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The Gift of Dr Robt: Hardy to his Nephew John Malcolm

A SHORT SYSTEM OF OPTICS,

PRINCIPALLY DESIGNED
FOR THE
USE OF UNDERGRADUATES
IN THE
UNIVERSITY OF DUBLIN.

By the Rev. JOHN STACK, A. M.
Late F. of T. C. D.

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M,DCC,XCIII.

CONTENTS.

SECTION I.

MARKS OR ABBREVIATIONS.

$\because$	ſignify	therefore.
$=$		equal to.
$a : b :: c : d$		four proportionals.
$+$		added to.
$-$		ſubducted from.
$\times$		multiplied by.
I		ſine of the angle of incidence.
R		ſine of the angle of refraction.
q. p.		quam proxime.

A
SHORT SYSTEM
OF
OPTICS.

SECTION I.

Of the Nature and Laws of Reflexion and Refraction.

SUBSECT. 1. A RAY of light AB is ſaid to be reflected, when having paſſed through one medium and fallen on the Fig. 1. ſurface of another, it *does not enter* this medium, but is turned back into the former.

2. A ray

2. A ray A B is ſaid to be refracted when having paſſed through one medium and fallen *obliquely* on the ſurface of a ſecond, it proceeds through this medium, not in its former direction, but in one more or leſs oblique to the refracting ſurface than the former, according as *the ſecond medium* is *rarer* or *denſer* than the firſt. Fig. 2.

3. The *angle of incidence* is that which the incident ray A B makes with B C, the perpendicular to the ſurface, raiſed from the point of incidence B. Fig. 1 & 2.

4. The *angles of reflexion* C B F, *or refraction* O B F, are thoſe made by the reflected or refracted rays reſpectively with the perpendicular. Fig. 1 & 2.

5. The *refracted angle* F B L is that contained by the refracted ray B F with the incident ray produced B L; or its ſupplement A B F. Fig. 2.

6. The

6. The reflected angle A B F is that contained under the incident and reflected rays A B, B F; or its ſupplement F B O.

Fig. 1.

7. A pencil of rays is any parcel of rays O S that are conceived to flow from or to the ſame point F.

Fig 3, 4, & 5.

8. The focus of a pencil is that point F to which the rays converge, or from which they diverge*.

9. The axis of a pencil O S is a right line F B drawn from the focus perpendicularly to a reflecting or refracting ſurface C P, or through a point B in a lens M L, *called* its centre.

Fig. 3, 4.

Fig. 5.

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* It is to be here remarked that if a pencil of rays fall on a plane reflecting ſurface, they ſhall after reflexion all converge to, or diverge from the ſame point of the axis accurately (as will appear from Subſ. 18). But if the pencil fall on a ſpheric reflecting ſurface, or on a refracting ſurface, whether plane or ſpherical, the rays at different diſtances from the axis ſhall, after reflexion or refraction, meet the ray that is in the axis in different points; and the centre of the ſmalleſt ſpace into which all the rays are collected is eſteemed the focus of the reflected or refracted pencil, and may be called its virtual focus.

10. The geometric focus of a pencil of rays reflected or refracted is the focus of thoſe rays that were in, and indefinitely near to the axis in their incidence*.

Fig. 6 & 7. 11. Foci, A and B, belonging to the ſame pencil before and after reflexion or refraction, are ſaid to be *conjugate* to each other.

12. The axis of a lens is a right line Fig. 5. F B. drawn through it perpendicular to both its ſurfaces.

13. The axis of a reflecting or refracting ſpheric ſurface is a right line paſſing through the middle point of the ſurface, and perpendicular thereto.

14. The principal focus of a lens or ſpheric ſurface is that geometric focus to which rays that fall parallel to its axis are reflected or refracted.

15. The focal length of a lens or ſpheric ſurface is the perpendicular diſtance

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* Since the nearer that the incident rays are to the axis, the ſmaller will be the ſpace into which a given number can be collected (Vide Barrow's Lect. Opt. L. 4. Art. 15.) it is evident that the virtual geometric focus of a pencil muſt be in its axis.

tance of the principal focus from the lens or ſurface.

16. Sir Iſaac Newton having abundantly demonſtrated that each ray of ſolar light is compounded of ſeveral rays different in their properties (e. g. in colour and refrangibility) it becomes neceſſary to make a diſtinction between the different kinds of light. *Homogeneous light* is that which forms any of thoſe rays ſeparately taken, which compoſe ſolar light when taken together. *Heterogeneous light* is the common light which reſults from the compoſition of the different kinds of the former.

17. The general and fundamental laws of reflexion and refraction are—1ſt, *In all kinds* * *of light, the angle of reflexion* B D C *is equal to the angle of incidence* A D B. This appears from

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* As all kinds of homogeneous rays are incident in the ſame angle when a ſolar ray falls on a reflecting ſurface, they muſt be all reflected in the ſame angle, therefore in reflexion no ſeparation of the homogeneous rays can take place.

the compoſition and reſolution of motion*.

Fig. 8. Let the motion of the incident ray be expounded by A D; A B will expound the parallel motion while A E expounds the perpendicular one; the perpendicular motion after reflexion will be equal to that before reflexion, and therefore will be expounded by D B = A E. The parallel motion being unaffected by reflexion continues uniform, and therefore will be expounded by B C = A B, therefore the courſe of the ray is D C; and by the 4th of book 1. Elements, the angle A D B = B D C.

Fig. 9. 2. In *homogeneous light only*, I : R *in the ſame proportion* †, however the angle of

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* The motion of light is proved by Mr. Romer's obſervations on the eclipſes of Jupiter's ſatellites. Vide Keil's Aſtronomy, Lect. 16.

† As the angle of incidence vaniſhes when a ray falls in the perpendicular, ſo muſt all the angle of reflexion or refraction vaniſh, and therefore the whole courſe of this ray muſt be in one and the ſame right line before and after reflexion or refraction.

incidence

incidence be varied *, upon a given refracting medium.—This is proved in fluids by the following experiment.

To the ſide of a ſtrong beam accurately planed, erect at the diſtance of eight or ten feet from each other two plates, exactly perpendicular to the beam: Adjoining to one of the plates, which is perforated, place a priſmatic glaſs veſſel filled with water, which ſhall reſt on the plate when the beam is inclined. On the oppoſite plate mark that point which is at the ſame diſtances from the upper ſurface and ſides of the beam, as the ſmall hole in the perforated plate.—In-

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* As each homogeneous ray has a degree of refrangibility different from the reſt, it is plain that where the incidence of them all is the ſame (i. e. when a ſolar ray is incident on a refracting ſurface) the angles of refraction ſhall be different, and therefore a ſeparation of the rays muſt enſue in a ſingle refraction.

cline the beam toward the ſun, and let that coloured light which you intend to examine, after refraction by the water, fall on the mark in the oppoſite plate—at that inſtant take the altitude of the ſun, whoſe complement is equal to the angle of incidence, as alſo the inclination of the beam to the horizon, whoſe complement is the angle of refraction.

The angles being thus found, you have their ſines, which are found to be always in one and the ſame proportion, whatever be the ſun's altitude at the time of making the experiment, i. e. however the incidence be varied*.

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* To ſee this proved otherwiſe in fluids by means of hollow priſms, and in ſolids by means of glaſs priſms of various angles, conſult Newton's Lect Opt. Part 1. Sect 2.—Theſe modes are here omitted, as involving too many geometrical lemmas.

S E C T. II.

Of the Poſition of the Conjugate Foci of Rays reflected and refracted.

18. IF a pencil of rays fall on a plane reflecting ſurface, *the perpendicular diſtances of the conjugate foci from the plane ſhall be equal.* Fig. 10.

1 *If the incident rays* A C *be parallel* —then as the angles of incidence A C B of all the rays are equal, the angles of reflexion B C D ſhall be equal ‡, ∵ the reflected rays C D are equally inclined to the ſurface, and ∵ are parallel to each other, i. e. the foci are both infinitely, and ∵ equally diſtant from the plane. Fig. 10. ‡ Subſ. 17.

2. *If*

2. *If the incident rays diverge*; then becauſe the angles of incidence A C D are equal to the angles of reflexion D C E reſpectively, the inclinations A C M of the incident rays, are equal to the inclinations E C L of the reflected, i. e. to the angles B C M vertically oppoſite—wherefore by 26 Lib. 1. El. the triangles A C M are equal to the triangles B C M reſpectively, and ∴ A M = B M. Fig. 7.

N. B. The ſame demonſtration will apply to the caſe of converging rays.

19. Of homogeneal rays that fall on a plane refracting ſurface; *the perpendicular diſtance of the geometric focus of the incident rays from the ſurface is to that of the refracted ones as* R : I.

1. *If the incident rays diverge*, produce the refracted ray L B until it meets the perpendicular in D—then in Fig. 12, 13.

the

the triangle A B D, as the ſine of the angle A is to the ſine of the angle D* (i. e.) as I : R ſo is B D to B A, i. e. when the rays fall (q. p.) perpendicularly, ſo is D M to A M.

N. B. The ſame demonſtration will apply to the caſe of converging rays, or even to that of parallel rays, by conſidering the focus A as infinitely diſtant—for as there is a conſtant proportion between A M and D M, if the former be infinite ſo ſhall the latter, i. e. the refracted rays ſhall be parallel, but for greater clearneſs this caſe is delivered ſeparately,—thus,

2. As the angles of incidence A B C are equal, and as their ſines have a given proportion to the ſines of the angles of refraction D B F (which are of the ſame affection) theſe angles ſhall be alſo equal Fig. 11.

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[*] It is here proper to remark, that the ſine of an angle is the ſame with the ſine of its ſupplement to two right angles.

equal, ∵ the refracted rays are alſo parallel, i. e. the conjugate foci ſhall be both infinitely diſtant from the ſurface*.

20. If a pencil of rays fall on a reflecting ſpheric ſurface, *the ſemi-radius* BD *of the ſphere is always a mean proportional between* Fig. 14, 15. *the diſtances* A D, *and* C D *of the geometric foci of the incident and reflected rays, from the point of biſection* D, *in that radius which coincides with the axis of the pencil.*

1ſt. *If the incident rays diverge from a focus* A—Draw B F and G B reſpectively parallel to the incident and reflected rays, then becauſe the angle of reflexion B E F = G E B, the angle of incidence, i. e. = E B F, the triangle B F E is an iſoſceles; ∵ when the point E coincides with P, the ſides B F, F E ſhall coincide with

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[*] N. B. It is plain that if the line into which a ray is reflected or refracted be taken to expreſs the direction of an incident ray, *this* ſhall be reflected or refracted into the line in which the former ray was incident.

the

the ſemi-radii B D, D P—and ſince from the nature of the conſtruction, the triangles B F E, B G E are always mutually equilateral, the points F and G will approach each other while E moves toward P, and will ultimately coincide at D. But becauſe of the ſimilar triangles ABG, B C F, F C : F B :: G B : A G, (i. e. when A E is q. p. perpendicular) D C : D B :: D B : D A.

N. B. A ſimilar demonſtration will apply to the caſe of converging rays.

2. *If the incident rays be parallel, the geometric focus of the reflected rays will be in that point where the radius is biſected*—for of the extreme proportionals in the analogy above, A D being infinitely increaſed, the other D C is infinitely diminiſhed, and vaniſhes—*Or it may be alſo thus demonſtrated*—The triangle B C D is an iſoſceles, whoſe ſides D C, D B, when taken together, are Fig. 16.

ultimately

ultimately equal to and coincident with the radius C F, ∵ the point D will then be the point of biſection.

21. Let G I be one of a pencil of Fig. 17, 18, 19, 20. homogeneous rays parallel to the axis of a refracting ſpheric ſurface—*The diſtance of the geometric focus of the refracted rays from the ſurface, is to its diſtance from the centre* C *of the ſphere as* I : R. For I F : F C :: ſine of I C F (or I C V = *angle of incidence)* ſine of C I F *(the angle of refraction or its ſupplement)*—But when the rays fall near the perpendicular, I F coincides with F V, ∵ by ſubſtituting for I F in the analogy its equal F V, the truth of the propoſition is evident.

Coroll. By diviſion of proportion we have hence *the radius* V C : F V *(the diſtance of the principal focus from the ſurface) :: difference of the ſines : ſine of incidence*, ∵ given the radius of the ſphere, the principal focus is alſo given.

22. Given

PLATE 1.

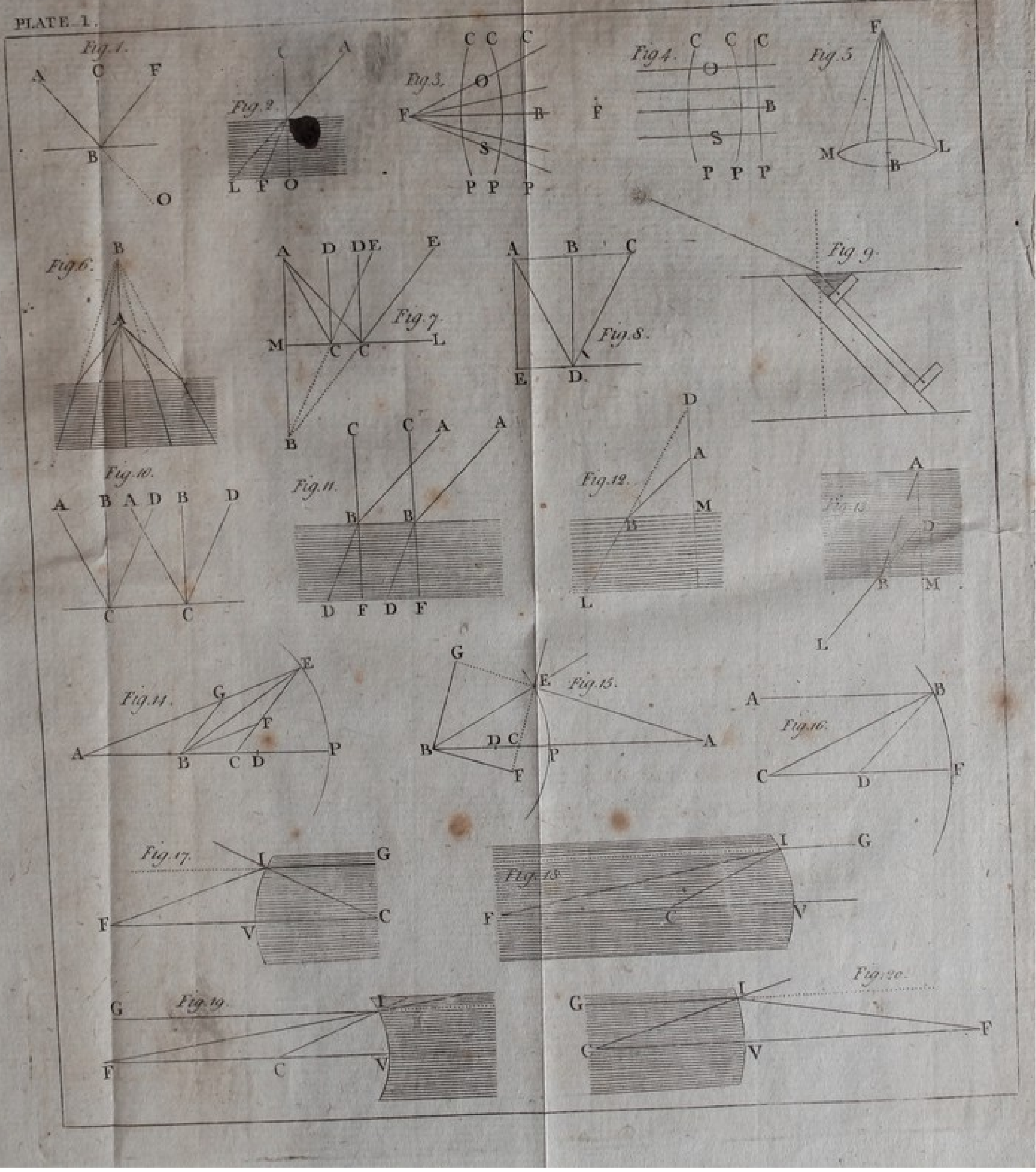
Fig. 1.
A C F
B
O
Fig. 2.
C A
L F O
Fig. 3.
C C C
O
F
B
S
P P P
F
Fig. 4.
C C C
O
B
S
P P P
Fig. 5
F
M L
B
Fig. 6.
B
A
A D DE E
Fig. 7
M C C L
B
A B C
Fig. 8.
E D
Fig. 9.
Fig. 10.
A B A D B D
C C
Fig. 11.
C C A A
B B
D F D F
Fig. 12.
D
A
B M
L
Fig. 13.
A
D
B M
L
Fig. 14.
E
G
F
A B C D P
G
E
Fig. 15.
B D C P A
F
A B
Fig. 16.
C D F
Fig. 17.
I G
F V C
Fig. 18.
I G
F C V
Fig. 19.
G I
F C V
Fig. 20.
G I
C V F

22. Given a focus A of homogeneous rays which fall converging or diverging on a ſpheric refracting ſurface, its conjugate is thus found——Take from the centre D a part of the axis D F equal to the principal focal diſtance*; *that point whoſe diſtance from the given focus* A *is a fourth proportional to the diſtances* A F, A C, A D, *(of the aſſumed point, the ſurface and the centre, from the ſame focus* A) will be its conjugate ſought.

Fig. 21, 22, 23, 24.

For from D deſcribe an arch with D F as its radius; if a ray (as B E) iſſued from B, it would be refracted into E G parallel ‡ to B D the axis of its pencil, then becauſe of the ſimilar triangles ABD, AEG, AB : AE :: AD : AG

‡ Vide Note to Subſ. 19.

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* At the ſame ſide with the given focus when in paſſing from a rarer to a denſer medium, the rays fall on a convex ſurface—at the other ſide if they fall on a concave. The contrary poſitions of the principal focus are to be aſſumed, when the rays paſs from a denſer medium to a rarer.

—But when the ray A E falls q. p. perpendicularly, A B and A E coincide reſpectively with A F and A C, ∵ A F : A C :: A D : A G.

Coroll. Hence by converſion A F : F C :: A D : D G—(and ſince by Cor. Subſ. 21. the radius being given, the principal focus is given) we ſhall here have the three firſt terms of the analogy if the radius be known, and therefore the fourth, which gives G the focus conjugate to A.

23. Lenſes are of ſix kinds, 1, Plano-convex. 2, Plano-concave. 3, Double convex. 4. Double concave. 5, Meniſcus. 6. The other kind of concavo-convex glaſs—They are beſt explained by their figures. Fig. 25.

N. B. The curved ſurfaces are ſpherical.

The

The centre of a lens is *that point thro' which if a ray paſſes, its whole courſe will be* q. p. *a right line*—It is thus found: Draw the axis of the glaſs B C, and alſo any two parallel radii B D and C E of its ſurfaces—If D E be drawn it will cut the axis in the point required—For ſuppoſing rays to paſs both ways along D E, becauſe of the parallels D B and C E, the angles of incidence B D E, C E D are equal, ∴ the angles of refraction are equal, i. e. the ray O E will emerge in D V parallel to its incidence—and as the thickneſs of the lens is uſually inconſiderable, the interval of the parallels is evaneſcent, and ∴ the courſe of the ray will not ſenſibly deviate from a right line. Fig.

24. The principal focus of a lens is thus found:

Let the axis L C of the incident pencil of parallel homogeneous rays be a little oblique to A B, the axis of the lens—parallel to L C draw A G, a radius of the firſt ſurface S G,—find § in A G the geometric focus X of the pencil after its refraction by that ſurface, i. e. the focus of the rays incident upon the ſecond ſurface P Q—draw X B, the axis of theſe incident rays. The geometric focus of the emergent rays is placed in this axis, as it is alſo in L C, the axis of the original pencil, ∵ the interſection F of thoſe axes is the point ſought*.

§ Subſ. 21.

Fig. 27.

Coroll. 1. Hence given the radii of the ſpheric ſurfaces, the principal focus is found by this analogy, A B : C B :: A X : C F. But A B and C B are conſtant, by the hypotheſis, and ſo alſo is A X *by Subſect* 21. ∵ C F is equal to the

focal

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[*] As rays parallel to the axis F C would be refracted to the focus F ſo rays diverging from the focus F will emerge parallel to F C, and ∵ to each other. Vid. note to Subſ. 19.

focal diſtance of rays that fall q. p. parallel to the axis of the lens.

Coroll 2. Hence it appears that if the radii of the ſpheric ſurfaces be equal, the focal length of the lens is equal to one of them—for as I : R from air into glaſs is found by experiment to be as 3 to 2, $AX = 2\,AC = AB$, † $\therefore$ $CF = CB$. † Cor. of Subſ. 21.

25. Given the focus of homogeneous rays incident on a lens the geometric focus conjugate to it is thus found :

From the centre of the lens C take* (in the axis of the pencil) F C the principal focal length of the lens—from the given focus A toward F take a point L, whoſe diſtance from A is a third proportional to A F, A C ;—L is the point required. For with C as a centre, and C F as an in- Fig. 28, 29.

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* Toward the given focus if the lens be convex ; to the contrary ſide of the lens if it be concave.

terval,

§ Vid. Note Subſ. 24.

terval, deſcribe an arch F G,—a ray § as G E iſſuing from G, will be refracted into E L parallel to G E the axis of its pencil—then becauſe of the ſimilar triangles A C G, A L E, A G : A C :: A E : A L, but when the rays fall q. p. perpendicularly, G coincides with F, and E with C, ∴ A F : A C :: A C ; A L.

Coroll. Hence given the principal focal length F C, and the diſtance A C, the focus L is found by this analogy derived from the laſt mentioned proportion by converſion—A F : F C :: A C : CL.

SECT.

SECT. III.

Of Colours and the different Refrangibility of Light.

26. THE origin of colours is owing to the compoſition which takes place in the rays of light, each heterogeneous ray conſiſting of innumerable rays of different colours—this is evident from the ſeparation that enſues in the well known experiment of the priſm.

Exp. 1. A ray being let into a darkened room through a ſmall round aperture, and falling on a triangular glaſs priſm, is by the refraction of the priſm conſiderably dilated, and will exhibit on the oppoſite wall an oblong image (uſually called *a ſpectrum*) variouſly coloured, whoſe Fig. 30.

whoſe extremities are bounded by ſemicircles, and whoſe ſides are rectilinear. The colours are commonly divided into ſeven, which however have various ſhades, gradually intermixing at their juncture:—their order, beginning from the ſide of the refracting angle of the priſm, is—*red*, *orange*, *yellow*, *green*, *blue*, *purple*, *violet*. The obvious concluſion from this experiment is, that the ſeveral component parts of ſolar light have different degrees of refrangibility, and that each ſubſequent ray in the order above-mentioned is more refrangible than the preceding—this will be preſently more unequivocally evinced; but firſt it is neceſſary (for a more deciſive refutation of objections, and ſtrengthening ſome concluſions in this ſubject) to attend to the mixture of the different coloured rays with each other in the ſpectrum, and alſo to point out the mode of diminiſhing it.

27. As a circular image would be depicted by the ſolar ray unrefracted by the priſm, ſo each ray that ſuffers no dilatation by the priſm would mark out a circular image—hence it appears that the ſpectrum is compoſed of *innumerable circles* of different colours. The mixture therefore is proportionable to Fig. 31. the number of circles mixed together; but all ſuch circles are mixed together, whoſe centres lie between thoſe of two contingent circles, ∵ the mixture is proportionable to the interval of thoſe centres, i. e. to the breadth of the ſpectrum——If therefore the breadth can be diminiſhed, retaining the length of the rectilinear ſides, the mixture will be leſſened proportionably, but this is done by the following proceſs[*]:

28. At

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[*] Obſerve here, that the breadth of the ſpectrum is equal to a line, which (at the diſtance of the wall from the hole) ſubtends an angle equal to the apparent diameter of the ſun, together with another line

equal

Fig. 32.

28. At a conſiderable diſtance from the hole place a double convex lens whoſe focal length is equal to half that diſtance, and place the priſm behind the lens—at a diſtance behind the lens, equal to the diſtance of the lens from the hole, will be formed a ſpectrum, the length of whoſe rectilinear ſides is the ſame as before, but its breadth much leſs; for the undiminiſhed breadth was equal to a line ſubtending (at the diſtance of the ſpectrum from the hole) an angle equal to the apparent diameter of the ſun, together with a line equal to the diameter of the hole—but the reduced breadth is equal to the diameter of the hole only; for (as it will appear hereafter)

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equal to the diameter of the hole—for the diameter of the circular unrefracted image would be equal to the ſum of theſe lines, as is plain from the figure 31; therefore the diameter of the circles compoſing the ſpectrum (i. e. the breadth of the ſpectrum) is equal to the ſame.

the

the image of the hole formed by the lens at the diſtance of double its focal length, is equal to the hole, ∴ its ſeveral images in the different kinds of rays are equal to the ſame, i. e. the breadth of the reduced ſpectrum is equal to the diameter of the hole.

29. It may be objected againſt the concluſion deduced from the priſmatic experiment in ſubſection 26, " that the length of the ſpectrum ariſes from a caſual diſperſion of the rays in paſſing through the priſm ;" or " that by the action of the glaſs they are cleft aſunder without any difference of refrangibility"—To obviate ſuch objections the following experiments are adduced ; but firſt it is worth obſerving, that the ſides of the ſpectrum being always rectilinear, ſeem to denote a regularity in the law that takes place in the diſperſion of the rays.

Exp. 1. A priſm A B C placed in an horizontal poſition, would project the ray into an oblong form, as has been ſeen; apply another horizontal priſm A D B, ſimilar to to the former, to receive the refracted light emerging from the firſt, and having its refracting angle turned the contrary way from that of the former Fig. 33. —The light, after paſſing through both priſms, will aſſume a circular form, as if it had not been at all refracted—whereas if the diſperſion were fortuitous, it would be reaſonable to ſuppoſe that by the double refraction the ſpectrum would be conſiderably lengthened.

Exp. 2. If the light emerging from the firſt priſm be received by a ſecond whoſe axis is perpendicular to that of the former, it will be refracted by this tranſverſe priſm into a poſition inclined Fig. 34. to the former, the *red* extremity being leaſt and the *violet* moſt removed from its former poſition; but it will not be

Fig. 21.

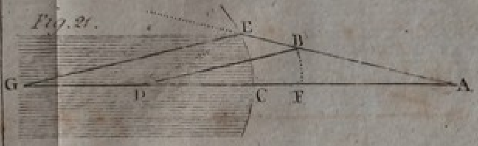


Fig. 22.

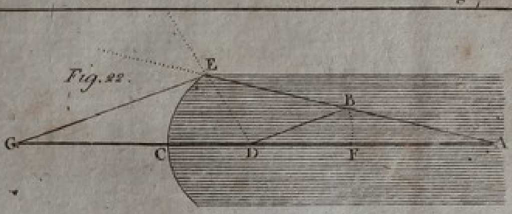


Fig. 23.

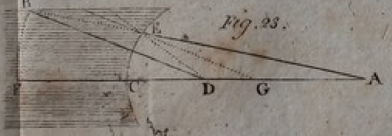


Fig. 24.

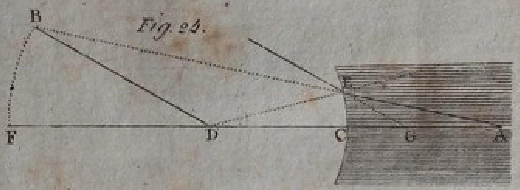


Fig. 25.

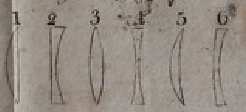


Fig. 26.

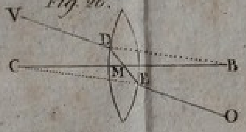


Fig. 27.

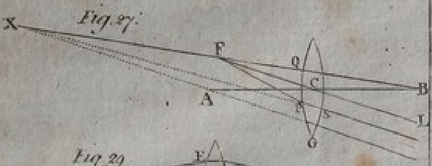


Fig. 28.

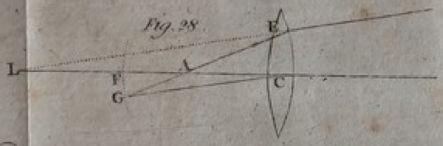


Fig. 29.

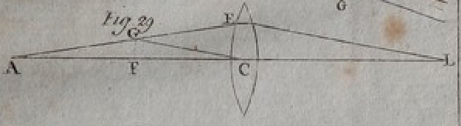


Fig. 30.

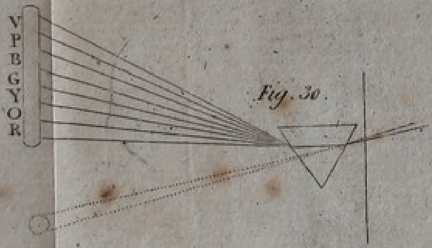


Fig. 31.

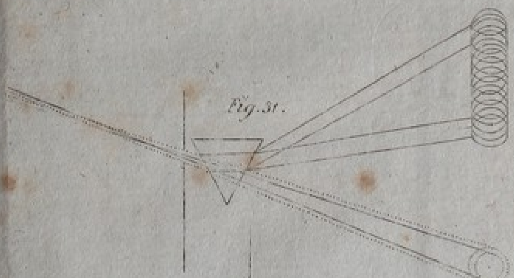
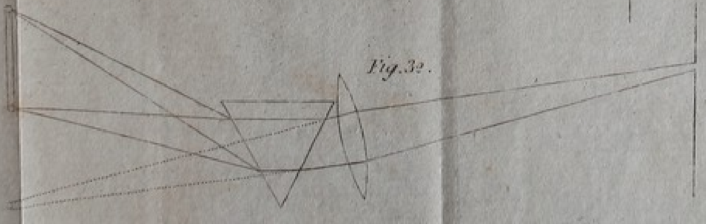


Fig. 32.



at all altered in breadth—whereas if the diſperſion proceeded from a diviſion of the rays into others of the ſame kind by the action of the glaſs, the image formed by the ſecond priſm would be ſquare, each ray in the firſt ſpectrum being as much dilated (on this hypotheſis) by the refraction of the tranſverſe priſm, as the ſolar ray was by the refraction of the firſt priſm. But leſt objections ſhould be made concerning the inequality of the incidences of the different coloured rays on the ſecond priſm in this experiment, the following deciſive experiment will demonſtrate unqueſtionably that the different homogeneous rays are differently refrangible.

Exp. 3. Cloſe to the priſm place a perforated board, and let the refracted light (having paſſed thro' the ſmall hole) be received on a ſecond board parallel to the firſt and perforated in like manner; behind that hole in the ſecond board place a priſm, with its refracting angle downward—turn the firſt priſm ſlowly about its axis, and the light will move up and down the ſecond board * ; let the colours be tranſmitted ſucceſſively, and mark the places of the different Fig 35. coloured rays on the wall after their refraction by the ſecond priſm—the *red* will appear loweſt, the *violet* higheſt, the reſt in the intermediate places in order. Here then the light being very much ſimplified †, and the incidences of all the rays on the ſecond priſm exactly the ſame ; the red was leaſt refracted, the violet moſt, &c. ‡

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[*] Vid. Newton's Lect. Opt. P. 1. Sect. 4.

[†] For in this experiment the diameter of the coloured circles is reduced to the diameter of the hole, as in Subſ. 28. —Vide Newt. Opt. P. 1. Prop. 2.

[‡] N. B. No ſenſible alteration takes place in the appearance of any one of the colours by the refraction of the ſecond priſm, nor even if any number of others be applied to receive it ſucceſſively.

30. The permanency of theſe original colours appear from hence, that they ſuffer no manner of change by any number of refractions, as is evident from the laſt mentioned experiment; nor yet by reflexion—for if any coloured body be placed in ſimplified homogeneous light, it will always appear of the ſame colour of the light in which it is placed, whether that differ from the colour of the body or not—e. g. if ultra-marine and vermillion be placed in a red light, both will appear red; in a green light, green, in a blue light, blue, &c.—It is however to be allowed, that a body appears brighter when in a light of its own colour than in another—and from this we ſee that the colours of natural bodies ariſe from an aptitude in them to reflect ſome rays more copiouſly and ſtrongly than others—but leſt this phænominon ſhould produce a doubt of the conſtancy of the primary colours, it is proper to

aſſign

aſſign the reaſon of it, which is this,—that when placed in its own coloured light, the body reflects the rays of the predominant colour more ſtrongly than any of thoſe intermixed with it [*], ∵ the proportion of the rays of the predominant colour to thoſe of the others, in the reflected light, will be greater than in the incident light—but when the body is placed in a light of a different colour from its own, for a ſimilar reaſon the contrary effect will follow, i. e. the proportion of the predominant colour to the others will be leſs in the reflected than in the incident light, and therefore as its ſplendour would be greater in the former caſe, and would be leſs in the latter than if all the rays were equally reflected, the ſplendour of the predominant colour will be

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[*] For in priſmatic light, however decompounded, ſome mixture will always remain.

much greater in the former caſe than in the latter.

31. As a ſolar ray was ſeparated into ſeveral others of different colours, ſo on the contrary, from thoſe homogeneous rays a ray of heterogeneous light may be compounded, perfectly correſponding both in appearance and properties with the ſolar rays.

Exp. The coloured rays diverging from the priſm are received by a double convex lens, at the diſtance of twice its focal length from the whole—at the ſame diſtance * behind the lens, where they are collected by its refraction, they are received on a ſecond priſm, whoſe refracting angle is equal to that of the former; the divergence of the homogeneous rays that would otherwiſe enſue, is counteracted by the ſecond priſm; and they are made to proceed parallel to each other from the place of their interſection, Fig. 36.

* Vide Subſ. 25.

and

and therefore are all compounded and mixed together in the emergent ray A B, which is exactly of the ſame appearance with the ſolar rays, and, by experiments made on it ſimilar to thoſe uſually made in ſolar light, is found to poſſeſs the ſame properties.

32. Since then, 1ſt, A ſolar ray may be reſolved into ſeveral differently coloured rays; 2dly, Since their colours are immutable either by reflexion or refraction, and therefore probably not generated in thoſe operations; and 3dly, Since from the mixture of thoſe coloured rays ſolar light may be formed, it ſeems an indiſputable concluſion, that the differently coloured rays do exiſt in ſolar light previous to any ſeparation that takes place in experiments.

33. *White* is compounded of all the primary colours mixed in their due proportions—for if a ſolar ray be ſeparated by

by the priſm into its component parts, and at a proper diſtance a lens be ſo placed as to collect the diverging coloured rays again into a focus, a paper placed perpendicularly to the rays in this point will exhibit whiteneſs. Exp. 1.

The ſame concluſion may be drawn from the experiment of mixing together paints of the ſame colours as the parts of the ſpectrum, and in the ſame proportion; the mixture will be white, though not of a reſplendent whiteneſs—becauſe the colours mixed are leſs bright than the primary ones—and the reaſons why they are ſo are theſe,—1ſt, That coloured bodies abſorb a great deal of the light incident on them; 2dly, That as they reflect all kinds of rays *, the luſtre of the predominant colour is diminiſhed by the mixture of the others. Exp. 2.

* Subſ. 30.

Black, and all grey colours are of the ſame ſpecies of white; for white, if Exp placed in a deep ſhade, approaches in appearance to black.

Exp. 4. On the contrary, if grey colours are ſtrongly illuminated, they appear of a full whiteneſs, whence it follows that they only differ in the quantity and not in the kind of light which they reflect.

Exp. 5. And black when ſtrongly illuminated, and viewed through a priſm, appears tinged at the edges with priſmatic colours, ſuch as would appear at the edges of a white body in the ſame circumſtances.

If in the light reflected from a body, the other colours bear a very great proportion to the principal one, a ſenſible change is produced in its ſpecies, and a compound colour ariſes—hence the varieties in the colours of natural bodies*.

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* A variety of compound colours may be generated by intercepting one or more of the priſmatic colours at the lens (in the firſt Exp. of Subſ. 33.) the colour exhibited at the focus will be compounded of the remainder.

34. As the colour of a body therefore proceeds from a certain combination of the primary rays which it reflects;—the combination of rays flowing from any point of an object will, when collected by a glaſs, exhibit the ſame compound colour in the correſponding point of the image—hence appears the reaſon why the images formed by glaſſes have the ſame colours of the objects they repreſent †.

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† N. B. The images of the points of an object are at the geometric foci q. p. for the rays collected into a given ſpace there are far more in number than thoſe collected into an equal ſpace at the foci of the oblique pencils.——Vide Barrow's Lect. Opt. Lect. 4. from Art. 15 to 20, incluſive.

SECT. IV.

Of the Images of Objects formed by Reflexion and Refraction.

35. IF A B be ſuppoſed an object, and C D its image, formed by a plane ſpeculum; *They ſhall be equally diſtant from the ſpeculum, at oppoſite ſides*; 2. *They ſhall be equal*; 3. *And ſimilarly ſituated.*

Fig. 37. For, 1. The geometric foci of rays flowing from the ſeveral points of the object, are at the ſame diſtances from the ſpeculum on one ſide, as their correſponding points are on the other, § ∵ the image C D, and object A B, are at equal diſtances from the ſpeculum at oppoſite ſides.

§ Subſ. 13.

2. Draw L B and L D, becauſe of the equal triangles L B F, L F D, L B = L D; and taking from two right angles the equals F L D, F L B, the angle C L D = A L B, and becauſe C L = A L, the triangle C L D = A L B, $\because$ C D = A B.

3. Becauſe the axes of the different pencils never interſect each other, the poſition of the different points in the image is ſimilar to that of the correſponding points of the object with reſpect to each other, $\because$ the object and its image are ſimilarly ſituated.

36. If an object A B be placed in a denſer medium, its image formed by the plane refracting ſurface is, 1. *Nearer to the ſurface than the object;* 2. *Is leſs than the object* (except when this is parallel to the ſurface); 3. *And is ſimilarly placed* §.

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§ "The contrary of the two firſt of theſe heads is true in the caſe of a refraction made from a rarer medium into a denſer."

Fig. 38, 39.

For, 1. The diſtances of the points of the image from the ſurface are to the diſtances of the correſponding points of the object from the ſame § as I : R, that is, in a ratio of leſſer inequality, ∵ the image is nearer to the ſurface than the object.

§ Vide Subſ. 19.

Fig. 39.

2. If the object A B be oblique to the ſurface, the image C D (intercepted between the ſame perpendiculars A L, B F,) ſhall be more * oblique to the ſurface, and ∵ leſs oblique to the perpendiculars A L and B F, and therefore leſs than the object A B.

3. As the axes of the different pencils do not ever interſect each other, the poſition of the points in the object will be ſimilar to the poſition of the correſponding points in the image.

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* For as A L : C L :: (R : I ::) B F : D F, by Subſ. 19, (dividendo & alternando) A C : B D :: A L : B F, i. e. A C is leſs than B D, ∵ C D is more oblique to the ſurface than A B.

[It is eaſy to transfer theſe demonſtrations to the caſe of refraction out of a rarer into a denſer medium.]

37. 1. The mean * diſtance of an image formed by a ſpheric ſpeculum from the principal focus *is a third proportional to the diſtances of the object and centre from the ſame focus*—for the diſtances of the ſeveral points in the image from the points of biſection, in the reſpective radii that coincide with the axes of the pencils, are third proportionals to the diſtances of the correſponding points in the object, and of the centre from the ſame points of biſection reſpectively || ∵ the mean diſtance of the image is a third proportional, &c.

Fig. 40, 41.

2. *The lineal magnitude of the object* L M, *is to that of the image* F D, *as their*

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* The mean diſtance of an object from a ſpheric ſpeculum or lens, is the diſtance of its middle point from the vortex of the former, or centre of the latter.

|| Vide Subſ. 20.

mean diſtances from the centre reſpectively, q. p[*].

For as they are baſes of ſimilar triangles C F D, C L M, they ſhall be as the ſides C F : C L, or C P : C Q.

3. *Their poſitions are ſimilar when they are on the ſame ſide of the centre; diſſimilar when on contrary ſides.*

For the axes of the ſeveral different pencils interſect each other in the centre, ∵ the images of the different points in the object lie at the ſame ſide of the axis of the ſpeculum with the points themſelves in the former caſe, and at the contrary ſide in the latter.

38. Hence the images formed by all convex ſpeculums are in poſitions ſimilar to thoſe of their objects; as alſo thoſe formed by concave ſpeculums, when the

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[*] The image does not accurately correſpond in form with the object, for it is affected by the curvature of the ſpeculum.

object is between the ſurface and the principal focus :—in theſe two caſes alſo the image is only imaginary, as the reflected rays never come to the foci from whence they ſeem to diverge. In all other caſes of reflexion from concave ſpeculums, the images are in poſitions contrary to thoſe or their objects, and theſe images are real, for the rays after reflexion do come to their reſpective foci. —Theſe things are clearly evident on inſpection of the figures, from the principles premiſed.

39. 1. The mean diſtance of an image (formed by a lens) from the object *is a third proportional to the mean diſtances of the object, from the principal focus, and the centre of the lens*; (the principal focus being taken at the ſame ſide of the lens with the object, if the lens be convex, but on the contrary ſide, if it be concave). For the diſtances of the ſeveral points in the image from the correſponding points in Fig. 42, 43, 44.

in the object, are third proportionals to the diſtances of theſe points, from the *geometric foci of parallel rays* *, (taken in the axes of the reſpective pencils as above directed) and from the *centre of the lens*; therefore the mean diſtance is a third proportional, &c.

2. *The lineal magnitudes of the object and image are reſpectively as their diſtances from the centre of the lens.*

Fig. 24, 43, 44. Becauſe the axes of the extreme pencils interſect each other in the centre, the lineal magnitudes are the baſes of ſimilar triangles, ∵ they ſhall be as the ſides, that is, as the mean diſtances from the centre.

3. *The image and object are ſimilarly ſituated, if both at the ſame ſide of the lens—diſſimilarly, if at oppoſite ſides.*

Becauſe as the axes of the extreme pencils interſect each other at the centre of the lens, the points of the object, and the correſponding points of the image, lie

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* Vide Subſ. 24.

lie at the ſame ſide of the axis of the lens Fig. 43, 44.
in the former caſe, and at oppoſite ſides
of it in the latter, ∵ &c. Fig. 42.

40. Hence any images formed by a concave lens, or thoſe formed by a convex lens where the object is within its principal focus, are in the ſame poſition with the objects they repreſent:—they are alſo only imaginary, for the refracted rays never meet at the foci whence they ſeem to diverge.

But the images of objects placed beyond the focus of a convex lens are reverſed, and alſo real, for the refracted rays do meet at their proper foci.

SECT.

SECT. V.

Of the Eye and the Nature of Viſion.

41. THE eye is nearly of a ſpherical ſhape, and is compoſed of three different ſubſtances, called, 1ſt, the *Aqueous*; 2d, the *Chryſtalline*; and 3d, the *Vitreous* humours, encloſed by three principal coats, which are formed by the expanſion of the different component parts of the optic nerve, viz. the * *Sclerotica* S S; 2d, the *Choroides* D D; and 3d, the *Retina* T T. Fig. 45. The *Sclerotica* is outermoſt; it is very ſtrong, and the fore-part, which is tranſ-

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* Over the Sclerotica is ſpread the *tunica adnata*, which forms what is commonly called the white of the eye; it is inſerted into the Sclerotica at the Cornea; but is not reckoned properly a part of the eye.

parent

parent and ſomewhat prominent, is called the *Cornea* C.—The Choroides is next in order, and has a circular perforation P, called the *Pupil**, immediately behind the middle of the Cornea :—the part II. of the Choroides viſible behind the Cornea is flat; it is called the *Iris*, or *Uvea*, and is differently coloured in different perſons. The Retina is the inmoſt coat, it extends round the eye 'till it meets the *ciliary ligaments* Q Q, membranes proceeding from the Choroides, and attached to the *capſula* or filament, which encloſes the Chryſtalline humour R.—The Chryſtalline is the moſt denſe of the three humours, and is in the ſhape of a double convex lens, whoſe fore-part has the leſs curvature ; the cavity between the cornea and the Chryſtalline is occupied by the aqueous humour, which has rather the Exp. 43.

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* The pupil is capable of contraction and dilatation, which are uſeful to procure greater diſtinctneſs and ſplendour in the picture on the *Retina*, when either is required.

leaſt denſity of the three, and the ſpace between the bottom of the eye and the Chryſtalline is filled by the vitreous humour V.

42. Objects preſented to the eye have their images painted on the back part of the retina, the rays of the incident pencils converging to their proper foci there by the refraction of the different humours :—and for this office they are admirably adapted; for as the diſtance between the back and front of the eye is very ſmall, and the rays of each of the pencils that form the image fall parallel, or elſe diverging on the eye, a ſtrong refractive power is neceſſary for bringing them to their foci at the retina—but each of the humours, by its peculiar form and denſity, contributes to cauſe a convergence of the rays;—the aqueous from its convex form; the Chryſtalline by its double convexity and greater denſity

than the aqueous *; and the Vitreous by a leſs denſity than the Chryſtalline joined to its concave form.

Theſe things are manifeſt from Subſ. 21ſt and 22d. The ſtructure of the eye is in general adapted to the reception of parallel rays—But as the diſtances of viſible objects are various, ſo the eye has powers of accommodating itſelf to rays proceeding from different diſtances, by altering the diſtance of the Chryſtalline from the retina †, which is done by the action of the ciliary ligaments.

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* Beſide this the Chryſtalline is much more denſe toward the middle than toward the edges, by which means the rays of any pencil that fall on the middle, and therefore almoſt perpendicularly, are brought to their focus as ſoon as thoſe that fall toward the edges, and therefore more obliquely—the exceſs of refraction ariſing from the greater denſity toward the centre ſupplying the defect from the want of obliquity in the incidence.

† This is amply proved by Dr. Porterfield, in his treatiſe on the eye; where it is ſhewn that in conſe-

43. That this change of ſituation in the Chryſtalline is adequate to ſuch accommodation may be thus ſhewn.—Suppoſe a pencil of rays to diverge from a point A, at a diſtance from the eye leſs than that which admits diſtinct viſion in the uſual ſituation of the humours :—the Fig. 46. rays would come to a focus V behind the retina L M ;—Let the Chryſtalline O P be brought forward, and C V the diſtance of the focus from the Chryſtalline will be increaſed ‡ ; but (becauſe of the great

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quence of the change of place that the Chryſtalline ſuffers, there will alſo enſue an alteration in the ſhape of the Cornea, which contributes to produce the effect required. To this latter caſe ſolely it is attributed by Dr. Helſham ; but Dr. Porterfield's ſyſtem ſeems much better ſupported and more convincing.

† By Cor. to Subſ. 25, we have this proportion, Fig. 46. A F : A C :: F C : C G, but if the lens be moved nearer to A, the proportion of A F to A C will be diminiſhed, and ∵ the proportion of F C to C G will be diminiſhed alſo ; and ∵ as F C is conſtant, C G will be encreaſed.

proportion

proportion that A C the ſmalleſt diſtance that admits diſtinct viſion has to F C the focal length of the Chryſtalline,) the diſtance C G of the Chryſtalline from the retina will be more encreaſed * than C V, ſo that C G and C V may become equal, and thus the focus made to fall exactly on the retina. The caſe of viewing remote objects need not be conſidered, as the common ſtructure of the eye is adapted to the admiſſion of rays coming from diſtant points—but if it were neceſſary to be explained, the converſe of this reaſoning would apply to it.

Theſe powers of accommodation are however limited; and the ſight is ſaid to be perfect when the eye can adapt itſelf to any diſtance within the uſual limits—and when it cannot viſion is indiſtinct.

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* e. g. Let A C be = 15, and F C be = 3; then A F = 12, and C V = $3\frac{3}{4}$. Diminiſh A F and A C by 3, and C V will be = 4; i. e. the increaſe of C G will be 3, while that of C V is only $\frac{1}{4}$.

44. Defective ſight ariſes from an incapacity of altering the poſition of the Chryſtalline within the uſual limits.—1. When it cannot be brought cloſe enough to the cornea, near objects appear indiſtinct—to this defect people in years are generally ſubject. 2. Where Exp. 48. the Chryſtalline cannot be drawn ſufficiently near to the retina, remote objects appear indiſtinct—this is the defect under which Myopes or ſhort-ſighted people labour. In each of theſe caſes the images of the different points in the object would be diffuſed over ſmall circles on the retina; and ſo being intermixed and confounded with each other, would there form a very confuſed picture of the object: for in the former caſe, the image of any point would be formed behind the retina, as the refraction of the eye is not ſufficiently ſtrong to bring the rays (diverging ſo much as they do in proceeding from a near point) to a focus at the retina.

This

This defect will therefore be remedied by a convex glaſs, which makes the point whence the rays now proceed more diſtant‡ than the object; ∵ the rays falling on the eye ſhall now diverge leſs than before, or elſe be parallel*, and ∵ ſhall be brought to a nearer focus, viz. at the retina.

‡ Vide Subſ. 25.

In the latter caſe the image is formed before the retina, becauſe the refractive power of the eye is too great to permit rays ſo little diverging (as they do in proceeding from a diſtant point) to reach the retina before they are collected into a focus—in this caſe the defect is ſupplied by a concave glaſs, which makes the point where the rays diverge, nearer † than the object; ∵ the rays falling on the eye will now diverge more than before, ſo as when refracted through the humours not to come to their focus before they reach the retina.

Fig. 47.

† Vide Subſ. 25.

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* Provided the diſtance of the object from the lens be not greater than its focal length.

Spectacles are conſtructed on the above principles, concave for ſhort-ſighted, and convex for long-ſighted people.

The theory of adapting the curvature of the glaſſes to the different degrees and kinds of defective ſight is laid down very clearly by Dr. Helſham, in his chapter on Dioptrics.

SECT.

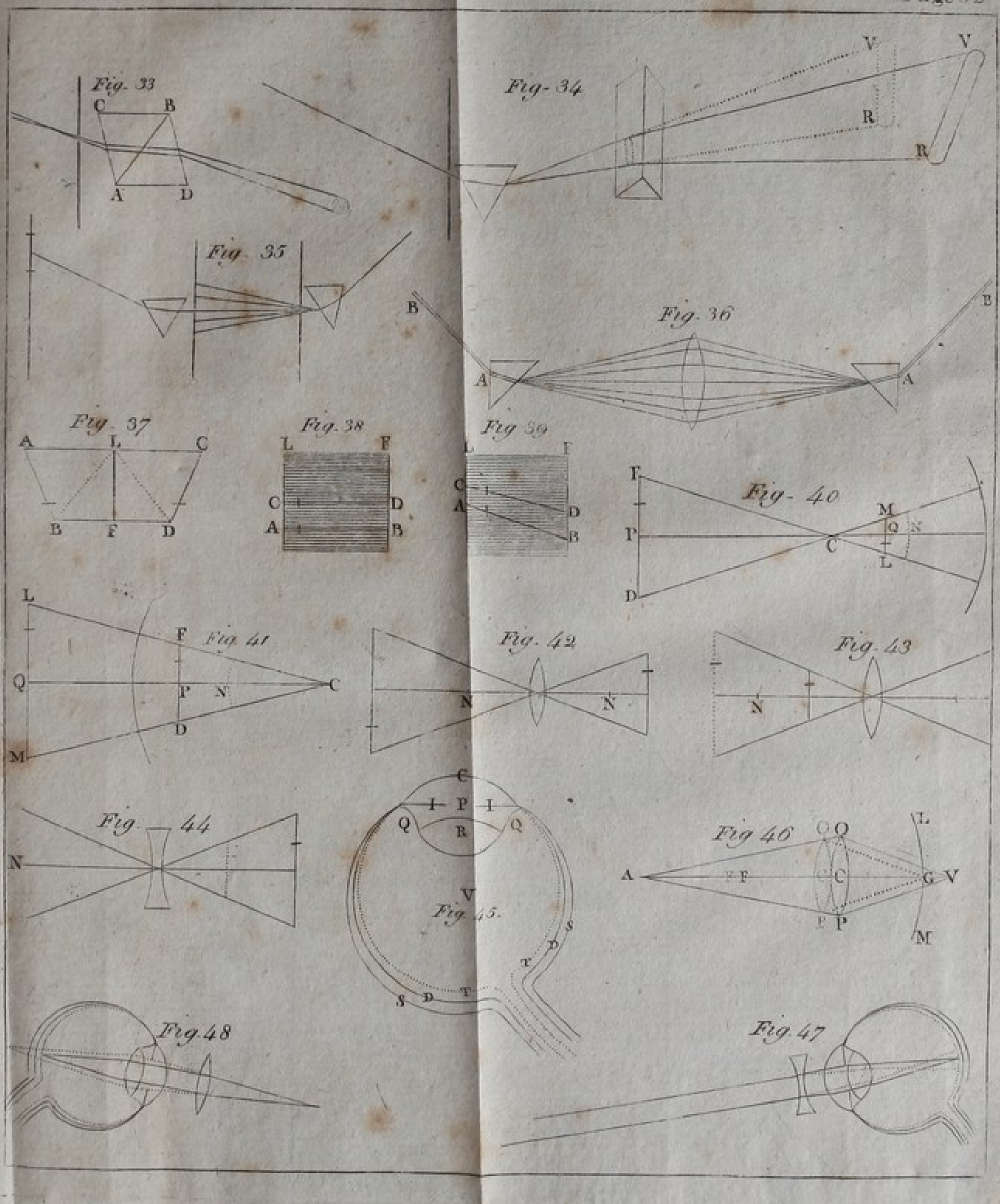
Fig. 33
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Fig. 34
V
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Fig. 35
Fig. 36
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Fig. 37
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Fig. 38
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Fig. 39
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Fig. 40
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Fig. 41
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Fig. 42
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Fig. 43
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Fig. 44
N
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Fig. 45
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Fig. 46
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O
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Fig. 48
Fig. 47

SECT. VI.

Of the apparent Magnitude, Situation, and Diſtance of Objects.

45. THE apparent magnitude * of objects, whether ſeen by the naked eye, or with glaſſes, is always proportioned to the magnitude of the image on the retina, i. e. to the angle formed by the axes of the extreme pencils that enter the pupil, q. p. Fig. 58.

Hence the apparent magnitude of any body will be *inverſely* as its diſtance from the eye, and *directly* as its real magnitude;—and as the appearance of any object ſeen with one or more glaſſes is the ſame as that of its laſt image to the

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[*] N. B. When magnitudes are ſpoken of, the lineal magnitudes are to be underſtood, unleſs the contrary is expreſſed.

naked eye, "*the apparent magnitude of an object ſeen with one or more glaſſes is as the real magnitude of the laſt image directly, and as its diſtance inverſely.*"

46. Hence objects ſeen through concave or convex lenſes, where the eye is at the lens—or in a concave mirror when * the eye is at the centre of the ſphere, appear of the ſame ſize as they do at the naked eye—for in theſe caſes the magnitude of the object is to that of its image as their diſtances from the eye. Fig. 49, 50, 51, 52.

The apparent magnitude of an object ſeen by the naked eye is likewiſe equal to its apparent magnitude ſeen with a glaſs, where the object is adjacent to the glaſs—for in this caſe the image and object coincide, as is eaſily ſeen from Subſ. 37, and Coroll. of Subſ. 25. But (the ob-

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* N. B. In the caſes where the image and object are at different ſides of the centre of the glaſs the object appears of the ſame magnitude that the image would if viewed by the eye turned toward it, as is explained in a ſubſequent note.

ject being at ſome diſtance from the glaſs) ſuppoſe the eye to be removed beyond the convex lens, or centre of the ſpheric concave, and whatever be the diſtance of the object from the glaſs it will appear* magnified in every caſe *except one* §, for the image is to the object in a greater ratio than that of their reſpective diſtances from the eye, whenever they are at the ſame ſide of the lens or centre of the ſpheric ſurface:—and alſo when they are at different ſides of it, if the eye be between the image † and the lens, or centre of the

§ See the next page ☞

Fig. 55, 56.

Fig. 53, 54.

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* The reverſe is true with reſpect to a concave lens, for if both the eye and object be removed from the lens, the object will always appear diminiſhed.

† It is proper to obſerve that in this caſe the object appears of the ſame magnitude that the image would if the eye viewed it directly from C— for the extremities of the object are ſeen in the direction of L C and M C, the principal viſual rays of their reſpective pencils; but theſe rays form an angle equal to the angle Q C R formed by Q C, R C, the rays in whoſe directions the extremities of the image would appear if the eye viewed it from C directly.

Fig. 57. ſpheric ſurface; or if the eye be beyond the image, if this be greater than or equal to the object—or even though it be leſs, while the eye is within a certain diſtance; for in all theſe caſes the image is to the object in a greater proportion than the reſpective diſtance from the eye; therefore in all theſe caſes the object will appear magnified.—But if the ☞ eye recedes ſtill farther from the image, the object will firſt appear in the glaſs of the ſame magnitude as it would to the naked eye, and afterwards appear diminiſhed—for the image and object firſt become proportionable to their diſtances, and after this the image is to the object in a proportion leſs than that of their reſpective diſtances.

By a proper application of the rule at the end of the preceding Subſection, which the principles premiſed will eaſily direct us to, we may in all caſes find when an object will appear magnified, and when diminiſhed.

47. The

47. The pictures of objects ſeen by the naked eye are inverted on the retina †, but the objects appear erect.—Various ſolutions are given for this apparent repugnance between the cauſe and effect, among which the following ſeems to be the moſt natural.

Inverſe or direct poſition is only the different ſituation of objects with reſpect to the earth; when the earth therefore, in viſion, ſuffers the ſame change of ſituation with the objects, the relative poſition of theſe with reſpect to the earth remains unaltered, i. e. objects are ſeen erect whoſe pictures are inverted on the retina; and, on the contrary, thoſe objects appear inverted whoſe pictures are erect on the retina.

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† Experience ſhews this to be fact—for the images of external objects appear inverted on the retina of the eye of an animal, when it is turned toward the light and ſtripped of the outſide coats. It is accounted for in theory in the ſame manner as the inverſion of the images formed by a convex lens.

Hence an object ſeen through a concave lens always appears erect, for its image is erect§, ∵ the picture on the retina is inverted, ∵ the object appears erect—The ſame reaſoning and concluſion will apply to the caſe of objects ſeen in a convex ſpeculum—and alſo to that of objects ſeen through a convex lens or in a concave ſpeculum when the image is imaginary (i. e. † when the object is placed within the principal focus)—or even though the image be real (i. e. ‡ when the object is placed beyond the focus) if the eye be placed between the image and the lens, or centre of the ſpheric ſpeculum*.

§ Subſ. 39.

† Subſ. 40.

‡ Subſ. 40.

But if the eye be placed beyond a real image formed by the concave ſpeculum

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* In this laſt caſe the object will be leſs diſtinct than in others, for the rays of each pencil fall on the eye converging—whereas (as was obſerved before) the eye is both by its form and by habit accommodated to receive pencils of rays parallel to, or rather a little diverging from each other.

Of

or convex lens, ſince the image is inverted, the picture on the retina will be erect, and ∴ the object will appear inverted.

48. The apparent diſtance of objects depends on their apparent magnitude, ſplendor and diſtinctneſs;—remote objects being found to appear ſmaller and more faint as their diſtance is greater, and near objects more confuſed and larger as their diſtance is leſs. We hence conclude, that (of objects which we are acquainted with) thoſe that appear of leſs magnitude and ſplendor than uſual are more remote, and that thoſe whoſe magnitude and confuſion are greater than uſual, are nearer to the place of obſervation.

The ſame general rules will hold for determining the apparent diſtance of objects ſeen with glaſſes—It would however be improper to enlarge upon theſe obſervations, as they rather belong to metaphyſical enquiry than to the ſubject under conſideration—and may be found treated of with great ingenuity and acuteneſs in Berkley's Eſſay on Viſion.

SECT. VII.

Of Teleſcopes.

49. THE two obſtacles to the accurate viſion of remote objects (viz. 1, faintneſs; and 2, want of ſufficient apparent magnitude) are remedied by the teleſcope; the former, becauſe the object-glaſs by its breadth collects a much greater quantity of the rays flowing from the object than the pupil of the eye can poſſibly do; and the latter, becauſe the object-glaſs forms near the eye a diſtinct image of the object, which the eye-glaſs enables us to view with every advantage of

of cloſeneſs and diſtinctneſs, as will be ſeen in the explanation of the inſtrument.

50. Teleſcopes are of two kinds; 1, *refractors*; and 2, *reflectors*. Of each there are various ſpecies.

The moſt ſimple ſpecies of the former conſiſts of two lenſes inſerted into the oppoſite extremities of a tube—the lens (which is placed next the object in obſervations, and therefore called the *object-glaſs*) is broad and convex; and will form the image E F, (as the object is ſuppoſed very remote) q. p. at its principal focus Q.—If the conſtruction of the eye enabled it to view this image within a ſufficiently ſmall diſtance, it would ſee the object clearly and much magnified: but as the pupil is ſmall; and as the eye is adapted to the admiſſion of rays nearly parallel, it cannot of itſelf cloſely view this image, which is uſually of great breadth in compariſon of the pupil, and Fig. 59, 60.

where ſo great a degree of divergence takes place in the pencils proceeding to the eye from each point in the image*. The eye-glaſs G H ſupplies this defect—for being placed at the diſtance of its own focal length C Q from the image E F, the rays of each ſeparate pencil paſſing through it, will emerge parallel to each other †.

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* The quantity of any object ſeen in a teleſcope, or the extent of view it can command, depends on *the field*, i. e. on the greateſt viſual angle G B H, it admits. This in the aſtronomical teleſcope is as the breadth of the eye-glaſs directly, and inverſely as the diſtance between the eye-glaſs and object-glaſs. But in the Gallilean it is as the breadth of the pupil directly, and inverſely as the diſtance of the eye-glaſs from the object-glaſs, as is plain from their figures. Fig. 59, 60.

† It is evident that to adapt any teleſcope to ſhort-ſighted perſons, the diſtance of the eye-glaſs from the image ſhould be leſs than its focal length, in order to make the rays of each pencil diverge to the eye; and for long-ſighted perſons, that diſtance ſhould be greater than the focal length of the eye-glaſs, that the rays of each pencil may fall converging on the eye. Vide Subſ. 25 and 44.

Fig. 59, 60. This will be the caſe whether the eye-glaſs be convex (as in the common aſtronomical teleſcope) or concave as in the Gallilean; but in the latter the obſtacle to a cloſe view of the image, ariſing from the ſmallneſs of the pupil, is not remedied (becauſe the pencils diverge from each other after paſſing * the eye-glaſs) as it is in the former, where the ſeveral pencils emerging from the eye-glaſs interſect; and the eye being placed at their interſection, receives all the pencils that fall from the image on the eye-glaſs. Hence we ſee how teleſcopes produce diſtinctneſs and vividneſs in the appearance of very remote objects.

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[*] By which means, 1, the field of view is contracted, (as all the parallel rays belonging to ſome of the exterior pencils will in many caſes eſcape the pupil); and 2, the apparent ſplendor of the object is diminiſhed, eſpecially toward the extremity of the field; for ſome of the parallel rays belonging to the extreme pencils which fall on the eye, paſs by the pupil while others enter it.

51. Their magnifying powers are eſtimated by the following rule:

"The *lineal* magnitude of an object ſeen with a teleſcope, is to that ſeen by the naked eye as the focal length of Fig. 59, 60. the object-glaſs to the focal length of the eye-glaſs."

For the latter magnitude is meaſured by the angle L B M formed at the eye, or (q. p.) at the * object-glaſs, by the axes of the extreme pencils: i. e. by the angle F B E which the image ſubtends at the object glaſs. But the former magnitude is meaſured by the angle G O H under the extreme pencils of parallel rays emerging from the eye-glaſs, i. e. by the angle E C F under the axes of the extreme pencils incident on the eye-glaſs §, ∵ the former magnitude : the latter, :: angle E C F : angle E B F

§ Vide Note to Subſ. 24.

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* Becauſe the length of the teleſcope is inconſiderable with reſpect to the diſtance of remote objects.

:: B Q

∷ B Q † the focal length of the object-glaſs, : C Q the focal length of the eye-glaſs *.

52. The poſition of objects ſeen with a Gallilean teleſcope is erect; for ſince by the operation of the concave eye-glaſs the rays tend to the ſame ſides of the eye as they do of the image (which is inverted) the picture on the retina will be inverted, ∴ the object will appear erect. But in the aſtronomical teleſcope, the rays coming from the extremities of the image interſect each other, and ∴ tend to the contrary ſides of the eye—

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† For ſmall angles are directly as their ſubtenſes and inverſely as the perpendiculars let fall from their vertices on the ſubtenſes, q. p. ∵ given the ſubtenſe, as in this caſe the angles ſhall be inverſely as the perpendiculars.

* Hence we ſee the reaſon why objects appear magnified if the broad and leſs convex lens be turned toward the object (as is uſually done) but diminiſhed, if the narrow and more convex lens be uſed as the object-glaſs of the teleſcope.

conſequently (as the image is inverted) the picture on the retina will be erect, ∴ the object will appear inverted. This inverſion is of no moment in aſtronomical obſervations; but as it is convenient in viewing terreſtrial objects that they ſhould appear erect, this purpoſe is effected by the addition of two eye-glaſſes, whoſe diſtance from each other ſhould be ſufficient to permit the interſection of the extreme pencils of parallel rays* proceeding from the firſt eye glaſs. At the focus of the ſecond an erect image ſhall be formed, from whoſe extremities the rays proceeding to the laſt eye-glaſs will emerge parallel from that glaſs, and tend to the contrary extremities of the eye, ∴ the picture on the retina is inverted, and ∴ the object ſhall appear erect.

Fig. 61.

It is obvious from what has been ſaid, that the diſtance between the object-glaſs and firſt eye-glaſs is equal to the ſum of their focal lengths, and alſo

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* Vide Note to Subſ. 24.

alſo that the diſtance of the ſecond and third eye-glaſs, from each other, is equal to the ſum of their focal lengths.

To prevent the neceſſity of a large aperture * in the principal eye-glaſs, it is convenient that the diſtance of the ſecond eye-glaſs, from the concourſe of the pencils proceeding from the firſt, ſhould not be leſs than the focal length of the ſecond eye-glaſs, otherwiſe (the pencils diverging from each other after refraction,) the ſecond image would be ſo much enlarged as to be incapable of being received on a third and principal eye-glaſs of ſufficiently ſmall aperture. This teleſcope with three eye-glaſſes is called the *common terreſtrial teleſcope.*

53. *Reflectors* are of three kinds, the Newtonian, the Gregorian and Caſſegrain's. It was by experience diſcovered that

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* The inconvenience of ſo large an aperture will appear hereafter.

that *refractors*, in which great magnifying power and light were required, cauſed a material confuſion in the objects that could no way be remedied, unleſs by unmanageable lengths; and Newton having diſcovered the chief cauſe of this imperfection to ariſe from the different refrangibility of light; and ſuppoſing it was impoſſible to refract the rays proceeding from the object, to their foci at the image, without an enſuing diſperſion of the colours, thought the perfection of refractors was not to be hoped for—He therefore applied himſelf to combine the powers of reflection and refraction in the conſtruction of a teleſcope, and attained his object with the greateſt ſucceſs.—His invention was as follows:

Into the end of a ſhort tube he inſerted a concave ſpeculum of about ſix inches in lineal aperture—before the principal focus (where the image of a remote Fig. 61.

remote object would be formed) a plane * ſpeculum, inclined to the axis of the concave in an angle of forty-five degrees, receives the rays proceeding to the image and reflecting them obliquely, forms another image equal to the preceding †—the rays diverging from each point of this image are received by a convex eye-glaſs at its own focal diſtance from the image.

† Subſ. 35.

The object appears inverted in this teleſcope, but may be erected by the addition of two convex eye-glaſſes as in the refractor.—The lineal magnitude of an object to the naked eye is to its lineal magnitude as ſeen in this teleſcope, as the focal length of the eye-

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* Newton originally uſed a rectangular iſoſceles priſm for this purpoſe; the incident rays, after paſſing through one of the perpendicular ſides, was reflected by the baſe and emerged uncoloured through the other ſide. The image was erected by making the ſides of the priſm a little convex.

glaſs to the focal length of the concave ſpeculum.—This will appear as in the computation of the magnifying powers of the common refractor; when it is conſidered that the image ſeen is equal to that which would be formed q. p. at the focus; and that the angle meaſuring the apparent magnitude to the naked eye is equal to that which the image in the focus ſubtends, at the centre of the ſpheric ſurface of which the ſpeculum is a ſegment.

54. The Gregorian reflector is thus conſtructed:

Into the extremity of a ſhort and wide tube is inſerted a concave ſpheric ſpeculum perforated at the vertex—beyond the focus of the ſpeculum, where the firſt image is formed, is placed another concave ſpeculum, whoſe diſtance from[*] Fig. 63.

the

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[*] For if it were not greater than the focal length,

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the focus of the principal ſpeculum is greater than its own focal length, but leſs than its radius—at the perforation of the firſt ſpeculum is inſerted a tube, in which the eye-glaſs is placed to receive the rays proceeding from the ſecond image (formed by the ſmaller ſpeculum)—its diſtance from the place of this image ſhould be equal to its focal length, by which means the rays of each pencil emerge parallel.

It is clear that the object appears erect in this teleſcope, for the firſt image is inverted †, and the ſecond image is inverted in reſpect of the firſt, i. e. it is erect in reſpect of the object—conſe-

† Vide Subſ. 37.

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the rays of each pencil would never come to a focus, and therefore no ſecond image could be formed; and if it were greater than the radius, the ſecond image would be farther from the eye-glaſs than the firſt, and conſequently both the field and power of the inſtrument would be too much diminiſhed.

quently its picture on the retina is inverted, and ∵ it will appear erect*.

55. Caſſegrain's teleſcope is the ſame with the Gregorian, except in the form and poſition of the leſſer ſpeculum, which is convex, and placed before the focus

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* The lineal magnitude of an object ſeen with this teleſcope, is to its apparent magnitude at the naked eye, in a ratio compounded of that of G F, (the focal length of the object ſpeculum) to om, (the focal length of the eye-glaſs) and of ro, (the diſtance of the ſecond image from the focus of the leſſer ſpeculum), to rz (the focal length of that ſpeculum)— for theſe magnitudes are to each other, as the angles $pmq : LGS :: \| \left\{\begin{matrix} GF : om \\ pq : LS \end{matrix}\right\} :: \left\{\begin{matrix} GF : om \\ zo : fz \end{matrix}\right\} :: \left\{\begin{matrix} GF : om \\ ro\text{-}rz : rz\text{-}rF, \end{matrix}\right.$ but ſince ro, rz, rF are continually proportional, adding $rz : rF$ reſpectively to their proportionals, $ro\text{-}rz : rz\text{-}rf$, it will be, as $ro\text{-}rz : rz\text{-}rF :: ro : rz$, ∵ by ſubſtituting the two latter terms in the place of the two former in the compound ratio above deduced, we have the apparent magnitude in the teleſcope to that at the naked eye $:: \left\{\begin{matrix} GF : om \\ ro : rz \end{matrix}\right\} :: GF \times ro : om \times rz$: &c.

|| Vide Note to Subſ. 51.

of

Fig. 64. of the object ſpeculum, at a diſtance leſs than its own focal length.

The object will appear inverted in this teleſcope; for the ſecond image is inverted with reſpect to the object, ∴ its picture on the retina will be erect, ∴ it will appear inverted.

Its magnifying power is computed as in the Gregorian.

56. Reflecting teleſcopes are ſuperior in their magnifying powers to refractors, for in the latter great magnifying power is produced, either, 1, by making the object-glaſs of a great focal length, which is in practice exceedingly inconvenient*; or, 2, by making the eye-glaſs of a very ſmall focal length, which would make the object appear very confuſed, as the errors generated in the

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[*] This appears from conſidering that the lineal magnifying power is as the focal length of the object-glaſs directly, and inverſely as the focal length of the eye-glaſs. Vide Subſ. 51.

image by the different refrangibility of light would thereby be too much magnified to admit diſtinct viſion.—Neither of theſe inconveniencies, then, can be avoided in a refractor of great magnifying power, without incurring the other.

But as [*] the image formed by the concave ſpeculum in the reflector is much more accurate than that formed by an object-glaſs of the ſame focal length and of a ſufficient aperture, therefore an eye-glaſs of leſs focal length, and conſequently of greater magnifying power, may be applied in the reflector than in the refractor [‡].

Though

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[*] For the circle of aberration formed at the focus of each pencil by the ſphericity of the ſpeculum, is far leſs than that proceeding from the different refrangibility of light paſſing through the object-glaſs.——*Vide Smith's Opt. B. 2. C. 6. or Newt. Lect. Opt. P. 1. Sect. 4th, ad fin.*

[‡] If it be deſired to conſider this matter in a leſs popular way, conſult Smith's Opt. B. 2. C. 7. where

Though the reflector has the above mentioned advantage over the refractor, it is counterbalanced in ſome degree by the inconveniencies it is ſubject to—as 1, More light is loſt in reflexion at the mirrors than in traſmiſſion through the glaſſes of the refractor. 2, A more frequent loſs of poliſh happens in the ſpeculums than in the lenſes, and a greater difficulty in reſtoring it. 3, The ſhape of the ſpeculums is much more liable to change than that of lenſes, either by warping or by inequalities produced in the grinding, or by the frequent cleanſing they require*. And this change of ſhape is of peculiar ill conſequence in the re-

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where it is ſhewn that the lineal amplifications are ∷ the ſquare roots of the lengths in the refractors, and ∴ that a lengthening of the teleſcope muſt conſtantly attend any increaſe of its power.

[*] There is an imperfection in all reflectors not generally remarked, viz. the valuable rays near the axis are loſt by the obſtruction of the leſſer ſpeculum.

ecting teleſcope, as the errors ariſing from unequal incidences at reflecting ſurfaces are ſix times greater than ſimilar errors in refractions between air and glaſs—for the angle C B H under the Fig. 65. reflected rays is equal to twice D B E, the difference of the incidences §; while the Angle G B H under the refracted Fig. 66. rays is but a third part of M B N, the difference of the incidences *.

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§ Suppoſe the poſition of the reflecting ſurface be changed from S O to F G, then as the angle of incidence A B D, is encreaſed by D B E, the difference of the incidences, the new angle of reflection E B H ſhall exceed D B C, the former angle of reflection, by an angle = to D B E; therefore C B H (the exceſs of A B E + E B H above A B D + D B C) ſhall be equal to twice D B E.

* As into a glaſs out of air I : R :: 3 : 2, ſmall angles of incidence and refraction will be in the ſame proportion, ∴ the refracted angles E B G and E B H are but the third parts of A B M, A B N, their reſpective angles of incidence, ∵ the difference G B H of theſe thirds will be equal to M B N, the third of the difference of the incidences.

N. B. A ſimilar proof may be applied where the incidence is diminiſhed.

But the late invention of compound object-glaſſes has procured for refracting teleſcopes all the advantages of amplifying and light that reflectors poſſeſs, without ſubjecting them to any of the imperfections juſt mentioned, except ſome loſs of light.

SECT. VIII.

Of the Errors cauſed by the Object-Glaſſes of Teleſcopes, and the Methods of correcting them.

57. THE great utility of this invention of teleſcopes is too apparent to require remark; but the imperfections it laboured under limited very much the advantages to be expected from it.

Thoſe imperfections principally relate to the errors cauſed by the *form* and *material*

material of the object-glaſs in the image formed at its focus *.

1ſt, It is plain, that of rays parallel or diverging that fall all over the ſurface of a lens, thoſe fall more obliquely that are neareſt to the extremities; they will therefore ſuffer the greateſt refraction, and will meet the axis of their pencil at a leſs diſtance from the lens than thoſe which fall nearer to that axis— Fig. 67, 68. the image I of the lucid point O, formed by the moſt remote rays, is called the extreme image; and its image P, formed by the rays neareſt to the axis, is called the principal image, which will neceſſarily be confuſed by the divergence of the rays flowing from the extreme and intermediate images. The ſpace I P between the principal and extreme images is called the *ſpace of diffuſion*, and is occupied

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* The errors occaſioned by the eye-glaſs will be treated of in another place.

cupied by other images of the lucid point, which are formed by the ſucceſſive annuli from the extremities of the lens to its centre; the ſmalleſt ſpace (*r p*) into which the rays are collected is evidently at the interſection of the rays neareſt to, and moſt remote from the axis; and this ſpace is called *the circle of aberration, ariſing from ſphericity.* The ratio of its diameter * to the lineal aperture of a plano-convex lens, which was four inches, and the radius of its ſpheric ſurface, one hundred feet, was found by calculation to be as § 1 to 299,695; and it is varied in different lenſes as the cubes † of the linear apertures directly, and inverſely as the ſquares of the focal lengths; hence it appears that this error always is augmented as the aperture is enlarged for a more copious admiſſion of light, or as the length of the teleſcope

§ Vide Newt. Op. 7th Prop. P. 1.

† Vide Newt. Lect. Opt. P. 1. Sect. 4.

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* When the incident rays are parallel to the axis of the lens.

is diminiſhed for greater conveniency of obſervation.

58. The limitations hence ariſing have far leſs impeded the perfection of the inſtrument than the confuſion occaſioned in the image by the different refrangibility of light; and, that great confuſion muſt reſult from this cauſe will be obvious to any one who views an object through a priſm; for its extremities will appear very indiſtinct, and tinged with a variety of compound priſmatic colours; a ſimilar appearance muſt enſue in viewing objects through a lens whoſe ſhape correſponds to that of a priſm. The only difference in the effects of theſe two inſtruments ariſes from the diſparity of their refracting angles; to which cauſe alſo it is to be attributed that the colouring occaſioned by the central parts of a lens is leſs than that cauſed by its exterior parts. This error is ſo great,

that the diameter of the circle of aberration proceeding from this cauſe is equal to $\frac{1}{55}$th part of the diameter of the aperture*.

This aberration, ſo far greater than the former, cauſed Sir I. Newton to deſpair of the poſſibility of making refractors with any conſiderable magnifying powers. He therefore ſubſtituted in their place the reflectors above deſcribed. Theſe, though free from the confuſion owing to the different refrangibility, and capable of being much ſhortened, ſtill were ſubject to the other diſadvantages mentioned in Subſ. 56; and beſide, the errors from ſphericity were common to them with refractors, and obſerved the ſame law, viz. : : $\frac{Lin.\ Apert.^3}{Foc.\ lengths^4}$ Both errors, however, were at once corrected by the very ingenious invention of compound glaſſes by Mr. Dolland.

The

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* Vide Smith's Optics, Book 2. Chap. 6.

The proceſs of this diſcovery, ſo intereſting to ſcience, is a ſtriking example of the gradations, by which the obſervation of a ſimple phenomenon may be purſued to purpoſes of a very complicated nature and general utility.

59. Newton's deſpair of bringing refracting teleſcopes to perfection aroſe from a perſuaſion that the diſperſion of the rays was always proportioned to their mean refraction, and of conſequence that no change in the direction of a ray could be made by refraction through any number of mediums, without an enſuing dilatation; hence in a teleſcope where ſuch a change of direction muſt happen in order to the formation of the image, that the image of each point in the object would neceſſarily be diffuſed over a circle of aberration whoſe diameter is equal to $\frac{1}{55}$th part of the diameter of the aperture, and from the

mixture

mixture of thoſe different circles, that a conſiderable confuſion muſt always exiſt in the image, if the object-glaſs be of large aperture.

The ground of Newton's concluſion was an experiment not inſtituted, nor perhaps obſerved, with the great accuracy ſo conſpicuous in the works of that illuſtrious philoſopher.—It was this: In a priſmatic veſſel whoſe ſides containing the refracting angle were moveable at pleaſure, a glaſs priſm was placed with its refracting angle upward; and the veſſel being filled with water, a ray was tranſmitted through both priſms, which, whenever it emerged in a direction parallel to that of its incidence, appeared white, and if inclined to it was always coloured; from ſuch phænomena the deduction above mentioned was fairly drawn—But on a repetition of this experiment by Mr. Dolland, the reſult was quite

quite the reverſe of that mentioned by Newton; for when the ray emerged in a direction parallel to that of its incidence, it appeared ſtrongly tinged with priſmatic colours.—On the other hand, by a proper adjuſtment of the refracting angles of the veſſel and priſm, while the direction of the ray was changed by the exceſs of refraction of the [†] water priſm, the ray emerged white; its diſperſion being counteracted by the contrary action of the glaſs priſm, which therefore appeared from this experiment to have an equal power of diſperſing the rays, (or collecting them when diſperſed) though with a ſmaller power of mean refraction, and conſequently that glaſs has a greater power of diſperſion in proportion to its mean refraction than water.

[†] This is ſolely owing to the exceſs of the *refracting angle* of the water priſm above that of the glaſs priſm.

60. Mr.

60. Mr. Dolland then endeavoured to apply this principle to practice, by conſtructing lenſes of glaſs with water encloſed, ſo that the mean refraction of the water ſhould be greater than that cauſed by the glaſs, and contrary thereto, for the purpoſe of counteracting the diſperſion made by the glaſs; while at the ſame time the rays of the ſeveral pencils were collected to their reſpective foci at the image, by the difference of the mean refractions.—But this method was found ſo extremely difficult, from the depth of the lenſes neceſſary in the conſtruction, &c. that he was obliged to relinquiſh it.

He ſuſpected, however, that different kinds of glaſs might alſo have different powers of diſperſion in proportion to their mean refractive powers; and experiment abundantly confirmed his conjecture, for by joining two priſms of ſmall angles made of crown-glaſs and white flint-glaſs, with their refracting angles in oppoſite directions, he produced effects ſimilar to thoſe that appeared in the experiment of the priſmatic veſſel; viz. 1ſt, with one pair of priſms, an emergence of the ray in a direction parallel to that of its incidence, with a ſtrong tincture of priſmatic colours;—and 2dly, with a different pair of priſms, an emergence of the ray perfectly uncoloured, in a direction inclined to that of its incidence—From theſe experiments white flint-glaſs appears to have the greater power of mean refraction, and alſo a greater power of diſperſion, in proportion to its mean refractive power, than crown-glaſs.

61. This immediately led him to the conſtruction of a double object-glaſs, compounded of a double concave of white

white flint, and a double convex of crown glaſs—the exceſs of mean refraction was in the latter, to bring the rays of each pencil to a focus at the image, and by its *form* to counteract ſufficiently the greater diſperſing power in the *ſubſtance* of the concave glaſs. Any ray paſſing through theſe two glaſſes was untinged by priſmatic colours, though it interſected the axis of its pencil after refraction; no image, therefore, whether extreme, principal, or intermediate, was confuſed by the different refrangibility: neither did any confuſion of moment ariſe from the ſphericity of the glaſs; becauſe the ſpace of diffuſion was made nearly to vaniſh, and the extreme image to coincide with the principal image: q. p. —for the aberration of the rays remote from the axis, which would be cauſed by the convex glaſs, were counteracted by the contrary aberrations produced by the

the concave; each aberration encreaſing as the incidence was more remote from the vertex of the object-glaſs, but ſtill in all parts compenſating each other very nearly.

62. But though the error from ſphericity is much diminiſhed by the conſtruction of this double object-glaſs, yet as the exceſs of refraction is in the convex lens, the aberration from the geometric focus cauſed by this lens muſt exceed the contrary one produced by the concave, and diſturb the diſtinctneſs of the image.

In order, therefore, entirely to exterminate this error, and yet to retain the correction of the diſperſion, Mr. Dolland conſtructed a triple object-glaſs, compounded of a concave encloſed between two convex lenſes. The ſum of the refracting as well as diſperſing powers of the two latter was equal to thoſe of the convex in the double object-glaſs; the rays, therefore, of the reſpective pencils were brought to their proper foci, and yet the image was not confuſed by the different refrangibility—the aberration from ſphericity was totally removed, becauſe the ſum of the errors cauſed by the two convex lenſes was much leſs * than the whole error made by

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* The lineal errors ariſing from ſphericity are : : $\frac{\text{Lin. Apert.}^3}{\text{Foc. length}^2}$. The refractions of lenſes are as the $\frac{\text{Lin. Apert.}}{\text{Foc. length}}$. In the caſe before us the appertures are given, ∴ the errors are reciprocally as the ſquares of the focal length — Let the convex lenſes be A, B, and C, — and let the refraction of A be equal to thoſe of B and C together — Let their lineal errors from ſphericity be E, e, q, reſpectively—their focal lengths F, f, Q.—If the refraction of A be = 3, and that of B = 1, that of C will be = 2—As the apertures are given, the focal lengths of A, B and C will be reciprocally as thoſe numbers.

∴ E : e

by the convex (of the double object-glaſs) whoſe refraction was only equal to theirs, and was completely deſtroyed by the contrary and equal error occaſioned by the ſtructure of the concave placed between them.

The very accurate adjuſtment of the lenſes, the proper proportions of their focal lengths, and of the curvatures of their ſurfaces. are in a great meaſure to be attained by trial, for it is found impoſſible to aſcertain the exact ratio either of the mean refractive or of the diſperſing powers in different kinds of glaſs, as they

$$\left.\begin{array}{r} \therefore E : e :: f^2 : F^2 :: 9 : 1 \\ E : q :: Q^2 : F^2 :: 9 : 4 \\ \therefore 2E : e + q :: 18 : 5 \\ \text{or } E : e + q :: 9 : 5 \end{array}\right\}$$

i. e. the error of the firſt lens will always be to the ſum of thoſe of the other two, as the ſquare of the whole to the ſum of the ſquares of the parts, i. e. always in a ratio of greater inequality.

frequently vary even in diſtinct pieces of the ſame pot.

N. B. Notwithſtanding the accuracy of the triple object-glaſs, the loſs of light ſuffered in ſo many refractions occaſions frequently a preference of the double one.

63. Mr. Hamilton, in his ingenious letters on the coaſt of Antrim, has taken occaſion to remark how much the wiſdom of nature has ſurpaſſed the ſagacity, and anticipated the invention of man, in the admirable ſtructure of the human eye. He obſerves, that though the conſtruction of the triple achromatic object-glaſs is the utmoſt perfection to which our long experience and reſearch in this ſcience has been able to conduct us, yet the human eye preſents an inſtrument ſurprizingly reſembling it in its ſtructure, though more perfect

in its uſe; a complete achromatic, compounded of three lenſes of different ſhapes and ſubſtances, correcting as well the errors of ſphericity as of refrangibility, and adapted to viſion at various diſtances. It may alſo be added, that as it has been found neceſſary to prevent the diſturbance of viſion in teleſcopes ariſing from the reflexion of the erratic light againſt the ſides of the tube, by blackening them; ſo it is diſcovered that the ſame inconvenience is guarded againſt in the eye by a black pigment ſpread all over thoſe parts of its inſide, which are by their poſition unfit to receive images of external objects.

Perhaps it will not be condemned as a fanciful purſuit of this analogy to remark the correſpondence of the Iris to the eye-ſtop * in teleſcopes. It is certain that

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* A plate with a circular hole placed before the principal image in teleſcopes and microſcopes.

they are partly ſimilar in their uſes, for in the teleſcope ſome rays of the oblique pencils which have no part in forming the image would be reflected by the ſides, (notwithſtanding the abſorption by the blacking) and thus diſturb the diſtinctneſs of the image; and as the *eye-ſtop* is placed before the principal image to intercept them, ſo the *Iris* intercepts ſuch rays proceeding through the cornea, as would be uſeleſs in forming the image at the retina, or would introduce confuſion into it.

Theſe obſervations ſuggeſt a reflection upon the great advantages which a well regulated *analyſis* of the works of nature may afford, not only to theoretical knowledge, but alſo to many mechanical operations uſeful to the purpoſes of life.

The principal aim of ſo many acute philoſophers as heretofore engaged in this ſcience was the formation of a perfect teleſcope.—The chief deſideratum in this undertaking was the conſtruction of an object-glaſs which ſhould form the images of remote objects with perfect accuracy, and various attempts to effect this were made without ſucceſs. —It was long ſince known by experiment that the humours of the eye formed accurate pictures on the retina. If attention had been paid to this phænomenon, and cloſe ſcrutiny made into its cauſes, it does not ſeem unreaſonable to conclude that the great object of optical enquiry had been probably much ſooner attained, and that ſeveral former ages would have enjoyed the benefits of this uſeful invention, to which they were total ſtrangers.

SECT. IX.

Of the improvements in the Eye-glaſſes of Teleſcopes.

64. THE uſe of the *principal* eye-glaſs, as before mentioned, is to view the laſt image under an angle greater than that in which it can diſtinctly appear to the naked eye.

Additional eye-glaſſes have three uſes, 1, erecting the image; 2, enlarging the field; 3, correcting the errors as well from ſphericity as diſperſion generated by the principal eye-glaſs.

1, The firſt method of erecting the image was by a ſingle lens placed at Fig 69. a diſtance from the image, greater than its own focal length, and uſually double of it.—At the focus conjugate to the place of the image, the erect image is formed :—as the extent of the field is diminiſhed, while the diſtance of the object-glaſs from the neareſt eye-glaſs encreaſes, (See Note on Subſ. 50.) it is plain that the field is more contracted in this teleſcope than in the *aſtronomical* one (where the image is inverted) or than in the common *terreſtrial* one, (where the image is erected by two eye-glaſſes). This latter teleſcope, therefore, has univerſally taken place of that above deſcribed.

65. In the *aſtronomical* teleſcope the Fig 70. field is enlarged by the inſertion of a plano-convex lens between the object-glaſs

glaſs and the image; the refraction of this glaſs brings the converging rays of the ſeveral pencils ſooner to their foci, and thus diminiſhing the length of the teleſcope, and conſequently the image, it diminiſhes the magnifying power—but as it receives ſeveral of the more oblique pencils that could never reach an eyeglaſs of ſufficiently ſmall aperture, it will enlarge the number of viſible points in the object; i. e. the field is increaſed.

In the *terreſtrial* teleſcope two lenſes (beſide thoſe neceſſary for erecting the image, viz. M N and P Q) are uſed for increaſing the field — e. g. if A B be an oblique pencil of parallel rays falling on an object glaſs Y T, after refraction, they would converge to F, and there the firſt image F S would be formed—the ſemi-aperture of M N muſt in this caſe be larger than the image F S—and as this lens muſt be very convex for producing Fig. 71.

ducing ſo great a change as is required in the direction of the pencil, conſiderable errors would ariſe from its great aperture and convexity—either this muſt enſue, or elſe the field muſt be contracted ſo as to loſe the pencil A B, &c. The neceſſity of this alternative is avoided by placing a lens R V of ſmall convexity between the firſt image and the object-glaſs. This reduces the image S F to L O, whereby neither ſo great an aperture or convexity is required in the lens M N (nor conſequently in † P Q) as before; yet the pencil A B is retained in the field—R H is the ſecond real image formed by the refraction of P Q; the eye-glaſs W X muſt have a large aperture to keep within the field all the pencils of this image; whence if its magnifying power be conſiderable great errors will ariſe; but by interpoſing the lens U E of ſmall convexity between the lens P Q and

† Vide Note, P. 103.

and the image R U formed by it, this image is reduced to I G, ſtill retaining the extreme pencil A B within the field; Fig. 71. therefore a leſs aperture of W X than what was neceſſary before will now ſerve to view this image.

It is to be obſerved that the magnifying power is diminiſhed in this conſtruction, as the image is ſucceſſively contracted by the contraction of R V and U E; yet the errors being likewiſe leſſened (as we ſhall preſently ſee) the principal eye-glaſs may have ſuch a magnifying power as to compenſate the diminution of the image*.

66. 1, The errors of *diſperſion* ariſing from the eye-glaſs are corrected by ſuch an adjuſtment of the additional glaſſes as

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[*] If a more ample explanation of this ſix-glaſs teleſcope (the invention of which we owe to Mr. Dolland) be deſired, it may be found in a paper inſerted in Mr. Ludlam's aſtronomical obſervations.

ſhall make their diſperſing powers mutually to correct each other; thoſe homogeneous rays that were diſperſed by one eye-glaſs being collected or elſe made to emerge parallel by the contrary refraction of the next glaſs, ſo however as to make them emerge from the principal eye-glaſs parallel to each other, and therefore to be collected at the retina by the refracting humours of the eye.

2, If the neceſſary refraction of the extreme pencils of the field be performed by a ſingle eye-glaſs, the errors from ſphericity will $:: \frac{\textit{Lin. Apert.}^3}{\textit{Focal length}^2}$. If it be performed by two or more eye-glaſſes of the ſame aperture, the errors generated by theſe will be far leſs than that generated by a ſingle one, as may be eaſily
Fig. 71. collected from the note on Subſ. 62—ſtill more will the errors be reduced if the glaſſes that principally perform this refraction

fraction can be diminiſhed in their aperture; but this is done in the ſix glaſs teleſcope of Dolland, where the inſertion of the lens R V diminiſhes the aperture of M N, and therefore * of P Q, and the inſertion of U E reduces the aperture of the principal eye-glaſs W X. The errors of ſphericity which theſe glaſſes R V and U E produce are much leſs than thoſe they remove, as from the nature of their poſition their apertures are not large, and their convexity is inconſiderable.

67. The moſt important improvement however in eye-glaſſes we owe to Mr. Ramſden, and he derived his firſt idea of it from a phænomenon obſerved by Sir I. Newton.—In the chapter of his Lectures Opticæ *de luce per priſma ad oculum tranſmiſſâ*—he remarks that the colouring

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* For theſe, in terreſtrial, teleſcopes are generally ſimilar and equal lenſes.

of objects ſeen through a priſm depends on the diſtance of the object from the priſm, and that when the object and priſm are in contact, the object is entirely uninfected by priſmatic colours.—Mr. Ramſden applied this principle to correct the errors of the eye-glaſs in the following manner:

Fig. 72. At the focus of an object-glaſs perfectly achromatic he let the image fall *near* the plane-ſide of a plano-convex lens—the lens having the ſame effect on the image as the priſm in Newton's experiment had on the object, the emergent rays were not ſenſibly infected by priſmatic colours, and were made to proceed to the eye parallel by a ſmall plano-convex placed nearer to the former than its own focal length †, its plane ſide being

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‡ Becauſe the rays after paſſing the firſt eye-glaſs diverge from the imaginary image C D, which is more

being turned to the eye, could be generated by it; but whatever errors, whether of ſphericity or diſperſion, were occaſioned by the diſtance of the image from the larger plano-convex, or by the refraction of the ſmaller (though they muſt be very inconſiderable) might be corrected by conſtructing the object-glaſs ſo as to admit aberrations equal to theſe, but in contrary directions, to take place in the image.

The reaſon why the image is not placed directly on the plane ſide of the lens A, is, that in this caſe any dirt or motes on the interior eye-glaſs will be ſeen diſtinctly and magnified, as being

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more diſtant from the lens A than the real image E F, (Vide Subſ. 39.) but in order to make the rays emerge parallel from B, it ſhould be at the diſtance of its own focal length from C D, and ∵ at a leſs diſtance from the lens A.

then at the focus of the exterior eyeglaſs.

For a fuller explanation of this improvement, conſult a paper of Mr. Ramſden's, in the philoſophical tranſactions of 1782.

Fig. 49.

Fig. 50.

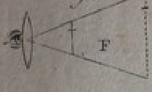


Fig. 51.

Fig. 52.

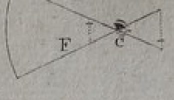


Fig. 53.

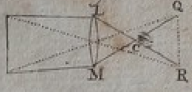


Fig. 54.

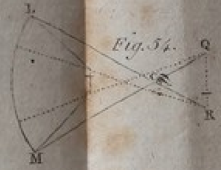


Fig. 55.

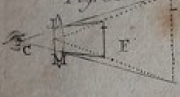


Fig. 56.

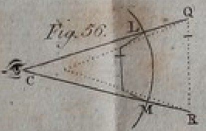


Fig. 57.

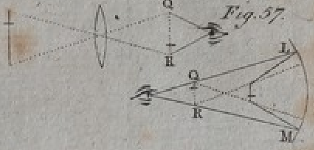


Fig. 58.

Fig. 59.

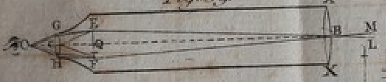


Fig. 60.

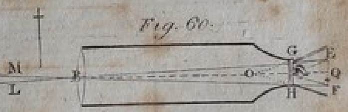
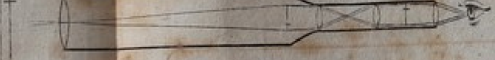


Fig. 61.



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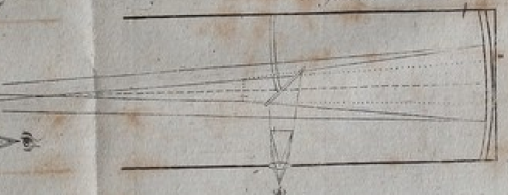

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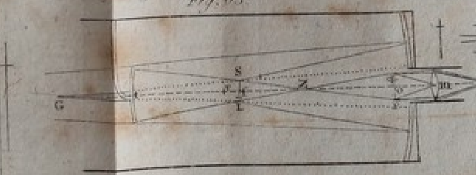

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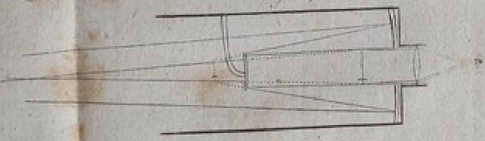

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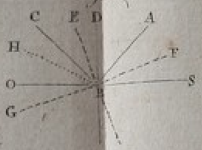

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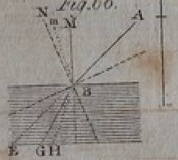

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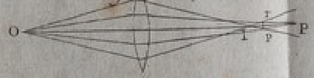

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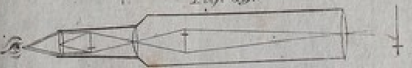

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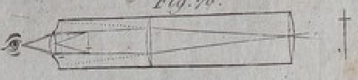

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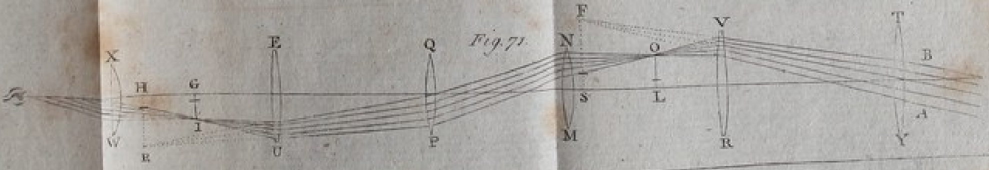

SECT. X.

Of Microſcopes and other Optical Inſtruments.

68. THE impediments to the viſion of very near objects ariſe from too great a divergence of the rays in each pencil incident on the eye, and are remedied by the *microſcope.* This inſtrument is of two kinds; 1, *refracting*; and 2, *reflecting.*

The *refracting* microſcope is either, 1, ſingle, or 2, compound; the former is a ſmall double convex lens of a ſhort focal length;

length; the object is placed in its focus, by which diſpoſition the rays of each pencil emerging from the lens become parallel, and ſo are brought to their reſpective foci on the retina by the humours of the eye: the magnifying power of the inſtrument appears from hence. Fig. 73.

The apparent lineal magnitude of an object ſeen with this inſtrument, is to its lineal magnitude ſeen with the naked eye, as the leaſt diſtance that admits of diſtinct viſion with the naked eye, to the focal length of the lens; for theſe magnitudes are as the angles under which the object appears*, i. e. inverſely as the diſtances at which it is viewed.

69. A compound microſcope is compoſed of two double convex glaſſes, the

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[*] And becauſe the rays of the extreme pencils emerge parallel to their axes, the angle at the eye is equal to that contained by theſe axes; i. e. to the angle ſubtended by the object at the focal length of the lens.

broader

broader next the eye; in this inſtrument the diſtance of the object from the object- Fig. 74. glaſs is to be made greater than the focal length of that lens; then the image will be formed at the focus conjugate to the place of the object, and the eye-glaſs being placed at its own focal diſtance from the image, ſhall make the rays emerge parallel to each other, and conſequently produce diſtinct viſion[*].

[†] The magnifying power of this inſtrument is thus ſhewn: Let D repreſent the ſmalleſt diſtance that admits of diſtinct viſion to the naked eye. The apparent magnitude of an object ſeen with this inſtrument is to the apparent magnitude when ſeen diſtinctly by the naked eye :: f L (the diſtance of the image from the object-glaſs) × D to N L (the diſtance of the object-glaſs from the object) × f R (the focal length of the eye-glaſs)—For the former of theſe Fig. 74. magnitudes is to the latter :: the angle (B G C or) Q R P to the angle under which the object would appear at the diſtance D (i. e.) $:: \frac{Q\,p}{f\,R} : \frac{A\,M}{D} :: \frac{L\,f}{f\,R} : \frac{N\,L}{D} :: L\,f \times D : N\,L \times f\,R.$

To enlarge the field of the compound microſcope, it is uſual to inſert a broad lens (as in the aſtronomical teleſcope) between the object-glaſs and the image.

70. The reflecting microſcope is thus conſtructed:

Vide Fig. 70. In the extremity of a broad tube inſert a concave ſpeculum N U; a point O in its axis, whoſe diſtance from the vertex V is greater than the focal length of the concave, is the place for the object, Fig. 75. whoſe image will conſequently be formed at the focus G conjugate to the point O: at the diſtance of its own focal length (GL) place a double convex lens, by which the image will be ſeen diſtinctly*.

N. B.

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* The magnifying power of the inſtrument is thus ſhewn: Let D repreſent the ſmalleſt diſtance that admits diſtinct viſion. The magnitude (M) of an object ſeen

N. B. The object is illuminated by light admitted into the tube through a ſpace P R adjoining to the ſpeculum; and the illuſtration of the object may be rendered more intenſe by a concave ſpeculum A B, which ſhall reflect the light ſo admitted to a focus at the place of the object.

71. A ſolar microſcope is conſtructed in the following manner: In the inſide

ſeen with this inſtrument is to its apparent magnitude (m) ſeen by the eye, as D × G V (the diſtance of the image from the vertex) : : O V (the diſtance of the object from the vertex) × G L (the focal diſtance of the eye-glaſs)—For they are as the angles W L X, and that under which S Q appears at the diſtance D. Then $M : m :: \frac{WX}{GL} : \frac{SQ}{D} :: \frac{CG}{GL} : \frac{CO}{D} :: \frac{FG - FV}{GL} : \frac{VF - OF}{D}$ add to the numerators reſpectively 2 F V and 2 O F which are proportional to them, and $M : m :: \frac{VG}{GL} : \frac{VO}{D} :: VG \times D :$ $O \times GL$.

of a tube is placed a convex lens AB and at a diſtance a little greater than its focal length, but leſs than double of it, is fixed ſome tranſparent coloured object Q P, whoſe image will be painted much enlarged at the focus conjugate to the place of the object—A broad lens C D, is placed before the object to collect the ſolar rays, for the purpoſe of illuminating it more ſtrongly, and conſequently making the image more diſtinct and vivid *. Fig. 77.

72. On the ſame principle a magic lanthorn is conſtructed; in a tube A G that projects from the body of the lanthorn is fixed a double convex lens GL— beyond its focus F is placed a plate A Q, Fig. 76.

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* This inſtrument is extremely uſeful, as it permits us to examine every minute objects without the ſame exertion of the eyes that they are ſubject to in other microſcopes.

with

with tranſparent coloured figures painted on it; theſe being illuminated by the candle in the lanthorn, and placed in an inverted poſition, their images will be painted erect, and magnified at M, the focus conjugate to P, as is clear from the principles heretofore demonſtrated.— The illumination of the objects is ſtrengthened by collecting the rays with the concave ſpeculum XY, or the convex lens R S.

73. A ſolar teleſcope is the ſame with the aſtronomical one, except that the diſtance of the eye-glaſs from the focus of the object-glaſs is made ſomewhat greater than its focal length. By this means the ſmall image of the ſun, formed in the focus of the object-glaſs, is projected much enlarged on a wall placed at a proper diſtance behind the eye-glaſs; and thus, eclipſes and the ſolar ſpots may be obſerved with great advantage.

74. The camera obſcura is an inſtrument uſed to facilitate the delineation of proſpects—It is conſtructed in the following manner :

Fig. 78. A C repreſents a box of about a foot and a half ſquare, ſhut on every ſide except D C ; O P is a ſmaller box placed on the top of the greater ; M N is a double convex lens, whoſe axis makes an angle of 45° with B L, a plane mirror fixed in the box O P ; the focal length of the lens is nearly equal to C S + S T, i. e. to the ſum of the diſtances of the lens from the middle of the mirror, and of the middle of the mirror from the bottom of the larger box.—The lens being turned toward the proſpect would form a picture of it, nearly at its focus ; but the rays being intercepted by the mirror will form the picture as far before the ſurface as the focus is behind it*, i. e. at the bottom of the larger box—(a communication being made between the boxes

* Subſ. 35.

boxes by the vacant ſpace Q O).—The draughtſman then putting his head and hands into the box through the open ſide D C, and drawing a curtain round to prevent the admiſſion of the light, which would diſturb the operation, may trace a diſtinct outline of the picture that appears on the bottom of the box.

75. There is another kind of camera obſcura, conſtructed thus. In the extremity of the arm P Q, that extends from the ſide of a ſmall ſquare box B L, Fig. 79. is placed a double convex lens whoſe axis is inclined in an angle of 45° to a plane mirror B O: The focal length of the lens is equal to its diſtance from the ſide of the box O T; therefore when the lens is turned towards the illuminated proſpect it would project the image on the ſide O T, if the mirror were removed, but this will reflect the image to the ſide M L, which is as far diſtant

from

from the middle of the mirror as this is from the ſide O T; it is there received on a piece of glaſs, rough at the upper ſide and ſmooth at the lower, and appears in its proper colours on the upper ſide of the plate. It is evident that in each of theſe inſtruments the image is inverted with reſpect to the object.

M S is a lid to prevent the admiſſion of light during the delineation of the picture, and others for the ſame purpoſe are applied to the ſides M R and N L.

APPENDIX.

Of the Rainbow.

THE rainbow is a circular image of the ſun, variouſly coloured. It is thus produced: The ſolar rays entering the drops of falling rain are refracted to their farther ſurfaces, and thence by one or more reflexions tranſmitted to the eye: At their emergence from the drop as well as at their entrance they ſuffer a refraction, by which the rays are ſeparated

ted into their different colours, and theſe therefore are exhibited to an eye properly placed to receive them.

That this is the true account of the formation of the rainbow, appears from the following conſiderations:

1. That a bow is never ſeen but when rain is falling and the ſun ſhining at the ſame time, and that the ſun and bow are always in oppoſite quarters of the heavens. This every one's experience can teſtify.

2. That the ſame appearance can be artificially repreſented by means of water thrown into the air, when the ſpectator is placed in a proper poſition with his back turned to the ſun. Experiment will ſhew this.

3. That its formation as above deſcribed can be clearly explained from the properties of light already demonſtrated.

This I ſhall proceed to ſhew in as full a manner as the deſign of this tract will allow.

Let A B be a drop of water, and C D a pencil of ſolar rays incident thereon; if all the rays of any one colour, e. g. red, belonging to the pencil C D, be refracted to the ſame point G, and thence reflected, they ſhall fall on the ſpace R Q with the ſame obliquity and at the ſame diſtances from each other as the refracted rays if proceeding backward from G would fall on the ſpace T S—but theſe at their refraction would emerge into T D, C S, &c. parallel to each other; ∵ the rays G R, G Q, ſhall emerge from the drop parallel to each other—and therefore will enter an eye properly placed copiouſly enough to cauſe a ſenſation; a red colour ſhall therefore appear in the direction of theſe rays, and ſo of others.—But if (as in figure Fig. 80.

81) the refracted rays do not meet in the ſame point, the reflected rays I V, P Q, will not fall on the ſurface at the ſame diſtance from each other that P T and I S do, though their obliquity to the ſurface be equal to that of the latter— ∴ the refracted rays ſhall emerge, diverging from each other, and conſequently ſhall not enter the eye copiouſly enough to cauſe a perception of their colour.

It is plain that where the rays of any colour emerge parallel, all theſe emerging rays ſhall be inclined to the incident rays in the ſame angle.—And by calculation * it is found that the red rays when they emerge parallel to each other make with the incident rays an angle A B O of 42° 2′, and the violet an angle A C O of 40° 17′, and the rays of the Fig. 82.

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* Vide Newt. Lect. Opt. P. 1. Sect. 4.

other

other colours, angles greater than the latter, and leſs than the former.

If through the eye which receives the emerging rays there be drawn a line * AX parallel to the incident rays it ſhall make with the emerging rays of each colour, angles R A X and V A X, &c. equal to the above. This line A X is called the axis of viſion. Fig 82.

The ſeveral drops placed in the lines A R, AV, &c. will exhibit to the eye at A the ſeveral priſmatic colours reſpectively, as appears from what has been ſaid; and if thoſe lines be ſuppoſed to revolve with a conical motion round the axis of viſion, it is evident, for the ſame reaſon, that all the drops placed in each of the conic ſurfaces ſo generated will tranſmit the rays of each colour reſpectively to the

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* Which will paſs through the ſun, as the diſtance of the eye from the rain is evaneſcent in compariſon of the ſun's diſtance from both.

eye, and therefore that a number of circular concentric arches of the priſmatic colours adjoining to each other will be exhibited to the eye.

This explanation relates to the interior bow, whoſe colours beginning from the outſide are red, orange, &c. as in the priſmatic ſpectrum. This bow can never be ſeen if the ſun be elevated more than 42° 2′ above the horizon; for the hori-
Fig. 83. zon H O always makes with the axis of viſion A X an angle equal to the elevation of the ſun, ∴ in the caſe here ſtated, the line A Q, marking the vertex of the rainbow, would fall entirely below the ohrizon.

As the interior bow is formed by one reflexion and two refractions, the exterior bow is formed by two reflections and two refractions at the ſurfaces of the
Fig. 84. drops of falling rain. If the red rays of any pencil C D of ſolar rays after refraction interſect each other at R, ſo that

when

when reflected at T V they may proceed parallel within the drop, after a ſecond reflection at X Q they will proceed to L M, interſecting each other at S equally diſtant from X Q , as R is from T V. And as the rays Q T, X V, if they proceeded backward, would after reflexion ſo fall on the ſurface N O as to be refracted into air parallel to each other; ſo X M, Q L, falling on the ſurface preciſely in the ſame circumſtances, ſhall be refracted to the eye parallel to each other, and therefore will enter it copiouſly enough to cauſe a perception of their colour (and ſo of the reſt.) The red rays, when * emerging parallel after two reflexions, are by calculation found to make with the incident rays, and therefore with the axis of vion, an angle of 50° 57′. The violet rays when emerging parallel are found to make with their incident rays, and therefore with the axis of viſion, an angle of 54° 7′:

The

* Vide New. Lec. Opt. P. 1, Sect. 4.

The other emerging rays meet the axis of viſion in the intermediate angles. Fig. 82. From hence it is eaſy to explain the generation of the exterior bow, in the ſame manner as that of the interior. It is to be remarked, that the order of colours in the exterior bow is the reverſe of that in the interior, and the reaſon of this appears in the above explanation; Fig. 82. for A E, which marks the direction of the violet rays in the outer bow, contains with A X, the axis of viſion, a greater angle than A D, which marks the direction of the red rays, contains with the ſame axis. The reverſe is the caſe in the interior bow.

It is evident (for a reaſon ſimilar to that given in the caſe of the interior bow) that an exterior bow cannot be ſeen when the elevation of the ſun is above 54° 7′.

FINIS.

PLATE.V.

Fig. 79.

Fig. 84.

The other emerging rays meet the axis of viſion in the intermediate angles. Fig. 82. From hence it is eaſy to explain the generation of the exterior bow, in the ſame manner as that of the interior. It is to be remarked, that the order of colours in the exterior bow is the reverſe of that in the interior, and the reaſon of this appears in the above explanation; Fig. 82. for A E, which marks the direction of the violet rays in the outer bow, contains with A X, the axis of vſion, a greater angle than A D, which marks the direction of the red rays, contains with the ſame axis. The reverſe is the caſe in the interior bow.

It is evident (for a reaſon ſimilar to that given in the caſe of the interior bow) that an exterior bow cannot be ſeen when the elevation of the ſun is above 54° 7′.

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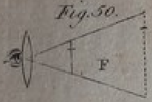
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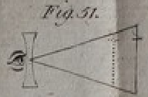
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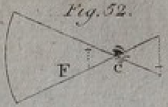
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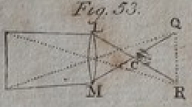
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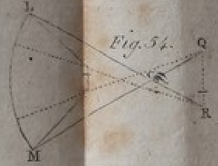
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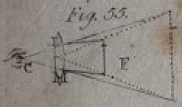
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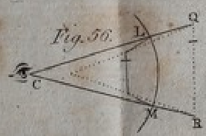
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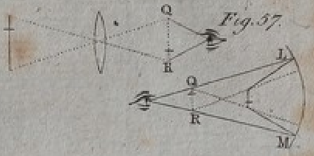
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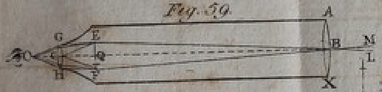
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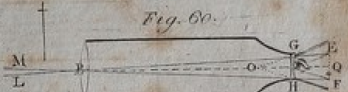
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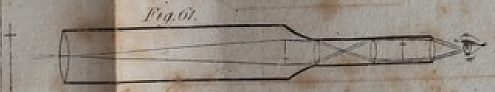
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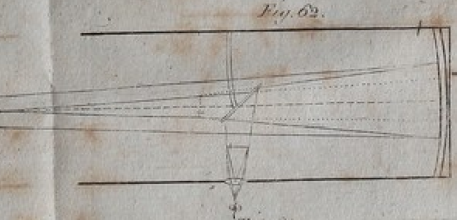
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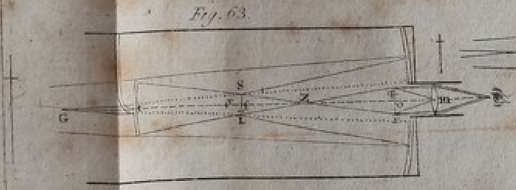
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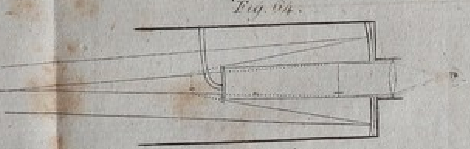
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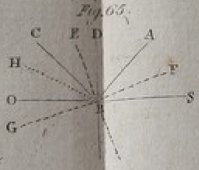
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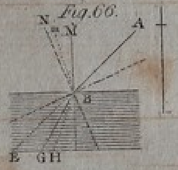
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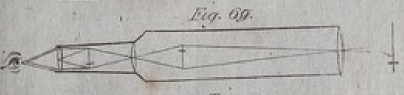
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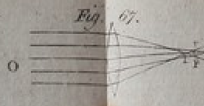
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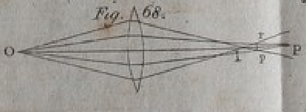
Fig. 68.

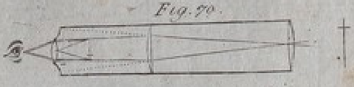
Fig. 70.

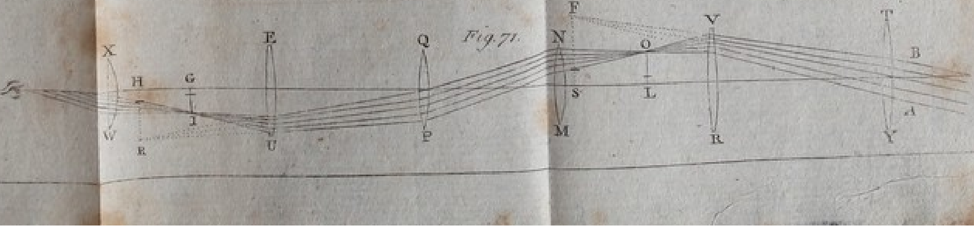
Fig. 71.