AIDS
TO THE
MATHEMATICS OF HYGIENE

FERGUSON

SECOND EDITION

2/6

BAILLIÈRE, TINDALL & COX.

AIDS

TO

THE MATHEMATICS OF HYGIENE.

BY

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PREFACE TO THE SECOND EDITION.

THE issue of a second edition of this little book has afforded me the opportunity of revising it throughout. A certain amount of new matter and some additional examples have been introduced, whilst the chapter on Vital Statistics has been re-written and enlarged. On the other hand, such portions have been deleted as appeared to stray beyond the limits I originally laid down for myself—viz., to deal with the study of Hygiene solely from the *mathematical* side of the subject.

This side has always seemed to me to be treated rather inadequately by even the standard text-books. In these, a formula is inserted in its entirety, without a hint as to how such a result has been arrived at, or by what method it was compiled. Without this knowledge, the only alternative is to commit it to memory —with the risk of forgetting it—before entering for an examination on the subject.

It is also generally taken for granted that the reader is well acquainted with all the methods of chemical and physical calculations ; but since these subjects are amongst the earliest of one's scientific studies, it is quite within the range of possibility that these details may have passed somewhat from recollection ; so that, should he be preparing for examination in Public Health, they must be all sought out at infinite trouble from, possibly, a dozen different sources (chemistry, physics, algebra, statics, dynamics, trigonometry, etc.).

It has been my endeavour to collect these formulæ, and to put them together in these pages, chiefly in the form of examples. I have also attempted to show, in as simple a manner as possible, how they may be applied and made use of, in the solution of the various problems so frequently met with during the course of one's study. The book is not meant to be a guide to Hygiene, nor yet to embrace every branch of the subject. It is only what its title claims for it—viz., a guide to those portions of Public Health which require mathematical treatment. As such, it is gratifying to find that it has already been of some assistance ; and it is hoped that it will continue to prove useful, especially to those who have not the time, nor yet, perhaps, the inclination, to commence their mathematical studies afresh.

The calculations connected with the practical examination of milk, water, etc., have been omitted, since these are so inseparably connected with the actual laboratory work, that to include them here would necessitate the introduction of the whole methods of analysis—a subject already fully dealt with in the many special text-books.

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AIDS

TO THE

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CHAPTER I.

HYGROMETRY.

GENERAL CONSIDERATIONS.

THE weight of a given volume of any gas or vapour can readily be calculated if we know:

(i.) The weight of a given volume of some gas which we can take as our standard;

(ii.) The relative density of the gas, compared with this standard.

In Chemistry, it is usual to take, as the standard, the weight of a given volume of *hydrogen* at a certain temperature and pressure; and the weight of any other gas, at the same temperature and pressure, is obtained by multiplying this by the relative density of the gas whose weight we are ascertaining.

E.g., we may take as our standard:

> 1 litre of hydrogen at 0° C. and 760 mm. weighs 0·08958 gramme.

Now, the densities of all **elements** in the gaseous state are identical with their atomic weights; and since O = 16, and S = 32, therefore

> 1 litre of oxygen at 0° C. and 760 mm. weighs 16 × 0·08958 = 1·43 grammes;
> and 1 litre of sulphur vapour at 0° C. and 760 mm. weighs 32 × 0·08958 = (2·86) grammes.

The density of a **compound** gas is one-half its molecular weight; and since CO_2 = 44, and H_2O = 18, therefore the density of CO_2 = 22, and the density of water-vapour (steam) = 9, compared with that of H as unity.

∴ 1 litre of CO_2 at 0° C. and 760 mm. weighs 22 × 0·08958 = 1·97 grammes.

In Physics, it is usual to take *air* as the standard, and the density of a gas or vapour is the relation between the weight of a given volume of this gas or vapour, and that of the same volume of air at the same temperature and pressure.

Knowing that air is 14·47 times heavier than hydrogen, we may find the density of any gaseous element, or compound, as follows:

> Density of oxygen : density of air : : 16 : 14·47;
> Density of water-vapour : density of air : : 9 : 14·47;

and so for any other gas.

The relative density of aqueous vapour is one which we shall have occasion to use frequently (*e.g.*, in all calculations in Hygrometry, etc.).

We may write it as follows :

$$\frac{\text{density of aqueous vapour}}{\text{density of air}} = \frac{9}{14\cdot47} = 0\cdot622\ ;$$

i.e., density of aqueous vapour = density of air × 0·622 ; or, density of aqueous vapour is, approximately, $\frac{5}{8}$ that of air.

Again :

$$\frac{\text{density of air}}{\text{density of oxygen}} = \frac{14\cdot47}{16}\ ;$$

$$\therefore \text{density of air} = \frac{\text{density of oxygen} \times 14\cdot47}{16}.$$

Now, we have seen that 1 litre of oxygen at 0° C. and 760 mm. weighs 1·43 grammes ;

$\therefore$ 1 litre of air at 0° C. and 760 mm. weighs

$$\frac{1\cdot43 \times 14\cdot47}{16} = 1\cdot293 \text{ grammes.}$$

Since the co-efficient of expansion of all gases is the same as that of air, it does not signify at what temperature or pressure the relative densities are compared, provided they are both at the same temperature and pressure ; *e.g.*, the relative densities of air and aqueous vapour (the pressure being constant) would be the same at 60° F. as at 32° F., provided both are at 60° F. or 32° F. respectively ; but the same fraction would not represent the relative density between air at 32° F. and aqueous vapour at 60° F.

We shall adopt as our standard (from which the weight of any other gas may be obtained) the following :

1 cubic foot of dry air at 32° F. and 760 mm. weighs 566·85 grains.

Consequently:

1 cubic foot of oxygen at 32° F. and 760 mm. weighs

$$\frac{16}{14{\cdot}47} \times 566{\cdot}85 \text{ grains.}$$

In this country the barometric pressure is usually stated in inches instead of millimetres. The ordinary atmospheric pressure is 760 mm. = 29·92 inches (approximately 30 inches); and one may be converted into the other by a simple rule of three; *e.g.*, find the value of 15 mm. in inches:

$$760 \text{ mm.} : 15 \text{ mm.} :: 30 \text{ inches} : x \text{ inches.}$$
$$x = 0{\cdot}59 \text{ inch.}$$

DRY AIR.

To find the Weight of a Given Volume of Dry Air at a Given Temperature and Pressure.

Boyle's (or Mariotte's) law tells us that, the temperature remaining the same, the volume of a given quantity of gas is inversely proportional to the pressure to which it is subjected. And as the quantity of gas remains the same, its density must obviously increase as its volume diminishes; therefore it follows, that for the same temperature, the density of a gas, and therefore its weight, is proportional to its pressure.

This may be expressed mathematically as follows:

$$P \propto \frac{1}{V}, \quad D \propto \frac{1}{V}, \quad \therefore D \propto P.$$

Also, we must remember that all gases expand $\frac{1}{273}$ part of their volume at 0° C. for every increase in temperature of 1° C., or $\frac{1}{491}$ part of their volume at

32° F. for every increase in temperature of 1° F.; *e.g.*, 1 vol. at 0° C. becomes at 10° C. $1+\frac{10}{273}$, or $\frac{273+10}{273}$ vols., and 1 vol. at 32° F. becomes at 60° F. $1+\frac{60-32}{491}$, or $\frac{491+(60-32)}{491}$ vols.

Now, it has already been stated (p. 3) that 1 cubic foot of dry air at 32° F. and 760 mm. = 566·85 grains; and if

W = weight (in grains) of air required,
V = volume (in cubic feet),
P = pressure,
t = temperature,

then, since:

(i.) Weight varies directly as volume;
(ii.) Density (and therefore weight) varies directly as pressure;
(iii.) Density (and therefore weight) varies inversely as absolute temperature;

therefore, from:

(i.) $W : 566{\cdot}85 :: V : 1$;
(ii.) $:: P : 760$;
(iii.) $:: 1 : \frac{491+(t-32)}{491}$;

$$\therefore \frac{W}{566{\cdot}85} = \frac{V \times P \times 491}{760 \times [491+(t-32)]};$$

$$\therefore W = 566{\cdot}85 \times V \times \frac{P}{760} \times \frac{491}{491+(t-32)} \text{ grains};$$

or, if we use the metrical system and the Centigrade scale, we have seen (p. 3) that 1 litre of dry air at 0° C. and 760 mm. = 1·293 grammes—

$$\therefore\ W=\frac{1{\cdot}293 \times V \times P \times 273}{760 \times (273+t)} \text{ grammes.}$$

Example :

Find weight of 1 cubic foot of dry air at 60° F. and 735 mm.

$$W=\frac{566{\cdot}85 \times 1 \times 735 \times 491}{760 \times [491+(60-32)]}=518 \text{ grains ;}$$

or, if we convert millimetres into inches (*vide* p. 4), we have 735 mm. = 29 inches ;

$$\text{and } W=\frac{566{\cdot}85 \times 1 \times 29 \times 491}{30 \times [491+(60-32)]}=518 \text{ grains.}$$

AQUEOUS VAPOUR.

To find the Weight of a Given Volume of Aqueous Vapour at a Given Temperature and Pressure.

To find the weight of a vapour, the weight of the same volume of dry air **at the same temperature and pressure** must be sought, and this is then to be multiplied by the relative density of the vapour.

Example :

Find the weight of a cubic foot of aqueous vapour at 60° F.

From a table of vapour-tensions (p. 93), we find that the maximum pressure which aqueous vapour can exert at 60° F. = 13·167 mm.

We have, therefore, to first find the weight of a cubic foot of dry air at 60° F. and 13·167 mm. as follows (*vide* p. 4) :

$$W=\frac{566{\cdot}85 \times 13{\cdot}167 \times 491}{760 \times [491+(60-32)]}=9{\cdot}29 \text{ grains ;}$$

and this result must be multiplied by the relative density of aqueous vapour compared with air—viz., by 0·622 (p. 3).

Therefore, 1 cubic foot of aqueous vapour at 60° F. weighs $9·29 \times 0·622 = 5·77$ grains.

Example:

What weight of aqueous vapour is contained in a cubic foot of air which is saturated at a temperature of 60° F.?

Dalton's law states that 'the tension, and consequently the quantity of vapour which saturates a given space, are the same for the same temperature, whether this space contains a gas or is a vacuum.' Therefore, the weight of aqueous vapour in a cubic foot of air is the same as if the space had been empty of air; so the question resolves itself into, What is the weight of a cubic foot of aqueous vapour at 60° F.?

[Ans.: 5·77 grains.]

SATURATED AIR.

To find the Weight of a Given Volume of Saturated Air at a Given Temperature and Pressure.

The mass of air may be divided into two parts—viz., a volume of dry air, and a volume of aqueous vapour; and the sum of the weights of these two volumes is the weight required.

Let P = the pressure of the moist air;
and p = the elastic force of the vapour which saturates it.

Then the **air** alone in the mixture only supports a pressure of $P - p$.

Example :

Find the weight of 1 cubic foot of saturated air, at 60° F. and ordinary atmospheric pressure.

(i.) Find the weight of 1 cubic foot of dry air at 60° F. and pressure $P-p$ (*vide* p. 4).

$P=760$, and $p=13{\cdot}167$ (*vide* p. 93).

$$\therefore W=566{\cdot}85\times\frac{760-13{\cdot}167}{760}\times\frac{491}{491+(60-32)}$$

$$=526{\cdot}97 \text{ grains.}$$

(ii.) Find the weight of 1 cubic foot of aqueous vapour at 60° F. and pressure p (*vide* p. 6).

$$W=\frac{566{\cdot}85\times 13{\cdot}167\times 491}{760\times[491+(60-32)]}\times 0{\cdot}622=5{\cdot}77 \text{ grains ;}$$

$\therefore$ Weight of 1 cubic foot of saturated air at 60° F. and 760 mm.

$$=526{\cdot}97+5{\cdot}77=532{\cdot}74 \text{ grains.}$$

Note.—We see that 1 cubic foot of **saturated** air at 60° F. and ordinary atmospheric pressure weighs 532·74 grains, whereas 1 cubic foot of **dry** air at 60° F. and the same pressure weighs $\frac{566{\cdot}85\times 491}{491+(60-32)}=536{\cdot}27$ grains That is to say, the saturated air weighs less than an equal volume of dry air. The explanation of this is as follows :

Dry air expands on taking up moisture, and when 1 cubic foot of dry air at 60° F., weighing 536·27 grains, takes up 1 cubic foot of aqueous vapour (at the same temperature and pressure) weighing 5·77 grains, the weight of the resulting moist air will be 536·27 + 5·77 = 542·04 grains, but the volume of the mixture will be, not 1 cubic foot, but somewhat more than 1 cubic foot. The moist air is under a pressure of 760 mm., and we have already seen that the elastic force of the vapour which saturates the air is 13·167 mm. Therefore, the air in the mixture supports a pressure of 760 − 13·167 = 746·833 mm. only, and, as volume is inversely as pressure, if V = volume required, then

$$V : 1 \text{ cubic foot} :: 760 : 746{\cdot}833;$$

$$\therefore V = \frac{760}{746{\cdot}833} = 1{\cdot}0176 \text{ cubic feet.}$$

That is, the resulting mixture of the dry air and the vapour produces 1·0176 cubic feet of saturated air, weighing 542·04 grains, and 1 cubic foot of this same moist air would only weigh

$$\frac{542{\cdot}04}{1{\cdot}0176} = 532{\cdot}7 \text{ grains.}$$

To find the Dew-point.

For any given temperature, air will only hold a certain quantity of aqueous vapour; the higher the temperature of the air, the greater will be the amount of vapour which it can contain; when the air contains its greatest possible amount, it is said to be 'saturated,' and the temperature at which saturation occurs is called the 'dew-point.'

The dew-point may be obtained **directly** by means of a Hygrometer (Daniell's, Regnault's, or Dine's), or **indirectly** by the dry-and-wet-bulb Hygrometer.

In the case of the former, the dew-point is the temperature at which the thermometer in the blackened bulb stands, at the moment when deposition of moisture on the bulb occurs. Whereas, in the case of the dry-and-wet-bulb, the wet-bulb does *not* indicate the dew-point, which, therefore, cannot be ascertained by simple inspection of the thermometer.

If the air be saturated no evaporation is possible, and the two thermometers will read alike; if not saturated, the wet-bulb will read lower than the dry-bulb, but not so low as the dew-point; in fact, the temperature of the wet-bulb is always above the dew-point. When the dry-bulb stands at 53° F. the dew-point is as much below the wet-bulb as the wet-bulb is below the dry-bulb. Above this temperature, the wet-bulb approaches nearer the dew-point, and the reverse is the case below that temperature.

Glaisher has empirically compiled some tables, whereby the difference between the dew-point and the wet-bulb bears a constant ratio to the difference between the wet-bulb and the dry bulb; so that, if the reading of the dry-bulb be given, the dew-point can be calculated.

According to him, the temperature of the dew-point is obtained by multiplying the difference between the wet-and-dry-bulb temperatures by a constant factor ('Glaisher's factor'), and subtracting the product thus obtained from the dry-bulb temperature, thus:

$$\text{Dew-point} = T_d - [(T_d - T_w) \times F].$$

Where T_d = temperature of dry-bulb (Fahrenheit).

T_w = temperature of wet-bulb „

F = factor (found opposite the dry-bulb temperature in the table).

If $T_d = 60°$ F. and $T_w = 54°$ F., then (from table) $F = 1{\cdot}88$ and

$$\text{dew-point} = 60 - [(60 - 54) \times 1{\cdot}88] = 48{\cdot}72° \text{ F.}$$

From this formula Glaisher calculated his tables, which give the dew-point on inspection.

The dew-point may also be found by Apjohn's formula. For this purpose we require a table of vapour-tensions (page 93).

His formula is as follows :

$$F = f - \left(\frac{d}{87} \times \frac{h}{30}\right) \text{ for temperatures above } 32^\circ \text{ F.,}$$

$$\text{and } F = f - \left(\frac{d}{96} \times \frac{h}{30}\right) \text{ for temperatures below } 32^\circ \text{ F.}$$

Where F = tension of vapour at dew-point,

f = tension of vapour at temperature of wet-bulb.

d = difference (Fahrenheit) between the wet-and-dry-bulb thermometers.

h = height of barometer (in inches).

Near the sea-level, the fraction $\frac{h}{30}$ differs very little from unity, and may be neglected ; so the formula may be simplified thus :

$$F = f - \frac{d}{87} \quad \text{or } F = f - \frac{d}{96}.$$

The constant 87 (or 96) represents the specific heat of air and vapour.

Having found F, we must refer again to the table of vapour-tensions, and the temperature opposite the tension F will be the dew-point.

To recapitulate the more important points—

The dew-point may be found in two different ways :

(i.) By direct observation of thermometer, in Daniell's, Regnault's, or Dine's Hygrometer.

(ii.) Indirectly, by the dry-and-wet-bulb Hygrometer ;

(*a*) By means of Glaisher's tables.

(*b*) By means of Apjohn's formula.

It must be noted that Glaisher's formula gives the dew-point directly, no table of vapour-tensions being required; whereas Apjohn's formula requires the use of a vapour-tension table, the result being *not* the dew-point, but the vapour-tension at dew-point, from which the dew-point can be ascertained. And we would again draw attention to the fact, that the wet-bulb thermometer does not give the dew-point (except when the air is saturated), and therefore the wet-and-dry-bulb Hygrometer must not be confounded with any of the *direct* Hygrometers, *e.g.*, Daniell's, where the thermometer *does* give the dew-point.

To find the Humidity and the Drying Power of the Air.

Example:

The temperature of a room is 60° F., and the dew-point is 50° F. Find the degree of humidity of the room, and the drying power of the air at the time of observation.

$$\text{R.H.} = \frac{\text{weight of water actually present in given vol. of air}}{\text{weight of water which would \textbf{saturate} the same vol.}}$$

The water-vapour actually present is enough to saturate the air at 50° F., but not enough to do so at 60° F. This gives us the actual tension of the water-vapour present in the air at 60° F.; for it must be the same as the **maximum tension** at 50° F. In other words, the actual vapour pressure in any portion of air is equal to the maximum vapour pressure at dew-point.

Let P_{50} = maximum tension at 50° F., and

P_{60} = „ „ „ 60° F.

Then, the weight of vapour actually present (*e.g.*, in a cubic foot) will be (*vide* p. 7):

$$566{\cdot}85 \times \frac{P_{50}}{760} \times \frac{491}{491+(60-32)} \times 0{\cdot}622 \text{ grains,}$$

and the weight of vapour which would saturate the same volume will be :

$$566{\cdot}85 \times \frac{P_{60}}{760} \times \frac{491}{491+(60-32)} \times 0{\cdot}622 \text{ grains.}$$

$$\therefore \text{R. H.} = \frac{\dfrac{566{\cdot}85 \times P_{50} \times 491 \times 0{\cdot}622}{760 \times [491+(60-32)]}}{\dfrac{566{\cdot}85 \times P_{60} \times 491 \times 0{\cdot}622}{760 \times [491+(60-32)]}} = \frac{P_{50}}{P_{60}}.$$

$$\text{So R. H.} = \frac{P_{50}}{P_{60}} = \frac{\text{maximum tension at } 50^{\circ} \text{ F.}}{\text{maximum tension at } 60^{\circ} \text{ F.}}$$

$$= \frac{\text{maximum tension at dew-point}}{\text{maximum tension at existing temperature}},$$

which, from the table (p. 93), will be found to be $\frac{9{\cdot}165}{13{\cdot}167} = 0{\cdot}696$, or 69·6 (approximately 70) per cent.

So the **relative humidity** of the air can always be found from the tables of maximum vapour-tensions, if only we know the dew-point and the temperature of the air at the time of observation.

The **drying power** of the air means the additional weight of vapour necessary to cause saturation.

If relative humidity = 70 per cent., then amount of vapour actually present in a given volume of air is 70 per cent. of saturation, and the difference (or 30 per cent.) will represent the drying power.

Thus, if W = weight of vapour required to **saturate** a cubic foot of air at 60° F., and relative humidity = 70 per cent., then the amount of vapour actually present will be $\frac{70W}{100}$, and the drying power of the same air will be :

$$W - \frac{70W}{100}, \text{ or } \frac{30W}{100};$$

or, if saturation $=1$, then amount of vapour actually present $= W \times 0{\cdot}7$, and drying power $= W\ (1 - 0{\cdot}7)$; and if $h=$ relative humidity, then the drying power of the air may be expressed by the general formula :

$$W(1 - h).$$

To return to our example.

We have found that the relative humidity in the room is (approximately) 0·7 ; we have also seen that the weight of water required to saturate a cubic foot of air at 60° F. $=5{\cdot}77$ grains ; therefore, the amount of vapour actually present is

$$5{\cdot}77 \times 0{\cdot}7 = 4{\cdot}03 \text{ grains},$$

and the drying power is the difference between these,

$$\text{viz., } 5{\cdot}77 - 4{\cdot}03 = 1{\cdot}7 \text{ grains};$$

or it may be stated thus :

$$\text{Drying power} = W(1 - h) = 5{\cdot}77(1 - 0{\cdot}7) = 5{\cdot}77 \times 0{\cdot}3$$
$$= 1{\cdot}7 \text{ grains.}$$

To find the Weight of a Given Volume of Moist (non-saturated) Air at a Given Temperature and Pressure, the Hygrometric State or Relative Humidity being given.

Example :

Find the weight of 1 cubic foot of moist air at 60° F. and ordinary atmospheric pressure, the relative humidity of the air being 60 per cent.

We have seen (p. 7) that 1 cubic foot of moist air at 60° F. is nothing more than a mixture of (i.) 1 cubic foot of dry air at 60° F., under the existing barometric pressure **minus** the tension of the vapour present, and (ii.) 1 cubic foot of aqueous vapour at 60° F., the tension of which must be found from the hygrometric state, or relative humidity.

We have also seen (p. 13) that :

$$\text{Relative humidity} = \frac{\text{max. tension at dew-point}}{\text{max. tension at existing temp.}}.$$

The existing temperature in this example is 60° F., and from the table (p. 93) we find that the maximum tension of vapour at 60° F. = 13·167 mm.

$$\therefore \text{ Relative humidity} = \frac{\text{max. tension at dew-point}}{13{\cdot}167};$$

$$\therefore \text{ max. tension at dew-point} = \text{rel. hum.} \times 13{\cdot}167,$$

and relative humidity = 60 per cent. ;

$$\therefore \text{ max. tension at dew-point} = \frac{60 \times 13{\cdot}167}{100} = 7{\cdot}9 \text{ mm.}$$

Now, the actual vapour-pressure in any portion of air is equal to the maximum vapour-pressure at dew-point.

Therefore :

Actual vapour-pressure in air under observation
= 7·9 mm.

The question, then, resolves itself into :

(i.) Find weight of 1 cubic foot of dry air at 60° F. and 760 − 7·9 = 752·1 mm. (*vide* p. 4).

$$W = 566{\cdot}85 \times \frac{752{\cdot}1}{760} \times \frac{491}{491 + (60 - 32)} = 530{\cdot}69 \text{ grains.}$$

(ii.) Find weight of 1 cubic foot of aqueous vapour at 60° F. and 7·9 mm. pressure (*vide* p. 6).

$$W = 566{\cdot}85 \times \frac{7{\cdot}9}{760} \times \frac{491}{491 + (60 - 32)} \times 0{\cdot}622$$

$$= 3{\cdot}46 \text{ grains.}$$

Therefore, the weight of 1 cubic foot of air (containing 60 per cent. of moisture) at 60° F.=

$$530{\cdot}69 + 3{\cdot}46 = 534{\cdot}15 \text{ grains.}$$

Incidentally, we may remark that the temperature at which the aqueous vapour would exert a maximum pressure of 7·9 mm. will be the dew-point, which from the table (p. 93) will be seen to be about 46° F.

To Graduate a Rain-Gauge.

Carefully measure the area of the top of the gauge or receiving surface. Suppose this to be 50 square inches. If, now, this area be covered with water to the height of 1 inch, the quantity of water would be 50 cubic inches. So, to graduate the gauge, we must put 50 cubic inches of water into the glass vessel, and put a mark at the level of the top of the fluid ; this mark will represent 1 inch rainfall. The space below may be divided into numerous equal parts, each representing equal fractions of an inch.

The 50 cubic inches of water may be obtained by measurement, as follows :

At 4° C. (or 39·2° F.), or the maximum density point of water, 1,000 fluid ounces = 1 cubic foot = 1,728 cubic inches ;

$$\therefore\ 50 \text{ cubic inches} = \frac{1{,}000 \times 50}{1{,}728} = 28{\cdot}9 \text{ fluid ounces}$$

If the area of the receiving surface had been 100 square inches instead of 50, then 100 cubic inches of water would represent 1 inch of rainfall. We may thus make the receiving surface any size we choose ; or, on the other hand, we may make any quantity of water represent any fraction of an inch—*e.g.*, what should be the diameter of the receiving surface so that 1 fluid ounce represents $\frac{1}{8}$ inch?

Let d=diameter in inches,

Then area of receiving surface $=\frac{\pi d^2}{4}$ (*vide* p. 92),

and quantity of water standing $\frac{1}{8}$ inch high on receiving surface will be $\frac{\pi d^2}{4}\times\frac{1}{8}=\frac{\pi d^2}{32}$ cubic inches, and this is equal to 1 fluid ounce.

$$\therefore \frac{\pi d^2}{32}=1 \text{ fluid ounce}=1{\cdot}728 \text{ cubic inches};$$

$$\therefore d^2=\frac{32\times 1{\cdot}728}{\pi}=\frac{32\times 1{\cdot}728}{3{\cdot}1416}=17{\cdot}6;$$

$$\therefore d=\sqrt{17{\cdot}6}=4{\cdot}19 \text{ inches.}$$

THE BAROMETER AND THE THERMOMETER.

Relationship between the Different Varieties of Barometers.

Example:

A mercurial barometer falls 1 inch. What is the corresponding fall in a glycerine, a water, and also an alcohol barometer?

Specific gravity of water			= 1,
"	"	of mercury	= 13·6,
"	"	of glycerine	= 1·26,
"	"	of pure alcohol	= 0·8.

The atmospheric pressure=29·92 inches Hg.; and since height varies inversely as density, we have (where x=height required):

For water barometer :

$$x : 29{\cdot}92 :: 13{\cdot}6 : 1$$
$$x = 407 \text{ inches} = 33{\cdot}9 \text{ feet.}$$

For glycerine barometer :

$$x : 29{\cdot}92 :: 13{\cdot}6 : 1{\cdot}26$$
$$x = 323 \text{ inches} = 26{\cdot}9 \text{ feet.}$$

For alcohol barometer :

$$x : 29{\cdot}92 :: 13{\cdot}6 : 0{\cdot}8$$
$$x = 508{\cdot}8 \text{ inches} = 42{\cdot}4 \text{ feet.}$$

So the atmospheric pressure supports—

29·92 inches mercury,
33·9 feet water,
26·9 feet glycerine,
42·4 feet alcohol.

Now, we have seen that :

$$29{\cdot}92 \text{ in. Hg.} = 407 \text{ in. water,}$$
$$\therefore 1 \text{ in. Hg.} = \frac{407}{29{\cdot}92} = 13{\cdot}6 \text{ in. water.}$$

Again, $29{\cdot}92$ in. Hg. $= 323$ in. glycerine,

$$\therefore 1 \text{ in. Hg.} = \frac{323}{29{\cdot}92} = 10{\cdot}8 \text{ in. glycerine.}$$

Also, $29{\cdot}92$ in. Hg. $= 508{\cdot}8$ in. alcohol,

$$\therefore 1 \text{ in. Hg.} = \frac{508{\cdot}8}{29{\cdot}92} = 17 \text{ in. alcohol.}$$

That is, a fall of 1 inch in a mercurial barometer corresponds to a fall of 13·6 inches, 10·8 inches, and 17 inches in the case of a water, glycerine, and alcohol barometer respectively.

To Convert Thermometer Scales.

Freezing-point $= 0°$ C., $32°$ F., and $0°$ R.
Boiling-point $= 100°$ C., $212°$ F., and $80°$ R.

$$\therefore C : F - 32 :: 100 : 212 - 32$$
$$:: 100 : 180;$$
$$\therefore 180\,C = 100\,(F - 32)$$
$$C = \frac{100\,(F-32)}{180} = \frac{5\,(F-32)}{9}.$$

Also, $100\,(F-32) = 180\,C$;

$$\therefore F - 32 = \frac{180\,C}{100} = \frac{9}{5}\,C;$$
$$\therefore F = \frac{9}{5}\,C + 32.$$

Similarly:

$$C : R :: 100 : 80;$$
$$\therefore C = \frac{100\,R}{80} = \frac{5}{4}\,R;$$
$$\text{and, } R = \frac{80\,C}{100} = \frac{4}{5}\,C.$$

Similarly:

$$R : F - 32 :: 80 : 180;$$
$$\therefore R = \frac{80\,(F-32)}{180} = \frac{4}{9}\,(F-32):$$
$$\text{and, } F - 32 = \frac{180\,R}{80} = \frac{9}{4}\,R.$$
$$\therefore F = \frac{9}{4}\,R + 32.$$

Examples:

(1) Find the equivalent of $80°$ F. on the other scales.

$$C = \frac{5\,(F-32)}{9} = \frac{5\,(80-32)}{9} = \frac{5 \times 48}{9} = 26 \cdot 7.$$
$$R = \frac{4\,(F-32)}{9} = \frac{4\,(80-32)}{9} = \frac{4 \times 48}{9} = 21 \cdot 3.$$
$$\therefore 80°\ F. = 26° \cdot 7\ C. = 21° \cdot 3\ R.$$

(2) Find the equivalent of 10° C. on the other scales.

$$F=\frac{9}{5}C+32=\frac{9\times 10}{5}+32=18+32=50.$$

$$R=\frac{4}{5}C=\frac{4}{5}\times 10=8\,;$$

$$\therefore 10^\circ C.=50^\circ F.=8^\circ R.$$

(3) At what temperature is the number on the Centigrade and Fahrenheit thermometers the same?

Let x=the temperature on F. scale,

then $\frac{5(x-32)}{9}$=temperature on C. scale.

$$\therefore \frac{5(x-32)}{9}=x,$$

and $x=-40^\circ$.

Correction of Barometric Height for Altitude.

As a rule, the barometer falls 1 inch in ascending 900 feet. If x=number of feet above sea-level of station where observation is taken, then, in ascending that height, the barometer will fall $\frac{1}{900}$=0·001 inch for every foot, or 0·001 x inch in x feet.

In correcting for altitude, therefore, 0·001x inch must be added to the reading taken. That is,

Barometric reading at sea-level = observed height + 0·001 x inch.

For strict accuracy, the difference in the temperature of the air at the elevated station, and at sea-level, should be taken into account, but for ordinary observations the temperature is assumed to be that of the external air at the station where the barometer is placed.

To Ascertain the Altitude by the Barometer.

If x = barometric height (in inches) at lower station, and y = barometric height (in inches) at upper station, then $(x-y)$ inches = difference between the two readings; *i.e.*, $(x-y)$ inch represents the fall.

Since barometer falls 1 inch for every 900 feet ascended, it will fall $(x-y)$ inches in 900 $(x-y)$ feet.

That is, there is a difference of 900 $(x-y)$ feet between the two stations.

To find the altitude, therefore, take barometric readings at both stations, and multiply their difference (in inches) by 900. The product will be the altitude (in feet).

Example:

Barometer at lower station =	- -	29·92 inches.
Barometer at upper station =	- -	29·21 "
	Difference =	0·71 "

∴ Altitude of upper station = 900 × 0·71 = 639 feet.

Correction of Barometric Height for Temperature.

It is usual to reduce all barometric readings to the freezing-point, in order to render them comparable in different places and at different times.

Let a = co-efficient of absolute expansion of mercury,
= 0·00018018 for every 1° C.,
and 0·0001001 for every 1° F.

Let h = height at 0° C., or 32° F.,
h_t = height at t° C., or t° F.

If volume of mercury at 0° C., or 32° F. = 1, then

$$\text{vol. at } t^\circ \text{ C.} = 1 + at, \text{ and}$$
$$\text{vol. at } t^\circ \text{ F.} = 1 + a(t-32).$$

Now, height of column varies inversely as the density, and density inversely as the volume; therefore, the height must vary directly as the volume, and therefore:

For the Centigrade scale:

$$\frac{h}{h_t}=\frac{1}{1+at};$$

$$\therefore h=\frac{h_t}{1+at}=\frac{h_t}{1+0{\cdot}00018t}.$$

This equation may be further simplified as follows:

Multiply both numerator and denominator by $(1-0{\cdot}00018t)$; then

$$h=\frac{h_t(1-0{\cdot}00018t)}{(1+0{\cdot}00018t)(1-0{\cdot}00018t)}$$

$$=\frac{h_t(1-0{\cdot}00018t)}{1-(0{\cdot}00018t)^2}.$$

Now, $(0{\cdot}00018t)^2$ is such an infinitely small number that it may be neglected, without in any appreciable degree affecting the result; so the equation becomes reduced to:

$$h=h_t(1-0{\cdot}00018t).$$

For the Fahrenheit scale:

$$\frac{h}{h_t}=\frac{1}{1+a(t-32)};$$

$$\therefore h=\frac{h_t}{1+0{\cdot}0001(t-32)}.$$

This equation may also be written:

$$h=h_t[1-0{\cdot}0001(t-32)].$$

Example:

If the barometer stands at 30·267 inches, and the thermometer at 60° F., what is the barometric reading reduced to 32° F.?

$$h_t = 30{\cdot}267, \text{ and } t - 32 = 60 - 32 = 28\ ;$$

$$\therefore\ h = h_t[1 - 0{\cdot}0001(t - 32)]$$
$$= 30{\cdot}267[1 - (28 \times 0{\cdot}0001)]$$
$$= 30{\cdot}267[1 - 0{\cdot}0028]$$
$$= 30{\cdot}267 \times 0{\cdot}9972$$
$$= 30{\cdot}182 \text{ inches.}$$

The above correction is simply for the expansion of the mercury due to heat, and does not correct for the expansion of the brass scale.

The **double correction** for temperature (viz., for the expansion of the mercury and also of the brass scale) may be made as follows :

For the Centigrade scale :

Let β=co-efficient of expansion of brass for each degree=0·00001878.

If each division on the scale at 0° C.=1 inch, then each division at 1° C. will measure $1+\beta$, and „ „ t° C. „ „ $1+\beta t$;

and if n divisions be the height read off on the scale, then n divisions at t° C. will become $n(1+\beta t)$ inches. So we have

$$h_t = n(1+\beta t)\ ;$$

but we have already seen that

$$h = \frac{h_t}{1+at}, \text{ or } h_t = h(1+at)\ ;$$

$$\therefore\ n(1+\beta t) = h(1+at)\ ;$$

$$\therefore\ h = \frac{n(1+\beta t)}{1+at}.$$

This formula may be simplified, as the one above, by multiplying both numerator and denominator by $(1 - at)$, thus :

$$h=\frac{n(1+\beta t)(1-\alpha t)}{(1+\alpha t)(1-\alpha t)}=\frac{n(1+\beta t)(1-\alpha t)}{1-(\alpha t)^2}.$$

The fraction $(\alpha t)^2$ may be neglected, and we have :

$$h=n(1+\beta t)(1-\alpha t)$$
$$=n-n\alpha t+n\beta t-n\alpha\beta t^2$$
$$=n-nt(\alpha-\beta)\ [\text{neglecting } (n\alpha\beta t^2)].$$

Now, $\alpha=0{\cdot}00018018$, and

$\beta=0{\cdot}00001878$;

$\therefore\ (\alpha-\beta)=0{\cdot}0001614$,

and the formula $h=n-nt(\alpha-\beta)$ becomes

$$h=n-0{\cdot}0001614nt\ ;$$

that is to say, if n=number of divisions read off on the scale, whilst the thermometer stands at $t°$ C., the reading must be diminished by $0{\cdot}0001614nt$.

For the Fahrenheit scale:

The formula will be the same, but with $(t-32)$ substituted for t ; thus :

$$h=n-n(t-32)(\alpha-\beta).$$

Of course $(\alpha-\beta)$ will have a different value.

We have seen that $\alpha=0{\cdot}0001001$.

We have also seen that for each degree Centigrade, $\beta=0{\cdot}00001878$, and the coefficient of expansion for each degree Fahrenheit is $\frac{5}{9}$ of the coefficient for each degree Centigrade. Therefore, coefficient of expansion of brass for Fahrenheit will be :

$$\frac{5}{9}\times 0{\cdot}00001878=0{\cdot}00001043=\beta\ ;$$

therefore $(\alpha-\beta)=0{\cdot}0001001-0{\cdot}00001043$

$=0{\cdot}00008967$,

and the formula $h=n-n(t-32)(\alpha-\beta)$ becomes

$$h=n-0{\cdot}00008967n(t-32).$$

These figures assume that the brass scale is correct at 0° C. or 32° F.—that is, that each division on the scale registers a true inch at the freezing-point.

The correction for temperature is usually made by the aid of special tables, and these are often constructed by means of a formula which assumes that the divisions on the scale are true at 62° F. instead of 32° F.

We have seen that for the Centigrade scale

n divisions at $t°$ C. will become $n(1+\beta t)$ divisions.

Similarly, for Fahrenheit

n divisions at $t°$ F. will become $n[1+(t-32)\beta]$.

But if we assume the scale correct at 62° F., instead of 32° F.,

n divisions at $t°$ F. will become $n[1+(t-62)\beta]$;

and, by proceeding in a manner similar to the one employed above, we can make a new expression for the value of h. Thus:

$$h_t=n[1+(t-62)\beta];$$

but we have seen (p. 22) that

$$h_t=h[1+\alpha(t-32)];$$
$$\therefore\ n[1+(t-62)\beta]=h[1+\alpha(t-32)];$$
$$\therefore\ h=\frac{n[1+(t-62)\beta]}{1+\alpha(t-32)}.$$

Simplify as before, and we get

$$h=n[1+(t-62)\beta][1-\alpha(t-32)];$$
$$=n[1-\alpha(t-32)+\beta(t-62)];$$
$$\text{or, } h=n-n[\alpha(t-32)-\beta(t-62)].$$

THE VERNIER.

The principle of the vernier is as follows :

A given length of the brass scale is taken, containing n divisions ; the same length of the vernier is taken and divided into $n+1$ divisions—that is, the given length contains n divisions on the fixed scale, and $n+1$ divisions on the vernier.

If S = length of one division on the scale,
and V = length of one division on the vernier,
then $(n+1)V = nS$;

$$\therefore V = \frac{n}{n+1} S.$$

If each scale division $= \frac{1}{x}$ inch, then $S = \frac{1}{x}$;

$$\text{and } V = \frac{n}{n+1} \times \frac{1}{x} = \frac{n}{x(n+1)} \text{ inch,}$$

$$\text{and } S - V = \frac{1}{x} - \frac{n}{x(n+1)} = \frac{1}{x(n+1)} \text{ inch.}$$

We thus obtain the following :

Length of scale-division $= \frac{1}{x}$ inch,

„ „ vernier „ $= \frac{n}{x(n+1)}$ inch, and

difference between S and V division $= \frac{1}{x(n+1)}$ inch.

A barometer scale is usually divided into inches, tenths of inches, and twentieths of inches ; that is, the distance between the smallest divisions is $\frac{1}{20}$ inch, or $\frac{1}{x} = \frac{}{20}$.

In graduating the vernier, it is usual to take, for the length, 24 scale-divisions, *i.e.*, $n=24$; then an equal length of the vernier is divided into $n+1=25$ divisions.

A vernier division will thus be less than a scale-division by $\frac{1}{x(n+1)}=\frac{1}{20\times 25}=0\cdot002$ inch.

Therefore, in reading a barometer, the number of divisions read off on the vernier must be multiplied by 0·002, and the product added to the already observed height on the brass scale.

The usual construction of the vernier has been here taken; sometimes, however, n divisions on the scale are divided into $n-1$ divisions on the vernier; in other words, a certain length containing n divisions on the scale may be divided into $n\pm1$ divisions on the vernier.

CHAPTER II.

VENTILATION.

In the Delivery of Air through an Inlet, having given Two of the Three following Data (viz., Velocity, Delivery, and Sectional Area), to find the Third.

THE greater the velocity of the air, the greater will be the outflow, or amount delivered, in a given time; that is, velocity varies directly as outflow.

Also, velocity varies inversely as the sectional area. This is not always obvious at first, but on a little consideration it will be seen to be true.

Take, by way of example, a narrow river suddenly widening its bed for a distance, and then narrowing again. Exactly the same quantity of water which enters the widened portion must leave it (for if more entered than left, the water would be dammed back; if more left than entered, the widened portion would run dry). Therefore, the same amount of water must pass the widened portion as passes an equal area of the narrow portion in the same time; but as the bed is widened, the velocity must be diminished in order to fulfil this condition. So the greater the sectional area, the less need the velocity be, in order to deliver a given quantity in a given time; on the other hand, the less the sectional area, the greater must be the velocity, in order to deliver the required amount in the time.

Therefore, since velocity varies directly as outflow, and inversely as the sectional area, we have the following :

$$\text{Velocity} = \frac{\text{outflow}}{\text{sectional area}}.$$

We may write this as follows :

$$V = \frac{O}{S}.$$

Examples :

(i.) Find the velocity of air required to deliver 3,000 cubic feet per hour through an inlet, whose sectional area is 24 square inches.

We have to find V, having given O=3,000 cubic feet per hour, and S=24 square inches, or $\frac{24}{144} = \frac{1}{6}$ square foot.

$$V = \frac{O}{S} = \frac{3{,}000 \text{ cubic feet}}{\frac{1}{6} \text{ square foot}} = 18{,}000 \text{ feet per hour,}$$

$$\text{or } \frac{18{,}000}{60 \times 60} = 5 \text{ feet per second.}$$

(or 3·4 miles per hour).

(ii.) Find sectional area of inlet required to deliver 3,000 cubic feet of air per hour, with a velocity of 5 feet per second.

O=3,000 cubic feet per hour.

V=5 feet per second, or 18,000 feet per hour.

$$V = \frac{O}{S}; \therefore S = \frac{O}{V} = \frac{3{,}000 \text{ cubic feet}}{18{,}000 \text{ feet}} = \frac{1}{6} \text{ square foot,}$$

$$= \frac{144}{6} = 24 \text{ square inches.}$$

(iii.) An inlet, having a sectional area of 24 square inches, delivers air at the rate of 5 feet per second; find total delivery per hour.

$$V = 5 \text{ feet per second.}$$

$$S = 24 \text{ square inches} = \frac{24}{144} = \frac{1}{6} \text{ square foot.}$$

$$V = \frac{O}{S}\ ;\ \therefore O = V \times S = 5 \text{ feet} \times \frac{1}{6} \text{ square foot ;}$$

$$= \frac{5}{6} \text{ cubic foot per second ;}$$

$$= \frac{5 \times 60 \times 60}{6} = \left\{ \begin{array}{l} 3{,}000 \text{ cubic feet} \\ \text{per hour.} \end{array} \right.$$

For the sake of simplicity, the same figures have been taken in each of the above three examples, from which we see that an inlet whose area is 24 square inches, will deliver 3,000 cubic feet of air per hour, with a velocity of 5 feet per second.

There are one or two points to notice in the solution of the above examples. In (i.) and (iii.) one of the data given is in *inches*, the other in *feet*. These must, of course, be both brought either to inches or feet before proceeding further. In (ii.) O = feet per *hour*, S = feet per *second*. Here, one of them must be brought to the same terms as the other.

Also, we may point out that:

$$\frac{\text{cubic feet}}{\text{square feet}} = \text{feet}\ ;\ \frac{\text{cubic feet}}{\text{feet}} = \text{square feet}\ ;$$

$$\frac{\text{square feet}}{\text{feet}} = \text{feet}\ ;\ \text{feet} \times \text{square feet} = \text{cubic feet.}$$

It will be noticed that no allowance has been made for friction (for further reference to this subject, *vide* p. 39). It is usual, for an ordinary inlet, to deduct one-fourth for friction.

Thus, in example (i.) 5 feet per second becomes $5 - \frac{5}{4} = 3{\cdot}75$ feet per second; and this lessening of the velocity will lessen the total delivery by one-fourth, which would then be only $3{,}000 - \frac{3{,}000}{4} = 2{,}250$ cubic feet per hour.

With the lessened velocity, the same sized inlet (viz., 24 square inches) would suffice to give the proportionately diminished delivery; but, in order to produce the 3,000 cubic feet per hour, allowing the velocity to be diminished by friction, the inlet would have to be enlarged in the same proportion as the increased delivery required; thus,

$$2250 : 3000 :: 24 \text{ sq. in.} : x \text{ sq. in.};$$

from which we find that $x = 32$ square inches.

The above methods do not take into consideration the difference (if any) in the temperature of the inside and outside air, nor yet the difference in level between inlet and outlet. (In this case the velocity must be calculated by Montgolfier's formula, p. 41.)

The following is a simple method of calculating the total delivery and velocity of the air, and the sectional area of the inlet.

If the sectional area be 1 square foot and the velocity 1 foot per second, the delivery must obviously be 1 cubic foot per second.

Sectional Area of Opening.	*Velocity.*	*Delivery.*
1 sq. ft., or 144 sq. in.	1 ft. per sec. ...	1 cub. ft. per sec., or 3,600 cub. ft. per hour.
ditto ...	5 ft. per sec. ...	$5 \times 3{,}600 = 18{,}000$ cub. ft. per hour.
$\frac{144}{6} = 24$ sq. in.	ditto ...	$\frac{18{,}000}{6} =$ 3,000 cub. ft. per hr.

The remarks we have made above regarding friction, etc., apply equally to this method.

To Calculate the Amount of CO_2 expired by an Adult per Hour.

Inspired air contains 4 parts per 10,000, or 0·04 per cent. of CO_2. Expired air contains 4·04 per cent.;

$\therefore$ $4{\cdot}04 - 0{\cdot}04 = 4$ per cent. of CO_2 is given off in each breath.

In the case of an adult male, at each breath 30·5 cubic inches of air pass in and out of the lungs, containing, when expired, an additional 4 per cent.,

$$\text{or } \frac{4 \times 30{\cdot}5}{100} = 1{\cdot}22 \text{ cubic inches } CO_2.$$

17 respirations per minute, or 1,020 respirations per hour, would produce $1{\cdot}22 \times 1020 = 1244{\cdot}4$ cubic inches per hour,

$$\text{or, } \frac{1244{\cdot}4}{1728} = 0{\cdot}72 \text{ cubic foot per hour.}$$

Women and children exhale less than this, and 0·6 cubic foot per hour is about the average per head for a mixed assembly of people.

To Calculate the Necessary Air-supply and the Impurity Present.

Let D = delivery of air (in cubic feet), or amount of air available.

E = total amount of CO_2 exhaled.

r = *added* respiratory impurity in 1 cubic foot of air.

Then,

$$\left.\begin{array}{c}\text{added im-}\\\text{purity in}\\\text{1 cub. ft.}\end{array}\right\} : \left\{\begin{array}{c}\text{1 cub.}\\\text{ft. of}\\\text{air}\end{array}\right\} : : \left\{\begin{array}{c}\text{total im-}\\\text{purity}\\\text{added}\end{array}\right\} : \left\{\begin{array}{c}\text{total de-}\\\text{livery of}\\\text{air.}\end{array}\right.$$

$$\therefore r : 1 : : E : D;$$

$$\therefore D = \frac{E}{r}.$$

The following data should be borne in mind :

On an average, each person exhales 0·6 cubic foot of CO_2 per hour, and this figure is usually taken in all calculations.

The total CO_2 in a room should not exceed 0·6 part per 1,000, and since atmospheric air contains 0·4 part per 1,000, it follows that the amount of *added* respiratory impurity should not exceed $0·6 - 0·4 = 0·2$ part per 1,000.

How much fresh air should each person be allowed per hour, in order that the above conditions may be fulfilled ?

$E = 0·6$ cubic foot per hour ;

$r = 0·2$ per 1,000, or 0·0002 per cubic foot ;

$$\therefore D = \frac{E}{r} = \frac{0·6}{0·0002} = 3{,}000.$$

That is, each person should be allowed 3,000 cubic feet of fresh air per hour.

Example :

The air of a room containing 20,000 cubic feet, in which 10 persons have been working for 5 hours, is found to contain 10 parts of carbonic acid in 10,000 parts. How much fresh air is entering per head per hour ?*

* D.P.H. Exam., Roy. Coll. Phys. and Surg., Dec. 1887.

Ten persons exhale $10 \times 0{\cdot}6 = 6$ cubic feet CO_2 per hour ;
Air of the room yields 10 parts CO_2 per 10,000 ;
CO_2 naturally present in air = 4 parts per 10,000.

Therefore, the added respiratory impurity = 6 parts per 10,000, or 0·0006 cubic foot per cubic foot of air.

That is :

$E = 6$ cubic feet ;
$r = 0{\cdot}0006$ cubic foot ;

$$\therefore D = \frac{E}{r} = \frac{6}{0{\cdot}0006} = 10{,}000 \text{ cubic feet per hour.}$$

As the room contains 20,000 cubic feet of space, there is sufficient air for the first two hours, but for each of the three hours afterwards, 10,000 cubic feet per hour (or 30,000 for the three hours) must be admitted, or 1,000 cubic feet per hour **for each person.**

In the above example, we have taken E = impurity exhaled **per hour**, and consequently D = delivery per hour. If we take E = total CO_2 exhaled in **five hours,** viz., $6 \times 5 = 30$ cubic feet, then

$$D = \frac{E}{r} = \frac{30}{0{\cdot}0006} = 50{,}000 \text{ for the \textbf{five} hours.}$$

That is, 50,000 cubic feet are required for the five hours ; but as the room already contains 20,000 cubic feet, an additional 30,000 cubic feet will be the total delivery necessary.

Example :

(i.) Six persons occupy a room of 5,000 cubic feet space continually for six hours ; calculate the percentage of CO_2 present in the air at the end of the time, supposing 10,000 cubic feet of fresh air have been supplied per hour. (ii.) In what time would the permissible limit of impurity be reached if there were no ventilation?

(i.) CO_2 originally present in room $= 0{\cdot}4$ per 1,000, or $0{\cdot}4 \times 5 = 2$ cubic feet in the 5,000.

One man exhales $0{\cdot}6$ cubic foot CO_2 per hour,

$\therefore$ Six men exhale $0{\cdot}6 \times 6 \times 6 = 21{\cdot}6$ cubic feet of CO_2 in 6 hours.

Fresh air added in 6 hours $= 10{,}000 \times 6 = 60{,}000$ cubic feet, containing 4 parts per 10,000 of CO_2,

or $4 \times 6 = 24$ cubic feet CO_2 in the 60,000.

Therefore, total CO_2 at end of 6 hours =

Originally present	... ...	2·0
Added by respiration	... ...	21·6
Added in fresh air	... ...	24·0
	Total ...	47·6

Amount of air available =

Originally present in room	...	5,000
Fresh air added	... ...	60,000
	Total ...	65,000

Total amount of CO_2 present at end of 6 hours $= 47{\cdot}6$ cubic feet in 65,000 cubic feet of air,

or $\frac{47{\cdot}6 \times 100}{65{,}000} = 0{\cdot}073$ per cent.

Or, the result may be obtained as follows:

$$E = 21{\cdot}6 \qquad D = 65{,}000$$

$$D = \frac{E}{r} \therefore r = \frac{E}{D} = \frac{21{\cdot}6}{65{,}000} = 0{\cdot}00033 \text{ per cubic foot,}$$

or $0{\cdot}033$ per cent.

That is, the *added* respiratory impurity $= 0{\cdot}033$ per cent.

But air of room, at the commencement, contained $0{\cdot}04$ per cent. CO_2 (p. 32);

$\therefore$ Total impurity $= 0{\cdot}033 + 0{\cdot}04 = 0{\cdot}073$ per cent.

(ii.) In what time would the permissible limit be reached, if there were no ventilation?

The permissible total impurity is 0·6 per 1,000 (*vide* p. 33),

$$\text{or } 0·6 \times 5 = 3 \text{ cubic feet in the } 5,000.$$

The room already contains 0·4 per 1,000,

$$\text{or } 0·4 \times 5 = 2 \text{ cubic feet in the } 5,000.$$

Therefore, the permissible *added* impurity $= 3 - 2 = 1$ cubic foot.

How long will it take the six persons to add this 1 cubic foot?

In 1 hour 6 persons exhale $0·6 \times 6 = 3·6$ cubic feet CO_2;

$$\therefore \text{ in } \frac{1}{3·6} \text{ hour they will add 1 cubic foot } CO_2,$$

$$\text{and } \frac{1}{3·6} \text{ hour} = \frac{60}{3·6} = 16·6 \text{ minutes.}$$

Or, the result may be obtained as follows:

Let x = time (in hours);
CO_2 in room = 2 cubic feet;
Six men in x hours add $3·6\,x$ cubic feet;
$\therefore$ Total CO_2 in x hours $= (3·6x + 2)$ cubic feet.
Available air = 5,000 cubic feet,
And permissible limit of impurity = 6 per 10,000.

Since this limit must not be surpassed, the total CO_2 and the available air must be to each other in the proportion of 6 to 10,000, or

$$\text{Total } CO_2 : \text{available air} :: 6 : 10,000;$$

$$\therefore 3·6x + 2 : 5,000 :: 6 : 10,000,$$

$$3·6x + 2 = \frac{5,000 \times 6}{10,000} = 3$$

$$x = \frac{3 - 2}{3·6} = \frac{1}{3·6} \text{ hours} = 16·6 \text{ minutes.}$$

Impurity produced by Artificial Illumination.

Combustion of Oil:

It is estimated that a lamp, with one moderately good wick, burns 154 grains of oil per hour. Oil (paraffin) contains about 85 per cent. of carbon.

Therefore:

1 grain of oil contains $\frac{85}{100}$ grains C., or 1 grain C. is contained in $\frac{100}{85}$ grains of oil.

Now, from the equation $C + O_2 = CO_2$, we know that 12 grains C. produce 44 grains CO_2;

$$\therefore 1 \text{ grain C. produces } \frac{44}{12} \text{ grains } CO_2;$$

$$\therefore \frac{100}{85} \text{ grains oil produce } \frac{44}{12} \text{ grains } CO_2;$$

$$\therefore 1 \text{ grain oil produces } \frac{44 \times 85}{12 \times 100} = 3{\cdot}11 \text{ grains } CO_2;$$

and 154 grains oil produce $3{\cdot}11 \times 154 = 479$ grains CO_2.

Now, 1 cubic foot of dry air at 60° and 760 mm. weighs $\frac{566{\cdot}85 \times 491}{491 + (60 - 32)} = 536{\cdot}27$ grains (p. 5);

$\therefore$ 1 cubic foot CO_2 (at same temperature and pressure) weighs $\frac{536{\cdot}27 \times 22}{14{\cdot}47} = 815{\cdot}3$ grains (p. 2);

$$\therefore 479 \text{ grains } CO_2 = \frac{479}{815{\cdot}3} = 0{\cdot}58 \text{ cubic foot.}$$

That is, the combustion of the oil in the lamp produces 0·58 cubic foot, or rather more than ½ cubic foot of CO_2 per hour. And since a human being exhales 0·6 cubic foot per hour, we see that an ordinary lamp produces, each hour, about the same vitiation of the air as each occupant of the room.

Example:

A room is fitted with four Tobin's tubes, each having a sectional area of 50 square inches, and in the room are two persons and a paraffin lamp, which burns 1 ounce of oil per hour. Find the velocity of the air current necessary to keep the impurity down to 0·6 part per 1,000.

154 grains oil produce 0·58 cubic foot CO_2 (p. 37);

$\therefore$ 1 ounce oil produces $\frac{0 \cdot 58 \times 437 \cdot 5}{154} = 1 \cdot 6$ cubic feet CO_2 per hour.

Two persons exhale $2 \times 0 \cdot 6 = 1 \cdot 2$ cubic feet CO_2 per hour;

$\therefore$ total CO_2 produced $= 1 \cdot 6 + 1 \cdot 2 = 2 \cdot 8$ cubic feet.

Total permissible amount of CO_2 = 6 per 10,000.

Amount present in air = 4 per 10,000.

$\therefore$ permissible respiratory impurity $= 6 - 4 = 2$ per 10,000; or, 0·0002 per cubic foot.

$$D = \frac{E}{r} = \frac{2 \cdot 8}{0 \cdot 0002} = 14{,}000 \text{ cubic feet per hour;}$$

$\therefore$ each 'Tobin' must deliver $\frac{14{,}000}{4} = 3{,}500$ cubic feet per hour.

$$\text{Now, } V = \frac{O}{S} \text{ (p. 29)} = \frac{3{,}500 \text{ cubic ft.}}{50 \text{ square ins.}} = \frac{3{,}500 \text{ cubic ft.}}{\frac{50}{144} \text{ square ft.}}$$

$= 10{,}080$ feet per hour, or 2·8 feet per second.

[No deduction has been here made for friction.]

FRICTION IN VENTILATION.

Example:

Compare the ventilation in the two rooms, A and B.

The room A is ventilated by a straight shaft, circular in section, 30 feet long, with a sectional area of 1 square foot.

The room B is ventilated by four shafts, square in section, each 30 feet long, and each having a sectional area of $\frac{1}{4}$ square foot.

It will be noticed that the sum of the sectional areas of the four smaller shafts in B, is equal to the sectional area of the larger shaft in A, and the lengths are the same—viz., 30 feet. Therefore, the cubic contents, or capacity of the four shafts in B, are together equal to the capacity of the larger shaft in A, and would be capable, therefore, of supplying the same quantity of air to B as the single shaft does to A, were it not for friction, which must now be considered.

In two tubes of similar shape and equal sectional area, the friction will vary directly as the length—*e.g.*, in two similar tubes, one 20 feet and the other 30 feet long, the friction in the longer tube will be increased by one-half, since its length is increased by one-half. If a shaft 40 feet long be increased $\frac{1}{4}$ of its length (*i.e.*, become 50 feet long), it will have an increased friction of $\frac{1}{4}$.

In two tubes of different shape, but of the same sectional area, the friction varies directly as the periphery.

For example, take a circular and a square tube, each with a sectional area of 1 square foot. The periphery of the circle, with a sectional area of 1 square foot or 144 square inches, may be found as follows (*vide* p. 92):

$$\pi r^2 = 144 \text{ sq. in.}, \therefore r^2 = \frac{144}{\pi} = \frac{144}{3{\cdot}1416} = 45{\cdot}8365\,;$$

$$\therefore r = \sqrt{45{\cdot}8365} = 6{\cdot}77\,;$$

therefore, diameter $= 6{\cdot}77 \times 2 = 13{\cdot}5$ inches.

If diameter $= 13{\cdot}5$, then periphery $= 13{\cdot}5 \times 3{\cdot}1416 = 42$ inches, or $3\frac{1}{2}$ feet.

A square with sectional area of 1 square foot has sides 1 foot long, and, therefore, periphery = 4 feet.

So we see :

$$\left.\begin{matrix}\text{Friction in}\\ \text{circular tube}\end{matrix}\right\} : \left\{\begin{matrix}\text{friction in}\\ \text{square tube}\end{matrix}\right\} : : 3\tfrac{1}{2} : 4.$$

To return to the comparison of the rooms :

In A, shaft = 30 feet, and periphery = $3\frac{1}{2}$ feet ; so total friction may be represented as :

$$30 \times 3\tfrac{1}{2} = 105 \text{ units.}$$

In B, the sectional area of the small square shafts being $\frac{1}{4}$ square foot, each side will be $\frac{1}{2}$ foot long, and the total periphery of the four sides $= 4 \times \frac{1}{2} = 2$ feet.

Each shaft = 30 feet in length, and periphery = 2 feet ;

$\therefore$ total friction in each shaft $= 30 \times 2 = 60$ units, and total friction in the four shafts $= 60 \times 4 = 240$ units.

So we obtain this result :

Friction in A : friction in B : : 105 : 240,
: : 7 : 16.

That is, the friction in B is more than twice that in A.

If, in each shaft in B, we place a right-angled bend, the friction will be doubled; or,

$$A : B :: 7 : 16 \times 2$$
$$:: 7 : 32;$$

or friction in B is between four and five times that in A. Therefore, to get the same ventilation in B as in A, we must either increase the number of the smaller shafts, or the size of them, between four and five times if we use the bent tubes, or more than double them if we use the straight small shafts.

MONTGOLFIER'S FORMULA.

When bodies fall to the earth, we find that different bodies fall through equal spaces from rest, in a given time; and the space fallen through varies as the square of the time.

Any deviation from this law is due to the resistance of the air.

If V = velocity in feet per second,
g = acceleration due to gravity,
h = height fallen (in feet),
then $V^2 = 2gh$.

The value of g increases from the equator to the poles; in the latitude of London, $g = 32{\cdot}19$ feet per second. For practical purposes, however, we may consider the value of g to be *constant*.

Suppose $h = 5$ miles $= 26{,}400$ feet,
Then $V^2 = 2gh = 2 \times 32{\cdot}19 \times 26{,}400 = 1{,}699{,}632$, and
$V = \sqrt{1{,}699{,}632} = 1{,}303$ feet per second.

If we assume the atmosphere to be of uniform density throughout, it would extend upwards to a height of about five miles; and the velocity with which air enters a vacuum is the same as a body would acquire in falling through a corresponding height; therefore, the velocity of air entering a vacuum = 1,303 feet per second. If the air be not entering a vacuum, but a space containing air at a different pressure, its velocity will depend on the difference between their respective pressures. Usually these pressures cannot be directly observed, but must be inferred from the difference between their temperatures. The difference in pressure can be obtained by ascertaining the difference in level between inlet and outlet, and multiplying this by the expansion of air due to the difference in temperature; and this difference of pressure may be taken to represent the height fallen.

The equation $V^2 = 2gh$ may be expressed as follows:

$$V^2 = 2gh = 2 \times 32{\cdot}19 \times h = 64{\cdot}38 \times h;$$

$$\therefore V = \sqrt{64{\cdot}38} \times \sqrt{h} = 8{\cdot}02 \times \sqrt{h}$$

(approximately $8\sqrt{h}$).

That is, the velocity in feet per second of falling bodies $= 8 \times \sqrt{\text{height fallen}}$.

Example:

The thermometer in a room stands at 60° F., whilst outside it stands at 50° F. The difference in the level between inlet and outlet is 10 feet; 3,000 cubic feet of fresh air per head are required every hour. What should be the size of the outlet?

Since difference in level = 10 feet, and expansion of air due to an increase of temperature of 10° F.

$$= \frac{10}{491} \text{ (\textit{vide} p. 4)};$$

$$\therefore \text{height fallen} = \frac{10 \times 10}{491} = \frac{100}{491} = 0{\cdot}2036 \text{ foot};$$

$$\therefore V = 8\sqrt{h} = 8\sqrt{0.2036} = 8 \times 0.45 = 3.6 \text{ feet per second, or 216 feet per minute.}$$

Deduct $\frac{1}{4}$ for friction, viz. : $\frac{216}{4} = 54$;

$$\therefore V = 216 - 54 = 162 \text{ feet per minute, or 9,720 feet per hour.}$$

Since delivery required = 3,000 cubic feet per hour, and velocity = 9,720 feet per hour, we can find the sectional area of the outlet by the formula $S = \frac{O}{V}$ (*vide* p. 29, example ii.)

$$S = \frac{O}{V} = \frac{3{,}000 \text{ cubic feet}}{9{,}720 \text{ feet}} = 0.308 \text{ square foot}$$
$$= 0.308 \times 144 = 44 \text{ square inches.}$$

That is to say, there must be 44 square inches of outlet *for each person*, and there ought also to be an inlet of equal size.

Example :

(i.) A room 30 feet long, 20 feet wide, and 12 feet high, containing 3 gas-burners, is occupied by 5 men. If there be no ventilation, in what time will the air have reached its permissible limit of impurity? (ii.) What delivery of air is required to prevent the permissible limit being exceeded? (iii.) If the only outlet be a chimney 15 feet high, and the temperature of the room 10° F. higher than the outside air, find the sectional area of the chimney required.

(i.) Five men exhale $5 \times 0.6 = 3$ cubic feet CO_2 per hour.

Say that 1 gas-burner produces 3·5 cubic feet CO_2 per hour; then 3 burners produce 10·5 cubic feet CO_2, and total CO_2 produced $= 3 + 10.5 = 13.5$ cubic feet per hour.

The room $=30\times 20\times 12=7{,}200$ cubic feet.

$$r=\frac{E}{D}\ (\text{p. }33)=\frac{13\cdot 5}{7{,}200}=0\cdot 00187.$$

If at the end of hour $r=0\cdot 00187$, how long will it take r to reach the permissible limit of $0\cdot 0002$?

Let x=number of minutes;

Then $0\cdot 00187 : 0\cdot 0002 :: 60$ min. : x min.

$x=6\cdot 4$ minutes.

(ii.) $D=\frac{E}{r}=\frac{13\cdot 5}{0\cdot 0002}=67{,}500$ cubic feet per hour.

(iii.) Velocity in feet per sec. $=8\sqrt{\frac{15\times 10}{491}}=4\cdot 4216$ per sec. $=15{,}918$ feet per hour. After deducting $\frac{1}{4}$ for friction, this is reduced to 11,940 feet per hour.

$$S=\frac{O}{V}\ (\textit{vide}\ \text{p. }29)=\frac{67{,}500}{11{,}940}=5\cdot 6 \text{ square feet;}$$

therefore, the chimney must have a sectional area of 5·6 square feet.

VENTILATION BY PROPULSION.

To Calculate Delivery of Air by a Revolving Fan.

The revolution of the fan sets the air in contact with it in motion. Each part of the fan is not, of course, revolving with the same velocity, which varies from a maximum at the extremities to *nil* at the centre. The velocity of the air is usually taken to be $\frac{3}{4}$ of that of the circumference of the fan, and is called the 'effective velocity.' So, if we know the speed of revolution of the extremity of the fan, the rate of movement of the air will be $\frac{3}{4}$ of this.

Example :

Suppose fan = 12 feet in diameter, then the circumference will be $12 \times 3{\cdot}1416 = 37{\cdot}70$ feet (*vide* p. 91). Therefore, in one complete revolution, the extremity of the fan travels 37·70 feet. If the fan be revolving 60 times per minute, the velocity of the extremity will be $37{\cdot}70 \times 60 = 2{,}262$ feet per minute, and the 'effective velocity' $= \frac{3 \times 2{,}262}{4} = 1{,}696$ feet per minute, or 101,760 feet per hour, which is the rate of movement of the air.

If the sectional area of the conduit, through which the air is delivered into the room, be known, then the discharge in cubic feet can be at once calculated by the formula $O = V \times S$ (*vide* p. 30).

By reversing this process, we could calculate the size of the fan necessary to ventilate a room, if we knew the size of the conduit and the total delivery per hour of fresh air required. These two data would give us the velocity per hour of the entering air, since $V = \frac{O}{S}$ (*vide* p. 29).

Suppose this velocity to be 84,840 feet per hour; then the rotatory velocity of the fan must be such as to impart to the air a rate of movement of 84,840 feet per hour. That is, the 'effective velocity' = 84,840 feet per hour, or $\frac{3}{4}$ of the velocity at the extremity of the fan. Therefore,

Velocity at extremity of fan $= \frac{4 \times 84{,}840}{3} = 113{,}120$ feet per hour, or 1,885 feet per minute.

Now, if D = diameter of fan (in feet),
then $D \times 3{\cdot}1416$ = circumference of fan (in feet);
and if R = number of revolutions per minute,

then $D \times 3{\cdot}1416 \times R$ = distance travelled by circumference of fan per minute ; but this = 1,885 ;

$$\therefore D \times 3{\cdot}1416 \times R = 1{,}885\ ;$$

$$\therefore D \times R = \frac{1{,}885}{3{\cdot}1416} = 600 \text{ feet}\ ;$$

$$\therefore D = \frac{600}{R}, \text{ and } R = \frac{600}{D}\ ;$$

So, if $D = 5$, then $R = 120$;
if $D = 10$, then $R = 60$;
if $D = 15$, then $R = 40$.

That is to say, a fan of 5 feet diameter with 120 revolutions per minute—*or* one of 10 feet diameter with 60 revolutions per minute ; *or* one of 15 feet diameter, having 40 revolutions per minute—would each of them give the required ventilation.

CHAPTER III.

RAINFALL AND SEWERAGE.

To Calculate the Amount of Water-supply available from Rainfall.

(i.) Amount of roof-space.

We have only to ascertain the area of horizontal surface covered by the roof, the slope of the roof not affecting the result. Whatever its slope, the roof simply catches the rain which would have fallen on the area of horizontal ground now covered by the roof, if the roof had not been there.

(ii.) Average amount of rainfall per annum is about 30 inches.

(iii.) The loss by evaporation is about $\frac{1}{5}$ of the total rainfall.

Example :

The roof-space in a town is 55 square feet per head, and the annual rainfall 27 inches ; find the amount of water available per head per annum.

Roof-space $=55$ square feet ; rainfall $=27$ inches $=2\frac{1}{4}$ feet ;

$\therefore$ amount of water per head per annum $=55\times 2\frac{1}{4}$ $=124$ cubic feet.

Deducting $\frac{124}{5}=25$ for evaporation, we have :

$$124-25=99 \text{ cubic feet}$$
$$=99\times 6{\cdot}23=617 \text{ gallons.}$$

So the amount of water available for each individual during the year is 617 gallons, or 1·69 gallons per diem.

General Formula for Rainfall.

Let R = annual rainfall (in inches),
E = annual evaporation (in inches) ;

Then (R − E) inches, or $\frac{R-E}{12}$ feet = available rainfall.

Let A = number of acres of collecting ground,
Or (A × 4,840 × 9) = number of square feet of ground

$$\text{Total water}=(A\times 4{,}840\times 9)\times\frac{R-E}{12} \text{ cubic feet}$$
$$=3{,}630\ A\times(R-E) \text{ cubic feet}$$
$$=3{,}630\ A\times(R-E)\times 6{\cdot}23 \text{ gallons}$$
$$=22{,}615\times A\times(R-E) \text{ gals. per annum}$$
$$=\frac{22{,}615\times A\times(R-E)}{365}$$
$$=62\ A\ (R-E) \text{ gallons per diem.}$$

The Hydraulic Mean Depth in a Circular Sewer running Full, or Half-full, is One-fourth the Diameter.

$$\text{H.M.D.}=\frac{\text{sectional area of fluid}}{\text{wetted perimeter}}.$$

(i.) In a sewer **running full**, the sectional area of the fluid will be the same as that of the pipe, and the wetted perimeter will obviously be identical with the circumference;

$$\therefore \text{H.M.D.} = \frac{\pi r^2}{2\pi r} = \frac{r}{2} = \frac{d}{4} \text{ (pp. 91, 92).}$$

(ii.) In a sewer **running half-full**, the sectional area of the fluid will be exactly half that of the pipe, and the wetted perimeter will similarly be one-half the circumference;

$$\therefore \text{H.M.D.} = \frac{\frac{\pi r^2}{2}}{\frac{2\pi r}{2}} = \frac{\pi r^2}{2\pi r} = \frac{r}{2} = \frac{d}{4}.$$

That is, H.M.D.=one-half the radius, or one-fourth the diameter.

FLOW IN SEWERS.

Having given two of the following—(*a*) velocity of flow; (*b*) diameter of sewer; (*c*) gradient —to find the third.

We will now work out three examples, and (as was done in the similar case of air, p. 29), we will use the same figures in each of the three.

(i.) A circular house drain, running full, has a diameter of 4 inches; the gradient is 64 feet to the mile. What is the velocity of the discharge?

We find this from the formula:

$$V = 55\sqrt{2DF},$$

where V=velocity in feet per minute,
D=hydraulic mean depth (in feet),
F=fall in feet per mile.

$$D=\frac{\text{diameter}}{4}\ (\textit{vide}\ \text{p. }48)=1\ \text{inch}=\tfrac{1}{12}\ \text{foot};$$

$$F=64.$$

$$\therefore V=55\sqrt{2\times\tfrac{1}{12}\times 64}=55\sqrt{10{\cdot}6}=55\times 3{\cdot}27$$
$$=180\ \text{feet per minute,}$$
$$=3\ \text{feet per second.}$$

(ii.) A circular house drain, running full, has a diameter of 4 inches. What should the gradient be, in order that it may discharge at the rate of 3 feet per second?

$$V=3\ \text{feet per second}=180\ \text{feet per minute,}$$
$$D=\tfrac{1}{12}\ \text{foot.}$$

We have to find F.

$$V=55\sqrt{2DF}=180;$$

$$\therefore \sqrt{2DF}=\frac{180}{55}=\frac{36}{11};$$

$$\therefore 2DF=\left(\frac{36}{11}\right)^2=\frac{1{,}296}{121},\ \text{and}\ F=\frac{1{,}296}{121\times 2D};$$

$$\text{but}\ D=\tfrac{1}{12},\ \therefore F=\frac{1{,}296}{121\times\frac{2}{12}}=64;$$

that is, fall in feet per mile $=64$,

or 64 feet in 5,280 feet (since 5,280 feet $=1$ mile);

$$\text{and}\ \frac{64}{5{,}280}=\frac{1}{82}.$$

The necessary gradient is therefore

64 feet per mile, or 1 in 82.

(iii.) A circular drain is laid with a fall of 64 feet per mile. What should be its diameter in order that, when running full, it may discharge at the rate of 3 feet per second?

Let d = diameter required (in feet).

F=64, and V=3 feet per second=180 feet per minute.

Hydraulic mean depth$=\frac{\text{diameter}}{4}$ (*vide* p. 48);

$$\text{therefore } D=\frac{d}{4}.$$

$$V=55\sqrt{2DF}=55\sqrt{2\times\frac{d}{4}\times 64};$$

$$\therefore 180=55\sqrt{2\times\frac{d}{4}\times 64}=55\sqrt{32d};$$

$$\therefore \sqrt{32d}=\frac{180}{55}=\frac{36}{11}; \therefore 32d=\left(\frac{36}{11}\right)^2=\frac{1{,}296}{121},$$

$$\text{and } d=\frac{1{,}296}{121\times 32}\text{ feet}=\frac{1{,}296\times 12}{121\times 32}=4 \text{ inches.}$$

Knowing the velocity and the sectional area, we can calculate the discharge from the sewer.

Example:

What is the discharge from a drain running full, 4 inches in diameter, with a gradient of 64 feet per mile?

$$V=\frac{O}{S} \text{ (p. 29)}, \therefore O=V\times S.$$

If diameter=4 inches, then

Sectional area (S)$=4^2\times 0{\cdot}7854$ (*vide* p. 92)

$=12{\cdot}56$ square inches$=\frac{12{\cdot}56}{144}$ square feet;

and we know (p. 49, example i.) that V=3 feet per second;

$$\therefore O=V\times S=3\times\frac{12{\cdot}56}{144}=0{\cdot}2616 \text{ cubic foot per second,}$$

$=15{\cdot}69$ cubic feet per minute,

$=941$ cubic feet per hour;

or, $941\times 6{\cdot}23=5{,}862$ gallons per hour.

Example:

Over an area of 1,000 acres a rainfall of $\frac{1}{6}$ inch per hour occurs. What must be the diameter of a drain to carry this away, the gradient being 10 feet per mile?

The diameter can be ascertained from the sectional area, and the sectional area may be found if the velocity and outflow are known, since $S=\frac{O}{V}$ (p. 29); therefore:

(i.) Find the outflow.

$$1{,}000 \text{ acres} = 1{,}000 \times 4{,}840 \text{ square yards,}$$
$$= 1{,}000 \times 4{,}840 \times 9 = 43{,}560{,}000 \text{ square feet;}$$
$$\text{and } \frac{1}{6} \text{ inch} = \frac{1}{72} \text{ feet;}$$

$\therefore$ Amount of rain falling $= 43{,}560{,}000 \times \frac{1}{72} = 605{,}000$ cubic feet per hour;

or, outflow $= 10{,}083$ cubic feet per minute.

(ii.) Find the velocity of flow.

Let d = diameter of drain required (in feet);

$$V = 55\sqrt{2DF}\ ;\ D = \frac{d}{4} \text{ (p. 48)}\ ;\ F = 10\ ;$$
$$\therefore V = 55\sqrt{2 \times \frac{d}{4} \times 10} = 55\sqrt{5d} = 55 \times 2{\cdot}236\sqrt{d}$$
$$= 122{\cdot}98\sqrt{d} \text{ feet per minute.}$$

(iii.) Find sectional area of drain.

$$S = \frac{O}{V} = \frac{10{,}083}{122{\cdot}98\sqrt{d}} = \frac{81{\cdot}98}{\sqrt{d}} \text{ square feet.}$$

(iv.) Find the diameter.

$$S = d^2 \times 0{\cdot}7854 \text{ (p. 92)};$$
$$\therefore d^2 = \frac{S}{0{\cdot}7854} = \frac{81{\cdot}98}{0{\cdot}7854 \times \sqrt{d}};$$
$$\therefore d^2 \times \sqrt{d} = \frac{81{\cdot}98}{0{\cdot}7854} = 104{\cdot}39;$$

$$\text{that is, } d^{\frac{5}{2}} = 104{\cdot}39\,;$$

$$\therefore \frac{5}{2}\, log\ d = log\ 104{\cdot}39 = 2{\cdot}0186589\,;$$

$$\therefore log\ d = \frac{2 \times 2{\cdot}0186589}{5} = 0{\cdot}8074636 = log\ 6{\cdot}4\,;$$

$$\therefore d = 6{\cdot}4 \text{ feet.}$$

[No deduction has been made for loss by evaporation.]

Example :

How much water will 1 inch of rain deliver on 1 acre of ground?

One inch (or $\frac{1}{12}$ foot) of rain over 1 square yard (or 9 square feet) will give

$$9 \times \frac{1}{12} = \frac{3}{4} \text{ cubic foot of water;}$$

$$= \frac{3}{4} \times 6{\cdot}23 = 4{\cdot}6725 \text{ gallons.}$$

One acre will give $4{,}840 \times 4{\cdot}6725 = 22{,}615$ gallons,

$$= 22{,}615 \times 10 = 226{,}150 \text{ pounds;}$$

$$= \frac{226{,}150}{2{,}240} = 101 \text{ tons (nearly).}$$

[1 acre = 4,840 square yards.
1 cubic foot = 6·23 gallons.
1 gallon = 10 pounds.
2,240 pounds = 1 ton.]

EXCRETA.

Example :

How much solid and liquid excreta are passed by an average adult man **per diem**, and how much water-free solids does the amount represent? How much

per annum would a mixed community of 50,000 persons pass, and how much ammonia would be contained in the total bulk?*

An average adult man passes daily:

4 ounces fæces and 50 ounces urine;

fæces contain 23·4 per cent., and urine 4·2 per cent. solids;

∴ 4 ounces fæces contain $\frac{4 \times 23.4}{100} = 0.936$ ounce water-free solids, and 50 ounces urine contain $\frac{50 \times 4.2}{100} = 2.1$ ounces water-free solids.

∴ Total **water-free solids** per diem for an adult man $= 0.936 + 2.1 = 3.036$ ounces.

In a mixed community, the quantities per head per diem average (according to Parkes):

2·5 ounces fæces } containing together 153 grains
40 ounces urine } of nitrogen.

In a year, 50,000 persons would pass:

$$2.5 \times 365 \times 50{,}000 \text{ ounces};$$

$$\text{or, } \frac{2.5 \times 365 \times 50{,}000}{16 \times 2{,}240} = 1{,}273 \text{ tons fæces};$$

$$\text{and } \frac{40 \times 365 \times 50{,}000}{16 \times 2{,}240} = 20{,}368 \text{ tons urine}$$

[These quantities are *not* water-free.]

Total excreta per annum =

$$20{,}368 + 1{,}273 = 21{,}641 \text{ tons.}$$

The amount of nitrogen will be:

$$153 \times 365 \times 50{,}000 \text{ grains};$$

$$\text{or } \frac{153 \times 365 \times 50{,}000}{437.5 \times 16 \times 2{,}240} = 178 \text{ tons.}$$

---

* D.P.H. Exam., Cambridge, 1884.

Now, from the molecular weight of ammonia, we know that 17 parts contain 14 parts by weight of nitrogen (since $N=14$, and $H=1$);

$$\text{or, } N : NH_3 :: 14 : 17;$$

$$\therefore NH_3 = \frac{N \times 17}{14};$$

and, since $N=178$ tons,

$$NH_3 = \frac{178 \times 17}{14} = 216 \text{ tons.}$$

We have seen that an adult man, on an average, passes per diem 3·036 ounces of water-freed solids.

If these solids, instead of being dry, contain 25 per cent. of moisture, then the remaining 75 per cent. will represent the solids, which we know weigh 3·036 ounces; therefore, the total weight in this case will be

$$\frac{3 \cdot 036 \times 100}{75} = 4 \cdot 048 \text{ ounces.}$$

If there be 60 per cent. of moisture, then 3·036 ounces will only represent 40 per cent. of the total weight; therefore,

$$\text{total weight will be } \frac{3 \cdot 036 \times 100}{40} = 7 \cdot 59 \text{ ounces.}$$

Similarly, if 85 per cent. of moisture be present, the weight of the solids will represent 15 per cent. of the total weight, which, therefore, will be

$$\frac{3 \cdot 036 \times 100}{15} = 20 \cdot 2 \text{ ounces.}$$

CHAPTER IV.

ENERGY AND EXERCISE.

THE fact that any agent is capable of doing work is usually expressed by saying that it possesses energy, and the quantity of energy it possesses is measured by the amount of work it can do. The 'unit of work' is generally taken to be the quantity of work which is done in lifting 1 pound through a height of 1 foot, and this quantity of work is called 1 'foot-pound.' The product of the weight lifted (expressed in pounds), into the height through which it is lifted (expressed in feet), gives the amount of work done (in foot-pounds).

Thus, a weight of 20 pounds lifted through a distance of 1 foot = 20 foot-pounds; or, a weight of 1 pound lifted through a distance of 20 feet = 20 foot-pounds.

In the same way, we can ascertain the 'work done' by a man during exercise.

If W = his weight (in pounds) and D = the vertical height (in feet) to which he lifts his body, then the work done = $W \times D$ foot-pounds; or if D = height in **miles**, then $5{,}280D$ = height in **feet**, and work done

$$= W \times 5{,}280D \text{ foot-pounds, or } \frac{W \times 5{,}280D}{2{,}240} \text{ foot-tons.}$$

We have, so far, spoken of a man raising his weight vertically upwards. Let us now see what the work will amount to if, instead, he propel his weight horizontally along level ground. A great deal will depend upon the velocity with which he walks. It has been calculated that at an ordinary rate of three miles per hour, a man, walking along level ground, does work equivalent to raising his own weight, vertically, through $\frac{1}{20}$ the distance travelled; or, what is the same thing, raises $\frac{1}{20}$ of his weight through the whole distance travelled.

We have already seen that raising his whole weight through the distance D miles requires an expenditure of $\frac{W \times 5{,}280D}{2{,}240}$ foot-tons; therefore, to raise $\frac{1}{20}$ of his weight through the distance D, will only require an expenditure of $\frac{W \times 5{,}280D}{2{,}240} \times \frac{1}{20}$ foot-tons; which number, therefore, gives us the work done in walking a distance of D miles (or 5,280D feet) on level ground, at the rate of three miles per hour.

We have taken W to represent the total weight—*i.e.*, weight of body + weight carried.

If W = weight of body, and W′ = weight carried, then our formula becomes

$$\frac{(W + W') \times 5{,}280D}{2{,}240} \times \frac{1}{20} \text{ foot-tons.}$$

The fraction '$\frac{1}{20}$' is spoken of as the 'coefficient of resistance' (or traction), and varies with the rate of walking. At three miles per hour, on level ground, it is equivalent to $\frac{1}{20 \cdot 59}$ $\left(\text{approximately } \frac{1}{20}\right)$, at four miles $= \frac{1}{16 \cdot 75}$, and at five miles $= \frac{1}{14 \cdot 10}$.

If C = coefficient of resistance, then our formula may be written

$$\frac{(W+W')\times 5{,}280D}{2{,}240}\times C \text{ foot-tons.}$$

EXERCISE.

Example:

A man, weighing 11 stones and carrying a weight of 30 pounds, walks on level ground at the rate of three miles per hour. How many miles should he walk in order to accomplish an average 'ordinary day's work'?

As we have just seen, the formula for ascertaining the amount of work done is:

$$\frac{(W+W')\times 5{,}280D}{2{,}240}\times C = \text{foot-tons,}$$

where W = man's weight (in pounds),
W′ = weight carried ,,
D = distance walked (in miles),
C = coefficient of resistance.

An ordinary day's work = 300 foot-tons.

At three miles an hour, $C=\frac{1}{20{\cdot}59}$; and 11 stones = 154 pounds.

$$\therefore \frac{(154+30)\times 5{,}280D}{2{,}240}\times\frac{1}{20{\cdot}59}=300\,;$$

$$\therefore 184\times 5{,}280D = 300\times 20{\cdot}59\times 2{,}240,$$

$$\text{and } D=\frac{300\times 20{\cdot}59\times 2{,}240}{184\times 5{,}280}=14{\cdot}2 \text{ miles.}$$

If, instead of walking on level ground, there be a rise of 1 in 300, we can find the distance walked, in order to produce an ordinary day's work, as follows:

Distance walked = D miles, or 5,280D feet.

If ground rises 1 foot in 300 feet, it will rise $\frac{5,280D}{300} = 17 \cdot 6D$ feet in the total distance walked. Therefore, in addition to walking D miles on level ground, he has to raise his whole weight through a height of 17·6D feet; that is, he has to lift 184 pounds, or $\frac{184}{2,240}$ tons, through a height of 17·6D feet. The energy expended in this portion of the work will be:

$$17 \cdot 6D \times \frac{184}{2,240} \text{ foot-tons.}$$

And we have already seen that in traversing the distance on level ground, the work done is:

$$\frac{184 \times 5,280D}{2,240 \times 20 \cdot 59} \text{ foot-tons};$$

and the total work is to be equivalent to 300 foot-tons;

$$\therefore \frac{184 \times 5,280D}{2,240 \times 20 \cdot 59} + \left(17 \cdot 6D \times \frac{184}{2,240}\right) = 300,$$

And D = 13·3 miles.

So the 'ordinary day's work' is accomplished by walking 14·2 miles on level ground, or 13·3 miles up an incline of 1 in 300, either distance being done at the rate of three miles per hour.

Note.—The distance travelled on level ground is equivalent to raising $\frac{1}{20}$ of the weight through the whole distance, whilst the upward distance is equivalent to raising the **whole** weight through the vertical height ascended.

METHODS OF CALCULATING DIETS.

Method (i.) :

We require a table showing the percentage composition of different foods, and also a 'standard diet,' showing the total amount of each variety of food necessary under different conditions.

The following are the usual 'standard diets' adopted :

	Subsistence (*Playfair*).	*Ordinary Work* (*Moleschott*).	*Laborious Work* (*Playfair*).
Proteids ...	2·0	4·6	6·5
Fats	0·5	3·0	4·0
Carbohydrates	12·0	14·0	17·0
Salts	0·5	1·0	1·3
Total (in oz.)	15·0	22·6	28·8

The above quantities are **water-free.**

Example :

Find the amount of bread and cheese sufficient for the diet of a man at **ordinary work.**

Let x=amount of cheese (in ounces).
y=amount of bread "

From a table, we find that ordinary cheese contains 31·0 per cent. of proteids, and 28·5 per cent. of fats.

Bread contains 8·0 per cent. proteids, 1·5 per cent. fats, and 49·2 per cent. carbohydrates.

Therefore, x ounces of cheese contain

$\frac{31x}{100}$ ounces proteids, and $\frac{28\cdot5x}{100}$ ounces fats ;

and y ounces of bread contain

$\frac{8y}{100}$ ounces proteids, $\frac{1\cdot5y}{100}$ ounces fats, and $\frac{49\cdot2y}{100}$ ounces carbohydrates.

Adopting the standard given for ordinary work, the proteids must be equal to a total of 4·6 ounces, the fats 3 ounces, and the carbohydrates 14 ounces.

$$\therefore \text{ proteids} = \frac{31x}{100} + \frac{8y}{100} = 4{\cdot}6\,;$$

$$\text{fats} = \frac{28{\cdot}5x}{100} + \frac{1{\cdot}5y}{100} = 3\,;$$

$$\text{carbohydrates} = \frac{49{\cdot}2y}{100} = 14.$$

From which we find that

$$x = 8{\cdot}23 \text{ ounces, and } y = 28{\cdot}45 \text{ ounces.}$$

That is, $8\frac{1}{4}$ ounces of cheese and $28\frac{1}{2}$ ounces of bread will supply the requisite amounts of proteids, fats, and carbohydrates.

Method (ii.) :

We require a table giving the amount of carbon and nitrogen in one ounce of the different foods, and we also require to know the total amount of carbon and nitrogen necessary for each individual per diem. The nitrogen should be to the carbon in the ratio of 1 to 15; and for ordinary work, 300 grains N. and 4,500 grains C. may be taken as a standard.

Example :

1 oz. bread contains 116 grs. C. and 5·5 grs. N.
1 oz. cheese contains 161 grs. C. and 21 grs. N.
$\therefore$ x oz. cheese contain $161x$ grs. C. and $21x$ grs. N.
y oz. bread contain $116y$ grs. C. and $5{\cdot}5y$ grs. N.

Total N. = 300; total C. = 4,500;

$$\therefore\ 21x + 5{\cdot}5y = 300,$$

$$\text{and } 161x + 116y = 4{,}500\,;$$

$$\therefore\ x = 6{\cdot}5 \text{ ounces cheese,}$$

$$\text{and } y = 29{\cdot}7 \text{ ounces bread.}$$

This method gives less cheese and more bread than the first one ; but the standards adopted in the two methods are slightly different—*e.g.*, if Moleschott's diet, which we used in method (i.), be reduced to carbon and nitrogen, we shall find C = 4,737, and N = 321 ; whereas in method (ii.) C = 4,500, and N = 300.

Are the salts in the right proportion?

From a table we find that

1 ounce cheese contains 20 grains of salts,
1 ounce bread contains 6 grains of salts ;

∴ 8·23 ozs. cheese contain 8·23 × 20 = 164·6 grs. salts,
and 28·45 ozs. bread contain 28·45 × 6 = 170·7 grs. salts.

Total salts ... 335·3 grains.

But the salts should be 1 ounce (437·5 grains) ; so we see that this diet is somewhat deficient in this respect.

What is the energy available from this diet?

When oxidized in the body, the energy developed by
1 ounce of cheese = 150 foot-tons ; and by
1 ounce of bread = 88 foot-tons.

Therefore,

28½ ozs. of bread will give 28½ × 88 = 2,508 ft.-tons,
and 8¼ ozs. of cheese will give 8¼ × 150 = 1,238 ft.-tons.

Total ... 3,746 ft.-tons.

What amount of energy is the 'standard diet' for ordinary work capable of yielding?

1 ounce	of water-free	proteid	yields	173 foot-tons.
1 „	„	fat	„	378 „
1 „	„	carbohydrate	„	135 „

Therefore,

4·6 ounces proteids	yield	$4·6 \times 173 =$ 796	foot-tons.
3 „ fat	„	$3 \times 378 =$ 1,134	„
14 „ carbohydrate	„	$14 \times 135 =$ 1,890	„
	Total ...	3,820	„

So we may say that the average diet, for ordinary daily work, yields nearly 4,000 foot-tons of potential energy. Of these 4,000 foot-tons, 300 foot-tons, on an average, are expended in external mechanical work (p. 58), and 260 foot-tons on internal bodily work (being utilized by the heart and respiratory organs, etc.).

Thus the total for internal and external work $= 300 + 260 = 560$, which is rather less than $\frac{1}{7}$ of the total energy available from the food.

Example:

What is the amount of mechanical energy expended by a man in doing an average day's work, and how much must be obtainable from the food to enable him to perform it? How much do you allow for the internal work of the body?*

* D.P.H. Examination, Roy. Coll. Phys. and Surg., December, 1887.

CHAPTER V.

THE CONSTRUCTION OF A HOSPITAL WARD.

SUPPOSING we wish to construct a ward to accommodate twenty patients, allowing 1,000 cubic feet of space per head—that is to say, a ward whose cubic capacity is 20,000 cubic feet. The variations in its possible dimensions are endless—*e.g.*, the following would satisfy the conditions as to capacity :

	Length.		*Width.*		*Height.*
1.	100 feet	...	20 feet	...	10 feet
2.	50 „	...	40 „	...	10 „
3.	40 „	...	25 „	...	20 „
4.	32 „	...	25 „	...	25 „
	etc.		etc.		etc.

But some of these would be useless for the accommodation of twenty patients—*e.g.*, if we place ten beds on each side of a ward constructed as No. 4, each bed (3 feet wide) would only have $\frac{32}{10} = 3{\cdot}2$ feet of wall-space, and the enormous height (25 feet) is simply wasted space. Obviously, then, the total amount of cubic space required will not help us much ; and it may be remarked at the outset, that it is not sufficient for the ward, *taken as a whole*, to satisfy (as the above examples do) the total gross conditions as to capacity, etc., but that it is necessary for each individual patient's portion of the ward to satisfy certain conditions. What these are will now be inquired into more fully.

Number of Patients:

The *number* of patients to be accommodated is of less importance than the consideration how *each* patient should be accommodated. The maximum number, however, should not exceed 30.

Conditions as to Height:

Anything above 14 or 15 feet should be neglected in calculating the necessary cubic space per head, since organic impurities tend to collect in the lower portions of the atmosphere, and excessive height will not counterbalance this, whereas increased space in other dimensions would dilute these.

Width:

The minimum width should be 24 feet; allowing $6\frac{1}{2}$ feet for the length of each bed, two beds on opposite sides of the ward would take up 13 feet, leaving $24-13=11$ feet for the passage down the middle of the ward between the two rows.

Length:

No limits within reason. If we have a ward containing twenty patients, and wish to enlarge it into one accommodating thirty, it is obvious that the only way it could be done would be by adding to the length; for, as we have already seen, increase in height should not be allowed to provide for the extra amount of cubic space required, and increase in width would only widen the passage down the middle of the ward between the two rows of beds, without providing any further bed accommodation.

Floor-Space :

From what has been said as to height, it will be seen that dilution of respiratory impurities can only be thoroughly carried out by each patient having a certain amount of floor-space, and the minimum should be 100 square feet per head, and should not be less than $\frac{1}{12}$ of the cubic space.

Amount of Cubic Space :

For a general hospital, each patient should have a minimum of 1,200 cubic feet; if the air be renewed three times per hour, this would give 3,600 cubic feet of fresh air per head per hour.

Windows :

It is generally allowed that there should be 1 square foot of window to every 70 cubic feet of space; they should reach downwards to within $2\frac{1}{2}$ or 3 feet of the floor, and upwards to within 1 foot of the ceiling. The space between the windows should be at least 1 foot wider than the width of the bed; as the beds are usually 3 feet wide, the space between the windows must be at least 4 feet.

Beds :

Three feet should be the minimum distance between adjoining beds.

Let us now see how to apply these data in constructing a ward of the usual oblong form to accommodate, *e.g.*, twenty-eight ordinary medical patients.

Floor-space must be a minimum of $100 \times 28 = 2{,}800$ square feet.

Cubic contents must be a minimum of $1{,}200 \times 28 = 33{,}600$ cubic feet.

It will be noticed that this exactly fulfils the condition that floor-space should be at least $\frac{1}{12}$ of the cubic space, since $\frac{33{,}600}{12} = 2{,}800$. Knowing the cubic space to be 33,600 cubic feet, and the floor-space 2,800 square feet, we see that a height of $\frac{33{,}600}{2{,}800} = 12$ feet will satisfy the conditions so far. Having obtained the height and the floor-space, the latter must now be divided up into length and breadth. Taking the minimum permissible width—viz., 24 feet—we are left with $\frac{2{,}800}{24} = 117{\cdot}5$ feet for the length. Will this length be sufficient? As there are twenty-eight beds, or fourteen on each side, each bed would thus have $\frac{117{\cdot}5}{14} = 8{\cdot}4$ feet of wall-space, which would provide for a bed of 3 feet in width, and a space of $8{\cdot}4 - 3{\cdot}0 = 5{\cdot}4$ feet between each. As we have fixed a minimum of only 3 feet between each bed, 117 feet will amply satisfy the conditions as to length. But we may remark incidentally, that although we might put the beds closer together, without overstepping the limit laid down, yet it would not be permissible on that account either to put more beds into the ward, or to shorten its length, otherwise each individual would be deprived of a portion of his floor-space and cubic space. The deficiency in the latter could be overcome by raising the height of the ward to 14 or 15 feet (not beyond this); but the deficiency in floor-space could not thus be overcome, except by increasing the width of the ward, which, of course, could be done. So the conditions are fulfilled in a ward 117·5 feet long, 24 feet wide, and 12 feet high, which provides a distance of more than 5 feet between each bed. If we choose to make the ward 25 feet wide, the length would then be $\frac{2,800}{25} = 112$ feet, which would provide a wall-space of $\frac{112}{14} = 8$ feet for each bed; this would be well over the minimum limit, and satisfy all conditions laid down.

Window-Space:

1 square foot to every 70 cubic feet of space. We must, therefore, have a minimum of $\frac{33,600}{70} = 480$ square feet of window-space. If we put the windows on each side of the ward, we shall require 240 square feet of window-space on each side. If we place a window between each bed, fourteen beds on each side will require thirteen windows, each having a minimum area of $\frac{240}{13} = 18·4$ square feet. If each window commences 3 feet from the floor, and reaches to within 1 foot of the ceiling, since height of ward is 12 feet, height of window must be 8 feet, and $\frac{18·4}{8} = 2·3$ feet will be the necessary width of window. These are the lowest limits allowable, and they might well be made somewhat wider. To what limit in width may we go? We have said that there must be 4 feet of wall-space

between the windows (or 1 foot more than the width of the bed). If width of bed is 3 feet, we should leave the required space between the windows by bringing them to within 6 inches of the bed on each side; and since space between the beds = 5·4 feet, the maximum width of window would be 5·4 feet − (2 × 6 inches) = 4·4 feet.

The **minimum** dimensions of the ward which will satisfy all conditions are, therefore:

> Length, 117·5 feet; breadth, 24 feet; height, 12 feet; window-space, 480 square feet.

If we decide to have a window between each bed (*i.e.*, twenty-six windows), the minimum size must be 8 feet high and 2·3 feet wide, and the maximum width 4·4 feet.

There are numerous other ways in which the ward could be constructed; moreover, the matter of ventilation has not been touched upon, but our object was to point out, in as simple a manner as possible, the chief features to be borne in mind, and to suggest a method of 'setting to work.'

CHAPTER VI.

CHEMICAL CALCULATIONS.

To calculate the Amount of SO_2 in a Room after Disinfection with Sulphur (1 pound of Sulphur being Burnt for every 1,000 Cubic Feet of Air).

1 litre of SO_2 at 32° F. and ordinary atmospheric pressure weighs $0{\cdot}08958 \times 32 = 2{\cdot}866$ grammes (*vide* p. 2).

Therefore, at 60° F. (the average temperature of a room) and the same pressure, 1 litre will weigh

$$\frac{2{\cdot}866 \times 491}{491 + (60 - 32)} = 2{\cdot}7119 \text{ grammes (p. 5).}$$

That is, at 60° F.

2·7119 grammes SO_2 occupy 1 litre, or 1,000 c.c.,
or 2·7119 grammes occupy 61 cubic inches;

$$\therefore \frac{2{\cdot}7119}{453{\cdot}6} \text{ lb. occupies } \frac{61}{1{,}728} \text{ cubic foot;}$$

$$\therefore 1 \text{ lb. } SO_2 = \frac{61 \times 453{\cdot}6}{1{,}728 \times 2{\cdot}7119} = 5{\cdot}9 \text{ cubic feet.}$$

Now, from the equation

$$S + O_2 = SO_2,$$

we see that 32 parts by weight of S combine with 32 parts of O, to produce 64 parts by weight of SO_2; but the density of SO_2 is 32 (or one-half its molecular weight, *vide* p. 2); therefore, 1 volume of S produces 2 volumes of SO_2, or

1 lb. of S produces 2 lbs. of SO_2;

but we have seen that

2 lbs. SO_2 occupy $5{\cdot}9 \times 2 = 11{\cdot}8$ cubic feet;

therefore, 1 lb. of S produces 11·8 cubic feet of SO_2; and, as 1 lb. of S is burnt for every 1,000 cubic feet of air, there must be 11·8 cubic feet of SO_2 in 1,000 cubic feet of air,

or 1·18 cubic feet SO_2 in 100 cubic feet of air;
that is, 1·18 per cent. of SO_2.

Or, the result may be obtained as follows:

1 cubic foot of dry air at 32° F. and ordinary pressure = 566·85 grains (p. 3);
therefore, 1 cubic foot of dry air at 60° F. =

$$\frac{566{\cdot}85 \times 491}{491 + (60 - 32)} = 536{\cdot}27 \text{ grains.}$$

Therefore, 1 cubic foot of SO_2 at 60° F. weighs

$$\frac{536{\cdot}27 \times 32}{14{\cdot}47} = 1185{\cdot}95 \text{ grains } (\textit{vide} \text{ p. 2})$$

$$= \frac{1185{\cdot}95}{7{,}000} \text{ lb.}$$

$$\therefore 1 \text{ lb. } SO_2 = \frac{7{,}000}{1185{\cdot}95} = 5{\cdot}9 \text{ cubic feet;}$$

$$\therefore 2 \text{ lbs. } SO_2 = 11{\cdot}8 \text{ cubic feet.}$$

That is, 1 pound of sulphur produces 11·8 cubic feet of SO_2; or, room contains 1·18 per cent. of SO_2.

STANDARD SOLUTIONS.

Silver Nitrate Solution for testing for Chlorine :

$$AgNO_3 + NaCl = AgCl + NaNO_3.$$

Since Ag = 108, Cl = 35·5, and $AgNO_3$ = 170, we know that 108 mgrs. of Ag combine with 35·5 mgrs. Cl, to produce AgCl. But 108 mgrs. Ag are contained in 170 mgrs. $AgNO_3$; therefore, it requires 170 mgrs. $AgNO_3$ to supply the necessary amount of Ag, which will combine with 35·5 mgrs. Cl to produce AgCl.

That is, 35·5 mgrs. Cl require 170 mgrs. $AgNO_3$,

and 1 mgr. Cl requires $\frac{170}{35·5} = 4·788$ mgrs. $AgNO_3$.

For convenience of calculation, the solution is generally made so that 1 c.c. exactly neutralizes 1 mgr. Cl.

Therefore, each c.c. of solution must contain 4·788 mgrs. $AgNO_3$, or 1,000 c.c. (1 litre) must contain 4·788 grammes $AgNO_3$.

The standard solution, therefore, is made by dissolving 4·788 grammes $AgNO_3$ in 1 litre of distilled water.

Solution for Estimation of CO_2 in Air.

Example :

In estimating the CO_2 in air by Pettenkofer's method, the oxalic acid solution is made of such a strength that 1 c.c. neutralizes 1 mgr. CaO. (i.) How is the solution made? and (ii.) How much CO_2 does each c.c. represent?

(i.) To make the solution :

$$CaO : H_2C_2O_4 + 2H_2O :: 56 : 126$$
$$:: 1 : 2·25$$

that is, 1 mgr. CaO is neutralized by 2·25 mgrs. oxalic acid.

But 1 c.c. of solution neutralizes 1 mgr. CaO;

∴ 1 c.c. of solution = 2·25 mgrs. oxalic acid.

So each c.c. of solution must contain 2·25 mgrs. oxalic acid, or 2·25 grammes per litre.

(ii.) How much CO_2 does each c.c. represent?

$$CaO + CO_2 = CaCO_3\ ;$$

and since $CaO = 56$, and $CO_2 = 44$,
therefore, 56 mgrs. CaO combine with 44 mgrs. CO_2,

and 1 mgr. CaO combines with $\frac{44}{56} = \frac{11}{14}$ mgr. CO_2.

Convert $\frac{11}{14}$ mgr. into c.c., thus:

1 litre of CO_2 weighs 1·97 grammes (p. 2),
or 1,000 c.c. CO_2 weigh 1,970 mgrs;

$$\therefore \frac{1{,}000 \times 11}{1{,}970 \times 14} = 0{\cdot}4 \text{ c.c. } CO_2 \text{ weighs } \frac{11}{14} \text{ mgr.}$$

Therefore, 1 mgr. CaO combines with 0·4 c.c. CO_2.

Now, since each c.c. of solution neutralizes 1 mgr. CaO, it follows that each c.c. of solution corresponds to 0·4 c.c. CO_2.

If baryta-water be used instead of lime-water, we have:

$$CaO : BaO :: 56 : 153$$
$$:: 1 : 2{\cdot}73.$$

Therefore, if 1 c.c. oxalic acid solution neutralizes 1 mgr. CaO, it will also neutralize 2·73 mgrs. BaO.

So 1 c.c. oxalic acid solution neutralizes either 1 mgr. CaO, or 2·73 mgrs. BaO; and in either case 1 c.c. represents 0·4 c.c. CO_2.

Ammonium Chloride Solution for 'Nesslerizing.'

Molecular weight of $NH_3 = 17$

" " $NH_4Cl = 53{\cdot}5$;

$$\therefore NH_3 : NH_4Cl :: 17 : 53{\cdot}5;$$

$$\therefore NH_4Cl = \frac{NH_3 \times 53{\cdot}5}{17}.$$

Suppose we wish to make the solution of such a strength that 1 c.c. = 0·01 mgr. of NH_3; then

1 c.c. must contain $\frac{0{\cdot}01 \times 53{\cdot}5}{17} = 0{\cdot}03147$ mgr. NH_4Cl,

or 1,000 c.c. must contain 0·03147 gramme NH_4Cl.

So the solution is made by dissolving 0·03147 gramme of ammonium chloride in 1 litre of distilled water.

To Convert 'Grains per Gallon' into 'Parts per 100,000.'

1 gallon of water = 70,000 grains.

If there are x grains per gallon, there are

x grains in 70,000 grains,

or $\frac{x \times 100{,}000}{70{,}000} = \frac{10x}{7}$ grains in 100,000 grains,

or $\frac{10x}{7}$ parts per 100,000 parts;

that is, x grains per gallon $= \frac{10x}{7}$ parts per 100,000 parts.

To Convert 'Parts per 100,000' into 'Grains per Gallon.'

x parts per 100,000 parts,

or $\frac{x \times 70,000}{100,000} = \frac{7x}{10}$ parts per 70,000 parts,

or $\frac{7x}{10}$ grains per 70,000 grains,

or $\frac{7x}{10}$ grains per gallon.

that is, x parts per 100,000 parts $= \frac{7x}{10}$ grains per gallon.

So, if 'grains per gallon' be multiplied by 10, and divided by 7, the result is 'parts per 100,000'; and if 'parts per 100,000' be multiplied by 7 and divided by 10, the result is 'grains per gallon.'

CHAPTER VII.

VITAL STATISTICS.

Estimation of Population.

THE increase in a population takes place in Geometrical Progression.

[NOTE.—(*a*) Quantities are said to be in *arithmetical progression* when they increase or decrease by a common difference; thus, the numbers 2, 4, 6, 8, 10 are in A.P., the common difference being 2.

(*b*) Quantities are said to be in *geometrical progression* when each is equal to the product of the preceding and some constant factor; thus, the numbers 2, 4, 8, 16, 32 are in G.P., the constant factor (or common ratio) being 2].

Suppose that in the census year of 1891 the population of a town was 10,000, and 13,000 in 1901. There is an increase of 3,000 in the ten years, or an *average* of 300 annually. But it would be incorrect to say that 10,000 persons become 10,300 by the end of the first year, 10,600 by the end of the second year, and so on, to 13,000 in the tenth year; for the population is increasing year by year, owing to increased number of parents each year, and increased number attaining a marriageable age, and yet this increasing number only produces the same stationary annual increase of 300. For instance, if in the first year 10,000 produce an additional 300, making a total of 10,300, the 10,300 commencing the second year (if increasing in the same ratio, which we presume to be the case) ought to produce more that 300—*viz.*, $\frac{300 \times 10{,}300}{10{,}000} = 309$, and similarly for any other year. In the accompany-

Year.	*Example* 1 (*in A.P.*)		*Example* 2 (*in G.P.*).	
	Population.	*Annual Increase.*	*Population.*	*Annual Increase.*
1891	10,000		10,000	
		300		266
1892	10,300		10,266	
		300		272
1893	10,600		10,538	
		300		281
1894	10,900		10,819	
		300		287
1895	11,200		11,106	
		300		296
1896	11,500		11,402	
		300		303
1897	11,800		11,705	
		300		311
1898	12,100		12,016	
		300		319
1899	12,400		12,335	
		300		329
1900	12,700		12,664	
		300		336
1901	13,000		13,000	

ing table, the population in Example 1 is obtained by inserting 9 arithmetical means and that in Example 2 by inserting 9 geometrical means between 10,000 and 13,000. The column headed 'Annual Increase' shows the difference between the population of any year and that of the preceding one. It will be noticed in Example 2 that the annual addition to the population increases with each succeeding year, as we have shown would naturally be the case, and this example gives the correct estimation of the population, whilst Example 1 shows an incorrect one. We thus see that the increase in population takes place in Geometrical Progression.

The following are the three methods employed in estimating a population :

Method 1 :

Assumes that the rate of increase remains constant, and is the same as existed during the previous intercensal period of ten years.

Let P = population in any census year, and
r = annual increase per unit of population.

Then, at end of year, one person becomes $1+r$, and P persons will become $P(1+r)$. Now, for the second year we start with this increased population, and say that

if 1 person becomes $1+r$, then
$P(1+r)$ persons become $P(1+r)\times(1+r)=P(1+r)^2$.

Similarly, at the end of the ninth year there will be $P(1+r)^9$ persons, and at the end of the tenth year they will have become $P(1+r)^9\times(1+r)=P(1+r)^{10}$, or after n years $P(1+r)^n$.

So, if P' = population required, then

$$P'=P(1+r)^n.$$

Example :

According to the census returns, the population of Friern Barnet (omitting Colney Hatch Lunatic Asylum, as should be done for statistical purposes) in 1891 was

6,716, and in 1901 was 10,101. Find the population for 1903.

The census is taken at the beginning of April, but all populations should be estimated to the middle of the year, or three months later. We must first find the value of r for the previous intercensal period (1891-1901). Here $P=6{,}716$, $P'=10{,}101$, $n=10$.

$$P'=P(1+r)^n\,;$$

$$\therefore\ \log P'=\log P+n\log(1+r)\,;$$

$$\therefore\ \log(1+r)=\frac{\log P'-\log P}{10}=\frac{\log 10{,}101-\log 6{,}716}{10}=$$

$$\frac{1}{10}(4{\cdot}0043644-3{\cdot}8271107)=0{\cdot}0177253=\log 1{\cdot}0416\,;$$

$$\therefore\ 1+r=1{\cdot}0416,\ \text{and}\ r=0{\cdot}0416.$$

Having found the value of r for the last intercensal period, we now assume for r the same value during the next period of ten years, and make use of the same formula, but remembering that now

$P=10{,}101$, $n=2\frac{1}{4}$, and $P'=$ the required population for 1903.

$$P'=P(1+r)^n\,;$$

$$\therefore\ \log P'=\log P+n\log(1+r)$$

$$=\log 10{,}101+\frac{9}{4}\log 1{\cdot}0416,$$

from which we find that

$$P'=11{,}072\,;$$

that is, the estimated population for the middle of 1903 is 11,072.

The following modification of the formula enables the required population to be found in one stage, without the necessity of first finding the value of r:

If P'' = required population,
P' = population of last census,
P = population of census previous to that,
then, as before, $P'' = P'(1+r)^n$;

$$\therefore \ log P'' = log P' + n \ log(1+r).$$

But we have already seen that

$$log\ (1+r) = \frac{log P' - log P}{10};$$

$$\therefore \ log P'' = log P' + n \frac{log P' - log P}{10}.$$

By giving to n successive values of $\frac{1}{4}$, $1\frac{1}{4}$, $2\frac{1}{4}$, etc., we get the population for each year; or, what is the same thing, the population of any year can be obtained by adding $\frac{log P' - log P}{10}$ to the log of that of the preceding year.

In our example the above formula would become

$$log P'' = log\ 10{,}101 + \left(2\tfrac{1}{4} \times \frac{log\ 10{,}101 - log\ 6{,}716}{10}\right).$$

This method is the one which the Registrar-General employs in estimating the populations, upon which he bases his statistics.

Method 2:

This method estimates the population from the average birth-rate.

Let x = average birth-rate per 1,000 for last ten years,
y = actual number of births registered in the year under observation;

Then, if there are x births in 1,000 people,

there is 1 birth in $\frac{1{,}000}{x}$ people,

and y births in $\frac{1{,}000 \times y}{x}$ people;

that is, population for the year $= \frac{1{,}000 \times y}{x}$

$$= \frac{1{,}000 \times \text{registered births for the year}}{\text{average birth-rate for last ten years}}.$$

Example:

The average birth-rate of Friern Barnet for the ten years 1884-1893 was 32·1 per 1,000. The actual number of births registered in 1893 was 209; the estimated population, therefore, for 1893 is

$$\frac{1{,}000 \times 209}{32{\cdot}1} = 6{,}511.$$

This method assumes that the birth-rate remains constant; moreover, birth-rates are stated annually as so many per 1,000 of an *estimated* population only; if we estimate the population wrongly, the birth-rate will be fallacious, and, therefore, any statistics based upon the birth-rate will be erroneous.

The effect upon the birth-rate of an inaccurate estimate of the population will be shown later.

Method 3:

Estimates the population from the number of inhabited houses.

In the case of Friern Barnet, the number of inhabited houses in 1893 (as ascertained from the rate-book) was 1,216. By the census of 1891 the population was 6,716, and the number of inhabited houses 1,117, giving $\frac{6{,}716}{1{,}117} = 6{\cdot}012$ persons to each house. So, if we multiply the number of inhabited houses in 1893 by the average number of persons to each house, we get $1{,}216 \times 6{\cdot}012 = 7{,}310$ as the estimated population for 1893. The advantage of this method is, that the calculation is founded upon exact and accurate figures from the census returns and the rate-book, and not upon estimates only. But it is beyond the scope of this book to discuss the relative merits of the three methods above enumerated, or to suggest which method might with advantage be employed in different localities.

If the same Rate of Increase continues, how long will a Population take to Double Itself?

$P' = P(1+r)^n$, and the population will be doubled when $P' = 2P$;

$$\text{that is, } P(1+r)^n = 2P, \quad \therefore (1+r)^n = 2,$$
$$\therefore n \, log(1+r) = log\ 2,$$
$$\therefore n = \frac{log\ 2}{log(1+r)}.$$

Applying this to our previous example (p. 79):

$$log(1+r) = 0{\cdot}0177253$$
$$log\ 2 = 0{\cdot}3010300$$
$$\therefore n = \frac{0{\cdot}3010300}{0{\cdot}0177253} = 17 \text{ years (nearly)}.$$

BIRTH-RATES AND DEATH-RATES.

Birth-rates and death-rates are reckoned as so many births and deaths per 1,000 inhabitants living, at all ages.

Now

$$\left.\begin{matrix}\text{Total births}\\ \text{registered}\\ \text{in year}\end{matrix}\right\} : \left\{\begin{matrix}\text{Total}\\ \text{popula-}\\ \text{tion}\end{matrix}\right\} :: \left\{\begin{matrix}\text{Births}\\ \text{per 1,000}\\ \text{persons}\end{matrix}\right\} : \left\{\begin{matrix}\text{1,000}\\ \text{per-}\\ \text{sons.}\end{matrix}\right.$$

$$\therefore \text{annual birth-rate per 1,000} = \frac{\text{registered births} \times 1{,}000}{\text{population}}.$$

Similarly,

$$\text{annual death-rate per 1,000} = \frac{\text{registered deaths} \times 1{,}000}{\text{population.}}$$

Note the effect of wrongly estimating the population. The greater the denominator of a fraction, the less is its value. Therefore, the greater the population in the fraction $\frac{\text{registered births} \times 1{,}000}{\text{population}}$, the less the value of such fraction will be, and consequently the less the birth-rate. Therefore, if a population be over-estimated, the birth-rate and death-rate will each be stated too low, and *vice versâ*.

The Infantile Mortality Rate is stated as an annual rate of so many deaths under 1 year of age to 1,000 births registered during the year.

Now,

$$\left\{\begin{matrix}\text{Regis-}\\\text{tered}\\\text{births}\end{matrix}\right\} : \left\{\begin{matrix}\text{1,000}\\\text{births}\end{matrix}\right\} :: \left\{\begin{matrix}\text{deaths}\\\text{under 1 year}\end{matrix}\right\} : \left\{\begin{matrix}\text{deaths}\\\text{per 1,000}\\\text{births}\end{matrix}\right\}$$

$$\therefore \text{Infantile mortality rate} = \frac{\text{deaths under 1 year} \times 1{,}000}{\text{registered births}}.$$

To find the mean annual birth-rate for a period of 10 years.

Estimate the population for each of the 10 years, add them together, and divide by 10. The result is the 'mean annual population.' Add together the number of births during the 10 years, and divide by 10; this gives the mean number of births per annum.

Then (as in the case of the annual birth-rate, p. 82),

$$\left.\begin{matrix}\text{mean annual birth-rate}\\\text{for the period of 10 years}\end{matrix}\right\} = \frac{\text{mean births} \times 1{,}000}{\text{mean annual population}};$$

and similarly for the mean annual death-rate.

ANNUAL RATES FOR SHORT PERIODS.

A 'weekly death-rate' of 15·6 per 1,000 does not mean that 15·6 deaths per 1,000 inhabitants occurred during that week; but that if the deaths that week continue at the same rate throughout the year, they would produce an *annual* death-rate of 15·6 per 1,000.

A year consists of 365 days, 5 hours, 48 minutes, 57 seconds.

i.e., 1 year = 365·24226 days,
= 52·17747 weeks;
a quarter = from 90 to 92 days;
a month = from 28 to 31 days.

The following general formula may be used for ascertaining annual rates from (*a*) weekly, (*b*) monthly, and (*c*) quarterly returns.

(*a*) **Weekly:**

Let b = births in a week;
x = number of days in a week (viz., 7).

Then b births in x days $= \frac{b}{x}$ births per diem,

and $\frac{b \times 365{\cdot}24226}{x}$ births per annum,

and $\frac{b \times 365{\cdot}24226 \times 1{,}000}{x \times \text{population}}$ births per 1,000 per annum.

In the case of monthly and quarterly returns, the above formula will apply, with the following alterations:

(*b*) **Monthly:**

Let b = number of births in the month,
x = number of days in the month (28 to 31).

(*c*) **Quarterly**:

Let b = number of births in the quarter,

x = number of days in the quarter (90 to 92).

And similarly for death-rates.

Example:

Thirty deaths occurred in one week in a population of 100,000; what is the annual death-rate?

$$\frac{30 \times 365{\cdot}24226 \times 1{,}000}{7 \times 100{,}000} = 15{\cdot}6 \text{ per } 1{,}000.$$

To find the death-rate of a combined district, where the death-rates of the individual districts are known.

Let A = population in one district,

and x = its death-rate per 1,000,

then $\frac{x \times A}{1{,}000}$ = total deaths in A.

Let B = population in second district,

and y = its death-rate per 1,000,

then $\frac{y \times B}{1{,}000}$ = total deaths in B;

$\therefore \frac{x \times A}{1{,}000} + \frac{y \times B}{1{,}000} = \frac{Ax + By}{1{,}000}$ = total deaths in combined district (A + B),

and $\frac{Ax + By}{1{,}000(A + B)}$ = death-rate per unit in combined district (A + B),

and $\frac{Ax + By}{A + B}$ = death-rate per 1,000 in combined district (A + B).

Example:

A has a population of 6,000, and death-rate of 10 per 1,000; B has a population of 14,000, and death-rate of 15 per 1,000. Find the death-rate of the combined district.

$$\frac{Ax+By}{A+B}=\frac{(6{,}000\times 10)+(14{,}000\times 15)}{6{,}000+14{,}000}=13·5 \text{ per } 1{,}000.$$

That this is the correct death-rate for the combined district may be readily proved; for a death-rate of 10 per 1,000 in A, means 60 deaths in the 6,000; and one of 15 per 1,000 in B, means 210 deaths in the 14,000, or a total of 60+210 deaths in the 6,000+14,000 inhabitants—that is, 270 deaths in 20,000, or 13·5 per 1,000.

We may here point out the error which would have occurred had we assumed that the death-rate of the combined district was the average of the death-rates of the individual districts—viz., $\frac{10+15}{2}=12·5$. This is only true when the populations of the individual districts are equal, for then A=B, and the expression $\frac{Ax+By}{A+B}$ becomes $\frac{x+y}{2}$.

The death-rates 10 and 15 per 1,000 are simply **averages**, and 'when an average is deduced from two or more averages—that is, when an average of averages is taken—there must be the same number of numerical units in each' (Parkes).

If the population of A and B had been equal—*i.e.*, had contained the same number of units—then the average of 10 and 15—viz., 12·5—would be the correct death-rate for the combined district, but not otherwise.

If the combined district consists of three parts, C being the population of the third part, and z its death-rate per 1,000, then $\frac{Ax+By+Cz}{A+B+C}$ = death-rate of combined district.

Example :

$$\text{Let } A=6{,}000,\ x=10,$$
$$B=8{,}000,\ y=15,$$
$$C=4{,}000,\ z=25,$$

then the death-rate of combined district will be found to be 15·5 per 1,000;

(but average of 10, 15, and 25 = 16·7).

The same formula may be used when the population and death-rate of one part of the district and the whole district are given, and it is required to find the death-rate of the other part.

Example :

The population of a combined district is 14,000, and its death-rate 13 per 1,000; the population of one part is 6,000, and its death-rate 10 per 1,000; find the death-rate of the other part.

$$x=10,\ A=6{,}000,\ A+B=14{,}000,$$
$$\therefore B=8{,}000;\ \text{Find } y.$$
$$\frac{Ax+By}{A+B}=13,\ \therefore Ax+By=13(A+B)$$
$$y=\frac{13(A+B)-Ax}{B}=\frac{(13\times 14{,}000)-(6{,}000\times 10)}{8{,}000}=15{\cdot}25.$$

POISSON'S RULE.

For statistics to be of any value, it is essential that the number of the units from which they are compiled shall be sufficiently large.

If M = number of cases,
m = number that recover,
n = number that die,

then $\frac{m}{M}$ = proportion of recoveries, and

$\frac{n}{M}$ = proportion of deaths.

Now, if we assume that in a second series of cases the fractions $\frac{m}{M}$ and $\frac{n}{M}$ will have the same value as in the first series, the error we shall fall into will vary, according to **Poisson's rule**, between $+2\sqrt{\frac{2mn}{M^3}}$ and $-2\sqrt{\frac{2mn}{M^3}}$. That is, the fraction $\frac{m}{M}$ in the first series will, in the second, lie somewhere between $\left(\frac{m}{M}+2\sqrt{\frac{2mn}{M^3}}\right)$ and $\left(\frac{m}{M}-2\sqrt{\frac{2mn}{M^3}}\right)$. Now, the greater the denominator, the less the value of a fraction. Therefore, the greater M^3 is, the less will be the value of $2\sqrt{\frac{2mn}{M^3}}$. That is, the greater the number of recorded cases, the less will be the error when applying the statistics to subsequent similar series.

If $M = 10$ cases,
$m = 8$ that recover,
$n = 2$ that die,

then the possible error is $2\sqrt{\frac{2mn}{M^3}} = 2\sqrt{\frac{2.8.2}{(10)^3}}$ $= 2\sqrt{\frac{32}{1{,}000}} = 2\sqrt{0{\cdot}032} = 0{\cdot}3576$ to unity, or $35{\cdot}76$ per cent.

The cases that die are 2 in 10, or 20 per cent. Therefore, in a second series of cases they may be either $20 + 35{\cdot}76 = 55{\cdot}76$ per cent., or $20 - 35{\cdot}76 = -15{\cdot}76$ per cent.

Similarly, the recoveries are 8 in 10, or 80 per cent.; therefore, in a second series they may be either

$$80+35{\cdot}76=115{\cdot}76 \text{ per cent., or}$$
$$80-35{\cdot}76=44{\cdot}24 \text{ per cent.}$$

That is to say, the mortality may be nearly 16 per cent. less than nothing ($-15{\cdot}76$), or the recoveries may be nearly 16 per cent. greater than the total number of cases ($115{\cdot}76$), which obviously cannot be.

Now, if we take another similar, but much larger, series of cases, where

$$M=100{,}000,$$
$$m=80{,}000,$$
$$n=20{,}000,$$

the possible error will be

$$2\sqrt{\frac{2mn}{M^3}}=0{\cdot}003576 \text{ to unity, or } 0{\cdot}3576 \text{ per cent.}$$

The cases that die are 20 per cent.; therefore, in a second series they may be either

$$20+0{\cdot}3576=20{\cdot}3576 \text{ per cent., or}$$
$$20-0{\cdot}3576=19{\cdot}6424 \text{ per cent.}$$

We thus see how small the error is, where a large number of cases is under consideration.

To find the Relative Values of Two or More Series of Observations.

Adopting the figures in the two previous examples, we have:

error in first series $=35{\cdot}76$ per cent.,
" second " $=0{\cdot}3576$

Now,

$$0{\cdot}3576 : 35{\cdot}76 :: 1 : 100,$$
$$:: \sqrt{(1)^2} : \sqrt{(100)^2},$$
$$:: \sqrt{1} : \sqrt{10{,}000},$$
$$:: \sqrt{10} : \sqrt{100{,}000},$$

and in the first series there were 10 cases,
" " second " " " 100,000 cases.

Therefore,

$$\left.\begin{matrix}\text{error}\\ \text{in 2nd}\\ \text{series}\end{matrix}\right\} : \left\{\begin{matrix}\text{error}\\ \text{in 1st}\\ \text{series}\end{matrix}\right\} :: \left\{\begin{matrix}\text{sq. root}\\ \text{of No. of}\\ \text{cases in}\\ \text{1st series}\end{matrix}\right\} : \left\{\begin{matrix}\text{sq. root}\\ \text{of No. of}\\ \text{cases in}\\ \text{2nd series}\end{matrix}\right.$$

That is,

the error varies inversely as the square root of the number of cases; and as the value will vary inversely as the error, it follows that the values of two series vary directly as the square roots of the number of cases in the respective series.

Example:

In 100 cases, 70 recover, and 30 die. Find the error in the similar, but larger, series of 10,000 cases.

$$\text{error in first series} = 2\sqrt{\frac{2mn}{M^3}} = 2\sqrt{\frac{2.70.30}{(100)^3}} = 0{\cdot}13 \text{ to unity, or 13 per cent.}$$

In second series, let x = error,

$$\text{then } x : 0{\cdot}13 :: \sqrt{100} : \sqrt{10{,}000};$$

$$\therefore x = \frac{0{\cdot}13 \times \sqrt{100}}{\sqrt{10{,}000}} = 0{\cdot}013 \text{ to unity, or } 1{\cdot}3 \text{ per cent.}$$

From the first series we see that recoveries may vary between $70+13=83$, and $70-13=57$; but in the case of the second series, only between $70+1{\cdot}3=71{\cdot}3$, and $70-1{\cdot}3=68{\cdot}7$.

The respective values of the two series are as

$$\sqrt{100} : \sqrt{10{,}000} = 1 : 10.$$

CHAPTER VIII.

MENSURATION, ETC.

[ABBREVIATIONS USED: d=diameter, r=radius, l=length, b=breadth, Ch=chord, h=height.]

Linear:

The ratio of the circumference of a circle to its diameter is **constant**, and is usually denoted by the symbol 'π.' Its numerical value, however, cannot be stated exactly. Approximately, it is equal to $\frac{22}{7}$, or, more exactly, $\pi = 3{\cdot}1416$.

So we have:

$$\pi = \frac{\text{circumference}}{\text{diameter}} = \frac{\text{circumference}}{2 \times \text{radius}};$$

$$\therefore \text{circumference} = \pi \times 2 \times \text{radius} = 2\pi r.$$

Or we may state it as follows:

$$\pi = \frac{\text{circumference}}{\text{diameter}} = 3{\cdot}1416;$$

$$\therefore \text{circumference} = d \times 3{\cdot}1416.$$

circumference of an ellipse =

$$\pi \times \frac{1}{2}\,(\text{major axis} + \text{minor axis}).$$

Superficial Area :

Sectional area of a circle $=\pi r^2=\pi\left(\frac{d}{2}\right)^2=\frac{\pi d^2}{4}$

$=\frac{d^2\times 3\cdot1416}{4}=d^2\times 0\cdot7854.$

Square: $l\times b=l^2$.

Rectangle: $l\times b$.

Parallelogram: length of one side × perpendicular between that side and the one parallel to it.

Triangle: $\frac{1}{2}$ (base × height)—*i.e.*, one-half the parallelogram on same base and of same height.

Triangle (when length of sides only are given): let a, b, c be the sides, and let $\frac{1}{2}(a+b+c)=s$: then area $=\sqrt{s(s-a)(s-b)(s-c)}$.

Ellipse: major axis × minor axis × 0·7854.

Sphere: $\pi d^2=4\pi r^2$.

Segment of circle: $\left(Ch\times h\times\frac{2}{3}\right)+\frac{h^3}{2Ch}$.

Sides of cylinder: $2\pi rh$—*i.e.*, circumference of base (circle) × height.

Curved surface of cone: $\pi r\sqrt{r^2+h^2}$, or

$\frac{1}{2}$ (circumference of base × length of slant side).

Irregular figures bounded by straight lines: divide into triangles, and take the sum of their areas.

Volume, Cubic Capacity, or Solid Contents :

Cube $=l\times b\times h=l^3$.

Rectangle $=l\times b\times h$.

**Triangle* (*e.g.*, a prism) = area of section of triangle × height.

Cylinder : $\pi r^2 h$ = area of base (circle) × height.

**Cone* (*or pyramid*) : $\frac{1}{3} \times \pi r^2 h = \frac{1}{3}$ (area of base × height) $= \frac{1}{3}$ of a cylinder on same base, and of same height.

Sphere : $\frac{4}{3}\pi r^3 = \frac{4\pi}{3}\left(\frac{d}{2}\right)^3 = d^3 \times 0{\cdot}5236$.

Dome : $\frac{2}{3} \times \pi r^2 h = \frac{2}{3}$ (area of base × height) $= \frac{2}{3}$ of a cylinder on same base, and of same height.

TENSIONS OF AQUEOUS VAPOUR.

The following table (by Regnault) gives the tensions of aqueous vapour at different temperatures. For convenience, the corresponding temperatures on the Fahrenheit scale have been added :

C.	*mm.*	*F.*	*C.*	*mm.*	*F.*	*C.*	*mm.*	*F.*
0°	4·600	32·0°	11°	9·792	51·8°	25°	23·550	77°
1	4·940	33·8	12	10·457	53·6	30	31·548	86
2	5·302	35·6	13	11·062	55·4	35	41·827	95
3	5·687	37·4	14	11·906	57·2	40	54·906	104
4	6·097	39·2	15	12·699	59·0	50	91·982	122
5	6·534	41·0	16	13·635	60·8	60	148·791	140
6	6·998	42·8	17	14·421	62·6	70	233·093	158
7	7·492	44·6	18	15·357	64·4	80	354·643	176
8	8·017	46·4	19	16·346	66·2	90	525·450	194
9	8·574	48·2	20	17·391	68·0	100	760·000	212
10	9·165	50·0						

* A triangle has three sides; its base (or section) is triangular, **but its sides are quadrilateral.** A cone, or

For intermediate temperatures, take the mean of the tensions at the temperatures above and below; the result, however, will not be absolutely correct, but approximate only; *e.g.*, 60° F. = 15·5° C. Take the means of the tensions at 15° C. and 16° C.

$$\begin{aligned} \text{Thus, } 15^\circ \text{ C.} &= 12{\cdot}699 \\ 16^\circ \text{ C.} &= 13{\cdot}635 \\ \hline & \quad 26{\cdot}334 \end{aligned}$$

$$\text{mean} = \frac{26{\cdot}334}{2} = 13{\cdot}167 = \text{tension at } 15{\cdot}5^\circ \text{ C. or } 60^\circ \text{ F.}$$

WEIGHTS AND MEASURES.

Length :

1 metre = 10 decimetres = 100 centimetres = 1,000 millimetres = 39·37 inches = 1·093 yards.
1 inch = 25·3 millimetres.
1 kilometre = 1,000 metres.
1 mile = 1,760 yards = 5,280 feet.

Surface :

144 square inches = 1 square foot.
9 square feet = 1 square yard.
4,840 square yards = 1 acre.
640 acres = 1 square mile.
1 square metre = 10·764 square feet = 1,550 square inches.

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pyramid, has triangular sides meeting in an apex. A pyramid may have **any number of sides,** therefore its base may be triangular, square, hexagonal, etc. The base of a cone is circular.

Capacity :

1,728 cubic inches = 1 cubic foot.
27 cubic feet = 1 cubic yard.
1 litre = 1 cubic decimetre = 1,000 cubic centimetres = 35·3 fluid ounces = 61 cubic inches = 0·22 gallon = 1·7617 pints.
1 cubic foot = 6·23 gallons = 1,728 cubic inches.
1 gallon = 4·54 litres = 276·94 cubic inches.

At 4° C., or 39·2° F. (the maximum density point of water),

1 cubic foot of water = 1,000 fluid ounces.

Weight :

1 cubic centimetre of distilled water at 4° C., or 39·2° F., weighs 1 gramme = 1,000 milligrammes.
1 litre of water weighs 1,000 grammes = 1 kilogramme.
1 gallon of water weighs 70,000 grains = 10 lbs. (Avoir.).
1 cubic foot of water = 61·25 lbs.
1 gramme = 15·432 grains.
1 ounce (Avoir.) = 437·5 grains.
1 ounce (Troy) = 480 grains.
1 ton = 2,240 lbs.
1 lb (Avoir.) = 453·6 grammes.
1 kilogramme = 2·2046 lbs.
1 minim weighs 0·91 grain of water.
1 fluid ounce weighs 437·5 grains of water.
1 lb. (Troy) = 5,760 grains.
1 lb. (Avoir.) = 7,000 grains.

By apothecaries' weight, 8 drachms = 1 ounce = 437·5 grains; therefore, 1 drachm = 54·7 grains, NOT 60 grains, as is often stated. In prescribing, however, the symbol '℥j' (1 drachm) is often used to denote 60 grains, but when thus employed, it does not represent one-eighth of an ounce.

Density :

1 lb. per cubic foot = 0·016019 gramme per cubic centimetre.

1 gramme per cubic centimetre = 62·4 lbs. per cubic foot.

Atmospheric Pressure :

= 760 millimetres of mercury.

= 29·922 inches of mercury.

= 1·033 kilogrammes per square centimetre.

= 14·73 lbs. per square inch.

THE END.

Baillière, Tindall and Cox, 8, *Henrietta Street, Covent Garden.*