Morrison

Lent to [illegible]

A

INTRODUCTION

TO

MENSURATION,

AND

PRACTICAL GEOMETRY,

WITH NOTES, CONTAINING THE REASON OF EVERY RULE.

By JOHN BONNYCASTLE,

OF THE ROYAL MILITARY ACADEMY, WOOLWICH.

The FIFTH EDITION, corrected and improved.

LONDON: Printed for J. JOHNSON, N° 72, ST. PAUL'S CHURCH YARD. 1798.

By Bye and Law, St. John's Square, Clerkenwell.

PREFACE.

THE ART OF MEASURING, like all other uſeful inventions, appears to have been the offspring of want and neceſſity; and to have had its origin in thoſe remote ages of antiquity, which are far beyond the reach of credible and authentic hiſtory. Egypt, the fruitful mother of almoſt all the liberal ſciences, is imagined likewiſe to have given birth to GEOMETRY OF MENSURATION; it being to the inundations of the Nile that we are ſaid to be indebted for this moſt perfect and delightful branch of human knowledge.

After the overflowings of the river had deluged the country, and all artificial boundaries and land-marks were deſtroyed, there could have been no other method of aſcertaining individual property, than by a previous knowledge of its figure and dimenſions. From this circumſtance, it appears highly probable, that Geometry was firſt known and cultivated by the ancient Egyptians; as being the only ſcience which could adminiſter to their wants, and furniſh them with the aſſiſtance they required. The name itſelf ſignifies properly the *art of meaſuring the earth*; which ſerves ſtill further to confirm this opinion; eſpecially as it is well known that many of the ancient mathematicians applied their geometrical knowledge entirely to that purpoſe; and that even the Elements of Euclid, as they now ſtand, are only the theory from whence we obtain the rules and precepts of our preſent more mechanical practice.

But to trace the ſciences to their firſt rude beginnings, is a matter only of learned curioſity, which could afford but little gratification to readers in general. It is of much more consequence to the rising generation to be informed that, in their present improved state, they are exceedingly useful and important. And in this respect, the art I have undertaken to elucidate is inferior to none, arithmetic only excepted. Its use in most of the different branches of the Mathematics is so general and extensive, that it may be justly considered as the mother and mistress of all the rest, and the source from whence were derived the various properties and principles to which they owe their existence.

As a testimony of this superior excellence, I need only mention a few of those who have studied and improved it; in which illustrious catalogue we have the names of Euclid, Archimedes, Thales, Anaxagoras, Pythagoras, Plato, Apollonius, Philo and Ptolemy, amongst the ancients; and Huygens, Wallis, Gregory, Halley, the Bernoullies, Euler Liebnitz, and Newton, among the moderns; all of whom applied themselves to particular parts of it, and greatly enlarged and improved the subject. To the latter especially we are indebted for many valuable discoveries in the higher branches of the art; which have not only enhanced its dignity and importance, but rendered the practical application of it more general and extensive.

The degree of estimation in which the art was held by these and other eminent characters, will, in general, it is apprehended, be thought a sufficient encomium on its merits. But, for the sake of young people, and those of a confined education, it may not be amiss to give a few more instances of its advantage, and shew that its importance in trade and business is not inferior to its dignity as a science. Artificers of almost all denominations are indebted to this invention for the establishment of their several occupations, and the perfection and value of their workmanship. Without its assistance all the great and noble works

of

of Art would have been imperfect and useless. By this means the architect lays down his plan, and erects his edifice; bridges are built over large rivers; ships are constructed; and property of all kinds is accurately measured, and justly estimated. In short, most of the elegancies and conveniencies of life owe their existence to this art, and will be multiplied in proportion as it is well understood, and properly practised.

From this view of the subject, it is hardly to be accounted for, that, in a commercial nation, like our own, an art of such general application should have been so greatly neglected. Mechanics of all kinds, it is well known, are but ill acquainted with its principles; and those who have been the best qualified to afford them any assistance, have thought it beneath their attention. Till within a few years past there could not be found a regular treatise upon this subject in the English language. Some particular branches, it is true, had been greatly cultivated and improved; but these were only to be found in their miscellaneous state, interspersed through a number of large volumes, in the possession of but few, and in a form and language totally unintelligible to those for whom they were more immediately necessary.

Dr. Hutton was the first person, in this country, who undertook to collect these scattered fragments, and to treat of the subject in a scientific, methodical manner. A small treatise by Hawney, and some others of little note, had indeed been long in the hands of the public; but these were extremely defective, both in matter and method; neither the principles nor practice of the art being properly or clearly explained. Before the publication of the treatise abovementioned, Mr. Robertson's may be considered as the only book, of any value, that could be consulted, either by the artizan or mathematician; and had he given the theory as well as the practice of the

art,

art, and divested his rules and examples of their algebraical form, there would have been no want of any other elementary treatise.

To these two writers I am greatly indebted for many things in the following pages, and am ready to acknowledge, that I have used an unreserved freedom in selecting from their works, wherever I found them to answer my purpose. To Dr. Hutton I am particularly obliged, and am so far from desiring to supercede the use of his performance by this publication, that I only wish it to be thought a useful introduction to it. His treatise is excellent in its kind; and had it been as well calculated for the use of the uninformed Artist as it is for the Mathematician, the following compendium had certainly never been published.

The method I have observed, in composing this work, is that which was used in the "*Scholar's Guide to Arithmetic*;" and, as my object has been to facilitate the acquirement of the same kind of useful knowledge, I am not without hopes of its being received with equal candour and approbation.

In school-books, and those designed for the use of learners, it has always appeared to me, that plain and concise rules, with proper exercises, are entirely sufficient for the purpose. In science, as well as in morals, example will ever inforce and illustrate precept; for this reason an operation, wrought out at length, will be found of more service to beginners than all the tedious directions and observations that can possibly be given them. From constant experience I have been confirmed in this idea; and it is in pursuance of it that I have formed the plan of this publication. I have not been ambitious of adding much new matter to the subject; but only to arrange and methodize it in a manner more easy and rational than had been done before.

The text part of the work contains the rules in words at length, with examples to exercise them; and,

and, in order that the learner may not be perplexed and interrupted in his progreſs, the remarks and demonſtrations are confined to the notes, and may be conſulted or not, as ſhall be thought neceſſary. To thoſe who would wiſh not to take things upon truſt, but to be acquainted with the grounds and *rationale* of the operations they perform, they will be found extremely ſerviceable; and for this purpoſe I have endeavoured to make them as eaſy as the nature of the ſubject would admit. But they can be conſulted only by ſuch as have made a previous acquaintance with ſeveral other branches of mathematical learning.

Some of the moſt difficult rules relating to the ſurfaces of ſolids, &c. could not be conveniently given, but by means of algebraical theorems; and as this was foreign to my purpoſe, I have not ſcrupled to omit them; being well perſuaded that what is done upon that head will be fully ſufficient to anſwer moſt practical purpoſes. In the practical Geometry likewiſe, which is prefixed to this treatiſe, ſuch problems only are introduced as were known to be moſt intimately connected with the ſubject. And as this part of the work is a proper and neceſſary introduction to the reſt, I have ſpared no pains in making it as clear and intelligible as poſſible.

Upon the whole, I have endeavoured to conſult the wants of the learner, more than thoſe of the man of ſcience. And if I have ſucceeded in this reſpect, my purpoſe is anſwered. I have not ſought for reputation as a mathematician, but only to be uſeful as a tutor.

N. B. The favourable reception this Work has met with, has induced me in this Edition to make ſuch Alterations and Additions as have ſince occurred to me, and which are ſuch as I hope will render it ſtill more acceptable to the Public.

Royal Academy, Woolwich,
March 1, 1794.

CONTENTS.

To

CONTENTS.

TABLES

TABLES

OF THE

DIFFERENT MEASURES USED IN THIS WORK.

Lineal Measures.

12 inches	make	1 foot.
3 feet	- - -	1 yard.
6 feet	- - -	1 fathom.
16½ feet, or 5½ yards,	- -	1 pole, or rod.
40 poles	- -	1 furlong.
8 furlongs	- -	1 mile.

Square Measures.

144 inches	make	1 foot.
9 feet	- - -	1 yard.
36 feet	- - -	1 fathom.
272¼ feet, or 30¼ yds.	- -	1 pole, or rod.
1600 poles	- - -	1 furlong.
64 furlongs	- -	1 mile.

Note, The chain made uſe of in meaſuring land, commonly called Gunter's chain, is 4 poles, or 22 yards in length, and conſiſts of 100 equal links, each link being $\frac{22}{100}$ of a yard, or 7.92 inches long.

An acre of land is alſo equal to 10 ſquare chains; that is, 10 chains in length, and 1 in breadth; or it is 4840 ſquare yards, or 160 ſquare poles, or 100,000 ſquare links.

Note alſo, that in Land Meaſure,

40 perches, or ſquare poles	make	1 rood.
4 roods	- - - - - -	1 acre.

And in Cubic Meaſure,

1728 inches	make	1 foot.
27 feet	- - -	1 yard.
166⅜ yards	- - -	1 pole.

PRAC-

PRACTICAL GEOMETRY.

DEFINITIONS.

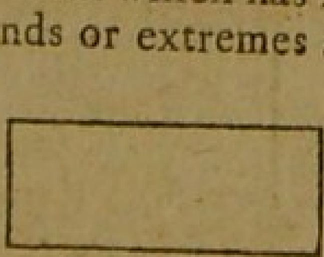
1. *GEOMETRY* is that ſcience which treats of the deſcriptions and properties of magnitudes in general.

2. A *point* is that which has no parts, or dimenſions.

3. A *line* is length without breadth; and its bounds or extremes are points.

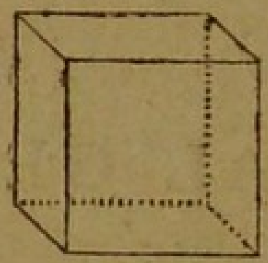
4. A *right* line is that which lies evenly between its extreme points.

5. A *ſuperficies* is that which has length and breadth only; and its bounds or extremes are lines.

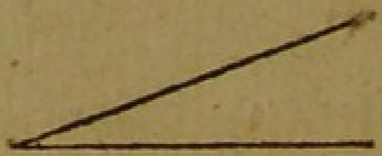


6. A *plane ſuperficies* is that which touches, in every part, any right line that can be drawn in that ſuperficies.

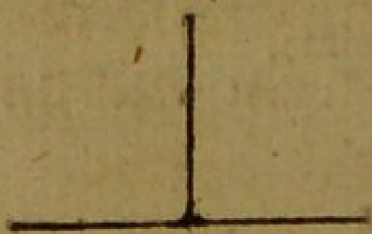
7. A *ſolid* is that which has length, breadth and thickneſs; and its bounds, or extremes, are ſuperficies.



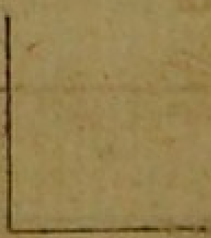
8. A *plane rectilineal angle* is the inclination or opening of two right lines which meet in a point.



9. One line is ſaid to be *perpendicular* to another, when it makes the angles on both ſides of it equal to each other.



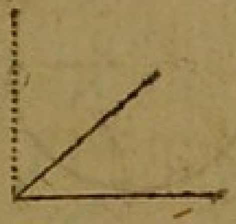
10. A *right angle* is that which is formed by two lines that are perpendicular to each other.*



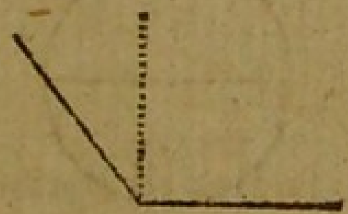
* Any angle differing from a right one, is, by ſome writers, called an *oblique angle*.

11. An

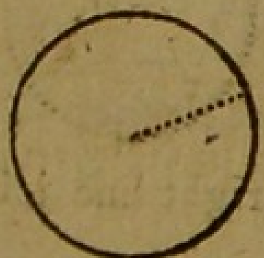
11. An *acute angle* is that which is lefs than a right angle.



12. An *obtufe angle* is that which is greater than a right angle.



13. A *circle* is a plane figure, formed by the revolution of a right line, about one of its extremities, which remains fixed.*

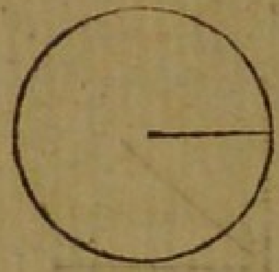


14. The centre of a circle, is the point about which it is defcribed; and the circumference is the line or boundary by which it is contained.

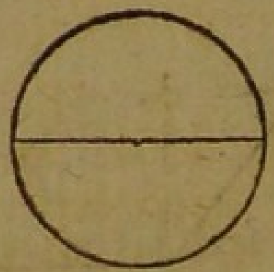
* N.B. The circumference itfelf, for the fake of concifenefs, is fometimes called a circle.

15. The

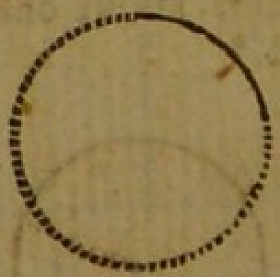
15. The *radius* of a circle is a right line drawn from the centre to the circumference.



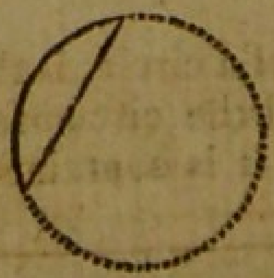
16. The *diameter* of a circle is a right line paſſing through the centre, and terminated both ways by the circumference.



17. An *arc* of a circle is any part of its periphery or circumference.



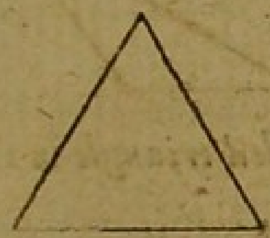
18. A *chord* is a right line joining the extremities of an arc.



N.B. A ſemicircle is half the circle, and a quadrant the quarter of it.

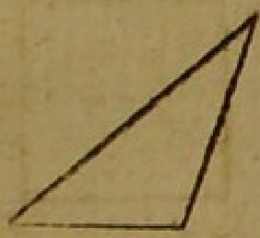
19. All plane figures bounded by three right lines are called *triangles*

20. An *equilateral triangle* is that whoſe three ſides are all equal.

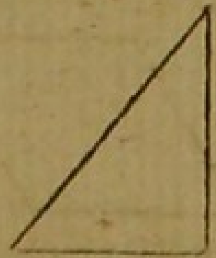


21. An *iſoſceles triangle* is that which has only two of its ſides equal.

22. A *ſcalene triangle* is that which has all its three ſides unequal.

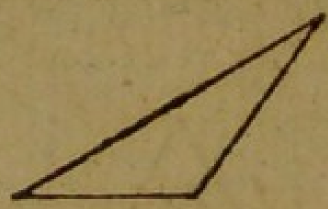


23. A *right-angled triangle* is that which has one right angle.*

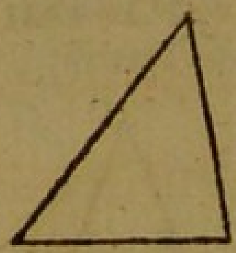


* Any triangle differing from a right angled one, is called *an oblique angled triangle.*

24. An *obtuſe-angled triangle* is that which has one obtuſe angle.



25. An *acute-angled triangle* is that which has all its angles acute.



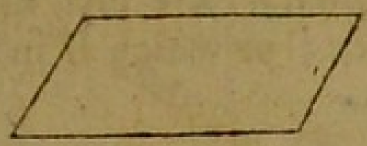
26. All plane figures, bounded by four right lines, are called *quadrangles*, or *quadrilaterals*.

27. A *ſquare* is a quadrilateral, whoſe ſides are all equal, and its angles all right angles.

28. A *rhombus* is a quadrilateral whoſe ſides are all equal, but its angles not right angles.*

* This figure, by working mechanics, is ſometimes called a lozenge.

29. A *parallelogram* is a quadrilateral whoſe oppoſite ſides are parallel.

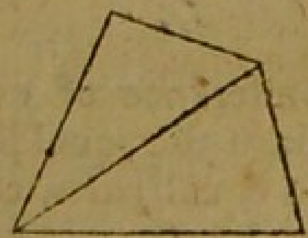


30. A *rectangle* is a parallelogram whoſe angles are all right angles.

31. A *rhomboid* is a parallelogram whoſe angles are not right angles.

32. All other four-ſided figures, beſides theſe, are called *trapeziums*.

33. A right line joining any two oppoſite angles of a four-ſided figure is called the *diagonal*.

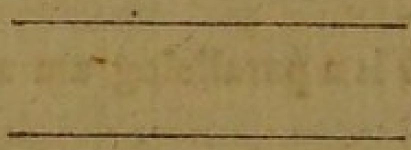


34. All plane figures contained under more than four ſides are called *polygons*.

35. Polygons having five ſides, are called *pentagons*; thoſe of ſix ſides, *hexagons*; thoſe of ſeven, *heptagons*; and ſo on.

36. A *regular polygon* is that whofe angles, as well as fides, are all equal.

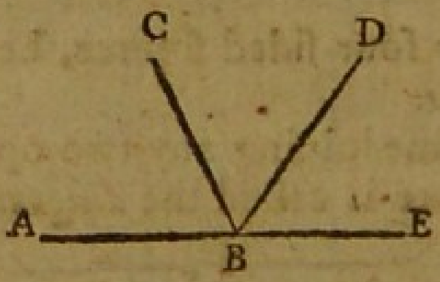
37. *Parallel right lines* are fuch as are every where at an equal diftance; or which if infinitely produced would never meet.



38. The *bafe* of any figure is that fide on which it is fuppofed to ftand; and the *altitude* is the perpendicular falling upon it from the oppofite angle.

39. In a right-angled triangle the fide oppofite to the right angle is called the *hypothenufe*; and the other two fides are called *legs*.

40. An angle is ufually denoted by three letters, the one which ftands at the angular point being always to be read in the middle.



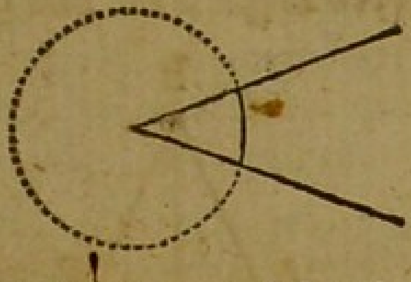
*41. The circumference of every circle is fuppofed to be divided into 360 equal parts, called *degrees*; each degree into 60 equal parts, called *minutes*; and fo on.

42. The *meafure* of any right-lined angle is an arc of a circle contained between the two lines

* This and the following definition are ufed only in Practical Geometry.

which

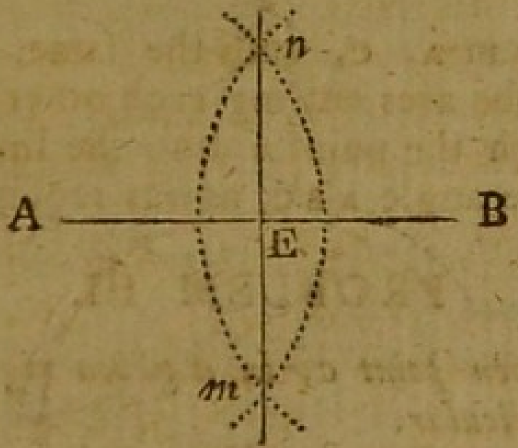
which form that angle, the angular point being the centre.



Note. The angle is eſtimated by the number of degrees contained in the arc; whence a right angle is an angle of 90 degrees, or $\frac{1}{4}$ of the circumference.

PROBLEM I.*

To divide a given line A B *into two equal parts.*



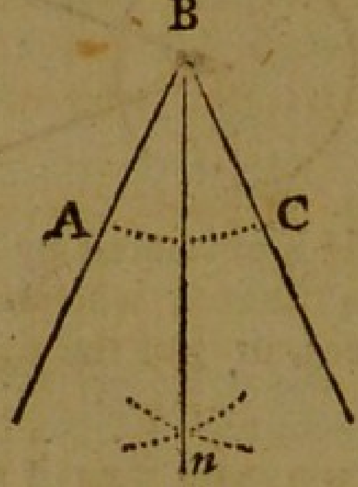
1. From the points A and B, as centres, with any diſtance greater than half A B, deſcribe arcs cutting each other in *n* and *m*.

2. Through theſe points draw the line *n* E *m*, and the point E, where it cuts A B, will be the middle of the line required.

* The demonſtrations of moſt of theſe problems may be found in Euclid's Elements.

PROBLEM II.

To divide a given angle A B C *into two equal parts.*



1. From the point B, with any radius, describe the arc A C.

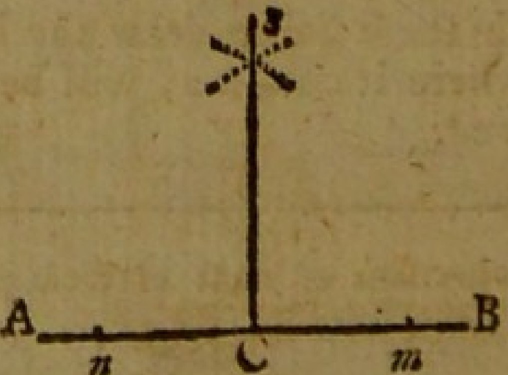
2. And from A, C, with the same, or any other radius, describe arcs cutting each other in *n*.

3. Through the point *n* draw the line B *n*, and it will bisect the angle A B C, as was required.

PROBLEM III.

From a given point C, *in a given right line* A B, *to erect a perpendicular.*

CASE I. *When the point is near the middle of the line.*



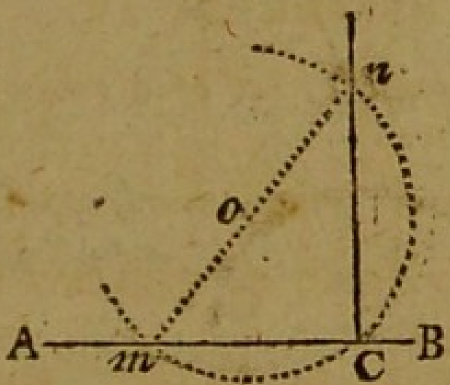
1. On

1. On each side of the point C take any two equal distances C*n*, C*m*.

2. From *n* and *m*, with any radius greater than C*n* or C*m*, describe arcs cutting each other in *s*.

3. Through the point *s*, draw the line *s*C, and it will be the perpendicular required.

CASE II. *When the point is at, or near, the end of the line.*

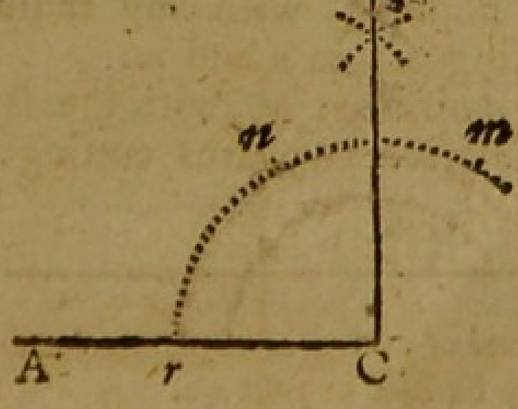


1. Take any point *o*, and with the radius or distance *o*C, describe the arc *m*C*n*, cutting AB in *m* and C.

2. Through the centre *o*, and the point *m*, draw the line *mon*, cutting the arc *m*C*n* in *n*.

3. From the point *n*, draw the line *n*C, and it will be the perpendicular required.

Another method.



1. From the point C, with any radius, defcribe the arc *rnm*, cutting the line A C in *r*.

2. With the fame radius, and *r* as a centre, crofs the arc in *n*; and from *n*, in like manner, crofs it in *m*.

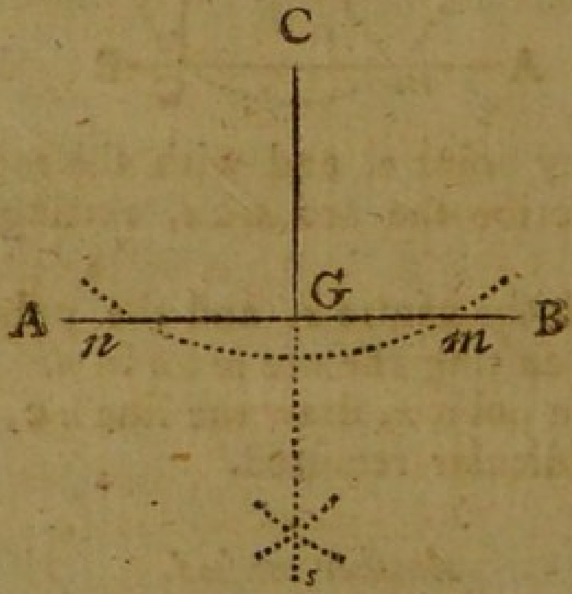
3. From the points *n* and *m*, with the fame, or any other radius, defcribe arcs cutting each other in *s*.

4. Through the point *s*, draw the line *s* C, and it will be the perpendicular required.*

PROBLEM IV.

From a given point C, *out of a given line* A B, *to let fall a perpendicular.*

CASE I. *When the point is nearly oppofite to the middle of the line.*



1. From the point C, with any radius, defcribe the arc *nm*, cutting A B in *n* and *m*.

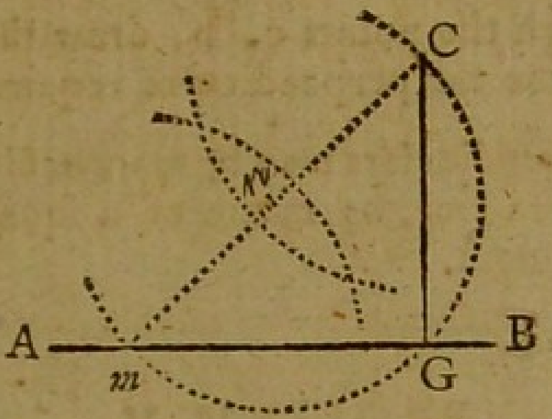
2. From the points *n*, *m*, with the fame, or any other radius, defcribe two arcs cutting each other in *s*.

* Another method of raifing a perpendicular from any point in a given line may be feen at page 40.

3. Through

3. Through the points C, *s*, draw the line C G *s*, and C G will be the perpendicular required.

CASE II. *When the point is nearly opposite to the end of the line.*



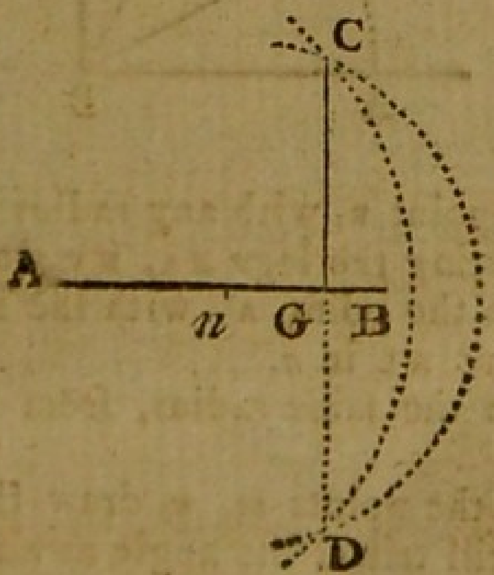
1. To any point *m*, in the line A B, draw the line C *m*.

2. Bisect the line C *m*, or divide it into two equal parts, in the point *n*.

3. From *n*, with the radius *n m*, or *n* C, describe the arc C G *m*, cutting A B in G.

4. Through the point C, draw the line C G, and it will be the perpendicular required.

Another method.



1. From

1. From A, or any other point in A B, with the radius A C, defcribe the arc C D.

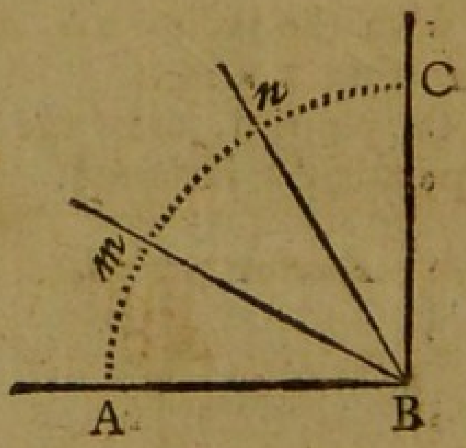
2. And from any other point n, in A B, with the radius n C, defcribe another arc cutting the former in C, D.

3. Through the points C, D, draw the line C G D, and C G will be the perpendicular required.

N. B. Perpendiculars may be more eafily raifed, and let fall in practice, by means of a fquare, or other proper inftrument.

PROBLEM V.

To trifect, or divide a right angle A B C into three equal parts.



1. From the point B, with any radius B A, defcribe the arc A C, cutting the legs B A, B C, in A, C.

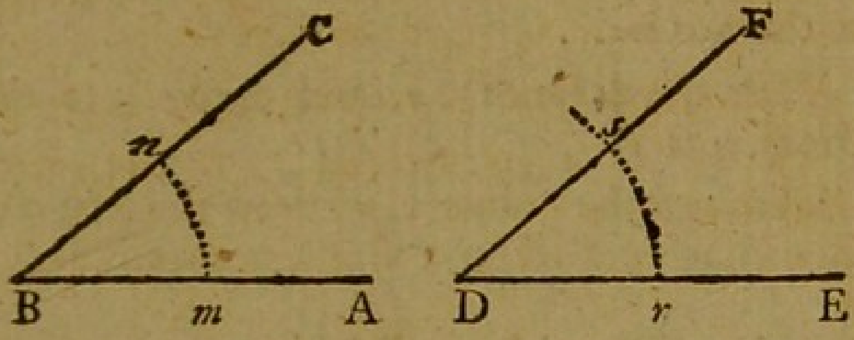
2. And from the point A, with the radius A B or B C, crofs the arc A C in n.

3. Alfo with the fame radius, from the point C, crofs it in m.

4. Through the points m, n, draw the lines B m, B n, and they will trifect the angle as was required.

PROBLEM VI.

At a given point D *to make an angle equal to a given angle* ABC.



1. From the point B, with any radius, describe the arc *nm*, cutting the legs BA, BC, in the points *m*, *n*.

2. Draw the line DE, and from the point D, with the same radius as before, describe the arc *rs*.

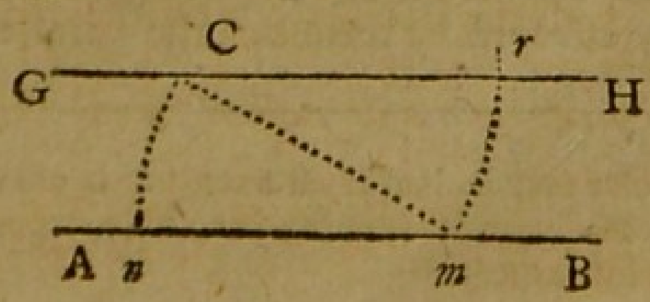
3. Take the distance *mn*, on the former arc, and apply it to the arc *rs*, from *r* to *s*.

4. Through the points D*s*, draw the line DF, and the angle EDF will be equal to the angle ABC as was required.

PROBLEM VII.

To draw a line parallel to a given line AB.

CASE I. *When the parallel line is to pass through a given point* C.



1. To

1. To A B, from the point C, draw any right line C *m*.

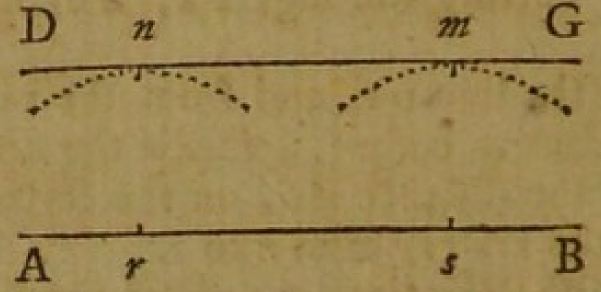
2. From the point *m*, with the radius *m* C, describe the arc C *n*, cutting A B in *n*.

3. And with the same radius, from the point C, describe the arc *mr*.

4. Take the distance C *n*, and apply it to the arc *mr*, from *m* to *r*.

5. Through the points C, *r*, draw the line G C *r* H, and it will be parallel to A B as was required.

CASE II. *When the parallel line is to be at a given distance from A B.*



1. From any two points *r*, *s*, in the line A B, with a radius equal to the given distance, describe the arcs *n*, *m*.

2. Draw the line D G, to touch those arcs without cutting them, and it will be parallel to A B as was required.

N. B. The former case of this problem, as well as several other operations in Practical Geometry, may be more easily effected by means of the parallel ruler.*

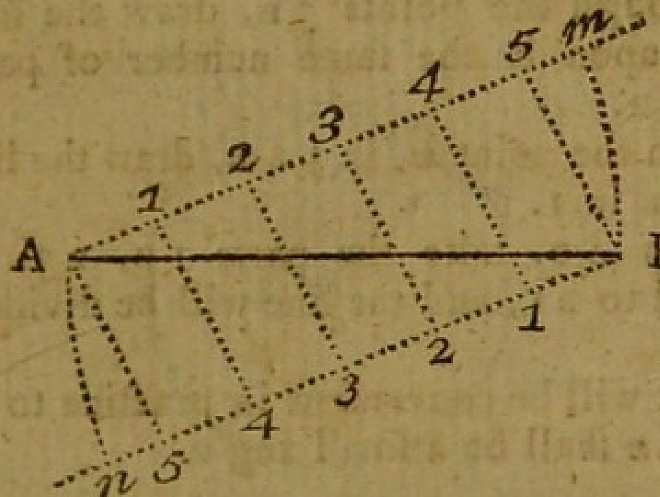
---

* This ruler may be had of all sizes, but is usually put into a portable case, with a drawing-pen, scale, compasses, and other useful instruments.

PRO-

PROBLEM VIII.

To divide a given line A B *into any propoſed number of equal parts.*



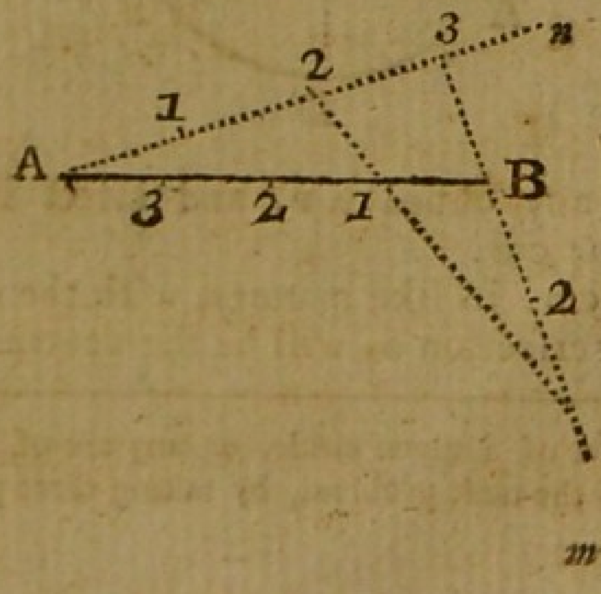
1. From one end of the line A, draw A *m*, making any angle with A B; and from B, the other end, draw B *n*, making an equal angle A B *n*.

2. In each of the lines A *m*, B *n*, beginning at A and B, ſet off as many equal parts, of any length, as A B is to be divided into.

3. Join the points A 5, 1 4, 2 3, &c. and A B will be divided as was required.

Note. B *n* may be drawn parallel to A *m*, by means of a parallel ruler.

Another method.



1. From

1. From the point A draw any line A n, and ſet on it as many equal parts, wanting one, as A B is to be divided into.

2. Through the points 3 B, draw the line 3 B m, and take upon it the ſame number of parts, each equal to 3 B.

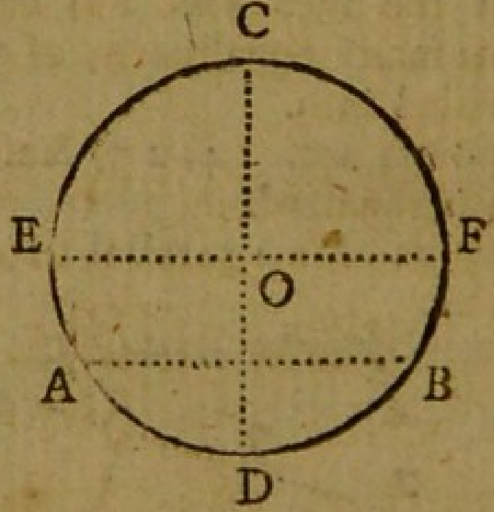
3. From the point m, in 3 B m, draw the line m 1 2, cutting A B in 1.

4. Upon A B, take the parts 1 2, 2 3, and 3 A, each equal to B 1, and the line will be divided as was required.

Note. It will be convenient in practice to draw A n ſo that B A n ſhall be a ſmall angle.

PROBLEM IX.

*To find the centre of a given circle, or one already deſcribed.**



1. Draw any chord A B, and biſect it with the perpendicular C D.

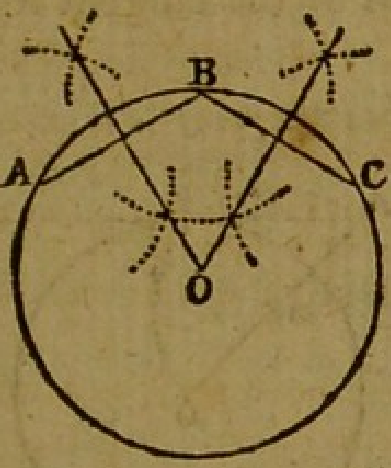
2. Biſect C D, in like manner, with the chord E F, and their interſection O, will be the centre required.

* The centre of a given circle, or any arc of it, may alſo be found as in the next problem, by taking three points in the circumference.

PRO-

PROBLEM X.

To describe the circumference of a circle through three given points, A, B, C.



1. From the middle point B, draw the lines, or chords, B A and B C.

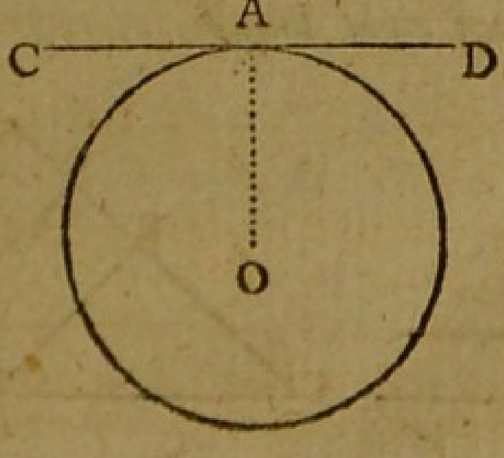
2. Bisect these chords perpendicularly, with lines meeting each other in O.

3. From the point of intersection O, with the distance O A, O B, or O C, describe the circle A B C, and it will be that required.

PROBLEM XI.

To draw a tangent to a given circle, that shall pass through a given point A.

CASE I. *When the point* A *is in the circumference of the circle.*

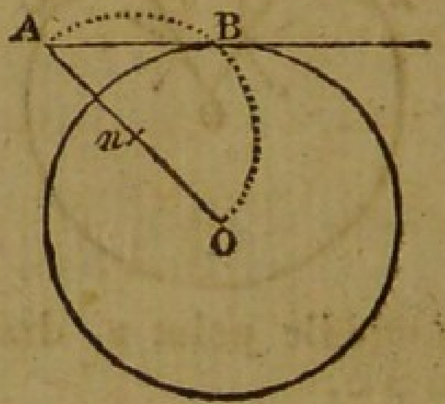


1. From

1. From the given point A, to the centre of the circle, draw the radius O A.

2. Through the point A, draw C D perpendicular to O A, and it will be the tangent required.

CASE II. *When the point A is without the circle.*



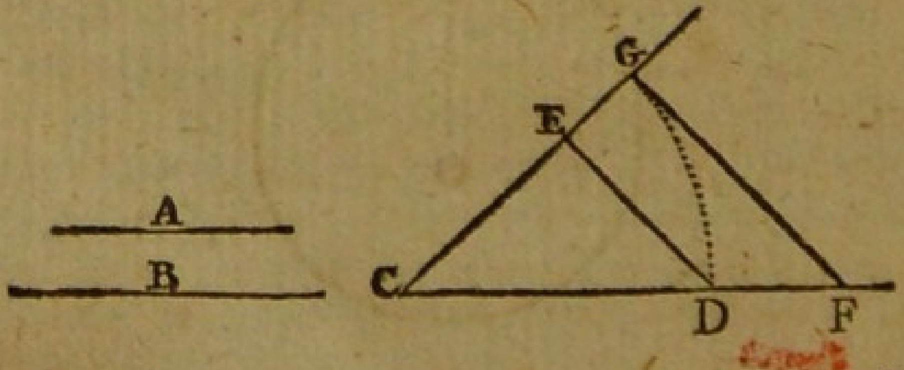
1. To the point A, from the centre O, draw the line O A, and biſect it in *n*.

2. From the point *n*, with the radius *n* A, or *n* O, deſcribe the ſemi-circle A B O, cutting the given circle in B.

3. Through the points A, B, draw the line A B, and it will be the tangent required.

PROBLEM XII.

To two given right lines A, B, to find a third proportional.



1. From

1. From the point C draw two right lines, making any angle F C G.

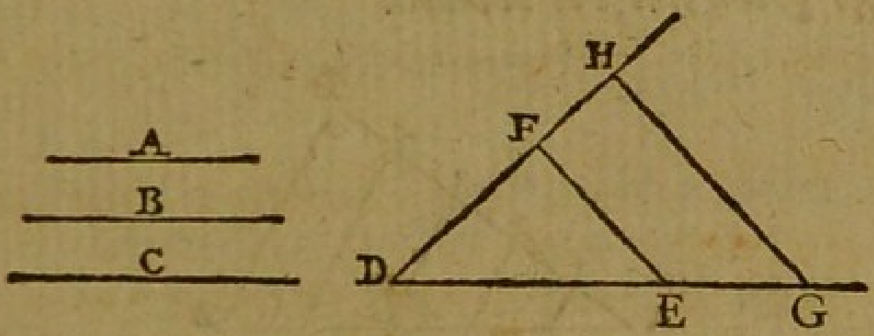
2. In these lines take C E equal to the first term A, and C G, C D, each equal to the second term B.

3. Join E D, and draw G F parallel to it; and C F will be the third proportional required.

That is C E (A) : C G (B) :: C D (B) : C F.

PROBLEM XIII.

To three given right lines A, B, C, to find a fourth proportional.



1. From the point D draw two right lines, making any angle G D H.

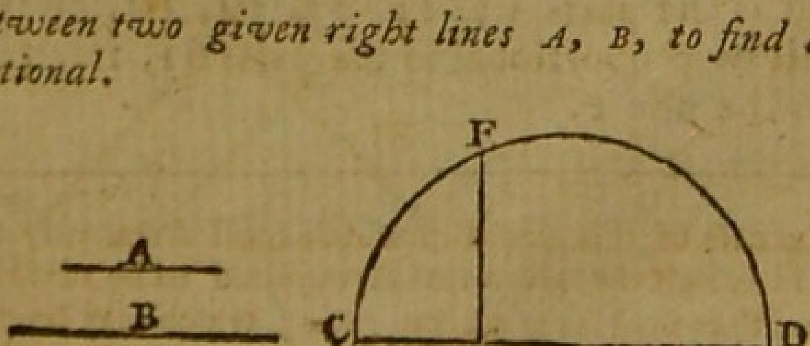
2. In these lines take D F equal to the first term A, D E equal to the second B, and D H equal to the third C.

3. Join F E, and draw H G parallel to it, and D G will be the fourth proportional required.

That is D F (A) : D E (B) :: D H (C) : D G.

PROBLEM XIV.

Between two given right lines A, B, to find a mean proportional.



1. Draw

1. Draw any right line, in which take C E equal to A, and E D equal to B.

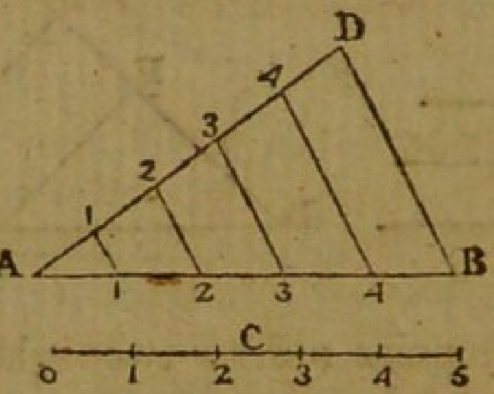
2. Bisect C D, in O, and with O D or O C, as radius, describe the semi circle C F D.

3. From the point E draw E F perpendicular to C D, and it will be the mean proportional required.

That is C E (A) : E F :: E F : E D (B).

PROBLEM XV.

*To divide a given line A B in the same proportion that another given line C is divided.**



1. From the point A draw A D equal to the given line C, and making any angle with A B.

2. To A D apply the several divisions of C, and join D B.

3. Draw the lines 4 4, 3 3, &c. each parallel to D B, and the line A B will be divided as was required.

That is the parts A 1, 1 2, 2 3, 3 4, 4 B, on the line A B, will be proportional to the parts O 1, 1 2, 2 3, 3 4, 4 5, on the line C.

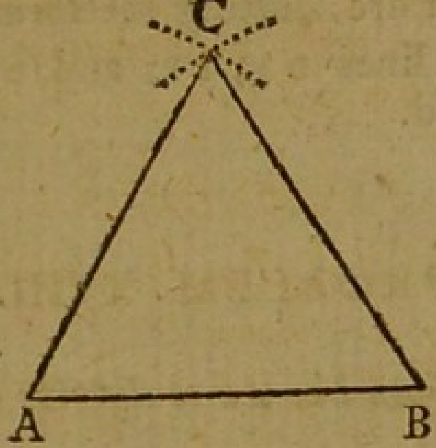
---

* The case of this problem which most frequently occurs, is that in which the given line is required to be divided into two parts that shall have a given ratio; which may be done in nearly the same manner as above.

PRO-

PROBLEM XVI.

Upon a given right line A B, *to make an equilateral triangle.*



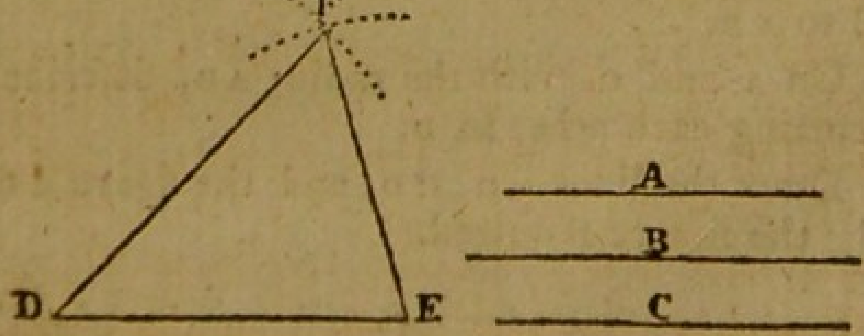
1. From the points A and B, with a radius equal to A B, defcribe arcs cutting in C.

2. Draw the lines A C, B C, and the figure A C B will be the triangle required.

Note. An ifofceles triangle may be formed in the fame manner, by taking any diftance for radius.

PROBLEM XVII.

To make a triangle whofe three fides fhall be refpectively equal to three given lines A, B, C.*



* The three given lines muft be each of fuch a length that any two of them taken together fhall be greater than the third.

1. Draw

1. Draw a line DE equal to one of the given lines C.

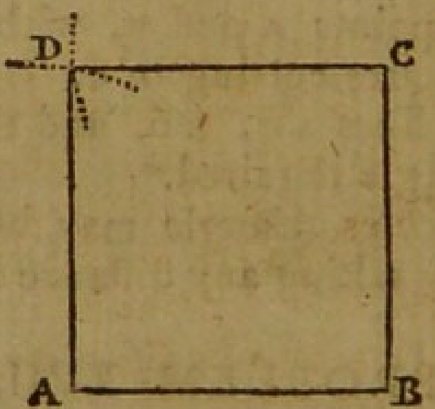
2. On the point D, with a radius equal to B, deſcribe an arc.

3. And on the point E, with a radius equal to A, deſcribe another arc, cutting the former in F.

4. Draw the lines DF, EF, and DFE will be the triangle required.

PROBLEM XVIII.

Upon a given line AB to deſcribe a ſquare.



1. From the point B, draw BC perpendicular, and equal to AB.

2. On A and C, with the radius AB, deſcribe two arcs cutting each other in D.

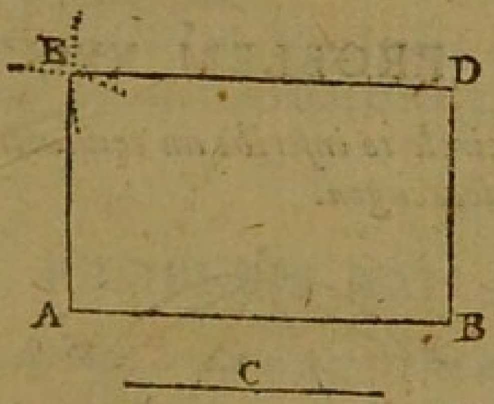
3. Draw the lines AD, CD, and the figure ABCD will be the ſquare required.

Note. A rhombus may be made on the given line AB in exactly the ſame manner, if BC be drawn with the proper obliquity, inſtead of perpendicularly.

PRO-

PROBLEM XIX.

To describe a rectangle, whose length and breadth shall be equal to two given lines A B *and* C.



1. At the point B, in the given line A B, erect the perpendicular B D, and make it equal to C.

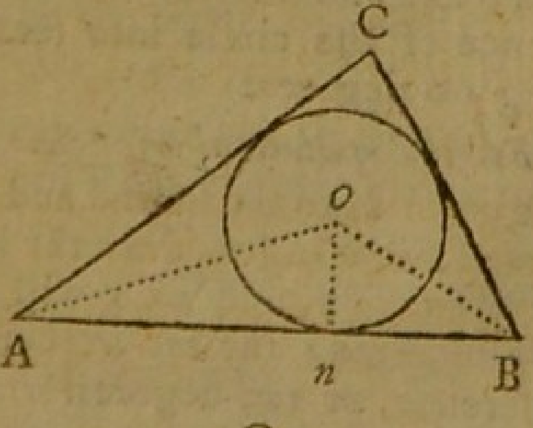
2. From the points D, A, with the radii A B and C, describe two arcs cutting each other in E.

3. Join E A and E D, and A B D E will be the rectangle required.

Note. A parallelogram may be described in nearly the same manner.

PROBLEM XX.

To a given triangle A B C *to inscribe a circle.*



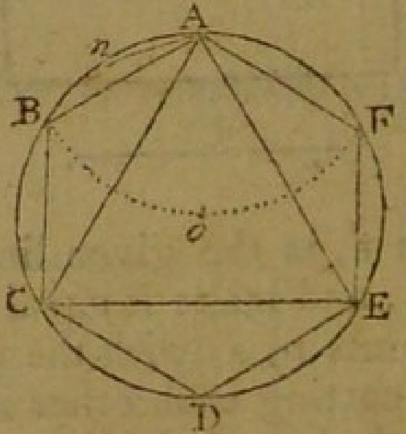
C 1. Bisect

1. Bisect the angles A and B with the lines AO and BO.

2. From the point of intersection O let fall the perpendicular O*n*, and it will be the radius of the circle required.

PROBLEM XXI.

In a given circle to inscribe an equilateral triangle, an hexagon, or a dodecagon.



For the hexagon.

1. From any point A as a centre, with a distance equal to the radius A*o*, describe the arc B*o*F.

2. Join the points AB, or AF, and either of these lines being carried six times round the circle will form the hexagon required.

That is, the radius of the circle is equal to the side of the hexagon; and the sides of the hexagon divide the circumference of the circle into six equal parts, each containing 60 degrees.

For the equilateral triangle.

1. From the point A, to the second and fourth divisions, or angles of the hexagon, draw the lines AC, AE.

2. Join the points CE, and ACE will be the equilateral triangle required; the arc AC being one third of the circumference, or 120 degrees.

For

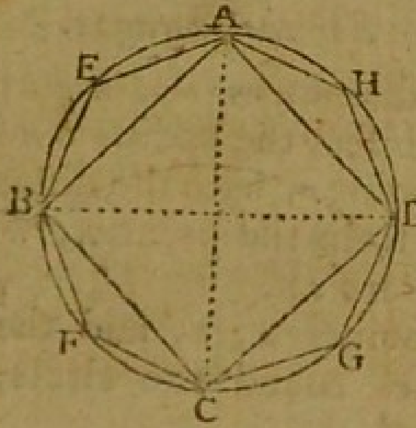
For the dodecagon.

Bisect the arc A B of the hexagon in the point *n*, and the line A *n* being carried twelve times round the circumference, will form the dodecagon required, the arc A *n* being 30 degrees.

If A *n* be again bisected, a polygon may be formed of 24 sides; and by another bisection a polygon of 48 sides; and so on.

PROBLEM XXII.

To describe a square, or an octagon, in a given circle.



For the square.

1. Draw the diameters B D and A C, intersecting each other at right angles.

2. Join the points A B, B C, C D, and D A, and A B C D will be the square required.

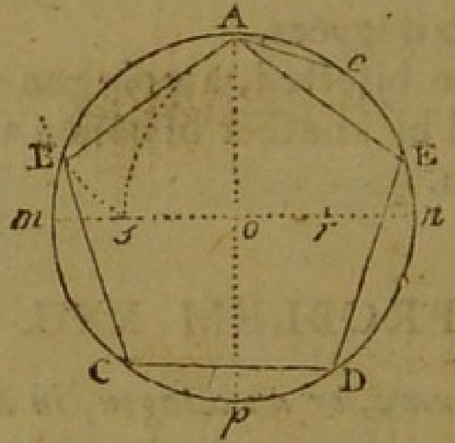
For the octagon.

Bisect the arc A B of the square in the point E, and the line A E being carried eight times round the circumference, will form the octagon.

If the arc A C be again bisected, a polygon may be formed of 16 sides; and by another bisection a polygon of 32 sides; and so on.

PROBLEM XXIII.*

To inscribe a pentagon, or decagon, in a given circle.



For the pentagon.

1. Draw the diameters A *p*, *nm* at right angles to each other, and bisect the radius O *n* in *r*.

2. From the point *r*, with the distance *r*A, describe the arc A *s*, and from the point A, with the distance A *s*, describe the arc *s* B.

3. Join the points A, B, and the line A B being carried five times round the circle, will form the pentagon required.

For the decagon.

Bisect the arc A E of the pentagon in *c*, and the line A *c* being carried ten times round the circumference will form the decagon required.

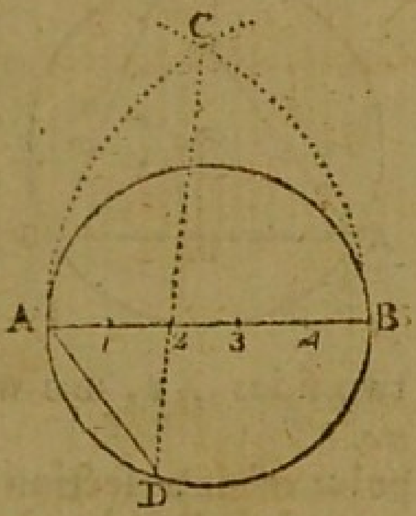
If the arc A *c* be again bisected, a polygon of 20 sides may be formed; and by another bisection, a polygon of 40 sides; and so on.

* Besides the figures here constructed, and those arising from thence by continual bisections, or taking the differences, no other regular polygon can be described, from any known method *purely geometrical.*

PRO-

PROBLEM XXIV.*

In a given circle to inſcribe any regular polygon.



1. Draw the diameter A B, which divide into as many equal parts as the figure has ſides.

2. From the points A B, as centres, with the radius A B, deſcribe arcs croſſing each other in C.

3. From the point C, through the ſecond diviſion of the diameters, draw the line C D.

4. Join the points A, D, and the line A D will be the ſide of the polygon required.

Note. In this conſtruction A D is the ſide of a pentagon.

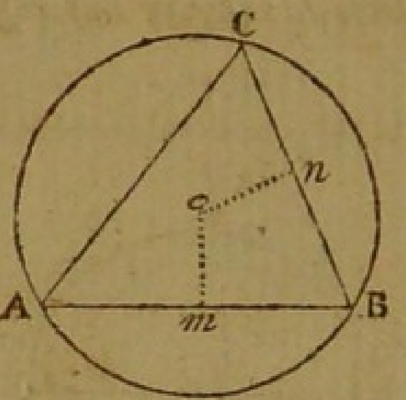
Another method, ſomething more accurate, is by erecting a perpendicular from the centre, of ſuch a length that the part without the circle ſhall be equal to $\frac{1}{4}$ of that within, and drawing a line from its extremity through the ſecond diviſion as before.

---

* This conſtruction is the invention of *Renaldinus*, and was firſt given in his 2d Book *De Reſol. &c. Comp. Mathem.* page 367. The rule for polygons in general is only an approximation, but holds true in the equilateral triangle and hexagon.

PROBLEM XXV.

About a given triangle ABC *to circumscribe a circle.*



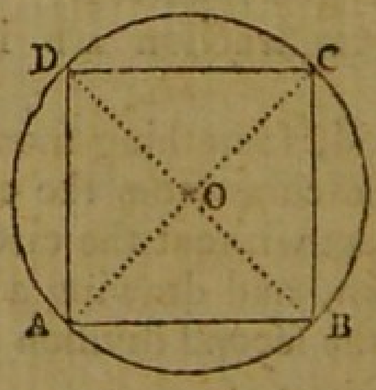
1. Bisect the two sides AB, BC with the perpendiculars *mo* and *no*.

2. From the point of intersection *o*, with the distance OA, OB or OC, describe the circle ACB, and it will be that required.

If any two of the angles be bisected, instead of the sides, the intersection of the lines will also give the centre of the circle.

PROBLEM XXVI.

About a given square ABCD *to circumscribe a circle.*



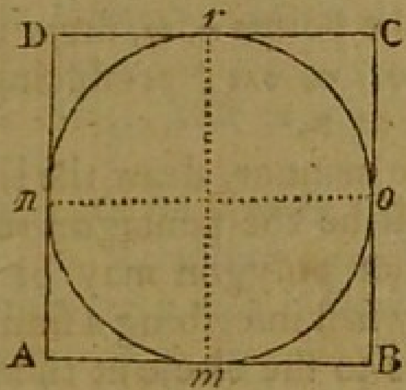
1. Draw the two diagonals AC and BD, intersecting each other in O.

2. From the point O, with the distance OA, OB, OC, or OD, describe the circle ABCD, and it will be that required.

PRO-

PROBLEM XXVII.

To circumscribe a square about a given circle.

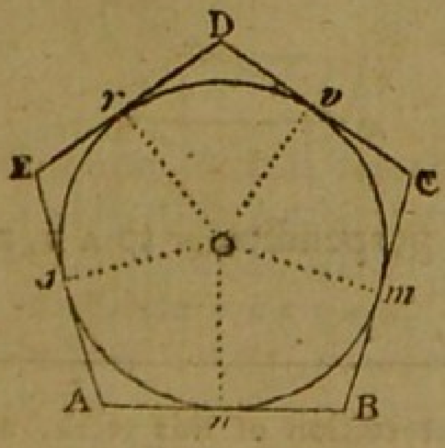


1. Draw any two diameters *no* and *rm* at right angles to each other.

2. Through the points *m o r n*, draw the lines A B, B C, C D, and D A, perpendicular to *r m* and *n o*, and A B C D will be the ſquare required.*

PROBLEM XXVIII.

About a given circle to circumscribe a pentagon.



---

* If each of the quadrants *r n*, *m n*, *m o* and *o r* be biſected, and tangents be drawn to thoſe points, the circumſcribing figure will be an octagon.

1. Infcribe a pentagon in the circle; or, which is the fame thing, find the points *n, m, v, r, s*, as in Problem XXIII.

2. From the centre *o*, to each of thefe points, draw the radii *on, om, ov, or* and *os*.

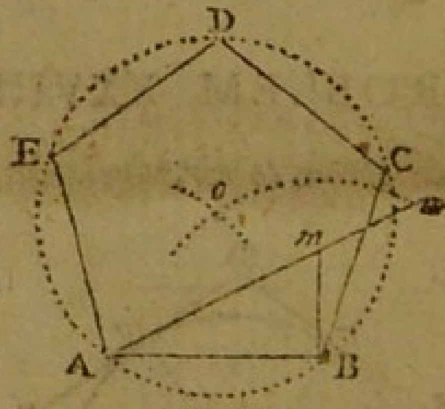
3. Through the points *n, m*, draw the lines A B, B C perpendicular to *on, om*; producing them till they meet each other at B.

4. In the fame manner, draw the lines C D, D E, E A, and A B C D E will be the pentagon required.

Note. Any other polygon may be made to circumfcribe a circle, by firft infcribing a fimilar one, and then drawing tangents to the circle at the angular points.

PROBLEM XXIX.*

On a given line A B *to make a regular pentagon.*



1. Make B *m* perpendicular to A B, and equal to one half of it.

---

* In the former edition of this work, another method of defcribing a pentagon was given, as firft propofed by *Albertus Durer*, in his Geometry, p. 55, printed 1532; but as that is only an approximation, and is not more eafy in practice than the prefent one, which is perfectly accurate, it is here omitted.

2. Draw

2. Draw A *m*, and produce it till the part *mn* is equal to B *m*.

3. From A and B as centres, with the radius B *n*, deſcribe arcs cutting each other in *o*.

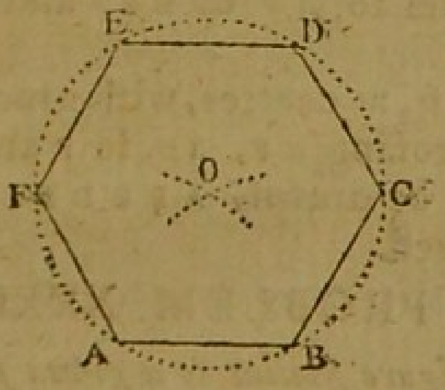
4. And from the point *o*, with the ſame radius, or with *o* A, or *o* B, deſcribe the circle A B C D E.

5. Apply the line A B five times round the circumference of this circle, and it will form the pentagon required.

Note. If tangents be drawn through the angular points A, B, C, D, E, a pentagon circumſcribing the circle will be formed; and if the arcs be biſected, a circumſcribing decagon may be formed.

PROBLEM XXX.

On a given line A B to make a regular hexagon.



1. From the points A, B as centres, with the radius A B, deſcribe arcs cutting each other in *o*.

2. And from the point O, with the diſtance O A or O B, deſcribe the circle A B C D E F.

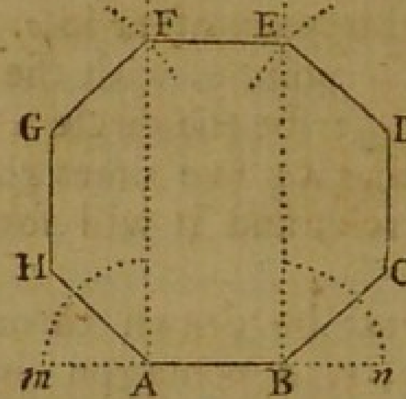
3. Apply the line A B ſix times round the circumference, and it will form the hexagon required.*

---

* This conſtruction is founded on the principle, that the radius of every circle is equal to the ſide of its inſcribed hexagon, or the chord of 60°.

PROBLEM XXXI.

On a given line A B *to form a regular octagon.*



1. On the extremes of the given line A B erect the indefinite perpendiculars A F and B E.

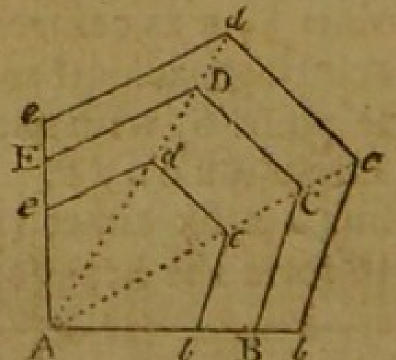
2. Produce A B both ways to *m* and *n*, and bisect the angles, *m* A F and *n* B E with the lines A H and B C.

3. Make A H and B C each equal to A B, and draw H G, C D parallel to A F or B E, and also each equal to A B.

4. From G, D, as centres, with a radius equal to A B, describe arcs crossing A F, B E, in F and E; and if G F, F E, and E D, be drawn, A B C D E F G H will be the octagon required.

PROBLEM XXXII.

To make a figure similar to a given figure A B C D E.



1. Take A *b* equal to the side of the figure required, and from the angle A draw the diagonals, A C, A D.

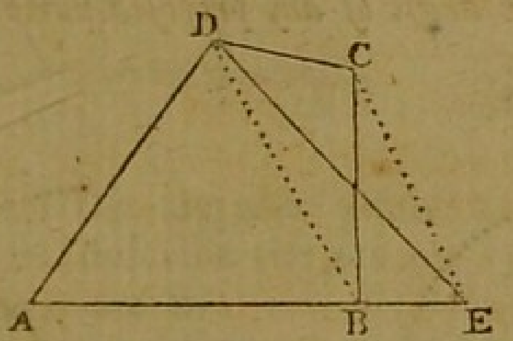
 2. From

2. From the points *b, c, d* draw *bc, cd, de* parallel to BC, CD, DE, and A*bcde* will be ſimilar to ABCDE.

The ſame thing may alſo be done by making the angles *b, c, d, e* reſpectively equal to the angles B, C, D, E.

PROBLEM XXXIII.

To make a triangle equal to a given trapezium ABCD.

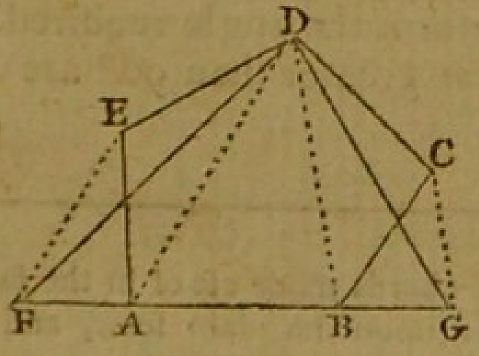


1. Draw the diagonal DB, and make CE parallel to it, meeting the ſide AB produced in E.

2. Join the points D, E, and ADE will be the triangle required.

PROBLEM XXXIV.

To make a triangle equal to any right lined figure ABCDEA.



1. Produce the ſide AB both ways at pleaſure.

2. Draw the diagonals DA, DB, and parallel to them the lines EF and CG.

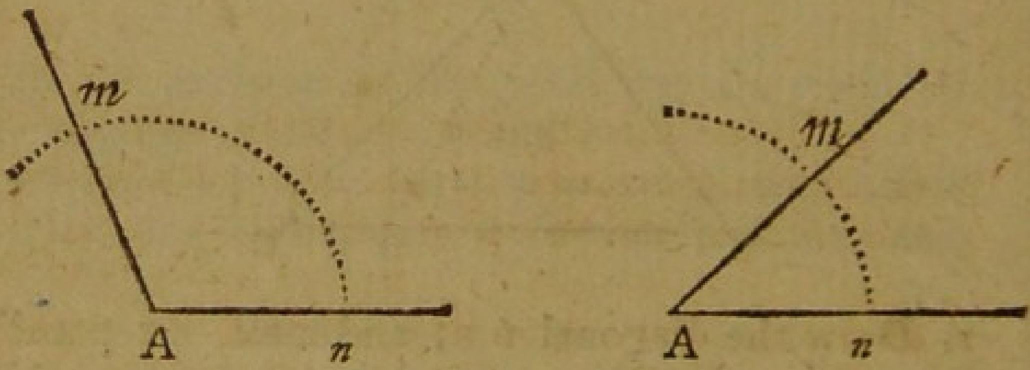
 3. Join

3. Join the points D F, D G, and D F G will be the triangle required.

And in nearly the ſame manner may any right lined figure whatever be reduced to a triangle.

PROBLEM XXXV.*

To make an angle of any propoſed number of degrees.



1. Take the firſt 60 degrees from the ſcale of chords, and from the point A, with this radius, deſcribe the arc *n m*.

2. Take the chord of the propoſed number of degrees from the ſame ſcale, and apply it from *n* to *m*.

3. From the point A draw the lines A *n* and A *m*, and they will form the angle required.

Note. Angles greater than 90° are uſually taken at twice.

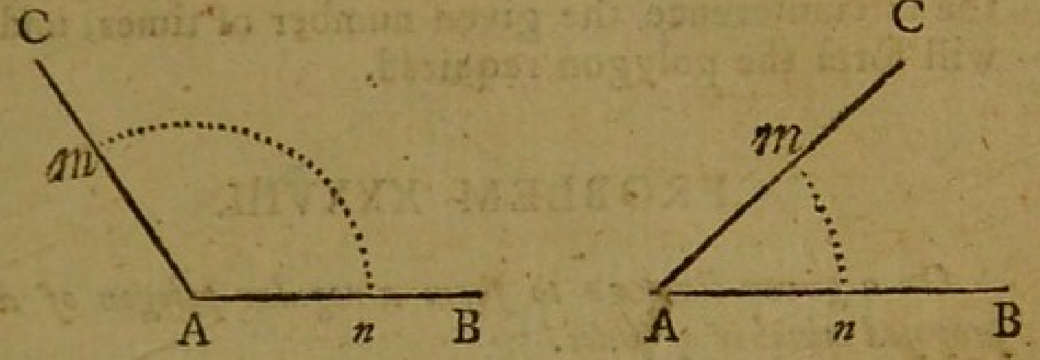
---

* The line of chords made uſe of in the following problems, is commonly put upon the plain ſcale, and is adapted to 90 degrees, or the fourth part of a circle.

For a deſcription of this, and other inſtruments made uſe of in Practical Geometry, ſee Mr. *Robertſon's Treatiſe on ſuch mathematical inſtruments as are uſually put into a portable caſe.*

PROBLEM XXXVI.

Any angle B A C *being given, to find the number of degrees it contains.**



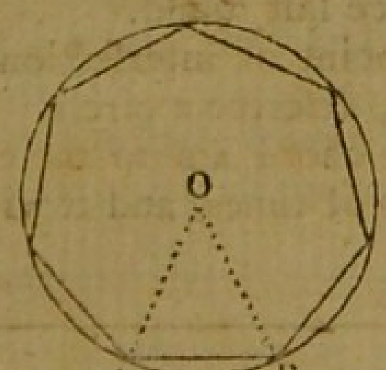
1. From the angular point A, with the chord of 60 degrees, defcribe the arc *n m*, cutting the legs in the points *n* and *m*.

2. Take the diftance *n m*, and apply it to the fcale of chords, and it will fhew the degrees required.

Note. When the diftance *n m* is greater than 90°, it muft be taken at twice, as before.

PROBLEM XXXVII.

In a given circle to infcribe a polygon of any propofed number of fides.



* Both this and the laft problem may be performed by means of a *protractor*, which is a graduated arc defigned for that purpofe.

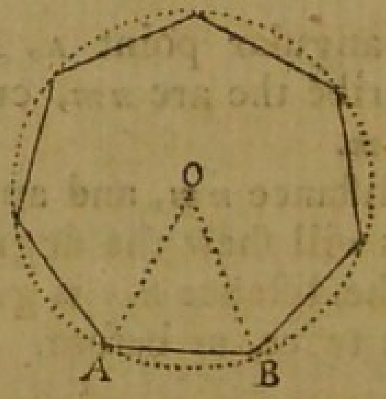
1. Divide

1. Divide 360° by the number of ſides, and make an angle A O B, at the centre, whoſe meaſure ſhall be equal to the degrees in the quotient.

2. Join the points A B, and apply the chord A B to the circumference the given number of times, and it will form the polygon required.

PROBLEM XXXVIII.

On a given line A B to form a regular polygon of any propoſed number of ſides.



1. Divide 360° by the number of ſides, and ſubtract the quotient from 180 degrees.

2. Make the angles A B O and B A O each equal to half the difference laſt found.

3. From the point of interſection O, with the diſtance O A or O B, deſcribe a circle.

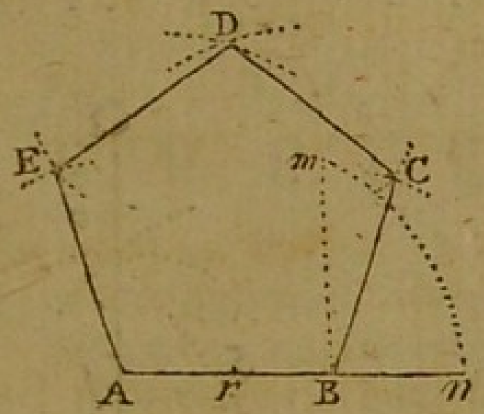
4. Apply the chord A B to the circumference the propoſed number of times, and it will form the polygon required.*

---

* By this method the circumference of a circle may alſo be divided into any number of equal parts; for if 360° be divided by the number of parts, and the angle A O B be made equal to the degrees in the quotient, the arc A B will be one of the equal parts required.

PROBLEM XXXIX.*

Upon a given right line A B *to describe a regular pentagon.*



1. Produce A B towards *n*, and at the point B make the perpendicular B *m* equal to A B.

2. Bisect A B in *r*, and from *r* as a centre, with the radius *r m*, describe the arc *m n*, cutting A B in *n*.

3. From the points A and B, with the radius A *n*, describe arcs cutting each other in D.

4. And from the points A, D and B, D, with the radius A B, describe arcs cutting each other in C and E.

5. Join E C, D C, D E and E A, and A B C D E will be the pentagon required.

This method differs but little from that of Problem XXIX, and is equally easy and convenient in practice.

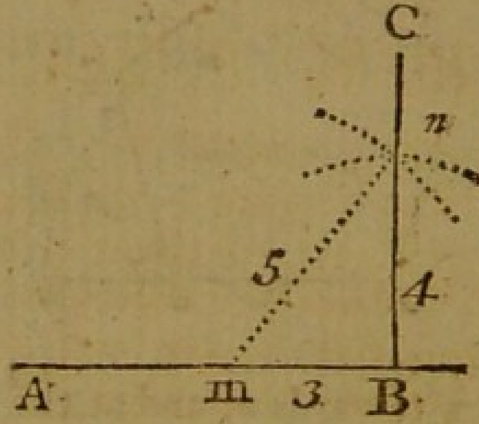
---

* This and the following problem were not given in the first Edition of this work, but are now added on account of their elegance and utility. The second is derived from the 47th Prop. B. I. Euclid's Elements, and the first is proposed for a demonstration in the Ladies Diary for the year 1786.

PRO-

PROBLEM XL.

To raiſe a perpendicular from any point B *in a given line* A B.



1. From any ſcale of equal parts take a diſtance equal to 3 diviſions, and ſet it from B to *m*.

2. And from the points B and *m*, with the diſtances 4 and 5, taken from the ſame ſcale, deſcribe arcs cutting each other in *n*.

3. Through the points *n*, B, draw the line B C, and it will be the perpendicular required.

Explanation of the characters made uſe of in the following part of the Work.

+	Is the ſign of addition.
—	———— of ſubtraction.
×	———— of multiplication.
÷	———— of diviſion.
√	———— of the ſquare root.
³√	———— of the cube root.
=	———— of equality.
: :: :	———— of proportion.

OF

OF THE MENSURATION OF SUPERFICIES.

THE *area* of any figure is the meaſure of its ſurface, or the ſpace contained within the bounds of that ſurface, without any regard to thickneſs.

A ſquare whoſe ſide is one inch, one foot, or one yard, &c. is called the *meaſuring unit*, and the area or content of any figure is computed by the number of thoſe ſquares contained in that figure.

PROBLEM I.

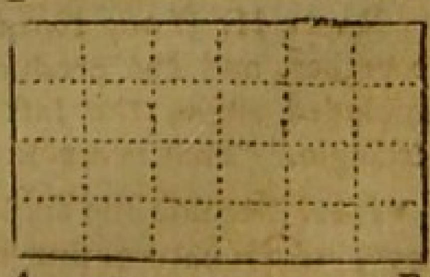
To find the area of a parallelogram; whether it be a ſquare, a rectangle, a rhombus, or a rhomboides.

RULE.*

Multiply the length by the perpendicular height, and the product will be the area.

EXAM-

D C

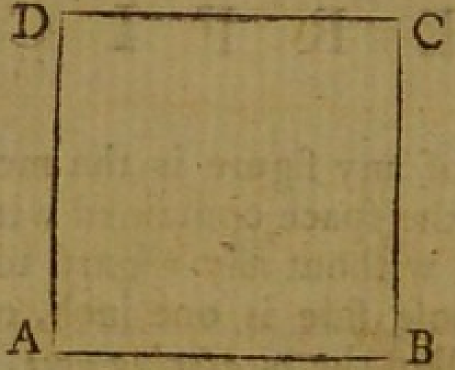


A B

* Take any rectangle A B C D, and divide each of its ſides, reſpectively, into as many equal parts as is expreſſed by the number of times they contain the linear meaſuring unit, and let all the oppoſite points of diviſion be connected by right lines. Then, it is evident, that theſe lines divide the rectangle

EXAMPLES.

1. Required the area of the ſquare ABCD whoſe ſide is 5 feet 9 inches.



Here 5 *fe.* 9 *in.* = 5.75; *and* $\overline{5.75}^2 = 5.75 \times 5.75 =$ 33.0625 *feet* = 33 *fe.* 0 *in.* 9 *pa.* = *area required.*

---

rectangle into a number of ſquares each equal to the ſuperficial meaſuring unit, and that the number of theſe ſquares, or the area of the figure, is equal to the number of linear meaſuring units in the length, as often repeated as there are linear meaſuring units in the breadth or height, that is equal to the length multiplied by the height, *which is the rule.*

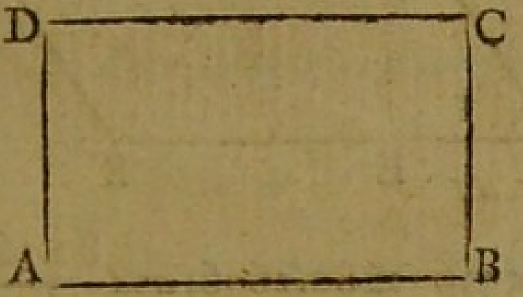
And ſince a rectangle is equal to an oblique parallelogram ſtanding upon the ſame baſe, and between the ſame parallels, (Euc. I. 35,) the rule is true for any parallelogram in general. Q. E. D.

Rule II. If any two ſides of a parallelogram be multiplied together, and the product again by the natural ſine of their included angle, the laſt product will give the area of the triangle. That is AB × BC × nat. ſine of the angle B = area.

Note. Becauſe the angles of a ſquare and rectangle are each 90°, whoſe natural ſine is unity, or 1, the rule in this caſe is the ſame as that given in the text.

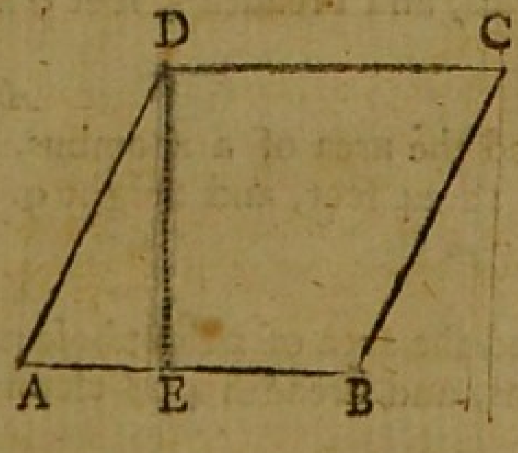
2. Re-

2. Required the area of the rectangle ABCD, whoſe length AB is 13.75 chains, and breadth BC 9.5 chains.



Here 13.75 × 9.5 = 130.625; *and* $\frac{130.625}{10}$ = 13.0625 *acres* = 13 *ac.* 0 *ro.* 10 *po.* = *area required.*

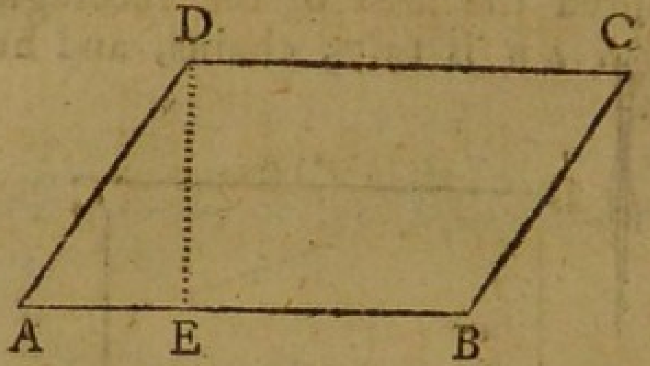
3. Required the area of the rhombus ABCD, whoſe length AB is 12 *fe.* 6 *in.* and its height DE 9 *fe.* 3 *in.*



Here 12 *fe.* 6 *in.* = 12.5, *and* 9 *fe.* 3 *in.* = 9.25;
Whence 12.5 × 9.25 = 115.625 *feet.* = 115 *fe.* 7 *in.* 6 *pa.* = *area required.*

4. What is the area of the rhomboides ABCD, whoſe length AB is 10.52 chains, and height DE 7.63 chains?

Here



Here $10.52 \times 7.63 = 80.2676$; *and* $\frac{80.2676}{10} = 80.2676$ *acres* = 8 *ac.* 0 *ro.* 4 *po.* = *area required.*

5. What is the area of a ſquare whoſe ſide is 35.25 chains?

ac. ro. po.
Anſ. 124 1 1

6. What is the area of a ſquare whoſe ſide is 8 feet 4 inches?

fe. in. pa.
Anſ. 69 5 4

7. What is the area of a rectangle whoſe length is 14 feet 6 inches, and breadth 4 feet 9 inches?

fe. in. pa.
Anſ. 68 10 6

8. Required the area of a rhombus, the length of whoſe ſide is 12.24 feet, and height 9.16 feet.

fe. in. pa.
Anſ. 112 1 5

9. Required the area of a rhomboides whoſe length is 10.51 chains, and breadth 4.28 chains.

ac. ro. p.
Anſ. 4 1 39

10. What is the area of a rhomboides whoſe length is 7 feet 9 inches, and height 3 feet 6 inches?

fe. in. pa.
Anſ. 17 1 6

11. To find the area of a rectangular board, whoſe length is $12\frac{1}{2}$ feet, and breadth 9 inches.

Anſ. $9\frac{3}{8}$ *feet.*

PRO-

PROBLEM II.

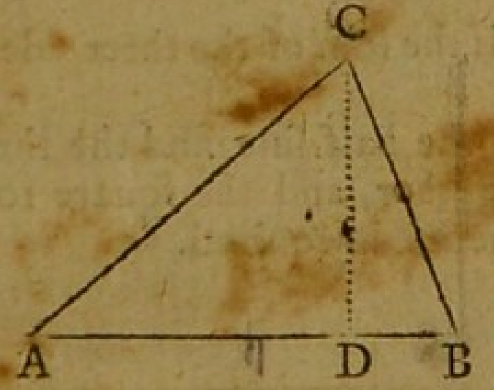
To find the area of a triangle.

RULE.*

Multiply the base by the perpendicular height, and half the product will be the area.

EXAMPLES.

1. Required the area of the triangle A B C, whose base A B is 10 feet 9 inches, and height D C 7 feet 3 inches.



Here 10 *fe.* 9 *in.* $=10.75$, *and* 7 *fe.* 3 *in.* $=7.25$;

Whence $10.75 \times 7.25 = 77.9375$, *and* $\frac{77.9375}{2} =$

38.96875 *feet* = 38 *fe.* 11 *in* $7\frac{1}{2}$ *pa.* = *area required.*

2. What is the area of a triangle whose base is 18 feet 4 inches, and height 11 feet 10 inches?

fe. in. pa.
Ans. 108 5 8

---

* A triangle is half a parallelogram of the same base and altitude, (Euc. I. 41.) and therefore the truth of the rule is evident.

3. What

3. What is the area of a triangle whose base is 16.75 feet, and height 6.24 feet? *fe. in. pa.*
Ans. 52 3 1

4. Required the area of a triangle whose base is 12.25 chains, and height 8.5 chains. *ac. ro. po.*
Ans. 5 0 33

5. What is the area of a triangle whose base is 20 feet, and height 10.25? *Ans.* 102.5 *fe.*

PROBLEM III.

*To find the area of a triangle whose three sides only are given.**

RULE.

1. From half the sum of the three sides subtract each side severally.

2. Multiply the half sum and the three remainders continually together, and the square root of the product will be the area required.

EXAM-

---

* *Demon.* Let AC $= a$, AB $= b$, BC $= c$, and AD $= x$: (See preceding fig.) Then, since BD $= b - x$, we shall have $c^2 - (b-x)^2 =$ CD$^2 = a^2 - x^2$, or $c^2 - b^2 + 2bx - x^2 = a^2 - x^2$ from which x is found $= \frac{a^2 + b^2 - c^2}{2b}$ by transf. and reduction.

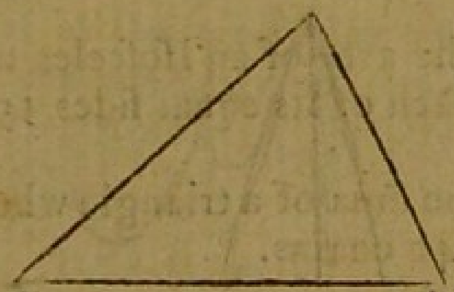
But CD$^2 =$ AC$^2 -$ AD$^2 = \overline{AC + AD} \times \overline{AC - AD} =$ $(a + \frac{a^2 + b^2 - c^2}{2b}) \times (a - \frac{a^2 + b^2 - c^2}{2b}) = \frac{2ab + a^2 + b^2 - c^2}{2b}$ $\times \frac{2ab - a^2 - b^2 + c^2}{2b} = \frac{(a+b)^2 - c^2}{2b} \times \frac{c^2 - (a-b)^2}{2b}$;

Whence CD $= \frac{1}{2b} \sqrt{(\overline{a + b}^2 - c^2) \times (c^2 - \overline{a - b}^2,)}$ and

the

EXAMPLES.

1. Required the area of the triangle A B C, whose three sides B C, C A and A B, are 24, 36, and 48 chains respectively.



Here $\frac{24+36+48}{2} = \frac{108}{2} = 54 = \frac{1}{2}$ *sum of the sides* ;

Also $54-24=30$ *first diff.* $54-36=18$ *second diff.* *and* $54-48=6$ *third diff.*

Whence $\sqrt{54\times30\times18\times0} = \sqrt{174960} = 418.043 =$ *area required.*

2. Re-

---

the area $\frac{1}{2}$ AB × CD $= \frac{1}{4}\sqrt{:(\overline{a+b}^2-c^2)\times(c^2-\overline{a-b}^2)}$ $= \frac{1}{4}\sqrt{(a+b+c\times a+b-c\times c+a-b\times c-a+b)}$ $= \sqrt{(\frac{a+b+c}{2}\times\frac{a+b-c}{2}\times\frac{c+a-b}{2}\times\frac{c-a+b}{2},)}$ which, by making $s = \frac{1}{2}\times(a+b+c)$ becomes $= \sqrt{(s\times \overline{s-c}\times\overline{s-b}\times\overline{s-a})} =$ algebraical expression for the rule, as was to be demonstrated.

Cor. 1. If s be put equal to $a+b$ and $d = b \backsim c$, the rule is $\sqrt{(s^2-a^2)\times(a^2-d^2.)}$

Cor.

2. Required the area of a triangle whose three sides are 13, 14 and 15 feet. *Ans.* 84 *square feet.*

3. How many acres are there in a triangle whose three sides are 49.00, 50.25 and 25.69 chains?

Ans. 61.498 *ac.*

4. Required the area of a right angled triangle, whose hypothenuse is 50, and the other two sides 30 and 40. *Ans.* 600.

5. Required the area of an equilateral triangle whose side is 25. *Ans.* 270.625.

6. Required the area of an Isosceles triangle whose base is 20, and each of its equal sides 15.

Ans. 111.803.

7. Required the area of a triangle whose three sides are 20, 30, and 40 chains.

Ans. 29 *ac.* 7 *po.*

PROBLEM IV.

Any two sides of a right angled triangle being given to find the third side.

Cor. 2. If all the sides be equal, the rule will become $\frac{1}{4} a^2 \sqrt{3}$, or $\frac{1}{4} a^2 \times 1.732$ for the equilateral triangle whose side is a.

Cor. 3. If the triangle be right angled, a being the hypothenuse, the rule will be $\frac{a+b+c}{2} \times - \frac{a+b+c}{2}$, or $\frac{1}{2} p \times \overline{\frac{1}{2} p - a}$, putting p for the perimeter.

RULE II. Any two sides of a triangle being multiplied together, and the product again by half the natural sine of their included angle, will give the area of the triangle.

That is, AC × CB × nat. sine of the angle C = twice area.

RULE.

RULE.*

1. *When the two legs are given, to find the hypothenuse.*

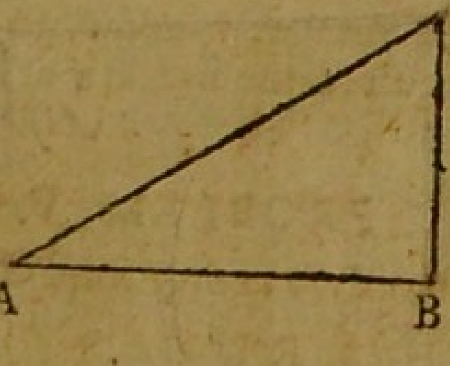
Add the ſquare of one of the legs to the ſquare of the other, and the ſquare root of the ſum will be equal to the hypothenuſe.

2. *When the hypothenuſe and one of the legs are given, to find the other leg.*

From the ſquare of the hypothenuſe take the ſquare of the given leg, and the ſquare root of the remainder will be equal to the other leg.

EXAMPLES.

1. In the right angled triangle ABC, the baſe AB is 56, and the perpendicular BC 33: what is the hypothenuſe?



Here $56^2 + 33^2 = 3136 + 1089 = 4225$; *and* $\sqrt{4225} = 65 =$ *hypothenuſe* AC.

2. If the hypothenuſe AC be 53, and the baſe AB 45: what is the perpendicular BC?

---

* By Euc. 47. I. $\overline{AB^2 + BC^2} = AC^2$, or $\overline{AC^2 - AB^2} = BC$; and therefore $\sqrt{AB^2 + BC^2} = AC$, or $\sqrt{AC^2 - AB^2} = BC$, or $\sqrt{AC^2 - BC^2} = AB$ which *is the ſame as the rule.*

Here $53^2-45^2=2809-2025=784$; *and* $\sqrt{784}=28=$*perpendicular* BC.

3. The baſe of a right angled triangle is 77, and the perpendicular 36: what is the hypothenuſe?

Anſ. 85.

4. The hypothenuſe of a right angled triangle is 109, and the perpendicular 60: what is the baſe?

Anſ. 91.

5. It is required to find the length of a ſhoar, which ſtrutting 12 feet from the upright of a building, will ſupport a jaumb 20 feet from the ground.

Anſ. 23.32380 *feet.*

6. The height of a precipice, ſtanding cloſe by the ſide of a river, is 103 feet, and a line of 320 feet will reach from the top of it to the oppoſite bank; required the breadth of the river. *Anſ.* 302.9703 *feet.*

7. A ladder 50 feet long, being placed in a ſtreet, reached a window 28 feet from the ground, on one ſide; and by turning it over, without removing the foot, it reached another window, 36 feet high on the other ſide: required the breadth of the ſtreet.

Anſ. 76.1233335 *feet.*

PROBLEM V.

To find the area of a trapezium.

RULE.*

Multiply the diagonal by the ſum of the two perpendiculars falling upon it from the oppoſite angles, and half the product will be the area.

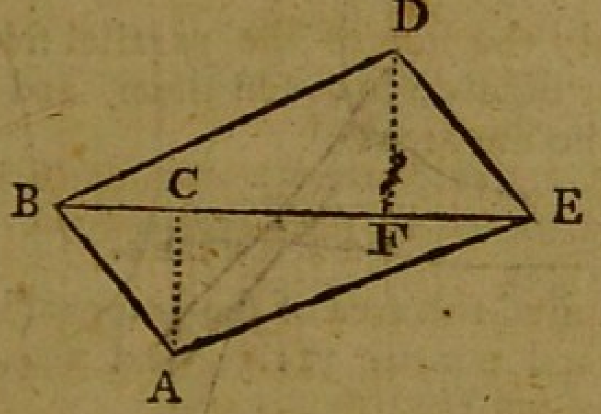
EXAM-

---

* *Demon.* The area of the triangle BDE is $=\frac{BE \times DF}{2}$; and the area of the triangle BAE is $=\frac{BE \times AC}{2}$; and there

EXAMPLES.

1. Required the area of the trapezium BAED, whoſe diagonal BE is 84, the perpendicular AC 21, and DF 28.



Here $\overline{28+21}\times 84 = 49\times 84 = 4116$; *and* $\frac{4116}{2} =$ 2058 *the area required.*

2. Required the area of a trapezium whoſe diagonal is 80.5, and the two perpendiculars 24.5 and 30.1.

Anſ. 2197.65.

3. What is the area of a trapezium whoſe diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches, and 60 feet 9 inches? *Anſ.* 6347 *fe.* 3 *in.*

---

therefore the ſum of theſe areas, or the area of the whole trapezium, is $= \frac{BE\times DF}{2} + \frac{BE\times AC}{2} = \frac{DF+AC}{2}\times BE$. Q.E.D.

If the trapezium can be inſcribed in a circle; that is, if the ſum of two of its oppoſite angles is equal to two right angles, or 180°, the area may be found thus:

Rule. From half the ſum of the four ſides ſubtract each ſide ſeverally; then multiply the four remainders continually together, and the ſquare root of the product will be the area.

PROBLEM VI.

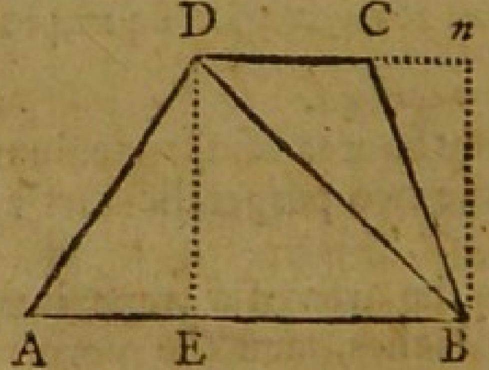
To find the area of a trapezoid, or a quadrangle, two of whose opposite sides are parallel.

RULE.*

Multiply the sum of the parallel sides by the perpendicular distance between them, and half the product will be the area.

EXAMPLES.

1. Required the area of the trapezoid ABCD, whose sides AB and DC are 321.51 and 214.24, and perpendicular DE 171.16.



Here 321.51 + 214.24 = 535.75 = *sum of the parallel sides* AB, DC.

---

* *Demon.* The △ ABD is $= \frac{AB \times DE}{2}$, and the △ BCD $= \frac{DC \times Bn}{2}$, or (because $Bn = DE$) $= \frac{DC \times DE}{2}$. Therefore △ ABD + △ BCD, or the whole trapezoid ABCD, is $= \frac{AB \times DE}{2} + \frac{DC \times DE}{2} = \frac{AB + DC}{2} \times DE$. Q.E.D.

Whence

Whence 535.75×171.16 (*the perp.* DE) $= 91698.9700$;

And $\frac{91698.9700}{2} = 45849.485$ *the area required.*

2. The parallel ſides of a trapezoid are 12.41 and 8.22 chains, and the perpendicular diſtance 5.15 chains; required the area. *ac. ro. po.*
Anſ. 5 1 9

3. Required the area of a trapezoid whoſe parallel ſides are 25 feet 6 inches and 18 feet 9 inches, and the perpendicular diſtance 10 feet 5 inches? *fe. in. pa.*
Anſ. 230 5 7

4. Required the area of a trapezoid whoſe parallel ſides are 20.5 and 12.25, and perpendicular diſtance 10.75. *Anſ.* 176.03125.

PROBLEM VII.

To find the area of a regular polygon.

RULE.*

Multiply half the perimeter of the figure by the perpendicular falling from its centre upon one of the ſides, and the product will be the area.

Note. The perimeter of any figure is the ſum of all its ſides.

EXAM-

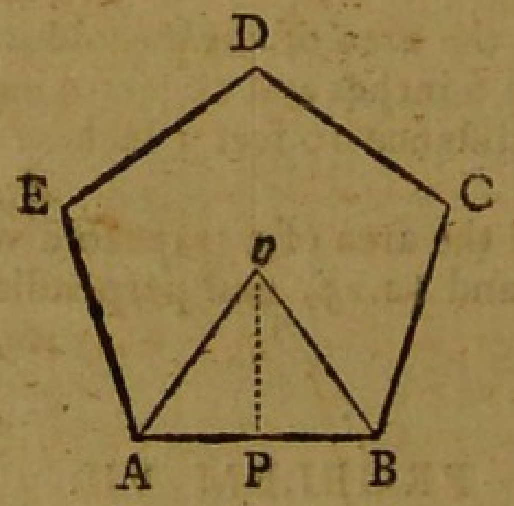
---

* *Demon.* Every regular polygon is compoſed of as many equal triangles as it has ſides, conſequently the area of one of thoſe triangles being multiplied by the number of ſides muſt give the area of the whole figure; but the area of either of the triangles is equal to the rectangle of the perpendicular and half the baſe, and therefore the rectangle of the perpendicular

 and

EXAMPLES.

1. Required the area of the regular pentagon ABCDE whose side AB, or BC, &c. is 25 feet, and perpendicular OP 17.2 feet?



Here $\frac{25 \times 5}{2} = 62.5 =$ *half perimeter*; *and* $62.5 \times 17.2 = 1075$ *square feet* = *area required.*

2. Required the area of a hexagon whose side is 14.6 feet, and perpendicular 12.64 feet?

Ans. 553.632 *square feet.*

3. Required the area of a heptagon whose side is 19.38, and perpendicular from the centre 20?

Ans. 1356.6.

4. Required the area of an octagon whose side is 9.941, and perpendicular 12? *Ans.* 477.168.

---

and half the sum of the sides is equal to the area of the whole polygon: thus, $OP \times \frac{AB}{2}$ is = area of the △ AOB, and $OP \times \frac{5AB}{2}$ = area of the polygon ABCDE. Q.E.D.

PRO-

PROBLEM VIII.

To find the area of a regular polygon, when the ſide only is given.

RULE.*

Multiply the ſquare of the ſide of the polygon, by the number ſtanding oppoſite to its name in the following table, and the product will be the area.

N° of ſides.	Names.	Multipliers.
3	Trigon or equil. △	0.433013—
4	Tetragon or ſquare	1.000000+
5	Pentagon	1.720477+
6	Hexagon	2.598076+
7	Heptagon	3.633912+
8	Octagon	4.828427+
9	Nonagon	6.181824+
10	Decagon	7.694209—
11	Undecagon	9.365640—
12	Duodecagon	11.196152+

EXAM-

---

* *Demon.* The multipliers in the table are the areas of the polygons to which they belong when the ſide is unity or 1.

Whence as all regular polygons, of the ſame number of ſides, are ſimilar to each other, and as ſimilar figures are as the ſquares of their like ſides, (*Euc.* VI. 20.) 1^2 : multiplier in the table :: ſquare of the ſide of any polygon : area of that polygon; or, which is the ſame thing, the ſquare of the ſide of any polygon × by its tabular number is = area of the polygon. Q. E. D.

The table is formed by trigonometry thus: As radius = 1 : tang. ∠ OBP :: BP $(\frac{1}{2})$: PO $= \frac{\text{BP} \times \text{tang.} \angle \text{OBP}}{\text{radius}} = \frac{1}{2}$ tang. ∠ OBP; whence OP × BP = $\frac{1}{4}$ tang. ∠ OBP = area

 of

EXAMPLES.

1. Required the area of a pentagon whoſe ſide is 15.

15
15
———
75
15
———
225 = *ſquare of the ſide.*
1.720477 = *area, when the ſide is* 1.
225
———
8602385
3440954
3440954
———
387.107325 = *area required.*

---

of the △ AOB; and ¼ tang. ∠ OBP × number of ſides = tabular number, or the area of the polygon.

The angle OBP, together with its tangent, for any polygon of not more than 12 ſides, is ſhewn in the following table.

Nº of ſides.	Names.	Angle OBP	Tangents.
3	Trigon	30°	$.57735+ = \frac{1}{3}\sqrt{3}$
4	Tetragon	45°	$1.00000+ = 1 \times 1$
5	Pentagon	54°	$1.37638+ = \sqrt{1+\frac{2}{5}\sqrt{5}}$
6	Hexagon	60°	$1.73205+ = \sqrt{3}$
7	Heptagon	$64^\circ\frac{2}{7}$	$2.07652+$
8	Octagon	$67^\circ\frac{1}{2}$	$2.41421+ = 1+\sqrt{2}$
9	Nonagon	70°	$2.74747+$
10	Decagon	72°	$3.07768+ = \sqrt{5+2\sqrt{5}}$
11	Undecagon	$73^\circ\frac{7}{11}$	$3.40568+$
12	Duodecagon	75°	$3.73205+ = 2+\sqrt{3}$

2. The

2. The ſide of a hexagon is 5 feet 4 inches; what is the area? *Anſ.* 73.9.

3. Required the area of an octagon whoſe ſide is 16? *Anſ.* 1236.0773.

4. Required the area of a decagon whoſe ſide is 20.5? *Anſ.* 3233.4913.

PROBLEM IX.

The diameter of a circle being given to find the circumference; or, the circumference being given to find the diameter.

RULE I.*

As 7 is to 22, ſo is the diameter to the circumference. Or, as 22 is to 7, ſo is the circumference to the diameter.

EXAM-

---

* The proportion of the diameter of a circle to its circumference has never yet been exactly attained. Nor can a ſquare, or any other right lined figure, be found, that ſhall be equal to a given circle. This is the celebrated problem called the *ſquaring of the circle*, which has exerciſed the abilities of the greateſt mathematicians for ages, and been the occaſion of ſo many endleſs diſputes. Several perſons of conſiderable eminence have, at different times, pretended that they had diſcovered the exact quadrature; but their errors have ſoon been detected, and it is now generally looked upon as a thing impoſſible to be done.

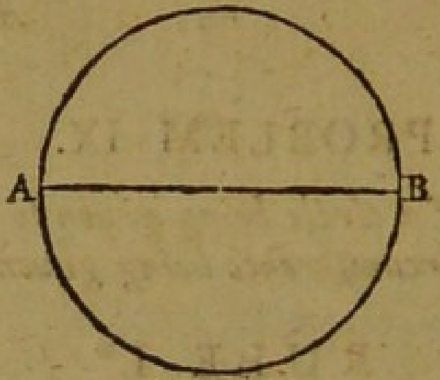
But though the relation between the diameter and circumference cannot be accurately expreſſed in known numbers, it may yet be approximated to any aſſigned degree of exactneſs. And in this manner was the problem ſolved by the great *Archimedes*, about two thouſand years ago, who diſcovered the proportion to be nearly as 7 is to 22, *which is the ſame as our firſt rule.*

This he effected by ſhewing that the perimeter of a circumſcribed regular polygon of 192 ſides is to the diameter in a leſs

 ratio

EXAMPLES.

1. If the diameter AB of a circle be 9, what is the circumference?



Here $7 : 22 :: 9 : 28\frac{2}{7}$; *Or* $\frac{22 \times 9}{7} = 28\frac{2}{7} =$ *circumference required.*

2. If

---

ratio than that of $3\frac{10}{70}$ to 1, and that the perimeter of an inscribed polygon of 96 sides is to the diameter in a greater ratio than that of $3\frac{10}{71}$ to 1, and from thence inferred the ratio above-mentioned; as may be seen in his book *de dimensione circuli.* The same proportion was also discovered by *Philo Gedarensis,* and *Apollonius Pergæus,* at a still earlier period, as we are informed by *Eutocius,* in his observations on a work not come to our hands, called *Ocyteboos.*

The proportion of *Vieta* and *Metius* is that of 113 to 355, which is something more exact than the former, *and is the same as the second rule.*

This is a very commodious proportion; for being reduced into decimals, it agrees with the truth as far as the sixth figure inclusively. It was derived from the pretended quadrature of a M. Van-eick, which first gave rise to the discovery.

But the first who ascertained this ratio to any great degree of exactness was *Van Ceulen,* a *Dutchman,* in his book *de Circulo & Adscriptis.* He found that if the diameter of a circle was 1, the circumference would be 3,141592653589793238462643383279502884 *nearly*; which is exactly true to 36 places

2. If the circumference of a circle ADBC be 36 feet, what is the diameter?

$$22 : 7 :: 36$$

```
                 36
                  7
               ----
           22)252(11 10/22 feet, the diameter.
              22
              --
               32
               22
               --
               10
```

Or $\frac{36 \times 7}{22} = \frac{18 \times 7}{11} = \frac{126}{11} = 11\frac{5}{11}$ *as before.*

RULE

---

places of decimals, and was effected by means of the continual bisection of an arc of a cirle, a method so exceedingly troublesome and laborious, that it must have cost him incredible pains. It is said to have been thought so curious a performance, that the numbers were cut on his *tomb-stone*, in *St. Peter's Church yard*, at *Leyden*. This last number has since been confirmed, and extended to double the number of places, by the late ingenious *Mr. Abraham Sharp*, of *Little Horton*, near *Bradford*, in *Yorkshire*.

But since the invention of *Fluxions*, and the *Summation of Infinite Series*, there have been several methods found out for doing the same thing with much more ease and expedition. The late *Mr. John Machin, Professor of Astronomy in Gresham College*, has, by these means, given a quadrature of the circle which is true to 100 places of decimals; and *M. De Lagny*, *M. Euler*, &c. have carried it still farther. All of which proportions are so extremely near the truth, that except the ratio could be completely obtained, we need not wish for a greater degree of accuracy.

The method of obtaining this proportion from the doctrine of fluxions may be shewn as follows:

RULE II.

As 113 is to 355 ſo is the diameter to the circumference. Or, as 355 is to 113 ſo is the circumference to the diameter.

EXAMPLES.

1. The diameter of a circle is 9 feet; what is the circumference?

Here, as $113 : 355 :: 9 : 28\frac{31}{113}$.

Or $\frac{355\times 9}{113}=\frac{3195}{113}=28.27=$ *circumference.*

2. If

Let $r=$ radius, $x=$ abciſſa, or verſed ſine, $y=$ ordinate, or ſine, and $z=$ length of the arc.

Then $\overline{2r-x}\times x=2rx-x^2=y^2$ by the property of the circle; from which x is found $=r+\sqrt{r^2-y^2}$, and $\dot{x}=-y\dot{y}\times\overline{r^2-y^2}|^{-\frac{1}{2}}$.

But $\dot{z}=\sqrt{\dot{x}^2+\dot{y}^2}$ as is ſhewn by the writers on fluxions; whence by ſubſtitution $\dot{z}=r\dot{y}\times\overline{r^2-y^2}|^{-\frac{1}{2}}$, which thrown into an infinite ſeries gives $\dot{z}=r\dot{y}\times(\frac{1}{r}+\frac{y^2}{2r^2}+\frac{3y^4}{8r^5}+\frac{5y^6}{16r^7}+\frac{35y^3}{128r^9}$ &c.); the fluent which is $z=y\times(1+\frac{y^2}{2.3r^2}+\frac{3y^4}{2.4.5r^4}+\frac{3.5y^6}{2.4.6.7r^6}+\frac{3.5.7y^8}{2.4.6.8.9r^8}$ &c. $=$ length of the arc in terms of the ſine; which, when y is equal to r, becomes $r\times(1+\frac{1}{2.3}+\frac{3}{2.4.5}+\frac{3.5}{2.4.6.7}+\frac{3.5.7}{2.4.6.8.9}$ &c. $=\frac{1}{4}$ of the circumference.

And if inſtead of y in this ſeries be ſubſtituted its value $\frac{rt}{\sqrt{r^2+t^2}}$, we ſhall have $z=t-\frac{t^3}{3r^2}+\frac{t^5}{5t^4}-\frac{t^7}{7r^6}+\frac{t^9}{9r^8}$ &c. $=$ length of the arc in terms of the tangent. Where, by taking the arc ſo that the tangent can be found in terms of the radius, the ſeries will become known, and may be determined to any degree of exactneſs.

Thus,

2. If the diameter of a circle be 10 feet, what is the circumference?

As 113 : 355 : : 10
10

113)3550(31$\frac{47}{113}$, *feet the circumference.*
339
160
113
47

$$Or\ \frac{355\times 10}{113}=\frac{3550}{113}=31.41=\textit{circumference.}$$

3. What

Thus, if the radius be 1, and the arc be $\frac{1}{8}$ part of the circumference, or 45°, its tangent will be equal to the radius, and the series will become $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\frac{1}{13}$ &c. = arc of 43°, and $8\times(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\frac{1}{13}$ &c.) = whole circumference.

This series is the simplest form that can possibly be obtained, but in order to get another that will converge faster, we must take a smaller arc; as for instance, suppose that of 30°, or $\frac{1}{12}$ part of the circumference.

Then since the tangent of 30°, to radius 1, is $\sqrt{\frac{1}{3}}$, the general series will become $\sqrt{\frac{1}{3}}\times(1-\frac{1}{3\cdot 3}+\frac{1}{5\cdot 3^2}-\frac{1}{7\cdot 3^3}+\frac{1}{9\cdot 3^4}$ &c.) = arc of 30°; and $12\sqrt{\frac{1}{3}}\times(1-\frac{1}{3\cdot 3}+\frac{1}{5\cdot 3^2}-\frac{1}{7\cdot 3^3}+\frac{1}{9\cdot 3^4}$ &c.) = whole circumference; *and so for any other arc whatever.*

Those

3. What is the diameter of a circle whoſe circumference is 50?

As 355 : 113 :: 50

```
         113
          50
      ------
355)5650(15 the diameter.
    355
    ----
    2100
    1775
    ----
     325
```

RULE III.

Multiply the diameter by 3.1416, and the product will be the circumference; or

Divide the circumference by 3.1416, and the quotient will be the diameter.

EXAMPLES.

1. If the diameter of a circle be 17, what is the circumference?

```
 3.1416
     17
 ------
 219912
 31416
 ------
53.4072 = circumference.
```

Thoſe who would wiſh to ſee the methods of *Machin*, *Euler*, &c. may conſult Dr. Hutton's Menſuration, and a paper of his in the Philoſophical Tranſactions upon this ſubject.

2. If

2. If the circumference of a circle be 354, what is the diameter?

By Rule I.

As 22 : 7 :: 354 : $\frac{354 \times 7}{22}$ = 122.639 = *circumference.*

By Rule II.

As 355 : 113 :: 354 : $\frac{354 \times 113}{355}$ = 112.681 = *circumference.*

By Rule III.

As 3.1416 : 1 :: 354 : $\frac{354}{3.1416}$ = 112.681 = *circumference, agreeing with the former as far as the third place of decimals.*

3. What is the circumference of a circle whoſe diameter is 40 feet? *Anſ.* 125.6640.

4. If the circumference of a circle be 12, what is the diameter? *Anſ.* 3.81972.

5. The earth is a globe, and its circumference is known to be 25000 miles, what is its diameter? *Anſ.* 7958 *nearly.*

If d be put = diameter of any circle, and c = circumference, the three rules above given, may be expreſſed thus:

$$\frac{22d}{7} = c; \frac{355d}{113} = c; 3.1416d = c,$$

$$\frac{7c}{22} = d; \frac{113c}{355} = d; \frac{c}{3.1416} = d.$$

And if r, or the radius, be uſed inſtead of the diameter, theſe rules become

$$\frac{44r}{7} = c; \frac{710r}{113} = c; 6.2832r = c,$$

$$\frac{7c}{44} = r; \frac{113c}{710} = r; \frac{c}{6.2832} = r.$$

PRO-

PROBLEM X.

To find the length of any arc of a circle.

RULE I.*

From 8 times the chord of half the arc ſubtract the chord of the whole arc, and $\frac{1}{3}$ of the remainder will be the length of the arc *nearly*.

EXAMPLES.

1. The chord of the whole arc DE is 48, and the verſed ſine BC of half the arc is 18: what is the length of the arc DCE?

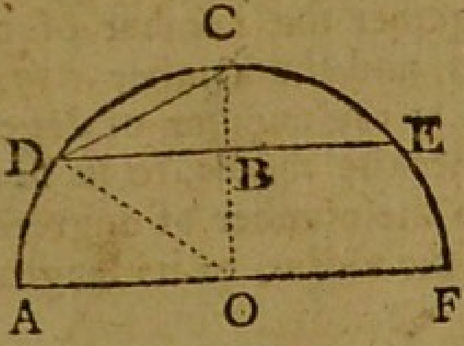
---

* *Demon.* Let the radius OD $= r$, and ſine DB $= s$. Then will the chord DC $= \sqrt{s + (r - \sqrt{r^2 - s^2})^2} = s + \frac{s^3}{8r^2} + \frac{7s^5}{128r^4}$ &c. Whence 8 times the chord DC $= 8s + \frac{s^3}{r^2} + \frac{7s^5}{16r^4}$ &c. And 8DC − DE $= 6s + \frac{s^3}{r^2} + \frac{7s^5}{16r^4}$ &c. whoſe $\frac{1}{3}$ part is $2s + \frac{s^3}{3r^2} + \frac{7s^5}{48r^4}$ &c.

But the length of the arc DC, whoſe ſine is s, is known to be $s + \frac{s^3}{6r^2} + \frac{3s^5}{40r^4}$ &c. and therefore the arc DE $= 2s + \frac{s^3}{3r^2} + \frac{6s^5}{40r^4}$ &c. which differs from $2s + \frac{s^3}{3r^2} + \frac{7s^5}{48r^4}$ &c. only by a ſmall quantity, and ſhews the rule to be very near the truth. Q. E. D.

Rule 2. If d be put = to the diameter, and v = verſed ſine of half the arc, then will $(5d\sqrt{\frac{5d}{5d-3v}} + 4\sqrt{dv}) \times \frac{2}{9}$ = length of the arc *nearly*.

Here



Here $24^2 + 18^2 = 576 + 324 = 900$; *and* $\sqrt{900} = 30 =$ DG. *Whence* $\frac{30 \times 8 - 48}{3} = \frac{240 - 48}{3} = \frac{192}{3} = 64$ *the length of the arc required.*

2. The chord of the whole arc is 50.8, and the chord of half the arc is 30.6: required the length of the arc. *Anſ.* 64.6.

3. The length of the whole chord is 6, and the length of the chord of half the arc 3.0538: what is the length of the arc? *Anſ.* 6.14768.

RULE II.*

1. Add the ſquare of half the chord to the ſquare of the verſed ſine of half the arc, and this ſum divided by the verſed ſine will give the diameter.

2. Take $\frac{4\frac{1}{5}}{}$ of the verſed ſine from the diameter, and divide $\frac{2}{3}$ of the verſed ſine by the remainder.

3. To the quotient, laſt found, add 1; and this ſum multiplied by the chord of the whole arc, will give the length of the arc *nearly*.

Note.

---

* *Demon.* Let the diameter AF $= d$, the verſed ſine CB $= v$, and the chord DE $= c$.

Then

Note. The length of the arc may alſo be found by multiplying together the number of degrees it contains, the radius, and the number .01745329.

Or, as 180 is to the number of degrees in the arc, ſo is 3.1416 times the radius, to its length.

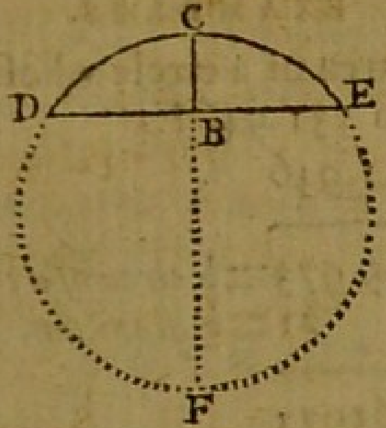
Or, as 3 is to the number of degrees in the arc, ſo is .05236 times the radius, to its length.

EXAMPLES.

1. If the chord DE be 48, and the verſed ſine CB 18: what is the length of the arc?

Then will the rule be expreſſed by $(\frac{2}{3}v \div \overline{d-\frac{41}{50}v}+1)$ $\times c$ or by $(\frac{2}{3}v \div \overline{d-\frac{41}{50}v}+1)\times 2\sqrt{dv-v^2}$, where $d=\frac{c^2}{40}+v$. But $\frac{\frac{2}{3}v}{d-\frac{41}{50}v}+1=1+\frac{2v}{3d}+\frac{41v^2}{75d^2}+\frac{41\times 41v^3}{75\times 50d^3}$ &c. and $2\sqrt{dv-v^2}=2\sqrt{dv}\times:1-\frac{v}{2d}-\frac{v^2}{8d^2}$ &c. Whence $(\frac{2}{3}v\div\overline{d-\frac{41}{50}v}+1)\times 2\sqrt{dv-v^2}=(1+\frac{2v}{3d}+\frac{41v^2}{75d^2}$ &c$)$. $\times(2\sqrt{dv}\times:1-\frac{v}{2d}-\frac{v^2}{8d^2}$&c.$)=2\sqrt{dv}\times:1+\frac{v}{6d}+$ $\frac{53v^2}{600d^2}$ &c. Now $2\sqrt{dv}\times:1+\frac{v}{6d}+\frac{3v^2}{40d^2}$ &c. is known to be the length of an arc whoſe diameter is d, and the verſed ſine of the half thereof v; and this differs from $2\sqrt{dv}\times:+\frac{v}{6d}+$ $\frac{53v^2}{600d^2}$ &c. in the third term only by $\frac{v^2}{75d^2}$, which ſhews the rule to be near an approximation. Q. E. I.

Here



Here $\frac{24^2+18^2}{18} = \frac{576+324}{18} = \frac{900}{18} = 50 =$ *diameter* C F.

And $18 \times \frac{2}{3} \div (50 - \frac{41}{50} \times 18) = 12 \div \overline{50 - 14.70} =$ $12 \div 35.24 = .3405$.

Whence $(1 + .3405) \times 48 = 1.3405 \times 48 = 64.3440 =$ *length of the arc required.*

2. The chord of the whole arc is 7, and the versed sine 2: what is the length of the arc?

Ans. 8.8439.

3. The chord of the whole arc is 40, and the versed sine 15: what is the length of the arc?

Ans. 53.62.

PROBLEM XI.

To find the area of a circle.

RULE I.*

Multiply half the circumference by half the diameter, and the product will be the area.

Or take $\frac{1}{4}$ of the product of the whole circumference and diameter.

EXAM-

---

* *Demon.* A circle may be considered as a regular polygon of an infinite number of sides, the circumference being equal to

EXAMPLES.

1. What is the area of a circle whofe diameter is 42, and circumference 131.946?

2)131.946

65.973 = ½ *circumference.*
21 = ½ *diameter.*

65973
131946

1385.433 = *area required.*

2. What is the area of a circle whofe diameter is 10 feet 6 inches, and circumference 31 feet 6 inches?

fe. in.
15 9 = 15.75 = ½ *circumference.*
5 3 = 5.25 = ½ *diameter.*

7875
3150
7875

82.6875
12

8.2500

Anf. 82 *feet,* 8 *inches.*

---

to the perimeter, and the radius to the perpendicular. But the area of a regular polygon is equal to half the perimeter multiplied by the perpendicular, and confequently the area of a circle is equal to half the circumference multiplied by the radius, or half the diameter. Q. E. D.

This rule may be otherwife demonftrated by the doctrine of fluxions.

3. What

3. What is the area of a circle whofe diameter is 1, and circumference 3.14159? *Anſ.* .7854.

4. What is the area of a circle whofe diameter is 7, and circumference 22? *Anſ.* $38\frac{1}{2}$.

RULE II.*

Multiply the ſquare of the diameter by .7854, and the product will be the area.

EXAM-

---

* *Demon.* All circles are to each other as the ſquares of their diameters. (Euc. XII. 2.)

But the area of a circle whofe diameter is 1, is .7854 &c. (by Rule I.) Therefore $1^2 : d^2 :: .7854\,\&c. : \frac{.7854\,\&c. \times d^2}{1}$ $= .7854\,\&c. \times d^2 =$ area of a circle whofe diameter is d.

Q. E. D.

The following proportions are thofe of *Metius* and *Archimedes.*

As 452 : 355 :: ſquare of the diameter : Area.
As 14 : 11 :: ſquare of the diameter : Area.

If the circumference be given, inſtead of the diameter, the area may be found as follows:

The ſquare of the circumference × .07958 = Area.
As 88 : 7 :: ſquare of the circumference : Area.
As 1420 : 113 :: ſquare of the circumference : Area.

And if d be the diameter, c the circumference, a the area, and $p = 3.14159$ &c. then:

$$1.\ d = \frac{c}{p} = \frac{4a}{c} = 2\sqrt{\frac{a}{p}}$$

$$2.\ c = pd = \frac{4a}{d} = 2\sqrt{pa}$$

$$3.\ a = \frac{pd}{4} = \frac{c}{4p} = \frac{dc}{4}$$

The

EXAMPLES.

1. What is the area of a circle whose diameter is 5?

$$\begin{array}{r} .7854 \\ 25 = \textit{square of the diameter.} \\ \hline 39270 \\ 15708 \\ \hline 19.6350 = \textit{the answer.} \end{array}$$

2. What is the area of a circle whose diameter is 7? *Ans.* 38.4846.

3. What is the area of a circle whose diameter is 4.5? *Ans.* 15.90435.

4. How many square yards are there in a circle whose diameter is $3\frac{1}{2}$ feet? *Ans.* 1.069016.

PROBLEM XII.

To find the area of a sector, or that part of a circle which is bounded by any two radii and their included arc.

The following table will also shew most of the useful problems, relating to the circle and its equal or inscribed square.

1. diameter × .8862 = side of an equal square.
2. circumf. × .2821 = side of an equal square.
3. diameter × .7071 = side of the inscribed square.
4. circumf. × .2251 = side of the inscribed square.
5. area × .6366 = side of the inscribed square.
6. side of a square × 1.4142 = diameter of its circumf. circle.
7. side of a square × 4.443 = circumf. of its circumf. circle.
8. side of a square × 1.128 = diameter of an equal circle.
9. side of a square × 3.545 = circumf. of an equal circle.

RULE.

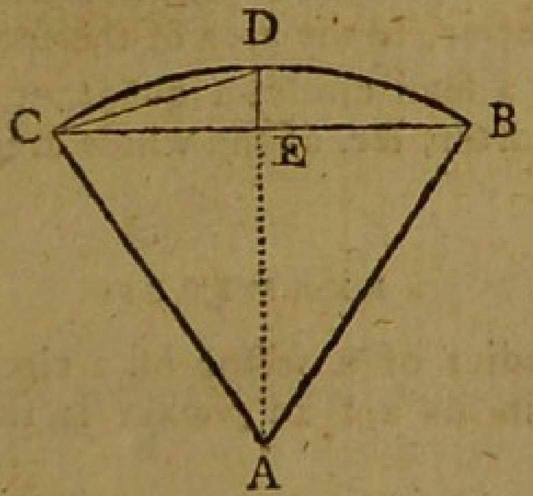
RULE.*

Multiply the radius, or half the diameter, by half the length of the arc of the ſector, and the product will be the area.

Note. The length of the arc may be found by either of the laſt problems.

EXAMPLES.

1. The radius AB is 40, the chord BC of the whole arc 50, and the chord CD of half the arc 27; required the area of the ſector.



Here $\frac{27\times 8-50}{3}=\frac{216-50}{3}=\frac{166}{3}=55\cdot\dot{3}=$ *length of the arc* CDB.

And $\frac{55\cdot\dot{3}}{2}\times 40=27.\dot{6}\times 40=1109.\dot{6}=$ *area of the ſector required.*

---

* The rule for finding the area of the ſector, is, evidently, the ſame as that for finding the area of the whole circle.

2. Required the area of a ſector of a circle, whoſe radius is 25, and the length of the arc 21.5.

Anſ. 268.75.

3. What is the area of a ſector whoſe radius is 10, the chord of the whole arc 20, and the chord of half the arc 11? *Anſ.* 113.3.

4. Find the area of the ſector whoſe radius is 9, and the chord of whoſe arc is 6.

Anſ. 27.5267835.

RULE II.*

As 360 is to the degrees in the arc of a ſector, ſo is the area of the whole circle, whoſe radius is equal to that of the ſector, to the area of the ſector required.

Note. For a ſemi-circle, a quadrant, &c. take one half, one quarter, &c. of the whole area.

EXAMPLES.

1. The radius of a ſector of a circle is 20, and the degrees in its arc 22; what is the area of the ſector?

---

* *Demon.* Let $r =$ radius, $d =$ number of degrees in the arc of the ſector, and $A =$ its area.

Then will $4r^2 \times .7854 = r^2 \times 3.1416 =$ area of the whole circle, and $2r \times 3.1416 =$ its circumference.

Alſo $360 : 2r \times 3.1416 :: d : \frac{2dr \times 3.1416}{360} =$ length of the arc of the ſector. But $\frac{2dr \times 3.1416}{360} \times \frac{1}{2} \times r = \frac{dr^2 \times 3.1416}{360} = A$, by the laſt rule. And conſequently $360 : d :: r^2 \times 3.1416 : A$.

Q. E. D.

```
                 .7854
                  1600=square of the diameter.
              ---------
              4712400
              7854
              ---------
360 : 22 :: 1256.6400=area of the whole circle.
                 22
              ---------
              251328
             251328
             ---------
      36.0)2764.608(7.6=area of the sector.
             252
             ----
              244
              216
             ----
               286
               &c.
```

2. Required the area of a sector whose radius is 25, and the length of its arc 147 degrees 29 minutes.

Ans. 804.4017.

3. Required the area of a semicircle whose radius is 13.

Ans. 265.4652.

4. Required the area of a quadrant whose radius is 21.

Ans. 346.3614.

PROBLEM XIII.

To find the area of a segment of a circle.

RULE I.

1. Find the area of the sector, having the same arc with the segment, by one of the last problems.

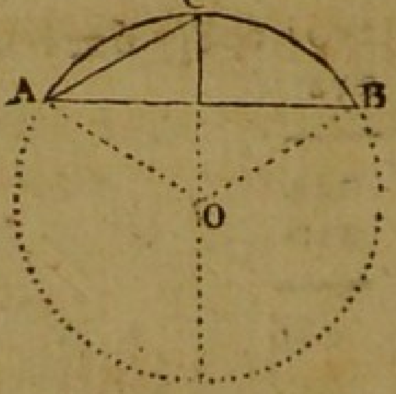
2. Find

2. Find the area of the triangle formed by the chord of the segment, and the radii of the sector.

3. Then the sum, or difference, of these areas, according as the segment is greater or less than a semicircle, will be the area required.

EXAMPLES.

1. The radius OB is 10, the chord AB is 18, and the chord AC 10; what is the area of the segment ABC?



Here $\frac{10\times8-18}{3}=\frac{80-18}{3}=\frac{62}{3}=20.\dot{6}=$ *arc* ACB.

And $\frac{20.\dot{6}}{2}\times10=10.\dot{3}\times10=103.\dot{3}=$ *area of the sector* OACB.

Also $\frac{18+10+10}{2}=\frac{38}{2}=19=\frac{1}{2}$ *sum of the sides* AO, OB, BA.

Whence $19-10=9=$ 1st *and* 2d *diff. and* $19-18=1=$ 3d *diff.*

And $\sqrt{19\times9\times9\times1}=\sqrt{1539}=39.23=$ *area of the* $\triangle$ BOA.

And $103.\dot{3}-39.23=64.10=$ *area of the segment required.*

It being in this case less than a semicircle.

2. Re-

2. Required the area of a ſegment whoſe height is 2, and chord 20. *Anſ.* 26.878.

3. Required the area of a ſegment of a circle whoſe radius is 24, the chord of the whole arc 20, and the chord of half the arc 10.2. *Anſ.* 28.07.

4. Required the area of a ſegment of a circle whoſe chord is 12, and the radius of the circle 10.

Anſ. 16.3274.

5. Required the area of a ſegment of a circle whoſe chord is 16, and the diameter of the circle 20.

Anſ. 44.7292.

6. What is the area of a ſegment, whoſe arc is a quadrant, the diameter being 18 feet? *Anſ.* 63.1174.

7. What is the area of a ſegment of a circle whoſe arc contains 280 degrees, the diameter being 50?

Anſ. 1834.9191.

RULE II.*

1. Add the ſquare of half the chord of the ſegment to the ſquare of its height, and multiply the ſquare root of the ſum by 4.

2. To

---

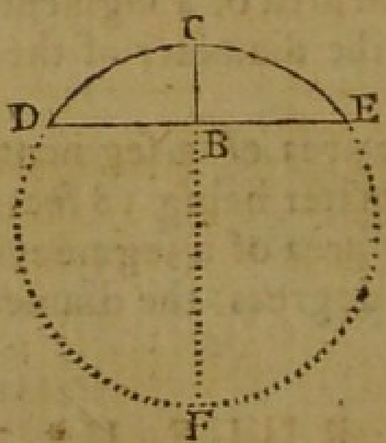
* *Demon.* Let c = BE, v = CB, and d = diameter CF; then will the rule be expreſſed by $(2c + \frac{4}{3}\sqrt{c^2+v^2}) \times \frac{2}{5}v$. But $(2c + \frac{4}{3}\sqrt{c^2+v^2})$ $\frac{2}{5}v = (\sqrt{dv-v^2} + \frac{2}{5}\sqrt{dv}) \times \frac{4}{5}v$ $= \frac{4}{5}v \times \sqrt{dv} + \frac{4}{5}v\sqrt{dv} \times (1 - \frac{v}{2d} - \frac{v^2}{8d^2} - \frac{3v^3}{48d^3}$ &c.$) = \frac{4}{3}v$ $\sqrt{dv} - \frac{2v^2\sqrt{dv}}{5d} - \frac{2v^3\sqrt{dv}}{20d^2}$ &c. $= 2v\sqrt{dv} \times (\frac{2}{3} - \frac{v}{5d}$ $- \frac{v^2}{20d^2} - \frac{v^3}{72d^3}$ &c.) And ſince BE $= \sqrt{dv-v^2}$, and CB $= v$, therefore $\dot{v}\sqrt{dv} \times : 1 - \frac{v}{2d} - \frac{v^2}{8d^2} - \frac{3v^3}{48d^3}$ &c. will be the fluxion of the ſegment; and its fluent, $v\sqrt{dv}$

×

2. To $\frac{1}{3}$ of the number laſt found, add the whole chord of the ſegment, and this ſum multiplied by $\frac{2}{5}$ of the height will give the area required.

EXAMPLES.

1. The chord D E of the ſegment D E C is 40, and its height, or verſed ſine, B C is 10: what is the area of the ſegment?



Here $4\sqrt{20^2+10^2}=4\sqrt{400+100}=4\sqrt{500}=4\times22.36=89.44$.

And $(\frac{89.44}{3}+40)\times\frac{2\times10}{5}=\overline{29.81+40}\times4=69.81\times4=279.24=$ *area required.*

$\times:\frac{2}{3}-\frac{v}{5d}-\frac{v^2}{8d^2}$ &c. will be the value of half the ſegment. Conſequenty its double, $2v\sqrt{dv}\times(\frac{2}{3}-\frac{v}{5d}-\frac{v^2}{28d^2}-\frac{v^3}{72d^3}$ &c.) will be the value of the whole ſegment; which differs from the expreſſion for the rule only in the third term, and therefore ſhews the *approximation* required. Q. E. D.

This rule was firſt given by *Sir Iſaac Newton*, and publiſhed by *Jones*, *Wallis*, and *Colſon*.

If $c=$ chord of the ſegment, and $v=$ its height; then $(7\sqrt{\frac{1}{4}c^2+\frac{2}{7}v^2}+\frac{4}{3}\sqrt{\frac{1}{4}c^2+v^2})\times\frac{4}{25}v=$ area *very nearly*, which is another approximating rule, ſufficiently eaſy in its application.

2. The

2. The chord is 20, and the height, or verſed ſine 2; required the area of the ſegment.

Anſ. 26.85908.

3. The length of the chord is 48, and the height of the ſegment 18; what is the area? *Anſ.* 633.6.

RULE III.*

1. Divide the height, or verſed ſine, by the diameter, and find the quotient in the table of verſed ſines.

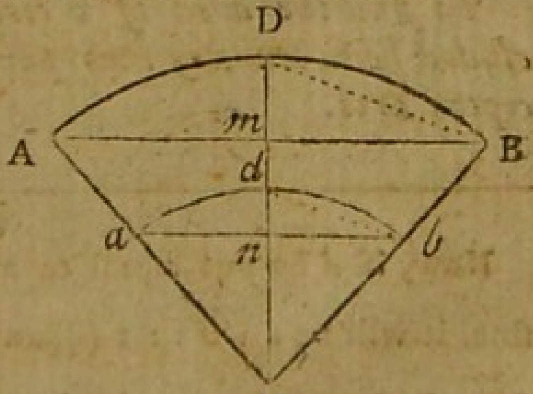
2. Mul-

---

* The table to which this rule refers, is formed of the areas of the ſegments of a circle whoſe diameter is 1; and which is ſuppoſed to be divided by perpendicular chords into 1000 equal parts.

The reaſon of the rule itſelf depends upon this property—That the verſed ſines of ſimilar ſegments are as the diameters of the circles to which they belong, and the areas of thoſe ſegments as the ſquares of the diameters; which may be thus demonſtrated.

Let ADBA and *adba* be any two ſimilar ſegments, cut off from the ſimilar ſectors ADBCA and *adbca*, by the chords AB and *ab*; and let the perpendicular CD biſect them.



Then, by ſimilar triangles, DB : *db* : : BC : *bc*, and DB : *db* : : D*m* : *dn*; whence, by equality, BC : *bc* : : D*m* : *dn*, or 2BC : 2*bc* : : D*m* : *dn*.

Again, ſince ſimilar ſegments are as the ſquares of their chords, it will be $AB^2 : ab^2$: : ſeg. ADBA : ſeg. *adba*; but $AB^2 : ab^2 :: CB^2 : cb^2$, whence, by equality, ſeg. ADBA : ſeg. *adba* : : $CB^2 : cb^2$, or ſeg. ADBA : ſeg. *adba* : : $4CB^2 : 4cb^2$. Q. E. D.

Now,

2. Multiply the number on the right hand of the verfed fine, by the fquare of the diameter, and the product will be the area.

EXAMPLES.

1. If the chord of a circular fegment be 40, its verfed fine 10, and the diameter of the circle 50: what is the area?

5.0)1.0

.2 = *tabular verfed fine.*
.111823 = *tabular fegment.*
2500 = *fquare of* 50.

55911500
223646

279.557500 = *area required.*

2. The chord of the fegment is 20, the verfed fine 5, and the diameter 25: what is the area?

Anf. 69.889.375.

PROBLEM XIV.

To find the area of a circular zone, or the fpace included between any two parallel chords and their intercepted arcs.

Now, if d be put equal to any diameter, and v the verfed fine, it will be $d : v :: 1$ (diameter in the table) : $\frac{v}{d}$ = verfed fine of a fimilar fegment in the table, whofe area let be called a.

Then $1^2 : d^2 :: a : ad^2$ = area of the fegment whofe height is v, and diameter d, as in the rule.

RULE.

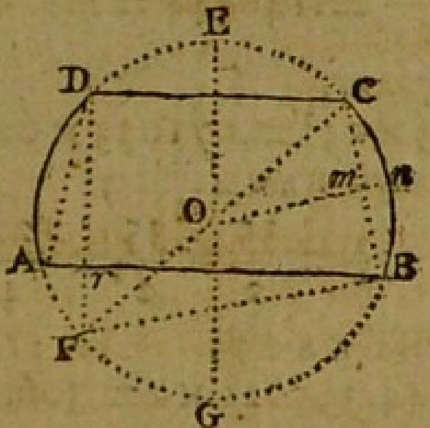
RULE.*

1. Find the area of that part of the zone which forms a trapezoid by problem 6, and the area of the segment B*n*CB by problem 13, rule 3.

2. Add the area of the trapezoid to twice the area of the segment, and it will give the area of the zone required.

EXAMPLES.

1. The greater chord AB is 20, the less DC 15, and their distance D*r* $17\frac{1}{2}$: required the area of the zone ABCD.



* The reason of this rule is too obvious to require a demonstration.

Note. When the two parallel sides of the zone are equal, the chord of the small segment will be equal to the breadth of the zone, and its height will be equal to $\sqrt{\frac{1}{4}AB^2 + \frac{1}{4}as^2} - \frac{1}{2}AB$.

And when one of the sides is the diameter of the circle, the chord of the same segment will be $\sqrt{^2as + D^2}$, and its height $= \frac{1}{2}AB - \sqrt{\frac{1}{4}AB^2 - \frac{1}{4}as^2 - \frac{1}{4}D^2}$ where $D = AB - DC$.

Another method, is by finding the areas of each of the segments ABEA, DCED, the difference of which is the area of the zone required; but this rule, when the segments are large, and any of the approximating theorems are used, is not so accurate as the former.

Here $\sqrt{Dr^2+rA^2}=\sqrt{17.5^2+2.5^2}=\sqrt{306.25+6.25}$ $=\sqrt{312.5}=17.6776=$ AD *or* CB.

And Dr $(17\frac{1}{2})$: rA $(2\frac{1}{2})$ :: Br $(17\frac{1}{2})$: $2\frac{1}{2}=$ rF; *or* $2\frac{1}{2}+17\frac{1}{2}=20=$ DF.

Also $\sqrt{DF^2+DC^2}=\sqrt{20^2+15^2}=\sqrt{400+225}$ $=\sqrt{625}=25=$ CF; *or* $\frac{25}{2}=12\frac{1}{2}=$ OC *or* On.

And rD $(17\frac{1}{2})$: AD (17.6776) :: rB $(17\frac{1}{2})$: 17.6776 $=$ FB; *or* $\frac{17.6776}{2}=8.8388=$ Om; *and* $12.5-8.8388$ $=3.6612=$ mn $=$ *height of the segment.*

Therefore $\frac{3.6612}{25}=.146448=$ *tab. versed sine*; *and* .071033 $=$ *tab. segment.*

Whence $.071033\times25^2=.071033\times625=44.395625$ $=$ *area of the segment* BnCB.

Also $\frac{15+20}{2}\times17\frac{1}{2}=\frac{35}{2}\times\frac{35}{2}=\frac{1225}{4}=306.25=$ *area of the trapezoid* ABCD.

And $306.25+44.395625\times2=306.25+88.79025$ $=394.04025=$ *area of the zone required.*

2. The greater side is 96, the lesser 60, and the breadth $17\frac{1}{2}$: what is the area of the zone?

Ans. 2136.7712.

3. One end of a circular zone is 48, the other end is 30, and the breadth is 13; what is the area of the zone?* *Ans.* 534.4249.

PROBLEM XV.

To find the area of a circular ring, or the space included between the circumferences of two concentric circles.

---

* In any of these questions, when the radius or diameter of the circle is given, the operation is much more easily performed.

RULE.

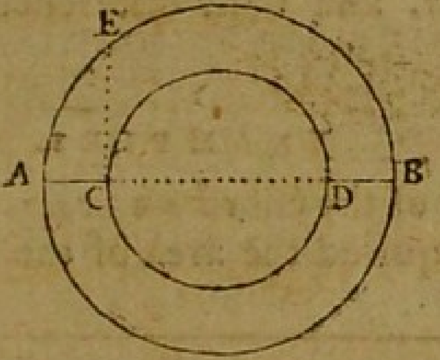
RULE.*

The difference between the areas of the two circles will be the area of the ring.

Or, Multiply the ſum of the diameters by their difference, and this product again by .7854, and it will give the area required.

EXAMPLES.

1. The diameters AB and CD are 20 and 15: required the area of the circular ring, or the ſpace included between the circumferences of thoſe circles.



Here $\overline{AB + CD} \times \overline{AB - CD} = 35 \times 5 = 175$; *and* $175 \times .7854 = 137.4450 =$ *area of the ring required.*

---

* *Demon.* The area of the circle AEBA $= AB^2 \times .7854$, and the area of the ſmall circle CD is $= CD^2 \times .7854$; therefore the area of the ring $= AB^2 \times .7854 - CD^2 \times .7854 = \overline{AB + CD} \times \overline{AB - CD} \times .7854$. Q.E.D.

Coroll. If CE be a perpendicular at the point C, the area of the ring will be equal to that of a circle whoſe radius is CE.

Rule 2. Multiply half the ſum of the circumferences by half the difference of the diameters, and the product will be the area.

This rule will alſo ſerve for any part of the ring, uſing half the ſum of the intercepted arcs for half the ſum of the circumferences.

2. The diameters of two concentric circles are 16 and 10: what is the area of the ring formed by thoſe circles? *Anſ.* 122.5224.

3. The two diameters are 21.75 and 9.5: required the area of the circular ring. *Anſ.* 300.6609.

4. Required the area of the ring, the diameters of whoſe bounding circles are 6 and 4. *Anſ.* 15.708.

PROBLEM XVI.

To find the areas of lunes, or the ſpaces included between the interſecting arcs of two eccentric circles.

RULE.*

Find the areas of the two ſegments from which the lune is formed, and their difference will be the area required.

EXAMPLES.

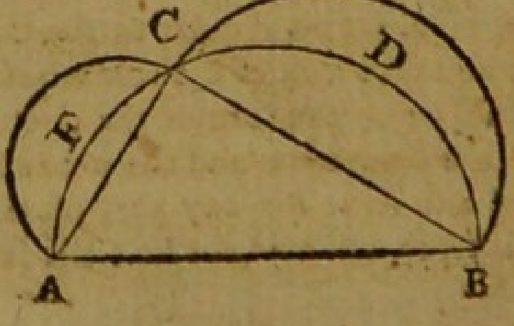
The length of the chord AB is 40, the height DC 10, and DE 4: required the area of the lune ACBEA.

---

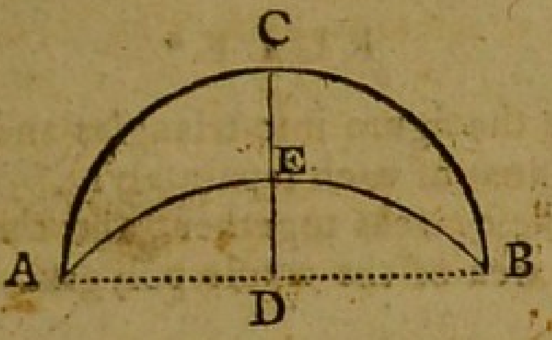
* Whoever wiſhes to be acquainted with the properties of lunes, and the various theorems ariſing from them, may conſult *Mr. Whiſton's Commentary on Tacquet's Euclid*, where they will find this ſubject very ingeniouſly managed.

The following property is one of the moſt curious.

If ABC be a right angled triangle, and ſemicircles be deſcribed on the three ſides as diameters, then will the ſaid triangle be equal to the two lunes D and F taken together.



4 ✓



Here $4\sqrt{AD^2 + DC^2} = 4\sqrt{400 + 100} = 22.36 \times 4$ $= 89.44$.

And $(\frac{89.44}{3} + 40) \times \frac{2 \times 10}{5} = \overline{29.81 + 40} \times 4 =$ $279.24 =$ *area of the segment* ABCA.

Also $4\sqrt{AD^2 + DE^2} = 4\sqrt{400 + 16} = 20.396 \times 4$ $= 81.854$.

And $(\frac{81.584}{3} + 40) \times \frac{2 \times 4}{5} = \overline{27.194 + 40} \times 1.6$ $= 107.5104 =$ *area of the segment* ABEA.

Whence $279.24 - 107.5104 = 171.7296 =$ *area of the lune required.**

2. The chord is 20, and the heights of the segments 10 and 2: required the area of the lune.

Ans. 128.555.

3. The length of the chord is 48, and the heights of the segments 18 and 7: what is the area of the lune?

Ans. 405,8676.

PROBLEM XVII.

To find the area of an irregular polygon, or a figure of any number of sides.

---

* *Note.* The areas of the segments in this and the following examples may be more accurately found by Rule 3, page 77.

RULE.*

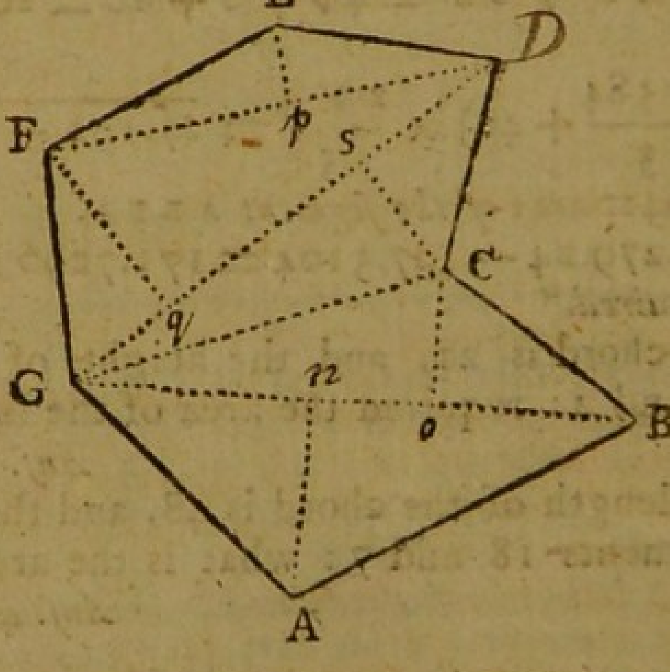
1. Divide the figure into triangles and trapeziums, and find the area of each ſeparately.

2. Add theſe areas together, and the ſum will be equal to the area of the whole polygon.

EXAMPLES.

1. Required the area of the irregular figure A B C D E F G A, the following lines being given.

G B = 30.5,	A n = 11.2,	C o = 6
G G = 29	F q = 11	C s = 6.6
F D = 24.1	E p = 4	....



---

* When any part of the figure is bounded by a curve, the area may be found as follows:

Rule. 1. Erect any number of perpendiculars upon the baſe, at equal diſtances, and find their lengths.

2. Add the lengths of the perpendiculars, thus found, together, and the ſum divided by their number will give the *mean breadth*.

3. Multiply the mean breadth by the length of the baſe, and it will give the area of that part of the figure required.

Here

Here $\frac{An + Co}{2} + GB = \frac{11.2+6}{2} \times 30.5 = 8.6 \times 30.5$ $= 262.3 =$ *area of the trapezium* ABCG.

And $\frac{Fq + Cs}{2} \times GD = \frac{11 + 6.6}{2} \times 2.9 = 8.8 \times 2.9 =$ $255.2 =$ *area of the trapezium* GCDF.

Also $\frac{FD \times Ep}{2} = \frac{24.8 \times 4}{2} = \frac{99.2}{2} = 49.6 =$ *area of the triangle* FDE.

Whence $262.3 + 255.2 + 49.6 = 567.1 =$ *area of the whole figure required.*

2. * In a pentangular field, beginning with the south side, and measuring round towards the east, the first, or south side, is 2735 links, the second 3115, the third 2370, the fourth 2925, and the fifth 2220; also the diagonal from the first angle to the third is 3800 links, and that from the third to the fifth 4010: required the area of the field.

Ans. 117 *ac.* 2 *ro.* 28 *po.*

To find the area of mixed or compound figures, or such as are composed of rectilineal and curvilineal figures together: the rule is to find the area of the several figures of which the whole figure is composed, then add all the areas together, and the sum will be the area of the whole compound figure. And in the same manner may any irregular field or piece of land be measured, by dividing it into trapeziums and triangles, and finding the area of each separately.

* *Note.* As this figure consists of three triangles, all of whose sides are given, by calculating their areas according to problem 3, the sum will be the area of the whole figure accurately, without drawing perpendiculars from the angles to the diagonals.

The same thing may also be done in most other cases of this kind.

OF

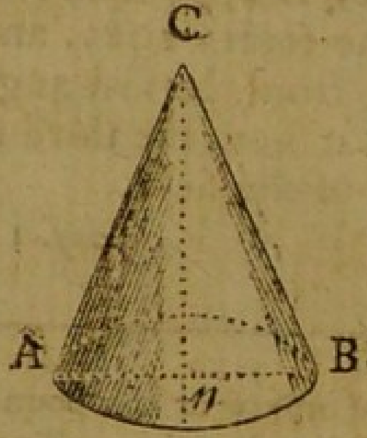
OF THE

CONIC SECTIONS.

DEFINITIONS.

1. THE *conic ſections* are ſuch plain figures as are formed by the cutting of a cone.

2. *A *cone* is a ſolid deſcribed by the revolution of a right-angled triangle about one of its legs, which remains fixed.



3. The *axis of a cone* is the right line about which the triangle revolves.

---

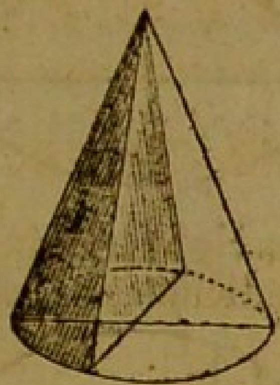
* This is Euclid's definition of a cone, and is that which is generally beſt underſtood by a learner; but the following one is more general.

Conceive the right line C B to move upon the fixed point C as a centre, and ſo as continually to touch the circumference of the circle A B, placed in any poſition, except in that of a plane which paſſes through the ſaid point; and then that part of the line which is intercepted between the fixed point and the periphery of the circle will generate the convex ſuperficies of a cone.

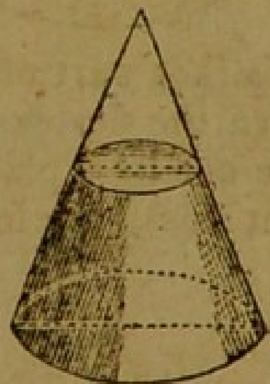
4. The

4. The *base of a cone* is the circle which is described by the revolving leg of the triangle.

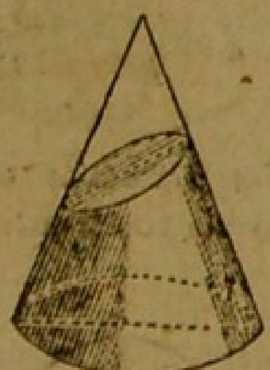
5. If a cone be cut through the vertex, by a plane perpendicular to that of the base, the section will be a *triangle*.



6. If a cone be cut into two parts, by a plane parallel to the base, the section will be a *circle*.



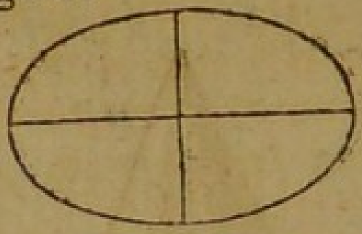
7. If a cone be cut by a plane which passes through its two slant sides in an oblique direction, the section will be an *ellipsis*.



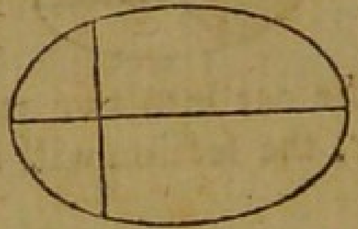
8. If two lines be drawn through the centre of an ellipsis, perpendicular to each other, and terminated both

both ways by the circumference, they are called the *tranſverſe* and *conjugate diameters*, or *axes*.

The longeſt line is the tranſverſe axis, and the ſhorteſt the conjugate.

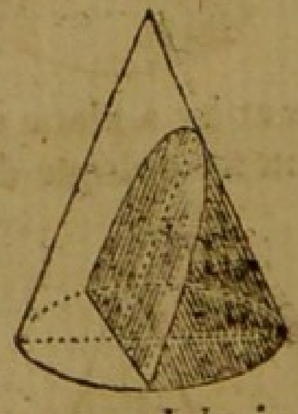


9. *An ordinate* is a right line drawn from any point of the curve, perpendicular to either of the diameters.

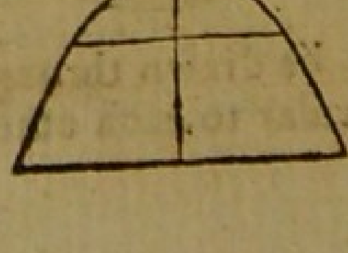


10. *An abſciſſa* is that part of the diameter which is contained between the vertex and the ordinate.

11. If a cone be cut by a plane, which is parallel to either of its ſlant ſides, the ſection will be *a parabola*.



12. *The axis of a parabola* is a right line drawn from the vertex, ſo as to divide the figure into two equal parts.



13. *The*

13. *The ordinate* is a right line drawn from any point in the curve perpendicular to the axis.

14. *The abscissa* is that part of the axis which is contained between the vertex and the ordinate.

15. *If a cone be cut into two parts, by a plane, which being continued, would meet the opposite cone, the section is called an *hyperbola*.

16. *The transverse diameter* or axis of an hyperbola is that part of the axis intercepted between the two opposite cones.

17. *The conjugate diameter* is a line drawn through the centre perpendicular to the transverse.

18. *An ordinate* is a line drawn from any point in the curve perpendicular to the axis; and the *abscissa* is the distance intercepted between that ordinate and the vertex.

---

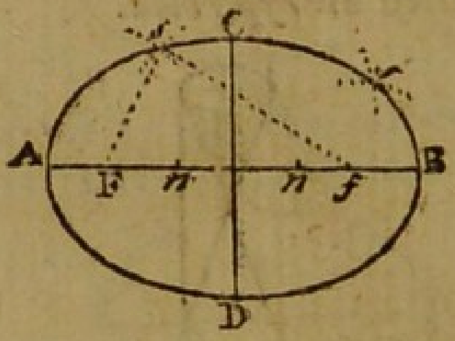
* The two opposite cones, in this definition, are supposed to be generated together, by the revolution of the same line.

All the figures which can possibly be formed by the cutting of a cone, are mentioned in these definitions, and are the five following ones; *viz.* a *triangle*, a *circle*, an *ellipsis*, a *parabola*, and an *hyperbola*; but the three last only are usually called the *conic sections*.

PRO-

PROBLEM I.

To describe an ellipsis, the transverse and conjugate diameters being given.



*Construction.** 1. Draw the transverse and conjugate diameters, A B, C D, bisecting each other perpendicularly in the centre *o*.

2. With the radius A *o*, and centre C, describe an arc, cutting A B in F *f*; and these two points will be the foci of the ellipse.

3. Take any number of points *n n*, &c. in the transverse diameter A B, and with the radii A *n*, *n* B,

---

* It is a known property of the ellipse, that the sum of two lines drawn from the foci, to meet in any point in the curve, is equal to the transverse diameter, and from this the truth of the construction is evident.

From the same principle is also derived the following method of describing an ellipse, by means of a string and two pins.

Having found the foci F, *f*, as before, take a thread of the length of the transverse diameter, and fasten its ends with two pins in the points F, *f*; then stretch the thread F *s* *f* to its greatest extent, and it will reach to the point *s* in the curve; and by moving a pencil round within the thread, keeping it always stretched, it will trace out the curve required.

and

and centres F f, describe arcs intersecting each other in s, s, &c.

4. Through the points s, s, &c. draw the curve A s C B D, and it will be the circumference of the ellipse required.

PROBLEM II.

In an ellipsis, any three of the four following terms being given, viz. the transverse and conjugate diameters, an ordinate and its abscissa, to find the fourth.

CASE I.

When the transverse, conjugate, and abscissa are known; to find the ordinate.

RULE.*

As the transverse diameter is to the conjugate,
So is the square root of the rectangle of the two abscissas,
To the ordinate which divides them.

EXAMPLES.

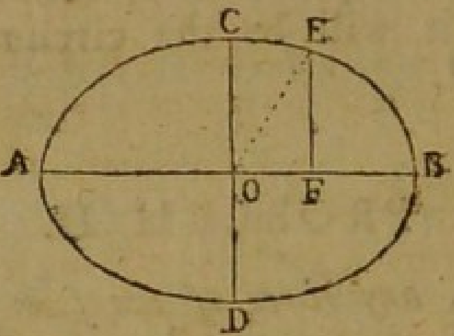
1. In the ellipsis A D B C, the transverse diameter A B is 120, the conjugate diameter C D is 40, and

---

* Let t = the transverse diameter, c = conjugate, x = any abscissa, and y = ordinate. Then will the general equation, expressing the property of the ellipse, be $t^2 : c^2 :: x \times (t-x) : y^2$; and from this the four rules here given are deduced, the one above being $y = \frac{c}{t}\sqrt{x \times \overline{t-x}}$.

the

the abſciſſa BF 24: what is the length of the ordinate EF?



Here 120 (AB) : 40 (CD) :: $\sqrt{96 \times 24}$ (AF × FB) : $\frac{40}{120}\sqrt{96 \times 24} = \frac{1}{3}\sqrt{2304} = \frac{1}{3} \times 48 = 16 =$ EF *the ordinate required.*

2. If the tranſverſe diameter be 35, the conjugate 25, and the abſciſſa 28: what is the ordinate?

Anſ. 10.

CASE II.

When the tranſverſe, conjugate, and ordinate are known, to find the abſciſſa.

RULE.*

As the conjugate diameter is to the tranſverſe,

So is the ſquare root of the difference of the ſquares of the ordinate and ſemi-conjugate,

To the diſtance between the ordinate and centre.

And this diſtance being added to and ſubtracted from the ſemi-tranſverſe, will give the two abſciſſas required.

---

* This rule in algebraic terms is as follows: The greater abſciſſa $x = t + \frac{t}{c}\sqrt{\frac{1}{4}c^2 - y^2}$, or the leſs $x = t - \frac{t}{c}\sqrt{\frac{1}{4}c^2 - y}$.

EXAM-

EXAMPLES.

1. The tranſverſe diameter AB is 120, the conjugate diameter CD is 40, and the ordinate FE is 16: what is the abſciſſa FB?

Here 40 (CD) : 120 (AB) :: $\sqrt{20^2 - 16^2}$ ($\sqrt{OB^2 - FE^2}$) : $\frac{120}{40} \sqrt{20^2 - 10^2} = 3\sqrt{400 - 250} = 3\sqrt{144} = 3 \times 12 = 36 =$ OF, *the diſtance from the centre.*

Whence 60 (OB) − 36 (OF) = 24 = BF }
And 60 (OA) + 36 (OF) = 96 = AF } = *two abſciſſas required.*

2. What are the two abſciſſas to the ordinate 10, the diameters being 35 and 23?

Anſ. 7 *and* 28.

CASE III.

When the conjugate, ordinate, and abſciſſa are known, to find the tranſverſe.

RULE.*

1. To, or from, the ſemi-conjugate, according as the leſs or great abſciſſa is uſed, add or ſubtract the ſquare root of the difference of the ſquares of the ordinate and ſemi-conjugate.

2. Then, as the ſquare of the ordinate, is to the rectangle of the conjugate and abſciſſa, ſo is the ſum or difference above found, to the tranſverſe diameter required.

---

* This rule in algebraic terms is as follows: $t = (c + 2\sqrt{\tfrac{1}{4}c^2 - y^2}) \times \frac{cx}{2y^2}$, or $t = (c - 2\sqrt{\tfrac{1}{4}c^2 - y^2}) + \frac{cx}{2y^2}$ according as the greater or leſs abſciſſa is uſed.

EXAM-

EXAMPLES.

1. The conjugate diameter C D is 40, the ordinate E F is 16, and the abſciſſa F B 24: required the tranſverſe A B.

Here 20 (O B) + $\sqrt{20^2-16^2}$ ($\sqrt{OB^2-EF^2}$) = 20 + $\sqrt{400-256}$ = 20 + $\sqrt{144}$ = 20 + 12 = 32.

And 16^2 (EF^2) : 40 × 24 (C D × B F) : : 32 : $\frac{40\times24\times32}{16^2} = \frac{40\times24\times2}{16} = \frac{5\times24\times2}{2} = 5\times24 =$ 120 *the tranſverſe diameter required.*

2. If an ordinate and its leſſer abſciſſa be 10 and 7, and the conjugate 25: what is the tranſverſe?

Anſ. 35.

CASE IV.

The tranſverſe, ordinate, and abſciſſa being given, to find the conjugate.

RULE.*

As the ſquare root of the product of the two abſciſſas, is to the ordinate,

So is the tranſverſe diameter to the conjugate.

EXAMPLES.

1. The tranſverſe A B is 120, the ordinate E F 16, and the abſciſſa F B 24: required the conjugate.

---

* The rule in algebraic terms is $ty \times \frac{1}{\sqrt{t-x.x}} = c$, the conjugate, or ſhorteſt diameter.

Here

Here $\sqrt{24\times96}$ ($\sqrt{\text{BF}\times\text{AF}}$) : 16 (PF) :: 120 (AB) : $16\times120\div\sqrt{24\times96}=16\times120\div\sqrt{2304}=16\times120\div48=\frac{15\times120}{48}=\frac{120}{3}=40$ *the conjugate diameter required.*

2. The tranſverſe diameter is 35, the ordinate is 10, and its abſciſſa 7: what is the conjugate?

Anſ. 25.

PROBLEM III.

To find the circumference of an ellipſe, the tranſverſe and conjugate diameters being known.

RULE.*

Multiply the ſquare root of half the ſum of the ſquares of the two diameters by 3.1416, and the product will be the circumference *nearly*.

EXAM-

---

* *Demon.* Let t = tranſverſe diameter, c = conjugate, p = 3.1416, and $d = 1-\frac{c^2}{t^2}$. Then will $pt\times(1-\frac{d}{2.2}-\frac{3d^2}{2.2.4.4}-\frac{3.3.5d^3}{2.2.4.4.6.6}$ &c.) = circumference of the ellipſe, as is ſhewn by the writers on fluxions.

Now the rule given above is $p\sqrt{\frac{t^2+c^2}{2}}=pt\sqrt{(\frac{1}{2}+\frac{c^2}{2t^2})}=pt\sqrt{(1-\frac{1}{2}+\frac{c^2}{2t^2})}$ = (by ſubſtitution) $pt\sqrt{(1-\frac{d}{2})}=pt\times:1-\frac{d}{2.2}-\frac{d^2}{2^3.4}-\frac{3d^3}{2^4.4.6}$ &c. But the three firſt

terms

EXAMPLES.

1. The tranfverfe diameter is 24 and the conjugate 20: required the circumference of the ellipfe.

Here $\sqrt{\frac{AB^2 - CD^2}{2}} = \sqrt{\frac{24^2 + 20^2}{2}} = \sqrt{\frac{576 + 400}{2}}$

$= \sqrt{288 + 200} = 488 = 22.09.$

And 22.09 × 3.1416 = 69.397944 *the circumference required.*

2. The axes are 24 and 18: what is the circumference? *Anf.* 66.6433.

---

terms of this feries differ from the three firft terms of the former only by $\frac{d^2}{64}$; therefore the rule is fhewn to be an approximation. Q. E. D.

Rule 2. Multiply ½ the fum of the two diameters by 3.1416, and the product will give the circumference *exact enough to anfwer moft practical purpofes.*

Rule 3. Find the circumference both from the laft rule and that given above, and ½ the fum of the refults will give the circumference *extremely near.*

Note. If a = femi tranfverfe BO, c = femi-conjugate CO, and x = diftance OF, of the ordinate EF from the centre, then will the arc CE be $= x \times (1 + \frac{c^2}{6a^4} x^2 + \frac{4a^2c^2 - c^4}{40a^8} x^4$

$+ \frac{8a4c^2 - 4a^2c^4 + c^6}{112a^{12}} x^6$ &c.

The following may ferve as a practical rule for finding the length of the arc CE.

Find the length of a circular arc intercepted by OE and OC, and whofe radius is ½ the fum of OE and OC, and it will be the elliptic arc CE *nearly.*

PRO-

PROBLEM IV.

To find the area of an ellipſe, the tranſverſe and conjugate diameters being given.

RULE.*

Multiply the tranſverſe diameter by the conjugate, and the product again by .7854, and the reſult will be the area.

Or multiply .7854 by one axe, and the product by the other.

EXAMPLES.

1. What is the area of an ellipſe whoſe tranſverſe diameter is 24, and the conjugate 18?

Here 24×18×.7854=339.2928=*area required.*

Or,

```
.7854
   24=tranſverſe.
------
 31416
15708
------
18.8496
     18=conjugate.
-------
1507968
188496
-------
339.2928=area before.
```

2. If the axes of an ellipſe be 35 and 25, what is the area? *Anſ.* 687.225.

3. Required the area of an ellipſis whoſe two axes are 70 and 50. *Anſ.* 2748.9.

* The demonſtration of this rule is contained in that of the next problem.

PROBLEM V.

To find the area of an elliptic ſegment, whoſe baſe is parallel to either of the axes of the ellipſe.

RULE.*

1. Divide the height of the ſegment by that axe of the ellipſe of which it is a part, and find, in the table, a circular ſegment whoſe verſed ſine is equal to the quotient.

2. Multiply the ſegment thus found, and the two axes of the ellipſe continually together, and the product will give the area required.

EXAMPLES.

1. Required the area of the elliptic ſegment EAF, whoſe height AG is 10, and the axes of the ellipſe 2AB and CD, 35 and 25 reſpectively.

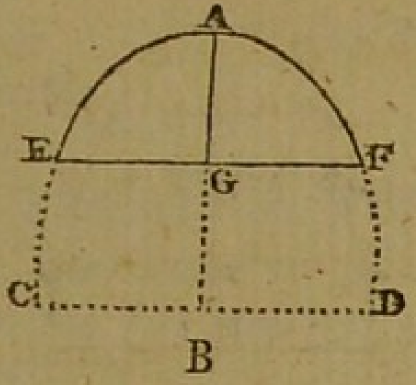
10.0000

---

* *Demon.* Let the tranſverſe diameter $2AB = t$, the conjugate $CD = c$, $AG = x$, and $EG = y$; then by the property of the curve we ſhall have $y = \frac{c}{t}\sqrt{tx - x^2}$, and the fluxion of the area $EAF = (y\dot{x}) = \frac{c}{t} \times \dot{x}\sqrt{tx - x^2}$. But $\dot{x} \times \sqrt{tx - x^2}$ is known to expreſs the fluxion of the correſponding circular ſegment, whoſe verſed ſine is x, and the diameter t. Let the fluent of this expreſſion therefore be denoted by A, and then the fluent of $\frac{c}{t} \times \dot{x}\sqrt{tx - x^2}$ will be $= \frac{c}{t} \times A$, from whence the rule is derived. Q. E. I.

Coroll. The ellipſe is equal to a circle whoſe diameter is a mean proportional between the two axes, and from hence the rule is formed for the whole ellipſe.

The



Here $\frac{10.0000}{35} = \frac{2.0000}{7} = .2857$ = *tabular versed sine.*

And the tabular segment belonging to this is .185153.

Whence .185153 × 35 (2 A B) × 25 (C D) = 6.480355 × 25 = 162.0210 = *area of the segment required.*

2. What is the area of an elliptic segment cut off by a double ordinate parallel to the conjugate axe, at the distance of 36 from the centre, the axes being 120 and 40? *Ans.* 536.7504.

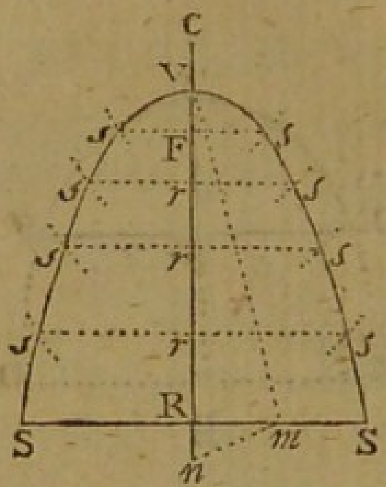
3. What is the area of a segment, cut off by an ordinate parallel to the transverse diameter, whose height is 5, the axes being 35 and 25? *Ans.* 97.845725.

PROBLEM VI.

To describe a parabola, any ordinate to the axe and its abscissa being given.

The area of an elliptic segment may also be found by the following rule.

Find the corresponding segment of the circle described upon the same axe to which the base of the segment is perpendicular. Then as this axe is to the other axe, so is the circular segment to the elliptic segment.



* *Construction.* 1. Bisect the given ordinate RS in *m*; join V*m*, and draw *mn* perpendicular to it, meeting the axe in *n*.

2. Make VC and VF each equal to R*n*, and F will be the focus of the curve.

3. Take any number of points *r*, *r*, &c. in the axe, through which draw the double ordinates S*r*S, S*r*S, &c. of an indefinite length.

4. With the radii CF, C*r*, &c. and centre F, describe arcs cutting the corresponding ordinates in the points *s*, *s*, &c. and the curve SVS drawn through all

---

* Since V*mn* is a right angled triangle, and *m*R is perpendicular to V*n*, $VR \times Rn = VR \times VF = Rm^2 = \frac{1}{4} RS^2$, which is a known property of the parabola when F is the focus. And, because $sF^2 = Cr^2 - rF^2 = \overline{Cr + rF} \times \overline{Cr - rF} = \overline{Cr + rF} \times CF = 2Vr \times 2VF = Vr \times 4VF$, therefore *s* is a point in the curve of a parabola, and the same may be shewn of any other point *s*.

Besides the methods above, for finding the focus, it may be found arithmetically as follows:

Divide the square of the ordinate by 4 times the abscissa, and the quotient will be the focal distance *v*F.

Several other methods of doing this, as well as of describing the curve itself, may also be found in Emerson's Conic Sections, and other performances.

all the points of interſection will be the parabola required.

Note. The line *s* F *s* paſſing through the focus F is called the parameter.

PROBLEM VII.

In a parabola, any three of the four following terms being given, viz. any two ordinates and their two abſciſſas, to find the fourth.

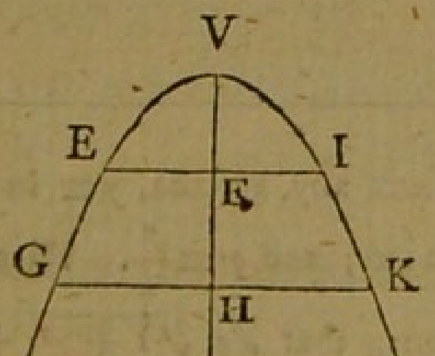
RULE.*

As any abſciſſa is to the ſquare of its ordinate, ſo is any other abſciſſa to the ſquare of its ordinate.

Or as the ſquare root of any abſciſſa is to its ordinate, ſo is the ſquare root of any other abſciſſa to its ordinate.

EXAMPLES.

1. The abſciſſa V F is 9, and its ordinate E F 6; required the ordinate G H, whoſe abſciſs V H is 16.



* If *x* and X be any two abſciſſas, and *y* and Y their correſponding ordinates, the equation of the curve will be $x Y^2 = $ Cap. X. y^2, which is the ſame as the rule.

Here $\sqrt{9}$ ($\sqrt{\text{VF}}$) : 6 (EF) :: $\sqrt{16}$ ($\sqrt{\text{VH}}$) : $\frac{6 \times \sqrt{16}}{\sqrt{9}} = \frac{6 \times 4}{3} = \frac{24}{3} = 8 =$ *ordinate* GH.

Or,

9 (VF) : 36 (EF^2) :: 16 (VH) : $\frac{16 \times 36}{9} = 16 \times 4 = 64 = GH^2$, *or* $8 = GH$ *as before.*

2. The two abſciſſas are 9 and 16, and their correſponding ordinates 6 and 8; from any three of theſe to find the fourth.

PROBLEM VIII.

To find the length of any arc of a parabola, cut off by a double ordinate.

RULE.*

To the ſquare of the ordinate add $\frac{4}{3}$ of the ſquare of the abſciſſa, and twice the ſquare root of the ſum will be the length of the curve required.

EXAM-

---

* *Demon.* Let $x =$ any abſciſſa, $y =$ its ordinate, $a = \frac{1}{2}$ the parameter of the axe, and $q = \frac{y}{a}$. Then it is ſhewn by the writers on fluxions, that $aq\sqrt{1+q^2} + a \times$ hyp. log. of $(q + \sqrt{1+q}) = 2y \times : 1 + \frac{q^2}{2.3} - \frac{q^4}{2.4.5} + \frac{3q^6}{2.4.6.7} - \frac{3.5q^8}{2.4.6.8.9}$ &c. $= c =$ length of the curve. But $\sqrt{1 + \frac{1}{3}q^2} = 1 + \frac{q^2}{2.3} - \frac{q^4}{2.4.9} + \frac{3q^6}{2.4.6.27}$ &c. agreeing with the former in the two firſt terms.

Therefore

EXAMPLES.

1. The abſciſſa VH is 2, and its ordinate GH 6: what is the length of the arc GVK?

Here $2^2\ (\text{VH}^2) \times \frac{4}{3} + 36\ (\text{GH}^2) = \frac{4 \times 4}{3} + 36 = \frac{16}{3} + 36 = 5.333$, &c. $+ 36 = 41.333333$.

```
And 41.3333333(6.429
    36              2
    ---          ------
124)533          12.858=length of the arc.
    496
    ----
1282)3733
     2564
     -------
12849)116933
      115941
      ------
        1292
```

PROBLEM IX.

To find the area of a parabola, its baſe and height being given.

RULE.*

Multiply the baſe by the height, and $\frac{2}{3}$ of the product will be the area required.

EXAM-

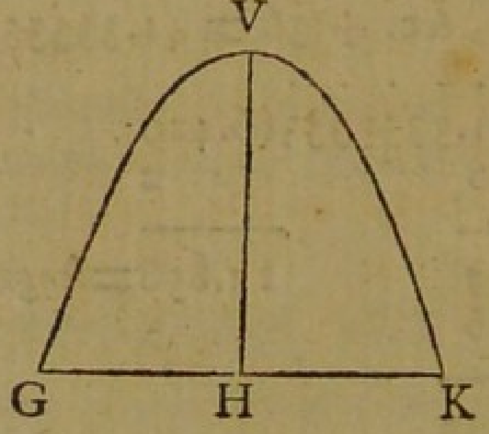
Therefore $\frac{c}{2y} = \sqrt{1 + \frac{1}{3}q^2}$ *nearly*; and conſequently $c = 2y\sqrt{1 + \frac{1}{3}q^2} = \sqrt{y^2 + \frac{4}{3}x^2}$ the ſame as the rule. Q. E. D.

* *Demon.* Let $\text{VH} = x$, $\text{GH} = y$, and the parameter $= p$. Then $px = y^2$ or $\sqrt{px} = y$ by the nature of the curve.

Whence

EXAMPLES.

1. What is the area of the parabola GVK, whose height VH is 12, and the base or double ordinate GK 16?



Here $16\ (GK) \times 12\ (VH) \times \frac{2}{3} = \frac{16 \times 12 \times 2}{3} = 16 \times 4 \times 2 = 128 =$ *area required.*

2. The abscissa is 12, and the double ordinate or base 38: what is the area? *Ans.* 304.

3. What is the area of a parabola whose abscissa is 10, and ordinate 8? *Ans.* $106\frac{2}{3}$.

PROBLEM X.

To find the area of the frustum of a parabola.

Whence the fluxion of the area ($= y\dot{x}$) $= \dot{x}\sqrt{px}$, and its fluent $= \frac{2}{3}x \times \sqrt{px}$.

But because $y = \sqrt{px}$, therefore $\frac{2}{3}x \times \sqrt{px} = \frac{2}{3}xy =$ area of the parabola, which is the same as the rule.

Coroll. Every parabola is $= \frac{2}{3}$ of its circumscribing parallelogram.

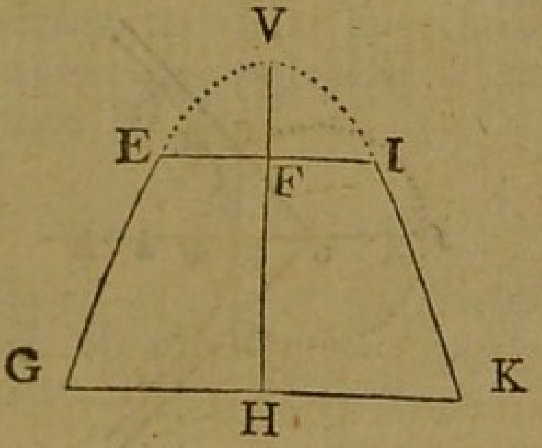
RULE.

RULE.*

Divide the difference of the cubes of the two ends of the fruſtum by the difference of their ſquares, and this quotient multiplied by $\frac{2}{3}$ of the altitude, will give the area required.

EXAMPLES.

1. In the parabolic fruſtum GEIK, the two parallel ends EI and GK are 6 and 10, and the altitude, or part of the abſciſſa FH, is 3: what is the area?



Here $\overline{10^3-6^3}$ $(GK^3-EI^3) \div \overline{10^2-6^2}$ (GK^2-EI^2) $= \frac{10^3-6^3}{10^2-6^2} = \frac{1000-216}{100-36} = \frac{784}{64} = \frac{98}{8} = \frac{49}{4} = 12.25.$

And $12.25 \times \frac{2 \times 3}{3} = 12.25 \times 2 = 24.5 =$ *area required.*

---

* *Demon.* Let $D = GK$, $d = EI$, and $a = FH$.
Then by the nature of the curve $D^2-d^2 : a :: D^2 : \frac{aD^2}{D^2-d^2} = VH$, and $D^2-d^2 : a :: d^2 : \frac{ad^2}{D^2-d^2} = VF$.

And therefore $\frac{2}{3} \times \frac{aD^2}{D^2-d} - \frac{2}{3} \times \frac{ad^2}{D^2-d^2} = \frac{2}{3} a \times \frac{D^3-d^3}{D^2-d^2} =$ area of the fruſtum. Q.E.D.

 2. The

2. The greater end of the frustum is 24, the lesser end is 20, and their distance $5\frac{1}{2}$: what is the area?

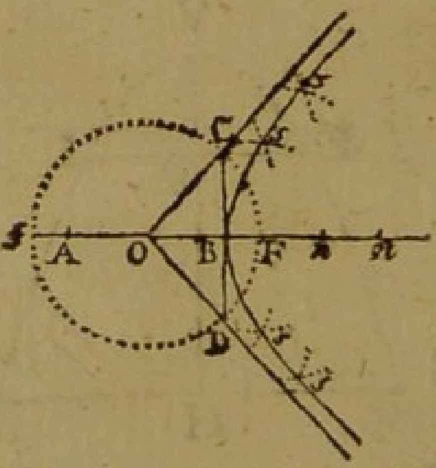
Ans. 121.3.

3. Required the area of the parabolic frustum, the greater end of which is 10, the less 6, and the height 4.

Ans. $32\frac{2}{3}$.

PROBLEM XI.

To construct an hyperbola, the transverse and conjugate diameters being given.



* *Construction.* 1. Make A B the transverse diameter, and C D perpendicular to it, the conjugate.

Note. Any parabolic frustum, is equal to a parabola of the same altitude, whose base is equal to one end of the frustum, increased by a third proportional to the sum of the two ends and the other end.

* The sum of two lines drawn from the foci of an ellipse to any point in the curve, is equal to its transverse diameter.

In like manner the difference of two lines drawn from the foci of any hyperbola to any point in the curve, is equal to its transverse diameter; as is shewn by the writers on *conics*.

But the arcs intersecting each other in *s*, *s*, &c. are described from the foci *f* and F, and with the distances A *n* and B *n*, whose difference is A B, and therefore the points *s*, *s*, &c. are in the curve of an hyperbola.

2. Bisect

2. Bisect A B in O, and from O with the radius O C, or O D, describe the circle D f C F, cutting A B produced in F and f, which points will be the two foci.

3. In A B produced take any number of points, n, n, &c. and from F and f, as centres, with the distances B n, A n, as radii, describe arcs cutting each other in s, s, &c.

4. Through the several points s, s, &c. draw the curve s B s, and it will be the hyperbola required.

5. If straight lines be drawn from the point O, through the extremities C D of the conjugate axis, they will be the asymptotes of the hyperbola, whose property it is to approach continually to the curve, and yet never to meet it.

PROBLEM XII.

In an hyperbola, any three of the four following terms being given, viz. the transverse and conjugate diameters, an ordinate ana its abscissa, to find the fourth.

CASE I.

The transverse and conjugate diameters, and the two abscissas being known, to find the ordinate.

RULE.*

As the transverse diameter,
Is to the conjugate;
So is the square root of the product of the two abscissas,
To the ordinate required.

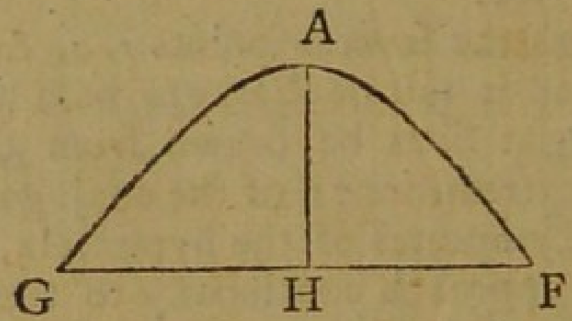
---

* Let t = transverse diameter, c = conjugate, x = abscissa, and y = ordinate. Then the general property of the curve is $t^2 : c^2 :: x \times (t + x) : y^2$; and from this analogy all the cases of this problem are deduced.

EXAM-

EXAMPLES.

1. In the hyperbola G A F, the tranſverſe diameter is 120, the conjugate 72, and the abſciſſa A H is 40: required the ordinate F H.



$$120\ (\textit{tranſ.}) : 72\ (\textit{conj.}) :: \surd(160 \times 40) : \frac{72 \times \surd(160 \times 40)}{120} = \frac{6 \times \surd(160 \times 40)}{10} = \frac{3}{5}\surd(160 \times 40) = \frac{3}{5}\surd 6400 = \frac{3}{5} \times 80 = \frac{3 \times 80}{5} = 3 \times 16 = 48 =$$ *ordinate* F H.

2. The tranſverſe diameter is 24, the conjugate 21, and the leſſer abſciſſa 8: what is its ordinate?

Anſ. 14.

3. The tranſverſe diameter of an hyperbola is 50, the conjugate 30, and the greater abſciſſa 12: required the ordinate.

CASE II.

The tranſverſe and conjugate diameters, and an ordinate being given, to find the two abſciſſas.

Note. In the hyperbola, the leſs abſciſſa added to the axis, gives the greater.

RULE.

RULE.

As the conjugate diameter is to the tranſverſe,
So is the ſquare root of the ſum of the ſquares of the ordinate and ſemi-conjugate,
To the diſtance between the ordinate and the centre, or half the ſum of the abſciſſas.

Then the ſum of this diſtance and the ſemi-tranſverſe will give the greater abſciſſa, and their difference the leſſer abſciſſa required.*

EXAMPLES.

The tranſverſe diameter is 120, the conjugate 72, and the ordinate 48: what are the two abſciſſas?

1296 = *ſquare of the ſemi-conjugate.*
2304 = *ſquare of the ordinate.*

3600(60 = *ſquare root.*
36

00

As 72 : 120 : : 60
120

72)7200(100 = ½ *ſum of the abſciſſas.*
72 60 = *ſemi-tranſverſe.*

160 = *greater abſciſſa.*

40 = *leſſer abſciſſa.*

---

* This rule in ſpecies is $\frac{t}{c}\sqrt{\frac{1}{4}c^2+y^2} = \frac{1}{2}t = x$, = greater or leſs abſciſſa, according as the upper or under ſign is uſed.

2. The

2. The tranſverſe and conjugate diameters are 24 and 21: required the two abſciſſas to the ordinate 14.

Anſ. 32 *and* 8.

3. The tranſverſe being 60, and the conjugate 36; required the two abſciſſas to the ordinate 24.

Anſ. 80 *and* 20.

CASE III.

The tranſverſe diameter, the two abſciſſas and the ordinate being given, to find the conjugate.

RULE.

As the ſquare root of the product of the two abſciſſas,
Is to the ordinate;
So is the tranſverſe diameter,
To the conjugate.*

EXAMPLES.

1. The tranſverſe diameter is 120, the ordinate is 48, and the two abſciſſas are 160 and 40: required the conjugate.

$$\begin{array}{r} 160 \\ 40 \\ \hline \dot{6}4\dot{0}0(80 \\ 64 \\ \hline 00 \end{array}$$

* This rule expreſſed algebraically is $ty \div \sqrt{x \times (t+x)} \times x = c =$ conjugate diameter.

As

As 80 : 48 :: 120 *the transverse axis.*

```
             48
            ---
            960
           480
          -----
     8.0)576.0
          -----
```

72 = *conjugate required.*

2. The transverse diameter is 24, the ordinate 14, and the abscissa 8 and 32: required the conjugate.

Ans. 21.

CASE IV.

The conjugate diameter, the ordinate and two abscissas being given, to find the transverse.

RULE.

1. Add the square of the ordinate to the square of the semi-conjugate, and find the square root of their sum.

2. Take the sum or difference of the semi-conjugate and this root, according as the less or greater abscissa is used, and then say,

As the square of the ordinate,
Is to the product of the abscissa and conjugate;
So is the sum, or difference, above found,
To the transverse required.*

---

* This rule in algebraic terms is $\frac{cx}{y^2} \times (\sqrt{\frac{1}{4}c^2 + y^2} = \frac{1}{2}c) = t$ = transverse diameter.

EXAM-

EXAMPLES.

1. The conjugate diameter is 72, the ordinate is 48, and the leſſer abſciſſa 40: what is the tranſverſe?

Here $\sqrt{48^2+36^2}=\sqrt{2304+1296}=\sqrt{3600}=60$, *and* $60+36=96$.

Alſo $72\times 40=2880=$ *product of the abſciſſa and conjugate. Whence,*

```
As 2304 : 2880 :: 96
                  96
               -----
               17280
              25920
              ------
        2304)276480(120=tranſverſe required.
              2304
              ----
               4608
               4608
               ----
```

2. The conjugate diameter is 21, the leſſer abſciſſa 8, and its ordinate 14: required the tranſverſe.

Anſ. 24.

3. Required the tranſverſe diameter of the hyperbola, whoſe conjugate is 36, the leſſer abſciſſa being 20. *Anſ.* 60.

PROBLEM XIII.

To find the length of any arc of an hyperbola, beginning at the vertex.

RULE.

RULE.*

1. To 19 times the tranſverſe, add 21 times the parameter of the axis, and multiply the ſum by the quotient of the abſciſſa divided by the tranſverſe.

2. To 9 times the tranſverſe, add 21 times the parameter, and multiply the ſum by the quotient of the abſciſſa divided by the tranſverſe.

3. To each of the products, thus found, add 15 times the parameter, and divide the former by the latter; then this quotient being multiplied by the ordinate will give the length of the arc *nearly*.

EXAMPLES.

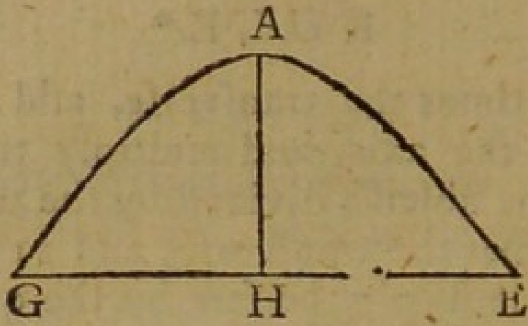
1. In the hyperbola GAE, the tranſverſe diameter is 80, the conjugate 60, the ordinate GH 10, and the abſciſſa AH 2.1637: required the length of the arc AG.

---

* *Demon.* Let $t =$ ſemi-tranſverſe axe, $c =$ ſemi-conjugate, $x =$ ordinate, and $y =$ abſciſſa. Then will $y \times (1 + \frac{t^2}{6c^4} y^2 - \frac{t^4 + 4t^2c^2}{40c^8} y^4 + \frac{t^6 + 4t^4c^2 + 8t^2c^4}{112c^{12}} y^6$, &c.) $=$ length of the arc, as is ſhewn by the writers on fluxions.

But $x = \frac{t}{c}\sqrt{c^2 + y^2} - a$, and $\frac{2c}{t} =$ parameter $= p$, by the nature of the curve. Conſequently the rule becomes $(15p + \overline{19t + 21p} \times \frac{x}{t}) \div (15p + \overline{9t + 21p} \times \frac{x}{t}) \times y = (15pt + 19tx + 21px) \div (15pt + 9tx + 21px) \times y = y \times : 1 + \frac{2x}{3p} - \frac{2x^2}{5p}$ &c. which by ſubſtituting the values of x and p, and expanding the terms, gives a ſeries, agreeing nearly in the three firſt terms with the former, and therefore the rule is an approximation.

If



Here $80 : 60 : 60 : \frac{6 \times 60}{80} = \frac{3 \times 60}{4} = 3 \times 15 = 45$ $=$ *parameter.*

And $\overline{80 \times 19 \times 45 \times 21} \times \frac{2.1637}{80} = \overline{1520 + 945} \times .02704 = 2465 \times .02704 = 66.6536.$

Also $\overline{80 \times 9 + 45 \times 21} \times \frac{2.1637}{83} = \overline{720 + 645} \times .0274 = 1665 \times .02704 = 45.0216.$

Whence $\overline{675 + 66.6536} \div \overline{675 \times 45.0216} = 741.6536 \div 720.0216 = 1.03004$; *and* $1.03004 \times 10 = 10.3004$ $=$ *length of the arc required.*

2. The transverse diameter of an hyperbola is 120, the conjugate 72, the ordinate 48, and the abscissa 40: required the length of the curve.

Ans. 62.6496.

3. Required the whole length of the curve of an hyperbola, to the ordinate 16; the transverse and conjugate axes being 10 and 60. *Ans.* 20.601.

PROBLEM XIV.

To find the area of an hyperbola, the transverse, conjugate and abscissa being given.

If $t =$ semi-transverse, $c =$ semi-conjugate, and $y =$ ordinate drawn from the end of the required arc; then $y \times (1 + \frac{t^2 y^2}{6c^4} A - \frac{t^2 + 4c^2}{c^2} \cdot \frac{3y^2}{20} B + \frac{t^4 + 4t^2c^2 + 8c^4}{t^2 + 4c^2} \cdot \frac{5y^2}{14c^4} C$ &c.$) =$ length of the arc.

RULE.

RULE.*

1. To the product of the tranſveſe and abſciſſa, add $\frac{5}{7}$ of the ſquare of the abſciſſa, and multiply the ſquare root of the ſum by 21.

2. Add 4 times the ſquare root of the product of the tranſverſe and abſciſſa, to the product laſt found, and divide the ſum by 75.

3. Divide 4 times the product of the conjugate and abſciſſa by the tranſverſe, and this laſt quotient multiplied by the former will give the area required *nearly*.

EXAMPLES.

In the hyperbola GAE, the tranſverſe axis is 30, the conjugate 18, and the abſciſſa or height AH is 10: what is the area?

Here

---

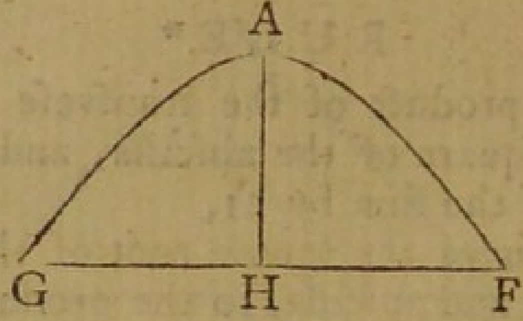
* *Demon.* Let $t =$ tranſverſe diameter, $c =$ conjugate, $x =$ abſciſſa, $y =$ ordinate, and $z = \frac{x}{t2 + x}$. Then it is well known that $4xy \times (\frac{1}{3} - \frac{1}{1 \cdot 3 \cdot 5} z - \frac{1}{3 \cdot 5 \cdot 7} z^2 - \frac{1}{5 \cdot 7 \cdot 9} z^3$, &c. $=$ area of the hyperbola.)

But $\frac{ty}{\sqrt{ta + x^2}} = c =$ conjugate axe, by the nature of the hyperbola. Conſequently the expreſſion for the rule $=$ $\frac{4cx}{t} \times \frac{21\sqrt{tx + \frac{5}{7}x + 4\sqrt{tx}}}{75} = 4xy \times \frac{21\sqrt{tx + \frac{5}{7}x^2 + 4\sqrt{tx}}}{\sqrt{tx - x^2}}$
And this thrown into a ſeries will very nearly agree with the former; which ſhews the rule to be an approximation.

Q. E. I.

Rule 2. If $2Y$, $2y =$ baſes, V, and v their diſtances from the centre, and the other letters as before, then will $VY - vy - \frac{tc}{4} \times$ hyp. log. of $\frac{Y + cV}{ty + cv} =$ area of the fruſtum of the hyperbola.

Rule



Here $21\sqrt{30\times10+\frac{5}{7}\times10^2}=21\sqrt{300+500\div7}=$ $21\sqrt{300+71.42857}=21\sqrt{371.42857}=21\times19.272$ $=404.712$.

And $(4\sqrt{30\times10}+404.712)\div75=(4\sqrt{300}+404.712)\div75=(4\times17.3205+404.712)\div75=(69.282+404.712)\div75=473.994\div75=6.3199$.

Whence $\frac{18\times19\times4}{30}\times6.3199=\frac{18\times4}{3}\times6.3199=6\times4\times6.3199=24\times6.3196=151.6776=$ *area required.*

2. The tranſverſe diameter is 100, the conjugate 60, and the leſſer abſciſs 50; what is the area of the hyperbola? *Anſ.* 3220.363472.

3. Required the area of the hyperbola to the abſciſſa 25, the two axes being 50 and 30. *Anſ.* 805.0909.

Rule 3. If t be put = tranſverſe axis, c = conjugate, and x = abſciſſa, the area of a ſegment of an hyperbola, cut off by a double ordinate, will be $=\frac{4\sqrt{tx+\frac{5}{4}x^2}+\sqrt{tx}}{15}\times\frac{4cx}{t}$ extremely near.

OF

OF THE

MENSURATION

OF

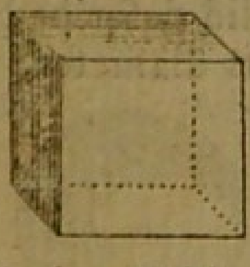
SOLIDS.

DEFINITIONS.

1. THE *meaſure* of any ſolid body, is the whole capacity or content of that body, when conſidered under the triple dimenſions of length, breadth, and thickneſs.

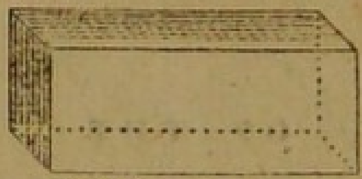
2. A *cube* whoſe ſide is one inch, one foot, or one yard, &c. is called the *meaſuring unit*; and the content or ſolidity of any figure, is computed by the number of thoſe cubes contained in that figure.

3. A *cube* is a ſolid contained by ſix equal ſquare ſides.

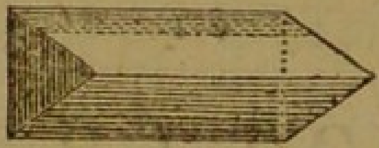


4. A *parallelopipedon* is a ſolid contained by ſix quadrilateral planes, every oppoſite two of which are equal and parallel.

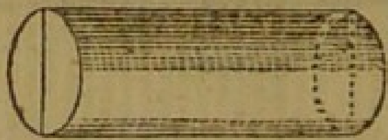
5. A



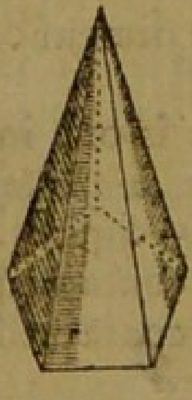
5. A *prifm* is a folid whofe ends are two equal, parallel, and fimilar plane figures, and its fides parallelograms.



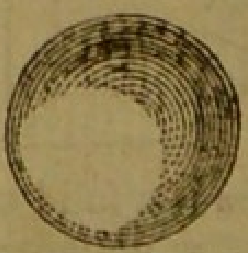
6. A *cylinder* is a folid defcribed by the revolution of a right angled parallelogram about one of its fides, which remains fixed.



7. A **pyramid* is a folid whofe fides are all triangles meeting in a point at the vertex, and the bafe any plane figure whatever.



8. A *fphere* is a folid defcribed by the revolution of a femi-circle about its diameter, which remains fixed.



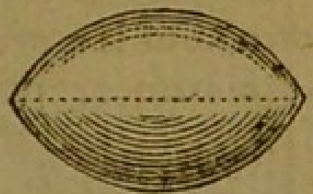
* The definition of a cone has been given already.

9. The

9. The centre of a ſphere is a point within the figure, every where equally diſtant from the convex ſurface of it.

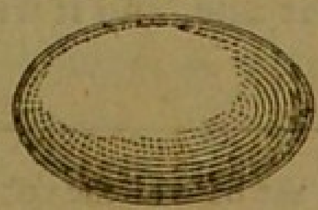
10. The diameter of the ſphere is a ſtraight line paſſing through the centre, and terminated both ways by the convex ſuperficies.

11. A *circular ſpindle* is a ſolid generated by the revolution of a ſegment of a circle about its chord, which remains fixed.



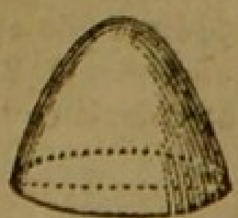
12. A *ſpheroid* is a ſolid generated by the revolution of a ſemi-ellipſis about one of its diameters, which is conſidered as quieſcent.

The ſpheroid is called *prolate*, when the revolution is made about the tranſverſe diameter, and *oblate* when it is made about the conjugate diameter.



13. *Elliptic, parabolic*, and *hyperbolic ſpindles*, are generated in the ſame manner as the circular ſpindle, the double ordinate of the ſection being always fixed or quieſcent.

14. *Parabolic* and *hyperbolic conoids*, are ſolids formed by the revolution of a ſemi-parabola or hyperbola about its tranſverſe axis, which is conſidered as quieſcent.



15. The

15. The *ſegment* of a pyramid, ſphere, or any other ſolid, is a part cut off from the top by a plane parallel to the baſe of that figure.

16. A *fruſtum* or *trunk*, is the part that remains at the bottom after the ſegment is cut off.

17. The *zone of a ſphere*, is that part which is intercepted between two parallel planes; and when thoſe planes are equally diſtant from the centre, it is called the middle zone of the ſphere.

18. The height of a ſolid is a perpendicular, drawn from its vertex to the baſe or plane on which it is ſuppoſed to ſtand.

PROBLEM I.

To find the ſolidity of a cube, the height of one of its ſides being given.

RULE.*

Multiply the ſide of the cube by itſelf, and that product again by the ſide, and it will give the ſolidity required.

---

* *Demon.* Conceive the baſe of the cube to be divided into a number of little ſquares, each equal to the *ſuperficial meaſuring unit.*

Then will thoſe ſquares be the baſes of a like number of ſmall cubes, which are each equal to the *ſolid meaſuring unit.*

But the number of little ſquares contained in the baſe of the cube are equal to the ſquare of the ſide of that baſe, as has been ſhewn already.

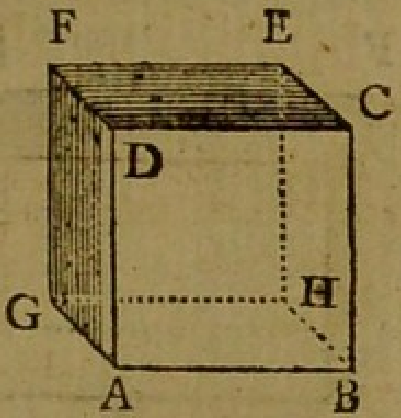
And conſequently, the number of ſmall cubes contained in the whole figure, muſt be equal to the ſquare of the ſide of the baſe multiplied by the height of that figure; or, which is the ſame thing, the ſquare of the ſide of the baſe multiplied by the baſe, is equal to the ſolidity. Q. E. D.

Note. The ſurface of the cube is equal to 6 times the ſquare of its ſide.

EXAM-

EXAMPLES.

8. The side AB, or BC of the cube ABCDFGHE, is 25.5: what is the solidity?



Here $AB^3 = \overline{25.5}|^3 = 25.5 \times 25.5 \times 25.5 = 25.5 \times 650.25 = 16581.375$ *the answer, or content of the cube.*

2. The side of a cube is 15 inches: what is the solidity?

so. in. pa.
Ans. 1 11 5

3. What is the solidity of a cube whose side is 17.5 inches? *Ans.* 3.101 *feet.*

PROBLEM II.

To find the solidity of a parallelopipedon.

RULE.*

Multiply the length by the breadth, and that product again by the depth or altitude, and it will give the solidity required.

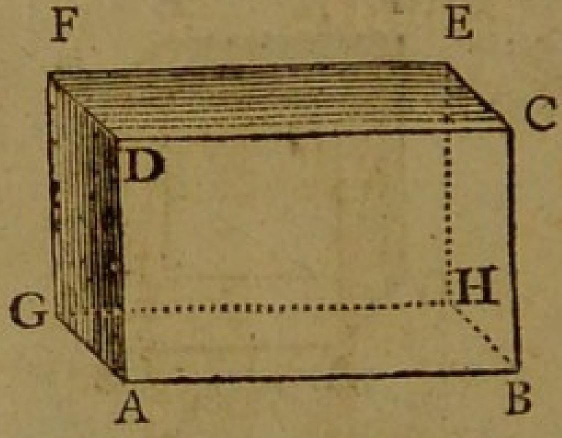
---

* The reason of this rule, as well as of the following ones for the prism and cylinder, is the same as that for the cube.

Note. The surface of the parallelopipedon is equal to the sum of the areas of each of its sides or ends.

EXAMPLES.

1. Required the solidity of the parallelopipedon ABCDFEGH, whose length AB is 8 feet, its breadth FD $4\frac{1}{2}$ feet, and the depth or altitude AD $6\frac{3}{4}$ feet?



Here AB × AD × FD $= 8 \times 6.75 \times 4.5 = 54 \times 4.5 =$ 243 *solid feet, the contents of the parallelopipedon required.*

2. The length of a parallelopipedon is 15 feet, and each side of its square base 21 inches: what is the solidity? *Ans.* 45.9 *feet.*

3. What is the solidity of a block of marble, whose length is 10 feet, its breadth $5\frac{3}{4}$ feet, and the depth $3\frac{1}{2}$ feet? *Ans.* 201.25 *feet.*

PROBLEM III.

To find the solidity of a prism.

RULE.*

Multiply the area of the base into the perpendicular height of the prism, and the product will be the solidity.

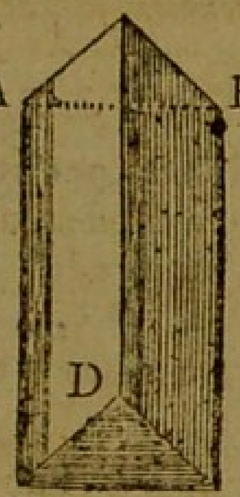
---

* The surface of a prism, is equal to the sum of the areas of the two ends, and each of its sides.

EXAM-

EXAMPLES.

1. What is the ſolidity of the triangular priſm ABCFED, whoſe length AB is 10 feet, and either of the equal ſides BC, CD, or DB, of one of its equilateral ends BCD, $2\frac{1}{2}$ feet?



* *Here* $\frac{1}{4} \times 2.5^2 \times \sqrt{3} = \frac{1}{4} \times 6.25 \times \sqrt{3} = 1.5625 \times \sqrt{3} = 1.5625 \times 1.732 = 2.70625 =$ *area of the baſe* BCD.

Or, $\frac{2.5+2.5+2.5}{2} = \frac{7.5}{2} = 3.75 = \frac{1}{2}$ *ſum of the ſides,* BC, CD, DB *of the triangle* CDB.

And $3.75 - 2.5 = 1.25$, $\therefore$ 1.25, 1.25 *and* $1.25 = 3$ *differences.*

Whence $\sqrt{3.75 \times 1.25 \times 1.25 \times 1.25} = \sqrt{3.75 \times 1.25^3} = \sqrt{7.32521875} = 2.7063 =$ *area of the baſe as before.*

And $2.7063 \times 10 = 27.063$ *ſolid feet, the content of the priſm required.*

2. What is the ſolidity of a triangular priſm, whoſe length is 18 feet, and one ſide of the equilateral end $1\frac{1}{2}$ feet?

Anſ. 17.50859 *feet.*

* This rule may be ſeen in the Notes to Problem III. p. 48.

2. Required the ſolidity of a priſm whoſe baſe is a hexagon, ſuppoſing each of the equal ſides to be 1 foot 4 inches, and the length of the priſm 15 feet.

Anſ. 69.282 *feet.*

PROBLEM IV.

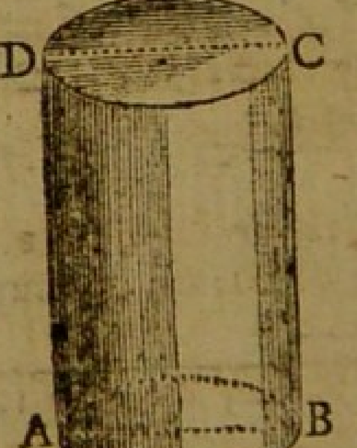
To find the convex ſurface of a cylinder.

RULE.*

Multiply the periphery or circumference of the baſe, by the height of the cylinder, and the product will be the convex ſurface required.

EXAMPLES.

1. What is the convex ſurface of the right cylinder A B C D, whoſe length B C is 20 feet, and the diameter of its baſe A B 2 feet?



* *Demon.* If the periphery of the baſe be conceived to move, in a direction parallel to itſelf, it will generate the convex ſuperficies of the cylinder; and therefore the ſaid periphery being multiplied by the length of the cylinder, will be equal to that ſuperficies. Q. E. D.

Note. If twice the area of either of the ends be added to the convex ſurface, it will give the whole ſurface of the cylinder.

Here

Here 3.14159 × 2 = 6.28318 = *periphery of the base* A. B.

And 6.28318 × 20 = 125.6636 *square feet, the convexity required.*

2. What is the convex surface of a right cylinder, the diameter of whose base is 30 inches, and the length 60 inches? *Ans.* 5654.862 *inches.*

3. Required the convex superficies of a right cylinder, whose circumference is 8 feet 4 inches, and its length 14 feet. *Ans.* 116.666 &c. *feet.*

PROBLEM V.

To find the solidity of a cylinder.

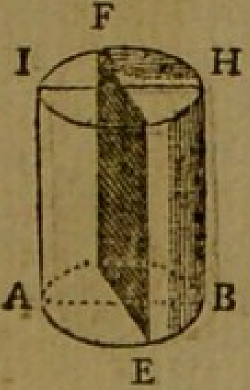
RULE.*

Multiply the area of the base by the perpendicular height of the cylinder, and the product will be the solidity.

EXAM-

* The four following cases contain all the rules for finding the superficies, and solidities of *cylindric ungulas.*

I. *When the section is parallel to the axis of the cylinder.*



Rule. 1. Multiply the length of the arc line of the base by the height of the cylinder, and the product will be the *curve surface.*

2. Multiply the area of the base by the height of the cylinder, and the product will be the *solidity.*

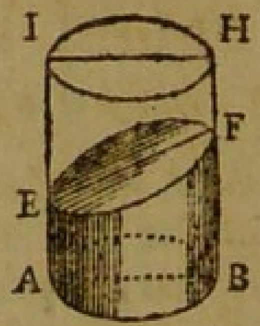
 II. *When*

EXAMPLES.

1. What is the ſolidity of the cylinder ABCD, the diameter of whoſe baſe AB is 30 inches, and the height BC 50 inches?

Here

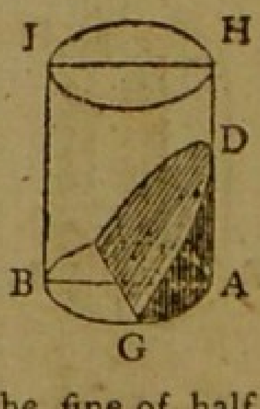
II. *When the ſection paſſes obliquely through the oppoſite ſides of the cylinder.*



Rule. 1. Multiply the circumference of the baſe of the cylinder by half the ſum of the greateſt and leaſt length, of the ungula, and the product will be the *curve ſurface.*

2. Multiply the area of the baſe of the cylinder by half the ſum of the greateſt and leaſt lengths of the ungula, and the product will be the *ſolidity.*

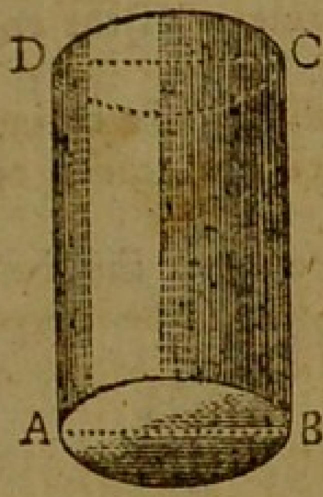
III. *When the ſection paſſes through the baſe of the cylinder, and one of its ſides.*



Rule. 1. Multiply the ſine of half the arc of the baſe by the diameter of the cylinder, and from this product ſubtract the product of the arc and coſine.

2. Multiply the difference, thus found, by the quotient of the height divided by the verſed ſine, and the product will be the *curve ſurface.*

1. From



Here $.7854 \times 30^2 = .7854 \times 900 = 706.86 =$ *area of the base* A B.

And $706.86 \times 50 = 35343$ *cubic inches*; *or* $\frac{35343}{1728} = 20.4531$ *solid feet, the answer required.*

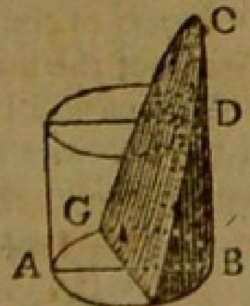
2. What

---

1. From ⅔ of the cube of the right sine of half the arc of the base, subtract the product of the area of the base and the cosine of the said half arc.

2. Multiply the difference, thus found, by the quotient arising from the height divided by the versed sine, and the product will be the *solidity*.

IV. *When the section passes obliquely through both ends of the cylinder.*



Rule. 1. Conceive the section to be continued, till it meets the side of the cylinder produced; then say, as the difference

 of

2. What is the solidity of a cylinder whose height is 5 feet, and the diameter of the end 2 feet?

Ans. 15.708 *feet.*

3. What is the solidity of a cylinder whose height is 20 feet, and the circumference of its base 20 feet also? *Ans.* 636.64 *feet.*

4. The circumference of the base of an oblique cylinder is 20 feet, and the perpendicular height 19.318: what is the solidity? *Ans.* 614.926 *feet.*

PROBLEM VI.

To find the convex surface of a right cone.

RULE.*

Multiply the circumference of the base by the slant height, or the length of the side of the cone, and half the product will be the surface required.

EXAM-

---

of the versed sines of half the arcs of the two ends of the ungula, is to the versed sine of half the arc of the lesser end, so is the height of the cylinder to the part of the side produced.

2. Find the surface of each of the ungulas, thus formed, by case the third, and their difference will be the *surface required.*

3. In like manner find the solidities of each of the ungulas, and their difference will be the *solidity required.*

* *Demon.* Let AB $= a$, BC $= b$, $3.1416 = p$, and ED $= y$.

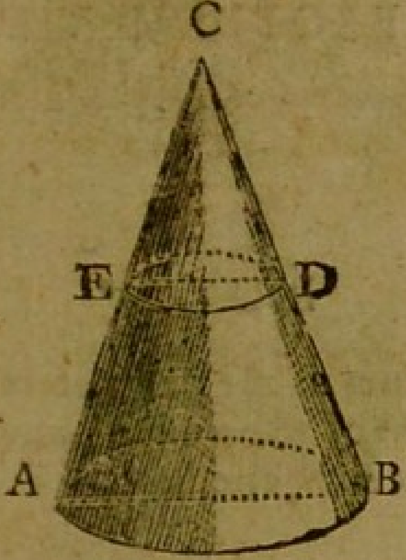
Then $a : b :: y : \frac{by}{a} =$ DC; and $py =$ circumference of the circle ED.

But

. From

EXAMPLES.

1. The diameter of the base A B is 3 feet, and the slant height A C or B C 15 feet: required the convex surface of the cone A C B.



Here $3.1416 \times 3 = 9.4248 =$ *circumference of the base* A B.

And, $\frac{9.4248 \times 15}{2} = \frac{141.3720}{2} = 70.686$ *square feet, the convex surface required.*

2. The diameter of a right cone is 4.5 feet, and the slant height 20 feet: required the convex surface.

Ans. 141.372 *feet.*

But $py \times \frac{b\dot{y}}{a} =$ fluxion of the surface of C E D, and its fluent $= \frac{pby^2}{2a}$; which, when $y = a$, becomes $\frac{pba}{2} =$ convex surface of the whole cone. Q. E. D.

To find the surface of a right pyramid.

Rule. Multiply the perimeter of the base by the length of the side, or slant height of the cone, and half the product will be the surface required.

 3. The

3. The circumference of the base is 10.75, and the slant height 18.25: what is the convex surface?

Ans. 98.09375.

PROBLEM VII.

To find the convex surface of the frustum of a right cone.

RULE.*

Multiply the sum of the perimeters of the two ends, by the slant height of the frustum, and half the product will be the surface required.

---

* *Demon.* Let the perimeter of the circle AB = P, that of ED = p, BD = b, and the rest as in the last problem.

Then P : p :: b (BC) : CD; and, by division, P − p :: b − CD (b) : CD = $\frac{pb}{P-p}$; but P × ($b + \frac{pb}{P-p}$) = twice the convex surface of the whole cone, by the last rule; and also $p \times \frac{pb}{P-p}$ = twice the convex surface of the part ECD. Therefore $P \times (b + \frac{pb}{P-p}) - p \times \frac{pb}{P-p} = bP + \overline{P-p} \times \frac{pb}{P-p} = bP + bp = \overline{P+p} \times b$ = twice the convex surface of the frustum ABDE; and the half thereof is $\frac{(P \times p) \times b}{2}$, which is the same as the rule. Q. E. D.

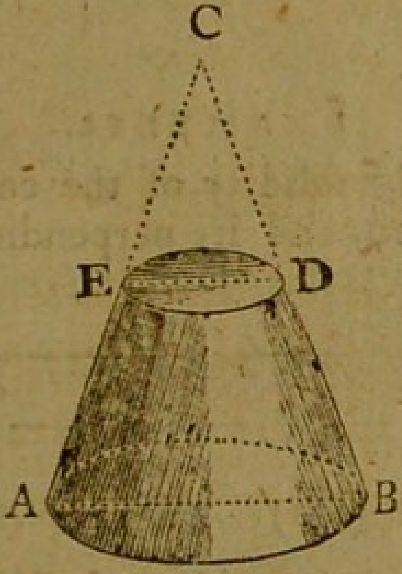
To find the surface of the frustum of a right pyramid.

Rule, Multiply the sum of the perimeters of the end by the slant height, and half the product will be the surface required.

EXAM-

EXAMPLES.

1. In the fruſtum ABED, the circumferences of the two ends AB and DE are 22.5 and 15.75 reſpectively, and the ſlant height BD is 26: what is the convex ſurface?



Here $\frac{(22.5+15.75) \times 26}{2} = \overline{22.5+15.75} \times 13 =$ $38.25 \times 15 = 497.25 =$ *convex ſurface required.*

2. What is the convex ſurface of the fruſtum of a right cone, the circumference of the greater end being 30 feet, that of the leſſer end 10 feet, and the length of the ſlant ſide 20 feet? *Anſ.* 400 *feet.*

3. What is the convex ſurface of the fruſtum of a right cone, the diameters of the ends being 8 and 4 feet, and the length of the ſlant ſide 20 feet?

Anſ. 376.992 *feet.*

4. If a ſegment of 6 feet ſlant height be cut off a cone whoſe ſlant height is 30 feet, and circumference of its baſe 10 feet: what is the ſurface of the fruſtum?

Anſ. 144 *feet.*

PROBLEM VIII.

To find the ſolidity of a cone or pyramid.

RULE.*

Multiply the area of the baſe by $\frac{1}{3}$ of the perpendicular height of the cone, and the product will be the ſolidity.

EXAMPLES.

1. Required the ſolidity of the cone A C B, whoſe diameter A B is 20, and its perpendicular height C S 24.

---

* *Demon.* Let S C $= a$, C $s = x$, and A $=$ area of the baſe of the cone A C B.

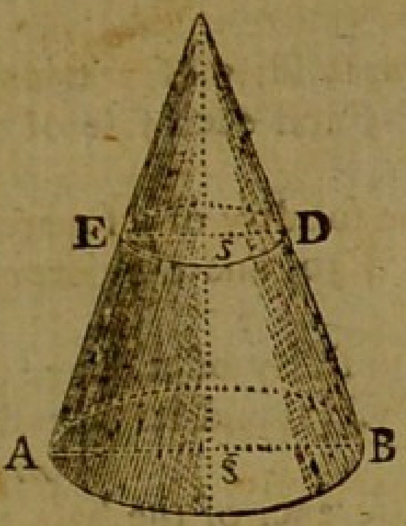
Then a^2 (C S^2) : x^2 (C s^2) : : A B^2 : E D^2 (by ſim. $\triangle s$) : : A : $\frac{A x^2}{a}$, ($=$ area of the circle E D) becauſe all the circles are as the ſquares of their diameters.

But $\frac{A x^2}{a^2} \times \dot{x} =$ fluxion of the cone E C D, and its fluent $=$ $\frac{A x^3}{3a^2}$; which, when $x = a$, becomes $\frac{A a}{3} = A \times \frac{a}{3}$ for the ſolidity of the whole cone. Q. E. D.

In the pyramid C E D B it will be a^2 (C S^2) : x^2 (C s^2) : : C E^2 : Ce^2 : : E D^2 : eo^2 (by ſim. $\triangle s$) : : A (area of the baſe E B) : $\frac{A x^2}{a^2}$ (area of the polygon eb) becauſe all ſimilar figures are as the ſquares of their like ſides.

But $\frac{A x^2}{a^2} \times \dot{x} =$ fluxion of the pyramid Ceob, and its correct fluent $= A \times \frac{a}{3}$ the ſame as in the cone; and this rule is general, let the figure of the baſe be what it will.

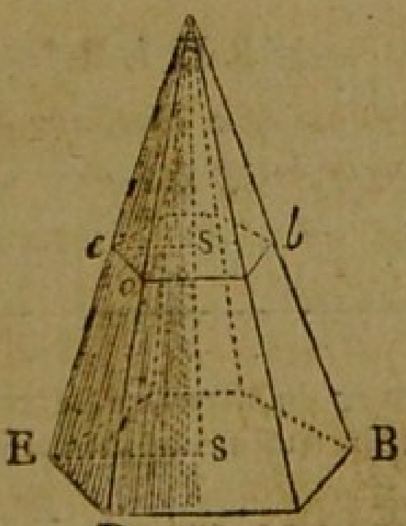
Here



Here $.7854 \times 20^2 = .7854 \times 400 = 314.16 =$ *area of the base* AB.

And $314.16 \times \frac{24}{3} = 314.16 \times 8 = 2513.28 =$ *solidity required.*

2. Required the solidity of the hexagonal pyramid ECBD, each of the equal sides of its base being 40, and the perpendicular height CS 60.



Here 2.598076 (*multiplier when the side is* 1) $\times 40^2 = 2.598076 \times 1600 = 4156.9216 =$ *area of the base.*

And,

And, $4156.9216 \times \frac{60}{3} = 4156.9216 \times 20 =$ 83138.432 *solidity required.*

3. Required the solidity of a triangular pyramid, whose height is 30, and each side of the base 3.

Ans. 38.97117.

4. Required the solidity of a square pyramid, each side of whose base is 30, and the perpendicular height 20.

Ans. 6000.

5. What is the solidity of a cone, the diameter of whose base is 18 inches, and its altitude 15 feet?

Ans. 8.83575 *feet.*

6. If the circumference of the base of a cone be 40 feet, and the height 50 feet; what is the solidity?

Ans. 2120 *feet.*

7. What is the content of a pentagonal pyramid, its height being 12 feet, and each side of its base 2 feet?

Ans. 27.527.

PROBLEM IX.

To find the solidity of the frustum of a cone or pyramid.

RULE.*

1. *For the frustum of a cone, the diameters of the two ends, and the height being given.*

Divide

---

* *Demon.* Let D = diameter AB, d = ED, p = .7854, and h = ss = height of the frustum of the cone ABDE, (see the last figure.)

Then D : d :: BS : Cs, and D − d : d :: CS − Cs (h) : $\frac{dh}{D-d}$ = Cs = height of the cone EDC.

But

Divide the difference of the cubes of the diameters of the two ends, by the difference of the diameters, and this quotient being multiplied by .7854 and again by $\frac{1}{3}$ of the height will give the solidity.

2. *For the frustum of a pyramid, the sides and height being given.*

To the areas of the two ends of the frustum add the square root of their product, and this sum being multiplied by $\frac{1}{3}$ of the height will give the solidity.

1. What is the solidity of the frustum of the cone EABD, the diameter of whose greater end AB is

---

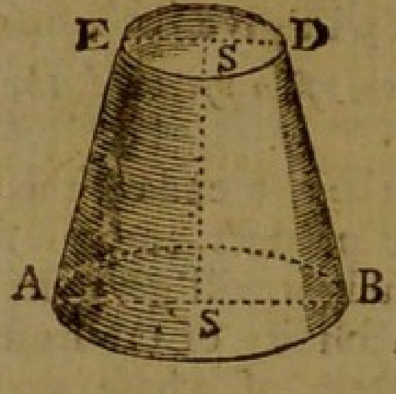
But $\frac{pD^2}{3} \times (b + \frac{db}{D-d}) =$ solidity of the whole cone ACB, and $\frac{pd^2}{3} \times \frac{db}{D-d} =$ that of the cone ECD. Therefore $\frac{pD^2}{3} \times (b + \frac{db}{D-b}) - \frac{pd^2}{3} \times \frac{db}{D-d} = (D^2 \times \frac{Db}{D-d} - d^2 \times \frac{db}{D-d}) \times \frac{p}{3} = (bD^2 + \overline{D^2 - d^2} \times \frac{db}{D-d}) \times \frac{p}{3} = \frac{D^3 - d^3}{D-d} \times \frac{bp}{3} =$ solidity of the frustum ABDE. Q. E. D.

Again, for the polygon, let $S =$ ED, $s = ed$, and $m =$ proper multiplier in the table of polygons; then $S : s :: cS : cs$, and $S - s : s :: cS - cs\ (b) : \frac{bs}{S-s}$.

But mS^2 and ms^2 are the areas of polygons whose sides are S and s respectively. And therefore $\frac{mS^2}{3} \times (b + \frac{bs}{S-s}) - \frac{ms^2}{3} \times \frac{bs}{S-s} = (mS^2 + \overline{mS^2 - ms^2} \times \frac{bS}{S-s} \times \frac{b}{3} = (mS + mSs + ms^2) \times \frac{b}{3} =$ solidity of the frustum c a D B b, which is the same as the rule.

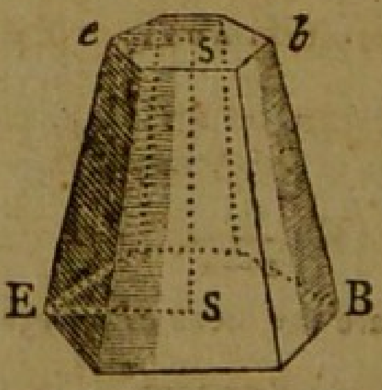
5 feet,

5 feet, that of the leſſer end E D 3 feet, and the perpendicular height *s* S 9 feet?



Here $\frac{5^3-3^3}{5-3}\times.7854\times\frac{9}{3}=\frac{125-27}{2}\times.7854\times3=$ $\frac{98}{2}\times2.3562=49\times2.3562=115.4538$ *ſolid feet, the content of the fruſtum.*

2. What is the ſolidity of the fruſtum *e* E D B *b*, of an hexagonal pyramid, the ſide E D of whoſe greater end is 4 feet, that *ed* of the leſſer end 3 feet, and the height S *s* 9 feet?



Here 2.598076 (*tab. mult.*) $\times3^2=2.598076\times9=$ $23.382624=$ *area of the polygon eb.*

And 2.598076 (*tab. mult.*) $\times4^2=2.598076\times16=$ $41.569216=$ *area of the polygon* E B.

Whence

Whence $\sqrt{41.569216 \times 23.382684} = \sqrt{971.999841}$ $= 31.176911$.

And $(41.569216 + 23.382684 + 31.176911) \times \frac{9}{3} =$ $96.128811 \times 3 = 288.386433$ *feet*, = *solidity required.*

3. What is the solidity of the fruſtum of a cone, the diameter of the greater end being 4 feet, that of the leſſer end 2 feet, and the altitude 9 feet?

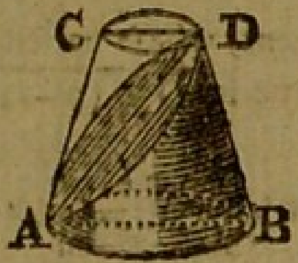
Anſ. 65.9736.

4. What

---

* The following caſes contain all the rules for finding the ſuperficies and ſolidities of conical ungulas.

1. *When the ſection paſſes through the oppoſite extremities of the ends of the fruſtum.*

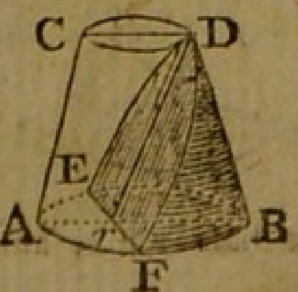


Let $D = AB$, the diameter of the greater end, $d = CD$ the diameter of the leſs end, b = perpendicular height of the fruſtum, and $n = .78539$, &c.

Then will $\overline{D^2 - d\sqrt{Dd}} \div \overline{D - d} \times \frac{1}{3} n d b$ = ſolidity of the elliptic ungula ADB.

And $\frac{n}{D - d}\sqrt{4b^2 + (D - d^2)} \times \overline{D^2 - (D - d)\sqrt{4Dd}}$ = curve ſurface of ADB.

2. *When the ſection cuts off part of the baſe, and makes the angle DrB leſs than the angle CAB.*



Let

4. What is the ſolidity of the fruſtum of a cone, the circumference of the greater end being 40, that of the leſſer end 20, and the length or height 50?

Anſ. 3713.64.

5. What is the ſolidity of the fruſtum of a ſquare pyramid, one ſide of the greater end being 18 inches, that of the leſſer end 15 inches, and the height 60 inches? *Anſ.* 16380 *inches.*

6. What is the ſolidity of the fruſtum of an hexagonal pyramid, the ſide of whoſe greater end is 3 feet, that of the leſſer end 2 feet, and the length 12 feet?

Anſ. 197.453776 *feet.*

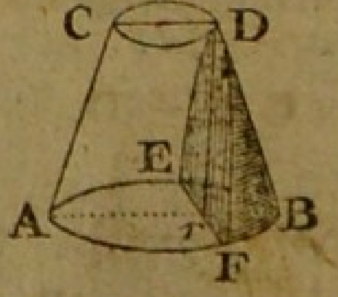
PRO-

Let S = tabular ſegment, whoſe verſed ſine is $\mathrm{B}r \div \mathrm{D}$, s = tab. ſeg. whoſe verſed ſine is $\overline{\mathrm{B}r - (\mathrm{D} - d)} \div d$, and the other letters as before.

Then $(\mathrm{S} \times \mathrm{D}^3 - s \times d^3 \times \frac{\mathrm{B}r}{\mathrm{B}r - \overline{\mathrm{D} - d}} \surd \frac{\mathrm{B}r}{\mathrm{B}r - \overline{\mathrm{D} - d}}) \times \frac{\frac{1}{3}b}{\mathrm{D} - d}$ = ſolidity of the elliptic hoof E F B D.

And $\frac{1}{\mathrm{D} - d} \surd \overline{4b^2 + (\mathrm{D} - d)^2} \times$ (ſeg. F B E $- \frac{d^2}{\mathrm{D}^2} \times \frac{\frac{1}{2} \times (\mathrm{D} + d) - \mathrm{A}r}{d - \mathrm{A}r} \times \surd \frac{\mathrm{B}r}{d - \mathrm{A}r} \times$ ſeg. of the circle A B, whoſe height is $\mathrm{D} \times \frac{d - \mathrm{A}r}{d}$ = convex ſurface of E F B D.

3. *When the ſection is parallel to one of the ſides of the fruſtum.*



Let A = area of the baſe F B E, and the other letters as before.

Then

PROBLEM X.

To find the solidity of a cuneus or wedge.

RULE.*

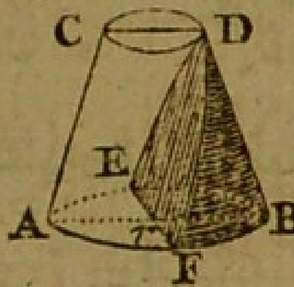
Multiply the ſum of twice the length of the baſe and the length of the edge, by the product of the height

---

Then $(\frac{A \times D}{D-d} - \frac{4}{3} d \sqrt{(D-d) \times d}) \times \frac{1}{3} b =$ ſolidity of the parabolic hoof EFBD.

And $\frac{1}{D-d} \sqrt{4b^2 + (D-d)^2} \times (\text{ſeg. } FBE - \frac{2}{3} \overline{D-d} \times \sqrt{d \times \overline{D-d}}) =$ convex ſurface of EFBD.

4. *When the ſection cuts off part of the baſe, and makes the angle DrB greater than the angle CAB.*



Let the area of the hyperbolic ſection EDF $=$ A, and the area of the circular ſeg. EBF $= a$.

Then $\frac{\frac{1}{3} b}{D-b} \times (a \times D - A \times \frac{d \times Br}{Cr}) =$ ſolidity of the hyperbolic ungula EFBD.

And $\frac{1}{D-d} \times \sqrt{4b^2 + (D-d)^2} \times (\text{cir. ſeg. } EBF - \frac{d^2}{D^2} \times \frac{Br - \frac{1}{2}(D-d)}{Br - \overline{D-d}} \sqrt{\frac{Br}{Br - \overline{D-d}}} =$ curve ſurface of EFBD.

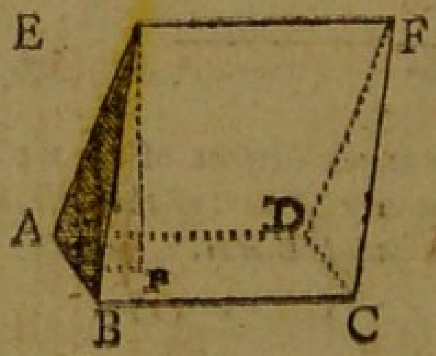
Note. The tranſverſe diameter of the hyp. ſeg. $= \frac{d \times Cr}{D - d - Br}$ and the conjugate $= d \sqrt{\frac{Br}{D - d - Br}}$, from which its area may be found by the former rules.

* *Demon.* When the length of the baſe is equal to that of the wedge, the wedge is evidently equal to half a priſm of the ſame baſe and altitude. And

height of the wedge and the breadth of the baſe, and $\frac{1}{6}$ of the laſt product will be the ſolidity.

EXAMPLES.

1. How many ſolid feet are there in a wedge, whoſe baſe is 5 feet 4 inches long, and 9 inches broad, the length of the edge being 3 feet 6 inches, and the perpendicular height 2 feet 4 inches?



Here $\frac{(64 \times 2 + 42) \times 28 \times 9}{6} = \frac{(128 + 42) \times 28 \times 9}{6} = \frac{170 \times 28 \times 9}{6} = \frac{170 \times 28 \times 3}{2} = 170 \times 14 \times 3 = 7140$ *ſolid inches.*

And according as the edge is ſhorter or longer than the baſe, the wedge is greater or leſs than half a priſm, by a pyramid of the ſame height and breadth at the baſe with the wedge, and the length of whoſe baſe is equal to the difference of the lengths of the edge and baſe of the wedge.

Therefore, let the length of the baſe BC $= L$; the length of the edge EF $= l$; the breadth of the baſe BA $= b$; and the height of the wedge EP $= b$; and we ſhall have by the former rules $\frac{blb}{2} \pm bb \times \frac{\pm L \mp l}{3} = \frac{blb}{2} + bb \times \frac{L - l}{3} = bb \times \frac{3l + 2L - 2l}{2} = bb \times \frac{2L + l}{6}$. Q. E. D.

And 7140÷1728=4.1319 *solid feet, the content required.*

2. The length and breadth of the base of a wedge are 35 and 15 inches, and the length of the edge is 55 inches: what is the solidity, supposing the perpendicular height to be 17.14508 inches?

Ans. 3.1006 *feet.*

PROBLEM XI.

To find the solidity of a prismoid.

RULE.*

To the sum of the areas of the two ends add four times the area of a section parallel to and equally distant from both ends, and this last sum multiplied by $\frac{1}{6}$ of the height will give the solidity.

Note. The length of the middle rectangle is equal to half the sum of the lengths of the rectangles of the two ends, and its breadth equal to half the sum of the breadths of those rectangles.

EXAM-

---

* *Demon.* The rectangular prismoid is evidently composed of two wedges, whose heights are equal to the height of the prismoid, and their bases its two ends. Wherefore, by the last problem, its solidity will be $= (\overline{2L+l} \times B + \overline{2l+L} \times b) \times \frac{h}{6}$, which, by putting $M = \frac{L+l}{2}$, and $m = \frac{B+b}{2}$, becomes $\overline{BL + bl + 4Mm} \times \frac{h}{6}$; which is the rule, as was to be shewn.

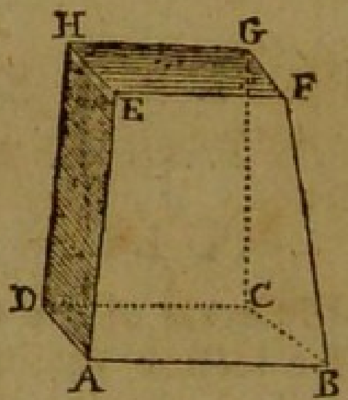
The solidities of the two parts, commonly called the ungulas, or hoofs, into which the frustum of a rectangular pyramid is divided, may be found by the two last rules, as they are only composed of wedges and prismoids.

A very elegant demonstration of this rule for the prismoid may be seen in *Simpson's Fluxions, page* 179, *2d Edition.*

If

EXAMPLES.

1. What is the solidity of a rectangular prismoid the length and breadth of one end being 14 and 12 inches, and the corresponding sides of the other 6 and 4 inches; and the perpendicular $30\frac{1}{2}$ feet?



Here $14 \times 12 + 6 \times 4 = 168 + 24 = 192 =$ *sum of the areas of the two ends.*

Also $\frac{14+6}{2} = \frac{20}{2} = 10 =$ *length of the middle rectangle.*

And $\frac{12+4}{2} = \frac{16}{2} = 8 =$ *breadth of the middle rectangle.*

Whence $10 \times 8 \times 4 = 80 \times 4 = 320 = 4$ *times the area of the middle rectangle.*

Or $(320 + 192) \times \frac{366}{6} = 512 \times 61 = 31232$ *solid inches.*

And $31242 \div 1728 = 18.074$ *solid feet, the content.*

2. What is the solid content of a prismoid, whose greater end measures 12 inches by 8, the lesser end 8 inches by 6, and the length, or height, 60 inches?

Ans. 2.453 *feet.*

If the bases of the prismoid are dissimilar rectangles, of which L, l and M, m are corresponding dimensions, and b the height; Then $(\overline{2L + l} . M + \overline{2l + L} . m) \times \frac{1}{6} b =$ solidity.

3. What

3. What is the capacity of a coal waggon, whoſe inſide dimenſions are as follow: at the top, the length is $81\frac{1}{2}$, and breadth 55 inches; at the bottom, the length is 41, and the breadth $29\frac{1}{2}$ inches; and the perpendicular depth is $47\frac{1}{4}$ inches?

Anſ. 126340.59375 *cubic inches; which is nearly equal to a chaldron of coals.*

PROBLEM XII.

To find the convex ſurface of a ſphere.

RULE.*

Multiply the diameter of the ſphere by its circumference, and the product will be the convex ſuperficies required.

Note. The curve ſurface of any zone or ſegment will alſo be found by multiplying its height by the whole circumference of the ſphere.

EXAM-

---

* *Demon.* Put the diameter BC $= d$, BA $= x$, AC $= y$, BC $= z$; and 3.14159, &c. $= p$.

Then, ſince the triangles AOC and CED are ſimilar, we ſhall have CA (y) : CO $(\frac{d}{2})$ :: CE $(\dot{x})$: CD $(\dot{x}) = \frac{d\dot{x}}{2y}$. But $2py\dot{z}$ is the general expreſſion for the fluxion of any ſurface; and therefore by ſubſtituting $\frac{d\dot{x}}{2y}$ for its equal $\dot{z}$, the fluxion will become $pd\dot{x}$; and conſequently $pdx =$ ſurface of any ſegment of a ſphere whoſe height is x, and $pdd =$ that of the whole ſphere. Q. E. D.

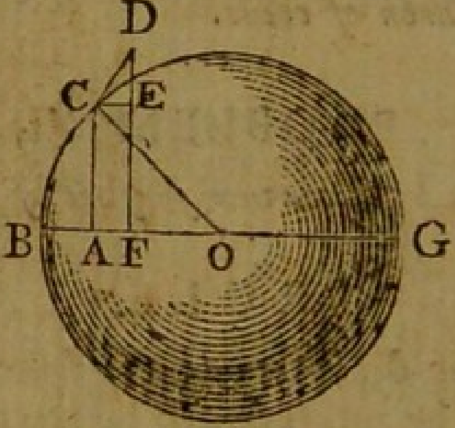
Cor. 1. The ſurface of the ſphere is equal to the curve ſurface of its circumſcribing cylinder.

Cor. 2. The ſurface of a ſphere is alſo equal to 4 times the area of a great circle of it.

1. To

EXAMPLES.

1. What is the convex ſuperficies of a globe BCG whoſe diameter BG is 17 inches?



Here $3.14159 \times 17 \times 17 = 53.40703 \times 17 = 907.91951$ *ſquare inches.*

And, $907.91951 \div 144 = 6.30499$ *ſquare feet the anſwer.*

---

1. To find the lunar ſurface included between two great circles of the ſphere.

RULE. Multiply the diameter into the breadth of the ſurface in the middle, and the product will be the ſuperficies required.

OR, As one right angle is to a great circle of the ſphere;
So is the angle made by the two great circles,
To the ſurface included by them.

2. To find the area of a ſpherical triangle, or the ſurface included by the interſecting arcs of three great circles of the ſphere.

RULE. As two right angles, or 180°,
Is to a great circle of the ſphere;
So is the exceſs of the three angles above two right angles,
To the area of the triangle.

2. What

2. What is the convex ſuperficies of a ſphere whoſe diameter is $1\frac{1}{3}$ feet, and the circumference 4.1888 feet? *Anſ.* 5.58506 *feet.*

3. If the diameter, or axis of the earth, be $7957\frac{3}{4}$ miles; what is the whole ſurface, ſuppoſing it to be a perfect ſphere? *Anſ.* 198943653 *ſquare miles.*

4. The diameter of a ſphere is 21 inches; what is the convex ſuperficies of that ſegment of it whoſe height is $4\frac{1}{2}$ inches? *Anſ.* 296.8802 *inches.*

5. What is the convex ſurface of a ſpherical zone, whoſe breadth is 4 inches, and the diameter of the ſphere, from which it was cut, 25 inches? *Anſ.* 314.16 *inches.*

PROBLEM XIII.

To find the ſolidity of a ſphere or globe.

RULE.*

Multiply the cube of the diameter by .5236, and the product will be the ſolidity.

EXAM-

---

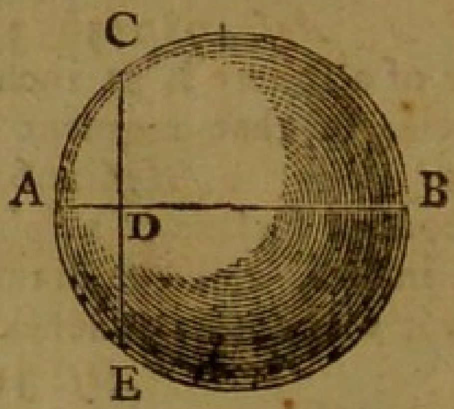
* *Demon.* Put AD $= x$, CD $= y$, the diameter AB $= d$, and $p = 3.14159$.

Then, by the property of the circle, $dx - x^2 = y^2$. But the general expreſſion for the fluxion of any ſolid is $py^2\dot{x}$; and therefore by writing $dx - x^2$ for its equal y^2, we ſhall have $p\dot{x} \times \overline{dx - x^2} = pdx\dot{x} - px^2\dot{x}$. The fluent of which is $\frac{pdx^2}{2} - \frac{px^3}{3} = \frac{3pdx^2 - 2px^3}{6} =$ content of the ſegment CAE.

And

EXAMPLES.

1. What is the ſolidity of the ſphere AEBC, whoſe diameter AB is 17 inches?



Here $17^3 \times .5236 = 17 \times 17 \times 17 \times .5236 = 289 \times 17 \times .5236 = 4913 \times .5236 = 5272.4468$ *ſolid inches*;

And $5272.4468 \div 1728 = 1.48868$ *ſolid feet the anſwer.*

2. What is the ſolidity of a ſphere whoſe diameter is $1\frac{1}{3}$ feet? *Anſ.* 1.2411 *feet.*

3. What is the ſolidity of the earth, ſuppoſing it to be perfectly ſpherical, and its diameter $7957\frac{4}{5}$ miles? *Anſ.* 263858149120 *miles.*

PROBLEM XIV.

To find the ſolidity of the ſegment of a ſphere.

And if d be ſubſtituted for x it will become $\frac{3p^3d - 2pd^3}{9} = \frac{pd^3}{6} = d^3 \times .5236$, or $.5236d^3$; which is the ſame as the rule.

Coroll. A ſphere, or globe, is equal to two-thirds of its circumſcribing cylinder.

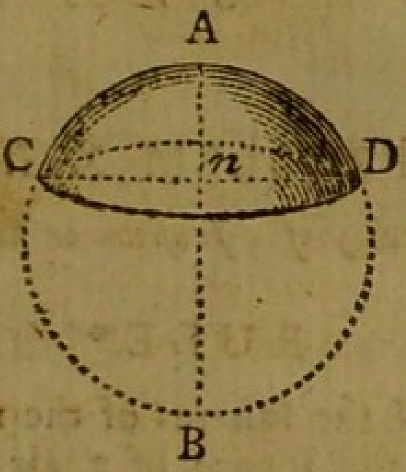
RULE.

RULE.*

To three times the ſquare of the radius of its baſe add the ſquare of its height; and this ſum multiplied by the height, and the product again by .5236 will give the ſolidity.

EXAMPLES.

1. The radius C*n* of the baſe of the ſegment CAD is 7 inches, and the height A*n* 4 inches: what is its ſolidity?



* *Demon.* Let r = radius of the baſe of the ſegment, b = height of the ſegment, and the other letters as before.

Then will $(3db^2-2b^3) \times \frac{p}{6}$ = ſolidity of the ſegment, as is ſhewn in the laſt problem.

B■t ſince $\frac{r^2+b^2}{b} = d$, by the property of the circle, we ſhall have $(\frac{3r^2b^2+3b^4}{b}-2b^3) \times \frac{p}{6} = \overline{3r^2+b^2} \times \frac{pb}{6}$ = ſolidity of the ſegment, which is the ſame as the rule.

Or if d=diameter of the ſphere, and b=height of the ſegment; then will $.5236b^2 \times (3d-2b)$ = ſolidity.

Here

Here $(7^2 \times 3+4^2) \times 4 \times .5236 = (49 \times 3+4^2) \times 4 \times .5236 = (147+4^2) \times 4 \times .5236 = (147+16) \times 4 \times .5236 = 163 \times 4 \times .5236 = 652 \times .5236 = 341.3872$ *solid inches, the answer.*

2. What is the solidity of the segment of a sphere, the diameter of whose base is 20, and its height 9?

Ans. 1799.6132.

3. What is the content of the spherical segment, whose height is 4 inches, and the radius of its base 8?

Ans. 435.6352.

4. What is the solidity of a spherical segment, the diameter of its base being 17.23368, and its height 4.5?

Ans. 572.5566.

PROBLEM XVI.

To find the solidity of a frustum or zone of a sphere.

RULE.*

To the sum of the squares of the radii of the two ends, add $\frac{1}{3}$ of the square of their distance, or the

---

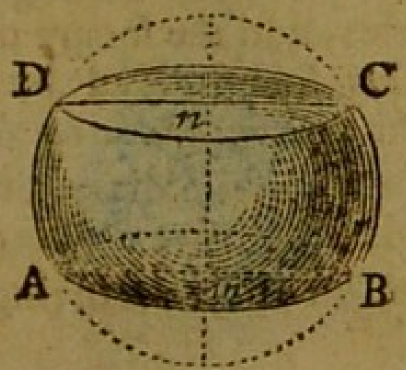
* *Demon.* The difference between two segments of a sphere whose heights are H and b, and the radii of whose bases are R and r, will, by the last problem $= \frac{p}{6} \times (3R^2H + H^3 - 3r^2b - b^3) =$ zone whose height is $H - b$. And therefore by putting a for the altitude of the frustum, and exterminating H and b by means of the two equations $\frac{3R^2+H^2}{H} = \frac{r^2+b^2}{b}$ and $H - b = a$, we shall have $(R^2 + r^2 + \frac{a^2}{3}) \times \frac{pa}{2}$, which is the rule.

breadth

breadth of the zone, and this sum multiplied by the said breadth, and the product again by 1.5708 will give the solidity.

EXAMPLES.

1. What is the solid content of the zone ABCD, whose greater diameter AB is 20 inches, the lesser diameter CD 15 inches, and the distance *nm* of the two ends 10 inches?



Here $(10^2+7.5^2+\frac{10^2}{3})\times 10\times 1.5708=(100+56.25+33.33)\times 10\times 1.5708=189.58\times 10\times 1.5708=1895.8\times 1.5708=2977.92264$ *solid inches, the answer.*

2. What is the solid content of a zone, whose greater diameter is 24 inches, the lesser diameter 20 inches, and the distance of the ends 4 inches?

Ans. 1566.6112 *inches.*

3. Required the solidity of the middle zone of a sphere, whose top and bottom diameters are each 3 feet, and the breadth of the zone 4 feet.

Ans. 61.7848 *feet.*

If it be the middle zone of the sphere, the solidity will be $=(d^2+\frac{2}{3}b^2)\times .7854b$; where $d=$ diameter of each end, and $b=$ its height.

PROBLEM XVII.

To find the ſurface of a circular ſpindle, the length and breadth, or middle diameter, being given.

RULE.*

1. To the ſquare of half the length of the ſpindle, or longeſt diameter, add the ſquare of half the middle diameter, and this ſum divided by the middle diameter will give the *radius of the circle.*

2. Take half the middle diameter from the radius of the circle, and it will give the *central diſtance.*

3. Find the length of the revolving arc by problem the 10th.

4. From the product of the longeſt diameter and the radius of the revolving arc, ſubtract the product of the ſaid arc and the central diſtance, and this remainder multiplied by 6.2832 will give the ſurface required.

EXAM-

---

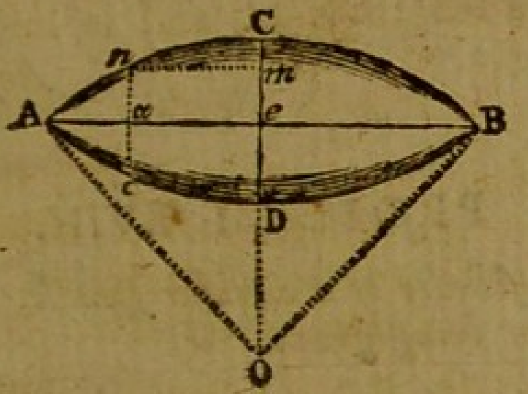
* *Demon.* Put $z =$ arc cn, $x =$ its ſine nm, $r =$ radius co $c =$ central diſtance oc, and $p = 3.14159$, &c.

Then $\sqrt{r^2 - x^2}$ $(om) : r :: \dot{x} : \dot{z}$, and $\dot{z}\sqrt{r^2 - x^2} = r\dot{x}$, by the property of the circle, as is ſhewn in the demonſtration to problem 13. But $2p\dot{z} \times na = 2p\dot{z}(\sqrt{r^2 - x^2} - c) = 2p \times (\dot{z}\sqrt{r^2 - x^2} - c\dot{z})$ is the general expreſſion for the fluxion, and therefore $2p \times (r\dot{x} - c\dot{z}) =$ fluxion of half the fruſtum cncD, and $2p \times (rx - cz) = 2p \times (r \times ae - c \times$ c$n) =$ to its ſurface. Q. E. D.

Coroll.

EXAMPLES.

1. What is the ſuperficial content of the circular ſpindle ADBC, whoſe length AB is 48, and the middle diameter CD 36?



Here $(24^2+18^2)\div 36=(576+324)\div 36=\frac{900}{36}=$

$\frac{100}{4}=25=OC$ *the radius of the circle* ACB.

Therefore $25-18=7=OC-Ce=$ *central diſtance* Oe.

And $\frac{2\times 18}{3}\div\left(50-\frac{41\times 18}{50}\right)=12\div 50-14.76=$

$12\div 35.24=.34052$.

Coroll. 1. When $ea=eA$, the rule becomes $2p\times(r\times eA-\overline{c\times CA})$ for the ſurface of half the ſpindle, and $2p\times\overline{r\times AB-c\times ACB}$ for that of the whole ſpindle.

Coroll. 2. If from the ſurface of the ſemi-ſpindle there be taken that of the fruſtum, there will remain $2p\times\overline{r\times aA-c\times An}$ for the ſegment nAc; ſo that the rule is general.

Coroll. 3. When e coincides with O, c vaniſhes, and the ſpindle becomes a ſphere; in which caſe the theorem alſo becomes barely $2prx$, the ſame expreſſion as was before found for the ſurface of the ſphere.

Whence

Whence $(1+.34052)\times 48=1.34052\times 48=64.34496$ $=$*length of the arc* ACB.

And $(48\times 25-64.34496\times 7)\times 6.2832=(1200-450.41472)\times 6.2832=749.58528\times 6.2832=4709.794231296=$*superficies required.*

2. What is the ſuperficial content of a circular ſpindle whoſe length is 48, and its middle diameter 30? *Anſ.* 4387.1644.

PROBLEM XVIII.

To find the ſolidity of a circular ſpindle, the length and middle diameter being given.

RULE.*

1. Find the area of the generating circular ſegment by problem the 13th; and the radius and central diſtance as in the laſt problem.

2. From $\frac{1}{3}$ of the cube of half the length of the ſpindle ſubtract the product of the central diſtance and

---

* *Demon.* Put $ae=x$, $oe=c$, $r=$radius, and $p=3.14159$, &c.

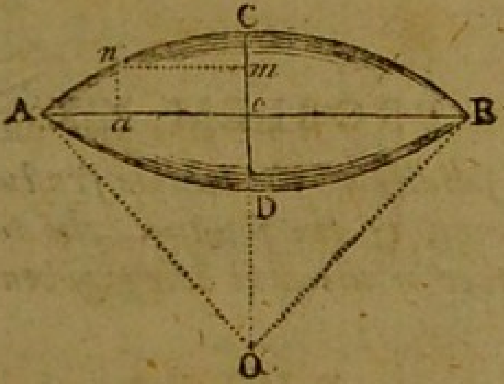
Then will the fluxion of the ſolid be $=p\dot{x}\times na^2=p\dot{x}\times(mo-c)^2=p\dot{x}\times\overline{mo^2-c\times 2mo-c}=p\dot{x}\times\overline{mo^2-c\times 2an+c}=p\dot{x}\times(r^2-x^2-c^2-2c\times an)$; whoſe fluent is $=px\times(r^2-\frac{x^2}{3}-c^2-\frac{2c}{x}\times ance)=p\times(\overline{Ae^2-\frac{1}{3}x^2}\times x-2c\times ance)=$ fruſtum generated by $ance$. And when $x=Ae$, we have $\overline{\frac{1}{3}Ae^3-c\times ACe}\times 2p$ for the half ſpindle CAD, and $\overline{\frac{1}{3}Ae^3-c\times ACe}\times 4p=$whole ſpindle ADBC. Q. E. D.

Coroll.

and half the area of the generating ſegment, and this remainder multiplied by 12.5664 will give the ſolidity.

EXAMPLES.

1. The longeſt diameter AB of the circular ſpindle ADBC is 48, and the middle diameter CD 36; what is the ſolidity of the ſpindle?



Here $\frac{48 \times 7}{2} = 24 \times 7$ (*7 being the central diſtance* oe, *as in the laſt prob.*) $= 168 =$ *area of the triangle* AOB.

And 32.17248 ($\frac{1}{2}$ *length of the arc* ACB *by laſt prob.*) $\times$ 25 (*radius* OC *by ſame prob.*) $= 804.312 =$ *area of the ſector* BCAO.

Coroll. 1. If the fruſtum be taken from the half ſpindle there will remain $p \times (Ae^2 \times Aa - \frac{1}{3} \times \overline{Ae^3 - ex^3} - 2c \times Ana) = p \times (\frac{1}{3} Aa^3 \times 3Ae - Aa - 2c \times Ana)$ for the ſegment of the ſpindle twice nAa.

Coroll. 2. When e coincides with o, c will vaniſh, and the theorem will become $\frac{4}{3} p \times OC^3 = \frac{1}{6} p \times D^3$, which is the ſolidity of the whole ſphere.

Which theorems agree with thoſe before given.

Also 804.312 − 168 = 636.312 = *area of the segment* ACBe.

And $(\frac{24^3}{3} - \frac{636.312}{2} \times 7) \times 12.5664 = (\frac{13824}{3} - 318.156 \times 7) \times 12.5664 = (4608 - 2227.092) \times 12.5664 = 2380.908 \times 12.5664 = 29919.4422912 =$ *solidity required.*

2. If the length of a circular ſpindle be 40, and its middle diameter 30: what is its ſolidity?

Anſ. 17310.458.

PROBLEM XIX.

To find the ſolidity of the middle fruſtum of a circular ſpindle, the length of the fruſtum, the middle diameter, and that of either of the ends being given.

RULE.*

1. Divide the ſquare of half the length of the fruſtum by half the difference of the middle diameter, and that of either of the two ends; and half this quotient added to $\frac{1}{4}$ of the ſaid difference will give the radius of the circle.

2. Find the central diſtance, and the revolving area, as in the laſt problem.

3. From the ſquare of the radius take the ſquare of the central diſtance, and the ſquare root of the remainder will give half the length of the ſpindle.

---

* The demonſtration is contained in that of the laſt problem.

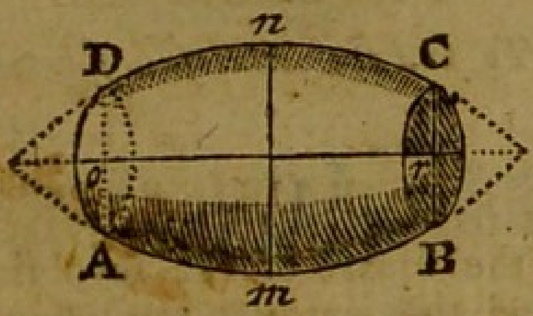
The ſolidity of the ſegment cannot be found independent of the length of the ſpindle. The rule may be ſeen in the Corollary to the laſt problem.

4. From the ſquare of half the length of the ſpindle take $\frac{1}{3}$ of the ſquare of half the length of the fruſtum, and multiply the remainder into the ſaid half length.

5. From this product take that of the generating area and central diſtance, and the remainder multiplied by 6.2832 will give the content of the fruſtum.

EXAMPLES.

1. What is the ſolidity of the fruſtum ABCD, whoſe middle diameter *nm* is 36, the diameter DA or CB 16, and the length *or* 40?



Here $\frac{1}{2} \times (20^2 \div \frac{36-16}{2}) + \frac{36-16}{4} = \frac{1}{2} \times (400 \div 10) + 5 = \frac{1}{2} \times 40 + 5 = 20 + 5 = 25 =$ *radius of the circle.*

Conſequently $25 - \frac{1}{2}$ *nm* $= 25 - 18 = 7 =$ *central diſtance.*

And $\frac{10}{50} = \frac{1}{5} = .2 =$ *tab. verſed ſine*; *and* .111823 = *tab. ſegment.*

Alſo $.111823 \times 50^2 = .111823 \times 2500 = 279.5575 =$ *area of the ſegment* D*n*C.

And $279.5575 + 320 = 599.5575 =$ *generating area* oD*n*C*r*.

Again $\sqrt{(25^2-7^2)} = \sqrt{(625-49)} = \sqrt{576} = 24$ $= \frac{1}{2}$ *length of the spindle.*

And $(24^2 - \frac{20^2}{3}) \times 20 - 599.5575 \times 7) \times 6.2832 =$ $(556 - 133.3) \times 20 - 4196.9025) \times 6.2832 =$ $(8853.334 - 4196.9025) \times 6.2832 = 4656.4135 \times$ $6.2832 = 29257.2904$ *solidity required.*

2. The middle diameter of the fruſtum of a circular ſpindle is 32, the diameter at the end 24, and the length 40: what is the ſolidity?

Anſ. 27287.5411256 *cubic inches.*

PROBLEM XX.

To find the ſolid of a ſpheroid.

RULE.*

Multiply the ſquare of the revolving axe by the fixed axe, and this product again by .5236, and it will give the ſolidity required.

Where note that .5236 is $= \frac{1}{6}$ of 3.14159.

EXAM-

---

* *Demon.* Let AC $= a$, DB $= b$, Ar $= x$, $rn = y$, and $p = 3.14159$, &c.

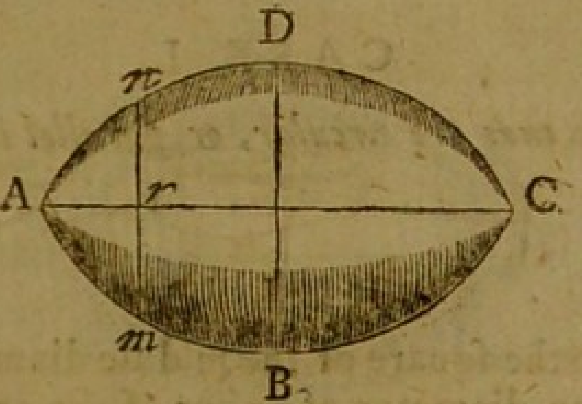
Then $a^2 : b^2 :: x \times (a-x) : \frac{b^2}{a^2} \times (ax - x^2) = y^2$ by the property of the ellipſis.

And therefore the fluxion of the ſolid $(= py^2\dot{x}) = \frac{pb}{a^2} \times$ $(ax\dot{x} - x^2\dot{x})$; and its fluent $\times \frac{pb^2}{a^2} \times (\frac{1}{2}ax^2 - \frac{1}{3}x^3) =$ ſegment

SAME.

EXAMPLES.

1. In the prolate ſpheroid ABCD, the tranſverſe or fixed axe AC is 90, and the conjugate or revolving axe DB is 70: what is the ſolidity?



Here $DB^2 \times AC \times .5237 = 70^2 \times 90 \times .5236 = 4900 \times 90 \times .5236 = 441000 \times .5236 = 230907.6 =$ *ſolidity required.*

2. What is the ſolidity of a prolate ſpheroid, whoſe fixed axe is 100, and its revolving axe 60?

Anſ. 188496.

3. What is the ſolidity of an oblate ſpheroid, whoſe fixed axe is 60, and its revolving axe 100?

Anſ. 314160.

NAm. Which, when $x = a$, becomes $\frac{pb^2}{a^2} \times (\frac{1}{2}a3 - \frac{1}{3}a^3) = \frac{pab^2}{6} =$ content of the whole ſpheroid. Q. E. D.

If f be put = fixed axe, r = revolving axe, $q = (f^2 \backsim r^2) \div f^2$, and $p = 3.1415$, &c.

Then will $prf\sqrt{1 + \frac{1}{3}q}$ = ſurface of the oblate ſpheroid, and $prf\sqrt{1 - \frac{1}{3}q}$ = that of the prolate ſpheroid.

PRO-

PROBLEM XXI.

To find the content of the middle fruſtum of a ſpheroid, its length, the middle diameter, and that of either of the ends being given.

CASE I.

When the ends are circular, or parallel in the revolving axis.

RULE.*

To twice the ſquare of the middle diameter add the ſquare of the diameter of either of the ends, and this ſum multiplied by the length of the fruſtum, and the product again by .2618, will give the ſolidity.

Where note that $.2618 = \frac{1}{12}$ of 3.14159.

EXAM-

---

* *Demon.* Let $AO = a$, $DO = b$, $En = b$, $no = c$, $ro = x$, $re = y$, and $p = 3.14159$, &c.

Then $a : b^2 :: a^2 - x^2 : \frac{b^2}{a^2} \times (a^2 - x^2) = b^2 - \frac{b^2 x^2}{a^2} =$ y^2 the property of the ellipſis.

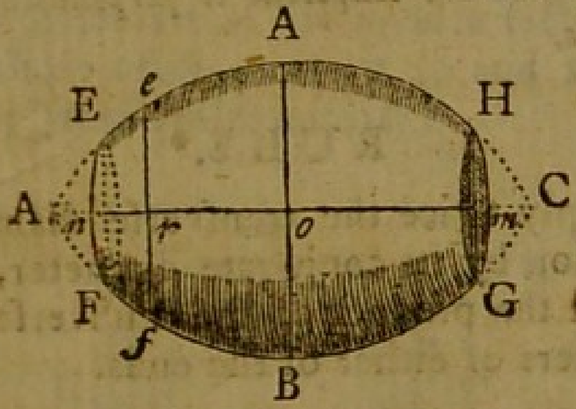
And alſo $a^2 : b^2 :: \overline{a^2 - c^2} : \frac{b^2 \times (a^2 - c^2)}{a^2} = b^2$; or $a^2 =$ $b^2 c^2 \div (b^2 - b^2.)$

Whence, by ſubſtituting this value of a^2 in the former equation, we ſhall have $y_2 = b_2 - \frac{b^4 x^2 - b^2 b^2 x^2}{b^2 c^2} = b^2 -$ $\frac{b^2 x^2 - b^2 x^2}{c^2} = b^2 - \frac{x^2}{c^2} \times (b^2 - b^2)$.

And

EXAMPLES.

1. In the middle fruſtum of a ſpheroid EFGH, the middle diameter DB is 50 inches, and that of either of the ends EF or GH 40 inches, and its length *nm* 18 inches: what is its ſolidity?



Here $(50^2\times2+40^2)\times18\times.2618=(2500\times2+1600)\times18\times.2618=(5000+1600)\times18\times.2618=6600\times18\times.2618=118800\times.2618=31101.84$ *cubic inches, the anſwer.*

2. What is the ſolidity of the middle fruſtum of a prolate ſpheroid, the middle diameter being 60, that of either of the two ends 36, and the diſtance of the ends 80? *Anſ.* 177940.224.

And conſequently the fluxion of the ſolid $(py^2\dot{x}) = pb^2\dot{x} - \frac{px^2\dot{x}}{c^2}\times(b^2-b^2)$; the fluent of which is $= pb^2x - \frac{px^3}{3c^2}\times(b^2-b^2)$; which, when $x=c$, becomes $pb^2c - \frac{pcb^2-pcb^2}{3} = \frac{pc\times(2b^2+b^2)}{3} = \frac{pc}{12}\times\overline{8b^2+4b^2}$. Q. E. D.

3. What

3. What is the solidity of the middle frustum of an oblate spheroid, the middle diameter being 100, that of either of the ends 80, and the distance of the ends 36?

Ans. 248814.72.

CASE II.

When the ends are elliptical or perpendicular to the revolving axis.

RULE.*

1. Multiply twice the transverse diameter of the middle section by its conjugate diameter, and to this product add the product of the transverse and conjugate diameters of either of the ends.

2. Mul-

---

* *Demon.* Put $so = a$, $Eo = b$, $om = r$, $on = x$, $An = y$, $nc = z$, and $p = 3.14159$, &c.

Then $a^2 : b^2 :: a^2 - x^2 : \frac{b^2}{a^2} \times (a^2 - x^2) = y^2$ by the property of the ellipsis.

And, since ACD is an ellipsis similar to EmF, it will be $b : r :: y : \frac{ry}{b} = z$; as is shewn by the writers on Conics.

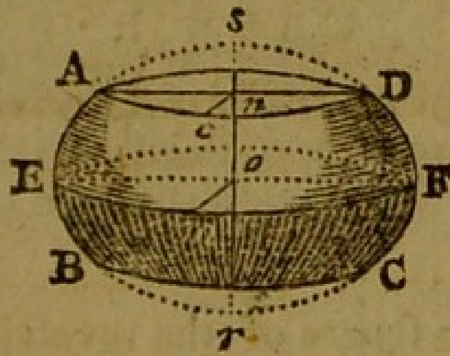
But the fluxion of the solid AEFD is $pyz\dot{x} = py\dot{x} \times \frac{ry}{b} = \frac{pry^2\dot{x}}{b} = \frac{pr\dot{x}}{b} \times \frac{b^2 \times (a^2 - x^2)}{a^2} = prb\dot{x} \times \frac{a^2 - x^2}{a^2}$. And the fluent $= prbx \times \frac{a^2 - \frac{1}{3}x^2}{a^2}$. Which, by substituting for a^2 its value $\frac{b^2 x^2}{b^2 - y^2}$, becomes $= prx \times \frac{2b^2 + y^2}{3b} = px \times \frac{2}{3}rb + \frac{ry^2}{3b}$.

And

2. Multiply the sum, thus found, by the distance of the ends, or the height of the frustum, and the product again by .2618, and it will give the solidity required.

EXAMPLES.

1. In the middle frustum ABCD of an oblate spheroid, the diameters of the middle section EF are 50 and 30; those of the end AD 40 and 24; and its height *ne* 18; what is the solidity?



Here $(50 \times 2 \times 30 + 40 \times 24) \times 18 \times .2618 = (3000 + 960) \times 18 \times .2618 = 3960 \times 18 \times .2618 = 71280 \times .2618 = 18661.104 =$ *solidity required.*

2. In the middle frustum of a prolate spheroid, the diameters of the middle section are 100 and 60; those of the end 80 and 48; and the length 36: what is the solidity? *Ans.* 149288.832.

And this again, by putting z for its equal $\frac{ry}{b}$, becomes $=$ $\frac{px}{3} \times \overline{2rb + yz} =$ frustum EFDA. Or $\frac{p \times ne}{12} \times (2EF \times 2om + AD \times 2nc) =$ middle frustum ABCD. Q.E.D.

3. In

3. In the middle fruſtum of an oblate ſpheroid, the diameters of the middle ſection are 100 and 60: thoſe of the end 60 and 36; and the length 80: what is the ſolidity of the fruſtum? *Anſ.* 296567.04.

PROBLEM XXII.

To find the ſolidity of the ſegment of a ſpheroid.

CASE I.

When the baſe is parallel to the revolving axis.

RULE.*

1. Divide the ſquare of the revolving axis by the ſquare of the fixed axe, and multiply the quotient by the difference between three times the fixed axe and twice the height of the ſegment.

2. Multiply the product, thus found, by the ſquare of the height of the ſegment, and this product again by .5236, and it will give the ſolidity required.

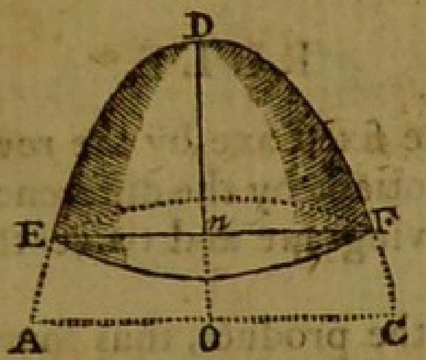
EXAMPLES.

1. In the prolate ſpheroid DEFD, the tranſverſe axis 2DO is 100, the conjugate AC 60, and the height D*n* of the ſegment EBF 10: what is the ſolidity?

---

* This rule is formed from the theorem for the ſegment in the demonſtration to problem the 20th.

Here



Here $(\frac{60^2}{100^2} \times \overline{300-10}) \times 10^2 \times .5236 = .36 \times 280 \times 10^2 \times .5236 = 100.80 \times 100 + .5236 = 10080 \times .5236 = 5277.888 =$ *solidity required.*

2. The axes of a prolate ſpheroid are 50 and 30: what is the ſolidity of that ſegment whoſe height is 5, and its baſe perpendicular to the fixed axe?

Anſ. 659.746.

3. The diameters of an oblate ſpheroid are 100 and 60; what is the ſolidity of that ſegment whoſe height is 12, and its baſe perpendicular to the conjugate axe? *Anſ.* 32672.64.

CASE II.

When the baſe is perpendicular to the revolving axis.

The content of the ſegment may alſo be found by the following theorem:

$(D^2 + 4d^2) \times \frac{1}{6} nb =$ content of the ſegment; D being the diameter of the baſe, $d =$ diameter in the middle, $b =$ height, and $n = .7854 =$ area of a circle whoſe diameter is 1.

RULE.

RULE.*

1. Divide the fixed axe by the revolving axe, and multiply the quotient by the difference between three times the revolving axe and twice the height of the ſegment.

2. Multiply the product, thus found, by the ſquare of the height of the ſegment, and this product again by .5236, and it will give the ſolidity required.

EXAMPLES.

1. In the oblate ſpheroid *a* E *b* F, the tranſverſe axe E F is 100, the conjugate *ab* 60, and the height *an*, of the ſegment *a* A D, 12: what is the ſolidity?

---

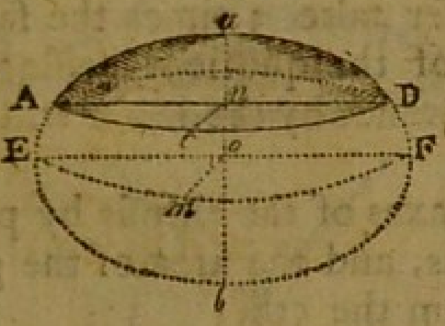
* *Demon.* Put $ao = a$, $Eo = b$, $om = r$, $an = x$, $An = y$, $ne = z$, and $p = 3.14159$, &c. Then will $a^2 : b^2 :: a^2 - (a-x)^2$ or $(2ax - x^2)$: $\frac{b^2 \times (2ax - x^2)}{a^2} = y^2$ by the property of the ellipſe.

And, ſince A *c* D is an ellipſe ſimilar to E *m* F, it will be $b : r :: y : \frac{ry}{b} = z$; as is ſhewn by the writers on Conics.

But the fluxion of the ſolid *a* A *c* D $= pyz\dot{x} = py\dot{x} \times \frac{ry}{b} = \frac{pry^2\dot{x}}{b} = \frac{pr\dot{x}}{b} \times \frac{b^2 \times (2ax - x^2)}{a^2}$; whoſe fluent is $= \frac{prb}{a}x^2 - \frac{prb}{3a^2}x^3$; which, when $x = b =$ the height of the ſegment, becomes $(3ab^2 - b^3) \times \frac{prb}{3a^2}$. Whence, ſince $r = a$, we ſhall have $(3ab^2 - b^3) \times \frac{pb}{3a} =$ ſolidity of the ſegment.

Q. E. D.

Here



Here 156 ($=$*dif. of* $3ab$ *and* $2an$) $\times 1\frac{2}{3}$ ($=$ EF, *or* $\frac{100}{ab}$) $\times 144$ ($=$*square of* an) $\times .5236 = \frac{156 \times 5}{3} \times 144 \times .5236 = 52 \times 5 \times 144 \times .5236 = 260 \times 144 \times 5236 = 37440 \times .5236 = 19603.584 =$ *solidity required.*

2. Required the content of the segment of a prolate spheroid; its height being 6, and the axes 40 and 24. *Ans.* 2450.44226.

PROBLEM XXIII.

To find the solidity of an elliptic spindle.

RULE.*

1. From three times the square of the middle diameter take 4 times the square of the diameter between the middle and the end; and from 4 times this

---

* *Demon.* Let $\text{F}v = a$, $\text{o}v = b$, $\frac{1}{2}$ the transverse diameter $= c$, $\text{Do} = d$, $va = x$, and vc or $an = y$.

Then, by the property of the ellipse, $c : d :: \sqrt{c^2 - x^2} : \frac{d}{c}\sqrt{c^2 - x^2} = co$; hence cv or $an = co - ov = \frac{a\sqrt{c^2 - x^2}}{c} - b = y$, and the fluxion of the solid $= py^2\dot{x} = p\dot{x} \times$ (d2 this laſt diameter take 3 times the ſaid middle diameter; and $\frac{1}{4}$ of the quotient ariſing from dividing the former difference by the latter will give the *central diſtance.*

2. Find the axes of the ellipſis by problem the 2d, in conic ſections, and the area of the generating ſegment by problem the 5th.

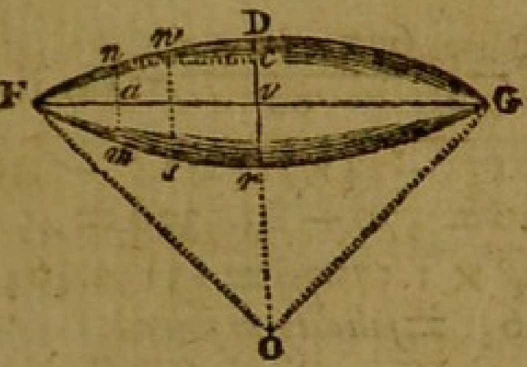
3. Divide 3 times the area, thus found, by the length of the ſpindle, and from the quotient ſubtract the middle diameter; then multiply the remainder by 4 times the central diſtance, and ſubtract the product from the ſquare of the middle diameter; and this difference multiplied by $\frac{1}{3}$ of the length of the ſpindle, and the product again by 1.57079 will give the ſolidity.

EXAMPLES.

1. What is the ſolidity of the elliptic ſpindle FrGD, whoſe length FG is 80, the middle diameter Dr 24, and the diameter ws at $\frac{1}{4}$ of the length 18.99094?

Here

$(d^2 - \frac{d^2x^2}{c} - \frac{2bd\sqrt{c^2-x^2}}{c} + b^2) = p\dot{x} \times (d^2 - b^2 - \frac{d^2x^2}{c^2} - \frac{2bd\sqrt{c^2-x^2}}{c}) - b = pd^2\dot{x} \times \frac{a^2-x^2}{c^2} - 2pby\dot{x}$; and its fluent $= pd^2x \times \frac{3a^2-x^2}{3c^2} - 2bp \times$ area $vanD$ = fruſtum $Dnmr$. And when $x = a$ the above theorem will become $\frac{2pd^2}{3c^2} \times a^3 - 2bp \times$ area FDv = DFr the half of the ſpindle.



Here $(\overline{3\times 24^2 - 18.99094^2\times 4} \div \overline{18.99094\times 4 - 3\times 4}) \times \frac{1}{4} = (\overline{1728 - 1442.62320833} \div \overline{75.96376 - 72}) \times \frac{1}{4} = (285.37679167 \div 3.96376) \times \frac{1}{4} = 71.996 \times \frac{1}{4} = 17.999 = 18$ *nearly, for the central diſtance* ov.

And $18 + \frac{24}{2} = 18 + 12 = 30\frac{1}{2}$ *conjugate* DO. *And* $\frac{30}{2} \times 30 \div \sqrt{30^2 - 18^2} = 40 \times 30 \div \sqrt{900 - 324} = 1200 \div \sqrt{576} = \frac{1200}{24} = 50 = \frac{1}{2}$ *tranſverſe.*

Alſo $\frac{12}{2\times 30} = \frac{12}{60} = \frac{1}{5} = .2 =$ *tab. verſed ſine; the circular ſegment to which is* $.111823 \therefore .111823 \times 100$ (AB) $\times 60$ (DF) $= 11.1823 \times 60 = 670.938 =$ *generating ſegment* FDG.

Whence

ſpindle. And if from the ſemi-ſpindle there be taken the fruſtum, there will remain $pd^2e^2 \times \frac{3a-e}{3c^2} - 2pb \times$ area $rna =$ ſegment nFm, e being the height Fa of the ſegment. But to convert theſe rules into thoſe given in the text, let $Dr = D$, $nm = d$, $ws = m$, $va = b$, $ov = c$, the ſegment $nDo = s$, and $n = .7854$.

Then, by the property of the ellipſe, $(c+\frac{1}{2}D)^2 - (c+\frac{1}{2}d) : 4 :: (c+\frac{1}{2}D)^2 - (c+\frac{1}{2}m)^2 : 1$; hence $4\times(c+\frac{1}{2}m)^2 - 3\times(c+\frac{1}{2}D)^2 = (c+\frac{1}{2}d)^2$, or $c = \frac{1}{4} \times \frac{3D^2 + d^2 - 4m^2}{-3D - d + 4m}$.

And

Whence $24^2 - [(\frac{670.038 \times 3}{80} - 24) \times 72$ *or* $40v] =$ $24^2 - (25.160175 - 24 \times 72) = 24^2 - 1.160175 \times 72 = 24^2 - 83.5326 = 576 - 83.5326 = 492.4674$. *And* $492.4674 \times \frac{80}{3} \times 1.57079 = 13132.464 \times 1.57079 = 20628.34312656 =$ *solidity required.*

2. The length of an elliptic ſpindle is 40, the middle diameter 12, and the diameter at $\frac{1}{4}$ of the length 9.49546: what is the ſolidity? *Anſ.* 2578.56.

PROBLEM XXIV.

To find the ſolidity of the middle fruſtum of an elliptic ſpindle; the length, the diameters of the middle and end, and another parallel thereto at $\frac{1}{4}$ of the length of the fruſtum, being given.

RULE.*

1. From the ſum of 3 times the ſquare of the middle diameter and the ſquare of that of the end

---

And the laſt two theorems become $\frac{1}{3} nb \times (\frac{1}{2}D^2 + d^2 - 2) \times \frac{3D^2 + d^2 - 4m}{-3D - d + 4m} \times (-D + d + \frac{3s}{b})$ for the fruſtum $Dnmr$; and $\frac{1}{3} nl \times (2D^2 - 2 \times \frac{2D^2 - 4m^2}{-3D + 4m} \times -D + \frac{3s}{l})$ for the ſemi-ſpindle DFr, where $l = vF$ and $s =$ area of FDv.

Q. E. D.

* The demonſtration of this rule is contained in that of the laſt problem.

The ſolidity of a ſegment of an elliptic ſpindle cannot be found independently of the axes of the ſpheroid.

take

take 4 times the ſquare of the diameter between the middle and the end; and from 4 times the laſt diameter take the ſum of the leaſt diameter and 3 times that of the middle, and $\frac{1}{4}$ of the quotient ariſing from dividing the former difference by the latter will give the *central diſtance*.

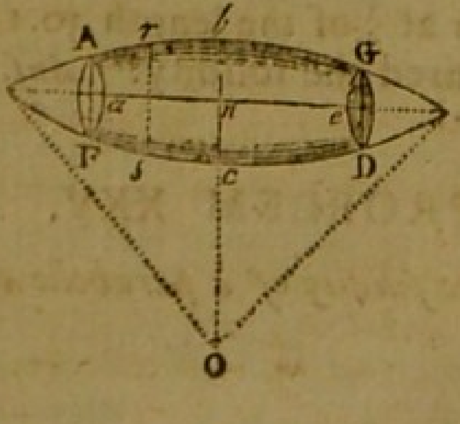
2. Find the axes of the ellipſe by problem the 2d, and the area of the elliptical ſegment whoſe chord is the length of the fruſtum by problem the 5th.

3. Divide three times the area thus found by the length of the fruſtum, and from the quotient ſubtract the difference between the middle diameter and that of the end, and multiply the remainder by 8 times the central diſtance.

4. Then from the ſum of the ſquare of the leaſt diameter, and twice the ſquare of that in the middle, take the product laſt found, and this difference multiplied by the length, and the product again by .261799, &c. will give the ſolidity required.

EXAMPLES.

1. What is the ſolidity of the fruſtum AFDG, whoſe length *ae* is 28, the middle diameter *bc* 24, the diameter AF of the end 21.6, and that *rs* at $\frac{1}{4}$ of the length 23.409?



Here $\frac{24^2\times 3+(21.6)^2-4\times(23.409)^2}{4\times 23.409-(21.6+3\times 24)}=\frac{2194.56-2191.925124}{93.63636-93.6}=\frac{2194.56-2191.925124}{.03636}=\frac{2.634876}{.03636}$ $=72.46.$

And $\frac{72.46}{4}=18.11=18$ *nearly* = *central distance* *on.*

Also $\frac{30\,(=bo)\times 14\,(=an)}{\sqrt{30^2(=bo^2)-28.8^2(=om^2)}}=\frac{420}{\sqrt{900-829.44}}$ $=\frac{420}{\sqrt{70.56}}=\frac{420}{8.4}=50=\frac{1}{2}$ *transverse diameter.*

And $\frac{1.2\,(=bm)}{60\,(=bp)}=.02=$ *tab. versed sine*; *and* .003748 = *circular segment belonging to* .02; ∴ .003748 × 100 (= LN) × 60 (=*bp*) = 22.488 = *elliptic segment, of* which ABG *is the arc.*

Whence $24^2\times 2+(21.6)^2-(\frac{3\times 22.488}{28}-2.4(=$ *diff. of bc and* AF$)\times 8\times 18=1152+466.56-(\overline{2.4094-2.4}\times 144)=1618.56-.0094\times 144=1618.56-1.3536$ $=1617.2064.$

And $1617.2064\times 28\,(=ae)\times .261799=45281.7792\times .261799=11854.7245127808=$ *solidity required.*

2. In the middle frustum of an elliptic spindle, the middle diameter is 32, the diameter at the end 24, and the diameter at $\frac{1}{4}$ of the length 30.15756, and the length 40: required the solidity. *Ans.* 27419.8219.

PROBLEM XXV.

To find the solidity of a parabolic conoid.

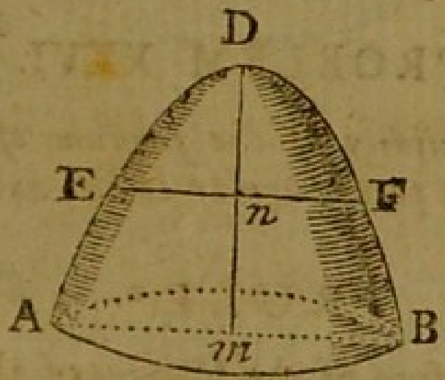
RULE.

RULE.*

Multiply the area of the baſe by half the altitude, and the product will be the content.

EXAMPLES.

1. What is the ſolidity of the paraboloid A D B, whoſe height D m is 84, and the diameter B A of its circular baſe 48?



---

* *Demon.* Let D$m = a$, B$m = b$, D$n = x$, E$n = y$, and $p = 3.14159$, &c.

Then by the nature of the parabola $a : b^2 :: x : y^2$, or $\frac{b^2 x}{a} = y^2$; wherefore $\frac{pb^2 x\dot{x}}{a}$ $(= py^2 \dot{x})$ = the fluxion of the ſolid, and $\frac{pb^2 x^2}{2a}$ = its fluent; which, when x becomes $= a$, is $\frac{1}{2} p a b^2$ for the whole ſolid, or for any ſegment whoſe height is $= a$, and the radius of its baſe $= b$. Q. E. D.

Coroll. The parabolic conoid is $= \frac{1}{2}$ its circumſcribing cylinder.

Note. The rule given above will hold for any ſegment of the paraboloid, whether the baſe be perpendicular or oblique to the axe of the ſolid.

2. What

Here $8^2 \times .7854 \times 42 \, (= \frac{1}{2} \text{D}m) = 2304 \times .7854 \times 42$ $= 1809.5616 \times 42 = 76001.5872 =$ *solidity required.*

2. What is the solidity of a paraboloid, whose height is 60, and the diameter of its circular base 100? *Ans.* 235620.

3. Required the solidity of a parabolic conoid, whose height is 30, and the diameter of its base 40? *Ans.* 18849.6.

4. Required the solidity of a parabolic conoid, whose height is 50, and the diameter of its base 100? *Ans.* 126350.

PROBLEM XXVI.

To find the solidity of the frustum of a paraboloid, when its ends are perpendicular to the axe of the solid.

RULE.*

Multiply the sum of the squares of the diameters of the two ends by the height of the frustum, and the product again by .3927, and it will give the solidity.

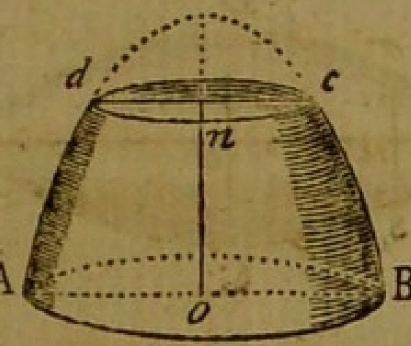
---

* *Demon.* The segment whose base is B, and altitude A, is $= \frac{1}{2}$ A B, and that whose base is b and altitude a is $= \frac{1}{2} ab$, by the last problem: wherefore the frustum, or the difference of the segment is $\frac{1}{2} \text{A B} - \frac{1}{2} ab$. But $\text{B} - b : \text{A} - a \; (d) ::$ $\text{B} : \text{A} = \frac{\text{B}d}{\text{B} - b}$; and $\text{B} - b : d :: b : a = \frac{bd}{\text{B} - b}$, by the nature of the paraboloid; and these values of A and a being substituted for them will make $\frac{1}{2} \text{A B} - \frac{1}{2} ab = \frac{d\text{B} - db}{2\text{B} - 2b} = \frac{1}{2} d \times$ $(\text{B} + b)$ which is the same as the rule. Q. E. D.

EXAM-

EXAMPLES.

1. Required the ſolidity of the parabolic fruſtum ABcd, the diameter AB of the greater end being 58, that of the leſſer end *dc* 30, and the height *no* 18.



Here $(58^2+30^2)\times 18\times .3927=(3364+900)\times 18\times .3927=4264\times 18\times .3927=76752\times .3927=$ 30140.5104 = *ſolidity required.*

2. What is the ſolidity of the fruſtum of a parabolic conoid, the diameter of the greater end being 60, that of the leſſer end 48, and the diſtance of the ends 18?

Anſ. 41733.0144.

PROBLEM XXVII.

To find the ſolidity of a parabolic ſpindle.

RULE.*

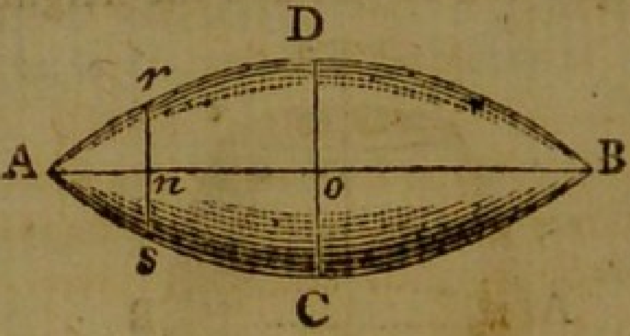
Multiply the ſquare of the middle diameter by the length of the ſpindle, and the product again by .418879, and it will give the ſolidity.

EXAM-

* *Demon.* Put $DO=a$, $AO=b$, $c=3.14159$, &c. $An=x$, and $rn=y$.

EXAMPLES.

1. The length of the parabolic ſpindle ACBD is 60, and the middle diameter DC 34: what is the ſolidity?



Here $34^2 \times 60 \times .418879 = 1156 \times 60 \times .418879 = 69360 \times .418879 = 29053.44744 =$ *ſolidity required.*

2. The length of a parabolic ſpindle is 9 feet, and the middle diameter 3 feet: what is the ſolidity?

Anſ. 33.929199.

Then it will be, by the property of the parabola, $b^2 : a :: 2bx - x^2$ (A$n \times n$B) : $a \times \frac{2bx - x^2}{b^2} = rn = y$; which, by putting $p = \frac{b^2}{a}$ (= parameter of Do,) becomes $\frac{2bx - x^2}{p}$; and hence the fluxion of the ſolid $= cy^2\dot{x} = \frac{c\dot{x}}{p^2} \times \overline{2bx - x}\,|^2$; the fluent of which is $cx^3 \times \frac{\frac{4}{3}b^2 - bx + \frac{1}{5}x^2}{p^2}$ = general expreſſion for the ſegment ras; and therefore when $x = b$, we ſhall have $\frac{8cb^5}{15p^2} = \frac{8}{15}ca^2b$ = ſemi-ſpindle DAC, and $a^2b \times .418879$ = whole ſpindle BDAC. Q. E. D.

PRO-

PROBLEM XXVIII.

To find the ſolidity of the middle fruſtum of a parabolic ſpindle.

RULE.*

Add 8 times the ſquare of the middle diameter, 3 times the ſquare of the leſs, and 4 times the product of thoſe diameters into one ſum; then this ſum being multiplied by the length, and the product again by .05236 will give ſolidity.

EXAMPLES.

1. In the middle fruſtum EFGH, of the parabolic ſpindle ACBD, the middle diameter DC is 36, the

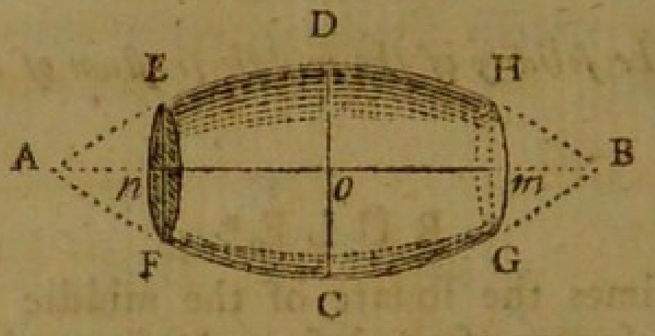
---

* *Demon.* Let $on = a - x = z$, and the other letters as in the laſt problem.

Then if from the value of the ſemi-ſpindle $\frac{8cb5}{15p^2}$ there be taken $cx^3 \times \frac{\frac{4}{3}b^2 - bx + \frac{1}{5}x^2}{p^2}$ the value of the ſegment EAF, there will remain $\frac{cz}{p^2} \times \overline{b4 - \frac{4}{3}b^2z^2 + \frac{1}{5}z^4} = \frac{ca^2z}{b4} \times (b4 - \frac{2}{3}b^2z^2 \times \frac{1}{5}z4) =$ the value of the fruſtum DEFC; and if inſtead of z there be ſubſtituted its value $b\sqrt{\frac{a-y}{a}}$ the value of the ſaid fruſtum will be denoted by $c \times no \times \frac{8DC^2 + 4no \times EF + 3EF^2}{60}$, and conſequently $nm \times .05236 \times 8DC^2 + 4no \times EF + 3EF^2 =$ whole fruſtum. Q. E. D.

 diameter

diameter of the end EF is 20, and the length nm 36: what is the solidity?



Here $(36^2 \times 8 + 20^2 \times 3 + 4 \times 36 \times 20) \times 36 \times .05236 = (10368 + 1200 + 2880) \times 36 \times .05236 = 14448 \times 36 \times .05236 = 520128 \times .0.5236 = 27233.90228 =$ *solidity required.*

2. Required the solidity of the middle frustum of a parabolic spindle, the middle diameter being 32, the diameter at the end 24, and the length 40?

Ans. 17210.448.

PROBLEM XXIX.

To find the solidity of an hyperboloid.

RULE.*

To the square of the radius of the base add the square of the middle diameter between the base and the

---

* *Demon.* Let $t =$ transverse, and $c =$ conjugate diameter of the generating hyperbola, $p = 3.14159$, y, Y, the ordinates, or semi-diameters of the ends of any frustum of the hyperboloid, $x =$ its altitude, and A $=$ distance of the less ordinate y from the vertex of the whole solid.

Then

the vertex; and this ſum multiplied by the altitude, and the product again by .5236 will give the ſolidity.

Then ſince $Y^2 = \frac{(t + A + x) \times (A + x)}{t^2} \times c^2$, we ſhall have the fluxion of the ſolid $= p Y^2 \dot{x} = p c^2 \dot{x} \times \frac{At + A^2 + 2Ax + tx + x^2}{t^2}$, and its fluent $= p c^2 x \times \frac{At + A^2 + Ax + \frac{1}{2}tx + \frac{1}{3}x^2}{t^2}$;

and this, by ſubſtituting $\frac{y^2}{c^2}$ for $\frac{At + A^2}{t^2}$, and $\frac{Y^2}{c^2}$ for $\frac{At + A^2 + 2Ax + tx + x^2}{t^2}$ becomes $(Y^2 + y^2 - \frac{c^2x^2}{3t^2}) \times \frac{1}{2}px =$ ſolidity of the fruſtum.

But to convert this into the rules given in the text, let D, δ, d, be the greateſt, middle, and leaſt diameters, $x =$ abſciſſa whoſe ordinate is δ, and $a =$ altitude. Then we ſhall have theſe three equations:

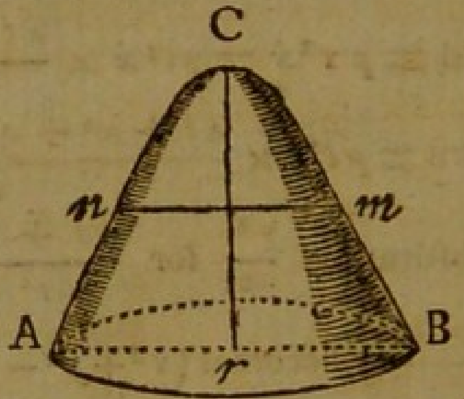
$$t^2\delta^2 = c^2 \times \overline{t + x} \times x$$
$$t^2 d^2 = c^2 \times \overline{t + x - \tfrac{1}{2}a} \times \overline{x - \tfrac{1}{2}a}$$
$$t^2 D^2 = c^2 \times \overline{t + x + \tfrac{1}{2}a} \times \overline{x + \tfrac{1}{2}a}$$

From the ſum of the two latter of which ſubtract the double of the former, and there will reſult $t^2 \times \overline{D^2 - 2\delta^2 + d^2} = \frac{1}{2}a^2c^2$; and hence $\frac{a^2c^2}{3t^2} = \frac{2D^2 - 4\delta^2 + 2d}{3}$. Which being ſubſtituted for it in the theorem above will give $\frac{D^2 + 4\delta^2 + d}{6} \times ap$ for the content of the fruſtum; which is the ſame as the following rule given in the text.

And if d the leaſt diameter be ſuppoſed to become infinitely little, or nothing, the rule will become $\frac{D^2 + 4\delta^2}{6} \times ap = \overline{D^2 + 4\delta^2} \times a \times .5236$. Q. E. D.

EXAMPLES.

2. In the hyperboloid A C B, the altitude C *r* is 10, the radius A *r* of the baſe 12, and the middle diameter *n m* 15.8745: what is the ſolidity?



Here $\overline{15.8745^2+12^2}\times 10\times .5236=\overline{251.99975+144}\times 10\times .5236=395.99975\times 10\times .5236=3959.9975\times .5236=2073.4558691=$ *ſolidity required.*

2. In an hyperboloid the altitude is 50, the radius of the baſe 52, and the middle diameter 68; what is the ſolidity? *Anſ.* 191847.

PROBLEM XXX.

To find the ſolidity of the fruſtum of an hyperbolic conoid.

RULE.*

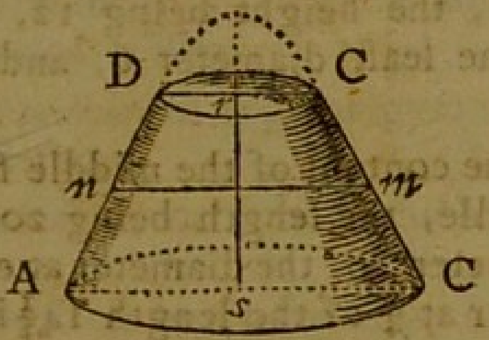
Add together the ſquares of the greateſt and leaſt ſemi-diameters, and the ſquare of the whole diameter in

* The demonſtration of this rule is contained in that of the laſt problem.

in the middle, then this ſum being multiplied by the altitude, and the product again by .52359 will give the ſolidity.

EXAMPLES.

1. In the hyperbolic fruſtum ADCB, the length rs is 20, the diameter AB of the greater end 32, that DC of the leſſer end 24, and the middle diameter nm 28.1708: required the ſolidity.



The rule for the hyperbolic ſpindle is the ſame as that for the elliptic ſpindle, page 175.

Or, if D = middle diameter, m = that at $\frac{1}{4}$ of the length, S = generating area of the hyperbola, L = length of the ſpindle, and $p = 3.14159$, &c.

Then will $\left(D + \frac{4m^2 - 3D^2}{4m - 3D} \times \frac{3S - D}{L}\right) \times \frac{1}{6} pL$ = ſolidity of the ſpindle. And if the generating hyperbola be equilateral, then will $\left(3S \times \frac{L^2 + D^2}{LD} - L^2\right) \times \frac{1}{6} pL$ = ſolidity of the ſpindle.

And, if l = length of the fruſtum, s = generating area, and the other letters as before; then will $\left(2D^2 + d^2 + \frac{4m^2 - 3D^2 - d^2}{4m - 3d - d} \times \frac{3s}{l} + d - D\right) \times \frac{1}{12} pl$ = ſolidity of the middle fruſtum of an hyperbolic ſpindle.

But if the generating hyperbola be equilateral, the fruſtum will be $= \left(\frac{3}{2} d^2 - l^2 + \frac{3s}{l} \times \frac{l^2 + D^2 - d^2}{D - d}\right) \times \frac{1}{6} pl$.

Here $(16^2+12^2+28.1708^2) \times 20 \times .52359 = (256 + 144 + 793.5939) \times 20 \times .5239 = 1193.5939 \times 20 \times .5239 = 23871.878 \times .5239 = 12499.07660202 =$ *solidity required.*

2. What is the solidity of the frustum of any hyperbolic conoid, whose greater diameter is 96, lesser diameter 54, the middle diameter 76.4264392, and the altitude 12.5? *Ans.* 116160.66.

3. Required the solidity of the frustum of an hyperbolic conoid, the height being 12, the greatest diameter 10, the least diameter 6, and the middle diameter $8\frac{1}{2}$? *Ans.* 667.59.

4. What is the content of the middle frustum of an hyperbolic spindle, the length being 20, the middle or greatest diameter 16, the diameter at each end 12, and the diameter at $\frac{1}{4}$ of the length $14\frac{1}{2}$? *Ans.* 3248.938.

5. Required the content of the segment of any spindle, its length being 10, the greatest diameter 8, and the middle diameter 6? *Ans.* 272.272.

Note. The content of any spindle formed by the revolution of a conic section about its axis may be found by the following rule:

Add together the squares of the greatest and least diameters, and the square of double the diameter in the middle between the two, and this sum multiplied by the length and the product again by .1309 will give the solidity.

And the rule will never deviate much from the truth when the figure revolves about any other line which is not the axis.

OF

OF THE

REGULAR BODIES.

A Regular body is a ſolid contained under a certain number of ſimilar and equal plane figures.

The whole number of regular bodies which can poſſibly be formed is five.

1. The *Tetraedron*, or regular pyramid, which has four triangular faces.

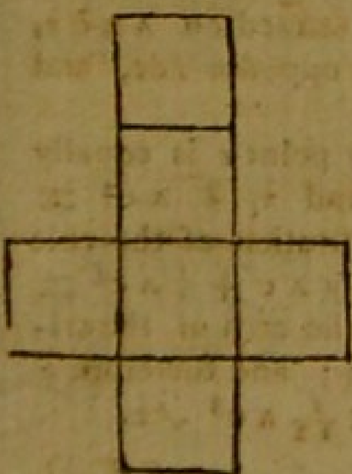
2. The *Hexaedron*, or cube, which has ſix ſquare faces.

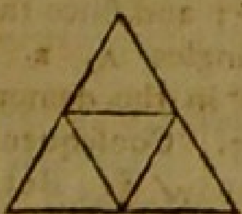
3. The *Octaedron*, which has eight triangular faces.

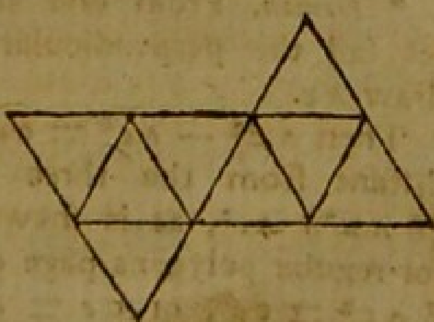
4. The *Dodecaedron*, which has twelve pentagonal faces.

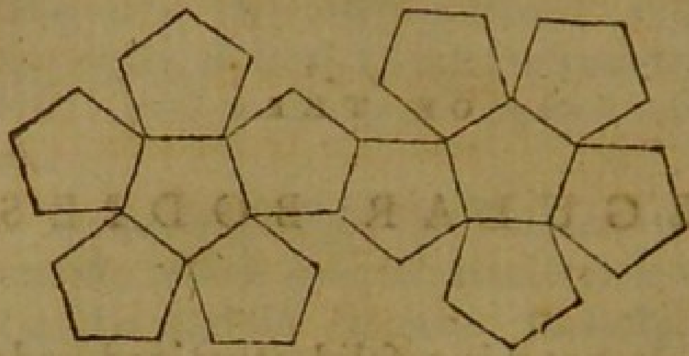
5. The *Icoſaedron*, which has twenty triangular faces.

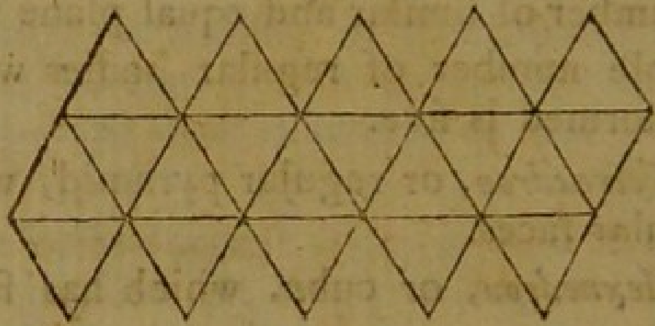
If the following figures are made of paſteboard, and the lines be cut half through, ſo that the parts may be turned up and glued together, they will repreſent the five regular bodies here mentioned.











PROBLEM I.

To find the ſolidity of a tetraedron.

RULE.*

Multiply $\frac{1}{12}$ of the cube of the linear ſide by the ſquare root of 2, and the product will be the ſolidity.

EXAM-

---

* *Demon.* From one angle C of the tetraedron $ABCn$, let fall the perpendicular Ce, upon the oppoſite ſide, and draw Ae.

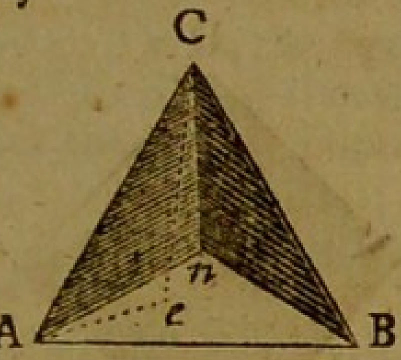
Then $AC^2 - Ae^2 = Ce^2$; and ſince the point e is equally diſtant from the three angles A, B and n, $\frac{1}{3}\,AC^2 = (\frac{1}{3}\,AB^2)\,Ae^2$, as is ſhewn in the demonſtration of the rule for regular polygons page 53. Conſequently $AC + \frac{1}{3}\,AC^2 = \frac{2}{3}\,AC^2 = Ce^2$, or $Ce = AC\,\sqrt{\frac{2}{3}}$. But the area of the triangle $AnB = \frac{1}{4}\,AB^2\,\sqrt{3} = \frac{1}{4}\,AC^2\,\sqrt{3}$; and therefore $\frac{1}{3}\,AC\,\sqrt{\frac{2}{3}}\,\frac{1}{3}Ce) \times \frac{1}{4}\,AC^2\,\sqrt{3}\,(\triangle AnB) = \frac{1}{12}\,AC^3\,\sqrt{2}$.

Q. E. D.

If

EXAMPLES.

1. The linear ſide of the tetraedron ABC*n* is 4: what is the ſolidity?



$\frac{4^3}{12}\times\sqrt{2}=\frac{4\times4\times4}{12}\times\sqrt{2}=\frac{4\times4}{3}\times\sqrt{2}=\frac{16}{3}\sqrt{2}=\frac{16}{3}\times1.414=\frac{22.624}{3}=7.5413=$ *ſolidity required.*

2. Required the ſolidity of a tetraedron whoſe ſide is 6? *Anſ.* 25.452.

PROBLEM II.

To find the ſolidity of an octaedron.

RULE.*

Multiply $\frac{1}{3}$ of the cube of the linear ſide by the ſquare root of 2, and the product will be the ſolidity.

EXAM-

If L be put = length of the linear edge, then will $L^2\sqrt{3}=$ whole *ſurface of the tetraedron.*

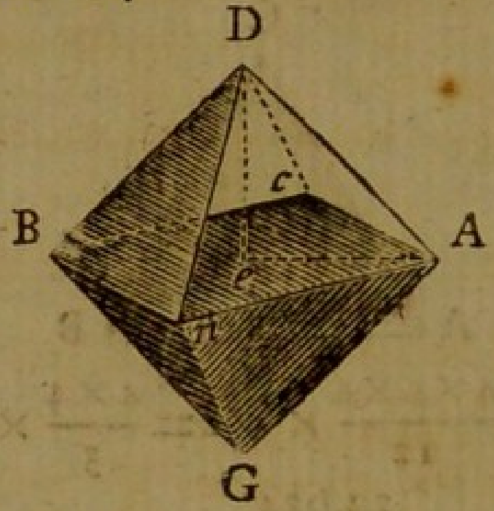
The rule for the hexaedron, or cube, has been given before.

* *Demon.* From the angle D of the octaedron DFGA let fall the perpendicular D*e*.

Then

EXAMPLES.

1. What is the ſolidity of the octaedron FGAD, whoſe linear ſide is 4?



$\frac{4^3}{3} \times \sqrt{2} = \frac{64}{3} \times \sqrt{2} = 21.333$, &c. $\times \sqrt{2} =$ 21.333, &c. $\times 1.452$, &c. $= 30.16486 =$ *ſolidity required.*

2. Required the ſolidity of an octaedron whoſe ſide is 8? *Anſ.* 243.3568.

PROBLEM III.

To find the ſolidity of a dodecaedron.

Then ſince the ſolid is compoſed of two equal ſquare pyramids, each of whoſe baſes $FnAc$ are equal to the ſquare of the linear ſide AG or AD, we ſhall have $FnAc \times \frac{2}{3} De = An^2 \times \frac{2}{3} De =$ content of the ſolid.

But De evidently biſects the diagonal FA, and is equal to Fe; therefore $An^2 \times \frac{2}{3} De = An^2 \times \frac{2}{3} Fe = An^2 \times \frac{1}{3} FA = \frac{1}{3} An^2 \times \sqrt{An^2 + Ac^2} = \frac{1}{3} An^2 \sqrt{2 An^2} = \frac{1}{3} An^3 \sqrt{2}$. Q. E. D.

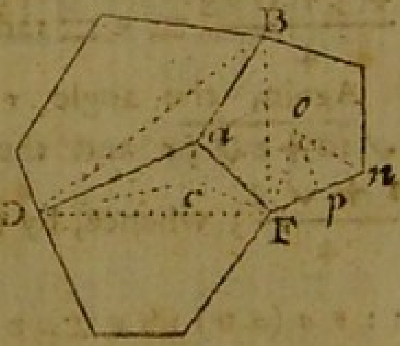
If L = linear ſide as before, then will $2L^2 \sqrt{3} =$ ſurface of the octaedron.

RULE

RULE.*

To 21 times the ſquare root of 5 add 47, and divide the ſum by 40: then the ſquare root of the quotient being multiplied by 5 times the cube of the linear ſide will give the ſolidity required.

EXAM-



* *Demon.* Let a be a ſolid angle of the dodecaedron, and ac a perpendicular falling on the equilateral plane BDF. Alſo join the points D, c and c, F.

Then the angle DaF contains 108 degrees, whoſe ſine is $\frac{1}{4}\sqrt{10+2\sqrt{5}}$, and the angle aFD contains 36 degrees, whoſe ſine is $\frac{1}{4}\sqrt{10-2\sqrt{5}}$, the radius in both caſes being taken equal to 1.

Therefore, by Trigonometry, $\frac{1}{4}\sqrt{10-2\sqrt{5}} : \frac{1}{4}\sqrt{10+2\sqrt{5}} :: aD : DF = aD\sqrt{\frac{5+\sqrt{5}}{5-\sqrt{5}}} = aD\frac{1+\sqrt{5}}{2} = \frac{1}{2}+\frac{1}{2}\sqrt{5}\times aD.$

Again, ſince c is the centre of DBF, the angles cDF and cFD are each 30°, and the angle $DcF = 120^\circ$; but the ſine of 30° is $\frac{1}{2}$; and the ſine of 120° is $\frac{1}{2}\sqrt{3}$; whence, by trigonometry, $\frac{1}{2}\sqrt{3} : DF :: \frac{1}{2} : Dc = \frac{DF}{\sqrt{3}} = aD\frac{1+\sqrt{5}}{2\sqrt{3}}$; and conſequently

$$AC = aD^2 - DC^2 = aD\sqrt{\frac{3-\sqrt{5}}{6}} = aD\sqrt{\frac{1}{2}-\frac{1}{6}\sqrt{5}}.$$

But a perpendicular from a upon the plane BDF muſt paſs through the center of the circumſcribing ſphere, and ac will be the verſed ſine of an arc whoſe chord is aD, and radius equal to that of the ſaid ſphere.

Whence $ac : aD :: aD : \frac{aD^2}{ac} = \frac{aD^2}{aD\sqrt{\frac{1}{2}-\frac{1}{6}\sqrt{3}}} = aD$

$\sqrt{3}$

EXAMPLES.

1. The linear fide of the dodecaedron ABCDE is 3: what is the folidity?

$\frac{\sqrt{3}+\sqrt{15}}{2}$ = diameter of the circumfcribing fphere, and aD $\frac{\sqrt{3}+\sqrt{15}}{4}$ = R = radius of the circumfcribing fphere.

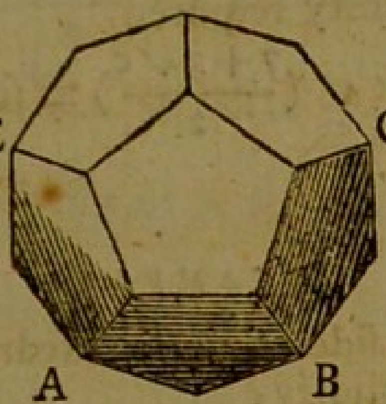
Again, the angle Fon contains 72°, whofe fine is $\frac{1}{4}\sqrt{10+2\sqrt{5}}$; and the angle oFn is 54°, whofe fine is $\frac{1+\sqrt{5}}{4}$; whence, by trigonometry, $\frac{1}{4}\sqrt{10+2\sqrt{5}} : \frac{1+\sqrt{5}}{4} :: Fn\,(aD) : oF = aD\,\frac{1+\sqrt{5}}{\sqrt{10+2\sqrt{5}}} = aD\sqrt{\frac{5+\sqrt{5}}{10}} = a\,D\sqrt{\frac{1}{2}+\frac{1}{20}\sqrt{5}}$.

But fince the radius of the circumfcribing fphere is the hypothenufe of a right angled triangle, whofe legs are oF and the radius of the infcribed fphere, we fhall have $\sqrt{R^2-oF^2} = \sqrt{(\frac{1}{4}\sqrt{3+\sqrt{15}})^2 aD^2 - \frac{1}{2}+\frac{1}{10}\sqrt{5}\,aD^2)} = aD\sqrt{\frac{25+11\sqrt{5}}{40}} = \sqrt{\frac{5}{8}+\frac{11}{40}\sqrt{5}}\,r$ = radius of the infcribed fphere.

And becaufe the folid is compofed of 12 equal pentagonal pyramids, each of whofe bafes are, by problem the 8th, $= \frac{5aD^2}{4}\sqrt{1+\frac{2}{5}\sqrt{5}}$; therefore $\frac{60aD^2}{4}\sqrt{1+\frac{2}{5}\sqrt{5}} \times \frac{1}{3}r = \frac{60aD^2}{4}\sqrt{1+\frac{2}{5}\sqrt{5}} \times \frac{aD}{3}\sqrt{\frac{25+11\sqrt{5}}{40}} = 5aD\sqrt{\frac{47+21\sqrt{5}}{40}}$ $\frac{21\sqrt{5}}{40}$ = folidity of the dodecaedron. Q. E. D.

If L be put for the linear fide, then will $15L^2\sqrt{\frac{5+2\sqrt{5}}{5}}$ = furface of the dodecaedron.

$\sqrt{21}$



$$\sqrt{\frac{21\sqrt{5}+47}{40}} \times 27 \times 5 = \sqrt{\frac{21 \times 2.23606+47}{40}}$$

$$\times 27 \times 5 = \sqrt{\frac{46.95726+47}{40}} \times 135 = 206.901$$ *soli-dity required.*

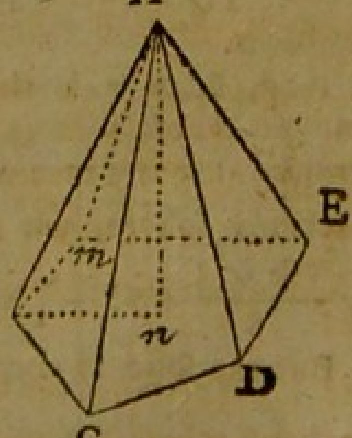
2. The linear side of a dodecaedron is 1; what is the solidity? *Ans.* 7.6631.

PROBLEM IV.

To find the solidity of an icosaedron.

RULE.*

To 3 times the square root of 5 add 7, and divide the sum by 2; then the square root of this quotient being



* *Demon.* Let A be a solid angle of the icosaedron, formed by 5 faces, or triangles, whose bases make the pentagon B C D E *m*.

Then, if a perpendicular be demitted from A upon the pentagonal plane B C D E *m*, it will fall into the centre *n*, and B *n*, by the demonstration of the last problem, will be $= \text{AB}\sqrt{\frac{5+\sqrt{5}}{10}}$, and the radius of the circle circumscribing one of the faces A B C $= \frac{1}{3}\text{AB}\sqrt{3}$.

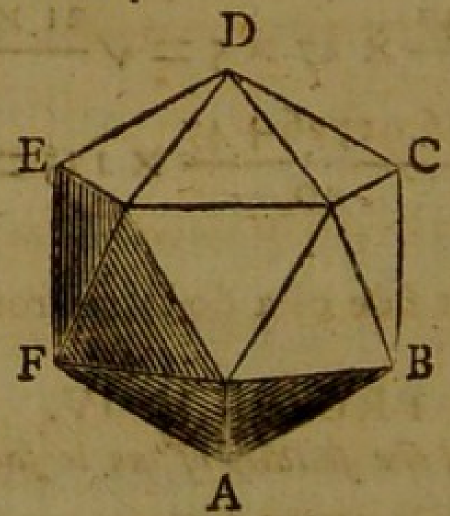
But

being multiplied by $\frac{5}{6}$ of the cube of the linear ſide will give the ſolidity required.

That is $\frac{5}{6}\ s^3 \times \sqrt{(\frac{7+3\sqrt{5}}{2})} =$ ſolidity when s = to the linear ſide.

EXAMPLES.

1. The linear ſide of the icoſaedron A B C, E F is 3: what is the ſolidity?



$\sqrt{3}$

But the radius of the circumſcribing ſphere is $R = \frac{BA^2}{2An} = \frac{BA^2}{2\sqrt{AB^2 - Bn^2}} = AB\sqrt{\frac{5+\sqrt{5}}{8}}$, found as in the laſt problem.

And, ſince R is the hypothenuſe of a right angled triangle, one of whoſe legs is $\frac{1}{3} AB \sqrt{3}$, the radius of the circle circumſcribing the face A B C, and the other r, the radius of the inſcribed ſphere, we ſhall have $r = \sqrt{R^2 - \frac{1}{3}(AB\sqrt{3})^2} = \sqrt{\frac{5+\sqrt{5}}{8} AB^2 - \frac{1}{3} AB^2} = AB\sqrt{\frac{7+3\sqrt{5}}{24}}$.

But the ſolid is compoſed of 20 equal triangular pyramids, each of whoſe baſes are $= \frac{AB^2}{4}\sqrt{3}$ by problem 8; there-

fore

$$\sqrt{\frac{3\sqrt{5}+7}{2}\times\frac{5\times 3^3}{6}}=\sqrt{\frac{3\times 2.23606+7}{2}\times\frac{5\times 27}{6}}=\sqrt{\frac{7.0818+7}{2}\times\frac{5\times 9}{2}}=\sqrt{\frac{14.0818}{2}\times\frac{45}{2}}$$

$=\sqrt{7.0409}\times 22.5=2.65\times 22.5=95.625=$ *ſolidity required.*

2. Required the ſolidity of an icoſaedron, whoſe linear ſide is 1? *Anſ.* 2.1816949905.

---

fore $\frac{20AB^2}{4}\sqrt{3}\times\frac{1}{3}r=5AB^2\sqrt{3}\times\frac{AB}{3}\sqrt{\frac{7+3\sqrt{5}}{24}}=\frac{5}{6}$ $AB^3\sqrt{\frac{7+3\sqrt{5}}{2}}=$ ſolidity of the icoſaedron. Q. E. D.

If L be put for the linear ſide, then will $5L^2\sqrt{3}=$ ſurface of the icoſaedron.

Note. The ſuperficies and ſolidity of any of the 5 regular bodies may be found as follows.

Rule. 1. Multiply the tabular area by the ſquare of the linear edge, and the product will be the ſuperficies.

2. Multiply the tabular ſolidity by the cube of the linear edge, and the product will be the ſolidity.

Surfaces and Solidities of the Regular bodies.

N° of Sides.	Names.	Surfaces.	Solidities.
4	Tetraedron	1.73205	0.11785
6	Hexaedron	6.00000	1.00000
8	Octaedron	3.46410	0.47140
12	Dodecaedron	20.64573	7.66312
20	Icoſaedron	8.66025	2.18169

OF

CYLINDRIC RINGS.

PROBLEM I.

To find the convex ſuperficies of a cylindric ring.

RULE.*

TO the thickneſs of the ring add the inner diameter, and this ſum being multiplied by the thickneſs, and the product again by 9.8696 will give the ſuperficies required.

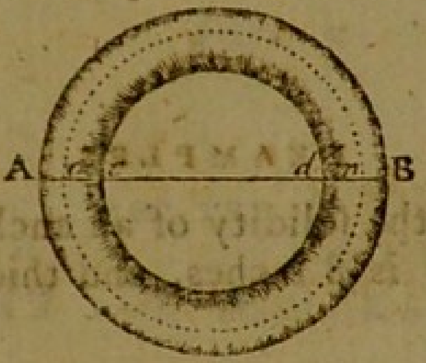
EXAMPLES.

1. The thickneſs of AC of a cylindric ring is 3 inches, and the inner diameter *cd* 12 inches: what is the convex ſuperficies?

---

* A ſolid ring of this kind is only a bent cylinder, and therefore the rules for obtaining its ſuperficies, or ſolidity, are the ſame as thoſe already given. For let AC be any ſection of the ſolid perpendicular to its axis *on*, and then $AC \times 3.14159$, &c. = circumference of that ſection, and $\overline{AC + cd}\,(on) \times 3.14159$, &c. = length of the axis *on*.

Whence $AC \times 3.14159$, &c. $\times \overline{AC + cd} \times 3.14159$, &c. $= \overline{AC + cd} \times AC \times \overline{3.14159, \&c.}|^2 = \overline{AC + cd} \times AC \times 9.8696$, &c. = ſuperficies required.



$\overline{12+3} \times 3 \times 9.8696 = 15 \times 3 \times 9.8696 = 45 \times 9.8696 = 444.132 =$ *superficies required.*

2. The thickneſs of a cylindric ring is 4 inches, and the inner diameter 18: what is the convex ſuperficies? *Anſ.* 868.52 *ſquare inches.*

3. The thickneſs of a cylindric ring is 2 inches, and the inner diameter 18: what is the convex ſuperficies? *Anſ.* 394.785 *ſquare inches.*

PROBLEM II.

To find the ſolidity of a cylindric ring.

RULE.*

To the thickneſs of the ring add the inner diameter, and this ſum being multiplied by the ſquare of half the thickneſs, and the product again by 9.8696 will give the ſolidity.

---

* *Demon.* $Ac^2 \times .78539$, &c. $= Ac^2 \times .\frac{3.14159}{4}$, &c. $= \frac{1}{4}Ac^2 \times 3.14159$, &c. = area of the ſection Ac; and $\overline{Ac+cd}$ $(on) \times 3.14159$, &c. = length of the axis on.

Therefore $\overline{Ac+cd} \times \frac{1}{4}Ac^2 \times \overline{3.14159, \&c.})^2 = \overline{Ac+cd} \times \frac{1}{4}Ac^2 \times 9.8698$. Q.E.D.

EXAM-

EXAMPLES.

1. What is the ſolidity of an anchor ring, whoſe inner diameter is 8 inches, and thickneſs in metal 3 inches?

$\overline{8+3\times\frac{3}{2}}|^2\times 9.8696 = 11\times 1.5^2\times 9.8696 = 11\times 2.25\times 9.8696 = 24.75\times 9.8696 = 244.2726 =$ *ſolidity required.*

2. The inner diameter of a cylindric ring is 18 inches, and its thickneſs 4 inches: what is the ſolidity? *Anſ.* 868.5248.

3. Required the ſolidity of a cylindric ring, whoſe thickneſs is 2 inches, and its inner diameter 12. *Anſ.* 138.1744.

4. What is the ſolidity of a cylindric ring, whoſe thickneſs is 4 inches, and inner diameter 26? *Anſ.* 789.568.

This figure being only a cylinder bent round into a ring, its ſurface and ſolidity may alſo be found as in the cylinder, namely, by multiplying the axis or length of the cylinder by the circumference of the ring, or ſection, for the ſurface, and by the area of a ſection for the ſolidity.

Thus, if $c =$ circumference of the ring, or ſection, $a =$ area of that ſection, and $l =$ length of the axis: then will $cl =$ ſurface of the ring, and $al =$ to its ſolidity. Which rules are the ſame as for the cylinder, and may be eaſily converted into thoſe given in the text.

Theſe rules are indeed ſo obvious, as to render any demonſtration of them altogether unneceſſary.

OF

OF THE

WEIGHT AND DIMENSIONS

OF

BALLS AND SHELLS.

THE weight and dimenſions of any ball or ſhell being had from experiments, the weight and dimenſions of any other ball or ſhell, of the ſame kind, may be found by the following rules.

PROBLEM I.

Having the diameter of an iron ſhot given, to find its weight.

RULE.*

Take $\frac{1}{8}$ of the cube of the diameter, and $\frac{1}{8}$ of that eighth, and the ſum will be the weight required in pounds, *exactly.*

EXAM-

---

* *Demon.* The weight of an iron ſhot whoſe diameter is 4 inches is 9lbs; and as globes are as the cubes of their diameters, therefore $4^3 : 9\text{lbs.} :: D^3$ (diameter being $= D$) : $\frac{9}{64} D^3 =$ weight. But $\frac{9}{64} = \frac{8}{64} + \frac{1}{64} = \frac{1}{8} + \frac{1}{8}$ of $\frac{1}{8}$. Wherefore the weight required is $\frac{1}{8} D^3 + \frac{1}{8}$ of $\frac{1}{8} D^3$.

Q. E. D.

Rule

EXAMPLES.

1. The diameter of an iron ſhot is 3.5 inches; what is its weight?

```
      3.5
      3.5
     ----
      175
     105
     ----
    12.25
      3.5
    -----
     6125
    3675
   ------
 8)42.875
   ------
  8)5.359
     .669
   ------
    6.028
```

6.028 = *lbs. nearly.*

2. The diameter of an iron ſhot is 6.7 inches; what is its weight? *Anſ.* 42.294*lbs.*

PROBLEM II.

Having the diameter of a leaden ball given, to find its weight.

Rule by Logarithms. To 3 times the log. of the diameter, add—1.1480625, and the ſum is the logarithm of the weight required.

RULE.

RULE.*

Take $\frac{1}{3}$ of the cube of the diameter, and from it ſubtract $\frac{1}{3}$ of this third, and the remainder is the weight required, *nearly*.

EXAMPLES.

1. What is the weight of a leaden ball, whoſe diameter is 3.3 inches?

3.3
3.3

99
99

10.89
3.3

3267
3267

3)35.937

3)11.979
3.993

7.986 = 8*lbs. weight, nearly.*

---

* The weight of a leaden ball, of $4\frac{1}{4}$ inches diameter, is 17lbs. therefore 4.25^3 : 17*lbs.* : : D^3 : $\frac{17}{76.765625} \times D^3 = \frac{2}{9} D^3$ *nearly*, $= (\frac{3}{9} - \frac{1}{9}) \times D^3 = \frac{1}{3} D^3 - \frac{1}{3}$ of $\frac{1}{3} D^3$.

Q. E. D.

Rule by Logarithms. To 3 times the log. of the diameter, add—1.3452821, and the ſum will be the logarithm of the weight in pounds *exactly*.

2. What

2. What is the weight of a leaden ball whose diameter is 5.24 inches? *Ans. 32lbs. nearly.*

PROBLEM III.

The weight of an iron ball being given, to find the diameter.

RULE.*

Multiply the weight by 7, and to the product add $\frac{1}{9}$ of the weight, and the cube root of the sum will be the diameter in inches.

EXAMPLES.

1. The weight of an iron ball is 24lbs. what is the diameter?

$$\begin{array}{r} 24 \\ 7 \\ \hline 168 \\ \tfrac{1}{9} \text{ of } 24 = \quad 2.666 \\ \hline 170.666 \end{array}$$

---

* *Demon.* By problem I. $\frac{9}{64}D^3 =$ weight $= w$; therefore $D^3 = \frac{64}{9} \times w = \frac{63}{9}w + \frac{1}{9}w = 7w + \frac{1}{9}w$; and consequently $D = \overline{7w + \frac{1}{9}w}|^{\frac{1}{3}}$. Q. E. D.

The Rule by Logarithms. To the log. of the weight in pounds, add 0.8519375, and $\frac{2}{3}$ of the sum will be the logarithm of the diameter in inches.

The cube root in any of the following questions may also be extracted by Logarithms.

Let

† *Let the supposed root be* 5, *whose cube is* 125; *then*

```
125              170.666
  2                    2
———              ———————
250              341.332
170.666          125.
———————          ———————
420.666    :     466.332    ::    5
                       5
                 ———————
420.666)2331.660(5.54 inches = cube root of
        2103330    170.666, or the dia. req.
        ———————
         2283300
         2103330
         ———————
          1799700
          1682664
          ———————
          117036
```

2. The weight of an iron ball is 12lbs. what is the diameter? *Ans.* 4.403 *inches.*

PROBLEM IV.

Having the weight of a leaden ball given, to find its diameter.

---

† *The rule here used for extracting the cube root is as follows:*

Find a root nearly equal to the true one by trial, and then say, as twice the cube of the supposed root added to the given number, is to twice the given number added to the cube of the supposed root, so is the supposed root to the true root, *nearly.*

For a further account of this method, together with its demonstration, see the *Scholar's Guide to Arithmetic,* page 170.

RULE.*

To 4 times the weight, add half the weight, and $\frac{3}{100}$ of half the weight, and the cube root of this sum will be the diameter, *nearly*.

EXAMPLES.

1. The weight of a leaden ball is 8lbs. what is the diameter?

$$\begin{array}{l} 8 \\ 4 \\ \hline 32 \\ 4 = \tfrac{1}{2} \text{ of } 8 \\ \hline 36 \\ .12 = \tfrac{3}{100} \text{ of } 4 \\ \hline 36.12 \end{array}$$

Let the supposed root be 3, whose cube is 27; then

---

* *Demon.* By problem II. $D^3 = \frac{76.765625}{17} \times w = 4w + \frac{1}{2}w + \frac{3}{100}$ of $\frac{1}{2}w$, *very nearly*; therefore $D = \overline{4w + \frac{1}{2}w + \frac{3}{100} \text{ of } \frac{1}{2}w}\,\big|^{\frac{1}{3}}$. Q. E. D.

The Rule by Logarithms. To the log. of the weight in pounds, add 0.6547179, and $\frac{1}{3}$ of the sum will be the logarithm of the diameter, in inches, *exactly*.

27

```
27            36.12
 2                2
——           ——————
54            72.24
36 12         27.
—————         ——————
90.12    :    99.24  ::  3
                  3
              ——————
        90.12)297.72(3.30= diam. nearly.
              27036
              ——————
               27360
               27036
              ——————
                 324
```

2. What is the diameter of a leaden ball whoſe weight is 12lbs? *Anſ.* 3.78 *inches*.

PROBLEM V.

Having the external and internal dimenſions of an iron ſhell given, to find its weight.

RULE.*

Take $\frac{1}{8}$ of the difference of the cubes of the diameter in inches, and $\frac{1}{8}$ of that eighth, and their ſum will be the weight in pounds.

---

* *Demon.* The weight is $\frac{9}{64} \times \overline{D^3-d^3} = (\frac{8}{64}+\frac{1}{64}) \times \overline{D^3-d^3} = \frac{1}{8} \times \overline{D^3-d^3} + \frac{1}{8}$ of $\frac{1}{8} \times \overline{D^3-d^3}$. Q.E.D.

EXAMPLES.

1. What is the weight of a 13 inch iron bomb-shell, the metal being 2 inches thick on a mean?

```
             13
             13
             --
             39
            13
            ---
            169
             13
            ---
            507
           169
           ----
           2197=cube of the external diam.
   9×9×9=729=cube of the internal diam.
           ----
        8)1468
          ----
        8)183.5
            22.9
          -----
          206.4=lbs. weight required.
```

2. What is the weight of a 9 inch iron bomb shell, the metal being $1\frac{1}{2}$ inches thick? *Ans.* 72.14*lbs.*

PROBLEM VI.

To find the number of pounds of powder that a hollow shell will hold.

RULE.

RULE.*

Cube the internal diameter in inches, and divide by 59.32, and the quotient is the weight, *nearly*.

EXAMPLES.

1. How many pounds of powder will a hollow shell hold, whose internal diameter is 9 inches?

```
        9
        9
       --
       81
        9
     ----
59.32)729.00(12.28 = pounds nearly.
      5932
      -----
      13580
      11864
      -----
      17160
      11864
      -----
       52960
       47456
       -----
        5504
```

2. How

* *Demon.* The content of the shell is $= D^3 \times .5236$; therefore 31.06 (*in. in 1lb.*) : 1 lb. : : $D^3 \times .5236$: $\frac{.5236}{31.06} \times D^3 = \frac{D^3}{59.32}$. Q. E. D.

Or the rule may be expressed thus. Cube the diameter, and take $\frac{1}{60}$ part of the result; also $\frac{1}{60}$ part of this $\frac{1}{60}$; and from

 the

2. How many pounds of powder will a hollow bomb-ſhell hold, whoſe internal diameter is 13 inches?

Anſ. 37*lbs. nearly.*

PROBLEM VII.

To find the dimenſions of a cubical box, to hold a given quantity of powder.

RULE.*

Multiply the weight in pounds by 31.06, and the cube root of the product will give the length of the ſide in inches.

EXAMPLES.

1. What muſt be the length of the ſide of a cubical box that is to hold 15lbs. of powder?

$$\begin{array}{r} 31.06 \\ 16 \\ \hline 15530 \\ 3106 \\ \hline 465.90 \end{array}$$

---

the ſum of theſe two quotients, ſubtract $\frac{1}{3}$ of the laſt quotient, and the remainder is the anſwer, *nearly*.

Rule by Logarithms. To 3 times the log. of the diameter in inches, add—2.2267989, and the ſum is the logarithm of the quantity of pounds required.

* *Demon.* As 31.06 inches : 1lb. : : s^3 (the cube of the ſide) : w (the weight); whence $s = \overline{31.06 \times w)}\,\frac{1}{3}$. Q. E. D.

Rule by Logarithms. To $\frac{1}{3}$ of the log. of the weight in pounds, add 0.4974005, and the ſum will be the logarithm of the ſide in inches.

Let

* *Let* 7.5 *be the supposed root, whose cube is* 421.875; then

```
421.875          465.9
      2              2
-------          -----
843.750          931.8
465.90           421.875
--------         --------
1309.650    :    1353.675   ::   7.5
                      7.5
                 --------
                  6768375
                 9475725
                 --------
1309.650)10152.5625(7.75=side req.
                 9167550
                 -------
                 9850125
                 9167550
                 -------
                  6825750
                  6548250
                  -------
                   277500
```

2. What is the side of a cubical box, that is to hold 12lbs. of powder? *Ans.* 7.19 *inches.*

PROBLEM VIII.

Given the length, breadth, and depth of a rectangular box, to find how many pounds of powder will fill it.

---

* The cube root may here also be found by Logarithms.

RULE.*

Multiply the length, breadth, and depth in inches together; and this product being multiplied again by .0322 will give the anſwer in pounds, *nearly*.

But if greater exactneſs be required, ſubtract $\frac{1}{4000}$ part of the product of the three dimenſions from the laſt reſult, and the remainder will be the anſwer, *very nearly*.

EXAMPLES.

1. The length of a rectangular box is 15 inches, the breadth 13 inches, and the depth 5 inches: how many pounds of powder will it hold?

```
      15
      13
    ----
      45
     15
    ----
     195
       5
    ----
     975
   .0322
  ------
    1950
   1950
  2925
  ------
31.3950=pounds required.
```

2. The

* *Demon.* Let L, B, D, = length, breadth, and depth reſpectively.

Then $\frac{LBD}{31.06}$ = weight in pounds = $LBD \times .03219574$ = $(.0322 - \frac{1}{4000}) \times LBD$. Q. E. D.

Rule

2. The length of a rectangular box is 1 foot, the breadth 9 inches, and the depth 4 inches: how many pounds of powder will it hold? *Anſ.* 13.9104*lbs.*

PROBLEM IX.

Given the diameter and length of a hollow cylinder, to find how many pounds of gunpowder will fill it.

RULE.*

Multiply the ſquare of the diameter by $\frac{1}{10}$ of the length; and then take $\frac{1}{4}$ of the product, $\frac{1}{90}$ part of this fourth, and $\frac{1}{90}$ part of this laſt, and the ſum of all theſe parts will be the anſwer in pounds, *nearly.*

Rule by Logarithms. Add the logarithms of the length, breadth, and depth in inches, and the conſtant logarithm —2.5077983 together; and the ſum will be the logarithm of the anſwer in pounds, *exactly.*

* *Demon.* $\frac{D^2 \times .78539 \times L}{31.06} = w = \frac{9}{356} D^2 L$, *nearly.*

But $\frac{9}{356} = \frac{9}{360-4} = \frac{9}{360} + \frac{9}{360} \times \frac{4}{360} + \frac{9}{360} \times \frac{4}{360} \times \frac{4}{360}$, &c. $= \frac{1}{10} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{90} + \frac{1}{4} \times \frac{1}{90} \times \frac{1}{90}$, &c.

Hence $w = D^2 \times \frac{L}{10} \times (\frac{1}{4} + \frac{1}{4} \times \frac{1}{90} + \frac{1}{4} \times \frac{1}{90} \times \frac{1}{90})$, *nearly.* Q. E. D.

The two firſt terms are near enough for common uſe.

Rule by Logarithms. To twice the log. of the diameter, add the logarithm of the length of the cylinder, and the conſtant logarithm—2.4028885, and the ſum will be the logarithm of the weight in pounds, *exactly.*

EXAMPLES.

1. The diameter of a hollow cylinder is 4 inches, and the length 1 foot: how many pounds of powder will it hold?

$$
\begin{array}{rl}
4 & \\
4 & \\
\hline
16 & = \textit{square of the diameter.} \\
1.2 & = \tfrac{1}{10} \textit{ of the length.} \\
\hline
32 & \\
16 & \\
\hline
4)\,19.2 & \\
\hline
4.8 & \\
.0533 & = \tfrac{1}{90} \textit{ of } 4.8 \\
.0005 & = \tfrac{1}{90} \textit{ of } .0533 \\
\hline
4.8538 & \textit{pounds the answer.}
\end{array}
$$

2. The diameter of a hollow cylinder is 6 inches, and its length 10 inches: how many pounds of powder will it hold? *Ans.* 9.1 *lbs.*

PROBLEM X.

Given the diameter of a hollow cylinder, to find what length of it will be filled by a given quantity of powder.

RULE.*

1. Divide the weight in pounds by the square of the diameter in inches. 2. Mul-

---

* *Demon.* By the last problem $L = \frac{356}{9} \times \frac{w}{D^2} = 39\frac{5}{9} \times \frac{w}{D^2} = (40 - \frac{4}{9}) \times \frac{w}{D^2} = (40 - \frac{3}{9} - \frac{1}{9}) \times \frac{w}{D^2} = \frac{40w}{D^2} - (\frac{1}{3} \times \frac{w}{D^2} - \frac{1}{3} \text{ of } \frac{1}{3} \times \frac{w}{D^2})$ *nearly.* Q. E. D.

2. Multiply the quotient by 40, and from the product subtract $\frac{1}{3}$ of the quotient.

3. From the remainder take $\frac{1}{3}$ of that third, and this last remainder will be the answer, *nearly*.

EXAMPLES.

1. The diameter of a hollow cylinder is 1 foot: what length of it will be filled by 10 pounds of powder?

12)10

12).8333

.0694416 = 10*lbs. divided by the square of* 12.
40

2.7776640
.0231472 = $\frac{1}{3}$ *of* .0694416

2.7545168
.0077157$\frac{1}{3}$ = *of* .0231472

2.7468011 = *height in inches required.*

2. The diameter of a hollow cylinder is 6 inches: what length of it will be filled by 12lbs. of powder?

Ans. 13.185.

Rule by Logarithms. To the constant logarithm 1.597111, add the logarithm of the given quantity of powder in pounds, add also the arithmetical complement of twice the logarithm of the diameter in inches; and the sum will be the logarithm of the length in pounds.

These rules, and their investigations, were given me by Mr. *Reuben Burrow*.

OF

OF THE

PILING

OF

BALLS AND SHELLS.

IRON *ſhot and ſhells* are uſually piled in horizontal courſes, in a pyramidical or wedge like form, the baſe being either an equilateral triangle, a ſquare or a rectangle. In the triangle and ſquare, the pile finiſhes in a ſingle ball; but in the rectangle, the top is a ſingle row of balls.

In triangular and ſquare piles the number of horizontal rows or courſes is always equal to the number of ſhot on one ſide, in the bottom row. But, in rectangular piles, the number of courſes is equal to the number of ſhot in the breadth of the bottom row; and the number in the top row, leſs one, is the difference between the number in the length and breadth of the bottom row.

PROBLEM I.

To find the number of ſhot in a finiſhed triangular pile.

RULE.*

Multiply the number in one ſide of the bottom row plus two, by that number plus one; and this product being

* *Demon.* In triangular piles, each horizontal courſe is a triangular number, produced by taking the ſucceſſive ſums of the

being again multiplied by $\frac{1}{6}$ of the number in the ſaid row, will give the anſwer required.

EXAMPLES.

1. Required the number of ſhot in a finiſhed triangular pile, the number in one ſide of the baſe being 20?

22 = *number plus* 2
21 = *number plus* 1
—
22
44
—
462
20
—
6)9240
—
1540 = *number of ſhot in the whole pile.*

2. Required the number of ſhot in a finiſhed triangular pile, the number in one ſide of the baſe being 40? *Anſ.* 11480.

---

the numbers 1; 1, 2; 1, 2, 3; 1, 2, 3, 4; 1, 2, 3, 4, 5, &c. and the whole number of ſhot in ſuch a pile is equal to the ſum of all thoſe triangular numbers, taken as far, or to as many terms, as are equal to the number in one ſide of the bottom courſe.

Thus, if 1, 2, 3, 4, 5, 6, 7, &c. be the natural numbers, then will 1, 3, 6, 10, 15, 21, 28, be the triangular numbers; or the number of ſhot in each courſe from the top.

But the ſum of the ſeries 1+3+6+10+15+21+28, &c. to n terms is $= \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3}$ (by *Simpſon*'s *Algebra*, page 216) $= \overline{n+2} \times \overline{n+1} \times \frac{n}{6}$; which is the ſame as the rule. Q.E.D.

PRO-

PROBLEM II.

To find the number of ſhot in a finiſhed ſquare pile.

RULE.*

Multiply the number in one ſide of the bottom row plus one, by double that number plus one; and this product being again multiplied by $\frac{1}{6}$ of the ſaid number, will give the anſwer required.

EXAMPLES.

1. Required the number of ſhot in a finiſhed ſquare pile, the number in one ſide of its baſe being 20.

41 = *double of* 20, *plus* 1
21 = 20 *plus* 1

```
  41
 82
----
 861
  20
------
6)17220
------
 2870
```

2870 = *number of ſhot in the pile.*

2. Required

---

* *Demon.* In ſquare piles, each horizontal courſe is a ſquare number, produced by taking the ſquare of the number in its ſide; and the whole number of ſhot in ſuch a pile is equal to the ſum of all thoſe ſquares, beginning at one and proceeding as far as the number in the ſide of the bottom courſe.

Thus, if 1, 2, 3, 4, 5, 6, 7, &c. be the natural numbers, then will 1, 4, 9, 16, 25, 36, 49, &c. be the ſquare numbers, or the number of ſhot in each courſe from the top.

But

2. Required the number of ſhot in a finiſhed ſquare pile, one ſide of the lower tier having 40 ſhot in it.

Anſ. 22140.

PROBLEM III.

To find the number of ſhot in a finiſhed rectangular pile.

RULE.*

1. To twice the length of the uppermoſt row add the number of courſes leſs one, and multiply the ſum by the number of courſes plus 1.

But the ſum of the ſeries $1+4+9+16+25+36+49$, &c. to n terms is $= \frac{n \times (n+1) \times (2n+2)}{6}$ (by *Simpſon*'s *Algebra*, page 206) $= \overline{n+1} \times \overline{2n+2} \times \frac{n}{6}$, which is the ſame as the rule. Q. E. D.

* *The inveſtigation of this rule* is given by *Mr. Simpſon*, in page 209 of his Algebra thus: Let $m+1$ and $p+1$ repreſent the length and breadth of the uppermoſt rank, and n the number of ranks one above another.

Then will $\overline{m+1} \times \overline{p+1} + \overline{m+2} \times \overline{p+2} + \overline{m+3} \times \overline{p+3}$, &c. be the number of ſhot in each of the rectangular courſes of the pile.

And $\overline{m+1} \times \overline{p+1} + \overline{m+2} \times \overline{p+2} + \overline{m+3} \times \overline{p+3} \ldots \ldots + \overline{m+n} \times \overline{p+n} =$ number of ſhot in the pile, whether it be whole, or broken.

But $\overline{m+1} \times \overline{p+1} + \overline{m+2} \times \overline{p+2} + \overline{m+3} \times \overline{p+3} \ldots\ldots + \overline{m+n} \times \overline{p+n} = n \times (mp + \frac{n+1}{2} \times \overline{m+p} + \frac{(n+1) \times (2n+1)}{6}$ $= \frac{n}{4} \times (\overline{2m+n+1} \times \overline{2p+n+1} + \frac{(n+1) \times (n-1)}{3}$ which is the ſame as the rule. Q. E. I.

2. Mul-

2. Multiply the number of courſes leſs one by that number more one, and add $\frac{1}{3}$ of this product to the former.

3. Take $\frac{1}{4}$ of this laſt ſum and multiply it by the number of courſes, and it will give the anſwer required.

EXAMPLES.

1. How many ſhot are there in a finiſhed rectangular pile of 15 courſes, the number in the uppermoſt row being 32?

32
2
—
64 = *twice the uppermoſt row.*
14 = *number of courſes leſs* 1.
—
78
16
—
468
78
—
1248
—
14 = *number of courſes leſs* 1.
16 = *number of courſes more* 1.
—
84
14
—
3)224
—
$74\frac{2}{3}$
1248
—
$1322\frac{2}{3}$

4)$1322\frac{2}{3}$

$$\begin{array}{r} 4)1322\frac{2}{3} \\ \hline 330\frac{2}{3} \\ 15 \\ \hline 1660 \\ 330 \\ \hline 4960 \end{array}$$

4960 = *number of ſhot in the pile.*

2. How many ſhot are there in a finiſhed rectangular pile of 20 courſes, the number in the top row being 40? *Anſ.* 11060.

The number of ſhot in any broken pile may be obtained by computing what the whole finiſhed pile would contain, and alſo what the pile taken away contained; in which caſe the remainder muſt be the number in the unfiniſhed part as required.

The rules for finding the number of ſhot in the different kinds of piles, may be expreſſed algebraically thus:

$\frac{n}{6} \times \overline{n+1} \times \overline{n+2} =$ N° in the triangular pile.

$\frac{n}{6} \times \overline{n+1} \times \overline{2n+1} =$ N° in the ſquare pile.

$\frac{n}{6} \times (n+1) \times (3m-n+1) =$ N° in the rectangular pile.

Where n denotes the number in one ſide of the bottom row, for the triangular and ſquare piles. And in the rectangular pile it denotes the breadth of the bottom row, and m its length. Which rule, for the latter caſe, is ſomething more convenient than that given in the text.

OF

ARTIFICERS WORK.

ARTIFICERS eſtimate, or compute, the value of their works by different meaſures, *viz* *.

1. *Glazing*, and *Maſon*'s flat work, &c. by the foot.

2. *Painting*, *Plaſtering*, *Paving*, &c. by the yard.

3. *Flooring*, *Partitioning*, *Roofing*, *Tiling*, &c. by the ſquare of 100 feet.

4. *Brickwork*, &c. by the rod of $16\frac{1}{2}$ feet, whoſe ſquare is $272\frac{1}{4}$.

The meaſures made uſe of in theſe works are contained in the following table:

12 inches	make	1 lineal foot
144 ſquare inches	make	1 ſquare foot
9 ſquare feet	make	1 ſquare yard
100 ſquare feet	make	a ſquare
$272\frac{1}{4}$ ſquare feet, or $30\frac{1}{4}$ ſquare yards	make	1 rod, perch, or ſquare pole.

* The beſt method of taking the dimenſions of all ſorts of artificers work is by feet, tenths and hundredths; becauſe the computations may then be performed by common multiplication, or by the ſliding rule, hereafter deſcribed.

OF

BRICKLAYERS WORK*.

BRICKLAYERS compute or value their work at the rate of a brick and a half thick, and, if a wall be more or leſs than this ſtandard, it muſt be reduced to it, as follows:

---

* *Note*. In practice it is uſual to divide the ſquare feet by 272 only, omitting the $\frac{1}{4}$.

The uſual way to take the dimenſions of a building, is to meaſure half round its middle, on the outſide, and half round it on the inſide; and this will give the true compaſs, in which the thickneſs of the wall is included.

When the height of the building is unequal, take ſeveral different altitudes, and their ſum being divided by the number you have taken, may be conſidered as the mean height.

To meaſure a chimney ſtanding by itſelf, without any party-wall adjoining; girt it about for the length, and reckon the height of the ſtory for the breadth; but if it ſtands againſt a wall, you muſt meaſure it round to the wall for the girt, and take the height as before.

When the chimney is wrought upright from the mantle-tree to the cieling, the thickneſs muſt always be the ſame with the jaumbs; and nothing is ever deducted for the vacancy between the floor and the mantle-tree, becauſe of the gathering of the breaſt and wings to make room for the hearth in the next ſtory.

To meaſure chimney ſhafts, or that part which appears above the roof; girt them with a line, about the leaſt place for the length, and take the height for the breadth; and if they be four inches thick, ſet down the thickneſs at one brick-work; but if they be nine inches thick, reckon it at a brick and a half, in conſideration of the plaſtering and ſcaffolding.

All

RULE.*

Multiply the ſuperficial content of the wall in feet, by the number of half bricks in the thickneſs, and $\frac{1}{3}$ of that product will be the content required.

EXAMPLES.

1. How many ſquare rods are there in a wall $52\frac{1}{2}$ feet long, 12 feet 9 inches high, and $2\frac{1}{2}$ bricks thick?

By Decimals.

52.5 *length*
12.75 = *height*

2625
3675
1050
525

272)669.375(2.4609
544   5 *half bricks*

1253.3)12.3045
1088   *ro. fe. in.*
4.1015 = 4 27 7 *the anſwer.*

1657
1632

2550
2448

102

By

---

* All windows, doors, &c. are to be deducted out of the contents of the walls in which they are placed. But this deduction is made only with regard to materials; for the value of

By Crofs Multiplication.

feet	in.	
52	6	
12	9	
630	0	
39	4	6
272)669	4	6
2 124	4	
	5	
3)12 82	8	
4 27	7	*as before.*

2. How many fquare rods are there in a wall $62\frac{1}{2}$ feet long, 14 feet 8 inches high, and $2\frac{1}{2}$ bricks thick?

ro. fe. in. p.
Anf. 5 167 9 4

3. If each fide wall of a building be 45 feet long on the outfide, each end wall 15 feet broad on the infide, the height of the building 20 feet, and the gable at each end of the wall 6 feet high, the whole being 2 bricks thick; what is the true content in ftandard rods? *Anf.* 12.1761.

---

of their workmanfhip is added to the bill at the ftated rate agreed on.

There are alfo other allowances to be made to the workman, fuch as thofe for returns or angles made by two adjoining walls, and double meafure for feathered gable ends, &c.

All ornamental work is generally valued by the foot fquare, fuch as arches, doors, architraves, frizes, cornices, &c. But carved mouldings, &c. are often agreed for by the running foot, or lineal meafure.

OF

MASONS WORK.

TO Maſonry belongs all ſorts of ſtone work, and the meaſure made uſe of is a foot, either ſuperficial or ſolid.

Walls, blocks of marble or ſtone, columns, &c. are meaſured by the ſolid foot; and pavements, ſlabs, chimney-pieces, &c. by the ſuperficial foot.*

EXAMPLES.

1. Required the ſolid content of a wall whoſe length is 48 feet 6 inches, its height 10 feet 9 inches, and thickneſs 2 feet.

By Decimals.

48.5
10.75
_____
2425
3395
4850
_____
521.375
2
_____
1042.750 *the anſwer.*

By

* Solid meaſure is principally uſed for materials, and the ſuperficial for workmanſhip.—In the ſolid meaſure the true length, breadth, and thickneſs, are taken, and multiplied continually together. And in the ſuperficial meaſure, the length and breadth of every part of the projection muſt be taken, as it appears without the general upright face of the building.

All

By Cross Multiplication.

feet	*in.*	
48	6	
10	9	
485	0	
36	4	6
521	4	6
2	0	0
1042	9	0 *the same as before.*

2. Required the solid content of a wall whose length is 53 feet 6 inches, its height 12 feet 3 inches, and its thickness 2 feet. *Ans.* 1310 *feet* 9 *in.*

3. What is a marble slab worth, whose length is 5 feet 7 inches, and breadth 1 foot 10 inches, at 6s. per foot? *Ans.* 3*l.* 1*s.* 5*d.*

4. What is the solid content of a wall, whose length is 60 feet 9 inches, its height 10 feet 3 inches, and its thickness $2\frac{1}{2}$ feet? *Ans.* 1556.71875 *feet.*

5. In a chimney-piece, suppose the

	fe.	*in.*
Length of the mantle and slab each, - -	4	6
Breadth of both together, - -	3	2

	fe.	*in.*
Length of each jamb, - -	4	4
Breadth of both together, -	1	9

What will be the content of the chimney-piece?

Ans. 21 *feet* 10 *in.*

All windows, doors, &c. are to be deducted out of the contents of the walls in which they are placed; but this deduction is only to be made with regard to materials, for the value of the workmanship is to be added to the workman's bill at the stated rate only.

OF

CARPENTERS AND JOINERS WORK*.

CARPENTERS and Joiners work is that of flooring, partitioning, roofing, &c. and is meafured by the fquare of 100 feet.

EXAM-

* *Note.* Large and plain articles are ufually meafured by the foot, or yard, &c. fquare; but inriched mouldings, and fome other articles, are often eftimated by running or lineal meafure, and fome things are rated by the piece.

In meafuring of joifts it is to be obferved, that only one of their dimenfions is the fame with that of the floor, and the other will exceed the length of the room by the thicknefs of the wall, and $\frac{1}{3}$ of the fame, becaufe each end is let into the wall about $\frac{2}{3}$ of its thicknefs.

No deductions are made for hearths, on account of the additional trouble and wafte of materials.

Partitions are meafured from wall to wall for one dimenfion, and from floor to floor, as far as they extend, for the other.

No deduction is made for door-ways, on account of the trouble of framing them.

In meafuring of Joiners work, the ftring is made to ply clofe to every part of the work over which it paffes.

The meafuring of centering for cellars is found by making a ftring pafs over the furface of the arch for the breadth, and taking the length of the cellar for the length; but in groin-centering, it is ufual to allow double meafure, on account of their extraordinary trouble.

In roofing, the length of the houfe in the infide, together with $\frac{2}{3}$ of the thicknefs of one gable, is to be confidered as the length, and the breadth is equal to double the length of a ftring which is ftretched from the ridge down to the rafter, along the eaves board, till it meets with the top of the wall.

For

EXAMPLES.

1. If a floor be 57 feet 3 inches long, and 28 feet 6 inches broad: how many squares will it contain?

By Decimals.

```
   57.25
    28.5
 -------
   28625
  45800
 11450
 -------
16|31.625
```

By

For ſtair-caſes, take the breadth of all the ſteps, and make a line ply cloſe over them, from the top to the bottom, and multiply the length of this line by the length of a ſtep for the whole area.—By the length of a ſtep is meant the length of the front and the returns at the two ends, and by the breadth is to be underſtood the girt of its two upper ſurfaces, or the tread and riſer.

For the baluſtrade, take the whole length of the upper part of the hand-rail, and girt over its end till it meet the top of the newel-poſt, for the length: and twice the length of the baluſter upon the landing, with the girt of the hand-rail, for the breadth.

For wainſcotting, take the compaſs of the room for the length; and the height from the floor to the cieling, making the ſtring ply cloſe into all the mouldings, from the breadth.—Out of this muſt be made deductions for windows, doors, and chimneys, &c. but workmanſhip is counted for the whole, on account of the extraordinary trouble.

For doors, it is uſual to allow for their thickneſs, by adding it into both the dimenſions of length and breadth, and then multiplying them together for the area.—If the door be pannelled on both ſides, take double its meaſure for the workmanſhip;

By Crofs Multiplication.

feet	*in.*	
57	3	
28	6	
1603	0	
28	7	6
16|31	7	6

2. A floor is 53 feet 6 inches long, and 47 feet 9 inches broad: how many fquares will it contain?

Anf. 25 *fq. and* 54 *feet.*

3. A partition is 91 feet 9 inches long, and 11 feet 3 inches broad: how many fquares will it contain?

Anf. 10 *fq. and* 32 *feet.*

4. If a houfe within the walls be 44 feet 6 inches long, and 18 feet 3 inches broad: how many fquares of roofing will cover it? *Anf.* 12 *fq. and* 18 *feet.*

5. If a houfe meafure within the walls 52 feet 8 inches in length, and 30 feet 6 inches in breadth, and the roof be of a true pitch, what will it coft roofing at 10*s*, 6*d*. per fquare? *Anf.* 12*l*. 12*s*. 11$\frac{3}{4}$*d.*

---

manfhip; but if one fide only be pannelled, take the area and its half for the workmanfhip.

For the furrounding architrave, gird it about the outermoft part for its length; and meafure over it, as far as it can be feen when the door is open, for the breadth.

Window fhutters, bafes, &c. are meafured in the fame manner.

In the meafuring of roofing for workmanfhip alone, all holes for chimney fhafts and fky-lights are generally deducted.

But in meafuring for work and materials, they commonly meafure in all fky-lights, luthern lights, and holes for the chimney fhafts, on account of their trouble and wafte of materials.

OF

OF

SLATERS AND TILERS WORK.

IN these works, the content of a roof is found by multiplying the length of the ridge by the girt from eave to eave; and, in slating, allowance must be made for the double row at the bottom.

In taking the girt, the line is made to ply over the lowest row of slates, and returned up the under-side till it meet with the wall or eaves-board; but in tiling, the line is stretched down only to the lowest part, without returning it up again.

Double measure is generally allowed for hips, vallies, gutters, &c. but no deductions are made for chimnies.

EXAMPLES.

1. The length of a slated roof is 45 feet 9 inches, and its girt 34 feet 3 inches: what is the content?

By Decimals.

```
        45·75
        34·25
        -----
        22875
        9150
       18300
      13725
     ---------
   9)1566.9375
     ---------
       174.104
```


By

By Crofs Multiplication.

feet	in.		
45	9		
34	3		
1555	6		
11	5	3	
9)1566	11	3	
174 0	11	3	

Anf. 174 *yards.*

2. What will the tiling a barn coft at 25*s.* 6*d.* per fquare, the length being 43 feet 10 inches, and the breadth 27 feet 5 inches, on the flat, the eave-boards projecting 16 inches on each fide? *Anf.* 24*l.* 9*s.* 5½*d.*

In angles formed in a roof, running from the ridge to the eaves, that angle of the roof, which bends inwards, is called a *valley*; and the angle bending outwards is called a *hip*. And in tiling and flating, it is common to add the length of the vallies to the content in feet; and fometimes alfo the hips are added.

In flating it is common to reckon the breadth of the roof 2 or 3 inches broader than what it meafures, becaufe the firft row is almoft covered by the fecond; and this is done fometimes when a roof is tiled.

Note. Sky-lights and chimney-fhafts are always deducted; but they feldom deduct luthern lights, or garret windows on the roof; for the covering them is reckoned equal to the hole in the roof.

In all works of this kind the content is computed, either in yards of 9 fquare feet, or in fquares of a hundred feet, and the fame allowance of hips and vallies is to be made as in roofing.

It is cuftomary to reckon the flat and half of any building within the walls, for the meafure of the roof of that building, when the roof is of a *true pitch*, or fo that the rafters are ¾ of the breadth of the building.

PLASTERERS

PLASTERERS WORK.

PLASTERERS work is of two kinds, viz. plaſtering upon laths, called cieling; and plaſtering upon walls, called rendering; and theſe different kinds muſt be meaſured ſeparately, and their contents collected into one ſum.

Note. Proper deductions muſt be made for doors, windows, &c.—And in meaſuring between quarters, there is commonly $\frac{1}{5}$ part of the whole area allowed; but when rendering between quarters is whitened or coloured, there is $\frac{1}{5}$ part to be added to the whole for the ſides of the quarters and braces.

EXAMPLES.

If a cieling be 59 feet 9 inches long, and 24 feet 6 inches broad: how many yards does it contain?

By Decimals.

$$\begin{array}{r} 59.75 \\ 24.5 \\ \hline 29857 \\ 23900 \\ 11950 \\ \hline 9)1463.875 \\ \hline 162.652 \end{array}$$

Plaſterers plain work is meaſured by the ſquare foot, or yard of 9 ſquare feet, and enriched mouldings, &c. by running or lineal meaſure.

By Cross Multiplication.

feet	*in.*		
59	9		
24	6		
1434	0		
29	10	6	
9)1453	10	6	
162 5	10	6	

Ans. 162 *yards* 5 *feet.*

2. If the partitions between rooms be 141 feet 6 inches about, and 11 feet 3 inches high; how many yards do they contain? *Ans.* 176.87.

3. There is a quantity of partitioning that measures 234 feet 8 inches about, and 14 feet 6 inches high, and is rendered between quarters; the lathing and plastering of which will be 8*d.* per yard, and the whiting 2*d.* per yard: what will the whole come to?

Ans. 13*l.* 17*s.* $2\frac{3}{4}$*d.*

4. The length of a room is 14 feet 5 inches, its breadth 13 feet 2 inches, and height 9 feet 3 inches to the under side of the cornice, whose girt is $8\frac{1}{2}$ inches, and its projection 5 inches from the wall on the upper part next the cieling: what will be the quantity of plastering, supposing there are no deductions but for one door, whose size is 7 feet by 4 feet? *Ans.* 53 *yards* 5 *feet* 3 *inches, of rendering,* 18 *yards* 5 *feet* 6 *inches, of cieling, and* 39 *feet* $0\frac{1}{11}$ *inches, of cornice.*

The work is chiefly of two kinds; viz. plastering upon laths, called cieling; and plastering upon walls, called rendering; all the different kinds are to be measured separately; and then the contents of the same kind, or the same number of coats must be collected together into one sum for the content of the whole.

OF

OF

PAINTERS WORK.

PAINTERS work is meaſured in the ſame manner as that of Joiners; and in taking the dimenſions, the line muſt be forced into all the mouldings and corners.

Windows are done at ſo much a-piece, and in carved mouldings, &c. it is cuſtomary to allow double the uſual meaſure.

EXAMPLES.

1. If a room be painted, whoſe height is 16 feet 6 inches, and its compaſs 97 feet 9 inches: how many yards does it contain?

By Decimals.

```
     97.75
      16.5
    ------
     48875
    58650
    9775
    ------
9)1612.875
    ------
   179.208
```

Baluſtrades and moſt other works of that kind are meaſured as in Joiners work, excepting for doors, window-ſhutters, &c. where the Joiner is only allowed the area and half area, but the Painter has always double the area of one ſide, becauſe every part that is painted muſt be meaſured.

By Cross Multiplication.

feet	in.	
97	9	
16	6	
1564	0	
48	10	6
9)1612	10	6
179 1	10	6

Ans. 179 *yards* 1 *foot*.

2. The height of a room is 14 feet 10 inches, and the circumference 21 feet 8 inches, how many square yards does it contain? *Ans*. 35.

3. Suppose a room, that was to be painted at 8*d*. *per* yard, measures as follows: The height is 11 *fe*. 7 *in*. the girt or compass 74 *fe*. 10 *in*. the door 7 *fe*. 6 *in*. by 3 *fe*. 9 *inches*; five window shutters, each 6 *fe*. 8 *in*. by 3 *fe*. 4 *in*. the breaks in the windows 14 *in*. deep, and 8 *fe*. high; the chimney 6 *fe*. 9 *in*. by 5 *fe*. a closet, the height of the room, 3½ *fe*. deep, and 4¾ *fe*. in front, with shelving, at 22 *fe*. 6 *in*. by 10 *in*. the shutters, doors and shelves, being all coloured on both sides: what will the whole come to?

Ans. 4*l*. 18*s*. 9*d*.

Note. Painters take their dimensions with a string, and measure from the top of the cornice to the floor, girting the string over all the mouldings and swellings; and their price is generally proportioned to the number of times they lay on their colour.

All work of this kind is done by the square yard, and every part where the colour lies must be measured, and estimated in the general account of the work.

Deductions are to be made for chimnies, casements, &c. and the price is generally proportioned to the number of times they lay on their colour.

OF

OF
GLAZIERS WORK.

GLAZIERS take their dimenſions in feet, inches, and parts, and eſtimate their work by the ſquare foot.

In taking the length and breadth of a window, the croſs-bars between the panes are always included.

Windows of every form are meaſured as if they were ſquares, and the greateſt lengths and breadths are conſtantly to be taken, on account of the waſte attending the cutting.

EXAMPLES.

1. If a pane of glaſs be 2 feet 8 inches and 3 quarters long, and 1 foot 4 inches 1 quarter broad: how many feet does it contain?

By Decimals.

The decimal of $8\frac{3}{4}$ *inches is* .729

And that of $4\frac{1}{2}$ *inches is* .354

```
    2.729
    1.354
  -------
    10916
   13645
   8187
   2729
  -------
 3.695066
       12
  -------
 8.340792
```

Glaziers generally meaſure their work to a quarter of an inch, and all circular, triangular, &c. windows, are meaſured as if they were ſquares.

By Cross Multiplication.

feet	in.	pa.		
2	8	9		
1	4	3		
2	8	9		
	10	11	0	
		8	2	3
3	8	4	2	3

Ans. 3 *feet* 8 *in.* 4 *pa.*

2. What will the glazing a triangular sky-light come to at 10*d.* per foot, supposing the base to be 12 feet 6 inches, and the perpendicular height 16 feet 9 inches? *Ans.* 4*l.* 7*s.* $2\frac{3}{4}$*d.*

3. There is a house with 3 tier of windows, three in a tier; the height of the first tier is 7 feet 10 inches; of the second 6 feet 8 inches; of the third 5 feet 4 inches, and the breadth of each 3 feet 11 inches: what will the glazing come to at 14*d.* per foot? *Ans.* 13*l.* 11*s.* $10\frac{1}{2}$*d.*

4. There is a house with 3 tier of windows, three in a tier; the height of those in the lower tier being 7 *fe.* 9 *in.* of those in the middle 6 *fe.* 6 *in.* and of those in the upper tier 5 *fe.* $5\frac{1}{4}$ *in.* and having also a circular window above the door, the greatest height of which is 1 foot $10\frac{1}{2}$ *in.* and the common breadth of all the windows 3 *fe.* 9 *in.* what will the glazing come to at $13\frac{1}{2}$*d.* per foot? *Ans.* 12*l.* 14*s.* $11\frac{1}{2}$*d.*

Note. Windows are sometimes measured by taking the dimensions of one pane, and multiplying it continually by the number of panes; and no allowance is ever made for round or oval windows, as the trouble of cutting them to those shapes is more than the value of the glass omitted.

OF
PAVIOURS WORK*.

PAVIOURS work is done by the ſquare yard, and the content is found by multiplying the length by the breadth.

Or if the dimenſions be taken in feet, and the area be found in the ſame meaſure, the reſult being divided by 9, will give the number of ſquare yards required.

EXAMPLES.

1. What will the paving a rectangular court-yard come to at 3*s.* 2*d.* per yard, ſuppcſing the length to be 27 feet 10 inches, and the breadth 14 feet 9 inches?

By Croſs Multiplication.

feet	*in.*		
27	10		
14	9		
389	8		
20	10	6	
9)410	6	6	
45	5	6	6

---

* Plumbers work is generally done by the pound, or hundred weight, and the price is regulated according to the value of the lead at the time the contract is made, or when the work is performed.

				45	5	6	6 *at* 3*s*. 2*d*.
	s.	*d.*					
	2	0	$\frac{1}{10}$	45			
	1	0	$\frac{1}{2}$	4	10		
		2	$\frac{1}{6}$	2	5		
f.	*in.*	*p.*					
3	0	0	$\frac{1}{3}$		7	6	
1	0	0	$\frac{1}{3}$		1	$0\frac{1}{2}$	
1	0	0	$\frac{1}{1}$		0	4	
0	6	0	$\frac{1}{2}$		0	4	
0	0	6	$\frac{1}{12}$		0	2	
					0	0	
				7	4	$4\frac{1}{2}$	*the anſwer.*

2. A rectangular court-yard is 42 feet 9 inches long, and 68 feet 6 inches in depth, and a foot-way goes quite through it, of five feet 6 inches in breadth; the foot-way is laid with ſtone at 3*s*. 6*d*. per yard, and the reſt with pebbles at 3*s*. per yard: what will the whole come to? *Anſ.* 49*l*. 17*s*. $0\frac{1}{4}$*d*.

Sheet-lead, uſed in roofing, guttering, &c. is generally between 7 and 12lbs. weight to the ſquare foot.

The following table will ſhew the weight of a ſquare foot to each of theſe thickneſſes.

Thick.	*lbs. ſq. foot.*	*Thick.*	*lbs. ſq. foot.*	*Thick.*	*lbs. ſq. foot.*
$\frac{1}{8}$	7.373	.15	8.848	.18	10.618
.13	7.668	.16	9.438	.19	11.207
.14	8.258	$\frac{1}{6}$	9.831	$\frac{1}{5}$	11.797
$\frac{1}{7}$	8.427	.17	10.028	.21	12.387

OF

OF

VAULTED AND ARCHED ROOFS.

ARCHED roofs are either *vaults, domes, saloons,* or *groins.*

Vaulted roofs are formed by arches ſpringing from the oppoſite walls, and meeting in a line at the top.

Domes are made by arches ſpringing from a circular or polygonal baſe, and meeting in a point at the top.

Saloons are formed by arches connecting the ſide walls to a flat roof, or cieling in the middle.

Groins are formed by the interſection of vaults with each other.

Domes and Saloons rarely occur in the practice of meaſuring, but vaults and groins cover the cellars of moſt houſes.

Vaulted roofs are generally one of the three following ſorts:

1. *Circular roofs*, or thoſe whoſe arch is ſome part of the circumference of a circle.

2. *Elliptical roofs*, or thoſe whoſe arch is ſome part of the circumference of an ellipſis.

3. *Gothic roofs*, or thoſe which are formed by two circular arcs that meet in a point directly over the middle of the breadth, or ſpan of the arch.

PROBLEM I.

To find the ſolid content of circular, elliptic, or gothic vaulted roofs.

RULE.

RULE.*

Multiply the area of one end by the length of the roof, and the product will be the ſolidity required.

EXAMPLES.

1. What is the ſolid content of a ſemi-circular vault, whoſe ſpan is 40 feet, and its length 120 feet?

$$
\begin{array}{rl}
.7854 & \\
1600 & = \textit{square of } 40 \\
\hline
4712400 & \\
7854 & \\
\hline
2)1256.6400 & \\
\hline
628.32 & = \textit{area of the end.} \\
120 & = \textit{length.} \\
\hline
75398.40 & = \textit{solidity required.}
\end{array}
$$

2. Required the ſolidity of an elliptic vault, whoſe ſpan is 40 feet, height 12 feet, and length 80?

Anſ. 30159.36 *feet.*

3. What is the ſolid content of a gothic vault, whoſe ſpan is 48, the chord of its arch 48, the diſtance of the arch from the middle of the chord 18, and the length of the vault 18? *Anſ.* 136228.044.

---

* *To find the ſolidity of the materials in either of the arches.*

Rule. From the ſolid content of the whole arch take the ſolid content of the void ſpace, and the remainder will be the ſolidity of the arch.

PRO-

PROBLEM II.

To find the concave, or convex surface, of circular, elliptic, or gothic vaulted roofs.

RULE.*

Multiply the length of the arch by the length of the vault, and the product will be the superficies required.

EXAMPLES.

1. What is the concave surface of a semi-circular vault, whose span is 40 feet, and its length 120?

$$\begin{array}{r} 3.1416 \\ 40 \\ \hline 2)125.6640 \\ \hline 62.832 = \textit{length of the arch.} \\ 120 \\ \hline 7539.840 = \textit{concave surface required.} \end{array}$$

PROBLEM III.

To find the solid content of a dome; its height, and the dimensions of its base being known.

---

* The convex surface of a vault may be found by stretching a string over it; but for the concave surface this method is not applicable, and therefore its length must be found from proper dimensions.

RULE.

RULE.*

Multiply the area of the baſe by $\frac{2}{3}$ of the height, and the product will be the ſolidity.

EXAMPLES.

1. What is the ſolid content of a ſpherical dome, the diameter of whoſe circular baſe is 60 feet?

$$\begin{array}{r} .7854 \\ 3600 = \textit{square of } 60. \\ \hline 4712400 \\ 23562 \\ \hline 2827.4400 = \textit{area of the base.} \\ 20 = \tfrac{2}{3} \textit{ of the height } (30). \\ \hline 56548.8000 = \textit{solidity required.} \end{array}$$

2. In an hexagonal ſpherical dome, one ſide of the baſe is 20 feet; what is the ſolidity?

Anſ. 12000 *feet.*

PROBLEM IV.

To find the ſuperficial content of a ſpherical dome.

RULE.†

Multiply the area of the baſe by 2, and the product will be the ſuperficial content required.

---

* Domes and ſaloons are of various figures, but they are things that ſeldom occur in the practice of meaſuring.

† *The practical rule for elliptical domes is as follows:*

Rule, Add the height to half the diameter of the baſe, and this ſum multiplied by 1.5708 will give the ſuperficial content *nearly.*

EXAM-

EXAMPLES.

1. What will the painting an hexagonal ſpherical dome come to at 1*s.* per yard; each ſide of the baſe being 20 feet?

2.598076=*area of a hexagon whoſe ſide is* 1
400=*ſquare of* 20.

1039.230400=*area of the baſe.*
2

9)2078.460800=*ſuperficial content required.*

2.0)230.940088

11.5470044 = 11*l.* 10*s.* 11*d.th expence of* [*painting*

PROBLEM V.

To find the ſolid content of a ſaloon.

RULE.*

1. Multiply the height of the arc, its projection, $\frac{1}{4}$ of the perimeter of the cieling, and 3.1416 continually together, and call the product A.

2. From

---

* *To find the ſuperficial content of a ſaloon.*

1. Find the area of the flat part of the cieling.

2. Find the convex ſurface of a cylinder, or cylindroid, whoſe length is equal to $\frac{1}{4}$ the perimeter of the cieling, and its diameters to twice the height and twice the projection of the arch.

3. Find the ſuperficial content of a dome of the ſame figure as the arch, and whoſe baſe is either a ſquare, or a figure ſimilar

2. From a ſide or diameter of the room take a like ſide or diameter of the cieling, and multiply the ſquare of the remainder by the proper factor (page 54) and this product again by $\frac{2}{3}$ of the height, and call the laſt product B.

3. Multiply the area of the flat cieling by the height of the arch, and this product added to the ſum of A and B will give the content required.

EXAMPLES.

1. What is the ſolid content of a ſaloon with a circular quadrantal arch of 2 feet radius, ſpringing over a rectangular room of 20 feet long and 16 feet wide?

Here the flat part of the cieling is 16 *feet by* 12; *and*

$$
\begin{array}{l}
4)56 \\
\overline{14} = \tfrac{1}{4} \textit{ of the perimeter.} \\
\;\;2 = \textit{height.} \\
\overline{28} \\
\;\;2 = \textit{projection.} \\
\overline{\cdot 55}
\end{array}
$$

ſimilar to that of the cieling; the ſide being equal to the difference of a ſide of the room, and a ſide of the cieling.

4. Add theſe three articles together, and the ſum will give the ſuperficial content required.

Note. In a rectangular, circular, or polygonal room, the baſe of the dome will be a ſquare, a circle, or a like polygon.

3.1416

3.1416
56
———
188496
157080
———
175.9296 = A.
20 = *side of the room.*
16 = *side of the cieling.*
——
4
4
——
16
1.000, &c. = *factor.*
———
16.000
$1\frac{1}{3} = \frac{2}{3}$ *of the height.*
———
16.000
5.333
———
21.333 = B.
16
12
——
192 = *area of the flat cieling.*
2 = *height of the arch.*
——
384
175.9296
21.3333
———
581.2629 = *solid content required.*

2. A cir-

2. A circular building of 40 feet diameter, and 25 feet high to the cieling, is covered with a ſaloon, whoſe circular arch is 5 feet radius; required the capacity of the room in cubic feet?

Anſ. 30779·45948 *feet.*

PROBLEM VI.

To find the ſolid content of the vacuity formed by a groin arch, either circular or elliptical.

RULE.

Multiply the area of the baſe by the height, and the product again by .904, and it will give the ſolidity required.

EXAMPLES.

1. What is the ſolid content of the vacuity formed by a circular groin, one ſide of its ſquare baſe being 12 feet?

$$
\begin{array}{r l}
12 & \\
12 & \\
\hline
144 & = \textit{area of the base.} \\
6 & = \textit{height.} \\
\hline
864 & \\
.904 & \\
\hline
3456 & \\
77760 & \\
\hline
781.056 & = \textit{solidity required.}
\end{array}
$$

2. What is the ſolid content of the vacuity formed by an elliptical groin, one ſide of its ſquare baſe being 20 feet, and the height 6 feet? *Anſ.* 2169.6.

PROBLEM VII.

To find the concave ſuperficies of a circular groin.

RULE.*

Multiply the area of the baſe by 1.1416, and the product will be the ſuperficies required.

EXAMPLES.

1. What is the curve ſuperficies of a circular groin arch, one ſide of its ſquare being 12 feet?

```
        12
        12
       ---
       144 = area of the baſe.
    1.1416
    ------
     45664
    45664
   11416
   -------
  164.3904 = ſuperficies required.
```

2. What

* This rule may alſo be obſerved in elliptical groins, the error being too ſmall to be regarded in practice.

In meaſuring works where there are many groins in a range, the cylindric pieces between the groins, and on their ſides, muſt be computed ſeparately.

2. What is the concave ſuperficies of a circular groin arch, one ſide of its ſquare being 9 feet?

Anſ. 92.4696.

And to find the ſolidity of the brick, or ſtone work, which forms the groin arches, obſerve the following

Rule. Multiply the area of the baſe by the height, including the work over the top of the groin, and this product leſſened by the ſolid content, found as before, will give the ſolidity rtquired.

The general rule for meaſuring all arches is this:

From the content of the whole, conſidered as ſolid, from the ſpringing of the arch to the outſide of it, deduct the vacuity contained between the ſaid ſpringing and the under ſide of it, and the remainder will be the content of the ſolid part.

And becauſe the upper ſides of all arches, whether vaults or groins, are built up ſolid, above the haunces, to the ſame height with the crown, it is evident that the area of the baſe will be the whole content above-mentioned, taking for its thickneſs the height from the ſpringing to the top. And for the content of the vacuity to be deducted, take the area of its baſe, accounting its thickneſs to be $\frac{2}{3}$ of the greateſt inſide height. But it may be noted that the area uſed in the vacuity, is not exactly the ſame with that uſed in the ſolid; for the diameter of the former is twice the thickneſs of the arch leſs than that of the latter.

And although I have mentioned the deduction of the vacuity as common to both the vault and the groin, it is reaſonable to make it only in the former, on account of the waſte of materials and trouble to the workman, in cutting and fitting them for the angles and interſections.

Whoever wiſhes to ſee this ſubject more fully handled, may conſult *La Théorie et la Pratique de la Géométrie*, par *M. l'Abbé Deidier*; a work in which ſeveral parts of Menſuration and Practical Geometry are ſkilfully handled, the examples being moſtly wrought out in an eaſy familiar manner, and illuſtrated with obſervations, and figures very neatly executed.

OF THE

CARPENTER's RULE.

THIS inſtrument is commonly called *Cogeſhall's* ſliding rule. It conſiſts of two pieces, of a foot in length each, which are connected together by means of a folding joint.

On one ſide of the rule, the whole length is divided into inches and half quarters, for the purpoſe of taking dimenſions. And on this face there are alſo ſeveral plain ſcales, divided by diagonal lines into twelfth parts, which are deſigned for planning ſuch dimenſions as are taken in feet and inches.

On one part of the other face there is a ſlider, and four lines marked A, B, C, and D; the two middle ones B and C being upon the ſlider.

Three of theſe lines A, B, C, are double ones, becauſe they proceed from one to 10 twice over; and the fourth line D is a ſingle one, proceeding from 4 to 40, and is called the *girt line*.

The uſe of the double lines, A and B, is for working proportions, and finding the areas of plane figures. And the uſe of the girt line D, and the other double line C, is for meaſuring ſolids.

When 1 at the beginning of any line is counted 1, then the 1 in the middle will be 10, and the 10 at the end 100. And when 1 at the beginning is counted 10, then the 1 in the middle is 100, and the 10 at the end 1000, &c. and all the ſmall diviſions are altered in value accordingly.

Upon the other part of this face there is a table of the value of a load of timber, at all prices, from 6*d.* to 2*s.* a foot.

Some rules have likewiſe a line of inches, or a foot divided decimally into 10th parts; as well as tables of board meaſure, timber meaſure, &c. but theſe will be beſt underſtood from a ſight of the inſtrument.

The uſe of the SLIDING RULE.

PROBLEM I.

To find the product of two numbers, as 7 *and* 26.

RULE.

Set 1 upon A, to one of the numbers (26) upon B; then againſt the other number (7) on A, will be found the product (182) upon B.

Note. If the third term runs beyond the end of the line, ſeek it on the other radius, or part of the line, and increaſe the product 10 times.

PROBLEM II.

To divide one number by another, as 510 *by* 12.

RULE.

Set the diviſor (12) on A, to 1 on B; then againſt the dividend (510) on A, is the quotient ($42\frac{1}{2}$) on B.

Note. If the dividend runs beyond the end of the line, diminiſh it 10 or 100 times to make it fall on A, and increaſe the quotient accordingly.

PROBLEM III.

To ſquare any number, as 27.

RULE.

Set 1 upon D to 1 upon C; then againſt the number (27) upon B, will be found the ſquare (729) upon C.

If you would ſquare 270, reckon the one on D to be 100; and then the 1 on C will be 1000, and the product 72900.

PROBLEM IV.

To extract the ſquare root of any number, as 4268.

RULE.

Set 1 upon C, to 1 upon D; then againſt (4268) the number on C, is (65.3) the root on D.

To value this right you muſt ſuppoſe the 1 on C to be ſome of theſe ſquares 1, 100, 1000, &c. which is the neareſt to the given number, and then the root correſponding will be the value of the 1 upon D.

PROBLEM V.

To find a mean proportional between any two numbers, as 27 and 450.

RULE.

Set one of the numbers (27) on C, to the ſame on D; then againſt the other number (450) on C, will be the mean (112) on D.

Note. If one of the numbers overruns the line, take the 100th part of it, and augment the anſwer 10 times.

PROBLEM VI.

Three numbers being given, to find a fourth proportional; suppose 12, 28, *and* 57.

RULE.*

Set the firſt number (12) upon A, to the ſecond (28) upon B; then againſt the third number (57) on A, is the fourth (133) on B.

Note. If one of the middle numbers runs off the line, take the tenth part of it only, and augment the anſwer 10 times.

The finding a third proportional is exactly the ſame, the ſecond number being twice repeated.

Thus, ſuppoſe a third proportional was required to 21 and 32.

Set the firſt 21 on B, to the ſecond 32 on A; then againſt the ſecond 32 on B, is 48.8 on A, which is the third proportional required.

---

* The uſe of the rule in board and timber meaſure will be ſhewn in what follows.

If the breadth of a board be given; to find how much in length will make a ſquare foot.

Rule. If the board be narrow, it will be found in the table of board meaſure on the rule; but, if not, ſhut the rule, and ſeek the breadth in the line of board meaſure, running along the rule, from that table; then over againſt it, on the oppoſite ſide, is the length in inches required.

The ſide of the ſquare of a piece of timber being given; to find how much in length will make a foot ſolid.

RULE. If the timber be ſmall, it will be found in the table of timber meaſure on the rule; but, if not, look for the ſide of the ſquare, in the line of timber meaſure, running along the rule, from that table, and againſt it in the line of inches is the length required.

OF

OF

TIMBER MEASURE.

PROBLEM I.

To find the area, or superficial content, of a board or plank.

RULE.

MULTIPLY the length by the breadth, and the product will be the content required.

Note. When the board is tapering, add the breadths of the two ends together, and take $\frac{1}{2}$ the sum for the mean breadth.

By the SLIDING RULE.

Set 12 on B to the breadth in inches on A, then against the length in feet on B, is the content on A, in feet and fractional parts, as required.

EXAMPLES.

1. What is the value of a plank, whose length is 8 feet 6 inches, and breadth 1 foot 3 inches throughout, at $2\frac{1}{2}d.$ per foot?

feet	*in.*		
8	6		
1	3		
8	6		
2	1	6	
10	7	6	*the content.*

	10 7 6
2*d.* *is* $\frac{1}{6}$	1 8
$\frac{1}{2}$ *is* $\frac{1}{4}$	5
in. 6 *is* $\frac{1}{2}$	$1\frac{1}{4}$
in. 1 *is* $\frac{1}{6}$	$\frac{1}{4}$
	2*s.* $2\frac{1}{2}$*d.* *the anſwer.*

By the SLIDING RULE.

As 12 *on B* : 15 *on A* : : $8\frac{1}{2}$ *on B* : $10\frac{1}{2}$ *on A.*

2. What is the content of a board, whoſe length is 5 feet 7 inches, and breadth 1 foot 10 inches?

feet in. pa.
Anſ. 10 2 10

3. At $1\frac{1}{2}$*d.* per foot, what is the value of a plank, whoſe length is 12 feet 6 inches, and breadth 11 inches throughout? *Anſ.* 1*s.* 5*d.*

4. Find the value of 5 oaken planks at 3*d.* per foot, each being $17\frac{1}{2}$ feet long, and their particular breadths as follows: viz. two of $13\frac{1}{2}$ inches in the middle, one of $14\frac{1}{2}$ inches in the middle, and the two remaining ones, each 18 inches at the broader end, and $11\frac{1}{4}$ at the narrower? *Anſ.* 1*l.* 5*s.* $9\frac{1}{4}$*d.*

PROBLEM II.

To find the ſolidity of ſquared or four-ſided timber.

RULE.

RULE.*

Multiply the mean breadth by the mean thicknefs, and this product again by the length, and it will give the folidity required.

By the SLIDING RULE.

As the length in feet on *C* : 22 on *D* : : quarter girt in inches on *D* : folidity on *C*.

---

* *Note* 1. If the ftick be equally broad and thick throughout, the breadth and thicknefs, any where taken, will be the mean breadth and thicknefs.

2. If the tree tapers regularly from one end to the other, the breadth and thicknefs, taken in the middle, will be the mean breadth and thicknefs.

3. If the ftick does not taper regularly, but is thicker in fome places than in others, let feveral different dimenfions be taken, and their fum divided by the number of them will give the mean dimenfions.

This method of finding the mean dimenfions is moftly ufed in practice, but, in many cafes, it is exceedingly erroneous.

The quarter girt, likewife, which is mentioned in the proportion by the fliding rule, is fubject to error. It is not the fourth part of the circumference, but the fquare root of the product arifing from multiplying the mean breadth by the mean thicknefs.

In order to fhew the fallacy of taking $\frac{1}{4}$ of the girt for the fide of a mean fquare, take the following example:

Suppofe a piece of timber to be 24 feet long, and a foot fquare throughout, and let it be flit into two equal parts, from end to end.

Then the fum of the folidities of the two parts, by the quarter girt method, will be 27 feet, but the true folidity is 24 feet; and if the two pieces were very unequal, the difference would be ftill greater.

EXAMPLES.

1. The length of a piece of timber is $20\frac{1}{2}$ feet, the breadth at the greater end is 1 foot 9 inches, and the thickneſs 1 foot 3 inches; and at the leſſer end the breadth is 1 foot 6 inches, and the thickneſs 1 foot: what is the ſolidity?

```
    1.75 = greater breadth.
    1.5  = leſſer breadth.
   -----
  2)3.25
   -----
    1.625 = mean breadth.
   -----
    1.25 = greater thickneſs.
    1.00 = leſſer thickneſs.
   -----
  2)2.25
   -----
    1.125 = mean thickneſs.
```

By Decimals.

```
       1.625
       1.125
     -------
        8125
       3250
      1625
     1625
     -------
    1.828125
        20.5
     -------
     9140625
    36562500
     -------
  37.4765625 = content.
```

By

By Cross Multiplication.

fe.	*in.*	*pa.*		
1	7	6		
1	1	6		
1	7	6		
	1	7	6	
		9	9	
1	9	11	3	
20	6			
36	6	9	0	
	10	11	7	6
37	5	8	7	6 = *content.*

By the SLIDING RULE.

As 1 *upon* B : $19\frac{6}{12}$ *upon* A : : $13\frac{6}{12}$ *upon* B : $263\frac{25}{100}$ *upon* A, *the mean square.*

As 16 *upon* C : 4 *upon* D : : 1.8 *upon* C : 16.2 *upon* D, *the side of the mean square.*

As $20\frac{1}{2}$ *upon* C : 12 *upon* D : : 16.2 *upon* D : $37\frac{5}{12}$ *upon* C, *the answer.*

2. The length of a piece of timber is 24.5 feet, and its ends are equal squares, whose sides are each 1.04 feet: what is the solidity? *Ans.* 26 *feet* 6 *inches.*

3. The length of a piece of timber is 20.38 feet, and the ends are unequal squares, the side of the greater being $19\frac{1}{8}$ inches, and that of the lesser $9\frac{1}{3}$ inches: what is the solidity? *Ans.* 29 *feet* 4 *inches.*

4. The length of a piece of timber is 27.36 feet; at the greater end the breadth is 1.78 feet, and the thickness 1.23 feet; and at the lesser end the breadth is 1.04 feet, and the thickness .91 feet: what is the solidity? *Ans.* 41.726 *feet.*

PROBLEM III.

To find the solidity of round or unsquared timber.

RULE I*.

Multiply the square of the quarter girt (or $\frac{1}{4}$ of the circumference) by the length, and the product will be the content, *according to the common practice.*

By the SLIDING RULE.

As the length upon C : 12 upon D : : $\frac{1}{4}$ girt upon D : content upon C.

EXAMPLES.

1. A piece of timber is $9\frac{3}{4}$ feet long, and the quarter girt is 39 inches: what is the solidity?

---

* Let c = girt or circumference, and l = length of the tree.

Then $\frac{c}{4} \times \frac{c}{4} \times l = \frac{c^2 l}{16}$ = content of the tree according to the rule.

And $\frac{c^2}{4 \times 3.1416} \times l = \frac{c^2 l}{12.5664}$ = true content, according to the rule for finding the content of a cylinder.

But $\frac{c^2 l}{12.5664}$ differs from $\frac{c^2 l}{16}$ by nearly $\frac{1}{4}$ part of the whole, and therefore the rule is exceedingly erroneous.

When the tree is tapering, the mean girt is found in the same manner as in board measure. Or if the tree be very irregular, the best way is to divide it into a certain number of lengths, and find the content of each part separately.

When trees have their bark on, an allowance is generally made, by deducting so much from the girt as is judged sufficient to reduce it to such a circumference as it would have without its bark. In oak this allowance is about $\frac{1}{10}$ or $\frac{1}{12}$ part of the girt; but for elm, beech, ash, &c. whose bark is not so thick, the deduction ought to be less.

By

By Decimals.

3.25=39 *inches.*
3.25
―――
1625
650
975
―――
10.5625
9.75=9¾ *feet.*
―――
528125
739375
950625
―――
102.984375=*solidity.*

By Cross Multiplication.

f.	*in.*			
3	3=39 *inches.*			
3	3			
9	9			
	9	9		
10	6	9		
9	9	=9¾ *feet.*		
95	0	9		
7	11	0	9	
102	11	9	9=*solidity.*	

By the SLIDING RULE.

As 9¾ *upon* C : 12 *upon* D : : 39 *upon* D : 103 *upon* C, *the content.*

2. The

2. The length of a tree is 25 feet, and the girt throughout $2\frac{1}{2}$ feet: what is its solidity?

Ans. 9 feet 9 inches.

3. The length of a tree is $14\frac{1}{2}$ feet, and its girt in the middle 3.15 feet: required the solidity?

Ans. 9 feet, nearly.

4. The girts of a tree in 4 different places are as follows: in the first place 5 feet 9 inches, in the second 4 feet 5 inches, in the third 4 feet 9 inches, and in the fourth 3 feet 9 inches; and the length of the whole tree is 15 feet: what is the solidity?

Ans. 20 feet 5 inches.

5. An oak tree is 45 feet 7 inches long, and its quarter girt 3 feet 8 inches; what is the solid content, allowing $\frac{1}{12}$ for the bark? *Ans. 515 feet, nearly.*

RULE II.*

Multiply the square of $\frac{1}{5}$ of the girt by twice the length, and the product will be the solidity, *extremely near the truth.*

By

---

* Let c = circumference, and l = length, as before.

Then $\frac{c}{5} \times \frac{c}{5} \times 2l = \frac{2c^2l}{25} = \frac{c^2l}{12.5}$ = content of the tree according to the rule.

And the true content is $= \frac{c^2l}{12.5664}$, as was before shown.

But $\frac{c^2l}{12.5}$ differs from $\frac{c^2l}{12.5664}$ by only about $\frac{1}{190}$ part of the whole; and is therefore sufficiently near the truth for any practical purpose.

This rule is full as easy in practice as the false one, and therefore ought to be generally used, since the ease of the other method is the only argument which is alledged for employing it.

The

By the SLIDING RULE.

As twice the length upon C : 12 upon D : : $\frac{1}{5}$ of the girt upon D : content upon C.

EXAM-

The following rule was given me by *Mr. Burrow*, and is a ſtill nearer approximation.

Rule. Multiply the ſquare of the circumference by the length, and take $\frac{1}{11}$ of the product; from this laſt number ſubtract $\frac{1}{8}$ of itſelf, and the remainder will be the anſwer.

For $\frac{c^2 l}{12.5664} = \frac{7}{88}$ of $c^2 l$ *very nearly*, $= \frac{8}{88} - \frac{1}{88} \times c^2 l = \frac{c^2 l}{11} - \frac{1}{8}$ of $\frac{c^2 l}{11}$, which is the ſame as the rule; and differs from the truth by only 1 foot in 2300.

The following problems, as well as many other things in this ſection, were taken from *Dr. Hutton*'s Menſuration. They are given to ſhew the artifices that may be uſed in meaſuring timber according to the falſe method now practiſed, and the abſolute neceſſity there is of aboliſhing it.

PROBLEM I.

To find where a tree muſt be cut, ſo that the two parts, meaſured ſeparately, ſhall produce a greater ſolidity than that of the whole tree, or any other two parts of it.

RULE. Cut it through exactly in the middle, or at $\frac{1}{2}$ of the length, and the two parts will meaſure the moſt poſſible.

Demon. Put G = greateſt girt, g = leaſt, x = girt at the ſection, L = whole length, and z = length to be cut off the leſs end.

Then, by ſimilar figures, L : z : : G $- g$: $x - g$; or $x = \frac{Gz - gz}{L} + g$.

But $\overline{g+x}|^2 \times z + \overline{G+x}|^2 \times \overline{L-z}$ = a maximum.

And if the fluxion of this expreſſion be put equal to nothing, and the value of x be ſubſtituted inſtead of it, there will reſult $z = \frac{1}{2}$ L. Q. E. D.

Example.

EXAMPLES.

1. A piece of timber is $9\frac{3}{4}$ feet long, and $\frac{1}{5}$ of the girt is 2.6 feet: what is the solidity?

By

Example. Suppofing a tree to girt 14 feet at the greater end, 2 feet at the lefs, and 8 feet in the middle, and that the length is 32 feet.

Then, by the common method, the whole tree meafures only 128.

But, when cut through the middle, the greater part meafures 121 feet, and the lefs part 25 feet.

And the fum of thefe two parts is 146 feet, which exceeds the whole by 18 feet, and is the moft that it can be made to meafure by cutting it into two parts.

PROBLEM II.

To find where a tree muft be cut, fo that the part next the greater end may meafure the moft poffible.

Rule. Cut it where the girt is $\frac{1}{3}$ of the greateft girt, and the greater end will meafure to the moft poffible.

Demon. By ufing the fame notation as in the laft problem, we fhall have $x = \frac{Gz - gz}{L} + g$, and $\overline{G + x)} \times \overline{L - z} =$ a maximum.

And, if this be put into fluxions, it will give $z = \frac{G - 3g}{G - g} \times \frac{1}{3} L$, and $x = \frac{G - g}{L} \times z + g = \frac{1}{3} G$. Q. E. D.

Example. Taking here the fame example as before, we fhall have $12 : 8 :: \frac{32}{3} : 7\frac{1}{9} =$ length to be cut off, $24\frac{8}{9} =$ length of the remaining part, and $4\frac{2}{3} =$ girt at the fection.

But the content of the whole tree is only 128 feet, and the content of the greater part $135\frac{41}{81}$ feet, which exceeds 128 by $7\frac{41}{81}$, and is the greateft poffible.

Note.

By Decimals.

```
      2.6
      2.6
     ----
      156
      52
     ----
     6.76
     9.75
     ----
     3380
    4732
   6084
   ------
   65.9100
         2
   ------
  131.8200=content.
```

By

Note. If the greateſt girt does not exceed 3 times the leaſt, the tree cannot be cut as is required by the problem; for when the leaſt girt is exactly equal to $\frac{1}{3}$ of the greater, the tree already meaſures to the moſt poſſible.

PROBLEM III.

To cut a tree, ſo that the part next the greater end may meaſure exactly the ſame as the whole tree.

Rule 1. Call the ſum of the girts of the two ends *s*, and their difference *d*.

2. Multiply *d* by the ſum of *d* and 4 *s*, and from the ſquare root of the product take the difference between *d* and 2 *s*.

3. Then as 2 *d* is to this remainder, ſo is the whole length of the tree, to the length to be cut off the ſmaller end.

Demon.

By the SLIDING RULE.

As 19.15 *upon* C : 12 *upon* D : : $31\frac{1}{5}$ *in. upon* D : 132, *the content upon* C.

2. If the length of a tree be 24 feet, and the girt throughout 8 feet: what is the content?

Anſ. 123 *feet, nearly.*

3. If a tree girt 14 feet at the thicker end, and 2 feet at the ſmaller end: required the ſolidity when the length is 24 feet? *Anſ.* $11\frac{1}{2}$ *feet, nearly.*

4. A tree girts in five different places as follows: in the firſt place 9.43 feet, in the ſecond 7.92 feet, in the third 6.15 feet, in the fourth 4.74 feet, and in the fifth 3.16 feet; and the whole length is $17\frac{1}{4}$ feet: what is the ſolidity? *Anſ.* 54.4065 *feet.*

Demon. Uſing ſtill the ſame notation, we ſhall have $s^2L = \overline{L-z} \times \overline{g+x})^2$; and by ſubſtituting for x its value $\frac{dz}{L} + g$, it will be $z = \frac{L}{2d} \times \sqrt{\overline{\overline{4s+d} \times d}} - 2s + d$, and $x = \frac{1}{2}\sqrt{\overline{\overline{4s+d} \times d}} - s$. Q. E. D.

The rule may be illuſtrated by taking the ſame example as in the laſt problems.

Thus, ſince $s = 16$, $d = 12$, and the length $L = 32$, $\frac{32}{24} \times \sqrt{\overline{76 \times 12}} - 20 = \frac{16}{3}\sqrt{57} - 26\frac{2}{3} = 13.599118 =$ length to be cut off; and therefore the length of the remaining part is 18.400882.

Alſo, $\frac{1}{2} \times \sqrt{\overline{76 \times 12}} - 8 = 2\sqrt{57} - 8 = 7.099669 =$ girt at the ſection. Whence the girt in the middle of the greater part is $\frac{14 + 7.099669}{2} = 10.549834$, whoſe $\frac{1}{4}$ part is 2.637458, and conſequently the content of this part is $\overline{2.637458})^2 \times 18.400882 = 128$ the ſame as the content of the whole tree.

OF

OF
SPECIFIC GRAVITY.

THE ſpecific gravities of bodies are their relative weights contained under the ſame given magnitude, as a cubic foot, a cubic inch, &c.

The ſpecific gravities of ſeveral ſorts of bodies are expreſſed by the numbers annexed to their names in the following table.

A table of the ſpecific gravities of bodies.

Fine gold - -	19640	Brick - - -	2000
Standard gold -	18888	Light earth - -	1984
Quickſilver -	14000	Solid gunpowder	1745
Lead - - -	11325	Sand - - - -	1520
Fine ſilver - -	11091	Pitch - - -	1150
Standard ſilver -	10535	Dry box wood -	1030
Copper - - -	9000	Sea water - -	1030
Gun metal - -	8784	Common water -	1000
Caſt braſs - -	8000	Dry oak - - -	925
Steel - - -	7850	Gunpowder, ſhaken	922
Iron - - -	7645	Dry aſh - - -	800
Caſt iron - -	7425	Dry maple - -	755
Tin - - - -	7320	Dry elm - - -	600
Marble - - -	2700	Dry fir - - -	550
Common ſtone -	2520	Cork - - - -	240
Loom - - -	2160	Air - - - - -	$1\frac{1}{4}$

Note. As a cubic foot of water weighs juſt 1000 ounces Avoirdupois, the numbers in this table expreſs not only the ſpecific gravities of the ſeveral bodies, but alſo the weight of a cubic foot of each, in Avoirdupois ounces; and hence, by proportion, the weight of any other quantity, or the quantity of any other weight, may be readily known.

PRO-

PROBLEM I.

To find the magnitude of a body from its weight being given.

RULE.

As the tabular ſpecific gravity of the body, is to its weight in Avoirdupois ounces,

So is one cubic foot, or 1728 cubic inches, to its content in feet, or inches, reſpectively.

EXAMPLES.

1. Required the content of an irregular block of common ſtone which weighs 1 *cwt.* or 112*lbs.*?

```
                112 lbs.
                 16
                ----
                672
                112
                ----
   2520  :     1792  ::  1728
               1728
              -----
              14336
              3584
             12544
             1792
            -------
   2520)3096576(1228 4/5 cubic inches the anſ.
        252
        ---
         576
         504
         ---
          725
          504
          ----
          2217
          2016
          ----
          2016
```

2. How

2. How many cubic inches of gunpowder are there in one pound weight? *Anſ.* 30 *nearly.*

3. How many cubic feet are there in a ton weight of dry oak? *Anſ.* $38\frac{138}{185}$.

PROBLEM II.

To find the weight of a body from its magnitude being given.

RULE.

As one cubic foot, or 1728 cubic inches, is to the content of the body,

So is its tabular ſpecific gravity, to the weight of the body.

EXAMPLES.

1. Required the weight of a block of marble, whoſe length is 63 feet, and its breadth and thickneſs each 12 feet; theſe being the dimenſions of one of the ſtones in the walls of Balbec.

```
            63
            12
           ---
           756
            12
           ---
  1  :    9072  : :  2700
          2700
       -------
       6350400
       18144
     ---------
16{4 | 24494400 }oz.
   {4 |  6123600 }
  112 |  1530900lbs.
   20 |    13668cwts.
             683 ton.
```

Anſ. $683\frac{2}{5}$ *tons, which is equal to the burthen of an Eaſt India ſhip.*

2. What

2. What is the weight of a pint of gunpowder ale-measure?

Ans. 19 *oz. nearly.*

3. What is the weight of a block of dry oak, which measures ten feet in length, 3 feet in breadth, and $2\frac{1}{4}$ feet deep?

Ans. $4335\frac{15}{16}$*lb.*

PROBLEM III.

To find the specific gravity of a body.

RULE.

Case 1. When the body is heavier than water, weigh it both in water and out of water, and the difference will be the weight lost in the water.

Then, as the weight lost in water, is to the whole weight,

So is the specific gravity of water, to the specific gravity of the body.

EXAMPLES.

A piece of stone weighed in air 10 pounds, but in water only $6\frac{3}{4}$lb. Required its specific gravity?

$$\begin{array}{l} 10 \\ \underline{6\frac{3}{4}} \\ 3\frac{1}{4} : 10 :: 1000 : \\ 13 : 40 :: 1000 : \\ \qquad\qquad\quad \underline{40} \\ 13)40000(3077 \\ \quad \underline{39} \\ \quad\ 100 \\ \quad\ \ \underline{91} \\ \quad\ \ \ 90 \end{array}$$

Case

Case. 2. When the body is lighter than water, so that it will not quite sink; affix to it another body heavier than water, so that the mass compounded of the two may sink together.

Weigh the heavier body and the compound mass separately both in water and out of it, and find how much each loses in water, by subtracting its weight in water from its weight in air.

Then as the difference of these remainders is to the weight of the light body in air,

So is the specific gravity of water to the specific gravity of the body.

EXAMPLE.

Suppose a piece of elm weighs in air 15lb. and that a piece of copper which weighs 18lb. in air, and 16lb. in water, is affixed to it, and that the compound weighs 6lb. in water; Required the specific gravity of the elm?

	18 *in air*	33	
	16 *in water*	6	
loss	2	27	*loss*
		2	
		25	

25 : 15 :: 1000 : 600 *Ans.*

PROBLEM IV.

To find the quantities of two ingredients in a given compound.

RULE.

RULE.

Take the differences of every pair of the three ſpecific gravities, *viz.* of the compound and each ingredient, and multiply the difference of every two by the third.

Then as the greateſt product is to the whole weight of the compound, ſo is each of the other products to the weights of the two ingredients.

EXAMPLES.

A compoſition of 112lb. being made of tin and copper, whoſe ſpecific gravity is found to be 8784; Required the quantity of each ingredient? The ſpecific gravity of tin being 7320, and of copper 9000.

9000	9000	8754
7320	8784	7320
1680	216	1464 *diff.*
8784	7320	9000
702720	4320	13176000
52704	648	
8784	1512	
14757120	1581120	

14757120 : 112 :: 13176000

112

26352000

13176

13176

14757120)1475712000(100

Anſ. 100*lb. of copper* } *in the compoſition.*
and 12*lb. of tin* }

MIS-

MISCELLANEOUS QUESTIONS.

1. WHAT difference is there between a floor 48 feet long, and 30 feet broad, and two others each of half the dimenſions? *Anſ.* 720 *feet.*

2. From a mahogany plank 26 inches broad, a yard and a half is to be ſawed off; what diſtance from the end muſt the line be ſtruck? *Anſ.* 6.23 *feet.*

3. A joiſt is $8\frac{1}{2}$ inches deep, and $3\frac{1}{2}$ broad: what will be the dimenſions of a ſcantling juſt as big again as the joiſt, that is $4\frac{3}{4}$ inches broad?

Anſ. 12.52 *inches deep.*

4. A roof is 24 feet 8 inches by 14 feet 6 inches, and is to be covered with lead at 8*lbs.* to the foot: what will it come to at 18*s.* per cwt?

Anſ. 22*l.* 19*s.* $10\frac{1}{4}$*d.*

5. What is the ſide of that equilateral triangle, whoſe area coſt as much paving at 8*d.* per foot, as the pallifading the three ſides did at a guinea per yard?

Anſ. 72.746 *feet.*

6. The two ſides of an obtuſe-angled triangle are 20 and 40 poles: what muſt the length of the third ſide be that the triangle may contain juſt an acre?

Anſ. 58.876, *or* 23·099.

7. If two ſides of a triangle, whoſe area is $60\sqrt{3}$, be 12 and 20: what is the third ſide? *Anſ.* 28.

8. If an area of 24 be cut off from a triangle, whoſe three ſides are 13, 14, and 15, by a line parallel to the longeſt ſide: what are the lengths of the ſides including that area?

Anſ. $\frac{13}{7}\sqrt{14}$, $2\sqrt{14}$, *and* $\frac{15}{7}\sqrt{14}$.

9. The

9. The diſtance of the centres of two circles, whoſe diameters are each 50, is equal to 30: what is the area of the ſpace incloſed by their circumference?

Anſ. 559.119.

10. The area of an equilateral triangle, whoſe baſe falls on the diameter, and its vertex in the middle of the arc of a ſemi-circle, is equal to 100: what is the diameter of the ſemi-circle?

Anſ. 26.3214.

11. The four ſides of a field, whoſe diagonals are equal to each other, are 25, 35, 31, and 19 poles, reſpectively: what is the area?

Anſ. 4 *ac.* 1 *ro.* 38 *poles.*

12. What is the length of a chord which cuts off $\frac{1}{3}$ of the area from a circle whoſe diameter is 289?

Anſ. 278.6716.

13. A cable which is 3 feet long, and 9 inches in compaſs, weighs 22*lbs.* what will a fathom of that cable weigh whoſe diameter is 9 inches?

Anſ. 434.26*lbs.*

14. A circular fiſh-pond is to be dug in a garden, that ſhall take up juſt half an acre, what muſt the length of the chord be that ſtrikes the circle?

Anſ. 27.75 *yards.*

15. A carpenter is to put an oaken curb to a round well, at 8*d.* per foot ſquare; the breadth of the curb is to be $7\frac{1}{4}$ inches, and the diameter within $3\frac{1}{2}$ feet: what will be the expence? *Anſ.* 5*s.* $2\frac{1}{4}$*d.*

16. Suppoſe the expence of paving a ſemi-circular plot, at 2*s.* 4*d.* per foot, amounted to 10*l.* what is the diameter of it? *Anſ.* 14.7737.

* 17. Seven men bought a grinding-ſtone of 60 inches in diameter, each paying $\frac{1}{7}$ part of the expence; what part of the diameter muſt each grind

---

* For an excellent geometrical conſtruction of this queſtion, ſee *Dr. Hutton*'s Diarian Miſcellany, page 54.

 down

down for his ſhare? *Anſ. The 1ſt,* 4.4508, *2d.* 4.8400, *3d.* 5.3535, *4th.* 6.0765, *5th.* 7.2079, *6th.* 9.3935, *and the 7th.* 22.6778.

18. A gentleman has a garden 100 feet long, and 80 feet broad, and a gravel walk is to be made of an equal width half round it: what muſt the width of the walk be ſo as to take up juſt half the ground?

Anſ. 25.968 *feet.*

19. In the midſt of a meadow well ſtored with graſs,
I took juſt an acre to tether my aſs;
How long muſt the cord be, that feeding all round,
He may'nt graze leſs or more than an acre of ground? *Anſ.* 39.25073 *yards.*

20. A maltſter has a kiln that is 16 feet 6 inches ſquare; now he wants to pull it down, and build a new one that will dry three times as much at a time as the old one did: what muſt be the length of its ſide?

Anſ. 28 *feet* 7 *inches.*

21. If a round ciſtern be 26.3 inches diameter, and 52.5 inches deep: how many inches diameter muſt a ciſtern be to hold twice the quantity, the depth being the ſame? *Anſ.* 37.19 *inches.*

22. A May-pole, whoſe top was broken off by a blaſt of wind, ſtruck the ground at 15 feet diſtance from the top of the pole: what was the height of the whole May-pole, ſuppoſing the length of the broken piece to be 39 feet? *Anſ.* 75 *feet.*

23. What will the diameter of a globe be, when the ſolidity and ſuperficial content thereof are equal to each other? *Anſ.* 6.

24. How many three inch cubes can be cut out of a 12 inch cube? *Anſ.* 64.

25. A farmer borrowed part of a hay-rick of his neighbour, which meaſured 6 feet every way, and paid him back again by two equal cubical pieces,

each of whoſe ſides were three feet: Query, whether the lender was fully paid?

Anſ. He was paid $\frac{1}{4}$ *part only.*

26. What will the painting a conical church-ſpire come to at 8*d.* per yard; ſuppoſing the circumference of the baſe to be 64 feet, and the altitude 118 feet?

Anſ. 14*l.* 0*s.* 8$\frac{3}{4}$*d.*

27. What will a marble fruſtum of a cone come to at 12*s.* per ſolid foot; the diameter of the greater end being 4 feet, that of the leſſer end 1$\frac{1}{2}$ feet, and the length of the ſlant ſide 8 feet? *Anſ.* 30*l.* 1*s.* 10*d.*

28. The diameter of a legal Wincheſter buſhel is 18$\frac{1}{2}$ inches, and its depth 8 inches: what muſt the diameter of that buſhel be whoſe depth is 7$\frac{1}{2}$ inches?

Anſ. 19.10671.

29. Three men bought a tapering piece of timber, which was the fruſtum of a ſquare pyramid; one ſide of the greater end was 3 feet, one ſide of the leſſer end 1 foot, and the length 18 feet: what is the length of each man's piece, ſuppoſing they paid equally, and are to have equal ſhares? *Anſ.* 1*ſt.* 3.269, 2*d.* 4.559. *and the* 3*d.* 10.172, *reckoning from the greater end to the leſs.*

30. Suppoſe the ball at the top of St. Paul's Church is 6 feet in diameter: what did the gilding of it come to at 3$\frac{1}{2}$*d.* per ſquare inch?

Anſ. 237*l.* 1*s.* 10$\frac{1}{4}$*d.*

31. A perſon wants a cylindric veſſel of 3 feet deep, that ſhall hold twice as much as a veſſel of 28 inches deep, and 46 inches in diameter: what muſt be the diameter of the veſſel required?

Anſ. 57.37 *inches.*

32. Two porters agreed to drink off a quart of ſtrong beer between them, at two pulls, or a draught each; now, the firſt having given it a black eye, as it is called, or drank till the ſurface of the liquor touched the oppoſite edge of the bottom, gave the remaining

remaining part of it to the other: what was the difference of their ſhares, ſuppoſing the pot was the fruſtum of a cone, the depth being 5.7 inches, the diameter at the top 3.7 inches, and that of the bottom 4.23 inches? *Anſ.* 7.07 *cubic inches.*

33. Three perſons having bought a ſugar-loaf, want to divide it equally amongſt them by ſections parallel to the baſe; it is required to find the altitude of each perſon's ſhare, ſuppoſing the loaf to be a cone, whoſe height is 20 inches? *Anſ.* 13.867 *the upper part,* 3.604 *the middle part, and* 2.528 *the lower part.*

34. How high above the ſurface of the earth muſt a perſon be raiſed to ſee $\frac{1}{3}$ of its ſurface?

Anſ. To the height of the earth's diameter.

35. A cubical foot of braſs is to be drawn into a wire of $\frac{1}{40}$ of an inch in diameter; what will be the length of the wire, allowing no leſs in the metal?

Anſ. 97784.797 *yards, or near* 56 *miles.*

35. A gentleman has a bowling-green, 300 feet long, and 200 feet broad, which he would raiſe one foot higher, by means of the earth to be dug out of a ditch that goes round it: to what depth muſt the ditch be dug, ſuppoſing its breadth to be every where 8 feet? *Anſ.* $7\frac{13}{266}$ *feet.*

36. Of what diameter muſt the bore of a cannon be, which is caſt for a ball of 24lbs. weight, ſo that the diameter of the bore may be $\frac{1}{10}$ of an inch more than that of the ball? *Anſ.* 5.757 *inches.*

37. At what height from the bottom muſt an upright cone be cut, ſo that the greateſt cylinder poſſible may be formed from the lower part of it?

Anſ. At $\frac{1}{3}$ *of the height.*

38. The ellipſe in Groſvenor ſquare meaſures 840 links acroſs the longeſt way, and 612 the ſhorteſt, within the rails: now the walls being 14 inches thick it is required to find what ground they incloſe, and

what they ſtand upon? *Anſ. They incloſe* 4 *ac.* 0 *ro.* 6 *po. and ſtand on* $1760\frac{1}{2}$ *ſquare feet.*

49. If a heavy ſphere whoſe diameter is 4 inches, be put into a conical glaſs, full of water, whoſe diameter is 5, and altitude 6 inches; it is required to find how much water will run over?

Anſ. $\frac{35}{47}$ *of a pint nearly.*

40. Suppoſe it be found, by meaſurement, that a man of war, with its ordnance, rigging and appointments, draws ſo much water as to diſplace 50000 cubic feet of water: required the weight of the veſſel?

Anſ. $1395\frac{1}{16}$ *tons.*

41. One ev'ning I chanc'd with a tinker to ſit,
Whoſe tongue ran a great deal too faſt for his wit;
He talk'd of his art with abundance of mettle;
So I aſk'd him to make me a flat-bottom'd kettle:
Let the top and the bottom diameters be,
In juſt ſuch proportion as five is to three:
Twelve inches the depth I propos'd, and no more;
And to hold in ale gallons ſeven leſs than a ſcore,
He promis'd to do it, and ſtrait to work went;
But when he had done it he found it too ſcant.
He alter'd it then, but too big he had made it;
For though it held right, the diameters fail'd it:
Thus making it often too big and too little,
The Tinker at laſt had quite ſpoilt his kettle;
But he ſwears he will bring his ſaid promiſe to paſs,
Or elſe that he'll ſpoil every ounce of his braſs.
Now to keep him from ruin, I pray find him out
The diameter's length, for he'll ne'er do it I doubt.

Anſ. The bottom diameter is 14.64017, *and the top diameter* 24.40028.

A TABLE

A

TABLE

OF THE

AREAS OF THE SEGMENTS OF A CIRCLE.

Whose Diameter is Unity, and supposed to be divided into 1000 equal Parts.

Versed Sine.	*Seg. Area.*	Versed. Sine	*Seg. Area.*	Versed Sine.	*Seg. Area.*
.001	.000042	.024	.004921	.047	.013392
.002	.000119	.025	.005230	.048	.013818
.003	.000219	.026	.005546	.049	.014247
.004	.000337	.027	.005867	.050	.014681
005	.000470	.028	.006194	.051	.015119
.006	.000618	.029	.006527	.052	.015561
.007	.000779	.030	.006865	.053	.016007
.008	.000951	.031	.007209	.054	.016457
.009	.001135	.032	.007558	.055	.016911
.010	.001329	.033	.007913	.056	.017369
.011	.001533	.034	.008273	.057	.017831
.012	.001746	.035	.008638	.058	.018296
.013	.001968	.036	.009008	.059	.018766
.014	.002199	.037	.009383	.060	.019239
.015	.002438	.038	.009763	.061	.019716
.016	.002685	.039	.010148	.062	.020196
.017	.002940	.040	.010537	.063	020680
.018	.003202	041	.010931	.064	.021168
.019	003471	.042	.011330	.065	.021659
.020	.003748	.043	.011734	.066	022154
.021	.004031	.044	.012142	.067	.022652
.022	.004322	.045	.012554	.068	023154
.023	.004618	.046	.012971	.069	.023659

The

The Areas of the Segments of a Circle.

Versed Sine.	*Seg. Area.*	Versed Sine.	*Seg. Area*	Versed Sine.	*Seg. Area.*
.070	.024168	.103	.042687	.136	.064074
.071	.024680	.104	.043296	.137	.064760
.072	.025195	.105	.043908	.138	.065449
.073	.025714	.106	.044522	.139	.066140
.074	.026236	.107	.045139	.140	.066833
.075	.026761	.108	.045759	.141	.067528
.076	.027289	.109	.046381	.142	.068225
.077	.027821	.110	.047005	.143	068924
.078	.028356	.111	.047632	.144	.069625
.079	.028894	.112	.048262	.145	.070328
.080	.029435	.113	.048894	.146	.071033
.081	029979	.114	.049528	.147	.071741
.082	.030526	.115	.050165	.148	.072450
.083	.031076	.116	.050804	.149	073161
.084	.031629	.117	.051446	.150	.073874
.085	.032186	.118	.052090	.151	.074589
.086	.032745	.119	.052736	.152	.075306
.087	.033307	.120	.053385	.153	.076026
.088	.033872	.121	.054036	.154	.076747
.089	.034441	.122	.054689	.155	.077469
.090	.035011	.123	.055345	.156	.078194
.091	.035585	.124	.056003	.157	.078921
.092	.036162	.125	.056663	.158	.079649
.093	.036741	.126	.057326	.159	.080380
.094	.037323	.127	.057991	.160	.081112
.095	.037909	.128	.058658	.161	.081846
.096	.038496	.129	.059327	.162	.082582
.097	.039087	.130	.059999	.163	.083320
.098	.039680	.131	.060672	.164	.084059
.099	.040276	.132	.061348	.165	.084801
.100	.040875	.133	.062026	.166	.085544
.101	.041476	.134	.062707	.167	.086289
.102	.042080	.135	.063389	.168	.087036

The

The Areas of the Segments of a Circle.

Versed Sine.	*Seg. Area.*	Ver. sed Sine.	*Seg. Area.*	Versed Sine.	*Seg. Area.*
.169	.087785	.202	.113426	.235	.140688
.170	.088535	.203	.114230	.236	.141537
.171	.089287	.204	.115035	.237	.142387
.172	.090041	.205	.115842	.238	.143238
.173	.090797	.206	.116650	.239	.144091
.174	.091554	.207	.117460	.240	.144944
.175	.092313	.208	.118271	.241	.145799
.176	.093074	.209	.119083	.242	.146655
.177	.093836	.210	.119897	.243	.147512
.178	.094601	.211	.120712	.244	.148371
.179	.095366	.212	.121529	.245	.149230
.180	.096134	.213	.122347	.246	.150091
.181	.096903	.214	.123167	.247	.150953
.182	.097674	.215	.123988	.248	.151816
.183	.098447	.216	.124810	.249	.152680
.184	.099221	.217	.125634	.250	.153546
.185	.099997	.218	.126459	.251	.154412
.186	.100774	.219	.127285	.252	.155280
.187	.101553	.220	.128113	.253	.156149
.188	.102334	.221	.128942	.254	.157019
.189	.103116	.222	.129773	.255	.157890
.190	.103900	.223	.130605	.256	.158762
.191	.104685	.224	.131438	.257	.159636
.192	.105472	.225	.132272	.258	.160510
.193	.106261	.226	.133108	.259	.161386
.194	.107051	.227	.133945	.260	.162263
.195	.107842	.228	.134784	.261	.163140
.196	.108636	.229	.135624	.262	.164019
.197	.109430	.230	.136465	.263	.164899
.198	.110226	.231	.137307	.264	.165780
.199	.111024	.232	.138150	.265	.166663
.200	.111823	.233	.138995	.266	.167546
.201	.112624	.234	.139841	.267	.168430

The

The Areas of the Segments of a Circle.

Versed Sine.	*Seg. Area.*	Versed Sine.	*Seg. Area.*	Versed Sine.	*Seg. Area.*
.268	.169315	.301	.199085	.334	.229801
.269	.170202	.302	.200003	.335	.230745
.270	.171089	.303	.200922	.336	.231689
.271	.171978	.304	.201841	.337	.232634
.272	.172867	.305	.202761	.338	.233580
.273	.173758	.306	.203683	.339	.234526
.274	.174649	.307	.204605	.340	.235473
.275	.175542	.308	.205527	.341	.236421
.276	.176435	.309	.206451	.342	.237369
.277	.177330	.310	.207376	.343	.238318
.278	.178225	.311	.208301	.344	.239268
.279	.179122	.312	.209227	.345	.240218
.280	.180019	.313	.210154	.346	.241169
.281	.180918	.314	211082	.347	.242121
.282	.181817	.315	.212011	.348	.243074
.283	.182718	.316	.212940	.349	.244026
.284	.183619	.317	.213871	.350	.244980
.285	.184521	.318	.214802	.351	.245934
.286	.185425	.319	.215733	.352	.246889
.287	.186329	.320	.216666	.353	.247845
.288	.187234	.321	.217599	.354	.248801
.289	.188140	.322	.218533	.355	.249757
.290	.189047	.323	.219468	.356	.250715
.291	.189955	.324	.220404	.357	.251673
292	.190864	.325	.221340	.358	.252631
.293	.191775	.326	.222277	.359	253590
.294	.192684	.327	.223215	.360	.254550
.295	.193596	.328	.224154	.361	.255510
.296	.194509	.329	.225093	.362	.256471
.297	.195422	.330	.226033	.363	.257433
.298	.196337	.331	.226974	.364	.258395
.299	.197252	.332	.227915	.365	.259357
.300	.198168	.333	.228858	.366	.260320

The

The Areas of the Segments of a Circle.

Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.
·367	·261284	·400	·293369	·433	·325900
·368	·262248	·401	·294349	·434	·326892
·369	·263213	·402	·295330	·435	·327882
·370	·264178	·403	·296311	·436	·328874
·371	·265144	·404	·297292	·437	·329866
·372	·266111	·405	·298273	·438	·330858
·373	·267078	·406	·299255	·439	·331850
·374	·268045	·407	·300238	·440	·332843
·375	·269013	·408	·301220	·441	·333836
·376	·269982	·409	·302203	·442	·334829
·377	·270951	·410	·303187	·443	·335822
·378	·271920	·411	·304171	·444	·336816
·379	·272890	·412	·305155	·445	·337810
·380	·273861	·413	·306140	·446	·338804
·381	·274832	·414	·307125	·447	·339798
·382	·275803	·415	·308110	·448	·340793
·383	·276775	·416	·309095	·449	·341787
·384	·277748	·417	·310081	·450	·342782
·385	·278721	·418	·311068	·451	·343777
·386	·279694	·419	·312054	·452	·344772
·387	·280668	·420	·313041	·453	·345768
·388	·281642	·421	·314029	·454	·346764
·389	·282617	·422	·315016	·455	·347759
·390	·283592	·423	·316004	·456	·348755
·391	·284568	·424	·316992	·457	·349752
·392	·285544	·425	·317981	·458	·350748
·393	·286521	·426	·318970	·459	·351745
·394	·287498	·427	·319959	·460	·352741
·395	·288476	·428	·320948	·461	·353739
·396	·289453	·429	·321938	·462	·354736
·397	·290432	·430	·322928	·463	·355732
·398	·291411	·431	·323918	·464	·356730
·399	·292390	·432	·324909	·465	·357727

The Areas of the Segments of a Circle.

Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.	Versed Sine.	Seg. Area.
.466	.358725	.478	.370706	.490	.382699
.467	.359723	.479	.371705	.491	.383699
.468	.360721	.480	.372764	.492	.384699
.469	.361719	.481	.373703	.493	.385699
.470	.362717	.482	.374702	.494	.386699
.471	.363715	.383	.375702	.495	.387699
.472	.364713	.484	.376702	.496	.388699
.473	.365712	.485	.377701	.497	.389699
.474	.366710	.486	.378701	.498	.390699
.475	.367709	.487	.379700	.499	.391699
.476	.368708	.488	.380700	.500	.392699
.477	.369707	.489	.381699		

THE END.

Published by the SAME AUTHOR,

1. THE SCHOLAR'S GUIDE TO ARITHMETIC. 5th Edit. Price 2s. bound.

2. AN INTRODUCTION TO ALGEBRA. 3d Edit.

3. AN INTRODUCTION TO ASTRONOMY, with many Plates. 2d Edit. Price 8s. bound.

4. ELEMENTS OF GEOMETRY. Containing the principal Propoſitions in the Firſt Six and the Eleventh and Twelfth Books of Euclid, with Notes Critical and Explanatory. Price 6s. bound.

Printed by Bye and Law, St. John's Square, Clerkenwell.

10779
05 4
5389 9

157079
1305 4
5395

Reg

Reg

700
574
1400
1225

137.4450
1.1.1.12
4.5400
12

4.0800
12

9.0002