LECTURES

ON

SELECT SUBJECTS.

LECTURES

ON

SELECT SUBJECTS

IN

MECHANICS,	PNEUMATICS,
HYDROSTATICS,	AND
HYDRAULICS,	OPTICS.

WITH

THE USE OF THE GLOBES,

THE ART OF DIALING,

AND

The Calculation of the Mean Times of NEW and FULL MOONS and ECLIPSES.

By JAMES FERGUSON, *F. R. S.*

Philosophia mater omnium bonarum artium est. CICERO. 1. Tusc.

THE SEVENTH EDITION.

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MDCCXC.

1790

TO HIS

ROYAL HIGHNESS

PRINCE EDWARD.

SIR,

AS Heaven has inſpired your ROYAL HIGHNESS with ſuch love of ingenious and uſeful arts, that you not only ſtudy their theory, but have often condeſcended to honour the profeſſors of mechanical and experimental philoſophy with your preſence and particular favour; I am thereby encouraged to lay myſelf and the following work at your ROYAL HIGHNESS's feet; and at the ſame time beg leave to expreſs that veneration with which I am,

SIR,

Your ROYAL HIGHNESS's

Moſt obliged,

And moſt obedient,

Humble Servant,

JAMES FERGUSON.

THE

PREFACE.

EVER since the days of the LORD CHANCELLOR BACON, *natural philoſophy hath been more and more cultivated in England.* THAT *great genius firſt ſet out with taking a general ſurvey of all the natural ſciences, dividing them into diſtinct branches, which he enumerated with great exactneſs. He inquired ſcrupulouſly into the degree of knowledge already attained to in each, and drew up a liſt of what ſtill remained to be diſcovered: this was the ſcope of his firſt undertaking. Afterward he carried his views much farther, and ſhewed the neceſſity of an experimental philoſophy, a thing never before thought of. As he was a profeſſed enemy to ſyſtems, he conſidered philoſophy no otherwiſe than as that part of knowledge which contributes to make men better and happier: he ſeems to limit it to the knowledge of things uſeful, recommending above all the ſtudy of nature, and ſhewing that no progreſs can be made therein, but by collecting facts, and* *comparing experiments, of which he points out a great number proper to be made.*

But notwithſtanding the true path to ſcience was thus exactly marked out, the old notions of the ſchools ſo ſtrongly poſſeſſed people's minds at that time, as not to be eradicated by any new opinions, how rationally ſoever advanced, until the illuſtrious Mr. Boyle, *the firſt who purſued* Lord Bacon's *plan, began to put experiments in practice with an aſſiduity equal to his great talents. Next, the* Royal Society *being eſtabliſhed, the true philoſophy began to be the reigning taſte of the age, and continues ſo to this day.*

The immortal Sir Isaac Newton *inſiſted, even in his early years, that it was high time to baniſh vague conjectures and hypotheſes from natural philoſophy, and to bring that ſcience under an entire ſubjection to experiments and geometry. He frequently called it* the experimental philoſophy, *ſo as to expreſs ſignificantly the difference between it and the numberleſs ſyſtems which had ariſen merely out of the conceits of inventive brains: the one ſubſiſting no longer than the ſpirit of novelty*

velty lasts; the other never failing while the nature of things remain unchanged.

The method of teaching and laying the foundation of physics, by public courses of experiments, was first undertaken in this kingdom, I believe, by Dr. JOHN KEILL, *and since improved and enlarged by Mr.* HAUKSBEE, *Dr.* DESAGULIERS, *Mr.* WHISTON, *Mr.* COTES, *Mr.* WHITESIDE, *Dr.* BRADLEY, *our late Regius and Savilian Professor of Astronomy, and Dr.* BLISS *his successor. Nor has the same been neglected by Dr.* JAMES, *and Dr.* DAVID GREGORY, *Sir* ROBERT STEWART, *and after him Mr.* MACLAURIN. ——*Dr.* HELSHAM *in Ireland, Messieurs* GRAVESANDE *and* MUSCHENBROEK, *and the Abbé* NOLLET *in France, have also acquired just applause thereby.*

The substance of my own attempt in this way of instrumental instruction, the following sheets (exclusive of the astronomical part) will shew: the satisfaction they have generally given, read as lectures to different audiences, affords me some hope that they may be favourably received in the same form by the public.

I ought

I ought to observe, that though the last five lectures cannot be properly said to concern experimental philosophy, I considered, however, that they were not of so different a class, but that they might, without much impropriety, be subjoined to the preceding ones.

*My apparatus (part of which is described here, and the rest in a * former work) is rather simple than magnificent, which is owing to a particular point I had in view at first setting out, namely, to avoid all superfluity, and to render every thing as plain and intelligible as I thought the subject would admit of.*

* Astronomy explained upon SIR ISAAC NEWTON'S principles, and made easy to those who have not studied mathematics.

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LECTURES

ON

SELECT SUBJECTS.

LECT. I.

Of Matter and its Properties.

AS the deſign of the firſt part of this courſe is to explain and demonſtrate thoſe laws by which the material univerſe is governed, regulated, and continued; and by which the various appearances in nature are accounted for; it is requiſite to begin with explaining the properties of matter.

By the word *matter* is here meant every thing that has length, breadth, and thickneſs, and reſiſts the touch. Matter, what.

The inherent properties of matter are ſolidity, inactivity, mobility, and diviſibility. Its properties.

The *ſolidity* of matter ariſes from its having length, breadth, thickneſs; and hence it is that all bodies are comprehended under ſome ſhape or other, and that each particular body hinders all others from occupying the ſame part of ſpace which it poſſeſſes. Thus, if a piece of wood or metal be ſqueezed ever ſo hard between two plates, they cannot be brought into contact. And even water or air has this property; for if a ſmall quantity of it be fixed between any other Solidity.

bodies,

bodies, they cannot be brought to touch one another.

Inactivity. A ſecond property of matter is *inactivity*, or *paſſiveneſs*; by which it always endeavours to continue in the ſtate that it is in, whether of reſt or motion. And therefore, if one body contains twice or thrice as much matter as another body does, it will have twice or thrice as much inactivity; that is, it will require twice or thrice as much force to give it an equal degree of motion, or to ſtop it after it hath been put into ſuch a motion.

That matter can never put itſelf into motion is allowed by all men. For they ſee that a ſtone, lying on the plane ſurface of the earth, never removes itſelf from that place, nor does any one imagine it ever can. But moſt people are apt to believe that all matter has a propenſity to fall from a ſtate of motion into a ſtate of reſt; becauſe they ſee that if a ſtone or a cannon-ball be put into ever ſo violent a motion, it ſoon ſtops; not conſidering that this ſtoppage is cauſed, 1. By the gravity or weight of the body, which ſinks it to the ground in ſpite of the impulſe; and, 2. By the reſiſtance of the air through which it moves, and by which its velocity is retarded every moment till it falls.

A bowl moves but a ſhort way upon a bowling-green; becauſe the roughneſs and unevenneſs of the graſſy ſurface ſoon creates friction enough to ſtop it. But if the green were perfectly level, and covered with poliſhed glaſs, and the bowl were perfectly hard, round, and ſmooth, it would go a great way farther; as it would have nothing but the air to reſiſt it; if then the air were taken away, the bowl would go on without any friction, and conſequently without any diminution of the velocity it had at ſetting out: and therefore, if the green were extended quite around the earth, the bowl would go on, round and round the earth, for ever.

If the bowl were carried ſeveral miles above the earth, and there projected in a horizontal direction, with ſuch a velocity as would make it move more than a ſemidiameter of the earth, in the time it would take to fall to the earth by gravity; in that caſe, and if there were no reſiſting medium in the way, the bowl would not fall to the earth at all; but would continue to circulate round it, keeping always in the ſame tract, and returning to the ſame point from which it was projected, with the ſame velocity as at firſt. In this manner the moon goes round the earth, although ſhe be as unactive and dead as any ſtone upon it.

The third property of matter is *mobility*; for we find that all matter is capable of being moved, if a ſufficient degree of force be applied to overcome its inactivity or reſiſtance. Mobility.

The fourth property of matter is *diviſibility*, of which there can be no end. For, ſince matter can never be annihilated by cutting or breaking, we can never imagine it to be cut into ſuch ſmall particles, but that if one of them be laid on a table, the uppermoſt ſide of it will be further from the table than the undermoſt ſide. Moreover, it would be abſurd to ſay that the greateſt mountain on earth has more halves, quarters, or tenth parts, than the ſmalleſt particle of matter has. Diviſibility.

We have many ſurpriſing inſtances of the ſmallneſs to which matter can be divided by art: of which the two following are very remarkable.

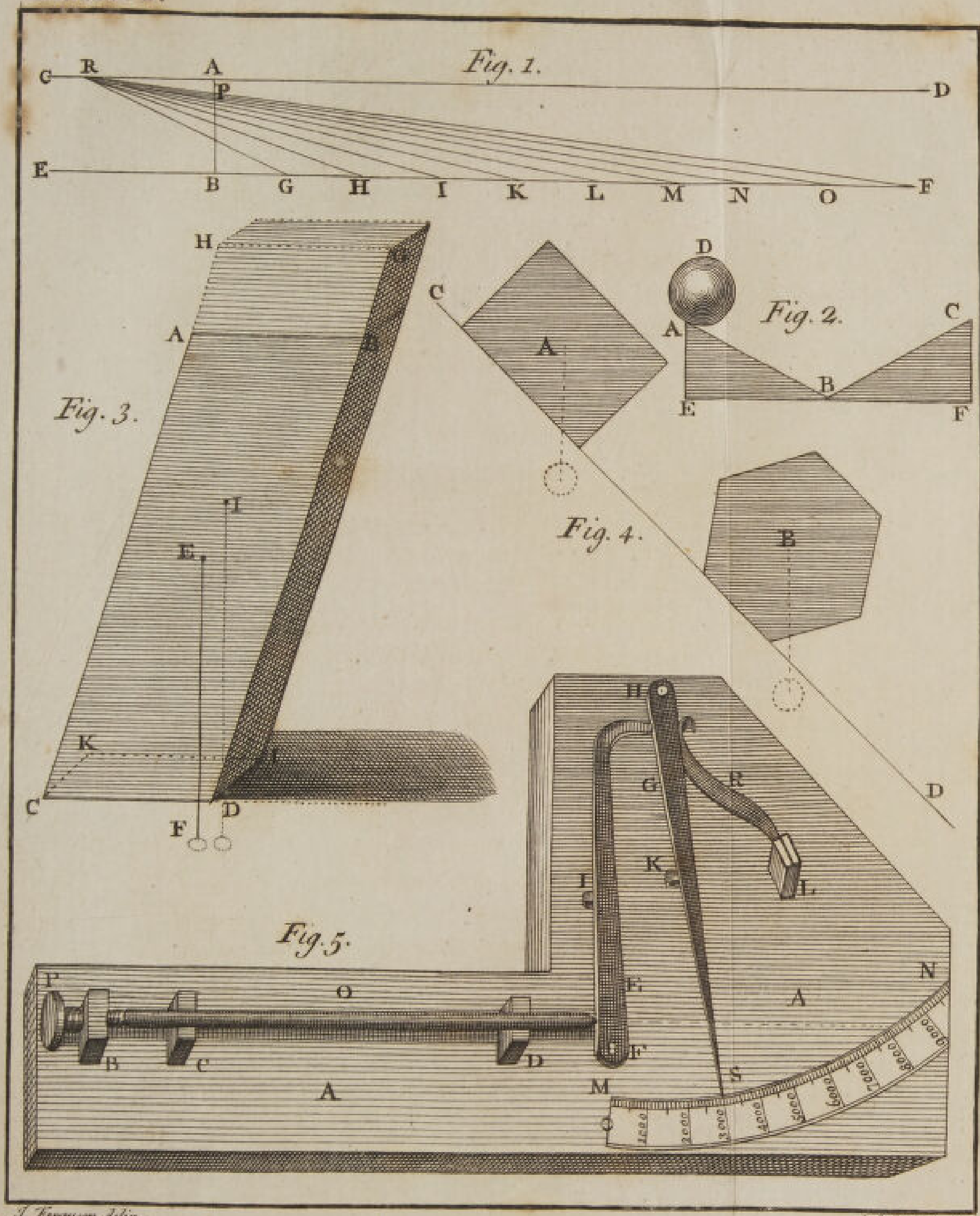
KC

1. If a pound of ſilver be melted with a ſingle grain of gold, the gold will be equally diffuſed through the whole ſilver; ſo that taking one grain from any part of the maſs (in which there can be no more than the 5760th part of a grain of gold) and diſſolving it in *aqua fortis*, the gold will fall to the bottom.

2. The gold beaters can extend a grain of gold into a leaf containing 50 ſquare inches; and this leaf may be divided into 500000 viſible parts. For an inch in length can be divided into 100 parts, every one of which will be viſible to the bare eye: conſequently a ſquare inch can be divided into 10000 parts, and 50 ſquare inches into 500000. And if one of theſe parts be viewed with a microſcope that magnifies the diameter of an object only 10 times, it will magnify the area 100 times; and then the 100th part of a 500000th part of a grain (that is, the 50 millionth part) will be viſible. Such leaves are commonly uſed in gilding; and they are ſo very thin, that if 124500 of them were laid upon one another, and preſſed together, they would not exceed one inch in thickneſs.

Yet all this is nothing in compariſon of the lengths that nature goes in the diviſion of matter. For Mr. *Leewenhoek* tells us, that there are more animals in the milt of a ſingle cod-fiſh, than there are men upon the whole earth: and that, by comparing theſe animals in a microſcope with grains of common ſand, it appeared that one ſingle grain is bigger than four millions of them. Now each animal muſt have a heart, arteries, veins, muſcles, and nerves, otherwiſe they could neither live nor move. How inconceivably ſmall then muſt the particles of their blood be, to circulate through the ſmalleſt ramifications

PLATE I.



J. Ferguson delin.

J. Mynde sc.

fications and joinings of their arteries and veins? It has been found by calculation, that a particle of their blood muſt be as much ſmaller than a globe of the tenth part of an inch in diameter, as that globe is ſmaller than the whole earth; and yet, if theſe particles be compared with the particles of light, they will be found to exceed them as much in bulk as mountains do ſingle grains of ſand. For, the force of any body ſtriking againſt an obſtacle is directly in proportion to its quantity of matter multiplied into its velocity; and ſince the velocity of the particles of light is demonſtrated to be at leaſt a million times greater than the velocity of a cannon-ball, it is plain, that if a million of theſe particles were as big as a ſingle grain of ſand, we durſt no more open our eyes to the light, than we durſt expoſe them to ſand ſhot point-blank from a cannon.

Plate I. Fig. 1.

The infinite diviſibility of matter proved.

That matter is infinitely diviſible in a mathematical ſenſe, is eaſy to be demonſtrated. For, let *AB* be the length of a particle to be divided; and let it be touched at oppoſite ends by the parallel lines *CD* and *EF*, which, ſuppoſe to be infinitely extended beyond *D* and *F*. Set off the equal diviſions *BG*, *GH*, *HI*, &c. on the line *EF*, toward the right hand from *B*; and take a point, as at *R*, any where toward the left hand from *A*, in the line *CD*: Then, from this point, draw the right lines *RG*, *RH*, *RI*, &c. each of which will cut off a part from the particle *AB*. But after any finite number of ſuch lines are drawn, there will ſtill remain a part, as *AP*, at the top of the particle, which can never be cut off: becauſe the lines *DR* and *EF* being parallel, no line can ever be drawn from the point *R* to any point of the line *EF* that will coincide with the line *R D.* Therefore the particle *A B* contains more than any finite number of parts.

Attraction. A fifth property of matter is *attraction*, which ſeems rather to be infuſed than inherent. Of this there are four kinds, viz. *coheſion*, *gravitation*, *magnetiſm*, and *electricity*.

Coheſion. The *attraction of coheſion* is that by which the ſmall parts of matter are made to ſtick and cohere together. Of this we have ſeveral inſtances, ſome of which follow.

1. If a ſmall glaſs tube, open at both ends, be dipt in water, the water will riſe up in the tube to a conſiderable height above its level in the baſon: which muſt be owing to the attraction of a ring of particles of the glaſs all round in the tube, immediately above thoſe to which the water at any inſtant riſes. And when it has riſen ſo high, that the weight of the column balances the attraction of the tube, it riſes no higher. This can be no ways owing to the preſſure of the air upon the water in the baſon; for, as the tube is open at top, it is full of air above the water, which will preſs as much upon the water in the tube as the neighbouring air does upon any column of an equal diameter in the baſon. Beſides, if the ſame experiment be made in an exhauſted receiver of the air pump, there will be found no difference.

2. A piece of loaf-ſugar will draw up a fluid, and a ſpunge will draw in water: and on the ſame principle ſap aſcends in trees.

3. If two drops of quickſilver be placed near each other, they will run together and become one large drop.

4. If two pieces of lead be ſcraped clean, and preſſed together with a twiſt, they will attract each

each other ſo ſtrongly, as to require a force much greater than their own weight to ſeparate them. And this cannot be owing to the preſſure of the air, for the ſame thing will hold in an exhauſted receiver.

5. If two poliſhed plates of marble or braſs be put together, with a little oil between them to fill up the pores in their ſurfaces, and prevent the lodgement of any air; they will cohere ſo ſtrongly, even if ſuſpended in an exhauſted receiver, that the weight of the lower plate will not be able to ſeparate it from the upper one. In putting theſe plates together, the one ſhould be rubbed upon the other, as a joiner does two pieces of wood when he glues them.

6. If two pieces of cork, equal in weight, be put near each other in a baſon of water, they will move equally faſt toward each other with an accelerated motion, until they meet: and then, if either of them be moved, it will draw the other after it. If two corks of unequal weights be placed near each other, they will approach with accelerated velocities inverſely proportionate to their weights: that is, the lighter cork will move as much faſter than the heavier, as the heavier exceeds the lighter in weight. This ſhews that the attraction of each cork is in direct proportion to its weight or quantity of matter.

This kind of attraction reaches but to a very ſmall diſtance; for, if two drops of quickſilver be rolled in duſt, they will not run together, becauſe the particles of duſt keep them out of the ſphere of each other's attraction.

Where the ſphere of attraction ends, a *repulſive force* begins; thus, water repels moſt bodies till they are wet; and hence it is, that a ſmall Repulſion.

 needle,

needle, if dry, ſwims upon water; and flies walk upon it without wetting their feet.

The repelling force of the particles of a fluid is but ſmall; and therefore, if a fluid be divided, it eaſily unites again. But if glaſs, or any other hard ſubſtance, be broke into ſmall parts, they cannot be made to ſtick together again without being firſt wetted: the repulſion being too great to admit of a re-union.

The repelling force between water and oil is ſo great, that we find it almoſt impoſſible to mix them ſo, as not to ſeparate again. If a ball of light wood be dipt in oil, and then put into water, the water will recede ſo as to form a channel of ſome depth all around the ball.

The repulſive force of the particles of air is ſo great, that they can never be brought ſo near together by condenſation as to make them ſtick or cohere. Hence it is, that when the weight of the incumbent atmoſphere is taken off from any ſmall quantity of air, that quantity will diffuſe itſelf ſo as to occupy (in compariſon) an infinitely greater portion of ſpace than it did before.

Gravitation. *Attraction of gravitation* is that power by which diſtant bodies tend toward one another. Of this we have daily inſtances in the falling of bodies to the earth. By this power in the earth it is, that bodies, on whatever ſide, fall in lines perpendicular to its ſurface; and conſequently, on oppoſite ſides, they fall in oppoſite directions; all toward the center, where the force of gravity is as it were accumulated: and by this power it is, that bodies on the earth's ſurface are kept to it on all ſides, ſo that they cannot fall from it. And as it acts upon all bodies in proportion to their reſpective quantities of matter, without any regard to their bulks or figures,

figures, it accordingly conſtitutes their weight. Hence,

If two bodies which contain equal quantities of matter, were placed at ever ſo great a diſtance from one another, and then left at liberty in free ſpace; if there were no other bodies in the universe to affect them, they would fall equally ſwift toward one another by the power of gravity, with velocities accelerated as they approached each other; and would meet in a point which was half-way between them at firſt. Or, if two bodies, containing unequal quantities of matter, were placed at any diſtance, and left in the ſame manner at liberty, they would fall toward one another with velocities which would be in an inverſe proportion to their reſpective quantities of matter; and moving faſter and faſter in their mutual approach, would at laſt meet in a point as much nearer to the place from which the heavier body began to fall, than to the place from which the lighter body began to fall, as the quantity of matter in the former exceeded that in the latter.

All bodies that we know of have gravity or weight. For, that there is no ſuch thing as poſitive levity, even in ſmoke, vapours, and fumes, is demonſtrable by experiments on the air-pump; which ſhews, that although the ſmoke of a candle aſcends to the top of a tall receiver when full of air, yet, upon the air's being exhauſted out of the receiver, the ſmoke falls down to the bottom of it. So, if a piece of wood be immerſed in a jar of water, the wood will riſe to the top of the water, becauſe it has a leſs degree of weight than its bulk of water has: but if the jar be emptied of water, the wood falls to the bottom.

Gravity demonstrated to be as the quantity of matter in bodies.

As every particle of matter has its proper gravity, the effect of the whole muſt be in proportion to the number of the attracting particles; that is, as the quantity of matter in the whole body. This is demonſtrable by experiments on pendulums; for, if they are of equal lengths, whatever their weights be, they vibrate in equal times. Now it is plain, that if one be double or triple the weight of another, it muſt require a double or triple power of gravity to make it move with the ſame celerity: juſt as it would require a double or triple force to project a bullet of twenty or thirty pounds weight, with the ſame degree of ſwiftneſs that a bullet of ten pounds would require. Hence it is evident, that the power or force of gravity is always proportional to the quantity of matter in bodies, whatever their bulks or figures are.

It decreaſes as the ſquare of the diſtance increaſes.

Gravity alſo, like all other virtues or emanations which proceed or iſſue from a center, decreaſes as the diſtance multiplied by itſelf increaſes: that is, a body at twice the diſtance of another, attracts with only a fourth part of the force; at thrice the diſtance, with a ninth part; at four times the diſtance, with a ſixteenth part; and ſo on. This too is confirmed by comparing the diſtance which the moon falls in a minute, from a right line touching her orbit, with the diſtance through which heavy bodies near the earth fall in that time. And alſo by comparing the forces which retain Jupiter's moons in their orbits, with their reſpective diſtances from Jupiter. Theſe forces will be explained in the next lecture.

The velocity which bodies near the earth acquire in deſcending freely by the force of gravity, is proportional to the times of their deſcent.

For,

For, as the power of gravity does not confist in a fingle impulfe, but is always operating in a conftant and uniform manner, it muft produce equal effects in equal times; and confequently in a double or triple time, a double or triple effect. And fo, by acting uniformly on the body, muft accelerate its motion proportionably to the time of its defcent.

To be a little more particular on this fubject, let us fuppofe that a body begins to move with a celerity conftantly and gradually increafing, in fuch a manner, as would carry it through a mile in a minute; at the end of this fpace it will have acquired fuch a degree of celerity, as is fufficient to carry it two miles the next minute, though it fhould then receive no new impulfe from the caufe by which its motion had been accelerated; but if the fame accelerating caufe continues, it will carry the body a mile farther; on which account, it will have run through four miles at the end of two minutes; and then it will have acquired fuch a degree of celerity, as is fufficient to carry it through a double fpace in as much more time, or eight miles in two minutes, even though the accelerating force fhould act upon it no more. But this force ftill continuing to operate in an uniform manner, will again, in an equal time, produce an equal effect; and fo, by carrying it a mile further, caufe it to move through five miles the third minute; for, the celerity already acquired, and the celerity ftill acquiring, will have each its complete effect. Hence we learn, that if the body fhould move one mile the firft minute, it would move three miles the fecond, five the third, feven the fourth, nine the fifth, and fo on in proportion.

And thus it appears, that the ſpaces deſcribed in ſucceſſive equal parts of time, by an uniformly accelerated motion, are always as the odd numbers 1, 3, 5, 7, 9, &c. and conſequently, the whole ſpaces are as the ſquares of the times, or of the laſt acquired velocities. For, the continued addition of the odd numbers yields the ſquares of all numbers from unity upward. Thus, 1 is the firſt odd number, and the ſquare of 1 is 1; 3 is the ſecond odd number, and this added to 1 makes 4, the ſquare of 2; 5 is the third odd number, which added to 4 makes 9, the ſquare of 3; and ſo on for ever. Since, therefore, the times and velocities proceed evenly and conſtantly as 1, 2, 3, 4, &c. but the ſpaces deſcribed in each equal times are as 1, 3, 5, 7, &c. it is evident that the ſpace deſcribed

In 1 minute will be - - - $1 =$ ſquare of 1
In 2 minutes - - - $1+3= 4=$ ſquare of 2
In 3 minutes - $1+3+5= 9=$ ſquare of 3
In 4 minutes $1+3+5+7=16=$ ſquare of 4, &c.

N. B. The character + ſignifies *more*, and = *equal*.

The deſcending velocity will give a power of equal aſcent.

As heavy bodies are uniformly accelerated by the power of gravity in their deſcent, it is plain that they muſt be uniformly retarded by the ſame power in their aſcent. Therefore, the velocity which a body acquires by falling, is ſufficient to carry it up again to the ſame height from whence it fell: allowance being made for the reſiſtance of the air, or other medium in which the body is moved. Thus, the body *D* in rolling down the inclined plane *A B* will acquire ſuch a velocity by the time it arrives at *B*, as

Fig. 2.

B, as will carry it up the inclined plane *B C*, almoſt to *C*; and would carry it quite up to *C*, if the body and plane were perfectly ſmooth, and the air gave no reſiſtance.—So, if a pendulum were put into motion, in a ſpace quite free of air, and all other reſiſtance, and had no friction on the point of ſuſpenſion, it would move for ever: for the velocity it had acquired in falling through the deſcending part of the arc, would be ſtill ſufficient to carry it equally high in the aſcending part thereof.

The center of gravity, The *center of gravity* is that point of a body in which the whole force of its gravity or weight is united. Therefore, whatever ſupports that point, bears the weight of the whole body: and while it is ſupported, the body cannot fall; becauſe all its parts are in a perfect equilibrium about that point.

and line of direction. An imaginary line drawn from the center of gravity of any body toward the center of the earth, is called the *line of direction*. In this line all heavy bodies deſcend, if not obſtructed.

Since the whole weight of a body is united in its center of gravity, as that center aſcends or deſcends, we muſt look upon the whole body to do ſo too. But as it is contrary to the nature of heavy bodies to aſcend of their own accord, or not to deſcend when they are permitted; we may be ſure that, unleſs the center of gravity be ſupported, the whole body will tumble or fall. Hence it is, that bodies ſtand upon their baſes when the line of direction falls within the baſe; for in this caſe the body cannot be made to fall, without firſt raiſing the center of gravity higher than it was before. Thus, the inclining body *ABCD*, whoſe center of gravity is *E*, ſtands firmly on its baſe *CDIK*, becauſe the line Fig. 3.

of

of direction *EF* falls within the base. But if a weight, as *ABGH*, be laid upon the top of the body, the center of gravity of the whole body and weight together is raised up to *I*; and then, as the line of direction *ID* falls without the base at *D*, the center of gravity *I* is not supported; and the whole body and weight tumble down together.

Hence appears the absurdity of people's rising hastily in a coach or boat when it is likely to overset: for, by that means they raise the center of gravity so far as to endanger throwing it quite out of the base; and if they do, they overset the vehicle effectually. Whereas, had they clapt down to the bottom, they would have brought the line of direction, and consequently the center of gravity, farther within the base, and by that means might have saved themselves.

The broader the base is, and the nearer the line of direction is to the middle or center of it, the more firmly does the body stand. On the contrary, the narrower the base, and the nearer the line of direction is to the side of it, the more easily may the body be overthrown, a less change of position being sufficient to remove the line of direction out of the base in the latter case than in the former. And hence it is, that a sphere is so easily rolled upon a horizontal plane; and that it is so difficult, if not impossible, to make things which are sharp-pointed to stand upright on the point.—From what hath been said, it plainly appears, that if the plane be inclined on which the heavy body is placed, the body will slide down upon the plane while the line of direction falls within the base; but it will tumble or roll down when that line falls without

the

the base. Thus, the body *A* will only slide down the inclined plane *CD*, while the body *B* rolls down upon it. Fig. 4.

When the line of direction falls within the base of our feet, we stand; and most firmly when it is in the middle: but when it is out of that base, we immediately fall. And it is not only pleasing, but even surprising, to reflect upon the various and unthought-of methods and postures which we use to retain this position, or to recover it when it is lost. For this purpose we bend our body forward when we rise from a chair, or when we go up stairs: and for this purpose a man leans forward when he carries a burden on his back, and backward when he carries it on his breast; and to the right or left side as he carries it on the opposite side. A thousand more instances might be added.

The quantity of matter in all bodies is in exact proportion to their weights, bulk for bulk. Therefore, heavy bodies are as much more dense or compact than light bodies of the same bulk, as they exceed them in weight.

All bodies porous.

All bodies are full of pores, or spaces void of matter: and in gold, which is the heaviest of all known bodies, there is perhaps a greater quantity of space than of matter. For the particles of heat and magnetism find an easy passage through the pores of gold; and even water itself has been forced through them. Besides, if we consider how easily the rays of light pass through so solid a body as glass, in all manner of directions, we shall find reason to believe that bodies are much more porous than is generally imagined.

The expansion of metals.

All bodies are some way or other affected by heat; and all metallic bodies are expanded in length,

length, breadth, and thicknefs thereby.—The proportion of the expanfion of feveral metals, according to the beft experiments I have been able to make with my pyrometer, is nearly thus: Iron and fteel, as 3, copper 4 and a half, brafs 5, tin 6, lead 7. An iron rod 3 feet long is about one 70th part of an inch longer in fummer than in winter.

The pyrometer. The pyrometer here mentioned being (for aught I know) of a new conftruction, a defcription of it may perhaps be agreeable to the reader.

Fig. 5. *A A* is a flat piece of mahogany, in which are fixed four brafs ftuds *B*, *C*, *D*, *L*; and two pins, one at *F* and the other at *H*. On the pin *F* turns the crooked index *E I*, and upon the pin *H* the ftraight index *G K*, againft which a piece of watch-fpring *R* bears gently, and fo preffes it toward the beginning of the fcale *M N*, over which the point of that index moves. This fcale is divided into inches and tenth parts of an inch: the firft inch is marked 1000, the fecond 2000, and fo on. A bar of metal *O* is laid into notches in the top of the ftuds *C* and *D*; one end of the bar bearing againft the adjufting fcrew *P*, and the other end againft the crooked index *E I*, at a 20th part of its length from its centre of motion *F*.—Now it is plain, that however much the bar *O* lengthens, it will move that part of the index *E I*, againft which it bears, juft as far: but the crooked end of the fame index, near *H*, being 20 times as far from the center of motion *F*, as the point is againft which the bar bears, it will move 20 times as far as the bar lengthens. And as this crooked end bears againft the index *G K* at only a 20th part of the whole length *G S* from its center of motion

motion *H*, the point *S* will move through 20 times the ſpace that the point of bearing near *H* does. Hence, as 20 multiplied by 20 produces 400, it is evident that if the bar lengthens but a 400th part of an inch, the point *S* will move a whole inch on the ſcale; and as every inch is divided into 10 equal parts, if the bar lengthens but the 10th part of the 400th part of an inch, which is only the 4000th part of an inch, the point *S* will move the tenth part of an inch, which is very perceptible.

To find how much a bar lengthens by heat, firſt lay it cold into the notches of the ſtuds, and turn the adjuſting ſcrew *P* until the ſpring *R* brings the point *S* of the index *GK* to the beginning of the diviſions of the ſcale at *M*: then, without altering the ſcrew any farther, take off the bar, and rub it with a dry woollen cloth till it feels warm; and then, laying it on where it was, obſerve how far it puſhes the point *S* upon the ſcale by means of the crooked index *EI*; and the point *S* will ſhew exactly how much the bar has lengthened by the heat of rubbing. As the bar cools, the ſpring *R* bearing againſt the index *KG*, will cauſe its point *S* to move gradually back toward *M* in the ſcale: and when the bar is quite cold, the index will reſt at *M*, where it was before the bar was made warm by rubbing. The indexes have ſmall rollers under them at *I* and *K*; which, by turning round on the ſmooth wood as the indexes move, make their motions the eaſier, by taking off a great part of the friction, which would otherwiſe be on the pins *F* and *H*, and of the points of the indexes themſelves on the wood.

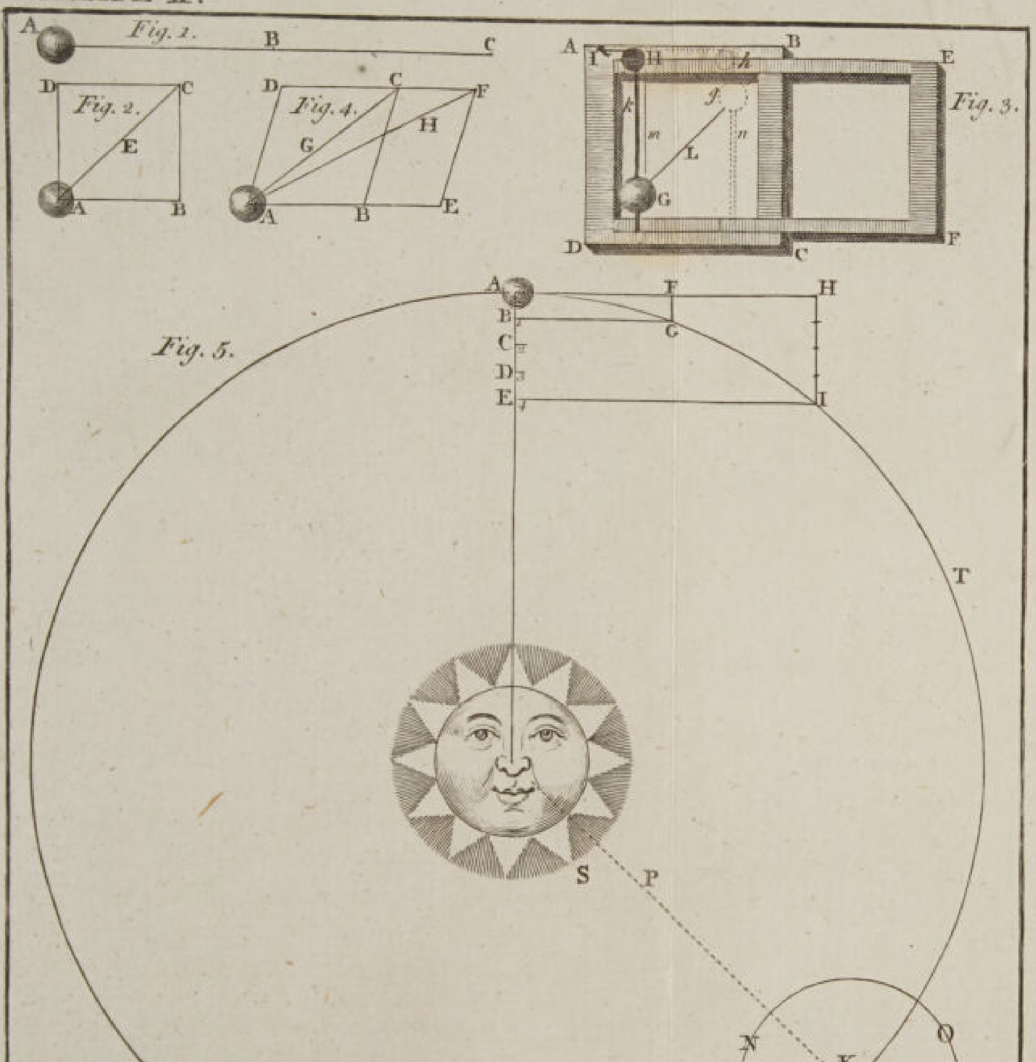
Beſide the univerſal properties above mentioned, there are bodies which have properties peculiar **Magnetiſm.**

peculiar to themſelves: ſuch as the loadſtone, in which the moſt remarkable are theſe: 1. It attracts iron and ſteel only. 2. It conſtantly turns one of its ſides to the north and another to the ſouth, when ſuſpended by a thread that does not twiſt. 3. It communicates all its properties to a piece of ſteel when rubbed upon it, without loſing any itſelf.

According to Dr. *Helſham*'s experiments, the attraction of the loadſtone decreaſes as the ſquare of the diſtance increaſes. Thus, if a loadſtone be ſuſpended at one end of a balance, and counterpoiſed by weights at the other end, and a flat piece of iron be placed beneath it, at the diſtance of four tenths of an inch, the ſtone will immediately deſcend and adhere to the iron. But if the ſtone be again removed to the ſame diſtance, and as many grains be put into the ſcale at the other end as will exactly counterbalance the attraction, then, if the iron be brought twice as near the ſtone as before, that is, only two tenth parts of an inch from it, there muſt be four times as many grains put into the ſcale as before, in order to be a juſt counterbalance to the attractive force, or to hinder the ſtone from deſcending and adhering to the iron. So, if four grains will do in the former caſe, there muſt be ſixteen in the latter. But from ſome later experiments, made with the greateſt accuracy, it is found that the force of magnetiſm decreaſes in a ratio between the reciprocal of the ſquare and the reciprocal of the cube of the diſtance; approaching to the one or the other, as the magnitudes of the attracting bodies are varied.

Electricity. Several bodies, particularly amber, glaſs, jet, ſealing-wax, agate, and almoſt all precious ſtones, have a peculiar property of attracting

PLATE II.



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and repelling light bodies when heated by rubbing. This is called *electrical attraction*, in which the chief things to be obſerved are, 1. If a glaſs tube about an inch and a half diameter, and two or three feet long, be heated by rubbing, it will alternately attract and repel all light bodies when held near them. 2. It does not attract by being heated without rubbing. 3. Any light body, being once repelled by the tube, will never be attracted again till it has touched ſome other body. 4. If the tube be rubbed by a moiſt hand, or any thing that is wet, it totally deſtroys the electricity. 5. Any body, except air, being interpoſed, ſtops the electricity. 6. The tube attracts ſtronger when rubbed over with bees-wax, and then with a dry woollen-cloth. 7. When it is well rubbed, if a finger be brought near it, at about the diſtance of half an inch, the effluvia will ſnap againſt the finger, and make a little crackling noiſe; and if this be performed in a dark place, there will appear a little flaſh of light.

LECT. II.

Of central Forces.

WE have already mentioned it as a neceſſary conſequence ariſing from the deadneſs or inactivity of matter, that all bodies endeavour to continue in the ſtate they are in, whether of reſt or motion. If the body *A* were placed in any part of free ſpace, and nothing either drew or impelled it any way, it would for ever remain in that part of ſpace, becauſe it could have no tendency of itſelf to remove any way from thence. If it receives a ſingle im-

All bodies equally indifferent to motion or reſt.

Plate II. Fig. 1.

pulſe any way, as ſuppoſe from *A* toward *B*, it will go on in that direction; for, of itſelf, it could never ſwerve from a right line, nor ſtop its courſe.—When it has gone through the ſpace *A B*, and met with no reſiſtance, its velocity will be the ſame at *B* as it was at *A*; and this velocity, in as much more time, will carry it through as much more ſpace, from *B* to *C*; and ſo on for ever. Therefore, when we ſee a body in motion, we conclude that ſome other ſubſtance muſt have given it that motion; and when we ſee a body fall from motion to reſt, we conclude that ſome other body or cauſe ſtopt it.

All motion naturally rectilineal.

As all motion is naturally rectilineal, it appears, that a bullet projected by the hand, or ſhot from a cannon, would for ever continue to move in the ſame direction it received at firſt, if no other power diverted its courſe. Therefore when we ſee a body move in a curve of any kind whatever, we conclude it muſt be acted upon by two powers at leaſt; one putting it in motion, and another drawing it off from the rectilineal courſe it would otherwiſe have continued to move in: and whenever that power, which bent the motion of the body from a ſtraight line into a curve, ceaſes to act, the body will again move on in a ſtraight line touching that point of the curve in which it was when the action of that power ceaſed. For example, a pebble moved round in a ſling ever ſo long a time, will fly off the moment it is ſet at liberty, by ſlipping one end of the ſling cord: and will go on in a line touching the circle it deſcribed before: which line would actually be a ſtraight one, if the earth's attraction did not affect the pebble, and bring it down to the ground. This ſhews that the natural tendency of the pebble, when put into motion, is to continue moving in a ſtraight line, although by the force that moves the ſling it be made to revolve in a circle.

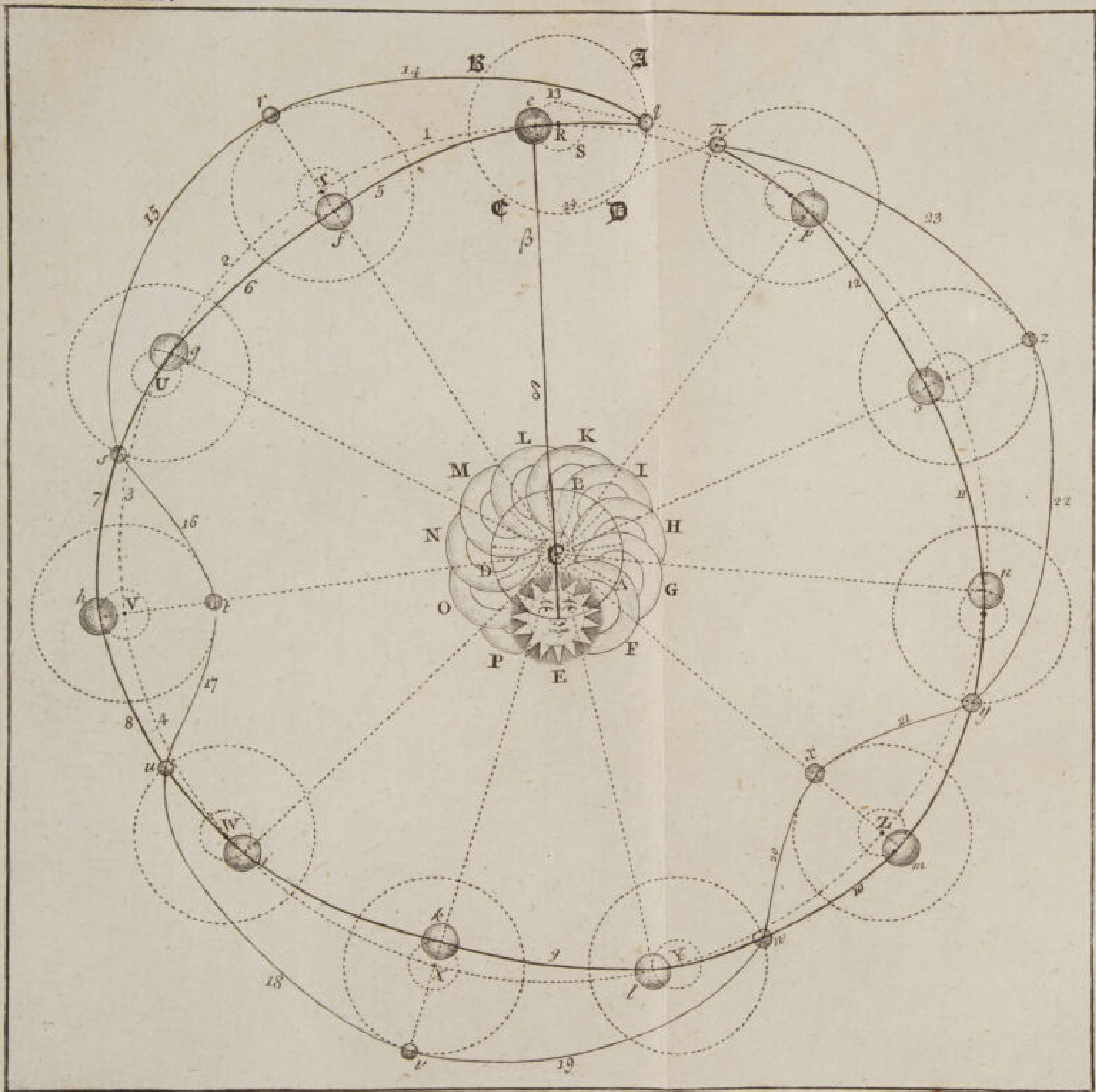
The change of motion produced is in proportion to the force impreſſed: for the effects of natural cauſes are always proportionate to the force or power of thoſe cauſes. The effects of combined forces.

By theſe laws it is eaſy to prove that a body will deſcribe the diagonal of a ſquare or parallelogram, by two forces conjoined, in the ſame time that it would deſcribe either of the ſides, by one force ſingly. Thus, ſuppoſe the body *A* to repreſent a ſhip at ſea; and that it is driven by the wind, in the right line *AB*, with ſuch a force as would carry it uniformly from *A* to *B* in a minute: then ſuppoſe a ſtream or current of water running in the direction *AD*, with ſuch a force as would carry the ſhip through an equal ſpace from *A* to *D* in a minute. By theſe two forces, acting together at right angles to each other, the ſhip will deſcribe the line *AEC* in a minute: which line (becauſe the forces are equal and perpendicular to each other) will be the diagonal of an exact ſquare. To confirm this law by an experiment, let there be a wooden ſquare *ABCD* ſo contrived, as to have the part *BEFC* made to draw out or puſh into the ſquare at pleaſure. To this part let the pully *H* be joined, ſo as to turn freely on an axis, which will be at *H* when the piece is puſhed in, and at *b* when it is drawn out. To this part let the ends of a ſtraight wire *k* be fixed, ſo as to move along with it, under the pulley; and let the ball *G* be made to ſlide eaſily on the wire. A thread *m* is fixed to this ball, and goes over the pulley to *I*; by this thread the ball may be drawn up on the wire, parallel to the ſide *AD*, when the Fig. 2. Fig. 3.

part *BEFC* is pushed as far as it will go into the square. But, if this part be drawn out, it will carry the ball along with it, parallel to the bottom of the square *DC*. By this means, the ball *G* may either be drawn perpendicularly upward by pulling the thread *m*, or moved horizontally along by pulling out the part *BEFC*, in equal times, and through equal spaces; each power acting equally and separately upon it. But if, when the ball is at *G*, the upper end of the thread be tied to the pin *I*, in the corner *A* of the fixed square, and the moveable part *BEFC* be drawn out, the ball will then be acted upon by both the powers together: for it will be drawn up by the thread toward the top of the square, and at the same time be carried with its wire *k* toward the right hand *BC*, moving all the while in the diagonal line *L*; and will be found at *g* when the sliding part is drawn out as far as it was before; which then will have caused the thread to draw up the ball to the top of the inside of the square, just as high as it was before, when drawn up singly by the thread without moving the sliding part.

If the acting forces are equal, but at oblique angles to each other, so will the sides of the parallelogram be: and the diagonal run through by the moving body will be longer or shorter, according as the obliquity is greater or smaller. Thus, if two equal forces act conjointly upon the Fig. 4. body *A*, one having a tendency to move it through the space *AB* in the same time that the other has a tendency to move it through an equal space *AD*; it will describe the diagonal *AGC* in the same time that either of the single forces would have caused it to describe either of the sides. If one of the forces be greater than the other,

PLATE III.



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other, then one ſide of the parallelogram will be ſo much longer than the other. For, if one force ſingly would carry the body through the ſpace *AE*, in the ſame time that the other would have carried it through the ſpace *AD*, the joint action of both will carry it in the ſame time through the ſpace *AHF*, which is the diagonal of the oblique parallelogram *ADEF*.

If both forces act upon the body in ſuch a manner, as to move it uniformly, the diagonal deſcribed will be a ſtraight line; but if one of the forces acts in ſuch a manner as to make the body move faſter and faſter, then the line deſcribed will be a curve. And this is the caſe of all bodies which are projected in rectilineal directions, and at the ſame time acted upon by the power of gravity; which has a conſtant tendency to accelerate their motions in the direction wherein it acts.

The laws of the planetary motions.

From the uniform projectile motion of bodies in ſtraight lines, and the univerſal power of gravity or attraction, ariſes the curvilineal motion of all the heavenly bodies. If the body *A* be projected along the ſtraight line *AFH* in open ſpace, where it meets with no reſiſtance, and is not drawn aſide by any power, it will go on for ever with the ſame velocity, and in the ſame direction. But if, at the ſame moment, the projectile force is given it at *A*, the body *S* begins to attract it with a force duly adjuſted *, and perpendicular to its motion at *A*, it will then be drawn from the ſtraight line *AFH*, and forced Fig. 5.

* To make the projectile force a juſt balance to the gravitating power, ſo as to keep the planet moving in a circle, it muſt give ſuch a velocity as the planet would acquire by gravity, when it had fallen through half the ſemidiameter of that circle.

to revolve about S in the circle $A\Upsilon W$; in the fame manner, and by the fame law, that a pebble is moved round in a fling. And if, when the body is in any part of its orbit (as fuppofe at K) a fmaller body as L, within the fphere of attraction of the body K, be projected in the right line LM, with a force duly adjufted, and perpendicular to the line of attraction LK; then, the fmall body L will revolve about the large body K in the orbit NO, and accompany it in its whole courfe round the yet larger body S. But then, the body K will no longer move in the circle $A\Upsilon W$; for that circle will now be defcribed by the common center of gravity between K and L. Nay, even the great body S will not keep in the center; for it will be the common center of gravity between all the three bodies S, K, and L, that will remain immoveable there. So, if we fuppofe S and K connected by a wire P that has no weight, and K and L connected by a wire q that has no weight, the common center of gravity of all thefe three bodies will be a point in the wire P near S; which point being fupported, the bodies will be all in *equilibrio* as they move round it. Though indeed, ftrictly fpeaking, the common center of gravity of all the three bodies will not be in the wire P but when thefe bodies are all in the right line. Here S may reprefent the fun, K the earth, and L the moon.

In order to form an idea of the curves defcribed by two bodies revolving about their common center of gravity, while they themfelves with a third body are in motion round the common center of gravity of all the three; let Plate III. us firft fuppofe E to be the fun, and e the earth going round him without any moon; and their

their moving forces regulated as above. In this case, while the earth goes round the sun in the dotted circle *RTUWX*, &c. the sun will go round the circle *ABD*, whose center *C* is the common center of gravity between the sun and earth: the right line β δ representing the mutual attraction between them, by which they are as firmly connected as if they were fixed at the two ends of an iron bar strong enough to hold them. So, when the earth is at *e*, the sun will be at *E*; when the earth is at *T*, the sun will be at *F*; and when the earth is at *g*, the sun will be at *G*, &c.

The curves described by bodies revolving about their common center of gravity.

Now, let us take in the moon *q* (at the top of the figure) and suppose the earth to have no progressive motion about the sun; in which case, while the moon revolves about the earth in her orbit 𝔄𝔅ℭ𝔇, the earth will revolve in the circle *S* 13, whose center *R* is the common center of gravity of the earth and moon; they being connected by the mutual attraction between them in the same manner as the earth and sun are.

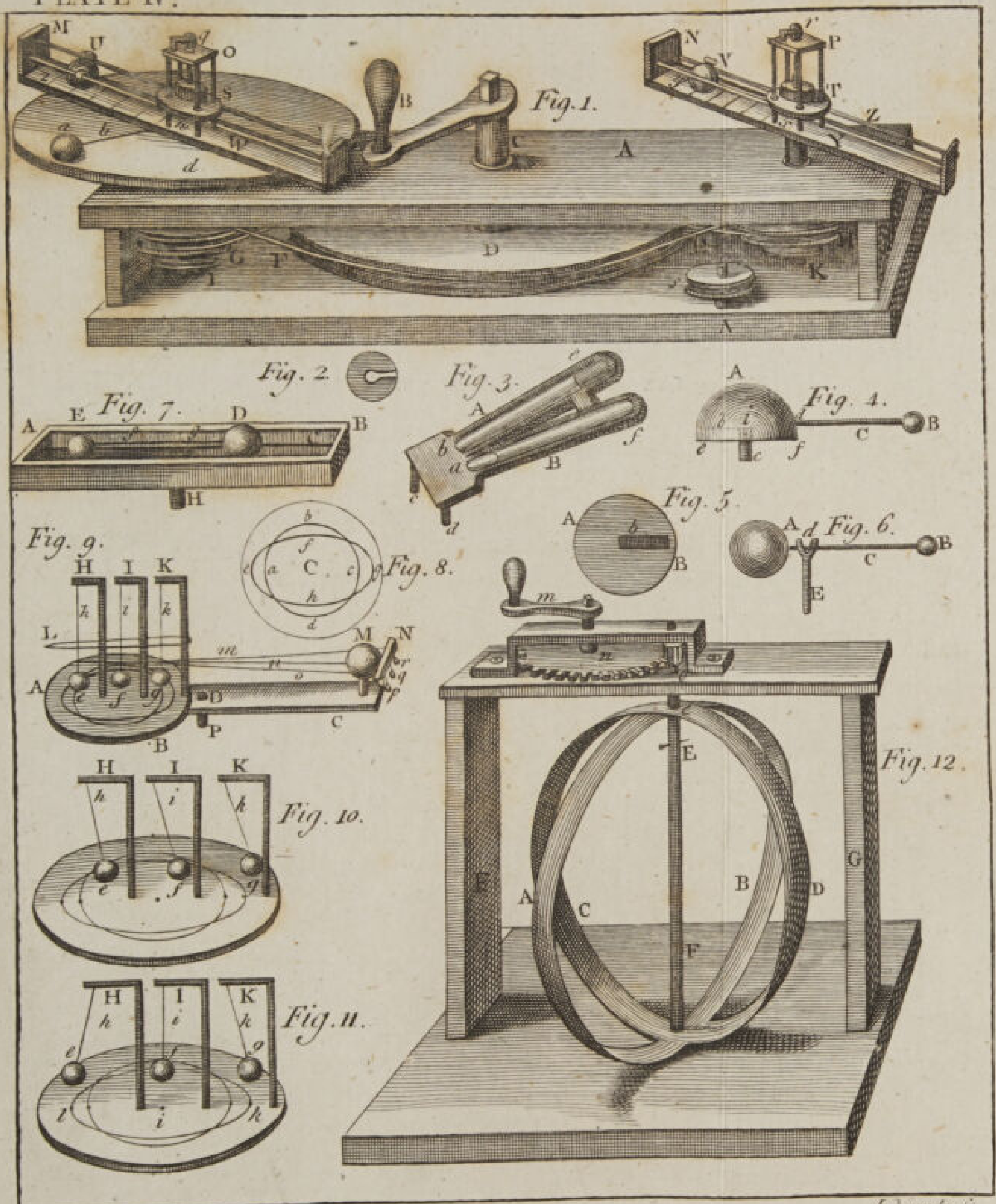
But the truth is, that while the moon revolves about the earth, the earth is in motion about the sun; and now, the moon will cause the earth to describe an irregular curve, and not a true circle, round the sun; it being the common center of gravity of the earth and moon that will then describe the same circle which the earth would have moved in, if it had not been attended by a moon. For, supposing the moon to describe a quarter of her progressive orbit about the earth in the time that the earth moves from *e* to *f*; it is plain, that when the earth comes to *f*, the moon will be found at *r*; in which time, their common center of gravity will have deſcribed the dotted arc *R* 1 *T*, the earth the curve *R* 5 *f*, and the moon the curve *q* 14 *r*. In the time that the moon deſcribes another quarter of her orbit, the center of gravity of the earth and moon will deſcribe the dotted arc *T* 2 *U*, the earth the curve *f* 6 *g*, the moon the curve *r* 15 *s*, and ſo on—And thus, while the moon goes once round the earth in her progreſſive orbit, their common center of gravity deſcribes the regular portion of a circle *R* 1 *T* 2 *U* 3 *V* 4 *W*, the earth the irregular curve *R* 5 *f* 6 *g* 7 *h* 8 *i*, and the moon the yet more irregular curve *q* 14 *r* 15 *s* 16 *t* 17 *u*; and then, the ſame kind of tracks over again.

The center of gravity of the earth and moon is 6000 miles from the earth's center toward the moon; therefore the circle *S* 13 which the earth deſcribes round that center of gravity (in every courſe of the moon round her orbit) is 12,000 miles in diameter. Conſequently the earth is 12,000 miles nearer the ſun at the time of full moon than at the time of new. [See the earth at *f* and at *h*.]

To avoid confuſion in ſo ſmall a figure, we have ſuppoſed the moon to go only twice and a half round the earth, in the time that the earth goes once round the ſun: it being impoſſible to take in all the revolutions which ſhe makes in a year, and to give a true figure of her path, unleſs we ſhould make the ſemidiameter of the earth's orbit at leaſt 95 inches; and then, the proportional ſemidiameter of the moon's orbit would be only a quarter of an inch.—For a true figure of the moon's path, I refer the reader to my treatiſe of aſtronomy.

If the moon made any complete number of revolutions about the earth, in the time that the earth

PLATE IV.



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earth makes one revolution about the ſun, the paths of the ſun and moon would return into themſelves at the end of every year; and ſo be the ſame over again; but they return not into themſelves in leſs than 19 years nearly; in which time, the earth makes nearly 19 revolutions about the ſun, and the moon 235 about the earth.

If the planet *A* be attracted toward the ſun, with ſuch a force as would make it fall from *A* to *B*, in the time that the projectile impulſe would have carried it from *A* to *F*, it will deſcribe the arc *AG* by the combined action of theſe forces, in the ſame time that the former would have cauſed it to fall from *A* to *B*, or the latter have carried it from *A* to *F*. But, if the projectile force had been twice as great, that is, ſuch as would have carried the planet from *A* to *H*, in the ſame time that now, by the ſuppoſition, it carries it only from *A* to *F*; the ſun's attraction muſt then have been four times as ſtrong as formerly, to have kept the planet in the circle *ATW*; that is, it muſt have been ſuch as would have cauſed the planet to fall from *A* to *E*, which is four times the diſtance of *A* from *B*, in the time that the projectile force ſingly would have carried it from *A* to *H*, which is only twice the diſtance of *A* from *F**. Thus, a double projectile force will balance a quadruple power of gravity in the ſame circle; as appears plain by the figure, and ſhall ſoon be confirmed by an experiment. Plate II. Fig. 5. **A double projectile force balances a quadruple power of gravity.**

The whirling-table is a machine contrived for ſhewing experiments of this nature. *AA* is a ſtrong frame of wood, *B* a winch or handle Plate IV. Fig. 1.

* Here the arcs *AG*, *AI*, muſt be ſuppoſed to be very ſmall; otherwiſe *AE*, which is equal to *HI*, will be more than quadruple to *AB*, which is equal to *FG*.

fixed

The whirling table described. fixed on the axis *C* of the wheel *D*, round which is the catgut ſtring *F*, which alſo goes round the ſmall wheels *G* and *K*, croſſing between them and the great wheel *D*. On the upper end of the axis of the wheel *G*, above the frame, is fixed the round board *d*, to which the bearer *MSX* may be faſtened occaſionally, and removed when it is not wanted. On the axis of the wheel *H* is fixed the bearer *NTZ*: and it is eaſy to ſee that when the winch *B* is turned, the wheels and bearers are put into a whirling motion.

Each bearer has two wires, *W*, *X*, and *Y*, *Z*, fixed and ſcrewed tight into them at the ends by nuts on the outſide. And when theſe nuts are unſcrewed, the wires may be drawn out in order to change the balls *U* and *V*, which ſlide upon the wires by means of braſs loops fixed into the balls, which keep the balls up from touching the wood below them. A ſtrong ſilk line goes through each ball, and is fixed to it at any length from the center of the bearer to its end, as occaſion requires, by a nut-ſcrew at the top of the ball; the ſhank of the ſcrew goes into the center of the ball, and preſſes the line againſt the under ſide of the hole that it goes through. —The line goes from the ball, and under a ſmall pulley fixt in the middle of the bearer; then up through a ſocket in the round plate (ſee *S* and *T*) in the middle of each bearer; then through a ſlit in the middle of the ſquare top (*O* and *P*) of each tower, and going over a ſmall pulley on the top, comes down again the ſame way, and is at laſt faſtened to the upper end of the ſocket fixt in the middle of the above-mentioned round plate. Theſe plates *S* and *T* have each four round holes near their edges for letting them

ſlide

ſlide up and down upon the wires which make the corners of each tower. The balls and plates being thus connected, each by its particular line, it is plain, that if the balls be drawn outward, or toward the ends *M* and *N* of their reſpective bearers, the round plates *S* and *T* will be drawn up to the top of their reſpective towers *O* and *P*.

There are ſeveral braſs weights, ſome of two ounces, ſome of three, and ſome of four, to be occaſionally put within the towers *O* and *P*, upon the round plates *S* and *T*: each weight having a round hole in the middle of it, for going upon the ſockets or axes of the plates, and is ſlit from the edge to the hole, for allowing it to be ſlipt over the aforeſaid line which comes from each ball to its reſpective plate. (See Fig. 2.)

The experiments to be made by this machine are as follows:

1. Take away the bearer *MX*, and take the ivory ball *a*, to which the line or ſilk cord *b* is faſtened at one end; and having made a loop on the other end of the cord, put the loop over a pin fixt in the center of the board *d*. Then, turning the winch *B* to give the board a whirling motion, you will ſee that the ball does not immediately begin to move with the board, but, on account of its inactivity, it endeavours to continue in the ſtate of reſt which it was in before.— Continue turning, until the board communicates an equal degree of motion with its own to the ball, and then turning on, you will perceive that the ball will remain upon one part of the board, keeping the ſame velocity with it, and having no relative motion upon it, as is the caſe with every thing that lies looſe upon the plane ſurface of the earth, which, having the motion of the earth communicated to it, never endeavours to remove from

Fig. 1.

The propenſity of matter to keep the ſtate it is in.

from that place. But ſtop the board ſuddenly by hand, and the ball will go on, and continue to revolve upon the board, until the friction thereof ſtops its motion: which ſhews, that matter being once put into motion would continue to move for ever, if it met with no reſiſtance. In like manner, if a perſon ſtands upright in a boat before it begins to move, he can ſtand firm; but the moment the boat ſets off, he is in danger of falling toward that place which the boat departs from: becauſe, as matter, he has no natural propenſity to move. But when he acquires the motion of the boat, let it be ever ſo ſwift, if it be ſmooth and uniform, he will ſtand as upright and firm as if he was on the plain ſhore; and if the boat ſtrikes againſt any obſtacle, he will fall toward that obſtacle; on account of the propenſity he has, as matter, to keep the motion which the boat has put him into.

2. Take away this ball, and put a longer cord to it, which may be put down through the hollow axis of the bearer *MX*, and wheel *G*, and fix a weight to the end of the cord below the machine; which weight, if left at liberty, will draw the ball from the edge of the whirling-board to its center.

Bodies moving in orbits have a tendency to fly out of theſe orbits.

Draw off the ball a little from the center, and turn the winch; then the ball will go round and round with the board, and will gradually fly off farther and farther from the center, and raiſe up the weight below the machine: which ſhews that all bodies revolving in circles have a tendency to fly off from theſe circles, and muſt have ſome power acting upon them from the center of motion, to keep them from flying off. Stop the machine, and the ball will continue to revolve for

for ſome time upon the board; but as the friction gradually ſtops its motion, the weight acting upon it will bring it nearer and nearer to the center in every revolution, until it brings it quite thither. This ſhews, that if the planets met with any reſiſtance in going round the ſun, its attractive power would bring them nearer and nearer to it in every revolution, until they fell upon it.

Bodies move faſter in ſmall orbits than in large ones.

3. Take hold of the cord below the machine with one hand, and with the other throw the ball upon the round board as it were at right angles to the cord, by which means it will go round and round upon the board. Then obſerving with what velocity it moves, pull the cord below the machine, which will bring the ball nearer to the center of the board, and you will ſee that the nearer the ball is drawn to the center, the faſter it will revolve; as thoſe planets which are neareſt the ſun revolve faſter than thoſe which are more remote; and not only go round ſooner, becauſe they deſcribe ſmaller circles, but even move faſter in every part of their reſpective circles.

Their centrifugal forces ſhewn.

4. Take away this ball, and apply the bearer *M X*, whoſe center of motion is in its middle at *w*, directly over the center of the whirling-board *d*. Then put two balls (*V* and *U*) of equal weights upon their bearing wires, and having fixed them at equal diſtances from their reſpective centers of motion *w* and *x* upon their ſilk cords, by the ſcrew nuts, put equal weights in the towers *O* and *P*. Laſtly, put the catgut ſtrings *E* and *F* upon the grooves *G* and *H* of the ſmall wheels, which being of equal diameters, will give equal velocities to the bearers above, when the winch *B* is turned: and the balls *U* and *V* will fly

fly off toward *M* and *N*; and will raiſe the weights in the towers at the ſame inſtant. This ſhews, that when bodies of equal quantities of matter revolve in equal circles with equal velocities, their centrifugal forces are equal.

5. Take away theſe equal balls, and inſtead of them put a ball of ſix ounces into the bearer *MX*, at a ſixth part of the diſtance *w z* from the center, and put a ball of one ounce into the oppoſite bearer, at the whole diſtance *x y*, which is equal to *w z* from the center of the bearer; and fix the balls at theſe diſtances on their cords, by the ſcrew nuts at top; and then the ball *U*, which is ſix times as heavy as the ball *V*, will be at only a ſixth part of the diſtance from its center of motion; and conſequently will revolve in a circle of only a ſixth part of the circumference of the circle in which *V* revolves. Now, let any equal weights be put into the towers, and the machine be turned by the winch; which (as the catgut ſtring is on equal wheels below) will cauſe the balls to revolve in equal times; but *V* will move ſix times as faſt as *U*, becauſe it revolves in a circle of ſix times its radius; and both the weights in the towers will riſe at once. This ſhews, that the centrifugal forces of revolving bodies (or their tendencies to fly off from the circles they deſcribe) are in direct proportion to their quantities of matter multiplied into their reſpective velocities; or into their diſtances from the centers of their reſpective circles. For, ſuppoſing *U*, which weighs ſix ounces, to be two inches from its center of motion *w*, the weight multiplied by the diſtance is 12: and ſuppoſing *V*, which weighs only one ounce, to be 12 inches diſtant from the center of motion *x*, the weight 1 ounce multiplied by the diſtance 12 inches is

is 12. And as they revolve in equal times, their velocities are as their diſtances from the center, namely, as 1 to 6.

If theſe two balls be fixed at equal diſtances from their reſpective centers of motion, they will move with equal velocities; and if the tower *O* has 6 times as much weight put into it as the tower *P* has, the balls will raiſe their weights exactly at the ſame moment. This ſhews that the ball *U* being ſix times as heavy as the ball *V*, has ſix times as much centrifugal force, in deſcribing an equal circle with an equal velocity.

A double velocity in the ſame circle, is a balance to a quadruple power of gravity.

6. If bodies of equal weights revolve in equal circles with unequal velocities, their centrifugal forces are as the ſquares of the velocities. To prove this law by an experiment, let two balls *U* and *V* of equal weights be fixed on their cords at equal diſtances from their reſpective centers of motion *w* and *x*; and then let the catgut ſtring *E* be put round the wheel *K* (whoſe circumference is only one half of the circumference of the wheel *H* or *G*) and over the pulley *s* to keep it tight; and let four times as much weight be put into the tower *P*, as in the tower *O*. Then turn the winch *B*, and the ball *V* will revolve twice as faſt as the ball *U* in a circle of the ſame diameter, becauſe they are equidiſtant from the centers of the circles in which they revolve; and the weight in the towers will both riſe at the ſame inſtant, which ſhews that a double velocity in the ſame circle will exactly balance a quadruple power of attraction in the center of the circle. For the weights in the towers may be conſidered as the attractive forces in the centers, acting upon the revolving balls; which, moving

moving in equal circles, is the fame thing as if they both moved in one and the fame circle.

Kepler's problem. 7. If bodies of equal weights revolve in unequal circles, in fuch a manner that the fquares of the times of their going round are as the cubes of their diftances from the centers of the circles they defcribe; their centrifugal forces are inverfely as the fquares of their diftances from thofe centers. For, the catgut ftring remaining as in the laft experiment, let the diftance of the ball *V* from the center *x* be made equal to two of the crofs divifions on its bearer; and the diftance of the ball *U* from the center *w* be three and a fixth part; the balls themfelves being of equal weights, and *V* making two revolutions by turning the winch, in the time that *U* makes one: fo that if we fuppofe the ball *V* to revolve in one fecond, the ball *U* will revolve in two feconds, the fquares of which are one and four: for the fquare of 1 is only 1, and the fquare of 2 is 4; therefore the fquare of the period or revolution of the ball *V*, is contained four times in the fquare of the period of the ball *U*. But the diftance of *V* is 2, the cube of which is 8, and the diftance of *U* is $3\frac{1}{6}$, the cube of which is 32 very nearly, in which 8 is contained four times; and therefore, the fquares of the periods of *V* and *U* are to one another as the cubes of their diftances from *x* and *w*, which are the centers of their refpective circles. And if the weight in the tower *O* be four ounces, equal to the fquare of 2, the diftance of *V* from the center *x*; and the weight in the tower *P* be ten ounces, nearly equal to the fquare of $3\frac{1}{6}$, the diftance of *U* from *w*; it will be found upon turning the machine by the winch, that the balls *U* and *V* will raife their refpective weights at the

the ſame inſtant of time. Which confirms that famous obſervation of KEPLER, viz. That the ſquares of the times in which the planets go round the ſun are in the ſame proportion as the cubes of their diſtances from him; and that the ſun's attraction is inverſely as the ſquare of the diſtance from his center: that is, at twice the diſtance, his attraction is four times leſs; and thrice the diſtance, nine times leſs; at four times the diſtance, ſixteen times leſs; and ſo on, to the remoteſt part of the ſyſtem.

8. Take off the catgut ſtring *E* from the great wheel *D* and the ſmall wheel *H*, and let the ſtring *F* remain upon the wheels *D* and *G*. Take away alſo the bearer *M X* from the whirling-board *d*, and inſtead thereof put the machine *AB* upon it, fixing this machine to the center of the board by the pins *c* and *d*, in ſuch a manner, that the end *e f* may riſe above the board to an angle of 30 or 40 degrees. In the upper ſide of this machine are two glaſs tubes *a* and *b*, cloſe ſtopt at both ends; and each tube is about three quarters full of water. In the tube *a* is a little quickſilver, which naturally falls down to the end *a* in the water, becauſe it is heavier than its bulk of water; and in the tube *b* is a ſmall cork which floats on the top of the water at *e*, becauſe it is lighter; and it is ſmall enough to have liberty to riſe or fall in the tube. While the board *b* with this machine upon it continues at reſt, the quickſilver lies at the bottom of the tube *a*, and the cork floats on the water near the top of the tube *b*. But, upon turning the winch, and putting the machine in motion, the contents of each tube will fly off toward the uppermoſt ends (which are fartheſt from the center of motion) the heavieſt

Fig. 3.

The abſurdity of the Carteſian vortexes.

 with

with the greateſt force. Therefore the quickſilver in the tube *a* will fly off quite to the end *f*, and occupy its bulk of ſpace there, excluding the water from that place, becauſe it is lighter than quickſilver; but the water in the tube *b* flying off to its higher end *e*, will exclude the cork from that place, and cauſe the cork to deſcend toward the lowermoſt end of the tube, where it will remain upon the loweſt end of the water near *b*; for the heavier body having the greater centrifugal force, will therefore poſſeſs the uppermoſt part of the tube; and the lighter body will keep between the heavier and the lowermoſt part.

This demonſtrates the abſurdity of the Carteſian doctrine of the planets moving round the ſun in vortexes: for, if the planet be more denſe or heavy than its bulk of the vortex, it will fly off therein, farther and farther from the ſun; if leſs denſe, it will come down to the loweſt part of the vortex, at the ſun: and the whole vortex itſelf muſt be ſurrounded with ſomething like a great wall, otherwiſe it would fly quite off, planets and all together.—But while gravity exiſts, there is no occaſion for ſuch vortexes; and when it ceaſes to exiſt, a ſtone thrown upward will never return to the earth again.

If one body moves round another, both of them muſt move round their common center of gravity.

9. If a body be ſo placed on the whirling-board of the machine (Fig. 1.) that the center of gravity of the body be directly over the center of the board, and the board be put into ever ſo rapid a motion by the winch *B*, the body will turn round with the board, but will not remove from the middle of it; for, as all parts of the body are in *equilibrio* round its center of gravity, and the center of gravity is at reſt in the center of motion, the centrifugal force of all parts of the

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the body will be equal at equal diſtances from its center of motion, and therefore the body will remain in its place. But if the center of gravity be placed ever ſo little out of the center of motion, and the machine be turned ſwiftly round, the body will fly off toward that ſide of the board on which its center of gravity lies. Thus, if the wire *C* with its little ball *B* be taken away from the demi-globe *A*, and the flat ſide *e f* of this demi-globe be laid upon the whirling-board of the machine, ſo that their centers may coincide; if then the board be turned ever ſo quick by the winch, the demi-globe will remain where it was placed. But if the wire *C* be ſcrewed into the demi-globe at *d*, the whole becomes one body, whoſe center of gravity is now at or near *d*. Let the pin *c* be fixed in the center of the whirling-board, and the deep groove *b* cut in the flat ſide of the demi-globe be put upon the pin, ſo as the pin may be in the center of *A* [See Fig. 5. where this groove is repreſented at *b*] and let the whirling-board be turned by the winch, which will carry the little ball *B* (Fig. 4.) with its wire *C*, and the demi-globe *A*, all round the center-pin *c i*; and then, the centrifugal force of the little ball *B*, which weighs only one ounce, will be ſo great, as to draw off the demi-globe *A*, which weighs two pounds, until the end of the groove at *e* ſtrikes againſt the pin *c*, and ſo prevents the demi-globe *A* from going any farther: otherwiſe, the centrifugal force of *B* would have been great enough to have carried *A* quite off the whirling-board. Which ſhews, that if the ſun were placed in the very center of the orbits of the planets, it could not poſſibly remain there; for the centrifugal forces of the planets would carry them quite off, and the ſun

Fig. 4.

Fig. 5.

with them; eſpecially when ſeveral of them happened to be in any one quarter of the heavens. For the ſun and planets are as much connected by the mutual attraction that ſubſiſts between them, as the bodies *A* and *B* are by the wire *C* which is fixed into them both. And even if there were but one ſingle planet in the whole heavens to go round ever ſo large a ſun in the center of its orbit, its centrifugal force would ſoon carry off both itſelf and the ſun. For, the greateſt body placed in any part of free ſpace might be eaſily moved: becauſe if there were no other body to attract it, it could have no weight or gravity of itſelf; and conſequently, though it could have no tendency of itſelf to remove from that part of ſpace, yet it might be very eaſily moved by any other ſubſtance.

10. As the centrifugal force of the light body *B* will not allow the heavy body *A* to remain in the center of motion, even though it be 24 times as heavy as *B*; let us now take the ball *A* (Fig.
Fig. 6. 6.) which weighs 6 ounces, and connect it by the wire *C* with the ball *B*, which weighs only one ounce; and let the fork *E* be fixed into the center of the whirling-board: then hang the balls upon the fork by the wire *C* in ſuch a manner, that they may exactly balance each other; which will be when the center of gravity between them, in the wire at *d*, is ſupported by the fork. And this center of gravity is as much nearer to the center of the ball *A*, than to the center of the ball *B*, as *A* is heavier than *B*, allowing for the weight of the wire on each ſide of the fork. This done, let the machine be put into motion by the winch; and the balls *A* and *B* will go round their common center of gravity *d*, keeping their balance, becauſe either will not allow

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the other to fly off with it. For, ſuppoſing the ball *B* to be only one ounce in weight, and the ball *A* to be ſix ounces; then, if the wire *C* were equally heavy on each ſide of the fork, the center of gravity *d* would be ſix times as far from the center of the ball *B* as from that of the ball *A*, and conſequently *B* will revolve with a velocity ſix times as great as *A* does; which will give *B* ſix times as much centrifugal force as any ſingle ounce of *A* has: but then, as *B* is only one ounce, and *A* ſix ounces, the whole centrifugal force of *A* will exactly balance the whole centrifugal force of *B*: and therefore, each body will detain the other ſo as to make it keep in its circle. This ſhews that the ſun and planets muſt all move round the common center of gravity of the whole ſyſtem, in order to preſerve that juſt balance which takes place among them. For, the planets being as unactive and dead as the above balls, they could no more have put themſelves into motion than theſe balls can; nor have kept in their orbits without being balanced at firſt with the greateſt degree of exactneſs upon their common center of gravity, by the Almighty hand that made them and put them in motion.

Perhaps it may be here aſked, that ſince the center of gravity between theſe balls muſt be ſupported by the fork *E* in this experiment, *what* prop it is that ſupports the center of gravity of the ſolar ſyſtem, and conſequently bears the weight of all the bodies in it; and by what is the prop itſelf ſupported? The anſwer is eaſy and plain; for the center of gravity of our balls muſt be ſupported, becauſe they gravitate toward the earth, and would therefore fall to it: but as the ſun and planets gravitate only toward one another, they have nothing elfe to fall to; and therefore have no occafion for any thing to fupport their common center of gravity: and if they did not move round that center, and confequently acquire a tendency to fly off from it by their motions, their mutual attractions would foon bring them together; and fo the whole would become one mafs in the fun: which would alfo be the cafe if their velocities round the fun were not quick enough to create a centrifugal force equal to the fun's attraction.

But after all this nice adjuftment, it appears evident that the Deity cannot withdraw his regulating hand from his works, and leave them to be folely governed by the laws which he has impreft upon them at firft. For if he fhould once leave them fo, their order would in time come to an end; becaufe the planets muft neceffarily difturb one another's motions by their mutual attractions, when feveral of them are in the fame quarter of the heavens; as is often the cafe: and then, as they attract the fun more toward that quarter than when they are in a manner difperfed equably around him, if he was not at that time made to defcribe a portion of a larger circle round the common center of gravity, the balance would then be immediately deftroyed; and as it could never reftore itfelf again, the whole fyftem would begin to fall together, and would in time unite in a mafs at the fun.—Of this difturbance we have a very remarkable inftance in the comet which appeared lately; and which, in going laft up before from the fun, went fo near to Jupiter, and was fo affected by his attraction, as to have the figure of its orbit much changed; and not only fo, but to have its period altered,

altered, and its courſe to be different in the heavens from what it was laſt before.

11. Take away the fork and balls from the whirling-board, and place the trough *AB* thereon, fixing its center to the center of the whirling-board by the pin *H*. In this trough are two balls *D* and *E*, of unequal weights, connected by a wire *f*; and made to ſlide eaſily upon the wire *C* ſtretched from end to end of the trough, and made faſt by nut-ſcrews on the outſide of the ends. Let theſe balls be ſo placed upon the wire *C*, that their common center of gravity *g* may be directly over the center of the whirling-board. Then, turn the machine by the winch, ever ſo ſwiftly, and the trough and balls will go round their center of gravity, ſo as neither of the balls will fly off; becauſe, on account of the equilibrium, each ball detains the other with an equal force acting againſt it. But if the ball *E* be drawn a little more toward the end of the trough at *A*, it will remove the center of gravity toward that end from the center of motion; and then, upon turning the machine, the little ball *E* will fly off, and ſtrike with a conſiderable force againſt the end *A*, and draw the great ball *B* into the middle of the trough. Or, if the great ball *D* be drawn toward the end *B* of the trough, ſo that the center of gravity may be a little toward that end from the center of motion, and the machine be turned by the winch, the great ball *D* will fly off, and ſtrike violently againſt the end *B* of the trough, and will bring the little ball *E* into the middle of it. If the trough be not made very ſtrong, the ball *D* will break through it. Fig. 7.

12. The reaſon why the tides riſe at the ſame abſolute time on oppoſite ſides of the earth, and Of the tides.

consequently in opposite directions, is made abundantly plain by a new experiment on the whirling-table. The cause of their rising on the side next the moon every one understands to be owing to the moon's attraction: but why they should rise on the opposite side at the same time, where there is no moon to attract them, is perhaps not so generally understood. For it would seem that the moon should rather draw the waters (as it were) closer to that side, than raise them upon it, directly contrary to her attractive force. Let
Fig. 8. the circle *a b c d* represent the earth, with its side *c* turned toward the moon, which will then attract the waters so, as to raise them from *c* to *g*, But the question is, why should they rise as high at that very time on the opposite side, from *a* to
Fig. 9. *e*? In order to explain this, let there be a plate *A B* fixed upon one end of the flat bar *D C*; with such a circle drawn upon it as *a b c d* (in Fig. 8.) to represent the round figure of the earth and sea; and such an ellipsis as *e f g h* to represent the swelling of the tide at *e* and *g*, occasioned by the influence of the moon. Over this plate *A B* let the three ivory balls *e*, *f*, *g*, be hung by the silk lines *h*, *i*, *k*, fastened to the tops of the crooked wires *H*, *I*, *K*, in such a manner, that the ball at *e* may hang freely over the side of the circle *e*, which is farthest from the moon *M* at the other end of the bar; the ball at *f* may hang freely over the center, and the ball at *g* hang over the side of the circle *g*, which is nearest the moon. The ball *f* may represent the center of the earth, the ball *g* some water on the side next the moon, and the ball *e* some water on the opposite side. On the back of the moon *M* is fixt the short bar *N* parallel to the horizon, and there are three holes in it above the little weights *p*, *q*, *r*. A silk

ſilk thread *o* is tied to the line *k* cloſe above the ball *g*, and paſſing by one ſide of the moon *M*, goes through a hole in the bar *N*, and has the weight *p* hung to it. Such another thread *n* is tied to the line *i*, cloſe above the ball *f*, and paſſing through the center of the moon *M* and middle of the bar *N*, has the weight *q* hung to it, which is lighter than the weight *p*. A third thread *m* is tied to the line *h*, cloſe above the ball *e*, and paſſing by the other ſide of the moon *M*, through the bar *N*, has the weight *r* hung to it, which is lighter than the weight *q*.

The uſe of theſe three unequal weights is to repreſent the moon's unequal attraction at different diſtances from her. With whatever force ſhe attracts the center of the earth, ſhe attracts the ſide next her with a greater degree of force, and the ſide fartheſt from her with a leſs. So, if the weights are left at liberty, they will draw all the three balls toward the moon with different degrees of force, and cauſe them to make the appearance ſhewn in Fig. 10; by which means they are evidently farther from each other than they would be if they hung at liberty by the lines *h*, *i*, *k*; becauſe the lines would then hang perpendicularly. This ſhews, that as the moon attracts the ſide of the earth which is neareſt her with a greater degree of force than ſhe does the center of the earth, ſhe will draw the water on that ſide more than ſhe draws the center, and ſo cauſe it to riſe on that ſide: and as ſhe draws the center more than ſhe draws the oppoſite ſide, the center will recede farther from the ſurface of the water on that oppoſite ſide, and ſo leave it as high there as ſhe raiſed it on the ſide next to her. For, as the center will be in the middle between the

Fig. 10.

the tops of the oppofite elevations, they muft of courfe be equally high on both fides at the fame time.

But upon this fuppofition the earth and moon would foon come together: and to be fure they would, if they had not a motion round their common center of gravity, to create a degree of centrifugal force fufficient to balance their mutual attraction. This motion they have; for as the moon goes round her orbit every month, at the diftance of 240000 miles from the earth's center, and of 234000 miles from the center of gravity of the earth and moon, fo does the earth go round the fame center of gravity every month at the diftance of 6000 miles from it; that is, from it to the center of the earth. Now as the earth is (in round numbers) 8000 miles in diameter, it is plain that its fide next the moon is only 2000 miles from the common center of gravity of the earth and moon; its center 6000 miles diftant therefrom; and its farther fide from the moon 10000. Therefore the centrifugal forces of thefe parts are as 2000, 6000, and 10000; that is, the centrifugal force of any fide of the earth, when it is turned from the moon, is five times as great as when it is turned toward the moon. And as the moon's attraction (expreft by the numbers 6000) at the earth's center keeps the earth from flying out of this monthly circle, it muft be greater than the centrifugal force of the waters on the fide next her; and confequently, her greater degree of attraction on that fide is fufficient to raife them; but as her attraction on the oppofite fide is lefs than the centrifugal force of the water there, the excefs of this force is fufficient to raife the water juft as high

high on the oppoſite ſide.—To prove this experimentally, let the bar *D C* with its furniture be fixed upon the whirling-board of the machine (Fig. 1.) by puſhing the pin *P* into the center of the board; which pin is in the center of gravity of the whole bar with its three balls *e*, *f*, *g*, and moon *M*. Now if the whirling-board and bar be turned ſlowly round by the winch, until the ball *f* hangs over the center of the circle, as in Fig. 11. the ball *g* will be kept toward the moon by the heavieſt weight *p* (Fig. 9.) and the ball *e*, on account of its greater centrifugal force, and the leſſer weight *r*, will fly off as far to the other ſide, as in Fig. 11. And thus, while the machine is kept turning, the balls *e* and *g* will hang over the end of the ellipſis *l f k*. So that the centrifugal force of the ball *e* will exceed the moon's attraction juſt as much as her attraction exceeds the centrifugal force of the ball *g*, while her attraction juſt balances the centrifugal force of the ball *f*, and makes it keep in its circle. And hence it is evident, that the tides muſt riſe to equal heights at the ſame time on oppoſite ſides of the earth. This experiment, to the beſt of my knowledge, is entirely new. Fig. 9.

The earth's motion demonſtrated.

From the principles thus eſtabliſhed, it is evident that the earth moves round the ſun, and not the ſun round the earth; for the centrifugal law will never allow a great body to move round a ſmall one in any orbit whatever; eſpecially when we find that if a ſmall body moves round a great one, the great one muſt alſo move round the common center of gravity between them two. And it is well known that the quantity of matter in the ſun is 227000 times as great as the quantity of matter in the earth. Now, as the ſun's diſtance

diſtance from the earth is at leaſt 81,000,000 of miles, if we divide that diſtance by 227,000, we ſhall have only 357 for the number of miles that the center of gravity between the ſun and earth is diſtant from the ſun's center. And as the ſun's ſemidiameter is ¼ of a degree, which, at ſo great a diſtance as that of the ſun, muſt be no leſs than 381500 miles, if this be divided by 357, the quotient will be nearly 1069, which ſhews that the common center of gravity between the ſun and earth is within the body of the ſun; and is only the 1069 part of his ſemidiameter from his center toward his ſurface.

All globular bodies, whoſe parts can yield, and which do not turn on their axes, muſt be perfect ſpheres, becauſe all parts of their ſurfaces are equally attracted toward their centers. But all ſuch globes which do turn on their axes will be oblate ſpheroids; that is, their ſurfaces will be higher, or farther from the center, in the equatorial than in the polar regions. For, as the equatorial parts move quickeſt, they muſt have the greateſt centrifugal force; and will therefore recede fartheſt from the axis of motion. Thus, if two circular hoops *A B* and
Fig. 12. *C D*, made thin and flexible, and croſſing one another at right angles, be turned round their axis *E F* by means of the winch *m*, the wheel *n*, and pinion *o*, and the axis be looſe in the pole or interſection *e*, the middle parts *A*, *B*, *C*, *D*, will ſwell out ſo as to ſtrike againſt the ſides of the frame at *F* and *G*, if the pole *e*, in ſinking to the pin *E*, be not ſtopt by it from ſinking farther: ſo that the whole will appear of an oval figure, the equatorial diameter being conſiderably longer than the polar. That our earth is of this figure, is demonſtrable from actual mea-

ſurement

ſurement of ſome degrees on its ſurface, which are found to be longer in the frigid zones than in the torrid: and the difference is found to be ſuch as proves the earth's equatorial diameter to be 36 miles longer than its axis.—Seeing then, the earth is higher at the equator than at the poles, the ſea, which like all other fluids naturally runs downward (or toward the places which are neareſt the earth's center) would run toward the polar regions, and leave the equatorial parts dry, if the centrifugal force of the water, which carried it to thoſe parts, and ſo raiſed them, did not detain and keep it from running back again toward the poles of the earth.

LECT. III.

Of the mechanical Powers.

The foundation of all mechanics.

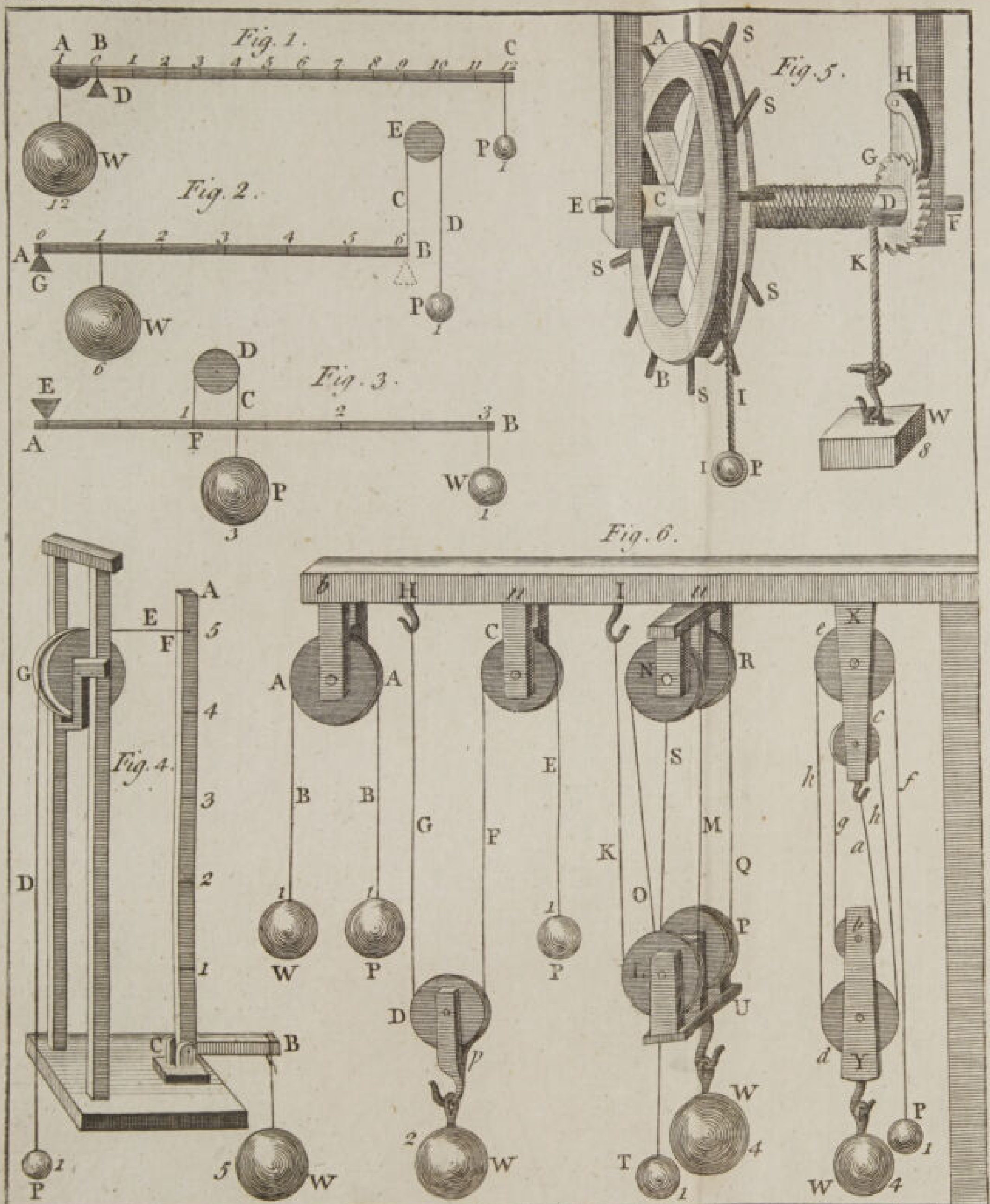
IF we conſider bodies in motion, and compare them together, we may do this either with reſpect to the quantities of matter they contain, or the velocities with which they are moved. The heavier any body is, the greater is the power required either to move it or to ſtop its motion: and again, the ſwifter it moves, the greater is its force. So that the whole *momentum* or quantity of force of a moving body is the reſult of its quantity of matter multiplied by the velocity with which it is moved. And when the products ariſing from the multiplication of the particular quantities of matter in any two bodies by their reſpective velocities are equal, the *momenta* or entire forces are ſo too. Thus, ſuppoſe a body, which we ſhall call *A*, to weigh 40 pounds, and to move at the rate of two miles in in a minute; and another body, which we ſhall call *B*, to weigh only four pounds, and to move 20 miles in a minute; the entire forces with which theſe two bodies would ſtrike againſt any obſtacle would be equal to each other, and therefore it would require equal powers to ſtop them. For 40 multiplied by 2 gives 80, the force of the body *A*: and 20 multiplied by 4 gives 80, the force of the body *B*.

Upon this eaſy principle depends the whole of mechanics: and it holds univerſally true, that when two bodies are ſuſpended on any machine, ſo as to act contrary to each other; if the machine be put into motion, and the perpendicular aſcent of one body multiplied into its weight, be equal to the perpendicular deſcent of the other body multiplied into its weight, thoſe bodies, how unequal ſoever in their weights, will balance one another in all ſituations: for, as the whole aſcent of one is performed in the ſame time with the whole deſcent of the other, their reſpective velocities muſt be directly as the ſpaces they move through; and the exceſs of weight in one body is compenſated by the exceſs of velocity in the other.—Upon this principle it is eaſy to compute the power of any mechanical engine, whether ſimple or compound; for it is but only finding how much ſwifter the power moves than the weight does (*i. e.* how much farther in the ſame time) and juſt ſo much is the power increaſed by the help of the engine.

How to compute the power of any mechanical engine.

In the theory of this ſcience, we ſuppoſe all planes perfectly even, all bodies perfectly ſmooth, levers to have no weight, cords to be extremely pliable, machines to have no friction; and in ſhort, all imperfections muſt be ſet aſide until the

PLATE V.



J. Ferguson delin. J. Mynde sculp.

the theory be eſtabliſhed; and then, proper allowances are to be made.

The ſimple *machines*, uſually called *mechanical powers*, are ſix in number, viz. the *lever*, the *wheel* and *axle*, the *pulley*, the *inclined plane*, the *wedge*, and the *ſcrew*.—They are called mechanical powers, becauſe they help us mechanically to raiſe weights, move heavy bodies, and overcome reſiſtances, which we could not effect without them. The mechanic powers, what.

1. A *lever* is a bar of iron or wood, one part of which being ſupported by a prop, all the other parts turn upon that prop as their center of motion: and the velocity of every part or point is directly as its diſtance from the prop. Therefore, when the weight to be raiſed at one end is to the power applied to the other to raiſe it, as the diſtance of the power from the prop is to the diſtance of the weight from the prop, the power and weight will exactly balance or counterpoiſe each other: and as a common lever has next to no friction on its prop, a very little additional power will be ſufficient to raiſe the weight. The *lever*.

There are four kinds of levers. 1. The common ſort, where the prop is placed between the weight and the power; but much nearer to the weight than to the power. 2. When the prop is at one end of the lever, the power at the other, and the weight between them. 3. When the prop is at one end, the weight at the other, and the power applied between them. 4. The bended lever, which differs only in form from the firſt ſort, but not in property. Thoſe of the firſt and ſecond kind are often uſed in mechanical engines; but there are few inſtances in which the third ſort is uſed.

A *com-*

The *balance.* A *common balance* is by ſome reckoned a lever of the firſt kind; but as both its ends are at equal diſtances from its center of motion, they move with equal velocities; and therefore, as it gives no mechanical advantage, it cannot properly be reckoned among the mechanical powers.

Plate V. Fig. 1. The firſt kind of lever. A lever of the firſt kind is repreſented by the bar *ABC*, ſupported by the prop *D*. Its principal uſe is to looſen large ſtones in the ground, or raiſe great weights to ſmall heights, in order to have ropes put under them for raiſing them higher by other machines. The parts *AB* and *BC*, on different ſides of the prop *D*, are called the *arms* of the lever: the end *A* of the ſhorter arm *AB* being applied to the weight intended to be raiſed, or to the reſiſtance to be overcome; and the power applied to the end *C* of the longer arm *BC*.

In making experiments with this machine, the ſhorter arm *AB* muſt be as much thicker than the longer arm *BC*, as will be ſufficient to balance it on the prop. This ſuppoſed, let *P* repreſent a power, whoſe gravity is equal to 1 ounce, and *W* a weight, whoſe gravity is equal to 12 ounces. Then, if the power be 12 times as far from the prop as the weight is, they will exactly counterpoiſe; and a ſmall addition to the power *P* will cauſe it to deſcend, and raiſe the weight *W*; and the velocity with which the power deſcends will be to the velocity with which the weight riſes, as 12 to 1: that is, directly as their diſtances from the prop; and conſequently, as the ſpaces through which they move. Hence, it is plain, that a man, who by his natural ſtrength, without the help of any machine, could ſupport a hundred weight, will by the help of this lever be enabled to ſupport twelve

twelve hundred. If the weight be leſs, or the power greater, the prop may be placed ſo much farther from the weight; and then it can be raiſed to a proportionably greater height. For, univerſally, if the intenſity of the weight multiplied into its diſtance from the prop be equal to the intenſity of the power multiplied into its diſtance from the prop, the power and weight will exactly balance each other; and a little addition to the power will raiſe the weight. Thus, in the preſent inſtance, the weight *W* is 12 ounces, and its diſtance from the prop is 1 inch; and 12 multiplied by 1 is 12; the power *P* is equal to 1 ounce, and its diſtance from the prop is 12 inches, which multiplied by 1 is 12 again; and therefore there is an equilibrium between them. So, if a power equal to 2 ounces be applied at the diſtance of 6 inches from the prop, it will juſt balance the weight *W*; for 6 multiplied by 2 is 12, as before. And a power equal to 3 ounces placed at 4 inches diſtance from the prop would be the ſame; for 3 times 4 is 12; and ſo on, in proportion.

The *ſtatera* or Roman *ſteelyard* is a lever of this kind, and is uſed for finding the weights of different bodies by one ſingle weight placed at different diſtances from the prop or center of motion *D*. For, if a ſcale hangs at *A*, the extremity of the ſhorter arm *A B*, is of ſuch a weight as will exactly counterpoiſe the longer arm *B C*; if this arm be divided into as many equal parts as it will contain, each equal to *A B*, the ſingle weight *P* (which we may ſuppoſe to be 1 pound) will ſerve for weighing any thing as heavy as itſelf, or as many times heavier as there are diviſions in the arm *B C*, or any quantity between its own weight and that quantity.

The ſteel-yard.

As for example, if P be 1 pound, and placed at the firſt diviſion 1 in the arm BC, it will balance 1 pound in the ſcale at A: if it be removed to the ſecond diviſion at 2, it will balance 2 pounds in the ſcale: if to the third, 3 pounds; and ſo on to the end of the arm BC. If each of theſe integral diviſions be ſubdivided into as many equal parts, as a pound contains ounces, and the weight P be placed at any of theſe ſubdiviſions, ſo as to counterpoiſe what is in the ſcale, the pounds and odd ounces therein will by that means be aſcertained.

To this kind of lever may be reduced ſeveral ſorts of inſtruments, ſuch as ſciſſars, pincers, ſnuffers; which are made of two levers acting contrary to one another: their prop or center of motion being the pin which keeps them together.

In common practice, the longer arm of this lever greatly exceeds the weight of the ſhorter: which gains great advantage, becauſe it adds ſo much to the power.

The ſecond kind of lever. A lever of the ſecond kind has the weight between the prop and the power. In this, as well as the former, the advantage gained is as the diſtance of the power from the prop to the diſtance of the weight from the prop: for the reſpective velocities of the power and weight are in that proportion; and they will balance each other when the intenſity of the power multiplied by its diſtance from the prop is equal to the intenſity of the weight multiplied by its diſtance from the prop. Thus, if AB be a lever on which the weight W of 6 ounces hangs at the diſtance of 1 inch from the prop G, and a power P equal to the weight of 1 ounce hangs at the end B, 6 inches from the prop, by the cord

Fig. 2.

CD

CD going over the fixed pulley *E*, the power will juſt ſupport the weight: and a ſmall addition to the power will raiſe the weight, 1 inch for every 6 inches that the power deſcends.

This lever ſhews the reaſon why two men carrying a burden upon a ſtick between them, bear unequal ſhares of the burden in the inverſe proportion of their diſtances from it. For it is well known, that the nearer any of them is to the burden, the greater ſhare he bears of it: and if he goes directly under it, he bears the whole. So, if one man be at *G*, and the other at *P*, having the pole or ſtick *AB* reſting on their ſhoulders; if the burden or weight *W* be placed five times as near the man at *G*, as it is to the man at *P*, the former will bear five times as much weight as the latter. This is likewiſe applicable to the caſe of two horſes of unequal ſtrength to be ſo yoked, as that each horſe may draw a part proportionable to his ſtrength; which is done by ſo dividing the beam they pull, that the point of traction may be as much nearer to the ſtronger horſe than to the weaker, as the ſtrength of the former exceeds that of the latter.

To this kind of lever may be reduced oars, rudders of ſhips, doors turning upon hinges, cutting-knives which are fixed at the point of the blade, and the like.

If in this lever we ſuppoſe the power and weight to change places, ſo that the power may be between the weight and the prop, it will become a lever of the third kind: in which, that there may be a balance between the power and the weight, the intenſity of the power muſt exceed the intenſity of the weight, juſt as much as the diſtance of the weight from the prop ex-

The third kind of lever.

Fig. 3. ceeds the diſtances of the power from it. Thus, let E be the prop of the lever AB, and W a weight of 1 pound, placed 3 times as far from the prop, as the power P acts at F, by the cord C going over the fixed pulley D; in this caſe, the power muſt be equal to three pounds, in order to ſupport the weight.

To this ſort of lever are generally referred the bones of a man's arm: for when we lift a weight by the hand, the muſcle that exerts its force to raiſe that weight, is fixed to the bone about one tenth part as far below the elbow as the hand is. And the elbow being the center round which the lower part of the arm turns, the muſcle muſt therefore exert a force ten times as great as the weight that is raiſed.

As this kind of lever is a diſadvantage to the moving power, it is never uſed but in caſes of neceſſity; ſuch as that of a ladder, which being fixed at one end, is by the ſtrength of a man's arms reared againſt a wall. And in clock-work, where all the wheels may be reckoned levers of this kind, becauſe the power that moves every wheel, except the firſt, acts upon it near the center of motion by means of a ſmall pinion, and the reſiſtance it has to overcome, acts againſt the teeth round its circumference.

The fourth kind of lever. Fig. 4.

The fourth kind of lever differs nothing from the firſt, but in being bended for the ſake of convenience. ACB is a lever of this ſort, bended at C, which is its prop, or center of motion. P is a power acting upon the longer arm AC at F, by means of the cord DE going over the pulley G; and W is a weight or reſiſtance acting upon the end B of the ſhorter arm BC. If the power is to the weight, as CB is to CF, they are *in equilibrio*. Thus, ſuppoſe W to be 5 pounds

pounds acting at the diſtance of one foot from the center of motion *C*, and *P* to be 1 pound acting at *F*, five feet from the center *C*, the power and weight will juſt balance each other. A hammer when uſed in drawing a nail is a lever of this ſort.

2. The ſecond mechanical power is the *wheel and axle*, in which the power is applied to the circumference of the wheel, and the weight is raiſed by a rope which coils about the axle as the wheel is turned round. Here it is plain that the velocity of the power muſt be to the velocity of the weight, as the circumference of the wheel is to the circumference of the axle: and conſequently, the power and weight will balance each other, when the intenſity of the power is to the intenſity of the weight, as the circumference of the axle is to the circumference of the wheel. Let *AB* be a wheel, *CD* its axle, and ſuppoſe the circumference of the wheel to be 8 times as great as the circumference of the axle; then, a power *P* equal to 1 pound hanging by the cord *I*, which goes round the wheel, will balance a weight *W* of 8 pounds hanging by the rope *K*, which goes round the axle. And as the friction on the pivets or gudgeons of the axle is but ſmall, a ſmall addition to the power will cauſe it to deſcend, and raiſe the weight: but the weight will riſe with only an eighth part of the velocity wherewith the power deſcends, and conſequently, though no more than an eighth part of an equal ſpace, in the ſame time. If the wheel be pulled round by the handles *S*, *S*, the power will be increaſed in proportion to their length. And by this means, any weight may be raiſed as high as the operator pleaſes.

The *wheel and axle.*

Fig. 5.

To this ſort of engine belong all cranes for raiſing great weights; and in this caſe, the wheel may have cogs all round it inſtead of handles, and a ſmall lantern or trundle may be made to work in the cogs, and be turned by a winch; which will make the power of the engine to exceed the power of the man who works it, as much as the number of revolutions of the winch exceed thoſe of the axle *D*, when multiplied by the exceſs of the length of the winch above the length of the ſemidiameter of the axle, added to the ſemidiameter or half thickneſs of the rope *K*, by which the weight is drawn up.—Thus, ſuppoſe the diameter of the rope and axle taken together, to be 13 inches, and conſequently, half their diameters to be $6\frac{1}{2}$ inches; ſo that the weight *W* will hang at $6\frac{1}{2}$ inches perpendicular diſtance from below the center of the axle. Now, let us ſuppoſe the wheel *A B*, which is fixt on the axle, to have 80 cogs, and to be turned by means of a winch $6\frac{1}{2}$ inches long, fixt on the axis of a trundle of 8 ſtaves or rounds, working in the cogs of the wheel.—Here it is plain, that the winch and trundle would make 10 revolutions for one of the wheel *A B*, and its axis *D*, on which the rope *K* winds in raiſing the weight *W*; and the winch being no longer than the ſum of the ſemidiameters of the great axle and rope, the trundle could have no more power on the wheel, than a man could have by pulling it round by the edge, becauſe the winch would have no greater velocity than the edge of the wheel has, which we here ſuppoſe to be ten times as great as the velocity of the riſing weight: ſo that, in this caſe, the power gained would be as 10 to 1. But if the length of the winch be 13 inches, the power gained

gained will be as 20 to 1: if $19\frac{1}{2}$ inches (which is long enough for any man to work by) the power gained would be as 30 to 1; that is, a man could raiſe 30 times as much by ſuch an engine, as he could do by his natural ſtrength without it, becauſe the velocity of the handle of the winch would be 30 times as great as the velocity of the riſing weight; the abſolute force of any engine being in proportion of the velocity of the power to the velocity of the weight raiſed by it.—But then, juſt as much power or advantage as is gained by the engine, ſo much time is loſt in working it. In this ſort of machines it is requiſite to have a ratchet-wheel *G* on one end of the axle, with a catch *H* to fall into its teeth; which will at any time ſupport the weight, and keep it from deſcending, if the perſon who turns the handle ſhould, through inadvertency or careleſſneſs, quit his hold while the weight is raiſing. And by this means, the danger is prevented which might otherwiſe happen by the running down of the weight when left at liberty.

3. The third mechanical power or engine conſiſts either of one *moveable pulley*, or a *ſyſtem of pulleys*; ſome in a block or caſe which is fixed, and others in a block which is moveable, and riſes with the weight. For though a ſingle pulley that only turns on its axis, and moves not out of its place, may ſerve to change the direction of the power, yet it can give no mechanical advantage thereto; but is only as the beam of a balance, whoſe arms are of equal length and weight. Thus, if the equal weights *W* and *P* hang by the cord *B B* upon the pulley *A*, whoſe frame *b* is fixed to the beam *H I*, they will counterpoiſe each other, juſt in the ſame manner as if the cord were cut in the middle, and its two

The *pulley*.

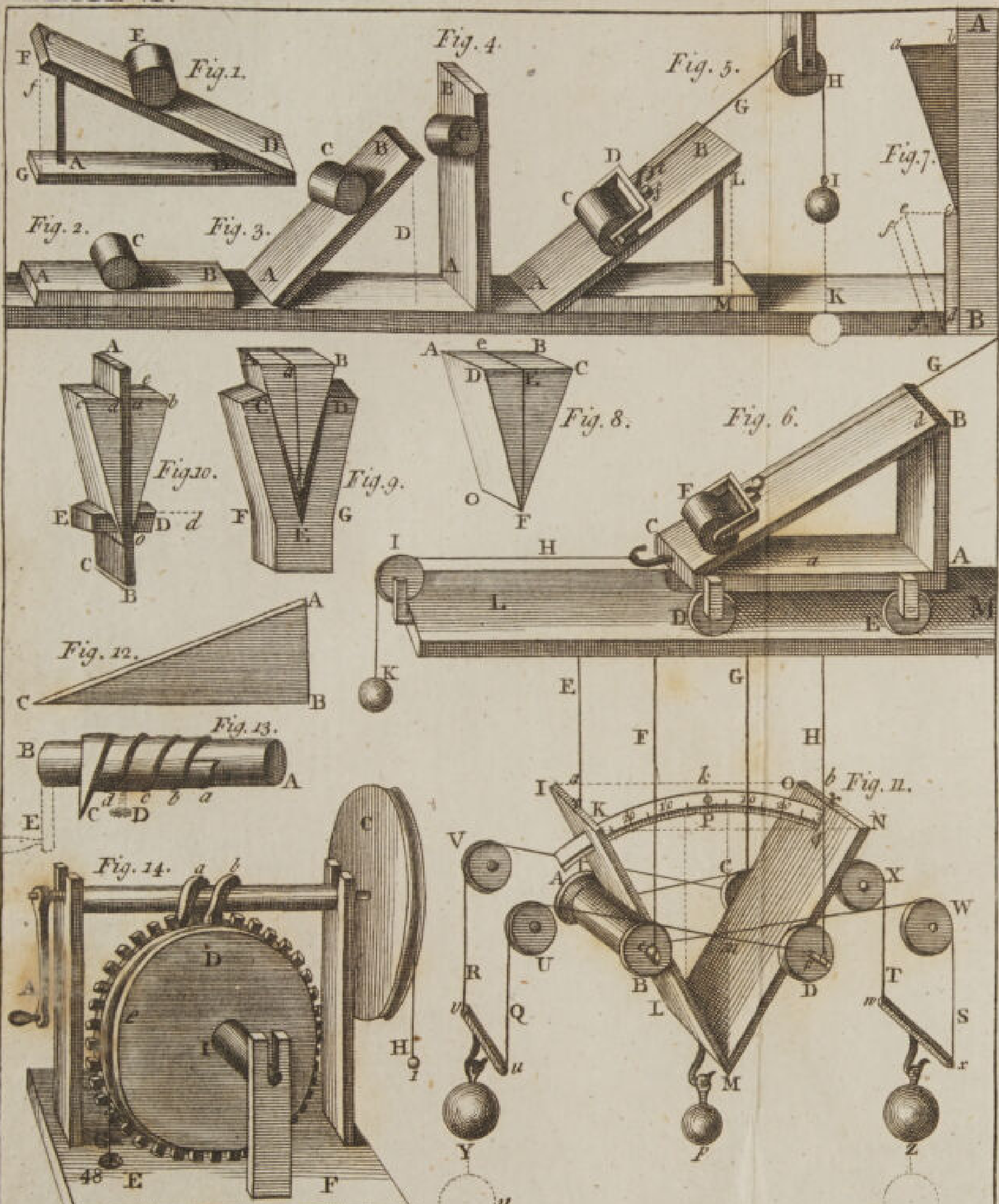
Fig. 6.

ends hung upon the hooks fixt in the pulley at *A* and *A*, equally diſtant from its center.

But if a weight *W* hangs at the lower end of the moveable block *p* of the pulley *D*, and the cord *G F* goes under that pulley, it is plain that the half *G* of the cord bears one half of the weight *W*, and the half *F* the other; for they bear the whole between them. Therefore, whatever holds the upper end of either rope, ſuſtains one half of the weight: and if the cord at *F* be drawn up ſo as to raiſe the pulley *D* to *C*, the cord will then be extended to its whole length, all but that part which goes under the pulley: and conſequently the power that draws the cord will have moved twice as far as the pulley *D* with its weight *W* riſes; on which account, a power whoſe intenſity is equal to one half of the weight will be able to ſupport it, becauſe if the power moves (by means of a ſmall addition) its velocity will be double the velocity of the weight; as may be ſeen by putting the cord over the fixt pulley *C* (which only changes the direction of the power, without giving any advantage to it) and hanging on the weight *P*, which is equal only to one half the weight *W*; in which caſe there will be an equilibrium, and a little addition to *P* will cauſe it to deſcend, and raiſe *W* through a ſpace equal to one half of that through which *P* deſcends.—Hence, the advantage gained will be always equal to twice the number of pulleys in the moveable or undermoſt block. So that, when the upper or fixt block *u* contains two pulleys, which only turn on their axis, and the lower or moveable block *U* contains two pulleys, which not only turn upon their axis, but alſo riſe with the block and weight; the advantage gained by this is as 4 to the

working

PLATE VI.



J. Ferguson delin. J. Mynde sc.

working power. Thus, if one end of the rope *KMOQ* be fixed to a hook at *I*, and the rope paſſes over the pulleys *N* and *R*, and under the pulleys *L* and *P*, and has a weight *T*, of one pound, hung to its other end at *T*, this weight will balance and ſupport a weight *W* of four pounds hanging by a hook at the moveable block *U*, allowing the ſaid block as a part of the weight. And if as much more power be added, as is ſufficient to overcome the friction of the pulleys, the power will deſcend with four times as much velocity as the weight riſes, and conſequently through four times as much ſpace.

The two pulleys in the fixed block *X*, and the two in the moveable block *Y*, are in the ſame caſe with thoſe laſt mentioned; and thoſe in the lower block give the ſame advantage to the power.

As a ſyſtem of pulleys has no great weight, and lies in a ſmall compaſs, it is eaſily carried about; and can be applied, in a great many caſes, for raiſing weights, where other engines cannot. But they have a great deal of friction on three accounts: 1. Becauſe the diameters of their axes bear a very conſiderable proportion to their own diameters; 2. Becauſe in working they are apt to rub againſt one another, or againſt the ſides of the block; 3. Becauſe of the ſtiffneſs of the rope that goes over and under them.

4. The fourth mechanical power is the *inclined plane*, and the advantage gained by it is as great as its length exceeds its perpendicular height. Let *AB* be a plane parallel to the horizon; and *CD* a plane inclined to it; and ſuppoſe the whole length *CD* to be three times as great as the perpendicular height *GfF*: in this caſe, the cylinder *E* will be ſupported upon the plane *CD*,

The *inclined plane.*

Plat. VI. Fig. 1.

CD, and kept from rolling down upon it, by a power equal to a third part of the weight of the cylinder. Therefore, a weight may be rolled up this inclined plane with a third part of the power which would be ſufficient to draw it up by the ſide of an upright wall. If the plane was four times as long as high, a fourth part of the power would be ſufficient; and ſo on, in proportion. Or, if a weight was to be raiſed from a floor to the height *GF*, by means of the machine *ABCD*, (which would then act as a half wedge, where the reſiſtance gives way only on one ſide) the machine and weight would be *in equilibrio* when the power applied at *GF* was to the weight to be raiſed, as *GF* to *GB*; and if the power be increaſed, ſo as to overcome the friction of the machine againſt the floor and weight, the machine will be driven, and the weight raiſed: and when the machine has moved its whole length upon the floor, the weight will be raiſed to the whole height from *G* to *F*.

The force wherewith a rolling body deſcends upon an inclined plane, is to the force of its abſolute gravity, by which it would deſcend perpendicularly in a free ſpace, as the height of the plane is to its length. For, ſuppoſe the plane
Fig. 2. *AB* to be parallel to the horizon, the cylinder *C* will keep at reſt upon any part of the plane where it is laid. If the plane be ſo elevated,
Fig. 3. that its perpendicular height *D* is equal to half its length *AB*, the cylinder will roll down upon the plane with a force equal to half its weight; for it would require a power (acting in the direction of *AB*) equal to half its weight, to keep
Fig. 4. it from rolling. If the plane *AB* be elevated, ſo as to be perpendicular to the horizon, the cylinder *C* will deſcend with its whole force of gravity,

gravity, becauſe the plane contributes nothing to its ſupport or hindrance; and therefore, it would require a power equal to its whole weight to keep it from deſcending.

Let the cylinder *C* be made to turn upon ſlender pivots in the frame *D*, in which there is a hook *e*, with a line *G* tied to it: let this line go over the fixed pulley *H*, and have its other end tied to the hook in the weight *I*. If the weight of the body *I*, be to the weight of the cylinder *C*, added to that of its frame *D*, as the perpendicular height of the plane *LM* is to its length *AB*, the weight will juſt ſupport the cylinder upon the plane, and a ſmall touch of a finger will either cauſe it to aſcend or deſcend with equal eaſe: then, if a little addition be made to the weight *I*, it will deſcend, and draw the cylinder up the plane. In the time that the cylinder moves from *A* to *B*, it will riſe through the whole height of the plane *ML*; and the weight will deſcend from *H* to *K*, through a ſpace equal to the whole length of the plane *AB*. Fig. 5.

If the machine be made to move upon rollers or friction-wheels, and the cylinder be ſupported upon the plane *CB* by a line *G* parallel to the plane, a power ſomewhat leſs than that which drew the cylinder up the plane will draw the plane under the cylinder, provided the pivots of the axes of the friction-wheels be ſmall, and the wheels themſelves be pretty large. For, let the machine *ABC* (equal in length and height to *ABM*, Fig. 5.) move upon four wheels, two whereof appear at *D* and *E*; and the third under *C*, while the fourth is hid from ſight by the horizontal board *a*. Let the cylinder *F* be laid upon the lower end of the inclined plane *CB*, and the line *G* be extended from the frame of the cylinder, about ſix feet parallel to the Fig. 6.

plane

plane CB; and, in that direction, fixed to a hook in the wall; which will fupport the cylinder, and keep it from rolling off the plane. Let one end of the line H be tied to a hook at C in the machine, and the other end to a weight K, fomewhat lefs than that which drew the cylinder up the plane before. If this line be put over the fixed pulley I, the weight K will draw the machine along the horizontal plane L, and under the cylinder F: and when the machine has been drawn a little more than the whole length CA, the cylinder will be raifed to d, equal to the perpendicular height AB above the horizontal part at A. The reafon why the machine muft be drawn further than the whole length CA is, becaufe the weight F rifes perpendicular to CB.

To the inclined plane may be reduced all hatchets, chifels, and other edge-tools which are chamfered only on one fide.

The *wedge.* 5. The fifth mechanical power or machine is the *wedge*, which may be confidered as two equally inclined planes DEF and CEF, joined

Fig. 8. together at their bafes $eEFO$: then DC is the whole thicknefs of the wedge at its back $ABCD$, where the power is applied: EF is the depth or heighth of the wedge: DF the length of one of its fides, equal to CF the length of the other fide; and OF is its fharp edge, which is entered into the wood intended to be fplit by the force of a hammer or mallet ftriking perpendicularly on its back. Thus ABb is a wedge driven into

Fig. 9. the cleft CDE of the wood FG.

When the wood does not cleave at any diftance before the wedge, there will be an equilibrium between the power impelling the wedge downward, and the refiftance of the wood acting againft the two fides of the wedge when the power is to the refiftance, as half the thicknefs of of the wedge at its back is to the length of either of its ſides; becauſe the reſiſtance then acts perpendicular to the ſides of the wedge. But, when the reſiſtance on each ſide acts parallel to the back, the power that balances the reſiſtances on both ſides will be as the length of the whole back of the wedge is to double its perpendicular height.

When the wood cleaves at any diſtance before the wedge (as it generally does) the power impelling the wedge will not be to the reſiſtance of the wood, as the length of the back of the wedge is to the length of both its ſides; but as half the length of the back is to the length of either ſide of the cleft, eſtimated from the top or acting part of the wedge. For, if we ſuppoſe the wedge to be lengthened down from *b* to the bottom of the cleft at *E*, the ſame proportion will hold; namely, that the power will be to the reſiſtance, as half the length of the back of the wedge is to the length of either of its ſides: or, which amounts to the ſame thing, as the whole length of the back is to the length of both the ſides.

In order to prove what is here advanced concerning the wedge, let us ſuppoſe the wedge to be divided lengthwiſe into two equal parts; and then it will become two equal inclined planes; one of which, as *a b c*, may be made uſe of as a half wedge for ſeparating the moulding *c d* from the wainſcot *A B*. It is evident, that when this half wedge has been driven its whole length *a c* between the wainſcot and moulding, its ſide *a c* will be at *e d*; and the moulding will be ſeparated to *f g* from the wainſcot. Now, from what has been already proved of the inclined plane, it appears, that to have an equilibrium between the power impelling the half wedge, and the reſiſtance of the moulding, the former muſt be to the latter, Fig. 7.

latter, as $a\,b$ to $a\,c$; that is, as the thickneſs of the back which receives the ſtroke is to the length of the ſide againſt which the moulding acts. Therefore, ſince the power upon the half wedge is to the reſiſtance againſt its ſide, as the half back $a\,b$ is to the whole ſide $a\,c$, it is plain, that the power upon which the whole wedge (where the whole back is double the half back) muſt be to the reſiſtance againſt both its ſides, as the thickneſs of the whole back is to the length of both the ſides; ſuppoſing the wedge at the bottom of the cleft: or as the thickneſs of the whole back to the length of both ſides of the cleft, when the wood ſplits at any diſtance before the wedge. For, when the wedge is driven quite into the wood, and the wood ſplits at ever ſo ſmall a diſtance before its edge, the top of the wedge then becomes the acting part, becauſe the wood does not touch it any where elſe. And ſince the bottom of the cleft muſt be conſidered as that part where the whole ſtickage or reſiſtance is accumulated, it is plain, from the nature of the lever, that the farther the power acts from the reſiſtance, the greater is the advantage.

Some writers have advanced, that the power of the wedge is to the reſiſtance to be overcome, as the thickneſs of the back of the wedge is to the length only of one of its ſides; which ſeems very ſtrange: for, if we ſuppoſe $A\,B$ to be a ſtrong inflexible bar of wood or iron fixt into the
Fig. 10. ground at $C\,B$, and D and E to be two blocks of marble lying on the ground on oppoſite ſides of the bar; it is evident that the block D may be ſeparated from the bar to the diſtance d, equal to $a\,b$, by driving the inclined plane or half wedge $a\,b\,o$ down between them; and the block E may be ſeparated to an equal diſtance on the other ſide, in like manner, by the half wedge $c\,d\,o$.

But the power impelling each half wedge will be to the reſiſtance of the block againſt its ſide, as the thickneſs of that half wedge is to its perpendicular height, becauſe the block will be driven off perpendicular to the ſide of the bar *A B*. Therefore the power to drive both the half wedges is to both the reſiſtances, as both the half backs is to the perpendicular height of each half wedge. And if the bar be taken away, the blocks put cloſe together, and the two half wedges joined to make one; it will require as much force to drive it down between the blocks, as is equal to the ſum of the ſeparate powers acting upon the half wedges when the bar was between them.

To confirm this by an experiment, let two cylinders, as *A B* and *C D*, be drawn toward one another by lines running over fixed pulleys, and a weight of 40 ounces hanging at the lines belonging to each cylinder: and let a wedge of 40 ounces weight, having its back juſt as thick as either of its ſides is long, be put between the cylinders, which will then act againſt each ſide with a reſiſtance equal to 40 ounces, while its own weight endeavours to bring it down and ſeparate them. And here, the power of the wedge's gravity impelling it downward, will be to the reſiſtance of both the cylinders againſt the wedge, as the thickneſs of the wedge is to double its perpendicular height; for there will then be an equilibrium between the weight of the wedge and the reſiſtance of the cylinders againſt it, and it will remain at any height between them; requiring juſt as much power to puſh it upward as to pull it downward.—If another wedge of equal weight and depth with this, and only half as thick, be put between the cylinders, it will require twice as much weight to be hung at the Fig. 11.

ends

ends of the lines which draw them together, to keep the wedge from going down between them. That is, a wedge of 40 ounces, whose back is only equal to half its perpendicular height, will require 80 ounces to each cylinder, to keep it in an equilibrium between them: and twice 80 is 160, equal to four times 40. So that the power will be always to the refiftance, as the thicknefs of the back of the wedge is to twice its perpendicular height, when the cylinders move off in a line at right angles to that perpendicular.

The beft way, though perhaps not the neateft, that I know of, for making a wedge with its appurtenances for fuch experiments, is as follows. Let *KILM* and *LMNO* be two flat pieces of wood, each about fifteen inches long and three or four in breadth, joined together by a hinge at *LM*; and let *P* be a graduated arch of brafs, on which the faid pieces of wood may be opened to any angle not more than 60 degrees, and then fixt at the given angle by means of the two fcrews *a* and *b*. Then, *IKNO* will reprefent the back of the wedge, *LM* its fharp edge which enters the wood, and the outfides of the pieces *KILM* and *LMNO* the two fides of the wedge againft which the wood acts in cleaving. By means of the faid arch, the wedge may be opened fo, as to adjuft the thicknefs of its back in any proportion to the length of either of its fides, but not to exceed that length: and any weight as *p* may be hung to the wedge upon the hook *M*, which weight, together with the weight of the wedge itfelf, may be confidered as the impelling power; which is all the fame in the experiment, whether it be laid upon the back of the wedge to pufh it down, or hung to its edge to pull it down.—Let *AB* and *CD* be two wooden cylinders, each about two inches thick, where

Fig. 11.

they

they touch the outsides of the wedge; and let their ends be made like two round flat plates, to keep the wedge from slipping off edgewise from between them. Let a small cord with a loop on one end of it, go over a pivot in the end of each cylinder, and the cords *S* and *T* belonging to the cylinder *AB* go over the fixt pulleys *W* and *X*, and be fastened at their other ends to the bar *w x*, on which any weight as *Z* may be hung at pleasure. In like manner, let the cords *Q* and *R* belonging to the cylinder *CD* go over the fixt pulleys *V* and *U* to the bar *v u*, on which a weight *Y* equal to *Z* may be hung. These weights, by drawing the cylinders toward one another, may be considered as the resistance of the wood acting equally against opposite sides of the wedge; the cylinders themselves being suspended near, and parallel to each other, by their pivots in loops on the lines *E*, *F*, *G*, *H*; which lines may be fixed to hooks in the cieling of the room. The longer these lines are, the better; and they should never be less than four feet each. The farther also the pulleys *V*, *U* and *X*, *W* are from the cylinders, the truer will the experiments be: and they may turn upon pins fixed into the wall.

In this machine, the weights *Y* and *Z*, and the weight *p*, may be varied at pleasure, so as to be adjusted in proportion of double the wedge's perpendicular height to the thickness of its back: and when they are so adjusted, the wedge will be *in equilibrio* with the resistance of the cylinders.

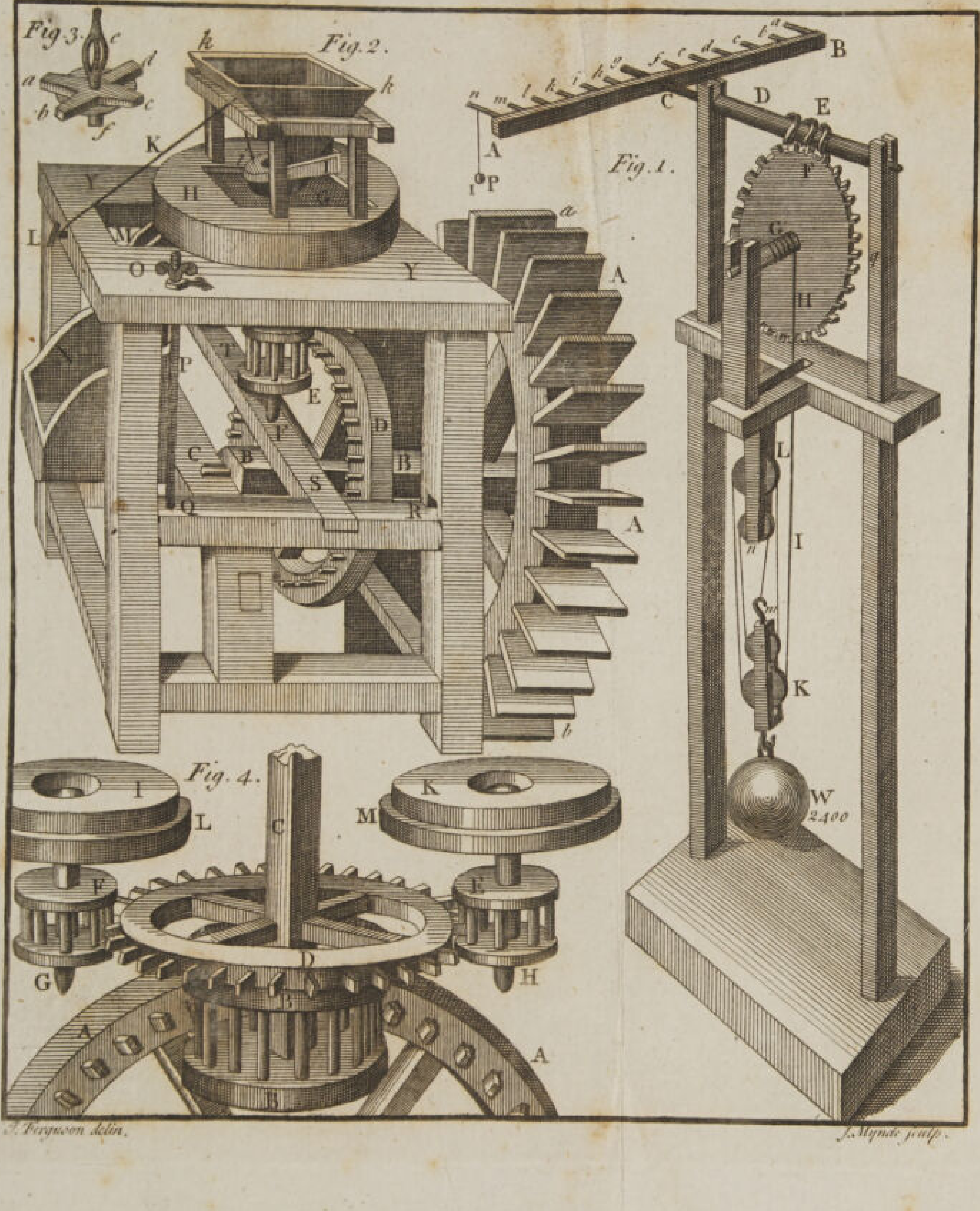
The wedge is a very great mechanical power, since not only wood but even rocks can be split by it; which would be impossible to effect by the lever, wheel and axle, or pulley: for the force of the blow, or stroke, shakes the cohering parts, and thereby makes them separate more easily.

The *screw.* 6. The sixth and last mechanical power is the *screw*; which cannot properly be called a simple machine, because it is never used without the application of a lever or winch to assist in turning it: and then it becomes a compound engine of a very great force either in pressing the parts of bodies closer together, or in raising great weights. It may be conceived to be made by cutting a piece of paper *ABC* (Fig. 12.) into the form of an inclined plane or half wedge, and then wrapping it round a cylinder *AB* (Fig. 13). And here it is evident, that the winch *E* must turn the cylinder once round before the weight of resistance *D* can be moved from one spiral winding to another, as from *d* to *c*: therefore, as much as the circumference of a circle, described by the handle of the winch, is greater than the interval or distance between the spirals, so much is the force of the screw. Thus, supposing the distance between the spirals to be half an inch, and the length of the winch to be twelve inches; the circle described by the handle of the winch where the power acts will be 76 inches nearly, or about 152 half inches, and consequently 152 times as great as the distance between the spirals: and therefore a power at the handle, whose intensity is equal to no more than a single pound, will balance 152 pounds acting against the screw; and as much additional force, as is sufficient to overcome the friction, will raise the 152 pounds; and the velocity of the power will be to the velocity of the weight, as 152 to 1. Hence it appears, that the longer the winch is, and the nearer the spirals are to one another, so much the greater is the force of the screw.

Fig. 12, 13.

A machine for shewing the force or power of the screw may be contrived in the following manner:

PLATE VII.



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manner: Let the wheel *C* have a ſcrew *a b* on its axle, working in the teeth of the wheel *D*, which ſuppoſe to be 48 in number. It is plain, that for every time the wheel *C* and ſcrew *a b* are turned round by the winch *A*, the wheel *D* will be moved one tooth by the ſcrew; and therefore, in 48 revolutions of the winch, the wheel *D* will be turned once round. Then, if the circumference of a circle deſcribed by the handle of the winch *A* be equal to the circumference of a groove *e* round the wheel *D*, the velocity of the handle will be 48 times as great as the velocity of any given point in the groove. Conſequently, if a line *G* (above number 48) goes round the groove *e*, and has a weight of 48 pounds hung to it below the pedeſtal *E F*, a power equal to one pound at the handle will balance and ſupport the weight.—To prove this by experiment, let the circumferences of the grooves of the wheels *C* and *D* be equal to one another; and then if a weight *H* of one pound be ſuſpended by a line going round the groove of the wheel *C*, it will balance a weight of 48 pounds hanging by the line *G*; and a ſmall addition to the weight *H* will cauſe it to deſcend, and ſo raiſe up the other weight. Fig. 14.

If the line *G*, inſtead of going round the groove *e* of the wheel *D*, goes round its axle *I*; the power of the machine will be as much increaſed, as the circumference of the groove *e* exceeds the circumference of the axle: which, ſuppoſing it to be ſix times, then one pound at *H* will balance 6 times 48, or 288 pounds hung to the line on the axle: and hence the power or advantage of this machine will be as 288 to 1. That is to ſay, a man, who by his natural ſtrength could lift a hundred weight, will be able to raiſe 288 hundred, or $14\frac{8}{20}$ ton weight by this engine.

But the following engine is ſtill more powerful, on account of its having the addition of four pulleys: and in it we may look upon all the mechanical powers as combined together, even if we take in the balance. For, as the axle *D* of the bar *A B* enters its middle at *C*, it is plain that if equal weights are ſuſpended upon any two pins equi-diſtant from the axis *C*, they will counterpoiſe each other.—It becomes a lever by hanging a ſmall weight *P* upon the pin *n*, and a weight as much heavier upon either of the pins *b*, *c*, *d*, *e*, or *f*, as is in proportion to the pins being ſo much nearer the axis. The wheel and axle *F G* is evident; ſo is the ſcrew *E* which takes in the inclined plane, and with it the half wedge. Part of a cord goes round the axle, the reſt under the lower pulleys *K*, *m*, over the upper pulleys *L*, *n*, and then it is tied to a hook at *m* in the lower or moveable block, on which the weight *W* hangs.

Plate VII. Fig. 1.

A combination of all the mechanical powers.

In this machine, if the wheel *F* has 30 teeth, it will be turned once round in thirty revolutions of the bar *A B*, which is fixt on the axis *D* of the ſcrew *E*: if the length of the bar is equal to twice the diameter of the wheel, the pins *a* and *n* at the ends of the bar will move 60 times as faſt as the teeth of the wheel do: and conſequently, one ounce at *P* will balance 60 ounces hung upon a tooth at *q* in the horizontal diameter of the wheel. Then, if the diameter of the wheel *F* is ten times as great as the diameter of the axle *G*, the wheel will have 10 times the velocity of the axle; and therefore one ounce *P* at the end of the lever *A C* will balance 10 times 60 or 600 ounces hung to the rope *H* which goes round

round the axle. Laſtly, if four pulleys be added, they will make the velocity of the lower block *K*, and weight *W*, four times leſs than the velocity of the axle: and this being the laſt power in the machine, which is four times as great as that gained by the axle, it makes the whole power of the machine 4 times 600, or 2400. So that a man who could lift one hundred weight in his arms by his natural ſtrength, would be able to raiſe 2400 times as much by this engine.—But it is here as in all other mechanical caſes; for the time loſt is always as much as the power gained, becauſe the velocity with which the power moves will ever exceed the velocity with which the weight riſes, as much as the intenſity of the weight exceeds the intenſity of the power.

The friction of the ſcrew itſelf is very conſiderable; and there are few compound engines, but what, upon account of the friction of the parts againſt one another, will require a third part more of power to work them when loaded, than what is ſufficient to conſtitute a balance between the weight and the power.

LECT. IV.

Of mills, cranes, wheel-carriages, and the engine for driving piles.

As theſe engines are ſo univerſally uſeful, it would be needleſs to make any apology for deſcribing them.

In a common *breaſt-mill*, where the fall of water may be about ten feet, *AA* is the great wheel, which is generally about 17 or 18 feet in diameter, Plate VII. Fig. 2. *A common mill.*

diameter, reckoned from the outermoſt edge of any float board at *a* to that of its oppoſite float at *b*. To this wheel the water is conveyed through a channel, and by falling upon the wheel, turns it round.

On the axis *B B* of this wheel, and within the mill-houſe, is a wheel *D*, about 8 or 9 feet diameter, having 61 cogs, which turn a trundle *E* containing ten upright ſtaves or rounds; and when theſe are the number of cogs and rounds, the trundle will make $6\frac{1}{10}$ revolutions for one revolution of the wheel.

The trundle is fixt upon a ſtrong iron axis called the ſpindle, the lower end of which turns in a braſs foot, fixt at *F*, in the horizontal beam *S T* called the bridge-tree; and the upper part of the ſpindle turns in a wooden buſh fixt into the nether millſtone which lies upon beams in the floor *Y Y*. The top part of the ſpindle above the buſh is ſquare, and goes into a ſquare hole in a ſtrong iron croſs *a b c d* (ſee Fig. 3.) called the rynd; under which, and cloſe to the buſh, is a round piece of thick leather upon the ſpindle, which it turns round at the ſame time as it does the rynd.

The rynd is let into grooves in the under ſurface of the running millſtone *G* (Fig. 2.) and ſo turns it round in the ſame time that the trundle *E* is turned round by the cog-wheel *D*. This millſtone has a large hole quite through its middle, called the eye of the ſtone, through which the middle part of the rynd and upper end of the ſpindle may be ſeen; while the four ends of the rynd lie hid below the ſtone in their grooves.

The end *T* of the bridge-tree *T S* (which ſupports the upper millſtone *G* upon the ſpindle) is fixed into a hole in the wall; and the end *S* is let into a beam *Q R* called the brayer, whoſe end *R* remains

remains fixt in a mortiſe: and its other end *Q* hangs by a ſtrong iron-rod *P* which goes through the floor *YY*, and has a ſcrew-nut on its top at *O*; by the turning of which nut, the end *Q* of the brayer is raiſed or depreſſed at pleaſure; and conſequently the bridge-tree *TS* and upper mill-ſtone. By this means, the upper millſtone may be ſet as cloſe to the under one, or raiſed as high from it, as the miller pleaſes. The nearer the millſtones are to one another, the finer they grind the corn, and the more remote from one another, the coarſer.

The upper millſtone *G* is incloſed in a round box *H*, which does not touch it any where; and is about an inch diſtant from its edge all around. On the top of this box ſtands a frame for holding the hopper *k k*, to which is hung the ſhoe *I* by two lines faſtened to the hind-part of it, fixed upon hooks in the hopper, and by one end of the crook-ſtring *K* faſtened to the fore-part of it at *i*; the other end being twiſted round the pin *L*. As the pin is turned one way, the ſtring draws up the ſhoe cloſer to the hopper, and ſo leſſens the aperture between them; and as the pin is turned the other way, it lets down the ſhoe, and enlarges the aperture.

If the ſhoe be drawn up quite to the hopper, no corn can fall from the hopper into the mill; if it be let a little down, ſome will fall: and the quantity will be more or leſs, according as the ſhoe is more or leſs let down. For the hopper is open at bottom, and there is a hole in the bottom of the ſhoe, not directly under the bottom of the hopper, but forwarder toward the end *i*, over the middle of the eye of the millſtone.

There is a ſquare hole in the top of the ſpindle, Fig. 3. in which is put the feeder *e*: this feeder (as the ſpindle turns round) jogs the ſhoe three times in each revolution, and ſo cauſes the corn to run conſtantly down from the hopper through the ſhoe, into the eye of the millſtone, where it falls upon the top of the rynd, and is, by the motion of the rynd, and the leather under it, thrown below the upper ſtone, and ground between it and the lower one. The violent motion of the ſtone creates a centrifugal force in the corn going round with it, by which means it gets farther and farther from the center, as in a ſpiral, in every revolution, until it be thrown quite out; and, being then ground, it falls through a ſpout *M*, called the mill-eye, into the trough *N*.

When the mill is fed too faſt, the corn bears up the ſtone, and is ground too coarſe; and beſides, it clogs the mill ſo as to make it go too ſlow. When the mill is too ſlowly fed, it goes too faſt, and the ſtones by their attrition are apt to ſtrike fire againſt one another. Both which inconveniences are avoided by turning the pin *L* backward or forward, which draws up or lets down the ſhoe; and ſo regulates the feeding as the miller ſees convenient.

The heavier the running millſtone is, and the greater the quantity of water that falls upon the wheel, ſo much the faſter will the mill bear to be fed; and conſequently ſo much the more it will grind. And on the contrary, the lighter the ſtone, and the leſs the quantity of water, ſo much ſlower muſt the feeding be. But when the ſtone is conſiderably wore, and become light, the mill muſt be fed ſlowly at any rate; otherwiſe the ſtone will be too much borne up by the corn under it, which will make the meal coarſe.

The quantity of power required to turn a heavy millſtone is but very little more than what is ſufficient to turn a light one: for as it is ſupported upon the ſpindle by the bridge-tree *S T*, and the end of the ſpindle that turns in the braſs foot therein being but ſmall, the odds ariſing from the weight is but very inconſiderable in its action againſt the power or force of the water. And beſides, a heavy ſtone has the ſame advantage as a heavy fly; namely, that it regulates the motion much better than a light one.

In order to cut and grind the corn, both the upper and under millſtones have channels or furrows cut into them, proceeding obliquely from the center toward the circumference. And theſe furrows are cut perpendicularly on one ſide and obliquely on the other into the ſtone, which gives each furrow a ſharp edge, and in the two ſtones they come, as it were, againſt one another like the edges of a pair of ſciſſars: and ſo cut the corn, to make it grind the eaſier when it falls upon the places between the furrows. Theſe are cut the ſame way in both ſtones when they lie upon their backs, which makes them run croſs ways to each other when the upper ſtone is inverted by turning its furrowed ſurface toward that of the lower. For, if the furrows of both ſtones lay the ſame way, a great deal of the corn would be driven onward in the lower furrows, and ſo come out from between the ſtones without being either cut or bruiſed.

When the furrows become blunt and ſhallow by wearing, the running ſtone muſt be taken up, and both ſtones new dreſt with a chiſel and hammer. And every time the ſtone is taken up, there muſt be ſome tallow put round the ſpindle upon the buſh, which will ſoon be melted by the heat the ſpindle acquires from its turning and rubbing againſt the buſh, and ſo will get in

between

between them: otherwiſe the buſh would take fire in a very little time.

The buſh muſt embrace the ſpindle quite cloſe, to prevent any ſhake in the motion, which would make ſome parts of the ſtones grate and fire againſt each other; while other parts of them would be too far aſunder, and by that means ſpoil the meal in grinding.

Whenever the ſpindle wears the buſh ſo as to begin to ſhake in it, the ſtone muſt be taken up, and a chiſel drove into ſeveral parts of the buſh; and when it is taken out, wooden wedges muſt be driven into the holes; by which means the buſh will be made to embrace the ſpindle cloſe all around it again. In doing this, great care muſt be taken to drive equal wedges into the buſh on oppoſite ſides of the ſpindle; otherwiſe it will be thrown out of the perpendicular, and ſo hinder the upper ſtone from being ſet parallel to the under one, which is abſolutely neceſſary for making good work. When any accident of this kind happens, the perpendicular poſition of the ſpindle muſt be reſtored by adjuſting the bridge-tree *S T* by proper wedges put between it and the brayer *Q R*.

It often happens, that the rynd is a little wrenched in laying down the upper ſtone upon it; or is made to ſink a little lower upon one ſide of the ſpindle than on the other; and this will cauſe one edge of the upper ſtone to drag all around upon the other, while the oppoſite edge will not touch. But this is eaſily ſet to rights, by raiſing the ſtone a little with a lever, and putting bits of paper, cards, or thin chips, between the rynd and the ſtone.

The diameter of the upper ſtone is generally about ſix feet, the lower ſtone about an inch more:

more: and the upper ſtone when new contains about $22\frac{1}{2}$ cubic feet, which weighs ſomewhat more than 19000 pounds. A ſtone of this diameter ought never to go more than 60 times round in a minute; for if it turns faſter, it will heat the meal.

The grinding ſurface of the under ſtone is a little convex from the edge to the center, and that of the upper ſtone a little more concave: ſo that they are fartheſt from one another in the middle, and come gradually nearer toward the edges. By this means, the corn at its firſt entrance between the ſtones is only bruiſed; but as it goes farther on toward the circumference or edge, it is cut ſmaller and ſmaller; and at laſt finely ground juſt before it comes out from between them.

The water-wheel muſt not be too large, for if it be, its motion will be too ſlow; nor too little, for then it will want power. And for a mill to be in perfection, the floats of the wheel ought to move with a third part of the velocity of the water, and the ſtone to turn round once in a ſecond of time.

In order to conſtruct a mill in this perfect manner, obſerve the following rules:

1. Meaſure the perpendicular height of the fall of water, in feet, above that part of the wheel on which the water begins to act; and call that, the height of the fall.

2. Multiply this conſtant number 64.2882 by the height of the fall in feet, and the ſquare root of the product ſhall be the velocity of the water at the bottom of the fall, or the number of feet that the water there moves *per* ſecond.

3. Divide the velocity of the water by 3, and the quotient ſhall be the velocity of the float-boards of the wheel; or the number of feet they

muſt

muſt each go through in a ſecond, when the water acts upon them ſo, as to have the greateſt power to turn the mill.

4. Divide the circumference of the wheel in feet by the velocity of its floats in feet *per* ſecond, and the quotient ſhall be the number of ſeconds in which the wheel turns round.

5. By this laſt number of ſeconds divide 60; and the quotient ſhall be the number of turns of the wheel in a minute.

6. Divide 60 (the number of revolutions the millſtone ought to have in a minute) by the number of turns of the wheel in a minute, and the quotient ſhall be the number of turns the millſtone ought to have for one turn of the wheel.

7. Then, as the number of turns of the wheel in a minute is to the number of turns of the millſtone in a minute, ſo muſt the number of ſtaves in the trundle be to the number of cogs in the wheel, in the neareſt whole numbers that can be found.

By theſe rules I have calculated the following table to a water-wheel 18 feet diameter, which I apprehend may be a good ſize in general.

To conſtruct a mill by this table, find the height of the fall of water in the firſt column, and againſt that height, in the ſixth column, you have the number of cogs in the wheel, and ſtaves in the trundle, for cauſing the millſtone to make about 60 revolutions in a minute, as near as poſſible, when the wheel goes with a third part of the velocity of the water. And it appears by the 7th column, that the number of cogs in the wheel, and ſtaves in the trundle, are ſo near the truth for the required purpoſe, that the leaſt number of revolutions of the millſtone in a minute is between 59 and 60, and the greateſt number never amounts to 61.

The

The MILL-WRIGHT's TABLE.

Height of the fall of water.	Velocity of the water *per* second.	Velocity of the wheel *per* second.	Revolutions of the wheel *per* minute.	Revolutions of the mill-stone for one of the wheel.	Cogs in the wheel and staves in the trundle.		Rev. of the mill-stone *per* min. by these staves and cogs.
Feet.	Feet. 100 parts of a foot.	Feet. 100 parts of a foot.	Rev. 100 parts of a Rev.	Rev. 100 parts of a Rev.	Cogs.	Staves.	Rev. 100 parts of a Rev.
1	8 .02	2 .67	2 .83	21 .20	127	6	59 .92
2	11 .34	3 .78	4 .00	15 .00	105	7	60 .00
3	13 .89	4 .63	4 .91	12 .22	98	8	60 .14
4	16 .04	5 .35	5 .67	10 .58	95	9	59 .87
5	17 .93	5 .98	6 .34	9 .46	85	9	59 .84
6	19 .64	6 .55	6 .94	8 .64	78	9	60 .10
7	21 .21	7 .07	7 .50	8 .00	72	9	60 .00
8	22 .68	7 .56	8 .02	7 .48	67	9	59 .67
9	24 .05	8 .02	8 .51	7 .05	70	10	59 .57
10	25 .35	8 .45	8 .97	6 .69	67	10	60 .09
11	26 .59	8 .86	9 .40	6 .38	64	10	60 .16
12	27 .77	9 .26	9 .82	6 .11	61	10	59 .90
13	28 .91	9 .64	10 .22	5 .87	59	10	60 .18
14	30 .00	10 .00	10 .60	5 .66	56	10	59 .36
15	31 .05	10 .35	10 .99	5 .46	55	10	60 .48
16	32 .07	10 .69	11 .34	5 .29	53	10	60 .10
17	33 .06	11 .02	11 .70	5 .13	51	10	59 .67
18	34 .02	11 .34	12 .02	4 .99	50	10	60 .10
19	34 .95	11 .65	12 .37	4 .85	49	10	60 .61
20	35 .86	11 .95	12 .68	4 .73	47	10	59 .59
1	2	3	4	5	6		7

Such

Such a mill as this, with a fall of water about $7\frac{1}{2}$ feet, will require about 32 hogſheads every minute to turn the wheel with a third part of the velocity with which the water falls; and to overcome the reſiſtance ariſing from the friction of the geers and attrition of the ſtones in grinding the corn.

The greater fall the water has, the leſs quantity of it will ſerve to turn the mill. The water is kept up in the mill-dam, and let out by a ſluice called the penſtock, when the mill is to go. When the penſtock is drawn up by means of a lever, it opens a paſſage through which the water flows to the wheel: and when the mill is to be ſtopt, the penſtock is let down, which ſtops the water from falling upon the wheel.

A leſs quantity of water will turn an overſhot-mill (where the wheel has buckets inſtead of float-boards) than a breaſt-mill, where the fall of the water ſeldom exceeds half the height *A b* of the wheel. So that, where there is but a ſmall quantity of water, and a fall great enough for the wheel to lie under it, the bucket (or overſhot) wheel is always uſed. But where there is a large body of water, with a little fall, the breaſt or float-board wheel muſt take place. Where the water runs only upon a little declivity, it can act but ſlowly upon the under part of the wheel at *b*; in which caſe, the motion of the wheel will be very ſlow: and therefore, the floats ought to be very long, though not high, that a large body of water may act upon them; ſo that what is wanting in velocity may be made up in power; and then the cog-wheel may have a greater number of cogs in proportion to the rounds in the trundle, in order to give the millſtone a ſufficient degree of velocity.

They who have read what is ſaid in the firſt lecture, concerning the acceleration of bodies falling

falling freely by the power of gravity acting constantly and uniformly upon them, may perhaps ask, Why should the motion of the wheel be equable, and not accelerated, seeing the water acts constantly and uniformly upon it? The plain answer is, That the velocity of the wheel can never be so great as the velocity of the water that turns it; for, if it should become so great, the power of the water would be quite lost upon the wheel, and then there would be no proper force to overcome the friction of the geers and attrition of the stones. Therefore, the velocity with which the wheel begins to move, will increase no longer than till its *momentum* or force is balanced by the resistance of the working parts of the mill; and then the wheel will go on with an equable motion.

A *hand-mill.*

[If the cog-wheel *D* be made about 18 inches diameter, with 30 cogs, the trundle as small in proportion, with 10 staves, and the millstones be each about two feet in diameter, and the whole work be put into a strong frame of wood, as represented in the figure, the engine will be a hand-mill for grinding corn or malt in private families. And then, it may be turned by a winch instead of the wheel *A A*: the millstone making three revolutions for every one of the winch. If a heavy fly be put upon the axle *B*, near the winch, it will help to regulate the motion.]

If the cogs of the wheel and rounds of the trundle could be put in as exactly as the teeth are cut in the wheels and pinions of a clock, then the trundle might divide the wheel exactly: that is to say, the trundle might make a given number of revolutions for one of the wheel, without a fraction. But as any exact number is not necessary in mill-work, and the cogs and rounds cannot be set in so truly as to make all the

the intervals between them equal; a fkilful mill-wright will always give the wheel what he calls a *hunting cog*; that is, one more than what will anfwer to an exact divifion of the wheel by the trundle. And then, as every cog comes to the trundle, it will take the next ftaff or round behind the one which it took in the former revolution: and by that means will wear all the parts of the cogs and rounds which work upon one another equally, and to equal diftances from one another in a little time; and fo make a true uniform motion throughout the whole work. Thus, in the above water-mill, the trundle has 10 ftaves, and the wheel 61 cogs.

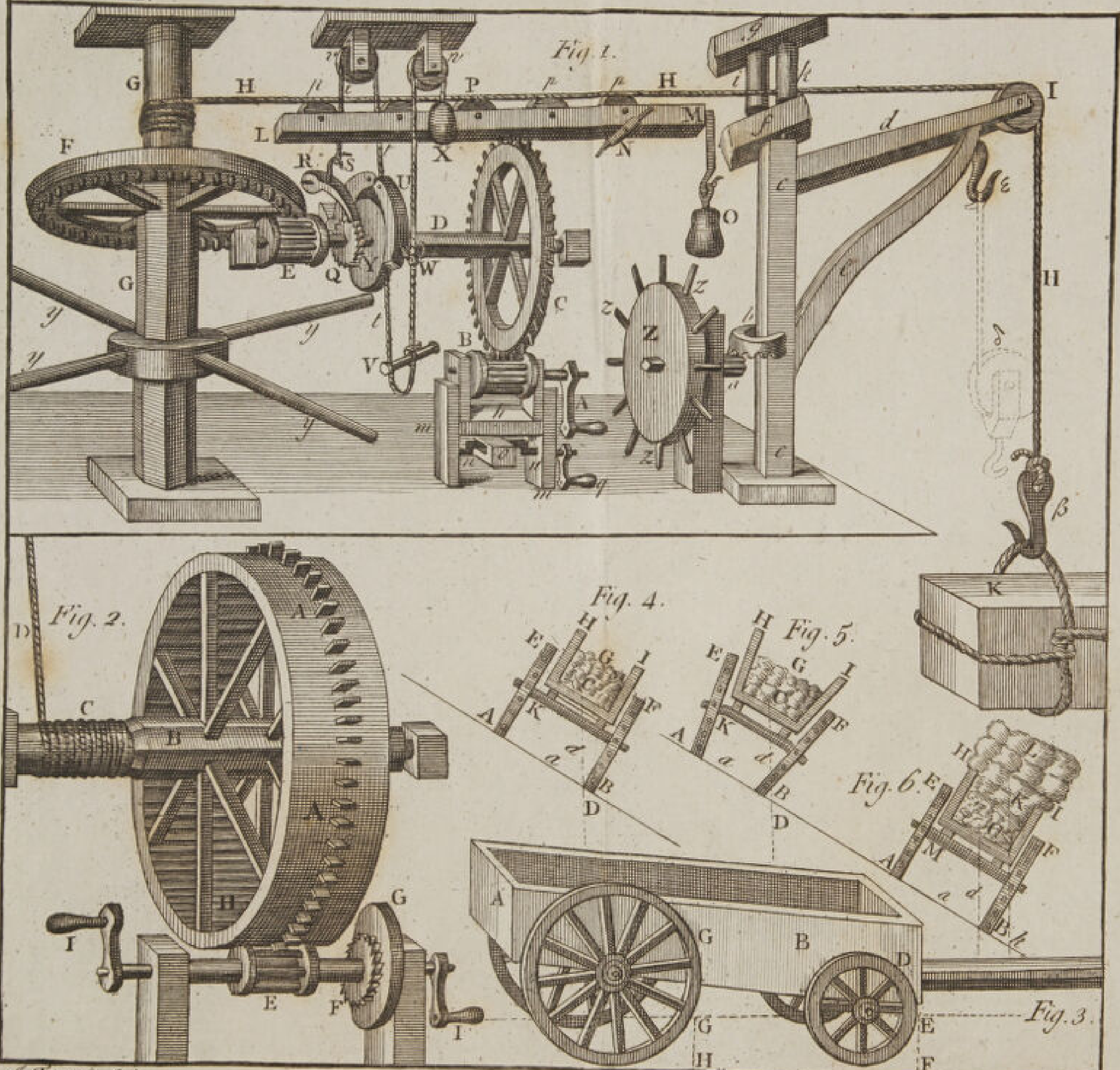
Sometimes, where there is a fufficient quantity of water, the cog-wheel *A A* turns a large trundle *B B*, on whofe axis *C* is fixed the horizontal wheel *D*, with cogs all around its edge, turning two trundles *E* and *F* at the fame time; whofe axes or fpindles *G* and *H* turn two millftones *I* and *K*, upon the fixed ftones *L* and *M*. And when there is not work for them both, either may be made to lie quiet, by taking out one of the ftaves of its trundle, and turning the vacant place toward the cog-wheel *D*. And there may be a wheel fixt on the upper end of the great upright axle *C* for turning a couple of boulting-mills; and other work for drawing up the facks, fanning and cleaning the corn, fharpening of tools, &c.

Fig. 4.

A *horfe-mill.* If, inftead of the cog-wheel *AA* and trundle *B B*, horizontal levers be fixed into the axle *C*, below the wheel *D*; then, horfes may be put to thefe levers for turning the mill: which is often done where water cannot be had for that purpofe.

A *wind-mill.* The working parts of a wind-mill differ very little from thofe of a water-mill; only the former is

PLATE . VIII.



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is turned by the action of the wind upon four ſails, every one of which ought (as is generally believed) to make an angle of $54\frac{3}{4}$ degrees with a plane perpendicular to the axis on which the arms are fixt for carrying them. It being demonſtrable, that when the ſails are ſet to ſuch an angle, and the axis turned endwiſe toward the wind, the wind has the greateſt power upon the ſails. But this angle anſwers only to the caſe of a vane or ſail juſt beginning to move *: for, when the vane has a certain degree of motion, it yields to the wind: and then that angle muſt be increaſed to give the wind its full effect.

Again, the increaſe of this angle ſhould be different, according to the different velocities from the axis to the extremity of the vane. At the axis it ſhould be $54\frac{3}{4}$ degrees, and thence continually decreaſe, giving the vane a twiſt, and ſo cauſing all the ribs of the vane to lie in different planes.

Laſtly, Theſe ribs ought to decreaſe in length from the axis to the extremity, giving the vane a curvilineal form; ſo that no part of the force of any one rib be ſpent upon the reſt, but all move on independent of each other. All this is required to give the ſails of a wind-mill their true form: and we ſee both the twiſt and the diminution of the ribs exemplified in the wings of birds.

It is almoſt incredible to think with what velocity the tips of the ſails move when acted upon by a moderate gale of wind. I have ſeveral times counted the number of revolutions made by the ſails in ten or fifteen minutes; and from the length of the arms from tip to tip, have computed, that if a hoop of that diameter was to run upon the ground with the ſame velo-

* See MACLAURIN's Fluxions, near the end.

city that it would move if put upon the ſail-arms, it would go upward of 30 miles in a hour.

As the ends of the ſails neareſt the axis cannot move with the ſame velocity that the tips or fartheſt ends do, although the wind acts equally ſtrong upon them; perhaps a better poſition than that of ſtretching them along the arms directly from the center of motion, might be to have them ſet perpendicularly acroſs the farther ends of the arms, and there adjuſted lengthwiſe to the proper angle. For, in that caſe, both ends of the ſails would move with the ſame velocity; and being farther from the center of motion, they would have ſo much the more power: and then, there would be no occaſion for having them ſo large as they are generally made; which would render them lighter, and conſequently, there would be ſo much the leſs friction on the thick neck of the axle where it turns in the wall.

A crane. A *crane* is an engine by which great weights are raiſed to certain heights, or let down to certain depths. It conſiſts of wheels, axles, pulleys, ropes, and a gib or gibbet. When the rope *H* is hooked to the weight *K*, a man turns the winch *A*, on the axis whereof is the trundle *B*, which turns the wheel *C*, an whoſe axis *D* is the trundle *E*, which turns the wheel *F* with its upright axis *G*, on which the great rope *H H* winds as the wheel turns; and going over a pulley *I* at the end of the arm *d* of the gib *c c d e*, it draws up the heavy weight *K*; which, being raiſed to a proper height, as from a ſhip to the quay, is then brought over the quay by pulling the wheel *Z* round by the handles *z*, *z*, which turns the gib by means of the half wheel *b* fixt on the gib-poſt *c c*, and the ſtrong pinion *a* fixt on the axis of the wheel *Z*. This wheel gives the man that turns it an abſolute command over

Plate VIII. Fig. 1.

9 the

the gib, ſo as to prevent it from taking any unlucky ſwing, ſuch as often happens when it is only guided by a rope tied to its arm *d*; and people are frequently hurt, ſometimes killed, by ſuch accidents.

The great rope goes between two upright rollers *i* and *k*, which turn upon gudgeons in the fixed beams *f* and *g*; and as the gib is turned toward either ſide, the rope bends upon the roller next that ſide. Were it not for theſe rollers, the gib would be quite unmanageable; for the moment it were turned ever ſo little toward any ſide, the weight *K* would begin to deſcend, becauſe the rope would be ſhortened between the pulley *I* and axis *G*; and ſo the gib would be pulled violently to that ſide, and either be broke to pieces, or break every thing that came in its way. Theſe rollers muſt be placed ſo, that the ſides of them, round which the rope bends, may keep the middle of the bended part directly even with the center of the hole in which the upper gudgeon of the gib turns in the beam *f*. The truer theſe rollers are placed, the eaſier the gib is managed, and the leſs apt to ſwing either way by the force of the weight *K*.

A ratchet-wheel *Q* is fixt upon the axis *D*, near the trundle *E*; and into this wheel the catch or click *R* falls. This hinders the machinery from running back by the weight of the burthen *K*, if the man who raiſes it ſhould happen to be careleſs, and ſo leave off working at the winch *A* ſooner than he ought to do.

When the weight *K* is raiſed to its proper height from the ſhip, and brought over the quay by turning the gib about, it is let down gently upon the quay, or into a cart ſtanding thereon, in the following manner: A man takes hold of the rope *t t* (which goes over the pulley *v* and is tied to a hook at *S* in the catch *R*) and ſo diſengages the catch from the ratchet-wheel *Q*; and then, the man at the winch *A* turns it backward, and lets down the weight *K*. But if the weight pulls too hard againſt this man, another lays hold of the ſtick *V*, and by pulling it downward, draws the gripe *U* cloſe to the wheel *Y*, which, by rubbing hard againſt the gripe. hinders the too quick deſcent of the weight; and not only ſo, but even ſtops it at any time, if required. By this means, heavy goods may be either raiſed or let down at pleaſure, without any danger of hurting the men who work the engine.

When part of the goods are craned up, and the rope is to be let down for more, the catch *R* is firſt diſengaged from the ratchet-wheel *Q*, by pulling the cord *t*; then the handle *q* is turned half round backward, which, by the crank *n n* in the piece *o*, pulls down the frame *h* between the guides *m* and *m* (in which it ſlides in a groove) and ſo diſengages the trundle *B* from the wheel *C*: and then, the heavy hook *β* at the end of the rope *H* deſcends by its own weight, and turns back the great wheel *F* with its trundle *E*, and the wheel *C*; and this laſt wheel acts like a fly againſt the wheel *F* and hook *β*; and ſo hinders it from going down too quick; while the weight *X* keeps up the gripe *U* from rubbing againſt the wheel *Y*, by means of a cord going from the weight, over the pulley *w* to the hook *W* in the gripe; ſo that the gripe never touches the wheel, unleſs it be pulled down by the handle *V*.

When the crane is to be ſet at work again, for drawing up another burthen, the handle *q* is turned half round forward; which, by the crank *n n*, raiſes up the frame *h*, and cauſes the trundle *B* to lay hold of the wheel *C*; and then, by turning the winch *A*, the burthen of goods *K* is drawn up as before.

The crank *n n* turns pretty ſtiff in the mortiſe near *o*, and ſtops againſt the farther end of it when it has got juſt a little beyond the perpendicular; ſo that it can never come back of itſelf: and therefore, the trundle *B* can never come away from the wheel *C*, until the handle *q* be turned half round backward.

The great rope runs upon rollers in the lever *L M*, which keeps it from bending between the axle at *G* and the pulley *I*. This lever turns upon the axis *N* by means of the weight *O*, which is juſt ſufficient to keep its end *L* up to the rope; ſo that, as the great axle turns, and the rope coils round it, the lever riſes with the rope, and prevents the coilings from going over one another.

The power of this crane may be eſtimated thus: ſuppoſe the trundle *B* to have 13 ſtaves or rounds, and the wheel *C* to have 78 ſpur cogs: the trundle *E* to have 14 ſtaves, and the wheel *F* 56 cogs. Then, by multiplying the ſtaves of the trundles, 13 and 14, into one another, their product will be 182; and by multiplying the cogs of the wheels, 78 and 56, into one another, their product will be 4368, and dividing 4368 by 182, the quotient will be 24; which ſhews that the winch *A* makes 24 turns for one turn of the wheel *F* and its axle *G* on which the great rope or chain *H I H* winds. So that, if the length or radius of the winch *A* were only equal to half the diameter of the great axle *G*, added to half the thickneſs of the rope *H*, the power of the crane would be as 24 to 1: but the radius of the winch being double the above length, it doubles the ſaid power, and ſo makes it as 48 to 1: in which caſe, a man may raiſe 48 times as much weight

 by

by this engine as he could do by his natural ſtrength without it, making proper allowance for the friction of the working parts.—Two men may work at once, by having another winch on the oppoſite end of the axis of the trundle under *B*; and this will make the power double.

If this power be thought greater than what may be generally wanted, the wheels may be made with fewer cogs in proportion to the ſtaves in the trundles; and ſo the power may be of whatever degree is judged to be requiſite. But if the weight be ſo great as will require yet more power to raiſe it, ſuppoſe a double quantity, then the rope *H* may be put under a moveable pulley, as δ, and the end of it tied to a hook in the gib at ε; which will give a double power to the machine, and ſo raiſe a double weight hooked to the block of the moveable pulley.

When only ſmall burthens are to be raiſed, this may be quickly done by men puſhing the axle *G* round by the long ſpokes *y*, *y*, *y*, *y*; having firſt diſengaged the trundle *B* from the wheel *C*: and then, this wheel will only act as a fly upon the wheel *F*; and the catch *R* will prevent its running back, if the men ſhould inadvertently leave off puſhing before the burthen be unhooked from β.

Laſtly, When very heavy burthens are to be raiſed, which might endanger the breaking of the cogs in the wheel *F*; their force againſt theſe cogs may be much abated by men puſhing round the long ſpokes *y*, *y*, *y*, *y*, while the man at *A* turns the winch.

I have only ſhewn the working parts of this crane, without the whole of the beams which ſupport them; knowing that theſe are eaſily ſuppoſed,

ſuppoſed, and that if they had been drawn, they would have hid a great deal of the working parts from ſight, and alſo confuſed the figure.

Another very good *crane* is made in the following manner. *A A* is a great wheel turned by men walking within it at *H*. On the part *C*, of its axle *B C*, the great rope *D* is wound as the wheel turns; and this rope draws up goods in the ſame way as the rope *H H* does in the above-mentioned crane, the gib-work here being ſuppoſed to be of the ſame ſort. But theſe cranes are very dangerous to the men in the wheel; for, if any of the men ſhould chance to fall, the burthen will make the wheel run back and throw them all about within it: which often breaks their limbs, and ſometimes kills them. The late ingenious Mr. *Padmore* of Briſtol (whoſe contrivance the forementioned crane is, ſo far as I can remember its conſtruction, after ſeeing it once about twelve years ago*) obſerving this dangerous conſtruction, contrived a method for remedying it, by putting cogs all around the outſide of the wheel, and applying a trundle *E* to turn it; which increaſes the power as much as the number of cogs in the wheel is greater than the number of ſtaves in the trundle: and by putting a ratchet-wheel *F* on the axis of the trundle (as in the above-mentioned crane) with a catch to fall into it, the great wheel is ſtopt from running back by the force of the weight, even if all the men in

Another *crane.* Fig. 2.

* Since the firſt edition of this book was printed, I have ſeen the ſame crane again; and do find, that though the working parts are much the ſame as above deſcribed, yet the method of raiſing or lowering the trundle *B*, and the catch *R*, are better contrived than I had deſcribed them.

it ſhould leave off walking. And by one man working at the winch *I*, or two men at the oppoſite winches when needful, the men in the wheel are much aſſiſted, and much greater weights are raiſed, than could be by men only within the wheel. Mr. *Padmore* put alſo a gripe-wheel *G* upon the axis of the trundle, which being pinched in the ſame manner as deſcribed in the former crane, heavy burthens may be let down without the leaſt danger. And before this contrivance, the lowering of goods was always attended with the utmoſt danger to the men in the wheel; as every one muſt be ſenſible of, who has ſeen ſuch engines at work.

And it is ſurpriſing that the maſters of wharfs and cranes ſhould be ſo regardleſs of the limbs, or even lives of their workmen, that excepting the late Sir *James Creed* of Greenwich, and ſome gentlemen at Briſtol, there is ſcarce an inſtance of any who has uſed this ſafe contrivance.

Wheel-carriages. The ſtructure of *wheel-carriages* is generally ſo well known, that it would be needleſs to deſcribe them. And therefore, we ſhall only point out ſome inconveniencies attending the common method of placing the wheels, and loading the waggons.

In coaches, and all other four-wheeled carriages, the fore-wheels are made of a leſs ſize than the hind ones, both on account of turning ſhort, and to avoid cutting the braces: otherwiſe, the carriage would go much eaſier if the fore-wheels were as high as the hind-ones, and the higher the better, becauſe they would ſink to leſs depths in little hollowings in the roads, and be the more eaſily drawn out of them.

them. But carriers and coachmen give another reaſon for making the fore-wheels much lower than the hind-wheels; merely, that when they are ſo, the hind-wheels help to puſh on the fore ones: which is too unphiloſophical and abſurd to deſerve a refutation, and yet for their ſatisfaction we ſhall ſhew by experiment that it has no exiſtence but in their own imaginations.

It is plain that the ſmall wheels muſt turn as much oftener round than the great ones, as their circumferences are leſs. And therefore, when the carriage is loaded equally heavy on both axles, the fore-axle muſt ſuſtain as much more friction, and conſequently wear out as much ſooner, than the hind-axle, as the fore-wheels are leſs than the hind-ones. But the great misfortune is, that all the carriers to a man do obſtinately perſiſt, againſt the cleareſt reaſon and demonſtration, in putting the heavier part of the load upon the fore-axle of the waggon; which not only makes the friction greateſt where it ought to be leaſt, but alſo preſſes the fore wheels deeper into the ground than the hind-wheels, notwithſtanding the fore-wheels, being leſs than the hind ones, are with ſo much the greater difficulty drawn out of a hole or over an obſtacle, even ſuppoſing the weights on their axles were equal. For the difficulty, with equal weights, will be as the depth of the hole Fig. 3. or height of the obſtacle is to the ſemidiameter of the wheel. Thus, if we ſuppoſe the ſmall wheel *D* of the waggon *AB* to fall into a hole of the depth *EF*, which is equal to the ſemidiameter of the wheel, and the waggon to be drawn horizontally along; it is evident, that the point *E* of the ſmall wheel will be drawn directly againſt the top of the hole; and therefore,

fore, all the power of horſes and men will not be able to draw it out, unleſs the ground gives way before it. Whereas, if the hind-wheel *G* falls into ſuch a hole, it ſinks not near ſo deep in proportion to its ſemidiameter; and therefore, the point *G* of the large wheel will not be drawn directly, but obliquely, againſt the top of the hole; and ſo will be eaſily got out of it. Add to this, that as a ſmall wheel will often ſink to the bottom of a hole, in which a great wheel will go but a very little way, the ſmall wheels ought in all reaſon to be loaded with leſs weight than the great ones; and then the heavier part of the load would be leſs jolted upward and downward, and the horſes tired ſo much the leſs, as their draught raiſed the load to leſs heights.

It is true, that when the waggon-road is much up hill, there may be danger in loading the hind part much heavier than the fore-part; for then the weight would overhang the hind-axle, eſpecially if the load be high, and endanger tilting up the fore-wheels from the ground. In this caſe, the ſafeſt way would be to load it equally heavy on both axles; and then, as much more of the weight would be thrown upon the hind-axle than upon the fore one, as the ground riſes from a level below the carriage. But as this ſeldom happens, and when it does, a ſmall temporary weight laid upon the pole between the horſes would overbalance the danger; and this weight might be thrown into the waggon when it comes to level ground; it is ſtrange that an advantage ſo plain and obvious as would ariſe from loading the hind-wheels heavieſt, ſhould not be laid hold of, by complying with this method.

To

To confirm theſe reaſonings by experiment, let a ſmall model of a waggon be made, with its fore-wheels $2\frac{1}{2}$ inches in diameter, and its hind-wheels $4\frac{1}{2}$; the whole model weighing about 20 ounces. Let this little carriage be loaded any how with weights, and have a ſmall cord tied to each of its ends, equally high from the ground it reſts upon; and let it be drawn along a horizontal board, firſt by a weight in a ſcale hung to the cord at the fore-part; the cord going over a pulley at the end of the board to facilitate the draught, and the weight juſt ſufficient to draw it along. Then, turn the carriage, and hang the ſcale and weight to the hind cord, and it will be found to move along with the ſame velocity as at firſt: which ſhews, that the power required to draw the carriage is all the ſame, whether the great or ſmall wheels are foremoſt; and therefore the great wheels do not help in the leaſt to puſh on the ſmall wheels in the road.

Hang the ſcale to the fore-cord, and place the fore wheels (which are the ſmall ones) in two holes, cut three eight parts of an inch deep into the board; then put a weight of 32 ounces into the carriage, over the fore-axle, and an equal weight over the hind one: this done, put 44 ounces into the ſcale, which will be juſt ſufficient to draw out the fore-wheels: but if this weight be taken out of the ſcale, and one of 16 ounces put into its place, if the hind-wheels are placed in the holes, the 16 ounce weight will draw them out; which is little more than a third part of what was neceſſary to draw out the fore-wheels. This ſhews, that the larger the wheels are, the leſs power will draw the carriage, eſpecially on rough ground.

Put

Put 64 ounces over the axle of the hind-wheels, and 32 over the axle of the fore ones, in the carriage; and place the fore-wheels in the holes: then, put 38 ounces into the ſcale, which will juſt draw out the fore-wheels; and when the hind ones come to the hole, they will find but very little reſiſtance, becauſe they ſink but a little way into it.

But ſhift the weights in the carriage, by putting the 32 ounces upon the hind-axle, and the 64 ounces upon the fore one; and place the fore-wheels in the holes: then, if 76 ounces be put into the ſcale, it will be found no more than ſufficient to draw out theſe wheels; which is double the power required to draw them out, when the lighter part of the load was put upon them: which is a plain demonſtration of the abſurdity of putting the heavieſt part of the load in the fore-part of the waggon.

Every one knows what an outcry was made by the generality, if not the whole body, of the carriers, againſt the broad-wheel act; and how hard it was to perſuade them to comply with it, even though the government allowed them to draw with more horſes, and carry greater loads, than uſual. Their principal objection was, that as a broad wheel muſt touch the ground in a great many more points than a narrow wheel, the friction muſt of courſe be juſt ſo much the greater; and conſequently, there muſt be ſo many more horſes than uſual, to draw the waggon. I believe that the majority of people were of the ſame opinion, not conſidering, that if the whole weight of the waggon and load in it bears upon a great many points, each ſuſtains a proportionably leſs degree of weight and friction, than when it bears only upon a few points; ſo that what

what is wanting in one, is made up in the other; and therefore will be juſt equal under equal degrees of weight, as may be ſhewn by the following plain and eaſy experiment.

Let one end of a piece of packthread be faſtened to a brick, and the other end to a common ſcale for holding weights: then, having laid the brick edgewiſe on a table, and let the ſcale hang under the edge of the table, put as much weight into the ſcale as will juſt draw the brick along the table. Then taking back the brick to its former place, lay it flat on the table, and leave it to be acted upon by the ſame weight in the ſcale as before, which will draw it along with the ſame eaſe as when it lay upon its edge. In the former caſe, the brick may be conſidered as a narrow wheel on the ground; and in the latter as a broad wheel. And ſince the brick is drawn along with equal eaſe, whether its broad ſide or narrow edge touches the table, it ſhews that a broad wheel might be drawn along the ground with the ſame eaſe as a narrow one (ſuppoſing them equally heavy) even though they ſhould drag, and not roll, as they go along.

As narrow wheels are always ſinking into the ground, eſpecially when the heavieſt part of the load lies upon them, they muſt be conſidered as going conſtantly up hill, even on level ground. And their ſides muſt ſuſtain a great deal of friction by rubbing againſt the ruts made by them. But both theſe inconveniencies are avoided by broad wheels; which, inſtead of cutting and ploughing up the roads, roll them ſmooth, and harden them; as experience teſtifies in places where they have been uſed, eſpecially either on wettiſh or ſandy ground: though after all it muſt be confeſſed, that they will not do in ſtiff clayey croſs roads;

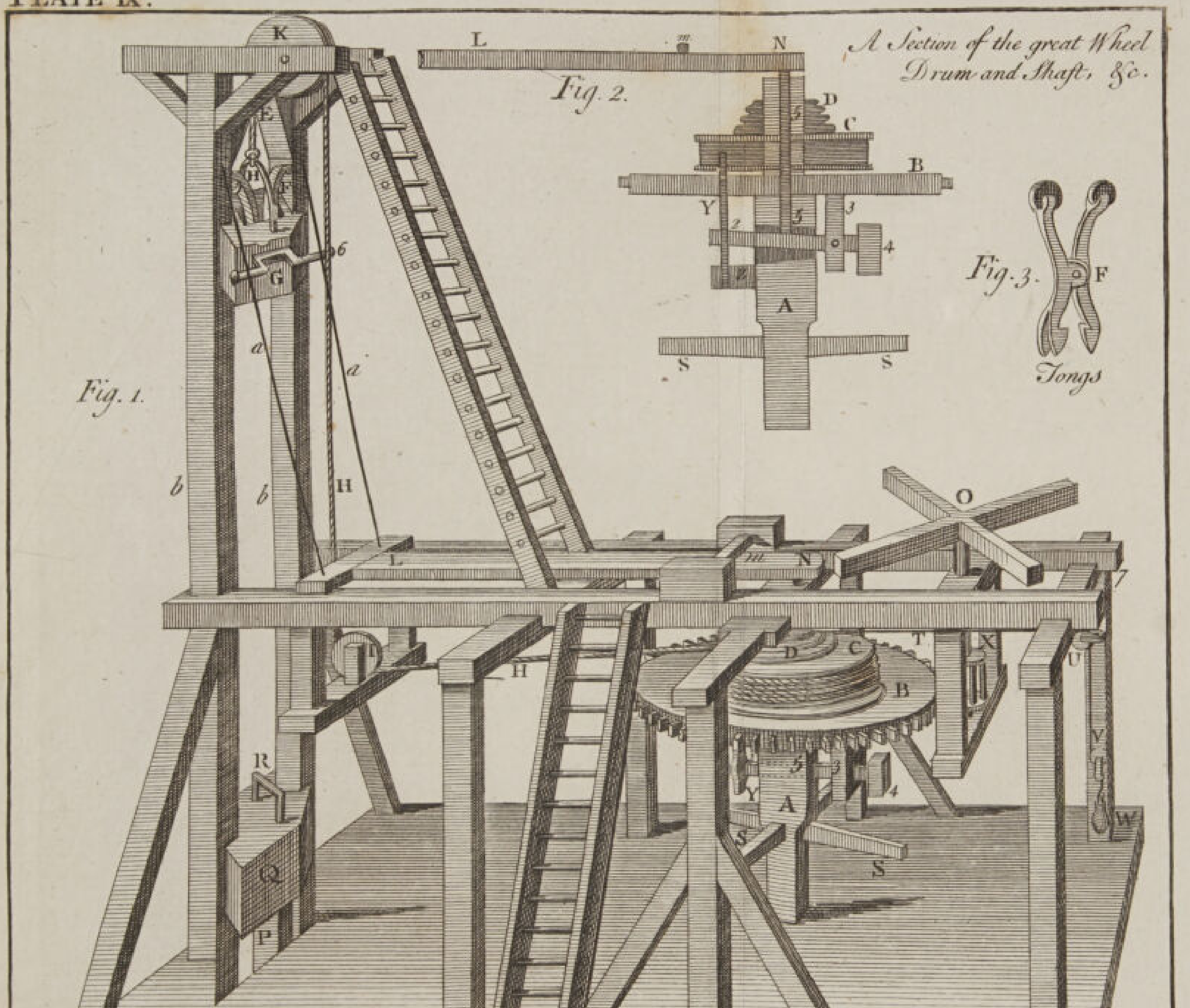
roads; becaufe they would foon gather up as much clay as would be almoft equal to the weight of an ordinary load.

If the wheels were always to go upon fmooth and level ground, the beft way would be to make the fpokes perpendicular to the naves; that is, to ftand at right angles to the axles; becaufe they would then bear the weight of the load perpendicularly, which is the ftrongeft way for wood. But becaufe the ground is generally uneven, one wheel often falls into a cavity or rut when the other does not; and then it bears much more of the weight than the other does: in which cafe, concave or difhing wheels are beft, becaufe when one falls into a rut, and the other keeps upon high ground, the fpokes become perpendicular in the rut, and therefore have the greateft ftrength when the obliquity of the load throws moft of its weight upon them; while thofe on the high ground have lefs weight to bear, and therefore need not be at their full ftrength. So that the ufual way of making the wheels concave is by much the beft.

The axles of the wheels ought to be perfectly ftraight, that the rim of the wheels may be parallel to each other; for then they will move eafieft, becaufe they will be at liberty to go on ftraight forward. But in the ufual way of practice, the axles are bent downward at their ends; which brings the fides of the wheels next the ground nearer to one another than their oppofite or higher fides are: and this not only makes the wheels to drag fidewife as they go along, and gives the load as much greater power of crufhing them than when they are parallel to each other; but alfo endangers the over-turning of the carriage when any wheel falls into a hole or rut; or

when

PLATE IX.


A Section of the great Wheel
Drum and Shaft, &c.
Fig. 1.
Fig. 2.
Fig. 3.
Tongs
J. Ferguson delin.
J. Mynde sc.

when the carriage goes in a road which has one ſide lower than the other, as along the ſide of a hill. Thus (in the hind view of a waggon or cart) let *A E* and *B F* be the great wheels parallel to each other, on their ſtraight axle *K*, and *HCI* the carriage loaded with heavy goods from *C* to *G*. Then, as the carriage goes on in the oblique road *A a B*, the center of gravity of the whole machine and load will be at *C**; and the line of direction *C d D* falling within the wheel *B F*, the carriage will not overſet. But if the wheels be inclined to each other on the ground, as *AE* and *BF* are, and the machine be loaded as before, from *C* to *G*, the line of direction *C d D* falls without the wheel *B F*, and the whole machine tumbles over. When it is loaded with heavy goods (ſuch as lead or iron) which lie low, it may travel ſafely upon an oblique road ſo long as the center of gravity is at *C*, and the line of direction *C d* falls within the wheels; but if it be loaded high with lighter goods (ſuch as woolpacks) from *C* to *L*, the center of gravity is raiſed from *C* to *K*, which throws the line of direction *K k* without the loweſt edge of the wheel *B F*, and then the load overſets the waggon.

Fig. 4.

* See page 13.

Fig. 5.

Fig. 4.

Fig. 6.

If there be ſome advantage from ſmall fore-wheels, on account of the carriage turning more eaſily and ſhort than it can be made to do when they are large; there is at leaſt as great a diſadvantage attending them, which is, that as their axle is below the level of the horſes breaſt, the horſes not only have the loaded carriage to draw along, but alſo part of its weight to bear; which tires them ſooner, and makes them grow much ſtiffer in their hams, than they would be if they drew on a level with the fore-axle. And for this reaſon, we find coach horſes

ſoon

ſoon become unfit for riding. So that on all accounts it is plain, that the fore-wheels of all carriages ought to be ſo high, as to have their axles even with the breaſt of the horſes; which would not only give the horſes a fair draught, but likewiſe keep them longer fit for drawing the carriage.

Plate IX. Fig. 1, 2. We ſhall conclude this lecture with a deſcription of Mr. *Vauloue*'s curious engine, which was made uſe of for driving the piles of Weſtminſter-bridge: and the reader may caſt his eyes upon the firſt and ſecond figures of the plate, in which the ſame letters of reference are annexed to the ſame parts, in order to explain thoſe in the ſecond, which are either partly or wholly hid in the firſt.

The *pile-engine.* *A* is the great upright ſhaft or axle, on which are the great wheel *B* and drum *C*, turned by horſes joined to the bars *S*, *S*. The wheel *B* turns the trundle *X*, on the top of whoſe axis is the fly *O*, which ſerves to regulate the motion, and alſo to act againſt the horſes, and keep them from falling when the heavy ram *Q* is diſcharged to drive the pile *P* down into the mud in the bottom of the river. The drum *C* is looſe upon the ſhaft *A*, but is locked to the wheel *B* by the bolt *Y*. On this drum the great rope *HH* is wound; one end of the rope being fixed to the drum, and the other to the follower *G*, to which it is conveyed over the pulleys *I* and *K*. In the follower *G* is contained the tongs *F* (ſee Fig. 3.) that takes hold of the ram *Q* by the ſtaple *R* for drawing it up. *D* is a ſpiral or fuſy fixt to the drum, on which is wound the ſmall rope *T* that goes over the pulley *U*, under the pulley *V*, and is faſtened to the top of the frame at 7. To the pulley block *V* is hung the counterpoiſe *W*, which

which hinders the follower from accelerating as it goes down to take hold of the ram: for, as the follower tends to acquire velocity in its defcent, the line *T* winds downward upon the fufy, on a larger and larger radius, by which means the counterpoife *W* acts ftronger and ftronger againft it; and fo allows it to come down with only a moderate and uniform velocity. The bolt *Y* locks the drum to the great wheel, being pufhed upward by the fmall lever 2, which goes through a mortife in the fhaft *A*, turns upon a pin in the bar 3 fixt to the great wheel *B*, and has a weight 4, which always tends to pufh up the bolt *Y* through the wheel into the drum. *L* is the great lever turning on the axis *m*, and refting upon the forcing bar 5, 5, which goes down through a hollow in the fhaft *A*, and bears up the little lever 2.

By the horfes going round, the great rope *H* is wound about the drum *C*, and the ram *Q* is drawn up by the tongs *F* in the follower *G*, until the tongs comes between the inclined planes *E*; which, by fhutting the tongs at the top, opens it at the foot, and difcharges the ram, which falls down between the guides *b b* upon the pile *P*, and drives it by a few ftrokes as far into the mud as it can go; after which, the top part is fawed off clofe to the mud, by an engine for that purpofe. Immediately after the ram is difcharged the piece 6 upon the follower *G* takes hold of the ropes *a*, *a*, which raife the end of the lever *L*, and caufe its end *N* to defcend and prefs down the forcing bar 5 upon the little lever 2, which by pulling down the bolt *Y*, unlocks the drum *C* from the great wheel *B*; and then, the follower, being at liberty, comes down by its own weight to the ram; and the lower ends of the tongs flip over the ſtaple *R*, and the weight of their heads cauſes them to fall outward, and ſhuts upon it. Then the weight 4 puſhes up the bolt *Y* into the drum, which locks it to the great wheel, and ſo the ram is drawn up as before.

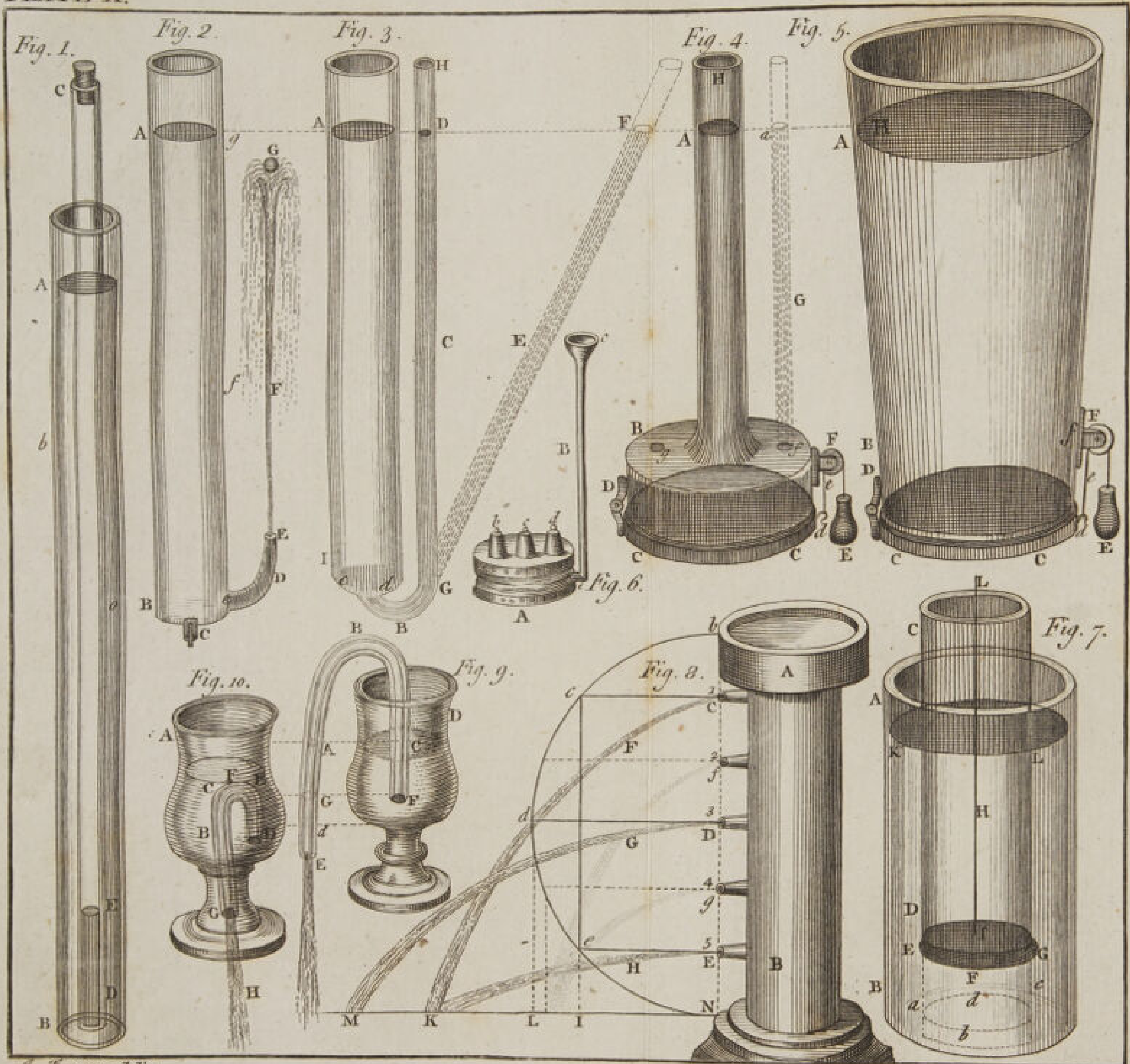
As the follower comes down, it cauſes the drum to turn backward, and unwinds the rope from it, while the horſes, great wheel, trundle, and fly, go on with an uninterrupted motion: and as the drum is turning backward, the counterpoiſe *W* is drawn up, and its rope *T* wound upon the ſpiral fuſy *D*.

There are ſeveral holes in the under ſide of the drum, and the bolt *Y* always takes the firſt one that it finds when the drum ſtops by the falling of the follower upon the ram; until which ſtoppage, the bolt has not time to ſlip into any of the holes.

This engine was placed upon a barge on the water, and ſo was eaſily conveyed to any place deſired.—I never had the good fortune to ſee it, but drew this figure from a model which I made from a print of it; being not quite ſatisfied with the view which the print gives. I have been told that the ram was a ton weight, and that the guides *b b*, between which it was drawn up and let fall down, were 30 feet high. I ſuppoſe the great wheel may have had 100 cogs, and the trundle 10 ſtaves or rounds; ſo that the fly would make 10 revolutions for one of the great wheel.

LECT.

PLATE X.



J. Ferguson delin. J. Mynde sc.

LECT. V.

Of hydrostatics, and hydraulic machines.

THE science of *hydrostatics* treats of the nature, gravity, pressure, and motion of fluids in general; and of weighing solids in them.

A fluid is a body that yields to the least pressure or difference of pressures. Its particles are so small, that they cannot be discerned by the best microscopes; they are hard, since no fluid, except air or steam, can be pressed into a less space than it naturally possesses; and they must be round and smooth, seeing they are so easily moved among one another. **Definition of a fluid.**

All bodies, both fluid and solid, press downward by the force of gravity: but fluids have this wonderful property, that their pressure upward and sidewise is equal to their pressure downward; and this is always in proportion to their perpendicular height, without any regard to their quantity; for, as each particle is quite free to move, it will move toward that part or side on which the pressure is least. And hence, no particle or quantity of a fluid can be at rest, till it is every way equally pressed.

To shew by experiment that fluids press upward as well as downward, let *A B* be a long upright tube filled with water near to its top; and *C D* a small tube open at both ends, and immersed into the water in the large one; if the immersion be quick, you will see the water rise in the small tube to the same height that it stands in the great one, or until the surfaces of the **Plate X. Fig. 1. Fluids press as much upward as downward.**

water in both are on the fame level: which fhews that the water is preffed upward into the fmall tube by the weight of what is in the great one; otherwife it could never rife therein, contrary to its natural gravity; unlefs the diameter of the bore were fo fmall, that the attraction of the tube would raife the water; which will never happen, if the tube be as wide as that in a common barometer. And, as the water rifes no higher in the fmall tube than till its furface be on a level with the furface of the water in the great one, this fhews that the preffure is not in proportion to the quantity of water in the great tube, but in proportion to its perpendicular height therein: for there is much more water in the great tube all around the fmall one, than what is raifed to the fame height in the fmall one, as it ftands within the great.

Take out the fmall tube, and let the water run out of it; then it will be filled with air. Stop its upper end with the cork *C*, and it will be full of air all below the cork: this done, plunge it again to the bottom of the water in the great tube, and you will fee the water rife up in it only to the height *E*; which fhews that the air is a body, otherwife it could not hinder the water from rifing up to the fame height as it did before, namely, to *A*; and in fo doing, it drove the air out at the top; but now the air is confined by the cork *C*: and it alfo fhews that the air is a compreffible body, for if it were not fo, a drop of water could not enter into the tube.

The preffure of fluids being equal in all directions, it follows that the fides of a veffel are as much preffed by a fluid in it, all around in any given ring of points, as the fluid below that ring is preffed by the weight of all that ftands above it.

it. Hence the preſſure upon every point in the ſides, immediately above the bottom, is equal to the preſſure upon every point of the bottom. To ſhew this by experiment, let a hole be made at *e* in the ſide of the tube *AB* cloſe by the bottom; and another hole of the ſame ſize in the bottom at *C*; then pour water into the tube, keeping it full as long as you chooſe the holes ſhould run, and have two baſons ready to receive the water that runs through the two holes, until you think there is enough in each baſon; and you will find by meaſuring the quantities, that they are equal; which ſhews that the water run with equal ſpeed through both holes: which it could not have done, if it had not been equally preſſed through them both. For, if a hole of the ſame ſize be made in the ſide of the tube, as about *f*, and if all three are permitted to run together, you will find that the quantity run through the hole at *f* is much leſs than what has run in the ſame time through either of the holes *C* or *e*. Fig. 2.

In the ſame figure, let a tube be turned up from the bottom at *e* into the ſhape *D E*, and the hole at *C* be ſtopt with a cork. Then, pour water into the tube to any height, as *Ag*, and it will ſpout up in a jet *EFG*, nearly as high as it is kept in the tube *AB*, by continuing to pour in as much there as runs through the hole *E*; which will be the caſe while the ſurface *Ag* keeps at the ſame height. And if a little ball of cork *G* be laid upon the top of the jet, it will be ſupported thereby, and dance upon it. The reaſon why the jet riſes not quite ſo high as the ſurface of the water *Ag*, is owing to the reſiſtance it meets with in the open air: for, if a tube, either great or ſmall, was ſcrewed upon the pipe at *E*, the water would rise in it until the surface of the water in both tubes were on the same level; as will be shewn by the next experiment.

The *hydrostatic paradox*. Any quantity of a fluid, how small soever, may be made to balance and support any quantity, how great soever. This is deservedly termed *the hydrostatical paradox*, which we shall first shew by an experiment, and then account for it upon the principle above-mentioned; namely, that *the pressure of fluids is directly as their perpendicular height, without any regard to their quantity.*

Fig. 3. Let a small glass tube *DCG*, open throughout, and bended at *B*, be joined to the end of a great one *AI* at *c d*, where the great one is also open; so that these tubes in their openings may freely communicate with each other. Then pour water through a small necked funnel into the small tube at *H*; this water will run through the joining of the tubes at *c d*, and rise up into the great tube; and if you continue pouring until the surface of the water comes to any part, as *A*, in the great tube, and then leave off, you will see that the surface of the water in the small tube will be just as high, at *D*; so that the perpendicular height of the water will be the same in both tubes, however small the one be in proportion to the other. This shews, that the small column *DCG* balances and supports the great column *A c d*: which it could not do if their pressures were not equal against one another in the recurved bottom at *B*.—If the small tube be made longer, and inclined in the situation *GEF*, the surface of the water in it will stand at *F*, on the same level with the surface *A* in the great tube; that is, the water will have the same perpendicular height in both tubes, although the column in the small tube is longer than that in the great one;

 the

the former being oblique, and the latter perpendicular.

Since then the preſſure of fluids is directly as their perpendicular heights, without any regard to their quantities, it appears that whatever the figure or ſize of veſſels be, if they are of equal heights, and if the areas of their bottoms are equal, the preſſures of equal heights of water are equal upon the bottoms of theſe veſſels; even though the one ſhould hold a thouſand or ten thouſand times as much water as would fill the other. To confirm this part of the hydroſtatical paradox by an experiment, let two veſſels be prepared of equal heights but very unequal contents, ſuch as *AB* in Fig. 4. and *AB* in Fig. 5. Let each veſſel be open at both ends, and their bottoms *D d*, *D d* be of equal widths. Let a braſs bottom *C C* be exactly fitted to each veſſel, not to go into it, but for it to ſtand upon; and let a piece of wet leather be put between each veſſel and its braſs bottom, for the ſake of cloſeneſs. Join each bottom to its veſſel by a hinge *D*, ſo that it may open like the lid of a box; and let each bottom be kept up to its veſſel by equal weights *E* and *E* hung to lines which go over the pulleys *F* and *F* (whoſe blocks are fixed to the ſides of the veſſels at *f*) and the lines tied to hooks at *d* and *d*, fixed in the braſs bottoms oppoſite to the hinges *D* and *D*. Things being thus prepared and fitted, hold the veſſel *AB* (Fig. 5.) upright in your hands over a baſon on a table, and cauſe water to be poured into the veſſel ſlowly, till the preſſure of the water bears down its bottom at the ſide *d*, and raiſes the weight *E*; and then part of the water will run out at *d*. Mark the height at which the ſurface *H* of the water ſtood in the veſſel, when the bot- Fig. 4, 5.

tom began to give way at d; and then, holding up the other vessel AB (Fig. 4.) in the same manner, cause water to be poured into it at H; and you will see that when the water rises to A in this vessel, just as high as it did in the former, its bottom will also give way at d, and it will lose part of the water.

The natural reason of this surprising phenomenon is, that since all parts of a fluid at equal depths below the surface are equally pressed in all manner of directions, the water immediately below the fixed part Bf (Fig. 4.) will be pressed as much upward against its lower surface within the vessel, by the action of the column Ag, as it would be by a column of the same height, and of any diameter whatever; (as was evident by the experiment with the tube, Fig. 3.) and therefore, since action and reaction are equal and contrary to each other, the water immediately below the surface Bf will be pressed as much downward by it, as if it was immediately touched and pressed by a column of the height gA, and of the diameter Bf: and therefore, the water in the cavity $BDdf$ will be pressed as much downward upon its bottom CC, as the bottom of the other vessel (Fig. 5.) is pressed by all the water above it.

Fig. 4. To illustrate this a little farther, let a hole be made at f in the fixed top Bf, and let a tube G be put into it; then, if water be poured into the tube A, it will (after filling the cavity Bd) rise up into the tube G, until it comes to a level with that in the tube A, which is manifestly owing to the pressure of the water in the tube A, upon that in the cavity of the vessel below it. Consequently, that part of the top Bf, in which the hole is now made, would, if corked up, be pressed

pressed upward with a force equal to the weight of all the water which is supported in the tube *G*: and the same thing would hold at *g*, if a hole were made there. And so if the whole cover or top *B f* were full of holes, and had tubes as high as the middle one *A g* put into them, the water in each tube would rise to the same height as it is kept into the tube *A*, by pouring more into it, to make up the deficiency that it sustains by supplying the others, until they are all full: and then the water in the tube *A* would support equal heights of water in all the rest of the tubes. Or, if all the tubes except *A*, or any other one, were taken away, and a large tube equal in diameter to the whole top *B f* were placed upon it, and cemented to it, and then if water were poured into the tube that was left in either of the holes, it would ascend through all the rest of the holes, until it filled the large tube to the same height that it stands in the small one, after a sufficient quantity had been poured into it: which shews, that the top *B f* was pressed upward by the water under it, and before any hole was made in it, with a force equal to that wherewith it is now pressed downward by the weight of all the water above it in the great tube. And therefore, the re-action of the fixed top *B f* must be as great, in pressing the water downward upon the bottom *C C*, as the whole pressure of the water in the great tube would have been, if the top had been taken away, and the water in that tube left to press directly upon the water in the cavity *B D d f*.

Perhaps the best machine in the world for demonstrating the upward pressure of fluids, is the hydrostatic bellows *A*; which consists of two thick oval boards, each about 16 inches broad, and 18 inches long, covered with leather, to open

Fig. 6. The *hydrostatic bellows.*

open and ſhut like a common bellows, but without valves; only a pipe *B*, about three feet high, is fixed into the bellows at *e*. Let ſome water be poured into the pipe at *c*, which will run into the bellows, and ſeparate the boards a little. Then lay three weights *b*, *c*, *d*, each weighing 100 pounds, upon the upper board, and pour more water into the pipe *B*, which will run into the bellows, and raiſe up the board with all the weights upon it; and if the pipe be kept full, until the weights are raiſed as high as the leather which covers the bellows will allow them, the water will remain in the pipe, and ſupport all the weights, even though it ſhould weigh no more than a quarter of a pound, and they 300 pounds: nor will all their force be able to cauſe them to deſcend and force the water out at the top of the pipe.

The reaſon of this will be made evident, by conſidering what has been already ſaid of the reſult of the preſſure of fluids of equal heights without any regard to the quantities. For, if a hole be made in the upper board, and a tube be put into it, the water will riſe in the tube to the ſame height that it does in the pipe: and would riſe as high (by ſupplying the pipe) in as many tubes as the board could contain holes. Now, ſuppoſe only one hole to be made in any part of the board, of an equal diameter with the bore of the pipe *B*; and that the pipe holds juſt a quarter of a pound of water; if a perſon claps his finger upon the hole, and the pipe be filled with water, he will find his finger to be preſſed upward with a force equal to a quarter of a pound. And as the ſame preſſure is equal upon all equal parts of the board, each part whoſe area is equal to the area of the hole, will be preſſed upward with a force equal to that of a quarter of a pound: the ſum

ſum of all which preſſures againſt the under ſide of an oval board 16 inches broad, and 18 inches long, will amount to 300 pounds; and therefore ſo much weight will be raiſed up and ſupported by a quarter of a pound of water in the pipe.

Hence, if a man ſtands upon the upper board, and blows into the bellows through the pipe *B*, he will raiſe himſelf upward upon the board: and the ſmaller the bore of the pipe is, the eaſier he will be able to raiſe himſelf. And then, by clapping his finger upon the top of the pipe, he can ſupport himſelf as long as he pleaſes; provided the bellows be air-tight ſo as not to loſe what is blown into it.

How a man may raiſe himſelf upward by his breath.

This figure, I confeſs, ought to have been much larger than any other upon the plate; but it was not thought of, until all the reſt were drawn; and it could not ſo properly come into any other plate.

Upon this principle of the upward preſſure of fluids, a piece of lead may be made to ſwim in water, by immerſing it to a proper depth, and keeping the water from getting above it. Let *C D* be a glaſs tube, open throughout, and *E F G* a flat piece of lead, exactly fitted to the lower end of the tube, not to go within it, but for it to ſtand upon; with a wet leather between the lead and the tube to make cloſe work. Let this leaden bottom be half an inch thick, and held cloſe to the tube by pulling the packthread *I H L* upward at *L* with one hand, while the tube is held in the other by the upper end *C*. In this ſituation, let the tube be immerſed in water in the glaſs veſſel *A B*, to the depth of ſix inches below the ſurface of the water at *K*: and then, the leaden bottom *E F G* will be plunged to the depth of ſomewhat more than eleven times its

How ſolid lead may be made to ſwim in water.

Fig. 7.

its own thickneſs: holding the tube at that depth, you may let go the thread at *L*; and the lead will not fall from the tube, but will be kept to it by the upward preſſure of the water below it, occaſioned by the height of the water at *K* above the level of the lead. For as lead is 11.33 times as heavy as its bulk of water, and is in this experiment immerſed to a depth ſomewhat more than 11.33 times its thickneſs, and no water getting into the tube between it and the lead, the column of water *EabcG* below the lead is preſſed upward againſt it by the water *KDEGL* all around the tube; which water being a little more than 11.33 times as high as the lead is thick, is ſufficient to balance and ſupport the lead at the depth *KE*. If a little water be poured into the tube upon the lead, it will increaſe the weight upon the column of water under the lead, and cauſe the lead to fall from the tube to the bottom of the glaſs veſſel, where it will lie in the ſituation *bd*. Or, if the tube be raiſed a little in the water, the lead will fall by its own weight, which will then be too great for the preſſure of the water around the tube upon the column of water below it.

How light wood may be made to lie at the bottom of water. Let two pieces of wood be planed quite flat, ſo as no water may get in between them when they are put together: let one of the pieces, as *bd*, be cemented to the bottom of the veſſel *AB* (Fig. 7.) and the other piece be laid flat and cloſe upon it, and held down to it by a ſtick, while water is poured into the veſſel; then remove the ſtick, and the upper piece of wood will not riſe from the lower one: for, as the upper one is preſſed down both by its own weight and the weight of all the water over it, while the contrary preſſure of the water is kept off by the wood

wood under it, it will lie as ſtill as a ſtone would do in its place. But if it be raiſed ever ſo little at any edge, ſome water will then get under it; which being acted upon by the water above, will immediately preſs it upward; and as it is lighter than its bulk of water, it will riſe, and float upon the ſurface of the water.

All fluids weigh juſt as much in their own elements as they do in open air. To prove this by experiment, let as much ſhot be put into a phial, as, when corked, will make it ſink in water: and, being thus charged, let it be weighed, firſt in air, and then in water, and the weights in both caſes wrote down. Then, as the phial hangs ſuſpended in water, and counterpoiſed, pull out the cork, that water may run into it, and it will deſcend, and pull down that end of the beam. This done, put as much weight into the oppoſite ſcale as will reſtore the equipoiſe; which weight will be found to anſwer exactly to the additional weight of the phial when it is again weighed in air, with the water in it.

The velocity of ſpouting water. The velocity with which water ſpouts out at a hole in the ſide or bottom of a veſſel, is as the ſquare root* of the depth or diſtance of the hole below the ſurface of the water. For, in order to make double the quantity of a fluid run through one hole as through another of the ſame ſize, it will require four times the preſſure of the other, and therefore muſt be four times the depth of the other below the ſurface of the water: and for the ſame reaſon, three times the quantity running in an equal time through the

* The ſquare root of any number is that which being multiplied by itſelf produces the ſaid number. Thus, 2 is the ſquare root of 4, and 3 is the ſquare root of 9: for 2 multiplied by 2 produces 4, and 3 multiplied by 3 produces 9, &c.

ſame

ſame ſort of hole, muſt run with three times the velocity, which will require nine times the preſſure; and conſequently muſt be nine times as deep below the ſurface of the fluid: and ſo on.—To prove this by an experiment, let two pipes, as *C* and *g*, of equal ſized bores, be fixed into the ſide of the veſſel *A B*; the pipe *g* being four times as deep below the ſurface of the water at *b* in the veſſel as the pipe *C* is: and while theſe pipes run, let water be conſtantly poured into the veſſel, to keep the ſurface ſtill at the ſame height. Then, if a cup that holds a pint be ſo placed as to receive the water that ſpouts from the pipe *C*, and at the ſame moment a cup that holds a quart be ſo placed as to receive the water that ſpouts from the pipe *g*, both cups will be filled at the ſame time by their reſpective pipes.

Fig. 8.

The horizontal diſtance to which water will ſpout from pipes.

The horizontal diſtance, to which a fluid will ſpout from a horizontal pipe, in any part of the ſide of an upright veſſel below the ſurface of the fluid, is equal to twice the length of a perpendicular to the ſide of the veſſel, drawn from the mouth of the pipe to a ſemicircle deſcribed upon the altitude of the fluid: and therefore, the fluid will ſpout to the greateſt diſtance poſſible from a pipe, whoſe mouth is at the center of the ſemicircle; becauſe a perpendicular to its diameter (ſuppoſed parallel to the ſide of the veſſel) drawn from that point, is the longeſt that can poſſibly be drawn from any part of the diameter to the circumference of the ſemicircle. Thus, if the veſſel *AB* be full of water, the horizontal pipe *D* be in the middle of its ſide, and the ſemicircle *N d c b* be deſcribed upon *D* as a center, with the radius or ſemidiameter *D g N*, or *D f b*, the perpendicular *D d* to the diameter *N D b* is the longeſt that can be drawn from

Fig. 8.

from any part of the diameter to the circumference $Ndcb$. And if the veſſel be kept full, the jet G will ſpout from the pipe D, to the horizontal diſtance NM, which is double the length of the perpendicular Dd. If two other pipes, as C and E, be fixed into the ſide of the veſſel at equal diſtances above and below the pipe D, the perpendiculars Cc and Ee, from theſe pipes to the ſemicircle, will be equal; and the jets F and H ſpouting from them will each go to the horizontal diſtance NK; which is double the length of either of the equal perpendiculars Cc or Ee.

How water may be conveyed over hills and vallies. Fluids by their preſſure may be conveyed over hills and vallies in bended pipes, to any height not greater than the level of the ſpring from whence they flow. But when they are deſigned to be raiſed higher than the ſprings, forcing engines muſt be uſed; which ſhall be deſcribed when we come to treat of pumps.

The *ſyphon*. A *ſyphon*, generally uſed for decanting liquors, is a bended pipe, whoſe legs are of unequal lengths; and the ſhorteſt leg muſt always be put into the liquor intended to be decanted, that the perpendicular altitude of the column of liquor in the other leg may be longer than the column in the immerſed leg, eſpecially above the ſurface of the water. For, if both columns were equally high in that reſpect, the atmoſphere, which preſſes as much upward as downward, and therefore acts as much upward againſt the column in the leg that hangs without the veſſel, as it acts downward upon the ſurface of the liquor in the veſſel, would hinder the running of the liquor through the ſyphon, even though it were brought over the bended part by ſuction. So that there is nothing left to

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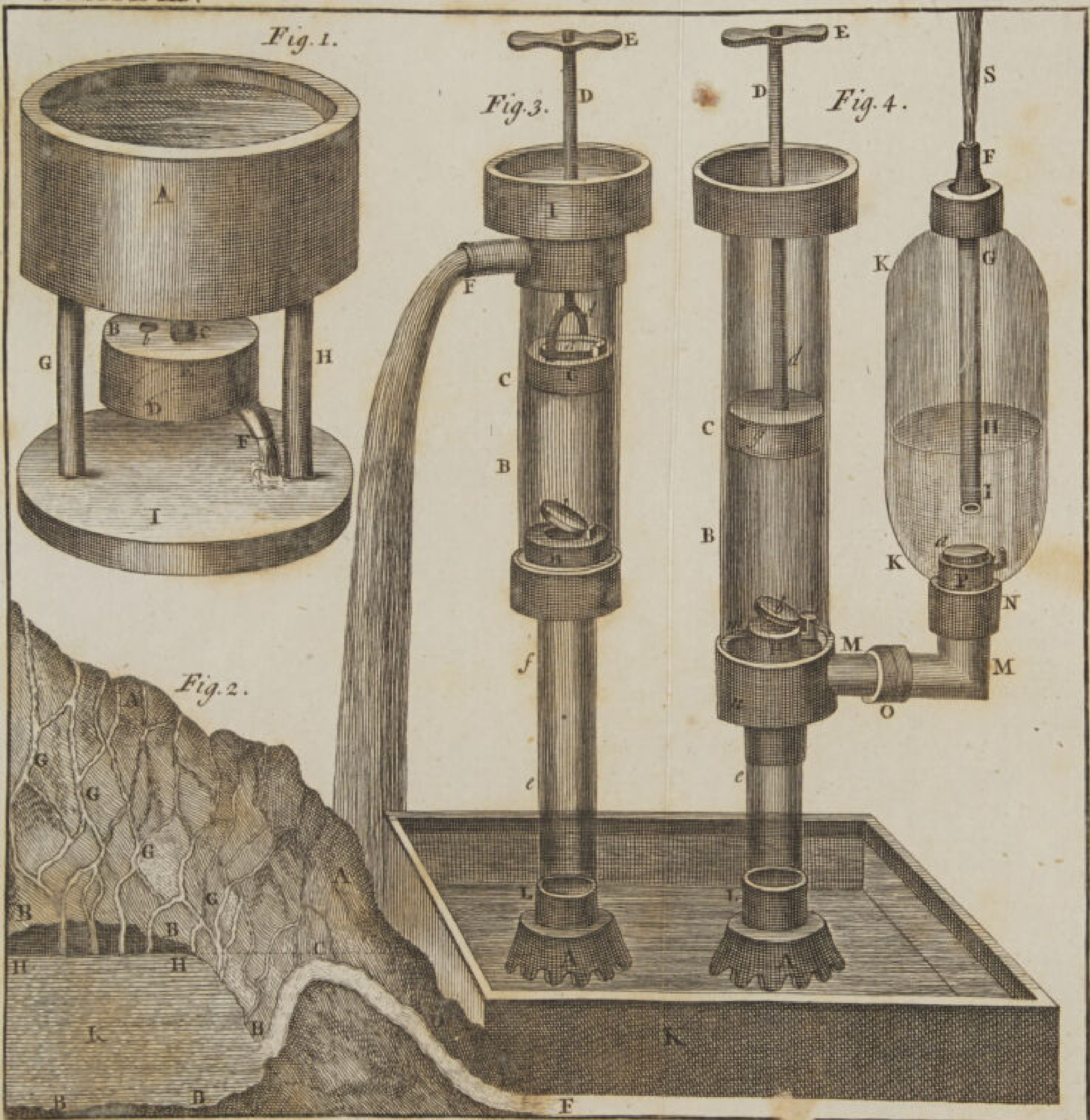
caufe the motion of the liquor, but the fuperior weight of the column, in the longer leg, on account of its having the greater perpendicular height.

Fig. 9. Let *D* be a cup filled with water to *C*, and *A B C* a fyphon, whofe fhorter leg *B C F* is immerfed in the water from *C* to *F*. If the end of the other leg were no lower than the line *A C*, which is level with the furface of the water, the fyphon would not run, even though the air fhould be drawn out of it at the mouth *A*. For although the fuction would draw fome water at firft, yet the water would ftop at the moment the fuction ceafed; becaufe the air would act as much upward againft the water at *A*, as it acted downward for it by preffing on the furface at *C*. But if the leg *A B* comes down to *G*, and the air be drawn out at *G* by fuction, the water will immediately follow, and continue to run, until the furface of the water in the cup comes down to *F*; becaufe, till then, the perpendicular height of the column *B A G* will be greater than that of the column *C B*; and confequently, its weight will be greater, until the furface comes down to *F*; and then the fyphon will ftop, though the leg *C F* fhould reach to the bottom of the cup. For which reafon, the leg that hangs without the cup is always made long enough to reach below the level of its bottom; as from *d* to *E*: and then, when the fyphon is emptied of air by fuction at *E*, the water immediately follows, and by its continuity brings away the whole from the cup; juft as pulling one end of a thread will make the whole clue follow.

If the perpendicular height of a fyphon, from the furface of the water to its bended top at *B*, be

PLATE XI.



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be more than 33 feet, it will draw no water, even though the other leg were much longer, and the ſyphon quite emptied of air; becauſe the weight of a column of water 33 feet high, is equal to the weight of as thick a column of air, reaching from the ſurface of the earth to the top of the atmoſphere; ſo that there will then be an equilibrium, and conſequently, though there would be weight enough of air upon the ſurface *C* to make the water aſcend in the leg *C B* almoſt to the height *B*, if the ſyphon were emptied of air, yet that weight would not be ſufficient to force the water over the bend; and therefore, it could never be brought over into the leg *B A G*.

Let a hole be made quite through the bottom of the cup *A*, and the longer leg of the bended ſyphon *D E B G* be cemented into the hole, ſo that the end *D* of the ſhorter leg *D E* may almoſt touch the bottom of the cup within. Fig. 10. *Tantalus's cup.*

Then, if water be poured into this cup, it will riſe in the ſhorter leg by its upward preſſure, driving out the air all the way before it through the longer leg: and when the cup is filled above the bend of the ſyphon at *F*, the preſſure of the water in the cup will force it over the bend of the ſyphon; and it will deſcend in the longer leg *C B G*, and run through the bottom, until the cup be emptied.

This is generally called *Tantalus's cup*, and the legs of the ſyphon in it are almoſt cloſe together; and a little hollow ſtatue, or figure of a man, is ſometimes put over the ſyphon to conceal it; the bend *E* being within the neck of the figure as high as the chin. So that poor thirſty *Tantalus* ſtands up to the chin in water, imagining it will riſe a little higher, and he may drink; but inftead of that, when the water comes up to his chin, it immediately begins to defcend, and fo, as he cannot ftoop to follow it, he is left as much pained with thirft as ever.

The fountain at command. Plate XI. Fig. 1. The device called *the fountain at command*, acts upon the fame principle with the fyphon in the cup. Let two veffels *A* and *B* be joined together by the pipe *C* which opens into them both. Let *A* be open at top, *B* clofe both at top and bottom, fave only a fmall hole at *b* to let the air get out of the veffel *B*, and *A* be of fuch a fize, as to hold about fix times as much water as *B*. Let a fyphon *D E F* be foldered to the veffel *B* at *e*, fo that the part *D E e* may be within the veffel, and *F* without it; the end *D* almoft touching the bottom of the veffel, and the end *F* below the level of *D*: the veffel *B* hanging to *A* by the pipe *C* (foldered into both) and the whole fupported by the pillars *G* and *H* upon the ftand *I*. The bore of the pipe muft be confiderably lefs than the bore of the fyphon.

The whole being thus conftructed, let the veffel *A* be filled with water, which will run through the pipe *C*, and fill the veffel *B*. When *B* is filled above the top of the fyphon at *E*, the water will run through the fyphon, and be difcharged at *F*. But as the bore of the fyphon is larger than the bore of the pipe, the fyphon will run fafter than the pipe, and will foon empty the veffel *B*; upon which the water will ceafe from running through the fyphon at *F*, until the pipe *C* refills the veffel *B*, and then it will begin to run as before. And thus the fyphon will continue to run and ftop alternately, until all the water in the veffel *A* has run

3 through

through the pipe *C*.—So that after a few trials, one may eaſily gueſs about what time the ſyphon will ſtop, and when it will begin to run: and then, to amuſe others, he may call out *ſtop*, or *run*, accordingly.

Upon this principle, we may eaſily account for *intermitting* or *reciprocrating ſprings*. Let *AA* be part of a hill, within which there is a cavity *BB*; and from this cavity a vein or channel running in the direction *BCDE*. The rain that falls upon the ſide of the hill will ſink and ſtrain through the ſmall pores and crannies *G*, *G*, *G*, *G*; and fill the cavity with water *K*. When the water riſes to the level *HHC*, the vein *BCDE* will be filled to *C*, and the water will run through *CDF* as through a ſyphon; which running will continue until the cavity be emptied, and then it will ſtop until the cavity be filled again. *Intermitting ſprings.* Fig. 2.

The *common pump* (improperly called the *ſucking pump*) with which we draw water out of wells, is an engine both pneumatic and hydraulic. It conſiſts of a pipe open at both ends, in which is a moveable piſton or bucket, as big as the bore of the pipe in that part wherein it works; and is leathered round, ſo as to fit the bore exactly; and may be moved up and down, without ſuffering any air to come between it and the pipe or pump barrel. The *common pump.*

We ſhall explain the conſtruction both of this and the forcing-pump by pictures of glaſs models, in which both the action of the piſtons and motion of the valves are ſeen.

Hold the model *DCBL* upright in the veſſel of water *K*, the water being deep enough to riſe at leaſt as high as from *A* to *L*. The valve *a* on the moveable bucket *G*, and the valve *b* Fig. 3.

I 2 on

on the fixed box *H* (which box quite fills the bore of the pipe or barrel at *H*) will each lie close, by its own weight, upon the hole in the bucket and box, until the engine begins to work. The valves are made of brafs, and lined underneath with leather for covering the holes the more clofely: and the bucket *G* is raifed and depreffed alternately by the handle *E* and rod *D d*, the bucket being fuppofed at *B* before the working begins.

Take hold of the handle *E*, and thereby draw up the bucket from *B* to *C*, which will make room for the air in the pump all the way below the bucket to dilate itfelf, by which its fpring is weakened, and then its force is not equivalent to the weight or preffure of the outward air upon the water in the veffel *K*: and therefore, at the firft ftroke, the outward air will prefs up the water through the notched foot *A*, into the lower pipe, about as far as *e*: this will condenfe the rarefied air in the pipe between *e* and *C* to the fame ftate it was in before; and then, as its fpring within the pipe is equal to the force or preffure of the outward air, the water will rife no higher by the firft ftroke; and the valve *b*, which was raifed a little by the dilatation of the air in the pipe, will fall, and ftop the hole in the box *H*; and the furface of the water will ftand at *e*. Then, deprefs the pifton or bucket from *C* to *B*, and as the air in the part *B* cannot get back again through the valve *b*, it will (as the bucket defcends) raife the valve *a*, and fo make its way through the upper part of the barrel *d* into the open air. But upon raifing the bucket *G* a fecond time, the air between it and the water in the lower pipe at *a* will be again left at liberty to

2 fill

fill a larger ſpace; and ſo its ſpring being again weakened, the preſſure of the outward air on the water in the veſſel *K* will force more water up into the lower pipe from *e* to *f*; and when the bucket is at its greateſt height *C*, the lower valve *b* will fall, and ſtop the hole in the box *H* as before. At the next ſtroke of the bucket or piſton, the water will riſe through the box *H* toward *B*, and then the valve *b*, which was raiſed by it, will fall when the bucket *G* is at its greateſt height. Upon depreſſing the bucket again, the water cannot be puſhed back through the valve *b*, which keeps cloſe upon the hole while the piſton deſcends. And upon raiſing the piſton again, the outward preſſure of the air will force the water up through *H*, where it will raiſe the valve, and follow the bucket to *C*. Upon the next depreſſion of the bucket *G*, it will go down into the water in the barrel *B*; and as the water cannot be driven back through the new cloſe valve *b*, it will raiſe the valve *a* as the bucket deſcends, and will be lifted up by the bucket when it is next raiſed. And now, the whole ſpace below the bucket being full, the water above it cannot ſink when it is next depreſſed; but upon its depreſſion, the valve *a* will riſe to let the bucket go down; and when it is quite down, the valve *a* will fall by its weight, and ſtop the hole in the bucket. When the bucket is next raiſed, all the water above it will be lifted up, and begin to run off by the pipe *F*. And thus, by raiſing and depreſſing the bucket alternately, there is ſtill more water raiſed by it; which getting above the pipe *F*, into the wide top *I*, will ſupply the pipe, and make it run with a continued ſtream.

So, at every time the bucket is raiſed, the valve *b* riſes, and the valve *a* falls; and at every time the bucket is depreſſed, the valve *b* falls, and *a* riſes.

As it is the preſſure of the air or atmoſphere which cauſes the water to riſe, and follow the piſton or bucket *G* as it is drawn up; and ſince a column of water 33 feet high is of equal weight with as thick a column of the atmoſphere, from the earth to the very top of the air; therefore, the perpendicular height of the piſton or bucket from the ſurface of the water in the well muſt always be leſs than 33 feet; otherwiſe the water will never get above the bucket. But, when the height is leſs, the preſſure of the atmoſphere will be greater than the weight of the water in the pump, and will therefore raiſe it above the bucket: and when the water has once got above the bucket, it may be lifted thereby to any height, if the rod *D* be made long enough, and a ſufficient degree of ſtrength be employed, to raiſe it with the weight of the water above the bucket.

The force required to work a pump will be as the height to which the water is raiſed, and as the ſquare of the diameter of the pump-bore, in that part where the piſton works. So that, if two pumps be of equal heights, and one of them be twice as wide in the bore as the other, the wideſt will raiſe four times as much water as the narroweſt; and will therefore require four times as much ſtrength to work it.

The wideneſs or narrowneſs of the pump, in any other part beſide that in which the piſton works, does not make the pump either more or leſs difficult to work, except what difference may ariſe from the friction of the water in the bore;

bore; which is always greater in a narrow bore than in a wide one, becaufe of the greater velocity of the water.

The pump-rod is never raifed directly by fuch a handle as *E* at the top, but by means of a lever, whofe longer arm (at the end of which the power is applied) generally exceeds the length of the fhorter arm five or fix times; and, by that means, it gives five or fix times as much advantage to the power. Upon thefe principles, it will be eafy to find the dimenfions of a pump that fhall work with a given force, and draw water from any given depth. But, as thefe calculations have been generally neglected by pump-makers (either for want of fkill or induftry) the following table was calculated by the late ingenious Mr. *Booth* for their benefit*. In this calculation, he fuppofed the handle of the pump to be a lever increafing the power five times; and had often found that a man can work a pump four inches diameter, and 30 feet high, and difcharge 27½ gallons of water (Englifh wine meafure) in a minute. Now, if it be required to find the diameter of a pump, that fhall raife water with the fame eafe from any other height above the furface of the *well*; look for that height in the firft column, and overagainft it in the fecond you have the diameter or width of the pump; and in the third, you find the quantity of water which a man of ordinary ftrength can difcharge in a minute.

* I have taken the liberty to make a few alterations in Mr. *Booth*'s numbers in the table, and to lengthen it out from 80 feet to 100.

Height of the pump above the surface of the well,	Diameter of the bore where the bucket works.		Water discharged in a minute, English wine measure.	
Feet.	Inches.	100 parts.	Gallons.	Pints.
10	6	.93	81	6
15	5	.66	54	4
20	4	.90	40	7
25	4	.38	32	6
30	4	.00	27	2
35	3	.70	23	3
40	3	.46	20	3
45	3	.27	18	1
50	3	.10	16	3
55	2	.95	14	7
60	2	.84	13	5
65	2	.72	12	4
70	2	.62	11	5
75	2	.53	10	7
80	2	.45	10	2
85	2	.38	9	5
90	2	.31	9	1
95	2	.25	8	5
100	2	.19	8	1

The *forcing pump.* Fig. 4.

The *forcing pump* raises water through the box *H* in the same manner as the common pump does, when the plunger or piston *g* is lifted up by the rod *D d*. But this plunger has no hole through it, to let the water in the barrel *B C* get above it, when it is depressed to *B*, and the valve *b* (which rose by the ascent of the water

water through the box *H* when the plunger *g* was drawn up) falls down and ſtops the hole in *H*, the moment that the plunger is raiſed to its greateſt height. Therefore, as the water between the plunger *g* and box *H* can neither get through the plunger upon its deſcent, nor back again into the lower part of the pump *L e*, but has a free paſſage by the cavity around *H* into the pipe *M M*, which opens into the air-veſſel *K K* at *P*; the water is forced through the pipe *M M* by the deſcent of the plunger, and driven into the air-veſſel; and in running up through the pipe at *P*, it opens the valve *a*; which ſhuts at the moment the plunger begins to be raiſed, becauſe the action of the water againſt the under ſide of the valve then ceaſes.

The water, being thus forced into the air-veſſel *K K* by repeated ſtrokes of the plunger, gets above the lower end of the pipe *G H I*, and then begins to condenſe the air in the veſſel *K K*. For, as the pipe *G H* is fixed air-tight into the veſſel below *F*, and the air has no way to get out of the veſſel but through the mouth of the pipe at *I*, and cannot get out when the mouth *I* is covered with water, and is more and more condenſed as the water riſes upon the pipe, the air then begins to act forcibly by its ſpring againſt the ſurface of the water at *H*: and this action drives the water up through the pipe *I H G F*, from whence it ſpouts in a jet *S* to a great height; and is ſupplied by alternately raiſing and depreſſing of the plunger *g*, which conſtantly forces the water that it raiſes through the valve *H*, along the pipe *M M*, into the air-veſſel *K K*.

The higher that the ſurface of the water *H* is raiſed in the air-veſſel, the leſs ſpace will the air

air be condenſed into, which before filled that veſſel; and therefore the force of its ſpring will be ſo much the ſtronger upon the water, and will drive it with the greater force through the pipe at *F:* and as the ſpring of the air continues while the plunger *g* is riſing, the ſtream or jet *S* will be uniform, as long as the action of the plunger continues: and when the valve *b* opens, to let the water follow the plunger upward, the valve *a* ſhuts, to hinder the water, which is forced into the air-veſſel, from running back by the pipe *MM* into the barrel of the pump.

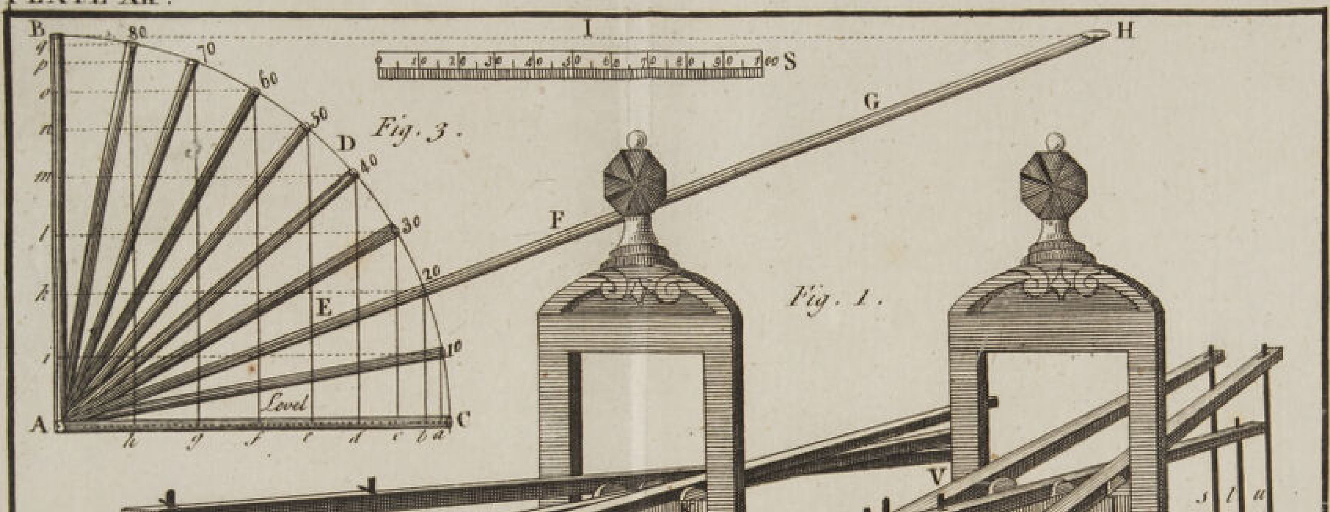
If there was no air-veſſel to this engine, the pipe *GHI* would be joined to the pipe *MMN* at *P*; and then, the jet *S* would ſtop every time the plunger is raiſed, and run only when the plunger is depreſſed.

Mr. *Newſham's* water-engine, for extinguiſhing fire, conſiſts of two forcing pumps, which alternately drive water into a cloſe veſſel of air; and by forcing the water into that veſſel, the air in it is thereby condenſed, and compreſſes the water ſo ſtrongly, that it ruſhes out with great impetuoſity and force through a pipe that comes down into it; and makes a continued uniform ſtream by the condenſation of the air upon its ſurface in the veſſel.

By means of forcing-pumps, water may be raiſed to any height above the level of a river or ſpring; and machines may be contrived to work theſe pumps, either by a running ſtream, a fall of water, or by horſes. An inſtance in each ſort will be ſufficient to ſhew the method.

Plate XII. Fig. 1. Firſt, by a running ſtream, or a fall of water. Let *AA* be a wheel, turned by the fall of water *BB*; and have any number of cranks (ſup-

PLATE XII.



J. Ferguson delin. J. Mynde sculp.

(ſuppoſe fix) as *C, D, E, F, G, H,* on its axis, according to the ſtrength of the fall of water, and the height to which the water is intended to be raiſed by the engine. As the wheel turns round, theſe cranks move the levers *c, d, e, f, g, h,* up and down, by the iron rods *i, k, l, m, n, o;* which alternately raiſe and depreſs the piſtons by the other iron rods *p, q, r, ſ, t, u, w, x, y,* in twelve pumps; nine whereof, as *L, M, N, O, P, Q, R, S, T,* appear in the plate; the other three being hid behind the work at *V.* And as pipes may go from all theſe pumps, to convey the water (drawn up by them to a ſmall height) into a cloſe ciſtern, from which the main pipe goes off, the water will be forced into this ciſtern by the deſcent of the piſtons. And as each pipe, going from its reſpective pump into the ciſtern, has a valve at its end in the ciſtern, theſe valves will hinder the return of the water by the pipes; and therefore, when the ciſtern is once full, each piſton upon its deſcent will force the water (conveyed into the ciſtern by a former ſtroke) up the main pipe, to the height the engine was intended to raiſe it: which height depends upon the quantity raiſed, and the power that turns the wheel. When the power upon the wheel is leſſened by any defect of the quantity of water turning it, a proportionable number of the pumps may be ſet aſide, by diſengaging their rods from the vibrating levers.

A pump engine to go by water.

This figure is a repreſentation of the engine erected at *Blenheim* for the Duke of *Marlborough,* by the late ingenious Mr. *Alderſea.* The water-wheel is 7½ feet in diameter, according to Mr. *Switzer*'s account in his Hydraulics.

When ſuch a machine is placed in a ſtream that runs upon a ſmall declivity, the motion of the

the levers and action of the pumps will be but flow; fince the wheel muft go once round for each ftroke of the pumps. But, when there is a large body of flow running water, a cog or fpur-wheel may be placed upon each fide of the water-wheel *A A*, upon its axis, to turn a trundle upon each fide; the cranks being upon the axis of the trundle. And by proportioning the cog-wheels to the trundles, the motion of the pumps may be made quicker, according to the quantity and ftrength of the water upon the firft wheel; which may be as great as the workman pleafes; according to the length and breadth of the float-boards or wings of the wheel. In this manner, the engine for raifing water at *London-Bridge* is conftructed; in which, the water-wheel is 20 feet diameter, and the floats 14 feet long.

A pump engine to go by horfes. Where a ftream or fall of water cannot be had, and gentlemen want to have water raifed, and brought to their houfes from a rivulet or fpring; this may be effected by a horfe-engine, working Fig. 2. three forcing pumps which ftand in a refervoir filled by the fpring or rivulet: the piftons being moved up and down in the pumps by means of a triple crank *A B C*, which, as it is turned round by the trundle *G*, raifes and depreffes the rods *D*, *E*, *F*. The trundle may be turned by fuch a wheel as *F* in Fig. 1. of Plate VIII. having levers *y*, *y*, *y*, *y*, on its upright axle, to which horfes may be joined for working the engine. And if the wheel has three times as many cogs as the trundle has ftaves or rounds, the trundle and cranks will make three revolutions for every one of the wheel: and as each crank will fetch a ftroke in the time it goes round, the three cranks will make nine ftrokes for every turn of the great wheel.

The

The cranks ſhould be made of caſt iron, becauſe *that* will not bend; and they ſhould each make an angle of 120 with both of the others, as at *a*, *b*, *c*; which is (as it were) a view of their *radii*, in looking endwiſe at the axis: and then there will be always one or other of them going downward, which will puſh the water forward with a continued ſtream into the main pipe. For, when *b* is almoſt at its loweſt poſition, and is therefore juſt beginning to loſe its action upon the piſton which it moves, *c* is beginning to move downward, which will by its piſton continue the propelling force upon the water: and when *c* is come down to the poſition of *b*, *a* will be in the poſition of *c*.

Plate XII. Fig. 2.

The more perpendicularly the piſton rods move up and down in the pumps, the freer and better will their ſtrokes be: but a little deviation from the perpendicular will not be material. Therefore, when the pump-rods *D*, *E*, and *F*, go down into a deep well, they may be moved directly by the cranks, as is done in a very good horſe-engine of this ſort at the late Sir *James Creed*'s at *Greenwich*, which forces up water about 64 feet from a well under ground, to a reſervoir on the top of his houſe. But when the cranks are only at a ſmall height above the pumps, the piſtons muſt be moved by vibrating levers, as in the above engine at *Blenheim*: and the longer the levers are, the nearer will the ſtrokes be to a perpendicular.

The quantity of water that may be raiſed by a horſe-engine.

Let us ſuppoſe, that in ſuch an engine as Sir *James Creed*'s, the great wheel is 12 feet diameter, the trundle 4 feet, and the radius or length of each crank 9 inches, working a piſton in its pump. Let there be three pumps in all, and the bore of each pump be four inches diameter. Then,

Then, if the great wheel has three times as many cogs as the trundle has ſtaves, the trundle and cranks will go three times round for each revolution of the horſes and wheel, and the three cranks will make nine ſtrokes of the pumps in that time, each ſtroke being 18 inches (or double the length of the crank) in a four-inch bore. Let the diameter of the horſe-walk be 18 feet, and the perpendicular height to which the water is raiſed above the ſurface of the well be 64 feet.

If the horſes go at the rate of two miles a hour (which is very moderate walking) they will turn the great wheel 187 times round in a hour.

In each turn of the wheel the piſtons make 9 ſtrokes in the pumps, which amount to 1683 in a hour.

Each ſtroke raiſes a column of water 18 inches long, and four inches thick, in the pump-barrels; which column, upon the deſcent of the piſton, is forced into the main pipe, whoſe perpendicular altitude above the ſurface of the well is 64 feet.

Now, ſince a column of water 18 inches long, and 4 inches thick, contains 226.18 cubic inches, this number multiplied by 1683 (the ſtrokes in a hour) gives 380661 for the number of cubic inches of water raiſed in a hour.

A gallon, in wine meaſure, contains 231 cubic inches, by which divide 380661, and it quotes 1468 in round numbers, for the number of gallons raiſed in a hour; which, divided by 63, gives $26\frac{1}{2}$ hogſheads.—If the horſes go faſter, the quantity raiſed will be ſo much the greater.

In this calculation it is ſuppoſed that no water is waſted by the engine. But as no forcing engine can be ſuppoſed to loſe leſs than a fifth part

part of the calculated quantity of water, between the piſtons and barrels, and by the opening and ſhutting of the valves, the horſes ought to walk almoſt $2\frac{1}{2}$ miles *per* hour, to fetch up this loſs.

A column of water 4 inches thick, and 64 feet high, weighs $349\frac{9}{18}$ pounds avoirdupoiſe, or $424\frac{5}{12}$ pounds troy; and this weight, together with the friction of the engine, is the reſiſtance that muſt be overcome by the ſtrength of the horſes.

The horſe-tackle ſhould be ſo contrived, that the horſes may rather puſh on than drag the levers after them. For if they draw, in going round the walk, the outſide leather ſtraps will rub againſt their ſides and hams; which will hinder them from drawing at right angles to the levers, and ſo make them pull at a diſadvantage. But if they puſh the levers before their breaſts, inſtead of dragging them, they can always walk at right angles to theſe levers.

It is no ways material what the diameter of the main or conduct pipe be: for the whole reſiſtance of the water therein, againſt the horſes, will be according to the height to which it is raiſed, and the diameter of that part of the pump in which the piſton works, as we have already obſerved. So that by the ſame pump, an equal quantity of water may be raiſed in (and conſequently made to run from) a pipe of a foot diameter, with the ſame eaſe as in a pipe of five or ſix inches: or rather with more eaſe, becauſe its velocity in a large pipe will be leſs than in a ſmall one; and therefore its friction againſt the ſides of the pipe will be leſs alſo.

And the force required to raiſe water depends not upon the length of the pipe, but upon the perpendicular height to which it is raiſed therein above the level of the ſpring. So that the ſame force,

force, which would raiſe water to the height *AB* in the upright pipe *Aiklmnopq B*, will raiſe it to the ſame height or level *B I H* in the oblique pipe *AEFGH.* For the preſſure of the water at the end *A* of the latter, is no more than its preſſure againſt the end *A* of the former.

Fig. 3.

The weight or preſſure of water at the lower end of the pipe, is always as the ſine of the angle to which the pipe is elevated above the level parallel to the horizon. For, although the water in the upright pipe *AB* would require a force applied immediately to the lower end *A* equal to the weight of all the water in it, to ſupport the water, and a little more to drive it up, and out of the pipe; yet, if that pipe be inclined from its upright poſition to an angle of 80 degrees (as in *A* 80) the force required to ſupport or to raiſe the ſame cylinder of water will then be as much leſs, as the ſine 80 *b* is leſs than the radius *AB*; or as the ſine of 80 degrees is leſs than the ſine of 90. And ſo, decreaſing as the ſine of the angle of elevation leſſens, until it arrives at its level *AC* or place of reſt, where the force of the water is nothing at either end of the pipe. For, although the abſolute weight of the water is the ſame in all poſitions, yet its preſſure at the lower end decreaſes, as the ſine of the angle of elevation decreaſes; as will appear plainly by a farther conſideration of the figure.

Let two pipes, *AB* and *AC*, of equal lengths and bores, join each other at *A*; and let the pipe *AB* be divided into 100 equal parts, as the ſcale *S* is; whoſe length is equal to the length of the pipe.—Upon this length, as a radius, deſcribe the quadrant *BCD*, and divide it into 90 equal parts or degrees.

Let the pipe *AC* be elevated to 10 degrees upon

upon the quadrant, and filled with water; then, part of the water that is in it will riſe in the pipe *A B*, and if it be kept full of water, it will raiſe the water in the pipe *A B* from *A* to *i*; that is, to a level *i* 10 with the mouth of the pipe at 10: and the upright line *a* 10, equal to *A i*, will be the ſine of 10 degrees elevation; which being meaſured upon the ſcale *S*, will be about 17.4 of ſuch parts as the pipe contains 100 in length: and therefore, the force or preſſure of the water at *A*, in the pipe *A* 10, will be to the force or preſſure at *A* in the pipe *A B*, as 17.4 to 100.

Let the ſame pipe be elevated to 20 degrees in the quadrant, and if it be kept full of water, part of that water will run into the pipe *A B*, and riſe therein to the height *A k*, which is equal to the length of the upright line *b* 20, or to the ſine of 20 degrees elevation; which, being meaſured upon the ſcale *S*, will be 34.2 of ſuch parts as the pipe contains 100 in length. And therefore, the preſſure of the water at *A*, in the full pipe *A* 20, will be to its preſſure, if that pipe were raiſed to the perpendicular ſituation *A B*, as 34.2 to 100.

Elevate the pipe to the poſition *A* 30 on the quadrant, and if it be ſupplied with water, the water will riſe from it, into the pipe *A B*, to the height *A l*, or to the ſame level with the mouth of the pipe at 30. The ſine of this elevation, or of the angle of 30 degrees, is *c* 30; which is juſt equal to half the length of the pipe, or to 50 of ſuch parts of the ſcale, as the length of the pipe contains 100. Therefore, the preſſure of the water at *A*, in a pipe elevated 30 degrees above the horizontal level, will be equal to one half of what it would be, if the ſame pipe ſtood upright in the ſituation *A B*.

And thus, by elevating the pipe to 40, 50, 60, 70, and 80 degrees on the quadrant, the ſines of theſe elevations will be *d* 40, *e* 50, *f* 60, *g* 70, and *h* 80; which will be equal to the heights *Am*, *An*, *Ao*, *Ap*, and *Aq*: and theſe

Sine of	Parts	Sine of	Parts	Sine of	Parts
D.1	17	D.31	515	D.61	875
2	35	32	530	62	883
3	52	33	545	63	891
4	70	34	559	64	899
5	87	35	573	65	906
6	104	36	588	66	913
7	122	37	602	67	920
8	139	38	616	68	927
9	156	39	629	69	934
10	174	40	643	70	940
11	191	41	656	71	945
12	208	42	669	72	951
13	225	43	682	73	956
14	242	44	695	74	961
15	259	45	707	75	966
16	276	46	719	76	970
17	292	47	731	77	974
18	309	48	743	78	978
19	325	49	755	79	982
20	342	50	766	80	985
21	358	51	777	81	988
22	375	52	788	82	990
23	391	53	799	83	992
24	407	54	809	84	994
25	423	55	819	85	996
26	438	56	829	86	997
27	454	57	839	87	998
28	469	58	848	88	999
29	485	59	857	89	1000
30	500	60	866	90	1000

heights

heights meaſured upon the ſcale *S* will be 64.[illegible], 76.6, 86.6, 94.0, and 98.5; which expreſs the preſſures at *A* in all theſe elevations, conſidering the preſſure in the upright pipe *AB* as 100.

Becauſe it may be of uſe to have the lengths of all the ſines of a quadrant from 0 degrees to 90, we have given the foregoing table, ſhewing the length of the ſine of every degree in ſuch parts as the whole pipe (equal to the radius of the quadrant) contains 1000. Then the ſines will be integral or whole parts in length. But if you ſuppoſe the length of the pipe to be divided only into 100 equal parts, the laſt figure of each part or ſine muſt be cut off as a decimal; and then thoſe which remain at the left hand of this ſeparation will be integral or whole parts.

Thus, if the radius of the quadrant (ſuppoſed to be equal to the length of the pipe *AC*) be divided into 1000 equal parts, and the elevation be 45 degrees, the ſine of that elevation will be equal to 707 of theſe parts: but if the radius be divided only into 100 equal parts, the ſame ſine will be only 70.7 or $70\frac{7}{10}$ of theſe parts. For, as 1000 is to 707, ſo is 100 to 70.7.

As it is of great importance to all engine-makers, to know what quantity and weight of water will be contained in an upright round pipe of a given diameter and height; ſo as by knowing what weight is to be raiſed, they may proportion their engines to the force which they can afford to work them; we ſhall ſubjoin tables ſhewing the number of cubic inches of water contained in an upright pipe of a round bore, of any diameter from one inch to ſix and

 a half;

a half; and of any height from one foot to two hundred: together with the weight of the said number of cubic inches, both in troy and avoirdupoise ounces. The number of cubic inches divided by 231, will reduce the water to gallons in wine measure; and divided by 282, will reduce it to the measure of ale gallons. Also, the troy ounces divided by 12, will reduce the weight to troy pounds: and the avoirdupoise ounces divided by 16, will reduce the weight to avoirdupoise pounds.

And here I must repeat it again, that the weight or pressure of the water acting against the power that works the engine, must always be estimated according to the perpendicular height to which it is to be raised, without any regard to the length of the conduct-pipe, when it has an oblique position; and as if the diameter of that pipe were just equal to the diameter of that part of the pump in which the piston works. Thus, by the following tables, the pressure of the water, against an engine whose pump is of a $4\frac{1}{2}$ inch bore, and the perpendicular height of the water in the conduct-pipe is 80 feet, will be equal to 8057.5 troy ounces, and to 8848.2 avoirdupoise ounces; which makes 671.4 troy pounds, and 553 avoirdupoise.

For any bore whose diameter exceeds $6\frac{1}{2}$ inches, multiply the numbers on the following page, against any height (belonging to 1 inch diameter) by the square of the diameter of the given bore, and the products will be the number of cubic inches, troy ounces, and avoirdupoise ounces of water, that the given bore will contain.

1 Inch

1 Inch diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoir-dupoise ounces.
1	9.42	4.97	5.46
2	11.85	9.95	10.92
3	28.27	14.92	16.38
4	37.70	19.89	21.85
5	47.12	24.87	27.31
6	56.55	29.84	32.77
7	65.97	34.82	38.23
8	75.40	39.79	43.69
9	84.82	44.76	49.16
10	94.25	49.74	54.62
20	188.49	99.48	109.24
30	282.74	149.21	163.86
40	376.99	198.95	218.47
50	471.24	248.69	273.09
60	565.49	298.43	327.71
70	659.73	348.17	382.33
80	753.98	397.90	436.95
90	848.23	447.64	491.57
100	942.48	497.38	546.19
200	1884.96	994.76	1092.38

EXAMPLE, *Required the number of cubic inches, and the weight of the water, in an upright pipe 278 feet high, and $1\frac{1}{2}$ inch diameter?*

Here the nearest single decimal figure is only taken into the account: and the whole being reduced by division, amounts to $25\frac{1}{2}$ wine gallons in measure; to $259\frac{1}{4}$ pounds troy, and to $213\frac{1}{2}$ pounds avoirdupoise.

Feet	Cubic inches	Troy oz.	Avoird. oz.
200	4241.1	2238.2	2457.8
70	1484.4	783.3	860.2
8	169.6	89.5	98.3
Ans. 278	5895.1	3111.0	3416.3

1 ½ Inch diameter.			
Feet high.	Quantity in cubic inches,	Weight in troy ounces.	In avoirdupoise ounces.
1	21.21	11.19	12.29
2	42.41	22.38	24.58
3	63.62	33.57	36.87
4	84.82	44.76	49.16
5	106.03	55.95	61.45
6	127.23	67.15	73.73
7	147.44	78.34	86.02
8	169.65	89.53	98.31
9	190.85	100.72	110.60
10	212.06	111.91	122.89
20	424.12	223.82	245.78
30	636.17	335.73	368.68
40	848.23	447.64	491.57
50	1060.29	559.55	614.46
60	1272.35	671.46	737.35
70	1484.40	783.37	860.24
80	1696.46	895.28	983.14
90	1908.52	1007.19	1106.03
100	2120.58	1119.10	1228.92
200	4241.15	2238.20	2457.84

These tables were at first calculated to fix decimal places for the sake of exactness; but in transcribing them there are no more than two decimal figures taken into the account, and sometimes but one; because there is no necessity for

2 Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoise ounces.
1	37.70	19.89	21.85
2	75.40	39.79	43.69
3	113.10	59.68	65.54
4	150.80	79.58	87.39
5	188.50	99.47	109.24
6	226.19	119.37	131.08
7	263.89	139.26	152.93
8	301.59	159.16	174.78
9	339.29	179.06	196.63
10	376.99	198.95	218.47
20	753.98	397.90	436.95
30	1130.97	596.85	655.42
40	1507.97	795.80	873.90
50	1884.96	994.75	1092.37
60	2261.95	1193.70	1310.85
70	2638.94	1392.65	1529.32
80	3015.93	1591.60	1747.80
90	3392.92	1790.56	1966.27
100	3769.91	1989.51	2184.75
200	7539.82	3979.00	4369.50

for computing to hundredth parts of an inch or of an ounce in practice. And as they never appeared in print before, it may not be amiſs to give the reader an account of the principles upon which they were conſtructed.

2½ Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoir-dupoise ounces.
1	58.90	31.08	34.14
2	117.81	62.17	68.27
3	176.71	93.26	102.41
4	235.62	124.34	136.55
5	294.52	155.43	170.68
6	353.43	186.52	204.82
7	412.33	217.60	238.96
8	471.24	248.69	273.09
9	530.14	279.77	307.23
10	589.05	310.86	341.37
20	1178.10	621.72	682.73
30	1767.15	932.58	1024.10
40	2356.20	1243.44	1365.47
50	2545.25	1554.30	1706.83
60	3534.29	1865.16	2048.20
70	4123.34	2176.02	2389.57
80	4712.39	2486.88	2730.94
90	5301.44	2797.74	3072.30
100	5890.49	3108.60	2413.67
200	11780.98	6217.20	4827.34

The folidity of cylinders are found by multiplying the areas of their bafes by their altitudes. And ARCHIMEDES gives the following proportion for finding the area of a circle, and the folidity of a cylinder raifed upon that circle:

As

3 Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoise ounces.
1	84.8	44.76	49.16
2	169.6	89.53	98.31
3	254.5	134.29	147.47
4	239.3	179.06	196.63
5	424.1	223.82	245.78
6	508.9	268.58	294.94
7	593.7	313.35	344.10
8	698.6	358.11	393.25
9	763.4	402.87	442.41
10	848.2	447.64	491.57
20	1696.5	895.28	983.14
30	2544.7	1342.92	1474.70
40	3392.9	1790.56	1966.27
50	4241.1	2238.19	2457.84
60	5089.4	2685.83	2949.41
70	5937.6	3133.47	3440.98
80	6785.8	3581.11	3932.55
90	7634.1	4028.75	4424.12
100	8482.3	4476.39	4915.68
200	16964.6	8952.78	9831.36

As 1 is to 0.785399, so is the square of the diameter to the area of the circle. And as 1 is to 0.785399, so is the square of the diameter multiplied by the height to the solidity of the cylinder. By this analogy the solid inches and parts

3½ Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoise ounces.
1	115.4	60.9	66.9
2	230.9	121.8	133.8
3	346.4	182.8	200.7
4	461.8	243.7	267.6
5	577.3	304.6	334.5
6	692.7	365.6	401.4
7	808.2	426.5	468.4
8	923.6	487.4	535.3
9	1039.1	548.4	602.2
10	1154.5	609.3	669.1
20	2309.1	1218.6	1338.2
30	3463.6	1827.9	2007.2
40	4618.1	2437.1	2676.3
50	5772.7	3046.4	3345.4
60	6927.2	3655.7	4014.5
70	8081.8	4265.0	4683.6
80	9236.3	4874.3	5352.6
90	10390.8	5483.6	6021.7
100	11545.4	6092.9	6690.8
200	23090.7	12185.7	13381.5

parts of an inch in the tables are calculated to a cylinder 200 feet high, of any diameter from 1 inch to 6½, and may be continued at pleasure.

And as to the weight of a cubic foot of running water, it has been often found upon trial, by Dr.

4 Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoise ounces.
1	150.8	79.6	87.4
2	301.6	159.2	174.8
3	452.4	238.7	262.2
4	603.2	318.3	349.6
5	754.0	397.9	436.9
6	904.8	477.5	524.3
7	1055.6	557.1	611.7
8	1206.4	636.6	699.1
9	1357.2	716.2	786.5
10	1508.0	795.8	873.9
20	3115.9	1591.6	1747.8
30	4523.9	2387.4	2621.7
40	6031.9	3183.2	3495.6
50	7539.8	3997.0	4369.5
60	9047.8	4774.8	5243.4
70	10555.8	5570.6	6117.3
80	12063.7	6366.4	6991.2
90	13571.7	7162.2	7865.1
100	15079.7	7958.0	8739.0
200	30159.3	15916.0	17478.0

Dr. *Wyberd* and others, to be 76 pounds troy, which is equal to 62.5 pounds avoirdupoise. Therefore, since there are 1728 cubic inches in a cubic foot, a troy ounce of water contains 1.8949 cubic inch; and an avoirdupoise

The weight of running water.

4½ Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoir-dupoise ounces.
1	190.8	100.7	110.6
2	381.7	201.4	221.2
3	572.6	302.2	331.8
4	763.4	402.9	442.4
5	954.3	503.6	553.0
6	1145.1	604.3	663.6
7	1338.0	705.0	774.2
8	1526.8	805.7	884.8
9	1717.7	906.5	995.4
10	1908.5	1007.2	1106.0
20	3817.0	2014.4	2212.1
30	5725.6	3021.6	3818.1
40	7634.1	4028.7	4424.1
50	9542.6	5035.9	5530.1
60	11451.1	6043.1	6636.2
70	13359.6	7050.3	7742.2
80	15268.2	8057.5	8848.2
90	17176.7	9064.7	9954.3
100	19085.2	10071.9	11060.3
200	38170.4	20143.8	22120.6

ounce of water 1.72556 cubic inch. Consequently, if the number of cubic inches contained in any given cylinder, be divided by 1.8949, it will give the weight in troy ounces; and divided by 1.72556, will give the weight in

5 Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoise ounces.
1	235.6	124.3	136.5
2	471.2	248.7	273.1
3	706.6	373.0	409.6
4	942.5	497.4	546.2
5	1178.1	621.7	682.7
6	1413.7	746.1	819.3
7	1649.3	870.4	955.8
8	1885.0	994.8	1092.4
9	2120.6	1119.1	1228.9
10	2356.2	1243.4	1365.5
20	4712.4	2486.9	2730.9
30	7068.6	3730.3	4096.4
40	9424.8	4973.8	5461.9
50	11780.0	6217.2	6827.3
60	14137.2	7460.6	8192.8
70	16493.4	8704.1	9558.3
80	18849.6	9947.5	10923.7
90	21205.8	11191.0	12289.2
100	23562.0	12434.4	13654.7
200	47124.0	24868.8	27309.3

n avoirdupoise ounces. By this method, the veights shewn in the tables were calculated; and re near enough for any common practice.

The *fire-engine* comes next in order to be explained; but as it would be difficult, even by the The *fire-engine*.

5½ Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoise ounces.
1	285.1	150.5	164.3
2	570.2	300.9	328.5
3	855.3	451.4	492.8
4	1140.4	601.8	657.1
5	1425.5	752.3	821.3
6	1710.6	902.7	985.6
7	1995.7	1053.2	1149.9
8	2280.8	1203.6	1314.2
9	2565.9	1354.1	1478.4
10	2851.0	1504.6	1642.7
20	5702.0	3009.1	3285.4
30	8553.0	4513.7	4928.1
40	11404.0	6018.2	6570.8
50	14255.0	7522.8	8213.5
60	17106.0	9027.4	9856.2
70	19957.0	10531.9	11498.9
80	22808.0	12036.5	13141.6
90	25659.0	13541.1	14784.3
100	28510.0	15045.6	16426.9
200	57020.0	30091.2	32853.9

the beſt plates, to give a particular deſcription of its ſeveral parts, ſo as to make the whole intelligible, I ſhall only explain the principles upon which it is conſtructed.

1. What-

6 Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoise ounces.
1	339.3	179.1	196.6
2	678.6	358.1	393.3
3	1017.9	537.2	589.9
4	1357.2	716.2	786.5
5	1696.5	895.3	983.1
6	2035.7	1074.3	1179.8
7	2375.0	1253.4	1376.4
8	2714.3	1432.4	1573.0
9	3053.6	1611.5	1769.6
10	3392.9	1790.6	1966.3
20	6785.8	3581.1	3932.5
30	10178.8	5371.7	5898.8
40	13571.7	7162.2	7865.1
50	16964.6	8952.8	9831.4
60	20357.5	10743.3	11797.6
70	23750.5	12533.9	13763.9
80	27143.4	14324.4	15730.2
90	30536.3	16115.0	17696.5
100	33929.2	17905.6	19662.7
200	67858.4	35811.2	39325.4

1. Whatever weight of water is to be raiſed, the pump-rod muſt be loaded with weights ſufficient for that purpoſe, if it be done by a forcing-pump, as is generally the caſe; and the power

6½ Inches diameter.			
Feet high.	Quantity in cubic inches.	Weight in troy ounces.	In avoirdupoise ounces.
1	398.2	210.1	230.7
2	797.4	420.3	461.4
3	1195.6	630.4	692.1
4	1593.8	840.6	922.8
5	1991.9	1050.8	1153.6
6	2390.1	1260.9	1384.3
7	2788.3	1471.1	1615.0
8	3186.5	1681.2	1845.7
9	3584.7	1891.3	2076.4
10	3982.9	2101.5	2307.1
20	7965.8	4202.9	4614.3
30	11948.8	6304.4	6921.4
40	15931.7	8405.9	9228.6
50	19914.6	10507.4	11535.7
60	23897.9	12608.9	13842.9
70	27880.5	14710.4	16150.0
80	31863.4	16811.8	18457.2
90	35846.3	18913.3	20764.3
100	39829.3	21014.8	23071.5
200	79658.6	42029.6	46143.0

power of the engine muſt be ſufficient for the weight of the rod, in order to bring it up.

2. It is known, that the atmoſphere preſſes upon the ſurface of the earth with a force equal to 15 pounds upon every ſquare inch.

3. When

3. When water is heated to a certain degree, the particles thereof repel one another, and conſtitute an elaſtic fluid, which is generally called *ſteam* or *vapour*.

4. Hot ſteam is very elaſtic; and when it is cooled by any means, particularly by its being mixed with cold water, its elaſticity is deſtroyed immediately, and it is reduced to water again.

5. If a veſſel be filled with hot ſteam, and then cloſed, ſo as to keep out the external air, and all other fluids; when that ſteam is by any means condenſed, cooled, or reduced to water, *that* water will fall to the bottom of the veſſel; and the cavity of the veſſel will be almoſt a perfect vacuum.

6. Whenever a vacuum is made in any veſſel, the air by its weight will endeavour to ruſh into the veſſel, or to drive in any other body that will give way to its preſſure; as may be eaſily ſeen by a common ſyringe. For, if you ſtop the bottom of a ſyringe, and then draw up the piſton, if it be ſo tight as to drive out all the air before it, and leave a vacuum within the ſyringe, the piſton being let go will be driven down with a great force.

7. The force with which the piſton is driven down, when there is a vacuum under it, will be as the ſquare of the diameter of the bore in the ſyringe. That is to ſay, it will be driven down with four times as much force in a ſyringe of a two-inch bore, as in a ſyringe of one inch: for the areas of circles are always as the ſquares of their diameters.

8. The preſſure of the atmoſphere being equal to 15 pounds upon a ſquare inch, it will be almoſt equal to 12 pounds upon a circular inch. So that if the bore of the ſyringe

be round, and one inch in diameter, the piſton will be preſt down into it by a force nearly equal to 12 pounds: but if the bore be two inches diameter, the piſton will be preſt down with four times that force.

And hence it is eaſy to find with what force the atmoſphere preſſes upon any given number either of ſquare or circular inches.

Theſe being the principles upon which this engine is conſtructed, we ſhall next deſcribe the chief working parts of it: which are, 1. A boiler. 2. A cylinder and piſton. 3. A beam or lever.

The *boiler* is a large veſſel made of iron or copper; and commonly ſo big as to contain about 2000 gallons.

The *cylinder* is about 40 inches diameter, bored ſo ſmooth, and its leathered piſton fitting ſo cloſe, that little or no water can get between the piſton and ſides of the cylinder.

Things being thus prepared, the cylinder is placed upright, and the ſhank of the piſton is fixed to one end of the *beam*, which turns on a center like a common balance.

The boiler is placed under the cylinder, with a communication between them, which can be opened and ſhut occaſionally.

The boiler is filled about half full of water, and a ſtrong fire is placed under it: then, if the communication between the boiler and the cylinder be opened, the cylinder will be filled with hot ſteam; which would drive the piſton quite out at the top of it. But there is a contrivance by which the beam, when the piſton is near the top of the cylinder, ſhuts the communication at the top of the boiler within.

This

This is no ſooner ſhut, than another is opened, by which a little cold water is thrown upward in a jet into the cylinder, which mixing with the hot ſteam, condenſes it immediately; by which means a vacuum is made in the cylinder, and the piſton is preſſed down by the weight of the atmoſphere; and ſo lifts up the loaded pump-rod at the other end of the beam.

If the cylinder be 42 inches in diameter, the piſton will be preſſed down with a force greater than 20000 pounds, and will conſequently lift up that weight at the oppoſite end of the beam: and as the pump-rod with its plunger is fixed to that end, if the bore where the plunger works were 10 inches diameter, the water would be forced up through a pipe of 180 yards perpendicular height.

But, as the parts of this engine have a good deal of friction, and muſt work with a conſiderable velocity, and there is no ſuch thing as making a perfect vacuum in the cylinder, it is found that no more than 8 pounds of preſſure muſt be allowed for, on every circular inch of the piſton in the cylinder, that it may make about 16 ſtrokes in a minute, about 6 feet each.

Where the boiler is very large, the piſton will make between 20 and 25 ſtrokes in a minute, and each ſtroke 7 or 8 feet; which, in a pump of 9 inches bore, will raiſe upward of 300 hogſheads of water in a hour.

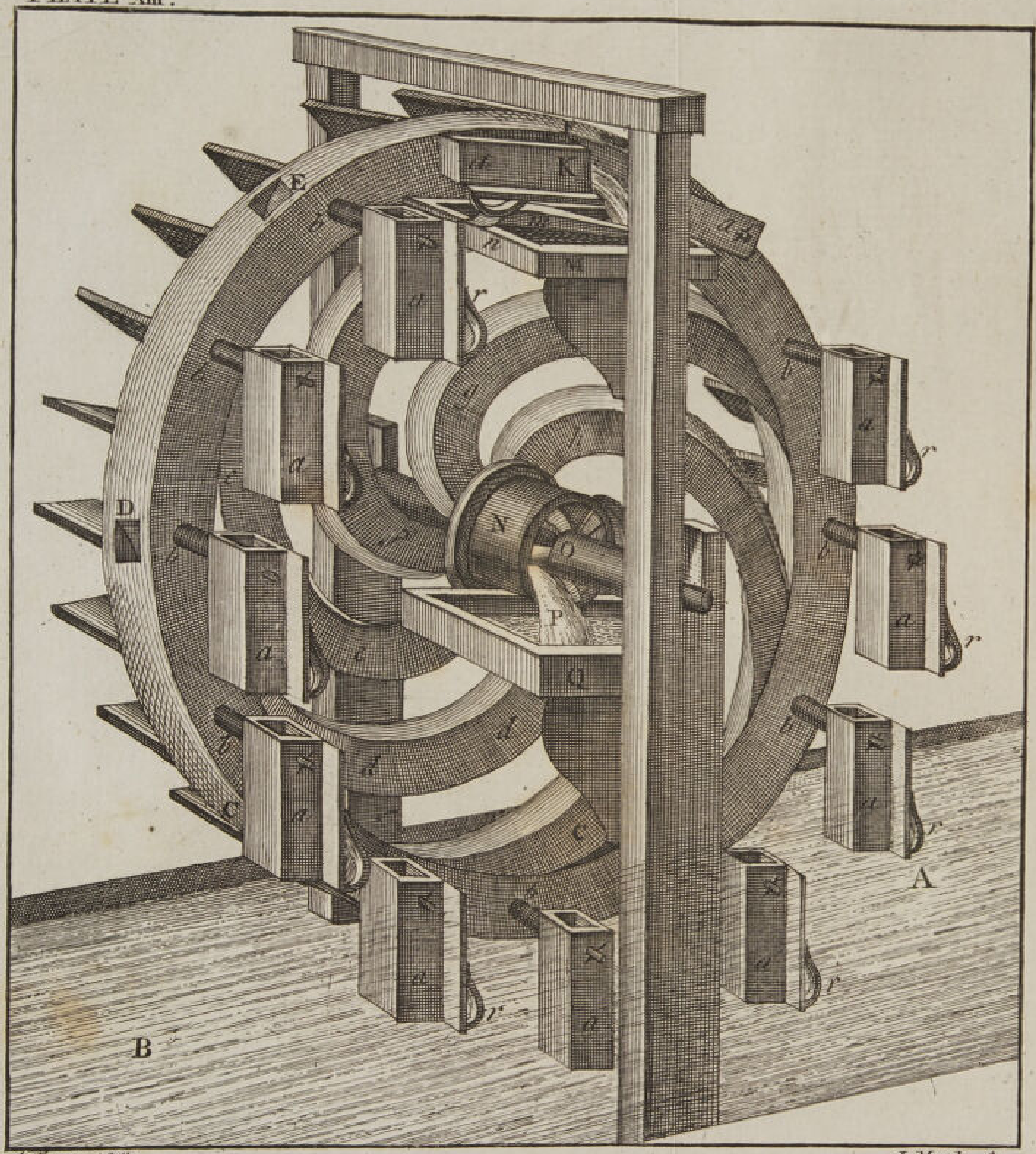
It is found by experience that a cylinder, 40 inches diameter, will work a pump 10 inches diameter, and 100 yards long: and hence we can find the diameter and length of a pump, that can be worked by any other cylinder.

For the convenience of thoſe who would make uſe of this engine for raiſing water, we ſhall ſubjoin part of a table calculated by Mr. *Beighton*, ſhewing how any given quantity of water may be raiſed in a hour, from 48 to 400 hogſheads; at any given depth, from 15 to 100 yards; the machine working at the rate of 16 ſtrokes *per* minute, and each ſtroke being 6 feet long.

One example of the uſe of this table will make the whole plain. Suppoſe it were required to draw 100 hogſheads *per* hour, at 90 yards depth: in the ſecond column from the right hand, I find the neareſt number, viz. 149 hogſheads 40 gallons, againſt which, on the right hand, I find the diameter of the bore of the pump muſt be 7 inches; and in the ſame collateral line, under the given depth 90, I find 27 inches, the diameter of the cylinder fit for that purpoſe.—And ſo for any other.

A Table

PLATE XIII.



J. Ferguson delin.

J. Mynde sc.

A Table shewing the power of the engine for raising water by fire.

This table is calculated to the measure of ale gallons, at 282 cubic inches *per* gallon.

The depth to be drawn in yards.													In one hour.		Diam. of pump
15	20	25	30	35	40	45	50	60	70	80	90	100	Hogsh.	Gal.	Inches.
18½	21¾	24	26¼	28½	30½	32½	34¼	37¼	40	—	—	—	440		12
17	19½	22	24	26¼	28	29¾	31¼	34½	37	39½	—	—	369	33	11
15½	18	20	22	23¾	25¼	27	28¾	31¼	33¾	36	38¼	40	304	48	10
14	16¼	18	20	21½	23	24¼	25	28	30½	33	35	36½	247	7	9
13½	15¼	17¼	19	20¼	21¾	23	24	26¼	28½	31	32½	35¼	221	15	8½
12½	14½	16½	18½	19	20½	21½	23	25	27	29	30½	32½	195	22	8
12	14	15½	17½	18¾	19¾	21	22	24¼	26	28	29½	31½	182	13	7¾
11	13¾	15	16½	18	19	20	21¼	23¼	25	27	28½	30¼	172	30	7½
10¾	13	14	15½	16¾	18¾	19	20½	22	24	25½	27	28½	149	40	7
10	12	13	14	15½	16¼	18	19	20	22	23	24½	26¼	128	54	6½
9½	11	12	13	14	15½	16	17	19	20½	22	23	24½	110	1	6
	10	11	12	13	14	15	15¾	17	19	20	21	22¼	94	30	5½
		10	11	11¼	13	13¾	14	15½	16¾	18½	19¼	20¼	66	61	5
			10	10	11¼	12	13½	14	15	16	17	18¼	60	60	4½
				9	10	11	11½	12	13½	14	15	16	48	51	4

Diameter of the cylinder in inches.

Plate XIII. The Persian wheel.

Water may be raised by means of a stream *AB* turning a wheel *CDE*, according to the order of the letters, with buckets *a, a, a, a,* &c. hung upon the wheel by strong pins *b, b, b, b,* &c, fixed in the side of the rim: but the wheel must be made as high as the water is intended to be raised above the level of that part of the stream in which the wheel is placed. As the wheel turns, the buckets on the right-hand go down into the water, and are filled therewith, and go up full on the left hand, until they come to the top at *K*; where they strike against the end *n* of the fixed trough *M*, and are thereby overset, and empty the water into the trough; from which it may be conveyed in pipes to the place which it is designed for: and as each bucket gets over the trough, it falls into a perpendicular position again, and goes down empty, until it comes to the water at *A*, where it is filled as before. On each bucket is a spring *r*, which going over the top or crown of the bar *m* (fixed to the trough *M*) raises the bottom of the bucket above the level of its mouth, and so causes it to empty all its water into the trough.

Sometimes this wheel is made to raise water no higher than its axle; and then, instead of buckets hung upon it, its spokes *C, d, e, f, g, h,* are made of a bent form, and hollow within; these hollows opening into the holes *C, D, E, F,* in the outside of the wheel, and also into those at *O* in the box *N* upon the axle. So that, as the holes *C, D,* &c. dip into the water, it runs into them; and as the wheel turns, the water rises in the hollow spokes, *c, d,* &c. and runs out in a stream *P* from the holes at *O*, and falls into the trough *Q*, from whence it is conveyed by pipes. And this is a very easy way of raising water,

water, because the engine requires no animal power to turn it.

The art of weighing different bodies in water, and thereby finding their specific gravities, or specific weights, bulk for bulk, was invented by ARCHIMEDES; of which we have the following account: Of the specific gravities of bodies.

Hiero, king of *Syracuse*, having employed a goldsmith to make a crown, and given him a mass of pure gold for that purpose, suspected that the workman had kept back part of the gold for his own use, and made up the weight by allaying the crown with copper. But the king, not knowing how to find out the truth of that matter, referred it to *Archimedes*; who having studied a long time in vain, found it out at last by chance. For, going into a bathing-tub of water, and observing that he thereby raised the water higher in the tub than it was before, he concluded instantly that he had raised it just as high as any thing else could have done, that was exactly of his bulk: and considering that any other body of equal weight, and of less bulk than himself, could not have raised the water so high as he did; he immediately told the king, that he had found a method by which he could discover whether there were any cheat in the crown. For, since gold is the heaviest of all known metals, it must be of less bulk, according to its weight, than any other metal. And therefore he desired that a mass of pure gold, equally heavy with the crown when weighed in air, should be weighed against it in water; and if the crown was not allayed, it would counterpoise the mass of gold when they were both immersed in water, as well as it did when they were weighed in air. But upon making the trial, he found that the maſs of gold weighed much heavier in water than the crown did. And not only ſo, but that, when the maſs and crown were immerſed ſeparately in one veſſel of water, the crown raiſed the water much higher than the maſs did; which ſhewed it to be allayed with ſome lighter metal that increaſed its bulk. And ſo, by making trials with different metals, all equally heavy with the crown when weighed in air, he found out the quantity of alloy in the crown.

The ſpecific gravities of bodies are as their weights, bulk for bulk; thus a body is ſaid to have two or three times the ſpecific gravity of another, when it contains two or three times as much matter in the ſame ſpace.

A body immerſed in a fluid will ſink to the bottom, if it be heavier than its bulk of the fluid. If it be ſuſpended therein, it will loſe as much of what it weighed in air, as its bulk of the fluid weighs. Hence, all bodies of equal bulks, which would ſink in fluids, loſe equal weights when ſuſpended therein. And unequal bodies loſe in proportion to their bulks.

The *hy-droſtatic balance.* The *hydroſtatic balance* differs very little from a common balance that is nicely made: only it has a hook at the bottom of each ſcale, on which ſmall weights may be hung by horſe-hairs, or by ſilk threads. So that a body, ſuſpended by the hair or thread, may be immerſed in water without wetting the ſcale from which it hangs.

How to find the ſpecific gravity of any body. If the body thus ſuſpended under the ſcale, at one end of the balance, be firſt counterpoiſed in air by weights in the oppoſite ſcale, and then immerſed into water, the equilibrium will be immediately deſtroyed. Then, if as much weight be be put into the ſcale from which the body hangs, as will reſtore the equilibrium (without altering the weights in the oppoſite ſcale) that weight, which reſtores the equilibrium, will be equal to the weight of a quantity of water as big as the immerſed body. And if the weight of the body in air be divided by what it loſes in water, the quotient will ſhew how much that body is heavier than its bulk of water. Thus, if a guinea ſuſpended in air, be counterbalanced by 129 grains in the oppoſite ſcale of the balance; and then, upon its being immerſed in water, it becomes ſo much lighter, as to require $7\frac{1}{4}$ grains put into the ſcale over it, to reſtore the equilibrium, it ſhews that a quantity of water, of equal bulk with the guinea, weighs $7\frac{1}{4}$ grains, or 7.25; by which divide 129 (the weight of the guinea in air) and the quotient will be 17.793; which ſhews that the guinea is 17.793 times as heavy as its bulk of water. And thus, any piece of gold may be tried, by weighing it firſt in air, and then in water; and if upon dividing the weight in air by the loſs in water, the quotient comes out to be 17.793, the gold is good; if the quotient be 18, or between 18 and 19, the gold is very fine; but if it be leſs than 17, the gold is too much allayed, by being mixed with ſome other metal.

If ſilver be tried in this manner, and found to be 11 times as heavy as water, it is very fine; if it be $10\frac{1}{2}$ times as heavy, it is ſtandard; but if it be of any leſs weight compared with water, it is mixed with ſome lighter metal, ſuch as tin.

By this method, the ſpecific gravities of all bodies that will ſink in water, may be found. But as to thoſe which are lighter than water, as

moſt

moſt ſorts of wood are, the following method may be taken, to ſhew how much lighter they are than their reſpective bulks of water.

Let an upright ſtud be fixed into a thick flat piece of braſs, and in this ſtud let a ſmall lever, whoſe arms are equally long, turn upon a fine pin as an axis. Let the thread which hangs from the ſcale of the balance be tied to one end of the lever, and a thread from the body to be weighed, tied to the other end. This done, put the braſs and lever into a veſſel, then pour water into the veſſel, and the body will riſe and float upon it, and draw down the end of the balance from which it hangs; then, put as much weight in the oppoſite ſcale as will raiſe that end of the balance, ſo as to pull the body down into the water by means of the lever; and this weight in the ſcale will ſhew how much the body is lighter than its bulk of water.

There are ſome things which cannot be weighed in this manner, ſuch as quickſilver, fragments of diamonds, &c. becauſe they cannot be ſuſpended in threads; and muſt therefore be put into a glaſs bucket, hanging by a thread from the hook of one ſcale, and counterpoiſed by weights put into the oppoſite ſcale. Thus, ſuppoſe you want to know the ſpecific gravity of quickſilver, with reſpect to that of water; let the empty bucket be firſt counterpoiſed in air, and then the quickſilver put into it and weighed. Write down the weight of the bucket, and alſo of the quickſilver; which done, empty the bucket, and let it be immerſed in water as it hangs by the thread, and counterpoiſed therein by weights in the oppoſite ſcale: then, pour the quickſilver into the bucket in the water, which will cauſe it to preponderate; and put as much weight into the opposite scale as will restore the balance to an equipoise; and this weight will be the weight of a quantity of water equal in bulk to the quicksilver. Lastly, divide the weight of the quicksilver in air, by the weight of its bulk of water, and the quotient will shew how much the quicksilver is heavier than its bulk of water.

If a piece of brass, glass, lead, or silver, be immersed and suspended in different sorts of fluids, the different losses of weight therein will shew how much it is heavier than its bulk of the fluid; the fluid being lightest in which the immersed body loses least of its aerial weight. A solid bubble of glass is generally used for finding the specific gravities of fluids.

Hence we have an easy method of finding the specific gravities both of solids and fluids, with regard to their specific bulks of common pump water, which is generally made a standard for comparing all others by.

In constructing tables of specific gravities with accuracy, the gravity of water must be represented by unity or 1.000, where three cyphers are added, to give room for expressing the ratios of other gravities in decimal parts, as in the following table.

N. B. Although guinea gold has been generally reckoned 17.798 times as heavy as its bulk of water, yet, by many repeated trials, I cannot say that I have found it to be more than 17.200 (or $17\frac{2}{10}$) as heavy.

A Table

A Table of the ſpecific gravities of ſeveral ſolid and fluid bodies.

A cubic inch of	Troy weight. oz.	pw.	gr.	Avoirdup. oz.	drams.	Comparative weight.
Very fine gold -	10	7	3.83	11	5.80	19.637
Standard gold -	9	19	6.44	10	14.90	18.888
Guinea gold -	9	7	17.18	10	4.76	17.793
Moidore gold -	9	0	19.84	9	14.71	17.140
Quickſilver - -	7	7	11.61	8	1.45	14.019
Lead - - -	5	19	17.55	6	9.08	11.325
Fine ſilver - -	5	16	23.23	6	6.66	11.087
Standard ſilver -	5	11	3.36	6	1.54	10.535
Copper - -	4	13	7.04	5	1.89	8.843
Plate braſs - -	4	4	9.60	4	10.09	8.000
Steel - - -	4	2	20.12	4	8.70	7.852
Iron - - -	4	0	15.20	4	6 77	7.645
Bl cktin - -	3	17	5.68	4	3.79	7.321
Speltar - - -	3	14	12.86	4	1.42	7.065
Lead ore - -	3	11	17.76	3	14.96	6.800
Glaſs of antimony	2	15	16.89	3	0.89	5.280
German antimony	2	2	4.80	2	5.04	4.000
Copper ore - -	2	1	11.83	2	4.43	3.775
Diamond - -	1	15	20.88	1	15.48	3.400
Clear glaſs - -	1	13	5.58	1	13.16	3.150
Lapis lazuli - -	1	12	5.27	1	12.27	3.054
Welch aſbeſtos -	1	10	17.57	1	10.97	2.913
White marble -	1	8	13.41	1	9.06	2.707
Black ditto - -	1	8	12.65	1	9.02	2.704
Rock cryſtal -	1	8	1.00	1	8.61	2.658
Green glaſs - -	1	7	15.38	1	8.26	2.620
Cornelian ſtone	1	7	1.21	1	7.73	2.568
Flint - - -	1	6	19.63	1	7.53	2.542
Hard paving ſtone	1	5	22.87	1	6.77	2.460
Live ſulphur -	1	1	2.40	1	2.52	2.000
Nitre - - -	1	0	1.08	1	1.59	1.900
Alabaſter - -	0	19	18.74	1	1.35	1.875
Dry ivory - -	0	19	6.09	1	0.89	1.825
Brimſtone - -	0	18	23.76	1	0.66	1.800
Alum - - -	0	17	21.92	0	15.72	1.714

The

The Table concluded.

A cubic inch of	Troy weight. oz.	pw.	gr.	Avoirdup. oz.	drams.	Comparative weight.
Ebony - - -	0	11	18.82	0	10.34	1.117
Human blood -	0	11	2.89	0	9.76	1.054
Amber - - -	0	10	20.79	0	9.54	1.030
Cow's milk - -	0	10	20.79	0	9.54	1.030
Sea water - -	0	10	20.79	0	9.54	1.030
Pump water -	0	10	13.30	0	9.26	1.000
Spring water -	0	10	12.94	0	9.25	0.999
Diſtilled water -	0	10	11.42	0	9.20	0.993
Red wine - -	0	10	11.42	0	9.20	0.993
Oil of amber -	0	10	7.63	0	9.06	0.978
Proof ſpirits -	0	9	19.73	0	8.62	0.931
Dry oak - -	0	9	18.00	0	8.56	0.925
Olive oil - -	0	9	15.17	0	8.45	0.913
Pure ſpirits - -	0	9	3.27	0	8.02	0.866
Spirit of turpentine	0	9	2.76	0	7.99	0.864
Oil of turpentine	0	8	8.53	0	7.33	0.772
Dry crabtree -	0	8	1.69	0	7.08	0.765
Saſſafras wood -	0	5	2.04	0	4.46	0.482
Cork - - -	0	2	12.77	0	2.21	0.240

Take away the decimal points from the numbers in the right-hand column, or (which is the ſame) multiply them by 1000, and they will ſhew how many avoirdupoiſe ounces are contained in a cubic foot of each body.

How to find out the quantity of adulteration in metals.

The uſe of the table of ſpecific gravities will beſt appear by an example. Suppoſe a body to be compounded of gold and ſilver, and it is required to find the quantity of each metal in the compound.

Firſt find the ſpecific gravity of the compound, by weighing it in air and in water, and dividing its aerial weight by what it loſes thereof in water, the quotient will ſhew its ſpecific gravity,

gravity, or how many times it is heavier than its bulk of water. Then, ſubtract the ſpecific gravity of ſilver (found in the table) from that of the compound, and the ſpecific gravity of the compound from that of gold; the firſt remainder ſhews the bulk of gold, and the latter the bulk of ſilver, in the whole compound: and if theſe remainders be multiplied by the reſpective ſpecific gravities, the products will ſhew the proportion of weights of each metal in the body. Example.

Suppoſe the ſpecific gravity of the compounded body be 13; that of ſtandard ſilver (by the table) is 10.5, and that of gold 19.63: therefore 10.5 from 13, remains 2.5, the proportional bulk of the gold; and 13 from 19.63, remains 6.63 the proportional bulk of ſilver in the compound. Then, the firſt remainder 2.5, multiplied by 19.63, the ſpecific gravity of gold, produces 49.075 for the proportional weight of gold; and the laſt remainder 6.63 multiplied by 10.5, the ſpecific gravity of ſilver, produces 69.615 for the proportional weight of ſilver in the whole body. So that for every 49.07 ounces or pounds of gold, there are 69.6 pounds or ounces of ſilver in the body.

Hence it is eaſy to know whether any ſuſpected metal be genuine, or allayed, or counterfeit; by finding how much it is heavier than its bulk of water, and comparing the ſame with the table: if they agree, the metal is good; if they differ, it is allayed or counterfeited.

How to try ſpirituous liquors. A cubical inch of good brandy, rum, or other proof ſpirits, weighs 235.7 grains: therefore, if a true inch cube of any metal weighs 235.7 grains leſs in ſpirits than in air, it ſhews the ſpirits are proof. If it loſes leſs of its aerial weight

weight in spirits, they are above proof; if it loses more, they are under. For, the better the spirits are, they are the lighter; and the worse, the heavier. All bodies expand with heat, and contract with cold, but some more and some less than others. And therefore the specific gravities of bodies are not precisely the same in summer as in winter. It has been found, that a cubic inch of good brandy is ten grains heavier in winter than in summer; as much spirit of nitre, 20 grains; vinegar 6 grains, and spring-water 3. Hence it is most profitable to buy spirits in winter, and sell them in summer, since they are always bought and sold by measure. It has been found, that 32 gallons of spirits in winter will make 33 in summer.

The expansion of all fluids is proportionable to the degree of heat; that is, with a double or triple heat a fluid will expand two or three times as much.

Upon these principles depends the construction of the thermometer, in which the globe or bulb, and part of the tube, are filled with a fluid, which, when joined to the barometer, is spirits of wine tinged, that it may be more easily seen in the tube. But when thermometers are made by themselves, quicksilver is generally used. The *thermometer.*

In the thermometer, a scale is fitted to the tube, to shew the expansion of the quicksilver, and consequently the degree of heat. And, as *Fahrenheit*'s scale is most in esteem at present, I shall explain the construction and graduation of thermometers according to that scale.

First, let the globe or bulb, and part of the tube, be filled with a fluid; then immerse the bulb in water just freezing, or snow just thawing;

ing; and even with that part in the ſcale where the fluid then ſtands in the tube, place the number 32, to denote the freezing point: then put the bulb under your arm-pit, when your body is of a moderate degree of heat, ſo that it may acquire the ſame degree of heat with your ſkin; and when the fluid has riſen as far as it can by that heat, there place the number 97: then divide the ſpace between theſe numbers into 65 equal parts, and continue thoſe diviſions both above 97 and below 32, and number them accordingly.

This may be done in any part of the world; for it is found that the freezing point is always the ſame in all places, and the heat of the human body differs but very little; ſo that the thermometers made in this manner will agree with one another; and the heat of ſeveral bodies will be ſhewn by them, and expreſſed by the numbers upon the ſcale, thus:

Air, in ſevere cold weather, in our climate, from 15 to 25. Air in winter, from 26 to 42. Air in ſpring and autumn, from 43 to 53. Air at midſummer, from 65 to 68. Extreme heat of the ſummer ſun, from 86 to 100. Butter juſt melting, 95. Alcohol boils with 174 or 175. Brandy with 190. Water 212. Oil of turpentine 550. Tin melts with 408, and lead with 540. Milk freezes about 30, vinegar 38, and blood 27.

A body ſpecifically lighter than a fluid will ſwim upon its ſurface, in ſuch a manner, that a quantity of the fluid, equal in bulk with the immerſed part of the body, will be as heavy as the whole body. Hence, the lighter a fluid is, the deeper a body will ſink in it; upon which depends the construction of the *hydrometer* or water-poise.

From this we can easily find the weight of a ship, or any other body that floats in water. For, if we multiply the number of cubic feet which are under the surface, by 62.5, the number of pounds in one cubic foot of fresh water; or by 64.4, the number of pounds in a cubic foot of salt water; the product will be the weight of the ship, and all that is in it. For, since it is the weight of the ship that displaces the water, it must continue to sink until it has removed as much water as is equal to it in weight; and therefore the part immersed must be equal in bulk to such a portion of the water as is equal to the weight of the whole ship.

How the weight of a ship may be estimated.

To prove this by experiment, let a ball of some light wood, such as fir or pear-tree, be put into water contained in a glass vessel; and let the vessel be put into a scale at one end of a balance, and counterpoised by weights in the opposite scale: then, marking the height of the water in the vessel, take out the ball; and fill up the vessel with water to the same height that it stood at when the ball was in it; and the same weight will counterpoise it as before.

From the vessel's being filled up to the same height at which the water stood when the ball was in it, it is evident that the quantity poured in is equal in magnitude to the immersed part of the ball: and from the same weight counterpoising, it is plain that the water poured in, is equal in weight to the whole ball.

In troy weight, 24 grains make a pennyweight, 20 pennyweights make an ounce, and 12 ounces a pound. In avoirdupoise weight, 16 drams make an ounce, and 16 ounces a pound. The troy pound contains 5760 grains, and the avoirdupoise pound 7000; and hence, the avoirdupoise dram weighs 27.34375 grains, and the avoirdupoise ounce 437.5.

Because it is often of use to know how much any given quantity of goods in troy weight do make in avoirdupoise weight; and the reverse; we shall here annex two tables for converting these weights into one another. Those from page 135 to page 146 are near enough for common hydraulic purposes; but the two following are better, where accuracy is required in comparing the weights with one another: and I find, by trial, that 175 troy ounces are precisely equal to 192 avoirdupoise ounces, and 175 troy pounds are equal to 144 avoirdupoise. And although there are several lesser integral numbers, which come very near to agree together, yet I have found none less than the above to agree exactly. Indeed 41 troy ounces are so nearly equal to 45 avoirdupoise ounces, that the latter contains only $7\frac{1}{2}$ grains more than the former: and 45 troy pounds weigh only $7\frac{3}{10}$ drams more than 37 avoirdupoise.

I have lately made a scale for comparing these weights with one another, and shewing the weight of pump-water, proof spirits, pure spirits, and guinea gold, taken in cubic inches, to any quantity less than a pound, both in troy and avoirdupoise; only by sliding one side of a square along the scale, and the other side crossing it.

A Table

A Table for reducing Troy weight into Avoirdupoise weight.

Troy Weight.		Avoirdupoise. lb.	oz.	drams.	Troy Weight.		Avoir Drams.
Pounds—	4000	3291	6	13.68	Pennywt.	19	16.67
	3000	2528	9	2.26		18	15.79
	2000	1645	11	6.84		17	14.92
	1000	822	13	11.42		16	14.04
	900	740	9	2.28		15	13.16
	800	658	4	9.14		14	12.29
	700	576	0	0.00		13	11.41
	600	493	11	6.85		12	10.53
	500	411	6	13.71		11	9.65
	400	329	2	4.57		10	8.78
	300	246	13	11.42		9	7.90
	200	164	9	2.28		8	7.02
	100	82	4	9.15		7	6.14
	90	74	0	13.62		6	5.27
	80	65	13	4.11		5	4.39
	70	57	9	9.60		4	3.51
	60	49	5	15.08		3	2.63
	50	41	2	4.57		2	1.75
	40	32	14	10.05		1	.88
	30	24	10	15.54	Grains —	23	.84
	20	16	7	5.03		22	.80
	10	8	3	10.52		21	.77
	9	7	6	7.86		20	.73
	8	6	9	5.21		19	.69
	7	5	12	2.56		18	.66
	6	4	14	15.90		17	.62
	5	4	1	13.25		16	.58
	4	3	4	10.60		15	.55
	3	2	7	7.95		14	.51
	2	1	10	5.30		13	.47
	1		13	2.65		12	.44
Ounces —	11		12	1.09		11	.40
	10		10	15.54		10	.36
	9		9	13.99		9	.33
	8		8	12.43		8	.29
	7		7	10.88		7	.26
	6		6	9.32		6	.22
	5		5	7.77		5	.18
	4		4	6.22		4	.15
	3		3	4.66		3	.11
	2		2	3.11		2	.07
	1		1	1.55		1	.04

A Table for reducing Avoirdupoise weight into Troy weight.

Averdupoise weight.	Troy weight.				Avoird. weight.	Troy weight.			
	lb.	oz.	pw.	gr.		lb.	oz.	pw.	gr.
Pounds 6000	7291	8	0	0	Ounces 15	1	1	13	10.50
5000	6076	4	13	8	14	1	0	15	5
4000	4861	1	6	16	13		11	16	23.50
3000	3645	10	0	0	12		10	18	18
2000	2430	6	13	8	11		10	0	12.50
1000	1215	3	6	16	10		9	2	7
900	1093	9	0	0	9		8	4	1.50
800	972	2	13	8	8		7	5	20
700	850	8	6	16	7		6	7	14.50
600	729	2	0	0	6		5	9	9
500	607	7	13	8	5		4	11	3.50
400	486	1	6	16	4		3	12	22
300	364	7	0	0	3		2	14	16.50
200	243	0	13	8	2		1	16	11
100	121	6	6	16	1			18	5.50
90	109	4	10	0	Drams 15			17	2.10
80	97	2	13	8	14			15	22.76
70	85	0	16	16	13			14	19.42
60	72	11	0	0	12			13	15.08
50	60	9	3	8	11			12	12.74
40	48	7	6	16	10			11	9.40
30	36	5	10	0	9			10	6.06
20	24	3	13	8	8			9	2.72
10	12	1	16	16	7			8	23.38
9	10	11	5	0	6			7	20.04
8	9	8	13	8	5			6	16.70
7	8	6	1	16	4			5	13.36
6	7	3	10	0	3			3	10.02
5	6	0	18	8	2			2	6.68
4	4	10	6	16	1			1	3.34
3	3	7	15	0	3/4				20.51
2	2	5	3	8	1/2				13.67
1	1	2	11	16	1/4				6.83

The

The two following examples will be ſufficient to explain theſe two tables, and ſhew their agreement.

Ex. I. *In* 6835 *pounds* 6 *ounces* 9 *pennyweights* 6 *grains Troy, Qu. How much Avoirdupoiſe weight?* (See page 165.)

		Avoirdupoiſe.		
		lb.	oz.	drams.
Pounds troy—	4000	3291	6	13.68
	2000	1645	11	6.84
	800	658	4	9.14
	20	16	7	5.03
	10	8	3	10.52
	5	4	1	13.25
oz.	6		6	9.32
pw.	9			7.90
gr.	6			.22
Anſwer.		5624	10	11.90

Ex. II. *In* 5624 *pounds* 10 *ounces* 12 *drams Avoirdupoiſe, Qu. How much Troy weight?* (See Page 166.)

		Troy.			
		lb.	oz.	pw.	gr.
Pounds avoird.	5000	6076	4	13	8
	600	729	2	0	0
	20	24	3	13	8
	4	4	10	6	16
oz.	10		9	2	7
dr.	12			13	15.08
Anſwer.		6835	6	9	6.08

LECT. VI.

Of Pneumatics.

THIS ſcience treats of the nature, weight, preſſure, and ſpring of the air, and the effects ariſing therefrom.

The properties of air.

The air is that thin tranſparent fluid body in which we live and breathe. It encompaſſes the whole earth to a conſiderable height; and, together with the clouds and vapours that float therein, it is called the atmoſphere. The air is juſtly reckoned among the number of fluids, becauſe it has all the properties by which a fluid is diſtinguiſhed. For, it yields to the leaſt force impreſſed, its parts are eaſily moved among one another, it preſſes according to its perpendicular height, and its preſſure is every way equal.

That the air is a fluid, conſiſting of ſuch particles as have no coheſion between them, but eaſily glide over one another, and yield to the ſlighteſt impreſſion, appears from that eaſe and freedom with which animals breathe in it, and move through it without any difficulty or ſenſible reſiſtance.

But it differs from all other fluids in the four following particulars: 1. It can be compreſſed into a much leſs ſpace than what it naturally poſſeſſes, which no other fluid can. 2. It cannot be congealed or fixed, as other fluids may. 3. It is of a different denſity in every part, upward from the earth's ſurface, decreaſing in its weight, bulk for bulk, the higher it riſes; and therefore muſt alſo decreaſe in denſity. 4. It is of an elaſtic

elaſtic or ſpringy nature, and the force of its ſpring is equal to its weight.

That air is a body, is evident from its excluding all other bodies out of the ſpace it poſſeſſes: for, if a glaſs jar be plunged with its mouth downward into a veſſel of water, there will but very little water get into the jar, becauſe the air of which it is full keeps the water out.

As air is a body, it muſt needs have gravity or weight: and that it is weighty, is demonſtrated by experiment. For, let the air be taken out of a veſſel by means of the air-pump, then, having weighed the veſſel, let in the air again, and upon weighing it when re-filled with air, it will be found conſiderably heavier. Thus, a bottle that holds a wine quart, being emptied of air and weighed, is found to be about 16 grains lighter than when the air is let into it again; which ſhews that a quart of air weighs 16 grains. But a quart of water weighs 14621 grains; this divided by 16, quotes 914 in round numbers; which ſhews, that water is 914 times as heavy as air near the ſurface of the earth.

As the air riſes above the earth's ſurface, it grows rarer, and conſequently lighter, bulk for bulk. For, becauſe it is of an elaſtic or ſpringy nature, and its lowermoſt parts are preſſed with the weight of all that is above them, it is plain that the air muſt be more denſe or compact at the earth's ſurface, than at any height above it; and gradually rarer the higher up. For, the denſity of the air is always as the force that compreſſes it; and therefore, the air toward the upper parts of the atmoſphere being leſs preſſed than that which is near the earth, it will expand itſelf, and thereby become thinner than at the earth's ſurface.

Dr. *Cotes* has demonſtrated, that if altitudes in the air be taken in arithmetical proportion, the rarity of the air will be in geometrical proportion. For inſtance,

At the altitude of	miles above the ſurface of the earth, the air is	times thinner and lighter than at the earth's ſurface.
7	4	
14	16	
21	64	
28	256	
35	1024	
42	4096	
49	16384	
56	65536	
63	262144	
70	1048576	
77	4194304	
84	16777216	
91	67108864	
98	268435456	
105	1073741824	
112	4294967296	
119	17179869184	
126	68719476736	
133	274877906944	
140	1099511627776	

And hence it is eaſy to prove by calculation, that a cubic inch of ſuch air as we breathe, would be ſo much rarefied at the altitude of 500 miles, that it would fill a hollow ſphere equal in diameter to the orbit of Saturn.

The weight or preſſure of the air is exactly determined by the following experiment:

The *Toricellian* experiment. Take a glaſs tube about three feet long, and open at one end; fill it with quickſilver, and putting your finger upon the open end, turn that end downward, and immerſe it into a ſmall veſſel

veſſel of quickſilver, without letting in any air: then take away your finger; and the quickſilver will remain ſuſpended in the tube $29\frac{1}{2}$ inches above its ſurface in the veſſel; ſometimes more, and at other times leſs, as the weight of the air is varied by winds and other cauſes. That the quickſilver is kept up in the tube by the preſſure of the atmoſphere upon that in the baſon, is evident; for, if the baſon and tube be put under a glaſs, and the air be then taken out of the glaſs, all the quickſilver in the tube will fall down into the baſon; and if the air be let in again, the quickſilver will riſe to the ſame height as before. Therefore the air's preſſure on the ſurface of the earth, is equal to the weight of $29\frac{1}{2}$ inches depth of quickſilver all over the earth's ſurface, at a mean rate.

A ſquare column of quickſilver, $29\frac{1}{2}$ inches high, and one inch thick, weighs juſt 15 pounds, which is equal to the preſſure of air upon every ſquare inch of the earth's ſurface; and 144 times as much, or 2160 pounds, upon every ſquare foot; becauſe a ſquare foot contains 144 ſquare inches. At this rate, a middle-ſized man, whoſe ſurface may be about 14 ſquare feet, ſuſtains a preſſure of 30240 pounds, when the air is of a mean gravity: a preſſure which would be inſupportable, and even fatal to us, were it not equal on every part, and counterbalanced by the ſpring of the air within us, which is diffuſed through the whole body; and re-acts with an equal force againſt the outward preſſure.

Now, ſince the earth's ſurface contains (in round numbers) 200,000,000 ſquare miles, and every ſquare mile 27,878,400 ſquare feet, there muſt be 5,575,680,000,000,000 ſquare feet

feet on the earth's ſurface; which multiplied by 2160 pounds (the preſſure on each ſquare foot) gives 12,043,468,800,000,000,000 pounds for the preſſure or weight of the whole atmoſphere.

When the end of a pipe is immerſed in water, and the air is taken out of the pipe, the water will riſe in it to the height of 33 feet above the ſurface of the water in which it is immerſed; but will go no higher; for it is found, that a common pump will draw water no higher than 33 feet above the ſurface of the well: and unleſs the bucket goes within that diſtance from the well, the water will never get above it. Now, as it is the preſſure of the atmoſphere, on the ſurface of the water in the well, that cauſes the water to aſcend in the pump, and follow the piſton or bucket, when the air above it is lifted up; it is evident, that a column of water 33 feet high, is equal in weight to a column of quickſilver of the ſame diameter, 29½ inches high; and to as thick a column of air, reaching from the earth's ſurface to the top of the atmoſphere.

The *barometer.*

In ſerene calm weather, the air has weight enough to ſupport a column of quickſilver 31 inches high; but in tempeſtuous ſtormy weather, not above 28 inches. The quickſilver, thus ſupported in a glaſs tube, is found to be a nice counterbalance to the weight or preſſure of the air, and to ſhew its alterations at different times. And being now generally uſed to denote the changes in the weight of the air, and of the weather conſequent upon them, it is called the *barometer*, or weather-glaſs.

The preſſure of the air being equal on all ſides of a body expoſed to it, the ſofteſt bodies ſuſtain

ſuſtain this preſſure without ſuffering any change in their figure; and ſo do the moſt brittle bodies without being broke.

The air is rarefied, or made to ſwell with heat; and of this property, *wind* is a neceſſary conſequence. For, when any part of the air is heated by the ſun, or otherwiſe, it will ſwell, and thereby affect the adjacent air: and ſo, by various degrees of heat in different places, there will ariſe various winds. The cauſe of *winds.*

When the air is much heated, it will aſcend toward the upper part of the atmoſphere, and the adjacent air will ruſh in to ſupply its place; and therefore, there will be a ſtream or current of air from all parts toward the place where the heat is. And hence we ſee the reaſon why the air ruſhes with ſuch force into a glaſs-houſe, or toward any place where a great fire is made. And alſo, why ſmoke is carried up a chimney, and why the air ruſhes in at the key-hole of the door, or any ſmall chink, when there is a fire in the room. So we may take it in general, that the air will preſs toward that part of the world where it is moſt heated.

Upon this principle, we can eaſily account for the *trade-winds*, which blow conſtantly from eaſt to weſt about the equator. For, when the ſun ſhines perpendicularly on any part of the earth, it will heat the air very much in that part, which air will therefore riſe upward, and when the ſun withdraws, the adjacent air will ruſh in to fill its place; and conſequently will cauſe a ſtream or current of air from all parts toward that which is moſt heated by the ſun. But as the ſun, with reſpect to the earth, moves from eaſt to weſt, the common courſe of the air will be that way too; continually preſſing after the ſun: and therefore, The *trade-winds.*

therefore, at the equator, where the ſun ſhines ſtrongly, there will be a continual wind from the eaſt; but, on the north-ſide, it will incline a little to the north, and on the ſouth-ſide, to the ſouth.

This general courſe of the wind about the equator, is changed in ſeveral places, and upon ſeveral accounts; as, 1. By exhalations that riſe out of the earth at certain times, and from certain places; in earthquakes, and from volcanoes. 2. By the falling of great quantities of rain, cauſing thereby a ſudden condenſation or contraction of the air. 3. By burning ſands, that often retain the ſolar heat to a degree incredible to thoſe who have not felt it, cauſing a more than ordinary rarefaction of the air contiguous to them. 4. By high mountains, which alter the direction of the winds in ſtriking againſt them. 5. By the declination of the ſun toward the north or ſouth, heating the air on the north or ſouth-ſide of the equator.

The *monſoons.* To theſe and ſuch like cauſes is owing, 1. The irregularity and uncertainty of winds in climates diſtant from the equator, as in moſt parts of *Europe*. 2. Thoſe periodical winds, called *monſoons*, which in the *Indian* ſeas blow half a year one way, and the other half another. 3. Thoſe winds which, on the coaſt of *Guinea*, and on the weſtern coaſts of *America*, blow always from weſt to eaſt. 4. The ſea-breezes, which, in hot countries, blow generally from ſea to land, in the day-time; and the land-breezes, which blow in the night; and, in ſhort, all thoſe ſtorms, hurricanes, whirlwinds, and irregularities, which happen at different times and places.

All

All common air is impregnated with a certain kind of *vivifying spirit* or quality, which is necessary to continue the lives of animals: and this, in a gallon of air, is sufficient for one man during the space of a minute, and not much longer. The *vivifying spirit* in air.

This spirit in air is destroyed by passing through the lungs of animals: and hence it is, that an animal dies soon, after being put under a vessel which admits no fresh air to come to it. This spirit is also in the air which is in water; for fish die when they are excluded from fresh air, as in a pond that is closely frozen over. And the little eggs of insects, stopped up in a glass, do not produce their young, though assisted by a kindly warmth. The seed also of plants mixed with good earth, and inclosed in a glass, will not grow.

This enlivening quality in air, is also destroyed by the air's passing through fire; particularly charcoal fire, or the flame of sulphur. Hence, smoking chimneys must be very unwholesome, especially if the rooms they are in be small and close.

Air is also vitiated, by remaining closely pent up in any place for a considerable time; or perhaps, by being mixed with malignant steams and particles flowing from the neighbouring bodies: or lastly, by the corruption of the vivifying spirit; as in the holds of ships, in oil-cisterns, or wine-cellars, which have been shut for a considerable time. The air in any of them is sometimes so much vitiated, as to be immediate death to any animal that comes into it.

Air that has lost its vivifying spirit, is called *damp*, not only because it is filled with humid or moist vapours, but because it deadens fire, extin- *Damps.*

extinguiſhes flame, and deſtroys life. The dreadful effects of damps are ſufficiently known to ſuch as work in mines.

If part of the vivifying ſpirit of air in any country begins to putrefy, the inhabitants of that country will be ſubject to an epidemical diſeaſe, which will continue until the putrefaction is over. And as the putrefying ſpirit occaſions the diſeaſe, ſo if the diſeaſed body contributes toward the putrefying of the air, then the diſeaſe will not only be epidemical, but peſtilential and contagious.

The atmoſphere is the common receptacle of all the effluvia or vapours ariſing from different bodies; of the ſteams and ſmoke of things burnt or melted; the fogs or vapours proceeding from damp watery places; and of the effluvia from ſulphureous, nitrous, acid, and alkaline bodies. In ſhort, whatever may be called volatile, riſes in the air to greater or leſs heights, according to its ſpecific gravity.

Fermentations. When the effluvia, which ariſe from acid and alkaline bodies, meet each other in the air, there will be a ſtrong conflict or *fermentation* between them; which will ſometimes be ſo great, as to produce a fire; then if the effluvia be combuſtible, the fire will run from one part to another, juſt as the inflammable matter happens to lie.

Any one may be convinced of this, by mixing an acid and an alkaline fluid together, as the ſpirit of nitre and oil of cloves; upon the doing of which, a ſudden ferment, with a fine flame, will ariſe; and if the ingredients be very pure and ſtrong, there will be a ſudden exploſion.

Thunder and lightning. Whoever conſiders the effects of fermentation, cannot be at a loſs to account for the dreadful

dreadful effects of *thunder* and *lightning*: for the effluvia of ſulphureous and nitrous bodies, and others that may riſe into the atmoſphere, will ferment with each other, and take fire very often of themſelves; ſometimes by the aſſiſtance of the ſun's heat.

If the inflammable matter be thin and light, it will riſe to the upper part of the atmoſphere, where it will flaſh without doing any harm: but if it be denſe, it will lie near the ſurface of the earth, where taking fire, it will explode with a ſurpriſing force; and by its heat rarefy and drive away the air, kill men and cattle, ſplit trees, walls, rocks, &c. and be accompanied with terrible claps of thunder.

The heat of lightning appears to be quite different from that of other fires; for it has been known to run through wood, leather, cloth, &c. without hurting them, while it has broken and melted iron, ſteel, ſilver, gold, and other hard bodies. Thus it has melted or burnt aſunder a ſword, without hurting the ſcabbard; and money in a man's pocket, without hurting his cloaths: the reaſon of this ſeems to be, that the particles of *that* fire are ſo fine, as to paſs through ſoft looſe bodies without diſſolving them; while they ſpend their whole force upon the hard ones.

It is remarkable, that knives and forks which have been ſtruck with lightning have a very ſtrong magnetical virtue for ſeveral years after; and I have heard that lightning ſtriking upon the mariner's compaſs, will ſometimes turn it round; and often make it ſtand the contrary way, the north-pole toward the ſouth.

Much of the ſame kind with lightning, are thoſe explofions, called *fulminating* or *fire-damps*, which *Fire-damps.*

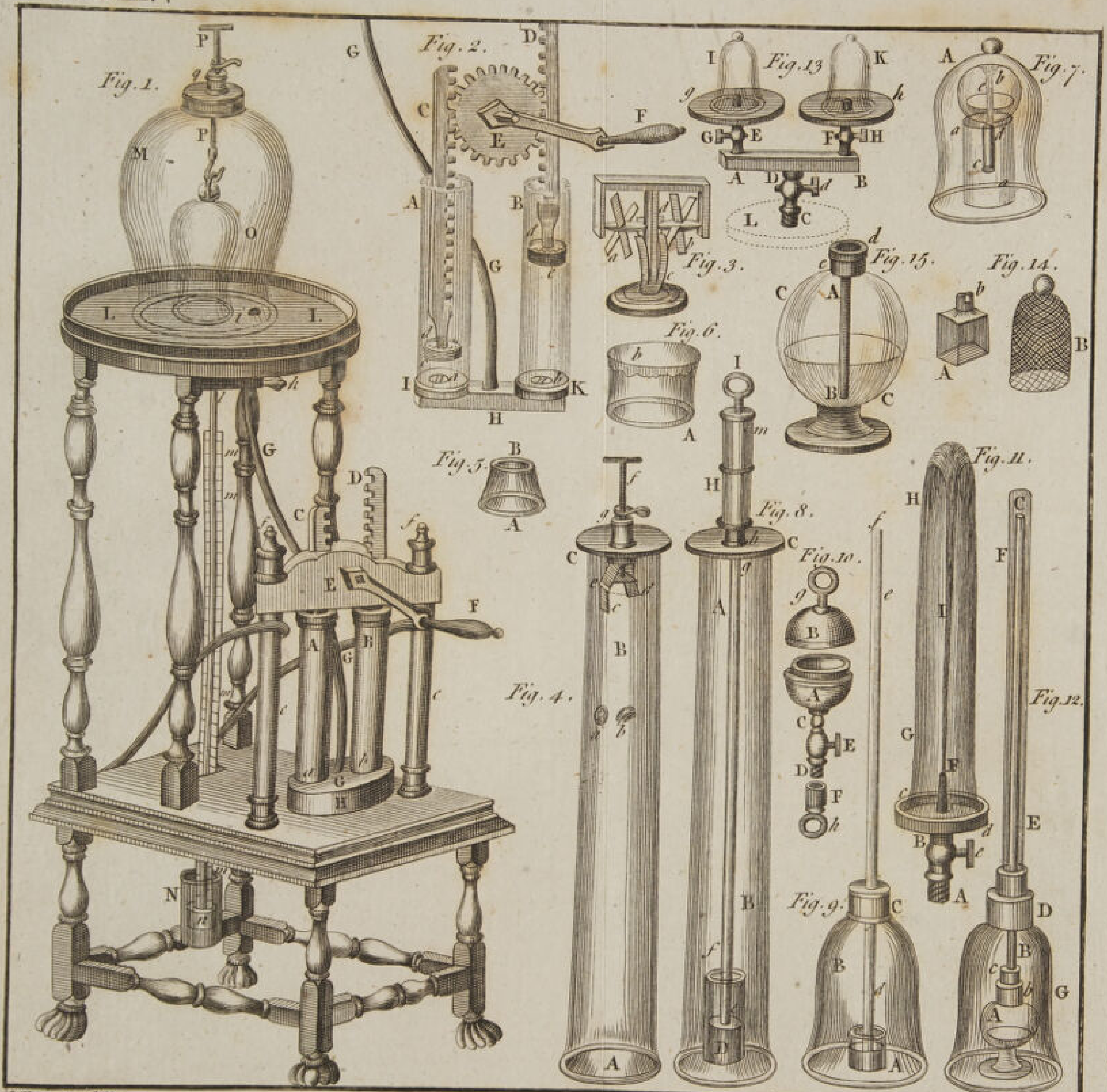
which ſometimes happen in mines; and are occaſioned by ſulphureous and nitrous, or rather oleaginous particles, riſing from the mine, and mixing with the air, where they will take fire by the lights which the workmen are obliged to make uſe of. The fire being kindled, will run from one part of the mine to another, like a train of gunpowder, as the combuſtible matter happens to lie. And as the elaſticity of the air is increaſed by heat, *that* in the mine will conſequently ſwell very much, and ſo, for want of room, will explode with a greater or leſs degree of force, according to the denſity of the combuſtible vapours. It is ſometimes ſo ſtrong, as to blow up the mine; and at other times ſo weak, that when it has taken fire at the flame of a candle, it is eaſily blown out.

Air that will take fire at the flame of a candle may be produced thus: Having exhauſted a receiver of the air-pump, let the air run into it through the flame of the oil of turpentine; then remove the cover of the receiver, and holding a candle to that air, it will take fire, and burn quicker or ſlower, according to the denſity of the oleaginous vapour.

Earthquakes. When ſuch combuſtible matter, as is abovementioned, kindles in the bowels of the earth, where there is little or no vent, it produces *earthquakes*, and violent ſtorms or hurricanes of wind when it breaks forth into the air.

An artificial earthquake may be made thus: Take 10 or 15 pounds of ſulphur, and as much of the filings of iron, and knead them with common water into the conſiſtency of a paſte: this being buried in the ground, will, in 8 or 10 hours time, burſt out in flames, and cauſe

PLATE XIV.



J. Ferguson delin. J. Mynde sculp.

the earth to tremble all around to a confiderable diftance.

From this experiment we have a very natural account of the fires of mount *Ætna*, *Vefuvius*, and other volcanos, they being probably fet on fire at firft by the mixture of fuch metalline and fulphureous particles.

The *air-pump* being conftructed the fame way as the water-pump, whoever underftands the one, will be at no lofs to underftand the other. The *air-pump.*

Having put a wet leather on the plate *L L* of the air-pump, place the glafs receiver *M* upon the leather, fo that the hole *i* in the plate may be within the glafs. Then, turning the handle *F* backward and forward, the air will be pumped out of the receiver; which will then be held down to the plate by the preffure of the external air, or atmofphere. For, as the handle *F* (Fig. 2.) is turned backward, it raifes the pifton *d e* in the barrel *B K*, by means of the wheel *E* and rack *D d*: and, as the pifton is leathered fo tight as to fit the barrel exactly, no air can get between the pifton and barrel; and therefore, all the air above *d* in the barrel is lifted up toward *B*, and a vacuum is made in the barrel from *b* to *e*; upon which, part of the air in the receiver *M* (Fig. 1.) by its fpring, rufhes through the hole *i*, in the brafs plate *L L*, along the pipe *G G*, which communicates with both barrels by the hollow trunk *I H K* (Fig. 2.) and pufhing up the valve *b*, enters into the vacant place *b e* of the barrel *B K*. For, wherever the refiftance or preffure is taken off, the air will run to that place, if it can find a paffage. — Then, if the handle *F* be turned forward, the pifton *d e* will be depreffed in the barrel; and, as the air which had got into the Plate XIV. Fig. 1.

barrel cannot be puſhed back through the valve *b*, it will aſcend through a hole in the piſton, and eſcape through a valve at *d*; and be hindered by that valve from returning into the barrel, when the piſton is again raiſed. At the next raiſing of the piſton, a vacuum is again made in the ſame manner as before, between *b* and *e*; upon which, more of the air that was left in the receiver *M*, gets out thence by its ſpring, and runs into the barrel *B K*, through the valve *B*. The ſame thing is to be underſtood with regard to the other barrel *A I*; and as the handle *F* is turned backward and forward, it alternately raiſes and depreſſes the piſtons in their barrels; always raiſing one while it depreſſes the other. And, as there is a vacuum made in each barrel when its piſton is raiſed, the particles of air in the receiver *M* puſh out another by their ſpring or elaſticity, through the hole *i*, and pipe *G G* into the barrels; until at laſt the air in the receiver comes to be ſo much dilated, and its ſpring ſo far weakened, that it can no longer get through the valves; and then no more can be taken out. Hence, there is no ſuch thing as making a perfect vacuum in the receiver; for the quantity of air taken out at any one ſtroke, will always be as the denſity thereof in the receiver: and therefore it is impoſſible to take it all out, becauſe, ſuppoſing the receiver and barrels of equal capacity, there will be always as much left as was taken out at the laſt turn of the handle.

There is a cock *k* below the pump-plate, which being turned, lets the air into the receiver again; and then the receiver becomes looſe, and may be taken off the plate. The barrels are fixed to the frame *E e e* by two ſcrew-nuts *f f*, which

which preſs down the top-piece *E* upon the barrels: and the hollow trunk *H* (in Fig. 2.) is covered by a box, as *GH* in Fig. 1.

There is a glaſs tube *l m m m n* open at both ends, and about 34 inches long; the upper end communicating with the hole in the pump-plate, and the lower end immerſed in quickſilver at *n* in the veſſel *N*. To this tube is fitted a wooden ruler *m m*, called the *gage*, which is divided into inches and parts of an inch, from the bottom at *n* (where it is even with the ſurface of the quickſilver) and continued up to the top, a little below *l*, to 30 or 31 inches.

As the air is pumped out of the receiver *M*, it is likewiſe pumped out of the glaſs tube *l m n*, becauſe that tube opens into the receiver through the pump-plate; and as the tube is gradually emptied of air, the quickſilver in the veſſel *N* is forced up into the tube by the preſſure of the atmoſphere. And if the receiver could be perfectly exhauſted of air, the quickſilver would ſtand as high in the tube as it does at that time in the barometer: for it is ſupported by the ſame power or weight of the atmoſphere in both.

The quantity of air exhauſted out of the receiver on each turn of the handle, is always proportionable to the aſcent of the quickſilver on that turn; and the quantity of air remaining in the receiver, is proportionable to the defect of the height of the quickſilver in the gage, from what it is at that time in the barometer.

I ſhall now give an account of the experiments made with the air-pump in my lectures; ſhewing the reſiſtance, weight, and elaſticity of the air.

I. *To shew the resistance of the air.*

1. There is a little machine, consisting of two Fig. 3. mills, *a* and *b*, which are of equal weights, independent of each other, and turn equally free on their axes in the frame. Each mill has four thin arms or sails, fixed into the axis: those of the mill *a* have their planes at right angles to its axis, and those of *b* have their planes parallel to it. Therefore, as the mill *a* turns round in common air, it is but little resisted thereby, because its sails cut the air with their thin edges: but the mill *b* is much resisted, because the broad sides of its sails move against the air when it turns round. In each axle is a pin near the middle of the frame, which goes quite through the axle, and stands out a little on each side of it: upon these pins the slider *d* may be made to bear, and so hinder the mills from going, when the strong spring *c* is set on bend against the opposite ends of the pins.

Having set this machine upon the pump-plate *L L* (Fig. 1.) draw up the slider *d* to the pins on one side, and set the spring *c* at bend upon the opposite ends of the pins: then push down the slider *d*, and the spring acting equally strong upon each mill, will set them both a going with equal forces and velocities: but the mill *a* will run much longer than the mill *b*, because the air makes much less resistance against the edges of its sails, than against the sides of the sails of *b*.

Draw up the slider again, and set the spring upon the pins as before; then cover the machine with the receiver *M* upon the pump-Fig. 1. plate, and having exhausted the receiver of air, push

push down the wire *P P* (through the collar of leathers in the neck *q*) upon the slider; which will disengage it from the pins, and allow the mills to turn round by the impulse of the spring: and as there is no air in the receiver to make any sensible resistance against them, they will both move a considerable time longer than they did in the open air; and the moment that one stops, the other will do so too.—This shews that air resists bodies in motion, and that equal bodies meet with different degrees of resistance, according as they present greater or less surfaces to the air, in the planes of their motions.

2. Take off the receiver *M*, and the mills; Fig. 4. and having put the guinea *a* and feather *b* upon the brass flap *c*, turn up the flap, and shut it into the notch *d*. Then, putting a wet leather over the top of the tall receiver *A B* (it being open both at top and bottom) cover it with the plate *C*, from which the guinea and feather tongs *e d* will then hang within the receiver. This done, pump the air out of the receiver; and then draw up the wire *f* a little, which by a square piece on its lower end will open the tongs *e d*; and the flap falling down as at *c*, the guinea and feather will descend with equal velocities in the receiver; and both will fall upon the pump-plate at the same instant. *N. B.* In this experiment, the observers ought not to look at the top, but at the bottom of the receiver; in order to see the guinea and feather fall upon the plate; otherwise on account of the quickness of their motion, they will escape the sight of the beholders.

II. *To shew the weight of the air.*

1. Having fitted a braſs cap, with a valve tied over it, to the mouth of a thin bottle or *Florence* flaſk, whoſe contents are exactly known, ſcrew the neck of this cap into the hole *i* of the pump-plate: then, having exhauſted the air out of the flaſk, and taken it off from the pump, let it be ſuſpended at one end of a balance, and nicely counterpoiſed by weights in the ſcale at the other end: this done, raiſe up the valve with a pin, and the air will ruſh into the flaſk with an audible noiſe: during which time, the flaſk will deſcend, and pull down that end of the beam. When the noiſe is over, put as many grains into the ſcale at the other end as will reſtore the equilibrium; and they will ſhew exactly the weight of the quantity of air which has got into the flaſk, and filled it. If the flaſk holds an exact quart, it will be found, that 16 grains will reſtore the equipoiſe of the balance, when the quickſilver ſtands at 29½ inches in the barometer: which ſhews, that when the air is at a mean rate of denſity, a quart of it weighs 16 grains: it weighs more when the quickſilver ſtands higher; and leſs when it ſtands lower.

2. Place the ſmall receiver *O* (Fig. 1.) over the hole *i* in the pump-plate, and upon exhauſting the air, the receiver will be fixed down to the plate by the preſſure of the air on its outſide, which is left to act alone, without any air in the receiver to act againſt it: and this preſſure will be equal to as many times 15 pounds, as there are ſquare inches in that part of the plate which the receiver covers; which will hold down the receiver ſo faſt, that it cannot be got off, until the

the air be let into it by turning the cock *k*; and then it becomes loofe.

3. Set the little glafs *A B* (which is open at both ends) over the hole *i* upon the pump-plate *L L*, and put your hand clofe upon the top of it at *B*: then, upon exhaufting the air out of the glafs, you will find your hand preffed down with a great weight upon it: fo that you can hardly releafe it, until the air be re-admitted into the glafs by turning the cock *k*; which air, by acting as ftrongly upward againft the hand as the external air acted in preffing it downward, will releafe the hand from its confinement. Fig. 5.

4. Having tied a piece of wet bladder *b* over the open top of the glafs *A* (which is alfo open at bottom) fet it to dry, and then the bladder will be tight like a drum. Then place the open end *A* upon the pump-plate, over the hole *i*, and begin to exhauft the air out of the glafs. As the air is exhaufting, its fpring in the glafs will be weakened, and give way to the preffure of the outward air on the bladder, which, as it is preffed down, will put on a fpherical concave figure, which will grow deeper and deeper, until the ftrength of the bladder be overcome by the weight of the air; and then it will burft with a report as loud as that of a gun.—If a flat piece of glafs be laid upon the open top of this receiver, and joined to it by a flat ring of wet leather between them; upon pumping the air out of the receiver, the preffure of the outward air upon the flat glafs will break it to pieces. Fig. 6.

5. Immerfe the neck *c d* of the hollow glafs ball *e b* in water, contained in the phial *a a*; then fet it upon the pump-plate, and cover it and the hole *i* with the clofe receiver *A*; and then begin Fig. 7.

 to

to pump out the air. As the air goes out of the receiver by its ſpring, it will alſo by the ſame means go out of the hollow ball *e b*, through the neck *d c*, and riſe up in bubbles to the ſurface of the water in the phial; from whence it will make its way, with the reſt of the air in the receiver, through the air-pipe *G G* and valves *a* and *b*, into the open air. When it has done bubbling in the phial, the ball is ſufficiently exhauſted; and then, upon turning the cock *k*, the air will get into the receiver, and preſs ſo upon the ſurface of the water in the phial, as to force the water up into the ball in a jet, through the neck *c d*; and will fill the ball almoſt full of water. The reaſon why the ball is not quite filled, is becauſe all the air could not be taken out of it; and the ſmall quantity that was left in, and had expanded itſelf ſo as to fill the whole ball, is now condenſed into the ſame ſtate as the outward air, and remains in a ſmall bubble at the top of the ball; and ſo keeps the water from filling that part of the ball.

Fig. 8. 6. Pour ſome quickſilver into the jar *D*, and ſet it on the pump-plate near the hole *i*; then ſet on the tall open receiver *A B*, ſo as to be over the jar and hole; and cover the receiver with the braſs plate *C*. Screw the open glaſs tube *f g* (which has a braſs top on it at *h*) into the ſyringe *H*, and putting the tube through a hole in the middle of the plate, ſo as to immerſe the lower end of the tube *e* in the quickſilver at *D*, ſcrew the end *h* of the ſyringe into the plate. This done, draw up the piſton in the ſyringe by the ring *I*, which will make a vacuum in the ſyringe, below the piſton; and as the upper end of the tube opens into the ſyringe, the air will be dilated in the tube, becauſe part of it, by its ſpring, gets

gets up into the ſyringe; and the ſpring of the undilated air in the receiver acting upon the ſurface of the quickſilver in the jar, will force part of it up into the tube: for the quickſilver will follow the piſton in the ſyringe, in the ſame way, and for the ſame reaſon, that water follows the piſton of a common pump when it is raiſed in the pump-barrel; and this, according to ſome, is done by ſuction. But to refute that erroneous notion, let the air be pumped out of the receiver *A B*, and then all the quickſilver in the tube will fall down by its own weight into the jar; and cannot be again raiſed one hair's breadth in the tube by working the ſyringe: which ſhews that ſuction had no hand in raiſing the quickſilver; and, to prove that it is done by preſſure, let the air into the receiver by the cock *k* (Fig. 1.) and its action upon the ſurface of the quickſilver in the jar will raiſe it up into the tube, although the piſton of the ſyringe continues motionleſs.—If the tube be about 32 or 33 inches high, the quickſilver will riſe in it very near as high as it ſtands at that time in the barometer. And, if the ſyringe has a ſmall hole, as *m*, near the top of it, and the piſton be drawn up above that hole, the air will ruſh through the hole into the ſyringe and tube, and the quickſilver will immediately fall down into the jar. If this part of the apparatus be air-tight, the quickſilver may be pumped up into the tube to the ſame height that it ſtands in the barometer; but it will go no higher, becauſe then the weight of the column of quickſilver in the tube is the ſame as the weight of a column of air, of the ſame thickneſs with the quickſilver, reaching from the earth to the top of the atmoſphere.

7. Having

Fig. 9. 7. Having placed the jar *A*, with ſome quickſilver in it, on the pump-plate, as in the laſt experiment, cover it with the receiver *B*; then puſh the open end of the glaſs tube *d e* through the collar of leathers in the braſs neck *C* (which it fits ſo as to be air-tight) almoſt down to the quickſilver in the jar. Then exhauſt the air out of the receiver, and it will alſo come out of the tube, becauſe the tube is cloſe at top. When the gauge *m m* ſhews that the receiver is well exhauſted, puſh down the tube, ſo as to immerſe its lower end into the quickſilver in the jar. Now, although the tube be exhauſted of air, none of the quickſilver will riſe into it, becauſe there is no air left in the receiver to preſs upon its ſurface in the jar. But let the air into the receiver by the cock *k*, and the quickſilver will immediately riſe in the tube; and ſtand as high in it, as it was pumped up in the laſt experiment.

Both theſe experiments ſhew, that the quickſilver is ſupported in the barometer by the preſſure of the air on its ſurface in the box, in which the open end of the tube is placed. And that the more denſe and heavy the air is, the higher does the quickſilver riſe; and, on the contrary, the thinner and lighter the air is, the more will the quickſilver fall. For if the handle *F* be turned ever ſo little, it takes ſome air out of the receiver, by raiſing one or other of the piſtons in its barrel; and conſequently, that which remains in the receiver is ſo much the rarer, and has ſo much the leſs ſpring and weight; and thereupon, the quickſilver falls a little in the tube: but upon turning the cock, and re-admitting the air into the receiver, it becomes as weighty as before, and the quickſilver riſes again to the ſame height.

height.—Thus we ſee the reaſon why the quick-ſilver in the barometer falls before rain or ſnow, and riſes before fair weather; for, in the former caſe, the air is too thin and light to bear up the vapours, and in the latter, too denſe and heavy to let them fall.

N. B. In all mercurial experiments with the air-pump, a ſhort pipe muſt be ſcrewed into the hole *i*, ſo as to riſe about an inch above the plate, to prevent the quickſilver from getting into the air-pipe and barrels, in caſe any of it ſhould be accidentally ſpilt over the jar: for if it once gets into the pipes or barrels, it ſpoils them, by looſening the ſolder, and corroding the braſs.

8. Take the tube out of the receiver, and put one end of a bit of dry hazel branch, about an inch long, tight into the hole, and the other end tight into a hole quite through the bottom of a ſmall wooden cup: then pour ſome quickſilver into the cup, and exhauſt the receiver of air, and the preſſure of the outward air, on the ſurface of the quickſilver, will force it through the pores of the hazel, from whence it will deſcend in a beautiful ſhower into a glaſs cup placed under the receiver to catch it.

9. Put a wire through the collar of leathers in the top of the receiver, and fix a bit of dry wood on the end of the wire within the receiver; then exhauſt the air, and puſh the wire down, ſo as to immerſe the wood into a jar of quickſilver on the pump-plate; this done, let in the air, and upon taking the wood out of the jar, and ſplitting it, its pores will be found full of quickſilver, which the force of the air, upon being let into the receiver, drove into the wood.

10. Join the two braſs hemiſpherical cups *A* and *B* together, with a wet leather between them, having Fig. 10.

ing a hole in the middle of it; then ſcrew the end D of the pipe CD into the plate of the pump at i, and turn the cock E, ſo as the pipe may be open all the way into the cavity of the hemiſpheres: then exhauſt the air out of them, and turn the cock a quarter round, which will ſhut the pipe CD, and keep out the air. This done, unſcrew the pipe at D from the pump; and ſcrew the piece Fh upon it at D; and let two ſtrong men try to pull the hemiſpheres aſunder by the rings g and h, which they will find hard to do: for if the diameter of the hemiſpheres be four inches, they will be preſſed together by the external air with a force equal to 190 pounds. And to ſhew that it is the preſſure of the air that keeps them together, hang them by either of the rings upon the hook P of the wire in the receiver M (Fig. 1.) and upon exhauſting the air out of the receiver, they will fall aſunder of themſelves.

11. Place a ſmall receiver O (Fig. 1.) near the hole i on the pump-plate, and cover both it and the hole with the receiver M; and turn the wire ſo by the top P, that its hook may take hold of the little receiver by a ring at its top, allowing that receiver to ſtand with its own weight on the plate. Then, upon working the pump, the air will come out of both receivers; but the large one M will be forcibly held down to the pump by the preſſure of the external air; while the ſmall one O, having no air to preſs upon it, will continue looſe, and may be drawn up and let down at pleaſure, by the wire PP. But, upon letting it quite down to the plate, and admitting the air into the receiver M, by the cock k, the air will preſs ſo ſtrongly upon the ſmall receiver O, as to fix it down to the plate; and at the

the ſame time, by counterbalancing the outward preſſure on the large receiver *M*, it will become looſe. This experiment evidently ſhews, that the receivers are held down by preſſure, and not by ſuction, for the internal receiver continued looſe while the operator was pumping, and the external one was held down; but the former became faſt immediately by letting in the air upon it.

12. Screw the end *A* of the braſs pipe *ABF* into the hole of the pump-plate, and turn the cock *e* until the pipe be open; then put a wet leather upon the plate *c d*, which is fixed on the pipe, and cover it with the tall receiver *GH*, which is cloſe at top: then exhauſt the air out of the receiver, and turn the cock *e* to keep it out; which done, unſcrew the pipe from the pump, and ſet its end *A* into a baſon of water, and turn the cock *e* to open the pipe; on which, as there is no air in the receiver, the preſſure of the atmoſphere on the water in the baſon will drive the water forcibly through the pipe, and make it play up in a jet to the top of the receiver. Fig. 11.

13. Set the ſquare phial *A* (Fig. 14.) upon the pump-plate, and having covered it with the wire cage *B*, put a cloſe receiver over it, and exhauſt the air out of the receiver; in doing of which, the air will alſo make its way out of the phial through a ſmall hole in its neck under the valve *b*. When the air is exhauſted, turn the cock below the plate, to re-admit the air into the receiver: and as it cannot get into the phial again, becauſe of the valve, the phial will be broke into ſome thouſands of pieces by the preſſure of the air upon it. Had the phial been of a round form, it would have ſuſtained this preſſure

preſſure like an arch, without breaking; but as its ſides are flat, it cannot.

To ſhew the elaſticity or ſpring of the air.

14. Tie up a very ſmall quantity of air in a bladder, and put it under a receiver; then exhauſt the air out of the receiver; and the ſmall quantity which is confined in the bladder (having nothing to act againſt it) will expand itſelf ſo by the force of its ſpring, as to fill the bladder as full as it could be blown of common air. But upon letting the air into the receiver again, it will overpower the air in the bladder, and preſs its ſides almoſt cloſe together.

15. If the bladder ſo tied up be put into a wooden box, and have 20 or 30 pound weight of lead put upon it in the box, and the box be covered with a cloſe receiver; upon exhauſting the air out of the receiver, that air which is confined in the bladder will expand itſelf ſo, as to raiſe up all the lead by the force of its ſpring.

Fig. 7. 16. Take the glaſs ball mentioned in the fifth experiment, which was left full of water all but a ſmall bubble of air at top, and having ſet it with its neck downward into the empty phial *a a*, and covered it with a cloſe receiver, exhauſt the air out of the receiver, and the ſmall bubble of air in the top of the ball will expand itſelf, ſo as to force all the water out of the ball into the phial.

Fig. 11. 17. Screw the pipe *A B* into the pump-plate, place the tall receiver *G H* upon the plate *c d*, as in the twelfth experiment, and exhauſt the air out of the recéiver; then, turn the cock *e* to keep out the air, unſcrew the pipe from the pump, and ſcrew it into the mouth of the copper veſſel

veſſel *C C* (Fig. 15.) the veſſel having firſt been about half filled with water. Then open the cock *e* (Fig. 11.) and the ſpring of the air which is confined in the copper veſſel will force the water up through the pipe *A B* in a jet into the exhauſted receiver, as ſtrongly as it did by its preſſure on the ſurface of the water in a baſon, in the twelfth experiment.

18. If a fowl, a cat, rat, mouſe, or bird, be put under a receiver, and the air be exhauſted, the animal will be at firſt oppreſſed as with a great weight, then grow convulſed, and at laſt expire in all the agonies of a moſt bitter and cruel death. But as this experiment is too ſhocking to every ſpectator who has the leaſt degree of humanity, we ſubſtitute a machine called the *lungs-glaſs* in place of the animal.

19. If a butterfly be ſuſpended in a receiver, by a fine thread tied to one of its horns, it will fly about in the receiver, as long as the receiver continues full of air; but if the air be exhauſted, though the animal will not die, and will continue to flutter its wings, it cannot remove itſelf from the place where it hangs in the middle of the receiver, until the air be let in again, and then the animal will fly about as before.

20. Pour ſome quickſilver into the ſmall bottle *A*, and ſcrew the braſs collar *c* of the tube *B C* into the braſs neck *b* of the bottle, and the lower end of the tube will be immerſed into the quickſilver, ſo that the air above the quickſilver in the bottle will be confined there, becauſe it cannot get out about the joinings, nor can it be drawn out through the quickſilver into the tube. This tube is alſo open at top, and is to be covered with the receiver *G* and large tube *E F*, which tube is fixed by braſs collars to the receiver, and is cloſe Fig. 12.

at

at the top. This preparation being made, exhauſt the air both out of the receiver and its tube; and the air will by the ſame means be exhauſted out of the inner tube *B C*, through its open top at *C*; and as the receiver and tubes are exhauſting, the air that is confined in the glaſs bottle *A* will preſs ſo by its ſpring upon the ſurface of the quickſilver, as to force it up in the inner tube as high as it was raiſed in the ninth experiment by the preſſure of the atmoſphere: which demonſtrates that the ſpring of the air is equivalent to its weight.

Fig. 13. 21. Screw the end *C* of the pipe *CD* into the hole of the pump-plate, and turn all the three cocks *d*, *G*, and *H*, ſo as to open the communications between all the three pipes *E*, *F*, *D C*, and the hollow trunk *A B*. Then, cover the plates *g* and *h* with wet leathers, which have holes in their middle where the pipes open into the plates; and place the cloſe receiver *I* upon the plate *g*: this done, ſhut the pipe *F* by turning the cock *H*, and exhauſt the air out of the receiver *I*. Then, turn the cock *d* to ſhut out the air, unſcrew the machine from the pump, and having ſcrewed it to the wooden foot *L*, put the receiver *K* upon the plate *h*; this receiver will continue looſe on the plate as long as it keeps full of air; which it will do until the cock *H* be turned to open the communication between the pipes *F* and *E*, through the trunk *A B*; and then the air in the receiver *K*, having nothing to act againſt its ſpring, will run from *K* into *I*, until it be ſo divided between theſe receivers, as to be of equal denſity in both; and then they will be held down with equal forces to their plates by the preſſure of the atmoſphere; though each receiver will then be kept down but with one half

half of preſſure upon it, that the receiver *I* had, when it was exhauſted of air; becauſe it has now one half of the common air in it which filled the receiver *K* when it was ſet upon the plate; and therefore a force equal to half the force of the ſpring of common air, will act within the receivers againſt the whole preſſure of the common air upon their outſides. This is called transferring the air out of one veſſel into another.

22. Put a cork into the ſquare phial *A*, and fix it in with wax or cement; put the phial upon the pump-plate with the wire cage *B* over it, and cover the cage with a cloſe receiver. Then, exhauſt the air out of the receiver, and the air that was corked up in the phial will break the phial by the force of its ſpring, becauſe there is no air left on the outſide of the phial to act againſt the air within it. Fig. 14.

23. Put a ſhrivelled apple under a cloſe receiver, and exhauſt the air; then the ſpring of the air within the apple will plump it out, ſo as to cauſe all the wrinkles to diſappear; but upon letting the air into the receiver again, to preſs upon the apple, it will inſtantly return to its former decayed and ſhrivelled ſtate.

24. Take a freſh egg, and cut off a little of the ſhell and film from its ſmalleſt end, then put the egg under a receiver, and pump out the air; upon which, all the contents in the egg will be forced out into the receiver, by the expanſion of a ſmall bubble of air contained in the great end, between the ſhell and film.

25. Put ſome warm beer into a glaſs, and having ſet it on the pump, cover it with a cloſe receiver, and then exhauſt the air. While this is doing, and thereby the preſſure more and more

taken off from the beer in the glafs, the air therein will expand itfelf, and rife up in innumerable bubbles to the furface of the beer; and from thence it will be taken away with the other air in the receiver. When the receiver is nearly exhaufted, the air in the beer, which could not difentangle itfelf quick enough to get off with the reft, will now expand itfelf fo, as to caufe the beer to have all the appearance of boiling; and the greateft part of it will go over the glafs.

26. Put fome warm water into a glafs, and put a bit of dry wainfcot or other wood into the water. Then, cover the glafs with a clofe receiver, and exhauft the air; upon which, the air in the wood having liberty to expand itfelf, will come out plentifully, and make all the water to bubble about the wood, efpecially about the ends, becaufe the pores lie lengthwife. A cubic inch of dry wainfcot has fo much air in it, that it will continue bubbling for near half an hour together.

Mifcellaneous Experiments.

27. Screw the fyringe *H* (Fig. 8.) to a piece of lead that weighs one pound at leaft; and, holding the lead in one hand, pull up the pifton in the fyringe with the other; then, quitting hold of the lead, the air will pufh it upward, and drive back the fyringe upon the pifton. The reafon of this is, that the drawing up of the pifton makes a vacuum in the fyringe, and the air, which preffes every way equally, having nothing to refift its preffure upward, the lead is thereby preffed upward, contrary to its natural tendency by gravity. If the fyringe, fo loaded, be

be hung in a receiver, and the air be exhaufted, the fyringe and lead will defcend upon the pifton-rod by their natural gravity; and, upon admitting the air into the receiver, they will be drove upward again, until the pifton be at the very bottom of the fyringe.

28. Let a large piece of cork be fufpended by a thread at one end of a balance, and counterpoifed by a leaden weight, fufpended in the fame manner, at the other. Let this balance be hung to the infide of the top of a large receiver; which being fet on the pump, and the air exhaufted, the cork will preponderate, and fhew itfelf to be heavier than the lead; but upon letting in the air again, the equilibrium will be reftored. The reafon of this is, that fince the air is a fluid, and all bodies lofe as much of their abfolute weight in it, as is equal to the weight of their bulk of the fluid, the cork being the larger body, lofes more of its real weight than the lead does; and therefore muft in fact be heavier, to balance it under the difadvantage of lofing fome of its weight: which difadvantage being taken off by removing the air, the bodies then gravitate according to their real quantities of matter, and the cork, which balanced the lead in air, fhews itfelf to be heavier when *in vacuo*.

29. Set a lighted candle upon the pump, and cover it with a tall receiver. If the receiver holds a gallon, the candle will burn a minute; and then, after having gradually decayed from the firft inftant, it will go out: which fhews, that a conftant fupply of frefh air is neceffary to feed flame; and fo it alfo is for animal life. For a bird kept under a clofe receiver will foon die, although no air be pumped out; and it is

found that, in the diving-bell, a gallon of air is ſufficient only for one minute for a man to breathe in.

The moment when the candle goes out, the ſmoke will be ſeen to aſcend to the top of the receiver, and there it will form a ſort of cloud: but upon exhauſting the air, the ſmoke will fall down to the bottom of the receiver, and leave it as clear at the top as it was before it was ſet upon the pump. This ſhews, that ſmoke does not aſcend on account of its being poſitively light, but becauſe it is lighter than air; and its falling to the bottom when the air is taken away, ſhews, that it is not deſtitute of weight. So moſt ſorts of wood aſcend or ſwim in water; and yet there are none who doubt of the wood's having gravity or weight.

30. Set a receiver, which is open at top, upon the air-pump, and cover it with a braſs plate, and wet leather; and having exhauſted it of air, let the air in again at top through an iron pipe, making it paſs through a charcoal flame at the end of the pipe; and when the receiver is full of that air, lift up the cover, and let down a mouſe or bird into the receiver, and the burnt air will immediately kill it. If a candle be let down into that air, it will go out directly; but, by letting it down gently, it will purify the air ſo far as it goes; and ſo, by letting it down more and more, the flame will drive out the bad air, and good air will get in.

31. Set a bell upon a cuſhion on the pump-plate, and cover it with a receiver; then ſhake the pump to make the clapper ſtrike againſt the bell, and the ſound will be very well heard: but, exhauſt the receiver of air, and then, if the clapper be made to ſtrike ever ſo hard againſt

the bell, it will make no ſound at all; which ſhews, that air is abſolutely neceſſary for the propagation of ſound.

32. Let a candle be placed on one ſide of a receiver, and viewed through the receiver at ſome diſtance; then, as ſoon as the air begins to be exhauſted, the receiver will be filled with vapours which riſe from the wet leather, by the ſpring of the air in it; and the light of the candle being refracted through that medium of vapours, will have the appearance of circles of various colours, of a faint reſemblance to thoſe in the rain-bow.

The air-pump was invented by *Otho Guerick* of *Magdeburg*, but was much improved by Mr. *Boyle*, to whom we are indebted for our greateſt part of the knowledge of the wonderful properties of the air, demonſtrated in the above experiments.

The elaſtic air which is contained in many bodies, and is kept in them by the weight of the atmoſphere, may be got out of them either by boiling, or by the air-pump, as ſhewn in the 25th experiment: but the fixed air, which is by much the greater quantity, cannot be got out but by diſtillation, fermentation, or putrefaction.

If fixed air did not come out of bodies with difficulty, and ſpend ſome time in extricating itſelf from them, it would tear them to pieces. Trees would be rent by the change of air from a fixt, to an elaſtic ſtate, and animals would be burſt in pieces by the exploſion of air in their food.

Dr. *Hales* found by experiment, that the air in apples is ſo much condenſed, that if it were let out into the common air, it would fill a ſpace

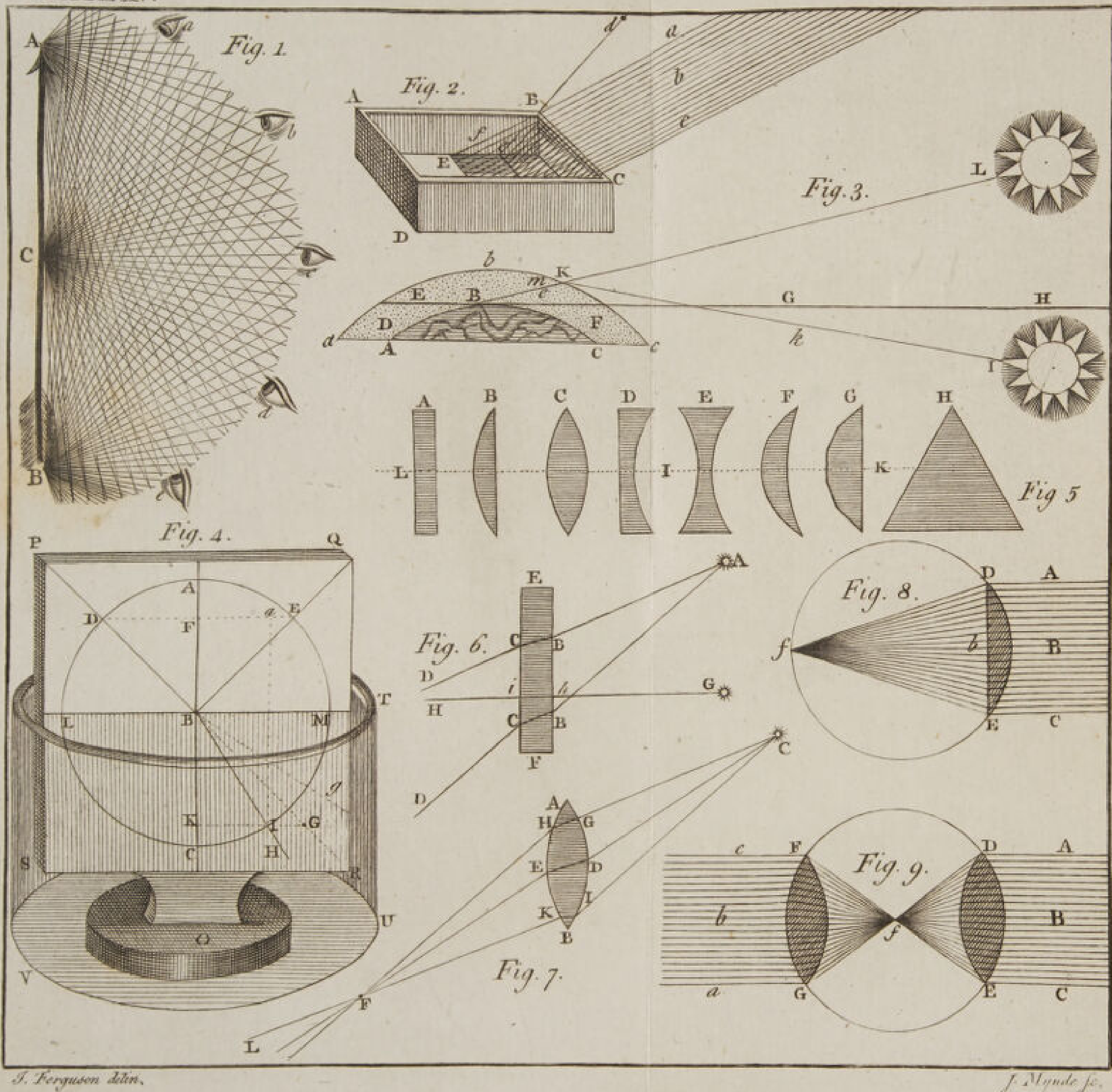
48 times as great as the bulk of the apples themſelves; ſo that its preſſure was equal to 11776lb. and in a cubic inch of oak, to 19860lb. againſt their ſides. So that if the air was let looſe at once in theſe ſubſtances, they would tear every thing to pieces about them with a force ſuperior to that of gunpowder. Hence, in eating apples, it is well that they part with the air by degrees, as they are chewed, and ferment in the ſtomach, otherwiſe an apple would be immediate death to him who eats it.

The mixing of ſome ſubſtances with others will releaſe the air from them, all of a ſudden, which may be attended with very great danger. Of this we have a remarkable inſtance in an experiment made by Dr. *Slare*; who having put half a dram of oil of carraway-ſeed into one glaſs, and a dram of compound ſpirit of nitre in another, covered them both on the air-pump with a receiver ſix inches wide, and eight inches deep, and then exhauſted the air, and continued pumping until all that could poſſibly be got both out of the receiver, and out of the two fluids, was extricated: then, by a particular contrivance from the top of the receiver, he mixed the fluids together; upon which they produced ſuch a prodigious quantity of air, as inſtantly blew up the receiver, although it was preſſed down by the atmoſphere with upward of 400 pounds weight.

N. B. In the 28th experiment, the cork muſt be covered all over with a piece of thin wet bladder glued to it, and not uſed until it be thoroughly dry.

LECT.

PLATE XV.



J. Ferguson delin.

J. Mynde sc.

LECT. VIII.

Of Optics.

LIGHT consists of an inconceivably great number of particles flowing from a luminous body in all manner of directions; and these particles are so small, as to surpass all human comprehension.

That the number of particles of light is inconceivably great, appears from the light of a candle; which, if there be no obstacle in the way to obstruct the passage of its rays, will fill all the space within two miles of the candle, every way, with luminous particles, before it has lost the least sensible part of its substance.

A ray of light is a continued stream of these particles, flowing from any visible body in a straight line: and that the particles themselves are incomprehensibly small, is manifest from the following experiment. Make a small pin-hole in a piece of black paper, and hold the paper upright on a table facing a row of candles standing by one another; then place a sheet of pasteboard at a little distance behind the paper, and some of the rays which flow from all the candles through the hole in the paper, will form as many specks of light on the pasteboard, as there are candles on the table before the plate: each speck being as distinct and clear, as if there was only one speck from one single candle: which shews, that the particles of light are exceedingly small, otherwise they could not pass through the hole from so many different candles without confusion.—Dr. *Niewentyt* has computed, that there flows more than 6,000,000,000,000 times as many

The amazing smallness of the particles of light.

many particles of light from a candle in one ſecond of time, as there are grains of ſand in the whole earth, ſuppoſing each cubic inch of it to contain 1,000,000.

Theſe particles, by falling directly upon our eyes, excite in our minds the idea of light. And when they fall upon bodies, and are thereby reflected to our eyes, they excite in us the ideas of theſe bodies. And as every point of a viſible body reflects the rays of light in all manner of directions, every point will be viſible in every part to which the light is reflected from it. Thus the object *ACB* is viſible to an eye in any part where the rays *Aa*, *Ab*, *Ac*, *Ad*, *Ae*, *Ba*, *Bb*, *Bc*, *Bd*, *Be*, and *Ca*, *Cb*, *Cc*, *Cd*, *Ce*, come. Here we have ſhewn the rays as if they were only reflected from the ends *A* and *B*, and from the middle point *C* of the object; every other point being ſuppoſed to reflect rays in the ſame manner. So that wherever a ſpectator is placed with regard to the body, every point of that part of the ſurface which is toward him will be viſible, when no intervening object ſtops the paſſage of the light.

Plate XV. Fig. 1.

Reflected light.

As no object can be ſeen through the bore of a bended pipe, it is evident that the rays of light move in ſtraight lines, while there is nothing to refract or turn them out of their rectilineal courſe.

While the rays of light continue in any * medium of an uniform denſity, they are ſtraight; but when they paſs obliquely out of one medium into another, which is either more denſe or more rare,

* Any thing through which the rays of light can paſs, is called a medium; as air, water, glaſs, diamond, or even a vacuum.

rare, they are refracted toward the denser medium: and this refraction is more or less, as the rays fall more or less obliquely on the refracting surface which divides the mediums.

To prove this by experiment, set the empty vessel *ABCD* into any place where the sun shines obliquely, and observe the part where the shadow of the edge *BC* falls on the bottom of the vessel at *E*; then fill the vessel with water, and the shadow will reach no farther than *e*; which shews, that the ray *a B E*, which came straight in the open air, just over the edge of the vessel at *B* to its bottom at *E*, is refracted by falling obliquely on the surface of the water at *B*; and instead of going on in the rectilineal direction *a B E*, it is bent downward in the water from *B* to *e*; the whole bend being at the surface of the water: and so of all the other rays *a b c*. Fig. 2. Refracted light.

If a stick be laid over the vessel, and the sun's rays be reflected from a glass perpendicularly into the vessel, the shadow of the stick will fall upon the same part of the bottom, whether the vessel be empty or full, which shews, that the rays of light are not refracted when they fall perpendicularly on the surface of any medium.

The rays of light are as much refracted by passing out of water into air, as by passing out of air into water. Thus, if a ray of light flows from the point *e*, under water, in the direction *e B*; when it comes to the surface of the water at *B*, it will not go on thence in the rectilineal course *B d*, but will be refracted into the line *B a*. Therefore,

To an eye at *e* looking through a plane glass in the bottom of the empty vessel, the point *a* cannot be seen, because the side *B c* of the vessel inter-

interpoſes; and the point *d* will juſt be ſeen over the edge of the veſſel at *B*. But if the veſſel be filled with water, the point *a* will be ſeen from *e*; and will appear as at *d*, elevated in the direction of the ray *e B* *.

The days are made longer by the refraction of the ſun's rays.

The time of ſun-riſing or ſetting, ſuppoſing its rays ſuffered no refraction, is eaſily found by calculation. But obſervation proves that the ſun riſes ſooner, and ſets later every day than the calculated time; the reaſon of which is plain, from what was ſaid immediately above. For, though the ſun's rays do not come part of the way to us through water, yet they do through the air or atmoſphere, which being a groſſer medium than the free ſpace between the ſun and the top of the atmoſphere, the rays, by entering obliquely into the atmoſphere, are there refracted, and thence bent down to the earth. And although there are many places of the earth to which the ſun is vertical at noon, and conſequently his rays can ſuffer no refraction at that time, becauſe they come perpendicularly through the atmoſphere: yet there is no place to which the ſun's rays do not fall obliquely on the top of the atmoſphere, at his riſing and ſetting; and conſequently, no clear day in which the ſun will not be viſible before he riſes in the horizon, and after he ſets in it: and the longer or ſhorter, as the atmoſphere is more or leſs replete with vapours. For, let *A B C* be part of the earth's ſurface, *D E F* the atmoſphere that covers it,

Fig. 3.

* Hence a piece of money lying at *e*, in the bottom of an empty veſſel, cannot be ſeen by an eye at *a*, becauſe the edge of the veſſel intervenes; but let the veſſel be filled with water, and the ray *e a* being then refracted at *B*, will ſtrike the eye at *a*, and ſo render the money viſible, which will appear as if it were raiſed up to *f* in the line *a B f*.

I and

and *E B G H* the ſenſible horizon of an obſerver at *B*. As every point of the ſun's ſurface ſends out rays of light in all manner of directions, ſome of his rays will conſtantly fall upon, and enlighten, ſome half of the atmoſphere; and therefore, when the ſun is at *I*, below the horizon *H*, thoſe rays which go on in the free ſpace *I k K* preſerve a rectilineal courſe until they fall upon the top of the atmoſphere, and thoſe which fall ſo about *K*, are refracted at their entrance into the atmoſphere, and bent down in the line *K m B*, to the obſerver's place at *B*: and therefore, to him, the ſun will appear at *L*, in the direction of the ray *B m K*, above the horizon *B G H*, when he is really below it at *I*.

The angle contained between a ray of light, and a perpendicular to the refracting ſurface, is called *the angle of incidence*; and the angle contained between the ſame perpendicular, and the ſame ray after refraction, is called *the angle of refraction*. Thus, let *L B M* be the refracting ſurface of a medium (ſuppoſe water) and *A B C* a perpendicular to that ſurface: let *D B* be a ray of light, going out of air into water at *B*, and therein refracted in the line *B H*; the angle *A B D*, is the angle of incidence, of which *D F* is the ſine; and the angle *K B H* is the angle of refraction, whoſe ſine is *K I*.

Angle of incidence.

Angle of refraction Fig. 4.

When the refracting medium is water, the ſine of the angle of incidence is to the ſine of the angle of refraction, as 4 to 3; which is confirmed by the following experiment, taken from Doctor Smith's Optics.

Deſcribe the circle *D A E C* on a plane ſquare board, and croſs it at right angles with the ſtraight lines *A B C*, and *L B M*; then, from the interſection *A*, with any opening of the compaſſes,

passes, set off the equal arcs AD and AE, and draw the right line DFE: then, taking Fa, which is three quarters of the length FE, from the point a, draw aI parallel to ABK, and join KI parallel to BM: so KI will be equal to three quarters of FE or of DF. This done, fix the board upright upon the leaden pedestal O, and stick three pins perpendicularly into the board, at the points D, B, and I: then set the board upright into the vessel VUT, and fill up the vessel with water to the line LBM. When the water has settled, look along the line DB, so as you may see the head of the pin B over the head of the pin D; and the pin I will appear in the same right line produced to G, for its head will be seen just over the head of the pin at B: which shews that the ray IB, coming from the pin at I, is so refracted at B, as to proceed from thence in the line BD to the eye of the observer; the same as it would do from any point G in the right line DBG, if there were no water in the vessel: and also shews that KI, the sine of refraction in water, is to DF, the sine of incidence in air, as 3 to 4 *.

Hence, if DBH were a crooked stick put obliquely into the water, it would appear a straight one, as DBG. Therefore, as the line BH appears at BG, so the line BG will appear at Bg; and consequently, a straight stick DBG put obliquely into water, will seem bent at the surface of the water in B, and crooked, as DBg.

When a ray of light passes out of air into glass, the sine of incidence is to the sine of re- fraction,

* This is strictly true of the red rays only, for the other coloured rays are differently refracted; but the difference is so small, that it need not be considered in this place.

fraction, as 3 to 2; and when out of air into a diamond, as 5 to 2.

Glass may be ground into eight different shapes at least, for optical purposes, viz. Fig. 5.

1. A *plane glass*, which is flat on both sides, and of equal thickness in all its parts, as *A*.

2. A *plano-convex*, which is flat on one side, and convex on the other, as *B*. Lenses.

3. A *double convex*, which is convex on both sides, as *C*.

4. A *plano-concave*, which is flat on one side, and concave on the other, as *D*.

5. A *double concave*, which is concave on both sides, as *E*.

6. A *meniscus*, which is concave on one side, and convex on the other, as *F*.

7. A *flat plano-convex*, whose convex side is ground into several little flat surfaces, as *G*.

8. A *prism*, which has three flat sides, and when viewed endwise, appears like an equilateral triangle, as *H*.

Glasses ground into any of the shapes *B, C, D, E, F*, are generally called *lenses*.

A right line *L I K*, going perpendicularly through the middle of a lens, is called *the axis of the lens*.

A ray of light *G h*, falling perpendicularly on a plane glass *E F*, will pass through the glass in the same direction *h i*, and go out of it into the air in the same right course *i H*. Fig. 6.

A ray of light *A B*, falling obliquely on a plane glass, will go out of the glass in the same direction, but not in the same right line; for in touching the glass, it will be refracted in the line *B C*, and in leaving the glass, it will be refracted in the line *C D*.

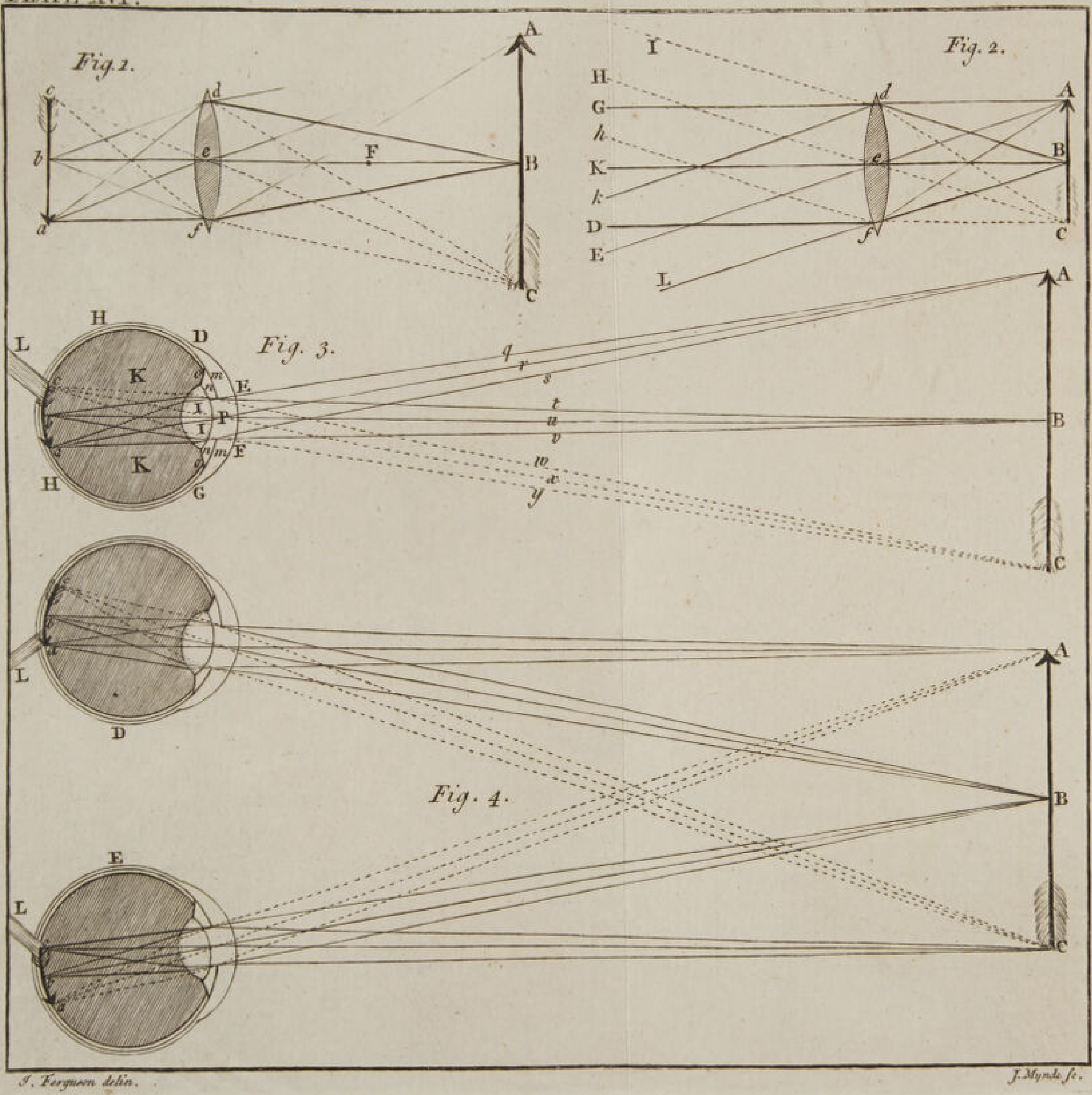
A ray

Fig. 7. A ray of light CD, falling obliquely on the middle of a convex glaſs, will go forward in the ſame direction DE, as if it had fallen with the ſame degree of obliquity on a plane glaſs; and will go out of the glaſs in the ſame direction with which it entered: for it will be equally refracted at the points D and E, as if it had paſſed through a plane ſurface. But the rays CG and CI will be ſo refracted, as to meet again at the point F. Therefore, all the rays which flow from the point C, ſo as to go through the glaſs, will meet again at F: and if they go farther onward, as to L, they croſs at F, and go forward on the oppoſite ſides of the middle ray $CDEF$, to what they were in approaching it in the directions HF and KF.

Fig. 8. The properties of different lenſes. When parallel rays, as ABC, fall directly upon a plano-convex glaſs DE, and paſs through it, they will be ſo refracted, as to unite in a point f behind it: and this point is called the *principal focus*: the diſtance of which, from the middle of the glaſs, is called the *focal diſtance*; which is equal to twice the radius of the ſphere of the glaſs's convexity. And,

Fig. 9. When parallel rays, as ABC, fall directly upon a glaſs DE, which is equally convex on both ſides, and paſs through it; they will be ſo refracted, as to meet in a point or principal focus f, whoſe diſtance is equal to the radius or ſemidiameter of the ſphere of the glaſs's convexity. But if a glaſs be more convex on one ſide than on the other, the rule for finding the focal diſtance is this; as the ſum of the ſemidiameters of both convexities is to the ſemidiameter of either, ſo is double the ſemidiameter of the other to the diſtance of the focus. Or, divide the

PLATE XVI.



the double product of the radii by their ſum, and the quotient will be the diſtance ſought.

Since all thoſe rays of the ſun which paſs through a convex glaſs are collected together in its focus, the force of all their heat is collected into that part; and is in proportion to the common heat of the ſun, as the area of the glaſs is to the area of the focus. Hence we ſee the reaſon why a convex glaſs cauſes the ſun's rays to burn after paſſing through it.

All theſe rays croſs the middle ray in the focus f, and then diverge from it, to the contrary ſides, in the ſame manner $F f G$, as they converged in the ſpace $D f E$ in coming to it.

If another glaſs $F G$, of the ſame convexity as $D E$, be placed in the rays at the ſame diſtance from the focus, it will refract them ſo, as that after going out of it, they will be all parallel, as $a b c$; and go on in the ſame manner as they came to the firſt glaſs $D E$, through the ſpace $A B C$; but on the contrary ſides of the middle ray $B f b$: for the ray $A D f$ will go on from f in the direction $f G a$, and the ray $C E f$ in the direction $f F c$; and ſo of the reſt.

The rays diverge from any radiant point, as from a principal focus: therefore, if a candle be placed at f, in the focus of the convex glaſs $F G$, the diverging rays in the ſpace $F f G$ will be ſo refracted by the glaſs, as, that after going out of it, they will become parallel, as ſhewn in the ſpace $c b a$.

If the candle be placed nearer the glaſs than its focal diſtance, the rays will diverge after paſſing through the glaſs, more or leſs, as the candle is more or leſs diſtant from the focus.

If the candle be placed farther from the glaſs than its focal diſtance, the rays will converge after

after paſſing through the glaſs, and meet in a point which will be more or leſs diſtant from the glaſs, as the candle is nearer to, or farther from its focus; and where the rays meet, they will form an inverted image of the flame of the candle; which may be ſeen on a paper placed in the meeting of the rays.

Plate XVI. Fig. 1. Hence, if any object *A B C* be placed beyond the focus *F* of the convex-glaſs *d e f*, ſome of the rays which flow from every point of the object, on the ſide next the glaſs, will fall upon it, and after paſſing through it, they will be converged into as many points on the oppoſite ſide of the glaſs, where the image of every point will be formed: and conſequently, the image of the whole object, which will be inverted. Thus, the rays *A d*, *A e*, *A f*, flowing from the point *A*, will converge in the ſpace *d a f*, and by meeting at *a*, will there form the image of the point *A*. The rays *B d*, *B e*, *B f*, flowing from the point *B*, will be united at *b* by the refraction of the glaſs, and will there form the image of the point *B*. And the rays *C d*, *C e*, *C f*, flowing from the point *C*, will be united at *c*, where they will form the image of the point *C*. And ſo of all the other intermediate points between *A* and *C*. The rays which flow from every particular point of the object, and are united again by the glaſs, are called *pencils of rays*.

If the object *A B C* be brought nearer to the glaſs, the picture *a b c* will be removed to a greater diſtance. For then, more rays flowing from every ſingle point, will fall more diverging upon the glaſs; and therefore cannot be ſo ſoon collected into the correſponding points behind it. Conſequently, if the diſtance of the object *A B C*

ABC be equal to the diſtance *eB* of the focus of the glaſs, the rays of each pencil will be ſo refracted by paſſing through the glaſs, that they will go out of it parallel to each other; as *dI*, *eH*, *fb*, from the point *C*; *dG*, *eK*, *fD*, from the point *B*; and *dK*, *eE*, *fL*, from the point *A*: and therefore, there will be no picture formed behind the glaſs. Fig. 2.

If the focal diſtance of the glaſs, and the diſtance of the object from the glaſs, be known, the diſtance of the picture from the glaſs may be found by this rule, viz. multiply the diſtance of the focus by the diſtance of the object, and divide the product by their difference; the quotient will be the diſtance of the picture.

The picture will be as much bigger or leſs than the object, as its diſtance from the glaſs is greater or leſs than the diſtance of the object. For, as *Be* is to *eb*, ſo is *AC* to *ca*. So that if *ABC* be the object, *cba* will be the picture; or, if *cba* be the object, *ABC* will be the picture. Fig. 1.

The manner of viſion.

Having deſcribed how the rays of light, flowing from objects, and paſſing through convex glaſſes, are collected into points, and form the images of the objects; it will be eaſy to underſtand how the rays are affected by paſſing through the humours of the eye, and are thereby collected into innumerable points on the bottom of the eye, and thereon form the images of the objects which they flow from. For, the different humours of the eye, and particularly the chryſtalline humour, are to be conſidered as a convex glaſs; and the rays in paſſing through them to be affected in the ſame manner as in paſſing through a convex glaſs.

The eye described. Fig. 3.

The eye is nearly globular. It confifts of three coats and three humours. The part $DHHG$ of the outer coat, is called the *fclerotica*, the reft $DEFG$ the *cornea*. Next within this coat is that called the *choroides*, which ferves as it were for a lining to the other, and joins with the *iris* $m\,n$, $m\,n$. The *iris* is compofed of two fets of mufcular fibres; the one of a circular form, which contracts the hole in the middle called the *pupil*, when the light would otherwife be too ftrong for the eye; and the other of radical fibres, tending every where from the circumference of the iris toward the middle of the pupil; which fibres, by their contraction, dilate and enlarge the pupil when the light is weak, in order to let in the more of its rays. The third coat is only a fine expanfion of the optic nerve L, which fpreads like net-work all over the infide of the choroides, and is therefore called the *retina*; upon which are painted (as it were) the images of all vifible objects, by the rays of light which either flow or are reflected from them.

Under the *cornea* is a fine tranfparent fluid, like water, which is therefore called the *aqueous humour*. It gives a protuberant figure to the cornea, fills the two cavities $m\,m$ and $n\,n$, which communicate by the pupil P, and has the fame limpidity, fpecific gravity, and refractive power as water. At the back of this lies the *chryftalline humour* $I\,I$, which is fhaped like a double convex glafs; and is a little more convex on the back than the fore-part. It converges the rays, which pafs through it from every vifible object to its focus at the bottom of the eye. This humour is tranfparent like chryftal, is much of the confiftence of hard jelly, and exceeds the fpecific ſpecific gravity of water in the proportion of 11 to 10. It is incloſed in a fine tranſparent membrane, from which proceed radial fibres *o o*, called the *ligamentum ciliare*, all around its edge; and join to the circumference of the iris. Theſe fibres have a power of contracting and dilating occaſionally, by which means they alter the ſhape or convexity of the chryſtalline humour, and alſo ſhift it a little backward or forward in the eye, ſo as to adapt its focal diſtance at the bottom of the eye to the different diſtances of objects; without which proviſion, we could only ſee thoſe objects diſtinctly, that were all at one diſtance from the eye.

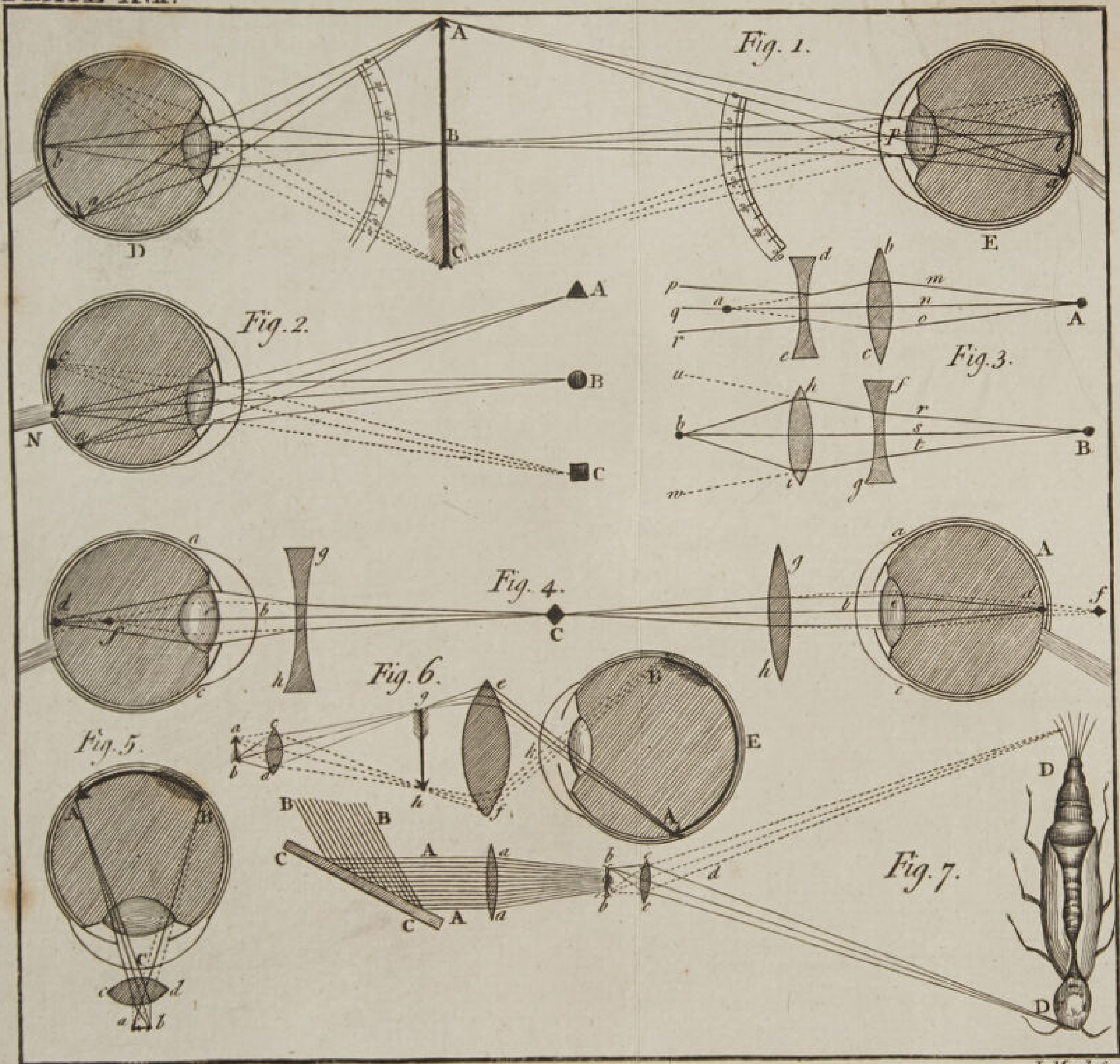
At the back of the chryſtalline, lies the *vitreous humour* K K, which is tranſparent like glaſs, and is largeſt of all in quantity, filling the whole orb of the eye, and giving it a globular ſhape. It is much of a conſiſtence with the white of an egg, and very little exceeds the ſpecific gravity and refractive power of water.

As every point of an object *A B C* ſends out rays in all directions, ſome rays, from every point on the ſide next the eye, will fall upon the cornea between *E* and *F*; and by paſſing on through the humours and pupil of the eye, they will be converged to as many points on the retina or bottom of the eye, and will thereon form a diſtinct inverted picture *c b a* of the object. Thus, the pencil of rays *q r s*, that flows from the point *A* of the object, will be converged to the point *a* on the retina; thoſe from the point *B* will be converged to the point *b*; thoſe from the point *C* will be converged to the point *c*; and ſo of all the intermediate points: by which means the whole image *a b c* is formed, and the object made viſible; although it muſt be owned, that the method by which this ſenſation is carried from the eye by the optic nerve to the common ſenſory in the brain, and there diſcerned, is above the reach of our comprehenſion.

But that viſion is effected in this manner, may be demonſtrated experimentally. Take a bullock's eye while it is freſh, and having cut off the three coats from the back part, quite to the vitreous humour, put a piece of white paper over that part, and hold the eye toward any bright object, and you will ſee an inverted picture of the object upon the paper.

Seeing the image is inverted, many have wondered why the object appears upright. But we are to conſider, 1. That *inverted* is only a relative term: and 2. That there is a very great difference between the real object and the means or image by which we perceive it. When all the parts of a diſtant proſpect are painted upon the retina, they are all right with reſpect to one another, as well as the parts of the proſpect itſelf; and we can only judge of an object's being inverted, when it is turned reverſe to its natural poſition, with reſpect to other objects which we ſee and compare it with.—If we lay hold of an upright ſtick in the dark, we can tell which is the upper or lower part of it, by moving our hand upward or downward; and know very well that we cannot feel the upper end by moving our hand downward. Juſt ſo we find by experience, that upon directing our eyes toward a tall object, we cannot ſee its top by turning our eyes downward, nor its foot by turning our eyes upward; but muſt trace the object the ſame way by the eye to ſee it from head to foot, as we do by the hand to feel it;

PLATE XVII.



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and as the judgment is informed by the motion of the hand in one caſe, ſo it is alſo by the motion of the eye in the other.

In Fig. 4. is exhibited the manner of ſeeing the ſame object *A B C*, by both the eyes *D* and *E* at once. Fig. 4.

When any part of the image *c b a* falls upon the optic nerve *L*, the correſponding part of the object becomes inviſible. On which account nature has wiſely placed the optic nerve of each eye, not in the middle of the bottom of the eye, but toward the ſide next the noſe; ſo that whatever part of the image falls upon the optic nerve of one eye, may not fall upon the optic nerve of the other. Thus the point *a* of the image *c b a* falls upon the optic nerve of the eye *D*, but not of the eye *E*; and the point *e* falls upon the optic nerve of the eye *E*, but not of the eye *D*: and therefore to both eyes taken together, the whole object *A B C* is viſible.

The nearer that any object is to the eye, the larger is the angle under which it is ſeen, and the magnitude under which it appears. Thus to the eye *D*, the object *A B C* is ſeen under the angle *A P C*; and its image *c b a* is very large upon the retina: but to the eye *E*, at a double diſtance, the ſame object is ſeen under the angle *A p C*, which is equal only to half the angle *A P C*, as is evident by the figure. The image *c b a* is likewiſe twice as large in the eye *D*, as the other image *c b a* is in the eye *E*. In both theſe repreſentations, a part of the image falls on the optic nerve, and the object in the correſponding part is inviſible. Plate XVII. Fig. 1.

As the ſenſe of ſeeing is allowed to be occaſioned by the impulſe of the rays from the viſible object upon the retina of the eye, and forming the image of the object thereon, and that the retina is only the expanſion of the optic nerve all over the choroides; it ſhould ſeem ſurpriſing that the part of the image which falls on the optic nerve ſhould render the like part of the object inviſible; eſpecially as that nerve is allowed to be the inſtrument by which the impulſe and image are conveyed to the common ſenſory in the brain. But this difficulty vaniſhes, when we conſider that there is an artery within the trunk of the optic nerve, which entirely obſcures the image in that part, and conveys no ſenſation to the brain.

That the part of the image which falls upon the middle of the optic nerve is loſt, and conſequently the correſponding part of the object is rendered inviſible, is plain by experiment. For, if

Fig. 2. a perſon fixes three patches, *A*, *B*, *C*, horizontally, upon a white wall, at the height of the eye, and the diſtance of about a foot from each other, and places himſelf before them, ſhutting the right eye, and directing the left toward the patch *C*, he will ſee the patches *A* and *C*, but the middle patch *B* will diſappear. Or, if he ſhuts his left eye, and directs the right toward *A*, he will ſee both *A* and *C*, but *B* will diſappear; and if he directs his eye toward *B*, he will ſee both *B* and *A*, but not *C*. For whatever patch is directly oppoſite to the optic nerve *N*, vaniſhes. This requires a little practice, after which he will find it eaſy to direct his eye, ſo as to loſe the ſight of which ever patch he pleaſes.

We are not commonly ſenſible of this diſappearance, becauſe the motions of the eye are ſo quick and inſtantaneous, that we no ſooner loſe the ſight of any part of an object, than we recover it again; much the ſame as in the twinkling of our eyes, for at each twinkling we are blinded; but it is ſo ſoon over, that we are ſcarce ever ſenſible of it.

Some eyes require the aſſiſtance of convex glaſſes to make them ſee objects diſtinctly, and others of concave. If either the cornea *a b c* or chryſtalline humour *e*, or both of them, be too flat, as in the eye *A*, their focus will not be on the retina, as at *d*, where it ought to be, in order to render viſion diſtinct; but beyond the eye, as at *f*. Conſequently thoſe rays which flow from the object *C*, and paſs through the humours of the eye, are not converged enough to unite at *d*; and therefore the obſerver can have but a very indiſtinct view of the object. This is remedied by placing a convex glaſs *g h* before the eye, which makes the rays converge ſooner, and imprints the image duly on the retina at *d*. Fig. 4. Why ſome eyes require ſpectacles.

If either the cornea, or chryſtalline humour, or both of them, be too convex, as in the eye *f*, the rays that enter in from the object *C*, will be converged to a focus in the vitreous humour, as at *f*; and by diverging from thence to the retina, will form a very confuſed image thereon: and ſo, of courſe, the obſerver will have as confuſed a view of the object, as if his eye had been too flat. This inconvenience is remedied by placing a concave glaſs *g h* before the eye; which glaſs, by cauſing the rays to diverge between it and the eye, lengthens the focal diſtance ſo, that if the glaſs be properly choſen, the rays will unite at the retina, and form a diſtinct picture of the object upon it.

Such eyes as have their humours of a due convexity, cannot ſee any object diſtinctly at a leſs diſtance than ſix inches; and there are numberleſs objects too ſmall to be ſeen at that diſtance, becauſe they cannot appear under any ſenſible angle. The method of viewing ſuch minute objects is by a *microſcope*, of which there are three ſorts, viz. the *ſingle*, the *double*, and the *ſolar*.

Fig. 5. The *ſingle micro-ſcope*. The *ſingle microſcope*, is only a ſmall convex glaſs, as *c d*, having the object *a b* placed in its focus, and the eye at the ſame diſtance on the other ſide; ſo that the rays of each pencil, flowing from every point of the object on the ſide next the glaſs, may go on parallel in the ſpace between the eye and the glaſs; and then, by entering the eye at *C*, they will be converged to as many different points on the retina, and form a large inverted picture *A B* upon it, as in the figure.

To find how much this glaſs magnifies, divide the leaſt diſtance (which is about ſix inches) at which an object can be ſeen diſtinctly with the bare eye, by the focal diſtance of the glaſs; and the quotient will ſhew how much the glaſs magnifies the diameter of the object.

Fig. 6. The *double mi-croſcope*. The *double* or *compound microſcope*, conſiſts of an object-glaſs *c d*, and an eye-glaſs *e f*. The ſmall object *a b* is placed at a little greater diſtance from the glaſs *c d* than its principal focus, ſo that the pencils of rays flowing from the different points of the object, and paſſing through the glaſs, may be made to converge and unite in as many points between *g* and *h*, where the image of the object will be formed: which image is viewed by the eye through the eye-glaſs *e f*. For the eye glaſs being ſo placed, that the image *g h* may be in its focus, and the eye much about the ſame diſtance on the other ſide, the rays of each pencil will be parallel, after going out of the eye-glaſs, as at *e* and *f*, till

till they come to the eye at *k*, where they will begin to converge by the refractive power of the humours; and after having croſſed each other in the pupil, and paſſed through the chryſtalline and vitreous humours, they will be collected into points on the retina, and form the large inverted image *A B* thereon.

The magnifying power of this microſcope is as follows. Suppoſe the image *g b* to be ſix times the diſtance of the object *a b* from the object-glaſs *c d*; then will the image be ſix times the length of the object: but ſince the image could not be ſeen diſtinctly by the bare eye at a leſs diſtance than ſix inches, if it be viewed by an eye-glaſs *e f*, of one inch focus, it will thereby be brought ſix times nearer the eye; and conſequently viewed under an angle ſix times as large as before; ſo that it will be again magnified ſix times; that is, ſix times by the object-glaſs, and ſix times by the eye-glaſs, which multiplied into one another, makes 36 times; and ſo much is the object magnified in diameter more than what it appears to the bare eye; and conſequently 36 times 36, or 1296 times in ſurface.

But becauſe the extent or field of view is very ſmall in this microſcope, there are generally two eye-glaſſes placed ſometimes cloſe together, and ſometimes an inch aſunder; by which means, although the object appears leſs magnified, yet the viſible area is much enlarged by the interpoſition of a ſecond eye-glaſs; and conſequently a much pleaſanter view is obtained.

The *ſolar microſcope*, invented by Dr. *Lieberkhun*, is conſtructed in the following manner. Having procured a very dark room, let a round hole be made in the window-ſhutter, about three inches

Fig. 7. The *ſolar microſcope.*

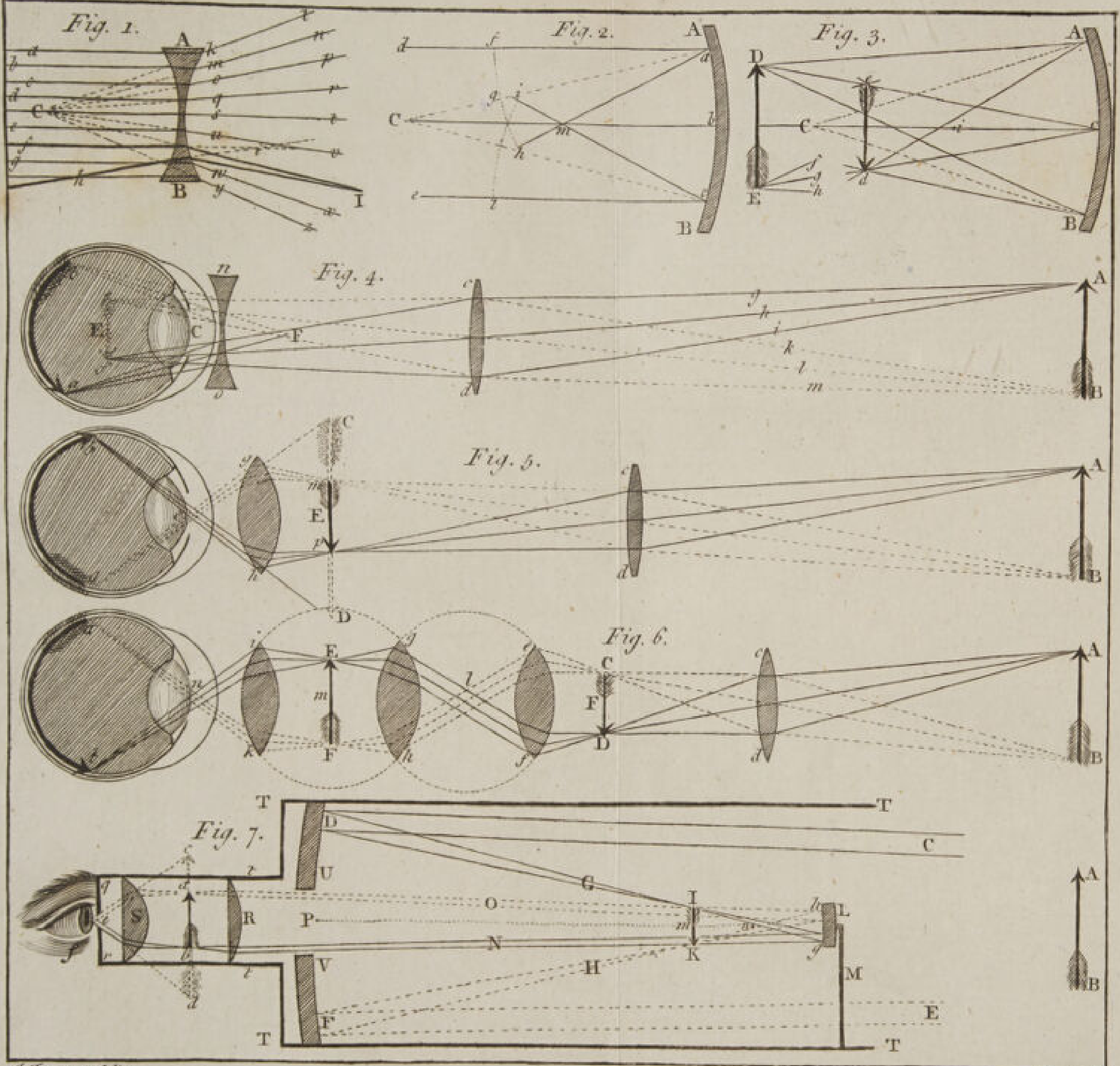
inches diameter, through which the ſun may caſt a cylinder of rays *A A* into the room. In this hole, place the end of a tube, containing two convex glaſſes and an object, *viz.* 1. A convex glaſs *a a*, of about two inches diameter, and three inches focal diſtance, is to be placed in that end of the tube which is put into the hole. 2. The object *b b*, being put between two glaſſes (which muſt be concave to hold it at liberty) is placed about two inches and a half from the glaſs *a a*. 3. A little more than a quarter of an inch from the object is placed the ſmall convex glaſs *c c*, whoſe focal diſtance is a quarter of an inch.

The tube may be ſo placed, when the ſun is low, that his rays *A A* may enter directly into it: but when he is high, his rays *B B* muſt be reflected into the tube by the plane mirror or looking-glaſs *C C*.

Things being thus prepared, the rays that enter the tube will be conveyed by the glaſs *a a* toward the object *b b*, by which means it will be ſtrongly illuminated; and the rays *d* which flow from it, through the magnifying glaſs *c c*, will make a large inverted picture of the object at *D D*, which, being received on a white paper, will repreſent the object magnified in length, in proportion of the diſtance of the picture from the glaſs *c c*, to the diſtance of the object from the ſame glaſs. Thus, ſuppoſe the diſtance of the object from the glaſs to be $\frac{3}{10}$ parts of an inch, and the diſtance of the diſtinct picture to be 12 feet or 144 inches, in which there are 1440 tenths of an inch; and this number divided by 3 tenths, gives 480; which is the number of times the picture is longer or broader than the object; and the length multiplied by the breadth, ſhews how much the whole ſurface is magnified.

Before

PLATE XVIII.



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Before we enter upon the defcription of tele-fcopes, it will be proper to fhew how the rays of light are affected by paffing through concave glaffes, and alfo by falling upon concave mirrors. *Telefcopes.*

When parallel rays, as *a b c d e f g h*, pafs directly through a glafs *A B*, which is equally concave on both fides, they will diverge after paffing through the glafs, as if they had come from a radiant point *C*, in the center of the glafs's concavity; which point is called the negative or virtual focus of the glafs. Thus the ray *a*, after paffing through the glafs *A B*, will go on in the direction *k l*, as if it had proceeded from the point *C*, and no glafs been in the way. The ray *b* will go on in the direction *m n*; the ray *c* in the direction *o p*, &c.—The ray *C*, that falls directly upon the middle of the glafs, fuffers no refraction in paffing through it; but goes on in the fame rectilineal direction, as if no glafs had been in its way. Plate XVIII. Fig. 1.

If the glafs had been concave only on one fide, and the other fide quite plane, the rays would have diverged, after paffing through it, as if they had come from a radiant point at double the diftance of *C* from the glafs; that is, as if the radiant had been at the diftance of a whole diameter of the glafs's concavity.

If rays come more converging to fuch a glafs, than parallel rays diverge after paffing through it, they will continue to converge after paffing through it; but will not meet fo foon as if no glafs had been in the way; and will incline toward the fame fide to which they would have diverged, if they had come parallel to the glafs. Thus the rays *f* and *h*, going in a converging ftate toward the edge of the glafs at *B*, and con-

converging more in their way to it than the parallel rays diverge after paffing through it, they will go on converging after they pafs through it, though in a lefs degree than they did before, and will meet at *I*: but if no glafs had been in their way, they would have met at *i*.

Fig. 2. When the parallel rays, as *d f a*, *C m b*, *e l c*, fall upon a concave mirror *A B* (which is not tranfparent, but has only the furface *A b B* of a clear polifh) they will be reflected back from that mirror, and meet in a point *m*, at half the diftance of the furface of the mirror from *C*, the center of its concavity: for they will be reflected at as great an angle from the perpendicular to the furface of the mirror, as they fell upon it, with regard to that perpendicular; but on the other fide thereof. Thus, let *C* be the center of concavity of the mirror *A b B*, and let the parallel rays *d f a*, *C m b*, and *e l c*, fall upon it at the points *a b*, and *c*. Draw the lines *C i a*, *C m b*, and *C h c*, from the center *C* to thefe points; and all thefe lines will be perpendicular to the furface of the mirror, becaufe they proceed thereto like fo many *radii* or fpokes from its center. Make the angle *C a h* equal to the angle *d a C*, and draw the line *a m h*, which will be the direction of the ray *d f a*, after it is reflected from the point *a* of the mirror: fo that the angle of incidence *d a C*, is equal to the angle of reflection *C a h*; the rays making equal angles with the perpendicular *C i a* on its oppofite fides.

Draw alfo the perpendicular *C h c* to the point *c*, where the ray *e l c* touches the mirror; and, having made the angle *C c i*, equal to the angle *C c e*, draw the line *c m i*, which will be the courfe course of the ray *e l c*, after it is reflected from the mirror.

The ray *C m b* passes through the center of concavity of the mirror, and falls upon it at *b*, the perpendicular to it; and is therefore reflected back from it in the same line *b m C*.

All these reflected rays meet in the point *m*; and in that point the image of the body which emits the parallel rays *d a*, *C b*, and *e c*, will be formed: which point is distant from the mirror equal to half the radius *b m C* of its concavity.

The rays which proceed from any celestial object may be esteemed parallel at the earth; and therefore, the images of that object will be formed at *m*, when the reflecting surface of the concave mirror is turned directly toward the object. Hence, the focus *m* of parallel rays is not in the center of the mirror's concavity, but half way between the mirror and that center.

The rays which proceed from any remote terrestrial object, are nearly parallel at the mirror; not strictly so, but come diverging to it, in separate pencils, or, as it were, bundles of rays, from each point of the side of the object next the mirror: and therefore they will not be converged to a point, at the distance of half the radius of the mirror's concavity from its reflecting surface; but into separate points at a little greater distance from the mirror. And the nearer the object is to the mirror, the farther these points will be from it; and an inverted image of the object will be formed in them, which will seem to hang pendent in the air; and will be seen by an eye placed beyond it (with regard to the mirror) in all respects

like

like the object, and as distinct as the object itself.

Fig. 3. Let *AcB* be the reflecting surface of a mirror, whose center of concavity is at *C*; and let the upright object *DE* be placed beyond the center *C*, and send out a conical pencil of diverging rays from its upper extremity *D*, to every point of the concave surface of the mirror *AcB*. But to avoid confusion, we only draw three rays of that pencil, as *DA*, *Dc*, *DB*.

From the center of concavity *C*, draw the three right lines *CA*, *Cc*, *CB*, touching the mirror in the same points where the foresaid rays touch it; and all these lines will be perpendicular to the surface of the mirror. Make the angle *CAd* equal to the angle *DAC*, and draw the right line *Ad* for the course of the reflected ray *DA*: make the angle *Ccd* equal to the angle *DcC*, and draw the right line *cd* for the course of the reflected ray *Dd*: make also the angle *CBd* equal to the angle *DBC*, and draw the right line *Bd* for the course of the reflected ray *DB*. All these reflected rays will meet in the point *d*, where they will form the extremity *d* of the inverted image *ed*, similar to the extremity *D* of the upright object *DE*.

If the pencils of rays *Ef*, *Eg*, *Eh*, be also continued to the mirror, and their angles of reflection from it be made equal to their angles of incidence upon it, as in the former pencil from *D*, they will all meet at the point *e* by reflection, and form the extremity *e* of the image *ed*, similar to the extremity *E* of the object *DE*.

And as each intermediate point of the object, between *D* and *E*, sends out a pencil of rays in like manner to every part of the mirror, the

rays

rays of each pencil will be reflected back from it, and meet in all the intermediate points between the extremities *e* and *d* of the image; and so the whole image will be formed, not at *i*, half the distance of the mirror from its center of concavity *C*; but at a greater distance, between *i* and the object *D E*; and the image will be inverted with respect to the object.

This being well understood, the reader will easily see how the image is formed by the large concave mirror of the reflecting telescope, when he comes to the description of that instrument.

When the object is more remote from the mirror than its center of concavity *C*, the image will be less than the object, and between the object and mirror: when the object is nearer than the center of concavity, the image will be more remote and bigger than the object: thus, if *D E* be the object, *e d* will be its image; for, as the object recedes from the mirror, the image approaches nearer to it; and as the object approaches nearer to the mirror, the image recedes farther from it; on account of the lesser or greater divergency of the pencils of rays which proceed from the object; for, the less they diverge, the sooner they are converged to points by reflection; and the more they diverge, the farther they must be reflected before they meet.

If the radius of the mirror's concavity and the distance of the object from it be known, the distance of the image from the mirror is found by this rule: divide the product of the distance and radius by double the distance made less by the radius, and the quotient is the distance required.

If

If the object be in the center of the mirror's concavity, the image and object will be coincident, and equal in bulk.

If a man places himſelf directly before a large concave mirror, but farther from it than its center of concavity, he will ſee an inverted image of himſelf in the air, between him and the mirror, of a leſs ſize than himſelf. And if he holds out his hand toward the mirror, the hand of the image will come out toward his hand, and coincide with it, of an equal bulk, when his hand is in the center of concavity; and he will imagine he may ſhake hands with his image. If he reaches his hand farther, the hand of the image will paſs by his hand, and come between his hand and his body: and if he moves his hand toward either ſide, the hand of the image will move toward the other; ſo that whatever way the object moves, the image will move the contrary way.

All the while a by-ſtander will ſee nothing of the image, becauſe none of the reflected rays that form it enter his eyes.

If a fire be made in a large room, and a ſmooth mahogany table be placed at a good diſtance near the wall, before a large concave mirror, ſo placed, that the light of the fire may be reflected from the mirror to its focus upon the table; if a perſon ſtands by the table, he will ſee nothing upon it but a longiſh beam of light: but if he ſtands at a diſtance toward the fire, not directly between the fire and mirror, he will ſee an image of the fire upon the table, large and erect. And if another perſon, who knows nothing of this matter beforehand, ſhould chance to come into the room, and ſhould look from the fire toward the table,

he

he would be ſtartled at the appearance; for the table would ſeem to be on fire, and by being near the wainſcot, to endanger the whole houſe. In this experiment there ſhould be no light in the room, but what proceeds from the fire; and the mirror ought to be at leaſt fifteen inches in diameter.

If the fire be darkened by a ſcreen, and a large candle be placed at the back of the ſcreen; a perſon ſtanding by the candle will ſee the appearance of a fine large ſtar, or rather planet, upon the table, as bright as Venus or Jupiter. And if a ſmall wax taper (whoſe flame is much leſs than the flame of the candle) be placed near the candle, a ſatellite to the planet will appear on the table: and if the taper be moved round the candle, the ſatellite will go round the planet.

For theſe two pleaſing experiments, I am indebted to the late reverend Dr. LONG, *Lowndes*'s profeſſor of aſtronomy at Cambridge, who favoured me with the ſight of them, and many more of his curious inventions.

In a *refracting teleſcope*, the glaſs which is neareſt the object in viewing it, is called the *object-glaſs*; and that which is neareſt the eye, is called the *eye-glaſs*. The object-glaſs muſt be convex, but the eye-glaſs may be either convex or concave: and generally, in looking through a teleſcope, the eye is in the focus of the eye-glaſs; though that is not very material: for the diſtance of the eye, as to diſtinct viſion, is indifferent, provided the rays of the pencils fall upon it parallel: only the nearer the eye is to the end of the teleſcope, the larger is the ſcope or area of the field of view. The *refracting teleſcope*.

Let $c\,d$ be a convex-glaſs fixed in a long tube, and have its focus at E. Then, a pencil of rays Fig. 4.

Q $g\,h\,i$,

g h i, flowing from the upper extremity *A* of the remote object *A B*, will be so refracted by passing through the glass, as to converge and meet in the point *f*; while the pencil of rays *k l m* flowing from the lower extremity *B*, of the same object *A B*, and passing through the glass, will converge and meet in the point *e*: and the images of the points *A* and *B*, will be formed in the points *f* and *e*. And as all the intermediate points of the object, between *A* and *B*, send out pencils of rays in the same manner, a sufficient number of these pencils will pass through the object glass *c d*, and converge to as many intermediate points between *e* and *f*; and so will form the whole inverted image *e E f*, of the distinct object. But because this image is small, a concave glass *n o* is so placed in the end of the tube next the eye, that its virtual focus may be at *F*. And as the rays of the pencils pass converging through the concave glass, but converge less after passing through it than before, they go on further, as to *b* and *a*, before they meet; and the pencils themselves being made to diverge by passing through the concave glass, they enter the eye, and form the large picture *a b* upon the retina, whereon it is magnified under the angle *b F a*.

But this telescope has one inconveniency which renders it unfit for most purposes, which is, that the pencils of rays being made to diverge by passing through the concave glass *n o*, very few of them can enter the pupil of the eye; and therefore the field of view is but very small, as is evident by the figure. For none of the pencils which flow either from the top or bottom of the object *A B* can enter the pupil of the eye at *C*, but are all stopt by falling upon the iris above and below the pupil: and therefore, only the middle part of the object can be ſeen when the teleſcope lies directly toward it, by means of thoſe rays which proceed from the middle of the object. So that to ſee the whole of it, the teleſcope muſt be moved upward and downward, unleſs the object be very remote; and then it is never ſeen diſtinctly.

This inconvenience is remedied by ſubſtituting a convex eye-glaſs, as *g h*, in place of the concave one; and fixing it ſo in the tube, that its focus may be coincident with the focus of the object-glaſs *c d*, as at *E*. For then, the rays of the pencils flowing from the object *A B*, and paſſing through the object-glaſs *c d*, will meet in its focus, and form the inverted image *m E p*: and as the image is formed in the focus of the eye-glaſs *g h*, the rays of each pencil will be parallel, after paſſing through that glaſs; but the pencils themſelves will croſs in its focus, on the other ſide, as at *e*: and the pupil of the eye being in this focus, the image will be viewed through the glaſs, under the angle *g e h*; and being at *E*, it will appear magnified, ſo as to fill the whole ſpace *C m e p D*. Fig. 5.

But, as this teleſcope inverts the image with reſpect to the object, it gives an unpleaſant view of terreſtrial objects; and is only fit for viewing the heavenly bodies, in which we regard not their poſition, becauſe their being inverted does not appear, on account of their being round. But whatever way the object ſeems to move, this teleſcope muſt be moved the contrary way, in order to keep ſight of it; for, ſince the object is inverted, its motion will be ſo too.

The magnifying power of this teleſcope is, as the focal diſtance of the object-glaſs to the focal diſtance of the eye-glaſs. Therefore, if the former be divided by the latter, the quotient will expreſs the magnifying power.

When we ſpeak of magnifying by a teleſcope or microſcope, it is only meant with regard to the diameter, not to the area or ſolidity of the object. But as the inſtrument magnifies the vertical diameter, as much as it does the horizontal, it is eaſy to find how much the whole viſible area or ſurface is magnified: for, if the diameters be multiplied into one another, the product will expreſs the magnification of the whole viſible area. Thus, ſuppoſe the focal diſtance of the object-glaſs be ten times as great as the focal diſtance of the eye-glaſs; then, the object will be magnified ten times, both in length and breadth: and 10 multiplied by 10, produces 100; which ſhews, that the area of the object will appear 100 times as big when ſeen through ſuch a teleſcope, as it does to the bare eye.

Hence it appears, that if the focal diſtance of the eye-glaſs, were equal to the focal diſtance of the object-glaſs, the magnifying power of the teleſcope would be nothing.

This teleſcope may be made to magnify in any given degree, provided it be of a ſufficient length. For, the greater the focal diſtance of the object-glaſs, the leſs may be the focal diſtance of the eye-glaſs; though not directly in proportion. Thus, an object-glaſs of 10 feet focal diſtance, will admit of an eye-glaſs whoſe focal diſtance is little more than $2\frac{1}{2}$ inches; which will magnify near 48 times: but an object-glaſs, of 100 feet focus, will require an eye-glaſs ſomewhat more than 6 inches; and will therefore magnify almoſt 200 times.

A teleſcope for viewing terreſtrial objects, ſhould be ſo conſtructed, as to ſhew them in their natural poſture. And this is done by one object-glaſs *c d*, and three eye-glaſſes *e f*, *g h*, *i k*, ſo placed, that the diſtance between any two, which are neareſt to each other, may be equal to the ſum of their focal diſtances; as in the figure, where the focus of the glaſſes *c d* and *e f* meet at *F*, thoſe of the glaſſes *e f* and *g h* meet at *l*, and of *g h* and *i k*, at *m*; the eye being at *n*, in or near the focus of the eye-glaſs *i k*, on the other ſide. Then, it is plain, that theſe pencils of rays, which flow from the object *A B*, and paſs through the object-glaſs *c d*, will meet and form an inverted image *C F D* in the focus of that glaſs; and the image being alſo in the focus of the glaſs *e f*, the rays of the pencils will become parallel, after paſſing through that glaſs, and croſs at *l*, in the focus of the glaſs *e f*; from whence they paſs on to the next glaſs *g h*, and by going through it they are converged to points in its other focus, where they form an erect image *E m F*, of the object *A B*: and as this image is alſo in the focus of the eye-glaſs *i k*, and the eye on the oppoſite ſide of the ſame glaſs; the image is viewed through the eye-glaſs in this teleſcope, in the ſame manner as through the eye-glaſs in the former one; only in a contrary poſition, that is, in the ſame poſition with the object. Fig. 6.

The three glaſſes next the eye, have all their focal diſtances equal: and the magnifying power of this teleſcope is found the ſame way as that of the laſt above; viz. by dividing the focal diſtance of the object-glaſs *c d*, by the focal diſtance of the eye-glaſs *i k*, or *g h*, or *e f*, ſince all theſe three are equal.

When the rays of light are ſeparated by refraction, they become coloured, and if they be united again, they will be a perfect white. But

Q 3 thoſe

Why the object appears coloured, when seen through a telescope.

those rays which pass through a convex glass, near its edges are more unequally refracted than those which are nearer the middle of the glass. And when the rays of any pencil are unequally refracted by the glass, they do not all meet again in one and the same point, but in separate points; which makes the image indistinct, and coloured, about its edges. The remedy is, to have a plate with a small round hole in its middle, fixed in the tube at *m*, parallel to the glasses. For, the wandering rays about the edges of the glasses will be stopt, by the plate, from coming to the eye: and none admitted but those which come through the middle of the glass, or at least at a good distance from its edges, and pass through the hole in the middle of the plate. But this circumscribes the image, and lessens the field of view, which would be much larger if the plate could be dispensed with.

The *reflecting telescope.*

The great inconvenience attending the management of long telescopes of this kind, has brought them much into disuse ever since the *reflecting telescope* was invented. For one of this sort, six feet in length, magnifies as much as one of the other a hundred feet. It was invented by Sir *Isaac Newton*, but has received considerable improvements since his time; and is now generally constructed in the following manner, which was first proposed by Dr. *Gregory*.

Fig. 7.

At the bottom of the great tube *TTTT* is placed the large concave mirror *DUVF*, whose principal focus is at *m*; and in its middle is a round hole *P*, opposite to which is placed the small mirror *L*, concave toward the great one; and so fixed to a strong wire *M*, that it may be moved farther from the great mirror, or nearer to it, by means of a long screw on the out-

outſide of the tube, keeping its axis ſtill in the ſame line *P m n* with that of the great one.—Now, ſince in viewing a very remote object, we can ſcarce ſee a point of it but what is at leaſt as broad as the great mirror, we may conſider the rays of each pencil, which flow from every point of the object, to be parallel to each other, and to cover the whole reflecting ſurface *D U V F*. But to avoid confuſion in the figure, we ſhall only draw two rays of a pencil flowing from each extremity of the object into the great tube, and trace their progreſs, through all their reflections and refractions, to the eye *f*, at the end of the ſmall tube *t t*, which is joined to the great one.

Let us then ſuppoſe the object *A B* to be at ſuch a diſtance, that the rays *C* may flow from its lower extremity *B*, and the rays *E* from its upper extremity *A*. Then the rays *C* falling parallel upon the great mirror at *D*, will be thence reflected, converging in the direction *D G*; and by croſſing at *I* in the principal focus of the mirror, they will form the upper extremity *I* of the inverted image *I K*, ſimilar to the lower extremity *B* of the object *A B*: and paſſing on to the concave mirror *L* (whoſe focus is at *n*) they will fall upon it at *g*, and be thence reflected converging in the direction *g N*, becauſe *g m* is longer than *g n*; and paſſing through the hole *P* in the large mirror, they would meet ſomewhere about *r*, and form the lower extremity *b* of the erect image *a b*, ſimilar to the lower extremity *B* of the object *A B*. But by paſſing through the plano-convex glaſs *R* in their way, they form that extremity of the image at *b*. In like manner, the rays *E*, which come from the top of the object *A B*, and fall parallel upon the great mirror at *F*, are thence reflected converging to its focus, where they form the lower extremity *K* of the inverted image *IK*, ſimilar to the upper extremity *A* of the object *AB*; and thence paſſing on to the ſmall mirror *L*, and falling upon it at *b*, they are thence reflected in the converging ſtate *bO*; and going on through the hole *P* of the great mirror, they will meet ſomewhere about *q*, and form there the upper extremity *a* of the erect image *ab*, ſimilar to the upper extremity *A* of the object *AB*: but by paſſing through the convex glaſs *R* in their way, they meet and croſs ſooner, as at *a*, where that point of the erect image is formed.—The like being underſtood of all thoſe rays which flow from the intermediate points of the object, between *A* and *B*, and enter the tube *TT*; all the intermediate points of the image between *a* and *b* will be formed: and the rays paſſing on from the image through the eye glaſs *S*, and through a ſmall hole *e* in the end of the leſſer tube *tt*, they enter the eye *f*, which ſees the image *ab* (by means of the eye-glaſs) under the large angle *ced*, and magnified in length, under that angle from *c* to *d*.

In the beſt reflecting teleſcopes, the focus of the ſmall mirror is never coincident with the focus *m* of the great one, where the firſt image *IK* is formed, but a little beyond it (with reſpect to the eye) as at *n*: the conſequence of which is, that the rays of the pencils will not be parallel after reflection from the ſmall mirror, but converge ſo as to meet in points about *q*, *e*, *r*; where they will form a larger upright image than *ab*, if the glaſs *R* was not in their way; and this image might be viewed by means of a ſingle eye-glaſs properly placed between the image and the eye: but then the field of view would be

leſs,

leſs, and conſequently not ſo pleaſant; for which reaſon, the glaſs *R* is ſtill retained, to enlarge the ſcope or area of the field.

To find the magnifying power of this teleſcope, multiply the focal diſtance of the great mirror by the diſtance of the ſmall mirror from the image next the eye, and multiply the focal diſtance of the ſmall mirror by the focal diſtance of the eye-glaſs: then, divide the product of the former multiplication by the product of the latter, and the quotient will expreſs the magnifying power.

I ſhall here ſet down the dimenſions of one of Mr. *Short*'s reflecting teleſcopes, as deſcribed in Dr. *Smith*'s Optics.

The focal diſtance of the great mirror 9.6 inches, its breadth 2.3; the focal diſtance of the ſmall mirror 1.5, its breadth 0.6: the breadth of the hole in the great mirror 0.5; the diſtance between the ſmall mirror and the next eye-glaſs 14.2; the diſtance between the two eye-glaſſes 2.4; the focal diſtance of the eye-glaſs next the metals 3.8; and the focal diſtance of the eye-glaſs next the eye 1.1.

One great advantage of the reflecting teleſcope is, that it will admit of an eye-glaſs of a much ſhorter focal diſtance than a refracting teleſcope will; and, conſequently, it will magnify ſo much the more: for the rays are not coloured by reflection from a concave mirror, if it be ground to a true figure, as they are by paſſing through a convex-glaſs, let it be ground ever ſo true.

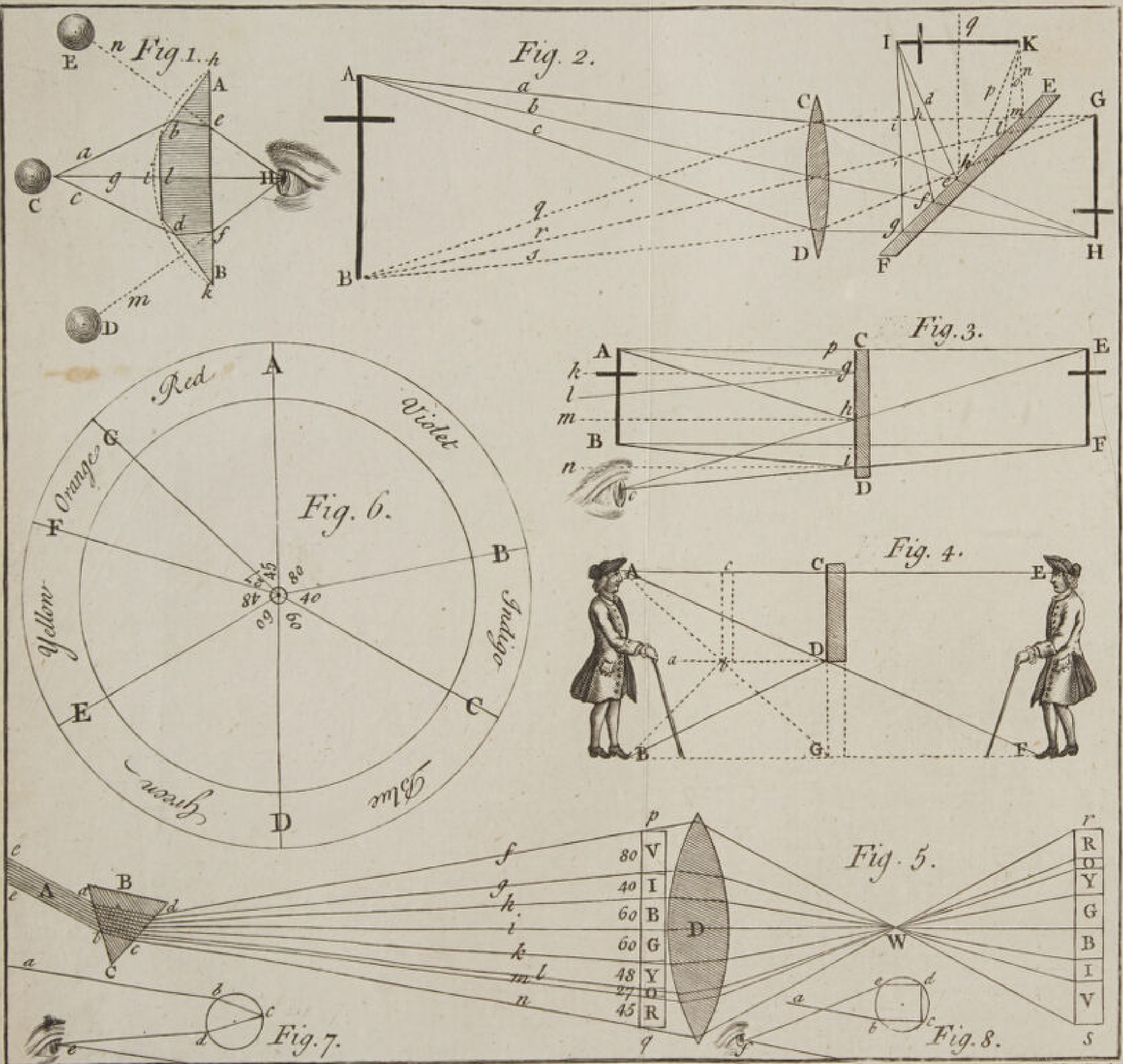
The adjuſting ſcrew on the outſide of the great tube fits this teleſcope to all ſorts of eyes, by bringing the ſmall mirror either nearer to the eye, or removing it farther; by which means,

means, the rays are made to diverge a little for ſhort-ſighted eyes, or to converge for thoſe of a long ſight.

The nearer an object is to the teleſcope, the more its pencils of rays will diverge before they fall upon the great mirror, and therefore they will be the longer of meeting in points after reflection; ſo that the firſt image *I K* will be formed at a greater diſtance from the large mirror, when the object is near the teleſcope, than when it is very remote. But as this image muſt be formed farther from the ſmall mirror than its principal focus *n*, this mirror muſt be always ſet at a greater diſtance from the large one, in viewing near objects, than in viewing remote ones. And this is done by turning the ſcrew on the outſide of the tube, until the ſmall mirror be ſo adjuſted, that the object (or rather its image) appears perfect.

In looking through any teleſcope toward an object, we never ſee the object itſelf, but only that image of it which is formed next the eye in the teleſcope. For, if a man holds his finger or a ſtick between his bare eye and an object, it will hide part (if not the whole) of the object from his view. But if he ties a ſtick acroſs the mouth of a teleſcope, before the object-glaſs, it will hide no part of the imaginary object he ſaw through the teleſcope before, unleſs it covers the whole mouth of the tube: for, all the effect will be, to make the object appear dimmer, becauſe it intercepts part of the rays. Whereas, if he puts only a piece of wire acroſs the inſide of the tube, between the eye-glaſs and his eye, it will hide part of the object which he thinks he ſees: which proves that he ſees not the real object, but its image. This is alſo confirmed by means of the ſmall

PLATE XIX.



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ſmall mirror *L*, in the reflecting teleſcope, which is made of opaque metal, and ſtands directly between the eye and the object toward which the teleſcope is turned; and will hide the whole object from the eye at *e*, if the two glaſſes *R* and *S* are taken out of the tube.

The multiplying glaſs is made by grinding down the round ſide *h i k* of a convex glaſs *A B* into ſeveral flat ſurfaces, as *b h*, *b l d*, *d k*. An object *C* will not appear magnified, when ſeen through this glaſs, by the eye at *H*; but it will appear multiplied into as many different objects as the glaſs contains plane ſurfaces. For, ſince rays will flow from the object *C* to all parts of the glaſs, and each plane ſurface will refract theſe rays to the eye, the ſame object will appear to the eye, in the direction of the rays which enter it through each ſurface. Thus, a ray *g i H*, falling perpendicularly on the middle ſurface, will go through the glaſs to the eye without ſuffering any refraction; and will therefore ſhew the object in its true place at *C*: while a ray *a b* flowing from the ſame object, and falling obliquely on the plane ſurface *b h*, will be refracted in the direction *b e*, by paſſing through the glaſs; and upon leaving it, will go on to the eye in the direction *e H*; which will cauſe the ſame object *C* to appear alſo at *E*, in the direction of the ray *H e*, produced in the right line *H e n*. And the ray *c d*, flowing from the object *C*, and falling obliquely on the plane ſurface *d k*, will be refracted (by paſſing through the glaſs and leaving it at *f*) to the eye at *H*; which will cauſe the ſame object to appear at *D*, in the direction *H f m*.— If the glaſs be turned round the line *g l H*, as an axis, the object *C* will keep its place, becauſe the ſurface *b l d* is not removed; but all the other

Plate XIX. Fig. 1. The *multiplying-glaſs.*

other objects will ſeem to go round *C*, becauſe the oblique planes, on which the rays *a b*, *c d* fall, will go round by the turning of the glaſs.

Fig. 2. The *camera obſcura*.

The *camera obſcura* is made by a convex glaſs *C D*, placed in a hole of a window-ſhutter. Then, if the room be darkened ſo as no light can enter but what comes through the glaſs, the pictures of all the objects (as fields, trees, buildings, men, cattle, &c.) on the outſide, will be ſhewn in an inverted order, on a white paper placed at *G H* in the focus of the glaſs; and will afford a moſt beautiful and perfect piece of perſpective or landſcape of whatever is before the glaſs; eſpecially if the ſun ſhines upon the objects.

If the convex glaſs *C D* be placed in a tube in the ſide of a ſquare box, within which is the plane mirror *E F*, reclining backward in an angle of 45 degrees from the perpendicular *k q*, the pencils of rays flowing from the outward objects, and paſſing through the convex glaſs to the plane mirror, will be reflected upward from it, and meet in points, as *I* and *K* (at the ſame diſtance that they would have met at *H* and *G*, if the mirror had not been in the way) and will form the aforeſaid images on an oiled paper ſtretched horizontally in the direction *I K*; on which paper, the outlines of the images may be eaſily drawn with a black lead pencil; and then copied on a clean ſheet, and coloured by art, as the objects themſelves are by nature.— In this machine, it is uſual to place a plane glaſs, unpoliſhed, in the horizontal ſituation *I K*, which glaſs receives the images of the outward objects; and their outlines may be traced upon it by a black-lead pencil.

N. B.

N. B. The tube in which the convex glaſs *C D* is fixed, muſt be made to draw out, or puſh in, ſo as to adjuſt the diſtance of that glaſs from the plane mirror, in proportion to the diſtance of the outward objects; which the operator does, until he ſees their images diſtinctly painted on the horizontal glaſs at *I K*.

The forming a horizontal image, as *I K*, of an upright object *A B*, depends upon the angles of incidence of the rays upon the plane mirror *E F*, being equal to their angles of reflection from it. For, if a perpendicular be ſuppoſed to be drawn to the ſurface of the plane mirror at *e*, where the ray *A a C e* falls upon it, that ray will be reflected upward in an equal angle with the other ſide of the perpendicular, in the line *e d I*. Again, if a perpendicular be drawn to the mirror from the point *f*, where the ray *A b f* falls upon it, that ray will be reflected in an equal angle from the other ſide of the perpendicular, in the line *f h I*. And if a perpendicular be drawn from the point *g*, where the ray *A c g* falls upon the mirror, that ray will be reflected in an equal angle from the other ſide of the perpendicular, in the line *g i I*. So that all the rays of the pencil *a b c*, flowing from the upper extremity of the object *A B*, and paſſing through the convex glaſs *C D*, to the plane mirror *E F*, will be reflected from the mirror and meet at *I*, where they will form the extremity *I* of the image *I K*, ſimilar to the extremity *A* of the object *A B*. The like is to be underſtood of the pencil *q r s*, flowing from the lower extremity of the object *A B*, and meeting at *K* (after reflection from the plane mirror) the rays form the extremity *K* of the image, ſimilar to the extremity *B* of the object: and ſo of all the pencils that flow from the intermediate

termediate points of the object to the mirror, through the convex glafs.

The opera-glafs. If a convex glafs, of a fhort focal diftance, be placed near the plane mirror, in the end of a fhort tube, and a convex glafs be placed in a hole in the fide of the tube, fo as the image may be formed between the laft-mentioned convex glafs, and the plane mirror, the image being viewed through this glafs will appear magnified. —In this manner the *opera-glaffes* are conftructed; with which a gentleman may look at any lady at a diftance in the company, and the lady know nothing of it.

The common looking-glafs. The image of any object that is placed before a plane mirror, appears as big to the eye as the object itfelf; and is erect, diftinct, and feemingly as far behind the mirror, as the object is before it: and that part of the mirror, which reflects the image of the object to the eye (the eye being fuppofed equally diftant from the glafs with the object) is juft half as long and half as broad as the object itfelf. Let AB be an object placed before the reflecting furface $g\,h\,i$ of the plane mirror CD; and let the eye be at o. Let Ah be a ray of light flowing from the top A of the object, and falling upon the mirror at h: and hm be a perpendicular to the furface of the mirror at h, the ray Ah will be reflected from the mirror to the eye at o, making an angle $m\,h\,o$ equal to the angle $A\,h\,m$: then will the top of the image E appear to the eye in the direction of the reflected ray $o\,h$ produced to E, where the right line $A\,p\,E$, from the top of the object, cuts the right line $o\,h\,E$, at E. Let $B\,i$ be a ray of light proceeding from the foot of the object at B to the mirror at i, and $n\,i$ a perpendicular to the mirror from the point i, where

Fig. 3.

where the ray *B i* falls upon it: this ray will be reflected in the line *i o*, making an angle *n i o*, equal to the angle *B i n*, with that perpendicular, and entering the eye at *o*: then will the foot *F* of the image appear in the direction of the reflected ray *o i*, produced to *F*, where the right line *BF* cuts the reflected ray produced to *F*. All the other rays that flow from the intermediate points of the object *AB*, and fall upon the mirror between *h* and *i*, will be reflected to the eye at *o*; and all the intermediate points of the image *EF* will appear to the eye in the direction-line of these reflected rays produced. But all the rays that flow from the object, and fall upon the mirror above *h*, will be reflected back above the eye at *o*; and all the rays that flow from the object, and fall upon the mirror below *i*, will be reflected back below the eye at *o*: so that none of the rays that fall above *h*, or below *i*, can be reflected to the eye at *o*; and the distance between *h* and *i* is equal to half the length of the object *AB*.

A man will see his image in a plane looking-glass, that is but half his height. Fig. 4.

Hence it appears, that if a man see his whole image in a plane looking-glass, the part of the glass that reflects his image must be just half as long and half as broad as himself, let him stand at any distance from it whatever; and that his image must appear just as far behind the glass as he is before it. Thus, the man *AB* viewing himself in the plane mirror *CD*, which is just half as long as himself, sees his whole image as at *EF*, behind the glass, exactly equal to his own size. For, a ray *AC* proceeding from his eye at *A*, and falling perpendicularly upon the surface of the glass at *C*, is reflected back to his eye in the same line *CA*; and the eye of his image will appear at *E*, in the same line produced

duced to *E*, beyond the glaſs. And a ray *B D*, flowing from his foot, and falling obliquely on the glaſs at *D*, will be reflected as obliquely on the other ſide of the perpendicular *a b D*, in the direction *D A*; and the foot of his image will appear at *F*, in the direction of the reflected ray *A D*, produced to *F*, where it is cut by the right line *B G F*, drawn parallel to the right line *A C E*. Juſt the ſame as if the glaſs were taken away, and a real man ſtood at *F*, equal in ſize to the man ſtanding at *B*: for to his eye at *A*, the eye of the other man at *E* would be ſeen in the direction of the line *A C E*; and the foot of the man at *F* would be ſeen by the eye *A*, in the direction of the line *A D F*.

If the glaſs be brought nearer the man *A B*, as ſuppoſe to *c b*, he will ſee his image as at *C D G*: for the reflected ray *C A* (being perpendicular to the glaſs) will ſhew the eye of the image at at *C*; and the incident ray *B b*, being reflected in the line *b A*, will ſhew the foot of his image as at *G*; the angle of reflection *a b A* being always equal to the angle of incidence *B b a*: and ſo of all the intermediate rays from *A* to *B*. Hence, if the man *A B* advances toward the glaſs *C D*, his image will approach toward it; and if he recedes from the glaſs, his image will alſo recede from it.

Having already ſhewn, that the rays of light are refracted when they paſs obliquely through different mediums, we come now to prove that ſome rays are more refrangible than others; and that, as they are differently refracted, they excite in our minds the ideas of different colours. This will account for the colours ſeen about the edges of the images of thoſe objects which are viewed through ſome teleſcopes.

Let

Let the sun shine into a dark room through a small hole, as at *e e*, in a window-shutter; and place a triangular prism *B C* in the beam of rays *A*, in such a manner, that the beam may fall obliquely on one of the sides *a b C* of the prism. The rays will suffer different refractions by passing through the prism, so that instead of going all out of it on the side *d c C*, in one direction, they will go on from it in the different directions represented by the lines *f*, *g*, *h*, *i*, *k*, *l*, *m*, *n*; and falling upon the opposite side of the room, or on white paper placed as at *p q* to receive them, they will paint upon it a series of most beautiful lively colours (not to be equalled by art) in this order, viz. those rays which are least refracted by the prism, and will therefore go on between the lines *n* and *m*, will be of a very bright intense red at *n*, degenerating from thence gradually into an orange colour, as they are nearer the line *m*: the next will be of a fine orange colour at *m*, and from thence degenerate into a yellow colour toward *l*: at *l* they will be of a fine yellow, which will incline toward a green, more and more, as they are nearer and nearer *k*: at *k* they will be a pure green, but from thence toward *i* they will incline gradually to a blue: at *i* they will be a perfect blue, inclining to an indigo colour from thence toward *h*: at *h* they will be quite the colour of indigo, which will gradually change toward a violet, the nearer they are to *g*: and at *g* they will be of a fine violet colour, which will incline gradually to a red as they come nearer to *f*, where the coloured image ends.

Fig. 5.

The *prism.*

The *colours of the light.*

There is not an equal quantity of rays in each of these colours; for, if the oblong image *p q* be divided into 360 equal parts, the red space

R *R* will

R will take up 45 of theſe parts; the orange *O*, 27; the yellow *Y*, 48; the green *G*, 60; the blue *B*, 60; the indigo *I*, 40; and the violet *V*, 80; all which ſpaces are as nearly proportioned in the figure as the ſmall ſpace *p q* would admit of.

If all theſe colours be blended together again, they will make a pure white; as is proved thus. Take away the paper on which the colours *p q* fell, and place a large convex glaſs *D* in the rays *f*, *g*, *h*, &c. which will refract them ſo, as to make them unite and croſs each other at *W*; where, if a white paper be placed to receive them, they will excite the idea of a ſtrong lively white. But if the paper be placed farther from the glaſs, as at *r s*, the different colours will appear again upon it, in an inverted order, occaſioned by the rays croſſing at *W*.

As white is a compoſition of all colours, ſo black is a privation of them all, and, therefore, properly no colour.

Fig. 6. Let two concentric circles be drawn on a ſmooth round board *A B C D E F G*, and the outermoſt of them divided into 360 equal parts or degrees: then, draw ſeven right lines, as ⊙ *A*, ⊙ *B*, &c. from the center to the outermoſt circle; making the lines ⊙ *A* and ⊙ *B* include 80 degrees of that circle; the lines ⊙ *B* and ⊙ *C* 40 degrees; ⊙ *C* and ⊙ *D* 60; ⊙ *D* and ⊙ *E* 60; ⊙ *E* and ⊙ *F* 48; ⊙ *F* and ⊙ *G* 27; ⊙ *G and* ⊙ *A* 45. Then, between theſe two circles, paint the ſpace *A G* red, inclining to orange near *G*; *G F* orange, inclining to yellow near *F*; *F E* yellow, inclining to green near *E*; *E D* green, inclining to blue near *D*; *D C* blue, inclining to indigo near *C*; *C B* indigo, inclining to violet near *B*; and *B A* violet, inclining to a ſoft red near *A*. This done, paint all that part of the board black which

which lies within the inner circle; and putting an axis through the center of the board, let it be turned very ſwiftly round that axis, ſo as the rays proceeding from the above colours, may be all blended and mixed together in coming to the eye; and then, the whole coloured part will appear like a white ring, a little greyiſh; not perfectly white, becauſe no colours prepared by art are perfect.

All the priſmatic colours blended together, make a white.

Any of theſe colours, except red and violet, may be made by mixing together the two contiguous priſmatic colours. Thus, yellow is made by mixing together a due proportion of orange and green; and green may be made by a mixture of yellow and blue.

All bodies appear of that colour, whoſe rays they reflect moſt; as a body appears red when it reflects moſt of the red-making rays, and abſorbs the reſt.

Any two or more colours that are quite tranſparent by themſelves, become opake when put together. Thus, if water or ſpirits of wine be tinged red, and put in a phial, every object ſeen through it will appear red; becauſe it lets only the red rays paſs through it, and ſtops all the reſt. If water or ſpirits be tinged blue, and put in a phial, all objects ſeen through it will appear blue, becauſe it tranſmits only the blue rays, and ſtops all the reſt. But if theſe two phials are held cloſe together, ſo as both of them may be between the eye and object, the object will no more be ſeen through them than through a plate of metal; for whatever rays are tranſmitted through the fluid in the phial next the object, are ſtopped by that in the phial next the eye. In this experiment, the phials ought not to be round, but ſquare; becauſe nothing but the

Tranſparent colours become opake, if put together.

 light

light itſelf can be ſeen through a round tranſparent body, at any diſtance.

As the rays of light ſuffer different degrees of refraction by paſſing obliquely through a priſm, or through a convex glaſs, and are thereby ſeparated into all the ſeven original or primary colours; ſo they alſo ſuffer different degrees of refraction by paſſing through drops of falling rain; and then, being reflected toward the eye, from the ſides of theſe drops which are fartheſt from the eye, and again refracted by paſſing out of theſe drops into the air, in which refracted directions they come to the eye; they make all the colours to appear in the form of a fine arch in the heavens, which is called the *rain-bow*.

There are always two rain-bows ſeen together, the interior of which is formed by the rays *a b*, which falling upon the upper part *b*, of the drop
Fig. 7. *b c d*, are refracted into the line *b c* as they enter the drop, and are reflected from the back of it at *c*, in the line *c d*, and then, by paſſing out of the drop into air, they are again refracted at *d*; and from thence they paſs on to the eye at *e*: ſo that to form the interior bow, the rays ſuffer two refractions, as at *b* and *d*; and one reflection, as at *c*.

The exterior bow is formed by rays which ſuffer two reflections, and two refractions; which is the occaſion of its being leſs vivid than the interior, and alſo of its colours being inverted with reſpect to thoſe of the interior. For, when a ray *a b* falls upon the lower part of the
Fig. 8. drop *b c d e*, it is refracted into the direction *b c* by entering the drop; and paſſing on to the back of the drop at *c*, it is thence reflected in the line *c d*, in which direction it is impoſſible for it to enter the eye at *f*: but by being again reflected

flected from the point *d* of the drop, it goes on in the drop to *e*, where it passes out of the drop into the air, and is there refracted downward to the eye, in the direction *e f*.

LECT. VIII. AND IX.

The description and use of the globes, and armillary sphere.

IF a map of the world be accurately delineated on a spherical ball, the surface thereof will represent the surface of the earth: for the highest hills are so inconsiderable with respect to the bulk of the earth, that they take off no more from its roundness, than grains of sand do from the roundness of a common globe; for the diameter of the earth is 8000 miles in round numbers, and no known hill upon it is three miles in perpendicular height. The *terrestrial globe.*

That the earth is spherical, or round like a globe, appears, 1. From its casting a round shadow upon the moon, whatever side be turned toward her when she is eclipsed. 2. From its having been sailed round by several persons. 3. From our seeing the farther, the higher we stand. 4. From our seeing the masts of a ship, while the hull is hid by the convexity of the water. Proof of the earth's being globul a.

The attractive power of the earth draws all terrestrial bodies toward its center; as is evident from the descent of bodies in lines perpendicular to the earth's surface, at the places whereon they fall; even when they are thrown off from the earth on opposite sides, and consequently, in opposite directions. So that the earth And that it may be peopled on all sides without any one's being in danger of falling away from it.

earth may be compared to a great magnet rolled in filings of ſteel, which attracts and keeps them equally faſt to its ſurface on all ſides. Hence, as all terreſtrial bodies are attracted toward the earth's center, they can be in no danger of falling from any ſide of the earth, more than from any other.

Up and *down*, what.

The heaven or ſky ſurrounds the whole earth: and when we ſpeak of *up* or *down*, we mean only with regard to ourſelves; for no point, either in the heaven, or on the ſurface of the earth, is *above* or *below*, but only with reſpect to ourſelves. And let us be upon what part of the earth we will, we ſtand with our feet toward its center, and our heads toward the ſky: and ſo we ſay, it is *up* toward the ſky, and *down* toward the center of the earth.

All objects in the heaven appear equally diſtant.

To an obſerver placed any where in the indefinite ſpace, where there is nothing to limit his view, all remote objects appear equally diſtant from him; and ſeem to be placed in a vaſt concave ſphere, of which his eye is the center. Every aſtronomer can demonſtrate, that the moon is much nearer to us than the ſun is; that ſome of the planets are ſometimes nearer to us, and ſometimes farther from us, than the ſun; that others of them never come ſo near us as the ſun always is; that the remoteſt planet in our ſyſtem, is beyond compariſon nearer to us than any of the fixed ſtars are; and that it is highly probable ſome ſtars are, in a manner, infinitely more diſtant from us than others; and yet all theſe celeſtial objects appear equally diſtant from us. Therefore, if we imagine a large hollow ſphere of glaſs to have as many bright ſtuds fixed to its inſide, as there are ſtars viſible in the heaven, and theſe ſtuds

The face of the heaven and earth

ſtuds to be of different magnitudes, and placed at the ſame angular diſtances from each other as the ſtars are; the ſphere will be a true repreſentation of the ſtarry heaven, to an eye ſuppoſed to be in its center, and viewing it all around. And if a ſmall globe, with a map of the earth upon it, be placed on an axis in the center of this ſtarry ſphere, and the ſphere be made to turn round on this axis, it will repreſent the apparent motion of the heavens round the earth.

Repreſented in a machine.

If a great circle be ſo drawn upon this ſphere, as to divide it into two equal parts, or hemiſpheres, and the plane of the circle be perpendicular to the axis of the ſphere, this circle will repreſent the *equinoctial*, which divides the heaven into two equal parts, called the *northern* and the *ſouthern hemiſpheres*; and every point of that circle will be equally diſtant from the *poles*, or ends of the axis in the ſphere. That pole which is in the middle of the northern hemiſphere, will be called the *north pole of the ſphere*, and that which is in the middle of the ſouthern hemiſphere, the *ſouth pole*.

The *equinoctial*.

The *poles*.

If another great circle be drawn upon the ſphere, in ſuch a manner as to cut the equinoctial at an angle of $23\frac{1}{2}$ degrees in two oppoſite points, it will repreſent the *ecliptic*, or circle of the ſun's apparent annual motion: one half of which is on the north ſide of the equinoctial, and the other half on the ſouth.

The *ecliptic*.

If a large ſtud be made to move eaſtward in this ecliptic, in ſuch a manner as to go quite round it, in the time that the ſphere is turned round weſtward 366 times upon its axis; this ſtud will repreſent the *ſun*, changing his place every day a 365th part of the ecliptic; and

The *ſun*.

going round weſtward, the ſame way as the ſtars do; but with a motion ſo much ſlower than the motion of the ſtars, that they will make 366 revolutions about the axis of the ſphere, in the time that the ſun makes only 365. During one half of theſe revolutions, the ſun will be on the north ſide of the equinoctial; during the other half, on the ſouth: and at the end of each half, in the equinoctial.

The *earth.*

If we ſuppoſe the terreſtrial globe in this machine to be about one inch in diameter, and the diameter of the ſtarry ſphere to be about five or ſix feet, a ſmall inſect on the globe would ſee only a very little portion of its ſurface; but it would ſee one half of the ſtarry ſphere; the convexity of the globe hiding the other half from its view. If the ſphere be turned weſtward round the globe, and the inſect could judge of the appearances which ariſe from that motion, it would ſee ſome ſtars riſing to its view in the eaſtern ſide of the ſphere, while others were ſetting on the weſtern: but as all the ſtars are fixed to the ſphere, the ſame ſtars would always riſe in the ſame points of view on the eaſt ſide, and ſet in the ſame points of view on the weſt ſide. With the ſun it would be otherwiſe, becauſe the ſun is not fixed to any point of the ſphere, but moves ſlowly along an oblique circle in it. And if the inſect ſhould look toward the ſouth, and call that point of the globe, where the equinoctial in the ſphere ſeems to cut it on the left ſide, the *eaſt point*; and where it cuts the globe on the right ſide, the *weſt point*; the little animal would ſee the ſun riſe north of the eaſt, and ſet north of the weſt, for $182\frac{1}{2}$ revolutions; after which, for as many more, the ſun would riſe ſouth of the eaſt, and ſet ſouth of the weſt,

The apparent motion of the heavens.

weſt. And in the whole 365 revolutions, the ſun would riſe only twice in the eaſt point, and ſet twice in the weſt. All theſe appearances would be the ſame, if the ſtarry ſphere ſtood ſtill (the ſun only moving in the ecliptic) and the earthly globe were turned round the axis of the ſphere eaſtward. For, as the inſect would be carried round with the globe, he would be quite inſenſible of its motion; and the ſun and ſtars would appear to move weſtward.

We are but very ſmall beings when compared with our earthly globe, and *the globe itſelf* is but a dimenſionleſs point compared with the magnitude of the ſtarry heavens. Whether the earth be at reſt, and the heaven turns round it, or the heaven be at reſt, and the earth turns round, the appearance to us will be exactly the ſame. And becauſe the heaven is ſo immenſely large, in compariſon of the earth, we ſee one half of the heaven as well from the earth's ſurface, as we could do from its center, if the limits of our view are not intercepted by hills.

Circles of the ſphere. We may imagine as many circles deſcribed upon the earth as we pleaſe; and we may imagine the plane of any circle deſcribed upon the earth to be continued, until it marks a circle in the concave ſphere of the heavens.

The horizon. The *horizon* is either *ſenſible* or *rational.* The *ſenſible* horizon is that circle, which a man ſtanding upon a large plane, obſerves to terminate his view all around, where the heaven and earth ſeem to meet. The plane of our ſenſible horizon continued to the heaven, divides it into two hemiſpheres; one viſible to us, the other hid by the convexity of the earth.

The

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The plane of the *rational horizon*, is ſuppoſed parallel to the plane of the ſenſible; to paſs through the center of the earth, and to be continued to the heavens. And although the plane of the ſenſible horizon touches the earth in the place of the obſerver, yet *this* plane, and that of the rational horizon, will ſeem to coincide in the heaven, becauſe the whole earth is but a point compared to the ſphere of the heaven.

The earth being a ſpherical body, the horizon, or limit of our view, muſt change as we change our place.

Poles. The *poles of the earth*, are thoſe two points on its ſurface in which its axis terminates. The one is called the *north pole*, and the other the *ſouth pole*.

The *poles of the heaven*, are thoſe two points in which the earth's axis produced terminates in the heaven: ſo that the *north pole* of the heaven is directly over the north pole of the earth; and the *ſouth pole* of the heaven is directly over the ſouth pole of the earth.

Equator. The *equator* is a great circle upon the earth, every part of which is equally diſtant from either of the poles. It divides the earth into two equal parts, called the *northern* and *ſouthern hemiſpheres*. If we ſuppoſe the plane of this circle to be extended to the heaven, it will mark the *equinoctial* therein, and will divide the heaven into two equal parts, called the *northern* and *ſouthern hemiſpheres* of the heaven.

Meridian. The *meridian* of any place is a great circle paſſing through that place and the poles of the earth. We may imagine as many ſuch meridians as we pleaſe, becauſe any place that is

ever

ever ſo little to the eaſt or weſt of any other place, has a different meridian from that place; for no one circle can paſs through any two ſuch places and the poles of the earth.

The *meridian* of any place is divided by the poles, into two ſemicircles: that which paſſes through the place is called the *geographical* or *upper meridian*; and that which paſſes through the opposite place, is called the *lower meridian*.

When the rotation of the earth brings the plane of the geographical meridian to the ſun, it is *noon* or *mid-day* to that place; and when our lower meridian comes to the ſun, it is *mid-night*. *Noon* and *mid-night*.

All places lying under the ſame geographical meridian, have their noon at the ſame time, and conſequently all the other hours. All thoſe places are ſaid to have the ſame *longitude*, becauſe no one of them lies either eaſtward or weſtward from any of the reſt.

If we imagine 24 ſemicircles, one of which is the geographical meridian of a given place, to meet at the poles, and to divide the equator into 24 equal parts; each of theſe meridians will come round to the ſun in 24 hours, by the earth's equable motion round its axis in that time. And, as the equator contains 360 degrees, there will be 15 degrees contained between any two of theſe meridians which are neareſt to one another: for 24 times 15 is 360. And as the earth's motion is eaſtward, the ſun's apparent motion will be weſtward, at the rate of 15 degrees each hour. Therefore, *Hour circles.*

They whoſe geographical meridian is 15 degrees eaſtward from us, have noon, and every other hour, a hour ſooner than we have. They whoſe meridian is fifteen degrees weſtward from *Longitude.*

us,

us, have noon, and every other hour, a hour later than we have: and ſo on in proportion, reckoning one hour for every fifteen degrees.

Ecliptic. As the earth turns round its axis once in 24 hours, and ſhews itſelf all round to the ſun in that time; ſo it goes round the ſun once a year, in a great circle called the *ecliptic*, which croſſes the equinoctial in two oppoſite points, making an angle of $23\frac{1}{2}$ degrees with the equinoctial on each ſide. So that one half of the ecliptic is in the northern hemiſphere, and the other in the ſouthern. It contains 360 equal parts, called degrees (as all other circles do, whether great or ſmall) and as the earth goes once round it every year, the ſun will appear to do the ſame, changing his place almoſt a degree, at a mean rate, every 24 hours. So that whatever place, or degree of the ecliptic, the earth is in at any time, the ſun will then appear in the oppoſite. And as one half of the ecliptic is on the north ſide of the equinoctial, and the other half on the ſouth; the ſun, as ſeen from the earth, will be half a year on the ſouth ſide of the equinoctial, and half a year on the north: and twice a year in the equinoctial itſelf.

Signs and *degrees.* The ecliptic is divided by aſtronomers into 12 equal parts, called *ſigns*, each ſign into 30 *degrees* and each degree into 60 *minutes:* but in uſing the globes, we ſeldom want the ſun's place nearer than half a degree of the truth.

The names and characters of the 12 ſigns are as follow; beginning at that point of the ecliptic where it croſſes the equinoctial to the northward, and reckoning eaſtward round to the ſame point again. And the days of the months on which the ſun now enters the ſigns, are ſet down below them.

Aries,

Aries,	*Taurus,*	*Gemini,*	*Cancer,*
♈	♉	♊	♋
March	April	May	June
20	19	20	21
Leo,	*Virgo,*	*Libra,*	*Scorpio,*
♌	♍	♎	♏
July	Auguſt	September	October
22	22	22	22
Sagittarius,	*Capricornus,*	*Aquarius,*	*Piſces,*
♐	♑	♒	♓
November	December	January	February
21	21	19	18

By remembering on what day the ſun enters any particular ſign, we may eaſily find his place any day afterward, while he is in that ſign, by reckoning a degree for each day; which will occaſion no error of conſequence in uſing the globes.

When the ſun is at the beginning of *Aries*, he is in the equinoctial; and from that time he declines northward every day, until he comes to the beginning of *Cancer*, which is 23½ degrees from the equinoctical: from thence he recedes ſouthward every day, for half a year; in the middle of which half, he croſſes the equinoctial at the beginning of *Libra*, and at the end of that half year, he is at his greateſt ſouth declination, in the beginning of *Capricorn*, which is alſo 23½ degrees from the equinoctial. Then, he returns northward from *Capricorn* every day, for half a year; in the middle of which half, he croſſes the equinoctial at the beginning of *Aries*; and at the end of it he arrives at *Cancer*.

The

The ſun's motion in the ecliptic is not perfectly equable, for he continues eight days longer in the northern half of the ecliptic, than in the ſouthern: ſo that the ſummer half year, in the northern hemiſphere, is eight days longer than the winter half year; and the contrary in the ſouthern hemiſphere.

Tropics. The *tropics* are leſſer circles in the heaven, parallel to the equinoctial; one on each ſide of it, touching the ecliptic in the points of its greateſt declination; ſo that each tropic is $23\frac{1}{2}$ degrees from the equinoctial, one on the north ſide of it, and the other on the ſouth. The northern tropic touches the ecliptic at the beginning of *Cancer*, the ſouthern at the beginning of *Capricorn*; for which reaſon the former is called the *tropic of Cancer*, and the latter the *tropic of Capricorn.*

Polar circles. The *polar circles* in the heaven, are each $23\frac{1}{2}$ degrees from the poles, all around. That which goes round the north pole, is called the *arctic circle*, from ἄρκτος, which ſignifies a *bear*; there being a collection or groupe of ſtars near the north pole, which goes by that name. The ſouth polar circle, is called the *antarctic circle*, from its being oppoſite to the arctic.

The ecliptic, tropics, and polar circles, are drawn upon the terreſtrial globe, as well as upon the celeſtial. But the ecliptic, being a great fixed circle in the heavens, cannot properly be ſaid to belong to the terreſtrial globe; and is laid down upon it only for the conveniency of ſolving ſome problems. So that, if this circle on the terreſtrial globe was properly divided into the months and days of the year, it would not only ſuit the globe better, but would alſo make the problems thereon much eaſier.

In

In order to form a true idea of the earth's motion round its axis every 24 hours, which is the caufe of day and night; and of its motion in the ecliptic round the fun every year, which is the caufe of the different lengths of days and nights, and of the viciffitude of feafons; take the following method, which will be both eafy and pleafant.

Let a fmall terreftrial globe, of about three inches diameter, be fufpended by a long thread of twifted filk, fixt to its north pole: then having placed a lighted candle on a table, to reprefent the fun, in the center of a hoop of a large cafk, which may reprefent the ecliptic, the hoop making an angle of $23\frac{1}{2}$ degrees with the plane of the table; hang the globe within the hoop near to it; and if the table be level, the equator of the globe will be parallel to the table, and the plane of the hoop will cut the equator at an angle of $23\frac{1}{2}$ degrees; fo that one half of the equator will be above the hoop, and the other half below it: and the candle will enlighten one half of the globe, as the fun enlightens one half of the earth, while the other half is in the dark. An idea of the feafons.

Things being thus prepared, twift the thread toward the left hand, that it may turn the globe the fame way by untwifting; that is, from weft, by fouth, to eaft. As the globe turns round its axis or thread, the different places of its furface will go regularly through the light and dark; and have, as it were, an alternate return of day and night in each rotation. As the globe continues to turn round, and to fhew itfelf all around to the candle, carry it flowly round the hoop by the thread, from weft, by fouth, to eaft; which is the way that the earth moves round the ſun, once a year, in the ecliptic: and you will ſee, that while the globe continues in the lower part of the hoop, the candle (being then north of the equator) will conſtantly ſhine round the north pole; and all the northern places which go through any part of the dark, will go through a leſs portion of it than they do of the light; and the more ſo, the farther they are from the equator: conſequently, their days are then longer than their nights. When the globe comes to a point in the hoop, mid-way between the higheſt and loweſt points, the candle will be directly over the equator, and will enlighten the globe juſt from pole to pole; and then every place on the globe will go through equal portions of light and darkneſs, as it runs round its axis; and conſequently, the day and night will be of equal length at all places upon it. As the globe advances thenceforward, toward the higheſt part of the hoop, the candle will be on the ſouth ſide of the equator, ſhining farther and farther round the ſouth pole, as the globe riſes higher and higher in the hoop; leaving the north pole as much in darkneſs, as the ſouth pole is then in the light, and making long days and ſhort nights on the ſouth ſide of the equator, and the contrary on the north ſide, while the globe continues in the northern or higher ſide of the hoop: and when it comes to the higheſt point, the days will be at the longeſt, and the nights at the ſhorteſt, in the ſouthern hemiſphere; and the reverſe in the northern. As the globe advances and deſcends in the hoop, the light will gradually recede from the ſouth pole, and approach toward the north pole, which will cauſe the northern days to lengthen, and the ſouthern days to ſhorten in the

the ſame proportion. When the globe comes to the middle point, between the higheſt and loweſt points of the hoop, the candle will be over the equator, enlightening the globe juſt from pole to pole, when every place of the earth (except the poles) will go through equal portions of light and darkneſs; and conſequently, the day and night will be then equal, all over the globe.

And thus, at a very ſmall expence, one may have a delightful and demonſtrative view of the cauſe of days and nights, with their gradual increaſe and decreaſe in length, through the whole year together, with the viciſſitudes of ſpring, ſummer, autumn, aad winter, in each annual courſe of the earth round the ſun.

If the hoop be divided into 12 equal parts, and the ſigns be marked in order upon it, beginning with *Cancer* at the higheſt point of the hoop, and reckoning eaſtward (or contrary to the apparent motion of the ſun) you will ſee how the ſun appears to change his place every day in the ecliptic, as the globe advances eaſtward along the hoop, and turns round its own axis: and that when the earth is in a low ſign, as at *Capricorn*, the ſun muſt appear in a high ſign, as at *Cancer*, oppoſite to the earth's real place: and that while the earth is in the ſouthern half of the ecliptic, the ſun appears in the northern half, and *vice verſâ:* that the farther any place is from the equator, between it and the polar circle, the greater is the difference between the longeſt and ſhorteſt day at that place; and that the poles have but one day and one night in the whole year.

Theſe things premiſed, we ſhall proceed to the deſcription and uſe of the terreſtrial globe,

 and

and explain the geographical terms as they occur in the problems.

The *ter-restrial globe* de-scribed.

This globe has the boundaries of land and water laid down upon it, the countries and kingdoms divided by dots, and coloured to distinguish them, the islands properly situated, the rivers and principal towns inserted, as they have been ascertained upon the earth by measurement and observation.

The equator, ecliptic, tropics, polar circles, and meridians, are laid down upon the globe in the manner already described. The ecliptic is divided into 12 signs, and each sign into 30 degrees, which are generally subdivided into halves, and into quarters if the globe is large. Each tropic is $23\frac{1}{2}$ degrees from the equator, and each polar circle $23\frac{1}{2}$ degrees from its respective pole. Circles are drawn parallel to the equator, at every ten degrees distance from it on each side to the poles: these circles are called *parallels of latitude*. On large globes there are circles drawn perpendicularly through every tenth degree of the equator, intersecting each other at the poles: but on globes of or under a foot diameter, they are only drawn through every fifteenth degree of the equator: these circles are generally called *meridians*, sometimes *circles of longitude*, and at other times *hour-circles*.

The globe is hung in a brass ring, called the *brasen meridian*; and turns upon a wire in each pole sunk half its thickness into one side of the meridian ring: by which means, *that* side of the ring divides the globe into two equal parts, called the *eastern* and *western hemispheres*; as the equator divides it into two equal parts, called the *northern* and *southern hemispheres*. This ring is divided

divided into 360 equal parts or degrees, on the side wherein the axis of the globe turns. One half of these degrees are numbered, and reckoned, from the equator to the poles, where they end at 90: their use is to shew the latitudes of places. The degrees on the other half of the meridian ring are numbered from the poles to the equator, where they end at 90: their use is to shew how to elevate either the north or south pole above the horizon, according to the latitude of any given place, as it is north or south of the equator.

The brasen meridian is let into two notches made in a broad flat ring, called the *wooden horizon*, the upper surface of which divides the globe into two equal parts, called the *upper* and *lower hemispheres*. One notch is in the north point of the horizon, and the other in the south. On this horizon are several concentric circles, which contain the months and days of the year, the signs and degrees answering to the sun's place for each month and day, and the 32 points of the compass.—The graduated side of the brass meridian lies toward the east side of the horizon, and should be generally kept toward the person who works problems by the globes.

There is a small *horary circle*, so fixed to the north part of the brazen meridian, that the wire in the north pole of the globe is in the center of that circle; and on the wire is an *index*, which goes over all the 24 hours of the circle, as the globe is turned round its axis. Sometimes there are two horary circles, one between each pole of the globe and the brasen meridian; which is the contrivance of the late ingenious Mr. *Joseph Harris*, master of the Assay-office in the Tower of London; and makes it very convenient for putting the poles of the globe through the horizon, and for elevating the pole to ſmall latitudes, and declinations of the ſun; which cannot be done where there is only one horary circle fixed to the outer edge of the braſen meridian.

There is a thin ſlip of braſs, called the *quadrant of altitude*, which is divided into 90 equal parts or degrees, anſwering exactly to ſo many degrees of the equator. It is occaſionally fixed to the uppermoſt point of the braſen meridian by a nut and ſcrew. The diviſions end at the nut, and the quadrant is turned round upon it.

As the globe has been ſeen by moſt people, and upon the figure of which, in a plate, neither the circles nor countries can be properly expreſſed, we judge it would ſignify very little to refer to a figure of it; and ſhall therefore only give ſome directions how to chooſe a globe, and then deſcribe its uſe.

Directions for chooſing of globes.

1. See that the papers be well and neatly paſted on the globes, which you may know, if the lines and circles thereon meet exactly, and continue all the way even and whole; the circles not breaking into ſeveral arches, nor the papers either coming ſhort, or lapping over one another.

2. See that the colours be tranſparent, and not laid too thick upon the globe to hide the names of places.

3. See that the globe hang evenly between the braſen meridian and the wooden horizon; not inclining either to one ſide or to the other.

4. See that the globe be as cloſe to the horizon and meridian as it conveniently may; otherwiſe, you will be too much puzzled to find againſt

againſt what part of the globe any degree of the meridian or horizon is.

5. See that the equinoctial line be even with the horizon all around, as the north or ſouth pole is elevated 90 degrees above the horizon.

6. See that the equinoctial line cuts the horizon in the eaſt and weſt points, in all elevations of the pole from 0 to 90 degrees.

7. See that the degree of the braſen meridian marked with 0, be exactly over the equinoctial line of the globe.

8. See that there be exactly half of the braſen meridian above the horizon; which you may know, if you bring any of the decimal diviſions on the meridian to the north point of the horizon, and find their complement to 90 in the ſouth point.

9. See that when the quadrant of altitude is placed as far from the equator, or the braſen meridian, as the pole is elevated above the horizon, the beginning of the degrees of the quadrant reaches juſt to the plane ſurface of the horizon.

10. See that while the index of the hour-circle (by the motion of the globe) paſſes from one hour to another, 15 degrees of the equator paſs under the graduated edge of the braſen meridian.

11. See that the wooden horizon be made ſubſtantial and ſtrong: it being generally obſerved, that in moſt globes, the horizon is the firſt part that fails, on account of its having been made too ſlight.

In uſing the globes, keep the eaſt ſide of the horizon toward you (unleſs your problem requires the turning of it) which ſide you may know by the word Eaſt upon the horizon; for Directions for uſing them.

then you have the graduated ſide of the meridian toward you, the quadrant of altitude before you, and the globe divided exactly into two equal parts, by the graduated ſide of the meridian.

In working ſome problems, it will be neceſſary to turn the whole globe and horizon about, that you may look on the weſt ſide thereof; which turning will be apt to jog the ball ſo, as to ſhift away that degree of the globe which was before ſet to the horizon or meridian: to avoid which inconvenience, you may thruſt in the feather-end of a quill between the ball of the globe and the braſen meridian; which, without hurting the ball, will keep it from turning in the meridian, while you turn the weſt ſide of the horizon toward you.

PROBLEM I.

*To find the * latitude and † longitude of any given place upon the globe.*

Turn the globe on its axis, until the given place comes exactly under that graduated ſide of

* The latitude of a place is its diſtance from the equator, and is north or ſouth, as the place is north or ſouth of the equator. Thoſe who live at the equator have no latitude, becauſe it is there that the latitude begins.

† The longitude of a place is the number of degrees (reckoned upon the equator) that the meridian of the ſaid place is diſtant from the meridian of any other place from which we reckon, either eaſtward or weſtward, for 180 degrees, or half round the globe. The Britiſh reckon the longitude from the meridian of London, and the French from the meridian of Paris. The meridian of that place, from which the longitude is reckoned, is called the *firſt meridian*. The places upon this meridian have no longitude, becauſe it is there that the longitude begins.

the brasen meridian, on which the degrees are numbered from the equator; and observe what degree of the meridian the place then lies under; which is its latitude, north or south, as the place is north or south of the equator.

The globe remaining in this position, the degree of the equator, which is under the brasen meridian, is the longitude of the place (from the meridian of *London* on the *English* globes) which is east or west, as the place lies on the east or west side of the first meridian of the globe.—All the *Atlantic Ocean*, and *America*, is on the west side of the meridian of *London*; and the greatest part of *Europe*, and of *Africa*, together with all *Asia*, is on the east side of the meridian of *London*, which is reckoned the *first meridian* of the globe by the *British* geographers and astronomers.

PROBLEM II.

The longitude and latitude of a place being given, to find that place on the globe.

Look for the given longitude in the equator (counting it eastward or westward from the first meridian, as it is mentioned to be east or west) and bring the point of longitude in the equator to the brasen meridian, on that side which is above the south point of the horizon: then count from the equator, on the brazen meridian, to the degree of the given latitude, toward the north or south pole, according as the latitude is north or south; and under that degree of latitude on the meridian, you will have the place required.

PROBLEM III.

To find the difference of longitude, or difference of latitude, between any two given places.

Bring each of these places to the brasen meridian, and see what its latitude is: the lesser latitude substracted from the greater, if both places are on the same side of the equator, or both latitudes added together, if they are on different sides of it, is the difference of latitude required. And the number of degrees contained between these places, reckoned on the equator, when they are brought separately under the brasen meridian, is their difference of longitude; if it be less than 180: but if more, let it be substracted from 360, and the remainder is the difference of longitude required. Or,

Having brought one of the places to the brasen meridian, and set the hour index to XII, turn the globe until the other place comes to the brasen meridian, and the number of hours and parts of an hour, past over by the index, will give the longitude in time; which may be easily reduced to degrees, by allowing 15 degrees for every hour, and one degree for every four minutes.

N. B. When we speak of bringing any place to the brasen meridian, it is the graduated side of the meridian that is meant.

PRO-

PROBLEM IV.

Any place being given, to find all those places that have the same longitude or latitude with it.

Bring the given place to the brasen meridian, then all those places which lie under that side of the meridian, from pole to pole, have the same longitude with the given place. Turn the globe round its axis, and all those places which pass under the same degree of the meridian that the given place does, have the same latitude with that place.

Since all latitudes are reckoned from the equator, and all longitudes are reckoned from the first meridian, it is evident, that the point of the equator which is cut by the first meridian, has neither latitude nor longitude.—The greatest latitude is 90 degrees, because no place is more than 90 degrees from the equator. And the greatest longitude is 180 degrees, because no place is more than 180 degrees from the first meridian.

PROBLEM V.

To find the antœci *, periœci †, *and* antipodes ‡, *of any given place.*

Bring the given place to the brasen meridian, and having found its latitude, keep the globe in that situation, and count the same number of degrees

* The *antœci* are those people who live on the same meridian, and in equal latitudes, on different sides of the equator. Being on the same meridian, they have the same hours; that is, when it is noon to the one, it is also noon to the other; and when it is mid-night to the one, it is also mid-night to the other, &c. Being on different sides of the equator,

degrees of latitude from the equator toward the contrary pole, and where the reckoning ends, you have the *antœci* of the given place upon the globe. Thofe who live at the equator have no *antœci*.

The globe remaining in the fame pofition, fet the hour-index to the upper XII, on the horary circle, and turn the globe until the index comes to the lower XII; then, the place which lies under the meridian, in the fame latitude with the given place, is the *periœci* required. Thofe who live at the poles have no *periœci*.

As the globe now ftands (with the index at the lower XII) the *antipodes* of the given place will be under the fame point of the brafen meridian where its *antœci* ftood before. Every place upon the globe has its *antipodes*.

tor, they have different or oppofite feafons at the fame time; the length of any day to the one is equal to the length of the night of that day to the other; and they have equal elevations of the different poles.

† The *periœci* are thofe people who live on the fame parallel of latitude, but on oppofite meridians: fo that though their latitude be the fame, their longitude differs 180 degrees. By being in the fame latitude, they have equal elevations of the fame pole (for the elevation of the pole is always equal to the latitude of the place) the fame length of days or nights, and the fame feafons. But being on oppofite meridians, when it is noon to the one, it is midnight to the other.

‡ The *antipodes* are thofe who live diametrically oppofite to one another upon the globe, ftanding with feet toward feet, on oppofite meridians and parallels. Being on oppofite fides of the equator, they have oppofite feafons, winter to one, when it is fummer to the other; being equally diftant from the equator, they have their contrary poles equally elevated above the horizon; being on oppofite meridians, when it is noon to the one, it muft be mid-night to the other; and as the fun recedes from the one when he approaches to the other, the length of the day to one muft be equal to the length of the night at the fame time to the other.

PRO-

PROBLEM VI.

To find the distance between any two places on the globe.

Lay the graduated edge of the quadrant of altitude over both the places, and count the number of degrees intercepted between them on the quadrant; then multiply these degrees by 60, and the product will give the distance in geographical miles: but to find the distance in English miles, multiply the degrees by 69½, and the product will be the number of miles required. Or, take the distance between any two places with a pair of compasses, and apply that extent to the equator; the number of degrees, intercepted between the points of the compasses, is the distance in degrees of a great circle *; which may be reduced either to geographical miles, or to English miles, as above.

* Any circle that divides the globe into two equal parts, is called a *great circle*, as the equator or meridian. Any circle that divides the globe into two unequal parts (which every parallel of latitude does) is called a *lesser circle*. Now, as every circle, whether great or small, contains 360 degrees, and a degree upon the equator or meridian contains 60 geographical miles, it is evident, that a degree of longitude upon the equator, is longer than a degree of longitude upon any parallel of latitude, and must therefore contain a greater number of miles. So that, although all the degrees of latitude are equally long upon an artificial globe (though not precisely so upon the earth itself) yet the degrees of longitude decrease in length, as the latitude increases, but not in the same proportion. The following table shews the length of a degree of longitude, in geographical miles, and hundredth parts of a mile, for every degree of latitude, from the equator to the poles: a degree on the equator being 60 geographical miles.

Great circle. *Lesser circle.*

PRO-

PROBLEM VII.

A place on the globe being given, and its distance from any other place, to find all the other places upon the globe which are at the same distance from the given place.

Bring the given place to the brasen meridian, and screw the quadrant of altitude to the meridian, directly over that place; then keeping the globe in that position, turn the quadrant quite round upon it, and the degree of the quadrant that touches the second place, will pass over all the other places which are equally distant with it from the given place.

This is the same as if one foot of a pair of compasses was set in the given place, and the other foot extended to the second place, whose distance is known; for if the compasses be then turned round the first place as a center, the moving foot will go over all those places which are at the same distance with the second from it.

A TABLE

A TABLE *shewing the number of miles in a degree of longitude, in any given degree of latitude.*

Deg.	Miles. Parts.	Deg.	Miles. Parts.	Deg.	Miles. Parts.
1	59.99	31	51.43	61	29.09
2	59.96	32	50.88	62	28.17
3	59.92	33	50.32	63	27.24
4	59.85	34	49.74	64	26.30
5	59.77	35	49.15	65	25.36
6	59.67	36	48.54	66	24.41
7	59.56	37	47.92	67	23.44
8	59.42	38	47.28	68	22.48
9	59.26	39	46.63	69	21.50
10	59.09	40	45.97	70	20.52
11	58.89	41	45.28	71	19.53
12	58.69	42	44.59	72	18.54
13	58.46	43	43.88	73	17.54
14	58.22	44	43.16	74	16.53
15	57.95	45	42.43	75	15.52
16	57.67	46	41.68	76	14.51
17	57.38	47	40.92	77	13.50
18	57.06	48	40.15	78	12.48
19	56.73	49	39.36	79	11.45
20	56.38	50	38.57	80	10.42
21	56.02	51	37.76	81	9.38
22	55.63	52	36.94	82	8.35
23	55.23	53	36.11	83	7.32
24	54.81	54	35.27	84	6.28
25	54.38	55	34.41	85	5.24
26	53.93	56	33.55	86	4.20
27	53.46	57	32.68	87	3.15
28	52.96	58	31.79	88	2.10
29	52.47	59	30.90	89	1.05
30	51.96	60	30.00	90	0.00

PROBLEM VIII.

The hour of the day at any place being given, to find all those places where it is noon at that time.

Bring the given place to the brasen meridian, and set the index to the given hour; this done, turn the globe until the index points to the upper XII, and then, all the places that lie under the brasen meridian have noon at that time.

N. B. The upper XII always stands for noon; and when the bringing of any place to the brasen meridian is mentioned, the side of that meridian on which the degrees are reckoned from the equator is meant, unless the contrary side be mentioned.

PROBLEM IX.

The hour of the day at any place being given, to find what time it then is at any other place.

Bring the given place to the brasen meridian, and set the index to the given hour; then turn the globe, until any place where the time is required comes to the brasen meridian, and the index will point out the time at that place.

PROBLEM X.

*To find the sun's place in the ecliptic, and his declination *, for any given day of the year.*

Look on the horizon for the given day, and right against it you have the degree of the sign in which the sun is (or his place) on that day

* The sun's declination is his distance from the equinoctial in degrees, and is north or south, as the sun is between the equinoctial and the north or south pole.

at noon. Find the ſame degree of that ſign in the ecliptic line upon the globe, and having brought it to the braſen meridian, obſerve what degree of the meridian ſtands over it; for that is the ſun's declination, reckoned from the equator.

PROBLEM XI.

The day of the month being given, to find all thoſe places of the earth over which the ſun will paſs vertically on that day.

Find the ſun's place in the ecliptic for the given day, and having brought it to the braſen meridian, obſerve what point of the meridian is over it; then turning the globe round its axis, all thoſe places which paſs under that point of the meridian are the places required; for as their latitude is equal, in degrees and parts of a degree, to the ſun's declination, the ſun muſt be vertical (or directly over head) to each of them at its reſpective noon.

PROBLEM XII.

*A place being given in the torrid zone *, to find thoſe two days of the year, on which the ſun ſhall be vertical to that place.*

Bring the given place to the braſen meridian, and mark the degree of latitude that is exactly over

* The globe is divided into five zones; one torrid, two temperate, and two frigid. The *torrid zone* lies between the two tropics, and is 47 degrees in breadth, or $23\frac{1}{2}$ on each ſide of the equator: the *temperate zones* lie between the tropics and polar circles, or from $23\frac{1}{2}$ degrees of latitude, to $66\frac{1}{2}$, on each

over it on the meridian; then turn the globe round its axis, and obſerve the two degrees of the ecliptic which paſs exactly under that degree of latitude: laſtly, find on the wooden horizon the two days of the year on which the ſun is in thoſe degrees of the ecliptic, and they are the days required: for on them, and none elſe, the ſun's declination is equal to the latitude of the given place; and conſequently, he will then be vertical to it at noon.

PROBLEM XIII.

To find all thoſe places of the north frigid zone, where the ſun begins to ſhine conſtantly without ſetting, on any given day, from the 20th of March, to the 22d of September.

On theſe two days, the ſun is in the equinoctial, and enlightens the globe exactly from pole to pole: therefore, as the earth turns round its axis, which terminates in the poles, every place upon it will go equally through the light and the dark, and ſo make the day and night equal to all places of the earth. But as the ſun declines from the equator, toward either pole, he will ſhine juſt as many degrees round that pole, as are equal to his declination from the equator; ſo that no place within the diſtance of the pole will then go through any part of the dark, and conſequently the ſun will not ſet to it. Now, as

each ſide of the equator; and are each 43 degrees in breadth: the *frigid zones* are the ſpaces included within the polar circles, which being each $23\frac{1}{2}$ degrees from their reſpective poles, the diameter of each of theſe zones is 47 degrees. As the ſun never goes without the tropics, he muſt every moment be vertical to ſome place or other in the torrid zone.

the

the ſun's declination is northward, from the 20th of March to the 22d of September, he muſt conſtantly ſhine round the north pole all that time; and on the day that he is in the northern tropic, he ſhines upon the whole north frigid zone; ſo that no place within the north polar circle goes through any part of the dark on that day. Therefore,

Having brought the ſun's place for the given day to the braſen meridian, and found his declination (by Prob. IX.) count as many degrees on the meridian, from the north pole, as are equal to the ſun's declination from the equator, and mark that degree from the pole where the reckoning ends: then, turning the globe round its axis, obſerve what places in the north frigid zone paſs directly under that mark; for they are the places required.

The like may be done for the ſouth frigid zone, from the 22d of September to the 20th of March, during which time the ſun ſhines conſtantly on the ſouth pole.

PROBLEM XIV.

To find the place over which the ſun is vertical, at any hour of a given day.

Having found the ſun's declination for the given day (by Prob. IX.) mark it with a chalk on the braſen meridian: then bring the place where you are (ſuppoſe London) to the braſen meridian, and ſet the index to the given hour; which done, turn the globe on its axis, until the index points to XII at noon; and the place on the globe, which is then directly under the point of the ſun's declination marked upon the meridian, has the ſun that moment in the zenith, or directly over head.

PROBLEM XV.

The day and hour at any place being given, to find all thoſe places where the ſun is then riſing, or ſetting, or on the meridian: conſequently, all thoſe places which are enlightened at that time, and thoſe which are in the dark.

This problem cannot be ſolved by any globe fitted up in the common way, with the hour circle fixed upon the braſs meridian; unleſs the ſun be on or near ſome of the tropics on the given day. But by a globe fitted up according to Mr. *Joſeph Harris*'s invention (already mentioned) where the hour-circle lies on the ſurface of the globe, below the meridian, it may be ſolved for any day in the year, according to his method; which is as follows.

Having found the place to which the ſun is vertical at the given hour, if the place be in the northern hemiſphere, elevate the north pole as many degrees above the horizon, as are equal to the latitude of that place; if the place be in the ſouthern hemiſphere, elevate the ſouth pole accordingly; and bring the ſaid place to the braſen meridian. Then, all thoſe places which are in the weſtern ſemicircle of the horizon, have the ſun riſing to them at that time; and thoſe in the eaſtern ſemicircle have it ſetting: to thoſe under the upper ſemicircle of the braſs meridian, it is noon; and to thoſe under the lower ſemicircle, it is midnight. All thoſe places which are above the horizon, are enlightened by the ſun, and have the ſun juſt as many degrees high to them, as they themſelves are above the horizon: and this height may be known, by fixing the quadrant of altitude on the braſen meridian over the place to which the ſun is vertical; and then, laying it over any other place, obſerve what number of degrees on the quadrant are intercepted between the ſaid place and the horizon. In all thoſe places that are 18 degrees below the weſtern ſemicircle of the horizon, the morning twilight is juſt beginning; in all thoſe places that are 18 degrees below the eaſtern ſemicircle of the horizon, the evening twilight is ending; and all thoſe places that are lower than 18 degrees, have dark night.

If any place be brought to the upper ſemicircle of the braſen meridian, and the hour index be ſet to the upper XII or noon, and then the globe be turned round eaſtward on its axis; when the place comes to the weſtern ſemicircle of the horizon, the index will ſhew the time of ſun-riſing at that place; and when the ſame place comes to the eaſtern ſemicircle of the horizon, the index will ſhew the time of ſun-ſet.

To thoſe places which do not go under the horizon, the ſun ſets not on that day: and to thoſe which do not come above it, the ſun does not riſe.

PROBLEM XVI.

The day and hour of a lunar eclipſe being given; to find all thoſe places of the earth to which it will be viſible.

The moon is never eclipſed but when ſhe is full, and ſo directly oppoſite to the ſun, that the earth's ſhadow falls upon her. Therefore, whatever place of the earth the ſun is vertical to at that time, the moon muſt be vertical to the antipodes of that place: ſo that the ſun will be then viſible to one half of the earth, and the moon to the other.

Find the place to which the ſun is vertical at the given hour (by Prob. XIV.) elevate the pole to the latitude of that place, and bring the place to the upper part of the braſen meridian, as in the former problem: then, as the ſun will be viſible to all thoſe parts of the globe which are above the horizon, the moon will be viſible to all thoſe parts of the globe which are below it, at the time of her greateſt obſcuration.

But with regard to an eclipſe of the ſun, there is no ſuch thing as ſhewing to what places it will be viſible, with any degree of certainty, by a common globe; becauſe the moon's ſhadow covers but a ſmall portion of the earth's ſurface; and her latitude, or declination from the ecliptic, throws her ſhadow ſo variouſly upon the earth, that to determine the places on which it falls, recourſe muſt be had to long calculations.

PROBLEM XVII.

*To rectify the globe for the latitude, the zenith *, and the ſun's place.*

Find the latitude of the place (by Prob. I.) and if the place be in the northern hemiſphere, raiſe the north pole above the north point of the horizon,

* The *zenith*, in this ſenſe, is the higheſt point of the braſen meridian above the horizon; but in the proper ſenſe it is that point of the heaven which is directly vertical to any given place, at any given inſtant of time.

as many degrees (counted from the pole upon the brasen meridian) as are equal to the latitude of the place. If the place be in the southern hemisphere, raise the south pole above the south point of the horizon, as many degrees as are equal to the latitude. Then, turn the globe till the place comes under its latitude on the brasen meridian, and fasten the quadrant of altitude so, that the chamfered edge of its nut (which is even with the graduated edge) may be joined to the zenith, or point of latitude. This done, bring the sun's place in the ecliptic for the given day, (found by Prob. X.) to the graduated side of the brasen meridian, and set the hour-index to XII at noon, which is the uppermost XII on the hour-circle; and the globe will be rectified.

The latitude of any place is equal to the elevation of the nearest pole of the heaven above the horizon of that place; and the poles of the heaven are directly over the poles of the earth, each 90 degrees from the equinoctial line. Let us be upon what place of the earth we will, if the limits of our view be not intercepted by hills, we shall see one half of the heaven, or 90 degrees every way round, from that point which is over our heads. Therefore, if we were upon the equator, the poles of the heaven would lie in our horizon, or limit of our view; if we go from the equator, toward either pole of the earth, we shall see the corresponding pole of the heaven rising gradually above our horizon, just as many degrees as we have gone from the equator: and if we were at either of the earth's poles, the corresponding pole of the heaven would be directly over our head. Consequently, the elevation or height of the pole in Remark.

degrees above the horizon, is equal to the number of degrees that the place is from the equator.

PROBLEM XVIII.

*The latitude of any place, not exceeding * 66½ degrees, and the day of the month, being given; to find the time of sun-rising and setting, and consequently the length of the day and night.*

Having rectified the globe for the latitude, and for the sun's place on the given day (as directed in the preceding problem) bring the sun's place in the ecliptic to the eastern side of the horizon, and the hour-index will shew the time of sun-rising; then turn the globe on its axis, until the sun's place comes to the western side of the horizon, and the index will shew the time of sun-setting.

The hour of sun-setting doubled, gives the length of the day; and the hour of sun-rising doubled gives the length of the night.

PROBLEM XIX.

The latitude of any place, and the day of the month, being given; to find when the morning twilight begins, and the evening twilight ends, at that place.

This problem is often limited; for, when the sun does not go 18 degrees below the horizon, the twilight continues the whole night; and for several

* All places whose latitude is more than 66½ degrees, are in the frigid zones: and to those places the sun does not set in summer, for a certain number of diurnal revolutions, which occasions this limitation of latitude.

ſeveral nights together in ſummer, between 49 and 66$\frac{1}{2}$ degrees of latitude: and the nearer to 66$\frac{1}{2}$, the greater is the number of theſe nights. But when it does begin and end, the following method will ſhew the time for any given day.

Rectify the globe, and bring the ſun's place in the ecliptic to the eaſtern ſide of the horizon; then mark that point of the ecliptic with a chalk which is in the weſtern ſide of the horizon, it being the point oppoſite to the ſun's place: this done, lay the quadrant of altitude over the ſaid point, and turn the globe eaſtward, keeping the quadrant at the chalk-mark, until it is juſt 18 degrees high on the quadrant; and the index will point out the time when the morning twilight begins: for the ſun's place will then be 18 degrees below the eaſtern ſide of the horizon. To find the time when the evening twilight ends, bring the ſun's place to the weſtern ſide of the horizon; and the point oppoſite to it, which was marked with the chalk, will be riſing in the eaſt; then, bring the quadrant over that point, and keeping it thereon, turn the globe weſtward, until the ſaid point be 18 degrees above the horizon on the quadrant, and the index will ſhew the time when the evening twilight ends; the ſun's place being then 18 degrees below the weſtern ſide of the horizon.

PROBLEM XX.

To find on what day of the year the ſun begins to ſhine conſtantly without ſetting, on any given place in the north frigid zone; and how long he continues to do ſo.

Rectify the globe to the latitude of the place, and turn it about until ſome point of the ecliptic, between *Aries* and *Cancer*, coincides with the north point of the horizon where the braſen meridian cuts it: then find, on the wooden horizon, what day of the year the ſun is in that point of the ecliptic; for that is the day on which the ſun begins to ſhine conſtantly on the given place, without ſetting. This done, turn the globe until ſome point of the ecliptic, between *Cancer* and *Libra*, coincides with the north point of the horizon, where the braſen meridian cuts it; and find, on the wooden horizon, on what day the ſun is in that point of the ecliptic; which is the day that the ſun leaves off conſtantly ſhining on the ſaid place, and riſes and ſets to it as to other places on the globe. The number of natural days, or complete revolutions of the ſun about the earth, between the two days above found, is the time that the ſun keeps conſtantly above the horizon without ſetting: for all the portion of the ecliptic, that lies between the two points which interſect the horizon in the very north, never ſets below it: and there is juſt as much of the oppoſite part of the ecliptic that never riſes; therefore, the ſun will keep as long conſtantly below the horizon in winter, as above it in ſummer.

Whoever

Whoever confiders the globe, will find, that all places of the earth do equally enjoy the benefit of the fun, in refpect of time, and are equally deprived of it. For, the days and nights are always equally long at the equator: and in all places that have latitude, the days at one time of the year are exactly equal to the nights at the oppofite feafon.

PROBLEM XXI.

To find in what latitude the fun fhines conftantly without fetting, for any length of time lefs than * $182\frac{1}{2}$ *of our days and nights.*

Find a point in the ecliptic half as many degrees from the beginning of *Cancer* (either toward *Aries* or *Libra*) as there are natural days † in the time given; and bring that point to the north fide of the brafen meridian, on which the degrees are numbered from the pole toward the equator: then, keep the globe from turning on its axis, and flide the meridian up or down, until the aforefaid point of the ecliptic comes to the north point of the horizon, and then, the elevation of the pole will be equal to the latitude required.

* The reafon of this limitation is, that $182\frac{1}{2}$ of our days and nights make half a year, which is the longeft time that the fun fhines without fetting, even at the poles of the earth.

† A natural day contains the whole 24 hours: an artificial day, the time that the fun is above the horizon.

PRO-

PROBLEM XXII.

The latitude of a place, not exceeding 66½ degrees, and the day of the month being given: to find the sun's amplitude, or point of the compass on which he rises or sets on that day.

Rectify the globe, and bring the sun's place to the eastern side of the horizon; then observe what point of the compass on the horizon stands right against the sun's place, for that is his amplitude at rising. This done, turn the globe westward, until the sun's place comes to the western side of the horizon, and it will cut the point of his amplitude at setting. Or, you may count the rising amplitude in degrees, from the east point of the horizon, to that point where the sun's place cuts it; and the setting amplitude, from the west point of the horizon, to the sun's place at setting.

PROBLEM XXIII.

*The latitude, the sun's place, and his altitude *, being given; to find the hour of the day, and the sun's azimuth, or number of degrees that he is distant from the meridian.*

Rectify the globe, and bring the sun's place to the given height upon the quadrant of altitude; on the eastern side of the horizon, if the time be in the forenoon; or the western side, if it

* The sun's altitude, at any time, is his height in degrees above the horizon at that time.

it be in the afternoon: then, the index will ſhew the hour; and the number of degrees in the horizon intercepted between the quadrant of altitude and the ſouth point, will be the ſun's true azimuth at that time.

N. B. Always when the quadrant of altitude is mentioned in working any problem, the graduated edge of it is meant.

If this be done at ſea, and compared with the ſun's azimuth, as ſhewn by the compaſs, if they agree, the compaſs has no variation in that place: but if they differ, the compaſs does vary; and the variation is equal to this difference.

PROBLEM XXIV.

The latitude, hour of the day, and the ſun's place, being given; to find the ſun's altitude and azimuth.

Rectify the globe, and turn it until the index points to the given hour; then lay the quadrant of altitude over the ſun's place in the ecliptic, and the degree of the quadrant cut by the ſun's place is his altitude at that time above the horizon; and the degree of the horizon cut by the quadrant in the ſun's azimuth, reckoned from the ſouth.

PROBLEM XXV.

The latitude, the sun's altitude, and his azimuth being given; to find his place in the ecliptic, the day of the month, and hour of the day, though they had all been lost.

Rectify the globe for the latitude and zenith *, and set the quadrant of altitude to the given azimuth in the horizon; keeping it there, turn the globe on its axis until the ecliptic cuts the quadrant in the given altitude: that point of the ecliptic which cuts the quadrant there, will be the sun's place; and the day of the month answering thereto, will be found over the like place of the sun on the wooden horizon. Keep the quadrant of altitude in that position, and having brought the sun's place to the brasen meridian, and the hour index to XII at noon, turn back the globe, until the sun's place cuts the quadrant of altitude again, and the index will shew the hour.

Any two points of the ecliptic which are equidistant from the beginning of *Cancer* or of *Capricorn*, will have the same altitude and azimuth at the same hour, though the months be different; and therefore it requires some care in this problem, not to mistake both the month, and the day of the month; to avoid which, observe, that from the 20th of March to the 21st of June, that part of the ecliptic which is be-

* By rectifying the globe for the zenith, is meant screwing the quadrant of altitude to the given latitude on the brass meridian.

tween

tween the beginning of *Aries* and beginning of *Cancer* is to be uſed: from the 21ſt of June to the 22d of September, between the beginning of *Cancer* and beginning of *Libra:* from the 22d of September to the 21ſt of December, between the beginning of *Libra* and the beginning of *Capricorn*; and from the 21ſt of December to the 20th of March, between the beginning of *Capricorn* and beginning of *Aries*. And as one can never be at a loſs to know in what quarter of the year he takes the ſun's altitude and azimuth, the above caution with regard to the quarters of the ecliptic, will keep him right as to the month and day thereof.

PROBLEM XXVI.

To find the length of the longeſt day at any given place.

If the place be on the north ſide of the equator, find its latitude (by Prob. I.) and elevate the north pole to that latitude; then, bring the beginning of *Cancer* ♋ to the braſen meridian, and ſet the hour-index to XII at noon. But if the given place be on the ſouth ſide of the equator, elevate the ſouth pole to its latitude, and bring the beginning of *Capricorn* ♑ to the braſs meridian, and the hour-index to XII. This done, turn the globe weſtward, until the beginning of *Cancer* or *Capricorn* (as the latitude is north or ſouth) comes to the horizon; and the index will then point out the time of ſunſetting, for it will have gone over all the afternoon hours, between mid-day and ſun-ſet; which

which length of time being doubled, will give the whole length of the day, from fun-rifing to fun-fetting. For, in all latitudes, the fun rifes as long before mid-day, as he fets after it.

PROBLEM XXVII.

To find in what latitude the longeft day is of any given length lefs than 24 hours.

If the latitude be north, bring the beginning of *Cancer* to the brafen meridian, and elevate the north pole to about $66\frac{1}{2}$ degrees; but if the latitude be fouth, bring the beginning of *Capricorn* to the meridian, and elevate the fouth pole to about $66\frac{1}{2}$ degrees; becaufe the longeft day in north latitude, is when the fun is in the firft point of *Cancer*; and in fouth latitude, when he is in the firft point of *Capricorn*. Then fet the hour-index to XII at noon, and turn the globe weftward, until the index points at half the number of hours given: which done, keep the globe from turning on its axis, and flide the meridian down in the notches, until the aforefaid point of the ecliptic (viz. *Cancer* or *Capricorn*) comes to the horizon; then, the elevation of the pole will be equal to the latitude required.

PRO-

PROBLEM XXVIII.

*The latitude of any place, not exceeding $66\frac{1}{2}$ degrees, being given; to find in what climate * the place is.*

Find the length of the longeſt day at the given place by Prob. XXVI. and whatever be the number of hours whereby it exceeds twelve, double that number, and the ſum will anſwer to the climate in which the place is.

PROBLEM XXIX.

The latitude, and the day of the month, being given; to find the hour of the day when the ſun ſhines.

Set the wooden horizon truly level, and the braſen meridian due north and ſouth by a mariner's compaſs: then, having rectified the globe, ſtick a ſmall ſewing needle into the ſun's place in the ecliptic, perpendicular to that part of the ſurface of the globe: this done, turn the globe on its axis, until the needle comes to the braſen meridian, and ſet the hour-index to XII

* A *climate* from the equator to either of the polar circles, is a tract of the earth's ſurface, included between two ſuch parallels of latitude, that the length of the longeſt day in the one exceeds that in the other by half an hour; but from the polar circles to the poles, where the ſun keeps long above the horizon without ſetting, each climate differs a whole month from the one next to it. There are twenty-four climates between the equator and each of the polar circles; and ſix from each polar circle to its reſpective pole.

at

at noon; then, turn the globe on its axis, until the needle points exactly toward the ſun (which it will do when it caſts no ſhadow on the globe) and the index will ſhew the hour of the day.

PROBLEM XXX.

A pleaſant way of ſhewing all thoſe places of the earth which are enlightened by the ſun, and alſo the time of the day when the ſun ſhines.

Take the terreſtrial ball out of the wooden horizon, and alſo out of the braſen meridian; then ſet it upon a pedeſtal in ſun-ſhine, in ſuch a manner, that its north pole may point directly toward the north pole of the heaven, and the meridian of the place where you are be directly toward the ſouth. Then, the ſun will ſhine upon all the like places of the globe, that he does on the real earth, riſing to ſome when he is ſetting to others; as you may perceive by that part where the enlightened half of the globe is divided from the half in the ſhade, by the boundary of the light and darkneſs: all thoſe places, on which the ſun ſhines, at any time, having day; and all thoſe, on which he does not ſhine, having night.

If a narrow ſlip of paper be put round the equator, and divided into 24 equal parts, beginning at the meridian of your place, and the hours be ſet to thoſe diviſions in ſuch a manner, that one of the VI's may be upon your meridian; the ſun being upon that meridian at noon, will then ſhine exactly to the two XII's; and at one to the two I's, &c. So that the place, where the enlightened half of the globe is parted from the ſhaded half, in this circle of hours, will ſhew the time of the day.

The principles of dialing ſhall be explained farther on, by the terreſtrial globe. At preſent we ſhall only add the following obſervations upon it; and then proceed to the uſe of the celeſtial globe.

1. *The latitude of any place is equal to the elevation of the pole above the horizon of that place, and the elevation of the equator is equal to the complement of the latitude, that is, to what the latitude wants of* 90 *degrees.*

2. *Thoſe places which lie on the equator, have no latitude, it being there that the latitude begins; and thoſe places which lie on the firſt meridian have no longitude, it being there that the longitude begins. Conſequently,* that *particular place of the earth where the firſt meridian interſects the equator, has neither longitude nor latitude.*

3. *At all places of the earth, except the poles, all the points of the compaſs may be diſtinguiſhed in the horizon: but from the north pole, every place is ſouth; and from the ſouth pole, every place is north. Therefore, as the ſun is conſtantly above the horizon of each pole for half a year in its turn, he cannot be ſaid to depart from the meridian of either pole for half a year together. Conſequently, at the north pole it may be ſaid to be noon every moment for half a year; and let the winds blow from what part they will, they muſt always blow from the ſouth; and at the ſouth pole, from the north.*

4. *Becauſe one half of the ecliptic is above the horizon of the pole, and the ſun, moon, and planets move in (or nearly in) the ecliptic; they will all*

rise and set to the poles. But, because the stars never change their declinations from the equator (at least not sensibly in one age) those which are once above the horizon of either pole, never set below it; and those which are once below it, never rise.

5. *All places of the earth do equally enjoy the benefit of the sun, in respect of time, and are equally deprived of it.*

6. *All places upon the equator have their days and nights equally long, that is,* 12 *hours each, at all times of the year. For although the sun declines, alternately, from the equator toward the north and toward the south, yet, as the horizon of the equator cuts all the parallels of latitude and declination in halves, the sun must always continue above the horizon for one half a diurnal revolution about the earth, and for the other half below it.*

7. *When the sun's declination is greater than the latitude of any place, upon either side of the equator, the sun will come twice to the same azimuth or point of the compass in the forenoon, at that place, and twice to a like azimuth in the afternoon; that is, he will go twice back every day, while his declination continues to be greater than the latitude. Thus, suppose the globe rectified to the latitude of* Barbadoes, *which is* 13 *degrees north; and the sun to be any where in the ecliptic, between the middle of Taurus and middle of Leo; if the quadrant of altitude be set to about* 18 *degrees* north of the east in the horizon, the sun's place be marked with a chalk upon the ecliptic, and the globe be then turned westward on its axis, the said mark will rise in the horizon a little to the north of the quadrant, and thence ascending, it will cross the quadrant toward*

* From the middle of Gemini to the middle of Cancer, the quadrant may be set 20 degrees.

the south; but before it arrives at the meridian, it will cross the quadrant again, and pass over the meridian northward of Barbadoes. *And if the quadrant be set about* 18 *degrees north of the west, the sun's place will cross it twice, as it descends from the meridian toward the horizon, in the afternoon.*

8. *In all places of the earth between the equator and poles, the days and nights are equally long,* viz. 12 *hours each, when the sun is in the equinoctial: for, in all elevations of the pole, short of* 90 *degrees (which is the greatest) one half of the equator or equinoctial will be above the horizon, and the other half below it.*

9. *The days and nights are never of an equal length at any place between the equator and polar circles, but when the sun enters the signs* ♈ *Aries and* ♎ *Libra. For in every other part of the ecliptic, the circle of the sun's daily motion is divided into two unequal parts by the horizon.*

10. *The nearer any place is to the equator, the less is the difference between the length of the days and nights in that place; and the more remote, the contrary. The circles which the sun describes in the heaven every* 24 *hours, being cut more nearly equal in the former case, and more unequally in the latter.*

11. *In all places lying upon any given parallel of latitude, however long or short the day or night be at any one of these places, at any time of the year, it is then of the same length at all the rest; for in turning the globe round its axis (when rectified according to the sun's declination) all these places will keep equally long above or below the horizon.*

12. *The sun is vertical twice a year to every place between the tropics; to those under the tropics,*

once a year, but never any where else. For, there can be no place between the tropics, but that there will be two points in the ecliptic, whose declination from the equator is equal to the latitude of that place; and but one point of the ecliptic which has a declination equal to the latitude of places on the tropic which that point of the ecliptic touches; and as the sun never goes without the tropics, he can never be vertical to any place that lies without them.

13. *To all places in the torrid zone *, the duration of the twilight is least, because the sun's daily motion is the most perpendicular to the horizon. In the frigid zones †, greatest; because the sun's daily motion is nearly parallel to the horizon; and therefore he is the longer of getting* 18 *degrees below it, till which time the twilight always continues. And in the temperate zones ‡ it is at a medium between the two, because the obliquity of the sun's daily motion is so.*

14. *In all places lying exactly under the polar circles, the sun, when he is in the nearest tropic, continues* 24 *hours above the horizon without setting; because no part of that tropic is below their horizon. And when the sun is in the farthest tropic, he is for the same length of time without rising; because no part of that tropic is above their horizon. But, at all other times of the year, he rises and sets there, as in other places; because all the circles that can be drawn parallel to the equator, between the tropics, are more or less cut by the horizon, as they are farther from, or nearer to, that tropic which is all above the horizon: and*

* Between the tropics.
† Between the polar circles and poles.
‡ Between the tropics and polar circles.

when

when the sun is not in either of the tropics, his diurnal course must be in one or other of these circles.

15. *To all places in the northern hemisphere, from the equator to the polar circle, the longest day and shortest night is when the sun is in the northern tropic; and the shortest day and longest night is when the sun is in the southern tropic; because no circle of the sun's daily motion is so much above the horizon, and so little below it, as the northern tropic; and none so little above it, and so much below it, as the southern. In the southern hemisphere, the contrary.*

16. *In all places between the polar circles and poles, the sun appears for some number of days (or rather diurnal revolutions) without setting; and at the opposite time of the year without rising; because some part of the ecliptic never sets in the former case, and as much of the opposite part never rises in the latter. And the nearer unto, or the more remote from the pole, these places are, the longer or shorter is the sun's continuing presence or absence.*

17. *If a ship sets out from any port, and sails round the earth eastward to the same port again, let her take what time she will to do it in, the people in that ship, in reckoning their time, will gain one complete day at their return, or count one day more than those who reside at the same port; because, by going contrary to the sun's diurnal motion, and being forwarder every evening than they were in the morning, their horizon will get so much the sooner above the setting sun, than if they had kept for a whole day at any particular place. And thus, by cutting off a part proportionable to their own motion, from the length of every day, they will gain a complete day of that sort at their return; without gaining one moment of absolute time more*

 than

than is elapsed during their course, to the people at the port. If they sail westward, they will reckon one day less than the people do who reside at the said port, because, by gradually following the apparent diurnal motion of the sun, they will keep him each particular day so much longer above their horizon, as answers to that day's course; and by that means, they cut off a whole day in reckoning, at their return, without losing one moment of absolute time.

Hence, if two ships should set out at the same time from any port, and sail round the globe, one eastward and the other westward, so as to meet at the same port on any day whatever; they will differ two days in reckoning their time, at their return. If they sail twice round the earth, they will differ four days; if thrice, then six, &c.

LECT. IX.

The use of the celestial globe, and armillary sphere.

The *celestial globe.* HAVING done for the present with the terrestrial globe, we shall proceed to the use of the celestial; first premising, that as the equator, ecliptic, tropics, polar circles, horizon, and brasen meridian, are exactly alike on both globes, all the former problems concerning the sun are solved the same way by both globes. To *rectify it.* The method also of rectifying the celestial globe is the same as rectifying the terrestrial, viz. Elevate the pole according to the latitude of your place, then screw the quadrant of altitude to the zenith, on the brass meridian; bring the sun's place in the ecliptic to the graduated edge of the brass meridian, on the side

ſide which is above the ſouth point of the wooden horizon, and ſet the hour-index to the uppermoſt XII, which ſtands for noon.

N. B. The ſun's place for any day of the year ſtands directly over that day on the horizon of the celeſtial globe, as it does on that of the terreſtrial.

Latitude and longitude of the ſtars. The *latitude* and *longitude* of the ſtars, and of all other celeſtial phenomena, are reckoned in a very different manner from the latitude and longitude of places on the earth: for all terreſtrial latitudes are reckoned from the equator; and longitudes from the meridian of ſome remarkable place, as of London by the Britiſh, and of Paris by the French; though moſt of the French maps begin their longitude at the meridian of the iſland *Ferro.*——But the aſtronomers of all nations agree in reckoning the *latitudes* of the moon, ſtars, planets, and comets, from the *ecliptic*; and their *longitudes* from the *equinoctial colure* *, in that ſemicircle of it which cuts the ecliptic at the beginning of *Aries* ♈; and thence eaſtward, quite round, to the ſame ſemicircle again. Conſequently thoſe ſtars which lie between the equinoctial and the northern half of the ecliptic, have north declination and ſouth latitude; thoſe which lie between the equinoctial and the ſouthern half of the ecliptic, have ſouth declination and north latitude; and

* *The great circle* that paſſes through the *equinoctial points* at the beginning of ♈ and ♎, and through the *poles of the world* (which are two oppoſite points, each 90 degrees from the equinoctial) is called the *equinoctial colure*: *Colures.* and the *great circle* that paſſes through the *beginning of* ♋ and ♑, and alſo through the *poles of the ecliptic*, and poles of the world, is called the *ſolſtitial colure.*

all those which lie between the tropics and poles, have their declinations and latitudes of the same denomination.

There are six great circles on the celestial globe, which cut the ecliptic perpendicularly, and meet in two opposite points in the polar circles; which points are each ninety degrees from the ecliptic, and are called its poles. These polar points divide those circles into 12 semicircles; which cut the ecliptic at the beginnings of the 12 signs. They resemble so many meridians on the terrestrial globe; and as all places which lie under any particular meridian semicircle on that globe, have the same longitude, so all those points of the heaven, through which any one of the above semicircles are drawn, have the same longitude.—And as the greatest latitudes on the earth are at the north and south poles of the earth, so the greatest latitudes in the heaven, are at the north and south poles of the ecliptic.

Constellations. In order to distinguish the stars, with regard to their situations and positions in the heaven, the ancients divided the whole visible firmament of stars into particular systems, which they called *constellations*; and digested them into the forms of such animals as are delineated upon the celestial globe. And those stars which lie between the figures of those imaginary animals, and could not be brought within the compass of any of them, were called *unformed stars*.

Because the moon and all the planets were observed to move in circles or orbits which cross the ecliptic (or line of the sun's path) at small angles, and to be on the north side of the ecliptic for one half of their course round the heaven of stars, and on the south side of it for the other

other half, but never to go quite 8 degrees from it on either side, the ancients distinguished that space by two lesser circles, parallel to the ecliptic (one on each side) at 8 degrees distance from it. And the space included between the circles, they called the *zodiac*, because most of the 12 constellations placed therein resemble some living creature.—These constellations are, 1. *Aries* ♈, the ram; 2. *Taurus* ♉, the bull; 3. *Gemini* ♊, the twins; 4. *Cancer* ♋, the crab; 5. *Leo* ♌, the lion; 6. *Virgo* ♍, the virgin; 7. *Libra* ♎, the balance; 8. *Scorpio* ♏, the scorpion; 9. *Sagittarius* ♐, the archer; 10. *Capricornus* ♑, the goat; 11. *Aquarius* ♒, the water-bearer; and 12. *Pisces* ♓, the fishes. *Zodiac.*

It is to be observed, that in the infancy of astronomy, these twelve constellations stood at or near the places of the ecliptic, where the above characteristics are marked upon the globe: but now, each constellation has got a whole sign forwarder, on account of the recession of the equinoctial points from their former places. So that the constellation of *Aries*, is now in the former place of *Taurus*; that of *Taurus*, in the former place of *Gemini*; and so on. **Remark.**

The stars appear of different magnitudes to the eye; probably because they are at different distances from us. Those which appear brightest and largest, are called *stars of the first magnitude*; the next to them in size and lustre, are called *stars of the second magnitude*; and so on to the *sixth*, which are the smallest that can be discerned by the bare eye.

Some of the most remarkable stars have names given them, as *Castor* and *Pollux* in the heads of the *Twins*, *Sirius* in the mouth of the *Great Dog*, *Procyon* in the side of the *Little Dog*, *Rigel* in in the left foot of *Orion*, *Arcturus* near the right thigh of *Bootes*, &c.

These things being premised, which I think are all that the young *Tyro* need be acquainted with, before he begins to work any problem by this globe, we shall now proceed to the most useful of those problems; omitting several which are of little or no consequence.

PROBLEM I.

*To find the right ascension * and declination † of the sun, or any fixed star.*

Bring the sun's place in the ecliptic to the brasen meridian, then that degree in the equinoctial which is cut by the meridian, is the sun's *right ascension*; and that degree of the meridian which is over the sun's place, is his *declination*. Bring any fixed star to the meridian, and its *right ascension* will be cut by the meridian in the equinoctial; and the degree of the meridian that stands over it, is its *declination*.

So that *right ascension* and *declination*, on the celestial globe, are found in the same manner as *longitude* and *latitude* on the terrestrial.

* The degree of the equinoctial, reckoned from the beginning of *Aries*, that comes to the meridian with the sun or star, is its *right ascension*.

† The distance of the sun or star in degrees from the equinoctial, toward either of the poles, north or south, is its declination, which is north or south accordingly.

PROBLEM II.

To find the latitude and longitude of any star.

If the given ſtar be on the north ſide of the ecliptic, place the 90th degree of the quadrant of altitude on the north pole of the ecliptic, where the twelve ſemicircles meet; which divide the ecliptic into the 12 ſigns: but if the ſtar be on the ſouth ſide of the ecliptic, place the 90th degree of the quadrant on the ſouth pole of the ecliptic: keeping the 90th degree of the quadrant on the proper pole, turn the quadrant about, until its graduated edge cuts the ſtar: then, the number of degrees in the quadrant, between the ecliptic and the ſtar, is its latitude; and the degree of the ecliptic cut by the quadrant is the ſtar's longitude, reckoned according to the ſign in which the quadrant then is.

PROBLEM III.

To repreſent the face of the ſtarry firmament, as ſeen from any given place of the earth, at any hour of the night.

Rectify the celeſtial globe for the given latitude, the zenith, and ſun's place, in every reſpect, as taught by the 17th problem, for the terreſtrial; and turn it about, until the index points to the given hour: then, the upper hemiſphere of the globe will repreſent the viſible half of the heaven for that time: all the ſtars upon

upon the globe being then in ſuch ſituations, as exactly correſpond to thoſe in the heaven. And if the globe be placed duly north and ſouth, by means of a ſmall ſea-compaſs, every ſtar on the globe will point toward the like ſtar in the heaven: by which means, the conſtellations and remarkable ſtars may be eaſily known. All thoſe ſtars which are in the eaſtern ſide of the horizon, are then riſing in the eaſtern ſide of the heaven: all in the weſtern, are ſetting in the weſtern ſide; and all thoſe under the upper part of the braſen meridian, between the ſouth point of the horizon and the north pole, are at their greateſt altitude, if the latitude of the place be north: but if the latitude be ſouth, thoſe ſtars which lie under the upper part of the meridian, between the north point of the horizon and the ſouth pole, are at their greateſt altitude.

PROBLEM IV.

The latitude of the place, and day of the month being given; to find the time when any known ſtar will riſe, or be on the meridian, or ſet.

Having rectified the globe, turn it about until the given ſtar comes to the eaſtern ſide of the horizon, and the index will ſhew the time of the ſtar's riſing; then turn the globe weſtward, and when the ſtar comes to the braſen meridian, the index will ſhew the time of the ſtar's coming to the meridian of your place; laſtly, turn on, until the ſtar comes to the weſtern ſide of the horizon, and the index will ſhew the time of the ſtar's ſetting.

N. B.

N. B. In northern latitudes, those stars which are less distant from the north pole, than the quantity of its elevation above the north point of the horizon, never set; and those which are less distant from the south pole, than the number of degrees by which it is depressed below the horizon, never rise: and *vice versâ* in southern latitudes.

PROBLEM V.

To find at what time of the year a given star will be upon the meridian, at a given hour of the night.

Bring the given star to the upper semicircle of the brass meridian, and set the index to the given hour; then turn the globe, until the index points to XII at noon, and the upper semicircle of the meridian will then cut the sun's place, answering to the day of the year sought; which day may be easily found against the like place of the sun among the signs on the wooden horizon.

PROBLEM VI.

The latitude, day of the month, and azimuth of any known star being given; to find the hour of the night.*

Having rectified the globe for the latitude, zenith, and sun's place; lay the quadrant of

* The number of degrees, that the sun, moon, or any star, is from the meridian, either to the east or west, is called its *azimuth*.

altitude

altitude to the given degree of azimuth in the horizon: then turn the globe on its axis, until the ſtar comes to the graduated edge of the quadrant; and when it does, the index will point out the hour of the night.

PROBLEM VII.

*The latitude of the place, the day of the month, and altitude * of any known ſtar, being given; to find the hour of the night.*

Rectify the globe as in the former problem, gueſs at the hour of the night, and turn the globe until the index points at the ſuppoſed hour; then lay the graduated edge of the quadrant of altitude over the known ſtar, and if the degree of the ſtar's height in the quadrant upon the globe, anſwers exactly to the degree of the ſtar's obſerved altitude in the heaven, you have gueſſed exactly: but if the ſtar on the globe is higher or lower than it was obſerved to be in the heaven, turn the globe backward or forward, keeping the edge of the quadrant upon the ſtar, until its center comes to the obſerved altitude in the quadrant; and then, the index will ſhew the true time of the night.

* The number of degrees that the ſtar is above the horizon, as obſerved by means of a common quadrant, is called its *altitude*.

PRO-

PROBLEM VIII.

An easy method for finding the hour of the night by any two known stars, without knowing either their altitude or azimuth; and then, of finding both their altitude and azimuth, and thereby the true meridian.

Tie one end of a thread to a common musket bullet; and, having rectified the globe as above, hold the other end of the thread in your hand, and carry it slowly round between your eye and the starry heaven, until you find it cuts any two known stars at once. Then, guessing at the hour of the night, turn the globe until the index points to the time in the hour-circle; which done, lay the graduated edge of the quadrant over any one of these two stars on the globe, which the thread cut in the heaven. If the said edge of the quadrant cuts the other star also, you have guessed the time exactly; but if it does not, turn the globe slowly backward or forward, until the quadrant (kept upon either star) cuts them both through their centers: and then, the index will point out the exact time of the night; the degree of the horizon, cut by the quadrant, will be the true azimuth of both these stars from the south; and the stars themselves will cut their true altitudes in the quadrant. At which moment, if a common azimuth compass be so set upon a floor or level pavement, that these stars in the heaven may have the same bearing upon it (allowing for the variation of the needle) as the quadrant of altitude has in the wooden horizon of the globe, a thread extended over the north and south points of that compass will

will be directly in the plane of the meridian: and if a line be drawn upon the floor or pavement, along the course of the thread, and an upright wire be placed in the southernmost end of the line, the shadow of the wire will fall upon that line, when the sun is on the meridian, and shines upon the pavement.

PROBLEM IX.

To find the place of the moon, or of any planet; and thereby to shew the time of its rising, southing, and setting.

Seek in the Nautical Almanac or *White's* Ephemeris, the geocentric place * of the moon or planet in the ecliptic for the given day of the month, and, according as its longitude and latitude is found, mark the same with a chalk upon the globe. Then, having rectified the globe, turn it round its axis westward; and as the said mark comes to the eastern side of the horizon, to the brasen meridian, and to the western side of the horizon, the index will shew at what time the planet rises, comes to the meridian, and sets, in the same manner as it would do for a fixed star.

PROBLEM X.

To explain the phenomena of the harvest moon.

In order to do this, we must premise the following things: 1. That as the sun goes only

* The place of the moon or planet, as seen from the earth, is called its geocentric place.

once

once a year round the ecliptic, he can be but once a year in any particular point of it: and that his motion is almost a degree every 24 hours, at a mean rate. 2. That as the moon goes round the ecliptic once in 27 days and 8 hours, she advances $13\frac{1}{6}$ degrees in it, every day at a mean rate. 3. That as the sun goes through part of the ecliptic in the time the moon goes round it, the moon cannot at any time be either in conjunction with the sun, or opposite to him, in that part of the ecliptic where she was so the last time before; but must travel as much forwarder, as the sun has advanced in the said time: which being $29\frac{1}{2}$ days, makes almost a whole sign. Therefore, 4. The moon can be but once a year opposite to the sun, in any particular part of the ecliptic. 5. That the moon is never full but when she is opposite to the sun, because at no other time can we see all that half of her, which the sun enlightens. 6. That when any point of the ecliptic rises, the opposite point sets. Therefore, when the moon is opposite to the sun, she must rise at * sun set. 7. That the different signs of the ecliptic rise at very different angles or degrees of obliquity with the horizon, especially in considerable latitudes; and that the smaller this angle is, the greater is the portion of the ecliptic that rises in any small part of time; and *vice versâ*. 8. That, in northern latitudes, no part of the ecliptic rises at so small an angle with the horizon, as *Pisces* and *Aries* do; therefore, a greater portion of the ecliptic rises in

* This is not always strictly true, because the moon does not keep in the ecliptic, but crosses it twice every month. However, the difference need not be regarded in a general explanation of the cause of the harvest moon.

one hour, about these signs, than about any of the rest. 9. That the moon can never be full in *Pisces* and *Aries* but in our autumnal months, for at no other time of the year is the sun in the opposite signs *Virgo* and *Libra*.

These things premised, take $13\frac{1}{6}$ degrees of the ecliptic in your compasses, and beginning at *Pisces*, carry that extent all round the ecliptic, marking the places with a chalk, where the points of the compasses successively fall. So you will have the moon's daily motion marked out for one complete revolution in the ecliptic; according to § 2 of the last paragraph.

Rectify the globe for any considerable northern latitude (as suppose that of London) and then, turning the globe round its axis, observe how much of the hour circle the index has gone over, at the rising of each particular mark on the ecliptic; and you will find that seven of the marks (which take in as much of the ecliptic as the moon goes through in a week) will all rise successively about *Pisces* and *Aries* in the time that the index goes over two hours. Therefore, while the moon is in *Pisces* and *Aries*, she will not differ in general above two hours in her rising for a whole week. But if you take notice of the marks on the opposite signs, *Virgo* and *Libra*, you will find that seven of them take nine hours to rise; which shews, that when the moon is in these two signs, she differs nine hours in her rising within the compass of a week. And so much later as every mark is of rising than the one that rose next before it, so much later will the moon be of rising on any day than she was on the day before, in the corresponding part of the heaven. The marks about *Cancer* and *Capricorn* rise at a mean difference of time between those about *Aries* and *Libra*.

Now, although the moon is in *Pisces* and *Aries* every month, and therefore must rise in those signs within the space of two hours later for a whole week, or only about 17 minutes later every day than she did on the former; yet she is never full in these signs, but in our autumnal months, *August* and *September*, when the sun is in *Virgo* and *Libra*. Therefore, no full moon in the year will continue to rise so near the time of sun set for a week or so, as these two full moons do, which fall in the time of harvest.

In the winter months, the moon is in *Pisces* and *Aries* about her first quarter; and as these signs rise about noon in winter, the moon's rising in them passes unobserved. In the spring months, the moon changes in these signs, and consequently rises at the same time with the sun; so that it is impossible to see her at that time. In the summer months she is in these signs about her third quarter, and rises not until mid-night, when her rising is but very little taken notice of; especially as she is on the decrease. But in the harvest months she is at the full, when in these signs, and being opposite to the sun, she rises when the sun sets (or soon after) and shines all the night.

In southern latitudes, *Virgo* and *Libra* rise at as small angles with the horizon, as *Pisces* and *Aries* do in the northern; and as our spring is at the time of their harvest, it is plain their harvest full moons must be in *Virgo* and *Libra*; and will therefore rise with as little difference of time, as ours do in *Pisces* and *Aries*.

For a fuller account of this matter, I must refer the reader to my Astronomy, in which it is described at large.

PROBLEM XI.

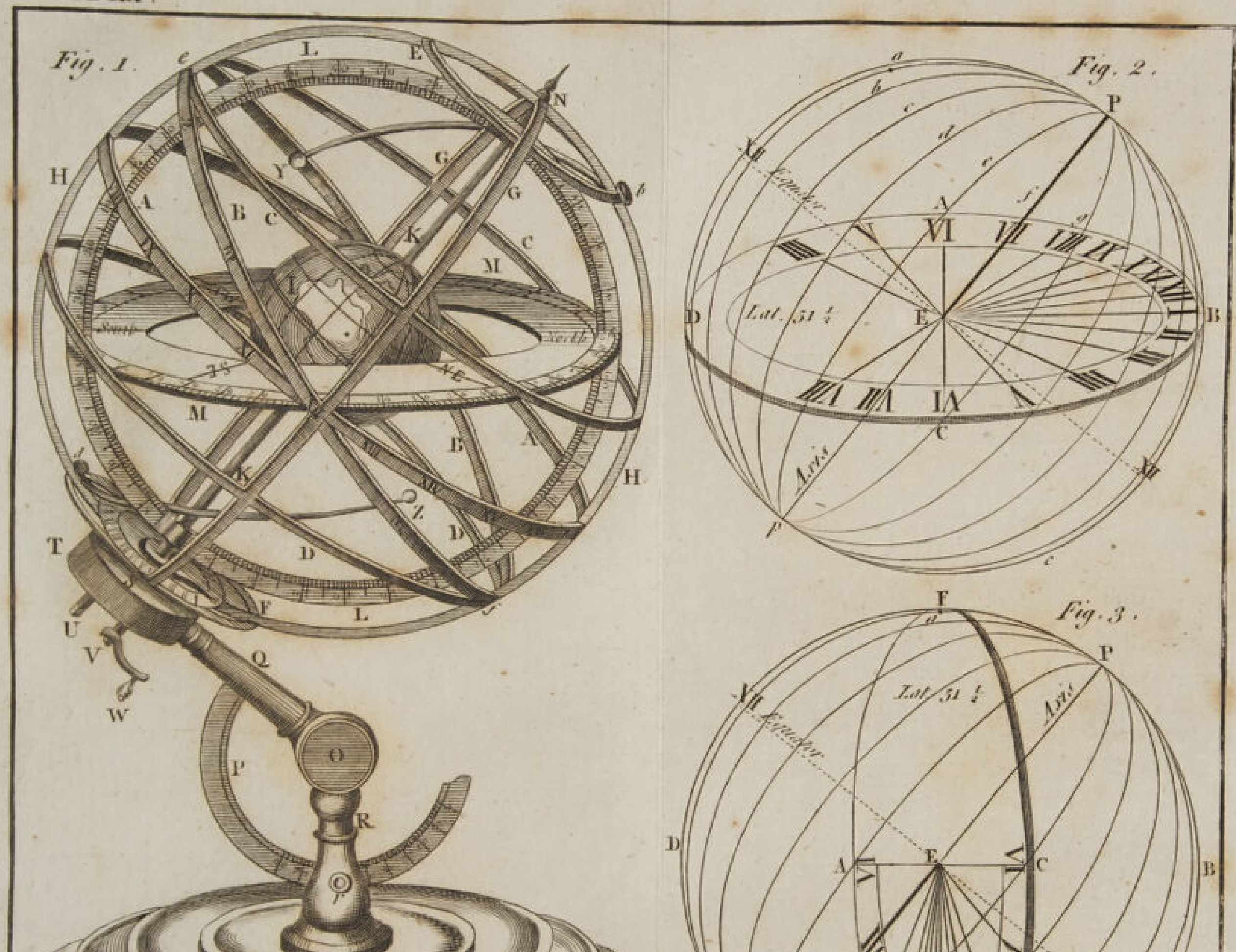
To explain the equation of time, or difference of time between well regulated clocks and true sun-dials.

The earth's motion on its axis being perfectly equable, and thereby causing an apparent equable motion of the starry heaven round the same axis, produced to the poles of the heaven; it is plain that equal portions of the celestial equator pass over the meridian in equal parts of time, because the axis of the world is perpendicular to the plane of the equator. And therefore, if the sun kept his annual course in the celestial equator, he would always revolve from the meridian to the meridian again in 24 hours exactly, as shewn by a well-regulated clock.

But as the sun moves in the ecliptic, which is oblique both to the plane of the equator and axis of the world, he cannot always revolve from the meridian to the meridian again in 24 equal hours; but sometimes a little sooner, and at other times a little later, because equal portions of the ecliptic pass over the meridian in unequal parts of time on account of its obliquity. And this difference is the same in all latitudes.

To shew this by a globe, make chalk-marks all round the equator and ecliptic, at equal distances from one another (suppose 10 degrees) beginning at *Aries* or at *Libra*, where these two circles intersect each other. Then turn the globe

PLATE XX.



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globe round its axis, and you will ſee that all the marks in the firſt quadrant of the ecliptic, or from the beginning of *Aries* to the beginning of *Cancer*, come ſooner to the braſen meridian than their correſponding marks do on the equator; thoſe on the ſecond quadrant, or from the beginning of *Cancer* to the beginning of *Libra*, come later: thoſe in the third quadrant, from *Libra* to *Capricorn*, ſooner; and thoſe in the fourth, from *Capricorn* to *Aries*, later. But thoſe at the beginning of each quadrant come to the meridian at the ſame time with their correſponding marks on the equator.

Therefore, while the ſun is in the firſt and third quadrants of the ecliptic, he comes ſooner to the meridian every day than he would do if he kept in the equator; and conſequently he is faſter than a well regulated clock, which always keeps equable or equatorial time: and while he is in the ſecond and fourth quadrants, he comes later to the meridian every day than he would do if he kept in the equator; and is therefore ſlower than the clock. But at the beginning of each quadrant, the ſun and clock are equal.

And thus, if the ſun moved equably in the ecliptic, he would be equal with the clock on four days of the year, which would have equal intervals of time between them. But as he moves faſter at ſome times than at others (being eight days longer in the northern half of the ecliptic than in the ſouthern) this will cauſe a ſecond inequality; which combined with the former, ariſing from the obliquity of the ecliptic to the equator, makes up that difference, which is ſhewn by the common equation tables to be between good clocks and true ſun-dials.

The description and use of the armillary sphere.

PlateXX. Fig. 1. Whoever has ſeen a common *armillary ſphere*, and underſtands how to uſe it, muſt be ſenſible that the machine here referred to, is of a very different, and much more advantageous conſtruction. And thoſe who have ſeen the curious glaſs ſphere invented by Dr. Long, or the figure of it in his Aſtronomy, muſt know that the furniture of the terreſtrial globe in this machine, the form of the pedeſtal, and the manner of turning either the earthly globe or the circles which ſurround it, are all copied from the Doctor's glaſs ſphere; and that the only difference is, a parcel of rings inſtead of a glaſs celeſtial globe; and all the additions are a moon within the ſphere, and a ſemicircle upon the pedeſtal.

The *armillary ſphere*. The exterior parts of this machine are a compages of braſs rings, which repreſent the principal circles of the heaven, viz. 1. The equinoctial *A A*, which is divided into 360 degrees (beginning at its interſection with the ecliptic in *Aries*) for ſhewing the ſun's right aſcenſion in degrees; and alſo into 24 hours, for ſhewing his right aſcenſion in time. 2. The ecliptic *B B*, which is divided into 12 ſigns, and each ſign into 30 degrees, and alſo into the months and days of the year; in ſuch a manner, that the degree or point of the ecliptic in which the ſun is, on any given day, ſtands over that day in the circle of months. 3. The tropic of *Cancer C C*, touching the ecliptic at the beginning of *Cancer* in *e*, and the tropic of *Capricorn D D*, touching the ecliptic at the beginning of *Capricorn* in *f*; each $23\frac{1}{2}$ degrees from

from the equinoctial circle. 4. The arctic circle *E*, and the antarctic circle *F*, each 23½ degrees from its respective pole at *N* and *S*. 5. The equinoctial colure *G G*, passing through the north and south poles of the heaven at *N* and *S*, and through the equinoctial points *Aries*, and *Libra* in the ecliptic. 6. The solstitial colure *H H*, passing through the poles of the heaven, and through the solstitial points *Cancer* and *Capricorn*, in the ecliptic. Each quarter of the former of these colures is divided into 90 degrees, from the equinoctial to the poles of the world, for shewing the declination of the sun, moon, and stars; and each quarter of the latter, from the ecliptic at *e* and *f*, to its poles *b* and *d*, for shewing the latitude of the stars.

In the north pole of the ecliptic is a nut *b*, to which is fixed one end of a quadrantal wire, and to the other end a small sun *Y*, which is carried round the ecliptic *B B*, by turning the nut: and in the south-pole of the ecliptic is a pin at *d*, on which is another quadrantal wire, with a small moon *Z* upon it, which may be moved round by hand: but there is a particular contrivance for causing the moon to move in an orbit which crosses the ecliptic at an angle of $5\frac{1}{3}$ degrees, in two opposite points called the *moon's nodes*; and also for shifting these points backward in the ecliptic, as the *moon's nodes* shift in the heaven.

Within these circular rings is a small terrestrial globe *I*, fixt on an axis *K K*, which extends from the north and south poles of the globe at *n* and *s*, to those of the celestial sphere at *N* and *S*. On this axis is fixt the flat celestial meridian *L L*, which may be set directly over the meridian of any place on the globe, and then turned round with the globe, so as to keep over the same

 meridian

meridian upon it. This flat meridian is graduated the ſame way as the braſs meridian of a common globe, and its uſe is much the ſame. To this globe is fitted the moveable horizon *M M*, ſo as to turn upon two ſtrong wires proceeding from its eaſt and weſt points to the globe, and entering the globe at oppoſite points of its equator, which is a moveable braſs ring let into the globe in a groove all around its equator. The globe may be turned by hand within this ring, ſo as to place any given meridian upon it, directly under the celeſtial meridian *L L*. The horizon is divided into 360 degrees all around its outermoſt edge, within which are the points of the compaſs, for ſhewing the amplitude of the ſun and moon, both in degrees and points. The celeſtial meridian *L L* paſſes through two notches in the north and ſouth points of the horizon, as in a common globe: but here, if the globe be turned round, the horizon and meridian turn with it. At the ſouth pole of the ſphere is a circle of 24 hours, fixt to the rings, and on the axis is an index which goes round that circle, if the globe be turned round its axis.

The whole fabric is ſupported on a pedeſtal *N*, and may be elevated or depreſſed upon the joint *O*, to any number of degrees from 0 to 90, by means of the arc *P*, which is fixed in the ſtrong braſs arm *Q*, and ſlides in the upright piece *R*, in which is a ſcrew at *r*, to fix it at any proper elevation.

In the box *T* are two wheels (as in Dr. *Long*'s ſphere) and two pinions, whoſe axes come out at *V* and *U*; either of which may be turned by the ſmall winch *W*. When the winch is put upon the axis *V*, and turned backward, the terreſtrial

restrial globe, with its horizon and celestial meridian, keep at rest; and the whole sphere of circles turns round from east, by south, to west, carrying the sun *Y*, and moon *Z*, round the same way, and causing them to rise above, and set below the horizon. But when the winch is put upon the axis *U*, and turned forward, the sphere with the sun and moon keep at rest; and the earth, with its horizon and meridian, turn round from west, by south, to east; and bring the same points of the horizon to the sun and moon, to which these bodies came when the earth kept at rest, and they were carried round it; shewing that they rise and set in the same points of the horizon, and at the same time in the hour-circle, whether the motion be in the earth or in the heaven. If the earthly globe be turned, the hour-index goes round its hour-circle; but if the sphere be turned, the hour-circle goes round below the index.

And so, by this construction, the machine is equally fitted to shew either the real motion of the earth, or the apparent motion of the heaven.

To rectify the sphere for use, first slacken the screw *r* in the upright stem *R*, and taking hold of the arm *Q*, move it up or down until the given degree of latitude for any place be at the side of the stem *R*; and then the axis of the sphere will be properly elevated, so as to stand parallel to the axis of the world, if the machine be set north and south by a small compass: this done, count the latitude from the north pole, upon the celestial meridian *L L*, down toward the north notch of the horizon, and set the horizon to that latitude; then, turn the nut *b* until the sun *Y* comes to the given day of the year in the

the ecliptic, and the ſun will be at its proper place for that day: find the place of the moon's aſcending node, and alſo the place of the moon, by an Ephemeris, and ſet them right accordingly: laſtly, turn the winch *W*, until either the ſun comes to the meridian *L L*, or until the meridian comes to the ſun (according as you want the ſphere or earth to move) and ſet the hour-index to the XII, marked noon, and the whole machine will be rectified.—Then turn the winch, and obſerve when the ſun or moon riſe and ſet in the horizon, and the hour-index will ſhew the times thereof for the given day.

As thoſe who underſtand the uſe of the globes will be at no loſs to work many other problems by this ſphere, it is needleſs to enlarge any farther upon it.

L E C T. X.

The principles and art of dialing.

Preliminaries. A Dial is a plane, upon which lines are deſcribed in ſuch a manner, that the ſhadow of a wire, or of the upper edge of a plate ſtile, erected perpendicularly on the plane of the dial, may ſhew the true time of the day.

The edge of the plate by which the time of the day is found, is called the ſtile of the dial, which muſt be parallel to the earth's axis; and the line on which the ſaid plate is erected, is called the ſubſtile.

The angle included between the ſubſtile and ſtile, is called the elevation, or height of the ſtile.

Thoſe dials whoſe planes are parallel to the plane of the horizon, are called horizontal dials; and

and thofe dials whofe planes are perpendicular to the plane of the horizon, are called vertical, or erect fun-dials.

Thofe erect dials, whofe planes directly front the north or fouth, are called direct north or fouth dials; and all other erect dials are called decliners, becaufe their planes are turned away from the north or fouth.

Thofe dials, whofe planes are neither parallel nor perpendicular to the plane of their horizon, are called inclining, or reclining dials, according as their planes make acute or obtufe angles with the horizon; and if their planes are alfo turned afide from facing the fouth or north, they are called declining-inclining, or declining-reclining dials.

The interfection of the plane of the dial, with that of the meridian, paffing through the ftile, is called the meridian of the dial, or the hour-line of XII.

Thofe meridians, whofe planes pafs through the ftile, and make angles of 15, 30, 45, 60, 75, and 90 degrees with the meridian of the place (which marks the hour-line of XII) are called hour-circles; and their interfections with the plane of the dial, are called hour-lines.

In all declining dials, the fubftile makes an angle with the hour-line of XII; and this angle is called the diftance of the fubftile from the meridian.

The declining plane's difference of longitude, is the angle formed at the interfection of the ftile and plane of the dial, by two meridians; one of which paffes through the hour-line of XII, and the other through the fubftile.

This

This much being premiſed concerning dials in general, we ſhall now proceed to explain the different methods of their conſtruction.

PlateXX. Fig. 2. The univerſal principle on which *dialing* depends.

If the whole earth $a P c p$ were tranſparent, and hollow, like a ſphere of glaſs, and had its equator divided into 24 equal parts by ſo many meridian ſemicircles, *a*, *b*, *c*, *d*, *e*, *f*, *g*, &c. one of which is the geographical meridian of any given place as London, which is ſuppoſed to be at the point *a*; and if the hours of XII were marked at the equator, both upon that meridian and the oppoſite one, and all the reſt of the hours in order on the reſt of the meridians, thoſe meridians would be the hour-circles of London: then, if the ſphere had an opaque axis, as $P E p$, terminating in the poles P and p, the ſhadow of the axis would fall upon every particular meridian and hour, when the ſun came to the plane of the oppoſite meridian, and would conſequently ſhew the time at London, and at all other places on the meridian of London.

Horizontal dial.

If this ſphere was cut through the middle by a ſolid plane $A B C D$, in the rational horizon of London, one half of the axis $E P$ would be above the plane, and the other half below it; and if ſtraight lines were drawn from the center of the plane, to thoſe points where its circumference is cut by the hour-circles of the ſphere, thoſe lines would be the hour-lines of a horizontal dial for London: for the ſhadow of the axis would fall upon each particular hour-line of the dial, when it fell upon the like hour-circle of the ſphere.

Fig. 3.

If the plane which cuts the ſphere be upright, as $A F C G$, touching the given place (London) at F, and directly facing the meridian of London,

don, it will then become the plane of an erect direct ſouth dial: and if right lines be drawn from its center *E*, to thoſe points of its circumference where the hour-circles of the ſphere cut it, theſe will be the hour-lines of a vertical or direct ſouth dial for London, to which the hours are to be ſet as in the figure (contrary to thoſe on a horizontal dial) and the lower half *Ep* of the axis will caſt a ſhadow on the hour of the day in this dial, at the ſame time that it would fall upon the like hour-circle of the ſphere, if the dial plane was not in the way. *Vertical dial.*

If the plane (ſtill facing the meridian) be made to incline, or recline, by any given number of degrees, the hour-circles of the ſphere will ſtill cut the edge of the plane in thoſe points to which the hour-lines muſt be drawn ſtraight from the center; and the axis of the ſphere will caſt a ſhadow on theſe lines at the reſpective hours. The like will ſtill hold, if the plane be made to decline by any given number of degrees from the meridian, toward the eaſt or weſt: provided the declination be leſs than 90 degrees, or the reclination be leſs than the co-latitude of the place: and the axis of the ſphere will be a gnomon, or ſtile, for the dial. But it cannot be a gnomon, when the declination is quite 90 degrees, nor when the reclination is equal to the co-latitude; becauſe in theſe two caſes, the axis has no elevation above the plane of the dial. *Inclining and reclining dials.* *Declining dials.*

And thus it appears, that the plane of every dial repreſents the plane of ſome great circle upon the earth; and the gnomon the earth's axis, whether it be a ſmall wire, as in the above figures, or the edge of a thin plate, as in the common horizontal dials.

The

The whole earth, as to its bulk, is but a point, if compared to its diſtance from the ſun: and therefore, if a ſmall ſphere of glaſs be placed upon any part of the earth's ſurface, ſo that its axis be parallel to the axis of the earth, and the ſphere have ſuch lines upon it, and ſuch planes within it, as above deſcribed: it will ſhew the hours of the day as truly as if it were placed at the earth's center, and the ſhell of the earth were as tranſparent as glaſs,

Fig. 2, 3. But becauſe it is impoſſible to have a hollow ſphere of glaſs perfectly true, blown round a ſolid plane; or if it was, we could not get at the plane within the glaſs to ſet it in any given poſition; we make uſe of a wire ſphere to explain the principles of dialing, by joing 24 ſemicircles together at the poles, and putting a thin flat plate of braſs within it.

Dialing by the common *terreſtrial globe.* A common globe, of 12 inches diameter, has generally 24 meridian ſemicircles drawn upon it. If ſuch a globe be elevated to the latitude of any given place, and turned about until any one of theſe meridians cuts the horizon in the north point, where the hour of XII is ſuppoſed to be marked, the reſt of the meridians will cut the horizon at the reſpective diſtances of all the other hours from XII. Then, if theſe points of diſtance be marked on the horizon, and the globe be taken out of the horizon, and a flat board or plate be put into its place, even with the ſurface of the horizon; and if ſtraight lines be drawn from the center of the board, to thoſe points of diſtance on the horizon which were cut by the 24 meridian ſemicircles, theſe lines will be the hour-lines of a horizontal dial for that latitude, the edge of whoſe gnomon muſt be in the very ſame ſituation that the axis of the globe

globe was, before it was taken out of the horizon: that is, the gnomon muſt make an angle with the plane of the dial, equal to the latitude of the place for which the dial is made.

If the pole of the globe be elevated to the co-latitude * of the given place, and any meridian be brought to the north point of the horizon, the reſt of the meridians will cut the horizon in the reſpective diſtances of all the hours from XII, for a direct ſouth dial, whoſe gnomon muſt make an angle with the plane of the dial, equal to the co-latitude of the place; and the hours muſt be ſet the contrary way on this dial, to what they are on the horizontal.

But if your globe have more than 24 meridian ſemicircles upon it, you muſt take the following method for making *horizontal and ſouth dials by it.*

To conſtruct a *horizontal dial.*

Elevate the pole to the latitude of your place, and turn the globe until any particular meridian (ſuppoſe the firſt) comes to the north point of the horizon, and the oppoſite meridian will cut the horizon in the ſouth. Then, ſet the hour-index to the uppermoſt XII on its circle; which done, turn the globe weſtward until 15 degrees of the equator paſs under the braſen meridian, and then the hour-index will be at I (for the ſun moves 15 degrees every hour) and the firſt meridian will cut the horizon in the number of degrees from the north point, that I is diſtant from XII. Turn on, until other 15 degrees of the equator paſs under the braſen meridian, and the hour-index will then be at II, and the firſt me-

* If the latitude be ſubtracted from 90 degrees, the remainder is called the co-latitude, or complement of the latitude.

ridian

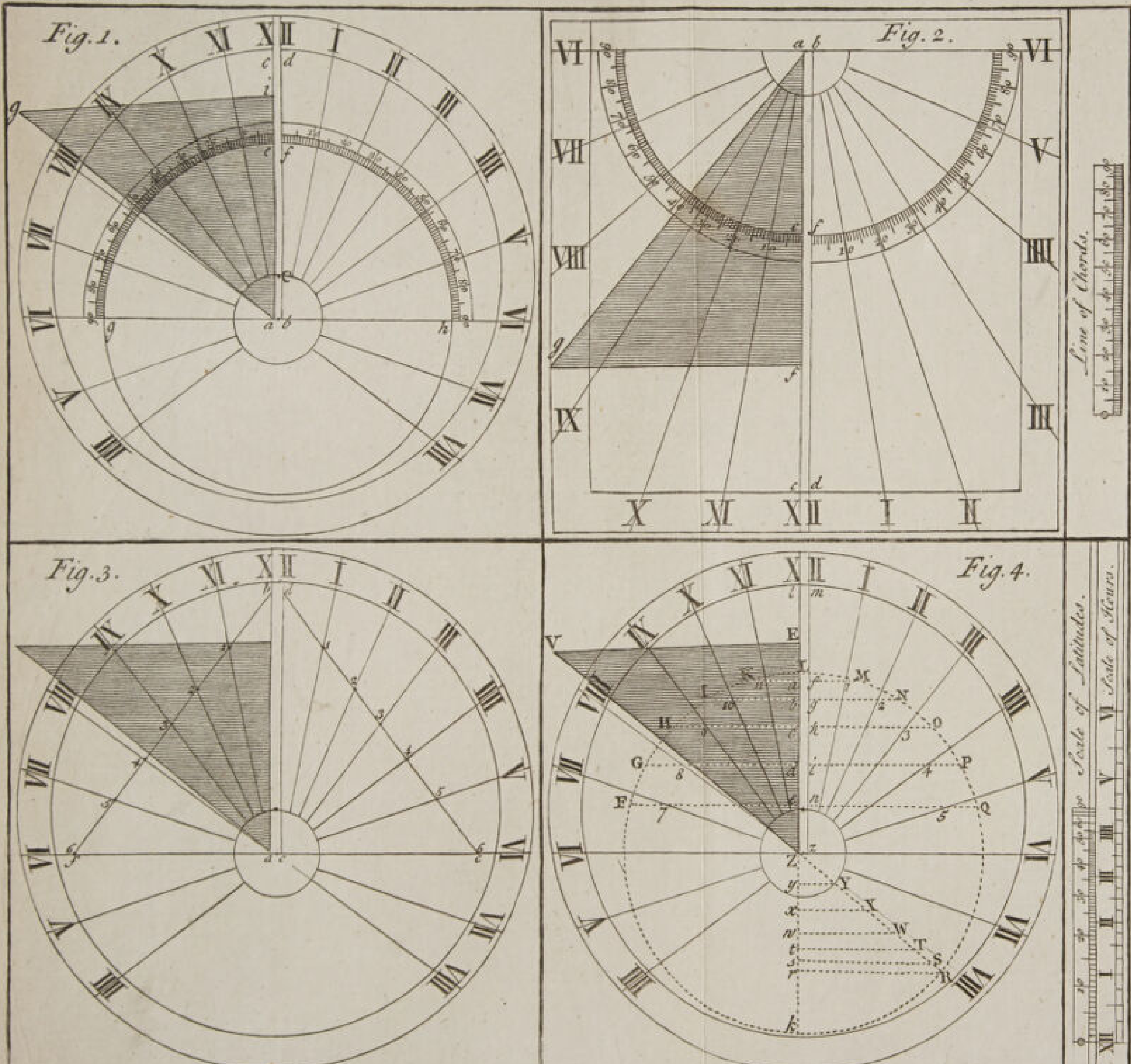
ridian will cut the horizon in the number of degrees that II is diſtant from XII: and ſo, by making 15 degrees of the equator paſs under the braſen meridian for every hour, the firſt meridian of the globe will cut the horizon in the diſtances of all the hours from XII to VI, which is juſt 90 degrees; and then you need go no farther, for the diſtances of XI, X, IX, VIII, VII, and VI, in the forenoon, are the ſame from XII, as the diſtances of I, II, III, IV, V, and VI, in the afternoon: and theſe hour-lines continued through the center, will give the oppoſite hour-lines on the other half of the dial: but no more of theſe lines need be drawn, than what anſwer to the ſun's continuance above the horizon of your place on the longeſt day, which may be eaſily found by the 26th problem of the foregoing lecture.

Thus, to make a horizontal dial for the latitude of London, which is $51\frac{1}{2}$ degrees north, elevate the north pole of the globe $51\frac{1}{2}$ degrees above the north point of the horizon, and then turn the globe, until the firſt meridian (which is that of London on the Engliſh terreſtrial globe) cuts the north point of the horizon, and ſet the hour-index to XII at noon.

Then, turning the globe weſtward until the index points ſucceſſively to I, II, III, IV, V, and VI, in the afternoon; or until 15, 30, 45, 60, 75, and 90 degrees of the equator paſs under the braſen meridian, you will find that the firſt meridian of the globe cuts the horizon in the following numbers of degrees from the north toward the eaſt, viz. $11\frac{2}{3}$ $24\frac{1}{4}$, $38\frac{1}{12}$ $53\frac{1}{2}$, $71\frac{1}{15}$, and 90; which are the reſpective diſtances of the above hours from XII upon the plane of the horizon.

To

PLATE XXI.



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To transfer these, and the rest of the hours, to a horizontal plane, draw the parallel right lines *a c* and *b d* upon that plane, as far from each other as is equal to the intended thickness of the gnomon or stile of the dial, and the space included between them will be the meridian or twelve o'clock line on the dial. Cross this meridian at right angles with the six o'clock line *g h*, and setting one foot of your compasses in the intersection *a*, as a center, describe the quadrant *g e* with any convenient radius or opening of the compasses: then, setting one foot in the intersection *b*, as a center, with the same radius describe the quadrant *f h*, and divide each quadrant into 90 equal parts or degrees, as in the figure. Plate XXI. Fig. 1.

Because the hour-lines are less distant from each other about noon, than in any other part of the dial, it is best to have the centers of these quadrants at a little distance from the center of the dial-plane, on the side opposite to XII, in order to enlarge the hour distances thereabout under the same angles on the plane. Thus, the center of the plane is at *C*, but the centers of the quadrants at *a* and *b*.

Lay a ruler over the point *b* (and keeping it there for the center of all the afternoon hours in the quadrant *f h*) draw the hour-line of I, through $11\frac{2}{3}$ degrees in the quadrant; the hour-line of II, through $24\frac{1}{4}$ degrees; of III, through $38\frac{1}{12}$ degrees; IIII, through $53\frac{1}{2}$, and V through $71\frac{1}{15}$: and because the sun rises about four in the morning, on the longest days at London, continue the hour-lines of IIII and V, in the afternoon, through the center *b* to the opposite side of the dial.—This done, lay the ruler to the center *a*, of the quadrant *e g*, and through the

like

like divifions or degrees of that quadrant, viz. $11\frac{2}{3}$, $24\frac{1}{4}$, $38\frac{1}{12}$, $53\frac{1}{2}$, and $71\frac{1}{15}$, draw the forenoon hour-lines of XI, X, IX, VIII, and VII; and becaufe the fun fets not before eight in the evening on the longeft days, continue the hour-lines of VII and VIII in the forenoon, through the center *a*, to VII and VIII in the afternoon; and all the hour-lines will be finifhed on this dial; to which the hours may be fet, as in the figure.

Laftly, through $51\frac{1}{2}$ degrees of either quadrant, and from its center draw the right line *a g* for the hypothenufe or axis of the gnomon *a g i*; and from *g*, let fall the perpendicular *g i*, upon the meridian line *a i*, and there will be a triangle made, whofe fides are *a g*, *g i*, and *i a*. If a plate fimilar to this triangle be made as thick as the diftance between the lines *a c* and *b d*, and fet upright between them, touching at *a* and *b*, its hypothenufe *a g* will be parallel to the axis of the world, when the dial is truly fet; and will caft a fhadow on the hour of the day.

N. B. The trouble of dividing the two quadrants may be faved, if you have a fcale with a line of chords upon it, fuch as that on the right hand of the plate: for if you extend the compaffes from 0 to 60 degrees of the line of chords, and with that extent, as a radius, defcribe the two quadrants upon their refpective centers, the above diftances may be taken with the compaffes upon the line, and fet off upon the quadrants.

Fig. 2. To conftruct an *erect direct fouth dial.*

To make an erect direct fouth dial. Elevate the pole to the co-latitude of your place, and proceed in all refpects as above taught for the horizontal dial, from VI in the morning to VI in the afternoon, only the hours muft be reverfed, as in the figure; and the hypothenufe *a g*, of the gnomon

gnomon *a g f*, muſt make an angle with the dial-plane equal to the co-latitude of the place. As the ſun can ſhine no longer on this dial, than from ſix in the morning until ſix in the evening, there is no occaſion for having any more than twelve hours upon it.

To conſtruct an *erect de-clining dial.*

To make an erect dial, declining from the ſouth toward the eaſt or weſt. Elevate the pole to the latitude of your place, and ſcrew the quadrant of altitude to the zenith. Then, if your dial declines toward the eaſt (which we ſhall ſuppoſe it to do at preſent) count in the horizon the degrees of declination, from the eaſt point toward the north, and bring the lower end of the quadrant to that degree of declination at which the reckoning ends. This done, bring any particular meridian of your globe (as ſuppoſe the firſt meridian) directly under the graduated edge of the upper part of the braſen meridian, and ſet the hour-index to XII at noon. Then, keeping the quadrant of altitude at the degree of declination in the horizon, turn the globe eaſtward on its axis, and obſerve the degrees cut by the firſt meridian in the quadrant of altitude (counted from the zenith) as the hour-index comes to XI, X, IX, &c. in the forenoon, or as 15, 30, 45, &c. degrees of the equator paſs under the braſen meridian at theſe hours reſpectively; and the degrees then cut in the quadrant by the firſt meridian, are the reſpective diſtances of the forenoon hours from XII on the plane of the dial.—Then, for the afternoon hours, turn the quadrant of altitude round the zenith until it comes to the degree in the horizon oppoſite to that where it was placed before; namely, as far from the weſt point of the horizon toward the ſouth, as it was ſet at firſt from the eaſt point toward the north; and turn the globe weftward on its axis, until the firft meridian comes to the brafen meridian again, and the hour-index to XII: then, continue to turn the globe weftward, and as the index points to the afternoon hours I, II, III, &c. or as 15, 30, 45, &c. degrees of the equator pafs under the brafen meridian, the firft meridian will cut the quadrant of altitude in the refpective number of degrees from the zenith, that each of thefe hours is from XII on the dial.—And note, that when the firft meridian goes off the quadrant at the horizon, in the forenoon, the hour-index fhews the time when the fun will come upon this dial: and when it goes off the quadrant in the afternoon, the index will point to the time when the fun goes off the dial.

Having thus found all the hour-diftances from XII, lay them down upon your dial-plate, either by dividing a femicircle into two quadrants of 90 degrees each (beginning at the hour-line of XII) or by the line of chords, as above directed.

In all declining dials, the line on which the ftile or gnomon ftands (commonly called the *fubftile-line*) makes an angle with the twelve o'clock line, and falls among the forenoon hour-lines, if the dial declines toward the eaft; and among the afternoon hour-lines, when the dial declines toward the weft; that is, to the left hand from the twelve o'clock line in the former cafe, and to the right hand from it in the latter.

To find the diftance of the fubftile from the twelve o'clock line; if your dial declines from the fouth toward the eaft, count the degrees of that declination in the horizon from the eaft point toward the north, and bring the lower end of the quadrant of altitude to that degree of declination

declination where the reckoning ends: then, turn the globe until the firſt meridian cuts the horizon in the like number of degrees, counted from the ſouth point toward the eaſt; and the quadrant and firſt meridian will then croſs one another at right angles, and the number of degrees of the quadrant, which are intercepted between the firſt meridian and the zenith, is equal to the diſtance of the ſubſtile-line from the twelve o'clock line; and the number of degrees of the firſt meridian, which are intercepted between the quadrant and the north pole, is equal to the elevation of the ſtile above the plane of the dial.

If the dial declines weſtward from the ſouth, count that declination from the eaſt point of the horizon toward the ſouth, and bring the quadrant of altitude to the degree in the horizon at which the reckoning ends; both for finding the forenoon hours, and the diſtance of the ſubſtile from the meridian: and for the afternoon hours, bring the quadrant to the oppoſite degree in the horizon, namely, as far from the weſt toward the north, and then proceed in all reſpects as above.

Thus, we have finiſhed our declining dial; and in ſo doing, we made four dials, viz.

1. A north dial, declining eaſtward by the ſame number of degrees. 2. A north dial, declining the ſame number weſt. 3. A ſouth dial, declining eaſt. And, 4. A ſouth dial, declining weſt. Only, placing the proper number of hours, and the ſtile or gnomon reſpectively, upon each plane. For (as above-mentioned) in the ſouth-weſt plane, the ſubſtile-line falls among the afternoon hours; and in the ſouth-eaſt, of the ſame declination among the forenoon

 hours,

hours, at equal diſtances from XII. And ſo, all the morning hours on the weſt decliner will be like the afternoon hours on the eaſt decliner: the ſouth-eaſt decliner will produce the north-weſt decliner; and the ſouth-weſt decliner, the north-eaſt decliner, by only extending the hour-lines, ſtile and ſubſtile, quite through the center: the axis of the ſtile (or edge that caſts the ſhadow on the hour of the day) being in all dials whatever parallel to the axis of the world, and conſequently pointing toward the north pole of the heaven in north latitudes, and toward the ſouth pole, in ſouth latitudes. *See more of this in the following lecture.*

An eaſy method for conſtructing of *dials.* But becauſe every one who would like to make a dial, may perhaps not be provided with a globe to aſſiſt him, and may probably not underſtand the method of doing it by logarithmic calculation; we ſhall ſhew how to perform it by the plain dialing lines, or ſcale of latitudes and hours; ſuch as thoſe on the right hand of Fig. 4. in Plate XXI, or at the top of Plate XXII, and which may be had on ſcales commonly ſold by the mathematical inſtrument-makers.

This is the eaſieſt of all mechanical methods, and by much the beſt, when the lines are truly divided: not only the half hours and quarters may be laid down by all of them, but every fifth minute by moſt, and every ſingle minute by thoſe where the line of hours is a foot in length.

Fig. 3. Having drawn your double meridian line *a b*, *c d*, on the plane intended for a horizontal dial, and croſſed it at right angles by the ſix o'clock line *f e* (as in Fig. 1.) take the latitude of your place with the compaſſes, in the ſcale of latitudes, and ſet that extent from *c* to *e*, and from *a* to *f*, on the ſix o'clock line: then, taking the whole ſix hours

hours between the points of the compaſſes in the ſcale of hours, with that extent ſet one foot in the point *e*, and let the other foot fall where it will upon the meridian line *c d*, as at *d*. Do the ſame from *f* to *b*, and draw the right lines *e d* and *f b*, each of which will be equal in length to the whole ſcale of hours. This done, ſetting one foot of the compaſſes in the beginning of the ſcale at XII, and extending the other to each hour on the ſcale, lay off theſe extents from *d* to *e* for the afternoon hours, and from *b* to *f* for thoſe of the forenoon: this will divide the lines *d e* and *b f* in the ſame manner as the hour-ſcale is divided, at 1, 2, 3, 4, 5, and 6; on which the quarters may alſo be laid down, if required. Then, laying a ruler on the point *c*, draw the firſt five hours in the afternoon, from that point, through the dots at the numeral figures 1, 2, 3, 4, 5, on the line *d e*; and continue the lines of IIII and V through the center *c* to the other ſide of the dial, for the like hours of the morning; which done, lay the ruler on the point *a*, and draw the laſt five hours in the forenoon through the dots 5, 4, 3, 2, 1, on the line *f b*; continuing the hour-lines of VII and VIII through the center *a* to the other ſide of the dial, for the like hours of the evening; and ſet the hours to their reſpective lines as in the figure. Laſtly, make the gnomon the ſame way as taught above for the horizontal dial, and the whole will be finiſhed.

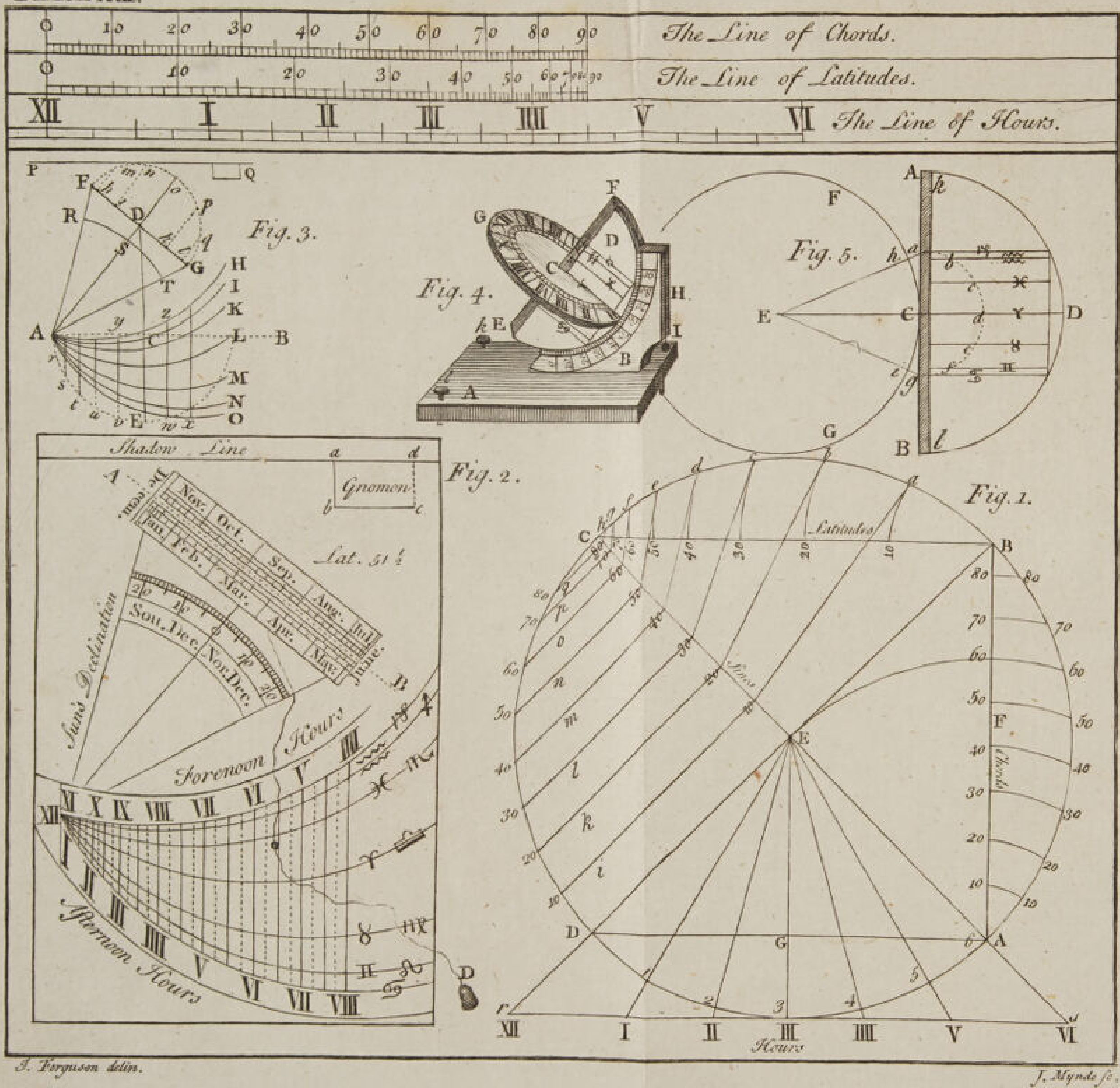
To make an erect ſouth dial, take the co-latitude of your place from the ſcale of latitudes, and then proceed in all reſpects for the hour-lines, as in the horizontal dial; only reverſing the hours, as in Fig. 2; and making the angle of the ſtile's height equal to the co-latitude.

I have drawn out a ſet of dialing lines upon the top of Plate XXII, large enough for making a dial of nine inches diameter, or more inches if required; and have drawn them tolerably exact for common practice, to every quarter of a hour. This ſcale may be cut off from the plate, and paſted upon wood, or upon the inſide of one of the boards of this book; and then it will be ſomewhat more exact than it is on the plate, for being rightly divided upon the copper-plate, and printed off on wet paper, it ſhrinks as the paper dries; but when it is wetted again, it ſtretches to the ſame ſize as when newly printed; and if paſted on while wet, it will remain of that ſize afterward.

But leſt the young dialiſt ſhould have neither globe nor wooden ſcale, and ſhould tear or otherwiſe ſpoil the paper one in paſting, we ſhall now ſhew him how he may make a dial without any of theſe helps. Only, if he has not a line of chords, he muſt divide a quadrant into 90 equal parts or degrees for taking the proper angle of the ſtile's elevation, which is eaſily done.

Fig. 4. With any opening of the compaſſes, as $Z L$, deſcribe the two ſemicircles $L F k$ and $L Q k$, upon the centers Z and z, where the ſix o'clock line croſſes the double meridian line, and divide *Horizontal dial.* each ſemicircle into 12 equal parts, beginning at L; though, ſtrictly ſpeaking, only the quadrants from L to the ſix o'clock line need be divided: then connect the diviſions which are equidiſtant from L, by the parallel lines KM, IN, HO, GP, and FQ. Draw VZ for the hypothenuſe of the ſtile, making the angle VZE equal to the latitude of your place; and continue the line VZ to R. Draw the line Rr parallel to the ſix o'clock line, and ſet off the diſtance aK from Z to Y, the

PLATE XXII.



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the diſtance *b I* from *Z* to *X*, *c H* from *Z* to *W*, *d G* from *Z* to *T*, and *e F* from *Z* to *S*. Then draw the lines *S s*, *T t*, *W w*, *X x*, and *Y y*, each parallel to *R r*. Set off the diſtance *y Y* from *a* to 11, and from *f* to 1; the diſtance *x X* from *b* to 10, and from *g* to 2; *w W* from *c* to 9, and from *h* to 3; *t T* from *d* to 8, and from *i* to 4; *s S* from *e* to 7, and from *n* to 5. Then laying a ruler to the center *Z*, draw the forenoon hour lines through the points 11, 10, 9, 8, 7; and laying it to the center *z*, draw the afternoon lines through the points 1, 2, 3, 4, 5; continuing the forenoon lines of VII and VIII through the center *Z*, to the oppoſite ſide of the dial, for the like afternoon hours; and the afternoon lines IIII and V through the center *z*, to the oppoſite ſide, for the like morning hours. Set the hours to theſe lines as in the figure, and then erect the ſtile or gnomon, and the horizontal dial will be finiſhed.

To conſtruct a ſouth dial, draw the line *V Z*, making an angle with the meridian *Z L* equal to the co-latitude of your place; and proceed in all reſpects as in the above horizontal dial for the ſame latitude, reverſing the hours as in Fig. 2. and making the elevation of the gnomon equal to the co-latitude. *South dial.*

Perhaps it may not be unacceptable to explain the method of conſtructing the dialing lines, and ſome others; which is as follows.

With any opening of the compaſſes, as *E A*, according to the intended length of the ſcale, deſcribe the circle *A D C B*, and croſs it at right angles by the diameters *C E A* and *D E B*. Divide the quadrant *A B* firſt into 9 equal parts, and then each part into 10; ſo ſhall the quadrant be divided into 90 equal parts or degrees. Draw the Plate XXII. Fig. 1. *Dialing lines*, how conſtructed.

the right line *AFB* for the chord of this quadrant, and ſetting one foot of the compaſſes in the point *A*, extend the other to the ſeveral diviſions of the quadrant, and transfer theſe diviſions to the line *AFB* by the arcs, 10 10, 20 20, &c. and this will be a line of chords, divided into 90 unequal parts; which, if transferred from the line back again to the quadrant, will divide it equally. It is plain by the figure, that the diſtance from *A* to 60 in the line of chords, is juſt equal to *AE*, the radius of the circle from which that line is made; for if the arc 60 60 be continued, of which *A* is the center, it goes exactly through the center *E* of the arc *AB*.

And therefore, in laying down any number of degrees on a circle, by the line of chords, you muſt firſt open the compaſſes, ſo as to take in juſt 60 degrees upon that line, as from *A* to 60: and then, with that extent, as a radius, deſcribe a circle which will be exactly of the ſame ſize with that from which the line was divided: which done, ſet one foot of the compaſſes in the beginning of the chord line, as at *A*, and extend the other to the number of degrees you want upon the line, which extent, applied to the circle, will include the like number of degrees upon it.

Divide the quadrant *CD* into 90 equal parts, and from each point of diviſion draw right lines as *ikl*, &c. to the line *CE*; all perpendicular to that line, and parallel to *DE*, which will divide *EC* into a line of ſines; and although theſe are ſeldom put among the dialing lines on a ſcale, yet they aſſiſt in drawing the line of latitudes. For, if a ruler be laid upon the point *D*, and over each diviſion in the line of ſines, it will divide the quadrant *CB* into 90 unequal parts,

as

as *B a*, *a b*, &c. ſhewn by the right lines 10 *a*, 20 *b*, 30 *c*, &c. drawn along the edge of the ruler. If the right line *B C* be drawn, ſubtending this quadrant, and the neareſt diſtances *B a*, *B b*, *B c*, &c. be taken in the compaſſes from *B*, and ſet upon this line in the ſame manner as directed for the line of chords, it will make a line of latitudes *B C*, equal in length to the line of chords *A B*, and of an equal number of diviſions, but very unequal as to their lengths.

Draw the right line *D G A*, ſubtending the quadrant *D A*; and parallel to it, draw the right line *r s*, touching the quadrant *D A* at the numeral figure 3. Divide this quadrant into ſix equal parts, as 1, 2, 3, &c. and through theſe points of diviſion draw right lines from the center *E* to the line *r s*, which will divide it at the points where the ſix hours are to be placed, as in the figure. If every ſixth part of the quadrant be ſubdivided into four equal parts, right lines drawn from the center through theſe points of diviſion, and continued to the line *r s*, will divide each hour upon it into quarters.

In Fig. 2. we have the repreſentation of a portable dial, which may be eaſily drawn on a card, and carried in a pocket-book. The lines *a d*, *a b* and *b c* of the gnomon muſt be cut quite through the card; and as the end *a b* of the gnomon is raiſed occaſionally above the plane of the dial, it turns upon the uncut line *c d* as on a hinge. The line dotted *A B* muſt be ſlit quite through the card, and the thread muſt be put through the ſlit, and have a knot tied behind, to keep it from being eaſily drawn out. On the other end of this thread is a ſmall plummet *D*, and on the middle of it a ſmall bead for ſhewing the time of the day.

A dial on a card. Fig. 2.

To

To rectify this dial, ſet the thread in the ſlit right againſt the day of the month, and ſtretch the thread from the day of the month over the angular point where the curve lines meet at XII; then ſhift the bead to that point on the thread, and the dial will be rectified.

To find the hour of the day, raiſe the gnomon (no matter how much or how little) and hold the edge of the dial next the gnomon toward the ſun, ſo as the uppermoſt edge of the ſhadow of the gnomon may juſt cover the *ſhadow-line*; and the bead then playing freely on the face of the dial, by the weight of the plummet, will ſhew the time of the day among the hour-lines, as it is forenoon or after-noon.

To find the time of ſun-riſing and ſetting, move the thread among the hour-lines, until it either covers ſome one of them, or lies parallel betwixt any two; and then it will cut the time of ſun-riſing among the forenoon hours, and of ſun-ſetting among the afternoon hours, on that day of the year for which the thread is ſet in the ſcale of months.

To find the ſun's declination, ſtretch the thread from the day of the month over the angular point at XII, and it will cut the ſun's declination, as it is north or ſouth, for that day, in the arched ſcale of north and ſouth declination.

To find on what days the ſun enters the ſigns: when the bead, as above rectified, moves along any of the curve lines which have the ſigns of the zodiac marked upon them, the ſun enters thoſe ſigns on the days pointed out by the thread in the ſcale of months.

The conſtruction of this dial is very eaſy, eſpecially if the reader compares it all along with

with Fig. 3. as he reads the following explanation of that figure.

Draw the occult line *AB* parallel to the top of the card, and crofs it at right angles with the fix o'clock line *ECD*; then upon *C*, as a center, with the radius *CA*, defcribe the femicircle *AEL*, and divide it into 12 equal parts (beginning at *A*) as *Ar*, *As*, &c. and from thefe points of divifion, draw the hour lines *r*, *s*, *t*, *u*, *v*, *E*, *w*, and *x*, all parallel to the fix o'clock line *EC*. If each part of the femicircle be divided into four equal parts, they will give the half-hour lines and quarters, as in Fig. 2. Draw the right line *ASDo*, making the angle *SAB* equal to the latitude of your place. Upon the center *A* defcribe the arch *RST*, and fet off upon it the arcs *SR* and *ST*, each equal to $23\frac{1}{2}$ degrees, for the fun's greateft declination; and divide them into $23\frac{1}{2}$ equal parts, as in Fig. 2. Through the interfection *D* of the lines *ECD* and *ADo*, draw the right line *FDG* at right angles to *ADo*. Lay a ruler to the points *A* and *R*, and draw the line *ARF* through $23\frac{1}{2}$ degrees of fouth declination in the arc *SR*; and then laying the ruler to the points *A* and *T*, draw the line *ATG* through $23\frac{1}{2}$ degrees of north declination in the arc *ST*: fo fhall the lines *ARF* and *ATG* cut the line *FDG* in the proper length for the fcale of months. Upon the center *D*, with the radius *DF*, defcribe the femicircle *FoG*; and divide it into fix equal parts, *Fm*, *mn*, *no*, &c. and from thefe points of divifion draw the right lines *mh*, *ni*, *pk*, and *ql*, each parallel to *oD*. Then fetting one foot of the compaffes in the point *F*, extend the other to *A*, and defcribe the arc *AzH* for the tropic of ♑: with the fame extent, fetting one foot in *G*, de- Fig. 3.

fcribe

fcribe the arc *AEO* for the tropic of ♑. Next fetting one foot in the point *b*, and extending the other to *A*, defcribe the arc *ACI* for the beginnings of the figns ♒ and ♐; and with the fame extent, fetting one foot in the point *l*, defcribe the arc *AN* for the beginnings of the figns ♊ and ♌. Set one foot in the point *i*, and having extended the other to *A*, defcribe the arc *AK* for the beginnings of the figns ♓ and ♏; and with the fame extent, fet one foot in *k*, and defcribe the arc *AM* for the beginnings of the figns ♉ and ♍. Then, fetting one foot in the point *D*, and extending the other to *A*, defcribe the curve *AL* for the beginnings of ♈ and ♎; and the figns will be finifhed. This done, lay a ruler from the point *A* over the fun's declination in the arch *RST* (found by the following table) for every fifth day of the year; and where the ruler cuts the line *FDG*, make marks; and place the days of the months right againft thefe marks, in the manner fhewn by Fig. 2. Laftly, draw the fhadow line *PQ* parallel to the occult line *AB*; make the gnomon, and fet the hours to their refpective lines, as in Fig. 2. and the dial will be finifhed.

Fig. 4. There are feveral kinds of dials, which are called *univerfal*, becaufe they ferve for all latitudes. Of thefe, the beft one that I know, is Mr. *Pardie*'s, which confifts of three principal parts: the firft whereof is called the *horizontal plane* (*A*) becaufe in the practice it muft be parallel to the horizon. In this plane is fixt an upright pin, which enters into the edge of the fecond part *B D*, called the *meridional plane*; which is made of two pieces, the loweft whereof (*B*) is called the *quadrant*, becaufe it contains a quarter of a circle, divided into 90 degrees; and it

An *univerfal dial.*

it is only into this part, near *B*, that the pin enters. The other piece is a *semicircle* (*D*) adjusted to the quadrant, and turning in it by a groove, for raising or depressing the diameter (*EF*) of the semicircle, which diameter is called the *axis* of the instrument. The third piece is a *circle* (*G*) divided on both sides into 24 equal parts, which are the hours. This circle is put upon the meridional plane so, that the axis (*EF*) may be perpendicular to the circle; and the point *C* be the common center of the circle, semicircle, and quadrant. The straight edge of the semicircle is chamfered on both sides to a sharp edge, which passes through the center of the circle. On one side of the chamfered part, the first six months of the year are laid down, according to the sun's declination for their respective days, and on the other side the last six months. And against the days on which the sun enters the signs, there are straight lines drawn upon the semicircle, with the characters of the signs marked upon them. There is a black line drawn along the middle of the upright edge of the quadrant, over which hangs a thread (*H*) with its plummet (*I*) for levelling the instrument. *N. B.* From the 22d of September to the 20th of March, the upper surface of the circle must touch both the center *C* of the semicircle, and the line of ♈ and ♎; and from the 20th of March to the 22d of September, the lower surface of the circle must touch that center and line.

To find the time of the day by this dial. Having set it on a level place in sun-shine, and adjusted it by the levelling screws *k* and *l*, until the plumb line hangs over the back line upon the edge of the quadrant, and parallel to the said edge; move the semicircle in the quadrant, until the line of ♈ and ♎ (where the circle touches) comes to the latitude of your place in the quadrant: then, turn the whole meridional plane *B D*, with its circle *G*, upon the horizontal plane *A*, until the edge of the ſhadow of the circle falls preciſely on the day of the month in the ſemicircle; and then, the meridional plane will be due north and ſouth, the axis *E F* will be parallel to the axis of the world, and will caſt a ſhadow upon the true time of the day, among the hours on the circle.

N. B. As, when the inſtrument is thus rectified, the quadrant and ſemicircle are in the plane of the meridian, ſo the circle is then in the plane of the equinoctial. Therefore, as the ſun is above the equinoctial in ſummer (in northern latitudes) and below it in winter; the axis of the ſemicircle will caſt a ſhadow on the hour of the day, on the upper ſurface of the circle, from the 20th of March to the 22d of September: and from the 22d of September, to the 20th of March, the hour of the day will be determined by the ſhadow of the ſemicircle, upon the lower ſurface of the circle. In the former caſe, the ſhadow of the circle falls upon the day of the month, on the lower part of the diameter of the ſemicircle; and in the latter caſe on the upper part.

The method of laying down the months and ſigns upon the ſemicircle, is as follows. Draw the right line *A C B*, equal the diameter of the ſemicircle *A D B*, and croſs it in the middle at
Fig. 5. right angles with the line *E C D*, equal in length to *A D B*; then *E C* will be the radius of the circle *F C G*, which is the ſame as that of the ſemicircle. Upon *E*, as a center, deſcribe the circle *F C G*, on which ſet off the arcs *C h* and *C i*, each equal to $23\frac{1}{2}$ degrees, and divide them accordingly into that number for the ſun's declination

clination. Then, laying the edge of a ruler over the center *E*, and alſo over the ſun's declination for every fifth day * of each month (as in the card dial) mark the points on the diameter *AB* of the ſemicircle from *a* to *g*, which are cut by the ruler; and *there* place the days of the months accordingly, anſwering the ſun's declination. This done, ſetting one foot of the compaſſes in *C*, and extending the other to *a* or *g*, deſcribe the ſemicircle *a b c d e f g*; which divide into ſix equal parts, and through the points of diviſion draw right lines, parallel to *CD*, for the beginning of the ſines (of which one half are on one ſide of the ſemicircle, and the other half on the other ſide) and ſet the characters of the ſigns to their proper lines, as in the figure.

The following table ſhews the ſun's place and declination, in degrees and minutes, at the noon of every day of the ſecond year after leap-year; which is a mean between thoſe of leap-year itſelf, and the firſt and third years after. It is uſeful for inſcribing the months and their days on ſun-dials; and alſo for finding the latitudes of places, according to the methods preſcribed after the table.

* The intermediate days may be drawn in by hand, if the ſpaces be large enough to contain them.

A Table shewing the sun's place and declination.

	January.					February.			
Days.	Sun's Pl.		Sun's Dec.		Days.	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	11 ♑	5	23 S	1	1	12 ♒	38	17 S	2
2	12	6	22	55	2	13	39	16	45
3	13	8	22	49	3	14	40	16	27
4	14	9	22	43	4	15	41	16	10
5	15	10	22	37	5	16	41	15	51
6	16	11	22	29	6	17	42	15	33
7	17	12	22	22	7	18	43	15	14
8	18	13	22	14	8	19	43	14	55
9	19	14	22	5	9	20	44	14	36
10	20	16	21	56	10	21	45	14	17
11	21	17	21	47	11	22	45	13	57
12	22	18	21	37	12	23	46	13	37
13	23	19	21	27	13	24	46	13	17
14	24	20	21	17	14	25	47	12	57
15	25	21	21	6	15	26	47	12	36
16	26	22	20	54	16	27	48	12	15
17	27	24	20	43	17	28	48	11	54
18	28	25	20	30	18	29	48	11	33
19	29	26	20	18	19	0 ♓	49	11	12
20	0 ♒	27	20	5	20	1	49	10	51
21	1	28	19	52	21	2	50	10	29
22	2	29	19	38	22	3	50	10	7
23	3	30	19	24	23	4	50	9	45
24	4	31	19	10	24	5	51	9	23
25	5	32	18	55	25	6	51	9	0
26	6	33	18	40	26	7	51	8	38
27	7	34	18	24	27	8	51	8	16
28	8	35	18	9	28	9	51	7	53
29	9	35	17	53					
30	10	36	17	36					
31	11	37	17	19					

In these tables N signifies north declination, and S south.

A Table

A Table shewing the sun's place and declination.

March.					April.				
Days.	Sun's Pl.		Sun's Dec.		Days.	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	10♓	52	7 S	30	1	11 ♈	38	4 N	36
2	11	52	7	7	2	12	37	4	59
3	12	52	6	44	3	13	36	5	22
4	13	52	6	21	4	14	35	5	45
5	14	52	5	58	5	15	34	6	8
6	15	52	5	35	6	16	33	6	31
7	16	51	5	12	7	17	31	6	53
8	17	51	4	48	8	18	30	7	16
9	18	51	4	25	9	19	29	7	38
10	19	51	4	2	10	20	28	8	0
11	20	51	3	38	11	21	27	8	22
12	21	50	3	14	12	22	25	8	44
13	22	50	2	51	13	23	24	9	6
14	23	50	2	27	14	24	23	9	28
15	24	49	2	4	15	25	21	9	49
16	25	49	1	40	16	26	20	10	11
17	26	48	1	16	17	27	18	10	32
18	27	48	0	53	18	28	17	10	53
19	28	48	0	29	19	29	15	11	14
20	29	47	0	5	20	0 ♉	14	11	34
21	0	47	0 N	19	21	1	12	11	55
22	1	46	0	42	22	2	11	12	15
23	2	45	1	6	23	3	9	12	35
24	3	45	1	29	24	4	7	12	55
25	4	44	1	53	25	5	6	13	14
26	5	43	2	17	26	6	4	13	34
27	6	42	2	40	27	7	2	13	53
28	7	42	3	3	28	8	0	14	12
29	8	41	3	27	29	8	59	14	30
30	9	40	3	50	30	9	57	14	49
31	10	39	4	13					

A Table shewing the sun's place and declination.

May.					June.				
Days.	Sun's Pl.		Sun's Dec		Days.	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	10 ♉	55	15N	7	1	10 ♊	44	22N	5
2	11	53	15	25	2	11	41	22	13
3	12	51	15	43	3	12	39	22	21
4	13	49	16	0	4	13	36	22	28
5	14	47	16	18	5	14	34	22	35
6	15	45	16	35	6	15	31	22	41
7	16	43	16	51	7	16	28	22	47
8	17	41	17	8	8	17	26	22	53
9	18	39	17	24	9	18	23	22	58
10	19	36	17	40	10	19	20	23	3
11	20	34	17	55	11	20	18	23	7
12	21	32	18	10	12	21	15	23	11
13	22	30	18	25	13	22	12	23	15
14	23	28	18	40	14	23	9	23	18
15	24	25	18	54	15	24	7	23	20
16	25	23	19	8	16	25	4	23	22
17	26	21	19	22	17	26	1	23	24
18	27	19	19	35	18	26	58	23	26
19	28	16	19	48	19	27	56	23	27
20	29	14	20	1	20	28	53	23	28
21	0 ♊	11	20	13	21	29	50	23	28
22	1	9	20	25	22	0 ♋	47	23	28
23	2	7	20	37	23	1	45	23	28
24	3	4	20	48	24	2	42	23	27
25	4	2	20	59	25	3	39	23	26
26	4	59	21	10	26	4	36	23	24
27	5	57	21	20	27	5	33	23	21
28	6	54	21	30	28	6	31	23	19
29	7	52	21	39	29	7	28	23	16
30	8	49	21	49	30	8	25	23	12
31	9	47	21	57					

A Table shewing the sun's place and declination.

	July.					August.			
Days.	Sun's Pl.		Sun's Dec.		Days.	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	9 ♋	22	23N	8	1	8 ♌	58	18N	2
2	10	19	23	4	2	9	55	17	47
3	11	16	23	0	3	10	53	17	32
4	12	14	22	55	4	11	50	17	16
5	13	11	22	49	5	12	48	17	0
6	14	8	22	43	6	13	45	16	43
7	15	5	22	37	7	14	43	16	26
8	16	0	22	30	8	15	41	16	9
9	17	2	22	23	9	16	38	15	52
10	17	57	22	16	10	17	36	15	25
11	18	54	22	8	11	18	33	15	17
12	19	51	22	0	12	19	31	14	59
13	20	49	21	52	13	20	29	14	41
14	21	46	21	43	14	21	26	14	23
15	22	43	21	33	15	22	24	14	4
16	23	40	21	22	16	23	22	13	45
17	24	38	21	14	17	24	20	13	26
18	25	35	21	3	18	25	17	13	7
19	26	32	20	52	19	26	15	12	47
20	27	29	20	41	20	27	13	12	27
21	28	27	20	30	21	28	11	12	7
22	29	24	20	18	22	29	9	11	47
23	0 ♌	21	20	6	23	0 ♍	7	11	27
24	1	19	19	54	24	1	5	11	6
25	2	16	19	41	25	2	3	10	46
26	3	13	19	28	26	3	1	10	25
27	4	11	19	14	27	3	59	10	4
28	5	8	19	1	28	4	57	9	43
29	6	6	18	46	29	5	55	9	21
30	7	3	18	32	30	6	53	9	0
31	8	0	18	17	31	7	51	8	38

A Table shewing the sun's place and declination.

September.					October.				
Days.	Sun's Pl.		Sun's Dec.		Days.	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	8 ♍	49	8 N	16	1	8 ♎	8	3 S	14
2	9	47	7	55	2	9	7	3	37
3	10	46	7	33	3	10	7	4	1
4	11	44	7	10	4	11	6	4	24
5	12	42	6	48	5	12	5	4	47
6	13	40	6	26	6	13	4	5	10
7	14	39	6	3	7	14	4	5	33
8	15	37	5	41	8	15	3	5	56
9	16	35	5	18	9	16	3	6	19
10	17	34	4	55	10	17	2	6	42
11	18	32	4	32	11	18	1	7	5
12	19	31	4	9	12	19	1	7	27
13	20	29	3	46	13	20	0	7	50
14	21	28	3	23	14	21	0	8	12
15	22	26	3	0	15	22	0	8	35
16	23	25	2	37	16	23	0	8	57
17	24	24	2	14	17	23	59	9	19
18	25	22	1	50	18	24	59	9	41
19	26	21	1	27	19	25	58	10	3
20	27	20	1	4	20	26	58	10	24
21	28	19	0	40	21	27	58	10	46
22	29	17	0	17	22	28	58	11	7
23	0 ♎	16	0 S	6	23	29	58	11	28
24	1	15	0	30	24	0 ♏	58	11	49
25	2	14	0	53	25	1	58	12	10
26	3	13	1	17	26	2	58	12	31
27	4	12	1	40	27	3	58	12	51
28	5	11	2	4	28	4	58	13	12
29	6	10	2	27	29	5	58	13	32
30	7	9	2	50	30	6	58	13	51
					31	7	58	14	11

A Table ſhewing the ſun's place and declination.

November.					December.				
Days.	Sun's Pl.		Sun's Dec.		Days.	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	8 ♏	58	14 S	30	1	9 ♐	16	21 S	52
2	9	58	14	50	2	10	17	22	1
3	10	59	15	8	3	11	18	22	10
4	11	59	15	27	4	12	19	22	18
5	12	59	15	45	5	13	20	22	26
6	13	59	16	3	6	14	21	22	33
7	15	0	16	21	7	15	22	22	40
8	16	0	16	39	8	16	23	22	46
9	17	0	16	56	9	17	24	22	52
10	18	1	17	13	10	18	25	22	58
11	19	1	17	30	11	19	26	23	3
12	20	2	17	46	12	20	27	23	8
13	21	2	18	2	13	21	28	23	12
14	22	3	18	18	14	22	29	23	15
15	23	4	18	34	15	23	30	23	18
16	24	4	18	49	16	24	31	23	21
17	25	5	19	4	17	25	33	23	24
18	26	5	19	18	18	26	34	23	26
19	27	6	19	32	19	27	35	23	27
20	28	7	19	46	20	28	36	23	28
21	29	7	19	59	21	29	37	23	28
22	0 ♐	8	20	12	22	0 ♑	38	23	28
23	1	9	20	25	23	1	40	23	28
24	2	10	20	37	24	2	41	23	27
25	3	11	20	49	25	3	42	23	25
26	4	11	21	1	26	4	43	23	23
27	5	12	21	12	27	5	44	23	21
28	6	13	21	23	28	6	46	23	18
29	7	14	21	33	29	7	47	23	15
30	8	15	21	43	30	8	48	23	11
					31	9	49	23	6

To find the latitude of any place by obſervation.

The latitude of any place is equal to the elevation of the pole above the horizon of that place. Therefore it is plain, that if a ſtar was fixt in the pole, there would be nothing required to find the latitude, but to take the altitude of that ſtar with a good inſtrument. But although there is no ſtar in the pole, yet the latitude may be found by taking the greateſt and leaſt altitude of any ſtar that never ſets: for if half the difference between theſe altitudes be added to the leaſt altitude, or ſubtracted from the greateſt, the ſum or remainder will be equal to the altitude of the pole at the place of obſervation.

But becauſe the length of the night muſt be more than 12 hours, in order to have two ſuch obſervations; the ſun's meridian altitude and declination are generally made uſe of for finding the latitude, by means of its complement, which is equal to the elevation of the equinoctial above the horizon; and if this complement be ſubtracted from 90 degrees, the remainder will be the latitude, concerning which, I think, the following rules take in all the various caſes.

1. If the ſun has north declination, and is on the meridian, and to the ſouth of your place, ſubtract the declination from the meridian altitude (taken by a good quadrant) and the remainder will be the height of the equinoctial or complement of the latitude north.

EXAM-

EXAMPLE.

Suppose	The sun's meridian altitude	42°	20′	South
	And his declination, subt.	10	15	North
	Rem. the complement of the lat.	32	5	
	Which subtract from — —	90	0	
	And the remainder is the latitude	57	55	North

2. If the sun has south declination, and is southward of your place at noon, add the declination to the meridian altitude; the sum, if less than 90 degrees, is the complement of the latitude north: but if the sum exceeds 90 degrees, the latitude is south; and if 90 be taken from that sum, the remainder will be the latitude.

EXAMPLES.

The sun's meridian altitude —	65°	10′	South
The sun's declination, add —	15	30	South
Complement of the latitude —	80	40	
Subtract from — —	90	0	
Remains the latitude —	9	20	North

The sun's meridian altitude —	80°	40′	South
The sun's declination, add —	20	10	South
The sum is — — —	100	50	
From which subtract — —	90	0	
Remains the latitude — —	10	50	South

3. If

3. If the ſun has north declination, and is on the meridian north of your place, add the declination to the north meridian altitude; the ſum, if leſs than 90 degrees, is the complement of the latitude ſouth: but if the ſum is more than 90 degrees, ſubtract 90 from it, and the remainder is the latitude north.

EXAMPLES.

Sun's meridian altitude	—	60°	30′	North
Sun's declination, add	—	20	10	North
Complement of the latitude	—	80	40	
Subtract from	— —	90	0	
Remains the latitude	—	9	20	South

Sun's meridian altitude	—	70°	20′	North
Sun's declination, add	—	23	20	North
The ſum is	— —	93	40	
From which ſubtract	—	90	0	
Remains the latitude	— —	3	40	North

4. If the ſun has ſouth declination, and is north of your place at noon, ſubtract the declination from the north meridian altitude, and the remainder is the complement of the latitude ſouth.

EXAM-

EXAMPLE.

Sun's meridian altitude —	52°	30′	North
Sun's declination, ſubtract	20	10	South
Complement of the latitude	32	20	
Subtract this from — —	90	0	
And the remainder is the latitude	57	40	South

5. If the ſun has no declination, and is ſouth of your place at noon, the meridian altitude is the complement of the latitude north: but if the ſun be then north of your place, his meridian altitude is the complement of the latitude ſouth.

EXAMPLES.

Sun's meridian altitude —	38°	30′	South
Subtract from — — —	90	0	
Remains the latitude —	51	30	North

Sun's meridian altitude —	38°	30′	North
Subtract from — —	90	0	
Remains the latitude —	51	30	South

6. If you obſerve the ſun beneath the pole, ſubtract his declination from 90 degrees, and add the remainder to his altitude; and the ſum is the latitude.

EXAM-

EXAMPLE.

Sun's declination — —	20°	30′	
Subtract from — —	90	0	
Remains — — —	69	30	add
Sun's altitude below the pole	10	20	
The sum is the latitude —	79	50	

Which is north or south, according as the sun's declination is north or south: for when the sun has south declination, he is never seen below the north pole; nor is he ever seen below the south pole, when his declination is north.

7. If the sun be in the zenith at noon, and at the same time has no declination, you are then under the equinoctial, and so have no latitude.

8. If the sun be in the zenith at noon, and has declination, the declination is equal to the latitude, north or south. These two cases are so plain, that they require no examples.

LECT. XI.

Of Dialing.

HAVING shewn in the preceding Lecture how to make sun-dials by the assistance of a good globe, or of a dialing scale, we shall now proceed to the method of constructing dials arithmetically; which will be more agreeable to those who have learnt the elements of trigo-

trigonometry, becauſe globes and ſcales can never be ſo accurate as the logarithms, in finding the angular diſtances of the hours. Yet, as a globe may be found exact enough for ſome other requiſites in dialing, we ſhall take it in occaſionally.

The conſtruction of ſun-dials on all planes whatever, may be included in one general rule: intelligible, if that of a horizontal dial for any given latitude be well underſtood. For there is no plane, however obliquely ſituated with reſpect to any given place, but what is parallel to the horizon of ſome other place; and therefore, if we can find that other place by a problem on the terreſtrial globe, or by a trigonometrical calculation, and conſtruct a horizontal dial for it; that dial, applied to the plane where it is to ſerve, will be a true dial for that place.—Thus, an erect direct ſouth dial in $51\frac{1}{2}$ degrees north latitude, would be a horizontal dial on the ſame meridian, 90 degrees ſouthward of $51\frac{1}{2}$ degrees north latitude; which falls in with $38\frac{1}{2}$ degrees of ſouth latitude; but if the upright plane declines from facing the ſouth at the given place, it would ſtill be a horizontal plane 90 degrees from that place; but for a different longitude: which would alter the reckoning of the hours accordingly.

CASE I.

1. Let us ſuppoſe that an upright plane at London declines 36 degrees weſtward from facing the ſouth; and that it is required to find a place on the globe, to whoſe horizon the ſaid plane is parallel; and alſo the difference of longitude between London and that place.

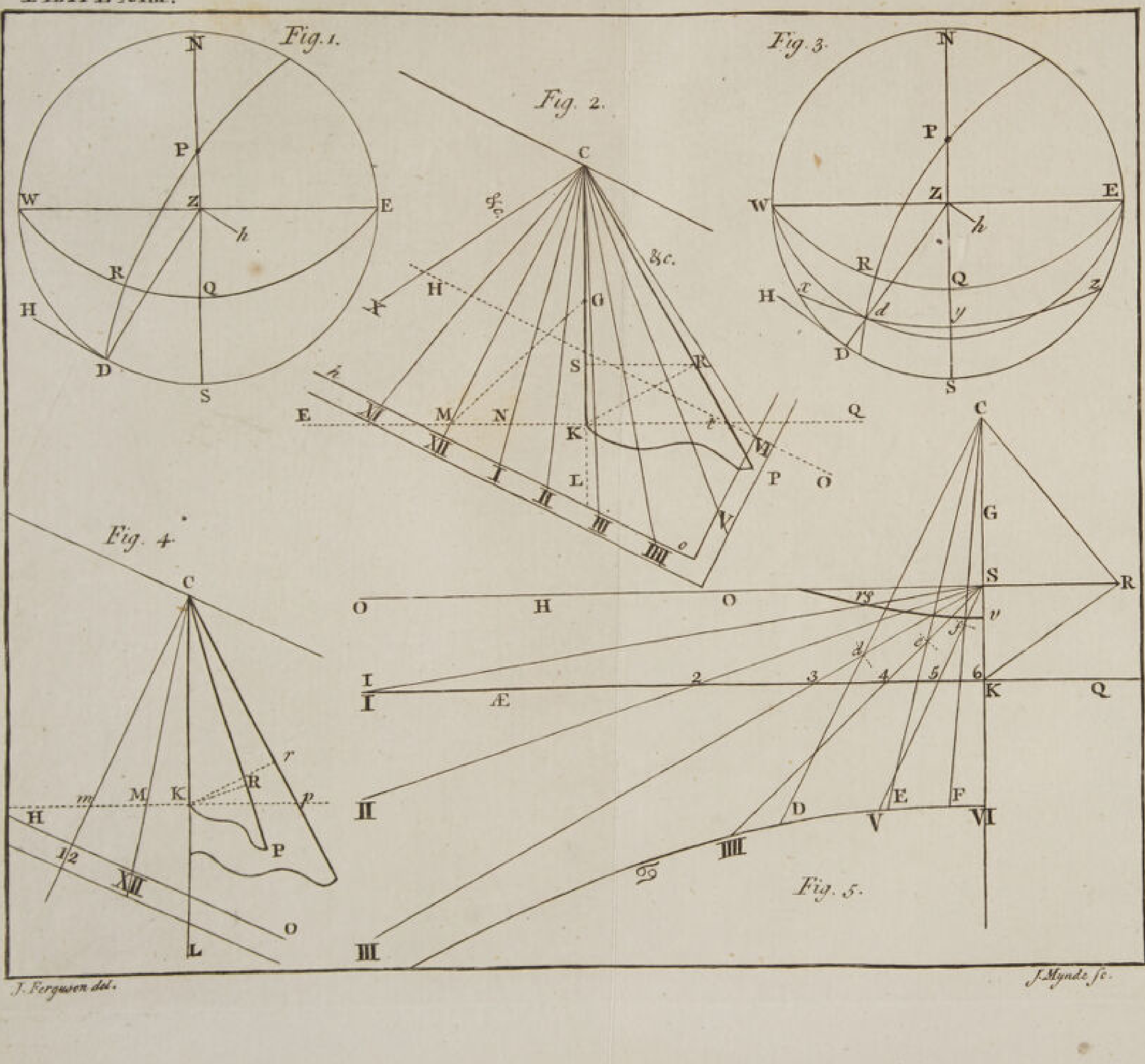
Rectify

Rectify the globe to the latitude of London, and bring London to the zenith under the braſs meridian, then that point of the globe which lies in the horizon at the given degree of declination (counted weſtward from the ſouth point of the horizon) is the place at which the above-mentioned plane would be horizontal.—Now, to find the latitude and longitude of that place, keep your eye upon the place, and turn the globe eaſtward, until it comes under the graduated edge of the braſs meridian; then, the degree of the braſs meridian that ſtands directly over the place, is its latitude; and the number of degrees in the equator, which are intercepted between the meridian of London and the braſs meridian, is the place's difference of longitude.

Thus, as the latitude of London is $51\frac{1}{2}$ degrees north, and the declination of the place is 36 degrees weſt; I elevate the north pole $51\frac{1}{2}$ degrees above the horizon, and turn the globe until London comes to the zenith, or under the graduated edge of the meridian; then, I count 36 degrees on the horizon weſtward from the ſouth point, and make a mark on that place of the globe over which the reckoning ends, and bringing the mark under the graduated edge of the braſs meridian, I find it to be under $30\frac{1}{4}$ degrees in ſouth latitude: keeping it there, I count in the equator the number of degrees between the meridian of London and the braſen meridian (which now becomes the meridian of the required place) and find it to be $42\frac{1}{4}$. Therefore an upright plane at London, declining 36 degrees weſtward from the ſouth, would be a horizontal plane at that place, whoſe latitude is $30\frac{1}{4}$ degrees ſouth of the equator, and longitude $42\frac{3}{4}$ degrees weſt of the meridian of London.

Which

PLATE XXIII.



J. Ferguson del.

J. Mynde sc.

Which difference of longitude being converted into time, is 2 hours 51 minutes.

The vertical dial declining weſtward 36 degrees at London, is therefore to be drawn in all reſpects as a horizontal dial for ſouth latitude 30¼ degrees; ſave only, that the reckoning of the hours is to anticipate the reckoning on the horizontal dial, by 2 hours 51 minutes: for ſo much ſooner will the ſun come to the meridian of London, than to the meridian of any place whoſe longitude is 42¾ degrees weſt from London.

2. But to be more exact than the globe will ſhew us, we ſhall uſe a little trigonometry.

Let *N E S W* be the horizon of London, whoſe zenith is *Z*, and *P* the north pole of the ſphere; and let *Z h* be the poſition of a vertical plane at *Z*, declining weſtward from *S* (the ſouth) by an angle of 36 degrees; on which plane an erect dial for London at *Z* is to be deſcribed. Make the ſemidiameter *Z D* perpendicular to *Z h*, and it will cut the horizon in *D*, 36 degrees weſt of the ſouth *S*. Then, a plane in the tangent *H D*, touching the ſphere in *D*, will be parallel to the plane *Z h*; and the axis of the ſphere will be equally inclined to both theſe planes. Plate XXIII. Fig. 1.

Let *W Q E* be the equinoctial, whoſe elevation above the horizon of *Z* (London) is 38½ degrees; and *P R D* be the meridian of the place *D*, cutting the equinoctial in *R*. Then, it is evident, that the arc *R D* is the latitude of the place *D* (where the plane *Z h* would be horizontal) and the arc *R Q* is the difference of longitude of the planes *Z h* and *D H*.

In the ſpherical triangle *W D R*, the arc *W D* is given, for it is the complement of the plane's decli-

declination from *S* the ſouth; which complement is 54° (viz. 90°—36°): the angle at *R*, in which the meridian of the place *D* cuts the equator, is a right angle; and the angle *R W D* meaſures the elevation of the equinoctial above the horizon of *Z*, namely 38½ degrees. Say therefore, as radius is to the co-ſine of the plane's declination from the ſouth, ſo is the co-ſine of the latitude of *Z* to the ſine of *R D* the latitude of *D*: which is of a different denomination from the latitude of *Z*, becauſe *Z* and *D* are on different ſides of the equator.

As radius	— — — —	10.00000
To co-ſine	36° 0′=*R* ♒	9.90796
So co-ſine	51° 30′=♒ *Z*	9.79415
To ſine	30° 14′=*D R*	(9.70211) =

the latitude of *D*, whoſe horizon is parallel to the vertical plane *Z b* at *Z*.

N. B. When radius is made the firſt term, it may be omitted, and then, by ſubtracting it mentally from the ſum of the other two, the operation will be ſhortened. Thus, in the preſent caſe,

To the logarithmic ſine of *W R*=*	54° 0′	9.90796
Add the logarithmic ſine of *R D*=†	38° 30′	9.79415
Their ſum—radius	— — —	9.70211

gives the ſame ſolution as above. And we ſhall keep to this method in the following part of the work.

* The co-ſine of 36° 0′, or of *R* ♒.
† The co-ſine of 51° 30′, or of ♒ *Z*.

To

To find the difference of longitude of the places *D* and *Z*, ſay, as radius is to the co-ſine of 38½ degrees, the height of the equinoctial at *Z*, ſo is the co-tangent of 36 degrees, the plane's declination, to the co-tangent of the difference of longitudes. Thus,

To the logarithmic ſine of *	51° 30′	9.89354
Add the logarithmic tang. of †	54° 0′	10.13874
Their ſum—radius - - - -		10.03228

is the neareſt tangent of 47° 8′ = *W R*; which is the co-tangent of 42° 52′ = *R Q*, the difference of longitude ſought. Which difference, being reduced to time, is 2 hours 51½ minutes.

3. And thus having found the exact latitude and longitude of the place *D*, to whoſe horizon the vertical plane at *Z* is parallel, we ſhall proceed to the conſtruction of a horizontal dial for the place *D*, whoſe latitude is 30° 14′ ſouth; but anticipating the time at *D* by 2 hours 51 minutes (neglecting the ½ minute in practice) becauſe *D* is ſo far weſtward in longitude from the meridian of London; and this will be a true vertical dial at London, declining weſtward 36 degrees.

Aſſume any right line *C S L* for the ſubſtile of the dial, and make the angle *K C P* equal to the latitude of the place (viz. 30° 14′) to whoſe horizon the plane of the dial is parallel; then *C R P* will be the axis of the ſtile, or edge that caſts the ſhadow on the hours of the day, in the dial. This done, draw the contingent line *E Q*, cutting the ſubſtilar line at right angles in *K*; Fig. 2.

* The co-ſine of 38° 30′, or of *W D R*.
† The co-tangent of 36°, or of *D W*.

A a and

and from *K* make *K R* perpendicular to the axis *C R P*. Then *K G* (=*K R*) being made radius, that is, equal to the chord of 60° or tangent of 45° on a good ſector, take 42° 52′ (the difference of longitude of the places *Z* and *D*) from the tangents, and having ſet it from *K* to *M*, draw *C M* for the hour-line of XII. Take *K N* equal to the tangent of an angle leſs by 15 degrees than *K M*; that is, the tangent 27° 52′; and through the point *N* draw *C N* for the hour-line of I. The tangent of 12° 52′ (which is 15° leſs than 27° 52′) ſet off the ſame way, will give a point between *K* and *N*, through which the hour-line of II is to be drawn. The tangent of 2° 8′ (the difference between 45° and 42° 52′) placed on the other ſide of *C L*, will determine the point through which the hour-line of III is to be drawn: to which 2° 8′, if the tangent of 15° be added, it will make 17° 8′; and this ſet off from *K* toward *Q* on the line *E Q*, will give the point for the hour-line of IV: and ſo of the reſt.—The forenoon hour-lines are drawn the ſame way, by the continual addition of the tangents 15°, 30°, 45°, &c. to 42° 52′ (=the tangent of *K M*) for the hours of XI, X, IX, &c. as far as neceſſary; that is, until there be five hours on each ſide of the ſubſtile. The ſixth hour, accounted from that hour or part of the hour on which the ſubſtile falls, will be always in a line perpendicular to the ſubſtile, and drawn through the center *C*.

4. In all erect dials, *C M*, the hour-line of XII, is perpendicular to the horizon of the place for which the dial is to ſerve: for that line is the interſection of a vertical plane with the plane of the meridian of the place, both which are perpendicular to the plane of the horizon:

horizon: and any line HO, or ho, perpendicular to CM, will be a horizontal line on the plane of the dial, along which line the hours may be numbered: and CM being ſet perpendicular to the horizon, the dial will have its true poſition.

5. If the plane of the dial had declined by an equal angle toward the eaſt, its deſcription would have differed only in this, that the hour-line of XII would have fallen on the other ſide of the ſubſtile CL, and the line HO would have a ſubcontrary poſition to what it has in this figure.

6. And theſe two dials, with the upper points of their ſtiles turned toward the north pole, will ſerve for the other two planes parallel to them; the one declining from the north toward the eaſt, and the other from the north toward the weſt, by the ſame quantity of angle. The like holds true of all dials in general, whatever be their declination and obliquity of their planes to the horizon.

CASE II.

7. If the plane of the dial not only *declines*, Fig. 3.
but alſo *reclines*, or *inclines*. Suppoſe its declination from fronting the ſouth S be equal to the arc SD on the horizon; and its reclination be equal to the arc Dd of the vertical circle DZ: then it is plain, that if the quadrant of altitude ZdD, on the globe, cuts the point D in the horizon, and the reclination is counted upon the quadrant from D to d; the interſection of the hour-circle PRd, with the equinoctial WQE, will determine Rd, the latitude of the place d,

 whoſe

whose horizon is parallel to the given plane *Z b* at *Z*; and *R Q* will be the difference in longitude of the planes at *d* and *Z*.

Trigonometrically thus: let a great circle pass through the three points *W*, *d*, *E*; and in the triangle *W D d*, right-angled at *D*, the sides *W D* and *D d* are given; and thence the angle *D W d* is found, and so is the hypothenuse *W d*. Again, the difference, or the sum, of *D W d* and *D W R*, the elevation of the equinoctial above the horizon of *Z*, gives the angle *d W R*; and the hypothenuse of the triangle *W R d* was just now found; whence the sides *R d* and *W R* are found, the former being the latitude of the place *d*, and the latter the complement of *R Q*, the difference of longitude sought.

Thus, if the latitude of the place *Z* be 52° 10′ north; the declination *S D* of the plane *Z b* (which would be horizontal at *d*) be 36°, and the reclination be 15°, or equal to the arc *D d*; the south latitude of the place *d*, that is, the arc *R d*, will be 15° 9′; and *R Q*, the difference of the longitude, 36° 2′. From these data, therefore, let the dial (Fig. 4.) be described, as in the former example.

8. Only it is to be observed, that in the reclining or inclining dials, the horizontal line will not stand at right angles to the hour-line of XII, as in erect dials; but its position may be found as follows.

Fig. 4. To the common substilar line *C K L*, on which the dial for the place *d* was described, draw the dial *C r p m* 12 for the place *D*, whose declination is the same as that of *d*, viz. the arc *S D*; and *H O*, perpendicular to *C m*, the hour-line of XII on this dial, will be a horizontal line on the dial *C P R M* XII. For the declination of

of both dials being the ſame, the horizontal line remains parallel to itſelf, while the erect poſition of one dial is reclined or inclined with reſpect to the poſition of the other.

Or, the poſition of the dial may be found by applying it to its plane, ſo as to mark the true hour of the day by the ſun, as ſhewn by another dial; or by a clock, regulated by a true meridian line and equation table.

9. There are ſeveral other things requiſite in the practice of dialing; the chief of which I ſhall give in the form of arithmetical rules, ſimple and eaſy to thoſe who have learnt the elements of trigonometry. For in practical arts of this kind, arithmetic ſhould be uſed as far as it can go; and ſcales never truſted to, except in the final conſtruction, where they are abſolutely neceſſary in laying down the calculated hour-diſtances on the plane of the dial. And although the inimitable artiſts of this metropolis have no occaſion for ſuch inſtructions, yet they may be of ſome uſe to ſtudents, and to private gentlemen who amuſe themſelves this way.

RULE I.

To find the angles which the hour-lines on any dial make with the ſubſtile.

To the logarithmic ſine of the given latitude, or of the ſtile's elevation above the plane of the dial, add the logarithmic tangent of the hour diſtance * from the meridian, or from the ſub-

* That is, of 15, 30, 45, 60, 75°, for the hours of I, II, III, IV, V in the afternoon: and XI, X, IX, VIII, VII in the forenoon.

fubftile †; and the fum *minus* radius will be the logarithmic tangent of the angle fought.

For, in Fig. 2. KC is to KM in the ratio compounded of the ratio of KC to KG ($=KR$) and of KG to KM; which, making CK the radius, 10,000000, or 10,0000, or 10 or 1, are the ratio of 10,000000, or of 10,0000, or of 10, or of 1, to $KG \times KM$.

Thus, in a horizontal dial, for latitude 51° 30′, to find the angular diftance of XI in the forenoon, or I in the afternoon, from XII.

To the logarithmic fine of 51° 30′	9.89354‡
Add the logarithmic tang. of 15° 0′	9.42805

The fum—radius is - - - 9.32159 = the logarithmic tangent of 11° 50′, or of the angle which the hour-line of XI or I makes with the hour of XII.

And by computing in this manner, with the fine of the latitude, and the tangents of 30, 45, 60, and 75°, for the hours of II, III, IV, and V in the afternoon; or of X, IX, VIII, and VII in the forenoon; you will find their angular diftances from XII to be 24° 18′, 38° 3′, 53° 35′, and 71° 6′: which are all that there is occafion to compute for.——And thefe diftances may be fet off from XII by a line of chords; or rather, by taking 1000 from a fcale of equal parts, and fetting that extent as a radius from C to XII: and then, taking 209 of

† In all horizontal dials, and erect north or fouth dials, the fubftile and meridian are the fame; but in all declining dials, the fubftile line makes an angle with the meridian.

‡ In which cafe, the radius CK is fuppofed to be divided into 1000000 equal parts.

the

the ſame parts (which, in the tables, are the natural tangent of 11° 50′) and ſetting them from XII to XI and to I, on the line *b o*, which is perpendicular to *C* XII: and ſo for the reſt of the hour-lines, which in the table of natural tangents, againſt the above diſtances, are 451, 782, 1355, and 2920, of ſuch equal parts from XII, as the radius *C* XII contains 1000. And laſtly, ſet off 1257 (the natural tangent of 51° 30′) for the angle of the ſtile's height, which is equal to the latitude of the place. Fig. 2.

The reaſon why I prefer the uſe of the tabular numbers, and of a ſcale decimally divided, to that of the line of chords, is becauſe there is the leaſt chance of miſtake and error in this way; and likewiſe, becauſe in ſome caſes it gives us the advantage of a *nonius'* diviſion.

In the univerſal ring-dial, for inſtance, the diviſions on the axis are the tangents of the angles, of the ſun's declination placed on either ſide of the center. But inſtead of laying them down from a line of tangents, I would make a ſcale of equal parts, whereof 1000 ſhould anſwer exactly to the length of the ſemi-axis, from the center to the inſide of the equinoctial ring; and then lay down 434 of theſe parts toward each end from the center, which would limit all the diviſions on the axis, becauſe 434 are the natural tangent of 23° 29′. And thus by a *nonius* affixed to the ſliding piece, and taking the ſun's declination from an Ephemeris, and the tangent of that declination from the table of natural tangents, the ſlider might be always ſet true to within two minutes of a degree.

And this ſcale of 434 equal parts might be placed right againſt the 23½ degrees of the ſun's declination, on the axis, inſtead of the ſun's

 place,

place, which is there of very little uſe. For then, the ſlider might be ſet in the uſual way, to the day of the month, for common uſe; but to the natural tangent of the declination, when great accuracy is required.

The like may be done wherever a ſcale of ſines or tangents is required on any inſtrument.

RULE II.

The latitude of the place, the ſun's declination, and his hour diſtance from the meridian, being given; to find (1.) *his altitude*; (2.) *his azimuth.*

Fig. 3. 1. Let d be the ſun's place, dR, his declination: and in the triangle PZd, Pd the ſum, or the difference, of dR, and the quadrant PR being given by the ſuppoſition, as alſo the complement of the latitude PZ, and the angle dPZ, which meaſures the horary diſtance of d from the meridian; we ſhall (by Caſe 4. of *Keill*'s Oblique ſpheric Trigonometry) find the baſe Zd, which is the ſun's diſtance from the zenith, or the complement of his altitude.

And (2.) As ſine Zd : ſine Pd :: ſine dPZ : dZP, or of its ſupplement DZS, the azimuthal diſtance from the ſouth.

Or, the practical rule may be as follows.

Write A for the ſine of the ſun's altitude, L and l for the ſine and co-ſine of the latitude, D and d for the ſine and co-ſine of the ſun's declination, and H for the ſine of the horary diſtance from VI.

Then the relation of H to A will have three varieties.

1, When

1. When the declination is toward the elevated pole, and the hour of the day is between XII and VI; it is $A = LD + Hld$, and $H = \frac{A-LD}{ld}$.

2. When the hour is after VI, it is $A = LD - Hld$, and $H = \frac{LD+A}{ld}$.

3. When the declination is toward the depreſſed pole, we have $A = Hld - LD$, and $H = \frac{A+LD}{ld}$.

Which theorems will be found uſeful, and expeditious enough for ſolving thoſe problems in geography and dialing, which depend on the relation of the ſun's altitude to the hour of the day.

EXAMPLE I.

Suppoſe the latitude of the place to be $51\frac{1}{2}$ degrees north; the time five hours diſtant from XII, that is, an hour after VI in the morning, or before VI in the evening: and the ſun's declination 20° north. *Required the ſun's altitude?*

Then, to log. L = log. ſine	51°	30′	1.89354*
add log. D = log. ſine	20°	0′	1.53405
Their ſum - - -			1.42759

gives LD = logarithm of 0.267664, in the natural ſines.

* Here we conſider the radius as unity, and not 10,00000, by which, inſtead of the index 9, we have—1, as above: which is of no farther uſe, than making the work a little eaſier.

And,

And, to log. H = log. ſine † 15° 0′ 1.41300
add { log. l = log. ſine ‡ 38° 0′ 1.79414
log. d = log. ſine ‖ 70° 0′ 1.97300

Their ſum - - - 1.18014
gives Hld = logarithm of 0.151408, in the natural ſines.

And theſe two numbers (of 0.267664 and 0.151408) make 0.419072 = A; which, in the table, is the neareſt natural ſine of 24° 47′, the ſun's altitude ſought.

The ſame hour-diſtance being aſſumed on the other ſide of VI, then $LD-Hld$ is 0.116256, the ſine of 6° 40′$\frac{1}{2}$; which is the ſun's altitude at V in the morning, or VII in the evening, when his north declination is 20°.

But when the declination is 20° ſouth, (or toward the depreſſed pole) the difference $Hld-LD$ becomes negative, and thereby ſhews that, a hour before VI in the morning, or paſt VI in the evening, the ſun's center is 6° 40′$\frac{1}{2}$ below the horizon.

EXAMPLE II.

In the ſame latitude and north declination, from the given altitude to find the hour.

Let the altitude be 48°; and becauſe, in this caſe $H=\frac{A-LD}{ld}$, and A (the natural ſine of 48°) = .743145, and LD = .267664, $A-LD$

† The diſtance of one hour from VI.
‡ The co-latitude of the place.
‖ The co-declination of the ſun.

will

will be 0.475481, whose logarithmic
sine is - - - 1.6771331
from which taking the logarithmic
sine of $l + d =$ - - 1.7671354

Remains - - - 1.9099977
the logarithmic sine of the hour-distance sought, viz. of 54° 22′; which, reduced to time, is 3 hours $37\frac{1}{2}$ min. that is, IX h. $37\frac{1}{2}$ min. in the forenoon, or II h. $22\frac{1}{2}$ min. in the afternoon.

Put the altitude = 18°, whose natural sine is .3090170; and thence $A - LD$ will be =.0491953; which divided by $l + d$, gives .0717179, the sine of 4° $6'\frac{1}{2}$, in time $16\frac{1}{2}$ minutes nearly, before VI in the morning or after VI in the evening, when the sun's altitude is 18°.

And, if the declination 20° had been toward the south pole, the sun would have been depressed 18° below the horizon at $16\frac{1}{2}$ minutes after VI in the evening; at which time, the twilight would end; which happens about the 22d of November, and 19th of January, in the latitude of $51°\frac{1}{2}$ north. The same way may the end of twilight, or beginning of dawn, be found for any time of the year.

NOTE 1. If in theorem 2 and 3 (page 36[illegible]) A is put = 0, and the value of H is computed, we have the hour of sun-rising and setting for any latitude, and time of the year. And if we put H=0, and compute A, we have the sun's altitude or depression at the hour of VI. And lastly, if H, A, and D are given, the latitude may be found by the resolution of a quadratic equation; for $l = \sqrt{1 - L^2}$.

NOTE

NOTE 2. When A is equal 0, H is equal $\frac{LD}{ld} = TL \times TD$, the tangent of the latitude multiplied by the tangent of the declination.

As, if it was required, *what is the greatest length of day in latitude* 51° 30′?

To the log. tangent of 51° 30′ 0.0993948
Add the log. tangent of 23° 29′ 1.6379563

Their sum - - - 1.7373511 is the log. sine of the hour-distance 33° 7′; in time 2 h. 12½ m. The longest day therefore is 12 h. + 4 h. 25 m. = 16 h. 25 m. And the shortest day is 12 h. — 4 h. 25 m. = 7 h. 35 m.

And if the longest day is given, the latitude of the place is found; $\frac{H}{TD}$ being equal to TL. Thus, if the longest day is 13½ hours = 2 × 6 h. + 45 m. and 45 minutes in time being equal to 11¼ degrees.

From the log. sine of 11° 15′ 1.2902357
Take the log. tang. of 23° 29′ 1.6379562

Remains - - - 1.6522795
= the logarithmic tangent of lat. 24° 11′.

And the same way, the latitudes, where the several geographical *climates* and parallels begin, may be found; and the latitudes of places, that are assigned in authors from the length of their days, may be examined and corrected.

NOTE 3. The same rule for finding the longest day in a given latitude, distinguishes the hour-lines that are necessary to be drawn on any dial from those which would be superfluous.

In lat. 52° 10′ the longest day is 16 h. 32 m. and the hour-lines are to be marked from 44 m.

 after

after III in the morning, to 16 m. after VIII in the evening.

In the ſame latitude, let the dial of Art. 7. Fig. 4. be propoſed; and the elevation of its ſtile (or the latitude of the place *d*, whoſe horizon is parallel to the plane of the dial) being 15° 9′; the longeſt day at *d*, that is, the longeſt time that the ſun can illuminate the plane of the dial, will (by the rule $H = \Upsilon L \times \Upsilon D$) be twice 6 hours 27 minutes = 12 h. 54 m. The difference of longitude of the planes *d* and *Z* was found in the ſame example to be 36° 2′; in time, 2 hours 24 minutes; and the declination of the plane was from the ſouth toward the weſt. Adding therefore 2 h. 24 min. to 5 h. 33 m. the earlieſt ſun-riſing on a horizontal dial at *d*, the ſum 7 h. 57 m. ſhews that the morning hours, or the parallel dial at *Z*, ought to begin at 3 min. before VIII. And to the lateſt ſun-ſetting at *d*, which is 6 h. 27 m. adding the ſame 2 h. 24 m. the ſum 8 h. 51 m. exceeding 6 h. 16 m. the lateſt ſun-ſetting at *Z*, by 35 m. ſhews that none of the afternoon hour-lines are ſuperfluous. And the 4 h. 13 m. from III h. 44 m. the ſun-riſing at *Z* to VII h. 57 m. the ſun-riſing at *d*, belong to the other face of the dial; that is, to a dial declining 36° from north to eaſt, and inclining 15°.

EXAMPLE III.

From the ſame *data* to find the ſun's *azimuth*.

If *H*, *L*, and *D* are given, then (by Art. 2. of Rule II.) from *H*, having found the altitude and its complement *Z d*; and the arc *P D* (the diſtance

diſtance from the pole) being given; ſay, As the co-ſine of the altitude is to the ſine of the diſtance from the pole, ſo is the ſine of the hour-diſtance from the meridian to the ſine of the azimuth diſtance from the meridian.

Let the latitude be 51° 30′ north, the declination 15° 9′ ſouth, and the time II h. 24 m. in the afternoon, when the ſun begins to illuminate a vertical wall, and it is required to find the poſition of the wall.

Then, by the foregoing theorems, the complement of the altitude will be 81° 32′½, and *P d* the diſtance from the pole being 109° 5′, and the horary diſtance from the meridian, or the angle *d P Z*, 36°.

To log. ſine 74° 51′	-	1.98464
Add log. ſine 36° 0′	-	1.76922
And from the ſum	-	1.75386
Take the log. ſine 81° 32′½		1.99525
Remains	- -	1.75861 = log.

ſine 35°, the azimuth diſtance ſouth.

When the altitude is given, find from thence the hour, and proceed as above.

This praxis is of ſingular uſe on many occaſions: in finding the declination of vertical planes more exactly than in the common way, eſpecially if the tranſit of the ſun's center is obſerved by applying a ruler with ſights, either plane or teleſcopical, to the wall or plane, whoſe declination is required.—In drawing a meridian-line, and finding the magnetic variation.—In finding the bearings of places in terreſtrial ſurveys; the tranſits of the ſun over any place, or his horizontal diſtance from it being obſerved, together with the altitude and hour.——And

5 thence

thence determining ſmall differences of longitude.—In obſerving the variation at ſea, &c.

The learned Mr. *Andrew Reid* invented an inſtrument ſeveral years ago, for finding the latitude at ſea from two altitudes of the ſun, obſerved on the ſame day, and the interval of the obſervations, meaſured by a common watch. And this inſtrument, whoſe only fault was that of its being ſomewhat expenſive, was made by Mr. *Jackſon.* Tables have been lately computed for that purpoſe.

But we may often, from the foregoing rules, reſolve the ſame problem without much trouble; eſpecially if we ſuppoſe the maſter of the ſhip to know within 2 or 3 degrees what his latitude is. Thus,

Aſſume the two neareſt probable limits of the latitude, and by the theorem $H=\frac{A+LD}{ld}$, compute the hours of obſervation for both ſuppoſitions. If one interval of thoſe computed hours coincides with the interval obſerved, the queſtion is ſolved. If not, the two diſtances of the intervals computed, from the true interval, will give a proportional part to be added to, or ſubtracted from, one of the latitudes aſſumed. And if more exactneſs is required, the operation may be repeated with the latitude already found.

But whichever way the queſtion is ſolved, a proper allowance is to be made for the difference of latitude ariſing from the ſhip's courſe in the time between the two obſervations.

Of

Of the double horizontal dial; and the Babylonian *and* Italian *dials.*

To the *gnomonic* projection, there is sometimes added a *stereographic* projection of the hour-circles, and the parallels of the sun's declination, on the same horizontal plane; the upright side of the gnomon being sloped into an edge, standing perpendicularly over the center of the projection: so that the dial, being in its due position, the shadow of *that* perpendicular edge is a vertical circle passing through the sun, in the stereographic projection.

The months being duly marked on the dial, the sun's declination, and the length of the day at any time, are had by inspection; as also his altitude, by means of a scale of tangents. But its chief property is, that it may be placed true, whenever the sun shines, without the help of any other instrument.

Fig. 3. Let d be the sun's place in the stereographic projection, $x\,d\,y\,z$ the parallel of the sun's declination, $Z\,d$ a vertical circle through the sun's center, $P\,d$ the hour-circle; and it is evident, that the diameter NS of this projection being placed duly north and south, these three circles will pass through the point d. And therefore, to give the dial its due position, we have only to turn its gnomon toward the sun, on a horizontal plane, until the hour on the common gnomonic projection coincides with that marked by the hour-circle $P\,d$, which passes through the intersection of the shadow $Z\,d$ with the circle of the sun's present declination.

The *Babylonian* and *Italian* dials reckon the hours, not from the meridian, as with us, but from

from the sun's rising and setting. Thus, in *Italy*, one hour before sun-set is reckoned the 23d hour; two hours before sun-set the 22d hour; and so of the rest. And the shadow that marks them on the hour-lines, is *that* of the point of a stile. This occasions a perpetual variation between their dials and clocks, which they must correct from time to time, before it arises to any sensible quantity, by setting their clocks so much faster or slower. And in *Italy* they begin their day, and regulate their clocks, not from sun-set, but from about mid-twilight, when the *Ave Maria* is said; which corrects the difference that would otherwise be between the clock and the dial. Plate XXIII.

The improvements which have been made in all sorts of instruments and machines for measuring time, have rendered such dials of little account. Yet, as the theory of them is ingenious, and they are really, in some respects, the best contrived of any for vulgar use, a general idea of their description may not be unacceptable.

Let Fig. 5. represent an erect direct south wall, on which a *Babylonian dial* is to be drawn, shewing the hours from sun-rising; the latitude of the place, whose horizon is parallel to the wall, being equal to the angle KCR. Make, as for a common dial $KG = KR$ (which is perpendicular to CR) the radius of the equinoctial $Æ\,Q$, and draw $R\,S$ perpendicular to $C\,K$ for the stile of the dial; the shadow of whose point R is to mark the hours, when $S\,R$ is set upright on the plane of the dial.

Then it is evident, that in the contingent line $Æ\,Q$, the spaces $K\,1$, $K\,2$, $K\,3$, &c. being taken equal to the tangents of the hour-distances from the meridian, to the radius $K\,G$, one, two, three, &c. hours after sun-rising, on the equinoctial day; the shadow of the point R will be

 found,

found, at theſe times, reſpectively in the points 1, 2, 3, &c.

Draw, for the like hours after ſun-riſing, when the ſun is in the tropic of Capricorn ♑ *v*, the like common lines *CD*, *CE*, *CF*, &c. and at theſe hours the ſhadow of the point *R* will be found in thoſe lines reſpectively. Find the ſun's altitudes above the plane of the dial at theſe hours, and with their co-tangents *S d*, *S e*, *S f*, &c. to radius *S R*, deſcribe arcs interſecting the hour-lines in the points *d*, *e*, *f*, &c. ſo ſhall the right lines 1 *d*, 2 *e*, 3 *f*, &c. be the lines of I, II, III, &c. hours after ſun-riſing.

The conſtruction is the ſame in every other caſe, due regard being had to the difference of longitude of the place at which the dial would be horizontal, and the place for which it is to ſerve. And likewiſe, taking care to draw no lines but what are neceſſary; which may be done partly by the rules already given for determining the time that the ſun ſhines on any plane; and partly from this, that on the tropical days, the hyperbola deſcribed by the ſhadow of the point *R*, limits the extent of all the hour-lines.

The moſt uſeful however, as well as the ſimpleſt of ſuch dials, is that which is deſcribed on the two ſides of a meridian plane.

That the *Babylonian* and *Italic* hours are truly enough marked by right lines, is eaſily ſhewn. Mark the three points on a globe, where the horizon cuts the equinoctial, and the two tropics, toward the eaſt or weſt; and turn the globe on its axis 15°, or 1 hour; and it is plain, that the three points which were in a great circle (viz. the horizon) will be in a great circle ſtill; which will be projected geometrically into a ſtraight line. But theſe three points are univerſally the ſun's ſun's places, one hour after ſun-ſet (or one hour before ſun-riſe) on the equinoctial and ſolſtitial days. The like is true of all other circles of declination, beſide the tropics; and therefore, the hours on ſuch dials are truly marked by ſtraight lines limited by the projections of the tropics; and which are rightly drawn, as in the foregoing example.

Note 1. The ſame dials may be delineated without the hour-lines *CD*, *CE*, *CF*, &c. by ſetting off the ſun's azimuths on the plane of the dial, from the center *S*, on either ſide of the ſubſtile *CSK*, and the correſponding co-tangents of altitude from the ſame center *S*, for I, II, III, &c. hours before or after the ſun is in the horizon of the place for which the dial is to ſerve, on the equinoctial and ſolſtitial days.

2. One of theſe dials has its name from the hours being reckoned from ſun-riſing, the beginning of the *Babylonian* day. But we are not thence to imagine that the *equal* hours, which it ſhews, were thoſe in which the aſtronomers of that country marked their obſervations. Theſe, we know with certainty, were unequal, like the *Jewiſh*, as being twelfth parts of the natural day: and a hour of the night was, in like manner, a twelfth part of the night; longer or ſhorter, according to the ſeaſon of the year. So that a hour of the day, and a hour of the night, at the ſame place, would always make $\frac{1}{12}$ of 24, or 2 equinoctial hours. In *Paleſtine*, among the *Romans*, and in ſeveral other countries, 3 of theſe unequal nocturnal hours were a *vigilia* or *watch*. And the reduction of equal and unequal hours into one another, is extremely eaſy. If, for inſtance, it is found, by a foregoing rule, that in a certain latitude, at a given time of the year, the

length of a day is 14 equinoctial hours, the unequal hour is then $\frac{14}{12}$ or $\frac{7}{6}$ of a hour, that is, 70 minutes; and the nocturnal hour is 50 minutes. The firſt watch begins at VII (ſun-ſet); the ſecond at three times 50 minutes after, viz. IX h. 30 m. the third always at midnight; the morning watch at $\frac{1}{2}$ hour paſt II.

If it were required to draw a dial for ſhewing theſe unequal hours, or 12th parts of the day, we muſt take as many declinations of the ſun as are thought neceſſary, from the equator toward each tropic: and having computed the ſun's altitude and azimuth for $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$th parts, &c. of each of the diurnal arcs belonging to the declinations aſſumed: by theſe, the ſeveral points in the circles of declination, where the ſhadow of the ſtile's point falls, are determined: and curve lines drawn through the points of a homologous diviſion will be the hour-lines required.

Of the right placing of dials, and having a true meridian line for the regulating of clocks and watches.

The plane on which the dial is to reſt, being duly prepared, and every thing neceſſary for fixing it, you may find the hour tolerably exact by a large equinoctial ring-dial, and ſet your watch to it. And then the dial may be fixed by the watch at your leiſure.

If you would be more exact, take the ſun's altitude by a good quadrant, noting the preciſe time of obſervation by a clock or watch. Then, compute the time for the altitude obſerved (by the rule, page 364) and ſet the watch to agree with that time, according to the ſun. A *Hadley's* quadrant is very convenient for this purpofe; for, by it you may take the angle between the fun and his image, reflected from a bafon of water: the half of which angle, fubtracting the refraction, is the altitude required. This is beft done in fummer, and the nearer the fun is to the prime vertical (the eaft or weft azimuth) when the obfervation is made, fo much the better.

Or, in fummer, take two equal altitudes of the fun in the fame day; one any time between 7 and 10 in the morning, the other between 2 and 5 in the afternoon; noting the moments of thefe two obfervations by a clock or watch: and if the watch fhews the obfervations to be at equal diftances from noon, it agrees exactly with the fun; if not, the watch muft be corrected by half the difference of the forenoon and afternoon intervals; and then the dial may be fet true by the watch.

Thus, for example, fuppofe you have taken the fun's altitude when it was 20 minutes paft VIII in the morning by the watch; and found, by obferving in the afternoon, that the fun had the fame altitude 10 minutes before IV; then it is plain, that the watch was 5 minutes too faft for the fun: for 5 minutes after XII is the middle time between VIII h. 20 m. in the morning, and III h. 50 m. in the afternoon; and therefore, to make the watch agree with the fun, it muft be fet back five minutes.

A good *meridian line*, for regulating clocks or watches, may be had by the following method. *A meridian line.*

Make a round hole, almoft a quarter of an inch diameter, in a thin plate of metal; and fix the plate in the top of a fouth window, in fuch a manner, that it may recline from the zenith at an angle equal to the co-latitude of your place, as nearly as you can guefs; for then, the plate will face the fun directly at noon on the equinoctial days. Let the fun fhine freely through the hole into the room; and hang a plumb-line to the ceiling of the room; at leaft five or fix feet from the window, in fuch a place as that the fun's rays, tranfmitted through the hole, may fall upon the line when it is noon by the clock; and having marked the faid place on the ceiling, take away the line.

Having adjufted a fliding bar to a dove-tail groove, in a piece of wood about 18 inches long, and fixed a hook into the middle of the bar, nail the wood to the above-mentioned place on the ceiling, parallel to the fide of the room in which the window is: the groove and bar being toward the floor. Then, hang the plumb-line upon the hook in the bar, the weight or plummet reaching almoft to the floor; and the whole will be prepared for farther and proper adjuftment.

This done, find the true folar time by either of the two laft methods, and thereby regulate your clock. Then, at the moment of next noon by the clock, when the fun fhines, move the fliding bar in the groove until the fhadow of the plumb-line bifects the image of the fun (made by his rays tranfmitted through the hole) on the floor, wall, or on a white fcreen placed on the north fide of the line; the plummet or weight at the end of the line hanging freely in a pail of water placed below it on the floor.—But becaufe this may not be quite correct for the firft time, on account that the plummet will not fettle immediately, even in water; it may be farther corrected on the following days, by the above method, with the ſun and clock; and ſo brought to a very great exactneſs.

N. B. The rays tranſmitted through the hole, will caſt but a faint image of the ſun, even on a white ſcreen, unleſs the room be ſo darkened that no ſun ſhine may be allowed to enter, but what comes through the ſmall hole in the plate. And always, for ſome time before the obſervation is made, the plummet ought to be immerſed in a jar of water, where it may hang freely; by which means the line will ſoon become ſteady, which otherwiſe would be apt to continue ſwinging.

As this meridian line will not only be ſufficient for regulating of clocks and watches to the true time by equation tables, but alſo for moſt aſtronomical purpoſes, I ſhall ſay nothing of the magnificent and expenſive meridian lines at *Bologne* and *Rome*, nor of the better methods by which aſtronomers obſerve preciſely the tranſits of the heavenly bodies on the meridian.

LECT. XII.

Shewing how to calculate the mean time of any New or Full Moon, or Eclipſe, from the creation of the world to the year of CHRIST 5800.

IN the following tables, the mean lunation is about a 20th part of a ſecond of time longer than its meaſure as now printed in the laſt edition of my aſtronomy; which makes the difference of a hour and 30 minutes in 8000 years. —But this is not material, when only the mean times are required.

PRECEPTS.

To find the mean time of any New or Full Moon in any given year and month after the Christian Æra.

1. If the given year be found in the third column of the *Table of the moon's mean motion from the ſun*, under the title, *Years before and after* CHRIST; write out that year, with the mean motions belonging to it, and thereto join the given month with its mean motions. But, if the given year be not in the table, take out the next leſſer one to it that you find, in the ſame column; and thereto add as many *complete years*, as will make up the given year: then, join the given month, and all the reſpective mean motions.

2. Collect theſe mean motions into one ſum of ſigns, degrees, minutes, and ſeconds; remembering, that 60 ſeconds (″) make a minute, 60 minutes (′) a degree; 30 degrees (°) a ſign, and 12 ſigns (^{s}) a circle. When the ſigns exceed 12, or 24, or 36 (which are whole circles) reject them, and ſet down only the remainder; which, together with the odd degrees, minutes and ſeconds already ſet down, mnſt be reckoned the whole ſum of the collection.

3. Subtract the reſult, or ſum of this collection, from 12 ſigns; and write down the remainder. Then, look in the table, under *Days*, for the next leſs mean motions to this remainder, and

and ſubtract them from it, writing down their remainder.

This done, look in the table under *hours* (marked H.) for the next leſs mean motions to this laſt remainder, and ſubtract them from it, writing down their remainder.

Then, look in the table under *minutes* (marked M.) for the next leſs mean motions to this remainder, and ſubtract them from it, writing down their remainder.

Laſtly, look in the table under *ſeconds* (marked S.) for the next leſs mean motions to this remainder, either greater or leſs; and againſt it you have the ſeconds anſwering thereto.

4. And theſe times collected, will give the mean time of the *required new moon*; which will be right in common years; and alſo in January and February in leap years; but always one day too late in leap years after February.

EXAMPLE

EXAMPLE I.

Required the time of new moon in September, 1764? (a year not inserted in the table)

		Moon from ſun.			
		s	°	′	″
To the year after *Chriſt's* birth -	1753	10	9	24	56
Add compleat years	11	0	10	14	20
(ſum 1764)					
And join September -		2	22	21	8
The ſum of theſe mean motions is		1	12	0	24
Which, being ſub. from a circle, or - - - -		12	0	0	0
Leaves remaining -		10	17	59	36
Next leſs mean mot. for 26 days, ſub. - - - -		10	16	57	34
And there remains - -			1	2	2
Next leſs mean mot. for 2 hours, ſub. - -			1	0	57
And the remainder will be -				1	5
Next leſs mean mot. for 2 min. ſub. - -				1	1
Remains the mean mot. of 12 ſec.					4

Theſe times, being collected, would ſhew the mean time of the required new moon in September 1764, to be on the 26th day, at 2 hours 2 min. 12 ſec. paſt noon. But, as it is in a leap-year, and after February, the time is one day too late. So, the true mean time is September the 25th, at 2 m. 12 ſec. paſt II in the afternoon.

N. B.

N. B. The tables always begin the day at noon, and reckon thenceforward, to the noon of the day following.

To find the mean time of full moon in any given year and month after the Christian Æra.

Having collected the moon's mean motion from the ſun for the beginning of the given year and month, and ſubtracted their ſum from 12 ſigns (as in the former example) add 6 ſigns to the remainder, and then proceed in all reſpects as above.

EXAMPLE II.

Required the mean time of full moon in September 1764?

	Moon from ſun.			
	s	°	′	″
To the year after *Chriſt*'s birth - 1753	10	9	24	56
Add complete years 11	0	10	14	20
(ſum 1764)				
And join September -	2	22	21	8
The ſum of theſe mean motions is - - -	1	12	0	24
Which, being ſubtracted from a circle, or - -	12	0	0	0
Leaves remaining -	10	17	59	36
To which remainder add	6	0	0	0
And the ſum will be -	4	17	59	36

Brought

	Moon from fun. s ° ′ ″
Brought over -	4 17 59 36
Next lefs mean mot. for 11 days, fubt. -	4 14 5 54
And there remains - - -	3 53 42
Next lefs mean mot. for 7 hours, fubt. -	3 33 20
And the remainder will be -	20 22
Next lefs mean mot. for 40 minutes, fubt. - -	20 19
Remains the mean mot. for 8 feconds - -	3

So, the mean time, according to the tables, is the 11th of September, at 7 hours 40 minutes 8 feconds paft noon. One day too late, being after February in a leap-year.

And thus may the mean time of any new or full moon be found, in any year after the Chriftian Æra.

To find the mean time of new or full moon in any given year and month before the Chriftian Æra.

If the given year before the year of CHRIST 1 be found in the third column of the table, under the title *Years before and after* CHRIST, write it out, together with the given month, and join the mean motions. But, if the given year be not in the table, take out the next greater one to it that you find; which being ftill farther back than the given year, add as many compleat years to it as will bring the time forward to the given year; then join the month, and proceed in all refpects as above.

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EXAMPLE III.

Required the mean time of new moon in May, the year before Christ 585?

The next greater year in the table is 600; which being 15 years before the given year, add the mean motions for 15 years to thoſe of 600, together with thoſe for the beginning of May.

	Moon from ſun.			
	s	°	′	″
To the year before *Chriſt* 600 -	5	11	6	16
Add compleat years motion 15	6	0	55	24
And the mean motions for May	0	22	53	23
The whole ſum is - -	0	4	55	3
Which, being ſubt. from a circle, or - -	12	0	0	0
Leaves remaining -	11	25	4	57
Next leſs mean mot. for 29 days, ſubt. - - -	11	23	31	54
And there remains -		1	33	3
Next leſs mean mot. for 3 hours ſubt. - -		1	31	26
And the remainder will be -			1	37
Next leſs mean mot. for 3 min. ſubt. - - -			1	31
Rem. the mean mot. of 14 ſeconds - - -				6

So,

So, the mean time by the tables, was the 29th of May, at 3 hours 3 min. 14 ſec. paſt noon. A day later than the truth, on account of its being in a leap-year. For as the year of CHRIST 1 was the firſt after a leap-year, the year 585 before the year 1 was a leap-year of courſe.

If the given year be after the Chriſtian Æra, divide its date by 4, and if nothing remains, it is a leap-year in the old ſtyle. But if the given year was before the Chriſtian Æra (or Year of CHRIST 1) ſubtract one from its date, and divide the remainder by 4; then, if nothing remains, it was a leap-year; otherwiſe not.

To find whether the ſun is eclipſed at the time of any given change, or the moon at any given full.

Of *eclipſes.* From the *Table of the ſun's mean motion* (or diſtance) *from the moon's aſcending node*, collect the mean motions anſwering to the given time; and if the reſult ſhews the ſun to be within 18 degrees of either of the nodes at the time of new moon, the ſun will be eclipſed at that time. Or, if the reſult ſhews the ſun to be within 12 degrees of either of the nodes at the time of full moon, the moon will be eclipſed at that time, in or near the contrary node; otherwiſe not.

EXAM-

EXAMPLE IV.

The moon changed on the 26th of September 1764, at 2 h. 2 m. (neglecting the seconds) after noon. (See Example I.) *Qu. Whether the sun was eclipsed at that time?*

		Sun from node.			
		s	°	′	″
To the year after *Christ*'s birth - -	1753	1	28	0	19
Add compleat years -	11	7	2	3	56
	(sum 1764)				
And September -		8	12	22	49
And 26 days - -			27	0	13
And 2 hours - -				5	12
And 2 minutes -		-			5
Sun's distance from the ascending node - - -		6	9	32	34

Now, as the descending node is just opposite to the ascending (viz. 6 signs distant from it) and the tables shew only how far the sun has gone from the ascending node, which, by this example, appears to be 6 signs 9 degrees 32 minutes 34 seconds, it is plain that he must have then been eclipsed; as he was then only 9° 32′ 34″ short of the descending node.

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EXAMPLE V.

The moon was full on the 11th of September, 1764, at 7 h. 40 min. past noon. (See Example II.) *Qu. Whether she was eclipsed at that time?*

	Sun from node. s ° ′ ″
To the year after *Christ's* birth - - 1753	1 28 0 19
Add compleat years 11	7 2 3 56
(sum 1764)	
And September - -	8 12 22 49
And 11 days - -	11 25 29
And 7 hours - - -	18 11
And 40 minutes - - -	1 44
Sun's distance from the ascending node - - -	5 24 12 28

Which being subtracted from 6 signs, leaves only 5° 47′ 32″ remaining; and this being all the space that the sun was short of the descending node, it is plain that the moon must then have been eclipsed, because she was just as near the contrary node.

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EXAMPLE VI.

Q. Whether the sun was eclipsed in May, the year before CHRIST 585? (See Example III.)

	Sun from node.			
	s	°	′	″
To the year before *Christ* 600 -	9	9	23	51
Add the mean motion of 15 complete years — —	9	19	27	49
Add { May — —	4	4	37	57
Add { 29 days — —	1	0	7	10
Add { 3 hours — —			7	48
Add { 3 minutes (neglecting the seconds) — —				8
Sun's distance from the ascending node — —	0	3	44	43

Which being less than 18 degrees, shews that the sun was eclipsed at that time.

Thales's eclipse.

This eclipse was foretold by *Thales*, and is thought to be the eclipse which put an end to the war between the Medes and Lydians.

When *eclipses must happen.*

The times of the *sun's conjunction with the nodes*, and consequently the *eclipse months* of any given year, are easily found by the *Table of the sun's mean motion from the moon's ascending node*; and much in the same way as the mean conjunctions of the sun and moon are found by the table of the moon's mean motion from the sun. For, collect the sun's mean motion from the node (which is the same as his distance gone from it) for the beginning of any given *year*, and subtract it from 12 signs; then, from the remainder, ſubtract the next leſs mean motions belonging to whatever *month* you find them in the table; and from their remainder ſubtract the next leſs mean motion for *days*, and ſo on for *hours* and *minutes*; the reſult of all which will ſhew the time of the ſun's mean conjunction with the *aſcending node* of the moon's orbit.

EXAMPLE VII.

Required the time of the ſun's conjunction with the aſcending node in the year 1764?

		Sun from node.			
		s	°	′	″
To the year after *Chriſt's* birth — —	1753	1	28	0	19
Add compleat years —	11	7	2	3	56
Mean diſt. at beg. of A. D. 1764		9	0	4	15
Subtract this diſtance from a circle, or — —		12	0	0	0
And there remains —		2	29	55	45
Next leſs mean motion for March, ſubtract —		2	1	16	39
And the remainder will be			28	39	6
Next leſs mean motion for 27 days, ſubtract — —			28	2	32
And there remains — —				36	34
Next leſs mean motion for 14 hours, ſubtracted —				36	21
Remains (nearly) the mean motion of 5 minutes —					13

Hence

Hence it appears, that the ſun will paſs by the moon's *aſcending node* on the 27th of March, at 14 hours 5 minutes paſt noon; viz. on the 28th day, at 5 minutes paſt II in the morning, according to the tables: but this being in a leap-year, and after February, the time is one day too late. Conſequently, the true time is at 5 min. paſt II in the morning on the 27th day; at which time, the deſcending node will be directly oppoſite to the ſun.

If 6 ſigns be added to the remainder ariſing from the firſt ſubtraction (viz. from 12 ſigns) and then the work carried on as in the laſt example, the reſult will give the mean time of the ſun's conjunction with the deſcending node. Thus, in

EXAMPLE VIII.

To find when the ſun will be in conjunction with the deſcending node *in the year* 1764?

		Sun from node.			
		s	°	′	″
To the year after *Chriſt*'s birth —	1753	1	28	0	19
Add compleat years	11	7	2	3	56
M. d. ſr. aſc. n. at beg. of 1764		9	0	4	15
Subtract this diſtance from a circle, or — —		12	0	0	0
And the remainder will be —		2	29	55	45
To which add half a circle, or		6	0	0	0
And the ſum will be —		8	29	55	45

	Sun fr. node.			
	s	°	′	″
Brought over —	8	29	55	45
Next leſs mean mot. for Sept. ſubt.	8	12	22	49
And there remains — —		17	32	56
Next leſs mean mot. for 16 days, ſubt. — —		16	37	4
And the remainder will be —			55	52
Next leſs mean mot. for 21 hours, ſubtracted — —			54	32
Rem. (nearly) the mean mot. of 31 minutes — —			1	20

So that, according to the tables, the ſun will be in conjunction with the *deſcending node* on the 16th of September, at 21 hours 31 minutes paſt noon: one day later than the truth, on account of the leap-year.

The limits of *eclipſes.*

When the moon changes within 18 days before or after the ſun's conjunction with either of the nodes, the ſun will be eclipſed at that change: and when the moon is full within 12 days before or after the time of the ſun's conjunction with either of the nodes, ſhe will be eclipſed at that full: otherwiſe not.

Their period and reſtitution.

If to the mean time of any eclipſe, either of the ſun or moon, we add 557 Julian years 21 days 18 hours 11 minutes and 51 ſeconds (in which there are exactly 6890 mean lunations) we ſhall have the mean time of another eclipſe. For at the end of that time, the moon will be either new or full, according as we add it to the time of new or full moon; and the ſun will be only 45″ farther from the ſame node, at the end of

the

the ſaid time, than he was at the beginning of it; as appears by the following example *.

The period.		Moon fr. ſun.				Sun fr. node.			
		s	°	′	″	s	°	′	″
Compleat Years	500—	3	5	32	47—	10	14	45	8
	40—	8	26	50	37—	1	23	58	49
	17—	3	2	21	39—	10	28	40	55
days	21—	8	16	0	21—		21	48	38
hours	18—		9	8	35—			46	44
minutes	11—			5	35—				29
ſeconds	51—				26—				2
Mean motions	—	0	0	0	0—	0	0	0	45

And this period is ſo very near, that in 6000 years it will vary no more from the truth, as to the reſtitution of eclipſes, than 8¼ minutes of a degree; which may be reckoned next to nothing. It is the ſhorteſt in which, after many trials, I can find ſo near a conjunction of the ſun, moon, and the ſame node.

* Dr. HALLEY's period of eclipſes contains only 18 years 11 days 7 hours 43 minutes 20 ſeconds; in which time, according to his tables, there are juſt 223 mean lunations: but, as in that time, the ſun's mean motion from the node is no more than $11^s\ 29^\circ\ 31'\ 49''$, which wants $28'\ 11''$ of being as nearly in conjunction with the ſame node at the end of the period, as it was at the beginning; this period cannot be of conſtant duration for finding eclipſes, becauſe it will in time fall quite without their limits. The following tables make this period 31 ſeconds ſhorter, as appears by the following calculation.

The period.		Moon fr. ſun.				Sun fr. node.			
		s	°	′	″	s	°	′	″
Compleat years	18—	7	11	59	4—	11	17	46	18
days	11—	4	14	5	54—		11	25	29
hours	7—		3	33	20—			18	11
minutes	42—			21	20—			1	49
ſec.	44—				22—				2
Mean motions	—	0	0	0	0—	11	29	31	49

 This

This table is made by the continual addition of a mean lunation, viz. $29^d\ 12^h\ 44^m\ 3^s\ 6^{th}\ 21^{iv}\ 14^v\ 24^{vi}\ 0^{vii}$.

Lun.		Days.	H.	M.	S.	Th.
1	Contain	29	12	44	3	6
2		59	1	28	6	13
3		88	14	12	9	19
4		118	2	56	12	25
5		147	15	40	15	32
6		177	4	24	18	38
7		206	17	8	21	44
8		236	5	52	24	51
9		265	18	36	27	57
10		295	7	20	31	3
20		590	14	41	2	7
30		885	22	1	33	11
40		1181	5	22	4	14
50		1476	12	42	35	18
100		2953	1	25	10	35
200		5906	2	50	21	11
300		8859	4	15	31	46
400		11812	5	40	42	22
500		14756	7	5	52	57
1000		29530	14	11	45	54
2000		59061	4	23	31	48
3000		88591	18	35	17	42
4000		118122	8	47	3	36
5000		147652	22	58	49	30
10000		295305	21	57	39	0
20000		590611	19	55	18	0
30000		885917	17	52	57	0
40000		1181223	15	50	36	0
50000		1476529	13	48	15	0
100000		2953059	3	36	39	0

In 100000 mean lunations, there are 8085 Julian years 12 days 21 hours 36 minutes 30 seconds = 2953059 days 3 hours 36 minutes 30 seconds.

Proof of the Table.

In		Moon from sun. s	°	′	″
Jul. years	4000	1	14	22	12
	4000	1	14	22	12
	80	5	23	41	15
	5	10	0	18	28
Days	12	4	26	17	20
Hours	21		10	40	1
Min.	36			18	17
Sec.	20				15
M. fr. sun.		0	0	0	0

Having by the former precepts computed the mean time of new moon in January, for any given year, it is easy, by this Table, to find the mean time of new moon in January for any number of years afterward: and by means of a small table of lunations for 12 or 13 months, to make a general table for finding the mean time of new or full moon in any given year and month whatever.

	D.	H.	M.	S.	Th.
In 11 lunations there are	324	20	4	34	10.
In 12 lunations ——	354	8	48	37	16.
In 13 lunations ——	383	21	32	40	23.

But then it would be best to begin the year with March, to avoid the inconvenience of losing a day by mistake in leap-year.

Years

Years of the Julian period.	Years of the World.	Years before and after CHRIST.	Moon from sun. s	°	'	"
706	0	4008	5	28	1	17
714	8	4000	5	9	23	24
1714	1008	3000	11	20	28	57
2714	2008	2000	6	1	34	30
3714	3008	1000	0	12	40	3
3814	3108	900	10	19	46	36
3914	3208	800	8	26	53	9
4014	3308	700	7	3	59	43
4114	3408	600	5	11	6	16
4214	3508	500	3	18	12	49
4314	3608	400	1	25	19	23
4414	3708	300	0	2	25	56
4514	3808	200	10	9	32	29
4614	3908	100	8	16	39	3
4714	4008	1	6	23	45	36
4814	4108	101	5	0	52	9
4914	4208	201	3	7	58	43
5014	4308	301	1	15	5	16
5114	4408	401	11	22	11	49
5214	4508	501	9	29	18	23
5714	5008	1001	1	4	51	5
6414	5708	1701	0	24	37	2
6466	5760	1753	10	9	24	56
6514	5808	1801	6	5	26	15

Before the Year of CHRIST 1.—After CHRIST.

Com-pleat years.	Moon from sun. s	°	'	"
11	0	10	14	20
12	5	2	3	11
13	9	11	40	[illegible]
14	1	21	18	0
15	6	0	55	24
16	10	22	44	15
17	3	2	21	39
18	7	11	59	4
19	11	21	36	27
20	4	13	25	19
40	8	26	50	37
60	1	10	15	56
80	5	23	41	15
100	10	7	6	33
200	8	14	13	7
300	6	21	19	40
400	4	28	26	13
500	3	5	32	47
1000	6	11	5	33
2000	0	22	11	6
3000	7	3	16	39
4000	1	14	22	12

The 4008th year before the year of CHRIST 1, was the 4007th year before the year of his birth; and is supposed to have been the year of the creation.

Compleat years.	Moon from sun. s	°	'	"
1	4	9	37	24
2	8	19	14	48
3	0	28	52	13
4	5	20	41	4
5	10	0	18	28
6	2	9	55	52
7	6	19	33	17
8	11	11	22	7
9	3	20	59	32
10	8	0	36	55

Julian years, 3 of which have 365 days, and the 4th 366.

Months.	Moon from sun. s	°	'	"
Jan.	0	0	0	0
Feb.	0	17	54	48
Mar.	11	29	15	16
April	0	17	10	3
May	0	22	53	23
June	1	10	48	11
July	1	16	31	32
Aug.	2	4	26	20
Sept.	2	22	21	8
Oct.	2	28	4	29
Nov.	3	15	59	17
Dec.	3	21	42	7

This table agrees with the *old stile* until the year 1753; and after that, with the *new*.

Days.	Moon from sun.				Moon from sun.				Moon from sun.			
	s	°	′	″	H M S	° ′ ″	′ ″ ‴	″ ‴ ⁗	M S Th	′ ″ ‴	″ ‴ ⁗	‴ ⁗ v
1	0	12	11	27								
2	0	24	22	53								
3	1	6	34	20	1	0	30	29	31	15	44	47
4	1	18	45	47	2	1	0	57	32	16	15	16
5	2	0	57	13	3	1	31	26	33	16	45	44
6	2	13	8	40	4	2	1	54	34	17	16	13
7	2	25	20	7	5	2	32	23	35	17	46	42
8	3	7	31	34	6	3	2	52	36	18	17	10
9	3	19	43	0	7	3	33	20	37	18	47	39
10	4	1	54	27	8	4	3	49	38	19	18	7
11	4	14	5	54	9	4	34	18	39	19	48	36
12	4	26	17	20	10	5	4	46	40	20	19	5
13	5	8	28	47	11	5	35	15	41	20	49	33
14	5	20	40	14	12	6	5	43	42	21	20	2
15	6	2	51	40	13	6	36	12	43	21	50	31
16	6	15	3	7	14	7	6	41	44	22	20	59
17	6	27	14	34	15	7	37	9	45	22	51	28
18	7	9	26	0	16	8	7	38	46	23	21	56
19	7	21	37	27	17	8	38	6	47	23	52	25
20	8	3	48	54	18	9	8	35	48	24	22	54
21	8	16	0	21	19	9	39	4	49	24	53	22
22	8	28	11	47	20	10	9	32	50	25	23	51
23	9	10	23	14	21	10	40	1	51	25	54	19
24	9	22	34	41	22	11	10	30	52	26	24	48
25	10	4	46	7	23	11	40	58	53	26	55	17
26	10	16	57	34	24	12	11	27	54	27	25	45
27	10	29	9	1	25	12	41	55	55	27	56	14
28	11	11	20	27	26	13	12	24	56	28	26	43
29	11	23	31	54	27	13	42	53	57	28	57	11
30	0	5	43	21	28	14	13	21	58	29	27	40
31	0	17	54	47	29	14	43	50	59	29	58	8
32	1	0	6	15	30	15	14	18	60	30	28	37

1 Lunation $= 29^{d}\ 12^{h}\ 44^{m}\ 3^{s}\ 6^{th}\ 21^{iv}\ 14^{v}\ 24^{vi}\ 0^{vii}$

In leap years, after February, a day and its motion must be added to the time for which the moon's mean distance from the sun is given. But, when the mean time of any new or full moon is required in leap-year after February, a day must be subtracted from the mean time thereof, as found by the tables. In common years they give the day right.

Years

A Table of the Sun's mean Motion from the Moon's Ascending Node.

Years of the Julian period.	Years of the World.	Years before and after CHRIST.	Sun from node. s	°	'	"	Compleat years.	Sun from node. s	°	'	"
706	0	4008	7	6	17	9	11	7	2	3	56
714	8	4000	0	11	4	55	12	7	22	11	39
1714	1008	3000	9	10	35	11	13	8	11	17	2
2714	2008	2000	6	10	5	28	14	9	0	22	25
3714	3008	1000	3	9	35	44	15	9	19	27	49
3814	3108	900	7	24	32	46	16	10	9	35	31
3914	3208	800	0	9	29	48	17	10	28	40	55
4014	3308	700	4	24	26	49	18	11	17	46	18
4114	3408	600	9	9	23	51	19	0	6	51	43
4214	3508	500	1	24	20	53	20	0	26	59	24
4314	3608	400	6	9	17	54	40	1	23	58	49
4414	3708	300	10	24	14	56	60	2	20	58	13
4514	3808	200	3	9	11	58	80	3	17	57	37
4614	3908	100	7	24	8	59	100	4	14	57	2
4714	4008	1	0	9	6	1	200	8	29	54	3
4814	4108	101	4	24	3	3	300	1	14	51	5
4914	4208	201	9	9	0	4	400	5	29	48	7
5014	4308	301	1	23	57	6	500	10	14	45	8
5114	4408	401	6	8	54	8	1000	8	29	30	17
5214	4508	501	10	23	51	9	2000	5	29	0	33
5714	5008	1001	9	8	36	18	3000	2	28	30	50
6414	5708	1701	4	23	15	30	4000	11	28	1	6
6466	5760	1753	1	28	0	19					
6514	5808	1801	8	25	44	44					

Before the year of CHRIST I.—After CHRIST.

Compleat Years.	Sun from node. s	°	'	"
1	0	19	5	23
2	1	8	10	47
3	1	27	16	10
4	2	17	23	53
5	3	6	29	16
6	3	25	34	40
7	4	14	40	3
8	5	4	47	46
9	5	23	53	9
10	6	12	58	33

Julian years, 3 of which have 365 days, and the 4th 366.

Months.	Sun from node. s	°	'	"
Jan.	0	0	0	0
Feb.	1	2	11	48
Mar.	2	1	16	39
April	3	3	28	27
May	4	4	37	57
June	5	6	49	45
July	6	7	59	14
Aug.	7	9	11	1
Sept.	8	12	22	49
Oct.	9	13	32	18
Nov.	10	15	44	5
Dec.	11	16	53	34

The 4008th year before the year of CHRIST I, was the 4007th year before the year of his birth; and is ſuppoſed to have been the year of the creation.

This table agrees with the *old ſtile* until the year 1753; and after that, with the *new*.

Days.	Sun from node.				Sun from node.				Sun from node.			
	s	°	'	"	H.	°	'	"	M.	'	"	'''
					M.	'	"	'''	S.	"	'''	''''
					S.	"	'''	''''	Th.	'''	''''	v
1	0	1	2	19								
2	0	2	4	38								
3	0	3	6	57	1	0	2	36	31	1	20	31
4	0	4	9	16	2	0	5	12	32	1	23	7
5	0	5	11	36	3	0	7	48	33	1	25	43
6	0	6	13	54	4	0	10	23	34	1	28	9
7	0	7	16	13	5	0	12	59	35	1	31	55
8	0	8	18	32	6	0	15	35	36	1	33	31
9	0	9	20	51	7	0	18	11	37	1	36	6
10	0	10	23	10	8	0	20	47	38	1	38	42
11	0	11	25	29	9	0	23	23	39	1	41	18
12	0	12	27	48	10	0	25	58	40	1	43	54
13	0	13	30	7	11	0	28	33	41	1	46	36
14	0	14	32	26	12	0	31	9	42	1	49	5
15	0	15	34	15	13	0	33	45	43	1	51	41
16	0	16	37	4	14	0	36	21	44	1	54	17
17	0	17	39	23	15	0	38	57	45	1	56	53
18	0	18	41	41	16	0	41	32	46	1	59	29
19	0	19	44	0	17	0	44	8	47	2	2	5
20	0	20	46	19	18	0	46	44	48	2	4	41
21	0	21	48	38	19	0	49	20	49	2	7	17
22	0	22	50	57	20	0	51	56	50	2	9	53
23	0	23	53	16	21	0	54	32	51	2	12	29
24	0	24	55	35	22	0	57	8	52	2	15	5
25	0	25	57	54	23	0	59	43	53	2	17	41
26	0	27	0	13	24	1	2	19	54	2	20	17
27	0	28	2	32	25	1	4	55	55	2	22	53
28	0	29	4	51	26	1	7	31	56	2	25	29
29	1	0	7	10	27	1	10	7	57	2	28	4
30	1	1	9	29	28	1	12	43	58	2	30	40
31	1	2	11	48	29	1	15	9	59	2	33	16
32	1	3	14	47	30	1	17	55	60	2	35	52

In leap years, after February, add one day and one day's motion to the time at which the sun's mean distance from the ascending node is required.

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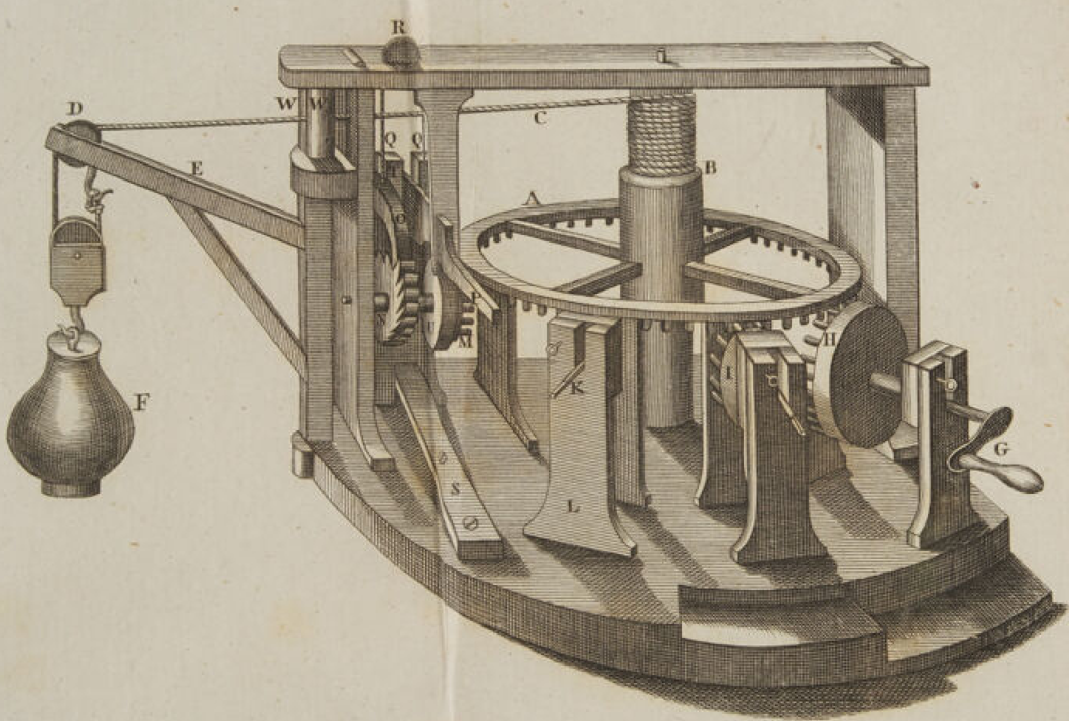


T. Ferguson delin.

J. Mynde sc.

A

SUPPLEMENT

TO THE PRECEDING

LECTURES.

MECHANICS.

The Description of a new and safe Crane, which has four different Powers, adapted to different Weights.

THE common crane consists only of a large wheel and axle; and the rope, by which goods are drawn up from ships, or let down from the quay to them, winds or coils round by the axle, as the axle is turned by men walking in the wheel. But, as these engines have nothing to stop the weight from running down, if any of the men happen to trip or fall in the wheel, the weight descends, and turns the wheel rapidly backward, and tosses the men violently about within it; which has produced melancholy instances, not only of limbs broke, but even of lives loſt, by this ill-judged conſtruction of cranes. And beſides, they have but one power for all ſorts of weights; ſo that they generally ſpend as much time in raiſing a ſmall weight, as in raiſing a great one.

Theſe imperfections and dangers induced me to think of a method of remedying them. And for that purpoſe, I contrived a crane with a proper ſtop to prevent the danger, and with different powers ſuited to different weights; ſo that there might be as little loſs of time as poſſible: and alſo, that when heavy goods are let down into ſhips, the deſcent may be regular and deliberate.

This crane has four different powers: and, I believe, it might be built in a room eight feet in width: the gib being on the outſide of the room.

Three trundles, with different numbers of ſtaves, are applied to the cogs of a horizontal wheel with an upright axle; and the rope, that draws up the weight, coils round the axle. The wheel has 96 cogs, the largeſt trundle 24 ſtaves, the next largeſt has 12, and the ſmalleſt has 6. So that the largeſt trundle makes 4 revolutions for one revolution of the wheel; the next makes 8, and the ſmalleſt makes 16. A winch is occaſionally put upon the axis of either of theſe trundles, for turning it; the trundle being then uſed that gives a power beſt ſuited to the weight: and the handle of the winch deſcribes a circle in every revolution equal to twice the circumference of the axle of the wheel. So that the length

length of the winch doubles the power gained by each trundle.

As the power gained by any machine, or engine whatever, is in direct proportion as the velocity of the power is to the velocity of the weight; the powers of this crane are easily estimated, and they are as follows.

If the winch be put upon the axle of the largest trundle, and turned four times round, the wheel and axle will be turned once round: and the circle described by the power that turns the winch, being, in each revolution, double the circumference of the axle, when the thickness of the rope is added thereto; the power goes through eight times as much space as the weight rises through: and therefore (making some allowance for friction) a man will raise eight times as much weight by the crane as he would by his natural strength without it: the power, in this case, being as eight to one.

If the winch be put upon the axis of the next trundle, the power will be as sixteen to one, because it moves 16 times as fast as the weight moves.

If the winch be put upon the axis of the smallest trundle, and turned round; the power will be as 32 to one.

But, if the weight should be too great, even for this power to raise, the power may be doubled by drawing up the weight by one of the parts of a double rope, going under a pulley in the moveable block, which is hooked to the weight below the arm of the gib; and then the power will be as 64 to one. That is, a man could then raiſe 64 times as much weight by the crane as he could raiſe by his natural ſtrength without it; becauſe, for every inch that the weight riſes, the working power will move through 64 inches.

By hanging a block with two pullies to the arm of the gib, and having two pullies in the moveable block that riſes with the weight, the rope being doubled over and under theſe pullies, the power of the crane will be as 128 to one. And ſo, by increaſing the number of pullies, the power may be increaſed as much as you pleaſe: always remembering, that the larger the pullies are, the leſs is their friction.

While the weight is drawing up, the ratch-teeth of a wheel ſlip round below a catch or click that falls ſucceſſively into them, and ſo hinders the crane from turning backward, and detains the weight in any part of its aſcent, if the man who works at the winch ſhould accidentally happen to quit his hold, or chooſe to reſt himſelf before the weight be quite drawn up.

In order to let down the weight, a man pulls down one end of a lever of the ſecond kind, which lifts the catch of the ratchet-wheel, and gives the weight liberty to deſcend. But, if the deſcent be too quick, he pulls the lever a little farther down, ſo as to make it rub againſt the outer edge of a round wheel; by which means he lets down the weight as ſlowly as he pleaſes: and, by pulling a little harder, he may ſtop the weight, if needful, in any part of its deſcent.

If

If he accidentally quits hold of the lever, the catch immediately falls, and ſtops both the weight and the whole machine.

This crane is repreſented in PLATE I. where *A* is the great wheel, and *B* its axle on which the rope *C* winds. This rope goes over a pulley *D* in the end of the arm of the gib *E*, and draws up the weight *F*, as the winch *G* is turned round. *H* is the largeſt trundle, *I* the next, and *K* is the axis of the ſmalleſt trundle, which is ſuppoſed to be hid from view by the upright ſupporter *L*. A trundle *M* is turned by the great wheel, and on the axis of this trundle is fixed the ratchet-wheel *N*, into the teeth of which the catch *O* falls. *P* is the lever, from which goes a rope *QQ*, over a pulley *R* to the catch; one end of the rope being fixed to the lever, and the other end to the catch. *S* is an elaſtic bar of wood, one end of which is ſcrewed to the floor: and, from the other end goes a rope (out of ſight in the figure) to the further end of the lever, beyond the pin or axis on which it turns in the upright ſupporter *T*. The uſe of this bar is to keep up the lever from rubbing againſt the edge of the wheel *U*, and to let the catch keep in the teeth of the ratchet-wheel: But a weight hung to the farther end of the lever would do full as well as the elaſtic bar and rope.

When the lever is pulled down, it lifts the catch out of the ratchet-wheel, by means of the rope *QQ*, and gives the weight *F* liberty to deſcend: but if the lever *P* be pulled a little farther down than what is ſufficient to lift the catch *O* out of the ratchet-wheel *N*, it will rub againſt the edge of the wheel *U*, and thereby hinder the too quick deſcent of the weight; and will quite ſtop the weight if pulled hard. And if the man who pulls the lever, ſhould happen inadvertently to let it go; the elaſtic bar will ſuddenly pull it up, and the catch will fall down and ſtop the machine.

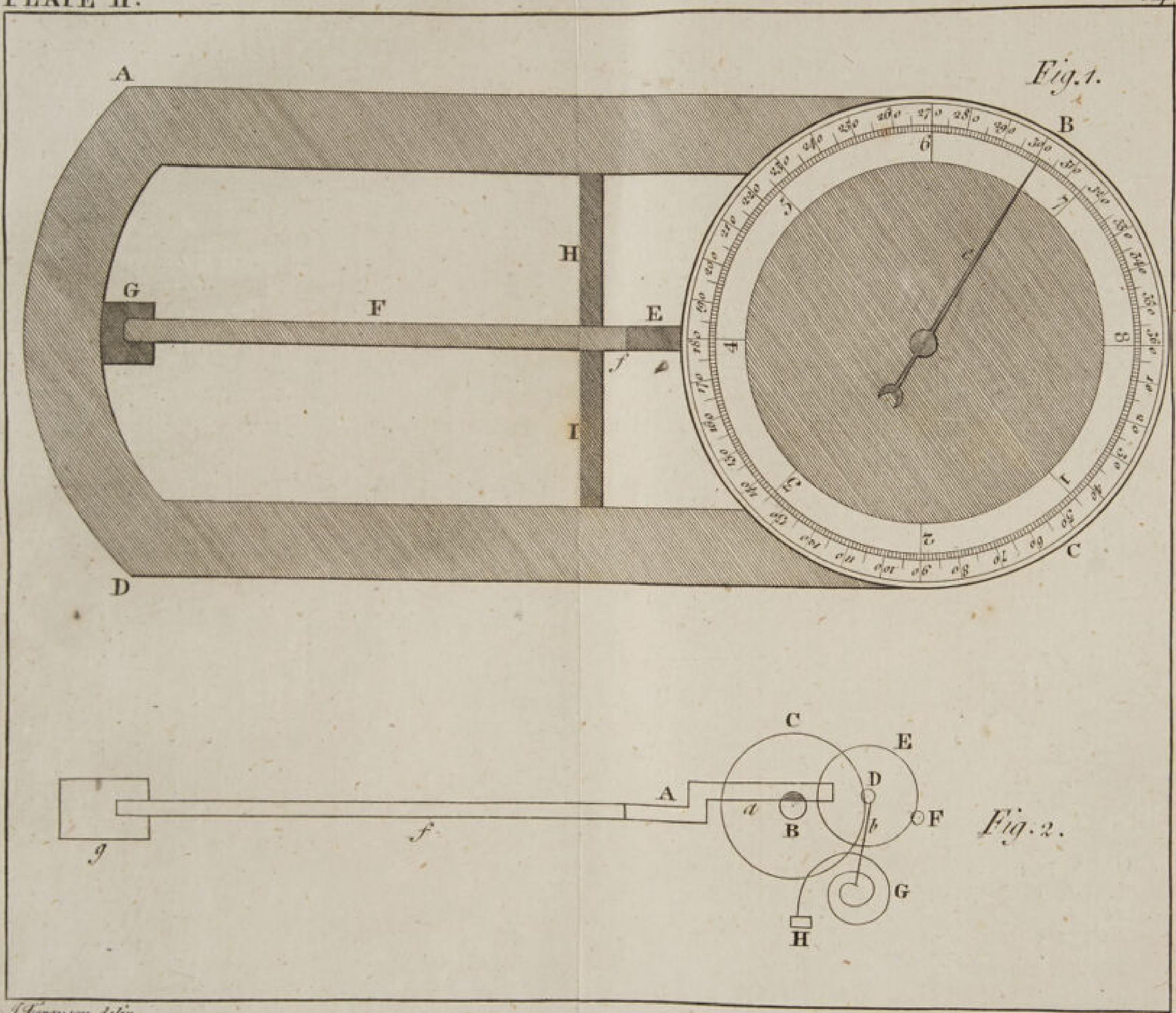
WW are two upright rollers above the axis or upper gudgeon of the gib *E:* their uſe is to let the rope *C* bend upon them, as the gib is turned to either ſide, in order to bring the weight over the place where it is intended to be let down.

N. B. The rollers ought to be ſo placed, that if the rope *C* be ſtretched cloſe by their utmoſt ſides, the half thickneſs of the rope may be perpendicularly over the center of the upper gudgeon of the gib. For then, and in no other poſition of the rollers, the length of the rope between the pulley in the gib and the axle of the great wheel will be always the ſame, in all poſitions of the gib: and the gib will remain in any poſition to which it is turned.

When either of the trundles is not turned by the winch in working the crane, it may be drawn off from the wheel, after the pin near the axis of the trundle is drawn out, and the thick piece of wood is raiſed a little behind the outward ſupporter of the axis of the trundle. But this is not material: for, as the trundle has no friction on its axis but what is occaſioned by its weight, it will be turned by the wheel without any ſenſible reſiſtance in working the crane.

A Pyro-

PLATE II.



J. Ferguson delin.

J. Mynde sc.

A Pyrometer, that makes the Expanſion of Metals by Heat viſible to the five and forty thouſandth Part of an Inch.

The upper ſurface of this machine is repreſented by *Fig.* 1. of Plate II. Its frame *ABCD* is made of mahogany wood, on which is a circle divided into 360 equal parts; and within that circle is another, divided into 8 equal parts. If the ſhort bar *E* be puſhed one inch forward (or toward the center of the circle) the index *e* will be turned 125 times round the circle of 360 parts or degrees. As 125 times 360 is 45,000, 'tis evident, that if the bar *E* be moved only the 45,000dth part of an inch, the index will move one degree of the circle. But as in my pyrometer, the circle is 9 inches in diameter, the motion of the index is viſible to half a degree, which anſwers to the ninety thouſandth part of an inch in the motion or puſhing of the ſhort bar *E*.

One end of a long bar of metal *F* is laid into a hollow place in a piece of iron *G*, which is fixed to the frame of the machine; and the other end of this bar is laid againſt the end of the ſhort bar *E*, over the ſupporting croſs bar *HI*: and, as the end *f* of the long bar is placed cloſe againſt the end of the ſhort bar, it is plain, that if *F* expands, it will puſh *E* forward, and turn the index *e*.

The machine ſtands on four ſhort pillars, high enough from a table, to let a ſpirit-lamp be put on the table under the bar *F*; and when that is done, the heat of the flame of the lamp expands the bar, and turns the index.

There

There are bars of different metals, as ſilver, braſs, and iron; all of the ſame length as the bar *F*, for trying experiments on the different expanſions of different metals, by equal degrees of heat applied to them for equal lengths of time; which may be meaſured by a pendulum, that ſwings ſeconds. Thus,

Put on the braſs bar *F*, and ſet the index to the 360th degree: then put the lighted lamp under the bar, and count the number of ſeconds in which the index goes round the plate, from 360 to 360 again; and then blow out the lamp, and take away the bar.

This done, put on an iron bar *F* where the braſs one was before, and then ſet the index to the 360th degree again. Light the lamp, and put it under the iron bar, and let it remain juſt as many ſeconds as it did under the braſs one; and then blow it out, and you will ſee how many degrees the index has moved in the circle: and by that means you will know in what proportion the expanſion of iron is to the expanſion of braſs; which I find to be as 210 is to 360, or as 7 is to 12.—By this method, the relative expanſions of different metals may be found.

The bars ought to be exactly of equal ſize; and to have them ſo, they ſhould be drawn, like wire, through a hole.

When the lamp is blown out, you will ſee the index turn backward; which ſhews that the metal contracts as it cools.

The inſide of this pyrometer is conſtructed as follows.

In

In *Fig.* 2. *Aa* is the ſhort bar which moves between rollers; and, on the ſide *a* it has 15 teeth in an inch, which take into the leaves of a pinion *B* (12 in number) on whoſe axis is the wheel *C* of 100 teeth, which take into the 10 leaves of the pinion *D*, on whoſe axis is the wheel *E* of 100 teeth, which take into the 10 leaves of the pinion *F*, on the top of whoſe axis is the index above-mentioned.

Now, as the wheels *C* and *E* have 100 teeth each, and the pinions *D* and *F* have ten leaves each; it is plain, that if the wheel *C* turns once round, the pinion *F* and the index on its axis will turn 100 times round. But, as the firſt pinion *B* has only 12 leaves, and the bar *Aa* that turns it has 15 teeth in an inch, which is 12 and a fourth part more; one inch motion of the bar will cauſe the laſt pinion *F* to turn a hundred times round, and a fourth part of a hundred over and above, which is 25. So that, if *Aa* be puſhed one inch, *F* will be turned 125 times round.

A ſilk thread *b* is tied to the axis of the pinion *D*, and wound ſeveral times round it; and the other end of the thread is tied to a piece of ſlender watch-ſpring *G* which is fixed into the ſtud *H*. So that, as the bar *f* expands, and puſhes the bar *Aa* forward, the thread winds round the axle, and draws out the ſpring; and as the bar contracts, the ſpring pulls back the thread, and turns the work the contrary way, which puſhes back the ſhort bar *Aa* againſt the long bar *f*. This ſpring always keeps the teeth of the wheels in contact with the leaves of

 the

the pinions, and ſo prevents any ſhake in the teeth.

In *Fig.* 1. the eight diviſions of the inner circle are ſo many thouſandth parts of an inch in the expanſion or contraction of the bars; which is juſt one thouſandth part of an inch for each diviſion moved over by the index.

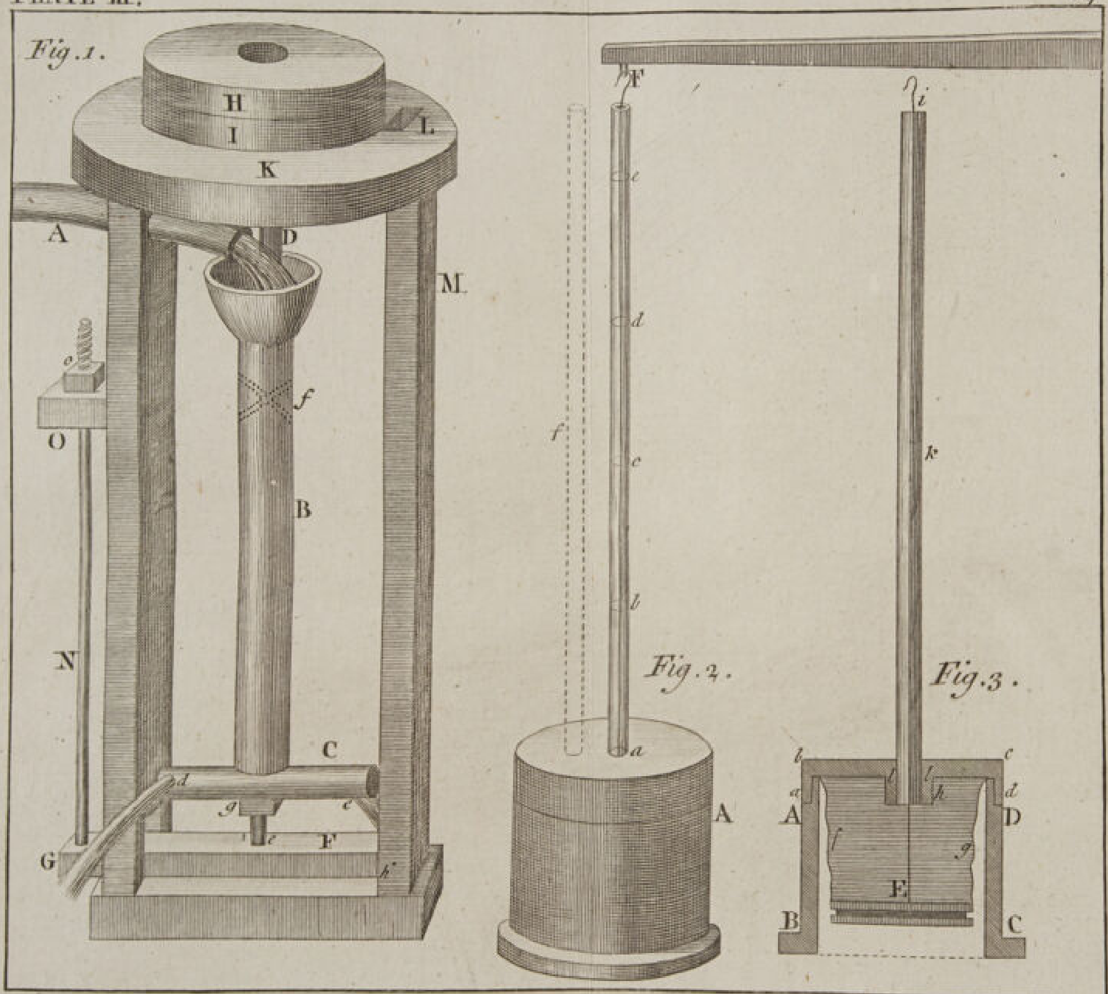
A Water-Mill, invented by Dr. Barker, *that has neither Wheel nor Trundle.*

This machine is repreſented by *Fig.* 1. of Plate III. in which, *A* is a pipe or channel that brings water to the upright tube *B*. The water runs down the tube, and thence into the horizontal trunk *C*, and runs out through holes at *d* and *e* near the ends of the trunk on the contrary ſides thereof.

The upright ſpindle *D* is fixt in the bottom of the trunk, and ſcrewed to it below by the nut *g*; and is fixt into the trunk by two croſs bars at *f*: ſo that, if the tube *B* and trunk *C* be turned round, the ſpindle *D* will be turned alſo.

The top of the ſpindle goes ſquare into the rynd of the upper mill-ſtone *H*, as in common mills; and, as the trunk, tube, and ſpindle turn round, the mill-ſtone is turned round thereby. The lower, or quieſcent mill-ſtone is repreſented by *I*; and *K* is the floor on which it reſts, and wherein is the hole *L* for letting the meal

PLATE III. Sup.



J. Ferguson delin.

J. Mynde sc.

meal run through, and fall down into a trough which may be about *M.* The hoop or caſe that goes round the mill-ſtone reſts on the floor *K*, and ſupports the hopper, in the common way. The lower end of the ſpindle turns in a hole in the bridge-tree *G F*, which ſupports the mill-ſtone, tube, ſpindle, and trunk. This tree is moveable on a pin at *b*, and its other end is ſupported by an iron rod *N* fixt into it, the top of the rod going through the fixt bracket *O*, and having a ſcrew-nut *o* upon it, above the bracket. By turning this nut forward or backward, the mill-ſtone is raiſed or lowered at pleaſure.

While the tube *B* is kept full of water from the pipe *A*, and the water continues to run out from the ends of the trunk; the upper mill-ſtone *H*, together with the trunk, tube, and ſpindle turns round. But, if the holes in the trunk were ſtopt, no motion would enſue; even though the tube and trunk were full of water. For,

If there were no hole in the trunk, the preſſure of the water would be equal againſt all parts of its ſides within. But, when the water has free egreſs through the holes, its preſſure there is entirely removed: and the preſſure againſt the parts of the ſides which are oppoſite to the holes, turns the machine.

HYDROSTATICS.

A Machine for demonſtrating that, on equal Bottoms, the Preſſure of Fluids is in Proportion to their perpendicular Heights, without any regard to their Quantities.

THIS is termed *The Hydroſtatical Paradox:* and the machine for ſhewing it is repreſented in *Fig.* 2. of Plate III. In which *A* is a box that holds about a pound of water, *a b c d e* a glaſs-tube fixt in the top of the box, having a ſmall wire within it; one end of the wire being hooked to the end *F* of the beam of a balance, and the other end of the wire fixt to a moveable bottom, on which the water lies, within the box; the bottom and wire being of equal weight with an empty ſcale (out of ſight in the figure) hanging at the other end of the balance. If this ſcale be pulled down, the bottom will be drawn up within the box, and that motion will cauſe the water to riſe in the glaſs-tube.

Put one pound weight into the ſcale, which will move the bottom a little, and cauſe the water to appear juſt in the lower end of the tube at *a*; which ſhews that the water preſſes with the force of one pound on the bottom: put another pound into the ſcale, and the water will riſe from *a* to *b* in the tube, juſt twice as high above the bottom as it was when at *a*; and then, as its preſſure on the bottom ſupports two pound weight in the ſcale, it is plain that the preſſure on the bottom is then equal to two pounds. Put a third pound weight in the ſcale, and the water

water will be raiſed from *b* to *c* in the tube, three times as high above the bottom as when it began to appear in the tube at *a*; which ſhews, that the ſame quantity of water that preſſed, but with the force of one pound on the bottom, when raiſed no higher than *a*, preſſes with the force of three pounds on the bottom when raiſed three times as high to *c* in the tube. Put a fourth pound weight into the ſcale, and it will cauſe the water to riſe in the tube from *c* to *d*, four times as high as when it was all contained in the box, which ſhews that its preſſure then upon the bottom is four times as great as when it lay all within the box. Put a fifth pound weight into the ſcale, and the water will riſe in the tube from *d* to *e*, five times as high as it was above the bottom before it roſe in the tube; which ſhews that its preſſure on the bottom is then equal to five pounds, ſeeing that it ſupports ſo much weight in the ſcale. And ſo on, if the tube was ſtill longer; for it would ſtill require an additional pound put into the ſcale, to raiſe the water in the tube to an additional height equal to the ſpace *de*; even if the bore of the tube was ſo ſmall as only to let the wire move freely within it, and leave room for any water to get round the wire.

Hence we infer, that if a long narrow pipe or tube was fixed in the top of a caſk full of liquor, and if as much liquor was poured into the tube as would fill it, even though it were ſo ſmall as not to hold an ounce weight of liquor; the preſſure ariſing from the liquor in the tube would be as great upon the bottom, and

and be in as much danger of burfting it out, as if the cafk was continued up, in its full fize, to the height of the tube, and filled with liquor.

In order to account for this furprifing affair, we muft confider that fluids prefs equally in all manner of directions; and confequently that they prefs juft as ftrongly upward as they do downward. For, if another tube, as *f*, be put into a hole made into the top of the box, and the box be filled with water; and then, if water be poured in at the top of the tube *a b c d e*, it will rife in the tube *f* to the fame height as it does in the other tube: and if you leave off pouring, when the water is at *c*, or any other place in the tube *a b c d e*, you will find it juft as high in the tube *f*: and if you pour in water to fill the firft tube, the fecond will be filled alfo.

Now it is evident that the water rifes in the tube *f*, from the downward preffure of the water in the tube *a b c d e*, on the furface of the water, contiguous to the infide of the top of the box; and as it will ftand at equal heights in both tubes, the upward preffure in the tube *f* is equal to the downward preffure in the other tube. But, if the tube *f* were put in any other part of the top of the box, the rifing of the water in it would ftill be the fame: or, if the top was full of holes, and a tube put into each of them, the water would rife as high in each tube as it was poured into the tube *a b c d e*; and then the moveable bottom would have the weight of the water in all the tubes to bear, befide the weight of all the water in the box.

And

And ſeeing that the water is preſſed upward into each tube, it is evident that, if they be all taken away, excepting the tube *abcde*, and the holes in which they ſtood be ſtopt up; each part, thus ſtopt, will be preſſed as much upward as was equal to the weight of water in each tube. So that, the upward preſſure againſt the inſide of the top of the box, on every part equal in breadth to the width of the tube *abcde*, will be preſſed upward with a force equal to the whole weight of water in the tube. And conſequently, the whole upward preſſure againſt the top of the box, ariſing from the weight or downward preſſure of the water in the tube will be equal to the weight of a column of water of the ſame height with that in the tube, and of the ſame thickneſs as the width of the inſide of the box: and this upward preſſure againſt the top will re-act downward againſt the bottom and be as great thereon, as would be equal to the weight of a column of water as thick as the moveable bottom is broad, and as high as the water ſtands in the tube. And thus, the paradox is ſolved.

The moveable bottom has no friction againſt the inſide of the box, nor can any water get between it and the box. The method of making it ſo, is as follows:

In *Fig.* 3. *ABCD* repreſents a ſection of the box, and *abcd* is the lid or top thereof, which goes on tight, like the lid of a common paper ſnuff-box. *E* is the moveable bottom, with a groove around its edge, and it is put into a bladder *fg*, which is tied cloſe around it in the groove by a ſtrong waxed thread; the bladder coming up like a purſe within the box, and put over the top of it at *a* and *d* all round, and then the lid preſſed on. So that, if water be poured in through the hole *ll* of the lid, it will lie upon the bottom *E*, and be contained in the ſpace *fEgh* within the bladder; and the bottom may be raiſed by pulling the wire *i*, which is fixed to it at *E*: and by thus pulling the wire, the water will be lifted up in the tube *k*, and as the bottom does not touch againſt the inſide of the box, it moves without friction.

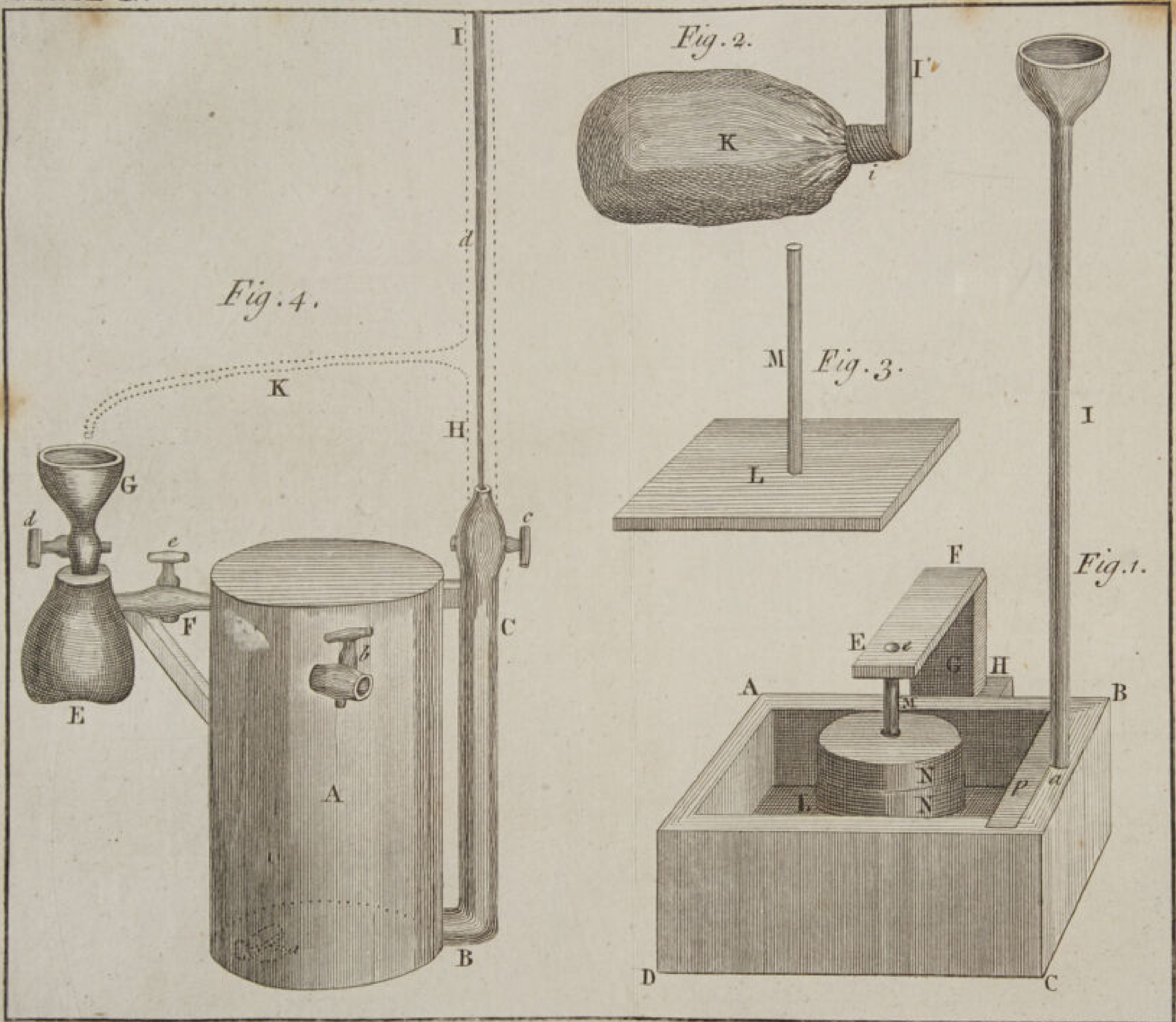
Now, ſuppoſe the diameter of this round bottom to be three inches (in which caſe, the area thereof will be 9 circular inches) and the diameter of the bore of the tube to be a quarter of an inch; the whole area of the bottom will be 144 times as great as the area of the top of a pin that would fill the tube like a cork.

And hence it is plain, that if the moveable bottom be raiſed only the 144th part of an inch, the water will thereby be raiſed a whole inch in the tube; and conſequently, that if the bottom be raiſed one inch, it would raiſe the water to the top of a tube 144 inches, or 12 feet, in height.

N. B. The box muſt be open below the moveable bottom, to let in the air. Otherwiſe, the preſſure of the atmoſphere would be ſo great upon the moveable bottom, if it be three inches in diameter, as to require 108 pounds in the ſcale, to balance that preſſure, before the bottom could begin to move.

A Machine,

PLATE IV. Supp.



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A Machine, to be ſubſtituted in place of the common Hydroſtatical Bellows.

In *Fig.* 1. of PLATE IV. *ABCD* is an oblong ſquare box, in one end of which is a round groove, as at *a*, from top to bottom, for receiving the upright glaſs tube *I*, which is bent to a right angle at the lower end (as at *i* in *Fig.* 2.) and to that part is tied the neck of a large bladder *K* (*Fig.* 2.) which lies in the bottom of the box. Over this bladder is laid the moveable board *L* (*Fig.* 1 and 3.), in which is fixt an upright wire *M*; and leaden weights *NN*, to the amount of 16 pounds, with holes in their middle, which are put upon the wire, over the board, and preſs upon it with all their force.

The croſs bar *p* is then put on, to ſecure the tube from falling, and keep it in an upright poſition: And then the piece *EFG* is to be put on, the part *G* ſliding tight into the dove-tailed groove *H*, to keep the weights *NN* horizontal, and the wire *M* upright; there being a round hole *e* in the part *EF* for receiving the wire.

There are four upright pins in the four corners of the box within, each almoſt an inch long, for the board *L* to reſt upon: to keep it from preſſing the ſides of the bladder below it cloſe together at firſt.

The whole machine being thus put together, pour water into the tube at top; and the water will run down the tube into the bladder below the board; and after the bladder has been filled up to the board, continue pouring water into the tube, and the upward preſſure which it will excite in the bladder, will raiſe the board with all the weight upon it, even though the bore of the tube ſhould be ſo ſmall, that leſs than an ounce of water would fill it.

This machine acts upon the ſame principle, as the one laſt deſcribed, concerning the *Hydroſtatical paradox*. For, the upward preſſure againſt every part of the board (which the bladder touches) equal in area to the area of the bore of the tube, will be preſſed upward with a force equal to the weight of the water in the tube; and the ſum of all theſe preſſures, againſt ſo many areas of the board, will be ſufficient to raiſe it with all the weights upon it.

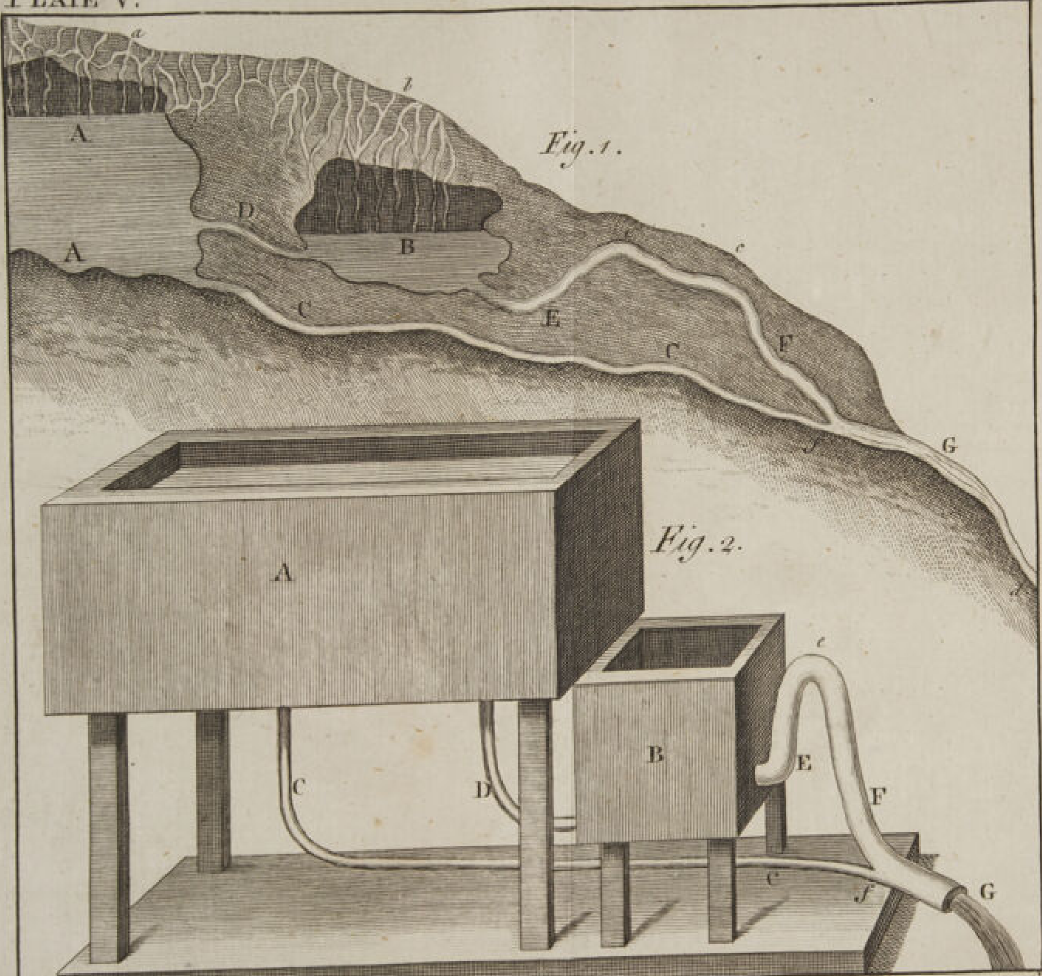
In my opinion, nothing can exceed this ſimple machine, in making the upward preſſure of fluids evident to ſight.

The Cauſe of reciprocating Springs, and of ebbing and flowing Wells, explained.

In *Fig.* 1. of PLATE V. let *abcd* be a hill, within which is a large cavern *AA* near the top, filled or fed by rains and melted ſnow on the top *a*, making their way through chinks and crannies into the ſaid cavern, from which proceeds a ſmall ſtream *CC* within the body of the hill, and iſſues out in a ſpring at *G* on the ſide of the hill, which will run conſtantly while the cavern is fed with water.

From the ſame cavern *AA*, let there be a ſmall channel *D*, to carry water into the cavern *B*;

PLATE V. Sup



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B; and from that cavern let there be a bended channel *E e F*, larger than *D*, joining with the former channel *C C*, as at *f* before it comes to the ſide of the hill: and let the joining at *f* be below the level of the bottom of both theſe caverns.

As the water riſes in the cavern *B*, it will riſe as high in the channel *E e F*: and when it riſes to the top of that channel at *e*, it will run down the part *e F G*, and make a ſwell in the ſpring *G*, which will continue till all the water is drawn off from the cavern *B*, by the natural ſyphon *E e F* (which carries off the water faſter from *B* than the channel *D* brings water to it) and then the ſwell will ſtop, and only the ſmall channel *C C* will carry water to the ſpring *G*, till the cavern *B* is filled to *B* again by the rill *D*; and then the water being at the top *e* of the channel *E e F*, that channel will act again as a ſyphon, and carry off all the water from *B* to the ſpring *G*, and ſo make a ſwelling flow of water at *G* as before.

To illuſtrate this by a machine (*Fig.* 2.) let *A* be a large wooden box, filled with water; and let a ſmall pipe *C C* (the upper end of which is fixed into the bottom of the box) carry water from the box to *G*, where it will run off conſtantly, like a ſmall ſpring. Let another ſmall pipe *D* carry water from the ſame box to the box or well *B*, from which let a ſyphon *E e F* proceed, and join with the pipe *C C* at *f*: the bore of the ſyphon being larger than the bore of the feeding-pipe *D*. As the water from this pipe riſes in the well *B*, it will alſo riſe as high in the ſyphon *E e F*; and when the ſyphon is

 full

full to the top *e*, the water will run over the bend *e*, down the part *e F*, and go off at the mouth *G*; which will make a great ſtream at *G*: and that ſtream will continue, till the ſyphon has carried off all the water from the well *B*; the ſyphon carrying off the water faſter from *B* than the pipe *D* brings water to it: and then the ſwell at *G* will ceaſe, and only the water from the ſmall pipe *CC* will run off at *G*, till the pipe *D* fills the well *B* again; and then the ſyphon will run, and make a ſwell at *G* as before.

And thus, we have an artificial repreſentation of an ebbing and flowing well, and of a reciprocating ſpring, in a very natural and ſimple manner.

HYDRAULICS.

An Account of the Principles by which Mr. Blakey *propoſes to raiſe Water from Mines, or from Rivers, to ſupply Towns and Gentlemen's Seats, by his new invented Fire-Engine, for which he has received* His MAJESTY'*s Patent.*

ALTHOUGH I am not at liberty to deſcribe the whole of this ſimple engine, yet I have the patentee's leave to deſcribe ſuch a one as will ſhew the principles by which it acts.

In *Fig.* 4. of PLATE IV. let *A* be a large, ſtrong, cloſe veſſel; immerſed in water up to the cock *b*, and having a hole in the bottom, with a valve *a* upon it, opening upward within the veſſel. A pipe *B G* riſes from the bottom of of this veſſel, and has a cock *c* in it near the top, which is ſmall there, for playing a very high jet *d*. *E* is the little boiler (not ſo big as a common tea-kettle) which is connected with the veſſel *A* by the ſteam-pipe *F*; and *G* is a funnel, through which a little water muſt be occaſionally poured into the boiler, to yield a proper quantity of ſteam. And a ſmall quantity of water will do for that purpoſe, becauſe ſteam poſſeſſes upward of 14,000 times as much ſpace or bulk as the water does from which it proceeds.

The veſſel *A* being immerſed in water up to the cock *b*, open that cock, and the water will ruſh in through the bottom of the veſſel at *a*, and fill it as high up as the water ſtands on its outſide; and the water, coming into the veſſel, will drive the air out of it (as high as the water riſes within it) through the cock *b*. When the water has done ruſhing into the veſſel, ſhut the cock *b*, and the valve *a* will fall down, and hinder the water from being puſhed out that way, by any force that preſſes on its ſurface. All the part of the veſſel above *b*, will be full of common air, when the water riſes to *b*.

Shut the cock *c*, and open the cocks *d* and *e*; then pour as much water into the boiler *E* (through the funnel *G*) as will about half fill the boiler; and then ſhut the cock *d*, and leave the cock *e* open.

This done, make a fire under the boiler *E*, and the heat thereof will raiſe a ſteam from the water in the boiler; and the ſteam will make its way thence, through the pipe *F*, into the veſſel *A*; and the ſteam will compreſs the air (above *b*) with a very great force upon the ſurface of the water in *A*.

When the top of the veſſel *A* feels very hot by the ſteam under it, open the cock *c* in the pipe *C*; and the air being ſtrongly compreſſed in *A*, between the ſteam and the water therein, will drive all the water out of the veſſel *A*, up the pipe *B C*, from which it will fly up in a jet to a very great height.—In my fountain, which is made in this manner after Mr. Blakey's, three tea-cup-fulls of water in the boiler will afford ſteam enough to play a jet 30 feet high.

When all the water is out of the veſſel *A*, and the compreſſed air begins to follow the jet, open the cocks *b* and *d* to let the ſteam out of the boiler *E* and veſſel *A*, and ſhut the cock *e* to prevent any more ſteam from getting into *A*; and the air will ruſh into the veſſel *A* through the cock *b*, and the water through the valve *a*; and ſo the veſſel will be filled up with water to the cock *b* as before. Then ſhut the cock *b* and the cocks *c* and *d*, and open the cock *e*; and then, the next ſteam that riſes in the boiler will make its way into the veſſel *A* again; and the operation will go on, as above.

When all the water in the boiler *E* is evaporated, and gone off into ſteam, pour a little more into the boiler, through the funnel *G*.

In order to make this engine raiſe water to any gentleman's houſe; if the houſe be on the bank of a river, the pipe *B C* may be continued

up

up to the intended height, in the direction *HI*. Or, if the houſe be on the ſide or top of a hill, at a diſtance from the river, the pipe, through which the water is forced up, mav be laid along on the hill, from the river or ſpring to the houſe.

The boiler may be ſed by a ſmall pipe *K*, from the water that riſes in the main pipe *BCHI*; the pipe *K* being of a very ſmall bore, ſo as to fill the funnel *G* with water in the time that the boiler *E* will require a freſh ſupply. And then, by turning the cock *d*, the water will fall from the funnel into the boiler. The funnel ſhould hold as much water as will about half fill the boiler.

When either of theſe methods of raiſing water, perpendicularly or obliquely, is uſed, there will be no occaſion for having the cock *c* in the main pipe *BCHI*: for ſuch a cock is requiſite only, when the engine is uſed as a fountain.

A contrivance may be very eaſily made, from a lever to the cocks *b*, *d*, and *e*; ſo that, by pulling the lever, the cocks *b* and *d* may be opened when the cock *e* muſt be ſhut; and the cock *e* be opened when *b* and *d* muſt be ſhut.

The boiler *E* ſhould be incloſed in a brick wall, at a little diſtance from it, all around; to give liberty for the flames of the fire under the boiler to aſcend round about it. By which means (the wall not covering the funnel *G*) the force of the ſteam will be prodigiouſly increaſed by the heat round the boiler; and the funnel and water in it will be heated from the boiler; ſo that, the boiler

boiler will not be chilled by letting cold water into it; and the rifing of the fteam will be fo much the quicker.

Mr. Blakey is the only perfon who ever thought of making ufe of air as an intermediate body between fteam and water: by which means, the fteam is always kept from touching the water, and confequently from being condenfed by it. And, on this new principle, he has obtained a patent: fo that no one (vary the engine how he will) can make ufe of air between fteam and water, without infringing on the patent, and being fubject to the penalties of the law.

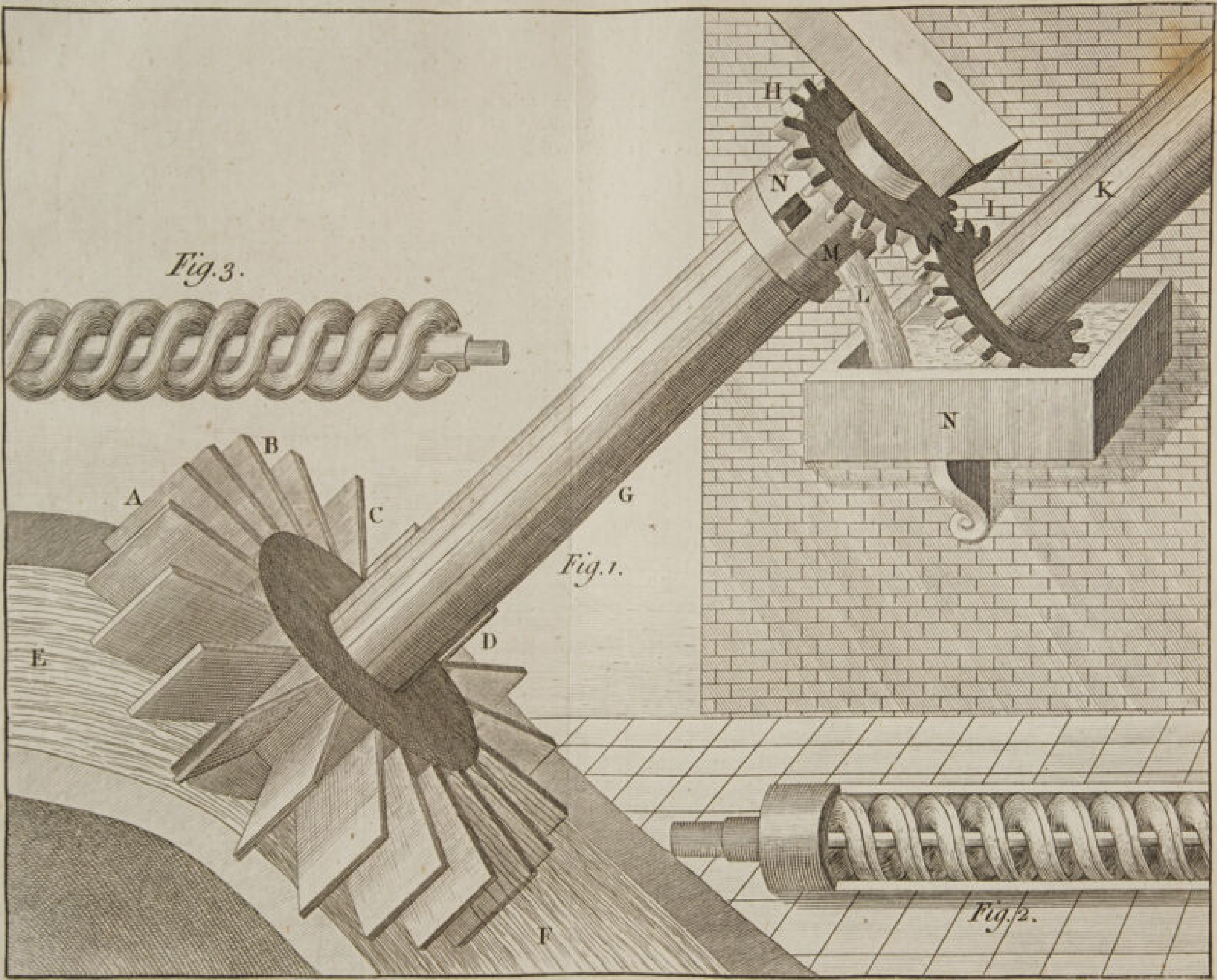
This engine may be built for a trifling expence, in comparifon of the common fire engine now in ufe: it will feldom need repairs, and will not confume half fo much fuel. And as it has no pumps with piftons, it is clear of all their friction: and the effect is equal to the whole ftrength or compreffive force of the fteam: which the effect of the common fire engine never is, on account of the great friction of the piftons in their pumps.

ARCHIMEDES's *Screw-Engine for raifing Water.*

In *Fig.* 1. of PLATE VI. *ABCD* is a wheel, which is turned round, according to the order of the letters, by the fall of water *EF*, which need not be more than three feet. The axle *G* of the wheel is elevated fo, as to make an angle of about 44 degrees with the horizon; and on the top of that axle is a wheel *H*, which turns fuch another wheel *I* of the fame number

of

PLATE VI. Sup:



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of teeth: the axle *K* of this laſt wheel being parallel to the axle *G* of the two former wheels.

The axle *G* is cut into a double-threaded ſcrew (as in *Fig.* 2.) exactly reſembling the ſcrew on the axis of the fly of a common jack, which muſt be (what is called) a right-handed ſcrew, like the wood-ſcrews, if the firſt wheel turns in the direction *ABCD*; but muſt be a left-handed ſcrew, if the ſtream turns the wheel the contrary way. And, which-ever way the ſcrew on the axle *G* be cut, the ſcrew on the axle *K* muſt be cut the contrary way; becauſe theſe axles turn in contrary directions.

The ſcrews being thus cut, they muſt be covered cloſe over with boards, like thoſe of a cylindrical caſk; and then they will be ſpiral tubes. Or, they may be made of tubes of ſtiff leather, and wrapt round the axles in ſhallow grooves cut therein; as in *Fig.* 3.

The lower end of the axle *G* turns conſtantly in the ſtream that turns the wheel, and the lower ends of the ſpiral tubes are open into the water. So that, as the wheel and axle are turned round, the water riſes in the ſpiral tubes, and runs out at *L*, through the holes *M*, *N*, as they come about below the axle. Theſe holes (of which there may be any number, as four or ſix) are in a broad cloſe ring on the top of the axle, into which ring, the water is delivered from the upper open ends of the ſcrew-tubes, and falls into the open box *N*.

The lower end of the axle *K* turns on a gudgeon, in the water in *N*; and the ſpiral tubes in that axle take up the water from *N*, and deliver it into ſuch another box under the top of *K*; on which there may be ſuch another wheel as *I*, to turn a third axle by ſuch a wheel upon it.—And in this manner, water may be raiſed to any given height, when there is a ſtream ſufficient for that purpoſe to act on the broad float boards of the firſt wheel.

A quadruple Pump-Mill for raiſing Water.

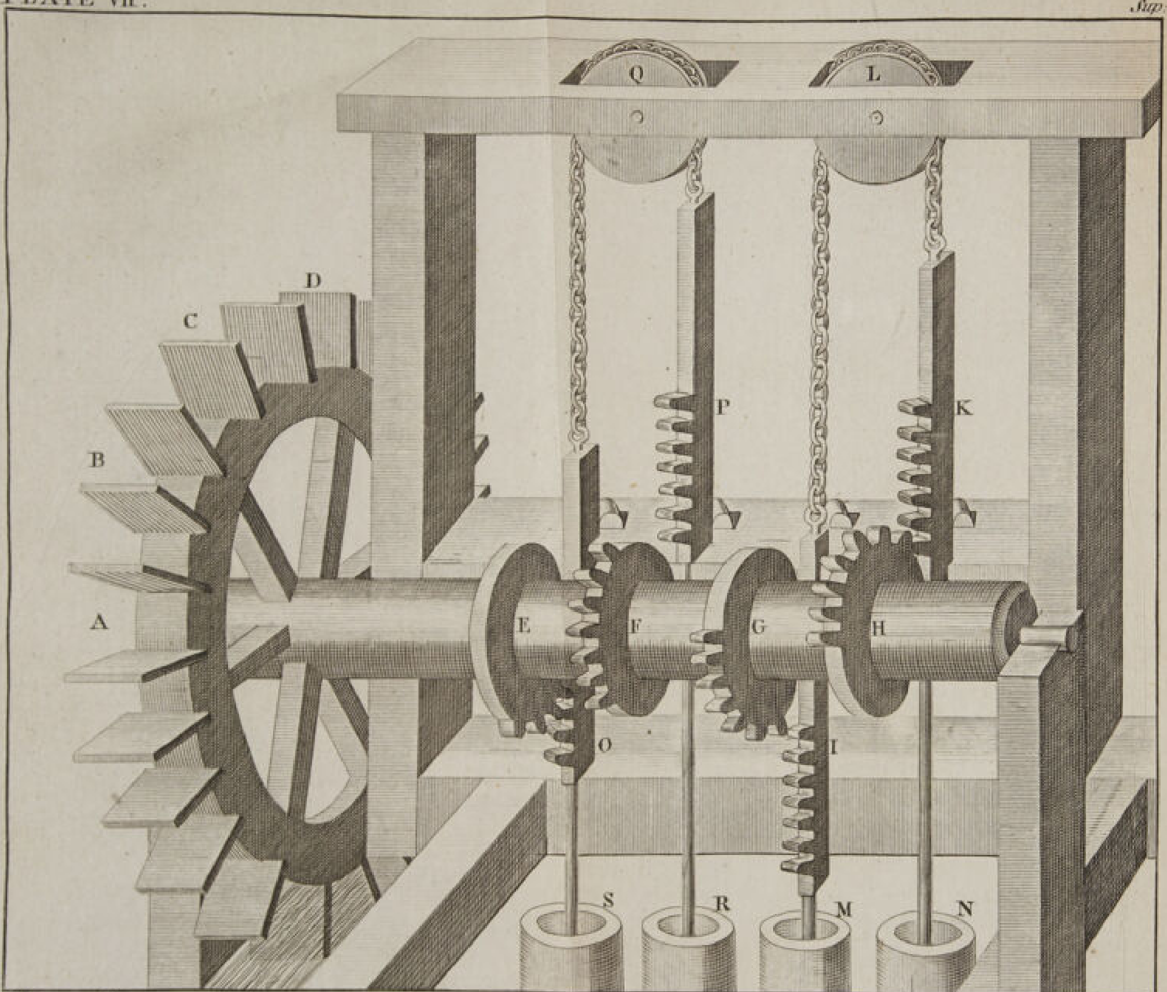
This engine is repreſented in Plate VII. in which *ABCD* is a wheel, turned by water according to the order of the letters. On the horizontal axis are four ſmall wheels, toothed almoſt half round: and the parts of their edges on which there are no teeth are cut down ſo, as to be even with the bottoms of the teeth where they ſtand.

The teeth of theſe four wheels take alternately into the teeth of four racks, which hang by two chains over the pullies *Q* and *L*; and to the lower ends of theſe racks there are four iron rods fixed, which go down into the four forcing pumps, *S*, *R*, *M* and *N*. And, as the wheels turn, the racks and pump-rods are alternately moved up and down.

Thus, ſuppoſe the wheel *G* has pulled down the rack *I*, and drawn up the rack *K* by the chain: as the laſt tooth of *G* juſt leaves the uppermoſt tooth of *I*, the firſt tooth of *H* is ready to take into the lowermoſt tooth of the rack *K* and pull it down as far as the teeth go; and

PLATE VII.

Sup:



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and then the rack *I* is pulled upward through the whole ſpace of its teeth, and the wheel *G* is ready to take hold of it, and pull it down again, and ſo draw up the other.——In the ſame manner, the wheels *E* and *F* work the racks *O* and *P*.

Theſe four wheels are fixed on the axle of the great wheel in ſuch a manner, with reſpect to the poſitions of their teeth; that, while they continue turning round, there is never one inſtant of time in which one or other of the pump-rods is not going down, and forcing the water. So that, in this engine, there is no occaſion for having a general air-veſſel to all the pumps, to procure a conſtant ſtream of water flowing from the upper end of the main pipe.

The piſtons of theſe pumps are ſolid plungers, the ſame as deſcribed in the fifth Lecture of my book, to which this is a Supplement. *See Plate* XI. *Fig.* 4. *of that book, with the deſcription of the figure.*

From each of theſe pumps, near the loweſt end, in the water, there goes off a pipe; with a valve on its fartheſt end from the pump; and theſe ends of the pipes all enter one cloſe box, into which they deliver the water: and into this box, the lower end of the main conduct pipe is fixed. So that, as the water is forced or puſhed into this box, it is alſo puſhed up the main pipe to the height that it is intended to be raiſed.

There is an engine of this ſort, deſcribed in *Ramelli*'s work: but I can truly ſay, that I never

never ſaw it till ſome time after I had made this model.

The ſaid model is not above twice as big as the figure of it, here deſcribed. I turn it by a winch fixed on the gudgeon of the axle behind the water wheel; and, when it was newly made, and the piſtons and valves in good order, I put tin pipes 15 feet high upon it, when they were joined together, to ſee what it could do. And I found, that in turning it moderately by the winch, it would raiſe a hogſhead of water in a hour, to the height of 15 feet.

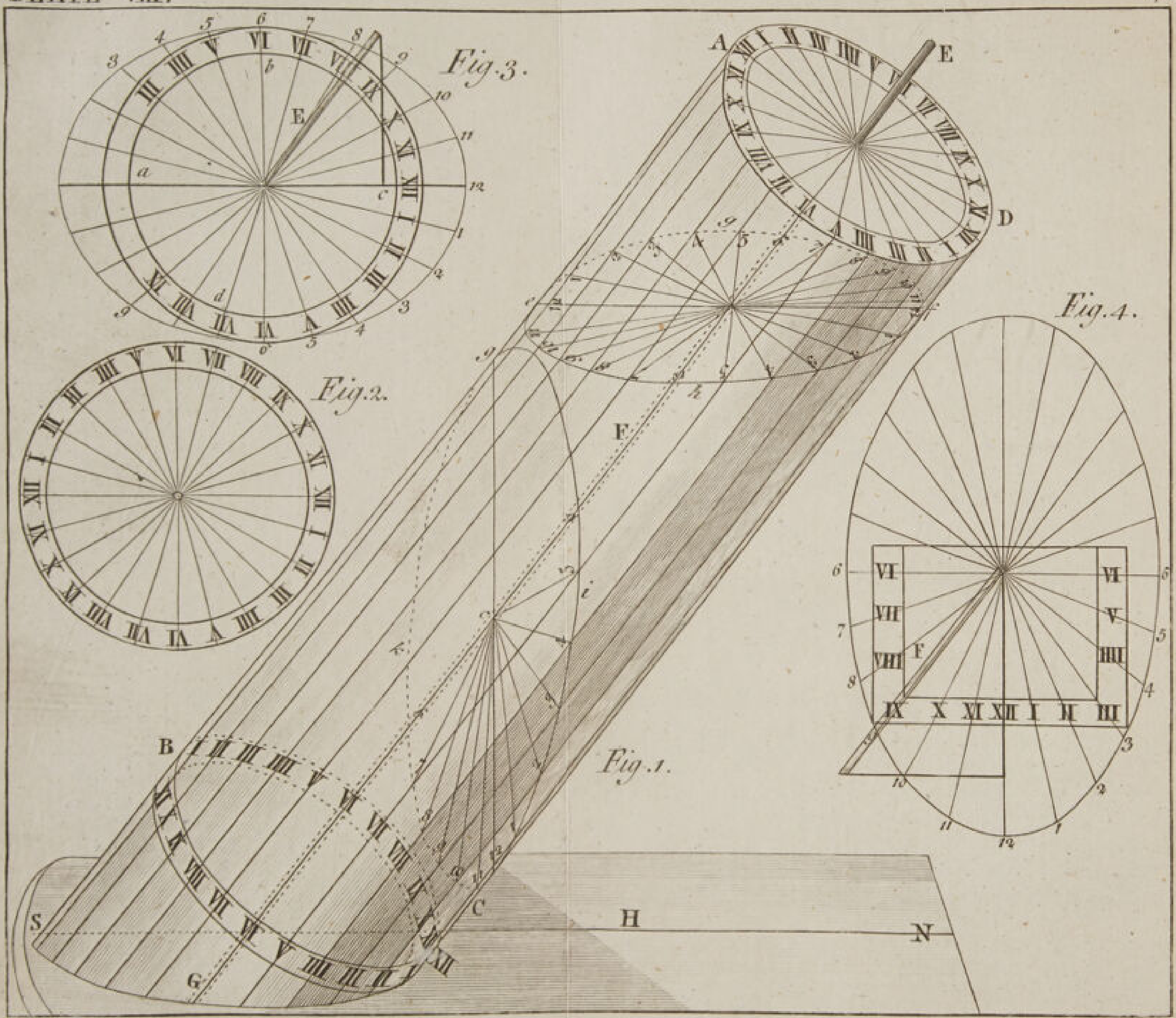
DIALING.

The univerſal Dialing Cylinder.

IN *Fig.* 1. of PLATE VIII. *ABCD* repreſents a cylindrical glaſs tube, cloſed at both ends with braſs plates, and having a wire or axis *EFG* fixt in the centers of the braſs plates at top and bottom. This tube is fixed to a horizontal board *H*, and its axis makes an angle with the board equal to the angle of the earth's axis with the horizon of any given place, for which the cylinder is to ſerve as a dial; And it muſt be ſet with its axis parallel to the axis of the world in that place; the end *E* pointing to the elevated pole. Or, it may be made to move upon a joint; and then it may be elevated for any particular latitude.

There are 24 ſtraight lines, drawn with a diamond, on the outſide of the glaſs, equidiſtant from each other, and all of them parallel to the axis. Theſe are the hour-lines; and the hours are

PLATE VIII. Sup:



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are ſet to them as in the figure: the XII next *B* ſtands for midnight, and the oppoſite XII, next the board *H*, ſtands for mid-day or noon.

The axis being elevated to the latitude of the place, and the foot-board ſet truly level, with the black line along its middle in the plane of the meridian, and the end *N* toward the north; the axis *EFG* will ſerve as a ſtile or gnomon, and caſt a ſhadow on the hour of the day, among the parallel hour-lines when the ſun ſhines on the machine. For, as the ſun's apparent diurnal motion is equable in the heavens, the ſhadow of the axis will move equably in the tube; and will always fall upon *that* hour-line which is oppoſite to the ſun, at any given time.

The braſs plate *AD*, at the top, is parallel to the equator, and the axis *EFG* is perpendicular to it. If right lines be drawn from the center of this plate, to the upper ends of the equidiſtant parallel lines on the outſide of the tube; theſe right lines will be the hour-lines on the equinoctial dial *AD*, at 15 degrees diſtance from each other: and the hour-letters may be ſet to them as in the figure. Then, as the ſhadow of the axis within the tube comes on the hour-lines of the tube, it will cover the like hour-lines on the equinoctial plate *AD*.

If a thin horizontal plate *ef* be put within the tube, ſo as its edge may touch the tube all around; and right lines be drawn from the center of that plate to thoſe points of its edge which are cut by the parallel hour-lines on the tube; theſe right lines will be the hour-lines of a horizontal dial, for the latitude to which the tube is elevated.

vated. For, as the ſhadow of the axis comes ſucceſſively to the hour-lines of the tube, and covers them, it will then cover the like hour-lines on the horizontal plate *ef*, to which the hours may be ſet; as in the figure.

If a thin vertical plate *g C*, be put within the tube, ſo as to front the meridian or 12 o'clock line thereof, and the edge of this plate touch the tube all around; and then, if right lines be drawn from the center of the plate to thoſe points of its edge which are cut by the parallel hour-lines on the tube; theſe right lines will be the hour-lines of a vertical ſouth-dial: and the ſhadow of the axis will cover them at the ſame times when it covers thoſe of the tube.

If a thin plate be put within the tube ſo, as to decline, or incline, or recline, by any given number of degrees; and right lines be drawn from its center to the hour-lines of the tube; theſe right lines will be the hour-lines of a declining, inclining, or reclining dial, anſwering to the like number of degrees, for the latitude to which the tube is elevated.

And thus, by this ſimple machine, all the principles of dialing are made very plain, and evident to the ſight. And the axis of the tube (which is parallel to the axis of the world in every latitude to which it is elevated) is the ſtile or gnomon for all the different kinds of ſun-dials.

And laſtly, if the axis of the tube be drawn out, with the plates *AD*, *ef*, and *gC* upon it; and ſet it up in ſun-ſhine, in the ſame poſition as they were in the tube; you will have an equi-

noctial

noctial dial *AD*, a horizontal dial *ef*, and a vertical ſouth dial *gC*; on all which, the time of the day will be ſhewn by the ſhadow of the axis or gnomon *EFG*.

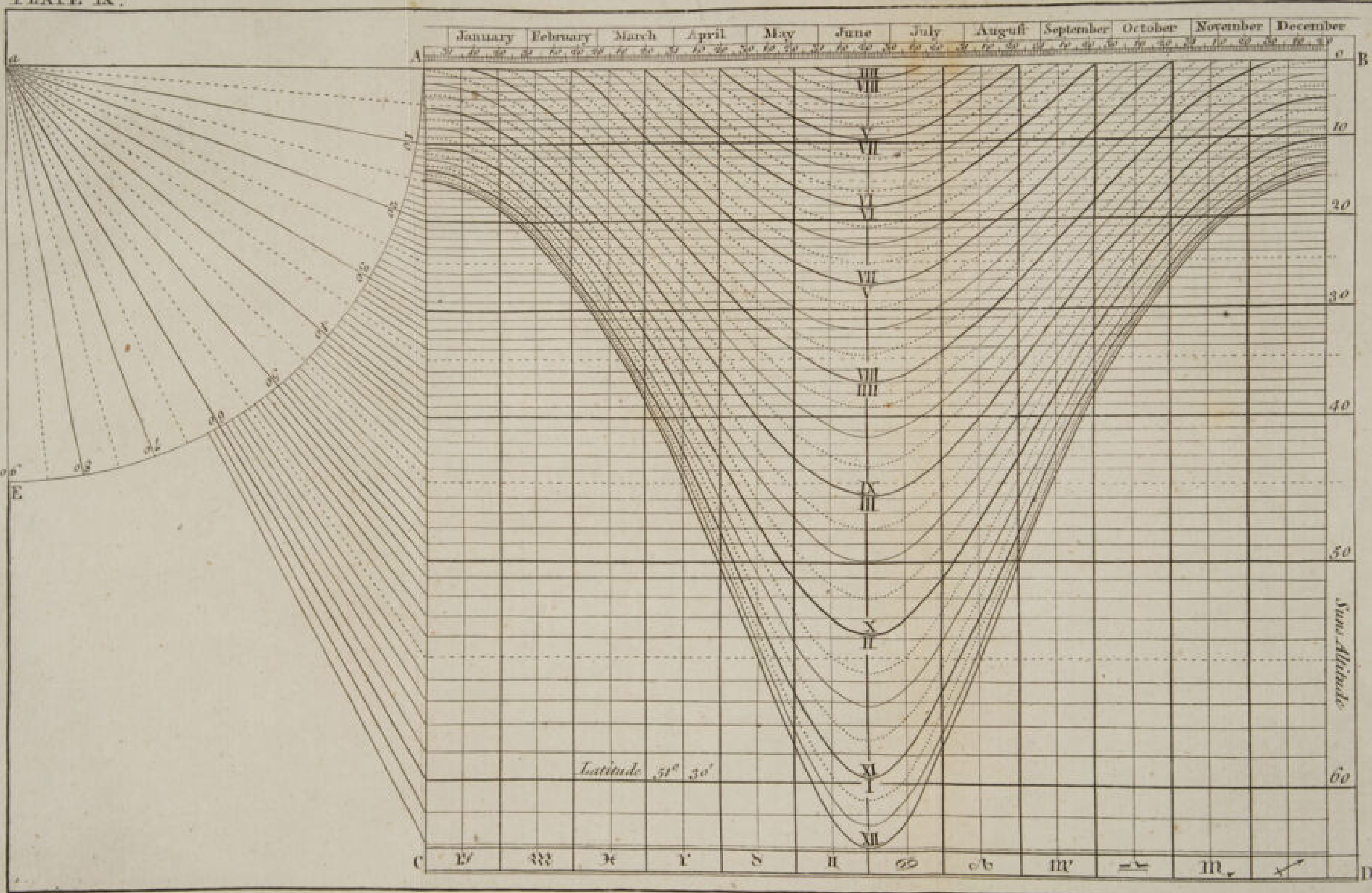
Let us now ſuppoſe that, inſtead of a glaſs tube, *ABCD* is a cylinder of wood; on which the 24 parallel hour-lines are drawn all around, at equal diſtances from each other; and that, from the points at top, where theſe lines end, right lines are drawn toward the center, on the flat ſurface *AD*. Theſe right lines will be the hour-lines on an equinoctial dial, for the latitude of the place to which the cylinder is elevated above the horizontal foot or pedeſtal *H*; and they are equidiſtant from each other, as in *Fig.* 2. which is a full view of the flat ſurface or top *AD* of the cylinder, ſeen obliquely in *Fig.* 1. And the axis of the cylinder (which is a ſtraight wire *EFG* all down its middle) is the ſtile or gnomon; which is perpendicular to the plane of the equinoctial dial, as the earth's axis is perpendicular to the plane of the equator.

To make a horizontal dial, by the cylinder, for any latitude to which its axis is elevated; draw out the axis and cut the cylinder quite through, as at *ehfg*, parallel to the horizontal board *H*, and take off the top part *eADfe*; and the ſection *ehfge* will be of an elliptical form, as in *Fig.* 3. Then, from the points of this ſection (on the remaining part *eBCf*) where the parallel lines on the outſide of the cylinder meet it, draw right lines to the center of the ſection; and they will be the true hour-lines for a horizontal dial, as *abcda* in *Fig.* 3. which may be included in a circle drawn on that ſection.

Then put the wire into its place again, and it will be a ſtile for caſting a ſhadow on the time of the day, on that dial. So, *E* (*Fig.* 3.) is the ſtile of the horizontal dial, parallel to the axis of the cylinder.

To make a vertical ſouth dial by the cylinder, draw out the axis, and cut the cylinder perpendicularly to the horizontal board *H*, as at *g i C k g*, beginning at the hour-line (*B g e A*) of XII, and making the ſection at right angles to the line *S H N* on the horizontal board. Then, take off the upper part *g A D C*, and the face of the ſection thereon will be elliptical, as ſhewn in *Fig.* 4. From the points in the edge of this ſection, where the parallel hour-lines on the round ſurface of the cylinder meet it, draw right lines to the center of the ſection; and they will be the true hour-lines on a vertical direct ſouth dial, for the latitude to which the cylinder was elevated: and will appear as in *Fig.* 4. on which the vertical dial may be made of a circular ſhape, or of a ſquare ſhape as repreſented in the figure. And *F* will be its ſtile parallel to the axis of the cylinder.

And thus, by cutting the cylinder any way, ſo as its ſection may either incline, or decline, or recline, by any given number of degrees; and from thoſe points in the edge of the ſection; where the outſide parallel hour-lines meet it, draw right lines to the center of the ſection; and they will be the true hour-lines, for the like declining, reclining, or inclining dial: and the axis of the cylinder will always be the gnomon or ſtile of the dial. For, which-ever way the plane of the dial lies, its ſtile (or the edge thereof that



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that casts the shadow on the hours of the day) must be parallel to the earth's axis, and point toward the elevated pole of the heaven.

To delineate a Sun-Dial on Paper; which, when pasted round a Cylinder of Wood, shall shew the Time of the Day, the Sun's Place in the Ecliptic, and his Altitude, at any Time of Observation. See PLATE IX.

Draw the right line *a A B*, parallel to the top of the paper; and, with any convenient opening of the compasses, set one foot in the end of the line at *a*, as a center, and with the other foot describe the quadrantal arc *A E*, and divide it into 90 equal parts or degrees. Draw the right line *A C*, at right angles to *a A B*, and touching the quadrant *A E* at the point *A*. Then, from the center *a*, draw right lines through as many degrees of the quadrant as are equal to the sun's altitude at noon, on the longest day of the year, at the place for which the dial is to serve; which altitude, at London, is 62 degrees: and continue these right lines till they meet the tangent line *A C*; and, from these points of meeting, draw straight lines across the paper, parallel to the first right line *A B*, and they will be the parallels of the sun's altitude, in whole degrees, from sun-rise till sun-set, on all the days of the year.—These parallels of altitude must be drawn out to the right line *B D*, which must be parallel to *A C*, and as far from it as is equal to the intended circumference of the cylinder on which the paper is to be pasted, when the dial is drawn upon it.

Divide the space between the right lines *A C* and *B D* (at top and bottom) into twelve equal

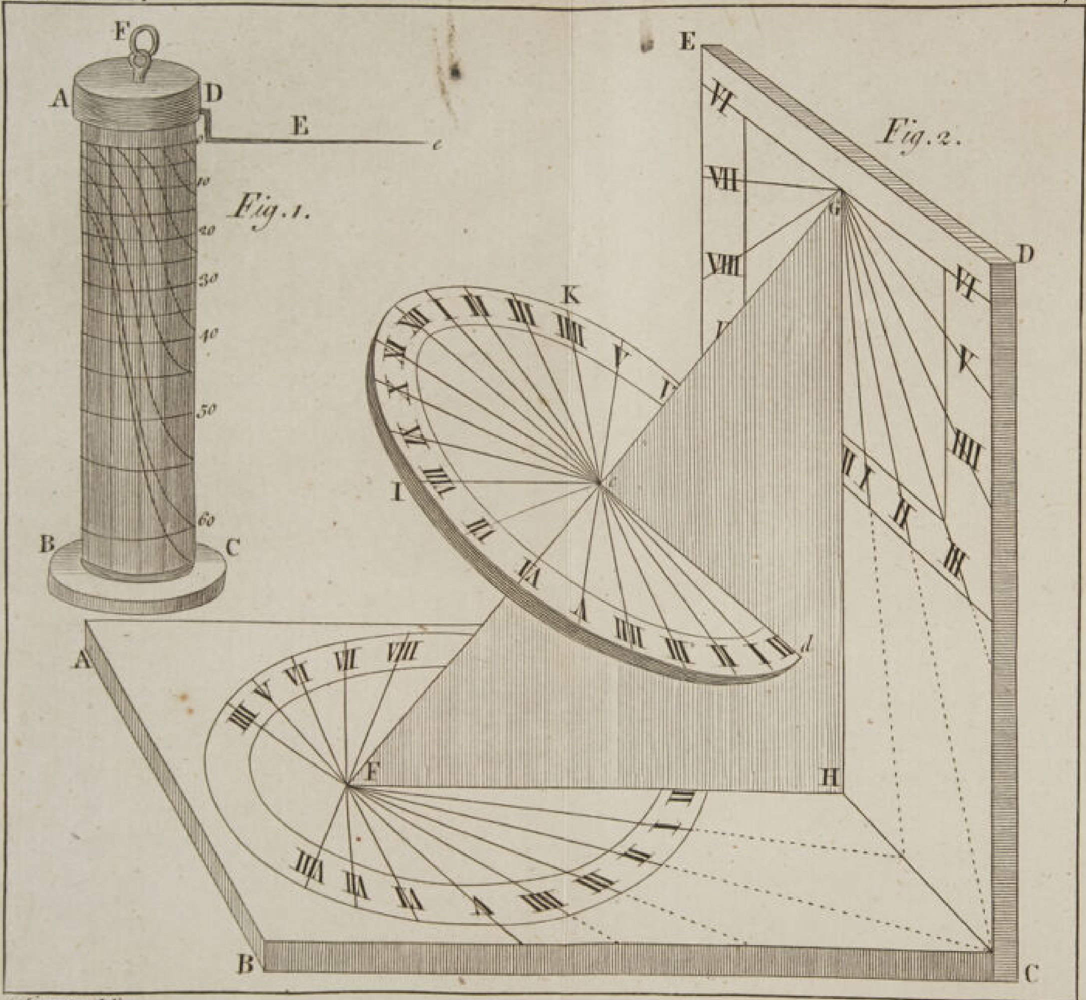
 parts,

parts, for the twelve ſigns of the ecliptic; and, from mark to mark of theſe diviſions at top and bottom, draw right lines parallel to *AC* and *BD*; and place the characters of the 12 ſigns in theſe twelve ſpaces, at the bottom, as in the figure: beginning with ♑ or Capricorn, and ending with ♓ or Piſces. The ſpaces including the ſigns ſhould be divided by parallel lines into halves; and if the breadth will admit of it without confuſion, into quarters alſo.

At the top of the dial, make a ſcale of the months and days of the year, ſo as the days may ſtand over the ſun's place for each of them in the ſigns of the ecliptic. The ſun's place, for every day of the year, may be found by any common ephemeris: and here it will be beſt to make uſe of an ephemeris for the ſecond year after leap-year; as the neareſt mean for the ſun's place on the days of the leap-year, and on thoſe of the firſt, ſecond, and third year after.

Compute the ſun's altitude for every hour (in the latitude of your place) when he is in the beginning, middle, and end of each ſign of the ecliptic; his altitude at the end of each ſign being the ſame as at the beginning of the next. And, in the upright parallel lines, at the beginning and middle of each ſign, make marks for theſe computed altitudes among the horizontal parallels of altitude, reckoning them downward, according to the order of the numeral figures ſet to them at the right hand, anſwering to the like diviſions of the quadrant at the left. And, through theſe marks, draw the curve hour-lines, and ſet the hours to them, as in the figure, reckoning the forenoon hours downward, and the

PLATE X. Sup:



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the afternoon hours upward.—The ſun's altitude ſhould alſo be computed for the half hours; and the quarter-lines may be drawn, very nearly in their proper places, by eſtimation and accuracy of the eye. Then, cut off the paper at the left hand, on which the quadrant was drawn, cloſe by the right line *AC*, and all the paper at the right hand cloſe by the right line *BD*; and cut it alſo cloſe by the top and bottom horizontal lines; and it will be fit for paſting round the cylinder.

This cylinder is repreſented in miniature by *Fig.* 1. PLATE X. It ſhould be hollow, to hold the ſtile *DE* when it is not uſed. The crooked end of the ſtile is put into a hole in the top *AD* of the cylinder; and the top goes on tightiſh, but muſt be made to turn round on the cylinder, like the lid of a paper ſnuff box. The ſtile muſt ſtand ſtraight out, perpendicular to the ſide of the cylinder, juſt over the right line *AB* in PLATE IX, where the parallels of the ſun's altitude begin: and the length of the ſtile, or diſtance of its point *e* from the cylinder, muſt be equal to the radius *aA* of the quadrant *AE* in PLATE IX.

The method of uſing this dial is as follows.

Place the horizontal foot *BC* of the cylinder on a level table where the ſun ſhines, and turn the top *AD* till the ſtile ſtands juſt over the day of the then preſent month. Then turn the cylinder about on the table, till the ſhadow of the ſtile falls upon it, parallel to thoſe upright lines which divide the ſigns; that is, till the ſhadow be parallel to a ſuppoſed axis in the middle of the cylinder: and then, the point, or loweſt end of the ſhadow, will fall upon the time of the day, as it is before or after noon, among the curve hour-lines; and will ſhew the ſun's altitude at that time, among the croſs parallels of his altitude, which go round the cylinder: and, at the ſame time, it will ſhew in what ſign of the ecliptic the ſun then is, and you may very nearly gueſs at the degree of the ſign, by eſtimation of the eye.

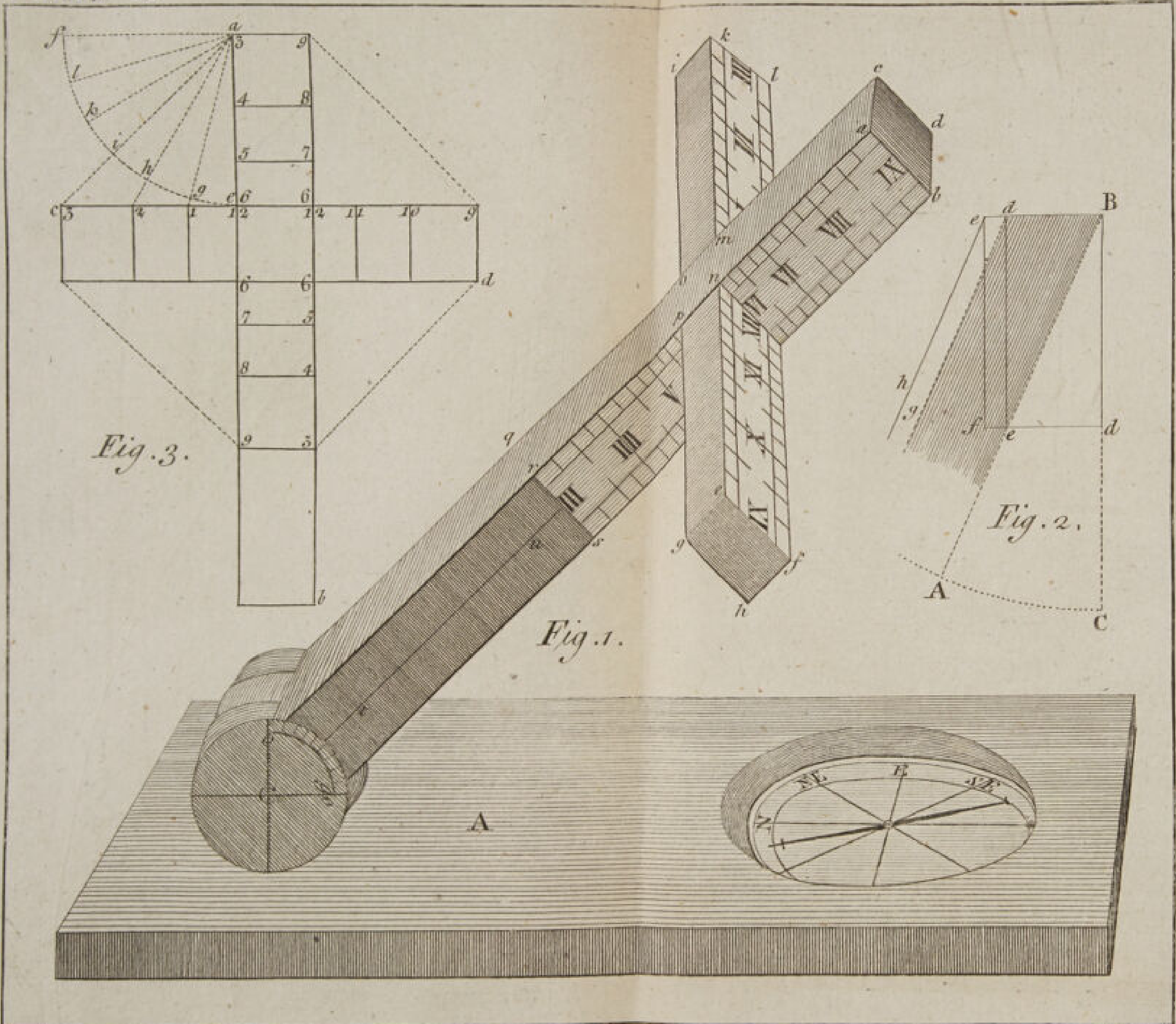
The ninth plate, on which this dial is drawn, may be cut out of the book, and paſted round a cylinder whoſe length is 6 inches and 6 tenths of an inch below the moveable top; and its diameter 2 inches and 24 hundred parts of an inch.—Or, I ſuppoſe the copper-plate prints of it may be had of the bookſellers in London. But it will only do for London, and other places of the ſame latitude.

When a level table cannot be had, the dial may be hung by the ring *F* at the top. And when it is not uſed, the wire that ſerves for a ſtile may be drawn out, and put up within the cylinder; and the machine carried in the pocket.

To make three Sun-dials upon three different Planes, ſo as they may all ſhew the Time of the Day by one Gnomon.

On the flat board *ABC*, deſcribe a horizontal dial, according to any of the rules laid down in the Lecture on Dialing; and to it fix its gnomon *FGH*, the edge of the ſhadow from the ſide *FG* being that which ſhews the time of the day.

To this horizontal or flat board, join the upright board *EDC*, touching the edge *GH* of the gnomon. Then, making the top of the gnomon



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gnomon at *G* the center of the vertical ſouth dial, deſcribe a ſouth dial on the board *E D C*.

Laſtly, on a circular plate *I K* deſcribe an equinoctial dial, all the hours of which dial are equidiſtant from each other, and making a ſlit *c d* in that dial, from its edge to its center, in the XII o'clock line; put the ſaid dial perpendicularly on the gnomon *F G*, as far as the ſlit will admit of; and the triple dial will be finiſhed; the ſame gnomon ſerving all the three, and ſhewing the ſame time of the day on each of them.

An univerſal Dial on a plain Croſs.

This dial is repreſented by *Fig.* 1. of PLATE XI, and is moveable on a joint *C*, for elevating it to any given latitude, on the quadrant *C* o 90, as it ſtands upon the horizontal board *A*. The arms of the croſs ſtand at right angles to the middle part; and the top of it from *a* to *n*, is of equal length with either of the arms *n e* or *m k*.

Having ſet the middle line *t u* to the latitude of your place, on the quadrant, the board *A* level, and the point *N* northward by the needle; the plane of the croſs will be parallel to the plane of the equator; and the machine will be rectified.

Then, from III o'clock in the morning, till VI, the upper edge *k l* of the arm *i o* will caſt a ſhadow on the time of the day on the ſide of the arm *c m*: from VI till IX the lower edge *i* of the arm *i o* will caſt a ſhadow on the hours on the ſide *o q*. From IX in the morning to XII at noon, the edge *a b* of the top part *a n* will caſt a ſhadow on the hours on the arm *n e f*: from XII to III in the afternoon, the edge *c d* of the top part will caſt a ſhadow on the hours on the arm *k l m*: from III to VI in the evening, the edge *g h* will caſt a ſhadow on the hours on the part *p s*; and from VI till IX, the ſhadow of the edge *e f* will ſhew the time on the top part *a n*.

The breadth of each part, *a b*, *e f*, &c. muſt be ſo great as never to let the ſhadow fall quite without the part or arm on which the hours are marked, when the ſun is at his greateſt declination from the equator.

To determine the breadth of the ſides of the arms which contain the hours, ſo as to be in juſt proportion to their length; make an angle *A B C* (Fig. 2.) of $23\frac{1}{2}$ degrees, which is equal to the ſun's greateſt declination: and ſuppoſe the length of each arm, from the ſide of the long middle part, and alſo the length of the top part above the arms, to be equal to *B d*.

Then, as the edges of the ſhadow from each of the arms, will be parallel to *B e*, making an angle of $23\frac{1}{2}$ degrees with the ſide *B d* of the arm when the ſun's declination is $23\frac{1}{2}$ degrees; it is plain, that if the length of the arm be *B d*, the leaſt breadth that it can have, to keep the edge *B e* of the ſhadow *B e g d* from going off the ſide of the arm *d e* before it comes to the end *e d* thereof, muſt be equal to *e d* or *d B*. But in order to keep the ſhadow within the quarter diviſions of the hours, when it comes near the end of the arm, the breadth thereof ſhould be ſtill greater, ſo as to be almoſt doubled, on account of the diſtance between the tips of the arms.

To

To place the hours right on the arms, take the following method:

Lay down the crofs *a b c d* (*Fig.* 3.) on a fheet of paper; and with a black lead pencil, held clofe to it, draw its fhape and fize on the paper. Then, taking the length *a e* in your compaffes, and fetting one foot in the corner *a*, with the other foot defcribe the quadrantal arc *e f*.—— Divide this arc into fix equal parts, and through the divifion marks draw right lines *a g*, *a h*, &c. continuing three of them to the arm *c e*, which are all that can fall upon it; and they will meet the arm in thefe points through which the lines that divide the hours from each other (as in *Fig.* 1.) are to be drawn right acrofs it.

Divide each arm, for the three hours it contains, in the fame manner; and fet the hours to their proper places (on the fides of the arms) as they are marked in *Fig.* 3. Each of the hour fpaces fhould be divided into four equal parts, for the half hours and quarters, in the quadrant *e f*; and right lines fhould be drawn through thefe divifion marks in the quadrant, to the arms of the crofs, in order to determine the places thereon where the fub-divifions of the hours muft be marked.

This is a very fimple kind of univerfal dial; it is very eafily made, and will have a pretty uncommon appearance in a garden.—I have feen a dial of this fort, but never faw one of the kind that follows:

An

An universal Dial, shewing the Hours of the Day by a terrestrial Globe, and by the Shadows of several Gnomons at the same Time: together with all the Places of the Earth which are then enlightened by the Sun; and those to which the Sun is then Rising, or on the Meridian, or Setting.

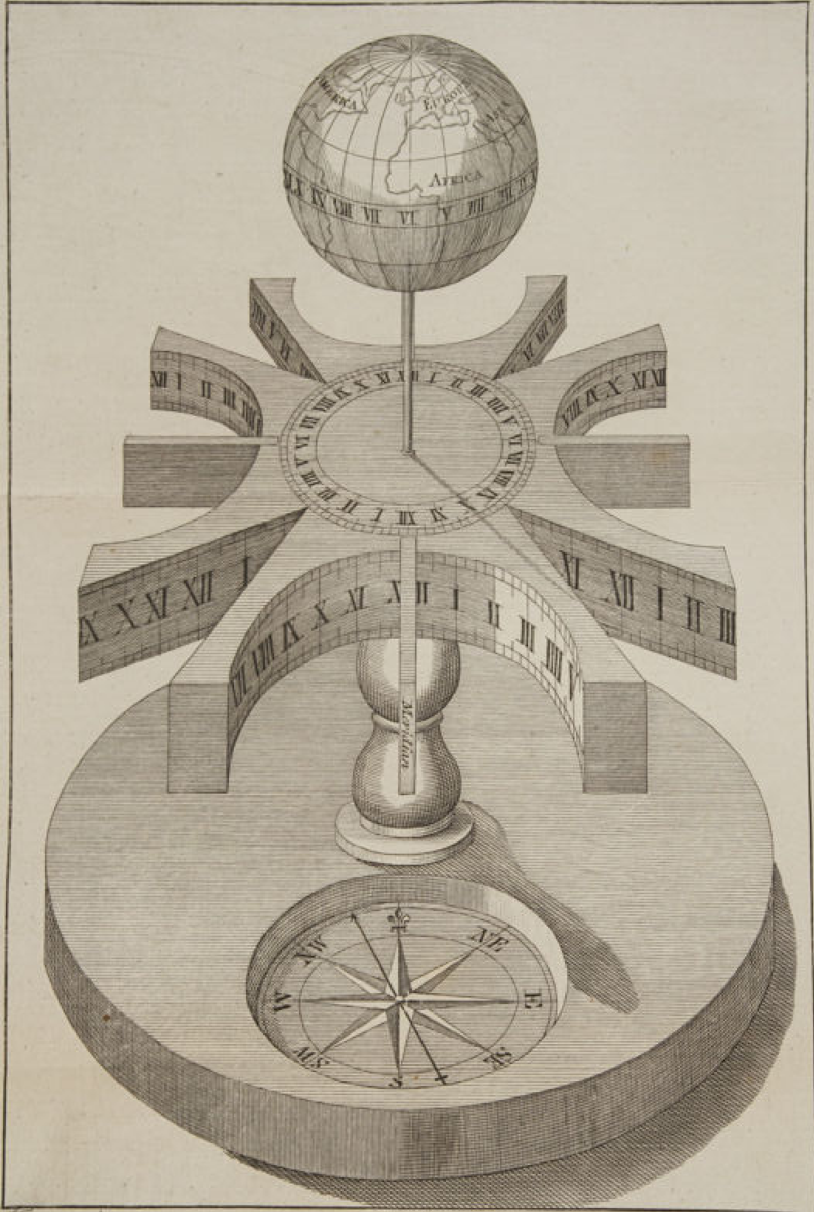
This dial (*See* PLATE XII.) is made of a thick square piece of wood, or hollow metal. The sides are cut into semicircular hollows, in which the hours are placed; the stile of each hollow coming out from the bottom thereof, as far as the ends of the hollows project. The corners are cut out into angles, in the insides of which, the hours are also marked; and the edge of the end of each side of the angle serves as a stile for casting a shadow on the hours marked on the other side.

In the middle of the uppermost side or plane, there is an equinoctial dial; in the center whereof, an upright wire is fixt, for casting a shadow on the hours of that dial, and supporting a small terrestrial globe on its top.

The whole dial stands on a pillar, in the middle of a round horizontal board, in which there is a compass and magnetic needle, for placing the *meridian* stile toward the south. The pillar has a joint with a quadrant upon it, divided into 90 degrees (supposed to be hid from sight under the dial in the figure) for setting it to the latitude of any given place; the same way as already described in the dial on the cross.

The equator of the globe is divided into 24 equal parts, and the hours are laid down upon it at

PLATE XII. Sup.



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at theſe parts. The time of the day may be known by theſe hours, when the ſun ſhines upon the globe.

To rectify and uſe this dial, ſet it on a level table, or ſole of a window, where the ſun ſhines, placing the meridian ſtile due ſouth, by means of the needle; which will be, when the needle points as far from the north fleur-de-lis toward the weſt, as it declines weſtward, at your place. Then bend the pillar in the joint, till the black line on the pillar comes to the latitude of your place in the quadrant.

The machine being thus rectified, the plane of its dial-part will be parallel to the equator, the wire or axis that ſupports the globe will be parallel to the earth's axis, and the north pole of the globe will point toward the north pole of the heaven.

The ſame hour will then be ſhewn in ſeveral of the hollows, by the ends of the ſhadows of their reſpective ſtiles. The axis of the globe will caſt a ſhadow on the ſame hour of the day, in the equinoctial dial, in the center of which it is placed, from the 20th of March to the 22d of September; and, if the meridian of your place on the globe be ſet even with the meridian ſtile, all the parts of the globe that the ſun ſhines upon, will anſwer to thoſe places of the real earth which are then enlightened by the ſun. The places where the ſhade is juſt coming upon the globe, anſwer all to thoſe places of the earth to which the ſun is then ſetting; as the places where it is going off, and the light coming on, anſwer to all the places of the earth where the ſun is then riſing. And, laſtly, if the hour of VI be be marked on the equator in the meridian of your place (as it is marked on the meridian of London in the figure) the divifion of the light and fhade on the globe will fhew the time of the day.

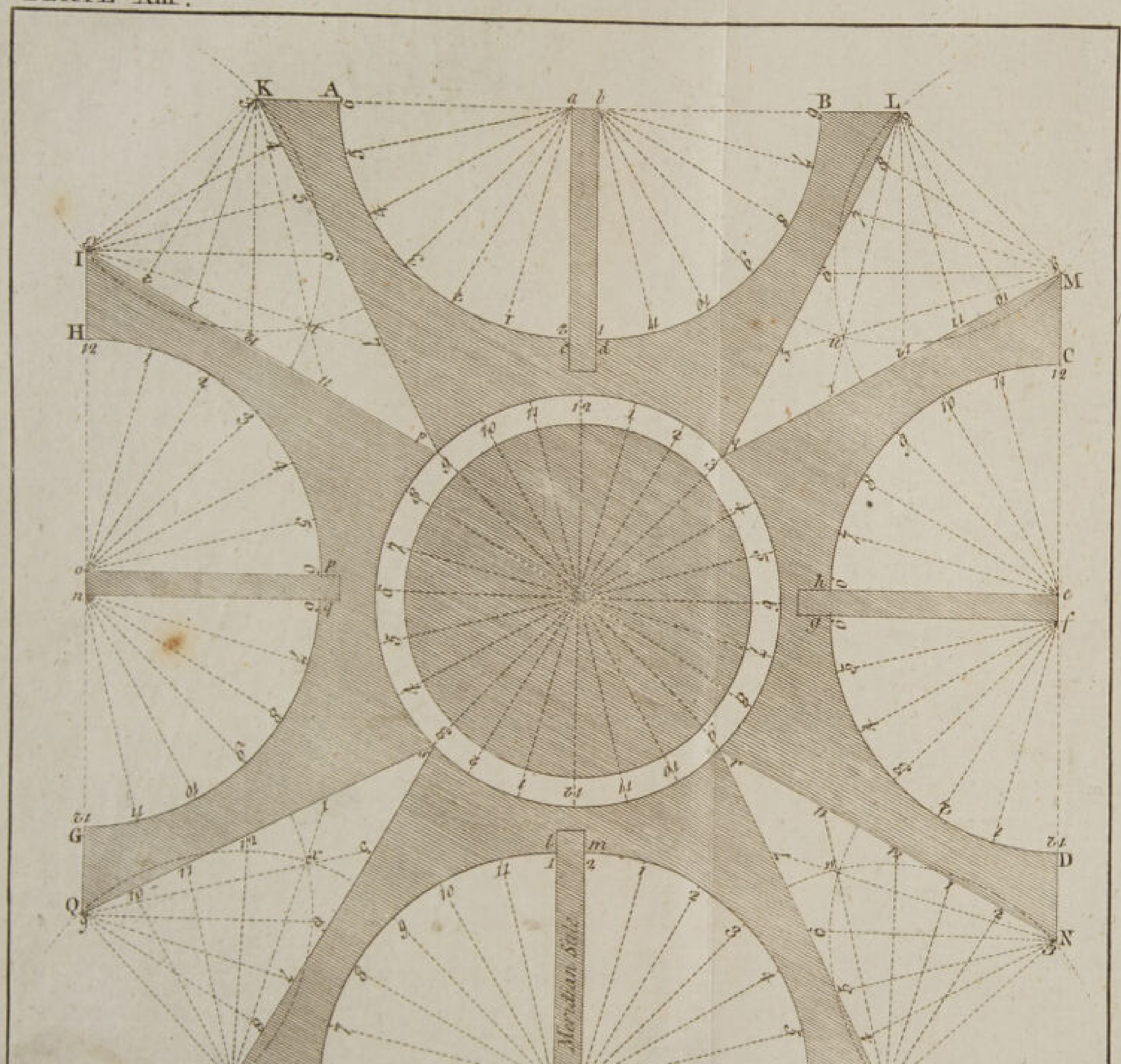
The northern ftile of the dial (oppofite to the fouthern or meridian one) is hid from fight in the figure, by the axis of the globe. The hours in the hollow to which that ftile belongs, are alfo fuppofed to be hid by the oblique view of the figure: but they are the fame as the hours in the front hollow. Thofe alfo in the right and left hand femicircular hollows are moftly hid from fight; and fo alfo are all thofe on the fides next the eye of the four acute angles.

The conftruction of this dial is as follows. *See* PLATE XIII.

On a thick fquare piece of wood, or metal, draw the lines *a c* and *b d*, as far from each other as you intend for the thicknefs of the ftile *a b c d*, and in the fame manner, draw the like thicknefs of the other three ftiles, *e f g h*, *i k l m*, and *n o p q*, all ftanding outright as from the center.

With any convenient opening of the compaffes, as *a A* (fo as to leave proper ftrength of ftuff when *K I* is equal to *a A*) fet one foot in *a*, as a center, and with the other foot defcribe the quadrantal arc *A c*. Then, without altering the compaffes, fet one foot in *b* as a center, and with the other foot defcribe the quadrant *d B*. All the other quadrants in the figure muft be defcribed in the fame manner, and with the

PLATE XIII.



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the ſame opening of the compaſſes, on their centers *e*, *f*; *i*, *k*; and *n*, *o*: and each quadrant divided into 6 equal parts, for ſo many hours, as in the figure; each of which parts muſt be ſub-divided into 4, for the half hours and quarters.

At equal diſtances from each corner, draw the right lines *Ip* and *Kp*, *Lq* and *Mq*, *Nr* and *Or*, *Ps* and *Qs*; to form the four angular hollows *IpK*, *LqM*, *NrO*, and *PsQ*: making the diſtances between the tips of theſe hollows, as *IK*, *LM*, *NO*, and *PQ*, each equal to the radius of the quadrants; and leaving ſufficient room within the angular points, *p*, *q*, *r*, and *s*, for the equinoctial circle in the middle.

To divide the inſides of theſe angles properly for the hour-ſpaces thereon, take the following method:

Set one foot of the compaſſes in the point *I*, as a center; and open the other to *K*, and with that opening, deſcribe the arc *Kt*: then, without altering the compaſſes, ſet one foot in *K*, and with the other foot deſcribe the arc *It*. Divide each of theſe arcs, from *I* and *K* to their interſection at *t*, into four equal parts; and from their centers *I* and *K*, through the points of diviſion, draw the right lines *I* 3, *I* 4, *I* 5, *I* 6, *I* 7; and *K* 2, *K* 1, *K* 12, *K* 11; and they will meet the ſides *Kp* and *Ip* of the angle *IpK* where the hours thereon muſt be placed. And theſe hour-ſpaces in the arcs muſt be ſub-divided into four equal parts, for the half hours and quarters.——Do the like for the other three angles, and draw the dotted lines, and ſet the hours in the infides where thofe lines meet them, as in the figure: and the like hour-lines will be parallel to each other in all the quadrants and in all the angles.

Mark points for all thefe hours, on the upper fide and cut out all the angular hollows, and the quadrantal ones quite through the places where their four gnomons muft ftand; and lay down the hours on their infides, as in Plate XII, and then fet in their four gnomons, which muft be as broad as the dial is thick; and this breadth and thicknefs muft be large enough to keep the fhadows of the gnomons from ever falling quite out at the fides of the hollows, even when the fun's declination is at the greateft.

Laftly, draw the equinoctial dial in the middle, all the hours of which are equidiftant from each other; and the dial will be finifhed.

As the fun goes round, the broad end of the fhadow of the ftile *a b c d* will fhew the hours in the quadrant *A c*, from fun rife till VI in the morning; the fhadow from the end *M* will fhew the hours on the fide *L q* from V to IX in the morning; the fhadow of the ftile *e f g h* in the quadrant *D g* (in the long days) will fhew the hours from fun-rife till VI in the morning; and the fhadow of the end *N* will fhew the morning hours, on the fide *O r*, from III to VII.

Juft as the fhadow of the northern ftile *a b c d* goes off the quadrant *A c*, the fhadow of the fouthern ftile *i k l m* begins to fall within the quadrant *F l*, at VI in the morning; and fhews the time, in that quadrant, from VI till XII at noon;

noon; and from noon till VI in the evening in the quadrant *m E*. And the ſhadow of the end *O* ſhews the time from XI in the forenoon till III in the afternoon, on the ſide *r N*; as the ſhadow of the end *P* ſhews the time from IX in the morning till 1 o'clock in the afternoon, on the ſide *Q s*.

At noon, when the ſhadow of the eaſtern ſtile *e f g h* goes off the quadrant *h C* (in which it ſhewed the time from VI in the morning till noon, as it did in the quadrant *g D* from ſun-riſe till VI in the morning) the ſhadow of the weſtern ſtile *n o p q* begins to enter the quadrant *H p*; and ſhews the hours thereon from XII at noon till VI in the evening; and after *that* till ſun-ſet, in the quadrant *q G*: and the end *Q* caſts a ſhadow on the ſide *P s* from V in the evening till IX at night, if the ſun be not ſet before that time.

The ſhadow of the end *I* ſhews the time on the ſide *K p* from III till VII in the afternoon; and the ſhadow of the ſtile *a b c d* ſhews the time from VI in the evening till the ſun ſets.

The ſhadow of the upright central wire, that ſupports the globe at top, ſhews the time of the day, in the middle or equinoctial dial, all the ſummer half year, when the ſun is on the north ſide of the equator.

In this Supplement to my book of Lectures, all the machines that I have added to my apparatus, ſince that book was printed, are deſcribed, excepting two; one of which is a model of of a mill for ſawing timber, and the other is a model of the great engine at London-bridge, for raiſing water. And my reaſons for leaving them out are as follow:

Firſt, I found it impoſſible to make ſuch a drawing of the ſaw-mill as could be underſtood; becauſe, in whatever view it be taken, a great many parts of it hid others from ſight. And, in order to ſhew it in my Lectures, I am obliged to turn it into all manner of poſitions.

Secondly, Becauſe any perſon who looks on *Fig.* 1. of PLATE XII in the book, and reads the account of it in the fifth Lecture therein, will be able to form a very good idea of the London-bridge engine, which has only two wheels and two trundles more than there are in Mr. *Alderſea*'s engine, from which the ſaid figure was taken.

FINIS.