ON

from the Author

A NEW CASE

OF

THE INTERFERENCE OF LIGHT.

BY

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XV. *On a new Case of the Interference of Light.*

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(1.) In the state of advance at which the theory of light has now arrived, a single case of interference directly explicable by the ordinary principles of undulations, even though occurring under new conditions, could hardly be deemed of sufficient importance to form the subject of a separate communication to the Royal Society.

But in the present instance though the case presented, in its more *general* features, is easily accounted for on the acknowledged doctrine of interference and retardation, yet it offers many particulars in its *details*, in reference to which such explanation is, at least, not equally obvious: while some points require for their elucidation investigations of a more extended character.

Again, it is found to be a case which by no means stands isolated, but offers analogies with other classes of phenomena which have excited considerable interest and discussion, especially with regard to what has been termed, perhaps improperly, a "polarity" in the prismatic rays,—a new instance of which is here exhibited. In these respects, then, I trust the subject may not appear unworthy of the notice of the Royal Society.

Having arrived at the primary results in July 1847, I communicated them to my friend Mr. G. G. Stokes of Pembroke College, Cambridge, who has gone extensively into the whole theory of these and other allied cases, and has afforded me much valuable aid in the investigation. In the present paper I propose only to describe my own experiments, with the general application of the undulatory theory to the explanation of them, supported by some numerical comparisons.

(2.) The main experiment is as follows:—in a *hollow glass prism*, or rather *trough*, containing some highly refractive and dispersive liquid, such as oil of sassafras, anise, or cassia, *a plate of glass is inserted* with its lower edge parallel to the edge of the prism, and so that its plane nearly bisects the angle of the prism, while it extends only through the upper half of the liquid, leaving the lower, or thinner part, clear (see fig. 1). Light being admitted through a narrow horizontal slit in the usual manner, *the spectrum thus formed is seen crossed by a number of dark bands parallel to the slit or edge of the prism.*

(3.) I have tried various combinations of oils and other media with plates of different thicknesses, both of glass and of other transparent substances. In these different instances some remarkable distinctions are exhibited. In some cases the bands are sensibly equidistant, in others increasing in number and fineness towards one

Fig. 1.

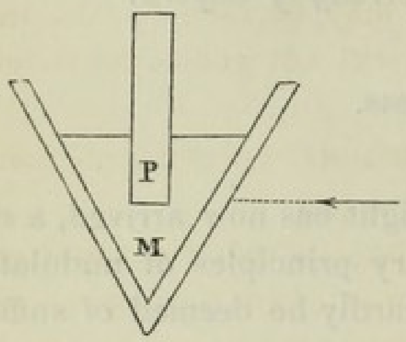


Fig. 2. Position 1.

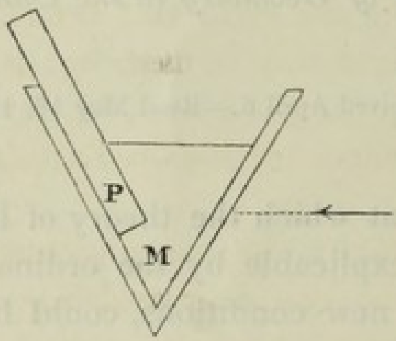
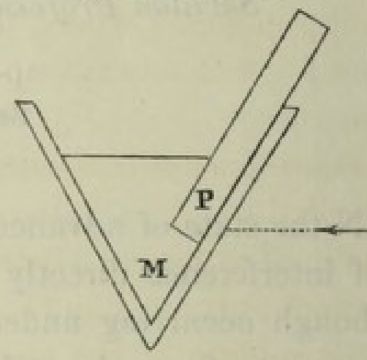


Fig. 3. Position 2.



Arrangement I.

end of the spectrum: in most cases extending throughout, but in some deficient at one part.

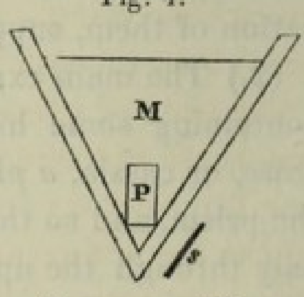
(4.) With the same oil or medium, if the plate exceed a certain thickness, the bands become too numerous and fine to be seen: if less than a certain other limit, they become too few, broad and faint: and for some intermediate thickness they appear most vivid and distinct.

(5.) With plates and media of different refractive and dispersive powers as well as different thicknesses, changes in the number and vividness of the bands take place, as well as in the limits of their visibility, in a manner evidently dependent on the thickness and relative refractions jointly; though not in any such obvious relation as can be stated by a simple experimental law.

(6.) It is not *necessary* that the plate should have precisely the vertical position just described; if *inclined* either way, even to being in contact with either side of the prism, the bands are still seen; but they undergo a slight shifting downwards as the plate is inclined, and are perhaps less vivid (see figs. 2, 3).

(7.) Some combinations of a medium and a plate, such as glass with oils of turpentine or angelica, or water, &c., *give no bands with this arrangement*. But Mr. Stokes pointed out from theory, that in these cases bands might be expected to appear with a *reverse* arrangement; that is, by placing a narrow slip of glass, &c. to intercept the *thinner* part of the prism, leaving the upper or thicker part clear, and of course cutting off any portion of light, which might otherwise pass below the plate (see fig. 4, where *s* is a small screen for intercepting the light below).

Fig. 4.



Arrangement II.

On using such an arrangement for the cases just mentioned, I accordingly found bands produced.

(8.) In any case to see the bands well, especially towards the violet end of the spectrum, a strong light is requisite: that of the sun direct, shaded by a blue glass, or at least that of the bright part of the sky near the sun, was usually thrown in by the shutter apparatus; and though a small telescope was usually employed, yet the bands, in cases where they are vivid and broad, may be seen by the naked eye; and for such cases lamp-light may be used, but it will not suffice for the more delicate.

(9.) When *crystallized* substances are employed as *plates* other peculiar phenomena are presented.

A plate of calc-spar (formed by the natural cleavage) with oil of cassia gives two distinct sets of bands; the one finer and narrower than the other, which about the middle of the spectrum may be seen distinctly superimposed one on the other.

On applying a Nicol-prism each set disappears alternately, leaving the other visible at each quarter of a revolution of the analyser; showing them due to the two oppositely polarized pencils. It is easily ascertained that the *finer* bands belong to the *extraordinary*, the *broader* to the *ordinary ray*.

(10.) A plate of quartz, cut perpendicular to the axis, with oil of sassafras, gives very distinct bands, which may be seen to be in fact composed of two sets superimposed and nearly coinciding, since when the plate is inclined (as in position 1, see fig. 2) at intervals throughout the spectrum the near coincidence of the dark spaces of one set, with the bright of the other, occasions an extreme faintness in the bands. In this case the two pencils deviate sufficiently from the axis to approximate to plane polarized light, and thus the Nicol-prism causes the indistinctness to disappear at each quarter of a revolution by stopping one of them alternately.

(11.) For an *explanation* of the *general* phenomena of the formation of bands under the conditions specified, the simple interference-theory suffices.

Of the homogeneous pencil going to form any one ray of the spectrum, that half which passes through the thicker part of the prism is more retarded than that through the thinner; but uniformly, and in proportion to the difference of refraction, throughout the spectrum.

The plate of glass, however, having different indices for the several primary rays from those of the medium, interrupts this uniformity, and causes the one part of the pencil to be always retarded in an increasing ratio with respect to the other, throughout the spectrum; and as this difference of retardation amounts successively to an odd or an even multiple of a half wave-length, the rays will be in discordance or accordance, or give a dark or bright band accordingly.

(12.) But to account for the different conditions which determine the number and character of the bands, and the limits of thickness, as well as which of the two arrangements before described, will produce them, it is necessary to consider more precisely the relative refractions of the plate and medium, or the amount and direction of the retardation ; and it is found that on such considerations we obtain an expression which includes these conditions ;—the difference of *sign* corresponding to the *two arrangements* : while it assigns the *number* of bands which will be formed between any two given rays, or throughout the whole spectrum, with a given thickness of the plate.

(13.) In the comparison of theory and observation, the broad facts,—that bands will be produced in the respective cases by the one or the other arrangement,—as well as their general character,—show an entire agreement with theory.

The more precise comparison of the *number* of bands formed throughout the spectrum, or within certain definite spaces of it, though in some cases unavoidably imperfect from the difficulty of distinguishing the bands, yet upon the whole gives accordances as good as perhaps can be expected.

(14.) When plates of doubly-refracting crystals are employed, the calculation for the extraordinary ray in particular directions in the crystal, becomes more complex, involving the laws of crystalline refraction.

In all cases a small error in the index causes a comparatively large difference in the calculated number of bands. But, from whatever cause, in these last instances the application of theory is, as yet, less satisfactory than in others ; at least in the instance of calc-spar.

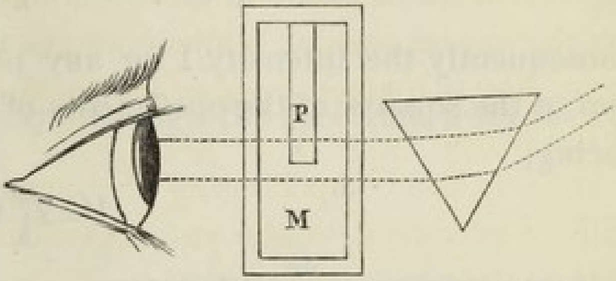
(15.) The first observation of the general phenomenon reminded me of the precisely *analogous* result obtained by Baron von Wrede, though in a manner so totally different*; in which two portions of light, unequally retarded the one by reflexion from the *first*, the other from the *second* surface of a plate of mica, impinge jointly on a prism, whence results a spectrum crossed with bands ; while the author's profound analysis (founded on the hypothesis of the internal reflexions of a ray among the molecules of a medium) opens an extended analogy between these cases and those of the absorption of definite rays by media in general.

(16.) There is also obviously a general analogy between these phenomena and those observed by Mr. Fox Talbot and Sir D. Brewster, on partially intercepting the spectrum by a plate of mica covering half the pupil of the eye ;—especially in the circumstance, that here also the plate must be applied towards *one side* of the prism, which corresponds to what has been described as a species of *polarity*. In those experiments the retardation is the difference of the retardations of the plate of mica and of a plate of air which would be contained between the surfaces of the mica produced : in mine it is the difference of the retardation of the glass and of an equal thickness of the liquid medium of the prism.

* Taylor's Foreign Scientific Memoirs, vol. i. part 3, p. 487.

(17.) Hence Mr. STOKES illustrated the case by supposing the glass plate inserted in a vessel with parallel sides filled with the medium; when if the spectrum formed by any prism were viewed through this combination, with the plate towards the thick part of the prism, the effect would be the same; and if the prism were of the same substance, the cases would be identical; the liquid prism in my experiment serving the double office of the prism and the flat vessel. (See fig. 5, where P is the plate and M the medium.)

Fig. 5.



(18.) Another remark here offers itself:—
If it should be considered that the theory is sufficiently established to give confidence in deductions from it, it may be applied to the *inverse problem* of finding the refractive indices of a plate, those of the medium, and the number of bands, being known. This may be important for many substances which occur in the form of plates, but cannot be cut into prisms.

(19.) There remain also other features of the case to be accounted for, which, if slight in appearance, are yet not unimportant in a theoretical point of view; such as relate to the changes in the vividness of the bands, especially as affected by the thickness or inclination of the plate, by enlarging or contracting the aperture, or breadth of the prism,—and some other points,—to which the simple interference-theory cannot apply.

In all experiments of this kind it is now generally understood that there must be, theoretically at least, even if it be practically insignificant, another species of action concerned, dependent on *the diffraction of the lens*, whether that of the eye, or of the object-glass of the telescope, producing that "diffusion," as it has been termed, in the optical image, which, if of sensible amount, may influence the phenomena.

The method of investigating this species of action in general is equally well understood, though in some parts the theory has been found susceptible of improvement. In the present instance such an investigation is necessary to include some of the peculiar modifications of the phenomena observed; and this constitutes the subject of Mr. STOKES's researches, which he will give in a separate form.

The remainder of this paper is devoted to the details of the observations and the theoretical investigations.

Theoretical Investigation.

(20.) The intensity of the incident light being unity, and the direct vibration for any point in the spectrum being

$$\sin \frac{2\pi}{\lambda}(vt-x);$$

that for the part retarded by r will be

$$\sin\frac{2\pi}{\lambda}(vt-x-r);$$

or,

$$\sin\frac{2\pi}{\lambda}(vt-x)\cos\frac{2\pi}{\lambda}r-\cos\frac{2\pi}{\lambda}(vt-x)\sin\frac{2\pi}{\lambda}r.$$

Consequently the intensity I for any point of the whole resulting wave will be the sum of the squares of the coefficients of $(vt-x)$, or we shall find on squaring and reducing,

$$I=2\left(1+\cos\frac{2\pi r}{\lambda}\right);$$

or if we assume $p=\frac{4r}{\lambda}$, it becomes

$$I=2\left(1+\cos\frac{\pi}{2}p\right).$$

(21.) And if we suppose p to be originally an even number and to increase by unity for successive rays of the spectrum, we shall have the corresponding values,

p	$\cos\frac{\pi}{2}p=-1$, and therefore	$I=0$
$p+1$	$=\ \ 0$	$I=2$
$p+2$	$=+1$	$I=4$
$p+3$	$=\ \ 0$	$I=2$
$p+4$	$=-1$	$I=0$
&c.	&c.	&c.

Thus for any two values p_1, p_2, if $p_1-p_2=4$, they correspond to a change from one dark band to another, and consequently, if for two rays $p_1-p_2=4n$, n will be the number of bands in the space comprised between those two rays: and if they be the extremes of the spectrum, n will be the whole number of bands. Its value may be assigned by considering the nature of the retardation, or obtaining an expression for r, as follows:—

(22.) On inserting the plate P of thickness τ whose index is μ_p, as above, into the medium whose index is μ_m, the retardation of the light which passes through the plate being the difference of the retardations of the plate and of an equal thickness of the liquid, will be expressed by

$$r=(\mu_p-\mu_m)\tau.$$

But since $\frac{p}{4}=\frac{r}{\lambda}$ (20.), we shall have

$$\frac{p}{4}=\left(\frac{\mu_p-\mu_m}{\lambda}\right)\tau.$$

And for any two rays whose indices are μ_1, μ_2, and wave-lengths λ_1, λ_2, we have

$$\frac{p_1-p_2}{4}=n=\left[\left(\frac{\mu_p-\mu_m}{\lambda}\right)_1-\left(\frac{\mu_p-\mu_m}{\lambda}\right)_2\right]\tau.$$

This formula may apply to the whole length of the spectrum, taking the two extreme rays. And the thickness of the plate being known, we may find the number of bands by computing the coefficient in terms of μ and λ which are known.

(23.) Writing this coefficient $=q$ or $n=q\tau$, since here n is supposed a whole positive number, and q may be either $+$ or $-$ according to the values of μ and λ, we have,

$$\frac{n}{q}=\pm\tau.$$

Hence if, taking successive rays, we have always through the spectrum from the red to the violet,

$$\left(\frac{\mu_p-\mu_m}{\lambda}\right)_1<\left(\frac{\mu_p-\mu_m}{\lambda}\right)_2,$$

or the value of q negative, the effect of the plate will be such that the arrangement I. will give bands : if q be positive, the effect of the plate will be such that arrangement II. will give bands.

If for any combination of a plate and a medium $\left(\frac{\mu_p-\mu_m}{\lambda}\right)$ should have a *maximum* or *minimum* value at any ray, the difference would *change signs*, and bands be formed towards that end of the spectrum where it was $-$ with arrangement I.; and towards that end where it was $+$ with arrangement II.

(24.) The general principles of the "diffraction-theory," as applicable to the present case, are precisely the same as in Mr. AIRY's paper*. But it will not be necessary here to go into the subject any further, since Mr. STOKES has greatly generalized and improved this theory so as to lead to other important results, the whole of which are discussed in his paper, in the present part of the Philosophical Transactions.

Observations.

(25.)

Arrangement.	Plate.	Medium.	Number of bands.				
	Glass.	Oil of Sassafras.	B to D.	D to F.	F to G.	G to H.	Total.
I.	inch. $\tau=$·5 ·34 ·17 ·08 ·04 ·015	No bands visible. Very fine and close. Fine. Clear. Broad and clear. Very broad and faint.	6	14	21	24	65
II.		No bands.					
(26.)							
		Oil of Cassia.					
I.	·08 ·04 ·015	Too fine to count. (·08, ·04) Fine.	15	29	32	Faint. 40 ?	116
II.		No bands.					

* Philosophical Transactions, 1841, Part I.

(27.)

Arrangement.	Plate.	Medium.	Number of bands.					
	Glass.	Oil of Cummin.	B to D.	D to E.	E to F.	F to G.	G to H.	Total.
I.	inch. $\tau=\cdot04$	Broad.	0	0	5	11	{ Faint. 15 ? }	31
II.		No bands.						

(28.)

		Oil of Turpentine.					
I.		No bands.					
II.	·08 ·04	Fine. { Clear in the red; broader towards blue. }	14	20	15	10	59

(29.)

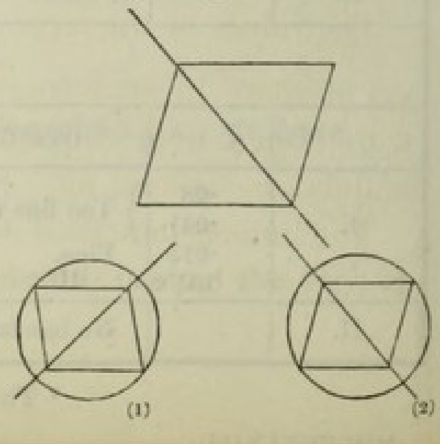
		Water.					
I.		No bands.					
II.	·04 ·015	Too fine to count. { Fine and faint; difficult to count. }	19	30 ?	22 ?	15 ?	86

(30.)

Arrangement.	Plate.	Medium.	Number of bands.			
	Calc-spar (by natural cleavage).	Oil of Cassia.	B to D.	D to F.	F to G.	G to H.
I.	inch. $\tau=\cdot04$	Two sets of bands superposed.				
	Position vertical.	With Nicol prism. Broad set.	0	Broad and faint. 14	30 ?	40 ?
		With Nicol prism. Fine set.	20	36 ?	Too fine to count.	
	Position 1. Position 2.	Both sets broader. Both sets finer.				
	$\tau=\cdot08$	Both sets too fine to count.				

(31.) To determine which set of bands belongs to the ordinary and which to the extraordinary pencil, we may proceed as follows:—Placing a rhomb of calc-spar with a small aperture behind it, so as to give two images, in the same relative position as the plate (see fig. 6) when the section of the Nicol-prism has its short diagonal perpendicular to that of the rhomb (1.), E disappears; when parallel (2.) O disappears. In the spectrum, in the former case the finer bands disappear, in the latter the broader. The fine bands therefore belong to E, the broad to O.

Fig. 6.



Or the same result might perhaps be deduced more directly from considering that as O is polarized parallel, and E perpendicular to the principal section, and that the *short* diagonal of the Nicol-prism is *horizontal* when light polarized by reflexion from an *horizontal* surface is *transmitted*,—or that the *short* diagonal is perpendicular to the plane of polarization of the *transmitted* ray. Then since when the short diagonal is perpendicular to the principal section of the calc-spar, the *fine* bands are stopped and the broad transmitted, it follows that the broad bands are polarized *parallel* to the principal section, or belong to O.

(32.)

Arrangement.	Plate.	Medium.	Number of bands.			
	Quartz cut perpendicular to axis.	Oil of Sassafras.	B to D.	D to F.	F to G.	G to H.
I.	inch. τ=·15 Position vertical.	Bands fine and clear, broader towards red end	8	43	Too fine to count; estimated at 70 ?	80 ?
	Position 1 ...... Position 2	Bands rather finer.				

With the plate inclined in position 1, eight intervals of indistinctness (extending over four or five bands each) occur from H to about E; from E to D the bands altogether become very faint: perhaps two such intervals may be discerned: from D to B no bands appear. On applying a Nicol-prism, the intervals of indistinctness disappear at each quarter of a revolution.

Calculation.

(33.) In all the following calculations for n by the formula (22.), the values employed for the reciprocals of the wave-lengths of primary rays are as follows:—

Ray.	λ (decimals of 1 French inch).	$\frac{1}{\lambda}$.
B.	·00002541	39354
D.	·00002175	45977
E.	·00001945	51413
F.	·00001794	55741
G.	·00001587	63011
H.	·00001464	68306

The values of the term $\left(\frac{\mu_p - \mu_m}{\lambda}\right)\tau$ are interesting as expressing the absolute interval of route in wave-lengths of the two interfering rays, and thus, when compared with observation, conveying an idea of the extent to which the regularity of the undulations is kept up.

The indices here used are those contained in my "Report on Refractive Indices"*. For glass I have assumed the indices of FRAUNHOFER's crown glass, No. 9.

* British Association Report, 1839.

(34.) Glass and Oil of Sassafras, $\tau = \cdot 04$ inch.

Ray.	μ_p.	μ_m.	$\mu_p - \mu_m$.	$\frac{\mu_p-\mu_m}{\lambda}$.	$\left(\frac{\mu_p-\mu_m}{\lambda}\right)\tau$.	Diff. = n.	Remarks.
B.	1·526	1·526	0	0	0		
D.	1·529	1·532	−·003	− 138	− 5·5	− 5	
F.	1·536	1·545	−·009	− 501	−20	−15	Hence in this case arrangement I. will give bands.
G.	1·542	1·557	−·015	− 945	−37·8	−18	
H.	1·546	1·569	−·023	−1571	−62·8	−25	
						−65	

(35.) Glass and Oil of Cassia (Report, No. ii.), $\tau = \cdot 015$ inch.

Ray.	μ_p.	μ_m.	$\mu_p - \mu_m$.	$\frac{\mu_p-\mu_m}{\lambda}$.	$\left(\frac{\mu_p-\mu_m}{\lambda}\right)\tau$.	Diff. = n.	Remarks.
B.	1·526	1·594	−·068	− 2676	− 40		
D.	1·529	1·607	−·078	− 3586	− 54	− 14	
F.	1·536	1·636	−·100	− 5574	− 84	− 30	Here arrangement I. will give bands.
G.	1·542	1·667	−·125	− 7867	−118	− 34	
H.	1·546	1·702	−·156	−10656	−160	− 42	
						−120	

(36.) Glass and Oil of Cummin, $\tau = \cdot 04$ inch.

Ray.	μ_p.	μ_m.	$\mu_p - \mu_m$.	$\frac{\mu_p-\mu_m}{\lambda}$.	$\left(\frac{\mu_p-\mu_m}{\lambda}\right)\tau$.	Diff. = n.	Remarks.
B.	1·526	1·502	+·024	+ 944	+37·7		
D.	1·529	1·507	+·022	+1011	+40·4	+ 2	
E.	1·533	1·513	+·020	+1028	+40·8	+ 1	Here arrangement II. would give bands from B. to E, and arrangement I. from E to H.
F.	1·536	1·520	+·016	+ 891	+35	− 6	
G.	1·542	1·533	+·009	+ 567	+22·6	−12	
H.	1·546	1·543	+·003	+ 205	+ 8	−14	
						−32	

(37.) We may remark that the indices for oil of cummin are all open to some uncertainty*, and it is easily found that a change in the indices of D and E of ·001 only would give *no bands* between B and E with either arrangement; for we should have on this supposition—

Ray.	μ_p.	μ_m.	$\mu_p - \mu_m$.	$\frac{\mu_p-\mu_m}{\lambda}$.	$\left(\frac{\mu_p-\mu_m}{\lambda}\right)\tau$.	Diff. = n.	Remarks.
B.	......	......	......	......	+37·7		
D.	......	1·508	+·021	+564	+38·5	0	No bands.
E.	......	1·514	+·019	+976	+38·8	0	

(38.) Glass and Oil of Turpentine, $\tau = \cdot 04$ inch.

Ray.	μ_p.	μ_m.	$\mu_p - \mu_m$.	$\frac{\mu_p-\mu_m}{\lambda}$.	$\left(\frac{\mu_p-\mu_m}{\lambda}\right)\tau$.	Diff. = n.	Remarks.
B.	1·526	1·470	+·056	+2203	+ 88		
D.	1·529	1·474	+·055	+2528	+101	+13	
F.	1·536	1·482	+·054	+3009	+120	+19	Hence arrangement II. will give bands.
G.	1·542	1·488	+·054	+3402	+136	+16	
H.	1·546	1·494	+·053	+3619	+145	+ 9	
						+57	

* See Report, British Association, 1839, p. 12.

(39.) Glass and Water, $\tau = \cdot 015$ inch.

Ray.	μ_p.	μ_m.	$\mu_p - \mu_m$.	$\frac{\mu_p - \mu_m}{\lambda}$.	$\left(\frac{\mu_p - \mu_m}{\lambda}\right)\tau$.	Diff. = n.	Remarks.
B.	1·526	1·331	+195	+ 7672	+115		
D.	1·529	1·333	+196	+ 9009	+135	+20	
F.	1·536	1·337	+199	+11092	+166	+31	Hence arrangement
G.	1·542	1·341	+201	+12663	+190	+24	II. will give bands.
H.	1·546	1·344	+202	+13796	+207	+17	
						+92	

(40.) Calc-spar and Oil of Cassia, ordinary ray. RUDBERG's indices.

Ray.	$\mu_{p\,O}$.	μ_m.	$\mu_p - \mu_m$.	$\frac{\mu_p - \mu_m}{\lambda}$.	$\left(\frac{\mu_p - \mu_m}{\lambda}\right)\cdot\tau$ $\tau = \cdot 04$.	Diff.	Remarks.
B.	1·6531	1·5945	+·0586	+2306	+92		Hence with arrange-
D.	1·6585	1·6073	+·0512	+2353	+94	+ 2	ment I. there will be no bands from
F.	1·6680	1·6358	+·0322	+1794	+71	− 23	B to D, but bands
G.	1·6762	1·6671	+·0091	+ 573	+23	− 48	from D to H.
H.	1·6833	1·7025	−·0192	+1311	−52	− 75	
						146	

(41.) For the *extraordinary ray*, in a plate bounded by the planes of cleavage, Mr. STOKES has calculated the results as follows:—

"The incidence is supposed to be perpendicular; that is, strictly, the rays B, D, &c. are supposed to be received in succession at a perpendicular incidence: but the results may be applied with very little error to the case in which rays of mean refrangibility are incident perpendicularly.

"The dihedral angle of the rhombohedron of calc-spar is 105° 5′*.

"If i be the inclination of the axis to the normal of the plate, we get by a spherical triangle,

$$\sin i = \cos 52^\circ\ 32'\cdot 5 \operatorname{cosec} 60^\circ$$

whence $$i = 44^\circ\ 36'\cdot 6.$$

Now† we have

$$v = \sqrt{a^2 \cos^2 i + c^2 \sin^2 i} = a \cos i \sec \theta$$

where $$\tan \theta = \frac{c}{a} \tan i = \frac{\mu'_o}{\mu'_E} \tan i,$$

and if $$\mu' = \frac{a}{v}\mu'_o \quad \mu' = \mu'_o \sec i \cos \theta.$$

Also $$\log \tan i = 9\cdot 99409, \quad \text{and } \log \sec i = 10\cdot 14758.$$

Hence $$\log \tan \theta = \log \mu'_o - \log \mu'_E + 9\cdot 99409$$
$$\log \mu' = \log \mu'_o + \log \cos \theta + 10\cdot 14758 - 20.$$

* PHILLIPS's Mineralogy. † AIRY's Tract, Art. 151.

"In this manner we obtain the following table:—

(42.) Calc-spar and Oil of Cassia. Extraordinary ray.

Ray.	Calc-spar μ'_O.	Calc-spar μ'_E.	θ.	μ'.	Oil of Cassia, μ.	$\mu'-\mu$.	$\frac{\mu'-\mu}{\lambda}$.	$\frac{\mu'-\mu}{\lambda}\times\cdot 0$	Diff.
B.	1·6531	1·4839	47° 42′·0	1·5628	1·5945	—·0317	—1248	— 49·9	
D.	1·6585	1·4863	44·8	1·5665	1·6073	—·0408	—1876	— 75·0	— 25·1
F.	1·6680	1·4907	49·5	1·5731	1·6358	—·0627	—3495	—139·8	— 64·8
G.	1·6762	1·4945	53·6	1·5787	1·6671	—·0884	—5570	—222·8	— 83·0
H.	1·6833	1·4978	57·0	1·5837	1·7025	—·1188	—8115	—324·6	—101·8
									275

"The direction considered being about 45° from the axis, μ' ought to be nearly equal to the mean of μ'_O μ'_E. Now the values of μ' found above, fall short of the mean of μ'_O and μ'_E by the following quantities:—

For B . . . . . . ·0057
D . . . . . . ·0059
F . . . . . . ·0062
G . . . . . . ·0066
H . . . . . . ·0068

"The smallness and regularity of these numbers is a test of the correctness of the arithmetic."

(43.) *Quartz and Oil of Sassafras.*

The angle of the prism being 60° and the ray F at the minimum deviation, and ϕ, ϕ' being the angles of incidence and refraction for the first surface, we have for either pencil,

$$\phi'_F=30°, \quad \sin\phi_F=\mu_F \sin\phi'_F,$$

which gives ϕ_F, and therefore ϕ for the other rays;

also $$\sin\phi'=\frac{\sin\phi}{\mu} \text{ gives } \phi' \text{ for the other rays.}$$

If i be the angle of incidence *on the plate* then in position (1) (see figs. 2, 3) $i=60°-\phi'$, and in position (2) $i=\phi'$.

Then (as in Airy's Tract, Art. 151) if i' be the angle of refraction referring to the *normal of the wave,* we have

$$\tan i'=\frac{a\sin i}{\sqrt{v^2-c^2\sin^2 i}}$$

where $$\frac{a}{c}=\frac{\mu_E}{\mu_O}$$

and $$v' = \sqrt{a^2 \cos^2 i' + c^2 \sin^2 i'},$$

and if V be the velocity in air, so that $\frac{V}{v} = \mu$, and we take $\frac{v'}{v} = \frac{\mu}{\varkappa}$, then $\frac{V}{v'} = \varkappa$;

and M being the retardation expressed by the number of waves' lengths, we get

$$M = \frac{\tau}{\lambda \cos i'} \left(\frac{V}{v'} - \mu \cos (i - i') \right).$$

Also i' is found to differ but little from 30° (about 30′ for the extreme rays) and $\cos (i - i') = \cdot 9999$, which may be put $= 1$. Thus

$$M = \frac{\tau}{\lambda} \sec i' \, (\varkappa - \mu).$$

For the ordinary ray we have simply

$$\sin i' = \frac{\mu}{\mu_o} \sin i$$

and $$M = \frac{\tau}{\lambda} \sec i' (\mu_o - \mu).$$

(44.) In this way the following results are obtained, $\tau = \cdot 15$ inch.

Ray.	Quartz μ_{PO}.	Oil of Sassafras, μ_m.	$\mu_p - \mu_m$.	$\frac{\mu_p - \mu_m}{\lambda}$.	$M' = \left(\frac{\mu_p - \mu_m}{\lambda}\right) \tau$.	Position vertical diff. $= n$.	$M = M' \sec i'$.	Position 1. Diff. $= n'$.
B.	1·5409	1·5257	+·0152	+598	+ 89·7		+103·4	
D.	1·5441	1·5321	+·0120	+552	+ 82·8	− 7	+ 95·7	− 7
E.	1·5471	1·5387	+·0084	+432	+ 64	− 18	+ 73·9	− 22
F.	1·5496	1·5448	+·0048	+267	+ 40	− 24	+ 46	− 28
G.	1·5542	1·5575	−·0033	−208	− 31	− 71	− 35·8	− 82
H.	1·5582	1·5693	−·0111	−758	−113	− 82	−131	95
						202		234

(45.) For the extraordinary ray Mr. STOKES has made a calculation, of which the following are the principal steps and results. From the expression above,

$$v' = a \cos i' \sqrt{1 + \frac{c^2}{a^2} \tan^2 i'};$$

and assuming $$\tan \theta = \frac{c}{a} \tan i' = \frac{\mu_o}{\mu_e} \tan i',$$

whence $$v' = a \cos i' \sec \theta,$$

$$\varkappa = \frac{V}{v'} = \mu_o \sec i' \cos \theta.$$

(46.) To obtain κ, assuming i' approximately as differing little from i, which is near enough for this purpose, the following values result :—

Ray.	μ_o.	μ_e.	κ. Pos. (1).	κ. Pos. (2).	μ. Sassafras.	$\sin\phi=\mu\sin\phi'$. ϕ'.	Pos. 1. $i=60-\phi'$.	Pos. 2. $i=\phi$.
						° ′ ″	° ′ ″	° ′ ″
B.	1·5409	1·5499	1·5430	1·5432	1·52575	30 25	29 35	30 25
D.	1·5442	1·5533	1·5464	1·5465	1·53215	30 16 30	29 43 30	30 16 30
E.	1·5471	1·5563	1·5494	1·5494	1·53870	30 30 80	29 52 0	30 30 80
F.	1·5496	1·5589	1·5519	1·5519	1·54485	30 0 0	30 0 0	30 0 0
G.	1·5542	1·5636	1·5566	1·5565	1·55750	29 44 0	30 16 0	29 44 0
H.	1·5582	1·5677	1·5607	1·5605	1·56935	29 29 0	30 31 0	29 29 0

(47.) Then with more accurate values of i' we obtain, successively,

Ray.	$\sin i'=\frac{\mu}{\kappa}\sin i$. i'. Position 1.	$\sin i'=\frac{\mu}{\kappa}\sin i$. i'. Position 2.	$\kappa-\mu$. Pos. 1.	$\kappa-\mu$. Pos. 2.	$\frac{\kappa-\mu}{\lambda\cos i'}$. Pos. 1.	$\frac{\kappa-\mu}{\lambda\cos i'}$. Pos. 2.	$\left(\frac{\kappa-\mu}{\lambda\cos i'}\right)\tau=$M. Pos. 1.	$\left(\frac{\kappa-\mu}{\lambda\cos i'}\right)\tau=$M. Pos. 2.	Diff. $=n'$. Pos. 1.	Diff. $=n'$. Pos. 2.
	° ′ ″	° ′ ″								
B.	29 13 0	30 2 0	+·0173	+·0175	+779	+795	+117	+119		
D.	29 25 30	29 58 0	+·0143	+·0144	+754	+762	+113	+114	− 4	− 5
E.	29 38 30	29 54	+·0107	+·0107	+633	+634	+ 95	+ 95	−18	−19
F.	29 51	29 51	+·0071	+·0071	+451	+451	+ 68	+ 68	−27	−27
G.	30 17	29 45	−·0009	−·0010	− 65	− 73	− 10	− 11	−78	−79
H.	30 42	29 40	−·0086	−·0088	−683	−694	−102	−104	−92	−93
									219	223

Hence arrangement I. gives bands.

(48.) For the ordinary ray with the plate vertical, the number of bands agrees well with observation as far as it goes.

It is obvious in general, that if two sets of bands differing in number be superposed, there will be a number of coincidences equal to their difference, separated by spaces of partial obliteration, which if the bands be of sensible breadth, will extend over several bands.

On comparing the above values of p for O and for E in position 1, we find—

From B to E . . . 29 − 22 = 7
From E to H . . . 205 − 197 = 8
From B to H . . . . = 15
} = number of intervals of indistinctness.

From E to H this agrees with observation, and may do so from B to E, occasioning a total disappearance of bands.

Postscript.—At the time the foregoing paper was communicated to the Royal Society, I had not seen Mr. Stokes's paper; nor in writing his, had he seen mine at length; hence it will be found that there are some repetitions in the latter, of points mentioned in mine, but usually put in so much clearer a light that the reader will not regret the repetition.