the table (Fig. 181) is divided into degrees, etc., and a vernier attached to the telescope indicates on this scale every position to which the telescope is moved, important *measurements* can be made with the instrument, which thus

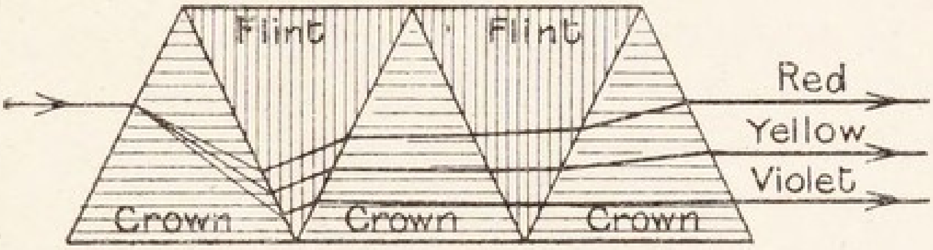


Fig. 182.—Prisms in direct-vision spectroscope.

becomes a *spectrometer*. The eye-piece is then furnished with cross wires whose intersection provides a fixed point of reference in the field of view. The adjustments required in practice before using the instrument are referred to later (p. 419).

A more convenient instrument for many purposes is the **direct-vision spectroscope** (Fig. 182), which consists of a number of alternate prisms of crown and flint glass with their edges turned in opposite directions and so selected that the refraction of the light by the crown glass is exactly corrected by that of the flint glass in the opposite direction. We have already seen (Fig. 179) that, if two exactly similar prisms are employed in this way, the white light which traverses them is unchanged in direction

and is still white; the second prism counteracts both the *deviation* and the *dispersion* produced by the first. These two effects, however, are not due to exactly the same factor in a prism or lens; it is therefore possible by employing prisms, or lenses, of different substances and of different angles (1) to produce deviation without dispersion—as in an achromatic lens; (2) to produce dispersion without deviation—as in the single straight tube of this direct-vision spectroscope. When, therefore, we look at the slit of this instrument illuminated by white light we see a continuous spectrum although the tube is straight.

As we have found (p. 272), the dispersion $= \frac{\Delta}{f} \times x$; we can equalize this for a combined convex (1) and concave (2) lens by making

$$\frac{\Delta_1}{f_1} \times x_1 = \frac{\Delta_2}{f_2} \times x_2$$

or, neglecting the thickness of the lens, and therefore calling $x_1 = x_2$, we must have

$$\frac{\Delta_1}{f_1} = \frac{\Delta_2}{f_2}$$

or *the dispersive powers must vary directly as the focal lengths*. We shall then have no dispersion, and yet, as the focal lengths are different, we shall secure deviation and therefore form an image. If we want the achromatic lens to be converging, f_1 must be shorter than f_2, and therefore Δ_1 must also be less than Δ_2; to secure this we can use crown glass ($\Delta = 0{\cdot}033$) for the convex lens (1), and flint glass ($\Delta = 0{\cdot}052$) for the concave lens (2). Similarly, by a suitable combination of different prisms of these materials (Fig. 182) it is evidently possible to make the total deviation zero without at the same time abolishing the dispersion.

When the spectroscope is directed towards the sky on a bright day, the continuous spectrum is seen to be crossed by a number of fine black lines—*Fraunhofer's lines*—which are parallel to the direction of the slit. To see them the spectroscope slit must be very narrow.

One, in the brightest yellow, coincides in position with the sodium line; it is called the D line (*see* Frontispiece). There are three prominent bands in the red, A, B, and C; then in the green will be noticed two, E and *b*, one at the beginning of the blue F, and one towards the violet G, etc. These lines are produced by the absorption of certain constituent vibrations from the colourless white light emitted by the sun's photosphere. The D line is due to the white light traversing masses of sodium vapour, probably in the sun's atmosphere, during its passage from the sun to the eye. Many of these lines have been identified as belonging to the vapours of metals—iron, etc.—which are known on the earth. When a powerful spectroscope—that is, one with many prisms—is used, the number of these lines is enormous.

Coloured solutions also absorb portions of the continuous spectrum. If we interpose a solution of potassium bichromate between a gas flame and the spectroscope, all the blue light is stopped; a solution of copper sulphate and ammonium hydrate transmits the blue, but stops the other colours. If we combine the two, the whole spectrum is cut out, and we have darkness. A solution of potassium permanganate stops the yellow, showing in its place a black absorption band, with the transmitted red and violet on either side of the black band. A solution of blood, if dilute (*see* Frontispiece), cuts out two bands of colour near the yellow red, and

gives the characteristic absorption spectrum of oxy-hæmoglobin.

We distinguish therefore—

(1) The *continuous spectrum* emitted by bodies at a white heat.

(2) *Absorption spectra* produced by the interposition of coloured fluid, substances in a state of vapour, etc., between a body giving a continuous spectrum and the spectroscope.

(3) *Bright line* or *emission spectra*, emitted by glowing masses of metallic vapours, sodium, calcium, etc., which give bright coloured lines in various parts of the spectrum.

Colour due to reflected or transmitted light.—Substances in general owe their colour to those constituents of white light which they do *not* absorb, but either reflect or transmit. Thus the surface of a red poppy absorbs from white light all colours but red, a blue gentian all colours but blue; in each case it is the *reflected* colour which we see.

A blue glass looks blue because it *transmits* the blue rays and stops the red ones. Gold leaf, when held between the eye and a source of white light, looks green, but when laid on the table and viewed from above looks yellow. The transmitted light is green but the reflected light is yellow. If a black photographic dish be filled with a solution of potassium bichromate, it is impossible to tell whether the solution is coloured or not, but if we sink in it a piece of white opal glass the yellow colour is at once seen by the light reflected from the white surface.

Colouring substances, or pigments, and coloured lights are often confused: e.g. green light is a pure

colour, and cannot be made by mixing, but every child who has a paint-box knows that blue and yellow paints form green. The explanation is that the yellow paint is not pure, but is a mixture of yellow and green; similarly the blue is a mixture of blue and green. Now, if we mix pure blue and yellow lights we get white light. Blue and yellow are termed *complementary colours*, so in mixing paints the pure blue and yellow neutralize each other, to form a greyish white, and the green from both appears.

The fact that objects derive their colour from the light which falls on them explains the familiar experience that when seen by artificial light they "do not look the same" as when seen by day. A red geranium blossom, when illuminated by a light which has no red in it, appears black. This explains the ghastly corpse-like hue shown by the tongue, lips, etc., when illuminated by a sodium flame.

The **rainbow** is caused by a combination of refraction, total reflection, and dispersion.

Parallel rays from the sun strike a spherical raindrop at A (Fig. 183). Some of the rays are refracted, and some reflected, passing out at B. The incident and reflected rays meet at P; the angle of deviation is D. If S A C B be in the position of minimum deviation, the angle S P B is about 42° for red light and 40° for violet light, so an observer facing the rain-cloud with the sun shining behind him will receive a series of red impressions from rays similar to B F, and inside these he will have a circle of yellow, green, blue, and violet rays.

Double refraction.—When a black dot on a piece of white paper is viewed through a rhomb of Iceland spar, two images of the dot are seen. Since the images are caused by refraction we must conclude

that this takes place in two directions, and gives rise to two refracted rays. One is called the extraordinary ray and the other the ordinary ray. The ordinary ray is the more highly refracted, and the image produced by it therefore appears to be raised rather above the other; the index of refraction of the ordinary ray is 1·66 nearly. The index of refraction of the extraordinary ray is 1·49 nearly. As we turn the crystal round above the dot, the extra-

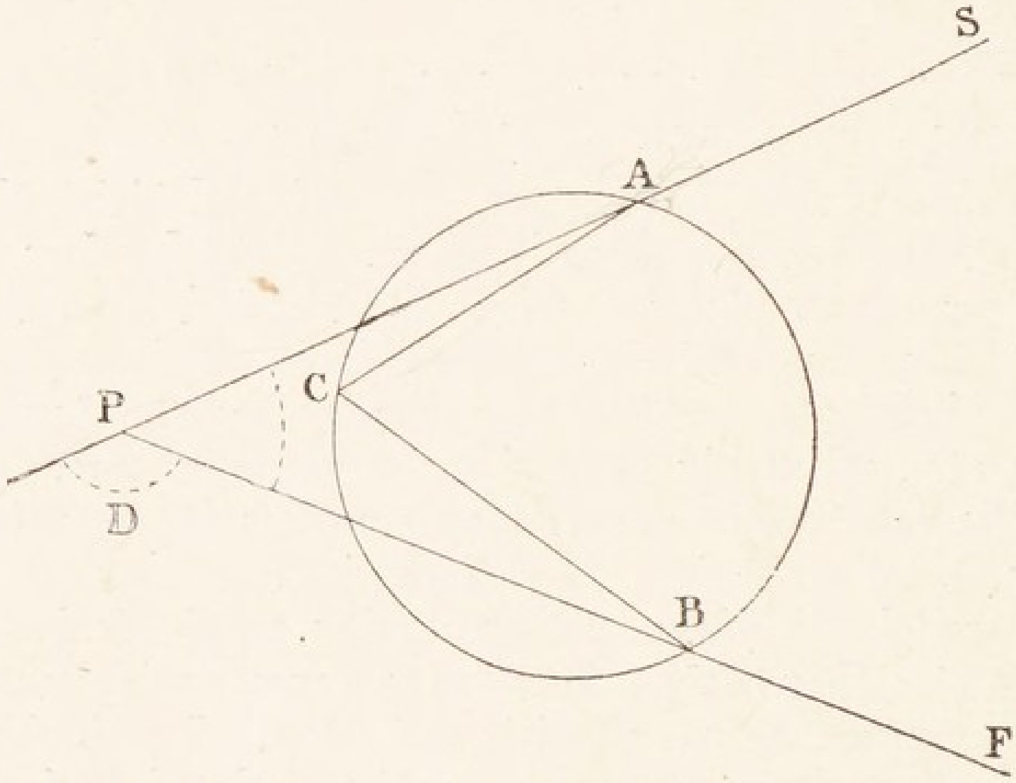


Fig. 183.—Path of sun's ray through drop of water in a rainbow. (*After Aldous.*)

ordinary image rotates round the ordinary one, which remains fixed. Just as, in wood, properties are modified by their relation to the direction of the grain, so, in crystals like Iceland spar, we recognize a certain direction or axis of symmetry which has a similar influence on properties. The elasticity is greater in a direction parallel to this axis than it is in a direction perpendicular to it. The velocity with which light travels in the crystal is therefore

greater in the first direction than in the second. Light incident on the spar divides into two portions whose vibrations are respectively restricted to these two directions. By dividing the crystal across the principal axis, so that this axis is perpendicular to the plane of section, and interposing between the two halves a layer of Canada balsam, one portion of the light is totally reflected (p. 245). The index of refraction of the balsam is 1·53 nearly, and the ordinary ray, if incident on the balsam at an angle exceeding the critical angle, will therefore be totally reflected. The extraordinary ray passes on and emerges parallel to its direction before incidence.

A crystal of Iceland spar modified in the way described is known as a **Nicol prism.** The light which has passed through it is due to transverse vibrations executed in one plane only, and is said to be *plane polarized light*. Ordinary light is due to transverse vibrations executed in various planes. The difference between ordinary and polarized light is not always apparent. If we look at a source of light through one Nicol prism, we see nothing to indicate that the light has been altered by its passage through the Nicol. If, however, we look *through a second Nicol* at light which has passed through the first, we soon become aware that the light has some special properties. If, for instance, while looking through the two Nicols we rotate the second one, we find that in some positions much more light appears to come through both Nicols than in others ; indeed, in a complete revolution of the second Nicol we find two positions in which the light is extinguished. These positions occur when the Nicols are *crossed ;* the plane of vibration in one prism is perpendicular to that in the other ; the two positions are separated by 180°. If the second Nicol is now

turned through a right angle we obtain the maximum illumination because the planes of vibration are then parallel, and the wave travels through the second as easily as through the first. In such a pair of Nicols the first—the one nearer the source of light —is often distinguished as the *polarizer*; the second, the one nearer to the observer's eye, as the *analyser*.

The two together form a **polariscope.** For convenience they are often mounted at the two ex-

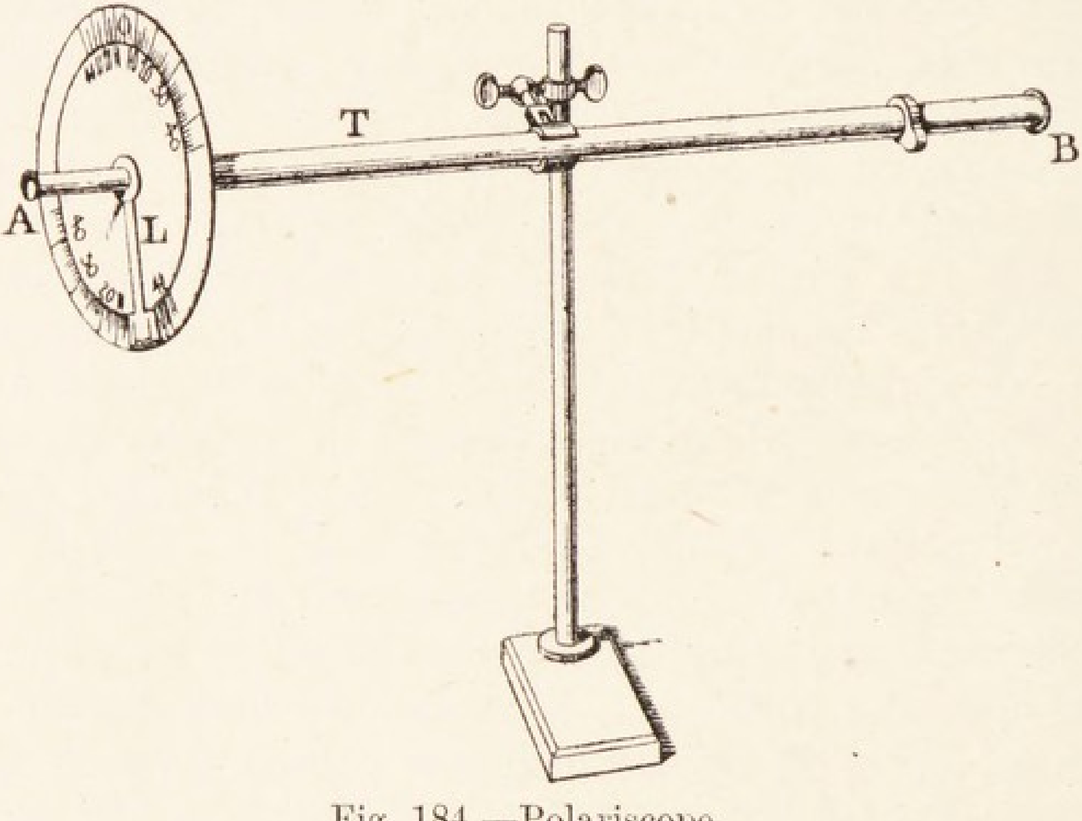


Fig. 184.—Polariscope.

tremities of a brass tube T (Fig. 184) which carries, at the end where the analyser is placed, a circle divided into degrees, etc. The brass mount in which the analyser A is fixed is provided with an arm L, by which it can be rotated by the observer while looking through the tube; the rotation of this arm is indicated by an attached vernier which travels round the divided circle at the same time. When a transparent tube filled with distilled water is inter-

posed between the two Nicols, the optical effects described are not altered, but when an aqueous solution of glucose is substituted for the distilled water, observation shows that a change has occurred. It is not possible now completely to extinguish the light unless we employ a monochromatic light like the sodium flame. If we do this, and note the position of darkness as indicated by the travelling vernier on the scale of the divided circle, we find that it is not the same as before. To obtain darkness in the second case the analyser must be rotated through a certain angle by moving L in a direction similar to the hand of a clock.

We conclude that the plane of polarization of the light has been rotated in passing through the solution, and the analyser therefore requires to be rotated through a corresponding angle in order that the position of darkness may again be secured. Substances which produce this rotation are said to be *optically active.* If the rotation is in the same sense as that of glucose they are *dextro*-rotatory; if in the opposite sense, *lævo*-rotatory. We find that at any temperature the angle of rotation varies with—

(1) The strength of the solution.
(2) The length of the column of liquid through which the light passes.
(3) The nature of the substance in solution.

Each substance produces, therefore, a *specific rotation.* For the sodium flame this is represented by $[\alpha_D]$, and is referred to certain standard conditions:—

1 grm. in 1 c.c. in a tube 1 decimetre long produces $[\alpha_D]$.

$\therefore$ p grm. in 100 c.c. in a tube l decimetre long produces $\dfrac{l \times p \times [\alpha_D]}{100}$.

For glucose $[\alpha_D] = 52{\cdot}7°$. If, therefore, a solution of glucose of unknown strength, when examined in a 2-decimetre tube, produces an *observed* rotation α, we have

$$\alpha = \frac{2 \times p \times 52{\cdot}7}{100}$$

from which we can at once find p, the number of grammes of glucose per 100 c.c.

When the polariscope is employed especially to determine the strength of sugar solutions it is termed a *saccharimeter*.

As it is sometimes inconvenient to work only by sodium light, the polariscope has been adapted for use with ordinary white light by the addition of a thin plate of quartz. This substance also produces rotation. Some crystals are right-handed and some left-handed, and the plate employed consists of two halves, one right-handed and the other left-handed. As white light is composite, darkness cannot be secured for all colours in one position. Hence in most positions the two halves show the complementary colours, red and blue, and change colours as the analyser is rotated. But two positions are found when both halves show the same colour and cannot be distinguished—in one position the colour is a pale yellow, in the other it is a greyish violet—the latter is the one chosen for reference. On either side of this position the transition tint is immediately lost, and the two halves show opposite complementary colours. When the instrument is set in this position, if a sugar solution is inserted it will be seen that the complementary colours have again appeared, and the analyser must be rotated through an angle, α, till the transition tint is again obtained. We can then calculate the strength of the solution as before, but the specific rotation has

a different value when this light is employed and is denoted by $[\alpha_J]$. For glucose $[\alpha_J] = 58{\cdot}3$.

We should therefore in this case have

$$\alpha \text{ (observed)} = \frac{2 \times p \times 58{\cdot}3}{100}$$

whence p is found as before.

Exercises

1. The refracting angle of a glass prism is 60°, and the angle of minimum deviation for sodium light passing through it is 40°. Calculate the refracting index of the glass, given that sin 50° = 0·766.

2. A solution of glucose was found to give a rotation of 7·3° when placed in a 2-decimetre tube and adjusted to the transition tint. Calculate the percentage of glucose present.

(For Answers, *see* p. 389.)

PART V.—ELECTRICITY AND MAGNETISM

CHAPTER I

STATIC ELECTRICITY

Production of Electricity by Friction—Induction—Electroscope—Conduction—Distribution of Electricity on Conductors—Potential—Capacity—Electrophorus—Electrical Machines—Thunderstorms — Lightning Conductors — Condensers — Leyden Jar — High-Frequency Currents—Exercises.

IF a piece of dry amber, which has been rubbed with a piece of dry flannel, be held near small fragments of paper, cork, etc., these light bodies will be attracted by the amber, and will move towards it. They must, therefore, be acted upon by some force in the direction of the amber (p. 19). The phenomenon seems to have been observed in very early times, and is mentioned by early classical writers. It was, however regarded by them as peculiarly characteristic of amber. Even now the modern science of electricity derives its name and many technical terms from *electron*, the Greek name for amber (ἤλεκτρον), and thus acknowledges its early origin in that discovery.

At the close of the sixteenth century, William Gilbert, M.D.,* showed that this attractive power could be developed by friction in many other sub-

* President of the Royal College of Physicians, 1600.

stances besides amber. The scientific study of electrical phenomena, which may be said to have commenced with Dr. Gilbert, has continued ever since. Nevertheless, we are even now unable to answer the simple question, What is Electricity ? We know that it is not matter, for a charge of electricity possesses neither weight nor extension. It appears, however, to reside in matter, and can be transferred from one mass to another. We also recognize two kinds of electricity, **positive** and **negative.** They appear to be developed simul-

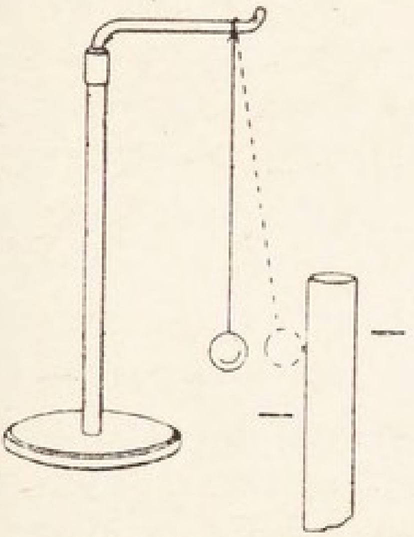


Fig. 185.—Attraction of pith ball by rubbed sealing-wax.

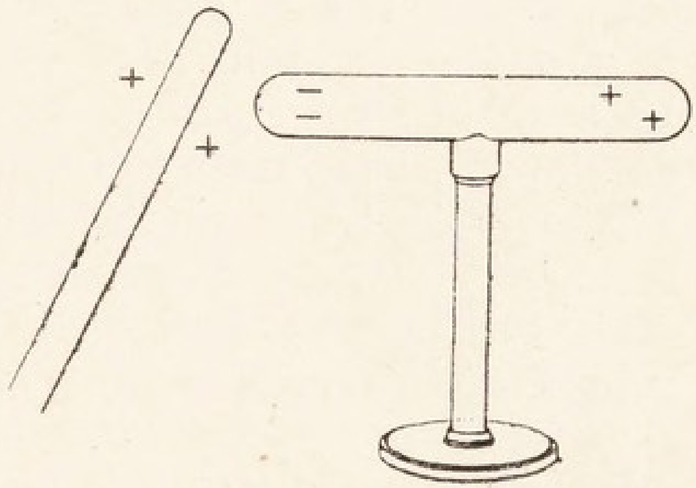


Fig. 186.—Induction.

taneously in equal amounts, or possibly they are separated from each other, and so become manifest to us, just as we recognize the hydrogen and the oxygen which are combined in water when they are separated from combination. Two opposite kinds of electricity attract each other, but two charges of the same kind repel each other. These conclusions are suggested by many familiar experiments.

If we rub a stick of sealing-wax or vulcanite with dry flannel, and then bring it near a pith ball suspended by a silk string (Fig. 185), the pith ball is at

first attracted towards the stick, but after contact with it is violently repelled. The same phenomenon occurs if we rub a dry glass rod with dry silk and employ it in a similar experiment. If, however, we bring the rubbed glass rod near a pith ball *which has touched the rubbed sealing-wax*, we find that, although the pith ball is repelled by the sealing-wax, it is strongly attracted by the rubbed glass.

These phenomena are usually explained as follows: The friction of the flannel on the sealing-wax disturbs the normal electrical condition of both. *Negative* or resinous electricity is said to be developed on the sealing-wax, and at the same time an equal amount of positive electricity is developed on the flannel. In a similar way, *positive* or vitreous electricity is developed on the glass rod and negative on the silk.

Now, electricities of the same name, as we have seen, repel, while those of unlike names attract, each other. The pith ball, by contact with the rubbed sealing-wax, became *negatively* charged, and therefore was repelled by the negatively charged wax, but strongly attracted by the positively charged glass rod. If a rubbed glass rod be brought near a metal cylinder, mounted on a vulcanite or dry glass rod, the electrical equilibrium of the cylinder is disturbed; some of its negative electricity is attracted to the end nearest the positively charged glass rod, and a corresponding quantity of positive electricity will be repelled (Fig. 186). This process is called **induction.** If the rod touch the cylinder the negative on the cylinder will neutralize some of the positive on the glass, and the cylinder will remain charged with positive electricity. Induction really affects the pith ball of Fig. 185, and precedes the attraction of the ball by the sealing-wax. The four stages of the experiment are therefore (1) induction;

(2) attraction between the *unlike* charges, inducing and induced ; (3) contact ; (4) repulsion between the like charges.

The **gold-leaf electroscope** (Fig. 187) is a very useful instrument for detecting charges of electricity developed by friction. It consists of a square box with glass sides or a round glass bell-jar, in which slips of gold leaf, A, A, are attached to a brass rod B, which passes through a thick vulcanite collar D, and ends in a circular brass plate C. The plate and leaves therefore constitute an insulated metal con-

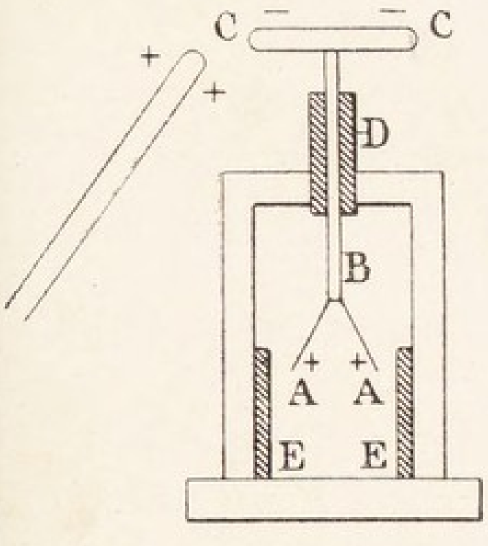


Fig. 187.—Gold-leaf electroscope.

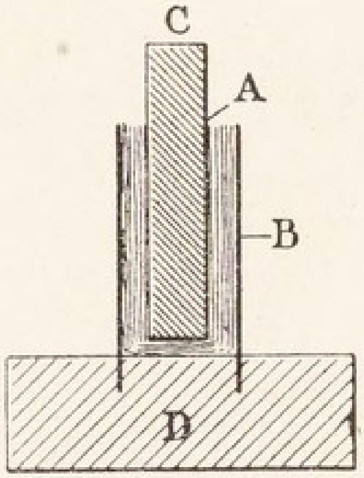


Fig. 188.—Charge on rubber.

ductor similar to that of Fig. 186. Two strips of tinfoil, E, E, are cemented on the inside of the case ; the leaves touch these, and so discharge themselves to earth if the charge is too strong. If we bring a rubbed glass rod into actual contact with the plate C (Fig. 187), some of the charge is distributed to the metal conductor and the leaves diverge with positive electricity ; but if the rubbed glass rod be brought very near without touching, as in the same figure, electrical equilibrium on the metal is disturbed—the plate becomes negative, while the

leaves both become positive and therefore diverge. On removing the charged rod, equilibrium is restored and the leaves collapse. If, however, before the charged rod is removed, the plate be touched with the finger, the positive electricity escapes to earth and the leaves collapse ; but if the glass rod be now withdrawn the leaves again diverge, because the negative electricity, previously confined to C by the inductive effect of the glass rod, is now distributed over the conductor and both leaves become negative. The leaves can, therefore, be charged with electricity of the *same* kind as the electrified body by contact, or with the opposite kind by induction.

Conduction.—If a brass tube, *held in the hand*, be rubbed with flannel, no charge will be developed on the brass. This is due to the fact that metals *conduct* electricity, which thus passes from the brass to the hand, and so, through the body of the operator, to the earth. If the brass tube be mounted on a vulcanite rod, then, on rubbing, a charge will be found on the brass. Vulcanite, a non-conductor, insulates the brass, and the charge remains. A very convenient method of proving that a charge is developed on metals by friction is to strike the brass plate of the gold-leaf electroscope with a feather brush. The brass plate is insulated by the glass jar, and the leaves diverge. Water is also a conductor, so that if a glass rod be damp it cannot be charged. For this reason the apparatus employed in these experiments must be quite dry.

Bad conductors are often called **insulators.** Of common substances, metals are the best conductors and paraffin wax is the best insulator. Between these extremes, other substances may be arranged in the following order :—

Conductors	*Non-conductors*
Metals	Indiarubber
Carbon	Silk
Acids	Glass
Water	Shellac
Ice	Ebonite
Marble	Paraffin wax

Development of charges.—To prove that a charge is developed on the rubber, a convenient plan is to mount a short cylinder of flannel A (Fig. 188) inside a *metal* tube B, bedded on a block of paraffin wax D. If a vulcanite rod C be twisted quickly inside the flannel bag, the tin tube in contact with the flannel will be found charged with positive electricity.

The **sign of the electricity depends on the nature of the rubber, as well as on the substance rubbed.** Glass rubbed with flannel becomes negative, but on friction with silk it is positive.

The following is a list of substances arranged in order so that each becomes positively electrified when rubbed with any substance which comes after it; the latter, at the same time, becomes negatively electrified:—

(+)	Metals
Cat-skin	Indiarubber
Flannel	Sealing-wax
Glass	Sulphur
Silk	Gun-cotton
Dry hand	(—)
Wood	

The charge of electricity developed on a body **resides on the surface.** This can be shown by Faraday's butterfly net (Fig. 189), the apex of which is attached to two long silk threads. It is mounted on an insulating stand and charged. If a

proof plane, a small piece of sheet metal on a vulcanite handle (Fig. 189), be placed in contact with the *outside* of the net, and then brought near an electroscope, it will be found to have taken a small charge from the net. If the proof plane be applied only to the *inside* it acquires no charge. If, however, the net be inverted by pulling the silk thread, the charge will again be found on the outside.

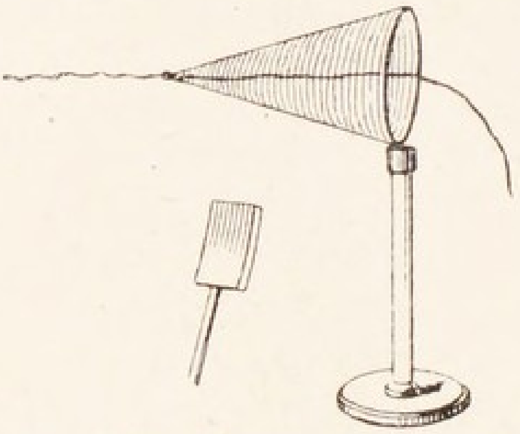


Fig. 189.—Proof plane and Faraday's butterfly net.

The same fact can be shown by charging an insulated brass sphere, and then surrounding it with two brass cups with glass handles. On the cups being removed by their glass handles, they will be found to have the charge, none being left on the sphere (Fig. 190).

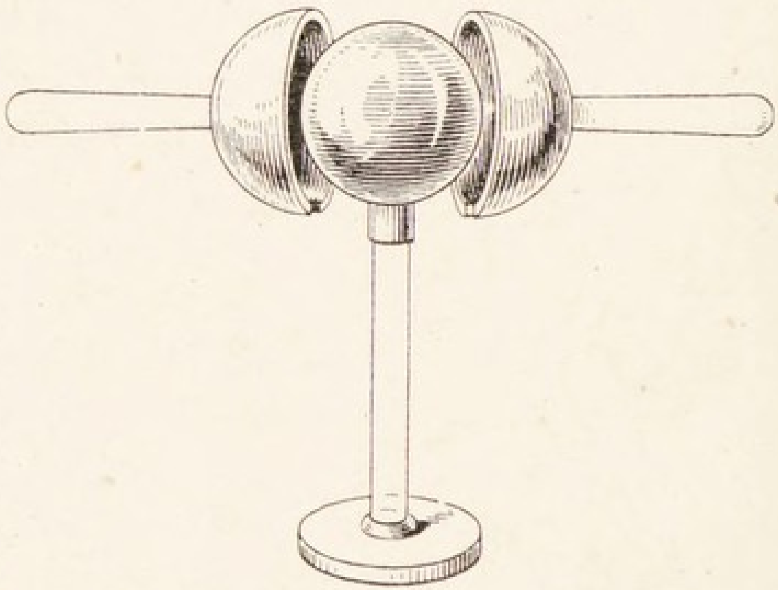


Fig. 190.—Charge always outside.

Charges of electricity may be small or large.— A unit charge, when placed at a distance of 1 cm. from a similar and equal charge, repels it with a force of 1 dyne. The force varies inversely as the square of the distance between the charges, and would be only $\frac{1}{4}$ dyne if they were 2 cm. apart. The force between a charge of Q units and another charge of Q' units at a distance of r cm.

is therefore $\frac{Q \cdot Q'}{r^2}$ dynes. If the charges are unlike, the force is one of attraction.

If a quantity, Q, of electricity be uniformly distributed over a surface of area S, then the **density** of the electricity on the surface is $\frac{Q}{S}$. The density is often not uniform, and will obviously be greatest at those parts of a conductor where the surface area is most limited. Hence the charge tends to accumulate in greater density *round sharp edges and points*, whereas on a sphere the density of the charge is uniform (Fig. 191). In fact, if a body has any sharp points, the electricity escapes from it so readily

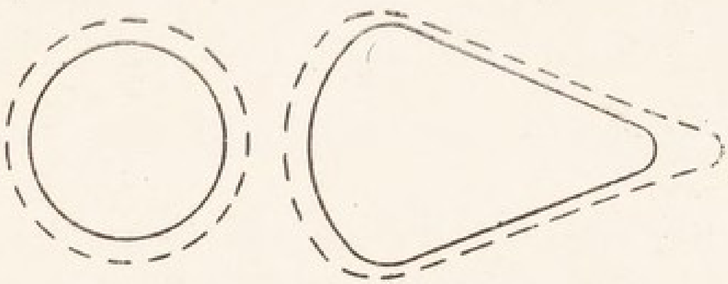


Fig. 191.—Distribution of charge.

that it is impossible to charge it to any great extent. Lightning conductors therefore always terminate in sharp points, so that when a positively charged thunder-cloud tends to charge the building by induction, the negative electricity escapes from the pointed end so readily that it prevents a charge of any great density accumulating on the top of the building.

The wind produced by this escaping electricity can be felt by the hand, and demonstrated by a candle flame, which is visibly blown about when held in the current. The discharge is utilized for therapeutic purposes in the **"breeze" treatment** with static electricity; when a point electrode,

connected with earth, is brought near the body of a patient who is seated on an insulated platform connected with the positive knob of a Wimshurst machine (p. 296) in use, he feels this *negative breeze* as a real breeze on the bare skin, or as a prickly sensation if the surface approached is covered by clothing. The treatment has been found to be valuable in the relief of neuritis, etc.

Potential and quantity.—Just as two vessels may each contain the same amount of water and yet not be equally *full*, so two conductors may each have the same quantity of electricity and yet not be charged to the same degree, or *potential*. The

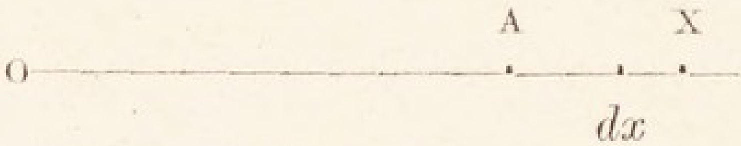


Fig. 192.—To illustrate potential.

conductors, like the vessels, may have different capacities. The distinction between potential and quantity of electricity is somewhat analogous to that between temperature and quantity of heat (p. 107). In the measurement of potential, the potential of the earth is usually chosen as zero. All bodies when connected with the earth, or "earthed," have therefore the same potential. The capacity, C, of a conductor is the quantity of electricity required to charge it to unit potential. The quantity, Q, required to charge it to a potential of V units is therefore $C \times V$, so that the charge, capacity, and potential are connected by the relation

$$Q = C\,V$$

The potential at a point A (Fig. 192), due to a charge Q situated at the point O, is measured by the work done in bringing a unit of electricity from an infinitely

distant point to the point A against the repelling electric force due to Q. When the unit charge is at X, if O X $= x$ cm., this repulsive force is $\frac{Q \times 1}{x^2}$ dynes (p. 293). If we suppose the force to keep this value while the unit charge is moved through the very small distance dx, we know that the work done in this movement is $\frac{Q \times 1}{x^2} \times dx$ (p. 27). The total work done in coming from infinity to A is the sum of a series of terms like this, and can be shown* to be equal to $\frac{Q}{O\,A}$.

In the case of a spherical conductor, of radius R, which has a charge Q, the potential at the centre due to the whole charge will evidently be $\frac{Q}{R}$. But this will be the potential, V, of the whole conductor, as there is no force *inside* a closed conductor. Therefore for the *sphere*, $V = \frac{Q}{R}$, or

$$Q = R\,V$$

but we have seen that for *any* conductor (p. 294)

$$Q = C\,V$$

therefore, for the *sphere*,

$$C = R$$

or the capacity of a sphere is measured by its radius.

Electrophorus. — This instrument (Fig. 193)

* With the notation of the integral calculus it is $Q \int_{\infty}^{OA} \frac{dx}{x^2}$ $= -Q\left[\frac{1}{x}\right]_{\infty}^{OA} = Q\left[\frac{1}{OA} - \frac{1}{\infty}\right] = Q\left[\frac{1}{OA}\right] = \frac{Q}{OA}$

usually consists of a flat circular plate, the "sole," made of vulcanite or other non-conducting material, and coated on the under-side with tinfoil. On this sole rests a somewhat smaller plate of brass furnished with a glass or vulcanite handle. The sole, when struck with fur or cat-skin, becomes charged with negative electricity. The brass plate is now placed on the vulcanite, which it only touches at a few separate points, and induction therefore occurs (Fig. 193, 1). When the brass plate is touched and "earthed" the repelled negative electricity escapes (2), and the brass plate, when lifted (3) by the insulating handle, is found to be charged with positive electricity. A small spark may be obtained from it on presenting the knuckle, or it may be used to light a gas burner. As the plate touches the sole in relatively few points, the charge on the sole remains almost undiminished, and the plate can be recharged many times by repeating the above process.

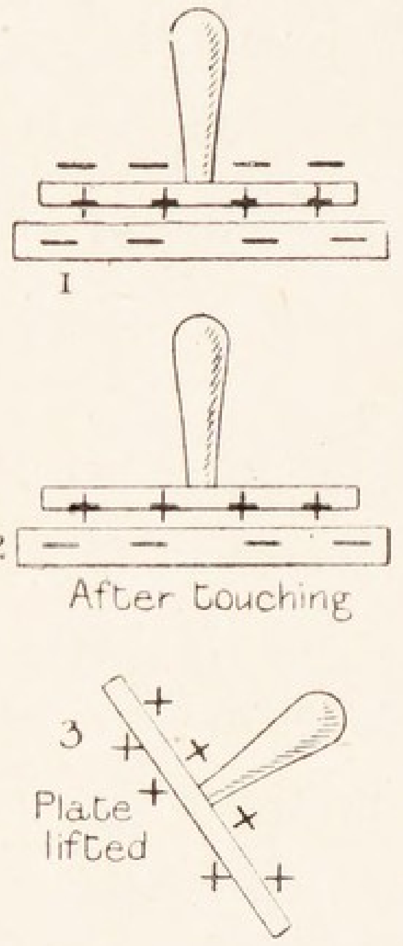


Fig. 193.—Electrophorus.

One of the best machines for producing electricity by friction and induction is the **Wimshurst machine** (Fig. 194). In this machine, at least two thin circular plates of vulcanite, or glass, are mounted so as to rotate in opposite directions. On the surface of the plates are cemented, at regular intervals, tongues of tinfoil B B, which, as they rotate, touch the brass brushes A A. C C are

two U-shaped rods of metal, the sides of which towards the plate terminate in a sharp edge or in points. The brushes A A are connected by a brass rod D. The second vulcanite plate is similarly fitted.

The action can be explained with the help of the next diagram (Fig. 195). For the sake of simplicity the discs are shown as cylinders revolving in opposite directions. A small charge of negative electricity is developed by the friction of the brushes on one of the tongues of tinfoil A; this acts by induction on the opposite tinfoil B of the other vulcanite plate. As B rotates to the left it touches brush C, and the negative electricity, repelled by induction, passes over and charges D with negative. B, remaining positive, rotates until it comes opposite the U-shaped piece X, when, acting by induction, it draws off negative electricity from the sharp spikes of X, and repels positive electricity to the rounded knob in which X terminates. The positive charge on B is neutralized by the negative charge from the spikes. Passing on, after neutralization, B becomes subject to induction from E, and when it touches the brush D positive electricity passes over by the brush D through the cross-wire to C. If we trace the progress of a tinfoil tongue on the other disc, say A, which is negative when it arrives at the U-shaped piece connected with Y, it draws off + from Y, and becomes neutral, while negative electricity is

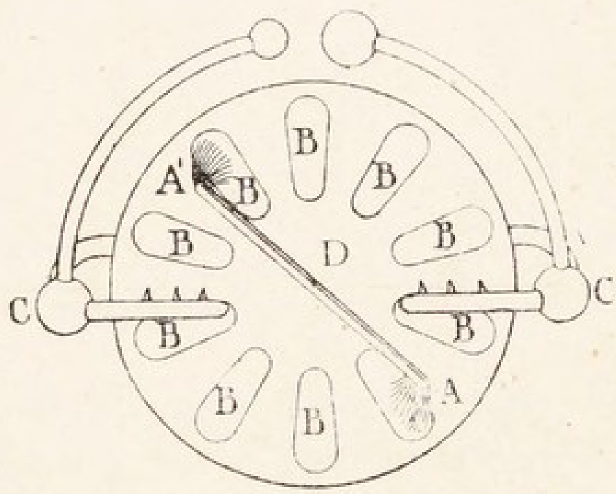


Fig. 194.—One plate of Wimshurst machine.

repelled to the terminal knob of Y. A is subsequently exposed to induction at F, and when it touches the brush G the negative electricity is repelled over to H, and so on.

It will be noticed that the tongues of tinfoil on the upper half of the outer cylinder are constantly positive, and those on the inner cylinder negative, the reverse being the case in the lower half. The result

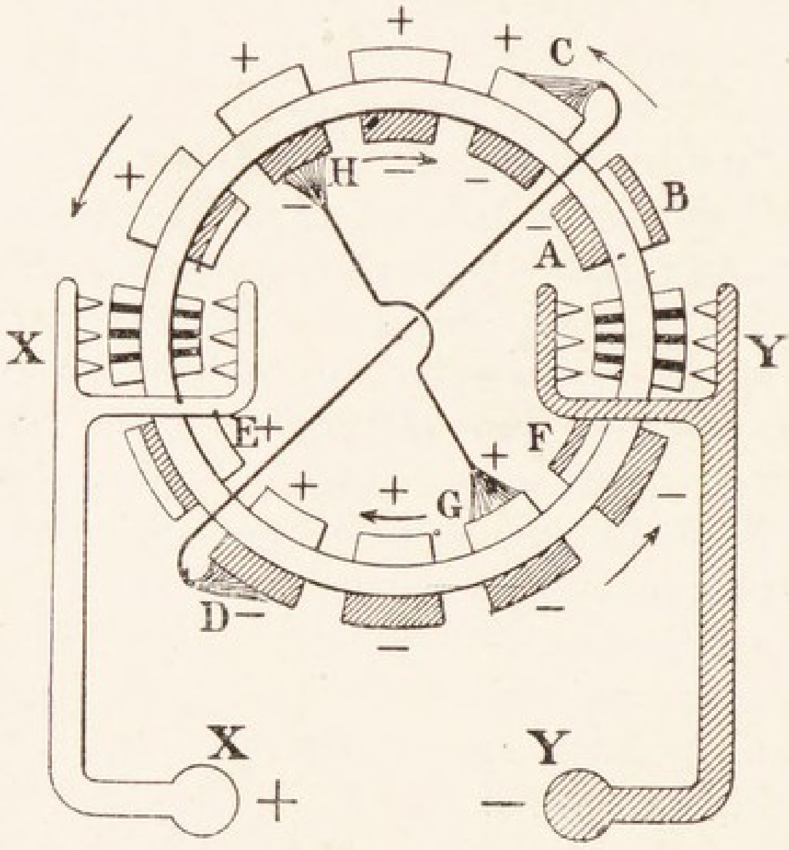


Fig. 195.—Wimshurst machine. (*After Silvanus Thompson, "Manual of Electricity."*)

is that X is constantly being drained of negative electricity, and Y of positive, so that a charge of positive electricity accumulates on the knob X and negative on Y until a spark crosses.

The machines used for medical treatment (p. 293) are fitted with more than one pair of plates, and generally have from four to six pairs; the first and last plates revolve independently, but the intervening plates are grouped in twos, each group re-

volving in the opposite direction to the group, or plate, on either side of it; we shall therefore have, with eight plates, three groups and the two terminal plates, and shall require five driving bands, the alternate ones being crossed. Such a machine is conveniently driven by an electromotor of $\frac{1}{4}$ h.p. to $\frac{1}{2}$ h.p. The *length* of the spark is some indication of the voltage or potential difference of the machine, and is influenced by the *diameter* of the plates, which should be from 30 in. to 36 in.; the total output of electricity, and therefore the frequency and *volume* of the spark, is influenced by the *number* of the plates. The speed of revolution of course affects both voltage and output.

Thunderstorms.—The cause of the electricity in the air is not known. It may be due to the friction of air, or when aqueous vapour condenses to water some of the energy may take the form of electricity. The clouds are usually positive, but in wet weather may be negative. The electrical state of the air is ascertained by allowing water to drop from a carefully insulated can. If the air be negative it draws positive from the drops of water and the can until the can is at nearly the same potential as the air.

Franklin first demonstrated that lightning was due to an electrical discharge, by flying a kite during a thunderstorm at Philadelphia, in June, 1752. At first he could get no sparks, the dry silk thread which held the kite being a non-conductor; but when the rain fell and the string became wet, plenty of sparks could be obtained by presenting his knuckle. In 1753 Richmann was killed while making a similar experiment at St. Petersburg.

The lightning may pass between clouds charged with opposite kinds of electricity, or between a charged cloud and the earth by induction.

Lightning conductors.—It is usual to protect buildings by lightning conductors. These, as we have seen (p. 293), should end in points. The effect of points in preventing the charging of a surface is well shown by the following experiment: A head of hair on a doll's head is connected with a machine and electrified, when it will be seen that the hairs all repel each other and stand out like a brush. If a knuckle be presented to them they are all attracted, but if the point of a needle be held to them they are all, as it were, blown away. The electricity of the opposite name escapes so readily that the hairs are surcharged with electricity of the same name, and so are repelled.

Lightning conductors should therefore end in points, preferably gilded, and should be connected by stout iron or copper rods with the earth. If the earth is not damp where the rod enters the soil, the end of the conductor should be buried in a pit tightly packed with coke. All masses of metal, lead roofs, pipes, etc., should be in metallic connection with the conductor so as to neutralize induction.

The **return shock** is a curious phenomenon by which people have been killed by electricity when no thunderstorm has taken place in the immediate vicinity. A cloud charged with (say) negative electricity extends for some distance, and subjects the building A (Fig. 196) to induction, but the distance is too great for a flash to strike across. A thunderstorm is raging at B, and the cloud is discharged by striking to earth there, and the restoration of A to its normal electrical state may be so sudden and violent as to produce fatal results.

The condenser.—By means of induction the capacity of a conductor may be greatly increased. Apparatus by which this is effected constitutes a

condenser. It includes (1) an insulated conductor to which the charge is given; (2) a conductor connected with earth; (3) a *dielectric* or non-conducting medium between (1) and (2).

If we mount a circular brass plate on a glass stand, and present to it a charged electrophorus plate, a spark will pass. If we repeat this two or three times we shall find that no spark will pass, indicating that the plate is fully charged. If against this plate we place a thin, slightly larger sheet of *glass* A (Fig. 197), and on the other side a second brass plate *connected*

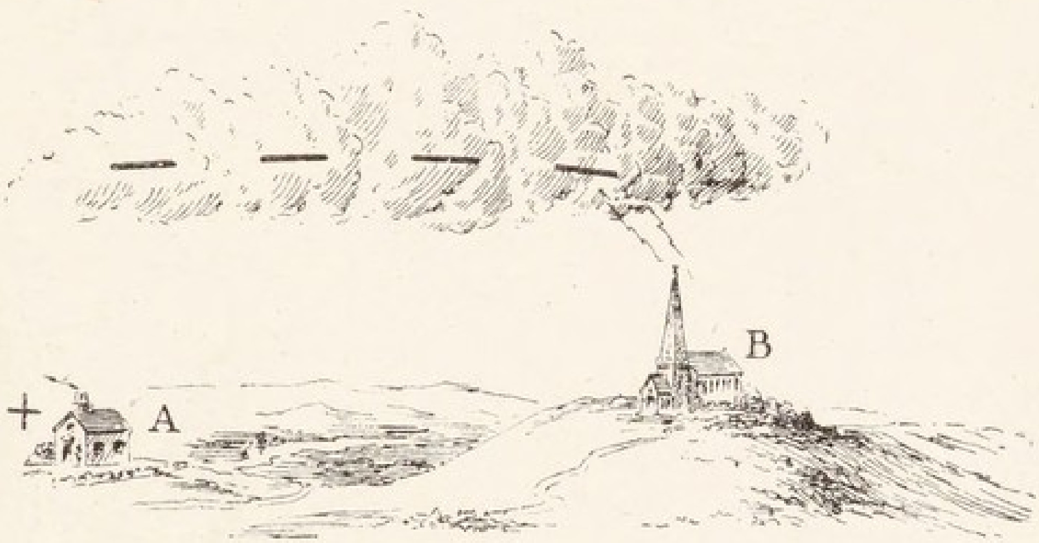


Fig. 196.—Return lightning shock.

with the earth by a wire, we shall find that the capacity of the brass plate is enormously increased. The electrophorus plate is charged with positive electricity, and charges B with positive; this acts by induction through the glass plate attracting negative electricity to the near side of C, and repelling positive electricity to the earth. Hence B has practically no *free* charge, and can therefore receive another quantity from the electrophorus, and this in its turn is similarly neutralized and held at the face of A by the process of induction. So the action goes on until we have comparatively enormous charges of + and − on the

plates B and C, attracting each other powerfully, but prevented from combining by the intervention of a *dielectric*, the plate of glass.

The Leyden jar.—The action of this form of condenser was discovered by accident by a philosopher of the eighteenth century, who, wishing to electrify water, took a bottle of water and, holding it in his hand, placed in it a chain from the prime conductor of an electrical machine. After working the machine for some time, as apparently nothing had happened he proceeded to lift out the chain, when he received

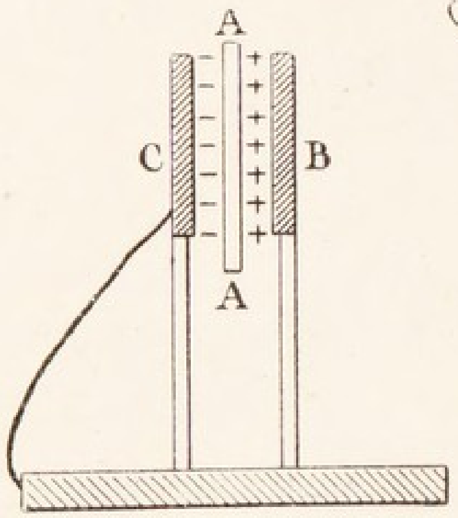


Fig. 197.—Condenser.

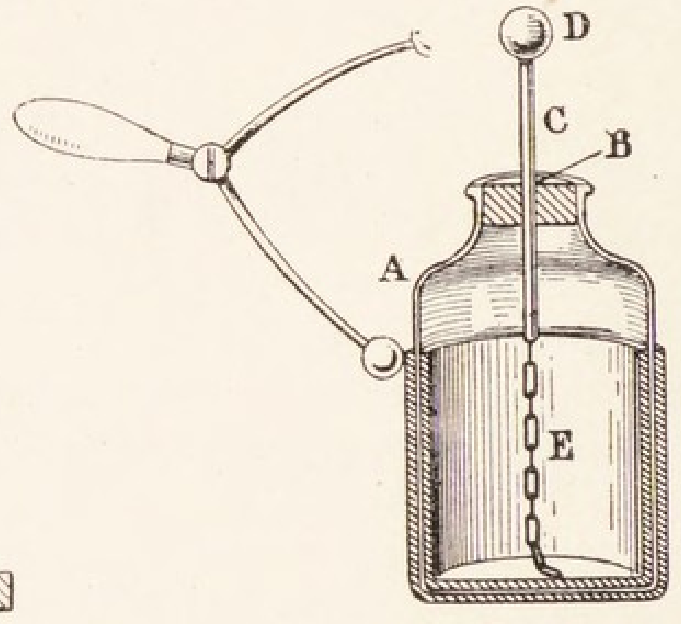


Fig. 198.—Leyden jar.

a severe shock. The water represented the insulated conductor, the hand was the earth-connected conductor; the bottle was the dielectric.

The Leyden jar (Fig. 198) consists of a glass bottle or jar A, which is lined below with tinfoil on both sides. The upper portion of the jar, for about two inches, is free from tinfoil and carefully lacquered with shellac varnish. A circular piece of varnished wood rests on the top of the jar B, and from the centre of it passes a brass rod C, ending in a knob D. Internally, C is connected with the inside coating

of tinfoil by a brass chain E. The outside coating rests on the table, so that it is connected with the earth. The knob D is connected with an electrical machine and the inner coating is thus charged with (say) negative electricity; this acts by induction on the outer coating, attracting positive electricity towards the glass and repelling negative to the earth. Powerful charges of opposite kinds thus accumulate on opposite sides of the glass dielectric, and when the two coatings are connected, as in Fig. 198, by a discharger with a glass handle, a bright flash passes. If the discharger be presented a second time a much smaller spark will pass.

The jar cannot be charged if placed on a glass support unless the knob is connected to earth. The functions of outer and inner coating are then reversed. The charges are really on the two surfaces of the glass, as can be shown by having the two metal coatings movable and made of tinplate instead of tinfoil. The brass rod C is surrounded by a glass tube, so that it can be lifted, and with it the internal coating, which is soldered to it, without getting a shock. The jar having been charged as usual, the inside coating is lifted out by the glass tube, then the glass jar is removed from the outside tin can, and the two coatings are made to touch each other so as to discharge any free electricity. On the jar, etc., being replaced, and the two coatings brought into contact by a discharger, a flash will pass. If Q, C, V represent the charge, capacity, and potential of the inner coating, the energy of the discharge is $\frac{1}{2}\, C.V^2$.

Leyden jars can be coupled together to form batteries, either by placing them in a box and coupling all the inner coatings together by brass rods, or by placing each of them on an insulated support

and connecting the inner coating of one with the outer coating of the next, the outer coating of the last jar being "earthed." This latter method is called coupling in "cascade."

A Leyden jar can be discharged slowly (Fig. 199) by a ball A suspended by a silk thread oscillating between the knob and some object, as a bell, connected with earth. Indeed, the spark itself is not the single discharge it appears to be, but is really a succession of exceedingly rapid oscillatory discharges, the period of oscillation being sometimes but a fraction of a micro-second.

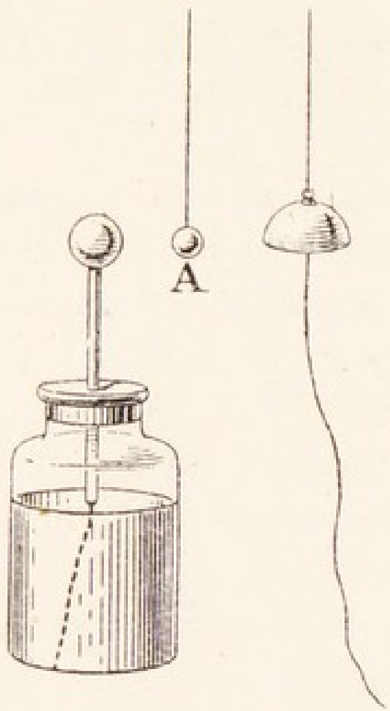


Fig. 199.—Slow discharge of Leyden jar.

The duration of the whole spark is about $\frac{1}{24000}$th of a second; the period of a single oscillation is perhaps one-twentieth of this. The duration of a flash of lightning is about $\frac{1}{10000}$th of a second, so that death by lightning is practically instantaneous and a person never really sees the flash which kills him; he is dead before the brain has time to translate the image on the retina into vision.

The velocity of the discharge of a jar through copper wire has been given as 288,000 miles per second. This oscillating discharge is the source of **high-frequency currents,** sometimes employed in medical practice. For this purpose the inner coatings of two large jars are attached to the terminals of a large induction coil (p. 379); the outer coatings are connected by a helix or solenoid having a few turns of thick copper wire, or *tubing*—for this high-

potential electricity keeps mainly, like static charges, on the surface of conductors. The working of the coil results in the usual sparks across the gap provided between the knobs of the jars, but, simultaneously, oscillating currents of high frequency, and of voltage 100,000 or even more, circulate in the helix, and also in any derived parallel circuit in which the patient may be included. The propagation of similar oscillations in the ether gives rise to the Hertz waves and makes possible wireless telegraphy; the oscillations vary widely in frequency (n) and in wave length (λ), but $n\,\lambda = 3 \times 10^{10}$ cm. per sec., which is practically the velocity of light.

The Atlantic cable and all submarine cables act as Leyden jars. The copper wire represents the inside coating, the gutta-percha and other insulating material the glass jar, and the ocean the outer coating; so that it takes a sensible interval of time—one second or so—to charge a long cable before any electrical disturbance is noted at the other end.

The capacity of a condenser of any given form depends upon—

(1) The surface area of the plate.

(2) The thickness of the dielectric: the thinner the dielectric, the greater the capacity.

(3) The nature of the dielectric: each subtance has its *specific inductive capacity.*

The capacity of the plate condenser of Fig. 197 is

$k \times \dfrac{S}{4\pi\, t}$, when S is the area of the face of B,

t is the thickness of A, and k is the specific inductive capacity of the substance of which A is made.

Unit of capacity.—The unit of capacity, a *farad*, is the capacity of a condenser which is charged by 1 *coulomb* (p. 339) to a potential of 1 *volt* (p. 342). The corresponding absolute C.G.S. electro-magnetic unit of capacity is equal to 10^9 farads (pp. 338, 352). The farad is inconveniently large, and the *microfarad*, one-millionth of a farad, is commonly employed in practice.

Exercises

1. Calculate the force between two electric charges, of 10 units and 20 units respectively, separated by a distance of 1 metre.

2. If a charge of 100 units is uniformly distributed over the surface of an insulated spherical conductor of diameter 4 cm., what is (i.) the surface density of the charge, (ii.) the surface potential ?

3. Calculate the capacity of a plate condenser (Fig. 197) if the plate B is circular and has a diameter 12 cm., the dielectric glass of thickness 1·5 mm., and the specific inductive capacity of the glass 8.

4. When the plate condenser of the previous example has a charge of 8 units, what is the potential of the plate B, and what energy would be liberated by the discharge of the condenser ?

5. What is the potential at a point P due to a charge of 10 units situated at a distance of 1 metre ?

(For Answers, *see* p. 390.)

CHAPTER II

MAGNETISM

Magnets, Natural and Artificial—Magnetic Field—Lines of Force—Magnetic Induction—Terrestrial Magnetism—Declination — Inclination — Magnetometer — Dip Circle—Exercises.

THERE is so much resemblance between the phenomena of electricity and those of magnetism that we cannot study the one without reference to the other. In Gilbert's day the relationship was clearly recognized, though even now it is not fully understood.

The term *magnetism* has been derived from Magnesia, in Lydia, where the lodestone, a magnetic oxide of iron (Fe_3O_4), was found in classical times. This oxide of iron has the power of attracting iron filings, and if cut and suspended in a suitable manner will point nearly north and south. If such a lodestone be placed underneath a thin sheet of paper and iron filings be sprinkled on the paper, it will be observed that the filings chiefly cluster round two spots, one at each end of the lodestone, where the magnetic force seems concentrated. These spots are termed the *poles* of the magnet. The one which points to the north when the magnet is suspended is called the *north-seeking* or, more shortly, the *north* pole, and in steel magnets is usually marked with a file mark or stamped with an N; the other end is the *south* pole.

If a needle be drawn lengthwise over the north pole of a magnet, the end which leaves it last becomes a south pole. If the needle be afterwards thrust through

a piece of cork and floated on water, it will point north and south. If a second needle be magnetized, the following experiments can be made: The north

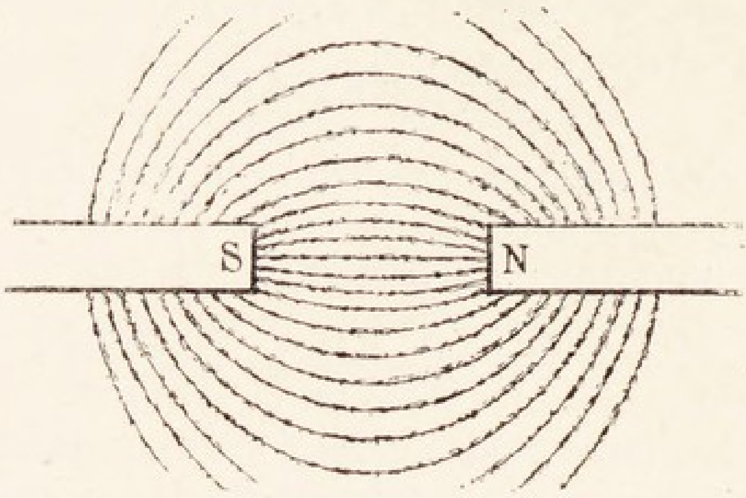


Fig. 200.—Lines of force, N. and S. (Diagrammatic.)

pole of one needle being presented to the north pole of the needle floating on the water, it will be seen that the two north poles repel each other, and that

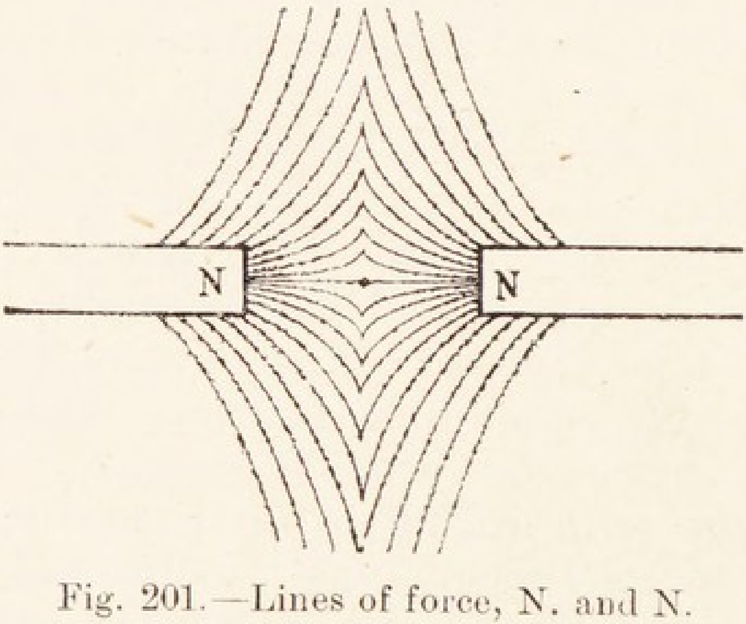


Fig. 201.—Lines of force, N. and N.

the two south poles also repel each other, but that the south pole attracts the north pole, and vice versa. The same effects may be observed with a steel magnet

and a compass needle. As was the case with electric charges (p. 287), *like* repel and *unlike* attract.

Lines of force.—The space which surrounds a magnet is termed the *magnetic field.* This magnetic field is traversed by lines of force, which can be rendered visible by scattering iron filings over a magnet placed under a sheet of paper. The lines indicate the direction in which the magnetic force acts at any point of the field. If we place a north opposite a south pole, the lines resemble those in Fig. 200. If two north or two south poles face each

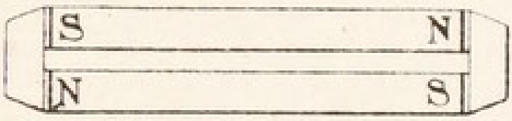


Fig. 202.—Bar magnets.

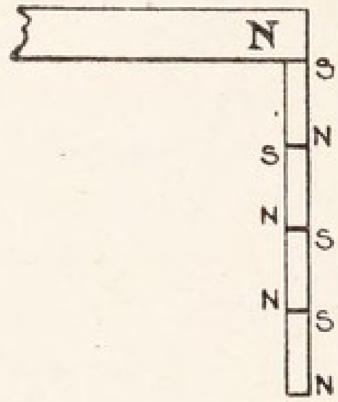


Fig. 203.—Magnetic induction.

other, the lines present the appearance shown in Fig. 201. Magnets may be in the form of bars, in which case they are usually made in pairs and kept in a box with the north pole of one opposite the south pole of the other, and a piece of soft iron, called a keeper, at each end (Fig. 202). Sometimes the bar is bent into the well-known horseshoe form, which also has a keeper. The keeper serves to concentrate the lines of force and prevent loss of magnetism.

Magnetic induction.—Like electricity, magnetism can be *induced* in a magnetic substance. A piece of soft iron wire is not necessarily a magnet. It does not attract another piece of soft iron. If, however,

it be brought near the north end of a bar magnet it becomes, for the time, a magnet, the end nearest the bar magnet being a south pole and the lower end a north pole. If a second piece of iron wire be brought into contact with the lower end of the first, it also becomes a magnet, and so on (Fig. 203). All the pieces of iron wire will fall if the south pole of a second magnet be brought over the north pole of the first. Directly the pieces of iron wire are removed from the magnet they lose their magnetism (*cf.* Electrical Induction, p. 288), so that if the top one be detached from the magnet the chain spontaneously falls to pieces.

If a piece of *hard steel* wire be brought near the pole of a magnet, it is at first not so powerfully attracted as the soft iron, but when it has been in contact a short time, especially if it be drawn over the magnet, it will be found, on withdrawal, to be permanently magnetic. It seems as if more energy were required to twist the molecules of the steel into the position proper to a magnet, but when once set in that position they retain it, whereas the molecules of the soft iron do not. This is attributed to the *coercive force of steel.* Other substances besides iron are susceptible of magnetic influence in a lesser degree, and sometimes of an opposite character. A bar of nickel or cobalt, for instance, suspended between the poles of a horseshoe magnet tends to set itself lengthwise from pole to pole; a bar of bismuth, antimony, or copper, on the other hand, tends to set itself at right angles to this direction. Iron, nickel, etc., are therefore called *para*-magnetic substances, while copper, etc., are called *dia*-magnetic.

Effect of breaking a magnet.—If a magnetized needle be broken in half, each half will be found to be a magnet. On again breaking each half, four

magnets are obtained (Fig. 204). In this respect magnetic differs from electrical induction. As we have seen, positive electricity can be isolated on one

Fig. 204.—Effect of breaking a magnet.

conductor and negative on another; but no one has yet separated north from south magnetism.

The magnetic force can be exerted through glass and many other substances. At a red heat all magnetism is lost. The student must be careful to

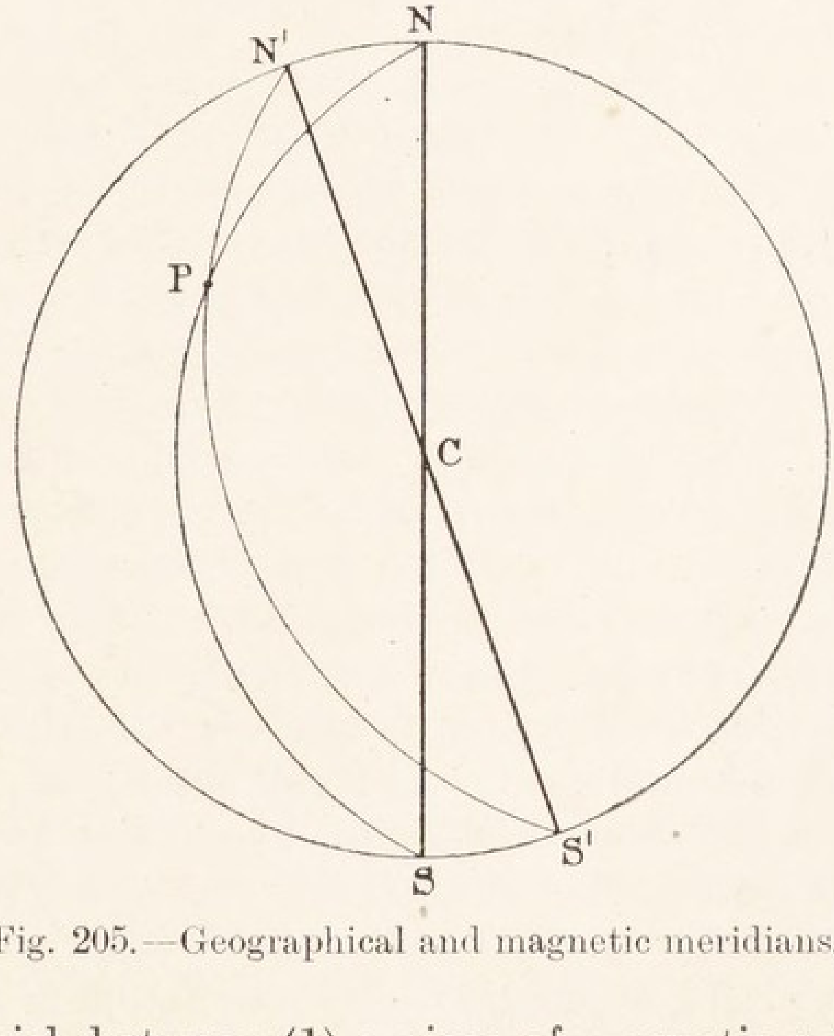


Fig. 205.—Geographical and magnetic meridians.

distinguish between (1) a piece of magnetic substance and (2) a magnet: (1) is only *attracted* by either pole of a magnet, (2) is both attracted and repelled, each extremity showing opposite behaviour when

presented to either pole of a second magnet. Repulsion is therefore the true test for a magnet.

The earth a magnet.—If a small bar magnet, placed on cork, be allowed to float freely on water, and be not influenced by another magnet, it will behave like a compass needle and point nearly north and south. When at rest the magnetic axis lies in the plane of the magnetic meridian (p. 311) of the place. If turned into any other position, the magnet when released will always return to the position of rest. The force which causes it to do so is that of terrestrial magnetism. The earth is itself a magnet. The magnetic poles, N′, S′ (Fig. 205), do not coincide with the geographical poles, N, S, but the magnetic axis, like the geographical axis, passes through the earth's centre, C. Moreover, just as the geographical meridians on the earth's surface are great circles whose common diameter is N C S, so the magnetic meridians are similar great circles whose common diameter is N′ C S′. At any place P (Fig. 205) on the earth's surface, these two meridians will evidently intersect at a certain angle. This angle is called the **magnetic declination** of the place P, and is often denoted by δ. If at P we draw a tangent to the circle N′ P S′ and also a tangent to the circle N P S, the angle between these two tangents will be δ. The direction of the first tangent is the direction in which all compass needles at P will point. At London the value of δ in 1911 was about 16° West; N′ therefore lies to this extent to the west of N (Fig. 205), but appears from the observations recorded in the table on p. 313 to be gradually moving in an easterly direction and approaching N. If we regard the magnetic axis as executing a slow vibration, it is clear that only a portion of the complete period has yet been observed.

TABLE OF VARIATIONS IN DECLINATION *

Year	*Declination* (δ)	*Year*	*Declination* (δ)
1580	11° 15′ E.	1860	21° 38·9′ W.
1622	6° 0′ ,,	1865	20° 58·7′ ,,
1657	0° 0′ ,,	1870	20° 18·3′ ,,
1672	2° 30′ W.	1875	19° 35·6′ ,,
1692	6° 0′ ,,	1880	18° 52·1′ ,,
1795	23° 57′ ,,	1890	17° 50·6′ ,,
1805	24° 8′ ,,	1900	16° 52·7′ ,,
1820	24° 34′ ,,	1905	16° 32·9′ ,,

Before 1860 the values in the above table refer to different places in the south-east of England, but may be taken as nearly true of London. From 1860 onwards they refer to Kew.

The compass or declination needle is mounted on a vertical axis and rotates about this axis in a horizontal plane. The force which directs its motion is the horizontal component of the earth's magnetic force, and is generally denoted by H. It acts upon a magnet pole of unit strength with a force of about 0·185 dynes at the present time in London. As the force produces rotation and not translation, it is evidently a couple (p. 46). The magnitude of the force has undergone a gradual increase, amounting to nearly 0·01 in the last fifty years. The values at intervals during that period are given in the table (p. 316).

Inclination.—Unlike the compass needle, the *dip needle* is so mounted that it can turn freely in a

* The values in the table are quoted from the "Encyclopædia Britannica," 11th ed., 1911.

vertical plane about a horizontal axis which passes through the centre of gravity of the needle. When placed in the magnetic meridian at London, this needle does not rest in a horizontal position. The north end *dips* downwards and the line joining the

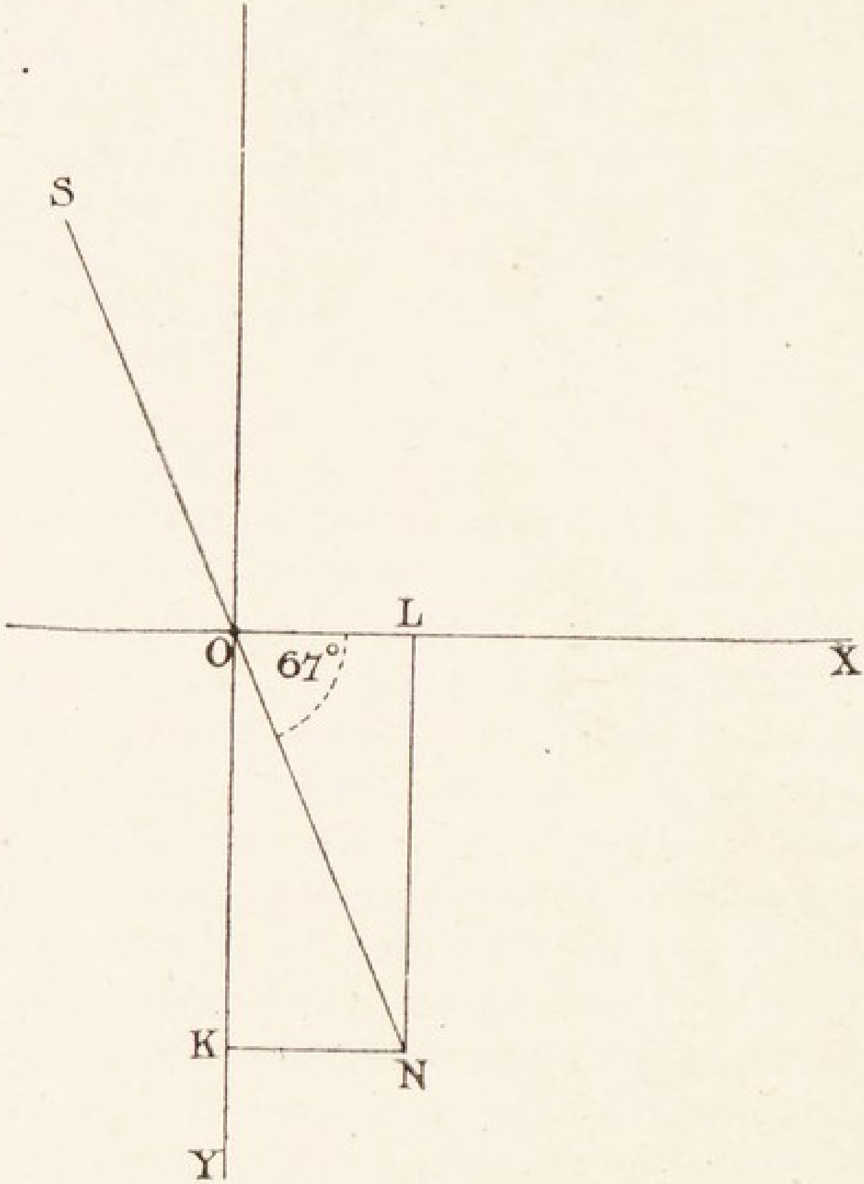


Fig. 206.—Approximate position of the dip needle at London in 1911.

poles is inclined to the horizon at an angle of nearly 67° (Fig. 206). This is termed the *dip* or *inclination* (θ). At the magnetic north pole the dip is 90° ; the needle is vertical. At the magnetic equator it is horizontal, there is no dip. As we travel southwards the *south* end of the needle dips downwards.

These effects can be reproduced by passing a dipping needle along a bar magnet (Fig. 207). The earth thus behaves like a huge magnet. The downward dip or vertical rotation is produced by the *vertical* component, V, of the earth's magnetic force. This is also a couple. If the vertical plane in which the magnet moves be at right angles to the magnetic meridian, the effect of H will be nil, and the needle will therefore be vertical. This enables us to locate the magnetic meridian in the laboratory. The total intensity, I, of the earth's magnetic force is the resultant of H and V, and as we know the direction

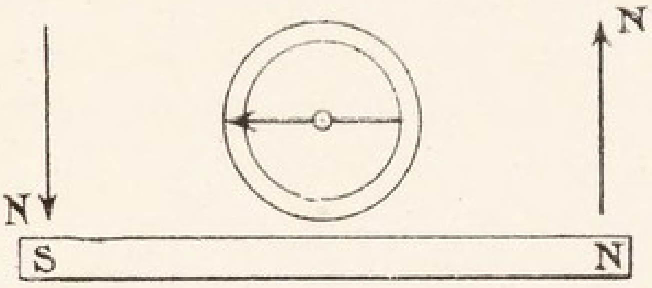


Fig. 207.—Magnet and dipping needle.

of I we can draw the parallelogram of forces (p. 34) and find V and I if H is known.

If from N (Fig. 206) we draw N L parallel to O Y, to meet O X in L, and N K parallel to O X, to meet O Y in K, the sides O L, O K of the parallelogram O L N K will represent the forces H, V respectively in magnitude and direction, and the diagonal O N will represent their resultant, I. They are evidently connected by the following relations :—

$$H = I \cos \theta$$
$$V = I \sin \theta$$
$$H^2 + V^2 = I^2$$

At London θ is now nearly 67° N. and is slowly declining in value. In this connection "London,"

generally means Greenwich or Kew. The values in the following table refer to Kew :*—

Date	*Dip* (θ)	*H*	*Date*	*Dip* (θ)	*H*
1857	68° 24·9′	0·17474	1891	67° 33·2′	0·18193
1860	68° 19·8′	0·17550	1895	67° 25·4′	0·18278
1865	68° 8·7′	0·17662	1900	67° 11·8′	0·18428
1870	67° 58·6′	0·17791	1905	67° 3·8′	0·18510
1874	67° 50·0′	0·17903	1908	67° 0·9′	0·18515

The difference in the values of the magnetic elements at these two London observatories in the year 1909 is shown in the following table :*—

Place	δ *West*	θ	*H*	*V*
Kew . .	16° 10·8′	66° 59·7′	0·18506	0·43588
Greenwich .	15° 47·6′	66° 53·9′	0·18526	0·43432

By the courtesy of the Superintendent, I am able to add the values of these elements at Kew for the year 1915 :—

Kew (1915) .	15° 18·4′	66° 56·6′	0·18463	0·43376

Since 1909 the values of these elements have continued to diminish, and comparison shows that at Kew, during this interval, δ has decreased by about 8·7′ per annum, θ by about 0·5′ per annum, H by

* Quoted from the "Encyclopædia Britannica," 11th ed., 1911.

about $0{\cdot}0_47$ per annum, and V by about $0{\cdot}0_335$ per annum.

If, however, we compare the values of δ and θ for 1915 with those for 1865 (p. 313), we find that for this period of *fifty* years the mean annual decrease in θ is $1{\cdot}442'$, and in δ is $6{\cdot}806'$; comparison suggests that the declination needle is perhaps moving more rapidly because it is moving *towards* its centre of oscillation, and the dip needle less rapidly because it is moving *from* its centre of oscillation.

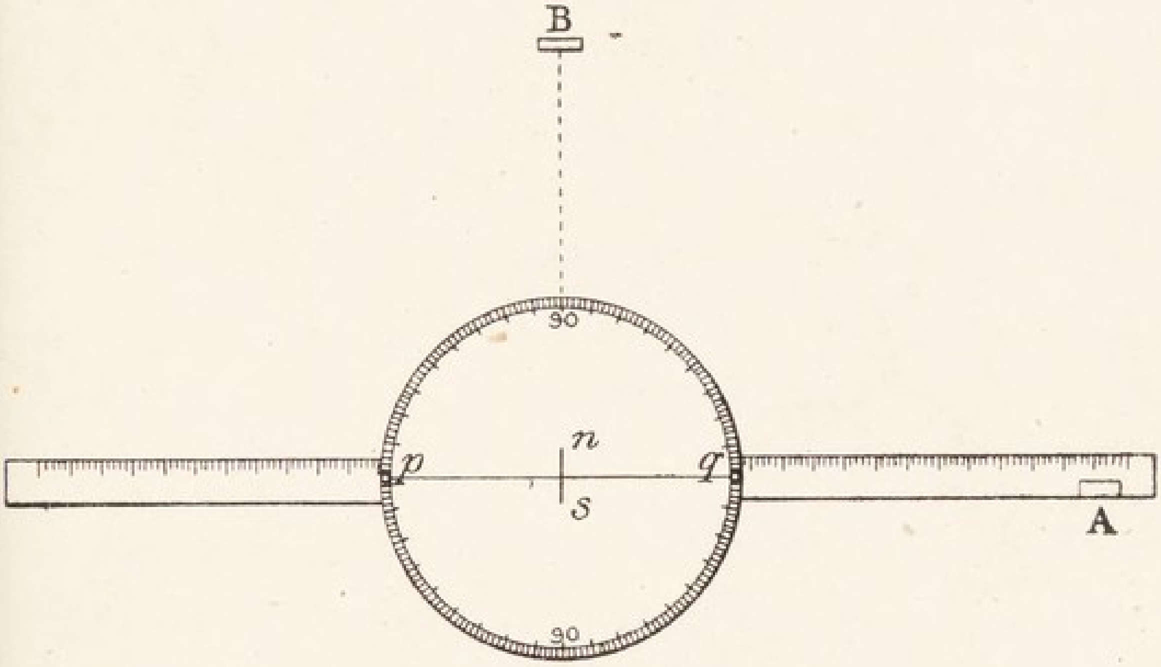


Fig. 208.—Magnetometer showing magnet in each of the A and B positions of Gauss.

Determination of H and θ.—The relations already stated show that if H and θ are known, I and V can be immediately found. H can be compared with the force exerted by a given magnet, by means of the **magnetometer.** This instrument (Fig. 208) consists essentially of a small compass needle $n\,s$ to which is attached a light pointer $p\,q$ whose extremities travel over a graduated circular scale as the needle turns about a vertical axis which passes through the centre of the circle. When the needle is in the

magnetic meridian, under the action of the earth's magnetism alone, the scale is so placed that the ends of the pointer are both at zero. A straight wooden scale projects on either side as shown. On this scale the distances marked are measured from the centre of the circle. If a magnet, A, be placed on this scale, at right angles to the magnetic meridian, and due east (or west) of the needle *n s*, it is then in what is known as the *A position of Gauss*. If, on the other hand, it be placed with its centre due north (or south) of *n s*, as B in the figure, it is then in the *B position of Gauss*. The effect of the magnet, in either position, will be to deflect the needle. The final position of the needle will be the result of equilibrium between the couple due to H and the couple due to the magnet. We may take as a working formula—

$$\frac{M}{H} = \tfrac{1}{2}\, r^3 \tan \delta, \textit{ in the A position,}$$

$$\text{and } \frac{M}{H} = r^3 \tan \delta, \textit{ in the B } \quad ,,$$

In these formulæ M is the *moment* of the magnet, and is therefore equal to the product obtained by multiplying the strength, m, of one of its poles by the distance, $2l$, between the poles (this distance may be taken as approximately equal to five-sixths of the length of the magnet); r is the distance between centre of compass needle and centre of magnet; δ is the deflection. A *small* magnet is employed in the experiment because the formulæ are most nearly true when the value of l is insignificant compared with that of r.

This experiment evidently enables us to compare M_1 and M_2, the moments of two different magnets. Place the two magnets in the A position, but on

opposite sides of the compass needle, and with their similar poles now opposing each other. Adjust their distance from the needle so that it remains at rest in the magnetic meridian. Each magnet must then be producing equal and opposite deflection, δ. Let the distances be now r_1 and r_2, then we know

$$\frac{M_1}{M_2} = \frac{{r_1}^3}{{r_2}^3}$$

If the deflection, δ, obtained in the A, or B, position is 45°, we have $\tan \delta = 1$, and then $\frac{M}{H} = \frac{r^3}{2}$, or r^3, respectively. In this midway position the earth and the magnet are really producing equal and opposite deflections. This will be also the case if we lay the bar magnet on the bench, in the magnetic meridian, with its N. pole pointing north, trace a straight line on the bench bisecting the axis of the magnet at right angles, and then find a point (P) on this line where a very short compass needle is not deflected from the meridian but remains parallel to the magnet. In this case also

$$\frac{M}{H} = d^3$$

where d is the distance from either pole of the magnet to the *neutral point* P.

Another relation between M and H can be obtained by suspending the magnet in a horizontal stirrup, in the magnetic meridian. Slightly displace the magnet from its position of rest—by the momentary approach of another magnet, for instance—and carefully determine the period of the resulting oscillation. If $2t$ be the time of a *complete* vibration, we know

$$t = \pi \sqrt{\frac{I}{M\,H}}$$

and therefore $\quad M\,H = \dfrac{\pi^2.I}{t^2}$

In this formula I is the *moment of inertia* of the magnet. In the case of a simple rectangular bar magnet, if b and c are the lengths of the horizontal sides, and W is the weight of the magnet,

$$I = \frac{W\,(b^2 + c^2)}{12}$$

This oscillation experiment will evidently enable us to compare the values of H at two different places. Suppose that at the first place the magnet makes $\boldsymbol{n_1}$ complete oscillations per second, so that $2\,t_1 = \dfrac{1}{n_1}$, then

$$M\,H_1 = \frac{\pi^2.I}{t_1^{\,2}} = 4\pi^2.I.n_1^{\,2}$$

Similarly, if the same magnet makes n_2 complete oscillations per second at the second place, then

$$M\,H_2 = \frac{\pi^2.I}{t_2^{\,2}} = 4\pi^2.I.n_2^{\,2}$$

therefore

$$\frac{H_1}{H_2} = \frac{n_1^{\,2}}{n_2^{\,2}}$$

From the *two* equations discussed in this section we can evidently find the two unknowns, M and H.

The value of θ is directly determined by readings of a **dip circle.** This instrument (Fig. 209) consists essentially of a vertical circle which serves as a background to a dipping needle, whose inclination is indicated by a scale graduated on the border of the circle. The face of the circle against which the

needle is seen is covered with looking-glass so that a reflection of the needle is seen by the observer if his line of vision is *not*—as it *should be*—truly normal to the plane of the needle and of the circle. In reading the inclination, therefore, the observer must so direct his vision that he sees no reflection of the needle in the face of the circle. In order that the reading then correctly taken shall be the true

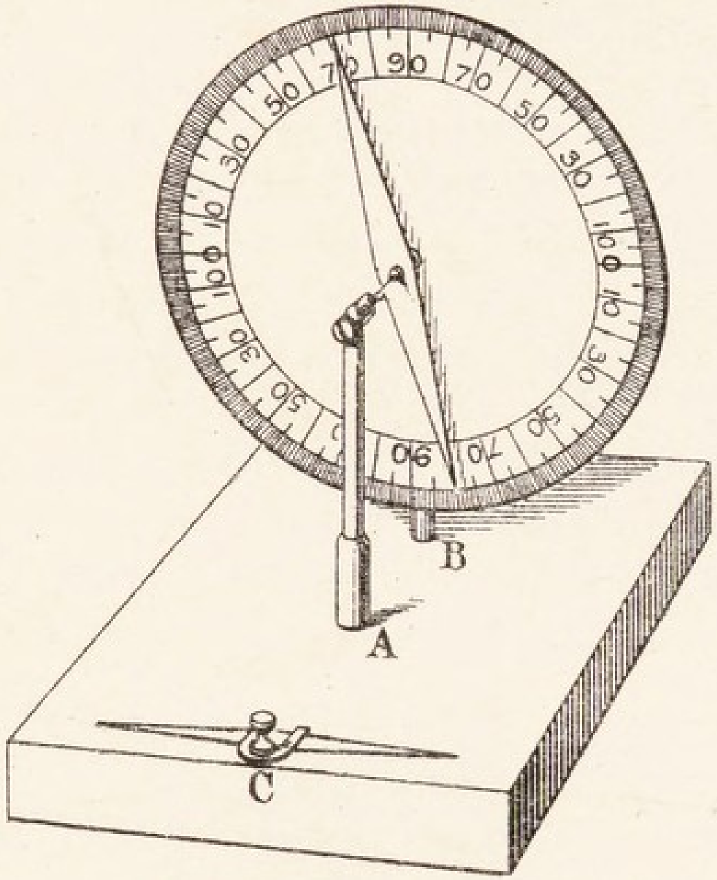


Fig. 209.—The dip circle.

dip, many other precautions must be adopted. The plane of the circle, or needle, must be parallel to the magnetic meridian. To secure this we find the po ition in which the needle is vertical (p. 315), and then turn the instrument through a right angle. The needle can now rotate in the magnetic meridian about a horizontal axis which rests on smooth supports held by the two pillars A B (Fig. 209). It is possible, however, that owing to a little rust on one

end of the needle, or to error in the position of the axis, one end, from mechanical causes, may tend to dip more than the other, and so reduce or exaggerate the magnetic effect. To eliminate these and other errors the following readings must be taken :—

	Upper end of needle.	*Lower end of needle.*	*Mean.*
When the graduated face is turned to the east—			
Front of needle facing circle	……	……	……
Back of needle facing circle	……	……	……
When the graduated face is turned to the west—			
Front of needle facing circle	……	……	……
Back of needle facing circle	……	……	……

We must now reverse the polarity of the needle. To do this, clamp it in the position indicated (C, Fig. 209). Place the opposite poles of two bar magnets on the needle (Fig. 210), and stroke the

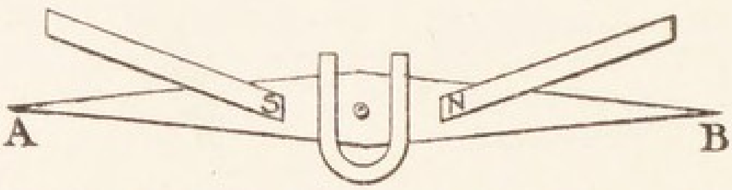


Fig. 210.—Magnetization of the dip needle.

needle from clamp to point on each side with the respective magnet. Repeat this three or four times. Then release the needle, turn the under side uppermost, replace the needle thus in the clamp, and again stroke it with the *same* poles as before. The end A will now be the north-seeking pole of the needle, and will therefore *dip*. This must be the end that was previously the upper end of the needle. The series of eight readings already described must now

be repeated with the reversed needle. The final mean of all the sixteen readings is the value of the dip.

Lines of force due to the earth's magnetism permeate the space around us. Their direction at any point is that of the dip needle at the same point (S N, Fig. 206).

If a steel poker be held pointing in this direction and struck smartly two or three times with a hammer, it will become a magnet, and the lower end will be the north-seeking pole. The polarity can be reversed by reversing the poker and re-striking. In this position the earth's lines of force traverse the iron, and the effect is due to their inductive action aided by the vibration produced by the blow. Steel ships, if built with their long axis in the magnetic meridian, become powerful magnets, and their effect on the compass needle has to be neutralized by placing pieces of iron or small magnets close to the binnacle.

Exercises

1. If the needle of the magnetometer (Fig. 208) finally shows no deflection when between two magnets, in the A position, whose centres are respectively 80 and 84 cm. distant from the needle, compare the moments of the two magnets.

2. When the nearer magnet of the previous example is removed, the deflection of 5° is obtained; calculate the moment of each magnet, assuming that $H = 0{\cdot}18$ dyne.

3. One of the magnets of the previous example, when suspended in the horizontal stirrup (p. 319), executed 12 *complete* vibrations in a minute at one end of the laboratory, and only 9 per minute at the other end. Compare the values of H at the two places.

(For Answers, *see* p. 390.)

CHAPTER III

CURRENT ELECTRICITY

Galvani and Volta—The Voltaic Cell—Polarization—Primary Batteries—Contact Keys—Commutators—Electrolysis—Equivalent Weights—Voltameters—Accumulators—Electrolytic Theory of Solutions—Resistance—Ohm's Law—Grouping of Cells—Exercises.

Galvani and Volta.—The stationary charges of electricity which have been considered in Chapter I. play a very subordinate part in the modern applications of electricity. These are mainly due to electricity in motion, or *current electricity*. The terms *galvanic* and *voltaic* are often used to denote this electricity, and will always serve to remind us of its discovery. In 1790 Galvani observed that some frog legs, which were hung up on an iron balcony with copper hooks, twitched when the legs touched the iron. He had previously noticed similar movements when the legs were in contact with metal in an electrical atmosphere. He regarded these movements as manifestations of the *animal electricity* which he believed to be inherent in the tissues, and the occurrence of the movements in a normal atmosphere seemed to confirm his view. Volta, however, an eminent Italian physicist, soon afterwards showed that the exciting cause was really external, and resided in the contact of dissimilar metals in presence of moisture. His views were communicated to the Royal Society of England on January 31st, 1793. In 1800 he constructed what

is now known as the *voltaic pile*, an apparatus in which an electric current is produced by these means.

Since that time many a frog has been dedicated to science as witness of the accuracy of Galvani's observations and of the truth of Volta's explanations. For that purpose the upper part of the body is removed from a freshly killed frog, and a fine copper wire A is inserted (Fig. 211) under the large nerves which supply the legs. The legs are then flexed and placed on a platform of zinc, the toes being in contact with the ledge B, to which a flexible copper wire C has been soldered. When the wire A makes contact with the wire C, the legs jump. This stimulation of nerve, and resultant muscular response, is known as **galvanism;** it is now recognized as a valuable agent in the diagnosis and treatment of nervous or muscular injury. Of course, in medical practice the wire is not in direct contact with the nerve, or even with the skin; contact is completed through the patient, but a moistened pad of absorbent cotton or similar material intervenes between the metal and the patient's skin.

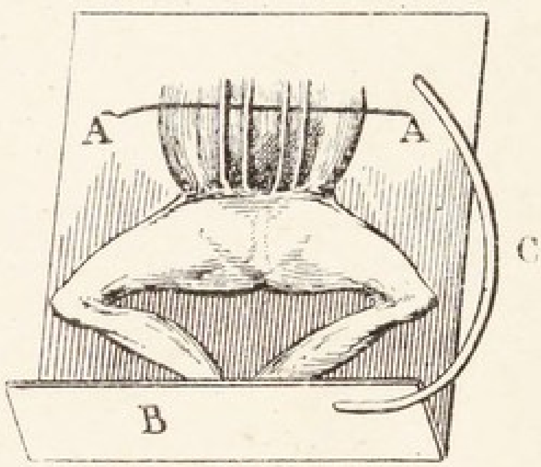


Fig. 211.—Galvani's experiment.

Simple voltaic cell.—This still retains the essential features of Volta's pile, two dissimilar metals and a conducting solution, or *electrolyte*. A plate of zinc and a plate of copper are immersed in 10 per cent. sulphuric acid. Dilute sulphuric acid has no action on copper, but readily attacks and dissolves common zinc. If, however, the surface of the zinc is rubbed, when clean, with a little mercury, it

soon becomes *amalgamated*, and is then not acted on by the dilute acid in ordinary circumstances. On this account the zinc plate of a voltaic cell is amalgamated. When, however, the amalgamated zinc plate is connected to the copper plate by a copper wire, the zinc begins to dissolve, and bubbles of hydrogen are given off at the *copper* plate. In this cell it is usual to say that the current flows *from the copper to the zinc outside the battery*, and returns inside the battery, through the sulphuric acid, from the zinc plate to the copper plate. The terminals of the battery, whether plates or wires connected with plates, are often called the *poles* of the battery; or,

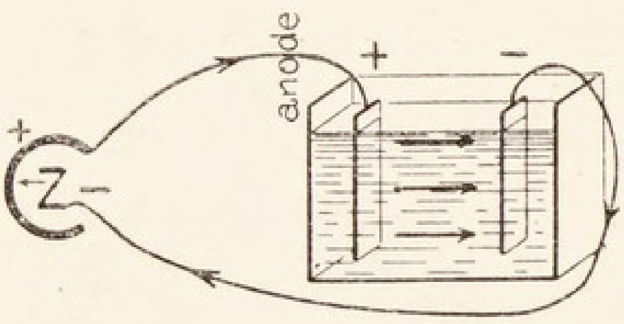


Fig. 212.—Direction of current.

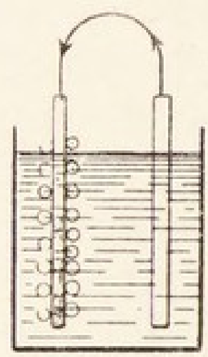


Fig. 213.—Copper plate behaving like zinc.

if used for plating, decomposing water, exciting a nerve, etc., they may be called *electrodes*. The positive electrode, *from* which the current flows, is called the *anode*, and the negative the *kathode* (Fig. 212).

A voltaic cell therefore consists of two substances, one of which is usually zinc, and the other may be copper, platinum, carbon, silver, etc., and of a solution in which, as a rule, the zinc dissolves while the cell is in use. The plates are named the opposite to the poles; thus, the zinc plate is the positive plate. A plate of zinc placed in a solution of copper sulphate will displace the copper from its combination with

the acid, or negative, radicle. Zinc is, therefore, regarded as more positive than copper.

If a simple cell, as described above, be coupled up with an electric bell so as to ring it continuously, it will be noticed after a short time that the bell ceases to ring. If the battery be examined it will be found that the copper plate is covered with a creamy effervescence of bubbles of hydrogen, being, in fact, superficially converted into a hydrogen plate; and if it be carefully taken out and placed in fresh sulphuric acid, it will for a short time act like a *zinc* plate to a clean copper plate (Fig. 213). This phenomenon is called **polarization.** The original current is weakened principally because the hydrogen film renders the copper plate more electro-positive, and therefore more like a zinc plate, but also because the film of gas causes additional resistance to the passage of the current.

Many remedies for polarization have been invented. The hydrogen may be (*a*) mechanically removed, as in the Smee cell; (*b*) replaced by the less positive metal, as in the Daniell cell; (*c*) oxidized to water, as in the Grove cell. The simplest mechanical plan is to remove the hydrogen by a stream of bubbles of air from a blower. The **Smee cell** consists of a platinum plate stretched in a wooden frame, on which are clamped two zinc plates; the elements are immersed in 10 per cent. sulphuric acid. The platinum plate is coated with spongy platinum, which absorbs hydrogen, and rough points on the surface facilitate the escape of the gas. The **Daniell cell** (Fig. 214) consists of a rod of zinc immersed in 10 per cent. sulphuric acid contained in a porous pot. This is surrounded by a copper vessel containing a saturated solution of copper sulphate. The hydrogen which tends to be evolved at the

copper plate acts upon a strong solution of copper sulphate, forming metallic copper and free sulphuric acid :

$$\underset{\text{Copper sulphate}}{CuSO_4} + \underset{\text{Hydrogen}}{H_2} = \underset{\text{Copper}}{Cu} + \underset{\text{Sulphuric acid.}}{H_2SO_4}$$

The copper vessel forms the negative plate of the battery, and copper is deposited upon it in place of hydrogen. The strength of the copper sulphate solution is maintained by some crystals of this salt placed in the perforated gallery.

In the Grove cell, and in the Bunsen cell, the

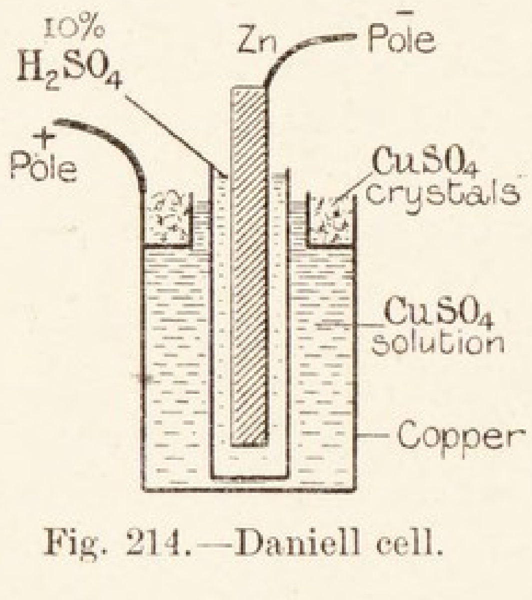


Fig. 214.—Daniell cell.

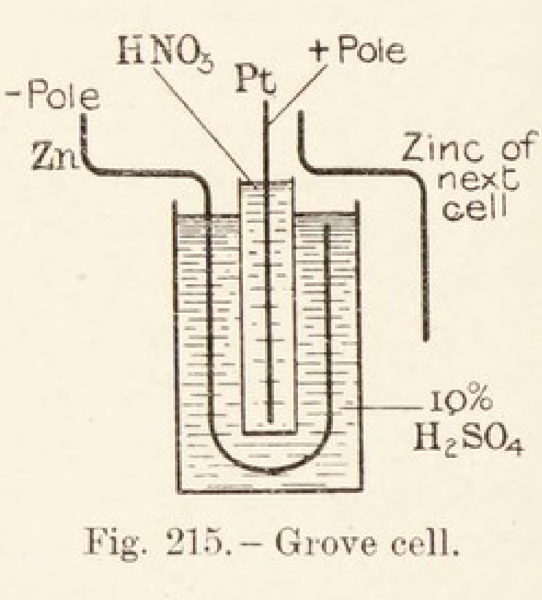


Fig. 215.– Grove cell.

hydrogen is oxidized to water by strong nitric acid contained in the inner pot, which is made of *porous* earthenware. The outer pot is made of *glazed* earthenware, and contains the dilute sulphuric acid.

The **Grove cell** contains a zinc plate bent as seen in Fig. 215, immersed in 10 per cent. sulphuric acid, and enclosing in the bend the porous pot and strong nitric acid, in which is immersed a platinum plate. The hydrogen which tends to come off at the platinum plate is at once oxidized by the strong nitric acid :

$$\underset{\text{Hydrogen}}{H_2} + \underset{\text{Nitric acid}}{2HNO_3} = \underset{\text{Nitrogen peroxide}}{2NO_2} + \underset{\text{Water.}}{H_2O}$$

The red fumes of nitrogen peroxide which escape render this cell objectionable unless the cells are placed in a draught cupboard.

The **Bunsen cell** (Fig. 216) has a cylindrical plate of zinc, and the round, porous cell contains a stick of gas carbon immersed in strong nitric acid. In other respects it resembles the Grove cell.

In the **bichromate cell** a plate of zinc is immersed in a saturated solution of potassium bichromate containing 10 per cent. sulphuric acid, or a solution

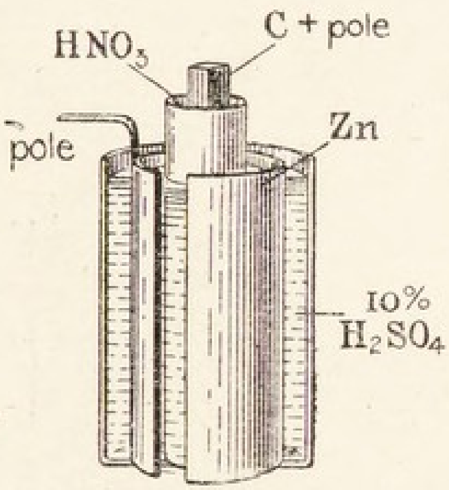


Fig. 216.—Bunsen cell.

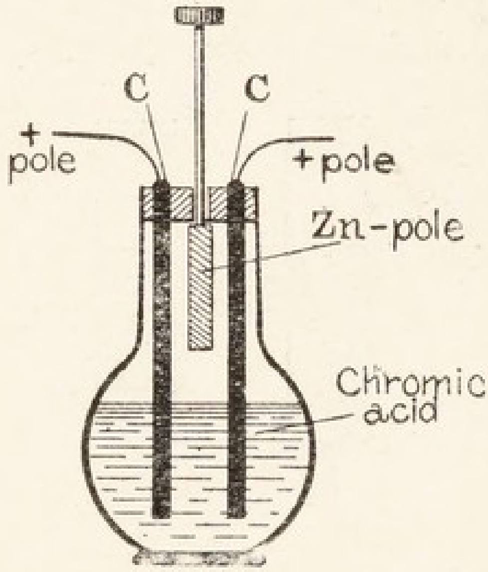


Fig. 217.—Bichromate cell.

of so-called chromic acid can be used. The zinc plate has a carbon plate fixed on either side. The hydrogen is oxidized, the orange colour of the bichromate changing to the purple green of chrome alum. The reaction is:

$$\underset{\text{Potassium bichromate}}{K_2Cr_2O_7} + \underset{\text{Sulphuric acid}}{4H_2SO_4} + \underset{\text{Hydrogen}}{3H_2} + \underset{\text{Water}}{17H_2O}$$

$$= \underbrace{K_2SO_4 + Cr_2(SO_4)_3 + 24H_2O}_{\text{Chrome alum.}}$$

In this form of cell (Fig. 217) the zinc must be so arranged that it can be lifted out of the solution

when the battery is not at work, because chromic acid attacks zinc even if it is amalgamated.

The **De La Rue cell** has a rod of zinc immersed in a solution of ammonium chloride and a strip of silver surrounded by a stick of fused silver chloride. The hydrogen reduces the silver chloride:

$$\underset{\text{Hydrogen}}{H_2} + \underset{\text{Silver chloride}}{2AgCl} = \underset{\text{Hydrochloric acid}}{2HCl} + \underset{\text{Silver.}}{2Ag}$$

Latimer-Clark cell.—This pattern is often employed as a standard cell for measuring purposes. It consists (Fig. 218) of a rod of pure zinc or a

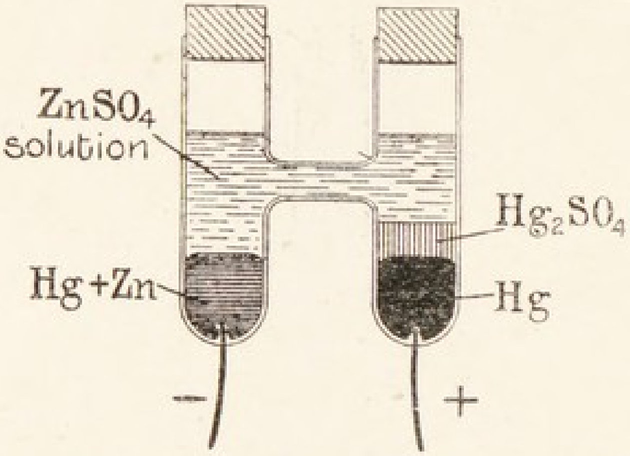


Fig. 218.—Latimer-Clark cell.

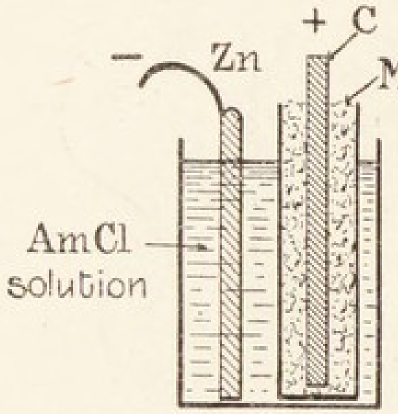


Fig. 219.—Leclanché cell.

mass of amalgam of pure zinc and pure mercury for the negative pole, and a mass of pure mercury for the positive. The mercury is covered with a layer of pure mercurous sulphate, and then the cell is filled up with a saturated solution of pure zinc sulphate. Finally, the cell is sealed to avoid evaporation.

The **Leclanché cell** (Fig. 219) contains an amalgamated zinc rod immersed in a solution of ammonium chloride, and a carbon plate surrounded by a mass of black oxide of manganese. The hydrogen, which tends to be evolved at the carbon plate, reduces the manganese to a lower state of oxidation, being

itself converted into water. This oxidation is slow, so that if short-circuited the cell polarizes, but recovers if disconnected and allowed to stand :

$$\underset{\text{Zinc}}{Zn} + \underset{\substack{\text{Ammonium}\\ \text{chloride}}}{2NH_4Cl} = \underset{\substack{\text{Zinc}\\ \text{chloride}}}{ZnCl_2} + \underset{\text{Ammonia}}{2NH_3} + \underset{\text{Hydrogen}}{H_2}$$

and then

$$2MnO_2 + H_2 = Mn_2O_3 + H_2O$$

or

$$3MnO_2 + 2H_2 = Mn_2O_4 + 2H_2O$$

Dry cells.—Most dry cells contain plates of zinc and carbon, the latter surrounded by black oxide of manganese, the exciting fluid being ammonium or zinc chloride rendered more or less solid by admixture

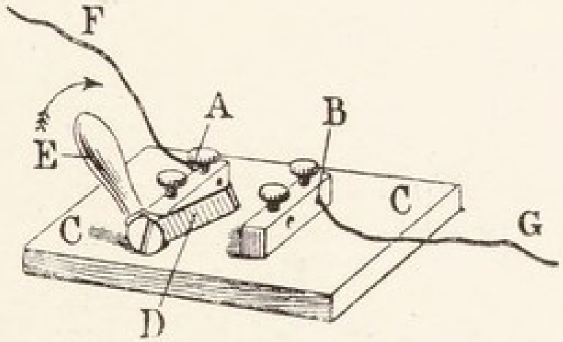


Fig. 220.—Du Bois-Reymond key.

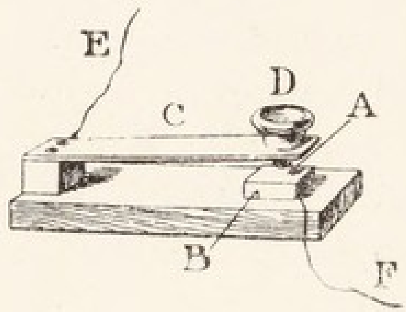


Fig. 221.—Spring key.

with plaster of Paris, flour, etc. They are essentially Leclanché cells working with a minimum of moisture, and in this portable form are very convenient and make up into batteries very suitable for general practice whenever treatment requiring a continuous current is desired.

Keys.—To prevent unnecessary expenditure of electrical energy, the circuit should include an instrument by which it may be instantly closed, or opened, at will. Such an instrument is technically known as a *key*. A form much used in physiological work is the **Du Bois-Reymond key** (Fig. 220). A piece of flat brass D, having a vulcanite handle E,

is screwed to a brass block A, which is mounted on a vulcanite block C. On the handle being turned over to the right, D rubs against the end of the brass block B and establishes contact between the wires F and G.

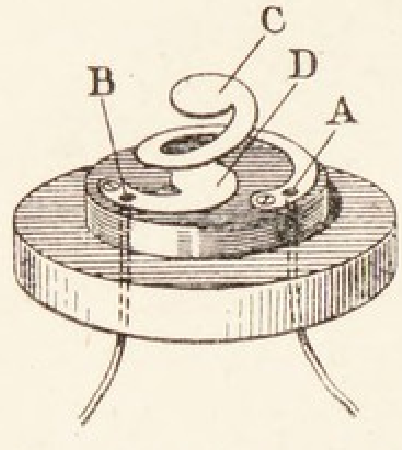


Fig. 222.—Bell push.

Another form of key is the **spring key** (Fig. 221). A springy strip of brass C ends underneath in a metallic point A. On the disc of vulcanite D being pressed down, A comes into contact with the metallic plate B and establishes connection between the wires E and F. The *electric bell push* (Fig. 222) is a spring key. On the ivory button being pushed down, the spiral spring C comes into contact with the metal D, and the wires A and B are connected.

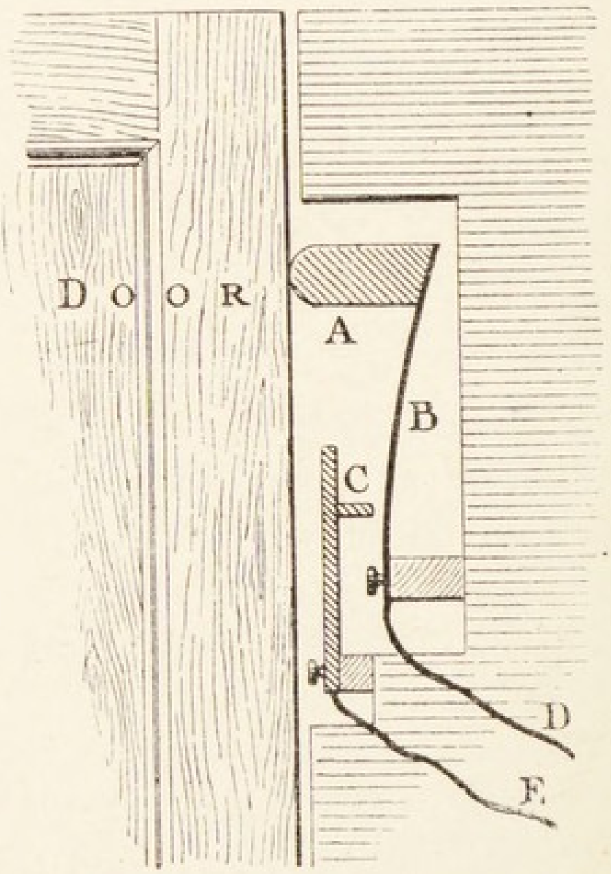


Fig. 223.—Burglar-alarm key.

One other kind of key is seen in Fig. 223 — the **"burglar alarm."** This is fitted where the hinges of the door are usually placed. When the door is shut, as in the figure, the edge of the "stile" of the door forces the vulcanite stud A back into the recess cut out for it, so that the spring B does not touch the fixed point C. When the door is opened,

the spring B moves to the left and makes contact with C, thus establishing connection between the wires D and E.

Commutator or reverser.—This is an instrument for changing or reversing the direction of a current in a circuit. Fig. 224 shows one form of the instrument. Six circular holes, 1 to 6, are drilled in a block of vulcanite and filled with mercury; 1 and 6 are connected by a copper wire, also 2 and 5. There are six brass binding screws S, whose ends are screwed quite through the vulcanite, so that they project into the mercury. B is a movable bridge of copper wire, with a glass handle C, by means of which 3 can be connected with 5, and 4 with 6; or, by turning the bridge over, 3 can be connected with 1, and 4 with 2. The central section of C consists of sealing-wax, or other non-conducting substance, so that 3 and 4 are not connected *via* C. Suppose the battery current enters at 3; it passes over the bridge to 5, thence into the circuit, round which it travels (as seen by the arrows) to 6, passes over the bridge to 4, where it leaves the commutator and returns to the battery. If the bridge is turned over, so that 3 is connected with 1, the current enters as before at 3, passes by the bridge to 1, and thence along the cross wire to 6. It now travels round the circuit *from* 6 *to* 5; passes from 5 to 2 by the cross wire, and then to 4 by the copper bridge in its new position. From 4

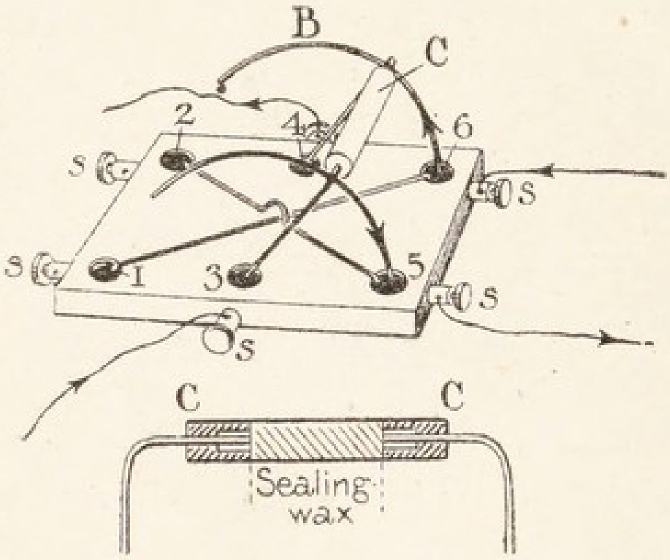


Fig. 224.—Commutator or reverser.

it returns to the battery. The main circuit between 5 and 6 is therefore traversed by the current in opposite directions in the two journeys.

Sometimes a key and reverser are combined in one instrument as in the key on the **Ruhmkorff coil** (Fig. 225). This consists of a circular block of vulcanite V; in it are fixed two brass axles X and Y; with X is connected a handle H by which the whole can be turned. Two brass plates A and A′ are fixed so as to cover only a small portion of the circum-

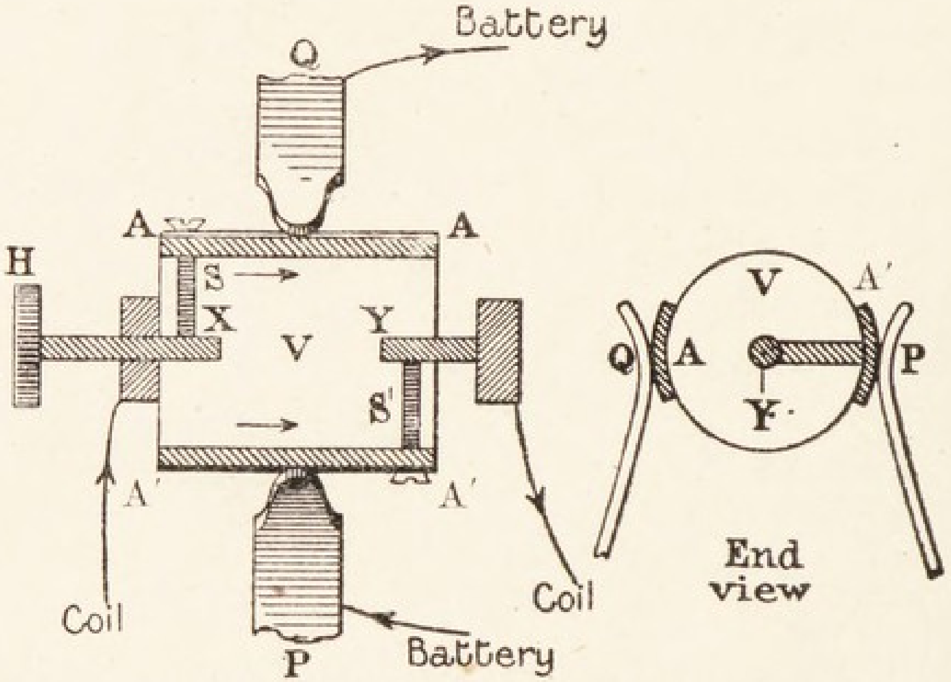


Fig. 225.—Key and reverser combined on Ruhmkorff coil.

ference of the vulcanite; two brass screws S and S′ connect these plates respectively with the brass axles X and Y. Two brass springs P and Q press against the circumference of the vulcanite; when they touch only vulcanite the *current* is blocked. The battery wires are connected with P and Q, the wires of the primary coil with X and Y. If the key be turned as in Fig. 225, the current passes in at P through A′ S′ to Y. If, however, the vulcanite block be turned half round so that Q touches A′, the

current passes in by P along A through S to X, and its direction through the primary coil is reversed.

Electrolysis. — Aqueous solutions of metallic salts conduct electricity, but the passage of the current is accompanied by a separation of the two constituent radicles of the salt, which usually make their appearance at the electrodes (p. 326). Sometimes, however, a secondary chemical reaction occurs there which results in the appearance of other products instead. The conducting solution is termed the *electrolyte;* the process is called *electrolysis.*

One group of elements and radicles is liberated at the electrode by which the current leaves the electrolyte—that is, the negative electrode, the one connected with the zinc end of the battery—and these are called *electro-positive* elements and radicles, or sometimes *kations,* as they are liberated at the kathode; they include hydrogen and the metals. The other class, liberated at the positive electrode, or *anode,* where the current enters the fluid, are termed *electro-negative,* or *anions,* and include the non-metals, chlorine, bromine, and the acid radicles SO_4, NO_3, etc.

If a solution of copper sulphate be placed in the U-tube (Fig. 226) and the wires be connected with a battery, the platinum plate connected with the zinc end of the battery will be found after a few minutes to be coated with red metallic copper. If the direction of the current be reversed, the copper will dissolve off the platinum plate and be deposited on the other, which is now the kathode. Copper, being a kation, is always deposited on the kathode.

If a solution of potassium iodide, to which a little starch solution has been added, be placed in the U-tube, instead of the copper sulphate, the passage of

the current is almost immediately marked by the appearance of a blue colour in the neighbourhood of the *anode*, where the electro-negative iodine is always liberated.

If a neutral solution of sodium sulphate be electrolysed in the U-tube (Fig. 226), the liquid surrounding the kathode will be found to become alkaline, while that surrounding the anode will become acid. The sodium sulphate dissociates into its kation, sodium, and its anion, SO_4. The sodium, liberated at the kathode, decomposes the water there, thus forming alkaline caustic soda, and liberating hydrogen. The

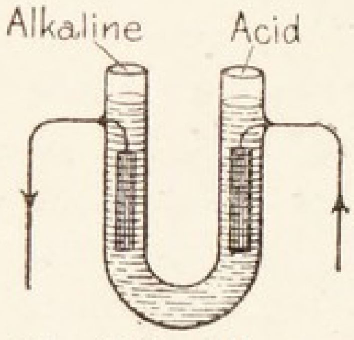


Fig. 226.—Electrolysis of sodium sulphate.

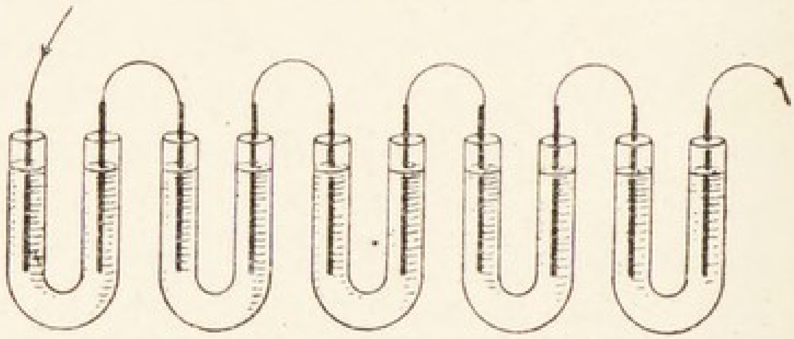


Fig. 227.—Series of U-tubes arranged for electrolysis.

anion, SO_4, decomposes water, forming sulphuric acid, and liberates oxygen.

The decomposition of metallic solutions by electrolysis finds numerous applications at the present day for electroplating (coating with silver), electrogilding, electrotyping (reproducing objects in copper), nickel-plating, etc. All that is necessary is to render the article to be plated thoroughly clean, immerse it in a suitable metallic solution, and make it the kathode by connecting it with the zinc end of a battery, when the metallic coating is deposited upon it. The anode should be a plate of the metal which is being deposited, gold for gilding, silver for electroplating, etc.

Electrolysis is also used in the manufacture of chlorine and caustic soda by the electrolysis of brine, and in the preparation of chlorates, etc.

Equivalent weights.—When a current is passed which liberates 1 grm. of hydrogen, it will set free 8 grm. of oxygen, 108 of silver, $\frac{63 \cdot 5}{2} = 31 \cdot 75$ of copper, $\frac{197}{3} = 65 \cdot 7$ of gold, 80 of bromine, 35·5 of chlorine. In other words, elements are liberated in proportion to their equivalent weights (equivalent weight $= \frac{\text{atomic weight}}{\text{valency}}$). So that if we pass a current through a series of U-tubes containing the following solutions—viz. (1) acidulated water, (2) copper sulphate, (3) silver cyanide, (4) gold cyanide, (5) salt (Fig. 227)—for every grm. of hydrogen set free there will be liberated 8 grm. of oxygen, 31·75 of copper, 49 of sulphuric acid, 108 of silver, 26 of cyanogen, 65·7 of gold, 40 of caustic soda, 35·5 of chlorine.

The weight of an element liberated depends, therefore, (1) upon its nature; it also depends (2) upon the strength of the current, and (3) upon the duration of the current. If we choose for (1) the element hydrogen, and for (3) the unit of time, 1 sec., it is clear that the weight of hydrogen liberated per second by any current will be simply proportional to the strength of that current. We have, therefore, a direct method of estimating this strength. When the electrolytic apparatus (Fig. 226) is adapted to this purpose, it is called a *voltameter*. If the element deposited and weighed is copper it is called a *copper voltameter*. The *silver* voltameter is generally preferred for accurate measurements. Current strength and other electrical quantities can be expressed in

absolute C.G.S. units related to the fundamental and derived units already defined (p. 4). These units are often inconveniently large or small for practical purposes, and certain practical units are therefore more generally employed. The practical unit of current strength is 1 ampere. The strength of the therapeutic current employed in galvanism is only a fraction of this unit and is usually stated in terms of the *milliampere ;* a range of 3 ma. to 20 ma. will meet most cases, subject to such variation with the size of the electrode as will keep the surface density approximately to 1 ma. per sq. in. The weight of silver liberated in 1 sec. by a current of 1 ampere is 0·001118 grm. The weight of hydrogen liberated in 1 sec. by a current of 1 ampere is 0·001118 divided by the *chemical equivalent* * of silver, and is found to be $0{\cdot}0_4104$. The weight of copper liberated from a solution of cupric sulphate in the same time by the same current is therefore $0{\cdot}0000104 \times \frac{63}{2}$ grm., and is called the *electro-chemical equivalent* of copper. If we denote the electro-chemical equivalent of any element by z we may say that the weight w of this element, liberated by a current of C amperes in t seconds, is given by the equation :

$$w = z.C.t$$

This is really the quantitative expression of the three relations already stated. They were established by Faraday, and are known as *Faraday's laws of electrolysis*.

It will be convenient to mention here that the absolute C.G.S. unit of current strength is equal to 10 amperes.

The quantity of electricity carried past any specified

* *See* "Manual of Chemistry" (Luff and Candy).

point in the circuit by 1 ampere in 1 sec. is called 1 *coulomb*. It is clear that 1 coulomb liberates the electro-chemical equivalent of an element.

It is sometimes convenient to remember the reciprocal relation, namely, that $\frac{1}{0{\cdot}0_4104}$, or 96,150, coulombs (or 9,615 C.G.S. units) liberate 1 grm. of hydrogen or the chemical equivalent, in grammes, of any other element. This quantity of electricity is conveniently called a *faraday*.

Accumulators, storage cells, secondary batteries.—One special application of electrolysis is the storing of electrical energy by converting it into chemical energy.

A secondary battery consists of two sheets of lead painted with red lead, Pb_3O_4, or of gratings of lead, with their interstices filled up with a paste of red lead and dilute sulphuric acid, immersed in dilute sulphuric acid. The two plates are connected with the poles of a battery or dynamo, when the lead plates act as electrodes, hydrogen being evolved at the kathode and reducing the red lead on the kathode to a mass of spongy metallic lead, while the oxygen evolved at the anode oxidizes the red lead there to peroxide of lead, PbO_2. When all the red lead has thus been converted on the one plate to metallic lead, on the other to peroxide, the hydrogen and oxygen come off in bubbles, indicating that the accumulator is charged. It is then disconnected from the dynamo. If the two lead plates prepared as above be now connected by a wire, a current will flow in the opposite direction, until the spongy lead and the peroxide have both been reconverted into red lead.

The current in a wire which connects the terminals of an accumulator is a continuous one, similar to

the galvanic current derived from the primary batteries already described. Faraday subsequently discovered that by magnetic methods (p. 374) a current of electricity might be produced in a wire which was not connected to any battery at all: this *faradic* current is not continuous but *alternating* or *interrupted*, and it would therefore not be suitable for electrolysis.

Electrolytic theory of solutions.—It was formerly believed that the current actually decomposed the salt molecules in solution into ions. The view now generally adopted is that a certain proportion of the molecules always are so dissociated or *ionized* in dilute aqueous solutions, and that the current directs them to their proper electrodes—the positive ions to the kathode and the negative ions to the anode. Here they become atoms and manifest their individual chemical characters. The extensive use of electrolysis at the present time in electro-therapeutics is due to the belief that the beneficial local effects observed are produced by these liberated ions. Experiment has shown that drugs can in this way be administered through the skin (*kataphoresis*), and that the ions do move in the directions already indicated. Thus, anions like iodine move from kathode towards anode, and therefore the absorbent kathode is soaked in an aqueous solution of potassium iodide; while kations like the alkaloids move from anode towards kathode, and their kataphoresis is effected from a suitable solution applied with the *anode*. Cocaine so administered produces only local anæsthesia, although when injected subcutaneously the same alkaloid enters the general circulation and produces quite different effects; the electrolytic method may therefore be very useful for minor operations. Sub-

stances like sugar, which are not ionized in aqueous solution, are not electrolytes.

Conductors differ greatly in the quantity of electricity which they allow to pass in the same circumstances. This difference in velocity is due to what is technically termed the **resistance** of the conductor. This is generally denoted by R, and depends (1) upon the material of which the conductor is composed—each substance has its own *specific resistance*,* r, just as it has its own specific heat, etc.; (2) upon the area, s, of the cross section of the conductor—the larger the area, the smaller the resistance; (3) upon the length, l, of the conductor—the longer the conductor the greater the resistance. These relations are expressed by the equation

$$R = \frac{r\,l}{s}$$

We see that if $l = 1$ cm. and $s = 1$ sq. cm., we shall have $r = R$. Hence the specific resistance—or, as it is sometimes called, the *resistivity*—of a substance is the resistance, expressed in absolute units, which is actually offered by 1 cm. of a conductor, composed of that substance, whose cross section has an area of 1 sq. cm. The specific resistances of a few substances are stated in the following table:—

SPECIFIC RESISTANCE, EXPRESSED IN OHMS, OF VARIOUS SUBSTANCES AT 0°

Silver	$1{\cdot}47 \times 10^{-6}$
Copper	$1{\cdot}56 \times 10^{-6}$
Aluminium	$2{\cdot}66 \times 10^{-6}$
Pure water	$25{\cdot}00 \times 10^{+6}$
Platinum	$10{\cdot}92 \times 10^{-6}$
German silver	$20{\cdot}00 \times 10^{-6}$
Mercury	$94{\cdot}07 \times 10^{-6}$
Mica	$1{\cdot}00 \times 10^{+14}$

The practical unit of resistance is the *ohm*, and is equal to the resistance of a column of mercury 1 sq. mm. in section and 106 cm. long at 0° C. The ohm is equal to 10^9 absolute C.G.S. units of resistance. The *internal* resistance of a cell is evidently the resistance of a column of the cell fluid whose length (l) is the distance between the plates and whose cross section (s) is the area of the immersed portion of a plate; hence in any particular type of cell we can only diminish this resistance by increasing the immersed plate area or by placing the plates nearer together.

The force causing the passage of electricity in the circuit is called the *electromotive force.* It is due to the difference in potential existing between the two terminals which the circuit connects. The practical unit of electromotive force is the *volt,* which is equal to 10^8 absolute C.G.S. units of potential. This difference in potential, or E.M.F., depends only on the elements used in the cell, and is independent of the *size* of the cell; the numerical values assigned to some of the cells already described are—

Bunsen	1·8 volts.
Grove	1·9 ,,
Daniell	1·07 ,,
Latimer-Clark . . .	1·433 ,, (at 15° C.)
Leclanché	1·5 ,,

If two bodies at different potentials be connected by a wire, the current flows from the body at higher potential to the body at the lower, until equilibrium of potential is established. If no current flows between two points, or bodies, so connected, they must be *at the same potential.* The potential of the earth is the zero of the potential scale (p. 294).

Ohm's law.—The strength, C, of a current flowing in a circuit varies (1) directly as the potential difference, or electromotive force, E, and (2) inversely as the resistance, R. This statement is known as Ohm's law, and is expressed by the equation

$$C = \frac{E}{R}$$

In the consideration of resistance it is often convenient to divide the resistance into R, the resistance of the external circuit, and r, the internal resistance of the battery or current producer. The equation will then be written

$$C = \frac{E}{R + r}$$

Coupling of cells in series and in parallel circuit.—Two or more cells can be combined to form a battery. For this purpose they may be grouped (1) *in series*, when the zinc of one cell is coupled to the copper of the next; or (2) *in parallel circuit*, when all the zincs are joined and all the coppers (Fig. 228); or (3) a combination of (1) and (2) may be used.

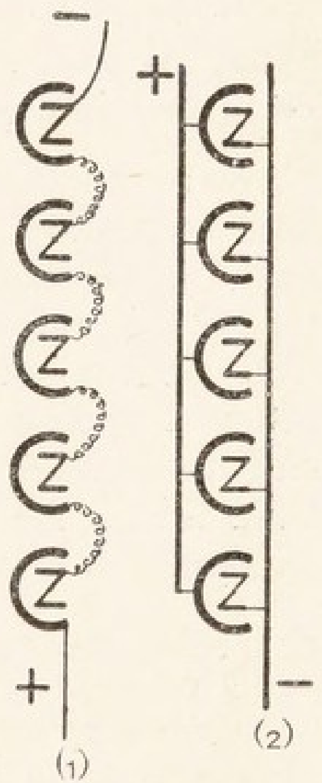


Fig. 228.—Coupling of cells (1) in series, (2) in parallel.

By coupling two cells in series we double the E.M.F., but at the same time we double the internal resistance, and the same holds with any number of cells so arranged. If we couple two cells in parallel circuit, we have practically one cell, with plates double the size. In this case the E.M.F. is unaltered, but the internal resistance is halved.

If the *external* resistance is *large*, as in the common

applications of galvanism to the human body, whose resistance may be about 3,000 ohms, it is more advantageous to connect the cells in series, since we cannot avoid the large external resistance and the internal resistance is relatively unimportant. On the other hand, when the *external* resistance is *small*, as in the galvano-cautery, where a short platinum wire, whose resistance is only about 0·05 ohm, has to be heated to dull redness, it is better to reduce the internal resistance by connecting the cells in parallel circuit. With any given number of cells the most efficient arrangement is the one that makes the external and internal resistances as nearly equal as possible ($R = r$).

Ex.: What arrangement of 20 Leclanché cells, each cell having an E.M.F. of 1·5 volts and an internal resistance of 0·3 ohm, will give most current (1) when the external resistance $R = 3{,}000$ ohms, (2) when $R = 0{\cdot}05$ ohm?

(1) When all the cells are connected in series, the internal resistance of the battery is $20 \times 0{\cdot}3 = 6$ ohms, and no arrangement can make it more nearly equal to R than this. This arrangement will therefore give the best current; the E.M.F. will then be $20 \times 1{\cdot}5 = 30$ volts, the total resistance 3,006 ohms, and the current $\frac{30}{3006}$ $= \frac{10}{1002}$ amperes $= 10$ ma. (nearly).

(2) We have now to group the cells so that the internal resistance of the battery shall be, as nearly as possible, $= 0{\cdot}05$ ohm. It will be found that this is the case if we divide the 20 cells into 10 pairs, the two cells in each pair being connected in series, but the 10 pairs being connected in parallel; each pair is then equivalent to a cell with E.M.F. $2 \times 1{\cdot}5 = 3$ volts, and internal resistance $2 \times 0{\cdot}3$ $= 0{\cdot}6$ ohm; and the battery is equivalent to 10 such cells in parallel and has therefore E.M.F. 3 volts and internal resistance $\frac{0{\cdot}6}{10} = 0{\cdot}06$ ohm. No other arrange-

ment will make the internal resistance so nearly equal to R. The current obtained from this arrangement is

$$\frac{3}{0{\cdot}05 + 0{\cdot}06} = \frac{300}{11} = 27{\cdot}27 \text{ amperes.}$$

Copper and silver, when pure, are the best conductors; but even traces of impurities seriously impair the conductivity. Iron is not nearly so good a conductor as copper, but by taking a larger-sized iron wire we increase the cross-section, and so compensate for its inferior conductivity. The earth is of such an immense cross-section that it may be looked upon as a perfect conductor. An aluminium wire of the *same size* as a copper wire

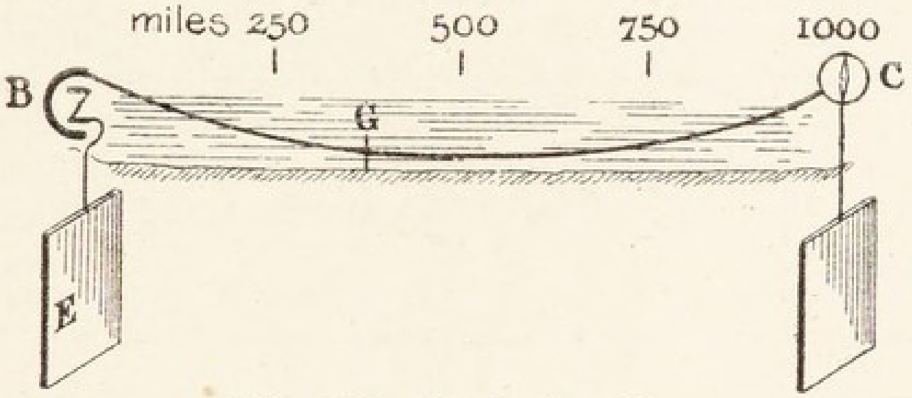


Fig. 229.—Fault in cable.

has only 60 per cent. of its conductivity, but if the size of the aluminium wire be increased until it *weighs* the *same* per metre it is twice as good a conductor as copper.

The measurement of resistance may enable us to locate a fault in a submarine cable (Fig. 229). The current flows from the battery at B through the cable to the receiving instrument at C, passing by the earth plate back through the earth to the plate E, and so home. Now suppose the cable is 1,000 miles long, and that each mile has a resistance of 5 ohms. As long as the cable is intact, it interposes a resistance of 5,000 ohms. Suppose it is broken at G, then the

broken end is earthed, and the resistance will be found to have fallen to 1,800 ohms. The fault is therefore distant $\frac{1800}{5} = 360$ miles from B.

Exercises

1. Calculate the weight of silver deposited from a solution of silver nitrate in 3 hours by a current of 5 amperes. [*First Professional.*]

2. Two cells of E.M.F. 1·1 and 1·4 volts, and resistance 0·5 and 0·2 ohm respectively, are arranged in series; the free terminals are connected by a wire of resistance 1 ohm. What current flows through the wire and through each cell? How would the current in the wire be affected if the connections of the second cell were inverted? [*Ibid.*]

3. Two platinum wires, equal in length but different in diameter, are joined in series with a battery which causes a difference of potential of 8 volts between the extreme points. The resistance of the thicker wire is 1 ohm, and of the other 3 ohms. Calculate the current flowing through the wires, and the quantity of electricity carried in 5 minutes. Find also the differences of potential between the ends of *each* wire. [*Ibid.*]

4. Calculate the volume of hydrogen liberated in 10 minutes by a current of 3 amperes, if the density of the gas be 0·0896 grm. per litre. [*Ibid.*]

5. A battery of E.M.F. 6 volts and internal resistance 2·5 ohms is used to supply current for two electric lamps, each of resistance 10 ohms. Calculate the current in each lamp when they are arranged (i.) in series, (ii.) in parallel. Compare the amount of heat produced in a lamp in the two cases. [*Ibid.*]

6. Calculate the current strength when a tangent galvanometer with a coil of 10 turns, of mean radius 10 cm., gives a deflection whose tangent is 0·75, and the strength of the earth's magnetic field is 0·16 dynes. [*Ibid.*]

7. A wire 1 metre long and 0·6 mm. diameter is found to have a resistance of 1·16 ohms. Calculate the specific resistance of the wire substance. [*Ibid.*]

(For Answers, *see* p. 390.)

CHAPTER IV

ELECTRIC MEASUREMENT

Measurement of Current—Galvanometers—Measurement of Resistance—Wheatstone's Bridge—Divided Circuits—Shunts—Measurement of Electromotive Force—Potentiometer — Capillary Electrometer — Thermo-Electricity—Exercises.

MEASUREMENT OF CURRENT STRENGTH

Effect of a current on a magnetic needle.—The measurement of the strength of a current by means of a voltameter (p. 337) depends upon the effect of the current on chemical combination. The measurement is, however, more frequently accom-

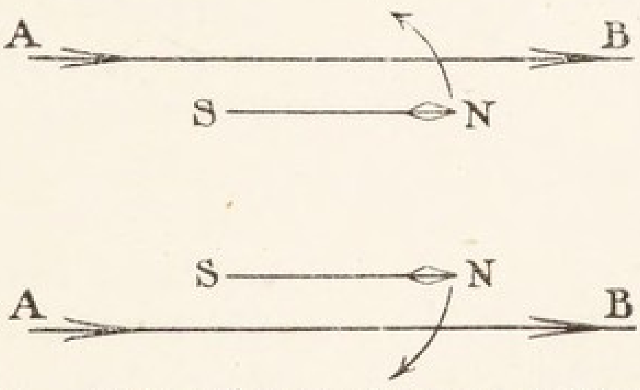


Fig. 230.—Action of current on magnetic needle.

plished by means of the **galvanometer,** an instrument which registers the quantitative effect of the current on a magnetic needle. If a copper wire conveying a current is held lengthwise over a magnetic needle resting in the magnetic meridian (p. 311), the needle immediately turns and tends to set itself at right angles to the length of the wire and to the magnetic meridian. If the current passes from A to B over the needle (Fig. 230), the north end of the

needle turns to the left; if the wire is placed *under* the needle, the north end moves to the right. If the direction of the current be reversed, the movement of the needle is reversed. The key to the movements is supplied by **Ampère's rule:** Suppose that a man swimming in the wire *with* the current turns so as to face the needle, then the north-seeking pole of the needle will be deflected towards his *left* hand. If a wire pass over a needle and then below, as in Fig. 231, the swimmer, moving with the current along A B, swims on his breast to face the needle, and the north end is deflected to his left. As he turns round the bend along C D he has to swim on his back to keep his face towards the needle, and his left hand will point in the same direction as before. Thus, twice the effect on the needle is produced by the current, and by making many turns of insulated wire round a needle the effect of a small current is multiplied and may produce a considerable deflection. The high resistance of the long coil may, however, be disadvantageous in some cases.

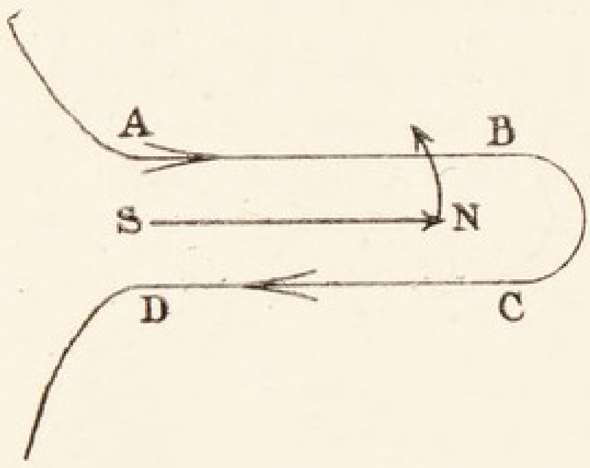


Fig. 231.—Effect of wire over and under magnetic needle.

The force exerted by the current may be represented by the couple F (Fig. 232) acting at right angles to the plane of the coil which is in the magnetic meridian. This is more or less counteracted by the horizontal component of the earth's magnetism (p. 313), which may be represented by the couple mH acting parallel to N S, if m be the strength of a pole of the needle used. If the needle comes to rest

in the position A B, making an angle, θ, with N S, the moments of these two couples must be equal and opposite, therefore

$$F \times BC = mH \times AC$$
$$\therefore \quad F = mH \times \frac{AC}{BC}$$
$$= mH \times \tan\theta$$

But $m\ H$ does not alter for the same needle and the same place; therefore the force, F, exerted by the current is simply proportional to the tangent of the deflection. Hence the name *tangent* galvanometer.

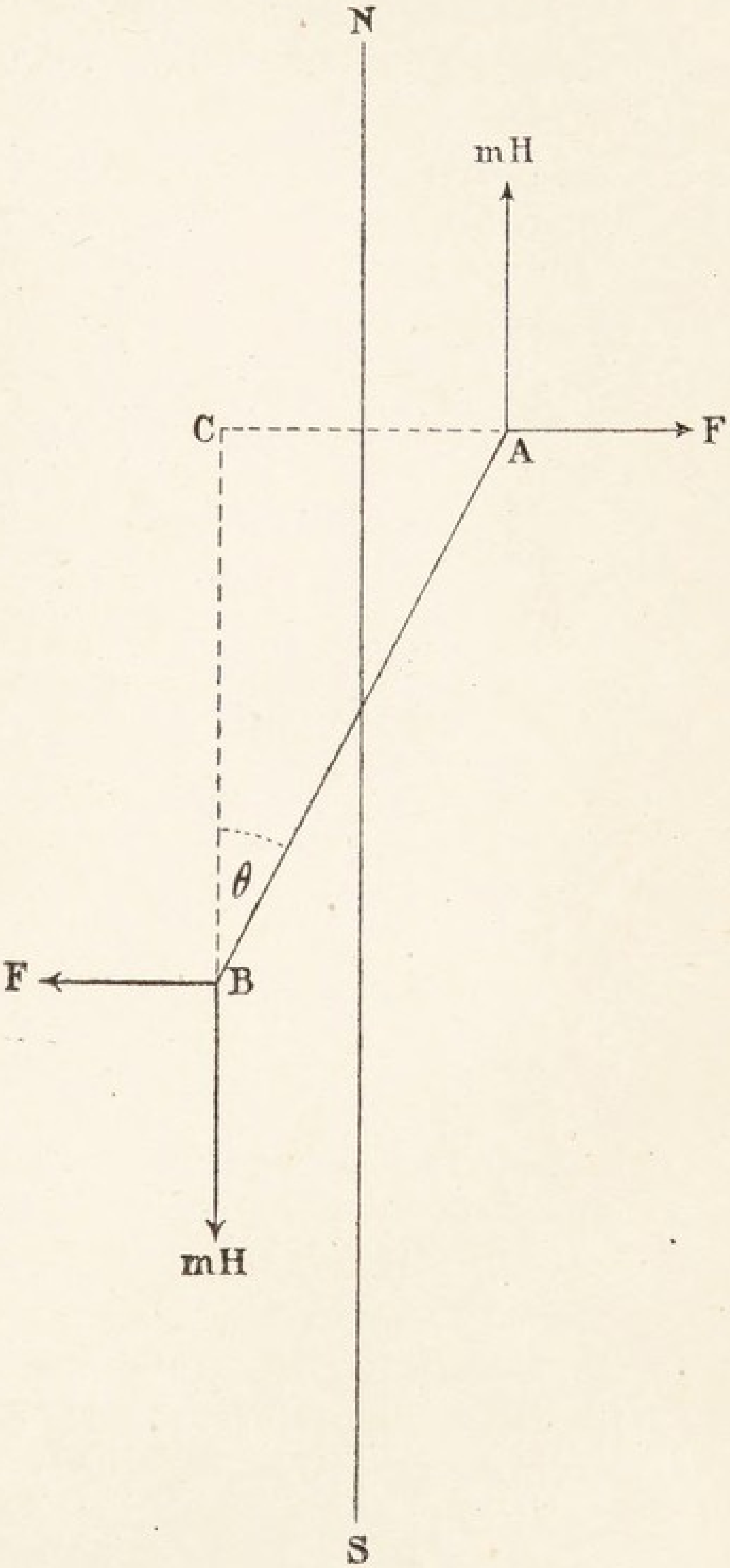


Fig. 232.—Assumption by magnetic needle of position of equilibrium.

Tangent galvanometer.—There are many forms of this instrument. Perhaps the simplest form consists of a thick copper wire coiled in a large open ring A (Fig. 233). There may, however, be two, or several, coils, and the diameter of a coil may vary considerably. The magnetic needle B must not be more than 1 in. long, and is suspended exactly in the centre of the ring. A light, long index of aluminium C C is

attached at right angles to the needle. The position of the index can be read off on a circular horizontal scale. The coil is set in the magnetic meridian, the short needle lying in the plane of the coil.

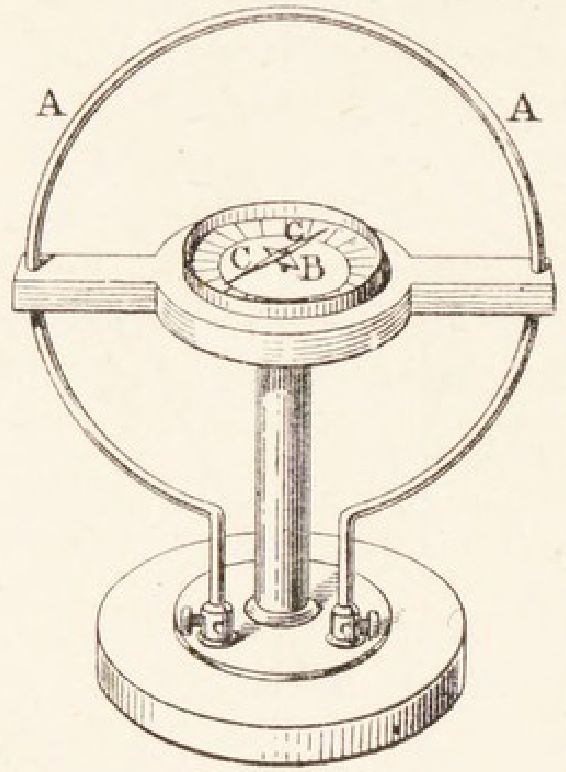


Fig. 233.—Tangent galvanometer.

Sine galvanometer.—If the coil of a tangent galvanometer can be turned about a vertical diameter and thus made to follow the deflected needle till both are again in the same plane, then since F (Fig. 232) is now at right angles to A B, the condition of equilibrium is:

$$F \times A\,B = mH \times A\,C$$

$$\therefore \quad F = mH \times \frac{A\,C}{A\,B}$$

$$= mH \times \sin\theta$$

So that the current strength is now proportional to the *sine* of the deflection, and the instrument becomes a *sine galvanometer*.

Astatic needle.—Instead of multiplying the effect of the current by using many turns of wire, we may diminish the opposing effect of the earth's magnetism by connecting it with a second needle of equal strength, but having its poles reversed, so that the north pole of one needle shall be over the south pole of the second, as in Fig. 234. If the two needles were perfectly symmetrical, and exactly balanced as to their magnetism, the combination would set

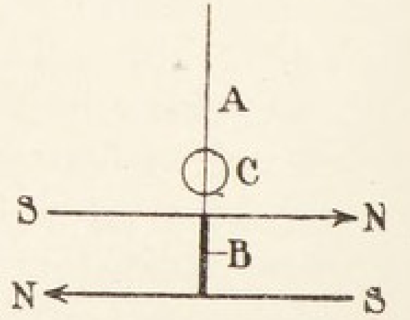


Fig. 234.—Astatic needle.

indifferently in any position. As a matter of fact, if the magnets are equally magnetized, the needle sets east and west. By using such an astatic needle the current has not to overcome the directive force of the earth's magnetism, and we have thus a much more delicate indicator. The needle is suspended by a silk fibre A, and the two needles are firmly fixed together by the rod B. In order to render the least movement of the needle visible, an exceedingly thin silvered glass mirror C (Fig. 235) is often attached to the upper needle; a beam of light from a slit at A falls on the mirror C, and is received, after

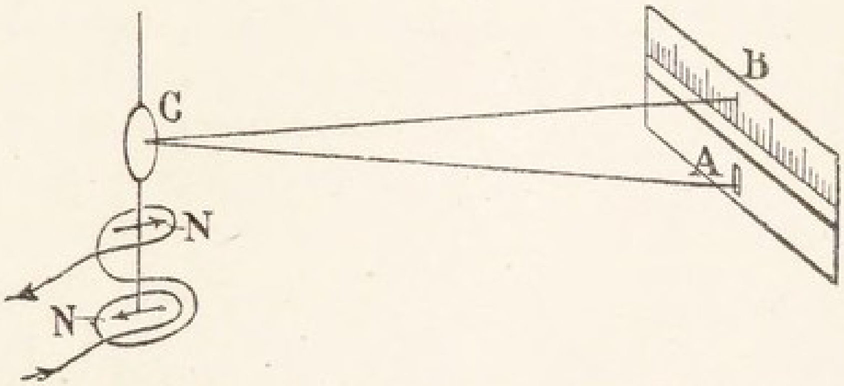


Fig. 235.—Light reflected from mirror above astatic needle.

reflection, on a scale B. Very small movements of the needle are thus rendered visible. Fig. 235 shows the mirror C attached to the astatic needle N N, and the way in which the light is reflected. This type of instrument is known as the Thomson galvanometer.

These reflecting or *mirror* galvanometers are often made with a single needle, and then are frequently furnished with a curved *control* magnet which can be adjusted above the coil by sliding on a vertical rod. This avoids the inconvenience of having to place the instrument in the magnetic meridian, and enables the observer to control the sensitiveness of the galvanometer.

In all the galvanometers hitherto mentioned, the coil conveying the current is stationary and the magnet moves. In another type of instrument the magnet is fixed while the coil is suspended in the field of the magnet, and therefore moves when the current is in circuit. The movement is transferred to a scale by a mirror or pointer as usual. This type of galvanometer is therefore called a *suspended coil*, or *moving coil*, galvanometer; it is also known as the D'Arsonval galvanometer. The principle upon which it depends is explained later (p. 376).

In order that the formula $F = mH \times \tan\theta$ may give not merely a relative but an absolute value of the current measured, we must make a suitable choice of **unit current.** We have already referred (p. 338) to two units, the absolute C.G.S. unit and the ampere, and have stated that the C.G.S. unit is equal to 10 amperes. We now define the C.G.S. unit to be a current which, flowing in a wire 1 cm. long, forming an arc of a circle of 1 cm. radius, exerts a force of 1 dyne on a unit magnetic pole placed at the centre (Fig. 236).

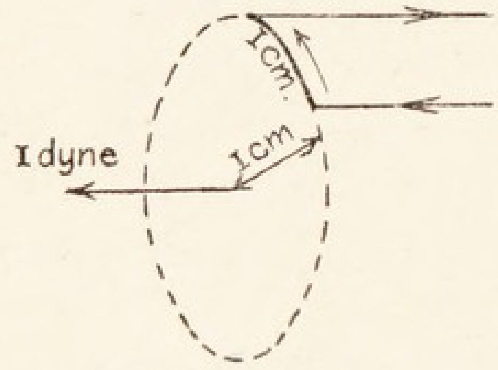


Fig. 236.—C.G.S. current.

If there are I units of current flowing in the wire the force exerted will be I dynes.

If the length of the wire is l cm. the force on the needle is l times as great $= l\,I$ dynes.

If the distance from the pole of the magnet, that is, the radius of the circle, is r cm., the force diminishes as the square of the distance, and therefore is divided by r^2, and $= \frac{l\,I}{r^2}$ dynes.

If the wire consists of n complete turns, l will be $n \times 2\pi r$, and the force becomes $\frac{2n\pi}{r} \times I$ dynes.

Ex.: Find the force in dynes exerted by a current of 80 amperes (8 C.G.S. units), the radius of coil being 20 cm. and having 10 turns.

$$\text{Force in dynes} = \frac{\left(2 \times 10 \times \frac{22}{7}\right) \times 8}{20} = \frac{176}{7}$$
$$= 25{\cdot}14 \text{ dynes}$$

We have already found (p. 349) that the force, F, of a current which produces a deflection, θ, of a needle of *unit* pole strength ($m = 1$) is given by $F = H \tan \theta$. If H is in dynes ($= 0{\cdot}185$) we must therefore have

$$\frac{2n\pi}{r} \times I = H \tan \theta$$

$$\therefore \quad I = \frac{r}{2n\pi} \,.\, H \tan \theta$$

Thus the actual value of the current in C.G.S. units may be obtained by multiplying the tangent of the galvanometer reading by the factor $\left[\frac{r}{2n\pi} \,.\, H\right]$ This is therefore called the reduction factor of the galvanometer. The quantity $\frac{r}{2\,n\pi}$ is independent of H, and is known as the galvanometer constant. Since we know (p. 338) that $w = z\,I\,t$ we can, by passing a current through a galvanometer and a voltameter at the same time, obtain both values of I, and by equating them we have

$$\frac{w}{z\,t} = \left[\frac{r}{2n\pi} \,.\, H\right] \tan \theta$$

In this way the reduction factor is experimentally determined.

If we wish to express the value of the current in amperes we must multiply the galvanometer constant by 10, and the reduction factor will then become $\frac{5r}{n\pi} H$. Since we can now convert our deflection readings to amperes, we might make this conversion once for all and calibrate the galvanometer accordingly ; the instrument then becomes an *ammeter*. In use the ammeter forms part of the circuit of the current it has to measure, and, since this current varies inversely as the resistance in the circuit, the ammeter itself should have a low resistance.

A dead-beat *milli*ammeter is included in the circuit for medical galvanism.

MEASUREMENT OF RESISTANCE

The simplest method of measuring the resistance of a conductor is the method of *substitution*. The conductor is placed in circuit with a battery and galvanometer, and the deflection noted. The conductor is then removed, and an adjustable resistance substituted for it by means of a **rheostat,** or a set of **resistance coils,** and the resistance is adjusted till the same deflection as at first is obtained. The resistance then in circuit must be the same as that of the conductor.

In one form of rheostat the resistance can be varied by winding or unwinding an uncovered German-silver wire (Fig. 237) from a vulcanite reel A on to a brass cylinder B, the resistance being determined by the length of wire on the vulcanite.

A resistance coil consists of two brass blocks A A′ (Fig. 238), connected underneath by a fine insulated wire C, which has a resistance of 1, 10, 1,000, etc., ohms, as marked on the instrument. The brass blocks are

mounted on vulcanite, and can be connected by pushing in a slightly conical brass plug D, and thus cutting out the resistance. When the plug is out

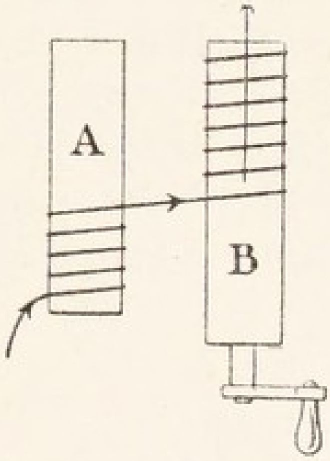


Fig. 237.—Rheostat.

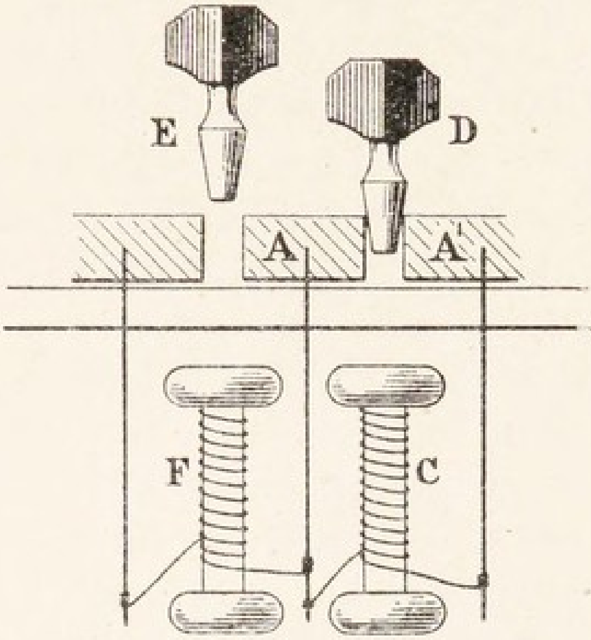


Fig. 238.—Resistance coil.

(E), the current passes through the resistance F; when the plug is replaced (D), the current passes through the *plug*, whose resistance is relatively nil. These coils are used singly and in sets.

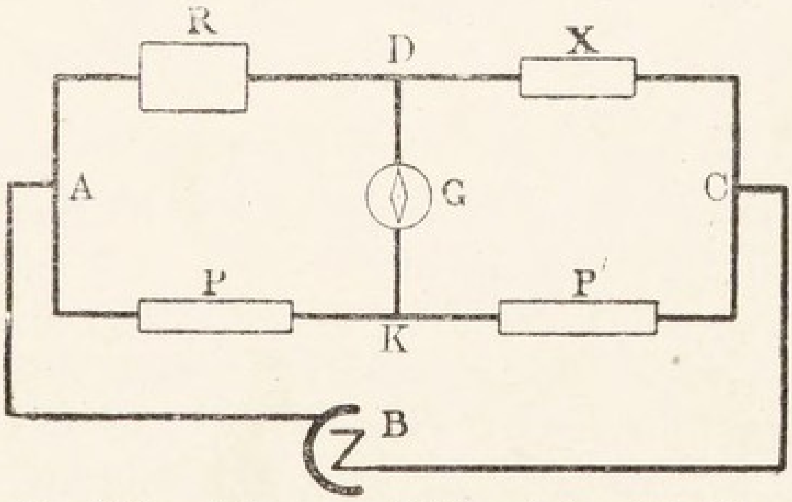


Fig. 239.—Diagram of Wheatstone's bridge.

Wheatstone's bridge.—The resistance of a conductor may also be measured by Wheatstone's bridge. The principle of this instrument is illustrated by Fig. 239. R is a resistance which can be varied,

X is an unknown resistance to be determined, P and P′ are resistances which can be made of some convenient ratio to each other, as 1 : 1, 1 : 10, 1 : 100. The connections are all made with thick copper wire of negligible resistance. A galvanometer G and a battery B are connected as seen in the figure. As seen in the diagram, the current enters the bridge at A and leaves it at C. In the bridge the circuit is divided; part of the current flows along the path A D C through the resistances R and X, and part flows along A K C through P and P′. Two points, D and K, one on each path, are found which are at the same potential, as shown by the fact that when connected no current flows from one to the other (p. 342). Suppose the total fall of potential from A to C to be $E_1 + E_2$ and let E_1 of this be the fall from A to D, or from A to K, then E_2 must be the fall from D to C, or from K to C. But the resistances in the two sections of either path must be proportional to the fall in potential they produce; therefore

$$\frac{R}{X} = \frac{E_1}{E_2} \text{ and also } \frac{P}{P'} = \frac{E_1}{E_2}$$

therefore $$\frac{R}{X} = \frac{P}{P'}$$

As the ratio P : P′ is known, and R is also known, X can readily be calculated.

In one common form of this bridge, a German-silver wire *a b*, of high resistance, is stretched between two thick copper plates C C (Fig. 240). At R a known resistance is inserted, and at X the unknown resistance. One wire from the galvanometer can be made to touch the wire *a b* at any spot. The battery is connected as shown at F F, which correspond to the points A C in the previous explanation. Contact is made between the galvanometer and *a b* until a

point is found at which no deflection of the needle is observed on touching. This point corresponds to K in Fig. 239. Then, as before, P : P′ :: R : X. *a b* is furnished with a scale, so that the lengths P and P′ can be read off; these lengths are proportional to the resistances, as the wire is of uniform resistance throughout.

Another very convenient arrangement is the "**post-office box,**" which is shown in Fig. 241. P and P′ are two sets of resistance coils of 10, 100, and 1,000 ohms each, so that P : P′ may be made 1 : 1, 1 : 10, or 1 : 100. The galvanometer is connected to the key

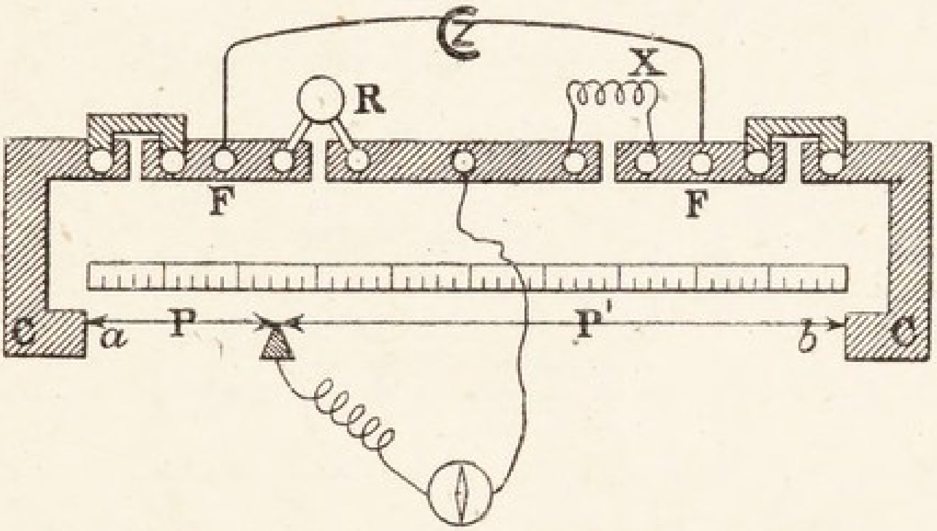


Fig. 240.—Wire form of Wheatstone bridge.

K, which, when put down, connects to the end of P. The other terminal of the galvanometer is connected to G L. The battery is coupled to the key K′, which, when down, connects it with B, the brass block connecting P and P′; the other end of the battery is attached to B′ E; X, the unknown resistance, joins B′ E with G L. Compared with the diagram of Fig. 239, the points B and B′ correspond to A and C, while the block G L and the key K correspond to the points D, K. We have therefore

$$\frac{P'}{X} = \frac{P}{R}$$

which leads to the same result as before. In determining X, P and P′ are first made equal by taking out the plugs 10 : 10. Suppose R is then adjusted by taking out plugs till, on making contact with the keys K and K′, no deflection, if possible, of the galvanometer occurs; then, since P = P′, we know that R = X.

It will often happen, however, that we cannot at first adjust R to produce no deflection. Suppose

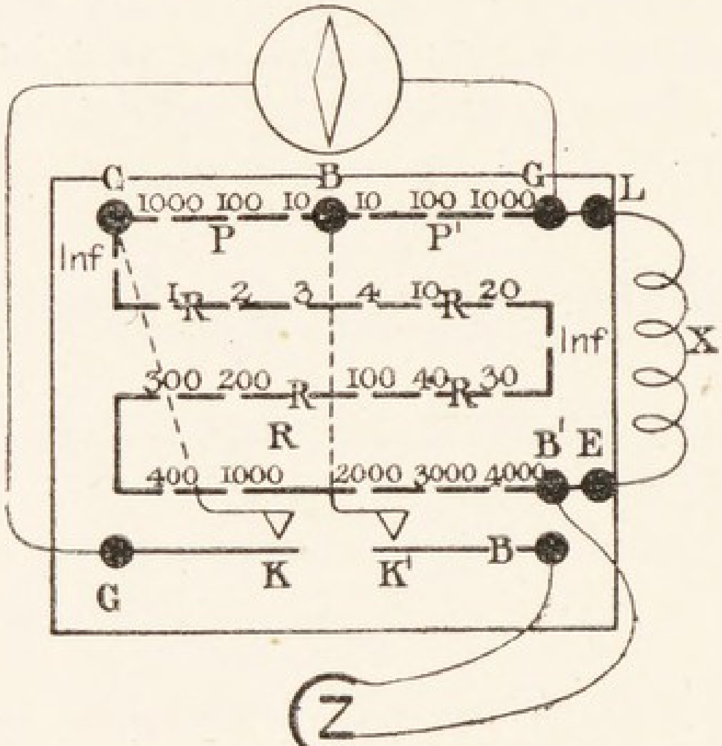


Fig. 241.—"Post-office box."

that when R is 3 the deflection is in one direction, and that when R is 4 the deflection is in the other. We know, then, that the true value of X is between 3 and 4. To determine the first decimal place, we replace the 10 plug in P and remove the 100 plug, so that P is now 10 times P′. We know therefore that R must be 10 times X, and will consequently be between 30 and 40. Begin with R = 30, and then try 31, 32, etc., till a value which gives no deflection is obtained. Suppose that then

R = 37; this gives X = 3·7. If, as at first, we cannot find a value which gives quite no deflection, but find that small deflections in opposite directions are obtained with values 37 and 38, we then make P : P′ :: 1,000 : 10, and proceed as before R will now be 100 times X, and will therefore be between 370 and 380. Suppose 372 gives no deflection, then X = 3·72 ohms.

The values of specific resistance tabulated on p. 341 show how widely substances differ in this property. It is this enormous difference between conductors like copper and non-conductors like mica that enables us to guide electricity along any path

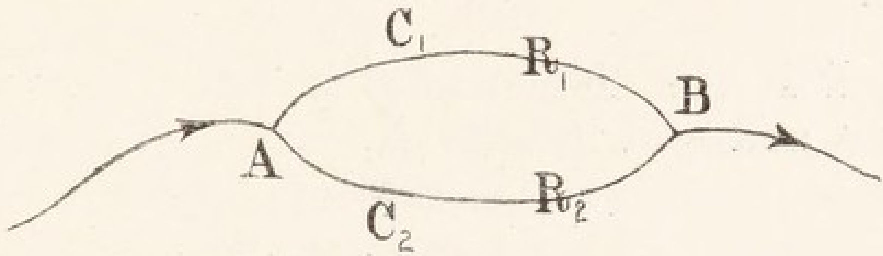


Fig. 242.—Divided circuit.

we choose. Thus, Lehfeldt states that a current would rather travel right round the earth by a copper wire about $\frac{1}{50}$ in. in diameter than pass through a thin piece of mica.

When a current passes through several resistances in *series*, it is evident that the total resistance is the *sum* of the resistances. Thus, if a current passes through a lamp with resistance A, a heating stove with resistance B, and a second lamp with resistance C, the total resistance would be $A + B + C$.

Conductors in parallel circuit.—When a current divides into two portions carried by two conductors in parallel circuit, the relative current strengths in the branches will be inversely proportional to their resistance, but directly proportional

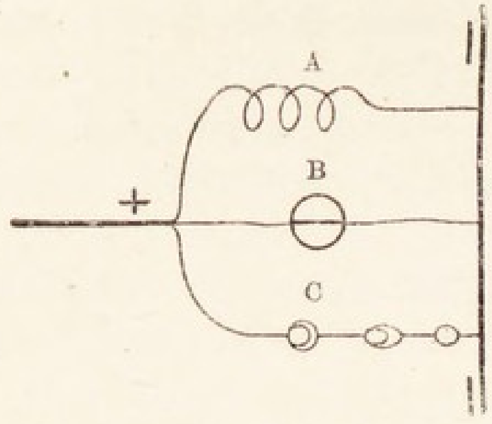


Fig. 243.—Divided current.

to their conductivity or conductance, which is the reciprocal of the resistance. If the current arriving at A (Fig. 242) divides and proceeds to B by two paths which have the respective resistances R_1 and R_2, the currents C_1 and C_2 carried by these paths are by Ohm's law

$$C_1 = \frac{E}{R_1}$$

$$C_2 = \frac{E}{R_2}$$

where R is the difference of potential between the points A and B of the circuit,

$$\therefore \quad C_1 R_1 = E = C_2 R_2$$

$$\therefore \quad C_1 : C_2 :: R_2 : R_1$$

but if C be the total current, and R be the *joint* resistance, between A and B, we know that

$$C = \frac{E}{R}$$

$$\text{and } C = C_1 + C_2$$

$$\therefore \frac{E}{R} = \frac{E}{R_1} + \frac{E}{R_2}$$

$$\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

$$\therefore R = \frac{R_1 R_2}{R_1 + R_2}$$

This is easily extended to three or more conductors in parallel circuit; thus, if A, B, C (Fig. 243) be the resistances of the three branches, and E be the potential difference between the two ends of the

divided circuit, and R the *joint* resistance of the divided circuit, we know that

$$\frac{E}{R} = \frac{E}{A} + \frac{E}{B} + \frac{E}{C}$$

$$\therefore \quad \frac{1}{R} = \frac{1}{A} + \frac{1}{B} + \frac{1}{C}$$

We have also shown that

$$C_2 R_2 = C_1 R_1 = C R$$

$$C_1 = \frac{R}{R_1} \times C$$

$$= \frac{R_2}{R_1 + R_2} \times C$$

and, similarly, $C_2 = \dfrac{R_1}{R_1 + R_2} \times C$

Shunts.—This division of a current enables us to send an aliquot portion of a current, instead of the whole current, through a galvanometer. For this purpose a wire of known resistance is joined in parallel to the terminal screws of the galvanometers. This wire is called a *shunt*. If the resistance of the shunt is S, and of the galvanometer G, the total resistance R of the shunted galvanometer is given by

$$\frac{1}{R} = \frac{1}{G} + \frac{1}{S}$$

the fraction of the whole current going through the galvanometer is $\dfrac{S}{S+G}$, and the fraction going through the shunt is $\dfrac{G}{S+G}$.

It is usually convenient to make $\dfrac{S}{G} = \dfrac{1}{9}$ or $\dfrac{1}{99}$, etc. If $\dfrac{S}{G} = \dfrac{1}{9}$, then $\frac{1}{10}$th of the total current goes through

the galvanometer, and $\frac{9}{10}$ths go through the shunt; if $\frac{S}{G} = \frac{1}{99}$, only $\frac{1}{100}$th of the total current goes through the galvanometer. A shunt of this kind is associated with the milliammeter included in the galvanism circuit (p. 354), so that, although the instrument only reads to 5 ma., it can really measure up to 50 ma.

The equation from Ohm's law can also be written $R = \frac{E}{C}$, so that the resistance in practical units can be stated as volts per ampere. Sometimes the reciprocal of the ohm is employed as unit to express the conductivity or **conductance**, and is called a *mho* (*mho* is *ohm* spelt backwards). Since $\frac{1}{R} = \frac{C}{E}$, a conductance of 1 ampere per volt is equal to 1 mho. If a 20-candle electric lamp take 0·7 ampere at an electrical pressure of 100 volts, the resistance is $\frac{100}{0{\cdot}7} = 143$ ohms, and the conductance $\frac{1}{143}$ mhos.

This principle comes into daily use in determining the resistance and conductance in the domestic use of electricity. The lights, stoves, etc., are usually placed in parallel circuit, i.e. crossing from the positive wire of the supply to the main wire connected with the negative end.

Ex.: If a supply of 200 volts feeds 20 lamps of 200 ohms each, a cooker of 30 ohms, and one larger lamp of 40 ohms, coupled in parallel circuit, find the joint conductance, etc. The conductivities are 20 at $\frac{1}{200}$

$= 0{\cdot}1$ mho; the large lamp $\frac{1}{40} = 0{\cdot}025$ mho; the cooker $\frac{1}{30} = 0{\cdot}033$ mho; the total mhos $= 0{\cdot}1 + 0{\cdot}025 + 0{\cdot}033 = 0{\cdot}158$ mho, and resistance $= \frac{1}{0{\cdot}158} = 6{\cdot}33$ ohms; the current flowing through when all lights were connected would be $\frac{200}{6{\cdot}33} = 31{\cdot}5$ amperes.

Board of Trade unit.—This unit of energy, for which in London no greater charge than 8d. can be demanded—the usual charge varies from 2d. to 6d.—equals 3,600,000 joules or 1,000 watt hours, or 1·34 horse-power, or 2,653,200 foot-pounds (*see* p. 28).

A joule = the work done per second when a current of 1 ampere flows through a circuit between the terminals of which a potential difference of 1 volt is maintained, and, roughly, = 0·737 foot-pounds.

$$\begin{aligned} \text{So if } A &= \text{current in amperes,} \\ V &= \text{potential in volts,} \\ S &= \text{number of seconds,} \\ J &= A\,V\,S. \end{aligned}$$

Ex.: 1. A pressure of 110 volts is maintained between the electric light mains of a house, and 20 glow lamps in parallel circuit, each taking a current of 0·3 amperes, are turned on for five hours each night for 20 nights. Find the energy in joules.

$20 \times 0{\cdot}3 \times 110 \times 5 \times 3{,}600 \times 20 = 237{\cdot}6$ million joules.

2. Find the cost per hour of a 16-candle lamp which takes 2·5 watts per candle, the price of the Board of Trade unit being 6d.

$\frac{2{\cdot}5}{1000} \times 16 \times 6 = 0{\cdot}24$ of a penny, or about one farthing.

Measurement of Electromotive Force

Since $C = \frac{E}{R}$, the measurement of E can be made to depend upon that of C and R (which we have already considered) by passing a current through a circuit of known resistance in which an ammeter is included. If we connect a high-resistance galvanometer, as a shunt, to two points of this circuit, we should be able to see what deflections corresponded to different known values of C (amperes) and R (ohms), and therefore to known values of E (volts). We could in this way calibrate the shunted galvanometer to read volts, and thus convert it into a *volt meter.* By giving to the shunted galvanometer a relatively high resistance we prevent any appreciable change in the joint resistance, or in the current, of the circuit. E can also be determined by comparison with the E.M.F. of a standard cell, E_1. If the two cells joined in series produce a deflection θ, and when opposed, in the same circuit, produce a deflection θ_1, then we know

$$\frac{E + E_1}{E - E_1} = \frac{\tan \theta}{\tan \theta_1}$$

and therefore

$$\frac{E}{E_1} = \frac{\tan \theta + \tan \theta_1}{\tan \theta - \tan \theta_1}$$

whence E is calculated.

The comparison can also be made by the potentiometer.

Measurement of a current by the potentiometer (Fig. 244).—$a\,b$ is a long wire of German silver, with which a battery of two Grove cells is connected, so that the current flows from b to a. The potential in the wire falls from a maximum at

b to a minimum at a. Much in the same way, if we attach a long narrow tube $a\,b$ to a tall cylinder full of water (Fig. 245), the pressure, owing to the friction against the sides of the tube, will gradually

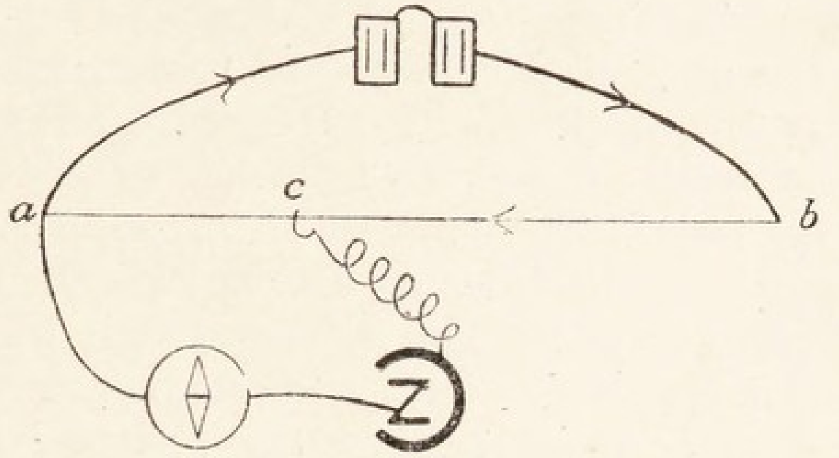


Fig. 244.—Potentiometer.

diminish as we get farther away; so that if we insert vertical glass tubes T T T, we see that the level of the water steadily sinks as we approach the end.

Now, if we take a standard Latimer-Clark cell

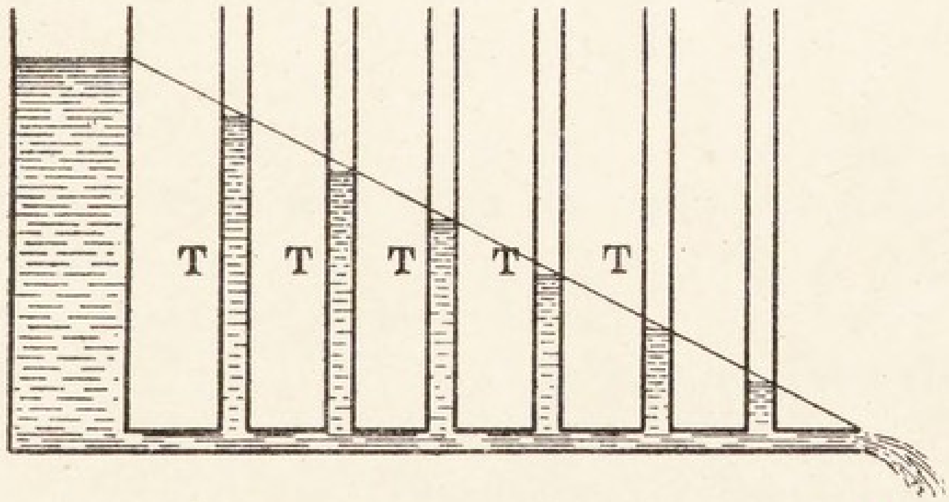


Fig. 245.—Fall of water pressure in long tube.

and couple one pole with a, so that the current flows in a direction opposite to that of the Grove cell, introducing a galvanometer into the circuit (Fig. 244), and then with the wire from the other end of the

Latimer-Clark cell touch the wire $a\,b$, until we find a spot, c, where the galvanometer indicates no current in the Latimer-Clark, we have tapped off just enough current to balance the standard cell. We then measure the distance $a\,c$. The experiment is repeated with the cell whose strength is unknown, and we find the distance is $a\,c'$; then $a\,c : a\,c' :: 1{\cdot}43$: voltage of the unknown cell (*see* Fig. 218, and p. 342).

This is really a special instance of the divided circuit (p. 359). The current C flowing in the fine wire from b to c divides at c into two portions, C_1 in the branch $c\,a$ of the fine wire, and C_2 in the branch circuit containing the cell and galvanometer; we therefore know that if r_1 be the resistance of $c\,a$, and r_2 that of the branch circuit,

$$C_2 = \frac{r_1}{r_1 + r_2} \times C$$

but if E is the E.M.F. of the cell in the branch circuit, we must also have, when no current passes through the galvanometer,

$$C_2 = \frac{E}{r_1 + r_2}$$

and therefore $\quad E = r_1 \times C$

Similarly, when we substitute another cell whose E.M.F. is E', and find that the contact slider must be moved to c' for the galvanometer to show no deflection, we have

$$E' = r_1' \times C$$

where r_1' is the resistance of $a\,c'$; but C is constant,

therefore $\quad \dfrac{E}{E'} = \dfrac{r_1}{r_1'} = \dfrac{a\,c}{a\,c'}$

since the wire $a\,b$ is of uniform resistance throughout.

Since the E.M.F. really depends upon the difference between the potentials of the two terminals, it can

be found by separately determining each potential. Instruments for this purpose are called electrometers. The quadrant electrometer is a standard form, but a description of it is rather outside the scope of the present manual. The **capillary electrometer** can be used to detect a small difference of potential between two points. This instrument (Fig. 246) was originally suggested by Lippmann. A piece of small glass tubing is thoroughly cleansed with 10 per cent. sulphuric acid, washed out, and dried. It is then drawn out into a very fine capillary tube A; the bore should be so small that it will stand a pressure of a metre of mercury without leaking, and yet be pervious. This is filled with mercury and connected with a bottle of mercury B, which can be elevated so as to force the mercury into the capillary. The capillary dips into a thin-walled glass tube C containing 10 per cent. sulphuric acid and some mercury; two platinum wires P and P′, fused into the glass, establish contact with the two masses of mercury.

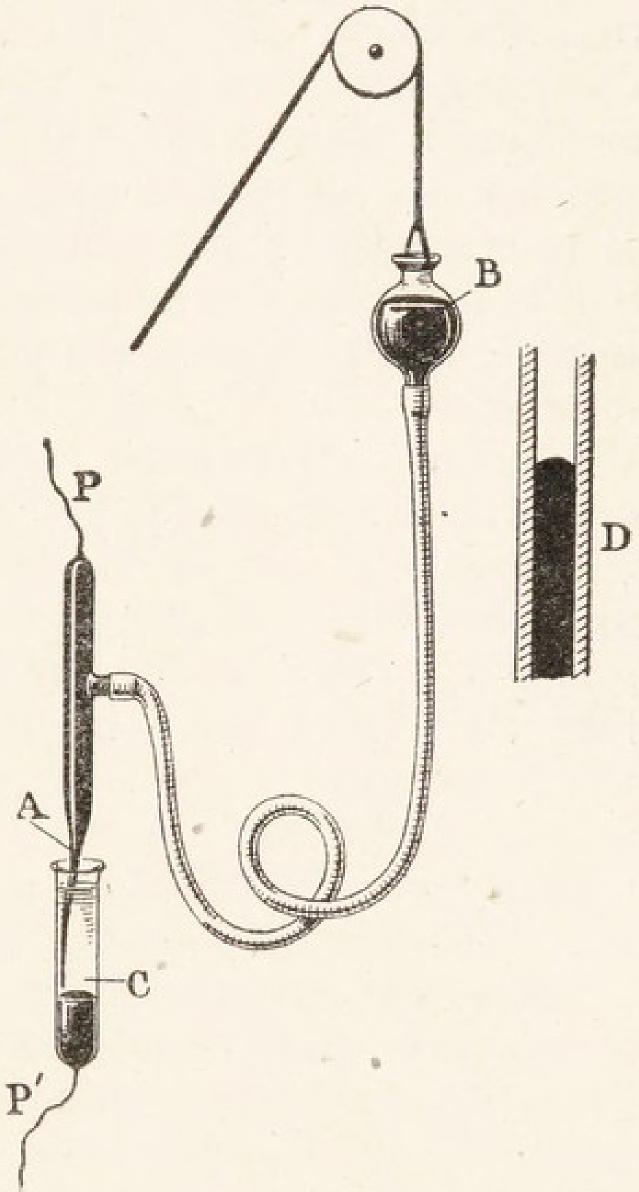


Fig. 246.—Lippmann's capillary electrometer.

The capillary tube is so placed that it lies close to the thin wall of the glass tube, where it can be viewed by a microscope; it presents the appearance seen in Fig. 246, D.

The capillary electrometer is an indicator of potential, not of current. If P and P′ are connected with the apex and base of a frog's heart, the mercury meniscus is seen to move with the electrical variation which takes place with each beat of the heart. If P is positive the mercury moves towards the point; if P is negative the motion is away from the point. For certain purposes, especially in physiology, this instrument is invaluable; its movements are instantaneous, and there is no back-swing. By its aid the currents developed by the voice when speaking to the telephone can be easily demonstrated, and the movements of the mercury meniscus can be photographed. When not in use it is most important to keep the wires P and P′ in metallic connection so that the instrument is short-circuited. The cause of the movement is that the surface tension between the mercury and the sulphuric acid is altered when there is a difference of potential between P and P′.

Thermal effects.—When a current passes through a resistance, part of the electricity is converted into heat. Thus, in the ordinary incandescent electric lamp the current passes in by two platinum wires fused into the glass bulb, through the delicate carbon filament of high resistance, which, owing to its resistance, becomes white hot, giving out light and heat. The intense white light of the *Nernst lamp* comes from a filament of yttria and zirconia (two rare earths) raised to a white heat by the electric current, to which it offers great resistance. In the *galvanic cautery* also the platinum loop is heated by its own resistance to the current.

The work W done by a current C in t sec. is the product of the quantity ($C t$) of electricity carried and the electromotive force ($E = C R$).

$$\therefore \quad W = C t \times C R = C^2.R.t$$

but, if this work is entirely converted into heat and produces H calories, we know (p. 187) that

$$W = H.J$$

$$\therefore \quad H = \frac{C^2.R.t}{J}$$

where $J = 4{\cdot}2 \times 10^7$, when C and R are in absolute units. If, however, the current is A amperes and the resistance is O ohms, then we know (p. 342) that $R = O \times 10^9$, and (p. 338) that $C = \frac{A}{10}$, and by substituting these values we find

$$H = \frac{A^2 \times O \times 10^9 \times t}{10^2 \times 4{\cdot}2 \times 10^7} = \frac{A^2.O.t}{4{\cdot}2}$$

Or, reverting to the usual notation, we find that the heat H in calories produced by a current of C amperes flowing for t seconds through a resistance of R ohms $= \frac{C^2 R t}{4{\cdot}2}$.

C can therefore be determined from this formula, if a coil of known resistance R is placed for a known time t in a water calorimeter, and the rise of temperature of the water observed.

Besides this frictional generation of heat which always occurs throughout the circuit, other thermal effects have been recognized. In 1822 T. J. Seebeck noticed that a current was produced in a circuit composed of two dissimilar metals when one junction was hotter than the other.

If a bar of antimony A be joined to a bar of bismuth B at one end, and the two free ends be connected with a galvanometer, then when the junction is warmed a current will be set up, flowing from the antimony to the bismuth, through the galvanometer. If a number of such couples be connected with a

galvanometer we have a delicate indicator of temperature, the **thermopile** (Fig. 247). A similar arrangement can be used, instead of a battery, to produce a current.

The E.M.F. due to this cause, or the thermoelectric power of the couple, varies with the metals, and also with the temperature difference of the junctions. Indeed, the *direction* of the current may undergo a change (*thermo-electric inversion*). Thus, copper is negative to iron, that is, the current flows from iron to copper across the *colder* junction, and the E.M.F. increases so long as the temperature of

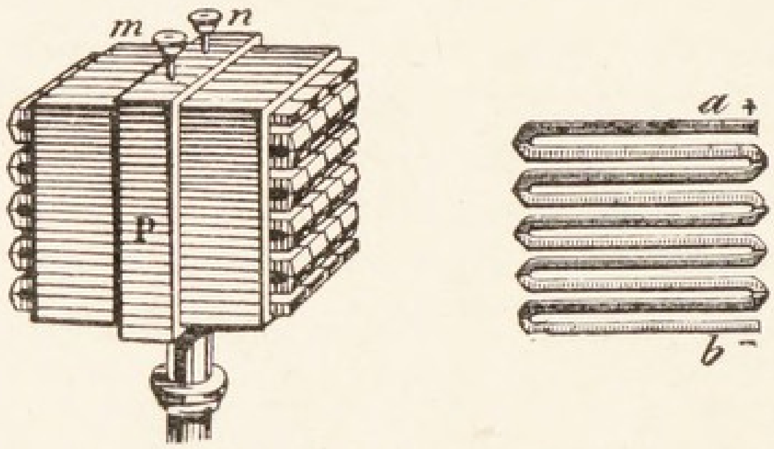


Fig. 247.—Thermopile.

the hotter junction does not exceed 270° C. Above this temperature, the *neutral point*, the E.M.F. diminishes, and vanishes entirely when the hot junction is as much above 270° C. as the cold junction is below 270° C. When the temperature is still further raised, copper is positive to iron. Other pairs of metals have their corresponding neutral points. In 1834 Peltier observed that, when a current is passed round a circuit of two dissimilar metals, heat is absorbed at a junction, and the junction is therefore *cooled*, if the current is in the same direction as would have been produced by heating this junction. If the current is reversed the thermal effect is reversed

and the junction is warmed. This *Peltier effect* is confined to the *junctions*, but Thomson (Lord Kelvin) realized that the conservation of energy necessitated compensating thermal effects elsewhere, and was able to show that a current from hot to cold *absorbs* heat in iron but *evolves* heat in copper. This *Thomson effect* occurs in the *wires*.

Exercises

1. If a current of 1 ampere is passed through a wire of resistance 4·2 ohms, heat is produced at the rate of 1 calorie per second. Calculate the rate at which heat is produced (i.) if the resistance be doubled but the current unchanged, (ii.) if the current be doubled but the resistance unchanged. Calculate also the difference of potential between the ends of the wire in the three cases. [*First Professional.*]

2. A tungsten-filament electric lamp carries a current of 0·2 ampere when the potential difference between its terminals is 20 volts. If this difference is increased to 100 volts the current only increases to 0·25 ampere, but the wire becomes white hot. What conclusions can be drawn from these facts? [*Ibid.*]

3. An ammeter is tested by sending a current through it, and through a solution of copper sulphate, at the same time. A steady current of nominally 1 ampere is maintained for half an hour, and 0·55 grm. of copper is found to have been deposited in this time. If the electro-chemical equivalent of copper be 0·00033 grm. per coulomb, what is the error of the ammeter? [*Ibid.*]

4. An ammeter of resistance 3·6 ohms is provided with an external shunt of resistance 0·4 ohm. The ammeter indicates 1 ampere; what is the total current flowing through the circuit? [*Ibid.*]

5. A battery of 2·1 volts in series with a tangent galvanometer and 200 ohms resistance gives a deflection of 45°. By putting into the circuit an additional resistance of 300 ohms the deflection is reduced to 30°. Find the reduction factor of the galvanometer. [*First M.B.*]

(For Answers, *see* p. 390.)

CHAPTER V

ELECTRO-MAGNETISM

Magnetic Action of a Current—Induced Currents—Lenz's Law—Dynamo—Telephone—Electro-Magnet—Induction Coil—Discharge in Vacuum Tubes—Röntgen Rays—Exercises.

Magnetic action of a current.—A copper wire in which a current of electricity is flowing not only exerts magnetic force upon a compass needle as

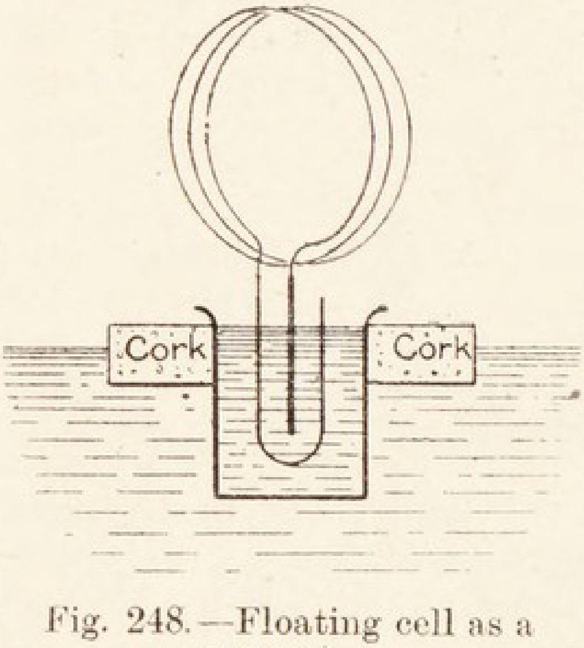


Fig. 248.—Floating cell as a magnet.

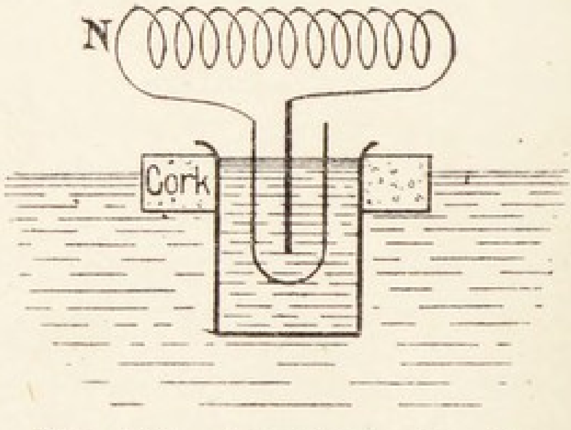


Fig. 249.—Floating cell with spiral acting as a compass needle.

already described (p. 347), but also behaves like a magnet in other respects.

If a small single cell be floated in water and its poles connected by a hoop of insulated copper wire (Fig. 248), the cell behaves like a magnet and sets one face of the hoop to the north and the other to the south. If the north pole of a bar magnet be

brought near the face of the hoop which looks north, the little cell will be repelled and float away; then turning itself round, it rushes on the magnet with its south-seeking face foremost.

The magnetic properties of a live copper wire are also very clearly shown if the poles of the floating cell are connected by a horizontal spiral of copper wire. This sets itself north and south and behaves just like a compass needle (Fig. 249). The position of the poles as regards the direction of the current agrees with Ampère's rule (p. 348). The north-seeking face of the loop and the north-seeking end of the spiral will be found to be on the left-hand of a man swimming in the wire *with* the current, as shown by the arrow (Fig. 250), with his face always directed towards the centre of the circle.

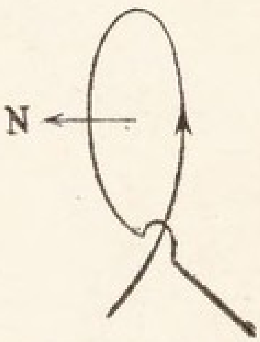


Fig. 250.—Relation between current and pole.

Not only can the introduction of a current into a coil make it a magnet, but, conversely, the introduction of a magnet into the helix of a coil causes a current to appear in the coil. This is an **induced current.** It is momentary, and is produced by making the circuit wire cut the lines of force in the field of the magnet. If the magnet is held *at rest* in the coil no further current is produced.

If now we suddenly remove the magnet, a second induced current is produced in the coil. This current is also momentary, but is in the *opposite* direction to the first in the coil.

In these and all similar cases the direction of the induced current is such as to oppose the motion which induced it. This is known as **Lenz's law.** It

is a particular example of Newton's Third Law (p. 28). The current induced, for instance, by thrusting a north pole *towards* one face, or end, of the coil must be such as would repel a north pole, and therefore must be such as to make that end a *north* pole; hence its direction can be determined by Ampère's rule. When, however, the north pole of the magnet is *withdrawn* from the coil, the current

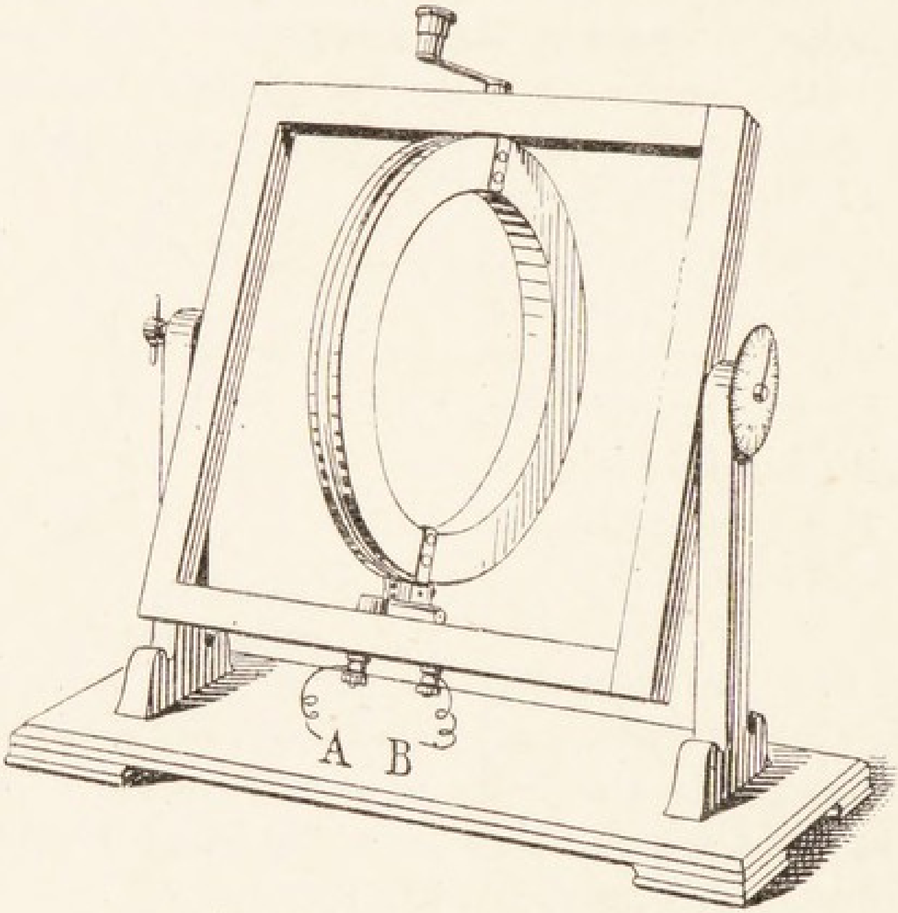


Fig. 251.—Delezenne's circle.

induced must be such as to attract the north pole back again, and therefore must be such as to make this end of the coil a *south* pole. Hence, it must be opposite to the direction of the first induced current.

Currents induced by the needle's own movements are utilized to bring it more quickly to rest in the *dead-beat* type of galvanometer.

The same effects are produced if the magnet is

stationary and the coil of wire is moved backwards and forwards through the magnetic field; or if the coil is made to rotate so that the wires cut the lines of force of the magnet; and thus we arrive at the principle of the **dynamo,** so largely used at the present day for generating electricity. Whenever a copper wire cuts these lines of force an electric current is generated in the wire. The larger the number of lines cut in a given time and the more powerful the magnetic field, the stronger is the current produced.

The lines of force due to the earth's magnetism are parallel to the *dip-needle* (p. 313). A copper

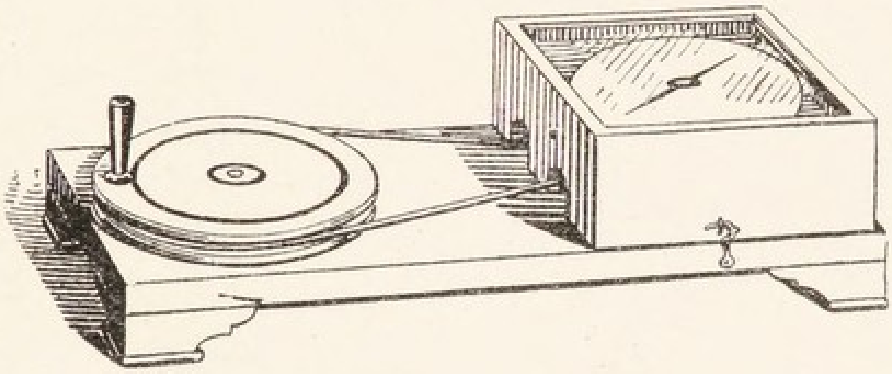


Fig. 252.—Arago's disc.

hoop held with its plane at right angles to the dip-needle will inclose the maximum number of these lines, and when rotated about a diameter will cut these lines, and a current will be induced in the wire. If the wire be mounted, as in Delezenne's circle (Fig. 251), so that the currents in one direction only can be run off, this can be detected by a suitable galvanometer.

If a copper disc (Fig. 252) be made to rotate beneath the magnet, N., S., and therefore cuts the lines of force round the magnet, a current is thereby induced in the disc tending to make the disc rotate in the opposite direction (Lenz's law), but to urge the magnet pole in the same direction as that which

the disc is obliged to take; the magnet accordingly soon follows the disc with increasing rapidity.

A dynamo is reversible. If, instead of rotating the masses of copper wire in the magnetic field and so producing a current, we pass a current into it the coil of copper wire will rotate.

Every wire conveying a current when placed in a

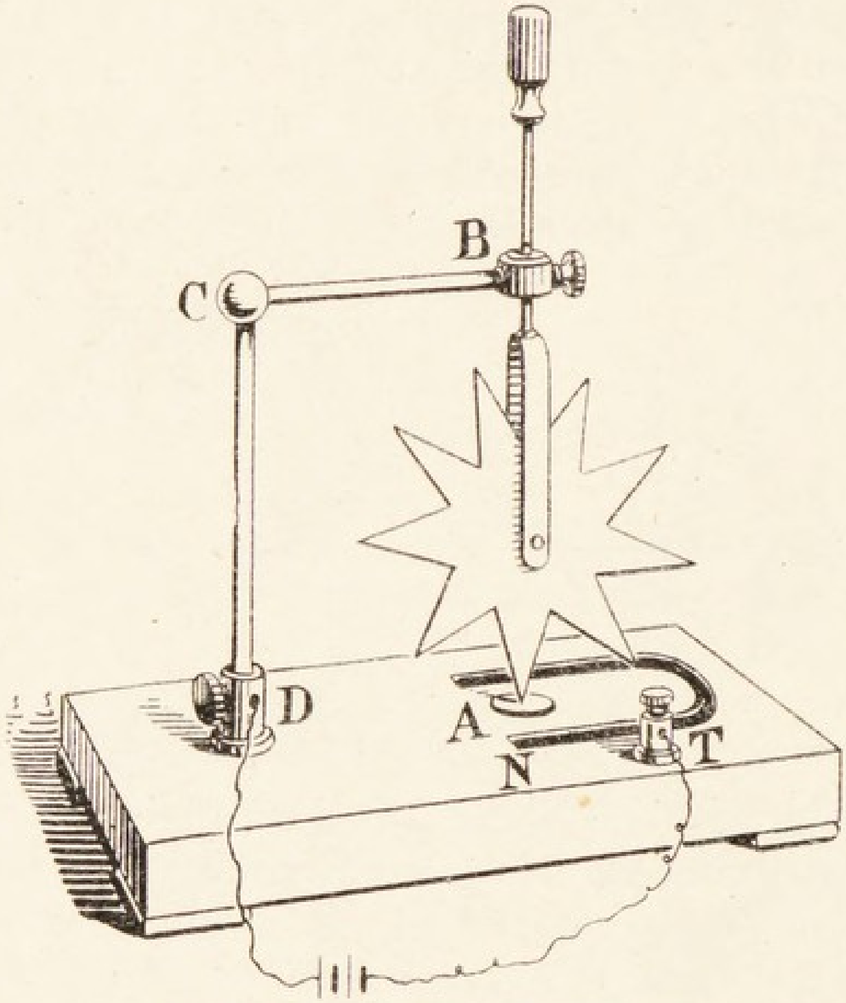


Fig. 253.—Barlow's spur wheel.

magnetic field experiences a force urging it in a definite direction. This direction is at right angles to the lines of magnetic force and also to the wire. The wire, in fact, is urged in the opposite direction to that in which the pole would be moved which Ampère's swimmer faces. In Barlow's spur wheel (Fig. 253), when the current enters at T, which is connected with the mercury in the little trough A,

and passes *up* the copper spoke, if the magnet is placed as in the figure the spoke will be urged from A towards D; as each spoke in turn, when it touches the mercury, experiences this force, the wheel rotates clockwise, as seen by the reader. On reversing the current the direction of rotation is reversed. If the position of the magnet be reversed the respective rotations are reversed. The same force is utilized in the *electromotor* and in the *moving-coil galvanometer*.

The **Graham-Bell telephone** (Fig. 254) is one of the most useful applications of these induced currents. A permanently magnetized steel rod A is placed in a wooden case. At one end of the magnet is a coil of insulated copper wire C, the two ends of which are brought through the wooden case to the two binding screws S S. Close to the end of the magnet is firmly clamped a circular iron plate B. If this plate is suddenly moved backwards and forwards, induced currents are set up in the coil of wire. These currents pass down the wires and produce corresponding movements in the iron disc of a second similar telephone connected by the wires with the first. The reproduction is so perfect that if words be spoken to the first telephone they will be audibly reproduced by the vibrations of the plate of the second telephone. The currents so generated can be demonstrated by the aid of the capillary electrometer.

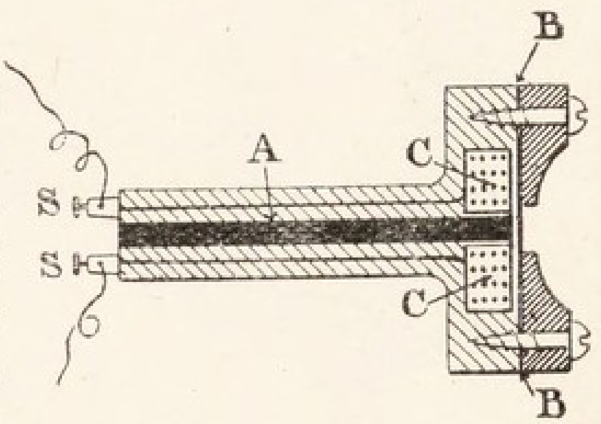


Fig. 254.—Graham-Bell telephone.

Electro-magnets.—A piece of soft iron, round which an insulated copper wire is suitably wound,

becomes a powerful electro-magnet (Fig. 255) when a current is sent through the wire, but only remains one while the current lasts. The polarity of its ends varies with the direction of the current and accords with Ampère's rule.

A live circuit can also behave like a magnet in inducing a current in another circuit. If we connect a copper wire A B with a battery and a key (Fig. 256), and bring near it a second wire C D, running parallel to A B, whose ends are connected with a galvanometer

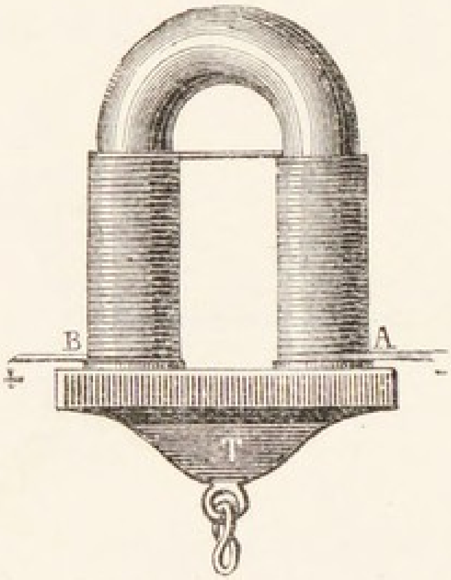


Fig. 255.—Electro-magnet.

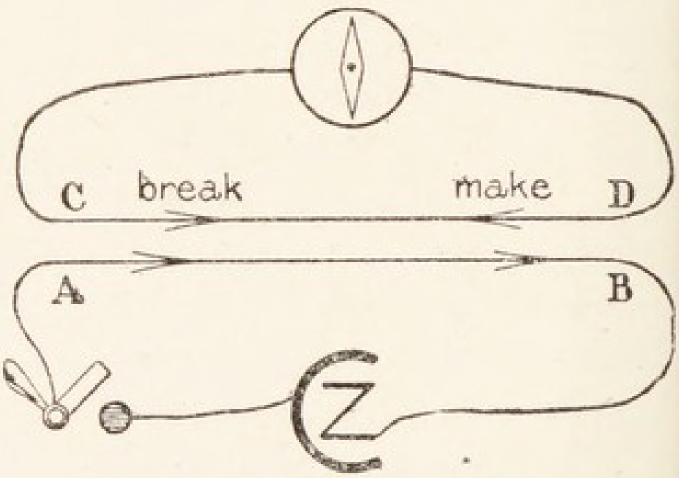


Fig. 256.—Induction by current.

or a capillary electrometer, we shall see that, at the moment we put down the key (*make* the circuit), and the current flows from A to B, a momentary current in a direction *opposite* to that in A B is induced in the wire D C. As long as the current in A B remains constant, nothing further will happen; but when we lift the key and *break* the circuit, another momentary current, in the *same* direction as that of the primary current in A B, is induced in C D. The strength of these *induced currents* in C D can be greatly increased by coiling the insulated wires A B and C D.

This induction of a secondary current, in a closed coil of wire, by the passage of a current through a neighbouring coil, is the principle of the **induction coil.** One form much used in the physiological laboratory is that of Du Bois-Reymond. The *primary coil* is composed (Fig. 257) of comparatively few turns of thick insulated copper wire. A key is inserted in the primary circuit so that the current may be made or broken. The *secondary coil* consists of many turns of thin insulated copper wire, and is so arranged that its distance from the primary coil can be altered (*sledge* type). If the ends of the two

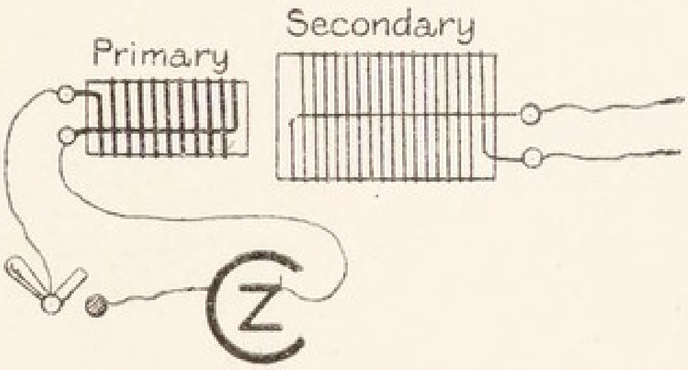


Fig. 257.—Induction coil for single shocks.

wires of the secondary coil be placed on the tongue, a distinct shock will be felt when the current in the primary circuit is made, and again when it is broken; the break shock being the stronger.

In this arrangement the make and break are made by hand, but by a simple automatic device a continuous and regular series of makes and breaks is made by the current itself—the "interrupted current." This is known as **faradization.** The current passes from the battery C (Fig. 258) to the screw S, thence by the steel spring P to the electro-magnet M, thence to the primary coil A, and finally home to the battery. When the battery is connected, the current causes the electro-magnet

to attract the little block of iron I fixed on the spring P. This breaks the contact between S and P, and the current ceases; M therefore ceases to attract I, and the spring re-establishes contact between P and S, when the cycle of events is repeated. Thus an automatic series of makes and breaks is set up, and the rate at which they take place is governed by the rate of vibration of the spring P. The primary coil is usually filled with soft-iron wires, which increase the inductive effect by becoming magnetized and demagnetized at each make and break. In some coils these iron wires are utilized to attract the vibrating spring, instead of having a special electro-magnet as above. Wires are better than a solid rod, as in the latter local magnetic effects would tend to delay the *sudden* changes in magnetism required to produce the maximum effect.

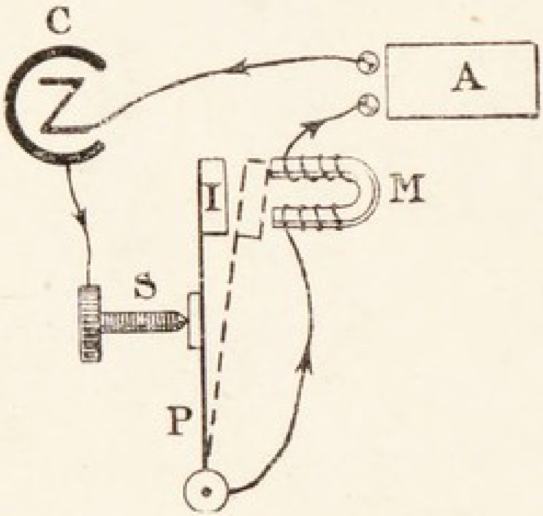


Fig. 258.—Faradization.

The general appearance of the coil is seen in Fig. 259. For single shocks the wires from the battery are inserted in screws 1 and 2; for the automatic break or faradization the screws 3 and 4 are used. A modified effect can be obtained by joining 1 and 3 with a thick wire, screwing up screw 7 into contact with the spring and withdrawing 5. The primary circuit is then never broken completely, but only short-circuited when the vibrator touches the screw 7 (*Helmholtz modification*). The secondary coil is movable, and its distance from the primary can be read off on the

scale, the maximum effect being attained when the coils overlap.

Extra current.—When a current is sent through a wire which has been wound into a coil, the successive layers of the coil act by induction on each other and produce a momentary current in the coil, which is opposed to the primary current at the "make." A second current would be generated at "break," in the same direction as the primary, if the circuit were not broken when the key was opened.

This extra current explains the difference between the strengths of the make and break in the Du Bois-Reymond coil. When the current is made in the

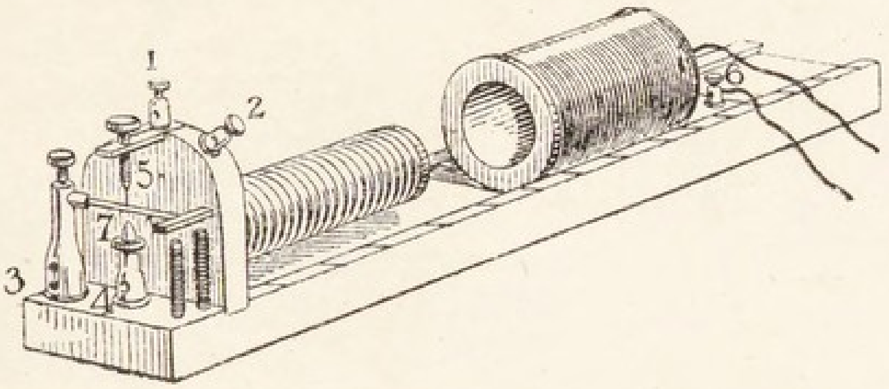


Fig. 259.—Du Bois-Reymond induction coil.

primary coil, it is opposed by its own self-induced extra current, running in the opposite direction, and so does not attain its full strength at once; but, on breaking, the potential falls *suddenly* from a maximum to zero. Now, the strength of the induced current depends not only on the change in the potential in the primary, but also on the rapidity with which this change is established. If the potential mounts slowly, the secondary current developed is much weaker than when the increase or decrease is instantaneous. So at the make, as the current is opposed by its own extra current, the full potential is not immediately attained; but at the break the

potential falls at once to zero, and the break shock in the secondary coil is much stronger. Medical treatment in which these induced, or *faradic*, currents are used is called **faradism.**

In the **Helmholtz modification** (Fig. 260) the extra current at break is not abolished, and being in the same direction as the primary it tends to tail off the fall of the potential. This *primary faradic* current is occasionally employed in faradism; having a much higher voltage than the battery current, it is able to flow through a derived circuit, including the

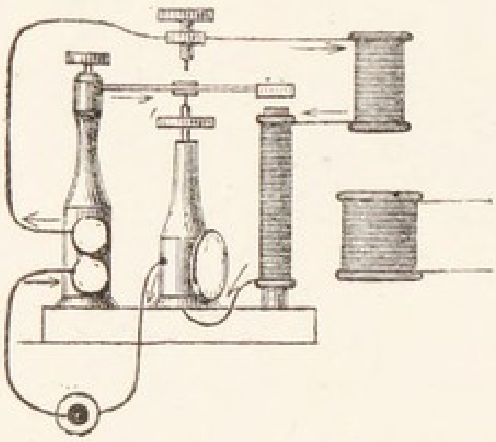


Fig. 260.—Helmholtz modification.

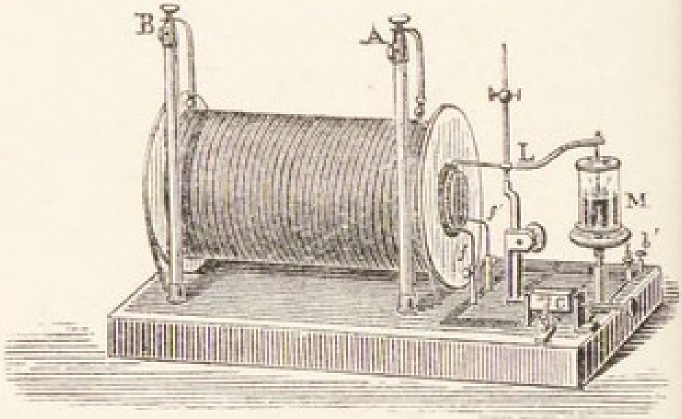


Fig. 261.—Ruhmkorff coil.

patient, though the battery current keeps to the primary coil.

In the **Ruhmkorff coil** the secondary coil is not movable, but is fixed in the position of maximum efficiency. By this means an E.M.F. of 20 volts can be increased to several thousand or more volts, the increase depending roughly on the ratio of the number of turns of wire in the two coils. In a huge coil made for the late Mr. Spottiswoode the copper wire of the primary coil was 660 yards long, whilst that of the secondary was 280 miles. The diameter of the wire was: in the primary 0·096 in., in the

secondary 0·0001 in. This gave a spark over 40 in. long.

One form of Ruhmkorff coil is shown in Fig. 261. The current from the battery enters at b' and passes to the commutator and key (*see* Fig. 225), thence through the primary coil by the wires f and f', thence to the lever centred at L, which forms the automatic breaker and maker of the current. One end of this lever dips into the mercury cup M; the other end has a piece of soft iron attached to it, which is pulled down, when the current passes, by the iron core of the coil; from the mercury the current passes to the screw b, and then to the battery. The ends of the primary coil are also associated with the two coatings of a condenser which lies in the wooden box on which the coil rests. This condenser is composed of layers of tinfoil separated by layers of paper soaked in paraffin wax. Alternate sheets of tinfoil are connected together and correspond to one coating of a Leyden jar. The condenser is charged by the direct current at make, and this swings back, as it were, in the form of an inverse current at break. Thus, the condenser suppresses the extra current at break but not at make, and so renders the induced current at break the more effective of the two. A and B are the ends of the secondary coil, and are connected, when the coil is in use, with the two points between which the spark is required to pass.

An induction coil may be looked upon as an instrument for transforming a moderate E.M.F. into an enormous E.M.F., transforming upwards. It is obvious that we might use it the other way round and transform a potential of some thousands of volts down to 100 or 200 volts. Transformers are actually used in this way for domestic purposes. It is found more economical to transmit currents of

high voltage and transform them down to 100 or 200 volts before they enter the houses, to be used for domestic lighting, etc.

The discharge from a powerful induction coil resembles a miniature lightning flash, and is accompanied by a sharp snapping sound. If the discharge passes through a tube partly exhausted, it takes the form of a narrow pale violet ribbon of light, which connects the two electrodes. As the exhaustion proceeds, we have the tube filled with an aurora-like glow, the colour depending on the nature of the minute quantity of the residual gas, as in the well-known Geissler tubes. If a more perfect pump, as a Sprengel, or double Fleuss, be used, we notice, when the pressure is reduced to something like 0·01 mm. of mercury, that the dark space surrounding the negative electrode seems to increase in area until it occupies the whole of the tube. The walls of the tube then appear to be phosphorescent, but the colour of the light varies with the nature of the glass. Tubes of ordinary soda glass have a yellowish-green glow. If the tube be made of lead glass the glow is bluish. This phosphorescence can be imparted to many other substances besides glass. It appears to be produced by the impact of minute negatively-charged particles (*electrons*) travelling with very high velocities. The charge on an electron has been estimated at $-\frac{15{\cdot}7}{10^{20}}$ coulombs. These particles seem to be projected from the kathode in directions normal to its surface. Their emission constitutes *kathode rays*. If we enclose in the tube some specimens of crystallized alumina (in the form of sapphires or rubies) and interpose them in the path of the kathode rays, they glow with a rich red light.

The property of endowing certain substances with

phosphorescence is useful in detecting and demonstrating the presence of the rays. They are, however, stopped by some opaque objects, and a metal plate in the path of the rays throws its shadow on the wall of the tube (Fig. 262). The obstacle is heated by the impact of the particles. The rays can be deflected by a neighbouring magnet.

These rays had been frequently observed and described before it was discovered that they were not always stopped by a solid substance, but could pass through very thin aluminium foil, for instance.

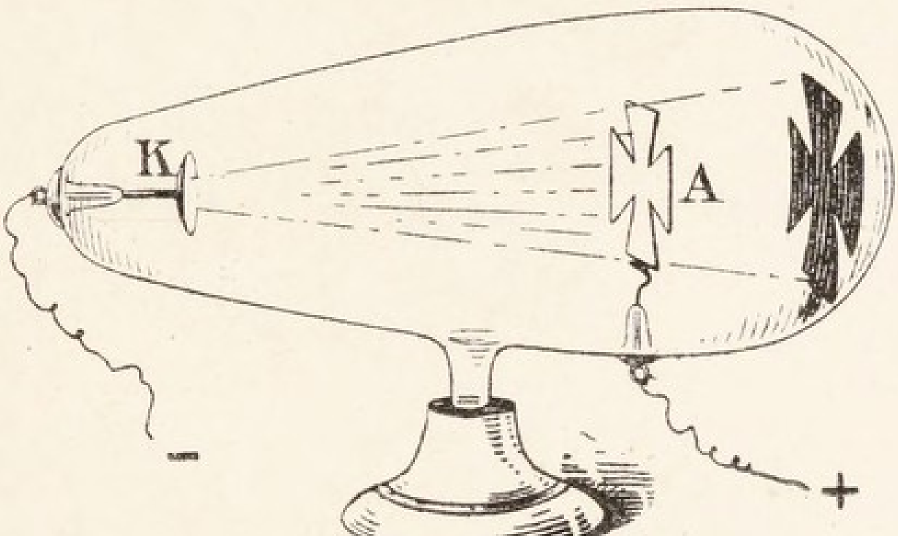


Fig. 262.—Vacuum tube, showing shadow.
A, Anode; K, kathode.

The fact seems to have been first observed by Hertz, and was afterwards completely confirmed and established by Lenard. Hence, kathode rays which have emerged in this way through a suitable *gate* in the vacuum tube are sometimes known as *Lenard rays*.

Even when the kathode ray is stopped by the solid wall of the tube, the destruction of the kinetic energy in the impact does apparently give rise to another variety of radiation, *external* to the tube. This was detected in 1895 by Röntgen, and these rays, at first called **X-rays**, are now also termed **Röntgen rays**,

They excite phosphorescence in many substances, such as barium platino-cyanide, but they differ in many respects from the ordinary kathode rays. They possess a considerable power of penetrating many solid substances. They pass easily through the flesh of the hand or arm, for instance, but not nearly so well through the bones. Hence, when the hand is held between a barium platino-cyanide screen and a Röntgen tube (Fig. 263), a dark, well-defined shadow of the bones is seen by an observer on the other side of the screen, *through* which he sees the

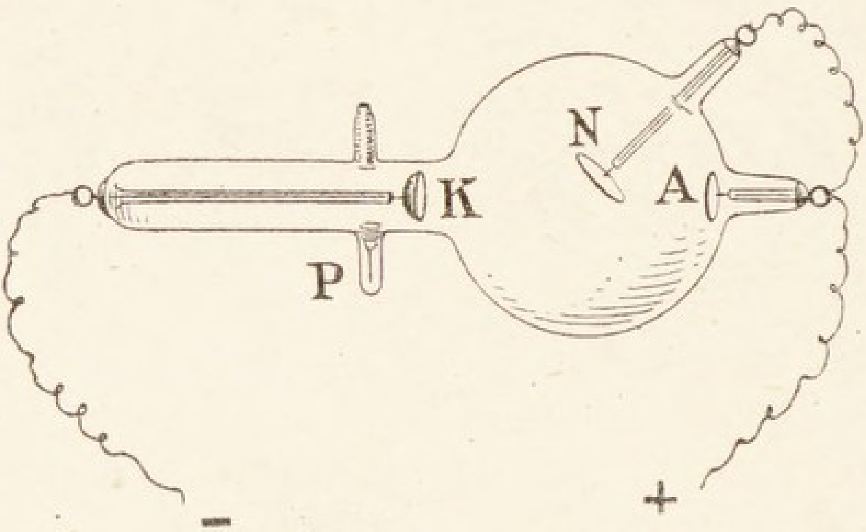


Fig. 263.—X-ray tube.
A, Anode; N, antikathode; K, kathode; P, palladium wire.

hand. At the same time the presence of any opaque foreign body, such as a needle or a bullet, is clearly demonstrated, and the apparatus is now in general use for locating such bodies.

The kathode and anode (Fig. 263, K, A) are made of aluminium; the antikathode (Fig. 263, N) is of copper coated with platinum (or tantalum); the concave kathode focuses the kathode rays approximately on N, and their bombardment of the antikathode gives rise to the X-rays.

These rays affect a photographic plate even when the plate is wrapped in light-proof paper, or enclosed

in a wooden box. If such a plate be substituted for the screen, and afterwards developed in the ordinary way, permanent radiographs may be printed from them as usual.

Röntgen rays are not refracted; they do not seem to be deflected by a magnet, but when they fall upon a charged electroscope the gold leaves collapse. The rays apparently discharge the electroscope by endowing the air with the power to conduct electricity. This power is not usually possessed by air or any gas.

After continued use the focus tube becomes even more highly vacuous, or *hard;* it is therefore often provided with a palladium wire (Fig. 263 P), which, when warmed, has the property of absorbing gas from the flame, and, when cold, giving it up in the tube, which thus becomes *soft.*

EXERCISE

Calculate the horse-power of the engine required to drive a dynamo designed to supply a house with enough current at 100 volts pressure to light 100 lamps in parallel, each of 200 ohms resistance, together with 2 motors in series taking 2 amperes. (1 horse-power = 746 watts.)

(For Answer, *see* p. 390.)

ANSWERS TO EXERCISES

Part I

Chapter I

1. (a) 80 ft. per sec.; (b) 48 ft.- sec. (a) 80 ft.; (b) 144 ft. 2. (a) 1 yd. per min. = 0·05 ft.-sec.; (b) 1 yd. per min. per min. = $\frac{1}{1200}$ ft. per sec. 3. $a = -105$ (nearly) ft.-sec. per sec.

Chapter II

1. 120 poundals; 16×10^5 dynes. 2. 4 poundals. 3. 60·8125 grm.; 61·8125 grm. 4. $m = 12{\cdot}5$ grm.; $v =$ 40 cm. per sec. 5. (1) $\frac{6}{1000}$ sec.; (2) $\dfrac{10^6}{6g}$ times the weight of the bullet. 6. 300 ft.-lb.; 9,000 ft.-poundals; 300 poundals.

Chapter III

1. $16\sqrt{3}$ at 30° to the vertical. 2. 100π; $\dfrac{10}{\pi}$ kilos = 3·183 kilos, nearly. 3. 60 ft.-lb. per sec. = 0·109 h.p., nearly.

Chapter IV

1. $46{\cdot}\dot{6}$ lb. 2. $\frac{4}{5}$; 13·7 grm., nearly. 3. 2·4; 0·81, nearly. 4. 25·93 tons wt. 5. $666{\cdot}\dot{6}$ cub. metres.

Chapter V

1. 5·7 cm. 2. 71 dynes per cm.

Chapter VI

1. 0·0049 grm., nearly. 2. 12 cm. 3. 0·1125 grm. per litre. 4. 27 ft., nearly. 5. 2·76 lb. per sq. in., nearly.

Part II

Chapter I

1. ° Fahr., 98·4, 39·2, −37·84, 674·06, 60; ° Cent., 36·89, 4, −38·8, 356·7, 15·55; ° Réaumur, 29·51, 3·2, −31·04, 285·36, 12·44. 3. 74·06°, nearly.

Chapter II

1. 0·0008 per deg. Centigrade. 2. $4\frac{1}{30}$ grm. 3. 74·696° C. 4. Steel, 60 cm.; brass, 40 cm. 5. The clock gains 20·736 secs. per day of 24 hours.

Chapter III

1. 2,851,200 calories. 2. $\theta = 0·45°$ C. 3. $k = 0·0_6 5184$. 4. 32,400 large Calories; 635·3 kilogrm. of steam.

Chapter IV

1. 14·81 grm.; 92·8 grm., nearly. 2. 11·0 grm. 3. 124 grm. 4. 540·3, nearly. 6. 1817·5. 7. $933·\dot{3}$ grm.

Chapter V

4. 7·25°, nearly.

Chapter VI

1. 48,000 lb.-° C.-calories. 2. 1·7 kilometres, nearly. 3. 5,840, nearly.

Part III

1. 343·04 metres per sec. 2. 1129·3 ft. per sec., nearly. 3. 6·56 in., 19·68 in., nearly. 4. 331·4 metres per sec., nearly. 5. 572, nearly.

Part IV

Chapter I

1. Concave; $f = 16$ cm. 2. The new position is the centre of curvature; object and image will now be equal. 3. At 41·42 cm. and 58·58 cm. distances when on *opposite* sides of the screen; at 241·42 cm. and 341·42 cm. distances when on the *same* side of the screen.

Chapter II

1. 3·45 cm. from the lens; $\times\ 7\frac{1}{4}$. 2. 7·2 in. from lens; inverted, $\times$ 5. 3. *See* Fig. 4 (p. 21), OP = 10, AO = 16, PAO is the critical angle. 4. (i.) 50 cm.; (ii.) ∞; (iii.) -25 cm. 5. $f = 42\frac{6}{7}$ cm.; $2\frac{1}{3}$ dioptres; convex. 6. Concave glasses, $f = -6$ in.

Chapter III

1. 1·532. 2. 6·25.

Part V

Chapter I

1. 0·02 dynes. 2. (i.) 1·99 ; (ii.) 50. 3. 480. 4. $\frac{1}{60}$; $\frac{1}{15}$. 5. $\frac{1}{10}$.

Chapter II

1. 1·166 : 1. 2. 4674·5 ; 4009. 3. 16 : 9.

Chapter III

1. 60·372 grm. 2. 1·47 amperes ; 0·18, nearly, in reversed direction. 3. 2 amperes ; 600 coulombs ; 2 (thick), 6 (thin). 4. 2,089 c.c. 5. (i.) 0·26 amperes ; (ii.) 0·8 amperes ; quantities of heat produced in the two cases are as 1 to 3. 6. 0·191 amperes, nearly. 7. 32·8 microhms, nearly.

Chapter IV

1. (i.) 2 cals. per sec. ; (ii.) 4 cals. per sec. ; 4·2, 8·4, 8·4 volts. 2. The resistance of the lamp unlit is 100 ohms, but the working resistance when incandescent is 400 ohms. 3. The error is 8 per cent. and is positive. 4. 10 amperes. 5. 0·005124.

Chapter V

7 h.p.

PART VI.—PRACTICAL PHYSICS

In the study of Physics a course of experiments is as indispensable as is a course of dissection in the study of Anatomy. In either case the knowledge obtained without practical work has no reality or permanence. The student is therefore earnestly advised to seek every opportunity not only of carefully examining the instruments employed in physical measurements, but also of making these measurements himself with the utmost possible precision. He should also make a clear, illustrated record of each measurement, and will find it convenient to do this in a quarto note-book of squared paper.

Section I.—Measurement of a linear magnitude (length, thickness, etc.)

This is one of the earliest exercises undertaken by the student, and is, indeed, so familiar that the full importance of it is hardly realized. It is the foundation of many subsequent measurements, and its accurate estimation is of the greatest value. A length is generally measured in the first instance by comparison with a suitable scale. The student may use, for instance, a metre, or $\frac{1}{2}$-metre, scale, showing centimetres, and may find that the required length is between 29 cm. and 30 cm. It is therefore 29 cm. and some fraction of a centimetre. The determination of this fraction of the smallest scale division is effected (a) by eye estimation, or (b) by the use of a *vernier*. The student should practise both methods, record the *two* estimates, and note how (a) improves with practice. The **vernier**—so named after the inventor—is a small sliding scale graduated in divisions which bear a special relation to those on the larger scale, which the vernier is intended to supplement. Fig. 264 shows a common type of vernier, associated with the scale of cm. and mm. The large scale shows that the height of the column to

be measured is rather more than 29·2 cm. It is not 29·3 cm.; it is not even 29·25 cm. A practised eye would see that it was barely 29·23 cm., but the vernier is intended to determine this second decimal place quickly and accurately. For that purpose it must read to $\frac{1}{100}$ cm. or $\frac{1}{10}$ mm., that is to $\frac{1}{10}$ of the smallest division on the large scale. *This length must therefore be,* as we shall presently see, *the difference between a scale division and a vernier division.* To construct this vernier, then, a length is measured equal to 9 scale divisions (mm.), and this is separated into 10 divisions; thus,

$$10 \text{ vernier divisions} = 9 \text{ scale divisions} = 9 \text{ mm.}$$
$$\therefore 1 \text{ vernier division} = \tfrac{9}{10} \text{ scale division} = 0{\cdot}9 \text{ mm.}$$
$$\therefore \left.\begin{array}{l} 1 \text{ scale division} \\ -1 \text{ vernier division} \end{array}\right\} = \tfrac{1}{10} \text{ ,, ,,} = 0{\cdot}1 \text{ mm.}$$

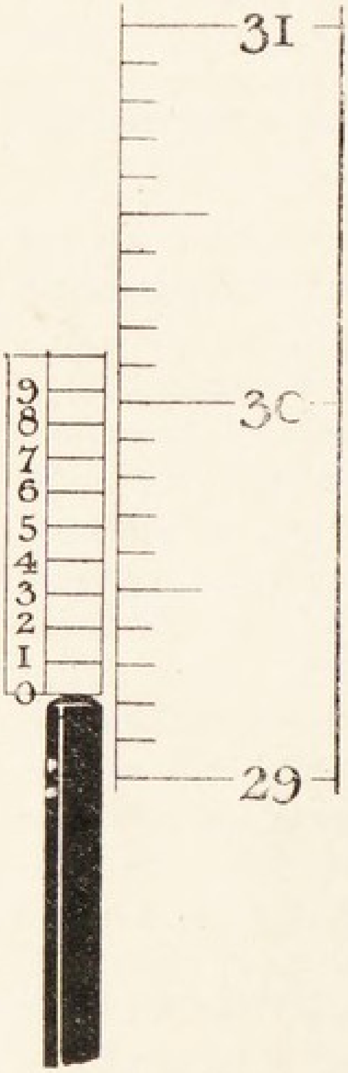


Fig. 264.—Vernier. Cm. scale × $\frac{5}{2}$.

To use this vernier, we now bring the zero of the vernier to the point to be measured, and then note which division on the vernier most nearly coincides horizontally with a division on the scale. In Fig. 264 this is the division numbered 2. The additional length over and above 29·2 cm. which we could not read accurately without the vernier is now seen to be the difference between 2 scale divisions and 2 vernier divisions, and therefore to be 0·2 mm. The second decimal place is therefore 2 and the height of the column is 29·22 cm.

The vernier used with the inch scale of standard English barometers is of a somewhat special type. On the main scale (Fig. 265), each inch is subdivided by longer lines into tenths, and by shorter lines into twentieths, of an inch. A length equal to 24 of these smallest scale

divisions is graduated into 25 divisions for the vertnier, therefore

$$\begin{aligned}
25 \text{ vernier divisions} &= 24 \text{ scale divisions} = \tfrac{24}{20} \text{ in.} \\
\therefore\ 1 \text{ vernier division} &= \tfrac{24}{25} \text{ scale division} = \tfrac{24}{500} \text{ in.} \\
\therefore\ \left.\begin{array}{l} 1 \text{ scale division} \\ -1 \text{ vernier division} \end{array}\right\} &= \tfrac{1}{25} \quad ,, \qquad ,, \quad = \tfrac{1}{500} \text{ in.} \\
&= 0{\cdot}002 \text{ in.}
\end{aligned}$$

This vernier therefore reads to 0·002 in., and for every single vernier division included in the reading we count 2 in the *third* decimal place. For every *five* vernier divisions so included we must therefore count 5 times this, or 1 in the *second* decimal place. As seen in the figure, only every fifth division is numbered, and these numbers therefore give the second decimal in the reading. The intervening unnumbered divisions represent 2, 4, 6, or 8 in the *third* place. The reading in the figure is shown by the main scale to be rather more than 29·25, but less than 29·3. The twelfth division on the vernier appears to coincide most closely with a scale division. The addition to the reading is therefore $12 \times 0{\cdot}002 = 0{\cdot}024$; a 2 in the *second* place for the two fives, and a 4 in the *third* place for the two single divisions. The complete reading is therefore 29·274 in. The limit of error allowed in a measurement will generally not exceed 1 per cent. It follows that if the length to be measured is small, the instrument employed must be so constructed as to show *proportionately* small differences. To measure, for instance, the thickness of a coverslip, or the

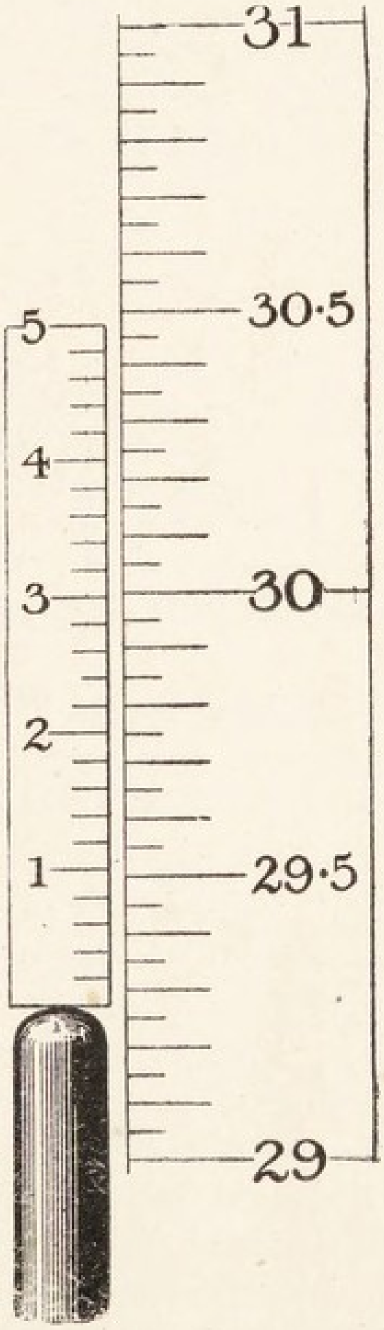


Fig. 265.—Vernier of standard barometer.

diameter of a fine wire, the scale and vernier of Fig. 264 would be much too coarse. To measure lengths of this order the **micrometer screw** (Fig. 266) is employed. In one often used the *pitch* (p. 42) is 0·5 mm., so that one complete turn of the screw alters the distance between the fixed point A and the end of the screw by 0·5 mm. When these are in contact, the zero line on B should join the line C.

Suppose we wish to ascertain the thickness of a cover-glass. The screw is withdrawn until we can introduce the glass between A and the end of the screw; the graduated head is turned until the pressure of the screw is just sufficient to support the weight of the glass; the thickness is then ascertained as follows: The number of

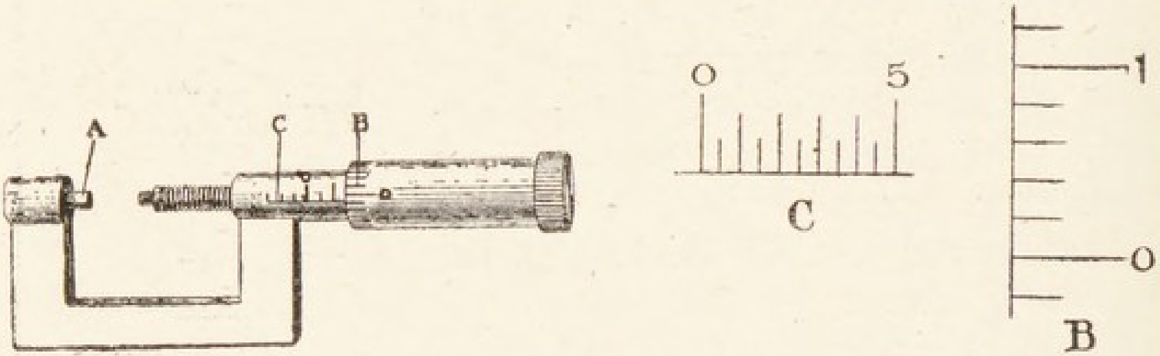


Fig. 266.—Micrometer screw gauge, with enlargement of B and C.

complete turns is indicated on the scale at C, which is uncovered as the screw is withdrawn. Each division so uncovered represents 0·5 mm. The circumference of B is often separated into 25 divisions, and of these every fifth division from zero is numbered 1, 2, etc. Each *numbered* division represents one-fifth of a turn, and therefore alters the distance between A and the screw end by 0·1 mm. Each *unnumbered* division therefore represents one-fifth of this = 0·02 mm. In Fig. 266 the screw has completed three turns all but half of an unnumbered division, and the length measured is therefore 1·5 mm.—0·01 mm. = 1·49 mm.

By increasing the length of the circumference B, we are able to increase the number of graduations and make the instrument still more delicate. This plan is adopted

in the **spherometer.** This is a little tripod, the micrometer screw A (Fig. 267) forming a fourth leg. The *triangle* formed by joining the extremities of the three fixed legs is *equilateral*, and the point of the screw, in the zero position, is on the plane of this triangle and at its centre of gravity. If the micrometer screw projects beyond this plane the instrument rocks when touched. By gently withdrawing the screw the rocking ceases, so that this rocking forms a delicate test as to when the micrometer screw and the three legs all touch the surface, whether plane or curved, on which the instrument rests.

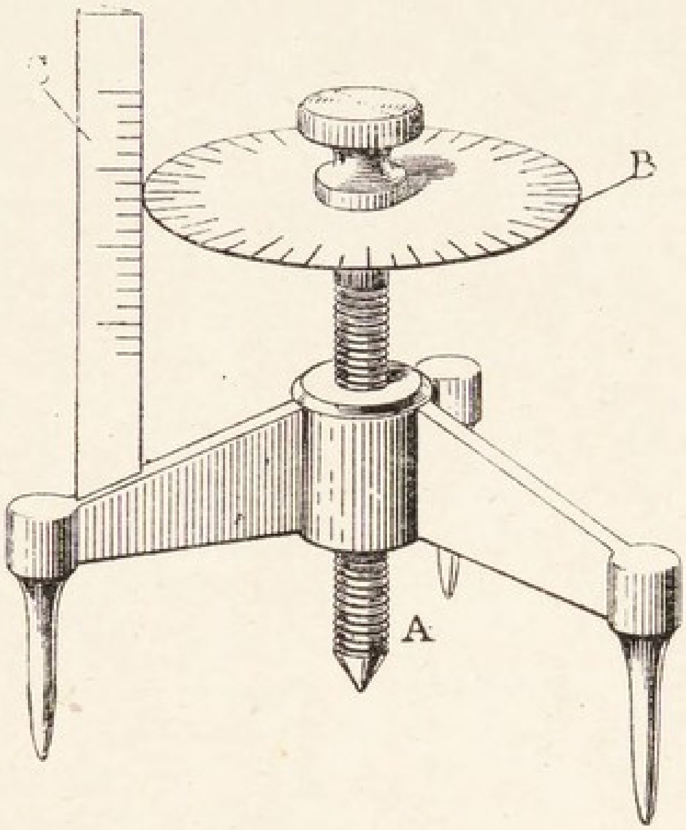


Fig. 267.—Spherometer.

The spherometer is first tested by placing it on a flat

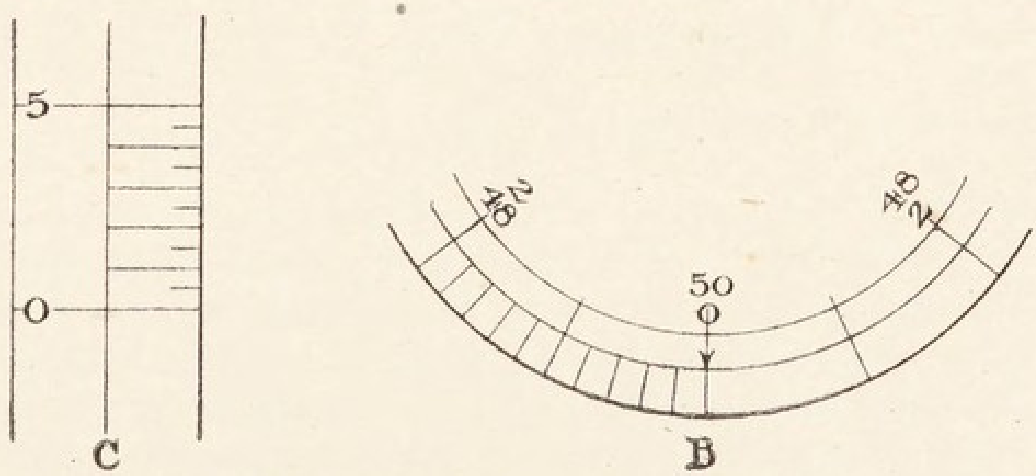


Fig. 268.—Enlargement of B and C in Fig. 267.

surface, such as a piece of good plate-glass. When the micrometer screw is withdrawn so that the rocking just

ceases, the zero line on the divided scale B should join the zero line on the vertical scale C. Usually two complete turns raise or lower the screw one millimetre, so that one turn = 0·5 mm. The head is graduated into 50 *numbered* divisions, so that each division = 0·01 mm., and each of these 50 divisions is subdivided into five *unnumbered* ones, each of which = 0·002 mm.

In order to determine the radius of curvature of a convex lens, the screw is withdrawn, so that the three fixed legs of the instrument rest upon the curved surface of the lens. The micrometer screw is then gently advanced until the rocking indicates contact, when the height of the screw above the plane of the equilateral triangle is read off on the two scales. From this reading, the radius of the sphere, of which the lens may be supposed to form a part, can be calculated. If d be the reading, a the length of a side of the fixed equilateral triangle, and r the radius of the sphere, we can prove by Euclid VI. 8 that $\left(\frac{a\sqrt{3}}{3}\right)^2 = d(2r - d)$ and, therefore, that

$$r = \frac{a^2}{6d} + \frac{d}{2}$$

We can thus measure the *surface* $(4\pi r^2)$, the *volume* $\left(\frac{4}{3}\pi r^3\right)$, and the *curvature* $\left(\frac{1}{r}\right)$ of the sphere. Hence the name of the instrument. With the spherometer we can therefore determine the *focal length* $\left(-\frac{r}{2}\right)$ of a convex, or concave, spherical mirror. Similarly, we can determine the focal length of a convex or concave lens, by finding the radii of curvature of its faces, if we know also the refractive index of the glass of which the lens is made. In calculating r from the given formula the student must always be careful to express the quantities a, d, r in terms of the same unit of length.

The following exercises will illustrate the use of these instruments. Both the practical and the intellectual value of such exercises is much increased by their appli-

cation to definite problems. Such applications are therefore suggested in many cases, and the student is advised to discover others for himself. Every instrument should be carefully handled and replaced in its case after use. In turning the screw, the milled head should be held lightly between finger and thumb, and the motion stopped directly the desired contact is felt; undue pressure will strain the screw and introduce a zero error. If the instrument already has an error of this kind, it must be noted and the reading corrected accordingly. The entry in the note-book should show *both* readings.

1. Measure both in *centimetres* and in *inches* the diameter of a penny, and the length and width of a visiting card.

From your measurements, find—

(*a*) The number of centimetres in a foot,
(*b*) ,, ,, ,, inches ,, metre,

and compare your values with those stated on p. 4 of this Manual.

2. Find with the micrometer screw—

(*a*) The thickness of a single visiting card;
(*b*) The combined thickness of 5 or 10 similar cards.

Calculate from (*b*) the average thickness (*c*) of a card. Compare (*c*) with (*a*) and note the variation from the mean value, in the individual measured. Expressed as a percentage of the mean value, this variation is $\frac{c-a}{c} \times 100$, and is positive or negative according as the individual is thicker or thinner than the average.

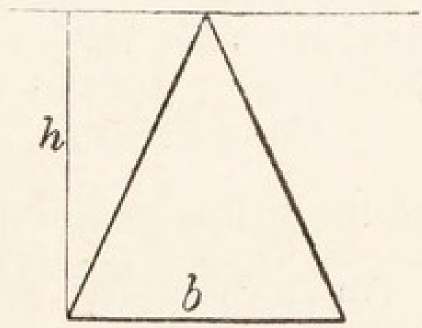


Fig. 269.—Calculating area of triangle.

3. Repeat, with the spherometer, the measurements (*a*) and (*b*) of the previous exercise, and compare the results obtained with the two instruments.

4. Find by means of the spherometer the radii of curvature of both faces of a double convex lens. Calling

these radii r_1, r_2, and assuming that the refractive index, μ, of the glass is 1·5, calculate the focal length (f) of the lens from the formula—

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

5. Mark the rim of a penny with a knife-blade or a small three-square file. Hold the coin vertical with the mark in contact with some point on a piece of paper; roll it, without slipping, along a straight line on the paper, till the mark again makes contact with the paper. Note the second point of contact, measure the distance between the two points, and so obtain the length of the circumference of the coin.

Calculate the length of the circumference of the same coin from its diameter by the formula—

$$\text{Circumference} = 2\pi r.$$

$$r\text{, the radius,} = \frac{\text{diameter}}{2}, \pi = \frac{22}{7}$$

An **area** is the product of two linear magnitudes, and therefore involves no new measurement. The following are useful examples:—

6. Calculate the area of one face of a penny from the formula—

$$\text{Area of a circle} = \pi r^2$$

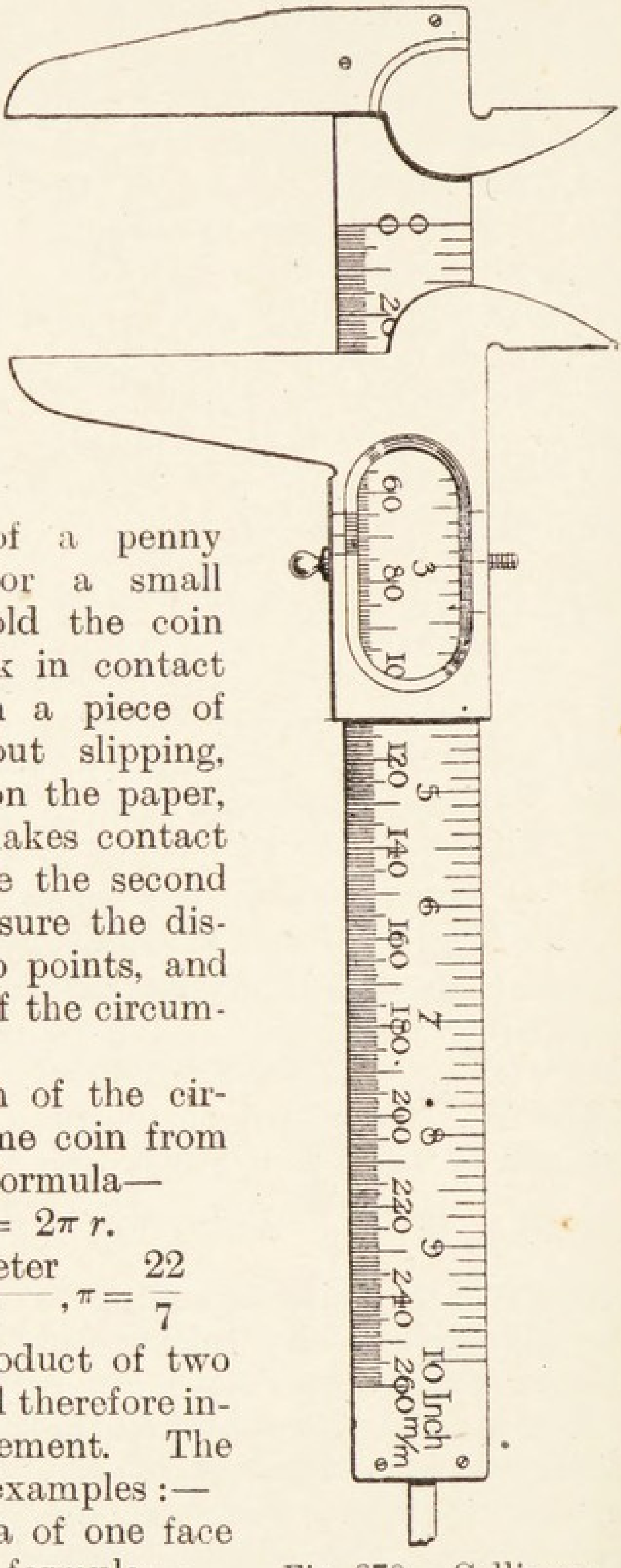


Fig. 270.—Callipers.

Express the area both in square centimetres and in square inches.

Find *from your results*—

(*a*) The factor for converting sq. in. into sq. cm. [= 6·4516], and

(*b*) The reciprocal of this, namely, the factor for converting sq. cm. into sq. in. [= 0·155].

7. Calculate the area of a visiting card.

$$\text{Area} = \text{length} \times \text{width}$$

8. Determine the area of a triangular piece of paper.

$$\text{Area} = \frac{\text{base} \times \text{vertical height}}{2} = \frac{b \times h}{2} \quad \text{(Fig. 269.)}$$

9. Also calculate the areas in Ex. 6, 7, 8 by laying the coin, card, etc., on your squared paper and counting the number of squares covered.

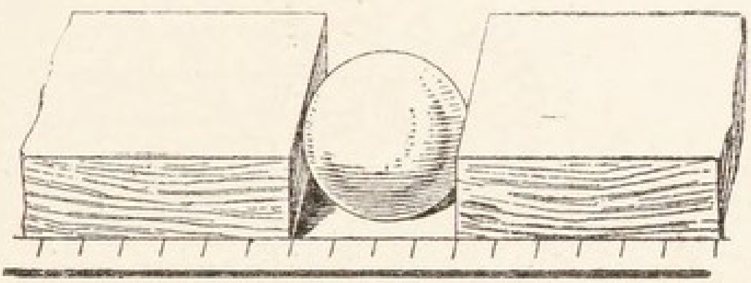


Fig. 271.—Measuring diameter of a sphere.

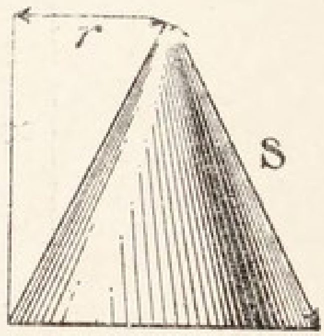


Fig. 272.—Calculating surface of cone.

10. Calculate the area of the curved surface of a cylinder, brass tube, etc.

$$\text{Area} = 2\pi\ r \times \text{height}$$

11. Calculate the area of the surface of a sphere.

$$\text{Area} = 4\pi\ r^2$$

The diameter of a sphere can be determined by callipers (Fig. 270), or by means of two square blocks of wood and a scale (Fig. 271).

12. Calculate the area of the surface of a cone.

$$\text{Area} = \pi\ r\ s \quad \text{(Fig. 272.)}$$

A **volume** is the product of three linear magnitudes, or of one area and one linear magnitude, and therefore involves no new measurement. The following are useful examples of bodies having a *regular* figure :—

13. Calculate the volume of a sphere (*a*) in c.c.; (*b*) in c. in.

$$\text{Volume} = \frac{4\,\pi\,r^3}{3}$$

Find from your results the factors for conversion of (*a*) to (*b*), and vice versa [0·061 and 16·387].

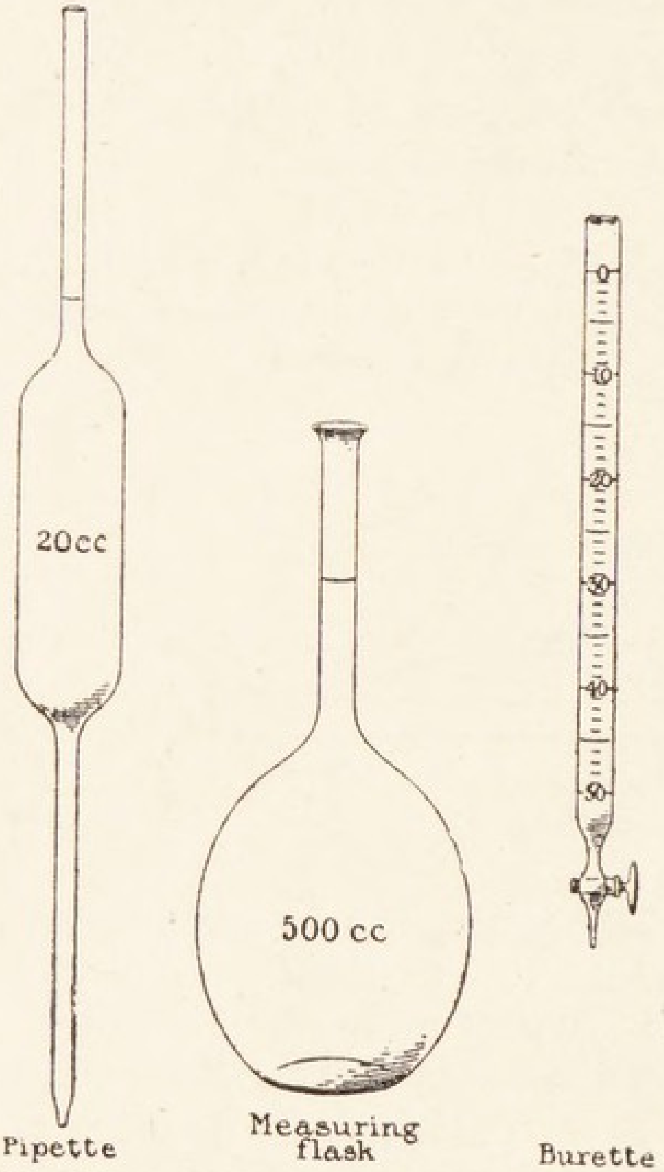


Fig. 273.—Pipette, measuring flask, and burette.

14. Calculate the volume of a cylinder.

$$\text{Volume} = \text{base} \times \text{height} = \pi\,r^2\,h$$

15. Calculate the volume of a cone (Fig. 272).

$$\text{Volume} = \frac{\pi\,r^2\,h}{3}$$

16. Also calculate the volume of one or more of the above solids by the displacement method (2, p. 55).

In the volumetric manipulation of liquids **pipettes** are employed to *deliver*, and **measuring-flasks** to *contain*, the volumes specified on the vessels (Fig. 273). By noting the initial and final graduations at which the level of a liquid stands in a **burette** the volume of *any* portion withdrawn can be measured.

The accuracy of the graduations on these instruments is tested by weighing the volumes of distilled water, at known temperature, which they respectively contain or deliver. The true volume corresponding to this weight can be found from tables.

The *calibration* of the burette is effected similarly by weighing, say, 5 c.c. from different places. The water is placed in a weighed stoppered bottle for weighing, since it tends to evaporate and lose weight in the process.

The next exercise requires the use of the **balance.** This instrument deserves to be treated with the utmost consideration by the student of science, for there is no single instrument to which science is more deeply indebted. The student must, however, remember that to preserve the accuracy and therefore the value of a balance, certain precautions must never be neglected. No substance must be placed on the scale pan that might act on the metal of the pan, and so alter its weight. A clean, dry dish, or other vessel, or a single block of metal like a brass weight, may be placed upon the pan; but, as a rule, the substance to be weighed must be placed not upon the pan itself but upon a weighed or counterpoised dish, watch-glass, or other convenient receptacle. If the substance is a liquid, it is weighed in a stoppered bottle to prevent evaporation. This precaution must also be taken in weighing a solid like iodine, which emits a vapour injurious to the metal of the balance. It is usual to place the substance to be weighed in the left pan and the weights in the right; no alteration must ever be made in the weight on either pan while the pans are actually in suspense; this must only be done when the beam has been lowered, and the pans are resting on the supports in the floor of the balance case. It need hardly be added

that the balance and its case must be kept scrupulously clean; any particles of dust, etc., accidentally admitted should be carefully removed with a camel-hair brush kept for this purpose. When the pans are nearly balanced the oscillations are slow, and, *ceteris paribus*, the more delicate the balance the slower the swing. The student should therefore learn to weigh by the method of oscillations. To do this he must note the sensibility of the balance. As the pans oscillate, a pointer attached to the beam travels over a scale at the foot of the balance pillar. When exactly balanced the pointer would in time come to rest over the central, or zero, division on the scale. If, however, the balance is not quite exact, but the weight on the *right* pan is a milligramme too much, the pointer will come to rest somewhere to the *left* of the true zero. The exact point will vary with the sensibility of the particular balance. Suppose this displacement of the zero position amounts to two scale divisions. Having determined this, we can weigh quite as accurately and far more quickly by reference to the displaced zero instead of adjusting the weights till they balance about the true zero. Suppose, for instance, that, when the pans nearly balance, we see that the pointer oscillates between the fourth division on the right and the sixth on the left; it would, therefore, in time come to rest one division to the left of the true zero—and therefore on *this* balance the weight in the right pan is half a milligramme too much. Hence the true weight is known.

Section II.—Exercises in the Determination of Specific Gravity

17. Find the weight in grammes of the regular solids measured in Ex. 13, 14, 15, and then deduce the specific gravity of the material of each by method 1 on p. 55.

18. Weigh 20 to 50 grm. of lead shot and determine the specific gravity of lead by dropping the shot into a burette containing water, and noting the rise in the level of the water (displacement method, p. 55).

19. Find the specific gravity of a glass stopper, a

piece of lead, or iron, by the method of Archimedes (p. 56).

20. Find the specific gravity of lead shot by weighing in a specific-gravity bottle filled with water (p. 59).

21. Find the specific gravity of a florin by Nicholson's hydrometer (p. 60).

22. Assuming that the given brass weight consists of an alloy of pure zinc with pure copper, calculate the percentage composition of the alloy (*a*) by weight, and (*b*) by volume, using the table values of the specific gravities of zinc and copper.

23. Determine the specific gravity of alcohol by the common hydrometer (p. 62).

24. Find the specific gravity of a specimen of alcohol by the specific-gravity bottle (p. 63).

25. Determine the specific gravity of alcohol by Hare's apparatus (p. 64).

26 Bend a glass U-tube, place mercury in one limb and water in the other. Measure the heights of the columns of mercury and of water above a horizontal line drawn through their common surface. Calculate their respective gravities, knowing that the heights vary inversely as the specific gravities.

Section III.—Measurement of Gaseous Pressure —Verification of Boyle's Law—Measurement of *g* — Correcting the Reading of a Barometer — Capillarity — Diffusion of Gases

27. Place some water in the U-tube used in Ex. 26, and attach one limb of the U-tube to the gas supply by an indiarubber tube. Turn on the gas and measure the pressure in millimetres of water.

28. Verify Boyle's law by the apparatus (Fig. 60, p. 104).

Introduce a convenient quantity of air in bulb A, level the mercury in the two tubes A and B, and read the volume,

V_1, of gas; then raise B, read off the difference of level, h, in the mercury in the two tubes, and add the atmospheric pressure, H, to this difference. Tabulate the results in three columns, as below:—

V	P	$V \times P$
V_1	H	$V_1 \times H$
V_2	$H + h$	$V_2 \times (H + h)$

Care should be taken not to handle the tube A, so as to avoid errors from alterations in temperature.

Make several determinations, and note that the values obtained for the product $V\ P$ in the third column are nearly constant.

29. Make a series of experiments with Atwood's machine as described on pp. 22 and 23. Calculate in each case the value of g from the equation $R\,g = (2P + R)\,a$.

30. Determine the value of g from the time of vibration of a simple pendulum, as described on p. 49.

By changing the length, l, of a simple pendulum obtain different values of t, the period of vibration, and show that $\frac{t^2}{l}$ is constant. Tabulate your results thus:—

l	t	$\frac{t^2}{l}$

31. Verify the parallelogram of forces by means of the apparatus shown in Fig. 10 (p. 34). Two weights are

attached to the ends of a string which passes over two pulleys. A third weight is attached to the string at some point between the pulleys. The sum of any two of the weights must be greater than the third. When the three weights are in equilibrium, place a drawing-board in a vertical position behind the string, and trace on the board the directions of O A and O B. From O draw a vertical upward straight line, and on it measure, from O, a length representing the weight R on any convenient scale. From the point so obtained draw straight lines parallel to A O, B O, completing the parallelogram of forces. Measure two adjacent sides of this parallelogram. Their lengths should represent in magnitude the forces P Q, acting at O, to which they are respectively parallel, on the same scale as the vertical diagonal represents their resultant, R.

32. When one of the three weights of Ex. 31 has a given value, apply the parallelogram of forces to find the values of the other two.

33. Calibrate a given rod to be used as a steelyard with movable fulcrum.

34. Find the coefficient of sliding friction of wood against wood.

35. Reading the Barometer (p. 91)

(1) See that the barometer tube is vertical.

(2) Adjust the level of the mercury in the cistern till its surface just touches the ivory point which ends the scale.

(3) Adjust the bottom of the vernier to touch the top of the mercury column, taking care that the eye is at the same level.

(4) Read the height, with the vernier, in inches and in millimetres.

(5) Read the temperature.

(6) Correct the reading for temperature by Table IV. (p. 430); that is to say, find the true height of the mercury column at 0° C. In order that the student may also be able to make this correction without the assistance of the Table, we shall now explain the calculation in detail.

If H_t be the true height of the mercury column at $t°$ C. and H_o the height of the same column at 0° C., then

$$\frac{1}{t} \times \frac{H_t - H_o}{H_o} = \left\{\begin{array}{l}\text{the coefficient of cubical ex-}\\ \text{pansion of mercury for 1° C.}\end{array}\right.$$

$$= 0{\cdot}00018$$

$$\text{whence, } H_t = H_o\,(1 + 0{\cdot}00018t)$$

For 1° F. the coefficient is $0{\cdot}00018 \times \frac{5}{9} = 0{\cdot}0001$, and if the temperature is $t°$ F. the formula becomes

$$H_t = H_o\,[1 + 0{\cdot}0001\,(t - 32)]$$

By either formula we can make the required correction for the temperature of the *mercury* above zero. If, however, the height of the barometer is registered on a *brass* scale which was correctly graduated at 0° C. and extends to the foot of the column, a further correction will be required for the expansion of the *brass*. If l_t is the actual reading at $t°$ C., a length of l_t brass at 0° C. must have increased to a length H_t at $t°$, and therefore

$$\frac{1}{t} \times \frac{H_t - l_t}{l_t} = \left\{\begin{array}{l}\text{the coefficient of linear ex-}\\ \text{pansion of brass for 1° C.}\end{array}\right.$$

$$= 0{\cdot}00001894$$

$$\text{whence, } H_t = l_t\,(1 + 0{\cdot}00001894t)$$

For 1° F. the coefficient is $0{\cdot}00001894 \times \frac{5}{9} = 0{\cdot}000010522\dot{7}$, and therefore the formula is not simplified in this case.

By equating the two values of H_t we combine the two corrections, and obtain

$$H_o = \frac{l_t\,\{1 + 0{\cdot}00001894t\}}{\{1 + 0{\cdot}00018t\}}$$

Ex.: The barometer reading is 30 in. when the temperature of the mercury is 15° C. (= 59° F.). Find the reduced reading at 0°.

Since $l_t = 30$ in., and $t = 15$, we have

$$H_o = \frac{30\,\{1{\cdot}0002841\}}{1{\cdot}0027} = 29{\cdot}928 \text{ in.}$$

Hence the correction to be applied to the reading in this case is 30 − 29·928 = 0·082 in.

Table IV. (p. 430) is constructed in this way, and this correction will be found in the vertical column under the reading 30 in., and in a horizontal line with the temperature 59° F. The correction has to be *subtracted* from the reading, and therefore has a *minus* sign prefixed to it in the table. If l_t is expressed in centimetres, the corresponding correction will be obtained.

36. Measure by the siphon barometer, fitted up as in Fig. 274, the vacuum produced by the air-pump.

37. Draw a graph of Table II. (p. 428).

Capillarity

38. Heat a piece of glass tube in the blowpipe flame till it is quite soft, then *take it out of the flame* and pull it into a capillary tube. In this way make some capillary tubes of various sizes. Break off lengths of 2 in. to 3 in. and observe the heights to which various fluids —ink, alcohol, etc.—will rise. Observe also the effect of inclining the tubes (p. 80).

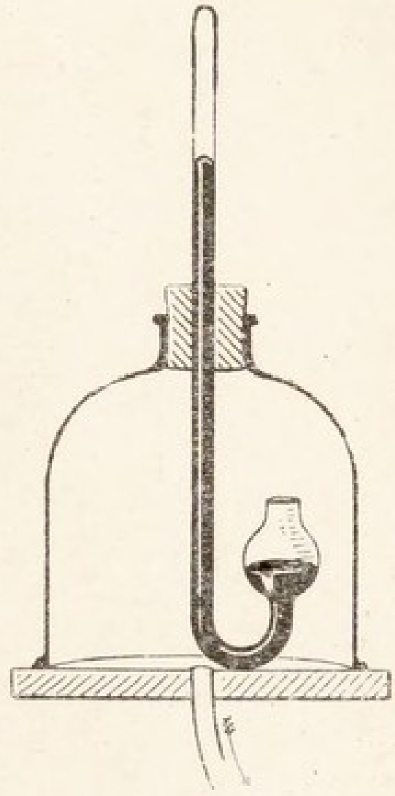


Fig. 274.—Measuring vacuum of air-pump.

39. Given the surface tension of water, determine the radius of a capillary tube by measuring the height that water rises in the tube.

40. Compare the surface tensions of two liquids by the capillary-tube method.

Diffusion of Gases

41. Fill a gas jar with carbon dioxide, cover it with a plate, and invert a jar of ordinary air over it (p. 75). Remove the plate, and allow the two jars to remain in communication for half an hour. Test for CO_2 in the upper jar with solution of calcium hydrate.

Section IV.—Heat

Expansion of Solids, Liquids, and Gases

1. Demonstrate the increase in length of a brass or iron bar or rod when heated as shown in Fig. 275. One end of the bar B rests on a ledge against a firm support at A, the other end rests on a needle N supported by a

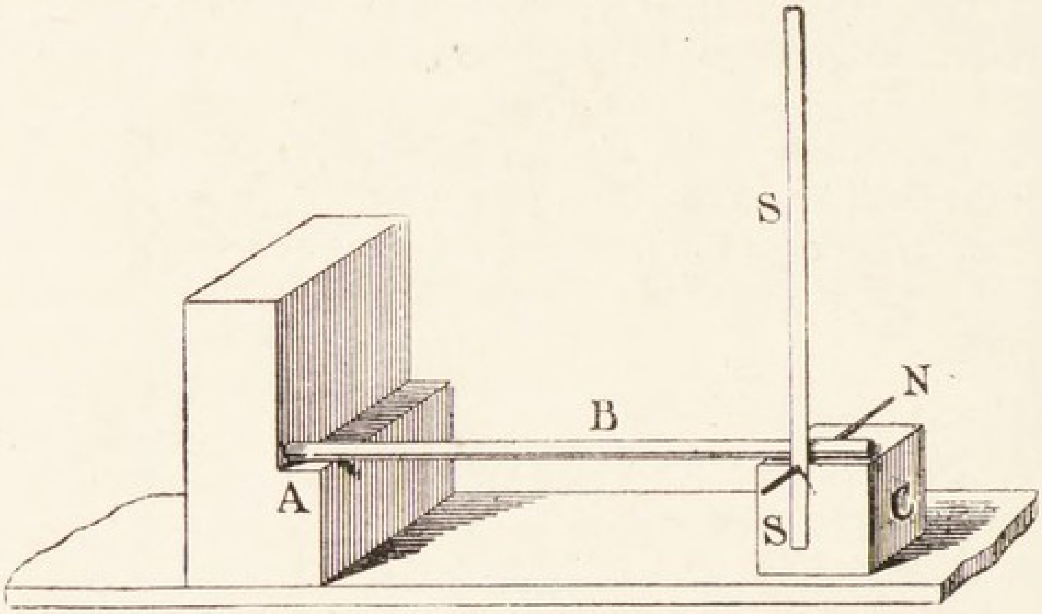


Fig. 275.—Expansion of brass rod.

block C. One end of the needle is thrust through the straw S. On warming the bar B with a Bunsen burner its length increases and the needle is rolled over, the movement being magnified by the straw, the upper end of which moves to the right.

2. Prove by Gravesande's ball and ring (p. 109) that a sphere increases in size when heated.

3. Heat a compound bar of brass and iron and observe that the more expansible metal takes the longer side of the curve (p. 125).

4. Determine the exact increase in the length of a rod of brass 1 metre long for a rise of temperature of 50° (20°–70°) with the aid of a spherometer (pp. 122 and 395).

5. Determine the mean coefficient of expansion of water, alcohol, etc., between two temperatures by filling a specific-gravity bottle with the liquid at these two temperatures, and weighing the bottle with its contents (pp. 63 and 126).

6. Fill a large-bulbed thermometer with boiled distilled water (p. 111), and observe carefully, with the aid of a scale, the alteration in the volume of the water when cooled from 10° C. to 0° C. (p. 131). Note carefully at what temperature water attains its smallest volume or maximum density.

7. Fit a small flask with a cork and upright glass tube; fill the flask with water and insert the cork and tube; mark the position of the surface of the liquid in the tube; then plunge the flask suddenly into hot water. Note the *drop* of the fluid in the tube owing to the expansion of the glass flask before the fluid begins to expand.

8. Fill a graduated glass gas-tube one-third full with mercury, add an equal volume of water, close the mouth of the tube with the thumb and invert in a trough of mercury. After a time note the temperature of the tube, and then calculate the real volume of the dry air at 0° and 760 mm. (pp. 132 and 428).

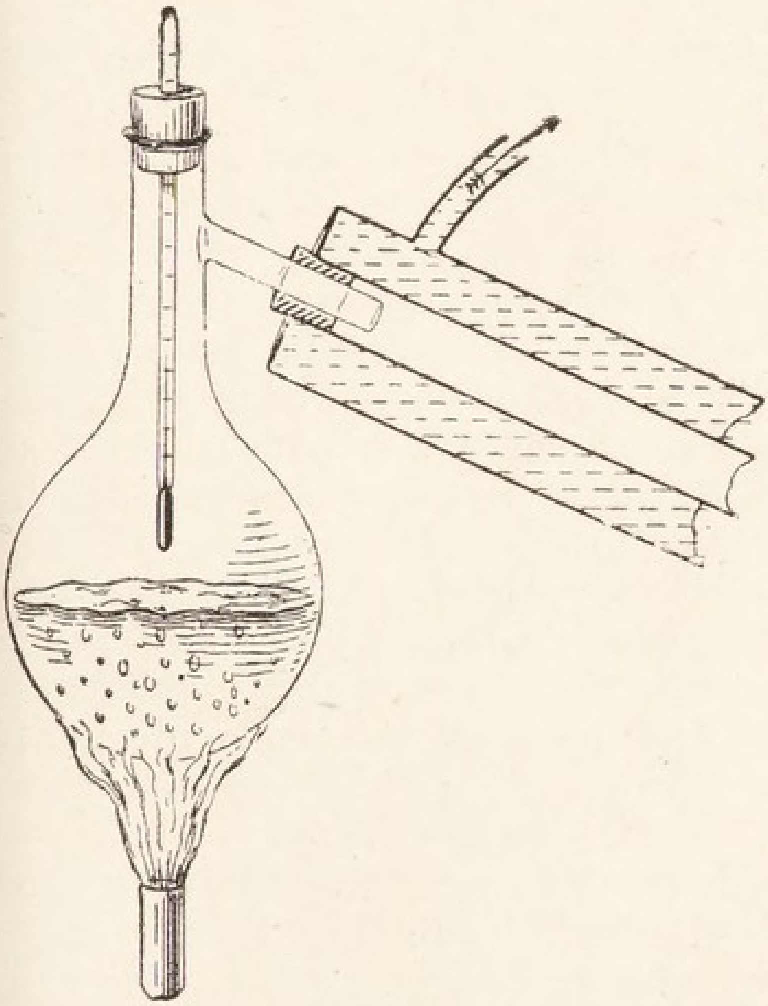


Fig. 276.—Determination of boiling-point.

Determination of Melting-Point and Boiling-Point

9. Determine the melting - point of various substances — wax, paraffin wax, naphthalene, etc.—as described on p. 149.

10. Plot a cooling curve for the given solid, and mark the *melting-point* on your curve.

11. Verify the zero point of a Centigrade thermometer (p. 111).

12. Assuming the bore of the given thermometer to be uniform, find the error in the reading at the temperature of the laboratory.

13. Determine the boiling-point of various liquids—water, alcohol, etc.—by the apparatus shown in Fig. 276. Note the height of the barometer.

14. Insert a little ether in a bent tube closed at one end (Fig. 106 and p. 107), warm the water in the beaker, note the temperature of the water when the surfaces of the mercury in the two limbs of the tube are level. This temperature is the boiling-point (p. 170).

Conduction

15. Stir some boiling water with a copper wire and a German silver wire, of the same diameter; notice the different rates at which the heat is conducted. A similar difference will be felt if a copper wire and a platinum wire be held in the fingers while the other ends are placed in a Bunsen flame.

16. Turn on the gas to a Bunsen burner, place a piece of fine copper gauze about 2 in. above the burner and apply a light *above* the gauze. The gas will burn above, but the gas under the gauze will not be ignited (p. 137).

17. Coil some copper bell-wire round a thick pencil so as to make a close coil about one inch long. Place it over the wick of a lighted candle without touching the wick. The candle will be extinguished.

18. Coat rods of various substances with wax and introduce them into an Ingenhousz trough, fill with boiling water, and measure the various lengths of wax melted (p. 138).

19. Repeat the experiment shown on p. 139 to prove that water is a bad conductor of heat.

Radiation and Absorption

20. Fill a Leslie's cube with boiling water (Fig. 94 and p. 144) and observe the varying amounts of heat given off by the blackened and polished sides, either with a

Leslie's differential air thermometer (p. 109) or a thermopile and galvanometer (Fig. 247, p. 370).

21. Place a piece of bright tin-plate, and a similar piece smoked over a paraffin lamp, in front of a fire for a few minutes, and note the difference in temperature (p. 144).

22. Place the hand near the side of an ordinary Bunsen flame, then cut off the air and notice how much more heat is radiated by the luminous flame.

23. Place a sheet of glass in front of a fire for a few minutes and notice how it absorbs the heat radiated from the fire (p. 147); most of the heat rays coming from the fire, being non-luminous, are absorbed by the glass.

Vapour Pressure

24. Measure the vapour pressure of water and of alcohol at various temperatures by the apparatus shown on p. 169. Compare your results with those on p. 170.

Latent Heat

25. Find the water equivalent of a beaker (or other calorimeter) and thermometer by the method fully described on p. 153.

26. With the calorimeter and thermometer employed in the last exercise, find the latent heat of fusion of ice as described on p. 154.

27. Place about 100 c.c. of water in the flask B (Fig. 100, p. 157) and 200 c.c. in the calorimeter C. Weigh C with its contents, and note their temperature. Boil the water until a rise of about 15° C. is observed in the water in C; note the temperature accurately. Then calculate from your data the latent heat of steam.

28. To standardize a burner, place a known weight of water in a beaker and heat it over a burner (Fig. 98). Stir the water well and take its temperature at regular intervals. Hence determine the number of calories supplied by the burner per minute.

29. Continue to heat the water till it begins to boil and for a measured interval, 10 or 15 minutes, after boiling commenced. Determine by difference the weight

of water converted into steam by the known amount of heat supplied in this interval. Hence calculate the latent heat of steam.

Specific Heat

30. Determine the specific heat of a metal by the method of mixtures (p. 161). Weigh carefully a mass of iron, zinc, lead, or aluminium (about 100–200 grm.), attach some fine string or cotton, and heat the metal in a saucepan of boiling water. Weigh the dry calorimeter and find its water value by multiplying the weight by its specific heat (iron 0·113, copper 0·095, brass 0·094) (p. 159). Measure 200 c.c. of water into the calorimeter (p. 154), and take its temperature carefully. Note the temperature of the boiling water, withdraw the heated metal and cool it in the calorimeter with constant stirring. Note the rise in temperature. Calculate the specific heat of the metal (p. 163) and compare the result with the numbers on p. 160.

31. Find the specific heat of glycerol by the method of cooling (p. 164).

Dew-Point

32. Determine this temperature—

(*a*) by stirring small pieces of ice into some water contained in a polished metal vessel until dew forms on the surface. Read off the temperature when this first takes place, then allow the vessel to stand till the dew disappears, and again note the temperature. The mean of the two readings is the dew-point;

(*b*) by Daniell's hygrometer, p. 178;

(*c*) by Dines' hygrometer, p. 179;

(*d*) by wet and dry bulb thermometer and Glaisher's factors (pp. 180 and 429).

33. Having determined the dew-point, calculate the relative humidity (pp. 177 and 428) of the air.

34. Find also the weight of aqueous vapour contained in 1 litre of the air in the room at the time of the experiment (p. 180).

Section V.—Sound

1. Study the vibrating segments and nodes of a long rope (p. 198). Time your shakings to produce one vibrating segment, two vibrating segments with one central node, three vibrating segments with two dividing nodes, etc.

2. Take a tracing of a vibrating tuning-fork on a revolving smoked surface (p. 206); determine the rate f movement of the surface by timing the number of evolutions in 5 minutes; then count the number of vibrations of the tuning-fork in a second.

3. Determine the wave-length of a tuning-fork by filling up a narrow jar with water (p. 207); hence calculate the velocity of sound in the air if the pitch of the note is known; or, conversely, find the number of vibrations of the fork, assuming the velocity of sound to be known.

Try the effect of using a wide instead of a narrow jar.

4. Demonstrate the nodes and vibrating segments in a string attached to one prong of an electro-magnetic tuning-fork as described on p. 198.

5. Pinch a stretched steel wire in the middle so as to cause it when " bowed " to divide into two vibrating segments and one central node, and observe that it gives the octave above the fundamental note.

Verify as far as possible the ratios given on p. 199.

6. With the electro-magnetic fork used in Ex. 4, or with the sonometer (Fig. 130), make a series of experiments to verify the formula $n = \frac{1}{2l}\sqrt{\frac{t}{m}}$.

Section VI.—Light

1. Compare the illuminating powers of a gas flame and a candle, as described on p. 218.

2. Prove that the angle of reflection is equal to the angle of incidence, with the aid of some pins and a piece of looking-glass as described on p. 221 (*see* Figs. 140, 141).

3. Put a stiff card, or metal sheet, in which a small arrow has been cleanly cut, just in front of a bright source of light. Some distance in front of the arrow, place a concave mirror. The centre of the mirror should be nearly on a level with the centre of the arrow. If the arrow is well illuminated by the light, an observer will now be able to see a bright inverted image of the arrow in the mirror. This is a real image and can be thrown on a white screen held at a convenient distance from the mirror. Slightly adjust the distance till the image is most sharply defined. Measure the distance $L\,A$ from arrow to mirror; measure the distance $l\,A$ from image to mirror; calculate $\frac{1}{L\,A} + \frac{1}{l\,A}$; find several positions and tabulate results (p. 232).

$L\,A$	$l\,A$	$\frac{1}{L\,A} + \frac{1}{l\,A}$

4. Determine the focal length of a concave mirror. Place the mirror opposite some brightly illuminated object, as a small arrow cut in cardboard, move the mirror backwards and forwards until a sharp image of the arrow is obtained on the cardboard, just by the side of the arrow. $L\,A$ and $l\,A$ are now equal (ii., p. 232), and the distance between the arrow and the mirror is therefore the radius of curvature, r, of the mirror, and the focal length $f = \frac{r}{2}$.

5. Verify Snell's law of sines, as described on p. 242 (Fig. 156).

6. Trace the course of a ray refracted through a glass prism (p. 247).

Place the prism on a sheet of white paper on a drawing-

board, and trace the outline of its base A B C (Fig. 277) with a pencil. Insert a pin H close to the prism. Viewed through the prism this pin appears in the direction I K. Mark this direction by pins at I and G, so that I G H are apparently in line. In a similar way place the pin L, so that L H G are in line. Similarly, if we look along L H we shall see the pins G I in the direction of L H produced. Remove the prism, produce I G to K, cutting the prism at D, and produce L H to M, cutting the prism at E. The incident ray strikes the prism at D, is refracted along D E, and emerges in the direction E L.

K M L is the angle of deviation; measure this angle

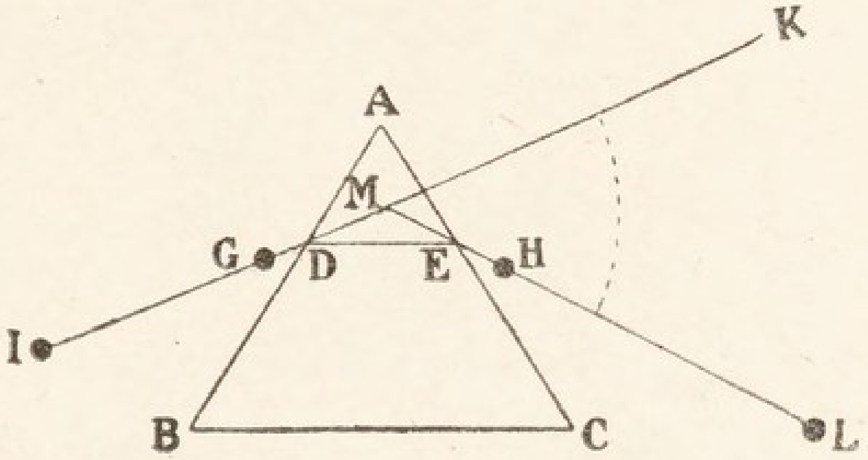


Fig. 277.—Path of ray through prism.

when the prism is in the position of *minimum deviation* (p. 247).

7. Through D (Fig. 277) draw N D N′ at right angles to A B. With centre D and any convenient radius describe a circle cutting D I and D E, produced if necessary, in two points. From the points draw perpendiculars on N N′. Measure these perpendiculars and so find (*see* p. 242) the refractive index of the prism.

8. Trace the path of a totally reflected ray through a prism (Fig. 278). Trace the base, A B C, of the prism with a pencil. Insert pins D and E at equal distances from B. Look along F D for the reflected image of E. Insert pin F so that F D E appear in line. In a similar way insert pin at G, so that G E D appear in line. Join F D, produce to K, produce G E to F′; from F′ drop a

perpendicular to A C, F″ F′; make F″ M = F′ M, and join F″ K, then K H F′ is the path of the ray through the prism. It will be noticed that the ray, totally reflected, is without colour, while the refracted ray is surrounded by a fringe of colours.

9. To determine the angle of a prism (Fig. 279). Trace the perimeter of the prism A B C, place a pin P some distance away, in the direction G A, so that a reflected image of P can be seen in the face B A by an observer looking in the direction D A, and a similar image in the

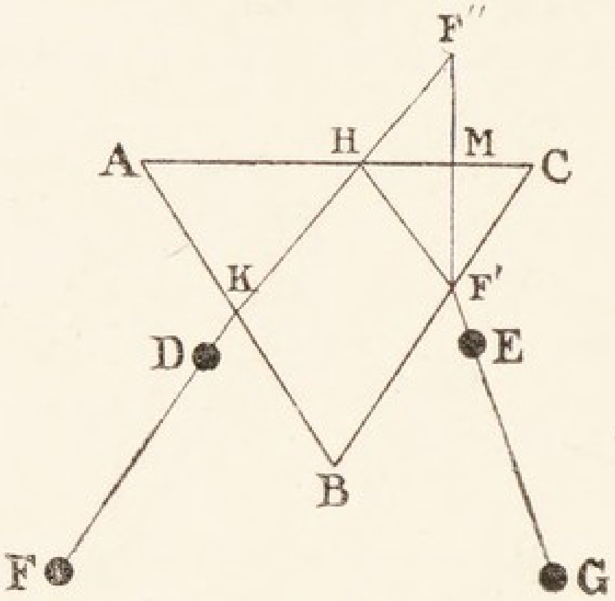


Fig. 278.—Path of totally reflected ray.

face C A when the observer looks in the direction E A. Mark these two directions by pins, as usual. When produced to meet, they include an angle which is double of the angle of the prism. This follows from the proposition already proved (p. 224); for, if we produce B A to F, it is clear that we may regard the reflecting surface, C A, as having been rotated through the angle C A F = 180° − A; but the reflected ray is rotated in the *same* direction through 360° − D A E; we know therefore that

$$360° - \text{DAE} = 2(180° - \text{A}) = 360° - 2\text{A}$$
$$\therefore\ \text{DAE} = 2\text{A}$$

We therefore determine A by measurement of D A E.

10. Determine the position of the image of a convex

lens by the method of parallax (p. 223). The method may be illustrated and tested by placing a short focus

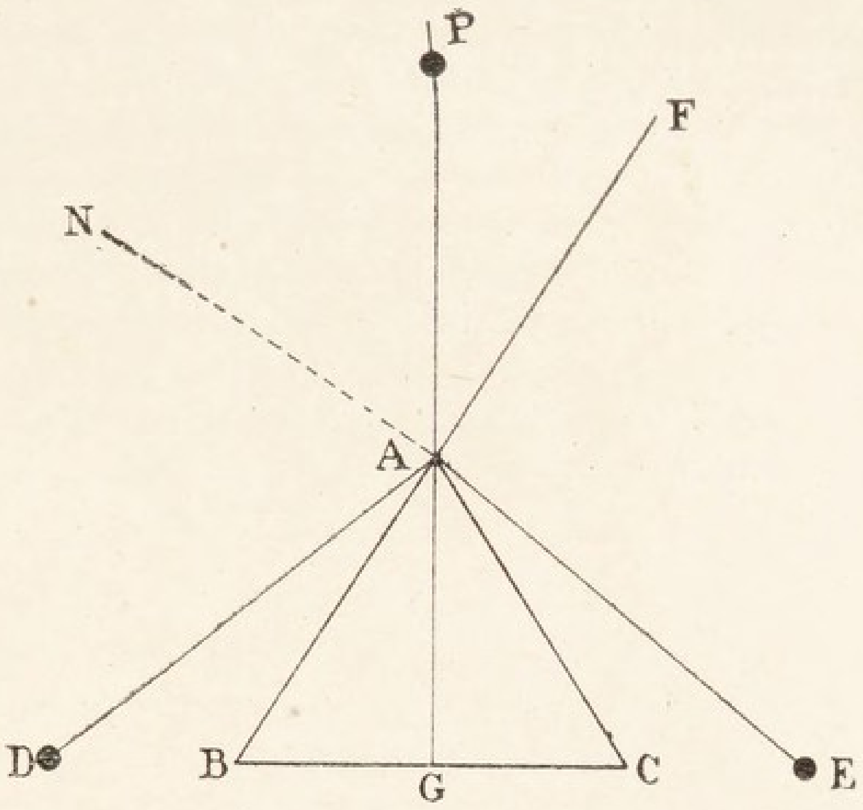


Fig. 279.—Determining angle of prism.

lens 6 in. in front of a card on which some black letters are printed (Fig. 280). On looking through the lens an inverted image of the letters will be seen, the

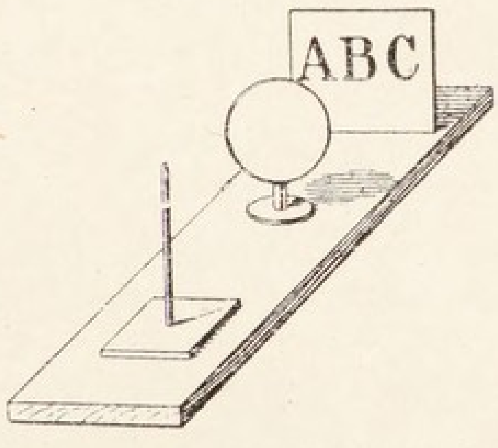


Fig. 280.—Position of image by parallax.

eye being some distance away. Now place a vertical rod between the lens and the eye, and move the head sideways. If the image of the letters is farther away

than the rod, the rod will appear to move over the letters in the reverse direction to the movement of the head; if, on the other hand, the rod is farther away, it will move over the letters in the same direction as the movement of the head. When they are at exactly the same distance the rod will remain on one letter, and move with it to the right or left as the head moves. When this position of the rod is found, substitute a candle for the letters and a screen for the rod; a sharp image of the candle should appear on the screen.

11. With the apparatus of Fig. 175 find various positions of object and image with a convex lens. Tabulate the values and draw a graph from them (p. 253).

u	v	$\frac{1}{u} + \frac{1}{v}$

Note the constant value obtained in the third column. The reciprocal of this constant is the focal length, f. Hence, find f. When u and v are equal, their value is $2f$. Hence find f.

12. Also find f by method 1 on p. 261, and determine the magnifying power (p. 265).

13. Mount two convex lenses to form an astronomical telescope, and measure its magnifying power (Fig. 170, p. 257).

14. Determine the focal length of some concave lenses by the methods described on p. 263 (Fig. 176).

15. Observe with a direct-vision spectroscope—

(*a*) a continuous spectrum of a gas flame, etc.;
(*b*) Fraunhofer's lines when the spectroscope is directed to the sky;

(*c*) a bright-line spectrum of sodium, calcium, etc.;

(*d*) an absorption spectrum of dilute blood solution, chlorophyll in alcohol, solutions of potassium permanganate, didymium salt, etc. (pp. 273, 277; *see also* Frontispiece).

16. The angle of minimum deviation, D, and the angle, A, of a prism are accurately determined by the spectrometer (p. 275). The eye-piece of the telescope is adjusted by the observer so that he can see the cross wires distinctly when looking at a distant object. The telescope is then directed to a distant object and the object glass so focused that the image of the distant object is seen in the plane of the cross wires. This is the case when their relative position is not altered by slight movements of the observer's eye (Ex. 10). The remaining adjustments are more easily made in a room which can be readily darkened, and where a sodium flame can be obtained. For this purpose a piece of asbestos impregnated with salt can be held in the flame of a Bunsen burner. The slit, *s*, of the collimator is now directed to the yellow flame, and the observer will see the illuminated slit on looking down the collimator. If it appears too broad the slit should be narrowed by the little screw at its side. The slit should now be viewed through the telescope *and* collimator. Both telescope and collimator are firmly clamped to the table of the instrument, but by slightly adjusting certain screws in the clamps the direction of the axis about which the telescope or collimator turns is slightly changed—the tube is, in fact, slightly *tilted* up or down. In this way the image of the slit can be brought to the centre of the cross wires. The collimator can also be focused till this image is sharp and clear. The instrument is then ready for use. The exact position of the telescope is now noted on the scale. The prism is next placed on the table and the slit cannot then be seen, but on turning the telescope slowly round while keeping the eye in position, the slit should reappear when the tube reaches a position corresponding to M H L (Fig. 277). I G (Fig. 277)

then corresponds to the collimator. The angle K M L through which the telescope has been turned is the deviation, and is registered on the scale. It may not be at first in the position of minimum deviation. If not, this position must be secured as described on p. 247. Moreover, the slit may not now appear at the centre of the cross wires. In that case the screw adjustments will have to be repeated till the slit is seen at the centre in both initial and final positions.

17. To measure the angle, A, of the prism, we place it in the position of Fig. 279, in which P may be said to represent the slit, P A the direction of the collimator, A D and A E two directions of the telescope in which a *reflected* image of the slit is seen from each face of the prism. The angle D A E through which the telescope moves from one position to the other is registered by the scale, and A is the half of this angle.

18. Find the number of grammes of glucose in an aqueous solution of that sugar by means of the saccharimeter, as described on p. 284.

19. Arrange apparatus to project a pure spectrum on a screen.

20. Determine the refractive index of a transparent solid or liquid by measurement of both the real and the apparent depth of an object when viewed normally through the transparent substance (p. 240).

Section VII.—Electricity and Magnetism

Frictional Electricity

1. Rub a stick of sealing-wax with dry flannel, suspend it by a silk thread from a glass support, and note that it is repelled by a second stick of rubbed sealing-wax, but attracted by a rod of glass rubbed with silk (p. 287).

2. Prove that electricity is developed on the rubber (p. 291) by the apparatus shown in Fig. 188 (p. 289).

3. Charge an electrophorus (p. 296), and with it light a gas-burner.

4. Study the action of a Wimshurst machine (p. 297), and with it charge a Leyden jar (p. 302).

5. Prove, by means of a Leyden jar with movable coatings, that the charge resides on the glass (p. 303).

6. Show that a Leyden jar cannot be charged in the usual manner if it stands on a sheet of glass.

7. Charge a gold-leaf electroscope (p. 289), by means of a lightly rubbed glass rod, (*a*) with positive electricity by contact, (*b*) with negative electricity by induction (p. 290).

Magnetism

8. Magnetize some steel needles by drawing them once or twice over the pole of a magnet.

9. Thrust the needles through pieces of cork and float them on some water. Observe that they point north and south.

10. Bring the north end of one needle near the north end of another, and observe that they repel each other; similarly, two south poles repel, while north attracts south, and vice versa.

11. Magnetize a needle and show that the end of the needle which last leaves the pole of the magnet is of the opposite name to the pole which it has just left.

12. Observe that a piece of soft iron wire attracts *both* ends of a floating magnet.

13. Attach a piece of iron wire to a magnet; it becomes a magnet and attracts a second piece of iron wire (Fig. 203, p. 309). Also, the magnetic properties of the iron wire disappear when it is removed from the magnet.

14. Break a magnetized needle in halves and show that you have now two magnets, each possessing a north and a south pole.

15. Pass a dipping needle along a bar magnet (Fig. 207, p. 315). Observe that over the north pole of the magnet the dipping needle stands vertical with its south pole downwards; in the middle of the magnet the needle is horizontal; and when over the south pole the needle is vertical but with its north pole downwards.

16. Magnetize a poker (p. 323) by holding it in the

magnetic meridian and at the dipping angle, and striking it two or three sharp blows with a hammer. Test its polarity with a compass needle. Reverse its polarity by holding it the other way up and striking half a dozen blows with the hammer.

17. Map out the lines of force and magnetic field by placing (*a*) two similar poles, (*b*) two dissimilar poles, of a couple of bar magnets under a sheet of thin paper and evenly scattering iron filings on the paper (Figs. 200, 201, p. 308).

18. Similarly map out the field of force of a small bar magnet. Measure the distance between the poles, as shown by the filings. Note that this distance is rather *less* than the length of the magnet.

19. Compare the moments of a small bar magnet, and of a small horseshoe magnet, by means of the magnetometer, as described on p. 318.

20. Find the neutral points in the neighbourhood of the given bar magnet, and deduce the value of $\frac{M}{H}$ (p. 319).

21. Find the dip (θ) by means of the sixteen readings described on pp. 322-3.

22. Connect the terminals of a single cell with a key, by covered copper wires.

(*a*) Lead part of the wire above, and parallel to, a declination needle which is at rest in the magnetic meridian. Note that the needle is not affected. Press down the key; the needle instantly turns: note in which direction the N. pole turns. Release the key and reverse the connections; on now pressing the key the current flows in the opposite direction: note that the N. pole also turns in the opposite direction.

(*b*) Repeat the experiments with the wire *below*, but still parallel to, the needle.

(*c*) Lead the wire over the centre of the needle, but *at right angles to its length*, so that the current flows east and west when the key is

down; the needle is now not affected, because it already lies at right angles to the current.

23. Connect a cell with an electro-magnet and identify the N. pole of the magnet. Reverse the connections, and note that this pole is now the S. pole. Trace the course of the current and see that the rule stated on p. 348 is verified.

Measurement of Current, etc.

24. Connect a single cell with a tangent galvanometer. Insert in the circuit both a key and a commutator (or a reversing key), and also a resistance box or rheostat, by which a known resistance, r, can be introduced into the circuit.

Let resistance of battery be represented by B,
and ,, ,, galvanometer be represented by G,
,, ,, ,, circuit wires ,, ,, ,, w,
then the total resistance R is $B + G + w + r$.

To find $B + G + w$:

Let $r = o$. See that the coil of the galvanometer is in the magnetic meridian, and the needle at rest.

Pass a current and note the deflection α.

Reverse the current and note the deflection α'.

Take the mean $\dfrac{\alpha + \alpha'}{2}$ and record this as δ.

Find tan δ from the tables, and record this in a parallel column. The current is simply proportional to tan δ; but, *ceteris paribus*, the current is inversely proportional to the resistance. If, therefore, while making no other change we can introduce a value of r, such that the value of tan δ is halved, we shall know that the current has been halved, and the total resistance doubled. This value of r must therefore be the value of $B + G + w$.

Perform experiments with this object, and tabulate your results in the columns below. Gradually increase the value of r till the value of tan δ is halved:—

R	$\delta \left[= \frac{\alpha + \alpha'}{2}\right]$	$tan\ \delta$	$R \times tan\ \delta$
$B + G + w$			
$B + G + w + r_1$			
$B + G + w + r_2$			
Etc.			
Etc.			

Having thus obtained the value of $B + G + w$, all the values of R in the first column are now known. Multiply each value of R by the corresponding value of tan δ, and enter the products in the fourth column. Note that the product is nearly constant in value.

25. With similar apparatus, but with three or more cells, perform experiments designed to show that the current—and therefore tan δ—is directly proportional to the E.M.F., or that $\frac{E}{\tan \delta}$ is constant. In this case the resistance, B, is also slightly varied, but the error due to this will be small if we work with a total value of R, compared to which B is nearly negligible. We also assume that the value of E is the same for each cell. Keep δ below 20° (p. 432), shunting the galvanometer if necessary. Tabulate results as below:—

	E.M.F.	R	δ	tan δ	$\frac{\text{E.M.F.}}{\tan \delta}$
		ohms			
1 cell . . .	E	500			
2 cells in *series* .	2 E	,,			
3 ,, ,, ,, .	3 E	,,			
2 ,, ,, *parallel* .	E	,,			
3 ,, ,, ,, .	E	,,			

26. With similar cells, but with a *low* external resistance, compare the currents obtained from two cells in series and in parallel :

	δ	$\tan \delta$
2 cells in series . .		
,, ,, parallel . .		

27. Measure e, the back E.M.F. of a water voltameter, by connecting it in series with an accumulator (or a Grove cell) whose E.M.F. has been previously measured alone (E). In this case $E + e$ will be found to be less than E, showing that e is negative.

28. With the apparatus employed in Ex. 24, find the resistance of a wire by the method of substitution described on p. 354.

Repeat the experiment with wires

(*a*) of different length, but same material and diameter ;
(*b*) of different diameter, but same material and length ;
(*c*) of different material, but same diameter and length.

Note the results and the conclusions to be drawn from them.

29. From the results found in the previous Exercise, and by measuring the dimensions of the wires, calculate the specific resistance of the substance of which the wire is made from the formula (p. 341)—

$$R = \frac{r\,l}{s}$$

30. With a Wheatstone's bridge, or a "post-office box" and a mirror galvanometer, determine the resistance (*a*) of the *primary* coil, and (*b*) of the *secondary* coil, of an

induction coil. Each coil must be entirely removed from the other for the experiments, and the "core" must be removed from the primary coil.

31. Compare the E.M.F.s of two cells by the method described on p. 364.

32. Find the strength, C, of a current by means of a voltameter (p. 337).

33. Find the strength, C, of a current by means of the heat developed in a wire of known resistance through which the current flows (p. 369).

34. Find the reduction factor of a galvanometer (p. 353).

35. Find the constant of a tangent galvanometer by means of a resistance box and a voltameter (p. 353).

36. Find the resistance of a given coil at 0° C. and 100° C.; hence deduce the room temperature of the laboratory (p. 356).

37. Compare the E.M.F.s of two cells by the potentiometer (p. 366).

38. Plot the relation between the candle-power and the wattage (i.e. amperes × volts) of the given electric lamp (p. 363).

39. Compare a series of corresponding readings of a given ammeter (p. 354) and a given galvanometer; represent the results by a graph. Find a mean value of the galvanometer constant.

APPENDIX

TABLE I

Pressure of Aqueous Vapour, at a Temperature from 0° F. to 88° F., in Inches of Mercury

0° F.	0·044 in.	32° F.	0·181 in.	59° F.	0·500 in.
2	0·048	33	0·188	60	0·518
4	0·052	34	0·196	61	0·537
6	0·057	35	0·204	62	0·556
8	0·062	36	0·212	63	0·576
10	0·068	37	0·220	64	0·596
11	0·071	38	0·229	65	0·617
12	0·074	39	0·238	66	0·639
13	0·078	40	0·247	67	0·661
14	0·082	41	0·257	68	0·684
15	0·086	42	0·267	69	0·708
16	0·090	43	0·277	70	0·733
17	0·094	44	0·288	71	0·759
18	0·098	45	0·299	72	0·785
19	0·103	46	0·311	73	0·812
20	0·108	47	0·323	74	0·840
21	0·113	48	0·335	75	0·868
22	0·118	49	0·348	76	0·897
23	0·123	50	0·361	77	0·927
24	0·129	51	0·374	78	0·958
25	0·135	52	0·388	79	0·990
26	0·141	53	0·403	80	1·023
27	0·147	54	0·418	82	1·092
28	0·153	55	0·433	84	1·165
29	0·160	56	0·449	86	1·242
30	0·167	57	0·465	87	1·282
31	0·174	58	0·482	88	1·323

TABLE II *

Pressure of Aqueous Vapour at Intervals of 1° C. from 0° C. to 30° C., at Intervals of 5° C. from 30° C. to 100° C., and at Intervals of 0·1° from 99° to 101°, in Millimetres of Mercury

t° C.	mm.	t° C.	mm.	t° C.	mm.
0	4·60	22	19·66	99·0	733·21
1	4·94	23	20·89	99·1	735·85
2	5·30	24	22·18	99·2	738·50
3	5·69	25	23·55	99·3	741·16
4	6·10	26	24·99	99·4	743·83
5	6·53	27	26·51	99·5	746·50
6	7·00	28	28·10	99·6	749·18
7	7·49	29	29·78	99·7	751·87
8	8·02	30	31·55	99·8	754·57
9	8·57	35	41·83	99·9	757·28
10	9·17	40	54·91	100·0	760·00
11	9·79	45	71·39	100·1	762·73
12	10·46	50	91·98	100·2	765·46
13	11·16	55	117·48	100·3	768·20
14	11·91	60	148·79	100·4	771·95
15	12·70	65	186·94	100·5	773·71
16	13·54	70	233·08	100·6	776·48
17	14·42	75	288·50	100·7	779·26
18	15·36	80	354·62	100·8	782·04
19	16·35	85	433·00	100·9	784·83
20	17·39	90	525·39	101·0	787·59
21	18·50	95	633·69		

* Abstracted from a table in "Numerical Tables and Constants," by Sydney Lupton, 1896 edition (Macmillan & Co.).

TABLE III

Glaisher's Factors

Dry-bulb.	Factor.	Dry-bulb.	Factor.	Dry-bulb.	Factor.
10°F.	8·78	40°F.	2·29	70°F.	1·77
12	8·78	42	2·23	72	1·75
14	8·76	44	2·18	74	1·73
16	8·70	46	2·14	76	1·71
18	8·50	48	2·10	78	1·69
20	8·14	50	2·06	80	1·68
22	7·60	52	2·02	82	1·67
24	6·92	54	1·98	84	1·66
26	6·08	56	1·94	86	1·65
28	5·12	58	1·90	88	1·64
30	4·15	60	1·88	90	1·63
32	3·32	62	1·86	92	1·62
34	2·77	64	1·83	94	1·60
36	2·50	66	1·81	96	1·59
38	2·36	68	1·79	98	1·58
				100	1·57

TABLE IV

Corrections to be applied to Barometers with Brass Scales to reduce the Observations to 32° F.

The brass scale is assumed to extend to the whole length of the mercury column, and the divisions on it are assumed to be correct at at 0° C. (32° F.).

Attached Thermo-meter.	Inches. 27	Inches. 28	Inches. 29	Inches. 30	Inches. 31
30°F.	– 0·004	– 0·004	– 0·004	– 0·004	– 0·004
31	0·006	0·006	0·007	0·007	0·007
32	0·008	0·009	0·009	0·009	0·010
33	0·011	0·011	0·012	0·012	0·012
34	0·013	0·014	0·014	0·015	0·015
35	0·016	0·016	0·017	0·018	0·018
36	0·018	0·019	0·020	0·020	0·021
37	0·021	0·021	0·022	0·023	0·024
38	0·023	0·024	0·025	0·026	0·026
39	0·025	0·026	0·027	0·028	0·029
40	0·028	0·029	0·030	0·031	0·032
41	0·030	0·031	0·033	0·034	0·035
42	0·033	0·034	0·035	0·036	0·037
43	0·035	0·036	0·038	0·039	0·040
44	0·037	0·039	0·040	0·042	0·043
45	0·040	0·041	0·043	0·044	0·046
46	0·042	0·044	0·045	0·047	0·049
47	0·045	0·046	0·048	0·050	0·051
48	0·047	0·049	0·051	0·052	0·054
49	0·050	0·051	0·053	0·055	0·057
50	0·052	0·054	0·056	0·058	0·060
51	0·054	0·056	0·058	0·060	0·062
52	0·057	0·059	0·061	0·063	0·065
53	0·059	0·061	0·064	0·066	0·068
54	0·062	0·064	0·066	0·068	0·071
55	0·064	0·066	0·069	0·071	0·076
56	– 0·066	– 0·069	– 0·071	– 0·074	– 0·076

TABLE IV—*continued*

Attached Thermo-meter.	Inches. 27	Inches. 28	Inches. 29	Inches. 30	Inches. 31
57°F.	− 0·069	− 0·071	− 0·074	− 0·076	− 0·079
58	0·071	0·074	0·077	0·079	0·082
59	0·074	0·076	0·079	0·082	0·085
60	0·076	0·079	0·082	0·085	0·087
61	0·078	0·081	0·084	0·087	0·090
62	0·081	0·084	0·087	0·090	0·093
63	0·083	0·086	0·089	0·093	0·096
64	0·086	0·089	0·092	0·095	0·098
65	0·088	0·091	0·095	0·098	0·101
66	0·090	0·094	0·097	0·101	0·104
67	0·093	0·096	0·100	0·103	0·107
68	0·095	0·099	0·102	0·106	0·109
69	0·098	0·101	0·105	0·109	0·112
70	0·100	0·104	0·108	0·111	0·115
71	0·102	0·106	0·110	0·114	0·118
72	0·105	0·109	0·113	0·117	0·120
73	0·107	0·111	0·115	0·119	0·123
74	0·110	0·114	0·118	0·122	0·126
75	0·112	0·116	0·120	0·125	0·129
76	0·114	0·119	0·123	0·127	0·131
77	0·117	0·121	0·126	0·130	0·134
78	0·119	0·124	0·128	0·133	0·137
79	0·122	0·126	0·131	0·135	0·140
80	0·124	0·129	0·133	0·138	0·143
81	0·126	0·131	0·136	0·141	0·145
82	0·129	0·134	0·138	0·143	0·148
83	− 0·131	− 0·136	− 0·141	− 0·146	− 0·151

TABLE V

Natural Sines and Tangents

Angle in degrees.	Angle in radians.	Sine.	Tangent.	
0	0·0000	0·000	0·000	90
1	0·0175	0·017	0·017	89
2	0·0349	0·035	0·035	88
3	0·0524	0.052	0·052	87
4	0·0698	0·070	0·070	86
5	0·0873	0·087	0·087	85
6	0·1047	0·105	0·105	84
7	0·1222	0·122	0·123	83
8	0·1396	0·139	0·141	82
9	0·1571	0·156	0·158	81
10	0·1745	0·174	0·176	80
15	0·2618	0·259	0·268	75
20	0·3491	0·342	0·364	70
25	0·4363	0·423	0·466	65
30	0·5236	0·500	0·577	60
35	0·6109	0·574	0·700	55
40	0·6981	0·643	0·839	50
45	0·7854	0·707	1·000	45
50	0·8727	0·766	1·192	40
55	0·9599	0·819	1·428	35
60	1·0472	0·866	1·732	30
65	1·1345	0·906	2·145	25
70	1·2217	0·940	2·747	20
75	1·3090	0·966	3·732	15
80	1·3963	0·985	5·671	10
81	1·4137	0·988	6·314	9
82	1·4312	0·990	7·115	8
83	1·4486	0·993	8·144	7
84	1·4661	0·995	9·57	6
85	1·4835	0·996	11·43	5
86	1·5010	0·998	14·30	4
87	1·5184	0·999	19·08	3
88	1·5359	0·999	28·64	2
89	1·5533	0·999	57·29	1
90	1·5708	1·000	Infin.	0
		Co-sine.	Co-tangent.	Angle in degrees.

TABLE VI

Relation between Mass and Volume of Water at Various Temperatures

(1 millilitre = 1·000028 c.c.)

Mass of water at t° C. contained in 1 millilitre.				Volume, in millilitres, filled by 1 grm. of water at t° C.			
t° C.	Grm.*	t° C.	Grm.*	t° C.	Mils.	t° C.	Mils.
0	0·999871	20	0·998259	0	1·000129	20	1·001744
1	0·999928	21	0·998047	1	1·000072	21	1·001957
2	0·999969	22	0·997828	2	1·000031	22	1·002177
3	0·999991	23	0·997601	3	1·000009	23	1·002405
4	1·000000	24	0·997367	4	1·000000	24	1·002641
5	0·999990	25	0·997120	5	1·000010	25	1·002888
6	0·999970	26	0·996866	6	1·000030	26	1·003144
7	0·999933	27	0·996603	7	1·000067	27	1·003408
8	0·999886	28	0·996331	8	1·000114	28	1·003682
9	0·999824	29	0·996051	9	1·000176	29	1·003965
10	0·999747	30	0·995770	10	1·000253	30	1·00425
11	0·999655	40	0·99235	11	1·000345	40	1·00770
12	0·999549	50	0·98819	12	1·000451	50	1·01195
13	0·999430	60	0·98338	13	1·000570	60	1·01691
14	0·999299	70	0·97794	14	1·000701	70	1·02256
15	0·999160	80	0·97194	15	1·000841	80	1·02887
16	0·999002	90	0·96556	16	1·000999	90	1·03567
17	0·998841	100	0·95866	17	1·001160	100	1·04312
18	0·998654			18	1·001348		
19	0·998460			19	1·001542		

* The numbers in this column represent the ratio

$$\frac{\text{density of water at } t^\circ \text{ C.}}{\text{density of water at } 4^\circ \text{ C.}}$$

they also represent in grammes the absolute density of water *per millilitre*; if multiplied by the factor 0·999972 they would represent in grammes the absolute density of water *per cubic centimetre* at each corresponding temperature.

TABLE VII

$$\frac{\text{Density of Water at } t^\circ \text{ C.}}{\text{Density of Water at } 60^\circ \text{ F.}}$$

This relative density, multiplied by 0·999044, gives the absolute density, in grammes per cubic centimetre, at the corresponding temperature.

t	Rel. density	t	Rel. density	t	Rel. density
5	1·000918	13·5	1·000292	22	0·998755
5·5	1·000908	14	1·000227	22·5	0·998641
6	1·000898	14·5	1·000157	23	0·998528
6·5	1·000879	15	1·000088	23·5	0·998410
7	1·000861	15·5	1·000009	24	0·998293
7·5	1·000837	16	0·999930	24·5	0·998169
8	1·000814	16·5	0·999849	25	0·998046
8·5	1·000783	17	0·999769	25·5	0·997920
9	1·000752	17·5	0·999675	26	0·997794
9·5	1·000713	18	0·999582	26·5	0·997661
10	1·000675	18·5	0·999484	27	0·997529
10·5	1·000629	19	0·999387	27·5	0·997392
11	1·000583	19·5	0·999286	28	0·997256
11·5	1·000530	20	0·999186	28·5	0·997116
12	1·000477	20·5	0·999080	29	0·996976
12·5	1·000417	21	0·998974	29·5	0·996835
13	1·000358	21·5	0·998864	30	0·996695

TABLE VIII

To find the Mass, m Gramme, of Water which fills v C.C.

v	At 4° C.	At 0° C.	At 60° F.
c.c.	grm.	grm.	grm.
1	0·999972	0·999843	0·999044
2	1·999944	1·999686	1·998088
3	2·999916	2·999529	2·997132
4	3·999888	3·999372	3·996176
5	4·999860	4·999215	4·995220
6	5·999832	5·999058	5·994264
7	6·999804	6·998901	6·993308
8	7·999776	7·998744	7·992352
9	8·999748	8·998587	8·991396

Ex.: To find the mass of 123 c.c. of water at 60° F.

	Mass of	100 c.c.	at 60°	=	99·9044 grm.
	,,	20 ,,	,,	=	19·98088 ,,
	,,	3 ,,	,,	=	2·997132 ,,
∴	,,	123 ,,	,,	=	122·882412 ,,

TABLE IX

To find the Volume, v C.C., occupied by m Grm. of Water

m.	At 4° C.	At 0° C.	At 60° F.
grm.	c.c.	c.c.	c.c.
1	1·000028	1·000157	1·000956
2	2·000056	2·000314	2·001912
3	3·000084	3·000471	3·002868
4	4·000112	4·000628	4·003824
5	5·000140	5·000785	5·004780
6	6·000168	6·000942	6·005736
7	7·000196	7·001099	7·006692
8	8·000224	8·001256	8·007648
9	9·000252	9·001413	9·008604

Ex.: To find the volume occupied by 123 grm. of water at 60° F.

100 grm. of water at 60° F. fill	..	100·0956 c.c.
20 ,, ,, ,, ,,	..	20·01912 ,,
3 ,, ,, ,, ,,	..	3·002868 ,,
∴ 123 ,, ,, ,, ,,	..	123·117588 ,,

TABLE X

To convert Degrees Fahr. to Degrees Cent.				To convert Degrees Cent. to Degrees Fahr.			
° F.	° C.	° F.	° C.	° C.	° F.	° C.	° F.
45	$7{\cdot}\dot{2}$	61	$16{\cdot}\dot{1}$	25	77	41	105·8
46	$7{\cdot}\dot{7}$	62	$16{\cdot}\dot{6}$	26	78·8	42	107·6
47	$8{\cdot}\dot{3}$	63	$17{\cdot}\dot{2}$	27	80·6	43	109·4
48	$8{\cdot}\dot{8}$	64	$17{\cdot}\dot{7}$	28	82·4	44	111·2
49	$9{\cdot}\dot{4}$	65	$18{\cdot}\dot{3}$	29	84·2	45	113
50	10	66	$18{\cdot}\dot{8}$	30	86	46	114·8
51	$10{\cdot}\dot{5}$	67	$19{\cdot}\dot{4}$	31	87·8	47	116·6
52	$11{\cdot}\dot{1}$	68	20	32	89·6	48	118·4
53	$11{\cdot}\dot{6}$	69	$20{\cdot}\dot{5}$	33	91·4	49	120·2
54	$12{\cdot}\dot{2}$	70	$21{\cdot}\dot{1}$	34	93·2	50	122
55	$12{\cdot}\dot{7}$	71	$21{\cdot}\dot{6}$	35	95	51	123·8
56	$13{\cdot}\dot{3}$	72	$22{\cdot}\dot{2}$	36	96·8	52	125·6
57	$13{\cdot}\dot{8}$	73	$22{\cdot}\dot{7}$	37	98·6	53	127·4
58	$14{\cdot}\dot{4}$	74	$23{\cdot}\dot{3}$	38	100·4	54	129·2
59	15	75	$23{\cdot}\dot{8}$	39	102·2	55	131
60	$15{\cdot}\dot{5}$	76	$24{\cdot}\dot{4}$	40	104	56	132·8

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