SOUND

~~B. Daniel~~

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VOLUME I.

A TEXT-BOOK OF SOUND.

WITH NUMEROUS DIAGRAMS AND EXAMPLES.

BY

EDMUND CATCHPOOL, B.Sc. LOND.,

FIRST CLASS HONOURS IN PHYSICS AT B.SC.,
SENIOR LECTURER IN PHYSICS AT UNIVERSITY TUTORIAL COLLEGE.

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PREFACE TO THE FIRST EDITION.

In writing this text-book I have tried to keep in view the following aims:—

To include all the facts of which a knowledge is expected in elementary examinations.

To present a clear picture of the external physical processes which cause the sensation of sound. As the mathematical symbols for a physical process are easily made a substitute for a clear conception of the process, the mathematical symbols have been avoided as far as possible.

To keep before the reader the distinction between phrases which describe actual processes or conditions and phrases which, while they facilitate the prediction of real processes and real phenomena, do not themselves stand for any physical condition or event. Such phrases are used in every department of physics, and are only mathematical symbols in disguise. Where I have departed from, or added to, the usual forms of explanation, it has usually been with the intention of making this distinction clearer.

As the book is intended as a physical treatise, it is the physical processes which cause the sensations of sound, and not the sensations themselves, which form its subject-matter. Only those peculiarities of the sensation are considered which throw light on the external physical processes which are taking place.

The following portions of the book may be taken as a suitable course of first reading:—

Arts. 1, 2, 11-16, 21-24, 26-35, 60, 62-64, 77-79, 84-86, 90-92, 94-97, 99, 102, 104, 105.

E. C.

November, 1894.

Note to the Third Edition.

All known errors have been removed, and several paragraphs have been re-written in the hope of distinctly improving the book.

CONTENTS.

CHAPTER I.

VIBRATORY MOTION.

1. Cause of Sound.—In every case in which the sensation of sound can be traced to an external cause, we find that the cause is something in a state of vibration. This vibration is often sufficiently great to be evident to the eye; not that the movements can be actually watched, for they are always too rapid for that, but they often produce the blurred indistinctness of outline, which we know from experience to result from rapid movement. This is well seen in the string of a harp, or along the edge of a large bell, when these are producing loud sounds. When the vibration has become too small to be visible, it can still be detected by touching the string or bell with the tip of the finger, which is able to perceive, as separate and successive, movements which follow one another much too rapidly for the eye to distinguish. If sound is still heard when even touch detects no vibration, we find that a suspended pith ball, held so as to hang just in contact with the sounding body, is driven away every time it touches the surface, and we naturally attribute this to blows or taps given to the ball by the sounding body, and infer that there is still vibration.

The vibration of the string or bell cannot, of course, be the immediate cause of the sensation of sound. It is a fact so familiar as to seem an axiom that "nothing acts except where it is"; the immediate cause of the sensation must be something in contact with the sense organ, the ear. If we stretch a thin membrane, such as goldbeater's skin, on a vertical ring of metal, so as to form a sort of tambourine, and hang a pith ball just in contact with the membrane, it will be found that the pith ball is driven away whenever the arrangement, which is called an *acoustic pendulum*, is in any place where sound can be heard; by special devices for detecting the movement of the pith ball, it is found that this is true even for the faintest audible sounds. We infer that sound is only heard in places where a stretched membrane would be in vibration.

The vibration of the membrane depends on some special condition of the air surrounding it, for if the "acoustic pendulum" is under the exhausted receiver of an air pump, it does not vibrate even when loud sounds are audible everywhere round the receiver. And this condition of the air not only requires for its production a vibrating body; the vibrating body must be in contact with the air. Thus, if one of the deep-toned clocks, which strike on a coiled wire, is suspended by pieces of cotton inside the exhausted air-pump receiver, no sound is heard when the hammer strikes the wire, although the wire visibly vibrates. If the clock, instead of being suspended, stands on the air-pump plate, it will be heard to strike, but in this case it can be shown that the plate itself is vibrating, and the plate is in contact with the air.

A vibrating body does not produce this condition in the whole of the air at the same moment. If we stand near a large clock bell, we cannot detect that there is any interval between the fall of the hammer and the perception of the sound, but if we are two hundred yards from the bell the interval is very noticeable.

Though movements of the air cannot be seen or felt, like those of solid bodies, it seems obvious that a condition of the air which is produced by the vibrations of a body in contact with it, which spreads from the air near the vibrating body to that at a distance, and which enables the air to set in vibration light membranes which expose a large surface to it, must be a vibratory motion of the air itself, and this becomes a certainty when we find that the velocity with which the condition travels is exactly that with which it can be calculated that a vibratory movement would spread from one part of the air to another.

This condition of vibratory movement spreads not only through air, but through all known kinds of molecular matter, and air is not in all cases the substance whose movement produces the sensation of sound. Thus persons whose ears are so defective in structure that vibrating air does not affect them, can often hear a watch held between their teeth, and in this case it is through the bones, not through air, that the vibration spreads.

The sensation of sound is never produced by vibrations of the ether of space, nor does the vibratory condition which causes sound ever spread from one material substance to another through a region which contains ether only ; this has been shown in the two air-pump experiments just described. It is also proved by the fact that the tremendous explosions which constantly take place on the sun are quite inaudible on the earth.

Though vibratory movements which are very slow, like that of a pendulum, or extremely rapid, like those of the air in some very small whistles, do not produce any sensation of sound, yet, as such vibrations do not differ in their physical nature from those which are audible, it is convenient to include all in the same department of physics. The physical study of sound then includes all kinds of vibration of molecular matter, but not the vibrations of the ether.

2. Meaning of "Vibration."—Any point is said to vibrate when it goes through the same, or nearly the same, series of movements at regular or nearly regular intervals. In popular language the term is confined to motion backwards and forwards along the same straight line, but the scientific use of the term is not so limited, and a point moving in a circle or a figure of 8 is also said to be vibrating. The time occupied by the complete series of movements which constitute a vibration, from the moment when the point passes any given position, to the moment when it passes the same position, in the same direction, to go through the same series of movements again, is called the *period* of the vibration, and is usually denoted by t. The number of such complete periods in one second is called the *frequency* of the vibration, and is usually denoted by n. Evidently n is always equal to $\frac{1}{t}$.

When a point moves backwards and forwards in a straight line, its vibrations are called *rectilinear*. In the case of rectilinear vibrations, most French, and some English, writers, count the time taken in moving from one end of the line to the other as the period of the vibration; we shall follow the usual English custom, and consider it only half a period.

3. Curves of Displacement.—A point may vibrate in a straight line in a great variety of ways; for instance, it may move rapidly in one direction and slowly in the other, like the piston of a Cornish pumping-engine; or quickly at one end of its path and slowly at the other, like an elastic ball dropped on the ground and rebounding again and again; or quickly in the middle of its path and slowly at the ends, like a pendulum-bob, or a ball dancing up and down at the end of a piece of elastic; and so on. These differences constitute the *character* of a vibration. In all these cases, if we marked down on paper the actual track of the vibrating body, that track would be simply a straight line, and there would be nothing to indicate the differences in the characters of the movements. In order to show these differences graphically, the following device is often used: Suppose the body is vibrating vertically; imagine that a sheet of paper is drawn horizontally at a uniform rate from right to left behind the body, and that the body leaves a trace on the paper. The form of this trace will depend on the relative velocities of the body at different times of its vibration. Thus, a ball dancing up and down at the end of a piece of elastic would leave a trace like Fig. 1, while a ball dropped

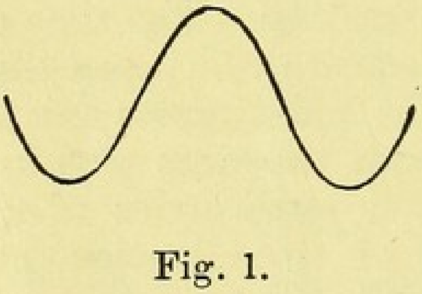

Fig. 1.

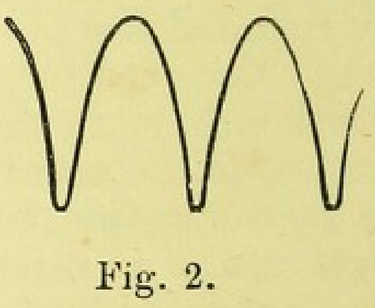

Fig. 2.

on a level surface and rebounding, would mark out Fig. 2. (In each of these cases successive vibrations are drawn exactly similar in extent and period; the gradual loss of velocity due to imperfect elasticity is neglected.) Suppose in this last instance the ball moved up and down a fixed line

YZ, Fig. 3,* Z being the point of the level surface against which it rebounded, and let the ball have left a trace, shown by the dotted curve, on a sheet of paper drawn uniformly from right to left behind it. A will be a point of the paper which was behind Z when the ball was at Y, and B will be a point on the paper which was behind Z when the ball was at Y

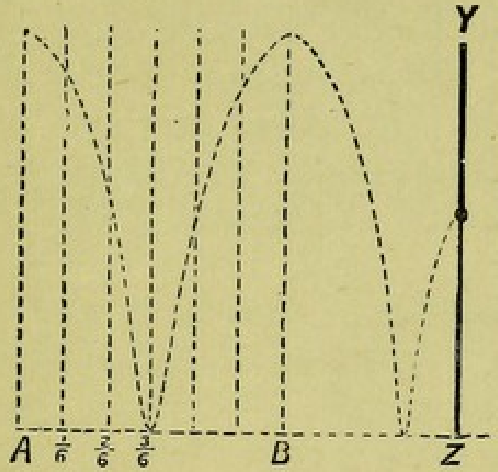

Fig. 3.

again after an interval t, which is the period of the vibration. If we divide AB into say six equal parts, and draw on the paper verticals through the points of division, these verticals are lines which coincided with YZ at instants $\frac{t}{6}$, $\frac{2t}{6}$, &c. after the instant when A passed behind Z, and the distance up any vertical from AB to the curve is the distance that the ball was from Z at the moment when that vertical coincided with YZ. So that, instead of supposing the curve actually traced by the moving body, we may construct it geometrically in this way. Mark off any length AB on a horizontal line, and divide it into, say, six equal parts. At the first point of division put up a perpendicular called an *ordinate*, equal in height to the distance of the moving body from some fixed point, when it has been moving for $\frac{1}{6}$ of its period, and so on. The curve can then be drawn through the ends of these ordinates. Or, if the movements of the

* In all curves in this book which represent the condition of the same body at different times, a point to the *left* of another represents the condition at an *earlier* moment.

point are inconveniently long or short for such a diagram, the ordinates may be made longer or shorter than the actual distances in any convenient ratio ; the curve will still indicate the character, though no longer the actual extent, of the vibratory movement. Such a curve, in which equal distances along a horizontal line represent equal intervals of time, while the distance of the curve from the line at each point is proportional to the distance of the moving point from a fixed one at the corresponding instant of time, is called a *curve of displacement*. Practical methods of determining the frequencies and displacement curves of points on vibrating bodies will be described in Chapter XII.

4. Curve of Velocities.—Another way of distinguishing between rectilinear vibrations of different characters, which, though not quite so simple, is of much greater value in the study of sound, is to draw a curve whose ordinate at each point is proportional to the *velocity* instead of the *displacement* of the vibrating body at the moment represented by the position of the ordinate along the horizontal line ; velocities in one direction being represented by heights above that line, velocities in the opposite direction by distances below it. As an instance, we will take again the case of a ball falling and rebounding vertically, whose curve of displacements was given in Fig. 2, and construct its curve of velocities. Mark off along a

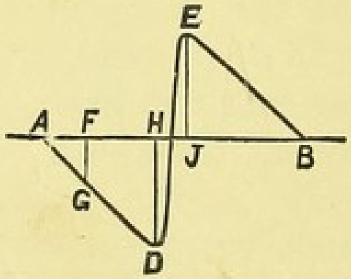

Fig. 4.

horizontal line any length, such as AB (Fig. 4), to represent the time of a complete vibration, and let A represent the moment when the ball is at its highest point and just about to begin falling, while B represents the moment when it has just reached the highest point again. At the moment corresponding to A the ball has no velocity, and the length of the ordinate to the curve at A is zero. After the lapse of $\frac{1}{6}$ of a complete period the ball has a downward velocity, represented by FG, and this downward velocity increases uniformly with the time, as shown by the straight portion AD. Just before half the period is completed, at the moment represented by H, the ball touches the ground, and in the short time represented by HJ, the downward velocity represented by HD is changed into the upward one represented by JE. At the instant represented by J, the ball ceases to touch the ground, and from this moment till the end of the period the upward velocity uniformly diminishes, as shown by the straight line EB.

The character of a rectilinear vibration is fully defined by either its displacement curve or its velocity curve, and either curve can be easily drawn when the other is given. The advantage of the velocity curve will appear later. (Art. 15.)

5. Harmonic Vibration.—There is one special kind of rectilinear vibration which is very important in the study of sound. If a point A (Fig. 6) moves to and fro along a straight line CD in such a way that the line from A to a point B, which is moving uniformly round a circle, remains parallel to the same direction, then A is said to execute a

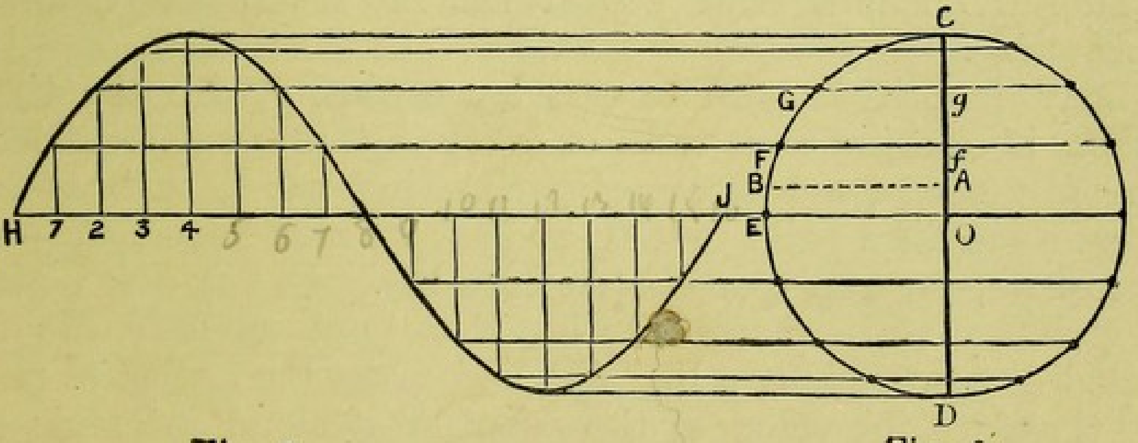

Fig. 5. Fig. 6.

Harmonic vibration. It is sometimes called *Pendular* motion, because any point of a pendulum moves very nearly in this manner when its swings are short, and sometimes *Simple* vibration, for a reason explained in Chap. VI.

Suppose B is moving uniformly round the circle CGD, and that the point *A* is at the same time moving up and down the straight line *CD*, so as always to be on the same level as *B*, *A* then vibrates harmonically. If we divide the circle into, say, sixteen equal parts, so that *B* takes equal times in moving from *E* to *F*, *F* to *G*, and so on, we see that *A* will take equal times in moving from *O* to *f*, *f* to *g*, &c., so that it will move slowly at the ends, quickly in the middle of its path.

To draw the displacement curve for this movement, we may mark off sixteen equal spaces along a horizontal line *HJ*, to represent the equal intervals of time occupied by *A* in passing from *O* to *f*, *f* to *g*, &c., and at these points draw ordinates equal (or, if preferred, proportional) to the distances of *O*, *f*, *g*, &c., from some fixed point, which may conveniently be the middle point *O* of *CD*. The distances from *O* of the points below *O* must be considered as of opposite sign to the distances of those above, and the corresponding ordinates drawn in the opposite direction. If we start at the moment when *A* is passing through *O* on its way upwards, the curve drawn through the ends of the ordinates will be like the one shown in Fig. 5. Such a curve is called a *harmonic* or *sine* curve, the latter name being given because, as can easily be shown, the ordinate at each point of *HJ* is proportional to the sine of the angle which *OB* makes with *OE* at the instant represented by that point.

Any quantity is said to vary harmonically with the time when it changes so as to be at every instant proportional to the distance of a harmonically vibrating point, such as *A*, from some fixed point in *CD*; so that we may have harmonic velocities, pressures, or electric currents. In any case where a quantity varies harmonically, if we put up ordinates at equal distances along a line *HJ*, and make the ordinates proportional to the value of the quantity at equal intervals of time, the curve through the ends of the ordinates is a harmonic or sine curve.

We have seen that, in general, the velocity curve of a vibrating point is quite different to its displacement curve. But if a point vibrates harmonically, we can easily show (see Appendix A) that its velocity also varies harmonically, so that its velocity curve is also a sine curve. Its velocity at any moment is, however, not proportional to its displacement at that moment, but to the displacement which it will have a quarter of a period afterwards. So the curve of velocities is a quarter of a period behind the curve of displacement, as shown in Fig. 7.

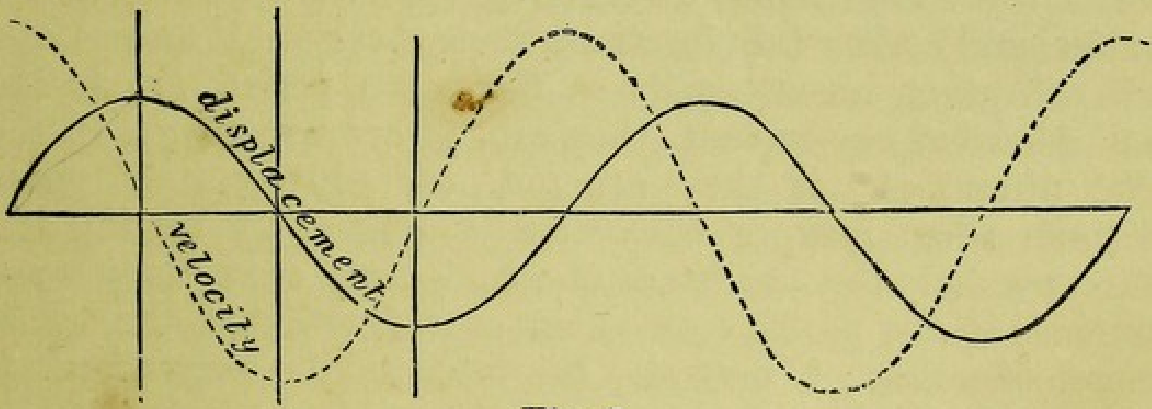

Fig. 7.

When a point vibrates harmonically, it may be shown (see Appendix A) that its acceleration is always towards, and proportional to its displacement from, its mean position. So that if the force on a material particle, when the particle is displaced from its position of equilibrium, is at every stage of displacement directed towards the position of equilibrium, and proportional to the amount of displacement, the particle will vibrate harmonically. We shall see later that, even when a vibration is not harmonic in character, it may be conveniently treated as the resultant of two or more harmonic vibrations. It is to this fact that the importance of harmonic vibrations is due.

The distance from the middle point to either end of a harmonic vibration is called the *amplitude* of the vibration; it is evidently one-fourth of the total distance traversed by the moving point in one complete vibration.

When the force on a particle is proportional to its distance from a given point, the time of vibration about that point is independent of the amplitude. (See Appendix A.) So that when, for instance, a steel rod is fixed in a vice, bent aside, and let go, the period of its vibration does not alter as the vibration itself dies away. Such vibrations are called *isochronous*. Isochronous vibrations need not be harmonic, but harmonic vibrations are practically always isochronous.

6. Phase.—The *phase* of a harmonically vibrating point usually means the fraction of a whole period of its vibration which has elapsed since it last passed its mean position in the direction which we are counting as positive. Some writers, however, count the time which has elapsed since it started from the positive end of its vibration. Upwards and to the right are usually counted the positive directions for vertical and horizontal vibrations respectively. If two points, both vibrating harmonically in the same period, are in the same phase at one instant, they are, of course, always in the same phase, and if they are not, the difference of phase between them may be conveniently defined by stating the difference between the times at which they cross their mean positions in the positive direction as a fraction of the whole period of either; it may also be stated as the corresponding angle of revolution of the uniformly revolving point, either in degrees or in radian measure. Thus, in Fig. 5, if we suppose f and g to be positions at the same instant of two points vibrating harmonically in the same period, and that they are moving in the same direction, their phase-difference may be called $\frac{1}{16}$ or $\frac{\pi}{8}$ or $22\frac{1}{2}°$, as we prefer.

When the two vibrations are of different periods, their phases change at different rates, and there is no constant phase-difference between them, and there will be instants at which the two vibrations are in the same phase. The "phase-difference" is, in this case, usually taken to mean the difference between the times at which the points cross their mean positions in the positive direction, expressed as a fraction of the greatest common measure of their periods, but other notations are also used.

7. Composition of Harmonic Vibrations.—A point may execute two or more harmonic vibrations at the same time. This does not mean, as it appears to, that a point A can be moving, at any one instant, with respect to a given point of reference such as the earth, with more than one velocity or in more than one direction. A conventional meaning is attached to the phrase. Harmonic motion, like all motion, is, of course, relative; thus the piston of the engine of a steamer which travels up and down the cylinder in such a way that its distance from the end varies harmonically, is vibrating harmonically with respect to the cylinder, though if the ship is rolling at the same time the movements of the piston with respect to the earth may be very complex. If, then, a point *A* is moving harmonically with respect to a body *B*, while *B* is moving harmonically with respect to the earth, the movement of *A* with respect to the earth is said to be compounded, or the resultant, or the sum, of the two harmonic motions, or, more shortly, *A* is said to be executing both movements at once. This is not very accurate, but it is not misleading, since there is no other sense in which a point can be moving in two ways at once.

When a point is executing two rectilinear harmonic vibrations at once, its motion need not be either rectilinear or harmonic; if the periods are incommensurable, it is not even a vibration. Of this we shall have many instances.

8. Lissajous' Figures.—An important case occurs when the two harmonic motions are at right angles to one another; for instance, let *A* vibrate horizontally with respect to a point *B*, while *B* vibrates vertically with respect to the paper. If the periods of the two vibrations are incommensurable, the motion of *A* will never exactly repeat itself; but if *m* horizontal vibrations occur in exactly the same time as *n* vertical ones, then at the end of this period the movements of *A*, both horizontal and vertical, will exactly repeat themselves, and the point will describe the same closed curve over and over again. The form of this curve will depend upon, and may therefore be used to determine, the amplitudes, relative periods, and phase-difference of the two vibrations.

When these are given, the curve may easily be constructed by a method which will be best understood from an example. Suppose we require to know what curve will be traced by a point which vibrates vertically and horizontally at the same time, the periods of the vertical and horizontal vibration being as 4 : 3, and their amplitudes 1 and $\frac{3}{4}$ inch respectively, and the two vibrations being in the same phase. Imagine that there is a sheet of glass lying on the page, with a dot on it which we will call *A*, and a horizontal line through *A*. If a point *B* moves harmonically right and left along this line, with an amplitude of $\frac{3}{4}$ inch on each side of *A*, and at the same time the plate of glass moves harmonically up and down the page with an amplitude of 1 inch, then *B* is said to be vibrating both vertically and horizontally with respect to the paper, and the line on the paper over which *B* passes is the curve required. Suppose that *L* (Fig. 8) is the point on the paper which is under the dot *A* at the moment when *A* and *B* are both at the

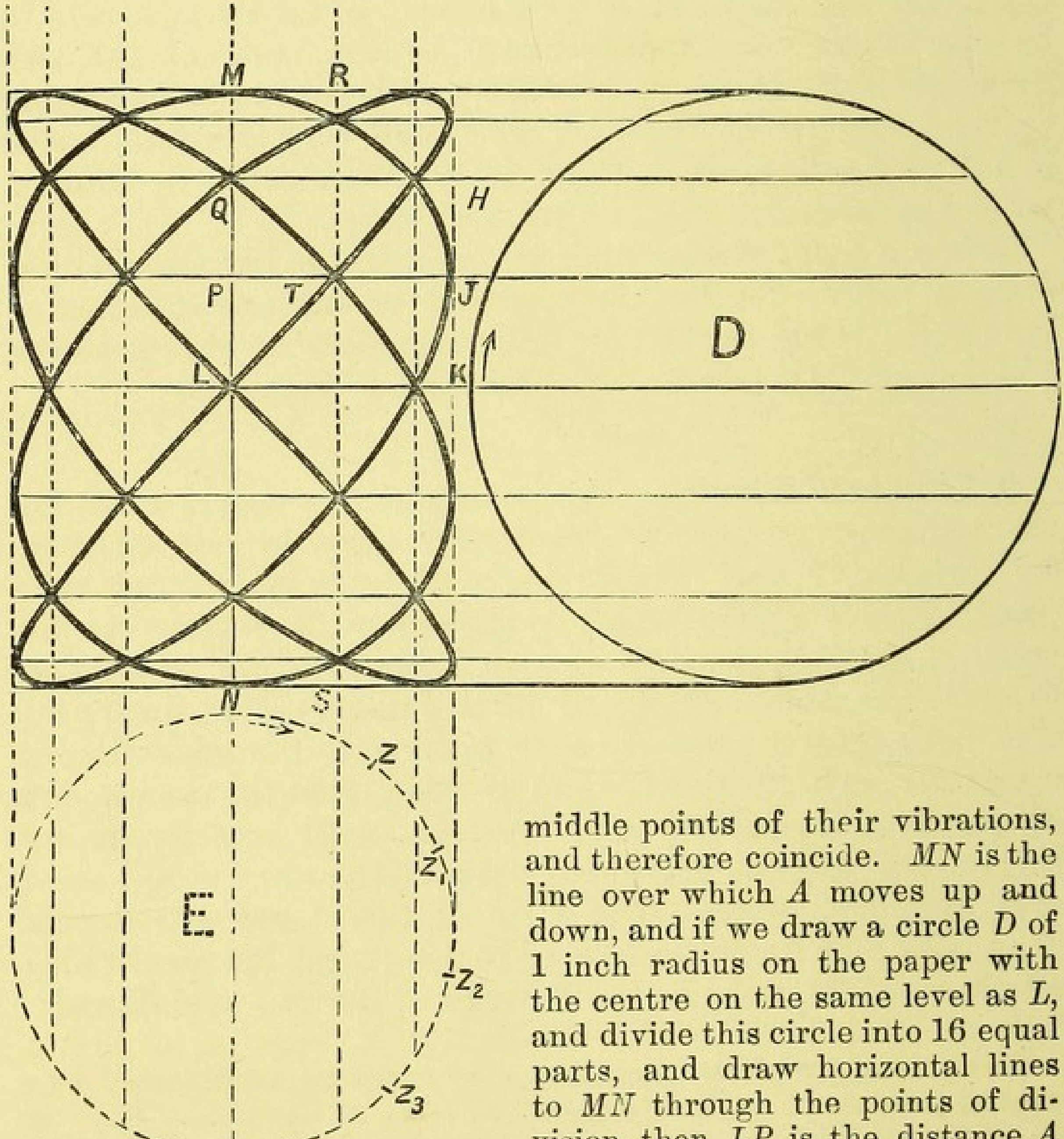

Fig. 8.

middle points of their vibrations, and therefore coincide. *MN* is the line over which *A* moves up and down, and if we draw a circle *D* of 1 inch radius on the paper with the centre on the same level as *L*, and divide this circle into 16 equal parts, and draw horizontal lines to *MN* through the points of division, then *LP* is the distance *A* will move in $\frac{1}{16}$ of its period, and so on. If we draw *on the glass* another circle *E* of $\frac{3}{4}$ inch radius, with its centre directly below *A*, and divide it into 12 equal parts, and draw vertical lines such as *RS* on the glass through the points of division, then, as *B* starts moving to the right from *A* on the glass at the same moment that

A starts upwards from L, B will reach the line RS on the glass at the same moment that A is above the point P on the paper, for $\frac{1}{16}$ of A's period is the same as $\frac{1}{12}$ of B's. B is therefore at this moment above the point T on the paper, for B is always on the same level as A; and LT is the line on the paper over which B has passed. Similarly, B crosses each of the rectangles diagonally, so that it moves over a curve on the paper like the one shown in the figure.

In Fig. 8 the complete lines are supposed drawn on the paper, and to remain stationary, while the system of dotted lines is supposed drawn on the glass and to be moving up and down with it.

If there is a phase-difference, say $\frac{1}{8}$, between the vibrations, and the horizontal vibration is in advance, then (since $\frac{1}{3}$ of the G.C.M. of the periods represented by 3 and 4 is $\frac{1}{9}$ of the shorter or $\frac{1}{12}$ of the longer) at the moment when A is over L, B will have arrived at a vertical line on the glass passing through Z, to a point $\frac{1}{9}$ of the circumference of E from N, and will be where this line cuts LK. If we divide E into 12 parts, beginning at Z, and number these Z_1, Z_2, &c., then B will pass through the point on PJ, which is exactly above Z_1, the point on QH which is exactly above Z_2, and so on, and a different curve will be traversed.

The circles might have been divided into any other numbers of parts in the ratio 4 : 3, instead of into 16 and 12.

These curves are known as *Lissajous' figures*. Their forms, for some of the most important ratios and phase-differences, are shown in Fig. 9, where the numbers at the left hand show the ratio of the period of the horizontal to that of the vertical vibration. As the figures stand on the page, they correspond to the phase-differences from 0 to $\frac{1}{2}$ as indicated above them; if the page is inverted, the same figures will correspond to the phase-differences from $\frac{1}{2}$ to 1, as shown by the fractions which will then be above them.

If n horizontal vibrations take very nearly but not exactly the same time as m vertical ones, the curve traced will be very nearly one corresponding to the ratio $m : n$, but when m vertical vibrations are finished, rather more or less than exactly n horizontal ones have been performed, and the phase-difference is not exactly the same as at starting. A slightly different curve, corresponding to a different phase-relation, is therefore traced in the next cycle, and as the phase-difference alters during each repetition of the figure, a series of curves corresponding to gradually increasing or diminishing phase-differences are traced in turn.

If the horizontal and vertical vibrations are nearly equal in period and are at first in the same phase, the curve traced in successive vibrations will gradually change, assuming in turn the forms of the first line of Fig. 9 from left to right

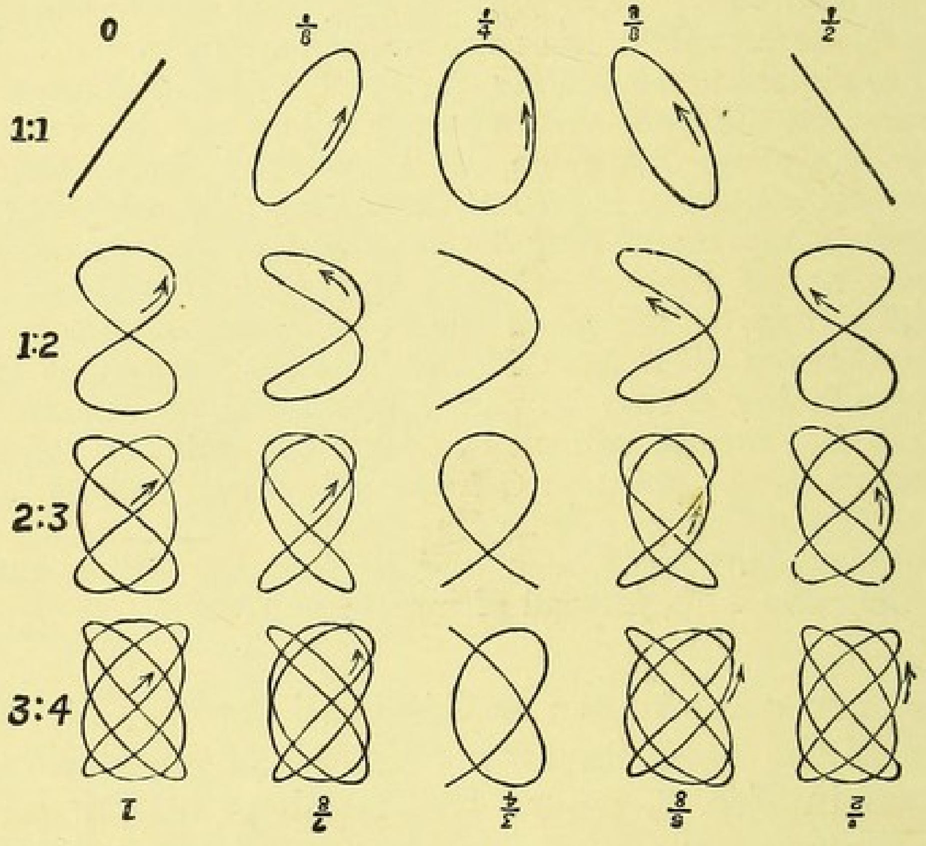

Fig. 9.

and back again, and returning to its original form when one movement has been repeated once more than the other. This gives a very accurate method of determining the difference between the frequencies of two vibrations. The practical details are given in Chapter XII.

9. Methods of producing Lissajous' Figures.—Though the connection with the subject is rather remote, it is usual in text-books on sound to mention several mechanical and optical devices for showing such curves as we have just described. A simple device is *Wheatstone's Kaleidophone*, a good form of which consists of a straight strip of steel, such as a piece of clock spring, twisted at the middle, so that the planes of the two halves are at right angles

(Fig. 10). If the lower end of the rod is fixed in a vice, the upper end can vibrate either parallel to the jaws, in

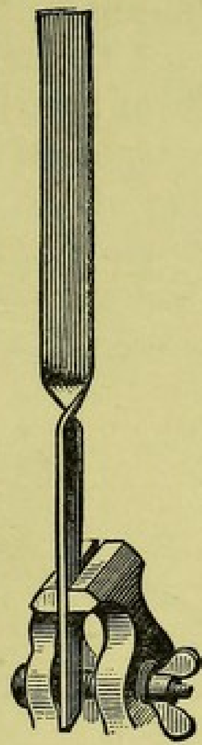

Fig. 10.

which case the lower part remains straight, and the time of vibration depends on the length and stiffness of the upper half; or at right angles to the jaws, and in this case only the lower part bends, and the time of vibration depends on the length of that part.

The vibrations of a spring fixed at one end soon become harmonic, as will be explained in Chapter XI., and by adjusting the position of the rod in the vice the periods of the two vibrations may be made to have any desired ratio. If the ratio is made a simple one, and the top of the rod displaced obliquely to the plane of the jaws, the extremity will describe the curve corresponding to the ratio. If a globule of mercury is attached by a little grease to the tip of the rod to form a brilliant point, the whole curve traced will be visible at once, by persistence of vision, if the vibrations of the rod are rapid enough.

If two tuning-forks are fixed so that the vibrations of one take place in a vertical, and those of the other in a horizontal, plane (Fig. 11), and if a small mirror is fixed either on the side or the end of one prong of each fork, so that, when the prong vibrates, the surface of the mirror is tilted through a corresponding angle, and if a convergent beam of light falls on one of the mirrors, is reflected from it to the other mirror, and from that to a screen, then, if only the first fork vibrates, the spot of light on the screen will vibrate harmonically up and down; if only the second fork vibrates, the spot will vibrate harmonically right and left; if both vibrate together, the movement of the spot will be the resultant of these two vibrations, which, if their periods

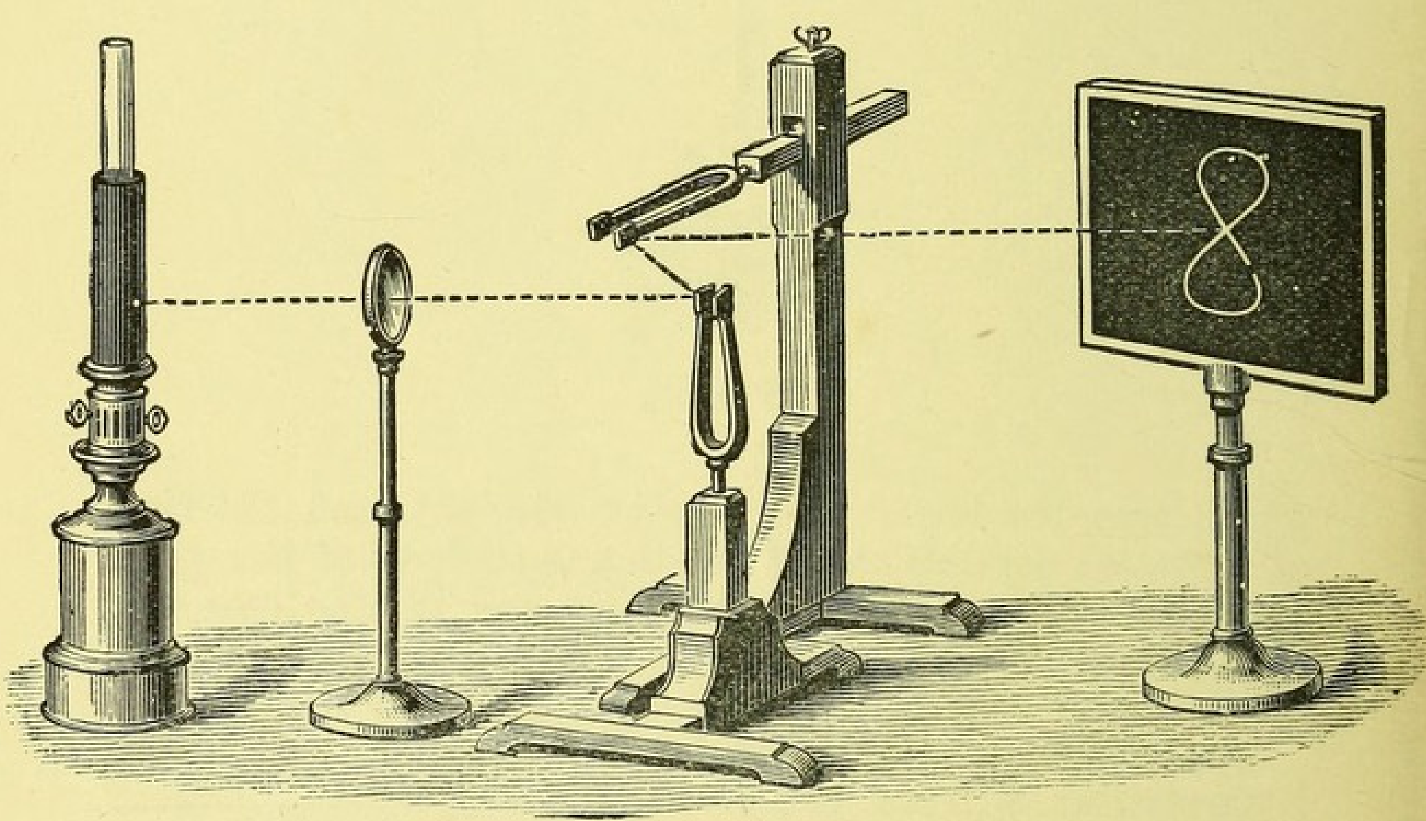

Fig. 11.

have a simple ratio, will be the corresponding Lissajous' figure. It was in this way that Lissajous produced the figures.

Instead of projecting the spot on a screen, we may watch, with the eye or a telescope, the image of a bright point formed by successive reflection in both mirrors; in this case the figure will be visible even when the vibrations of the forks are very minute.

In these two methods, the track described by the spot is only visible by persistence of vision, and the vibrations must therefore be rapid. *Blackburn's Pendulum* is a contrivance for leaving a permanent trace, so that the vibrations may be much slower. A funnel, *F*, filled with sand (Fig. 12), is suspended from one of three strings, knotted together at *C*,

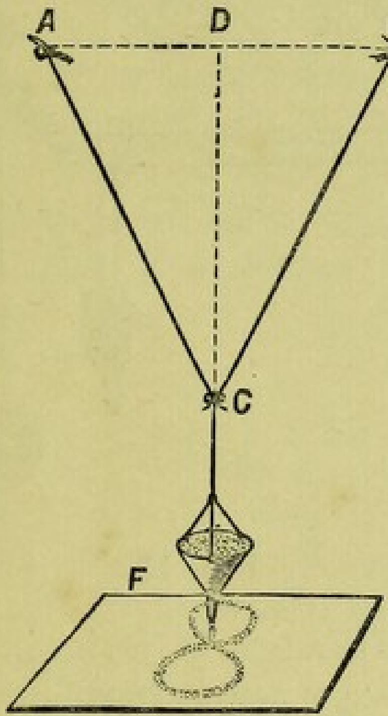

Fig. 12.

and the other two strings are fastened to two points, *A* and *B*, on the same level. If the funnel is set swinging in the vertical plane which contains *A* and *B*, only the part *CF* swings, and the period is that of a pendulum of length *CF*; if the movement of *F* is perpendicular to the plane through *AB*, the whole system of strings swings together about the line *AB*, in the period of a pendulum of length *DF*. By adjusting the length of *CF*, any desired ratio can be given to these periods, and if this ratio is made a simple one, and *F* is then displaced obliquely to the plane of *AB*, it will describe one of Lissajous' figures. If the sand runs out slowly, it will leave a sand-trace of the figure on a table placed below.

Tisley's Harmonograph is a much better device. It consists of two heavy pendulums, which swing in planes at right angles to each other, and whose rods extend a few inches above their centres of suspension. In one form, shown in Fig. 13, two horizontal strips of wood, each of which rests at one end on the pointed top of one of the pendulum rods, are hinged together at the other by a glass pen, which passes through

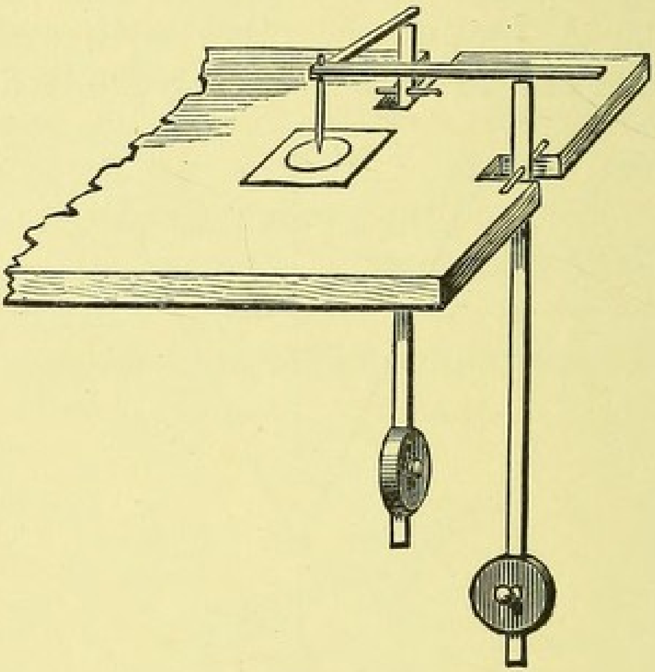

Fig. 13.

both strips. Either pendulum, swinging without the other, would give the pen a very nearly harmonic rectilinear motion; when they swing together, the pen traces one of Lissajous' figures.

In all these methods, the periods of vibration in the two directions are independent, and, therefore, they can never be adjusted quite exactly to any simple ratio; the curve described, therefore, always changes slowly. It is easy, by means of rods connected to toothed-wheels working in one another, to give to a pencil harmonic motions at right angles, which have exactly a simple ratio, and so to trace always the same curve, but it is not usual to describe such methods in works on Sound.

If the figure compounded of two vibrations at right angles is given, the ratio of the periods of the vibrations can at once be found by inspection; it is the ratio of the number of times any vertical line is cut by the curve to the number of times any horizontal line is cut by the curve.

10. Harmonic Vibrations in the same line.—Another important case is that in which the same point executes at the same time two or more harmonic vibrations in the same line. For instance, suppose that *B* vibrates harmonically, with respect to the paper, along *XY*, while *A* vibrates harmonically, with respect to *B*, in the same direction. The motion of *A*, with respect to the paper, will be rectilinear, and its character can therefore only be shown by some device such as a displacement curve. If *O* is the middle point of the vibration of *B* (or any other point fixed with respect to the paper), then the distances of *B* from *O* at the successive instants represented by 1, 2, &c., will be equal to the ordinates at those points of some harmonic curve, such as *CD* (Fig. 15), the length of a double bend of which repre-

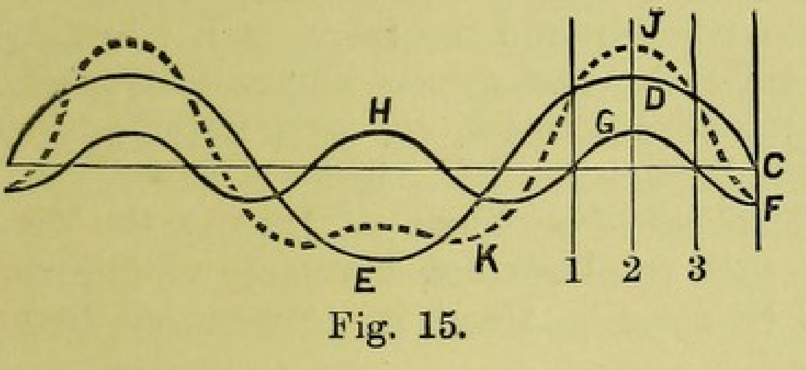

Fig. 15.

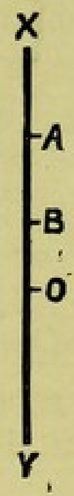

Fig. 14.

sents the period of *B*'s vibration. And the distances of *A* from *B* at the same instants will be equal to the ordinates of another harmonic curve, such as *FGH*, the length of a complete double undulation of which represents the period of *A*'s vibration. As the distance of *A* from *O* is always the algebraic sum of the distance of *B* from *O*, and that of *A* from *B*, the displacement curve of *A*'s vibration with respect to the paper will be found by drawing at each point ordinates equal to the algebraic sum of the ordinates to *CDE* and *FGH* at the same point, and drawing the curve through the ends of these ordinates. The dotted curve in Fig. 15 shows the displacement curve of the vibration compounded of the two harmonic vibrations whose displacement curves are *CDE* and *FGH*.

It is evident at once that this is not a harmonic curve, so that a point which executes two harmonic vibrations at once may be vibrating non-harmonically. The importance of this fact will appear in Chapter V.

A spot of light may be made to execute on a screen a vibration which is the resultant of two or more harmonic vibrations in the same direction, by successively reflecting a beam of light from mirrors carried by tuning-forks in exactly the same way as described for Lissajous' figures, except that all the forks must vibrate so as to displace the spot in the same direction, *e.g.*, all in vertical planes. The vibrating spot will, however, simply appear as a straight line of light; there will be nothing to show the character of its movement. The displacement curve corresponding to the vibration may be demonstrated by placing between the last fork and the screen a mirror rotating continuously on a vertical axis; the beam, being reflected from this after reflection by all the forks, will sweep round and round the room, and, vibrating vertically while it traverses the screen horizontally, will trace on the screen the displacement curve of its vibration, so rapidly that the whole curve will be visible at once.

There are many mechanical methods of tracing the displacement curves of such resultant non-harmonic vibrations, but it is not usual to describe them in elementary text-books.

CHAPTER II.

PROGRESSIVE UNDULATION.

11. Transmission of Condensations and Rarefactions through Air.—Imagine a flat plate, held straight in front of you, vertically, with its edge towards you, and then suppose it rather suddenly displaced a very short distance towards your left. As the plate moves, the layer of air immediately to the left of it will, of course, be driven to the left with a velocity equal to that of the plate, and the air in that layer will also be compressed, the increase in pressure and in density being, in the early stages of the movement, proportional to the velocity of the plate (Appendix D). But though the velocity and the additional density of this layer of air both depend on the velocity with which the plate is displaced, these conditions are rapidly transferred from the layer of air in contact with the plate to a layer a little further off, and from that to the next, and so on, with a velocity which is practically independent of the velocity of the plate, and depends only on the properties of the air. In just the same way, a layer of air immediately to the right of the plate will move with it to the left, and at the same time its pressure and density will be diminished; and these conditions of motion to the left, reduced pressure, and reduced density, are transferred from that layer to other layers of air more and more distant from the plate with the same velocity as the compressed condition on the other side.

The simplest way of explaining this process is to suppose the air divided, by purely imaginary surfaces, into small blocks, or *particles*, in contact with one another; these particles being, not molecules, but portions of air each containing many millions of molecules at least. Such a particle of air behaves, as regards compression, very much like an elastic solid; like a sponge, or ball of wool, to use Boyle's illustration; and the air of a room transmits displacements and condensations very much as a mass of balls of wool in contact might be imagined to do. The way in which a series of elastic bodies transmits a condensed or compressed condition to a distance, while the bodies themselves are only very slightly displaced, is well illustrated by a row of railway carriages, standing with their buffers in contact, when one of the carriages is pushed. We will call the carriages in order, from right to left, *A*, *B*, *C*, &c. (Fig. 16, line 1), and suppose that the engine suddenly moves a short distance, say a foot, towards *A* (2). The buffer-springs of *A* are driven in, so that the carriage (if we measure to the ends of the buffers) is

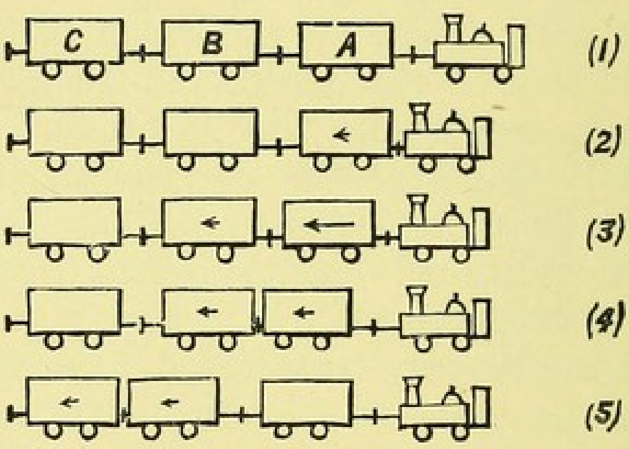

Fig. 16.

shortened, or, as we may call it, condensed, and the force due to the compressed springs makes *A* move with increasing velocity towards *B*. When it reaches the half-way point between the engine and *B*, the pressures at the two ends of *A* will be equal (3), but, owing to its inertia, it will not stop at that point, but will continue to move to the left till the increasing pressure between it and *B* brings it to rest. This will happen when *A* has moved just one foot, so that the distance between it and the engine is the same as at first (4). Meanwhile, *B* has begun to move to the left, and exactly repeats the movements of *A*, and when it has been displaced one foot in its turn, it stops at its original distance from *A* (5), and the buffers between *A* and *B* cease to extend. The whole of *A* is then at rest, including the buffers, and it has its original length. *C* repeats the movements of *B*, just as *B* repeated those of *A*, and so a condition of condensation and movement is transferred from carriage to carriage all along the train.*

In this process several points are noteworthy. First, that though a condition, that of compression or condensation of the carriages, travelled continuously along the train from right to left, yet each carriage moved only a short distance, and then stopped, so that it is a *condition*, and not any material substance, which travelled along the train. Secondly, not only this transfer of a condition, but the existence of the condition, depends on the fact that each carriage repeats the movement of one next it *a little later;* if they all moved together there would be no condensed condition anywhere, and the direction in which the condition travels is from the earlier-moving to the later-moving carriages. Third, the condensed condition is to be found always, and only, in carriages which are moving; when any carriage comes completely to rest (buffers included) it is exactly its original length. Fourth, the condition may travel much faster than the carriages themselves move even at their quickest; in our example, the condensed condition is transferred from one carriage to another in the time in which the carriage itself has moved only a foot. In all these particulars the transmission of a condensation through air or any elastic substance resembles that through a row of railway carriages.

We shall find, as we proceed, that many different conditions can be transferred besides that of condensation, and the four statements just given are true of all of them. In the case described, we may notice also that the movement of the carriages themselves was in the *same* direction as the movement of the condition, but this is only true when it is a condition of condensation which is transferred.

If instead of a momentary impulse the engine gives to *A* a push lasting for an appreciable time, several carriages will have begun to move before *A* comes to rest, and as the

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* In this description the movement of the carriages has been slightly modified to make it illustrate more exactly the movements of the air. The carriages would not move just as described.

condition travels, there will always be several carriages moving at once. But it will still be true that each carriage moves exactly like the one next nearer the engine, but arrives at each stage of its movement a little later; and also that each carriage is compressed as long as it is moving, and recovers its original length as it comes to rest. The movement may be considered as due to a succession of momentary impulses given to *A* one after the other, and each passed on in turn.

In the case of a row of carriages, each communicates its movement to one of equal mass, and the movement of an elastic substance does not correspond exactly to that of the carriages except when the portions which are successively in motion are equal. This is not the case when a plate is waved in free air, so it will be simpler first to consider a column of air confined in a tube. Let *AB*, Fig. 17, be a long tube full of air, and *D* a piston. We may imagine the

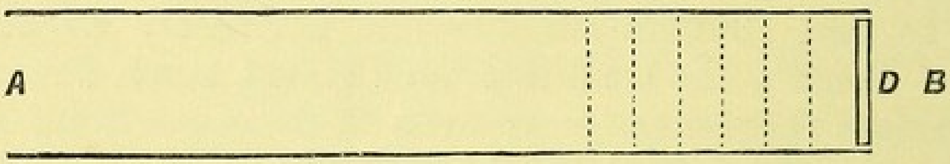

Fig. 17.

air in *AB* divided by imaginary planes, transverse to the tube, into equal discs or layers, and we will suppose these planes to move with the air, if it moves, so that the air between the same two planes is always the same air. If *D* is displaced a short distance to the left, the first of these layers, that next to *D*, becomes compressed and moves to the left, and then the next, and so on, just like the railway carriages. As with the carriages, the movement may have extended to a number of layers before *D* comes to rest, so that a number are moving at once; but, in any case, each moves exactly like the one next nearer to *D*, but begins to move, and reaches each stage of its movement, a little later. As with the carriages, also, every layer is condensed while it is moving, and comes to rest as it is restored to its original length. The only differences to be noticed (and these are only apparent) are, first, that the air-discs become condensed all through, instead of shortening only at the ends like the carriages, and, secondly, that, however thin we make our imaginary layers, each will always be a little later in its movements than the one next nearer *D*, so that no finite length of the air-column begins to move absolutely at the same instant, as a whole railway carriage appears to do.

If we suppose the carriages *A*, *B*, *C*, &c. all held compressed between an engine to the right of *A* and the end of a siding to the left of *Z*, then, if the engine moved one foot away from *A*, allowing the buffers at that end of *A* to expand, *A* would be, for the moment, longer than the other carriages, or rarefied, as we may call it, and would begin to move towards the engine. As this would diminish the force on the end of *B* next *A*, *B* would follow and become rarefied, while *A* would stop when it reached its original distance from the engine, and would have been compressed to its original length again when *B* stopped after moving one foot in its turn. In this way a condition of *rarefaction* passes from *A to Z*, while each carriage in turn moves a foot in the direction *from Z towards A.* In every travelling rarefaction the movement of the matter is in the *opposite* direction to the movement of the rarefied condition.

Air is always in a compressed condition, for there is no limit to the extent to which it expands if pressure is removed: and particles of air move like compressed railway carriages. If the piston *D* was displaced to the right, first the air in the layer just to the left of *D*, and then that in layer after layer further to the left, would become rarefied and move to the right, each in turn coming to rest at the same moment at which it was restored to its original thickness. The air to the right of *D* is of course condensed by the movement of *D* to the right, and a condensation travels away on that side, but for the present we consider only the air to the left of *D*.

Whether we move *D* to the right or to the left, then, the movement is repeated, first, by the air in contact with *D*, and then, layer after layer, by the air further and further off, so that the condition of being in motion travels away from *D* in either case. When *D* moves towards the air on the side we are considering, it condenses the air, and each layer moves in turn away from *D*, and is condensed while it is moving; when D moves away from the air on that side, it rarefies the air, and each layer (beginning with the one close to D) moves in turn towards D, and is rarefied while it is moving.

If we move D backwards and forwards, in any manner, exactly the same movement is performed by each layer, but later and later the further the layer is from D, so that D and layers near it may have begun to move one way, while more distant layers are still moving the other. All the layers which are moving away from D are condensed, and all those which are moving towards D are rarefied.

12. Progressive Undulation.—This process, in which a condensed or rarefied condition is transmitted through the carriages of a train, or air in a tube, is an instance of what is called *progressive undulation*, which may be defined as **the continuous transference in the same direction of a condition of altered relative position of adjacent particles by similar movements performed successively by consecutive particles.** In the instances we have so far considered, whether the condition transmitted is one of condensation or rarefaction, the movements of the particles are in the line along which the condition is transferred (though they may be in the opposite direction along that line). When this is the case, the progressive undulation is called *longitudinal*. In longitudinal progressive undulation the condition that is transferred is always one of altered *distance* between successive particles, *i.e.* of condensation or rarefaction; it is practically the only kind which occurs in air and gases. In Chapter XI. we shall treat of other kinds of progressive undulation in which the condition transmitted is an altered relative *direction* instead of an altered relative distance.

Since the sensation of sound is usually caused by the arrival through the air of condensations and rarefactions transmitted by the process of longitudinal progressive undulation, this process is often conveniently called *sound;* travelling condensations and rarefactions are called *sound waves*, and the velocity with which such waves are transmitted in any substance is called the *velocity of sound* in the substance. The term *pulse* is often used to denote either a condensation or a rarefaction, and avoids the frequent repetition of both terms.

13. Undulation and Vibration.—It is important to distinguish clearly between undulation and vibration. When the same point repeats the same movements over and over again, the point is vibrating. When different parts of the same substance perform a similar movement one after the other, the substance is undulating. If it happens that the movement which the parts perform in turn is a vibratory movement, then the substance is both vibrating and undulating. This is what is usually happening when sound passes through the air. But there is no necessary connection between the two; a substance may undulate without vibrating, like the train after a push, or it may vibrate without undulating if all the parts of it move at the same time, like a pendulum.

14. Relation between Velocity and Condensation.—It can be shown (Appendix D) that in any longitudinal progressive undulation there is a constant ratio, depending only on the nature and condition of the substance conveying the undulation, between the velocities with which different parts are moving, and the amounts by which the densities of those parts differ from the average density. If we call the difference from the average density the "condensation" or "rarefaction,"* the velocity at any point is proportional to the condensation or rarefaction there.

This is, of course, true close to *D* as at any other point, and the velocity of the air close to *D* is the velocity of *D*; the degree of condensation which exists close to *D* (and therefore the excess of pressure on the side towards which *D* is moving) is always proportional to the velocity with which *D is moving*, and does not depend at all on how *D* has previously moved.

* Strictly, "condensation" means not the difference between the actual and average densities, but the ratio of this difference to the average density. But these are proportional to one another, so that the velocity with which the air is moving is proportional to the "condensation" in either sense. Rarefaction may be conveniently included in the term condensation, and considered merely as the negative variety.

15. Wave-form.—Suppose that when D has been moving, in any manner, for some time, we draw a horizontal line, OX, Fig. 18 (1), along the tube, and from it draw ordinates at different points, proportional to the velocities of the air at

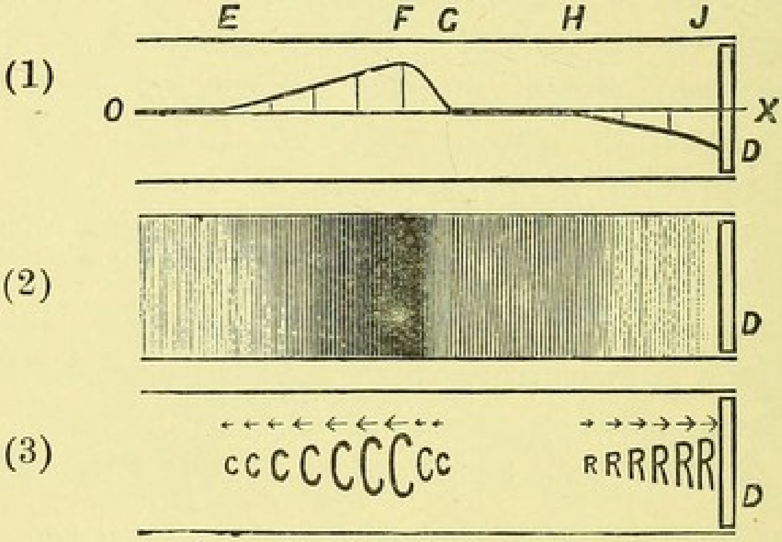

Fig. 18.

those points, at some given moment, drawing the ordinates above OX to represent velocity in the direction in which the waves are moving, and below for velocity in the opposite direction. Then the same ordinates are also proportional to the degrees of condensation or rarefaction at the same points, ordinates above OX representing condensation. So that the curve in Fig. 18 (1) denotes the same condition which is represented by shading in (2), and by the relative heights of the letters C and R (Condensation and Rarefaction) in (3). In (3) the relative velocities are represented by the lengths of the arrows over the letters; this is unnecessary in the case of a progressive undulation, as the velocities are proportional to the condensation or rarefaction, but we shall find it useful in other cases.

A curve like that in Fig. 18 (1), whose ordinates, at equidistant points along a horizontal line, are proportional to the velocities with which the air is moving at a number of equidistant points is said to show the *wave-form* of the undulation, the wave-form meaning the way in which the velocity of the air varies from one part to another of the wave. It will, of course, be understood that waves of condensation and rarefaction in air have not anything which can be termed *form* in the ordinary sense, as waves of the sea have; the term wave-form is not to be taken literally.

In air, condensations and rarefactions are transmitted at about 1100 feet per second. If the wave-form shown in Fig. 18 (1) represents the velocities of the air at different points of the tube at a moment denoted by *T*, then the ordinate which represents the velocity at the point 1 foot to the right of any point, *F* for instance, represents the velocity which will exist at the point *F* at a moment $\frac{1}{1100}$ second later than *T* (written, the moment $T + \frac{1}{1100}$), and so on. The same curve which represents the simultaneous velocities of the air at different points in a wave which is travelling from right to left is therefore also a velocity curve in which the velocities of the air at any one point, at successive instants, are represented by the successive ordinates read from left to right. The same length along *OX*, which represents one foot when the curve is taken as representing the simultaneous velocities at different points in the air, represents $\frac{1}{1100}$ second when the curve is taken as representing the successive velocities of the air at the same point. Successive ordinates of the part of the curve to the left of any point represent the successive velocities which have, in turn, existed at that point. As the velocity of the air close to *D* is the same as that of *D* itself, the wave-form of the air in the tube is a velocity curve of the movements already performed by *D*.

The difference between the actual pressure at a point in the air and the average pressure (a difference which we shall call the *pressure-difference*) is proportional in all waves to the condensation or rarefaction, and therefore, in travelling waves, to the velocity of the undulating substance (Art. 14). The successive ordinates of the wave-form, read from left to right, may therefore be taken as representing the successive pressure-differences, as well as the successive velocities, which exist at the same point at successive instants. This is the most useful of the many meanings of the wave-form, as will appear in Chapter VI.

16. Wave-length.—If *D vibrates*, or repeats the same movements at regular intervals, the same series of conditions will be produced at its surface over and over again, and travel along the tube. Suppose, for instance, that it repeats its whole movement in $\frac{1}{1100}$ second, then the air, 1 foot, 2 feet, &c., from the surface of *D* is in the same condition as at the surface, and if we divide the tube into lengths of 1 foot, the air in any of these sections, at any instant, is moving in the same way as the air in any other section. One of these sections is called a *complete wave*, and its length is called a *wave-length*. It is evident that the wave-length is the distance which a given condition travels before *D* comes round to the same stage of its vibration again; in other words, it is the velocity of the undulation multiplied by the period of the vibration. If V denotes the velocity of sound and λ the wave-length, $\lambda = Vt$, or $\lambda = \frac{V}{n}$.

17. Harmonic Waves.—If *D* vibrates harmonically, the wave-form of the undulation in the tube will be a harmonic curve. Such waves are called *harmonic waves*. Two complete harmonic waves travelling, like those in Fig. 18, from right to left, are represented in Fig. 19, the heights of

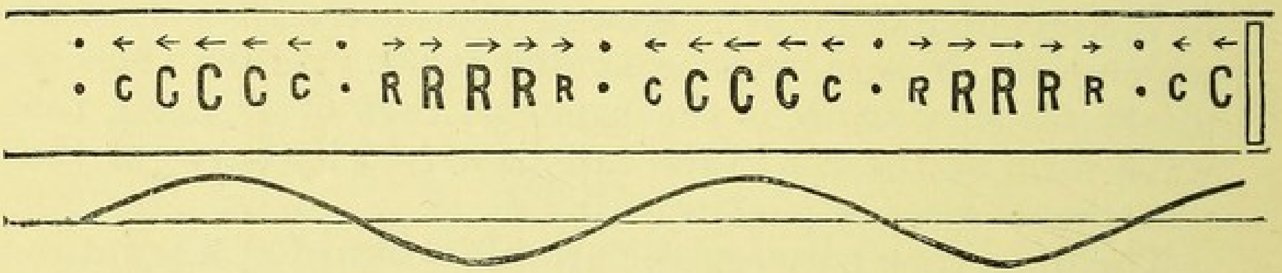

Fig. 19.

the letters *C* and *R* being proportional to the condensation or rarefaction. The corresponding wave-form is shown below.

The way in which a condition of condensation or rarefaction can move always in the same direction while the air itself moves backwards and forwards, is illustrated by successive rows, 1, 2, &c., in Fig. 20. In this figure the successive rows represent successive stages of a movement in which each of the vertical lines vibrates harmonically to right and left of a fixed point, from which it never departs more than $\frac{1}{8}$ of an inch, *a* vibrating about a point under *A*, and so on; but each line of a row, in order from right to left, is a little behind the line to the right of it in its movement, and the result is, as seen, that though the mean positions of the lines are equi-

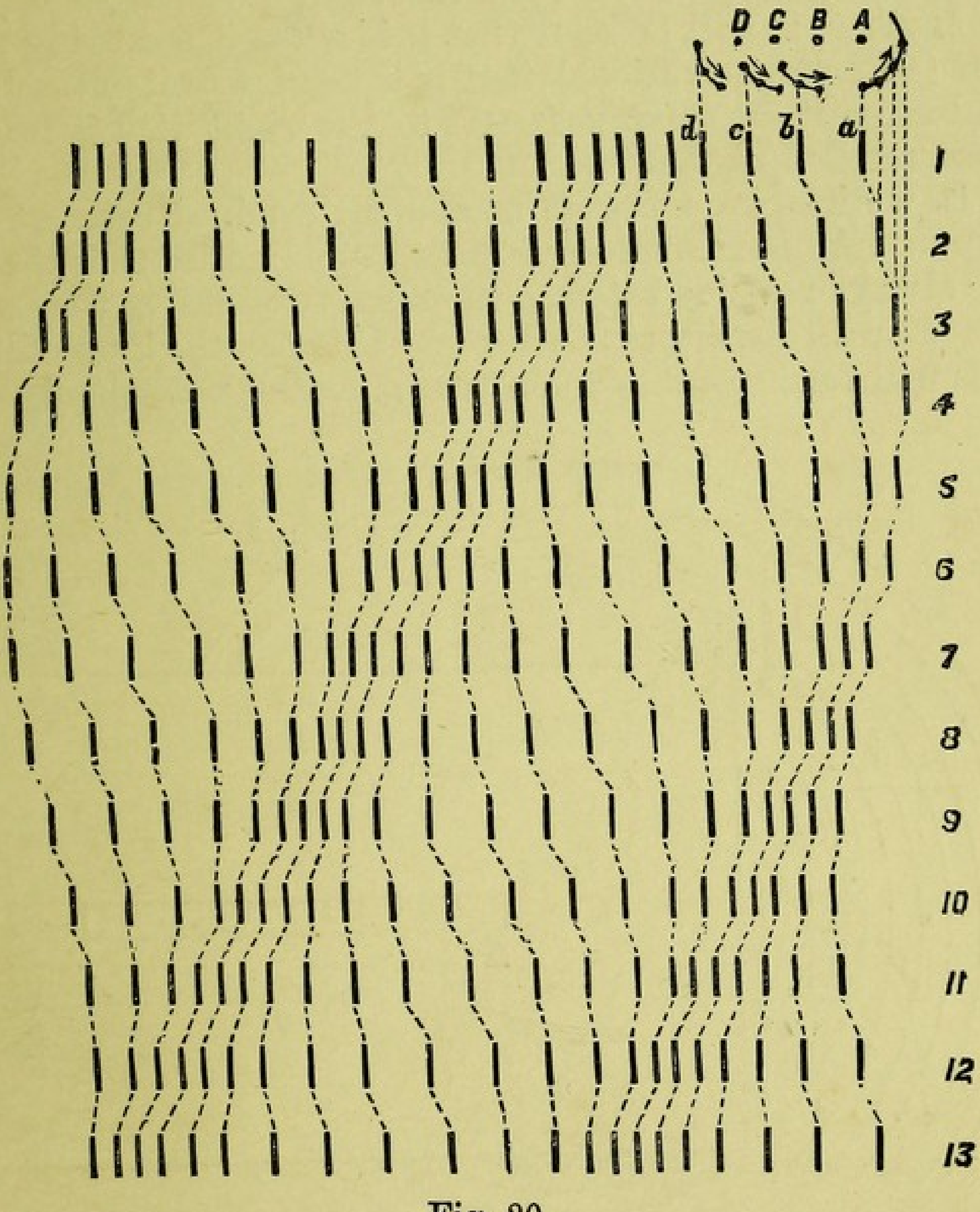

Fig. 20.

distant, the lines themselves are at any one time closer in some places and further apart in others, and that these conditions are found further to the left at each successive stage of the motion. The construction for the harmonic motion of a few lines is given, and it will be seen that in any one row, each line is $\frac{1}{12}$ of its period behind the one to the right of it, and that in each row all the movements are $\frac{1}{12}$ of a period further advanced than in the previous row. The thirteenth row is the same as the first, every line having returned to its original position.

If any line is traced (by the dotted lines) from one row to another, it will be seen that it is when it is moving to the left (with the conditions) that it is nearest to its neighbours, and when it is moving fastest to the right that it is in the most rarefied region.

If we imagine an equal mass of air confined between each line and the next, and moving with the lines so that the *same* air always remains between the same two lines as they move, but alters in density as the distance between the lines changes, this will correspond to the actual movement of the air.

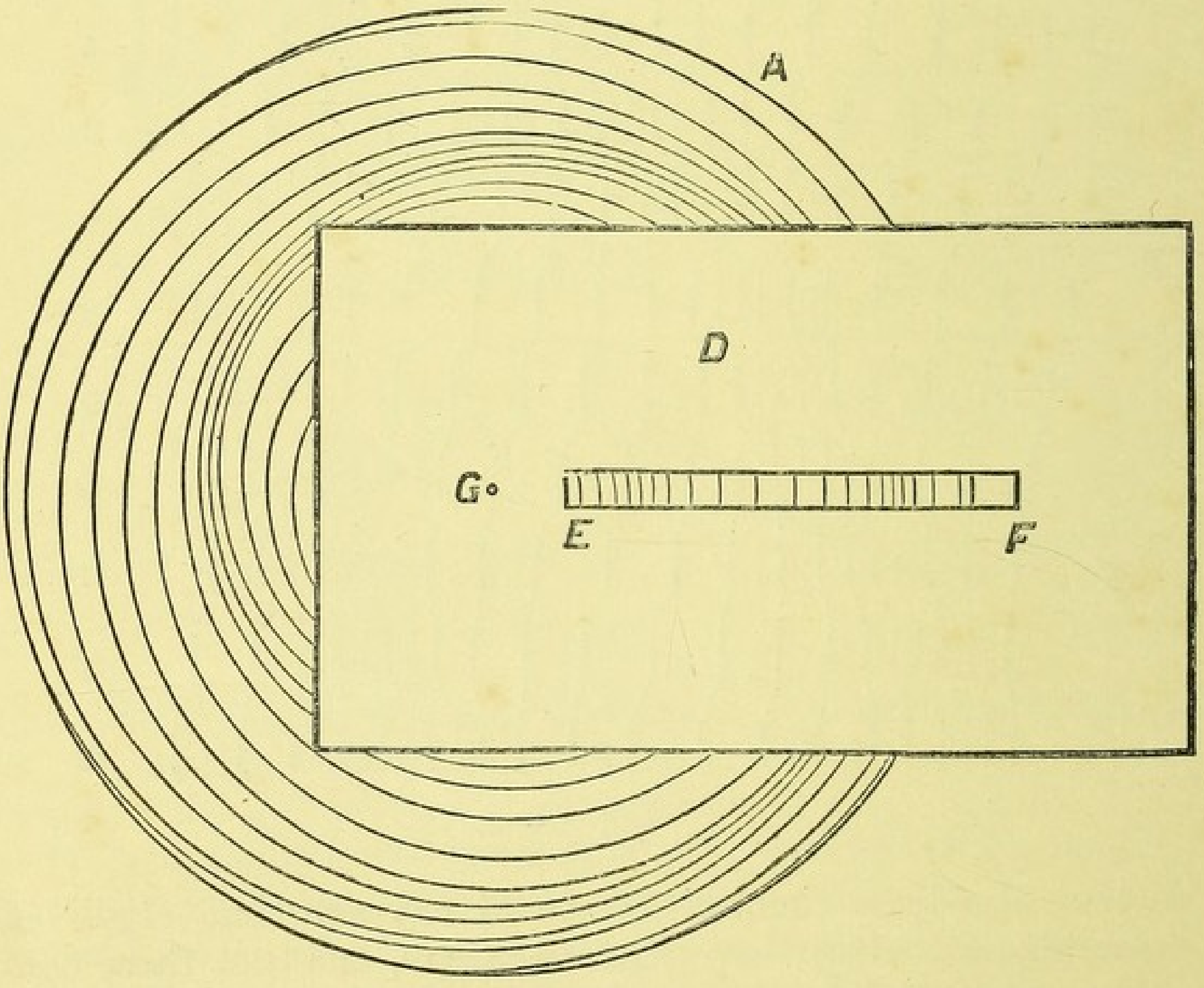

Fig. 21.

18. Crova's Disc.—A good and simple device for illustrating the movements of the air or other substance when a harmonic progressive undulation is passing through it, is

Crova's disc (Fig. 21); this the student can easily construct, and should do so if he finds any difficulty in realizing the motion.

In the middle of a circle of cardboard, *A*, 8 inches in diameter, draw a small circle of $\frac{1}{8}$ inch radius, and divide its circumference into twelve equal parts. With each of these points of division in turn as centre, describe a circle in ink, making the radius of the first $\frac{3}{4}$ inch, of the second $\frac{7}{8}$ inch, and so on, increasing $\frac{1}{8}$ inch each time. When you get to the last of the centres go on with the one you used first, and so round the circle again, making twenty or more circles in all. In another piece of card cut a slit, *EF*, 3 inches long and $\frac{1}{8}$ inch wide, and make a pinhole at *G* in a line with *EF* and 1 inch from *E*. Push a pin exactly through the centre of the disc, and put its point through *G*; then, holding *D* in one hand, rotate the disc behind *D* by means of the pin.

The portions of the circles visible through the slit are practically straight lines, close in some places, and wider apart in others, like those in Fig. 21; as the disc rotates these condensations and rarefactions will be seen to travel continuously along *EF*, but if the disc is rotated slowly and the motion of any one of the lines carefully followed, it will be seen that it simply moves harmonically to the right and left, and never departs more than $\frac{1}{8}$ inch from its mean position. We can also easily verify the fact that where the lines are near together they are moving in the same direction as the condensations and rarefactions, but not so fast, but that where the lines are far apart they are moving in the opposite direction, and that the quickest motion of the lines is to be found where there is the greatest degree of condensation and rarefaction.

19. Spiral Wire.—The same motion can be shown by means of a spiral wire. Get about 20 yards of thick copper wire (14 gauge is suitable) and wind it in a close spiral round a tube or cylinder $2\frac{1}{2}$ to 3 inches in diameter. Stretch this spiral till it is about 2 yards long, so that the turns

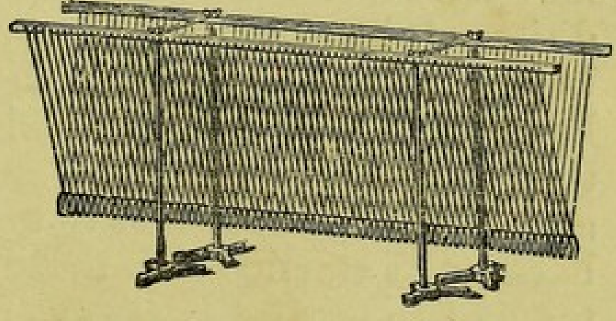

Fig. 21 (*a*).

are about an inch apart. Suspend the highest point of each turn by two threads not less than 2 feet long (the longer the better) from two horizontal wooden bars about 18 inches apart, as shown in Fig. 21 *a*. This is Weinhold's Wave Machine. If a slight push is given to one end of the spiral, in the direction of the length of the spiral, some of the turns at that end will be pressed closer together, and this condition will be transferred from them to the next turns, and so on, a condition of unusual closeness of turns travelling continuously to the other end of the spiral, while each turn of wire, successively, moves a little way and then stops. The condition of unusual closeness leaves one turn as it includes another, so that it always includes the same number of turns; it does not extend so as to include more than when it was originally produced.

20. Some difficulty is sometimes found in understanding how it can be true that the air in a condensation is always moving away from the source, and yet true that the air through which sound is passing moves alternately towards and from the source, and never departs more than a very small distance from its mean position. To understand this it must be remembered that the movement of the air is usually extremely slow compared with that of the condensed and rarefied conditions. Consider a cubic centimetre of air which we will call *A* quite at the left-hand end of the tube in Fig. 19; it is at rest, and neither condensed nor rarefied. But a condensed condition is advancing to the left, and when it reaches *A*, *A* will diminish in volume and at the same time begin to move to the left. But the air moves no faster than *D* moved, while the condensed condition rushes on with the speed of a rifle ball, so that before *A* has moved far to the left, the condensed condition has passed it, and the following rarefied condition has got to it. *A* now expands to more than its original volume, and begins to move slowly to the right, but has only got back to its starting point when the next condensation comes up, and so on. *A* is always moving the same way as the waves when it is in a condensed condition, but it is only in

a condensed condition a little while at a time, and when it is rarefied it moves the other way.

Another form of the difficulty is to understand how the air is a rarefaction and that in the following condensation can always be moving towards each other (see Fig. 19). As before, the explanation is that the air in the same condensation is not always the same air. The air that is now rarefied will an instant hence be condensed, and the air which is now condensed will then be rarefied, so that the parts which are now approaching each other will then be receding, so that no great change in the distance of two portions of the air occurs.

21. Energy Transmitted by Progressive Undulation.—We have already pointed out that in progressive undulation there is no continuous transference of any thing *material*, but it would perhaps hardly be correct to say that *nothing* except a condition travels from one place to another. Usually, progressive undulation is a process by which *energy* is transferred from one place to another, and it is now becoming quite usual to speak of energy as a *thing*, rather than a condition. When a succession of particles perform similar movements in turn, it is nearly always because the movement of each *causes* the movement of the next, and one body can cause the movement of another only by transferring to that other some of its own energy.

In the case of air, through which waves of condensation and rarefaction are travelling, we can easily see that the wave as a whole—the condensed and rarefied portions taken together—contains more energy than an equal volume of undisturbed air. For both the air in the rarefied portion and that in the condensed portion is moving relatively to the earth, and work could be done by stopping this motion. When this energy was exhausted, and all the air at rest, we could still get work done by allowing the air in the condensed portions to expand through a suitable engine into the rarefied portions;* this work would be done by using part of the heat of the compressed portions. The air would then be in its

* It is not, of course, meant that this is a practicable experiment; that it is an imaginable one proves the existence of energy in the wave.

ordinary condition, so that it must, while undulating, have contained more than the usual amount of energy. It is usual to distinguish the part of this energy which depends on the motion of the air relatively to the earth as its kinetic energy, and the part which depends on the differences of pressure in different parts as the potential energy. If we make this distinction, it can be shown that half the energy of a complete wave is kinetic and half potential. The kinetic energy is greatest in the most condensed and most rarefied portions of the wave; the regions where there is average density are also at rest with respect to the earth, and do not differ in any way from undisturbed air, so that there is no extra energy in them.

We shall see later that this is rather an artificial way of looking at the energy of a sound-wave in air, since the whole energy is really kinetic energy of the movements of the molecules. But it is convenient for the present to regard the air simply as an elastic substance, and to divide the extra energy due to its undulation into kinetic and potential portions, as we are obliged to do in the case of solids and liquids where we do not know the real nature of the "potential" part of the energy.

It will be seen that though sound is a special mode of transmission of energy, it is not a distinct kind of energy, for part of the energy of a sound-wave is ordinary kinetic energy of moving air, and the other part is heat.

22. Sound Waves in Free Air. The statement at the end of Art. 19, that in a tube any movement of one layer of air is exactly repeated by those more distant from the source, is only roughly true if applied to considerable distances, for owing to friction between the air and the sides of the tube, and still more owing to exchanges of heat between the air and the sides of the tube, the energy of a wave travelling along a tube is rather rapidly converted into heat, and left behind in that form in the air through which the wave has passed and in the walls of the tube, and so the movements of the air became less and less as we get further from the source. This will be more fully explained in the next chapter. If a body vibrates, not in a tube but in free air, no transformation of the energy of the waves after they have once started can occur from these causes, and though the energy is still ultimately converted into heat, the transformation is almost incomparably slower. In some other respects,

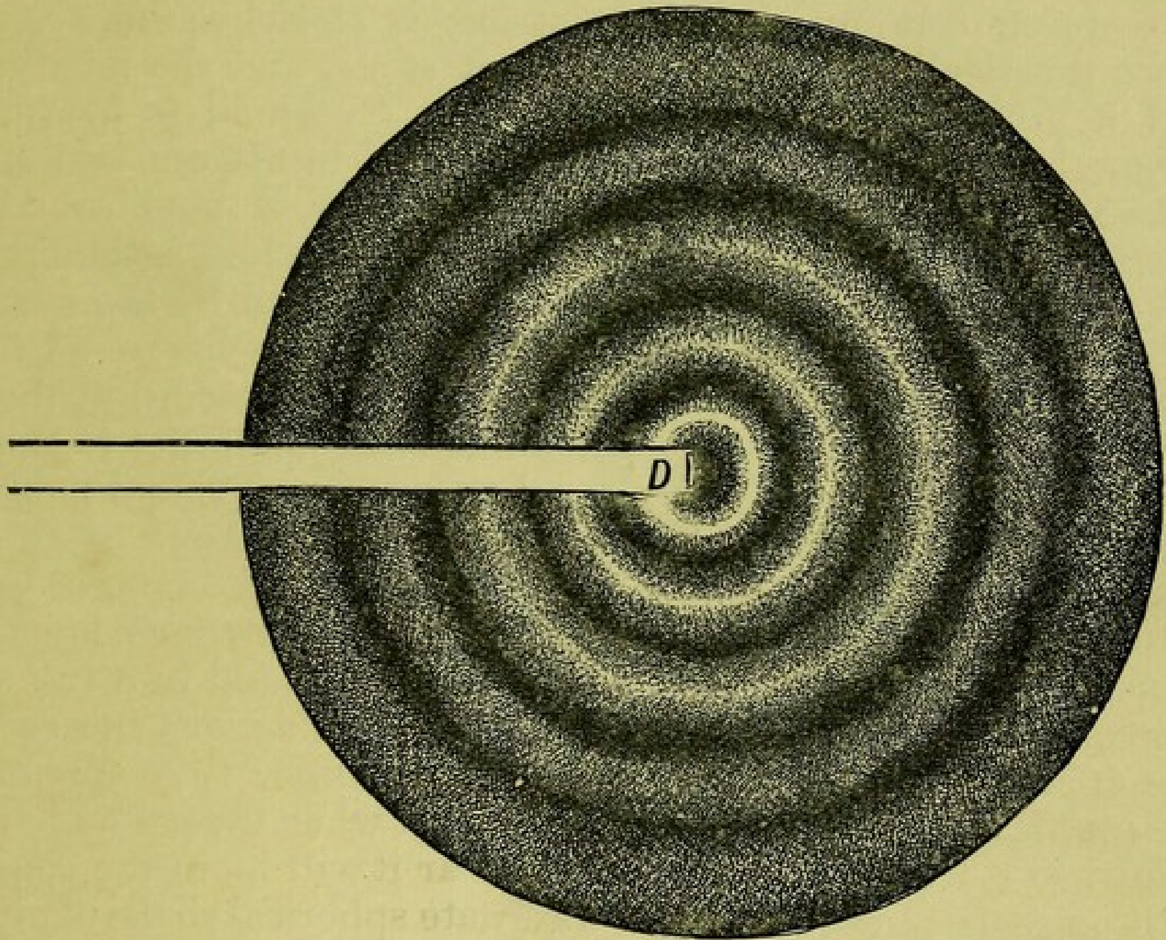

Fig. 22.

also, waves produced in free air differ from those sent along a tube. Suppose that *D*, Fig. 22, is, as before, a piston vibrating at the end of a tube,* but this time we will consider the free air to the right of *D*. First we must notice that though any movement of a piston would

* A moving plate surrounded on all sides by free air would produce condensation on one side and rarefaction on the other, at the same time, and these would spread through the *same air*, and the resulting effect depends on principles to be explained in Chapter IV. The plate is, therefore, described as moving at the end of a tube so that only the effect of the condensations and rarefactions produced on one side of it need be considered.

send a wave travelling along a tube, waves are not produced in free air, to any appreciable extent, unless the vibrating surface is large, or the vibrations of high frequency. If neither of these conditions is fulfilled, the air in front of the moving surface simply slips away, parallel to the surface, till it gets to the edges, instead of becoming compressed. If a string is stretched between two practically immovable blocks of stone, and the middle of it pulled to one side and let go, no sound is heard, though the vibration is seen to be large, but if one end of the string is fastened to a panel of a door, a loud sound comes from the panel when the string vibrates, although the movements of the panel cannot be detected by the eye. The air in front of the moving string simply slips round it to the back, without being appreciably condensed, but the air in front of the moving panel has not time to get to the edge of the panel while the panel moves one way, and must become condensed and rarefied alternately. Even if the surface is large enough, or its frequency high enough, to produce waves, it does not communicate nearly as much energy to the air in each vibration as it would in a tube. Secondly, when the condensations and rarefactions succeed each other rapidly enough to be transmitted, they spread through the air in *all directions,* so that when the piston has finished a few vibrations, the air near it will be in the condition represented in Fig. 22, alternate spherical shells of air being condensed while the others are rarefied. As in the tube, wherever the air is condensed it is moving away from the piston, and wherever it is rarefied it is moving in towards the piston; the movement of the air is everywhere at right angles to the shells. The air in each shell keeps transferring its condition to the air outside it, so that the condensed and rarefied conditions alternately spread continuously outwards from shell to shell of air, like the circular wave-crests produced when we throw a stone into a pond. Thirdly, as the movement and energy of one shell of air are always communicated to a larger one, the extent of the movement and the degree of condensation and rarefaction become less and less as we go outwards from the piston, or the wave-form becomes flatter. Fourthly, in the immediate neighbourhood of the piston very complex actions occur, with the result that the movement of the air a little distance from the piston may differ considerably in character, as well as in amplitude, from the previous movements of the air close to the piston, so that the wave-form changes as the wave spreads, and at a little distance is not the same as the velocity-curve of the movement of the piston. One instance of this change of character is that, although a single movement of a piston in a tube, to left or right, not followed by an opposite movement, would send a condensed or a rarefied condition, not followed by its opposite, along the tube, a single movement of a surface in the open air produces a complete wave, consisting of a condensed and a rarefied shell. In fact we can easily show that the two principles of conservation of energy and conservation of momentum require that a wave, when expanded to many times its original radius, shall consist of two opposite layers, the excess of air in one of which is nearly equal to the defect of air in the other; more and more nearly equal the larger the wave. So that a source may produce nothing but condensations, like a harmonium or a siren (Arts. 89, 102), and yet the waves which arrive at a point a little way off will be alternately condensed and rarefied. But though a single movement of the source produces a complete wave, a double movement of the source only produces one complete wave, not two, at least in ordinary cases. If the movement of the piston is harmonic and very small, the movements in all the surrounding air are harmonic, differing only in amplitude, and not in character, from those of the source. This is an example of a universal principle, that if any physical quantity varies harmonically, and if its variations are small enough, then all variations which depend on it are also harmonic.

All the above statements about the waves produced by a piston like *D* are roughly true about the waves produced by any surface vibrating in free air, such as a spring or tuning-fork. The slight modifications necessary are explained in Chapter IV.

Though the movement of the air near a vibrating body such as a spring is a vibration similar to that of the spring itself, the causes of these similar movements are very different. When we bend a spring and let it go, it returns of itself to its original position, passes it, returns again, and so on, each movement being a consequence of its previous movement; it vibrates in a particular way, and with a particular period. But if we displace a part of the air, either in an infinitely long tube, or in open space, it shows not the slightest tendency to return to its original position. The air, therefore, cannot be made to vibrate, like a spring, by simply displacing a part of it and then leaving it free; each movement that it makes is the result, not of its own previous movements, but of a previous movement of something else, transmitted to it through the air between. The air vibrates only as a pump-handle vibrates; it requires a separate push from outside for every movement. It therefore vibrates just as easily in one way as another, and in one period as another; in fact, it simply copies the movements of some other vibrating body.

We shall see in Chap. X. that in a tube of finite length the air may vibrate merely through having been displaced, without any other vibrating body to cause each movement.

At different distances from the piston the air will be in different stages of vibration, but it will be possible to draw round *D* continuous surfaces, in each of which all the air is moving in the same way at the same moment; these will be, approximately, spherical surfaces, having *D* as their centre. Such **a continuous surface, at every point of which the air is in the same stage of its vibration at the same moment, is called a wave.** In the case of the waves produced by a piston in a tube, the waves were planes transverse to the tube: in the open air, at a distance from the source, they are generally roughly spherical, as far as they are complete. The form of these waves must not be confused with, or called, the wave-form, a term which, as explained above, is used in an entirely different sense.

We may now define a *wave-length* rather more accurately as **the distance, measured at right angles to the waves, between two consecutive wave-fronts in which the air is at the same stage of its movement.** If the movement is harmonic, the wave-length is the distance between two surfaces in which the air is in the same *phase*.

23. Law of Inverse Squares.—If, through any point in the air, we suppose an imaginary surface drawn at right angles to the line along which the air is vibrating, then the amount of energy which passes, in each second, through a square centimetre of this surface round the point, is called the *intensity of sound* at that point. It is not usual to measure it in absolute units, but only to compare the intensities of the same

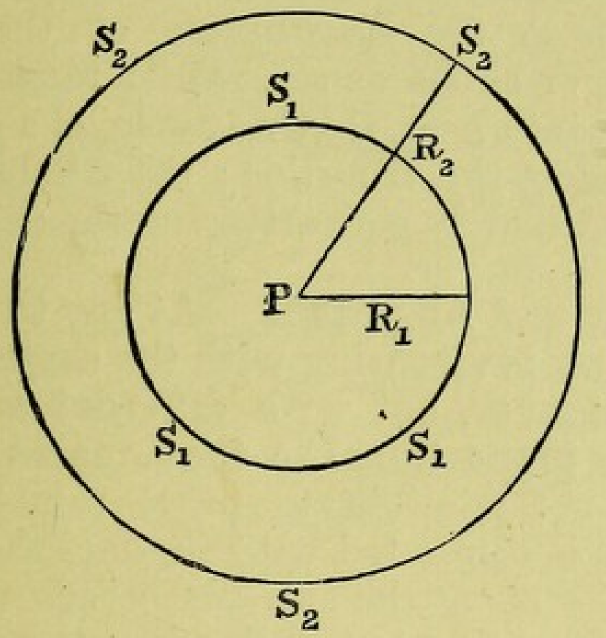

Fig. 23.

sound at different places. **If sound is travelling outwards in all directions from a source P, the intensities at two points S_1, S_2 must be inversely as the squares of their distances from P.** For, suppose imaginary spherical surfaces drawn through S_1, S_2 (Fig. 23). Then, if R_1, R_2 are the distances o S_1, S_2 from P, the areas of these spherical surfaces are as R to R_2^2. And the sound energy which starts from P in (say) a second, takes a second in passing $S_1S_1S_1$, and also a second in passing S_2 S_2 S_2. Therefore, the amounts of energy which pass in a second through *one square centimetre* of $S_1S_1S_1$ and $S_2S_2S_2$ respectively must be inversely as the areas of these surfaces, that is, as $\frac{1}{R_1^2}$ is to $\frac{1}{R_2^2}$.

In this proof we have assumed that all the sound-energy which starts from P passes $S_1S_1S_1$ and $S_2S_2S_2$ unchanged in form. This is nearly true; the energy of each shell is passed on to the next almost unaltered in total amount for a very long distance, and though a very little remains behind (in the form of heat) in every part of the air through which the wave has passed, yet, many miles from the source, by far the greater part of it is still travelling on. It is because of the diminution of the intensity of the vibration, due to the distribution of the energy over larger and larger shells of air, rather than because it has been converted into heat, that we cease, at a great distance, to be able to detect the vibration; the whole of the energy is, however, untimately converted into heat.

24. Intensity and Amplitude.—At any two points in the air, where the air is vibrating with the same frequency and in the same manner, but with different amplitudes, the intensities are proportional to the squares of the amplitudes. It is not possible to prove this by elementary methods, but it is connected with the fact that the average velocities of the air at the two places are proportional to these amplitudes (since at each place the air moves four times its amplitude in each vibration, and the number of vibrations per second is the same), and that the average kinetic energies of equal masses, moving similarly, are proportional to the squares of their average velocities.

Since *when sound spreads in all directions* the intensity varies inversely as the square of the distance from the source, and since, whether the sound spreads in all directions or not, the intensity is directly proportional to the square of the amplitude, it follows that **when sound spreads in all directions the amplitude of vibration of the air must vary inversely as the distance from the source**, so that at 100 yards from a bell the air is moving backwards and forwards half as far as at 50 yards.

The circumstances which determine the intensity of sound at a point in the air can be stated in various ways, which sometimes are not obviously equivalent. For instance, we may state the intensity at a point P entirely in terms of what is going on at that point. It would be a complete statement to say that it depended on the frequency and amplitude of the vibration of the air at the point P, and the density of that air, for if these were given we could calculate the intensity at P without any other data. So that if we give these as the circumstances on which the intensity depends, we must not include any others, such as the distance from the source. But we could say, with equal correctness, that the intensity at the point P depends on the frequency and amplitude of the vibrating source, on the density of the air in which the source vibrates and the dimensions of the vibrating surface in contact with it, on the distance of the point P from the source, and on whether the sound has spread equally in all directions or not. This again would be a complete statement, and we cannot include in it any other conditions, such as those which were included in the other statement, without making it incorrect. There are many other ways in which we could make a complete statement different from either of these, the essential point being that the statement must include sufficient data to calculate the intensity, and no more.

The illustrations given to explain the nature of progressive undulation, being chosen because of the obvious movement of the undulating substance, are liable to suggest an exaggerated idea of the extent of the movements of air conveying sound. There is no theoretical limit either to the largeness or smallness of the movements which can be passed on through the air, all are passed on in the same way, and (unless *very* large) with practically the same velocity. But the backward and forward swingings of the air by which ordinary conversation is transmitted to us are of very small extent, usually less than a thousandth of an inch, and often less than a millionth. Thus, though the movement of the air is reversed hundreds or thousands of times per second, its velocity, even at its quickest moments, may be very small; the sound of a whistle is quite audible at a place where the air at it quickest moment is not moving as fast as 2 inches an hour. The velocity with which the conditions travel may thus be millions of times as great as the velocity of the movements of the matter which transmits them. The difference in density between the condensations and the rarefactions is correspondingly small.

25. Doppler's Principle.—The velocity with which a pulse travels *relatively to the air*, when it has once been produced, is the same whether the vibrating body which produced it was at rest in the air or moving through it, and the same whether the air is moving relatively to the earth or not. Suppose that a source of sound, say a whistle, produces n condensations per second, and let a be the velocity with which it is moving, relatively to the air, towards an observer. (If it is not moving directly towards the observer, a is to mean the component, in the direction towards the observer, of the velocity, so that if it is moving directly away from the observer, a will be the speed with a negative sign.) Let V centimetres per second be the velocity with which the waves travel relatively to the air. When the whistle begins to sound, a condensation starts away from it in every direction with a velocity V; at the end of $1/n$ second, when the whistle is just producing the next condensation, the first has travelled V/n centimetres while the whistle has travelled a/n towards the observer, and the part of the first which travels towards the observer has therefore a start of $(V-a)/n$ centimetres in front of the part of the second which travels the same way. This is therefore the length of the waves travelling towards the observer.

Let b be the velocity with which the observer is moving relatively to the air, away from the source, reckoned in the same way as a. Then a total length $V-b$ of successive waves will pass the observer in each second; as the length of each wave is $(V-a)/n$, the number of waves which pass the observer per second is $(V-b) \div \{(V-a)/n\}$ or

$$n(V-b)/(V-a).$$

From this expression we easily deduce :—

(1) If the source and the observer move the same way with the same velocity, the frequency of the waves received is the same as that of the waves sent out.

(2) If the source and observer approach (*i.e.* if $a > b$) the frequency of the waves received is greater than that

of the waves sent out, and *vice versâ*, but in this case the frequency of arrival depends on the velocities of both the source and observer relatively to the air and not simply on the rate of approach $a - b$. The change of frequency produced by a certain velocity of the source relatively to the air is not the same as that produced by an equal velocity of the observer relatively to the air.

If a and b are very small compared with V,

$$(V-b)/(V-a) = 1 + (a-b)/V$$

nearly, so that the frequency of arrival is in this case nearly the same whether it is the source or the observer which is moving relatively to the air. The reason of the alteration of frequency is, however, quite different in the two cases; in one the length of the waves is altered, while in the other nothing is altered but the rate at which they pass the observer.

(3) The ratio of the frequencies of the waves received by an observer before and after passing a source at rest relatively to the air, is the same as the ratio before and after a source, moving with the same speed, passes an observer at rest, though the actual frequencies are different.

We shall see (Art. 60) that the frequency with which waves arrive at the ear determines the pitch of the sound heard, greater frequency corresponding to higher pitch, and that the ratio of two frequencies determines the musical interval between the sounds (Art. 66). So that if our distance from a source of sound is increasing, we hear a sound of lower pitch, and *vice versâ*. The change in the pitch of the sound heard when a whistling locomotive, which has been approaching us, passes and begins to recede, is a familiar instance of this. From (3) we see that the musical interval between the notes heard is the same for the same speed, whether it is the whistle or the observer that is moving relatively to the air, though the notes themselves are different.

The altered pitch due to movement of the observer relatively to the air begins and ends with the movement, but that due to movement of the source does not begin till the waves, which left the source as its motion began, have reached the observer.

Movement of the air relatively to the earth makes no difference to the formulae given above provided that a and b are measured relatively to the air, but if we wish to state the frequency of arrival of the waves in terms of the velocities of source and observer relatively to the earth, the formulae are different. Let a', b', w, be the velocities of source, observer, and wind, *relatively to the earth*, in the direction from source to observer (or their components in this direction), while V is the velocity of sound *relatively to the air*. Then $a' - w$, $b' - w$, are the velocities relatively to the air, and must be substituted for a and b. Hence

$$\text{frequency of arrival} = n\left(\frac{V - b' + w}{V - a' + w}\right).$$

From this we see that wind, in the same direction as the sound, lessens the change of pitch due to a given speed of source or observer, and *vice versa*. It comes to the same thing as a change in the speed of the sound.

The fact that the frequency of the arrival of waves at a point is affected by movements of the point, or of the source of the waves, is called *Doppler's Principle.*

26. Sound Waves in Liquids and Solids.—So far we have spoken only of sound waves travelling in air. But condensations and rarefactions are transmitted through liquids and solids in exactly the same way, by exactly similar movements. If one end of a rod of wood or metal is pushed or pulled, successive layers move, and are compressed or rarified, in turn, just like the layers of air in a tube, and if any point of a solid block is made to vibrate, condensation and rarefaction spread through it in widening spheres, as they do through air.

It must not be supposed that this handing-on of condensations and rarefactions by successive movements of different portions is a process specially connected with audible sound; it is the only way in which a pull or a push can be transmitted at all. When a signalman moves a distant signal by pulling at a handle, which is connected with the signal by a rod or wire, the movement of the signal appears to be simultaneous with the movement of the lever, but this is only because of the great velocity with which pulses travel in iron, not because any pull or push can be transmitted in a different way or with a different velocity from sound. When the man begins to move the lever, he produces an elongation or rarefaction of a piece of the wire at his end, and this state of rarefaction travels along the wire at the rate of about three miles a second, and it is only when the rarefaction reaches the other end of the wire that the signal begins to move.

26a. Experimental Verification.—From the nature of the case, there is no absolutely direct evidence that the air in sound-waves moves, and alters in density, in the way described in this chapter. That there is no continuous movement of the air may be inferred from the fact that the sound from any source passes through a cloud of smoke without causing any visible movement of the smoke. That the air does move backwards and forwards seems proved by the fact that a light membrane in it moves backwards and forwards. That the movement of the air is in a line passing through the source, seems proved by the fact that an acoustic pendulum is very little or not at all affected when it is edgewise to the source. That the passage of sound waves through the air is accompanied by changes of pressure at any point is shown by the fact that if a hole, cut in one side of a tube, has a thin membrane stretched over it, against which a pith-ball hangs, sound waves sent along the tube make the pith-ball dance; a mere movement of the air along the tube, without any change of pressure, could not produce a movement of the membrane, since the air moves parallel to the membrane. There is no experiment to show that changes of density occur, but it is impossible to alter the pressure of a gas suddenly without changing its density, and the changes of pressure must be sudden, since they make the membrane vibrate hundreds of times a second. Also it can be shown (by acoustic pendulums in a long tube) that the air near the source moves sooner than that at a distance, and it is obvious that if the air in one part of a tube moves in the direction of the length of the tube, while the air in another part does not, the air between must change in density.

CHAPTER III.

VELOCITY OF SOUND.

27. Velocity of a Pulse along a Rod.—If we take a long uniform rod AB of any elastic substance, and apply at one end A a force of f dynamical units, directed towards B, a portion of the rod close to A shortens, till it exerts a force f on the next portion, and so the compressed condition extends along the rod, each original unit length of the rod shortening by a certain amount l. If m is the mass of a piece of the rod of unit length, it can be shown (Appendix D) that the velocity with which the compressed condition extends along the rod is $\sqrt{\frac{f}{lm}}$. This is true whatever the circumstances under which the compression takes place, and whether the "rod" is solid or fluid, but only if f is measured in the *dynamical* units based on the units of length, mass, and time employed in the other measurements. Thus, if l and m are in centimetres and grammes, f must be in *dynes*, and the velocity will be in centimetres per second; or, if l and m are in feet and pounds, f must be in *poundals*.

For moderate forces, l is proportional to f, so that $\sqrt{\frac{f}{lm}}$ is the same whether the force is smaller or larger, if not very large. Also, if we double the sectional area of the rod, we require twice the force to produce the same change in the length of a centimetre, but a centimetre has twice the mass, so that $\sqrt{\frac{f}{lm}}$ is the same for a thick rod as for a thin one of the same material. We do not need, therefore, to know either the force applied or the diameter of the rod to calculate the velocity with which the condensed condition extends along the rod, for it would be the same for any diameter and any force. If we suppose the rod of unit sectional area, f is numerically equal to the pressure* applied, and if a rod of unit area is free to expand sideways, $\frac{f}{l}$ is equal to the quantity called Young's Modulus for the substance of the rod, which is the ratio of a change of stress applied to the ends of a rod of that substance to the change it produces in each unit of length of the rod, when the rod is free to expand sideways. Also, if the rod is of unit sectional area, each unit of length of it contains unit volume, and m is numerically equal to the density of the substance. So that for such a rod, and therefore for any rod,

$$\text{velocity} = \sqrt{\frac{\text{Young's Modulus}}{\text{density}}}.$$

If the force at A is a pull instead of a push, the velocity is still $\sqrt{\frac{f}{lm}}$ where l is the *increase* in length of each centimetre produced by the force f. If f is small, this increase is equal to the decrease produced by an equal push, so that the stretched or rarefied condition extends at the same rate as the condensed condition.

The change of length produced by a given *change* of stress is nearly the same whatever the actual stress may be, and the density of a solid is only slightly altered by any ordinary stress. The velocity of a condensation or rarefaction along a rod or wire, therefore, depends hardly at all on whether the rod or wire is stretched or not.

28. Velocity in a Fluid in a Tube.—If instead of a solid rod we had a fluid column contained in a tube, and applied a force of f dynes to a piston at one end, the condensation

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* In this book, "pressure" always means stress, *i.e.*, the ratio of the force to the area on which it is exerted. Some writers call the force itself the "pressure," and the ratio of force to area the "intensity of pressure."

would extend along the tube with a velocity $\sqrt{\frac{f}{lm}}$ just as before. If our tube were of unit sectional area, we should be applying an increase of pressure f not only to the end, but to every side, of the condensed portion of the fluid, and l would be not only the shortening in each unit length, but also the diminution in each unit of volume of the fluid column. So $\frac{f}{l}$ would be the ratio of a change of hydrostatic pressure to the change produced by it in each unit volume of a fluid; a quantity which is called the *volume elasticity** of the fluid. The velocity V with which a condensed or rarefied condition spreads along a liquid or gaseous column is therefore $\sqrt{\frac{\text{volume elasticity}}{\text{density}}}$.

29. The two ways of measuring Elasticity.—The volume elasticity of a fluid depends on the conditions under which the change of volume takes place. If we could have a cubic centimetre of air confined under a piston C, Fig. 24, in a tube AB of unit sectional area, both piston and tube being of very low thermal conductivity, and applied to the piston a very small downward force of f dynes, the piston would instantly descend, diminishing the volume of the air by an amount x.

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* Strictly, Young's Modulus is defined not as the ratio of change of stress to change produced in unit length, but as the limit which this ratio approaches when the change of stress is indefinitely diminished; and, similarly, the volume elasticity is the limit of the ratio of change of hydrostatic pressure to change produced in each unit of volume, when the change of pressure is indefinitely diminished. So that the formulæ $V = \sqrt{\frac{\text{Young's Modulus}}{\text{density}}}$ and $V = \sqrt{\frac{\text{volume elasticity}}{\text{density}}}$ are only absolutely true for indefinitely small changes of stress. But, even for much larger changes of stress than those which occur in sound waves, the ratio of change of stress to change of length or volume is practically the same as for indefinitely small changes of stress.

At the same time a quantity of heat equal to the work done in the descent of the piston would be produced in the compressed air, which would raise its temperature. As the air cooled to its original temperature, the piston would descend

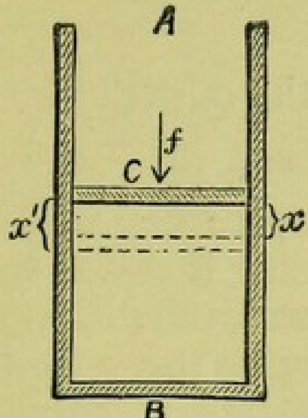

Fig. 24.

further, till the volume of the air had been diminished altogether by a volume x'. Similarly, if we applied an upward pull f to the piston, it would instantly rise till the volume of the air under it had been increased by x, and the temperature of this air would fall; then, as the air returned to its original temperature, the piston would rise still further till the total increase of volume was x'. The ratio $\frac{f}{x}$ is called the *adiabatic elasticity* (x being measured before any heat has entered or left the air), and $\frac{f}{x'}$ is called the *isothermal elasticity*. As x' is greater than x, the isothermal elasticity is smaller than the adiabatic.

The experiment as described above is impracticable, as the gas loses or gains heat much too quickly to allow of determining x, but when a pulse travels along a tube, the air in each centimetre of the tube changes from its original to its altered volume in less than $\frac{1}{30000}$ of a second, which would be too short a time for it to lose or gain much heat, even if air were a good conductor and radiator instead of an extremely bad one, so that the change of each centimetre length of the air column is the same as the instantaneous movement which would be produced by the same force applied to the piston of our imaginary experiment. The quantity $\sqrt{\frac{f}{lm}}$ in such a case is therefore equal to $\sqrt{\frac{\text{adiabatic elasticity}}{\text{density}}}$. The same is true for a liquid.

It will be seen that the quantity $\sqrt{\frac{f}{lm}}$ is greater in the case either of a rarefaction or a condensation than it would be if no change of temperature took place. This is sometimes expressed by saying that the rise of temperature produced by condensation and the fall of temperature produced by rarefaction both increase the velocity of the sound. That is not a good way of putting it, because it suggests that if these changes of temperature did not occur the sound would travel with the smaller velocity $\sqrt{\frac{\text{isothermal elasticity}}{\text{density}}}$; in reality it would not travel at all. Indeed, we shall see later that the change of temperature and the movement of the air are different ways of expressing the same fact. Newton, who was the first to calculate the velocity of sound from dynamical principles, did not know of the changes of temperature, but supposed that the velocity would be equal to

$$\sqrt{\frac{\text{isothermal elasticity}}{\text{density}}};$$

it was Laplace who first pointed out that the adiabatic elasticity should be used.

Similarly, in the formula $V = \sqrt{\frac{f}{lm}}$ for the velocity of pulses along a rod, l should strictly be the change in unit length *instantly* produced by a force f, before any heat has left or entered, and therefore

$$V = \sqrt{\frac{\text{Young's Modulus}}{\text{density}}}$$

is not exactly true unless the adiabatic value of Young's Modulus is taken. But in solids and liquids the difference between adiabatic and isothermal changes is very small.

30. Indirect Formulæ for Velocity.—There is no simple method of measuring the adiabatic elasticity directly, but it can be shown theoretically that it bears a constant ratio to the isothermal elasticity, and that this ratio is the same as that of the specific heat at constant pressure to the specific heat at constant volume (*Text-book of Heat*, 42), a ratio usually denoted by γ. So

$$V = \sqrt{\frac{\gamma \times \text{isothermal elasticity}}{\text{density}}}.$$

This is true for all substances, and as the isothermal elasticity can be directly measured, and γ can be calculated, by thermodynamic methods, from the specific heat at constant pressure and the coefficient of expansion, this formula can be used to find V. For most *gases* a simpler formula is very nearly true. If any gas exactly obeyed Boyle's law, it can be shown that its isothermal elasticity would be equal to its pressure, and this is very nearly the case with all gases which are far from their condensing temperatures. So that for such gases

$$V = \sqrt{\frac{\gamma P}{D}} \text{ nearly.}$$

The value of γ depends on the number of atoms in a molecule of the gas: for one-atom molecules, like mercury gas, it is 1·66; for two-atom molecules, like oxygen and nitrogen, it is 1·41; and for three-atom molecules, like water vapour, it is about 1·32.

The velocity of sound really depends, of course, on the changes which actually take place in the air as the wave travels through it, not on the changes which would take place under quite different circumstances. The isothermal elasticity and the specific heats have therefore no direct connection with the velocity, for the air is neither compressed without change of temperature nor heated without change of pressure, nor heated without change of volume, when a sound wave passes through it. But the physical properties of gas on which the velocity of sound depends (the chief of which, as we shall shortly see, is the velocity of its molecules) are also factors in determining its elasticities and specific heats, and therefore it is quite possible to express the velocity of sound in terms of these quantities.

31. Velocity in Free Air.—The velocity of sound along a rod which is free to expand sideways is much less than it is when the transverse expansion is prevented, the ratio $\frac{f}{l}$ being much larger in the latter case. In a column of air which was free to expand sideways sound would not be propagated at all, as in that case the ratio $\frac{f}{l}$ would be zero. But in open air the velocity, except within a very short distance from the source, is the same as along the air in a tube. For as the condensed condition travels outwards in all directions from the source, the condensed air does not move except in the line of propagation of the sound, since in all directions at right angles to this line it is in contact with air condensed as much as itself. The velocity with which the condensation travels is therefore the same as if transverse expansion were prevented by a rigid tube. For the same reason the velocity of a sound wave through a large mass of a solid substance, a cliff for instance, is much greater than along a rod.

32. Velocity Independent of Pressure.—Let D be the density of a gas at 0° C. and any standard pressure H dynes per sq. cm.; then $\frac{D}{H}$ is a constant for the gas, and does not depend on what the pressure H was at which D was measured. We will denote $\frac{D}{H}$ by d; it is the density which the gas would have at 0° C. and a pressure of 1 dyne per sq. cm. Then the density of the gas at a pressure P dynes and a temperature t° C. is $Pd\left(\frac{273}{273+t}\right)$; see *Text-book of Heat*, 32. So the velocity of sound in the gas at this pressure and temperature is

$$\sqrt{\frac{\gamma P}{Pd\left(\frac{273}{273+t}\right)}} \quad \text{or} \quad \sqrt{\frac{\gamma\,(273+t)}{273d}}.$$

It is therefore *independent of the pressure*, a fact which may be explained by saying that the adiabatic elasticity and the density are both proportional to the pressure, and that therefore their ratio, on which the velocity depends (29), is the same for all pressures; but this is a mathematical rather than a physical explanation, and the physical reason will appear later.

In the formulæ just given it must be noticed that d is not the density of the gas actually conveying the sound, but the density it would have at 0° C. and at a pressure of 1 dyne, or the constant ratio of its density to its pressure at 0° C.

33. Velocity and Temperature.—If V_1, V_2 are the velocities of sound in the *same* gas at two different temperatures t_1, t_2, we have

$$\frac{V_1}{V_2} = \frac{\sqrt{\frac{\gamma\,(273+t_1)}{273d}}}{\sqrt{\frac{\gamma\,(273+t_2)}{273d}}} = \sqrt{\frac{273+t_1}{273+t_2}},$$

or the ratio of the velocities is the square root of the ratio of the absolute air-thermometer temperatures. (*Text-book of Heat*, 35.) This is true whether the pressures are the same or not.

34. Velocities in different Gases.—If V_x, V_y are the velocities of sound *at the same temperature* in two different gases, X and Y, *which have the same number of atoms to the molecule*, and therefore the same value of γ

$$\frac{V_x}{V_y} = \frac{\sqrt{\frac{\gamma\,(273+t)}{273d_x}}}{\sqrt{\frac{\gamma\,(273+t)}{273d_y}}} = \sqrt{\frac{d_y}{d_x}},$$

or the velocities are inversely as the square roots of the densities of the gases at 0° C. and 1 dyne pressure, and therefore inversely as the square roots of the densities at any pressure and temperature (the same for both). These densities are proportional to the atomic weights, so the velocities are inversely proportional to the square roots of the atomic weights. This is true whether the pressures are the same or not.

For instance, if X is hydrogen and Y oxygen,

$$\frac{V_x}{V_y} = \sqrt{\frac{16}{1}} = 4,$$

so that sound travels four times as fast in hydrogen as in oxygen at the same temperature.

Sound travels more quickly in water vapour than in air, for, though γ is smaller for water vapour, d is also smaller, and $\frac{\gamma}{d}$ is greater for water vapour. The presence of water vapour in the air therefore increases the velocity of sound slightly.

The density of dry air at 0° C. and a million dynes per sq. cm. pressure (a "c.g.s. atmosphere") is ·0012759, so that d for air is ·0000000012759. The velocity of sound in air at any temperature t° C. is therefore

$$\sqrt{\frac{1{\cdot}41(273+t)}{273\,({\cdot}000000001275)}},$$

or $\sqrt{\frac{1.41}{{\cdot}000000001275}\left(1+\frac{t}{273}\right)}$, or $33{,}240\sqrt{1+\frac{t}{273}}$.

If t is small, $\sqrt{1+\frac{t}{273}} = 1+\frac{t}{546}$ nearly, and the velocity is $33{,}240\left(1+\frac{t}{546}\right)$, or $33{,}240+60t$ approximately. This agrees very closely with experimental determinations.

35. Experimental Determination of Velocity.—The velocity of sound can be determined by direct experiment; this is one of the most accurate methods of finding the adiabatic elasticity, which can be calculated from the formula given for V if V is known independently. A simple method of finding the velocity is for two observers, at a distance of some miles, each to fire a cannon, as nearly as possible at the same time, and for each to notice the interval between seeing the flash and hearing the report of the distant cannon. The intervals will not be equal unless the night is absolutely windless, for the sound travels relatively to the air with the same velocity whether the air is moving relatively to the earth or not, so that the velocity of the sound relative to the earth, which is what is measured in the experiment described, is greater than the velocity relative to the air when the sound travels in the direction of the wind, and smaller in the opposite direction. The average of the two velocities found by dividing the distance between the observers by the observed intervals will be free from this error.

Another error arises from the fact that we do not perceive either the flash or the report exactly at the moment when the light and sound reach us, the processes of perception requiring appreciable times, which are unequal for sight and hearing. This can be avoided in several ways, of which the best is perhaps to dispense with the observer altogether, and let an electric current which fires the cannon mark the moment of its occurrence on a revolving cylinder, while the arrival of the sound at the distant station makes another mark on the same cylinder by setting in vibration an acoustic pendulum (1). For this purpose the membrane and pith-ball are gilt to make them conducting, and form part of an electric circuit; the breaking of this circuit on the arrival of the sound releases the armature of an electro-magnet, which marks the cylinder.

By this and similar methods the time taken by the sound in travelling from one station to the other can be very exactly ascertained, but as it is not possible to determine at all accurately the average temperature of the air through which the sound passed, and as the velocity depends greatly on temperature, there is not much value in such determinations.

36. Relation of Velocity to Molecular Structure.—So far we have considered the air as a continuous elastic substance, of which every portion exerts a pressure on all the surrounding portions. This is not an incorrect way of regarding the matter, but it is rather a superficial one, for the transmission of sound through a substance is closely connected with its molecular structure. It is well known that air, for instance, is not a continuous substance, but consists of very minute detached bodies, called molecules, of which there are, very roughly, a million billions in a cubic centimetre. These fly about in every direction with an average velocity which depends on the temperature (more accurately, perhaps, the temperature depends on the average velocity), but which at ordinary temperatures is about 50,000 cm. (1,700 feet) per second. Though the space actually filled by the molecules is certainly less, and probably much less, than $\frac{1}{1200}$ of the whole space, the number of molecules is so great that the average distance traversed by a molecule before coming into collision with another is only $\frac{1}{200000}$ of a centimetre, and, though there is of course no regularity in these collisions, it must be a very rare thing for a molecule to move $\frac{1}{20000}$ of a centimetre ($\frac{1}{100}$ of a hair's-breadth) without an encounter which quite changes its direction. Thus, though the molecules move as fast as cannon-balls, they do not make rapid continuous progress, and most of those which are in a particular cubic centimetre of the air at one moment would still be found in a compact group a second or two later, though the position of this group, relatively to the earth, may have changed considerably. A "particle" of air is thus like a swarm of bees, which may be at rest, or moving in a constant direction, though every bee is moving much faster, and continually changing its direction. Where we have spoken in the preceding chapter of the velocity of the air, it is this swarm-velocity that is meant, and it is equal to the average velocity of the molecules reckoned algebraically, velocities in one direction being counted as of opposite sign to those in the other.

To simplify the explanation of the way in which the transmission of sound is related to molecular movements, we will suppose that all the molecules have the same mass (which is not the case in a mixed gas like air), and have the same velocity, which we will take as 50,000 cm. per second. We will also suppose that they all move in horizontal lines running east and west, and that all the collisions which happen are direct, so that two molecules still move after collision in the same line as before, and that the molecules behave like perfectly elastic equal balls in direct collision, so that each moves after the collision in the same direction and with the same velocity as the other did before collision. Suppose a solid plate in the air moving from east to west, with a velocity of 10 cm. per second. Then all the molecules on the west side of the plate which strike it do so with a velocity, *relative to the plate*, of 50,010 cm., and rebound with the same relative velocity; that is, they move westwards with a velocity of 50,020 cm. Each of these soon encounters a molecule coming eastwards at 50,000 cm. per second, and sends this molecule off westwards at 50,020 cm. per second, while it returns eastwards at 50,000 cm. per second, till it meets the plate again, and so on. Meanwhile the molecule to which it has passed on its velocity passes it on to one still further west, and returns eastwards at 50,000 cm. per second. Thus in the air near the plate all the eastward-moving molecules have a velocity of 50,000 cm. per second, while the westward-moving ones have a velocity of 50,020 cm. per second, and this condition of the air is spreading westwards with the velocity of *the molecules themselves.* The swarm-velocity of this air is the average of +50,020 and −50,000, or 10 cm. per second, the same as that of the plate, and it is in the same direction as the condition is spreading. It is a condensed condition, for there is all the air between the plate and the furthest point to which the condition has extended westwards that there would have been if the plate had been at rest, and the distance is smaller than it would have been in that case. Also it is a condition of raised temperature, for the relative velocity of the molecules moving eastwards to those moving westwards is 100,020, while in the rest of the air it is 100,000, and the temperature of a gas is an expression for the relative velocity of its molecules among themselves. Thus all that we have previously learnt about a travelling condensation is seen to be a natural result of the molecular constitution of the air. It will be obvious how the description must be altered to make it apply to a rarefaction.

Roughly speaking, then, the velocity of sound in a gas is the velocity of the molecules, but in an actual gas the condensed or rarefied condition spreads outwards only about two-thirds as fast as the average velocity of the molecules. This is chiefly because most of the collisions which happen are oblique, so that each molecule rebounds in a different direction to that in which the other one was going before the collision, and thus the extra velocity communicated to one molecule by the moving plate is handed on in a zig-zag manner. There are also other reasons, which cannot be explained satisfactorily in an elementary book.

The reason why the velocity of sound is independent of pressure may now be more clearly explained. The temperature of a gas is an expression for (though not simply proportional to) the average kinetic energy of each of the molecules, and this depends on their velocity and mass, not on their number. So that for any one gas the velocity of its molecules, and therefore the velocity of sound in it, is always the same at the same temperature, however other conditions may vary.

It will now be seen that the two statements that "the air in a condensed region is moving in the same direction as the waves," and that "the air in a condensed region is at a higher temperature than the undisturbed air," are two ways, each incomplete, of stating the fact that in a condensed region the molecules which are moving in the direction of the sound are moving with more than usual velocity, while those which are moving in the opposite direction have their ordinary velocity. It is only this particular kind of molecular movement which transfers itself from one part of the air to another, so that raised temperature is an essential part of the condition of travelling condensation, and not merely a

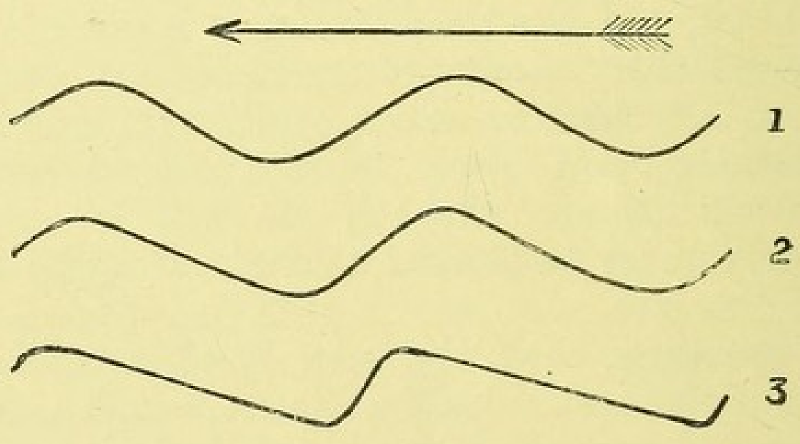

Fig. 25.

cause increasing the velocity with which it travels. The same is of course true of the lowered temperature of a travelling rarefaction.

From the molecular point of view it seems as if the condensed condition ought to travel a little faster than the rarefied one, for each condition is handed on by the molecules which travel in the same direction as the condition, and these have more than the average velocity in a condensation, less in a rarefaction. The formula of Art. 33 suggests the same thing, since the condensations are at a higher temperature than the rarefactions. The matter is not nearly so simple as it appears, and the real explanation cannot be given here, but it is a fact that condensation does travel faster than rarefaction. So that, in a wave-system, the wave-form gradually changes as the waves get further from the source, just as sea-waves change in form when, reaching shallow water, the crests travel perceptibly faster than the troughs. Some stages in the transformation of waves whose wave-form was originally harmonic are shown in Fig. 25; the waves, like those in previous diagrams, are supposed to be travelling from right to left. The greater the degree of condensation and rarefaction the greater the difference in the velocities, and the more rapid the change in wave-form.

CALCULATIONS.

The most important formulæ for the velocity of sound are—

(1) $$V = \sqrt{\frac{1 \cdot 41\,(273 + t)}{273d}} \qquad \text{(Art. 32)},$$

where t is the centigrade temperature, and d the quotient obtained by dividing the density of the gas at 0° C. and any pressure by that pressure expressed in dynes per sq. cm.

(2) For the same gas at different temperatures, t_1, t_2,

$$\frac{V_1}{V_2} = \sqrt{\frac{273 + t_1}{273 + t_2}} \qquad \text{(Art. 33)}.$$

(3) For different gases, both having the same number of atoms to the molecule, at the same temperature,

$$\frac{V_x}{V_y} = \sqrt{\frac{\text{atomic weight of } y}{\text{atomic weight of } x}} \qquad \text{(Art. 34)}.$$

EXAMPLES I.

Elementary.

1. At what temperature is the velocity of sound in nitrogen (atomic weight, 14) equal to its velocity in oxygen at 10° C. (atomic weight, 16)?

2. At what temperature is the velocity of sound in air twice as great as in air at 20° C.?

3. The velocity of sound in air is 34,000 centimetres per second when the thermometer is at 13° C. and the barometer at 78 centimetres. What will be the velocity if the thermometer rises to 22° and the barometer falls to 72 centimetres?

4. The velocity of sound along a tube full of air is found to be 1100 feet per second at 0°. What will be its velocity in the same tube (1) if the air in the tube is raised to 20° without letting any escape from the tube; (2) if the air in the tube is raised to 20°, and the pressure kept constant?

Advanced.

5. A whistle makes 200 waves a second. What number of waves will a person receive per second (*a*) if the whistle comes towards him with the velocity of sound; (*b*) if the whistle goes away from him with the velocity of sound; (*c*) if he goes towards the whistle with the velocity of sound; (*d*) if he goes away from the whistle with the velocity of sound?

6. A cylindrical rod is 2 metres long when laid down, and 2 centimetres in diameter, and weighs 2000 grams. When it stands upright, with one end on the ground, it is $\frac{1}{10}$ millimetre shorter than when laid down. What is the velocity of sound in the substance of the rod?

7. Calculate the velocity of sound in hydrogen at 20° C. and 1,100,000 dynes pressure, given that the density of hydrogen at 0° and 1,000,000 dynes pressure is ·0000884, and that the ratio of the specific heats of hydrogen is 1·408.

8. Given that the velocity of sound in air at 0° is 33,240 cm. per second, and that the density of air at 0° and a pressure of 10^6 dynes is ·001275, find the adiabatic elasticity of air at a pressure of 10^6 dynes.

CHAPTER IV.

INTERFERENCE.

37. Principle of Interference.—Suppose that we have two sources of sound waves, two tuning-forks, for instance; we will call them *A* and *B*. Consider a region of air *C*, so situated that waves of quite small amplitude pass through it if either *A* or *B* vibrates alone. Let *A* and *B* be vibrating together, but suppose that we know what the condensation and velocity of the air would be at each point of the region *C*, at a given moment *T*, if *A* was vibrating as it actually is, but *B* was not vibrating, and that we also know what the condensation and velocity at each point would be, at the same moment *T*, if *B* was vibrating as it actually is, but *A* was not vibrating. Then it can be shown from the principles of dynamics that the actual condensation and velocity of the air at each point of the region *C*, at the given moment *T*, are very nearly the same as would be found by adding the condensation and velocity which there would be at that point if only *A* was vibrating, and the condensation and velocity which there would be if only *B* was vibrating.

In this addition, rarefaction and condensation must be counted of opposite signs and added algebraically, and velocities must be added in the only way in which quantities which have direction as well as magnitude can be added, by finding the diagonal of the parallelogram whose sides represent in magnitude and direction the velocities whose sum is required.

Similarly, the displacement of any particle of the air from its mean position is the same as would be found by adding the displacement which it would have if *A* was vibrating alone and the displacement which it would have if *B* was vibrating alone, displacements being added like velocities.

The principle that, when the condensation, velocity, and displacement of the air are small, they are equal to the sums of the condensations, velocities, and displacements which there would be if each of the sources was vibrating without the others, is called the principle of *interference*. It is a singularly ill-chosen name, because the fact which is especially to be noticed is that no interference occurs, but that each source of sound produces the same difference in the condition of the air that it would do if the other sources were absent. As Lord Rayleigh says, "if this is interference, it is difficult to see what non-interference would be." But the term is firmly established, and must be used.

In the ordinary physical sense of the terms, neither the condensations and rarefactions which *A* would produce by itself, nor those which *B* would produce by itself, exist in the region *C* when *A* and *B* vibrate together. For instance, at a point where there would be a certain degree of condensation if *A* was vibrating alone, and an equal degree of rarefaction if *B* was vibrating alone, the principle of interference shows that the air is at its average density, and is neither condensed nor rarefied. But the easiest way of finding the actual condition of the air in the region *C* is to *imagine* the condensations and rarefactions which there *would be* there if *A* was vibrating alone, and then those which there would be if *B* was vibrating alone, and then add these imaginary conditions to find the real condition of the air. It is usual, and convenient, to speak of these imaginary wave-systems as "existing" at the same time in the region *C*, and to call its actual condition their resultant. This should not cause any confusion, because there cannot, in any physical sense, be two wave-systems at once in the same air, any more than there can be two different winds at once at the same place; the same air cannot have two densities at once. But it is important to remember that, whenever we speak of two wave-systems in the same air, these wave-systems are imaginary, and that what really exists in the air is one wave-system different from either of the imaginary ones.

When the imaginary wave-systems in a given region travel in directions which cross each other, the real movements of the air are very complicated, each particle moving in curves something like Lissajous' figures. It is more important to consider the real condition of the air when the imaginary wave-systems travel along the same line, either in the same or in opposite directions.

38. Wave-systems in the same Direction.—Suppose, as before, that two bodies, A and B, are vibrating together. Let A and B be so situated that, if either of them vibrated alone, waves would travel in the same direction through a given region, say the direction XO (Fig. 26). Let $A'A'A'$

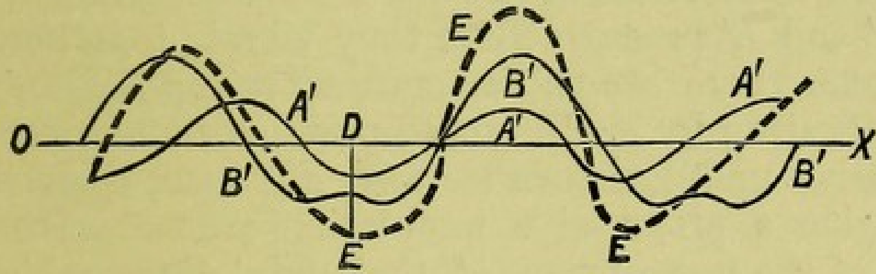

Fig. 26.

be the wave-form of the waves which would be travelling through the given region if A was vibrating alone; $B'B'B'$ that of the waves which would be travelling through the same region if B was vibrating alone. Then at any point, such as D, the actual condensation is proportional to the algebraic sum of the ordinates from D to $A'A'A'$ and $B'B'B'$, so that, if we draw a curve EEE whose ordinate at each point is the algebraic sum of the ordinates to $A'A'A'$ and $B'B'B'$, the ordinates of EEE will be proportional to the actual condensations.

Since the two wave-systems which there would be if A and B vibrated separately would both travel in the direction XO, ordinates to either curve above the axis OX indicate velocities which the air would have in the direction XO, and ordinates below the axis indicate velocities which the air would have in the direction OX. The actual velocity of the air at any point is therefore proportional to the algebraic sum of the ordinates to $A'A'A'$ and $B'B'B'$ at that point, *i.e.*, it is proportional to the ordinate to EEE, and it is a velocity in the direction XO where the ordinate to EEE is above the axis.

The actual condition of the air, then, is a condition in which both the condensation and the velocity are proportional to the ordinates to *EEE*, and are therefore proportional to each other. Now, this, as we have seen, is the distinctive feature of progressive undulation. The actual condition of the air, then, is a progressive wave-system, travelling in the direction *XO*, and having a wave-form *EEE*. A progressive system having this wave-form might be produced by suitable movements of a single body, such as the piston in Chapter II., and there would be no physical difference whatever between such a wave-system and the one which A' and B' produce when they vibrate together.

Whenever there are two or more vibrating bodies, each of which, vibrating by itself, would produce in a given region a progressive undulation *in the same direction*, there is in the given region a progressive undulation whose wave-form is the sum of the wave-forms of the undulations which there would be if the bodies vibrated separately, and such a progressive undulation bears no physical trace of having originated from several sources, but is exactly like an undulation which might be produced by a single vibrating body.

If, on the other hand, the undulations which the bodies would produce separately would travel in different directions through any region, the actual condition of the air in that region is one in which condensation and velocity are not proportional to each other, and is therefore a condition which could not be produced by any possible vibrations of a single source.

39. Harmonic Waves of Equal Length.—If the imaginary wave-systems (those which would actually exist if the different sources vibrated separately) are all harmonic, and all in the same direction, it does not follow that the actual wave-system is harmonic, for the sum of two harmonic curves is not always a harmonic curve (Art. 10). But it can be shown that the curve obtained by adding the ordinates of harmonic curves *whose bends are of the same length* is always a harmonic curve with bends of that length, whatever the relative positions of the bends. Since harmonic curves whose bends are of the same length are the wave-forms of harmonic wave-systems of equal wave-length, it follows that, where there are several imaginary harmonic wave-systems of *equal wave-length*, travelling in the same direction, the actual undulation is a harmonic one, and of that wave-length. There are several important special cases of this.

(1) If the imaginary wave-systems are in the same phase (that is, if the wave-system which would exist at a given moment if one source was vibrating alone has its maximum condensations in the same positions as the maximum condensations of the wave-system which would exist at the same moment if another source vibrated alone), the maximum ordinates of the actual wave-form are the sums of the maximum ordinates of the imaginary wave-systems, and the actual maximum velocity is therefore the sum of the imaginary maximum velocities. The amplitudes of harmonic vibrations of equal frequency are proportional to their maximum velocities (Appendix A), so that the actual amplitude of vibration is the sum of the amplitudes of the vibrations which the air would execute if the sources vibrated separately.

This case is shown in Fig. 27, where the continuous curves

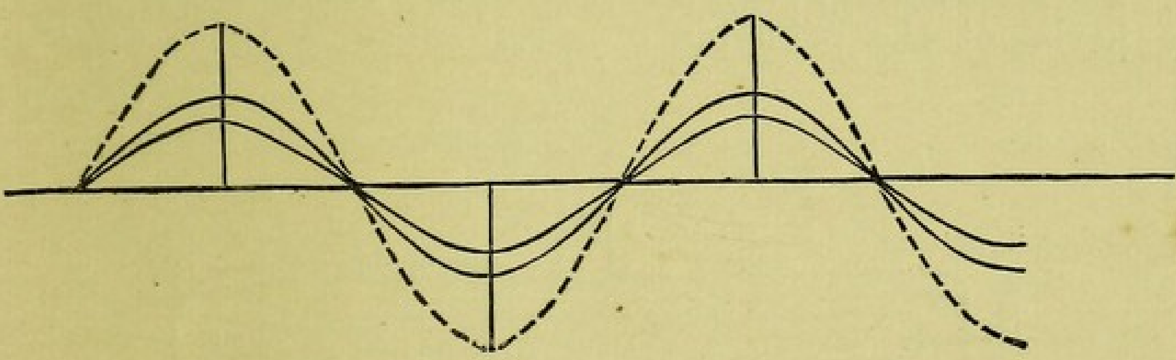

Fig. 27.

are the wave-forms of two imaginary wave-systems in the same phase, and the dotted curve, obtained by adding the ordinates of the others, is the wave-form of the actual condition of the air.

(2) If the imaginary wave-systems are in opposite phases (that is, if the maximum condensations of one imaginary system coincide with the maximum rarefactions of the other), the actual amplitude of the vibrating air is the difference of

the amplitudes which it would have if the sources vibrated separately. (Fig. 28.)

(3) In case 2, if the imaginary systems are equal, the real wave-form is a straight line; the air is of uniform density,

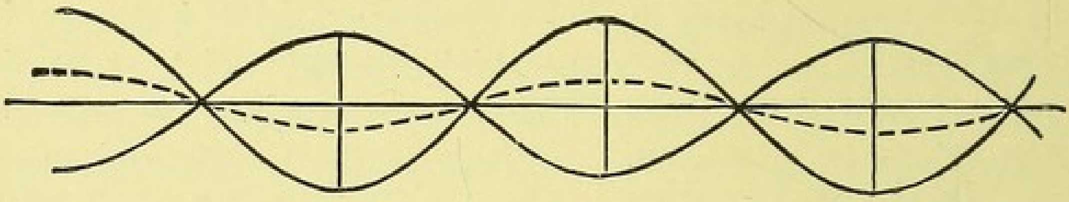

Fig. 28.

and there is no real wave-system. The condition of the air is exactly as if the sources were not vibrating.

(4) If the condensations of one imaginary system do not coincide in position either with the condensations or with the rarefactions of the other imaginary system, the amplitude of the real vibration is less than the sum and greater than the difference of the amplitudes of the imaginary vibrations.

In all these cases the intensity is of course proportional to the square of the amplitude.

40. Illustrative Experiments.—In order to verify by experiment the principle of interference in these different

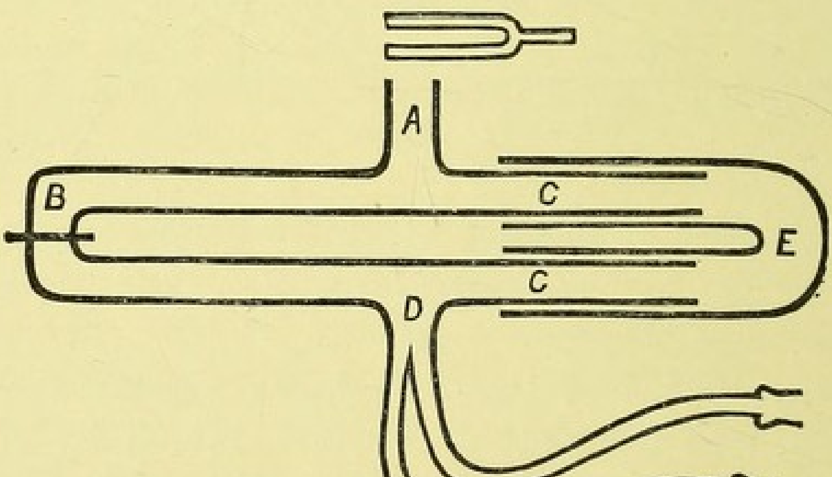

Fig. 29.

cases, we shall require some instrument for roughly comparing the intensities of sounds; the most convenient is the ear. All that we require to know about its indications at present is that vibrations of the air of the same frequency but different intensity cause sensations of different loudness, the more intense vibrations producing the louder sensations.

All the cases may be illustrated by means of an instrument, shown in section in Fig. 29, which is slightly altered from a form devised by Prof. Quincke. A tube *A* divides into two branches *B* and *C*, which re-unite at *D*. The length of *C* can be altered by drawing out the sliding portion *E* (on the trombone principle). *D* ends in a flexible pipe, or, better, in two flexible pipes, one of which is placed in each ear. In the branch *B* is a sliding door by which the branch can be closed.

If the branch *CE* is adjusted till it is the same length as *B*, and a vibrating fork held opposite the opening at *A*, it is evident that the condensations which there would be in *D* if waves travelled by one branch would be in the same places as the condensations which there would be, at the same instant, if waves travelled only by the other branch. The actual vibration in *D* is therefore greater than if waves travelled only by one branch, as is shown by the diminution of loudness when the sliding door in *B* is closed.

The same occurs if the difference between the lengths of *ABD* and *ACECD* is made any exact number of wave-lengths of the waves from the fork.

If *ACECD* is made half a wave-length longer than *B*, the condensations of the waves which there would be in *D* if waves travelled only by *CE* would be in the same places as the rarefactions of the waves which there would be if waves travelled only by *B*, and the amplitudes of these two systems are nearly equal. There is, therefore, little vibration of the air in *D* when the sliding door in *B* is open, and hardly any sound is heard. When the sliding door is closed, the sound is heard clearly.

The same occurs if the difference between the lengths of *B* and *CE* is made any odd number of half wave-lengths.

This apparatus may evidently be used to determine the length of the waves from a fork, but it is not a good method, as there is no point of absolute silence, and the points of faintest sound are not easy to determine exactly. For absolute silence it would be necessary, not only that the maximum condensations of one imaginary system should exactly coincide with the maximum rarefactions of the other, but that the condensation of one system should be equal at *every point* to the rarefaction of the other. If the wave-forms of the two systems are alike (as in the experiment just described), this requires that the wave-form should be exactly symmetrical, the part below the axis being exactly like the part above, inverted. Harmonic waves would, of course, fulfil this condition, but neither perfectly harmonic waves nor waves of any other perfectly symmetrical form are easy to produce.

If a whistle is fitted into the middle of the bottom of a box lined with felt (to prevent reflection from, and transmission through, the sides of the box), and two holes are made in the top of the box at equal distances from the whistle, similar pulses start from these openings simultaneously when the whistle is sounded. At points in the external air above the box, which are equidistant from the openings, a membrane stretched on a ring, and held horizontally, is strongly affected, and sand scattered on it is instantly thrown off, but it is easy to find points, nearer to one hole than to the other, where the membrane is very little affected; these are, of course, points whose distances from the two holes differ by an odd number of half wave-lengths.

In these instances the two imaginary wave-systems are produced in the same phase, and the difference in their phases, at the points where the air is undisturbed, is due to the different distances they have travelled. When the imaginary wave-systems in a region are in opposite phases, it is usually either for this reason, or because they were originally produced in opposite phases by parts of the same vibrating body which move in opposite directions at the same time. This is a very common cause, for nearly every vibrating body has parts which move simultaneously in opposite directions, like the prongs of a tuning fork or opposite sides of a bell; and, even in the simple case of a spring held in a vice, one side of the spring is producing a condensation while the other side is producing a rarefaction. A good illustration is furnished by a circular plate of glass or metal (A, Fig. 30) fixed in a horizontal position to a stand by a screw through the middle, but free everywhere else. When a violin bow is drawn across the edge of such a plate, the plate vibrates in an even number of sectors—four if it is not touched anywhere except by the bow—of which at a given

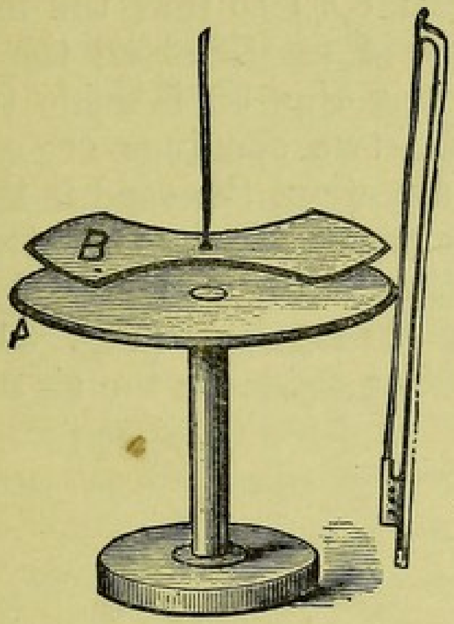

Fig. 30.

moment half are moving upward and the alternate ones downward, and we have, therefore, condensations starting from one set and rarefactions from the other at the same moment, and at a little distance above the plate, where the effects due to each of these wave-systems would be about equal, we have practically no changes of density at all. If we cut a circle of cardboard of the same size as the plate, divide it into the same number of sectors that the plate is vibrating in, cut away alternate sectors nearly to the middle, and hold the remaining part of the card *B* so as to cover, but not to touch, alternate sectors of the plate, we shall find that the sound at a few feet above the plate is much louder than before, since the waves from one set of sectors are absorbed by the card and those which pass through the openings are all in agreement.

The number and position of the sectors of the plate may be ascertained by scattering on it a little sand, which quickly collects on the lines which divide the oppositely vibrating sectors.

Hopkins's Forked Tube is used in conjunction with a vibrating plate to illustrate the principle of interference. It is a branched tube shaped like a letter Y. The ends of the branches are open, but a piece of gold-beater's skin is stretched over the end of the main stem. The tube is held in an inverted position (⅄) so that the membrane is at the top, and a little sand is scattered on the membrane. If a plate like that shown in Fig. 30 is made to vibrate, and the tube held so that the two openings are over sectors which vibrate in the same direction, the sand is thrown off; but, if the openings are symmetrically over sectors which move in opposite directions, the sand is only slightly disturbed.

The prongs of a tuning-fork (Fig. 31) move outwards together; they therefore condense the air outside them while

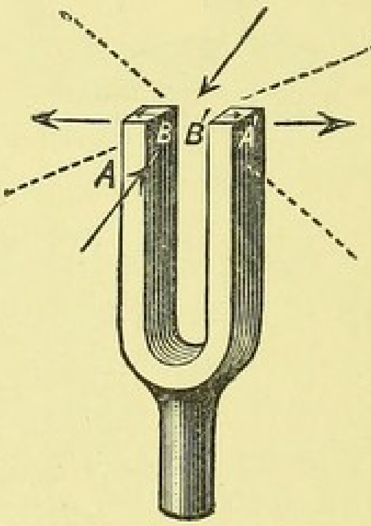

Fig. 31.

they rarefy the air between them, and condensation starts off in all directions from the outer surfaces A, A' at the same time that rarefaction is starting from the inner surfaces B, B'. The air in the direction of the line joining the prongs is, to a large extent, shielded from the waves from B, B by the prongs themselves, while along a line at right angles to this the effect of the waves from the outer surfaces A, A' is small, but in four directions which make angles of about 45° with the line joining the prongs the waves from the outer surface A of one prong are almost exactly balanced by the waves from the inner surface B' of the other prong, and in these four directions very little sound is heard. The simplest way to show this is to strike the fork, and then to twist the stem of it round between your fingers so as to make the fork present each side to your ear in turn; the sound will be heard to swell out and die away four times in each rotation of the fork.

If, when your ear is in one of the four directions from the fork in which the sound is faintest, you hold a paper tube so as to enclose, but not to touch, one of the prongs, the sound becomes much louder, showing that the waves produced by the two prongs vibrating together are of less intensity than either would produce by itself.

For reasons given above, it is very difficult to arrange an experiment in which the imaginary wave-systems are so exactly opposite that there is complete silence. A very near approach to success can be made, however, by mounting two equal organ pipes, open at the upper ends (as well as at the lower, or mouthpiece, ends), close side by side on the same wind-chest. When these are blown, they always start in opposite phases, so that a condensation leaves the mouth of one at the same moment that a rarefaction leaves the mouth of the other, and, as the waves from open pipes are almost perfectly harmonic, the condensation of one imaginary system is almost exactly equal to the rarefaction of the other at every point in the surrounding space. If there is any vibration of the air at all due to the two pipes, it cannot be heard among the rushing sound from the air at the mouthpieces. Yet, if we stretch a thin membrane on a ring, and lower this horizontally, with a little sand on it, into either of the pipes, the sand is at once thrown off, showing that the air-columns are vibrating.

41. Energy of Waves from two Sources.—When the condensations of two equal imaginary wave-systems agree in any region, the actual amplitude of vibration should, on the principle of interference, be twice as great as either of the sources would produce separately, and, therefore, the actual intensity four times as great as the intensity of the sound which either of the sources would produce separately. There is, therefore, four times as much energy passing per second through the region as there would be if only one source vibrated. This looks at first like a creation of energy, but of course that never occurs. If the sources of the two imaginary systems are several wave-lengths apart, there are some regions in the surrounding space where the imaginary wave-systems are in the same phase, and some regions where they are in opposite phases; in the former the intensity is greater, and in the latter less, than the sum of the intensities of the imaginary systems. The total energy travelling away from the sources is the sum of the amounts which would be travelling away if the sources vibrated separately, though it is differently distributed; in some directions more energy travels away, and in other directions less, than the sum of the amounts which would travel in those directions if the sources vibrated separately.

If the sources are very close together, and vibrate in the same phase, the imaginary wave-systems are in the same phase in every part of the space round the sources; in this case either the actual amplitude must be less than the sum of the imaginary amplitudes (so that the principle of interference does not hold), or the sources must give out energy faster than they would do if they vibrated separately. Usually it is the latter that occurs; each vibrating body, moving in air which is already moving in the same direction, gives up energy to it faster than it would to undisturbed air, just as a man pushing at a truck gives energy to it much faster when it has acquired some speed than he does when it has only just started and is moving very slowly.

42. Waves of unequal Length.—So far we have consi-

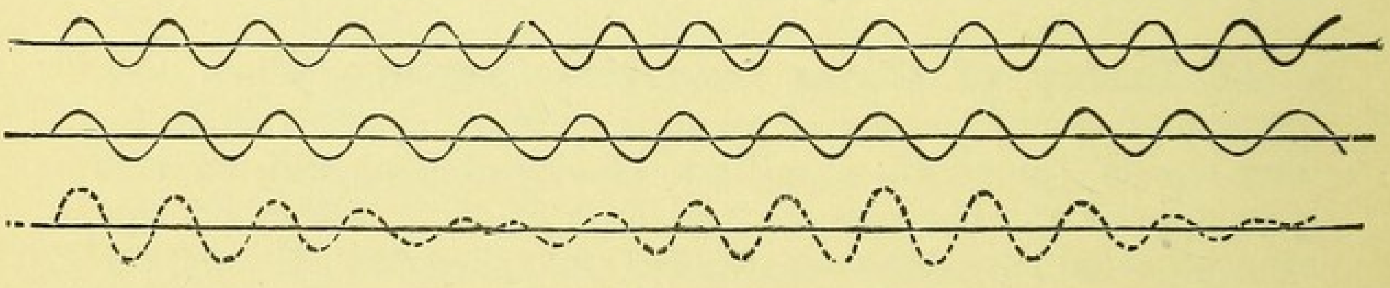

Fig. 32.

dered only cases in which the imaginary wave-systems are of the same wave-length. Fig. 32 illustrates the actual condition of the air when there are in it two imaginary wave-systems, both harmonic and travelling in the same direction, but of different wave-lengths. As in previous figures, the complete lines are the wave-forms of the imaginary wave-systems, and the dotted curve, obtained by adding the ordinates of the others, is the wave-form of the actual wave-system.* Its bends are not harmonic in form, and their maximum ordinates are greater in some places than in others, so that in some regions the condensations and rarefactions are intense, in others only slight. As the conditions indicated by this wave-form pass any point the air there moves backwards and forwards through a distance which alters at each movement, being greatest when the most intense regions of the wave-system are passing.

In this case the actual wave-system contains as much energy as the two imaginary systems would contain, but differently distributed; it is nearly all in the regions where the condensations and rarefactions are intense.†

The sound heard by an ear at which such a wave-system arrives presents peculiarities which will be discussed later, but the most obvious is the alternation of increased and diminished loudness as more intense and less intense regions of the travelling system arrive at the ear.

43. Beats.—These changes in loudness are called *beats.* They are readily heard when two tuning forks, whose frequencies differ by three or four, are sounded at the same time, or two organ-pipes, which have nearly the same frequency.

If V is the velocity of sound, and n_1, n_2 the frequencies of two sources of sound, then there are n_1 complete waves of one imaginary system, and n_2 complete waves of the other, in

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* For greater clearness, the imaginary and real wave-forms are here drawn on separate axis-lines, instead of as in the previous diagrams, but the three curves denote conditions of the air in the same region.

† The name "interference" is, by some writers, reserved for the cases in which the actual energy is very differently distributed from that of the imaginary systems; that is, for the cases where the imaginary systems are of equal, or nearly equal, wave-lengths.

a distance V, and, therefore, $n_1 - n_2$ places in that distance where the phases of the imaginary systems agree; these are the regions of maximum intensity. The distance between two regions of maximum intensity is, therefore, $\frac{V}{n_1 - n_2}$, and the number of such maxima which pass any given point in a second is $n_1 - n_2$. The number of beats heard per second is, therefore, the difference of the frequencies of the sources.

These beats or variations in the *intensity* of the vibration at a point must be carefully distinguished from the condensations or variations of density at the point. The beats in fact are variations in the amount of variation of density; the mean density is the same in the more intense parts of the wave-system as in the less intense parts.

The principle of interference is very nearly true when the condensations and rarefactions are slight; but in cases where the density-differences are not very small, compared with the mean density of the air, the actual wave-form is different from the one found by adding the ordinates of the wave-forms of the wave-systems which the sources would produce if they vibrated separately. Such cases will be considered in Chapter VI.

CHAPTER V.

FORCED VIBRATION.

44. Free and Forced Vibration.—As the series of conditions which constitutes a wave-system travels through the air, the density at any fixed point increases and diminishes alternately, and the pressure changes proportionally to the changes of density. If there is a solid body stationary in the air, the pressure on its surface rises and falls as the waves pass it, varying harmonically (Art. 5) if the waves are harmonic, or, in any case, rising and falling proportionally to successive ordinates of their wave-form. As all bodies yield more or less to pressure, the body will vibrate. Such vibrations, caused by continuous changes of external pressure, are called *forced vibrations*, to distinguish them from the *free vibrations* which take place when a body is displaced from a position of equilibrium and then left to itself. In most cases it is not possible to determine by elementary methods the exact character of the forced vibrations which a given wave-system will cause in a given body; we can only say that, as the differences of pressure in different parts of an ordinary sound wave are very small, the vibration produced, if it depends on the changes of pressure actually taking place at the moment, will usually be very small also.

45. Resonant Forced Vibration.—But if the successive waves are all exactly equal and similar, so that the pressure-change is exactly periodic (*i.e.*, repeats itself exactly at regular intervals), and if the period of this pressure change is related in a certain way, to be explained presently, to one of the periods of free vibration of the body (it may have several, according to the way in which it is displaced), the effects of a number of successive changes of pressure may be added together, and so vibratory movements of the body may be produced, which so greatly exceed in amplitude those which would be caused by the pressure-changes actually occurring at the moment, that the latter may be neglected in comparison. Such forced vibration, greatly exceeding in extent that which would be due to the pressure-changes actually occurring, and resulting chiefly from past pressure-changes, is called *resonant forced vibration.* We shall sometimes call it simply *resonant vibration*, though this term is often applied also to free vibration when caused by a previous sound.

46. Experiments on Resonant Vibration.—We have next to determine under what circumstances such resonant forced vibration is produced, and for this purpose we may try some simple experiments.

If we hang up two heavy weights, *A* and *B*, by strings of equal length, and set them swinging, we shall find it easy by slightly altering the length of one string to get them to swing in almost exactly equal times, so that if started in the same direction they will keep time with each other for some hundreds of consecutive swings. Now stop *B*, and tie a fine thread to it, and every time *A* passes the middle point of its swing in one direction, say from right to left, give a very slight momentary pull to the thread, not enough to produce any visible movement of *B*. After a few seconds, we shall find that *B* is swinging very perceptibly, and at the end of a minute or two we shall have it swinging through quite a large arc. The weight swings to a distance on each side of its mean position much greater than the displacement which would be due to the actual pull of the thread; often to a greater distance than the thread could pull it aside without breaking.

The same thing happens in many other cases. The regular tramp of soldiers crossing a bridge will break the bridge down if its period of oscillation agrees with the interval between the steps; and the vibrations of the air caused by an organ (it is said even by the voice) will break a pane of glass in a window if the pane is of such a size that it vibrates with the same frequency as the waves reach it.

47. Different Effects of Harmonic and Non-harmonic Pressure-Changes.—If we alter the length of *A*'s string a few inches, so that the times of swing are slightly unequal, and repeat the experiment, no large vibration of *B* will be produced. But if we lengthen *A*'s string till *A* makes only one vibration to two of *B* (or, which is simpler, if we make *A* swing with the same period as *B*, but pull the thread only at the end of every two double swings of *A*) we shall gradually increase the movement of *B*, till it becomes nearly as large as before. The same occurs if we pull the thread every third time *A* passes its lowest point from right to left, or at any regular intervals which are exact multiples of *B*'s free-vibration period. So that in some cases periodic impulses can cause a resonant vibration whose period is an exact submultiple of their own. But there is a very important exception to this. If the periodic impulses, instead of being forces which begin and stop suddenly, like the pull of the thread, are forces which increase and diminish *harmonically*, they will not set in resonant vibration a body whose natural period is an exact submultiple of their own. This can easily be shown by a slight variation of our experiment. Hang up *A* and *B* (still by strings of equal length) a long distance, say 20 feet, apart, and tie one end of a piece of very thin elastic, about 2 feet shorter than this distance, to *B*. Hold the other end of this elastic in your hand, exactly under *A*, so that it is slightly stretched. Set *A* swinging, with an amplitude of about a foot, along a line passing through *B*, and move your hand as *A* swings, so as to keep the end of the elastic exactly under *A*. The pull of the elastic on *B* will then increase and diminish harmonically, or very nearly, and it will be found that *B* will begin to vibrate, and keep increasing its vibration, if its free-vibration period is the same as that of *A*. But if we lengthen the string of *A* till it makes only one swing to two of *B*, and then keep the end of the elastic under *A* as it swings, no considerable movement of *B* will be produced. *B* does move, of course, but the movement does not increase beyond that due to a single pull. Now, we saw that a slight pull on a thread tied to *B*, occurring once in every two swings of *B*, would produce a large vibration. The difference between the two cases is this:—The pull of the thread is a force which, at regular intervals, increases very suddenly from zero to its maximum, and, after lasting a very short time, falls very rapidly to zero. In other words, the thread exerts on *B* a force which varies periodically but *not harmonically*, while the pull of the elastic varies harmonically. We conclude that a periodic force which increases and diminishes harmonically does not cause resonant vibrations of periods which are submultiples of its own, though a force which varies non-harmonically may do so.

48. Period and Amplitude of Forced Vibrations.—Now adjust the lengths of the strings till *A* swings nearly, but not exactly, with the same frequency as *B*. Suppose 30 swings of *A* require the same time as 31 of *B*. If we repeat the experiment with the elastic, we shall find that *B* will still be set vibrating, and that its amplitude of vibration will increase for a time, but will soon reach a limit which is much smaller than when the periods agree more exactly. We shall find also that the vibration of *B* agrees in period exactly with that of the pulls given to the elastic, and does not depend on its own natural period of vibration. If the period of *A*'s vibration is 2 seconds, the forced vibrations of *B* will have a period of 2 seconds also, whatever the natural period of *B*, but, unless that natural period is about 2 seconds, the vibration of *B* never becomes very much greater than that which one of the pulls would produce.

Though the vibration produced in a body by a periodic force, which only nearly agrees in period with the free vibrations, is always of smaller amplitude than it would be if the periods agreed exactly, the extent of the difference depends very much on the mass of the vibrating body and the amount of friction resisting its motion. If the mass is large and the friction small, as in the case of a heavy pendulum or tuning fork, the resonant vibration produced by impulses of *nearly* the natural period of the vibrating body is very small compared with that produced by equal impulses of *exactly* the right period. But if the mass is small and the friction large, as in the case of a boat rolling in the waves, or air vibrating in a narrow tube, impulses of nearly the right period will produce nearly as strong resonant vibration as impulses of exactly the right period.

Practically, when we have to do with a body which is not very light, and periodic pressure-changes which are not very violent, we may assume that no *perceptible* movement will be caused by a *harmonic* pressure-change of any period which is not almost exactly one of the periods in which the body can vibrate freely.* So *harmonic* waves do not cause strong vibration of bodies which they pass, unless the waves arrive with frequencies which are nearly free-vibration frequencies of the bodies.

We have not yet discovered what are the conditions under which *non-harmonic* waves cause resonant vibration, though we have seen that a non-harmonic rise and fall of pressure, such as non-harmonic waves would produce, sometimes causes resonant vibration where harmonic waves, arriving with the same frequency, would fail. To determine what non-harmonic waves will cause resonant vibration of a given body, we must resort to a mathematical device, explained in the next chapter.

* Under exceptional circumstances this may be untrue; an instance is given in Art. 99.

CHAPTER VI.

FOURIER'S THEOREM.

49. Components.—We saw in the last chapter how the rise and fall of pressure at any point can be represented by a curve. Suppose that the dotted line *ACBDZ* (Fig. 33 or 34) represents the rise and fall of pressure at some point *P*, on a scale of, say, an inch to a second, the ordinate *A'A* representing the amount by which the pressure is below the average at a given moment *T*; an ordinate $\frac{1}{8}$ inch to the

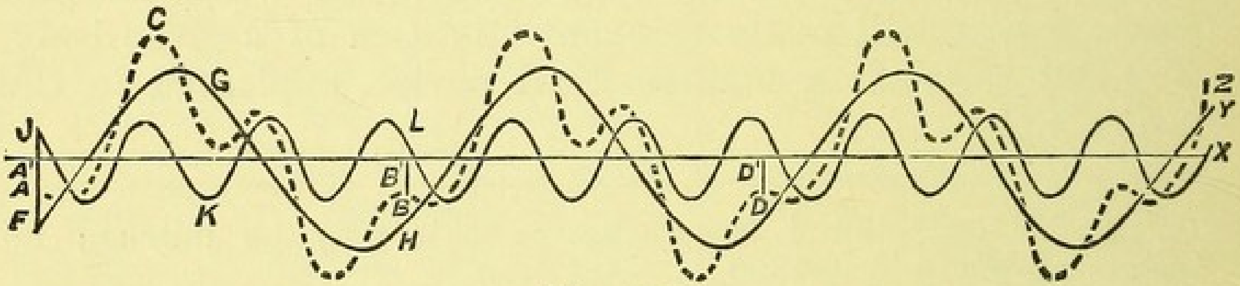


Fig. 33.

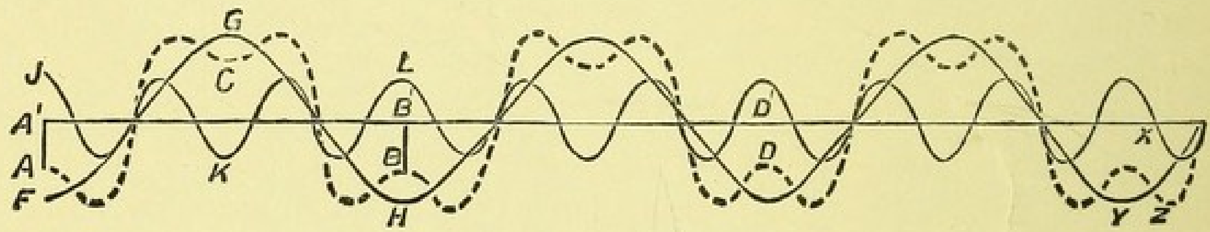


Fig. 34.

right of *A'A* representing the difference from the average pressure $\frac{1}{8}$ second later, and so on. Now, there is no physical difference between a force *f* and two forces, acting together, whose sum is equal to *f*. So that, instead of saying that the pressure-difference* at the moment *T* is *A'A*, we

* The term "pressure-difference" will be used to mean the difference between the actual pressure of the air at a given moment and its average pressure.

can say that it is the sum of pressure-differences represented by *A′F* and *A′J*. So that, if *FGHY* and *JKLX* are any two curves whose ordinates everywhere add up to those of *ACBDX*, there is no difference between a rise and fall of pressure represented by *ACBDX* and two pressure-changes, happening at the same time, represented by *FGHY* and *JKLX*.

A dynamical proposition, too difficult for this book, shows that if the force on any body, due to a displacement from its position of rest, is proportional to the displacement (which is usually very nearly the case in the small displacements which we are considering), then the movement of this body produced by several pressure-changes happening together is the sum (in the sense explained in Art. 7) of the movements which the same pressure-changes would produce separately.

Pressure-changes such as those represented by *FGHY* and *JKLX*, which add up to a given pressure-change such as that represented by *ACBZ*, are called *components* of the pressure-change represented by *ACBZ*.

In Figs. 33, 34, *FGHY* and *JKLX* are drawn as harmonic curves, and so represent harmonic pressure-changes, but the proposition just given is just as true for any other kind. And we could evidently draw as many sets of curves as we pleased whose ordinates would add up to those of *ACBZ*, so that any pressure-change has an unlimited number of sets of components, and the dynamical proposition just given shows that, to predict the movements which will be produced by any pressure-change such as that represented by *ACBZ*, we may first find what movements would be produced by the members of any set of components separately, and then add these movements.

All this is true whatever kind of pressure-changes the components are. But, unless the components are *harmonic* pressure-changes, the device of components does not help us, for it is no easier to tell what movements they will produce than it is to tell what movements the pressure-change represent d by *ACBZ* will produce.

If we could find *harmonic* pressure-changes which would add up to the pressure-change whose effect is to be found, then, as we saw in the last chapter, it would be very easy to say whether any of these, separately, would produce *resonant* vibration of a given body whose free-vibration frequency is known. It would not be easy to tell exactly what movements the others would produce separately, but we can be sure that they would be very small compared with the resonant vibrations. So that practically, if any of these harmonic pressure-changes would produce *resonant* vibration of the given body, we may neglect the effect which the others would produce.

50. Fourier's Theorem.—Figs. 15, 33, and 34 show that there are *some* non-harmonic curves whose ordinates are the sums of the ordinates of harmonic curves, so that if a rise and fall of pressure corresponds to one of these non-harmonic curves, its effect in producing resonant vibration can be easily calculated by finding the effects of pressure-changes corresponding to the harmonic curves. A very important theorem, due to Fourier, shows that this can be done not only in some cases, but in all.

For our purpose we may state Fourier's Theorem in this way. Let there be any given line, such as the dotted line *ACB* in Fig. 33 or 34, which does not overhang anywhere, so that no ordinate cuts it more than once, and let *A*, *B* be two points on it. Then it is always possible to find sets of *harmonic* curves whose ordinates will add up to those of the line *ACB*. And (which is the important point for our purpose), *if A and B are points whose ordinates are equal*, there is, among the sets of harmonic curves whose ordinates add up to those of the line *ACB*, *one* set (and only one) which consists entirely of harmonic curves whose lengths (the length of a double bend measured along the axis) are contained an exact number of times between the ordinates to *A* and *B*.

Figs. 33, 34 illustrate this. The ordinates at *A* and *B* are equal, and the ordinates at every point of *ACB* are the sums of the ordinates of the two harmonic curves *FGH* and *JKL*, whose lengths are each contained an exact number of times in the length *A'B'*. We could find other sets of harmonic curves which would add up to *ACB*, but there is only this one set of harmonic curves *whose lengths are con-*

tained an exact number of times between A' and B', which will add up to ACB.

To simplify the diagram, we have illustrated a case in which this set of harmonic curves consists of only two. Usually there are more than two, and the number may be infinite.

51. Application to Sound.—Fourier's Theorem is purely a mathematical one. Its connection with sound arises from the fact that, as the pressure of the air cannot have more than one value at any one point at the same moment, any possible rise and fall of pressure can be represented by a curve which does not overhang anywhere, and that if this rise and fall of pressure is *periodic*, or exactly repeats itself at equal intervals, this curve may be divided into lengths which are all alike, so that the first and last ordinate of each length are equal. Suppose the dotted line $ACBZ$ represents a periodic rise and fall of pressure, and that ACB represents one complete cycle of changes of pressure, so that the rest of the curve consists of repetitions of ACB. Let FGH and JKL be the set of harmonic curves whose lengths are contained an exact number of times between A' and B', and which add up to ACB. Then, if FGH, JKL are continued to the right, they repeat themselves in the next period $B'D'$ and these portions add up to the same curve as before —that is, to the form BD. So that, however far we continue $ACBDZ$, the harmonic curves $FGHX$, $JKLY$, also continued, will everywhere add up to the curve $ACBDZ$.

As the harmonic curves FGH, JKL repeat themselves exactly an exact number of times in the length $A'B'$, they represent harmonic pressure-changes which repeat themselves an exact number of times in the period represented by $A'B'$, that is, they are harmonic pressure-changes whose frequencies are exact multiples of the frequency of the non-harmonic pressure-change represented by $ACBZ$. In the figure, FGH represents a harmonic rise and fall of pressure occurring once, and JKL a harmonic rise and fall of pressure occurring three times, in one complete period of the non-harmonic pressure-change represented by the dotted line.

52. Harmonic Components.—If there is a non-harmonic pressure-change repeating itself with frequency n, then Fourier's Theorem shows us that we can always find a series of *harmonic* pressure-changes, of frequencies which are exact multiples of n (n itself being included), such that, occurring together, they are the given non-harmonic pressure-change. These harmonic pressure-changes are called the *harmonic components* of the non-harmonic pressure-change.

It should be clearly understood that the statement that a non-harmonic pressure-change of frequency n is always the sum of harmonic pressure-changes of frequencies which are multiples of n, does not at all imply that any actions are taking place which could cause pressure-changes of these frequencies, or that these pressure-changes have any separate existence. It is only true in the same sense that anything 5 feet long is the sum of two parts, one 3 feet and the other 2 feet; it does not mean that there is any physical distinction between the parts. For instance, if a man rows a boat, making (we will suppose, for convenience, that it is possible) one double stroke per second, he exerts on the oar a push-and-pull whose frequency is 1 per sec. This push-and-pull is not harmonic; instead of gradually rising to a maximum and then gradually diminishing, it is nearly uniform till near the end of the stroke. Fourier's Theorem shows us that it is the sum of a number of harmonic pushes and pulls whose frequencies are 1, 2, 3, 4, 5, &c., per second, happening together. This does not mean that the man is really giving pushes and pulls at these rates; he pulls continuously for more than half-a-second at a time. But if we replaced the man by a number of men, of whom one pushed and pulled harmonically once a second, one twice a second, one three times a second, and so on, then, if the intensities of these harmonic pushes and pulls, and the moments of beginning them, were rightly chosen, the push-and-pull on the oar would be the same as the non-harmonic push-and-pull exerted by the actual rower. And this is all that is meant by calling these harmonic pushes-and-pulls the components of his non-harmonic push-and-pull.

The proposition of Art. 49 shows that the effect of a non-harmonic change of pressure in producing vibration is the sum of the effects which its harmonic components (or any set of components) would produce separately. And we saw in the last chapter that a harmonic rise and fall of pressure produces *resonant* vibration in a body if the frequency of the pressure-change is one of the free-vibration frequencies of the body. Putting these facts together, we see that **a non-harmonic rise and fall of pressure on a body will set it in resonant vibration if, and not unless, one of the harmonic components of the non-harmonic pressure-change has a frequency which agrees with one of the free-vibration frequencies of the body.**

Of course, *any* periodic pressure-change on a body sets it in vibration. The statement just given relates only to *resonant* vibration. The changes of pressure, as sound-waves pass, are so small that, unless the effects of a number of changes are added by resonance, we may generally neglect the movements produced.

We now see why, in the experiment described in Art. 46, a momentary pull every 4 seconds sets a pendulum, whose free-vibration frequency is 2 seconds, in resonant vibration. The pull of the string on the pendulum rises and falls non-harmonically once in 4 seconds, and one of the harmonic components of this non-harmonic variation of force is a pull rising and falling harmonically twice in 4 seconds. We do not actually pull twice in 4 seconds, but a harmonic pull whose frequency is twice in 4 seconds, occurring at the same time with other harmonic pulls whose periods are contained exactly in 4 seconds, would be the non-harmonic pull (increasing and diminishing again very rapidly once every 4 seconds and then remaining zero the rest of the period) which the string really gives to the pendulum.

53. Fundamental and its Harmonics.—If a non-harmonic pressure-change has a frequency n, then a harmonic pressure-change of frequency n is called the *fundamental* pressure-change of the non-harmonic one, and harmonic pressure-changes whose frequencies are $2n$, $3n$, $4n$, &c., are called the 2nd, 3rd, 4th, &c., *harmonics* of that fundamental. The harmonic components of a non-harmonic pressure-change are all included among its fundamental and the harmonics of that fundamental. If the fundamental is one of the harmonic components, it is called the fundamental component.

The same method and the same terms may be applied to anything which varies with the time, and which can therefore be represented by a curve. For instance, a non-harmonic vibration of a point of frequency n is always the sum (Art. 7) of harmonic vibrations of frequencies which are multiples of n, and these harmonic vibrations are called the harmonic components of the non-harmonic vibration.

54. Harmonic Components of a Wave-System.—We may also apply the same theorem to the wave-system which causes the rise and fall of pressure. Instead of representing a pressure-change, suppose now that the curve *ACBZ* (Fig. 33 or 34) is the wave-form of any non-harmonic wave-system, so that *ACB* is the wave-form of one wave-length. Then Fourier's Theorem shows us that we can always find a set of imaginary harmonic wave-systems whose wave-forms, *FGHY* and *JKLX*, add up to the wave-form of the non-harmonic system, and whose wave-lengths are contained an exact number of times in one wave-length of the non-harmonic system. These imaginary harmonic wave-systems are called the harmonic components of the real non-harmonic wave-system. Each wave-system would produce, as it passed any point, a rise and fall of pressure represented by a curve which is the same as the wave form of the system, so that the effect of any non-harmonic wave-system is the sum of the effects which its harmonic components, if they were real wave-systems, would produce separately.

A non-harmonic wave-system, therefore, sets a body in resonant vibration if any one of the harmonic components of the non-harmonic system is one whose waves, if they were real, would arrive with a frequency corresponding to one of the free-vibration frequencies of the body.

A given non-harmonic wave-system is often said to "consist of" its harmonic components. This is rather misleading, because it suggests the idea that this particular set of components has a real existence in some different sense to other sets of components, harmonic or other, whose wave-forms would also add to that of the actual wave-system. This is not the case; we could always find sets of non-harmonic waves, and even sets of non-harmonic waves all of the same kind, whose wave-forms would add up to the wave-form of any given wave-system, and the real waves "consist of" any of these sets of non-harmonic components exactly in the same sense as they "consist of" the harmonic ones. All "components," harmonic or other, are imaginary, and the only reason why we do not usually refer to any other set except the harmonic ones is that the effect of a succession of similar harmonic waves in producing *resonant* vibration is so easy to predict. It is only when we want to know what *resonant* vibrations a given wave-system can produce (and in one other case which will be considered later) that there is any advantage in Fourier's device of adding the effects which the imaginary harmonic components would produce separately, to find the effect of the real waves.

We shall see later that, owing to the nature of the mechanism of hearing, a separate *sensation* corresponds to each of the *harmonic* components of a system of non-harmonic waves. This of course does not in any way indicate that the harmonic components have an external physical existence, any more than the seven distinct colour sensations which we receive from the spectrum indicate that there are seven physically different kinds of light. It only shows that the mechanism of hearing is such that each distinct sensation depends on the *resonant* vibration of a distinct body, so that the sensations which a wave-system can cause depend on the bodies it can set in *resonant* vibration. We shall see in the next chapter how this mechanism works.

54. Harmonic Analysis.—As the harmonic components of a wave-system, and the harmonic components of the rise and fall of pressure produced by it, are represented by the same harmonic curves, to find the harmonic components of the pressure-change is equivalent to finding the harmonic components of the wave-system. If we knew the character of the rise and fall of pressure exactly, so that we could draw the curve representing it, we could find by geometrical methods the harmonic curves which add up to that curve, and these represent the harmonic components of the pressure-change. But it is not practically possible to determine either the wave-form of a series of waves or the rise and fall of pressure (represented by the same curve) which the waves produce. It is much easier to determine the harmonic components of the pressure-change by finding experimentally what bodies the pressure-change will set in resonant vibration. It is not necessary to have bodies of every possible free-vibration period for this. The harmonic components of a pressure-change are all pressure-changes whose frequencies are exact multiples of the frequency of the actual rise and fall of pressure. If, for instance, the complete cycle of changes of pressure is repeated 100 times per second, the harmonic components of the pressure-change are all pressure-changes whose frequencies are multiples of 100. So that, if we expose to this rise and fall of pressure bodies whose free-vibration frequencies are 100, 200, 300, 400, &c., some of these will be set in resonant vibration, and the frequencies of these are the frequencies of the harmonic components of the actual rise and fall of pressure.

For this experiment it is evidently desirable that the bodies used should be bodies which have only one free-vibration frequency. There are very few such bodies; the best for the purpose is the air contained in a globular or cylindrical vessel which has an opening much narrower than the greatest diameter of the vessel. Such a vessel is called a "resonator." The details of the experiment are given in Chapter XII.

The determination of the harmonic components of a pressure-change, vibration, wave-system, &c., is called the *harmonic analysis* of it.

55. Relation of Harmonic Components to Source of Waves.—In some cases, as shown in Art. 39, a non-harmonic wave-system is produced by two or more sources, each of which alone would produce a harmonic system. In this case, as we saw, if the vibrations are of small amplitude, the wave-form of the actual waves is the sum of the harmonic wave-forms of the wave-systems which the sources would produce separately. These harmonic wave-systems are therefore the harmonic components of the actual wave-system.

Even where there are not any sources which really vibrate harmonically, the harmonic components of a wave-system, though quite imaginary, are often similar to real harmonic wave-systems which might be produced by the actual source. It often happens that a body vibrates so that every part of it moves harmonically in the same period; in this case it usually produces nearly harmonic waves. If there are several different ways in which the body could vibrate harmonically, with different periods, there are several harmonic wave-systems of different wave-lengths which it could produce. If such a body vibrates in any other way, it is usually one in which its movement is the *sum* (Art. 7) of two or more of the harmonic vibrations which it could perform; in this case the waves from it have a wave-form which is the sum of the wave-forms of the harmonic waves which these vibrations would produce, so that these harmonic waves are the harmonic components of the actual wave-system.

In these cases the harmonic components of the waves produced are similar to real waves which might be produced by the same sources. It must not be supposed that this is always the case, still less that the possibility of finding imaginary harmonic systems whose wave-forms would add up to that of the waves from a given source depends at all on whether the source of sound could vibrate so as to produce such harmonic systems. In many cases there is no relation between the harmonic components of a wave-system and any real harmonic waves which the source could possibly produce. For instance, if air is blown from a pipe against a revolving disc having 100 holes drilled in a circle at the same distance from the axis as the end of the pipe, a puff of air will come through each hole as it passes—100 puffs per second if the disc rotates once in a second. At the disc, every process which occurs at all is repeated 100 times a second, and not oftener; there is no operation being repeated 200 or 300 times a second. A hundred similar waves, each about 11 feet long, start from the disc in each second. These waves are far from harmonic in character, the pressure of the air on the side of the disc away from the pipe rising very suddenly as a hole comes opposite the pipe and falling very suddenly when the hole has passed, instead of the gradual harmonic rise and fall. But the wave-form of these waves, as Fourier's theorem shows, might be made by adding the ordinates of the wave-forms of harmonic waves 11 feet, $\frac{11}{2}$ feet, $\frac{11}{3}$ feet, &c., in length. So that harmonic waves of these lengths are harmonic components of the waves from the disc, although to produce real harmonic waves $\frac{11}{3}$ feet long would require a movement repeating itself exactly 300 times in each second, and nothing that happens at the source is repeated with this frequency.

56. Resultant Tones.—We may go further than this, and say that there are always, among the imaginary harmonic components of a wave-system, some which could not really be produced by any vibration which the source of sound could execute. The phenomenon of "resultant tones" is one instance of this. As stated in Art. 37, the principle of superposition is not *exactly* true except for infinitely small vibrations. If there are two sources, A and B, each of which would by itself produce intense harmonic waves, the wave-form of the actual waves can only be approximately made by adding the harmonic wave-forms of the waves which A and B would produce separately. It can, as Fourier's theorem shows, be *exactly* made by adding harmonic wave-forms, but some others are required besides those of the waves which could really be produced by A and B separately. The most important of these additional harmonic curves are wave-forms of waves which might be produced by two other sources vibrating harmonically with frequencies which are respectively the sum and difference of the frequencies of A and B. The actual waves have therefore four imaginary harmonic components, and the frequencies with which the waves of these harmonic components would arrive, if they were real, are (if we call the frequencies of A, B, a and b respectively) a, b, $a+b$, $a-b$. As stated above, waves may produce a distinct sensation for each of their harmonic components; so that there are two additional sensations when A and B vibrate together, which cannot be produced by either separately.

These additional sounds are called respectively the summation and difference tones.

57. Change of Components.—As stated in Art. 36, waves change their form a little as they travel. At some distance from the source their form cannot be made by adding the same harmonic curves which add up to their form near the source. In other words, the harmonic components change as the waves travel; the components at a distance are not the same as near the source. But if the fundamental is one of the components at one place it always is so at any other place.

58. Different Wave-systems with same Components.—As shown in Figs. 33, 34, the same harmonic curves, in different relative positions, add up to quite different forms. So that two wave-systems, of quite different wave-forms, may have the same harmonic components, in different relative positions. Two such wave-systems set the same bodies in resonant vibration, and therefore cannot be distinguished from each other by merely observing which of a series of resonators they excite.

In this chapter we have considered chiefly the rise and fall of pressure on a body as waves pass it, because it is to changes of pressure that the resonant vibration of the body is directly due, and the chief use of Fourier's device is to determine what bodies will be set in resonant vibration. But the vibration of the particles of the air as the waves pass may also be represented by a curve, and may therefore be considered as the sum of a number of harmonic vibrations, of frequencies which are multiples of the frequencies of the actual vibration. The curve which represents the movement of a particle of air (the displacement curve, Art. 3) is not the same as the curve which represents the rise and fall of pressure in that particle, which is the same as its velocity curve (Art. 15). The two curves, however, can always be made by adding the same harmonic curves in different relative positions. So the harmonic components of the vibration have the same frequencies as the harmonic components of the change of pressure.

The *constitution* of a non-harmonic pressure-change means the frequencies and amplitudes of its harmonic components without reference to their relative phases, and the same term is applied to vibrations, wave-systems, &c. So that the vibrations, pressure-changes, or wave-systems represented by the dotted curves in Figs. 33 and 34 are said to have the same constitution, though not the same character.

By the aid of Fourier's Theorem we can often simplify problems in interference. Thus in the first experiment of Art. 40, instead of saying that absolute silence can only be produced if the waves are symmetrical, we can say that it can only be produced if the difference between the paths ABD and ACECD is an odd number of half wave-lengths for each component of the actual waves, which obviously can only occur if the components are all odd harmonics of the fundamental. By comparing Figs. 33, 34, 15, we see that waves with only odd harmonic components are symmetrical, the wave form of the rarefaction being that of the condensation inverted, but that this is not the case if an even component is present.

CHAPTER VII.

THE EAR AND HEARING.

59. Structure of the Ear.—The rise and fall of pressure which occur as sound waves arrive at the ear, produce, by their effect on its structures, the sensation of sound, and the nature of this sensation depends on the nature of the rise and fall of pressure; in other words, on the character of the sound waves. We may, therefore, learn much about sound waves from the sensations they produce. But, to understand what physical facts about the waves are indicated by different sensations, we must know something about the structure of the ear.

This apparatus is a very complex one, but it is constructed on quite simple principles. The external ear, or *pinna*, has, in ourselves, no connection with hearing, which is not affected by its removal; but in many animals it is funnel-shaped, and it then serves to increase the intensity of the vibration, in a way explained in the next chapter. A funnel-shaped pinna is also useful for ascertaining the direction from which the waves come. From the pinna a tube, called the *meatus*, in man about 1¼ in. long, leads directly inwards, and is closed at the inner end by a stretched membrane, the *tympanum* or *drum of the ear*. This membrane is connected by a chain of three small bones to another membrane, the *fenestra ovalis*, which forms part of the wall of the hearing chamber, or *cochlea*.

The principle of this hearing chamber may be understood if we imagine a long, low, narrow building of two storeys (a ground floor and one above it), having at one end two doors, one into the ground floor, the other into the upper storey. The floor which separates the storeys is not complete; it does not extend quite to the further end of the building from the doors, and along one side of the building it does not reach, as a solid floor, up to the wall; along this side, however, but not at the end, it is completed by two sheets of membrane, one completing the ceiling of the lower room, the other the floor of the upper one; between these membranes there is a space, which is entirely shut off from the rooms, since the membranes meet and join at the end furthest from the doors. The lower and upper rooms are in free communication at the end furthest from the doors, where the floor between them is entirely wanting. The doors are thin and flexible. The whole space inside the building is filled with water.

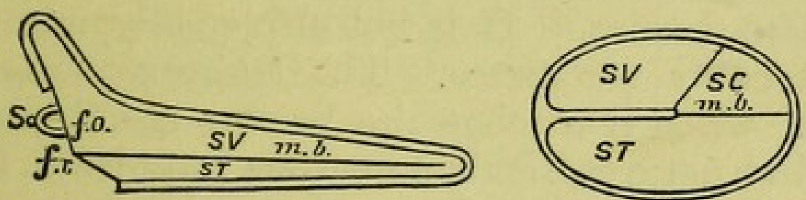
In the space between the two membranes there are about 3000 rods of different lengths; these stand on the lower membrane, in two parallel rows running its whole length; they are fastened at their lower ends to the membrane on which they stand. The rods in each row lean towards the other row, and at their upper ends each touches, and is joined to, its fellow in the other row; each such pair is free except where the lower ends are attached to the lower membrane. The whole series forms "a sort of gable roof."

If in such a building the thin material of the door of the upper storey was pushed in, a wave, very similar to an ordinary water wave on a canal, would travel all along the upper room to the further end, depressing all portions of the membranous part of the floor in turn as it passed along; as the liquid is incompressible, the membranous ceiling of the lower room would be forced down at the same time, and the flexible door of the lower room would be forced out. Every movement of the door of the upper room produces a movement of the membranous part of the ceiling of the lower and of the rods supported by it, and if the door is made to vibrate regularly, any rod whose natural vibration period corresponds will be set in resonant vibration.

In the actual ear, a chamber constructed on a principle similar to the building above described is coiled into a spiral so close that one side of the building, where the solid floor is complete, becomes a mere central pillar. The chamber resembles a snail shell in form; hence the name *cochlea*. The upper door is the *fenestra ovalis* already mentioned; the other is called the *fenestra rotunda*. The rods are called the *fibres or rods of Corti.* A nerve-filament from the brain is connected to each pair of rods.

If the *cochlea* was uncoiled and straightened out, a longitudinal section of it would be something like Fig. 35 (*a*), and a transverse section something like Fig. 35 (*b*); but these figures merely indicate the general arrangement, not the proportions of the parts.

When sound waves reach the *tympanum*, it vibrates, and its movements cause corresponding, but smaller, movements



(*a*) Longitudinal section. (*b*) Transverse section.

Fig. 35.

SV, *ST*, *Scala vestibuli*, and *Scala tympani*, the two storeys; *f.o.*, *fenestra ovalis*; *S*, *stapes*, the last of the chain of bones connecting the *fenestra ovalis* to the *tympanum*; *f.r.*, *fenestra rotunda*; *SC*, *Scala cochleae*, the space between the membranes; *m.b.*, *membrana basilaris*, to which the rods are attached.

of the *fenestra ovalis*, with which it is connected. These movements are transmitted to the liquid which fills the *cochlea*, and to the rods which are bathed in this liquid. The sensation of sound depends on the vibration of these rods; usually on their resonant vibration, since ordinary sound waves do not produce sufficiently great vibration unless the effects of a number of waves are added by resonance. There is no way of ascertaining whether the resonant vibration of each rod produces a separate sensation, because we cannot make one rod vibrate alone. As the rods are of small mass, and move with considerable friction in the liquid, the resonant vibration caused by the movements of the liquid is not much stronger in the rod whose free-vibration period exactly corresponds than in a number of others whose periods are not very different (Art. 48). Harmonic waves, therefore, set in resonant vibration not one rod, but a group of consecutive rods. The resulting sensation is, however, always a simple one; it is only when two groups of rods vibrate, while intermediate ones do not, that we have the sensation of hearing two sounds at once, and this cannot be caused by harmonic waves.

Several objections have been raised to the explanation given above. It has been stated that the organ of Corti is absent in birds (though the contrary is also maintained), and also that fibres so short as the rods of Corti cannot have such low frequencies as are required by the explanation. Many persons believe that the fibres of the basilar membrane itself, and not the rods of Corti, are the resonant bodies. It is not universally admitted that the ear acts by resonance. The *telephonic theory* is that each wave, when it reaches the basilar membrane and its associated structures, sends a nervous impulse to the brain, and that the sensation depends on the frequency with which these impulses are received. It will be seen that this view differs radically from the resonant theory. On the resonant theory periodicity extends only as far as the basilar membrane; the nervous impulse sent to the brain is not periodic, and the pitch of the sensation depends on which nerve-fibre is stimulated, not on how it is stimulated, so that if the nerve-fibre belonging to the rod of Corti whose natural frequency is 200 could be stimulated in any other way, for instance by a non-periodic stimulus such as an electric current, we should hear the note which is usually produced by 200 waves per second, though nothing would really be repeated with that frequency or repeated periodically at all. On the telephonic theory the nervous impulses transmitted to the brain have the same frequency as the waves, and the sensation depends on the frequency of these nervous impulses, so that no non-periodic stimulation could produce a sensation of definite pitch. As the telephonic theory does not profess to explain any of the observed relations between waves and sensations of sound, it hardly admits of experimental proof or disproof, but the resonant theory is more in accordance with analogy, since in other cases the stimulation of a particular nerve produces a particular kind of sensation, irrespective of the nature of the stimulus; a blow on the eye produces the sensation of light.

In the following pages we speak of the sensation as due to the resonant vibration of the rods of Corti, without meaning to imply that this theory is the correct one.

60. Pitch.—The sensation differs according to the frequency of the vibration. This difference is called a difference of *pitch*, and the sensations produced by harmonic waves of greater and smaller frequency are said to be sensations of *higher* and *lower* pitch respectively. There seems no relation between the ordinary meanings of the words "high" and "low" and the sensations produced by waves of different frequencies arriving at the ear; probably the names are derived from the sensations felt in the vocal organs in producing such waves.

Strictly, the term pitch is applied only to the sensation, not to the vibration which produces it, but the term is often loosely used as equivalent to frequency.

That pitch depends on frequency of vibration may be shown by fitting two toothed wheels, with different numbers of teeth, on the same uniformly revolving axle (the axle of a heavy top answers well). If a card is held against the wheels in turn, the one with more teeth gives a note of higher pitch.

Non-harmonic waves produce in the ear the same effect as on any other system of resonators; that is, they set in resonant vibration all the rods whose frequencies of free vibration correspond with the frequencies of any of their harmonic components. The resulting sensation differs very much in different persons, and even in the same person, according to the amount of attention he gives. A person who has been musically trained, and listens attentively, will generally be conscious of hearing, at the same time, the different sounds which real harmonic waves, similar to the harmonic components, would produce separately. With less training or attention, only a single sensation is perceived, which usually seems to be of rather higher pitch than would be produced by harmonic waves of the same frequency, but is not exactly like the sensation produced by harmonic waves of any frequency. There is then a difference between sensations produced by waves of different forms, even if they are of the same frequency, and this is called a difference in the *quality* of the sound. The terms *character*, *timbre*, *clang-tint*, are used by different writers as equivalent to quality.

61. Quality.—It is this difference in the quality of the sensations which enables us to distinguish between continuous notes of the same frequency produced by different instruments, a flute and a harmonium, for instance, though we may be quite unable to say which is of higher pitch. But where the sound is not continuous we are aided in our judgment about its origin, by the way in which the sound begins and ends.

Two wave-systems which are exactly similar in frequency, amplitude, and wave-form must give rise to the same sensation, for there is no respect in which the two can differ. But it does not follow that two wave-systems of different wave-form will produce different sensations.

Quite different curves may be produced (Art. 58) by adding the ordinates of the same harmonic curves in different relative positions, and waves of these different forms set the same resonators in vibration. If the rods of the ear were *independent* resonators, it is probable that two wave-systems of different wave-forms would produce exactly the same sensation, if their wave-forms were such as might be formed by adding the same harmonic curves in different relative positions. If this was the case, we could say that the quality of the sensation produced by non-harmonic waves depended on the frequencies and amplitudes, but *not* on the relative phases, of their harmonic components.

It was Helmholtz's view that this actually was the case, and he designed the following experiment to prove it. He had 13 tuning-forks, whose frequencies were in the ratios 1 : 2 : 3 : 4 : &c., mounted each in front of a resonator (Art. 54) of corresponding frequency; the openings of these resonators could be closed by sliding doors. All the forks were kept in continuous vibration by the same momentary electric currents passing at regular intervals round electro-magnets placed close to the prongs; the interval was an exact multiple of the period of the slowest, and therefore of every other, fork; and, as the force varied non-harmonically, it produced resonant vibration in them all (Art. 46). The sound of any fork was hardly audible, except when the door of its resonator was open. Helmholtz first determined, by the method described in Art. 54, the harmonic components of some sound of marked and peculiar quality (say that of a trumpet) which was of the same frequency as the slowest of the forks. He then opened the doors of the resonators opposite the forks whose frequencies corresponded to the harmonic components of the sound of the trumpet, and found that the sound from these resonators vibrating together was similar in quality to that of the trumpet. Now in this experiment the harmonic waves from the resonators would not, unless by accident, be in the same relative positions as the harmonic components of the waves from the trumpet; there was, therefore, usually no resemblance between the wave-forms of the two wave-systems. The fact that the sensations were similar seemed to prove that waves of different form produced the same sensation if they could be made of the same harmonics in different relative positions.

This experiment is often described as the synthesis of a given non-harmonic sound. It is, however, only the sensation, not the wave-system, which is reproduced, so that the experiment is purely a physiological (and psychological) one. There is no physical resemblance whatever (except in length) between the original waves and Helmholtz's copy of them, and if we *saw* sound waves instead of hearing them, it would be obvious that they were totally unlike. The apparent resemblance is due simply to the very imperfect way in which the ear distinguishes between waves of different form, just as the apparent resemblance between the white lights produced by adding different pairs of complementary colours is due to the imperfect way in which the eye distinguishes between non-harmonic light waves.

Though Helmholtz failed to detect any difference in the sensations produced by the trumpet and by the forks, it is not impossible that resonant bodies which, like the rods of Corti, are all connected to the same membrane may vibrate differently according to the relative phases of the different harmonic components, and though Helmholtz has shown that this difference will be small, there is some experimental evidence to show that waves having the same harmonic components in different relative positions are sometimes distinguishable by the ear. It is therefore best to state that the quality of the sensation depends on the wave-form of the waves which produce it, but that waves of different wave-form produce *nearly* the same sensation if there is no difference except in relative position (phase) between the harmonic curves of which the different wave-forms can be built up. The sensation produced by harmonic waves is by some called a *tone*, that due to non-harmonic but periodic waves a *note*. A note may be regarded as a number of tones heard at the same time.

62. Limits of Audible Sound.—Waves arriving with a frequency greater than 38,000 per second produce no sensation of sound. Harmonic waves of smaller frequency than about 33 per second also produce no sensation,* but non-harmonic waves of lower frequency than 33, if they have a harmonic component of greater frequency than 33, may produce the sensation corresponding to the frequency of that component. These limits vary considerably in different persons.

The ear is very sensitive to differences of frequency between sounds of frequencies such as are commonly produced by the voice; a change of $\frac{1}{4}$ per cent. in the frequency of such sounds is easily detected. Outside the limits of the voice, the sensitiveness of the ear to changes of frequency is much smaller.

63. Musical Sound and Noise.—When a series of travelling condensations and rarefactions is irregular, and cannot be divided into waves equal in length and similar in wave-form, the sensation produced is that of *noise*, and has no recognisable pitch. This may be shown by pressing a card against a revolving wheel notched at the edge with irregular teeth of different sizes.

64. Loudness.—Differences of intensity in the waves reaching the ear produce differences of *loudness* in the sound heard, but no simple relation between the intensity and the loudness can be stated, loudness, indeed, like all sensations, not admit-

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* It is sometimes stated that the waves, when very slow, are heard as separate shocks, but there seems no evidence of this, at any rate in the case of harmonic waves.

ting of quantitative measurement. Of two wave-systems of the same frequency and wave-form, the one of greater intensity sounds the louder, but this is not necessarily the case if the frequencies or wave-forms are different. Waves of very low and very high frequency (near the limits of audible frequency) sound not nearly as loud as waves of the same intensity which are nearer the middle of the audible range.

65. Discord.—When two sources sound together which would, separately, produce harmonic waves of nearly equal length, the actual wave-system consists of alternate groups of more and less intense waves (Art. 42). As these reach the ear, the sound heard keeps increasing and diminishing in loudness; these variations are called *beats*. These beats may be considered a consequence of the fact, explained above, that there is not much difference between the intensity of the resonant vibration of the rods of Corti caused by waves of exactly their own free-vibration period and that caused by waves of nearly their own period. So that, if two sources of sound have nearly equal vibration frequencies, there are rods which either of the sources would set in resonant vibration, the frequency of this vibration being always that of the source, not the free-vibration frequency of the rod (Art. 48). The movements which these rods would be executing if one of the sources was vibrating alone are sometimes in the same direction, sometimes in the opposite direction, to those which they would be executing if the other source was vibrating alone, and the actual movement of these rods keeps increasing and diminishing.

From this it will be seen that no beats will be heard if the frequencies of the sources are so different that they do not cause resonant vibration of the same rods. This is found to be the case when the difference of frequencies is more than $\frac{1}{5}$ of the smaller frequency.

These constant variations in the intensity of the sound, like the variations in the light of a flickering or "bobbing" flame, are very unpleasant within certain limits of frequency. These limits are different for sounds of different pitch. With very low or very high notes, the beats are hardly noticed. For notes of medium pitch (within the range of the voice) they are hardly distinguished if less frequent than 2 per second. From 2 to 10 per second, each beat is heard separately, but the effect is not very unpleasant. Above 10 per second the beats are no longer heard separately, but produce the peculiar jarring sensation known as *discord.* The unpleasantness of this increases with the frequency of the beats up to a certain point, about 30 beats per second for the middle of the voice-range. The effect then becomes less unpleasant, with increasing frequency, and becomes imperceptible when the beats are more than 1 to every 4 vibrations, or when they are more than about 80 per second, whichever is first reached.

As each rod is set in vibration by harmonic components nearly agreeing with itself in frequency, just as if these components were the whole sound, we have the sensation of beats or discord when two non-harmonic wave-systems arrive together, if any two among their harmonic components have frequencies whose differences are within the limits just given.

In consequence of this, non-harmonic wave-systems are often discordant when harmonic systems of the same frequencies would be concordant. A tuning fork vibrating gently 200 time a second sounds quite harmonious with one of any frequency above 240, since they do not affect the same fibres. But a note of frequency 200 and one of frequency 280 on a piano would be discordant, since the third harmonic, 600, in the sound from the first would be discordant with the second harmonic, 560, in the sound from the second. Indeed, if we assume that all the possible harmonics are present in each note, we shall find that there will be discord between almost any two notes of moderate frequency, and even between the different harmonics of the same note if it is low enough. But if only the three or four lowest harmonics of each note are intense enough to affect the ear, we shall find that a combination of two notes of moderate frequencies is usually concordant if the frequencies are in a simple ratio, and discordant if not. Thus 200 and 280 are discordant if the harmonics up to the third are intense enough to affect the ear, but 200 and 250 have no discordant harmonics below 800 and 750, so that there is no discord unless the fourth harmonic of the lower note is fairly strong, while 200 and 300 or 200 and 400 have no discordant harmonics at all.

Though the concord or discord of two notes thus depends chiefly on their frequency-ratio and the harmonics present, it also depends to some extent (owing to the fact that beats more frequent than about 80 per second do not produce noticeable discord) on absolute frequency. Thus 200 and 300 are concordant whatever harmonics are present, but 100 and 150, the same ratio, are discordant if the first four harmonics of each are present, and in general the lower the notes the simpler must be the ratio, or the fewer the harmonics present, to avoid discord.

When the frequency of one note is double that of another, the first is said to be an *octave* above the second, for a reason which will appear shortly, and all notes whose frequencies lie between any number n and $2n$ (inclusive) are said to be "in the same octave."

When one note is an octave above another, all the harmonic components of the first are among the possible harmonic components of the second; if the two notes contain many harmonics, there is a strong similarity between the sensations produced, so that from the point of view of sensation the first is often considered to be not a different note from the second, but "the same note an octave higher." This may be shown by fixing on a revolving axle three toothed wheels, having respectively say 60, 120, and 130 teeth. If a card is held in quick succession against the first and second, a resemblance will be noted in the sounds produced which is quite absent if we press the card successively against the first and third, or the second and third, although in the last case the notes are much nearer in pitch than those produced by the first and second. For this reason, two such notes are called in music by the same name and are for most purposes regarded as the same note.

Any notes of the same name can be sounded together without discord; indeed, this simply amounts to strengthening some of the harmonic components of the lowest, so that the result is a note of the same pitch as the lowest, but of different quality. But any other combination of notes within the same octave has harmonic components whose frequencies are not multiples of the lowest, and is different in its effect from a note of any quality.

66. Musical Scales.—A combination of two or more notes sounded together is called a *chord*. If it produces the jarring effect described above it is called a discord, if not, a *concord*. Though a discord by itself, or if long continued, is unpleasant, a series of discords following each other according to certain rules may be pleasant, and modern musical compositions usually consist largely of discords. But it is essential that a series of discords should lead up to a concord; concords form an essential part of all musical compositions in which more than one note is sounded at once. The *scale*, a series of notes used in a musical composition, must therefore be so chosen that concords can be formed from it. All musical scales consist of notes related to each other, directly or indirectly, by simple frequency-ratios. This is not entirely due to the difficulty of combining other notes without discord, for before harmony, or the pleasing combination of notes, was attempted it had been found that melody, or the pleasing succession of notes, required the choice of notes which are now known to be related by simple frequency-ratios. This, of course, cannot depend on beats, and is probably due to the fact that when the notes used are related in this way, the same frequencies often recur, in various combinations, among the harmonics. Both for melody and for harmony, then, we must have simple relations between the frequencies of the notes used, but the conditions for the two are by no means identical, and a scale may be quite suitable for melody, and not for harmony. As we saw above, it is also a condition for harmony (though not for melody) that we should be able to produce the notes used without intense harmonics higher than the third; notes without any harmonics higher than the fundamental, however, are wanting in expression.

In any musical scale the frequencies of all the notes are related, directly or through other notes, to the frequency of one note which is called the *key-note* or *tonic* of the scale. All scales, ancient and modern, include the notes whose frequencies are $\frac{3}{2}$ and $\frac{4}{3}$ of that of the tonic; these are called the *dominant* and *subdominant* respectively. The other notes of the scale have been very variously chosen, both as to number and relation, in different times and countries; most are more directly related to the dominant and subdominant than to the tonic. The scales now used in western Europe are made, with various slight modifications, from a series of notes whose frequencies are in the ratios of the numbers

24, 27, 30, 32, 36, 40, 45, 48,

the last of these being the lowest note of a similar series having the same ratios, and so on, both upwards and downwards. On the system called the "moveable Do" system, these notes are called (whatever their actual frequencies) by the names

Do, Re, Mi, Fa, Sol, La, Si, Do,

the names being repeated in the octaves above and below. (Other spellings of these names are also used, and Te is often written for Si, and Ut for Do, the latter especially in foreign works.) When it is necessary to distinguish between notes of the same name in differest octaves, this is done by dashes above or below the name; thus Mi″ means a note two octaves above the note indicated by Mi.

Any one of these notes might be chosen as tonic, but the scales which have survived as those in which the Do and the La are respectively chosen as tonic. If the Do is chosen as tonic, the scale is called the *major diatonic scale.* Its notes have to the key-note the ratios

Do	Re	Mi	Fa	Sol	La	Si	Do
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

If the La is chosen as key-note, there is no note having a frequency exactly $\frac{4}{3}$ that of the key-note, so the Re is slightly modified. The ratios of the notes to the key-note are then

La	Si	Do	Re	Mi	Fa	Sol	La
1	$\frac{9}{8}$	$\frac{6}{5}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{8}{5}$	$\frac{9}{5}$	2

This is the original form of the *minor diatonic scale.* In modern music the $\frac{9}{5}$ is usually changed to $\frac{15}{8}$, and the $\frac{8}{5}$

sometimes changed to $\frac{5}{3}$, making these notes the same as in the major scale.

When two notes whose frequencies are in a simple ratio are sounded, either together or in turn, a special relation is perceived between them, owing to the fact that some of their harmonics are identical. This relation is always recognisably the same for the same ratio, when the actual frequencies are changed, and is different for each different ratio; it is called the interval between the notes. The chief intervals occur among the relations of the notes of the diatonic scales to the key-note, and are named from the positions of these notes in the scales, the key-note being counted as 1.

The interval between any two notes of the diatonic scale is named according to the number of notes of that scale from one to the other, both inclusive; thus the interval between Mi and La is called a fourth. When two different intervals have the same name, the larger is called *major* and the smaller *minor*; thus the interval between Do and Mi (ratio 5 : 4) is called a major third, while that between Mi and Sol (ratio 6 : 5) is called a minor third. Where one of two nearly equal intervals is much simpler than the other, the simpler is distinguished as *perfect;* thus the interval between Do and Sol (ratio 3 : 2) is called a perfect fifth, while the interval between Si and Fa′ (ratio 64 : 45) is a diminished fifth. The diatonic scale with its notes in the exact ratios given above, and the intervals of that scale, are called *just* to distinguish them from tempered scales and intervals such as those described in the next section. The terms eighth and second are not used; 2 : 1 is called an octave, 9 : 8 a major tone, 10 : 9 a minor tone (*i.e.* the relation between two notes of the same frequency) unison.

If there are any three notes, x, y, z, in ascending order, the interval between x and z is called the *sum* of the interval between x and y, and the interval between y and z. If N_x, N_y, N_z are the frequencies of the three notes, $\frac{N_z}{N_x} = \frac{N_y}{N_x} \times \frac{N_z}{N_y}$, or the ratio corresponding to the sum of two intervals is the *product* of the ratios corresponding to the intervals themselves. Thus the sum of a perfect fifth and a perfect fourth is an octave, since $\frac{3}{2} \times \frac{4}{3} = \frac{2}{1}$. For many purposes it is more convenient to take the *logarithm* of the ratio of the frequencies as the numerical measure of the interval. Then the logarithmic measure of the sum of two intervals is the sum of the logarithmic measures of the intervals themselves. Thus a major third added to a perfect fourth makes a major sixth, since $\log \frac{5}{4} + \log \frac{4}{3} = \log \frac{5}{3}$, and a major ninth is twice as great as a perfect fifth, since $\log \frac{9}{4} = 2 \log \frac{3}{2}$.

The only combinations of more than two notes within the same octave which involve no dissonance, if the first three harmonics of each note are present, are in the ratios 4 : 5 : 6 and 10 : 12 : 15. The former is called the *major triad*, the latter the *minor triad*. The diatonic series includes (in each octave) three major triads, viz.:— Do, Mi, Sol; Fa, La, Do′; Sol, Si, Re′, and three minor triads, La_1, Do, Mi; Re, Fa, La; Mi, Sol, Si; the second of these being very slightly inaccurate unless the Re is modified. In either the major or minor scale the tonic, subdominant, and dominant are each the lowest note of the triad, major or minor; these triads are called the tonic, subdominant, and dominant triads respectively. In either scale every note forms part of at least one of these triads. In the major scale these triads are all major; in the minor scale the tonic triad is minor, and the others may be so.

Each triad consists of a major and a minor third, but in the major triad the major third is below the minor, and *vice versa*.

Of the many other scales which were used before the development of harmony led to the disuse of scales which did not allow a sufficient variety of concords, the most important was the Pythagorean, whose notes have to the key-note the ratios

$$1 \quad \frac{9}{8} \quad \frac{81}{64} \quad \frac{4}{3} \quad \frac{3}{2} \quad \frac{27}{16} \quad \frac{243}{128} \quad 2,$$

every note of which is derived from the key-note by taking octaves and fifths upwards or downwards. An approximation to this scale is often used by soloists, using instruments without fixed notes, like the violin.

The musical effect of a composition depends chiefly on the order and duration of the chords and intervals which succeed each other, *i.e.* on the *relative*, not the absolute, frequencies of the notes, at least to the extent that the composition is recognisably the same when the frequencies of all the notes are raised or lowered in the same ratio. Thus any series of notes of proper durations, whose frequencies have the ratios of 1, $\frac{15}{16}$, $\frac{5}{6}$, $\frac{15}{16}$, 1, $\frac{9}{8}$, $\frac{5}{4}$, 1, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{9}{8}$, 1, $\frac{15}{16}$, $\frac{5}{6}$, $\frac{15}{16}$, $\frac{3}{4}$, etc., is recognised as the air "Men of Harlech." All these are diatonic intervals, so that if we had an instrument whose notes formed a diationic scale (having, for instance, notes of the frequencies 240, 270, 300, 320, 360, 400, 450, with the doubles and halves of each of these, and so on), we could produce on it the air "Men of Harlech" and many other simple compositions. But an instrument with only these notes would be very restricted in its use. We could produce on it the air "Men of Harlech" by sounding notes of frequencies 240, 225, 200, 225, etc., or if we preferred it higher, we might use notes of twice these frequencies, but we cannot get notes, having the right ratios, of any intermediate frequencies. The pleasurable effect of a musical composition depends largely on the appropriateness of the absolute pitch of the notes used, and absolute pitch is of still greater importance when the instrument is to accompany the voice, since a small change of absolute pitch may make some of the notes too high or too low for the singer to produce. It is therefore very probable that neither the lower nor the higher series of notes given above would be satisfactory, though either would give the required air. To be satisfactory in this respect an instrument would require to be able to produce the notes of any one of at least six diatonic scales. Besides this, for any but the simplest musical effects, it is necessary to be able to pass from one scale to another whose key-note is related to that of the first, and so on, and in most music notes are used occasionally which do not belong to the diatonic scale. To fulfil all these conditions, and yet retain the exact ratios of the notes in each scale, we should require not less than twenty notes in each octave, a number not easily practicable in instruments with fixed notes. In such instruments, therefore, a tempered scale is used.

67. Tempered Scales.—A tempered scale is one in which the frequencies of the notes are not exactly in simple ratios; by sacrificing this advantage it is possible to make a sufficient variety of scales out of only twelve notes to the octave. The temperament most in use is the equal temperament.

The equal temperament scale consists of a series of notes having the ratios

	Do	Re	Mi	Fa	Sol	La	Si	Do
	1	$2^{\frac{2}{12}}$	$2^{\frac{4}{12}}$	$2^{\frac{5}{12}}$	$2^{\frac{7}{12}}$	$2^{\frac{9}{12}}$	$2^{\frac{11}{12}}$	2
or	1	1·123	1·260	1·335	1·498	1·682	1·888	2

If we compare these with the just diatonic ratios

(1) 1·125 1·250 1·333 1·500 1·667 1·875 2,

we see that none of them differ by as much as 1 per cent., so that when two notes of the tempered scale which correspond to a perfectly harmonious combination on the just scale are sounded together the beats are not rapid enough to be disagreeable. It is this very remarkable approximation of the powers of $\sqrt[12]{2}$ to simple fractions which makes an equally tempered scale possible. The most important intervals, the fourth and the fifth, are, by a curious coincidence, the nearest to those of the just scale, being within about $\frac{1}{5}$ per cent.

In key-board instruments a series of notes is provided, the frequency of each of which is $2^{\frac{1}{12}}$ times that of the note below it; any one of these notes may be taken as the key-note of either a major or a minor scale.

Thus if we take any note of the series as Do, and count upwards, reckoning it as first, the third, fifth, sixth, eighth, tenth, twelfth, and thirteenth notes of the series will correspond very nearly to the Re, Mi, Fa, Sol, La, Si and Do′ of the just distance scale, so nearly that most persons do not detect the difference. (The Re of this scale is between the Re of the just major scale and the modified Re of the just minor scale, and serves for either.) So that from this series of notes we can make twelve major and twelve minor scales, whose key-notes differ in frequency, each from the next, by less than six per cent., and we can take any note of any scale which we are using, as the keynote of a fresh scale, which may be either major or minor.

It is usual to let one of the notes have a frequency about 260, and to denote it by C, and the notes of the major scale which has it as key-note by the letters D, E, F, G, A, B. (The notes of this scale are the white notes of the piano.) The remaining notes are denoted by the letter of the note below with the sign ♯ (sharp) or by that of the note above with the sign ♭ (flat.)

68. Resultant Tones.—It was shown in Art. 55 that if there are two bodies, vibrating in the same air with frequencies n_1 and n_2, the pressure-change in the air will have components of frequencies $n_1 + n_2$ and $n_1 - n_2$, besides multiples of n_1 and n_2, and these will of course produce the corresponding sensations. These sensations are called *physical resultant tones*, that corresponding to $n_1 - n_2$ being called the *difference tone*, and the other, which is less intense, being called the *summation tone*.

Since the physical resultant tones depend on the amplitude of vibration being too large for the principle of superposition to be applied, they must be of extremely small intensity except when the same air is strongly agitated by both the sources of sound. In fact the tuning-fork method, described in Art. 106, failed to detect them in any case in which the sources were really independent, even though near together, though they were easily detected when two strings vibrated on the same sounding box, or when air was blown from the same air-chamber through two rows of holes in a rotating disc. The tuning-fork can detect sounds far too faint to affect the ear, yet the ear can often hear resultant tones which the fork does not show. It seems to follow that resultant tones are sometimes produced in the ear itself, and Helmholtz had pointed out that this would be the case. It can be shown that if a pressure-change, whose harmonic components are n_1 and n_2 occurs at the surface of a membrane which is more easily displaced one way than the other, the forced vibrations of the membrane will have harmonic components of frequencies $n_1 - n_2$ and $n_1 + n_2$ as well as n_1 and n_2. The tympanum is such a membrane. The new harmonic components introduced by it are called *physiological resultant tones.* It seems to be proved by the experiments just mentioned that the summation and difference tones heard when two independent sources vibrate are almost entirely ear-made.

Both physical and physiological resultant tones increase in intensity in a much greater ratio than the sounds due to the sources separately, so that they are much more easily detected when the sounds themselves are loud.

Owing to resultant tones, it often happens that two notes are discordant when sounded together, which on the principles explained in Art. 65 we should expect to be concordant. Thus closed cylindrical organ pipes (which produce only the odd harmonics) of frequencies 400 and 780 have no harmonics which can clash, but the difference tone 380 is discordant with 400, and the summation tone 1180 with the odd harmonic 1200. Discord depending on resultant tones differs from discord depending on harmonics in being hardly noticeable unless the sound is loud.

The summation tone may be audible when both notes are below the limits of audible frequency, and the difference tone when both notes are above the limit.

The difference tone is sometimes used in music when a lower note is required than the instrument can produce. Thus organ pipes of frequencies 80 and 120, sounded together, produce a note of frequency 40. This method can only be applied with loud sounds.

69. **Difference Tones and Beats.**—The difference tone agrees in frequency with the beats due to the two sources, and, before Helmholtz's investigation, it was usual to explain the difference tone by the supposition that beats (variations of intensity) when they became too rapid to be heard separately, produced the same sensation of tone as variations of pressure of the same frequency. This of course left the summation tone unexplained. The statement that beats produce a tone is still sometimes made, but it is difficult to attach a distinct meaning to it. If we produce beats of frequency n by sounding together two sources whose frequencies differ by n, we necessarily

produce a difference tone of that frequency, while there is no other known mode of making the intensity of a sound vary n times a second in which the pressure change produced has not a harmonic component of frequency n. For instance, if the sound from a tuning-fork of frequency 1000 is allowed to pass through a hole which is opened and partly closed ten times a second, the sound on the other side of the hole becomes louder and fainter ten times a second, so that beats of that frequency are heard. The pressure-change in the air beyond the hole repeats itself completely only ten times a second, since though the pressure actually rises and falls 1000 times a second all these pressure-changes are not equal. The actual pressure-change, repeated completely ten times per second, has a harmonic component of that frequency (Art. 52). If the hole is opened and closed 100 times a second, the beats are too rapid to be heard separately, and a note of frequency 100 is heard together with the note of the fork, but as there is now a harmonic component of frequency 100 present it is not clear that the note should be attributed to the beats. The only clear meaning of the statement that beats produce a tone is that when two sources vibrate together with frequencies whose difference is n, n being within the limits of audible frequency for vibrations, a note is heard, corresponding in pitch to n vibrations, which does not increase in loudness, when the sounds are increased, more rapidly than the sounds, as the difference tone does. But there seems no evidence of this.

If, instead of adding two harmonic curves where bends are of equal heights, as in Fig. 32, we add two harmonic curves one of which has higher bends than the other, the resultant curve is like the third line in Fig. 32, but the crests are not equidistant, but are closer in the more intense parts than in the less intense parts, or *vice versa*. It has sometimes been supposed from this that there would be a difference of pitch, as well as of loudness, between the more intense and the less intense parts of the waves from two unequal sources of different frequencies. On the resonant theory of the ear, there should be no difference due to this cause, since the less and more intense parts have the same harmonic components, and the effect depends on the harmonic components of the pressure change, not on how many times a second the actual pressure rises and falls. The difference seems to be actually heard, but another possible explanation is that the resonant bodies of the ear (like stretched strings) have not the same free frequency for large as for small vibrations, so that the fibre which resounds most strongly to a loud sound is not the fibre which resounds most strongly to a faint sound of the same frequency.

CHAPTER VIII.

REFLECTION OF SOUND.

70. Reflection with Change of Sign. — In Art. 11 we described the transmission of a condensed or rarefied condition along a row of elastic bodies such as the carriages of a train. We will now consider what will happen when the pulse reaches the last carriage, which we will call *Z*. As in the latter part of Art. 11, we will suppose *Z* in contact with a fixed obstacle, such as the end of a siding, and that all the carriages are to some degree compressed between the engine and this obstacle. We will also suppose ourselves looking from a position such that the direction from *A* to *Z* is from right to left, as in Fig. 16.

When a condensation, produced in *A* by a push of the engine, reaches *Z*, *Z* moves in its turn to the left, compressing the buffers between it and the end of the siding. When *Z* comes to rest, these are more compressed than those between *Y* and *Z*, and therefore *Z* begins to move back again, and stops only when it has transferred its condensed condition to *Y*. Then *Y* moves in the same direction, and in this way the condensation travels back again to *A*, each carriage moving in turn a short distance towards the engine.

In the same way, if a rarefaction travelled from *A* to *Z*, *Z* would move in its turn towards *A*, but, as the end of the siding is fixed, this leaves the pressure between *Z* and the end of the siding less than that between *Z* and *Y*, and so *Z* moves back, *Y* follows, and a rarefaction travels back to *A*, each carriage moving a short distance away from the engine.

In these cases it will be noticed that the condition that travels back is of the *same* kind as that which travelled to *Z* from *A*, but the *direction* in which the carriages themselves move is *reversed* when the wave travels back again. When this is the case, the wave is said to be reflected *with change of sign*. It should, however, be noticed, that as the direction of movement of a wave is determined when the direction of movement of the substance and the kind of pulse (condensation or rarefaction) are given, the wave can only be reflected by reversing the sign of one of these. So that "reflection with change of sign" is really reflection in which it is the velocity and not the condensation of the undulating substance which is reversed in sign.

Reflection with change of sign takes place not only at a fixed obstacle, but at any point where the movement of a smaller mass is transferred to a larger one. If there is a row of trucks of which, say, those from A to F are empty, and the rest loaded, then, when a condensation produced in A reaches F, a smaller condensation travels on along the rest of the train, while another condensation travels back from F to A. The energy in these two condensations is equal to that in the original condensation.

In exactly the same way condensations or rarefaction travelling through the air (or any substance) are reflected when they reach a substance of greater density. The greater and more sudden the change of density the larger the proportion of the energy which is reflected.

71. Reflection without Change of Sign.—When the movement of one portion of an undulating substance is communicated to another of smaller mass, a different kind of reflection takes place. Suppose that there is a series of trucks of which those from A to F are loaded, while the rest are unloaded, and that a push is given to A which compresses it. Each truck from A to E moves in turn to the left, and is brought to rest by the increasing pressure between it and the next. When F moves on in its turn, and the pressure between F and G begins to increase, G, being lighter, moves on more quickly than the heavier trucks did, so that the pressure between F and G does not increase so fast as that between E and F did; F therefore moves further than the other loaded trucks before it comes to rest, and G moves the same distance as F. This evidently leaves F in an expanded or rarefied condition, and also leaves a greater space between F and E

than between E and D. E therefore begins to move towards F, expanding its buffers as it does so, and then D towards E, and so on, and in this way a rarefied condition travels back from F to A, while a condensed condition travels forward from G along the rest of the train. As before, these two contain together the energy of the original condensation.

In this case it is to be noticed that the reflected pulse is of the opposite kind to the original one, rarefaction instead of condensation, but that the movement of the trucks when the reflected pulse reaches them is in the *same* direction as it was when the original condensation reached them. The condensation is reversed in sign, the velocity is not. This is called reflection *without change of sign*. A rarefaction arriving at F starts a condensation back in the same way.

Reflection of this kind happens when a wave which has been travelling in a dense medium reaches a rarer one, for instance, when sound waves produced under water reach the surface, or when waves travelling in air reach the surface of a gas flame. Every condensation that arrives starts a rarefaction back, and *vice versâ*.

In either kind of reflection, each pulse which arrives at the second medium sends a pulse forward into the second medium as well as one back again through the first, though the forward pulse may be of very small intensity if the difference of density is great. This forward pulse in the second medium is of the same kind as the original pulse in the first medium, neither the kind of pulse nor the direction of movement of the substance being reversed in it.

72. Reflection in Tubes.—As might be expected, the first kind of reflection occurs when sound waves travelling along a tube reach its closed end; each pulse as it arrives starts one of the same kind back. The second kind of reflection occurs when sound waves passing along a tube reach its *open* end.

It is difficult at first to see why this should happen, since the air beyond the end of the tube is not less dense than that in the tube, but the reason is somewhat as follows:—In the case described in Art. 71 the reason why a condensation reaching F started a rarefaction back again was that when F moved on, the pressure in front did not increase so fast as it had done with the other heavy trucks, so that *F* moved further than they did. The same occurs when a pulse comes to the open end of a tube. Let *AB* (Fig. 36) be a tube

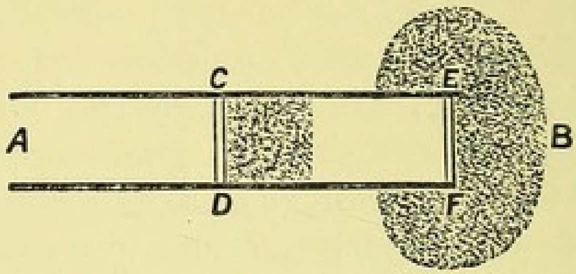


Fig. 36.

of 1 sq. cm. area of cross section, and let *CD* be a thin layer of air in the tube. Suppose *CD* displaced 1 mm. to the right in $\frac{1}{33000}$ sec. At the end of this interval the effect will not have spread beyond 1 cm. of the tube on each side of *CD*, so that an extra hundred cubic millimetres of air have been forced into the cubic centimetre of space immediately to the right of *CD*, and the pressure there increased accordingly. But if a layer *EF* at the end of the tube moved a millimetre to the right in $\frac{1}{33000}$ sec., at the end of this interval the condensation would affect all the air within a radius of 1 cm. from the end of the tube, and only the same volume of extra air, 100 cubic mm., has been added to this much larger space. The rise of pressure to the right of *EF* is therefore not nearly so great as it was in the case of *CD*. So that when a wave of condensation passes along *AB*, and each layer of air passes on its energy to the next and comes to rest itself, *EF* will not have expended all the energy passed on to it by the previous layer when it has moved the same distance as the other layers moved. *EF* will therefore move further than the other layers did, and so, while it transmits a condensation to the air in front, it will leave a rarefaction behind it, which will travel back along the tube just as in the case of the railway trucks. In the same way, a rarefaction reaching the end of the tube spreads a rarefaction through the air beyond the end, but starts a condensation back along the tube.

73. Illustrative Experiments.—Take a coil of hard brass

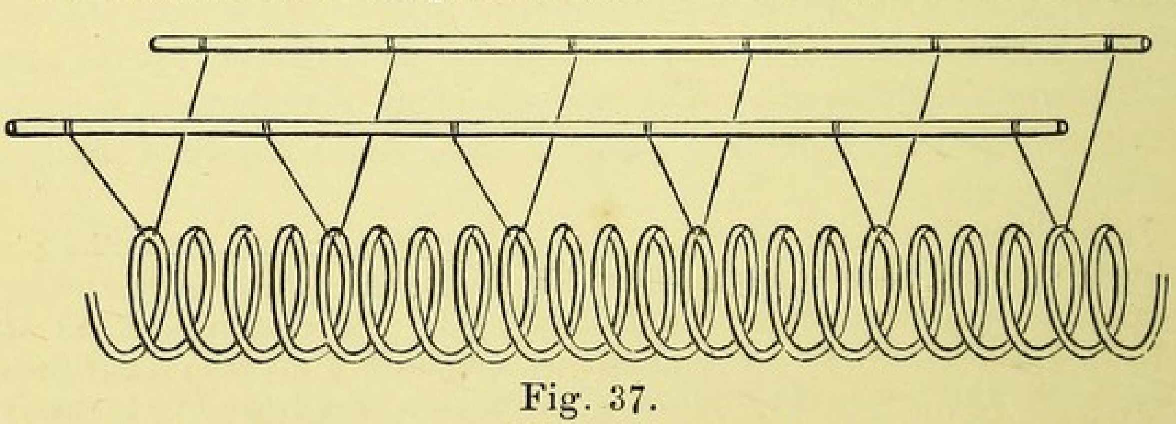


Fig. 37.

wire about 20 gauge, having (as it usually has when sold) a diameter of about 5 inches. Separate the coils so as to form a close spiral, and suspend every fourth or fifth ring by two threads to two parallel wooden rods 8 or 10 feet long, as shown in Fig. 37, arranging the threads so that the turns of wire hang about an inch apart. The longer the threads are, the better; the distance between the rods may be about equal to the length of the threads. (To make the figure clearer the size and distance of the coils of wire are much exaggerated.)

If one end of this spiral is fixed to a wall, and condensations and rarefactions sent along it as described in Art. 19, each pulse when it reaches the wall sends a pulse of the same kind back. But when this pulse reaches the free end from which it started, it is reflected again as a pulse of the opposite kind.

The reflection of sound waves from the surface of a denser medium hardly needs experimental illustration; echoes are a familiar instance of it. Reflection from a rarer medium may be illustrated by the following experiment:—

Arrange two tubes, each about 3 inches in diameter and 3 or 4 feet long, at right angles to each other, as shown in Fig. 38. (Tubes of thick paper are sufficient.) Place a

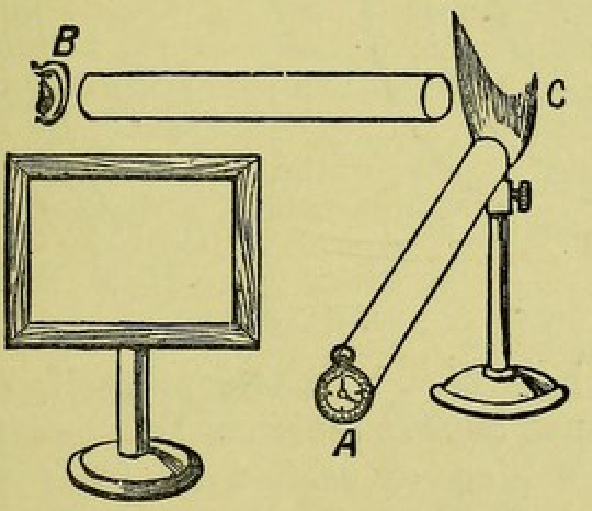
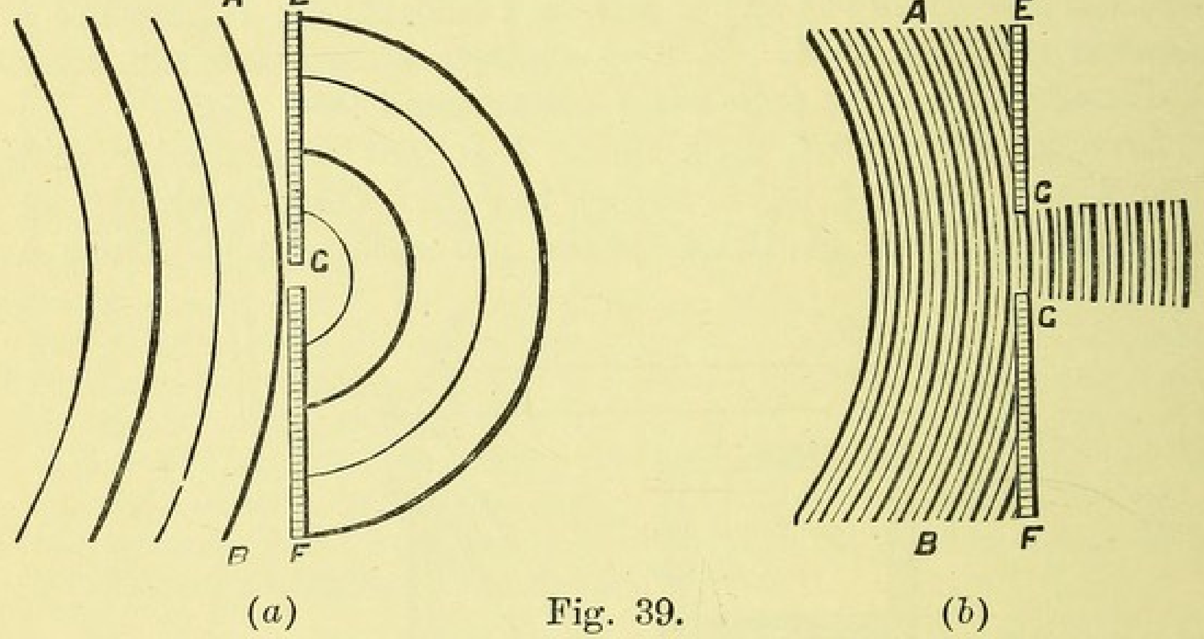


Fig. 38.

watch at *A*, and your ear at *B*, with a large book or some newspapers between *A* and *B* to prevent sound travelling direct. The watch will be nearly inaudible, but the ticking becomes very distinct if a large flat gas flame is placed at *C*, with its plane vertical and inclined at 45° to *AC* and *CB*.

74. Reflection in Open Air.—In the cases of wave-propagation which we have so far considered, the *wave-fronts*, or continuous surfaces drawn through those points where the air is in the same phase of its vibration, have been either complete spheres or have been bounded at the edges by the walls of a tube; in either case the wave-front can only travel at right angles to itself; expansion in its own plane is impossible. But when wave-fronts, expanding through the air, reach a fixed obstacle, only the pieces of the wave-fronts which are stopped by the obstacle are reflected. We require therefore to know how pieces of wave-front travel through the air when they are not confined at the edges by the walls of a tube.

Suppose we have a screen or wall *EF* (Fig. 39) of some material which does not transmit sound, and that in this



(*a*) Fig. 39. (*b*)

screen there is a hole *G*. If waves or shells of condensation and rarefaction, such as *AB*, arrive at this screen from a source to the left of it, a piece of each wave-front passes through the hole, and spreads through the air to the right of the screen. How it spreads depends on the relation between the diameter of the hole and the distance between successive shells of maximum condensation (wave-length). As each shell of condensation arrives at the hole, the air in the hole becomes condensed and moves to the right, exactly as it would do if a solid piston fitting the hole moved to the right, and, if the distance between a wave-front of maximum condensation and one of maximum rarefaction is larger than the diameter of the hole, a shell of condensation spreads spherically in every direction through the air to the right of the screen, exactly like the waves produced by the vibrating piston in Fig. 22 ; rarefactions spread similarly in their turn. This is illustrated in Fig. 39 (*a*) ; the thick circles are wave-fronts of maximum condensation, the thin ones those of maximum rarefaction. In this case the sound is audible at any point. But, if the wave-length is much smaller than the diameter of the hole, the piece of each shell which passes through hardly spreads at all after reaching the other side, but travels at right angles to its own wave-fronts* as shown in Fig. 39 (*b*). In this case no sound is audible except at points from which straight lines can be drawn to the source of the sound ; we have in fact on the further side of the hole a *beam* of sound in a definite direction from the source.

This may be considered as an instance of "interference" (Chap-

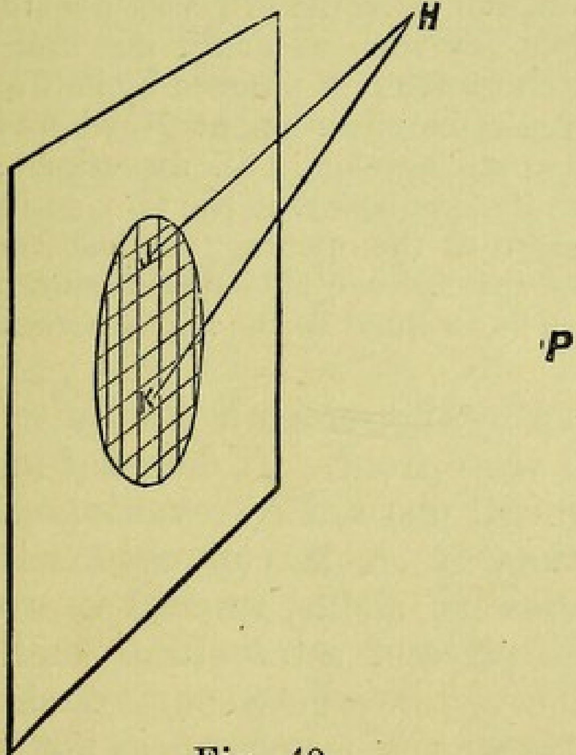


Fig. 40.

ter III.). Let *H* (Fig. 40) be any point from which the source of

* This would also be the case with the waves produced by a vibrating solid surface if they were short compared to the surface, but practically solid surfaces cannot be made to vibrate fast enough for this, so that the waves spread from them in all directions.

sound would not be visible, and let waves be arriving from the left whose wave-length is much smaller than the diameter of the hole. Then the distances of *H* from different parts of the hole differ by more than half a wave-length. We may suppose the hole divided by cross lines into small holes, and we can pair off each of these small holes with another whose distance from *H* is half a wave-length greater or less. (It is not self-evident that they can all be paired off in this way, but this is proved in works on Physical Optics, where the subject of transmission of waves is more fully treated.) Let *J*, *K* be such a pair. Then, if all the opening except *J* was blocked up, waves would spread from *J* in all directions to the right, and a condensation would reach *H* at the same moment that a rarefaction would reach *H* if all the opening except *K* was blocked up, so that, if *J* and *K* are open together and the rest of the opening blocked, there is no vibration at *H*. As this applies to every pair of small holes, there is no vibration at *H* when the whole opening is free.

It is evident that this argument does not apply to a point such as *P*, from which the source would be visible through the opening, for the different parts of the opening are all practically at the same distance from *P*. So that there are waves from the opening in the direction of *P*, but not in the direction of *H*. Also the argument would not apply if the wave-length was greater than twice the diameter of the hole, for then we could not find two parts of the opening whose distances from *H* differed by half a wave-length. In that case there would be vibration at *H*, wherever *H* was taken, that is, the waves would spread in all directions from the opening. There is not a definite line between the two cases; the smaller the wave-length compared to the opening, the less the pieces of wave-surface which come through the hole spread sideways, and the more nearly the vibration is confined to the regions from which the source would be visible.

The part played by the screen is simply to limit the size of the piece of each wave-front. It does not matter what limits the size ; any limited piece of a travelling shell of condensation or rarefaction, if small compared with the distance between two successive shells, spreads in every direction in front of it, but a piece of a travelling shell which is large compared with the distance between two shells advances at right angles to its own surface, and does not spread. As each piece of shell is accompanied by a nearly constant quantity of energy, the intensity of the sound diminishes very slowly with distance when the pieces of shells do not increase in size.

The different behaviour of waves of different lengths is easily shown experimentally. In a sheet of roofing felt, say 3 feet square, cut a round hole 6 inches in diameter. It will be found that the sound of a humming top, placed 2 feet from the hole on one side of the felt, is quite audible at any point on the other, and is not perceptibly louder at points from which the top is visible through the hole than at points in quite a different direction from the hole; the sound is, however, nowhere nearly as loud as if the felt was removed. But if for the top, which gives waves perhaps 4 feet long, we substitute as source of sound a watch, which gives waves from 2 inches to half an inch in length, we shall find that at any point from which the watch is visible through the hole the sound is nearly as loud as if no screen was interposed, while at points from which the watch is not visible it is hardly audible at all.

There is a similar difference between the behaviour of waves of different lengths when an obstacle is interposed between a source of sound and the ear. If the interval between successive shells of condensation is greater, or even not much smaller, than the diameter of the obstacle, the waves close in round the edge into the space behind the obstacle, and the sound is heard at any point. But waves which are very close compared with the diameter of the obstacle do not close in much, but advance at right angles to their own surfaces, each shell advancing with a hole in it where part has been stopped by the obstacle. The obstacle thus casts a *sound shadow* whenever the wave-length of the sound is much less than the dimensions of the obstacle. This, like the corresponding case of the opening in a screen, may be shown to be an instance of interference.

The short waves from a watch are almost entirely cut off from the ear by a quarto magazine interposed, but the longer waves from a small clock are not. This is most strikingly shown by placing the watch between the clock and the ear, so that both are heard at once, but in such a position that the watch sounds much the louder. If a thick quarto magazine is placed in the line from the ear to the two sources of sound, the clock is heard much louder than the watch, even if the watch is audible at all. A large screen (such as a pile of open newspapers) cuts off the sound from both; a packet of post-cards from neither.

When a wave of condensation reaches a plane solid or liquid surface, a wave of condensation begins to travel away from each point of that surface as soon as the original shell of condensation reaches that point. The result is that a reflected shell of condensation is formed, which makes the same angle with the plane surface as the original shell, but slopes in the opposite direction. Some successive stages in the formation

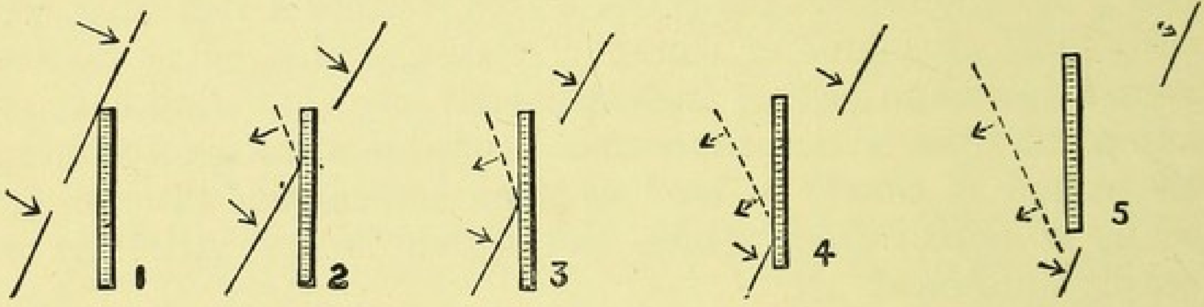


Fig. 41.

of such a reflected shell are shown in Fig. 41, the reflected part being shown by a broken line.

If the piece of each shell which is reflected is large compared with the distance between two condensed shells, the reflected pieces advance at right angles to their own surfaces without spreading, and the effect of the reflected sound is confined to a reflected beam, which follows the same law as the reflection of light. If the condensed waves are farther apart than the diameter of the reflecting surface, the reflected waves spread in every direction from the reflecting surface, and diminish so rapidly in intensity that they cannot be detected at a short distance. The same is true when the reflection is "without change of sign," as from the surface of a gas flame; hence in the experiment shown in Fig. 38 the source of sound must produce very short waves.

A reflected sound is called an *echo*. An echo, to be audible, must be formed by reflection against a surface whose dimensions are very large compared with a wave-length of the sound, for otherwise the sound energy intercepted by the surface is dispersed in all directions, and soon becomes inaudible. The sound of a gun requires a cliff or high wall to form a good echo, but a much smaller surface will give a clear echo of a shrill whistle.

Echoes afford a rough method of finding the velocity of sound. If we shout, and observe the interval before the echo is heard, this interval is the time taken by sound to travel twice the distance from us to the cliff or wall.

When a shell of condensation or rarefaction arrives at a concave solid surface, and is turned back, the reflected shell is concave to the direction in which it is going, if the convexity of the original shell was not too great. Some stages of such reflection are shown in Fig. 42, where the lines

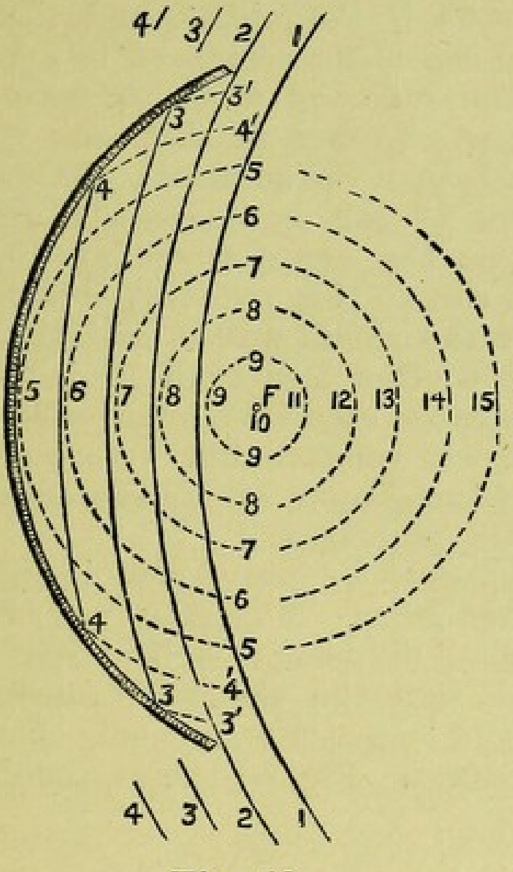


Fig. 42.

marked 1, 2, 3, 4, &c., are successive positions of a shell of (say) condensation which originally started from a source *C*. As each part of the shell reaches the concave surface, a condensed condition starts back, so that when one part of the shell of condensation has reached the position 33, other parts have already started back and reached the position 33′, so that 3′333′ is a continuous shell of condensation, of which the part shown by the complete line is still travelling towards the mirror, while the broken line parts are travelling away. If these reflected portions are large compared with the distance between a condensed and a rarefied shell, they will advance at right angles to their own wave-fronts without spreading, assuming the successive positions 4′444′, 555, &c.,

so that the reflected wave may (if the reflecting surface is rightly shaped) converge towards a point *F*. At this point the intensity of the sound is very great, for the diminishing pieces of shells have always the same energy; the sound is brought to a focus just as light is. After reaching *F* the wave diverges again, on the other side, assuming successively the positions 11, 12, &c.

All this is only true if the mirror, and therefore the reflected pieces of the shells, are large compared to a wave-length. If this is not the case, the reflected wave spreads in every direction, instead of converging. In fact, in that case it makes no practical difference what the form of the mirror is. This seems to be lost sight of by some makers of ear trumpets, who attempt, by parabolic reflectors, to converge the sound waves into the ear. No reflector, whatever its shape, can do this unless it is many times larger than the wave-length of the sound waves, and the waves produced in conversation are not often less than 18 inches long. A funnel-shaped tube concentrates sound to some extent, but it acts not by making the wave-fronts concave, but by communicating the movement and energy of each layer of air to a smaller one, so that the amplitude continually increases.

In Fig. 42, the reflected condensed shell is represented converging till it becomes a mere point. This does not really happen, because before it gets so small it ceases to be large compared with the distance between it and the shells of rarefaction in front and behind, and then it no longer advances only at right angles to itself, but spreads at the edges. Fig. 43 shows, roughly, some successive

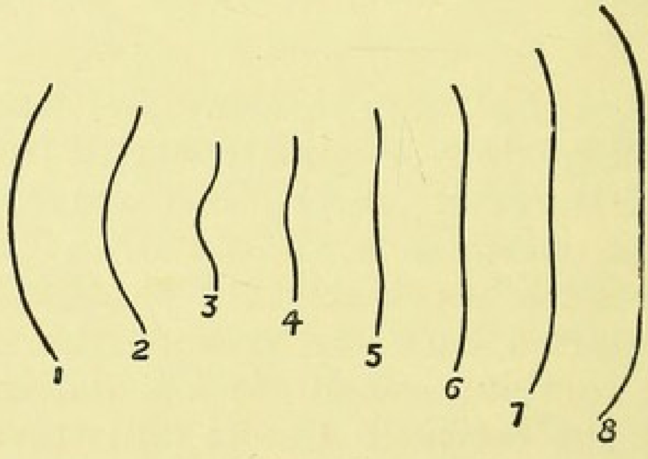


Fig. 43.

stages in the advance of a concave wave-surface which is no longer large compared to a wave-length.

If *F* is the centre of the reflected shells as long as they remain spherical, it is evident that the total distance from *C* to any point of the mirror, and thence to *F*, must be constant, for every part of the same reflected wave-front (999, for instance) started at the same time from *C*, has been travelling with the same velocity ever since, and is now at the same distance from *F*. It can easily be shown that this is equivalent to saying that lines from *C* and *F* to any given point of the mirror make equal angles with the reflecting surface; *i.e.*, *F* and *C* are conjugate foci for light (*Text-book of Light*, Art. 32), so that the point where the sound is most intense is the point where an image of the source is formed if the mirror is polished. In fact, light waves are reflected just as sound waves are when the sound waves are short compared to the reflecting surface.

If the concave surface is a yard across, a watch placed at *C* is heard as loudly by an ear at *F* as if the watch was an inch instead of perhaps several feet away, but there is no increase of loudness due to the mirror at a point a few inches from *F*. If a large tuning-fork is used instead of a watch, the sound will not be perceptibly louder at *F* than at other points.

As before, we may consider the intensity of the sound at *F*, and its absence at points near *F*, to be due to "interference"; to any point except *F* there are routes from *C* (*viâ* the reflector) of different lengths, so that condensation would arrive by one route at the same time as rarefaction by another. As long as all routes from *C* to *F* *viâ* the reflector are too nearly equal for this to happen, it makes little difference to the intensity at *F* whether they are *exactly* equal or not. So that roughnesses or irregularities of the mirror are unimportant if they are much smaller than a wave-length; the mirror need not be polished, but may be of gutta-percha or sheet lead.

75. Refraction of Sound.—Sound waves may also be converged by means of a lens, but the lens must be of gas, not of any denser substance, or nearly all the energy will be reflected at its first surface. If two convex circular sheets of collodion film are cut from a large collodion balloon and attached by their edges to a metal hoop, and the space between them filled with carbon dioxide gas, we have a lens which will answer the purpose. Sound travels more slowly in carbon dioxide than in air, so that when each shell reaches the lens it advances more slowly at its centre than at the edges, as shown in Fig. 44, so that when it comes out at the

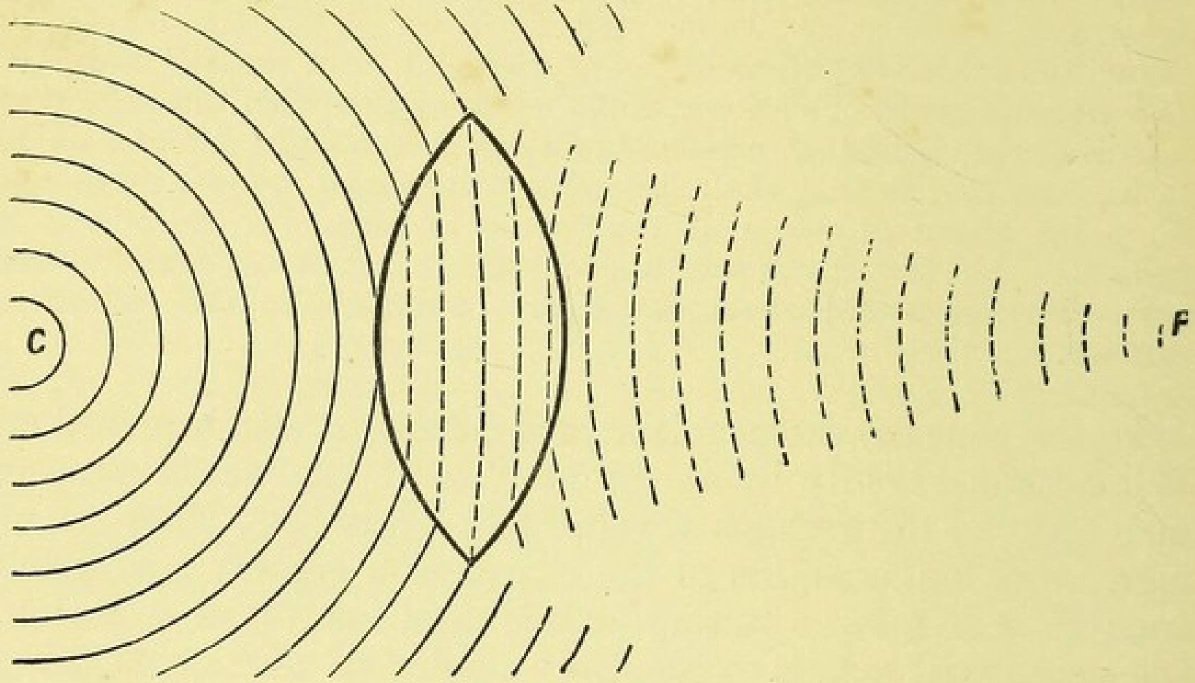


Fig. 44.

other side it is concave instead of convex towards the direction in which it is advancing. If these concave pieces of shells are large compared with the distance between successive waves, they will converge towards a point *F*, though, as explained above, they do not really become points. As in the case of the concave mirror, it is only sound whose wavelength is short compared with the diameter of the lens which can thus be converged; waves of greater length spread in all directions after passing through the lens, as if the lens was a mere hole, and do not converge.

A similar effect is produced by a concave lens filled with a lighter gas than air; coal-gas is best. Hydrogen does not answer well, the difference of density between air and hydrogen being so great that a large part of the energy is reflected on reaching the lens (Art. 72).

Prisms of collodion film filled with carbon dioxide have also been made, and give a deviated beam of sound if the wave-length is short compared to the prism, but there is no dispersion of sound of different wave-lengths, as in the case of light, since all sound waves travel with the same velocity. (The spectrum analysis of sound has, however, been performed with a gigantic diffraction grating).

The statements made above are strictly true only for harmonic waves. If the waves are non-harmonic, we may conveniently apply Fourier's device, and calculate the result of the reflected or refracted waves as if the "harmonic components" had a separate physical existence, and were reflected or refracted each according to its wave-length. The different harmonic components are of course quite differently reflected and refracted; thus a concave mirror converges the shorter harmonic components nearly to one point, while it distributes the longer ones in every direction. The character of non-harmonic waves is therefore often entirely altered by reflection or refraction, or by simply passing an obstacle, having at one place a larger proportion and at another a smaller proportion of harmonic components of very short wave-length.

A simple experiment shows this very well. Hold a watch at arm's length, and interpose a large sheet of card between it and the ear. The sound is practically cut off. Now try with a post-card instead of the large sheet. The sound is not much fainter when the card is interposed, but it is a much lower note, or rather, as a trained ear will recognise, all the higher components, which give the peculiar sharp click, are cut off, and only the lowest of the notes produced is heard. The longer harmonic components come round the card to the ear; the shorter ones are absent behind the card (Art. 74).

76. Effect of Wind.—When there is a wind blowing, the air close to the ground travels more slowly than that higher up, so that on the side of the source towards which the wind

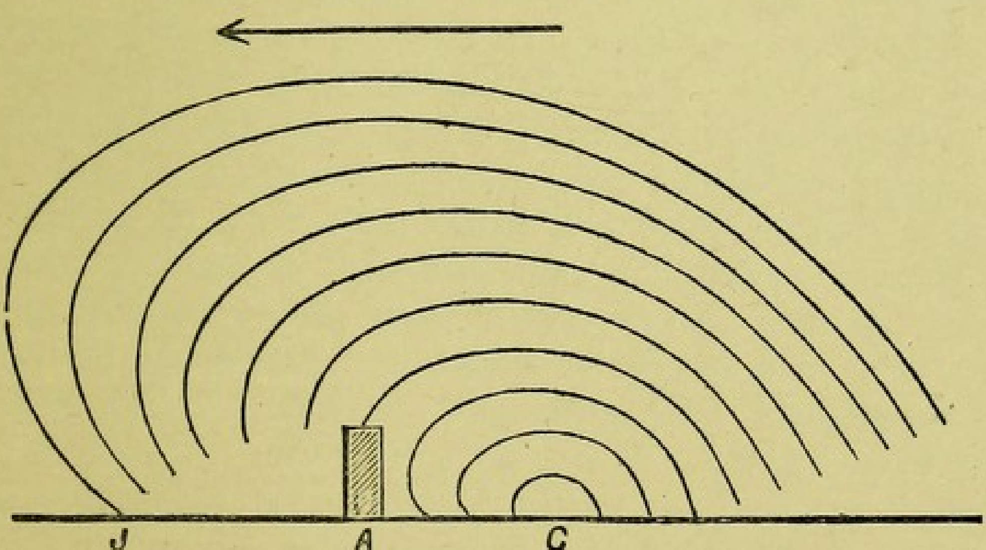


Fig. 45.

blows the waves advance more rapidly above than below.

As the wave-fronts, being large compared with a wave-length, advance at right angles to themselves, an obstacle such as *A*, Fig. 45, will not cut off the sound coming from a source *C*, from an observer at *J*, for the higher parts of the waves, which have passed over *A*, soon begin to advance in a slightly downward direction, and so reach the earth at a point beyond. It is for this reason that sound is much better heard on the side of the source towards which the wind blows, especially if there are obstacles between the source and the listener. In the other direction the waves advance less rapidly above than below, and soon leave the earth entirely, as shown in the figure, where the wind is supposed blowing from right to left. (The difference of velocity above and below is very greatly exaggerated in the figure.)

CHAPTER IX.

STATIONARY UNDULATION.

77. In Chapter IV. we found that, if two sources of sound vibrate at the same time, the air is in the condition of progressive undulation only in the regions in which the waves from the sources, if the sources vibrated one at a time, would travel in the same direction. Of the other vibratory conditions of the air which may exist, the most important is the condition of Stationary Undulation. This is produced in any region of the air if two sources vibrate together which, vibrating separately, would send through the region, in exactly opposite directions, waves of the same length, amplitude, and wave-form. We will first suppose that this wave-form is harmonic.

Let each of the lines *Y*, *Z*, 11, 12, &c., in Fig. 46, represent the same region of air, in different conditions. Suppose that there are two similar vibrating springs *A* and *B* (not shown), one to the left and the other to the right of the region. Let the top line *Y* represent, on the plan explained in Art. 15, the condition in which the air in this region *would have been*, at a given instant *T*, if *A* had been vibrating exactly as it is, while *B* was at rest, and let the second line *Z* represent the condition in which the same air *would have been*, at the same instant *T*, if *B* had been vibrating exactly as it is, while *A* was at rest. *Y* represents a progressive wave-system travelling from left to right, while *Z* represents an exactly similar wave-system travelling from right to left. The actual condition of the air in the region represented, at the instant *T*, will be (very nearly) the *resultant* of the two conditions represented by *Y* and *Z*.

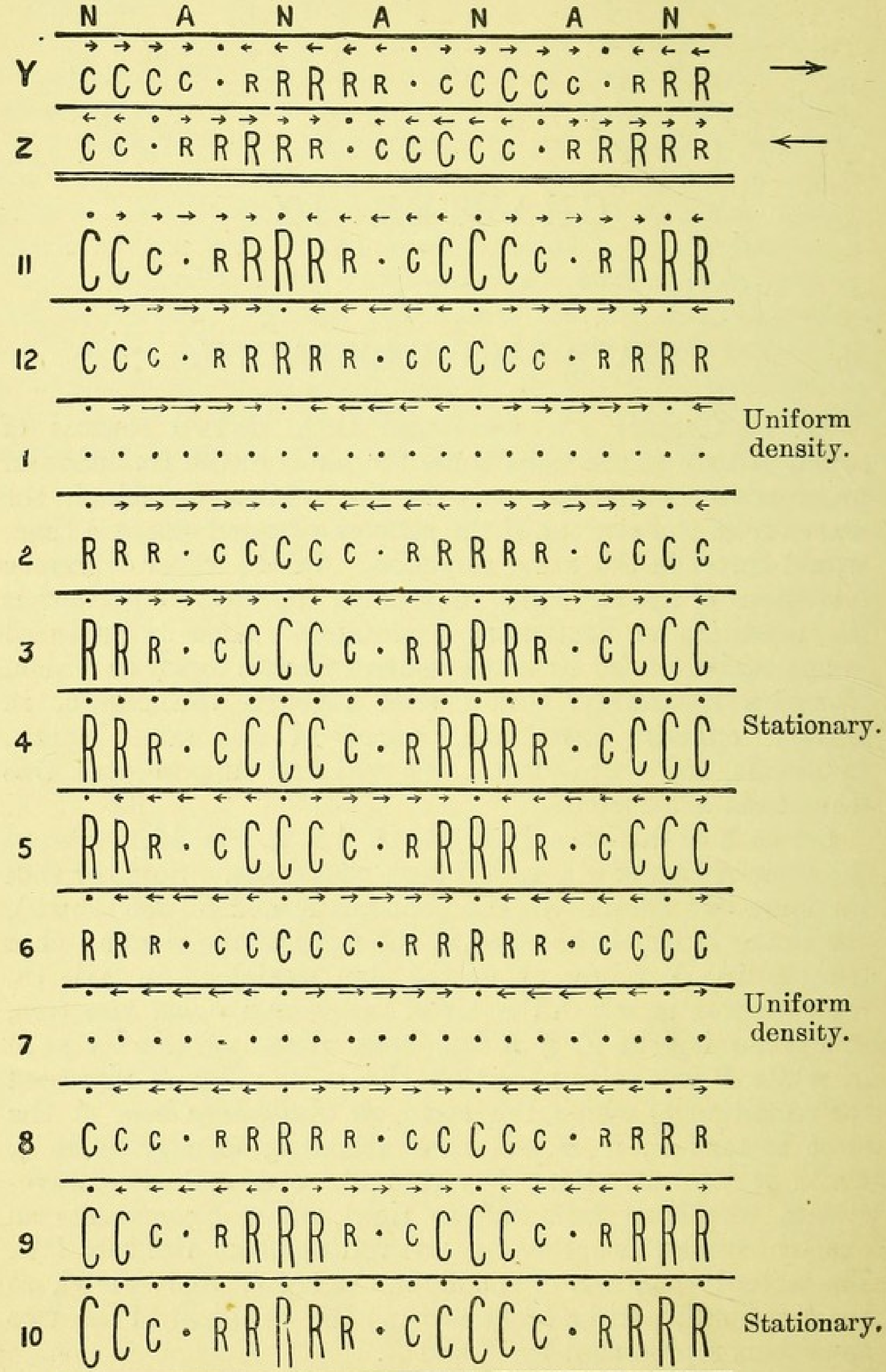

N A N A N A N
Y
Z
11
12
1
Uniform density.
2
3
4
Stationary.
5
6
7
Uniform density.
8
9
10
Stationary.

Fig. 46.

This condition is represented in the third line marked 11. The height of the letter C or R at each point, which denotes the degree of condensation or rarefaction there, is the sum of the heights of the letters at the corresponding points of Y and Z if they are both C or both R, and the difference of their heights if one is C and one R; and the length of the arrow over each letter, which denotes the velocity with which the air is moving at each point, is the sum or difference of the lengths of the arrows at the corresponding points of Y and Z, according as these are in the same or opposite directions.

On examining line 11, we see that we have a condition of the air quite different from any kind of progressive undulation. In every progressive undulation the velocity of the air is greatest where the condensation or rarefaction is most extreme, but here the greatest velocity occurs at the parts which are neither condensed nor rarefied, and the greatest density-differences occur where the air is at rest. So that, though there are regions where the air is condensed and others where it is rarefied, these do not correspond with the "condensations" and "rarefactions" of progressive undulation. In progressive undulation a "condensation" means a region where the air is all condensed, and all moving one way. In the condensed region in line 11, half the air is moving one way and half the other.

Next let us consider what will be the condition of the air in this region an instant later than the moment for which Y and Z are drawn. We can represent the condition in which the air would have been an instant later if A had been vibrating alone by shifting the line Y a little to the right, and the condition in which it would have been if B had been vibrating alone by shifting the line Z the same distance to the left. Suppose each shifted the distance of between two letters, and find the resultant condition again. This time we get line 12. We notice that, though we have supposed lines Y and Z altered only in position, the resultant condition has not been displaced either to right or left, but has changed in degree; the condensations and rarefactions have become everywhere less pronounced, while the velocity of the air has everywhere increased. We see, in fact, that in this mode of vibration a given condition of the air does not move along as it does in progressive undulation. The greatest degree of rarefaction which existed at the moment shown in line 11 is not to be found in line 12 a little to right or left of its former place; it no longer exists anywhere. And, though there is nowhere so great a degree of rarefaction as in line 11, the greatest degree of rarefaction which exists anywhere in line 12 is in the same position as the greatest degree of rarefaction in line 11.

At certain points, those under the *A*'s in the top line, the degree of condensation which would be due to one of the progressive wave-systems *Y*, *Z* is exactly equal to the degree of rarefaction which would be due to the other, so that the actual condition of the air at these points, as shown in line 11, is one of average density. It is easy to see that, if each of the lines *Y*, *Z* is shifted onward the same distance, to represent the conditions which would be due to the two wave-systems a moment later, the rarefactions and condensations under the *A*'s are still equal, so that this condition of average density is a permanent one at these points, which are called *Antinodes.*

We see also that there are certain other points, those under the *N*'s in the top line, where the *velocity* of the air which would be due to one of the wave-systems *Y*, *Z* is equal and opposite to the *velocity* which would have been due to the other, so that at these points the air is at rest; and, as before, we see that this is a permanent condition at these points, which are called *Nodes.*

The antinodes are called by some writers "ventral segments or loops." This name is more properly applied to the whole region between one node and the next, an antinode being the middle point of a ventral segment.

The existence of points fixed with respect to the air, at which a definite condition of the air is always to be found, is the most striking peculiarity of this mode of vibration; hence the name Stationary Undulation.

To get a more exact idea of this condition, we will trace the changes in the region shown in the figure through some further stages, at each stage advancing *Y* and *Z* one letter as before. The condition of the air at these stages is shown in the lines 1 to 10. These complete a cycle, as the next stage after that shown in line 10 is line 11 again.

By comparing the successive lines of Fig. 46 we can make out the following important points.

(1) Nodes occur at the points where the maximum condensations of one imaginary progressive wave-system (*i.e.*, the wave-system which one of the sources would produce if it vibrated alone) arrive at the same moments as the maximum condensations of the other. Antinodes occur where a maximum condensation of one imaginary system arrives at the same moment as a maximum rarefaction of the other.

(2) The distance between two successive nodes, or two successive antinodes, is half the wave-length of the waves which the sources would produce separately. From a node to an antinode is one-fourth of this wave-length.

(3) The period in which the stationary undulation goes through all its changes is the period in which the waves which the sources would produce separately would advance their own wave-length, *i.e.*, it is the vibration period of either source.

(4) Condensation and rarefaction do not move along as in progressive undulation; they simply appear and disappear again, to be succeeded by the opposite condition in the same place.

(5) The nodes are not places of greater *average* density than the rest of the air, but of greatest variation of density; each node is a point of maximum and minimum density in turn. The average density is the same at nodes as elsewhere.

(6) There is an instant, twice in each complete vibration, when all the air is stationary at the same moment (line 4 or 10). This may be called the stationary instant. At a stationary instant every point has the maximum degree of condensation or rarefaction which it ever has, and this is greatest at the nodes and diminishes to zero at the antinodes, alternate nodes being condensed and rarefied. After the stationary instant all the air, except at the nodes, begins to move from the condensed nodes towards the rarefied ones (lines 5, 11); its velocity at any one instant is greatest at the antinodes and diminishes to zero at the nodes. The

velocity increases everywhere (line 6 or 12), the velocities at the different points always keeping the same ratio, so that it is always greatest at the antinodes. Meanwhile the condensation and rarefaction everywhere diminish, but fastest at the nodes, so that the degrees of condensation and rarefaction at different points remain in the same ratio. The velocity increases, and the condensation and rarefaction diminish, till we reach an instant (line 7 or 1) when the air has everywhere the same density, which is that of the external air. This may be called the moment of uniform density. At this moment the air has at each point the maximum velocity which it ever has at that point. The air continues to move in the same direction, but with diminishing velocity, and the nodes towards which it is moving, which were previously the rarefied ones, now become condensed (line 2 or 8) and those which were previously condensed become rarefied. The velocity everywhere diminishes, and the degree of condensation and rarefaction everywhere increases (line 3 or 9), till all the air comes to rest at the same moment, and we have a stationary instant again. Then all the movements begin again, but in the reverse direction, and so on.

(7) At any given moment all the air between two consecutive nodes is moving in the same direction, and all the air between two consecutive antinodes is in the same condition (all rarefied or all condensed).

(8) Since the velocity of the air at each point varies harmonically, the air at each point moves harmonically, and its amplitude at each point is proportional to the maximum velocity of the air there; this amplitude is therefore greatest at the antinodes and zero at the nodes. Each particle of air passes its mean position at the moment (line 1 or 7) when it has its maximum velocity.

78. Energy of Stationary Undulation.—The total energy in the stationary undulation is the sum of the energies of the undulations which the sources would produce separately, but it is in a different form. For, while in any progressive undulation half the energy is at any one moment kinetic and the other half potential, in the stationary wave-system the whole energy keeps changing from one form to the other.

When the air is in the condition shown in line 1, Fig. 46, there are no differences of density, and the whole energy is kinetic, depending on the velocity with which the air is moving. At this moment most of the energy is near the antinodes, where the air is moving fastest. When the air is in the stage of stationary undulation shown in line 4, there is no kinetic energy, as all the air is at rest; it is all potential, depending on differences of pressure, and none is at the antinodes. In intermediate stages the energy is partly in one form, partly in the other.

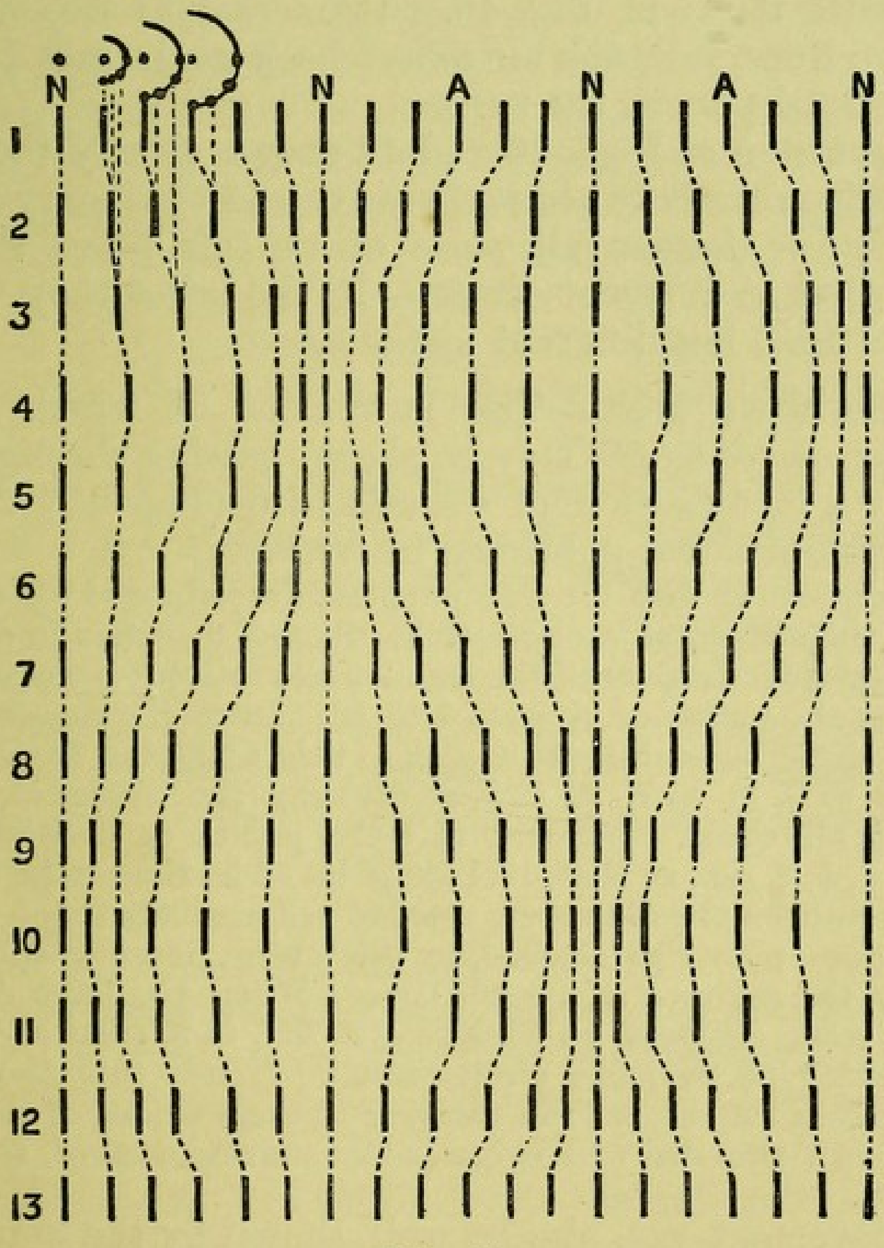


Fig. 47.

79. Fig. 47 shows in the same form as Fig. 20 the movements which take place in a region of stationary undulation.

The vertical lines in each row represent, as before, plane surfaces, seen edgewise, which were equidistant in the undisturbed air, and the air is to be supposed moving in such a way that the same air always remains between the same two planes. The lines vibrate harmonically about their mean positions; those at the antinodes have the greatest amplitude of vibration, and those at the nodes have zero amplitude. They all cross their mean positions together, but those on opposite sides of the same node are moving in opposite directions. The numbers of different stages correspond with those in Fig. 46. It is easy to see, from the successive lines, how the air sways backwards and forwards between fixed planes, the nodes.

A comparison of Figs. 20 and 47 shows clearly the fundamental difference between progressive and stationary undulation. In the former all parts of the air move the same distance, but at different times. In the latter they move at the same time, but different distances.

80. Cheshire's Disc.—The movement of which Fig. 47 shows successive stages may be shown passing through these stages by a modification of Crova's Disc (Fig. 21), designed by Mr. Cheshire and published in *Nature*.

To construct the disc for this purpose, describe a circle of $\frac{1}{8}$ inch radius in the middle of a circle of cardboard 8 inches in diameter. Divide the circumference of the small circle into twelve equal parts, numbering the points of division 1 to 12. Draw a diameter from 3 to 9, and draw lines at right angles to this diameter from 2 to 4, 1 to 5, 12 to 6, 11 to 7, 10 to 8, so dividing the diameter into six parts, not all equal. We will call the points of division of the diameter *a*, *b*, *c*, &c., so that, including its ends, there will be seven marked points on the diameter, marked respectively 9, *a*, *b*, *c*, *d*, *e*, 3. Taking these points as centres, in this order, describe ink circles, increasing the radius $\frac{1}{8}$ inch each time. When you have described the circle with centre 3, go back to *e*, *d*, &c., to 9, then *a*, &c., back again, making 20 or more circles in all. Mount this disc exactly like the one in Fig. 21, and, on rotating it, the portions of the circles seen through the slit will execute the movements of stationary undulation.

The movement can also be well shown by the spiral wire of Art. 73, as described in the next chapter.

81. Stationary Undulation caused by Reflection.—It does not often happen that the condition of stationary undulation is due to the simultaneous vibration of two sources. Much more usually it is due to reflection of waves from a single source. In this case the two imaginary component wave-systems are the waves which the source would produce if no reflection occurred, and the reflected wave which each of these would produce if it arrived by itself at the reflecting surface. The real condition of the air between the source and the reflecting surface (say a tuning fork and a wall) can be found by adding these imaginary conditions, and it is evidently nearly stationary undulation along a line from the fork at right angles to the wall, if the circumstances are such that an incident wave is reflected without much spreading and consequent loss of intensity. This, as we saw in the last chapter, requires that the waves shall be short compared to the reflecting surface; a shrill whistle, or a squeaker, such as is used in many toys, is therefore better than a tuning fork. If such a squeaker is blown with a steady pressure of air (so as to give always waves of the same length) at a distance of a few feet from the wall of a room, there is stationary undulation between.

At the wall itself there is a node, for the part of the reflected wave which is just starting back is the reflection of, and similar in condensation to, the part of the incident wave which arrived an infinitely short time ago, and therefore differs infinitely little in condensation from the part of the incident wave which is just arriving; and a node is a point where the degrees of condensation of the components are always equal. Other nodes occur every half wave-length of the incident waves from the wall.

It is only along a line from the source perpendicular to the wall that the direct and reflected waves are in exactly opposite directions, so that the condition of stationary undulation strictly exists only along this line, but there will be a condition which is very nearly nodal at all points, not very far from this line, which are at the same distance from the wall as the true nodes, and similarly for the antinodes, so that the nodes and antinodes are surfaces parallel to the wall. Their position depends only on the position of the wall and the length of the waves, not at all on the position of the source, which is not, unless by accident, either at a node or at an antinode.

82. Experiments on Stationary Undulation.—The existence of nodes and antinodes may be shown, and their positions found, in two ways. One end of a flexible tube may be inserted into the ear, and the other end moved along the perpendicular from the source to the wall, keeping the plane of the opening of the tube parallel to this line. A series of points are found where the sound is fainter than at intermediate points; these are the antinodes, for the movement of the air backwards and forwards across the opening, without change of density, sends no waves up the tube. When the open end of the tube is at a node, each change in the density of the air sends a corresponding wave up the tube, and loud sound is heard. This is not a very good method. It is clear from Fig. 46 that the absence of change of density at the antinodes depends on the exact equality of the condensations of one component wave-system and the rarefactions of the other. Now the rarefactions of the reflected waves are produced by the rarefactions of the incident waves, and do not correspond in wave-form with the condensations of the incident waves unless the incident waves are symmetrical, which never is the case in practice. So that unsymmetrical waves, like those from the squeaker, do not, when reflected "with change of sign," form perfect antinodes at all, though they do form true nodes, or places of no movement. So the antinodes are indicated, not by silence, but only by faintest sound, and the nodes by loudest sound; neither is easy to determine. A sensitive flame, which directly determines the nodes, is much better.

A sensitive flame is produced by burning, at a burner perforated with a single pin-hole, coal-gas at a pressure equal to that of 8 or 10 inches of water. It will be found that, as the tap is turned on, the flame, which is like a much elongated candle flame, increases in length to about 16 or 20 inches, and then suddenly shortens to half that length, flaring at the top and producing a loud noise. If the supply is adjusted so that the flame is just on the point of flaring, it is very sensitive to movement of the air just above the burner, which makes the flame flare as long as the movement lasts, but it is not at all sensitive to changes of pressure in the air apart from movement. Such a flame, held in a region of stationary undulation, flares everywhere except at the nodes, and thus the nodes, if they are at all perfect, may be very exactly determined.

This is, of course, a method of finding the length of the waves from the source, which is twice the distance between two consecutive nodes (Art. 77). For very short waves it is one of the best methods.

This form of sensitive flame requires a gas-bag or gas-holder, since the ordinary gas supply is at too low a pressure, but sensitive flames may be more simply obtained. If gas at the ordinary supply pressure is allowed to issue from a pin-hole burner, and a piece of wire gauze fixed a little distance above the jet, the gas may be lit above the gauze without lighting that between the gauze and the burner, and the flame above the gauze will be blue at the bottom and yellow above. If we increase the distance of the gauze from the burner, the blue part increases and the yellow diminishes, and by trying different distances a position may be found at which the flame is sensitive, the yellow tip entirely disappearing while any vibratory movement of the air, of high frequency, is taking place above the burner. A wide glass tube or lamp-chimney round the flame, standing on the gauze, makes the flame still more sensitive and the effect more visible, as a much longer flame is then obtained, which shortens to less than half its length, and becomes much less luminous, when there is any vibration in the air. These flames, like the other, are unaffected at a node, but disturbed at any other point in a stationary undulation.

Sensitive flames may also be used for showing to an audience the phenomena of reflection and refraction of sound—for instance, its concentration at the focus of a concave mirror.

83. As reflected waves are not quite as intense as the incident ones, stationary undulation produced by reflection is more or less imperfect. It is convenient to consider the incident wave-system as the sum of two systems, one of the same intensity as the reflected waves, the other making up the actual intensity. The first of these, with the reflected system, gives true stationary undulation, so that the actual condition of the air is the sum of a stationary undulation, and a very feeble progressive undulation in the direction of the incident waves.

Very perfect stationary undulation occurs in organ-pipes, but there it is complicated by resonance. We consider it in the next chapter.

CHAPTER X.

VIBRATIONS OF AIR IN PIPES.

84. Wave in a Tube.—Take a wide tube, say about 3 inches in diameter and 6 inches long, and stand it on a table, so that its lower end is closed. Hold a strip of paper, an inch wide, with its free end over the open mouth of the tube, and tap the upper side of the paper sharply, near the free end, with a penholder, so that the paper moves suddenly towards the tube, but does not reach it. In addition to the noise which would be produced in any case by striking the paper, another sound will be heard which is not produced when the paper is held at a distance from the tube and struck in the same way. This additional sound will be found to be always the same for the same tube, and quite independent of the size of the paper or the way in which it is held and struck, and most persons will recognise that it is a musical note of definite pitch, which can be matched on the piano or by the voice, while the tap of the penholder on the paper is a mere noise, and has no definite pitch.

Any large wide-mouthed hollow vessel, a jug for instance, may replace the tube in this experiment, and the sound produced is so characteristic of hollow vessels that any sound which produces the same effect on the ear is commonly termed a "hollow sound."

The sensation of definite pitch is found, in all cases which can be investigated, to depend on condensations and rarefactions reaching the ear at *regular* intervals (Art. 63). Now there is nothing regular about the movements of a piece of paper which has been struck; this is shown by the sound being a mere noise. The tube, then, has in some way the effect of converting the single wave produced by striking the paper into a succession of waves starting at regular intervals.

And it is not the tube itself which does this, but the air contained in it, for, if we try the experiment with a metal and a pasteboard tube of the same size and shape, we find no difference in the hollow sounds they give back.

The way in which this happens is as follows:—The condensation produced by tapping the paper travels down the tube to the table, where (Art. 72) it is reflected, and a condensation travels up the tube again. When it arrives at the open end, a slight condensation starts off through the outside air, but (Art. 72) by far the greater part of the energy travels down the tube again in the form of a rarefaction. This is reflected, still as a rarefaction, at the closed end, and, when it reaches the mouth again, a slight rarefaction starts off through the air, while a condensation travels back along the tube, and so on. Only a small fraction of the energy leaves the tube each time the wave reaches the mouth, so that the wave may travel hundreds of times up and down the tube, changing its sign each time it reaches the mouth, before it becomes imperceptible. A condensation starts off from the mouth at the end of every fourth single journey of the wave, and a rarefaction half way between each two condensations, and, as these travel away through the air at the same speed as the wave travels in the tube, we have waves, travelling away through the air, whose wave-length is four times the length of the tube.

A similar action will take place if we tap a strip of paper over the upper end of a tube which is also open at the lower end. In this case the condensation produced by the sudden movement of the paper travels down to the bottom of the tube and there starts a slight condensation off through the outside air, while a rarefaction travels back up the tube. When this reaches the upper end, a rarefaction starts off through the air and a condensation travels down again, and so on. In this case a condensation starts away from the tube through the outside air at the end of every second single journey of the wave in the tube, so that the waves in the air are only twice as long as the tube. An open tube, under these circumstances, gives waves of about the same length as a tube of half its length closed at one end (called a "closed tube").

Though a hollow vessel is often said to "resound" to a sudden blow like that of the penholder on the paper, the sound produced is not an instance of resonant vibration in the sense in which we have used the term, for the air in the tube was set in vibration by a single violent impulse, not a succession of properly timed small impulses. But, as the air in the tube has, as we have seen, a natural period of vibration, it can be set in resonant forced vibration by impulses of the same period.

85. Resonant Vibration of Air Columns.—Suppose we take a tube open at the top and closed at the bottom, and make a flat spring vibrate over the mouth. Each movement of the spring towards the mouth of the tube sends a condensation down the tube, and each movement away from the mouth, a rarefaction. When the first condensation returns to the top, it would of itself start a rarefaction down. If at this moment the spring is moving upwards, a rarefaction produced by its movement travels down the tube together with the rarefaction which would in any case be produced by the arrival of the condensation at the top ; we have a rarefaction of nearly double the amplitude of the first wave. When this returns to the top, it would of itself send a condensation down, and, if the spring is at this moment moving downwards, this condensation will be increased by that due to the spring, and so on. Evidently the effect of a large number of impulses can be added up in this way if the period of the spring is exactly such that, each time the wave in the tube returns to the mouth, it finds the spring moving in the opposite direction to that in which it was moving the previous time.

The condition that the air in a closed tube may be set in resonant vibration by a spring is, therefore, that the spring must make an odd number of half vibrations in the time that a pulse takes to travel twice the length of the tube.

In the case of an open tube, a condensation sent down returns as a rarefaction, which would of itself start a condensation down again. In order that this may be increased by the movement of the spring, the spring must be at that moment moving downwards again, so that it must have completed a whole number of (double) vibrations while a pulse has travelled twice the length of the tube.

In trying the experiment it is more convenient to use a tuning fork than a spring, because, unless the spring is very firmly fixed, it shakes its support, and then it does not vibrate very long or regularly.

A closed tube, then, will be set in resonant vibration by a fork which makes either 1, 3, 5, or any other odd number of half vibrations while a pulse travels twice the length of the tube. In any of these cases each pulse of condensation or rarefaction, on reaching the mouth, gives off a small fraction of its energy in the form of a wave of its own kind, which travels away through the outside air, while the rest of the energy travels back down the tube as a pulse of the opposite kind, increased by the wave which the fork was sending down at the same moment. The wave in the tube keeps on increasing till the energy sent off at each return equals that received from the fork. As the tube sends a pulse off for each one that the fork sends down the tube, the waves start from the tube with the same frequency as from the fork, and are of the same wave-length, but they are of much greater intensity, so that the sound heard, though of the same pitch as that heard when the fork is sounded without the tube, is very much louder. It is not at first clear how this increased loudness can be produced, as of course the tube cannot send out more energy than it receives from the fork, but we shall see presently that, when the air in the tube is in resonant vibration, the air near the mouth moves up and down, keeping time with the fork, and under these circumstances the fork communicates its energy much more rapidly to the air, as explained in Art. 41. Of course the fork comes to rest much sooner when it makes the tube resound, but while it lasts the sound is much louder.

86. Condition of the Air in a Resounding Tube. If we produce at the mouth of the tube a single condensation, as in the experiment with the strip of paper, a real wave travels up and down the tube, and would be seen to do so if the air was visible. When a fork vibrates continuously at the mouth of the tube, producing condensations and rarefactions alternately, it is convenient to speak as if these actually travelled down the tube, while previous waves, reflected from the lower end, were travelling up again. It has, however, been explained (Art. 37) that this is merely a convenient way of saying that the real condition of the air is one which can be found by adding two such wave-systems, and does not mean that these systems really exist in any physical sense. So that, if we could make the air in a tube, which is resounding to the tuning fork, visible, we should not *see* waves travelling down and other waves travelling up, or indeed anything moving continuously along the tube either way; what we should see would be simply the process of stationary undulation described in the last chapter, since this is the condition which we find when we add two imaginary equal wave-systems in opposite directions.

It is very important to keep in mind that the real physical condition of the air in a resounding tube is the condition of stationary undulation, and that the waves travelling up and down are a mathematical fiction, not a physical fact. To describe the condition of the air as one in which waves are travelling along it in opposite directions is as physically incorrect (and as mathematically correct) as to describe the condition of a man who is standing still, by saying that he is walking forwards and walking backwards at the same time.

All harmonic stationary longitudinal undulation is of the same kind, so that the general description of this process given in Art. 77 applies to the air in any resounding tube if the vibrations are harmonic. To make it a complete account of the movement of the air in a resounding tube it only remains to state where the nodes and antinodes are situated in the tube. This is easily found in any given case. Suppose, for instance, that we hold over the mouth of a tube, closed at the bottom, a fork which makes $2\frac{1}{2}$ vibrations while a pulse would travel up and down the tube, so that a pulse travels $\frac{4}{5}$ of the length of the tube while the fork makes a vibration. Fig. 48 shows the position of the fork at the moment when it is producing a maximum of condensation, and the positions which each maximum of condensation and rarefaction previously produced would occupy at that moment if it was solitary. The maximum condensation produced one period ago would be $\frac{4}{5}$ of the way to the bottom, and the one before that would have reached the bottom and come $\frac{3}{5}$ of the way up again, while the maximum rarefaction which started $2\frac{1}{2}$ periods ago would have just reached the top, so that the maximum condensation due to it would be just ready to start down with that now being produced by the fork. Those pulses which started still earlier would in the same way have positions coincident with later ones, so that the points marked are all the places where there would be

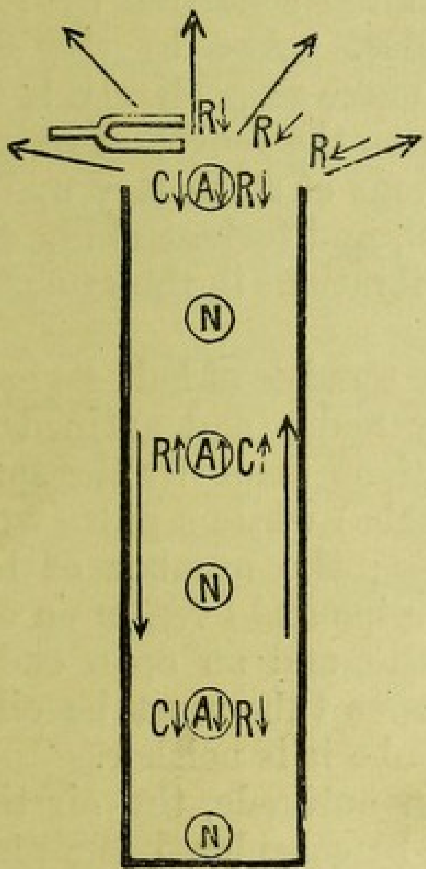


Fig. 48.

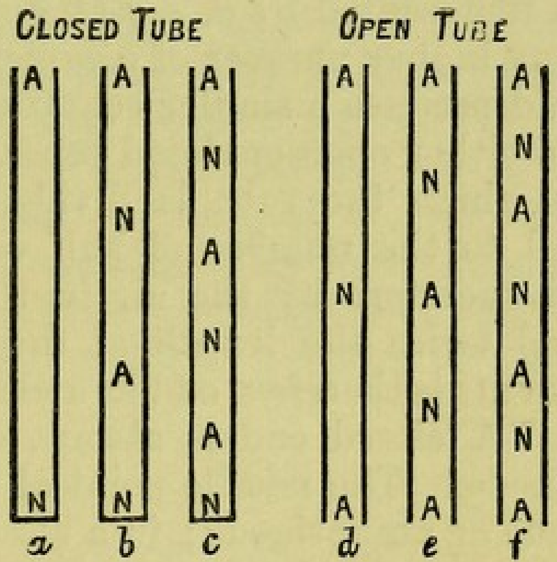


Fig. 49.

maxima of condensation and rarefaction if each pulse travelled independently. The actual condition of the air can be inferred from the positions of these imaginary pulses. *N*, *N*, *N*, where an imaginary maximum of condensation travelling down passes another travelling up, are nodes (Art. 77); *A*, *A*, *A*, where imaginary pulses of opposite kinds cross, are antinodes. In Fig. 48 the imaginary waves travelling down the tube are shown on the left, and the imaginary reflected waves travelling up again on the right, the directions in which the air would be moving in such waves being shown by short arrows. The actual condition of the air in the tube (*i.e.*, nodal or antinodal) with the actual direction of movement of air is shown by encircled letters in the middle column. The moment chosen is the "uniform density instant."

In the same way we find that forks which make respectively $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$ vibrations while a pulse travels twice the length of a closed tube throw the air into the conditions of stationary undulation represented by *a*, *b*, *c* of Fig. 49, and forks making $\frac{7}{2}$, $\frac{9}{2}$, $\frac{11}{2}$ or any higher odd number of half-vibrations in the same time will produce a similar condition with still shorter intervals between the nodes. And by the same method we find that forks which make respectively 1, 2, 3 vibrations while a pulse travels twice the length of an open tube throw the air into the conditions of stationary undulation represented by *d*, *e*, *f* of Fig. 49, and forks making 4, 5, or any higher number of complete vibrations in the same time would produce a similar condition.

In either open or closed tubes, the number of half-segments into which the tube is divided by nodes and antinodes is equal to the number of half-vibrations made by the spring (and therefore by the air in the tube) while a pulse would travel twice the length of the tube; the number of half-segments is therefore odd in a closed pipe and even in an open one. A closed end is always a node, and an open end an antinode. The middle point of an open tube must be either a node or an antinode; in a closed tube it is neither.

The mouth of the tube being an antinode, the air there simply moves in and out, and may be considered as a piston vibrating at the end of the tube like that in Fig. 22. Condensation and rarefaction are therefore produced in turn in the air just outside the mouth, and travel away (by the process of progressive undulation) in all directions through the external air. The condensation just outside the mouth is at its maximum when the air just inside the mouth is moving outwards most rapidly, which is at one of the moments of uniform density of the stationary undulation in the tube; the maximum degree of rarefaction just outside the tube occurs at the other uniform density instant (Art. 77). It is this latter instant which is represented in Fig. 48, and the letters and arrows round the mouth are intended to indicate that the air there is rarefied and moving inwards towards the mouth, while the rarefied condition is travelling away in all directions. It will be seen that this condition of the air outside the mouth might be considered as a continuation into the external air, with diminished amplitude, of the imaginary return waves which have been reflected at the bottom. The changes of density produced in the air outside, though it is to them that the sound we hear is due, are very small compared to those which occur in the air in the tube itself.

As a condensation starts off through the external air each time the air just inside the mouth of the tube moves outwards, the length of the waves in the air outside is the distance a pulse travels in one period of the stationary undulation. And we showed in Art. 77 that the period of a stationary undulation was four times the time required for a pulse to travel a half-segment (from a node to the nearest antinode). So that the waves produced in the external air are always four times the length of a half-segment of the stationary undulation in the tube. If we examine Fig. 49, keeping this in mind, we can easily make out the following table, in which l is the length of the tube, and t the time required by a pulse to travel this length.

Kind of Tube.	Closed.				Open.			
Mode of Vibration. Fig. 49.	Slowest or Fundamental. *a*	2nd. *b*	3rd. *c*	&c.	Slowest or Fundamental. *d*	2nd. *e*	3rd. *f*	&c.
No. of half-segments	1	3	5	&c.	2	4	6	&c.
Length of half-segment	l	$\frac{1}{3}l$	$\frac{1}{5}l$	&c.	$\frac{1}{2}l$	$\frac{1}{4}l$	$\frac{1}{6}l$	&c.
Length of waves in external air	$4l$	$\frac{4}{3}l$	$\frac{4}{5}l$	&c.	$2l$	l	$\frac{2}{3}l$	&c.
Period of vibration	$4t$	$\frac{4}{3}t$	$\frac{4}{5}t$	&c.	$2t$	t	$\frac{2}{3}t$	&c.
Ratio of frequency to slowest vibration	1	3	5	&c.	1	2	3	&c.

Some of the statements just given are only approximately true, for the mouth of a tube is not strictly an antinode.

The stationary undulation in the tube is not divided from the progressive undulation which starts from it by a definite line; the reflection of the wave at the open end does not take place exactly at any one point, so that there is an intermediate region, partly in the pipe and partly outside, where the undulation is not exactly either stationary or progressive. There is thus no true antinode at the mouth, nor is the distance from the mouth to the first node quite as great as the distance from a node to an antinode in the tube. The difference is shown, both by experiment and calculation, to be about equal to $\frac{4}{5}$ of the radius of the tube, so that we must add this quantity to the length of a closed tube, and twice this quantity to that of an open one, to get the length of an exact number of half-segments. Strictly, therefore, these increased lengths should be substituted for l in the table.

The length of the waves from a closed tube vibrating in its slowest mode is thus $4(l+a)$ where a is about $\frac{4}{5}$ of the radius of the tube; from an open tube it is $2(l+2a)$. The waves from the open tube are therefore not exactly half as long as those from a closed tube of the same length, but rather more; the note produced by the open tube is rather less than an octave higher than that of the closed one.

The slowest resonant harmonic vibration possible for a pipe is called its fundamental vibration, and the others are called its harmonics, or overtones. We see from the table that the overtones of a closed pipe have frequencies which are odd multiples of that of its fundamental, while the overtones of an open pipe have frequencies which include every exact multiple of its fundamental.

The air in a tube closed at both ends may be set in resonant vibration, as in Kundt's experiment described in Chapter XII. The modes of vibration possible in this case are like those of an open tube with nodes and antinodes interchanged. As no waves are given off from a tube closed at both ends, the resonance is not audible in this case unless the observer is inside the tube, which may be a long room or passage; but in smaller tubes the fact of the resonant vibration may be shown in other ways, to be explained in Art. 93.

So far, we have considered only pipes of uniform bore. Pipes of varying diameter can also be thrown into resonant vibration; in this case they divide into segments whose natural vibration periods are equal, but whose lengths are unequal. A conical pipe, stopped at the apex, may be made to vibrate with 1, 3, 5, &c., half-segments like a cylindrical stopped tube, the apex being of course a node. But, when it vibrates as one half-segment, its period is only half that of a cylindrical stopped tube of the same length, or the same as that of a cylindrical open tube of that length. Also, when it vibrates with 3 half-segments, the first of these (beginning at the apex) is half the length of the tube, and when it vibrates with 5 half-segments, the first is $\frac{1}{3}$ of the length of the tube, and so on, so that its successive harmonics have to its fundamental the ratios 2, 3, &c. Thus both its fundamental and its more rapid modes of vibration correspond *in frequency* with those of an open cylindrical tube of the same length, though the actual mode of vibration is quite different.

87. Vibration produced by Air Blast.—The air in a tube may be set in resonant vibration in many other ways. One is to send a blast of air across an open end of the tube.

It is not very clear how this produces resonant vibration, and various explanations are given. It is sometimes stated that the rushing noise produced by the blast striking the edge of the hole has among its harmonic components a vibration of the frequency to which the tube resounds; but, if that was the cause, a tube ought also to resound when a rushing sound is produced near its mouth by blowing across the edge of something else, without the blast itself reaching the tube, and this does not occur. The following is perhaps nearer to the true explanation. A very slight difference in the direction of the blast of air determines whether the air goes into the tube, so producing a condensation, or simply passes across the opening, in which case it exhausts air from the tube, by an action similar to that of spray and scent diffusers. If, as is usually the case, this exhausting action first takes place, the air inside the mouth is rarefied, till the pressure inside the tube becomes so much less than that outside, that the air blast is deflected inwards, so producing condensation; and so on. These conditions travel down the tube, and are reflected, and each condensation, as it reaches the mouth again, deflects the air blast outwards, so that its action increases the rarefaction which would in any case be produced by a condensation reaching the mouth; and similarly for a rarefaction. The stronger the blast, the more rapidly it exhausts or condenses the air, and the larger the number of rarefactions and condensations which start down the tube before the first returns, after which the action of the blast is simply to increase the waves each time they return to the mouth.

A blast of proper intensity will throw the air in a tube into any of the modes of stationary undulation into which it might be thrown by a fork; a very weak blast causing the fundamental vibration, and blasts of increasing strength producing higher and higher harmonics. It is, however, difficult to produce any but the fundamental mode of vibration in wide tubes.

The blast of air may be across the end, as in whistling with a key, or across a hole in the side, as in a flute, or directed by means of a passage so that the jet strikes a sharp

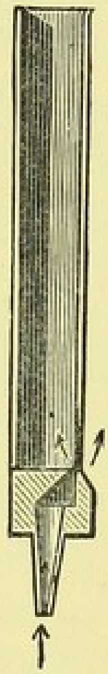


Fig. 50.

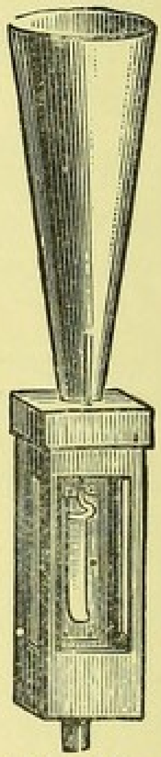


Fig. 51.

edge, as in a whistle, or organ pipe with a flute mouthpiece; a section of the latter is given in Fig. 50.

Instead of producing any required mode of vibration by adjusting the strength of the blast, we can produce one mode to the exclusion of others by opening additional holes in the side of the tube at points which are antinodes for the particular mode of vibration we require, for no mode of vibration is possible which has not an antinode at every opening. This principle is used in the flute.

88. Non-harmonic Vibration.—The impulses given by the wavering air-blast to the air-column are periodic but not harmonic, and may therefore be considered as the sums of harmonic pressure-changes of frequencies which are multiples of the real movement of the blast. (Art. 52.) Each such pressure-change would produce resonant vibration of its own period if the air column had a free-vibration period not very different. The vibration produced is therefore not harmonic, and may have harmonic components of any frequencies (multiples of that of the vibration) which are nearly equal to possible free-vibration frequencies of the air column. Thus the vibration in a closed tube, caused by a blast, may have harmonic components whose frequencies are odd multiples of its own, but not even ones; that of an open tube may have components which are any exact multiples of its frequency. The harmonic components of the waves sent out are the same as those of the air column itself.

Any air column, open or closed, if set in resonant vibration by a single tuning fork, vibrates harmonically like the fork, and under these circumstances the resonant sounds from an open and a closed tube are exactly alike. But, when either tube, if not very wide, is set in resonant vibration by a blast of air, it usually vibrates non-harmonically, and in that case the sounds are of different quality, because, as explained above, the closed tube gives waves whose harmonic components are all odd multiples of their fundamental, while the open tube has even multiples as well. Very wide tubes are not easily set in non-harmonic vibration by a blast of air, so that there is not so much difference in the quality of the sounds from open and from closed pipes when they are wide as when they are narrow.

There is an important difference between the vibration produced by a fork and that produced by a blast of air. The air column in a tube is of small mass, and there is considerable loss of energy in each vibration owing to the waves sent off; forced resonance, of a period different to its natural vibration, is therefore easily produced. (Art. 48.) A fork has a natural period not easily altered, and accordingly sets an air column in resonant vibration of this period, even when the natural period of the air column is considerably different. The air blast, on the contrary, has no natural period of vibration, and is controlled by the return of the waves it produced, so that the period of the resonant vibration is exactly that in which a pulse would travel the length of four half-segments of the tube. (Art. 77.)

89. Reed pipes.—An air column may also be thrown into resonant vibration by means of a *reed*. The term "reed" is applied sometimes to the whole and sometimes to a part of an arrangement consisting of a strip or tongue, usually of thin metal, fixed at one end to a plate, so that it covers a rectangular opening in the plate, as shown in Fig. 51. The tongue may be either a little larger than the opening, in which case it is a *striking reed*, or a little smaller, when it is a *free reed*. In either case the tongue is bent so that its free end stands a little away from the plate, and a blast of air is blown through the opening from the side on which the tongue is fixed. As the air increases in speed, it carries the tongue with it, and so blocks the opening; when the rush of air stops, the tongue springs back. Thus a succession of puffs of air escape through the opening, producing waves. These may either be allowed to escape into the air, as in the harmonium, in which case the frequency of the puffs is that of the natural vibrations of the tongue; or they may escape into a pipe whose length is such that it can be set in resonant vibration of the same frequency, as in the clarinet and several kinds of organ pipes. In this case, the tongue being of small mass as well as the air column, each forces the vibrations of the other; the vibration period is a compromise between that natural to the reed and that natural to the air.

The tongue of a free reed is not a freely vibrating spring, being affected by the changing pressure of the air blast; its frequency is not independent of this pressure, as is often stated, but increases with it: this is easily shown on a concertina. The striking tongue, which rebounds from the plate, has its frequency still more increased by an increase of air pressure. The striking form is the one practically used to cause vibration in tubes.

Fig. 51 shows a reed in conjunction with a conical tube, the box which contains the reed being provided with windows for observing the vibration. The air is blown from below into this box, passes through the reed in the direction away from the reader, and escapes behind the metal plate into the conical tube. The vibration period of the tongue is adjusted by a sliding wire which allows a shorter or longer portion to vibrate.

An air blast against an edge of a hole in a tube sets the air column in vibration whether the other end of the tube is open or closed ; in either case the hole across which the air is blown is an antinode, and the air vibrates in and out through it, but the blast sucks out in one half-vibration the air that it has just blown in in the other ; there is no continuous current of air along the tube, and if smoky air is used for the blast the air in the tube remains clear for a long time. The case is quite different when a tube is made to vibrate by a reed at one end, as in the clarinet. Here the air never passes out of the tube through the reed, so that the tube must be open at the other end ; and as a condensation, returning to the reed end, does not escape there, it is reflected as a condensation, so that the reed end is a node. A reed tube is therefore always a closed tube, and, if cylindrical, can give only the odd harmonics of its fundamental. As pointed out above, this may be avoided by making the tube conical, as in the French horn, in which, as in many other instruments, the lips of the performer take the place of a mechanical reed.

Vocal Sounds.—The vowel sounds produced by the voice are due to the vibrations of two cartilaginous plates, the *vocal chords*, placed at the top of the windpipe, edge to edge, with a narrow slit between them ; air blown through this slit from the lungs keeps the plates vibrating. The apparatus is really a free reed. The vocal chords have muscles attached to them, which can vary the frequency of the vibration, and the pitch of the sound produced. The different vowel sounds are produced by varying the size and shape of the mouth cavity, but it is uncertain what effect this produces. Some believe that the mouth reinforces by resonance certain harmonics in the sound produced by the chords, a sound containing among its harmonics the same multiples of the fundamental being recognised as the same vowel, whatever the absolute frequency of the fundamental. Thus a note whose harmonic components are the fundamental and its octave is said to give the sound ō, while the fundamental with the first five harmonic overtones gives the sound ā. On this view each vowel is a note of particular quality. Another view is that each vowel is distinguished by the addition to a note which may have any pitch, of one or two other notes whose absolute frequencies determine which vowel is heard ; thus the sound ō requires a note of frequency about 980 added to the louder and lower note. The note which determines the vowel is not a harmonic of the note which constitutes the greater part of the sound. The two views are often held together, each vowel sound being considered to require certain harmonics of the fundamental note, and also certain notes of fixed frequency independent of the fundamental. There seems no direct evidence that these notes are produced, and if they are, they probably exist only for a short period at the beginning of the sound. The consonant sounds are produced by the tongue, teeth, and lips. Most of them are *noises*. Others, *e.g.* *P*, *J*, *S*, are notes with accompanying noises. Several sounds usually called consonants, *e.g.* *L* and *M*, are really vowels with their quality modified by the resonance of the cavities of the nose, or of parts of the mouth cavity partitioned off by the tongue.

90. Effect of Change of Temperature.—As the vibration period of an air column depends on the time taken by a pulse to travel its length, the vibration frequency is proportional to the velocity of a pulse. Rise of temperature therefore increases the vibration frequency of the air column, the frequency being proportional to the square root of the absolute temperature (Art. 33). The increase of frequency is really a little less than this, because the pipes lengthen with rise of temperature; this effect is greater with metal than with wood pipes. In reed pipes the increase of frequency with rise of temperature is much less than with flute pipes, because the stiffness of the tongue diminishes as the temperature rises, so that, while the rise of temperature shortens the natural period of the air column, it lengthens that of the reed. Reeds without pipes, as used in the harmonium and concertina, diminish slightly in frequency with rise of temperature.

91. Vibration of Liquid Columns.—Liquid columns may also be set in resonant vibration. If a common tin whistle is immersed entirely in water in a jar, and connected by a tube to a high pressure water supply (such as the ordinary water pipes of a house), the water in the whistle is set in resonant vibration. Not much sound is heard, as sound does not pass easily from water to air (Art. 71) but the trembling of the jar is easily felt. If the observer puts his ear under the water, the sound is well heard.

92. Vibrations of Solid Rods.—As condensations and rarefactions travel along rods of elastic material exactly as along the air in tubes, and are reflected at free ends of rods exactly as at open ends of tubes, and at fixed or loaded points of rods exactly as at closed ends of tubes, rods can be set in resonant stationary undulation exactly like that of air columns, but different means must be adopted to give the successive impulses. The simplest way is to draw a resined cloth along the rod.

Resin, like other viscous substances, adheres the more strongly to surfaces over which it moves the more slowly it travels along them. The cloth sticks to the rod, pulling the part with which it is contact along with it, and so producing condensation in front and rarefaction behind. These conditions travel to the ends of the rod, or to any point of it which is fixed or loaded, and are reflected; and thus travel up and down the rod, and as they pass any part of the rod that part moves a short distance. As a wave passes the cloth, if it is one in which the particles of the rod move in the same direction as the cloth, the relative velocity of the cloth along the rod is diminished, and the cloth adheres more strongly to the rod, and gives the surface a pull in the direction in which it is already moving, while a contrary action takes place if the wave is one in which the particles of the rod are moving the opposite way to the cloth. Each wave is therefore increased each time it passes the cloth, and the rod is set in resonant vibration.

The resonant vibration of a rod fixed or loaded at one end is exactly similar to that of the air in a closed tube, for the waves are reflected in just the same way. The vibration of a rod not firmly fixed anywhere, for instance held in one hand and rubbed with the other, is similar to that of the air in an open tube. In the vibration of a rod clamped in the middle, each half vibrates like any other rod clamped at one end, but the two halves keep time with each other, so that points at equal distances from the middle always move in opposite directions at the same time. A rod fixed at both ends (which may be a stretched wire) vibrates like the air in a tube closed at both ends. If pieces of lead are fixed by clamps to two points on a stretched wire, and a piece of sandpaper, or resined cloth, drawn along the part of the wire between them, a loud sound is produced, whose pitch depends on the distance between the pieces of lead, but not on the tightness of the wire. This sound is due to the resonant longitudinal vibration of the part of the wire between the lead blocks, which are nodes. The blocks need not be fixed, except to the wire, as the waves are almost totally reflected on arriving at a portion of the wire of so much greater density than the rest (Art. 70).

Owing to the great number of successive impulses whose energy may be added to cause a resonant vibration, the movement of the air in a resounding tube, or of the material of a rod in resonant vibration, may be very large compared with the ordinary movements of progressive undulation. In an air column the movement of the air may amount to an actual wind, capable of carrying along cork filings or other light powders. By drawing, with one hand, a resined cloth along a steel bar, we can make the bar lengthen and shorten to an extent which it would require a direct pull of many tons' weight to effect. Thick glass rods capable of supporting a ton or more, may easily be pulled to pieces in this way.

93. Experimental Illustrations.—The movement of the vibrating air in a tube may be studied in various ways. If the tube, or one side of it, is of glass, we may place it vertically, and let down into it a thin membrane stretched on a horizontal wire ring (as described in Art. 40), with a little sand on the membrane; the sand dances everywhere except at nodes. Or the tube may be placed horizontally and a light powder such as cork dust scattered in it; this, being blown about everywhere else, soon collects at the nodes. The same method may be used for a vibrating liquid column, a heavier powder, such as precipitated silica, being used instead of cork dust.

For demonstrating to a large audience the different modes of stationary undulation of the air in an organ pipe, König's *manometric capsules* (Fig. 52) are very useful. A manometric capsule is a box, shaped like a large pill-box, of which one end consists of a stretched membrane. It is fitted into a hole cut in the side of the organ pipe, with the membrane side inwards, in contact with the air in the pipe. Coal-gas

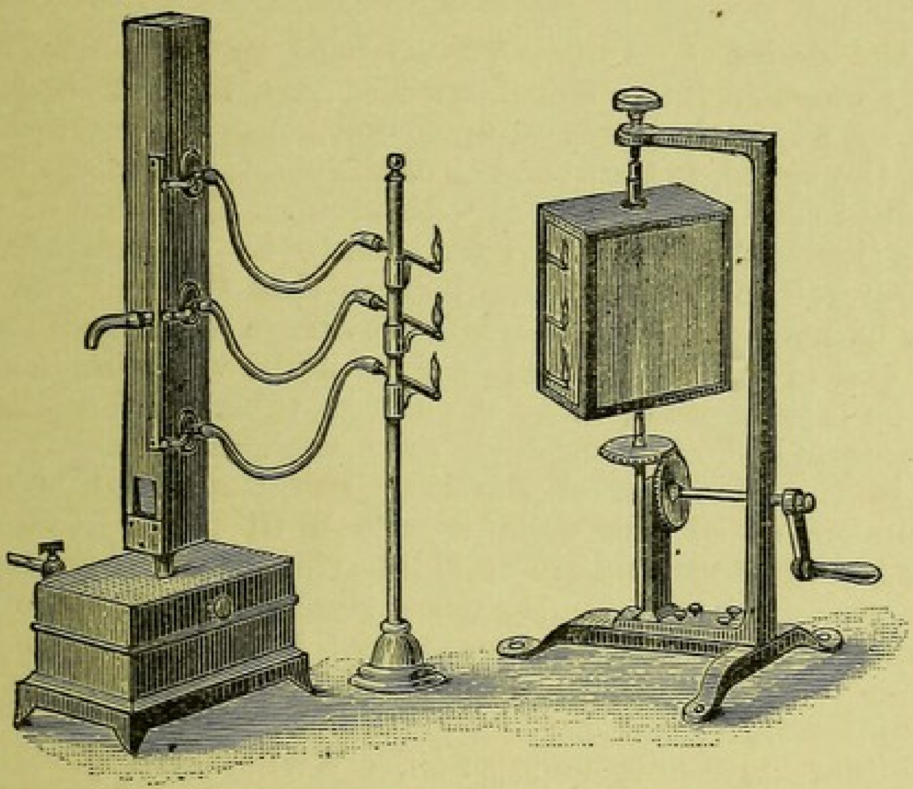


Fig. 52.

is conveyed into the capsule by a tube, and out by another to a pin-hole burner, where it burns as a small flame. If any rapid changes take place in the pressure of the air in the part of the organ pipe where the capsule is inserted, the membrane vibrates and the flame flickers, and, though this flickering is too rapid to be easily observed directly, it may be detected by watching the reflection of the flame in a rapidly rotating mirror. It then appears as a band of light toothed along its upper edge, the teeth being the images of the flame at the moments when it is highest. (A similar effect can be produced, without a rotating mirror, by rapidly turning the head from side to side while looking at the flame.) If a number of such capsules are inserted in the side of an organ pipe, the flames of all of them flicker except of those at antinodes, and the positions of these are therefore easily seen. The methods previously given detect the nodes.

The nodes of a rod in stationary undulation may be shown either by scattering sand on it or by putting a number of card or wire rings on the rod and holding it nearly, but not quite, horizontally. When the rod is set in resonant vibration, the rings slip down it till they reach a node.

Any of the modes of resonant vibration of an air column may be imitated by means of the spiral wire of Art. 73. A short heavy pendulum, of adjustable length, must be suspended vertically over one end of the spiral, so that the bob of the pendulum hangs between the horizontal rods and on a level with them. The pair of strings supporting that end of the spiral are then to be disconnected from the rods and tied to the pendulum bob, so that the end coil of the spiral now hangs from the bob, while the others hang from the rods. The other end of the spiral must be fixed if the vibration in a closed tube is to be represented; free for an open tube. If the pendulum, which represents the tuning fork used with an air column, is set swinging in the direction of the length of the spiral, only an irregular movement of the coils results until the period of the pendulum is adjusted so that it fulfils the condition necessary in order that periodic impulses may cause resonant vibration of a rod or fluid column; the condition explained in Art. 85. The spiral then begins to vibrate quite regularly, with definite nodes where the coils do not move, and forms a good illustration of the process of longitudinal stationary undulation. With a little practice, the hand, moved regularly and rapidly backwards and forwards, may replace the pendulum bob, and gives better results.

The resonant vibration of gas columns and rods affords a means of determining the velocity of sound in the respective gases and solids; these methods are explained in Chapter XII.

EXAMPLES II.

In the following examples the velocity of sound in air is to be taken as $33{,}240 + 60t$ cm. per second, t being the centigrade temperature. Unless otherwise stated, the temperature is to be taken as 0° C.

Elementary.

1. Give the lengths of the three shortest closed tubes, and of the three shortest open tubes, which would resound to a tuning fork making 200 vibrations per second.

2. A whistle making 2000 vibrations per second is placed a metre from a wall. At what points between the whistle and the wall are antinodes to be found?

3. A whistle is sounded near a wall, and the nearest node to the wall is 3 cm. from it. Determine the frequency of the whistle.

4. Give in each case the four slowest vibration frequencies possible in a tube 3·324 metres long (*a*) when open at both ends, (*b*) when open at one end, (*c*) when closed at both ends. [Neglect the correction for radius.]

5. If a tube makes 340 vibrations per second when the temperature is 16° C., what is its frequency, in the same mode of vibration, when the temperature is 51° C.? [Neglect expansion of tube.]

6. The air in a closed tube 34 cm. long is vibrating with two nodes and two antinodes, and its temperature is 51° C. What is the wave-length of the waves produced in the air outside the tube, if the temperature of that air is 16° C.?

7. A closed tube 15 cm. long resounds, when full of oxygen, to a given fork. Give the length of a closed tube, full of hydrogen, which will resound to the same fork.

8. If the velocity of sound in hydrogen is 126,000 cm. per second, and in air 33,300 cm. per second, what is the length of the waves which will be produced in the surrounding air by blowing an open organ pipe, a metre long, with hydrogen, the pipe being also full of hydrogen.

Advanced.

1. A vertical tube 1 metre long and 4 cm. in diameter is gradually filled with water, while a tuning fork, making 500 vibrations per second, is held over the upper end. At what positions of the water surface will the tube resound (taking the correction for diameter of the pipe into account)?

2. What must be the diameter of a closed tube 2 ft. long in order that it may resound to the lowest note given by an open tube 4 ft. long and 8 ins. in diameter?

3. Where must a conical tube, closed at the apex, be cut in two in order that each part may resound to the same note? What is the ratio between the frequency of this note and the note to which the whole tube would resound?

4. A wooden rod 1 yd. in length floats (when compelled to float vertically) with 2 ins. of its length out of water, and if rubbed longitudinally, without being firmly fixed anywhere, the lowest note it can be made to give has a frequency of 256. Find Young's Modulus for the wood in poundals per square foot.

5. A brass wire, 2 metres long and 1 sq. mm. in sectional area weighs 16 gm., and when it is hung up by one end, and 20 kilog. are suspended from the other, it elongates by 6 mm. What note will it give when rubbed in the direction of its length?

CHAPTER XI.

TRANSVERSE UNDULATION.

94. Transverse Wave in a Cord.—Take a long rope AB (Fig. 53) and fasten one end B firmly to a wall at a point about six feet from the ground. Hold the other end A in your hand, about four feet from the ground, and stretch the

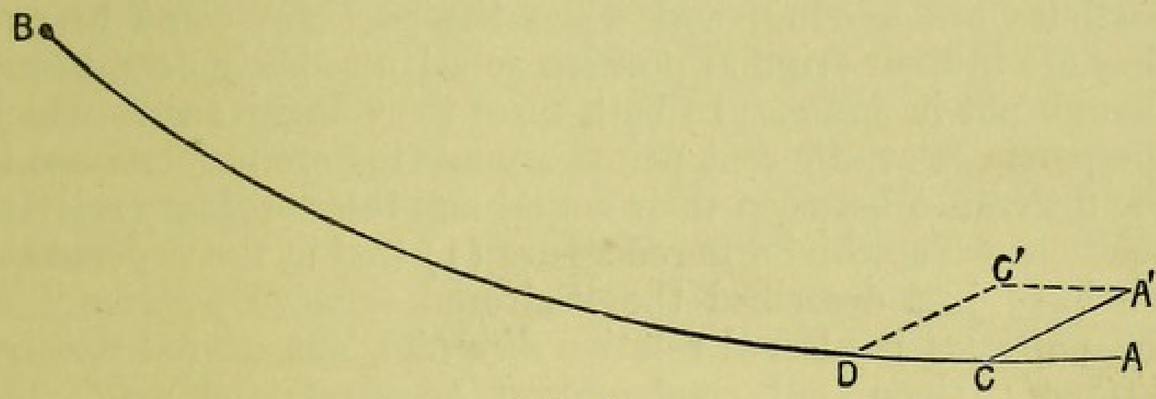


Fig. 53.

rope till the part nearest to your hand is about horizontal. It will not be quite straight; but we neglect that at present. Raise your hand suddenly a few inches, so that A is at A'. The immediate result of this is that a short portion $A'C$, close to your hand, is in an altered condition, not rarefied or condensed, but sloping instead of horizontal, the rope having the form $A'CB$. The point C is now acted on by two forces, due to the stretched condition of the string, along CA' and CB, and the resultant of these is upwards. C therefore begins to move up, and it stops only when it has moved as far as A moved, so that a portion of the rope $A'C'$ is horizontal again, while the sloping condition exists in another portion $C'D$, the rope having now the form $A'C'DB$. D then begins to move up in the same way, and so the sloping condition which was first produced in $A'C$ travels all along the rope.

Now this continuous movement of a condition of slope along the rope has been effected by each portion of the rope moving in turn a short distance in a direction at right angles to the rope: A to A', C to C', and so on. Further, each portion moved only while it formed part of the sloping section; as soon as the sloping condition had passed it, it was at rest again.

The whole process is in many respects very similar to the transmission of a pulse of condensation or rarefaction, as described in Art. 11. In both cases a *condition* of altered relative position of the particles travels continuously, while each particle in turn moves a short distance and then stops. In both cases the altered relative position exists where the particles are moving, and when the particles come to rest they are in their original position relative to their neighbours, though not in space. In both cases the velocities with which the particles at different points are moving are proportional to the difference between their actual and their ordinary relative position. The chief differences are (1) that in the experiment we have just described the "altered relative position" of the particles is altered relative *direction*, not altered relative *distance;* slope, not condensation or rarefaction; (2) the short movement which each particle executes in turn is at right angles to the direction in which the condition travels, not in that line, as in Art. 11. For this reason this kind of motion is called *transverse progressive undulation.*

95. Velocity of Transverse Waves.—It can be shown (see Appendix D) that the slope produced by the movement of A (*i.e.*, the angle between the changed direction and the original one) is proportional (as long as it is small) to the velocity with which A was displaced, but that the velocity with which the sloping condition travels along the rope does not depend on how A moved, but only on the mass of each unit length of the string and on the force with which the string is stretched. Even without investigating the exact relation, it is evident that, the more tightly the string is stretched, the greater the resultant force on C, and therefore the quicker C will move up to C', and the sooner D will begin to move, and so on. So that the velocity of a transverse wave depends on the force with which the string is stretched. The velocity of a longitudinal wave, on the other hand, is the same whatever the tightness of the string, as explained in Art. 27. Even if a string or wire is stretched to the point of breaking, the velocity of a transverse wave along it is always much less than that of a longitudinal one.

A fuller investigation shows (see Appendix D) that the velocity with which a sloping condition travels along a rope is

$$\sqrt{\frac{\text{force with which the rope is stretched}}{\text{mass of unit length of the rope}}},$$

the stretching force being measured in *dynamical* units, as explained in Art. 27.

96. Reflection of Transverse Waves.—When the sloping condition arrives at the fixed point B, it is reflected, and travels back again to A. The direction of the slope is the same in the reflected as in the original wave, but the movement of each particle of the rope while it forms part of the reflected wave is in the opposite direction to its motion while it formed part of the original wave. This corresponds to the reflection of a condensation or rarefaction at the closed end of a tube, and is reflection with change of sign.

If such a transverse wave arrives at a free end of a rope, it is also reflected, but in this case the slope is reversed, and the motion of the particles is not. This corresponds to the reflection of condensations and rarefactions at the open end of a tube. The wave produced by cracking a whip is reflected in this way.

Every movement which we cause A to execute at right angles to the length of the rope is repeated in turn by each particle; later and later the further from A. So that, until the waves reach B, the past displacements of A are the present displacements of the successive points of the rope. The rope is in fact a displacement curve representing the history of the movements of A.

If we move A up and down harmonically, the rope itself is thus thrown into harmonic waves, which travel along it away from A. The continuous line in Fig. 54 shows the form of the rope at an instant when A has been vibrating harmonically for some time; the arrows show the relative velocities of the material of the rope at different points. These velocities are proportional at each point to the slope of the rope there, and are in opposite directions at points where

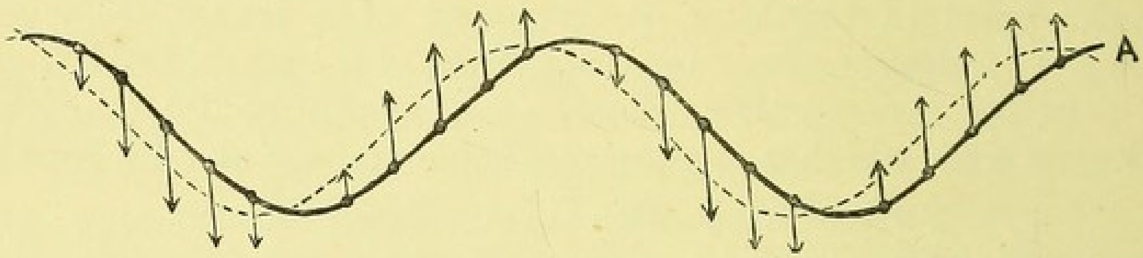


Fig. 54.

the rope slopes oppositely. An instant later every point of the rope has moved a short distance in the direction of the arrow attached to it, and the form of the rope is the dotted curve. Thus, while each particle of the rope has moved up or down, the form of the rope has moved to the left.

If the stretching force is different in different parts of the string, the velocities with which the waves travel along different parts are proportional to the square roots of the stretching forces, as shown above. The number of waves which pass any point in a second is of course equal to the number that start in a second, and is the same in all parts of the string. The waves are therefore longer where they travel quicker, the length of a wave varying as its velocity varies as it goes along, as in longitudinal waves.

97. Transverse Stationary Undulation.—Next suppose we move both A and B up and down harmonically with the same frequency. A series of harmonic waves, of equal wave-length, will start from each end towards the other. After these wave-systems have met in the middle of the rope, the principle of superposition shows that the actual displacement at any given moment, of every part of the rope, can be found by adding the displacements which would be due at that moment to the wave-systems separately.

Let X, Fig. 55, be the form which the rope would have, at a given moment T, if only the waves from A travelled along it, and let Y be the form which it would have at the same moment if only the waves from B travelled along it. The actual form of the rope at the moment T is that found by adding the ordinates of X and Y; it is shown in line 8. The forms which would be due an instant later to the waves from A and from B respectively can be found by shifting X to the left and Y an equal distance to the right. If we suppose each advanced $\frac{1}{8}$ of a wave-length, and add the

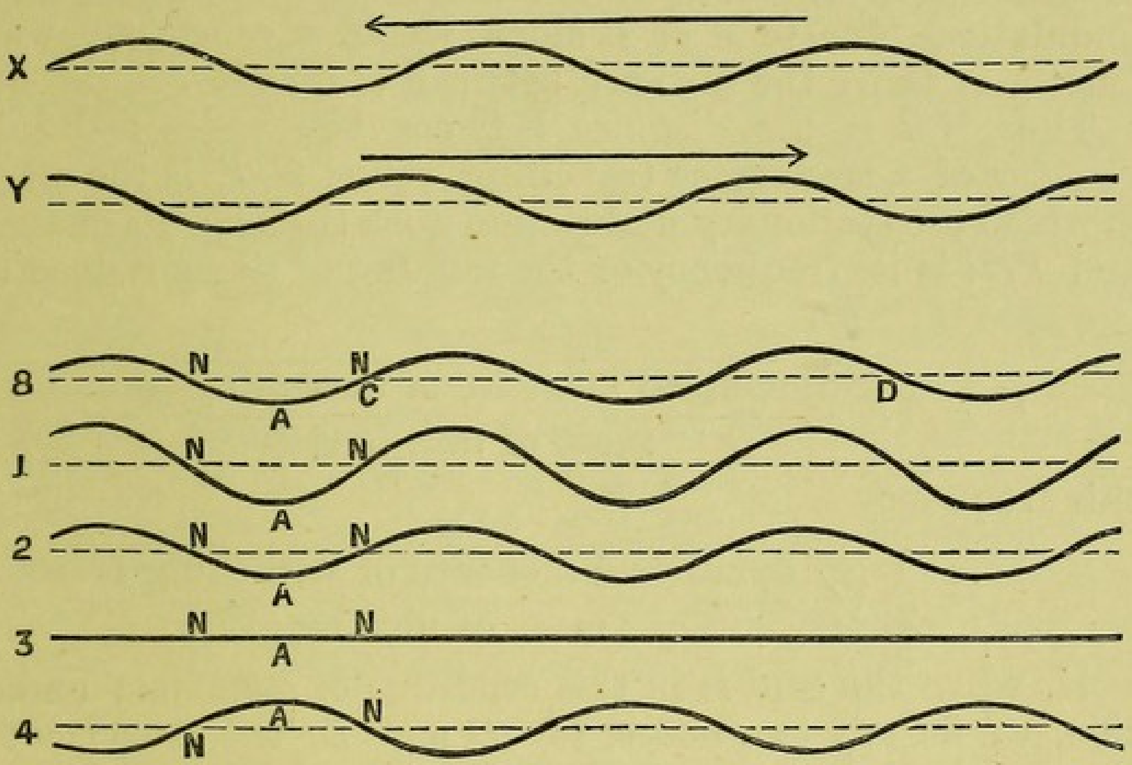


Fig. 55.

ordinates again, we get line 1. By advancing X and Y again each $\frac{1}{8}$ of a wave-length, we get line 2, and so on. (Only 5 out of the 8 such stages of a complete cycle are shown, the remaining 3 being simply 1, 2, and 3 inverted.) We see that the actual movement of the cord is one in which its form is always a harmonic curve, and that, twice in each complete cycle, there is a moment when every part of the cord has, simultaneously, its maximum displacement, and is therefore at rest (stationary instant) while twice in each cycle the curve becomes a straight line. We see also that there are certain fixed points—nodes—through which the cord always passes, so that the displacement at these points is always zero, but that it is at these points that the greatest changes of slope occur. Also that between these there are other points—antinodes—where the greatest displacements occur, but where the cord is always parallel to its original position, so that there are no changes of slope. The movement of the cord is thus one of *transverse stationary undulation*. We see also that the distance from one node to the next is half a wave-length of either of the wave-systems which would be caused by the movement of A or B alone, and that the stationary undulation goes through the complete cycle of movements in the time in which either of the progressive undulations, due to A or B alone, would advance one wave-length, or twice the distance between two nodes.

Thus, if l is the distance between two nodes and V the velocity of a transverse travelling wave, $2l/V$ is the period in which the stationary undulation goes through its changes, and $V/2l$ is its frequency or the number of times it does this is a second. As

$$V = \sqrt{\frac{\text{dynamical measure of stretching force}}{\text{mass of unit length}}},$$

this frequency

$$= \frac{1}{2l}\sqrt{\frac{\text{dynamical measure of stretching force}}{\text{mass of unit length}}}.$$

If, when the cord is in this condition of stationary undulation, we fix any two nodes, say C and D, the movement of the cord is of course unaffected, since C and D were stationary already. The condition of stationary undulation will therefore continue till the energy of the string has been partly communicated to the air in sound waves, and partly converted into heat in the string itself. When two points on a string are fixed, a solitary travelling wave in the part between would run backwards and forwards between them, being reflected each time it reached either, and the stationary undulation of a cord fixed at two points may be conveniently considered as the resultant of two fictitious wave-systems travelling in opposite directions and continually reflected in the same way.

We saw above that, if we had any stretched string, we could, by sending harmonic waves of length $2l$ along it from both ends, throw it into a condition of stationary undulation with nodes l apart, and that if the mass per unit length of this string is m, and the force with which it is stretched f, the frequency of its stationary undulation is $\frac{1}{2l}\sqrt{\frac{f}{m}}$. Also that, if we fix any two nodes xl apart (x being any whole number), the part of the string between them will continue to vibrate with the same frequency in x loops separated by nodes. At each stationary instant such a string is a motionless harmonic curve having x single bends. So that, if we took a similar string, similarly stretched, fixed it in the form of a harmonic curve having x single bends of length l, and then let it go everywhere at once except the two ends (that this is impracticable does not matter for our present purpose), it also would vibrate in the same way and with the same frequency. (This can also be proved directly, without considering the movement as the resultant of two fictitious travelling waves.) The more bends we made, the shorter l would be, and the greater the frequency, the frequency being proportional to the number of bends. If, for instance, we could bend a string into the form of the continuous harmonic curve F, Fig. 56, and let go, it would vibrate from that position to that of the dotted one and back, with a frequency

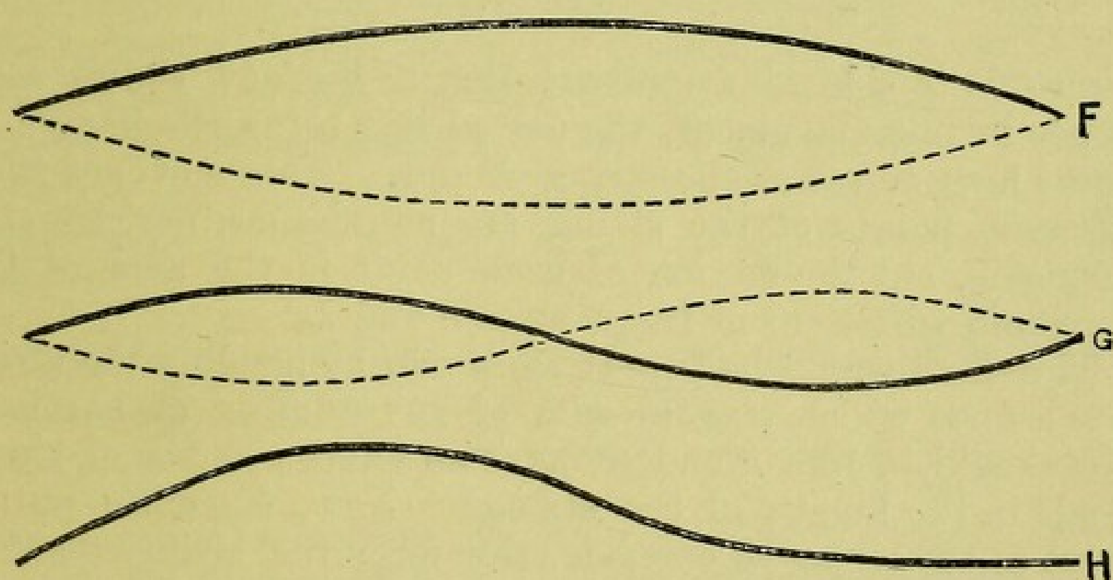


Fig. 56.

$\frac{1}{2l}\sqrt{\frac{f}{m}}$, l being in this case the whole length of the string. We will call this frequency n. If we could bend the same string into the form of the continuous line G and let go, it would vibrate in two loops, with a frequency $2n$, since l is now only half the length of the string. Similarly for three or more bends. In any case, if the string starts from a harmonic form, every point of it vibrates harmonically with the same frequency.

98. Quality of Sound from String.—Next suppose that we could bend the string into the form *H*, whose ordinates are the sums of the ordinates of *F* and *G*. When we let it go, the movement of each point of the string will be the sum (in the sense explained in Art. 7.) of the movement which it would have executed if it had been bent in the form *F* and that which it would have executed if it had been bent into the form *G*. In other words, the string vibrates in two loops from side to side of an imaginary line which vibrates like *F*, in the same way as the string *G* vibrates in two loops

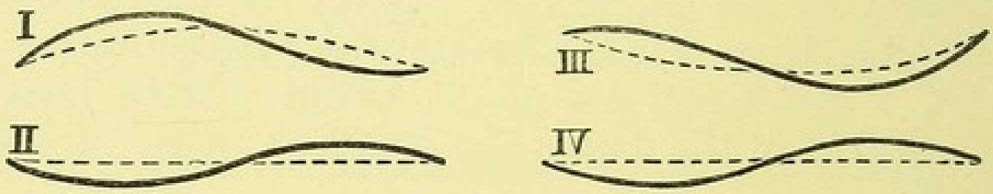


Fig. 57.

from side to side of a straight line. Fig. 57 shows some stages in this movement, the dotted line being the imaginary line which moves as the string *F* did. The movements of different points of the string are not similar, nor, usually, harmonic, but the motion of each point is the sum of two harmonic vibrations of frequencies n and $2n$.

The same would be true if we began by bending the string into a form which was the sum of any number of harmonic curves which, like *F* and *G*, have an exact number of single bends in the length of the string. Now, when we pull a point of the string to one side, the form the string assumes is that of two straight lines meeting at an obtuse angle at that point. This is not a form which looks likely to be the sum of a number of harmonic curves, but Fourier's Theorem, or an easy extension of it, shows that any of the angular forms which can be produced by pulling a point of the string to one side, is a form which might be made by adding the ordinates of harmonic curves of which the length of the string contains an exact number of single bends. It shows further that, of the infinite number of such curves, all are required except those which cut their axis line at the point at which the string was pulled aside. Those which cut the axis very near this point are required only of small amplitude.

Suppose, for instance, we pull the string *CD* (Fig. 58) to one side at a point *E*, $\frac{1}{5}$ of the way from *C* to *D*. The form *CED* is one which might be produced by adding the ordinates of harmonic curves which have 1, 2, 3, 4, 6, 7, 8, 9, 11, &c., half-waves in the length *CD*, those which have any

Fig. 58.

multiple of 5 half-waves being omitted, because they would cross the axis under the point *E*. So that, if n is the frequency with which the string would vibrate if bent into the form of a single harmonic half-wave, the motion of each point of the string, when it has been pulled into the form *CED* and released, is the sum of harmonic movements whose frequencies are n, $2n$, $3n$, $4n$, $6n$, &c. Vibrations of these frequencies are the harmonic components of the real movement of the string.

Similarly, if we pull a string aside at a point $\frac{a}{b}$ of its length from one end, $\frac{a}{b}$ being a fraction in its lowest terms, any harmonic curve which has xb single bends in the length of the string (x being any whole number) cuts the axis at the point where the string was pulled aside. The harmonic components of the vibration produced when the string is let go are vibrations whose frequencies are all the multiples of n which are not multiples of bn, n having the same meaning as above.

If we strike a string, as in the piano, the harmonic components of the vibration produced have the same frequencies as if we had pulled it to one side at the same point, but their relative amplitudes are different. A hard hammer, which touches only a very short piece of the string, and remains in contact with it a very short time, produces a vibration whose high-frequency harmonics are intense, and conversely.

Next, suppose we touch the vibrating string at some other point with a light object such as a feather or camel's-hair brush. This stops all modes of vibration of the string except those which have a node at the point touched, and the subsequent motion of the string is the sum of those harmonic components of its previous movement, which have a node at this point. If, for instance, after plucking the string CD at $\frac{1}{5}$ of its length from one end we touch it at a point $\frac{1}{3}$ of its length from one end, all harmonic vibrations become impossible except in 3, 6, 9, 12, 15, &c., loops. Of these its motion included all except that in 15 loops, so that the subsequent motion is the sum of the harmonic vibrations which it could execute in 3, 6, 9, 12, 18, &c., loops, all multiples of 3 which are not multiples of 5 being included. The harmonic components of this vibration have of course frequencies $3n$, $6n$, $9n$, $12n$, $18n$, &c.; n having the same meaning as before.

Similarly, if a string which has been plucked or struck at a point $\frac{a}{b}$ of its length from one end is then touched at a point $\frac{c}{d}$ from one end ($\frac{a}{b}$ and $\frac{c}{d}$ being fractions in their lowest terms), the harmonic components of the subsequent vibration have frequencies which include all the multiples of n which are multiples of dn but are not multiples of bn.

The ends of a stretched string must be fastened to some solid body, for instance, to pegs or screws in a board, and when the string vibrates its pull on the pegs varies and the board vibrates also, and so produces air waves. The movements of the air so produced correspond nearly to those of the board, and the movement of the board, though it does not correspond closely to that of the string, is the sum of the movements which the harmonic components of the vibration of the string would produce separately, so that the movement of the board has a harmonic component corresponding in frequency to each harmonic component of the movement of the string. (The relative amplitudes of the components may be very different for the board and string, since some components may cause resonant vibration of the board, and not others. There may also be components of the vibration of the board which are not components of that of the string.) The string itself also produces waves in the air, but for the reason explained in Art. 22 they are insignificant in intensity compared with those from the larger moving surface of the board, and a vibrating string would hardly be audible if it could be fastened to absolutely immovable points.

99. Resonant Vibrations of Strings.—The harmonic components of the air waves, and therefore the quality of the sensation they produce, thus depend on the point at which the string is pulled aside or struck, but we cannot make the string vibrate harmonically or produce harmonic air waves by pulling or striking it. But it may be set in *resonant* vibration by impulses agreeing in frequency with any of its modes of vibration; if these impulses are harmonic in character, the resonant vibration will be harmonic. For instance, if we attach a thread to one prong of a tuning fork, as shown in Fig. 59, it will be found that when one of certain definite

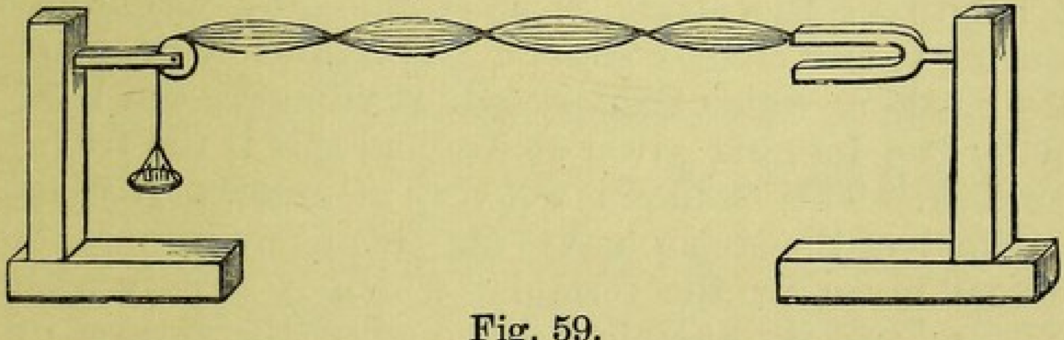


Fig. 59.

loads is placed in the scale-pan, and the fork excited by drawing a violin bow across one prong, the thread is thrown into strong stationary undulation, the number of loops depending on the weight in the pan. This occurs only if the weight is such that one of the modes of free stationary undulation of the thread is nearly of the same frequency as the fork; the weights which are required to make the same string vibrate in 1, 2, 3, &c., loops are inversely proportional to the squares of these numbers. This is "Melde's experiment."

The fork may also be placed so that it vibrates in the direction of the length of the thread, instead of at right angles to that direction, so that the fork pulls the thread instead of shaking it up and down. In this case there is resonant vibration when the weight is such that one of the modes of vibration of the thread has *half* the frequency of the fork, for the thread is straight when the prong is at one end of its swing, and loosest when the prong is at the other, so that a quarter vibration of the thread takes place in half a vibration of the fork.

Merely bringing the stem of a vibrating fork into contact with the board on which a string is stretched will set the string in resonant vibration if the frequency of one of its modes agrees nearly with that of the fork. This may be used to find the frequency of a fork, if the string is stretched by means of a weight so that the stretching force is known. This weight, or the length of the string, is altered till the fork throws the string into resonant vibration, which is detected by placing a little folded piece of paper astride the string; the paper is thrown off when resonance occurs. The mode of vibration is determined by placing a large number of such "riders" at different points along the string; those which are at or near nodes are not thrown off. The mass per unit length of the string is ascertained by weighing and measuring it or a similar piece. The frequency of the string in the mode in which the fork sets it vibrating can be calculated by the formula given above, and this is the frequency of the fork. This method is not very accurate, partly because the rigidity of the string makes its frequency rather greater than that given by the formula, which is strictly accurate only for a perfectly flexible string, and partly because, owing to the small mass of the string, it is not easy to determine exactly when the resonance is at its maximum, and the free-vibration frequency of the string therefore equal to that of the fork. The latter cause of error may be avoided by adjusting the string until the note produced by twanging it or drawing a violin bow across it is the same as that of the fork, as shown by the absence of beats, or, as very slow beats are not easily heard, till the beats have a frequency which is easily counted. The frequency of the beats must of course be added to or subtracted from the calculated frequency of the string. The action of the bow, which is resined, is the same as that of the resined cloth in Art. 92; though it produces resonant vibration, its impulses depend on the vibration already existing, and the vibration it produces has the frequency of the free vibration of the string.

This experiment is most easily carried out on a *sonometer*, which is an instrument for experimenting on the vibrations of strings. Its usual form is shown in Fig. 60. AA is a long

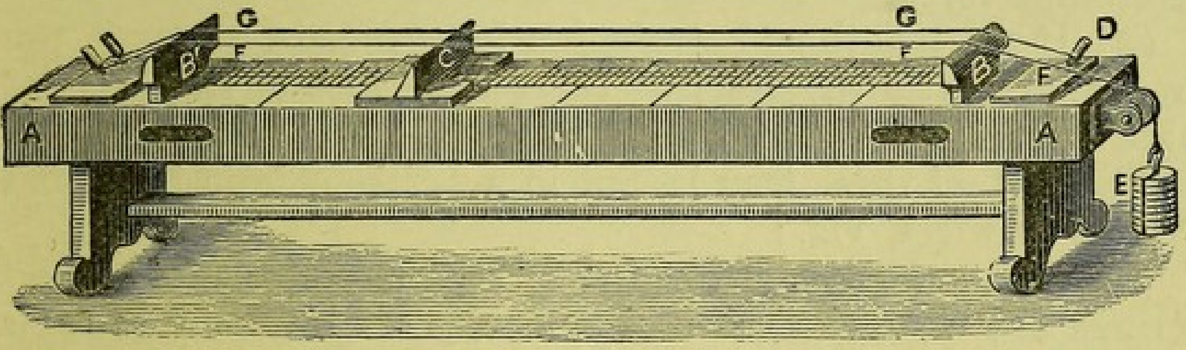


Fig. 60.

box with holes in its sides. Two wires are stretched over the two fixed bridges B', B, the upright faces of which are a metre apart. One of the wires passes over a pulley and is stretched by weights; the other is stretched by a wrest pin, D, which is turned by a key. A third bridge C is movable, and a little higher than the others, so that it presses the string and reflects transverse waves.

A simpler form, with one string, is called a *monochord*.

The formula $n = \frac{1}{2l}\sqrt{\frac{f}{m}}$ is most easily proved experimentally by means of the same apparatus, the string being stretched by a weight. l, f, m can be observed, and the calculated value of n compared with that determined by direct measurement. For this, if we wish to avoid all assumptions, we can use the photographic method, or the method of tracing on a revolving drum, or the vibration microscope. If we assume that the pitch of the note produced corresponds to the frequency of the vibrations of the string, the frequency may also be measured by the siren.

These methods are described in the next chapter. The calculated and observed frequencies agree very closely.

From the formula $n = \frac{1}{2l}\sqrt{\frac{f}{m}}$ several facts at once follow which are sometimes called the "laws" of the transverse vibration of strings. The most important are:

(1) If the stretching force and mass of unit length remain constant, the frequency varies inversely as the length.

(2) If the length and mass per unit length remain constant, the frequency varies directly as the square root of the stretching force.

(3) If the length and stretching force remain constant, the frequency varies inversely as the square root of the mass of unit length.

The last of these may be put in a different form for round strings or wires, viz.:

(3*a*) If the length, stretching force, and material of several strings are the same, their frequencies are inversely as their radii (or diameters).

These laws, being merely proportional, are true whatever units are used to measure the different quantities. In this, as already pointed out, they differ from the absolute formula $n = \frac{1}{2l}\sqrt{\frac{f}{m}}$, which cannot be deduced from these laws, as it contains more than they do.

The "laws" are of course completely proved by the experiments which prove the absolute formula, but they may also be illustrated by less elaborate experiments which do not involve the measurement of an absolute frequency. These experiments are often said to "prove" the laws, but the proof is far from conclusive, and involves several assumptions.

We shall evidently require some means of determining whether the changes of frequency produced by changing the length, stretching force, or thickness of the string correspond with the changes predicted by the laws given above. As explained in Art. 65, most persons easily recognise whether the interval between two notes is exactly an octave, and it was shown by an experiment that when it is an octave, the frequency of one vibration is twice that of the other. We shall assume, then, that the perception of this relation between the sounds is sufficient proof that the frequencies of the strings are as 2 : 1. With this assumption, the following experiments may be considered to illustrate the laws, though it would be easy to devise quite different laws with which the experiments are equally consistent.

Law 1.—Adjust the tension of the string which is stretched by the wrest-pin till it gives the same note as the other string when made to vibrate, which is best done by drawing a resined violin bow across it. Insert the movable bridge *C* under the string *F*, but not under *G*, and adjust the distance *CB* till the portion of the string *F* between *B* and *C* gives, when plucked or bowed, a note an octave higher than that given by *G*. It will then be found that the length *BC* is almost exactly half the length *BB′*.

Law 2.—Next remove the bridge *C* and increase the weight *E* till *F* gives a note an octave higher than *G*, and therefore an octave higher than *F* did originally. It will then be found that the total weight at *E* has been increased very nearly to four times its original amount, allowing for the weight of the rod in each case.

Law 3.—Next substitute for *F* a brass wire of 20 gauge, and load *E* till this wire gives the same note as *G*. Then replace it by a brass wire of 25 gauge, and load this till it gives a note an octave higher than *G*. It will be found that the weight required is about the same as that used for the 20-gauge wire. By weighing equal lengths of the two wires, or by measuring with a screw-gauge, it can be shown that the mass of unit length of 20-gauge wire is about 4 times as great as of 25-gauge wire, or that the diameter of the former is twice that of the latter.

The quantity $\sqrt{\dfrac{\text{stretching force}}{\text{mass per unit length}}}$ which appears in the expressions for velocity of a transverse wave and frequency of a transverse vibration may be put in a different form. The ratio $\frac{\text{stretching force}}{\text{area of cross section}}$ is the tension* of the string. The ratio $\frac{\text{mass per unit length}}{\text{area of cross section}}$ is the density of the material. The ratio $\frac{\text{stretching force}}{\text{mass per unit length}}$ is therefore equal to, though not the same as $\frac{\text{tension}}{\text{density}}$; the latter may therefore be substituted for the former. The mass of unit length of a string is sometimes called its *linear density*.

100. Transverse Vibration of Rods and Plates.—A wave of transverse displacement may be sent along a rod of any elastic material in the same way as along a stretched cord, but the rod need not be stretched, and it is the elasticity of the material, not tension, which restores the successive portions of the rod to their original relative positions. The velocity of such a transverse wave, if harmonic, is independent of the thickness b of the rod in the direction perpendicular to the displacement; proportional directly to the thickness t in the direction of the displacement, and to the square root of Young's Modulus Y for the substance; proportional inversely to the square root of the density d and to the length of the wave λ. A non-harmonic wave has no definite velocity, but the movement is the sum of the movements which would be due to the harmonic components, each travelling with the velocity proper to its wave-length. This dependence of velocity on wave-length makes the laws of transverse vibration of elastic rods much more complex than those of strings. In elastic rods stationary transverse undulation is produced, as in strings, by the interference of wave systems, which are travelling in opposite directions, and which have been reflected from the ends of the rod. The modes of stationary transverse undulation possible in a bar depend on what points, if any, are fixed, and whether the bar at these points is fixed both in direction and in position, as when a rod is held in a vice, or fixed only in position, as when it merely rests on fixed supports, as in the toy called the *harmonicon*. In any of these cases there may be nodes and antinodes, and an end which is fixed in position but not in direction is always a node. An end fixed in direction is not a true node, since it is not

* "Tension" is still sometimes used as equivalent to "stretching force," but it is better to avoid this and use the term only for the stress, or ratio of stretching force to area of cross section. Compare footnote to Art. 27.

a point of maximum change of slope (Art. 97); nor is a free end a true antinode, since it is not a place where the rod moves parallel to itself. The point of no transverse motion nearest to a free end is also not a true node, since the change of slope there is not as great as at the free end itself. All the true nodes are equidistant; the distance from an end fixed in direction to the nearest node, or from a free end to the nearest true node, is $1\frac{1}{4}$ times the distance between two true nodes; the distance from a free end to the nearest point of no transverse motion is a little less than a third of the distance between two true nodes. From these rules the different possible modes of harmonic vibration in any given case are easily determined; the simpler ones, for a bar entirely free (or supported at two nodes) are shown in Fig. 61, for a bar fixed in direction at one end in Fig. 62.

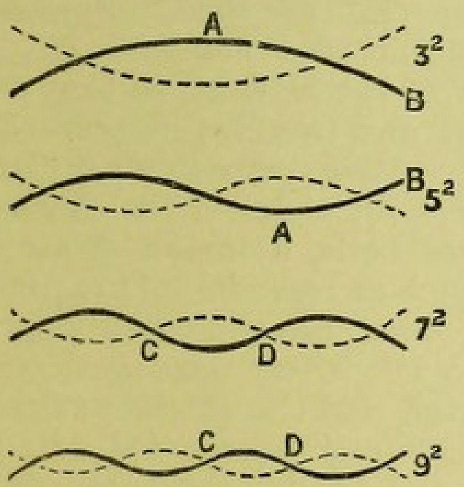


Fig. 61.

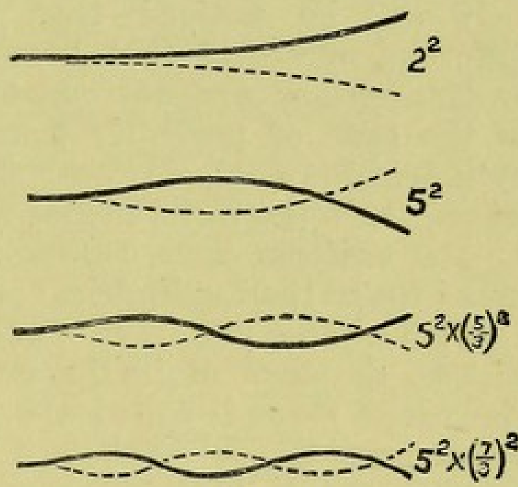


Fig. 62.

The frequency of the vibration, being the reciprocal of the time required for two travelling waves to pass each other whose length λ is twice the distance D between two true nodes, is independent of b, proportional to t and $\sqrt{Y}$, and inversely to $\sqrt{d}$ and D^2. From this it follows that, (1) if two rods vibrate in the same mode, their frequencies are proportional directly to t and $\sqrt{Y}$, and inversely to $\sqrt{d}$ and to the squares of their lengths; (2) if the same rod vibrates in two different modes, the frequencies are inversely proportional to the squares of the lengths of pieces which vibrate in the same way, for instance, the pieces A, B or the pieces C, D, in Fig. 62. The frequency of each mode of vibration, relatively to the other modes of the *same* figure, is shown by the number to the right of the diagram.

If a rod is bent into a U-shape, and made to vibrate with an even number of nodes, two of these nodes are very close together at the bend, and at the antinode between them there is not only no change of direction, but very little transverse motion, as shown in Fig. 63. If the vibrating rod is held by this point, it shakes the holder very slightly, and so loses very little energy except by communicating it directly to the air; it thus vibrates a very long time. The *tuning-fork* (Fig. 31) is an application of this principle. It may be made to vibrate by striking one prong on an inelastic substance such as lead, or by drawing a violin bow across one prong, or (if the prongs are nearer together at the points than at the bend) by drawing between the points of the prongs a rod of wood just too large to pass without bending them. The relative frequencies of the different modes of vibration do not differ much from those of a straight bar fixed at one end (Fig. 62). The slowest mode (that shown in Fig. 63) lasts much longer than the others, so that the vibrations of a tuning-fork which has been vibrating for some time are always in this mode, and are (practically) perfectly harmonic.

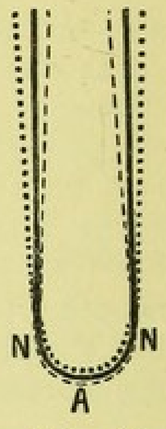


Fig. 63.

It will be noticed that the frequencies of the different modes of free harmonic vibration of a bar or tuning-fork are not exact multiples of the slowest, as they are in the case of perfectly flexible strings and (nearly) in perfectly cylindrical or conical tubes. The following laws apply to most cases where the more rapid modes are not exact multiples of the slowest, *e.g.*, to uniform rods, tuning-forks, plates, bells, columns of air of other forms than cylindrical or conical, such as resonators (Art. 106).

(1) The free vibration of such a body, if not harmonic, is the sum of two or more of its harmonic modes occurring together. The slowest of these is called the fundamental, and the others *partials*, *upper partials*, or *overtones*. These terms can be applied to any components of higher frequency than the fundamental, whether exact multiples of the fundamental or not. The term harmonics is restricted to exact multiples.

(2) When such a body is set in forced vibration by a periodic force, all the components of its motion are exact multiples of the frequency of the force, so that all the upper partials are harmonics of the fundamental. The components which have any considerable amplitude correspond to components of the periodic force which nearly agree in frequency with possible free vibrations of the body.

It must be remembered that, even when a bar or fork vibrates quite harmonically, the air-waves from it are not exactly harmonic (Art. 22), and therefore they have harmonic components which are exact multiples of the frequency of the bar. So that, when a bar is in free non-harmonic vibration, the waves from it have two distinct sets of harmonic components: one set which correspond in frequencies to different modes in which the bar can vibrate, but are not exact multiples of the fundamental frequency; the other set having frequencies which are exact multiples of the fundamental frequency, but do not correspond to anything in the movement of the bar. The former set are called the *non-harmonic overtones*, the latter the *harmonic overtones*. The latter are still present when the motion of a fork has become quite harmonic.

If a tuning-fork vibrates in front of a spherical resonator whose slowest vibration frequency is the fundamental frequency of the fork, none of the overtones of the fork correspond with free-vibration frequencies of the resonator, and therefore the forced vibration of the resonator has no intense overtones, but is nearly harmonic. The waves so produced are the most nearly harmonic waves which can be made.

Plates of glass, metal, or any elastic solid, clamped at one point and free everywhere else, can be made to vibrate transversely, as already explained in Art. 40. In the stationary undulation produced, there are certain lines on the surface which have no movement in space, but where the surface slopes first one way, then the other. These lines are called nodal lines, and the parts of the surface separated by a nodal line are moving, at any one moment, in opposite directions. The antinodes, or places where the surface moves parallel to itself, are also lines. The nodal lines can be found by scattering sand on the surface and drawing a violin bow across the edge of the plate, which for this purpose must be fixed horizontally; the sand collects along the nodal lines, as it is thrown into the air from every other point. The figures so produced are called *Chladni's figures*. In the case of a circular disc fixed at its centre the nodal lines are radii, and divide the disc into an even number of equal sectors. There are four such sectors if the edge of the disc is touched only by the bow, but if, while the plate is bowed, we touch with our fingers two points on the edge whose distance apart is contained an even number of times in the circumference, there will be that number of radial nodal lines, and two of them will run from the centre to the points touched by the fingers. There may also be circular nodal lines concentric with the plate.

The frequencies of the same plate vibrating with different numbers of radial nodes are proportional to the squares of these numbers. The frequencies of plates of the same material vibrating with the same number of radial nodes are inversely as the squares of the radii and directly as the thicknesses of the plates.

Circular stretched membranes vibrate in modes which are very similar, though the vibrations depend on the force with which they are stretched, not on elasticity, and the relative frequencies are not those of plates.

Plates of other forms can also be made to vibrate, their nodal lines being often very complex. In the case of square and rectangular plates they have a general resemblance to Lissajous' figures. The following device, due to Wheatstone, enables us in many cases to predict their forms :—A rectangular plate D, Fig. 64, might be supposed divided into a series of equal rods by parallel cuts in either of two directions. We will call these the A and B series respectively. Among the modes of transverse vibration possible for these imaginary rods there is sure to be one mode for those of the A series, and one for those of the B series, of about equal frequency. Suppose the rods made to vibrate in these modes, but with exactly equal frequencies and amplitudes, all the rods of the same series executing simultaneously the same movement, and the rods of both series reaching their stationary instants simultaneously. In the figure,

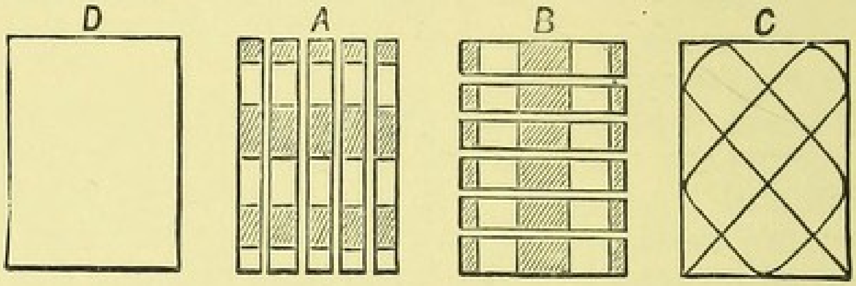


Fig. 64.

the shaded parts of the rods are parts which might be moving all in one direction at the same time; while the unshaded parts were moving in the other direction. The actual movement of any part of the plate is the sum of the movements of the corresponding portions of the two fictitious rod-systems vibrating as above, and the nodal line of the actual vibration is one like that in C, at every point of which the movements of the fictitious rods would either both be zero or equal and opposite.

A *bell*, which is simply a concave circular plate, bears to a flat plate the same relation as a tuning-fork to a straight bar, the advantage gained by the bent form being the same as explained above in the case of the fork. The modes of vibration of a bell are the same as those of a circular plate, the edge being divided by nodal points into any even number, not less than 4, of oppositely moving segments. This may be tested by holding a suspended pith

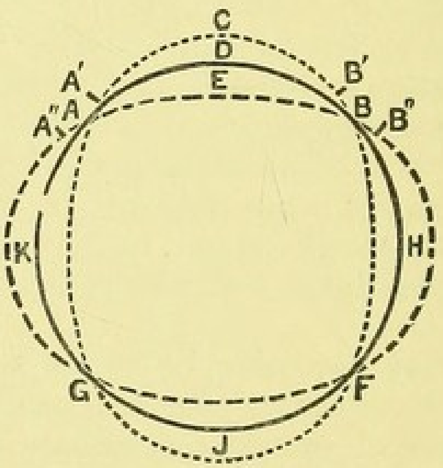


Fig. 65.

ball against different parts of the edge, or by inverting the bell, filling it with water, and bowing the edge; ripples will be seen to proceed from all points except the nodes.

The curved form makes a difference which in the case of a bell is of some importance. The edge of a bell vibrating in four segments keeps changing its form from that of the dotted line in Fig. 65 to that of the broken one and back again, and its edge always passes through the points *ABFG*, which are therefore considered nodes. These nodes, however, are not points where the substance of the edge is not displaced, as they are in a flat plate. The arc *ACB* is longer than the arc *ADB*, so that particles of the rim which were at *A*, *B* before the vibration began are at A', B' when the rim has the dotted form and at A'', B'' when it has the broken one. (The distances AA', &c., are exaggerated in the figure.) There is therefore a tangential vibration of the material of the bell at *A*, *B*, *F*, *G*, as well as a radial vibration at *D*, *H*, *J*, *K*, and a vibration making an oblique angle with the edge at intermediate points. As one of these movements cannot take place without the other (unless the edge alters in length), the whole movement is produced if we either make *D* vibrate radially or make *A* vibrate tangentially, so that striking the edge of the bell at *D*, or drawing a violin bow across that part of the edge, produces exactly the same vibration as applying a resined finger at *A* and drawing it along the edge. So that, when we run a wet finger round the edge of a tumbler, it vibrates just as if we struck it at a point 45° from the finger, and as the finger moves round, the mode of vibration moves round also, the finger being always at a point of purely tangential vibration, or node.

If one end of a rod is twisted, the next portion twists, and a wave of twist or torsion travels along the rod. The velocity of such torsional waves depends on the rigidity of the substance, and their chief interest is as a means of determining this. They are reflected and form nodes and antinodes, exactly like longitudinal vibrations of rods.

For a rod of circular section the velocity is

$$\sqrt{\frac{\text{dynamical measure of rigidity}}{\text{density}}}.$$

101. Law of Linear Dimensions.—A very important law, which applies to every kind of vibration of solids or fluids, is due to Bernouilli and is called the Law of Linear Dimensions. It states that bodies of geometrically similar form, and the same material, differing only in dimensions, when they vibrate in the same manner, have periods proportional to their linear dimensions.

EXAMPLES III.

Elementary.

1. If four strings of the same length and material, but of diameters in the ratios 1 : 2 : 3 : 4, are all stretched to half their breaking stress, compare their vibration frequencies.

2. A string is stretched by the weight of 60 lbs. and it is found that a hump (or small transverse wave) produced by striking it travels along it at 30 feet per second. By what weight must it be stretched to make the hump travel 90 feet per second?

3. A string stretched by the weight of 10 lbs. gives a certain note. By what weight must a second string of the same material, but of twice the length and twice the diameter, be stretched to make it give a note an octave higher than that of the first string?

4. Four violin strings, all of the same length and material, but of diameters in the ratios 4 : 3 : 2 : 1 are to be stretched so that each gives a note a fifth above the preceding. Compare the forces necessary. (For meaning of "a fifth above" see Art. 66.)

Advanced.

5. A string whose mass is 500 grams hangs vertically, and a mass of 400 grams is fastened to its lower end. A transverse wave an inch in length is produced by shaking the lower end, and travels to the top of the string. What is the length of the wave at the middle and at the top?

6. A rope weighing half a pound to the foot is stretched horizontally by a force equal to the weight of 100 lbs. If the rope is shaken at one end, with what velocity do the waves run along it?

7. A wire a metre long, weighing $2\frac{1}{2}$ grams, is stretched on a sonometer by a weight of 18 kilograms, and when its length is adjusted to 80 centimetres and a vibrating fork applied, the string throws off a rider placed at any point except two, besides the ends. Find the frequency of the fork.

8. A bar 2 feet long and $\frac{1}{10}$ inch square, fixed at one end in a vice, makes 20 vibrations per second. How many vibrations will a bar of the same material, 1 foot long, $\frac{1}{4}$ inch wide, and $\frac{1}{8}$ inch thick, make per second if made to vibrate transversely in the direction of its thickness?

9. In performing Melde's experiment, it was found that the string vibrated in 5 loops when 10 grams was placed in the scale-pan. What mass must be placed in the scale-pan to make the string vibrate in 7 loops? [Neglect the weight of the scale-pan.]

CHAPTER XII.

ACOUSTIC MEASUREMENTS.

102. **Frequency.**—The frequency of a solid vibrating body is best determined by means of a *Stroboscope.* This is a disc which can be made to rotate on its axis by means of a band passing round a small wheel on the axis and a larger wheel turned by hand. Radial slits are cut in the disc at equal intervals, and a counting mechanism, records its rotations. If we fix a bright point to a vibrating part of the body and look at the point through the slits while we allow the disc to slacken gradually from a very high speed, we shall see the point as a line of light, except when the disc reaches certain speeds when the bright line changes for a few seconds into a number of separate points. When the frequency of the slits is only a little greater than that of the point, a single slowly moving point is seen, which becomes stationary when the frequencies are exactly equal. The rotation is now maintained by the handle at such a speed that the point seems to remain stationary, and the counting mechanism set in action for a measured time; from the number of rotations recorded the frequency of the slits is easily found, and this is the frequency of the point. Care must be taken that the speed of the disc is the *highest* speed at which a single stationary point is seen, as the same appearance is presented if the frequency of the slits is an exact submultiple of that of the point. If the amplitude of the vibration is small, the bright point may be viewed through a microscope; the stroboscope disc revolving between the microscope and the point.

Graphic methods, in which the vibrating body traces its movements, are less accurate, since the necessary friction always affects the frequency of the vibration. The *Vibroscope* (Fig. 66) is a good example of this method. The vibrating body is fixed so that the points whose movements are to be counted moves near to the surface of the cylinder, and parallel to its axis. The cylinder is covered with smoked paper, and a fine wire, attached with wax to the vibrating body, just touches the paper. As the handle is turned, and the body vibrates, the wire traces a wavy line on the paper; this line forms a screw thread

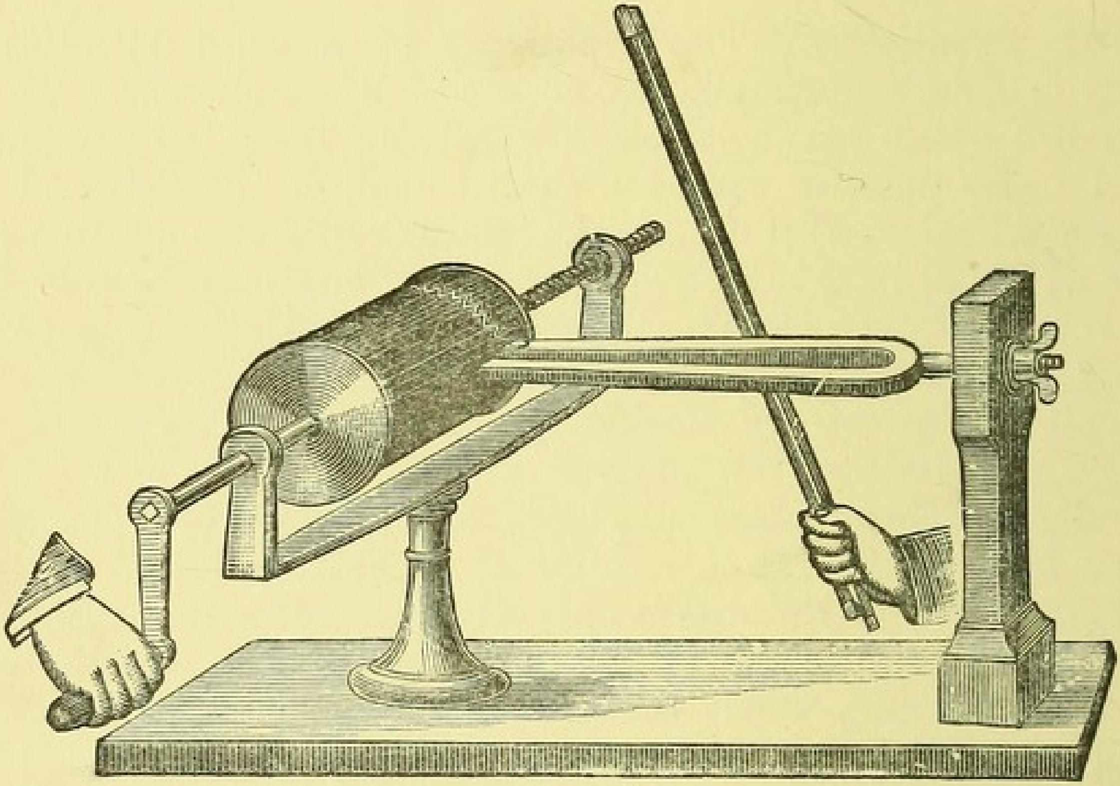


Fig. 66.

round the cylinder, which is made to advance in the direction of its length by a screw thread cut on the axle. When this apparatus is used for measuring frequencies, the vibrating body is insulated and connected to one end of the secondary of an induction coil, while the other end is connected to the revolving cylinder. The pendulum of a (seconds) clock is arranged to make contact, as it passes its lowest point, with a drop of mercury, and so to close and break, once in each second, a circuit which includes pendulum, mercury drop, the primary of the induction coil, and a battery. The spark which passes between the wire and the smoked paper, each time this circuit is broken, knocks off a little spot of the soot at the point in contact with the wire at the moment; the number of double bends between two successive spots on the wavy line traced by the wire gives the frequency.

Both these methods give the displacement curve of the movement of a point, so that they may be used to determine the character as well as the frequency. A fairly accurate measurement of the frequency of a tuning fork may be made by means of the simple

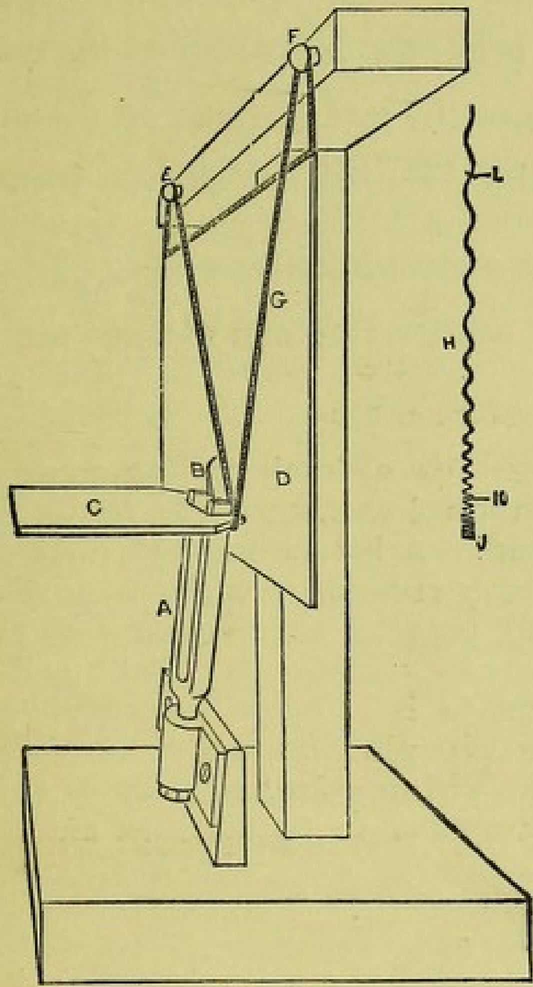


Fig. 67.

apparatus shown in Fig. 67. The tuning fork *A* is fixed in a nearly vertical position, and a bristle or pointed piece of thin sheet metal *B* is fastened with wax to one of its prongs. The prongs are pressed together till they are close enough to enter a square notch in the metal plate *C*, which holds them in this compressed position. A smoked plate of glass *D* is suspended from two pins *E*, *F* by a silk thread *G* hanging over them in two loops, one of which is cemented to the upper edge of the glass plate, while the other is caught under one of the points of the metal plate *C*. When *C* is withdrawn, which should be done quite horizontally, the plate *D* falls freely (the friction of the silk thread on the pins being negligible), and at the same instant the prongs, suddenly allowed to fly apart, begin to vibrate. The point *B* thus traces on the falling plate a wavy line similar to *H*, the bends of which at the lower end are too close to be distinguished, because traced when the plate was falling very slowly, but widen out as we go up. In any part of the trace where the bends are quite clear and distinct we mark three points, *X*, *Y*, *Z*, such that there is some exact whole number a of double bends between X and Y, and the same number between Y and Z. Let the distance XY be d_1, and YZ, d_2. Then the *average* velocities of the plate while the parts XY, YZ passed the tracing point, were $\frac{d_1 n}{a}$ and $\frac{d_2 n}{a}$ respectively (n being the frequency of the fork), and these were the actual velocities at the mid-instants of these intervals. So the plate gained a velocity $\frac{(d_2 - d_1)n}{a}$ while the pointer traced a waves, in $\frac{a}{n}$ seconds, so its acceleration was $\frac{(d_2 - d_1)n^2}{a^2}$. This $= g$,* which is 981 in England if centimetres and seconds are the units. Hence $n = a\sqrt{\frac{981}{d_2 - d_1}}$.

It is difficult to get on a freely falling plate a continuous trace representing more than $\frac{1}{6}$ sec., and this limits the accuracy with which n can be found. A better method is to fix the glass plate to the bob of a heavy seconds pendulum, so that the face of the glass is vertical and parallel to the plane of swing. The fork is held so that the bristle vibrates vertically and touches the plate. The trace on the plate is a wavy line, the bends of which are of unequal lengths, as the plate moves with different velocities in different parts of the swing. In any part of this trace where the waves are not too crowded to be distinct, mark off any three consecutive lengths co taining each the same number a of waves. Let d_1, d_2, d_3 be these lengths. The longer the portion of the trace occupied by them, the more accurate the result. Let θ be the angle whose cosine is $\frac{d_1 + d_3}{2d_2}$, and T the time of a double swing of the pendulum. Then the frequency of the fork can be shown to be $\frac{360° \times a}{\theta T}$.

In all these graphic methods the attachment of a tracing point, by increasing the mass, diminishes the frequency of vibration, and this must be allowed for. It is best done by comparing the frequency of the vibrating body, before and after the attachment of the tracing point, with the nearly equal frequency of a fork which remains in the same condi-

* This experiment is also used as a method for finding g, the frequency of the fork being assumed known, but, as g is a quantity which can easily be measured many times more accurately than the frequency of the fork can possibly be known, this cannot be considered as more than a very rough method for finding g.

tion in both experiments; the change due to the mass of the tracing point is thus determined.

There are several good methods of making this comparison between the frequencies of nearly equal forks. One is simply to count the beats heard when the two are sounded together; the number of beats per second is the difference of the frequencies. If two forks, *A* and *B*, are very nearly equal in frequency, the beats cannot be heard; in this case take a third fork, *C*, of about the same frequency, and load its prongs with wax till it gives about four beats per second when sounded with *A*. Then determine by counting beats how much faster *A* and *B* are, respectively, than *C*; the difference between these two differences is of course the difference between *A* and *B*.

Where a third fork is not used, it will be necessary to determine not only the difference between *A* and *B*, but which of them has the higher frequency. Most persons can decide this by ear; if there is any difficulty, the prongs of one fork must be loaded with a little wax, and the beats counted. If they are now faster than before, the fork that has been loaded was the slower, and *vice versâ*.

The device of a third fork may be made to give very accurate results by employing Lissajous' figures instead of beats to determine the difference of frequency. In this method, if the nearly equal frequencies of *A* and *B* are to be compared, a third fork, *C* (Fig. 68), has a convex lens fixed to one prong, and is adjusted by sliding movable masses along the prongs till its frequency is a little less than those of *A* and *B*. It is fixed so that its prongs, and their plane of motion, are horizontal. One of the forks to be compared, say *A*, has a small bright dot scratched with a diamond on the smoked end of one prong *m*, and is fixed vertically, with the dot immediately under the lens of *C*, and so that the movements of the dot are at right angles to those of the lens. The dot is illuminated by a condensing lens *c*. If *A* and *C* are both made to vibrate, the virtual image of the dot seen from above through the lens has a movement which is the sum of the movements of the dot and lens; it describes, according to the relative phases of these movements, one of the forms of the top line of Fig. 9, the curve described (which, by persistence of vision, appears as a continuous bright line) assumes in turn all the forms of the series, going through the whole cycle in the period in which

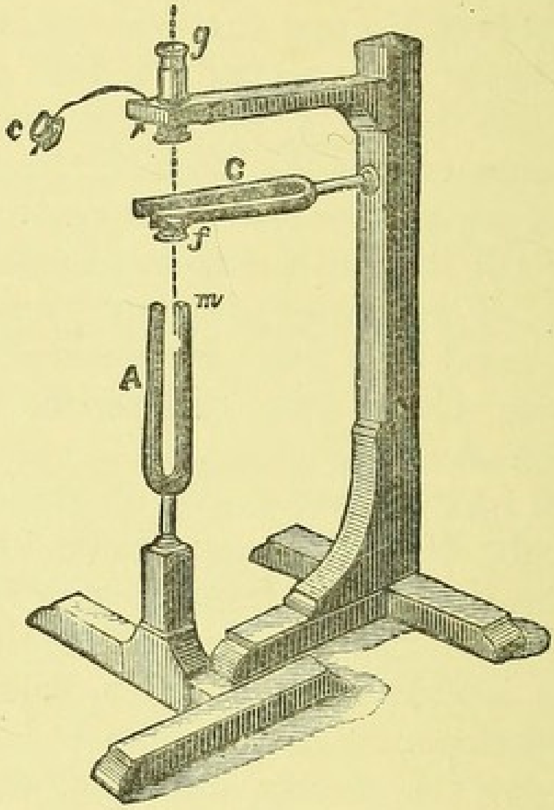


Fig. 68.

A gains one complete vibration on *C*. As the moments at which the curve becomes a straight line can be observed very exactly, the difference of frequency between *A* and *C*, which is the number of complete cycles per second, can be found within $\frac{1}{50}$ of a vibration per second. Similarly, *B* can be compared with *C*, and the difference between *A* and *B* deduced from the two results.

The superiority in exactness of this method to that of audible beats results from the great accuracy with which we can ascertain when one fork has gained an exact whole number of vibrations on the other, and the fact that the vibrations can be observed for a much longer time than they are audible.

A still better arrangement is to place the lens *f* at such a distance above *m* that a real image of *m* is formed, and to examine this real image through a fixed eyepiece *g*, as shown in the figure.

The character, as well as the frequency, of the non-harmonic vibration of a point may be determined by the use of a lens-fork. If a globule of mercury is attached with grease to a string, and observed through the vibrating lens after the string has been plucked or struck, a curve is visible if there is a simple ratio between the two periods, but the curve is not one of Lissajous' figures, since one of the movements, that of the string, is not harmonic. It is an easy matter to determine what movement of the string, combined with the harmonic movement of the fork, would give the observed curve.

By means of this *vibration microscope* we can adjust a fork very exactly to a frequency which has any simple ratio to that of *C*. The fork can be made quicker by filing it near the points, or slower by filing it at the bend, and is to be adjusted till a Lissajous' figure corresponding to the required ratio is seen, and changes only very slowly. Two forks can be adjusted to have a given ratio to each other by adjusting them, in turn, to any convenient ratios to the lens-fork which have the required ratio to each other.

The vibratory movements of the air caused by a vibrating body at any point which is always at the same distance from the body must agree in frequency with those of the body itself; the exact repetition of the same causes must produce the same effects. Frequency is indeed the only particular in which vibrations of the air necessarily agree with those of the body causing them. Any of the methods for determining the frequency of the movements of the air can therefore be used to determine the frequency, though not the character, of the vibrations of a solid body.

To determine the frequency of the movements of the air at a point, an apparatus called a Phonautograph (Fig. 69) may be used. A large funnel, usually made parabolic in form, is closed at the smaller end by a stretched membrane; below the centre of this membrane, at right angles to its surface, is fixed a short bristle. The funnel is fixed so that the membrane is parallel and very close to the surface of a smoked cylinder exactly like that of the Vibroscope. A spring, which touches a point of the membrane exactly above the centre, insures the membrane always vibrating with a vertical nodal line through the centre; the part of the membrane which carries the style is therefore never displaced in the direction of the axis of the funnel, but merely alters its direction, vibrating on a fixed vertical axis. The point of the bristle therefore moves parallel to the axis of the cylinder, and always remains just in contact with the smoked surface, and its movements correspond in frequency, though not in any other respect, with those of the air.

Seconds can be marked on the trace left by the bristle by the electrical method described above for the vibroscope, or a tuning-fork of known frequency can be made to trace a wavy line on the cylinder by the side of that traced by the bristle, and the frequency of the bristle determined by comparing its movements with those of the fork.

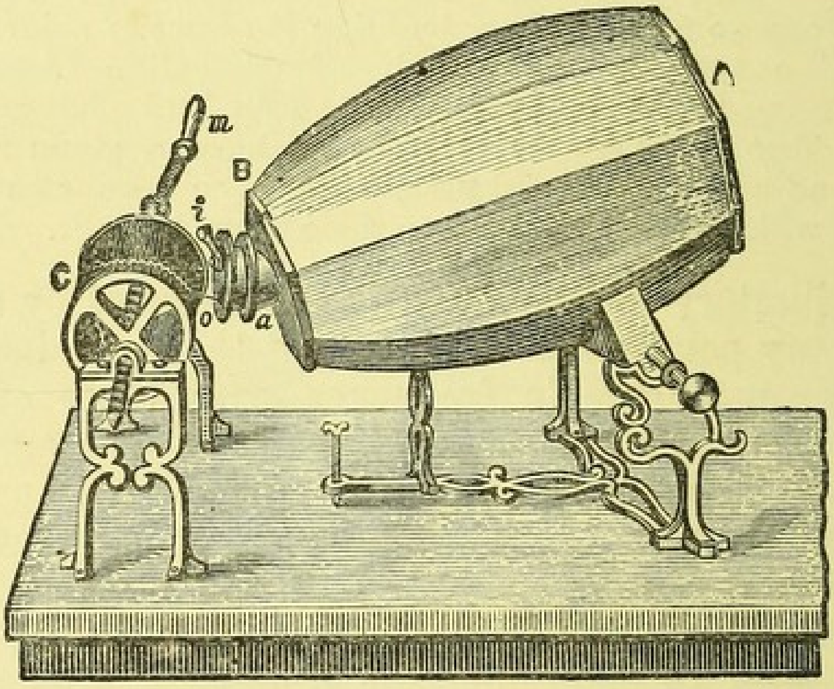


Fig. 69.

The frequency of air waves can also be found by means of a counting siren. Any instrument in which a sound is produced by puffs of air escaping at regular intervals through holes or notches in a rotating plate or cylinder, is called a *siren*. The name was given because most forms of such an instrument may be blown with water in water instead of with air in air; this is however not peculiar to the siren, but is equally true of nearly all wind instruments (Art. 91), and Homer's Sirens differed from the mechanical ones in this respect. A common but not very good form of counting siren is shown in Fig. 70. The lid of the box *C* and the revolving plate *D* contain the same number of holes, and a puff issues simultaneously from all the holes each time those in the plate coincide with those in the lid, so that there are as many successive puffs in each rotation of the disc as there are holes in the circle. The rotations of the disc are counted by wheels, similar to the counting apparatus of a gas meter, which are driven by a screw on the axle of the disc, and can be made to begin counting by pressing a knob, and stopped by releasing it. Air is blown into the box *A*, and the velocity of the disc increased till the pitch of the note heard is nearly that of the sound whose frequency is to be determined. There is nothing harmonic about the

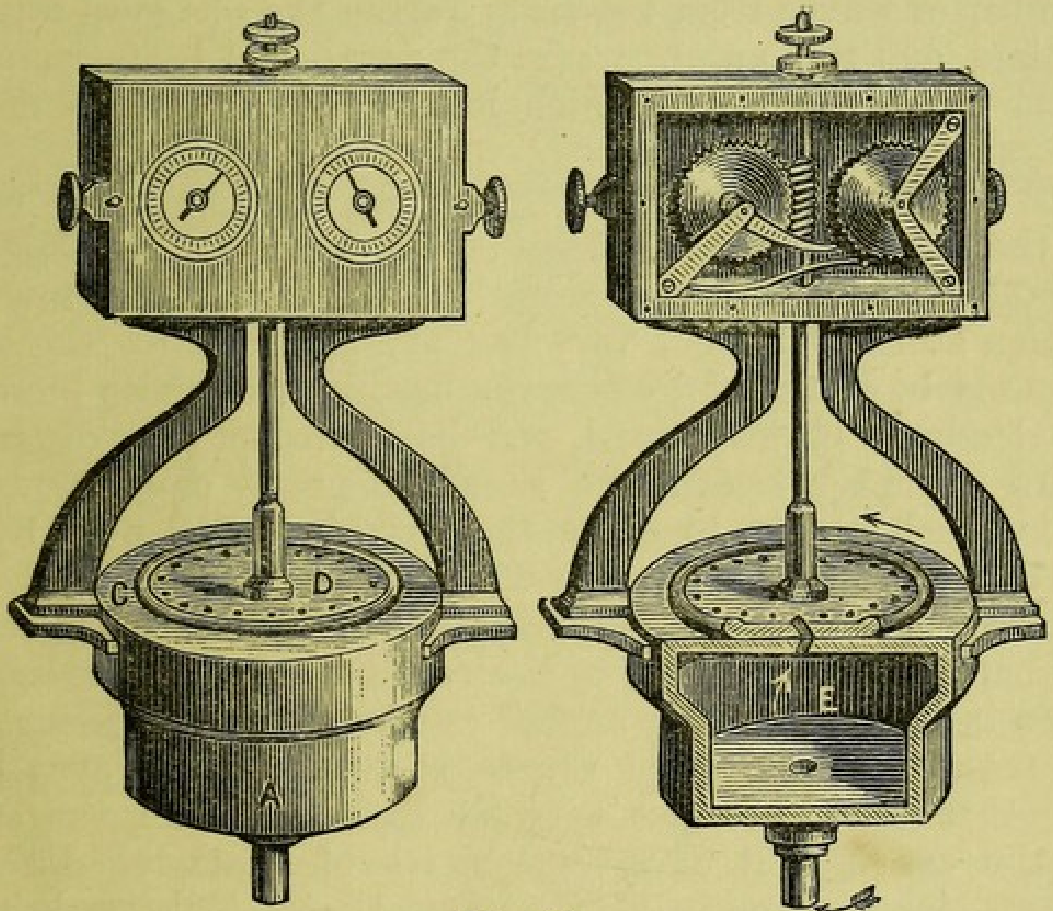


Fig. 70.

process by which the waves from a siren are produced, and the waves, being not at all harmonic in form (Art. 55), have many harmonic components, and can excite in the ear or other resonators vibrations of higher frequencies than that of the actual puffs, and it is only after much practice that it is possible to be certain whether it is the frequency of the puffs, or one of its multiples, which is equal to the frequency of the waves from the other source. When we are sure that it is the lowest in pitch of the sensations due to the siren (which is often not the loudest), which corresponds to the lowest of the sensations due to the other source, the velocity of rotation should be slightly diminished till beats are heard at the rate of about four per second. The counting mechanism is then set in action for a definite time, say thirty seconds, while the observer keeps count of the total number of beats heard during this interval, and regulates the rotation so that the beats do not become too rapid to count or too slow to distinguish. The total number of rotations recorded, multiplied by the number of holes in the disc, gives the number of waves from the siren during the observed interval of time, and this number, *plus* the number of beats counted, is the number of waves which left the other source during the same interval.

In the siren shown in the figure, the disc is made to rotate by the air issuing from the holes, which are drilled obliquely in both lid and disc, but slope opposite ways, as shown in section at *E.* This is a very bad arrangement, for the speed can only be increased by blowing harder, which also increases the loudness of the sound, and beats are not very distinctly heard except between two sounds of nearly equal loudness. It is much better to drive the disc independently by an electromotor or other easily controlled mechanism.

It must not be forgotten that beats are also heard if any of the harmonic components of the sound from the siren nearly agree in frequency with any of the harmonic components of the sound from the other source, so that, if the waves from the other source are also very far from harmonic, the effect on the ear is that of several series of beats, of different frequencies, happening at the same time. This renders the counting of the beats of the fundamental vibration very difficult.

103. König's Wave Siren.—By substituting for a disc with holes a disc with its edge cut in harmonic waves, the siren may be made to give more nearly harmonic sounds. In this case, the air is blown from a single radial slit, which is entirely or only partly covered by the disc, according to the position of the latter. The amount of air issuing from the slit varies nearly harmonically if the teeth of the disc are of harmonic form, so that the waves as they start are approximately harmonic. They become less so, however, as they proceed, so that the difficulty of beats between the harmonic components is not entirely overcome.

By cutting the edge of such a disc into waves or shallow teeth of different forms, waves of different wave-form can be produced, and M. König showed that discs whose edges are cut into different curves which have the same harmonic components (Art. 58) may produce sounds of different quality, and he considered that this disproved Helmholtz's view (Art. 61) that the quality of the sound due to wave systems having different forms but the same components is the same. M. König's experiment, however, in no way disproves Helmholtz's view unless we assume that the harmonic components of the waves produced are the same as the harmonic components of the form of the teeth of the disc, and there is no reason to believe that this is the case. There are, however, more satisfactory arguments against Helmholtz's view.

These are the most important methods of measuring frequency directly. If the velocity of sound in a medium is known, and the length of the waves is measured, the frequency can be at once determined, since

$$\text{frequency} \times \text{wave-length} = \text{velocity}.$$

104. Wave-length.—No method is in use for directly measuring the wave-length of a travelling undulation; it would, perhaps, be difficult to devise one. A method based on the interference of two systems travelling in the same direction was given in Art. 40. More usually the travelling undulation is converted, by reflection, into a stationary undulation, and the wave-length of the travelling undulation determined by finding the distance between the nodes and antinodes of the stationary one.

One method of doing this has been given in Art. 82; it is the best when the waves are very short. For longer waves, such as those from a tuning-fork, a long glass tube about 2 inches wide, and closed at one end, may be fixed vertically, and water poured in until the air column above the water is of such a length that it resounds when the vibrating fork is held over the mouth of the tube. If the tube is long enough, a number of lengths can be found which resound to the same fork. As explained in Art. 85, a closed tube resounds whenever the distance from the point of reflection, just outside the open end, to the closed end, is an odd number of quarter wave-lengths of the waves arriving from outside. The distance between any two consecutive levels of the surface of the water which leave air columns of lengths which will resound is half a wave-length of the waves from the fork, or the distance from the highest of such water-levels to the point of reflection (which is about ·8 of the radius of the tube above the level of the opening) is a quarter of the wave-length. It is from the last fact that the wave-length is most easily found; the resonance of the longer columns being less distinct.

105. Velocity.—If the frequency of the fork is already known, this experiment gives the velocity of sound, which is wave-length × frequency. By surrounding the tube with a larger one, and filling the space between with water of different temperatures, we can determine the change of velocity due to a given change of temperature in the air of the inner tube. In this experiment, mercury should be used to adjust the length of the air column in the inner tube instead of water, as, if water was used, the air would be mixed with a large proportion of water vapour at high temperatures.

The tube can be filled with other gases instead of air, and the velocities determined. For gases lighter than air, the open end of the tube must be turned downwards, and the length of the column adjusted by means of a sliding piston instead of liquid.

Owing to the small mass of the air column, its vibrations are easily controlled, so that it resounds nearly as loudly when it is a centimetre too long or too short as when it is of a length which would vibrate freely with the same frequency as the waves arriving from outside. Any single determination of the wave-length by this method is therefore very uncertain, though the average of a large number of independent determinations is fairly reliable. A much better way is to use a closed organ-pipe whose length can be adjusted by a sliding piston. The tube is kept sounding by an air-blast, and the piston adjusted till the note is the same as that of the fork, as shown by the absence of beats. In this case the air column is vibrating with its free-vibration frequency. But an organ-pipe does not work well unless its mouth is much smaller than the cross section of the tube, and the narrowness of the opening increases the time required for the air to move in and out, so that an air column much shorter than a quarter of the wave-length of the fork vibrates with the same frequency as the fork. In an organ-pipe, therefore, the distance from the mouth to the nearest node differs from a quarter of a wave-length by a considerable, and rather uncertain, amount, so that we cannot determine the wave-length of a note from a knowledge of any one length of the column which will give the note. Two observations are necessary. First blow gently, so that the air column vibrates in its fundamental mode (Fig. 49, *a*), and observe what length gives the same note as the fork. Then blow harder so as to make the column vibrate in its second mode (Fig. 49, *b*), and increase the length of the column till it again gives the note of the fork. In each experiment there is a node at the piston, and in the second experiment there is also a node at the point where the piston was in the first, since the frequency is the same in each case. So the difference between the lengths of the column in the two experiments is half a wave-length of the waves from the fork.

A whistle fitted by means of a cork into one end of a long glass tube 2 inches in diameter answers well for this experiment ; it can be blown by foot-bellows.

The difficulty about the exact position of the point of reflection is avoided by another method, due to Kundt. Though a wave-length is actually measured, it is not really a method of determining the wave-length of a given wave-system, but of comparing the lengths of waves of the same frequency in different substances, and so the velocities of sound in those substances. One form of the apparatus is shown in Fig. 71. A wide glass tube BB', about 4 feet long and 2 inches in diameter, is closed at one end by a tight-fitting

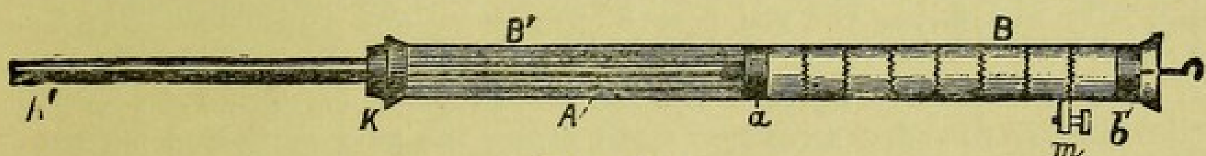


Fig. 71.

cork K. This cork grasps tightly the middle of a metal or glass rod AA', on the inner end of which is fixed a disc a of wood or card fitting the tube very closely without being tight. A cork b' sliding in the other end of the tube serves to adjust the length of the air space between itself and the card disc. In this space is scattered lycopodium powder or fine cork dust,

A cloth, dusted with resin, or (for a glass rod) moistened with alcohol, is drawn along the outer half of the rod, and when this squeaks the rod is in longitudinal stationary undulation, the middle being a node and the ends antinodes. If the lycopodium is distributed over the walls of the tube by shaking, and then the tube is placed horizontally and the rod made to squeak, again and again, the position of the cork b' being changed each time, a position of b' will soon be found such that, when the rods squeaks, the lycopodium slips right to the bottom of the tube, forming a line, except at a series of equi-distant points, where it only slips down each side part of the way to the bottom, forming a double line, so that the powder forms along the bottom a pattern like this 
This happens because the air is in stationary undulation, and everywhere except close to the nodes it keeps moving the powder backwards and forwards; each time the powder moves it slips a little downhill till it reaches the lowest position. The oval loops are at the nodes, and their centres always divide the distance from the cork b' to the disc a into an exact number of equal parts; the single-line parts of the pattern are narrowest when each of these parts is exactly half the length of the waves which the disc would produce in an infinitely long tube of the same diameter, so that the length of such waves is found by adjusting the distance ab' till the single-line parts of the pattern are as narrow as possible, and dividing it by half the number of single lines.

Though this is the arrangement usually described, there is some difficulty in holding the rod firmly enough by one point. The experiment is much easier if the rod is firmly held in two clamps, at distances of a quarter of the length of the rod from each end. The rod is set in vibration by drawing a resined cloth along the part of therod between the clamps. The clamps are nodes, the ends of the rod and its middle points antinodes.

Twice the distance between two antinodes in the rod is the distance a wave would travel in the rod while the rod makes one vibration. The distance a wave would travel in the air of the tube while the rod makes one vibration is found as explained above. The ratio of these is the ratio of the velocity of sound in the rod to the velocity of sound in the air in the tube, so that either of these velocities can be determined if the other is known. Or if the frequency of the rod is determined by the vibroscope or siren, or by calculation,* the absolute velocity of sound in air can be found. By filling the tube with different gases in turn, the relative velocities of sound in them can be found: they are proportional to the spaces between consecutive nodes; If liquids be used instead of gases a heavier powder, such as fine silica, must be used. As the walls of the tube are not perfectly rigid, the velocity of sound along the liquid in a tube is not nearly so great as in a large volume of the same liquid (Art. 31), so the results are not of great value.

Rods of different materials may be tried in place of the glass one, the tube being always filled with air. The velocities of sound in the different rods are proportional to the numbers obtained by dividing the length of each rod by the length of the segments into which the ridges divide the air space when that rod is sounding; for these numbers are the ratios of the velocities of sound in the rods to velocity in air.

For any of these purposes, except the comparison of different rods, the rod and disc may be dispensed with, and the wide tube itself made to vibrate longitudinally. This is done by holding it by the middle, and drawing a cloth, moistened with alcohol, along one half. The tube is closed at both ends by corks, one at least of which must be adjustable in position as in the last experiment. When the distance between the corks is exactly or nearly an exact number of half wave-lengths in air of the frequency of the vibrations of the glass, the air is thrown into resonant stationary undulation, the corks being nodes (very nearly, like the disc in the other form).

106. Wave-form.—No accurate method seems to be known of determining the wave-form of an undulation. The phonautograph described above gives traces which are different for waves of different character, and these traces are sometimes loosely spoken of as the wave-forms, but neither this method nor any method which depends on the yielding of

* If Young's Modulus for the glass, and its density, are known, the velocity of a travelling wave in it can be found from Art. 27, and the period of the stationary undulation of the rod, when vibrating with one node as in this experiment, is the time that a travelling wave would require to travel twice the length of the rod.

a membrane can give the wave-form as defined in Art. 15. If a membrane could be massless and perfectly loose, it would move exactly as the air moved, and might be made to trace a displacement curve of the movement of the air. If a membrane could be stretched infinitely tightly, its displacement at each instant would be proportional to the pressure on it at that instant, and it might be made to trace a true pressure-curve or wave-form, though the amplitude of this would be indefinitely small. The movement of any real membrane is a compromise between them, modified considerably by the mass of the membrane and its liability to be thrown into resonant vibration if any of its own very numerous modes of free vibration agree in frequency with any of the harmonic components of the waves reaching it.

The wave-forms of undulations then cannot at present be determined with any accuracy, and are, in fact, unknown, except in the regions close to large vibrating surfaces, where they cannot differ much from the velocity curves of the surfaces themselves. We can, however, determine, by the method explained in Art. 54, the frequencies of the harmonic waves whose wave-forms would add up to that of the actual waves. That is not at all the same as knowing the actual wave-form, because we cannot find either the relative amplitudes or the relative phases of these harmonic waves at all accurately, and harmonic waves of the same frequencies, but of different amplitudes and relative phases, may add up to quite different wave-forms. Still, for many purposes, to know the frequencies of the harmonic waves whose wave-forms would add up to the actual wave-form, is as useful as to know the wave-form itself, and this can be ascertained by means of resonators. A resonator is a globular or cylindrical box with a wide hole at one end, and a small one, fitted with a tube, at the other. Its frequency of free vibration is found by blowing across the wide hole, and determining the frequency of the sound produced, by the siren, phonautograph, or other method. This frequency can be altered by slightly altering the size of the larger hole, for a tube whose diameter is diminished at an open end has a much longer vibration period than an ordinary open tube. In the case of cylindrical resonators, which are made in two pieces to slide one inside the other like the joints of a telescope, the frequency can also be adjusted by altering the length. A set of such resonators is prepared whose free-vibration frequencies are exact multiples of the frequency of the waves to be analysed, and, if the tubes from the smaller openings of these resonators are placed in turn in the ear, we hear a loud sound from those whose free-vibration frequencies correspond to those of the harmonic components of the arriving waves.

As the components can only be detected by this method one at a time, it is only suitable for the analysis of sounds which can be produced continuously. For those of short duration we must be able to determine which resonators are vibrating at a given moment. For this purpose the smaller opening of each resonator is fitted with a manometric capsule, and the flames from all the capsules are arranged close one above another in front of a rotating mirror, as in Fig. 53. The appearance of the flames in the mirror shows which resonators are vibrating. In this way the analysis can be shown to a large audience.

This method is not suitable for the detection of feeble harmonic components in a loud sound, as, on account of the small mass of the air in a resonator, a strong component of a wrong frequency will produce more vibration than a feeble component of the right frequency, so that the presence or absence of the latter cannot be ascertained. Tuning forks are much better detectors of weak components, since the forced vibration produced in a fork by even a very strong component of different frequency cannot be detected. The resonant forced vibration produced by a weak harmonic component of the same frequency as the fork is small, but it can be detected by a device due to Professor Rücker. A beam of light is partly transmitted and partly reflected by a plate of glass placed at 45° to it; these two parts are reflected back along their own courses by silvered mirrors, one fixed, the other fastened to the prong of the fork as in Fig. 11. When these reflected beams reached the plate of unsilvered glass again, part of the beam which was originally transmitted is reflected, and part of the beam which was reflected is transmitted, and these parts coincide and can be received on the same screen. The apparatus is arranged so that the light reaching the centre of the screen by the two routes has travelled the same distance when the fork is still; the centre of the screen is bright, but the parts of the screen to which the two paths differ by an odd number of half wave lengths of light are dark; there is a system of approximately hyperbolic dark and bright bands. A movement of the prong to the extent of a quarter wave length makes the dark parts bright and *vice versa*; a trembling of the prong to this or any greater extent interchanges dark and light so rapidly that no bands are visible. [This effect is produced by an extremely feeble sound of exactly the frequency of the fork, but not by very loud sounds of other frequencies. The method has been successfully used to investigate the question whether resultant tones exist outside the ear. (Art. 68).]

107. Phonograph.—Edison's Phonograph is similar to the phonautograph in principle, but a cylinder covered with wax is substituted for the smoked one, and a very narrow chisel for the tracing point. The centre of the membrane is in this case made an antinode, and the point of the chisel therefore moves perpendicularly to the surface of the cylinder, not parallel to it, and so digs a trench of varying depth as the membrane vibrates. A section of this trench, parallel to its length and perpendicular to the surface of the cylinder, shows the form of the bottom, which depends on the quality of the sound, but is not the "wave-form" in any accurate sense of the term. It is, however, (like the trace of the phonautograph) a curve whose harmonic components are nearly the same as those of the waves. If the chisel is now replaced by a blunt point, and the cylinder, after being replaced in its original position, turned again in the same direction as at first the blunt point presses on the undulating bottom of the trench already cut by the chisel, and the membrane to which the point is connected repeats the movements by which the trench was originally cut. It thus produces waves in the air similar to those which, by their arrival, caused the original movements of the membrane, though they are reversed, the condensations of the original waves being represented by rarefactions in those afterwards produced by the membrane. Though in this and other respects there is considerable difference between the wave-forms of the original sound and the sound reproduced by the membrane, the two have nearly the same harmonic components, and therefore seem to the ear so nearly of the same quality that, if the original sound was that of the speaking voice, the different vowels (which are only special qualities) are easily distinguished in the reproduction.

It is worth noticing that it is the imperfection of the ear as a detector of differences in air waves that makes the phonograph and telephone possible. The waves they give out are in form often so different from the original waves that, if the ear had as great a power of distinguishing between different wave-forms as the eye has of distinguishing different outlines, it would be impossible to recognise any resemblance to the original sound.

108. Amplitude.—The amplitude of the vibrations of any point of a solid body can of course be found from its trace on the vibroscope cylinder. No method is known of measuring the amplitude of the vibrations of gases; such estimates as those in Art. 24 are arrived at by assuming that all the energy observed to be expended in keeping the source in vibration is converted into sound, and determining the amplitude of vibration necessary at any given distance from the source in order that the air at that distance may transmit energy at the same rate as it leaves the source. The true amplitude, of course, cannot be greater than this, but it is on theoretical grounds that it is believed to be not much less.

APPENDIX.

A. (Art. 5).

Let B (Fig. 72) be a point revolving uniformly round a circle of radius a in a period t, and A a point moving up and down a diameter CD so as to be always on the same level as B, so that A moves harmonically. Let B_1 be the position of B when OB has moved through an

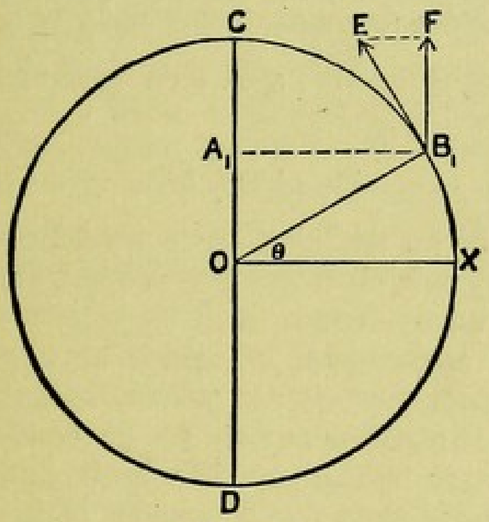


Fig. 72.

angle θ from the position at right angles to CD. At this moment the displacement OA_1 of A from its mean position O is $a \sin \theta$. The velocity of B is $\frac{2\pi a}{t}$ in the direction BE, and the component of this velocity in the direction parallel to OC is BF, or $BE \cos \theta$, or $\frac{2\pi a}{t} \sin (\theta + 90°)$. This is the velocity of A when it is at A_1. Now $\theta + 90°$ is the angle through which OB will have rotated from OX a quarter of a period later. So the velocity of a harmonically vibrating point at any instant is $\frac{2\pi}{t}$ times the displacement which it will have a quarter of a period later.

Now the velocity of A is the rate of change of the displacement of A, so that, if we take the displacement of A to represent any quantity which varies harmonically, the velocity of A at the same instant represents the rate at which that quantity is changing. Hence the rate of change of any quantity that varies harmonically, and has a mean value $= 0$, is $\frac{2\pi}{t}$ times the value that the quantity itself will have a quarter of a period later. And the acceleration of A is the rate of change of the velocity of A. The acceleration of A when at A_1 is therefore $\frac{2\pi}{t}$ times the velocity it will have a quarter of a period later, or $\frac{4\pi^2 a}{t^2}\sin(\theta + 180°)$, or $-\frac{4\pi^2 a}{t^2}\sin\theta$. It is therefore proportional to the displacement of A, but in the opposite direction.

The ratio of the acceleration to the displacement is $-\frac{4\pi^2}{t^2}$, and is therefore independent of the amplitude. If, therefore, the force on a body, due to displacement, is such as to produce an acceleration which is always $-k$ times the displacement, the body will vibrate harmonically in a period t such that $k = \frac{4\pi^2}{t^2}$, and this period is independent of the amplitude.

B. (Art. 23).

The term *intensity* of an undulation is used by some writers to mean the quantity of energy which flows per second through a square centimetre parallel to the wave-fronts, and by others to mean the quantity of energy in a cubic centimetre of space at a given moment. We have adopted the former definition, which has the high authority of Lord Rayleigh; but the other seems to be coming more into favour. It makes some difference to the proofs which definition is adopted; if the first, it follows easily that the intensity is inversely as the square of the distance, but it is difficult to give a satisfactory elementary proof that the intensity is proportional to the square of the amplitude. From the second definition this latter proposition follows at once, but it is then difficult to prove the former. In many books the definition is given so loosely that it may be taken to have either meaning, and then both propositions deduced from it.

If the definition we have adopted is used, the intensities of the transmitted and the reflected wave, when waves reach the surface between two media, are together equal to the intensity of the incident waves, but this is not true on the second definition.

C. (Art 37).

The name *interference* is applied by some writers to all cases of superposition, and limited by others either (a) to those cases in which the actual distribution of energy is very different from that of the imaginary systems (see footnote to Art. 42), or (b) to those cases in which the vibration due to the sources together is less intense than that which would be due to one of them alone. We adopt (a) as being usual in England, though (b) has the high authority of Lord Rayleigh.

D. (Arts. 14, 27, 95).

Velocity of a Longitudinal Wave along a Rod or Fluid Column (Art. 27).

Let a force of f dynes be applied to one end A of a very long rod AB, and let the force be directed towards B. The first centimetre of the rod shortens till it exerts a force of f dynes on the next centimetre, and then this shortens till it exerts the same force on the next, and so on. As the process is exactly the same for each successive centimetre, the compressed condition extends, with uniform velocity, along the rod. Let this velocity be V, and let the mass of each centimetre be m, and let the amount by which each (original) centimetre is diminished in length, when there is a force f at each end of it, be l. As each centimetre becomes compressed in its turn, all the centimetres between it and A, which are compressed already, advance towards B through a distance l. As V fresh centimetres become compressed in each second, Vl is the velocity with which the portion already compressed moves towards B. In each second V fresh centimetres are added to this portion, and each of these, when compressed, has a momentum Vlm. So V^2lm is the increase of momentum of this portion per second. But rate of change of momentum is equal to the force producing it. Hence $f = V^2lm$ or $V = \sqrt{\dfrac{f}{lm}}$.

Let A, B, be two points in a fluid column of unit sectional area, and let the line joining them be parallel to the length of the column, and the distance, x, between them very small. Let p, d, v be the pressure (dynamical measure) density, and velocity of the fluid at A, and p_1, d_1, v_1 those at B, and let all these conditions be travelling along the column with the velocity V. Then the rate at which the velocity of the fluid between A and B is changing is $\dfrac{(v_1 - v)\,V}{x}$, while the mass of fluid between two transverse planes through A and B is xd (since d_1 is very nearly the same as d) and the forces on the ends of this layer are p and p_1. Hence

$$\frac{p_1 - p}{xd} = \frac{(v_1 - v)\,V}{x} \text{ or } V = \frac{(p_1 - p)\,d}{v_1 - v}.$$

Also the rate at which the density of the fluid between the planes is changing is $\dfrac{(d_1 - d)\,V}{x}$, while it is also $\dfrac{(v_1 - v)\,d}{x}$, since $\dfrac{v_1 - v}{x}$ is the fractional change per second of volume of the space between the planes. Hence

$$v_1 - v = \frac{(d_1 - d)\,V}{d}.$$

Substituting,

$$V^2 = \frac{p_1 - p}{d_1 - d} \text{ or } V = \sqrt{\frac{p_1 - p}{d_1 - d}}.$$

RELATION BETWEEN CONDENSATION AND VELOCITY (ART. 14).

$$\frac{\text{Density of compressed portion of rod}}{\text{Density of uncompressed portion of rod}} = \frac{1}{1 - l} = 1 + l \text{ nearly.}$$

$$\therefore \frac{\text{Change of density}}{\text{Original density}} = l \text{ nearly.}$$

$$\frac{\text{Velocity of movement of compressed material}}{\text{Velocity of advance of compressed condition}} = \frac{Vl}{V} = l.$$

$$\therefore \frac{\text{Change of density}}{\text{Original density}} = \frac{\text{velocity of substance}}{\text{velocity of wave}}.$$

As the denominators are constants,

velocity of undulating substance

$\propto$ difference between its actual and average density.

VELOCITY OF A TRANSVERSE WAVE ALONG A STRING (ART. 95).

Let AB, Fig. 73, be a very long string, stretched horizontally by a force of s dynes at each end, so that a small portion A is in equilibrium

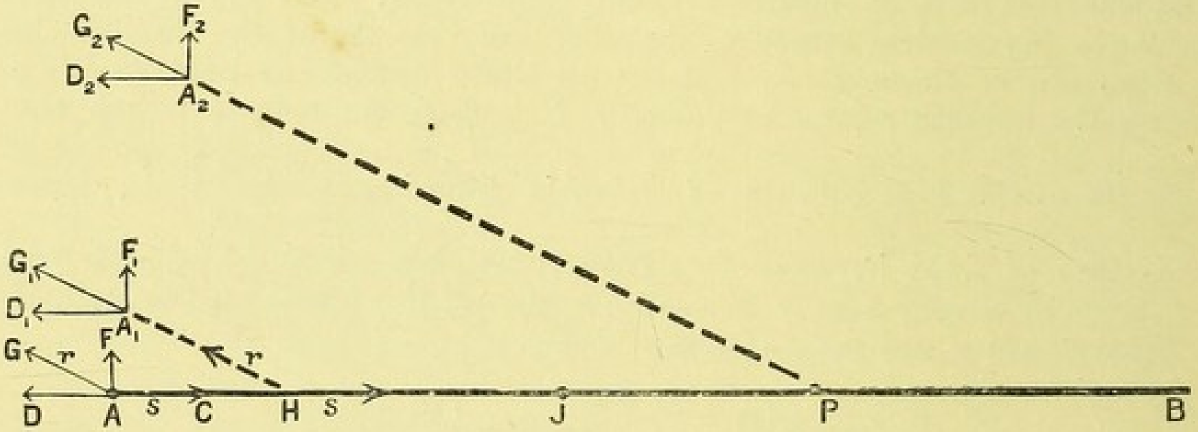


Fig. 73.

between two forces of s dynes, one AC along AB, and the other AD in the opposite direction. Apply at A another force AF of f dynes in an upward direction, and let the resultant of AF and AD be a force r in the direction AG. A will then move in the direction AF, and it can never be in equilibrium until the part of the string close to A is in the direction opposite to AG. When a portion of the string has thus assumed the position A_1H, the forces on a small portion at H are a force r in the direction HA_1, and a force s in the direction HB.

These are equal and parallel to the forces r, s on A when the force was first applied; H therefore begins to move upwards exactly as A did, and another portion HJ becomes equally sloping, and so on. As the forces on A_1 would accelerate it if A_1H was more horizontal than A_1G_1, and retard A_1 if A_1H was less horizontal than A_1G_1, A moves in such a way that AH is always opposite to AG, and similarly for each succeeding portion. Thus, as point after point of the string begins to move sideways, A continues to move in the direction AA_1 in such a way that the part of the string between A and the point of the string which is just beginning to move is parallel to the line GA. Let A_2 be the position A has reached when the sloping condition has extended to P. AA_2P is really isosceles, but in practice AA_2 is very short compared with the other sides, and we may consider AA_2 as perpendicular to AP. Hence $\frac{AA_2}{AP} = \frac{A_2F_2}{A_2D_2} = \frac{f}{s}$. If V is the velocity with which the sloping condition extends along the string, $\frac{Vf}{s}$ is the velocity with which the part of the string which is already sloping moves upwards. If m is the mass of one centimetre of the string, $\frac{Vfm}{s}$ is the momentum of each centimetre of the sloping portion, and, as V fresh centimetres are added to this portion per second, $\frac{V^2fm}{s}$ is the increase of momentum per second. This is equal to the force which produces it. Hence $f = \frac{V^2fm}{s}$ or $V = \sqrt{\frac{s}{m}}$.

Just as in the corresponding proof for a longitudinal wave, every variation of f produces a corresponding change of slope in the part of the string close to A, which slope is always proportional to f, and every change of slope produced in the part close to A extends along the string with the velocity $\sqrt{\frac{s}{m}}$.

The reader who has not studied dynamics may be surprised that a constant force applied to A gives A instantly a velocity which does not afterwards increase, not an increasing velocity. It must be remembered that the effect of a force is to cause a momentum which increases as long as the force lasts. If the mass in movement is constant, its velocity must increase, but if, as in the cases considered above, the mass in movement keeps increasing as long as the force lasts, the velocity of the part in motion may be constant.

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